COORDINATE GEOMETRY

CHANGE OF AXES

EXERCISE

1. The point to which the origin should be shifted in order to eliminate x and y terms in the equation $4x^2+9y^2-8x+36y+4=0$ is

   1) (1,3) 2) (–4,3) 3) (–1,2) 4) (1,–2)

2. In order to eliminate the first degree terms from the equation $2x^2+4xy+5y^2-4x-22y+7=0$, the point to which origin is to be shifted is

   1) (1,–3) 2) (2,3) 3) (–2,3) 4) (1,3)

3. The point to which the origin should be shifted in order to eliminate x and y terms in the equation $14x^2+4xy+11y^2-36x+48y+41=0$ is

   1) (1,3) 2) (–4,3) 3) (–1,2) 4) (1,–2)

4. The point to which the axes are to translated to eliminate y term and constant term in the equation $y^2+8x+4y-2=0$ is

   1) (3,–2) 2) (3,–2/3) 3) (3/4,–2) 4) (2/3,–4)

5. If the axes are tranlated to the circumcentre of the triangle formed by (9,3), (–1,7), (–1,3) then the centroid of the triangle in the new system is

   1) (5,5/3) 2) (4,3)
   3) (–5/3, –2/3) 4) (0,0)

6. The transformed equation of $x^2+2y^2+2x-4y+2=0$ when the axes are translated to the point (–1,1) is.

   1) $X^2+2Y^2=1$ 2) $X^2+3Y^2=1$
   3) $X^2-Y^2+3=0$ 4) $4X^2-9Y^2=36$

7. If the first degree terms of $x^2+4xy+y^2-2x+2y-6=0$ are eliminated by translation of axes then the transformed equation is .

   1) $X^2+4XY+Y^2=8$ 2) $X^2+4XY+Y^2=6$
   3) $X^2+4XY+Y^2=4$ 4) $X^2+4XY+Y^2=2$

8. If the transformed equation of a curve is $3X^2+XY-Y^2-7X+Y+7=0$ when the axes are translated to the point (1,2), then the original equation of the curve is

   1) $3x^2+xy-y^2+15x+4y+13=0$ 2) $3x^2+xy-y^2-15x+4y+13=0$
   3) $3x^2+xy+y^2-15x+4y+13=0$ 4) $3x^2+xy-y^2+15x-4y+13=0$

9. The origin is shifted to (1,2). The equation $y^2-8x-4y+12=0$ changes to $y^2=4ax$ then a =

   1) 1 2) 2 3) –2 4) –1

10. By translating the axes the equation $xy - x + 2y = 6$ has changed to $xy = c$, then $c =$

1) 4 2) 5 3) 6 4) 7

11. If the axes are rotated through an angle 45°, the coordinates of $\left(2\sqrt{2}, -3\sqrt{2}\right)$ in the new system are

1) $\left(3\sqrt{3}, -5\right)$ 2) $(-1, -5)$

3) $\left(5\sqrt{3}, -7\right)$ 4) $\left(7 - \sqrt{3}\right)$

12. If the coordinates of a point P are transformed to $\left(4, -6\sqrt{3}\right)$ when the axes are rotated through an angle 30°, then P =

1) $\left(3\sqrt{3}, -5\right)$ 2) $(-1, -5)$

3) $\left(5\sqrt{3}, -7\right)$ 4) $\left(7 - \sqrt{3}\right)$

13. If the axes are rotated through an angle 45° in the positive direction without changing the origin, then the coordinates of the point $\left(\sqrt{2}, 4\right)$ in the old system are

1) $\left(1 - 2\sqrt{2}, 1 + 2\sqrt{2}\right)$ 2) $\left(1 + 2\sqrt{2}, 1 - 2\sqrt{2}\right)$

