भारतीय प्रौद्योगिकी संस्थान तिरुपति
IIT TIRUPATI

ADVANCED SIGNAL ANALYSIS AND PROCESSING LAB

Lab sheet. No: 07

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Q1. **Let H(z) = 6 + z^−1 – z^−2**

- **(a) Specify all possible causal systems that have the same magnitude response as H(z). Although these systems may have different phase response.**
- **(b) Plot the magnitude and phase response of all these equivalent systems using MATLAB. Use freqz command to get frequency response.**
- **(c) Among all these systems which one is having minimum phase value at all frequencies.**

AIM: To find the all-possible causal system transfer functions that have the same magnitude response of given H(Z). And plot the magnitude and phase response of those transfer functions using freqz command.

Short Theory:

The system is described by a rational transfer function with real coefficients

$$H(z) = \frac{b_0 + b_1 z^{-1} + \cdots + b_M z^{-M}}{a_0 + a_1 z^{-1} + \cdots + a_N z^{-N}}.$$

- For a given transfer function H(z) , the we can get the poles and zeros location . The magnitude response For the conjugate reciprocal locations of a poles and zeros also same as the magnitude response of H(z).
- a system is considered to be minimum phase if all of its zeros as well as all poles are inside the unit circle.

Key Commands:

Filt % filt(num,den) creates a discrete-time transfer function sys with numerator(s) num and denominator(s) den

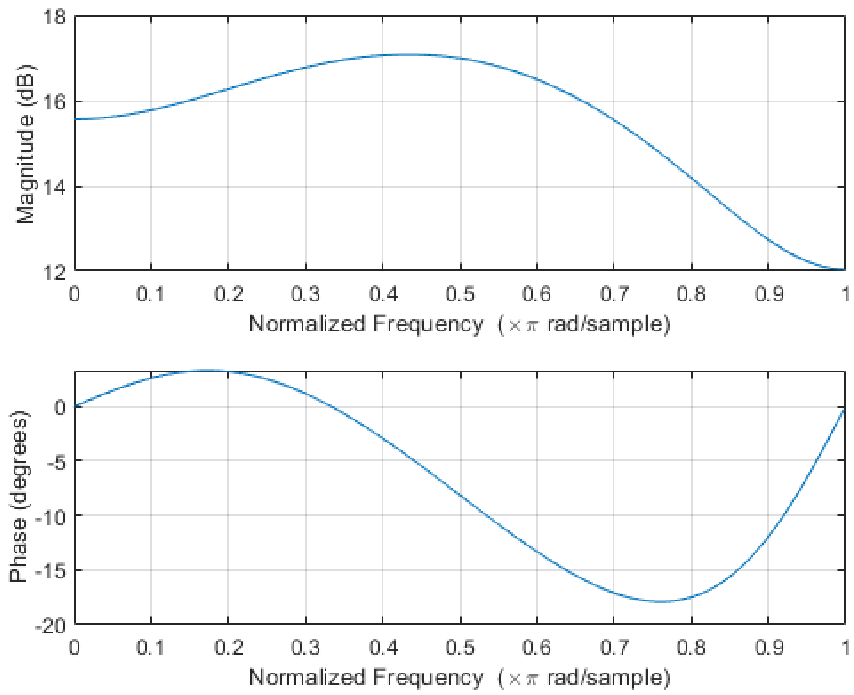
Freqz % freqz(b,a,n) returns the n-point frequency response vector h and the corresponding angular frequency vector w for the digital filter with transfer function coefficients stored in b and a.

tf2zpk % tf2zpk(b,a) finds the matrix of zeros z, the vector of poles p, and the associated vector of gains k from the transfer function parameters b and a

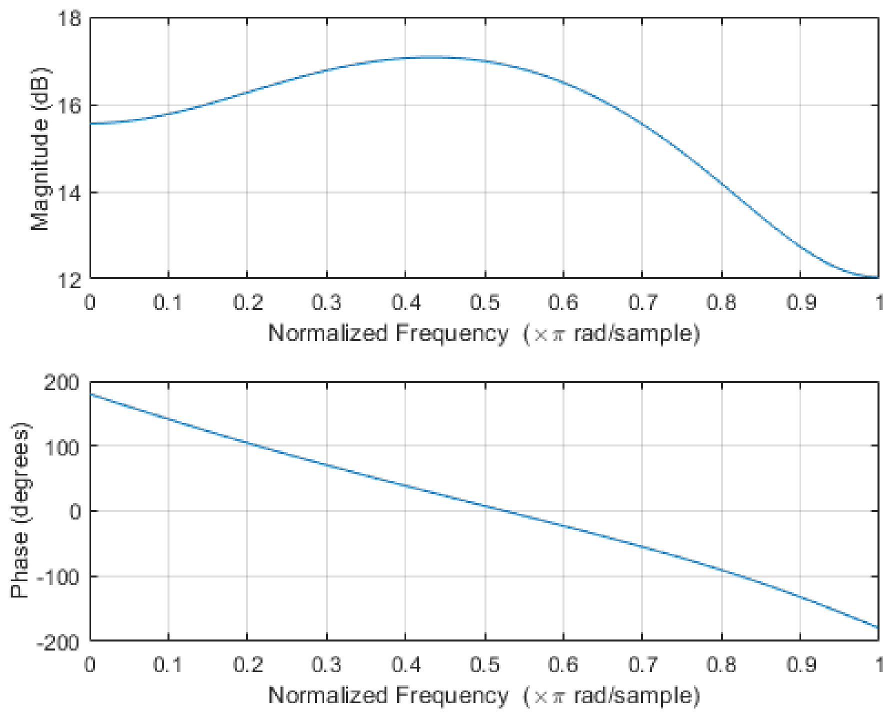
zp2tf % zp2tf(z,p,k) converts a factored transfer function representation

