

FUNCTIONS

OBJECTIVE PROBLEMS

1. Let A be a set of n distinct elements. Then the total number of distinct functions from A to A is
a) n^2 b) n^n c) $2n$ d) $n!$
2. Set A has 3 elements and set B has 4 elements. The number of injections that can be defined from A into B is
a) 144 b) 12 c) 24 d) 64
3. Let $n(A) = 4$ and $n(B) = K$. The number of all possible injections from A to B is 120 then $k =$
a) 9 b) 24 c) 5 d) 6
4. Let $n(A) = 4$ and $n(b) = 5$. The number of all possible many-one functions from A to B is
a) 625 b) 20 c) 120 d) 505
5. Set A contains 3 elements and set B contains 2 elements. The number of onto functions from A onto B is
a) 3 b) 6 c) 8 d) 9
6. Let $A = \{1, 2, 3, \dots, n\}$ and $B = \{a, b, c\}$, the number of functions from A to B that are onto is
a) $3^n - 2^n$ b) $3^n - 2^n = 1$ c) $3(2^n - 1)$ d) $3^n - 3(2^n - 1)$
7. Set A has 3 elements, Set B has 4 elements. The number of surjections that can be defined from A to B is
a) 144 b) 12 c) 0 d) 64

8. Let A, B are two sets each with 10 elements, then the number of all possible bijections from A to B is
- a) 20 b) 10! c) 100 d) 1000
9. The number of one-one onto functions that can be defined from (1, 2, 3, 4) onto set B is 24 then $n(B) =$
- a) 4 b) 2 c) 3 d) 6
10. $A = \{1, 2, 3, 4\}$, $B = \{a, b, c, d, e\}$, then the number of all possible constant functions from A to B is
- a) 9 b) 4 c) 5 d) 16
11. If $f(x) = \cos(\log x)$, then $f(x)f(y) - \frac{1}{2}[f(x/y) + f(xy)] =$
- (a) -1 (b) $\frac{1}{2}$ (c) -2 (d) None of these
12. If $f(x) = \sin \log x$, then the value of $f(xy) + f\left(\frac{x}{y}\right) - 2f(x) \cdot \cos \log y$ is equal to
- (a) 1 (b) 0 (c) -1 (d) $\sin \log x \cdot \cos \log y$
13. If $f(x) = \cos(\log x)$, then $f(x^2)f(y^2) - \frac{1}{2}\left[f\left(\frac{x^2}{2}\right) + f\left(\frac{x^2}{y^2}\right)\right]$ has the value
- (a) -2 (b) -1 (c) 1/2 (d) None of these
14. If $f(x) = \cos(\log x)$, then the value of $f(x) \cdot f(4) - \frac{1}{2}\left[f\left(\frac{x}{4}\right) + f(4x)\right]$
- (a) 1 (b) -1 (c) 0 (d) ± 1
15. Given the function $f(x) = \frac{a^x + a^{-x}}{2}$, ($a > 2$). Then $f(x+y) + f(x-y) =$
- (a) $2f(x) \cdot f(y)$ (b) $f(x) \cdot f(y)$ (c) $\frac{f(x)}{f(y)}$ (d) None of these
16. Let $f: R \rightarrow R$ be defined by $f(x) = 2x + |x|$, then $f(2x) + f(-x) - f(x) =$
- (a) $2x$ (b) $2|x|$ (c) $-2x$ (d) $-2|x|$
17. If $e^{f(x)} = \frac{10+x}{10-x}$, $x \in (-10, 10)$ and $f(x) = kf\left(\frac{200x}{100+x^2}\right)$, then $k =$
- (a) 0.5 (b) 0.6 (c) 0.7 (d) 0.8

18. If $f(x) = \log \left[\frac{1+x}{1-x} \right]$, then $f \left[\frac{2x}{1+x^2} \right]$ is equal to
 (a) $[f(x)]^2$ (b) $[f(x)]^3$ (c) $2f(x)$ (d) $3f(x)$
19. If $f(x) = \cos[\pi^2]x + \cos[-\pi^2]x$, then
 (a) $f\left(\frac{\pi}{4}\right) = 2$ (b) $f(-\pi) = 2$ (c) $f(\pi) = 1$ (d) $f\left(\frac{\pi}{2}\right) = -1$
20. If $y = f(x) = \frac{ax+b}{cx-a}$, then x is equal to
 (a) $1/f(x)$ (b) $1/f(y)$ (c) $yf(x)$ (d) $f(y)$
21. The function equivalent to $\log x^2$ is
 (a) $2 \log x$ (b) $2 \log |x|$ (c) $|\log x^2|$ (d) $(\log x)^2$
22. The graph of the function $y = f(x)$ is symmetrical about the line $x = 2$, then
 (a) $f(x) = -f(-x)$ (b) $f(2+x) = f(2-x)$ (c) $f(x) = f(-x)$ (d) $f(x+2) = f(x-2)$
23. If $f(x) = \frac{1}{\sqrt{x+2\sqrt{2x-4}}} + \frac{1}{\sqrt{x-2\sqrt{2x-4}}}$ for $x > 2$, then $f(11) =$
 (a) $7/6$ (b) $5/6$ (c) $6/7$ (d) $5/7$
24. If $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = 3x - 4$, then $f^{-1}: \mathbb{R} \rightarrow \mathbb{R}$ is
 (a) $4 - 3x$ (b) $\frac{x+4}{3}$ (c) $\frac{1}{3x-4}$ (d) $\frac{3}{x+4}$
25. Which of the following function is invertible?
 (a) $f(x) = 2^x$ (b) $f(x) = x^3 - x$ (c) $f(x) = x^2$ (d) None of these
26. The inverse of the function $\frac{10^x - 10^{-x}}{10^x + 10^{-x}}$ is
 (a) $\frac{1}{2} \log_{10} \left(\frac{1+x}{1-x} \right)$ (b) $\frac{1}{2} \log_{10} \left(\frac{1-x}{1+x} \right)$ (c) $\frac{1}{4} \log_{10} \left(\frac{2x}{2-x} \right)$ (d) None of these
27. If $f(x) = \frac{x}{1+x}$, then $f^{-1}(x)$ is equal to
 (a) $\frac{(1+x)}{x}$ (b) $\frac{1}{(1+x)}$ (c) $\frac{(1+x)}{(1-x)}$ (d) $\frac{x}{(1-x)}$
28. If $f(x) = \frac{x}{\sqrt{1+x^2}}$, then $(f \circ f \circ f)(x) =$
 (a) $\frac{3x}{\sqrt{1+x^2}}$ (b) $\frac{x}{\sqrt{1+3x^2}}$ (c) $\frac{3x}{\sqrt{1+x^2}}$ (d) None of these

