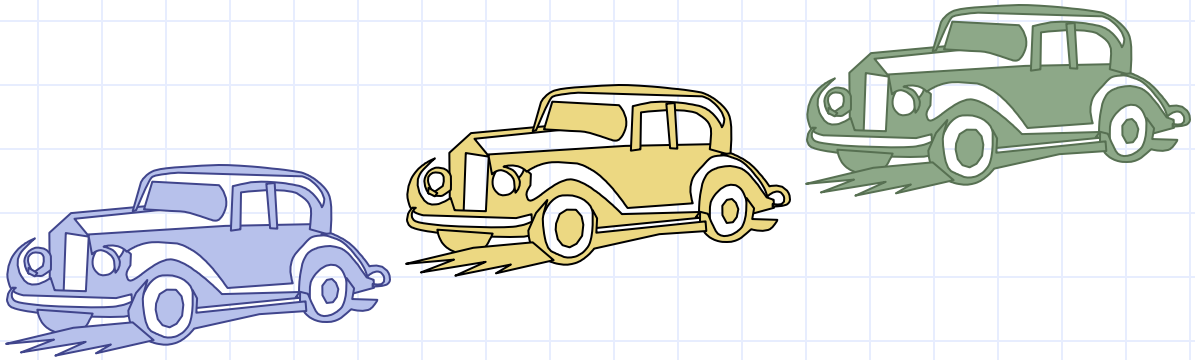
Queues



The Queue ADT

- The Queue ADT stores arbitrary objects
- Insertions and deletions follow the first-in first-out scheme
- Insertions are at the rear of the queue and removals are at the front of the queue
- Main queue operations:
  - enqueue(object): inserts an element at the end of the queue
  - object dequeue(): removes and returns the element at the front of the queue

- Auxiliary queue operations:
  - object first(): returns the element at the front without removing it
  - integer len(): returns the number of elements stored
  - boolean is_empty(): indicates whether no elements are stored
- Exceptions
  - Attempting the execution of dequeue or front on an empty queue throws an EmptyQueueException

Example

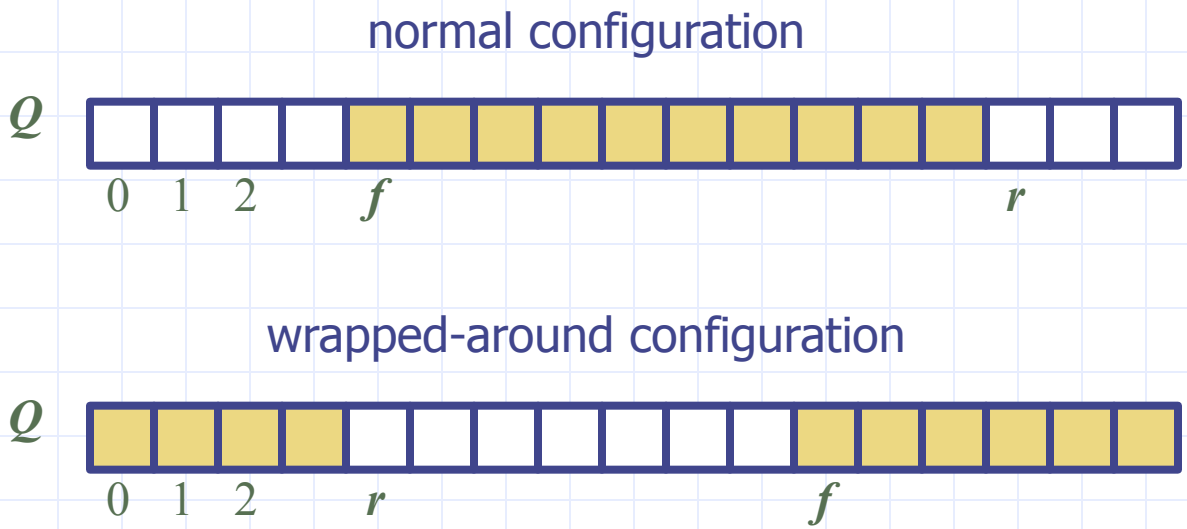
Operation	Return Value	first $\leftarrow Q \leftarrow$ last
Q.enqueue(5)	–	[5]
Q.enqueue(3)	–	[5, 3]
len(Q)	2	[5, 3]
Q.dequeue( )	5	[3]
Q.is_empty( )	False	[3]
Q.dequeue( )	3	[ ]
Q.is_empty( )	True	[ ]
Q.dequeue( )	"error"	[ ]
Q.enqueue(7)	–	[7]
Q.enqueue(9)	–	[7, 9]
Q.first( )	7	[7, 9]
Q.enqueue(4)	–	[7, 9, 4]
len(Q)	3	[7, 9, 4]
Q.dequeue( )	7	[9, 4]

