HYPERBOLIC FUNCTIONS

OBJECTIVES

1. **If** $x = \log(y + \sqrt{y^2 + 1})$, **then** $y =$

(a) $\tanh x$ (b) $\coth x$

(c) $\sinh x$ (d) $\cosh x$

2. **Find real part of** $\cos^{-1}\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)$

(a) $\pi/3$ (b) $\pi/4$

(c) $\log\left(\frac{\sqrt{3}-1}{2}\right)$ (d) None of these

3. **The value of** $\cosh^{-1}(\sec x)$ **is**

(a) $\log\left(\frac{1+\sin x}{\cos x}\right)$ (b) $\log\left(\frac{1-\sin x}{\cos x}\right)$

(c) $\log\left(\frac{1+\cos x}{\sin x}\right)$ (d) $\log\left(\frac{1-\cos x}{\sin x}\right)$

4. **The period of** $\sinh\left(\frac{x}{2}\right)$ **is**

(a) $2\pi i$ (b) 2π

(c) $4\pi i$ (d) 4π

5. **Find real part of** $\tan^{-1}(1+i)$

(a) $-\frac{1}{2}\tan^{-1}(2)$ (b) $\frac{1}{2}\tan^{-1}(2)$

(c) $-\frac{1}{2}\tan^{-1}\left(\frac{1}{2}\right)$ (d) 0

6. $\sinh^{-1}(\sinh^{-1}\theta)$ **is equal to**

(a) $i\theta$ (b) θ

(c) $-i\theta$ (d) $\pi + i\theta$

7. **If** $-\frac{\pi}{2} < x < \frac{\pi}{2}$, **then the value of** $\log \sec x$ **is**

(a) $2\coth^{-1}\left(\text{cosec}^2 \frac{x}{2} - 1\right)$ (b) $2\coth^{-1}\left(\text{cosec}^2 \frac{x}{2} + 1\right)$

(c) $2\text{cosech}^{-1}\left(\cot^2 \frac{x}{2} - 1\right)$ (d) $2\text{cosech}^{-1}\left(\cot^2 \frac{x}{2} + 1\right)$

8. $\sinh^2 x$ **equals**

(a) $\cosh 2x - 1$ (b) $\cosh^2 x + 1$

(c) $\frac{1}{2}(\cosh 2x - 1)$ (d) $\frac{1}{2}(\cosh 2x + 1)$

9. $\frac{1 + \tanh x}{1 - \tanh x}$ **is equal to**

(a) e^{2x} (b) e^{-2x}

(c) i (d) -1

10. **If** $u = \log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$, **then the value of** $\tanh \frac{u}{2}$ **is**

(a) $\cot \frac{x}{2}$ (b) $-\cot \frac{x}{2}$

(c) $-\tan \frac{x}{2}$ (d) $\tan \frac{x}{2}$

11. $\text{cosech}^{-1} x$ **equals**

(a) $\log\left(\frac{1 + \sqrt{1 + x^2}}{x}\right)$ (b) $\log\left(\frac{1 + \sqrt{1 - x^2}}{x}\right)$

(c) $\log\left(\frac{1 - \sqrt{1 - x^2}}{x}\right)$ (d) $\log\left(\frac{1 - \sqrt{1 + x^2}}{x}\right)$

12. **If** $\tanh x = \frac{3}{4}$, **then the value of** x **is**

(a) $\sqrt{7}$ (b) $-\sqrt{7}$

(c) $\log \sqrt{7}$ (d) $-\log \sqrt{7}$

13. **If** $x = \log\left(\frac{1}{y} + \sqrt{\frac{1}{y^2} + 1}\right)$, **then** y **is equal to**

(a) $\tanh x$ (b) $\cosh x$

(c) $\sinh x$ (d) $\text{cosech} x$

14. $\cosh^{-1} x =$

(a) $\log(x+\sqrt{x^2+1})$ (b) $\log(x-\sqrt{x^2+1})$

(c) $\log(x-\sqrt{x^2-1})$ (d) $\log(x+\sqrt{x^2-1})$

15. $\tanh^{-1} x =$

(a) $\frac{1}{2}\log\left(\frac{x+1}{x-1}\right)$ (b) $\frac{1}{2}\log\left(\frac{x-1}{x+1}\right)$

(c) $\frac{1}{2}\log\left(\frac{1-x}{1+x}\right)$ (d) $\frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$

