THEORY OF EQUATIONS

OBJECTIVE PROBLEMS

1. **If the equation $x^3+6x+20=0$ has one imaginary root 1+3i, then its real root is**

   1) 2 2) -2 3) 3 4) -3

2. **If the sum of two roots of the equation $x^3-3x^2-16x+k=0$ is zero, then value of k is 1) -48 2) 36 3) 48 4) 24**

3. **If the product of two roots of $x^3-5x^2-kx+24=0$ is 12, then k =**

   1) 4 2) -4 3) 2 4) -2

4. **If one root of $x^3-5x^2+kx-4=0$ is the reciprocal of another, then the value of k is**

   1) 5 2) 4 3) 3 4) 2

5. **If two roots of $x^3+px^2+qx+r=0$ are connected by the relation $\alpha\beta+1=0$, then the condition is**

   1) $r^2-pr+q+1=0$ 2) $r^2+pr+q+1=0$

   3) $p^2+pr+q+1=0$ 4) $q^2+pr+q+1=0$

6. **If two of the roots of $x^3+qx+r=0$ are equal, the condition is**

   1) $27r^2+4q^3=0$ 2) $4r^2-27q^3=0$ 3) $27r^2+4q^3=0$ 4) $4r^2+27q^3=0$

7. **If one of the roots of $2x^3+6x^2+5x+k=0$ is equal to half the sum of the other two, then k =**

   1) -1 2) 1 3) 2 4) -2

8. **If α,β,γ are the roots of $x^3-px^2+qx-r=0$, then the equation whose roots are $\beta+\gamma,\gamma+\alpha,\alpha+\beta$ is**

   1) $x^3-2px^2+\left(p^2+q\right)x+\left(r-qp\right)=0$ 2) $x^3+2px^2+\left(p^2-q\right)x+\left(r+qp\right)=0$

   3) $x^3+2px^2+\left(p^2+q\right)x+\left(r+qp\right)=0$ 4) $x^3+2px^2-\left(p^2+q\right)x-\left(r+qp\right)=0$

9. If the product of the two roots of $x^4+px^3+qx^2+rx+s=0$ is equal to the product of the other two, then

1) $ps^2=r$ 2) $p^2s=r^2$

3) $ps=r^3$ 4) $p^2s=r^3$

10. The condition that the equation $x^3-px^2+qx-r=0$ may have two roots equal in magnitude but opposite in sign is

1) pq+r=0 2) pq-r=0 3) 2pq-r=0 4) 2pq-r=0

11. If the sum of two roots of $x^3-px^2+qx-r=0$, then pq =

1) r^2 2) $\frac{1}{r^2}$ 3) r 4) $\frac{1}{r}$

12. If α,β,γ are the roots of $x^3-px^2+qx-r=0$, then $\frac{1}{\alpha^2\beta^2}+\frac{1}{\beta^2\gamma^2}+\frac{1}{\gamma^2\alpha^2}=$

1) $\frac{p^2+2q}{r^2}$ 2) $\frac{p^2-2q}{r^2}$

3) $\frac{q^2-2p}{r^2}$ 4) $\frac{q^2+2p}{r^2}$

13. If α,β,γ are the roots of $x^3-2x^2+3x+4=0$, then $\frac{1}{\beta^2\gamma^2}+\frac{1}{\gamma^2\alpha^2}+\frac{1}{\alpha^2\beta^2}=$

