QUADRATIC EQUATIONS

OBJECTIVE PROBLEMS

1. The solution of the equation $x+\frac{1}{x}=2$ **will be**

(a) 2, –1 (b) 0, –1, $-\frac{1}{5}$

(c) $-1, -\frac{1}{5}$ (d) None of these

2. The roots of the given equation $(p-q)x^2+(q-r)x+(r-p)=0$ **are**

(a) $\frac{p-q}{r-p}, 1$ (b) $\frac{q-r}{p-q}, 1$

(c) $\frac{r-p}{p-q}, 1$ (d) $1, \frac{q-r}{p-q}$

3. If $x^2+y^2=25, xy=12$ **, then** $x=$

(a) {3, 4} (b) {3, –3}

(c) {3, 4, –3, –4} (d) {–3, –3}

4. The roots of the equation $a(x^2+1)-(a^2+1)x=0$ **are**

(a) $a, \frac{1}{a}$ (b) a, 2a

(c) $a, \frac{1}{2a}$ (d) None of these

5. The value of $2+\cfrac{1}{2+\cfrac{1}{2+..........\infty}}$ **is**

(a) $1-\sqrt{2}$ (b) $1+\sqrt{2}$

(c) $1\pm\sqrt{2}$ (d) None of these

6. The number of real solutions of the equation $|x|^2-3|x|+2=0$ **are**

(a) 1 (b) 2

(c) 3 (d) 4

7. The roots of the equation $x^4-8x^2-9=0$ **are**

(a) $\pm3, \pm1$ (b) $\pm3, \pm i$

(c) $\pm2, \pm i$ (d) None of these

8. Let one root of $ax^2+bx+c=0$ where a,b,c are integers be $3+\sqrt{5}$, then the other root is

(a) $3-\sqrt{5}$ (b) 3

(c) $\sqrt{5}$ (d) None of these

9. The roots of the equation $\sqrt{3x+1}+1=\sqrt{x}$ are

(a) 0 (b) 1

(c) 0, 1 (d) None

10. The value of $x=\sqrt{2+\sqrt{2+\sqrt{2+.....}}}$ is

(a) –1 (b) 1

(c) 2 (d) 3

11. If $P(x)=ax^2+bx+c$ and $Q(x)=-ax^2+dx+c$ where $ac\neq 0$, then $P(x).Q(x)=0$ has at least

(a) Four real roots (b) Two real roots

(c) Four imaginary roots (d)None of these

12. The real roots of the equation $x^2+5|x|+4=0$ are

(a) – 1, 4 (b) 1, 4

(c) – 4, 4 (d) None of these

13. If the roots of the equation $(p^2+q^2)x^2-2q(p+r)x+(q^2+r^2)=0$ be real and equal, then p,q,r will be in

(a) A.P. (b) G.P.

(c) H.P. (d) None of these

14. Let α and β be the roots of the equation $x^2+x+1=0$ The equation whose roots are α^{19},β^7 is

(a) $x^2-x-1=0$ (b) $x^2-x+1=0$

(c) $x^2+x-1=0$ **(d)** $x^2+x+1=0$

15. If the product of the roots of the equation $2x^2+6x+\alpha^2+1=0$ is $-\alpha$, then the value of α will be

(a) –1 (b) 1

(c) 2 (d) –2

16. If $x^{2/3}-7x^{1/3}+10=0$, then $x=$

(a) {125} (b) {8}

(c) ϕ (d) {125, 8}

17. The number of roots of the equation $|x|^2 - 7|x| + 12 = 0$ **is**

(a) 1 (b) 2

(c) 3 (d) 4

18. The equation $\sqrt{(x+1)} - \sqrt{(x-1)} = \sqrt{(4x-1)}$ **has**

(a) No solution (b) One solution (c)Two solutions (d) More than two solutions

19. The number of solutions of $\log_4(x-1) = \log_2(x-3)$

(a) 3 (b) 1

(c) 2 (d) 0

20. If the roots of the given equation $2x^2 + 3(\lambda - 2)x + \lambda + 4 = 0$ **be equal in magnitude but opposite in sign, then** $\lambda =$

(a) 1 (b) 2

(c) 3 (d) 2/3

21. If a root of the equation $x^2 + px + 12 = 0$ **is 4, while the roots of the equation** $x^2 + px + q = 0$ **are same, then the value of** q **will be**

(a) 4 (b) 4/49

(c) 49/4 (d) None of these

22. The equation $e^x - x - 1 = 0$ **has**

(a) Only one real root $x = 0$

(b) At least two real roots

(c) Exactly two real roots

(d) Infinitely many real roots

23. The number of solutions for the equation $x^2 - 5|x| + 6 = 0$ **is**

(a) 4 (b) 3

(c) 2 (d) 1

24. The values of '*a*' and '*b*' for which equation $x^4 - 4x^3 + ax^2 + bx + 1 = 0$ **have four real roots**

(a) – 6, – 4 (b) – 6, 5

(c) – 6, 4 (d) 6, – 4

25. If $a + b + c = 0,\ a \neq 0, a, b, c \in Q$**, then both the roots of the equation** $ax^2 + bx + c = 0$ **are**

(a) Rational (b) Non-real

(c) Irrational (d) Zero

26. If $a+b+c=0$, then the roots of the equation $4ax^2+3bx+2c=0$ are

(a) Equal (b) Imaginary

(c) Real (d) None of these

27. If the roots of the given equation $(\cos p-1)x^2+(\cos p)x+\sin p=0$ are real, then

(a) $p\in(-\pi,0)$ (b) $p\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ (c) $p\in(0,\pi)$ (d) $p\in(0,2\pi)$

28. The roots of the equation

$(a^2+b^2)t^2-2(ac+bd)t+(c^2+d^2)=0$ are equal, then

(a) $ab=dc$ (b) $ac=bd$

(c) $ad+bc=0$ (d) $\frac{a}{b}=\frac{c}{d}$

29. The expression $x^2+2bx+c$ has the positive value if

(a) $b^2-4c>0$ (b) $b^2-4c<0$

(c) $c^2<b$ (d) $b^2<c$

30. If the roots of the equations $px^2+2qx+r=0$

and $qx^2-2\sqrt{pr}x+q=0$ be real, then

(a) $p=q$ (b) $q^2=pr$

(c) $p^2=qr$ (d) $r^2=pq$

31. If l,m,n are real and $l\neq m$, then the roots of the equation $(l-m)x^2-5(l+m)x-2(l-m)=0$ are

(a) Complex (b) Real and distinct

(c) Real and equal (d) None of these

32. The least integer *k* which makes the roots of the equation $x^2+5x+k=0$ imaginary is

(a) 4 (b) 5

(c) 6 (d) 7

33. The condition for the roots of the equation,

$(c^2-ab)x^2-2(a^2-bc)x+(b^2-ac)=0$ **to be equal is**

(a) $a=0$ (b) $b=0$

(c) $c=0$ (d) None of these

34. Roots of $ax^2+b=0$ are real and distinct if

(a) $ab>0$ (b) $ab<0$

(c) $a,b>0$ (d) $a,b<0$

35. The expression $y = ax^2 + bx + c$ **has always the same sign as *c* if**

(a) $4ac < b^2$ (b) $4ac > b^2$

(c) $ac < b^2$ (d) $ac > b^2$

36. $x^2 + x + 1 + 2k(x^2 - x - 1) = 0$ **is a perfect square for how many values of *k***

(a) 2 (b) 0

(c) 1 (d) 3

37. Let $p, q \in \{1, 2, 3, 4\}$**. The number of equations of the form** $px^2 + qx + 1 = 0$ **having real roots is**

(a) 15 (b) 9

(c) 7 (d) 8

38. If the roots of equation $x^2 + a^2 = 8x + 6a$ **are real, then**

(a) $a \in [2, 8]$ (b) $a \in [-2, 8]$

(c) $a \in (2, 8)$ (d) $a \in (-2, 8)$

39. If a root of the equation $ax^2 + bx + c = 0$ **be reciprocal of a root of the equation then** $a'x^2 + b'x + c' = 0$**, then**

(a) $(cc' - aa')^2 = (ba' - cb')(ab' - bc')$

(b) $(bb' - aa')^2 = (ca' - bc')(ab' - bc')$

(c) $(cc' - aa')^2 = (ba' + cb')(ab' + bc')$

(d) None of these

40. If α **and** β **are the roots of the equation** $4x^2 + 3x + 7 = 0$**, then** $\frac{1}{\alpha} + \frac{1}{\beta}$ **=**

