# PAIR OF LINES

## OBJECTIVES

**1.** **The equation** $x^2 + ky^2 + 4xy = 0$ **represents two coincident lines, if** ***k*** **=**

(a) 0 (b) 1

(c) 4 (d) 16

**2.** **The equation of one of the line represented by the equation** $x^2 + 2xy \cot\theta - y^2 = 0$**, is**

(a) $x - y\cot\theta = 0$

(b) $x + y\tan\theta = 0$

(c) $x\sin\theta + y(\cos\theta + 1) = 0$

(d) $x\cos\theta + y(\sin\theta + 1) = 0$

**3.** **The nature of straight lines represented by the equation** $4x^2 + 12xy + 9y^2 = 0$ **is**

(a) Real and coincident (b) Real and different

(c) Imaginary and different (d) None of the above

**4.** **If the equation** $ax^2 + 2hxy + by^2 = 0$ **represents two lines** $y = m_1x$ **and** $y = m_2x$**, then**

(a) $m_1 + m_2 = \frac{-2h}{b}$ and $m_1m_2 = \frac{a}{b}$

(b) $m_1 + m_2 = \frac{2h}{b}$ and $m_1m_2 = \frac{-a}{b}$

(c) $m_1 + m_2 = \frac{2h}{b}$ and $m_1m_2 = \frac{a}{b}$

(d) $m_1 + m_2 = \frac{2h}{b}$ and $m_1m_2 = -ab$

**5.** **If the equation** $2x^2 - 2hxy + 2y^2 = 0$ **represents two coincident straight lines passing through the origin, then** $h =$

(a) $\pm 6$ (b) $\sqrt{6}$

(c) $-\sqrt{6}$ (d) $\pm 2$

**6.** **If one of the line represented by the equation** $ax^2 + 2hxy + by^2 = 0$ **is coincident with one of the line represented by** $a'x^2 + 2h'xy + b'y^2 = 0$**, then**

(a) $(ab' - a'b)^2 = 4(ah' - a'h)(hb' - h'b)$ (b) $(ab' + a'b)^2 = 4(ah' - a'h)(hb' - h'b)$

(c) $(ab' - a'b)^2 = (ah' - a'h)(hb' - h'b)$ (d) None of these

7. **Difference of slopes of the lines represented by equation** $x^2(\sec^2\theta - \sin^2\theta) - 2xy\tan\theta + y^2\sin^2\theta = 0$ **is**

(a) 4 (b) 3

(c) 2 (d) None of these

8. **The gradient of one of the lines of** $ax^2 + 2hxy + by^2 = 0$ **is twice that of the other, then**

(a) $h^2 = ab$ (b) $h = a + b$

(c) $8h^2 = 9ab$ (d) $9h^2 = 8ab$

9. **If the ratio of gradients of the lines represented by** $ax^2 + 2hxy + by^2 = 0$ **is 1 : 3, then the value of the ratio** $h^2 : ab$ **is**

(a) $\frac{1}{3}$ (b) $\frac{3}{4}$

(c) $\frac{4}{3}$ (d) 1

10. **If** $\frac{x^2}{a} + \frac{y^2}{b} + \frac{2xy}{h} = 0$ **represent pair of straight lines and slope of one line is twice the other. Then** $ab : h^2$ **is**

(a) 9 : 8 (b) 8 : 9

(c) 1 : 2 (d) 2 : 1

11. **If the sum of slopes of the pair of lines represented by** $4x^2 + 2hxy - 7y^2 = 0$ **is equal to the product of the slopes, then the value of *h* is**

(a) – 6 (b) – 2

(c) – 4 (d) 4

12. **If the slope of one line of the pair of lines represented by** $ax^2 + 4xy + y^2 = 0$ **is 3 times the slope of the other line, then *a* is**

(a) 1 (b) 2

(c) 3 (d) 4

13. **If the sum of the slopes of the lines given by** $x^2 - 2cxy - 7y^2 = 0$ **is four times their product, then *c* has the value**

(a) – 2 (b) – 1

(c) 2 (d) 1

**14. Difference of slopes of the lines represented by equation** $x^2(\sec^2\theta - \sin^2\theta) - 2xy\tan\theta + y^2\sin^2\theta = 0$ **is**

(a) 4 (b) 3 (c)2 (d) **None** of these

**15. The pair of straight lines passes through the point (1, 2) and perpendicular to the pair of straight lines** $3x^2 - 8xy + 5y^2 = 0$**, is**

(a) $(5x+3y+11)(x+y+3)=0$

(b) $(5x+3y-11)(x+y-3)=0$

(c) $(3x+5y-11)(x+y+3)=0$

(d) $(3x-5y+11)(x+y-3)=0$

**16. If the equation** $ax^2+by^2+cx+cy=0$ **represents a pair of straight lines, then**

(a) $a(b+c)=0$ (b) $b(c+a)=0$

(c) $c(a+b)=0$ (d) $a+b+c=0$

**17. If the equation** $Ax^2+2Bxy+Cy^2+Dx+Ey+F=0$ **represents a pair of straight lines, then** $B^2-AC$

(a) < 0 (b) = 0

(c) > 0 (d) None of these

**18. If** $4ab=3h^2$**, then the ratio of slopes of the lines represented by the equation** $ax^2+2hxy+by^2=0$ **will be**

(a) $\sqrt{2}:1$ (b) $\sqrt{3}:1$

(c) $2:1$ (d) $1:3$

**19. If** $6x^2+11xy-10y^2+x+31y+k=0$ **represents a pair of straight lines, then** $k=$

(a) – 15 (b) 6

(c) – 10 (d) – 4

**20. The lines represented by the equation** $ax^2(b-c)-xy(ab-bc)+cy^2(a-b)=0$ **are**

(a) $a(b-c)x-c(a-b)y=0$, $x+y=0$

(b) $x+y=0$, $x-y=0$

(c) $a(b-c)x-c(a-b)y=0$, $x-y=0$

(d) None of these

**21. If the slope of one of the line represented by the equation** $ax^2+2hxy+by^2=0$ **be** $\lambda$ **times that of the other, then**

