COMPOUND ANGLES

OBJECTIVES

1. If $\sin A = \sin B$ and $\cos A = \cos B$, then

(a) $\sin\frac{A-B}{2} = 0$ (b) $\sin\frac{A+B}{2} = 0$

(c) $\cos\frac{A-B}{2} = 0$ (d) $\cos(A+B) = 0$

2. $\cos^2 48° - \sin^2 12° =$

(a) $\frac{\sqrt{5}-1}{4}$ (b) $\frac{\sqrt{5}+1}{8}$

(c) $\frac{\sqrt{3}-1}{4}$ (d) $\frac{\sqrt{3}+1}{2\sqrt{2}}$

3. If $\tan\alpha = \frac{m}{m+1}$ and $\tan\beta = \frac{1}{2m+1}$, then $\alpha+\beta =$

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{6}$ (d) None of these

4. If $\cos(\alpha+\beta) = \frac{4}{5}, \sin(\alpha-\beta) = \frac{5}{13}$ and α, β lie between 0 and $\frac{\pi}{4}$, then $\tan 2\alpha =$

(a) $\frac{16}{63}$ (b) $\frac{56}{33}$

(c) $\frac{28}{33}$ (d) None of these

5. If $\tan A = -\frac{1}{2}$ and $\tan B = -\frac{1}{3}$, then $A+B =$

(a) $\frac{\pi}{4}$ (b) $\frac{3\pi}{4}$

(c) $\frac{5\pi}{4}$ (d) None of these

6. The value of $\sin 47^o + \sin 61° - \sin 11° - \sin 25° =$

(a) $\sin 36°$ (b) $\cos 36°$

(c) $\sin 7°$ (d) $\cos 7°$

7. If $A+B = \frac{\pi}{4}$, then $(1+\tan A)(1+\tan B) =$

(a) 1 (b) 2 (c) ∞ (d) – 2

8. $\frac{1}{\sin 10°} - \frac{\sqrt{3}}{\cos 10°} =$

(a) 0 (b) 1 (c) 2 (d) 4

9. $\cos^2\left(\frac{\pi}{6}+\theta\right)-\sin^2\left(\frac{\pi}{6}-\theta\right)=$

(a) $\frac{1}{2}\cos 2\theta$ (b) 0

(c) $-\frac{1}{2}\cos 2\theta$ (d) $\frac{1}{2}$

10. If $m\tan(\theta-30°)=n\tan(\theta+120°)$, then $\frac{m+n}{m-n}=$

(a) $2\cos 2\theta$ (b) $\cos 2\theta$

(c) $2\sin 2\theta$ (d) $\sin 2\theta$

11. The value of $\tan 20^o+2\tan 50^o-\tan 70^o$ is equal to

(a) 1 (b) 0

(c) $\tan 50^o$ (d) None of these

12. The value of $\dfrac{\tan 70^o-\tan 20^o}{\tan 50^o}=$

(a) 1 (b) 2

(c) 3 (d) 0

13. If $b\sin\alpha=a\sin(\alpha+2\beta)$, then $\frac{a+b}{a-b}=$

(a) $\frac{\tan\beta}{\tan(\alpha+\beta)}$ (b) $\frac{\cot\beta}{\cot(\alpha-\beta)}$

(c) $\frac{-\cot\beta}{\cot(\alpha+\beta)}$ (d) $\frac{\cot\beta}{\cot(\alpha+\beta)}$

14. If $\frac{\sin A-\sin C}{\cos C-\cos A}=\cot B$, then A,B,C are in

(a) A.P. (b) G.P.

(c) H.P. (d) None of these

15. $\cos A+\cos(240°+A)+\cos(240°-A)=$

(a) $\cos A$ (b) 0

(c) $\sqrt{3}\sin A$ (d) $\sqrt{3}\cos A$

16. $\sin(\beta+\gamma-\alpha)+\sin(\gamma+\alpha-\beta)+\sin(\alpha+\beta-\gamma)-\sin(\alpha+\beta+\gamma)=$

(a) $2\sin\alpha\sin\beta\sin\gamma$ (b) $4\sin\alpha\sin\beta\sin\gamma$ (c) $\sin\alpha\sin\beta\sin\gamma$ (d) None of these

