Astronomy
Problem of The Day

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Note. I would like to express my heartfelt gratitude to #Bombom, who initiated this work—all problem-making credit goes to him. My role is merely to convert it into a LaTeX document for enhanced readability while providing detail/alternative solutions where possible, along with some insightful remarks.

Most *Day* problems are relatively easy and can be solved with national-level preparation, making them suitable for IOAA Junior. However, every *PoTW* problem is at the IOAA level, typically ranging between $\boldsymbol{T6 - T10}$.

The original work can be found here: https://www.instagram.com/astro.potd/

Contents

Topic Wise Contents

1 Week 1

Day 1

It is known that a spacecraft is orbiting its parent star. The orbit of this spacecraft has a semi-major axis (a) of 0.8 AU, and a semi-minor axis (b) of 0.6 AU. It is also known that the mass of the parent star is 2 $M_\odot$. Determine the velocity of the spacecraft at its' periastron. [Solution]

Day 2

James is an astronomer trying to measure the distance of a star to our very own star, The Sun. He measured the angle of parallax to be $0.03''$. Use the information above to determine this star's distance to The Sun! [Solution]

Day 3

Sirius **A** is an A-type star located 8.6 lightyears away. It is known to have a radius of 1.71 $R_\odot$. The Sun has a magnitude of -26.8, the Solar Constant ($S_\odot$) is $1370\,\mathrm{W/m^2}$, and the Solar Radius ($R_\odot$) is 6.96×10^8 meters. Given that Sirius **A**'s magnitude is -1.61, determine its' effective temperature in terms of solar effective temperature. [Solution]

Day 4

Suppose we are living in a flat universe with $H_0 = 70$ km/s/Mpc, $T_0 = 2.73$ K , and composed of matter ($\Omega_{m,0} = 0.25$), radiation ($\Omega_{r,0} = 8.4 \times 10^{-5}$), and dark energy. Calculate the Hubble Parameter (H) at the time of recombination era, given that Hydrogen recombination occurs at $T = 3000$ K. [Solution]

Day 5

An observer on Earth observes a star and finds that the peak of the spectrum is at 410 nm, and the Hydrogen-α line is at 842 nm. Bombom lives on a planet orbiting that star. Calculate the surface brightness of that star as seen by Bombom.
[Solution]

Day 6

Jack is a 15-year-old who is very interested in Astronomy, so he tries to study where the stars will rise and where the stars will set. In doing so, he chose his favorite star, Rigel (β Ori). He knows that Rigel has an equatorial coordinate of $(05^h14^m32.3^s, -08°12'05.9'')$. Given that Jack lives in Melbourne ($37°50'27.3660''$ S, $144°56'47.2452''$ E), and the date is currently 10th of April, Determine the civil time and Azimuth of Rigel when it rises. [Solution]

Day 7: Problem of The Week 1

The Great Square of Pegasus is an Asterism that consists of four stars

$$\begin{aligned}
\alpha \text{ Peg: } & 23^h05^m,\ 15°18' \\
\beta \text{ Peg: } & 23^h04^m,\ 28°11' \\
\gamma \text{ Peg: } & 0^h14^m,\ 15°17' \\
\delta \text{ Peg: } & 23^h09^m,\ 29°11'
\end{aligned}$$

Find the coordinates for the intersection of the two diagonals. [Solution]

2 Week 2

Day 8

Star **A** has a mass of 3 $M_\odot$. and a radius of 2 $R_\odot$. Assuming the density of Star **A** is homogenous. Determine the pressure in the core of Star **A**.
[Solution]

Day 9

Name the Constellations! [Solution]

Day 10

The Galactic longitude of a star is $l = 34°$. Its radial velocity concerning the Sun is $v_r = 76$ km/s. Assuming that the stars in the disk of the Galaxy are orbiting around the galactic center in circular orbits, the Sun is 8 kpc from the center of the Galaxy. It has a constant velocity of $v_0 = 250$ km/s. Calculate the distance of the star from the center of the Galaxy.
[Solution]

Day 11

Consider a smooth runway along the moon's equator. A rocket ship began to move in an attempt to take off along the runway. Even though there is no atmosphere on the moon to lift the plane's wings, at some point the plane starts to lift off the runway. What was the velocity of the craft at that time (relative to the Moon's ground surface)? [Solution]

Day 12

The Galactic Center is believed to contain a supermassive black hole with a mass $M = 4 \times 10^6\, M_\odot$. The astronomy community is trying to resolve its event horizon, which is a challenging task. For a non-rotating black hole, this is the Schwarzschild radius. Assume that we have an Earth-sized telescope (using Very Long Baseline Interferometry). What wavelengths should we adopt in order to resolve the event horizon of the black hole? [Solution]

Day 13

The apparent magnitude of the galaxy is 9, and the absolute magnitude is -25 mag. If there is an absorption of 0.15 mag/kpc. Determine the distance to the galaxy. [Solution]

Day 14: PoTW 2

Alice is looking at the sky. To her west, there is an infinitely long wall, 5 meters in height and 3 meters away from Alice, going from north to south. Alice observes that star **number 1** reaches the top of the wall at an azimuthal angle of $A = 20°$ W and also observes **star 2** reach the top of the wall at an azimuthal angle of $A = 60°$ W, both at the same time.

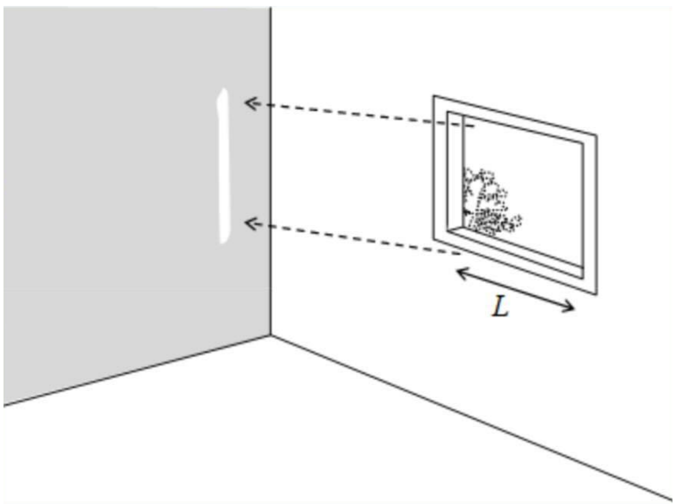
Now, consider all cities on Earth that observe **stars 1** and **2** with the same altitude. Out of all of these cities, how far away is the city closest to Alice? [Solution]

3 Week 3

Day 15

The solar wind hits the Earth with a velocity of 400 km/s and a proton density of around $7\,\text{cm}^{-3}$.
The mass loss rate of the Sun due to the solar wind is... [Solution]

Day 16



An astronomer who lives in a city on the equator is building a house. They want to have a particular corner of a room that receives direct sunlight through a window when the sun is rising on the (northern) summer solstice day. The house will have a rectangular shape oriented such that the front side is facing 16 degrees to the north from true east, which makes this house look odd compared to neighboring houses. The thickness of the wall is 15 cm. What is L, the width of the window? [Solution]

Day 17

A neutron star has a jet width of 5°, and the jet's poles are 20° off of its rotational axis. Determine the probability that this pulsar is detected from Earth. [Solution]

Day 18

During Winter Solstice, Venus seems to have reached the greatest eastern elongation. If given that Venus is on the ecliptic plane, determine the equatorial coordinates of Venus. [Solution]

Day 19

A main sequence star with the radius and mass of $R = 4\ R_\odot$, $M = 6\ M_\odot$ has an average magnetic field of 1×10^{-4} T. Calculate the strength of the magnetic field of the star when it evolves to a neutron star with the radius of 20 km. [Solution]

Day 20

Consider a flat and matter-dominated universe, in this case, the age of the universe can be modeled as

$$t = \frac{2}{3H_0}\, a^{\frac{3}{2}}$$

If the current temperature of the universe is 2.73 K, calculate the duration for which the universe will have a temperature between 0° C and 100° C!
Use $H_0 = 70$ km/s/Mpc. [Solution]

Day 21: PoTW 3

An asteroid is at a distance of 625000 km from the Earth and it is initially not in motion relative to the Earth. To prevent the asteroid from falling to the Earth, Bombom planted a nuclear explosive at the center of the asteroid and proceeded to use the device to explode the asteroid such that the asteroid is shattered isotropically into small pieces that moved radially in every direction with the same velocity of 0.5 km/s.

a. Find how much of the asteroid would still hit the Earth.
b. Find how much of the Earth's surface still got hit by the asteroid chunks.
c. What shape is formed by the envelope of all of the pieces' orbit?

4 Week 4

Day 22

An asteroid orbits the Sun with an eccentricity of 0.7. The asteroid's rotation is fast enough so that the temperature is homogeneous, and the asteroid can also be considered a black body. If the maximum temperature of this asteroid is 250 K, then the minimum surface temperature of this asteroid is... [Solution]

Day 23

A star with absolute magnitude 2.31 is at a distance of 1.5 kpc. Due to absorption of interstellar matter, when seen from Earth this star has a magnitude of 14.59. If interstellar matter (a collection of dust particles) between the star and the observer is evenly distributed and the size of the dust is about 0.3 μm, determine the average density of dust particles in that viewing direction. Ignore the scattering effect. [Solution]

Day 24

Due to light aberrations, the coordinates observed in the moving frame will shift compared to the stationary frame. As a result of the Earth's revolution, the observed coordinates of the Sun will shift. Determine the Sun's ecliptic longitude as observed on Earth at the summer solstice. [Solution]

Day 25

Assuming Earth and Mars' orbits are both circular, calculate the minimum phase of Mars as seen from Earth. [Solution]

Day 26

Hydrogen a line from an object is observed on 7500 Å. If the redshift fully came from the object's peculiar motion, calculate its radial velocity! [Solution]

Day 27

Determine the sunset duration for an observer at the North Pole. [Solution]

Day 28: PoTW 4

On October 7th, Bombom found a new asteroid that is at the coordinate $\lambda = 30°$, $\beta = 0°$. As seen from the Earth, the angular diameter of that asteroid is $0.275''$ and from 2 PM until 5 PM, the asteroid has drifted $90''$ to the West relative to the fixed star on the ecliptic plane. Bombom also uses radar with a frequency of 100 MHz, and he detects its reflection 21 minutes later at a frequency of 99.9963 MHz. From photometric observation, the magnitude of the asteroid is 10.52.

a. Calculate the distance of that asteroid from the Sun at that time.
b. Calculate the asteroid's heliocentric velocity at that time.
c. Find the semimajor and eccentricity of the asteroid's orbit.
d. Assuming the asteroid is spherical, and its reflection is isotropic, calculate its albedo.
e. Compute its average surface temperature when it is closest to the Sun.

5 Week 5

Day 29

Within a certain galaxy, a star is orbiting the center of the galaxy with a perfectly circular orbit at a distance of 10 kpc, and it is also known that the orbital velocity of this star is 300 km/s. If it is known that the luminosity of this galaxy is $4 \times 10^{10}\, L_\odot$, determine its *mass-to-light ratio*.
(State your answer in $M_\odot/L_\odot$) [Solution]

Day 30

Jupiter's Great Red Spot has an angular diameter of 1.5 arcminutes. If the spot is photographed with a [60 cm, $f/18$] diameter telescope, what is the size of the image on the plate? [Solution]

Day 31

At a certain moment, Mars appeared to be exactly at the first point of Aries. At that time, Mars was also located in the Eastern quadrature. The next day, how far will Mars deviate from the first point of Aries? [Solution]

Day 32

A certain hypothetical spiral galaxy contains only Sun-like stars. We are within that galaxy, and we measure that our solar system's orbital velocity around the disk is about 220 km/s, our distance to the center of the galaxy is about 8 kpc, and the thickness of the disk is about 0.5 kpc, determine the Schwarzschild Radius of the supermassive black hole.

