# Assignment
# Ncert Exampler

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## I. Question:- GATE 2023.54

Suppose that $X$ is a discrete random variable with the following probability mass

$$P(X = 0) = \frac{1}{2}\left(1 + e^{-1}\right) \tag{1}$$

$$P(X = k) = \frac{e^{-1}}{2k!} \; for \; k = 1, 2, 3, \ldots . \tag{2}$$

Which of the following is/are true?

1) $E(X) = 1$
2) $E(X) < 1$
3) $E(X|X > 0) < \frac{1}{2}$
4) $E(X|X > 0) > \frac{1}{2}$

**Solution:**

1) **Theory:**
   a) As we know,

$$E(X) = \sum k p_X(k) \tag{3}$$

Therefore,

$$\begin{aligned} E(X) &= 0 \cdot \frac{1}{2}\left(1 + e^{-1}\right) + \sum_{k=1}^{\infty} \frac{ke^{-1}}{2k!} && (4) \\ &= \sum_{k=1}^{\infty} \frac{e^{-1}}{2(k-1)!} && (5) \\ &= \frac{1}{2e} \sum_{k=1}^{\infty} \frac{1}{(k-1)!} && (6) \\ &= \frac{1}{2e} \cdot e && \text{(Using standard result of exponential series)} \\ &= \frac{1}{2} && (7) \end{aligned}$$

b) To find $E(X|X>0)$, first we need to find $\Pr(X|X>0)$ which can be given as:

$$\Pr(X|X>0) = \frac{\Pr(X=k)}{1-\Pr(X=0)} \tag{8}$$
$$= \frac{e^{-1}}{2k!}\cdot\frac{1}{(1-\frac{1}{2}(1+e^{-1}))} \tag{9}$$
$$= \frac{e^{-1}}{2k!}\cdot\frac{2}{(1-e^{-1})} \tag{10}$$
$$= \frac{e^{-1}}{k!(1-e^{-1})} \tag{11}$$
$$= \frac{1}{k!(e-1)} \tag{12}$$

Therefore,

$$E(X|X>0) = \sum_{k=1}^{\infty} k\frac{1}{k!(e-1)} \tag{13}$$
$$= \frac{1}{e-1}\sum_{k=1}^{\infty}\frac{1}{(k-1)!} \tag{14}$$
$$= \frac{1}{e-1}\cdot e \qquad \text{(Using standard result of exponential series)}$$
$$= 1.582 \tag{15}$$

Refering to equations (7) and (15), we get that option (2) and (4) are correct.

2) **Simulation:**

To make the simulation of the given question, generate a large set of random variables, say $X$, with the probability:

$$P(X=0) = \frac{1}{2}\left(1+e^{-1}\right) \tag{16}$$
$$P(X=k) = \frac{e^{-1}}{2k!}\ for\ k = 1,2,3,\ldots. \tag{17}$$

a) To get $E(X)$, take the weighted sum of all possible values of X including 0, each multiplied by its respective probability. Expression for this can be given as:

$$E(X) = \sum kp_X(k) \tag{18}$$

b) As given in theory, to get $E(X|X>0)$, take the weighted sum of all possible values of $X$ excluding 0, each multiplied by its respective probability and dividing it by the probability of getting random variables greater than 0. Expression for this can be given as:

$$\Pr(X|X>0) = \frac{\Pr(X=k)}{1-\Pr(X=0)} \tag{19}$$

(20)