Plots:

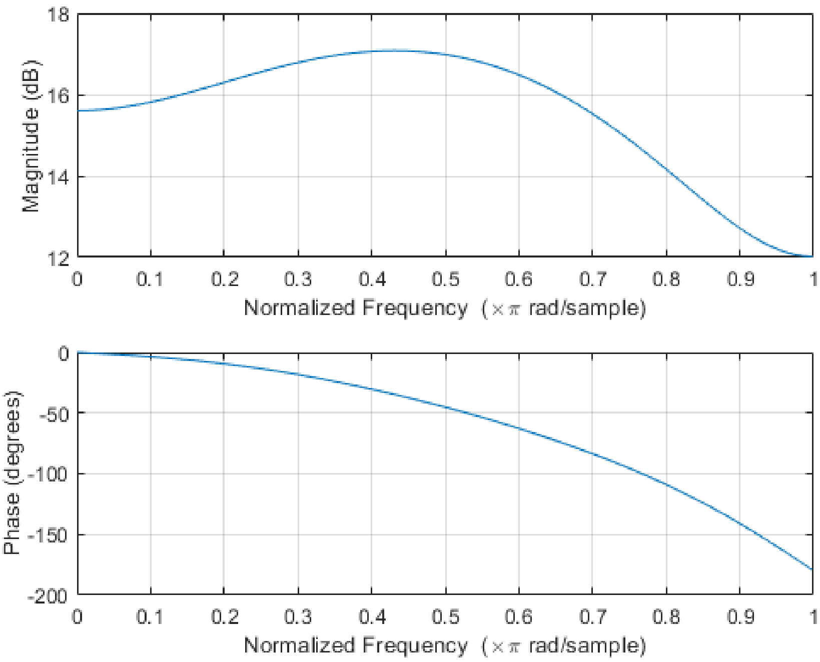
magnitude response of H1(z)=**(6 + z^-1 - z^-2)**



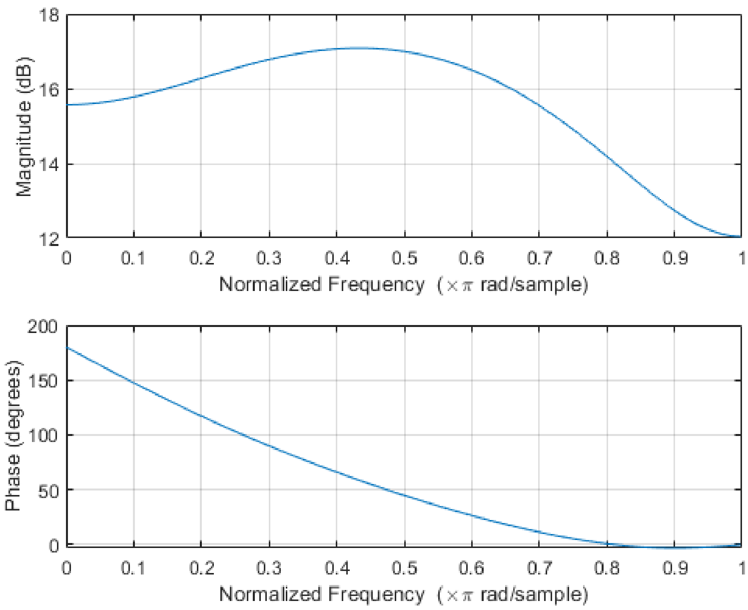
magnitude response of H2(z)= **(1 - z^-1 - 6 z^-2)**



magnitude response of H3(z)=(`3 + 5.01 z^-1 - 1.98 z^-2`)



magnitude response of H4(z)= (`2 - 5 z^-1 - 3 z^-2`)



Inferences/comments:

1) The conjugate reciprocal locations of a poles and zeros of a H(z) also same as the magnitude response of H(z) even though the phase response is different.
2) A system function H(z) is said to be a minimum phase system if all of its poles and zeros are within the unit circle. I,e. H1(z)=(6 + z^-1 - z^-2) has zeros at z= -0.5 and 1/3 .

