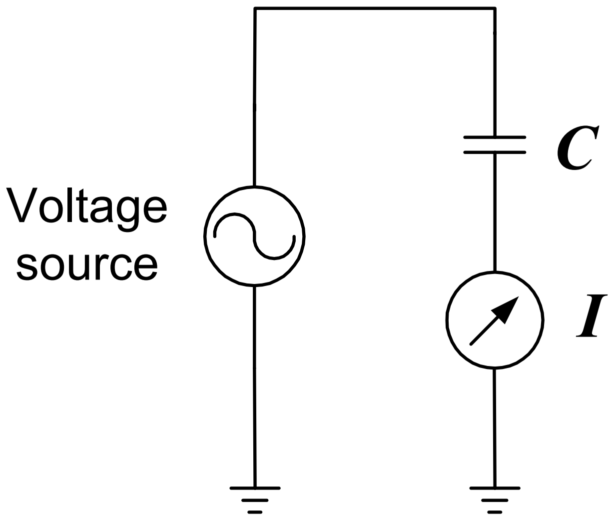
Inductance and Capacitance Measurements

Capacitance Measurements

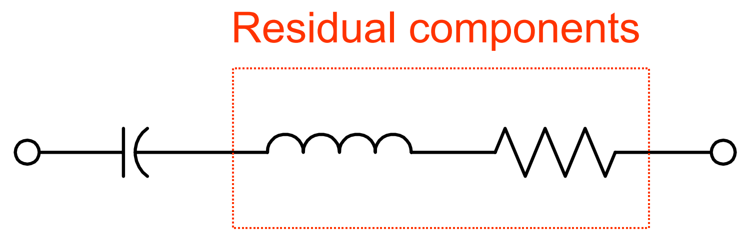


$$I = \frac{V}{X_C} = V(2\pi fC)$$

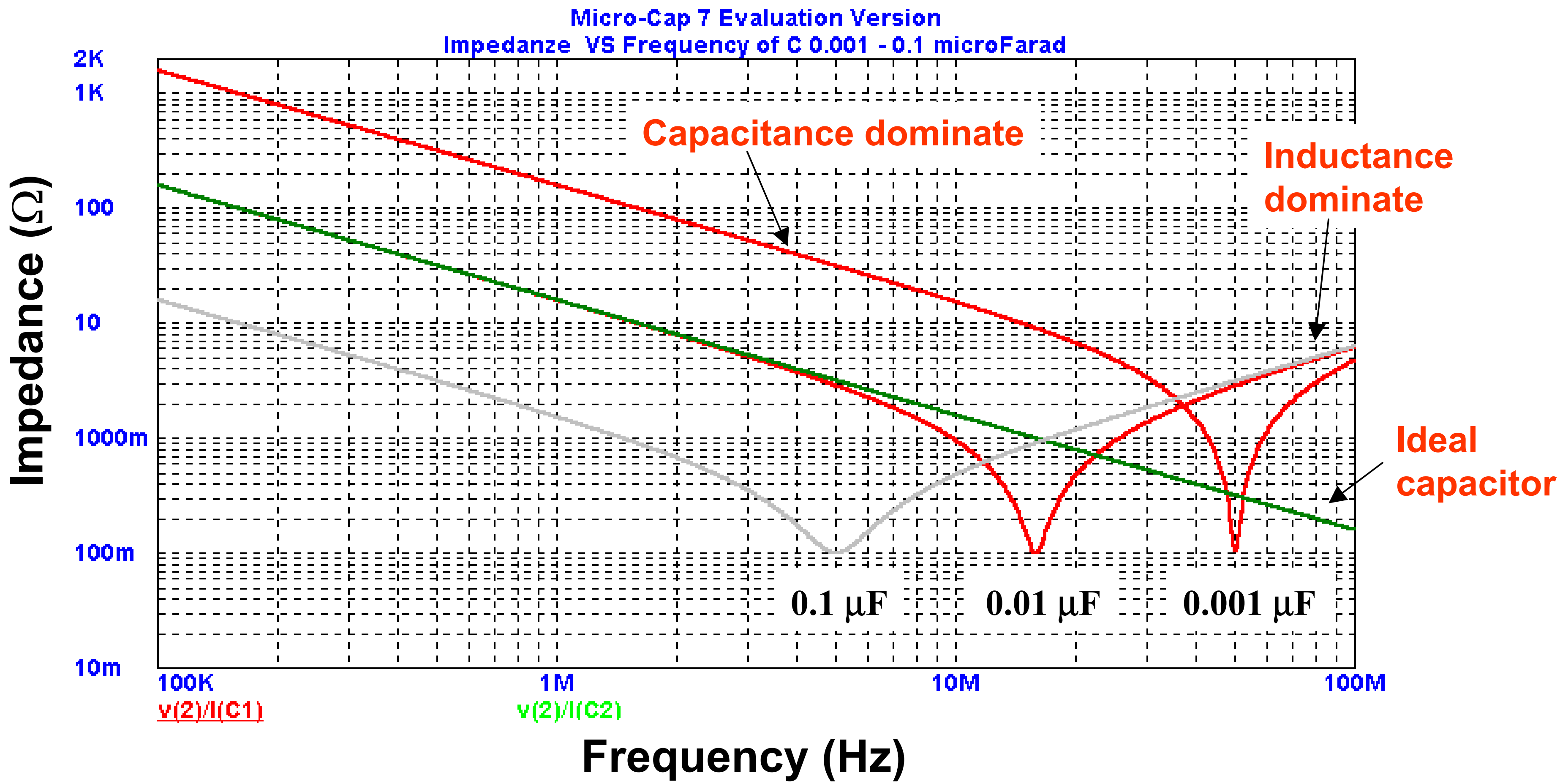
Example: C = 10 pF, V = 10 V (limited by breakdown voltage of the capacitance), I = 100 mA; give f = 1600 MHz $\longrightarrow$ parasitic components are dominate the measurements

