TRIGONOMETRIC EQUATIONS

OBJECTIVES

1. **The most general value of θ satisfying $\sin^2\theta = \frac{1}{4}$ is**

   1) $n\pi \pm \frac{\pi}{6}$ 2) $2n\pi \pm \left(\frac{\pi}{6} \text{ or } \frac{5\pi}{6}\right)$ 3) $(2n-1)\pi \pm \frac{\pi}{6}$ 4) $2n\pi \pm \frac{\pi}{3}$

2. **The solution of $7\sin^2 x + 3\cos^2 x = 4$ is**

   1) $2n\pi \pm \left(\frac{\pi}{6} \text{ or } \frac{5\pi}{6}\right)$ 2) $2n\pi \pm \left(\frac{\pi}{3} \text{ or } \frac{2\pi}{3}\right)$ 3) $n\pi \pm \frac{\pi}{3}$ 4) $n\pi \pm \frac{2\pi}{3}$

3. **The solution of the equation $\tan^2\theta + \cot^2\theta = 2$ is**

   1) $\theta = n\pi \pm \frac{\pi}{4}, n \in z$ 2) $\theta = n\pi \pm \frac{\pi}{3}, n \in z$

   3) $\theta = 2n\pi \pm \frac{\pi}{4}, n \in z$ 4) $\theta = 2n\pi \pm \frac{\pi}{3}, n \in z$

4. **The general solution of the equation $\sin\theta + \cos\theta = -\sqrt{2}$ is**

   1) $n\pi + (-1)^n \frac{\pi}{2} - \frac{\pi}{4}$ 2) $n\pi - \frac{3\pi}{4}$ 3) $2n\pi - \frac{3\pi}{4}$ 4) $2n\pi + \frac{3\pi}{4}$

5. **If $\sqrt{3}\cos\theta - \sin\theta$ is positive and $\theta \in (-\pi, \pi)$ the value of θ lies in**

   1) $\left(-\frac{\pi}{6}, \frac{5\pi}{6}\right)$ 2) $\left(-\frac{\pi}{3}, \frac{2\pi}{3}\right)$ 3) $\left(-\frac{\pi}{4}, \frac{3\pi}{4}\right)$ 4) $\left(\frac{-2\pi}{3}, \frac{\pi}{3},\right)$

6. **The equation $\sqrt{3}\sin x + \cos x = 4$ has**

   1) only one solution 2) Two Solutions

   3) infinitely many solutions 4) No Solution

7. **The general solution of the equation $\tan\theta + \tan 4\theta + \tan 7\theta = \tan\theta\tan 4\theta\tan 7\theta$ is**

   1) $\theta = \frac{n\pi}{2}, n \in z$ 2) $\theta = \frac{n\pi}{12}, n \in z$ 3) $\theta = \frac{n\pi}{7}, n \in z$ 4) $\theta = \frac{n\pi}{4}$

8. **The general value of 'θ' satisfying $\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta\tan 2\theta = \sqrt{3}$ is**

   1) $(n+1)\frac{\pi}{9}$ 2) $(n+1)\frac{\pi}{3}$ 3) $(3n+1)\frac{\pi}{3}$ 4) $(3n+1)\frac{\pi}{9}$

9. The general value of 'θ' satisfying the equation $\tan\theta \tan(120^0 + \theta) \tan(120^0 - \theta) = \frac{1}{\sqrt{3}}$ is

1) $(6n+1)\frac{\pi}{18}$ 2) $(3n+1)\frac{\pi}{3}$ 3) $(6n+1)\frac{\pi}{6}$ 4) $(2n+1)\frac{\pi}{6}$

10. The number of solutions of the equation $\sin 5\theta = \frac{1}{2}$ lying in $[0, \pi]$ is

1) 3 2) 6 3) 9 4) 10

11. If $2\tan^2\theta = \sec^2\theta$, then the general value of θ is

(a) $n\pi + \frac{\pi}{4}$ (b) $n\pi - \frac{\pi}{4}$ (c) $n\pi \pm \frac{\pi}{4}$ (d) $2n\pi \pm \frac{\pi}{4}$

12. If $\cot\theta + \tan\theta = 2\operatorname{cosec}\theta$, the general value of θ is

(a) $n\pi \pm \frac{\pi}{3}$ (b) $n\pi \pm \frac{\pi}{6}$ (c) $2n\pi \pm \frac{\pi}{3}$ (d) $2n\pi \pm \frac{\pi}{6}$

13. If $\tan m\theta = \tan n\theta$, then the general value of θ will be in

(a) A. P. (b) G. P. (c) H. P. (d) None of these

14. If $\sin\left(\frac{\pi}{4}\cot\theta\right) = \cos\left(\frac{\pi}{4}\tan\theta\right)$, then $\theta =$

(a) $n\pi + \frac{\pi}{4}$ (b) $2n\pi \pm \frac{\pi}{4}$ (c) $n\pi - \frac{\pi}{4}$ (d) $2n\pi \pm \frac{\pi}{6}$

15. General value of θ satisfying the equation $\tan^2\theta + \sec 2\theta = 1$ is

(a) $m\pi, n\pi + \frac{\pi}{3}$ (b) $m\pi, n\pi \pm \frac{\pi}{3}$ (c) $m\pi, n\pi \pm \frac{\pi}{6}$ (d) None of these

