MULTIPLE AND SUBMULTIPLE ANGLES

OBJECTIVES

1. $\frac{1}{\tan 3A - \tan A} - \frac{1}{\cot 3A - \cot A} =$

(a) $\tan A$ (b) $\tan 2A$

(c) $\cot A$ (d) $\cot 2A$

2. $\sqrt{2+\sqrt{2+2\cos 4\theta}} =$

(a) $\cos\theta$ (b) $\sin\theta$

(c) $2\cos\theta$ (d) $2\sin\theta$

3. $\frac{\cot^2 15° - 1}{\cot^2 15° + 1} =$

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) $\frac{3\sqrt{3}}{4}$ (d) $\sqrt{3}$

4. $1 - 2\sin^2\left(\frac{\pi}{4}+\theta\right) =$

(a) $\cos 2\theta$ (b) $-\cos 2\theta$

(c) $\sin 2\theta$ (d) $-\sin 2\theta$

5. If $a\tan\theta = b$, then $a\cos 2\theta + b\sin 2\theta =$

(a) a (b) b

(c) $-a$ (d) $-b$

6. $(\sec 2A + 1)\sec^2 A =$

(a) $\sec A$ (b) $2\sec A$

(c) $\sec 2A$ (d) $2\sec 2A$

7. **If** $\tan\frac{A}{2} = \frac{3}{2}$, **then** $\frac{1+\cos A}{1-\cos A} =$

(a) -5 (b) 5

(c) $9/4$ (d) $4/9$

8. If $\tan x = \frac{b}{a}$, then $\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}} =$

(a) $\frac{2\sin x}{\sqrt{\sin 2x}}$ (b) $\frac{2\cos x}{\sqrt{\cos 2x}}$ (c) $\frac{2\cos x}{\sqrt{\sin 2x}}$ (d) $\frac{2\sin x}{\sqrt{\cos 2x}}$

9. If $\cos 2B = \frac{\cos(A+C)}{\cos(A-C)}$**, then** $\tan A, \tan B, \tan C$ **are in**

(a) A.P. (b) G.P.

(c) H.P. (d) None of these

10. $\frac{\sin 2A}{1+\cos 2A} \cdot \frac{\cos A}{1+\cos A} =$

(a) $\tan \frac{A}{2}$ (b) $\cot \frac{A}{2}$

(c) $\sec \frac{A}{2}$ (d) $\operatorname{cosec} \frac{A}{2}$

11. If $\cos(\theta-\alpha), \cos\theta$ **and** $\cos(\theta+\alpha)$ **are in H.P., then** $\cos\theta \sec\frac{\alpha}{2}$ **is equal to**

(a) $\pm\sqrt{2}$ (b) $\pm\sqrt{3}$

(c) $\pm 1/\sqrt{2}$ (d) None of these

12. If θ **and** ϕ **are angles in the 1st quadrant such that** $\tan\theta = 1/7$ **and** $\sin\phi = 1/\sqrt{10}$ **.Then**

(a) $\theta + 2\phi = 90°$ (b) $\theta + 2\phi = 60°$

(c) $\theta + 2\phi = 30°$ (d) $\theta + 2\phi = 45°$

13. If $90° < A < 180°$ **and** $\sin A = \frac{4}{5}$**, then** $\tan \frac{A}{2}$ **is equal to**

(a) $1/2$ (b) $3/5$

(c) $3/2$ (d) 2

14. For $A = 133°$, $2\cos\frac{A}{2}$ **is equal to**

(a) $-\sqrt{1+\sin A} - \sqrt{1-\sin A}$ (b) $-\sqrt{1+\sin A} + \sqrt{1-\sin A}$

(c) $\sqrt{1+\sin A} - \sqrt{1-\sin A}$ (d) $\sqrt{1+\sin A} + \sqrt{1-\sin A}$

15. Which of the following number(s) is/are rational

(a) $\sin 15°$ (b) $\cos 15°$

(c) $\sin 15° \cos 15°$ (d) $\sin 15° \cos 75°$

16. If θ **is an acute angle and** $\sin\frac{\theta}{2} = \sqrt{\frac{x-1}{2x}}$ **, then** $\tan\theta$ **is equal to**

