9.3.12

EE24BTECH11028 - Jadhav Rajesh

Question: Which of the following differential equations has $y = x$ as one of its particular solution?

$$(C)\, \frac{d^2y}{dx^2} - x^2 \frac{dy}{dx} + xy = 0 \tag{0.1}$$

Solution: By first principle of derivatives,

$$y'(t) = \lim_{h \to 0} \frac{y(t+h) - y(t)}{h} \tag{0.2}$$

$$y(t+h) = y(t) + hy'(t) \tag{0.3}$$

$$\frac{d^2y}{dx^2} - x^2 \frac{dy}{dx} + xy = 0 \tag{0.4}$$

Rewriting the given equation , we get:

$$\frac{d^2y}{dx^2} = x^2 \frac{dy}{dx} - xy \tag{0.5}$$

To solve this equation numeriacally,we apply Euler's method. we start by introducing the following substitution:
Let:

$$y_1 = y, y_2 = \frac{dy}{dx} \tag{0.6}$$

Thus, the system become:

$$y_1' = y_2 \tag{0.7}$$

$$y_2' = x^2 y_2 - xy_1 \tag{0.8}$$

The system of equation in matrix form:

$$y' = \begin{pmatrix} y_2 \\ x^2 y_2 - xy_1 \end{pmatrix} \tag{0.9}$$

Using Euler's method, the update formulas become:

$$y_1(x+h) = y_1 + h \cdot y_2 \tag{0.10}$$

$$y_2(x+h) = y_2(x) + h \cdot \left(x^2 y_2 - xy_1\right) \tag{0.11}$$

This can expressed in matrix form as:

$$y_{n+1} = y_n + h \begin{pmatrix} 0 & 1 \\ x & -x^2 \end{pmatrix} \cdot y_n \tag{0.12}$$

We will use the initial condition:

$$y_1 = 1, y_2 = 0 \tag{0.13}$$

We ,applying Euler's method and iterating.The following plot represent the solution based on these initial condition.

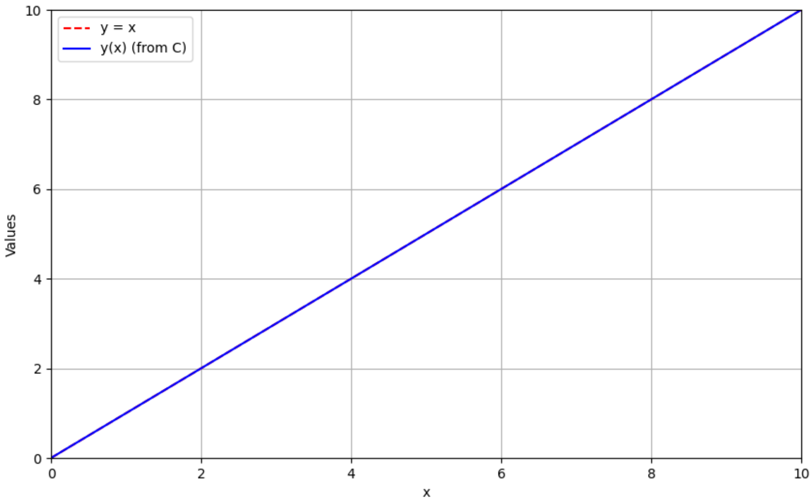

Fig. 0.1: Numerical Solution