(a) $4\lambda h=ab(1+\lambda)$ (b) $\lambda h=ab(1+\lambda)^2$

(c) $4\lambda h^2=ab(1+\lambda)^2$ (d) None of these

**22. The equation of the lines passing through the origin and having slopes 3 and $-\frac{1}{3}$ is**

(a) $3y^2+8xy-3x^2=0$ (b) $3x^2+8xy-3y^2=0$

(c) $3y^2-8xy+3x^2=0$ (d) $3x^2+8xy+3y^2=0$

**23. If the slope of one of the lines represented by $ax^2+2hxy+by^2=0$ be the square of the other, then**

(a) $a^2b+ab^2-6abh+8h^3=0$

(b) $a^2b+ab^2+6abh+8h^3=0$

(c) $a^2b+ab^2-3abh+8h^3=0$

(d) $a^2b+ab^2-6abh-8h^3=0$

**24. If the lines represented by the equation $6x^2+41xy-7y^2=0$ make angles $\alpha$ and $\beta$ with *x*-axis, then $\tan\alpha.\tan\beta$ =**

(a) – 6/7 (b) 6/ 7

(c) 7/6 (d) – 7/6

**25. The gradient of one of the lines $x^2+hxy+2y^2=0$ is twice that of the other, then *h* =**

(a) $\pm 3$ (b) $\pm\frac{3}{2}$

(c) $\pm 2$ (d) $\pm 1$

**26. The equation of the locus of foot of perpendiculars drawn from the origin to the line passing through a fixed point (*a*, *b*), is**

(a) $x^2+y^2-ax-by=0$ (b) $x^2+y^2+ax+by=0$

(c) $x^2+y^2-2ax-2by=0$ (d)None of these

**27. If the bisectors of the lines $x^2-2pxy-y^2=0$ be $x^2-2qxy-y^2=0$, then**

(a) $pq+1=0$ (b) $pq-1=0$

(c) $p+q=0$ (d) $p-q=0$

**28. The figure formed by the lines $x^2+4xy+y^2=0$ and $x-y=4$, is**

(a) A right angled triangle (b)An isosceles triangle

(c) An equilateral triangle (d)None of these

**29. Area of the triangle formed by the lines $y^2-9xy+18x^2=0$ and $y=9$ is**

(a) $\frac{27}{4}$ *sq . units* (b) 27 *sq. units* (c) $\frac{27}{2}$ *sq. units* (d)None of these

**30. The area (in square units) of the quadrilateral formed by the two pairs of lines** $l^2x^2 - m^2y^2 - n(lx + my) = 0$ and $l^2x^2 - m^2y^2 + n(lx - my) = 0$ is

(a) $\frac{n^2}{2|lm|}$ (b) $\frac{n^2}{|lm|}$

(c) $\frac{n}{2|lm|}$ (d) $\frac{n^2}{4|lm|}$

**31. If the pair of straight lines given by** $Ax^2 + 2Hxy + By^2 = 0$**,** $(H^2 > AB)$ **forms an equilateral triangle with line** $ax + by + c = 0$**, then** $(A + 3B)(3A + B)$ **is**

(a) $H^2$ (b) $-H^2$

(c) $2H^2$ (d) $4H^2$

**32. If one of the lines of the pair** $ax^2 + 2hxy + by^2 = 0$ **bisects the angle between positive directions of the axes, then *a*, *b*, *h* satisfy the relation**

(a) $a + b = 2|h|$ (b) $a + b = -2h$

(c) $a - b = 2|h|$ (d) $(a - b)^2 = 4h^2$

**33. The equation of the pair of straight lines, each of which makes an angle** $\alpha$ **with the line** $y = x$**, is**

(a) $x^2 + 2xy\sec 2\alpha + y^2 = 0$

(b) $x^2 + 2xy\,\mathrm{cosec}\, 2\alpha + y^2 = 0$

(c) $x^2 - 2xy\,\mathrm{cosec}\, 2\alpha + y^2 = 0$

(d) $x^2 - 2xy\sec 2\alpha + y^2 = 0$

**34. The circum centre of the triangle formed by the lines** $xy + 2x + 2y + 4 = 0$ **and** $x + y + 2 = 0$

(a)(0, 0) (b) (–2, – 2)

(c)(–1, –1) (d) (–1, –2)

**35. If the lines** $ax^2 + 2hxy + by^2 = 0$ **represents the adjacent sides of a parallelogram, then the equation of second diagonal if one is** $lx + my = 1$**, will be**

(a) $(am + hl)x = (bl + hm)y$ (b) $(am - hl)x = (bl - hm)y$

(c) $(am - hl)x = (bl + hm)y$ (d)None of these

**36. The orthocenter of the triangle formed by the lines** $xy = 0$ **and** $x + y = 1$ **is**

(a) $(0,0)$ (b) $\left(\frac{1}{2},\frac{1}{2}\right)$

(c) $\left(\frac{1}{3},\frac{1}{3}\right)$ (d) $\left(\frac{1}{4},\frac{1}{4}\right)$

**37. If the pair of lines $ax^2 + 2(a+b)xy + by^2 = 0$ lie along diameters of a circle and divide the circle into four sectors such that the area of one of the sectors is thrice the area of another sector then**

(a) $3a^2 + 10ab + 3b^2 = 0$ (b) $3a^2 + 2ab + 3b^2 = 0$

(c) $3a^2 - 10ab + 3b^2 = 0$ (d) $3a^2 - 2ab + 3b^2 = 0$

**38. The equations to a pair of opposite sides of a parallelogram are $x^2 - 5x + 6 = 0$ and $y^2 - 6y + 5 = 0$. The equations to its diagonals are**

(a) $x + 4y = 13$ and $y = 4x - 7$

(b) $4x + y = 13$ and $4y = x - 7$

(c) $4x + y = 13$ and $y = 4x - 7$

(d) $y - 4x = 13$ and $y + 4x = 7$

**39. The equation of the pair of straight lines parallel to *x*-axis and touching the circle $x^2 + y^2 - 6x - 4y - 12 = 0$**

(a) $y^2 - 4y - 21 = 0$ (b) $y^2 + 4y - 21 = 0$

(c) $y^2 - 4y + 21 = 0$ (d) $y^2 + 4y + 21 = 0$

**40. The equation $x^2 - 3xy + \lambda y^2 + 3x - 5y + 2 = 0$ when $\lambda$ is a real number, represents a pair of straight lines. If $\theta$ is the angle between the lines, then $\operatorname{cosec}^2\theta =$**

(a) 3 (b) 9

(c) 10 (d) 100

**41. The angle between the pair of straight lines $y^2 \sin^2\theta - xy \sin^2\theta + x^2(\cos^2\theta - 1) = 1$, is**