Applications of Queues

- Direct applications
  - Waiting lists, bureaucracy
  - Access to shared resources (e.g., printer)
  - Multiprogramming
- Indirect applications
  - Auxiliary data structure for algorithms
  - Component of other data structures

Array-based Queue

- Use an array of size N in a circular fashion
- Two variables keep track of the front and rear
  - f index of the front element
  - r index immediately past the rear element
- Array location r is kept empty

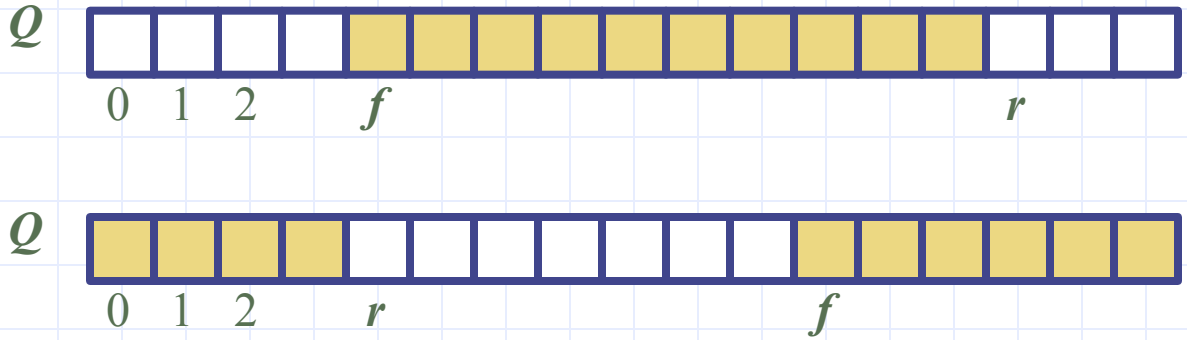


Queue Operations

- We use the modulo operator (remainder of division)

Algorithm *size*()
  return $(N - f + r) \bmod N$

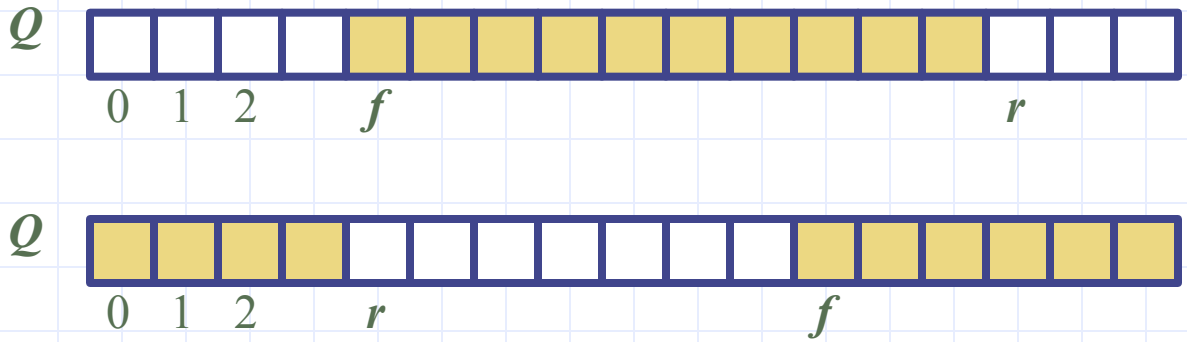
Algorithm *isEmpty*()
  return $(f = r)$



Queue Operations (cont.)

