INDEFINITE INTEGRATION

OBJECTIVE PROBLEMS

1. $\int \frac{\sin x}{\sin(x-\alpha)} dx =$

(a) $x \cos\alpha - \sin\alpha \log \sin(x-\alpha) + c$
(b) $x \cos\alpha + \sin\alpha \log \sin(x-\alpha) + c$
(c) $x \sin\alpha - \sin\alpha \log \sin(x-\alpha) + c$
(d) None of these

2. $\int \left(1 + x + \frac{x^2}{2\,!} + \frac{x^3}{3\,!} + \ldots\ldots\ldots\right) dx =$

(a) $-e^x + c$
(b) $e^x + c$
(c) $e^{-x} + c$
(d) $-e^{-x} + c$

3. $\int \frac{x-1}{(x+1)^2} dx =$

(a) $\log(x+1) + \frac{2}{x+1} + c$
(b) $\log(x+1) - \frac{2}{x+1} + c$
(c) $\frac{2}{x+1} - \log(x+1) + c$
(d) None of these

4. $\int \frac{dx}{\sin x + \cos x} =$

(a) $\log\tan\left(\frac{\pi}{8} + \frac{x}{2}\right) + c$
(b) $\log\tan\left(\frac{\pi}{8} - \frac{x}{2}\right) + c$
(c) $\frac{1}{\sqrt{2}}\log\tan\left(\frac{\pi}{8} + \frac{x}{2}\right) + c$
(d) None of these

5. If $\int (\sin 2x - \cos 2x)\, dx = \frac{1}{\sqrt{2}}\sin(2x - a) + b$, then

(a) $a = \frac{\pi}{4}, b = 0$

(b) $a = -\frac{\pi}{4}, b = 0$

(c) $a = \frac{5\pi}{4}, b =$ any constant

(d) $a = -\frac{5\pi}{4}, b =$ any constant

6. $\int \frac{\cos x - 1}{\cos x + 1} dx =$

(a) $2\tan\frac{x}{2} - x + c$
(b) $\frac{1}{2}\tan\frac{x}{2} - x + c$
(c) $x - \frac{1}{2}\tan\frac{x}{2} + c$
(d) $x - 2\tan\frac{x}{2} + c$

7. $\int (\sin^{-1} x + \cos^{-1} x)\, dx =$

(a) $\frac{1}{2}\pi x + c$ (b) $x(\sin^{-1} x - \cos^{-1} x) + c$

(c) $x(\cos^{-1} x - \sin^{-1} x) + c$ (d) $\frac{\pi}{2} + x + c$

8. $\int \frac{dx}{\sin x + \sqrt{3}\cos x} =$

(a) $\log \tan\left(\frac{x}{2} + \frac{\pi}{2}\right) + c$ (b) $\frac{1}{2}\log \tan\left(\frac{x}{2} + \frac{\pi}{6}\right) + c$

(c) $\log \cot\left(\frac{x}{2} + \frac{\pi}{6}\right) + c$ (d) $\frac{1}{2}\log \cot\left(\frac{x}{2} + \frac{\pi}{6}\right) + c$

9. $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x}\, dx =$

(a) $\tan x + \cot x + c$ (b) $\tan x - \cot x + c$

(c) $\operatorname{cosec} x - \cot x + c$ (d) $\sec x - \operatorname{cosec} x + c$

10. $\int \frac{dx}{\sqrt{1+x} + \sqrt{x}} =$

(a) $\frac{2}{3}(1+x)^{2/3} - \frac{2}{3}x^{2/3} + c$

(b) $\frac{3}{2}(1+x)^{2/3} + \frac{3}{2}x^{2/3} + c$

(c) $\frac{3}{2}(1+x)^{3/2} + \frac{3}{2}x^{3/2} + c$

(d) $\frac{2}{3}(1+x)^{3/2} - \frac{2}{3}x^{3/2} + c$

11. $\int \{1 + 2\tan x(\tan x + \sec x)\}^{1/2}\, dx =$

(a) $\log(\sec x + \tan x) + c$

(b) $\log(\sec x + \tan x)^{1/2} + c$

(c) $\log \sec x(\sec x + \tan x) + c$

(d) None of these

12. $\int \frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}}\, dx =$

(a) $e \,.\, 3^{-3x} + c$ (b) $e^3 \log x + c$

(c) $\frac{x^3}{3} + c$ (d) None of these

13. $\int e^{\log(\sin x)} dx =$

(a) $\sin x + c$ (b) $-\cos x + c$

(c) $e^{\log(\cos x)} + c$ (d) None of these

14. $\int \frac{1}{\sqrt{1+\cos x}} dx =$

(a) $\sqrt{2} \log\left(\sec\frac{x}{2} + \tan\frac{x}{2}\right) + K$

(b) $\frac{1}{\sqrt{2}} \log\left(\sec\frac{x}{2} + \tan\frac{x}{2}\right) + K$

(c) $\log\left(\sec\frac{x}{2} + \tan\frac{x}{2}\right) + K$

(d) None of these

15. $\int \left(\sin^4 x - \cos^4 x\right) dx =$

(a) $-\frac{\cos 2x}{2} + c$ (b) $-\frac{\sin 2x}{2} + c$

(c) $\frac{\sin 2x}{2} + c$ (d) $\frac{\cos 2x}{2} + c$

16. If $\int \frac{f(x)\,dx}{\log \sin x} = \log \log \sin x$, then $f(x) =$

(a) $\sin x$ (b) $\cos x$

(c) $\log \sin x$ (d) $\cot x$

17. $\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx =$

(a) $2\sec x + c$ (b) $2\tan x + c$

(c) $\tan x + c$ (d) None of these

18. $\int \frac{\sin x + \cos x}{\sqrt{1+\sin 2x}} dx =$

(a) $\sin x + c$ (b) $\cos x + c$

(c) $x + c$ (d) $x^2 + c$

19. $\int \frac{\tan x}{\sec x + \tan x} dx =$

(a) $\sec x + \tan x - x + c$ (b) $\sec x - \tan x + x + c$

(c) $\sec x + \tan x + x + c$ (d) $-\sec x - \tan x + x + c$

20. $\int \frac{dx}{x + x\log x} =$

(a) $\log(1+\log x)$ (b) $\log\log(1+\log x)$ (c) $\log x + \log(\log x)$ (d) None of these

21. $\int \frac{dx}{e^{x}+e^{-x}} =$

(a) $\tan^{-1}(e^{-x})$ (b) $\tan^{-1}(e^{x})$

(c) $\log(e^{x}-e^{-x})$ (d) $\log(e^{x}+e^{-x})$

22. $\int \frac{x^{e-1}+e^{x-1}}{x^{e}+e^{x}} dx =$

(a) $\log(x^{e}+e^{x})+c$ (b) $e\log(x^{e}+e^{x})+c$

(c) $\frac{1}{e}\log(x^{e}+e^{x})+c$ (d) None of these

23. $\int \frac{dx}{e^{x}-1} =$

(a) $\ln(1-e^{-x})+c$ (b) $-\ln(1-e^{-x})+c$

(c) $\ln(e^{x}-1)+c$ (d) None of these

24. $\int \frac{e^{x}(x+1)}{\cos^{2}(xe^{x})} dx =$

(a) $\tan(xe^{x})+c$ (b) $\sec(xe^{x})\tan(xe^{x})+c$

(c) $-\tan(xe^{x})+c$ (d) None of these

25. $\int \frac{\sin x \cos x}{a\cos^{2}x+b\sin^{2}x} dx =$

(a) $\frac{1}{2(b-a)}\log(a\cos^{2}x+b\sin^{2}x)+c$

(b) $\frac{1}{b-a}\log(a\cos^{2}x+b\sin^{2}x)+c$

(c) $\frac{1}{2}\log(a\cos^{2}x+b\sin^{2}x)+c$

(d) None of these

26. $\int \sec x \log(\sec x+\tan x)\, dx =$

(a) $[\log(\sec x+\tan x)]^{2}+c$

(b) $\frac{1}{2}[\log(\sec x+\tan x)]^{2}+c$

(c) $\sec^{2}x+\tan x\sec x+c$

(d) None of these

27. $\int \frac{\sin x\, dx}{a^{2}+b^{2}\cos^{2}x} =$

(a) $\log(a^{2}+b^{2}\cos^{2}x)+c$ (b) $\frac{1}{ab}\tan^{-1}\left(\frac{a\cos x}{b}\right)+c$ (c) $\frac{1}{ab}\cot^{-1}\left(\frac{b\cos x}{a}\right)+c$ (d) $\frac{1}{ab}\cot^{-1}\left(\frac{a\cos x}{b}\right)+c$

