Now, $\dfrac{1}{D-in}\sec nx = e^{inx}\displaystyle\int \frac{e^{-inx}}{\cos nx}\,dx$

$$= e^{inx}\int \frac{\cos nx - i\sin nx}{\cos nx}\,dx$$

$$= e^{inx}\left(x + i\cdot\frac{1}{n}\log\cos nx\right).$$

Similarly, $\dfrac{1}{D+in}\sec nx = e^{-inx}\left(x - i\cdot\dfrac{1}{n}\log\cos nx\right).$

Therefore the particular integral is

$$= \frac{1}{2in}\left\{e^{inx}\left(x + i\cdot\frac{1}{n}\log\cos nx\right) - e^{-inx}\left(x - i\cdot\frac{1}{n}\log\cos nx\right)\right\}$$

$$= \frac{1}{n}\left\{x\sin nx + \frac{1}{n}\cos nx\log\cos nx\right\}.$$

Hence the complete solution is

$$y = A\cos nx + B\sin nx + \frac{x}{n}\sin nx + \frac{\cos nx}{n^2}\log\cos nx.$$

Examples V (B)

1. Evaluate the following :

(a) $\dfrac{1}{D^2-1}xe^x$. (b) $\dfrac{1}{(D-2)^2}x^3e^{2x}$. (c) $\dfrac{1}{D^2-1}\sin^2 x$.

(d) $\dfrac{1}{D^2+4}\sin 2x$. (e) $\dfrac{1}{D^2(D-1)^2}x^3$. (f) $\dfrac{1}{D^2-2D+4}e^x\cos^2 x$.

Solve the following equations (2 – 32) :

2. (a) $(D^2-4D+4)y = x^3$. (b) $(D^3-D^2-6D)y = 1+x^2$.

3. (a) $(D^3-1)y = x^3-x^2$. (b) $D^2(D^2+D+1)y = x^2$.

[C. Ч. 1980]

4. $\dfrac{d^2y}{dx^2} - 5\dfrac{dy}{dx} + 6y = e^{4x}$. 5. $\dfrac{d^2y}{dx^2} + 2a\dfrac{dy}{dx} + (a^2+b^2)y = e^{px}$.

6. $\dfrac{d^3y}{dx^3} + y = (e^x+1)^2$.

7. (a) $(D^3+4D^2+4D)y = 8e^{-2x}$.

(b) $(D^2+4D+4)y = 2\sinh 2x$.

8. $(D^3+3D^2+3D+1)y = e^{-x}$.

9. $(D^3+2D^2+D)y = e^{2x}+x^2+x$.

10. (a) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x$. (b) $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = x(x + e^x)$. [B. H. 1981]

11. (a) $(D^3 - 3D^2 + 3D - 1)y = xe^x + e^x$. [C. H. 1996]

(b) $(D^4 - 4D^3 + 6D^2 - 4D + 1)y = e^{2x}\cos x$. [V. H. 1987]

12. $(D^3 - 7D - 6)y = e^{2x}(1 + x)$.

13. (a) $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = x^2e^{3x}$. (b) $(D^2 - a^2)y = p\sinh ax$.

14. $(D - 1)^2(D^2 + 1)^2 y = e^x + x$. [C. H. 1983]

15. (a) $(D^2 + aD)y = \sin ax$. (b) $(D^3 + 1)y = \cos 2x$.

(c) $(D^2 - D + 1)y = 2\sin 3x$. [B. H. 2002]

16. (a) $(D^2 + a^2)y = \cos ax + \cos bx$. (b) $(D^3 + a^2D)y = \sin ax$.

17. $(D^3 + 1)y = \sin 3x - \cos^2\frac{1}{2}x$.

18. $(D^2 - 1)y = x\sin x + (1 + x^2)e^x$.

19. $\frac{d^2y}{dx^2} - y = xe^x\sin x$. 20. $\frac{d^2y}{dx^2} - y = x^2\cos x$. [N. B. H. 2004]

21. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x\sin x$. [N. B. H. 1986]

22. $(D^3 - 1)y = x\sin x$.

23. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = e^x\cos x$. [V. H. 1988]

24. $\frac{d^2y}{dx^2} + 2y = x^2e^{3x} + e^x\cos 2x$. [V. H. 1992]

25. $\frac{d^4y}{dx^4} - y = e^x\cos x$. [B. H. 1991; N. B. H. 2003]

26. $\frac{d^4y}{dx^4} - y = x\sin x$.

27. $(D^5 - D)y = e^x + \sin x - x$. [C. H. 198]

28. $(D^5 - D^4 + 2D^3 - 2D^2 + D - 1)y = \cos x$.

29. $(D^4 + 10D^2 + 9)y = 96\sin 2x\cos x$.

30. $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$. [B. H. 19]

31. $\frac{d^2y}{dx^2} + 4y = 4\tan 2x$. 32. $(D^2 + 4)^2(D - 2)^2 y = \cos^2 x$.

