COORDINATE GEOMETRY

LOCUS

EXERCISE

1. The locus of P(x,y) such that its distance from A(0,0) is less than 5 units is

   1) $x^2 + y^2 < 5$ 2) $x^2 + y^2 < 10$

   3) $x^2 + y^2 < 25$ 4) $x^2 + y^2 < 20$

2. The equation of the locus of the point whose distance from x-axis is twice its distance from the y-axis, is

   1)$y^2=4x^2$ 2)$4y^2=x^2$ 3)$y=3x$ 4)$4x+y=0$

3. The equation to the locus of points equidistant from the points (2,3), (–2,5) is

   1) $2x - y + 4 = 0$ 2) $2x - y - 1 = 0$

   3) $2x + y - 4 = 0$ 4) $2x + y + 1 = 0$

4. If the equation of the locus of a point equidistant from the points (a_1, b_1) and (a_2, b_2) is $(a_1 - a_2)x + (b_1 - b_2)y + c = 0$ then the value of c is

   1) $\frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2)$ 2) $a_1^2 - a_2^2 + b_1^2 - b_2^2$

   3) $\frac{1}{2}(a_1^2 + a_2^2 + b_1^2 + b_2^2)$ 4) $\sqrt{a_1^2 + b_1^2 - a_2^2 - b_2^2}$

5. $A(-9,0), B(-1,0)$ are two points. If P is a point such that PA:PB = 3 : 1, then the locus of P is

   1) $x^2 + y^2 = 9$ 2) $x^2 + y^2 + 9 = 0$

   3) $x^2 - y^2 = 9$ 4) $x^2 - y^2 + 9 = 0$

6. The locus of the moving point P, such that 2PA = 3PB where A(0,0), B(4,–3) is

   1) $5x^2 + 5y^2 - 72x + 54y + 225 = 0$ 2) $5x^2 + 5y^2 + 72x - 54y - 225 = 0$

   3) $3x^2 + 3y^2 - 70x + 52y + 225 = 0$ 4) None

7. Sum of the squares of the distance from a point to (c,0) and (–c,0) is $4c^2$. It's locus is

   1) $x^2 + y^2 + c^2 = 0$ 2) $x^2 + y^2 = 4c^2$

   3) $x^2 + y^2 = c^2$ 4) $x^2 - y^2 = c^2$

8. A(2,3), B(1,5),C(–1,2) are three points. If P is a point moves such that $PA^2 + PB^2 = 2PC^2$, then the locus of P is

   1) $10x - 8y + 29 = 0$ 2) $10x + 8y - 29 = 0$

   3) $10x + 8y + 29 = 0$ 4) $10x - 8y - 29 = 0$

9. $A(2,3), B(-1,1)$ are two points. If P is a point such that $\angle APB = 90°$, then the locus of P is

1) $x^2+y^2-x-4y+1=0$ 2) $x^2+y^2+x+4y-1=0$

3) $x^2+y^2-x+4y-1=0$ 4) $x^2+y^2+x-4y+1=0$

10. The locus of P such that are of Δ PAB is 12 square units where A= (2,3) and B=(–4,5) is

1) $x^2+6xy+9y^2+22x+66y+23=0$ 2) $x^2-6xy+9y^2+22x+66y+23=0$

3) $x^2+6xy+9y^2-22x-66y-23=0$ 4) $x^2-6xy+9y^2-22x-66y-23=0$

11. $0(0,0), A(4,0), B(0,6)$ are three points. If P is a point such that area of Δ POB is twice the area of Δ POA, then the locus of P is

1) $4x^2-6y^2=0$ 2) $3x^2-4y^2=0$

3) $9x^2-16y^2=0$ 4) $4x^2-9y^2=0$

12. A(2,3), B(2,–3) are two points. The equation to the locus of P such that PA+PB=8 is

1) $16x^2+7y^2-64x-48=0$ 2) $16x^2+7y^2-64x+48=0$

3) $16x^2-7y^2+64x-48=0$ 4) $16x^2-7y^2+64x+48=0$

13. A(2,3), B(–2,3) are two points. The locus of P which moves such that PA–PB = 4 is

1) $y+3=0$ 2) $y-3=0$

3) $y^2+3=0$ 4) $y^2-3=0$

14. The perimeter of a triangle is 20 and the points $(-2,-3)$ and $(-2,3)$ are two of the vertices of it. The locus of the third vertex is

