Time Complexity :- Rate of growth :-

$= \lim_{\text{change in input} \to 0} \frac{\text{change in no. of operation}}{\text{change in input}}$

$n = 65$

$n + \delta n = 65 + 0 \cdots\cdots 1$

$= \lim_{\Delta n \to 0} \frac{\Delta \text{operation}}{\Delta n} = \frac{d\,\text{ops}}{dn}$

No of operation

$f(N) = 3N + 3$ → Sum of Natural No.

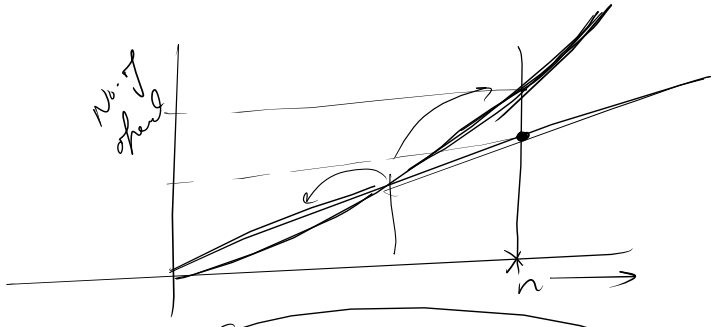
① $\frac{df(N)}{dN} = 3$

$N = 5 \longrightarrow 18$

$N = 6 \longrightarrow 21$ (+3)

②

n is



$f(n) = 3n + 3$

$g(x)$ → Rate of growth

$O(g(x))$

$f(n) = n^2 + n$

↳ $O(n^2)$

AP → Arithmetic prog.

AP $a,\ a+d,\ a+2d,\ a+3d,\ a+4d,\ \dots\ a+(n-1)d$

Common → d

→ $a_n = a + (n-1)d$

$S_n = \frac{n}{2}(2a+(n-1)d]$ (No. of terms)

$= \frac{n}{2}[a + a+(n-1)d]$

$= \frac{n}{2}[\text{first term} + \text{last term}]$

Series

1, 2, 3, 4, 5, 6, 7, 8, 9

(i) 2 + 4 + 6 + 8 + 10 + 12 ------ 2n (first term, last term, N terms)

$S_n = \frac{n}{2}(2+2n)$

$= \frac{n(1+n)2}{2}$

$= n(n+1)$

$= n^2+n$

$S_n = \frac{N}{2}(2+10)$ (No. of terms)

$= \frac{5}{2} \times 12^6 = 30$

1st term, 2nd term, 3rd term, 4th, 5th

2, 4, 6, 8, 10

N = 5

(ii) 1 + 3 + 5 + 7 + 9 + 11 (2, 2)

$S_n \Rightarrow \frac{6}{2}[1+11] = \frac{6}{2} \times 12^6 = 36$

GP

(a) ar, ar^2, ar^3, ar^4 (ratio, ratio, Ratio)

$a_n = ar^{n-1}$

$S_n = \frac{a(r^N-1)}{r-1}$ (No. of term)

$\frac{ar^2}{ar}$

(i) $2^1, 2^2, 2^3, 2^4, 2^5$

2, 4, 8, 16, 32, ------ 2^n (nth term)

1st, 2nd, 3rd, 4th, 5th

$a = 2$

$r = 2$

$2^1 \cdot 2^{n-1}$

2^{n+1}

2^n

(#) Sum → $1 + 2 + 4 + 8 + 16 + \cdots\cdots 2^n$

$1 = 2^0$, $2 = 2^1$, $4 = 2^2$, $8 = 2^3$, 2^n ← $(n+1)^{th}$ term

$a = 1$

$r = 2$

No. of term = ~~n~~ $n+1$

$$S_n = \frac{a(r^N - 1)}{r - 1} = \frac{1(2^{n+1} - 1)}{2 - 1} = 2^{n+1} - 1$$

① 2^{n-1}

② $2^n + 1$

③ $2^n - 1$

✓ ④ $2^{n+1} - 1$

(#) → $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \cdots \frac{1}{n}$

discrete → 1, 2, 3, 4, 5, 6, 7, 8, 9, ...

