

Practical -8

Plot Integral Surface of First Order PDE

Question 1. Solve the PDE $u_x + u_y = 1/2$. With the initial condition $u(s,s) = s/4$, $0 \leq s \leq 1$

Solution :

$$x = s + st/4 = (t^2)/4, \quad y = s + t, \quad u = s/4 + t/2$$

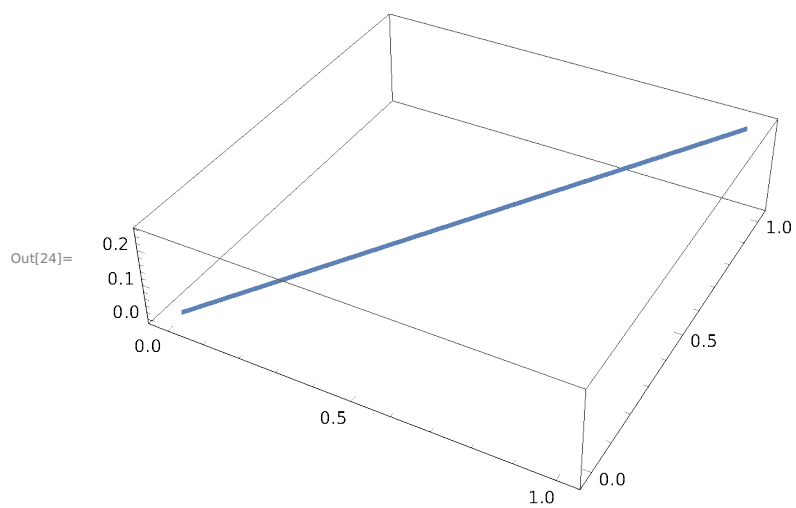
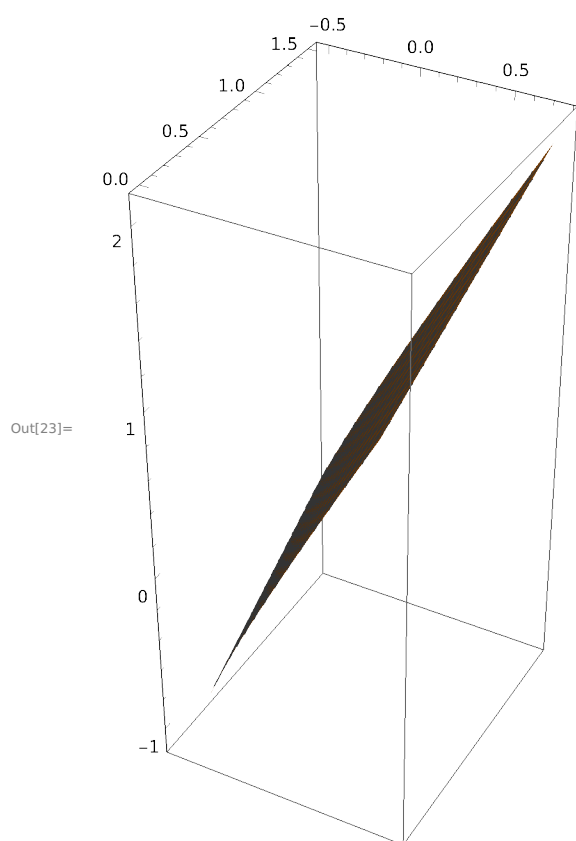
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In[19]:= Sol = DSolve[
  {x'[t] == u[t], y'[t] == 1, u'[t] == 1/2, x[0] == s, y[0] == s, u[0] == s/4}, {x[t], y[t], u[t]}, t]