Assume that the average number density is around 0.495 stars per parsec and is homogenous. [Solution]

Day 33

Sally owns a telescope, and after further observations, she concluded that the diameter of the beam that comes out of the eyepiece is the same as her pupil's diameter, which is 5 mm.

Telescope Specifications

- $D = 400\,\text{mm}$
- $d_{\text{telescope}} = 1620\,\text{mm}$

CCD Specifications

- $\text{Size} = 2160\,\text{px} \times 2160\,\text{px}$
- $\text{Pixel Size} = 9\,\mu\text{m} \times 9\,\mu\text{m}$

One day, Sally bought a CCD with the specifications above, and she took off the eyepiece that she previously had and swapped it out for the CCD. Using a Solar Filter, determine how many pixels were activated when Sally pointed her telescope at the Sun. (Assume that the entire Sun is within the scope of her telescope.)

Note: $d_{\text{telescope}}$ is the distance between the two lenses, the object lens, and the eyepiece lens. [Solution]

Day 34

Determine the length and direction of the shadow that is cast by the Tower of Pisa ($43°43'23''$ N, $10°23'47.10''$ E) on the 19th of October, 1 hour before the Sun sets. It is known that the Tower of Pisa is 52 meters in length and it is tipping towards the North by 5.5°. [Solution]

Day 35: PoTW 5

Bombom puts three identical objects with mass M at the corners of an equilateral triangle such that each of them is separated by a distance of R

a. Then Bombom gave them all an initial velocity of v_o so that all of them were orbiting their center of mass in a circular orbit. Determine the value of v_o and the period of the system!
b. If the initial velocity was $0.5\,v_o$ instead, all of them would have an elliptical orbit. Find the period of the system, and also determine its eccentricity.

6 Week 6

Day 36

Assume a flat Universe, and Hubble constant of 70 km/s/Mpc, Determine the energy density.

Day 37

On the 4th of August 2066, we developed a telescope with enough resolution to see other star systems as clearly as we can see ours. The first-ever trial for this telescope was on a solar system named FJX-1392. The first observation was of an unidentified object, assumed as a satellite orbiting the parent star for 2 years. A couple of years after the initial observation, we see that the same object is now on a different orbit with a period of 5 years. It is speculated that an alien lifeform is controlling the satellite and changed its orbital period.

Determine how much time did that satellite take to perform the orbit transfer, assuming that the maneuver used was a Hohmann Maneuver.

Note. that the satellite is first on a circular orbit, and transferred to a different circular orbit, with a different orbital period

Day 38

Sir Nodnod is living on a planet within a certain star system, it is known that the orbital period of this planet around its parent star is 36.05 years, and its orbital radius is 100 AU. Suddenly, the star collapsed into a black hole. Determine its density.

Note. The mass of the star is conserved.

Day 39

A binary star system has an inclination of $0°$, given that its maximum separation angle is $6''$, and its minimum separation angle is $1.5''$. Determine its orbital eccentricity.

Day 40

Determine the velocity of an asteroid that is 30% into its revolution period, given that its semimajor axis and eccentricity are 1.6 AU and 0.57, respectively.

Note. The revolution period begins at perihelion.

Day 41

Determine the phase (%) of the moon, 8 days before the full moon.

Day 42: PoTW 6

Suppose Bombom is living in a universe that is filled by radiation only. This universe is expanding with Hubble constant H_0, and its temperature at the present is T_0.

a. Find the condition for Ho in therms of To so that this universe has: flat curvature ($\kappa = 0$), positive curvature ($\kappa = 1$), and negative curvature ($\kappa = -1$).
b. Find the radius of curvature at the present (R_0).
c. Find the age of the universe (t_0) in terms of radiation density parameter (Ω_0) and H_0.
d. Using $H_0 = 70$ km/s/Mpc and $T_0 = 2.73$ K calculate R_0, t_0, and also the temperature of the universe at $t = t_0/2$.

7 Week 7

Day 43

Given the average temperature in space is T_{space}, and the average cooked chicken temperature is T_{eat}, assume a cylindrical chicken drumstick.

The cylindrical chicken is in space, at a distance d from the Sun. How long does it take for a chicken drumstick to be cooked? The diameter of the drumstick is D. Assume an average density of ρ_{chicken}, a specific heat capacity of C_{chicken}, and the chicken does not re-emit any radiation.

The orientation of the drumstick is such that it exposes the drumstick to the largest projected area and Also assume almost instantaneous heat transfer so that its temperature is always uniform.

Day 44

Consider a flat and matter-dominated universe. At a certain time t, the universe, instead of expanding at a rate of Ho, shrank at a rate of $H_0 = 67.4$ km/s/Mpc. The current average temperature of the universe is 2.73 K, which determines the amount of time that it takes for the universe to shrink such that the average temperature of the universe is 273 K.

Day 45

A star at $[\alpha = 30°,\ \delta = +20°]$ is observed to have its hydrogen alpha spectral line shift by +2.85 Å, and it is also observed to have a proper motion with a position angle of $63°1'47.02''$. Due to the proper motion, the right ascension of the star increases by $0.01''$/year. It is also known from the records that this star has a parallax value of $0.02''$. Determine its true velocity.

Day 46

An observer at a latitude of 30°N observed a star rising on the horizon at Az = 50° to determine its azimuth and altitude when the hour angle of that star is 3^h5^m.

Day 47

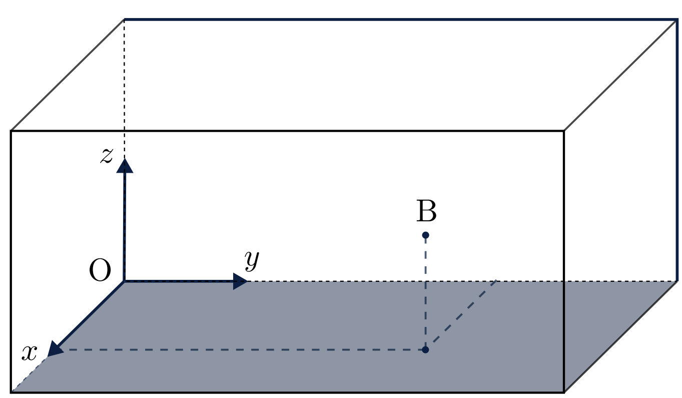
The radius of the Earth and the Moon are $R_{\oplus} = 6378$ km and $R_{\text{moon}} = 1737$ km. The mass of the Earth and the Moon are $M_{\oplus} = 5.97 \times 10^{24}$ kg and $M_{\text{moon}} = 7.35 \times 10^{22}$ kg. Determine the roche limit between the Earth and the Moon.

Day 48: APAO 15

According to estimations of scientists, there are about 1800 galaxies in the cluster of galaxies in the constellation Coma Berenices, which show redshift of 0.023. The region in our sky covered by this cluster is about 90 degrees^2. Estimate the average distance between the galaxies in this cluster.

Day 49: PoTW 7

Within a corner of the Backrooms, Bombom is eating an ice cream on a table (**Point B**). Measured from one of the corners (**Point O**), Bombom is at coordinate (x, y, z). From Bombom's point of view, the edge of the x axis and the edge of the z axis will form an obtuse angle θ as seen on the image below. Derive an expression for θ in terms of x, y, and z.



8 Week 8

Day 50

One of the most common things used by Hellenistic and western astrology is the birth chart/natal chart. In this chart, there are three parts: Planets, Zodiac Sign, and Houses. We divide the ecliptic plane into twelve equal parts called houses, with the 1st house at the ascendant (which is the zodiac sign that is rising at that time), and the 2nd house is next to the 1st house (measured westward). These are the interpretations of all houses, planets, and signs.

TWELVE HOUSES	THE PLANETS	THE ZODIAC SIGN
1st House: Physical Appearance	The Sun: Purpose	Aries: Passion
2nd House: Money	The Moon: Emotion	Taurus: Loyalty
3rd House: Friends	Mercury: Mind	Gemini: Curiosity
4th House: Family	Venus: Value	Cancer: Creativity
5th House: Children	Mars: Motivation	Leo: Courage
6th House: Health	Jupiter: Luck	Virgo: Organization
7th House: Marriage	Saturn: Maturity	Libra: Justice
8th House: Transformation	Uranus: Innovation	Scorpio: Power
9th House: Education	Neptune: Dreams	Sagittarius: Adventure
10th House: Career	Pluto: Rebirth	Capricorn: Ambition
11th House: Humanitarianism		Aquarius: Innovation
12th House: Spirituality		Pisces: Imagination

Table 1: Reading Example: If when you are born, Mars is in Taurus in the third house, this means you will be motivated (Mars) through loyalty (Taurus) of interpersonal relationships (3rd house).

One day, Bombom's friend is planning to have a baby with these perks:
"Have a strong purpose through ambition for having lots of money." If he is at latitude 30° N, find the date and time (local time) of the birth so that the baby has those perks.

Day 51

Determine the longest and shortest wavelength of the Balmer Series.

Day 52

Determine the duration for which the sun is above the horizon for an observer at 35° N, on the 17th of July. Ignore atmospheric refractions.

Day 53

It is known that Sir Nodnod has to sleep 8 hours a day, and on 4th of April 2024, 22:56 LT, he is still making a PoTD Problem. Given that the illumination phase of the Moon is currently 0.23, determine the phase of the Moon when Sir Nodnod wakes up in the morning.

Day 54

Given a comet is orbiting the sun with

$$\vec{v} = \begin{pmatrix} 50 \\ 24 \\ 2 \end{pmatrix} \text{ km/s},$$

and is at

$$\vec{r} = \begin{pmatrix} 10 \\ 2 \\ 14 \end{pmatrix} \text{ AU}.$$

Determine the inclination of the orbit.
Note: the xy plane makes the plane of the ecliptic.

Day 55

Determine the number of photons in the visible spectrum that pass through a telescope every second it observes Sirius. It is known that the diameter of the telescope is 96 mm wide, and the magnitude of Sirius when observed with the naked eye is 1.46.

Day 56: PoTW 8

Bombom is observing an elliptical galaxy at redshift $z = 1.25$.
From photometric observation, its apparent magnitude is $m = -22.75$.
From spectroscopic observation, its radial velocity dispersion is $\sigma_r = 220\,\mathrm{km/s}$.
From astrometric observation, its angular diameter is $\theta = 6.5''$.
If Bombom lives in a flat, matter-dominated universe, with Hubble Constant $H_0 = 70\,\mathrm{km/s/Mpc}$,

a. Calculate the galaxy's proper distance, angular diameter distance, and luminosity distance.
b. Calculate the luminosity of the galaxy.
c. Calculate the mass of the galaxy (Virial mass). Assume that the elliptical galaxy is spherical with uniform mass distribution.
d. Calculate the mass-to-light ratio (M/L) in terms of $M_\odot/L_\odot$.

9 Week 9

Day 57

Determine the Thermal Time Scale of the Sun.

Day 58

When Venus is at an elongation 20°, determine its distance from the Earth.

Day 59

Determine the limiting magnitude of a telescope that has a focal ratio of 20 mm and a focal length of 1600 mm.

Day 60

The flux density of the sun on the frequency $f = 110$ MHz is ..... Jy. Determine the amount of photons that a 10 m radio telescope will receive if it were to observe the sun for 10 seconds on $f = 108$ MHz til $f = 112$ MHz.

Day 61

An electron is moving with a constant velocity of 0.6c. Determine the kinetic energy and the momentum of the electron.

Day 62

Determine the maximum eccentricity of an asteroid's orbit entirely within the Goldilocks zone.

Day 63: PoTW 9

Initially a spacecraft is orbiting the Sun at orbital radius r_1. This spacecraft can go to any other circular orbit with radius r_2 (with $r_2 > r_1$) using Hohmann transfer, that is by using transfer orbit which is tangent to the initial and final orbit.

a. Total impulse needed to the orbital transfer is Δv, and we denote y as $y = \Delta v/v_1$, and x as $x = r_2/r_1$. Write an expression for y as a function of x.
b. Find the maximum value for y and the corresponding x value. Also, calculate the maximum Δv and the corresponding r_2 if the initial orbit is the Earth orbit.
c. Bombom is making a spacecraft. The structure of the spacecraft is 20 tons, it carries 500 tons of fuel, and the exhaust velocity of the thruster is 5000 m/s. Could this spacecraft access all possible circular orbit $(r_2 > r_1)$ using Hohmann transfer? If yes, calculate the maximum payload it could carry so that it could still access all the possible orbits. For this problem, just consider the heliocentric orbit (neglecting Earth's gravity).

