ADDITION OF VECTORS

OBJECTIVES

1. **The point having position vectors** $2\mathbf{i}+3\mathbf{j}+4\mathbf{k},\ \ 3\mathbf{i}+4\mathbf{j}+2\mathbf{k},\ \ 4\mathbf{i}+2\mathbf{j}+3\mathbf{k}$ **are the vertices of**

   (a) Right angled triangle (b) Isosceles triangle

   (c) Equilateral triangle (d) Collinear

2. If $\mathbf{a}=\mathbf{i}+2\mathbf{j}+3\mathbf{k},\ \mathbf{b}=-\mathbf{i}+2\mathbf{j}+\mathbf{k}$ and $\mathbf{c}=3\mathbf{i}+\mathbf{j},$ then the unit vector along its resultant is

   (a) $3\mathbf{i}+5\mathbf{j}+4\mathbf{k}$ (b) $\frac{3\mathbf{i}+5\mathbf{j}+4\mathbf{k}}{50}$

   (c) $\frac{3\mathbf{i}+5\mathbf{j}+4\mathbf{k}}{5\sqrt{2}}$ (d) None of these

3. **If ABCDEF is a regular hexagon and** $\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}=\lambda\overrightarrow{AD},$ **then** $\lambda=$

   (a)2 (b) 3 (c)4 (d)6

4. **A unit vector a makes an angle** $\frac{\pi}{4}$ **with** ***z*****-axis. If** $\mathbf{a}+\mathbf{i}+\mathbf{j}$ **is a unit vector, then a is equal to**

   (a) $\frac{\mathbf{i}}{2}+\frac{\mathbf{j}}{2}+\frac{\mathbf{k}}{\sqrt{2}}$ (b) $\frac{\mathbf{i}}{2}+\frac{\mathbf{j}}{2}-\frac{\mathbf{k}}{\sqrt{2}}$

   (c) $-\frac{\mathbf{i}}{2}-\frac{\mathbf{j}}{2}+\frac{\mathbf{k}}{\sqrt{2}}$ (d) None of these

5. **The perimeter of the triangle whose vertices have the position vectors** $(\mathbf{i}+\mathbf{j}+\mathbf{k}),\ (5\mathbf{i}+3\mathbf{j}-3\mathbf{k})$ **and** $(2\mathbf{i}+5\mathbf{j}+9\mathbf{k}),$ is given by

   (a) $15+\sqrt{157}$ (b) $15-\sqrt{157}$

   (c) $\sqrt{15}-\sqrt{157}$ (d) $\sqrt{15}+\sqrt{157}$

6. **In a trapezium, the vector** $\overrightarrow{BC}=\lambda\overrightarrow{AD}.$ **We will then find that** $\mathbf{p}=\overrightarrow{AC}+\overrightarrow{BD}$ **is collinear with** $\overrightarrow{AD},$ **If** $\mathbf{p}=\mu\overrightarrow{AD},$ then

   (a) $\mu=\lambda+1$ (b) $\lambda=\mu+1$

   (c) $\lambda+\mu=1$ (d) $\mu=2+\lambda$

7. **If** ***OP*** **= 8 and** $\overrightarrow{OP}$ **makes angles** 45^o **and** 60^o **with** ***OX*****-axis and** ***OY*****-axis respectively, then** $\overrightarrow{OP}=$

   (a) $8(\sqrt{2}\mathbf{i}+\mathbf{j}\pm\mathbf{k})$ (b) $4(\sqrt{2}\mathbf{i}+\mathbf{j}\pm\mathbf{k})$

   (c) $\frac{1}{4}(\sqrt{2}\mathbf{i}+\mathbf{j}\pm\mathbf{k})$ (d) $\frac{1}{8}(\sqrt{2}\mathbf{i}+\mathbf{j}\pm\mathbf{k})$

8. **The position vectors of two points *A* and *B* are $\mathbf{i}+\mathbf{j}-\mathbf{k}$ and $2\mathbf{i}-\mathbf{j}+\mathbf{k}$ respectively. Then** $|\overrightarrow{AB}| =$

(a) 2 (b) 3

(c) 4 (d) 5

9. **The direction cosines of the resultant of the vectors $(\mathbf{i}+\mathbf{j}+\mathbf{k})$, $(-\mathbf{i}+\mathbf{j}+\mathbf{k})$, $(\mathbf{i}-\mathbf{j}+\mathbf{k})$ and $(\mathbf{i}+\mathbf{j}-\mathbf{k})$, are**

(a) $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{6}}\right)$ (b) $\left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$

(c) $\left(-\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}\right)$ (d) $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

10. **The position vectors of *A* and *B* are $2\mathbf{i}-9\mathbf{j}-4\mathbf{k}$ and $6\mathbf{i}-3\mathbf{j}+8\mathbf{k}$ respectively, then the magnitude of $\overrightarrow{AB}$ is**

(a) 11 (b) 12

(c) 13 (d) 14

11. **If the position vectors of *A* and *B* are $\mathbf{i}+3\mathbf{j}-7\mathbf{k}$ and $5\mathbf{i}-2\mathbf{j}+4\mathbf{k}$, then the direction cosine of $\overrightarrow{AB}$ along *y*-axis is**