17. The value of $\sin\frac{\pi}{16}\sin\frac{3\pi}{16}\sin\frac{5\pi}{16}\sin\frac{7\pi}{16}$ is

(a) $\frac{1}{16}$ (b) $\frac{\sqrt{2}}{16}$

(c) $\frac{1}{8}$ (d) $\frac{\sqrt{2}}{8}$

18. $\cos^2\alpha+\cos^2(\alpha+120°)+\cos^2(\alpha-120°)$ is equal to

(a) 3/2 (b) 1

(c) 1/2 (d) 0

19. $\cos^2 76^o+\cos^2 16^o-\cos 76^o\cos 16^o=$

(a) – 1/4 (b) 1/2

(c) 0 (d) 3/4

20. $\frac{\sin(B+A)+\cos(B-A)}{\sin(B-A)+\cos(B+A)}=$

(a) $\frac{\cos B+\sin B}{\cos B-\sin B}$ (b) $\frac{\cos A+\sin A}{\cos A-\sin A}$

(c) $\frac{\cos A-\sin A}{\cos A+\sin A}$ (d) None of these

21. $\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{16\pi}{15}=$

(a) 1/2 (b) 1/4

(c) 1/8 (d) 1/16

22. $\cos\alpha.\sin(\beta-\gamma)+\cos\beta.\sin(\gamma-\alpha)+\cos\gamma.\sin(\alpha-\beta)=$

(a) 0 (b) 1/2

(c) 1 (d) $4\cos\alpha\cos\beta\cos\gamma$

23. If $\sin A+\sin 2A=x$ and $\cos A+\cos 2A=y$, then $(x^2+y^2)(x^2+y^2-3)=$

(a) $2y$ (b) y

(c) $3y$ (d) None of these

24. If $\sin\theta=\frac{12}{13},(0<\theta<\frac{\pi}{2})$ and $\cos\phi=-\frac{3}{5},\left(\pi<\phi<\frac{3\pi}{2}\right)$. Then $\sin(\theta+\phi)$ will be

(a) $\frac{-56}{61}$ (b) $\frac{-56}{65}$

(c) $\frac{1}{65}$ (d) – 56

25. $\sin 12°\sin 24°\sin 48°\sin 84° =$

(a) $\cos 20°\cos 40°\cos 60°\cos 80°$ (b) $\sin 20°\sin 40°\sin 60°\sin 80°$

(c) $\frac{3}{15}$ (d) None of these

26. If $\cos A = m\cos B$, then

(a) $\cot\frac{A+B}{2} = \frac{m+1}{m-1}\tan\frac{B-A}{2}$ (b) $\tan\frac{A+B}{2} = \frac{m+1}{m-1}\cot\frac{B-A}{2}$

(c) $\cot\frac{A+B}{2} = \frac{m+1}{m-1}\tan\frac{A-B}{2}$ (d) None of these

27. The expression $2\cos\frac{\pi}{13}.\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$ is equal to

(a) – 1 (b) 0

(c) 1 (d) None of these

28. $\sin 36°\sin 72°\sin 108°\sin 144° =$

(a) 1/4 (b) 1/16

(c) 3/4 (d) 5/16

29. The sum $S = \sin\theta + \sin 2\theta + ... + \sin n\theta,$ equals

(a) $\sin\frac{1}{2}(n+1)\theta\sin\frac{1}{2}n\theta/\sin\frac{\theta}{2}$ (b) $\cos\frac{1}{2}(n+1)\theta\sin\frac{1}{2}n\theta/\sin\frac{\theta}{2}$

(c) $\sin\frac{1}{2}(n+1)\theta\cos\frac{1}{2}n\theta/\sin\frac{\theta}{2}$ (d) $\cos\frac{1}{2}(n+1)\theta\cos\frac{1}{2}n\theta/\sin\frac{\theta}{2}$

30. $\frac{\cos 10^o + \sin 10^o}{\cos 10^o - \sin 10^o} =$

(a) $\tan 55^o$ (b) $\cot 55^o$

(c) $-\tan 35^o$ (d) $-\cot 35^o$

31. If $\tan\alpha$ equals the integral solution of the inequality $4x^2 - 16x + 15 < 0$ and $\cos\beta$ equals to the slope of the bisector of first quadrant, then $\sin(\alpha+\beta)\sin(\alpha-\beta)$ is equal to

