PROPERTIES OF TRIANGLES

OBJECTIVES

1. **In triangle** ABC **,** $(b+c)\cos A+(c+a)\cos B\ +(a+b)\cos C=$

(a) 0 (b) 1

(c) $a+b+c$ (d) $2(a+b+c)$

2. **In** ΔABC, **if** $\cos A+\cos C=4\sin^2\frac{1}{2}B,$ **then** a,b,c **are in**

(a) A. P. (b) G. P.

(c) H. P. (d) None of these

3. **If the angles of a triangle** ABC **be in A.P., then**

(a) $c^2=a^2+b^2-ab$ (b) $b^2=a^2+c^2-ac$

(c) $a^2=b^2+c^2-ac$ (d) $b^2=a^2+c^2$

4. **In** ΔABC, $b^2\cos 2A-a^2\cos 2B=$

(a) b^2-a^2 (b) b^2-c^2

(c) c^2-a^2 (d) $a^2+b^2+c^2$

5. **In** $\Delta ABC, \frac{\sin(A-B)}{\sin(A+B)}=$

(a) $\frac{a^2-b^2}{c^2}$ (b) $\frac{a^2+b^2}{c^2}$

(c) $\frac{c^2}{a^2-b^2}$ (d) $\frac{c^2}{a^2+b^2}$

6. **In** ΔABC, $a\sin(B-C)+b\sin(C-A)+c\sin(A-B)=$

(a) 0 (b) $a+b+c$

(c) $a^2+b^2+c^2$ (d) $2(a^2+b^2+c^2)$

7. **In** $\Delta ABC,(a-b)^2\cos^2\frac{C}{2}+(a+b)^2\sin^2\frac{C}{2}=$

(a) a^2 (b) b^2

(c) c^2 (d) None of these

8. **If in a triangle** ABC, $(s-a)(s-b)=s\,(s-c)$**, then angle** C **is equal to**

(a) 90^o (b) 45^o

(c) 30^o (d) 60^o

9. In $\Delta ABC, \left(\cot\frac{A}{2}+\cot\frac{B}{2}\right)\left(a\sin^2\frac{B}{2}+b\sin^2\frac{A}{2}\right)=$

(a) $\cot C$ (b) $c\cot C$ (c) $\cot\frac{C}{2}$ (d) $c\cot\frac{C}{2}$

10. In ΔABC, **if** $(a+b+c)(a-b+c)$ **=3*ac*, then**

(a) $\angle B = 60^o$ (b) $\angle B = 30^o$ (c) $\angle C = 60^o$ (d) $\angle A+\angle C = 90^o$

11. If $\cos^2 A+\cos^2 C=\sin^2 B$, **then** ΔABC **is**

(a) Equilateral (b) Right angled (c) Isosceles (d) None of these

12. If in a triangle ABC **,** $\angle C = 60^o$, **then** $\frac{1}{a+c}+\frac{1}{b+c}=$

(a) $\frac{1}{a+b+c}$ (b) $\frac{2}{a+b+c}$

(c) $\frac{3}{a+b+c}$ (d) None of these

13. If $\tan\frac{B-C}{2}=x\cot\frac{A}{2}$, **then** $x=$

(a) $\frac{c-a}{c+a}$ (b) $\frac{a-b}{a+b}$

(c) $\frac{b-c}{b+c}$ (d) None of these

14. In ΔABC **,** $a^2(\cos^2 B-\cos^2 C)+\ b^2(\cos^2 C-\cos^2 A)+\ c^2(\cos^2 A-\cos^2 B)=$

(a) 0 (b) 1

(c) $a^2+b^2+c^2$ (d) $2(a^2+b^2+c^2)$

15. If the sides of a triangle are in the ratio $2:\sqrt{6}:(\sqrt{3}+1)$ **, then the largest angle of the triangle will be**

(a) 60^o (b) 75^o

(c) 90^o (d) 120^o

16. If the sides of a triangle are in A. P., then the cotangent of its half the angles will be in

(a) H. P. (b) G . P.

(c) A. P. (d) No particular order

17. If in a triangle, $a\cos^2\frac{C}{2}+c\cos^2\frac{A}{2}=\frac{3b}{2}$, **then its sides will be in**

(a) A. P. (b) G. P.

(c) H. P. (d) A. G.

18. In triangle ABC if $A+C=2B$, then $\frac{a+c}{\sqrt{a^2-ac+c^2}}$ is equal to

(a) $2\cos\frac{A-C}{2}$ (b) $\sin\frac{A+C}{2}$

(c) $\sin\frac{A}{2}$ (d) None of these

19. If in ΔABC, $2b^2=a^2+c^2$, then $\frac{\sin 3B}{\sin B}=$

(a) $\frac{c^2-a^2}{2ca}$ (b) $\frac{c^2-a^2}{ca}$

(c) $\left(\frac{c^2-a^2}{ca}\right)^2$ (d) $\left(\frac{c^2-a^2}{2ca}\right)^2$

20. In a triangle ABC, $\frac{2\cos A}{a}+\frac{\cos B}{b}+\frac{2\cos C}{c}=\frac{a}{bc}+\frac{b}{ca}$, then the value of angle *A* is

(a) 45^o (b) 30^o

(c) 90^o (d) 60^o

21. In a triangle ABC if $2a^2b^2+2b^2c^2=a^4+b^4+c^4$, then angle *B* is equal to

(a) 45^o Or 135^o (b) 135^o or 120^o

(c) 30^o Or 60^o (d) None of these

22. If in a triangle the angles are in A. P. and $b:c=\sqrt{3}:\sqrt{2}$, then $\angle A$ is equal to

(a) 30^o (b) 60^o

(c) 15^o (d) 75^o

23. In a triangle ABC, $a=4, b=3$, $\angle A=60^o$. Then *c* is the root of the equation

(a) $c^2-3c-7=0$ (b) $c^2+3c+7=0$

(c) $c^2-3c+7=0$ (d) $c^2+3c-7=0$

24. If $a=2, b=3, c=5$ in ΔABC, then *C* =

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$ (d) None of these

25. If in the $\Delta ABC, AB=2BC$, then $\tan\frac{B}{2}:\cot\left(\frac{C-A}{2}\right)$

(a) 3 :1 (b) 2 : 1

(c) 1 : 2 (d) 1 : 3

26. If in a triangle ABC, $\cos A\cos B+\sin A\sin B\sin C=1$, **then the sides are proportional to**

(a) $1:1:\sqrt{2}$ (b) $1:\sqrt{2}:1$

(c) $\sqrt{2}:1:1$ (d) None of these

27. The smallest angle of the triangle whose sides are $6+\sqrt{12},\sqrt{48},\sqrt{24}$ **is**

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{6}$ (d) None of these

28. In a ΔABC, $\frac{\cos A}{a}=\frac{\cos B}{b}=\frac{\cos C}{c}$ **and the side** $a=2$, **then area of the triangle is**

