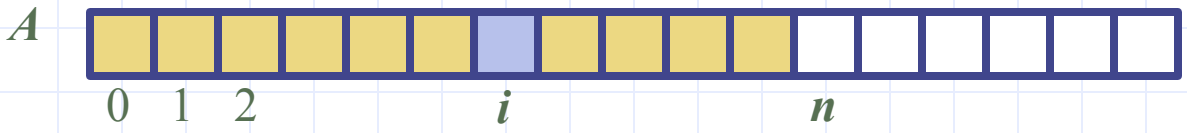
Array-Based Sequences

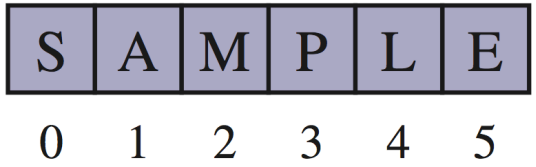
Python Sequence Classes

- Python has built-in types, **list**, **tuple**, and **str**.
- Each of these **sequence** types supports indexing to access an individual element of a sequence, using a syntax such as A[i]
- Each of these types uses an **array** to represent the sequence.
  - An array is a set of memory locations that can be addressed using consecutive indices, which, in Python, start with index 0.

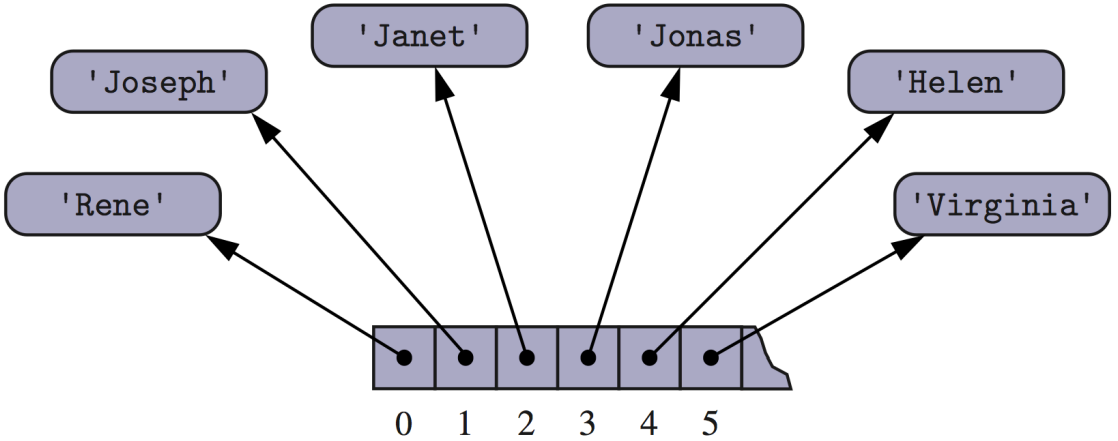


Arrays of Characters or Object References

- An array can store primitive elements, such as characters, giving us a **compact array.**



- An array can also store references to objects.



Compact Arrays

- Primary support for compact arrays is in a module named **array**.
  - That module defines a class, also named array, providing compact storage for arrays of primitive data types.
- The constructor for the array class requires a type code as a first parameter, which is a character that designates the type of data that will be stored in the array.

```
primes = array('i', [2, 3, 5, 7, 11, 13, 17, 19])
```

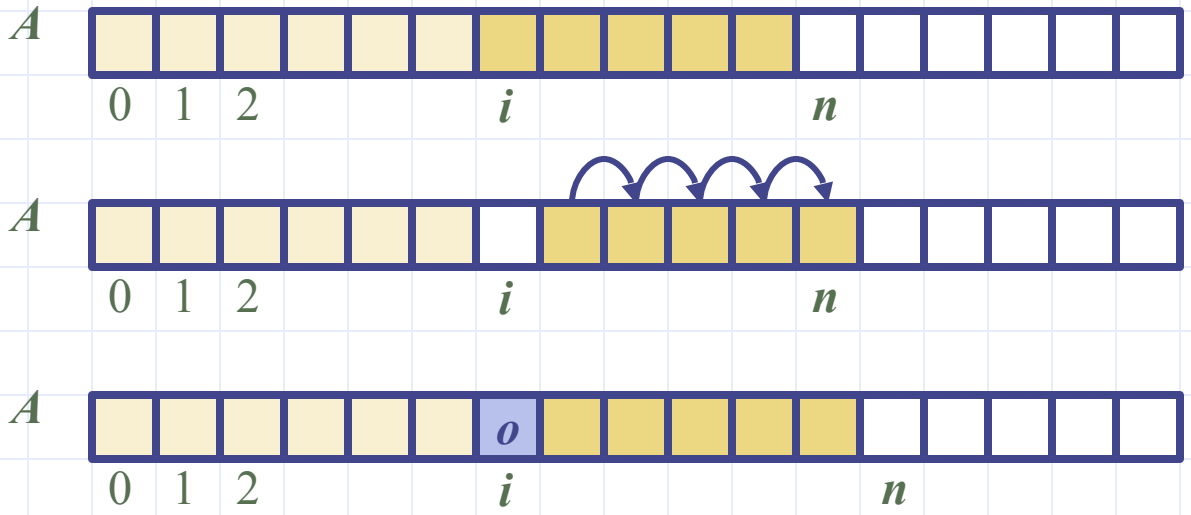
Type Codes in the array Class

- Python's array class has the following type codes:

Code	C Data Type	Typical Number of Bytes
`'b'`	signed char	1
`'B'`	unsigned char	1
`'u'`	Unicode char	2 or 4
`'h'`	signed short int	2
`'H'`	unsigned short int	2
`'i'`	signed int	2 or 4
`'I'`	unsigned int	2 or 4
`'l'`	signed long int	4
`'L'`	unsigned long int	4
`'f'`	float	4
`'d'`	float	8

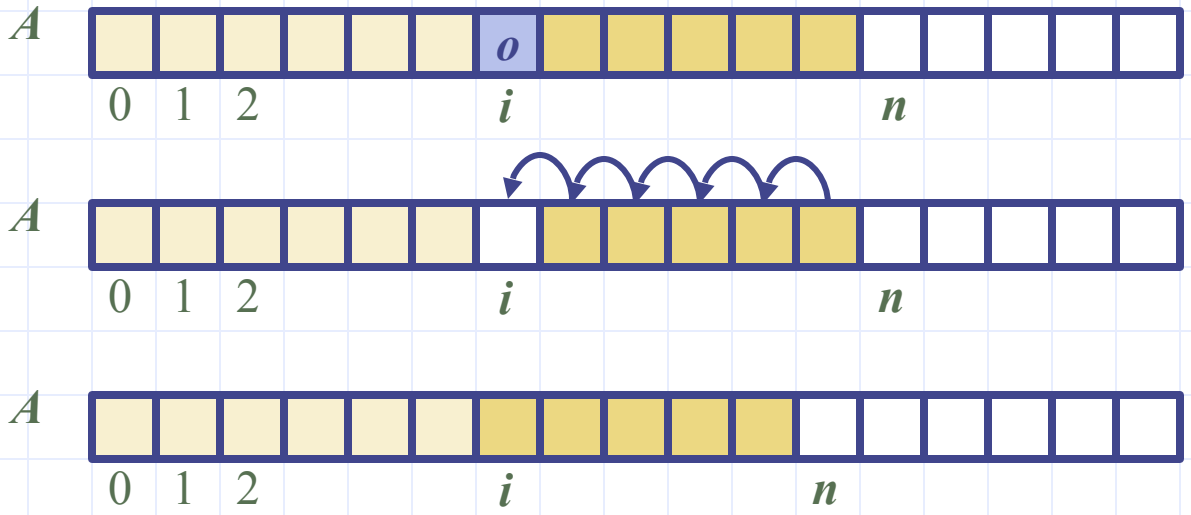
Insertion

- In an operation $add(i, o)$, we need to make room for the new element by shifting forward the $n - i$ elements $A[i], \ldots, A[n-1]$
- In the worst case ($i = 0$), this takes $O(n)$ time



Element Removal

- In an operation ***remove***(i), we need to fill the hole left by the removed element by shifting backward the $n - i - 1$ elements $A[i + 1], \ldots, A[n - 1]$
- In the worst case ($i = 0$), this takes $O(n)$ time



Performance

- In an array based implementation of a dynamic list:
  - The space used by the data structure is $O(n)$
  - Indexing the element at I takes $O(1)$ time
  - ***add*** and ***remove*** run in $O(n)$ time in worst case
- In an ***add*** operation, when the array is full, instead of throwing an exception, we can replace the array with a larger one…

Growable Array-based Array List

- In an add(o) operation (without an index), we could always add at the end
- When the array is full, we replace the array with a larger one
- How large should the new array be?
  - Incremental strategy: increase the size by a constant c
  - Doubling strategy: double the size

```
Algorithm add(o)
    if t = S.length – 1 then
        A ← new array of
                size …
        for i ← 0 to n–1 do
            A[i] ← S[i]
        S ← A
    n ← n + 1
    S[n–1] ← o
```

Comparison of the Strategies

- We compare the incremental strategy and the doubling strategy by analyzing the total time $T(n)$ needed to perform a series of n add(o) operations
- We assume that we start with an empty stack represented by an array of size 1
- We call amortized time of an add operation the average time taken by an add over the series of operations, i.e., $T(n)/n$

Incremental Strategy Analysis

- We replace the array $k = n/c$ times
- The total time $T(n)$ of a series of n add operations is proportional to

$$n + c + 2c + 3c + 4c + \ldots + kc =$$
$$n + c(1 + 2 + 3 + \ldots + k) =$$
$$n + ck(k + 1)/2$$

- Since c is a constant, $T(n)$ is $O(n + k^2)$, i.e., $O(n^2)$
- The amortized time of an add operation is $O(n)$

Doubling Strategy Analysis

- We replace the array $k = \log_2 n$ times
- The total time $T(n)$ of a series of n add operations is proportional to

$$n + 1 + 2 + 4 + 8 + \ldots + 2^k =$$
$$n + 2^{k+1} - 1 =$$
$$3n - 1$$

- $T(n)$ is $O(n)$
- The amortized time of an add operation is $O(1)$

geometric series