Practical-5

Solutions of Ordinary Differential Equations

Question 1: Find the general solution of the following linear system
2dx/dt - 2dy/dt -3x=t, 2dx/dt + 2dy/dt +3x + 8y =2

In[1]:=
```
DSolve[{2 x'[t] - 2 y'[t] - 3 x[t] == t, 2 x'[t] + 2 y'[t] + 3 x[t] + 8 y[t] == 2}, {x[t], y[t]}, t]
```

Out[1]=
$$\left\{\left\{x[t] \to \frac{1}{64} e^{-3t}\left(1+3e^{4t}\right)\left(e^{-t}(-7-5t)+e^{3t}\left(\frac{19}{9}-\frac{t}{3}\right)\right)-\right.\right.$$
$$\frac{1}{64} e^{-3t}\left(-1+e^{4t}\right)\left(e^{3t}\left(\frac{19}{3}-t\right)+e^{-t}(7+5t)\right)+\frac{1}{4} e^{-3t}\left(1+3e^{4t}\right)c_1-\frac{1}{2} e^{-3t}\left(-1+e^{4t}\right)c_2,$$
$$y[t] \to -\frac{3}{128} e^{-3t}\left(-1+e^{4t}\right)\left(e^{-t}(-7-5t)+e^{3t}\left(\frac{19}{9}-\frac{t}{3}\right)\right)+$$
$$\left.\left.\frac{1}{128} e^{-3t}\left(3+e^{4t}\right)\left(e^{3t}\left(\frac{19}{3}-t\right)+e^{-t}(7+5t)\right)-\frac{3}{8} e^{-3t}\left(-1+e^{4t}\right)c_1+\frac{1}{4} e^{-3t}\left(3+e^{4t}\right)c_2\right\}\right\}$$

Question 2: Find the general solution of the following linear system
dx/dt + dy/dt -2x -4y =e^t, dx/dt + dy/dt -y =e^4t

In[8]:=
```
DSolve[{x'[t] + y'[t] - 2 x[t] - 4 y[t] == Exp[t], x'[t] + y'[t] - y[t] == Exp[4 t]}, {x[t], y[t]}, t]
```

Out[8]=
$$\left\{\left\{x[t] \to -e^{t}\left(-1+e^{3t}\right)+\frac{1}{3}\times\left(3e^{-2t}\left(-e^{3t}+e^{6t}\right)+e^{-2t}c_1\right),\right.\right.$$
$$\left.\left.y[t] \to e^{t}\left(-1+e^{3t}\right)-\frac{2}{9}\times\left(3e^{-2t}\left(-e^{3t}+e^{6t}\right)+e^{-2t}c_1\right)\right\}\right\}$$

Question 3: Find the general solution of the following linear system
dx/dt + dy/dt -x = -2t, dx/dt + dy/dt -3x -y =t^2

In[3]:=
```
DSolve[{x'[t] + y'[t] - x[t] == -2 t, x'[t] + y'[t] - 3 x[t] - y[t] == t^2}, {x[t], y[t]}, t]
```

Out[3]=
$$\left\{\left\{x[t] \to -2t-t^2+\frac{1}{4}\times\left(4\times\left(-2+2t+t^2\right)-e^{-t}c_1\right),\ y[t] \to 2t+t^2+\frac{1}{2}\times\left(-4\times\left(-2+2t+t^2\right)+e^{-t}c_1\right)\right\}\right\}$$

Question 4: Find the general solution of the following linear system
dx/dt + dy/dt -x -3y= e^t, dx/dt + dy/dt +x =e^3t