(a) $\frac{3}{5}$ (b) $-\frac{3}{5}$

(c) $\frac{2}{\sqrt{5}}$ (d) $\frac{4}{5}$

32. $\frac{\cos 9° + \sin 9°}{\cos 9° - \sin 9°} =$

(a) $\tan 54°$ (b) $\tan 36°$

(c) $\tan 18°$ (d) None of these

33. $\cos^2 22\frac{1}{2}^0 - \cos^2 52\frac{1}{2}^0 =$

a) $\frac{\sqrt{3}+1}{4\sqrt{2}}$ b) $\frac{-(\sqrt{3}+1)}{4\sqrt{2}}$ c) $\frac{\sqrt{3}-1}{4\sqrt{2}}$ d) $\frac{\sqrt{5}+1}{8}$

34. $\frac{\cos 72^0}{\sin^2 24^0 - \sin^2 6^0} =$

a) 1 b) 2 c) 3 d) 4

35. **$\tan 30^0 + \tan 15^0 + \tan 30^0 \tan 15^0 =$**

a) 0 b) 1 c) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$ d) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

36. **If cos(A+B) = 4/5, sin(A-B) = 5/13 and A+B, A-B are acute, then tan2A =**

a) 33/56 b) 56/33 c) 16/63 d) 63/16

37. **In triangle ABC,** $\sum \frac{\cot A + \cot B}{\tan A + \tan B} =$

1) 1 2) 1/2 3) -1 4) 2 If cosx + cosy = 1/2,

38. **sinx + siny = 1/3, then the value of cos(x-y) =**

1) $\frac{59}{72}$ 2) $-\frac{59}{72}$ 3) $\frac{59}{2}$ 4) $-\frac{59}{2}$

39. **If tanA - tanB = x, cotB - cotA = y, then cot(A-B) =**

1) $\frac{x}{y}+\frac{y}{x}$ 2) $\frac{1}{x}+\frac{1}{y}$ 3) $\frac{1}{x}-\frac{1}{y}$ 4) x + y

40. **I : In ΔABC, if cot A + cot B + cot C = $\sqrt{3}$, then the triangle is equilateral.**

II : f(θ) = $\frac{\cot\theta}{1+\cot\theta}$ and α+β = $\frac{5\pi}{4}$, then f(α) f(β)= $\frac{1}{2}$

1) Only I is true 2) Only II is true

3) Both I & II are true 4) Neither I nor II are true

41. $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha =$

(a) $\tan\alpha$ (b) $\tan 2\alpha$

(c) $\cot\alpha$ (d) $\cot 2\alpha$

42. $\sqrt{3}\ \text{cosec}\ 20^o - \sec 20^o =$

(a) 2 (b) $\dfrac{2\sin 20^o}{\sin 40^o}$

(c) 4 (d) $\dfrac{4\sin 20^o}{\sin 40^o}$

43. If $a\cos 2\theta + b\sin 2\theta = c$ has α and β as its solution, then the value of $\tan\alpha + \tan\beta$ is

(a) $\dfrac{c+a}{2b}$ (b) $\dfrac{2b}{c+a}$

(c) $\dfrac{c-a}{2b}$ (d) $\dfrac{b}{c+a}$

44. If $a\sin^2 x + b\cos^2 x = c,\ \ b\sin^2 y + a\cos^2 y = d$ and $a\tan x = b\tan y,$ then $\dfrac{a^2}{b^2}$ is equal to

(a) $\dfrac{(b-c)\,(d-b)}{(a-d)\,(c-a)}$ (b) $\dfrac{(a-d)\,(c-a)}{(b-c)\,(d-b)}$

(c) $\dfrac{(d-a)\,(c-a)}{(b-c)\,(d-b)}$ (d) $\dfrac{(b-c)\,(b-d)}{(a-c)\,(a-d)}$

45. If $\tan(A+B) = p,\ \tan(A-B) = q,$ then the value of $\tan 2A$ in terms of p and q is

(a) $\dfrac{p+q}{p-q}$ (b) $\dfrac{p-q}{1+pq}$

(c) $\dfrac{p+q}{1-pq}$ (d) $\dfrac{1+pq}{1-p}$

46. $\sin^4\dfrac{\pi}{4} + \sin^4\dfrac{3\pi}{8} + \sin^4\dfrac{5\pi}{8} + \sin^4\dfrac{7\pi}{8} =$