(Where *m* and *n* are integers)

16. The solution of the equations $x + y = \frac{2\pi}{3}$ and $\cos x + \cos y = \frac{3}{2}$ where x and y are real is

1) $x = -\frac{\pi}{3}, y = \pi$ 2) $x = \pi, y = \frac{-\pi}{3}$ 3) $x = \pi, y = \frac{\pi}{2}$ 4) doesn't exist

17. If $4\sin^4 x + \cos^4 x = 1$, then x =

(a) $n\pi$ (b) $n\pi \pm \sin^{-1}\frac{2}{5}$ (c) $n\pi + \frac{\pi}{6}$ (d) None of these

18. The solution of the equation $\begin{vmatrix} \cos\theta & \sin\theta & \cos\theta \\ -\sin\theta & \cos\theta & \sin\theta \\ -\cos\theta & -\sin\theta & \cos\theta \end{vmatrix} = 0$, is

(a) $\theta = n\pi$ (b) $\theta = 2n\pi \pm \frac{\pi}{2}$ (c) $\theta = n\pi \pm (-1)^n\frac{\pi}{4}$ (d) $\theta = 2n\pi \pm \frac{\pi}{4}$

19. If $\sin 5x + \sin 3x + \sin x = 0$ **, then the value of *x* other than 0 lying between** $0 \le x \le \frac{\pi}{2}$ **is**

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{12}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

20. If $\tan(\pi \cos\theta) = \cot(\pi \sin\theta)$**, then** $\sin\left(\theta + \frac{\pi}{4}\right)$ **equals**

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{1}{2\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

21. If $(1+\tan\theta)(1+\tan\phi) = 2$ **, then** $\theta + \phi$ **=**

(a) 30^o (b) 45^o (c) 60^o (d) 75^o

22. The solutions of the equation 4 cosθ - 3 secθ = 2 tanθ are

1) $\frac{\pi}{10}, \frac{3\pi}{10}$ **2)** $-\frac{\pi}{10}, \frac{3\pi}{10}$ **3)** $\frac{\pi}{10}, -\frac{3\pi}{10}$ **4)** $-\frac{\pi}{10}, -\frac{3\pi}{10}$

23. The general value of 'θ' satisfying tanθ + 4 cot2θ + 1 = 0 is

1) $n\pi + \frac{\pi}{4}, n\pi + \tan^{-1} 2$ 2) $n\pi - \frac{\pi}{4}, n\pi + \tan^{-1} 2$

3) $n\pi + \frac{\pi}{4}, n\pi - \tan^{-1} 2$ 4) $2n\pi \pm \frac{\pi}{4}$

24. The equation $\sin x + \sin y + \sin z = -3$ **for** $0 \le x \le 2\pi,\ \ 0 \le y \le 2\pi,\ \ 0 \le z \le 2\pi$ **, has**

(a) One solution (b) Two sets of solutions

(c) Four sets of solutions (d) No solution

25. If $2\sin^2\theta = 3\cos\theta,$ **where** $0 \le \theta \le 2\pi$ **, then** $\theta =$

(a) $\frac{\pi}{6}, \frac{7\pi}{6}$ (b) $\frac{\pi}{3}, \frac{5\pi}{3}$ (c) $\frac{\pi}{3}, \frac{7\pi}{3}$ (d) None of these

26. The value of θ **in between** 0^o **and** 360^o **and satisfying the equation** $\tan\theta + \frac{1}{\sqrt{3}} = 0$ **is equal to**

(a) $\theta = 150^o$ and 300^o (b) $\theta = 120^o$ and 300^o (c) $\theta = 60^o$ and 240^o (d) $\theta = 150^o$ and 330^o

27. The most general value of θ **satisfying the equations** $\tan\theta = -1$ **and** $\cos\theta = \frac{1}{\sqrt{2}}$ **is**

(a) $n\pi + \frac{7\pi}{4}$ (b) $n\pi + (-1)^n \frac{7\pi}{4}$ (c) $2n\pi + \frac{7\pi}{4}$ (d) None of these

28. **The value of** θ **lying between 0 and** $\pi/2$ **and satisfying the equation**

$$\begin{vmatrix} 1+\sin^2\theta & \cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & 1+\cos^2\theta & 4\sin 4\theta \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$$

(a) $\frac{7\pi}{24}$ or $\frac{11\pi}{24}$ (b) $\frac{5\pi}{24}$ (c) $\frac{\pi}{24}$ (d) None of these

29. $\sqrt{1+\sin 2A} - \sqrt{1-\sin 2A} = -2\sin A$ **is true if A lies in the intervals**

1) $\left(2n\pi - \frac{\pi}{4},\ 2n\pi + \frac{\pi}{4}\right)$ 2) $\left(2n\pi + \frac{\pi}{4},\ 2n\pi + \frac{3\pi}{4}\right)$

3) $\left(2n\pi + \frac{3\pi}{4},\ 2n\pi + \frac{5\pi}{4}\right)$ 4) $\left(2n\pi + \frac{5\pi}{4},\ 2n\pi + \frac{7\pi}{4}\right)$

30. **General solution of** $\sin^2\theta \sec\theta + \sqrt{3}\tan\theta = 0$ **is**