(a) $x^2 - 1$ (b) $\sqrt{x^2-1}$

(c) $\sqrt{x^2+1}$ (d) $x^2 + 1$

17. $\cos^4\frac{\pi}{8}+\cos^4\frac{3\pi}{8}+\cos^4\frac{5\pi}{8}+\cos^4\frac{7\pi}{8}=$

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{3}{2}$ (d) $\frac{3}{4}$

18. $\sqrt{\frac{1-\sin A}{1+\sin A}}=$

(a) $\sec A+\tan A$ (b) $\tan\left(\frac{\pi}{4}-A\right)$

(c) $\tan\left(\frac{\pi}{4}+\frac{A}{2}\right)$ (d) $\tan\left(\frac{\pi}{4}-\frac{A}{2}\right)$

19. If α is a root of $25\cos^2\theta+5\cos\theta-12=0$, $\pi/2<\alpha<\pi$, then $\sin 2\alpha$ is equal to

(a) $24/25$ (b) $-24/25$

(c) $13/18$ (d) $-13/18$

20. $\cos 2(\theta+\phi)-4\cos(\theta+\phi)\sin\theta\sin\phi+2\sin^2\phi=$

(a) $\cos 2\theta$ (b) $\cos 3\theta$

(c) $\sin 2\theta$ (d) $\sin 3\theta$

21. Given that $\cos\left(\frac{\alpha-\beta}{2}\right)=2\cos\left(\frac{\alpha+B}{2}\right)$, then $\tan\frac{\alpha}{2}\tan\frac{\beta}{2}$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) $\frac{1}{8}$

22. If $\sin\theta+\cos\theta=x$, then $\sin^6\theta+\cos^6\theta=\frac{1}{4}[4-3(x^2-1)^2]$ for

(a) All real x (b) $x^2\le 2$

(c) $x^2\ge 2$ (d) None of these

23. If $2\tan A=3\tan B$, then $\frac{\sin 2B}{5-\cos 2B}$ is equal to

(a) $\tan A-\tan B$ (b) $\tan(A-B)$

(c) $\tan(A+B)$ (d) $\tan(A+2B)$

24. If $\sin\theta+\sin\phi=a$ and $\cos\theta+\cos\phi=b$, then $\tan\frac{\theta-\phi}{2}$ is equal to

(a) $\sqrt{\frac{a^2+b^2}{4-a^2-b^2}}$ (b) $\sqrt{\frac{4-a^2-b^2}{a^2+b^2}}$

(c) $\sqrt{\frac{a^2+b^2}{4+a^2+b^2}}$ (d) $\sqrt{\frac{4+a^2+b^2}{a^2+b^2}}$

25. If $\tan\beta = \cos\theta\tan\alpha$, **then** $\tan^2\frac{\theta}{2} =$

(a) $\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}$ (b) $\frac{\cos(\alpha-\beta)}{\cos(\alpha+\beta)}$ (c) $\frac{\sin(\alpha-\beta)}{\sin(\alpha+\beta)}$ (d) $\frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}$

26. If $\frac{2\sin\alpha}{\{1+\cos\alpha+\sin\alpha\}} = y$, **then** $\frac{\{1-\cos\alpha+\sin\alpha\}}{1+\sin\alpha} =$

(a) $\frac{1}{y}$ (b) y

(c) $1-y$ (d) $1+y$

27. $\frac{\sec 8A - 1}{\sec 4A - 1} =$

(a) $\frac{\tan 2A}{\tan 8A}$ (b) $\frac{\tan 8A}{\tan 2A}$

(c) $\frac{\cot 8A}{\cot 2A}$ (d) None of these

28. $\frac{\cos A}{1-\sin A} =$

(a) $\sec A - \tan A$ (b) $\operatorname{cosec} A + \cot A$

(c) $\tan\left(\frac{\pi}{4} - \frac{A}{2}\right)$ (d) $\tan\left(\frac{\pi}{4} + \frac{A}{2}\right)$

29. Let $0 < x < \frac{\pi}{4}$. **Then** $\sec 2x - \tan 2x =$

(a) $\tan\left(x - \frac{\pi}{4}\right)$ (b) $\tan\left(\frac{\pi}{4} - x\right)$

(c) $\tan\left(x + \frac{\pi}{4}\right)$ (d) $\tan^2\left(x + \frac{\pi}{4}\right)$