1) $\frac{1}{8}$ 2) $\frac{1}{4}$ 3) $\frac{-1}{8}$ 4) $\frac{-1}{4}$

14. If α,β,γ are the roots of $x^3-px^2+qx-r=0$, then $\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}=$

1) $\frac{q^2+2pr}{r^2}$ 2) $\frac{2pr-q^2}{r^2}$

3) $\frac{q^2-2pr}{r^2}$ 4) $\frac{q^2+2pr}{-r^2}$

15. If the roots of $x^3-9x^2+20x-12=0$ are in the ratio 1:3, then the roots are

1) 1, 2, 6 2) 2, 6, 3 3) $3,\frac{1}{3},12$ 4) 1, 3, 5

16. The roots of $x^3-9x^2+x-2=0$ are

1) $-2\pm 2i$ 2) $2,\pm i$ 3) $1,\pm 2i$ 4) $-1,\pm 2i$

17. If, 1, α,β are the roots of $x^3+2x^2-5x+6=0$, then

1) $\alpha=3,\beta=2$ 2) $\alpha=-3,\beta=2$

3) $\alpha=-3,\beta=-2$ 4) $\alpha=3,\beta=-2$

18. The equation whose roots are -1, 2+i is

1) $x^3+3x^2-x+5=0$ 2) $x^3-3x^2+x-5=0$

3) $x^3-3x^2+x+5=0$ 4) $x^3+3x^2+x+5=0$

19. The cubic equation having, the roots $2+\sqrt{3},1$ is

1) $x^3+5x^2+5x+1=0$

2) $x^3-5x^2+5x-1=0$

3) $x^3-5x^2-5x-1=0$

4) $x^3-5x^2-5x+1=0$

20. The roots of the equation $x^3+3px^2+3qx+r=0$ are in A.P. then

1) p^3-pq-r 2) $3p^3-3pq=r$

3) $2p^3+r=3pq$ 4) $2p^3-3r=pq$

21. If the roots of the equation $ax^3+3bx^2+3cx+d=0$ are in A.P., then the condition is

1) $2b^3-3abc+a^2d=0$ 2) $2b^3+3abc+a^2d=0$

3) $2b^3+3abc-a^2d=0$ 4) $2b^3-3abc-a^2d=0$

22. If the roots of the equation $x^3-7x^2+14x-8=0$ are in G.P., then the roots are

1) 2, 4, 8 2) 1,1/2,1/4 3) 1,2,4 4) 3,5,4

23. If the roots of the equation $8x^3-14x^2+7x-1=0$ are in G.P., then the roots are

1) $1,\frac{1}{3},\frac{1}{9}$ 2) $1,\frac{1}{2},\frac{1}{3}$ 3) $\frac{1}{2},\frac{1}{4},1$ 4) 1,2,4

24. If the roots of $x^3+x^2+kx+8=0$ are in G.P., the value of k is

1) 2 2) -2 3) 1 4) -1

25. The roots of the equation $x^3+3px^2+3qx+r=0$ are in G.P. Then

1) $rp^2=q$ 2) $r^2p=q^2$ 3) $rp^3=q$ 4) $rp^3=q^3$

26. If the roots of the equation $x^3-px^2+qx-r=0$ are in H.P., then the value of the mean root is

1) $\frac{3q}{r}$ 2) $\frac{3r}{q}$ 3) $\frac{3p}{q}$ 4) $\frac{3q}{p}$

27. If the roots of $x^3+px^2+qx+r=0$ are in H.P., then

1) $2q^3+27r^2=9pqr$ 2) $2q^2+27r=9pq$

3) $2q^2-27r^2=9rqr$ 4) $2q^2-27r=9pq$

28. The roots of the equation $x^3-3ax^2+3bx-c=0$ are in H.P. then the mean root is

1) $\frac{a}{b}$ 2) $\frac{b}{c}$ 3) $\frac{c}{a}$ 4) $\frac{c}{b}$

29. If α,β,γ are the roots of $x^3-x^2+8x-6=0$, then $\alpha^2,\beta^2,\gamma^2$ are the roots of the equation

1) $x^3-15x^2+52x-30=0$ 2) $x^3-5x^2+52x-30=0$

3) $x^3-15x^2-52x-36=0$ 4) $x^3+15x^2+52x-36=0$

30. If α,β,γ are the roots of $x^3-2x^2+5x-3=0$, then the equation whose roots are $\beta\gamma+\frac{1}{\alpha},\gamma\alpha+\frac{1}{\beta}\alpha\beta+\frac{1}{\gamma}$ is

1) $3x^3+20x^2+32x+64=0$ 2) $3x^3-20x^2+32x-64=0$

3) $3x^3+20x^2+32x-64=0$ 4) $3x^3-20x^2-32x+64=0$

31. If α,β,γ are the roots of the equation $x^3+qx+r=0$, the equation whose roots are $\frac{\beta+\gamma}{\alpha^2},\frac{\gamma+\alpha}{\beta^2},\frac{\alpha+\beta}{\gamma^2}$ is

1) $rx^3+qx^2-1=0$ 2) $rx^3-qx^2-1=0$

3) $rx^3+qx^2-1=0$ 4) $rx^3+qx^2+1=0$

32. If $f(x)\equiv 5x^3-13x^2-12x+7=0$, then $f(x)$ expressed in power series of (x-2) is

1) $5x^3+17x^2+4x+29=0$ 2) $5x^3+17x^2-4x-29=0$

3) $5x^3-17x^2-4x+29=0$ 4) $5x^3-17x^2+4x-29=0$

33. The equation whose roots are the roots of $x^4-5x^3+7x^2-17x+11=0$ each diminished by 4 is

1) $x^4-11x^3+43x^2+55x+9=0$ 2) $x^4+11x^3-43x^2+55x+9=0$

3) $x^4+11x^3+43x^2+55x-9=0$ 4) $x^4-11x^3-43x^2-55x+9=0$

34. The equation whose roots are k times the roots of $3x^4-\frac{5}{2}x^3+\frac{7}{6}x^2-x+\frac{7}{18}=0$ is an equation with integral coefficients. Then k =

1) 2 2) 3 3) 5 4) 6

35. If -4,-1,-1 are the roots of $x^3+6x^2+9x+4=0$, then the roots of the equation $\left(x-\frac{2}{3}\right)^3+6\left(x-\frac{2}{3}\right)^2+9\left(x-\frac{2}{3}\right)+4=0$ are