33. Find the solution of the equation
$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x = 3\sin 2t,$$
which will satisfy $x = 0$, $\frac{dx}{dt} = 0$ at $t = 0$. [N. B. H. 1983]

34. Find the solution of the equation
$$\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 6y = 2e^{3x},$$
which satisfies the conditions $y = 1$, $\frac{dy}{dx} = 0$ at $x = 0$. [B. H. 1988]

35. Find the solution of the equation
$$\frac{d^2x}{dt^2} - x = 2,$$
which satisfies the conditions $\frac{dx}{dt} = 3$ when $x = 1$ and $t = 2$ when $x = -1$.

Answers

1. (a) $\frac{1}{4}e^{x}(x^2 - x + \frac{1}{2})$. (b) $\frac{1}{20}e^{2x}x^5$. (c) $\frac{1}{10}\cos 2x - \frac{1}{2}$.
(d) $-\frac{1}{4}x\cos 2x$. (e) $\frac{x^5}{20} + \frac{x^4}{2} + 3x^3 + 12x^2$. (f) $\frac{1}{6}e^{x} - \frac{1}{2}e^{x}\cos 2x$.

2. (a) $y = (C_1 + C_2x)e^{2x} + \frac{1}{8}(2x^3 + 6x^2 + 9x + 6)$.
(b) $y = A + Be^{-2x} + Ce^{3x} - \frac{1}{18}x^3 + \frac{1}{36}x^2 - \frac{25}{108}x$.

3. (a) $y = Ae^{x} + e^{-\frac{x}{2}}(B\sin\frac{1}{2}\sqrt{3}\,x + C\cos\frac{1}{2}\sqrt{3}\,x) - x^3 + x^2 - 6$.
(b) $y = C_1 + C_2x + \left(C_3\cos\frac{\sqrt{3}}{2}x + C_4\sin\frac{\sqrt{3}}{2}x\right)e^{-\frac{1}{2}x} + \frac{1}{12}x^4 - \frac{1}{3}x^3$.

4. $y = Ae^{3x} + Be^{2x} + \frac{1}{2}e^{4x}$.

5. $y = e^{-ax}(A\cos bx + B\sin bx) + \dfrac{e^{px}}{(a+p)^2 + b^2}$.

6. $y = Ae^{-x} + \left(B\cos\frac{\sqrt{3}}{2}x + C\sin\frac{\sqrt{3}}{2}x\right)e^{\frac{1}{2}x} + \frac{1}{9}e^{2x} + e^{x} + 1$.

7. (a) $y = C_1 + e^{-2x}(C_2 + C_3x) - 2x^2e^{-2x}$.
(b) $y = (C_1 + C_2x)e^{-2x} + \frac{1}{16}e^{2x} - \frac{1}{2}x^2e^{-2x}$.

8. $y = (C_1 + C_2x + C_3x^2)e^{-x} + \frac{1}{6}e^{-x}x^3$.

9. $y = A + (B + Cx)e^{-x} + \frac{x}{6}(2x^2 - 9x + 24) + \frac{1}{18}e^{2x}$.

10. (a) $y = e^x(C_1 + C_2x) + \frac{1}{6}x^3e^x$.

(b) $y = C_1e^{2x} + C_2e^{3x} + \frac{18x^2 + 30x + 19}{108} + \frac{e^x}{4}(2x + 3)$.

11. (a) $y = (C_1 + C_2x + C_3x^2)e^x + \frac{1}{24}e^x(x^4 + 4x^3)$.

(b) $y = (C_1 + C_2x + C_3x^2 + C_4x^3)e^x - \frac{1}{4}e^{2x}\cos x$.

12. $y = Ae^{-x} + Be^{-2x} + Ce^{3x} - \frac{1}{12}e^{2x}\left(x + \frac{17}{12}\right)$.

13. (a) $y = C_1e^{2x} + C_2e^{3x} + e^{3x}\left(\frac{1}{3}x^3 - x^2 + 2x\right)$.

(b) $y = \left(C_1 + \frac{px}{2a}\right)\cosh ax + C_2\sinh ax$.

14. $y = (C_1 + C_2x)e^x + (C_3 + C_4x)\cos x + (C_5 + C_6x)\sin x$
$+ \frac{1}{8}x^2e^x + x$

15. (a) $y = A + Be^{-ax} - \frac{1}{2a^2}(\sin ax + \cos ax)$.

(b) $y = C_1e^{-x} + \left(C_2\cos\frac{\sqrt{3}}{2}x + C_3\sin\frac{\sqrt{3}}{2}x\right)e^{\frac{1}{2}x} + \frac{1}{65}(\cos 2x + 8\sin$

(c) $y = \left(C_1\cos\frac{\sqrt{3}}{2}x + C_2\sin\frac{\sqrt{3}}{2}x\right)e^{\frac{1}{2}x} + \frac{2}{73}(3\cos 3x - 8\sin 3x)$.

16. (a) $y = A\cos(ax + \beta) + \frac{x}{2a}\sin ax + \frac{\cos bx}{a^2 - b^2}$.