(a) 1 (b) 2

(c) $\frac{\sqrt{3}}{2}$ (d) $\sqrt{3}$

29. If angles of a triangle are in the ratio of 2 : 3: 7, then the sides are in the ratio of

(a) $\sqrt{2}:2:(\sqrt{3}+1)$ (b) $2:\sqrt{2}:(\sqrt{3}+1)$

(c) $\sqrt{2}:(\sqrt{3}+1):2$ (d) $2:(\sqrt{3}+1):\sqrt{2}$

30. The perimeter of a ΔABC **is 6 times the arithmetic mean of the sines of its angles. If the side *a* is 1, then the angle *A* is**

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$ (d) π

31. *ABC* is a triangle such that $\sin(2A+B)=\sin(C-A)=-\sin(B+2C)=\frac{1}{2}$. **If *A*, *B* and *C* are in A.P., then *A*, *B* and *C* are**

(a) $30^o,60^o,90^o$ (b) $45^o,60^o,75^o$

(c) $45^o,45^o,90^o$ (d) $60^o,60^o,60^o$

32. If a^2,b^2,c^2 **are in A. P. then which of the following are also in A.P.**

(a) $\sin A,\sin B,\sin C$ (b) $\tan A,\tan B,\tan C$

(c) $\cot A,\cot B,\cot C$ (d) None of these

33. If in a triangle *ABC* side $a=(\sqrt{3}+1)$ ***cms*** **and** $\angle B=30^o$, $\angle C=45^o$, **then the area of the triangle is**

(a) $\frac{\sqrt{3}+1}{3}cm^2$ (b) $\frac{\sqrt{3}+1}{2}cm^2$ (c) $\frac{\sqrt{3}+1}{2\sqrt{2}}cm^2$ (d) $\frac{\sqrt{3}+1}{3\sqrt{2}}cm^2$

34. In ΔABC, if $\sin^2\frac{A}{2}, \sin^2\frac{B}{2}, \sin^2\frac{C}{2}$ be in H. P. then *a, b, c* will be in

(a) A. P. (b) G. P.

(c) H. P. (d) None of these

35. In ΔABC, if $2(bc\cos A + ca\cos B + ab\cos C) =$

(a) 0 (b) $a+b+c$

(c) $a^2+b^2+c^2$ (d) None of these

36. In ΔABC, if $\tan\frac{A}{2}\tan\frac{C}{2} = \frac{1}{2}$, then a, b, c are in

(a) A. P. (b) G. P.

(c) H. P. (d) None of these

37. In ΔABC, $(b^2-c^2)\cot A + (c^2-a^2)\cot B + (a^2-b^2)\cot C =$

(a) 0 (b) $a^2+b^2+c^2$

(c) $2(a^2+b^2+c^2)$ (d) $\frac{1}{2abc}$

38. In a triangle *ABC*, $a^3\cos(B-C) + b^3\cos(C-A) + c^3\cos(A-B) =$

(a) abc (b) $3abc$

(c) $a+b+c$ (d) None of these

39. In a ΔABC, side *b* is equal to

(a) $c\cos A + a\cos C$ (b) $a\cos B + b\cos A$

(c) $b\cos C + c\cos B$ (d) None of these

40. In triangle ABC, $\frac{1+\cos(A-B)\cos C}{1+\cos(A-C)\cos B} =$

(a) $\frac{a-b}{a-c}$ (b) $\frac{a+b}{a+c}$

(c) $\frac{a^2-b^2}{a^2-c^2}$ (d) $\frac{a^2+b^2}{a^2+c^2}$

41. In ΔABC, $(b-c)\cot\frac{A}{2} + (c-a)\cot\frac{B}{2} + (a-b)\cot\frac{C}{2}$ **is equal to**

(a) 0 (b) 1

(c) ± 1 (d) 2

42. If $A = 30^o, a = 7, b = 8$ in ΔABC, then *B* has

(a) One solution (b) Two solutions (c) No solution (d) None of these

43. If in a ΔABC, $\cos A + 2\cos B + \cos C = 2$, then a, b, c are in

(a) A. P. (b) H. P.

(c) G. P. (d) None of these

44. In a ΔABC, $2a\sin\left(\frac{A-B+C}{2}\right)$ is equal to

(a) $a^2+b^2-c^2$ (b) $c^2+a^2-b^2$

(c) $b^2-c^2-a^2$ (d) $c^2-a^2-b^2$

45. In a ΔABC, if $b = 20, c = 21$ and $\sin A = 3/5$, then $a =$

(a) 12 (b) 13

(c) 14 (d) 15

46. The lengths of the sides of a triangle are $\alpha-\beta, \alpha+\beta$ and $\sqrt{3\alpha^2+\beta^2}$, $(\alpha > \beta > 0)$. Its largest angle is

(a) $\frac{3\pi}{4}$ (b) $\frac{\pi}{2}$

(c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{6}$

47. If α, β, γ are angles of a triangle, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma - 2\cos\alpha\cos\beta\cos\gamma$ is

(a) 2 (b) –1

(c) –2 (d) 0

48. In a triangle ABC, if $a\sin A = b\sin B$, then the nature of the triangle

(a) $a > b$ (b) $a < b$

(c) $a = b$ (d) $a + b = c$

49. The ratio of the sides of triangle *ABC* is $1:\sqrt{3}:2$. The ratio of $A:B:C$ is

(a) $3:5:2$ (b) $1:\sqrt{3}:2$

(c) 3 : 2 : 1 (d) 1: 2 : 3

50. In a ΔABC, if $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$, then $\cos C =$

(a) $\frac{7}{5}$ (b) $\frac{5}{7}$

(c) $\frac{17}{36}$ (d) $\frac{16}{17}$

51. In a triangle ABC, right angled at *C*, the value of $\tan A + \tan B$ is

(a) $a+b$ (b) $\frac{a^2}{bc}$ (c) $\frac{b^2}{ac}$ (d) $\frac{c^2}{ab}$

52. If the lengths of the sides of a triangle are 3, 5, 7, then the largest angle of the triangle is

(a) $\pi/2$ (b) $5\pi/6$

(c) $2\pi/3$ (d) $3\pi/4$

53. If in triangle $ABC, \frac{a^2-b^2}{a^2+b^2}=\frac{\sin(A-B)}{\sin(A+B)}$**, then the triangle is**

(a) Right angled (b)Isosceles

(c) Right angled or isosceles (d) Right angled isosceles

54. If in triangle $ABC, \cos A=\frac{\sin B}{2\sin C}$**, then the triangle is**

(a) Equilateral (b) Isosceles

(c) Right angled (d) None of these

55. If in a triangle ABC**,** $\cos A+\cos B+\cos C=\frac{3}{2}$**, then the triangle is**

(a) Isosceles (b) Equilateral

(c) Right angled (d) None of these

56. In any triangle *ABC*, the value of $a(b^2+c^2)\cos A+b(c^2+a^2)\cos B+c(a^2+b^2)\cos C$ **is**

(a) $3abc^2$ (b) $3a^2bc$

(c) $3abc$ (d) $3ab^2c$

57. In a triangle PQR**,** $\angle R=\frac{\pi}{2}$**. If** $\tan\left(\frac{P}{2}\right)$ **and** $\tan\left(\frac{Q}{2}\right)$ **are the roots of the equation** $ax^2+bx+c=0 (a\neq 0)$.