(a) $\frac{4}{\sqrt{162}}$ (b) $-\frac{5}{\sqrt{162}}$

(c) – 5 (d) 11

12. **The position vectors of the points *A, B, C* are $(2\mathbf{i}+\mathbf{j}-\mathbf{k})$, $(3\mathbf{i}-2\mathbf{j}+\mathbf{k})$ and $(\mathbf{i}+4\mathbf{j}-3\mathbf{k})$ respectively. These points**

(a) Form an isosceles triangle

(b) Form a right-angled triangle

(c) Are collinear

(d) Form a scalene triangle

13. $3\,\overrightarrow{OD}+\overrightarrow{DA}+\overrightarrow{DB}+\overrightarrow{DC} =$

(a) $\overrightarrow{OA}+\overrightarrow{OB}-\overrightarrow{OC}$ (b) $\overrightarrow{OA}+\overrightarrow{OB}-\overrightarrow{BD}$

(c) $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}$ (d) None of these

14. **The vectors $\overrightarrow{AB} = 3\mathbf{i}+4\mathbf{k}$, and $\overrightarrow{AC} = 5\mathbf{i}-2\mathbf{j}+4\mathbf{k}$ are the sides of a triangle *ABC*. The length of the median through *A* is**

(a) $\sqrt{18}$ (b) $\sqrt{72}$ (c) $\sqrt{33}$ (d) $\sqrt{288}$

15. **The magnitudes of mutually perpendicular forces a, b and c are 2, 10 and 11 respectively. Then the magnitude of its resultant is**

(a) 12 (b) 15

(c) 9 (d) None

16. ***ABC* is an isosceles triangle right angled at *A*. Forces of magnitude $2\sqrt{2}, 5$ and 6 act along $\overrightarrow{BC}, \overrightarrow{CA}$ and $\overrightarrow{AB}$ respectively. The magnitude of their resultant force is**

(a) 4 (b) 5 (c) $11+2\sqrt{2}$ (d)30

17. **If a, b and c be three non-zero vectors, no two of which are collinear. If the vector $\mathbf{a}+2\mathbf{b}$ is collinear with c and $\mathbf{b}+3\mathbf{c}$ is collinear with a, then (λ being some non-zero scalar) $\mathbf{a}+2\mathbf{b}+6\mathbf{c}$ is equal to**

(a) $\lambda\mathbf{a}$ (b) $\lambda\mathbf{b}$ (c) $\lambda\mathbf{c}$ (d)**0**

18. **In a regular hexagon *ABCDEF*, $\overrightarrow{AE} =$**

(a) $\overrightarrow{AC}+\overrightarrow{AF}+\overrightarrow{AB}$ (b) $\overrightarrow{AC}+\overrightarrow{AF}-\overrightarrow{AB}$

(c) $\overrightarrow{AC}+\overrightarrow{AB}-\overrightarrow{AF}$ (d) None of these

19. **If $\mathbf{a}=2\mathbf{i}+5\mathbf{j}$ and $\mathbf{b}=2\mathbf{i}-\mathbf{j}$, then the unit vector along $\mathbf{a}+\mathbf{b}$ will be**

(a) $\frac{\mathbf{i}-\mathbf{j}}{\sqrt{2}}$ (b) $\mathbf{i}+\mathbf{j}$ (c) $\sqrt{2}(\mathbf{i}+\mathbf{j})$ (d) $\frac{\mathbf{i}+\mathbf{j}}{\sqrt{2}}$

20. **In the triangle *ABC*, $\overrightarrow{AB}=\mathbf{a}, \overrightarrow{AC}=\mathbf{c}, \overrightarrow{BC}=\mathbf{b}$, then**

(a) $\mathbf{a}+\mathbf{b}+\mathbf{c}=\mathbf{0}$ (b) $\mathbf{a}+\mathbf{b}-\mathbf{c}=\mathbf{0}$

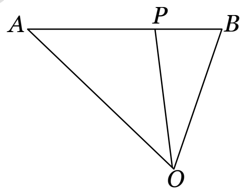
(c) $\mathbf{a}-\mathbf{b}+\mathbf{c}=\mathbf{0}$ (d) $-\mathbf{a}+\mathbf{b}+\mathbf{c}=\mathbf{0}$

21. **If the position vectors of the point *A, B, C* be i, j, k respectively and *P* be a point such that $\overrightarrow{AB}=\overrightarrow{CP}$, then the position vector of *P* is**

(a) $-\mathbf{i}+\mathbf{j}+\mathbf{k}$ (b) $-\mathbf{i}-\mathbf{j}+\mathbf{k}$

(c) $\mathbf{i}+\mathbf{j}-\mathbf{k}$ (d) None of these

22. **If in the given figure $\overrightarrow{OA}=\mathbf{a}, \overrightarrow{OB}=\mathbf{b}$ and $AP:PB=m:n$, then $\overrightarrow{OP}=$**



