

Time Complexity :- Rate of growth

$$\approx \frac{\text{Change in no. of operation}}{\text{Change in input}}$$

$$= \lim_{\Delta n \rightarrow 0} \frac{\Delta \text{operation}}{\Delta n} = \frac{\text{d operation}}{\text{d } n}$$

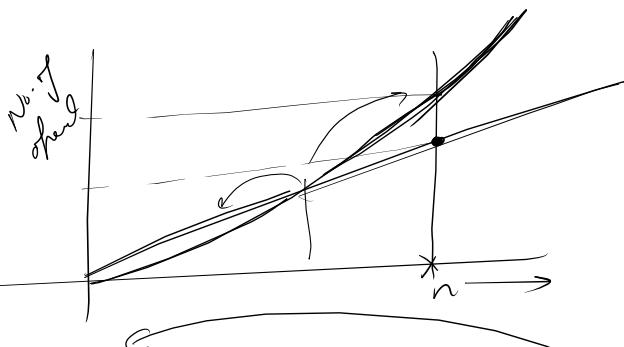
No. of operations

$f(n) \sim 3n^1 + 3$ → Sum of Natural No.

$$\textcircled{1} \quad \frac{df(n)}{dn} = 3$$

$$N^{1.5} \longrightarrow 18 \\ N=6 \longrightarrow 21^2 + 3$$

\textcircled{2}



\# $f(n) \sim 3n^1 + 3$

$g(x)$ → Rate of growth

$O(g(x))$

$f(n) \sim n^2 + n$

$\hookrightarrow O(n^2)$

AP \rightarrow Arithmetic progression

$$\text{AP} \quad a, \underbrace{a+d, a+2d}_{\text{Common diff}} , a+3d, a+4d, \dots, a+(n-1)d$$

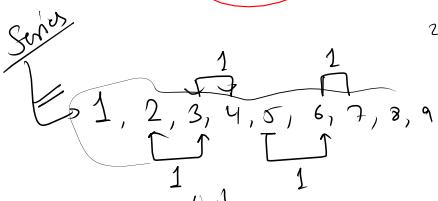
$$\hookrightarrow a_n = a + (n-1)d$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$= \frac{n}{2} [a + a + (n-1)d]$$

No. of terms

$$= \frac{n}{2} [first term + last term]$$



$$\textcircled{\#} \quad 2 + 4 + 6 + 8 + 10 + 12 + \dots + \underbrace{2n}_{\text{last term}} \rightarrow S_n = \frac{n}{2} (2 + 2n)$$

$$\textcircled{\times} \quad S_n = \frac{n}{2} (2 + 2n)$$

No. of terms

1st term

2nd term

3rd term

4th term

5th term

6th term

7th term

8th term

9th term

10th term

11th term

12th term

13th term

14th term

15th term

16th term

17th term

18th term

19th term

20th term

21st term

22nd term

23rd term

24th term

25th term

26th term

27th term

28th term

29th term

30th term

31st term

32nd term

33rd term

34th term

35th term

36th term

37th term

38th term

39th term

40th term

41st term

42nd term

43rd term

44th term

45th term

46th term

47th term

48th term

49th term

50th term

51st term

52nd term

53rd term

54th term

55th term

56th term

57th term

58th term

59th term

60th term

61st term

62nd term

63rd term

64th term

65th term

66th term

67th term

68th term

69th term

70th term

71st term

72nd term

73rd term

74th term

75th term

76th term

77th term

78th term

79th term

80th term

81st term

82nd term

83rd term

84th term

85th term

86th term

87th term

88th term

89th term

90th term

91st term

92nd term

93rd term

94th term

95th term

96th term

97th term

98th term

99th term

100th term

$$\textcircled{\#} \quad 1 + \underbrace{3 + 5}_{2} + 7 + 9 + 11 \rightarrow$$

$$S_n \Rightarrow \frac{6}{2} [1 + 11] + \frac{6}{2} \times 10, \quad \frac{3}{2} \times 12, \quad \frac{3}{2} \times 40$$

GP

a) a, ar, ar^2, ar^3, ar^4 ratio $\rightarrow a_n = ar^{n-1}$ No. of terms

ratio $\rightarrow \frac{ar^2}{ar} = r^2$ $\rightarrow r^2$ $\rightarrow r$

$$\hookrightarrow 2^1, 2^2, 2^3, 2^4, 2^5, \dots, \underbrace{2^n}_{n^{th} term} \rightarrow 2 \cdot 2^{n-1}$$

