MATH 801: Complex Variables and Transform Methods

Md. Toriqul Islam
Lecturer in Mathematics
Basic Science Division
World University of Bangladesh

Lecture Outline

- The Real Number System
- Graphical Representation of Real Numbers
- The Complex Number System
- Fundamental Operations with Complex Numbers

The Real Number System

The number system as we know it today is a result of gradual development as indicated in the following list.

(1) Natural numbers $1, 2, 3, 4, \ldots$, also called *positive integers*, were first used in counting. If a and b are natural numbers, the sum $a + b$ and product $a \cdot b, (a)(b)$ or ab are also natural numbers.

For this reason, the set of natural numbers is said to be *closed* under the operations of *addition* and *multiplication* or to satisfy the *closure property* with respect to these operations.

The Real Number System

(2) Negative integers and zero, denoted by $-1, -2, -3, \ldots$ and 0, respectively, permit solutions of equations such as $x + b = a$ where a and b are any natural numbers.

This leads to the operation of *subtraction*, or *inverse of addition*, and we write $x = a - b$.

The set of positive and negative integers and zero is called the set of *integers* and is closed under the operations of addition, multiplication, and subtraction.

The Real Number System

(3) **Rational numbers** or $fractions$ such as $3/4, -8/3, 1/2$ ... permit solutions of equations such as $bx = a$ for all integers a and b where $b \neq 0$. This leads to the operation of *division* or *inverse of multiplication*, and we write $x = a/b$ or $a \div b$ (called the $quotient$ of a and b) where a is the $numerator$ and b is the $denominator$.

The set of integers is a part or subset of the *rational numbers*, since integers correspond to rational numbers a/b where $b = 1$.

The set of rational numbers is closed under the operations of addition, subtraction, multiplication, and division, so long as division by zero is excluded.

The Real Number System

(4) **Irrational numbers** such as $\sqrt{2}$ and π are numbers that cannot be expressed as a/b where a and b are integers and $b \neq 0$.

The set of rational and irrational numbers is called the set of *real numbers*.

It is assumed that the student is already familiar with the various operations on *real numbers*.

Graphical Representation of Real Numbers

Real numbers can be represented by points on a line called the *real axis*, as indicated in the following figure. The point corresponding to zero is called the *origin*.

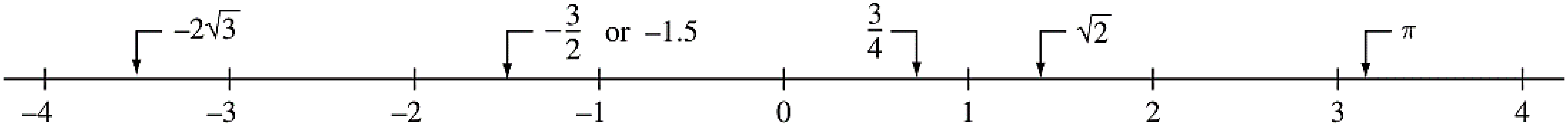

Figure: Number Line

Graphical Representation of Real Numbers

- Conversely, to each point on the line there is one and only one real number. If a point A corresponding to a real number a lies to the right of a point B corresponding to a real number b, we say that a is greater than b or b is less than a and write $a > b$ or $b < a$, respectively.

- The set of all values of x such that $a < x < b$ is called an *open interval* on the real axis while $a \leq x \leq b$, which also includes the endpoints a and b, is called a *closed interval*.

- The symbol x, which can stand for any real number, is called a *real variable*.

- The *absolute value* of a real number a, denoted by $|a|$, is equal to a if $a > 0$, to $-a$ if $a < 0$ and to 0 if $a = 0$.

- The distance between two points a and b on the real axis is $|a - b|$.

The Complex Number System

- There is no real number x that satisfies the polynomial equation $x^2 + 1 = 0$. To permit solutions of this and similar equations, the set of *complex numbers* is introduced.

- We can consider a *complex number* as having the form $a + bi$ where a and b are real numbers and i, which is called the *imaginary unit*, has the property that $i^2 = -1$. If $z = a + bi$, then a is called the *real part* of z and b is called the *imaginary part* of z and are denoted by $Re\{z\}$ and $Im\{z\}$, respectively.

- The symbol z, which can stand for any complex number, is called a *complex variable*.

The Complex Number System

- Two complex numbers $a + bi$ and $c + di$ are *equal* if and only if $a = c$ and $b = d$.
- We can consider real numbers as a subset of the set of complex numbers with $b = 0$.
- Accordingly the complex numbers $0 + 0i$ and $-3 + 0i$ represent the real numbers 0 and -3, respectively.
- If $a = 0$, the complex number $0 + bi$ or bi is called a *pure imaginary number*.
- The *complex conjugate*, or briefly *conjugate*, of a complex number $a + bi$ is $a - bi$. The complex conjugate of a complex number z is often indicated by $\bar{z}$ or z^*.

Fundamental Operations with Complex Numbers

In performing operations with complex numbers, we can proceed as in the algebra of real numbers, replacing i^2 by -1 when it occurs.

(1) Addition

$$(a+bi)+(c+di)=a+bi+c+di=(a+c)+(b+d)i$$

(2) Subtraction

$$(a+bi)-(c+di)=a+bi-c-di=(a-c)+(b-d)i$$

Fundamental Operations with Complex Numbers

(3) Multiplication

$$(a+bi)(c+di) = ac + adi + bci + bdi^2 = (ac - bd) + (ad + bc)i$$

(4) Division

If $c \neq 0$ and $d \neq 0$, then

$$\frac{a+bi}{c+di} = \frac{a+bi}{c+di} \cdot \frac{c-di}{c-di}$$

$$= \frac{ac - adi + bci - bdi^2}{c^2 - d^2 i^2}$$

$$= \frac{ac + bd + (bc - ad)i}{c^2 + d^2} = \frac{ac+bd}{c^2+d^2} + \frac{bc-ad}{c^2+d^2} i$$

Next Lecture

- Graphical Representation of Complex Numbers
- Polar Form of Complex Numbers
- De Moivre’s Theorem
- Vector Interpolation of Complex Number