Practical -2

Plotting of Second Differential Equations

Question 1: Solve the equation d^2y/dx^2 + 10dy/dx +y =0

Solution:

In[6]:=
```
DSolve[y''[x] + 10 y'[x] + y[x] == 0 , y[x], x]
```

Out[6]= $\left\{\left\{y[x] \to e^{(-5-2\sqrt{6})x} c_1 + e^{(-5+2\sqrt{6})x} c_2\right\}\right\}$

Question 2: Solve the equation d^2y/dx^2 +y =0

Solution:

In[1]:=
```
DSolve[y''[x] + y[x] == 0, y[x], x]
```

Out[1]= $\{\{y[x] \to c_1 \mathrm{Cos}[x] + c_2 \mathrm{Sin}[x]\}\}$

Question 3: Solve the equation d^2y/dx^2 + dy/dx -6 y =0

Solution :

In[3]:=
```
DSolve[y''[x] + y'[x] - 6 y[x] == 0, y[x], x]
```

Out[3]= $\{\{y[x] \to e^{-3x} c_1 + e^{2x} c_2\}\}$

Question 4: Solve the equation 4d^2y/dx^2 + 12dy/dx + 9y =0

Solution :

In[4]:=
```
DSolve[4 y''[x] + 12 y'[x] + 9 y[x] == 0, y[x], x]
```

Out[4]= $\{\{y[x] \to e^{-3x/2} c_1 + e^{-3x/2} x\, c_2\}\}$

Question 5: Solve the equation d^2y/dx^2 -6dy/dx + 13y =0

Solution :

In[5]:=
```
DSolve[y''[x] - 6 y'[x] + 13 y[x] == 0, y[x], x]
```

Out[5]= $\{\{y[x] \to e^{3x} c_2 \mathrm{Cos}[2x] + e^{3x} c_1 \mathrm{Sin}[2x]\}\}$

Question 6: Solve the equation d^2y/dx^2 -2dy/dx + y =0

Solution :

In[6]:=
```
DSolve[y''[x] - 2 y'[x] + y[x] == 0, y[x], x]
```

Out[6]= $\{\{y[x] \to e^{x} c_1 + e^{x} x\, c_2\}\}$

Plotting of Solutions of Second order Differential Equations

Question 1: Solve the equation d^2y/dx^2 + y =0 and plot its three solutions.

Solution :

In[1]:=
```
Sol = DSolve[y''[x] + y[x] == 0, y[x], x]
Sol1 = Evaluate[y[x] /. Sol[[1]] /. {C[1] → 1, C[2] → 2}]
Sol2 = y[x] /. Sol[[1]] /. {C[1] → 1 / 2, C[2] → 5}
Sol3 = y[x] /. Sol[[1]] /. {C[1] → -1, C[2] → -4}
Plot[{Sol1, Sol2, Sol3}, {x, -20, 20},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thick}, {Purple, Thickness[0.02]}},
 PlotLegends → {Sol1, Sol2, Sol3}]
```

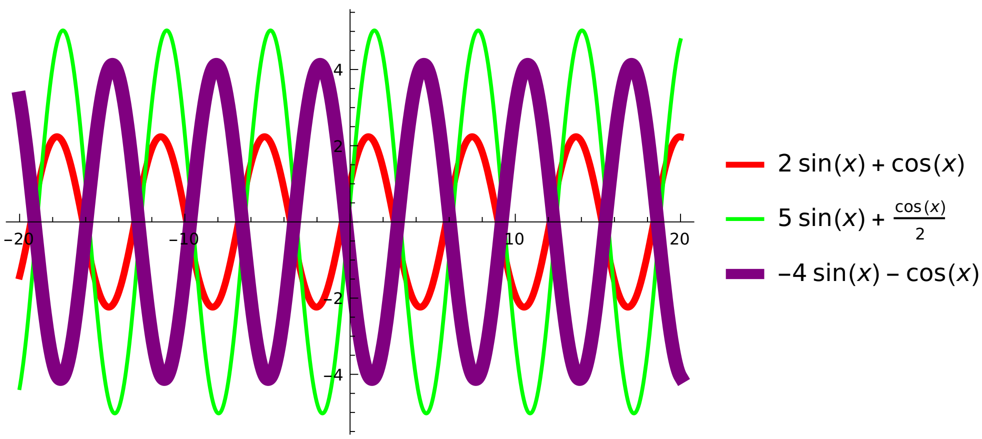
Out[1]= $\{\{y[x] \to c_1 \operatorname{Cos}[x] + c_2 \operatorname{Sin}[x]\}\}$

Out[2]= $\operatorname{Cos}[x] + 2 \operatorname{Sin}[x]$

Out[3]= $\frac{\operatorname{Cos}[x]}{2} + 5 \operatorname{Sin}[x]$

Out[4]= $-\operatorname{Cos}[x] - 4 \operatorname{Sin}[x]$

Out[5]=



Question 2: Solve the equation d^2y/dx^2 + dy/dx- 6y =0 and plot its three solutions.