Continuous

$\{1, 2, 3, 4, 5, 6\}$

1 — 1.00000001 — 2

$\leq$

$$\sum_{x=1}^{x=N} \frac{1}{x} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \cdots + \frac{1}{N}$$

$$\leq \int_1^N \frac{1}{x} = [\log x]_1^N$$

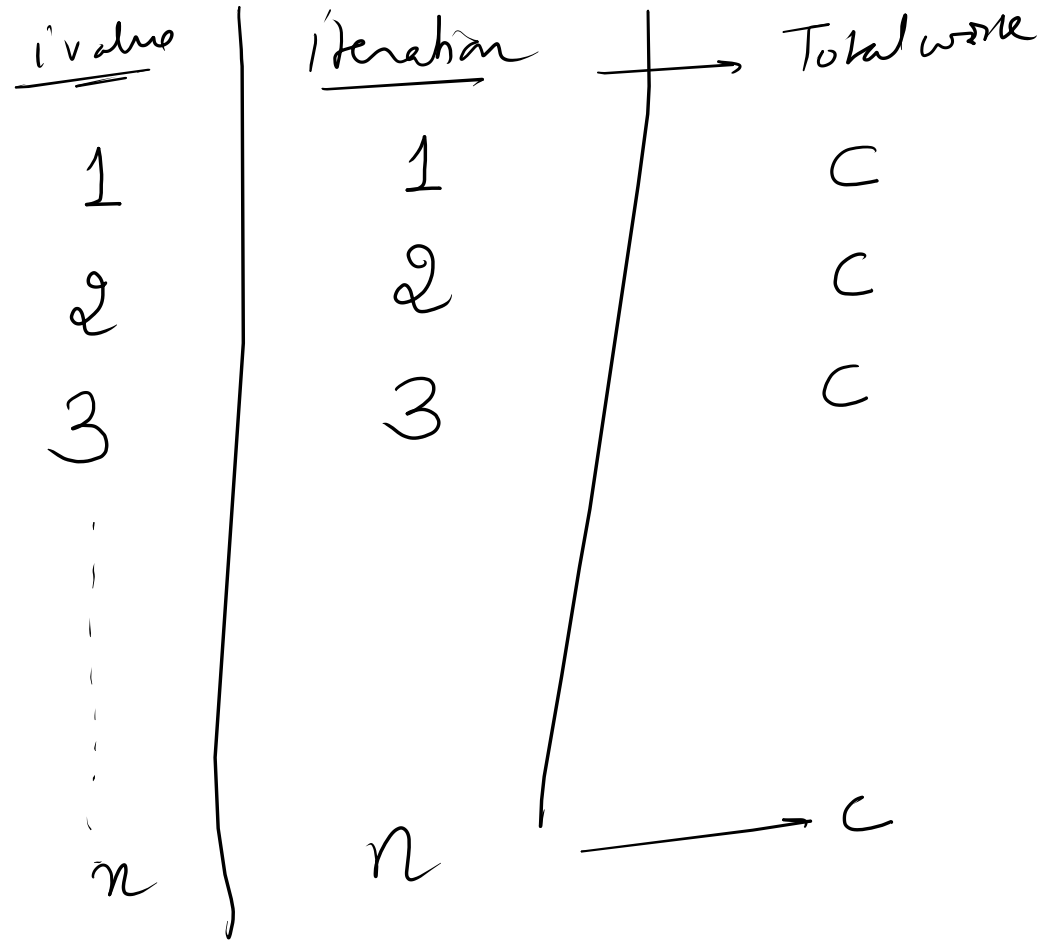
$= \log N - \log 1$ ($\log 1 \to 0$)

$= \log N$

$\log_{\text{anybase}} 1 = 0$

$\log_b a^c = c \log_b a$

```java
for(int i=1;i<=n;i++) {
    System.out.println("Hello");
}
```



Time complexity = Total

$= C + C + C + \cdots\cdots + C$ (n times)