1) $-\frac{14}{3},-\frac{5}{3},\frac{5}{3}$ 2) $-\frac{10}{3},-\frac{1}{3},-\frac{1}{3}$

3) $\frac{14}{3},\frac{5}{3},\frac{5}{3}$ 4) $-\frac{14}{3},\frac{5}{3},\frac{5}{3}$

36. The equation whose roots are less by 2 than the roots of $2x^2+4x-5=0$ is

1) $x^2+2x-3=0$ 2) $4x^2-8x-10=0$

3) $2x^2+12x+11=0$ 4) $x^2-2x-3=0$

37. If $\frac{3}{2},1,2$ are the roots of $2x^3-9x^2+13x-6=0$, then the roots of the equation $6x^3-13x^2+9x-2=0$ are

1) $\frac{2}{3},1,\frac{1}{2}$ 2) $\frac{1}{3},1,\frac{1}{2}$

3) $\frac{3}{2},1,2$ 4) $-\frac{2}{3},-1,-\frac{1}{2}$

38. The second term of the equation $2x^3+6x^2-x+1=0$ can be removed by diminishing its roots by

1) 2 2) -2 3) 1 4) -1

39. By removing the second term in the equation $x^3-3x^2+12x+16=0$ the transformed equation is

1) $y^3+9y+26=0$ 2) $y^3-9y+26=0$

3) $y^3+9y-26=0$ 4) $y^3-9y-26=0$

40. If $f(x)=x^3-3x^2+4=0$ has a repeated root, then that root is

1) 2 2) -2 3) 1 4) -1

41. The multiple root of $x^3-x^2-x+1=0$ is

1) 1 2) -1 3) -2 4) 2

42. If the equation $x^3-3qx+2r=0$ has two equal roots, then

1) $q^2=r^3$ 2) $q=r^3$ 3) $q^3=r$ 4) $q^3=r^2$

43. If α,β,γ are the roots of $x^3+4x^2+5x+2=0$, then $\sum\frac{1}{\alpha^2\beta^2}=$

1) $\frac{3}{2}$ 2) $\frac{2}{3}$ 3) $\frac{3}{4}$ 4) $\frac{4}{3}$

44. If α,β,γ are the roots of $ax^3+bx^2+cx+d=0$, then the value of $\sum\alpha^2\beta^2=$

1) $\frac{c^2+2bd}{a^2}$ 2) $\frac{2bd-c^2}{a^2}$ 3) $\frac{c^2-2bd}{a^2}$ 4) $\frac{c^2+bd}{a^2}$

45. **If** α,β,γ **are the roots of** $x^3-9x^2+26x-24=0$**, then the value of** $\Sigma(\alpha+3)(\beta+3)$ **is**

1) 107 2) 108 3) 128 4) 182

46. **The roots of** $x^3-21x^2+126x-216=0$ **are in ------**

1) A.P. 2) G.P. 3) A.G.P 4) H.P.

47. **The number of real roots of equation** $(x+1/x)^3+(x+1/x)=0$ **is**

1) 2 2) 3 3) 6 4) 0

48. **If** α **is an imaginary root of** $x^5-1=0$**, then the equation whose roots are** $\alpha+\alpha^4$ **and** $\alpha^2+\alpha^3$ **is**

1) $x^2-x-1=0$ 2) $x^2+x-1=0$

3) $x^2-x+1=0$ 4) $x^2+x+1=0$

THEORY OF EQUATIONS

HINTS AND SOLUTIONS

1. Ans (2)

Given 1+3i is one root $\Rightarrow$ 1-3i will also be a root.

Sum of the two roots = 1+3i+1-3i=2

s_1 = sum of all the roots = 0.

$\therefore$ The third root = 0-2 = -2

2. Ans.(3)

Let the roots be $\alpha, -\alpha, \beta \Rightarrow s_1 = \alpha + \beta = -(-3) = 3 \Rightarrow \beta = 3$

Since 3 is a root of the given equation, $3^3 - 3(3)^2 - 16(3) + k = 0 \Rightarrow k = 48$

3. Ans (3)

Let α, β, γ be the roots so that $\alpha\beta$ = 12. $s_3 = \alpha\beta\gamma = -24 \Rightarrow 12\gamma = -24 \Rightarrow \gamma = -2$

$\Rightarrow 2k = -24 + 28 = 4$ $\therefore$ k = 2

4. Ans.(1)

Let the roots be $\alpha, \frac{1}{\alpha}, \beta$

$s_3 = \alpha\left(\frac{1}{\alpha}\right)\beta = -(-4) = 4 \Rightarrow \beta = 4$. But β is a root of the G.E.

64-5(16)+4k-4=0 $\Rightarrow 4k = 20 \Rightarrow k = 5$

5. Ans.(2)

Let the roots be α, β, γ. Then $\alpha\beta\gamma = -r$. Given $\alpha\beta = -1$.

$\Rightarrow (-1)\gamma = -r \Rightarrow \gamma = r$. But γ is a root of G.E.