- Operation enqueue throws an exception if the array is full
- This exception is implementation-dependent

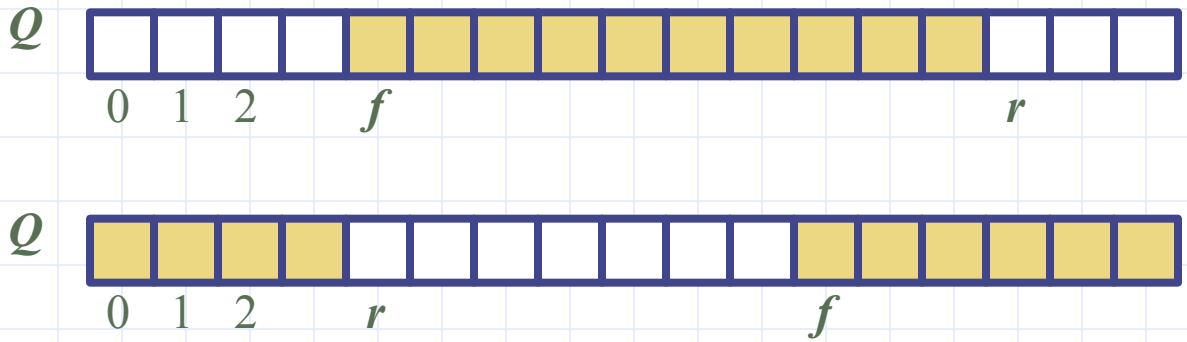
```
Algorithm enqueue(o)
    if size() = N – 1 then
        throw FullQueueException
    else
        Q[r] ← o
        r ← (r + 1) mod N
```



Queue Operations (cont.)

- Operation dequeue throws an exception if the queue is empty
- This exception is specified in the queue ADT

```
Algorithm dequeue()
    if isEmpty() then
        throw EmptyQueueException
    else
        o ← Q[f]
        f ← (f + 1) mod N
        return o
```



Queue in Python

- Use the following three instance variables:
  - _data: is a reference to a list instance with a fixed capacity.
  - _size: is an integer representing the current number of elements stored in the queue (as opposed to the length of the data list).
  - _front: is an integer that represents the index within data of the first element of the queue (assuming the queue is not empty).

Queue in Python, Beginning

```
1  class ArrayQueue:
2    """FIFO queue implementation using a Python list as underlying storage."""
3    DEFAULT_CAPACITY = 10          # moderate capacity for all new queues
4
5    def __init__(self):
6      """Create an empty queue."""
7      self._data = [None] * ArrayQueue.DEFAULT_CAPACITY
8      self._size = 0
9      self._front = 0
10
11   def __len__(self):
12     """Return the number of elements in the queue."""
13     return self._size
14
15   def is_empty(self):
16     """Return True if the queue is empty."""
17     return self._size == 0
18
```