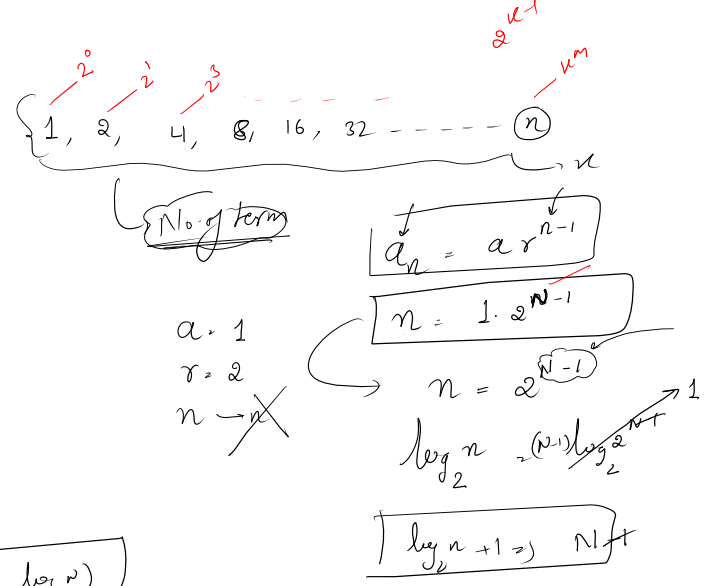
$T(n) = nc$

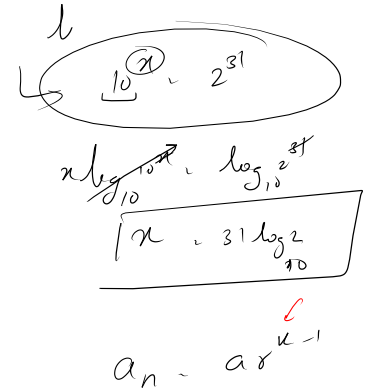
$g(n) = O(n)$ ← g(n)

```java
for(int i=1;i<=n;i*=2) {
    System.out.println("Hello");
}
```

$C + C + C + \cdots\cdots C$ ($\log_2 N + 1$ times) $= Cx$

$f(N) = C(\log_2 N + 1) = O(\log_2 N)$

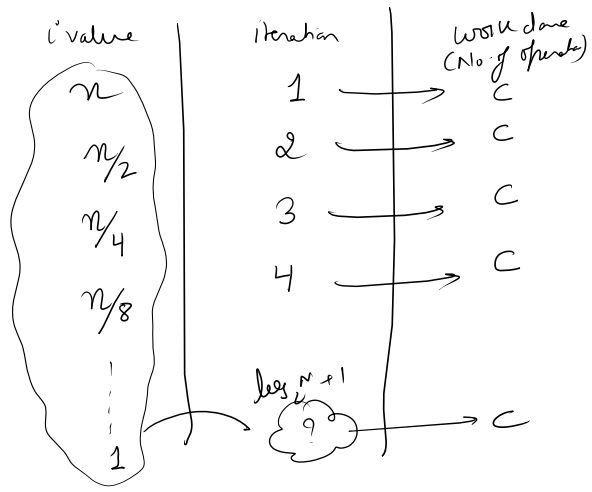




2 → 2 → 2

↳ 2 × 3

```java
for(int i=n;i>=1;i=i/2) {
    System.out.println("Hello");
}
```



Total work : $c + c + c + c + \cdots \log_2 N + 1$

$f(N) = c \times \log_2 N + 1$

$g(N)$ $O(\log_2 N)$

G.P: $n, n/2, n/4, n/8, n/16, n/32, \cdots 1$

k^m

$a_n = ar^{n-1}$

$a_k = ar^{k-1}$

$1 = n \cdot \left(\frac{1}{2}\right)^{k-1}$ $\left(\frac{1}{2}\right)^{k-1} = 1$

$1 = \frac{n}{2^{k-1}}$

$2^{k-1} = n$

$\log_2 2^{k-1} = \log_2 n$

$(k-1)\log_2 2 = \log_2 n$ ($\log_2 2 = 1$)