(b) $y = C_1 + C_2\cos ax + C_3\sin ax - \frac{x}{2a^2}\sin ax$.

17. $y = C_1e^{-x} + e^{\frac{1}{2}x}\left(C_2\cos\frac{\sqrt{3}}{2}x + C_3\sin\frac{\sqrt{3}}{2}x\right)$
$+ \frac{1}{730}(\sin 3x + 27\cos 3x) - \frac{1}{2} + \frac{1}{4}(\sin x - \cos x)$

18. $y = C_1e^x + C_2e^{-x} + \frac{1}{12}xe^x(2x^2 - 3x + 9) - \frac{1}{2}(x\sin x + \cos x)$.

19. $y = C_1e^x + C_2e^{-x} + \frac{1}{25}e^x\{(14 - 5x)\sin x - 2(5x + 1)\cos x\}$.

20. $y = A\cosh x + B\sinh x + \frac{1}{2}(1 - x^2)\cos x + x\sin x$.

21. $y = (C_1 + C_2x)e^x - (x\sin$

22. $y = C_1e^x + \left(C_2\cos\frac{\sqrt{3}}{2}x\right.$

23. $y = Ae^x\cos(\sqrt{3}x + a) +$

24. $y = A\cos(\sqrt{2}x + a) + \frac{e}{1}$

25. $y = C_1e^x + C_2e^{-x} + C_3$

26. $y = C_1e^x + C_2e^{-x} + (C$

27. $y = C_1 + C_2e^x + C_3e^{-x}$

28. $y = C_1e^x + (C_2 + C_3x$

29. $y = A\cos(x - \alpha) + B$

30. $y = A\cos x + B\sin x -$

31. $y = A\cos 2x + B\sin 2$

32. $y = (C_1 + C_2x)e^{2x} +$

33. $x = \frac{1}{8}(3 + 6t)e^{-2t}$

34. $y = \frac{7}{5}e^x - \frac{1}{15}e^{6x} -$

35. $x + 2 = e^{t-2}$.

5.11. Formation of solutions are specif

While solving li coefficients, we obser a root $m = \alpha$, then th such as $Ae^{\alpha x}$ occurre

21. $y = (C_1 + C_2 x)e^{x} - (x \sin x + 2\cos x)e^{x}$.

22. $y = C_1 e^{x} + \left(C_2 \cos \frac{\sqrt{3}}{2} x + C_3 \sin \frac{\sqrt{3}}{2} x \right) e^{-\frac{1}{2}x}$
$+ \frac{1}{2}(x \cos x - x \sin x - 3\cos x)$.

23. $y = Ae^{x} \cos(\sqrt{3}\, x + a) + \frac{1}{2} e^{x} \cos x$.

24. $y = A \cos(\sqrt{2}\, x + a) + \frac{e^{3x}}{121}\left(11x^2 - 12x + \frac{50}{11} \right) + \frac{e^{x}}{17}(4 \sin 2x - \cos 2x)$.

25. $y = C_1 e^{x} + C_2 e^{-x} + C_3 \sin(x + a) - \frac{1}{5} e^{x} \cos x$.

26. $y = C_1 e^{x} + C_2 e^{-x} + (C_3 \cos x + C_4 \sin x) + \frac{1}{8}(x^2 \cos x - 3x \sin x)$.

27. $y = C_1 + C_2 e^{x} + C_3 e^{-x} + C_4 \cos x + C_5 \sin x + \frac{1}{4} xe^{x} + \frac{1}{4} x \sin x + \frac{1}{2} x^2$.

28. $y = C_1 e^{x} + (C_2 + C_3 x)\cos x + (C_4 + C_5 x)\sin x + \frac{x^2}{16}(\cos x - \sin x)$.

29. $y = A\cos(x - \alpha) + B\cos(3x - \beta) - 3x \cos x + x \cos 3x$.

30. $y = A\cos x + B \sin x - x \cos x + \sin x \log \sin x$.

31. $y = A \cos 2x + B \sin 2x - \log \tan\left(\frac{\pi}{4} + x \right) \cos 2x$.

32. $y = (C_1 + C_2 x)e^{2x} + \{ (C_3 + C_4 x)\cos 2x + (C_5 + C_6 x)\sin 2x \}$
$+ \frac{1}{128} + \frac{x^2}{512} \sin 2x$.

33. $x = \frac{1}{8}(3 + 6t)e^{-2t} - \frac{3}{8}\cos 2t$.

34. $y = \frac{7}{5} e^{x} - \frac{1}{15} e^{6x} - \frac{1}{3} e^{3x}$.

35. $x + 2 = e^{t-2}$.

5.11. Formation of linear differential equations, whose solutions are specified.

While solving linear differential equations with constant coefficients, we observed that if the auxiliary equation $f(m) = 0$ had a root $m = \alpha$, then the operator $f(D)$ had a factor $(D - \alpha)$ and a term such as $Ae^{\alpha x}$ occurred in the general solution of the equation. In the