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$

(c) $\frac{2\pi}{3}$ (d) None of these

**42. The lines joining the origin to the points of intersection of the line $y = mx + c$ and the circle $x^2 + y^2 = a^2$ will be mutually perpendicular, if**

(a) $a^2(m^2 + 1) = c^2$ (b) $a^2(m^2 - 1) = c^2$

(c) $a^2(m^2 + 1) = 2c^2$ (d) $a^2(m^2 - 1) = 2c^2$

**43. The lines represented by the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ will be equidistant from the origin, if**

(a) $f^2 + g^2 = c(b - a)$ (b) $f^4 + g^4 = c(bf^2 + ag^2)$ **(c)** $f^4 - g^4 = c(bf^2 - ag^2)$ **(d)** $f^2 + g^2 = af^2 + bg^2$

**44. The area bounded by the angle bisectors of the lines** $x^2 - y^2 + 2y = 1$ **and the line** $x + y = 3$**, is**

(a) 2 (b) 3

(c) 4 (d) 6

**45. The pair of lines represented by** $3ax^2 + 5xy + (a^2 - 2)y^2 = 0$ **are perpendicular to each other for**

(a) Two values of $a$ (b) $\forall a$

(c) For one value of $a$ (d) For no value of $a$

**46. If the lines represented by the equation** $ax^2 - bxy - y^2 = 0$ **make angles** $\alpha$ **and** $\beta$ **with the *x*-axis, then** $\tan(\alpha + \beta) =$

(a) $\frac{b}{1+a}$ (b) $\frac{-b}{1+a}$

(c) $\frac{a}{1+b}$ (d) None of these

**47. Pair of straight lines perpendicular to each other represented by**

(a) $2x^2 = 2y(2x + y)$ (b) $x^2 + y^2 + 3 = 0$

(c) $2x^2 = y(2x + y)$ (d) $x^2 = 2(x - y)$

**48. Acute angle between the lines represented by** $(x^2 + y^2)\sqrt{3} = 4xy$ **is**

(a) $\pi/6$ (b) $\pi/4$

(c) $\pi/3$ (d) None of these

**49. The angle between the two straight lines** $2x^2 - 5xy + 2y^2 - 3x + 3y + 1 = 0$ **is**

(a) $45^o$ (b) $60^o$

(c) $\tan^{-1}\frac{4}{3}$ (d) $\tan^{-1}\frac{3}{4}$

**50. The angle between the lines represented by the equation** $x^2 - 2pxy + y^2 = 0$**, is**

(a) $\sec^{-1} p$ (b) $\cos^{-1} p$

(c) $\tan^{-1} p$ (d) None of these

**51. The angle between the lines represented by the equation** $4x^2 - 24xy + 11y^2 = 0$ **are**

(a) $\tan^{-1}\frac{3}{4}, \tan^{-1}\left(-\frac{3}{4}\right)$ (b) $\tan^{-1}\frac{1}{3}, \tan^{-1}\left(-\frac{1}{3}\right)$

(c) $\tan^{-1}\frac{4}{3}, \tan^{-1}\left(-\frac{4}{3}\right)$ (d) $\tan^{-1}\frac{1}{2}, \tan^{-1}\left(-\frac{1}{2}\right)$

**52. The angle between the pair of straight lines** $x^2 + 4y^2 - 7xy = 0$**, is**

(a) $\tan^{-1}\left(\frac{1}{3}\right)$ (b) $\tan^{-1} 3$ (c) $\tan^{-1}\frac{\sqrt{33}}{5}$ (d) $\tan^{-1}\frac{5}{\sqrt{33}}$

**53. If $(a+3b)(3a+b)=4h^2$, then the angle between the lines represented by $ax^2+2hxy+by^2=0$ is**

(a) $30^o$ (b) $45^o$

(c) $60^o$ (d) $\tan^{-1}\frac{1}{2}$

**54. The angle between the pair of lines given by equation $x^2+2xy-y^2=0$, is**

(a) $\pi/3$ (b) $\pi/6$

(c) $\pi/2$ (d) 0

**55. The lines $(lx+my)^2-3(mx-ly)^2=0$ and $lx+my+n=0$ form**

(a) An isosceles triangle (b)A right angled triangle

(c) An equilateral triangle (d)None of these

**56. Angle between the lines represented by the equation $x^2+2xy\sec\theta+y^2=0$ is**

(a) $\theta$ (b) $2\theta$

(c) $\frac{\theta}{2}$ (d) None of these

**57. The angle between the pair of lines represented by $2x^2-7xy+3y^2=0$, is**

(a) $60^o$ (b) $45^o$

(c) $\tan^{-1}\left(\frac{7}{6}\right)$ (d) $30^o$

**58. If the angle between the pair of straight lines represented by the equation $x^2-3xy+\lambda y^2+3x-5y+2=0$ is $\tan^{-1}\left(\frac{1}{3}\right)$, where '$\lambda$' is a non negative real number. Then $\lambda$ is**

(a) 2 (b) 0

(c) 3 (d) 1

**59. If the acute angles between the pairs of lines $3x^2-7xy+4y^2=0$ and $6x^2-5xy+y^2=0$ be $\theta_1$ and $\theta_2$ respectively, then**

(a) $\theta_1=\theta_2$ (b) $\theta_1=2\theta_2$

(c) $2\theta_1=\theta_2$ (d) None of these

**60. If two of the three lines represented by the equation $ax^3+bx^2y+cxy^2+dy^3=0$ are perpendicular, then**

(a) $a^2+d^2=2ac$ (b) $a^2+d^2=2bd$

(c) $a^2+ac+bd+d^2=0$ (d) $a^2+d^2=4bc$

61. **If the angle** $2\theta$ **is acute, then the acute angle between** $x^2(\cos\theta - \sin\theta) + 2xy\cos\theta + y^2(\cos\theta + \sin\theta) = 0$ **is**

(a) $2\theta$ (b) $\theta/3$

(c) $\theta$ (d) $\theta/2$

62. **Condition that the two lines represented by the equation** $ax^2 + 2hxy + by^2 = 0$ **to be perpendicular is**

(a) $ab = -1$ (b) $a = -b$

(c) $a = b$ (d) $ab = 1$

63. **If the bisectors of angles represented by** $ax^2 + 2hxy + by^2 = 0$ **and** $a'x^2 + 2h'xy + b'y^2 = 0$ **are same, then**