16. Which of the following functions is not defined at $x=0$

(a) $\tanh x$ (b) $\text{cosech}\, x$

(c) $\sin x$ (d) $\text{sech} x$

17. The value of $\sinh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ **is**

(a) $\tanh^{-1} x$ (b) $\coth^{-1} x$

(c) $\sinh^{-1}(2x)$ (d) $\cosh^{-1}(2x)$

18. $\tanh(x+y)$ **equals**

(a) $\frac{\tanh x+\tanh y}{1-\tanh x\tanh y}$ (b) $\frac{\tanh x+\tanh y}{1+\tanh x\tanh y}$

(c) $\frac{\tanh x-\tanh y}{1-\tanh x\tanh y}$ (d) $\frac{\tanh x-\tanh y}{1+\tanh x\tanh y}$

19. If $\cosh^{-1} x=\log(2+\sqrt{3})$, **then** $x=$

(a) 2 (b) 1

(c) 3 (d) 5

20. $\log(3+2\sqrt{2})=$

(a) $\sinh^{-1} 3$ (b) $\cosh^{-1} 3$

(c) $\tanh^{-1} 3$ (d) $\cosh^{-1} 3$

21. $\coth^{-1} x$ **equals**

(a) $\frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$ (b) $\frac{1}{2}\log\left(\frac{x+1}{x-1}\right)$

(c) $\frac{1}{2}\log\left(\frac{x-1}{x+1}\right)$ (d) None of these

22. **The general value of** $\cosh^{-1} x$ **is**

(a) $2\pi ri + \log(x + \sqrt{x^2 + 1})$

(b) $2\pi ri + \log(x + \sqrt{x^2 - 1})$

(c) $\pi ri + (-1)^r \log(x + \sqrt{x^2 + 1})$

(d) $2\pi ri + (-1)^r \log(x + \sqrt{x^2 - 1})$

23. **If** $u = \log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$, **then** $\cosh u$ **is equal to**

(a) $\sec x$ (b) $\operatorname{cosec} x$

(c) $\tan x$ (d) $\sin x$

24. **If** $\cosh z = \sec\theta$, **then** $\sinh z$ **equals**

(a) $\operatorname{cosec}\theta$ (b) $\cot\theta$

(c) $\tan\frac{\theta}{2}$ (d) $\tan\theta$

25. **If** $\tan\left(\frac{x}{2}\right)\coth\left(\frac{x}{2}\right) = 1$, **then the value of** $\cos x \cosh x$ **is**

(a) 1 (b) – 1

(c) $\cos^2 x$ (d) $\sinh^2 x$

26. **The value of** $2\coth^{-1}\left(\frac{z}{2}\right)$ **is**

(a) $\log\left(\frac{z-2}{z+2}\right)$ (b) $\frac{1}{2}\log\left(\frac{z-1}{z+1}\right)$

(c) $\frac{1}{2}\log\left(\frac{z+1}{z-1}\right)$ (d) $-\log\left(\frac{z-2}{z+2}\right)$

27. $\operatorname{sech}^{-1}(\sin x)$ **equals**

(a) $\log\cot\frac{x}{2}$ (b) $\log\tan\frac{x}{2}$

(c) $\log\cot x$ (d) None of these

28. **The value of** $\tanh^{-1}(2^{-1})$ **is**

(a) $\log 2$ (b) $\log\ 2^{-1}$

(c) $\log\sqrt{3}$ (d) None of these

29. The value of cosh (log 3) =

1) 10/3 2) 4/3 3) 5/3 4) 2/3

30. The value of sinh (log 2) =

1) 5/4 2) 3/4 3) 1/4 4) 1/2

31. **If u = log tan$\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$ then cosh u =**

1) sec θ 2) cosecθ 3) tanθ 4) cot θ

32. **If x = log $\left[\cot\left(\frac{\pi}{4}+\theta\right)\right]$ then sinhx =**

1) tan2θ 2) - tan2θ 3) sec2θ 4) - cosec2θ

33. **If $\tanh^2 x = \tan^2\theta$ then cosh2x =**

1) cos2θ 2) sec 2θ 3) sin2θ 4) cosθ

34. **If $\tan h^2 x = \sin^2\theta$ then sechx =**

1) sinθ 2) | cosθ | 3) tanθ 4) cos θ

35. **If coshx = secθ then $\tan h^2\frac{x}{2}$=**

1) $\tan^2\theta$ 2) $\cot^2\frac{\theta}{2}$ 3) $\cot^2\theta$ 4) $\tan^2\frac{\theta}{2}$

