Assignment
Ncert Exampler

Rajeev Kumar
EE22BTECH11042

I. Question:- GATE 2023.54

Suppose that X is a discrete random variable with the following probability mass

$$P(X=0) = \frac{1}{2}\left(1+e^{-1}\right) \tag{1}$$

$$P(X=k) = \frac{e^{-1}}{2k!} \; for \; k = 1,2,3,\ldots. \tag{2}$$

Which of the following is/are true?

1) $E(X) = 1$
2) $E(X) < 1$
3) $E(X|X>0) < \frac{1}{2}$
4) $E(X|X>0) > \frac{1}{2}$

Solution:

1) As we know,

$$E(X) = \sum X p_X(X) \tag{3}$$

Therefore,

$$E(X) = 0 \cdot \frac{1}{2}\left(1+e^{-1}\right) + \sum_{k=1}^{\infty} \frac{k e^{-1}}{2k!} \tag{4}$$

$$= 0 + \frac{1\cdot e^{-1}}{2\cdot 1!} + \frac{2\cdot e^{-1}}{2\cdot 2!} + \frac{3\cdot e^{-1}}{2\cdot 3!} + \frac{4\cdot e^{-1}}{2\cdot 4!} + \ldots \tag{5}$$

$$= e^{-1}\left(\frac{1}{2} + \frac{1}{2} + \frac{1}{4} + \frac{1}{12} + \ldots\right) \tag{6}$$

$$= 0.5 \tag{7}$$

2) To find $E(X|X>0)$, first we need to find $P(X|X>0)$ which can be given as:

$$p_X(X|X>0) = \frac{p_X(X=k)}{1-p_X(X=0)} \tag{8}$$

$$= \frac{e^{-1}}{2k!\left(1-\frac{1}{2}\left(1+e^{-1}\right)\right)} \tag{9}$$

$$= \frac{e^{-1}}{k!\left(1-e^{-1}\right)} \tag{10}$$

$$= \frac{1}{k!(e-1)} \tag{11}$$

Therefore,

$$E(X|X>0) = \sum_{k=1}^{\infty} k \frac{1}{k!(e-1)} \tag{12}$$

$$= \frac{1}{e-1}\left(1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + ...\right) \tag{13}$$

$$= 1.581 \tag{14}$$

Refering to equations (7) and (14), we get that option (2) and (4) are correct.