(a) $-\frac{3}{7}$ (b) $\frac{3}{7}$

(c) $-\frac{3}{5}$ (d) $\frac{3}{5}$

41. If the roots of the equation $Ax^2 + Bx + C = 0$ **are** α, β **and the roots of the equation** $x^2 + px + q = 0$ **are** α^2, β^2**, then value of *p* will be**

(a) $\frac{B^2 - 2AC}{A^2}$ (b) $\frac{2AC - B^2}{A^2}$

(c) $\frac{B^2 - 4AC}{A^2}$ (d) None of these

42. If α **and** β **are the roots of the equation** $x^2 - a(x + 1) - b = 0$ **then** $(\alpha + 1)(\beta + 1) =$

(a) b (b) $-b$

(c) $1 - b$ (d) $b - 1$

43. If the sum of the roots of the quadratic equation $ax^2+bx+c=0$ is equal to the sum of the squares of their reciprocals, then $a/c, b/a, c/b$ are in

(a) A.P. (b) G.P. (c) H.P. (d) None of these

44. If the roots of the equation $x^2+2mx+m^2-2m+6=0$ are same, then the value of *m* will be

(a) 3 (b) 0 (c) 2 (d) –1

45. If the sum of the roots of the equation $ax^2+bx+c=0$ be equal to the sum of their squares, then

(a) $a(a+b)=2bc$ (b) $c(a+c)=2ab$

(c) $b(a+b)=2ac$ (d) $b(a+b)=ac$

46. If α,β are the roots of $x^2+px+1=0$ and γ,δ are the roots of $x^2+qx+1=0$, then $q^2-p^2=$

(a) $(\alpha-\gamma)(\beta-\gamma)(\alpha+\delta)(\beta+\delta)$

(b) $(\alpha+\gamma)(\beta+\gamma)(\alpha-\delta)(\beta+\delta)$

(c) $(\alpha+\gamma)(\beta+\gamma)(\alpha+\delta)(\beta+\delta)$

(d) None of these

47. If $2+i\sqrt{3}$ is a root of the equation $x^2+px+q=0$, where *p* and *q* are real, then $(p,q)=$

(a) $(-4,7)$ (b) $(4,-7)$

(c) (4, 7) (d) $(-4,-7)$

48. If the roots of the equation $ax^2+bx+c=0$ be α and β, then the roots of the equation $cx^2+bx+a=0$ are

(a) $-\alpha,-\beta$ (b) $\alpha,\frac{1}{\beta}$

(c) $\frac{1}{\alpha},\frac{1}{\beta}$ (d) None of these

49. The quadratic in b, such that A.M. of its roots is A and G.M. is *G*, is

(a) $t^2-2At+G^2=0$ (b) $t^2-2At-G^2=0$

(c) $t^2+2At+G^2=0$ (d) None of these

50. If the sum of the roots of the equation $x^2+px+q=0$ is three times their difference, then which one of the following is true

(a) $9p^2=2q$ (b) $2q^2=9p$

(c) $2p^2=9q$ (d) $9q^2=2p$

51. A two digit number is four times the sum and three times the product of its digits. The number is

(a) 42 (b) 24

(c) 12 (d) 21

52. If the product of roots of the equation, $mx^2 + 6x + (2m - 1) = 0$ **is –1, then the value of *m* will be**

(a) 1 (b) – 1 (c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

53. If the roots of the equation $\frac{1}{x+p} + \frac{1}{x+q} = \frac{1}{r}$ **are equal in magnitude but opposite in sign, then the product of the roots will be**

(a) $\frac{p^2 + q^2}{2}$ (b) $-\frac{(p^2 + q^2)}{2}$

(c) $\frac{p^2 - q^2}{2}$ (d) $-\frac{(p^2 - q^2)}{2}$

54. If α, β **are the roots of the equation** $ax^2 + bx + c = 0$**, then** $\frac{\alpha}{a\beta + b} + \frac{\beta}{a\alpha + b} =$

(a) $\frac{2}{a}$ (b) $\frac{2}{b}$

(c) $\frac{2}{c}$ (d) $-\frac{2}{a}$

55. The equation formed by decreasing each root of $ax^2 + bx + c = 0$ **by 1 is** $2x^2 + 8x + 2 = 0,$ **then**

(a) $a = - b$ (b) $b = - c$

(c) $c = - a$ (d) $b = a + c$

56. If the ratio of the roots of $x^2 + bx + c = 0$ **and** $x^2 + qx + r = 0$ **be the same, then**

(a) $r^2c = b^2q$ (b) $r^2b = c^2q$

(c) $rb^2 = cq^2$ (d) $rc^2 = bq^2$

57. If the ratio of the roots of $ax^2 + 2bx + c = 0$ **is same as the ratio of the roots of** $px^2 + 2qx + r = 0$**, then**

(a) $\frac{b}{ac} = \frac{q}{pr}$ (b) $\frac{b^2}{ac} = \frac{q^2}{pr}$

(c) $\frac{2b}{ac} = \frac{q^2}{pr}$ (d) None of these

58. If the sum of the roots of the equation $x^2 + px + q = 0$ is equal to the sum of their squares, then

(a) $p^2 - q^2 = 0$ (b) $p^2 + q^2 = 2q$

(c) $p^2 + p = 2q$ (d) None of these

59. Let α, β be the roots of $x^2 - x + p = 0$ and γ, δ be the roots of $x^2 - 4x + q = 0$. If $\alpha, \beta, \gamma, \delta$ are in G.P., then integral values of p, q are respectively

(a) – 2, – 32 (b) – 2, 3 (c)– 6, 3 (d)– 6, – 32

60. If the roots of $ax^2 + bx + c = 0$ are α, β and the roots of $Ax^2 + Bx + C = 0$ are $\alpha - k, \beta - k,$ then $\frac{B^2 - 4AC}{b^2 - 4ac}$ is equal to

(a) 0 (b) 1

(c) $\left(\frac{A}{a}\right)^2$ (d) $\left(\frac{a}{A}\right)^2$

61. If α, β are the roots of $9x^2 + 6x + 1 = 0,$ then the equation with the roots $\frac{1}{\alpha}, \frac{1}{\beta}$ is

(a) $2x^2 + 3x + 18 = 0$ (b) $x^2 + 6x - 9 = 0$

(c) $x^2 + 6x + 9 = 0$ (d) $x^2 - 6x + 9 = 0$

62. If p and q are the roots of $x^2 + px + q = 0,$ then

(a) $p = 1, q = -2$ (b) $p = -2, q = 1$

(c) $p = 1, q = 0$ (d) $p = -2, q = 0$

63. If α, β are the roots of $ax^2 + bx + c = 0$ and $\alpha + \beta,\ \alpha^2 + \beta^2,\ \alpha^3 + \beta^3$ are in G.P., where $\Delta = b^2 - 4ac$, then

(a) $\Delta \neq 0$ (b) $b\Delta = 0$

(c) $cb \neq 0$ (d) $c\Delta = 0$

64. If $1 - i$ is a root of the equation $x^2 - ax + b = 0$, then $b =$

(a) – 2 (b) – 1

(c) 1 (d) 2

65. If α, β are the roots of the equation $x^2 + 2x + 4 = 0,$ then $\frac{1}{\alpha^3} + \frac{1}{\beta^3}$ is equal to

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$

(c) 32 (d) $\frac{1}{4}$

66. If *a* and *b* are roots of $x^2 - px + q = 0$**, then** $\frac{1}{a} + \frac{1}{b} =$

(a) $\frac{1}{p}$ (b) $\frac{1}{q}$

(c) $\frac{1}{2p}$ (d) $\frac{p}{q}$

67. Product of real roots of the equation $t^2x^2 + |x| + 9 = 0$

(a) Is always positive (b)Is always negative

(c) Does not exist (d) None of these

68. If α, β **are the roots of** $ax^2 + bx + c = 0$**, then the equation whose roots are** $2+\alpha, 2+\beta$ **is**

(a) $ax^2 + x(4a - b) + 4a - 2b + c = 0$

(b) $ax^2 + x(4a - b) + 4a + 2b + c = 0$

(c) $ax^2 + x(b - 4a) + 4a + 2b + c = 0$

(d) $ax^2 + x(b - 4a) + 4a - 2b + c = 0$

69. If one root of the equation $x^2 + px + q = 0$ **is the square of the other, then**

(a) $p^3 + q^2 - q(3p+1) = 0$ (b) $p^3 + q^2 + q(1+3p) = 0$

(c) $p^3 + q^2 + q(3p-1) = 0$ (d) $p^3 + q^2 + q(1-3p) = 0$

70. Let two numbers have arithmetic mean 9 and geometric mean 4. Then these numbers are the roots of the quadratic equation