(a) $\frac{m\mathbf{a}+n\mathbf{b}}{m+n}$ (b) $\frac{n\mathbf{a}+m\mathbf{b}}{m+n}$ (c) $m\mathbf{a}-n\mathbf{b}$ (d) $\frac{m\mathbf{a}-n\mathbf{b}}{m-n}$

23. **If** $\mathbf{a} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, $\mathbf{b} = 2\mathbf{i} - 4\mathbf{j} - 3\mathbf{k}$ **and** $\mathbf{c} = -\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, **then** $\mathbf{a} + \mathbf{b} + \mathbf{c}$ **is**

(a) $3\mathbf{i} - 4\mathbf{j}$ (b) $3\mathbf{i} + 4\mathbf{j}$

(c) $4\mathbf{i} - 4\mathbf{j}$ (d) $4\mathbf{i} + 4\mathbf{j}$

24. **If** A, B, C **are the vertices of a triangle whose position vectors are a, b, c and** ***G*** **is the centroid of the** ΔABC, **then** $\overrightarrow{GA} + \overrightarrow{GB} + \overrightarrow{GC}$ **is**

(a) **0** (b) $\vec{A} + \vec{B} + \vec{C}$

(c) $\frac{\mathbf{a} + \mathbf{b} + \mathbf{c}}{3}$ (d) $\frac{\mathbf{a} + \mathbf{b} - \mathbf{c}}{3}$

25. **If a and b are the position vectors of** ***A*** **and** ***B*** **respectively, then the position vector of a point** ***C*** **on** ***AB*** **produced such that** $\overrightarrow{AC} = 3\overrightarrow{AB}$ **is**

(a) $3\mathbf{a} - \mathbf{b}$ (b) $3\mathbf{b} - \mathbf{a}$

(c) $3\mathbf{a} - 2\mathbf{b}$ (d) $3\mathbf{b} - 2\mathbf{a}$

26. **If the position vectors of the points** ***A, B, C*** **be** $\mathbf{i} + \mathbf{j}$, $\mathbf{i} - \mathbf{j}$ **and** $a\,\mathbf{i} + b\,\mathbf{j} + c\,\mathbf{k}$ **respectively, then the points** ***A, B, C*** **are collinear if**

(a) $a = b = c = 1$

(b) $a = 1$, b and c are arbitrary scalars

(c) $a = b = c = 0$

(d) $c = 0$, $a = 1$ and b is arbitrary scalars

27. **In a triangle** ***ABC*****, if** $2\overrightarrow{AC} = 3\overrightarrow{CB}$, **then** $2\overrightarrow{OA} + 3\overrightarrow{OB}$ **equals**

(a) $5\overrightarrow{OC}$ (b) $-\overrightarrow{OC}$

(c) $\overrightarrow{OC}$ (d) None of these

28. **If** $ABCDEF$ **is regular hexagon, then** $\overrightarrow{AD} + \overrightarrow{EB} + \overrightarrow{FC} =$

(a) 0 (b) $2\overrightarrow{AB}$

(c) $3\overrightarrow{AB}$ (d) $4\overrightarrow{AB}$

29. **If** ***O*** **be the circumcentre and** ***O'*** **be the orthocentre of the triangle** ***ABC*****, then** $\overrightarrow{O'A} + \overrightarrow{O'B} + \overrightarrow{O'C} =$

(a) $\overrightarrow{OO'}$ (b) $2\overrightarrow{O'O}$ (c) $2\overrightarrow{OO'}$ (d) **0**

30. **If** ***ABCD*** **is a parallelogram and the position vectors of** ***A, B, C*** **are** $\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$, $\mathbf{i} + \mathbf{j} + \mathbf{k}$ **and** $7\mathbf{i} + 7\mathbf{j} + 7\mathbf{k}$, **then the position vector of** ***D*** **will be**

(a) $7\mathbf{i} + 5\mathbf{j} + 3\mathbf{k}$ (b) $7\mathbf{i} + 9\mathbf{j} + 11\mathbf{k}$ (c) $9\mathbf{i} + 11\mathbf{j} + 13\mathbf{k}$ (d) $8\mathbf{i} + 8\mathbf{j} + 8\mathbf{k}$

31. If $\overrightarrow{AO}+\overrightarrow{OB}=\overrightarrow{BO}+\overrightarrow{OC}$, then *A, B, C* form

(a) Equilateral triangle (b) Right angled triangle

(c) Isosceles triangle (d) Line

32. If D, E, F are respectively the mid points of AB, AC and BC in ΔABC, then $\overrightarrow{BE}+\overrightarrow{AF}=$

(a) $\overrightarrow{DC}$ (b) $\frac{1}{2}\overrightarrow{BF}$ (c) $2\overrightarrow{BF}$ (d) $\frac{3}{2}\overrightarrow{BF}$

33. If *G* and *G'* be the centroids of the triangles *ABC* and $A'B'C'$ respectively, then $\overrightarrow{AA'}+\overrightarrow{BB'}+\overrightarrow{CC'}=$

