AO Class

Prep Course for

TA Final Exam Questions

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Introduction

- This is a short note for summarizing TA's seminars to help students preparing the final exam questions that will be prepared by the TA.

- Note
  - All the problems from TA may have strange numbers in the questions.
  - But the answers should be very simple.
  - Example: answer $= ax + b \quad = 6.902 \times 0.123 + 2.151 \qquad\qquad = 3.000,$
  - Example: answer $=$ formula $= 12.334 - 0.034 + (1.111 + 2.33) \times 0.0 = 12.300,$

crazy numbers neat answer

- Contents of this file

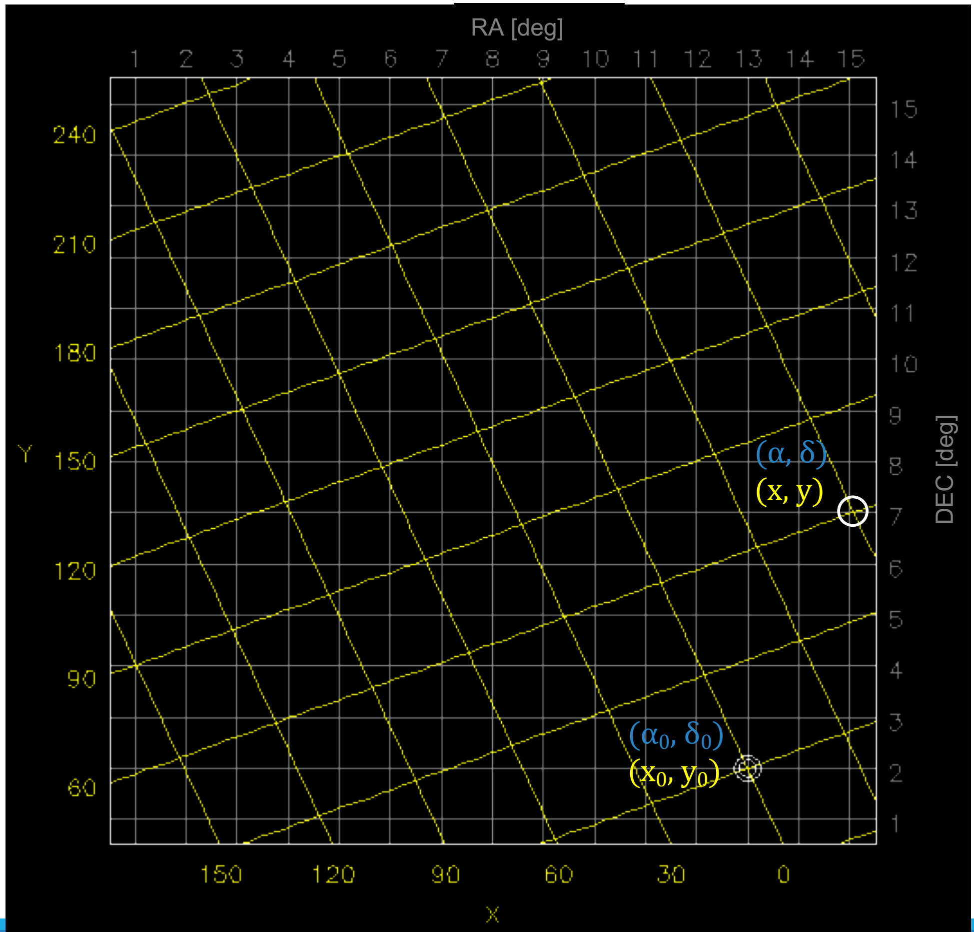
1. WCS Basics
2. Standardization

WCS Basics

WCS = World Coordinate System

WCS basics

- Header contains information.



https://www.atnf.csiro.au/people/mcalabre/WCS/Intro/WCS02.html

e.g.,

$$\alpha = \alpha_0 + \frac{\partial \alpha}{\partial x}(x - x_0) + \frac{\partial \alpha}{\partial y}(y - y_0)$$

$$\delta = \delta_0 + \frac{\partial \delta}{\partial x}(x - x_0) + \frac{\partial \delta}{\partial y}(y - y_0)$$

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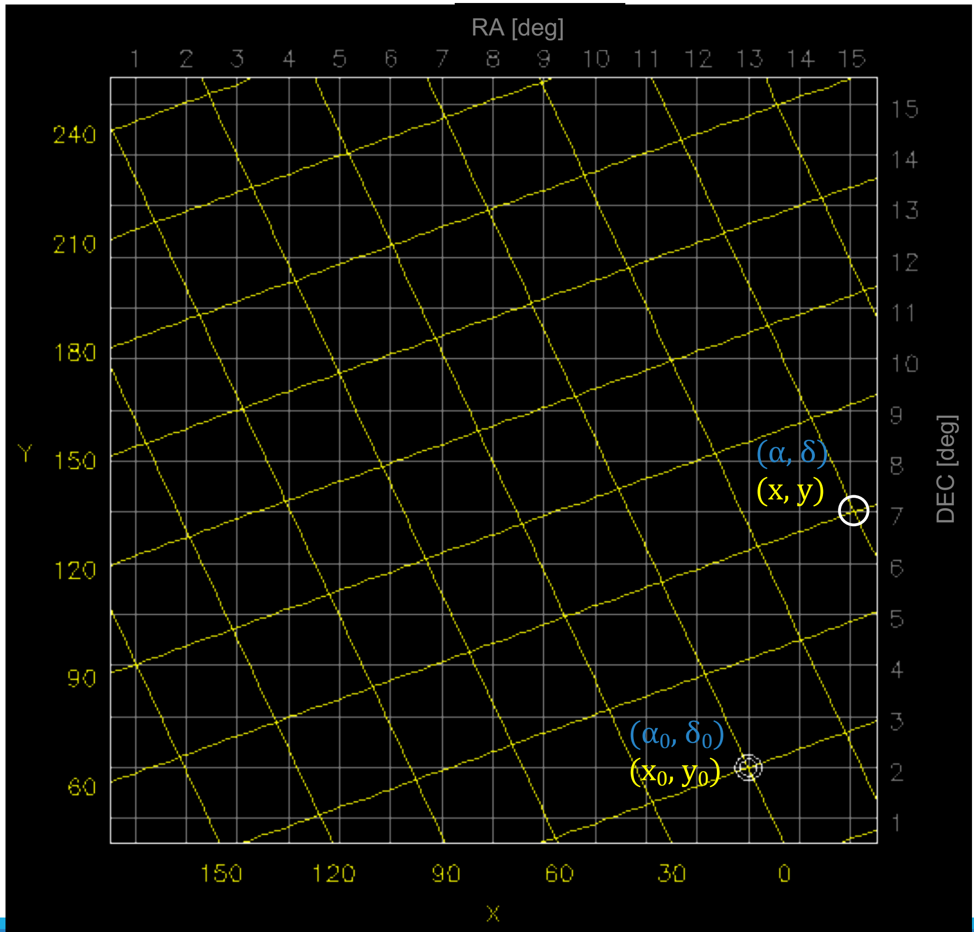

