CS F402 Computational Geometry

Proposal, Literature Survey and Initial work

Capacitated K-Supplier Problem

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1 Problem Statement

Given a set $\boldsymbol{F}$ of $\boldsymbol{m}$ points associated with non-negative integer capacity and a set $\boldsymbol{C}$ of $\boldsymbol{n}$ points in the plane, the objective is to open a set $\boldsymbol{F'} \subseteq \boldsymbol{F}$ of $\boldsymbol{k}$ facilities such that the maximum distance of any client in $\boldsymbol{C}$ to its nearest open facility in $\boldsymbol{F}$' is minimized. The number of clients assigned to a center can not exceed the center's capacity.

Let $d(f, \boldsymbol{F'})$ = minimum distance from point f to g where g is any point in $\boldsymbol{F}$'. Then,

$$d(f, \boldsymbol{F'}) = min\, d(f, g) : g \in \boldsymbol{F'} \tag{1}$$

The given problem can be expressed as,

minimize $max\ d(f,\boldsymbol{F}')$, where

$f \in \boldsymbol{C}$ and $\boldsymbol{F'} \subseteq \boldsymbol{F}$ such that $|\boldsymbol{F'}| = \boldsymbol{k}$

2 Introduction

Multiple applications of the capacitated K-supplier problem can be found. One obvious application is in the domain of facility location in supply - chain markets and businesses, commonly observed in large supermarket chains. Here the consumer outlets form the set $\boldsymbol{C}$ and locations of potential intermediate suppliers form the set $\boldsymbol{F}$.

Other applications include opening hospitals in a city with limited capacities, opening of centres for the disposal of obnoxious substances, setting up servers in a network, data mining and information retrieval such as data clustering.

Capacitated k-center problem is another NP-hard problem closely related to the capacitated k-supplier problem. In this problem, there is a single set $\boldsymbol{C}$ from which a set $\boldsymbol{C'}$ of k points are required to be chosen so as to minimize the maximum distance of a point in the set $\boldsymbol{C}$ to a point in the set $\boldsymbol{C'}$, while ensuring that the assignments to a center cannot exceed its capacity. Any solution to this problem has the possibility of extension to our problem.

3 Literature Review and Initial work

The k-center problems have been extensively studied in the literature with many variants like outliers, opening costs etc. A simple, tight 2-approximation algorithm was given by Hochbaum and Shmoys[1] in $O(n^2 \log_2 n)$ in general metrics as early as 1985. Several other variations of this problem have been studied. The k-supplier problem which is a generalization of k-center problem has tight approximation ratio of 3 in general metrics given by Hochbaum and Shmoys which runs in $O((n^2 + mn)\log_2(mn))$ time[2]. Later a capacitated version of the k-center problem was proposed by Bar-Ilan, Kortsarz and Peleg[3].They gave a 10-factor approximation algorithm with polynomial running time.It was later improved by Khuller and Sussmann[4] to 6 in the case of uniform capacities. When multiple centers can be opened in the same location the bound was improved to 5. Much later after a gap of 15 years, in 2012, Cygan, Hajiaghayi and Khuller[5] proposed a LP rounding algorithm with constant factor approximation algorithm for arbitrary capacities with a factor in hundreds. Later it was improved to a 9-approximation by An, Bhaskara and Svensson[6].

There are many different specializations of this problem such as the capacitated k-center with outliers and non-uniform capacities with a approximation factor of 25 (Kociumaka Cygan[7](2014)), the fault-tolerant capacitated k-center(Chechik and Peleg[8](2015)) where nodes can be reassigned to some other centers in the case of a failure with an approximation factor of 9, it was later improved by Fernandes et al.[9](2016) to 6. Recently Ding et al.(2017)[10] came with capacitated k-center problem with two sided bounds and outliers with a combinatorial algorithm as opposed to previous works which are mainly based on LP. A Heterogeneous variation of this problem where capacities to be installed is decided by the algorithm based on the locations is studied by Chakrabarty et al.[11](2016).

To the best of our knowledge, the problem of our interest i.e the capacitated k-supplier problem without any constraints on it not studied very deeply but An et al.[6] in their paper, extended their approach to capacitated k-supplier problem and achieved an approximation factor of 11. Capacitated k-supplier with bounds and outliers have also been studied ([7],[10]).

Initial work involved understanding the main idea of each of the cited research work and search for the prospect of improvement. NP-hard problems given as a part of competitive programming contests were collected and those analogous to capacitative K-supplier problem were identified. Efficient use of datastructures were studied in the solutions which gave best performance.

4 Proposal to improve the existing work

In our research, we hope to extend the capacitated k-supplier problem which has an approximation factor of 11 in general metrics to Euclidean metrics. We anticipate a decrease in approximation factor, by using the properties of Euclidean space. We draw inspiration mainly from Nagarajan et al.'s[12] paper where the authors have heavily used the properties of Euclidean space and got an approximation ratio of 2.74 in the case of Euclidean k-supplier problem where it is NP-hard to approximate it beyond a factor of 2.64 in Euclidean metrics and a factor of 3 in general space. We hope to extend LP rounding, centrality of trees concepts of An et al. 's[6] work to reach the desired goal.

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