1) $\theta = n\pi + (-1)^{n+1}\frac{\pi}{3},\ \theta = n\pi,\ n\ \varepsilon Z$ 2) $\theta = n\pi,\ n\ \varepsilon Z$

3) $\theta = n\pi + (-1)^{n+1}\frac{\pi}{3},\ n\ \varepsilon Z$ 4) $\theta = \frac{n\pi}{2},\ n\ \varepsilon Z$

31. **The most general values of x for which sinx + cosx =** $\min\limits_{a \in R}$ **{1, $a^2 - 4a + 6$ } are given by**

1) $n\pi$ 2) $2n\pi + \frac{\pi}{2}$ 3) $n\pi + (-1)^n\frac{\pi}{2} - \frac{\pi}{4}$ 4) $2n\pi + \frac{\pi}{4}$

32. **The only value of *x* for which** $2^{\sin x} + 2^{\cos x} > 2^{1-(1/\sqrt{2})}$ **holds, is**

(a) $\frac{5\pi}{4}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{2}$ (d) All values of x

33. **If** $\tan\theta + \tan 2\theta + \tan 3\theta = \tan\theta \tan 2\theta \tan 3\theta$**, then the general value of** θ **is**

(a) $n\pi$ (b) $\frac{n\pi}{6}$ (c) $n\pi \pm \frac{\pi}{3}$ (d) $\frac{n\pi}{2}$

34. **If** $\sin 3\alpha = 4\sin\alpha \sin(x+\alpha)\sin(x-\alpha)$, **then** $x =$

(a) $n\pi \pm \frac{\pi}{6}$ (b) $n\pi \pm \frac{\pi}{3}$ (c) $n\pi \pm \frac{\pi}{4}$ (d) $n\pi \pm \frac{\pi}{2}$

35. **The general solution of** $\sin^2\theta \sec\theta + \sqrt{3}\tan\theta = 0$ **is**

(a) $\theta = n\pi + (-1)^{n+1}\frac{\pi}{3}, \theta = n\pi, n \in Z$ (b) $\theta = n\pi, n \in Z$

(c) $\theta = n\pi + (-1)^{n+1}\frac{\pi}{3}, n \in Z$ (d) $\theta = \frac{n\pi}{2}, n \in Z$

36. **If** $\cos 3x + \sin\left(2x - \frac{7\pi}{6}\right) = -2$ **, then** $x =$ **(where** $k \in Z$**)**

(a) $\frac{\pi}{3}(6k+1)$ (b) $\frac{\pi}{3}(6k-1)$ (c) $\frac{\pi}{3}(2k+1)$ (d) None of these

37. **If tan(cotx) = cot(tanx) , then sin2x =**

1) $(2n+1)\frac{\pi}{4}$ **2)** $\frac{4}{(2n+1)\pi}$ **3)** $\frac{4\pi}{2n+1}$ **4)** $\frac{2}{(2n+1)\pi}(n \neq -1)$

38. **If tanmθ = cotnθ, then the G.S. of θ =**

1) $\frac{(k+1)\pi}{2(m+n)}$ 2) $\frac{(2k+1)\pi}{2(m+n)}$ 3) $\frac{(2n+1)\pi}{m+n}$ 4) $\frac{(n+1)\pi}{m+n}$

39. **The number of solutions of the given equation** $\tan\theta + \sec\theta = \sqrt{3},$ **where** $0 < \theta < 2\pi$ **is**

(a) 0 (b) 1 (c) 2 (d) 3

40. **If** $|k| = 5$ **and** $0^o \le \theta \le 360^o$ **, then the number of different solutions of 3** $\cos\theta + 4\sin\theta = k$ **is**

(a) Zero (b) Two (c) One (d) Infinite

41. **The solution of equation** $\cos^2\theta + \sin\theta + 1 = 0$ **lies in the interval**

(a) $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$ (b) $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$ (c) $\left(\frac{3\pi}{4}, \frac{5\pi}{4}\right)$ (d) $\left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$

42. **The number of solution of the equation** $2\cos(e^x) = 5^x + 5^{-x}$ **, are**

(a) No solution (b) One solution

(c) Two solutions (d) Infinitely many solutions

43. **If cot** $(\alpha + \beta) = 0,$ **then** $\sin(\alpha + 2\beta) =$

(a) $\sin\alpha$ (b) $\cos\alpha$ (c) $\sin\beta$ (d) $\cos 2\beta$

44. If sin2x cos2x cos4x = λ has a solution, then λ lies in the interval

1) [-1/2, 1/2] **2) [-1/4, 1/4]** **3) [-1/3, 1/3]** **4) [-3/4, ¾]**

45. **The equation** $\sin^6 x + \cos^6 x = a^2$ **has real solution, if**

1) a ε (-1, 1) ∪ (2, 3) 2) a ε [-1, ½] ∪ [1/2, 1]

3) a ε (-1/2, 1/2) ∪ (3/2, 2) 4) a ε (-1/2, 1)

46. **If 32** $\tan^8\theta = 2\cos^2\alpha - 3\cos\alpha$ **and 3cos2θ = 1, then the general value of α is**

1) $2n\pi,\ n\ \varepsilon Z$ 2) $2n\pi \pm \frac{2\pi}{3},\ n\ \varepsilon Z$ 3) $2n\pi \pm \frac{\pi}{3},\ n\ \varepsilon Z$ 4) $n\pi \pm \frac{\pi}{3},\ n\ \varepsilon Z$