28. $\int \frac{e^{2x}-1}{e^{2x}+1} dx =$

(a) $\frac{e^{2x}-1}{e^{2x}+1}+c$ (b) $\log(e^{2x}+1)-x+c$

(c) $\log(e^{2x}+1)+c$ (d) None of these

29. $\int \frac{e^{\tan^{-1}x}}{1+x^2} dx =$

(a) $\log(1+x^2)+c$ (b) $\log e^{\tan^{-1}x}+c$

(c) $e^{\tan^{-1}x}+c$ (d) $\tan^{-1} e^{\tan^{-1}x}+c$

30. $\int \frac{a^x}{\sqrt{1-a^{2x}}} dx =$

(a) $\frac{1}{\log a}\sin^{-1} a^x+c$ (b) $\sin^{-1} a^x+c$

(c) $\frac{1}{\log a}\cos^{-1} a^x+c$ (d) $\cos^{-1} a^x+c$

31. $\int \frac{1}{\cos^2 x(1-\tan x)^2} dx =$

(a) $\frac{1}{\tan x-1}+c$ (b) $\frac{1}{1-\tan x}+c$

(c) $-\frac{1}{3}\frac{1}{(1-\tan x)^3}+c$ (d) None of these

32. $\int \frac{x}{1+x^4} dx =$

(a) $\frac{1}{2}\cot^{-1} x^2+c$ (b) $\frac{1}{2}\tan^{-1} x^2+c$

(c) $\cot^{-1} x^2+c$ (d) $\tan^{-1} x^2+c$

33. $\int \frac{1}{\cos^{-1} x.\sqrt{1-x^2}} dx =$

(a) $\log(\cos^{-1} x)+c$ (b) $-\log(\cos^{-1} x)+c$

(c) $-\frac{1}{2(\cos^{-1} x)^2}+c$ (d) None of these

34. $\int \frac{\cos 2x}{(\cos x+\sin x)^2} dx =$

(a) $\log\sqrt{\cos x+\sin x}+c$ (b) $\log(\cos x-\sin x)+c$

(c) $\log(\cos x+\sin x)+c$ (d) $-\frac{1}{\cos x+\sin x}+c$

35. To evaluate $\int \frac{\sec^2 x}{(1+\tan x)(2+\tan x)} dx$, the most suitable substitution is

(a) $1+\tan x=t$ (b) $2+\tan x=t$ (c) $\tan x=t$ (d) None of these

36. $\int \frac{1}{\sqrt{x}} \sin\sqrt{x}\, dx =$

(a) $-\frac{1}{2}\cos\sqrt{x} + c$ (b) $-2\cos\sqrt{x} + c$

(c) $\frac{1}{2}\cos\sqrt{x} + c$ (d) $2\cos\sqrt{x} + c$

37. $\int \frac{\sin 2x}{\sin 5x \sin 3x}\, dx =$

(a) $\log\sin 3x - \log\sin 5x + c$

(b) $\frac{1}{3}\log\sin 3x + \frac{1}{5}\log\sin 5x + c$

(c) $\frac{1}{3}\log\sin 3x - \frac{1}{5}\log\sin 5x + c$

(d) $3\log\sin 3x - 5\log\sin 5x + c$

38. $\int \frac{\sec^2 x\, dx}{\sqrt{\tan^2 x + 4}} =$

(a) $\log\left[\tan x + \sqrt{\tan^2 x + 4}\right] + c$

(b) $\frac{1}{2}\log\left[\tan x + \sqrt{\tan^2 x + 4}\right] + c$

(c) $\log\left[\frac{1}{2}\tan x + \frac{1}{2}\sqrt{\tan^2 x + 4}\right] + c$

(d) None of these

39. $\int e^x \tan^2(e^x) dx =$

(a) $\tan(e^x) - x + c$ (b) $e^x(\tan e^x - 1) + c$

(c) $\sec(e^x) + c$ (d) $\tan(e^x) - e^x + c$

40. $\int \frac{\cos x - \sin x}{1 + \sin 2x}\, dx =$

(a) $-\frac{1}{\cos x + \sin x} + c$ (b) $\frac{1}{\cos x + \sin x} + c$

(c) $\frac{1}{\cos x - \sin x} + c$ (d) None of these

41. $\int \frac{x^3}{\sqrt{1 - x^8}} dx =$

(a) $\frac{1}{2}\sin^{-1}(x^4) + c$ (b) $\frac{1}{3}\sin^{-1}(x^4) + c$

(c) $\frac{1}{4}\sin^{-1}(x^4) + c$ (d) None of these

42. $\int \frac{1}{(x^2-1)\sqrt{x^2+1}}\,dx =$

(a) $\frac{1}{2\sqrt{2}}\log\left\{\frac{\sqrt{1+x^2}+x\sqrt{2}}{\sqrt{1+x^2}-x\sqrt{2}}\right\}+c$

(b) $\frac{1}{2\sqrt{2}}\log\left\{\frac{\sqrt{1+x^2}-\sqrt{2}}{\sqrt{1+x^2}+\sqrt{2}}\right\}+c$

(c) $\frac{1}{2\sqrt{2}}\log\left\{\frac{\sqrt{1+x^2}-x\sqrt{2}}{\sqrt{1+x^2}+x\sqrt{2}}\right\}+c$

(d) None of these

43. $\int \frac{(x+1)(x+\log x)^2}{x}\,dx =$

(a) $\frac{1}{3}(x+\log x)+c$ (b) $\frac{1}{3}(x+\log x)^2+c$

(c) $\frac{1}{3}(x+\log x)^3+c$ (d) None of these

44. $\int \sqrt{\frac{x}{a^3-x^3}}\,dx =$

(a) $\sin^{-1}\left(\frac{x}{a}\right)^{3/2}+c$ (b) $\frac{2}{3}\sin^{-1}\left(\frac{x}{a}\right)^{3/2}+c$

(c) $\frac{3}{2}\sin^{-1}\left(\frac{x}{a}\right)^{3/2}+c$ (d) $\frac{3}{2}\sin^{-1}\left(\frac{x}{a}\right)^{2/3}+c$

45. $\int \frac{\log(x+\sqrt{1+x^2})}{\sqrt{1+x^2}}\,dx =$

(a) $\frac{1}{2}[\log(x+\sqrt{1+x^2})]^2+c$ (b) $\log(x+\sqrt{1+x^2})^2+c$

(c) $\log(x+\sqrt{1+x^2})+c$ (d) None of these

46. $\int \frac{\cos x-\sin x}{\sqrt{\sin 2x}}\,dx$ equals

(a) $\cosh^{-1}(\sin x+\cos x)+c$

(b) $\sinh^{-1}(\sin x+\cos x)+c$

(c) $-\cosh^{-1}(\sin x+\cos x)+c$

(d) $-\sinh^{-1}(\sin x+\cos x)+c$

47. $\int x^x (1+\log x)\, dx$ is equal to

(a) x^x (b) x^{2x}

(c) $x^x \log x$ (d) $\frac{1}{2}(1+\log x)^2$

48. The value of $\int \frac{dx}{x\sqrt{x^4-1}}$ is

(a) $\frac{1}{2}\sec^{-1} x^2 + k$ (b) $\log x\sqrt{x^4-1} + k$

(c) $x \log\sqrt{x^4-1} + k$ (d) $\log\sqrt{x^4-1} + k$

49. $\int \frac{(x^4-x)^{1/4}}{x^5}\, dx$ is equal to

(a) $\frac{4}{15}\left(1-\frac{1}{x^3}\right)^{5/4} + c$ (b) $\frac{4}{5}\left(1-\frac{1}{x^3}\right)^{5/4} + c$

(c) $\frac{4}{15}\left(1+\frac{1}{x^3}\right)^{5/4} + c$ (d) None of these

50. The value of $\int \left(1+\frac{1}{x^2}\right) e^{\left(x-\frac{1}{x}\right)}\, dx$ equals

(a) $e^{x-\frac{1}{x}} + c$ (b) $e^{x+\frac{1}{x}} + c$

(c) $e^{x^2-\frac{1}{x}} + c$ (d) $e^{x^2+\frac{1}{x^2}} + c$

51. $\int \frac{1}{[(x-1)^3(x+2)^5]^{1/4}}\, dx$ is equal to

(a) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4} + c$ (b) $\frac{4}{3}\left(\frac{x+2}{x-1}\right)^{1/4} + c$

(c) $\frac{1}{3}\left(\frac{x-1}{x+2}\right)^{1/4} + c$ (d) $\frac{1}{3}\left(\frac{x+2}{x-1}\right)^{1/4} + c$

52. A primitive of $\frac{x}{x^2+1}$ is

(a) $\log_e(x^2+1)$ (b) $x\tan^{-1} x$

(c) $\frac{\log_e(x^2+1)}{2}$ (d) $\frac{1}{2}x\tan^{-1} x$

53. $\int \sqrt{\frac{1+x}{1-x}}\, dx =$

(a) $-\sin^{-1} x - \sqrt{1-x^2} + c$ (b) $\sin^{-1} x + \sqrt{1-x^2} + c$

(c) $\sin^{-1} x - \sqrt{1-x^2} + c$ (d) $-\sin^{-1} x - \sqrt{x^2-1} + c$

54. The value of $\int \frac{\sin x - \cos x}{\sin x + \cos x} dx$ is

(a) $\frac{1}{\sin x + \cos x} + c$ (b) $\frac{1}{\sin x - \cos x} + c$

(c) $\log(\sin x + \cos x) + c$ (d) $\log\left(\frac{1}{\sin x + \cos x}\right) + c$

55. $\int \left\{\frac{(\log x - 1)}{1 + (\log x)^2}\right\}^2 dx$ is equal to

(a) $\frac{xe^x}{1 + x^2} + c$ (b) $\frac{x}{(\log x)^2 + 1} + c$

(c) $\frac{\log x}{(\log x)^2 + 1} + c$ (d) $\frac{x}{x^2 + 1} + c$

56. $\int \frac{\sin x}{\sin x - \cos x} dx =$

(a) $\frac{1}{2}\log(\sin x - \cos x) + x + c$

(b) $\frac{1}{2}[\log(\sin x - \cos x) + x] + c$

(c) $\frac{1}{2}\log(\cos x - \sin x) + x + c$

(d) $\frac{1}{2}[\log(\cos x - \sin x) + x] + c$

57. Let $f(x) = \int \frac{x^2 dx}{(1 + x^2)\left(1 + \sqrt{1 + x^2}\right)}$ and $f(0) = 0$, then the value of $f(1)$ be

(a) $\log(1 + \sqrt{2})$ (b) $\log(1 + \sqrt{2}) - \frac{\pi}{4}$

(c) $\log(1 + \sqrt{2}) + \frac{\pi}{2}$ (d) None of these

58. $\int \frac{dx}{\sin(x - a)\sin(x - b)}$ is

(a) $\frac{1}{\sin(a - b)}\log\left|\frac{\sin(x - a)}{\sin(x - b)}\right| + c$

(b) $\frac{-1}{\sin(a - b)}\log\left|\frac{\sin(x - a)}{\sin(x - b)}\right| + c$

(c) $\log \sin(x - a)\sin(x - b) + c$

(d) $\log\left|\frac{\sin(x - a)}{\sin(x - b)}\right|$

59. $\int x\cos^2 x dx =$

(a) $\frac{x^4}{4}-\frac{1}{4}x\sin 2x-\frac{1}{8}\cos 2x+c$

(b) $\frac{x^2}{4}+\frac{1}{4}x\sin 2x+\frac{1}{8}\cos 2x+c$

(c) $\frac{x^4}{4}-\frac{1}{4}x\sin 2x+\frac{1}{8}\cos 2x+c$

(d) $\frac{x^4}{4}+\frac{1}{4}x\sin 2x-\frac{1}{8}\cos 2x+c$

60. $\int x\tan^{-1} x dx =$

(a) $\frac{1}{2}(x^2+1)\tan^{-1}x-\frac{1}{2}x+c$

(b) $\frac{1}{2}(x^2-1)\tan^{-1}x-\frac{1}{2}x+c$

(c) $\frac{1}{2}(x^2+1)\tan^{-1}x+\frac{1}{2}x+c$

(d) $\frac{1}{2}(x^2+1)\tan^{-1}x-x+c$

61. $\int \log x dx =$

(a) $x+x\log x+c$

(b) $x\log x-x+c$

(c) $x^2\log x+c$

(d) $\frac{1}{x}\log x+x+c$

62. The value of $\int\frac{\log x}{(x+1)^2}dx$ is

(a) $\frac{-\log x}{x+1}+\log x-\log(x+1)$

(b) $\frac{\log x}{(x+1)}+\log x-\log(x+1)$

(c) $\frac{\log x}{x+1}-\log x-\log(x+1)$

(d) $\frac{-\log x}{x+1}-\log x-\log(x+1)$

63. $\int\left(\frac{2+\sin 2x}{1+\cos 2x}\right)e^x dx =$

(a) $e^x\cot x+c$

(b) $-e^x\cot x+c$

(c) $-e^x\tan x+c$

(d) $e^x\tan x+c$

64. $\int \left[\log(\log x) + \dfrac{1}{(\log x)^2}\right] dx =$

(a) $x\log(\log x) + \dfrac{x}{\log x} + c$ (b) $x\log(\log x) - \dfrac{x}{\log x} + c$

