Assignment 1
Ncert Exampler

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I. Question 10.13.2.10

I toss three coins together. The possible outcomes are no heads, 1 heads, 2 heads and 3 heads. So, Isay that probability of no heads is 1/4. What's wrong with this conclusion

Solution:

Let, $p_X(X_i)$ be the sequence of independent Bernoulli random varibles.

$$X_i = \begin{cases} 1, & \text{coin toss result in a Heads} \\ 0, & \text{result in Tails} \end{cases} \tag{1}$$

which means

$$p = p_X(X_i = 1) = 0.5 \tag{2}$$

$$q = p_X(X_i = 0) = 0.5 \tag{3}$$

For number of trials be n and the pmf of getting k heads is given by:

$$p_X(k) = {}^nC_k \times (p)^k \times (q)^{n-k} \tag{4}$$

$$= {}^nC_k \times (0.5)^k \times (0.5)^{n-k} \tag{5}$$

The cdf for pmf of n no. of trails id given by:

$$F_X(k) = p_X(0) + p_X(1) + ... + p_X(k) \tag{6}$$

$$= {}^nC_0 (0.5)^n (0.5)^0 + ... + {}^nC_k (0.5)^{n-k} (0.5)^k \tag{7}$$

Then cmf of getting no head after three toss is given by :

$$F_X(0) = p_X(0) = {}^3C_0 \times \left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^0 \tag{8}$$

$$= \frac{3!}{(3-0)! \times 0!} \times \frac{1}{2}^3 \tag{9}$$

$$= \frac{3!}{3! \times 1} \times \frac{1}{8} \tag{10}$$

$$= \frac{1}{8} \tag{11}$$

$$\therefore p_x(\textit{getting no heads}) = \frac{1}{8} \tag{12}$$

Hence, the given statement is wrong$\left(\because \frac{1}{8} \neq \frac{1}{4}\right)$