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Christian-Albrechts-Universität zu Kiel
Faculty of Engineering

Intelligent Systems

Exercise 9 - Classification and Anomalies

Simon Reichhuber

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DBSCAN and Outlier Detection

A. **Calculate the *Local Outlier Factor* (*LOF*) of the points A_1 and N in the figure**

B. Draw the distribution of ascending minimum *kdists* of every point with $k = 1, 2, 3$.

C. How can you estimate the parameter ϵ by given a percentage of noise?

D. Find parameters $\epsilon > 0$, min_pts $\in \mathbb{N}$ s.t.
   - $A_i, i = 1, 2, 3$ is clustered as a cluster
   - $B_j, j = 1, 2$ is clustered as a cluster
   - N is marked as noise.

E. Find parameters $\epsilon > 0$, min_pts $\in \mathbb{N}$, and points C_k s.t.
   - $A_i, i = 1, 2, 3, B_j, j = 1, 2$ is clustered as a cluster.
   - N is marked as noise.

Calculate the *Local Outlier Factor* (*LOF*) of the points A_1 and N in the figure

What exactly is the *Local Outlier Factor?*

Novelty detection algorithms (11)

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LOF – Equations

- k = number of neighbours considered
- $kdist(x)$ = distance to k-th neighbour
- $N_k(x) =$ ordered set of k-nearest neighbours of x
- $reachability\_dist_k(x, y) = \max\big(kdist(y), dist(x, y)\big)$
- $lrd_k(x) = \frac{k}{\sum_{y \in N_k(x)} reachability\_dist_k(x,y)}$ = local reachability density
- $LOF_k(x) = \frac{\sum_{y \in N_k(x)} \frac{lrd_k(y)}{lrd_k(x)}}{k}$

Metric undefined $\rightarrow$ Choose Euclidean metric

$dist(x, y) = ||x - y||_2$

For $k = 1$ it follows:

$$1 - dist(A_1) = 1$$

$$N_1(A_1) = \{first : A_2\}$$

$$lrd_1(A_1) = \frac{1}{\sum_{y \in N_1(A_1)} reachability\_dist_1(A_1, y)}$$

$$= \frac{1}{reachability\_dist_1(A_1, A_2)} = \frac{1}{max(1 - dist(A_2), dist(A_1, A_2))}$$

$$= \frac{1}{max(1, 1)} = 1$$

Remark:

For $k = 1, k - dist$ is noted as $1 - dist$.

$$LOF_1(A_1) = \frac{\sum_{y \in N_1(A_1)} \frac{lrd_1(y)}{lrd_1(A_1)}}{1} = \frac{1}{\sum_{y \in N_1(A_2)} reachability\_dist_1(A_2, y)}$$

$$= \frac{1}{\sum_{y \in \{first:A_1\}} reachability\_dist_1(A_2, y)}$$

$$= \frac{1}{reachability\_dist_1(A_2, A_1)}$$

$$= \frac{1}{max(1 - dist(A_1), dist(A_2, A_1))} = \frac{1}{max(1, 1)} = 1$$

Remark:

The function $reachability\_dist_k(x, y)$ is non-symmetric, because $kdist(x, y)$ is non-symmetric.

$$LOF_1(N) = \frac{\sum_{y \in N_1(N)} \frac{lrd_1(y)}{lrd_1(N)}}{1}$$

$$\sum_{y \in \{first:A_2\}} \frac{lrd_1(y)}{lrd_1(N)}$$

$$\frac{lrd_1(A_2)}{lrd_1(N)} = \frac{\sum_{y \in N_1(N)} reachability\_dist_1(N, y)}{\sum_{y \in N_1(A_2)} reachability\_dist_1(A_2, y)}$$

$$= \frac{reachability\_dist_1(N, A_2)}{reachability\_dist_1(A_2, A_1)}$$

$$= \frac{max(1 - dist(A_2), dist(N, A_2))}{max(1 - dist(A_1), dist(A_2, A_1))}$$

$$= \frac{max(1, 3)}{max(1, 1)} = 3$$

→ Because $LOF_1(A_1) = 1$ and $LOF_1(N) = 3$ it is more likely that N is an outlier.