then

(a) $a+b=c$ (b) $b+c=a$

(c) $a+c=b$ (d) $b=c$

58. In a ΔABC, $a^2\sin 2C+c^2\sin 2A=$

(a) Δ (b) 2Δ

(c) 3Δ (d) 4Δ

59. If *A* is the area and 2*s* the sum of 3 sides of triangle, then

(a) $A\leq\frac{s^2}{3\sqrt{3}}$ (b) $A\leq\frac{s^2}{2}$

(c) $A>\frac{s^2}{\sqrt{3}}$ (d) None of these

60. If the median of ΔABC **through *A* is perpendicular to** AB**, then**

(a) $\tan A+\tan B=0$ (b) $2\tan A+\tan B=0$

(c) $\tan A+2\tan B=0$ (d) None of these

61. If $c^2 = a^2 + b^2$, then $4s(s-a)(s-b)(s-c) =$

(a) s^4 (b) b^2c^2

(c) c^2a^2 (d) a^2b^2

62. The sides of triangle are $3x+4y$, $4x+3y$ and $5x+5y$ units, where $x, y > 0$. The triangle is

(a) Right angled (b)Equilateral

(c) Obtuse angled (d) None of these

63. If in a ΔABC, the altitudes from the vertices *A, B, C* on opposite sides are in H.P. then $\sin A, \sin B, \sin C$ are in

(a) A.G.P. (b) H.P.

(c) G.P. (d) A.P.

64. Which of the following is true in a triangle *ABC*

(a) $(b+c)\sin\frac{B-C}{2} = 2a\cos\frac{A}{2}$

(b) $(b+c)\cos\frac{A}{2} = 2a\sin\frac{B-C}{2}$

(c) $(b-c)\cos\frac{A}{2} = a\sin\frac{B-C}{2}$

(d) $(b-c)\sin\frac{B-C}{2} = 2a\cos\frac{A}{2}$

65. If p_1, p_2, p_3 are altitudes of a triangle ABC from the vertices A, B, C and Δ the area of the triangle, then $p_1^{-2} + p_2^{-2} + p_3^{-2}$ is equal to

(a) $\frac{a+b+c}{\Delta}$ (b) $\frac{a^2+b^2+c^2}{4\Delta^2}$

(c) $\frac{a^2+b^2+c^2}{\Delta^2}$ (d) None of these

66. If the line segment joining the points $A(a,b)$ and $B(c,d)$ subtends an angle θ at the origin, then $\cos\theta$ is equal to

(a) $\frac{ab+cd}{\sqrt{(a^2+b^2)(c^2+d^2)}}$ (b) $\frac{ac+bd}{\sqrt{(a^2+b^2)(c^2+d^2)}}$

(c) $\frac{ac-bd}{\sqrt{(a^2+b^2)(c^2+d^2)}}$ (d) None of these

67. **If *a, b* and *c* are the sides of a triangle such that** $a^4+b^4+c^4=2c^2(a^2+b^2)$ **then the angles opposite to the side *C* is**

(a) $45°$ Or $135°$ (b) $30°$ or $100°$

(c) $50°$ Or $100°$ (d) $60°$ or $120°$

68. **The in radius of the triangle whose sides are 3, 5, 6, is**

(a) $\sqrt{8/7}$ (b) $\sqrt{8}$

(c) $\sqrt{7}$ (d) $\sqrt{7/8}$

69. **Which is true in the following**

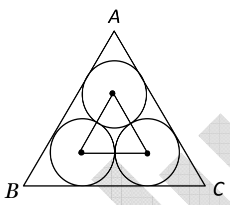
(a) $a\cos A+b\cos B+c\cos C=R\sin A\sin B\sin C$

(b) $a\cos A+b\cos B+c\cos C=2R\sin A\sin B\sin C$

(c) $a\cos A+b\cos B+c\cos C=4R\sin A\sin B\sin C$

(d) $a\cos A+b\cos B+c\cos C=8R\sin A\sin B\sin C$

70. **The area of the equilateral triangle which containing three coins of unity radius is**



(a) $6+4\sqrt{3}$ *sq. units*

(b) $8+\sqrt{3}$ *sq. units*

(c) $4+\frac{7\sqrt{3}}{2}$ *sq. units*

(d) $12+2\sqrt{3}$ *sq. units*

71. **If the length of the sides of a triangle are 3, 4 and 5 units, then *R* (the circum radius) is**

(a) 2.0 *unit* (b) 2.5 *unit*

(c) 3.0 *unit* (d) 3.5 *unit*

72. **If the radius of the circum circle of an isosceles triangle** PQR **is equal to** $PQ(=PR)$, **then the angle** *P* is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{2}$ (d) $\frac{2\pi}{3}$

73. **In** ΔABC, **if** $b=6, c=8$ **and** $\angle A=90°$, **then *R*=**

(a) 3 (b) 4

(c) 5 (d) 7

74. **If the sides of triangle are 13, 14, 15, then the radius of its in circle is**

(a) $\frac{67}{8}$ (b) $\frac{65}{4}$

(c) 4 (d) 2

75. **If** x, y, z **are perpendicular drawn** a, b **and** c**, then the value of** $\frac{bx}{c}+\frac{cy}{a}+\frac{az}{b}$ **will be**

(a) $\frac{a^2+b^2+c^2}{2R}$ (b) $\frac{a^2+b^2+c^2}{R}$

(c) $\frac{a^2+b^2+c^2}{4R}$ (d) $\frac{2(a^2+b^2+c^2)}{R}$

76. **The circum-radius of the triangle whose sides are 13, 12 and 5 is**

(a) 15 (b) 13/2

(c) 15/2 (d) 6

77. **In an equilateral triangle of side** $2\sqrt{3}$ ***cm*****, the circum-radius is**

(a) 1 *cm* (b) $\sqrt{3}$ *cm*

(c) 2 *cm* (d) $2\sqrt{3}$ *cm*

78. **radius of the circum circle to that of the in circle is**

(a) $\frac{16}{9}$ (b) $\frac{16}{7}$

(c) $\frac{11}{7}$ (d) $\frac{7}{16}$

79. $\frac{a\cos A+b\cos B+c\cos C}{a+b+c}=$

(a) $1/r$ (b) r/R

(c) R/r (d) $1/R$

80. **In an equilateral triangle the in radius and the circum-radius are connected by**

(a) $r=4R$ (b) $r=R/2$

(c) $r=R/3$ (d) None of these

81. **If** R **is the radius of the circum circle of the** ΔABC **and** Δ **is its area, then**