(a) $(a-b)h' = (a'-b')h$ (b) $(a-b)h = (a'-b')h'$

(c) $(a+b)h' = (a'-b')h$ (d) $(a-b)h' = (a'+b')h$

64. **If** $y = mx$ **be one of the bisectors of the angle between the lines** $ax^2 - 2hxy + by^2 = 0$**, then**

(a) $h(1+m^2) + m(a-b) = 0$

(b) $h(1-m^2) + m(a+b) = 0$

(c) $h(1-m^2) + m(a-b) = 0$

(d) $h(1+m^2) + m(a+b) = 0$

65. **The line** $x - 2y = 0$ **will be a bisector of the angle between the lines represented by the equation** $x^2 - 2hxy - 2y^2 = 0$**, if** $h =$

(a) 1/2 (b) 2

(c) $-2$ (d) –1/2

66. **If the bisectors of the angles between the pairs of lines given by the equation** $ax^2 + 2hxy + by^2 = 0$ **and** $ax^2 + 2hxy + by^2 + \lambda(x^2 + y^2) = 0$ **be coincident, then** $\lambda =$

(a) $a$ (b) $b$

(c) $h$ (d) Any real number

67. **The point of intersection of the lines represented by equation** $2(x+2)^2 + 3(x+2)(y-2) - 2(y-2)^2 = 0$ **is**

(a) (2, 2) (b) (–2, –2)

(c) (– 2, 2) (d) (2, –2)

68. **The equation of the bisectors of angle between the lines represented by equation** $(y-mx)^2=(x+my)^2$ **is**

(a) $mx^2+(m^2-1)xy-my^2=0$

(b) $mx^2-(m^2-1)xy-my^2=0$

(c) $mx^2+(m^2-1)xy+my^2=0$

(d) None of these

69. **One bisector of the angle between the lines given by** $a(x-1)^2+2h(x-1)y+by^2=0$ **is** $2x+y-2=0$**. The other bisector is**

(a) $x-2y+1=0$ (b) $2x+y-1=0$

(c) $x+2y-1=0$ (d) $x-2y-1=0$

70. **Distance between the pair of lines represented by the equation** $x^2-6xy+9y^2+3x-9y-4=0$ **is**

(a) $\frac{15}{\sqrt{10}}$ (b) $\frac{1}{2}$

(c) $\sqrt{\frac{5}{2}}$ (d) $\frac{1}{\sqrt{10}}$

71. **The lines joining the points of intersection of line** $x+y=1$ **and curve** $x^2+y^2-2y+\lambda=0$ **to the origin are perpendicular, then the value of** $\lambda$ **will be**

(a) 1/2 (b) –1/2

(c) $1/\sqrt{2}$ (d) 0

72. **The distance between the parallel lines** $9x^2-6xy+y^2+18x-6y+8=0$ **is**

(a) $1/\sqrt{10}$ (b) $2/\sqrt{10}$

(c) $4/\sqrt{10}$ (d) $\sqrt{10}$

73. **If the lines joining origin to the points of intersection of the line** $fx-gy=\lambda$ **and the curve** $x^2+hxy-y^2+gx+fy=0$ **be mutually perpendicular, then**

(a) $\lambda=h$ (b) $\lambda=g$

(c) $\lambda=fg$ (d) $\lambda$ may have any value

74. **The equation of second degree** $x^2+2\sqrt{2}xy+2y^2+4x+4\sqrt{2}y+1=0$ **represents a pair of straight lines. The distance between them is**

(a) 4 (b) $4/\sqrt{3}$

(c) 2 (d) $2\sqrt{3}$

**75. The lines joining the origin to the points of intersection of the curves** $ax^2+2hxy+by^2+2gx=0$ **and** $a'x^2+2h'xy+b'y^2+2g'x=0$ **will be mutually perpendicular, if**

(a) $g(a'-b')=g'(a+b)$ (b) $g(a'+b')=g'(a+b)$

(c) $g(a'+b')=g'(a-b)$ (d) $g(a'-b')=g'(a-b)$

**76. If the distance of two lines passing through origin from the point** $(x_1,y_1)$ **is** $'d'$**, then the equation of lines is**

(a) $(xy_1-yx_1)^2=d^2(x^2+y^2)$

(b) $(x_1y_1-xy)^2=(x^2+y^2)$

(c) $(xy_1+yx_1)^2=(x^2-y^2)$

(d) $(x^2-y^2)=2(x_1+y_1)$

# PAIR OF STRAIGHT LINES

## HINTS AND SOLUTIONS

1. **(c) To represent pair of coincident straight lines** $x^2 + ky^2 + 4xy = 0$ **must be perfect square.**

   **Therefore,** $k = 4$ **.**

2. (c) **The lines represented by the equation** $x^2 + 2xy\cot\theta - y^2 = 0$ **are** $ax + hy \pm y\sqrt{h^2 - ab} = 0$

   $\Rightarrow x + y\cot\theta \pm y\sqrt{\cot^2\theta + 1} = 0$

   $\Rightarrow x + y\left(\frac{\cos\theta}{\sin\theta} \pm \frac{1}{\sin\theta}\right) = 0 \Rightarrow x\sin\theta + y(\cos\theta \pm 1) = 0$

   Hence, one line is $x\sin\theta + y(\cos\theta + 1) = 0$ .

3. **(a)** $4x^2 + 12xy + 9y^2 = 0$

   Here $h^2 - ab = 36 - 36 = 0$ , $\left(\text{from } \tan\theta = \frac{\pm 2\sqrt{h^2 - ab}}{a + b}\right)$

   Hence, lines are real and coincident.

4. **(a) It is a fundamental concept.**

5. **(d) If it represents two coincident straight lines, then the condition** $h^2 - ab = 0$ **should apply as angle between them would be zero. Hence** $h^2 - 4 = 0$ **or** $h = \pm 2$ **.**

6. **(a) concept**

7. (c) We know that $m_1 - m_2 = \sqrt{(m_1 + m_2)^2 - 4m_1m_2}$

   $= \sqrt{\left(\frac{2\tan\theta}{\sin^2\theta}\right)^2 - 4\left(\frac{\sec^2\theta - \sin^2\theta}{\sin^2\theta}\right)}$

   $= \sqrt{\frac{4\tan^2\theta}{\sin^4\theta} - 4(\sec^2\theta\,\text{cosec}^2\theta - 1)} = 2$ .