29. If $f(x) = \log_a x$ and $F(x) = a^x$, then $F[f(x)]$ is
 (a) $f[F(x)]$ (b) $f[F(2x)]$ (c) $F[f(2x)]$ (d) $F[(x)]$
30. Let f and g be functions defined by $f(x) = \frac{x}{x+1}$, $g(x) = \frac{x}{1-x}$, then $(f \circ g)(x)$ is
 (a) $\frac{1}{x}$ (b) $\frac{1}{x-1}$ (c) $x-1$ (d) x
31. If $f(x) = \frac{\alpha x}{x+1}$, $x \neq -1$. Then, for what value of α is $f(f(x)) = x$
 (a) $\sqrt{2}$ (b) $-\sqrt{2}$ (c) 1 (d) -1
32. If $f(x) = (a - x^n)^{1/n}$, where $a > 0$ and n is a positive integer, then $f[f(x)] =$
 (a) x^3 (b) x^2 (c) x (d) None of these
33. If $f(x_1) - f(x_2) = f\left(\frac{x_1 - x_2}{1 - x_1 x_2}\right)$ for $x_1, x_2 \in [-1, 1]$, then $f(x)$ is
 (a) $\log \frac{(1-x)}{(1+x)}$ (b) $\tan^{-1} \frac{(1-x)}{(1+x)}$ (c) $\log \frac{(1+x)}{(1-x)}$ (d) all the above
34. If $f(x) = \begin{cases} x, & \text{when } x \text{ is rational} \\ 0, & \text{when } x \text{ is irrational} \end{cases}$; $g(x) = \begin{cases} 0, & \text{when } x \text{ is rational} \\ x, & \text{when } x \text{ is irrational} \end{cases}$ then $(f - g)$ is
 (a) One-one onto (b) One-one not onto
 (c) Not one-one but onto (d) Not one-one not onto
35. If $e^x = y + \sqrt{1 + y^2}$, then $y =$
 (a) $\frac{e^x + e^{-x}}{2}$ (b) $\frac{e^x - e^{-x}}{2}$ (c) $e^x + e^{-x}$ (d) $e^x - e^{-x}$
36. If $f(x) = \frac{x - |x|}{|x|}$, then $f(-1) =$
 (a) 1 (b) -2 (c) 0 (d) +2
37. If $f(x + ay, x - ay) = axy$, then $f(x, y)$ is equal to
 (a) xy (b) $x^2 - a^2 y^2$ (c) $\frac{x^2 - y^2}{4}$ (d) $\frac{x^2 - y^2}{a^2}$

38. Let $f(x) = \begin{cases} \frac{1}{2}, & \text{if } 0 \leq x \leq \frac{1}{2} \\ \frac{1}{3}, & \text{if } \frac{1}{2} < x \leq 1 \end{cases}$, then f is

- (a) A rational function (b) A trigonometric function
(c) A step function (d) An exponential function

39. Let $f : (2, 3) \rightarrow (0, 1)$ be defined by $f(x) = x - [x]$ then $f^{-1}(x)$ equals

- (a) $x - 2$ (b) $x + 1$ (c) $x - 1$ (d) $x + 2$

40. The domain of $\sin^{-1} \left[\log_3 \left(\frac{x}{3} \right) \right]$ is

- (a) $[1, 9]$ (b) $[-1, 9]$ (c) $[-9, 1]$ (d) $[-9, -1]$

41. The domain of the function $f(x) = \sin^{-1}[\log_2(x/2)]$ is

- (a) $[1, 4]$ (b) $[-4, 1]$ (c) $[-1, 4]$ (d) None of these

42. The domain of the function $f(x) = \log(\sqrt{x-4} + \sqrt{6-x})$ is

- (a) $[4, \infty)$ (b) $(-\infty, 6]$ (c) $[4, 6]$ (d) None of these

43. Domain of the function $f(x) = \left[\log_{10} \left(\frac{5x - x^2}{4} \right) \right]^{1/2}$ is

- (a) $-\infty < x < \infty$ (b) $1 \leq x \leq 4$ (c) $4 \leq x \leq 16$ (d) None

44. Domain of the function $f(x) = \sqrt{2 - 2x - x^2}$ is

- (a) $-\sqrt{3} \leq x \leq \sqrt{3}$ (b) $-1 - \sqrt{3} \leq x \leq -1 + \sqrt{3}$
(c) $-2 \leq x \leq 2$ (d) $-2 + \sqrt{3} \leq x \leq -2 - \sqrt{3}$

45. Domain of the function $\frac{\sqrt{1+x} - \sqrt{1-x}}{x}$ is

- (a) $(-1, 1)$ (b) $(-1, 1) - \{0\}$ (c) $[-1, 1]$ (d) $[-1, 1] - \{0\}$

46. The largest possible set of real numbers which can be the domain of $f(x) = \sqrt{1 - \frac{1}{x}}$ is

- (a) $(0, 1) \cup (0, \infty)$ (b) $(-1, 0) \cup (1, \infty)$ (c) $(-\infty, -1) \cup (0, \infty)$ (d) $(-\infty, 0) \cup (1, \infty)$

47. Domain of the function $f(x) = \frac{x^2 - 3x + 2}{x^2 + x - 6}$ is

- (a) $\{x : x \in R, x \neq 3\}$
 (b) $\{x : x \in R, x \neq 2\}$
 (c) $\{x : x \in R\}$
 (d) $\{x : x \in R, x \neq 2, x \neq -3\}$

48. The domain of the function $y = \frac{1}{\sqrt{|x| - x}}$ is

- (a) $(-\infty, 0)$ (b) $(-\infty, 0]$ (c) $(-\infty, -1)$ (d) $(-\infty, \infty)$

49. Function $\sin^{-1} \sqrt{x}$ is defined in the interval

- (a) $(-1, 1)$ (b) $[0, 1]$ (c) $[-1, 0]$ (d) $(-1, 2)$

50. The domain of the function $f(x) = \frac{\sin^{-1}(3-x)}{\ln(|x| - 2)}$ is

- (a) $[2, 4]$ (b) $(2, 3) \cup (3, 4]$ (c) $[2, \infty)$ (d) $(-\infty, -3) \cup [2, \infty)$

51. The domain of $f(x) = \frac{\log_2(x+3)}{x^2 + 3x + 2}$ is

- (a) $R - \{-1, -2\}$ (b) $(-2, +\infty)$ (c) $R - \{-1, -2, -3\}$ (d) $(-3, +\infty) - \{-1, -2\}$

52. The domain of the function $f(x) = \log_{3+x}(x^2 - 1)$ is

- (a) $(-3, -1) \cup (1, \infty)$ (b) $[-3, -1) \cup [1, \infty)$
 (c) $(-3, -2) \cup (-2, -1) \cup (1, \infty)$ (d) $[-3, -2) \cup (-2, -1) \cup [1, \infty)$

53. Domain of definition of the function $f(x) = \frac{3}{4 - x^2} + \log_{10}(x^3 - x)$, is

- (a) $(1, 2)$ (b) $(-1, 0) \cup (1, 2)$ (c) $(1, 2) \cup (2, \infty)$ (d) $(-1, 0) \cup (1, 2) \cup (2, \infty)$

54. Domain of the function $\sqrt{\log\{(5x - x^2)/6\}}$ is

- (a) $(2, 3)$ (b) $[2, 3]$ (c) $[1, 2]$ (d) $[1, 3]$

55. The domain of the function $\sqrt{\log(x^2 - 6x + 6)}$ is

- (a) $(-\infty, \infty)$
 (b) $(-\infty, 3 - \sqrt{3}) \cup (3 + \sqrt{3}, \infty)$
 (c) $(-\infty, 1] \cup [5, \infty)$
 (d) $[0, \infty)$