Q2. A causal stable all-pass system $H_1(z)$ with real valued impulse response has only one pole at $z = -0.5$

(a) write H1(z) and draw its pole-zero plot.

(b) Plot the magnitude and phase response of H1(ejw) equivalent systems using MATLAB. Use freqz command.

AIM: To design the all pass system that is having one real pole at z=-0.5 and plot the magnitude and phase response using freqz command.

Short Theory:

The system which allow all the frequencies to pass through them is all pass system. The poles and zeros of a discrete time all pass system are present on the circles with radii rk and 1/rk. If poles and zeros are complex, for every pole there will be conjugate reciprocal zero. All pass system is generally used to adjust the phase response of a n LTI system.

Key Commands:

Filt % filt(num,den) creates a discrete-time transfer function sys with numerator(s) num and denominator(s) den

Freqz % freqz(b,a,n) returns the n-point frequency response vector h and the corresponding angular frequency vector w for the digital filter with transfer function coefficients stored in b and a.

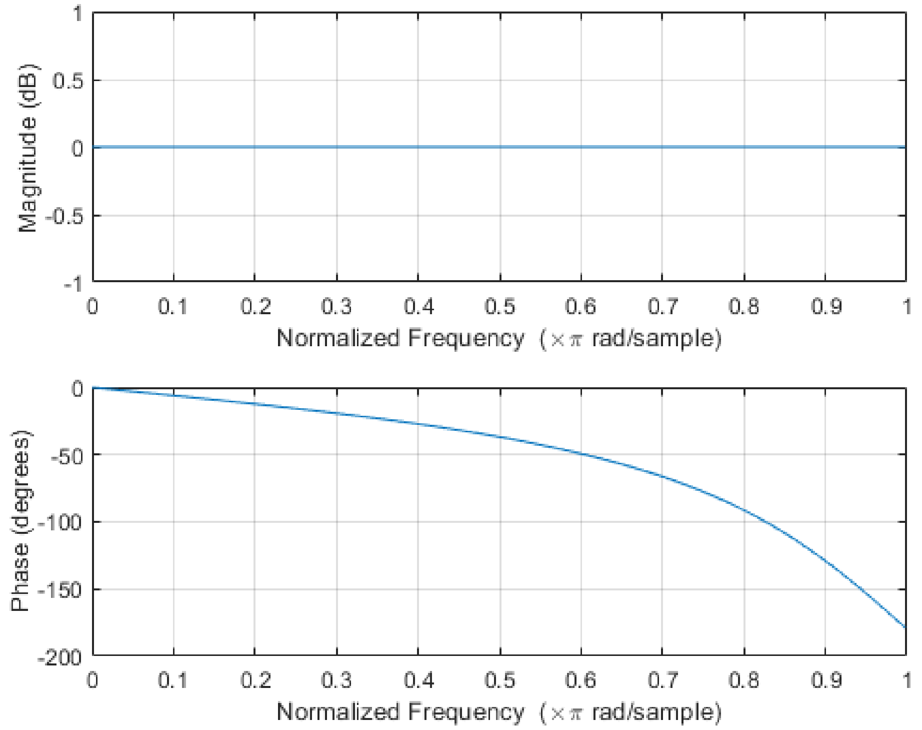
tf2zpk % tf2zpk(b,a) finds the matrix of zeros z, the vector of poles p, and the associated vector of gains k from the transfer function parameters b and a

zp2tf % zp2tf(z,p,k) converts a factored transfer function representation

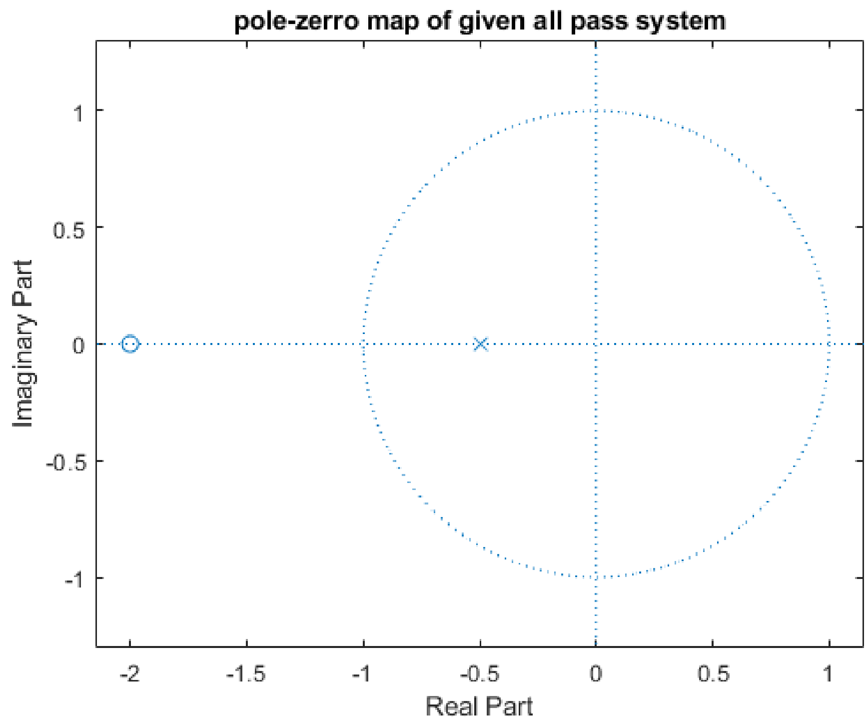
Plots:

magnitude response of

```
        0.5 + z^-1
H(z)= ------------
      1 + 0.5 z^-1
```