47. **If** $\cos 2\theta = (\sqrt{2}+1)\left(\cos\theta - \frac{1}{\sqrt{2}}\right)$**, then the value of** θ **is**

(a) $2n\pi + \frac{\pi}{4}$ (b) $2n\pi \pm \frac{\pi}{4}$ (c) $2n\pi - \frac{\pi}{4}$ (d) None of these

48. **If** $2\cos^2 x + 3\sin x - 3 = 0,\ 0 \le x \le 180^o$**, then** x **=**

(a) $30^o, 90^o, 150^o$ (b) $60^o, 120^o, 180^o$ (c) $0^o, 30^o, 150^o$ (d) $45^o, 90^o, 135^o$

49. **The values of** θ **satisfying** $\sin 7\theta = \sin 4\theta - \sin\theta$ **and** $0 < \theta < \frac{\pi}{2}$ **are**

(a) $\frac{\pi}{9}, \frac{\pi}{4}$ (b) $\frac{\pi}{3}, \frac{\pi}{9}$ (c) $\frac{\pi}{6}, \frac{\pi}{9}$ (d) $\frac{\pi}{3}, \frac{\pi}{4}$

50. **If** $0 \le x \le \pi$ **and** $81^{\sin^2 x} + 81^{\cos^2 x} = 30$**, then** x **=**

(a) $\pi/6$ (b) $\pi/2$ (c) $\pi/4$ (d) $3\pi/4$

51. **If** $\cos A \sin\left(A - \frac{\pi}{6}\right)$ **is maximum, then the value of** A **is equal to**

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) None of these

52. **If** $12\cot^2\theta - 31\operatorname{cosec}\theta + 32 = 0$**, then the value of** $\sin\theta$ **is**

(a) $\frac{3}{5}$ or 1 (b) $\frac{2}{3}$ or $\frac{-2}{3}$ (c) $\frac{4}{5}$ or $\frac{3}{4}$ (d) $\pm\frac{1}{2}$

53. **The general solution of** $a\cos x + b\sin x = c,$ **where** a, b, c **are constants**

(a) $x = n\pi + \cos^{-1}\left(\frac{c}{\sqrt{a^2+b^2}}\right)$ (b) $x = 2n\pi - \tan^{-1}\left(\frac{b}{a}\right)$

(c) $x = 2n\pi - \tan^{-1}\left(\frac{b}{a}\right) \pm \cos^{-1}\left(\frac{c}{\sqrt{a^2+b^2}}\right)$ (d) $x = 2n\pi + \tan^{-1}\left(\frac{b}{a}\right) \pm \cos^{-1}\left(\frac{c}{\sqrt{a^2+b^2}}\right)$

54. **If** $1 + \sin x + \sin^2 x + ...$ **to** $\infty = 4 + 2\sqrt{3}, 0 < x < \pi,$ **then**

(a) $x = \frac{\pi}{6}$ (b) $x = \frac{\pi}{3}$

(c) $x = \frac{\pi}{3}$ or $\frac{\pi}{6}$ (d) $x = \frac{\pi}{3}$ or $\frac{2\pi}{3}$

55. **If** $5\cos^2\theta + 7\sin^2\theta - 6 = 0$**, then the general value of** θ **is**

(a) $2n\pi \pm \frac{\pi}{4}$ (b) $n\pi \pm \frac{\pi}{4}$ (c) $n\pi + (-1)^n \frac{\pi}{4}$ (d) None of these

56. If the solution for θ **of** $\cos p\theta+\cos q\theta=0,\ p>0,\ q>0$ **are in A.P., then the numerically smallest common difference of A.P. is**

(a) $\frac{\pi}{p+q}$ (b) $\frac{2\pi}{p+q}$ (c) $\frac{\pi}{2(p+q)}$ (d) $\frac{1}{p+q}$

57. The general value of α for which $(1+\sin\alpha)(1+x^2)+x\cos\alpha=0$ is an identity in x is (for integral values of n)

1) $2n\pi+\frac{\pi}{2}$ 2) $2n\pi-\frac{\pi}{2}$ 3) $n\pi+\frac{\pi}{2}$ 4) $n\pi+\frac{3\pi}{2}$

58. If 1+cos(x-y) = 0 then

1) cosx – cos y = 0 2) cos x + cos y = 0 3) sinx + sin y = 0 4) cosx+siny = 0

59. If 'a' is any real number, the number of roots of cotx - tanx = a in the first quadrant is

1) 2 2) 0 3) 1 4) infinite

60. The values of x between 0 and 2π which satisfy the equation sinx $\sqrt{8\cos^2 x}=1$ are in A.P. The common difference of the A.P is

1) $\pi/8$ 2) $\pi/4$ 3) $3\pi/8$ 4) $\frac{5\pi}{8}$

61. The number of pairs (*x*, *y*) satisfying the equations $\sin x+\sin y=\sin(x+y)$ **and** $|x|+|y|=1$ **is**

(a) 2 (b) 4

(c) 6 (d) ∞

62. The equation $3\cos x+4\sin x=6$ **has**

(a) Finite solution (b) Infinite solution (c) One solution (d) No solution

63. The set of values of *x* for which the expression $\frac{\tan 3x-\tan 2x}{1+\tan 3x\tan 2x}=1$**, is**

(a) ϕ (b) $\frac{\pi}{4}$

(c) $\left\{n\pi+\frac{\pi}{4}: n=1,2,3.....\right\}$ (d) $\left\{2n\pi+\frac{\pi}{4}: n=1,2,3.....\right\}$

