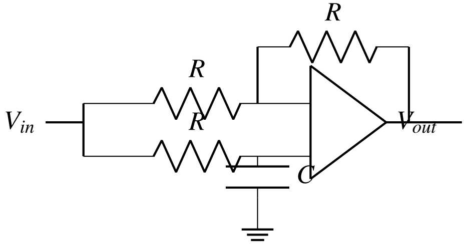
2020-GATE-PH-27-39

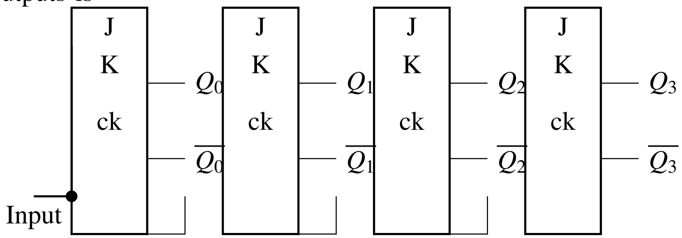
EE24BTECH11029- JANAGANI SHRETHAN REDDY

1) Let $|e_1\rangle = \begin{pmatrix}1\\0\\0\end{pmatrix}, |e_2\rangle = \begin{pmatrix}1\\1\\0\end{pmatrix}, |e_3\rangle = \begin{pmatrix}1\\1\\1\end{pmatrix}$.Let $S = \{\mathbf{e_1}, \mathbf{e_2}, \mathbf{e_3}\}$.Let $\mathbf{R}^3$ denote the three dimensional real vector space.Which one of the following is correct?
   a) S is an orthonormal set
   b) S is a linearly dependent set
   c) S is a basis for $\mathbf{R}^3$
   d) $\sum_{i=1}^{3} = |e_i\rangle\langle e_i| = \begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$
2) $\mathbf{S_x}$ denotes the spin operator defined as $S_x = \frac{h}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}$.Which one of the following is correct?
   a) The eigenstates spin operator S_x are $|\uparrow\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ and $|\downarrow\rangle_x = \begin{pmatrix}0\\1\end{pmatrix}$
   b) The eigenstates spin operator S_x are $|\uparrow\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-\end{pmatrix}$ and $|\downarrow\rangle_x = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$
   c) In the spin state $\frac{1}{2}\left\{\frac{1}{\sqrt{3}}\right\}$,upon the measurement of $\mathbf{S_x}$, the probability for obtaining $|\uparrow\rangle_x$ is $\frac{2+\sqrt{3}}{4}$
   d) In the spin state $\frac{1}{2}\left\{\frac{1}{\sqrt{3}}\right\}$,upon the measurement of $\mathbf{S_x}$, the probability for obtaining $|\uparrow\rangle_x$ is $\frac{1}{4}$
3) The input voltage ($\mathbf{V}_{in}$) to the circuit shown in the figure is $2\cos(100t)\,V$.The output voltage ($\mathbf{V}_{out}$) is $2\cos\left(100t - \frac{\pi}{2}\right) V$.If value of $C\,(in\mu F)$ is



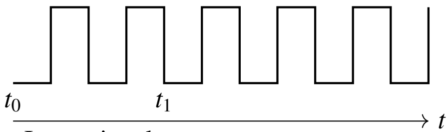
   a) 0.1
   b) 1
   c) 10
   d) 100

4) Consider a 4-bit counter constructed out of four flip-flops. It is formed by connecting the J and K inputs to logic high and feeding the Q output to the clock input of the following flip-flop . The input signal to the counter is a series of square pulses and the change of state is triggered by the falling edge. At time $t = t_0$ the outputs are in logic low state $(Q_0 = Q_1 = Q_2 = Q_3 = 0)$. Then at $t = t_1$, the logic state of the outputs is



1 (logic high)

4-bit ripple counter



Input signal

a) $Q_0 = 1, Q_1 = 0, Q_2 = 0, Q_3 = 0$
b) $Q_0 = 0, Q_1 = 0, Q_2 = 0, Q_3 = 1$
c) $Q_0 = 1, Q_1 = 0, Q_2 = 1, Q_3 = 0$
d) $Q_0 = 0, Q_1 = 1, Q_2 = 1, Q_3 = 1$

5) Consider the Lagrangian $L = a\left(\frac{dx}{dy}\right)^2 + b\left(\frac{dy}{dt}\right)^2 + cxy$, where a, b and c are constants.If P_x and P_y are the momenta conjugate to the coordinates x and y respectively, then the Hamiltonian is

a) $\frac{P^2x}{4a} + \frac{P^2y}{4b} - cxy$
b) $\frac{P^2x}{2a} + \frac{P^2y}{2b} - cxy$
c) $\frac{P^2x}{2a} + \frac{P^2y}{2b} + cxy$
d) $\frac{P^2x}{a} + \frac{P^2y}{b} + cxy$

6) Which one of the following matrices does NOT represent a proper rotation in a plane?