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For $k = 2$ it follows:

$$2-dist(A_1) = \sqrt{2}$$

$$N_2(A_1) = \{first : A_2, second : A_3\}$$

$$lrd_2(A_1) = \frac{2}{\sum_{y \in N_2(A_1)} reachability\_dist_2(A_1, y)}$$

$$= \frac{2}{\sum_{y \in \{first:A_2, second:A_3\}} reachability\_dist_2(A_1, y)}$$

$$= \frac{2}{reachability\_dist_2(A_1, A_2) + reachability\_dist_2(A_1, A_3)}$$

$$= \frac{2}{max(2-dist(A_2), dist(A_1, A_2)) + max(2-dist(A_3), dist(A_1, A_3))}$$

$$= \frac{2}{max(1, 1) + max(\sqrt{2}, \sqrt{2})} = \frac{2}{1+\sqrt{2}} \approx 0.828$$

1. A LOF CALCUATIONS (6)

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$$
\begin{aligned}
lrd_2(A_2) &= \frac{2}{\sum_{y \in N_2(A_2)} reachability\_dist_2(A_2, y)} \\
&= \frac{2}{\sum_{y \in \{first: A_1, second: A_3\}} reachability\_dist_2(A_2, y)} \\
&= \frac{2}{max(2 - dist(A_1), dist(A_2, A_1)) + max(2 - dist(A_3), dist(A_2, A_3))} \\
&= \frac{2}{max(\sqrt{2}, 1) + max(\sqrt{2}, 1)} = \frac{1}{\sqrt{2}} \approx 0.707
\end{aligned}
$$

1. A LOF calculations (7)

$$lrd_2(A_3) = \frac{2}{\sum_{y \in N_2(A_3)} reachability\_dist_2(A_3, y)}$$

$$lrd_2(A_3) = \frac{2}{\sum_{y \in \{first: A_2, second: A_1\}} reachability\_dist_2(A_3, y)}$$

$$lrd_2(A_3) = \frac{2}{max(2 - dist(A_2), dist(A_3, A_2)) + max(2 - dist(A_1), dist(A}$$

$$= \frac{2}{max(1, 1) + max(\sqrt{2}, \sqrt{2})} = \frac{2}{1 + \sqrt{2}} \approx 0.828$$

1. A LOF calculations (8)

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$$
\begin{aligned}
LOF_2(A_1) &= \frac{\sum_{y \in N_2(A_1)} \frac{lrd_2(y)}{lrd_2(A_1)}}{2} \\
&= \frac{\sum_{y \in \{first:A_2, second:A_3\}} \frac{lrd_2(y)}{lrd_2(A_1)}}{2} \\
&= \frac{\frac{lrd_2(A_2)}{lrd_2(A_1)} + \frac{lrd_2(A_3)}{lrd_2(A_1)}}{2} \\
&= \frac{\frac{0.707}{0.828} + \frac{0.828}{0.828}}{2} \approx 0.439
\end{aligned}
$$

$$2-dist(N)=3$$

$$N_2(N)=\{first:A_2, second:B_1\}$$

$$lrd_2(N)=\frac{2}{\sum_{y\in N_2(N)} reachability\_dist(N,y)}$$

$$=\frac{2}{\sum_{y\in\{first:A_2,second:B_1\}} reachability\_dist_2(N,y)}$$

$$=\frac{2}{max(2-dist(A_2),dist(N,A_2))+max(2-dist(B_1),dist(N,B_1))}$$

$$=\frac{2}{max(1,3)+max(3,3)}=\frac{1}{3}$$

1. A LOF calculations (10)

$$lrd_2(A_2) \approx 0.707$$

$$lrd_2(B_1) = \frac{2}{\sum_{y \in N_2(B_1) reachability\_dist_2(B_1, y)}}$$

$$= \frac{2}{\sum_{y \in \{first:B_2, second:N\}} reachability\_dist(B_1, y)}$$

$$= \frac{2}{max(2 - dist(B_2), dist(B_1, B_2)) + max(2 - dist(N), dist(B_1, N))}$$

