# Rajkumar Choudhury

# 8751

# 19055582140

# BSc(Prog.)Computer Science

**Question 1: Solve first order differential equation: dy/dx + 3y=e^x and plot its solutions for (c1=3,c2=-3; c1=1,c2=-6; c1=3,c2=4)**

**Solution:**

In[26]:=
```
sol = DSolve[y'[x] + 3 y[x] == Exp[x], y[x], x]
sol1 = Evaluate[y[x] /. sol⟦1⟧ /. {C[1] → 3 , C[2] → -3}]
sol2 = y[x] /. sol⟦1⟧ /. {C[1] → 1 , C[2] → -6}
sol3 = y[x] /. sol⟦1⟧ /. {C[1] → 3, C[2] → 4}
Plot[{sol1, sol2, sol3}, {x, -5, 3}, PlotRange → {-30, 30},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thick}}, PlotLegends → {sol1, sol2, sol3}]
```

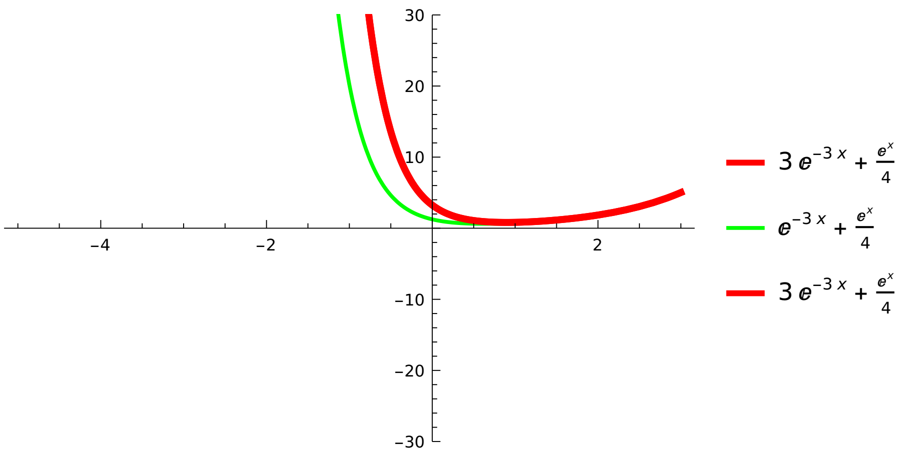
Out[26]= $\left\{\left\{y[x] \to \frac{e^{x}}{4} + e^{-3x}\, c_1\right\}\right\}$

Out[27]= $3 e^{-3x} + \frac{e^{x}}{4}$

Out[28]= $e^{-3x} + \frac{e^{x}}{4}$

Out[29]= $3 e^{-3x} + \frac{e^{x}}{4}$

Out[30]=




**Question 2: Solve third order differential equation: d^3y/dx^3 - 9d^2y/dx^2 +4y= sin(2x) and plot its two solutions for (c1=3,c2=1,c3=6;  c1=8,c2=9,c3=7)**

**Solution:**

In[31]:=
```
sol = DSolve[y'''[x] - 9 y''[x] + 4 y[x] == Sin[2 x], y[x], x]
sol1 = Evaluate[y[x] /. sol⟦1⟧ /. {C[1] → 3 , C[2] → 1, C[3] → 6}]
sol2 = y[x] /. sol⟦1⟧ /. {C[1] → 8 , C[2] → 9, C[3] → 7}
Plot[{sol1, sol2}, {x, -5, 3}, PlotRange → {-30, 30},
 PlotStyle → {{Red, Thickness[0.01]}, {Green, Thick}}, PlotLegends → {sol1, sol2}]
```

Out[31]=

$$\left\{\left\{y[x] \to e^{x\,(-0.644\,\ldots)}\,c_1 + e^{x\,(0.694\,\ldots)}\,c_2 + e^{x\,(8.95\,\ldots)}\,c_3 + \frac{\cdots 1 \cdots}{\left(-(-0.644\,\ldots)+(0.694\,\ldots)\right)\left(4+(0.694\,\ldots)^2\right)\cdots 4 \cdots\left((0.694\,\ldots)^2+2(-0.644\,\ldots)(8.95\,\ldots)\right)\left(4+(8.95\,\ldots)^2\right)}\right\}\right\}$$

large output | show less | show more | show all | set size limit ...