(c) $x\log(\log x) + \dfrac{\log x}{x} + c$ (d) $x\log(\log x) - \dfrac{\log x}{x} + c$

65. $\int e^{2x}(-\sin x + 2\cos x)\, dx =$

(a) $e^{2x}\sin x + c$ (b) $-e^{2x}\sin x + c$

(c) $-e^{2x}\cos x + c$ (d) $e^{2x}\cos x + c$

66. $\int \cos(\log_e x)\,dx$ is equal to

(a) $\dfrac{1}{2}x\{\cos(\log_e x) + \sin(\log_e x)\}$

(b) $x\{\cos(\log_e x) + \sin(\log_e x)\}$

(c) $\dfrac{1}{2}x\{\cos(\log_e x) - \sin(\log_e x)\}$

(d) $x\{\cos(\log_e x) - \sin(\log_e x)\}$

67. $\int x^n \log x\, dx =$

(a) $\dfrac{x^{n+1}}{n+1}\left\{\log x + \dfrac{1}{n+1}\right\} + c$ (b) $\dfrac{x^{n+1}}{n+1}\left\{\log x + \dfrac{2}{n+1}\right\} + c$

(c) $\dfrac{x^{n+1}}{n+1}\left\{2\log x - \dfrac{1}{n+1}\right\} + c$ (d) $\dfrac{x^{n+1}}{n+1}\left\{\log x - \dfrac{1}{n+1}\right\} + c$

68. $\int e^x \sin x\, dx =$

(a) $\dfrac{1}{2}e^x(\sin x + \cos x) + c$ (b) $\dfrac{1}{2}e^x(\sin x - \cos x) + c$

(c) $e^x(\sin x + \cos x) + c$ (d) $e^x(\sin x - \cos x) + c$

69. $\int \dfrac{xe^x}{(1+x)^2}dx =$

(a) $\dfrac{e^{-x}}{1+x} + c$ (b) $-\dfrac{e^{-x}}{1+x} + c$

(c) $\dfrac{e^x}{1+x} + c$ (d) $-\dfrac{e^x}{1+x} + c$

70. $\int e^x\left(\dfrac{1}{x} - \dfrac{1}{x^2}\right)dx =$

(a) $-\dfrac{e^x}{x^2} + c$ (b) $\dfrac{e^x}{x^2} + c$ (c) $\dfrac{e^x}{x} + c$ (d) $-\dfrac{e^x}{x} + c$

71. $\int e^x\left[\frac{1+x\log x}{x}\right]dx =$

(a) $e^x+\log x+c$ (b) $\frac{e^x}{\log x}+c$

(c) $e^x-\log x+c$ (d) $e^x\log x+c$

72. $\int[\sin(\log x)+\cos(\log x)]\,dx =$

(a) $x\cos(\log x)+c$ (b) $\sin(\log x)+c$

(c) $\cos(\log x)+c$ (d) $x\sin(\log x)+c$

73. $\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx =$

(a) $-\frac{e^x}{x^2}+c$ (b) $\frac{e^x}{x^2}+c$

(c) $\frac{e^x}{x}+c$ (d) $-\frac{e^x}{x}+c$

74. $\int\frac{x-\sin x}{1-\cos x}dx =$

(a) $x\cot\frac{x}{2}+c$ (b) $-x\cot\frac{x}{2}+c$

(c) $\cot\frac{x}{2}+c$ (d) None of these

75. $\int\cos^{-1}\left(\frac{1}{x}\right)dx$

(a) $x\sec^{-1}x+\cosh^{-1}x+C$ (b) $x\sec^{-1}x-\cosh^{-1}x+C$

(c) $x\sec^{-1}x-\sin^{-1}x+C$ (d) None of these

76. $\int x\sin^2 x\,dx =$

(a) $\frac{x^2}{4}+\frac{x}{4}\sin 2x+\frac{1}{8}\cos 2x+c$

(b) $\frac{x^2}{4}-\frac{x}{4}\sin 2x+\frac{1}{8}\cos 2x+c$

(c) $\frac{x^2}{4}+\frac{x}{4}\sin 2x-\frac{1}{8}\cos 2x+c$

(d) $\frac{x^2}{4}-\frac{x}{4}\sin 2x-\frac{1}{8}\cos 2x+c$

77. $\int e^x\frac{(x^2+1)}{(x+1)^2}dx =$

(a) $\left(\frac{x-1}{x+1}\right)e^x+c$ (b) $e^x\left(\frac{x+1}{x-1}\right)+c$ (c) $e^x(x+1)(x-1)+c$ (d) None of these

78. $\int \left[\frac{1}{\log x} - \frac{1}{(\log x)^2}\right] dx =$

(a) $\frac{1}{\log x} + c$ (b) $\frac{x}{\log x} + c$

(c) $\frac{x}{(\log x)^2}$ (d) None of these

79. $\int e^{2x} \frac{1+\sin 2x}{1+\cos 2x} dx =$

(a) $e^{2x} \tan x + c$ (b) $e^{2x} \cot x + c$

(c) $\frac{e^{2x} \tan x}{2} + c$ (d) $\frac{e^{2x} \cot x}{2} + c$

80. $\int \frac{(x+3)e^x}{(x+4)^2} dx =$

(a) $\frac{1}{(x+4)^2} + c$ (b) $\frac{e^x}{(x+4)^2} + c$

(c) $\frac{e^x}{x+4} + c$ (d) $\frac{e^x}{x+3} + c$

81. $\int \frac{\sin^{-1} x}{(1-x^2)^{3/2}} dx =$

(a) $\frac{x}{\sqrt{1-x^2}} \sin^{-1} x + \frac{1}{2}\log(1-x^2) + c$

(b) $\frac{x}{\sqrt{1-x^2}} \sin^{-1} x - \frac{1}{2}\log(1-x^2) + c$

(c) $\frac{1}{\sqrt{1-x^2}} \sin^{-1} x - \frac{1}{2}\log(1-x^2) + c$

(d) $\frac{1}{\sqrt{1-x^2}} \sin^{-1} x + \frac{1}{2}\log(1-x^2) + c$

82. If $\int \frac{e^x(1+\sin x)dx}{1+\cos x} = e^x f(x) + c$, then $f(x) =$

(a) $\sin\frac{x}{2}$ (b) $\cos\frac{x}{2}$

(c) $\tan\frac{x}{2}$ (d) $\log\frac{x}{2}$

83. $\int e^{\tan^{-1} x}\left(\frac{1+x+x^2}{1+x^2}\right) dx$ is equal to

(a) $xe^{\tan^{-1} x} + c$ (b) $x^2 e^{\tan^{-1} x} + c$

(c) $\frac{1}{x} e^{\tan^{-1} x} + c$ (d) None of these

84. $\int e^x(1-\cot x+\cot^2 x)\,dx$ equals

(a) $e^x \cot x + c$ (b) $e^x \operatorname{cosec} x + c$

(c) $-e^x \cot x + c$ (d) $-e^x \operatorname{cosec} x + c$

85. $\int \sin^{-1}(3x-4x^3)dx =$

(a) $x\sin^{-1}x+\sqrt{1-x^2}+c$ (b) $x\sin^{-1}x-\sqrt{1-x^2}+c$

(c) $2[x\sin^{-1}x+\sqrt{1-x^2}]+c$ (d) $3[x\sin^{-1}x+\sqrt{1-x^2}]+c$

86. $\int \frac{x-1}{(x-3)(x-2)}dx =$

(a) $\log(x-3)-\log(x-2)+c$

(b) $\log(x-3)^2-\log(x-2)+c$

(c) $\log(x-3)+\log(x-2)+c$

(d) $\log(x-3)^2+\log(x-2)+c$

87. $\int \frac{dx}{(x^2+1)(x^2+4)} =$

(a) $\frac{1}{3}\tan^{-1}x-\frac{1}{3}\tan^{-1}\frac{x}{2}+c$

(b) $\frac{1}{3}\tan^{-1}x+\frac{1}{3}\tan^{-1}\frac{x}{2}+c$

(c) $\frac{1}{3}\tan^{-1}x-\frac{1}{6}\tan^{-1}\frac{x}{2}+c$

(d) $\tan^{-1}x-2\tan^{-1}\frac{x}{2}+c$

88. $\int \frac{\cos x}{(1+\sin x)(2+\sin x)}dx =$

(a) $\log[(1+\sin x)(2+\sin x)]+c$

(b) $\log\frac{2+\sin x}{1+\sin x}+c$

(c) $\log\frac{1+\sin x}{2+\sin x}+c$

(d) None of these

89. $\int \frac{x\,dx}{(x^2-a^2)(x^2-b^2)} =$

(a) $\frac{1}{a^2-b^2}\log\left(\frac{x^2-a^2}{x^2-b^2}\right)+c$ (b) $\frac{1}{a^2-b^2}\log\left(\frac{x^2-b^2}{x^2-a^2}\right)+c$

(c) $\frac{1}{2(a^2-b^2)}\log\left(\frac{x^2-a^2}{x^2-b^2}\right)+c$ (d) $\frac{1}{2(a^2-b^2)}\log\left(\frac{x^2-b^2}{x^2-a^2}\right)+c$