$$= \frac{2}{max(4, 1) + max(3, 3)} = \frac{2}{7}$$

1. A LOF CALCULATIONS (11)

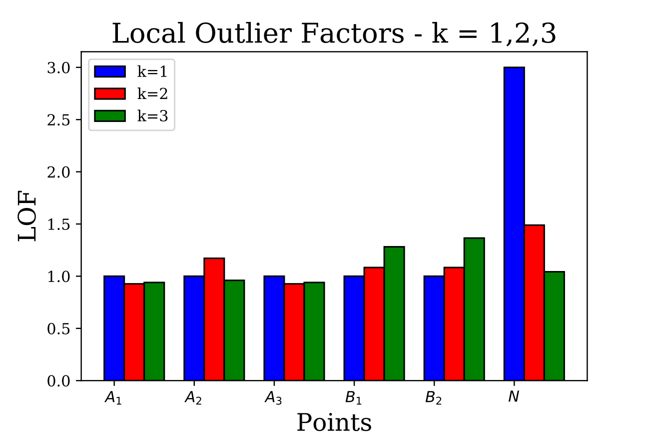
$$LOF_2(N) = \frac{\sum_{y \in N_2(N)} \frac{lrd_2(y)}{lrd_2(N)}}{2}$$

$$= \frac{\sum_{y \in \{first:A_2, second:B_1\}} \frac{lrd_2(y)}{lrd_2(N)}}{2}$$

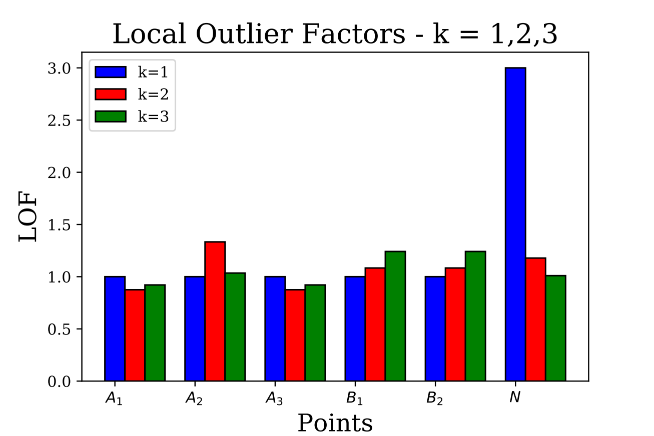
$$= \frac{\frac{0.707}{\frac{1}{3} + \frac{\frac{2}{7}}{\frac{1}{3}}}}{2} \approx 1.489$$

$\rightarrow LOF_2(A_1) < LOF_2(N)$, i.e. $0.439 < 1.489$, hence N is more likely an outlier.

Choosing Euclidean distance metric:



Choosing Euclidean distance metric:



A. Calculate the *Local Outlier Factor* (*LOF*) of the points A_1 and N in the figure
B. Draw the distribution of ascending minimum *kdists* of every point with $k = 1, 2, 3$.
C. How can you estimate the parameter ϵ by given a percentage of noise?
D. Find parameters $\epsilon > 0$, min_pts $\in \mathbb{N}$ s.t.
    - $A_i, i = 1, 2, 3$ is clustered as a cluster
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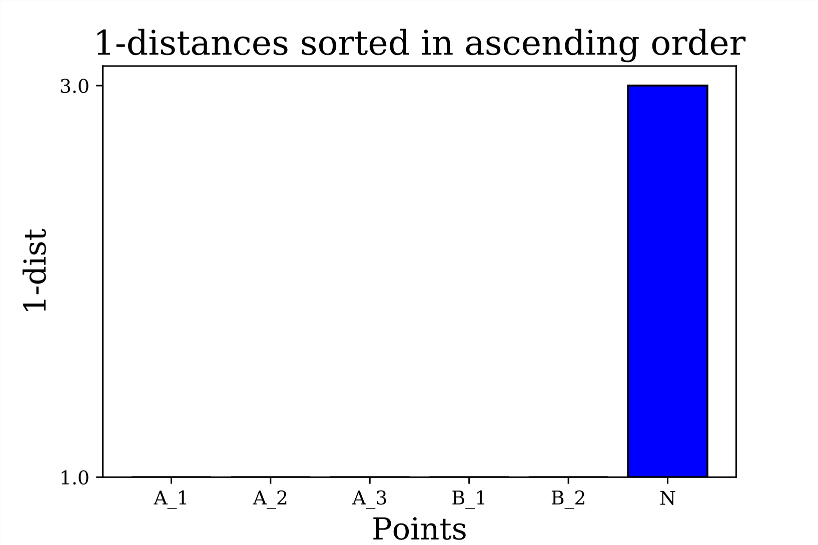
E. Find parameters $\epsilon > 0$, min_pts $\in \mathbb{N}$, and points C_k s.t.
    - $A_i, i = 1, 2, 3, B_j, j = 1, 2$ is clustered as a cluster.
    - N is marked as noise.