64. One root of the equation $\cos x-x+\frac{1}{2}=0$ **lies in the interval**

(a) $\left[0,\frac{\pi}{2}\right]$ (b) $\left[-\frac{\pi}{2},0\right]$ (c) $\left[\frac{\pi}{2},\pi\right]$ (d) $\left[\pi,\frac{3\pi}{2}\right]$

65. The number of values of θ in [0, 2π] satisfying the equation $2\sin^2\theta = 4 + 3\cos\theta$ are

(a) 0 (b) 1 (c) 2 (d) 3

66. For $0 \le x \le 2\pi$, match the following

Trigonometric equation	**Number of solutions**
I. $\tan^2 x + \cot^2 x = 2$	a) 2
II. $\sin^2 x - \cos x = 1/4$	b) 0
III. $4\sin^2\theta + 6\cos^2\theta = 10$	c) 1
IV. sin x = 1	d) 4

1) d, a, b, c 2) d, a, c, b 3) d, b, c, a 4) d, c, a, b

67. A : 3 sin x + 4 cos x = 7 has no solution

R : a cos x + b sin x = c has no solution if $|c| > \sqrt{a^2 + b^2}$

1) Both A and R are true and R is correct explanation of A

2) Both A and R are true and R is not correct explanation of A

3) A is true but R is false

4) A is false but R is true

HINTS AND SOLUTIONS

1. (a) $\theta = n\pi \pm \alpha;\ n \in Z.$

2. (a) $\cos^2 x = 1 - \sin^2 x$

3. (a)

4. (c)

5. (d)

6. (d)

$$\sqrt{3}\sin x + \cos x = 4$$
$$\Rightarrow \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x = 2$$
$$\Rightarrow \sin\left(x + 30^0\right) = 2$$

7. (b)

8. (d)

9. (a)

10. (b)

11. (c) $2\tan^2\theta = \sec^2\theta \Rightarrow 2\tan^2\theta = \tan^2\theta + 1$

$\Rightarrow \tan^2\theta = 1 = \tan^2\left(\frac{\pi}{4}\right) \Rightarrow \theta = n\pi \pm \frac{\pi}{4}$.

12. (c) $\cot\theta + \tan\theta = 2\,\text{cosec}\,\theta \Rightarrow \frac{2}{\sin\theta} = \frac{1}{\sin\theta\cos\theta}$

$\Rightarrow \cos\theta = \frac{1}{2}$ or $\sin\theta = 0 \Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}$ or $\theta = n\pi$.

13. (a) $\tan m\theta = \tan n\theta \Rightarrow m\theta = p\pi + n\theta \Rightarrow \theta = \frac{p\pi}{(m-n)}$

Hence different values of θ are in A.P. with $\frac{\pi}{m-n}$ as common difference.

14. (a) We have $\frac{\pi}{4}\cot\theta = \frac{\pi}{2} - \frac{\pi}{4}\tan\theta \Rightarrow \tan\theta + \cot\theta = 2$

$\Rightarrow \sin 2\theta = 1 = \sin\frac{\pi}{2} \Rightarrow \theta = n\pi + \frac{\pi}{4}$.

15. (b) $\sec 2\theta = \frac{1}{\cos 2\theta} = \frac{1+\tan^2\theta}{1-\tan^2\theta}$,

$\tan^2\theta + \frac{1+\tan^2\theta}{1-\tan^2\theta} = 1$.

$\Rightarrow \tan\theta = 0$ or $\tan\theta = \pm\sqrt{3}$

Now $\tan\theta = 0 \Rightarrow \theta = m\pi$, where m is an integer and $\tan\theta = \pm\sqrt{3} = \tan(\pm\pi/3) \Rightarrow \theta = n\pi \pm \frac{\pi}{3}$, where n is an integer. Thus $\theta = m\pi, n\pi \pm \frac{\pi}{3}$, where m and n are integers.

16. (d)

17. (a) $4\sin^4 x = 1 - \cos^4 x = (1-\cos^2 x)(1+\cos^2 x)$

$\Rightarrow \sin^2 x[4\sin^2 x - 1 - (1-\sin^2 x)] = 0$

$\Rightarrow \sin^2 x[5\sin^2 x - 2] = 0 \Rightarrow \sin x = 0$ or $\sin x = \pm\sqrt{2/5}$.

$\Rightarrow x = n\pi$

18. (b) After solving the determinant $2\cos\theta = 0$

$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{2}$.

19. (c) $\sin 5x + \sin 3x + \sin x = 0$

$\Rightarrow -\sin 3x = \sin 5x + \sin x = 2\sin 3x\cos 2x$

$\Rightarrow \sin 3x = 0 \Rightarrow x = 0$

or $\cos 2x = -\frac{1}{2} = -\cos\left(\frac{\pi}{3}\right) = \cos\left(\pi - \frac{\pi}{3}\right)$

$\Rightarrow 2x = 2n\pi \pm \left(\pi - \frac{\pi}{3}\right) \Rightarrow x = n\pi \pm \left(\frac{\pi}{3}\right)$

For x € $[0, \frac{\pi}{2}]$, $\Rightarrow x = \frac{\pi}{3}$.