$k = \log_2 n + 1$

$n, n/2, n/4, n/8, \cdots 1$

$1, 2, 4, 8, \cdots n$

```java
int i=1;
while(i<=n) {
    System.out.println("Hello");
    i=i*2;
    i=i*3;
}
```

i value | iteration | work done

1 | 1 | c

6 | 2 | c

36 | 3 | c

$(6)^3$ | 4 | c

n | $\log_6 n + 1$ | c

$a = 1$

$r = 6$

$a_n = a r^{n-1}$

$n = 1 \cdot 6^{n-1}$

$n = 6^{n-1}$

$\log_6 n = \log_6^{n-1}$

$\log_6 n = (n-1)\log_6^1$

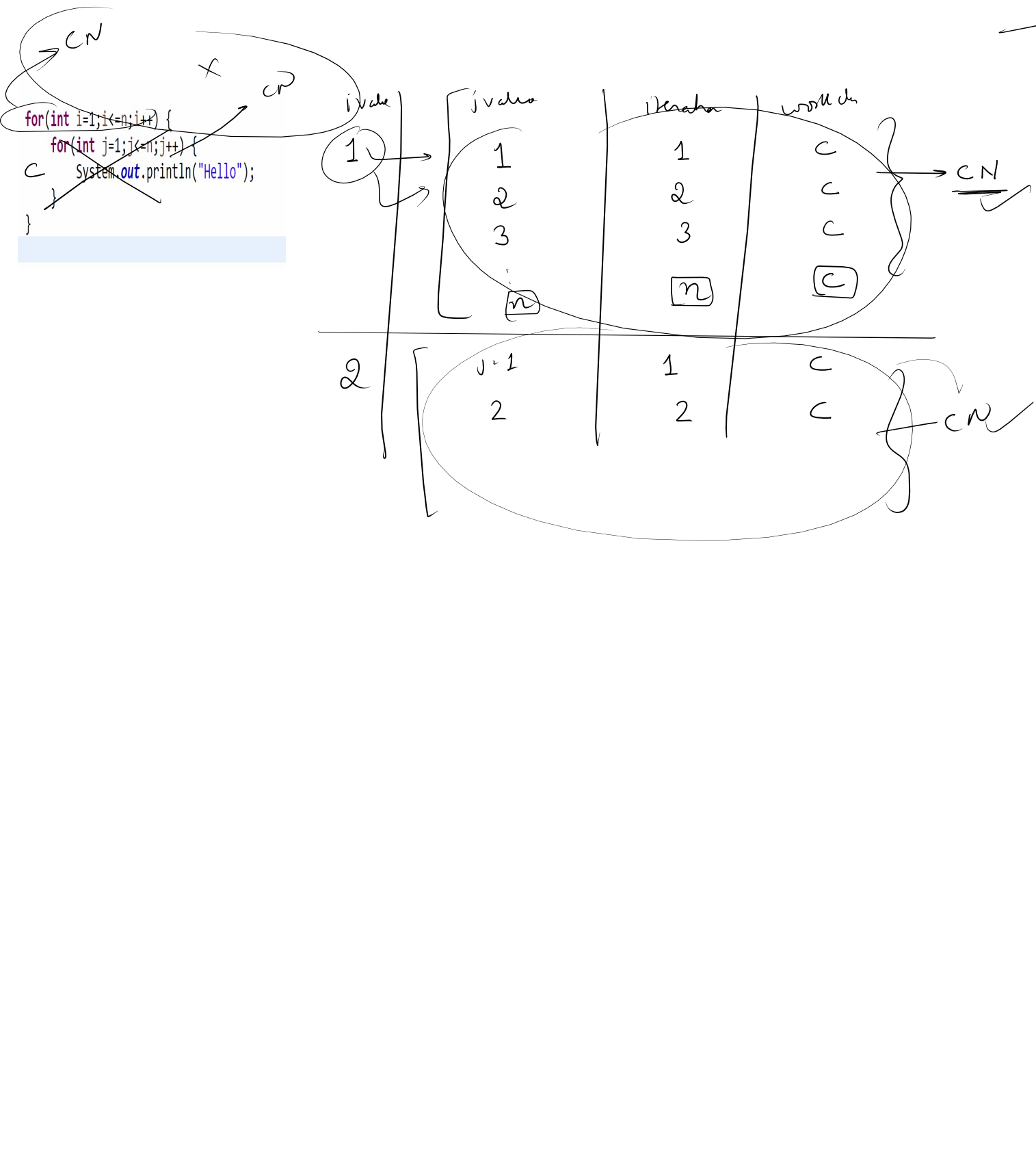
$k = \log_6^n + 1$

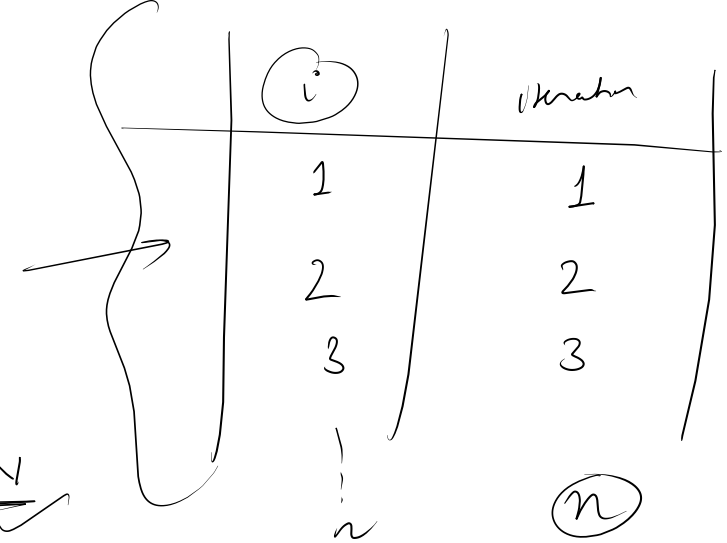
Total work $= c \times (\log_6^n + 1)$

$\hookrightarrow O(\log_6^n) = O\left(\frac{\log_2^n}{\log_2^6}\right)$

$\log_{a^b} = \frac{\log_2^b}{\log_c^a}$

```java
for(int i=1;i<=n;i++) {
    for(int j=1;j<=n;j++) {
        System.out.println("Hello");
    }
}
```

