Assignment
Ncert Exampler

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I. QUESTION:- GATE 2023.54

Suppose that X is a discrete random variable with the following probability mass

$$P(X = 0) = \frac{1}{2}\left(1 + e^{-1}\right) \tag{1}$$

$$P(X = k) = \frac{e^{-1}}{2k!} \; for \; k = 1, 2, 3, \ldots . \tag{2}$$

Which of the following is/are true?
1) $E(X) = 1$
2) $E(X) < 1$
3) $E(X|X > 0) < \frac{1}{2}$
4) $E(X|X > 0) > \frac{1}{2}$

Solution:
As we know,

$$E(X) = \sum XP(X) \tag{3}$$

Therefore,

$$E(X) = 0 \cdot \frac{1}{2}\left(1 + e^{-1}\right) + \sum_{k=1}^{\infty} \frac{ke^{-1}}{2k!} \tag{4}$$

$$= 0 + \frac{1 \cdot e^{-1}}{2 \cdot 1!} + \frac{2 \cdot e^{-1}}{2 \cdot 2!} + \frac{3 \cdot e^{-1}}{2 \cdot 3!} + \frac{4 \cdot e^{-1}}{2 \cdot 4!} + \ldots \tag{5}$$

$$= e^{-1}\left(\frac{1}{2} + \frac{1}{2} + \frac{1}{4} + \frac{1}{12} + \ldots\right) \tag{6}$$

$$= 0.5 \tag{7}$$

To find $E(X|X > 0)$, first we need to find $P(X|X > 0)$ which can be given as:

$$P(X|X > 0) = \frac{P(X = k)}{1 - P(X = 0)} \tag{8}$$

$$= \frac{e^{-1}}{2k!(1 - \frac{1}{2}\left(1 + e^{-1}\right))} \tag{9}$$

$$= \frac{e^{-1}}{k!(1 - e^{-1})} \tag{10}$$

$$= \frac{1}{k!(e - 1)} \tag{11}$$

Therefore,

$$E(X|X > 0) = \sum_{k=1}^{\infty} k \frac{1}{k!(e-1)} \tag{12}$$

$$= \frac{1}{e-1}\left(1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + ...\right) \tag{13}$$

$$= 1.581 \tag{14}$$

Refering to equations (7) and (14), we get that option (2) and (4) are correct.