30. If $\cos\theta = \frac{1}{2}\left(a + \frac{1}{a}\right)$, **then the value of** $\cos 3\theta$ **is**

(a) $\frac{1}{8}\left(a^3 + \frac{1}{a^3}\right)$ (b) $\frac{3}{2}\left(a + \frac{1}{a}\right)$

(c) $\frac{1}{2}\left(a^3 + \frac{1}{a^3}\right)$ (d) $\frac{1}{3}\left(a^3 + \frac{1}{a^3}\right)$

31. If $\tan^2\theta = 2\tan^2\phi + 1$, **then** $\cos 2\theta + \sin^2\phi$ **equals**

(a) –1 (b) 0

(c) 1 (d) None of these

32. $\sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$ is equal to

(a) $\cot 7\frac{1^o}{2}$ (b) $\sin 7\frac{1^o}{2}$

(c) $\sin 15^o$ (d) $\cos\ 15^o$

33. $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)=$

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{1}{8}$ (d) $\frac{1}{16}$

34. The value of $\frac{\tan x}{\tan 3x}$ whenever defined never lie between

(a) 1/3 and 3 (b) 1/4 and 4

(c) 1/5 and 5 (d) 5 and 6

35. $\sin^4\frac{\pi}{4}+\sin^4\frac{3\pi}{8}+\sin^4\frac{5\pi}{8}+\sin^4\frac{7\pi}{8}=$

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$

(c) $\frac{3}{2}$ (d) $\frac{3}{4}$

36. If $\sin 6\theta = 32\cos^5\theta\sin\theta - 32\cos^3\theta\sin\theta + 3x$, **then** $x=$

(a) $\cos\theta$ (b) $\cos 2\theta$

(c) $\sin\theta$ (d) $\sin 2\theta$

37. If $a\cos 2\theta + b\sin 2\theta = c$ **has α and β as its solution, then the value of** $\tan\alpha+\tan\beta$ **is**

(a) $\frac{c+a}{2b}$ (b) $\frac{2b}{c+a}$

(c) $\frac{c-a}{2b}$ (d) $\frac{b}{c+a}$

38. $\sqrt{3}\ \text{cosec}\ 20^o - \sec 20^o =$

(a) 2 (b) $\frac{2\sin 20^o}{\sin 40^o}$

(c) 4 (d) $\frac{4\sin 20^o}{\sin 40^o}$

39. If $\frac{x}{\cos\theta}=\frac{y}{\cos\left(\theta-\frac{2\pi}{3}\right)}=\frac{z}{\cos\left(\theta+\frac{2\pi}{3}\right)}$, **then** $x+y+z=$

(a) 1 (b) 0

(c) -1 (d) None of these

40. **If** $\tan(A+B)=p,\ \tan(A-B)=q,$ **then the value of** $\tan 2A$ **in terms of *p* and *q* is**

(a) $\frac{p+q}{p-q}$ (b) $\frac{p-q}{1+pq}$ (c) $\frac{p+q}{1-pq}$ (d) $\frac{1+pq}{1-p}$

41. **If** $\frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha}=y$, **then** $\frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}$ **equal to**

1) 1/y 2) y 3) 1-y 4) 1+y

42. $\left(\frac{\sqrt{3}+2\cos A}{1-2\sin A}\right)^{-3}+\left(\frac{1+2\sin A}{\sqrt{3}-2\cos A}\right)^{-3}=$

1) 1 2) $\sqrt{3}$ 3) 0 4) -1

43. **The equation whose roots are $\sin^2 18^0$, $\cos^2 36^0$ is**

1) $16x^2 - 12x - 1=0$ 2) $16x^2 - 12x + 1=0$ 3) $16x^2 + 12x+1=0$ 4) $16x^2 + 12x-1=0$

