

Code No: 151AA

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech I Year I Semester Examinations, March/April - 2023

MATHEMATICS - I

(Common to CE, EEE, ME, ECE, CSE, EIE, IT, MCT, MMT, ECM, AE, MIE, PTM, CSBS, CSIT, ITE, CE(SE), CSE(CS), CSE(AI&ML), CSE(DS), CSE(IOT), CSE(N), TTE, AI&DS, AI&ML, CSD)

Time: 3 Hours

Max. Marks: 75

Note: i) Question paper consists of Part A, Part B.

ii) Part A is compulsory, which carries 25 marks. In Part A, Answer all questions.

iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART – A**(25 Marks)**

- 1.a) Find the rank of the matrix by reducing to Echelon form $A = \begin{bmatrix} 1 & 4 & -2 & 1 \\ -2 & -3 & 4 & 3 \\ -3 & 3 & 6 & 12 \end{bmatrix}$ [2]
- b) Check whether the matrix $A = \begin{bmatrix} i & 0 & 0 \\ 0 & 0 & i \\ 0 & i & 0 \end{bmatrix}$ is Skew-Hermitian or not. [3]
- c) If $A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$, then find the eigenvalues of $3A^3 + 5A^2 - 6A + 2I$. [2]
- d) If $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$, then find A^3 using Cayley Hamilton theorem. [3]
- e) Test for convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)}$. [2]
- f) Test for convergence of the series $\sum_{n=1}^{\infty} \log\left(1 + \frac{1}{n}\right)$. [3]
- g) Evaluate $\int_0^{\infty} x^2 e^{-x^2} dx$ using Beta-Gamma function. [2]
- h) Find the value of 'c' using Cauchy's mean value theorem for the function $f(x) = x^2$ and $g(x) = x^3$ [1, 2]. [3]
- i) If $x^3 + y^3 - 3axy = 0$ then find the value of $\frac{dy}{dx}$. [2]
- j) Using Euler's theorem find the value of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$, if $u = \frac{x^3 y^3}{x^3 + y^3}$. [3]

PART - B**(50 Marks)**

2.a) Find rank of matrix $A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 \\ 1 & -1 & 4 & 0 \\ 2 & 2 & 8 & 0 \end{bmatrix}$ by reducing it to normal form.

b) Solve the system of equations $x - y + 2z = 4$, $3x + y + 4z = 6$, $x + y + z = 1$ by using Gauss elimination method. [5+5]

OR

3.a) Find the inverse of the matrix by using Gauss-Jordan method $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$.

b) Solve the system of equations $x + y + 5z = 110$; $27x + 6y - z = 85$; $6x + 15y + 2z = 72$ using Gauss Seidel method. [5+5]

4.a) Find the Eigen values and Eigen vectors of $A = \begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$.

b) Verify Cayley Hamilton theorem for the Matrix $A = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 9 & 1 \\ 3 & 7 & 0 \end{bmatrix}$ and hence find A^{-1} .

[5+5]

OR

5. Reduce the quadratic form $3x_1^2 + 3x_2^2 + 3x_3^2 + 2x_1x_2 + 2x_1x_3 - 2x_2x_3$ to canonical form using orthogonal transformation. Also find signature and rank of the quadratic form. [10]

6.a) Examine the convergence of $\sum \left[\frac{1.4.7....(3n-2)}{3.6.9....3n} \right]^2$.

b) Examine the convergence of $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots, (x > 0)$. [5+5]

OR

7.a) Test for convergence of $1 + \frac{3}{7}x + \frac{3.6}{7.10}x^2 + \frac{3.6.9}{7.10.13}x^3 + \dots$

b) Test for convergence of $\sum \left(\frac{n-1}{n} \right)^{n^2}$. [5+5]

8.a) If $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{\sqrt{x}}$ prove that 'c' of the Cauchy's generalized mean value theorem is the geometric mean of 'a' and 'b' for $a > 0, b > 0$.

b) Find Maclaurin's series expansion of the $f(x, y) = \sin^2 x$. [5+5]

OR

9.a) Find the area bounded by pair of curve $y = 2x$ and $y^2 = 2(2 - x)$.

b) Evaluate $\int_0^1 e^{-a^2 x^2} dx$. [5+5]

10.a) Find $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$ using Euler's theorem for the function $u = \cos^{-1} \left(\frac{x+y}{\sqrt{x} + \sqrt{y}} \right)$.

b) If $u = \frac{yz}{x}$, $v = \frac{xz}{y}$, $w = \frac{xy}{z}$ find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$. [5+5]

OR

11.a) Determine whether the functions $U = \frac{x}{y+z}$, $V = \frac{y}{z+x}$, $W = \frac{z}{x+y}$ are dependent. If dependent find the relationship between them.

b) Find the temperature at any point (x, y, z) in space is $f = 400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$. [5+5]

---ooOoo---