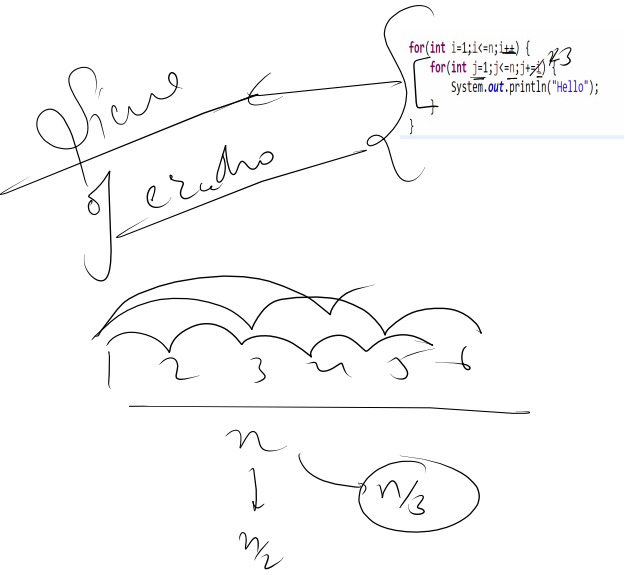


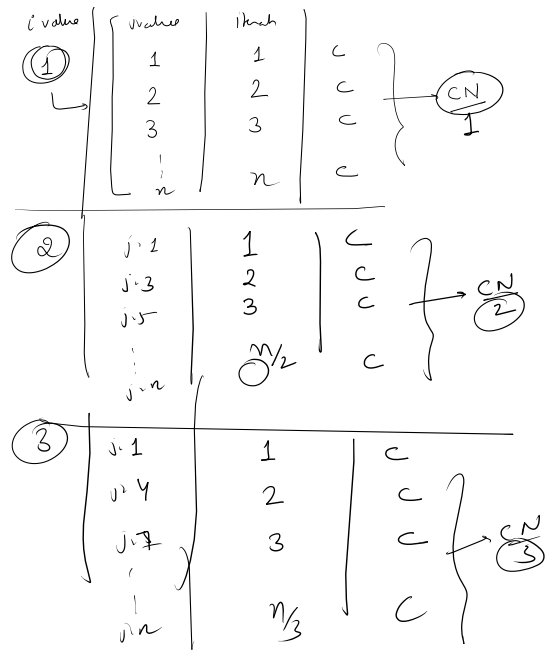


$CN + CN + CN + \cdots\cdots + CN$

$\therefore CN \times N = CN^2$

$O(N^2)$