(a) $R=\frac{a+b+c}{\Delta}$ (b) $R=\frac{a+b+c}{4\Delta}$

(c) $R=\frac{abc}{4\Delta}$ (d) $R=\frac{abc}{\Delta}$

82. **In a triangle ABC, if $b = 2, B = 30°$ then the area of circum circle of triangle *ABC* in *square units* is**

(a) π (b) 2π

(c) 4π (d) 6π

83. **The sum of the radii of inscribed and circumscribed circles for an *n* sided regular polygon of side *a*, is**

(a) $a\cot\left(\frac{\pi}{n}\right)$ (b) $\frac{a}{2}\cot\left(\frac{\pi}{2n}\right)$

(c) $a\cot\left(\frac{\pi}{2n}\right)$ (d) $\frac{a}{2}\cot\left(\frac{\pi}{2n}\right)$

84. **In a $\Delta ABC, r_1 < r_2 < r_3$, then**

(a) $a < b < c$ (b) $a > b > c$

(c) $b < a < c$ (d) $a < c < b$

85. **If the sides of a triangle are in ratio 3 : 7 : 8, then *R* : *r* is equal to**

(a) 2 : 7 (b) 7 : 2

(c) 3 : 7 (d) 7 : 3

86. **If the two angle on the base of a triangle are $\left(22\frac{1}{2}\right)^o$ and $\left(112\frac{1}{2}\right)^o$, then the ratio of the height of the triangle to the length of the base is**

(a) 1 : 2 (b) 2 : 1

(c) 2 : 3 (d) 1: 1

87. **In a ΔABC, if $a = 2x,\ b = 2y$ and $\angle C = 120°$, then the area of the triangle is**

(a) xy (b) $xy\sqrt{3}$

(c) $3xy$ (d) $2xy$

88. **In ΔABC, if $\sin A : \sin C = \sin(A-B) : \sin(B-C)$, then**

(a) a, b, c are in A.P. (b) a^2, b^2, c^2 are in A.P.

(c) a^2, b^2, c^2 are in G. P. (d) None of these

89. **In ΔABC, if $8R^2 = a^2 + b^2 + c^2$, then the triangle is**

(a) Right angled (b) Equilateral

(c) Acute angled (d) Obtuse angled

90. If the sides of a Δ be $(x^2+x+1),(2x+1)$ and (x^2-1), then the greatest angle is

(a) 105° (b) 120°

(c) 135° (d) None

91. In triangle ABC, if $\angle A=45°,\ \angle B=75°$, then $a+c\sqrt{2}$ =

(a) 0 (b) 1

(c) b (d) $2b$

92. In triangle ABC if $\frac{\cos A}{a}=\frac{\cos B}{b}=\frac{\cos C}{c}$, then the triangle is

(a) Right angled (b) Obtuse angled

(c) Equilateral (d) Isosceles

93. In a triangle, the length of the two larger sides are 10 *cm* and 9 *cm* respectively. If the angles of the triangle are in A.P., then the length of the third side in *cm* can be

(a) $5-\sqrt{6}$ only

(b) $5+\sqrt{6}$ only

(c) $5-\sqrt{6}$ or $5+\sqrt{6}$

(d) Neither $5-\sqrt{6}$ nor $5+\sqrt{6}$

94. In a triangle ABC, $\tan\frac{A}{2}=\frac{5}{6}$ and $\tan\frac{C}{2}=\frac{2}{5}$, then

(a) a, b, c are in A.P.

(b) $\cos A, \cos B, \cos C$ are in A.P.

(c) $\sin A, \sin B, \sin C$ are in A.P.

(d) (a) and (c) both

PROPERTIES OF TRIANGLES

HINTS AND SOLUTIONS

1. (c) $(b+c)\cos A+(c+a)\cos B+(a+b)\cos C=a+b+c$

2. (a) Standard problem

3. (b) A, B, C are in A. P. then angle $B=60^o$, $\cos B=\dfrac{a^2+c^2-b^2}{2ac}$

$\Rightarrow \dfrac{1}{2}=\dfrac{a^2+c^2-b^2}{2ac}\Rightarrow a^2+c^2-b^2=ac$

$\Rightarrow b^2=a^2+c^2-ac$.

4. (a) $b^2\cos 2A-a^2\cos 2B=b^2(1-2\sin^2 A)-a^2(1-2\sin^2 B)$

$=b^2-a^2-2(b^2\sin^2 A-a^2\sin^2 B)=b^2-a^2$.

5. (a) $\dfrac{\sin(A-B)}{\sin(A+B)}=\dfrac{\sin A\cos B-\sin B\cos A}{\sin C}$

6. (a) $a\sin(B-C)+b\sin(C-A)+c\sin(A-B)$

$=k\,(\Sigma\sin A\sin(B-C)\;=k\,\{\Sigma\sin(B+C)\sin(B-C)\}$

$=k\left\{\Sigma\dfrac{1}{2}(\cos 2C-\cos 2B)\right\}=0$.

7. (c) $(a^2+b^2-2ab)\cos^2\dfrac{C}{2}+(a^2+b^2+2ab)\sin^2\dfrac{C}{2}$

$=a^2+b^2+2ab\left(\sin^2\dfrac{C}{2}-\cos^2\dfrac{C}{2}\right)$

$=a^2+b^2-2ab\cos C=a^2+b^2-(a^2+b^2-c^2)=c^2$.

8. (a) $\tan\dfrac{C}{2}=\sqrt{\dfrac{(s-a)(s-b)}{s(s-c)}}=1=\tan\left(\dfrac{\pi}{4}\right)$, from given data. Hence $C=90^o$.

9. (d) $\left\{\cot\dfrac{A}{2}+\cot\dfrac{B}{2}\right\}\left\{a\sin^2\dfrac{B}{2}+b\sin^2\dfrac{A}{2}\right\}$

$=\left\{\dfrac{\cos\dfrac{C}{2}}{\sin\dfrac{A}{2}\sin\dfrac{B}{2}}\right\}\left\{a\sin^2\dfrac{B}{2}+b\sin^2\dfrac{A}{2}\right\}$

10. (a) $(a+c)^2-b^2=3ac\Rightarrow a^2+c^2-b^2=ac$

But $\cos B=\dfrac{a^2+c^2-b^2}{2ac}=\dfrac{1}{2}\Rightarrow B=\dfrac{\pi}{3}$.

11. (b) Concept.

12. (c) $\cos C = \frac{\pi}{3} \Rightarrow a^2 + b^2 - c^2 = ab$

$\Longrightarrow b^2 + bc + a^2 + ac = ab + ac + bc + c^2$

$\Longrightarrow b(b+c) + a(a+c) = (a+c)(b+c)$

13. (c) $\tan\frac{B-C}{2} = x\cot\frac{A}{2} \Rightarrow x = \frac{b-c}{b+c}$.