(a) $x^2 - 18x - 16 = 0$ (b) $x^2 - 18x + 16 = 0$

(c) $x^2 + 18x - 16 = 0$ **(d)** $x^2 + 18x + 16 = 0$

71. If $\alpha \neq \beta$ **but** $\alpha^2 = 5\alpha - 3$ **and** $\beta^2 = 5\beta - 3$**, then the equation whose roots are** α/β **and** β/α **is**

(a) $3x^2 - 25x + 3 = 0$ (b) $x^2 + 5x - 3 = 0$

(c) $x^2 - 5x + 3 = 0$ (d) $3x^2 - 19x + 3 = 0$

72. Difference between the corresponding roots of $x^2 + ax + b = 0$ **and** $x^2 + bx + a = 0$ **is same and** $a \neq b$**, then**

(a) $a + b + 4 = 0$ (b) $a + b - 4 = 0$

(c) $a - b - 4 = 0$ (d) $a - b + 4 = 0$

73. If 3 is a root of $x^2 + kx - 24 = 0$**, it is also a root of**

(a) $x^2 + 5x + k = 0$ (b) $x^2 - 5x + k = 0$

(c) $x^2 - kx + 6 = 0$ (d) $x^2 + kx + 24 = 0$

74. If x, y, z are real and distinct, then

$u = x^2 + 4y^2 + 9z^2 - 6yz - 3zx - zxy$ is always

(a) Non-negative (b) Non-positive

(c) Zero (d) None of these

75. If a root of the equations $x^2 + px + q = 0$ and $x^2 + \alpha x + \beta = 0$ is common, then its value will be (where $p \neq \alpha$ and $q \neq \beta$)

(a) $\frac{q-\beta}{\alpha-p}$ (b) $\frac{p\beta-\alpha q}{q-\beta}$ (c) $\frac{q-\beta}{\alpha-p}$ or $\frac{p\beta-\alpha q}{q-\beta}$ (d) None of these

76. If $x^2 - 3x + 2$ be a factor of $x^4 - px^2 + q$, then $(p,q) =$

(a) (3, 4) (b) (4, 5)

(c) (4, 3) (d) (5, 4)

77. If the two equations $x^2 - cx + d = 0$ and $x^2 - ax + b = 0$ have one common root and the second has equal roots, then $2(b+d) =$

(a) 0 (b) $a+c$

(c) ac (d) $-ac$

78. If x is real, the expression $\frac{x+2}{2x^2+3x+6}$ takes all value in the interval

(a) $\left(\frac{1}{13}, \frac{1}{3}\right)$ (b) $\left[-\frac{1}{13}, \frac{1}{3}\right]$

(c) $\left(-\frac{1}{3}, \frac{1}{13}\right)$ (d) None of these

79. If x is real, the function $\frac{(x-a)(x-b)}{(x-c)}$ will assume all real values, provided

(a) $a > b > c$ (b) $a < b < c$

(c) $a > c < b$ (d) $a < c < b$

80. If the equation $a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x = 0$, $a_1 \neq 0$, $n \geq 2$, has a positive root $x = \alpha$, then the equation $na_n x^{n-1} + (n-1)a_{n-1}x^{n-2} + ... + a_1 = 0$ has a positive root, which is

(a) Greater than or equal to α (b)Equal to α

(c) Greater than α (d) Smaller than α

81. If α and β $(\alpha < \beta)$ are the roots of the equation $x^2 + bx + c = 0$, where $c < 0 < b$, then

(a) $0 < \alpha < \beta$ (b) $\alpha < 0 < \beta < |\alpha|$

(c) $\alpha < \beta < 0$ (d) $\alpha < 0 < |\alpha| < \beta$

82. If x is real, then the maximum and minimum values of expression $\frac{x^2+14x+9}{x^2+2x+3}$ will be

(a) 4, – 5 (b) 5, – 4

(c) – 4, 5 (d) – 4, – 5

83. If $x^2 - hx - 21 = 0, x^2 - 3hx + 35 = 0$ $(h > 0)$ has a common root, then the value of h is equal to

(a) 1 (b) 2

(c) 3 (d) 4

84. If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real and less than 3, then

(a) $a < 2$ (b) $2 \le a \le 3$ (c) $3 < a \le 4$ (d) $a > 4$

85. If $b > a$, then the equation $(x-a)(x-b) = 1$ has

(a) Both roots in $[a, b]$

(b) Both roots in $(-\infty, a)$

(c) Both roots in $(b, +\infty)$

(d) One root in $(-\infty, a)$ and the other in $(b, +\infty)$

86. If S is a set of $P(x)$ is polynomial of degree ≤ 2 such that $P(0) = 0$, $P(1) = 1$, $P'(x) > 0 \ \forall x \in (0,1)$, then

(a) $S = 0$

(b) $S = ax + (1-a)x^2 \ \forall a \in (0, \infty)$

(c) $S = ax + (1-a)x^2 \ \forall a \in R$

(d) $S = ax + (1-a)x^2 \ \forall a \in (0, 2)$

87. The smallest value of $x^2 - 3x + 3$ in the interval $(-3, 3/2)$ is

(a) 3/4 (b) 5

(c) –15 (d) –20

88. The maximum possible number of real roots of equation $x^5 - 6x^2 - 4x + 5 = 0$ is

(a) 0 (b) 3

(c) 4 (d) 5

89. The solution set of the equation $pqx^2 - (p+q)^2 x + (p+q)^2 = 0$ is

(a) $\left\{\frac{p}{q}, \frac{q}{p}\right\}$ (b) $\left\{pq, \frac{p}{q}\right\}$

(c) $\left\{\frac{q}{p}, pq\right\}$ (d) $\left\{\frac{p+q}{p}, \frac{p+q}{q}\right\}$ (e) $\left\{\frac{p-q}{p}, \frac{p-q}{q}\right\}$

90. If x is real and satisfies $x+2>\sqrt{x+4}$, then

(a) $x<-2$ (b) $x>0$

(c) $-3<x<0$ (d) $-3<x<4$

91. If α, β and γ are the roots of equation $x^3-3x^2+x+5=0$ then $y=\sum\alpha^2+\alpha\beta\gamma$ satisfies the equation

(a) $y^3+y+2=0$ (b) $y^3-y^2-y-2=0$

(c) $y^3+3y^2-y-3=0$ (d) $y^3+4y^2+5y+20=0$

92. If α, β and γ are the roots of $x^3+8=0$, then the equation whose roots are α^2, β^2 and γ^2 is

(a) $x^3-8=0$ (b) $x^3-16=0$

(c) $x^3+64=0$ (d) $x^3-64=0$.

93. If $x^2+2ax+10-3a>0$ for all $x\in R$, then

(a) $-5<a<2$ (b) $a<-5$

(c) $a>5$ (d) $2<a<5$

94. If α,β,γ are the roots of the equation $x^3+4x+1=0$, then $(\alpha+\beta)^{-1}+(\beta+\gamma)^{-1}+(\gamma+\alpha)^{-1}=$

(a) 2 (b) 3

(c) 4 (d) 5

QUADRATIC EQUATIONS

HINTS AND SOLUTIONS

1. (d) $x+\frac{1}{x}=2 \Rightarrow x+\frac{1}{x}-2=0\ (\because x\neq 0)$

$\Rightarrow x^2-2x+1=0 \Rightarrow (x-1)^2=0 \Rightarrow x=1,1$.

2. (c) Given equation is $(p-q)x^2+(q-r)x+(r-p)=0$

$$x=\frac{(r-q)\pm\sqrt{(q-r)^2-4(r-p)(p-q)}}{2(p-q)}$$

$$\Rightarrow x=\frac{(r-q)\pm(q+r-2p)}{2(p-q)} \Rightarrow x=\frac{r-p}{p-q},1$$

3. (c) $x^2+y^2=25$ and $xy=12$

$\Rightarrow x^2+\left(\frac{12}{x}\right)^2=25 \Rightarrow x^4+144-25x^2=0$

$\Rightarrow (x^2-16)(x^2-9)=0 \Rightarrow x^2=16$ and $x^2=9$

$\Rightarrow x=\pm 4$ and $x=\pm 3$.

