Practical -3

Plotting of Third Order Differential Equations

Question 1: Solve third order differential equation d^3y/dx^3 -5(d^2y)/dx^2+8dy/dx-4y=0 and plot its any three solutions.

Solution :

In[45]:=
```
Sol = DSolve[y'''[x] - 5 y''[x] + 8 y'[x] - 4 y[x] == 0, y[x], x]
Sol1 = Evaluate[y[x] /. Sol[[1]] /. {C[1] → 1, C[2] → 0.5, C[3] → 2/3}]
Sol2 = y[x] /. Sol[[1]] /. {C[1] → -1/2, C[2] → 0, C[3] → 1}
Sol3 = y[x] /. Sol[[1]] /. {C[1] → -1, C[2] → -4, C[3] → 2}
Plot[{Sol1, Sol2, Sol3}, {x, -5, 3}, PlotRange → {-30, 30},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thick}, {Purple, Thickness[0.02]}},
 PlotLegends → {Sol1, Sol2, Sol3}]
```

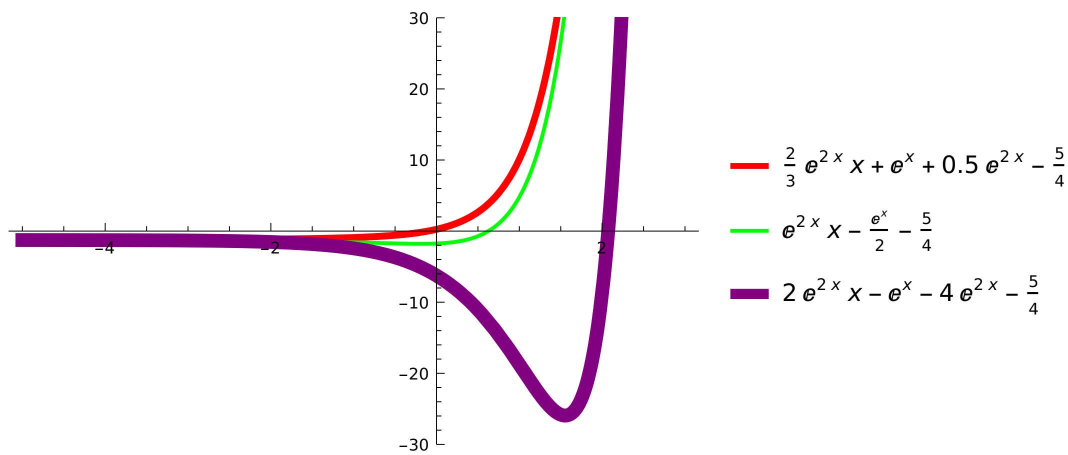
Out[45]= $\left\{\left\{y[x] \to -\frac{5}{4} + e^{x} c_1 + e^{2x} c_2 + e^{2x} x\, c_3\right\}\right\}$

Out[46]= $-\frac{5}{4} + e^{x} + 0.5\, e^{2x} + \frac{2}{3} e^{2x} x$

Out[47]= $-\frac{5}{4} - \frac{e^{x}}{2} + e^{2x} x$

Out[48]= $-\frac{5}{4} - e^{x} - 4 e^{2x} + 2 e^{2x} x$

Out[49]=



Question 2: Solve third order differential equation d^3y/dx^3 +3(d^2y)/dx^2 -25dy/dx +21y=0 and plot its any four solutions.