10 Week 10

Day 64

Determine today's (April 15th, 14 UT) Julian Date.

Day 65

The precession of the Earth's rotation axis causes the ecliptic longitude to change linearly but the ecliptic latitude remains constant. Given that Earth's precession period is 26,000 years, determine what the equatorial coordinates of Arcturus (14^h16^m, $19°5'$) will be in 9000 years. In this case, assume that obliquity is constant.

Day 66

The hydrostatic equilibrium within the Sun can be expressed with the following expression

$$\frac{dP}{dr} = -\frac{GM(r)}{r^2}\,\rho$$

Assuming that the density within the Sun is homogeneous, hence determine the pressure within the core of the Sun.

Day 67

An astronomer is trying to find the distance of 2 stars, all he has is the 2 stars' coordinates in 3D space relative to his position. Star **A** has coordinates (30, 23, 12) pc, Star **B** has coordinates (34, 12, 19) pc.

Day 68

If it is known that the planet Miller from the movie Interstellar (2014) is orbiting around a Supermassive Blackhole (SMBH) named Gargantua at $r = 2.97$ AU, and it was stated in the movie that the gravitational time dilation effect was so extreme that 1 hour spent on the planet equates to roughly 7 years on Earth. Determine the mass of Gargantua.

- The formula to determine gravitational time dilation within a circular orbit is

$$t = \frac{t_0}{\sqrt{1 - \frac{3}{2}\frac{r_s}{r}}}$$

Where t is the time experienced by a far observer, to is the time experienced by an observer within a strong gravitational field, and r_s is the Schwarzschild radius.

Day 69

A sorcerer accidently casted a spell that made the Sun shrink, shrinking it by the factor of 4. Determine Sun's new apparent magnitude.

Note. Assume constant temperature.

Day 70: PoTW 10

A star with radius 4 $R_\odot$ is observed at 4000 Å with a bandwidth of 500 Å. The star's parallax is $0.08''$, and its peak radiation is at 8000 Å. The observation is done with a $2.5\,m$ telescope, a CCD with quantum efficiency of 70%, and 25 seconds of exposure time.

a. Determine the star's temperature, and determine whether we could use Wien's Approximation for the observation wavelength or not.
b. Determine the luminosity of the star on the observed wavelength.
c. Determine the photon count for the setup mentioned above.
d. If we take into account the intrinsic noise from the instrument, determine the uncertainty of the photon count, as well as the Signal-to-Noise ratio (S/N).
e. Determine the exposure time needed to double the Signal-to-Noise ratio.

11 Week 11

Day 71

Given that the hydrogen particles within the Sun's Corona has a velocity of 160 km/s, determine the temperature of the Sun's Corona.

Day 72

Determine the minimum inclination of a binary star whose two stars are like the Sun, and are 1 AU apart from each other, to appear as an eclipsing binary star.

Day 73

Derive and determine the Roche limit of the Sun, in respect to the Earth.

Note. Assume Earth is a rigid body.

Day 74: 3rd IOAA, Iran 2009

Calculate how much the radius of the Earth's orbit increases as a result of the Sun losing mass due to the thermonuclear reactions in its center in 100 years. Assume the Earth's orbit remains circular during this period.

Day 75

Determine the size of the projection of the Sun on a piece of paper 1.5 meters away from the eyepiece of a telescope. The telescope has a focal length of 900 mm, and the eyepiece has a focal length of 10 mm, the diameter of the telescope is 30 mm.

Day 76

Determine the weight of average adult (70 kg) if the Earth was hollow with a thickness of 10 km.

Day 77: PoTW 11

An asteroid orbital velocity is $\vec{v} = -14\hat{i}+5\hat{j}+2\hat{k}$ km/s when it is at position $\vec{r} = 2\hat{i} - \hat{j} + \hat{k}$ AU. Calculate its semi-major (a), eccentricity (e), orbital inclination (i), longitude of ascending node (Ω), and argument of perihelion (ω) of that asteroid. ($\hat{i}$ is the direction of vernal equinox, and $\hat{k}$ is in the direction of the North Ecliptic Pole)

12 Week 12

Day 78

Determine the maximum value of doppler shift we would expect from a binary system where both stars are Sun-like and is 5 AU apart from each other. Assume that the barycenter of the system is stationary relative to us and circular orbit.

Day 79

On some days, the Moon are visible during the day, and on some, it's not even visible during the night. This is due to the fact that the Moon goes through phases in its orbit. Determine the duration in the Moon's orbit from the point of first quarter to the third, passing the Full Moon phase; Determine also the time from third quarter to the first, passing the New Moon phase. Determine the ratio between the two durations.

Note. $e_{\text{moon}} = 0.0549$, $a = 384.748$ km.

Day 80

Determine what day is 21st of October 451227, given that 31st of May, 2024 is a Friday.

Day 81

A star cluster has main-sequence stars of spectral type **B7** (surface temperature around 13,000 K, radius around 3.28 solar radii) turning off the main sequence. Estimate the age of this star cluster.

Day 82

Consider two neutron stars, both having a mass of 1.4 $M_\odot$. They are both in a circular orbit around a barycenter and both are initially 100 km apart. Determine the amount of time that it takes for them to collide due to their emission of gravitational waves.

Note. The rate of orbital decay due to gravitational wave emissions may be approximated using the following expression

$$\frac{dr}{dt} = -\frac{64}{5}\frac{G^3}{c^5}\frac{(m_1 m_2)(m_1 + m_2)}{r^3}$$

Day 83

Consider the Summer Triangle. Draw three edges such that all the stars within the previously mentioned asterism act as their vertices, effectively creating a spherical triangle. Determine the solid angle of this triangle.

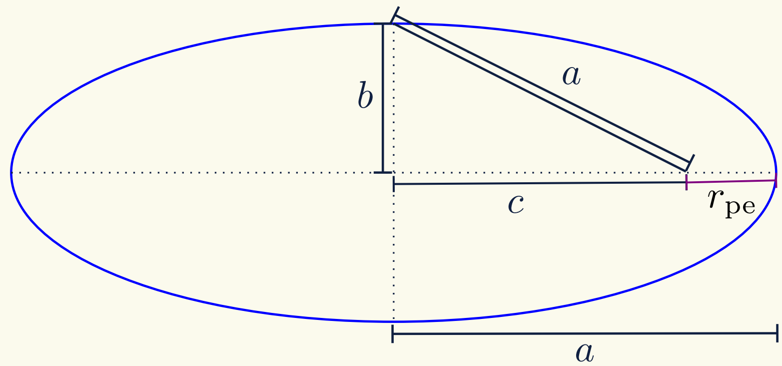
Below are the equatorial coordinates of the stars in the asterism:

- α *Lyr* ($18^h 36^m$, $+38^\circ 47'$)
- α *Cyg* ($20^h 41^m$, $+45^\circ 16'$)
- α *Aql* ($19^h 50^m$, $+08^\circ 52'$)

13 Solutions

★ Solution of Day 1 ★

Check Problem Description: Day 1.



Given,

1. $M_\star = 2\,M_\odot$
2. $a = 0.8$ AU,
3. $b = 0.6$ AU.

Using simple Pythagoras in the ellipse,

$$a^2 = b^2 + c^2$$
$$c^2 = a^2 - b^2$$
$$c = 0.529 \text{ AU}$$

Now, the distance of the periastron,

$$r_{\text{pe}} = a - c$$
$$= 0.271 \text{ AU}$$

From Vis-visa equation,

$$v = \sqrt{2GM_\star\left(\frac{1}{r_{\text{pe}}} - \frac{1}{2a}\right)}$$

$$\boxed{v = 104.48 \text{ km/s}}$$

★ Solution of Day 2 ★

Check Problem Description: Day 2
We know,

$$p('') = \frac{1}{d\ (\text{pc})} \implies d\ (\text{pc}) = \frac{1}{p('')}$$
$$\boxed{d = 33.3\,\text{pc}}$$

★ Solution of Day 3 ★

Check Problem Description: Day 3
Given,

1. $d_{\text{sirius}} = 8.6$ ly,
2. $R_{\text{sirius}} = 1.71\ R_\odot$,
3. $m_\odot = -26.8^m$,
4. $S_\odot = 1370\ \text{W/m}^2$,
5. $R_\odot = 6.96 \times 10^8$ m,
6. $m_{\text{sirius}} = -1.61^m$.

Question: T_{sirius}?
Answer:

$$L_\odot = 4\pi R_\odot^2 \sigma T_\odot^4$$

$$
\begin{aligned}
S_\odot &= \frac{L_\odot}{4\pi d^2} = \frac{\cancel{4\pi} R_\odot^2 \sigma T_\odot^4}{\cancel{4\pi} d^2} = \frac{R_\odot^2 \sigma T_\odot^4}{d^2} \\
m_{\text{sirius}} - m_\odot &= -2.5 \log\left(\frac{E_{\text{sirius}}}{S_\odot}\right) \\
\frac{E_{\text{sirius}}}{S_\odot} &= 8.39 \times 10^{-11} \\
\frac{R_{\text{sirius}}^2 \sigma T_{\text{sirius}}^4}{d_{\text{sirius}}^2} \cdot \frac{d^2}{R_\odot^2 \sigma T_\odot^4} &= 8.39 \times 10^{-11} \\
\left(\frac{T_{\text{sirius}}}{T_\odot}\right)^4 &= 8.45 \\
T_{\text{sirius}} &= \boxed{1.7\, T_\odot}
\end{aligned}
$$

★ Solution of Day 4 ★

Check Problem Description: Day 4

Given,

1. $H_0 = 70$ km/s/Mpc,
2. $T_0 = 2.73$ K,
3. $\Omega_{m,0} = 0.25$,
4. $\Omega_{r,0} = 8.4 \times 10^{-5}$,
5. $T_{rcmb} = 3000$ K,
6. Flat Universe $\implies \Omega = 1$

Question: What is the value of the Hubble parameter (H) at the time of recombination?

Solution:

Step 1

Calculate the Scale doctor (a) at the time of recombination.

$$
T = \frac{T_0}{a} \implies a = \frac{T_0}{T} = 9.1 \times 10^{-4}
$$

Step 2

Calculate the current amount of dark energy

$$
\Omega = \Omega_{m,0} + \Omega_{r,0} + \Omega_{\Lambda,0} = 1
$$

$$
\boxed{\Omega_{\Lambda,0} = 7.5 \times 10^{-1}}
$$

Step 3

Making H as a function of scale factor (for the flat universe, $\rho = \rho_c$)

$$
\begin{aligned}
\rho = \rho_c &= \frac{3H^2}{8\pi G} \\
H^2 &= \frac{8\pi G}{3} \rho \\
\frac{H^2}{H_0^2} &= \frac{8\pi G}{3H_0^2} (\rho_m + \rho_r + \rho_\Lambda) \\
\frac{H^2}{H_0^2} &= \frac{1}{\rho_{c,0}} (\rho_{m,0} \cdot a^{-3} + \rho_{r,0} \cdot a^{-4} + \rho_{\Lambda,0}) \\
\frac{H^2}{H_0^2} &= \Omega_{m,0} \cdot a^{-3} + \Omega_{r,0} \cdot a^{-4} + \Omega_{\Lambda,0}
\end{aligned}
$$

$$
\boxed{H = 4.82 \times 10^{-14}\, /\text{s} = 1.49 \times 10^6\, \text{km/s/Mpc}}
$$

★ Solution of Day 5 ★

Check Problem Description: Day 5

Given,

1. $\lambda_{\text{max}} = 410$ nm
2. $H_\alpha = 842$ nm
3. $R = 1.0974 \times 10^7$ m^{-1}

Question: What is the value of surface brightness S (in terms of mag/arcsec^2).