90. $\int \frac{1}{\cos x(1+\cos x)} dx =$

(a) $\log(\sec x + \tan x) + 2\tan\frac{x}{2} + c$

(b) $\log(\sec x + \tan x) - 2\tan\frac{x}{2} + c$

(c) $\log(\sec x + \tan x) + \tan\frac{x}{2} + c$

(d) $\log(\sec x + \tan x) - \tan\frac{x}{2} + c$

91. $\int \frac{1}{x - x^3} dx =$

(a) $\frac{1}{2}\log\frac{(1-x^2)}{x^2} + c$ (b) $\log\frac{(1-x)}{x(1+x)} + c$

(c) $\log x(1-x^2) + c$ (d) $\frac{1}{2}\log\frac{x^2}{(1-x^2)} + c$

92. $\int \frac{dx}{(x+1)(x+2)} =$

(a) $\log\frac{x+2}{x+1} + c$ (b) $\log(x+1) + \log(x+2) + c$

(c) $\log\frac{x+1}{x+2} + c$ (d) None of these

93. $\int \frac{e^x}{(1+e^x)(2+e^x)} dx =$

(a) $\log[(1+e^x)(2+e^x)] + c$ (b) $\log\left[\frac{1+e^x}{2+e^x}\right] + c$

(c) $\log[(1+e^x)\sqrt{2+e^x}] + c$ (d) None of these

94. $\int \frac{1}{1+\cos^2 x} dx =$

(a) $\frac{1}{\sqrt{2}}\tan^{-1}(\tan x) + c$ (b) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{1}{2}\tan x\right) + c$

(c) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{1}{\sqrt{2}}\tan x\right) + c$ (d) None of these

95. $\int \frac{dx}{x(x^5+1)} =$

(a) $\frac{1}{5}\log x^5(x^5+1) + c$ (b) $\frac{1}{5}\log x^5\left(\frac{1+x^5}{x^5}\right) + c$

(c) $\frac{1}{5}\log x^5\left(\frac{x^5}{x^5+1}\right) + c$ (d) None of these

96. $\int \frac{dx}{e^x + 1 - 2e^{-x}} =$

(a) $\log(e^x - 1) - \log(e^x + 2) + c$

(b) $\frac{1}{2}\log(e^x - 1) - \frac{1}{3}\log(e^x + 2) + c$

(c) $\frac{1}{3}\log(e^x - 1) - \frac{1}{3}\log(e^x + 2) + c$

(d) $\frac{1}{3}\log(e^x - 1) + \frac{1}{3}\log(e^x + 2) + c$

97. $\int \frac{1}{(x^2 + a^2)(x^2 + b^2)} dx =$

(a) $\frac{1}{(a^2 - b^2)}\left[\frac{1}{b}\tan^{-1}\left(\frac{x}{b}\right) - \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)\right] + c$

(b) $\frac{1}{(b^2 - a^2)}\left[\frac{1}{b}\tan^{-1}\left(\frac{x}{b}\right) - \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)\right] + c$

(c) $\frac{1}{b}\tan^{-1}\left(\frac{x}{b}\right) - \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$

(d) $\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) - \frac{1}{b}\tan^{-1}\left(\frac{x}{b}\right) + c$

98. $\int \frac{dx}{x[(\log x)^2 + 4\log x - 1]} =$

(a) $\frac{1}{2\sqrt{5}}\log\left[\frac{\log x + 2 - \sqrt{5}}{\log x + 2 + \sqrt{5}}\right] + c$

(b) $\frac{1}{\sqrt{5}}\log\left[\frac{\log x + 2 - \sqrt{5}}{\log x + 2 + \sqrt{5}}\right] + c$

(c) $\frac{1}{2\sqrt{5}}\log\left[\frac{\log x + 2 + \sqrt{5}}{\log x + 2 - \sqrt{5}}\right] + c$

(d) $\frac{1}{\sqrt{5}}\log\left[\frac{\log x + 2 + \sqrt{5}}{\log x + 2 - \sqrt{5}}\right] + c$

99. $\int \frac{dx}{\sqrt{2x - x^2}} =$

(a) $\cos^{-1}(x - 1) + c$ (b) $\sin^{-1}(x - 1) + c$

(c) $\cos^{-1}(1 + x) + c$ (d) $\sin^{-1}(1 - x) + c$

100. $\int \frac{x^2 + 1}{x^4 + 1} dx =$

(a) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2 - 1}{2x}\right) + c$ (b) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2 - 1}{\sqrt{2}x}\right) + c$

(c) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2 - 1}{2\sqrt{x}}\right) + c$ (d) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2 - 1}{\sqrt{2x}}\right) + c$

101. $\int \frac{dx}{7+5\cos x} =$

(a) $\frac{1}{\sqrt{6}}\tan^{-1}\left(\frac{1}{\sqrt{6}}\tan\frac{x}{2}\right)+c$ (b) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{1}{\sqrt{3}}\tan\frac{x}{2}\right)+c$

(c) $\frac{1}{4}\tan^{-1}\left(\tan\frac{x}{2}\right)+c$ (d) $\frac{1}{7}\tan^{-1}\left(\tan\frac{x}{2}\right)+c$

102. $\int \frac{1}{(x-1)(x^2+1)}dx =$

(a) $\frac{1}{2}\log(x-1)-\frac{1}{4}\log(x^2+1)-\frac{1}{2}\tan^{-1}x+c$

(b) $\frac{1}{2}\log(x-1)+\frac{1}{4}\log(x^2+1)-\frac{1}{2}\tan^{-1}x+c$

(c) $\frac{1}{2}\log(x-1)-\frac{1}{2}\log(x^2+1)-\frac{1}{2}\tan^{-1}x+c$

(d) None of these

103. $\int \frac{x^2-1}{x^4+x^2+1}dx =$

(a) $\frac{1}{2}\log\left(\frac{x^2+x+1}{x^2-x+1}\right)+c$ (b) $\frac{1}{2}\log\left(\frac{x^2-x-1}{x^2+x+1}\right)+c$

(c) $\log\left(\frac{x^2-x+1}{x^2+x+1}\right)+c$ (d) $\frac{1}{2}\log\left(\frac{x^2-x+1}{x^2+x+1}\right)+c$

104. $\int \frac{dx}{\cos(x-a)\cos(x-b)} =$

(a) $\operatorname{cosec}(a-b)\log\frac{\sin(x-a)}{\sin(x-b)}+c$ (b) $\operatorname{cosec}(a-b)\log\frac{\cos(x-a)}{\cos(x-b)}+c$

(c) $\operatorname{cosec}(a-b)\log\frac{\sin(x-b)}{\sin(x-a)}+c$ (d) $\operatorname{cosec}(a-b)\log\frac{\cos(x-b)}{\cos(x-a)}+c$

105. $\int \frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}dx =$

(a) $\sin 2x+c$ (b) $-\frac{1}{2}\sin 2x+c$

(c) $\frac{1}{2}\sin 2x+c$ (d) $-\sin 2x+c$

106. $\int \frac{x^2}{(9-x^2)^{3/2}}dx =$

(a) $\frac{x}{\sqrt{9-x^2}}-\sin^{-1}\frac{x}{3}+c$ (b) $\frac{x}{\sqrt{9-x^2}}+\sin^{-1}\frac{x}{3}+c$

(c) $\sin^{-1}\frac{x}{3}-\frac{x}{\sqrt{9-x^2}}+c$ (d) None of these

107. $\int \frac{dx}{4\sin^2 x + 5\cos^2 x} =$

(a) $\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{2\tan x}{\sqrt{5}}\right)+c$ (b) $\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{\tan x}{\sqrt{5}}\right)+c$

(c) $\frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{2\tan x}{\sqrt{5}}\right)+c$ (d) None of these

108. $\int \sqrt{\frac{a-x}{x}}\,dx =$

(a) $a\left[\sin^{-1}\sqrt{\frac{x}{a}}+\sqrt{\frac{x}{a}}\sqrt{\frac{a-x}{a}}\right]+c$ (b) $\sin^{-1}\frac{x}{a}+\frac{x}{a}\sqrt{a^2-x^2}+c$

(c) $a\left[\sin^{-1}\frac{x}{a}-\frac{x}{a}\sqrt{a^2-x^2}\right]+c$ (d) $\sin^{-1}\frac{x}{a}-\frac{x}{a}\sqrt{a^2-x^2}+c$

109. $\int \frac{x^2+1}{x^4-x^2+1}\,dx =$

(a) $\tan^{-1}\left(\frac{1+x^2}{x}\right)+c$ (b) $\cot^{-1}\left(\frac{1+x^2}{x}\right)+c$

(c) $\tan^{-1}\left(\frac{x^2-1}{x}\right)+c$ (d) $\cot^{-1}\left(\frac{x^2-1}{x}\right)+c$

110. $\int \frac{x-1}{(x+1)^3}e^x\,dx =$

(a) $\frac{-e^x}{(x+1)^2}+c$ (b) $\frac{e^x}{(x+1)^2}+c$ (c) $\frac{e^x}{(x+1)^3}+c$ (d) $\frac{-e^x}{(x+1)^3}+c$

111. $\int \frac{3\cos x+3\sin x}{4\sin x+5\cos x}\,dx =$

(a) $\frac{27}{41}x-\frac{3}{41}\log(4\sin x+5\cos x)$ (b) $\frac{27}{41}x+\frac{3}{41}\log(4\sin x+5\cos x)$

(c) $\frac{27}{41}x-\frac{3}{41}\log(4\sin x-5\cos x)$ (d) None of these

112. $\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx =$

(a) $\frac{1}{2}[\sin^{-1}x^2+\sqrt{1-x^4}]+c$ (b) $\frac{1}{2}[\sin^{-1}x^2+\sqrt{1-x^2}]+c$

(c) $\sin^{-1}x^2+\sqrt{1-x^4}+c$ (d) $\sin^{-1}x^2+\sqrt{1-x^2}+c$

113. If $\int \frac{4e^x+6e^{-x}}{9e^x-4e^{-x}}\,dx = Ax+B\log(9e^{2x}-4)+C$, then A, B and C are

(a) $A=\frac{3}{2}, B=\frac{36}{35}, C=\frac{3}{2}\log 3+$ constant (b) $A=\frac{3}{2}, B=\frac{35}{36}, C=\frac{3}{2}\log 3+$ constant

(c) $A=-\frac{3}{2}, B=-\frac{35}{36}, C=-\frac{3}{2}\log 3+$ constant (d) None of these

114. The value of $\int \frac{\sqrt{(x^2-a^2)}}{x}dx$ will be

(a) $\sqrt{(x^2-a^2)}-a\tan^{-1}\left[\frac{\sqrt{(x^2-a^2)}}{a}\right]$ (b) $\sqrt{(x^2-a^2)}+a\tan^{-1}\left[\frac{\sqrt{(x^2-a^2)}}{a}\right]$

(c) $\sqrt{(x^2-a^2)}+a^2\tan^{-1}[\sqrt{x^2-a^2}]$ (d) $\tan^{-1}x/a+c$

115. If $I_n=\int(\log x)^n\,dx$, then $I_n+nI_{n-1}=$

(a) $x(\log x)^n$ (b) $(x\log x)^n$

(c) $(\log x)^{n-1}$ (d) $n(\log x)^n$

116. $\int \frac{dx}{(\sin x+\sin 2x)}=$

(a) $\frac{1}{6}\log(1-\cos x)+\frac{1}{2}\log(1+\cos x)-\frac{2}{3}\log(1+2\cos x)$