14. (a) $\Sigma a^2(\cos^2 B - \cos^2 C) = \Sigma a^2(\sin^2 C - \sin^2 B)$

$= k^2 \Sigma a^2 (c^2 - b^2) = 0$.

15. (b) $\cos\theta = \frac{4+6-(\sqrt{3}+1)^2}{2.2.\sqrt{6}} \Rightarrow \theta = 75^o$.

16. (c) standard problem

17. (a) $a\frac{s(s-c)}{ab} + c\frac{s(s-a)}{bc} = \frac{3b}{2}$

$\Longrightarrow 2s(s-c+s-a) = 3b^2 \Longrightarrow 2s(b) = 3b^2 \Rightarrow 2s = 3b$

$\Longrightarrow a+b+c = 3b \Rightarrow a+c = 2b \Rightarrow a,b,c$ are in A.P.

18. (a) $A + C = 2B \Rightarrow B = 60^o$, $\cos B = \frac{a^2+c^2-b^2}{2ac}$

Since $B = 60^o \Rightarrow ac = a^2 + c^2 - b^2$

$\Longrightarrow b^2 = a^2 + c^2 - ac$

Therefore $\frac{a+c}{\sqrt{a^2 - ac + c^2}} = \frac{a+c}{b} = \frac{\sin A + \sin C}{\sin B}$

19. (d) $\frac{\sin 3B}{\sin B} = \frac{3\sin B - 4\sin^3 B}{\sin B} = 3 - 4\sin^2 B$

$= 3 - 4 + 4\cos^2 B = -1 + \frac{4(a^2+c^2-b^2)^2}{4(ac)^2}$

$= -1 + \frac{\left(\frac{a^2+c^2}{2}\right)^2}{(ac)^2} = -1 + \frac{(a^2+c^2)^2}{4(ac)^2}$

$= \frac{(a^2+c^2)^2 - 4a^2c^2}{4(ac)^2} = \left(\frac{c^2-a^2}{2ac}\right)^2$.

20. (c) standard problem

21. (a) $2a^2b^2 + 2b^2c^2 = a^4 + b^4 + c^4$

Also, $(a^2 - b^2 + c^2)^2 = a^4 + b^4 + c^4 - 2(a^2b^2 + b^2c^2 - c^2a^2)$

$\Rightarrow (a^2-b^2+c^2)^2 = 2c^2a^2 \Rightarrow \frac{a^2-b^2+c^2}{2ca} = \pm\frac{1}{\sqrt{2}} = \cos B$

$\Rightarrow B = 45^o$ Or 135^o.

22. (d) angles are in A.P., therefore $B = 60^o$

A $\frac{b}{c} = \frac{\sin B}{\sin C} = \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3}}{2\sin C} = \frac{\sqrt{3}}{\sqrt{2}}$

$\Rightarrow C = 45^o$ So that $A = 180^o - 60^o - 45^o = 75^o$.

23. (a) $\cos A = \frac{(b^2+c^2-a^2)}{2bc}$

$\Rightarrow \cos 60^o = \frac{1}{2} = \frac{9+c^2-16}{2.3c} \Rightarrow c^2-3c-7=0$.

24. (d) $\cos C = \frac{a^2+b^2-c^2}{2ab} = -1$

$\Rightarrow \angle C = 180^o$,

25. (d) We have, $\frac{\tan\left(\frac{B}{2}\right)}{\cot\left(\frac{C-A}{2}\right)} = \frac{\sin\frac{B}{2}\sin\left(\frac{C-A}{2}\right)}{\cos\frac{B}{2}\cos\left(\frac{C-A}{2}\right)}$

$= \frac{\sin C - \sin A}{\sin C + \sin A} = \frac{kc-ka}{kc+ka} = \frac{c-a}{c+a} = \frac{a}{3a} = \frac{1}{3}, \{\because c = 2a\}$.

26. (a) From the given relation $\sin C = \frac{1-\cos A\cos B}{\sin A\sin B} \le 1$

…..(i)

$\Rightarrow 1 \le \cos A\cos B + \sin A\sin B$

$\Rightarrow \cos(A-B) \ge 1$; $\because \cos\theta \not> 1$ …..(ii)

$\therefore A - B = 0$ Or $A = B$

27. (c) Let $A = 6+\sqrt{12}, b = \sqrt{48}, c = \sqrt{24}$

$\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{\sqrt{3}}{2} \Rightarrow C = \frac{\pi}{6}$.

28. (d) $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c} \Rightarrow \frac{\cos A}{k\sin A} = \frac{\cos B}{k\sin B} = \frac{\cos C}{k\sin C}$

$\Rightarrow \cot A = \cot B = \cot C \Rightarrow A = B = C = 60^o$

$\Rightarrow \Delta ABC$ is equilateral.

$\therefore \Delta = \frac{\sqrt{3}}{4}a^2 = \sqrt{3}$.

29. (a) Obviously, the angles are $30^o, 45^o, 105^o$.

$$\therefore\ a:b:c = \sin 30^o : \sin 45^o : \sin 105^o$$

$$= \frac{1}{2} : \frac{1}{\sqrt{2}} : \frac{\sqrt{3}+1}{2\sqrt{2}} = \sqrt{2} : 2 : (\sqrt{3}+1).$$

30. (a) We have $a+b+c = \dfrac{6(\sin A + \sin B + \sin C)}{3}$

$$\Rightarrow k(\sin A + \sin B + \sin C) = 2(\sin A + \sin B + \sin C),$$

Where $k = \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

31. (b) A, B, C are in A.P., therefore $B = 60^o$

Now, $\sin(2A+B) = \dfrac{1}{2}$ (given)

$\Rightarrow 2A + B = 30^o$ Or 150^o

But as $B = 60^o$, $2A + B \neq 30^o$.

Hence $2A + B = 150^o \Rightarrow A = 45^o$

Hence $A = 45^o$, $B = 60^o, C = 75^o$.

32. (c) $\sin^2 B - \sin^2 A = \sin^2 C - \sin^2 B$

$\therefore\ \sin(B+A)\sin(B-A) = \sin(C+B)\sin(C-B)$

Or $\sin C(\sin B \cos A - \cos B \sin A)$

$= \sin A(\sin C \cos B - \cos C \sin B)$

Divide by $\sin A \sin B \sin C$

$\therefore\ \cot A - \cot B = \cot B - \cot C$

33. (b) $\Delta = \dfrac{1}{2}\dfrac{a^2 \sin B \sin C}{\sin(B+C)} = \dfrac{1}{2}\dfrac{(\sqrt{3}+1)^2 . \frac{1}{2} \times \frac{1}{\sqrt{2}}}{\frac{(\sqrt{3}+1)}{2\sqrt{2}}} = \dfrac{\sqrt{3}+1}{2}$.

34. (c) $\dfrac{1}{\sin^2 \frac{A}{2}}, \dfrac{1}{\sin^2 \frac{B}{2}}, \dfrac{1}{\sin^2 \frac{C}{2}}$ are in A. P.

