

Assignment 1

Probability And Random Processes

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EE22BTECH11042

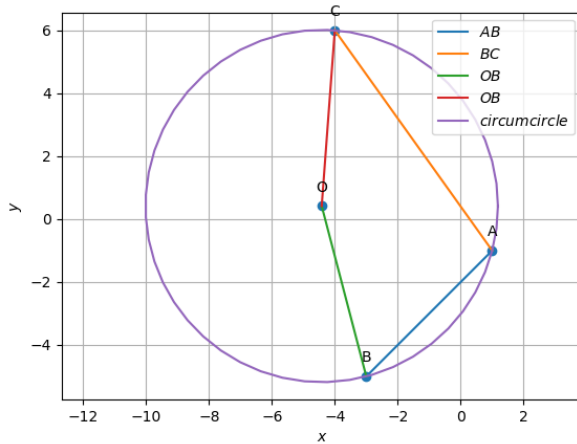


Fig. 0. plot of circumcircle O and points A, B and C.

I. QUESTION 1.4.6

Verify that

$$\angle BOC = 2\angle BAC$$

Solution: Given,

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$$

Also, we have a point $\mathbf{O} = \begin{pmatrix} -53 \\ 12 \\ 5 \end{pmatrix}$ which is intersection point of the perpendicular bisectors of AB and AC and is circumcentre of the triangle made by points A, B and C.

1) To find the value of $\angle BOC$:

$$\mathbf{B} - \mathbf{O} = \begin{pmatrix} 5 \\ 12 \\ 67 \end{pmatrix} \quad (1)$$

$$\mathbf{C} - \mathbf{O} = \begin{pmatrix} 17 \\ 12 \\ -65 \end{pmatrix} \quad (2)$$

calculating the norm of $\mathbf{B} - \mathbf{O}$ and $\mathbf{C} - \mathbf{O}$, we get:

$$\|\mathbf{B} - \mathbf{O}\| = \frac{\sqrt{4514}}{12} \quad (3)$$

$$\|\mathbf{C} - \mathbf{O}\| = \frac{\sqrt{4514}}{12} \quad (4)$$

by doing matrix multiplication, we get:

$$(\mathbf{B} - \mathbf{O})^\top (\mathbf{C} - \mathbf{O}) = \frac{-4270}{144} \quad (5)$$

to calculate the $\angle BOC$:

$$\cos BOC = \frac{(\mathbf{B} - \mathbf{O})^\top (\mathbf{C} - \mathbf{O})}{\|\mathbf{B} - \mathbf{O}\| \|\mathbf{C} - \mathbf{O}\|} \quad (6)$$

$$= \frac{\frac{-4270}{144}}{\frac{\sqrt{4514}}{12} \times \frac{\sqrt{4514}}{12}} \quad (7)$$

$$= \frac{-4270}{4514} \quad (8)$$

$$\Rightarrow \angle BOC = \cos^{-1} \left(\frac{-4270}{4514} \right) \quad (9)$$

$$= 161.07536^\circ \quad (10)$$

Taking the reflex of above angle, we get:

$$\angle BOC = 360^\circ - 161.07536^\circ \quad (11)$$

$$= 198.92464^\circ \quad (12)$$

2) To find the value of $\angle BAC$:

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -5 \\ 7 \end{pmatrix} \quad (13)$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} -4 \\ -4 \end{pmatrix} \quad (14)$$

calculating the norm of $\mathbf{B} - \mathbf{A}$ and $\mathbf{C} - \mathbf{A}$, we get:

$$\|\mathbf{B} - \mathbf{A}\| = \sqrt{74} \quad (15)$$

$$\|\mathbf{C} - \mathbf{A}\| = 4\sqrt{2} \quad (16)$$

by doing matrix multiplication, we get:

$$(\mathbf{B} - \mathbf{A})^\top (\mathbf{C} - \mathbf{A}) = -8 \quad (17)$$

to calculate the $\angle BAC$:

$$\cos BAC = \frac{(\mathbf{B} - \mathbf{A})^\top (\mathbf{C} - \mathbf{A})}{\|\mathbf{B} - \mathbf{A}\| \|\mathbf{C} - \mathbf{A}\|} \quad (18)$$

$$= \frac{-8}{\sqrt{74} \times 4 \sqrt{2}} \quad (19)$$

$$= \frac{-8}{4 \sqrt{148}} \quad (20)$$

$$\Rightarrow \angle BAC = \cos^{-1} \left(\frac{-8}{4 \sqrt{148}} \right) \quad (21)$$

$$= 99.46232^\circ \quad (22)$$

from equation (22):

$$\Rightarrow 2 \times \angle BAC = 198.92464^\circ \quad (23)$$

On comparing equation (12) and equation (23):

$$\angle BOC = 2 \times \angle BAC \quad (24)$$

Hence, verified.