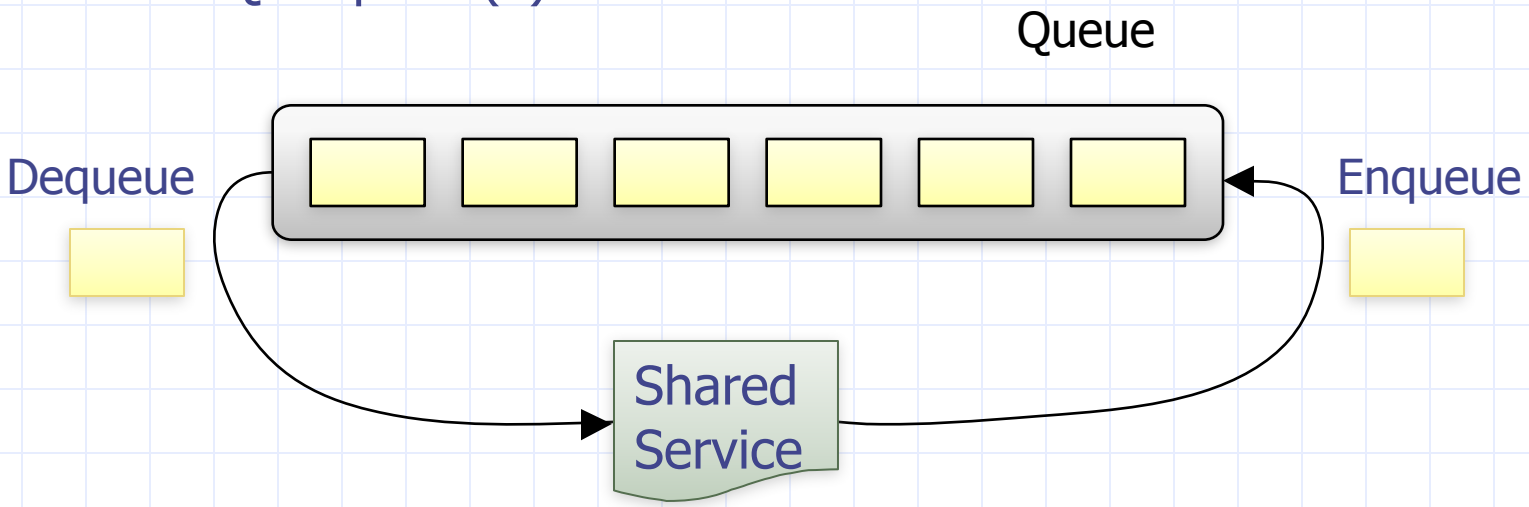
```
19   def first(self):
20     """Return (but do not remove) the element at the front of the queue.
21
22     Raise Empty exception if the queue is empty.
23     """
24     if self.is_empty():
25       raise Empty('Queue is empty')
26     return self._data[self._front]
27
28   def dequeue(self):
29     """Remove and return the first element of the queue (i.e., FIFO).
30
31     Raise Empty exception if the queue is empty.
32     """
33     if self.is_empty():
34       raise Empty('Queue is empty')
35     answer = self._data[self._front]
36     self._data[self._front] = None                    # help garbage collection
37     self._front = (self._front + 1) % len(self._data)
38     self._size -= 1
39     return answer
```

Queue in Python, Continued

```
40    def enqueue(self, e):
41      """Add an element to the back of queue."""
42      if self._size == len(self._data):
43        self._resize(2 * len(self.data))          # double the array size
44      avail = (self._front + self._size) % len(self._data)
45      self._data[avail] = e
46      self._size += 1
47
48    def _resize(self, cap):                       # we assume cap >= len(self)
49      """Resize to a new list of capacity >= len(self)."""
50      old = self._data                            # keep track of existing list
51      self._data = [None] * cap                   # allocate list with new capacity
52      walk = self._front
53      for k in range(self._size):                 # only consider existing elements
54        self._data[k] = old[walk]                 # intentionally shift indices
55        walk = (1 + walk) % len(old)              # use old size as modulus
56      self._front = 0                             # front has been realigned
```

