Exercise 2.1

1. Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer.

(i) $4x^2 - 3x + 7$

Solution:

The equation $4x^2 - 3x + 7$ can be written as $4x^2 - 3x^1 + 7x^0$

Since x is the only variable in the given equation and the powers of x (i.e., 2, 1 and 0) are whole numbers, we can say that the expression $4x^2 - 3x + 7$ is a polynomial in one variable.

(ii) $y^2 + \sqrt{2}$

Solution:

The equation $y^2 + \sqrt{2}$ can be written as $y^2 + \sqrt{2}\ y^0$

Since y is the only variable in the given equation and the powers of y (i.e., 2 and 0) are whole numbers, we can say that the expression $y^2 + \sqrt{2}$ is a polynomial in one variable.

(iii) $3\sqrt{t} + t\sqrt{2}$

Solution:

The equation $3\sqrt{t} + t\sqrt{2}$ can be written as $3t^{\frac{1}{2}} + \sqrt{2}t$

Though, t is the only variable in the given equation, the powers of t (i.e., $\frac{1}{2}$) is not a whole number. Hence, we can say that the expression $3\sqrt{t} + t\sqrt{2}$ is **not** a polynomial in one variable.

(iv) $y + \frac{2}{y}$

Solution:

The equation $y + \frac{2}{y}$ can be written as $y + 2y^{-1}$

Though, y is the only variable in the given equation, the powers of y (i.e.,-1) is not a whole number. Hence, we can say that the expression $y + \frac{2}{y}$ is **not** a polynomial in one variable.

(v) $x^{10} + y^3 + t^{50}$

Solution:

Here, in the equation $x^{10} + y^3 + t^{50}$

Though, the powers, 10, 3, 50, are whole numbers, there are 3 variables used in the expression $x^{10} + y^3 + t^{50}$. Hence, it is **not** a polynomial in one variable.

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2. Write the coefficients of x^2 in each of the following:

(i) $2 + x^2 + x$

Solution:

The equation $2 + x^2 + x$ can be written as $2 + (1)\,x^2 + x$

We know that, coefficient is the number which multiplies the variable.

Here, the number that multiplies the variable x^2 is 1

$\therefore$, the coefficients of x^2 in $2 + x^2 + x$ is 1.

(ii) $2 - x^2 + x^3$

Solution:

The equation $2 - x^2 + x^3$ can be written as $2 + (-1)\,x^2 + x^3$

We know that, coefficient is the number (along with its sign,i.e., - or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is -1

$\therefore$, the coefficients of x^2 in $2 - x^2 + x^3$ is -1.

(iii) $\frac{\pi}{2}x^2+x$

Solution:

The equation $\frac{\pi}{2}x^2+x$ can be written as $(\frac{\pi}{2})\,x^2 + x$

We know that, coefficient is the number (along with its sign,i.e., - or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is $\frac{\pi}{2}$

$\therefore$, the coefficients of x^2 in $\frac{\pi}{2}x^2+x$ is $\frac{\pi}{2}$.

(iv) $\sqrt{2}x-1$

Solution:

The equation $\sqrt{2}x-1$ can be written as $0x^2+\sqrt{2}x-1$ [Since $0x^2$ is 0]

We know that, coefficient is the number (along with its sign,i.e., - or +) which multiplies the variable.

Here, the number that multiplies the variable x^2 is 0

$\therefore$, the coefficients of x^2 in $\sqrt{2}x-1$ is 0.

3. Give one example each of a binomial of degree 35, and of a monomial of degree 100.

Solution:

Binomial of degree 35: A polynomial having two terms and the highest degree 35 is called a binomial of degree 35

Eg., $3x^{35}+5$

Monomial of degree 100: A polynomial having one term and the highest degree 100 is called a monomial of degree 100
Eg., $4x^{100}$

Exercise 2.1

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4. Write the degree of each of the following polynomials:

(i) $5x^3 + 4x^2 + 7x$

Solution:
The highest power of the variable in a polynomial is the degree of the polynomial.
Here, $5x^3 + 4x^2 + 7x = 5x^3 + 4x^2 + 7x^1$
The powers of the variable x are: 3, 2, 1
$\therefore$, the degree of $5x^3 + 4x^2 + 7x$ is 3 as 3 is the highest power of x in the equation.

