

8.EX-10

EE24BTECH11029 - J SHRETHAN REDDY

Question:

Find the area of the region enclosed between the two circles: $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$

Answer:

THEORETICAL SOLUTION

$$x^2 + y^2 = 4 \quad (0.1)$$

$$(x - 2)^2 + y^2 = 4 \quad (0.2)$$

equate equation (0.1) and (0.2).we get

$$x = 1 \quad y = \pm \sqrt{3} \quad (0.3)$$

The points of intersection of given circles are $A(1, \sqrt{3})$ and $A'(1, -\sqrt{3})$

Area of enclosed by two circles

$$area = 2 \left[\int_0^1 y \, dx + \int_1^2 y \, dx \right] \quad (0.4)$$

$$= 2 \left[\int_0^1 \sqrt{4 - (x - 2)^2} \, dx + \int_1^2 \sqrt{4 - x^2} \, dx \right] \quad (0.5)$$

$$= 2 \left[\frac{1}{2} (x - 2) \sqrt{4 - (x - 2)^2} + \frac{1}{2} \times 4 \sin^{-1} \frac{x - 2}{2} \right]_0^1 + 2 \left[\frac{1}{2} x \sqrt{4 - x^2} + \frac{1}{2} \times 4 \sin^{-1} \frac{x}{2} \right]_1^2 \quad (0.6)$$

$$= \left[(-\sqrt{3} - 4 \sin^{-1}) \left(\frac{-1}{2} \right) - 4 \sin^{-1}(-1) \right] + \left[4 \sin^{-1} 1 - \sqrt{3} - 4 \sin^{-1} \frac{1}{2} \right] \quad (0.7)$$

$$= \left(-\sqrt{3} - \frac{2\pi}{3} + 2\pi \right) + \left(2\pi - \sqrt{3} - \frac{2\pi}{3} \right) \quad (0.8)$$

$$= \frac{8\pi}{3} - 2\sqrt{3} \quad (0.9)$$

Computational Solution:

Using the trapezoidal rule to get the area

The trapezoidal rule is as follows.

$$A = \int_a^b f(x) dx \approx h \left(\frac{1}{2} f(a) + f(x_1) + f(x_2) \cdots + f(x_{n-1}) + \frac{1}{2} f(b) \right) \quad (0.10)$$

$$h = \frac{b-a}{n} \quad (0.11)$$

$$A = j_n, \quad \text{where,} \quad j_{i+1} = j_i + h \frac{f(x_{i+1}) + f(x_i)}{2} \quad (0.12)$$

$$j_{i+1} = j_i + h \left(\sqrt{x_{i+1}} + \sqrt{x_i} \right) \quad (0.13)$$

$$x_{i+1} = x_i + h \quad (0.14)$$

$$h = \frac{1}{30000} \quad (0.15)$$

$$n = 30000 \quad (0.16)$$

Using the code answer obtained is $A = sq.\text{units}$