- **Simple**
- **Not practical in the real applications**

$$\mathbf{Z} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$



Equivalent circuit of a capacitor at high frequency



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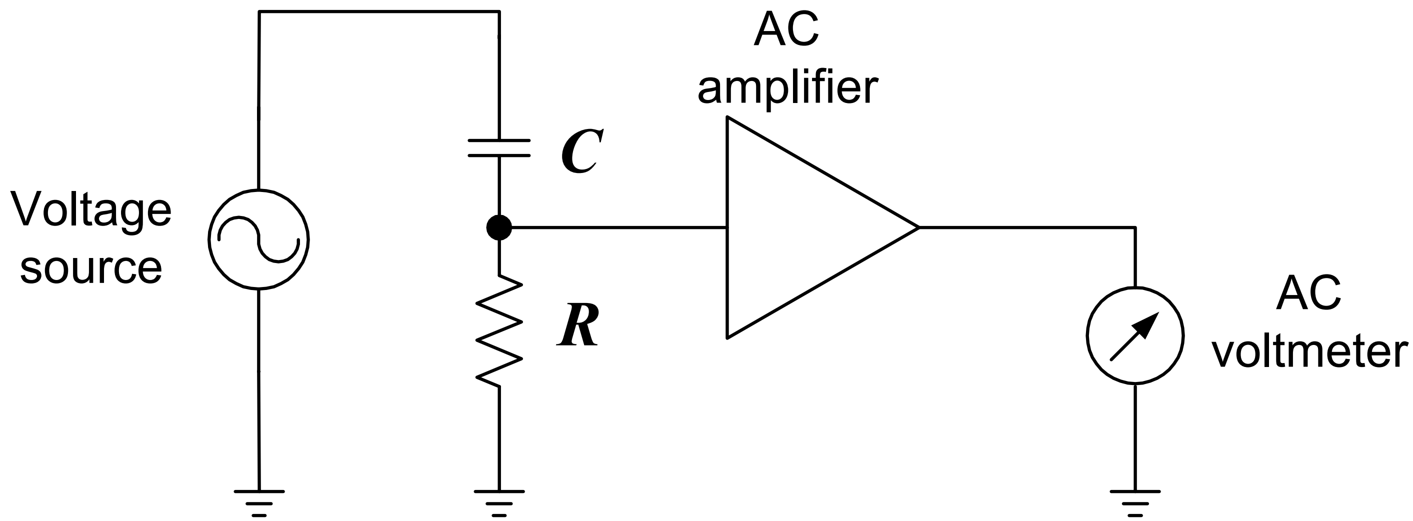
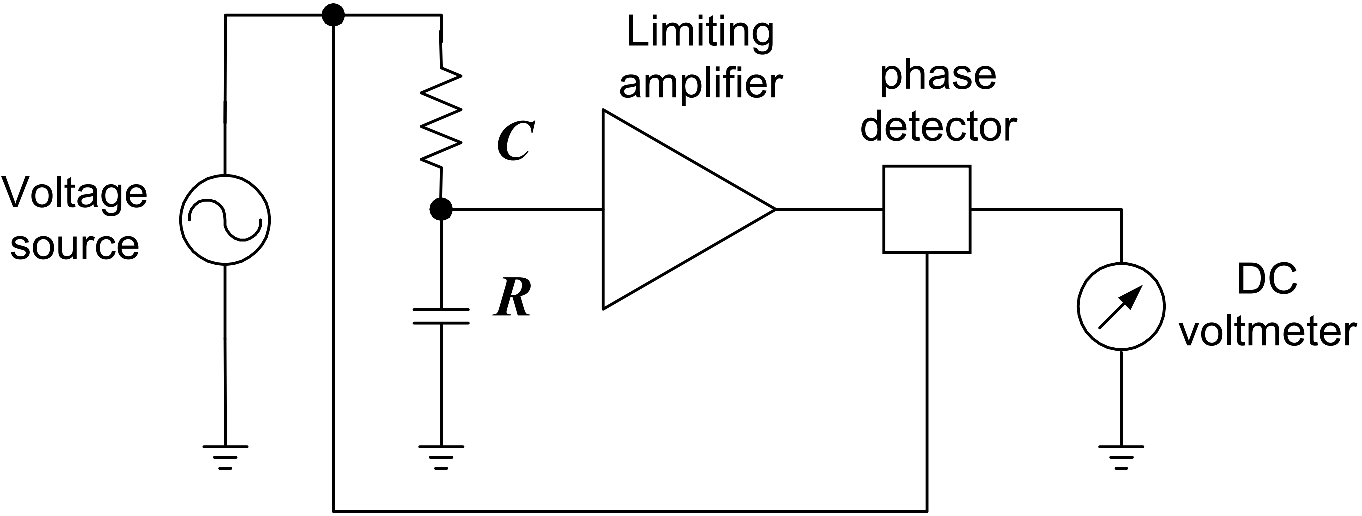


$$|\mathbf{V}| = A\frac{RV_{in}}{\sqrt{R^2 + X_C^2}} = A\frac{RV_{in}}{\sqrt{R^2 + \left(\frac{1}{2\pi fC}\right)^2}}$$

Where A = Amplifier gain

- **Non-linear relationship between V and C**
- **It is hard to keep the amplifier gain constant over the large input voltage**

Inductance and Capacitance Measurements



- In this scheme, we measure the phase difference between the input voltage, V_{in} and the voltage across the series resistance.

