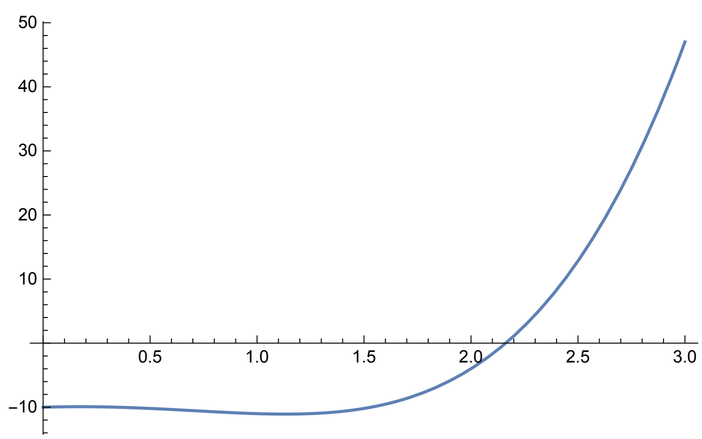
ASSIGNMENT

Q1 . Find an interval of unit length that Contains the smallest positive root of the function $f(x) = x^4 - 3x^2 + x - 10$ perform 12 iterations of Bisection Method starting with resulted interval.

In[5]:=
```
Bisection[a0_, b0_, n_, f_] :=
 Module [{a = N[a0], b = N[b0]},
  c = (a + b) / 2;
  i = 0
    If[f[a] * f[b] > 0,
     Print["we cannot continue with the bisection method as ",
      f, "(a).", f, "(b)>=0"];
     Return[]];
  output = {{i, a, c, b, f[c]}};
  While[i < n, If[Sign[f[b]] == Sign[f[c]], b = c, a = c;];
   c = (a + b) / 2;
   i = i + 1; output = Append[output, {i, a, c, b, f[c]}];];
  Print[NumberForm[TableForm[output,
     TableHeadings → {None, {"i", "aᵢ", "cᵢ", "bᵢ", "f[cᵢ]"}}], 16]];
  Print["Approximate root after ", i, " iterations is ",
   NumberForm[c, 16]];
  Print["Function value at approximated root,f[c]=",
   NumberForm[f[c], 16]];]
f[x_] = x^4 - 3 x^2 + x - 10;
Plot[f[x], {x, 0, 3}]
```

Out[7]=



we can choose the interval of unit length [2, 3] as the smallest positive root around 2.5, belongs to it

In[8]:=
```
Bisection[2, 3, 12, f];
```

i	a_i	c_i	b_i	$f[c_i]$
0	2.	2.5	3.	12.8125
1	2.	2.25	2.5	2.69140625
2	2.	2.125	2.25	−1.031005859375
3	2.125	2.1875	2.25	0.7297515869140625
4	2.125	2.15625	2.1875	−0.1749410629272461
5	2.15625	2.171875	2.1875	0.2712278962135315
6	2.15625	2.1640625	2.171875	0.0466114915907383
7	2.15625	2.16015625	2.1640625	−0.06454621977172792
8	2.16015625	2.162109375	2.1640625	−0.00906291579303797
9	2.162109375	2.1630859375	2.1640625	0.01875037580703065
10	2.162109375	2.16259765625	2.1630859375	0.004837755005667077
11	2.162109375	2.162353515625	2.16259765625	−0.002114073766389168
12	2.162353515625	2.1624755859375	2.16259765625	0.001361467229262558

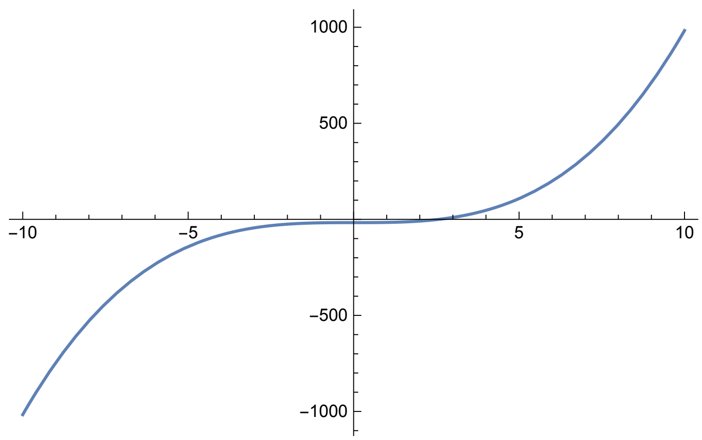
Approximate root after 12 iterations is 2.1624755859375

Function value at approximated root,f[c]=0.001361467229262558

Q2. Perform 4 iterations of the Newton Raphson Method to obtain approximate value of $(17)^{1/3}$ starting with the initial approximation x0 = 2.