Solution :

In[6]:=
```
Sol = DSolve[y''[x] + y'[x] - 6 y[x] == 0, y[x], x]
Sol1 = Evaluate[y[x] /. Sol[[1]] /. {C[1] → 0, C[2] → 2.5}]
Sol2 = y[x] /. Sol[[1]] /. {C[1] → 1, C[2] → 5}
Sol3 = y[x] /. Sol[[1]] /. {C[1] → -1/2, C[2] → 5}
Plot[{Sol1, Sol2, Sol3}, {x, -2, 2},
 PlotStyle → {{Pink, Thickness[0.01]}, {Green, Thick}, {Orange, Thickness[0.02]}},
 PlotLegends → {Sol1, Sol2, Sol3}]
```

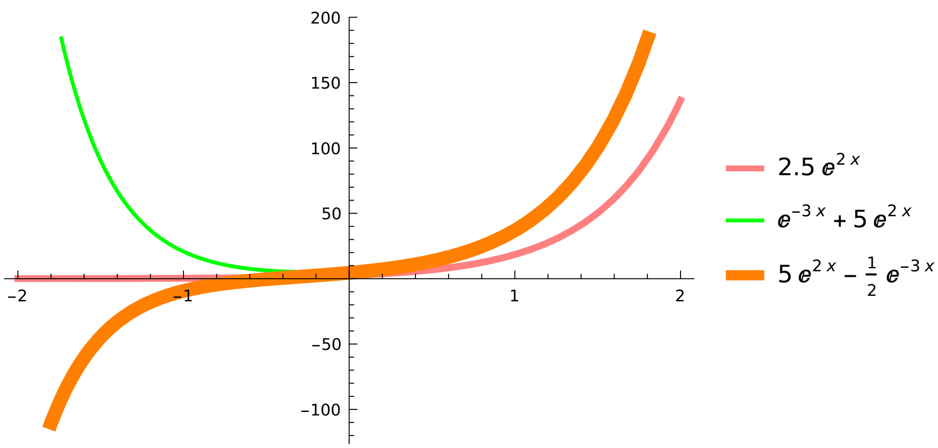
Out[6]= $\{\{y[x] \to e^{-3x} c_1 + e^{2x} c_2\}\}$

Out[7]= $2.5\, e^{2x}$

Out[8]= $e^{-3x} + 5\, e^{2x}$

Out[9]= $-\frac{1}{2} e^{-3x} + 5\, e^{2x}$

Out[10]=



Question 3: Solve the equation 4d^2y/dx^2 + 12dy/dx +9y =0 and plot its four solutions.

Solution :

In[11]:=
```
Sol = DSolve[4 y''[x] + 12 y'[x] + 9 y[x] == 0, y[x], x]
Sol1 = Evaluate[y[x] /. Sol[[1]] /. {C[1] → -1, C[2] → 4}]
Sol2 = y[x] /. Sol[[1]] /. {C[1] → 3, C[2] → 6}
Sol3 = y[x] /. Sol[[1]] /. {C[1] → 10, C[2] → 7}
Sol4 = y[x] /. Sol[[1]] /. {C[1] → -1.5, C[2] → -5}
Plot[{Sol1, Sol2, Sol3, Sol4}, {x, -2, 2},
 PlotStyle → {{Pink, Thickness[0.01]}, {Green, Thick}, {Orange, Thickness[0.02]}},
 PlotLegends → {Sol1, Sol2, Sol3, Sol4}]
```

Out[11]= $\{\{y[x] \to e^{-3x/2} c_1 + e^{-3x/2} x\, c_2\}\}$

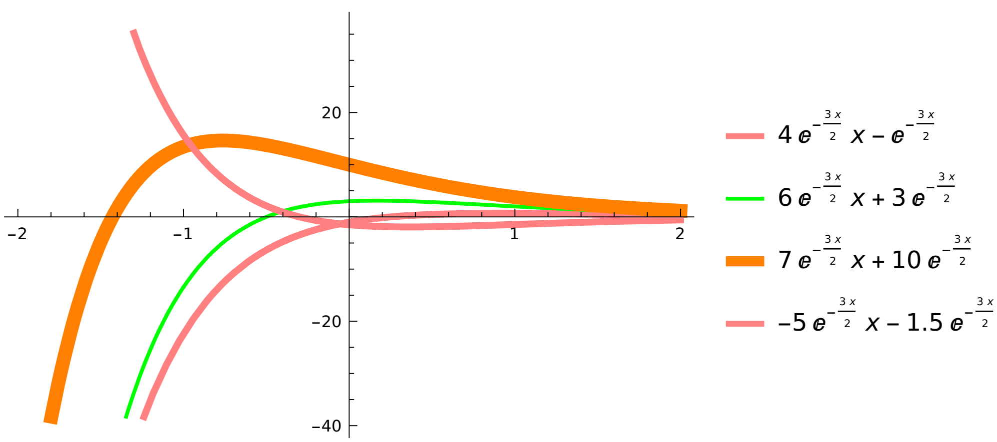
Out[12]= $-e^{-3x/2} + 4 e^{-3x/2} x$

Out[13]= $3 e^{-3x/2} + 6 e^{-3x/2} x$

Out[14]= $10 e^{-3x/2} + 7 e^{-3x/2} x$

Out[15]= $-1.5 e^{-3x/2} - 5 e^{-3x/2} x$

Out[16]=



Question 5: Solve the equation d^2y/dx^2 -2dy/dx +y =0 and plot its any five solutions.

Solution :

In[24]:=
```
Sol = DSolve[y''[x] - 2 y'[x] + y[x] == 0, y[x], x]
Sol1 = Evaluate[y[x] /. Sol[[1]] /. {C[1] → 0.5, C[2] → 3}]
Sol2 = y[x] /. Sol[[1]] /. {C[1] → -3, C[2] → -2}
Sol3 = y[x] /. Sol[[1]] /. {C[1] → -1, C[2] → 7}
Sol4 = y[x] /. Sol[[1]] /. {C[1] → -6, C[2] → 1}
Sol5 = y[x] /. Sol[[1]] /. {C[1] → 1 / 5, C[2] → 2 / 3}
Plot[{Sol1, Sol2, Sol3, Sol4, Sol5}, {x, -5, 5},
 PlotStyle → {Thickness[0.01], Thick, Thickness[0.02], Thickness[0.03], Thickness[0.04]},
 PlotLegends → {Sol1, Sol2, Sol3, Sol4, Sol5}]
```