(a) $\dfrac{1}{2}$ (b) $\dfrac{1}{4}$

(c) $\dfrac{3}{2}$ (d) $\dfrac{3}{4}$

47. If $A + C = B,$ then $\tan A\tan B\tan C =$

(a) $\tan A\tan B + \tan C$ (b) $\tan B - \tan C - \tan A$

(c) $\tan A + \tan C - \tan B$ (d) $-(\tan A\tan B + \tan C)$

COMPOUND ANGLES

HINTS AND SOLUTIONS

1. **(a)** $\sin A = \sin B$ **and** $\cos A = \cos B$

$$\frac{\sin A}{\sin B} = \frac{\cos A}{\cos B} \Rightarrow \sin A \cos B - \cos A \sin B = 0$$

$$\Rightarrow \sin(A-B) = 0$$

$$\sin\left(\frac{A-B}{2}\right) = 0.$$

2. **(b)** $\cos^2 A - \sin^2 B = \cos(A+B).\cos(A-B)$

$$\therefore \cos^2 48^o - \sin^2 12^o = \cos 60^o.\cos 36^o$$

$$= \frac{1}{2}\left(\frac{\sqrt{5}+1}{4}\right) = \frac{\sqrt{5}+1}{8}.$$

3. **(b)** $\tan\alpha = \frac{m}{m+1}$ **and** $\tan\beta = \frac{1}{2m+1}$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$$

$$= \frac{\frac{m}{m+1} + \frac{1}{2m+1}}{1 - \frac{m}{(m+1)}\frac{1}{(2m+1)}} = \frac{2m^2+m+m+1}{2m^2+m+2m+1-m}$$

$$= \frac{2m^2+2m+1}{2m^2+2m+1} = 1 \Rightarrow \tan(\alpha+\beta) = \tan\frac{\pi}{4}$$

Hence, $\alpha+\beta = \frac{\pi}{4}$.

4. **(b)** $\cos(\alpha+\beta) = \frac{4}{5}$ **,** $\sin(\alpha-\beta) = \frac{5}{13}$

$$\Rightarrow \sin(\alpha+\beta) = \frac{3}{5} \text{ and } \cos(\alpha-\beta) = \frac{12}{13}$$

$$\Rightarrow 2\alpha = \sin^{-1}\frac{3}{5} + \sin^{-1}\frac{5}{13}$$

$$= \sin^{-1}\left[\frac{3}{5}\sqrt{1-\frac{25}{169}} + \frac{5}{13}\sqrt{1-\frac{9}{25}}\right]$$

$$\Rightarrow 2\alpha = \sin^{-1}\left(\frac{56}{65}\right) \Rightarrow \sin 2\alpha = \frac{56}{65}$$

$$\tan 2\alpha = \frac{\sin 2\alpha}{\cos 2\alpha} = \frac{56/65}{33/65} = \frac{56}{33}.$$

5. (b) $\tan A = -\dfrac{1}{2}$ **and** $\tan B = -\dfrac{1}{3}$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{-\frac{1}{2} - \frac{1}{3}}{1 - \frac{1}{2}.\frac{1}{3}} = -1$$

$$\Rightarrow \tan(A+B) = \tan\frac{3\pi}{4}.$$

6. (d) $\sin 47^o + \sin 61^o - (\sin 11^o + \sin 25^o)$

$$= 2\sin 54^o \cos 7^o - 2\sin 18^o \cos 7^o$$

$$= 2\cos 7^o(\sin 54^o - \sin 18^o)$$

$$= 2\cos 7^o \,.\, 2\cos 36^o \,.\, \sin 18^o$$

$$= 4.\cos 7^o.\frac{\sqrt{5}+1}{4}.\frac{\sqrt{5}-1}{4} = \cos 7^o.$$

7. (b) $A + B = \dfrac{\pi}{4} \Rightarrow \tan(A+B) = \tan\dfrac{\pi}{4}$

$$\Rightarrow \frac{\tan A + \tan B}{1 - \tan A \tan B} = 1$$

$$\Rightarrow \tan A + \tan B + \tan A \tan B = 1$$

$$\Rightarrow (1 + \tan A)(1 + \tan B) = 2.$$

8. (d) $\dfrac{1}{\sin 10^\circ} - \dfrac{\sqrt{3}}{\cos 10^\circ}$

$$= \frac{2\left(\frac{1}{2}\cos 10^o - \frac{\sqrt{3}}{2}\sin 10^o\right)}{\frac{2}{2}\left(= \frac{\cos 10^o - \sqrt{3}\sin 10^o}{\sin 10^o \cos 10^o} \sin 10^o \cos 10^o\right)}$$