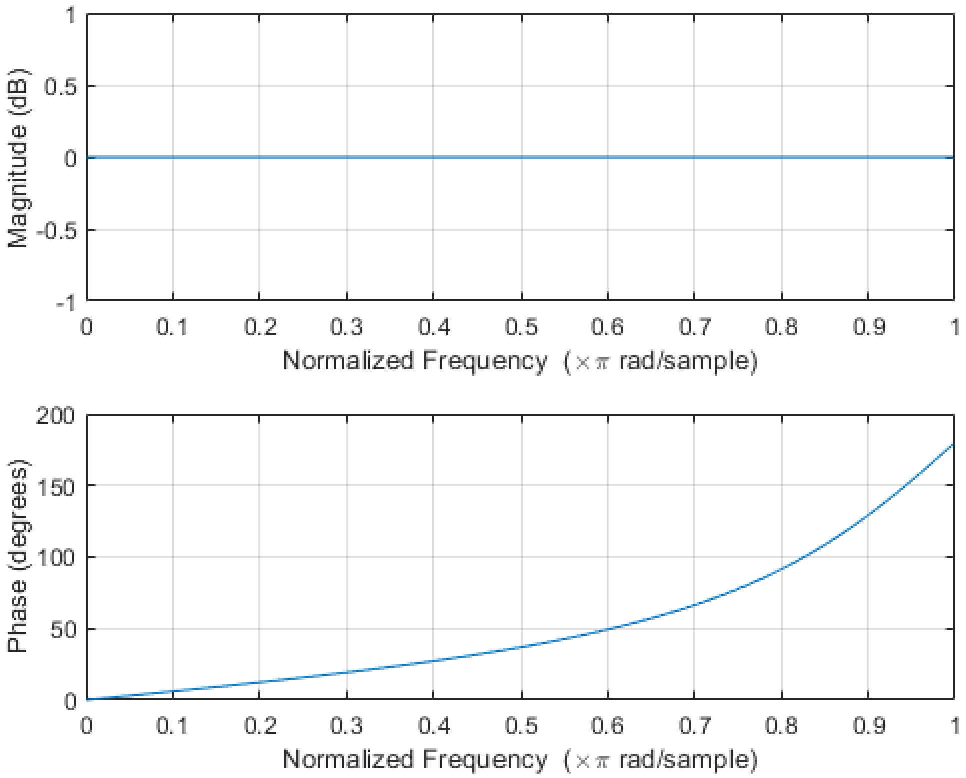


Magnitude (dB)
Normalized Frequency (×π rad/sample)
Phase (degrees)
Normalized Frequency (×π rad/sample)



pole-zerro map of given all pass system
Imaginary Part
Real Part

```
                                   2 + z^-1
frequency  response of   H2(z)=   ----------
                                   1 + 2 z^-1
```



Inferences/comments:

1) If the causal all pass system has a one pole at z=-0.5 then another zero at conjugate reciprocal location I,e, z=-2.
2) The magnitude response of all pass system is constant for all frequencies.
3) And the equivalent system for H(z) has the same magnitude response as H(z).

Q3. Repeat Q2 for a system H2(z) which has only one pole at $z = -2$

AIM: To design the all pass system that is having one real pole at z=-2 and plot the magnitude and phase response using freqz command.

Short Theory:

Consider an LSI system with transfer function

$$H(z) = (z^{-1} - a^*)/ (1 - az^{-1})$$

Note that the system above has a pole at $z = a$ and a zero at $z = (1/a)^*$. This means that if the pole is located at $z = re^{j\theta}$, the zero would be located at $z = (1/re^{-j\theta})$. If h[n] is real, the poles and zeros would each occur in complex-conjugate pairs. We note that even though the magnitude is constant, the phase does depend on frequency.

This property implies that you can invert the magnitude of a pole or zero location in an LSI system without changing the magnitude of the transfer function of that system.