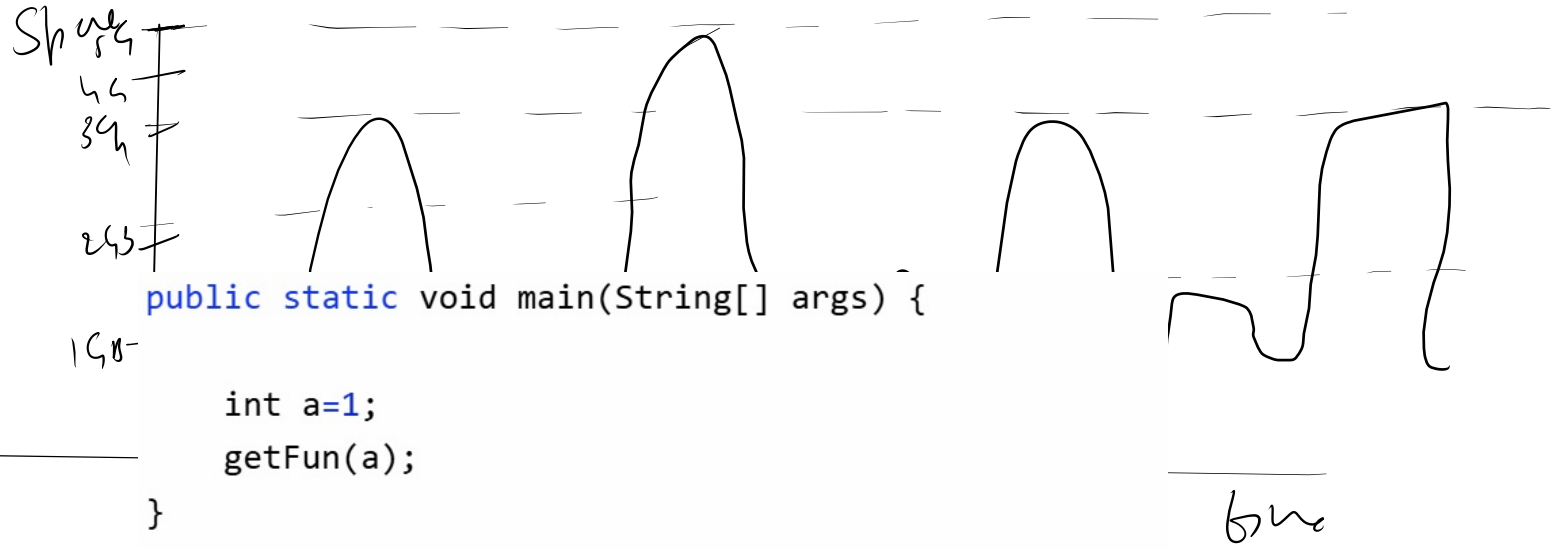




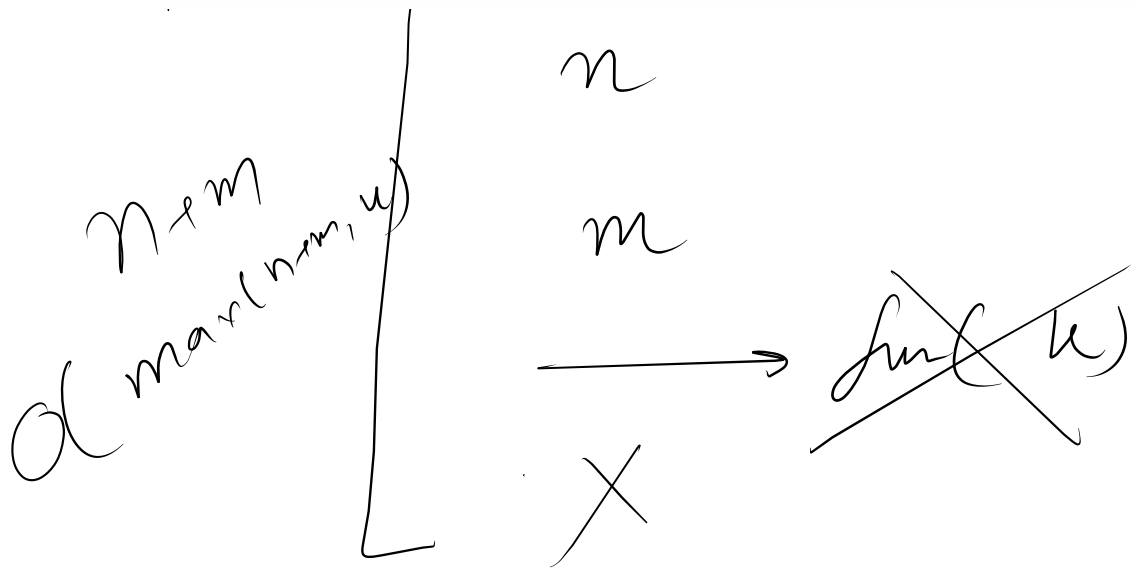
$CN + \frac{CN}{2} + \frac{CN}{3} + \frac{CN}{4} + \frac{CN}{5} + \frac{CN}{6} + \cdots + \frac{CN}{N}$