(a) $\frac{2}{3}\overrightarrow{GG'}$ (b) $\overrightarrow{GG'}$

(c) $2\overrightarrow{GG'}$ (d) $3\overrightarrow{GG'}$

34. If *D, E, F* be the middle points of the sides *BC, CA* and *AB* of the triangle *ABC*, then $\overrightarrow{AD}+\overrightarrow{BE}+\overrightarrow{CF}$ is

(a) A zero vector (b) A unit vector

(c) 0 (d) None of these

35. *A* and *B* are two points. The position vector of *A* is $6\mathbf{b}-2\mathbf{a}$. A point *P* divides the line *AB* in the ratio 1 : 2. If $\mathbf{a}-\mathbf{b}$ is the position vector of *P*, then the position vector of *B* is given by

(a) $7\mathbf{a}-15\mathbf{b}$ (b) $7\mathbf{a}+15\mathbf{b}$

(c) $15\mathbf{a}-7\mathbf{b}$ (d) $15\mathbf{a}+7\mathbf{b}$

36. The sum of two forces is 18 *N* and resultant whose direction is at right angles to the smaller force is 12*N*. The magnitude of the two forces are

(a) 13, 5 (b) 12, 6

(c) 14, 4 (d) 11, 7

37. If three points *A, B, C* are collinear, whose position vectors are $\mathbf{i}-2\mathbf{j}-8\mathbf{k}$, $5\mathbf{i}-2\mathbf{k}$ and $11\mathbf{i}+3\mathbf{j}+7\mathbf{k}$ respectively, then the ratio in which *B* divides *AC* is

(a) 1 : 2 (b) 2 : 3 (c) 2 : 1 (d) 1 : 1

38. The vectors $3\mathbf{i}+\mathbf{j}-5\mathbf{k}$ and $a\mathbf{i}+b\mathbf{j}-15\mathbf{k}$ are collinear, if

(a) $a=3, b=1$ (b) $a=9, b=1$

(c) $a=3, b=3$ (d) $a=9, b=3$

39. If a, b, c are three non-coplanar vectors such that $\mathbf{a}+\mathbf{b}+\mathbf{c}=\alpha\mathbf{d}$ and $\mathbf{b}+\mathbf{c}+\mathbf{d}=\beta\mathbf{a}$, then $\mathbf{a}+\mathbf{b}+\mathbf{c}+\mathbf{d}$ is equal to

(a) 0 (b) $\alpha\mathbf{a}$ (c) $\beta\mathbf{b}$ (d) $(\alpha+\beta)\mathbf{c}$

40. If $(x, y, z) \neq (0, 0, 0)$ **and** $(\mathbf{i}+\mathbf{j}+3\mathbf{k})x+(3\mathbf{i}-3\mathbf{j}+\mathbf{k})y+(-4\mathbf{i}+5\mathbf{j})z=\lambda(x\mathbf{i}+y\mathbf{j}+z\mathbf{k})$, **then the value of** λ **will be**

(a) – 2, 0 (b) 0, – 2

(c) – 1, 0 (d) 0, – 1

41. The vectors $\mathbf{i}+2\mathbf{j}+3\mathbf{k},\ \lambda\mathbf{i}+4\mathbf{j}+7\mathbf{k},\ -3\mathbf{i}-2\mathbf{j}-5\mathbf{k}$ **are collinear, if** λ **equals**

(a) 3 (b) 4

(c) 5 (d) 6

42. The points with position vectors $10\mathbf{i}+3\mathbf{j}, 12\mathbf{i}-5\mathbf{j}$ **and** $a\mathbf{i}+11\mathbf{j}$ **are collinear, if** $a=$

(a) – 8 (b) 4

(c) 8 (d) 12

43. If three points *A*, *B* and *C* have position vectors $(1, x, 3), (3, 4, 7)$ **and** $(y, -2, -5)$ **respectively and if they are collinear, then** $(x, y)=$

(a) (2, – 3) (b) (– 2, 3)

(c) (2, 3) (d) (– 2, – 3)

ADDITION OF VECTORS

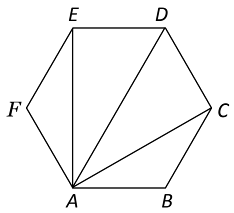
HINTS AND SOLUTIONS

1. (c) $\overrightarrow{AB} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}$, $\overrightarrow{BC} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$, $\overrightarrow{CA} = 2\mathbf{i} - \mathbf{j} - \mathbf{k}$

Clearly $|AB| = |BC| = |CA| = \sqrt{6}$

2. (c) $\mathbf{R} = 3\mathbf{i} + 5\mathbf{j} + 4\mathbf{k} \Rightarrow \hat{\mathbf{R}} = \dfrac{3\mathbf{i} + 5\mathbf{j} + 4\mathbf{k}}{5\sqrt{2}}$.