In[9]:=
```
NewtonRaphson[x0_, n_, f_] :=
  Module[{xk1, xk = N[x0]}, k = 0; Output = {{k, x0, f[x0]}};
   While[k < n, fPrimexk = f '[xk];
    If[fPrimexk == 0, Print["The derivative of function at ",
      k, "th iteration is zero,we can not proceed
        further with the iterative scheme"];
     Break[]];
    xk1 = xk - f[xk] / fPrimexk;
    xk = xk1;
    k ++;
    Output = Append[Output, {k, xk, f[xk]}];];
   Print[NumberForm[TableForm[Output,
      TableHeadings → {None, {"k", "xk", "f[xk]"}}], 10]];
   Print["Root after ", n, " iterations xk = ",
    NumberForm[xk, 10]];
   Print["Function value at approximated root,f[xk] = ",
    NumberForm[f[xk], 10]];];
f[x_] = x^3 - 17;
Plot[f[x], {x, -10, 10}]
NewtonRaphson[2, 4, f]
```

Out[11]=



k	xk	f[xk]
0	2	−9
1	2.75	3.796875
2	2.582644628	0.2263772599
3	2.571331512	0.0009901837441
4	2.571281592	$1.922353121 \times 10^{-8}$

Root after 4 iterations xk = 2.571281592

Function value at approximated root,f[xk] = $1.922353121 \times 10^{-8}$

Q3. Solve the system of equations

$$4x_1 + x_2 + x_3 = 2$$
$$x_1 + 5x_2 + 2x_3 = -6$$
$$x_1 + 2x_2 + 3x_3 = -4$$

with the inital vector $x^{(0)} = (0.5, -0.5, -0.5)$. Perform 15 iterations.

```
In[13]:= GaussJacobi[A0_, B0_, X0_, max_] :=
  Module[{A = N[A0], B = N[B0], i, j, k = 0, n = Length[X0], X = X0, Xk = X0},
   Print["X"_0, "=", X];
   While[k < max,
    For[ i = 1, i ≤ n, i++,
     X[[i]] = 1/A[[i,i]] (B[[i]] + A[[i,i]] Xk[[i]] - Sum[A[[i,j]] Xk[[j]], {j, 1, n}])];

    Print["X"_(k+1), "=", X];
    Xk = X;
    k = k + 1;];
   Print[" No. of iterarations performed ", max];
   Return[X];];
A = {{4, 1, 1}, {1, 5, 2}, {1, 2, 3}};
B = {2, -6, -4};
X0 = {0.5, -0.5, -0.5};
GaussJacobi[A, B, X0, 15]
```

$X_{0}=\{0.5, -0.5, -0.5\}$

$X_{1}=\{0.75, -1.1, -1.16667\}$

$X_{2}=\{1.06667, -0.883333, -0.85\}$

$X_{3}=\{0.933333, -1.07333, -1.1\}$

$X_{4}=\{1.04333, -0.946667, -0.928889\}$

$X_{5}=\{0.968889, -1.03711, -1.05\}$

$X_{6}=\{1.02178, -0.973778, -0.964889\}$

$X_{7}=\{0.984667, -1.0184, -1.02474\}$

$X_{8}=\{1.01079, -0.987037, -0.982622\}$

$X_{9}=\{0.992415, -1.00911, -1.01224\}$

$X_{10}=\{1.00534, -0.993588, -0.9914\}$

$X_{11}=\{0.996247, -1.00451, -1.00605\}$

$X_{12}=\{1.00264, -0.996828, -0.995744\}$

$X_{13}=\{0.998143, -1.00223, -1.00299\}$

$X_{14}=\{1.00131, -0.998431, -0.997894\}$

$X_{15}=\{0.999081, -1.0011, -1.00148\}$

No. of iterarations performed 15

Out[17]= {0.999081, -1.0011, -1.00148}