Draw the distribution of ascending minimum *kdists*

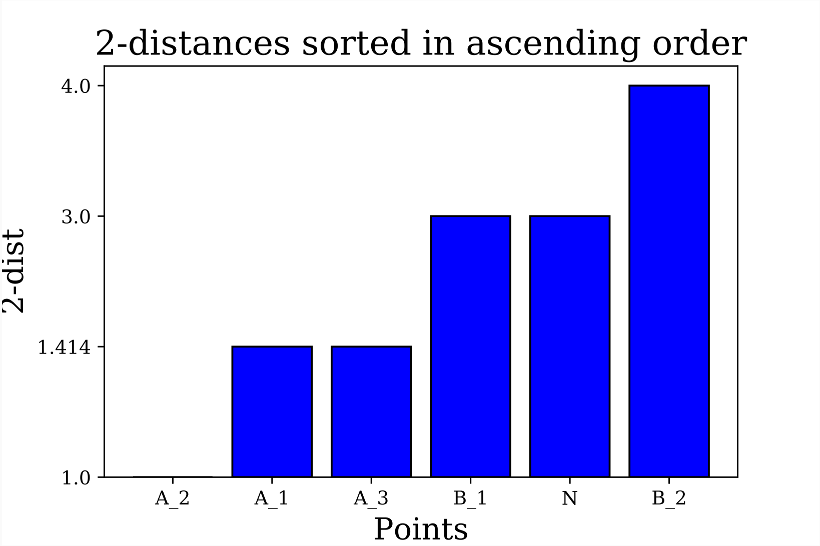
$$min_y(kdist(x, y)),$$

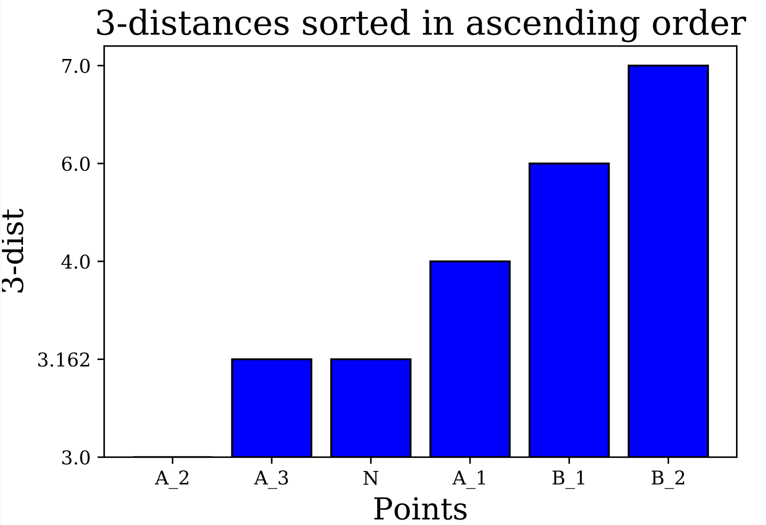
of every point with $k = 1, 2, 3$.

1. B 1-DISTANCES



Find the proportion of noise by using the ellbow method.


2-distances sorted in ascending order
2-dist
4.0
3.0
1.414
1.0
A_2
A_1
A_3
B_1
N
B_2
Points


3-distances sorted in ascending order
3-dist
7.0
6.0
4.0
3.162
3.0
A_2
A_3
N
A_1
B_1
B_2
Points

A. Calculate the *Local Outlier Factor* (*LOF*) of the points A_1 and N in the figure

B. Draw the distribution of ascending minimum *kdists* of every point with $k = 1, 2, 3$.

C. How can you estimate the parameter ϵ by given a percentage of noise?

D. Find parameters $\epsilon > 0$, min_pts $\in \mathbb{N}$ s.t.
- $A_i, i = 1, 2, 3$ is clustered as a cluster
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E. Find parameters $\epsilon > 0$, min_pts $\in \mathbb{N}$, and points C_k s.t.
- $A_i, i = 1, 2, 3, B_j, j = 1, 2$ is clustered as a cluster.
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How can you estimate the parameter ϵ by given a percentage of noise?