8. **(c) Here,** $m_1 + m_2 = \frac{-2h}{b}$ **and** $m_1m_2 = \frac{a}{b}$

   Given that $m_1 = 2m_2$

   $\therefore 3m_2 = \frac{-2h}{b}$ and $2m_2^2 = \frac{a}{b}$

   $\Rightarrow 2\left(\frac{-2h}{3b}\right)^2 = \frac{a}{b} \Rightarrow 8h^2 = 9ab$ .

**9. (c) If the gradients of two lines are in ratio** $1:n$**.**

Then $\frac{h^2}{ab}=\frac{(n+1)^2}{4n}=\frac{(3+1)^2}{4.3}=\frac{4}{3}$.

**10. (a) Let** $m_1, m_2$ **be the slopes**

$\therefore m_1+m_2=-\frac{2b}{h}$ and $m_1 m_2=\frac{b}{a}$

Again $m_2=2m_1$

$\therefore 3m_1=-\frac{2b}{h}$ and $2m_1^2=\frac{b}{a}$

$\therefore \frac{9m_1^2}{2m_1^2}=\frac{4b^2}{h^2}\times\frac{a}{b}\Rightarrow ab:h^2=9:8$.

**11. (b) Comparing the given equation with the standard equation, we get** $a=4$ **and** $b=-7$**. Let** $m_1$ **and** $m_2$ **are the slopes of given lines. Therefore sum of the slopes** $(m_1+m_2)=-\frac{2h}{b}=\frac{2h}{7}$ **and product of the slopes** $(m_1 m_2)=\frac{a}{b}=\frac{4}{-7}$**.**

$\because m_1+m_2=m_1m_2$, therefore $\frac{2h}{7}=\frac{4}{-7}$ or $h=-2$.

**12. (c) Here,** $m_1+m_2=-4$ **.....(i)**

And $m_1 m_2=a$ .....(ii)

Given that $m_1=3m_2$.

By (i), $3m_2+m_2=-4 \Rightarrow m_2=-1$

Hence, $m_1=-3$. Now, by (ii) $a=3$.

**13. (a) Given equation of pair of lines is**

$6x^2-xy+4cy^2=0$ .....(i)

Slope of line $3x+4y=0$ is $\frac{-3}{4}=m_1$ (say)

Product of slopes of lines $m_1m_2=\frac{a}{b}=\frac{6}{4c}=\frac{3}{2c}$

$\therefore m_2=\frac{3/2c}{-3/4}=\frac{-2}{c}$. Also, $m_1+m_2=\frac{-2h}{b}=\frac{1}{4c}$

$\Rightarrow \frac{-3}{4}-\frac{2}{c}=\frac{1}{4c}\Rightarrow -3c-8=1\Rightarrow c=-3$.

**14. (c) We know that** $m_1 - m_2 = \sqrt{(m_1+m_2)^2 - 4m_1m_2}$

$$= \sqrt{\left(\frac{2\tan\theta}{\sin^2\theta}\right)^2 - 4\left(\frac{\sec^2\theta - \sin^2\theta}{\sin^2\theta}\right)}$$

$$= \sqrt{\frac{4\tan^2\theta}{\sin^4\theta} - 4(\sec^2\theta\,\mathrm{cosec}^2\theta - 1)} = 2.$$

**15. (a) Equation of lines are** $(px+qy)(py-qx)=0$.

Hence, one line is $px+qy=0$.

**16. (c)** $ab(0) + 2\left(\frac{c}{2}\right)\left(\frac{c}{2}\right)(0) - a\left(\frac{c}{2}\right)^2 - b\left(\frac{c}{2}\right)^2 - 0(0)^2 = 0$

$\Rightarrow ac^2 + bc^2 = 0 \;\Rightarrow c^2(a+b) = 0 \;\Rightarrow c(a+b) = 0$.

**17. (d) Using the condition** $\Delta = 0$

$\Rightarrow ACF + 2.\frac{E}{2}.\frac{D}{2}.B - A.\left(\frac{E}{2}\right)^2 - C.\left(\frac{D}{2}\right)^2 - F.(B)^2 = 0$.

**18. (d) Here** $m_1 + m_2 = \frac{-2h}{b}$ **.....(i)**

And $m_1 m_2 = \frac{a}{b}$ .....(ii)

Also, given that $4ab = 3h^2$. now we have to find $\frac{m_1}{m_2}$,

Therefore with the help of (i) and (ii), we get

$$(m_1 - m_2)^2 = \frac{4h^2 - 4ab}{b^2} = \frac{4h^2 - 3h^2}{b^2} = \frac{h^2}{b^2}$$

$\Rightarrow m_1 - m_2 = \frac{h}{b}$ .....(iii)

Now on solving (i) and (iii), we get $m_1 = \frac{-h}{2b}$ and $m_2 = \frac{-3h}{2b}$; $\therefore m_1 : m_2 = 1:3$.

**19. (a)** $-6.10k + \frac{11.1.31}{4} - 6\left(\frac{31}{2}\right)^2 + 10\left(\frac{1}{2}\right)^2 - k\left(\frac{11}{2}\right)^2 = 0$

$\Rightarrow -k\frac{361}{4} = \frac{5415}{4} \Rightarrow k = -15$.

**20. (c) From options, proceed as to find the equation represented by them** $a(b-c)x - c(a-b)y = 0$ **and** $x - y = 0$.

**21. (c) It is given that** $m_2 = \lambda m_1 \Rightarrow m_1 + \lambda m_1 = \frac{-2h}{b}$

$\Rightarrow m_1 = \frac{-2h}{b(1+\lambda)}$ .....(i)

and $m_1 . \lambda m_1 = \frac{a}{b} \Rightarrow m_1 = \sqrt{\frac{a}{b\lambda}}$ .....(ii)

Hence, by (i) and (ii), $\sqrt{\frac{a}{b\lambda}} = \frac{-2h}{b(1+\lambda)}$

On squaring both sides, we get $4\lambda h^2 = ab(1+\lambda)^2$.