44. **If** $\frac{x^2+1}{2x}=\cos A$, then $\frac{x^6+1}{2x^3}=$

1) $\cos^3 A$ 2) cos3A 3) $\cos^2 A$ 4) cos2A

45. $\tan 9^0 - \tan 27^0 - \tan 63^0 + \tan 81^0 =$

1) 4 2) 2 3) -1 4) -2

46. **The value of $\cos 12^0 \cos 24^0 \cos 48^0 \cos 84^0$ is**

1) 1/16 2) 1/8 3) 1/4 4) ½

47. **The value of $\sin 6^0 \sin 42^0 \sin 66^0 \sin 78^0$ is**

1) 1/6 2) 1/8 3) 1 4) 1/16

48. **The value of $\tan 6^0 \tan 42^0 \tan 66^0 \tan 78^0$ is**

1) 1/16 2) 3/16 3) 1 4) 3

49. **$4\cos 9^0$ =**

1) $\sqrt{3+\sqrt{5}}+\sqrt{5-\sqrt{5}}$ **2)** $\sqrt{3+\sqrt{5}}-\sqrt{5-\sqrt{5}}$ **3)** $\sqrt{3-\sqrt{5}}+\sqrt{5-\sqrt{5}}$ **4)** $\sqrt{3+\sqrt{5}}-\sqrt{5+\sqrt{5}}$

50. **If $\tan 70^0 - \tan 20^0 - 2\tan 40^0 = k.\tan\theta$, then (k, θ) =**

1) $(4, 10^0)$ 2) $(4, 20^0)$ 3) $(2, 10^0)$ 4) $(2, 20^0)$

51. $\operatorname{Tan}^6\frac{\pi}{9}-33\tan^4\frac{\pi}{9}+27\tan^2\frac{\pi}{9}=$

1) 0 2) $\sqrt{3}$ 3) 3 4) 9

52. $\left(1+\sec 20^o\right)\left(1+\sec 40^o\right)\left(1+\sec 80^o\right)=$

1) 0 2) $\cot^2 10^0$ 3) .1 4) $\tan^2 10$

53. $(2\cos\theta-1)\,(2\cos 2\theta-1)\,(2\cos 4\theta-1)\,(2\cos 8\theta-1)=$

1) 0 2) 1 3) $\frac{2\cos 8\theta+1}{2\cos\theta+1}$ 4) $\frac{2\cos 16\theta+1}{2\cos\theta+1}$

MULTIPLE AND SUBMULTIPLE ANGLES

HINTS AND SOLUTIONS

1. (d) $\frac{1}{\tan 3A - \tan A} - \frac{1}{\cot 3A - \cot A}$

$$= \frac{1}{\tan 3A - \tan A} + \frac{\tan A \tan 3A}{\tan 3A - \tan A} = \frac{1}{\tan 2A} = \cot 2A.$$

2. (c) $\sqrt{2+\sqrt{2+2\cos 4\theta}} = \sqrt{2+\sqrt{2.2\cos^2 2\theta}}$

$$= \sqrt{2+2\cos 2\theta} = \sqrt{4\cos^2\theta} = 2\cos\theta.$$

3. (b) $\frac{\cot^2 15^o - 1}{\cot^2 15^o + 1} = \frac{\frac{\cos^2 15^o}{\sin^2 15^o} - 1}{\frac{\cos^2 15^o}{\sin^2 15^o} + 1}$

$$= \frac{\cos^2 15^o - \sin^2 15^o}{\cos^2 15^o + \sin^2 15^o} = \cos(30^o) = \frac{\sqrt{3}}{2}.$$

4. (d) $1 - 2\sin^2\left(\frac{\pi}{4} + \theta\right) = \cos\left(\frac{\pi}{2} + 2\theta\right) = -\sin 2\theta$.

5. (a) $\tan\theta = \frac{b}{a}$.

$$a\cos 2\theta + b\sin 2\theta = a\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) + b\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

6. (d) $(\sec 2A + 1)\sec^2 A = \left(\frac{1+\tan^2 A}{1-\tan^2 A} + 1\right)(1+\tan^2 A)$

$$= \frac{2(1+\tan^2 A)}{1-\tan^2 A} = 2\sec 2A.$$

7. (d) $\tan\frac{A}{2} = \frac{3}{2}$. ➔ $\frac{1+\cos A}{1-\cos A} = \frac{2\cos^2\frac{A}{2}}{2\sin^2\frac{A}{2}} = \cot^2\frac{A}{2} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$.

