PRACTICAL ASSIGNMENT : DSE (i) : DIFFERENTIAL EQUATIONS

Question 1: Solve first order differential equation: dy/dx+y=Sin(x) and plot its solutions for (c1=1,c2=2; c1=1/2,c2=5; c1=-1,c2=-4)

Solution:

In[30]:=
```
sol = DSolve[y'[x] + y[x] == Sin[x], y[x], x]
sol1 = Evaluate[y[x] /. sol[[1]] /. {C[1] → 1 , C[2] → 2}]
sol2 = y[x] /. sol[[1]] /. {C[1] → 1 / 2 , C[2] → 5}
sol3 = y[x] /. sol[[1]] /. {C[1] → -1, C[2] → -4}
Plot[{sol1, sol2, sol3}, {x, -5, 3}, PlotRange → {-30, 30},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thick}, {Purple, Thickness[0.02]}},
 PlotLegends → {sol1, sol2, sol3}]
```

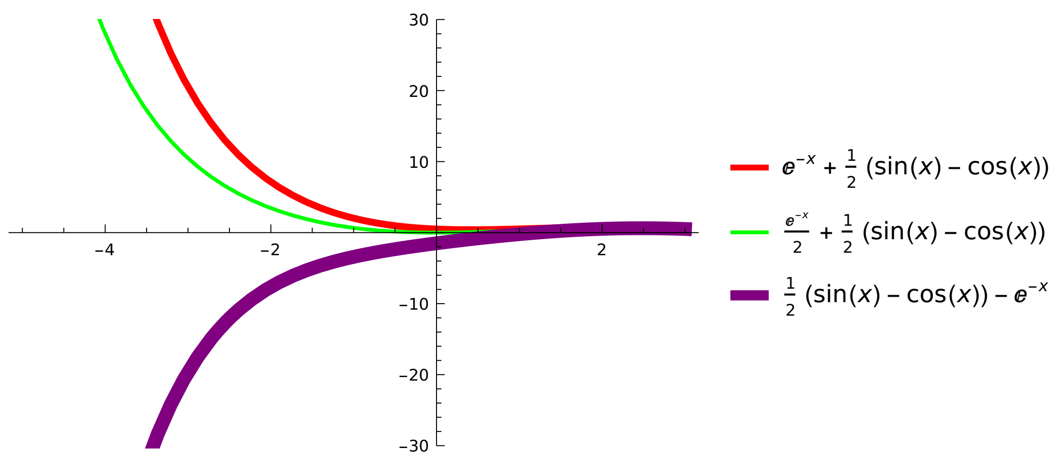
Out[30]= $\left\{\left\{y[x] \to e^{-x} c_1 + \frac{1}{2}(-\text{Cos}[x] + \text{Sin}[x])\right\}\right\}$

Out[31]= $e^{-x} + \frac{1}{2}(-\text{Cos}[x] + \text{Sin}[x])$

Out[32]= $\frac{e^{-x}}{2} + \frac{1}{2}(-\text{Cos}[x] + \text{Sin}[x])$

Out[33]= $-e^{-x} + \frac{1}{2}(-\text{Cos}[x] + \text{Sin}[x])$

Out[34]=



Question 2: Solve second order differential equation: d^2y/dx^2+8dy/dx-4y=0 and plot its three solutions for (c1=1,c2=0.5,c3=2/3; c1=-1/2,c2=0,c3=2/3; c1=-1,c2=-4,c3=2)

Solution:

In[35]:=
```
sol = DSolve[y''[x] + 8 y'[x] - 4 y[x] == 0, y[x], x]
sol1 = Evaluate[y[x] /. sol⟦1⟧ /. {C[1] → 1 , C[2] → 0.5, C[3] → 2 / 3}]
sol2 = y[x] /. sol⟦1⟧ /. {C[1] → -1 / 2 , C[2] → 0, C[3] → 2 / 3}
sol3 = y[x] /. sol⟦1⟧ /. {C[1] → -1, C[2] → -4, C[3] → 2}
Plot[{sol1, sol2, sol3}, {x, -5, 3}, PlotRange → {-30, 30},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thick}, {Purple, Thickness[0.02]}},
 PlotLegends → {sol1, sol2, sol3}]
```

