

Indian Institute of Technology Madras
Department of Data Science and Artificial Intelligence
DA5000: Mathematical Foundations of Data Science
Tutorial I

Problem

1. Suppose $A\mathbf{x} = \mathbf{b}$ and $C\mathbf{x} = \mathbf{b}$ have the same complete solutions for every vector \mathbf{b} . Is it true that $A = C$? Prove or disprove this statement.
2. Determine whether the following statements are true or false. Justify each answer.
 - (a) If the columns of a matrix are dependent, so are the rows.
 - (b) The column space of a 2×2 matrix is the same as its row space.
 - (c) The column space of a 2×2 matrix has the same dimension as its row space.
 - (d) The columns of a matrix are a basis for the column space.
3. Let A, B be $m \times n$ matrices. True or false (with a reason or a counterexample):
 - (a) If $m = n$ then the row space of A equals the column space.
 - (b) The matrices A and $-A$ share the same four subspaces.
 - (c) If A and B share the same four subspaces then A is a multiple of B .
4. Find a basis and the dimension for each of the following subspaces of 3×3 matrices:
 - (a) All diagonal matrices.
 - (b) All symmetric matrices ($A^T = A$).
 - (c) All skew-symmetric matrices ($A^T = -A$).
5. Prove that the column space $C(A)$ and the left null space $N(A^T)$ are orthogonal complements in \mathbb{R}^m .
6. If the entries of a 3×3 matrix are chosen randomly between 0 and 1, what are the most likely dimensions of the four subspaces? What if the matrix is 3×5 ?
7. Prove that for any matrix A , the dimensions of the column space $C(A)$ and the row space $C(A^T)$ are equal.