(ii) $4 - y^2$

Solution:
The highest power of the variable in a polynomial is the degree of the polynomial.
Here, in $4 - y^2$,
The power of the variable y is: 2
$\therefore$, the degree of $4 - y^2$ is 2 as 2 is the highest power of y in the equation.

(iii) $5t - \sqrt{7}$

Solution:
The highest power of the variable in a polynomial is the degree of the polynomial.
Here, in $5t - \sqrt{7}$,
The power of the variable y is: 1
$\therefore$, the degree of $5t - \sqrt{7}$ is 1 as 1 is the highest power of y in the equation.

(iv) 3

Solution:
The highest power of the variable in a polynomial is the degree of the polynomial.
Here, $3 = 3 \times 1 = 3 \times x^0$
The power of the variable here is: 0
$\therefore$, the degree of 3 is 0.

5. Classify the following as linear, quadratic and cubic polynomials:

Solution:
We know that,
Linear polynomial: A polynomial of degree one is called a linear polynomial.
Quadratic polynomial: A polynomial of degree two is called a quadratic polynomial.
Cubic polynomial: A polynomial of degree three a cubic polynomial.

(i) $x^2 + x$

Solution:

The highest power of $x^2 + x$ is 2

$\therefore$, the degree is 2

Hence, $x^2 + x$ is a quadratic polynomial

Exercise 2.1
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(ii) $x - x^3$

Solution:

The highest power of $x - x^3$ is 3

$\therefore$, the degree is 3

Hence, $x - x^3$ is a cubic polynomial

(iii) $y + y^2 + 4$

Solution:

The highest power of $y + y^2 + 4$ is 2

$\therefore$, the degree is 2

Hence, $y + y^2 + 4$ is a quadratic polynomial

(iv) $1 + x$

Solution:

The highest power of $1 + x$ is 1

$\therefore$, the degree is 1

Hence, $1 + x$ is a linear polynomial

(v) $3t$

Solution:

The highest power of $3t$ is 1

$\therefore$, the degree is 1

Hence, $3t$ is a linear polynomial

(vi) r^2

Solution:

The highest power of r^2 is 2

$\therefore$, the degree is 2

Hence, r^2 is a quadratic polynomial

(vii) $7x^3$

Solution:

The highest power of $7x^3$ is 3

$\therefore$, the degree is 3

Hence, $7x^3$ is a cubic polynomial

Exercise 2.2

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1. Find the value of the polynomial (x)=5x−4x²+3

(i) x= 0

(ii) x = – 1

(iii) x = 2

Solution:

Let $f(x)= 5x-4x^2+3$

(i) When x=0

$f(0)=5(0)+4(0)^2+3$
$=3$

(ii) When x= -1

$f(x)=5x-4x^2+3$
$f(-1)=5(-1) -4(-1)^2+3$
$=-5-4+3$
$=-6$

(iii) When x=2

$f(x)=5x-4x^2+3$
$f(2)=5(2) -4(2)^2+3$
$=10-16+3$
$=-3$

2. Find p(0), p(1) and p(2) for each of the following polynomials:

(i) $p(y)=y^2-y+1$

Solution:

$p(y)=y^2-y+1$
$\therefore p(0)=(0)^2-(0)+1=1$
$p(1)=(1)^2-(1)+1=1$
$p(2)=(2)^2-(2)+1=3$

(ii) $p(t)=2+t+2t^2-t^3$

Solution:

$p(t)= 2+t+2t^2-t^3$
$\therefore p(0)=2+0+2(0)^2-(0)^3=2$

$p(1)=2+1+2(1)^2-(1)^3=2+1+2-1=4$
$p(2)=2+2+2(2)^2-(2)^3=2+2+8-8=4$

(iii) **$p(x)=x^3$**

Solution:

$p(x)=x^3$
$\therefore p(0)=(0)^3=0$
$p(1)=(1)^3=1$
$p(2)=(2)^3=8$

Exercise 2.2

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(iv) $p(x)=(x-1)(x+1)$

Solution:

$p(x)=(x-1)(x+1)$
$\therefore p(0)=(0-1)(0+1)=(-1)(1)=-1$
$p(1)=(1-1)(1+1)=0(2)=0$
$p(2)=(2-1)(2+1)=1(3)=3$

3. Verify whether the following are zeroes of the polynomial, indicated against them.

(i) $p(x)=3x+1, x=-\frac{1}{3}$

Solution:

For, $x=-\frac{1}{3}$, $p(x)=3x+1$

$\therefore p(-\frac{1}{3})=3(-\frac{1}{3})+1=-1+1=0$

$\therefore -\frac{1}{3}$ is a zero of p(x).

(ii) $p(x)=5x-\pi, x=\frac{4}{5}$

Solution:

For, $x=\frac{4}{5}$ $p(x)=5x-\pi$

$\therefore p(\frac{4}{5})=5(\frac{4}{5})-\pi=4-\pi$

$\therefore \frac{4}{5}$ is not a zero of p(x).

(iii) **$p(x)=x^2-1, x=1, -1$**

Solution:

For, $x=1, -1$;
$p(x)=x^2-1$

$\therefore p(1)=1^2-1=$
$1-1=0$

$p(-1)=(-1)^2-1=1-1=0$

$\therefore 1, -1$ are zeros of p(x).

(iv) p(x)=(x+1)(x–2), x= −1, 2

Solution:

For, x=−1,2;

$p(x)=(x+1)(x-2)$

$\therefore p(-1)=(-1+1)(-1-2)$

$=((0)(-3))=0$

$p(2)=(2+1)(2-2)=(3)(0)=0$

$\therefore -1,2$ are zeros of p(x).

(v) p(x)=x², x=0

Solution:

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For, x=0 $p(x)= x^2$

$p(0)=0^2=0$

$\therefore 0$ is a zero of p(x).

(vi) p(x)=lx+m, x=$-\frac{m}{l}$

Solution:

For, x=$-\frac{m}{l}$; p(x)=lx+m

$\therefore p(-\frac{m}{l})=l(-\frac{m}{l})+m=-m+m=0$

$\therefore -\frac{m}{l}$ is a zero of p(x).

(vii) p(x)=3x²−1,x=$-\frac{1}{\sqrt{3}}$, $\frac{2}{\sqrt{3}}$

Solution:

For, x=$-\frac{1}{\sqrt{3}}$, $\frac{2}{\sqrt{3}}$; $p(x)=3x^2-1$

$\therefore p(-\frac{1}{\sqrt{3}})=3(-\frac{1}{\sqrt{3}})^2-1=3(\frac{1}{3})-1=1-1=0$

$\therefore p(\frac{2}{\sqrt{3}})=3(\frac{2}{\sqrt{3}})^2-1=3(\frac{4}{3})-1=4-1=3\neq 0$

$\therefore -\frac{1}{\sqrt{3}}$ is a zero of p(x) but $\frac{2}{\sqrt{3}}$ is not a zero of p(x).

(viii) $p(x)=2x+1, x=\frac{1}{2}$

Solution:

For, $x=\frac{1}{2}$ $p(x)=2x+1$

$\therefore p(\frac{1}{2})=2(\frac{1}{2})+1=1+1=2\neq 0$

$\therefore \frac{1}{2}$ is not a zero of p(x).

4. Find the zero of the polynomial in each of the following cases:

(i) p(x) = x + 5

Solution:

$p(x)=x+5$

$\Rightarrow x+5=0$

$\Rightarrow x=-5$

$\therefore$-5 is a zero polynomial of the polynomial p(x).

(ii) p(x) = x – 5

Solution:

$p(x)=x-5$

$\Rightarrow x-5=0$

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$\Rightarrow x=5$

$\therefore$5 is a zero polynomial of the polynomial p(x).