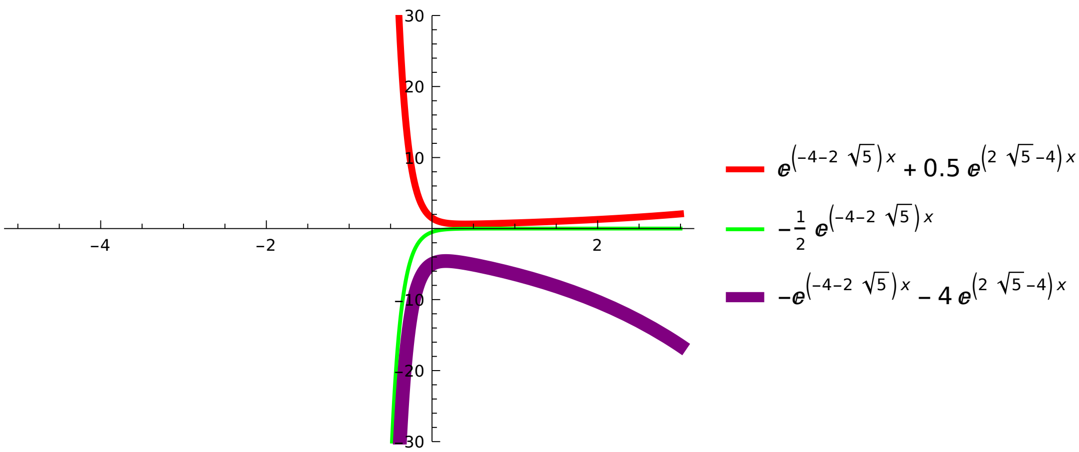
Out[35]= $\left\{\left\{y[x] \to e^{\left(-4-2\sqrt{5}\right)x} c_1 + e^{\left(-4+2\sqrt{5}\right)x} c_2\right\}\right\}$

Out[36]= $e^{\left(-4-2\sqrt{5}\right)x} + 0.5\, e^{\left(-4+2\sqrt{5}\right)x}$

Out[37]= $-\frac{1}{2}\, e^{\left(-4-2\sqrt{5}\right)x}$

Out[38]= $-e^{\left(-4-2\sqrt{5}\right)x} - 4\, e^{\left(-4+2\sqrt{5}\right)x}$

Out[39]=



Question 3: Solve third order differential equation: d^3y/dx^3-5d^2y/dx^2+8dy/dx-4y=0 and plot its three solutions for (c1=1,c2=0.5,c3=2/3; c1=-1/2,c2=0,c3=2/3;c1=-1,c2=-4,c3=2)

Solution:

```
In[40]:= sol = DSolve[y'''[x] - 5 y''[x] + 8 y'[x] - 4 y[x] == 0, y[x], x]
sol1 = Evaluate[y[x] /. sol⟦1⟧ /. {C[1] → 1 , C[2] → 0.5, C[3] → 2 / 3}]
sol2 = y[x] /. sol⟦1⟧ /. {C[1] → -1 / 2 , C[2] → 0, C[3] → 2 / 3}
sol3 = y[x] /. sol⟦1⟧ /. {C[1] → -1, C[2] → -4, C[3] → 2}
Plot[{sol1, sol2, sol3}, {x, -5, 3}, PlotRange → {-30, 30},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thick}, {Purple, Thickness[0.02]}},
 PlotLegends → {sol1, sol2, sol3}]
```

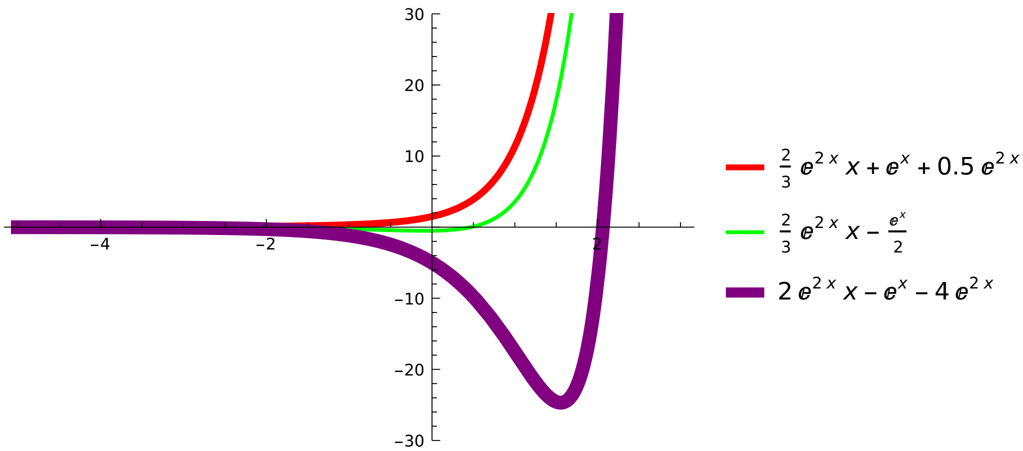
Out[40]= $\{\{y[x] \to e^{x} c_1 + e^{2x} c_2 + e^{2x} x c_3\}\}$

Out[41]= $e^{x} + 0.5\, e^{2x} + \frac{2}{3} e^{2x} x$

Out[42]= $-\frac{e^{x}}{2} + \frac{2}{3} e^{2x} x$

Out[43]= $-e^{x} - 4 e^{2x} + 2 e^{2x} x$

Out[44]=



Question 4: Solve second order differential equation: d^2y/dt^2+5dy/dt+6y=Tan(3t) by variation parameter method.