(b) $6\log(1-\cos x)+2\log(1+\cos x)-\frac{2}{3}\log(1+2\cos x)$

(c) $6\log(1-\cos x)+\frac{1}{2}\log(1+\cos x)+\frac{2}{3}\log(1+2\cos x)$

(d) None of these

117. $\int \tan^3 2x\sec 2x\,dx=$

(a) $\frac{1}{6}\sec^3 2x-\frac{1}{2}\sec 2x+c$ (b) $\frac{1}{6}\sec^3 2x+\frac{1}{2}\sec 2x+c$

(c) $\frac{1}{9}\sec^2 2x-\frac{1}{3}\sec 2x+c$ (d) None of these

118. $\int \frac{x^2dx}{(a+bx)^2}=$

(a) $\frac{1}{b^2}\left[x+\frac{2a}{b}\log(a+bx)-\frac{a^2}{b}\frac{1}{a+bx}\right]$

(b) $\frac{1}{b^2}\left[x-\frac{2a}{b}\log(a+bx)+\frac{a^2}{b}\frac{1}{a+bx}\right]$

(c) $\frac{1}{b^2}\left[x+\frac{2a}{b}\log(a+bx)+\frac{a^2}{b}\frac{1}{a+bx}\right]$

(d) $\frac{1}{b^2}\left[x+\frac{a}{b}-\frac{2a}{b}\log(a+bx)-\frac{a^2}{b}\frac{1}{a+bx}\right]$

119. $\int \frac{x^2}{(x\sin x+\cos x)^2}dx=$

(a) $\frac{\sin x+\cos x}{x\sin x+\cos x}$ (b) $\frac{x\sin x-\cos x}{x\sin x+\cos x}$ (c) $\frac{\sin x-x\cos x}{x\sin x+\cos x}$ (d) None of these

120. $\int \frac{dx}{\sqrt{x+a}+\sqrt{x+b}} =$

(a) $\frac{2}{3(b-a)}[(x+a)^{3/2}-(x+b)^{3/2}]+c$

(b) $\frac{2}{3(a-b)}[(x+a)^{3/2}-(x+b)^{3/2}]+c$

(c) $\frac{2}{3(a-b)}[(x+a)^{3/2}+(x+b)^{3/2}]+c$

(d) None of these

INDEFINITE INTEGRATION

HINTS AND SOLUTIONS

1. (b) $\int \frac{\sin x}{\sin(x-\alpha)}dx = \int \frac{\sin(x-\alpha+\alpha)}{\sin(x-\alpha)}dx$

$$= \int \frac{\{\sin(x-\alpha)\cos\alpha+\cos(x-\alpha)\sin\alpha\}}{\sin(x-\alpha)}dx$$

2. (b) $\int \left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+.......\right)dx = \int e^x dx = e^x + c.$

3. (a) $\int \frac{x-1}{(x+1)^2}dx = \int \frac{x+1-2}{(x+1)^2}dx$

$$= \int \frac{1}{x+1}dx - \int \frac{2}{(x+1)^2}dx = \log(x+1)+\frac{2}{(x+1)}+c\,.$$

4. (c) $\int \frac{dx}{\sin x+\cos x} = \frac{1}{\sqrt{2}}\int \frac{dx}{\sin x\cos\frac{\pi}{4}+\cos x\sin\frac{\pi}{4}}$

$$= \frac{1}{\sqrt{2}}\int \operatorname{cosec}\left(x+\frac{\pi}{4}\right)dx = \frac{1}{\sqrt{2}}\log\tan\left(\frac{\pi}{8}+\frac{x}{2}\right)+c.$$

5. (d) $\int (\sin 2x-\cos 2x)dx = \frac{1}{\sqrt{2}}\sin(2x-a)+b$

$$\Rightarrow -\frac{1}{2}(\sin 2x+\cos 2x) = \frac{1}{\sqrt{2}}\sin(2x-a)+b$$

$$\Rightarrow -\left[\frac{1}{\sqrt{2}}\sin 2x+\frac{1}{\sqrt{2}}\cos 2x\right] = \sin(2x-a)+b\sqrt{2}$$

$$\Rightarrow \sin\left(2x+\frac{5\pi}{4}\right) = \sin(2x-a)+b\sqrt{2}$$

6. d) $\int \frac{\cos x - 1}{\cos x + 1} dx = -\int \tan^2 \frac{x}{2} dx$

$= -\int \left(\sec^2 \frac{x}{2} - 1 \right) dx = \int \left(1 - \sec^2 \frac{x}{2} \right) dx = x - 2\tan \frac{x}{2} + c$.

7. (a) $\int (\sin^{-1} x + \cos^{-1} x) dx = \int \left(\frac{\pi}{2} \right) dx = \frac{\pi x}{2} + c$

8. (b) $\int \frac{dx}{\sin x + \sqrt{3}\cos x} = \frac{1}{2} \int \frac{dx}{\frac{\sin x}{2} + \frac{\sqrt{3}}{2}\cos x}$

$= \frac{1}{2} \int \frac{dx}{\sin\left(x + \frac{\pi}{3} \right)} = \frac{1}{2} \int \operatorname{cosec}\left(x + \frac{\pi}{3} \right)$

9. (d) $\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx = \int \left(\frac{\sin x}{\cos^2 x} + \frac{\cos x}{\sin^2 x} \right) dx$

10. (d) Rationalise

11. (c) $\int (1 + 2\tan^2 x + 2\tan x \sec x)^{1/2} dx$

$= \int (\sec^2 x + \tan^2 x + 2\tan x \sec x)^{1/2} dx$

$= \int (\sec x + \tan x) dx = \log(\sec x + \tan x) + \log \sec x + c$

$= \log \sec x (\sec x + \tan x) + c$.

12. (c) $\int \frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}} dx = \int \frac{x^5 - x^4}{x^3 - x^2} dx$

$= \int \frac{x^4(x-1)}{x^2(x-1)} dx = \int x^2 dx = \frac{x^3}{3} + c$.

13. (b) $\int e^{\log(\sin x)} dx = \int \sin x \, dx = -\cos x + c$.

14. (a) $\int \frac{1}{\sqrt{1+\cos x}} dx = \int \frac{dx}{\sqrt{2\cos^2(x/2)}} = \frac{1}{\sqrt{2}} \int \sec \frac{x}{2} dx$

$= \frac{1}{\sqrt{2}} \left\{ \log\left(\sec \frac{x}{2} + \tan \frac{x}{2} \right) \right\} . \frac{1}{1/2} = \sqrt{2} \log\left(\sec \frac{x}{2} + \tan \frac{x}{2} \right) + K$.

15. (b) $\int (\sin^4 x - \cos^4 x) dx = \int (\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) dx$

$= \int (\sin^2 x - \cos^2 x) dx = -\int (\cos^2 x - \sin^2 x) dx$

16. (d) $\int \frac{f(x)dx}{\log \sin x} = \log \log \sin x$

Differentiating both sides, we get

$\frac{f(x)}{\log \sin x} = \frac{\cot x}{\log \sin x} \Rightarrow f(x) = \cot x$.

17. (c) $\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx = \int \frac{2(\cos^2 x + \sin^2 x) - 1}{\cos^2 x} dx$

$$= \int \sec^2 x \, dx = \tan x + c .$$

18. (c) $\int \frac{\sin x + \cos x}{\sqrt{1+\sin 2x}} dx = \int \frac{\sin x + \cos x}{\sqrt{(\sin x + \cos x)^2}} dx = \int dx = x + c .$

19. (b) $\int \frac{\tan x}{(\sec x + \tan x)} dx = \int \frac{\tan x(\sec x - \tan x)}{(\sec x + \tan x)(\sec x - \tan x)} dx$

20. (a) $\int \frac{dx}{x + x \log x} = \int \frac{dx}{x(1+\log x)}$

Now putting $1 + \log x = t \Rightarrow \frac{1}{x} dx = dt,$

21. (b) $\int \frac{dx}{e^x + e^{-x}} = \int \frac{e^x}{e^{2x}+1} dx = \int \frac{dt}{t^2+1} = \tan^{-1}(t)$

$= \tan^{-1}(e^x) + c$, {Putting $e^x = t \Rightarrow e^x dx = dt$}.

22. (c) Put $x^e + e^x = t \Rightarrow e(x^{e-1} + e^{x-1})dx = dt,$

$$\int \frac{x^{e-1} + e^{x-1}}{x^e + e^x} dx = \frac{1}{e}\int \frac{dt}{t} = \frac{1}{e}\log t = \frac{1}{e}\log(x^e + e^x) + c .$$

23. (a) $\int \frac{dx}{e^x - 1} = \int \frac{e^{-x}}{1 - e^{-x}} dx$

24. (a) $\int \frac{e^x(x+1)}{\cos^2(xe^x)} = \int e^x(x+1)\sec^2(xe^x)dx$

Putting $xe^x = t \Rightarrow (x+1)e^x dx = dt$

25. (a) Put $a\cos^2 x + b\sin^2 x = t \Rightarrow 2(b-a)\sin x \cos x = dt,$

26. (b) Let $\log(\sec x + \tan x) = t \Rightarrow \sec x \, dx = dt$

Therefore $\int \sec x \log(\sec x + \tan x) dx = \int t \, dt$

$$= \frac{t^2}{2} + c = \frac{[\log(\sec x + \tan x)]^2}{2} + c.$$

27. (c) Put $b\cos x = t$.

