

Practical 8

Gauss Seidel Method

Gauss - Seidel Iteration Method: A general linear iterative method for the solution of the system of equations $Ax = b$ may be defined of the form.

$$x^{(k+1)} = H x^{(k)} + C$$

where $x^{(k+1)}$ and $x^{(k)}$ are approximations of x in the $(k+1)$ th and k th iterations.

H is called the **iteration matrix** of A and C is called **column vector**.

Here, $H = -D^{-1}(L+U)$

$$C = (D+L)^{-1}b$$

where, D = diagonal matrix

L = lower triangular matrix

U = upper triangular matrix.

Gauss Seidel method with number of iterations as stopping criteria:

Q1. Use the Gauss Seidel iteration method to solve the system of equations in 10 iterations

$$2x_1 - x_2 + 0x_3 = 7$$

$$-x_1 + 2x_2 - x_3 = 1$$

$$0x_1 - x_2 + 2x_3 = 1$$

with the initial vector $x^{(0)} = (0,0,0)$.

```

GaussSeidel[A0_, B0_, X0_, max_] :=
Module[{A = N[A0], B = N[B0], i, j, k = 0, n = Length[X0], X = X0, Xk = X0},
Print["X"0, "=", X];
While[k < max,
For[i = 1, i ≤ n, i++,
X[[i]] =  $\frac{1}{A[[i,i]]} \left( B[[i]] + A[[i,i]] X[[i]] - \sum_{j=1}^n A[[i,j]] X[[j]] \right)$ ;
Print["X"_{k+1}, "=", X];
Xk = X;
k = k + 1;];
Print["No. of iterations performed ", k];
Return[X];];
A = {{2, -1, 0}, {-1, 2, -1}, {0, -1, 2}};
B = {7, 1, 1};
X0 = {0, 0, 0};
GaussSeidel[A, B, X0, 10]

```

```

X0={0, 0, 0}
X1={3.5, 2.25, 1.625}
X2={4.625, 3.625, 2.3125}
X3={5.3125, 4.3125, 2.65625}
X4={5.65625, 4.65625, 2.82813}
X5={5.82813, 4.82813, 2.91406}
X6={5.91406, 4.91406, 2.95703}
X7={5.95703, 4.95703, 2.97852}
X8={5.97852, 4.97852, 2.98926}
X9={5.98926, 4.98926, 2.99463}
X10={5.99463, 4.99463, 2.99731}
No. of iterations performed 10
{5.99463, 4.99463, 2.99731}

```

Q2. Solve the system of equations

$$4x_1 + x_2 + x_3 = 2$$

$$x_1 + 5x_2 + 2x_3 = -6$$

$$x_1 + 2x_2 + 3x_3 = -4$$

with the initial vector $x^{(0)} = (0.5, -0.5, -0.5)$. Perform 11 iterations.

```

A = {{4, 1, 1}, {1, 5, 2}, {1, 2, 3}};
B = {2, -6, -4};
X0 = {0.5, -0.5, -0.5};
GaussSeidel[A, B, X0, 11]

```

$X_0 = \{0.5, -0.5, -0.5\}$
 $X_1 = \{0.75, -1.15, -0.816667\}$
 $X_2 = \{0.991667, -1.07167, -0.949444\}$
 $X_3 = \{1.00528, -1.02128, -0.987574\}$
 $X_4 = \{1.00221, -1.00541, -0.997129\}$
 $X_5 = \{1.00064, -1.00128, -0.999362\}$
 $X_6 = \{1.00016, -1.00029, -0.999862\}$
 $X_7 = \{1.00004, -1.00006, -0.999971\}$
 $X_8 = \{1.00001, -1.00001, -0.999994\}$
 $X_9 = \{1., -1., -0.999999\}$
 $X_{10} = \{1., -1., -1.\}$
 $X_{11} = \{1., -1., -1.\}$
No. of iterations performed 11
 $\{1., -1., -1.\}$