Solution:

Step 1

Calculate the H_α line at the resting condition,

$$\frac{1}{H_{\alpha,0}} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$
$$H_{\alpha,0} = 656.47\,\text{nm}$$

Step 2

Calculate the redshift,

$$z = \frac{H_\alpha - H_{\alpha,0}}{H_{\alpha,0}}$$
$$z = 0.2826$$

Step 3

Calculate the peak wavelength at the resting condition

$$z = \frac{\lambda_{\text{max}} - \lambda_{\text{max},0}}{\lambda_{\text{max},0}}$$

$$\frac{\lambda_{\text{max}}}{\lambda_{\text{max},0}} = 1 + z$$
$$\lambda_{\text{max},0} = 319.66\,\text{nm}$$

Step 4

Calculate the Star's surface temperature,

$$b = \lambda_{\text{max},0}\, T$$
$$T = \frac{b}{\lambda_{\text{max},0}}$$
$$T = 9065.96\,\text{K}$$

Step 5

Calculate the Surface brightness (B)

$$B = \frac{\sigma T^4}{\pi}$$
$$B = 1.23 \times 10^8\ \text{W/m}^2\text{/sterad}$$
$$B = 2.87 \times 10^{-3}\text{W/m}^2\text{/arcsec}^2$$

Step 6

Calculate the Surface brightness (S)

$$S - m_\odot = -2.5\log\left(\frac{B}{S_\odot}\right)$$
$$\boxed{S = -12.6\,\text{mag/arcsec}^2}$$

★ Solution of Day 6 ★

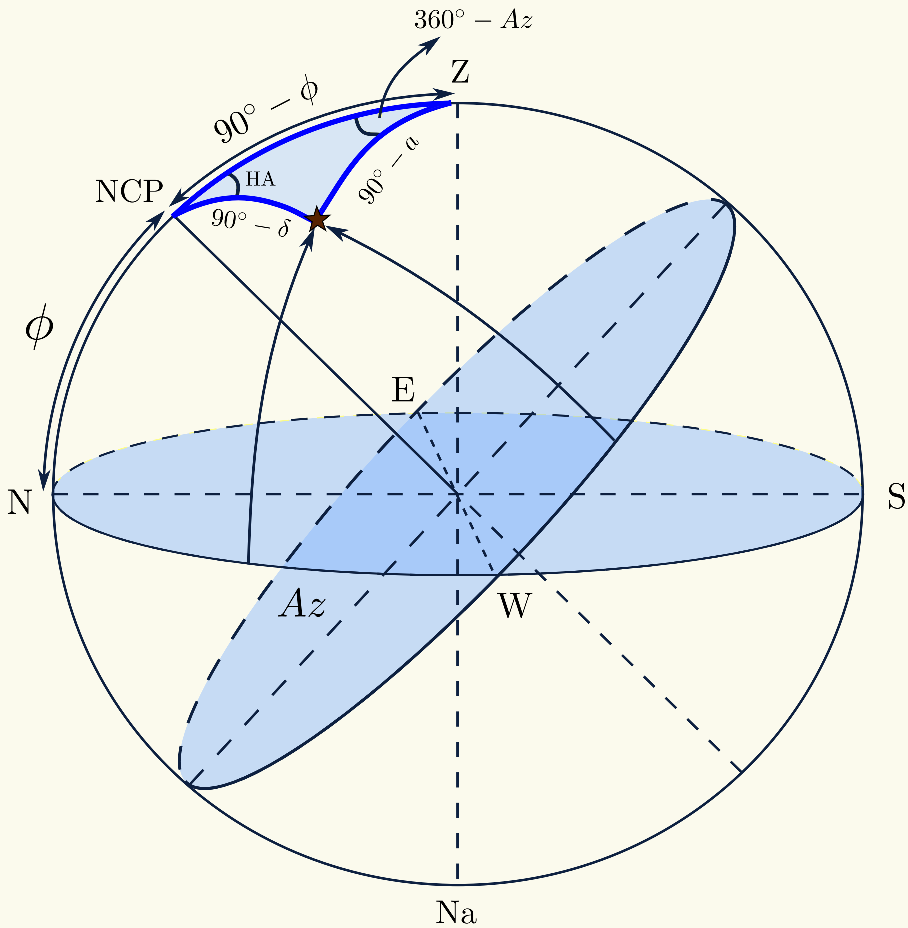
Check Problem Description: Day 6

Given,

1. $\alpha_{\text{rigel}} = 05^h 14^m 32.3^s$
2. $\delta_{\text{rigel}} = -0.8°12'05.9''$
3. $\phi_{\text{obs}} = -37°50'27.3660''$
4. $\lambda_{\text{obs}} = 144°56'47.2452''$ E
5. Date: 10th April
6. $a_{\text{rigel}} = 0°$

Question: Civil time and Azimuth when Rigel rises.

From Sunrise Equation,



$$\cos \mathrm{HA}_{\mathrm{rigel}} = -\tan \delta_{\mathrm{rigel}} \tan \phi_{\mathrm{obs}}$$
$$\mathrm{HA}_{\mathrm{rigel}} = -6^{\mathrm{h}}25^{\mathrm{m}}42.86^{\mathrm{s}}$$

$$\lambda_\odot = \frac{n}{365.25} \times 360^\circ$$
$$\lambda_\odot = 19^\circ 42' 45.09''$$
$$\tan \alpha_\odot = \cos \varepsilon \tan \lambda_\odot$$
$$\alpha_\odot = 18^\circ 11' 22.39'' = 1^{\mathrm{h}}12^{\mathrm{m}}45.49^{\mathrm{s}}$$

Using Local Sidereal Time relation,

$\mathrm{LST} = \mathrm{HA} + \alpha$

$$\mathrm{LST} = \mathrm{HA}_{\mathrm{rigel}} + \alpha_{\mathrm{rigel}} = -1^{\mathrm{h}}11^{\mathrm{m}}10.56^{\mathrm{s}}$$
$$\mathrm{LST} = \mathrm{HA}_\odot + \alpha_\odot \implies \mathrm{HA}_\odot = -2^{\mathrm{h}}23^{\mathrm{m}}56.05^{\mathrm{s}}$$
$$\mathrm{LT} = \mathrm{HA}_\odot + 12^{\mathrm{h}} \implies \mathrm{LT} = 9^{\mathrm{h}}36^{\mathrm{m}}3.95^{\mathrm{s}}$$
$$\mathrm{CT} = \mathrm{LT} - \frac{\lambda_{\mathrm{obs}} - \lambda_{\mathrm{timezone}}}{15} = \boxed{9^{\mathrm{h}}56^{\mathrm{m}}16.8^{\mathrm{s}}}$$

Using cosine law,

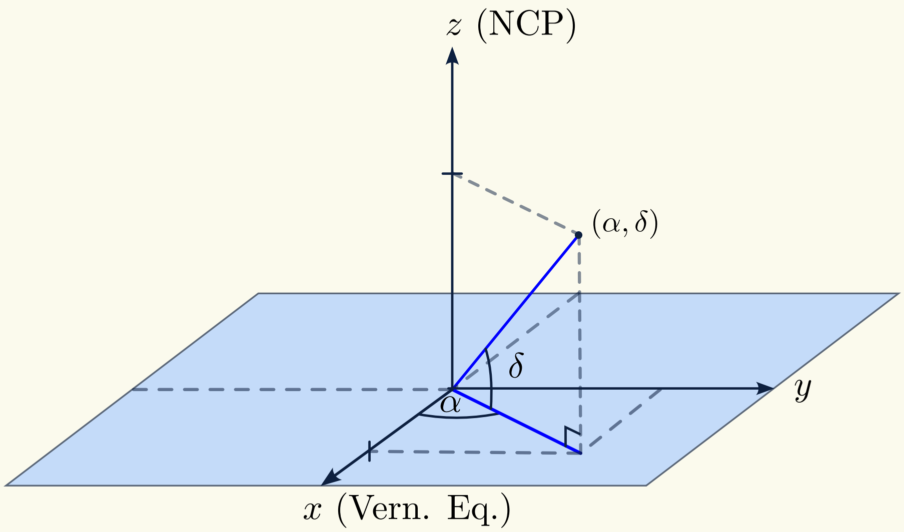
$$\sin \delta = \sin \phi \sin a + \cos \phi \cos a \cdot \cos Az$$
$$\cos Az = \frac{\sin \delta - \sin \phi \sin a}{\cos \phi \cos a} = \frac{\sin \delta}{\cos \phi} \; [\because a = 0]$$
$$Az = \boxed{100^\circ 24' 26.08''.}$$

★ Solution of Day 7 ★

Check Problem Description: Day 7

From the graph, we are able to deduce that the values of α and δ are to be expressed in terms of (x, y, z). Transformation between Cartesian to Celestial coordinates:

$$x = \cos \delta \cos \alpha, \quad y = \cos \delta \sin \alpha, \quad z = \sin \delta$$



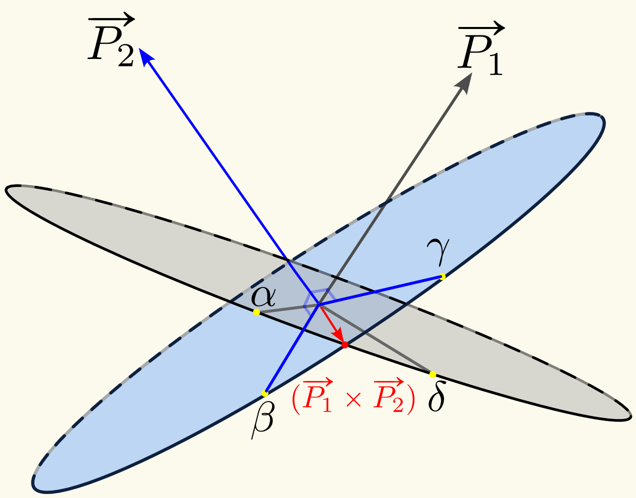
Therefore, we can also express (α, δ) in the standard $(\hat{\imath}, \hat{\jmath}, \hat{k})$ vector notation. So we can now express the coordinates of 4 stars in vector notation.

$$\alpha \text{ Peg: } 23^h05^m,\ 15°18'$$
$$\beta \text{ Peg: } 23^h04^m,\ 28°11'$$
$$\gamma \text{ Peg: } 0^h14^m,\ 15°17'$$
$$\delta \text{ Peg: } 23^h09^m,\ 29°11'$$

Note. that because the equatorial coordinate system is a direction-wise coordinate system, the r value for all the stars are the same, and we're practically making unit vectors.

Expressing in vector coordinates:

$$\overrightarrow{\alpha \text{ Peg}}: (0.937\hat{\imath} - 0.229\hat{\jmath} + 0.264\hat{k})$$
$$\overrightarrow{\beta \text{ Peg}}: (0.885\hat{\imath} - 0.213\hat{\jmath} + 0.472\hat{k})$$
$$\overrightarrow{\gamma \text{ Peg}}: (0.963\hat{\imath} + 0.059\hat{\jmath} + 0.264\hat{k})$$
$$\overrightarrow{\delta \text{ Peg}}: (0.872\hat{\imath} + 0.034\hat{\jmath} + 0.488\hat{k})$$



A great circle can be perfectly described by using its polar coordinates. The pole of the great circle passing through α_{peg} and δ_{peg} is P_1, and the pole of the great circle passing through β_{peg} and γ_{peg} is P_2. We can get $\vec{P_1}$ by using the cross product of $\overrightarrow{\alpha \text{ Peg}}$ and $\overrightarrow{\delta \text{ Peg}}$, and similarly $\vec{P_2}$ by using cross product of $\overrightarrow{\beta \text{ Peg}}$ and $\overrightarrow{\gamma \text{ Peg}}$.