28. (b) $\int \frac{e^{2x} - 1}{e^{2x} + 1} dx = \int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$

Now put $e^x + e^{-x} = t \Rightarrow (e^x - e^{-x})dx = dt,$

29. (c) Putting $t = \tan^{-1} x \Rightarrow dt = \frac{1}{1+x^2} dx,$

30. (a) Put $a^x = t \Rightarrow a^x \log_e a \, dx = dt,$

31. (b) $\int \frac{1}{\cos^2 x(1-\tan x)^2}dx = \int \frac{\sec^2 x\,dx}{(\tan x-1)^2}$

Put $\tan x - 1 = t \Rightarrow \sec^2 x\,dx = dt,$

32. (b) Put $t = x^2 \Rightarrow dt = 2x\,dx,$

33. (b) Put $\cos^{-1} x = t \Rightarrow -\frac{1}{\sqrt{1-x^2}}dx = dt,$

34. (c) $\int \frac{\cos 2x}{(\cos x + \sin x)^2}dx = \int \frac{(\cos x - \sin x)(\cos x + \sin x)}{(\cos x + \sin x)^2}dx$

$$= \int \frac{\cos x - \sin x}{\cos x + \sin x}dx$$

Put $t = \sin x + \cos x \Rightarrow dt = (\cos x - \sin x)dx$,

35. (c) PUT $\tan x = t.$

36. (b) Put $\sqrt{x} = t \Rightarrow \frac{1}{\sqrt{x}}dx = 2dt,$

37. (c) $\int \frac{\sin 2x}{\sin 5x \sin 3x}dx = \int \frac{\sin(5x-3x)}{\sin 5x \sin 3x}dx$

$$= \int \frac{\sin 5x \cos 3x - \cos 5x \sin 3x}{\sin 5x \sin 3x}dx$$

$$= \frac{1}{3}\log \sin 3x - \frac{1}{5}\log \sin 5x + c.$$

38. (a) Put $t = \tan x \Rightarrow dt = \sec^2 x\,dx,$

39. (d) Put $e^x = t \Rightarrow e^x dx = dt,$

40. (a) $\int \frac{\cos x - \sin x}{1+\sin 2x}dx = \int \frac{\cos x - \sin x}{(\sin x + \cos x)^2}dx$

Now put $\sin x + \cos x = t,$ then the required integral is

$-\frac{1}{\sin x + \cos x} + c$.

41. (c) $\int \frac{x^3}{\sqrt{1-x^8}}dx = \int \frac{x^3}{\sqrt{1-(x^4)^2}}$

Put $x^4 = t \Rightarrow 4x^3 dx = dt,$

42. (c) Put $x = \tan\theta \Rightarrow dx = \sec^2\theta\,d\theta,$ then

$$\int \frac{dx}{(x^2-1)\sqrt{x^2+1}} = \int \frac{\sec^2\theta\,d\theta}{(\tan^2\theta - 1)\sec\theta} = \int \frac{\cos\theta\,d\theta}{(2\sin^2\theta - 1)}$$

Again put $t = \sin\theta \Rightarrow dt = \cos\theta\,d\theta,$

43. (c) Put $t = x + \log x \Rightarrow dt = \left(1+\frac{1}{x}\right)dx,$

44. (b) Put $x = a(\sin\theta)^{2/3} \Rightarrow dx = \frac{2}{3}a(\sin\theta)^{-1/3}\cos\theta\, d\theta$

$$\therefore \int \sqrt{\frac{x}{a^3 - x^3}}\,dx = \int \frac{a^{1/2}(\sin\theta)^{1/3}\frac{2}{3}a(\sin\theta)^{-1/3}\cos\theta}{\sqrt{a^3 - a^3\sin^2\theta}}\,d\theta$$

$$= \frac{2}{3}a^{3/2}\int \frac{\cos\theta\, d\theta}{a^{3/2}\sqrt{1-\sin^2\theta}} = \frac{2}{3}\sin^{-1}\left(\frac{x}{a}\right)^{3/2} + c\,.$$

45. (a) Put $\log(x + \sqrt{1+x^2}) = t \Rightarrow \frac{1}{\sqrt{1+x^2}}dx = dt,$

46. (a) $I = \int \frac{\cos x - \sin x}{\sqrt{\sin 2x}}dx = \int \frac{\cos x - \sin x}{\sqrt{(\sin x + \cos x)^2 - 1}}dx$

Put $\sin x + \cos x = t \Rightarrow (\cos x - \sin x)dx = dt$

47. (a) $I = \int x^x(1+\log x)dx$.

Put $x^x = t$, then $x^x(1+\log x)dx = dt$

$\therefore I = \int dt \;\; \Rightarrow I = t + C \;\; \Rightarrow I = x^x + C$.

48. (a) $I = \int \frac{dx}{x\sqrt{x^4 - 1}}$

Put $x^2 = t \Rightarrow 2x\,dx = dt \Rightarrow dx = \frac{dt}{2x} = \frac{dt}{2\sqrt{t}}$

$\therefore\; I = \int \frac{dt}{2t\sqrt{t^2-1}} = \frac{1}{2}\sec^{-1}t + k = \frac{1}{2}\sec^{-1}x^2 + k$

49. (a) $\int \frac{(x^4 - x)^{1/4}}{x^5}dx = \int \frac{1}{x^4}\left(1 - \frac{1}{x^3}\right)^{1/4}dx$

$$= \frac{1}{3}\int t^{1/4}dt = \frac{4}{15}t^{5/4} + c = \frac{4}{15}\left(1 - \frac{1}{x^3}\right)^{5/4} + c$$

50. (a) $I = \int \left(1 + \frac{1}{x^2}\right)e^{x - \frac{1}{x}}dx$. Put $x - \frac{1}{x} = t \Rightarrow \left(1 + \frac{1}{x^2}\right)dx = dt$

$\therefore\; I = \int e^t dt = e^t + c = e^{x - \frac{1}{x}} + c$.

51. (a) $\int \frac{1}{[(x-1)^3(x+2)^5]^{1/4}}dx = \int \frac{1}{\left(\frac{x-1}{x+2}\right)^{3/4}(x+2)^2}dx$

$= \frac{1}{3}\int \frac{1}{t^{3/4}}dt$, $\quad \left\{\because \frac{x-1}{x+2} = t \Rightarrow \frac{3}{(x+2)^2}dx = dt\right\}$

$$= \frac{1}{3}\left(\frac{t^{1/4}}{1/4}\right) + c = \frac{4}{3}t^{1/4} + c = \frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4} + c\,.$$

52. (c) $f(x)=\dfrac{x}{1+x^2}$, $\therefore\ I=\int f(x)=\int \dfrac{x}{1+x^2}dx$

Put $1+x^2=t \Rightarrow 2x\,dx=dt \Rightarrow x\,dx=dt/2$

$\therefore\ I=\dfrac{1}{2}\int \dfrac{dt}{t}=\dfrac{1}{2}\log t+c$; $I=\dfrac{1}{2}\log(1+x^2)+c$.

53. (c) $I=\int\sqrt{\dfrac{1+x}{1-x}}dx=\int\dfrac{1+x}{\sqrt{1-x^2}}dx$

$=\int\dfrac{dx}{\sqrt{1-x^2}}+\int\dfrac{x}{\sqrt{1-x^2}}dx=\sin^{-1}x-\sqrt{1-x^2}+c$.

54. (d) Put $\sin x+\cos x=t\Rightarrow(\cos x-\sin x)dx=dt$

$\Rightarrow -(\sin x-\cos x)dx=dt$

55. (b) $\int\left\{\dfrac{\log x-1}{1+(\log x)^2}\right\}^2 dx$. Put $\log x=t\Rightarrow dx=e^t dt$

$\therefore$ Integral $=\int e^t\left[\dfrac{1}{1+t^2}-\dfrac{2t}{(1+t^2)^2}\right]dt$

56. (b) $\int\dfrac{\sin x\,dx}{\sin x-\cos x}=\dfrac{1}{2}\int\dfrac{2\sin x}{\sin x-\cos x}dx$

$=\dfrac{1}{2}\int\dfrac{(\sin x-\cos x+\sin x+\cos x)}{\sin x-\cos x}dx$

$=\dfrac{1}{2}\int\left(1+\dfrac{\sin x+\cos x}{\sin x-\cos x}\right)dx=\dfrac{1}{2}[x+\log(\sin x-\cos x)]+c$.

57. (b) $f(x)=\int\dfrac{x^2dx}{(1+x^2)\left(1+\sqrt{1+x^2}\right)}$

Let $x=\tan\theta, dx=\sec^2\theta\,d\theta=(1+x^2).d\theta$

58. (a) Let $\int\dfrac{dx}{\sin(x-a)\sin(x-b)}$

$=\dfrac{1}{\sin(a-b)}\int\dfrac{\sin\{(x-b)-(x-a)\}}{\sin(x-a)\sin(x-b)}dx$

$=\dfrac{1}{\sin(a-b)}\int\dfrac{\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)}{\sin(x-a)\sin(x-b)}dx$

$=\dfrac{1}{\sin(a-b)}\left[\int\cot(x-a)dx-\int\cot(x-b)dx\right]$

$=\dfrac{1}{\sin(a-b)}[\log\sin(x-a)-\log\sin(x-b)]+c$

$=\dfrac{1}{\sin(a-b)}\log\left|\dfrac{\sin(x-a)}{\sin(x-b)}\right|+c$.

59. (b) $\int x\cos^2 x\,dx = \frac{1}{2}\int x(1+\cos 2x)dx$

$$= \frac{x^2}{4} + \frac{1}{2}\left[\frac{x\sin 2x}{2} - \int \frac{\sin 2x}{2}dx\right] + c$$

$$= \frac{x^2}{4} + \frac{x\sin 2x}{4} + \frac{\cos 2x}{8} + c.$$

60. (a) $\int x.\tan^{-1} x\,dx = \frac{x^2}{2}\tan^{-1} x - \frac{1}{2}\int \frac{x^2+1-1}{1+x^2}dx$

$$= \frac{x^2}{2}\tan^{-1} x - \frac{1}{2}x + \frac{1}{2}\tan^{-1} x + c$$

$$= \frac{1}{2}\tan^{-1} x.(x^2+1) - \frac{1}{2}x + c\,.$$

61. (b) $\int \log x\,dx = x\log x - \int x.\frac{1}{x}dx + c = x\log x - x + c$

62. (a) $\int \frac{\log x}{(x+1)^2}dx = \int \log x\,(x+1)^{-2}dx$

$$= \log x.\left\{-(x+1)^{-1}\right\} - \int \frac{1}{x}.\{-(x+1)^{-1}\}dx$$

$$= \frac{-\log x}{(x+1)} + \int \frac{1}{x(x+1)}dx = \frac{-\log x}{(x+1)} + \int\left[\frac{1}{x} - \frac{1}{x+1}\right]dx$$

$$= \frac{-\log x}{x+1} + \log x - \log(x+1)\,.$$

63. (d) $\int \left(\frac{2+\sin 2x}{1+\cos 2x}\right)e^x\,dx = \int \left(\frac{2e^x}{1+\cos 2x}\right)dx + \int \frac{e^x\sin 2x}{1+\cos 2x}dx$

$$= \int e^x\sec^2 x\,dx + \int e^x\tan x\,dx = e^x\tan x + c\,.$$

64. (b) $\int \left[\log(\log x) + \frac{1}{(\log x)^2}\right]dx = \int \log(\log x)dx + \int \frac{1}{(\log x)^2}dx$

$$= x\log(\log x) - \int \frac{x}{x\log x}dx + \int \frac{1}{(\log x)^2}dx$$

$$= x\log(\log x) - \frac{x}{\log x} - \int \frac{1}{(\log x)^2}dx + \int \frac{1}{(\log x)^2}dx$$

$$= x\log(\log x) - \frac{x}{\log x} + c.$$

65. (d) $\int e^{2x}(2\cos x - \sin x)dx = e^{2x}\cos x + c$

66. (a) Let $I = \int \cos(\log_e x)dx = \int \cos(\log_e x).1\,dx$

67. (d) $\int x^n\log x\,dx = \log x.\frac{x^{n+1}}{n+1} - \int \frac{x^{n+1}}{n+1}.\frac{1}{x}dx$