Out[32]=

$$3\,e^{x\,(-0.644\,\ldots)} + e^{x\,(0.694\,\ldots)} + 6\,e^{x\,(8.95\,\ldots)} + \left(\cdots 231 \cdots + \cdots 1 \cdots - (0.694\,\ldots)^2\,(8.95\,\ldots)^6\,\mathrm{Sin}[2\,x]\right) \Big/ \left(\left(-(-0.644\,\ldots)+(0.694\,\ldots)\right)\left(4+(0.694\,\ldots)^2\right)\left((-0.644\,\ldots)-(8.95\,\ldots)\right)\cdots 3 \cdots\left((0.694\,\ldots)^2+2\,(-0.644\,\ldots)\,(8.95\,\ldots)\right)\left(4+(8.95\,\ldots)^2\right)\right)$$

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Out[33]=

$$8\,e^{x\,(-0.644\,\ldots)} + 9\,e^{x\,(0.694\,\ldots)} + 7\,e^{x\,(8.95\,\ldots)} + \left(\cdots 231 \cdots + \cdots 1 \cdots - (0.694\,\ldots)^2\,(8.95\,\ldots)^6\,\mathrm{Sin}[2\,x]\right) \Big/ \left(\left(-(-0.644\,\ldots)+(0.694\,\ldots)\right)\left(4+(0.694\,\ldots)^2\right)\left((-0.644\,\ldots)-(8.95\,\ldots)\right)\cdots 3 \cdots\left((0.694\,\ldots)^2+2\,(-0.644\,\ldots)\,(8.95\,\ldots)\right)\left(4+(8.95\,\ldots)^2\right)\right)$$

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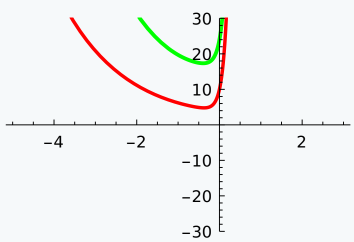
Out[34]=




$$-2\,(0.694\,\ldots)\,(8.95\,\ldots)^6\cos(2\,x)+2\,(-0.644\,\ldots)\,(8.95\,\ldots)^6\cos(2\,x)-2\,(0.694\,\ldots)^2$$

$$-2\,(0.694\,\ldots)\,(8.95\,\ldots)^6\cos(2\,x)+2\,(-0.644\,\ldots)\,(8.95\,\ldots)^6\cos(2\,x)-2\,(0.694\,\ldots)^2$$

large output | show less | show more | show all | set size limit ...

**Question 3: Find the solution for the system of ordinary differential equations.**

**dx/dt+ dy/dt -2x -4y=e^t      dx/dt+dy/dt-y=e^4t**

## Solution:

In[46]:=
```
DSolve[{x '[t] + y '[t] - 2 x[t] - 4 y[t] == Exp[t], x '[t] + y '[t] - y[t] == Exp[4 t]}, {x[t], y[t]}, t]
```

Out[46]=

$$\left\{\left\{x[t] \to -e^{t}\left(-1+e^{3t}\right)+\frac{1}{3}\times\left(3e^{-2t}\left(-e^{3t}+e^{6t}\right)+e^{-2t}c_1\right),\right.\right.$$
$$\left.\left.y[t] \to e^{t}\left(-1+e^{3t}\right)-\frac{2}{9}\times\left(3e^{-2t}\left(-e^{3t}+e^{6t}\right)+e^{-2t}c_1\right)\right\}\right\}$$

## Question 4: Solve second order differential equation: d^2y/dt^2+6y=Cot(t) by variation parameter method.

## Solution:

In[35]:=
```
sol = DSolve[y''[t] + 6 y[t] == 0, y[t], t]
sol1 = Evaluate[y[t] /. sol[[1]] /. {C[1] → 1, C[2] → 0}]
sol2 = y[t] /. sol[[1]] /. {C[1] → 0, C[2] → 1}
fs = {sol1, sol2}
wm = {fs, D[fs, t]}; wm // MatrixForm

wd = Simplify[Det[wm]]

u1 = (Integrate[- sol2 Cot[t], t]) / wd
u2 = (Integrate[sol1 Cot[t], t]) / wd

yc = DSolve[y''[t] + 6 y[t] == 0, y[t], t]
yp = Simplify[Evaluate[y[t] /. sol[[1]] /. {C[1] → u1, C[2] → u2}]]
yg = yc + yp
```

Out[35]= $\left\{\left\{y[t] \to c_1 \operatorname{Cos}\left[\sqrt{6}\, t\right] + c_2 \operatorname{Sin}\left[\sqrt{6}\, t\right]\right\}\right\}$

Out[36]= $\operatorname{Cos}\left[\sqrt{6}\, t\right]$

Out[37]= $\operatorname{Sin}\left[\sqrt{6}\, t\right]$

Out[38]= $\left\{\operatorname{Cos}\left[\sqrt{6}\, t\right], \operatorname{Sin}\left[\sqrt{6}\, t\right]\right\}$

Out[39]//MatrixForm=

$$\begin{pmatrix} \operatorname{Cos}\left[\sqrt{6}\, t\right] & \operatorname{Sin}\left[\sqrt{6}\, t\right] \\ -\sqrt{6}\operatorname{Sin}\left[\sqrt{6}\, t\right] & \sqrt{6}\operatorname{Cos}\left[\sqrt{6}\, t\right] \end{pmatrix}$$