(iii) p(x) = 2x + 5

Solution:

$p(x)=2x+5$

$\Rightarrow 2x+5=0$

$\Rightarrow 2x=-5$

$\Rightarrow x=-\frac{5}{2}$

$\therefore x=-\frac{5}{2}$ is a zero polynomial of the polynomial p(x).

(iv) p(x) = 3x – 2

Solution:

$p(x)=3x-2$

$\Rightarrow 3x-2=0$
$\Rightarrow 3x=2$
$\Rightarrow x=\dfrac{2}{3}$
$\therefore x=\dfrac{2}{3}$ is a zero polynomial of the polynomial p(x).

(v) p(x) = 3x
Solution:
$p(x)=3x$
$\Rightarrow 3x=0$
$\Rightarrow x=0$
$\therefore 0$ is a zero polynomial of the polynomial p(x).

(vi) p(x) = ax, a $\neq$ 0
Solution:
$p(x)=ax$
$\Rightarrow ax=0$
$\Rightarrow x=0$
$\therefore x=0$ is a zero polynomial of the polynomial p(x).

(vii) p(x) = cx + d, c $\neq$ 0, c, d are real numbers.
Solution:
$p(x)= cx + d$
$\Rightarrow cx + d =0$
$\Rightarrow x=\dfrac{-d}{c}$
$\therefore x=\dfrac{-d}{c}$ is a zero polynomial of the polynomial p(x).

Exercise 2.3

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1. **Find the remainder when x^3+3x^2+3x+1 is divided by**

(i) x+1
Solution:
$x+1=0$
$\Rightarrow x=-1$
$\therefore$Remainder:
$p(-1)=(-1)^3+3(-1)^2+3(-1)+1$
$=-1+3-3+1$
$=0$

(ii) $x-\dfrac{1}{2}$

Solution:

$x-\frac{1}{2}=0$

$\Rightarrow x=\frac{1}{2}$

$\therefore$Remainder:

$p(\frac{1}{2})=(\frac{1}{2})^3+3(\frac{1}{2})^2+3(\frac{1}{2})+1$

$=\frac{1}{8}+\frac{3}{4}+\frac{3}{2}+1$

$=\frac{27}{8}$

(iii) x

Solution:

$x=0$

$\therefore$Remainder:

$p(0)=(0)^3+3(0)^2+3(0)+1$

$=1$

(iv) $x+\pi$

Solution:

$x+\pi=0$

$\Rightarrow x=-\pi$

$\therefore$Remainder:

$p(0)=(-\pi)^3+3(-\pi)^2+3(-\pi)+1$

$=-\pi^3+3\pi^2-3\pi+1$

(v) 5+2x

Solution:

$5+2x=0$

$\Rightarrow 2x=-5$

$\Rightarrow x=-\frac{5}{2}$

Exercise 2.3

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$\therefore$Remainder:

$(-\frac{5}{2})^3+3(-\frac{5}{2})^2+3(-\frac{5}{2})+1=-\frac{125}{8}+\frac{75}{4}-\frac{15}{2}+1$

$=-\frac{27}{8}$

2. Find the remainder when x^3-ax^2+6x-a is divided by x-a.

Solution:

Let $p(x)=x^3-ax^2+6x-a$

$x-a=0$

$\therefore x=a$

Remainder:

$p(a)=(a)^3-a(a^2)+6(a)-a$

$=a^3-a^3+6a-a=5a$

3. Check whether 7+3x is a factor of $3x^3+7x$.

Solution:

$7+3x=0$

$\Rightarrow 3x=-7$ only if $7+3x$ divides $3x^3+7x$ leaving no remainder.