- Here, the magnitude of the amplifier is not a critical factor, its gain should be large enough to give an output that can be detected by the phase detector.

$$\mathbf{V} = A\frac{jX_C V_{in}}{R + jX_C} = A\frac{X_C(X_C + jR)V_{in}}{R^2 + X_c^2} \Rightarrow \boxed{\theta = \arctan\left(\frac{R}{X_C}\right) = \arctan(2\pi fRC)}$$

Taylor's Series: $\boxed{\theta = \arctan(2\pi fRC) = 2\pi fRC - \frac{1}{3}(2\pi fRC)^3 + \frac{1}{5}(2\pi fRC)^5 \ldots}$

If $2\pi fRC < 0.1$; that gives $\theta \sim 2\pi fRC$ within 0.3 % error $\Rightarrow \boxed{\theta \approx 2\pi fRC}$

Inductance and Capacitance Measurements

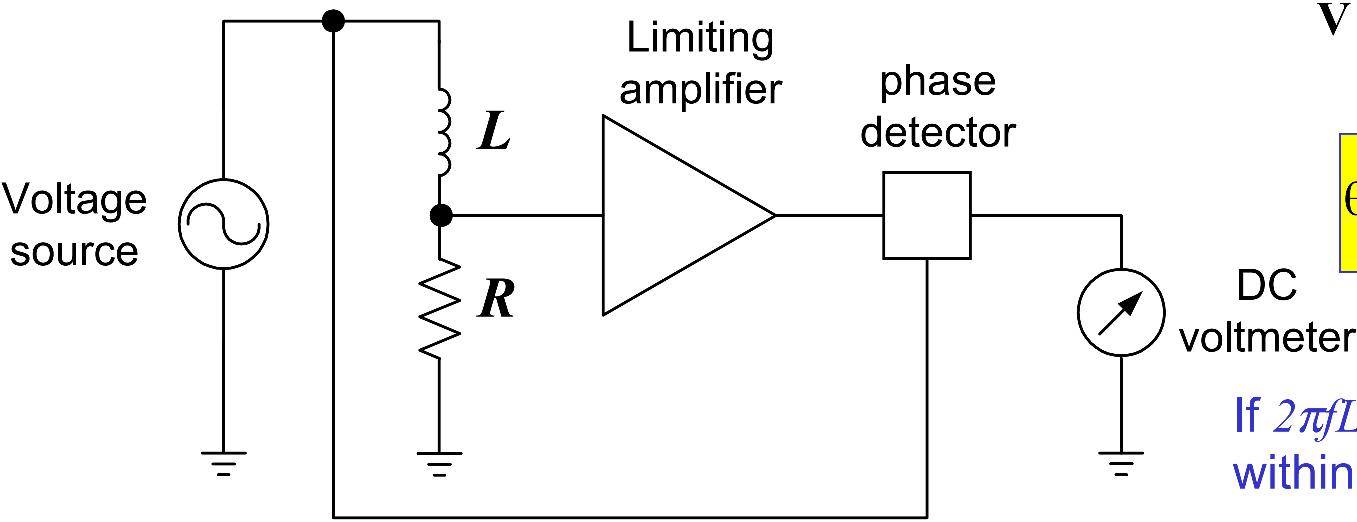
Example: to cover C from 0-100 pF with a source frequency of 1 MHz; how can we select the series resistance.

$$0.1 = 2\pi fRC = 2\pi \times 1\ \text{MHz} \times R \times 100\ \text{pF}$$

Solve for R:

$$R = \frac{0.1}{2\pi \times 10^{-4}} = 1590\ \Omega$$

- **Inductance Measurements**



$$\mathbf{V} = A\frac{RV_{in}}{R + jX_L} = A\frac{R(R - jX_L)V_{in}}{R^2 + X_L^2}$$

$$\boxed{\theta = \arctan\left(\frac{X_L}{R}\right) = \arctan\left(\frac{2\pi fL}{R}\right)}$$

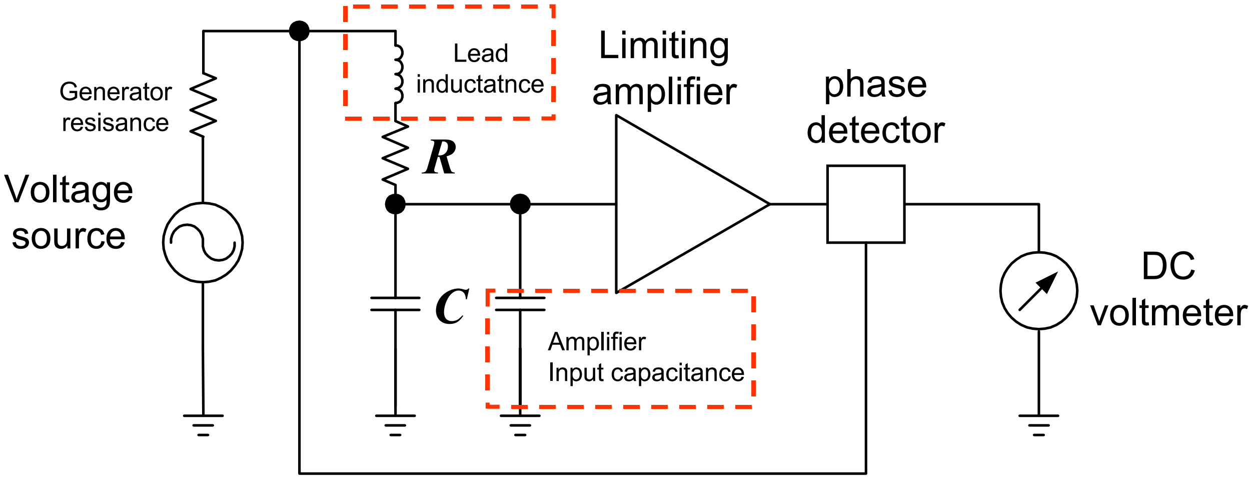
If $2\pi fL/R < 0.1$; that gives $\theta \sim 2\pi fL/R$ within 0.3 % error