$r^3 + pr^2 + qr + r = 0 \Rightarrow r^2 + pq + q + 1 = 0$ is the condition.

6. Ans (1)

Let the roots be α, α, β. Then $\alpha + \alpha + \beta = 0 \Rightarrow \beta = -2\alpha$

$\alpha.\alpha.\beta = -r \Rightarrow \alpha^2\beta = -r \Rightarrow 2\alpha^3 = -r \Rightarrow \alpha^3 = \frac{r}{2}$ But α is root of G.E.

$\Rightarrow \alpha^3 q\alpha + r = 0 \Rightarrow \frac{r}{2} + qa + r = 0$

$\Rightarrow q\alpha = \frac{3r}{2} \Rightarrow q^3\alpha^3 = -\frac{27r^3}{8} \Rightarrow q^3\left(\frac{r}{2}\right) = \frac{27r^3}{8} \Rightarrow 4q^3r + 27r^3 = 0$

$\therefore$ Required condition is $4q^3 + 27r^2 = 0$

7. Ans (2)

Given one root is equal to half the sum of the other two. $\Rightarrow$ The roots are in A.P.

Let the roots be a – c, a + d

$s_1 = a - d + a + a + d = -\frac{6}{2} = 3 \Rightarrow 3a = -3 \therefore a = -1$ (-1) is a root of f(x) = 0.

$\Rightarrow f(-1) = 0$

$2(-1)3 + 6(-1)2 + 5(-1) + k = 0 \Rightarrow k = 1$

8. Ans (1)

Given $\Sigma\alpha = p, \Sigma\alpha\beta = q\ \alpha\beta\gamma = r$

Let $y = \beta + \gamma = \alpha + \beta + \gamma - \alpha = p - \alpha = p - x\ (\therefore \alpha = x) \Rightarrow x = p - y)$

Required equation is $(p-y)^3 - p(p-y)^2 + q(p-y) - r = 0$

$\Rightarrow p^3 - y^3 - 3p^2y + 3py^2 - p^3 - py^2 + 2p^2y + pq - qy - r = 0 \Rightarrow y^3 - 2py^2 + (p^2+q)y + (r - pq) = 0$

$\Rightarrow x^3 - 2px^2 + (p^2+q)x + (r-pq) = 0$

9. Ans (2)

Given $\Sigma\alpha = -p, \Sigma\alpha\beta = q,\ \Sigma\alpha\beta\gamma = -r, \alpha\beta\gamma\delta = s$.

Also given $\alpha\beta = \gamma\delta$. Now $\alpha\beta(\alpha\beta) = s \Rightarrow \alpha^2\beta^2 = s$.

$\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta = -r$

$\alpha\beta(\gamma+\delta) + \gamma\delta(\alpha+\beta) = -r \Rightarrow \alpha\beta(\alpha+\beta+\gamma+\delta) = -r\ (\therefore \gamma\delta = \alpha\beta)$

$\Rightarrow \alpha\beta(-p) = -r \Rightarrow \alpha^2\beta^2p^2 = r^2 \Rightarrow sp^2 = r^2$.

10. Ans (2)

Let the roots be $\alpha, -\alpha, \beta$

$s_1 = \alpha - \alpha + \beta = -(-p) = p \Rightarrow \beta = p \Rightarrow x = p$

But β is a root of G.E. $\Rightarrow p^3 - p.p^2 + qp - r = 0 \Rightarrow pq - r = 0$

11. Ans (3) Let the roots be $\alpha, -\alpha\beta \Rightarrow S_1 = \alpha - \alpha + \beta = p \Rightarrow \beta = p$

But β is a root of the given equation

$\beta^3 - p\beta^2 + q\beta - r = 0 \Rightarrow p^3 - p(p^2) + qp - r = 0 \Rightarrow pq - r = 0 \Rightarrow pq = r$.

12. Ans (2)

Given $\alpha + \beta + \gamma = p, \alpha\beta + \beta\gamma + \gamma\alpha = q, \alpha\beta\gamma = -r$

G.E. $= \dfrac{\gamma^2 + \alpha^2 + \beta^2}{\alpha^2\beta^2\gamma^2} = \dfrac{(\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\beta\gamma+\gamma\alpha)}{(\alpha\beta\gamma)^2} = \dfrac{p^2 - 2q}{r^2}$

13. Ans (3)

Given $\alpha + \beta + \gamma = 2,\ \alpha\beta + \beta\gamma + \gamma\alpha = 3.\alpha\beta\gamma = 4$

$$\frac{1}{\beta^2\gamma^2} + \frac{1}{\gamma^2\alpha^2} + \frac{1}{\alpha^2\beta^2} = \frac{\alpha^2 + \beta^2 + \gamma^2}{\alpha^2\beta^2\gamma^2}$$

$$= \frac{(\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\beta\gamma+\gamma\alpha)}{(\alpha\beta\gamma)^2} = \frac{4 - 2(3)}{16} = -\frac{2}{16} = -\frac{1}{8}$$

14. Ans (3) $\alpha + \beta + \gamma = p, \alpha\beta + \beta\gamma + \gamma\alpha q, \alpha\beta\gamma = r$ $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2} + \dfrac{1}{\gamma^2} = \dfrac{\beta^2\gamma^2 + \gamma^2\alpha^2 + \alpha^2\beta^2}{\alpha^2\beta^2\gamma^2}$

$$\frac{(\beta\gamma + \gamma\alpha + \alpha\beta)^2 - 2\alpha\beta\gamma(\alpha+\beta+\gamma)}{(\alpha\beta\gamma)^2} = \frac{q^2 - 2rp}{r^2}$$

15. Ans (1)

Let the roots be $\alpha, 3\alpha, \beta$.

