Assignment 1
Probability And Random Processes

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EE22BTECH11042

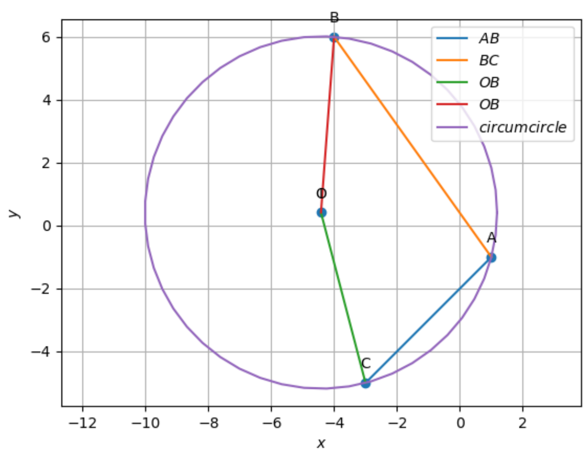

Fig. 0. Plot of circumcentre O and points A, B and C.

I. Question 1.4.6

Verify that

$$\angle BOC = 2\angle BAC$$

.

Solution: Given,

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$$

Also,we have a point $\mathbf{O} = \begin{pmatrix} \frac{-53}{12} \\ \frac{5}{12} \end{pmatrix}$ which is intersection point of the perpendicular bisectors of AB and AC and is circumcentre of the triangle made by points A,B and C.

1) To find the value of $\angle BOC$:

$$\mathbf{B} - \mathbf{O} = \begin{pmatrix} \frac{5}{12} \\ \frac{67}{12} \end{pmatrix} \tag{1}$$

$$\mathbf{C} - \mathbf{O} = \begin{pmatrix} \frac{17}{12} \\ \frac{-65}{12} \end{pmatrix} \tag{2}$$

calculating the norm of $\mathbf{B} - \mathbf{O}$ and $\mathbf{C} - \mathbf{O}$,we get:

$$\|\mathbf{B} - \mathbf{O}\| = \frac{\sqrt{4514}}{12} \tag{3}$$

$$\|\mathbf{C} - \mathbf{O}\| = \frac{\sqrt{4514}}{12} \tag{4}$$

by doing matrix multiplication, we get:

$$(\mathbf{B} - \mathbf{O})^\top (\mathbf{C} - \mathbf{O}) = \frac{-4270}{144} \tag{5}$$

to calcuate the $\angle BOC$:

$$\cos BOC = \frac{(\mathbf{B} - \mathbf{O})^\top (\mathbf{C} - \mathbf{O})}{\|\mathbf{B} - \mathbf{O}\| \, \|\mathbf{C} - \mathbf{O}\|} \tag{6}$$

$$= \frac{\frac{-4270}{144}}{\frac{\sqrt{4514}}{12} \times \frac{\sqrt{4514}}{12}} \tag{7}$$

$$= \frac{-4270}{4514} \tag{8}$$

$$\implies \angle BOC = \cos^{-1}\left(\frac{-4270}{4514}\right) \tag{9}$$

$$\implies \angle BOC = 161.07536^\circ \tag{10}$$

Taking the reflex of above angle,we get:

$$\angle BOC = 360° - 161.07536° = 198.92464° \tag{11}$$

Therefore $\angle BOC = 198.92464°$

2) To find the value of $\angle BAC$:

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -5 \\ 7 \end{pmatrix} \tag{12}$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} -4 \\ -4 \end{pmatrix} \tag{13}$$

calculating the norm of $\mathbf{B} - \mathbf{A}$ and $\mathbf{C} - \mathbf{A}$,we get:

$$\|\mathbf{B} - \mathbf{A}\| = \sqrt{74} \tag{14}$$

$$\|\mathbf{C} - \mathbf{A}\| = 4\sqrt{2} \tag{15}$$

by doing matrix multiplication, we get:

$$(\mathbf{B}-\mathbf{A})^\top(\mathbf{C}-\mathbf{A}) = -8 \tag{16}$$

to calcuate the $\angle BAC$:

$$\cos BAC = \frac{(\mathbf{B}-\mathbf{A})^\top(\mathbf{C}-\mathbf{A})}{\|\mathbf{B}-\mathbf{A}\|\,\|\mathbf{C}-\mathbf{A}\|} \tag{17}$$

$$= \frac{-8}{\sqrt{74}\times 4\sqrt{2}} \tag{18}$$

$$= \frac{-8}{4\sqrt{148}} \tag{19}$$

$$\implies \angle BAC = \cos^{-1}\left(\frac{-8}{4\sqrt{148}}\right) \tag{20}$$

$$\implies \angle BAC = 99.46232^\circ \tag{21}$$

$$\implies 2\times\angle BAC = 198.92464^\circ \tag{22}$$

As we can see,

$$\angle BOC = 2\times\angle BAC \tag{23}$$

Hence,verified.