Solution :

In[1]:=
```
eqn = y'''[x] + 3 y''[x] - 25 y'[x] + 21 y[x];
Sol = DSolve[eqn == 0, y[x], x]
Sol1 = Evaluate[y[x] /. Sol[[1]] /. {C[1] → 1, C[2] → 0, C[3] → 2}]
Sol2 = y[x] /. Sol[[1]] /. {C[1] → -1 / 2, C[2] → 0, C[3] → 1}
Sol3 = y[x] /. Sol[[1]] /. {C[1] → -1, C[2] → -4, C[3] → 2}
Sol4 = y[x] /. Sol[[1]] /. {C[1] → -0.5, C[2] → -2, C[3] → 1}
Plot[{Sol1, Sol2, Sol3, Sol4}, {x, -0.5, 0.5},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thick}, {Purple, Thickness[0.02]},
   {Orange, Thickness[0.01]}}, PlotLegends → {Sol1, Sol2, Sol3, Sol4}]
```

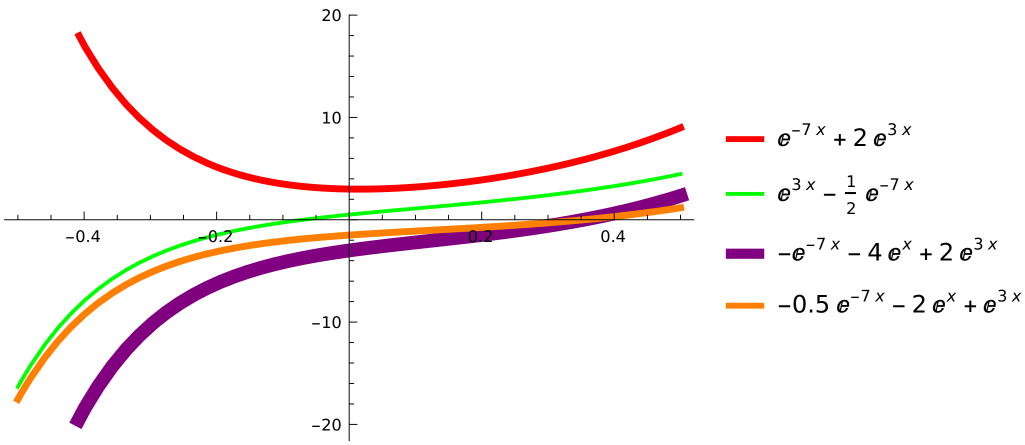
Out[2]= $\{\{y[x] \to e^{-7x} c_1 + e^{x} c_2 + e^{3x} c_3\}\}$

Out[3]= $e^{-7x} + 2 e^{3x}$

Out[4]= $-\frac{1}{2} e^{-7x} + e^{3x}$

Out[5]= $-e^{-7x} - 4 e^{x} + 2 e^{3x}$

Out[6]= $-0.5 e^{-7x} - 2 e^{x} + e^{3x}$

Out[7]=


Question 3: Solve third order differential equation d^3y/dx^3 -4(d^2y)/dx^2 -25dy/dx +28y=0 and plot its any four solutions.

Solution :

In[8]:=
```
eqn = y'''[x] - 4 y''[x] - 25 y'[x] + 28 y[x];
Sol = DSolve[eqn == 0, y[x], x]
Sol1 = Evaluate[y[x] /. Sol[[1]] /. {C[1] → 1, C[2] → 0, C[3] → 2}]
Sol2 = y[x] /. Sol[[1]] /. {C[1] → -2, C[2] → 10, C[3] → 3}
Sol3 = y[x] /. Sol[[1]] /. {C[1] → -1, C[2] → -4, C[3] → 20}
Sol4 = y[x] /. Sol[[1]] /. {C[1] → -0.5, C[2] → -2, C[3] → 1}
Plot[{Sol1, Sol2, Sol3, Sol4}, {x, -0.5, 0.5},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thick}, {Purple, Thickness[0.02]},
   {Orange, Thickness[0.01]}}, PlotLegends → {Sol1, Sol2, Sol3, Sol4}]
```