CRPIX1 CRPIX2
(coordinate reference pixel)

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CRVAL1 CRVAL2
(coordinate reference value)

CDi_j
Coordinate Description
matrix component
i (RA/DEC), j (x/y)

```
(see left figure and fill)
CTYPE1 = '?'          CRVAL1 = ?
CTYPE2 = '?'          CRVAL2 = ?
CUNIT1 = '?'          CRPIX1 = ?
CUNIT2 = '?'          CRPIX2 = ?
EQUINOX= 2000.0
```

WCS basics

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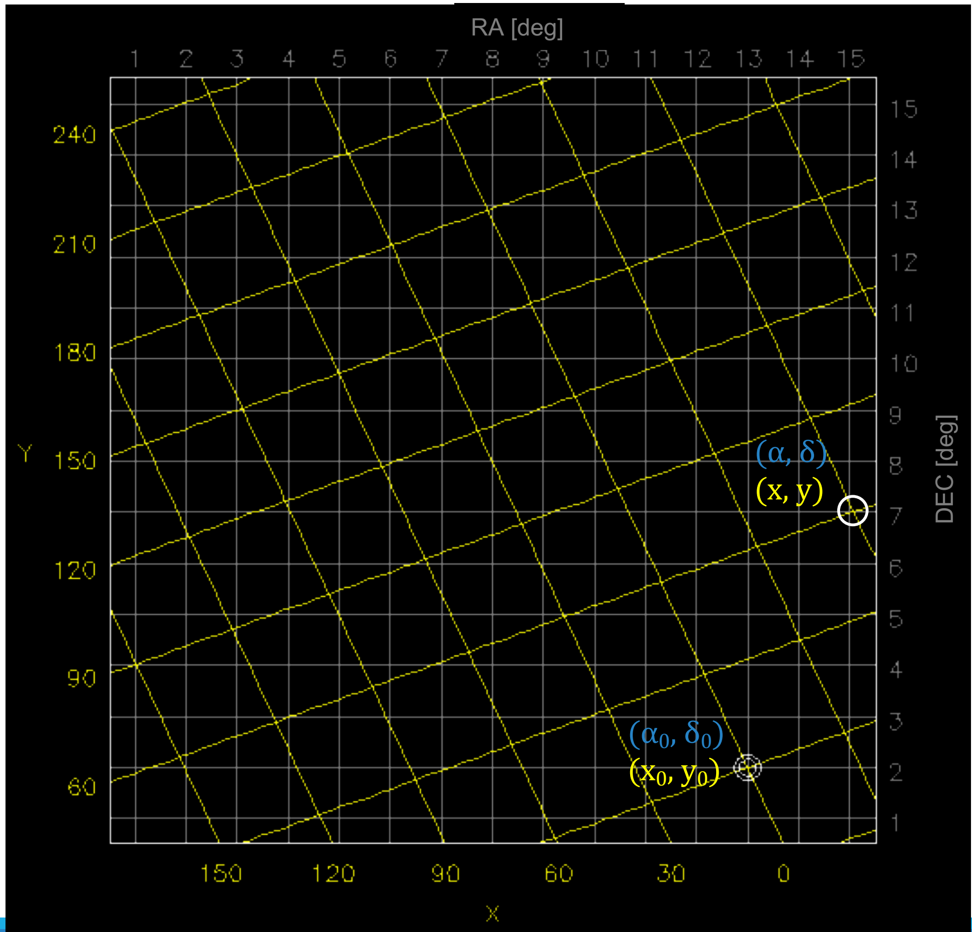

