Practical-6

Solution of Cauchy Problem for first order PDE

Question 1: Obtain the solution of the linear equation ux + x*uy = 0, with the Cauchy data u(0,y) = Sin[y].

In[5]:=
```
Eqn = D[u[x, y], x] + x * D[u[x, y], y] == 0
```

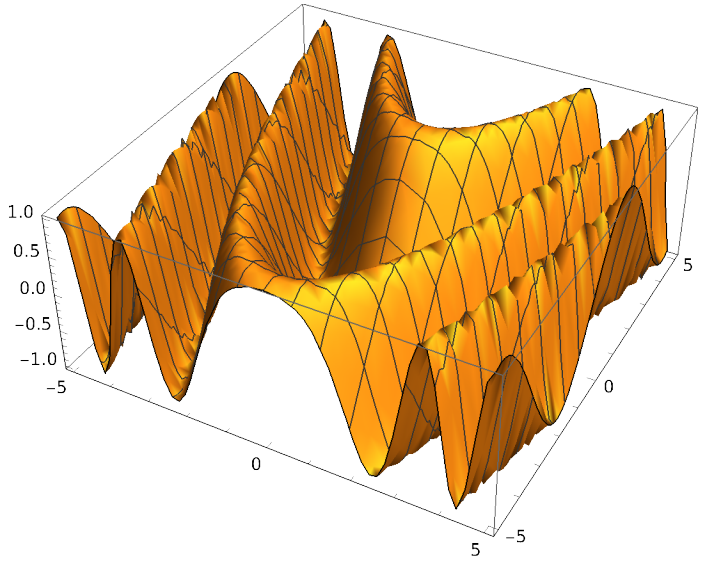
Out[5]=
$$x\, u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == 0$$

In[6]:=
```
Sol = u[x, y] /. DSolve[{Eqn, u[0, y] == Sin[y]}, u[x, y], {x, y}]
```

Out[6]=
$$\left\{-\text{Sin}\left[\frac{x^2}{2} - y\right]\right\}$$

In[7]:=
```
Plot3D[Sol, {x, -5, 5}, {y, -5, 5}]
```

Out[7]=



Question 2: Obtain the solution of the linear equation 3*ux + 2*uy = 0, with the Cauchy data u(x,0) = Sin[x].

In[8]:=
```
Eqn = 3 * D[u[x, y], x] + 2 * D[u[x, y], y] == 0
```

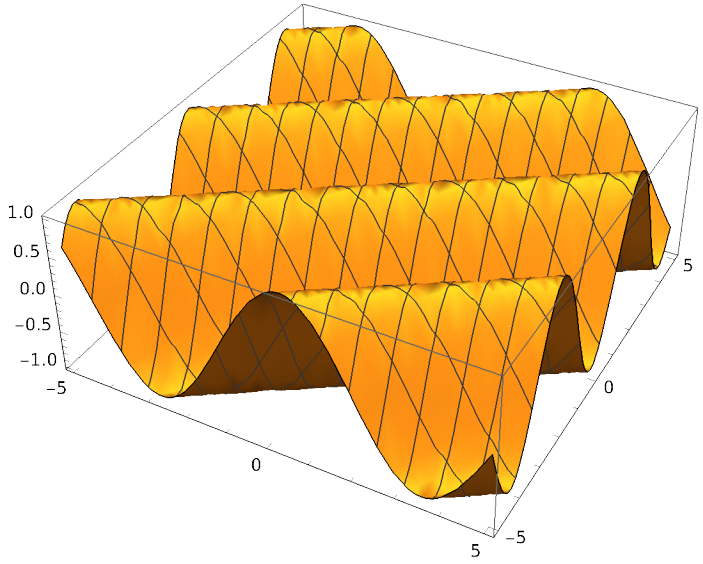
Out[8]=
$$2\, u^{(0,1)}[x, y] + 3\, u^{(1,0)}[x, y] == 0$$

In[9]:=
```
Sol = u[x, y] /. DSolve[{Eqn, u[x, 0] == Sin[x]}, u[x, y], {x, y}]
```

Out[9]=
$$\left\{-\text{Sin}\left[\frac{3}{2}\left(-\frac{2\,x}{3}+y\right)\right]\right\}$$