Solution:

In[67]:=
```
sol = DSolve[y''[t] + 5 y'[t] + 6 y[t] == 0, y[t], t]
sol1 = Evaluate[y[t] /. sol[[1]] /. {C[1] → 1, C[2] → 0}]
sol2 = y[t] /. sol[[1]] /. {C[1] → 0, C[2] → 1}
fs = {sol1, sol2}
wm = {fs, D[fs, t]}; wm // MatrixForm

wd = Simplify[Det[wm]]

u1 = (Integrate[- sol2 Tan[3 t], t]) / wd
u2 = (Integrate[sol1 Tan[3 t], t]) / wd

yc = DSolve[y''[t] + 5 y'[t] + 6 y[t] == 0, y[t], t]
yp = Simplify[Evaluate[y[t] /. sol[[1]] /. {C[1] → u1, C[2] → u2}]]
yg = yc + yp
```

Out[67]= $\{\{y[t] \to e^{-3t} c_1 + e^{-2t} c_2\}\}$

Out[68]= e^{-3t}

Out[69]= e^{-2t}

Out[70]= $\{e^{-3t}, e^{-2t}\}$

Out[71]//MatrixForm=

$$\begin{pmatrix} e^{-3t} & e^{-2t} \\ -3 e^{-3t} & -2 e^{-2t} \end{pmatrix}$$

Out[72]= e^{-5t}

Out[73]=
$$-\frac{1}{20} e^{3t} \left(-10 i \,\text{Hypergeometric2F1}\left[\frac{i}{3}, 1, 1+\frac{i}{3}, -e^{6 i t}\right] - (3-i) e^{6 i t} \,\text{Hypergeometric2F1}\left[1, 1+\frac{i}{3}, 2+\frac{i}{3}, -e^{6 i t}\right]\right)$$

Out[74]=
$$\frac{1}{15} e^{2t} \left(-5 i \,\text{Hypergeometric2F1}\left[\frac{i}{2}, 1, 1+\frac{i}{2}, -e^{6 i t}\right] - (2-i) e^{6 i t} \,\text{Hypergeometric2F1}\left[1, 1+\frac{i}{2}, 2+\frac{i}{2}, -e^{6 i t}\right]\right)$$

Out[75]= $\{\{y[t] \to e^{-3t} c_1 + e^{-2t} c_2\}\}$

Out[76]=
$$\frac{1}{60} \times \left(30 i \,\text{Hypergeometric2F1}\left[\frac{i}{3}, 1, 1+\frac{i}{3}, -e^{6 i t}\right] - 20 i \,\text{Hypergeometric2F1}\left[\frac{i}{2}, 1, 1+\frac{i}{2}, -e^{6 i t}\right] + (1-i) e^{6 i t} \left((6+3 i) \,\text{Hypergeometric2F1}\left[1, 1+\frac{i}{3}, 2+\frac{i}{3}, -e^{6 i t}\right] - (6+2 i) \,\text{Hypergeometric2F1}\left[1, 1+\frac{i}{2}, 2+\frac{i}{2}, -e^{6 i t}\right]\right)\right)$$

Out[77]=

$$\left\{\left\{\frac{1}{60}\times\left(30\,i\,\text{Hypergeometric2F1}\left[\frac{i}{3},1,1+\frac{i}{3},-e^{6it}\right]-20\,i\,\text{Hypergeometric2F1}\left[\frac{i}{2},1,1+\frac{i}{2},-e^{6it}\right]+(1-i)\,e^{6it}\left((6+3i)\,\text{Hypergeometric2F1}\left[1,1+\frac{i}{3},2+\frac{i}{3},-e^{6it}\right]-(6+2i)\,\text{Hypergeometric2F1}\left[1,1+\frac{i}{2},2+\frac{i}{2},-e^{6it}\right]\right)\right)+\left(y[t]\to e^{-3t}c_1+e^{-2t}c_2\right)\right\}\right\}$$

Question 5: Solve third order differential equation: d^3y/dt^3+5d^2y/dt^2+2dy/dt-7y=Cosec(4t) by variation parameter method.