Key Commands:

zp2tf % zp2tf(z,p,k) converts a factored transfer function representation

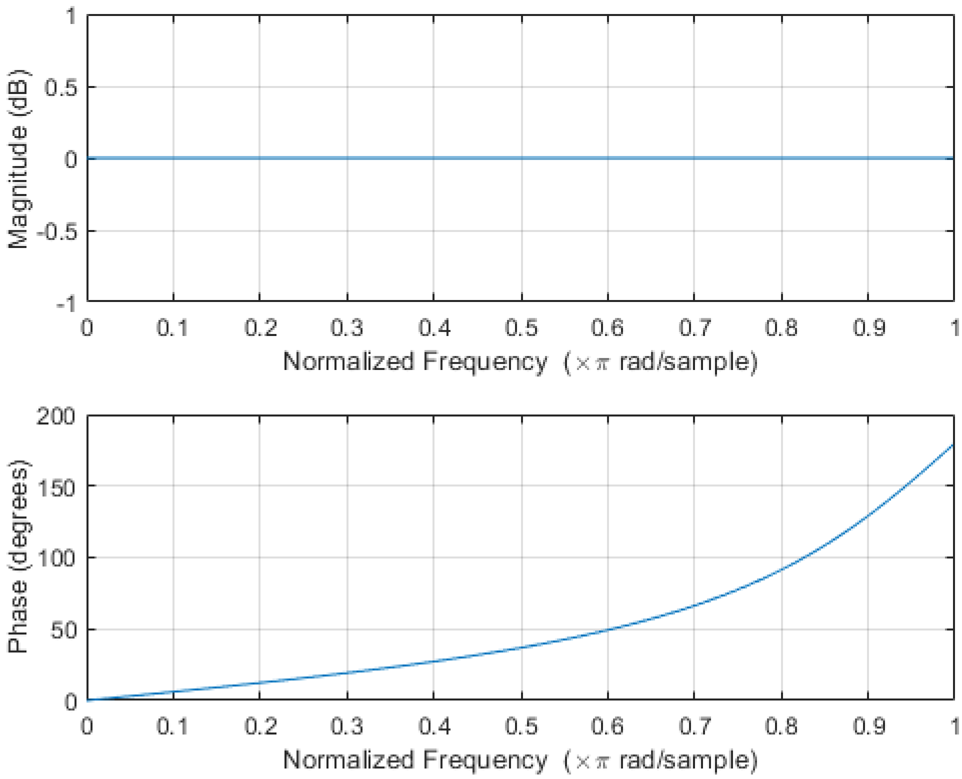
Freqz % freqz(b,a,n) returns the n-point frequency response vector h and the corresponding angular frequency vector w for the digital filter with transfer function coefficients stored in b and a.

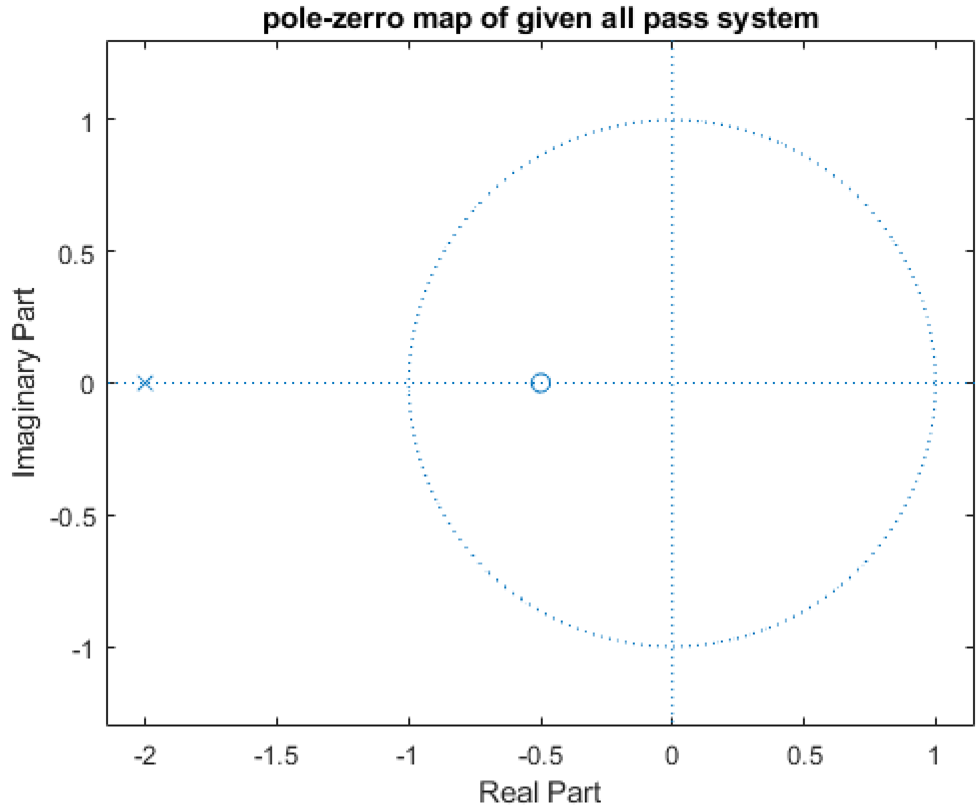
Filt % filt(num,den) creates a discrete-time transfer function sys with numerator(s) num and denominator(s) den

zplane % zplane(z,p) plots the zeros specified in column vector z and the poles specified in column vector p in the current figure window.