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(see left figure and fill)
CTYPE1 = 'RA'            CRVAL1 = 13.00
CTYPE2 = 'DEC'           CRVAL2 = 2.00
CUNIT1 = 'deg'           CRPIX1 = 0.00
CUNIT2 = 'deg'           CRPIX2 = 0.00
EQUINOX= 2000.0
```

Note: normally we put the "projection" algorithm in CTYPE, such as "RA--TAN": see https://fits.gsfc.nasa.gov/fits_wcs.html

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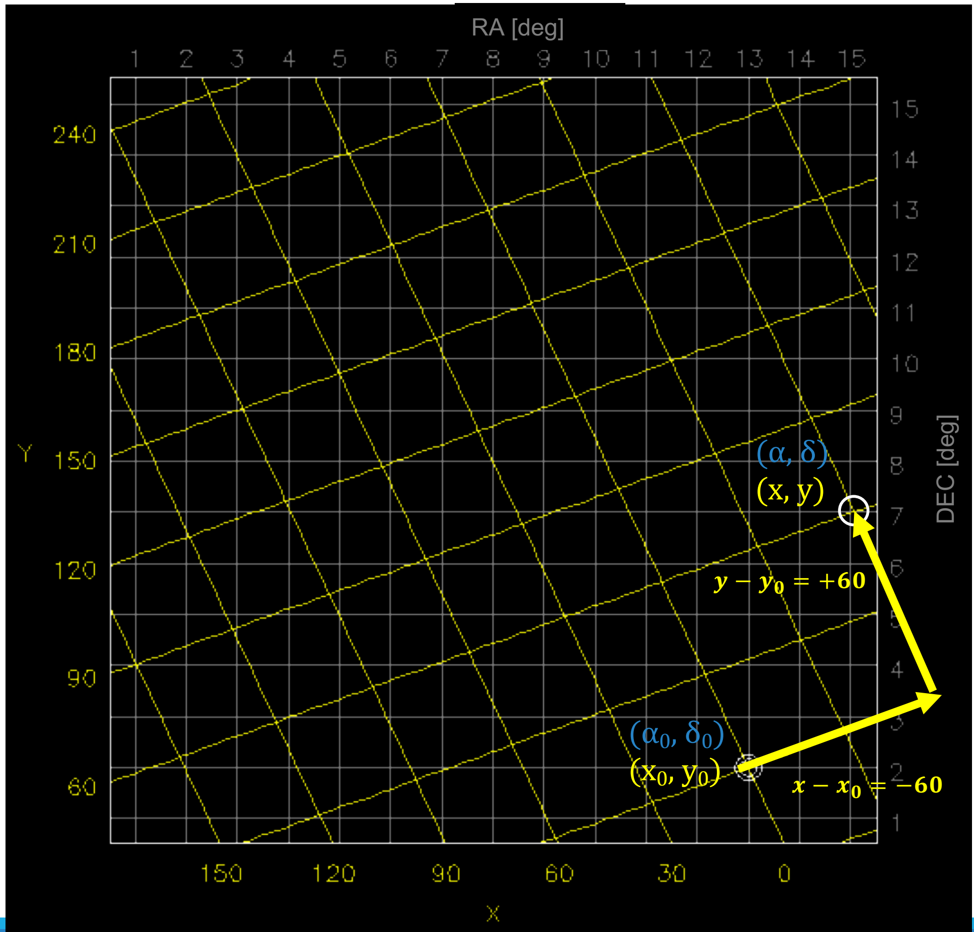

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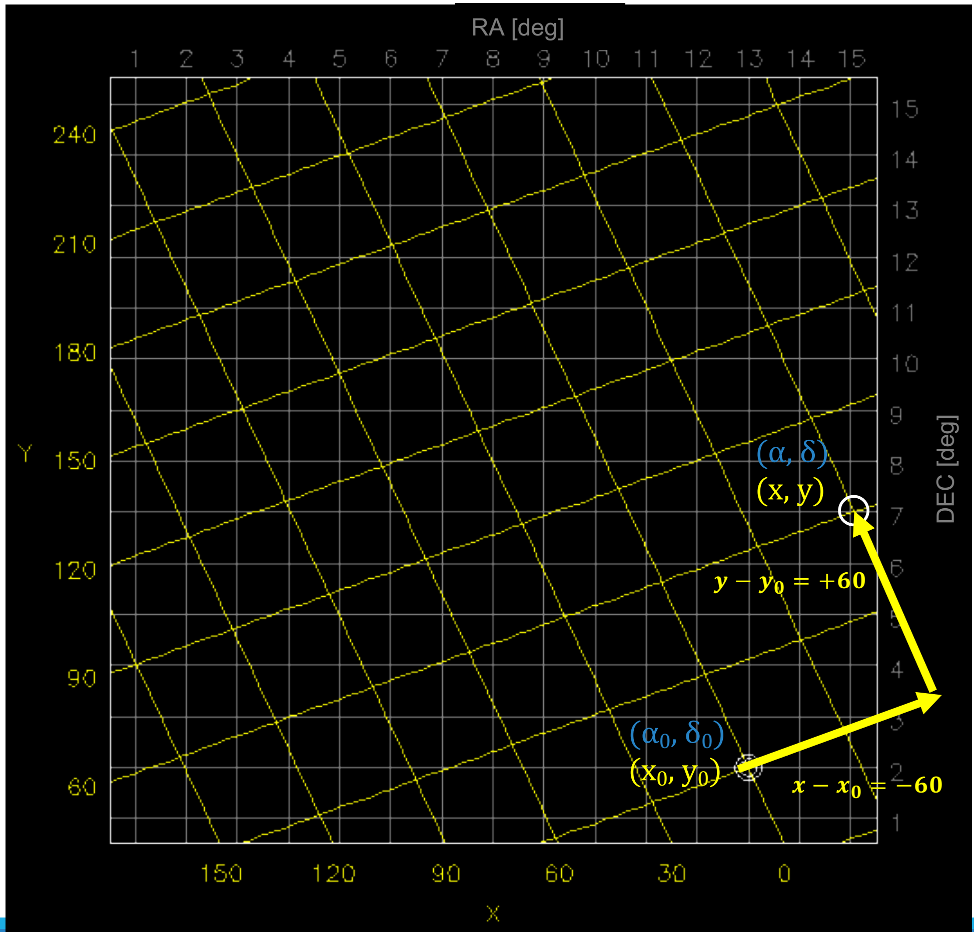

$$\begin{bmatrix} \text{CTYPE1} \\ \text{CTYPE2} \end{bmatrix} = \begin{bmatrix} \text{CRVAL1} \\ \text{CRVAL2} \end{bmatrix} + \begin{bmatrix} \text{CD1\_1} & \text{CD1\_2} \\ \text{CD2\_1} & \text{CD2\_2} \end{bmatrix} \begin{bmatrix} \text{X} - \text{CRPIX1} \\ \text{Y} - \text{CRPIX2} \end{bmatrix}$$

In the units of `CUNIT1` and `CUNIT2`
at the equinox position of J `EQUINOX` (i.e., J 2000.0 in this case)