Out[24]= $\{\{y[x] \to e^x c_1 + e^x x c_2\}\}$

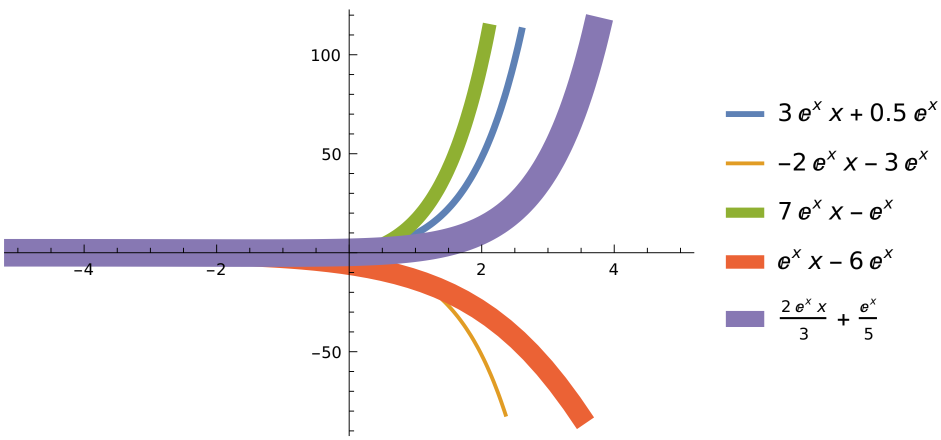
Out[25]= $0.5 e^x + 3 e^x x$

Out[26]= $-3 e^x - 2 e^x x$

Out[27]= $-e^x + 7 e^x x$

Out[28]= $-6 e^x + e^x x$

Out[29]= $\frac{e^x}{5} + \frac{2 e^x x}{3}$

Out[30]=


Question 6: Solve the equation x^2 d^2y/dx^2 - xdy/dx+ y = 2log(x) and plot its any five solutions.

Solution :

In[43]:=
```
Sol = DSolve[x ^ 2 y ''[x] - xy '[x] + y[x] == 2 log (x), y[x], x]
Sol1 = Evaluate [y[x] /. Sol⟦1⟧ /. {C[1] → 0.5 , C[2] → 3}]
Sol2 = y[x] /. Sol⟦1⟧ /. {C[1] → -3, C[2] → -2}
Sol3 = y[x] /. Sol⟦1⟧ /. {C[1] → -1, C[2] → 7}
Sol4 = y[x] /. Sol⟦1⟧ /. {C[1] → -6, C[2] → 1}
Sol5 = y[x] /. Sol⟦1⟧ /. {C[1] → 1 / 5, C[2] → 2 / 3}
Plot[{Sol1 , Sol2 , Sol3 , Sol4 , Sol5}, {x, -5, 5},
 PlotStyle → {Thickness [0.01], Thick , Thickness [0.02], Thickness [0.03], Thickness [0.04]},
 PlotLegends → {Sol1 , Sol2 , Sol3 , Sol4 , Sol5}]
```

Out[43]=

$$\Big\{\Big\{y[x] \to \sqrt{x}\, c_1 \operatorname{Cos}\Big[\frac{1}{2}\sqrt{3}\operatorname{Log}[x]\Big] + \sqrt{x}\, c_2 \operatorname{Sin}\Big[\frac{1}{2}\sqrt{3}\operatorname{Log}[x]\Big] + \sqrt{x}\operatorname{Cos}\Big[\frac{1}{2}\sqrt{3}\operatorname{Log}[x]\Big]\int_1^x\left(-\frac{4\log \operatorname{Sin}\big[\frac{1}{2}\sqrt{3}\operatorname{Log}[K[1]]\big]}{\sqrt{3}\sqrt{K[1]}} - \frac{2\operatorname{Sin}\big[\frac{1}{2}\sqrt{3}\operatorname{Log}[K[1]]\big]\, xy'[K[1]]}{\sqrt{3}\, K[1]^{3/2}}\right)dK[1] + \sqrt{x}\operatorname{Sin}\Big[\frac{1}{2}\sqrt{3}\operatorname{Log}[x]\Big]\int_1^x\left(\frac{4\log \operatorname{Cos}\big[\frac{1}{2}\sqrt{3}\operatorname{Log}[K[2]]\big]}{\sqrt{3}\sqrt{K[2]}} + \frac{2\operatorname{Cos}\big[\frac{1}{2}\sqrt{3}\operatorname{Log}[K[2]]\big]\, xy'[K[2]]}{\sqrt{3}\, K[2]^{3/2}}\right)dK[2]\Big\}\Big\}$$

Out[44]=

$$0.5\sqrt{x}\operatorname{Cos}\Big[\frac{1}{2}\sqrt{3}\operatorname{Log}[x]\Big] + 3\sqrt{x}\operatorname{Sin}\Big[\frac{1}{2}\sqrt{3}\operatorname{Log}[x]\Big] + \sqrt{x}\operatorname{Cos}\Big[\frac{1}{2}\sqrt{3}\operatorname{Log}[x]\Big]\int_1^x\left(-\frac{4\log \operatorname{Sin}\big[\frac{1}{2}\sqrt{3}\operatorname{Log}[K[1]]\big]}{\sqrt{3}\sqrt{K[1]}} - \frac{2\operatorname{Sin}\big[\frac{1}{2}\sqrt{3}\operatorname{Log}[K[1]]\big]\, xy'[K[1]]}{\sqrt{3}\, K[1]^{3/2}}\right)dK[1] + \sqrt{x}\operatorname{Sin}\Big[\frac{1}{2}\sqrt{3}\operatorname{Log}[x]\Big]\int_1^x\left(\frac{4\log \operatorname{Cos}\big[\frac{1}{2}\sqrt{3}\operatorname{Log}[K[2]]\big]}{\sqrt{3}\sqrt{K[2]}} + \frac{2\operatorname{Cos}\big[\frac{1}{2}\sqrt{3}\operatorname{Log}[K[2]]\big]\, xy'[K[2]]}{\sqrt{3}\, K[2]^{3/2}}\right)dK[2]$$

