# Assignment
# Ncert Exampler

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## I. Question:- GATE 2023.54

Suppose that $X$ is a discrete random variable with the following probability mass

$$P(X=0)=\frac{1}{2}\left(1+e^{-1}\right) \tag{1}$$

$$P(X=k)=\frac{e^{-1}}{2k!}\ for\ k=1,2,3,\dots. \tag{2}$$

Which of the following is/are true?

1) $E(X)=1$
2) $E(X)<1$
3) $E(X|X>0)<\frac{1}{2}$
4) $E(X|X>0)>\frac{1}{2}$

**Solution:**

1) **Theory:**
   a) As we know,

$$E(X)=\sum kp_X(k) \tag{3}$$

Therefore,

$$E(X)=0\cdot\frac{1}{2}\left(1+e^{-1}\right)+\sum_{k=1}^{\infty}\frac{ke^{-1}}{2k!} \tag{4}$$

$$=\sum_{k=1}^{\infty}\frac{e^{-1}}{2(k-1)!} \tag{5}$$

$$=\frac{1}{2e}\sum_{k=1}^{\infty}\frac{1}{(k-1)!} \tag{6}$$

$$=\frac{1}{2e}\cdot e \qquad \text{(Using standard result of exponential series)}$$

$$=\frac{1}{2} \tag{7}$$

b) To find $E(X|X>0)$, first we need to find $\Pr(X|X>0)$ which can be given as:

$$\begin{aligned}
\Pr(X|X>0) &= \frac{\Pr(X=k)}{1-\Pr(X=0)} && (8)\\
&= \frac{e^{-1}}{2k!}\cdot\frac{1}{(1-\frac{1}{2}(1+e^{-1}))} && (9)\\
&= \frac{e^{-1}}{2k!}\cdot\frac{2}{(1-e^{-1})} && (10)\\
&= \frac{e^{-1}}{k!(1-e^{-1})} && (11)\\
&= \frac{1}{k!(e-1)} && (12)
\end{aligned}$$

Therefore,

$$\begin{aligned}
E(X|X>0) &= \sum_{k=1}^{\infty} k\frac{1}{k!(e-1)} && (13)\\
&= \frac{1}{e-1}\sum_{k=1}^{\infty}\frac{1}{(k-1)!} && (14)\\
&= \frac{1}{e-1}\cdot e && \text{(Using standard result of exponential series)}\\
&= 1.582 && (15)
\end{aligned}$$

Refering to equations (7) and (15), we get that option (2) and (4) are correct.

2) **Simulation:**

To make the simulation of the given question, generate a large set of random variables, say $X$, with the probability:

$$P(X=0) = \frac{1}{2}\left(1+e^{-1}\right) \tag{16}$$

$$P(X=k) = \frac{e^{-1}}{2k!}\ for\ k = 1,2,3,\dots. \tag{17}$$

To simulate random variables, a random number generator is initiated using a seed, often the current time, ensuring distinct sequences with each program run. Subsequently, in a loop, a random floating-point number, say randomnum, is generated. The key principle at play is Inverse Transform Sampling. In this method, randomnum is compared to cumulative probabilities (CDF) for various values of the random variable $k$ until the CDF exceeds randomnum. This process continues until a match is found, and $k$ is returned as the generated random variable.

a) To get $E(X)$, take the weighted sum of all possible values of X including 0, each multiplied by its respective probability. Expression for this can be given as:

$$E(X) = \sum k p_X(k) \tag{18}$$

b) As given in theory, to get $E(X|X>0)$, take the weighted sum of all possible values of $X$ excluding 0, each multiplied by its respective probability and dividing it by the probability of getting random variables greater than 0. Expression for this can be given as:

$$\Pr(X|X>0) = \frac{\Pr(X=k)}{1-\Pr(X=0)} \tag{19}$$

(20)