8. (b) $\tan x = \frac{b}{a}$

$$\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}} = \sqrt{\frac{1+b/a}{1-b/a}} + \sqrt{\frac{1-b/a}{1+b/a}}$$

$$= \frac{2}{\sqrt{1-\frac{b^2}{a^2}}} = \frac{2}{\sqrt{1-\tan^2 x}} = \frac{2}{\sqrt{1-\frac{\sin^2 x}{\cos^2 x}}} = \frac{2\cos x}{\sqrt{\cos 2x}}.$$

9. (b) $\cos 2B = \frac{\cos(A+C)}{\cos(A-C)} = \frac{\cos A\cos C - \sin A\sin C}{\cos A\cos C + \sin A\sin C}$

$$\Rightarrow \frac{1-\tan^2 B}{1+\tan^2 B} = \frac{1-\tan A\tan C}{1+\tan A\tan C}$$

$\Rightarrow 1+\tan^2 B-\tan A\tan C-\tan A\tan C\tan^2 B$

$=1-\tan^2 B+\tan A\tan C-\tan A\tan C\tan^2 B$

$\Rightarrow 2\tan^2 B=2\tan A\tan C\Rightarrow \tan^2 B=\tan A\tan C$

10. (a) $\left(\frac{\sin 2A}{1+\cos 2A}\right)\left(\frac{\cos A}{1+\cos A}\right)$

$=\frac{2\sin A\cos A}{2\cos^2 A}\frac{\cos A}{1+\cos A}=\frac{\sin A}{1+\cos A}=\tan\frac{A}{2}$.

11. **(a) Given** $\cos(\theta-\alpha), \cos\theta$ **and** $\cos(\theta+\alpha)$ **are in H.P.**

$\Rightarrow \frac{1}{\cos(\theta-\alpha)},\frac{1}{\cos\theta},\frac{1}{\cos(\theta+\alpha)}$ Will be in A.P

Hence, $\frac{2}{\cos\theta}=\frac{1}{\cos(\theta-\alpha)}+\frac{1}{\cos(\theta+\alpha)}$

$=\frac{\cos(\alpha+\theta)+\cos(\theta-\alpha)}{\cos^2\theta-\sin^2\alpha}\Rightarrow \frac{2}{\cos\theta}=\frac{2\cos\theta\cos\alpha}{\cos^2\theta-\sin^2\alpha}$

$\Rightarrow \cos^2\theta-\sin^2\alpha=\cos^2\theta\cos\alpha$

$\Rightarrow \cos^2\theta(1-\cos\alpha)=\sin^2\alpha$

$\Rightarrow \cos^2\theta\left(2\sin^2\frac{\alpha}{2}\right)=4\sin^2\frac{\alpha}{2}\cos^2\frac{\alpha}{2}$

$\cos^2\theta\sec^2\frac{\alpha}{2}=2\Rightarrow \cos\theta\sec\frac{\alpha}{2}=\pm\sqrt{2}$.

12. **(d) Given,** $\tan\theta=\frac{1}{7}, \sin\phi=\frac{1}{\sqrt{10}}$

$\sin\theta=\frac{1}{\sqrt{50}}, \cos\theta=\frac{7}{\sqrt{50}}, \cos\phi=\frac{3}{\sqrt{10}}$

$\therefore \cos 2\phi=2\cos^2\phi-1=2.\frac{9}{10}-1=\frac{8}{10}$

$\sin 2\phi=2\sin\phi\cos\phi=2\times.\frac{1}{\sqrt{10}}\times\frac{3}{\sqrt{10}}=\frac{6}{10}$

$\therefore \cos(\theta+2\phi)=\cos\theta\cos 2\phi-\sin\theta\sin 2\phi$

$=\frac{7}{\sqrt{50}}\times\frac{8}{10}-\frac{1}{\sqrt{50}}.\frac{6}{10}$

13. (d) $\sin A=\frac{4}{5}\Rightarrow \tan A=-\frac{4}{3}$, $(90^o<A<180^o)$

$\tan A=\frac{2\tan\frac{A}{2}}{1-\tan^2\frac{A}{2}}$, (Let $\tan\frac{A}{2}=P$)

$\Rightarrow -\frac{4}{3}=\frac{2P}{1-P^2}\Rightarrow 4P^2-6P-4=0$

$\Rightarrow P = \frac{-1}{2}$ (impossible), hence $\tan\frac{A}{2} = 2$.