In[10]:=
```
Plot3D[Sol, {x, -5, 5}, {y, -5, 5}]
```

Out[10]=



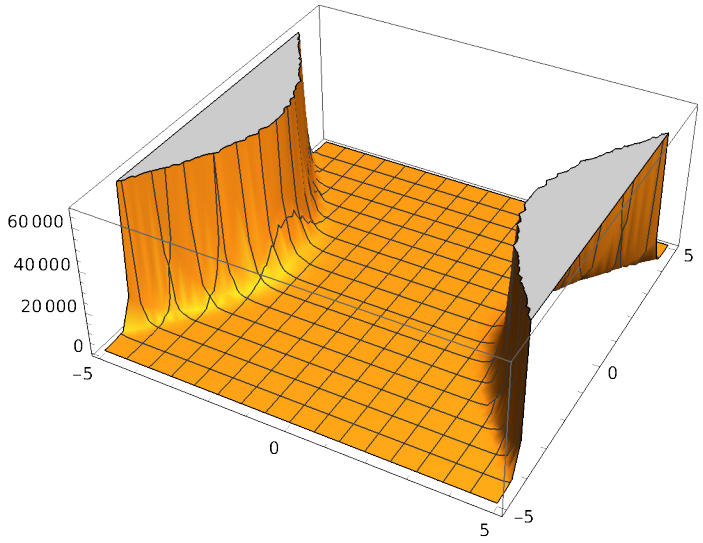
Question 3: Obtain the solution of the linear equation y*ux + x*uy = 0, with the Cauchy data u(0,y) = Exp[-y^2].

In[18]:=
```
Eqn = y * D[u[x, y], x] + x * D[u[x, y], y] == 0
Sol = u[x, y] /. DSolve[{Eqn, u[0, y] == Exp[-y ^ 2]}, u[x, y], {x, y}]
Plot3D[Sol, {x, -5, 5}, {y, -5, 5}]
```

Out[18]=
$$x\,u^{(0,1)}[x,\,y]+y\,u^{(1,0)}[x,\,y]==0$$

Out[19]=
$$\left\{e^{x^2-y^2}\right\}$$

Out[20]=



Question 4: ∂x * u[x,y] + 2* ∂y * u[x,y]= 1 + u[x,y]

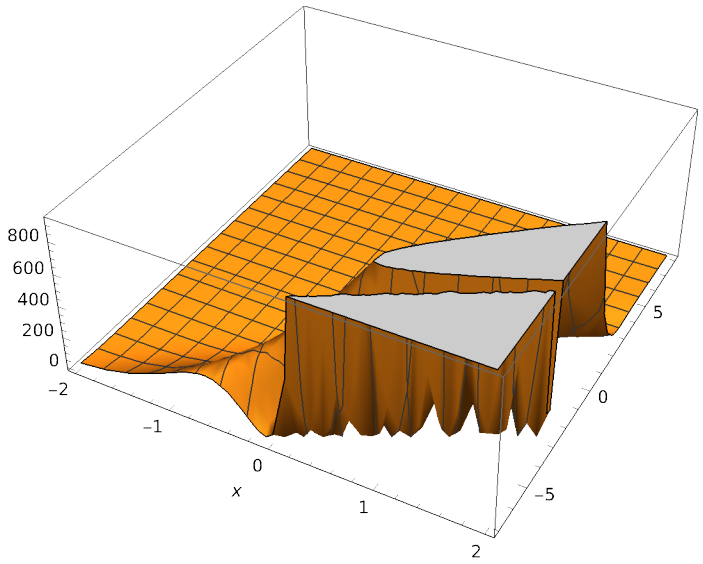
u[x,y] = Sin[x] on y = 3x +1

In[11]:=
```
Eqn = D[u[x, y], x] + 2 * D[u[x, y], y] == 1 + u[x, y]
Sol = DSolve[{Eqn, u[x, 3 * x + 1] == Sin[x]}, u[x, y], {x, y}]
Plot3D[u[x, y] /. Sol, {x, -2, 2}, {y, -7, 8}, AxesLabel → {Automatic}]
```

Out[11]=
$$2\,u^{(0,1)}[x,\,y] + u^{(1,0)}[x,\,y] == 1 + u[x,\,y]$$

Out[12]=
$$\{\{u[x,\,y] \to -e^{-y}\left(-e^{1+3\,x} + e^{y} + e^{1+3\,x}\,\text{Sin}[1 + 2\,x - y]\right)\}\}$$

Out[13]=



Question 5: Solve the PDE Subscript [u,x] + Subscript[u,y]=1/2 . With the initial condition u(s,s) = s/4, 0<=s<=1

Solution :

x = s+st/4 = (t^2)/4 ; y=s+t , u=s/4 + t/2

In[29]:=
```
Sol = DSolve[
   {x'[t] == u[t], y'[t] == 1, u'[t] == 1 / 2, x[0] == s, y[0] == s, u[0] == s / 4}, {x[t], y[t], u[t]}, t]
Print["u[t]=", Sol[[1, 1, 2]]]
Print["y[t]=", Sol[[1, 2, 2]]]
Print["x[t]=", Sol[[1, 3, 2]]]
map = ParametricPlot3D [
   {Sol[[1, 1, 2]], Sol[[1, 2, 2]], Sol[[1, 3, 2]]}, {t, -1, 1}, {s, 0, 1}, PlotPoints → 10]
```

Out[29]=
$$\left\{\left\{u[t] \to \frac{1}{4}(s + 2\,t),\; x[t] \to \frac{1}{4} \times (4\,s + s\,t + t^2),\; y[t] \to s + t\right\}\right\}$$

$$u[t]=\frac{1}{4}(s+2\,t)$$

$$y[t]=\frac{1}{4}\times(4\,s+s\,t+t^2)$$

$$x[t]=s+t$$

Out[33]=