3) $\left(2\sqrt{2}, \sqrt{2}\right)$ 4) $\left(\sqrt{2}, 2\right)$

14. The angle of rotation of axes in order to eliminate xy term in the equation $x^2 + 2\sqrt{3}xy - y^2 = 2a^2$ is

1) $\pi/6$ 2) $\pi/4$ 3) $\pi/3$ 4) $\pi/2$

15. The angle of rotation of axes to remove xy term in the equation $9x^2 - 2\sqrt{3}xy + 3y^2 = 0$ is

1) $\pi/6$ 2) $\pi/4$ 3) $\pi/3$ 4) $5\pi/12$

16. The angle of rotation of axes to remove xy term in the equation $x^2 + 4xy + y^2 - 2x + 2y - 6 = 0$ is

1) $\pi/12$ 2) $\pi/6$ 3) $\pi/3$ 4) $\pi/4$

17. The transformed equation of $x^2 + 6xy + 8y^2 = 10$ when the axes are rotated through an angle $\pi/4$ is

1) $15x^2 - 14xy + 3y^2 = 20$ 2) $15x^2 + 14xy - 3y^2 = 20$

3) $15x^2 + 14xy + 3y^2 = 20$ 4) $15x^2 - 14xy - 3y^2 = 20$

18. If the axes are rotated through an angle 30° about the origin then the transformed equation of $x^2 + 2\sqrt{3}xy - y^2 = 2a^2$ is.

1) $X^2 + Y^2 = a^2$ 2) $X^2 - Y^2 = a^2$

3) $X^2 + Y^2 = 2a^2$ 4) $X^2 - Y^2 = 2a^2$

19. The transformed equation of $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ when the axes are rotated through an angle 90° is

1) $bX^2 - 2hXY + aY^2 + 2fX - 2gY + c = 0$

2) $bX^2 + 2hXY + aY^2 + 2fX + 2gY + c = 0$

3) $bX^2 - 2hXY + aY^2 - 2fX + 2gY + c = 0$

4) $bX^2 + 2hXY + aY^2 - 2fX - 2gY + c = 0$

20. The transformed equation of $x^2 + y^2 - 4x + 6y - 12 = 0$ when the axes are rotated through an angle 180° is

1) $X^2 + Y^2 + 4X - 6Y + 12 = 0$

2) $X^2 + Y^2 + 4X - 6Y - 12 = 0$

3) $X^2 + Y^2 - 4X - 6Y - 12 = 0$

4) $X^2 + Y^2 - 4X - 6Y + 12 = 0$

21. If the transformed equation of a curve is $17X^2 - 16XY + 17Y^2 = 225$ when the axes are rotated through an angle 45°, then the original equation of the curve is

1) $25x^2 + 9y^2 = 225$ 2) $9x^2 + 25y^2 = 225$

3) $25x^2 - 9y^2 = 225$ 4) $9x^2 - 25y^2 = 225$

22. If the transformed equation of a curve is $X^2 - 2XY \tan 2\alpha - Y^2 = a^2$ when the axes are rotated through an angle α, then the original equation of the curve is

1) $x^2 + y^2 = a^2 \cos 2\alpha$ 2) $x^2 - y^2 = a^2 \cos 2\alpha$

3) $x^2 + a^2 = y^2 \cos 2\alpha$ 4) $x^2 - a^2 = y^2 \cos 2\alpha$

23. The angle of rotation of the axes so that the equation $\sqrt{3}x - y + 5 = 0$ may be reduced to the form Y = constant is

1) $\pi/6$ 2) $\pi/4$ 3) $\pi/3$ 4) $\pi/2$

24. A line L has intercepts a and b on the coordinate axes. Keeping the origin fixed, the axes are rotated through a fixed angle. Then the same line has intercepts p and q on the new axes. Then

1) $a^2 + p^2 = b^2 + q^2$ 2) $a^2 + b^2 = p^2 + q^2$ 3) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2} + \frac{1}{q^2}$ 4) $\frac{1}{a^2} + \frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{q^2}$

25. The line joining two points A(2,0), B(3,1) is rotated about A anticlock wise direction through an angle 15°. If B goes to C then C =

1) $\left(2+\frac{1}{\sqrt{2}},\sqrt{\frac{3}{2}}\right)$ 2) $\left(2-\frac{1}{\sqrt{2}},\sqrt{\frac{3}{2}}\right)$