4. (a) Equation $a(x^2+1)-(a^2+1)x=0$

$\Rightarrow ax^2-(a^2+1)x+a=0$

$\Rightarrow (ax-1)(x-a)=0 \Rightarrow x=a,\frac{1}{a}$.

5. (b) Let $x=2+\cfrac{1}{2+\cfrac{1}{2+.....\infty}}$

$\Rightarrow x=2+\frac{1}{x}$

$\Rightarrow x=1\pm\sqrt{2}$

But the value of the given expression cannot be negative or less than 2, therefore $1+\sqrt{2}$ is required answer.

6. (d) Given $|x|^2-3|x|+2=0$

$\Rightarrow (|x|-1)(|x|-2)=0$

$\Rightarrow |x|=1$ and $|x|=2 \Rightarrow x=\pm 1, x=\pm 2$.

7. (b) Equation $x^4-8x^2-9=0$

$\Rightarrow x^4-9x^2+x^2-9=0 \Rightarrow x^2(x^2-9)+1(x^2-9)=0$

$\Rightarrow (x^2+1)(x^2-9)=0 \Rightarrow x=\pm i, \pm 3$.

8. (a) If one root of a quadratic equation with rational coefficients is irrational and of the form $\alpha+\sqrt{\beta}$, then the other root must also be irrational and of the form $\alpha-\sqrt{\beta}$.

9. (d) Given equation is $\sqrt{3x+1}+1=\sqrt{x}$

$\Rightarrow \sqrt{3x+1}=\sqrt{x}-1$

Squaring on both sides, we get $3x+1=x+1-2\sqrt{x}$

$\Rightarrow 2\sqrt{x}+2x=0$ (Irrational function)

Thus $x\neq 0$ and $x\neq 1$, since equation is non-quadratic equation.

10. (c) $x=\sqrt{2+x} \Rightarrow x^2-x-2=0$

$\Rightarrow (x-2)(x+1)=0 \Rightarrow x=2,-1$

But $\sqrt{2+\sqrt{2+.....}} \neq -1$, so it is equal to 2.

11. (b) Let all four roots are imaginary. Then roots of both equations $P(x)=0$ and $Q(x)=0$ are imaginary.

Thus $b^2-4ac<0; d^2+4ac<0$, So $b^2+d^2<0$, which is impossible unless $b=0, d=0$.

So, if $b \neq 0$ or $d \neq 0$ at least two roots must be real.

If $b=0,\ d=0$, we have the equations.

$P(x)=ax^2+c=0$ and $Q(x)=-ax^2+c=0$

Or $x^2=-\frac{c}{a}; x^2=\frac{c}{a}$ as one of $\frac{c}{a}$ and $-\frac{c}{a}$ must be positive, so two roots must be real.

12. (d) $x^2+5|x|+4=0 \Rightarrow |x|^2+5|x|+4=0$

$\Rightarrow |x|=-1,-4$, which is not possible. Hence, the given equation has no real root.

13. (b) Given equation is $(p^2+q^2)x^2-2q(p+r)x+(q^2+r^2)=0$

Roots are real and equal, then

$4q^2(p+r)^2-4(p^2+q^2)(q^2+r^2)=0$

$\Rightarrow q^2(p^2+r^2+2pr)-(p^2q^2+p^2r^2+q^4+q^2r^2)=0$

$\Rightarrow q^2p^2+q^2r^2+2pq^2r-p^2q^2-p^2r^2-q^4-q^2r^2=0$

$\Rightarrow 2pq^2r-p^2r^2-q^4=0 \Rightarrow (q^2-pr)^2=0$

Hence $q^2=pr$. Thus *p, q, r* in G.P.

14. (d) Given $x^2+x+1=0$

$\therefore\ x=\frac{1}{2}[-1\pm i\sqrt{3}]=\frac{1}{2}(-1+i\sqrt{3}), \frac{1}{2}(-1-i\sqrt{3})=\omega, \omega^2$

But $\alpha^{19}=\omega^{19}=\omega$ and $\beta^7=\omega^{14}=\omega^2$.

Hence the equation will be same.

15. (a) According to condition $\frac{\alpha^2+1}{2}=-\alpha$

$\Rightarrow \alpha^2+2\alpha+1=0 \Rightarrow \alpha=-1,-1$.

16. (d) Given that $x^{2/3}-7x^{1/3}+10=0$. Given equation can be written as $(x^{1/3})^2-7(x^{1/3})+10=0$

Let $a=x^{1/3}$, then it reduces to the equation

$a^2-7a+10=0 \Rightarrow (a-5)(a-2)=0 \Rightarrow a=5,2$

Putting these values, we have $a^3=x \Rightarrow x=125$ and 8.

17. (d) The equation $(|x|-4)(|x|-3)=0$

$\Rightarrow\ |x|=4 \Rightarrow x=\pm 4 \ \Rightarrow |x|=3 \Rightarrow x=\pm 3$.

18. (a) Given $\sqrt{(x+1)}-\sqrt{(x-1)}=\sqrt{(4x-1)}$

Squaring both sides, we get $-2\sqrt{(x^2-1)}=2x-1$

Squaring again, we get $x=5/4$ which does not satisfy the given equation. Hence equation has no solution.

19. (b) $\log_4(x-1)=\log_2(x-3) \quad \Rightarrow x-1=(x-3)^2$

$\Rightarrow x^2-7x+10=0 \quad \Rightarrow (x-5)(x-2)=0$

$\therefore x=5,$ 2 but $x-3<0$ when $x=2$

$\therefore$ Only solution is $x=5$

$\therefore$ Hence number of solution is one.

20. (b) Let roots are α and $-\alpha$, then sum of the roots

$\alpha+(-\alpha)=\frac{3(\lambda-2)}{2} \Rightarrow 0=\frac{3}{2}(\lambda-2) \Rightarrow \lambda=2$

21. (c) Put $x=4$ in $x^2+px+12=0$, we get $p=-7$

Now second equation $x^2+px+q=0$ have equal roots. Therefore $p^2=4q \ \Rightarrow q=\frac{49}{4}$

22. (a) $e^x=x+1 \Rightarrow 1+\frac{x}{1!}+\frac{x^2}{2!}+......=x+1$

$\Rightarrow \frac{x^2}{2!}+\frac{x^3}{3!}+......=0$

$x^2=0, x^3=0,$ ...... $x^n=0$

Hence, $x=0$ only one real roots.

23. (a) Given equation $x^2-5|x|+6=0$

i.e., $x^2-5x+6=0$ and $x^2+5x+6=0$

$x^2-3x-2x+6=0$ and $x^2+3x+2x+6=0$

$(x-3)(x-2)=0$ and $(x+3).(x+2)=0$

$x=3, x=2$ and $x=-3, x=-2$.

i.e., Four solutions of this equation.

24. (d) Let for real roots are $\alpha,\beta,\gamma,\delta$ then equation is

$(x-\alpha)(x-\beta)(x-\gamma)(x-\delta)=0$

$$x^4-(\alpha+\beta+\gamma+\delta)x^3+(\alpha\beta+\beta\gamma+\gamma\delta+\alpha\delta+\beta\delta+\alpha\gamma)x^2-(\alpha\beta\gamma+\beta\gamma\delta+\alpha\beta\delta+\alpha\gamma\delta)x+\alpha\beta\gamma\delta=0$$

$$x^4-\sum\alpha.x^3+\sum\alpha\beta.x^2-\sum\alpha\beta\gamma.x+\alpha\beta\gamma\delta=0$$

On comparing with $x^4-4x^3+ax^2+bx+1=0$

$\sum\alpha=4, \sum\alpha\beta=a$

$\sum\alpha\beta\gamma=-b, \alpha\beta\gamma\delta=1$

Solving

$\therefore b=-4$; $\therefore a=6$ and $b=-4$.

25. (a) $D\equiv b^2-4ac=(-a-c)^2-4ac \qquad (\because a+b+c=0)$

$=(a+c)^2-4ac=(a-c)^2\geq 0$

Hence roots are rational.