Solution:

In[46]:=

```
Sol = DSolve[y'''[t] + 5 y''[t] + 2 y'[t] - 7 y[t] == 0, y[t], t]
sol1 = Evaluate[y[t] /. Sol[[1]] /. {C[1] → 1, C[2] → 0, C[3] → 0}]
sol2 = y[t] /. Sol[[1]] /. {C[1] → 0, C[2] → 1, C[3] → 0}
sol3 = y[t] /. Sol[[1]] /. {C[1] → 0, C[2] → 0, C[3] → 1}

fs = {sol1, sol2, sol3}
wm = {fs, D[fs, t], D[fs, {t, 2}]}; MatrixForm[wm]
wd = Simplify[Det[wm]]

a = (1 / wd (Det[{{0, sol2, sol3}, {0, D[sol2, t], D[sol3, t]},
        {Cosec[4 t], D[sol2, {t, 2}], D[sol3, {t, 2}]}}])) // Simplify
u1 = Integrate[a, t]

b = (1 / wd (Det[{{sol1, 0, sol3}, {D[sol1, t], 0, D[sol3, t]},
        {D[sol1, {t, 2}], Cosec[4 t], D[sol3, {t, 2}]}}])) // Simplify
u2 = Integrate[b, t]

c = (1 / wd (Det[{{sol1, sol2, 0}, {D[sol1, t], D[sol2, t], 0},
        {D[sol1, {t, 2}], D[sol2, {t, 2}], Cosec[4 t]}}])) // Simplify
u3 = Integrate[c, t]
yc = Evaluate[y[t] /. Sol[[1]]]
yp = Simplify[Evaluate[y[t] /. Sol[[1]] /. {C[1] → u1, C[2] → u2, C[3] → u3}]]
yg = yc + yp
```

Out[46]= $\left\{\left\{y[t]\to e^{t\,(-4.09\ldots)}c_1+e^{t\,(-1.84\ldots)}c_2+e^{t\,(0.931\ldots)}c_3\right\}\right\}$

Out[47]= $e^{t\,(-4.09\ldots)}$

Out[48]= $e^{t \boxed{-1.84\ldots}}$

Out[49]= $e^{t \boxed{0.931\ldots}}$

Out[50]= $\left\{e^{t \boxed{-4.09\ldots}}, e^{t \boxed{-1.84\ldots}}, e^{t \boxed{0.931\ldots}}\right\}$

Out[51]//MatrixForm=

$$\begin{pmatrix} e^{t \boxed{-4.09\ldots}} & e^{t \boxed{-1.84\ldots}} & e^{t \boxed{0.931\ldots}} \\ e^{t \boxed{-4.09\ldots}} \boxed{-4.09\ldots} & e^{t \boxed{-1.84\ldots}} \boxed{-1.84\ldots} & e^{t \boxed{0.931\ldots}} \boxed{0.931\ldots} \\ e^{t \boxed{-4.09\ldots}} \boxed{-4.09\ldots}^2 & e^{t \boxed{-1.84\ldots}} \boxed{-1.84\ldots}^2 & e^{t \boxed{0.931\ldots}} \boxed{0.931\ldots}^2 \end{pmatrix}$$

Out[52]= $-e^{-5\,t}\left(\boxed{-4.09\ldots} - \boxed{-1.84\ldots}\right)\left(\boxed{-4.09\ldots} - \boxed{0.931\ldots}\right)\left(\boxed{-1.84\ldots} - \boxed{0.931\ldots}\right)$

Out[53]= $$-\frac{e^{t\left(5+\boxed{-1.84\ldots}+\boxed{0.931\ldots}\right)}\ \mathrm{Cosec}[4\,t]}{\left(-\boxed{-4.09\ldots}+\boxed{-1.84\ldots}\right)\left(\boxed{-4.09\ldots}-\boxed{0.931\ldots}\right)}$$

Out[54]= $$-\frac{\int e^{t\left(5+\boxed{-1.84\ldots}+\boxed{0.931\ldots}\right)}\ \mathrm{Cosec}[4\,t]\,dt}{\left(-\boxed{-4.09\ldots}+\boxed{-1.84\ldots}\right)\left(\boxed{-4.09\ldots}-\boxed{0.931\ldots}\right)}$$

Out[55]= $$\frac{e^{-t \boxed{-1.84\ldots}}\ \mathrm{Cosec}[4\,t]}{-2+\boxed{-1.84\ldots}^2+2\,\boxed{-4.09\ldots}\,\boxed{0.931\ldots}}$$

