Objective: The objective of this lab sheet is to provide hands-on practice of the single perceptron algorithm for a two-class classification problem and to correlate these results with Bayesian classification.

Problem :

1.

Generate a dataset of 2000 samples of 2D feature vectors that stem from two equiprobable classes modeled by normal distributions with mean vectors $\mu_1 = [-2, 2]^T$ and $\mu_2 = [2, -2]^T$ and covariance matrices $S_1 = [1\ 0; 0\ 1]$ and $S_2 = [1\ 0;\ 0\ 1]$.
Write a code to perform two-class classification of the above simulated data using gradient descent-based single perceptron algorithm.

a) Initialize the weight vector to $w = [1, 1, -0.5]^T$ and empirically fix the learning rate. Plot the samples and draw the decision boundary obtained as a result of the above algorithm. Write your code such that the intermediate decision plane at the end of each iteration is also dynamically displayed while running your code.

b) Rerun your code developed in part-a with randomly initialized weight vector. Draw your inferences from these experiments.

c) Perform a Bayesian classification of the same data based on the value of the covariance matrices given above, and compute analytically the equation of the decision line. Compare this result by plotting this resulting line and the line obtained in part-a over the entire data samples.

d) Repeat part-a and part-c by selecting μ_1, μ_2 far apart and find the decision boundary by randomly fixing other parameters. Draw your inferences by comparing them with the results in part-c.