

**Note:** This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

**PART- A****(25 Marks)**

- 1.a) Define exact differential equation. Give an example. [2]
- b) Find a particular integral of  $y'' - 2y' + y = \frac{e^x}{x}$ . [3]
- c) Show that the matrix  $A = \begin{pmatrix} 3i & 2+i \\ -2+i & -i \end{pmatrix}$  is Skew-Hermitian. [2]
- d) Find the values of  $a$  and  $b$  such that the system  $2x+3y+5z=9$ ,  $7x+3y-2z=8$ ,  $2x+3y+az=b$  has no solution. [3]
- e) Find the sum and product of the Eigen values of the matrix  $A = \begin{pmatrix} 2 & 5 & 7 \\ 1 & 4 & 6 \\ 2 & -2 & 3 \end{pmatrix}$ . [2]
- f) Write the quadratic form corresponding to the matrix  $A = \begin{pmatrix} 1 & 5 & 7 \\ 5 & 4 & 6 \\ 7 & 6 & 3 \end{pmatrix}$ . [3]
- g) If  $u = f(x-y, y-z, z-x)$ , find  $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$ . [2]
- h) Expand  $f(x, y) = e^{xy}$  about origin up to 2<sup>nd</sup> degree terms. [3]
- i) Form a partial differential equation by eliminating the arbitrary function  $f$  from  $z = f(x^2 + y^2)$ . [2]
- j) Solve  $\sqrt{p} + \sqrt{q} = 1$ . [3]

**PART-B****(50 Marks)**

- 2.a) Solve  $(3xy^2 - y^3)dx - (2x^2y - xy^2)dy = 0$ .
- b) Solve  $y'' + y = x \sin x$ . [5+5]

**OR**

- 3.a) Apply the method of variation of parameters to solve  $y'' - y = x^2$ .
- b) If the temperature of the air is 30<sup>0</sup> C and the substance cools from 100<sup>0</sup> C to 70<sup>0</sup> C in 15 minutes, find when the temperature will be 40<sup>0</sup> C. [5+5]

4.a) Find the rank of the matrix  $A = \begin{pmatrix} 0 & 1 & -3 & -1 \\ 0 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{pmatrix}$  by reducing to echelon form.

b) Show that the system of equations  $5x + 3y + 7z = 4$ ,  $3x + 26y + 2z = 9$ ,  $7x + 2y + 10z = 5$  is consistent and hence solve it. [5+5]

**OR**

5. Solve the system of equations  $2x - 2y - 2z = -4$ ,  $-y + z = -1$ ,  $-x + 5y + 2z = 6$  by LU – decomposition method. [10]

6.a) Find the Eigen values of  $5A^5 - 2A^2 + 7A - 3A^{-1} + I$ , if  $A = \begin{pmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{pmatrix}$ .

b) Using Cayley-Hamilton theorem, find  $A^{-1}$  and  $A^{-2}$  if  $A = \begin{pmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{pmatrix}$ . [5+5]

**OR**

7. Reduce the quadratic form  $Q = 8x^2 + 7y^2 + 3z^2 + 12xy + 4xz - 8yz$  to canonical form and hence find its rank, nature, index and signature. [10]

8.a) If  $f(x, y) = \ln\left(\frac{x^4 + y^4}{x + y}\right)$ , show that  $xf_x + yf_y = 3$ .

b) Determine whether the functions  $u = \frac{x+y}{x-y}$ ,  $v = \frac{xy}{(x-y)^2}$  are dependent. If so, find the relation between them. [5+5]

**OR**

9.a) Find the Taylor series expansion of  $f(x, y) = e^x \cos y$  in powers of  $(x-1)$  and  $\left(y - \frac{\pi}{4}\right)$ .

b) Find the maximum and minimum values of the function  $f(x, y) = x^4 + y^4 - x^2 - y^2 + 1$ . [5+5]

10.a) Find all possible second order partial differential equations by eliminating the arbitrary constants  $a, b, c$  from  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ .

b) Solve  $(p-q)z = z^2 + (x+y)^2$ . [5+5]

**OR**

11.a) Reduce the equation  $p^2x^2 = z(z - qy)$  to  $F(p, q, z) = 0$  form and hence solve it.

b) Solve  $p^2y(1+x^2) = qx^2$ . [5+5]