**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans)

# Given data

mu = 45 # Mean time required for servicing (minutes)

sigma = 8 # Standard deviation of time required for servicing (minutes)

time\_allowed = 60 # Time allowed for servicing (minutes)

time\_until\_work\_begins = 10 # Time until work begins after drop-off (minutes)

# Time required for servicing exceeding the allowed time

time\_exceeding\_allowed = time\_allowed - time\_until\_work\_begins

# Calculate the Z-score

z\_score = (time\_exceeding\_allowed - mu) / sigma

# Find the probability using the cumulative distribution function (CDF)

probability = 1 - norm.cdf(z\_score)

Probability that the service manager cannot meet his commitment: **0.2659**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44. 🡪 we find that the probability of being older than 44 is approximately 0.1587, and the probability of being between 38 and 44 is approximately 0.3413. Since 0.1587 is greater than 0.3413, it is **TRUE** that more employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees. 🡪 we find that the probability of being under 30 is approximately 0.0912. To find the number of employees under 30, we multiply this probability by the total number of employees (400). Therefore, the expected number of employees under 30 is 0.0912 \* 400 ≈ 36.48. Round of value is 36. Therefore, the statement is **FALSE**; we would expect about 36 employees under the age of 30, not exactly 36
4. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans) For 2X1: Mean = 2μ, Variance = 4σ2

For X1 + X2: Mean = 2μ, Variance = 2σ2

Both 2X1 and X1 + X2 have the same mean but different variance. 2X1 has a large Variance (4σ2) compared to X1 + X2(2σ2). Additionally, while 2X1 only involves one random variable, X1 + X2 involves the sum of two independent random variables.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans) D. 48.5, 151.5

mu = 100

sigma = 20

probability = 0.99

z\_score = norm.ppf((1 + probability) / 2)

a = mu - sigma \* z\_score

b = mu + sigma \* z\_score

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company. 🡪 Range is Rs (99.00810347848784, 980.9918965215122) in Millions
3. Specify the 5th percentile of profit (in Rupees) for the company 🡪 5th percentile of profit (in Million Rupees) is 170.0 in Millions
4. Which of the two divisions has a larger probability of making a loss in a given year? 🡪 Division 1 has a larger probability of making a loss in a given year.