Out[40]= $\sqrt{6}$

Out[41]=

$$-\frac{1}{12\sqrt{6}} i e^{-i\sqrt{6}t}\left(\sqrt{6}\,\text{Hypergeometric2F1}\left[1,-\sqrt{\frac{3}{2}},1-\sqrt{\frac{3}{2}},e^{2it}\right]+\sqrt{6}\,e^{2i\sqrt{6}t}\,\text{Hypergeometric2F1}\left[1,\sqrt{\frac{3}{2}},1+\sqrt{\frac{3}{2}},e^{2it}\right]+3e^{2it}\left(\left(2+\sqrt{6}\right)\text{Hypergeometric2F1}\left[1,1-\sqrt{\frac{3}{2}},2-\sqrt{\frac{3}{2}},e^{2it}\right]+\left(-2+\sqrt{6}\right)e^{2i\sqrt{6}t}\,\text{Hypergeometric2F1}\left[1,1+\sqrt{\frac{3}{2}},2+\sqrt{\frac{3}{2}},e^{2it}\right]\right)\right)$$

Out[42]=

$$\frac{1}{12\sqrt{6}} e^{-i\sqrt{6}t}\left(\sqrt{6}\,\text{Hypergeometric2F1}\left[1,-\sqrt{\frac{3}{2}},1-\sqrt{\frac{3}{2}},e^{2it}\right]-\sqrt{6}\,e^{2i\sqrt{6}t}\,\text{Hypergeometric2F1}\left[1,\sqrt{\frac{3}{2}},1+\sqrt{\frac{3}{2}},e^{2it}\right]+3e^{2it}\left(\left(2+\sqrt{6}\right)\text{Hypergeometric2F1}\left[1,1-\sqrt{\frac{3}{2}},2-\sqrt{\frac{3}{2}},e^{2it}\right]-\left(-2+\sqrt{6}\right)e^{2i\sqrt{6}t}\,\text{Hypergeometric2F1}\left[1,1+\sqrt{\frac{3}{2}},2+\sqrt{\frac{3}{2}},e^{2it}\right]\right)\right)$$

Out[43]=

$$\left\{\left\{y[t]\to c_1\,\text{Cos}\left[\sqrt{6}\,t\right]+c_2\,\text{Sin}\left[\sqrt{6}\,t\right]\right\}\right\}$$

Out[44]=

$$\frac{1}{12\sqrt{6}} e^{-i\sqrt{6}t}\left(\sqrt{6}\,\text{Hypergeometric2F1}\left[1,-\sqrt{\frac{3}{2}},1-\sqrt{\frac{3}{2}},e^{2it}\right]+\sqrt{6}\,\text{Hypergeometric2F1}\left[1,\sqrt{\frac{3}{2}},1+\sqrt{\frac{3}{2}},e^{2it}\right]+3e^{2it}\left(\left(2+\sqrt{6}\right)\text{Hypergeometric2F1}\left[1,1-\sqrt{\frac{3}{2}},2-\sqrt{\frac{3}{2}},e^{2it}\right]+\left(-2+\sqrt{6}\right)\text{Hypergeometric2F1}\left[1,1+\sqrt{\frac{3}{2}},2+\sqrt{\frac{3}{2}},e^{2it}\right]\right)\right)\left(-i\,\text{Cos}\left[\sqrt{6}\,t\right]+\text{Sin}\left[\sqrt{6}\,t\right]\right)$$

Out[45]=

$$\left\{\left\{\left(y[t] \to c_1 \operatorname{Cos}\left[\sqrt{6}\ t\right] + c_2 \operatorname{Sin}\left[\sqrt{6}\ t\right]\right) + \frac{1}{12\sqrt{6}} e^{-i\sqrt{6}\ t} \left(\sqrt{6}\ \text{Hypergeometric2F1}\left[1, -\sqrt{\frac{3}{2}}, 1-\sqrt{\frac{3}{2}}, e^{2it}\right] + \sqrt{6}\ \text{Hypergeometric2F1}\left[1, \sqrt{\frac{3}{2}}, 1+\sqrt{\frac{3}{2}}, e^{2it}\right] + 3 e^{2it}\left(\left(2+\sqrt{6}\right) \text{Hypergeometric2F1}\left[1, 1-\sqrt{\frac{3}{2}}, 2-\sqrt{\frac{3}{2}}, e^{2it}\right] + \left(-2+\sqrt{6}\right) \text{Hypergeometric2F1}\left[1, 1+\sqrt{\frac{3}{2}}, 2+\sqrt{\frac{3}{2}}, e^{2it}\right]\right)\right)\left(-i \operatorname{Cos}\left[\sqrt{6}\ t\right] + \operatorname{Sin}\left[\sqrt{6}\ t\right]\right)\right\}\right\}$$

**Question 5: Obtain the solution of the linear equation y*ux - 2xy*uy = 2xu, with the Cauchy data u(0,y) = y^3.**

**Solution:**