$\alpha + 3\alpha + \beta = 0$ ....(1)

$\beta = 9 - 4\alpha$, From (2) : $3\alpha^2(9 - 4\alpha) = 12$

$\alpha.3\alpha.\beta = 12$ ....(2)

$9\alpha^2 - 4\alpha^3 = 4 \Rightarrow 4\alpha^3 - 9\alpha^2 4 = 0$.

By inspection : $\alpha = 2$

$\therefore \beta = 9 - 8 = 1$

Hence, the roots are 2,6,1 $\Rightarrow$ 1,2,6

16. Ans (2)

By inspection x = 2 is a root of the equation.

$\therefore (x-2)$ is a factor of $x^3 - 2x^2 + x - 2$

$\therefore x^3 - 2x^2 + x^2 - 2 = (x-2)(x^2+1) = 0$

$\Rightarrow x = 2,\ x = \pm i$

17. Ans(4)

$1 + \alpha + \beta = 2$.

$\Rightarrow \alpha + \beta = 1$ and $\alpha\beta = -6 \Rightarrow \alpha(1-\alpha) = -6$

$\alpha^2 - \alpha - 6 = 0 \Rightarrow (\alpha-3)(\alpha+2) = 0, \alpha = 3, -2$

$\beta = -2, 3 \quad \therefore \alpha = 3,\ \beta = -2$

18. Ans (3)

Given -1, 2+i are the roots of a given equation.

$\Rightarrow 2 - i$ is also a root of the given equation.

$\therefore$ Required equation is (x+1) (x-2-i) (x-2+i) = 0

$\Rightarrow (x+1)\left[(x-2)^2 + 1\right] = 0 \Rightarrow (x+1)(x^2 - 4x + 5) = 0$

$\Rightarrow x^3 - 3x^2 + x + 5 = 0$

19. Ans (2)

Irrational roots occur in pairs $\Rightarrow 2 - \sqrt{3}$ is also a root. The cubic equation is

$\left(x - 2 - \sqrt{3}\right)\left(x - 2 + \sqrt{3}\right)(x-1) = 0$

$\Rightarrow \left[(x-2)^2 - 3)\right](x-1) = 0 \Rightarrow (x^2 - 4x + 1)(x-1) = 0$

$\Rightarrow x^3 - 5x^2 + 5x - 1 = 0$

20. Ans(3)

Let the roots be a – d,a, a+d.

$a-d+a+a+d=-3p \Rightarrow 3a=-3p \Rightarrow a=-p$

But $a=-p+3p\left(-p^2\right)+3q\left(-p\right)+r=0$

$\Rightarrow -p^3+3p^3-3pq+r=0$

$\Rightarrow 2p^3+r=3pq$ is the required condition.

21. Ans (1)

Given equation is

$x^3+\frac{3b}{a}x^2+\frac{3c}{a}x+\frac{d}{a}=0$ ....(1)

Let the roots be $\alpha-\beta,\alpha$ and $\alpha+\beta$.

$s_1\alpha-\beta+\alpha+\alpha+\beta=\frac{3b}{a} \Rightarrow 3\alpha=-\frac{3b}{a} \Rightarrow \alpha=-\frac{b}{a}$ Since α is a root of (1), we have

$-\frac{b^3}{a^3}+\frac{3b}{a}\left(\frac{b^2}{a^2}\right)+\frac{3a}{a}\left(-\frac{b}{a}\right)+\frac{d}{a}=0$

$\Rightarrow \frac{2b^3}{a^3}-\frac{3bc}{a^2}+\frac{d}{a}=0 \Rightarrow 2b^3-3abc+a^2d=0$ is the condition.

22. Ans(3)

Let the roots be $\frac{a}{r},a,ar$

$s_3=\frac{a}{r}(a)(ar)=-(-8)=8 \Rightarrow a^3=8 \Rightarrow a=2$ $s_1=\frac{a}{r}+a+ar=-(-7)=7 \Rightarrow 2\left(\frac{1}{r}+1+r\right)=7$

$\Rightarrow 2r^2-5r+2=0 \Rightarrow (2r-1)(r-2)=0 \Rightarrow r=2$ or $1/2$

$\therefore$ The roots are 1,2,4.