**22. (b)** $m_1 = 3,\ m_2 = -\frac{1}{3}$**. Hence, the lines are** $y = 3x, y = -\frac{1}{3}x$**.**

Multiplying both the lines, we get

$(y-3x)(3y+x) = 0 \Rightarrow 3x^2 + 8xy - 3y^2 = 0.$

**23. (a) Here,** $m_1 = m_2^2 \Rightarrow m_2^2 + m_2 = \frac{-2h}{b}$ **.....(i)**

and $m_2^2 m_2 = \frac{a}{b} \Rightarrow m_2 = \left(\frac{a}{b}\right)^{1/3}$ .....(ii)

Putting this value of $m_2$ in (i), we get

$$\left\{\left(\frac{a}{b}\right)^{1/3}\right\}^2 + \left(\frac{a}{b}\right)^{1/3} = \frac{-2h}{b}$$

On cubing both sides, we get

$$\left(\frac{a}{b}\right)^2 + \frac{a}{b} + 3\left(\frac{a}{b}\right)^{2/3} . \left(\frac{a}{b}\right)^{1/3} . \left\{\left(\frac{a}{b}\right)^{2/3} + \left(\frac{a}{b}\right)^{1/3}\right\} = \frac{-8h^3}{b^3}$$

$$\Rightarrow \left(\frac{a}{b}\right)^2 + \frac{a}{b} - \frac{6ah}{b^2} = \frac{-8h^3}{b^3} \quad \left\{\because \left(\frac{a}{b}\right)^{2/3} + \left(\frac{a}{b}\right)^{1/3} = \frac{-2h}{b}\right\}$$

$\Rightarrow ab(a+b) - 6abh + 8h^3 = 0$.

**24. (a)** $\tan\alpha \tan\beta = m_1 m_2 = \frac{a}{b} = -\frac{6}{7}$**.**

**25. (a) Applying the condition,** $4\lambda h^2 = ab(1+\lambda)^2$

Here $\lambda = 2$, therefore

$4 \times 2 \times \left(\frac{h}{2}\right)^2 = 1 \times 2(1+2)^2 \Rightarrow h^2 = 9 \Rightarrow h = \pm 3$ .

**26. (a)** $\lambda(x-a)+(y-b)=0$ **is the equation of line.**

$r = -\left(\frac{-a\lambda - b}{\lambda^2+1}\right)$

Coordinates of point $\equiv \left\{-\lambda\left(\frac{-a\lambda-b}{\lambda^2+1}\right), -\left(\frac{-a\lambda-b}{\lambda^2+1}\right)\right\}$

$h = \lambda\left(\frac{a\lambda+b}{\lambda^2+1}\right), k = \frac{a\lambda+b}{\lambda^2+1}, \lambda = \frac{h}{k}$

$\therefore h = h\left(\frac{ah+kb}{h^2+k^2}\right) \Rightarrow x^2+y^2 = ax+by$.

**27. (a) Bisector of the angle between the lines** $x^2-2pxy-y^2=0$ **is** $\frac{x^2-y^2}{xy} = \frac{1-(-1)}{-p}$

$\Rightarrow px^2+2xy-py^2=0$

But it is represented by $x^2-2qxy-y^2=0$ .

Therefore $\frac{p}{1} = \frac{2}{-2q} \Rightarrow pq = -1$ .

**28. (c)** $S_1 = \frac{1}{-2+\sqrt{4-1}} = \frac{1}{-2+\sqrt{3}} = -(\sqrt{3}+2)$

$S_2 = \frac{1}{-2-\sqrt{4-1}} = \frac{1}{-2-\sqrt{3}} = (\sqrt{3}-2)$ and $S_3 = 1$ .

$\theta_{13} = \tan^{-1}\left|\frac{-(\sqrt{3}+2)-1}{1-(\sqrt{3}+2)}\right| = \tan^{-1}\left|\frac{-(\sqrt{3}+3)}{-(\sqrt{3}+1)}\right|$

$= \tan^{-1}(\sqrt{3}) = 60°$ .

$\theta_{23} = \tan^{-1}\left|\frac{\sqrt{3}-2-1}{1+\sqrt{3}-2}\right| = \tan^{-1}\left|\frac{\sqrt{3}-3}{\sqrt{3}-1}\right|$

$= \tan^{-1}(\sqrt{3}) = 60°$ .

**29. (a) Applying the formula given in the theory part, the required area is**

$\frac{(-9)^2\sqrt{(9/2)^2-18}}{18\times 1+9\times 0\times 1+1\times 0} = \frac{81}{18}\sqrt{\frac{81}{4}-18}$

$= \frac{81}{18}\times\frac{3}{2} = \frac{27}{4}$ *sq. units.*

**30. (a) Given lines are**

$lx+my=0, lx+my+n=0$

$lx-my=0, lx+my-n=0$

Area $=\left|\frac{(c_1-d_1)(c_2-d_2)}{(a_1b_2-a_2b_1)}\right|=\left|\frac{(0-n)(0+n)}{(-lm-lm)}\right|=\frac{n^2}{2|lm|}$.

**31. (d) We know that the pair of lines**

$(a^2-3b^2)x^2+8abxy+(b^2-3a^2)y^2=0$ With the line $ax+by+c=0$ form an equilateral triangle. Hence comparing with $Ax^2+2Hxy+By^2=0$, then

$A=a^2-3b^2, B=b^2-3a^2, 2H=8ab$.

Now, $(A+3B)(3A+B)=(-8a^2)(-8b^2)$

$=(8ab)^2=(2H)^2=4H^2$

**32. (b) Bisector of the angle between positive directions of the axes is $y=x$. Since it is one of the lines of the given pair $ax^2+2hxy+by^2=0,$ we have**

$x^2(a+2h+b)=0$ or $a+b=-2h$.

**33. (d) Any line through the origin is $y=mx$. If it makes an angle $\alpha$ with the line $y=x$, then we should have**

$\tan\alpha=\pm\left\{\frac{m_1-m_2}{1+m_1m_2}\right\}=\pm\frac{(m-1)}{1+m}$

Eliminate m from above eq.