35. (c) On putting the values of $\cos A, \cos B$ and $\cos C$, we get the required result *i.e.*, $a^2 + b^2 + c^2$.

36. (d) $\tan\dfrac{A}{2}\tan\dfrac{C}{2} = \dfrac{1}{2} \Rightarrow \sqrt{\dfrac{(s-b)(s-c)}{s(s-a)}\dfrac{(s-a)(s-b)}{s(s-c)}} = \dfrac{1}{2}$

$$\Rightarrow \frac{s-b}{s} = \frac{1}{2} \Rightarrow 2s - 2b - s = 0 \Rightarrow a + c - 3b = 0.$$

37. (a) $(b^2 - c^2)\cot A = (b^2 - c^2)\dfrac{\cos A}{\sin A} = \dfrac{(b^2 - c^2)(b^2 + c^2 - a^2)}{2bc.ka}$

38. (b) Standard Problem

39. (a) Concept

40. (d) $\frac{1+\cos C\cos(A-B)}{1+\cos(A-C)\cos B}=\frac{1-\cos(A+B)\cos(A-B)}{1-\cos(A-C)\cos(A+C)}$

$\Rightarrow \frac{1-\cos^2 A+\sin^2 B}{1-\cos^2 A+\sin^2 C}=\frac{\sin^2 A+\sin^2 B}{\sin^2 A+\sin^2 C}=\frac{a^2+b^2}{a^2+c^2}$.

41. (a) Standard Problem.

42. (b) Here $b\ \sin A=8\ \sin 30^o=4, a=7$

Thus, we have $b>a>b\sin A$.

Hence angle B has two solutions.

43. (a) Standard Problem

44. (b) $2ac\ \sin\frac{A-B+C}{2}=2ac\ \sin\frac{\pi-2B}{2}=2ac\ \cos B$

$=2ac\frac{c^2+a^2-b^2}{2ca}=c^2+a^2-b^2$.

45. (b) $a^2=b^2+c^2-2bc\cos A$

$\Rightarrow a^2=(20)^2+(21)^2-2.20.21.\frac{4}{5}=169 \Rightarrow a=13.$

46. (c) Let $a=\alpha-\beta, b=\alpha+\beta, c=\sqrt{3\alpha^2+\beta^2}$

$\therefore \cos C=\frac{a^2+b^2-c^2}{2ab}$

$\Rightarrow \cos C=\frac{\alpha^2+\beta^2-2\alpha\beta+\alpha^2+\beta^2+2\alpha\beta-3\alpha^2-\beta^2}{2(\alpha^2-\beta^2)}$

$\Rightarrow \cos C=-\frac{(\alpha^2-\beta^2)}{2(\alpha^2-\beta^2)}=\cos\left(\frac{2\pi}{3}\right)$

$\Rightarrow \angle C=\frac{2\pi}{3}$,

47. (a) Standard Problem

48. (c) $a^2=b^2\Rightarrow a=b$.

49. (d) We have , $a:b:c=1:\sqrt{3}:2$

i.e., $a=\lambda, b=\sqrt{3}\lambda, c=2\lambda$

$$\cos A = \frac{3\lambda^2 + 4\lambda^2 - \lambda^2}{2(\sqrt{3}\lambda)\,(2\lambda)} = \frac{6\lambda^2}{4\sqrt{3}\lambda^2} = \frac{\sqrt{3}}{2}$$

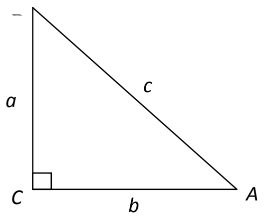
$$\cos A = \frac{\sqrt{3}}{2} \implies A = 30°$$

Similarly, $\cos B = \frac{1}{2} \Rightarrow B = 60°$, $\cos C = 0 \Rightarrow C = 90°$.

Hence $A : B : C = 1 : 2 : 3$.

50. (b) Standard Problem

51. (d) Given, A right–angled triangle *ABC* with right angled at *C*.



Let *a*, *b* and *c* be the lengths of sides *BC*, *CA* and *AB* respectively. We know from the Pythagoras theorem that $c^2 = a^2 + b^2$ and $\tan A = \frac{a}{b}$.

Similarly, $\tan B = \frac{b}{a}$.

Therefore, $\tan A + \tan B = \frac{a}{b} + \frac{b}{a} = \frac{a^2 + b^2}{ab} = \frac{c^2}{ab}$.

52. (c) Let sides of triangle a, b, c are respectively 3, 5 and 7.

$$\therefore \cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{9 + 25 - 49}{2 \times 3 \times 5} = \frac{-15}{30} = -\frac{1}{2}$$

$\angle C = \frac{2\pi}{3}$ (Largest angle)

53. (c) Standard Problem

54. (b) $\cos A = \frac{\sin B}{2 \sin C} \Rightarrow \frac{b^2 + c^2 - a^2}{2bc} = \frac{b}{2c}$

$\implies b^2 + c^2 - a^2 - b^2 = 0 \implies c^2 = a^2$.

55. (b) Standard Problem

56. (c) Standard Problem

57. (a) Standard Problem

58. (d) $a^2 \sin 2C + c^2 \sin 2A = a^2(2 \sin C \cos C) + c^2(2 \sin A \cos A)$

$$= 2a^2\left(\frac{2\Delta}{ab}\cos C\right) + 2c^2\left(\frac{2\Delta}{bc}\cos A\right)$$

$$= 4\Delta\left\{\frac{a\cos C + c\cos A}{b}\right\} = 4\Delta\left(\frac{b}{b}\right) = 4\Delta\,.$$

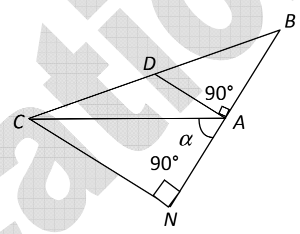
59. (a) We have, $2s = a + b + c, A^2 = s(s-a)(s-b)(s-c)$

$\because$ A.M $\geq$ G.M .

$$\Rightarrow \frac{s-a+s-b+s-c}{3} \geq \sqrt[3]{(s-a)(s-b)(s-c)}$$

$$\Rightarrow \frac{3s-2s}{3} \geq \frac{(A^2)^{1/3}}{s^{1/3}} \Rightarrow \frac{s^3}{27} \geq \frac{A^2}{s} \Rightarrow A \leq \frac{s^2}{3(\sqrt{3})}\,.$$

60. (c) We have $BD = DC$ and $\angle DAB = 90^o$. Draw CN perpendicular to BA produced, then in ΔBCN , we have $DA = \frac{1}{2}CN$ and $AB = AN$



Let $\angle CAN = \alpha$

$$\because \tan A = \tan(\pi - \alpha) = -\tan\alpha = -\frac{CN}{NA} = -2\frac{AD}{AB} = -2\tan B$$

$$\Rightarrow \tan A + 2\tan B = 0\,.$$

61. (d) Δ is Right Angled, $\angle C = 90^o$

$$\therefore 4\Delta^2 = 4\left(\frac{1}{2}ab\right)^2 = a^2b^2\,.$$

62. (c) Let $a = 3x + 4y, b = 4x + 3y$ and $c = 5x + 5y$.