3. (b) By triangle law, $\overrightarrow{AB} = \overrightarrow{AD} - \overrightarrow{BD}$, $\overrightarrow{AC} = \overrightarrow{AD} - \overrightarrow{CD}$



Therefore, $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF}$

$= 3\overrightarrow{AD} + (\overrightarrow{AE} - \overrightarrow{BD}) + (\overrightarrow{AF} - \overrightarrow{CD}) = 3\overrightarrow{AD}$

Hence $\lambda = 3$, [Since $\overrightarrow{AE} = \overrightarrow{BD}, \overrightarrow{AF} = \overrightarrow{CD}$].

4. (c) Let $\mathbf{a} = l\mathbf{i} + m\mathbf{j} + n\mathbf{k}$, where $l^2 + m^2 + n^2 = 1$.

$\mathbf{a}$ makes an angle $\frac{\pi}{4}$ with $z-$axis.

$\therefore n = \dfrac{1}{\sqrt{2}}, \quad l^2 + m^2 = \dfrac{1}{2}$ ….. (i)

$\therefore \mathbf{a} = l\,\mathbf{i} + m\,\mathbf{j} + \dfrac{\mathbf{k}}{\sqrt{2}}$

$\mathbf{a} + \mathbf{i} + \mathbf{j} = (l+1)\mathbf{i} + (m+1)\mathbf{j} + \dfrac{\mathbf{k}}{\sqrt{2}}$

Its magnitude is 1, hence $(l+1)^2 + (m+1)^2 = \dfrac{1}{2}$ .....(ii)

From (i) and (ii), $2lm = \dfrac{1}{2} \Rightarrow l = m = -\dfrac{1}{2}$

Hence $\mathbf{a} = -\dfrac{\mathbf{i}}{2} - \dfrac{\mathbf{j}}{2} + \dfrac{\mathbf{k}}{\sqrt{2}}$.

5. (a) $\mathbf{a} = 4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k} \Rightarrow |\mathbf{a}| = \sqrt{16 + 16 + 4} = 6$

$\mathbf{b} = -3\mathbf{i} + 2\mathbf{j} + 12\mathbf{k} \Rightarrow |\mathbf{b}| = \sqrt{144 + 4 + 9} = \sqrt{157}$

$\mathbf{c} = -\mathbf{i} - 4\mathbf{j} - 8\mathbf{k} \Rightarrow |\mathbf{c}| = \sqrt{64 + 16 + 1} = 9$

Hence perimeter is $15 + \sqrt{157}$.

6. (a) We have, $\mathbf{p} = \overrightarrow{AC} + \overrightarrow{BD} = \overrightarrow{AC} + \overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{AC} + \lambda\overrightarrow{AD} + \overrightarrow{CD}$

$$= \lambda\overrightarrow{AD} + (\overrightarrow{AC} + \overrightarrow{CD}) = \lambda\overrightarrow{AD} + \overrightarrow{AD} = (\lambda+1)\overrightarrow{AD}.$$

Therefore $\mathbf{p} = \mu\overrightarrow{AD} \Rightarrow \mu = \lambda + 1$.

7. (b) Here is the only vector $4(\sqrt{2}\mathbf{i} + \mathbf{j} \pm \mathbf{k})$, whose length is 8.

8. (b) $\overrightarrow{AB} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k} \Rightarrow |\overrightarrow{AB}| = 3$.

9. (d) Resultant vector $= 2\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$.

Direction cosines are $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$.

10. (d) $\overrightarrow{AB} = (6-2)\mathbf{i} + (-3+9)\mathbf{j} + (8+4)\mathbf{k} = 4\mathbf{i} + 6\mathbf{j} + 12\mathbf{k}$

$|\overrightarrow{AB}| = \sqrt{16+36+144} = 14$.

11. (b) $\overrightarrow{AB} = 4\mathbf{i} - 5\mathbf{j} + 11\mathbf{k}$

Direction cosine along $y-$axis $= \frac{-5}{\sqrt{16+25+121}} = \frac{-5}{\sqrt{162}}$.

12. (c) $\overrightarrow{AB} = \mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$

$\overrightarrow{BC} = -2\mathbf{i} + 6\mathbf{j} - 4\mathbf{k}$

$\overrightarrow{CA} = \mathbf{i} - 3\mathbf{j} - 2\mathbf{k}$

$|\overrightarrow{AB}| = \sqrt{1+9+4} = \sqrt{14}$

$|\overrightarrow{BC}| = \sqrt{4+36+16} = \sqrt{56} = 2\sqrt{14}$

$|\overrightarrow{CA}| = \sqrt{1+9+4} = \sqrt{14}$

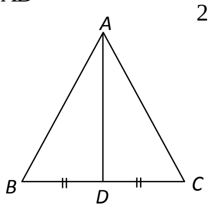
$|\overrightarrow{AB}| + |\overrightarrow{AC}| = |\overrightarrow{BC}|$

Hence A, B, C are collinear.

13. (c) $3\overrightarrow{OD} + \overrightarrow{DA} + \overrightarrow{DB} + \overrightarrow{DC}$

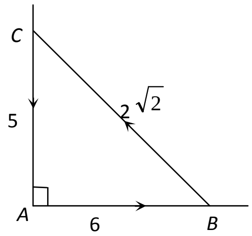
$= \overrightarrow{OD} + \overrightarrow{DA} + \overrightarrow{OD} + \overrightarrow{DB} + \overrightarrow{OD} + \overrightarrow{DC} = \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC}$.

