

MATHEMATICS-I**(Common to CE, EEE, ME, ECE, CSE, EIE, IT, MCT, MMT, AE, MIE, PTM, ITE)****Time: 2 hours****Max. Marks: 75****Answer any five questions
All questions carry equal marks**

1.a) Find the rank of $\begin{bmatrix} 9 & 7 & 3 & 6 \\ 4 & -1 & 4 & 1 \\ 6 & 8 & 2 & 4 \end{bmatrix}$ by using Normal form.

b) Find the inverse of the matrix $A = \begin{bmatrix} 2 & -6 & -2 & -3 \\ 5 & -13 & -4 & -7 \\ -1 & 4 & 1 & 2 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ by using the Gauss-Jordan method. [7+8]

2. Solve the following system of linear equations by using Gauss-Seidel method

$$2x_1 - x_2 = 7$$

$$-x_1 + 2x_2 - x_3 = 1$$

$$-x_2 + 2x_3 = 1$$

[15]

3.a) Find the Eigen values and Eigen vectors of $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$.

b) Show that the Eigen values of a Hermitian matrix are real and distinct. [8+7]

4. State the Cayley-Hamilton theorem. Verify the Cayley-Hamilton theorem for the matrix

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}.$$

[15]

5.a) Test the convergence of the series $1 + \frac{1^2 \cdot 2^2}{1 \cdot 3 \cdot 5} + \frac{1^2 \cdot 2^2 \cdot 3^2}{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9} + \dots$

b) Test the convergence of the series $\sum \frac{x^n}{n!}$. [8+7]

6.a) Examine the convergence of the series $1 + \frac{x}{2} + \frac{x^2}{3^2} + \frac{x^3}{4^3} + \dots$ ($x > 0$).

b) Examine the convergence of the series $\sum \frac{1}{\sqrt{n}}$. [8+7]

7.a) If $a < b$, prove that $\frac{b-a}{1+b^2} < \tan^{-1}b - \tan^{-1}a < \frac{b-a}{1+a^2}$.

b) Show that $\int_0^1 \sqrt{1-x^n} dx = \frac{1}{n} \beta\left(\frac{1}{n}, \frac{3}{2}\right)$. [8+7]

8.a) If $x^x y^y z^z = c$ show that $\frac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}$ if $x = y = z$.

b) Find the relative maximum and minimum values of the function $f(x, y) = 3x^2 y^2 - 3x^3 - 3y^3$. [7+8]

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