Plots:

```
                                      2 + z^-1
frequency  response of   H2(z)=      ----------
                                      1 + 2 z^-1
```





Inferences/comments:

1) If the causal all pass system has a one pole at z=-2 then another zero at conjugate reciprocal location I,e, z=-0.5.
2) The magnitude response of all pass system is constant for all frequencies.
3) And the equivalent system for H(z) has the same magnitude response as H(z).

Q4. Repeat Q2 for a system H3(z) which a cascade of H1(z) and H2(z) system defined earlier. comment on your observations.

AIM: To design the H3(z) transfer function from the above generated transfer functions H1(z) and H2(z) i.e, H3(z)=H1(z) * H2(z).

Short Theory:

Consider an LSI system with transfer function

$$H(z) = (z^{-1} - a^*) / (1 - az^{-1})$$

Note that the system above has a pole at z = a and a zero at $z = (1/a)^*$. This means that if the pole is located at $z = re^{j\theta}$, the zero would be located at $z = (1/re^{-j\theta})$. If h[n] is real, the poles and zeros would each occur in complex-conjugate pairs. We note that even though the magnitude is constant, the phase does depend on frequency.

This property implies that you can invert the magnitude of a pole or zero location in an LSI system without changing the magnitude of the transfer function of that system.

Key Commands:

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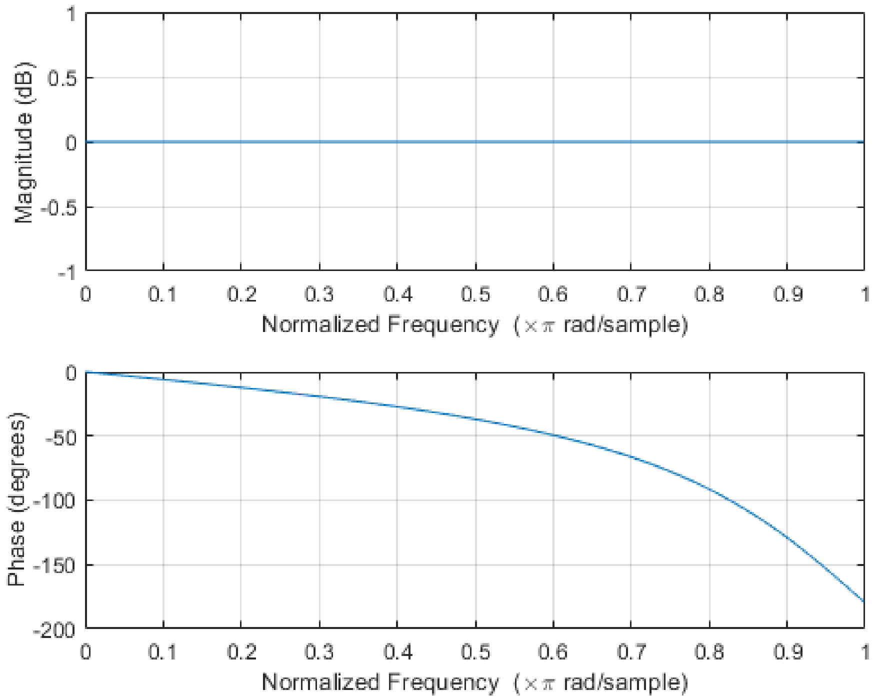
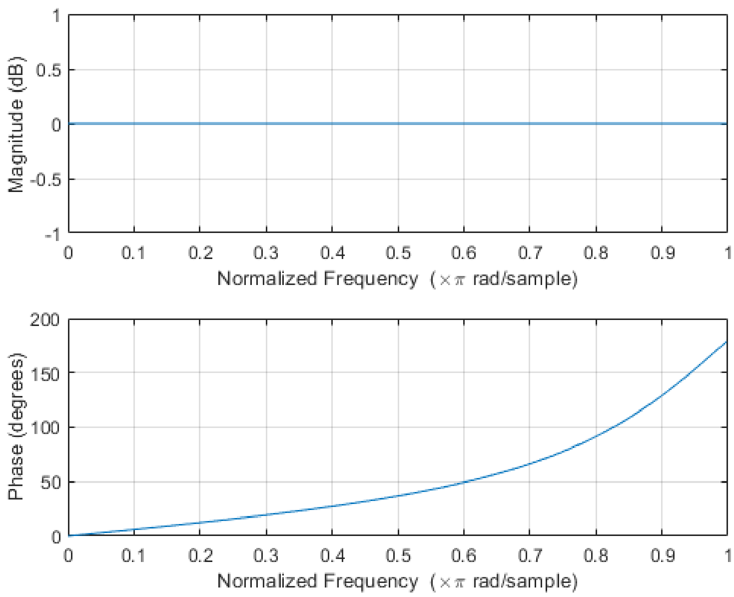
Plots:

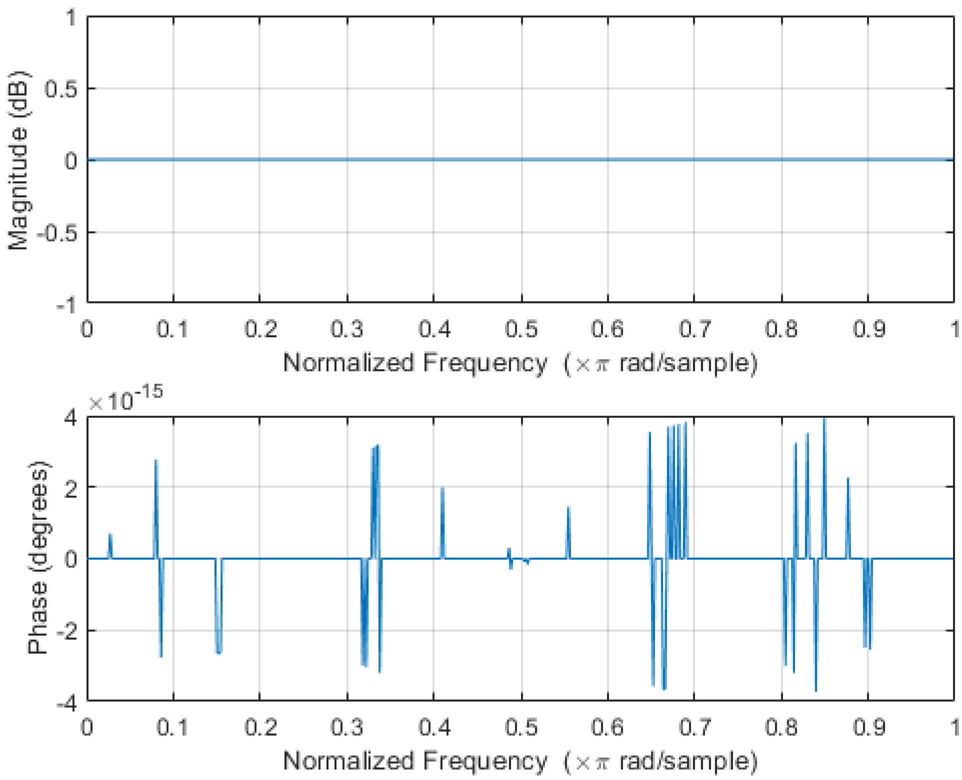
```
                                  0.5 + z^-1
frequency response of  H1(z)  =  ------------
                                  1 + 0.5 z^-1
```




```
                                 2 + z^-1
magnitude response of h2(z) =  ----------
                                 1 + 2 z^-1
```



```
                         1 + 2.5 z^-1 + z^-2
magnitude response of  H(z) = -------------------
                         1 + 2.5 z^-1 + z^-2
```



Inferences/comments:

1) **If you cascade the 2 all pass systems, then the result is also a all pass system.**
2) **If you cascade the 2 all pass systems, the resulting all pass system order is increases.**

Q5. Let H(z) = (1−2z^−1)/(1+ 1/3z−1). Express as a concatenation of Minimum phase & All-pass system.

AIM: To express the given H(z) in concatenation of minimum phase and All-pass system.

Short Theory:

Minimum Phase Systems

A system function H(z) is said to be a minimum phase system if all of its poles and zeros are within the unit circle.