26. (c) We have $4ax^2+3bx+2c=0$ Let roots are α and β

Let $D=B^2-4AC=9b^2-4(4a)(2c)=9b^2-32ac$

Given that, $(a+b+c)=0\Rightarrow b=-(a+c)$

Putting this value, we get

$=9(a+c)^2-32ac=9(a-c)^2+4ac$.

Hence roots are real.

27. (c) Given equation $(\cos p-1)x^2+(\cos p)x+\sin p=0$

Its discriminant $D\geq 0$ since roots are real

$\Rightarrow \cos^2 p-4(\cos p-1)\sin p\geq 0$

$\Rightarrow \cos^2 p-4\cos p\sin p+4\sin p\geq 0$

$\Rightarrow (\cos p-2\sin p)^2-4\sin^2 p+4\sin p\geq 0$

$\Rightarrow (\cos p-2\sin p)^2+4\sin p(1-\sin p)\geq 0$ …..(i)

Now $(1-\sin p)\geq 0$ for all real p, $\sin p>0$ for $0<p<\pi$. Therefore $4\sin p(1-\sin p)\geq 0$ when $0<p<\pi$ or $p\in(0,\pi)$

28. (d) Accordingly, $\{2(ac+bd)\}^2=4(a^2+b^2)(c^2+d^2)$

$\Rightarrow 4a^2c^2+4b^2d^2+8abcd=4a^2c^2+4a^2d^2+4b^2c^2+4b^2d^2$

$\Rightarrow 4a^2d^2+4b^2c^2-8abcd=0 \Rightarrow 4(ad-bc)^2=0$

$\Rightarrow ad=bc\Rightarrow \frac{a}{b}=\frac{c}{d}$.

29. (d) $x^2+2bx+c=(x+b)^2+c-b^2$

$\because (x+b)^2$ is a perfect square, therefore the given expression is positive if $c-b^2>0$ or $b^2<c$.

30. (b) Equations $px^2 + 2qx + r = 0$ and

$qx^2 - 2(\sqrt{pr})x + q = 0$ have real roots, then from first $4q^2 - 4pr \geq 0 \Longrightarrow q^2 - pr \geq 0 \Rightarrow q^2 \geq pr$ .....(i)

And from second $4(pr) - 4q^2 \geq 0$ (for real root)

$\Longrightarrow pr \geq q^2$ .....(ii)

From (i) and (ii), we get result $q^2 = pr$.

31. (b) Given equation is $(l-m)x^2 - 5(l+m)x - 2(l-m) = 0$

Its discriminant $D = 25\ (l+m)^2 + 8\ (l-m)^2$

Which is positive, since l, m, n are real and $l \neq m$.

Hence roots are real and distinct.

32. (d) Roots are non real if discriminant < 0

i.e. if $5^2 - 4.1k < 0$ *i.e.* if $4k > 25$ *i.e.* if $k > \frac{25}{4}$

Hence, the required least integer *k* is 7.

33. (a) According to question,

$4(a^2 - bc)^2 - 4(c^2 - ab)(b^2 - ac) = 0$

$\Longrightarrow a(a^3 + b^3 + c^3 - 3abc) = 0$

$\Longrightarrow a = 0$ or $a^3 + b^3 + c^3 = 3abc$

34. (b) $B^2 - 4AC > 0 \Longrightarrow 0 - 4ab > 0 \Longrightarrow ab < 0$.

35. (b) Let $f(x) = ax^2 + bx + c$. Then $f(0) = c$. Thus the graph of $y = f(x)$ meets *y*-axis at (0, *c*).

If $c > 0$, then by hypothesis $f(x) > 0$ This means that the curve $y = f(x)$ does not meet *x*-axis.

If $c < 0$, then by hypothesis $f(x) < 0$, which means that the curve $y = f(x)$ is always below *x*-axis and so it does not intersect with *x*-axis. Thus in both cases $y = f(x)$ does not intersect with *x*-axis *i.e.* $f(x) \neq 0$ for any real *x*. Hence $f(x) = 0$ *i.e.* $ax^2 + bx + c = 0$ has imaginary roots and so $b^2 < 4ac$.

36. (a) Given equation $(1+2k)x^2 + (1-2k)x + (1-2k) = 0$

If equation is a perfect square then roots are equal

i.e., $(1-2k)^2 - 4(1+2k)(1-2k) = 0$

i.e., $k = \frac{1}{2}, \frac{-3}{10}$. Hence total number of values = 2.

37. (c) For real roots, discriminant ≥ 0

$\Rightarrow q^2 - 4p \geq 0 \Rightarrow q^2 \geq 4p$

For $p=1, q^2 \geq 4 \Rightarrow q=2,3,4$

$p=2, q^2 \geq 8 \Rightarrow q=3,4$

$p=3, q^2 \geq 12 \Rightarrow q=4$

$p=4, q^2 \geq 16 \Rightarrow q=4$

Total seven solutions are possible.

38. (b) Since the roots $x^2-8x+a^2-6a=0$ are real.

$\therefore\ 64-4(a^2-6a) \geq 0$ Or $a^2-6a-16 \leq 0$

$\Rightarrow\ a \in [-2,8]$

39. (a) Let α be a root of first equation, and then $\frac{1}{\alpha}$ be a root of second equation.

Therefore $a\alpha^2+b\alpha+c=0$ and $a'\frac{1}{\alpha^2}+b'\frac{1}{\alpha}+c'=0$ or $c'\alpha^2+b'\alpha+a'=0$

Hence $\frac{\alpha^2}{ba'-b'c}=\frac{\alpha}{cc'-aa'}=\frac{1}{ab'-bc'}$

$(cc'-aa')^2=(ba'-cb')(ab'-bc')$.

40. (a) Given equation $4x^2+3x+7=0$, therefore

$\alpha+\beta=-\frac{3}{4}$ and $\alpha\beta=\frac{7}{4}$

Now $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{-3/4}{7/4}=\frac{-3}{4}\times\frac{4}{7}=-\frac{3}{7}$.

41. (b) α,β are the roots of $Ax^2+Bx+C=0$.

So, $\alpha+\beta=-\frac{B}{A}$ and $\alpha\beta=\frac{C}{A}$

Again α^2,β^2 are the roots of $x^2+px+q=0$ then

$\alpha^2+\beta^2=-p$ and $(\alpha\beta)^2=q$

Now $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$\Rightarrow\ \alpha^2+\beta^2=\left(-\frac{B}{A}\right)^2-2\frac{C}{A}$

$\Rightarrow\ -p=\frac{B^2-2AC}{A^2} \Rightarrow p=\frac{2AC-B^2}{A^2}$

42. (c) Given equation $x^2-a(x+1)-b=0$

$\Rightarrow\ x^2-ax-a-b=0 \Rightarrow \alpha+\beta=a, \alpha\beta=-(a+b)$

Now $(\alpha+1)(\beta+1)=\alpha\beta+\alpha+\beta+1$

$=-(a+b)+a+1=1-b$

43. (c) As given, if α,β be the roots of the quadratic equation, then $\alpha+\beta=\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha^2\beta^2}$

$\Rightarrow -\frac{b}{a}=\frac{(b^2/a^2)-(2c/a)}{(c^2/a^2)}=\frac{b^2-2ac}{c^2}$

$\Rightarrow \frac{2a}{c}=\frac{b^2}{c^2}+\frac{b}{a}=\frac{(ab^2+bc^2)}{ac^2}$

$\Rightarrow 2a^2c=ab^2+bc^2 \Rightarrow \frac{2a}{b}=\frac{b}{c}+\frac{c}{a}$

$\Rightarrow \frac{c}{a},\frac{a}{b},\frac{b}{c}$ are in A.P. $\Rightarrow \frac{a}{c},\frac{b}{a},\frac{c}{b}$ are in H.P.

44. (a) Let roots are α and α, then $\alpha+\alpha=-2m \Rightarrow \alpha=-m$

and $\alpha.\alpha=m^2-2m+6 \Rightarrow m^2=m^2-2m+6$

$\Rightarrow m=3$.

45. (c) Let α and β be two roots of $ax^2+bx+c=0$

Then $\alpha+\beta=-\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$

$\Rightarrow \alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta=\frac{b^2}{a^2}-2\frac{c}{a}$

So under condition $\alpha+\beta=a^2+\beta^2$

$\Rightarrow -\frac{b}{a}=\frac{b^2-2ac}{a^2} \Rightarrow b(a+b)=2ac$.