23. Ans(3)

Given $x^3-\frac{14}{8}x^2+\frac{7}{8}x-\frac{1}{8}=0$ ....(1)let Let the roots of (1) be $\frac{a}{r},a,ar$

$s_3=\frac{a}{r}.a.ar=a^3=-\left(-\frac{1}{8}\right) \Rightarrow a^3=\frac{1}{8} \Rightarrow a=\frac{1}{2}$

$s_1 = \frac{a}{r} + a + ar = -\left(-\frac{14}{8}\right) = \frac{14}{8} \Rightarrow \frac{1}{2}\left(\frac{1}{r} + 1 + r\right) = \frac{7}{4}$

$\Rightarrow 2\left(r^2 + r + 1\right) = 7r \Rightarrow 2r^2 - 5r + 2 = 0$

$\Rightarrow (2r-1)(r-2) = 0 \Rightarrow r = 2$ or $\frac{1}{2}$.

$\therefore$ The roots be $\frac{1}{4}, \frac{1}{2}, 1.$

24. Ans (1)

$\frac{a}{r} a.ar = -8 \Rightarrow a^3 = -8 \Rightarrow a = -2$

But 'a' is a root of the given equation.

$\therefore -8 + 4 - k(2) + 8 = 0 \Rightarrow 2k = 4 \Rightarrow k = 2$

25. Ans (4) Let the roots be $\frac{a}{r}, a, ar$

$\Rightarrow s_3 = a^3 = -r \Rightarrow a = (-r)^{1/3}$

But 'a' is a root $\Rightarrow a^3 + 3pa^2 + 3qa + r = 0$

$\Rightarrow -r + 3pr^{2/3} + 3q(-r)^{1/3} + r = 0$

$\Rightarrow qr^{1/3} \Rightarrow pr^{1/3} = q \Rightarrow p^3 r = q^3 \Rightarrow pr^{2/3} = qr^{1/3} \Rightarrow pr^{1/3} = q \Rightarrow p^3 r = q^3$

26. Ans (2)

Let the roots be $\frac{1}{a-d}, \frac{1}{a}, \frac{1}{a+d}$ $\frac{1}{a-d}.\frac{1}{a}.\frac{1}{a+d} = r \Rightarrow \frac{1}{(a-d)a(a+d)} = r$

$\frac{1}{(a-d)a} + \frac{1}{(a-d)(a+d)} + \frac{1}{a(a+d)} = q$

$\Rightarrow \frac{a+d+a+a-d}{(a-d)a(a+d)} = q \Rightarrow 3a(r) = q \Rightarrow a = \frac{q}{3r}$

$\therefore$ Mean root is $\frac{1}{a} = \frac{3r}{q}$

27. Ans (1)

Let $\frac{1}{a-d}, \frac{1}{a}, \frac{1}{a+d}$ be the roots

$$s^3 = \frac{1}{(a-d)a(a+d)} = -r,\ s_2 = \frac{1}{(a-d)a} + \frac{1}{(a-d)(a+d)} + \frac{1}{a(a+d)} = q$$

$$\Rightarrow \frac{a+d+a+a-d}{(a-d)a(a+d)} = q \Rightarrow 3a(-r) = q \Rightarrow \frac{1}{a} = \frac{3r}{q}$$

But $\frac{1}{a}$ is a root $\Rightarrow \frac{1}{a^2} + q.\frac{1}{a} + r = 0$

$$\Rightarrow \frac{-27r^3}{q^3} p.\frac{9r^2}{q^2} + q.\left(-\frac{3r}{q}\right) + r = 0$$

$$\Rightarrow -27r^3 + 9pqr^2 - 3q^3r + rq^3 = 0$$

$$\Rightarrow -27r^3 + 9pqr^2 - 3q^3r + rq^3 = 0$$

$$\Rightarrow -27r^2 + 9pqr - 2q^3 = 0 \Rightarrow 2q^3 + 27r^2 = 9pqr$$

28. Ans (4)

Put x = 1/y in G.E. Then $cy^3 - 3by^2 + 3ay - 1 = 0$ ....(1)

The roots of (2) are in A.P. let $\alpha-\beta, \alpha, \alpha+\beta$ be the roots of (1)

$$s_1 = \alpha - \beta + \alpha + \beta = \frac{3b}{c} \Rightarrow 3\alpha = \frac{3b}{c} \Rightarrow 3\alpha = \frac{3b}{c} \Rightarrow \alpha = \frac{b}{c}$$

The mean root of (1) $\frac{b}{c}$. $\therefore$ The mean root of G.E. is $\frac{c}{b}$.