1) $\frac{(x+2)^2}{40}+\frac{y^2}{49}=1$ 2) $\frac{(x-2)^2}{40}+\frac{y^2}{49}=1$

3) $\frac{(x+2)^2}{49}+\frac{y^2}{40}=1$ 4) none

15. The locus represented by $x=\frac{a}{2}\left(t+\frac{1}{t}\right), y=\frac{a}{2}\left(t-\frac{1}{t}\right)$ is

1) $x^2+y^2=a^2$ 2) $x^2-y^2=a^2$

3) $2x^2-y^2=a^2$ 4) $x^2-2y^2=a^2$

16. The locus of the point $\left(a\cos\theta + b\sin\theta, a\sin\theta - b\cos\theta\right)$ where $0 \le \theta < 2\pi$ is

1) $x^2 + y^2 = a^2 + b^2$ 2) $\left(x^2 - y^2\right)^2 = 16xy$

3) $x^2 - y^2 = a^2 + b^2$ 4) $x^2 - y^2 = a^2 - b^2$

17. If a point (x,y) = $\left(\tan\theta + \sin\theta, \tan\theta - \sin\theta\right)$, then the locus of (x,y) is

1) $\left(x^2y\right)^{2/3} + \left(xy^2\right)^{2/3} = 1$ 2) $x^2 - y^2 = 4xy$

3) $x^2 - y^2 = 12xy$ 4) $\left(x^2 - y^2\right)^2 = 16xy$

18. The locus of the point $\left(\operatorname{cosec}\theta - \sin\theta, \sec\theta - \cos\theta\right)$ where $0 \le \theta < 2\pi$ is

1) $\left(x^2y\right)^{2/3} + \left(xy^2\right)^{2/3} = 1$ 2) $\left(x^2y^2\right)^{2/3} + \left(xy^2\right)^{2/3} = 1$

3) $\left(x/a\right)^{2/3} + \left(y/b\right)^{2/3} = 1$ 4) $\left(x^2/a\right)^{2/3} + \left(y^2/b\right)^{2/3} = 1$

19. The locus of the point represented by $x = 3\left(\cos t + \sin t\right), y = 2\left(\cos t - \sin t\right)$ is

1) $\frac{x^2}{9} + \frac{y^2}{4} = 1$ 2) $\frac{x^2}{4} + \frac{y^2}{9} = 1$

3) $\frac{x^2}{18} + \frac{y^2}{8} = 1$ 4) $\frac{x^2}{8} + \frac{y^2}{18} = 1$

20. The locus of the point represented by $x = t^2 + t + 1$, $y = t^2 - t + 1$ is

1) $x^2 - 2xy + y^2 - 2x - 2y + 4 = 0$ 2) $x^2 + 2xy + y^2 - 2x - 2y + 4 = 0$

3) $x^2 - 2xy + y^2 + 2x + 2y + 4 = 0$ 4) $x^2 - 2xy - y^2 + 2x + 2y - 4 = 0$

21. If a point P moves such that its distances from the point A(1,1) and the line x+y+2 = 0 are equal then the locus of P is

1) a straight line 2) a pair of straight line

3) a parabola 4) an ellipse

22. If $p, x_1, x_2, x_3....$ and $q, y_1, y_2, y_3....$ form two infinite AP's with common differences a and b respectively, then locus of P $\left(\alpha, \beta\right)$, where $\alpha = \frac{x_1 + x_2 + ... + x_\infty}{n}$ and $\beta = \frac{y_1 + y_2 + ... + y_n}{n}$

1) $a\left(x - p\right) = b\left(y - q\right)$ 2) $p\left(x - a\right) = q\left(y - b\right)$

3) $p\left(x - p\right) = a\left(y - q\right)$ 4) $b\left(x - p\right) = a\left(y - q\right)$

23. A straight rod of length 9 unit, slides with its ends A,B always on the x and y axes respectively. Then the locus of the centroid of ΔOAB is

1) $x^2+y^2=3$ 2) $x^2+y^2=9$

3) $x^2+y^2=1$ 4) $x^2+y^2=81$

24. The ends of a rod of length / move on two mutually perpendicular lines. The locus of the point on the rod which divides it in the ratio 1:2 is

