Assignment 1
Probability And Random Processes

Rajeev Kumar
EE22BTECH11042

I. Question 1.4.6

Verify that

$$\angle BOC = 2\angle BAC$$

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II. Solution

Given,

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$$

Also,we have a point $\mathbf{O} = \begin{pmatrix} \frac{-53}{12} \\ \\ \frac{5}{12} \end{pmatrix}$ which is intersection point of the perpendicular bisectors of AB and AC.

1) To find the value of $\angle BOC$:

$$\mathbf{OB} = \mathbf{B} - \mathbf{O} \tag{1}$$
$$= \begin{pmatrix} \frac{5}{12} \\ \\ \frac{67}{12} \end{pmatrix} \tag{2}$$

$$\mathbf{OC} = \mathbf{C} - \mathbf{O} \tag{3}$$
$$= \begin{pmatrix} \frac{17}{12} \\ \\ \frac{-65}{12} \end{pmatrix} \tag{4}$$

calculating the norm of **OB** and **OC**,we get:

$$\|OB\| = \frac{\sqrt{4514}}{12} \tag{5}$$
$$\|OC\| = \frac{\sqrt{4514}}{12} \tag{6}$$

by doing matrix multiplication, we get:

$$\mathbf{OB} \cdot \mathbf{OC} = (\mathbf{OB})(\mathbf{OC})^{\intercal} \tag{7}$$
$$= \frac{-4270}{144} \tag{8}$$

to calcuate the $\angle BOC$:

$$\cos O = \frac{\mathbf{OB} \cdot \mathbf{OC}}{\|OB\| \, \|OC\|} \tag{9}$$
$$= \frac{\frac{-4270}{144}}{\frac{\sqrt{4514}}{12} \times \frac{\sqrt{4514}}{12}} \tag{10}$$
$$= \frac{-4270}{4514} \tag{11}$$

$$\Rightarrow \angle O = \cos^{-1}\left(\frac{-4270}{4514}\right) \tag{12}$$
$$\Rightarrow \angle O = \angle BOC = 161.075355593 \tag{13}$$

Therefore $\angle BOC = 161.075355593$.

2) To find the value of $\angle BAC$:

$$\mathbf{AB} = \mathbf{B} - \mathbf{A} \tag{14}$$
$$= \begin{pmatrix} -5 \\ 7 \end{pmatrix} \tag{15}$$

$$\mathbf{AC} = \mathbf{C} - \mathbf{A} \tag{16}$$
$$= \begin{pmatrix} -4 \\ -4 \end{pmatrix} \tag{17}$$

calculating the norm of **AB** and **AC**,we get:

$$\|AB\| = \sqrt{74} \, \|AC\| = 4\sqrt{2} \tag{18}$$

by doing matrix multiplication, we get:

$$\mathbf{AB} \cdot \mathbf{AC} = (\mathbf{AB})(\mathbf{AC})^{\intercal} \tag{19}$$
$$= -8 \tag{20}$$

to calcuate the $\angle BAC$:

$$\cos A = \frac{\mathbf{AB} \cdot \mathbf{AC}}{||AB||\,||AC||} \tag{21}$$

$$= \frac{-8}{\sqrt{74} \times 4\sqrt{2}} \tag{22}$$

$$= \frac{-8}{4\sqrt{148}} \tag{23}$$

$$\Rightarrow \angle A = \cos^{-1}\left(\frac{-8}{4\sqrt{148}}\right) \tag{24}$$

$$\Rightarrow \angle A = \angle BAC = 99.4623222077 \tag{25}$$

Therefore $\angle BAC = 99.4623222077$.

As we can see,

$\angle BOC \neq 2 \times \angle BAC$
Therefore given statement is wrong.