9-9.3-12

EE24BTECH11028 - Jadhav Rajesh

Question: Ifarea between the curve $x = y^2$ and $x = 4$ is diveded into two equalparts by the line $x = a$,then find the value of a.
Solution: : The given parameters are

Variable	**Description**
V, u, f	Parameters of Parabola
q_1, m_1, q_2, m_2	Parameters of lines
a_0, a_1, a_2, a_3	Points of intersection
A	Area between the conics

TABLE 0: Variables Used

$$V = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, u = \frac{-1}{2} e_1 f = 0 \tag{0.1}$$

the parameters of the lines are

$$q_2 = \begin{pmatrix} a \\ 0 \end{pmatrix}, m_2 = e_2 \tag{0.2}$$

$$\mu_i = \frac{1}{m^T V m}((-m^T(Vh + u) \pm \sqrt{(m^T(Vh + u))^2} - g(h)(m^T Vm)) \tag{0.3}$$

subtituting the above values in (0.3)

$$\mu_i = a, -a \tag{0.4}$$

yeilding the points of intersection as

$$a_0 = \begin{pmatrix} a \\ a \end{pmatrix}, a_1 = \begin{pmatrix} a \\ -a \end{pmatrix} \tag{0.5}$$

simailar,for the line $x - 4 = 0$,

$$q_1 = \begin{pmatrix} 4 \\ 0 \end{pmatrix}, m_1 = e_2 \tag{0.6}$$

yeilding

$$\mu_i = 2, -2 \tag{0.7}$$

and

$$a_3 = \begin{pmatrix} 4 \\ 2 \end{pmatrix}, a_2 = \begin{pmatrix} 4 \\ -2 \end{pmatrix} \tag{0.8}$$

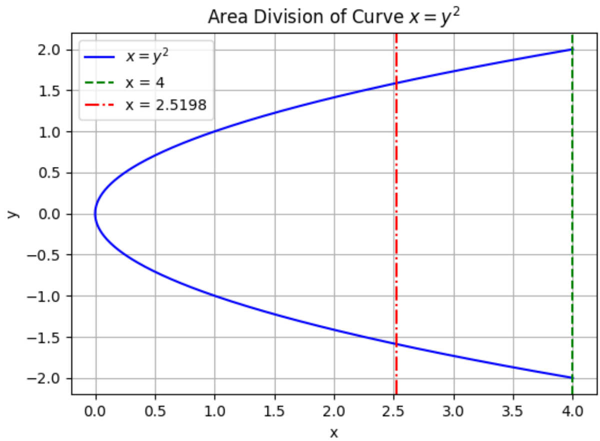
Area between parabola and the line $x = 4$ is divided equally by the line $x = a$.Thus,

$$A_1 = \int_0^a \sqrt{x}\mathrm{d}x \tag{0.9}$$

$$A_2 = \int_a 4\sqrt{x}\mathrm{d}x \tag{0.10}$$

$$and A_1 = A_2 \tag{0.11}$$

$$\Rightarrow a = 4^{\frac{2}{3}} \tag{0.12}$$



Area Division of Curve $x = y^2$
$x = y^2$
x = 4
x = 2.5198
y
x
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0

Fig. 0.1