8.EX-10

EE24BTECH11029 - J SHRETHAN REDDY

Question:
Find the area of the region enclosed between the two circles: $x^2+y^2 = 4$ and $(x-2)^2+y^2 = 4$
Answer:
THEORETICAL SOLUTION

$$x^2 + y^2 = 4 \tag{0.1}$$
$$(x-2)^2 + y^2 = 4 \tag{0.2}$$

equate equation (0.1) and (0.2).we get

$$x = 1 \quad y = \pm\sqrt{3} \tag{0.3}$$

The points of intersection of given circles are $A\left(1, \sqrt{3}\right)$ and $A'\left(1, -\sqrt{3}\right)$
Area of enclosed by two circles

$$area = 2\left[\int_0^1 y\,dx + \int_1^2 y\,dx\right] \tag{0.4}$$
$$= 2\left[\int_0^1 \sqrt{4-(x-2)^2}\,dx + \int_1^2 \sqrt{4-x^2}\,dx\right] \tag{0.5}$$
$$= 2\left[\frac{1}{2}(x-2)\sqrt{4-(x-2)^2} + \frac{1}{2}\times 4\sin^{-1}\frac{x-2}{2}\right]_0^1 + 2\left[\frac{1}{2}x\sqrt{4-x^2} + \frac{1}{2}\times 4\sin^{-1}\frac{x}{2}\right]_1^2 \tag{0.6}$$
$$= \left[\left(-\sqrt{3} - 4\sin^{-1}\right)\left(\frac{-1}{2}\right) - 4\sin^{-1}(-1)\right] + \left[4\sin^{-1}1 - \sqrt{3} - 4\sin^{-1}\frac{1}{2}\right] \tag{0.7}$$
$$= \left(-\sqrt{3} - \frac{2\pi}{3} + 2\pi\right) + \left(2\pi - \sqrt{3} - \frac{2\pi}{3}\right) \tag{0.8}$$
$$= \frac{8\pi}{3} - 2\sqrt{3} \tag{0.9}$$

Computational Solution:
Using the trapezoidal rule to get the area
The trapezoidal rule is as follows.

$$A = \int_a^b f(x)\, dx \approx h\left(\frac{1}{2} f(a) + f(x_1) + f(x_2) \cdots + f(x_{n-1}) + \frac{1}{2} f(b)\right) \tag{0.10}$$

$$h = \frac{b-a}{n} \tag{0.11}$$

$$A = j_n, \quad where, \quad j_{i+1} = j_i + h\frac{f(x_{i+1}) + f(x_i)}{2} \tag{0.12}$$

$$j_{i+1} = j_i + h\left(\sqrt{x_{i+1}} + \sqrt{x_i}\right) \tag{0.13}$$

$$x_{i+1} = x_i + h \tag{0.14}$$

$$h = \frac{1}{30000} \tag{0.15}$$

$$n = 30000 \tag{0.16}$$

Using the code answer obtained is $A = 1.369707 sq$.units

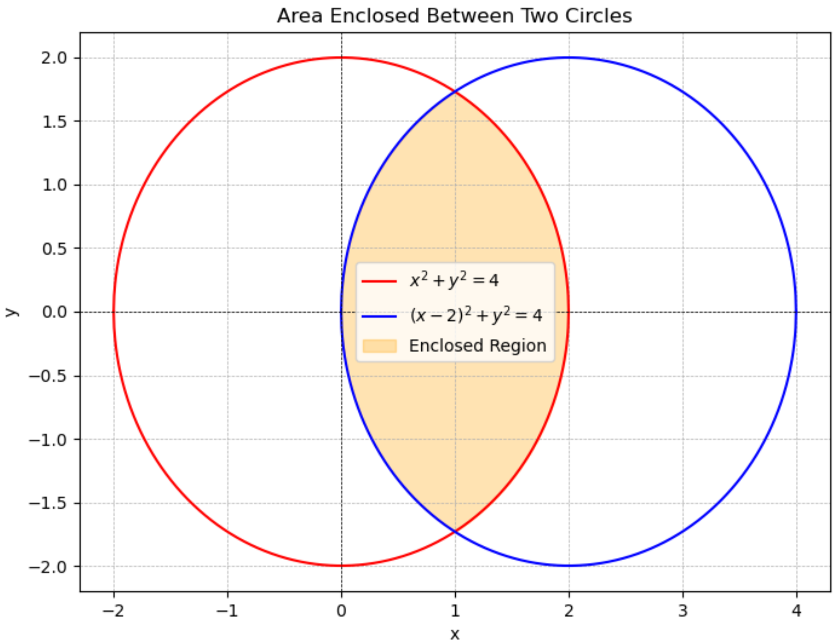

Fig. 0.1: plot