46. (a) As given, $\alpha+\beta=-p,\ \alpha\beta=1,\gamma+\delta=-q$ and $\gamma\delta=1$

Now, $(\alpha-\gamma)(\beta-\gamma)(\alpha+\delta)(\beta+\delta)$

$=\{\alpha\beta-\gamma(\alpha+\beta)+\gamma^2\}\{\alpha\beta+\delta(\alpha+\beta)+\delta^2\}$

$=(1+p\gamma+\gamma^2)(1-p\delta+\delta^2)=(p\gamma-q\gamma)(-p\delta-q\delta)$ (Since γ is a root of $x^2+qx+1=0$)

$\Rightarrow \gamma^2+q\gamma+1=0 \Rightarrow \gamma^2+1=-q\gamma$ and similarly $\delta^2+1=-q\delta=-\gamma\delta(p-q)(p+q)=q^2-p^2$.

47. (a) Since $2+i\sqrt{3}$ is a root, therefore $2-i\sqrt{3}$ will be other root. Now sum of the roots $=4=-p$ and product of roots $=7=q$. Hence $(p,q)=(-4,7)$.

48. (c) α,β are roots of $ax^2+bx+c=0$

$\Rightarrow \alpha+\beta=-\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$

Let the roots of $cx^2+bx+a=0$ be α',β', then

$\alpha'+\beta'=-\frac{b}{c}$ and $\alpha'\beta'=\frac{a}{c}$

but $\frac{\alpha+\beta}{\alpha\beta}=\frac{-b/a}{c/a}=\frac{-b}{c} \Rightarrow \frac{1}{\alpha}+\frac{1}{\beta}=\alpha'+\beta'$

Hence $\alpha' = \frac{1}{\alpha}$ and $\beta' = \frac{1}{\beta}$.

49. (a) If α, β are the roots, then

$$A = \frac{\alpha+\beta}{2} \Rightarrow \alpha+\beta = 2A \text{ and } G = \sqrt{\alpha\beta} \Rightarrow \alpha\beta = G^2$$

The required equation is $t^2 - 2At + G^2 = 0$.

50. (c) Let α, β are roots of $x^2 + px + q = 0$

So $\alpha + \beta = -p$ and $\alpha\beta = q$

Given that $(\alpha+\beta) = 3(\alpha-\beta) = -p \Rightarrow \alpha - \beta = \frac{-p}{3}$

Now $(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$

$\Rightarrow \frac{p^2}{9} = p^2 - 4q$ or $2p^2 = 9q$.

51. (b) It is obviously 24.

52. (c) According to condition

$$\frac{2m-1}{m} = -1 \Rightarrow 3m = 1 \Rightarrow m = \frac{1}{3}$$

53. (b) Given equation can be written as

$$x^2 + x(p+q-2r) + pq - pr - qr = 0 \qquad .....(i)$$

Whose roots are α and $-\alpha$, then the product of roots

$$-\alpha^2 = pq - pr - qr = pq - r(p+q) \qquad .....(ii)$$

And sum $0 = p + q - 2r \Rightarrow r = \frac{p+q}{2}$ .....(iii)

From (ii) and (iii), we get

$$-\alpha^2 = pq - \frac{p+q}{2}(p+q) = -\frac{1}{2}\left\{(p+q)^2 - 2pq\right\}$$

$$= -\frac{(P^2+q^2)}{2}.$$

54. (d) $\alpha + \beta = -\frac{b}{a}, \alpha\beta = \frac{c}{a}$

and $\alpha^2 + \beta^2 = \frac{(b^2 - 2ac)}{a^2}$

Now $\frac{\alpha}{a\beta + b} + \frac{\beta}{a\alpha + b} = \frac{\alpha(a\alpha+b) + \beta(a\beta+b)}{(a\beta+b)(a\alpha+b)}$

$$= \frac{a(\alpha^2+\beta^2)+b(\alpha+\beta)}{\alpha\beta a^2+ab(\alpha+\beta)+b^2} = \frac{a\frac{(b^2-2ac)}{a^2}+b\left(-\frac{b}{a}\right)}{\left(\frac{c}{a}\right)a^2+ab\left(-\frac{b}{a}\right)+b^2}$$

$$= \frac{b^2-ac-b^2}{a^2c-ab^2+ab^2} = \frac{-2ac}{a^2c} = -\frac{2}{a}.$$

55. (b) α, β be the roots of $ax^2+bx+c=0$

$\therefore \alpha+\beta=-b/a$, $\alpha\beta=c/a$

Roots are $\alpha-1, \beta-1$

Sum, $\alpha+\beta-2=(-b/a)-2=-8/2=-4$

Product, $(\alpha-1)(\beta-1)=\alpha\beta-(\alpha+\beta)+1 \quad = c/a+b/a+1=1$

$\because$ New equation is $2x^2+8x+2=0$

$\therefore b/a=2$ *i.e.* $b=2a$, also $c+b=0 \Rightarrow b=-c$.

56. (c) Let α, β be the roots of $x^2+bx+c=0$ and α', β' be the roots of $x^2+qx+r=0$.

Then $\alpha+\beta=-b, \alpha\beta=c, \alpha'+\beta'=-q, \alpha'\beta'=r$

It is given that $\frac{\alpha}{\beta}=\frac{\alpha'}{\beta'} \Rightarrow \frac{\alpha+\beta}{\alpha-\beta}=\frac{\alpha'+\beta'}{\alpha'-\beta'}$

$\Rightarrow \frac{(\alpha+\beta)^2}{(\alpha-\beta)^2}=\frac{(\alpha'+\beta')^2}{(\alpha'-\beta')^2} \Rightarrow \frac{b^2}{b^2-4c}=\frac{q^2}{q^2-4r}$

$\Rightarrow b^2r=q^2c$

57. (b) If the roots of equation $ax^2+2bx+c=0$ are in the ratio $m:n$, Then we have

$mn(2b)^2=(m+n)^2ac$ .....(i)

Also if the roots of the equation $px^2+2qx+r=0$ are also in the same ratio $m:n$, then

$mn(2q)^2=(m+n)^2pr$ .....(ii)

Dividing (i) and (ii), we get $\frac{b^2}{q^2}=\frac{(ac)}{(pr)}$ or $\frac{b^2}{ac}=\frac{q^2}{pr}$.

58. (c) Let the roots be α and $\beta \Rightarrow \alpha+\beta=-p$, $\alpha\beta=q$

Given, $\alpha+\beta=\alpha^2+\beta^2$

But $\alpha+\beta=(\alpha+\beta)^2-2\alpha\beta \Rightarrow -p=(-p)^2-2q$

$\Rightarrow p^2-2q=-p \Rightarrow p^2+p=2q$.

59. (a) Let r be the common ratio of the G.P. $\alpha, \beta, \gamma, \delta$ then $\beta = \alpha r,\ \gamma = \alpha r^2$ and $\delta = \alpha r^3$

$\therefore \alpha + \beta = 1 \Rightarrow \alpha + \alpha r = 1 \Rightarrow \alpha(1+r) = 1$ .....(i)

$\alpha\beta = p \Rightarrow \alpha(\alpha r) = p \Rightarrow \alpha^2 r = p$ .....(ii)

$\gamma + \delta = 4 \Rightarrow \alpha r^2 + \alpha r^3 = 4 \Rightarrow \alpha r^2(1+r) = 4$ .....(iii)

and $\gamma\delta = q \Rightarrow \alpha r^2.\alpha r^3 = q \Rightarrow \alpha^2 r^5 = q$ .....(iv)

$\Rightarrow (p, q) = (-2, -32)$.

60. (c) $(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta = (b^2 - 4ac)/a^2$ ......(i)

Also $\{(\alpha-k)-(\beta-k)\}^2$

$= \{(\alpha-k)+(\beta-k)\}^2 - 4(\alpha-k)(\beta-k)$

$= (-B/A)^2 - 4(C/A) = (B^2 - 4AC)/A^2$ .....(ii)

From (i) and (ii), $(b^2-4ac)/a^2 = (B^2-4AC)/A^2$

$\therefore \quad \frac{b^2-4AC}{b^2-4ac} = \left(\frac{A}{a}\right)^2$

61. (c) Given equation is $9x^2 + 6x + 1 = 0$

$\Rightarrow \alpha + \beta = \frac{-6}{9} = \frac{-2}{3}$ and $\alpha\beta = 1/9$

$\therefore \alpha - \beta = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = \sqrt{\frac{4}{9} - 4.\frac{1}{9}} = 0$

$\Rightarrow \alpha = \frac{-1}{3}, \beta = \frac{-1}{3}$

$\therefore$ Equation $x^2 - (\alpha+\beta)x + \alpha\beta = 0$

$\Rightarrow x^2 + 6x + 9 = 0$.