$$\boxed{\theta \approx 2\pi fL / R}$$

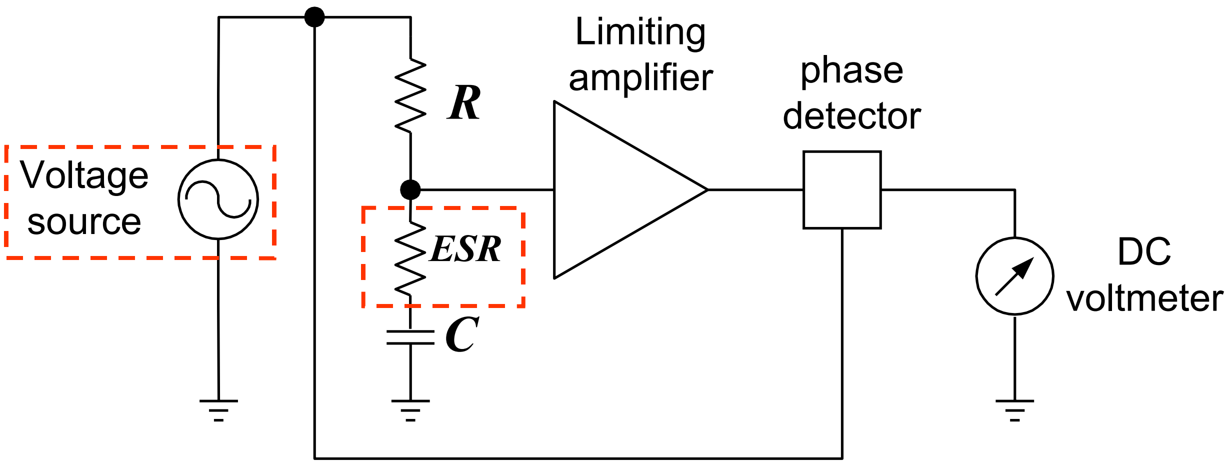
- Example: to cover L from 0-1µH with a source frequency of 1 MHz; the series resistance for full scale deflection is 62.8 Ω

Source of Errors

- **Parasitic components in the instruments**



- **Non-ideal characteristic of the device under test**



Therefore, This type of measurement is suitable for High Q inductance and Low D capacitance

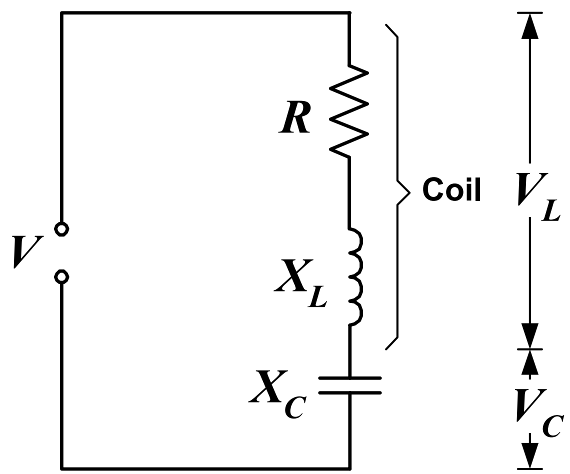
- **Harmonic distortion of the signal source**

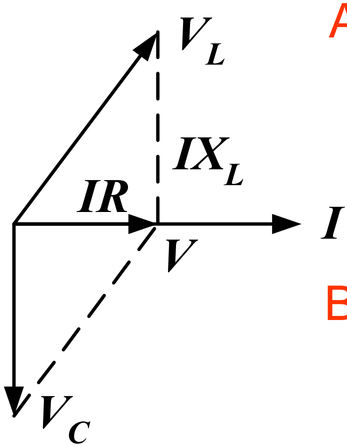
Q Meter

Q-meter is an instrument designed to measure the Q factor of a coil and for measuring inductance, capacitance, and resistance at RF.

Basic Q-meter Circuit

•The basic operation based on the well-known characteristics of series resonant circuits.





Series resonant circuit

At resonance:

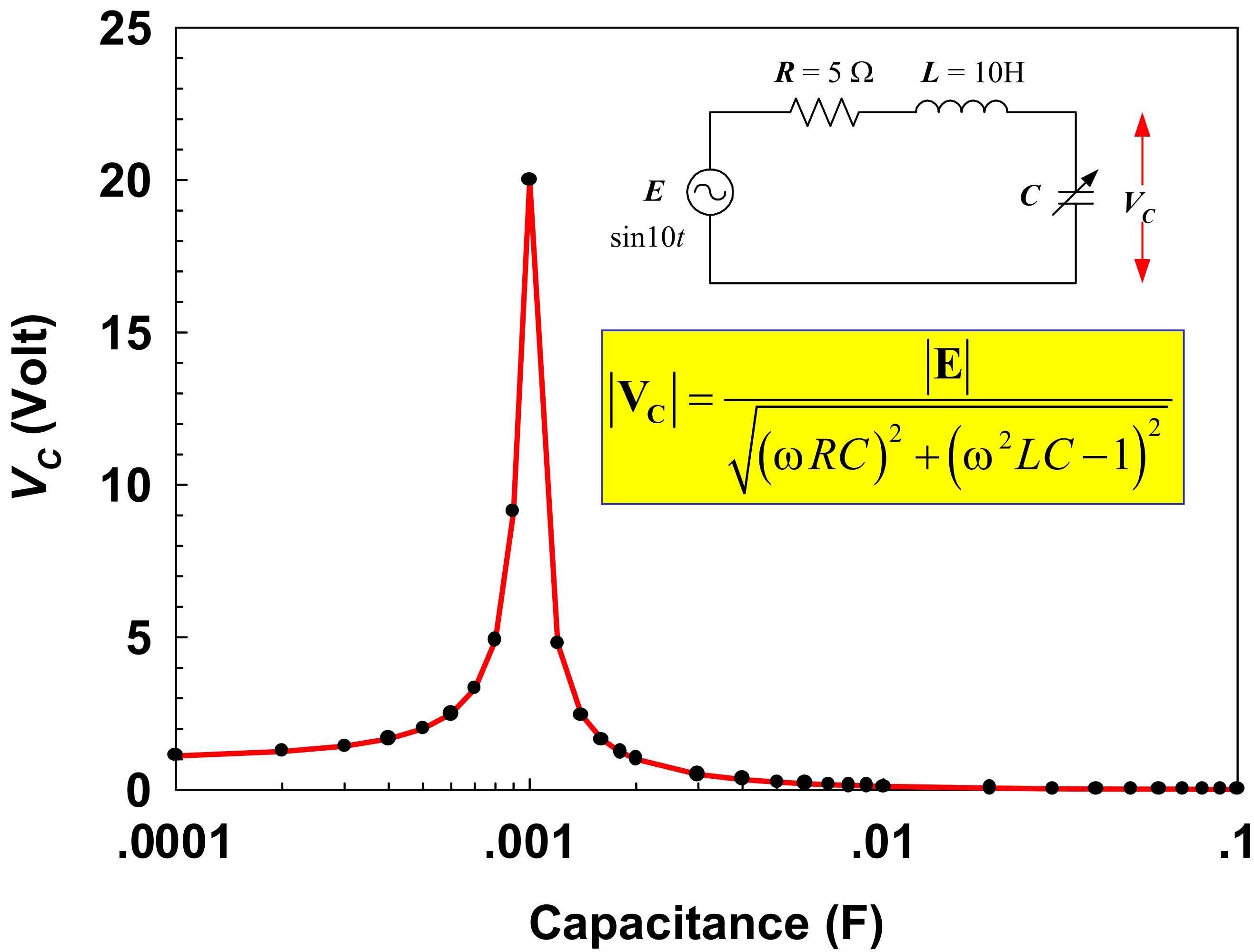
$$X_L = X_C$$
$$V_C = V_L = IX_C = IX_L$$
$$V = IR$$

By the definition of Q:

$$Q = \frac{X_L}{R} = \frac{X_C}{R} = \frac{V_C}{V}$$

Therefore, if V is a known constant, a voltmeter connected across the capacitor can be calibrated in term of the circuit Q

Q Meter



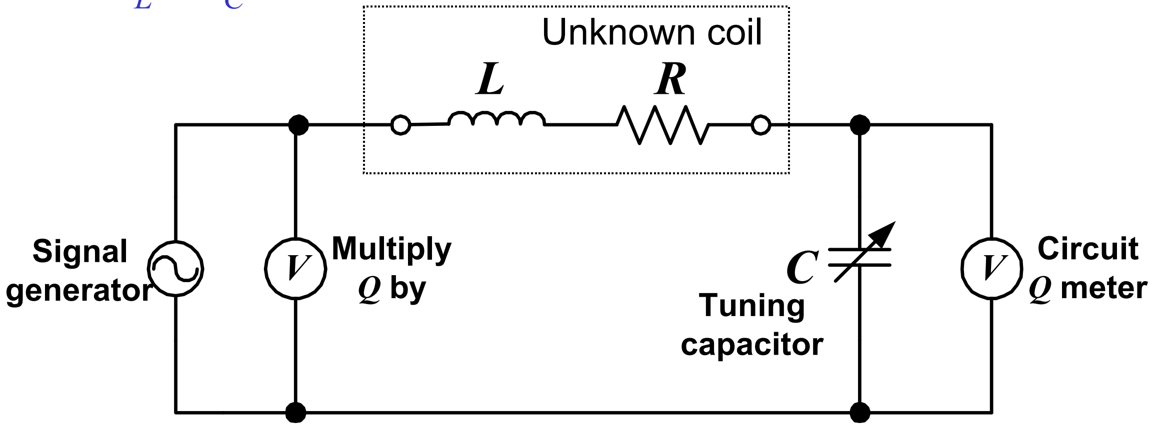
$R = 5\ \Omega$
$L = 10\mathrm{H}$
E
$\sin 10t$
C
V_C
$$|\mathbf{V_C}| = \frac{|\mathbf{E}|}{\sqrt{(\omega RC)^2 + (\omega^2 LC - 1)^2}}$$
25
20
15
10
5
0
V_C (Volt)
.0001
.001
.01
.1
Capacitance (F)

Q Meter

Example: when the below circuit is in the resonance, V = 100 mV, R = 5 Ω, and $X_L = X_C$ = 100 Ω

(a) Calculate the coil Q and the voltmeter indication.

