Practical-4

Solution of differential equation by variation of parameter method.

Question 1 :Solve second order differential equation (d^2y)/dx^2+ 9y=sec(3x) by variation of parameter method

Solution:

```
In[99]:= Sol = DSolve[y''[x] + 9 y[x] == 0, y[x], x]
sol1 = Evaluate[y[x] /. Sol[[1]] /. {C[1] → 1, C[2] → 0}]
sol2 = y[x] /. Sol[[1]] /. {C[1] → 0, C[2] → 1}
fs = {sol1, sol2}
wm = {fs, D[fs, x]}; wm // MatrixForm

wd = Simplify[Det[wm]]

u1 = (Integrate[- sol2 Sec[3 x], x])

u2 = (Integrate[sol1 Sec[3 x], x])

yc = DSolve[y''[x] + 9 y[x] == 0, y[x], x]
yp = Evaluate[y[x] /. Sol[[1]] /. {C[1] → u1, C[2] → u2}]
yg = yc + yp
```

Out[99]= $\{\{y[x] \to c_1 \text{Cos}[3x] + c_2 \text{Sin}[3x]\}\}$

Out[100]= Cos[3 x]

Out[101]= Sin[3 x]

Out[102]= {Cos[3 x], Sin[3 x]}

Out[103]//MatrixForm=

$$\begin{pmatrix} \text{Cos}[3x] & \text{Sin}[3x] \\ -3\,\text{Sin}[3x] & 3\,\text{Cos}[3x] \end{pmatrix}$$

Out[104]= 3

Out[105]= $\frac{1}{3}\text{Log}[\text{Cos}[3x]]$

Out[106]= x

Out[107]= $\{\{y[x] \to c_1 \text{Cos}[3x] + c_2 \text{Sin}[3x]\}\}$

Out[108]= $\frac{1}{3}\text{Cos}[3x] \times \text{Log}[\text{Cos}[3x]] + x\,\text{Sin}[3x]$

Out[109]= $\left\{\left\{\frac{1}{3}\text{Cos}[3x] \times \text{Log}[\text{Cos}[3x]] + (y[x] \to c_1 \text{Cos}[3x] + c_2 \text{Sin}[3x]) + x\,\text{Sin}[3x]\right\}\right\}$

Question 2 :Solve third order differential equation (d^3y)/dx^3 + 4 dy/dx=sec(2x) by variation of parameter method

Solution:

```
In[1]:= Sol = DSolve[y'''[x] + 4 y'[x] == 0, y[x], x]
        sol1 = Evaluate[y[x] /. Sol[[1]] /. {C[1] → 2, C[2] → 0, C[3] → 0}]
        sol2 = y[x] /. Sol[[1]] /. {C[1] → 0, C[2] → -2, C[3] → 0}
        sol3 = y[x] /. Sol[[1]] /. {C[1] → 0, C[2] → 0, C[3] → 1}

        fs = {sol1, sol2, sol3}
        wm = {fs, D[fs, x], D[fs, {x, 2}]}; MatrixForm[wm]
        wd = Simplify[Det[wm]]

        u1 = (1 / wd (Det[{{0, sol2, sol3}, {0, D[sol2, x], D[sol3, x]},
                {Sec[2 x], D[sol2, {x, 2}], D[sol3, {x, 2}]}}])) // Simplify
        a = Integrate[u1, x]

        u2 = (1 / wd (Det[{{sol1, 0, sol3}, {D[sol1, x], 0, D[sol3, x]},
                {D[sol1, {x, 2}], Sec[2 x], D[sol3, {x, 2}]}}])) // Simplify
        b = Integrate[u2, x]

        u3 = (1 / wd (Det[{{sol1, sol2, 0}, {D[sol1, x], D[sol2, x], 0},
                {D[sol1, {x, 2}], D[sol2, {x, 2}], Sec[2 x]}}])) // Simplify
        c = Integrate[u3, x]
        yc = Evaluate[y[x] /. Sol[[1]]]
        yp = Evaluate[y[x] /. Sol[[1]] /. {C[1] → a, C[2] → b, C[3] → c}]
        yg = yc + yp
```

Out[1]= $\left\{\left\{y[x] \to c_3 - \frac{1}{2} c_2 \, \mathrm{Cos}[2\,x] + \frac{1}{2} c_1 \, \mathrm{Sin}[2\,x]\right\}\right\}$

Out[2]= Sin[2 x]

Out[3]= Cos[2 x]

Out[4]= 1

Out[5]= {Sin[2 x], Cos[2 x], 1}

Out[6]//MatrixForm=

$$\begin{pmatrix} \mathrm{Sin}[2\,x] & \mathrm{Cos}[2\,x] & 1 \\ 2\,\mathrm{Cos}[2\,x] & -2\,\mathrm{Sin}[2\,x] & 0 \\ -4\,\mathrm{Sin}[2\,x] & -4\,\mathrm{Cos}[2\,x] & 0 \end{pmatrix}$$