Out[56]= $$\frac{\int e^{-t \boxed{-1.84\ldots}}\ \mathrm{Cosec}[4\,t]\,dt}{-2+\boxed{-1.84\ldots}^2+2\,\boxed{-4.09\ldots}\,\boxed{0.931\ldots}}$$

Out[57]= $$\frac{e^{-t \boxed{0.931\ldots}}\ \mathrm{Cosec}[4\,t]}{\left(-\boxed{-4.09\ldots}+\boxed{0.931\ldots}\right)\left(-\boxed{-1.84\ldots}+\boxed{0.931\ldots}\right)}$$

Out[58]= $$\frac{\int e^{-t \boxed{0.931\ldots}}\ \mathrm{Cosec}[4\,t]\,dt}{\left(-\boxed{-4.09\ldots}+\boxed{0.931\ldots}\right)\left(-\boxed{-1.84\ldots}+\boxed{0.931\ldots}\right)}$$

Out[59]= $e^{t \boxed{-4.09\ldots}}\,\mathbb{C}_1 + e^{t \boxed{-1.84\ldots}}\,\mathbb{C}_2 + e^{t \boxed{0.931\ldots}}\,\mathbb{C}_3$

Out[60]=

$$\frac{e^{t\,\text{-4.09}\dots}\int e^{t\left(5+\text{-1.84}\dots+\text{0.931}\dots\right)}\text{Cosec}[4\,t]\,dt}{\left(\text{-4.09}\dots-\text{-1.84}\dots\right)\left(\text{-4.09}\dots-\text{0.931}\dots\right)}+\frac{e^{t\,\text{0.931}\dots}\int e^{-t\,\text{0.931}\dots}\text{Cosec}[4\,t]\,dt}{\left(-\text{-4.09}\dots+\text{0.931}\dots\right)\left(-\text{-1.84}\dots+\text{0.931}\dots\right)}+\frac{e^{t\,\text{-1.84}\dots}\int e^{-t\,\text{-1.84}\dots}\text{Cosec}[4\,t]\,dt}{-2+\text{-1.84}\dots^2+2\,\text{-4.09}\dots\,\text{0.931}\dots}$$

Out[61]=

$$e^{t\,\text{-4.09}\dots}c_1+e^{t\,\text{-1.84}\dots}c_2+e^{t\,\text{0.931}\dots}c_3+\frac{e^{t\,\text{-4.09}\dots}\int e^{t\left(5+\text{-1.84}\dots+\text{0.931}\dots\right)}\text{Cosec}[4\,t]\,dt}{\left(\text{-4.09}\dots-\text{-1.84}\dots\right)\left(\text{-4.09}\dots-\text{0.931}\dots\right)}+\frac{e^{t\,\text{0.931}\dots}\int e^{-t\,\text{0.931}\dots}\text{Cosec}[4\,t]\,dt}{\left(-\text{-4.09}\dots+\text{0.931}\dots\right)\left(-\text{-1.84}\dots+\text{0.931}\dots\right)}+\frac{e^{t\,\text{-1.84}\dots}\int e^{-t\,\text{-1.84}\dots}\text{Cosec}[4\,t]\,dt}{-2+\text{-1.84}\dots^2+2\,\text{-4.09}\dots\,\text{0.931}\dots}$$

Question 6: Find the solution for the system of ordinary differential equations.
dx/dt+ dy/dt-x=-2t   dx/dt+dy/dt-3x-y=t^2

Solution:

In[17]:=

```
DSolve[{x '[t] + y '[t] - x[t] == -2 t, x '[t] + y '[t] - 3 x[t] - y[t] == t ^ 2}, {x[t], y[t]}, t]
```

Out[17]=

$$\left\{\left\{x[t]\to -2\,t-t^2+\frac{1}{4}\times\left(4\times\left(-2+2\,t+t^2\right)-e^{-t}c_1\right),\ y[t]\to 2\,t+t^2+\frac{1}{2}\times\left(-4\times\left(-2+2\,t+t^2\right)+e^{-t}c_1\right)\right\}\right\}$$

Question 7: Obtain the solution of the linear equation ux - uy = 1,
with the Cauchy data u(x,0) = x^2.