68. (b) Let $I = \int e^x\sin x\,dx = e^x\sin x - \int e^x\cos x\,dx + c$

69. (c) $\int \frac{xe^x}{(1+x)^2}dx = \int \frac{(x+1-1)}{(1+x)^2}e^x dx$

$$= \int e^x\left(\frac{1}{1+x}-\frac{1}{(1+x)^2}\right)dx = \frac{e^x}{1+x}+c.$$

70. (c) $I = \int e^x(1+\tan x+\tan^2 x)dx$

$\Rightarrow \int e^x(1+\tan x+\tan^2 x)dx = \int e^x(\tan x+\sec^2 x)dx.$ $\quad I = e^x \tan x + c$

71. (d) $\int e^x\left[\frac{1+x\log x}{x}\right]dx = \int e^x\left(\log x+\frac{1}{x}\right)dx = e^x \log x + c.$

72. (d) $\int \sin(\log x)dx + \int \cos(\log x)dx$

$$= x\sin(\log x) - \int \frac{x\cos(\log x)}{x}dx + \int \cos(\log x)dx + c$$

$$= x\sin(\log x) + c.$$

73. (c) $\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx = e^x\frac{1}{x}+c$

74. (b) $\int \frac{x-\sin x}{1-\cos x}dx = \int \frac{x}{1-\cos x}dx - \int \frac{\sin x}{1-\cos x}dx$

$$= \frac{1}{2}\int x\,\mathrm{cosec}^2\left(\frac{x}{2}\right)dx - \int \frac{2\sin(x/2)\cos(x/2)}{2\sin^2(x/2)}dx$$

$$= \frac{1}{2}\int x\,\mathrm{cosec}^2\left(\frac{x}{2}\right)dx - \int \cot\left(\frac{x}{2}\right)dx = -x\cot\left(\frac{x}{2}\right)+c.$$

75. (b) $I = \int \cos^{-1}\left(\frac{1}{x}\right)dx = \int \sec^{-1} x.1\, dx$

$$= \sec^{-1} x\int dx - \int\left[\frac{d}{dx}\sec^{-1} x\int dx\right]dx$$

$$= x\sec^{-1} x - \int \frac{1}{x\sqrt{x^2-1}}x.\, dx$$

$$= x\sec^{-1} x - \int \frac{1}{\sqrt{x^2-1}}dx = x\sec^{-1} x - \cosh^{-1} x + c.$$

76. (d) $\int x\sin^2 x\, dx = \int x.\frac{(1-\cos 2x)}{2}dx$

$$= \frac{1}{2}\left[\int x\, dx - \int x.\cos 2x\, dx\right] = \frac{x^2}{4}-\frac{x}{4}\sin 2x - \frac{1}{8}\cos 2x + c.$$

77. (a) $\int \frac{e^x(x^2+1)}{(x+1)^2}dx = \int \frac{e^x(x^2-1+2)}{(x+1)^2}dx$

$$= \int e^x\left[\frac{x-1}{x+1}+\frac{2}{(x+1)^2}\right]dx = \int e^x[f(x)+f'(x)]dx$$

78. (b) $\int \left[\frac{1}{\log x} - \frac{1}{(\log x)^2}\right] dx = \int \frac{1}{\log x} dx - \int \frac{1}{(\log x)^2} dx$

$= \frac{x}{\log x} + \int \frac{1}{(\log x)^2} \cdot \frac{1}{x} x\, dx - \int \frac{1}{(\log x)^2} dx + c = \frac{x}{\log x} + c$.

79. (c) $\int e^{2x} \frac{1+\sin 2x}{1+\cos 2x} dx = \int e^{2x} \left[\frac{1}{1+\cos 2x} + \frac{\sin 2x}{1+\cos 2x}\right] dx$

$= \int e^{2x} \left[\frac{\sec^2 x}{2} + \tan x\right] dx$

$= \frac{1}{2} \int e^{2x} \sec^2 x\, dx + \int e^{2x} \tan x\, dx$

80. (c) $I = \int \frac{(x+3)e^x}{(x+4)^2} dx = \int \frac{(x+4-1)e^x dx}{(x+4)^2}$

$\Rightarrow I = \int e^x \left(\frac{1}{x+4} - \frac{1}{(x+4)^2}\right) dx$

81. (a) Put $t = \sin^{-1} x \Rightarrow \sin t = x \Rightarrow \cos t\, dt = dx$,

82. (c) $I = \int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx = \int e^x \left[\frac{1+2\sin(x/2)\cos(x/2)}{2\cos^2(x/2)}\right] dx$

$I = \int e^x \left[\frac{1}{2}\sec^2(x/2) + \tan(x/2)\right] dx = e^x . \tan(x/2) + c$

83. (a) Put $\tan^{-1} x = t$ and $\frac{dx}{1+x^2} = dt$,

84. (c) $I = \int e^x (1 - \cot x + \cot^2 x) dx = \int e^x (-\cot x + \operatorname{cosec}^2 x) dx$

$= e^x(-\cot x) + c = -e^x \cot x + c$.

85. (d) Put $x = \sin\theta \Rightarrow dx = \cos\theta\, d\theta$,

86. (b) $\int \frac{x-1}{(x-3)(x-2)} dx$

$= \int \frac{x-3}{(x-3)(x-2)} dx + \int \frac{2}{(x-3)(x-2)} dx$

$= \log\left[\frac{(x-2)(x-3)^2}{(x-2)^2}\right] + c = \log\left[\frac{(x-3)^2}{(x-2)}\right] + c.$

87. (c) $\int \frac{dx}{(x^2+1)(x^2+4)} = \frac{1}{3}\left[\int \frac{dx}{x^2+1} - \int \frac{dx}{x^2+4}\right]$

$= \frac{1}{3}\left[\tan^{-1} x - \frac{1}{2}\tan^{-1}\frac{x}{2}\right] + c = \frac{1}{3}\tan^{-1} x - \frac{1}{6}\tan^{-1}\frac{x}{2} + c$.

88. (c) Put $\sin x = t \Rightarrow \cos x\, dx = dt$,

$\int \frac{\cos x}{(1+\sin x)(2+\sin x)} dx = \int \frac{dt}{(t+1)(t+2)} \quad = \int \frac{1}{t+1} dt - \int \frac{1}{t+2} dt = \log\left(\frac{t+1}{t+2}\right) + c = \log\left(\frac{\sin x + 1}{\sin x + 2}\right) + c$.

89. (c) $\int \frac{x}{(x^2-a^2)(x^2-b^2)}dx$

$$= \frac{1}{a^2-b^2}\left[\int \frac{x}{x^2-a^2}dx - \int \frac{x\,dx}{x^2-b^2}\right].$$

90. (d) $\int \frac{1}{\cos x(1+\cos x)}dx = \int \frac{dx}{\cos x} - \int \frac{dx}{1+\cos x}$

$$= \int \sec x\; dx - \frac{1}{2}\int \sec^2 \frac{x}{2}dx$$

91. (d) $\int \frac{1}{x-x^3}dx = \int \frac{1}{x(1+x)(1-x)}dx$

$$= \frac{1}{2}\int \left(\frac{2}{x} - \frac{1}{1+x} + \frac{1}{1-x}\right)dx$$

92. (c) $\int \frac{dx}{(x+1)(x+2)} = \int \left(\frac{1}{x+1} - \frac{1}{x+2}\right)dx$

93. (b) $\int \frac{e^x}{(1+e^x)(2+e^x)}dx = \int \left\{\frac{e^x}{1+e^x} - \frac{e^x}{2+e^x}\right\}dx$

94. (c) $\int \frac{dx}{1+\cos^2 x} = \int \frac{\sec^2 x\,dx}{\sec^2 x+1} = \int \frac{\sec^2 x}{\tan^2 x+2}dx$

$$= \int \frac{dt}{t^2+2} = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t}{\sqrt{2}}\right)+c \qquad \{\text{Putting } \tan x = t\}$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{1}{\sqrt{2}}\tan x\right)+c.$$

95. (d) We have $I = \int \frac{dx}{x(x^5+1)} = \int \frac{dx}{x^6\left(1+\frac{1}{x^5}\right)}$

Put $1+\frac{1}{x^5} = t \Rightarrow \frac{-5}{x^6}dx = dt$

$$\Rightarrow I = -\frac{1}{5}\int \frac{dt}{t} = -\frac{1}{5}\log t + c$$

$$I = -\frac{1}{5}\log\left(1+\frac{1}{x^5}\right)+c = -\frac{1}{5}\log\left(\frac{x^5+1}{x^5}\right)+c$$

$$\therefore\; I = \frac{1}{5}\log\left(\frac{x^5}{x^5+1}\right)+c.$$

96. (c) $\int \frac{e^x dx}{e^{2x}+e^x-2} = \int \frac{dt}{t^2+t-2} \qquad \{\because\; e^x = t \Rightarrow e^x dx = dt\}$

$$= \int \frac{dt}{(t+2)(t-1)} = \int \frac{1}{3}\left[\frac{1}{t-1} - \frac{1}{t+2}\right]dt$$

97. (a) $\int \frac{1}{(x^2+b^2)(x^2+a^2)}dx$

$$= \frac{1}{a^2-b^2}\int \left[\frac{1}{x^2+b^2} - \frac{1}{x^2+a^2}\right]dx$$

98. (a) Put $\log x = t \Rightarrow \frac{1}{x}dx = dt,$

99. (b) $\int \frac{dx}{\sqrt{2x-x^2}} = \int \frac{dx}{\sqrt{1-(x-1)^2}} = \sin^{-1}(x-1)+c.$

100. (d) $\int \frac{x^2+1}{x^4+1}dx = \int \frac{\left(1+\frac{1}{x^2}\right)}{\left(x^2+\frac{1}{x^2}\right)}dx = \int \frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x-\frac{1}{x}\right)^2+2}$