3) $\left(\sqrt{2}-1,\frac{\sqrt{3}}{2}\right)$ 4) $\left(\sqrt{2}-\frac{1}{2},\sqrt{\frac{2}{3}}\right)$

26. The point (4,1) undergoes the following three transformations successively i) reflection about the line $y = x$ ii) translation through a distance 2 unit along the positive direction of x axis. The final position of the point is

1) (3,4) 2) (4,3) 3) (–1,4) 4) none

27. The point (4,1) undergoes the following three transformations successively (i) Reflection about the line $y = x$ (ii) Translation through a distance 2 unit along the positive direction of x-axis. (iii) Rotation through an angle $\pi/4$ about the origin in the clockwise direction. The final position of the point is given by the coordinates

1) $\left(1/\sqrt{2},7/\sqrt{2}\right)$ 2) $\left(-2,7\sqrt{2}\right)$

3) $\left(-1\sqrt{2},7/\sqrt{2}\right)$ 4) $\left(\sqrt{2},7\sqrt{2}\right)$

28. The point P(1,1) is translated parallel to $2x = y$ in the first quadrant through a unit distance. The coordinates of the new position of P are

1) $\left(1\pm\frac{2}{\sqrt{5}},1\pm\frac{1}{\sqrt{5}}\right)$ 2) $\left(1\pm\frac{1}{\sqrt{5}},1\pm\frac{2}{\sqrt{5}}\right)$

3) $\left(\frac{1}{\sqrt{5}},\frac{2}{\sqrt{5}}\right)$ 4) $\left(\frac{2}{\sqrt{5}},\frac{1}{\sqrt{5}}\right)$

CHANGES OF AXES – SOLUTIONS

1. Ans.4

Sol: Here a = 4, b = 9, g = -4, f = 18. Required point is $\left(\frac{-g}{a}.\frac{-f}{b}\right)=\left(\frac{-(-4)}{4},\frac{-18}{9}\right)=(1,-2)$

2. Ans.3

Sol: Required point $=\left(\frac{hf-bg}{ab-h^2},\frac{gh-af}{ab-h^2}\right)=\left(\frac{2(-11)-5(-2)}{10-4}\frac{(-2)(2)-2(-11)}{10-4}\right)=\left(\frac{-12}{6}\frac{18}{6}\right)=(-2,3)$

3. Ans. 4

Sol: Here a = 14, h = -2, b = 11, g = -18, f = 24 Required point is

$$\left(\frac{hf-by}{ab-h^2},\frac{gh-af}{ab-h^2}\right)=\left(\frac{-2(24)-11(-18)}{14(11)-4},\frac{(-18)(-2)-14(24)}{14(11)-4}\right)$$

$$=\left(\frac{-48+198}{150},\frac{36-336}{150}\right)=(1-2)$$

4. Ans. 3

Sol: Given equation is $y^2+8x+4y-2=0 \Rightarrow (y+2)^2+8(x-3/4)=0$ ∴ Point of translation = (3/4,-2)

5. Ans.3

Sol: Given points from a right angled triangle right angled at (-1,3)

Circumcentre = Midpoint of (9,3), (-1,7) = (4,5)

Centroid of the triangle = (7/3, 13/3)

Centroid in the new system $=\left(\frac{7}{3}-4,\frac{13}{3}-5\right)=\left(-\frac{5}{3},-\frac{2}{3}\right)$

6. Ans.1

Sol: x = X-1, y = Y+1

The transformed equation is

$(X-1)^2+2(Y+1)^2+2(X-1)-4(Y+1)+2=0$

$\Rightarrow X^2+2Y^2=0$

7. Ans.3

Sol: Point of translation $= \left(\frac{2(1)-1(-1)}{1-4}, \frac{(-1)(2)-1(1)}{1-4} \right) = (-1,1)$

Transformed equation is $X^2 + 4XY + Y^2 - 1(-1) + 1(1) - 6 = 0 \Rightarrow X^2 + 4XY + Y^2 = 4$