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CTYPE1  = 'RA'          CRVAL1  = 13.00
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WCS basics

- pixel scale
  - $= \frac{\text{pixel size}}{\text{effective focal length}}$
  - unit: [rad/pix], but we usually convert it to arcsec/pix
  - Useful numbers: 1 [rad] = 57.29578[°] = 206265 [arcsec]
  - FOV = (number of pixels) × (pixel scale)

- Quick check if WCS is correct?
  - Roughly calculate pixel scale
  - Compare it with CD matrix in the header.

- Example:
  - Focal length is 2 m, pixel size is 12 μm.
  - Then pixel scale is 3.43×10^{-4}deg/pix.
  - Therefore, `CDi_j` must have an order of $\sim3\times10^{-4}$ unless severe distortion is there.

Standardization

Standardization

- Standardization formula:

$$\boxed{\begin{aligned} M_f &= m_f - k_f' X - k_f'' XC + z_f + k_f C \\ &= m_f + (z_f - k_f' X) + (k_f - k_f'' X) C \\ &= m_f + a(X) + b(X) C \end{aligned}}$$

  - f: The filter (V, B, g', etc).
  - X : airmass
  - M_f : The *standard* apparent magnitude (or the *true* apparent magnitude) at filter *f*.
  - m_f : The *instrumental* magnitude ($m_f \coloneqq -2.5 \log_{10} N$).
  - C : The *true* color index, e.g., B-V or g-r.
  - k_f' : The first order extinction coefficient at filter *f*.
  - k_f'' : The second order extinction coefficient at filter *f*.
  - z_f : The zero point at filter *f*.
  - k_f : The system transform coefficient at filter *f*.

$$m_f \coloneqq -2.5 \log_{10} count$$

To the 1st order Taylor series:

$$|dm_f| = \frac{2.5}{\ln 10} \frac{\Delta count}{count}$$

Note: lower- and upper-cased letters are used for the *instrumental* and *true* magnitudes, respectively.

- For example, v, b, $m_{g'}$ are instrumental magnitudes of an object and V, B, and $M_{g'}$ are true appparent magnitudes of it.

Standardization

at least two with different colors: called **blue-red pair**