(b) Determine the Q factor and voltmeter indication for another coil that R = 10 Ω, and $X_L = X_C$ = 100 Ω at resonance



Solution (a)

$$I = \frac{V}{R} = \frac{100\text{ mV}}{5\ \Omega} = 20\text{ mA}$$

$$V_L = V_C = IX_C = 20\text{ mA} \times 100\ \Omega = 2\text{ V}$$

$$Q = \frac{V_C}{V} = \frac{2\text{ V}}{100\text{ mV}} = 20$$

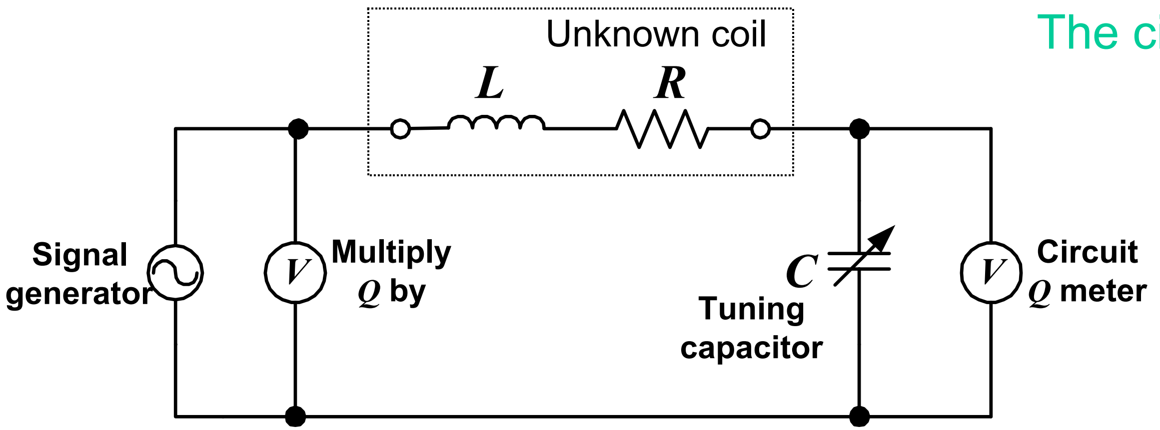
(b) For the second coil:

$$I = \frac{V}{R} = \frac{100\text{ mV}}{10\ \Omega} = 10\text{ mA}$$

$$V_L = V_C = IX_C = 10\text{ mA} \times 100\ \Omega = 1\text{ V}$$

$$Q = \frac{V_C}{V} = \frac{1\text{ V}}{100\text{ mV}} = 10$$

Measurement Method: Direct Connection

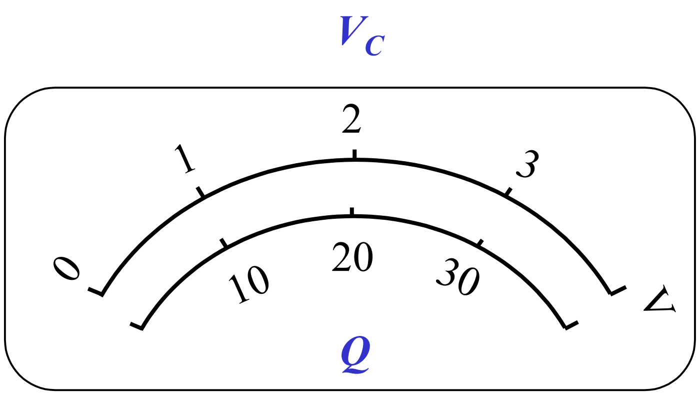


The circuit can be adjusted to the resonance by

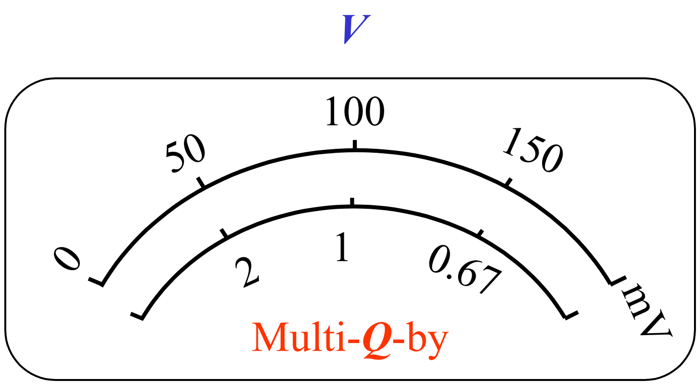
- ✓ Preset the source frequency, and then vary the tuning capacitor
- ✓ Preset the tuning capacitor and then adjust the source frequency

$$X_L = X_C \text{ and } L = \frac{1}{(2\pi f)^2 C} \text{ Henry}$$

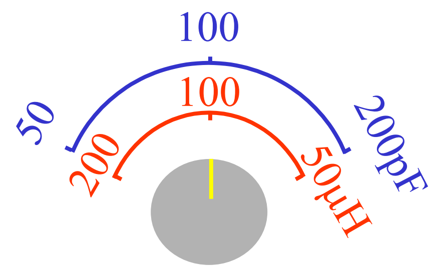
Practical Q meter:



(a) Capacitor voltmeter calibrated to monitor Q



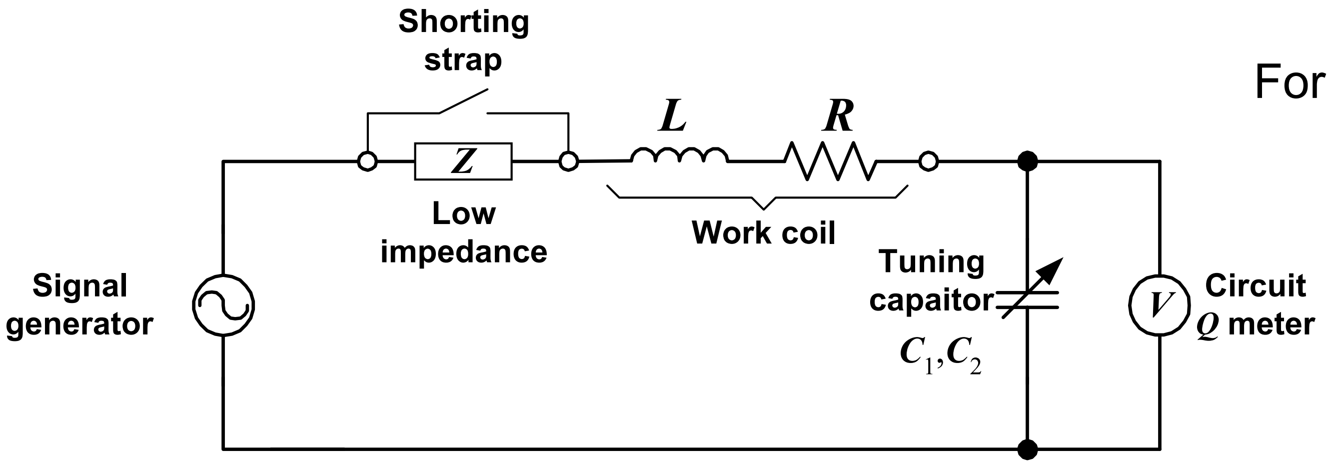
(b) Supply voltmeter calibrated as a multiply-Q-by meter



