So the general solution is

$$y = A\cos ax + B\sin ax - \frac{1}{a^2}\cos ax \ln \tan\left(\frac{\pi}{4} + \frac{ax}{2}\right).$$

EXERCISE

Solve the following differential equations

1. $(D^2 - 5D + 6)y = x^2$,
2. $(D^2 + 4)y = 2x + 3$,
3. $(D^2 - 3D + 2)y = e^{3x}$,
4. $(D^2 - 3D + 2)y = e^{2x}$,
5. $(D-2)^2 y = 2e^{2x}$,
6. $(D^2 - 3D + 2)y = \sin 3x$,
7. $(D^2 - D - 2)y = \sin 2x$,
8. $(D^2 - 2D + 5)y = \sin x$,
9. $(D^2 + 1)y = \sin x \sin 2x$,
10. $(D^2 + 4)y = \sin 2x$,
11. $(D^2 + 1)y = xe^{2x}$,
12. $(D^2 - 1)y = e^x(1 + x^2)$,
13. $(D^2 + 2D + 2)y = xe^{-x}$,
14. $(D+2)^2 y = 4\sinh 2x$,
15. $(D^2 - 1)y = x^2 \cos x$,
16. $(D^2 - 4)y = \cos^2 x$,
17. $(D+1)^2 y = x\cos x$,
18. $(D-1)^2 y = xe^x \sin x$,
19. $(D^2 + 3D + 2)y = e^{e^x}$,
20. $(D^2 - 1)y = \cosh x \cos x$,

[*Hints*: P.I. $= \frac{1}{2}\frac{1}{(D+1)(D-1)}e^x \cos x + \frac{1}{2}\frac{1}{(D+1)(D-1)}e^{-x}\cos x$

$= \frac{1}{2}e^x \frac{1}{D^2 + 2D}\cos x + \frac{1}{2}e^{-x}\frac{1}{D^2 - 2D}\cos x$

$= \frac{1}{2}e^x \frac{2D+1}{4D^2 - 1}\cos x - \frac{1}{2}e^{-x}\frac{2D-1}{4D^2 - 1}\cos x$

$= -\frac{1}{10}e^x(\cos x - 2\sin x) - \frac{1}{10}e^{-x}(2\sin x + \cos x)$]

21. $(D^2 + 1)y = \sin x \sin 2x$,
22. $(D^2 + 1)y = \cos x \sin 3x$,
23. $(D^2 - 2D)y = e^{2x} - e^x$,
24. $(D^2 + a^2)y = \sec ax$,
25. $(D^2 + a^2)y = \operatorname{cosec} ax$,
26. $(D^2 + a^2)y = \cot ax$,
27. $(D^2 - a^2)y = \cosh ax$,
28. $(D^2 - 3D + 2)y = \sin e^{-x}$,
29. $(D^2 - 9D + 18)y = e^{e^{-3x}}$,
30. $(D^2 - 4)y = x\sin x$,
31. $(D^2 - 1)y = x^2 \sin x$,
32. $(D^2 + 4)y = x\sin^2 x$,

15. $y = c_1e^{-x} + c_2e^{2x} + \frac{1}{2}(1 - x^2)\cos x + x\sin x,$

16. $y = c_1e^{-2x} + c_2e^{2x} - \frac{1}{16}(2 + \cos 2x),$

17. $y = (c_1 + xc_2)e^{-x} + \frac{1}{2}[(x - 1)\sin x + \cos x],$

18. $y = (c_1 + xc_2)e^{x} - (2\cos x + x\sin x)e^{x},$

19. $y = c_1e^{-2x} + c_2e^{-x} + e^{-2x}e^{e^x},$

20. $y = c_1e^{x} + c_2e^{-x} + \frac{2}{5}\sin x\sinh x - \frac{1}{5}\cos x\cosh x,$

21. $y = c_1\cos x + c_2\sin x + \frac{x}{4}\sin x + \frac{1}{16}\cos 3x,$

22. $y = c_1\cos x + c_2\sin x - \frac{1}{30}\sin 4x - \frac{1}{6}\sin 2x,$

23. $y = c_1 + \left(c_2 + \frac{x}{2}\right)^2 e^{2x} + e^{x},$

24. $y = c_1\cos ax + c_2\sin ax + \frac{x}{a}\sin ax + \frac{1}{a^2}\cos ax\ln\cos ax,$

25. $y = c_1\cos ax + c_2\sin ax - \frac{x}{a}\cos ax + \frac{1}{a^2}\sin ax\ln\sin ax,$

26. $y = c_1\cos ax + c_2\sin ax + \frac{1}{a^2}\sin ax\ln\tan\left(\frac{ax}{2}\right),$