Out[45]=

$$-3\sqrt{x}\operatorname{Cos}\Big[\frac{1}{2}\sqrt{3}\operatorname{Log}[x]\Big] - 2\sqrt{x}\operatorname{Sin}\Big[\frac{1}{2}\sqrt{3}\operatorname{Log}[x]\Big] + \sqrt{x}\operatorname{Cos}\Big[\frac{1}{2}\sqrt{3}\operatorname{Log}[x]\Big]\int_1^x\left(-\frac{4\log \operatorname{Sin}\big[\frac{1}{2}\sqrt{3}\operatorname{Log}[K[1]]\big]}{\sqrt{3}\sqrt{K[1]}} - \frac{2\operatorname{Sin}\big[\frac{1}{2}\sqrt{3}\operatorname{Log}[K[1]]\big]\, xy'[K[1]]}{\sqrt{3}\, K[1]^{3/2}}\right)dK[1] + \sqrt{x}\operatorname{Sin}\Big[\frac{1}{2}\sqrt{3}\operatorname{Log}[x]\Big]\int_1^x\left(\frac{4\log \operatorname{Cos}\big[\frac{1}{2}\sqrt{3}\operatorname{Log}[K[2]]\big]}{\sqrt{3}\sqrt{K[2]}} + \frac{2\operatorname{Cos}\big[\frac{1}{2}\sqrt{3}\operatorname{Log}[K[2]]\big]\, xy'[K[2]]}{\sqrt{3}\, K[2]^{3/2}}\right)dK[2]$$

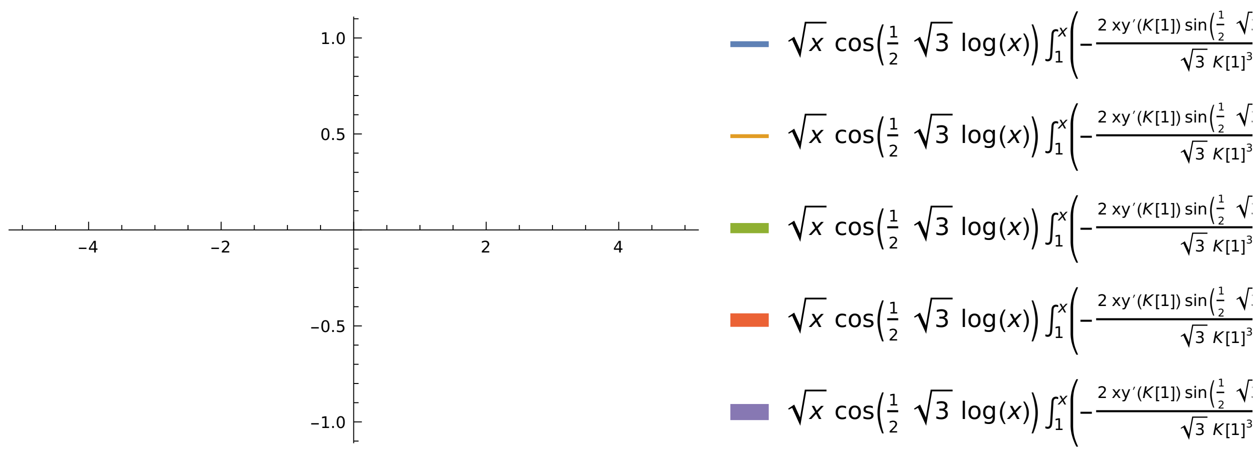
Out[46]=

$$-\sqrt{x}\operatorname{Cos}\Big[\frac{1}{2}\sqrt{3}\operatorname{Log}[x]\Big] + 7\sqrt{x}\operatorname{Sin}\Big[\frac{1}{2}\sqrt{3}\operatorname{Log}[x]\Big] + \sqrt{x}\operatorname{Cos}\Big[\frac{1}{2}\sqrt{3}\operatorname{Log}[x]\Big]\int_1^x\left(-\frac{4\log \operatorname{Sin}\big[\frac{1}{2}\sqrt{3}\operatorname{Log}[K[1]]\big]}{\sqrt{3}\sqrt{K[1]}} - \frac{2\operatorname{Sin}\big[\frac{1}{2}\sqrt{3}\operatorname{Log}[K[1]]\big]\, xy'[K[1]]}{\sqrt{3}\, K[1]^{3/2}}\right)dK[1] + \sqrt{x}\operatorname{Sin}\Big[\frac{1}{2}\sqrt{3}\operatorname{Log}[x]\Big]\int_1^x\left(\frac{4\log \operatorname{Cos}\big[\frac{1}{2}\sqrt{3}\operatorname{Log}[K[2]]\big]}{\sqrt{3}\sqrt{K[2]}} + \frac{2\operatorname{Cos}\big[\frac{1}{2}\sqrt{3}\operatorname{Log}[K[2]]\big]\, xy'[K[2]]}{\sqrt{3}\, K[2]^{3/2}}\right)dK[2]$$

Out[47]= $$-6\sqrt{x}\,\mathrm{Cos}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right]+\sqrt{x}\,\mathrm{Sin}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right]+$$
$$\sqrt{x}\,\mathrm{Cos}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right]\int_1^x\left(-\frac{4\log\mathrm{Sin}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[K[1]]\right]}{\sqrt{3}\sqrt{K[1]}}-\frac{2\,\mathrm{Sin}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[K[1]]\right]xy'[K[1]]}{\sqrt{3}\,K[1]^{3/2}}\right)dK[1]+$$
$$\sqrt{x}\,\mathrm{Sin}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right]\int_1^x\left(\frac{4\log\mathrm{Cos}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[K[2]]\right]}{\sqrt{3}\sqrt{K[2]}}+\frac{2\,\mathrm{Cos}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[K[2]]\right]xy'[K[2]]}{\sqrt{3}\,K[2]^{3/2}}\right)dK[2]$$