Out[9]= $\{\{y[x] \to e^{-4x} c_1 + e^{x} c_2 + e^{7x} c_3\}\}$

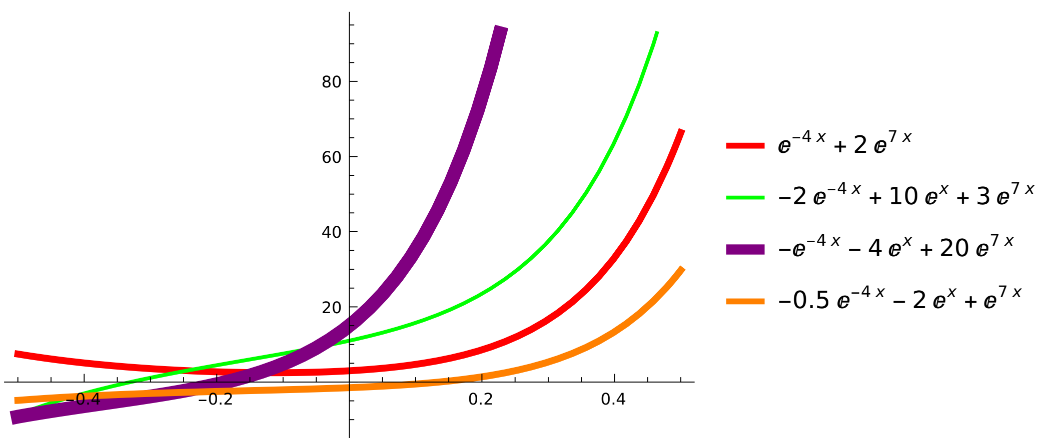
Out[10]= $e^{-4x} + 2 e^{7x}$

Out[11]= $-2 e^{-4x} + 10 e^{x} + 3 e^{7x}$

Out[12]= $-e^{-4x} - 4 e^{x} + 20 e^{7x}$

Out[13]= $-0.5 e^{-4x} - 2 e^{x} + e^{7x}$

Out[14]=



Question 4: Solve third order differential equation (d^3y)/dx^3 -13(d^2y)/dx^2 +19dy/dx +33y=cos(2x) and plot its any four solutions.

Solution :

In[13]:=
```
eqn = y'''[x] - 13 y''[x] + 19 y'[x] + 33 y[x];
Sol = DSolve[eqn == Cos[2 x], y[x], x]
Sol1 = Evaluate[y[x] /. Sol⟦1⟧ /. {C[1] → 1, C[2] → 0, C[3] → 2}]
Sol2 = y[x] /. Sol⟦1⟧ /. {C[1] → 10, C[2] → 3, C[3] → 6}
Sol3 = y[x] /. Sol⟦1⟧ /. {C[1] → -1, C[2] → -7, C[3] → 0.7}
Sol4 = y[x] /. Sol⟦1⟧ /. {C[1] → -10.5, C[2] → 2, C[3] → 1}
Plot[{Sol1, Sol2, Sol3, Sol4}, {x, 4, 6},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thick}, {Purple, Thickness[0.02]},
   {Orange, Thickness[0.01]}}, PlotLegends → {Sol1, Sol2, Sol3, Sol4}]
```

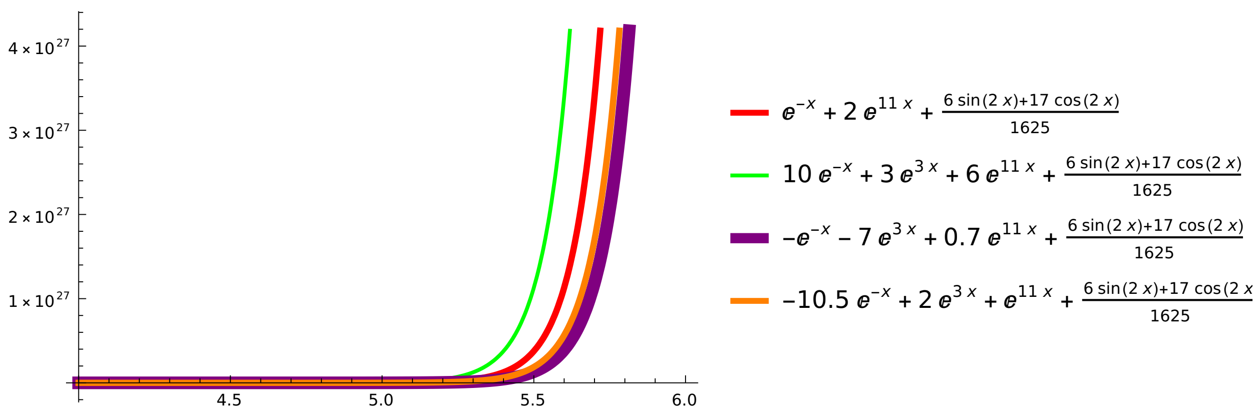
Out[14]= $$\left\{\left\{y[x] \to e^{-x}\, c_1 + e^{3x}\, c_2 + e^{11x}\, c_3 + \frac{17\,\mathrm{Cos}[2x] + 6\,\mathrm{Sin}[2x]}{1625}\right\}\right\}$$