$$= \frac{4\sin(30^o - 10^o)}{\sin 20^o} = \frac{4\sin 20^o}{\sin 20^o} = 4.$$

9. (a) $\cos^2\left(\dfrac{\pi}{6} + \theta\right) - \sin^2\left(\dfrac{\pi}{6} - \theta\right)$

$$= \cos\left(\frac{\pi}{6} + \theta + \frac{\pi}{6} - \theta\right)\cos\left(\frac{\pi}{6} + \theta - \frac{\pi}{6} + \theta\right)$$

$$= \cos\frac{2\pi}{6}\cos 2\theta = \frac{1}{2}\cos 2\theta.$$

10. (a) $\dfrac{m}{n} = \dfrac{\tan(120^o + \theta)}{\tan(\theta - 30^o)}$

$$\Rightarrow \frac{m+n}{m-n} = \frac{\tan(\theta + 120^o) + \tan(\theta - 30^o)}{\tan(\theta + 120^o) - \tan(\theta - 30^o)}$$

(By componendo and dividendo)

11. **(b)** $\tan 20^o + 2\tan 50^o - \tan 70^o$

$$= \frac{\sin 20^o}{\cos 20^o} - \frac{\sin 70^o}{\cos 70^o} + 2\tan 50^o$$

Simplify

12. (b) same as above

13. (c) $b\sin\alpha = a\sin(\alpha+2\beta) \Rightarrow \frac{a}{b} = \frac{\sin\alpha}{\sin(\alpha+2\beta)}$

$$\Rightarrow \frac{a+b}{a-b} = \frac{\sin\alpha + \sin(\alpha+2\beta)}{\sin\alpha - \sin(\alpha+2\beta)} = \frac{2\sin(\alpha+\beta)\cos\beta}{-2\cos(\alpha+\beta)\sin\beta}$$

14. (a) $\frac{\sin A - \sin C}{\cos C - \cos A} = \cot B \Rightarrow \frac{2\cos\frac{A+C}{2}\sin\frac{A-C}{2}}{2\sin\frac{A+C}{2}\sin\frac{A-C}{2}} = \cot B$

$$\Rightarrow \cot\frac{(A+C)}{2} = \cot B \Rightarrow B = \frac{A+C}{2}$$

Thus A, B, C are in A.P.

15. (b) $\cos A + \cos(240^o + A) + \cos(240^o - A)$

$$= \cos A + 2\cos 240^o \cos A$$

$$= \cos A\{1 + 2\cos(180^o + 60^o)\} = \cos A\left\{1 + 2\left(-\frac{1}{2}\right)\right\}$$

$$= 0.$$

16. (b) L.H.S. $= 2\sin\gamma\cos(\beta-\alpha) + 2\sin(-\gamma)\cos(\alpha+\beta)$

$$= 2\sin\gamma[\cos(\beta-\alpha) - \cos(\alpha+\beta)]$$

$$= 2\sin\gamma . 2\sin\alpha\sin\beta = 4\sin\alpha\sin\beta\sin\gamma.$$

17. (b) $\sin\frac{\pi}{16}.\sin\frac{3\pi}{16}.\sin\frac{5\pi}{16}.\sin\frac{7\pi}{16}$

$$= \frac{1}{4}\left[2\sin\frac{\pi}{16}\sin\frac{3\pi}{16}.2\sin\frac{5\pi}{16}\sin\frac{7\pi}{16}\right]$$

$$= \frac{1}{4}\left[\left(\cos\frac{\pi}{8} - \cos\frac{\pi}{4}\right)\left(\cos\frac{\pi}{8} - \cos\frac{3\pi}{4}\right)\right]$$

$$= \frac{1}{4}\left[\left(\cos\frac{\pi}{8} - \frac{1}{\sqrt{2}}\right)\left(\cos\frac{\pi}{8} + \frac{1}{\sqrt{2}}\right)\right]$$

$$= \frac{1}{4}\left[\left(\cos^2\frac{\pi}{8} - \frac{1}{2}\right)\right] = \frac{1}{8}\left[2\cos^2\frac{\pi}{8} - 1\right]$$

$$= \frac{1}{8}\left[\cos\frac{\pi}{4}\right] = \frac{1}{8}\times\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{16}.$$