Print["u[t]=", Sol[[1, 1, 2]]]
Print["y[t]=", Sol[[1, 2, 2]]]
Print["x[t]=", Sol[[1, 3, 2]]]
map = ParametricPlot3D [
  {Sol[[1, 1, 2]], Sol[[1, 2, 2]], Sol[[1, 3, 2]]}, {t, -1, 1}, {s, 0, 1}, PlotPoints -> 10]
map1 = ParametricPlot3D [{s, s, s/4}, {s, 0, 1}]
Show[map, map1, PlotLabel -> "Integral Surface through initial Curve"]
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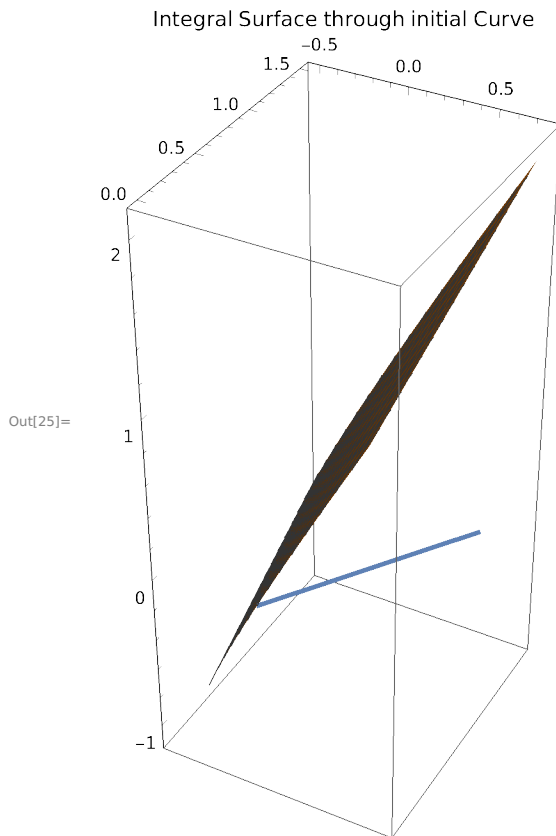