- If you observed standard stars in many airmasses (X), you know M_f and C from catalog and m_f from CCD image.

$$M_f - m_f = a(X) + b(X)C$$

Known

$$a(X) \coloneqq z_f - k'_f X$$
$$b(X) \coloneqq k_f - k''_f X$$

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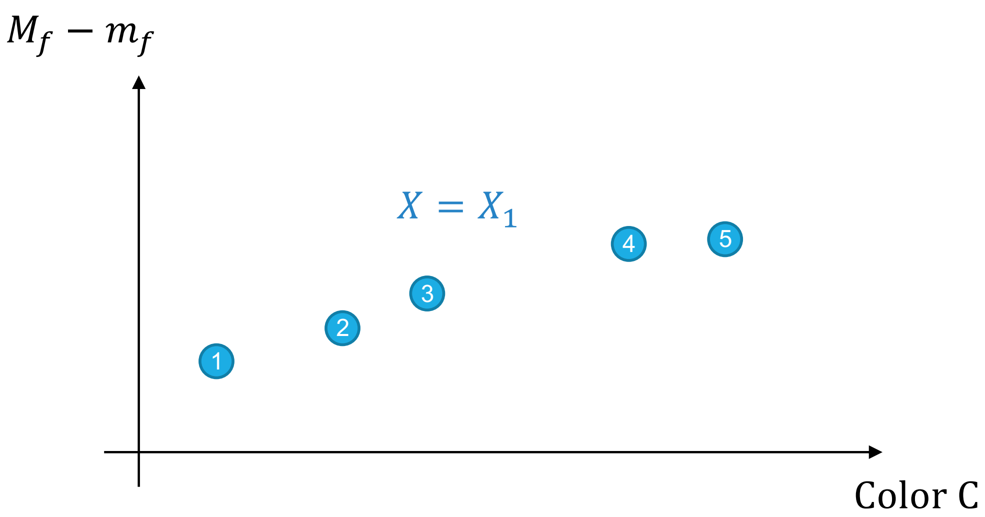

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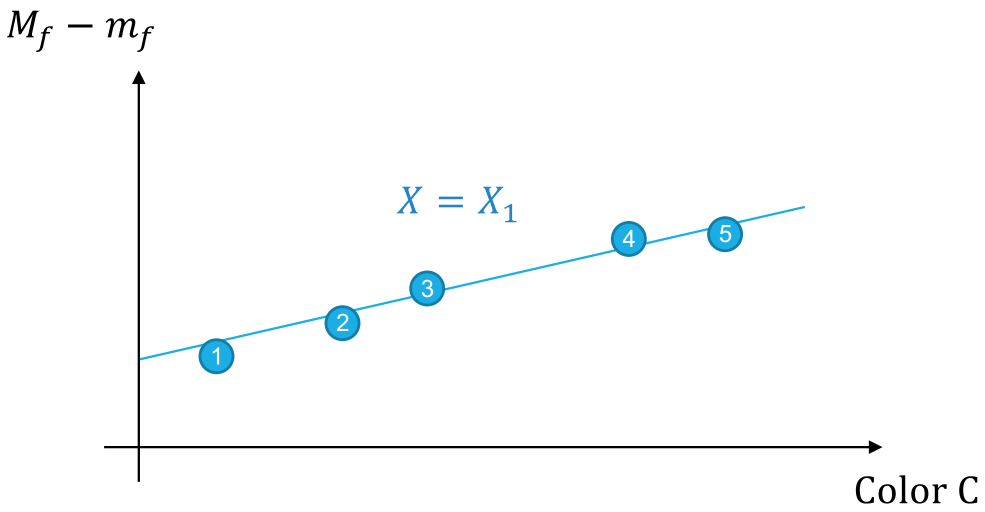

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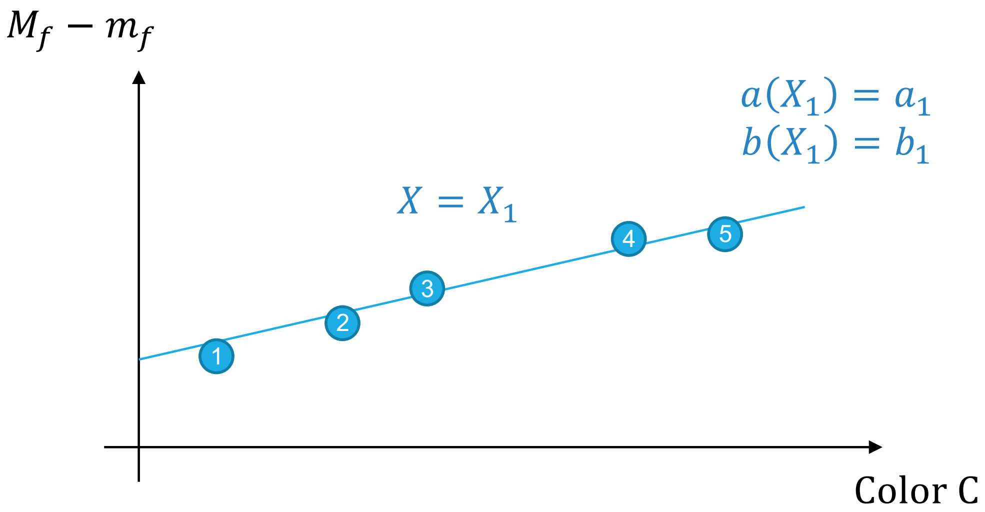

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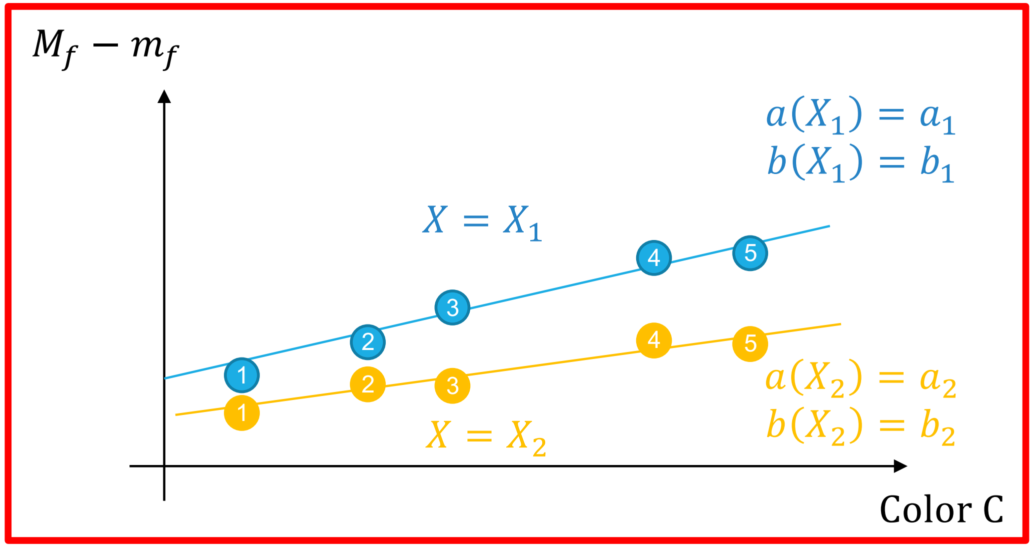

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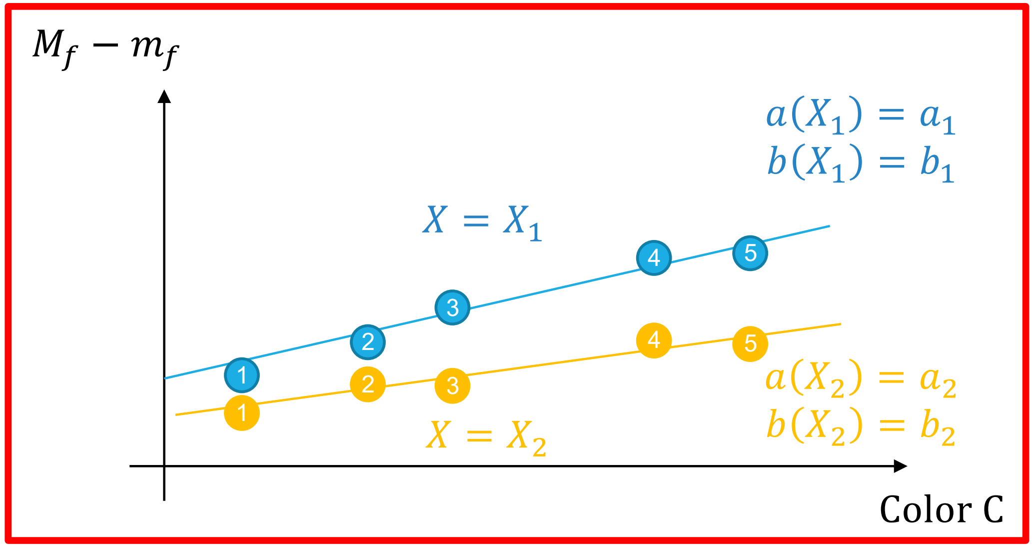

Quick quiz:
Say it's a graph from R_C-band with $C = V - R$ ranging from 0 to 1.
Which is larger, X_1 or X_2?

Standardization

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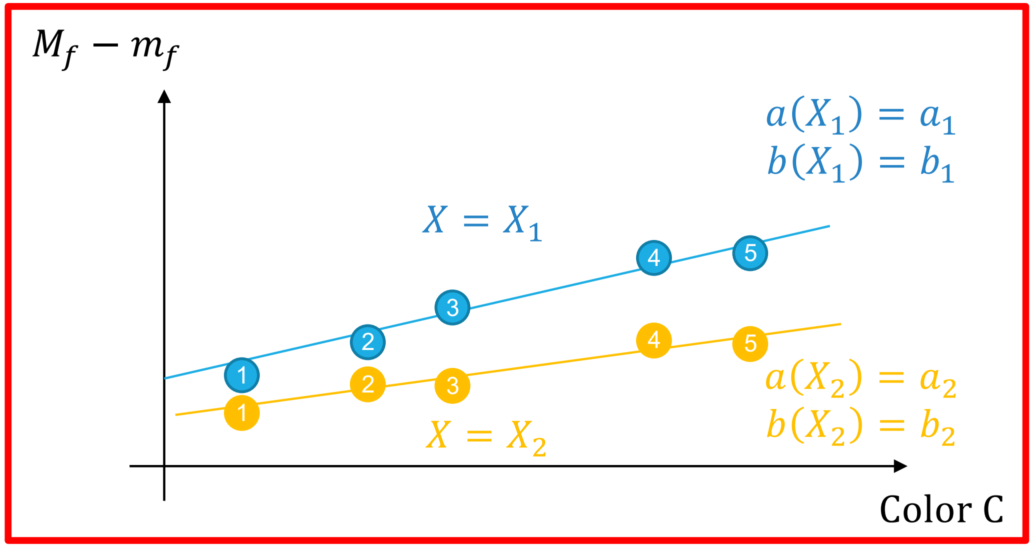

Repeat for all X values