- 56. The domain of the function $f(x) = \exp(\sqrt{5x-3-2x^2})$ is**
- (a) $\left[1, -\frac{3}{2}\right]$ (b) $\left[\frac{3}{2}, \infty\right]$ (c) $[-\infty, 1]$ (d) $\left[1, \frac{3}{2}\right]$
- 57. The domain of the function $f(x) = \frac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is**
- (a) $[1, 2)$ (b) $[2, 3)$ (c) $[1, 2]$ (d) $[2, 3]$
- 58. The domain of the function $f(x) = \sin^{-1}\{(1+e^x)^{-1}\}$ is**
- (a) $\left(\frac{1}{4}, \frac{1}{3}\right)$ (b) $[-1, 0]$ (c) $[0, 1]$ (d) $[-1, 1]$
- 59. Domain of the function $\sqrt{2-x} - \frac{1}{\sqrt{9-x^2}}$ is**
- (a) $(-3, 1)$ (b) $[-3, 1]$ (c) $(-3, 2]$ (d) $[-3, 1)$
- 60. Domain of the function $f(x) = \frac{x-3}{(x-1)\sqrt{x^2-4}}$ is**
- (a) $(1, 2)$ (b) $(-\infty, -2) \cup (2, \infty)$
 (c) $(-\infty, -2) \cup (1, \infty)$ (d) $(-\infty, \infty) - \{1, \pm 2\}$
- 61. The domain of the function $f(x) = \sqrt{\log \frac{1}{|\sin x|}}$ is**
- (a) $R - \{2n\pi, n \in I\}$ (b) $R - \{n\pi, n \in I\}$
 (c) $R - \{-\pi, \pi\}$ (d) $(-\infty, \infty)$
- 62. The function $f(x) = \frac{\sec^{-1} x}{\sqrt{x - [x]}}$, where $[.]$ denotes the greatest integer less than or equal to x is defined for all x belonging to**
- (a) R (b) $R - \{(-1, 1) \cup (n | n \in Z)\}$
 (c) $R^+ - (0, 1)$ (d) $R^+ - \{n | n \in N\}$
- 63. Domain of $f(x) = \log |\log x|$ is**
- (a) $(0, \infty)$ (b) $(1, \infty)$ (c) $(0, 1) \cup (1, \infty)$ (d) $(-\infty, 1)$
- 64. Domain of function $f(x) = \sin^{-1} 5x$ is**
- (a) $\left(-\frac{1}{5}, \frac{1}{5}\right)$ (b) $\left[-\frac{1}{5}, \frac{1}{5}\right]$ (c) R (d) $\left(0, \frac{1}{5}\right)$
- 65. The domain of the function $f(x) = \log(\sqrt{x-4} + \sqrt{6-x})$ is**
- (a) $[4, \infty)$ (b) $(-\infty, 6]$ (c) $[4, 6]$ (d) None of these

66. Domain of the function $f(x) = \sin^{-1}(1 + 3x + 2x^2)$ is

- (a) $(-\infty, \infty)$ (b) $(-1, 1)$ (c) $\left[-\frac{3}{2}, 0\right]$ (d) $\left(-\infty, \frac{-1}{2}\right) \cup (2, \infty)$

67. The range of the function $f(x) = \frac{x+2}{|x+2|}$ is

- (a) $\{0, 1\}$ (b) $\{-1, 1\}$ (c) R (d) $R - \{-2\}$

68. Range of the function $\frac{1}{2 - \sin 3x}$ is

- (a) $[1, 3]$ (b) $\left[\frac{1}{3}, 1\right]$ (c) $(1, 3)$ (d) $\left(\frac{1}{3}, 1\right)$

69. If x is real, then value of the expression $\frac{x^2 + 14x + 9}{x^2 + 2x + 3}$ lies between

- (a) 5 and 4 (b) 5 and -4 (c) -5 and 4 (d) None of these

70. If the function $f: [1, \infty) \rightarrow [1, \infty)$ is defined by $f(x) = 2^{x(x-1)}$, then $f^{-1}(x)$ is

- (a) $\left(\frac{1}{2}\right)^{x(x-1)}$ (b) $\frac{1}{2}(1 + \sqrt{1 + 4 \log_2 x})$
(c) $\frac{1}{2}(1 - \sqrt{1 + 4 \log_2 x})$ (d) Not defined

71. The range of function The range of function $f(x) = x^2 - 6x + 7$ is

- (a) $(-\infty, \infty)$ (b) $[-2, \infty)$ (c) $(-2, 3)$ (d) $(-\infty, -2)$

72. Range of the function $f(x) = \frac{x^2 + x + 2}{x^2 + x + 1}$; $x \in R$ is

- (a) $(1, \infty)$ (b) $(1, 11/7]$ (c) $(1, 7/3]$ (d) $(1, 7/5]$

73. The function $f: R \rightarrow R$ is defined by $f(x) = \cos^2 x + \sin^4 x$ for $x \in R$, then $f(R) =$

- (a) $\left[\frac{3}{4}, 1\right]$ (b) $\left[\frac{3}{4}, 1\right)$ (c) $\left[\frac{3}{4}, 1\right]$ (d) $\left(\frac{3}{4}, 1\right)$

74. If $f: R \rightarrow R$, then the range of the function $f(x) = \frac{x^2}{x^2 + 1}$ is

- (a) R^- (b) R^+ (c) R (d) $R \times R$

75. Range of the function $f(x) = 9 - 7 \sin x$ is

- (a) $(2, 16)$ (b) $[2, 16]$ (c) $[-1, 1]$ (d) $(2, 16]$

76. The range of $f(x) = \sec\left(\frac{\pi}{4} \cos^2 x\right)$, $-\infty < x < \infty$ is

- (a) $[1, \sqrt{2}]$ (b) $[1, \infty)$ (c) $[-\sqrt{2}, -1] \cup [1, \sqrt{2}]$ (d) $(-\infty, -1] \cup [1, \infty)$

77. If $f(x) = a \cos(bx + c) + d$, then range of $f(x)$ is

- (a) $[d + a, d + 2a]$ (b) $[a - d, a + d]$ (c) $[d + a, a - d]$ (d) $[d - a, d + a]$

78. Range of the function $f(x) = \frac{x^2}{x^2 + 1}$ is

- (a) $(-1, 0)$ (b) $(-1, 1)$
(c) $[0, 1)$ (d) $(1, 1)$

79. The domain of $\sin^{-1}(\log_3 x)$ is

- (a) $[-1, 1]$ (b) $[0, 1]$ (c) $[0, \infty]$ (d) R

80. For $\theta > \frac{\pi}{3}$, the value of $f(\theta) = \sec^2 \theta + \cos^2 \theta$ always lies in the interval

- (a) $(0, 2)$ (b) $[0, 1]$ (c) $(1, 2)$ (d) $[2, \infty)$

81. The Domain of function $f(x) = \log_e(x - [x])$ is

- (a) R (b) $R - Z$ (c) $(0, +\infty)$ (d) Z

82. If $f(x) = x^2 + 1$, then $f^{-1}(17)$ and $f^{-1}(-3)$ will be

- (a) 4, 1 (b) 4, 0 (c) 3, 2 (d) None of these

83. If $f(x) = |\cos x|$ and $g(x) = [x]$, then $g \circ f(x)$ is equal to

- (a) $|\cos [x]|$ (b) $| \cos x |$ (c) $[|\cos x|]$ (d) $| [\cos x] |$

84. Let $g(x) = 1 + x - [x]$ and $f(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$, then for all x , $f(g(x))$ is equal to

- (a) x (b) 1 (c) $f(x)$ (d) $g(x)$

85. A real valued function $f(x)$ satisfies the function equation $f(x - y) = f(x)f(y) - f(a - x)f(a + y)$ where a is a given constant and $f(0) = 1$, $f(2a - x)$ is equal to

- (a) $f(a) + f(a - x)$ (b) $f(-x)$ (c) $-f(x)$ (d) $f(x)$

86. Let $g(x) = 1 + x - [x]$ and

$f(x) = \begin{cases} -1, & \text{If } x < 0 \\ 0, & \text{If } x = 0 \\ 1, & \text{if } x > 0 \end{cases}$, then for all values of x the value of $f \circ g(x)$