**34. (c) The separate equations of the lines given by $xy+2x+2y+4=0$ are $(x+2)(y+2)=0$ or $x+2=0$, $y+2=0$. Solving the equations of the sides of the triangle, we obtain the coordinates of the vertices as $A(-2,0), B(0,-2)$ and $C(-2,-2)$. Clearly, $\Delta ABC$ is a right angled triangle with right angle at *C*. Therefore the centre of the circum-circle is the midpoint of *AB* whose coordinates are (–1, –1).**

**35. (b) Let the equation of lines represented by $ax^2+2hxy+by^2=0$ be $y-m_1x=0$ and $y-m_2x=0$**

**36. (a) Since the triangle is right angled at $O(0,0)$, therefore (0,0) is its orthocentre.**

**37. (b) Angle between the given lines is**

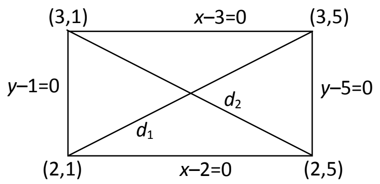
$\tan\frac{\pi}{4}=\frac{2\sqrt{(a+b)^2-ab}}{a+b}$

$\Rightarrow \frac{2\sqrt{(a+b)^2-ab}}{a+b}=1$

$\Rightarrow 3a^2+2ab+3b^2=0$

**38. (c) Equation of diagonal $d_1$ is $y-1=\frac{5-1}{3-2}(x-2)$**

$\Rightarrow y-1=\frac{4}{1}(x-2)\Rightarrow y=4x-7$




Equation of diagonal $d_2$ is $y-1=\frac{5-1}{2-3}(x-3)$

$\Rightarrow y-1=-4(x-3) \Rightarrow 4x+y=13$

So equations are, $4x+y=13$ and $y=4x-7$.

**39. (a) Let the lines are $y=m_1x+c_1$ and $y=m_2x+c_2$ since pair of straight lines parallel to *x*-axis,**

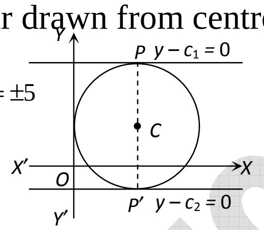
$\therefore m_1=m_2=0$

And the lines will be $y=c_1$ and $y=c_2$

Given circle is $x^2+y^2-6x-4y-12=0$, centre (3, 2) and radius = 5.

Here, the perpendicular drawn from centre to the lines are $CP$ and $CP'$.

$CP=\frac{2-c_1}{\sqrt{1}}=\pm 5 \Rightarrow 2-c_1=\pm 5$




$c_1=7$ and $c_1=-3$

Hence the lines are

$y-7=0, y+3=0$ *i.e.,* $(y-7)(y+3)=0$

$\therefore$ Pair of straight lines is $y^2-4y-21=0$.

**40. (c) The equation $x^2-3xy+\lambda y^2+3x-5y+2=0$ represents a pair of straight lines.**

$\therefore 2\lambda+2\left(-\frac{5}{2}\right)\left(\frac{3}{2}\right)\left(-\frac{3}{2}\right)-\frac{25}{4}-\frac{9\lambda}{4}-\frac{18}{4}=0 \Rightarrow \lambda=2$

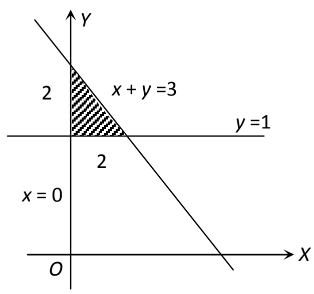
If $\theta$ is the angle between the lines, then

$\tan\theta=\frac{2\sqrt{h^2-ab}}{a+b}=\frac{2\sqrt{(9/4)-2}}{1+2}=\frac{1}{3}$

$\Rightarrow \operatorname{cosec}^2\theta=1+\cot^2\theta=1+9=10$.

**41. (d)** $\alpha=\tan^{-1}\left\{\frac{2\sqrt{\frac{\sin^2\theta}{4}-\sin^2\theta(\cos^2\theta-1)}}{\sin^2\theta+\cos^2\theta-1}\right\}$

$=\tan^{-1}\infty \Rightarrow \alpha=\frac{\pi}{2}$




**42. (c) Standard problem.**

**43. (c) Standard problem**

**44. (a) the lines are** $x=0$**,** $y=1$**.and x+y=3**

Required area

$$=\frac{1}{2}\times 2\times 2=2.$$

**45. (a)** $\because$ **The lines are perpendicular, if**

Coefficient of $x^2$ + coefficient of $y^2=0$

$\Rightarrow 3a+(a^2-2)=0 \Rightarrow a^2+3a-2=0$

$\because$ The equation is a quadratic equation in '$a$' and $B^2-4AC>0$.

$\therefore$ The roots of $a$ are real and distinct. Therefore, the lines are perpendicular to each other for two values of '$a$'.

**46. (b) Here the equation is** $ax^2-bxy-y^2=0$ **and given that** $m_1=\tan\alpha$ **and** $m_2=\tan\beta$ **and we know that**

$$m_1+m_2=\frac{b}{-1}=\tan\alpha+\tan\beta$$

And $m_1m_2=\frac{a}{-1}=\tan\alpha.\tan\beta$

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\frac{-b}{1-(-a)}=\frac{-b}{(1+a)}.$$

**47. (a) Since** $2x^2=2y(2x+y)\Rightarrow x^2-2xy-y^2=0$.

Hence, coefficient of $x^2$ + coefficient of $y^2=1-1=0$.

Hence the lines are perpendicular.

**48. (a)** $\tan\theta=\pm\frac{2\sqrt{4-3}}{2\sqrt{3}}=\frac{1}{\sqrt{3}}$ **or** $\theta=30°$

Hence, acute angle is 30° or $\frac{\pi}{6}$.

**49. (d)** $\theta=\tan^{-1}\left(\frac{2\sqrt{\frac{25}{4}-4}}{4}\right)=\tan^{-1}\frac{3}{4}$.

**50. (a)** $\tan\theta=\frac{\pm 2\sqrt{p^2-1}}{1+1}=\pm\sqrt{p^2-1}\Rightarrow\theta=\sec^{-1}p$.

**51. (c)** $\tan\theta=\pm 2\frac{\sqrt{h^2-ab}}{a+b}=\pm 2\frac{\sqrt{144-44}}{15}=\pm\frac{4}{3}$.

$\Rightarrow \theta = \tan^{-1}\left(\pm\frac{4}{3}\right)$ .

**52. (c)** $\alpha = \tan^{-1}\frac{2\sqrt{\frac{49}{4}-4}}{5} = \tan^{-1}\frac{\sqrt{33}}{5}$.

**53.** (c) $\theta = \tan^{-1}\left(\frac{2\sqrt{h^2-ab}}{a+b}\right) = \tan^{-1}\left(\frac{\sqrt{4h^2-4ab}}{a+b}\right)$

$= \tan^{-1}\left(\frac{\sqrt{3a^2+3b^2+10ab-4ab}}{a+b}\right) = 60°$ .