1) $9x^2+34y^2=2\ell^2$ 2) $9x^2-34y^2=\ell^2$

3) $9x^2+36y^2=4\ell^2$ 4) none of these

25. Locus of centroid of the triangle whose vertices are (a cos t, a sin t), (b sin t, –b cos t) and (1,0), where t is a parameter, is

1) $(3x-1)^2+(3y^2)=a^2-b^2$ 2) $(3x-1)^2+(3y^2)=a^2+b^2$

3) $(3x+1)^2+(3y^2)=a^2+b^2$ 4) $(3x+1)^2+(3y^2)=a^2-b^2$

26. A(a,0), B(–a,0) are two points. If a point P moves such that $\angle PAB-\angle PBA=\pi/2$ the locus of P is

1) $x^2+y^2=a^2$ 2) $x^2-y^2+a^2=0$

3) $x^2-2xy-y^2=a^2$ 4) $x^2-y^2=a^2$

27. A(a,0), B(–a,0) are two points. If a point P moves such that $\angle PAB-\angle PBA=2\alpha$ then the locus of P is

1) $x^2+2xy\tan 2\alpha-y^2=a^2$ 2) $x^2-2xy\cot\alpha+y^2=a^2$

3) $x^2-2xy\tan^2 2\alpha+y^2=a^2$ 4) $x^2+2xy\cot 2\alpha-y^2=a^2$

28. Equation $\sqrt{(x-2)^2+y^2}+\sqrt{(x+2)^2+y^2}=4$ represents

1) A circle 2) A pair of lines

3) A parabola 4) An ellipse

LOCUS- SOLUTIONS

1. Ans.3

Sol: $PA < 5 \Rightarrow \sqrt{x^2 + y^2} < 5 \Rightarrow x^2 + y^2 < 25$

2. Ans.1

Sol: Let P(x,y)be a point in the locus. Distance from x-axis = 2 (distance from y-axis)

$\Rightarrow |y| = 2|x| \Rightarrow y^2 = 4x^2$.

3. Ans.1

Sol: $A(2,3)B(-2,5)$. Let the point be $P(x,y)$

$PA = PB \Rightarrow PA^2 = PB^2 \Rightarrow (x-2)^2 + (y-3)^2$

$= (x+2)^2 + (y-5)^2 \Rightarrow -4x - 6y + 13 = 4x - 10y + 29 \Rightarrow 2x - y + 4 = 0$

4. Ans.1

Sol: Locus equation is $(a_1-a_2)x+(b_1-b_2)y = \frac{1}{2}(OA^2 - OB^2) \Rightarrow e = \frac{1}{2}(OB^2 - OA^2)$

$= \frac{1}{2}(a_2^2 + b_2^2 - a_1^2 - b_1^2)$

5. Ans.1

Sol: Let P +(x,y), Given that $PA:PB = 3:1 \Rightarrow PA = 3PB \Rightarrow PA^2 = 9PB^2$

$\Rightarrow (x+9)^2 + y^2 = 9\left[(x+1)^2 + y^2\right] \Rightarrow 8x^2 + 8y^2 = 72 \Rightarrow x^2 + y^2 = 9$

6. Ans.1

Sol: Let P(x,y) A(0,0) B(4,3)

$2PA = 3PB \Rightarrow 4PA^2 = 9PB^2 \Rightarrow 4(x^2 + y^2) = 9\left[(x-4)^2 + (y+3)^2\right]$

$\Rightarrow 5x^2 + 5y^2 - 72x + 54y + 225 = 0$

7. Ans.3

Sol: A(c,0) B(-c,0). Let the point be P(x,y)

$PA^2 + PB^2 = 4C^2 \Rightarrow (x-c)^2 + y^2 + (x+c)^2 + y^2 = 4c^2 \Rightarrow 2x^2 + 2y^2 = 2c^2 \Rightarrow x^2 + y^2 = c^2$

