2024

MATHEMATICS — MDC

Paper : CC-3

(Ordinary Differential Equations and Group Theory)

Full Marks : 75

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group - A

[Ordinary Differential Equations]

(Marks : 45)

Answer *any nine* questions.

1. (a) Determine the order and degree of the differential equation $\sqrt{y+\left(\frac{dy}{dx}\right)^2}=1+x$.

   (b) Obtain the differential equation of all circles each of which touches the axis of x at the origin. 2+3

2. Solve : $(x+2y-3)dx=(2x+y-3)dy$. 5

3. Solve : $(x^2y-2xy^2)dx+(3x^2y-x^3)dy=0$. 5

4. Solve : $x\,dx+y\,dy+\frac{x\,dy-y\,dx}{x^2+y^2}=0$. 5

5. Find both general solution and singular solution of the ODE : $x=py-p^2$, where $p=\frac{dy}{dx}$. 5

6. Solve : $\frac{dy}{dx}-\frac{3}{x+2}y=(x+2)^3$. 5

7. Solve : $y+px=x^4p^2$. 5

8. Solve : $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=x^2e^{3x}$. 5

9. Solve by the method of undetermined coefficients $(D^2-3D)y=2x^2+1$. 5

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10. Solve : $(x^2D^2 - 3xD + 5)y = x^2\sin(\log x)$. 5

11. Solve by the method of variation of parameters, the equation $\frac{d^2y}{dx^2} + a^2y = \sec ax$. 5

12. Show that e^{2x} and e^{3x} are linearly independent solutions of $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$. Find the solution $y(x)$ with the condition $y(0) = 0$ and $y'(0) = 1$. 5

13. Solve for x from the system of equations : 5

$$\frac{dx}{dt} + 4x + 3y = t$$

$$\frac{dy}{dt} + 2x + 5y = e^t.$$

14. Solve : $\frac{a\,dx}{yz(b-c)} = \frac{b\,dy}{zx(c-a)} = \frac{c\,dz}{xy(a-b)}$. 5

15. Show that the equation $x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = \frac{1}{(1-x)^2}, 0 < x < 1$ is exact and hence solve it. 5

16. Solve : $\frac{d^2y}{dx^2} = \sec^2 y \tan y$. 5

Group - B

[Group Theory]

(Marks : 30)

Answer ***any six*** questions.

17. Define a group. Does the set of all 2×2 non-singular matrices over integers form a group under matrix multiplication? Justify your answer. 2+3

18. Let (G, o) be a semi-group and for any two elements $a, b \in G$, each of the equation $a\,o\,x = b$ and $y\,o\,a = b$ has a solution in G. Then prove that (G, o) is a group. 5

19. (a) If a is an element of a group G such that $o(a) = 7$, then prove that there exists an element b in G such that $b^3 = a$.

(b) Give an example of a group G and subgroups H and K such that HK fails to be a subgroup of G. Justify your claim. 2+3

20. (a) Let $(G, *)$ be a group with identity element e. If $a^2 = e \forall a \in G$, show that G is commutative.

(b) In a group $(G, *)$, 'a' is an element of order 30. Find the order of a^{18}. 3+2

21. Show that a non-empty subset H of a group (G, o) is a subgroup of (G, o) if and only if $\forall a, b \in H, a^{-1} o b \in H$. 5

22. (a) Prove that the centre $Z(G)$ of a group G is a subgroup of G.

(b) Prove or disprove : $Z(S_3) = \{(1)\}$. 2+3

23. Define cyclic group. Let $S = \{1, i, -1, -i\}$. Then show that $(S, \cdot)$ is a cyclic group. Show that every cyclic group is abelian. 1+2+2

24. (a) Define cosets of a subgroup H in a group (G, o).

(b) The set $H = \{\bar{0}, \bar{3}, \bar{6}, \bar{9}\}$ is a subgroup of $\mathbb{Z}_{12}$. Find all cosets of H. 2+3

25. Prove that every group of prime order is cyclic. 5

26. If $\alpha = (1, 3, 4)(5, 6)(2, 7, 8, 9)$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 7 & 8 & 9 & 6 & 4 & 5 & 2 & 3 & 1 \end{pmatrix}$ be two permutations, find the product $\gamma = \beta^{-1}\alpha\beta$. Is γ an even permutation? 5