20. (c) $\tan(\pi\cos\theta) = \tan\left(\frac{\pi}{2} - \pi\sin\theta\right)$

$\therefore \sin\theta + \cos\theta = \frac{1}{2} \Rightarrow \sin\left(\theta + \frac{\pi}{4}\right) = \frac{1}{2\sqrt{2}}$.

21. (b) $(1+\tan\theta)(1+\tan\phi) = 2 \Rightarrow \frac{\tan\theta + \tan\phi}{1-\tan\theta\tan\phi} = 1$

$\Rightarrow \tan(\theta+\phi) = 1 \Rightarrow \theta + \phi = \frac{\pi}{4} = 45^\circ$.

22. (c)

23. (c)

24. (a) Given $\sin x + \sin y + \sin z = -3$ is satisfied only when $x = y = z = \frac{3\pi}{2}$, for $x, y, z \in [0, 2\pi]$.

25. (b) $2 - 2\cos^2\theta = 3\cos\theta$

$\Rightarrow \quad 2\cos^2 + 3\cos\theta - 2 = 0$

$\Rightarrow \quad \cos\theta = \frac{-3 \pm \sqrt{9+16}}{4} = \frac{-3 \pm 5}{4}$

Neglecting (–) sign, we get

$\cos\theta = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right) \Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}$.

26. (d) We have, $\tan\theta + \frac{1}{\sqrt{3}} = 0$ or $\tan\theta = -\frac{1}{\sqrt{3}}$

$\because \ \theta$ lies in between $0°$ and $360°$

$\therefore \ \theta = 150°$ and $330°$.

27. (c) $\tan\theta = -1 = \tan\left(2\pi - \frac{\pi}{4}\right)$, $\cos\theta = \frac{1}{\sqrt{2}} = \cos\left(2\pi - \frac{\pi}{4}\right)$

Hence general value is $2n\pi + \left(2\pi - \frac{\pi}{4}\right) = 2n\pi + \frac{7\pi}{4}$.

28. (a) determinant $= \begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ \sin^2\theta & \cos^2\theta & 1+4\sin 4\theta \end{vmatrix} = 0$

$\Rightarrow \quad 1 + 4\sin 4\theta + \cos^2\theta + \sin^2\theta = 0$

$\Rightarrow 4\sin 4\theta = -2 \Rightarrow \sin 4\theta = \frac{-1}{2}$

$\Rightarrow 4\theta = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$, $(0 < 4\theta < 2\pi)$

Since, $0 < \theta < \frac{\pi}{2} \Rightarrow 0 < 4\theta < 2\pi \Rightarrow \theta = \frac{7\pi}{24}, \frac{11\pi}{24}$.

29.(3)

30.(a)

31.(b)

32. (a) Since A.M. $\geq$ G.M. $\frac{1}{2}(2^{\sin x} + 2^{\cos x}) \geq \sqrt{2^{\sin x}.2^{\cos x}}$

$\Rightarrow \quad 2^{\sin x} + 2^{\cos x} \geq 2.2^{\frac{\sin x + \cos x}{2}}$

$\Rightarrow \quad 2^{\sin x} + 2^{\cos x} \geq 2^{1 + \frac{\sin x + \cos x}{2}}$

and we know that $\sin x + \cos x \geq -\sqrt{2}$

$\therefore \quad 2^{\sin x} + 2^{\cos x} > 2^{1-(1/\sqrt{2})}$, for $x = \frac{5\pi}{4}$.

33. (b) $\tan\theta + \tan 2\theta + \tan 3\theta = \tan\theta \tan 2\theta \tan 3\theta$

$$\tan 6\theta = \frac{\tan\theta + \tan 2\theta + \tan 3\theta - \tan\theta \tan 2\theta \tan 3\theta}{1 - \sum \tan\theta \tan 2\theta}$$

= 0, (from the given condition)

$\Rightarrow \quad 6\theta = n\pi \Rightarrow \theta = n\pi/6$.

34. (b) $3\sin\alpha - 4\sin^3\alpha = 4\sin\alpha(\sin^2 x - \sin^2\alpha)$

$\therefore \quad \sin^2 x = \left(\frac{\sqrt{3}}{2}\right)^2 \Rightarrow \sin^2 x = \sin^2 \pi/3$

$\Rightarrow \quad x = n\pi \pm \pi/3$.

35. (b) The given equation can be written as

$\Rightarrow \quad \frac{\sin^2\theta}{\cos\theta} + \sqrt{3}\tan\theta = 0 \Rightarrow \quad \tan\theta\sin\theta + \sqrt{3}\tan\theta = 0$

$\tan\theta(\sin\theta + \sqrt{3}) = 0 \quad \Rightarrow \quad \tan\theta = 0 \quad \Rightarrow \theta = n\pi, n \in Z$.