8. Ans.2

Sol: A(2,3) B(1,5)C(-1,2) Let P(x,y)

$$PA^2+PB^2=2PC^2 \Rightarrow (x-2)^2+(y-3)^2+(x-1)^2$$

$$+(y-5)^2=2\left[(x+1)^2+(y-2)^2\right]$$

$$\Rightarrow -4x-6y+13-2x-10y+26$$

$$=2(2x-4y+5) \Rightarrow 10x+8y-29=0$$

9. Ans. 1

Sol: A(2,3)B(-1,1) Let P=(x,y)

$$\angle APB=90° \Rightarrow PA^2+PB^2=AB^2$$

$$\Rightarrow (x-2)^2+(y-3)^2+(x+1)^2+(y-1)^2=3^2+2^2$$

$$\Rightarrow 2(x^2+y^2-x-4y+1)=0 \Rightarrow x^2+y^2-x-4y+1=0$$

10. Ans.3

Sol: Let P=(x,y), Given that area of Δ PAB = 12sq. Unit.

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 2+4 & 3-5 \\ 2-x & 3-y \end{vmatrix}=12 \Rightarrow \begin{vmatrix} 6 & -2 \\ (2-x) & (3-y) \end{vmatrix}=24$$

$$\Rightarrow |6(3-y)-(-2)(2-x)|=24$$

$$\Rightarrow |22-6y-2x|=24$$

$$\Rightarrow |-2(x+3y-11)|=24$$

$$\Rightarrow |-(x+3y-11)|=12 \Rightarrow (x+3y-11)^2=144 \Rightarrow x^2+6xy+9y^2-22x-66y-23=0$$

11. Ans.3

Sol: let P=(x,y). Given that area of ΔPOA

$$\Rightarrow \frac{1}{2}\begin{vmatrix} 0-0 & 0-6 \\ 0-x & 0-y \end{vmatrix}=2\times\frac{1}{2}\begin{vmatrix} 0-4 & 0-0 \\ 0-x & 0-y \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} 0 & -6 \\ -x & -y \end{vmatrix} = 2\begin{vmatrix} -4 & 0 \\ -x & -y \end{vmatrix} \Rightarrow |-6x| = 2|4y| \Rightarrow 36x^2 = 64y^2 \Rightarrow 9x^2 = 16y^2$$

$$\Rightarrow 9x^2 - 16y^2 = 0$$

12. Ans.1

Sol: The locus of P is

$$\frac{4(x-2)^2}{8^2 - 4(3)^2} + \frac{4y^2}{8^2} = 1 \quad 1 \Rightarrow 16(x^2 - 4x + 4) + 7y^2 = 112$$

$$\Rightarrow 16x^2 + 7y^2 - 64x - 48 = 0$$

13. Ans.2

Sol: The locus of P is $\frac{4x^2}{4^2} + \frac{4(y-3)^2}{4^2 - 4(2)^2} = 1 \Rightarrow y - 3 = 0$

14. Ans.1

Sol: $A(-2,-3), B(-2,3)$. Let the third vertex be P= (x, y). $AB = 6$

PA+PB+AB = 20 $\Rightarrow$ PA+PB=20-AB = 20-6 = 14 $\rightarrow (1)$

The locus of P is $\frac{4(x+2)^2}{14^2 - 4(3)^2} + \frac{4y^2}{14^2} = 1 \Rightarrow \frac{4(x+2)^2}{160} + \frac{4y^2}{196} = 1 \Rightarrow \frac{(x+2)^2}{40} + \frac{y^2}{49} = 1$

15. Ans.2

Sol: $x = \frac{a}{2}\left(t + \frac{1}{t}\right) \Rightarrow t + \frac{1}{t} = \frac{2x}{a}$

$$\therefore y = \frac{a}{2}\left(t - \frac{1}{t}\right) \Rightarrow t - \frac{1}{t} = \frac{2y}{a}$$

$$\left(t + \frac{1}{t}\right)^2 = \left(t - \frac{1}{t}\right)^2 + 4 \Rightarrow \left(\frac{2x}{a}\right)^2 = \left(\frac{2y}{a}\right)^2 + 4 \Rightarrow \frac{4x^2}{a^2} - \frac{4y^2}{a^2} = 4 \Rightarrow x^2 - y^2 = a^2$$