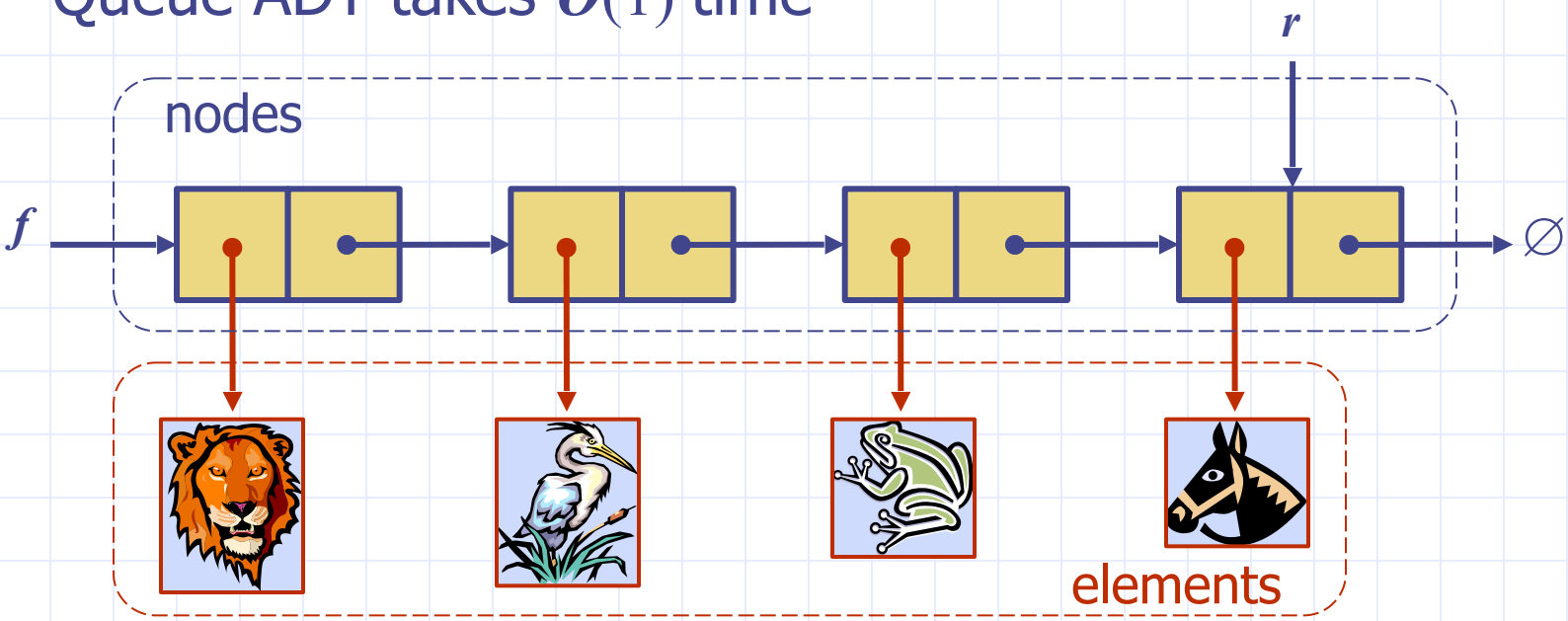
Application: Round Robin Schedulers

- We can implement a round robin scheduler using a queue Q by repeatedly performing the following steps:
  1. e = Q.dequeue()
  2. Service element e
  3. Q.enqueue(e)



Queue as a Linked List

- We can implement a queue with a singly linked list
  - The front element is stored at the first node
  - The rear element is stored at the last node
- The space used is $O(n)$ and each operation of the Queue ADT takes $O(1)$ time

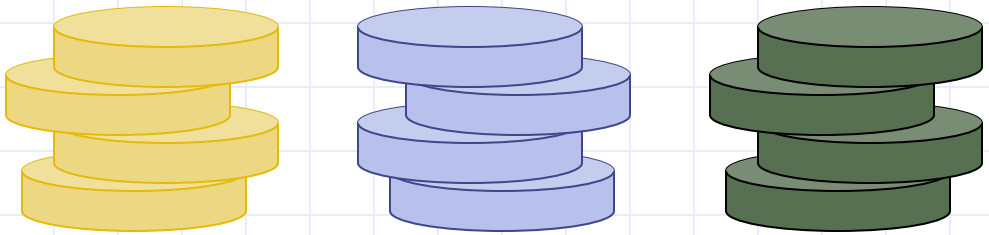


Linked-List Queue in Python

```
1  class LinkedQueue:
2    """FIFO queue implementation using a singly linked list for storage."""
3
4    class _Node:
5      """Lightweight, nonpublic class for storing a singly linked node."""
6      (omitted here; identical to that of LinkedStack._Node)
7
8    def __init__(self):
9      """Create an empty queue."""
10     self._head = None
11     self._tail = None
12     self._size = 0                          # number of queue elements
13
14   def __len__(self):
15     """Return the number of elements in the queue."""
16     return self._size
17
18   def is_empty(self):
19     """Return True if the queue is empty."""
20     return self._size == 0
21
22   def first(self):
23     """Return (but do not remove) the element at the front of the queue."""
24     if self.is_empty():
25       raise Empty('Queue is empty')
26     return self._head._element              # front aligned with head of list
```

```
27   def dequeue(self):
28     """Remove and return the first element of the queue (i.e., FIFO).
29
30     Raise Empty exception if the queue is empty.
31     """
32     if self.is_empty():
33       raise Empty('Queue is empty')
34     answer = self._head._element
35     self._head = self._head._next
36     self._size -= 1
37     if self.is_empty():                     # special case as queue is empty
38       self._tail = None                     # removed head had been the tail
39     return answer
40
41   def enqueue(self, e):
42     """Add an element to the back of queue."""
43     newest = self._Node(e, None)            # node will be new tail node
44     if self.is_empty():
45       self._head = newest                   # special case: previously empty
46     else:
47       self._tail._next = newest
48     self._tail = newest                     # update reference to tail node
49     self._size += 1
```