14. (c) For $A = 133^o, \frac{A}{2} = 66.5^o \Rightarrow \sin\frac{A}{2} > \cos\frac{A}{2} > 0$

Hence, $\sqrt{1+\sin A} = \sin\frac{A}{2} + \cos\frac{A}{2}$ .....(i)

and $\sqrt{1-\sin A} = \sin\frac{A}{2} - \cos\frac{A}{2}$ .....(ii)

Subtract (ii) from (i), $2\cos\frac{A}{2} = \sqrt{1+\sin A} - \sqrt{1-\sin A}$.

15. (c) $\sin 15^o = \sin(45^o - 30^o) = \frac{\sqrt{3}-1}{2\sqrt{2}} =$ irrational

$\cos 15^o = \cos(45^o - 30^o) = \frac{\sqrt{3}+1}{2\sqrt{2}} =$irrational

$\therefore \sin 15^o \cos 15^o = \frac{1}{2}(2\sin 15^o \cos 15^o)$

$= \frac{1}{2}\sin 30^o = \frac{1}{2}.\frac{1}{2} = \frac{1}{4} =$ rational

$\therefore \sin 15^o \cos 75^o = \sin 15^o \sin 15^o = \sin^2 15^o$

$= \left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^2 = \frac{4-2\sqrt{3}}{8} =$ irrational

16. (b) $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$$\tan\theta = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{1-2\sin^2\frac{\theta}{2}} = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$$

17. (c) standard problem

18. (d) $\sqrt{\frac{1-\sin A}{1+\sin A}} = \sqrt{\frac{1-\cos\left(\frac{\pi}{2}-A\right)}{1+\cos\left(\frac{\pi}{2}-A\right)}}$

$$= \sqrt{\frac{2\sin^2\left(\frac{\pi}{4}-\frac{A}{2}\right)}{2\cos^2\left(\frac{\pi}{4}-\frac{A}{2}\right)}} = \tan\left(\frac{\pi}{4}-\frac{A}{2}\right).$$

19. (b) Since α is a root of $25\cos^2\theta + 5\cos\theta - 12 = 0$

$\therefore 25\cos^2\alpha + 5\cos\alpha - 12 = 0$

$\Rightarrow \cos\alpha = \frac{-5 \pm \sqrt{25+1200}}{50} = \frac{-5 \pm 35}{50}$

$\Rightarrow \cos\alpha = -4/5$ $[\because \pi/2 < \alpha < \pi \Rightarrow \cos\alpha < 0]$

$\therefore \sin 2\alpha = 2\sin\alpha\cos\alpha = -24/25$.

20. (a) We have, $\cos 2(\theta+\phi) - 4\cos(\theta+\phi)\sin\theta\sin\phi + 2\sin^2\phi$

Now, put $\theta = \phi = \frac{\pi}{4}$

$\cos 2\left(\frac{\pi}{2}\right) - 4\cos\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right) + 2\sin^2\left(\frac{2\pi}{4}\right) = 0$ Put $\theta = \phi = \pi/4$ in option (a),

Then, $\cos 2\theta = \cos\pi/2 = 0$.

21. (b) $\cos\left(\frac{\alpha-\beta}{2}\right) = 2\cos\left(\frac{\alpha+\beta}{2}\right)$

$$\Rightarrow \cos\frac{\alpha}{2}\cos\frac{\beta}{2} + \sin\frac{\alpha}{2}\sin\frac{\beta}{2} = 2\cos\frac{\alpha}{2}\cos\frac{\beta}{2} - 2\sin\frac{\alpha}{2}\sin\frac{\beta}{2}$$

$$\Rightarrow 3\sin\frac{\alpha}{2}\sin\frac{\beta}{2} = \cos\frac{\alpha}{2}\cos\frac{\beta}{2} \Rightarrow \tan\frac{\alpha}{2}\tan\frac{\beta}{2} = \frac{1}{3}.$$