16. Ans.1

Sol: Let the point be (x,y)

$$\therefore (x, y) = (a\cos\theta + b\sin\theta, a\sin\theta - b\sin\theta) \Rightarrow x = a\cos\theta + b\sin\theta, y = a\sin\theta - b\cos\theta$$

$$x^2+y^2=(a\cos\theta+b\sin\theta)^2+(a\sin\theta-b\cos\theta)^2= a^2(\cos^2\theta+\sin^2\theta)+b^2(\sin^2\theta+\cos^2)$$

$$\Rightarrow x^2+y^2=a^2+b^2$$

17. Ans.4

Sol: $\therefore(x,y)=(\tan\theta+\sin\theta,\tan\theta-\sin\theta)\ \Rightarrow x=\tan\theta+\sin\theta, y=\tan\theta-\sin\theta$

$$xy=(\tan\theta+\sin\theta)(\tan\theta-\sin\theta)\ =\tan^2\theta-\sin^2\theta=\tan^2\theta(1-\cos^2\theta)=\tan^2\theta\sin^2\theta$$

$$x^2-y^2=(\tan\theta+\sin\theta)^2-(\tan\theta-\sin\theta)^2\ =4\tan\theta.\sin\theta\Rightarrow(x^2-y^2)^2=16\tan^2\theta.\sin^2\theta$$

$$\Rightarrow(x^2-y^2)^2=16xy$$

18. Ans.1

Sol: $\therefore(x,y)=(\cos ec\theta-\sin\theta,\sec\theta-\cos\theta)$

$$\Rightarrow x=\cos ec\theta-\sin\theta, y=\sec\theta-\cos\theta$$

$$x=\cos ec\theta-\sin\theta=\frac{1-\sin^2\theta}{\sin\theta}=\frac{\cos^2\theta}{\sin\theta};$$

$$y=\sec\theta-\cos\theta=\frac{1-\cos^2\theta}{\cos\theta}=\frac{\sin^2\theta}{\cos\theta}$$

$$x^2y=\frac{\cos^4\theta}{\sin^2\theta}\times\frac{\sin^2\theta}{\cos\theta}=\cos^3\theta\ \ xy^2=\frac{\cos^2\theta}{\sin\theta}\times\frac{\sin^4\theta}{\cos^2\theta}=\sin^3\theta$$

$$(x^2y)^{2/3}+(xy^2)^{2/3}=\cos^2\theta+\sin^2\theta\ \Rightarrow(x^2y)^{2/3}+(xy^2)^{2/3}=1$$

19. Ans.3

Sol: $\frac{x}{3}=\cos t+\sin t,\frac{y}{2}=\cos t-\sin t\ \Rightarrow\left(\frac{x}{3}\right)^2+\left(\frac{y}{2}\right)^2=2\Rightarrow\frac{x^2}{18}+\frac{y^2}{8}=1$

20. Ans.1

Sol: $x+y=2(t^2+1), x-y=2t\Rightarrow x+y=2\ \ 2\left[\left(\frac{x-y}{2}\right)^2+1\right]\Rightarrow 2x+2y=x^2-2xy+y^2+4$

$$\Rightarrow x^2-2xy+y^2-2x-2y+4=0$$

21. Ans.3

22. Ans.4

Sol:

$$\alpha = \frac{x_1 + x_2 + ... + x_n}{n} = \frac{\frac{n}{2}\left[2(p+a)+(n-1)a\right]}{n}$$

$$= \frac{p+a+(n-1)a}{2} \Rightarrow \alpha - p = \frac{a(n+1)}{2} \rightarrow (1)$$

$$\therefore \beta - q = b(n+1)/2 \rightarrow (2)$$

$$\frac{(1)}{(2)} \Rightarrow \frac{\alpha - p}{\beta - q} = \frac{a}{b} \Rightarrow b(\alpha - p) = a(\beta - q)$$

Locus of P is $b(x-p) = a(y-q)$

23. Ans.

Sol: Let a, b be the intercepts made by the rod of length 9.

Let $P(x_1 y_1)$ be the centroid of ΔOAB. Then $a = 3x_1, b = 3y_1$.

$a^2 + b^2 = 81 \Rightarrow 9x_1^2 + 9y_1^2 = 81 \Rightarrow x_1^2 + y_1^2 = 9$.

$\therefore$ The locus of P is $x^2+y^2 = 9$.

