

Neural Networks and Deep Learning – Summer Term 2018

Exercise sheet 2

Submission due: Tuesday, May 08, 11:30 sharp

Exercise 1 (Single-layer perceptron and Boolean functions with 2 inputs):

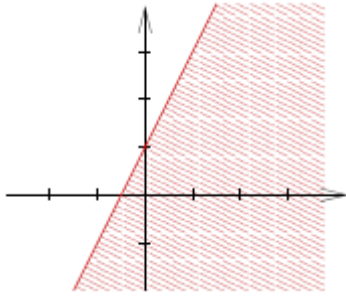
- a) Show that the Boolean function XOR cannot be realized by a (single-layer) perceptron (with 2 inputs).

Note: The output y of a single-layer perceptron with 2 inputs x_1 and x_2 , threshold θ and weights w_1 and w_2 is given by

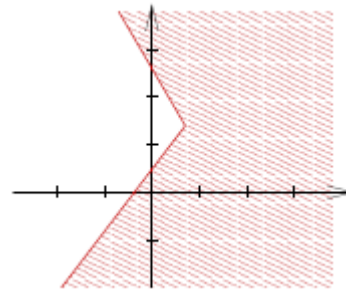
$$y = \Theta[x_1 w_1 + x_2 w_2 - \theta] \quad (\Theta \text{ is the Heaviside function})$$

- b) Give all Boolean functions with 2 inputs (i.e. for each Boolean function: the output for each input combination) and indicate whether they can be realized by a (single-layer) perceptron.
- a) Select three Boolean functions with two inputs and give values for the synaptic weights w_1 , w_2 and threshold θ so that the Boolean function is realized by a single-layer perceptron. Show for each of the three Boolean functions and each input pair that the Boolean function is indeed realized by the chosen combination of weights and threshold.
- d) Which of the following partitioning of \mathbb{R}^2 can be realized by a single-layer perceptron with two inputs? For those that can be realized, give weights and threshold of the perceptron. (Consider abscissa as x_1 and ordinate as x_2).

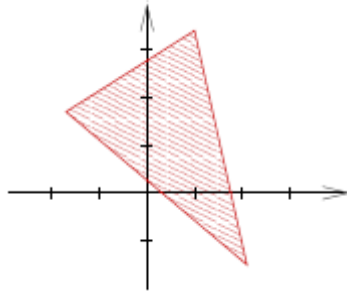
i)



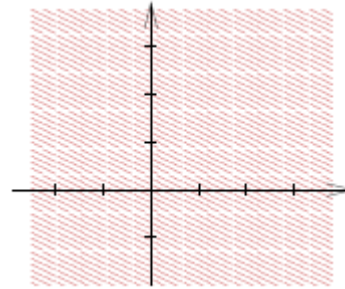
ii)



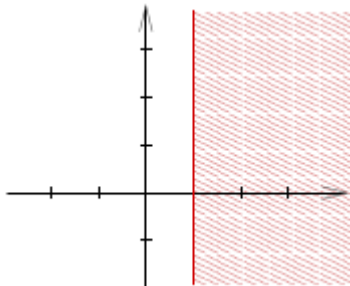
iii)



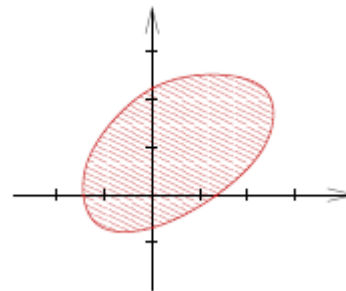
iv)



v)



vi)



(From: Riedmiller)

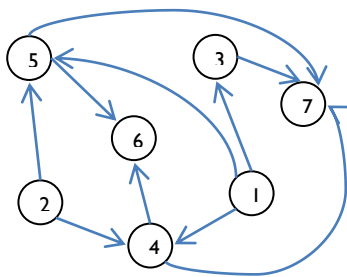
Exercise 2 (Types of neural networks, synaptic weight matrix):

a) Explain the following terms related to neural networks:

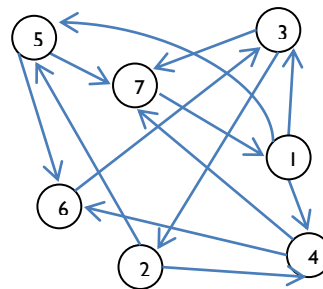
- Boolean function
- Feedforward neural network
- Recurrent neural network
- Multi-layer perceptron

b) Specify whether the following artificial neural networks are feedforward or recurrent neural networks and explain your selection.

i)



ii)