Out[15]= $$e^{-x} + 2\,e^{11x} + \frac{17\,\mathrm{Cos}[2x] + 6\,\mathrm{Sin}[2x]}{1625}$$

Out[16]= $$10\,e^{-x} + 3\,e^{3x} + 6\,e^{11x} + \frac{17\,\mathrm{Cos}[2x] + 6\,\mathrm{Sin}[2x]}{1625}$$

Out[17]= $$-e^{-x} - 7\,e^{3x} + 0.7\,e^{11x} + \frac{17\,\mathrm{Cos}[2x] + 6\,\mathrm{Sin}[2x]}{1625}$$

Out[18]= $$-10.5\,e^{-x} + 2\,e^{3x} + e^{11x} + \frac{17\,\mathrm{Cos}[2x] + 6\,\mathrm{Sin}[2x]}{1625}$$

Out[19]=


Question 5: Solve third order differential equation x^3(d^3y)/dx^3 +3x^2(d^2y)/dx^2 +xdy/dx +y=0 and plot its any three solutions.

Solution :

In[7]:=
```
eqn = (x ^ 3) (y '''[x]) + (3 x ^ 2) * (y ''[x]) + x * y '[x] + y[x];
Sol = DSolve[eqn == 0, y[x], x]
Sol1 = Evaluate[y[x] /. Sol[[1]] /. {C[1] → 1, C[2] → 0.5, C[3] → 2 / 3}]
Sol2 = y[x] /. Sol[[1]] /. {C[1] → -1 / 2, C[2] → 0, C[3] → 1}
Sol3 = y[x] /. Sol[[1]] /. {C[1] → -1, C[2] → -4, C[3] → 2}
Plot[{Sol1, Sol2, Sol3}, {x, -5, 3}, PlotRange → {-30, 30},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thick}, {Purple, Thickness[0.02]}},
 PlotLegends → {Sol1, Sol2, Sol3}]
```