**54. (c) Using condition** $a+b=0$ **.**

**55.** (c) standard problem

**56. (a) Let angle between both the lines is** $\alpha$**, then**

$\alpha = \tan^{-1}\left(\frac{2\sqrt{h^2-ab}}{a+b}\right) = \tan^{-1}\left(\frac{2\sqrt{\sec^2\theta-1}}{1+1}\right) = \theta$

**57. (b)** $\tan\theta = \frac{2\sqrt{(-7/2)^2-6}}{5} \Rightarrow \tan\theta = 1 \Rightarrow \theta = 45°$.

**58. (a) Given that** $\theta = \tan^{-1}\left(\frac{1}{3}\right) \Rightarrow \tan\theta = \frac{1}{3}$

Now, since $\tan\theta = \frac{2\sqrt{h^2-ab}}{a+b} \Rightarrow \frac{1}{3} = \frac{2\sqrt{\left(\frac{-3}{2}\right)^2-\lambda}}{\lambda+1}$

$\Rightarrow (\lambda+1)^2 = 9(9-4\lambda) \Rightarrow \lambda^2+38\lambda-80=0$

$\Rightarrow \lambda = \frac{-38\pm\sqrt{(38)^2+320}}{2} \Rightarrow \lambda = \frac{-38\pm 42}{2} \Rightarrow \lambda = 2$ .

**59. (b)** $\tan 45° = \frac{2\sqrt{\frac{1}{4}-ab}}{a+b}$

$\Rightarrow (a+b)^2 = (1-4ab) \Rightarrow a^2+b^2+6ab-1=0$ ,

**60. (c) Standard problem.**

**61.** (c) $\because \tan\phi = \frac{2\sqrt{h^2-ab}}{a+b}$

**62. (b) Coefficient of** $x^2$ + **coefficient of** $y^2 = 0$

$\Rightarrow a+b=0 \Rightarrow a=-b$ .

63. (a) **Since bisectors are same, therefore** $\frac{a-b}{h}=\frac{a'-b'}{h'}$

$\Rightarrow (a-b)h'=(a'-b')h$.

64. (c) **standard problem**

65. (c) **Here one equation of bisector is** $x-2y=0$. **We know that both bisectors are perpendicular, therefore second bisector will be** $2x+y=0$ **because it passes through origin.**

Hence the combined equations of bisectors is given by $(2x+y)(x-2y)=0 \Rightarrow -2x^2+3xy+2y^2=0$.

Now comparing it by $hx^2+3xy-hy^2=0$, we get $h=-2$.

66. (d) **Standard Problem**

67. (c) **Putting** $X=x+2,\ Y=y-2$

Equation becomes $2X^2+3XY-2Y^2=0$

Which cuts at $X=0, Y=0$

So, $x+2=0, y-2=0$

Point of intersection $(-2,2)$.

68. (a) **The equation is**

$y^2+m^2x^2-2mxy-x^2-m^2y^2-2mxy=0$

$\Rightarrow x^2(m^2-1)+y^2(1-m^2)-4mxy=0$

Therefore, the equation of bisectors is $\frac{x^2-y^2}{xy}$

$=\frac{(m^2-1)-(1-m^2)}{-2m} \Rightarrow mx^2+(m^2-1)xy-my^2=0$.

69. (d) **Standard Problem**

70. (c) **The distance between the pair of straight lines is** $2\sqrt{\frac{g^2-ac}{a(a+b)}}$.

71. (d) **Making the equation of curve homogeneous with the help of line** $x+y=1$**, we get**

$x^2+y^2-2y(x+y)+\lambda(x+y)^2=0$

$\Rightarrow x^2(1+\lambda)+y^2(-1+\lambda)-2yx=0$

Therefore the lines be perpendicular, if $A+B=0$.

$\Rightarrow 1+\lambda-1+\lambda=0 \Rightarrow \lambda=0$.

72. (b) **Distance** $=2\sqrt{\frac{g^2-ac}{a(a+b)}}=\frac{2}{\sqrt{10}}$.

**73. (d)** **Making the equation of curve homogeneous with the help of equation of line $\frac{fx-gy}{\lambda}=1$ and to be perpendicular to both the lines represented by this homogeneous equation** $a+b=0 \Rightarrow \lambda+gf-\lambda-gf=0 \Rightarrow 0=0$

**74. (c) Distance** $=2\sqrt{\frac{g^2-ac}{a(a+b)}}=2\sqrt{\frac{4-1}{1(1+2)}}=2$ **.**

**75. (b)** **The family of lines passing through point of intersection of the given curves will be**

$ax^2+2hxy+by^2+2gx+\lambda(a'x^2+2h'xy+b'y^2+2g'x)=0$

$\Rightarrow (a+a'\lambda)x^2+(2h+2h'\lambda)xy+(b+b'\lambda)y^2+(2g+2g'\lambda)x=0$

Now the condition for perpendicularity is $\Delta=0$ and $a+b=0$.

$\Rightarrow a+a'\lambda+b+b'\lambda=0 \Rightarrow \lambda=-\frac{a+b}{a'+b'}$

And $\Delta=abc+2fgh-af^2-bg^2-ch^2=0$

$\Rightarrow 0+0-0-(b+b'\lambda)(2g+2g'\lambda)^2-0=0$

$\Rightarrow 4(b+b'\lambda)(g+g'\lambda)^2=0$

Now on putting the value of $\lambda$, we get

$g(a'+b')=g'(a+b)$ **.**

**76. (c) Applying the formula, the distance between them is** $\left|2\sqrt{\frac{(k^2/4)-0}{1.(1+4)}}\right|=\left|\frac{k}{\sqrt{5}}\right|$ **(a)If the equation of line is** $y=mx$ **and the length of perpendicular drawn on it from the point** $(x_1,y_1)$ **is *d*, then** $\frac{y_1-mx_1}{\sqrt{1+m^2}}=\pm d \Rightarrow (y_1-mx_1)^2=d^2(1+m^2)$. **But** $m=\frac{y}{x}$, **therefore on eliminating '*m*' , the required equation is** $(xy_1-yx_1)^2=d^2(x^2+y^2)$**.**