EE24BTECH11028 - Jadhav Rajesh

Question: Which of the following differential equations has y = x as one of its particular solution?

$$(C)\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + xy = 0 {(0.1)}$$

Solution: NUMERICAL METHOD

Consider,

$$\frac{d^2y}{dx^2} - x^2 \frac{dy}{dx} + xy = 0 {(0.2)}$$

Assuming the initial conditions y(0) = 0 and y'(0) = 1.

HOMOGENEOUS PART:

The associated homogeneous equation is:

$$\frac{d^2y}{dx^2} - x^2 \frac{dy}{dx} + xy = 0. ag{0.3}$$

Assume a power series solution:

$$y_h = \sum_{n=0}^{\infty} a_n x^n. \tag{0.4}$$

The derivatives are:

$$\frac{dy_h}{dx} = \sum_{n=1}^{\infty} n a_n x^{n-1}, \quad \frac{d^2 y_h}{dx^2} = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}.$$
 (0.5)

Substitute into the homogeneous equation:

$$\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} - x^2 \sum_{n=1}^{\infty} na_n x^{n-1} + x \sum_{n=0}^{\infty} a_n x^n = 0.$$
 (0.6)

Rewriting terms, we derive the recurrence relation:

$$a_{n+2} = \frac{n-2}{(n+2)(n+1)} a_{n-1}.$$
 (0.7)

Apply the initial conditions

The initial conditions are: $y(0) = 0 \implies a_0 = 0$, $y'(0) = 1 \implies a_1 = 1$.

Using the recurrence relation:

$$a_{n+2} = \frac{n-2}{(n+2)(n+1)} a_{n-1}, \tag{0.8}$$

we compute the coefficients:

- 1) For n = 0: $a_0 = 0$
- 2) For n = 1: $a_1 = 1$
- 3) For n = 2: $a_2 = \frac{0-2}{(2+2)(2+1)}a_1 = \frac{-2}{12} = -\frac{1}{6}$
- 4) For n = 3: $a_3 = \frac{1-2}{(3+2)(3+1)}a_2 = \frac{-1}{20} \cdot \left(-\frac{1}{6}\right) = \frac{1}{120}$ 5) For n = 4: $a_4 = \frac{2-2}{(4+2)(4+1)}a_3 = 0$

The pattern is:

$$a_{2k} = 0, \quad a_{2k+1} = \frac{(-1)^k}{(2k+1)!}.$$
 (5.1)

Therefore, the homogeneous solution is:

$$y = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}.$$
 (5.2)

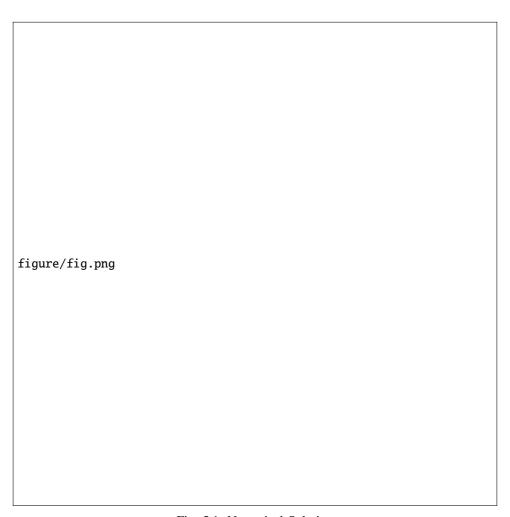


Fig. 5.1: Numerical Solution