Put $x-\frac{1}{x} = t \Rightarrow \left(1+\frac{1}{x^2}\right)dx = dt,$

101. (a) $I = \frac{dx}{7+5\cos x} = \int \frac{dx}{7+5\left(\frac{1-\tan^2(x/2)}{1+\tan^2(x/2)}\right)}$

$= \int \frac{\sec^2(x/2)dx}{7+7\tan^2(x/2)+5-5\tan^2(x/2)}$

$= \int \frac{\sec^2(x/2)dx}{12+2\tan^2(x/2)} = \int \frac{\frac{1}{2}\sec^2(x/2).dx}{6+\tan^2(x/2)}$

Put $\tan\frac{x}{2} = t \Rightarrow \frac{1}{2}\sec^2\frac{x}{2}dx = dt$

102. (a) Resolve In To Partia Fractions

103. (d) The given function can be written as $\int \frac{\left(1-\frac{1}{x^2}\right)}{\left(x+\frac{1}{x}\right)^2-1}dx$

Put $x+\frac{1}{x} = t \Rightarrow \left(1-\frac{1}{x^2}\right)dx = dt,$ then it reduces to

$\int \frac{dt}{t^2-1} = \frac{1}{2}\log\left|\frac{t-1}{t+1}\right|+c$

104. (b) $\int \frac{dx}{\cos(x-a)\cos(x-b)}$

$= \frac{1}{\sin(a-b)}\int \frac{\sin\{(x-b)-(x-a)\}}{\cos(x-a).\cos(x-b)}dx$

$= \frac{1}{\sin(a-b)}\int \left\{\frac{\sin(x-b)}{\cos(x-b)} - \frac{\sin(x-a)}{\cos(x-a)}\right\}dx$

$= \operatorname{cosec}(a-b)\log\frac{\cos(x-a)}{\cos(x-b)}+c$.

105. (b) $\int \frac{\sin^8 x-\cos^8 x}{1-2\sin^2 x\cos^2 x}dx$

$= \int \frac{(\sin^4 x+\cos^4 x)(\sin^4 x-\cos^4 x)}{(\sin^2 x+\cos^2 x)^2-2\sin^2 x\cos^2 x}dx$

$$= \int (\sin^4 x - \cos^4 x)\,dx$$

$$= \int (\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x)\,dx$$

$$= \int (\sin^2 x - \cos^2 x)\,dx = \int -\cos 2x\,dx = -\frac{\sin 2x}{2} + c.$$

106. (a) Put $x = 3\sin\theta \Rightarrow dx = 3\cos\theta\,d\theta,$ therefore

$$\int \frac{x^2}{(9 - x^2)^{3/2}}\,dx = \int \frac{9\sin^2\theta}{(9 - 9\sin^2\theta)^{3/2}}.3\cos\theta\,d\theta$$

$$= \int \frac{27\sin^2\theta\cos\theta}{27\cos^3\theta}\,d\theta = \int \tan^2\theta\,d\theta = \int (\sec^2\theta - 1)\,d\theta$$

107. (c) $\displaystyle\int \frac{dx}{4\sin^2 x + 5\cos^2 x} = \int \frac{\sec^2 x\,dx}{4\tan^2 x + 5} = \frac{1}{4}\int \frac{\sec^2 x\,dx}{\tan^2 x + \frac{5}{4}}$

Put $\tan x = t \Rightarrow \sec^2 x\,dx = dt,$ then it reduces to

$$\frac{1}{4}\int \frac{dt}{t^2 + \left(\frac{\sqrt{5}}{2}\right)^2} = \frac{2}{4\sqrt{5}}\tan^{-1}\left(\frac{2t}{\sqrt{5}}\right) + c$$

108. (a) $I = \displaystyle\int \sqrt{\frac{a - x}{x}}\,dx$.

Put $x = a\sin^2\theta \Rightarrow dx = 2a\sin\theta\cos\theta\,d\theta,$ then

$$I = \int \sqrt{\frac{\cos^2\theta}{\sin^2\theta}}\,.2a\sin\theta\cos\theta\,d\theta$$

$$= a\int 2\cos^2\theta\,d\theta = a\int (1 + \cos 2\theta)\,d\theta$$

$$= a\left[\sin^{-1}\sqrt{\frac{x}{a}} + \sqrt{\frac{x}{a}}.\sqrt{\frac{a - x}{a}}\right] + c\ .$$

109. (c) $\displaystyle\int \frac{x^2 + 1}{x^4 - x^2 + 1}\,dx = \int \frac{\left(1 + \frac{1}{x^2}\right)}{x^2 + \frac{1}{x^2} - 1}$

$$= \int \frac{1 + \frac{1}{x^2}}{\left(x - \frac{1}{x}\right)^2 + 1}\,dx = \int \frac{dt}{t^2 + 1} = \tan^{-1} t + c$$

110. (b) $\displaystyle\int \frac{x - 1}{(x + 1)^3}e^x\,dx = \int e^x\left(\frac{(x + 1)}{(x + 1)^3} - \frac{2}{(x + 1)^3}\right)dx$

$$= \int e^x\left(\frac{1}{(x + 1)^2} - \frac{2}{(x + 1)^3}\right)dx = \frac{e^x}{(x + 1)^2} + c\ .$$

111. (a) Standard Problem

112. (a) $\int x\sqrt{\frac{1-x^2}{1+x^2}}dx = \int \frac{x.(1-x^2)}{\sqrt{1-x^4}}dx \quad = \int \frac{x}{\sqrt{1-x^4}}dx - \int \frac{x^3}{\sqrt{1-x^4}}dx$

$= \frac{1}{2}[\sin^{-1}(x^2) + \sqrt{1-x^4}] + c$.

113. (d) $I = \int \frac{4e^x + 6e^{-x}}{9e^{2x} - 4e^{-x}}dx = \frac{4}{9}\int \frac{9e^{2x}dx}{9e^{2x}-4} + 6\int \frac{dx}{9e^{2x}-4}$

$\therefore \int \frac{dx}{9e^{2x}-4} = \frac{1}{8}\log(9e^{2x}-4) - \frac{1}{4}\log 3 - \frac{1}{4}x +$ const.

$\therefore \; I = \frac{35}{36}\log(9e^{2x}-4) - \frac{3}{2}x - \frac{3}{2}\log 3 +$ const.

Comparing with the given integral, we get

$A = -\frac{3}{2}, \quad B = \frac{35}{36}, \quad C = -\frac{3}{2}\log 3 +$ const.

114. (a) Let $\sqrt{(x^2-a^2)} = t \Rightarrow x^2 - a^2 = t^2 \Rightarrow x^2 = a^2 + t^2$

$\therefore xdx = tdt$

$\therefore \int \frac{\sqrt{(x^2-a^2)}}{x}dx = \int \frac{\sqrt{(x^2-a^2)}\,x}{x^2}dx$

$\Rightarrow \quad I = \int \frac{t}{a^2+t^2}tdt = \int \frac{t^2}{a^2+t^2}dt$

$\Rightarrow \quad I = \int \left(1 - \frac{a^2}{a^2+t^2}\right)dt = t - a^2\frac{1}{a}\tan^{-1}\left(\frac{t}{a}\right)$

$\Rightarrow \quad I = \sqrt{(x^2-a^2)} - a\tan^{-1}\left[\frac{\left\{\sqrt{(x^2-a^2)}\right\}}{a}\right]$.

115. (a) $I_n = \int (\log x)^n dx$ .....(i)

$\therefore I_{n-1} = \int (\log x)^{n-1} dx$ .....(ii)

Now, $I_n = \int (\log x)^n . dx = (\log x)^n x - n\int (\log x)^{n-1}\frac{1}{x}x\,dx$

$= x(\log x)^n - n\int (\log x)^{n-1}dx$

$I_n = x(\log x)^n - nI_{n-1}$; $\therefore I_n + n\,I_{n-1} = x(\log x)^n$.

116. (a) $I = \int \frac{dx}{\sin x(1+2\cos x)} = \int \frac{\sin x\,dx}{\sin^2 x(1+2\cos x)}$

$= \int \frac{\sin x\,dx}{(1-\cos x)(1+\cos x)(1+2\cos x)}$

Now differential coefficient of $\cos x$ is $-\sin x$ which is given in numerator and hence we make the substitution $\cos x = t \Rightarrow -\sin x\, dx = dt$

117. (a) Let $\sec 2x = t,$ then $\sec 2x \tan 2x\, dx = \frac{1}{2}dt$

$$\therefore \frac{1}{2}\int (t^2-1)dt = \frac{1}{6}t^3 - \frac{1}{2}t + c = \frac{1}{6}\sec^3 2x - \frac{1}{2}\sec 2x + c\,.$$

118. (d) Put $a+bx = t \Rightarrow x = \frac{t-a}{b}$ and $dx = \frac{dt}{b}$

$$\therefore\ I = \int \left(\frac{t-a}{b}\right)^2 \times \frac{1}{t^2}\frac{dt}{b}$$

119. (c) Differentiation of $x\sin x + \cos x$ is $x\cos x,$ then

$$I = \int \frac{x^2 dx}{(x\sin x+\cos x)^2} = \int \frac{x\cos x}{(x\sin x+\cos x)^2}.\frac{x}{\cos x}dx$$

Integrate by parts $\left[\int \frac{1}{t^2}dt = -\frac{1}{t}\right]$

$$\therefore\ I = \frac{-1}{(x\sin x+\cos x)}.\frac{x}{\cos x}$$

$$+\int \frac{1}{(x\sin x+\cos x)}.\frac{\cos x.1 - x(-\sin x)}{\cos^2 x}dx$$

$$= -\frac{1}{x\sin x+\cos x}.\frac{x}{\cos x} + \int \sec^2 x\, dx$$

$$= -\frac{1}{x\sin x+\cos x}.\frac{x}{\cos x} + \frac{\sin x}{\cos x}$$

$$= \frac{-x + x\sin^2 x + \sin x\cos x}{(x\sin x+\cos x)\cos x}$$

$$= \frac{\sin x\cos x - x(1-\sin^2 x)}{(x\sin x+\cos x)\cos x} = \frac{\sin x - x\cos x}{x\sin x+\cos x}\,.$$

120. (b) $\int \frac{dx}{\sqrt{x+a}+\sqrt{x+b}} = \int \frac{\sqrt{x+a}-\sqrt{x+b}}{(x+a)-(x+b)}dx$

$$= \frac{1}{(a-b)}\int (x+a)^{1/2}dx - \frac{1}{(a-b)}\int (x+b)^{1/2}dx$$

$$= \frac{2}{3(a-b)}[(x+a)^{3/2} - (x+b)^{3/2}] + c.$$