27. $y = c_1e^{ax} + c_2e^{-ax} + \frac{x}{2a}\sinh ax,$

28. $y = c_1e^{x} + c_2e^{-x} - e^{2x}\sin e^{-x},$

29. $y = c_1e^{2x} + c_2e^{6x} + \frac{1}{9}e^{6x}e^{e^{-3x}},$

30. $y = c_1\cos 2x + c_2\sin 2x + \frac{x}{3}\sin x - \frac{2}{9}\cos x,$

31. $y = c_1e^{x} + c_2e^{-x} - x\cos x - \frac{1}{2}(x^2 - 1)\sin x,$

32. $y = c_1\cos 2x + c_2\sin 2x + \frac{x}{8}\left(1 - \frac{x}{2}\sin 2x - \frac{1}{4}\cos 2x\right),$

33. $y = c_0 + c_1e^{-x} + c_2e^{-2x} + \frac{1}{6}x^3 - \frac{3}{4}x^2 + \frac{7}{4}x,$

34. $y = c_0 + c_1e^{x} + c_2e^{-x} + x\cosh x,$

35. $y = c_1e^{-x} + e^{\frac{x}{2}}\left[c_2\cos\frac{\sqrt{3}x}{2} + c_3\sin\frac{\sqrt{3}x}{2}\right] + \frac{1}{65}(\cos 2x - 8\sin 2x),$

36. $y = c_0 + c_1\sin ax + c_2\cos ax -$

37. $y = c_1 + (c_2 + xc_3)e^{-x} + \frac{e^{2x}}{18}$

38. $y = c_1e^{x} + (c_2 + xc_3)e^{-x} + \frac{x}{}$

39. $y = c_1e^{x} + (c_2 + xc_3)e^{-x} -$

40. $y = (c_1 + xc_2)e^{x} + c_3e^{3x}$

41. $y = (c_1 + xc_2 + x^2c_3)e^{x}$

42. $y = c_1e^{x} + (c_2 + xc_3$

43. $y = c_1e^{x} + c_2e^{3x} + c_$

44. $y = (c_1 + xc_2)e^{2x}$

45. $y = c_1e^{x} + e^{-\frac{x}{2}}[c$

46. $y = c_1 + xc_2 +$

47. $y = c_1 + xc_2$

48. $y = e^{x}[(c_1 +$

49. $y = c_1e^{2x} +$

50. $y = c_1 +$

51. $y = (c_1$

52. $y = e$

53. $y =$

54. $y =$

55. y

46. $D^2(D-1)^2 y = x$.

[*Hints*: P.I. $= \frac{1}{D^2(1-D)^2} x = \frac{1}{D^2}(1+2D+3D^2+\cdots)x = \frac{1}{D^2}(x+2) = \frac{x^3}{6} + x^2$.]

47. $(D^4 - 2D^3 + D^2)y = x^2$

48. $(D^4 - 4D^3 + 8D^2 - 8D + 4)y = e^x$

49. $(D^4 + D^3 - 3D^2 - 5D - 2)y = 3xe^{-x}$

50. $(D^4 - 8D)y = x^2 + e^{3x}$

51. $(D^4 - 4D^3 + 6D^2 - 4D + 1)y = e^{2x}\cos x$

52. $(D^4 + 2D^3 + 3D^2 + 2D + 1)y = xe^x$

53. $(D^4 + 10D^2 + 9)y = \cos(2x+3)$.

54. $(D^3 - 3D^2 - 6D + 8)y = xe^{-3x}$,

55. $(D^4 - 1)y = e^x \cos x$.

56. $(D^4 + 2D^2 + 1)y = x^2 \cos x$,

57. $(D-1)^2(D^2+1)^2 y = e^x + x$.

ANSWERS

1. $y = c_1 e^{2x} + c_2 e^{3x} + \frac{1}{6}\left(x^2 + \frac{5x}{3} + \frac{19}{18}\right)$

2. $y = c_1 \cos 2x + c_2 \sin 2x + \frac{1}{4}(2x+3)$,

3. $y = c_1 e^{2x} + c_2 e^x + \frac{1}{2}e^{3x}$,

4. $y = c_1 e^{2x} + c_2 e^x + xe^{2x}$,

5. $y = (c_1 + xc_2)e^{2x} + x^2 e^{2x}$,

6. $y = c_1 e^{2x} + c_2 e^x + \frac{1}{130}(9\cos 3x - 7\sin 3x)$,

7. $y = c_1 e^{2x} + c_2 e^{-x} + \frac{1}{20}(\cos 2x - 3\sin 2x)$,

8. $y = (c_1 \cos 2x + c_2 \sin 2x)e^x + \frac{1}{10}(2\sin x + \cos x)$,

9. $y = c_1 \cos x + c_2 \sin x + \frac{1}{4}x\sin x + \frac{1}{16}\cos 3x$,

10. $y = c_1 \cos 2x + c_2 \sin 2x - \frac{1}{4}x\cos 2x$,

11. $y = c_1 \cos x + c_2 \sin x + \frac{1}{25}(5x-4)e^{2x}$,

12. $y = c_1 e^{-x} + c_2 e^x + \frac{1}{12}xe^x(2x^2 - 3x + 9)$,

13. $y = (c_1 \cos x + c_2 \sin x + x)e^{-x}$,

14. $y = (c_1 + xc_2)e^{-2x} + \frac{1}{8}e^{2x} - x^2 e^{-2x}$