JEE MAINS

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1) The area of the region,enclosed by the circle $x^2 + y^2 = 2$ which is not common to the region bounded by the parabola $y^2 = x$ and straight line $y = x$, is
   a) $\left(\frac{1}{3}\right)(12\pi - 1)$
   b) $\left(\frac{1}{6}\right)(12\pi - 1)$
   c) $\left(\frac{1}{3}\right)(6\pi - 1)$
   d) $\left(\frac{1}{6}\right)(24\pi - 1)$

2) Total number of six-digit numbers in which only and all five digits $1, 3, 5, 7$ and 9 appear, is
   a) 5^6
   b) $\left(\frac{1}{2}\right)(6!)$
   c) $6!$
   d) $\left(\frac{5}{2}\right)6!$

3) An unbiased coin is tossed 5 times. Suppose that a variable X is assigned the value k when k consecutive heads are obtained for $k = 3, 4, 5$, otherwise X takes the value -1. The expected value of X, is
   a) $\frac{1}{8}$
   b) $\frac{3}{16}$
   c) $\frac{-1}{8}$
   d) $\frac{-3}{16}$

4) If Re $\frac{(z-1)}{(2z+i)} = 1$, where $z = x + iy$, then the point (x, y) lies on a
   a) circle whose centre is at $\left(\frac{-1}{2}, \frac{-3}{2}\right)$
   b) straight line whose slope is $\frac{3}{2}$
   c) circle whose diameter is $\frac{\sqrt{5}}{2}$
   d) straight line whose slope is $\frac{-2}{3}$

5) If $f(a + b + 1 - x) = f(x) \forall x$, where a and b are fixed positive real numbers,then $\frac{1}{(a+b)} \int_a^b x(f(x) + f(x + 1))\, dx$ is equal to
   a) $\int_{a-1}^{b-1} f(x)\, dx$
   b) $\int_{a+1}^{b+1} f(x + 1)\, dx$
   c) $\int_{a-1}^{b-1} f(x + 1)\, dx$
   d) $\int_{a+1}^{b+1} f(x)\, dx$

6) If the distance between the foci of an ellipse is 6 and the distance between its directrices is 12, then the length of its latus rectum is
   a) $2\sqrt{3}$
   b) $\sqrt{3}$
   c) $\frac{3}{\sqrt{2}}$
   d) $3\sqrt{2}$

7) The logical statement $(p \implies q) \wedge (q \implies \sim p)$ is equivalent to
   a) $\sim p$
   b) p
   c) q
   d) $\sim q$

8) Te greatest positive integer k, for which $49^k + 1$ is a factor of the sum $49^{125} + 49^{124} + \cdots + 49^2 + 49 + 1$, is
   a) 32
   b) 60
   c) 65
   d) 63

9) A vector $\mathbf{a} = \alpha\hat{i} + 2\hat{j} + \beta\hat{k}\,(\alpha, \beta \in \mathbf{R})$ lies in the plane of the vectors,$\mathbf{b} = \hat{i} + \hat{j}$ and $\mathbf{c} = \hat{i} - \hat{j} + 4\hat{k}$.If $\mathbf{a}$ bisects the angle between $\mathbf{b}$ and $\mathbf{c}$, then
   a) $\mathbf{a}\hat{i} + 3 = 0$
   b) $\mathbf{a}\hat{k} + 4 = 0$
   c) $\mathbf{a}\hat{i} + 1 = 0$
   d) $\mathbf{a}\hat{k} + 2 = 0$

10) If $y(\alpha) = \sqrt{2\left(\frac{\tan\alpha + \cot\alpha}{1 + \tan^2\alpha}\right) + \frac{1}{\sin^2\alpha}}$ where $\alpha \in \left(\frac{3\pi}{4}, \pi\right)$ then $\frac{dy}{d\alpha}$ at $\alpha = \frac{5\pi}{6}$ is
   a) $-\frac{1}{4}$
   b) $\frac{4}{3}$
   c) 4
   d) -4

11) $y = mx + 4$ is a tangent to both the parabolas, $y^2 = 4x$ and $x^2 = 2by$, then b is equal to
   a) -64
   b) 128
   c) -128
   d) -32

12) Let α be a root of the equation $x^2 + x + 1 = 0$ and the matrix $A = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha^4 \end{pmatrix}$ then the matrix A^{31} is equal to
   a) A
   b) A^2
   c) A^3
   d) l_3

13) If $g(x) = x^2 + x - 1$ and $(gof)(x) = 4x^2 - 10x + 5$,then $f\left(\frac{5}{4}\right)$ is equal to
   a) $\frac{-3}{2}$
   b) $\frac{-1}{2}$
   c) $\frac{1}{2}$
   d) $\frac{3}{2}$

14) let α and β are two real roots of the equation $(k+1)\tan^2 x - \sqrt{2}\lambda \tan x = 1 - k$, where $(k \neq -1)$ and are real numbers. If $\tan^2(\alpha + \beta) = 50$, then value of λ is
   a) $5\sqrt{2}$
   b) $10\sqrt{2}$
   c) 10
   d) 5

15) Let P be a plane passing through the points $(2, 1, 0), (4, 1, 1)$ and $(5, 0, 1)$ and R be any point $(2, 1, 6)$. Then the image of R in the plane P is:
   a) $(6, 5, 2)$
   b) $(6, 5, -2)$
   c) $(4, 3, 2)$
   d) $(3, 4, -2)$