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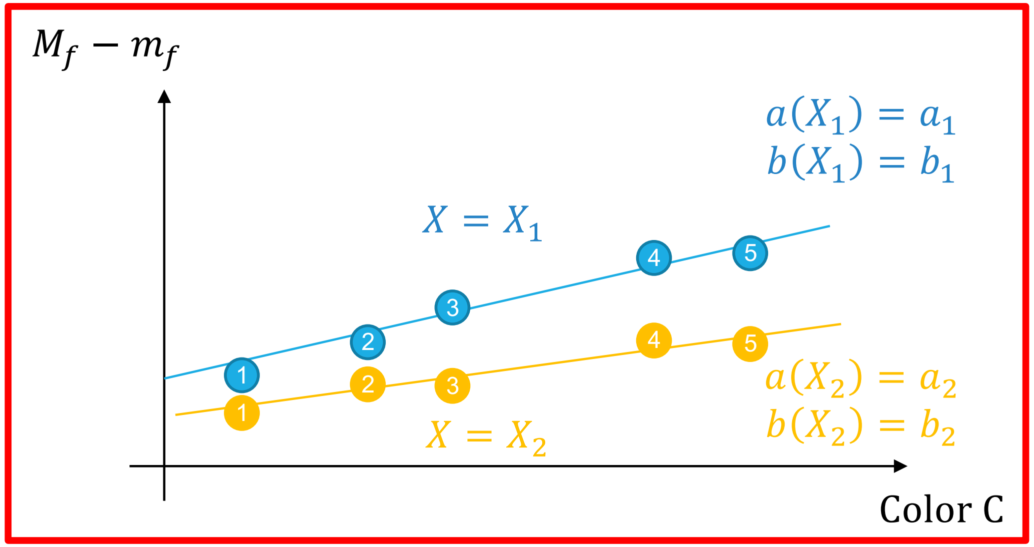

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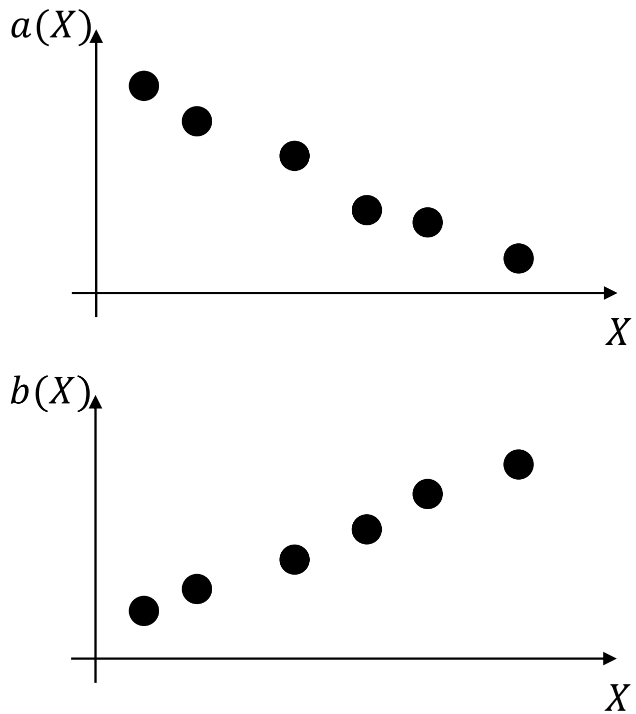

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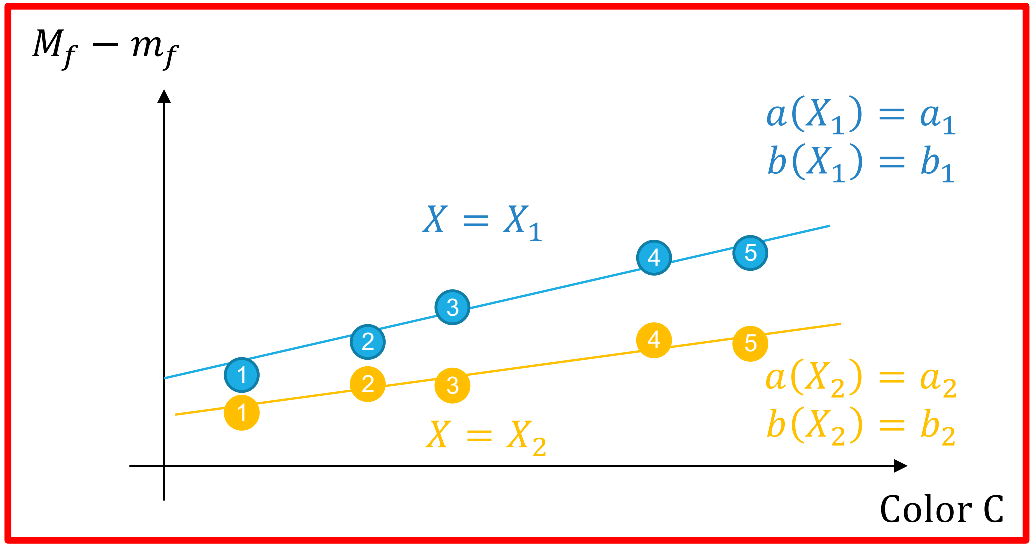

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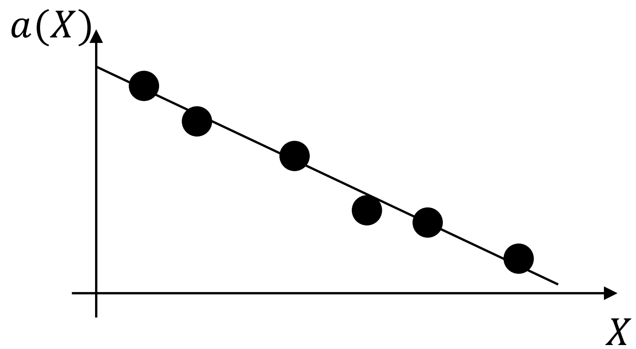

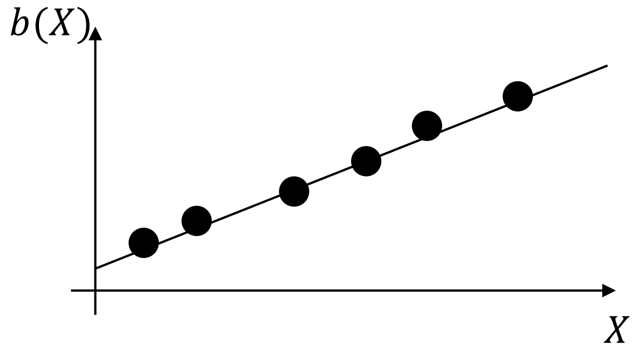

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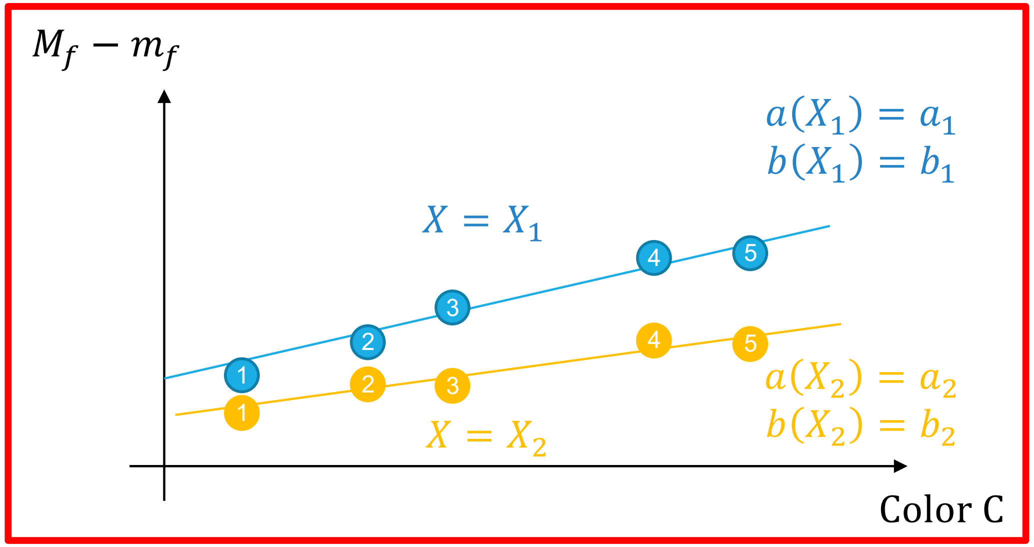

Repeat for all X values

$$a(X) \coloneqq z_f - k'_f X$$
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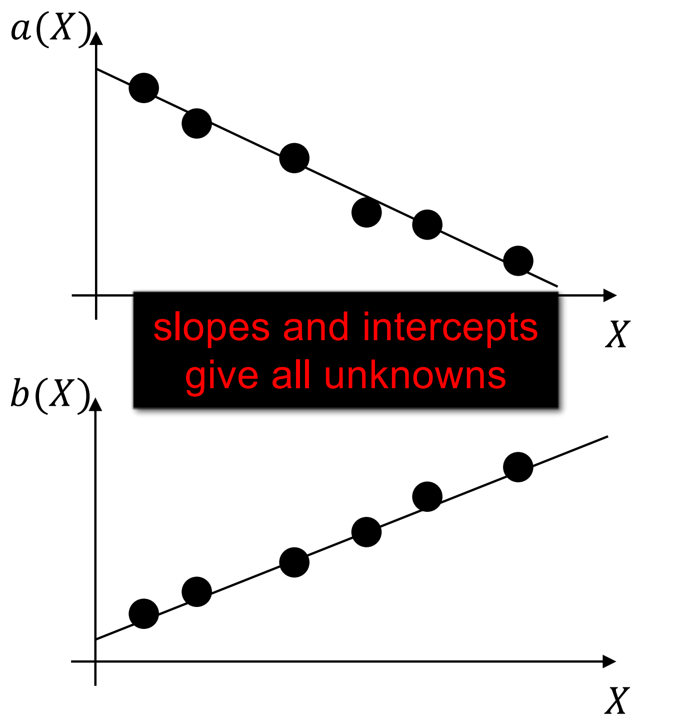

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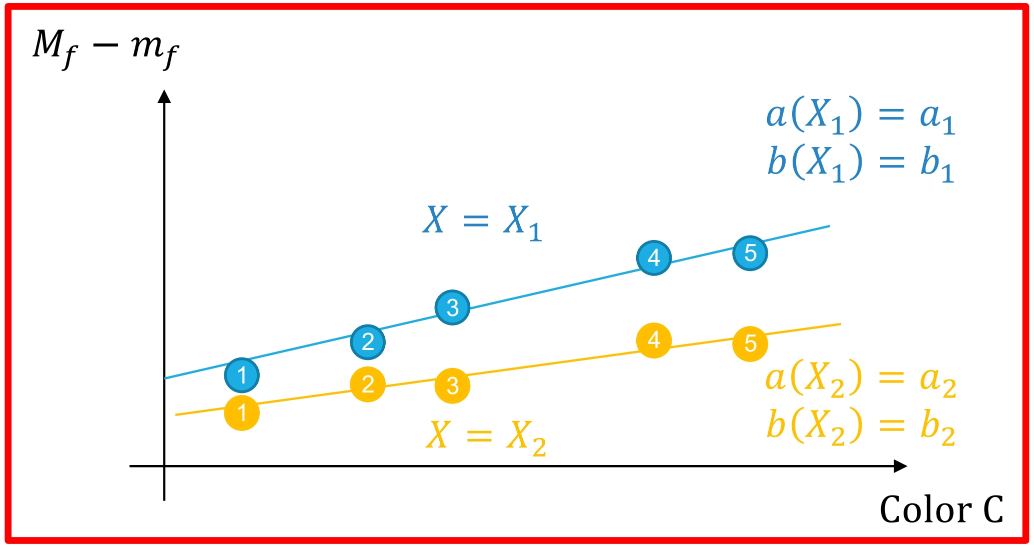

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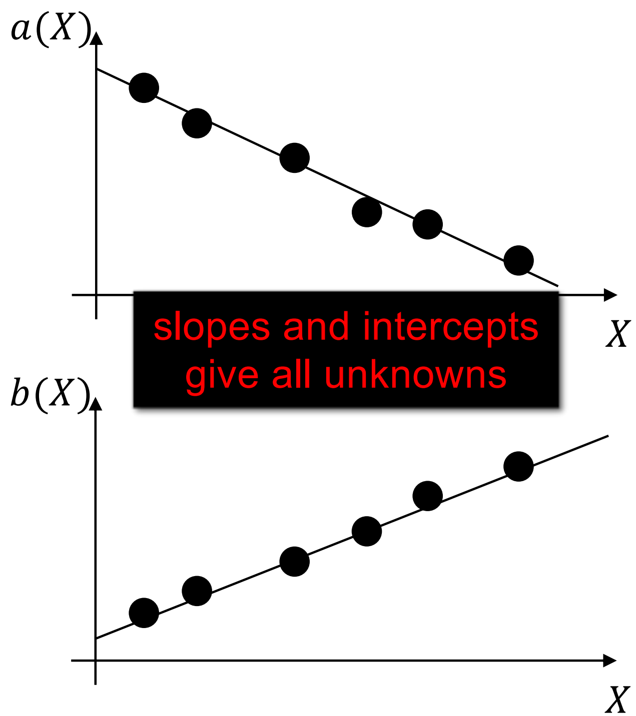


NOTE: z_f depends on the detector, so it will not depend on the filter unless you use different detector for each filter.