36. (a) We have $\cos 3x + \sin\left(2x - \frac{7\pi}{6}\right) = -2$

$\Rightarrow \quad 1 + \cos 3x + 1 + \sin\left(2x - \frac{7\pi}{6}\right) = 0$

$\Rightarrow \quad (1 + \cos 3x) + 1 - \cos\left(2x - \frac{2\pi}{3}\right) = 0$

$\Rightarrow \quad 2\cos^2\frac{3x}{2} + 2\sin^2\left(x - \frac{\pi}{3}\right) = 0$

$\Rightarrow \quad \cos\frac{3x}{2} = 0$ and $\sin\left(x - \frac{\pi}{3}\right) = 0$

$\Rightarrow \quad \frac{3x}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, ...$ and $x - \frac{\pi}{3} = 0, \pi, 2\pi ..... \Rightarrow x = \frac{\pi}{3}$

$\cos\frac{3x}{2} = 0$ and $\sin\left(x - \frac{\pi}{3}\right) = 0$ is $x = 2k\pi + \frac{\pi}{3} = \frac{\pi}{3}(6k+1)$, where $k \in Z$.

37. (b)

38. (b)

39. (c) $\sec\theta + \tan\theta = \sqrt{3}$

$\sec\theta - \tan\theta = \frac{1}{\sqrt{3}}$

$\tan\theta = \frac{1}{2}\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = \frac{1}{\sqrt{3}} = \tan\left(\frac{\pi}{6}\right)$

$\Rightarrow \quad \theta = n\pi + \frac{\pi}{6}$.

$\therefore$ Solutions for $0 \le \theta \le 2\pi$ are $\frac{\pi}{6}$ and $\frac{7\pi}{6}$.

40. (b) $3\cos\theta + 4\sin\theta = 5\left[\frac{3}{5}\cos\theta + \frac{4}{5}\sin\theta\right] = 5\cos(\theta - \alpha)$

Where $\cos\alpha = \frac{3}{5}$, $\sin\alpha = \frac{4}{5}$

Now $3\cos\theta + 4\sin\theta = k$

$\therefore \quad 5\cos(\theta - \alpha) = k \Rightarrow \cos(\theta - \alpha) = \pm 1$

$\Rightarrow \quad \theta - \alpha = 0^o, 180^o \Rightarrow \theta = \alpha,\ 180^o + \alpha$.

41. (d) We have, $\cos^2\theta + \sin\theta + 1 = 0$

$\Rightarrow \quad 1 - \sin^2\theta + \sin\theta + 1 = 0$

$\Rightarrow \quad \sin^2\theta - \sin\theta - 2 = 0 \Rightarrow (\sin\theta + 1)(\sin\theta - 2) = 0$

$\sin\theta = 2$, which is not possible and $\sin\theta = -1$.

Therefore, solution of given equation lies in the interval $\left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$.

42. (a) We know $\frac{5^x + 5^{-x}}{2} \ge 1$, (using A.M. $\ge$ G.M.)

But since $\cos(e^x) \le 1$

So, there does not exist any solution.

43. (a) Given, cot $(\alpha + \beta) = 0 \Rightarrow \cos(\alpha + \beta) = 0$

$\Rightarrow \quad \alpha + \beta = (2n+1)\frac{\pi}{2}, n \in I$

$\therefore \sin(\alpha + 2\beta) = \sin(2\alpha + 2\beta - \alpha) = \sin[(2n+1)\pi - \alpha]$

$= \sin(2n\pi + \pi - \alpha) = \sin(\pi - \alpha) = \sin\alpha$.

44. (b)

45. (b)

46. (b)

47. (b) $2\cos^2\theta - (\sqrt{2}+1)\cos\theta - 1 + \frac{(\sqrt{2}+1)}{\sqrt{2}} = 0$

$\Rightarrow \quad \cos\theta = \dfrac{(\sqrt{2}+1) \pm \sqrt{(\sqrt{2}+1)^2 - \frac{8}{\sqrt{2}}}}{4}$

$\Rightarrow \cos\theta = \cos\left(\frac{\pi}{4}\right) \Rightarrow \theta = 2n\pi \pm \frac{\pi}{4}$.

48. (a) $2 - 2\sin^2 x + 3\sin x - 3 = 0$

$\Rightarrow (2\sin x - 1)(\sin x - 1) = 0 \Rightarrow \sin x = \frac{1}{2}$ or $\sin x = 1$

$\Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}$ *i.e.*, $30°, 150°, 90°$.

49. (a) $\sin 7\theta + \sin\theta - \sin 4\theta = 0$

$\Rightarrow 2\sin 4\theta\cos 3\theta - \sin 4\theta = 0$

$\Rightarrow \sin 4\theta(2\cos 3\theta - 1) = 0 \Rightarrow \sin 4\theta = 0,\ \cos 3\theta = \frac{1}{2}$

Now $\sin 4\theta = 0 \Rightarrow 4\theta = \pi \Rightarrow \theta = \frac{\pi}{4}$.

and $\cos 3\theta = \frac{1}{2} \Rightarrow 3\theta = \frac{\pi}{3} \Rightarrow \theta = \frac{\pi}{9}$.