Out[7]= -8

Out[8]= $-\frac{1}{4}\,\mathrm{Tan}[2\,x]$

Out[9]= $\frac{1}{8}\,\mathrm{Log}[\mathrm{Cos}[2\,x]]$

Out[10]= $-\frac{1}{4}$

Out[11]= $-\frac{x}{4}$

Out[12]= $\frac{1}{4}\text{Sec}[2\,x]$

Out[13]= $\frac{1}{4}\times\left(-\frac{1}{2}\text{Log}[\text{Cos}[x]-\text{Sin}[x]]+\frac{1}{2}\text{Log}[\text{Cos}[x]+\text{Sin}[x]]\right)$

Out[14]= $c_3-\frac{1}{2}c_2\,\text{Cos}[2\,x]+\frac{1}{2}c_1\,\text{Sin}[2\,x]$

Out[15]= $\frac{1}{8}x\,\text{Cos}[2\,x]+\frac{1}{4}\times\left(-\frac{1}{2}\text{Log}[\text{Cos}[x]-\text{Sin}[x]]+\frac{1}{2}\text{Log}[\text{Cos}[x]+\text{Sin}[x]]\right)+\frac{1}{16}\text{Log}[\text{Cos}[2\,x]]\times\text{Sin}[2\,x]$

Out[16]= $c_3+\frac{1}{8}x\,\text{Cos}[2\,x]-\frac{1}{2}c_2\,\text{Cos}[2\,x]+$
$\frac{1}{4}\times\left(-\frac{1}{2}\text{Log}[\text{Cos}[x]-\text{Sin}[x]]+\frac{1}{2}\text{Log}[\text{Cos}[x]+\text{Sin}[x]]\right)+\frac{1}{2}c_1\,\text{Sin}[2\,x]+\frac{1}{16}\text{Log}[\text{Cos}[2\,x]]\times\text{Sin}[2\,x]$

Question 3 :Solve second order differential equation (d^2y)/dx^2+ y= tan x by variation of parameter method

Solution:

```
In[88]:= Sol = DSolve[y''[x] + y[x] == 0, y[x], x]
sol1 = Evaluate[y[x] /. Sol⟦1⟧ /. {C[1] → 1, C[2] → 0}]
sol2 = y[x] /. Sol⟦1⟧ /. {C[1] → 0, C[2] → 1}
fs = {sol1, sol2}
wm = {fs, D[fs, x]}; wm // MatrixForm

wd = Simplify[Det[wm]]

u1 = (Integrate[- sol2 Tan[x], x])

u2 = (Integrate[sol1 Tan[x], x])

yc = DSolve[y''[x] + y[x] == 0, y[x], x]
yp = Evaluate[y[x] /. Sol⟦1⟧ /. {C[1] → u1, C[2] → u2}]
yg = yc + yp
```

Out[88]= $\{\{y[x] \to c_1 \text{Cos}[x] + c_2 \text{Sin}[x]\}\}$

Out[89]= Cos[x]

Out[90]= Sin[x]

Out[91]= {Cos[x], Sin[x]}

Out[92]//MatrixForm=

$$\begin{pmatrix} \text{Cos}[x] & \text{Sin}[x] \\ -\text{Sin}[x] & \text{Cos}[x] \end{pmatrix}$$

Out[93]= 1

Out[94]= $\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right] + \text{Sin}[x]$

Out[95]= -Cos[x]

Out[96]= $\{\{y[x] \to c_1 \text{Cos}[x] + c_2 \text{Sin}[x]\}\}$

Out[97]= $-\text{Cos}[x] \times \text{Sin}[x] + \text{Cos}[x] \times \left(\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right] + \text{Sin}[x]\right)$

Out[98]= $\Big\{\Big\{(y[x] \to c_1 \text{Cos}[x] + c_2 \text{Sin}[x]) - \text{Cos}[x] \times \text{Sin}[x] + \text{Cos}[x] \times \left(\text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] - \text{Sin}\left[\frac{x}{2}\right]\right] - \text{Log}\left[\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right] + \text{Sin}[x]\right)\Big\}\Big\}$

Question 4 :Solve third order differential equation (d^3y)/dx^3 -6 dy^2/dx^2 +11dy/dx -6y =e^x by variation of parameter method

Solution:

In[110]:=
```
Sol = DSolve[y'''[x] - 6 y''[x] + 11 y'[x] - 6 y[x] == 0, y[x], x]
sol1 = Evaluate[y[x] /. Sol[[1]] /. {C[1] → 1, C[2] → 0, C[3] → 0}]
sol2 = y[x] /. Sol[[1]] /. {C[1] → 0, C[2] → 1, C[3] → 0}
sol3 = y[x] /. Sol[[1]] /. {C[1] → 0, C[2] → 0, C[3] → 1}

fs = {sol1, sol2, sol3}
wm = {fs, D[fs, x], D[fs, {x, 2}]}; MatrixForm[wm]
wd = Simplify[Det[wm]]

u1 = (1 / wd (Det[{{0, sol2, sol3}, {0, D[sol2, x], D[sol3, x]},
        {Exp[x], D[sol2, {x, 2}], D[sol3, {x, 2}]}}])) // Simplify
a = Integrate[u1, x]

u2 = (1 / wd (Det[{{sol1, 0, sol3}, {D[sol1, x], 0, D[sol3, x]},
        {D[sol1, {x, 2}], Exp[x], D[sol3, {x, 2}]}}])) // Simplify
b = Integrate[u2, x]

u3 = (1 / wd (Det[{{sol1, sol2, 0}, {D[sol1, x], D[sol2, x], 0},
        {D[sol1, {x, 2}], D[sol2, {x, 2}], Exp[x]}}])) // Simplify
c = Integrate[u3, x]
yc = Evaluate[y[x] /. Sol[[1]]]
yp = Evaluate[y[x] /. Sol[[1]] /. {C[1] → a, C[2] → b, C[3] → c}]
yg = yc + yp
```

Out[110]= $\{\{y[x] \to e^{x} c_1 + e^{2x} c_2 + e^{3x} c_3\}\}$

Out[111]= e^{x}

Out[112]= e^{2x}

Out[113]= e^{3x}

Out[114]= $\{e^{x}, e^{2x}, e^{3x}\}$

Out[115]//MatrixForm=

$$\begin{pmatrix} e^{x} & e^{2x} & e^{3x} \\ e^{x} & 2e^{2x} & 3e^{3x} \\ e^{x} & 4e^{2x} & 9e^{3x} \end{pmatrix}$$

Out[116]= $2 e^{6x}$

Out[117]= $\frac{1}{2}$

Out[118]= $\frac{x}{2}$

Out[119]= $-e^{-x}$

Out[120]= e^{-x}

Out[121]= $\frac{e^{-2x}}{2}$

Out[122]= $-\frac{1}{4}e^{-2x}$

Out[123]= $e^{x}\,\mathbb{c}_1 + e^{2x}\,\mathbb{c}_2 + e^{3x}\,\mathbb{c}_3$

Out[124]= $\frac{3e^{x}}{4} + \frac{e^{x}x}{2}$

Out[125]= $\frac{3e^{x}}{4} + \frac{e^{x}x}{2} + e^{x}\,\mathbb{c}_1 + e^{2x}\,\mathbb{c}_2 + e^{3x}\,\mathbb{c}_3$

Question 5 :Solve second order differential equation x^2 (d^2y)/dx^2 -2y= 4x-8 given condition is y[1]=4,y'[1]=-1 by variation of parameter method

Solution:

In[126]:=

```
Sol = DSolve[x^2 y''[x] - 2 y[x] == 0, y[x], x]
sol1 = Evaluate[y[x] /. Sol[[1]] /. {C[1] → 1, C[2] → 0}]
sol2 = y[x] /. Sol[[1]] /. {C[1] → 0, C[2] → 1}
fs = {sol1, sol2}
wm = {fs, D[fs, x]}; wm // MatrixForm

wd = Simplify[Det[wm]]

u1 = (Integrate[- sol2 (4 x - 8), x])

u2 = (Integrate[sol1 (4 x - 8), x])

yc = DSolve[x^2 y''[x] - 2 y[x] == 0, y[x], x]
yp = Evaluate[y[x] /. Sol[[1]] /. {C[1] → u1, C[2] → u2}]
yg = yc + yp
```

Out[126]= $\left\{\left\{y[x] \to \frac{\mathbb{c}_1}{x} + x^2\, \mathbb{c}_2\right\}\right\}$

Out[127]= $\frac{1}{x}$

Out[128]= x^2

Out[129]= $\left\{\frac{1}{x}, x^2\right\}$

Out[130]//MatrixForm=

$$\begin{pmatrix} \frac{1}{x} & x^2 \\ -\frac{1}{x^2} & 2\,x \end{pmatrix}$$

Out[131]= 3

Out[132]= $-4\left(-\frac{2\,x^3}{3} + \frac{x^4}{4}\right)$

Out[133]= $4\,x - 8\,\mathrm{Log}[x]$

Out[134]= $\left\{\left\{y[x] \to \frac{\mathbb{c}_1}{x} + x^2\, \mathbb{c}_2\right\}\right\}$

Out[135]= $-\frac{4\left(-\frac{2\,x^3}{3} + \frac{x^4}{4}\right)}{x} + x^2\,(4\,x - 8\,\mathrm{Log}[x])$

Out[136]= $\left\{\left\{-\frac{4\left(-\frac{2\,x^3}{3} + \frac{x^4}{4}\right)}{x} + x^2\,(4\,x - 8\,\mathrm{Log}[x]) + \left(y[x] \to \frac{\mathbb{c}_1}{x} + x^2\, \mathbb{c}_2\right)\right\}\right\}$