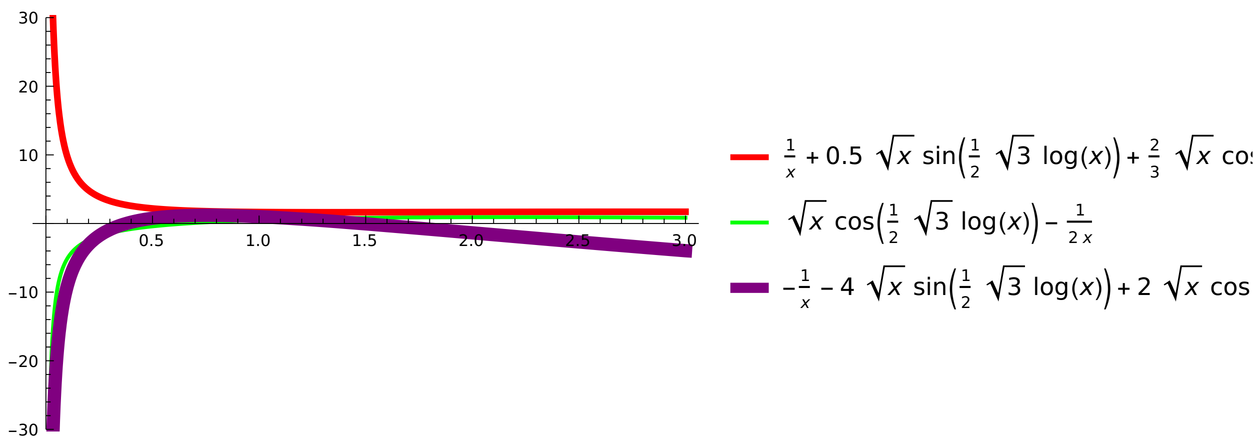
Out[8]= $\left\{\left\{y[x] \to \frac{c_1}{x} + \sqrt{x}\, c_3 \operatorname{Cos}\left[\frac{1}{2}\sqrt{3}\operatorname{Log}[x]\right] + \sqrt{x}\, c_2 \operatorname{Sin}\left[\frac{1}{2}\sqrt{3}\operatorname{Log}[x]\right]\right\}\right\}$

Out[9]= $\frac{1}{x} + \frac{2}{3}\sqrt{x}\operatorname{Cos}\left[\frac{1}{2}\sqrt{3}\operatorname{Log}[x]\right] + 0.5\sqrt{x}\operatorname{Sin}\left[\frac{1}{2}\sqrt{3}\operatorname{Log}[x]\right]$

Out[10]= $-\frac{1}{2x} + \sqrt{x}\operatorname{Cos}\left[\frac{1}{2}\sqrt{3}\operatorname{Log}[x]\right]$

Out[11]= $-\frac{1}{x} + 2\sqrt{x}\operatorname{Cos}\left[\frac{1}{2}\sqrt{3}\operatorname{Log}[x]\right] - 4\sqrt{x}\operatorname{Sin}\left[\frac{1}{2}\sqrt{3}\operatorname{Log}[x]\right]$

Out[12]=



Question 6: Solve third order differential equation x^3(d^3y)/dx^3 -4x^2(d^2y)/dx^2 +8xdy/dx -8y=4 logx and plot its any three solutions.

Solution :

In[26]:=

```
eqn = (x^3) y'''[x] + (-4 x^2) y''[x] + 8 x * y'[x] - 8 y[x];
Sol = DSolve[eqn == 4 Log[x], y[x], x]
Sol1 = Evaluate[y[x] /. Sol⟦1⟧ /. {C[1] → 1, C[2] → 0.5, C[3] → 2/3}]
Sol2 = y[x] /. Sol⟦1⟧ /. {C[1] → -1/2, C[2] → 0, C[3] → 1}
Sol3 = y[x] /. Sol⟦1⟧ /. {C[1] → -1, C[2] → -4, C[3] → 2}
Plot[{Sol1, Sol2, Sol3}, {x, -5, 3}, PlotRange → {-30, 30},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thick}, {Purple, Thickness[0.02]}},
 PlotLegends → {Sol1, Sol2, Sol3}]
```

Out[27]= $\left\{\left\{y[x] \to x\, c_1 + x^2\, c_2 + x^4\, c_3 + \frac{1}{8} \times (-7 - 4\,\mathrm{Log}[x])\right\}\right\}$

Out[28]= $x + 0.5\, x^2 + \frac{2\, x^4}{3} + \frac{1}{8} \times (-7 - 4\,\mathrm{Log}[x])$

Out[29]= $-\frac{x}{2} + x^4 + \frac{1}{8} \times (-7 - 4\,\mathrm{Log}[x])$

Out[30]= $-x - 4\, x^2 + 2\, x^4 + \frac{1}{8} \times (-7 - 4\,\mathrm{Log}[x])$

Out[31]=