- (a) x (b) 1 (c) $f(x)$ (d) $g(x)$

87. The function $f : R \rightarrow R$, $f(x) = x^2, \forall x \in R$ is

- (a) Injection but not surjection (b) Surjection but not injection
(c) Injection as well as surjection (d) Neither injection nor surjection

88. If $f: R \rightarrow R$, then $f(x) = |x|$ is

- (a) One-one but not onto (b) Onto but not one-one
(c) One-one and onto (d) None of these

89. Which one of the following is a bijective function on the set of real numbers

- (a) $2x - 5$ (b) $|x|$ (c) x^2 (d) $x^2 + 1$

90. If $(x, y) \in R$ and $x, y \neq 0$; $f(x, y) \rightarrow \frac{x}{y}$, then this function is a/an

- (a) Surjection (b) Bijection (c) One-one (d) None of these

91. The function $f(x) = \sin(\log(x + \sqrt{x^2 + 1}))$ is

- (a) Even function (b) Odd function
(c) Neither even nor odd (d) Periodic function

92. If $f(x) = \sin^2 x$ and the composite function $g\{f(x)\} = \sin x$, then the function $g(x)$ is equal to

- (a) $\sqrt{x-1}$ (b) \sqrt{x} (c) $\sqrt{x+1}$ (d) $-\sqrt{x}$

93. If $f(x) = 2x^6 + 3x^4 + 4x^2$ then $f'(x)$ is

- (a) Even function (b) An odd function
(c) Neither even nor odd (d) None of these

94. The function $f(x) = \log(x + \sqrt{x^2 + 1})$, is

- (a) An even function (b) An odd function
(c) A Periodic function (d) Neither an even nor odd function

95. The period of $f(x) = x - [x]$, if it is periodic, is

- (a) $f(x)$ is not periodic (b) $\frac{1}{2}$ (c) 1 (d) 2

96. A function f from the set of natural numbers to integers defined by

$$f(n) = \begin{cases} \frac{n-1}{2}, & \text{when } n \text{ is odd} \\ -\frac{n}{2}, & \text{when } n \text{ is even} \end{cases}, \text{ is}$$

- (a) One-one but not onto (b) Onto but not one-one
(c) One-one and onto both (d) Neither one-one nor onto

97. Let $f: N \rightarrow N$ defined by $f(x) = x^2 + x + 1$, $x \in N$, then f is

- (a) One-one onto (b) Many one onto
(c) One-one but not onto (d) None of these

98. The function $f: R \rightarrow R$ defined by $f(x) = (x-1)(x-2)(x-3)$ is

- (a) One-one but not onto (b) Onto but not one-one
(c) Both one-one and onto (d) Neither one-one nor onto

99. If $f: [0, \infty) \rightarrow [0, \infty)$ and $f(x) = \frac{x}{1+x}$, then f is

- (a) One-one and onto (b) One-one but not onto
(c) Onto but not one-one (d) Neither one-one nor onto

100. Let the function $f: R \rightarrow R$ be defined by $f(x) = 2x + \sin x$, $x \in R$. Then f is

- (a) One-to-one and onto (b) One-to-one but not onto
(c) Onto but not one-to-one (d) Neither one-to-one nor onto

101. $f(x) = x + \sqrt{x^2}$ is a function from $R \rightarrow R$, then $f(x)$ is

- (a) Injective (b) Surjective (c) Bijective (d) None of these

102. Range of $f(x) = \frac{x^2 + 34x - 71}{x^2 + 2x - 7}$ is

- (a) $[5, 9]$ (b) $(-\infty, 5] \cup [9, \infty)$ (c) $(5, 9)$ (d) None of these

103. Which of the following functions is inverse of itself

- (a) $f(x) = \frac{1-x}{1+x}$ (b) $f(x) = 5^{\log x}$ (c) $f(x) = 2^{x(x-1)}$ (d) None of these

104. If $f(x) = 3x - 5$, then $f^{-1}(x)$

- (a) Is given by $\frac{1}{3x-5}$ (b) Is given by $\frac{x+5}{3}$
(c) Does not exist because f is not one-one (d) Does not exist because f is not onto

105. If $f: R \rightarrow S$ defined by $f(x) = \sin x - \sqrt{3} \cos x + 1$ is onto, then the interval of S is

- (a) $[-1, 3]$ (b) $[1, 1]$ (c) $[0, 1]$ (d) $[0, -1]$

106. Let $f: R \rightarrow R$ be a function defined by $f(x) = \frac{x-m}{x-n}$, where $m \neq n$. Then

- (a) f is one-one onto (b) f is one-one into
(c) f is many one onto (d) f is many one into

107. Let $f(x) = \frac{x^2 - 4}{x^2 + 4}$ for $|x| > 2$, then the function $f : (-\infty, -2] \cup [2, \infty) \rightarrow (-1, 1)$ is

- (a) One-one into (b) One-one onto
(c) Many one into (d) Many one onto

108. Let X and Y be subsets of R , the set of all real numbers. The function $f : X \rightarrow Y$ defined by $f(x) = x^2$ for $x \in X$ is one-one but not onto if (Here R^+ is the set of all positive real numbers)

- (a) $X = Y = R^+$ (b) $X = R, Y = R^+$
(c) $X = R^+, Y = R$ (d) $X = Y = R$

109. Function $f : R \rightarrow R, f(x) = x^2 + x$ is

- (a) One-one onto (b) One-one into
(c) Many-one onto (d) Many-one into

FUNCTIONS

HINTS AND SOLUTIONS

1. (b).

$$n(A) = n(B) = n$$

$$\text{and no. functions} = n(B)^{n(A)}$$

2. (c) Synopsis 3. (c) Synopsis

4. (d) Synopsis 5. (b) Synopsis

6. (d) 7. (c) 8. (b)

9. (a) 10. (c) Synopsis

11. (d) $f(x) = \cos(\log x) \Rightarrow f(y) = \cos(\log y)$

$$f(x) \cdot f(y) - \frac{1}{2} \left[f\left(\frac{x}{y}\right) + f(xy) \right]$$

$$= \cos(\log x) \cos(\log y) - \frac{1}{2} \left[\cos\left(\log \frac{x}{y}\right) + \cos(\log xy) \right]$$

$$= \cos(\log x) \cos(\log y) - \frac{1}{2} [2 \cos(\log x) \cos(\log y)] = 0.$$

12. (b) $f(xy) = \sin \log xy = \sin(\log x + \log y) \dots (i)$

$$f(x/y) = \sin \log(x/y) = \sin(\log x - \log y) \dots (ii)$$

$$\therefore f(xy) + f(x/y) = 2 \sin \log x \cos \log y$$

$$2 \sin \log x \cos \log y - 2 \sin \log x \cos \log y = 0.$$

13. (d)

14. (c)