In[7]:=
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DSolve[{x'[t] + y'[t] - x[t] - 3 y[t] == Exp[t], x'[t] + y'[t] + x[t] == Exp[3 t]}, {x[t], y[t]}, t]
```

Out[7]=
$$\left\{\left\{x[t] \to -e^{t}\left(-1+e^{2t}\right)+\frac{3}{16}\times\left(\frac{4}{3}e^{t}\left(-3+4e^{2t}\right)+e^{-3t}c_1\right),\right.\right.$$
$$\left.\left.y[t] \to e^{t}\left(-1+e^{2t}\right)+\frac{1}{8}\times\left(-\frac{4}{3}e^{t}\left(-3+4e^{2t}\right)-e^{-3t}c_1\right)\right\}\right\}$$

Question 5: Find the general solution of the following linear system dy/dt=0, dx/dt + 10x = t^2, dz/dt + 24z =e^t

In[6]:=
```
DSolve[{y'[t] == 0, x'[t] + 10 x[t] == t^2, z'[t] + 24 z[t] == Exp[t]}, {x[t], y[t], z[t]}, t]
```

Out[6]=
$$\left\{\left\{y[t] \to c_1,\ x[t] \to \frac{1}{500}\times\left(1-10t+50t^2\right)+e^{-10t}c_2,\ z[t] \to \frac{e^{t}}{25}+e^{-24t}c_3\right\}\right\}$$

Question 6: Find the general solution of the following linear system d^2x/dt^2 + dy/dt -x +y= 1, d^2y/dt^2 + dx/dt -x +y =0

In[9]:=
```
DSolve[{x''[t] + y'[t] - x[t] + y[t] == 1, y''[t] + x'[t] - x[t] + y[t] == 0}, {x[t], y[t]}, t]
```

Out[9]=
$$\left\{\left\{x[t] \to \left(e^{t}-t\right)\left(1-e^{t}+t\right)+e^{-t}\left(-e^{-t}+e^{t}-t\right)\left(-1+e^{2t}-e^{t}t\right)+\right.\right.$$
$$e^{-t}\left(1+e^{t}t\right)\left(-e^{-t}-e^{t}+\frac{t^2}{2}\right)-e^{-t}\left(1-e^{t}+e^{t}t\right)\left(-e^{t}+t+\frac{t^2}{2}\right)+$$
$$e^{-t}\left(1+e^{t}t\right)c_1+e^{-t}\left(-1+e^{2t}-e^{t}t\right)c_2-e^{-t}\left(1-e^{t}+e^{t}t\right)c_3+\left(1-e^{t}+t\right)c_4,$$
$$y[t] \to \left(e^{t}-t\right)t-e^{-t}\left(-e^{-t}+e^{t}-t\right)\left(1-e^{t}+e^{t}t\right)+e^{-t}\left(1-e^{t}+e^{t}t\right)\left(-e^{-t}-e^{t}+\frac{t^2}{2}\right)-e^{-t}\left(1-2e^{t}+e^{t}t\right)$$
$$\left.\left.\left(-e^{t}+t+\frac{t^2}{2}\right)+e^{-t}\left(1-e^{t}+e^{t}t\right)c_1-e^{-t}\left(1-e^{t}+e^{t}t\right)c_2-e^{-t}\left(1-2e^{t}+e^{t}t\right)c_3+t\,c_4\right\}\right\}$$

Question 7: Find the general solution of the following linear system d^2x/dt^2 - dy/dt -x +y= e^t, dy/dt + dx/dt -4x -y = 2e^t

In[10]:=
```
DSolve[{x''[t] - y'[t] - x[t] + y[t] == Exp[t], y'[t] + x'[t] - 4 x[t] - y[t] == 2 Exp[t]}, {x[t], y[t]}, t]
```

Out[10]=

$$\left\{\left\{x[t] \to \frac{2\, e^{-\frac{t}{2}-\frac{\sqrt{21}\,t}{2}-\frac{1}{2}\times(-3+\sqrt{21})\,t}\left(-1+e^{\sqrt{21}\,t}\right)\times\left(7+3\sqrt{21}+\left(7-3\sqrt{21}\right)e^{\sqrt{21}\,t}\right)}{\sqrt{21}\left(-7+\sqrt{21}\right)\times\left(7+\sqrt{21}\right)}+\right.\right.$$

$$\frac{e^{-\frac{t}{2}-\frac{\sqrt{21}\,t}{2}-\frac{1}{2}\times(-3+\sqrt{21})\,t}\left(21-\sqrt{21}+21\,e^{\sqrt{21}\,t}+\sqrt{21}\,e^{\sqrt{21}\,t}\right)\times\left(3+\sqrt{21}+\left(-3+\sqrt{21}\right)e^{\sqrt{21}\,t}\right)}{3\sqrt{21}\left(-7+\sqrt{21}\right)\times\left(7+\sqrt{21}\right)}-$$