Out[48]= $$\frac{1}{5}\sqrt{x}\,\mathrm{Cos}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right]+\frac{2}{3}\sqrt{x}\,\mathrm{Sin}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right]+$$
$$\sqrt{x}\,\mathrm{Cos}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right]\int_1^x\left(-\frac{4\log\mathrm{Sin}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[K[1]]\right]}{\sqrt{3}\sqrt{K[1]}}-\frac{2\,\mathrm{Sin}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[K[1]]\right]xy'[K[1]]}{\sqrt{3}\,K[1]^{3/2}}\right)dK[1]+$$
$$\sqrt{x}\,\mathrm{Sin}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right]\int_1^x\left(\frac{4\log\mathrm{Cos}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[K[2]]\right]}{\sqrt{3}\sqrt{K[2]}}+\frac{2\,\mathrm{Cos}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[K[2]]\right]xy'[K[2]]}{\sqrt{3}\,K[2]^{3/2}}\right)dK[2]$$

Out[49]=



Question 7: Solve the equation x^2 d^2y/dx^2 + y = x^2 and plot its any five solutions.

Solution :

In[22]:=
```
Sol = DSolve[x ^ 2 y ''[x] + y[x] == x ^ 2, y[x], x]
Sol1 = Evaluate[y[x] /. Sol⟦1⟧ /. {C[1] → 0.5, C[2] → 3}]
Sol2 = y[x] /. Sol⟦1⟧ /. {C[1] → -3, C[2] → -2}
Sol3 = y[x] /. Sol⟦1⟧ /. {C[1] → -1, C[2] → 7}
Sol4 = y[x] /. Sol⟦1⟧ /. {C[1] → -6, C[2] → 1}
Sol5 = y[x] /. Sol⟦1⟧ /. {C[1] → 1 / 5, C[2] → 2 / 3}
Plot[{Sol1, Sol2, Sol3, Sol4, Sol5},
 {x, -15, 15}, PlotLegends → {Sol1, Sol2, Sol3, Sol4, Sol5}]
```

Out[22]= $$\left\{\left\{y[x] \to \sqrt{x}\, c_1 \,\mathrm{Cos}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right] + \sqrt{x}\, c_2 \,\mathrm{Sin}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right] + \frac{1}{3}x^2\left(\mathrm{Cos}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right]^2 + \mathrm{Sin}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right]^2\right)\right\}\right\}$$

Out[23]= $$0.5\sqrt{x}\,\mathrm{Cos}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right] + 3\sqrt{x}\,\mathrm{Sin}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right] + \frac{1}{3}x^2\left(\mathrm{Cos}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right]^2 + \mathrm{Sin}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right]^2\right)$$

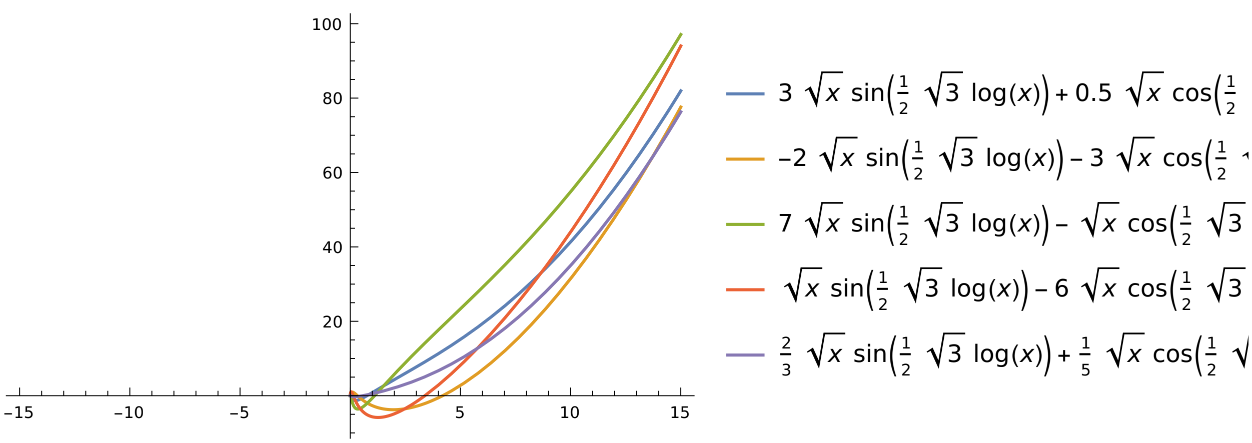
Out[24]= $$-3\sqrt{x}\,\mathrm{Cos}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right] - 2\sqrt{x}\,\mathrm{Sin}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right] + \frac{1}{3}x^2\left(\mathrm{Cos}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right]^2 + \mathrm{Sin}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right]^2\right)$$

Out[25]= $$-\sqrt{x}\,\mathrm{Cos}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right] + 7\sqrt{x}\,\mathrm{Sin}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right] + \frac{1}{3}x^2\left(\mathrm{Cos}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right]^2 + \mathrm{Sin}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right]^2\right)$$

Out[26]= $$-6\sqrt{x}\,\mathrm{Cos}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right] + \sqrt{x}\,\mathrm{Sin}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right] + \frac{1}{3}x^2\left(\mathrm{Cos}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right]^2 + \mathrm{Sin}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right]^2\right)$$

Out[27]= $$\frac{1}{5}\sqrt{x}\,\mathrm{Cos}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right] + \frac{2}{3}\sqrt{x}\,\mathrm{Sin}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right] + \frac{1}{3}x^2\left(\mathrm{Cos}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right]^2 + \mathrm{Sin}\left[\frac{1}{2}\sqrt{3}\,\mathrm{Log}[x]\right]^2\right)$$

Out[28]=



Question 8: Solve the equation d^2y/dx^2 = √1+(dy/dx)^2 and plot its any five solutions.