The parameter ϵ can be determined by using the the sorted $k - distances$. Points with $k - distance > \epsilon$ will be treated as noise. In the following the noise proportion is set to $1/6$

k=1:

$\epsilon = 2$

k=2:

$\epsilon = 3.5$

k=3:

$\epsilon = 6.5$

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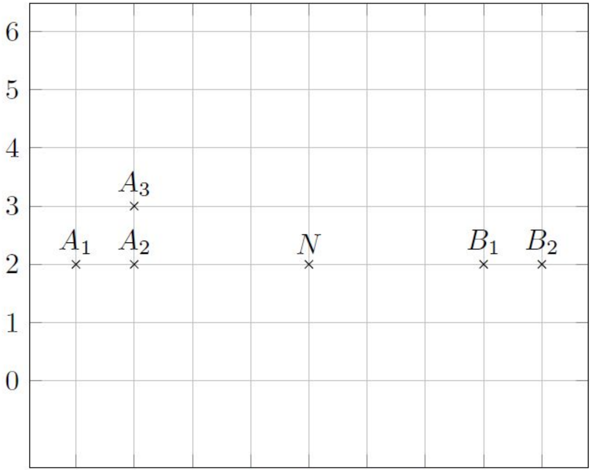
D. Find parameters $\epsilon > 0$, min_pts $\in \mathbb{N}$ s.t.
   - $A_i, i = 1, 2, 3$ **is clustered as a cluster**
   - $B_j, j = 1, 2$ **is clustered as a cluster**
   - N **is marked as noise.**
E. Find parameters $\epsilon > 0$, min_pts $\in \mathbb{N}$, and points C_k s.t.
   - $A_i, i = 1, 2, 3, B_j, j = 1, 2$ is clustered as a cluster.
   - N is marked as noise.

Find parameters $\epsilon > 0$, min_pts $\in \mathbb{N}$ s.t.
Cluster_A $= \{A_1, A_2, A_3\}$, Cluster_B $= \{B_1, B_2\}$, and N is marked as noise.



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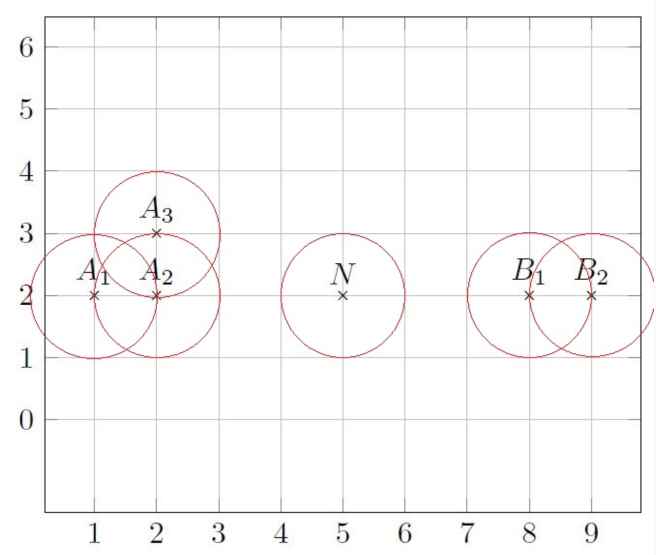
Choose Euclidean metric, *min_pts* $= 1, \epsilon = 1$