50. (a) We have, $81^{\sin^2 x} + 81^{\cos^2 x} = 30$

put $x = \frac{\pi}{6}$

Then $(81)^{\sin^2 \pi/6} + (81)^{\cos^2 \pi/6} = 30$

$\Rightarrow (81)^{1/4} + (81)^{3/4} = 30 \Rightarrow 30 = 30$

51. (a) $\cos A \sin\left(A - \frac{\pi}{6}\right) = \frac{1}{2}\left[\sin\left(2A - \frac{\pi}{6}\right) - \sin\frac{\pi}{6}\right]$

But $\sin\left(2A - \frac{\pi}{6}\right) - \frac{1}{2}$ attain maximum value at $2A - \frac{\pi}{6} = \frac{\pi}{2} \Rightarrow A = \frac{\pi}{3}$.

52. (c) $12\cot^2\theta - 31\,\mathrm{cosec}\,\theta + 32 = 0$

$12(\mathrm{cosec}^2\theta - 1) - 31\,\mathrm{cosec}\,\theta + 32 = 0$

$(4\,\mathrm{cosec}\,\theta - 5)(3\,\mathrm{cosec}\,\theta - 4) = 0$

$\mathrm{cosec}\,\theta = \frac{5}{4}, \frac{4}{3}$; $\therefore \sin\theta = \frac{4}{5}, \frac{3}{4}$.

53. Put $a = b = c = 1$, then $\cos\left(x - \frac{\pi}{4}\right) = \cos\frac{\pi}{4}$

$\Rightarrow x = 2n\pi + \frac{\pi}{4} \pm \frac{\pi}{4}$ which is given by option (d).

54. (d) $1 + \sin x + \sin^2 x + \ldots.\infty = 4 + 2\sqrt{3}$

$\Rightarrow \frac{1}{1 - \sin x} = 4 + 2\sqrt{3} \Rightarrow \sin x = 1 - \frac{1}{4 + 2\sqrt{3}}$

$\Rightarrow \quad \sin x = 1 - \frac{(4 - 2\sqrt{3})}{4} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

$\Rightarrow \quad x = \frac{\pi}{3}$ or $\frac{2\pi}{3}$.

55. (b) $5 - 5\sin^2\theta + 7\sin^2\theta = 6 \ \Rightarrow \ 2\sin^2\theta = 1$

$\Rightarrow \quad \sin^2\theta = \frac{1}{2} = \sin^2\left(\frac{\pi}{4}\right) \Rightarrow \theta = n\pi \pm \frac{\pi}{4}$.

56. (b) Given $\cos p\theta = -\cos q\theta = \cos(\pi + q\theta)$

$\Rightarrow \ p\theta = 2n\pi \pm (\pi + q\theta), n \in I$

$\Rightarrow \ \theta = \frac{(2n+1)\pi}{p-q}$ or $\frac{(2n-1)\pi}{p+q}, n \in I$

Both the solutions form an A.P. $\theta = \frac{(2n+1)\pi}{p-q}$ gives us an A.P. with common difference $\frac{2\pi}{p-q}$ and $\theta = \frac{(2n-1)\pi}{p+q}$ gives us an A.P. with common difference $= \frac{2\pi}{p+q}$. Certainly, $\frac{2\pi}{p+q} < \left|\frac{2\pi}{p-q}\right|$.

57. (b)

58. (b)

59. (c)

60. (b)

61. (c) $2\sin\frac{1}{2}(x+y)\cos\frac{1}{2}(x-y) = 2\sin\frac{1}{2}(x+y)\cos\frac{1}{2}(x+y)$

$\sin\frac{1}{2}(x+y) = 0$ or $\sin\frac{1}{2}x = 0$ or $\sin\frac{1}{2}y = 0$

Thus $x + y = -1, \ x - y = -1$.

When $x + y = 0,$ we have to reject $x + y = 1$ and check with the options or $x + y = -1$ and solve it with $x - y = 1$ or $x - y = -1$ which gives $\left(\frac{1}{2}, -\frac{1}{2}\right)$ or $\left(-\frac{1}{2}, \frac{1}{2}\right)$ as the possible solution. Again solving with $x = 0$, we get $(0, \pm 1)$ and solving with $y = 0$, we get $(\pm 1, 0)$ as the other solution. Thus we have six pairs of solution for x and y.

62. (d) $3\cos x + 4\sin x = 6$

$\Rightarrow \ \frac{3}{5}\cos x + \frac{4}{5}\sin x = \frac{6}{5} \Rightarrow \ \cos(x - \theta) = \frac{6}{5}$, [where $\theta = \cos^{-1}(3/5)$]

So, that equation has no solution.

63. (a) $\tan(3x - 2x) = \tan x = 1 \Rightarrow x = n\pi + \frac{\pi}{4}$

64. But this value does not satisfy the given equation. Hence option (a) is the correct answer(a)

$f(x) = \cos x - x + \frac{1}{2}$, $f(0) = \frac{3}{2} > 0$

$f\left(\frac{\pi}{2}\right) = 0 - \frac{\pi}{2} + \frac{1}{2} = \frac{1-\pi}{2} < 0$, $\left(\because \pi = \frac{22}{7} \text{ nearly}\right)$

$\therefore$ One root lies in the interval $\left[0, \frac{\pi}{2}\right]$.

65. (a) $2 - 2\cos^2\theta = 4 + 3\cos\theta \Rightarrow 2\cos^2\theta + 3\cos\theta + 2 = 0$

$\Rightarrow \cos\theta = \frac{-3 \pm \sqrt{9-16}}{4}$,

which is imaginary, hence no solution.

66. (a)

67. (a)