

Code No:151AA

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY, HYDERABAD**B.Tech I Year I Semester Examinations, December - 2018****MATHEMATICS-I****(Common to CE, EEE, ME, ECE, CSE, EIE, IT, MCT, MMT, AE, MIE, PTM)****Time: 3 hours****Max. Marks: 75****Note:** This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A. Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART - A**(25 Marks)**

- 1.a) If A is Hermitian matrix and B is a Skew- Hermitian matrix , prove that $(B+iA)$ is Skew -Hermitian matrix. [2]
- b) Let A be a square matrix of order 3 with Eigenvalues 2, 2 and 3 and A is diagonalizable then find rank of $(A-2I)$. [2]
- c) State Cauchy's root test. [2]
- d) Find the value of $\Gamma\left(-\frac{1}{2}\right)$ [2]
- e) Verify Euler's theorem for the function $xy+yz+zx$. [2]
- f) Prove that the transpose of a unitary matrix is unitary. [3]
- g) Find the Eigen values of the matrix $A = \begin{pmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{pmatrix}$ [3]
- h) Test for convergence $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$. [3]
- i) Discuss the applicability of Rolle's Theorem to the function $f(x) = 2 + (x-1)^{\frac{2}{3}}$ in the interval $[0, 2]$. [3]
- j) If $u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\frac{y}{x}$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$. [3]

PART - B**(50 Marks)**

- 2.a) Reduce the given matrix into normal form and hence find the rank

$$\begin{pmatrix} 2 & 3 & -2 & 5 & 1 \\ 3 & -1 & 2 & 0 & 4 \\ 4 & -5 & 6 & -5 & 7 \end{pmatrix}$$

- b) Solve the equations $x + y + z = 6$; $3x + 3y + 4z = 20$; $2x + y + 3z = 13$ using Gauss elimination method. [5+5]

OR

3.a) Find the rank of the matrix $\begin{pmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{pmatrix}$ by reducing it to Normal form.

b) Solve the system of equations by Gauss-Seidel method $20x + y - 2z = 17$;
 $3x + 20y - z = -18$; $2x - 3y + 20z = 25$. [5+5]

4.a) Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, and $A^{-1} = \alpha A^2 + \beta A + \gamma I$, $\alpha, \beta, \gamma \in R$, then find $\alpha + \beta + \gamma$.

b) Find the nature of the quadratic form $10x^2 + 2y^2 + 5z^2 - 4xy - 10xz + 6yz$. [5+5]

OR

5.a) Let A be a 3×3 matrix over R such that $\det(A) = 6$ and $\text{tr}(A) = 0$. If $\det(A+I) = 0$, where I is the identity matrix of order 3, then find the Eigen values of A.

b) Reduce the quadratic form $5x^2 + 26y^2 + 10z^2 + 4yz + 14zx + 6xy$ to canonical form. [5+5]

6.a) Test the convergence of the series $1 + \frac{x}{2} + \frac{2!}{3^2}x^2 + \frac{3!}{4^3}x^3 + \dots$.

b) Examine the following series for convergence $\sum \frac{(-1)^{n-1} \sin nx}{n^3}$. [5+5]

OR

7.a) Test for convergence of the series $\sum \frac{x^n}{(2n)!}$.

b) Examine for absolute convergence the series $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$. [5+5]

8.a) Show that $\frac{\tan x}{x} > \frac{x}{\sin x}$, $0 < x < \frac{\pi}{2}$.

b) Find the surface area of the solid generated by revolving the loop of the curve $9y^2 = x(x-3)^2$. [5+5]

OR

9.a) Show that $|\cos b - \cos a| \leq |b - a|$.

b) Show that $\int_0^{\infty} x^{m-1} (a-x)^{n-1} dx = a^{m+n-1} \beta(m, n)$. [5+5]

10.a) If $u = f(y-z, z-x, x-y)$ show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.

b) If $x = uv, y = \frac{u+v}{u-v}$ determine $\frac{\partial(u, v)}{\partial(x, y)}$. [5+5]

OR

11.a) If $U = x + y - z, V = x - y + z, W = x^2 + y^2 + z^2 - 2yz$, show that the functions are functionally dependent.

b) The temperature T at any point (x, y, z) in space is $T = 400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$. [5+5]