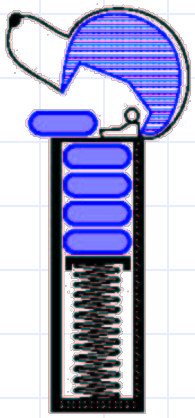
Stacks



Abstract Data Types (ADTs)

- An abstract data type (ADT) is an abstraction of a data structure
- An ADT specifies:
  - Data stored
  - Operations on the data
  - Error conditions associated with operations
- Example: ADT modeling a simple stock trading system
  - The data stored are buy/sell orders
  - The operations supported are
    - order buy(stock, shares, price)
    - order sell(stock, shares, price)
    - void cancel(order)
  - Error conditions:
    - Buy/sell a nonexistent stock
    - Cancel a nonexistent order

The Stack ADT



- The Stack ADT stores arbitrary objects
- Insertions and deletions follow the last-in first-out scheme
- Think of a spring-loaded plate dispenser
- Main stack operations:
  - push(object): inserts an element
  - object pop(): removes and returns the last inserted element
- Auxiliary stack operations:
  - object top(): returns the last inserted element without removing it
  - integer len(): returns the number of elements stored
  - boolean is_empty(): indicates whether no elements are stored

Example

Operation	Return Value	Stack Contents
S.push(5)	–	[5]
S.push(3)	–	[5, 3]
len(S)	2	[5, 3]
S.pop( )	3	[5]
S.is_empty( )	False	[5]
S.pop( )	5	[ ]
S.is_empty( )	True	[ ]
S.pop( )	"error"	[ ]
S.push(7)	–	[7]
S.push(9)	–	[7, 9]
S.top( )	9	[7, 9]
S.push(4)	–	[7, 9, 4]
len(S)	3	[7, 9, 4]
S.pop( )	4	[7, 9]
S.push(6)	–	[7, 9, 6]
S.push(8)	–	[7, 9, 6, 8]
S.pop( )	8	[7, 9, 6]

Applications of Stacks

- Direct applications
  - Page-visited history in a Web browser
  - Undo sequence in a text editor
  - Chain of method calls in a language that supports recursion
- Indirect applications
  - Auxiliary data structure for algorithms
  - Component of other data structures

Array-based Stack

- A simple way of implementing the Stack ADT uses an array
- We add elements from left to right
- A variable keeps track of the index of the top element

Array-based Stack (cont.)

- The array storing the stack elements may become full
- A push operation will then need to grow the array and copy all the elements over.

Performance and Limitations

- Performance
  - Let n be the number of elements in the stack
  - The space used is $O(n)$
  - Each operation runs in time $O(1)$ (amortized in the case of a push)