Solution :

In[15]:=
```
Sol = DSolve[y''[x] == Sqrt[1 + (y'[x])^2], y[x], x]
Sol1 = Evaluate[y[x] /. Sol[[1]] /. {C[1] → 0.5, C[2] → 3}]
Sol2 = y[x] /. Sol[[1]] /. {C[1] → -2, C[2] → -2}
Sol3 = y[x] /. Sol[[1]] /. {C[1] → -1, C[2] → 7}
Sol4 = y[x] /. Sol[[1]] /. {C[1] → -6, C[2] → 4}
Sol5 = y[x] /. Sol[[1]] /. {C[1] → 5, C[2] → 2/3}
Plot[{Sol1, Sol2, Sol3, Sol4, Sol5},
 {x, -15, 15}, PlotLegends → {Sol1, Sol2, Sol3, Sol4, Sol5}]
```

Out[15]= {{y[x] → c_2 + Cosh[x] × Cosh[c_1] + Sinh[x] × Sinh[c_1]}}

Out[16]= 3 + 1.12763 Cosh[x] + 0.521095 Sinh[x]

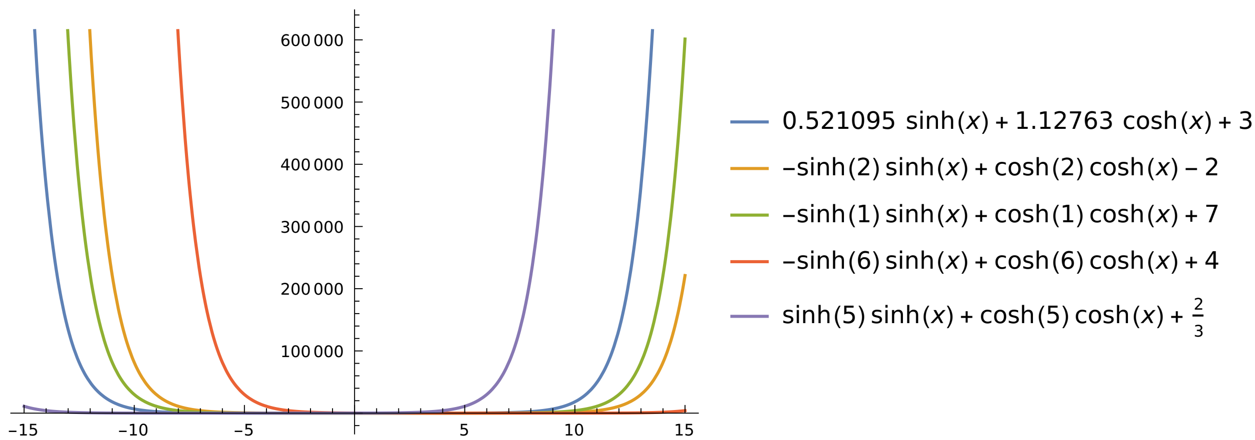
Out[17]= - 2 + Cosh[2] × Cosh[x] - Sinh[2] × Sinh[x]

Out[18]= 7 + Cosh[1] × Cosh[x] - Sinh[1] × Sinh[x]

Out[19]= 4 + Cosh[6] × Cosh[x] - Sinh[6] × Sinh[x]

Out[20]= $\frac{2}{3}$ + Cosh[5] × Cosh[x] + Sinh[5] × Sinh[x]

Out[21]=



Question 9: Solve the equation (1+x^2)d^2y/dx^2 +1+(dy/dx)^2= 0 and plot its any five solutions.

Solution :

```
In[50]:= Sol = DSolve[(1 + x^2) y''[x] + 1 + y'[x]^2 == 0, y[x], x]
Sol1 = Evaluate[y[x] /. Sol[[1]] /. {C[1] → 0.5, C[2] → 3}]
Sol2 = y[x] /. Sol[[1]] /. {C[1] → -2, C[2] → -2}
Sol3 = y[x] /. Sol[[1]] /. {C[1] → -1, C[2] → 7}
Sol4 = y[x] /. Sol[[1]] /. {C[1] → -6, C[2] → 4}
Sol5 = y[x] /. Sol[[1]] /. {C[1] → 5, C[2] → 2/3}
Plot[{Sol1, Sol2, Sol3, Sol4, Sol5},
 {x, -15, 15}, PlotLegends → {Sol1, Sol2, Sol3, Sol4, Sol5}]
```

Out[50]= $\{\{y[x] \to c_2 - x\,\mathrm{Cot}[c_1] + \mathrm{Csc}[c_1]^2\,\mathrm{Log}[-\mathrm{Cos}[c_1] - x\,\mathrm{Sin}[c_1]]\}\}$