8. Ans.2

Sol: $X = x - 1, Y = y - 2$

The original equation is

$3(x-1)^2 + (x-1)^2 + (x-1)(y-2) - (y-2)^2 - 7(x-1) + (y-2) + 7 = 0 \Rightarrow 3x^2 + xy - y^2 - 15x + 4y + 13 = 0$

9. Ans.2

Sol: x=X+1, y=Y+2

The transformed equation of $y^2 - 8x - 4y + 12 = 0$ is

$(Y+2)^2 - 8(X+1) - 4(Y+2) + 12 = 0 \Rightarrow Y^2 - 8X = 0 \Rightarrow Y^2 = 8X \quad \therefore a = 2$

10. Ans.1

Sol: x=X+h, y=Y+k

The transformed equation of $xy - x + 2 = 6$ is

$(X+h)(Y+k) - (X+h) + 2(Y+k) - 6 = 0$

$\Rightarrow XY + (k-1)X + (h+2)Y + (hk - h + 2k - 6) = 0$

comparing this equation with xy = c we get k – 1 = 0, h +2 = 0 & c = -(hk-h+2k-6)

$\Rightarrow$ k = 1, h = -2 & c = -[(-2)(1)+2+2-6]=4

11. Ans.2

Sol: $(x, y) = (2\sqrt{2}, -3\sqrt{2})\theta = 45°$

$X = 2\sqrt{2}\left(\frac{1}{\sqrt{2}}\right) - 3\sqrt{2}\left(\frac{1}{\sqrt{2}}\right) = -1, Y = -2\sqrt{2}\left(\frac{1}{\sqrt{2}}\right) - 3\sqrt{2}\left(\frac{1}{\sqrt{2}}\right) = -5$

$\therefore (X, Y) = (-1, -5)$

12. Ans.3

Sol: $(X, Y) = \left(4, -6\sqrt{3}\right), \theta = 30°$

$$y = X\sin\theta + Y\cos\theta = 4\left(\frac{1}{2}\right) - 6\sqrt{3}\left(\frac{\sqrt{3}}{2}\right) = 2 - 9 = -7$$

$$y = X\sin\theta + Y\cos\theta = 4\left(\frac{1}{2}\right) - 6\sqrt{3}\left(\frac{\sqrt{3}}{2}\right) = 2 - 9 = -7$$

$$\therefore P\left(5\sqrt{3}, -7\right)$$

13. Ans.1

Sol:

$$x = \sqrt{2}\cos 45° - 4\sin 45° = \sqrt{2}\left(1/\sqrt{2}\right) - 4\left(1/\sqrt{2}\right)$$

$$y = \sqrt{2}\sin 45° + 4\cos 45° = \sqrt{2}\left(1/\sqrt{2}\right) + 4\left(1/\sqrt{2}\right) = 1 + 2\sqrt{2}$$

$$\therefore \text{ Required point} = \left(1 - 2\sqrt{2}, 1 + 2\sqrt{2}\right)$$

14. Ans.1

Sol: Comparing the given equation with $ax^2 + 2hxy + by^2 = c$ we get $a = 1$, $h = \sqrt{3}, b = -1$

Angle of rotation is

$$\frac{1}{2}\text{Tan}^{-1}\left(\frac{2h}{a-b}\right) = \frac{1}{2}\text{Tan}^{-1}\left(\frac{2\sqrt{3}}{1+1}\right) = \frac{1}{2}\text{Tan}^{-1}\left(\sqrt{3}\right)$$

$$\frac{1}{2} \times \frac{\pi}{3} = \frac{\pi}{6}$$

15. Ans.4

Sol: Here $a = 9$, $b = 3$, $h = -\sqrt{3}$

Angle of rotation, $\theta = \frac{1}{2}\text{Tan}^{-1}\left(\frac{2h}{a-b}\right) = \frac{1}{2}\text{Tan}^{-1}\left(\frac{-2\sqrt{3}}{9-3}\right) = \frac{1}{2}\text{Tan}^{-1}\left(\frac{-1}{\sqrt{3}}\right) = \frac{1}{2}\left(-\frac{\pi}{6}\right) = \frac{-\pi}{12}$

$$\therefore \theta = \frac{n\pi}{2} - \frac{\pi}{12} = \frac{5\pi}{12} \quad \text{when } n = 1$$

16. Ans.4

Sol: Coefficient of $x^2 - 1 =$ coefficient of y^2.

