Practical 5
LU Decomposition Method

Definition: A non - singular matrix A has a LU decomposition if it can be expressed as the product of a lower triangular matrix L and an upper triangular matrix U i.e., A = LU

Q1. To find the LU decomposition

```
LUDecomp[A0_, n_] := Module[{A = A0, i, p}, U = A0; L = IdentityMatrix[n];
  Print[MatrixForm[L], MatrixForm[U], " = ",
   MatrixForm[A0]];
  For[p = 1, p ≤ n - 1, p++,
   For[i = p + 1, i ≤ n, i++,
     m = A[[i,p]] / A[[p,p]];
     L[[i,p]] = m;
     A[[i]] = A[[i]] - m A[[p]];
     U = A;
     Print[MatrixForm[L], MatrixForm[U],
      " = ", MatrixForm[A0]];];];]
Print["L", "=", MatrixForm[L]];
Print["U", "=", MatrixForm[U]];
A = {{4, 2, 3}, {-3, 1, 4}, {2, 4, 5}};
LUDecomp[A, 3]
```

$$L=\begin{pmatrix} 1 & 0 & 0 \\ -\frac{3}{4} & 1 & 0 \\ \frac{1}{2} & \frac{6}{5} & 1 \end{pmatrix}$$

$$U=\begin{pmatrix} 4 & 2 & 3 \\ 0 & \frac{5}{2} & \frac{25}{4} \\ 0 & 0 & -4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 4 & 2 & 3 \\ -3 & 1 & 4 \\ 2 & 4 & 5 \end{pmatrix} = \begin{pmatrix} 4 & 2 & 3 \\ -3 & 1 & 4 \\ 2 & 4 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ -\frac{3}{4} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 4 & 2 & 3 \\ 0 & \frac{5}{2} & \frac{25}{4} \\ 2 & 4 & 5 \end{pmatrix} = \begin{pmatrix} 4 & 2 & 3 \\ -3 & 1 & 4 \\ 2 & 4 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ -\frac{3}{4} & 1 & 0 \\ \frac{1}{2} & 0 & 1 \end{pmatrix}\begin{pmatrix} 4 & 2 & 3 \\ 0 & \frac{5}{2} & \frac{25}{4} \\ 0 & 3 & \frac{7}{2} \end{pmatrix} = \begin{pmatrix} 4 & 2 & 3 \\ -3 & 1 & 4 \\ 2 & 4 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ -\frac{3}{4} & 1 & 0 \\ \frac{1}{2} & \frac{6}{5} & 1 \end{pmatrix}\begin{pmatrix} 4 & 2 & 3 \\ 0 & \frac{5}{2} & \frac{25}{4} \\ 0 & 0 & -4 \end{pmatrix} = \begin{pmatrix} 4 & 2 & 3 \\ -3 & 1 & 4 \\ 2 & 4 & 5 \end{pmatrix}$$

Q2. Given $A = \begin{pmatrix} 1 & 2 & 6 \\ 4 & 8 & -1 \\ -2 & 3 & 5 \end{pmatrix}$. Can A be factorized as A = LU?

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 6 \\ 4 & 8 & -1 \\ -2 & 3 & 5 \end{pmatrix}$$

LUDecomp[A, 3]

{{1, 2, 6}, {4, 8, -1}, {-2, 3, 5}}

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 6 \\ 4 & 8 & -1 \\ -2 & 3 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 6 \\ 4 & 8 & -1 \\ -2 & 3 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 6 \\ 0 & 0 & -25 \\ -2 & 3 & 5 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 6 \\ 4 & 8 & -1 \\ -2 & 3 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 6 \\ 0 & 0 & -25 \\ 0 & 7 & 17 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 6 \\ 4 & 8 & -1 \\ -2 & 3 & 5 \end{pmatrix}$$

Power::infy : Infinite expression $\frac{1}{0}$ encountered. ≫

∞::indet : Indeterminate expression 0 ComplexInfinity encountered. ≫

∞::indet : Indeterminate expression 0 ComplexInfinity encountered. ≫

$$\begin{pmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ -2 & \text{ComplexInfinity} & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 6 \\ 0 & 0 & -25 \\ \text{Indeterminate} & \text{Indeterminate} & \text{ComplexInfinity} \end{pmatrix} = \begin{pmatrix} 1 & 2 & 6 \\ 4 & 8 & -1 \\ -2 & 3 & 5 \end{pmatrix}$$

Remark : A has no LU decomposition