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Out[19]= {{u[t] -> 1/4 (s + 2 t), x[t] -> 1/4 (4 s + s t + t^2), y[t] -> s + t}}
```

$$u[t] = \frac{1}{4} (s + 2 t)$$

$$y[t] = \frac{1}{4} (4 s + s t + t^2)$$

$$x[t] = s + t$$





Question 2. Solve the PDE $u_x - u_y = 2$. With the initial condition $u(s,s) = 2s$, $0 \leq s \leq 1$

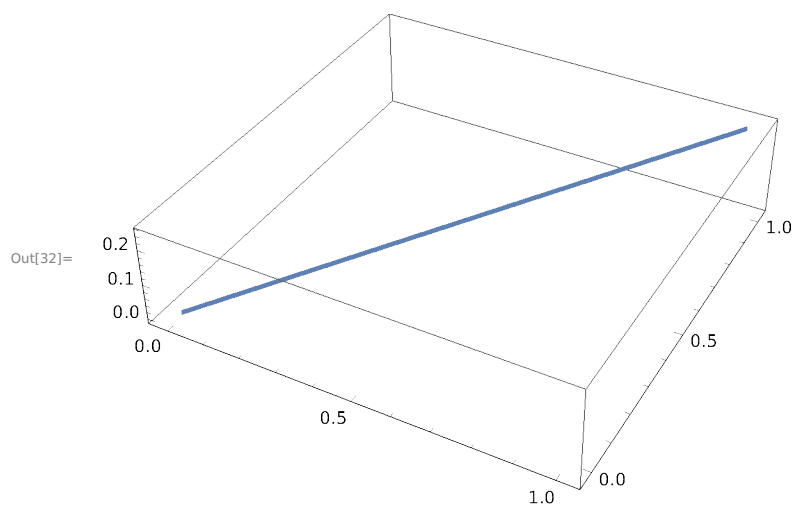
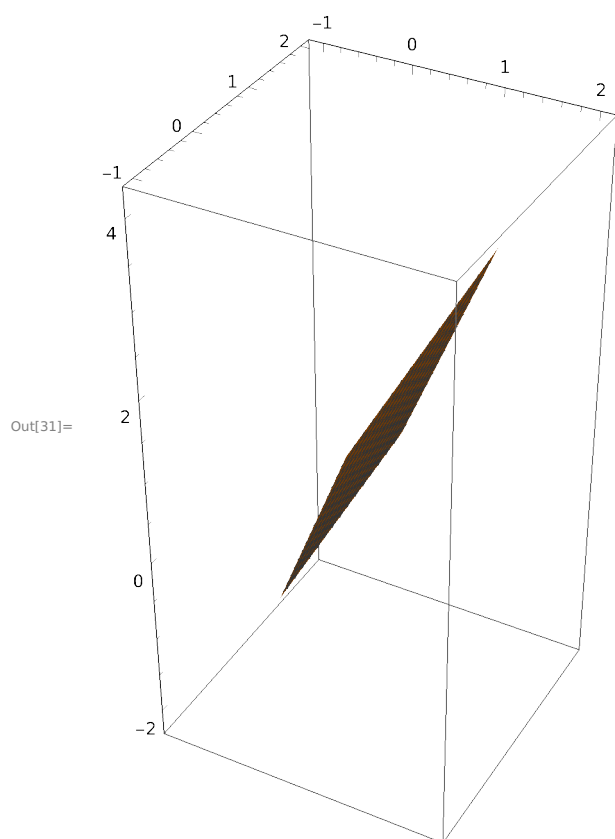
Solution :

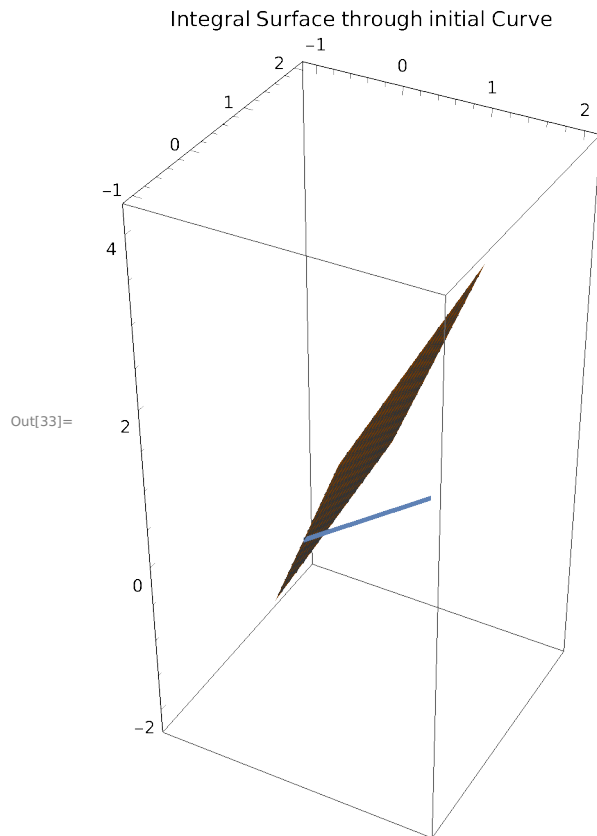
$$x = s + t, \quad y = s - t, \quad u = 2t + 2s$$

