Practical 14
Second Order Runge Kutta Method

(1) Using Runge – Kutta Method of second order, find approximate solution fo r th e initial value problem $x'(t) = 1 + \frac{x}{t}$ $1 \le t \le 6$, $x(t) = 1$. Us e n = 5 discrete points a t equal space. Compare the solution with the known analytic solution $x(t) = t(1 + \ln(t))$.

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RungeKutta2ndOrder[a0_, b0_, n0_, f_, alpha_, actualSolution_] :=
  Module[{a = a0, b = b0, n = n0, h, ti, k1, k2, k3, k4},
           h = (b - a) / n;
   ti = Table[a + (j - 1) h, {j, 1, n + 1}];
   wi = Table[0, {n + 1}]; wi[[1]] = alpha;
   actualSol = actualSolution[ti[[1]]];
   difference = Abs[actualSol - wi[[1]]];
   OutputDetails = {{0, ti[[1]], alpha, actualSol, difference}};
   For[i = 1, i ≤ n, i++,
    k1 = h f[ti[[i]], wi[[i]]];
    k2 = h f[ti[[i]] + h / 2, wi[[i]] + k1 / 2];
    wi[[i + 1]] = wi[[i]] + k2;
    actualSol = actualSolution[ti[[i + 1]]];
    difference = Abs[actualSol - wi[[i + 1]]];
    OutputDetails = Append[OutputDetails,
      {i, N[ti[[i + 1]]], N[wi[[i + 1]]], N[actualSol], N[difference]}];];
   Print[NumberForm[TableForm[OutputDetails, TableHeadings →
      {None, {"i", "ti", "wi", "actSol(ti)", "Abs(wi-actSol(ti))"}}], 6]];];
f[t_, x_] := 1 + x/t
actualSolution[t_] := t (1 + Log[t]);
RungeKutta2ndOrder[1, 6, 5, f, 1, actualSolution]
```

0	1	1	1	0
1	2.	3.33333	3.38629	0.052961
2	3.	6.2	6.29584	0.0958369
3	4.	9.40952	9.54518	0.135654
4	5.	12.873	13.0472	0.174174
5	6.	16.5385	16.7506	0.212029

(2) Using Runge – Kutta Method of second order, find approximate solution fo r th e initial value problem x ' (t) = t ^ 2 – x, 0 ≤ t ≤ 0.8, x (0) = 1. Us e n = 8 discrete points a t equal space. Compare the solution w ith the known analytic solution x (t) = 2 – e ^ {–t} – 2 t + t ^ 2.

```
f[t_, x_] := t^2 - x;
actualSolution[t_] := 2 - Exp[-t] - 2 t + t^2;
RungeKutta2ndOrder[0, 0.8, 8, f, 1, actualSolution]
```

0	0	1	1	0
1	0.1	0.90525	0.905163	0.000087418
2	0.2	0.821451	0.821269	0.000182003
3	0.3	0.749463	0.749182	0.000281602
4	0.4	0.690064	0.68968	0.000384406
5	0.5	0.643958	0.643469	0.000488906
6	0.6	0.611782	0.611188	0.000593849
7	0.7	0.594113	0.593415	0.000698206
8	0.8	0.591472	0.590671	0.000801141