14. (c) P.V. of $\overrightarrow{AD} = \frac{(3+5)i + (0-2)j + (4+4)k}{2} = 4\mathbf{i} - \mathbf{j} + 4\mathbf{k}$



$|\overrightarrow{AD}| = \sqrt{16+16+1} = \sqrt{33}$.

15. (b) $R = \sqrt{4+100+121} = 15$.



16. (b) $R\cos\theta = 6\cos 0^\circ + 2\sqrt{2}\cos(180^o - B) + 5\cos 270^o$

ABC is a right angled isosceles triangle

i.e., $\angle B = \angle C = 45^\circ$

$\therefore R^2 = 61 + 8(1) - 24\sqrt{2}\cdot\frac{1}{\sqrt{2}} - 20\sqrt{2}\cdot\frac{1}{\sqrt{2}} = 25$

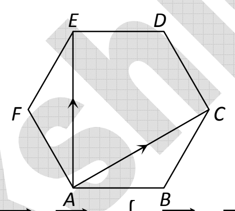
$\therefore R = 5$.

17. (d) Let $\mathbf{a}+2\mathbf{b} = x\mathbf{c}$ and $\mathbf{b}+3\mathbf{c} = y\mathbf{a}$, then $\mathbf{a}+2\mathbf{b}+6\mathbf{c} = (x+6)\mathbf{c}$ and $\mathbf{a}+2\mathbf{b}+6\mathbf{c} = (1+2y)\mathbf{a}$

So, $(x+6)\mathbf{c} = (1+2y)\mathbf{a}$

Since $\mathbf{a}$ and $\mathbf{c}$ are non-zero and non-collinear, we have $x+6=0$ and $1+2y=0$ *i.e.,* $x=-6$ and $y = -\frac{1}{2}$. in either case, we have $\mathbf{a}+2\mathbf{b}+6\mathbf{c} = \mathbf{0}$.

18. (b) $\overrightarrow{AE} = \overrightarrow{AC} + \overrightarrow{CD} + \overrightarrow{DE}$



$= \overrightarrow{AC} + \overrightarrow{AF} - \overrightarrow{AB}$, $\{\because \overrightarrow{CD} = \overrightarrow{AF} \text{ and } \overrightarrow{DE} = -\overrightarrow{AB}\}$.

19. (d) $\mathbf{a}+\mathbf{b} = 4\mathbf{i}+4\mathbf{j}$, therefore unit vector $\frac{4(\mathbf{i}+\mathbf{j})}{\sqrt{32}} = \frac{\mathbf{i}+\mathbf{j}}{\sqrt{2}}$.

20. (b) $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 0 \Rightarrow \mathbf{a}+\mathbf{b}-\mathbf{c} = 0$.

21. (a) Let the position vector of *P* is $x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$, then $\overrightarrow{AB} = \overrightarrow{CP} \Rightarrow \mathbf{j}-\mathbf{i} = x\mathbf{i}+y\mathbf{j}+(z-1)\mathbf{k}$

By comparing the coefficients of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$, we get $x=-1$, $y=1$ and $z-1=0 \Rightarrow z=1$

Hence required position vector is $-\mathbf{i}+\mathbf{j}+\mathbf{k}$.

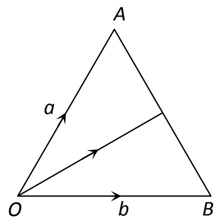
22. (b) Concept

23. (c) $\mathbf{a}+\mathbf{b}+\mathbf{c}=(3+2-1)\mathbf{i}+(-2-4+2)\mathbf{j}+(1-3+2)\mathbf{k}=4\mathbf{i}-4\mathbf{j}$.

24. (a) Position vectors of vertices *A*, *B* and *C* of the triangle *ABC* = **a, b** and **c**. We know that position vector of centroid of the triangle (*G*) $=\frac{\mathbf{a}+\mathbf{b}+\mathbf{c}}{3}$.

Therefore, $\overrightarrow{GA}+\overrightarrow{GB}+\overrightarrow{GC}=0$

25. (d) Since given that $\overrightarrow{AC}=3\overrightarrow{AB}$, it means that point C divides AB externally. Thus $\overrightarrow{AC}:\overrightarrow{BC}=3:2$



Hence $\overrightarrow{OC}=\frac{3.\mathbf{b}-2.\mathbf{a}}{3-2}=3\mathbf{b}-2\mathbf{a}$.

26. (d) Here $\overrightarrow{AB}=-2\mathbf{j}$, $\overrightarrow{BC}=(a-1)\mathbf{i}+(b+1)\mathbf{j}+c\mathbf{k}$

The points are collinear, then $\overrightarrow{AB}=k(\overrightarrow{BC})$

$$-2\mathbf{j}=k\{(a-1)\mathbf{i}+(b+1)\mathbf{j}+c\mathbf{k}\}$$

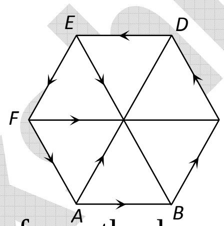
On comparing, $k(a-1)=0$, $k(b+1)=-2$, $kc=0$.