(c) Capacitance dial calibrated to indicate coil inductance

Measurement Method: Series Connection

For low-impedance components: low value resistors, small coils, and large capacitors



For the first measurement: short Z

$$X_L = X_C \text{ or } \omega L = \frac{1}{\omega C_1}$$

Thus $$Q_1 = \frac{\omega L}{R} = \frac{1}{\omega C_1 R}$$

Q meter for a low-impedance component

For the second measurement:

The capacitor is tuned again to the new resonance point.

$$X_S = X_{C2} - X_L \text{ or } X_S = \frac{1}{\omega C_2} - \frac{1}{\omega C_1} \quad \Longrightarrow \quad X_S = \frac{C_1 - C_2}{\omega C_1 C_2}$$

X_s is inductive if $C_1 > C_2$ and capacitive if $C_2 > C_1$

Measurement Method: Series Connection

The resistive components of the unknown impedance **Z**.

$$R_1 = \frac{X_1}{Q_1} \text{ and } R_2 = \frac{X_2}{Q_2}$$

$$R_S = R_2 - R_1 = \frac{1}{\omega C_1 Q_1} - \frac{1}{\omega C_2 Q_2}$$

Thus

$$\boxed{R_S = R_2 - R_1 = \frac{C_1 Q_1 - C_2 Q_2}{\omega C_1 Q_1 C_2 Q_2}}$$

Case I: **Z** = purely resistive, $C_1 = C_2$

$$\boxed{R_S = R_2 - R_1 = \frac{Q_1 - Q_2}{\omega C_1 Q_1 Q_2} = \frac{\Delta Q}{\omega C_1 Q_1 Q_2}}$$

Case II: **Z** = inductive, $C_1 > C_2$

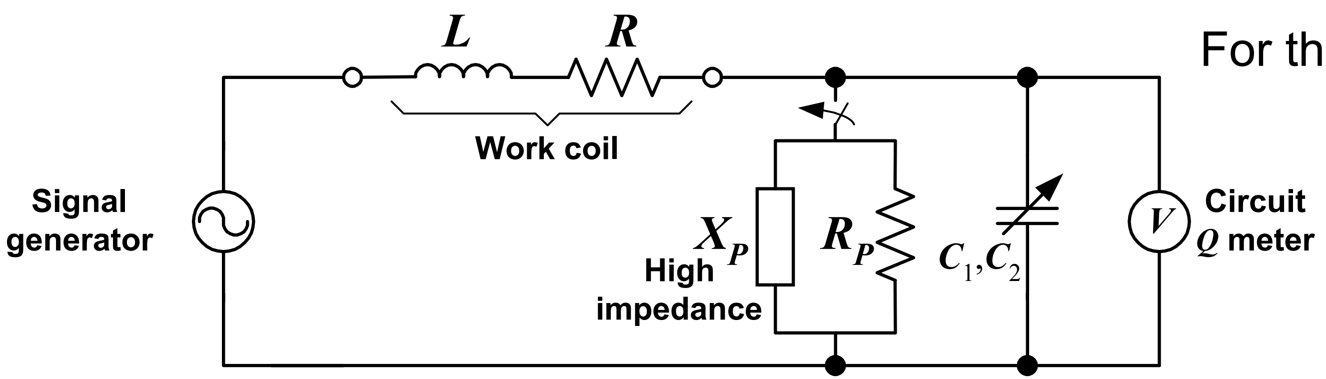
$$\boxed{L_S = \frac{C_1 - C_2}{\omega^2 C_1 C_2}} \qquad \boxed{Q_S = \frac{X_s}{R_S} = \frac{(C_1 - C_2) Q_1 Q_2}{C_1 Q_1 - C_2 Q_2}}$$

Case III: **Z** = capacitive, $C_1 < C_2$

$$\boxed{C_S = \frac{C_1 C_2}{C_2 - C_1}} \qquad \boxed{Q_S = \frac{X_s}{R_S} = \frac{(C_1 - C_2) Q_1 Q_2}{C_1 Q_1 - C_2 Q_2}}$$

Measurement Method: Parallel Connection

For high-impedance components: high value resistors, Large coils, and small capacitors



For the first measurement: open switch

$$X_L = X_C \text{ or } \omega L = \frac{1}{\omega C_1}$$

Thus $$Q_1 = \frac{\omega L}{R} = \frac{1}{\omega C_1 R}$$

Q meter for a high-impedance component

For the second measurement:

The capacitor is tuned again to the new resonance point.

$$X_L = \frac{X_{C2} X_P}{X_{C2} + X_P} \Rightarrow X_P = \frac{1}{\omega (C_1 - C_2)}$$

If the unknown is inductive, $X_P = \omega L_P$; $$L_P = \frac{1}{\omega^2 (C_1 - C_2)}$$