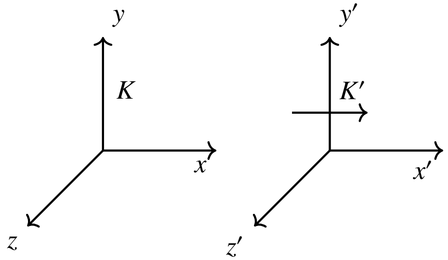
a) $\begin{pmatrix} -\sin\theta & \cos\theta \\ -\cos\theta & -\sin\theta \end{pmatrix}$
b) $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$
c) $\begin{pmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{pmatrix}$

d) $\begin{pmatrix} -\sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{pmatrix}$

7) A uniform magnetic field $\mathbf{B} = B_0\mathbf{y}$ exits in a inertial frame K. A perfect conducting sphere moves with a constant velocity $\mathbf{v} = v_0\mathbf{x}$ with respect to this inertial fame.The rest frame of the sphere is $K\prime$.The electrical and magnetic fields in k and $k\prime$ are related as

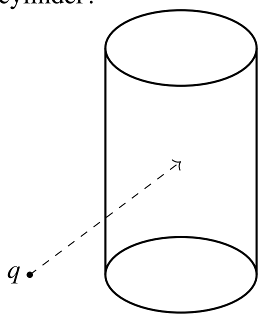
$$\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}, \begin{cases} \mathbf{E}'_{\|} = \mathbf{E}_{\|} & \mathbf{E}'_{\perp} = \gamma\left(\mathbf{E}_{\perp}\right) + \mathbf{v}\times\mathbf{B} \\ \mathbf{B}'_{\|} = \mathbf{B}_{\|} & \mathbf{B}'_{\perp} = \gamma\left(\mathbf{B}_{\perp}\right) + \mathbf{v}\times\mathbf{E} \end{cases}$$

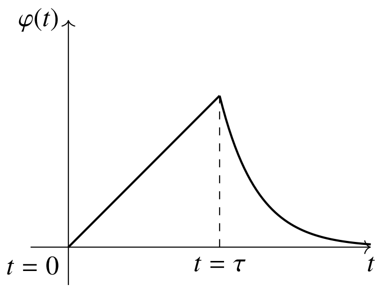
The induced surface charge density on the sphere in the fame K' is

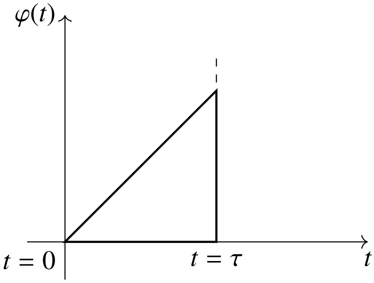


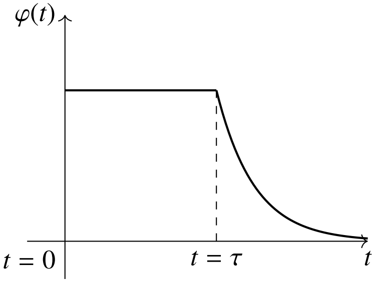
a) maximum along Z'
b) maximum along y'
c) maximum along x'
d) uniform over the sphere

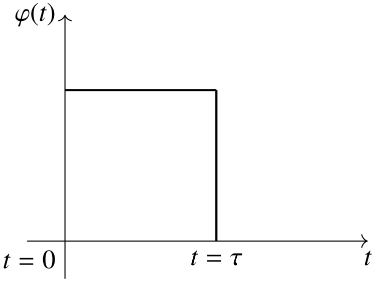
8) A charge q moving with uniform speed enters a cylinder region in free space at $t = 0$ and exits the region at $t = \tau$.Which one of the following options best describe the time dependence of the total electric flux $\phi(t)$, through the entire surface of the cylinder?



a) 

b) 

c) 

d) 

9) Consider a one-dimensional non-magnetic crystal with one atom per unit cell.Assume that the valence electrons (i) do not interact weakly with the ions. If n is number of valence electrons per unit cell, then at $0K$,
   a) the crystal is metallic for any value of n
   b) the crystal is non-metallic for any value of n
   c) the crystal is metallic for even value of n
   d) the crystal is metallic for odd value of n

10) According to the Fermi gas model of the nucleus, the nucleons move in a spherical volume of radius $R = R_0 A^{\frac{1}{3}}$,where A is the mass number and R_0 is an empirical constant with the dimensions of length.The Fermi energy of the nucleaus E_F is proportional to
   a) ${R_0}^2$
   b) $\frac{1}{R_0}$
   c) $\frac{1}{{R_0}^2}$
   d) $\frac{1}{{R_0}^3}$

11) Consider a two dimensional crystal with 3 atoms in the basis. The number of allowed

in optical branches (n) and acoustic branches (m) due to the lattice vibrations are

a) $(n, m) = (2, 4)$
b) $(n, m) = (3, 3)$
c) $(n, m) = (4, 2)$
d) $(n, m) = (1, 5)$

12) The internal energy U of a system is given by $U(S, V) = \lambda V^{-\frac{2}{3}} S^2$,where λ is a constant of appropriate dimensions; V and S denote the volume and entropy respectively. Which one of the following gives the correct equation of state of the system?

a) $\frac{PV^{\frac{1}{3}}}{T^2}$ =constant
b) $\frac{PV}{T^{\frac{1}{3}}}$ =constant
c) $\frac{PV}{v^{\frac{1}{3}}T}$ =constant
d) $\frac{PV^{\frac{2}{3}}}{T}$ =constant

13) The potential energy of a particle of mass m is given by
$U(x) = a \sin\left(k^2 x - \frac{\pi}{2}\right), \quad a>0, k^2>0.$
The angular frequency of small oscillations of the particle about $x = 0$ is

a) $k^2 \sqrt{\frac{2a}{m}}$
b) $k^2 \sqrt{\frac{a}{m}}$
c) $k^2 \sqrt{\frac{a}{2m}}$
d) $2k^2 \sqrt{\frac{a}{m}}$