62. (a) $p + q = -p$ and $pq = q \Rightarrow p = 1$

and $q = -2$.

63. (d) $(\alpha^2+\beta^2)^2 = (\alpha+\beta)(\alpha^3+\beta^3)$

$\left(\frac{b^2-2ac}{a^2}\right)^2 = \left(\frac{-b}{a}\right)\left(\frac{-b^2+3abc}{a^3}\right)$

$\Rightarrow \quad 4a^2c^2 = acb^2 \Rightarrow ac(b^2-4ac) = 0$

As $a \neq 0 \Rightarrow c\Delta = 0$

64. (d) $1-i$ is a root of the equation so $x = 1-i$

$\Rightarrow (x-1) = -i \Rightarrow (x-1)^2 = (-i)^2 \Rightarrow x^2 - 2x + 2 = 0$

By comparison, $a = 2$, $b = 2$.

65. (d) Here, $\alpha+\beta=-2$ and $\alpha\beta=4$

$$\therefore \frac{1}{\alpha^3}+\frac{1}{\beta^3}=\frac{\alpha^3+\beta^3}{(\alpha\beta)^3}=\frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{(\alpha\beta)^3}$$

$$=\frac{(-2)^3-3(-2)(4)}{(4)^3}=\frac{16}{64}=\frac{1}{4}.$$

66. (d) Roots of given equation $x^2-px+q=0$ is a and b

i.e., $a+b=p$ ……(i) and $ab=q$ ……(ii)

Then $\frac{1}{a}+\frac{1}{b}=\frac{a+b}{ab}=\frac{p}{q}$.

67. (c) Note that for $t\in R, t^2x^2+|x|+9\geq 9$ and hence the given equation cannot have real roots.

68. (d) We have $\alpha+\beta=\frac{-b}{a}$ and $\alpha\beta=\frac{c}{a}$

Now sum of the roots $=2+\alpha+2+\beta=4-\frac{b}{a}$

And product of the roots $=(2+\alpha)(2+\beta)$

$$=4+\frac{c}{a}-\frac{2b}{a}=\frac{4a+c-2b}{a}$$

Hence the required equation is

$$\mathbf{x^2-x\left(4-\frac{b}{a}\right)+\frac{4a+c-2b}{a}=0}$$

$$\Rightarrow ax^2-x(4a-b)+4a+c-2b=0$$

$$\Rightarrow ax^2+x(b-4a)+4a-2b+c=0.$$

69. (d) Let root of the given equation $x^2+px+q=0$ are α and α^2.

Now, $\alpha.\alpha^2=\alpha^3=q,\ \alpha+\alpha^2=-p$

Cubing both sides, $\alpha^3+(\alpha^2)^3+3\alpha.\alpha^2(\alpha+\alpha^2)=-p^3$

$$q+q^2+3q(-p)=-p^3$$

$$p^3+q^2+q(1-3p)=0.$$

70. (b) Let the two number is x_1 and x_2

$\frac{x_1+x_2}{2}=9$ and $x_1x_2=16$

$x_1+x_2=18$ and $x_1x_2=16$

Equation $x^2-(\text{Sum of roots})x+\text{Product of roots}=0$

Required equation $x^2-18x+16=0$.

71. (d) $\alpha^2 - 5\alpha + 3 = 0$ .....(i)

$\beta^2 - 5\beta + 3 = 0$ .....(ii)

From (i) – (ii),

$\Rightarrow (\alpha^2 - \beta^2) - 5\alpha + 5\beta = 0$

$\Rightarrow \alpha^2 - \beta^2 = 5(\alpha - \beta) \Rightarrow \alpha + \beta = 5$

From (i) + (ii),

$\Rightarrow (\alpha^2 + \beta^2) - 5(\alpha + \beta) + 6 = 0$

$\Rightarrow (\alpha^2 + \beta^2) - 5.5 + 6 = 0 \Rightarrow \alpha^2 + \beta^2 = 19$

Then $(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$

$\Rightarrow 25 = 19 + 2\alpha\beta \Rightarrow \alpha\beta = 3$

Then the equation, whose roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$, is

$$x^2 - x\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) + \frac{\alpha}{\beta}.\frac{\beta}{\alpha} = 0$$

$$\Rightarrow x^2 - x\left(\frac{\alpha^2 + \beta^2}{\alpha\beta}\right) + 1 = 0$$

$$\Rightarrow x^2 - x.\frac{19}{3} + 1 = 0 \Rightarrow 3x^2 - 19x + 3 = 0.$$

72. (a) Let α_1, β_1 are the roots of the eq^n $x^2 + ax + b = 0 \Rightarrow x = \frac{-a \pm \sqrt{a^2 - 4b}}{2}$

$$\Rightarrow \alpha_1 = \frac{-a + \sqrt{a^2 - 4b}}{2}, \beta_1 = \frac{-a - \sqrt{a^2 - 4b}}{2}$$

And α_2, β_2 are the roots of the equation $x^2 + bx + a = 0$

So, $\alpha_2 = \frac{-b + \sqrt{b^2 - 4a}}{2}$, $\beta_2 = \frac{-b - \sqrt{b^2 - 4a}}{2}$

Now $\alpha_1 - \beta_1 = \sqrt{a^2 - 4b}$; $\alpha_2 - \beta_2 = \sqrt{b^2 - 4a}$

Given, $\alpha_1 - \beta_1 = \alpha_2 - \beta_2 \Rightarrow \sqrt{a^2 - 4b} = \sqrt{b^2 - 4a}$

$\Rightarrow a^2 - b^2 = -4(a - b) \Rightarrow a + b + 4 = 0$.

73. (c) Equation $x^2 + kx - 24 = 0$ has one root is 3.

$\Rightarrow 3^2 - 3k - 24 = 0 \Rightarrow k = 5$

Put $x = 3$ and $k = 5$ in options, only (c) gives the correct answer.

74. (a) $x, y, z \in R$ and distinct.

Now, $u = x^2 + 4y^2 + 9z^2 - 6yz - 3zx - 2xy$

$= \frac{1}{2}(2x^2 + 8y^2 + 18z^2 - 12yz - 6zx - 4xy)$

$= \frac{1}{2}\{(x^2 - 4xy + 4y^2) + (x^2 - 6zx + 9z^2) + (4y^2 - 12yz + 9z^2)\}$

$= \frac{1}{2}\{(x - 2y)^2 + (x - 3z)^2 + (2y - 3z)^2\}$

Since it is sum of squares. So u is always non- negative

75. (c) Let the common root be *y*. Then $y^2 + py + q = 0$ and $y^2 + \alpha y + \beta = 0$

On solving by cross multiplication, we have

$$\frac{y^2}{p\beta - q\alpha} = \frac{y}{q - \beta} = \frac{1}{\alpha - p}$$

$\therefore\ y = \frac{q - \beta}{\alpha - p}$ and $\frac{y^2}{y} = y = \frac{p\beta - q\alpha}{q - \beta}$

76. (d) $x^2 - 3x + 2$ be factor of $x^4 - px^2 + q = 0$

Hence $(x^2 - 3x + 2) = 0 \Rightarrow (x - 2)(x - 1) = 0$

$\Rightarrow x = 2, 1,$ Putting these values in given equation

So $4p - q - 16 = 0$ .....(i)

And $p - q - 1 = 0$ .....(ii)

Solving (i) and (ii), we get $(p, q) = (5, 4)$

77. (c) Let roots of $x^2 - cx + d = 0$ be α, β then roots of $x^2 - ax + b = 0$ be α, α

$\therefore\ \alpha + \beta = c, \alpha\beta = d, \alpha + \alpha = a, \alpha^2 = b$

Hence $2(b + d) = 2(\alpha^2 + \alpha\beta) = 2\alpha(\alpha + \beta) = ac$

78. (b) If the given expression be *y*, then $y = 2x^2 y + (3y - 1)x + (6y - 2) = 0$