29. Ans(4)

Given $\alpha+\beta+\gamma = 1, \alpha\beta+\beta\gamma+\gamma\alpha = 8\alpha\beta\gamma = 6$

Required equation is $x^3 - x^2(\alpha^2+\beta^2+\gamma^2) + x(\alpha^2\beta^2+\gamma^2\alpha^2+\beta^2\gamma^2) - \alpha^2\beta^2\gamma^2 = 0 =$

$$x^3 - x^2\left[(\alpha+\beta+\gamma)^2 - 2\Sigma\alpha\beta\right] + x\left[(\Sigma\alpha\beta)^2 - 2\alpha\beta\gamma\Sigma\alpha\right] - (\alpha\beta\gamma)^2 = 0$$

$$\Rightarrow x^3 - x^2(1-16) + x(64-12.1) - 36 = 0 \Rightarrow x^3 + 15x^2 + 52x - 36 = 0$$

30. Ans. (2)

Given $\Sigma\alpha = 2, \Sigma\alpha\beta = 5, \alpha\beta\gamma = 3$

Let y = $\beta\gamma + \frac{1}{\alpha} = \frac{\alpha\beta\gamma+1}{\alpha} = \frac{3+1}{\alpha} = \frac{4}{\alpha} = \frac{4}{x} \; (\because a = x) \Rightarrow x = \frac{4}{y}$

Required equation is $\frac{64}{y^3} - 2\frac{16}{y^2} + \frac{20}{y} - 3 = 0$

$\Rightarrow 64 - 32 + 20y^2 - 3y^2 = 0$

$\Rightarrow 3y^3 - 20y^2 + 32y - 64 = 0$ or $3x^3 - 20x^2 - 64 = 0$.

31. Ans (2)

Given $\Sigma\alpha = 0, \Sigma\alpha\beta = q, \alpha\beta\gamma = -r$

Let $y = \frac{\beta+\gamma}{\alpha^2} = \frac{\alpha+\beta+\gamma}{\alpha^2} = \frac{0-\alpha}{\alpha^2} = \frac{\alpha}{\alpha^2}$

$= -\frac{1}{\alpha} = -\frac{1}{x} \Rightarrow x = -\frac{1}{y}$

Putting this value in G.E., we have: $\frac{1}{y^3} + q\left(-\frac{1}{y}\right) + r = 0$

$\Rightarrow ry^3 - qy^2 - 1 = 0$ or $rx^3 - qx^2 - 1 = 0$

32. Ans (2)

Given $f(x) = 5x^3 - 13x^2 - 12x + 7 = 0$

$f(x-2) = 0 \Rightarrow$ We have to diminish the roots by 2 Diminishing the roots of (1) by (2), we get

$$
\begin{array}{r|rrrr}
2 & 5 & -13 & -12 & 7 \\
 & & 10 & -6 & -36 \\
\hline
 & 5 & -3 & -18 & -29 \\
 & & 10 & 14 & \\
\hline
 & 5 & -7 & -4 & \\
 & & 10 & & \\
\hline
 & 5 & 17 & & \\
\end{array}
$$

The transformed equation is $5x^3 + 17x^2 - 4x - 29 = 0$

33. Ans (3)

4	1	−5	−7	−17	11
		4	-4	12	-20
	1	-1	3	-5	-9
		4	12	60	
	1	3	15	55	
		4	28		
	1	7	43		
	1	4			
	1	11			

The transformed equation is $x^4+11x^3+43x^2+55x-9=0$

34. Ans (6)

G.E is $x^4-\frac{5}{6}x^3+\frac{7}{18}x^2-\frac{1}{3}x+\frac{7}{54}=0$

$$x^4-\frac{5}{3^1 2^1}x^3+\frac{7}{3^1 2^1}x^2-\frac{1}{3^1 2^0}x+\frac{7}{3^3 2^1}=0$$

........(1)

Multiplying the roots of (1) by

2.3 = 6, we remove the fractional coefficients.

$\therefore k=6$

35. Ans (2) Diminishing the roots of the given equation by -2/3 we get the second equation. The roots of the second equation are : $+4+\frac{2}{3},-1+\frac{2}{3},-1+\frac{2}{3}$

$\Rightarrow \frac{-10}{3},\frac{-1}{3},\frac{-1}{3}$

36. Ans (2)

Diminish the roots of G.E. by 2, we have:

2	2	4	-5
		4	16
	2	8	11
		4	
	2	12	

Transformed equation is $2x^2+12x+11=0$.

37. Ans (1)

G.E. is $2x^3 - 9x^2 + 13x - 6 = 0$. Put 1/x in the place of x.

$\frac{2}{x^3} - \frac{9}{x^2} + \frac{13}{x} - 6 = 0 \Rightarrow 6x^3 - 13x^2 + 9x - 2 = 0$ is the reciprocal equation of G.E. whose roots are 3/2,1,2.

Hence, the roots of transformed equation 2/3, 1, 1 /2.

38. Ans (4)

G.E. is $2x^3 + 6x^2 - x + 1 = 0$ where $a_o = 2, a_1 = 6, a_2 = -1, a_3 = 1$

To remove the second term, diminish the roots by h = $\frac{-a_1}{na_0} = -\frac{6}{3(2)} = -1$

39. Ans (1)

Here $a_0 = 1 a_1 = -3, h = \frac{a_1}{na_0} = -\frac{-(-3)}{3(1)} = 1$

Diminishing the roots by 1 to remove the second term.