$a = 2$

$r = 2$

2^{n+1}

#

Sum

$$1 + 2 + 4 + 8 + 16 + \dots + 2^n = \frac{a(r^{n-1})}{r-1}$$

$a = 1$, $r = 2$

No. of terms = $\cancel{n+1}$

$$S_n = \frac{1(2^{n-1} - 1)}{2 - 1} = \cancel{2^{n-1}} - 1$$

④ $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \dots + \frac{1}{n}$

discrete \rightarrow Continuous

$$\{1, 2, 3, 4, 5, 6, 7, \dots\}$$

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

$$\sum_{n=1}^N \frac{1}{n} = \int_1^N \frac{1}{x} dx$$

$$\int_1^N \frac{1}{x} dx = [\log x]_1^N = \log N - \cancel{\log 1} = \log N$$

$\log_b 1 = 0$
any base

$\log_b a^c = c \log_b a$

```

for(int i=1; i<=n; i++) {
    System.out.println("Hello");
}

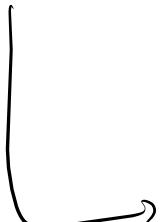
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<u>i value</u>	<u>Iteration</u>	Total work
1	1	C
2	2	C
3	3	C
⋮	⋮	⋮
n	n	C

Time complexity, Total

$$= C + C + C + \dots + C^n$$

$$\cancel{T(n)} = ne$$

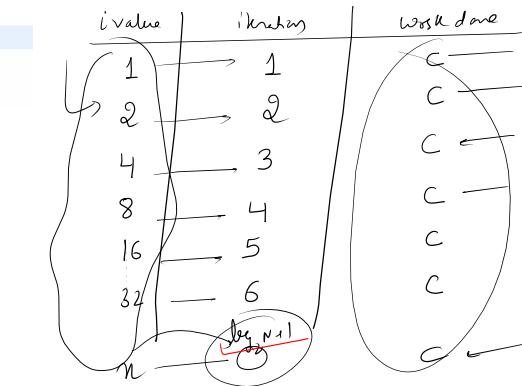


$$\cancel{g(n)}$$

$$= O(n)$$

$$C^n$$

```
for(int i=1; i<=n; i*=2) {
    System.out.println("Hello");
}
```



$$C + C + C + \dots + C \xrightarrow{\log_2 n + 1} C(\log_2 n)$$

$$O(\log_2 n)$$

Diagram illustrating the recurrence relation for the work done:

$$a_n = ar^{n-1}$$

$$n = 1 \cdot 2^{N-1}$$

$$n = 2^{N-1}$$

$$\log_2 n \xrightarrow{(N-1)\log_2 2} N-1$$

Diagram illustrating the recurrence relation for the work done:

$$a_n = ar^{n-1}$$

$$n \log_{10} r \xrightarrow{\log_{10} r} \frac{N}{\log_{10} r}$$

$$N = 31 \log_2 \frac{10}{r}$$

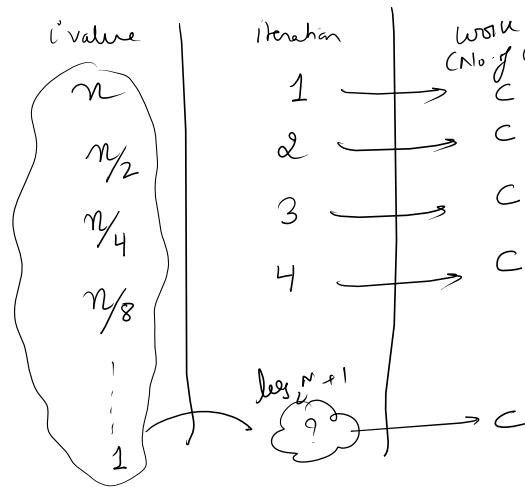
$$2 \leftarrow 2 \leftarrow 2$$

$$\hookrightarrow 2 \times 3$$

