RK PAL

Ordinary differential equation.

An equation consisting of two variables as one independent (x) & the other dependent (y) with their differential coefficient is known as differential equation.

Since the independent variable is 1 ~~one~~ it is known as the ordinary differential equation.

If the independent variable is more than 1 then the differential equation is known as partial differential equation.

Ordinary differential equation.

E.g 1) $\frac{dy}{dx} + 2x = 0.$

2) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2x = 0.$

Partial differential equation

$$u = f(x,y) = x^2 + xy + y^2.$$

$$\frac{\partial^2 u}{\partial x^2} \quad \cancel{\frac{\partial^2 u}{\partial x^2}} + \frac{\partial^2 u}{\partial y^2} = 0.$$

Imp for 2 marks

Order & degree of ordinary differential equation

The order of differential eq^n is the order of highest order differential coefficient involving in it while the degree of the diff eq^n is a greatest exponent of the highest order derivative.

Q) Find the order & degree of the following differential equation.

1) $\frac{dy}{dx} + xy = x^2.$ → order 1 → degree 1.

2) $\frac{d^2y}{dx^2} = \frac{x^2}{y(1+\sqrt{x})}.$ → order 2 → degree 1.

3) $\left(\frac{d^2y}{dx^2}\right)^2 = 4.$ → order → 2 → degree 2.

4) $\frac{d^2y}{dx^2} = 5\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}.$

a solution of

$= 0.$

$).$

$\frac{dy}{dx}$

0

y the differential

$0.$

$x \cdot 2\cos x + \sin x e^{x}$

$= 2e^{x}\cos x + y.$

$(-\sin x) + 2\cos x e^{x}$

$+ \frac{dy}{dx}$

$2x + 2e^{x}\cos x$

$+ \frac{dy}{dx}$

$- y) + \frac{dy}{dx}$

y

$= 0$

Q). Show that the substitution $z = \sin x$ transforms the eqn

$$\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y\cos^2 x = 0$$

into $\frac{d^2y}{dz^2} + y = 0.$

$$\frac{dz}{dx} = \cos x$$

$$\cancel{\frac{d^2y}{dx^2}} = \therefore \frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}.$$

$$= \frac{dy}{dz} \cos x.$$

$$\frac{d^2y}{dx^2} = \cancel{\frac{d^2y}{}} \frac{dy}{dx}\left(\frac{dy}{dz} \cdot \cos x\right).$$

$$= \cancel{\frac{d^2y}{dx}}\left(-\sin x \frac{dy}{dz} + \cos x \frac{d^2y}{dz^2} \times \frac{dz}{dx}.\right.$$

$$= -\sin x \frac{dy}{dz} + \cos x \frac{d^2y}{dz^2} \frac{dz}{dx}.$$

$$-\sin \frac{dy}{dz} + \cos x \frac{d^2y}{dz^2} \frac{dz}{dx} \cancel{= 0} + \tan x$$

$$\frac{dy}{dx} + y\cos^2 x = 0.$$

$$-\sin \frac{dy}{dz} + \cos x \frac{d^2y}{dz^2} \frac{dz}{x} + \frac{\sin x}{\cos x} \frac{dy}{dx} + y\cos^2 x$$

$$= -\sin x \frac{dy}{dz} + \cos^2 x \frac{d^2y}{dz^2} \quad \left(\frac{dz}{dx} = \cos x\right)$$

$$\therefore \frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y\cos x = 0$$

$$\Rightarrow -\sin x \frac{dy}{dz} + \cos^2 x \frac{d^2y}{dz^2} + \frac{\sin x}{\cos x} \cdot \cos x \frac{dy}{dz} + y\cos^2 x = 0$$

$$\Rightarrow \cos^2 x \frac{d^2y}{dz^2} + y\cos^2 x = 0$$

$$\Rightarrow \frac{d^2y}{dz^2} + y = 0$$

Q. verify that $y = a \log x + b$ is a solution of the differential eqn $x \frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$.

$$\frac{dy}{dx} = \frac{a}{x} + 0 \ldots\ldots (i).$$

$$\frac{d^2y}{dx^2} = \frac{-a}{x^2}.$$

$$\frac{d^2y}{dx^2} x^2 = -a \ldots\ldots (ii).$$

From (i).

$$a = x\, dy/dx.$$

$$\therefore \text{from } (2) \quad x^2 \frac{d^2y}{dx^2} = -x \frac{dy}{dx}$$

$$\therefore x \frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$$

Q. verify that $y = e^x \sin 2x$ satisfy the differential eqn $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 0$.

$$\frac{dy}{dx} = \cancel{e^x 2\cos 2x} \quad e^x \cdot 2\cos 2x + \sin 2x\, e^x$$

$$= 2e^x \cos 2x + y.$$

$$\cancel{\frac{d^2y}{dx^2} = -4e^x \sin 2x}.$$

$$\frac{d^2y}{dx^2} = 2 \cdot 2e^x \cdot (-\sin 2x) + 2\cos 2x\, e^x + \frac{dy}{dx}$$

$$= -4 e^x \sin 2x + 2e^x \cos 2x + \frac{dy}{dx}$$

$$= -4y + \left(\frac{dy}{dx} - y\right) + \frac{dy}{dx}$$

$$= 2\frac{dy}{dx} - 5y$$

$$\therefore \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 0$$

(x)
one independent
ntial coefficient

it is known as

than 1 then
tial differential

tial equation

of highest
in it while the
t exponent of

the following

1 → degree 1.

er 2 → degree 1.

der → 2 → degree 2.

$\}^{3/2}$.

Squaring both sides.

$$\left(\frac{d^2y}{dx^2}\right)^2 = 25\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^3.$$

Order 2 → degree → 2.

(Imp for 5 marks) → Formation of diff eq^n.

Q) Determine the differential equation which primitive is $y = ax+b$.

Differentiating w.r.t x.

$$\frac{dy}{dx} = a+0.$$

$$\frac{d^2y}{dx^2} = 0.$$

Imp Q) $y = Ae^{x} + Be^{-x} + x^2$

Form the differential equation.

$$\frac{dy}{dx} = Ae^{x}\cancel{0} - Be^{-x} + 2x.$$

$$\frac{d^2y}{dx^2} = Ae^{x} + Be^{-x} + 2.$$

$$= y - x^2 + 2.$$

$\therefore \frac{d^2y}{dx^2} - y = \cancel{x}\ 2 - x^2$ is the required differential eq^n.

Q) Find the differential eq^n.

$$y = A\cos x + B\cosh x.$$

$$\frac{dy}{dx} = \cancel{\sin} - A\sin x + B\sinh x.$$

$$\frac{d^2y}{dx^2} = -A\cos x + B\cosh x.$$

$$\cancel{y - B\cosh x =} -A\cos x$$

$$= -y + 2B\cosh x.$$