If the unknown is capacitive, $X_P = 1/\omega C_P$; $$C_P = C_1 - C_2$$

Measurement Method: Parallel Connection

In a parallel resonant circuit, the total resistance at resonance: $Q_2 = R_T/X_L$

While $X_2 = X_{C1}$, Therefore $R_T = Q_2 X_L = Q_2 X_{C1} = Q_2/C_1$

$$\frac{1}{R_T} = \frac{1}{R_P} + \frac{1}{R_{LP}}$$

Where $$R_{LP} = \frac{R}{R^2 + \omega^2 L^2}$$

$$\frac{1}{R_P} = \frac{1}{R_T} - \frac{1}{R_{LP}} = \frac{\omega C_1}{Q_2} - \frac{R}{R^2 + \omega^2 L^2}$$

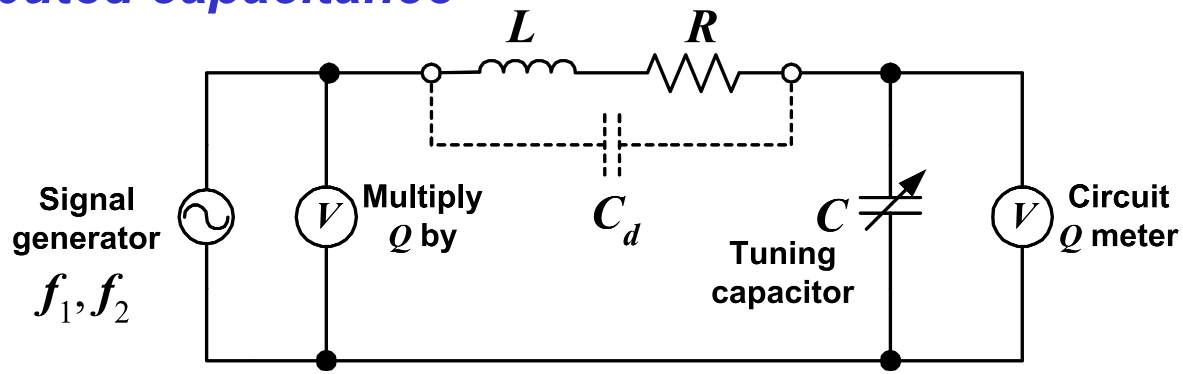
$$\frac{1}{R_P} = \frac{\omega C_1}{Q_2} - \left(\frac{1}{R}\right)\frac{1}{1 + \omega^2 L^2 / R^2} \approx \frac{\omega C_1}{Q_2} - \frac{1}{R Q_1^2}$$

since $Q_1 = 1/\omega C_1 R$ $$\frac{1}{R_P} = \frac{\omega C_1}{Q_2} - \frac{\omega C_1}{Q_1} \qquad R_P = \frac{Q_1 Q_2}{\omega C_1 (Q_1 - Q_2)} = \frac{Q_1 Q_2}{\omega C_1 \Delta Q}$$

$$\boxed{Q_P = \frac{R_P}{X_P} = \frac{(C_1 - C_2) Q_1 Q_2}{C_1 (Q_1 - Q_2)} = \frac{(C_1 - C_2) Q_1 Q_2}{C_1 \Delta Q}}$$

Source of Error

- ***Distributed capacitance***



One simple method of finding the distributed capacitance (C_d) is to make two measurements at different frequencies

$$\boxed{f = \frac{1}{2\pi\sqrt{LC}}}$$

At first measurement: f_1

$$f_1 = \frac{1}{2\pi\sqrt{L(C_1 + C_d)}}$$

At first measurement: f_2

$$f_2 = \frac{1}{2\pi\sqrt{L(C_2 + C_d)}}$$

If we set $f_2 = 2 f_1$

$$\frac{1}{2\pi\sqrt{L(C_2 + C_d)}} = \frac{2}{2\pi\sqrt{L(C_1 + C_d)}} \Rightarrow C_d = \frac{C_1 - 4C_2}{3}$$

Source of Error

The effective Q of a coil with distributed capacitance is less than the true Q

$$\boxed{\text{True } Q = Q_e\left(\frac{C+C_d}{C}\right)}$$

- ***Residual or insertion resistance:*** in the Q meter circuit can be an important source of error when the signal generator voltage is not metered.

If R_S is the source resistance, the circuit current resonance is

$$I = \frac{V}{R+R_s} \quad \text{Instead of} \quad I = \frac{V}{R}$$

Also, the indicated Q factor of the coil is $$Q = \frac{\omega L}{R+R_s}$$

Instead of the actual coil Q, which is $$Q = \frac{\omega L}{R}$$

Thus R_s must be kept minimized.

Source of Error

Example The self-capacitance of a coil is to be measured by using the procedure just outlined. The first measurement is at f_1 = 2 MHz, and C_1 = 460 pF. The second measurement, at f2 = 4 MHz, yields a new value of tuning capacitor, C_2 = 100 pF. Find the distributed capacitance, C_d.

Solution

$$C_d = \frac{C_1 - 4C_2}{3} = \frac{460 - 4\times 100}{3} = 20 \text{ pF}$$

Example A coil with a resistance of 10 Ω is connected in the " direction-measurement" mode. Resonance occurs when the oscillator frequency is 1.0 MHz, and the resonating capacitor is set at 65 pF. Calculate the percentage error in trounced in the calculated value of Q by the 0.02 Ω insertion resistance.

Solution The effective Q of the coil equals

$$Q_e = \frac{1}{\omega CR} = \frac{1}{2\pi \times 10^6 \times 65 \times 10^{-12} \times 10} = 245$$

The indicated Q of the coil equals

$$Q_e = \frac{1}{\omega C\left(R + 0.02\right)} = 244.5$$

The percentage error is then

$$\frac{245 - 244.5}{245} \times 100\% = 0.2\%$$