Hence $c=0$, $a=1$ and b is arbitrary scalar.

27. (a) $2\overrightarrow{OA}+3\overrightarrow{OB}=2(\overrightarrow{OC}+\overrightarrow{CA})+3(\overrightarrow{OC}+\overrightarrow{CB})$

$=5\overrightarrow{OC}+2\overrightarrow{CA}+3\overrightarrow{CB}=5\overrightarrow{OC}$, $\{\because 2\overrightarrow{CA}=-3\overrightarrow{CB}\}$.

28. (d) A regular hexagon *ABCDEF*.



We know from the hexagon that $\overrightarrow{AD}$ is parallel to $\overrightarrow{BC}$ or $\overrightarrow{AD}=2\overrightarrow{BC}$; $\overrightarrow{EB}$ is parallel to $\overrightarrow{FA}$ or $\overrightarrow{EB}=2\overrightarrow{FA}$, and $\overrightarrow{FC}$ is parallel to $\overrightarrow{AB}$ or $\overrightarrow{FC}=2\overrightarrow{AB}$.

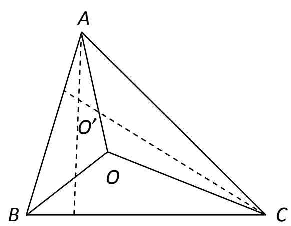
Thus $\overrightarrow{AD}+\overrightarrow{EB}+\overrightarrow{FC}=2\overrightarrow{BC}+2\overrightarrow{FA}+2\overrightarrow{AB}$

$=2(\overrightarrow{FA}+\overrightarrow{AB}+\overrightarrow{BC})=2(\overrightarrow{FC})=2(2\overrightarrow{AB})=4\overrightarrow{AB}$.

29. (b) $\overrightarrow{O'A}=\overrightarrow{O'O}+\overrightarrow{OA}$

$\overrightarrow{O'B}=\overrightarrow{O'O}+\overrightarrow{OB}$

$\overrightarrow{O'C}=\overrightarrow{O'O}+\overrightarrow{OC}$



$\Rightarrow \overrightarrow{O'A}+\overrightarrow{O'B}+\overrightarrow{O'C}$

$=3\overrightarrow{O'O}+\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}$

Since $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}=\overrightarrow{OO'}=-\overrightarrow{O'O}$

$\therefore\ \overrightarrow{O'A}+\overrightarrow{O'B}+\overrightarrow{O'C}=2\overrightarrow{O'O}$.

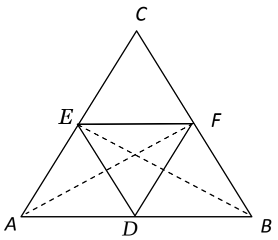
30. (b) Let position vector of D is $x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$, then $\overrightarrow{AB}=\overrightarrow{DC}$ $\Rightarrow -2\mathbf{j}-4\mathbf{k}=(7-x)\mathbf{i}+(7-y)\mathbf{j}+(7-z)\mathbf{k}$

$\Rightarrow x=7,\ y=9,\ z=11.$

Hence position vector of D will be $7\mathbf{i}+9\mathbf{j}+11\mathbf{k}$.

31. (c) $\overrightarrow{AB}=\overrightarrow{BC}$ (As given). Hence it is an isosceles triangle.

32. (a) $\overrightarrow{BE}+\overrightarrow{AF}=\overrightarrow{OE}-\overrightarrow{OB}+\overrightarrow{OF}-\overrightarrow{OA}$



$=\dfrac{\overrightarrow{OA}+\overrightarrow{OC}}{2}-\overrightarrow{OB}+\dfrac{\overrightarrow{OB}+\overrightarrow{OC}}{2}-\overrightarrow{OA}$

$=\overrightarrow{OC}-\dfrac{\overrightarrow{OA}+\overrightarrow{OB}}{2}=\overrightarrow{OC}-\overrightarrow{OD}=\overrightarrow{DC}$.

33. (d) $\overrightarrow{GA}+\overrightarrow{GB}+\overrightarrow{GC}=\mathbf{0}$ and $\overrightarrow{G'A'}+\overrightarrow{G'B'}+\overrightarrow{G'C'}=\mathbf{0}$

$\Rightarrow(\overrightarrow{GA}-\overrightarrow{G'A'})+(\overrightarrow{GB}-\overrightarrow{G'B'})+(\overrightarrow{GC}-\overrightarrow{G'C'})=\mathbf{0}$

$\Rightarrow(\overrightarrow{GA}+\overrightarrow{G'G}-\overrightarrow{G'A'})+(\overrightarrow{GB}+\overrightarrow{G'G}-\overrightarrow{G'B'})+(\overrightarrow{GC}+\overrightarrow{G'G}-\overrightarrow{G'C'})=3\overrightarrow{G'G}$

$\Rightarrow(\overrightarrow{GA}-\overrightarrow{GA'})+(\overrightarrow{GB}-\overrightarrow{GB'})+(\overrightarrow{GC}-\overrightarrow{GC'})=3\overrightarrow{G'G}$

$\Rightarrow\overrightarrow{A'A}+\overrightarrow{B'B}+\overrightarrow{C'C}=3\overrightarrow{G'G}\Rightarrow\overrightarrow{AA'}+\overrightarrow{BB'}+\overrightarrow{CC'}=3\overrightarrow{GG'}$.