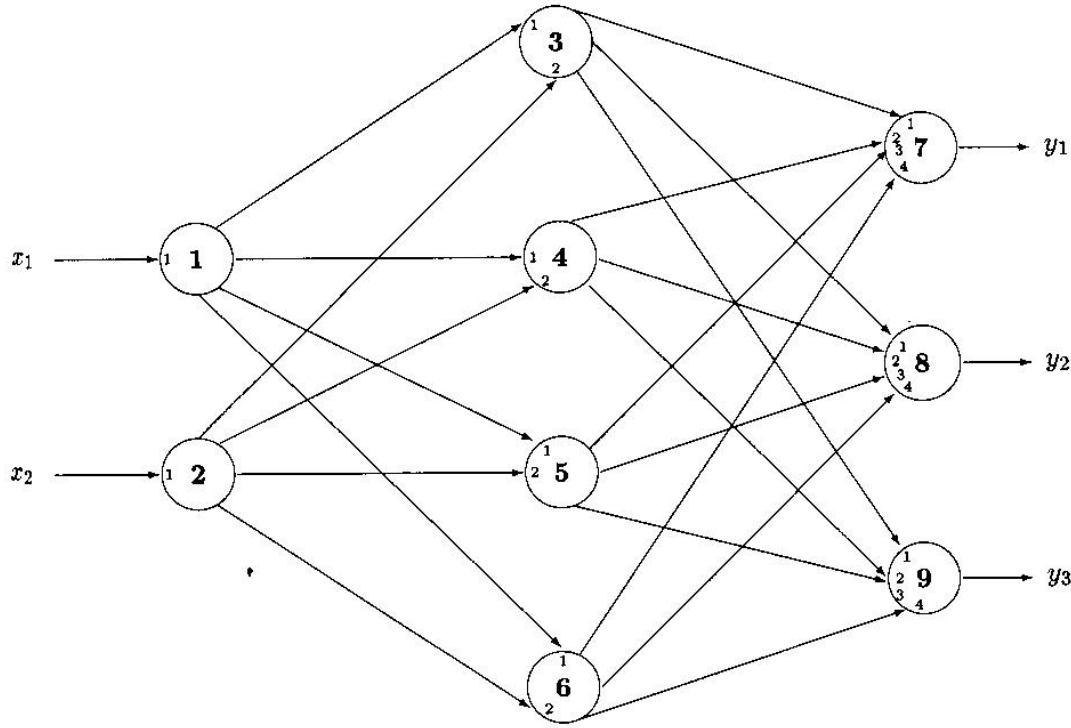


c) Using the neuron numbers from 1 to 7 given in the circles, fill out the following general weight matrix by marking the corresponding field entries. Example: Mark the field in row i and column j (weight w_{ij}) if there is a connection from neuron j to neuron i .

	1	2	3	4	5	6	7
1							
2							
3							
4							
5							
6							
7							

Exercise 3 (Computing the output of a feedforward neural network):

- a) Compute the output of the following feedforward neural network for the input $x_1=3; x_2=1$. Which neurons can be computed in parallel, which have to wait?



Note: The small numbers in each circle correspond to the components of the weight vector; see example below. In this part of the exercise, the threshold is set to $\theta=0$ for all neurons.

Neuron	Activation function of neuron	Weight vector
1	Linear; $c=1$	(1)
2		(1)
3	Threshold element; $\theta=0$	(1,-2)
4		(-1,0)
5		(3,2)
6		(0,2)
7	Linear; $c=1$	(0,2,-3,1)
8		(1,-2,3,8)
9		(0,2,3,-4)

c is the slope of the linear activation function: $f(h) = c \cdot h$

“Threshold element” means that the activation function is the Heaviside function

Example for weight vector of neuron 8:

1st component of weight vector (1) refers to connection neuron 3 \rightarrow neuron 8

2nd component of weight vector (-2) refers to connection neuron 4 \rightarrow neuron 8

3rd component of weight vector (3) refers to connection neuron 5 \rightarrow neuron 8

4th component of weight vector (8) refers to connection neuron 6 \rightarrow neuron 8

(Source: Stefan Hartmann, Cesar Research)