18. (a) $\cos^2\alpha + \cos^2(\alpha+120^o) + \cos^2(\alpha-120^o)$

$$= \cos^2\alpha + \{\cos(\alpha+120^o)+\cos(\alpha-120^o)\}^2 - 2\cos(\alpha+120^o)\cos(\alpha-120^o)$$

$$= \cos^2\alpha + \{2\cos\alpha\cos 120^o\}^2 - 2\{\cos^2\alpha - \sin^2 120^o\}$$

$$= \cos^2\alpha + \cos^2\alpha - 2\cos^2\alpha + 2\sin^2 120^o$$

$$= 2\sin^2 120^o = 2\times\frac{3}{4} = \frac{3}{2}.$$

19. (d) $\cos^2 76^o + \cos^2 16^o - \cos 76^o \cos 16^o$

$$= \frac{1}{2}[1+\cos 152^o + 1 + \cos 32^o - \cos 92^o - \cos 60^o]$$

$$= \frac{1}{2}\left[2 - \frac{1}{2} + \cos 152^o + \cos 32^o - \cos 92^o\right]$$

$$= \frac{1}{2}\left[\frac{3}{2} + 2\cos 92^o \cos 60^o - \cos 92^o\right]$$

$$= \frac{1}{2}\left[\frac{3}{2} + \cos 92^o - \cos 92^o\right] = \frac{3}{4}.$$

20. (b) $\dfrac{\sin(B+A)+\cos(B-A)}{\sin(B-A)+\cos(B+A)}$

$$= \frac{\sin(B+A)+\sin(90^o - \overline{B-A})}{\sin(B-A)+\sin(90^o - \overline{A+B})}$$

$$= \frac{2\sin(A+45^o)\cos(45^o - B)}{2\sin(45^o - A)\cos(45^o - B)}$$

$$= \frac{\sin(A+45^o)}{\sin(45^o - A)} = \frac{\cos A + \sin A}{\cos A - \sin A}.$$

21. (d) $\cos\dfrac{2\pi}{15}\cos\dfrac{4\pi}{15}\cos\dfrac{8\pi}{15}\cos\dfrac{16\pi}{15}$

$$= \frac{\sin 2^4 \frac{2\pi}{15}}{2^4 \sin\frac{2\pi}{15}} = \frac{\sin\frac{32\pi}{15}}{16\sin\frac{2\pi}{15}} = \frac{1}{16}\frac{\sin\frac{2\pi}{15}}{\sin\frac{2\pi}{15}} = \frac{1}{16}.$$

22. (a) $\cos\alpha\sin(\beta-\gamma)+\cos\alpha\sin(\gamma-\alpha)+\cos\gamma\sin(\alpha-\beta)$

Put $\alpha=\beta=\gamma=60^o \Rightarrow \frac{1}{2}(0)+\frac{1}{2}(0)+\frac{1}{2}(0)=0$.

23. (a) Squaring and adding, we get

$$x^2 + y^2 = 1 + 1 + 2\cos(2A - A)$$

$\therefore \dfrac{x^2+y^2-2}{2} = \cos A$ .....(i)

Also $\cos A + 2\cos^2 A - 1 = y$

Or $(\cos A + 1)(2\cos A - 1) = y$

Put for $\cos A$ from (i) and get the answer.