36. **If $\tan h^{-1}\frac{1}{2} = \frac{1}{2}$ log k then k =**

1) 2 2) 3 3) 2/3 4) 3/2

37. **If sinhx= $\frac{3}{4}$, sinh2x =**

1) $\frac{15}{4}$ 2) $\frac{15}{16}$ 3) $\frac{15}{8}$ 4) $\frac{15}{2}$

38. **If sinx .coshy = cosθ, cosx.sinhy = sinθ then $\sinh^2 y$ =**

1) $\cosh^2 x$ 2) $\cos^2 hy$ 3) $\cos^2 x$ 4) $\sec^2 x$

39. $\sinh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) =$

1) $\coth^{-1}x$ 2) $-\coth^{-1}x$ 3) $-\tanh^{-1}x$ 4) $\tanh^{-1}x$

40. **For x > 1, if $\cosh^{-1}x = 2\log_e(1+\sqrt{2})$ then x =**

1) 2 2) 3 3) ½ 4) 0

41. **Domain of $\text{sech}^{-1}(2x-1)$ is**

1) [0, 1] 2) (1/2, 1] 3) [1/2, 1] 4) R

42. $\tanh^{-1}\frac{1}{3} + \coth^{-1}3 =$

1) $\frac{1}{4}\log_e^2$ 2) $\frac{1}{2}\log_e^2$ 3) $\log_e^2$ 4) $2\log_e^2$

HYPERBOLIC FUNCTIONS

HINTS AND SOLUTIONS

1. (c) $x = \log(y + \sqrt{y^2+1}) = \sinh^{-1} y \Rightarrow y = \sinh x$.

2. (b) $\because$ Expression $\cos^{-1}(\cos\theta + i\sin\theta)$

$= \sin^{-1}\sqrt{\sin\theta} - i\log(\sqrt{\sin\theta} + \sqrt{1+\sin\theta})$, where $\theta = \frac{\pi}{6}$

$\therefore$ Real part of $\cos^{-1}\left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right) = \sin^{-1}\sqrt{\frac{1}{2}} = \frac{\pi}{4}$.

3. (a) Here, $\cosh^{-1}(\sec x) = \log(\sec x + \sqrt{\sec^2 x - 1})$

$= \log(\sec x + \tan x) = \log\left(\frac{1+\sin x}{\cos x}\right)$.

4. (c) Since period of $\sinh x$ is $2\pi i$, therefore period of $\sinh\left(\frac{x}{2}\right)$ will be $4\pi i$.

5. (a) Real part = $\frac{1}{2}\tan^{-1}\frac{2(1)}{1-1-1} = -\frac{1}{2}\tan^{-1}(2)$.

6. (c) Given that, $\sinh^{-1} x = \text{cosech}^{-1} y$

Or $\sinh^{-1} x = \sinh^{-1}\left(\frac{1}{y}\right)$

Or $x = \sinh\left\{\sinh^{-1}\left(\frac{1}{y}\right)\right\}$ or $x = \frac{1}{y} \Rightarrow xy = 1$.

7. (a) Let $\log\sec x = y$

$\therefore \frac{1}{\cos x} = \frac{e^{y/2}}{e^{-y/2}}$

By componendo and Dividendo rule,

$\frac{1+\cos x}{1-\cos x} = \frac{e^{y/2}+e^{-y/2}}{e^{y/2}-e^{-y/2}} \Rightarrow \cot^2\left(\frac{x}{2}\right) = \coth\left(\frac{y}{2}\right)$

$\Rightarrow y = 2\coth^{-1}\left(\text{cosec}^2\frac{x}{2} - 1\right)$.

8. (c) $\sinh^2 x = \frac{1}{2}(\cosh 2x - 1)$

9. (a) $\left(\frac{1+\tanh x}{1-\tanh x}\right)=\left(\frac{1+\frac{e^x-e^{-x}}{e^x+e^{-x}}}{1-\frac{e^x-e^{-x}}{e^x+e^{-x}}}\right)=\frac{e^x+e^{-x}+e^x-e^{-x}}{e^x+e^{-x}-e^x+e^{-x}}$

$=\left(\frac{2e^x}{2e^{-x}}\right)=e^{2x}$.