The intersection of the 2 diagonals made by $\alpha - \delta$ and $\beta - \gamma$, is the cross product of $\overrightarrow{P_1} \times \overrightarrow{P_2}$.

Let $\overrightarrow{N}$ be $\overrightarrow{P_1} \times \overrightarrow{P_2}$.

$$\overrightarrow{N}:\ (0.111\hat{\imath} - 0.011\hat{\jmath} + 0.0467\hat{k})$$

Therefore, we are able to deduce the values of α_N and δ_N.

$$\tan \alpha_N = \frac{N_y}{N_x}$$
$$\tan \delta_N = \frac{N_z}{\sqrt{N_x^2 + N_y^2}}$$

At last, we have found the value for the intersection, which is $\underline{(23^h36^m,\ 22°54')}$.

★ Solution of Day 8 ★

Check Problem Description: Day 8
Given,

1. $M_\star = 3M_\odot$
2. $R_\star = 2R_\odot$
3. ρ is Homogenous

Question: Pressure in the core of Star **A**.
Answer: Use the *Hydrostatic Equilibrium Equation*,

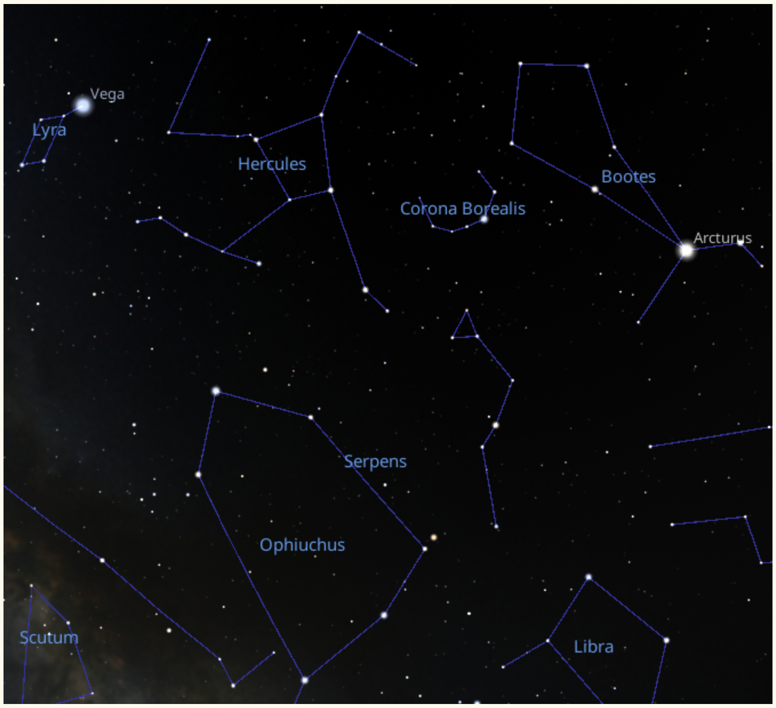
$$\frac{dP}{dr} = -\frac{GM(r)\rho}{r^2}$$
$$\int_0^{P_{\text{core}}} dP = -G\rho \int_R^0 \frac{M(r)}{r^2}\,dr$$
$$\int_0^{P_{\text{core}}} dP = -\frac{4}{3}\pi G\rho^2 \int_R^0 r\,dr$$
$$P = -\frac{2}{3}\pi G\rho^2\,(r^2|_R^0$$

$$\boxed{P = 7.64 \times 10^{13}\ \text{Pa}}$$

★ Solution of Day 9 ★

Check Problem Description: Day 9

A. Ophiachus
B. Serpens Caput
C. Corona Borealis
D. Hercules
E. Lyra



Recommendation: To better understand Observation problems and how to work with star charts, proceed to *Star Charts 101 and Practices* by Fahim Rajit Hossain (me).

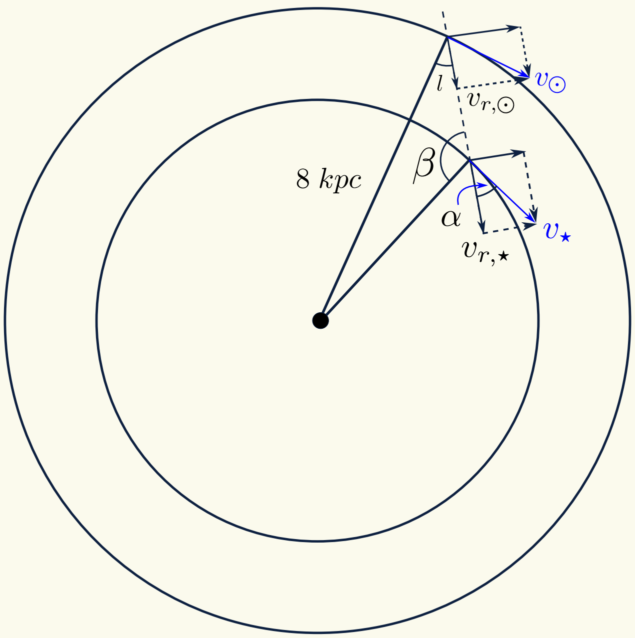
★ Solution of Day 10 ★

Check Problem Description: Day 10

Given:

1. $l = 34°$,
2. $v_r = 76$ km/s.
3. $v_0 = 250$ km/s

Question: What is the distance between the Galactic center to the star?



Answer:

$$v_\odot = v_\star = v_0$$
$$v_{r,\odot} = v_\odot \sin l$$
$$v_{r,\odot} = 139.8\,\text{km/s}$$
$$v_{r,\text{rel}} = v_{r,\star} - v_\odot \sin l$$
$$76\,\text{km/s} = v_{r,\star} - v_\odot \sin l$$
$$v_{r,\star} = 215.8\,\text{km/s}$$
$$v_{r,\star} = v_\star \cos\alpha$$
$$\cos\alpha = \frac{v_{r,\star}}{v_\star} = \frac{215.8\,\text{km/s}}{250\,\text{km/s}}$$
$$\alpha = 30.32°$$
$$\beta = 180° - (90° - \alpha)$$
$$\beta = 59.68°$$
$$\frac{\sin\beta}{8\,\text{kpc}} = \frac{\sin l}{d_\star} \implies d_\star = \frac{\sin l \cdot 8\,\text{kpc}}{\sin\beta}$$
$$d_\star = \boxed{5.18\,\text{kpc}}$$

★ Solution of Day 11 ★

Check Problem Description: Day 11

Known:

1. $M_{\text{moon}} = 7.35 \times 10^{22}$ kg,
2. $R_{\text{moon}} = 1737$ km.

Answer:

$$F_s + N - F_g = 0$$

The plane will start to lift when the normal force is equal to 0, and therefore,

$$F_s = F_g$$
$$\frac{mv^2}{R} = \frac{GMm}{R^2}$$
$$v_{\text{liftoff}} = \sqrt{\frac{GM_{\text{moon}}}{R_{\text{moon}}}}$$
$$v_{\text{liftoff}} = \boxed{1.68\text{ km/s}}$$

★ Solution of Day 12 ★

Check Problem Description: Day 12

Given:

1. $M_{BH} = 4 \times 10^6\ M_\odot$,
2. $d_{GC} = 8$ kpc.

Question: In what wavelength should we observe the black hole in?
Answer: We need the angular distance of the black hole to be smaller than the angular resolution of the telescope. We know the angular resolution,

$$\theta_{\text{tel}} = 1.22\frac{\lambda}{d}$$

So,

$$\begin{aligned}\theta_{BH} &\geq \theta_{\text{tel}}\\ \frac{2R_{BH}}{d_{GC}} &\geq \frac{\lambda}{2R_\oplus}\\ \lambda &\leq \frac{4 \cdot R_{BH} \cdot R_\oplus}{d_{GC}}\\ \lambda &\leq \frac{8GM_{BH} \cdot R_\oplus}{c_0^2 \cdot d_{GC}}\\ \lambda &\leq 1.224 \times 10^{-3}\ \text{m}\end{aligned}$$

We will need to observe in the Radio region.

★ Solution of Day 13 ★

Check Problem Description: Day 13

Given:

1. $m = 9^m$,
2. $M = -25^m$,
3. $\alpha = 0.15$ mag/kpc $= 0.00015$ mag/pc.

Question: Distance of the Galaxy.

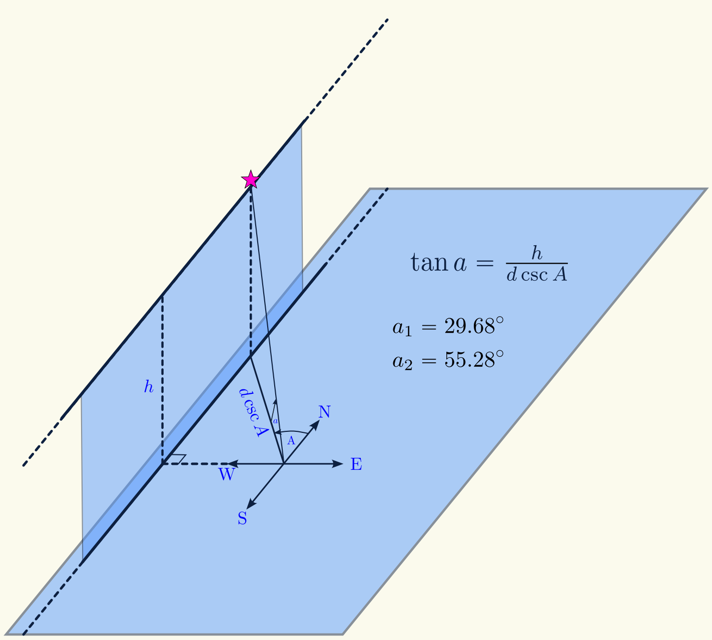
Answer:

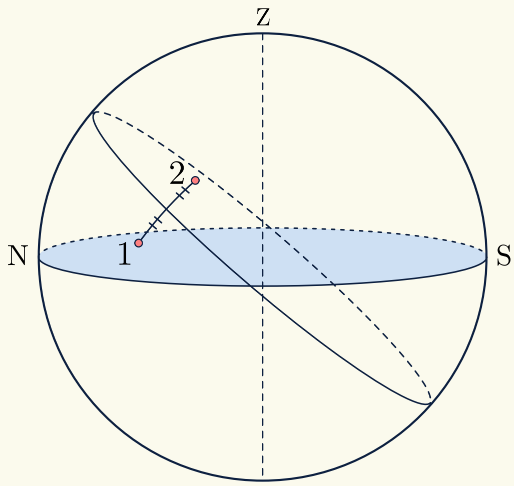
$$\begin{aligned}m - M &= -5 + 5\log d + \alpha d\\ d &= \frac{m - M + 5 - 5\log d}{\alpha}\end{aligned}$$

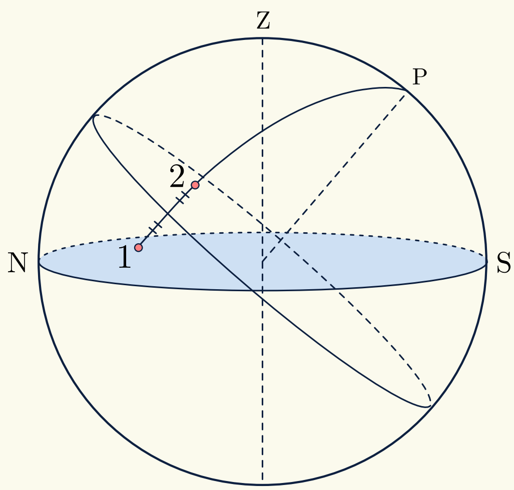
Through iteration: $\boxed{d = 94.2\ \text{kpc}}$

★ Solution of Day 14 ★

Check Problem Description: Day 14







The altitude of the 2 stars would only be the same if their angular distance to the local zenith is the same.

On the celestial sphere, we can map all of the possible zeniths (or you could think of it as mapping all possible locations on Earth) that satisfy the equidistance condition, and the result of the mapping would be of a great circle.