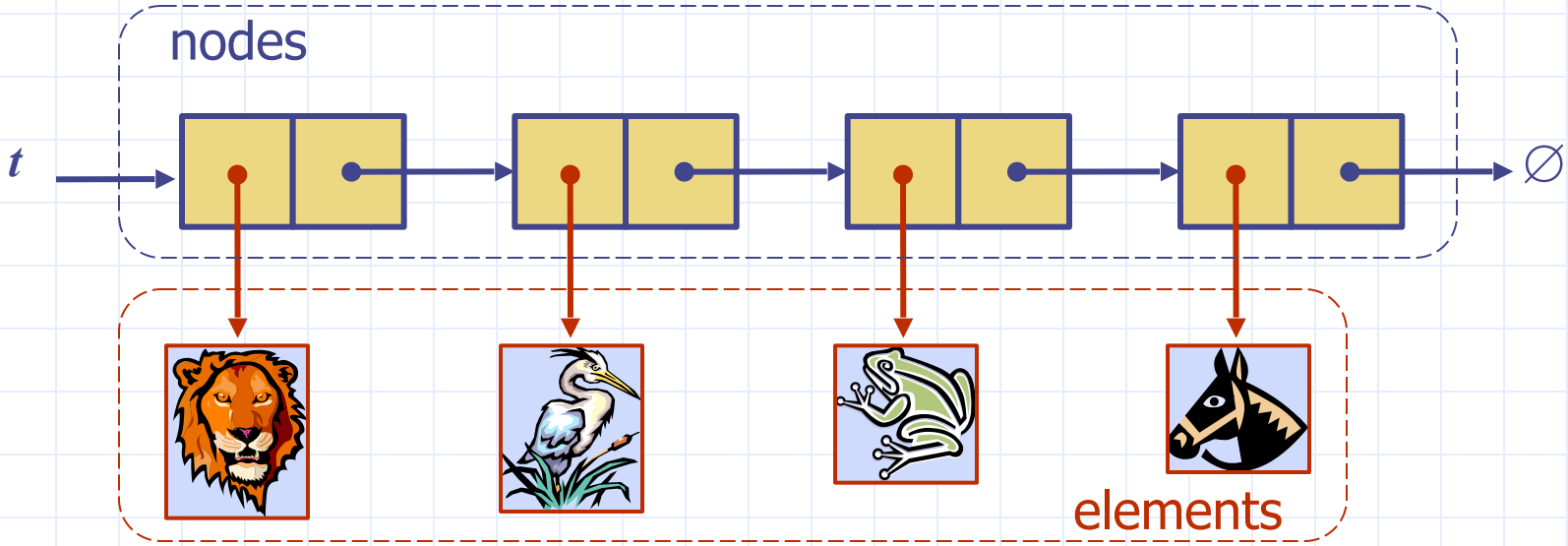
Array-based Stack in Python

```
1  class ArrayStack:
2    """LIFO Stack implementation using a Python list as underlying storage."""
3
4    def __init__(self):
5      """Create an empty stack."""
6      self._data = [ ]                       # nonpublic list instance
7
8    def __len__(self):
9      """Return the number of elements in the stack."""
10     return len(self._data)
11
12   def is_empty(self):
13     """Return True if the stack is empty."""
14     return len(self._data) == 0
15
16   def push(self, e):
17     """Add element e to the top of the stack."""
18     self._data.append(e)                   # new item stored at end of list
19
```

```
20   def top(self):
21     """Return (but do not remove) the element at the top of the stack.
22
23     Raise Empty exception if the stack is empty.
24     """
25     if self.is_empty( ):
26       raise Empty('Stack is empty')
27     return self._data[-1]                  # the last item in the list
28
29   def pop(self):
30     """Remove and return the element from the top of the stack (i.e., LIFO).
31
32     Raise Empty exception if the stack is empty.
33     """
34     if self.is_empty( ):
35       raise Empty('Stack is empty')
36     return self._data.pop( )               # remove last item from list
```

Stack as a Linked List

- We can implement a stack with a singly linked list
- The top element is stored at the first node of the list
- The space used is $O(n)$ and each operation of the Stack ADT takes $O(1)$ time