22. (b) squaring $\sin 2\theta = x^2 - 1 \le 1 \Rightarrow x^2 \le 2$

Or $-\sqrt{2} \le x \le \sqrt{2}$ $[\because \sin 2\theta \le 1]$

Now $\sin^6\theta + \cos^6\theta$

$$= (\sin^2\theta + \cos^2\theta)^3 - 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta)$$

$$= 1 - 3\sin^2\theta\cos^2\theta = 1 - \frac{3}{4}\sin^2 2\theta$$

$$= 1 - \frac{3}{4}(x^2-1)^2 = \frac{1}{4}\{4 - 3(x^2-1)^2\}$$

23. (b) $2\tan A = 3\tan B$

$\Rightarrow \tan A = \frac{3}{2}\tan B = \frac{3}{2}t$, [Let $\tan B = t$]

$$\Rightarrow \sin 2B = \frac{2t}{1+t^2}, \cos 2B = \frac{1-t^2}{1+t^2}$$

$$\therefore \frac{\left(\frac{2t}{1+t^2}\right)}{5 - \left(\frac{1-t^2}{1+t^2}\right)} = \frac{2t}{4+6t^2} = \frac{t}{2+3t^2} = \tan(A-B).$$

24. (b) Put $\theta = \frac{\pi}{2}, \phi = 0^o$, then $a = 1 = b$

$\therefore \tan\frac{\theta-\phi}{2} = 1$, which is given by (a) and (b)

Again putting $\theta = \frac{\pi}{4} = \phi$, we get $\tan\frac{\theta-\phi}{2} = 0$,

25. (c) $\tan^2\frac{\theta}{2} = \frac{1-\cos\theta}{1+\cos\theta} = \frac{\tan\alpha - \tan\beta}{\tan\alpha + \tan\beta} = \frac{\sin(\alpha-\beta)}{\sin(\alpha+\beta)}$.

26. (b) We have, $\frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha}=y$

Then $\frac{4\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{2\cos^2\frac{\alpha}{2}+2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}=y$

$\Rightarrow \frac{2\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}}\times\frac{\left(\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}\right)}{\left(\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}\right)}=y$

$\Rightarrow \frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}=y$.

27. (b) $\frac{\sec 8A-1}{\sec 4A-1}=\frac{1-\cos 8A}{\cos 8A}.\frac{\cos 4A}{1-\cos 4A}$

$=\frac{2\sin^2 4A}{\cos 8A}\frac{\cos 4A}{2\sin^2 2A}=\frac{2\sin 4A\cos 4A}{\cos 8A}\frac{\sin 4A}{2\sin^2 2A}$

$=\tan 8A\frac{2\sin 2A\cos 2A}{2\sin^2 2A}=\frac{\tan 8A}{\tan 2A}$.

28. (d) $\frac{\cos A}{1-\sin A}=\frac{\cos A(1+\sin A)}{\cos^2 A}=\frac{(1+\sin A)}{\cos A}$

$=\frac{\left(\cos\frac{A}{2}+\sin\frac{A}{2}\right)^2}{\left(\cos\frac{A}{2}+\sin\frac{A}{2}\right)\left(\cos\frac{A}{2}-\sin\frac{A}{2}\right)}=\frac{\cos\frac{A}{2}+\sin\frac{A}{2}}{\cos\frac{A}{2}-\sin\frac{A}{2}}$

$=\frac{1+\tan\frac{A}{2}}{1-\tan\frac{A}{2}}$,

29. (b) $\sec 2x-\tan 2x=\frac{1-\sin 2x}{\cos 2x}$

$=\frac{(\cos x-\sin x)^2}{(\cos^2 x-\sin^2 x)}=\frac{\cos x-\sin x}{\cos x+\sin x}=\frac{1-\tan x}{1+\tan x}$

$=\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\left(\frac{\pi}{4}\right)\sin x}=\tan\left(\frac{\pi}{4}-x\right)$.

30. (c) $\because \cos 3\theta=4\cos^3\theta-3\cos\theta$

$\therefore \cos 3\theta=4\frac{1}{2^3}\left(a+\frac{1}{a}\right)^3-3\frac{1}{2}\left(a+\frac{1}{a}\right)$

$\Rightarrow \cos 3\theta=\frac{1}{2}\left(a+\frac{1}{a}\right)\left[\left(a+\frac{1}{a}\right)^2-3\right]$

$\Rightarrow \cos 3\theta=\frac{1}{2}\left(a^3+\frac{1}{a^3}\right)$.