$\therefore$ Angle of rotation is $\pi/4$

17. Ans.3

Sol: (X,Y) be the new coordinates of (x,y) when the axes are rotated through an angle $\pi/4$

Then $x = \dfrac{X-Y}{\sqrt{2}}, y = \dfrac{X+Y}{\sqrt{2}}$

The transformed equation is $\left(\dfrac{X-Y}{\sqrt{2}}\right)^2 + 6\left(\dfrac{X-Y}{\sqrt{2}}\right)\left(\dfrac{X+Y}{\sqrt{2}}\right) + 8\left(\dfrac{X+Y}{\sqrt{2}}\right)^2 = 10$

$\Rightarrow X^2 + Y^2 - 2XY + 6\left(X^2 - Y^2\right) + 8\left(X^2 + Y^2 + 2XY\right) = 20$

$\Rightarrow 15X^2 + 14XY + 3Y^2 = 20$

18. Ans.2

Sol: $x = X\cos 30° - Y\sin 30° = \dfrac{\sqrt{3}X - Y}{2}$

$y = X\sin 30° + Y\cos 30° = \dfrac{X + \sqrt{3}Y}{2}$

Given equation is $x^2 + 2\sqrt{3}xy - y^2 = 2a^2$

$\left(\dfrac{\sqrt{3}X-Y}{2}\right)^2 + 2\sqrt{3}\left(\dfrac{\sqrt{3}X-Y}{2}\right)\left(\dfrac{X+\sqrt{3}Y}{2}\right) - \left(\dfrac{X+\sqrt{3}Y}{2}\right)^2 = 2a^2$

$3X^2 + Y^2 - 2\sqrt{3}XY + 2\sqrt{3}\left(\sqrt{3}X^2 + 3XY - XY - \sqrt{3}Y^2\right) - X^2 - 3Y^2 - 2\sqrt{3}XY = 8a^2$

$2X^2 - 2Y^2 - 4\sqrt{3}XY + 6X^2 + 6\sqrt{3}XY - 2\sqrt{3}XY - 6y^2 = 8a^2$

$2X^2 - 2Y^2 - 4\sqrt{3}XY + 6X^2 + 6\sqrt{3}XY - 2\sqrt{3}XY - 6Y^2 = 8a^2$

$8X^2 - 8Y^2 = 8a^2 \Rightarrow X^2 - Y^2 = a^2$

19. Ans.1

Sol: $x = X\cos 90° - Y\sin 90° = -Y, y = X\sin 90° + Y\cos 90° = X$ $\therefore$ The transformed equation is

$a(-Y)^2 + 2h(-Y)(X) + bx^2 - 2g(-Y) + 2f(X) + C = 0$

$\Rightarrow bX^2 - 2hXY + aY^2 + 2fX - 2gY + c = 0$

20. $x = X\cos 180° - Y\sin 180° = -X, y = X\sin 180° + Y\cos 180° = -Y$

$\therefore$ The transformed equation is $(-X)^2+(-Y)^2-4(-X)+6(-Y)-12=0$

$\Rightarrow X^2+Y^2+4X-6Y-12=0$

21. Ans. 1

Sol: $Y=-x\sin 45+y\cos 45°=\dfrac{y-x}{\sqrt{2}}$

$X=x\cos 45°+y\sin 45°=\dfrac{x+y}{\sqrt{2}},$

The original equation of the curve is

$$17\left(\frac{x+y}{\sqrt{2}}\right)^2-16\left(\frac{x+y}{\sqrt{2}}\right)\left(\frac{y-x}{\sqrt{2}}\right)+17\left(\frac{y-x}{\sqrt{2}}\right)^2=225$$

$$\Rightarrow \frac{17}{2}\left[(x+y)^2+(y-x)^2\right]-\frac{16}{2}\left(y^2-x^2\right)=225 \Rightarrow 225 \Rightarrow 25x^2+9y^2=225$$