To find the closest location to Alice, we need to find the closest angular distance of Alice's zenith to the great circle.

First, we should find the pole of the great circle of zenith.

Using spherical triangle Z − Star 1 − Star 2

- $\cos\theta = \sin a_1 \sin a_2 + \cos a_1 \cos a_2 \cos(A_2 - A_1)$

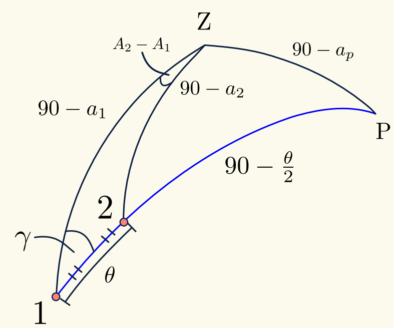
  $\theta = 38.18°$
- $\sin a_2 = \sin a_1 \cos\theta + \cos a_1 \sin\theta \cos\gamma$

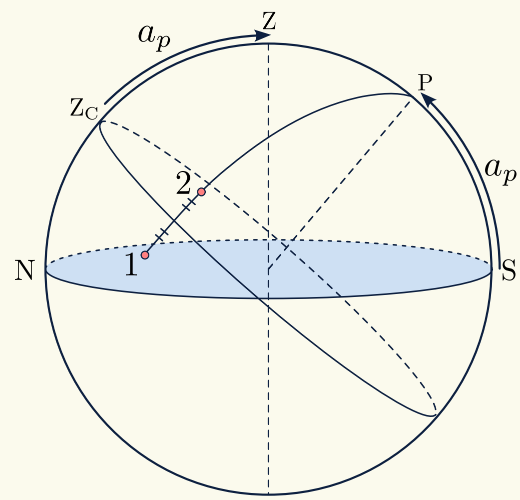
  $\gamma = 36.31°$

Using spherical triangle Z − Star 1 − P

- $\sin a_p = \sin a_1 \cdot \left(-\sin\frac{\theta}{2}\right) + \cos a_1 \cos\frac{\theta}{2} \cos\gamma$

  $a_p = 29.97°$





The angular separation of Alice's Zenith (Z) and the Zenith of the closes location that sees both the stars at the same time (Z_C) is a_p.

Therefore the Geodesic distance is:

$$S = \frac{a_p}{360°} \times 2\pi R_\oplus$$

$$S = \boxed{3336.37 \text{ km}}$$

★ Solution of Day 15 ★

Check Problem Description: Day 15

Given:

1. $v = 400$ km/s,
2. $n_p = 7/\text{cm}^3$,
3. $m_p = 1.6726 \times 10^{-27}$ kg.

Question: $dM_\odot/dt$ due to Solar Wind.
Answer:
$n_p = 7 \times 10^6$ proton/m^3

$$\begin{aligned}\text{Proton flux } (\phi_p) &= n_p \times v \\ &= (7 \times 10^6) \times (400,000\ m/s) \\ &= 2.8 \times 10^{12}\ (\text{proton/s})/\text{m}^2\end{aligned}$$

Number of protons released per second,

$$\begin{aligned}L_p &= \phi_p \times 4\pi d_\odot^2 \\ &= 2.8 \times 10^{12}\ (\text{proton/s})/\text{m}^2 \times 4\pi(1.5 \times 10^{11})^2\ \text{m}^2 \\ &= 7.92 \times 10^{35}\end{aligned}$$

Since L_p is in *protons/s*, we can write it as $\frac{dM_\odot}{dt}$ in *kg/s*

$$\frac{dM_\odot}{dt} = L_p \times m_p = \boxed{1.32 \times 10^9\ \text{kg/s}}$$

★ Solution of Day 16 ★

Check Problem Description: Day 16

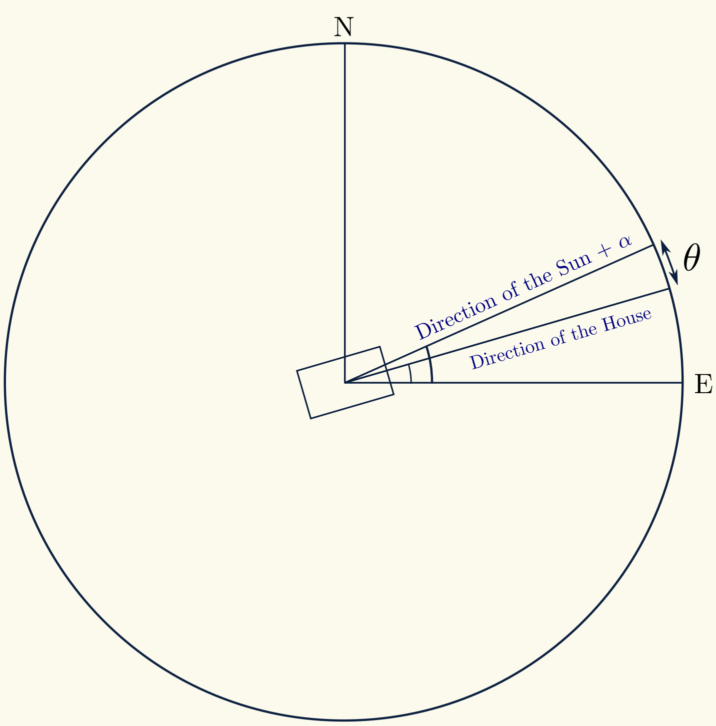
Given:

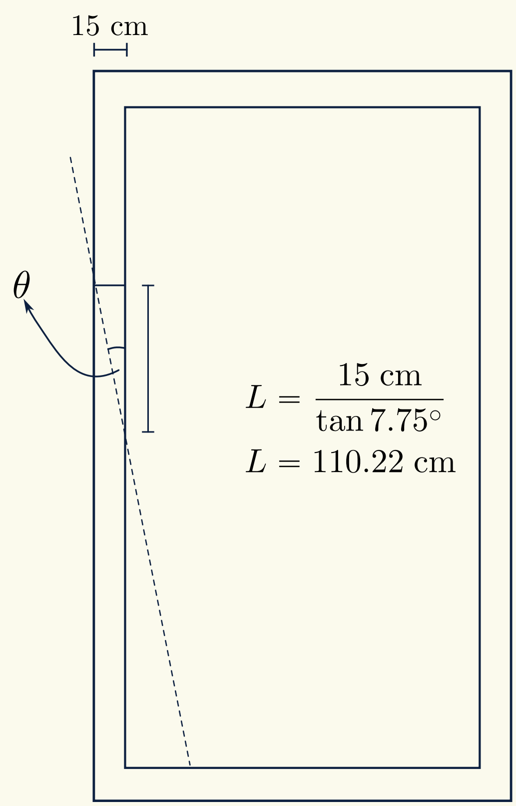
1. Offset from the true East $= 16°$,
2. Position of the center of the Sun $= 23.5°$,
3. Angular radius of the Sun: $0.25°$.

Question: What is the minimum value of L.

Answer: Let θ be the difference in angle from the edge of the Sun's disk to the direction of the house.

$$\theta = \delta_\odot + 0.25° - 16°$$
$$\theta = 7.75°$$





$$L = \frac{15\ \text{cm}}{\tan 7.75°}$$
$$L = 110.22\ \text{cm}$$

★ Solution of Day 17 ★

Check Problem Description: Day 17

Given:

1. Jet Width = 5°,
2. Jet Axis of Rotation = 20°.

Question: What is the probability that we detect the pulsar?

Answer: Using spherical caps,
Let θ_{out} and θ_{in} be the two angles that define 2 distinct spherical caps.

$$\theta_{out} = 20^\circ + \frac{5^\circ}{2} = 22.5^\circ$$

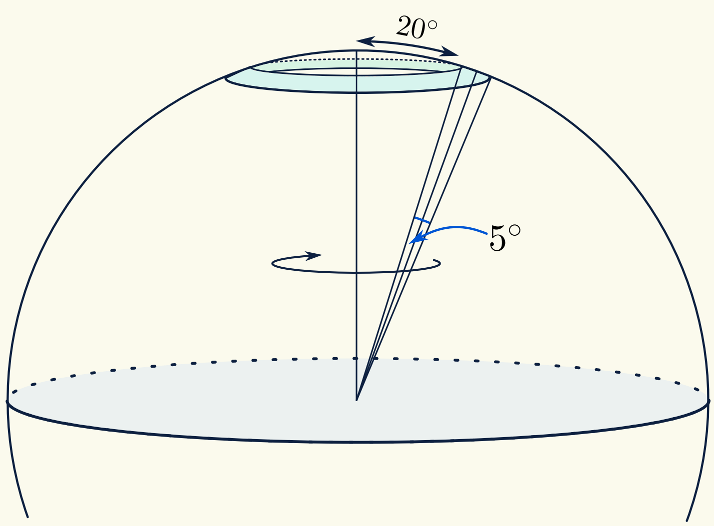
$$\theta_{in} = 20^\circ - \frac{5^\circ}{2} = 17.5^\circ$$

$$\Omega = 2\pi(1 - \cos\theta)$$

$$\Omega_{out} = 2\pi(1 - \cos\theta_{out}) = 0.478 \text{ steradians}$$
$$\Omega_{in} = 2\pi(1 - \cos\theta_{in}) = 0.291 \text{ steradians}$$
$$\Omega_{jet} = 2(\Omega_{out} - \Omega_{in}) = 0.375 \text{ steradians}$$

The probability that we are in the area shined by the jets is as follows:

$$P = \frac{\Omega_{jet}}{4\pi} \times 100\% = \boxed{2.98\%}$$



★ Solution of Day 18 ★

Check Problem Description: Day 18

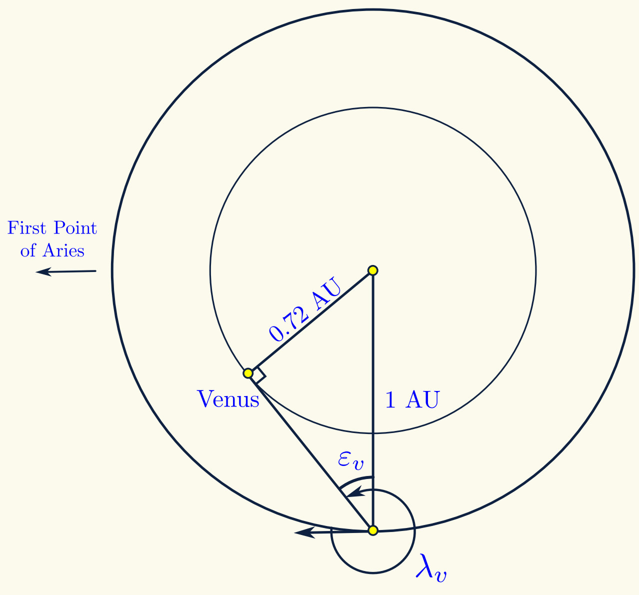
Given:

1. Winter Solstice (Northern 21/22 Dec),
2. Greatest Elongation of Venus, it should be left of the Sun,
3. Ecliptic Latitude: $\beta_v = 0°$.

Question: What is the equatorial coordinates $(\alpha_v,\ \delta_v,)$ of Venus?

Answer: From the relation of greatest elongation in triangle $\triangle SEV$:

$$\sin\varepsilon_v = SV$$
$$\sin\varepsilon_v = 0.72$$
$$\varepsilon_v = 46.05^\circ$$



Now,

$$\begin{aligned}\lambda_v &= 270^\circ + \varepsilon_v \\ &= 316.05^\circ \\ \beta_v &= 0^\circ\end{aligned}$$

Since the β_v value is 0°, we can use the following coordinate transformation formulae:

$$\begin{aligned}\tan\alpha_v &= \cos\varepsilon \cdot \tan\lambda_v \\ \sin\delta_v &= \sin\varepsilon \sin\lambda_v\end{aligned}$$

Therefore, the equatorial coordinates for Venus are: $(21^h 14^m,\ -16^\circ 03')$.