Clearly, c is the largest side and thus the largest angle C is given by

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{-2xy}{2(12x^2 + 25xy + 12y^2)} < 0$$

$\Rightarrow$ C is an obtuse angle.

63. (d) $\frac{2\Delta}{a}, \frac{2\Delta}{b}, \frac{2\Delta}{c}$ are in H.P.

$\Rightarrow \frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in H.P. $\Rightarrow a, b, c$ are in A.P.

$\Rightarrow \sin A, \sin B, \sin C$ are in A.P.

64. (c) $\frac{b-c}{a} = \frac{\sin B - \sin C}{\sin A} = \frac{2\sin\frac{B-C}{2}\cos\frac{B+C}{2}}{2\sin\frac{A}{2}\cos\frac{A}{2}} = \frac{\sin\frac{B-C}{2}}{\cos\frac{A}{2}}$

$\Rightarrow (b-c)\cos\frac{A}{2} = a\sin\frac{B-C}{2}$.

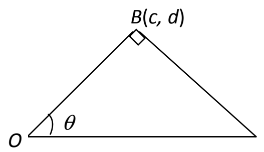
65. (b) We have $\frac{1}{2}ap_1 = \Delta, \frac{1}{2}bp_2 = \Delta, \frac{1}{2}cp_3 = \Delta$

$\Rightarrow p_1 = \frac{2\Delta}{a}, p_2 = \frac{2\Delta}{b}, p_3 = \frac{2\Delta}{c}$

$\therefore \frac{1}{p_1^2} + \frac{1}{p_2^2} + \frac{1}{p_3^2} = \frac{a^2+b^2+c^2}{4\Delta^2}$.

66. (b) Here $(AB)^2 = (a-c)^2 + (b-d)^2$

$(OA)^2 = (a-0)^2 + (b-0)^2 = a^2 + b^2$ and $(OB)^2 = c^2 + d^2$



Now from triangle $AOB, \cos\theta = \frac{(OA)^2 + (OB)^2 - (AB)^2}{2OA.OB}$

$= \frac{a^2+b^2+c^2+d^2-\{(a-c)^2+(b-d)^2\}}{2\sqrt{a^2+b^2}.\sqrt{c^2+d^2}}$

$= \frac{ac+bd}{\sqrt{(a^2+b^2)(c^2+d^2)}}$.

67. (a) $a^4+b^4+c^4-2a^2c^2-2b^2c^2+2a^2b^2 = 2a^2b^2$

$\Rightarrow (a^2+b^2-c^2)^2 = (\sqrt{2}ab)^2 \Rightarrow a^2+b^2-c^2 = \pm\sqrt{2}ab$

$\Rightarrow \frac{a^2+b^2-c^2}{2ab} = \pm\frac{\sqrt{2}ab}{2ab} = \pm\frac{1}{\sqrt{2}}$

$\Rightarrow \cos C = \cos 45^o$ or $\cos 135^o \Rightarrow C = 45^o$ or 135^o.

68. (a) $r = \frac{\Delta}{s} = \sqrt{\frac{8}{7}}$.

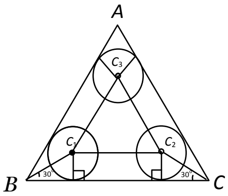
69. (c) $\because\ a = 2R\sin A,\ b = 2R\sin B,\ c = 2R\sin C$

$\therefore\ a\cos A + b\cos B + c\cos C$

$= R[(2\sin A\cos A)+(2\sin B\cos B)+(2\sin C\cos C)]$

$= R(\sin 2A+\sin 2B+\sin 2C) = 4R\sin A\sin B\sin C.$

70. (a) In ΔBC_1M ; $BM = (C_1M).\cot 30$



$\Rightarrow BM = \sqrt{3}$

$\Rightarrow$ Similarly, $CN = \sqrt{3}$ and $MN = C_1C_2 = 1+1 = 2$

Hence, side $BC = \sqrt{3}+\sqrt{3}+2 = 2(1+\sqrt{3})$

$\Rightarrow$ Area of equilateral triangle

$= \frac{\sqrt{3}}{4}[2(1+\sqrt{3})]^2 = 6+4\sqrt{3}$ *sq units*.

71. (b) Sides are 3, 4, 5 since $3^2+4^2=5^2$

So, triangle is a right angle triangle.

Hence, $R = 5/2 = 2.5$.

72. (d) In ΔPQR, the radius of circum circle is $PQ = PR$

$\therefore PQ = PR = \frac{PQ}{2\sin R} = \frac{QR}{2\sin P} = \frac{PR}{2\sin Q}$

$\Rightarrow \sin R = \sin Q = \frac{1}{2} \Rightarrow \angle R = \angle Q = \frac{\pi}{6}$

$\Rightarrow \angle P = \pi - \angle R - \angle Q = \frac{2\pi}{3}$.

73. (c) $\cos A = 0 \Rightarrow 36+64-a^2 = 0 \Rightarrow a = 10 \Rightarrow R = \frac{a}{2\sin A} = \frac{5}{1}$.

74. (c) $s = \frac{1}{2}(a+b+c) = 21$

$\Delta = \sqrt{[s(s-a)(s-b)(s-c)]} = 84$; $\therefore r = \frac{\Delta}{s} = 4$.

75. (a) Let area of triangle be Δ, then according to question, $\Delta = \frac{1}{2}ax = \frac{1}{2}by = \frac{1}{2}cz$

$\therefore \frac{bx}{c}+\frac{cy}{a}+\frac{az}{b} = \frac{b}{c}\left(\frac{2\Delta}{a}\right)+\frac{c}{a}\left(\frac{2\Delta}{b}\right)+\frac{a}{b}\left(\frac{2\Delta}{c}\right)$

$= \frac{2\Delta(b^2+c^2+a^2)}{abc} = \frac{2(a^2+b^2+c^2)}{abc}.\frac{abc}{4R} = \frac{a^2+b^2+c^2}{2R}$.

76. (b) $R = \frac{abc}{4\Delta}$, where $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$a = 13, b = 12, c = 5, s = \frac{30}{2} = 15$

$\Delta = \sqrt{15(2)(3)10} = 3\times 2\times 5 = 30$

$\therefore R = \frac{13\times 12\times 5}{4\times 30} = \frac{13}{2}$.