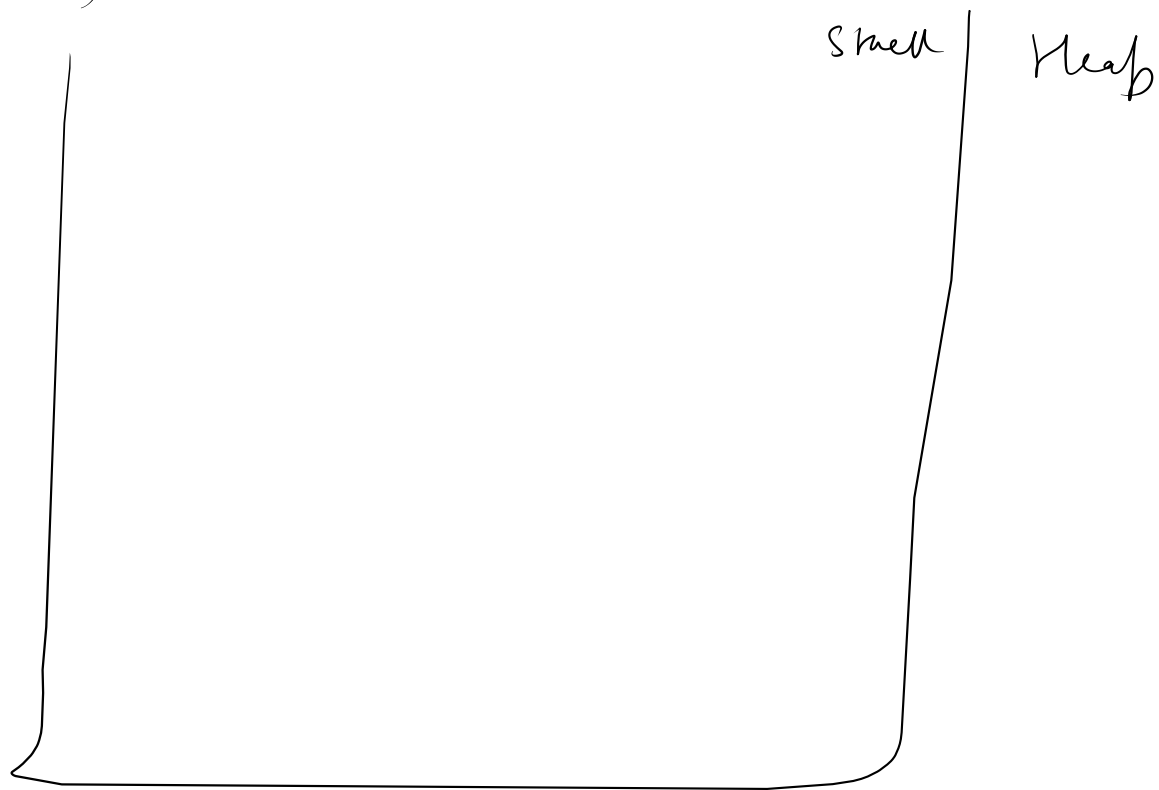
$CN\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \cdots + \frac{1}{N}\right)$

$CN \times \log_2 n = O(N \log_2 N)$



```
public static void main(String[] args) {
    int a=1;
    getFun(a);
}
```





```
public static void getFun(int a){
    if(a==3){
        return ;
    }
    System.out.println("Kya me chalunga ");
    getFun(a++);
}
```