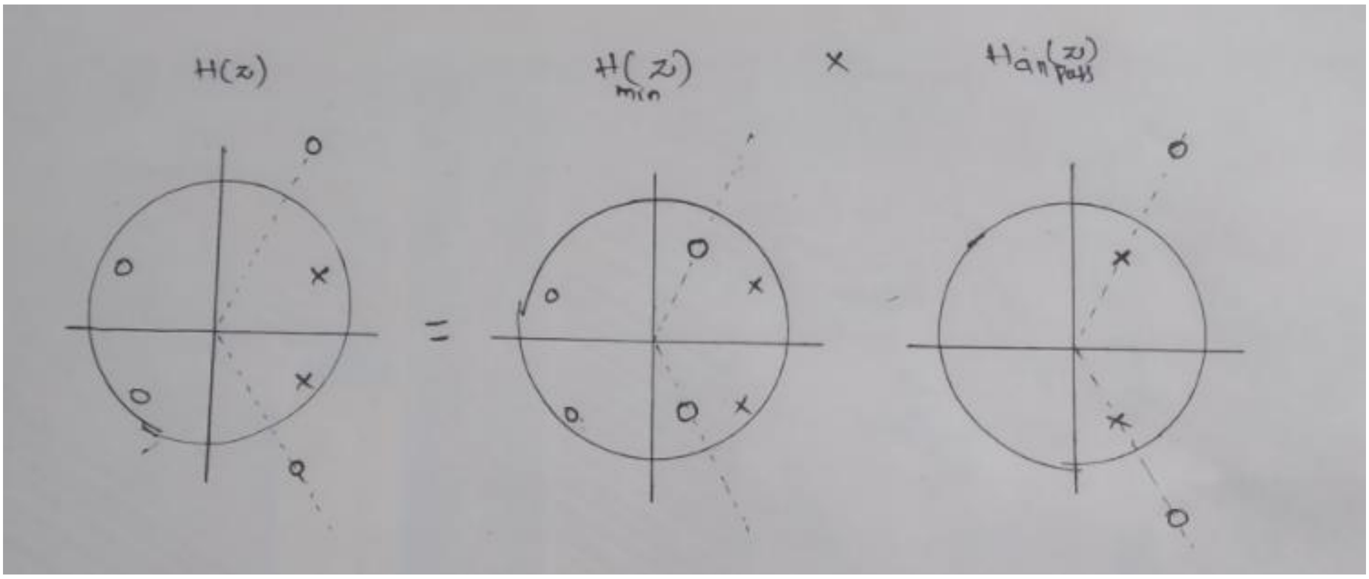
Allpass systems

An All-pass system is a system whose frequency response magnitude is constant for all frequencies, i.e.,

$$|H(ej\omega)| = \text{constant}\ ,\ \omega \in [-\pi, \pi].$$

Consider a stable and causal LTI system with system function H(z). Since the system is a stable system the poles of the system function are required to be inside the unit circle . The zeroes are however, free to wander outside.In case in our H(z) any pole is out side the unit circle also we can represent the H(z) into $H_{min}(z) * H_{ap}(z)$ As shown bellow.

For example,



This implies that system function H(z) can then be factorized into two parts as:

$$H(z) = H_{min}(z) * H_{ap}(z)$$

Key Commands:

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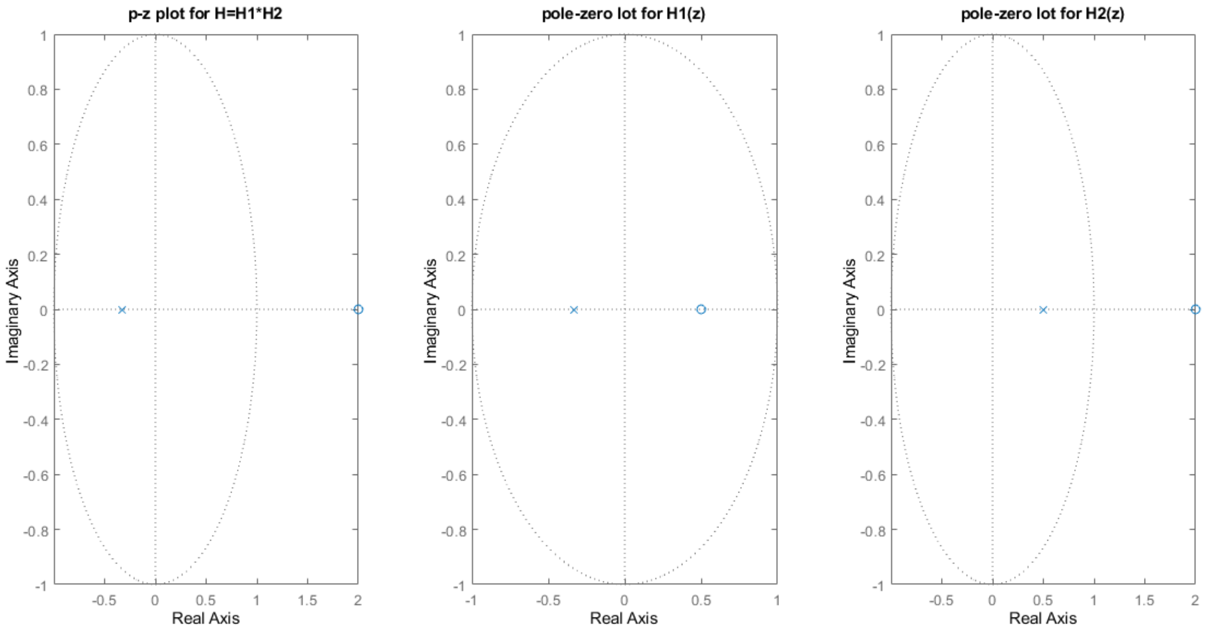
Filt % filt(num,den) creates a discrete-time transfer function sys with numerator(s) num and denominator(s) den

minreal % minreal(sys) eliminates uncontrollable or unobservable state in state-space models, or cancels pole-zero pairs in transfer functions or zero-pole-gain models

Plots:



Inferences/comments:

1) Non minimum phase system are hard to realizable .If the given H(z) is not a minimum phase system , to make it realizable , we convert given H(z) into concatenation of minimum phase system and all pass system.
2) Given H(z) is non minimum phase because it has a zero at z=2. So we can write H(z) as

$$H(z) = \underbrace{\frac{z^{-1} - 2}{1 + \frac{1}{3}z^{-1}}}_{H_{\min}(z)} \cdot \underbrace{\frac{1 - 2z^{-1}}{z^{-1} - 2}}_{H_{\mathrm{ap}}(z)}$$