Do the identical thing for many filters to get coefficients for all filters. Get M_f for all filters. Then you will get the standardized color, too.

Standardization

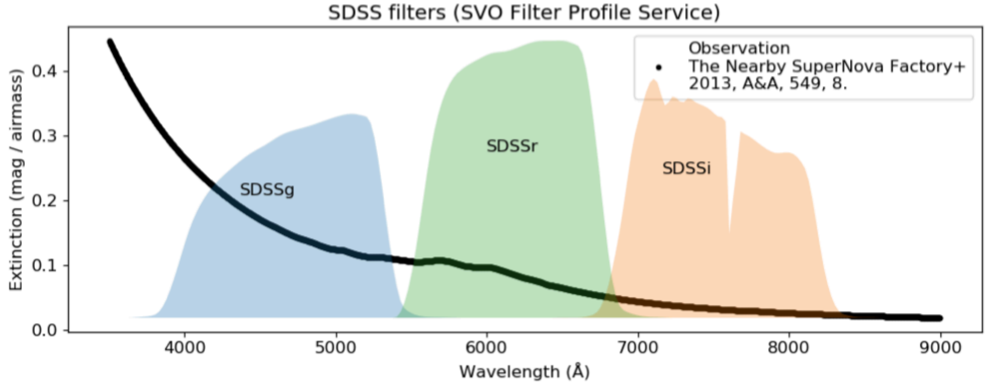
$$a(X) \coloneqq z_f - k'_f X$$
$$b(X) \coloneqq k_f - k''_f X$$

- Nominal values:

Table 4.1: The extinction coefficients of SDSS from SmithJA+ (2002, AJ, 123, 2121).

Parameter	u′	g′	r′	i′	z′
k'_f	$> +0.5$	$+0.20 \pm 0.05$	$+0.10 \pm 0.05$	$+0.05 \pm 0.05$	$+0.05 \pm 0.05$
k''_f method 1	-0.021 ± 0.003	-0.016 ± 0.003	-0.004 ± 0.003	$+0.006 \pm 0.003$	$+0.003 \pm 0.003$
k''_f method 2	-0.032	-0.015	0.000	$+0.005$	$+0.006$

- k_f: depends on instrument. $\lesssim 0.05$



- Example:
  - If the true color of target is expected to lie in [-1, 1]
  - $|b(X)C| = \left|\left(k_f - k''_f X\right)C\right| \sim |(0.05 + 0.01X)C| \lesssim 0.1$
  - Sometimes (actually quite often) it is better not to consider this color term, as it will only increase redundant degrees of freedom.