15. (a) $f(x+y) + f(x-y)$

$$= \frac{1}{2} [a^{x+y} + a^{-x-y} + a^{x-y} + a^{-x+y}]$$

$$= \frac{1}{2} [a^x (a^y + a^{-y}) + a^{-x} (a^y + a^{-y})]$$

$$= \frac{1}{2} (a^x + a^{-x}) (a^y + a^{-y}) = 2f(x)f(y).$$

16. (b) $f(2x) = 2(2x) + |2x| = 4x + 2|x|,$

$$y = x^2 + 1,$$

$$f(x) = 2x + |x| \Rightarrow f(2x) + f(-x) - f(x)$$

$$= 4x + 2|x| + |x| - 2x - 2x - |x| = 2|x|.$$

17. (a) $e^{f(x)} = \frac{10+x}{10-x} \Rightarrow f(x) = \log\left(\frac{10+x}{10-x}\right)$

$$\Rightarrow f\left(\frac{200x}{100+x^2}\right) = \log\left[\frac{10 + \frac{200x}{100+x^2}}{10 - \frac{200x}{100+x^2}}\right] = \log\left[\frac{10(10+x)}{10(10-x)}\right]^2$$

$$= 2 \log\left(\frac{10+x}{10-x}\right) = 2f(x)$$

$$\therefore f(x) = \frac{1}{2}f\left(\frac{200x}{100+x^2}\right) \Rightarrow k = \frac{1}{2} = 0.5.$$

18. (c) $f(x) = \log(x + \sqrt{x^2 + 1})$

$$\therefore f\left(\frac{2x}{1+x^2}\right) = \log\left[\frac{1 + \frac{2x}{1+x^2}}{1 - \frac{2x}{1+x^2}}\right] = \log\left[\frac{x^2 + 1 + 2x}{x^2 + 1 - 2x}\right]$$

$$= \log\left[\frac{1+x}{1-x}\right]^2 = 2 \log\left[\frac{1+x}{1-x}\right] = 2f(x).$$

19. (d) $f(x) = \cos[\pi^2]x + \cos[-\pi^2]x$

$$f(x) = \cos(9x) + \cos(-10x) = \cos(9x) + \cos(10x)$$

$$= 2 \cos\left(\frac{19x}{2}\right) \cos\left(\frac{x}{2}\right)$$

$$f\left(\frac{\pi}{2}\right) = 2 \cos\left(\frac{19\pi}{4}\right) \cos\left(\frac{\pi}{4}\right); f\left(\frac{\pi}{2}\right) = 2 \times \frac{-1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = -1.$$

20. (d) $y = \frac{ax+b}{cx-a} \Rightarrow x(cy-a) = b+ay \Rightarrow x = \frac{ay+b}{cy-a} = f(y).$

21. (b) domain of $\log x^2$ is $\mathbb{R}-\{0\}$ and the domain of $\log|x|$ is also $\mathbb{R}-\{0\}$.

$$\log x^2 \text{ and } 2 \log|x| \text{ are identical functions.}$$

22. (b) $f(x) = f(-x) \Rightarrow f(0+x) = f(0-x)$ is symmetrical about $x=0$.

$$\therefore f(2+x) = f(2-x) \text{ is symmetrical about } x=2.$$

23. (c) $f(x) = \frac{1}{\sqrt{x+2\sqrt{2x-4}}} + \frac{1}{\sqrt{x-2\sqrt{2x-4}}}$

$$f(11) = \frac{1}{\sqrt{11+2\sqrt{18}}} + \frac{1}{\sqrt{11-2\sqrt{18}}}$$

$$= \frac{1}{3+\sqrt{2}} + \frac{1}{3-\sqrt{2}} = \frac{3-\sqrt{2}}{7} + \frac{3+\sqrt{2}}{7} = \frac{6}{7}.$$

24. (b) $f(x) = 3x-4$. let $y = f^{-1}(x) \Rightarrow f(y) = x$

$$\Rightarrow 3y-4 = x \Rightarrow 3y = x+4$$

$$\Rightarrow y = \frac{x+4}{3} \Rightarrow f^{-1}(x) = \frac{x+4}{3}.$$

25. (a) A function is invertible if it is bijection.

26. (a) $y = \frac{10^x - 10^{-x}}{10^x + 10^{-x}} \Rightarrow x = \frac{1}{2} \log_{10} \left(\frac{1+y}{1-y} \right)$

Let $y = f(x) \Rightarrow x = f^{-1}(y)$

$$\Rightarrow f^{-1}(y) = \frac{1}{2} \log_{10} \left(\frac{1+y}{1-y} \right) \Rightarrow f^{-1}(x) = \frac{1}{2} \log_{10} \left(\frac{1+x}{1-x} \right)$$

27. (d) $f(x) = \frac{x}{1+x}$. Let $y = f(x) \Rightarrow x = f^{-1}(y)$

$$\therefore y = \frac{x}{1+x} \Rightarrow y + yx = x \Rightarrow x = \frac{y}{1-y}$$

$$\Rightarrow f^{-1}(y) = \frac{y}{1-y} \Rightarrow f^{-1}(x) = \frac{x}{1-x}.$$

28. (b) $(f \circ f \circ f)(x) = (f \circ f)(f(x)) = (f \circ f) \left(\frac{x}{\sqrt{1+x^2}} \right)$

$$= f \left(\frac{\left(\frac{x}{\sqrt{1+x^2}} \right)}{\sqrt{1 + \frac{x^2}{1+x^2}}} \right) = f \left(\frac{x\sqrt{1+x^2}}{\sqrt{1+x^2}\sqrt{1+2x^2}} \right)$$

$$= f \left(\frac{x}{\sqrt{1+2x^2}} \right) = \frac{\frac{x}{\sqrt{1+2x^2}}}{\sqrt{1 + \frac{x^2}{1+2x^2}}} = \frac{x}{\sqrt{1+3x^2}}.$$

29. (a) $F[f(x)] = F(\log_a x) = a^{\log_a x} = x$

$$f[F(x)] = f(a^x) = \log_a a^x = x \log_a a = x.$$

30. (d) $(f \circ g)(x) = f(g(x)) = f \left(\frac{x}{1-x} \right) = \frac{\frac{x}{1-x}}{\frac{x}{1-x} + 1} = \frac{x}{x+1-x} = x.$

31. (d) $f(f(x)) = \frac{\alpha f(x)}{f(x)+1} = \frac{\alpha \left(\frac{\alpha x}{x+1} \right)}{\left(\frac{\alpha x}{x+1} + 1 \right)} = \frac{\alpha^2 x}{\alpha x + x + 1}$

$$\therefore x = \frac{\alpha^2 x}{(\alpha+1)x+1} \text{ or } x((\alpha+1)x+1-\alpha^2) = 0$$

$$\text{Or } (\alpha+1)x^2 + (1-\alpha^2)x = 0.$$

$$\Rightarrow \alpha+1=0, 1-\alpha^2=0, \therefore \alpha=-1.$$

32. (c) $f[f(x)] = [a - \{f(x)\}^n]^{1/n} = [a - (a-x^n)]^{1/n} = x.$

33. (d) When $x_1 = -1$ and $x_2 = 1$, then

$$f(-1) - f(1) = f\left[\frac{-1-1}{1+1(1)}\right] = f(-1) \Rightarrow f(1) = 0$$

Which is satisfied when $f(x) = \tan^{-1}\left(\frac{1-x}{1+x}\right)$

When $x_1 = x_2 = 0$, then

$$f(0) - f(0) = f\left[\frac{0-0}{1-0}\right] = f(0) \Rightarrow f(0) = 0$$

When $x_1 = -1$ and $x_2 = 0$ then

$$f(-1) - f(0) = f\left(\frac{-1-0}{1-0}\right) = f(-1) \Rightarrow f(0) = 0$$

Which is satisfied when $f(x) = \log\left(\frac{1-x}{1+x}\right)$ and $f(x) = \log\left(\frac{1+x}{1-x}\right)$.