```

for(int i=n;i>1;i=i/2) {
    System.out.println("Hello");
}

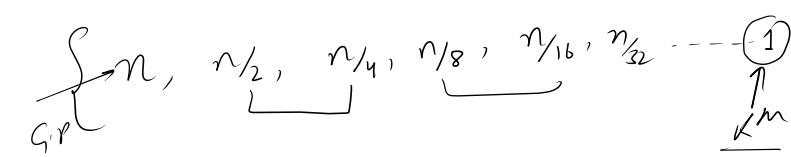
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Total work ~ $c + c + c + c + \dots + \log_2^{n+1}$

$$f(n) = \cancel{c + \log_2^{n+1}}$$

$$g(n) = \underline{\underline{\mathcal{O}(\log_2 n)}}$$



$$a_n = a \sigma^{n-1}$$

$$a_k = a \sigma^{k-1}$$

$$1 = n \cdot \left(\frac{1}{2}\right)^{k-1}$$

$$\left(\frac{1}{2}\right)^{k-1} = 1$$

$$1 = \cancel{n \cdot \frac{1}{2}^{k-1}}$$

$$f(x^{(k)}) = n$$

$$\log_2^{x^{(k)}} = \log_2^n$$

$$(k-1) \log_2^1 + \log_2^n = k \cdot \log_2^{n+1}$$

```

int i=1;
while(i<=n) {
    System.out.println("Hello");
    i=i*2;
    i=i*3;
}

```

$$\begin{array}{l}
 a^1 \\
 r = 6 \\
 a_n = a \cdot r^{n-1} \\
 n = 1 \cdot 6^{n-1}
 \end{array}$$

i	value	iteration	work done
1	1	1	C
6	6	2	C
36	36	3	C
(6) ³	(6) ³	4	C
⋮	⋮	⋮	⋮
n	$6^{\frac{n-1}{2}}$		C

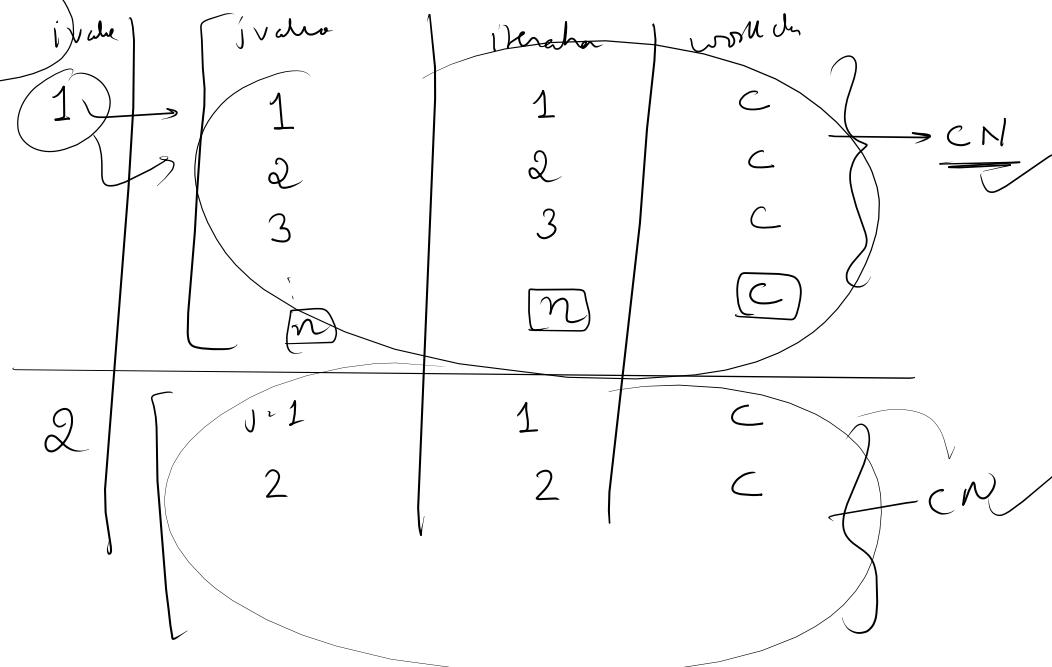
$$\begin{aligned}
 n &= 6^{n-1} \\
 \log_6 n &\sim \log_6 6^{n-1} \\
 &\sim (n-1) \log_6 6 \\
 k &\sim \log_6 n + 1
 \end{aligned}$$

Total work = ~~$O(\log_6^n + 1)$~~ .

$$\hookrightarrow O(\log_6^n) = O\left(\frac{\log_2^n}{\log_2 6}\right)$$

$$\log_a b = \frac{\log_2 b}{\log_2 a}$$

~~for(int i=1; i<=n; i++) {
 for(int j=1; j<=n; j++) {
 System.out.println("Hello");
 }
}~~



i	Kreuzen	
1	1	CN
2	2	CN
3	3	CN
...	...	
n		CN

$$\begin{aligned}
 & CN - CN - CN - \dots - CN \\
 & \sim CN \times n = O(n^2) \\
 & O(n^2)
 \end{aligned}$$