77. (c) $a = b = c = 2\sqrt{3}$

$$\Delta = \left(\frac{\sqrt{3}a^2}{4}\right) = 3\sqrt{3} sq.cm, \therefore R = \frac{abc}{4\Delta} = 2cm.$$

78. (b) We have $R = \frac{abc}{4\Delta}$ and $r = \frac{\Delta}{s}$

$$\Rightarrow \frac{R}{r} = \frac{abc}{4\Delta}.\frac{s}{\Delta} = \frac{abc}{4(s-a)(s-b)(s-c)}$$

Since $a:b:c = 4:5:6; \frac{a}{4} = \frac{b}{5} = \frac{c}{6} = k \text{ (say)}$

Thus $\frac{R}{r} = \frac{(4k)(5k)(6k)}{4\left(\frac{15k}{2} - 4k\right)\left(\frac{15k}{2} - 5k\right)\left(\frac{15k}{2} - 6k\right)} = \frac{16}{7}$.

79. (b) standard problem

80. (b) $r = 4R \sin\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{2}$

$$\Rightarrow r = 4R\sin^3 30^o, \quad \{\because A = B = C = 60^o\}$$

$$\Rightarrow r = \frac{R}{2}.$$

81. (c) Area of the triangle *ABC* $(\Delta) = \frac{bc}{2}\sin A$. From the sine formula, $a = 2R \sin A$ or $\sin A = \frac{a}{2R}$.

$$\Rightarrow \Delta = \frac{1}{2}bc.\frac{a}{2R} = \frac{abc}{4R} \text{ or } R = \frac{abc}{4\Delta}.$$

82. (c) Radius of circum-circle (R) $= \frac{a}{2\sin A} = \frac{b}{2\sin B} = \frac{c}{2\sin C}$

$$R = \frac{b}{2\sin B} = \frac{2}{2\sin 30°} = 2$$

Now, area of circle $= \pi R^2 = 4\pi$.

83. (b) $\tan\left(\frac{\pi}{n}\right) = \frac{a}{2r}$ and $\sin\left(\frac{\pi}{n}\right) = \frac{a}{2R}$

$$\Rightarrow r + R = \frac{a}{2}\left[\cot\frac{\pi}{n} + \text{cosec}\,\frac{\pi}{n}\right] = \frac{a}{2}\cot\left(\frac{\pi}{2n}\right).$$

84. (a) In a ΔABC, $r_1 < r_2 < r_3$

$$\Rightarrow \frac{1}{r_1} > \frac{1}{r_2} > \frac{1}{r_3} \Rightarrow \frac{s-a}{\Delta} > \frac{s-b}{\Delta} > \frac{s-c}{\Delta}$$

$$\Rightarrow s-a > s-b > s-c \Rightarrow -a > -b > -c \Rightarrow a < b < c.$$

85. (b) Let $a = 3k, b = 7k, c = 8k$

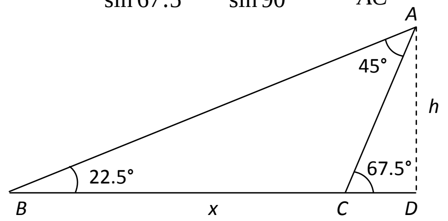
$$\therefore s = \frac{1}{2}(a+b+c) = 9k$$

Then $\frac{R}{r}=\frac{abc}{4\Delta}.\frac{s}{\Delta}=\frac{abc.s}{4s(s-a)(s-b)(s-c)}$

$$=\frac{3k.7k.8k}{4.6k.2k.k}=\frac{7}{2}$$

i.e., $R:r=7:2$.

86. (a) In $\Delta ACD, \frac{h}{\sin 67.5^o}=\frac{AC}{\sin 90^o} \Rightarrow \frac{h}{AC}=\sin 67.5^o$ ....(i)



In $\Delta ABC, \frac{AC}{\sin 22.5^o}=\frac{x}{\sin 45^o} \Rightarrow \frac{AC}{x}=\sqrt{2}\sin 22.5^o$ .....(ii)

From (i) and (ii), $\frac{h}{x}=\frac{1}{2}$.

87. (b) $\Delta=\frac{1}{2}ab\sin C=\frac{1}{2}\times 2x\times 2y\times\frac{\sqrt{3}}{2}=xy\sqrt{3}$.

88. (b) $\frac{\sin A}{\sin C}=\frac{\sin(A-B)}{\sin(B-C)}$

$\Rightarrow \sin(B+C)\sin(B-C)=\sin(A+B)\sin(A-B)$

$\Rightarrow \sin^2 B-\sin^2 C=\sin^2 A-\sin^2 B$

$\Rightarrow 2\sin^2 B=\sin^2 A+\sin^2 C \Rightarrow 2b^2=a^2+c^2$

Hence a^2,b^2,c^2 are in A.P.

89. (a) standard problem

90. (b) Sides are $(x^2+x+1),(2x+1),(x^2-1)$. The greatest side subtends the greatest angle. Hence x^2+x+1 is the greatest side.

Now $\cos\theta=\frac{(2x+1)^2+(x^2-1)^2-(x^2+x+1)^2}{2(2x+1)(x^2-1)}$

$\Rightarrow \theta=120^o$.

91. (d) $\angle C=180^o-45^o-75^o=60^o$

Therefore $a+c\sqrt{2}=k(\sin A+\sqrt{2}\sin C)$

$$= k\left(\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\sqrt{2}\right) = k\left(\frac{1+\sqrt{3}}{\sqrt{2}}\right)$$

and $k = \frac{b}{\sin B} \Rightarrow a + c\sqrt{2} = \frac{b}{\sin 75^o}\left(\frac{1+\sqrt{3}}{\sqrt{2}}\right) = 2b$.

92. (c) $\frac{\cos A}{\cos B} = \frac{a}{b} = \frac{\sin A}{\sin B} \Longrightarrow \sin A \cos B = \sin B \cos A$

$\Longrightarrow \sin(A-B) = 0 \Longrightarrow \sin(A-B) = \sin 0$

$\Longrightarrow A - B = 0 \quad \Longrightarrow A = B$

Similarly, $A = B = C$. Hence it is an equilateral triangle.

93. (c) We know that in triangle larger the side larger the angle. Since angles $\angle A, \angle B$ and $\angle C$ are in AP.

Hence $\angle B = 60^o$ $\quad \cos B = \frac{a^2 + c^2 - b^2}{2ac} \Rightarrow \frac{1}{2} = \frac{100 + a^2 - 81}{20a}$

$\Rightarrow a^2 + 19 = 10a \Rightarrow a^2 - 10a + 19 = 0$

$a = \frac{10 \pm \sqrt{100 - 76}}{2} \Rightarrow a + c\sqrt{2} = 5 \pm \sqrt{6}$.

94. (d) Here $\tan\frac{A}{2}\tan\frac{C}{2} = \frac{s-b}{s}$

$\frac{5}{6}.\frac{2}{5} = \frac{s-b}{s} \Rightarrow 3s - 3b = s \Rightarrow 2s = 3b$

$\Rightarrow a + b + c = 3b$ or $a + c = 2b$.

$\therefore$ a, b, c are in A.P., also sin$A$, sin$B$, sin$C$ are in A.P.