Assignment
Ncert Exampler

Rajeev Kumar
EE22BTECH11042

I. Question:- GATE 2023.54

Suppose that X is a discrete random variable with the following probability mass

$$P(X = 0) = \frac{1}{2}\left(1 + e^{-1}\right) \tag{1}$$

$$P(X = k) = \frac{e^{-1}}{2k!}\ for\ k = 1, 2, 3, \dots. \tag{2}$$

Which of the following is/are true?

1) $E(X) = 1$
2) $E(X) < 1$
3) $E(X|X > 0) < \frac{1}{2}$
4) $E(X|X > 0) > \frac{1}{2}$

Solution:

1) **Theory:**
   a) As we know,

$$E(X) = \sum k p_X(k) \tag{3}$$

Therefore,

$$E(X) = 0 \cdot \frac{1}{2}\left(1 + e^{-1}\right) + \sum_{k=1}^{\infty} \frac{ke^{-1}}{2k!} \tag{4}$$

$$= \sum_{k=1}^{\infty} \frac{e^{-1}}{2(k-1)!} \tag{5}$$

$$= \frac{1}{2e} \sum_{k=1}^{\infty} \frac{1}{(k-1)!} \tag{6}$$

$$= \frac{1}{2e} \cdot e \qquad \text{(Using standard result of exponential series)}$$

$$= \frac{1}{2} \tag{7}$$

b) To find $E(X|X>0)$, first we need to find $\Pr(X|X>0)$ which can be given as:

$$\Pr(X|X>0) = \frac{\Pr(X=k)}{1-\Pr(X=0)} \tag{8}$$
$$= \frac{e^{-1}}{2k!} \cdot \frac{1}{(1-\frac{1}{2}(1+e^{-1}))} \tag{9}$$
$$= \frac{e^{-1}}{2k!} \cdot \frac{2}{(1-e^{-1})} \tag{10}$$
$$= \frac{e^{-1}}{k!(1-e^{-1})} \tag{11}$$
$$= \frac{1}{k!(e-1)} \tag{12}$$

Therefore,

$$E(X|X>0) = \sum_{k=1}^{\infty} k \frac{1}{k!(e-1)} \tag{13}$$
$$= \frac{1}{e-1} \sum_{k=1}^{\infty} \frac{1}{(k-1)!} \tag{14}$$
$$= \frac{1}{e-1} \cdot e \qquad \text{(Using standard result of exponential series)}$$
$$= 1.582 \tag{15}$$

Refering to equations (7) and (15), we get that option (2) and (4) are correct.

2) **Simulation:**
To make the simulation of the given question, generate a large set of random variables, say X, with the probability:

$$P(X=0) = \frac{1}{2}\left(1+e^{-1}\right) \tag{16}$$
$$P(X=k) = \frac{e^{-1}}{2k!}\ for\ k = 1, 2, 3, \dots. \tag{17}$$

To do so, initialize a random number generator using a seed, such as the current time, to ensure different sequences of random numbers for each program run. Then, within a loop, you generate a random floating-point number between 0 and 1, denoted by say randomnum. Subsequently, you calculate the cumulative probability of a random variable taking a specific value k by repeatedly comparing randomnum to the cumulative probabilities for increasing values of k. The loop continues until the cumulative probability exceeds randomnum. When this condition is met, you return the value of k, representing the generated random variable.

a) To get $E(X)$, take the weighted sum of all possible values of X including 0, each multiplied by its respective probability. Expression for this can be given as:

$$E(X) = \sum k p_X(k) \tag{18}$$

b) As given in theory, to get $E(X|X>0)$, take the weighted sum of all possible values of X excluding 0, each multiplied by its respective probability and dividing it by the probability of getting random variables greater than 0. Expression for this can be given as:

$$\Pr(X|X>0) = \frac{\Pr(X=k)}{1-\Pr(X=0)} \tag{19}$$

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