★ Solution of Day 19 ★

Check Problem Description: Day 19

Given:

1. $R_0 = 4R_\odot$,
2. $M_0 = 6M_\odot$,
3. $B = 1 \times 10^{-4}$ T,
4. $R' = 16$ km.

Question: What is the magnetic field of star after it evolves into a Neutron star?

Answer: Using the conservation of magnetic flux, we're able to use

$$\begin{aligned}B_0 \cdot A_0 &= B' \cdot A' \\ (1 \times 10^{-4}) \cdot (\cancel{4\pi} R_0^2) &= B' \cdot (\cancel{4\pi} R'^2) \\ B' &= \left(\frac{R_0}{R'}\right)^2 \cdot (1 \times 10^{-4}) \\ B' &= \boxed{1.94 \times 10^6 \text{ T}}\end{aligned}$$

★ Solution of Day 20 ★

Check Problem Description: Day 20

Given:

1. $T_0 = 2.73$ K,
2. $t = \dfrac{2}{3H_0} a^{\frac{3}{2}}$,
3. $H_0 = 70$ km/s/Mpc$= 2.26 \times 10^{-18}/s$.

Question: What is the duration between $T = 0^\circ$ C and $T = 100^\circ$ C?

Answer:

$$\begin{aligned}0^\circ \text{ C} &= 273.15 \text{ K} \\ T &= T_0 \cdot a^{-1} \\ a &= \frac{T_0}{T}\end{aligned}$$

$$\begin{aligned}a_{0^\circ\text{C}} &= \frac{T_0}{273.15 \text{ K}} \\ a_{0^\circ\text{C}} &= 0.00999...\end{aligned}$$

Using the same method, we get:

$$\begin{aligned}a_{100^\circ\text{C}} &= \frac{T_0}{273.15 \text{ K}} \\ a_{100^\circ\text{C}} &= 0.00731...\end{aligned}$$

Now time interval,

$$\begin{aligned}\Delta t &= \frac{2}{3H_0}\left(a_{0^\circ\text{C}}^{\frac{3}{2}} - a_{100^\circ\text{C}}^{\frac{3}{2}}\right) \\ \Delta t &= \boxed{3.49 \text{ Myr}}\end{aligned}$$

★ Solution of Day 22 ★

Check Problem Description: Day 22

Given:

1. $e = 0.7$,
2. $T_{\max} = 250$ K,

Question: What is the minimum surface Temperature?

Answer: Notice that the minimum surface temperature is achievable only at aphelion.

$$L_{pe} = \frac{L_\odot}{4\pi[a(1-e)]^2}$$

$$L_{ap} = \frac{L_\odot}{4\pi[a(1+e)]^2}$$

Using the formula for planets 'Luminosity', we can also apply the same formula for this asteroid. Notice that the albedo is 1 due to the asteroid being a black body.

$$E \cdot \cancel{A} \cdot \cancel{\pi R^2_{ast}} = 4\pi \cancel{R^2_{ast}} \sigma T^4_{ast}$$

$$T_{ast} = \sqrt[4]{\frac{E}{4\sigma}}$$

$$T_{ast} = \sqrt[4]{\frac{\frac{L}{4\pi d^2}}{4\sigma}} \implies \sqrt[4]{\frac{L_\odot}{16\pi\sigma d^2}}$$

Therefore we can determine the minimum temperature:

$$\frac{T_{\max}}{T_{\min}} = \frac{\sqrt[4]{\frac{\cancel{L_\odot}}{\cancel{16\pi\sigma}[a(1+e)]^2}}}{\sqrt[4]{\frac{\cancel{L_\odot}}{\cancel{16\pi\sigma}[a(1+e)]^2}}}$$

$$\frac{T_{\max}}{T_{\min}} = \sqrt[4]{\frac{(1+e)^2}{(1-e)^2}} \implies 2.38$$

$$T_{\min} = \frac{T_{\max}}{2.38} = \boxed{105.02 \text{ K}}$$

★ Solution of Day 23 ★

Check Problem Description: Day 23

Given:

1. $M = 2.31^m$,
2. $d = 1.5$ kpc,
3. $m = 14.59^m$,
4. $D = 0.3\ \mu m$.

Question: What is the dust particles' average density in the observation direction?

Answer: Calculate the amount of absorption,

$$m - M = -5 + 5\log d + A$$

$$A = 1.4$$

Optical Depth (τ),

$$A = 2.5 \log e \cdot \tau \approx 1.086\tau$$

$$\tau = 1.29$$

Again for dust particles, we can write,

$$\tau = \sigma n d$$

neglecting the scattering effects, we can approximate the extinction cross-section (σ) equal to the geometric cross-section:

$$\tau = \left(\frac{1}{4}\pi D^2\right) nd$$

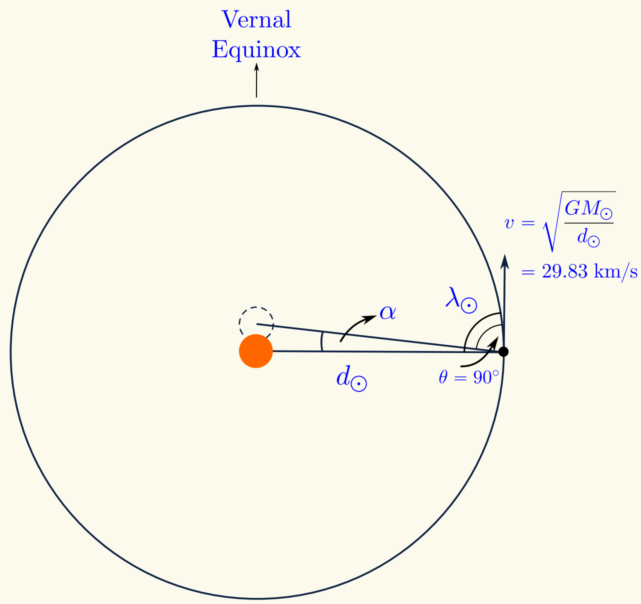
$$n = \frac{\tau}{\frac{1}{4}(\pi D^2)\, d}$$

$$n = \boxed{3.93 \times 10^{-7}/\text{m}^3}$$

★ Solution of Day 24 ★

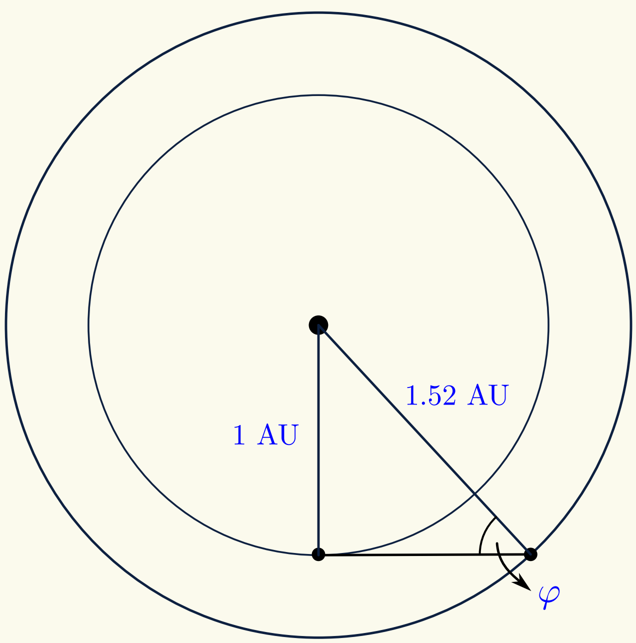
Check Problem Description: Day 24

$$\sin\alpha = \frac{v}{c}\sin\theta'$$
$$\theta' = \theta - \alpha$$
$$\theta' = 90^\circ - \alpha$$
$$\sin\alpha = \frac{v}{c}\sin(90^\circ - \alpha)$$
$$\tan\alpha = \frac{v}{c}$$
$$\alpha = 20.52''$$
$$\lambda'_\odot = \lambda_\odot - \alpha$$
$$\lambda'_\odot = 89^\circ 59' 39.48''$$



★ Solution of Day 25 ★

Check Problem Description: Day 25



The phase equation is given by,

$$f = \frac{1+\cos\varphi}{2}$$

To have the minimum value of f, the value of has to be maximized. Consider an observer on the surface of Mars, the angle φ would be the elongation angle. Now, notice that the maximum value of φ would only be achievable by having Earth on Mars' greatest elongation angle, which would require the Mars – Earth – Sun angle to be exactly 90°, or to have the line of sight of Mars – Earth be tangential to Earth's Orbit.

Therefore,

$$\sin\varphi = \frac{1}{1.52} \implies \varphi = 41.14^\circ$$
$$\boxed{f = 87.66\%}$$

★ Solution of Day 26 ★

Check Problem Description: Day 26

Given:

1. Observed wavelength of $\lambda_{H_\alpha} = 7500$ Å,

Question: What is the radial velocity v_r?

Using Rydberg formula for Hydrogen,

$$\frac{1}{\lambda_0} = R\left(\frac{1}{n^2} - \frac{1}{m^2}\right) \implies \lambda_0 = 6564.67 \text{ Å}$$
$$z = \frac{\lambda - \lambda_0}{\lambda} = 0.142$$

Answer: Due to the high values of Doppler Shifting, we have to use the relativistic Doppler Shift formula:

$$z = \frac{\Delta\lambda}{\lambda_0} = \sqrt{\frac{1+\frac{v_r}{c}}{1-\frac{v_r}{c}}} - 1$$

Solving for v_r:

$$z + 1 = \sqrt{\frac{1+\frac{v_r}{c}}{1-\frac{v_r}{c}}}$$
$$(z+1)^2 = \frac{1+\frac{v_r}{c}}{1-\frac{v_r}{c}}$$
$$(z+1)^2\left(1-\frac{v_r}{c}\right) = 1+\frac{v_r}{c}$$
$$(z+1)^2 - (z+1)^2\frac{v_r}{c} = 1+\frac{v_r}{c}$$
$$(z+1)^2 - 1 = \frac{(z+1)^2}{c}v_r + \frac{v_r}{c}$$
$$(z+1)^2 - 1 = \left((z+1)^2+1\right)\frac{v_r}{c}$$
$$v_r = \left(\frac{(z+1)^2-1}{(z+1)^2+1}\right)c$$
$$v_r = \boxed{0.132c}$$

★ Solution of Day 27 ★

Check Problem Description: Day 27

Given: Since the Sun's altitude determines when it is visible, we define the sunset duration as the time taken for the Sun to move from an altitude of

$$a_{\odot,1} = \frac{\theta_\odot}{2}, \quad a_{\odot,2} = -\frac{\theta_\odot}{2}$$

Notice that, for an observer on the North Pole the Sun's altitude is equal to its declination, $a_\star = \delta_\star$. Therefore, we can conclude that the sunset at the North Pole occurs when the Sun's altitude is between $\frac{\theta_\odot}{2}$ and $-\frac{\theta_\odot}{2}$.

Derived from the *Ecliptic–Equatorial* spherical triangle, for any stellar object that lies on the ecliptic plane ($\beta = 0$), follow the form:

$$\sin\delta = \sin\varepsilon\sin\lambda$$

Therefore we can perform the calculations to find $\Delta\lambda$, which will lead into finding Δt

- $\sin\delta_{\odot,1} = \sin\varepsilon\sin\lambda_{\odot,1}$
  $\delta_{\odot,1} = 0.266°$
  $\lambda_{\odot,1} = 179°19'59.77''$
- $\sin\delta_{\odot,2} = \sin\varepsilon\sin\lambda_{\odot,2}$
  $\delta_{\odot,2} = -0.266°$
  $\lambda_{\odot,2} = 180°40'0.83''$

The difference in ecliptic longitude between these two positions is:

$$\Delta\lambda_\odot = 1°20'0.45''$$

To convert this longitude change into time,

$$\lambda = \frac{n}{365.25} \times 360°$$
$$n = \frac{\lambda}{360°} \times 365.25$$
$$\Delta t = \frac{\Delta\lambda}{360°} \times 365.25$$
$$\Delta t = \boxed{32^h 28^m 11^s}$$

★ Solution of Day 29 ★

Check Problem Description: Day 29

Given:

1. $r_{orb} = 10$ kpc,
2. $v_{orb} = 300$ km/s,
3. $L_G = 4 \times 10^{10}\ L_\odot$.