Out[51]= $3 - 1.83049\,x + 4.35069\,\mathrm{Log}[-0.877583 - 0.479426\,x]$

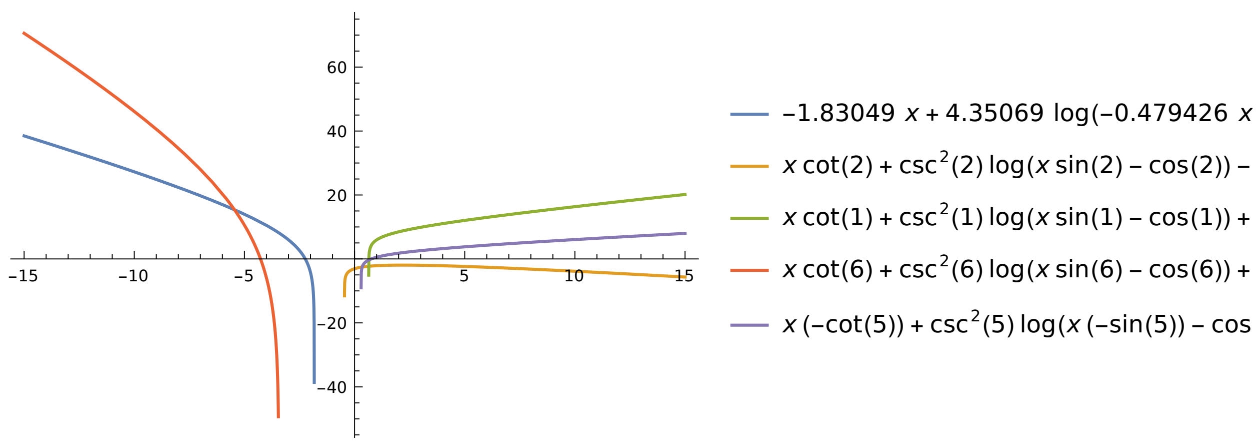
Out[52]= $-2 + x\,\mathrm{Cot}[2] + \mathrm{Csc}[2]^2\,\mathrm{Log}[-\mathrm{Cos}[2] + x\,\mathrm{Sin}[2]]$

Out[53]= $7 + x\,\mathrm{Cot}[1] + \mathrm{Csc}[1]^2\,\mathrm{Log}[-\mathrm{Cos}[1] + x\,\mathrm{Sin}[1]]$

Out[54]= $4 + x\,\mathrm{Cot}[6] + \mathrm{Csc}[6]^2\,\mathrm{Log}[-\mathrm{Cos}[6] + x\,\mathrm{Sin}[6]]$

Out[55]= $\frac{2}{3} - x\,\mathrm{Cot}[5] + \mathrm{Csc}[5]^2\,\mathrm{Log}[-\mathrm{Cos}[5] - x\,\mathrm{Sin}[5]]$

Out[56]=



Question 10: Solve the equation 2x^2 d^2y/dx^2 -xdy/dx + (1-x^2)y= x^2 and plot its any five solutions.

Solution :

```
In[57]:= Sol = DSolve[(2 x^2) y''[x] - xy'[x] + (1 - x^2) y[x] == x^2, y[x], x]
Sol1 = Evaluate[y[x] /. Sol[[1]] /. {C[1] → 0.5, C[2] → 3}]
Sol2 = y[x] /. Sol[[1]] /. {C[1] → -2, C[2] → -2}
Sol3 = y[x] /. Sol[[1]] /. {C[1] → -1, C[2] → 7}
Sol4 = y[x] /. Sol[[1]] /. {C[1] → -6, C[2] → 4}
Sol5 = y[x] /. Sol[[1]] /. {C[1] → 5, C[2] → 2/3}
Plot[{Sol1, Sol2, Sol3, Sol4, Sol5},
 {x, -15, 15}, PlotLegends → {Sol1, Sol2, Sol3, Sol4, Sol5}]
```

Out[57]=

$$\left\{\left\{y[x]\to\sqrt{x}\,\text{BesselJ}\left[\frac{i}{2},-\frac{i\,x}{\sqrt{2}}\right]c_1+\sqrt{x}\,\text{BesselY}\left[\frac{i}{2},-\frac{i\,x}{\sqrt{2}}\right]c_2+\sqrt{x}\,\text{BesselJ}\left[\frac{i}{2},-\frac{i\,x}{\sqrt{2}}\right]\int_1^x\left(-\frac{1}{4}\pi\,\text{BesselY}\left[\frac{i}{2},-\frac{i\,K[1]}{\sqrt{2}}\right]\sqrt{K[1]}-\frac{\pi\,\text{BesselY}\left[\frac{i}{2},-\frac{i\,K[1]}{\sqrt{2}}\right]xy'[K[1]]}{4\,K[1]^{3/2}}\right)dK[1]+\sqrt{x}\,\text{BesselY}\left[\frac{i}{2},-\frac{i\,x}{\sqrt{2}}\right]\int_1^x\left(\frac{1}{4}\pi\,\text{BesselJ}\left[\frac{i}{2},-\frac{i\,K[2]}{\sqrt{2}}\right]\sqrt{K[2]}+\frac{\pi\,\text{BesselJ}\left[\frac{i}{2},-\frac{i\,K[2]}{\sqrt{2}}\right]xy'[K[2]]}{4\,K[2]^{3/2}}\right)dK[2]\right\}\right\}$$

Out[58]=

$$0.5\sqrt{x}\,\text{BesselJ}\left[\frac{i}{2},-\frac{i\,x}{\sqrt{2}}\right]+3\sqrt{x}\,\text{BesselY}\left[\frac{i}{2},-\frac{i\,x}{\sqrt{2}}\right]+\sqrt{x}\,\text{BesselJ}\left[\frac{i}{2},-\frac{i\,x}{\sqrt{2}}\right]\int_1^x\left(-\frac{1}{4}\pi\,\text{BesselY}\left[\frac{i}{2},-\frac{i\,K[1]}{\sqrt{2}}\right]\sqrt{K[1]}-\frac{\pi\,\text{BesselY}\left[\frac{i}{2},-\frac{i\,K[1]}{\sqrt{2}}\right]xy'[K[1]]}{4\,K[1]^{3/2}}\right)dK[1]+\sqrt{x}\,\text{BesselY}\left[\frac{i}{2},-\frac{i\,x}{\sqrt{2}}\right]\int_1^x\left(\frac{1}{4}\pi\,\text{BesselJ}\left[\frac{i}{2},-\frac{i\,K[2]}{\sqrt{2}}\right]\sqrt{K[2]}+\frac{\pi\,\text{BesselJ}\left[\frac{i}{2},-\frac{i\,K[2]}{\sqrt{2}}\right]xy'[K[2]]}{4\,K[2]^{3/2}}\right)dK[2]$$