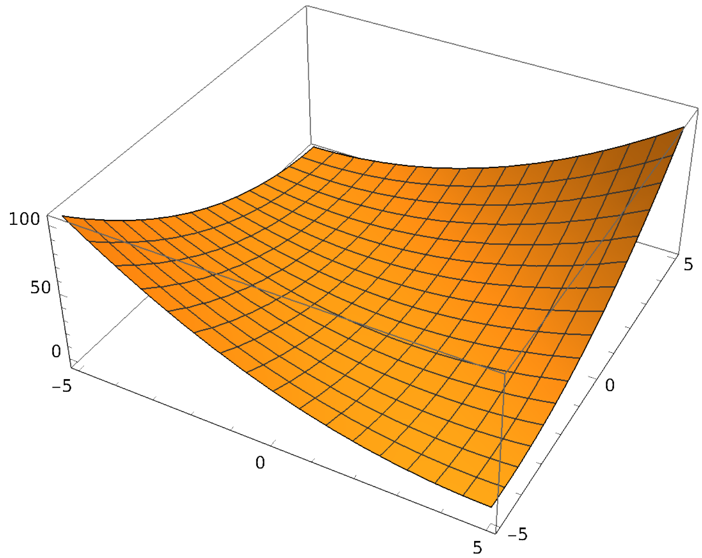
Solution:

In[18]:=
```
Eqn = D[u[x, y], x] - D[u[x, y], y] == 1
Sol = u[x, y] /. DSolve[{Eqn, u[x, 0] == x ^ 2}, u[x, y], {x, y}]
Plot3D[Sol, {x, -5, 5}, {y, -5, 5}]
```

Out[18]= $-u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == 1$

Out[19]= $\{x^2 - y + 2\,x\,y + y^2\}$

Out[20]=



Question 8: Find the characteristics of the equation and plot them.

(u-y)*ux+y*uy=x+y

dx/(u-y) = dy/y = du/(x+y)

Solution:

On taking I + III and II ,

we get (u+x) / y = C1

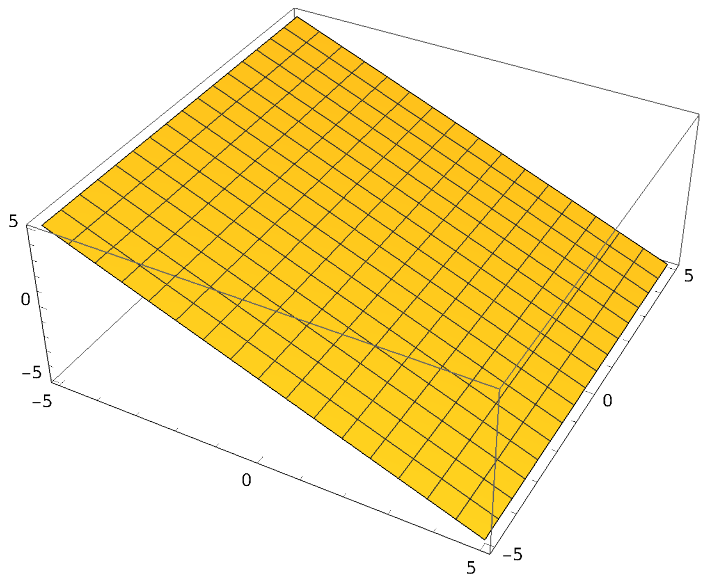
On taking I + II = III,

we get (x+y)^2 - u^2 = C2

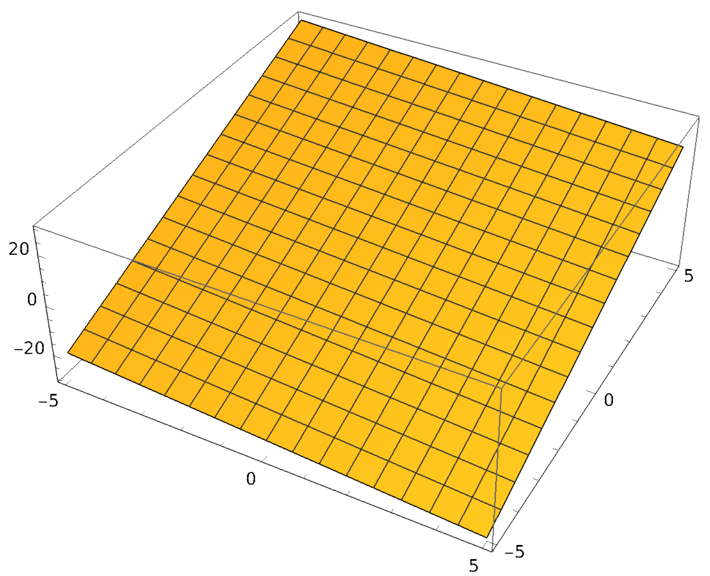
Now we Integrate to plot these for some particular values :