24. (b) $\sin\theta = \dfrac{12}{13}$ $\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(\dfrac{12}{13}\right)^2} = \dfrac{5}{13}$

and $\cos\phi = \dfrac{-3}{5}, \sin\phi = \sqrt{1-\dfrac{9}{25}} = \dfrac{-4}{5}$, $\left[\because \pi < \phi < \dfrac{3\pi}{2}\right]$

$\sin(\theta+\phi) = \sin\theta.\cos\phi + \cos\theta.\sin\phi$

$= \left(\dfrac{12}{13}\right)\left(\dfrac{-3}{5}\right) + \left(\dfrac{5}{13}\right)\left(\dfrac{-4}{5}\right) = \dfrac{-36}{65} - \dfrac{20}{65} = \dfrac{-56}{65}$.

25. (a) $\sin 12^o \sin 24^o \sin 48^o \sin 84^o$

$$= \frac{1}{4}(2\sin 12^o \sin 48^o)(2\sin 24^o \sin 84^o)$$

$$= \frac{1}{2}(\cos 36^o - \cos 60^o)(\cos 60^o - \cos 108^o)$$

$$= \frac{1}{4}\left(\cos 36^o - \frac{1}{2}\right)\left(\frac{1}{2} + \sin 18^o\right)$$

$$= \frac{1}{4}\left\{\frac{1}{4}(\sqrt{5}+1) - \frac{1}{2}\right\}\left\{\frac{1}{2} + \frac{1}{4}(\sqrt{5}-1)\right\} = \frac{1}{16}$$

And $\cos 20^o \cos 40^o \cos 60 \cos 80^o$

$$= \frac{1}{2}[\cos(60^o - 20^o)\cos 20^o \cos(60^o + 20^o)]$$

$= \dfrac{1}{2}\left[\dfrac{1}{4}\cos 3(20^o)\right] = \dfrac{1}{8}\cos 60^o = \dfrac{1}{2}\times\dfrac{1}{8} = \dfrac{1}{16}$.

26. (a) $\cos A = m \cos B \Rightarrow \dfrac{m}{1} = \dfrac{\cos A}{\cos B}$

$$\Rightarrow \frac{m+1}{m-1} = \frac{\cos A + \cos B}{\cos A - \cos B} = \frac{2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{B-A}{2}\right)}{2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right)}$$

$$= \cot\left(\frac{A+B}{2}\right)\cot\left(\frac{B-A}{2}\right)$$

Hence, $\cot\left(\dfrac{A+B}{2}\right) = \dfrac{m+1}{m-1}\tan\dfrac{B-A}{2}$.

27. (b) $2\cos\dfrac{\pi}{13}.\cos\dfrac{9\pi}{13} + \cos\dfrac{3\pi}{13} + \cos\dfrac{5\pi}{13}$

$$= 2\cos\frac{\pi}{13}.\cos\frac{9\pi}{13} + 2\cos\frac{4\pi}{13}\cos\frac{\pi}{13}$$

$$= 2\cos\frac{\pi}{13}\left[\cos\frac{9\pi}{13} + \cos\frac{4\pi}{13}\right]$$

$$= 2\cos\frac{\pi}{13}\left[2\cos\frac{\pi}{2}.\cos\frac{5\pi}{26}\right] = 0,$$

28. (d) $\sin 36^o \ \sin 72^o \sin 108^o \ \sin 144^o$

$$= \sin^2 36^o \ \sin^2 72^o = \frac{1}{4}\{(2\sin^2 36^o)(2\sin^2 72^o)\}$$ 0n

$$= \frac{1}{4}\{(1-\cos 72^o)(1-\cos 144^o)\}$$

$$= \frac{1}{4}\{(1-\sin 18^o)(1+\cos 36^o)\}$$

$$= \frac{1}{4}\left[\left(1-\frac{\sqrt{5}-1}{4}\right)\left(1+\frac{\sqrt{5}+1}{4}\right)\right] = \frac{20}{16}\times\frac{1}{4} = \frac{5}{16}.$$

29. (a) $S = \sin\theta + \sin 2\theta + \sin 3\theta + ... + \sin n\theta$

$\sin\theta + \sin(\theta+\beta) + \sin(\theta+2\beta) + ...n$ term

$$= \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}\sin\left[\frac{\theta+\theta+(n-1)\beta}{2}\right]$$

Put $\beta = \theta$, then $S = \dfrac{\sin\frac{n\theta}{2}.\sin\frac{\theta(n+1)}{2}}{\sin\frac{\theta}{2}}$.