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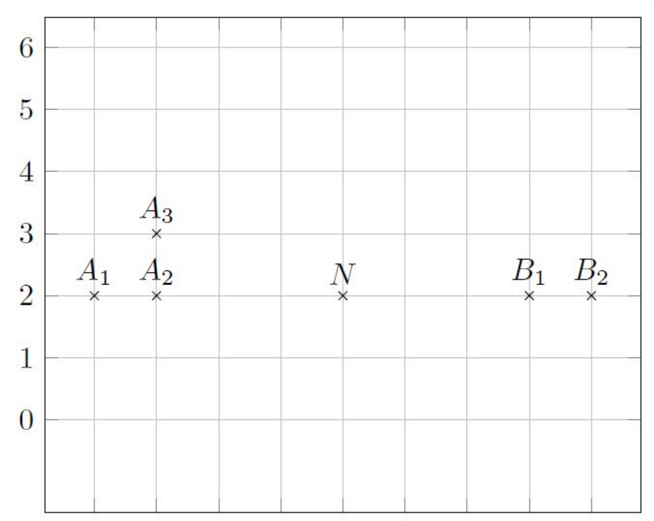
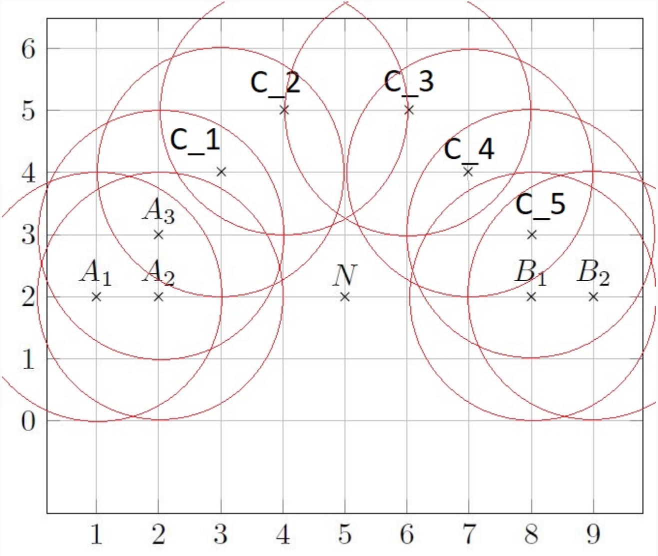
E. **Find parameters $\epsilon > 0$, min_pts $\in \mathbb{N}$, and points C_k s.t.**
   - $A_i, i = 1, 2, 3, B_j, j = 1, 2$ **is clustered as a cluster.**
   - N **is marked as noise.**

Find parameters $\epsilon > 0$, min_pts $\in \mathbb{N}$, and points $C_k \in \mathbb{R}^2$ s.t. *Cluster* $= \{A_1, A_2, A_3, B_1, B_2\}$, and N is marked as noise.



Choose Euclidean metric, *min_pts* $= 2, \epsilon = 2$



Classification algorithms

A. **Observe the data set in the table. First, create a 1-R Classifier that is able to predict whether a person is going to visit the party this evening by using the information of his/her amount of money, whether he/she writes an exam tomorrow, or if his/her heartthrob will come to the party.**
B. Extend your 1-R Classifier to a Decision Tree. Which features should be placed on higher levels of the tree?
C. Apply the Naïve Bayes Classifier on the same data set. Calculate also the probabilities $P(Yes|E1)$ and $P(No|E6)$.

2. A

Observe the data set in the table. First, create a 1-R Classifier that is able to predict whether a person is going to visit the party this evening by using the information of his/her amount of money, whether he/she writes an exam tomorrow, or if his/her heartthrob will come to the party.

Sample	**Money**	**Exam**	**Heartthrob**	**Party**
E1	10	Yes	Yes	Yes
E2	13	No	Yes	Yes
E3	11	Yes	No	No
E4	12	No	No	Yes
E5	7	Yes	Yes	Yes
E6	5	Yes	No	No
E7	6	No	Yes	Yes
E8	8	No	No	No

Sample	Manifestation	Yes	No
Money	>9	3	1
Money	<=9	2	2
Exam	Ja	2	2
Exam	Nein	3	1
Heartthrob	Ja	4	0
Heartthrob	Nein	1	3

Figure 6: 1R Feature selection

Money	>9	3	1
Money	<=9	2	2
Exam	Ja	2	2
Exam	Nein	3	1
Heartthrob	Ja	4	0
Heartthrob	Nein	1	3

Figure 7: 1R Feature selection

The feature Heartthrob produces a minimal number of prediction errors (1).

Rule: If Heartthrob == ”Yes” then Party, elsif Heartthrob == ”No” then no Party.