Linked-List Stack in Python

```
1  class LinkedStack:
2    """LIFO Stack implementation using a singly linked list for storage."""
3
4    #---------------------------- nested _Node class ----------------------------
5    class _Node:
6      """Lightweight, nonpublic class for storing a singly linked node."""
7      __slots__ = '_element', '_next'         # streamline memory usage
8
9      def __init__(self, element, next):      # initialize node's fields
10       self._element = element               # reference to user's element
11       self._next = next                     # reference to next node
12
13   #------------------------------ stack methods ------------------------------
14   def __init__(self):
15     """Create an empty stack."""
16     self._head = None                       # reference to the head node
17     self._size = 0                          # number of stack elements
18
19   def __len__(self):
20     """Return the number of elements in the stack."""
21     return self._size
22
```

```
23   def is_empty(self):
24     """Return True if the stack is empty."""
25     return self._size == 0
26
27   def push(self, e):
28     """Add element e to the top of the stack."""
29     self._head = self._Node(e, self._head)     # create and link a new node
30     self._size += 1
31
32   def top(self):
33     """Return (but do not remove) the element at the top of the stack.
34
35     Raise Empty exception if the stack is empty.
36     """
37     if self.is_empty():
38       raise Empty('Stack is empty')
39     return self._head._element                 # top of stack is at head of list
```

```
40   def pop(self):
41     """Remove and return the element from the top of the stack (i.e., LIFO).
42
43     Raise Empty exception if the stack is empty.
44     """
45     if self.is_empty():
46       raise Empty('Stack is empty')
47     answer = self._head._element
48     self._head = self._head._next              # bypass the former top node
49     self._size -= 1
50     return answer
```

Parentheses Matching

- Each “(”, “{”, or “[” must be paired with a matching “)”, “}”, or “[”
  - correct: ( )(( )){([( )])}
  - correct: ((( )(( )){([( )])}
  - incorrect:)(( )){([( )])}
  - incorrect: ({[ ])}
  - incorrect: (

Parentheses Matching Algorithm

Algorithm ParenMatch(*X,n*):
Input: An array *X* of *n* tokens, each of which is either a grouping symbol, a variable, an arithmetic operator, or a number
Output: **true** if and only if all the grouping symbols in *X* match
Let *S* be an empty stack
for *i*=0 to *n*-1 **do**
    if *X*[*i*] is an opening grouping symbol **then**
        S.push(*X*[*i*])
    else if *X*[*i*] is a closing grouping symbol **then**
        if *S*.is_empty() **then**
            return false {nothing to match with}
        if *S*.pop() does not match the type of *X*[*i*] **then**
            return false {wrong type}
if *S*.isEmpty() **then**
    return true {every symbol matched}
else return false {some symbols were never matched}

Parentheses Matching in Python

```
1  def is_matched(expr):
2    """Return True if all delimiters are properly match; False otherwise."""
3    lefty = '({['                                  # opening delimiters
4    righty = ')}]'                                 # respective closing delims
5    S = ArrayStack()
6    for c in expr:
7      if c in lefty:
8        S.push(c)                                  # push left delimiter on stack
9      elif c in righty:
10       if S.is_empty():
11         return False                             # nothing to match with
12       if righty.index(c) != lefty.index(S.pop()):
13         return False                             # mismatched
14   return S.is_empty( )                           # were all symbols matched?
```

Evaluating Arithmetic Expressions

Slide by Matt Stallmann included with permission.

$14 - 3 * 2 + 7 = (14 - (3 * 2)) + 7$

Operator precedence

* has precedence over +/–

Associativity

operators of the same precedence group evaluated from left to right

Example: $(x - y) + z$ rather than $x - (y + z)$

Idea: push each operator on the stack, but first pop and perform higher and equal precedence operations.

Algorithm for Evaluating Expressions

Slide by Matt Stallmann included with permission.

Two stacks:

- opStk holds operators
- valStk holds values
- Use $ as special "end of input" token with lowest precedence

Algorithm doOp()

```
x ← valStk.pop();
y ← valStk.pop();
op ← opStk.pop();
valStk.push( y op x )
```

Algorithm repeatOps(refOp):

```
while ( valStk.size() > 1 ∧
        prec(refOp) ≤
        prec(opStk.top())
  doOp()
```

Algorithm EvalExp()

Input: a stream of tokens representing an arithmetic expression (with numbers)

Output: the value of the expression

```
while there's another token z
    if isNumber(z) then
        valStk.push(z)
    else
        repeatOps(z);
        opStk.push(z)
repeatOps($);
return valStk.top()
```

Algorithm on an Example Expression

Slide by Matt Stallmann included with permission.

Operator ≤ has lower precedence than +/–