Question: What is the mass-to-light ratio of the galaxy?

Answer:

$$v_{orb} = \sqrt{\frac{GM_G}{r_{orb}}} \implies M_G = \frac{v_{orb}^2 \cdot r_{orb}}{G}$$
$$M_G = 2.09 \times 10^{11}\ M_\odot$$

Now,

$$\Upsilon = M/L = \frac{M_G}{L_G} = \frac{2.09 \times 10^{11}\ M_\odot}{4 \times 10^{10} L_\odot}$$
$$\Upsilon = \boxed{5.22}$$

Note. Masses are often calculated from the dynamics of the virialized system or from gravitational lensing. The original author simplified by using orbital velocity. Typical mass-to-light ratios for galaxies range from 2 to 10 $\Upsilon_\odot$ while on the largest scales, the mass-to-light ratio of the observable universe is approximately 100 $\Upsilon_\odot$, in concordance with the current best fit cosmological model.

★ Solution of Day 30 ★

Check Problem Description: Day 30

Given:

1. $\delta_{GRS} = 1.5' = 90''$,
2. $f/D = 18$ km/s,
3. $D = 60$ cm= 600 mm.

Question: What is the size of the image on the plate?

Answer: What we're trying to determine is the length of the projected image out of the telescope. therefore, we need the plate scale parameter of the telescope.

We know the mathematical expression for the plate scale is as follows

$$PS = \frac{206265''}{f} = \cdots\ ''/mm$$

knowing that we need to determine the f value of the telescope.

$$f/D = 18 \implies f = 18 \cdot D = 10800 \text{ mm}$$

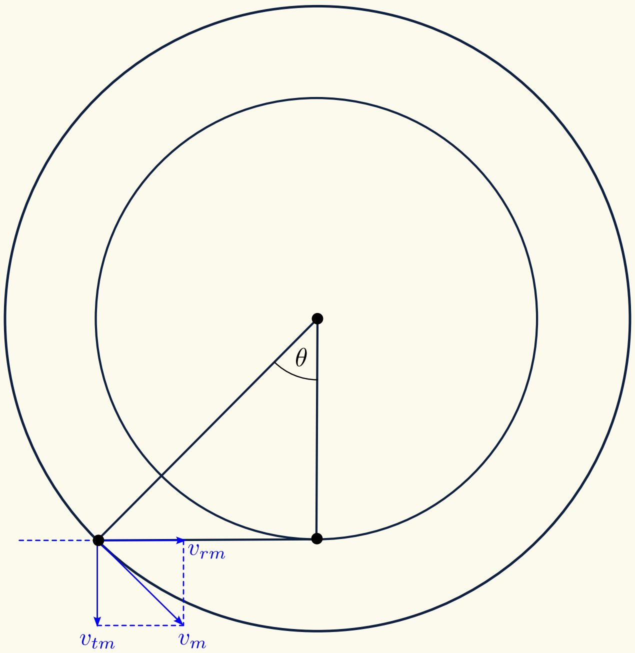
$$PS = \frac{206265''}{f} = 19.1''/mm$$

Deviding the δ_{GRS} by PS, we get the projected image length,

$$l = \frac{\delta_{GRS}}{PS} = \boxed{4.71 \text{ mm}}$$

★ Solution of Day 31 ★

Check Problem Description: Day 31



Note. To understand the problem statement one need to understand the name of each planetary position and especially notice if is it an Eastern or Western position (Elongation/Quadrature). Also one should understand the significance of the First Point of Aries. Here it means that it lies on the ecliptic plane $\beta = 0$, so we don't want to account for addition components of the velocity.

Because the value of Δt is very short, we're allowed to use the approximation using the relative angular velocity.

$$v_m = \sqrt{\frac{GM_\odot}{d_m}} = 24.2 \text{ km/s}$$

$$r_m = 1.52 \text{ AU}$$
$$v_{tm} = v_m \cdot \sin\theta$$
$$d_{m-\oplus} = r_m \cdot \sin\theta$$
$$\omega_m = \frac{v_{tm}}{d_{m-\oplus}} = \frac{v_m}{r_m} = 0.525°/day$$

Therefore the next day we would see Mars $\underline{31'31.21''}$ off the vernal equinox.

★ Solution of Day 32 ★

Check Problem Description: Day 32

Given:

1. $v_{orb} = 220$ km/s,
2. $d_G = 8$ kpc,
3. $y_G = 0.5$ kpc $\implies$ disk thickness.
4. $n = 0.495$ star/pc^3

Question: What is the value of R_s?

Answer:

- Calculating the mass inside the Sun's orbit

$$v_{orb} = \sqrt{\frac{GM_0}{d_G}}$$

$$M_0 = \frac{v_{orb}^2 \cdot d_G}{G} \implies M_0 = 8.98 \times 10^{10}\ M_\odot$$

- Calculating the predicted mass inside the Sun's orbit

$$M_{0,pred} = \pi d_G^2\, y_G\, n$$
$$M_{0,pred} = 4.98 \times 10^{10}\ M_\odot$$

- difference in mass is the mass of the Black-hole

$$M_{BH} = M_0 - M_{0,pred} = 4 \times 10^{10}\ \mathrm{M}_\odot$$

- Calculating the Schwarzschild Radius

$$R_S = \frac{2GM_{BH}}{c^2} = 1.19{\times}10^{14} \text{ m} = \boxed{791.86 \text{ AU}}$$

★ Solution of Day 33 ★

Check Problem Description: Day 33

Given:

1. $d = 40$ mm,
2. $d_{\text{telescope}} = 1620$ mm,
3. $d_{\text{exit}} = d_{\text{pupil}} = 5$ mm.
4. Pixel size $= 9\mu m \times 9\mu m$

Question: How many pixels are activated?

Answer:

- Calculating the focal length

$$d_{\text{exit}} = \frac{D}{M} \implies M = \frac{D}{d_{\text{exit}}}$$
$$\frac{f_{OB}}{f_{OC}} = M = \frac{400}{5}$$
$$f_{OB} = 80 \cdot f_{OC} \quad (1)$$

- Given:

$$d_{\text{telescope}} = 1620$$
$$f_{OB} + f_{OC} = 1620 \quad (2)$$

Eq. 1 → 2

$$81 \cdot f_{OC} = 1620\,\text{mm}$$
$$f_{OC} = 20 \text{ mm}$$
$$f_{OB} = 1600 \text{ mm}$$

- Calculating the plate scale,

$$PS = \frac{206265''}{f_{OB}} = 128.92''/mm$$

- Calculating the Sun's projected length,

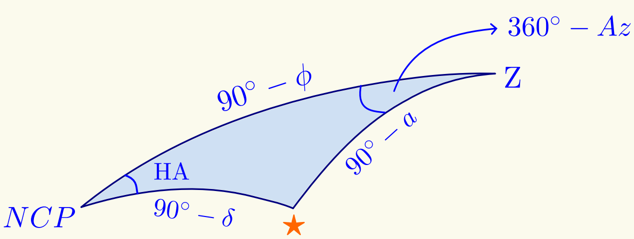
$$\delta_\odot = \frac{2 \times 6.69 \times 10^8\ mm}{1.5 \times 10^{11}\ m} \times 206265$$
$$\delta_\odot = 1914.144''$$
$$l = \frac{\delta_\odot}{PS} = 14.848\ mm$$

- Amount of pixels activated,

$$\left(\frac{l}{px.\,size}\right)^2 \approx \boxed{2722500\ px}$$

★ Solution of Day 34 ★

Check Problem Description: Day 34



Given:

1. $\phi = 34°43'23''$,
2. 19 October,
3. Observation: 1 Hour before Sunset
4. $h = 52$ m.

$$\lambda_\odot = \frac{n}{365.25} \times 360°$$
$$\lambda_\odot = \frac{212}{365.25} \times 360° \implies \lambda_\odot = 208°57'9.98''$$

$$\sin\delta_\odot = \sin\varepsilon \sin\delta_\odot \implies \delta_\odot = -11°7'46.76''$$

For $a = 0°$ the following equation is applicable,

$$\cos HA = -\tan\phi\tan\delta$$
$$\cos HA_\odot = -\tan\phi\tan\delta_\odot$$
$$\implies HA_\odot = 5^h16^m37.23^s$$

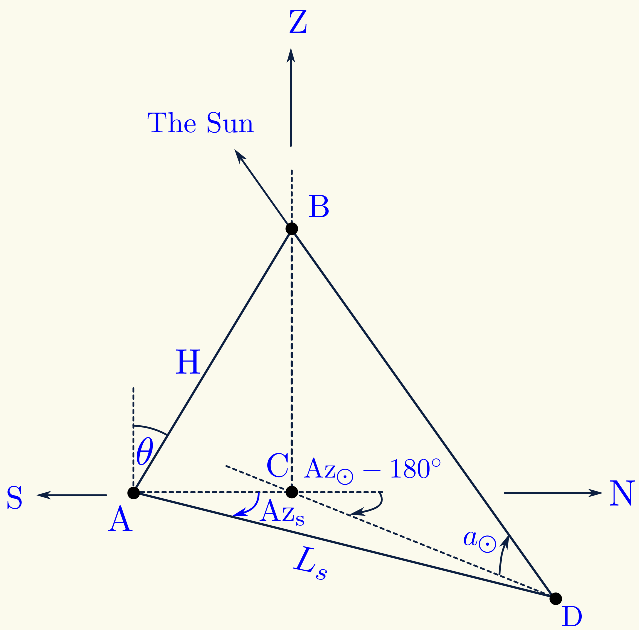
Therefore 1 hour before sunset would be $4^h16^m37.23^s$.

$$\sin a_\odot = \sin\phi\sin\delta_\odot + \cos a_\odot\cos\delta_\odot\cos HA_\odot$$
$$a_\odot = 10°7'10.68''$$

$$\sin\delta_\odot = \sin a_\odot \sin\phi + \cos a_\odot\cos\phi\cos(360° - Az)$$
$$\cos(360° - Az) = \frac{\sin\delta_\odot - \sin a_\odot\sin\phi}{\cos a_\odot\cos\phi}$$
$$\implies Az = 243°46'2.33''$$

Let AB be the tower, with tipping angle θ, and height H, then its shadow would be AD. Notice that B's shadow is D.

Also, notice that the Altitude of the Sun ($a_\odot$) and the Azimuth of the Sun ($Az_\odot$) affect both the direction AND the length of the shadow.



To determine L_s we can do the following

- Determine AC

$$AC = H \sin\theta$$
$$AC = 4.98 \text{ m}$$

- Determine CB

$$CB = H \cos\theta$$
$$AC = 51.76 \text{ m}$$

- Determine CD

$$CD = \frac{CB}{\tan a_\odot}$$
$$CD = 290 \text{ m}$$

- Determine $\angle ACD$

$$\angle ACD = 180^\circ - (Az_\odot - 180^\circ)$$
$$\angle ACD = 360^\circ - Az_\odot = 116^\circ 13' 57.67'$$

$$L_s = \sqrt{AC^2 + CD^2 - 2 \cdot AC \cdot CD \cdot \cos(\angle ACD)}$$
$$L_s = 292.24 \text{ m}$$

To determine Az_s, we can do the following:

$$\frac{L_s}{\sin(\angle ACD)} = \frac{CD}{\sin(Az_s)}$$
$$\sin(Az_s) = \frac{CD}{L_s} \cdot \sin(\angle ACD)$$
$$Az_s = 62^\circ 53' 26.83''$$

Therefore the final answers are

$$L_s = 292.24 \text{ m}$$
$$Az_s = 62^\circ 53' 26.83''$$