

Code No: 131AA

**JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD****B. Tech I Year I Semester Examinations, March/April - 2023****MATHEMATICS - I****(Common to CE, EEE, ME, ECE, CSE, EIE, IT, MCT, MMT, AE)****Time: 3 Hours****Max. Marks: 75****Note:** i) Question paper consists of Part A, Part B.

ii) Part A is compulsory, which carries 25 marks. In Part A, Answer all questions.

iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

**PART – A****(25 Marks)**

- 1.a) Define an exact differential equation. Write the solution of that exact differential-equation. [2]
- b) Write the general solution of  $(D^2 + a^2)y = \sin ax$ . [3]
- c) Define a Hermitian matrix and a skew-Hermitical matrix. [2]
- d) Define Echelon form of a matrix. What is the rank of a matrix which is in Echelon form. [3]
- e) Show that the sum of the eigen values is equal to its trace. [2]
- f) Define a quadratic form and give its real symmetric matrix. [3]
- g) If  $z = e^{xy}$ , find  $\frac{\partial z}{\partial x}$  and  $\frac{\partial z}{\partial y}$ . [2]
- h) If  $x = r \cos \theta$ ,  $y = r \sin \theta$ , find  $\frac{\partial(r, \theta)}{\partial(x, y)}$ . [3]
- i) Form a partial differential equation from  $z = f(x^2 + y^2)$  by the elimination of arbitrary function. [2]
- j) Solve  $px + qy = z$ . [3]

**PART - B****(50 Marks)**

- 2.a) Solve  $(\cos x \tan y + \cos(x + y))dx + (\sin x \sec^2 y + \cos(x + y))dy = 0$ .
- b) According to Newton's law of cooling, the rate at which a substance cools in moving air is proportional to the difference between the temperature of the substance and that of the air. If the temperature of the air is  $30^\circ\text{C}$  and the substance cools from  $100^\circ\text{C}$  to  $70^\circ\text{C}$  in 15 minutes, find when the temperature will be  $40^\circ\text{C}$ ? [5+5]

**OR**

3. Solve  $\frac{d^2 y}{dx^2} + 4y = \tan 2x$  by method of variation of parameters. [10]

4.a) Find the rank of the matrix  $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ .

b) Solve the system of equation  $3x + 3y + 2z = 1$ ;  $x + 2y = 4$ ;  $10y + 3z = -2$ ;  $2x - 3y - z = 5$ .  
[5+5]

**OR**

5. Solve the system  $\lambda x + y + z = 0$ ;  $x + \lambda y + z = 0$ ;  $x + y + \lambda z = 0$  for all values of  $\lambda$ , if it has a non-trivial solution.  
[10]

6. Verify Cayley-hamilton theorem for the matrix  $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$  and hence find the value

of  $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$ .  
[10]

**OR**

7. Reduce the quadratic form  $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$  to the canonical form and find its nature.  
[10]

8.a) If  $u = \log\left(\frac{x^4 + y^4}{x + y}\right)$ , show that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3$ .

b) If  $u = \sin^{-1}(x - y)$ ,  $x = 3t$ ,  $y = 4t^3$ , show that  $\frac{du}{dt} = \frac{3}{\sqrt{1-t^2}}$ .  
[5+5]

**OR**

9. Find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ .  
[10]

10.a) Form a partial differential equation from  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ .

b) Solve  $(x^2 - y^2 - z^2)p + 2xyq = 2zx$ .  
[5+5]

**OR**

11.a) For a partial differential equation from  $f(x + y + z, x^2 + y^2 + z^2) = 0$ .

b) Solve  $p^2 - q^2 = x - y$ .  
[5+5]

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