10. (d) $u=\log\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)=\log\left(\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right)$

$=2\tanh^{-1}\left(\tan\frac{x}{2}\right)\Rightarrow\tanh\left(\frac{u}{2}\right)=\tan\frac{x}{2}$.

11. (a) Concept.

12. (c) $x=\tanh^{-1}(3/4)=\frac{1}{2}\log\left(\frac{1+3/4}{1-3/4}\right)=\frac{1}{2}\log 7=\log\sqrt{7}$.

13. (d) Concept.

14. (d) Concept.

15. (d) Concept.

16. (b) $\text{cosech}x$ is not defined at $x=0$.

17. (a) Let $x=\tanh y$, then $\frac{x}{\sqrt{1-x^2}}=\frac{\tanh y}{\text{sec h}y}=\sinh y$

$\therefore\ \sinh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)=\sinh^{-1}(\sinh y)\Rightarrow y=\tanh^{-1}(x)$.

18. (b) Concept.

19. (a) $\cosh^{-1}x=\log(x+\sqrt{x^2-1})=\log(2+\sqrt{3})$

$\therefore\ x=2$

20. (b) $\log(3+2\sqrt{2})=\log(3+\sqrt{8})=\log(3+\sqrt{9-1})$

$=\log(3+\sqrt{3^2-1})=\cosh^{-1}3$.

21. (b)Concept.

22. (b) Concept.

23. (a) $u=\log\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$

$$\Rightarrow \frac{e^u}{1} = \frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}$$

$$\cosh u = \frac{e^u + e^{-u}}{2} = \frac{e^{2u}+1}{2e^u} = \frac{\left(\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right)^2 + 1}{2.\left(\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right)}$$

$$= \frac{2\left(1+\tan^2\frac{x}{2}\right)}{2\left(1-\tan\frac{x}{2}\right)\left(1+\tan\frac{x}{2}\right)} = \frac{1+\tan^2\frac{x}{2}}{1-\tan^2\frac{x}{2}}$$

$$= \frac{1}{\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}} = \frac{1}{\cos x} = \sec x\,.$$

24. (d) We know that $\cosh^2 z - \sinh^2 z = 1$

$$\sinh^2 z = \cosh^2 z - 1$$

$$\Rightarrow \sinh^2 z = \sec^2\theta - 1$$

$$\sinh^2 z = \tan^2\theta$$

25. (a) We have $\tan\frac{x}{2} = \tanh\frac{x}{2} \Rightarrow \frac{\tan^2 x/2}{1} = \frac{\tanh^2 x/2}{1}$

$$\Rightarrow \frac{1+\tan^2 x/2}{1-\tan^2 x/2} = \frac{1+\tanh^2 x/2}{1-\tanh^2 x/2}$$

$$\Rightarrow \frac{1}{\cos x} = \cosh x \Rightarrow \cos x \cosh x = 1\,.$$

26. (d) We know that $2\coth^{-1} x = \log\left(\frac{x+1}{x-1}\right)$

$$\therefore 2\cot h^{-1}\left(\frac{z}{2}\right) = \log\left(\frac{\frac{z}{2}+1}{\frac{z}{2}-1}\right) = \log\left(\frac{z+2}{z-2}\right) = -\log\left(\frac{z-2}{z+2}\right).$$

27. (a) $\sec h^{-1}(\sin x) = \log\left(\frac{1+\sqrt{1-(\sin x)^2}}{\sin x}\right) = \log\left(\frac{1+\cos x}{\sin x}\right)$

$$= \log\left(\frac{2\cos^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right) = \log\left(\cot\frac{x}{2}\right).$$

28. (c) We know that, $\tanh^{-1} x = \frac{1}{2}\log\left(\frac{1+x}{1-x}\right)$

$$\therefore \tanh^{-1}\left(\frac{1}{2}\right) = \frac{1}{2}\log\left(\frac{1+\frac{1}{2}}{1-\frac{1}{2}}\right) = \frac{1}{2}\log\left(\frac{\frac{3}{2}}{\frac{1}{2}}\right) = \frac{1}{2}\log(3)$$

$$= \log(3)^{\frac{1}{2}} = \log\sqrt{3}.$$

29. (c)

30.(b)

31.(a)

32.(b)

33.(b)

34.(b)

35.(d)

36.(b)

37.(c)

38.(c)

39.(d)

40.(b)

41.(b)

42.(c)