In[29]:=
```
F0 = Plot3D[-x, {x, -5, 5}, {y, -5, 5}, PlotPoints → 10]
F1 = Plot3D[5 * y - x, {x, -5, 5}, {y, -5, 5}, PlotPoints → 10]
F2 = Plot3D[10 * y - x, {x, -5, 5}, {y, -5, 5}, PlotPoints → 10]
G1 = Show[F0, F1, F2]
```

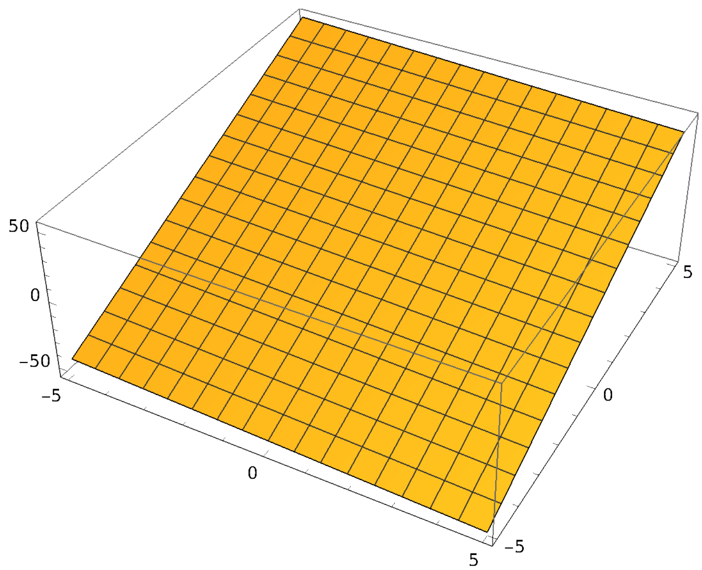
Out[29]=



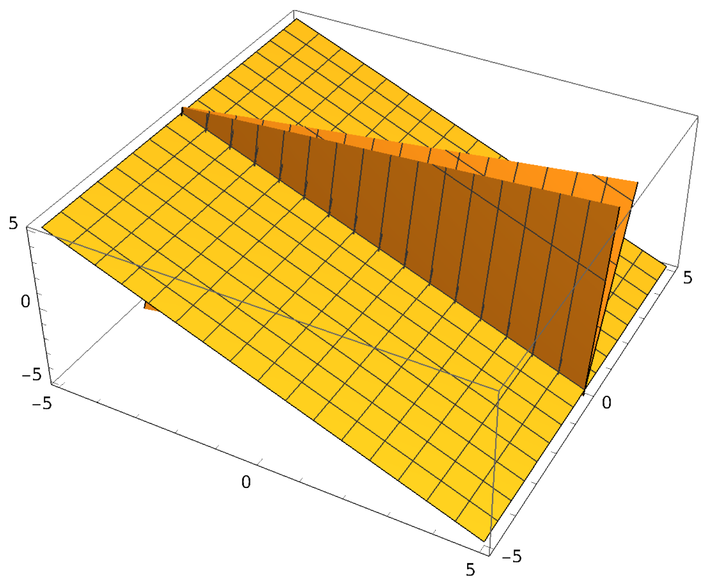
Out[30]=



Out[31]=



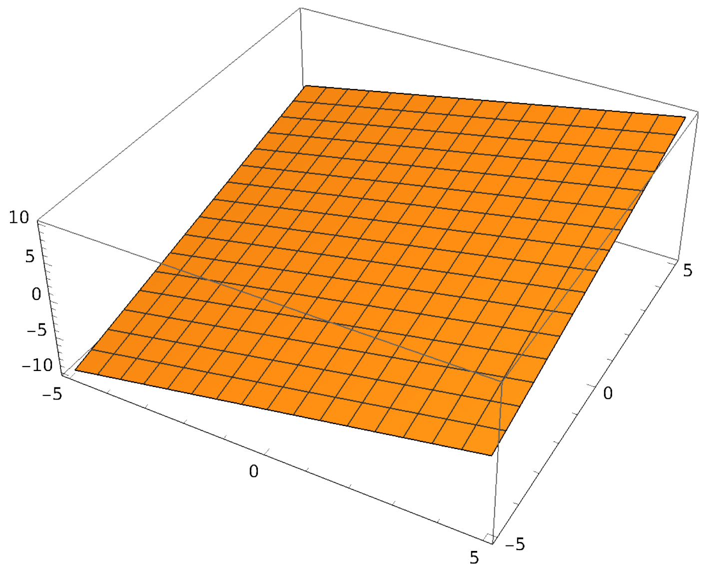
Out[32]=



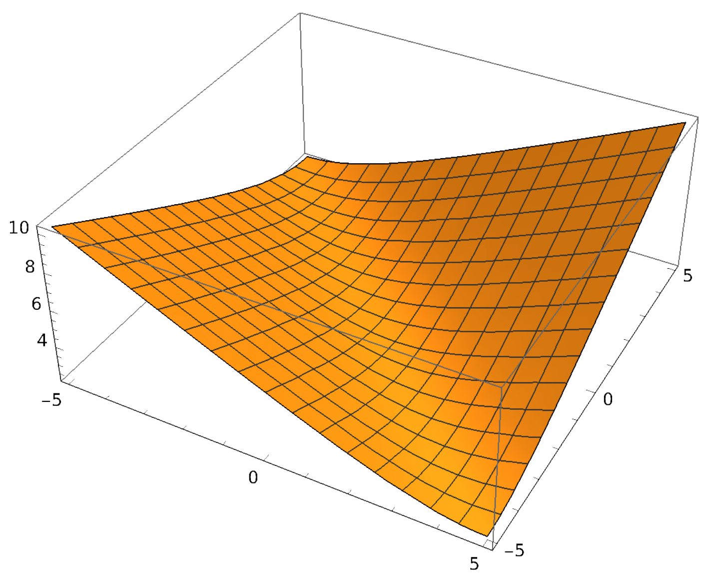