```
In[27]:= Sol =
  DSolve[{x'[t] == 1, y'[t] == -1, u'[t] == 2, x[0] == s, y[0] == s, u[0] == 2 * s}, {x[t], y[t], u[t]}, t]
Print["u[t]=", Sol[[1, 3, 2]]]
Print["y[t]=", Sol[[1, 2, 2]]]
Print["x[t]=", Sol[[1, 1, 2]]]
map = ParametricPlot3D[{Sol[[1, 1, 2]], Sol[[1, 2, 2]], Sol[[1, 3, 2]]},
  {t, -1, 1}, {s, 0, 1}, PlotPoints -> 100]
map1 = ParametricPlot3D[{s, s, s/4}, {s, 0, 1}]
Show[map, map1, PlotLabel -> "Integral Surface through initial Curve"]

Out[27]= {{x[t] -> s + t, y[t] -> s - t, u[t] -> 2 (s + t)}}

u[t]=2 (s + t)
y[t]=s - t
x[t]=s + t
```





Question 3. Solve the PDE $u_x + u_y = 1$. With the initial condition $u(s,s) = \sin(s)$, $0 \leq s \leq 1$

Solution :

$$x = s + t, \quad y = s + t, \quad u = t + \sin(s)$$

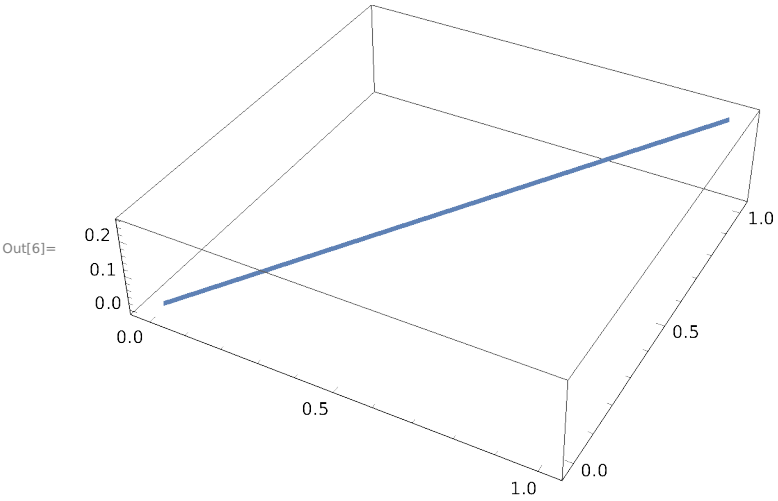
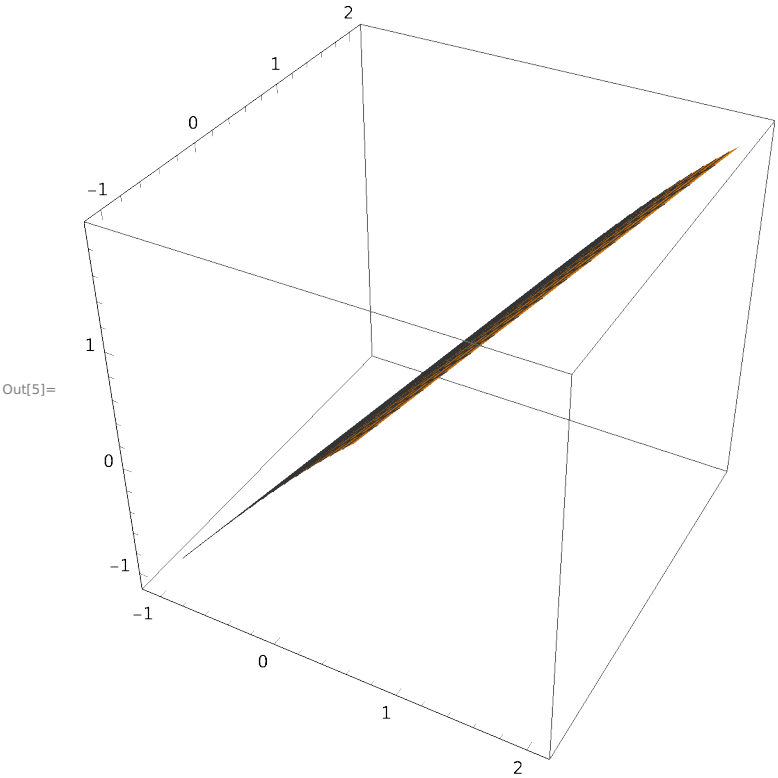
```
In[1]:= Sol =
  DSolve[{x'[t] == 1, y'[t] == 1, u'[t] == 1, x[0] == s, y[0] == s, u[0] == Sin[s]}, {x[t], y[t], u[t]}, t]
Print["u[t]=", Sol[[1, 3, 2]]]
Print["y[t]=", Sol[[1, 2, 2]]]
Print["x[t]=", Sol[[1, 1, 2]]]
map = ParametricPlot3D[{Sol[[1, 1, 2]], Sol[[1, 2, 2]], Sol[[1, 3, 2]]}, {t, -1, 1}, {s, 0, 1}]
map1 = ParametricPlot3D[{s, s, s/4}, {s, 0, 1}]
Show[map, map1, PlotLabel -> "Integral Surface through initial Curve"]