30. (a) $\dfrac{\cos 10^o + \sin 10^o}{\cos 10^o - \sin 10^o} = \tan(45^o + 10^o) = \tan 55^o$.

31. (d) We have $4x^2-16x+15<0 \Rightarrow \frac{3}{2}<x<\frac{5}{2}$

$\therefore$ Integral solution of $4x^2-16x+15<0$ is $x = 2$.

Thus $\tan\alpha = 2$. It is given that $\cos\beta = \tan 45^o = 1$

$\therefore \sin(\alpha+\beta)\sin(\alpha-\beta) = \sin^2\alpha - \sin^2\beta$

$$= \frac{1}{1+\cot^2\alpha} - (1-\cos^2\beta) = \frac{1}{1+\frac{1}{4}} - 0 = \frac{4}{5}.$$

32. (a) $\frac{1+\tan 9^o}{1-\tan 9^o} = \tan\left(45^o+9^o\right) = \tan 54^o$.

33. (a)

34. (b)

35. (b)

36. (b)

37. (a)

38. (b)

39. (b)

40. (c)

41. (c) $\tan\alpha + 2\tan 2\alpha + 4\tan 4\alpha + 8\cot 8\alpha$

$$= \tan\alpha + 2\tan 2\alpha + 4\left[\frac{\sin 4\alpha}{\cos 4\alpha} + 2\frac{\cos 8\alpha}{\sin 8\alpha}\right]$$

42. (c) $\sqrt{3}\operatorname{cosec} 20° - \sec 20° = \frac{\sqrt{3}}{\sin 20°} - \frac{1}{\cos 20°}$

$$= \frac{\sqrt{3}\cos 20° - \sin 20°}{\sin 20° \cos 20°} = \frac{2\left[\frac{\sqrt{3}}{2}\cos 20° - \frac{1}{2}\sin 20°\right]}{\frac{2}{2}\sin 20° \cos 20°}$$

$$= \frac{4\cos(20°+30°)}{\sin 40°} = \frac{4\cos 50°}{\sin 40°} = \frac{4\sin 40°}{\sin 40°} = 4.$$

43. (b) $a\cos 2\theta + b\sin 2\theta = c$

$$\Rightarrow a\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) + b\frac{2\tan\theta}{1+\tan^2\theta} = c$$

$$\Rightarrow a - a\tan^2\theta + 2b\tan\theta = c + c\tan^2\theta$$

$$\Rightarrow -(a+c)\tan^2\theta + 2b\tan\theta + (a-c) = 0$$

$$\therefore \tan\alpha + \tan\beta = -\frac{2b}{-(c+a)} = \frac{2b}{c+a}.$$

44. (b) $a\sin^2 x + b\cos^2 x = c \Rightarrow (b-a)\cos^2 x = c-a$

$\Rightarrow (b-a) = (c-a)(1+\tan^2 x)$

$b\sin^2 y + a\cos^2 y = d \Rightarrow (a-b)\cos^2 y = d-b$

$\Rightarrow (a-b) = (d-b)(1+\tan^2 y)$

$\therefore \tan^2 x = \frac{b-c}{c-a}, \tan^2 y = \frac{a-d}{d-b}$

$\therefore \frac{\tan^2 x}{\tan^2 y} = \frac{(b-c)(d-b)}{(c-a)(a-d)}$ .....(i)

But $a\tan x = b\tan y$, *i.e.*, $\frac{\tan x}{\tan y} = \frac{b}{a}$ .....(ii)

From (i) and (ii), $\frac{b^2}{a^2} = \frac{(b-c)(d-b)}{(c-a)(a-d)}$

$\Rightarrow \frac{a^2}{b^2} = \frac{(c-a)(a-d)}{(b-c)(d-b)}$.

45. (c) $2A = (A+B)+(A-B)$

$\Rightarrow \tan 2A = \frac{\tan(A+B)+\tan(A-B)}{1-\tan(A+B)\tan(A-B)} = \frac{p+q}{1-pq}$.

46. (c) standard problem

47. (b) $B = A+C \Rightarrow \tan B = \tan(A+C)$

$\Rightarrow \tan B = \frac{\tan A + \tan C}{1-\tan A\tan C}$

$\Rightarrow \tan A\tan B\tan C = \tan B - \tan A - \tan C$.