In[41]:=

```
H0 = Plot3D[x + y, {x, -5, 5}, {y, -5, 5}, PlotPoints → 10]
H1 = Plot3D[Sqrt[(x + y)^2 + 5], {x, -5, 5}, {y, -5, 5}, PlotPoints → 10]
H2 = Plot3D[Sqrt[(x + y)^2 + 10], {x, -5, 5}, {y, -5, 5}, PlotPoints → 10]
G2 = Show[H0, H1, H2]
```

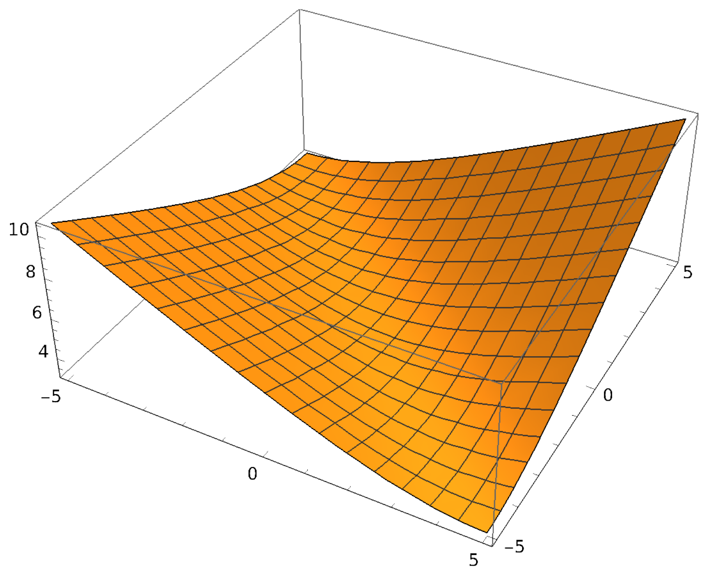
Out[41]=



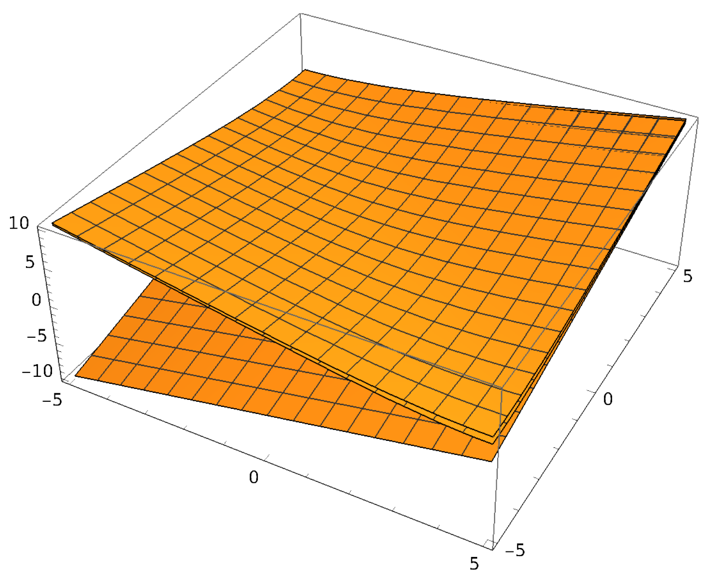
Out[42]=



Out[43]=



Out[44]=



In[45]:= **Show[GraphicsArray [{G1 , G2}]]**

Out[45]=