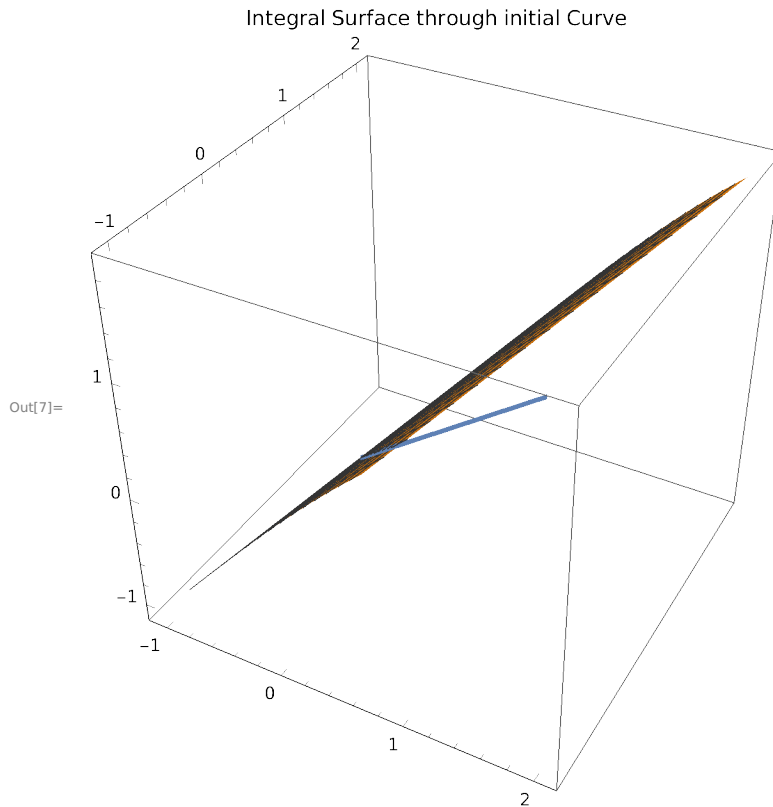
Out[1]= {{x[t] -> s + t, y[t] -> s + t, u[t] -> t + Sin[s]}}

u[t]=t + Sin[s]

y[t]=s + t

x[t]=s + t
```





Question 4. Solve the PDE $ux + 2uy = 0$. With the initial condition $u(0,s) = 4 e^{(-2s)}$, $0 \leq s \leq 1$

Solution :

$u = 4 e^{(-2s)}$, $y = s + 2t$, $x = t$

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In[8]:= Sol = DSolve[
  {x'[t] == 1, y'[t] == 2, u'[t] == 0, x[0] == 0, y[0] == s, u[0] == 4 * Exp[-2 s]}, {x[t], y[t], u[t]}, t]
Print["u[t]=", Sol[[1, 3, 2]]]
Print["y[t]=", Sol[[1, 2, 2]]]
Print["x[t]=", Sol[[1, 1, 2]]]
map = ParametricPlot3D [{Sol[[1, 1, 2]], Sol[[1, 2, 2]], Sol[[1, 3, 2]]}, {t, -1, 1}, {s, 0, 1}]
map1 = ParametricPlot3D [{s, s, s/4}, {s, 0, 1}]
Show[map, map1, PlotLabel -> "Integral Surface through initial Curve"]

Out[8]= {{x[t] -> t, y[t] -> s + 2 t, u[t] -> 4 e^{-2 s}}}
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$u[t]=4 e^{-2 s}$

$y[t]=s + 2 t$

$x[t]=t$

