R18

Code No: 151AA

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech I Year I Semester Examinations, June - 2022

MATHEMATICS - I

(Common to EEE, CSE, IT, CSIT, ITE, CE(SE), CSE(CS), CSE(DS), CSE(Networks), CSD)

Time: 3 Hours **Max. Marks: 75**

Answer any five questions

All questions carry equal marks

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1.a) Apply rank test to find whether the following system has any solution other than $x = y = z = w = 0$, $x + 2y + 3z + 4w = 0$, $8x + 5y + z + 4w = 0$, $5x + 6y + 8z + w = 0$, $8x + 3y + 7z + 2w = 0$.

b) By using Gauss's elimination method solve $5x$-y-$2z$=142, x-$3y$-z=-30, $2x$-y-$3z$ = -5. [8+7]

2.a) Find the characteristic polynomial of the matrix $A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ and verify Cayley- Hamilton theorem for this matrix. Hence find A^{-1}, if exist.

b) The matrix $A = \begin{pmatrix} a & h \\ h & b \end{pmatrix}$ is transformed to the diagonal form $D = T^{-1}AT$, where $T = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix}$. Find the value of θ which gives this diagonal transformation. [8+7]

3.a) Prove that the following series for conditional convergence:

i) $1 - \frac{1}{2^p} + \frac{1}{3^p} - \frac{1}{4^p} + \ldots\infty$ ii) $1 - \frac{1}{2} + \frac{1.3}{2.4} - \frac{1.3.5}{2.4.6} + \ldots\infty$

b) Prove that the series $\frac{\sin x}{1^3} - \frac{\sin 2x}{2^3} + \frac{\sin 3x}{3^3} - \ldots$ converges absolutely. [8+7]

4.a) Express the $\log(cosx)$ as Taylor series about $x = \frac{\pi}{3}$.

b) Find the volume of solid generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (0 < b < a)$ rotated about minor axis. [8+7]

5. If $u = xf(x,y) + yg(x,y)$, show that $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2}$=0. [15]

6.a) Determine whether u and v are functionally dependent, where u and v are defined by $u = sinx + cosy$ and $v = cosx + siny$.

b) Find the greatest value of $u = xyz$, if x, y and z are positive real numbers for which $4x + 2y + z = 12$. [8+7]

7. A function $f(x, y)$ is written in terms of new variables $u = e^{x}\cos y$, and $v = e^{x}\sin y$. Show that $\frac{\partial f}{\partial x} = u\frac{\partial f}{\partial u} + v\frac{\partial f}{\partial v}$ and $\frac{\partial f}{\partial y} = -v\frac{\partial f}{\partial u} + u\frac{\partial f}{\partial v}$, and so deduce that. [15]

8.a) By using techniques involving the Gamma function, find the value of $\int_0^{\infty} \sqrt{x}\, e^{-\sqrt{x}}\, dx$.

b) By using techniques involving the Beta function, find the exact value of $\int_0^{\pi/4} \sin^3 2x \cos^4 2x\, dx$.

[8+7]

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