34. (a) $\overrightarrow{AD}=\overrightarrow{OD}-\overrightarrow{OA}=\dfrac{\mathbf{b}+\mathbf{c}}{2}-\mathbf{a}=\dfrac{\mathbf{b}+\mathbf{c}-2\mathbf{a}}{2}$,

(Where O is the origin for reference)

Similarly, $\overrightarrow{BE}=\overrightarrow{OE}-\overrightarrow{OB}=\dfrac{\mathbf{c}+\mathbf{a}}{2}-\mathbf{b}=\dfrac{\mathbf{c}+\mathbf{a}-2\mathbf{b}}{2}$ and $\overrightarrow{CF}=\dfrac{\mathbf{a}+\mathbf{b}-2\mathbf{c}}{2}$.

35. (a) Standard problem.

36. (a) $P+Q=18,\ R=12,\ \theta=90^o$, (say)

$\tan\theta=\tan 90^o=\infty$

$\Rightarrow\ P+Q\cos\alpha=0$, $\therefore\ \cos\alpha=\dfrac{-P}{Q}$

Also, $(12)^2 = P^2 + Q^2 + 2PQ\cos\alpha$

Or $144 = P^2 + Q^2 + (2P)(-P)$

$\Rightarrow 144 = Q^2 - P^2 = (Q+P)(Q-P)$

Or $144 = 18(Q-P)$ or $Q - P = 8$

After solving $Q = 13$, $P = 5$.

37. (b) Let the B divide AC in ratio $\lambda : 1$, then

$$5\mathbf{i} - 2\mathbf{k} = \frac{\lambda(11\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}) + \mathbf{i} - 2\mathbf{j} - 8\mathbf{k}}{\lambda + 1}$$

$\Rightarrow 3\lambda - 2 = 0 \;\Rightarrow \lambda = \frac{2}{3}$ *i.e.*, ratio = 2 : 3.

38. (d) $\frac{3}{a} = \frac{1}{b} = \frac{-5}{-15} \Rightarrow a = 9,\, b = 3.$

39. (a) We have $\mathbf{a} + \mathbf{b} + \mathbf{c} = \alpha\,\mathbf{d}$ and $\mathbf{b} + \mathbf{c} + \mathbf{d} = \beta\,\mathbf{a}$

$\therefore\ \mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d} = (\alpha+1)\mathbf{d}$ and $\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d} = (\beta+1)\mathbf{a}$.

$\Rightarrow (\alpha+1)\mathbf{d} = (\beta+1)\mathbf{a}$

If $\alpha \neq -1$, then $(\alpha+1)\mathbf{d} = (\beta+1)\mathbf{a} \Rightarrow \mathbf{d} = \frac{\beta+1}{\alpha+1}\mathbf{a}$

$$\Rightarrow \mathbf{a} + \mathbf{b} + \mathbf{c} = \alpha\,\mathbf{d} \Rightarrow \mathbf{a} + \mathbf{b} + \mathbf{c} = \alpha\left(\frac{\beta+1}{\alpha+1}\right)\mathbf{a}$$

$$\Rightarrow \left(1 - \frac{\alpha(\beta+1)}{\alpha+1}\right)\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$$

$\Rightarrow \mathbf{a}, \mathbf{b}, \mathbf{c}$ are coplanar which is contradiction to the given condition, $\therefore\ \alpha = -1$ and so $\mathbf{a} + \mathbf{b} + \mathbf{c} + \mathbf{d} = 0$.

40. (d) From given equation

$(1-\lambda)x + 3y - 4z = 0$

$x - (\lambda+3)y + 5z = 0$

$3x + y - \lambda z = 0$

$$\Rightarrow \begin{vmatrix} (1-\lambda) & 3 & -4 \\ 1 & -(\lambda+3) & 5 \\ 3 & 1 & -\lambda \end{vmatrix} = 0 \Rightarrow \lambda = 0, -1.$$

41. (a) $\begin{vmatrix} 1 & 2 & 3 \\ \lambda & 4 & 7 \\ -3 & -2 & -5 \end{vmatrix} = 0 \Rightarrow \lambda = 3.$

42. (c) If given points be A, B, C then $\overrightarrow{AB} = k(\overrightarrow{BC})$ or $2\mathbf{i} - 8\mathbf{j} = k[(a-12)\mathbf{i} + 16\mathbf{j}] \Rightarrow k = \frac{-1}{2}$

Also, $2 = k(a-12) \Rightarrow a = 8.$

43. (a) If *A, B, C* are collinear. Then $\overrightarrow{AB} = \lambda\,\overrightarrow{BC}$