

Assignment

Ncert Exemplar

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I. QUESTION:- GATE 2023.54

Suppose that X is a discrete random variable with the following probability mass

$$P(X = 0) = \frac{1}{2} (1 + e^{-1}) \quad (1)$$

$$P(X = k) = \frac{e^{-1}}{2k!} \text{ for } k = 1, 2, 3, \dots \quad (2)$$

Which of the following is/are true?

- 1) $E(X) = 1$
- 2) $E(X) < 1$
- 3) $E(X|X > 0) < \frac{1}{2}$
- 4) $E(X|X > 0) > \frac{1}{2}$

Solution:

- 1) As we know,

$$E(X) = \sum X p_X(X) \quad (3)$$

Therefore,

$$E(X) = 0 \cdot \frac{1}{2} (1 + e^{-1}) + \sum_{k=1}^{\infty} \frac{k e^{-1}}{2k!} \quad (4)$$

$$= \sum_{k=1}^{\infty} \frac{e^{-1}}{2(k-1)!} \quad (5)$$

$$= \frac{1}{2e} \sum_{k=1}^{\infty} \frac{1}{(k-1)!} \quad (6)$$

$$= \frac{1}{2e} \left(\frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots \right) \quad (7)$$

$$= \frac{1}{2e} \cdot e \quad (\text{Using standard result of exponential series})$$

$$= 0.5 \quad (8)$$

2) To find $E(X|X > 0)$, first we need to find $\Pr(X|X > 0)$ which can be given as:

$$\Pr(X|X > 0) = \frac{\Pr(X = k)}{1 - \Pr(X = 0)} \quad (9)$$

$$= \frac{e^{-1}}{2k!(1 - \frac{1}{2}(1 + e^{-1}))} \quad (10)$$

$$= \frac{e^{-1}}{k!(1 - e^{-1})} \quad (11)$$

$$= \frac{1}{k!(e - 1)} \quad (12)$$

Therefore,

$$E(X|X > 0) = \sum_{k=1}^{\infty} k \frac{1}{k!(e - 1)} \quad (13)$$

$$= \frac{1}{e - 1} \sum_{k=1}^{\infty} \frac{1}{(k - 1)!} \quad (14)$$

$$= \frac{1}{e - 1} \left(\frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots \right) \quad (15)$$

$$= \frac{1}{e - 1} \cdot e \quad (\text{Using standard result of exponential series})$$

$$= 1.582 \quad (16)$$

Referring to equations (8) and (16), we get that option (2) and (4) are correct.