$\Rightarrow x=\frac{-7}{3}$

$\therefore$Remainder:

$3(\frac{-7}{3})^3+7(\frac{-7}{3})=-\frac{-343}{9}+\frac{-49}{3}$

$=\frac{-343-(49)3}{9}$

$=\frac{-343-147}{9}$

$=\frac{-490}{9}\neq 0$

$\therefore 7+3x$ is not a factor of $3x^3+7x$

Exercise 2.4

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1. Determine which of the following polynomials has (x + 1) a factor:

(i) x^3+x^2+x+1

Solution:

Let p(x)= x^3+x^2+x+1

The zero of x+1 is -1. [x+1=0 means x=-1]

$p(−1)=(−1)^3+(−1)^2+(−1)+1$
$=−1+1−1+1$
$=0$

∴By factor theorem, x+1 is a factor of x^3+x^2+x+1

(ii) $x^4 + x^3 + x^2 + x + 1$

Solution:

Let p(x)= $x^4 + x^3 + x^2 + x + 1$

The zero of x+1 is -1. . [x+1=0 means x=-1]

$p(−1)=(−1)^4+(−1)^3+(−1)^2+(−1)+1$
$=1−1+1−1+1$
$=1\neq 0$

∴By factor theorem, x+1 is a factor of $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$

Solution:

Let p(x)= $x^4 + 3x^3 + 3x^2 + x + 1$

The zero of x+1 is -1.

p(−1)=(−1)4+3(−1)3+3(−1)2+(−1)+1
$=1−3+3−1+1$
$=1\neq 0$

∴By factor theorem, x+1 is a factor of $x^4 + 3x^3 + 3x^2 + x + 1$

(iv) $x^3 – x^2 – (2 + \sqrt{2})x + \sqrt{2}$

Solution:

Let p(x)= $x^3 – x^2 – (2 + \sqrt{2})x + \sqrt{2}$

The zero of x+1 is -1.

$p(−1)=(−1)^3–(−1)^2–(2+\sqrt{2})(−1)+\sqrt{2}$
$=−1−1+2+\sqrt{2}+\sqrt{2}$
$=2\sqrt{2}$

∴By factor theorem, x+1 is not a factor of $x^3 – x^2 – (2 + \sqrt{2})x + \sqrt{2}$

Exercise 2.4

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2. Use the Factor Theorem to determine whether g(x) is a factor of p(x) in each of the following cases:

(i) $p(x)=2x^3+x^2–2x–1$, $g(x) = x + 1$

Solution:

$p(x)= 2x^3+x^2-2x-1$, $g(x) = x + 1$

$g(x)=0$

$\Rightarrow x+1=0$

$\Rightarrow x=-1$

$\therefore$Zero of g(x) is -1.

Now,

$p(-1)=2(-1)^3+(-1)^2-2(-1)-1$

$=-2+1+2-1$

$=0$

$\therefore$By factor theorem, g(x) is a factor of p(x).

(ii) $p(x)=x^3+3x^2+3x+1$, $g(x) = x + 2$

Solution:

p(x)=x3+3x2+3x+1, $g(x) = x + 2$

$g(x)=0$

$\Rightarrow x+2=0$

$\Rightarrow x=-2$

$\therefore$Zero of g(x) is -2.

Now,

$p(-2)=(-2)^3+3(-2)^2+3(-2)+1$

$=-8+12-6+1$

$=-1\neq 0$

$\therefore$By factor theorem, g(x) is not a factor of p(x).

(iii) $p(x)=x^3-4x^2+x+6$, $g(x) = x - 3$

Solution:

$p(x)= x^3-4x^2+x+6$, $g(x) = x -3$

$g(x)=0$

$\Rightarrow x-3=0$

$\Rightarrow x=3$

$\therefore$Zero of g(x) is 3.

Now,

$p(3)=(3)^3-4(3)^2+(3)+6$

$=27-36+3+6$

$=0$

$\therefore$By factor theorem, g(x) is a factor of p(x).

Exercise 2.4

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3. Find the value of k, if x – 1 is a factor of p(x) in each of the following cases:

(i) $p(x)=x^2+x+k$

Solution:

If x-1 is a
factor of p(x), then p(1)=0
By Factor Theorem
$\Rightarrow(1)^2+(1)+k=0$
$\Rightarrow 1+1+k=0$
$\Rightarrow 2+k=0$
$\Rightarrow k=-2$

(ii) $p(x)=2x^2+kx+\sqrt{2}$

Solution:

If x-1 is a factor of p(x), then p(1)=0
$\Rightarrow 2(1)^2+k(1)+\sqrt{2}=0$
$\Rightarrow 2+k+\sqrt{2}=0$
$\Rightarrow k=-(2+\sqrt{2})$

(iii) $p(x)=kx^2-\sqrt{2}x+1$

Solution:

If x-1 is a factor of p(x), then p(1)=0
By Factor Theorem
$\Rightarrow k(1)^2-\sqrt{2}(1)+1=0$
$\Rightarrow k=\sqrt{2}-1$

(iv) $p(x)=kx^2-3x+k$

Solution:

If x-1 is a factor of p(x), then p(1)=0
By Factor Theorem
$\Rightarrow k(1)^2-3(1)+k=0$
$\Rightarrow k-3+k=0$
$\Rightarrow 2k-3=0$
$\Rightarrow k=\frac{3}{2}$

4. Factorize:

(i) $12x^2-7x+1$

Solution:

Using the splitting the middle term method,
We have to find a number whose sum=-7 and product=$1\times 12=12$
We get -3 and -4 as the numbers [$-3+-4=-7$ and $-3\times -4=12$]

Exercise 2.4

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$12x^2-7x+1=12x^2-4x-3x+1$

$=4x(3x-1)-1(3x-1)$

$=(4x-1)(3x-1)$

(ii) $2x^2+7x+3$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum=7 and product=$2 \times 3 = 6$

We get 6 and 1 as the numbers [$6+1=7$ and $6 \times 1 = 6$]

$2x^2+7x+3 = 2x^2+6x+1x+3$

$=2x(x+3)+1(x+3)$

$=(2x+1)(x+3)$

(iii) $6x^2+5x-6$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum=5 and product=$6 \times -6 = -36$

We get -4 and 9 as the numbers [$-4+9=5$ and $-4 \times 9 = -36$]

$6x^2+5x-6=6x^2+9x-4x-6$

$=3x(2x+3)-2(2x+3)$

$=(2x+3)(3x-2)$

(iv) $3x^2-x-4$

Solution:

Using the splitting the middle term method,

We have to find a number whose sum=-1 and product=$3 \times -4 = -12$

We get -4 and 3 as the numbers [$-4+3=-1$ and $-4 \times 3 = -12$]

$3x^2-x-4 = 3x^2-x-4$

$=3x^2-4x+3x-4$

$=x(3x-4)+1(3x-4)$

$=(3x-4)(x+1)$

5. Factorize:

(i) x^3-2x^2-x+2

Solution:

Let $p(x)=x^3-2x^2-x+2$

Factors of 2 are ±1 and ± 2

By trial method, we find that

$p(1)=0$

So, $(x+1)$ is factor of $p(x)$

Now,

$p(x)= x^3–2x^2–x+2$

$p(−1)=(−1)^3–2(−1)^2–(−1)+2$

$=−1−1+1+2$

$=0$

Therefore, $(x+1)$ is the factor of $p(x)$

$$
\begin{array}{r|l}
 & x^2-3x+2 \\
\hline
x+1 & x^3-2x^2-x+2 \\
 & x^3+x^2 \\
 & - \quad - \\
\hline
 & -3x^2-x+2 \\
 & -3x^2-3x \\
 & + \quad + \\
\hline
 & 2x+2 \\
 & 2x+2 \\
 & - \quad - \\
\hline
 & 0
\end{array}
$$

Now, Dividend = Divisor × Quotient + Remainder

$(x+1)(x^2–3x+2) =(x+1)(x^2–x–2x+2)$

$=(x+1)(x(x−1)−2(x−1))$

$=(x+1)(x−1)(x+2)$

(ii) $x^3–3x^2–9x–5$

Solution:

Let $p(x) = x^3–3x^2–9x–5$

Factors of 5 are ±1 and ±5

By trial method, we find that

$p(5) = 0$

So, $(x-5)$ is factor of $p(x)$

Now,

$p(x) = x^3–3x^2–9x–5$

$p(5) = (5)^3–3(5)^2–9(5)–5$

$=125−75−45−5$

$=0$

Therefore, $(x-5)$ is the factor of $p(x)$