Out[59]=

$$-2\sqrt{x}\,\text{BesselJ}\left[\frac{i}{2},-\frac{i\,x}{\sqrt{2}}\right]-2\sqrt{x}\,\text{BesselY}\left[\frac{i}{2},-\frac{i\,x}{\sqrt{2}}\right]+\sqrt{x}\,\text{BesselJ}\left[\frac{i}{2},-\frac{i\,x}{\sqrt{2}}\right]\int_1^x\left(-\frac{1}{4}\pi\,\text{BesselY}\left[\frac{i}{2},-\frac{i\,K[1]}{\sqrt{2}}\right]\sqrt{K[1]}-\frac{\pi\,\text{BesselY}\left[\frac{i}{2},-\frac{i\,K[1]}{\sqrt{2}}\right]xy'[K[1]]}{4\,K[1]^{3/2}}\right)dK[1]+\sqrt{x}\,\text{BesselY}\left[\frac{i}{2},-\frac{i\,x}{\sqrt{2}}\right]\int_1^x\left(\frac{1}{4}\pi\,\text{BesselJ}\left[\frac{i}{2},-\frac{i\,K[2]}{\sqrt{2}}\right]\sqrt{K[2]}+\frac{\pi\,\text{BesselJ}\left[\frac{i}{2},-\frac{i\,K[2]}{\sqrt{2}}\right]xy'[K[2]]}{4\,K[2]^{3/2}}\right)dK[2]$$

Out[60]=

$$-\sqrt{x}\,\text{BesselJ}\left[\frac{i}{2},-\frac{i\,x}{\sqrt{2}}\right]+7\sqrt{x}\,\text{BesselY}\left[\frac{i}{2},-\frac{i\,x}{\sqrt{2}}\right]+\sqrt{x}\,\text{BesselJ}\left[\frac{i}{2},-\frac{i\,x}{\sqrt{2}}\right]\int_1^x\left(-\frac{1}{4}\pi\,\text{BesselY}\left[\frac{i}{2},-\frac{i\,K[1]}{\sqrt{2}}\right]\sqrt{K[1]}-\frac{\pi\,\text{BesselY}\left[\frac{i}{2},-\frac{i\,K[1]}{\sqrt{2}}\right]xy'[K[1]]}{4\,K[1]^{3/2}}\right)dK[1]+\sqrt{x}\,\text{BesselY}\left[\frac{i}{2},-\frac{i\,x}{\sqrt{2}}\right]\int_1^x\left(\frac{1}{4}\pi\,\text{BesselJ}\left[\frac{i}{2},-\frac{i\,K[2]}{\sqrt{2}}\right]\sqrt{K[2]}+\frac{\pi\,\text{BesselJ}\left[\frac{i}{2},-\frac{i\,K[2]}{\sqrt{2}}\right]xy'[K[2]]}{4\,K[2]^{3/2}}\right)dK[2]$$

Out[61]=
$$-6\sqrt{x}\,\text{BesselJ}\left[\frac{i}{2},-\frac{i\,x}{\sqrt{2}}\right]+4\sqrt{x}\,\text{BesselY}\left[\frac{i}{2},-\frac{i\,x}{\sqrt{2}}\right]+$$
$$\sqrt{x}\,\text{BesselJ}\left[\frac{i}{2},-\frac{i\,x}{\sqrt{2}}\right]\int_1^x\left(-\frac{1}{4}\pi\,\text{BesselY}\left[\frac{i}{2},-\frac{i\,K[1]}{\sqrt{2}}\right]\sqrt{K[1]}-\frac{\pi\,\text{BesselY}\left[\frac{i}{2},-\frac{i\,K[1]}{\sqrt{2}}\right]xy'[K[1]]}{4\,K[1]^{3/2}}\right)dK[1]+$$
$$\sqrt{x}\,\text{BesselY}\left[\frac{i}{2},-\frac{i\,x}{\sqrt{2}}\right]\int_1^x\left(\frac{1}{4}\pi\,\text{BesselJ}\left[\frac{i}{2},-\frac{i\,K[2]}{\sqrt{2}}\right]\sqrt{K[2]}+\frac{\pi\,\text{BesselJ}\left[\frac{i}{2},-\frac{i\,K[2]}{\sqrt{2}}\right]xy'[K[2]]}{4\,K[2]^{3/2}}\right)dK[2]$$

Out[62]=
$$5\sqrt{x}\,\text{BesselJ}\left[\frac{i}{2},-\frac{i\,x}{\sqrt{2}}\right]+\frac{2}{3}\sqrt{x}\,\text{BesselY}\left[\frac{i}{2},-\frac{i\,x}{\sqrt{2}}\right]+$$
$$\sqrt{x}\,\text{BesselJ}\left[\frac{i}{2},-\frac{i\,x}{\sqrt{2}}\right]\int_1^x\left(-\frac{1}{4}\pi\,\text{BesselY}\left[\frac{i}{2},-\frac{i\,K[1]}{\sqrt{2}}\right]\sqrt{K[1]}-\frac{\pi\,\text{BesselY}\left[\frac{i}{2},-\frac{i\,K[1]}{\sqrt{2}}\right]xy'[K[1]]}{4\,K[1]^{3/2}}\right)dK[1]+$$
$$\sqrt{x}\,\text{BesselY}\left[\frac{i}{2},-\frac{i\,x}{\sqrt{2}}\right]\int_1^x\left(\frac{1}{4}\pi\,\text{BesselJ}\left[\frac{i}{2},-\frac{i\,K[2]}{\sqrt{2}}\right]\sqrt{K[2]}+\frac{\pi\,\text{BesselJ}\left[\frac{i}{2},-\frac{i\,K[2]}{\sqrt{2}}\right]xy'[K[2]]}{4\,K[2]^{3/2}}\right)dK[2]$$

Out[63]=