34. (a) $(f-g)(x) = \begin{cases} x, & x \in Q \\ -x, & x \notin Q \end{cases}$

35. (b) $\because e^x = y + \sqrt{1+y^2}$

$$\therefore e^x - y = \sqrt{1+y^2}$$

$$(e^x - y)^2 = (1 + y^2)$$

$$e^{2x} + y^2 - 2ye^x = 1 + y^2 \Rightarrow e^{2x} - 1 = 2ye^x$$

$$\Rightarrow 2y = \frac{e^{2x} - 1}{e^x} \Rightarrow 2y = e^x - e^{-x}$$

$$y = \frac{e^x - e^{-x}}{2}.$$

36. (b) $f(-1) = \frac{-1 - |-1|}{|-1|} = \frac{-1-1}{1} = -2.$

37. (c) $f(x+ay, x-ay) = axy \dots\dots(i)$

Let $x+ay = u$ and $x-ay = v$

Then $x = \frac{u+v}{2}$ and $y = \frac{u-v}{2a}$

Substituting the value of x and y in (i), we obtain

$$f(u,v) = \frac{u^2 - v^2}{4} \Rightarrow f(x,y) = \frac{x^2 - y^2}{4}.$$

38. (c)

39. (d) $f(x) = x - [x]$, here $[x] = 2$

$$\therefore f(x) = y = x - 2 \Rightarrow x = y + 2 = f^{-1}(y) \Rightarrow f^{-1}(x) = x + 2.$$

40. (a) $y = \sin^{-1} \left[\log_3 \left(\frac{x}{3} \right) \right] \Rightarrow -1 \leq \log_3 \left(\frac{x}{3} \right) \leq 1$

$$\Rightarrow \frac{1}{3} \leq \frac{x}{3} \leq 3 \Rightarrow 1 \leq x \leq 9 \Rightarrow x \in [1, 9].$$

41. (a) $f(x) = \sin^{-1} [\log_2 (x/2)]$, Domain of $\sin^{-1} x$ is $x \in [-1, 1]$

$$\Rightarrow -1 \leq \log_2 (x/2) \leq 1 \Rightarrow \frac{1}{2} \leq \frac{x}{2} \leq 2 \Rightarrow 1 \leq x \leq 4$$

$$\therefore x \in [1, 4].$$

42. (c) $f(x) = \log(\sqrt{x-4} + \sqrt{6-x})$

$$\Rightarrow x-4 \geq 0 \text{ and } 6-x \geq 0 \Rightarrow x \geq 4 \text{ and } x \leq 6$$

$$\therefore \text{Domain of } f(x) = [4, 6].$$

43. (b) We have $f(x) = \left[\log_{10} \left(\frac{5x-x^2}{4} \right) \right]^{1/2}$ (i)

From (i), clearly $f(x)$ is defined for those values of x for which $\log_{10} \left[\frac{5x-x^2}{4} \right] \geq 0$

$$\Rightarrow \left(\frac{5x-x^2}{4} \right) \geq 10^0 \Rightarrow \left(\frac{5x-x^2}{4} \right) \geq 1$$

$$\Rightarrow x^2 - 5x + 4 \leq 0 \Rightarrow (x-1)(x-4) \leq 0$$

44. (b) $2 - 2x - x^2 \geq 0 \Rightarrow -1 - \sqrt{3} \leq x \leq -1 + \sqrt{3}.$

45. (d) $1+x \geq 0 \Rightarrow x \geq -1$; $1-x \geq 0 \Rightarrow x \leq 1$, $x \neq 0$

$$\text{Domain is } [-1, 1] - \{0\}.$$

46. (d) $1 - \frac{1}{x} > 0 \Rightarrow x > 1$. Also, $x \neq 0$.

$$\therefore \text{Required interval} = (-\infty, 0) \cup (1, \infty)$$

47. (d) $(x-2)(x+3) \neq 0$

$$\text{Domain is } \{x : x \in R, x \neq 2, x \neq -3\}.$$

48. (a) $|x| - x > 0$

$$|x| > x \text{ but } |x| = x \text{ for } x \text{ positive and } |x| > x \text{ for } x \text{ negative. Domain is } (-\infty, 0).$$

49. (b) Let $y = \sin^{-1} \sqrt{x} \Rightarrow \sqrt{x} = \sin y$

$$\Rightarrow x = \sin^2 y, \therefore 0 \leq x \leq 1$$

50. (b) $f(x) = \frac{\sin^{-1}(3-x)}{\log[|x|-2]}$

Let $g(x) = \sin^{-1}(3-x) \Rightarrow -1 \leq 3-x \leq 1$

Domain of $g(x)$ is $[2, 4]$

And let $h(x) = \log[|x|-2] \Rightarrow |x|-2 > 0$

$$\Rightarrow |x| > 2 \Rightarrow x < -2 \text{ Or } x > 2 \Rightarrow (-\infty, -2) \cup (2, \infty)$$

We know that

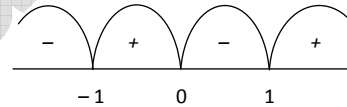
$$(f/g)(x) = \frac{f(x)}{g(x)} \forall x \in D_1 \cap D_2 - \{x \in R : g(x) = 0\}$$

$$\therefore \text{Domain of } f(x) = (2, 4] - \{3\} = (2, 3) \cup (3, 4].$$

51. (d) Here $x+3 > 0$ and $x^2+3x+2 \neq 0$

$$\therefore x > -3 \text{ and } (x+1)(x+2) \neq 0, \text{ i.e. } x \neq -1, -2$$

$$\therefore \text{Domain} = (-3, \infty) - \{-1, -2\}$$



52. (c) $x^2 - 1 > 0$

$$\Rightarrow x^2 > 1, \Rightarrow x < -1 \text{ or } x > 1 \text{ and } 3+x > 0$$

$$\therefore x > -3 \text{ and } x \neq -2$$

$$\therefore D_f = (-3, -2) \cup (-2, -1) \cup (1, \infty).$$

53. (d) $f(x) = \frac{3}{4-x^2} + \log_{10}(x^3-x)$. $4-x^2 \neq 0 \Rightarrow x \neq \pm\sqrt{4}$

$$x^3 - x > 0 \Rightarrow x(x^2 - 1) > 0 \Rightarrow x > 0, x > 1$$

54. (b) $\log\left\{\frac{5x-x^2}{6}\right\} \geq 0 \Rightarrow \frac{5x-x^2}{6} \geq 1$ or $x^2 - 5x + 6 \leq 0$ or $(x-2)(x-3) \leq 0$. Hence $2 \leq x \leq 3$.

55. (c) $\log(x^2 - 6x + 6) \geq 0$

$$\Rightarrow x^2 - 6x + 6 \geq 1 \Rightarrow (x-5)(x-1) \geq 0$$

$$\Rightarrow x \leq 1 \text{ or } x \geq 5. \Rightarrow \text{Domain is } (-\infty, 1] \cup [5, \infty).$$

56. (d) $5x - 3 - 2x^2 \geq 0 \Rightarrow (x-1)\left(x - \frac{3}{2}\right) \geq 0$

$$\therefore \text{Domain is } [1, 3/2].$$

57. (b) $9 - x^2 > 3 \Rightarrow -3 < x < 3$ (i)

$-1 \leq (x - 3) \leq 1 \Rightarrow 2 \leq x \leq 4$ (ii)

From (i) and (ii), $2 \leq x < 3$ i.e., $[2, 3)$

58. (a) $-1 \leq \frac{1}{1+e^x} \leq 1$

$2 < e^x < 3 \Rightarrow 3 < (e^x + 1) < 4 \Rightarrow \frac{1}{4} < \frac{1}{1+e^x} < \frac{1}{3}$

\therefore Domain of $f(x) = \left(\frac{1}{4}, \frac{1}{3}\right)$.