If $y \neq 0$ then $\Delta \geq 0$ for real *x i.e.* $B^2 - 4AC \geq 0$

Or $-39y^2 + 10y + 1 \geq 0$ or $(13y + 1)(3y - 1) \leq 0$

$\Rightarrow -1/13 \leq y \leq 1/3$

If $y = 0$ then $x = -2$ which is real and this value of y is included in the above range

79. (d) Let $y = \frac{(x - a)(x - b)}{(x - c)}$

Or $y(x - c) = x^2 - (a + b)x + ab$

Or $x^2 - (a + b + y)x + ab + cy = 0$

$$\Delta = (a+b+y)^2 - 4(ab+cy)$$

$$= y^2 + 2y(a+b-2c) + (a-b)^2$$

Since x is real and y assumes all real values, we must have $\Delta \geq 0$ for all real values of y. The sign of a quadratic in y is same as of first term provided its discriminant $B^2 - 4AC < 0$

This will be so if $4(a+b-2c)^2 - 4(a-b)^2 < 0$

Or $4(a+b-2c+a-b)(a+b-2c-a+b) < 0$

Or $16(a-c)(b-c) < 0$ or $16(c-a)(c-b) = -ve$

$\therefore$ c lies between a and b i.e., $a < c < b$ .....(i)

Where $a < b$, but if $b < a$ then the above condition will be $b < c < a$ or $a > c > b$ .....(ii)

Hence from (i) and (ii) we observe that (d) is correct answer.

80. (d) Let $f(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x$;

$f(0) = 0;\quad f(\alpha) = 0$

$\Rightarrow$ $f'(x) = 0$, has atleast one root between $(0, \alpha)$

i.e., equation $na_n x^{n-1} + (n-1)a_{n-1} x^{n-2} + ... + a_1 = 0$

Has a positive root smaller than α.

81. (b) Here $D = b^2 - 4c > 0$ because $c < 0 < b$. So roots are real and unequal.

Now, $\alpha + \beta = -b < 0$ and $\alpha\beta = c < 0$

$\therefore$ One root is positive and the other negative, the negative root being numerically bigger. As $\alpha < \beta, \alpha$ is the negative root while β is the positive root. So, $|\alpha| > \beta$ and $\alpha < 0 < \beta$.

82. (a) Let $y = \dfrac{x^2 + 14x + 9}{x^2 + 2x + 3}$

$\Rightarrow$ $y(x^2 + 2x + 3) - x^2 - 14x - 9 = 0$

$\Rightarrow$ $(y-1)x^2 + (2y-14)x + 3y - 9 = 0$

For real x, its discriminant ≥ 0

i.e. $4(y-7)^2 - 4(y-1)3(y-3) \geq 0$

$\Rightarrow$ $y^2 + y - 20 \leq 0$ or $(y-4)(y+5) \leq 0$

Now, the product of two factors is negative if these are of opposite signs. So following two cases arise:

Case I: $y - 4 \geq 0$ or $y \geq 4$ and $y + 5 \leq 0$ or $y \leq -5$

This is not possible.

Case II: $y - 4 \leq 0$ or $y \leq 4$ and $y + 5 \geq 0$ or $y \geq -5$ Both of these are satisfied if $-5 \leq y \leq 4$

83. (d) Subtracting, we get $2hx = 56$ or $hx = 28$

Putting in any, $x^2 = 49$

$\therefore \left[\frac{28}{h}\right]^2 = 7^2 \Rightarrow h = 4 (h > 0)$

84. (a) Given equation is $x^2 - 2ax + a^2 + a - 3 = 0$

If roots are real, then $D \geq 0$

$\Rightarrow 4a^2 - 4(a^2 + a - 3) \geq 0 \Rightarrow -a + 3 \geq 0$

$\Rightarrow a - 3 \leq 0 \Rightarrow a \leq 3$

As roots are less than 3, hence $f(3) > 0$

$9 - 6a + a^2 + a - 3 > 0 \Rightarrow a^2 - 5a + 6 > 0$

$\Rightarrow (a-2)(a-3) > 0 \Rightarrow$ either $a < 2$ or $a > 3$

Hence $a < 2$ satisfy all.

85. (d) The equation is $x^2 - (a+b)x + ab - 1 = 0$

$\therefore$ discriminant $= (a+b)^2 - 4(ab-1) = (b-a)^2 + 4 > 0$

$\therefore$ Both roots are real. Let them be α, β where

$\alpha = \frac{(a+b) - \sqrt{(b-a)^2 + 4}}{2}$, $\beta = \frac{(a+b) + \sqrt{(b-a)^2 + 4}}{2}$

Clearly, $\alpha < \frac{(a+b) - \sqrt{(b-a)^2}}{2} = \frac{(a+b) - (b-a)}{2} = a$

$(\because b > a)$

And $\beta > \frac{(a+b) + \sqrt{(b-a)^2}}{2} = \frac{a+b+b-a}{2} = b$

Hence, one root α is less than *a* and the other root β is greater than *b*.

86. (d) Let $P(x) = bx^2 + ax + c$

As $P(0) = 0 \Rightarrow c = 0$

As $P(1) = 1 \Rightarrow a + b = 1$

$P(x) = ax + (1-a)x^2$

Now $P'(x) = a + 2(1-a)x$

As $P'(x) > 0$ for $x \in (0,1)$

Only option (d) satisfies above condition

87. (a) $x^2 - 3x + 3 = \left(x - \frac{3}{2}\right)^2 + \frac{3}{4}$

Therefore, smallest value is $\frac{3}{4}$, which lie in $\left(-3, \frac{3}{2}\right)$

88. (b) Let $f(x) = x^5 - 6x^2 - 4x + 5 = 0$

Then the number of change of sign in $f(x)$ is 2 therefore $f(x)$ can have at most two positive real roots.

Now, $f(-x) = -x^5 - 6x^4 + 4x + 5 = 0$

Then the number of change of sign is 1.

Hence $f(x)$ can have at most one negative real root. So that total possible number of real roots is 3.

89. (d) Given equation $(pq)x^2 - (p+q)^2 x + (p+q)^2 = 0$

Let solution set is $\left\{\frac{p+q}{p}, \frac{p+q}{q}\right\}$

Sum of roots = $\frac{(p+q)^2}{pq} \Rightarrow \frac{p+q}{p} + \frac{p+q}{q} = \frac{(p+q)^2}{pq}$

Similarly, product of roots = $\frac{(p+q)^2}{pq}$

$\Rightarrow \frac{p+q}{p} \times \frac{p+q}{q} = \frac{(p+q)^2}{pq}$.

90. (b) Given , $x + 2 > \sqrt{x+4} \Rightarrow (x+2)^2 > (x+4)$

$\Rightarrow x^2 + 4x + 4 > x + 4 \Rightarrow x^2 + 3x > 0$

$\Rightarrow x(x+3) > 0 \Rightarrow x < -3$ or $x > 0 \Rightarrow x > 0$.

91. (b) Given equation $x^3 - 3x^2 + x + 5 = 0$.

Then $\alpha + \beta + \gamma = 3$, $\alpha\beta + \beta\gamma + \gamma\alpha = 1$, $\alpha\beta\gamma = -5$

$y = \Sigma\alpha^2 + \alpha\beta\gamma = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha) + \alpha\beta\gamma$

$= 9 - 2 - 5 = 2$

$\therefore\ y = 2$

It satisfies the equation $y^3 - y^2 - y - 2 = 0$.

92. (d) Let $y = x^2$. Then $x = \sqrt{y}$

$\therefore\ x^3 + 8 = 0 \Rightarrow y^{3/2} + 8 = 0$

$\Rightarrow y^3 = 64 \Rightarrow y^3 - 64 = 0$

Thus the equation having roots α^2, β^2 and γ^2 is $x^3 - 64 = 0$.

93. (a) According to given condition,

$$4a^2 - 4(10 - 3a) < 0 \Rightarrow a^2 + 3a - 10 < 0$$

$$\Rightarrow (a+5)(a-2) < 0 \Rightarrow -5 < a < 2.$$

94. (c) If α, β, γ are the roots of the equation.

$$x^3 - px^2 + qx - r = 0$$

$$\therefore (\alpha+\beta)^{-1} + (\beta+\gamma)^{-1} + (\gamma+\alpha)^{-1} = \frac{p^2+q}{pq-r}$$

Given, $p = 0,\ q = 4,\ r = -1$

$$\Rightarrow \frac{p^2+q}{pq-r} = \frac{0+4}{0+1} = 4.$$