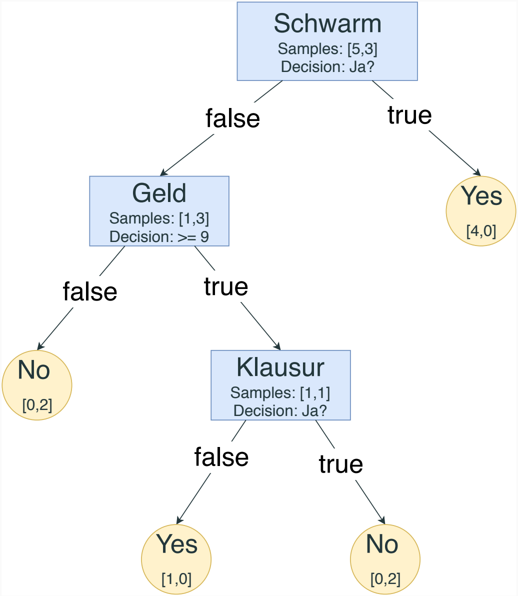
2. B

A. Observe the data set in the table. First, create a 1-R Classifier that is able to predict whether a person is going to visit the party this evening by using the information of his/her amount of money, whether he/she writes an exam tomorrow, or if his/her heartthrob will come to the party.

B. Extend your 1-R Classifier to a Decision Tree. Which features should be placed on higher levels of the tree?

C. Apply the Naïve Bayes Classifier on the same data set. Calculate also the probabilities $P(Yes|E1)$ and $P(No|E6)$.

Extend your 1-R Classifier to a Decision Tree. Which features should be placed on higher levels of the tree?


Schwarm
Samples: [5,3]
Decision: Ja?
false
true
Geld
Samples: [1,3]
Decision: >= 9
Yes
[4,0]
false
true
No
[0,2]
Klausur
Samples: [1,1]
Decision: Ja?
false
true
Yes
[1,0]
No
[0,2]

A. Observe the data set in the table. First, create a 1-R Classifier that is able to predict whether a person is going to visit the party this evening by using the information of his/her amount of money, whether he/she writes an exam tomorrow, or if his/her heartthrob will come to the party.

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Apply the Naïve Bayes Classifier on the same data set.
Calculate also the probabilities $P(Yes|E1)$ **and** $P(No|E6)$**.**

Apply Bayes Rule on **P(Ja|E1)**=

- $\frac{P(E1|Yes)*P(Yes)}{P(E1)} = \frac{P(Money=Viel|Yes)*P(Exam=Yes|Yes)*P(Heartthrob=Yes|Yes)*P(Yes)}{P(E1|Yes)*P(Yes)+P(E1|No)*P(No)}$
- $\frac{\frac{3}{5}*\frac{2}{5}*\frac{4}{5}*\frac{5}{8}}{P(E1|Yes)+P(E1|No)*P(No)}$
- $\frac{\frac{3}{25}}{\frac{3}{25}+P(E1|No)*P(No)}$

And for **P(No|E1)** =

- $\frac{\frac{1}{3}*\frac{2}{3}*\frac{0}{3}*\frac{5}{8}}{\frac{3}{25}+P(E1|No)*P(No)}$

Zero Frequency Problem: If any manifestation is missing in the data, we virtually add an artificial value to it and assume that it occurs at least one time. Usually, the value is one.

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Again, use Bayes rule: **P(Yes|E1)** =

- $\frac{\frac{3+1}{5+1}*\frac{2+1}{5+1}*\frac{4+1}{5+1}*\frac{5}{8}}{P(E1|Yes)*P(Yes)+P(E1|No)*P(No)}$
- $\frac{\frac{25}{144}}{\frac{25}{144}+\frac{9}{256}} = 0.832$

And for **P(No|E1)** =

- $\frac{\frac{1+1}{3+1}*\frac{2+1}{3+1}*\frac{0+1}{3+1}*\frac{3}{8}}{\frac{25}{144}+P(E1|No)*P(No)}$
- $\frac{\frac{9}{256}}{\frac{25}{144}+\frac{9}{256}} = 0.168$

Use Bayes rule: **P(No|E6)** =

- $\frac{\frac{2+1}{3+1} * \frac{1+1}{3+1} * \frac{3+1}{3+1} * \frac{3}{8}}{P(E6|No) * P(No) + P(E6|Yes) * P(Yes)}$
- $\frac{\frac{9}{64}}{\frac{9}{64} + \frac{5}{96}} = 0.73$

And for **P(Yes|E6)** =

- $\frac{\frac{2+1}{5+1} * \frac{2+1}{5+1} * \frac{1+1}{5+1} * \frac{5}{8}}{\frac{9}{64} + P(E1|Yes) * P(Yes)}$
- $\frac{\frac{5}{96}}{\frac{9}{64} + \frac{5}{96}} = 0.27$