59. (c) (i) $x \leq 2$ (ii) $\sqrt{9-x^2} > 0 \Rightarrow |x| < 3$ or $-3 < x < 3$.

Domain is $(-3, 2]$.

60. (b) $|x| > 2$ and $x \neq 1$

61. (b) $f(x) = \sqrt{\log \frac{1}{|\sin x|}} \Rightarrow \sin x \neq 0 \Rightarrow x \neq n\pi + (-1)^n 0$

$\Rightarrow x \neq n\pi$. Domain of $f(x) = R - \{n\pi, n \in I\}$.

62. (b) The function $\sec^{-1} x$ is defined for all $x \in R - (-1, 1)$ and the function $\frac{1}{\sqrt{x-[x]}}$ is defined for all $x \in R - Z$. So the given function is defined for all $x \in R - \{(-1, 1) \cup (n | n \in Z)\}$.

63. (c) $f(x) = \log |\log x|$, $f(x)$ is defined if $|\log x| > 0$ and $x > 0$ i.e., if $x > 0$ and $x \neq 1$ ($\because |\log x| > 0$ if $x \neq 1$)
 $\Rightarrow x \in (0, 1) \cup (1, \infty)$.

64. (b) $-1 \leq 5x \leq 1 \Rightarrow \frac{-1}{5} \leq x \leq \frac{1}{5}$. Domain is $\left[\frac{-1}{5}, \frac{1}{5}\right]$.

65. (c) $f(x) = \log(\sqrt{x-4} + \sqrt{6-x})$

$\Rightarrow x-4 \geq 0$ and $6-x \geq 0 \Rightarrow x \geq 4$ and $x \leq 6$

\therefore Domain = $[4, 6]$.

66. (c) $-1 \leq 1+3x+2x^2 \leq 1$

$2x^2+3x+1 \geq -1$; $2x^2+3x+2 \geq 0$

$x = \frac{-3 \pm \sqrt{9-16}}{6} = \frac{-3 \pm i\sqrt{7}}{6}$ Which is imaginary, not allowed

$2x^2+3x+1 \leq 1$

$\Rightarrow 2x^2+3x \leq 0 \Rightarrow 2x\left(x+\frac{3}{2}\right) \leq 0$

$$\Rightarrow \frac{-3}{2} \leq x \leq 0 \Rightarrow x \in \left[-\frac{3}{2}, 0\right]$$

$$\text{Domain of function} = \left[-\frac{3}{2}, 0\right].$$

67. (b) $f(x) = \frac{x+2}{|x+2|}$

$$f(x) = \begin{cases} -1, & x < -2 \\ 1, & x > -2 \end{cases}$$

\therefore Range of $f(x)$ is $\{-1, 1\}$.

68. (b) $f(x) = \frac{1}{2 - \sin 3x}$, $\sin 3x \in [-1, 1]$

Hence $f(x)$ lies in $\left[\frac{1}{3}, 1\right]$.

69. (c) $\frac{x^2 + 14x + 9}{x^2 + 2x + 3} = y \Rightarrow x^2 + 14x + 9 = x^2y + 2xy + 3y$

$$\Rightarrow x^2(y-1) + 2x(y-7) + (3y-9) = 0$$

Since x is real, $\therefore 4(y-7)^2 - 4(3y-9)(y-1) > 0$

$$\Rightarrow 4(y^2 + 49 - 14y) - 4(3y^2 + 9 - 12y) > 0$$

$$\Rightarrow 4y^2 + 196 - 56y - 12y^2 - 36 + 48y > 0$$

$$\Rightarrow 8y^2 + 8y - 160 < 0 \Rightarrow y^2 + y - 20 < 0$$

$$\Rightarrow (y+5)(y-4) < 0;$$

70. (b) Given $f(x) = 2^{x(x-1)} \Rightarrow x(x-1) = \log_2 f(x)$

$$\Rightarrow x^2 - x - \log_2 f(x) = 0 \Rightarrow x = \frac{1 \pm \sqrt{1 + 4 \log_2 f(x)}}{2}$$

Only $x = \frac{1 + \sqrt{1 + 4 \log_2 f(x)}}{2}$ lies in the domain

$$\therefore f^{-1}(x) = \frac{1}{2}[1 + \sqrt{1 + 4 \log_2 x}].$$

71. (b) $x^2 - 6x + 7 = (x-3)^2 - 2$

Minimum value is -2 and maximum ∞ .

Hence range of function is $[-2, \infty]$.

72. (c) $f(x) = 1 + \frac{1}{\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}} \Rightarrow \text{Range} = (1, 7/3]$.

73. (c) $y = f(x) = \cos^2 x + \sin^4 x$
 $\Rightarrow y = f(x) = \cos^2 x + \sin^2 x(1 - \cos^2 x)$
 $\Rightarrow y = \cos^2 x + \sin^2 x - \sin^2 x \cos^2 x$
 $\Rightarrow y = 1 - \sin^2 x \cos^2 x \Rightarrow y = 1 - \frac{1}{4} \sin^2 2x$
 $\therefore \frac{3}{4} \leq f(x) \leq 1, (\because 0 \leq \sin^2 2x \leq 1)$
 $\Rightarrow f(R) \in [3/4, 1]$.

74. (b) R^+ {as y is always positive $\forall x \in R$ }.

75. (b) $y = f(x) = 9 - 7 \sin x$. Range = $[2, 16]$.

76. (a) $f(x) = \sec\left(\frac{\pi}{4} \cos^2 x\right)$

$0 \leq \cos^2 x \leq 1$ at $\cos x = 0$, $f(x) = 1$ and at $\cos x = 1$, $f(x) = \sqrt{2}$; $\therefore 1 \leq x \leq \sqrt{2} \Rightarrow x \in [1, \sqrt{2}]$.

77. (d) $f(x) = a \cos(bx + c) + d$ (i)

For minimum $\cos(bx + c) = -1$

$\Rightarrow f(x) = -a + d = (d - a)$

For maximum $\cos(bx + c) = 1$

$\Rightarrow f(x) = a + d = (d + a)$

$\therefore \text{Range of } f(x) = [d - a, d + a]$.

78. (c) Let $y = \frac{x^2}{x^2 + 1}$

$\Rightarrow (y - 1)x^2 + 0x + y = 1, y \neq 1$ For real values of x ,

We have $D \geq 0 \Rightarrow -4y(y - 1) \geq 0 \Rightarrow y(y - 1) \leq 0 \Rightarrow y \in [0, 1)$

$0 \leq \frac{x^2}{x^2 + 1} < 1$.

79. (e) $-1 \leq \log_3 x \leq 1$; $3^{-1} \leq x \leq 3 \Rightarrow \frac{1}{3} \leq x \leq 3$

$\therefore \text{Domain of function} = \left[\frac{1}{3}, 3\right]$

80. (d) $-1 \leq \cos \theta \leq 1 \Rightarrow \cos^2 \theta \leq 1$

And $\sec^2 \theta \geq 1$ for $\theta > \frac{\pi}{3}$, $\sec \theta \geq 2$

$\Rightarrow \sec^2 \theta \geq 4$. \therefore Required interval $= [2, \infty)$

81. (a) The domain of $\log_e \{x - [x]\}$ is \mathbb{R} , because $[x] \leq 0$

82. (d) Let $y = x^2 + 1 \Rightarrow x = \pm\sqrt{y-1}$

$\Rightarrow f^{-1}(y) = \pm\sqrt{y-1} \Rightarrow f^{-1}(x) = \pm\sqrt{x-1}$

$\Rightarrow f^{-1}(17) = \pm\sqrt{17-1} = \pm 4$ and $f^{-1}(-3) = \pm\sqrt{-3-1} = \pm\sqrt{-4}$, which is not possible

83. (c) $g \circ f(x) = g\{f(x)\} = [|\cos x|]$.