1	1	-3	12	16
		1	-2	10
	1	-2	10	
		1	-1	
	1	-1	9	
		1	-1	
	1	-1	9	
		1		
	1	0		

The transformed equation is $y^3 + 9y + 26 = 0$

40. Ans (1)

Given $f(x) = x^3 - 3x^2 + 4$

$f(x) = 3x^2 - 6x = 3x(x-2) = 0 \Rightarrow x = 0, 0$

$f(2) = 8 - 12 + 4 = 0$. But $f(0) \neq 0$

$\therefore$ 2 is a repeated root .

41. Ans (1)

Let $f\left(x\right)=x^{3}-x^{2}-x+1 \Rightarrow f\left(x\right)=3x^{2}-2x-1$

$f\left(x\right)=0 \Rightarrow 3x^{2}-2x-1=0 \Rightarrow \left(x-1\right)\left(3x+1\right)=0 \Rightarrow x=1,-1/3$

But f(1) = 0 and $f\left(-1/3\right)\neq 0$. $\therefore 1$ is repeated root.

42. Ans (4)

Let the roots be α,α,β.

Then $\alpha+\alpha+\beta=0 \quad \alpha.\alpha.\beta=-2r$

$\beta=-2\alpha, \quad \alpha^{2}\left(-2\alpha\right)=-2r \Rightarrow \alpha^{3}=r$

43. Ans (1)

$\alpha+\beta+\gamma=-4,\ \alpha\beta+\beta\gamma+\gamma\alpha=5,\ \alpha\beta\gamma=-2$

$$\Sigma\frac{1}{\alpha^{2}\beta^{2}}=\frac{1}{\alpha^{2}\beta^{2}}+\frac{1}{\beta^{2}\gamma^{2}}+\frac{1}{\gamma^{2}\alpha^{2}}=\frac{\gamma^{2}+\alpha^{2}+\beta^{2}}{\alpha^{2}\beta^{2}\gamma^{2}}$$

$$=\frac{\left(\alpha+\beta+\gamma\right)^{2}-2\left(\alpha\beta+\beta\gamma+\gamma\alpha\right)}{\left(\alpha\beta\gamma\right)^{2}}=\frac{16-2(5)}{(-2)^{2}}=\frac{6}{4}=\frac{3}{2}$$

44. Ans (3)

G.E. is $x^{3}+\frac{b}{a}x^{2}+\frac{c}{a}x+\frac{d}{a}=0 \qquad ......(1)$

$$\Sigma\alpha=-\frac{b}{a}\Sigma\alpha\beta=\frac{c}{a},\alpha\beta\gamma=-\frac{d}{a}$$

$$\Sigma\alpha^{2}\beta^{2}=\alpha^{2}\beta^{2}+\beta^{2}\gamma^{2}+\gamma^{2}\alpha^{2}=\left(\alpha\beta+\beta\gamma+\gamma\alpha\right)^{2}-2\alpha\beta\gamma\left(\alpha+\beta+\gamma\right)$$

$$=\frac{c^{2}}{a^{2}}-2\left(-\frac{d}{a}\right)\left(-\frac{b}{a}\right)=\frac{c^{2}-2bd}{a^{2}}$$

45. Ans (1)

Let $y=\alpha+3=x+3 \Rightarrow x=y-3$. The transformed equation is

$\therefore\left(y-3\right)^{3}-9\left(y-3\right)^{2}+26\left(y-3\right)-24=0$

$\Rightarrow y^{3}+27-9y^{2}+27y-9y^{2}-81+54y+26y-78-24=0$

$\Rightarrow y^3 - 18y^2 + 107y - 210 = 0$ Whose roots are $\alpha+3, \beta+3, \gamma+3$

$s_2 = \Sigma(\alpha+3)(\beta+3) = 107$

46. Ans (2)

$x^3 - 21x^2 + 126x - 216$

$= (x^3 - 216) - 21x[x-6] = 0$

$\Rightarrow (x-6)(x^2 + 6x + 36 - 21x) = 0$

$\Rightarrow (x-6)(x^2 - 15x + 36) = 0$

$\Rightarrow (x-6)(x-3)(x-12) = 0$

$\Rightarrow x = 3, 6, 12$

These roots are in G.P.

47. Ans (4)

Given eq. is $\left(x+\frac{1}{x}\right)^3 + \left(x+\frac{1}{x}\right) = 0$

$\Rightarrow \left(x+\frac{1}{x}\right)\left[\left(x+\frac{1}{x}\right)^2 + 1\right] = 0 \Rightarrow \left(x+\frac{1}{x}\right)^2 + 1 = 0 \Rightarrow x + \frac{1}{x} = 0 (\text{or}) x^2 + 1 = 0$ (Or)

$\left(x+\frac{1}{x}\right)^2 + 1 = 0$

There are no real roots.

48. Ans (2)