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1) If $a \in |a|<1$, then the value of
$\frac{\left(1-|a^2|\right)}{\pi} \int_\Gamma \frac{|dz|}{|z+\alpha|^2},$
where Γ is simple closed curve $|z| = 1$ taken with the positive orientation is

2) Cosider $C[-1,1]$ equipped with the supremum norm given by $||f||_\infty = sup\{|f(t)| : t \in [-1,1]\}$ for $f \in C[-1,1]$.Define a linear functional T on $C[-1,1]$ by $T(f) = \int_0^1 f(t)\,dt - \int_0^1 f(t)\,dt$ for all $f \in C[-1,1]$.Then the value of $||T||$ is

3) Consider the vector space $C[0,1]$ over R. Cosider the following statements:
P : If the set $\{t, f_1, t^2 f_2, t^3 f_3\}$ is linearly independent, then the set $\{f_1, f_2, f_3\}$ is linearly independent, where $f_1, f_2, f_3 \in C[0,1]$ and t^n represents the polynomial function $t \to t^n, n \in N$
Q : If $F : C[0,1] \to R$ is given by $F(x) = \int_0^1 x\left(t^2\right) dt$ for each $x \in C[0,1]$, then F is a linear map.
Which of the above statements hold TRUE?
   a) Only P
   b) Only Q
   c) Both P and Q
   d) Neither P nor Q

4) Using the Newton-Raphson method with the initial guess $x^{(0)} = 6$, the apporximate value of the real root of $x \log_{10} x = 4.77$, after the second iteration, is

5) Let the following dicrete data be obtained from a curve $y = y(x)$;
x : 0 0.25 0.5 0.75 1.0
y : 1 0.9896 0.9589 0.9089 0.8415
Let S be the solid of revolution obtained by rotating the above curve about the $x-axis$ between $x = 0$ and $x = 1$ and let V denote its volume. The approximate value of V, obtained using Simpsons $\frac{1}{3}$ rule, is

6) The integral surface of the first order partial differential equation
$2y(z-3)\frac{dz}{dx} + (2x-z)\frac{dz}{dy} = y(2x-3)$
passing through the curve $x^2 + y^2 = 2x, z = 0$ is
   a) $x^2 + y^2 - z^2 - 2x + 4z = 0$
   b) $x^2 + y^2 - z^2 - 2x + 8z = 0$
   c) $x^2 + y^2 + z^2 - 2x + 16z = 0$

d) $x^2 + y^2 + z^2 - 2x + 8z = 0$

7) The boundary value problem $\frac{d^2\phi}{dx^2} + \lambda\phi = x, \phi(0) = 0$ and $\frac{d\phi}{dx}(1) = 0$, is converted into the integral equation $\phi(x) = g(x) + \lambda \int_0^1 k(x,\xi)\phi(\xi)\, d\xi$, where the kernel $k(x,\xi) = \begin{cases} \xi, 0<\xi<x \\ x, x<\xi<1 \end{cases}$ then $g\left(\frac{2}{3}\right)$ is

8) If $y_1(x) = x$ is a solution to the differential equation $\left(1-x^2\right)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$, then its general solution is
   a) $y(x) = c_1x + c_2\left(x\ln|1+x^2| - 1\right)$
   b) $y(x) = c_1x + c_2\left(\ln\frac{|1-x|}{|1+x|} + 1\right)$
   c) $y(x) = c_1x + c_2\left(\frac{x}{2}\ln\frac{|1+x|}{|1-x|} - 1\right)$
   d) $y(x) = c_1x + c_2\left(x\ln|1-x^2| - 1\right)$

9) The solution to the initial value problem $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = 3e^{-t}\sin t, y(0) = 0$ and $\frac{dy}{dt}(0) = 3$, is
   a) $y(t) = e^t(\sin t + \sin 2t)$
   b) $y(t) = e^{-t}(\sin t + \sin 2t)$
   c) $y(t) = 3e^t(\sin t)$
   d) $y(t) = 3e^{-t}(\sin t)$

10) The time to failure, in months, of light bulbs manufactured at two plants A and Bobey the exponential distribution with means 6 and 2 months respectively. Plant B produces four times as many bulbs as plant A does. Bulbs from these plants are indistinguishable. They are mixed and sold together. Given that a bulb purchased at random is working after 12 months, the probability that it was manufactured at plant A is

11) Let X, Y be continuous random variables with joint density function
$f_{x,y}(x,y) = \begin{cases} e^{-y}(1-e^{-x})\, if 0<x<y<\infty \\ e^{-x}(1-e^{-y})\, if 0<y<x<\infty \end{cases}$
The value of $E[X+Y]$ is

12) Let $X(0,1)(1,2)$ be the subspace of R, where R is equipped with the usual topology. Which of the following is FALSE?
   a) There exists a non-constant continitous function $f : X \to Q$
   b) X is homeomorphic to $(-\infty,-3) \cup [0,\infty)$
   c) There exists an onto continuous function : $[0,1] \to \mathbf{X}$, where $\mathbf{X}$ is closure of X in $\mathbf{R}$
   d) There exists an onto continuous function $f :\ [0,1] \to X$

13) Let $X = \begin{pmatrix} 2 & 0 & -3 \\ 3 & -1 & -3 \\ 0 & 0 & -1 \end{pmatrix}$.A matrix P such that $P^{-1}XP$ is a diagonal matrix, is

a) $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

b) $\begin{pmatrix} -1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

c) $\begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

d) $\begin{pmatrix} -1 & -1 & 1 \\ 0 & -1 & 1 \\ 1 & 1 & 0 \end{pmatrix}$