84. (b) Here $g(x) = 1 + n - n = 1, x = n \in \mathbb{Z}$

$1 + n + k - n = 1 + k$, $x = n + k$ (where $n \in \mathbb{Z}, 0 < k < 1$)

Now $f(g(x)) = \begin{cases} -1, & g(x) < 0 \\ 0, & g(x) = 0 \\ 1, & g(x) > 0 \end{cases}$

Clearly, $g(x) > 0$ for all x . So, $f(g(x)) = 1$ for all x .

85. (c) $f(a - (x - a)) = f(a)f(x - a) - f(0)f(x) \dots (i)$

Put $x = 0, y = 0$; $f(0) = (f(0))^2 - [f(a)]^2 \Rightarrow f(a) = 0$

$[\because f(0) = 1]$. From (i), $f(2a - x) = -f(x)$.

86. (b) $g(x) = 1 + \{x\}$; $f\{g(x)\} = f\{1 + \{x\}\} = f(k) = 1$

Where, $k = 1 + \{x\}, 1 \leq k < 2$

87. (d)

88. (d) $f(-1) = f(1) = 1 \Rightarrow f$ is not one

Range of the function is $\mathbb{R}^+ \cup \{0\}$ which is not equal to co domain. Therefore f is not onto.
so f is neither one-one nor onto.

89. (a) $|x|$ is not one-one; x^2 is not one-one;

$x^2 + 1$ is not one-one. But $2x - 5$ is one-one because $f(x) = f(y) \Rightarrow 2x - 5 = 2y - 5 \Rightarrow x = y$

Now $f(x) = 2x - 5$ is onto. $\therefore f(x) = 2x - 5$ is bijective.

90. (a)

91. (b) $f(x) = \sin\left(\log(x + \sqrt{1 + x^2})\right)$

$$\Rightarrow f(-x) + f(x) = 0$$

$\therefore f(x)$ is odd function.

92. (b) $(g \circ f)(x) = |\sin x|$ and $f(x) = \sin^2 x$

$$\Rightarrow g(\sin^2 x) = |\sin x|; \therefore g(x) = \sqrt{x}.$$

93. (b) $f(x) = 2x^6 + 3x^4 + 4x^2$

$$f(-x) = 2(-x)^6 + 3(-x)^4 + 4(-x)^2 = f(x)$$

$\Rightarrow f(x)$ is an even function and derivative of an even function is always odd.

94. (b) $f(x) = \log(x + \sqrt{x^2 + 1})$

and $f(-x) = -\log(x + \sqrt{x^2 + 1}) = -f(x)$

$f(x)$ is odd function.

95. (c)

96. (c) $f(1) = 0, f(2) = -1, f(3) = 1, f(4) = -2, f(5) = 2$

And $f(6) = -3$ so on.

No two elements of domain have same image. And range of the function is set of all integers. Hence f is one-one and onto function.

97. (a) Let $x, y \in N$ such that $f(x) = f(y)$

Then $f(x) = f(y) \Rightarrow x^2 + x + 1 = y^2 + y + 1$

$$\Rightarrow (x - y)(x + y + 1) = 0 \Rightarrow x = y \text{ Or } x = (-y - 1) \notin N$$

$\therefore f$ is one-one.

Again, since for each $y \in N$, there exist $x \in N$

$\therefore f$ is onto.

98. (b) $f(x) = (x - 1)(x - 2)(x - 3)$

$$\Rightarrow f(1) = f(2) = f(3) = 0 \Rightarrow f(x) \text{ is not one-one.}$$

For each $y \in R$, there exists $x \in R$ such that $f(x) = y$. Therefore f is onto. Hence $f: R \rightarrow R$ is onto but not one-one.

99. (b) $f'(x) = \frac{1}{(1+x)^2} > 0, \forall x \in [0, \infty)$ and range $\in [0, 1)$

\Rightarrow Function is one-one but not onto

100. (a) $f'(x) = 2 + \cos x > 0$. So, $f(x)$ is strictly monotonic increasing so, $f(x)$ is one-to-one and onto.

101. (d) $f(x) = x + \sqrt{x^2} = x + |x|$

Clearly f is not one-one as $f(-1) = f(-2) = 0 \Rightarrow f$ is not one-one

$f(x) \geq 0, \forall x \in R, \Rightarrow \text{Range of } f = (0, \infty) \subset R. \Rightarrow f$ is not onto.

102. (b) Let $\frac{x^2 + 34x - 71}{x^2 + 2x - 7} = y$

$$\Rightarrow x^2(1-y) + 2(17-y)x + (7y-71) = 0$$

$$\Rightarrow \Delta \geq 0$$

$$\Rightarrow y^2 - 14y + 45 \geq 0 \Rightarrow y \geq 9, y \leq 5.$$

103. (a) $f \circ f(x) = f(f(x)) = f\left(\frac{1-x}{1+x}\right) = \frac{1 - \frac{1-x}{1+x}}{1 + \frac{1-x}{1+x}} = x, \forall x$

$\therefore f \circ f = I \Rightarrow f$ is the inverse of itself.

104. (b) f is one-one and onto, so f^{-1} exists and is $f^{-1}(x) = \frac{x+5}{3}$.

105. (a) $-\sqrt{1+(-\sqrt{3})^2} \leq (\sin x - \sqrt{3} \cos x) \leq \sqrt{1+(-\sqrt{3})^2}$

$$-2 \leq (\sin x - \sqrt{3} \cos x) \leq 2$$

$$-2+1 \leq (\sin x - \sqrt{3} \cos x + 1) \leq 2+1$$

$$-1 \leq (\sin x - \sqrt{3} \cos x + 1) \leq 3 \text{ i.e., range } = [-1, 3]$$

\therefore For f to be onto $S = [-1, 3]$

106. (b) For any $x, y \in R$, we have

$$f(x) = f(y) \Rightarrow \frac{x-m}{x-n} = \frac{y-m}{y-n} \Rightarrow x = y$$

$\therefore f$ is one-one.

Let $\alpha \in R$ such that $f(x) = \alpha \Rightarrow \frac{x-m}{x-n} = \alpha \Rightarrow x = \frac{m-n\alpha}{1-\alpha}$

Clearly $x \notin R$ for $\alpha = 1$. So, f is not onto.

107. (c) Let $f(x) = f(y) \Rightarrow \frac{x^2-4}{x^2+4} = \frac{y^2-4}{y^2+4}$

$$\Rightarrow \frac{x^2-4}{x^2+4} - 1 = \frac{y^2-4}{y^2+4} - 1 \Rightarrow x^2+4 = y^2+4$$

$$\Rightarrow x = \pm y, \therefore f(x) \text{ is many-one.}$$

Now for each $y \in (-1,1)$, there does not exist $x \in X$ such that $f(x) = y$. Hence f is into.

108. (c) $f(x_1) = f(x_2) \Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = x_2$, [if $X = \mathbb{R}^+$]

$\Rightarrow f$ is one-one. Since $R_f = \mathbb{R}^+ \subseteq \mathbb{R} = Y$; $\therefore f$ is not onto.

109. (d) $\because f(0) = f(-1) = 0$ hence $f(x)$ is many one. But there is no pre-image of -1 . Hence $f(x)$ is into function. So function is many-one into.