24. Ans.3

Sol: Let A(a,0) B(0,b) be the end points of a rod of length 'ℓ'.

$\therefore$ AB = ℓ $\Rightarrow$ a2+b2= ℓ^2

Let P(h,k) divides $\overline{AB}$ in the ratio 1:2 $\Rightarrow$P=(2a/3,b/3)

$\therefore h = 2a/3 \Rightarrow a = 3h/2$ and $k = b/3 \Rightarrow b = 3k$

$a^2 + b^2 = \ell^2 \Rightarrow \frac{9h^2}{4} + 9k^2 = \ell^2 \Rightarrow 9(h^2 + 4k^2) = 4\ell^2$ Locus of P is $9x^2 + 36y^2 = 4\ell^2$.

25. Ans.2

Sol: 3G = A+B+C $\Rightarrow$ (3x, 3y) =

(a cost +bsint+1, a sin t-b cost)

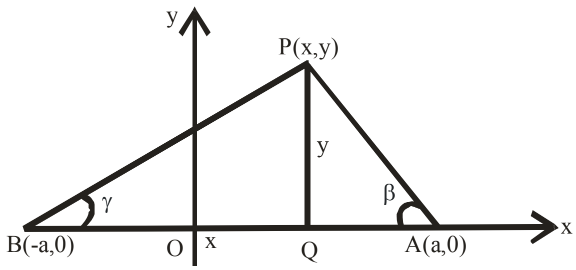
$\Rightarrow 3x = a\cos t + b\sin t + 1, 3y = a\sin t - b\cos t \Rightarrow$ $a\cos t + b\sin t, 3y = a\sin t - b\cos t$

$$\Rightarrow (3x-1)^2 + (3y)^2 = a^2 + b^2$$

26. Ans.4

Sol: $\tan\beta = \dfrac{y}{a-x}, \tan\gamma = \dfrac{y}{a+x}$

$$\cot(\beta-\gamma) = \cot\frac{\pi}{2} = 0 \Rightarrow \cot\beta\cot\gamma + 1 = 0 \quad \Rightarrow \left(\frac{a-x}{y}\right)\left(\frac{a+x}{y}\right) = -1 \Rightarrow a^2 - x^2 = -y^2$$



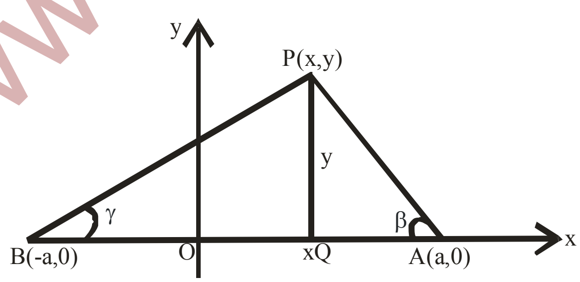
$$\Rightarrow x^2 - y^2 = a^2$$

27. Ans.4

Sol: $\tan\beta = \dfrac{y}{a-x}, \tan\gamma = \dfrac{y}{a+x}$

$$\tan 2\alpha = \tan(\beta-\gamma) = \frac{\tan\beta - \tan\gamma}{1+\tan\beta\tan\gamma}$$

$$= \frac{\dfrac{y}{a-x} - \dfrac{y}{a+x}}{1+\left(\dfrac{y}{a-x}\right)\left(\dfrac{y}{a+x}\right)} = \frac{y(a+x-a+x)}{a^2-x^2+y^2}$$



$$\Rightarrow a^2 - x^2 + y^2 = 2xy\cot 2\alpha \Rightarrow x^2 + 2xy\cot 2\alpha - y^2 = a^2$$

28. Ans.2

Sol: Since $\sqrt{(x-2)^2+y^2}+\sqrt{(x+2)^2+y^2}=4 \to (1)$

Assuming, $\left\{(x-2)^2+y^2\right\}-\left\{(x+2)^2+y^2\right\}=-8x \to (2)$

Dividing (2) by (1), we have

$\sqrt{(x-2)^2+y^2}-\sqrt{(x+2)^2+y^2}=-2x \to (3)$

Adding (1) & (3) , we have $2\sqrt{(x-2)^2+y^2}=4-2x \Rightarrow \left\{(x-2)^2+y^2\right\} \; =(2-x)^2$