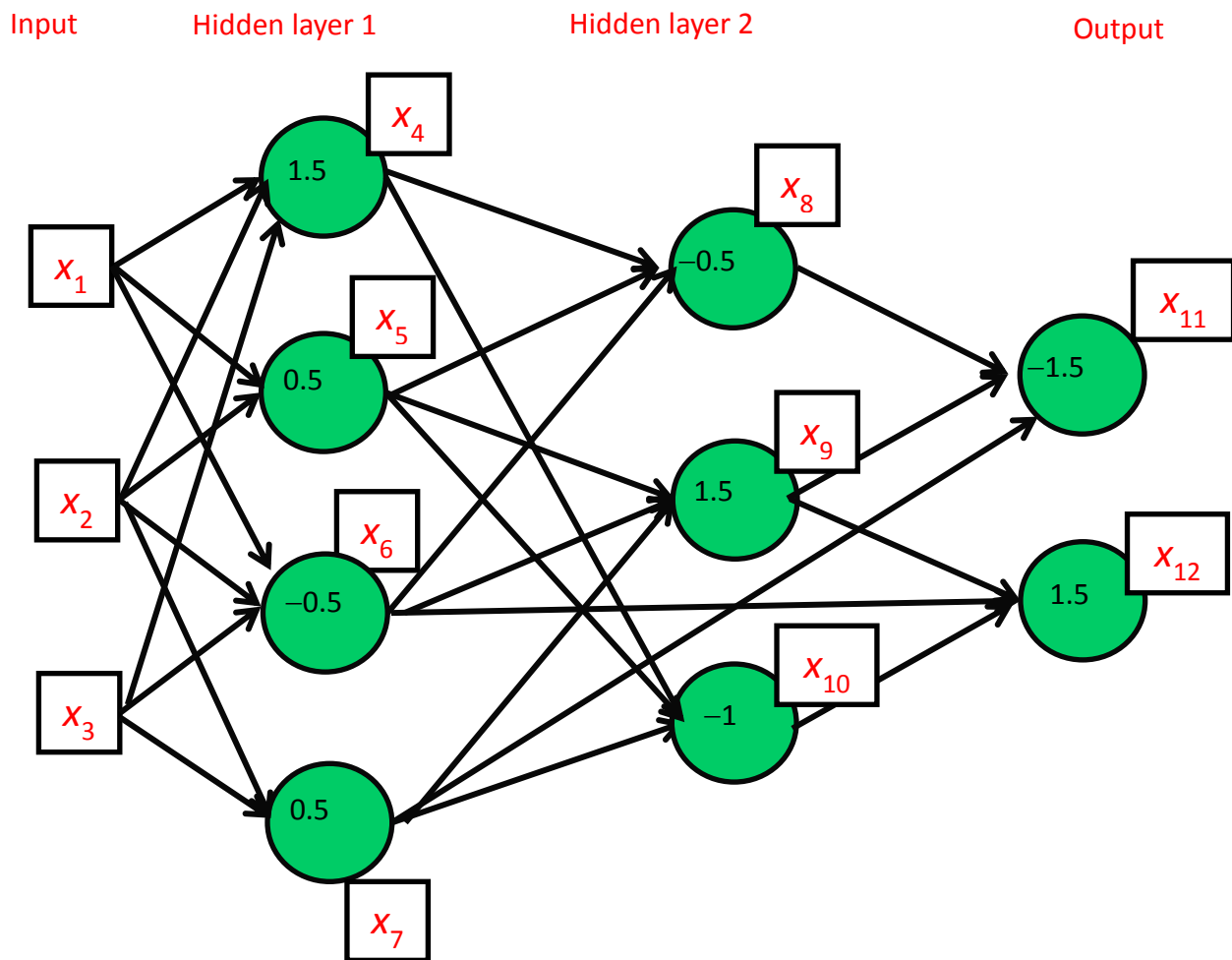
b) Assume the following weight matrix, where an entry w_{ij} (i th row, j th column) corresponds to the synaptic weight from neuron j to neuron i . (No entry means the synaptic weight is 0). Further assume that the activation function of the neurons of hidden layer 2 (neurons 8, 9 and 10) is linear (with slope $c=1$), whereas the activation function of all other neurons is a Heaviside step function. In this part of the exercise, the threshold θ of each node is indicated in the network graph as number in the corresponding neuron.

Compute the output of the following feedforward neural network for the inputs $x_1=1$, $x_2=0$, $x_3=1$ and $x_1=0$, $x_2=1$, $x_3=1$.

Weight matrix:

	1	2	3	4	5	6	7	8	9	10	11	12
1												
2												
3												
4	-2	5	-4									
5	1	-2										
6	3	-1	6									
7		7	1									
8				-1	4	-2						
9					-3	5	1					
10				8	2		-3					
11							6	1	-2			
12						1			-4	3		

Network:



(Note: this is a feedforward neural network of second order)

Exercise 4 (Multi-layer perceptron and XOR):

a) Find a multi-layer perceptron which realizes the Boolean function XOR. Demonstrate that the found perceptron indeed performs XOR on all possible input pairs.

b) Find a perceptron with two (binary) inputs which realizes the function

$$F(x_1, x_2) = \begin{cases} 1 & : x_1 + x_2 = 1 \\ 0 & : \text{else} \end{cases}$$

Note: “+” denotes mathematical addition.