31. (b) Let $\theta = 45^o$, then $\phi = 0$

$\therefore \cos(2\times 45^o) + \sin^2 0 = 0 + 0 = 0$.

32. (a) We have $\cot A = \frac{\cos A}{\sin A} = \frac{2\cos^2 A}{2\sin A\cos A} = \frac{1+\cos 2A}{\sin 2A}$

Putting $A = 7\frac{1^o}{2} \Rightarrow \cot 7\frac{1^o}{2} = \frac{1+\cos 15^o}{\sin 15^o}$ On simplification,

We get $\cot 7\frac{1^o}{2} = \sqrt{6}+\sqrt{2}+\sqrt{3}+\sqrt{4}$.

33. (c) standard problem

34. (a) Let $y = \frac{\tan x}{\tan 3x} = \frac{\tan x}{\frac{3\tan x - \tan^3 x}{1-3\tan^2 x}}$

$$y = \frac{1-3\tan^2 x}{3-\tan^2 x} = \frac{\frac{1}{3}-\tan^2 x}{1-\frac{1}{3}.\tan^2 x}$$

Hence, y should never lie between $\frac{1}{3}$ and 3 whenever defined.

35. (c)standard problem

$= \frac{1}{4}(3)+\frac{1}{4}(3) = \frac{3}{2}$.

36. (d) $\sin 6\theta = 2\sin 3\theta\cos 3\theta$

$= 2[3\sin\theta - 4\sin^3\theta][4\cos^3\theta - 3\cos\theta]$

$=24\sin\theta\cos\theta(\sin^2\theta+\cos^2\theta) - 18\sin\theta\cos\theta - 32\sin^2\theta\cos^2\theta$

$= 32\cos^5\theta\sin\theta - 32\cos^3\theta\sin\theta + 3\sin 2\theta$

On comparing, $x = \sin 2\theta$.

37. (b) $a\cos 2\theta + b\sin 2\theta = c$

$\Rightarrow a\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) + b\frac{2\tan\theta}{1+\tan^2\theta} = c$

$\Rightarrow \quad a - a\tan^2\theta + 2b\tan\theta = c + c\tan^2\theta$

$\Rightarrow -(a+c)\tan^2\theta + 2b\tan\theta + (a-c) = 0$

$\therefore \tan\alpha + \tan\beta = -\frac{2b}{-(c+a)} = \frac{2b}{c+a}$.

38. (c) $\sqrt{3}\text{cosec } 20° - \sec 20° = \frac{\sqrt{3}}{\sin 20°} - \frac{1}{\cos 20°}$

$$= \frac{\sqrt{3}\cos 20° - \sin 20°}{\sin 20° \cos 20°} = \frac{2\left[\frac{\sqrt{3}}{2}\cos 20° - \frac{1}{2}\sin 20°\right]}{\frac{2}{2}\sin 20° \cos 20°}$$

$$= \frac{4\cos(20° + 30°)}{\sin 40°} = \frac{4\cos 50°}{\sin 40°} = \frac{4\sin 40°}{\sin 40°} = 4 .$$

39. (b) $\frac{x}{\cos\theta} = \frac{y}{\cos\left(\theta - \frac{2\pi}{3}\right)} = \frac{z}{\cos\left(\theta + \frac{2\pi}{3}\right)} = k$

$$\Longrightarrow\ x = k\cos\theta,\ y = k\cos\left(\theta - \frac{2\pi}{3}\right),\ z = k\cos\left(\theta + \frac{2\pi}{3}\right)$$

$$\Longrightarrow\ x + y + z = k\left[\cos\theta + \cos\left(\theta - \frac{2\pi}{3}\right) + \cos\left(\theta + \frac{2\pi}{3}\right)\right]$$

$$= k[(0) = 0 \qquad \Rightarrow \quad x + y + z = 0 .$$

40. (c) $2A = (A + B) + (A - B) \ \Rightarrow\ \tan 2A = \frac{\tan(A + B) + \tan(A - B)}{1 - \tan(A + B)\tan(A - B)} = \frac{p + q}{1 - pq}$.

41.(b)

42.(c)

43.(b)

44. (b)

45. (a)

46. (a)

47. (d)

48. (c)

49. (a)

50. (a)

51. (c)

52. (c)

53. (d)