22. Ans.2

Sol: $X=x\cos\alpha+y\sin\alpha, Y=-x\sin\alpha+y\cos\alpha$

The original equation of the curve is

$$(x\cos\alpha+y\sin\alpha)^2-2(x\cos\alpha+y\sin\alpha)(-x\sin\alpha+y\cos\alpha\tan 2\alpha)-(-x\sin\alpha+y\cos\alpha)^2=a^2$$

$$\Rightarrow x^2\cos^2\alpha+y^2\sin^2\alpha+2xy\cos\alpha\sin\alpha+2x^2\cos\alpha\sin\alpha\tan 2\alpha-2xy\cos^2\alpha\tan 2\alpha$$

$$+2xy\sin^2\alpha\tan 2\alpha-2y^2\cos\alpha\sin\alpha\tan 2\alpha-x^2\sin^2\alpha-y^2\cos^2\alpha+2xy\sin\alpha\cos\alpha=a^2$$

$$\Rightarrow x^2\left(\cos^2\alpha+\sin 2\alpha\tan 2\alpha-\sin^2\alpha\right)-y^2\left(-\sin^2\alpha+\sin 2\alpha\tan 2\alpha+\cos^2\alpha\right)+xy$$

$$\left[\sin 2\alpha-2\cos^2\alpha\tan 2\alpha+2\sin^2\alpha\tan 2\alpha+\sin 2\alpha\right]=a^2$$

$$\Rightarrow \left(x^2-y^2\right)\left(\cos 2\alpha+\frac{\sin^2 2\alpha}{\cos 2\alpha}\right)=a^2$$

$$\Rightarrow x^2-y^2=a^2\cos 2\alpha$$

23. Ans.3

Sol: Angle of rotation $= \mathrm{Tan}^{-1}\left(-\frac{a}{b}\right) = \mathrm{Tan}^{-1}\left(\frac{-\sqrt{3}}{-1}\right) = \frac{\pi}{3}$

24. Ans.3

Sol: Equation of L with a,b as intercepts is $\frac{x}{a}+\frac{y}{b}=1 \to (1)$

Transformed equation to L with p, q as intercepts is $\frac{X}{p}+\frac{Y}{q}=1 \to (2)$

The distance from origin to (1) and (2) is the same

$$\Rightarrow \frac{|-1|}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}} = \frac{|-1|}{\sqrt{\frac{1}{p^2}+\frac{1}{q^2}}} \Rightarrow \frac{1}{a^2}+\frac{1}{b^2} = \frac{1}{p^2}+\frac{1}{q^2}$$

25. Ans.1

Sol: By changing the origin to A(2,0) the coordinates of B become (1,1).

Now $\theta = -15°$

$$X = \cos 15° - \sin 15° = \frac{1}{\sqrt{2}},\ Y = \sin 15° + \cos 15° = \frac{\sqrt{3}}{\sqrt{2}}$$

Changing the origin to the original place, then the coordinates of C are $\left(2+\frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{\sqrt{2}}\right)$

26. Ans. 1

Sol: Reflection of (4,1) with reference to $y = x$ is (1,4) The point (1,4) is translated through distance 2 along a horizontal line in the direction of x – axis.

$\therefore$ The new coordinates of (1,4) are (3,4).

27. Ans.3

Sol: Reflection of (4,1) about the line $y = x$ is (1,4). Translation through a distance 2 along positive direction of x-axis is (3,4). Rotation through an angle $\pi/4$ in the clockwise direction is

$$\left(3\cos\left(\frac{-\pi}{4}\right)+4\sin\left(\frac{-\pi}{4}\right) - 3\sin\left(\frac{-\pi}{4}\right)+4\cos\left(\frac{-\pi}{4}\right)\right) = \left(\frac{-1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$$

28. Ans.2

Sol: If θ is the inclination of the given line then $\tan\theta = 2 \Rightarrow \cos\theta = \frac{1}{\sqrt{5}}, \sin\theta = \frac{2}{\sqrt{5}}$