$$\frac{2\,e^{-\frac{t}{2}-\frac{\sqrt{21}\,t}{2}}\left(21-\sqrt{21}+21\,e^{\sqrt{21}\,t}+\sqrt{21}\,e^{\sqrt{21}\,t}\right)c_1}{3\times\left(-7+\sqrt{21}\right)\times\left(7+\sqrt{21}\right)}-\frac{4\sqrt{\frac{7}{3}}\,e^{-\frac{t}{2}-\frac{\sqrt{21}\,t}{2}}\left(-1+e^{\sqrt{21}\,t}\right)c_2}{\left(-7+\sqrt{21}\right)\times\left(7+\sqrt{21}\right)},$$

$$y[t] \to \left(e^{-\frac{t}{2}-\frac{\sqrt{21}\,t}{2}-\frac{1}{2}\times(-3+\sqrt{21})\,t}\left(7+3\sqrt{21}+\left(7-3\sqrt{21}\right)e^{\sqrt{21}\,t}\right)\times\right.$$

$$\left.\left(21-\sqrt{21}+21\,e^{\sqrt{21}\,t}+\sqrt{21}\,e^{\sqrt{21}\,t}-42\,e^{\frac{3t}{2}+\frac{\sqrt{21}\,t}{2}}\right)\right)\Big/$$

$$\left(21\times\left(-7+\sqrt{21}\right)\times\left(7+\sqrt{21}\right)\right)+\left(e^{-\frac{t}{2}-\frac{\sqrt{21}\,t}{2}-\frac{1}{2}\times(-3+\sqrt{21})\,t}\left(3+\sqrt{21}+\left(-3+\sqrt{21}\right)e^{\sqrt{21}\,t}\right)\times\right.$$

$$\left.\left(21-11\sqrt{21}+21\,e^{\sqrt{21}\,t}+11\sqrt{21}\,e^{\sqrt{21}\,t}-42\,e^{\frac{3t}{2}+\frac{\sqrt{21}\,t}{2}}\right)\right)\Big/\left(3\sqrt{21}\left(-7+\sqrt{21}\right)\times\left(7+\sqrt{21}\right)\right)-$$

$$\frac{1}{7}\,e^{t-\frac{1}{2}\times(-3+\sqrt{21})\,t}\left(7+2\sqrt{21}+\left(7-2\sqrt{21}\right)e^{\sqrt{21}\,t}+7\,e^{\frac{1}{2}\times(-3+\sqrt{21})\,t}\,t\right)-$$

$$\frac{2\,e^{-\frac{t}{2}-\frac{\sqrt{21}\,t}{2}}\left(21-11\sqrt{21}+21\,e^{\sqrt{21}\,t}+11\sqrt{21}\,e^{\sqrt{21}\,t}-42\,e^{\frac{3t}{2}+\frac{\sqrt{21}\,t}{2}}\right)c_1}{3\times\left(-7+\sqrt{21}\right)\times\left(7+\sqrt{21}\right)}-$$

$$\left.\left.\frac{2\,e^{-\frac{t}{2}-\frac{\sqrt{21}\,t}{2}}\left(21-\sqrt{21}+21\,e^{\sqrt{21}\,t}+\sqrt{21}\,e^{\sqrt{21}\,t}-42\,e^{\frac{3t}{2}+\frac{\sqrt{21}\,t}{2}}\right)c_2}{3\times\left(-7+\sqrt{21}\right)\times\left(7+\sqrt{21}\right)}+e^{t}\,c_3\right\}\right\}$$

Question 8: Find the general solution of the following linear system dx/dt +2x -3y= t, dy/dt -3x +2y = e^2t

In[11]:=
```
DSolve[{x'[t] + 2 x[t] - 3 y[t] == t, y'[t] - 3 x[t] + 2 y[t] == Exp[2 t]}, {x[t], y[t]}, t]
```

Out[11]=

$$\left\{\left\{x[t] \to \frac{1}{4}\,e^{-5\,t}\left(-1+e^{6\,t}\right)\left(e^{t}+\frac{e^{7\,t}}{7}+e^{-t}\,(-1-t)+e^{5\,t}\left(\frac{1}{25}-\frac{t}{5}\right)\right)+\right.\right.$$

$$\frac{1}{4}\,e^{-5\,t}\left(1+e^{6\,t}\right)\left(e^{t}-\frac{e^{7\,t}}{7}+e^{-t}\,(-1-t)+e^{5\,t}\left(-\frac{1}{25}+\frac{t}{5}\right)\right)+\frac{1}{2}\,e^{-5\,t}\left(1+e^{6\,t}\right)c_1+\frac{1}{2}\,e^{-5\,t}\left(-1+e^{6\,t}\right)c_2,$$

$$y[t] \to \frac{1}{4}\,e^{-5\,t}\left(1+e^{6\,t}\right)\left(e^{t}+\frac{e^{7\,t}}{7}+e^{-t}\,(-1-t)+e^{5\,t}\left(\frac{1}{25}-\frac{t}{5}\right)\right)+$$

$$\left.\left.\frac{1}{4}\,e^{-5\,t}\left(-1+e^{6\,t}\right)\left(e^{t}-\frac{e^{7\,t}}{7}+e^{-t}\,(-1-t)+e^{5\,t}\left(-\frac{1}{25}+\frac{t}{5}\right)\right)+\frac{1}{2}\,e^{-5\,t}\left(-1+e^{6\,t}\right)c_1+\frac{1}{2}\,e^{-5\,t}\left(1+e^{6\,t}\right)c_2\right\}\right\}$$