*Show
of errors*



$$\frac{n}{3}$$

for(int i=1;i<=n;i++) {
 for(int j=1;j<=n;j+=3) {
 System.out.println("Hello");
 }
}

i value	j values	k count	c	CN
①	1	1	c	CN ₁
	2	2	c	
	3	3	c	
	...	n	c	
②	j=1 j=3 j=5 ... j=n	1 2 3 ... $\frac{n}{2}$	c c c c	CN ₂
③	j=1 j=4 j=7 ... j=n	1 2 3 ... $\frac{n}{3}$	c c c c	CN ₃

i value	Item	Total
1	1	CN_1
2	2	CN_2
3	3	CN_3
...	...	$CN \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} \right)$
		$CN \times \log_2 n = O(n \log n)$

Stack

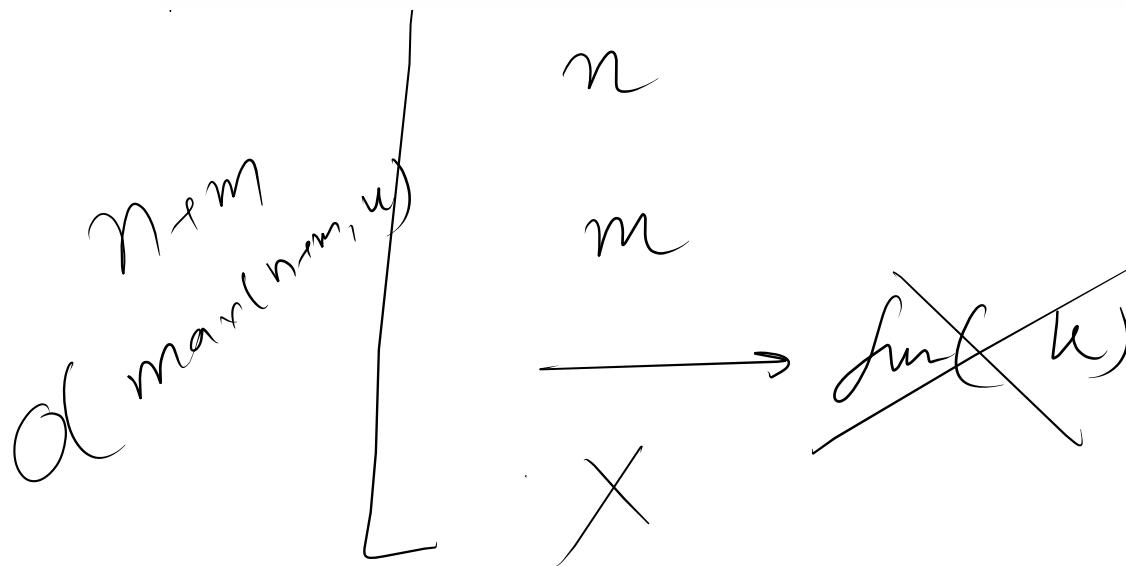
45
34
23

public static void main(String[] args) {

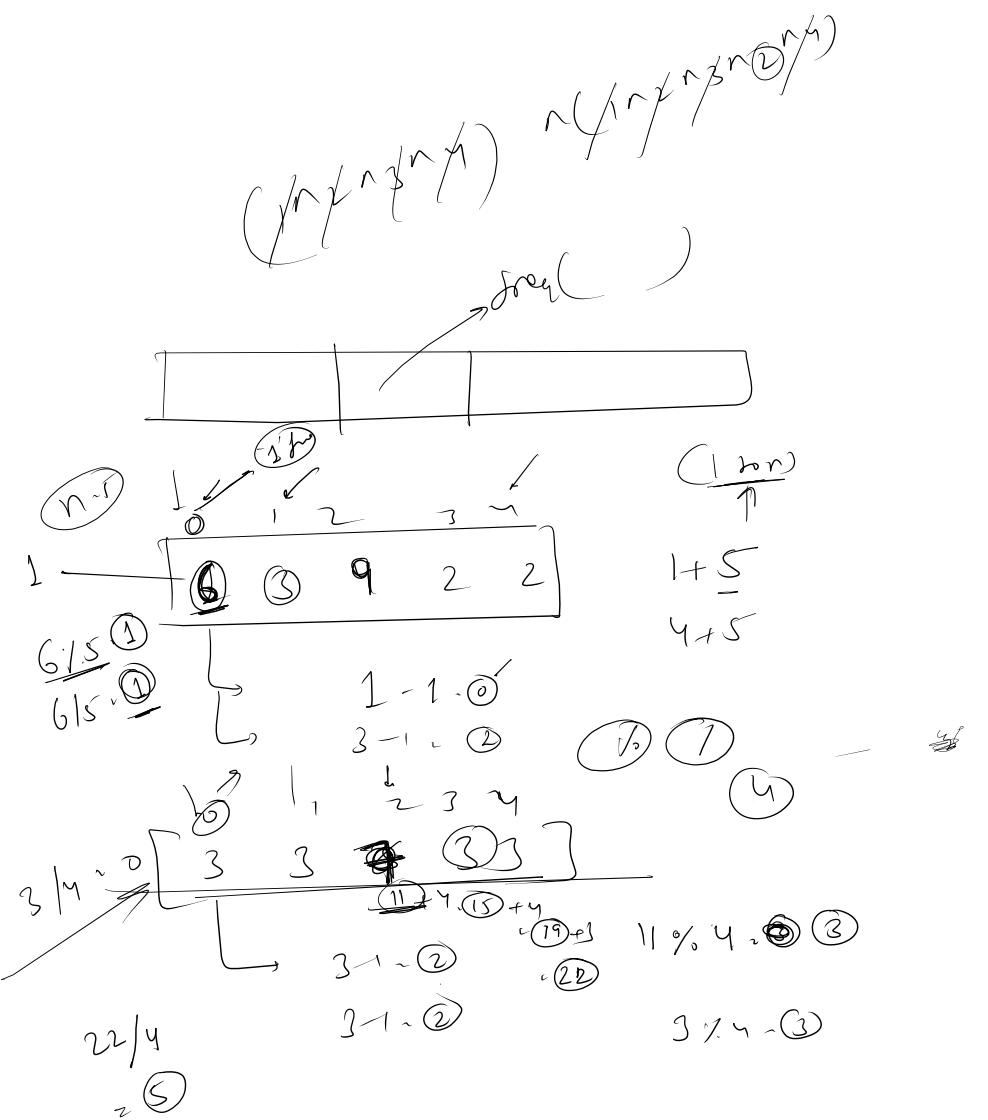
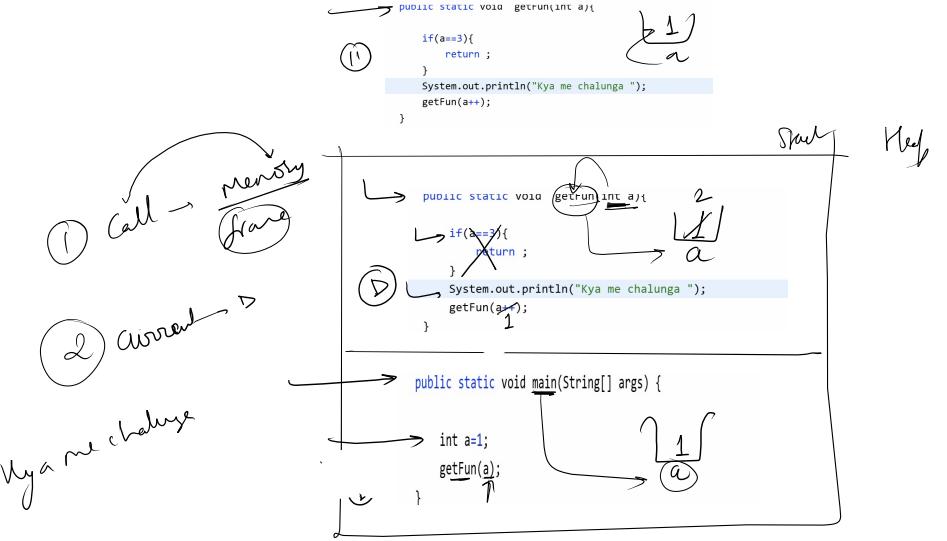
198-

int a=1;
getFun(a);
}

fun



Stack | Heap



$$\begin{aligned}
 l &\rightarrow n_c \\
 l \cdot 2 &\rightarrow n_c \quad \Rightarrow n_c + \frac{n_c}{2} + \frac{n_c}{3} + \dots + \frac{n_c}{n} \\
 l \cdot 3 &\rightarrow n_c \quad [l \cdot \frac{1}{2}, \frac{2}{3}, \dots, \frac{n-1}{n}]
 \end{aligned}$$