

Formal Specification

Software Engineering

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March 18, 2024

¹Based on material from Wolfgang Aherndt,...

This module

— This module —

- ▶ Formal Specification (**starting today**)
- ▶ Formal Program Verification (theory behind)

Formal Specification: Contents

Content

- ▶ Why specification is important.
- ▶ Writing formal specifications: **First Order Logic**.
- ▶ **Dafny**: A programming language with support for **automated checking of formal specifications**.
- ▶ Dafny supports **automated checking of method pre- and postconditions**.
- ▶ **Note**: Focus is on **writing good specifications**, not so much programming.
- ▶ With skills in Java, simple programming in Dafny is very similar.

Motivation

As motivating examples, let's consider two programs.

Example 1: method alwaysTrue()

```
// should always return true
public static boolean alwaysTrue(int i) {

    // Just 'return true;' is all too boring
    .
    // Instead:
    return ( Math.abs(i) >= 0 );

}
```

Example 1: Testing alwaysTrue()

```
Scanner sc = new Scanner(System.in);

while (true) {

    // read an integer from System.in
    int i = sc.nextInt();

    // this will print "true"
    System.out.println(alwaysTrue(i));
}
```

Demo: TestAlwaysTrue.java

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}
```

Demo: TestAlwaysTrue.java

Surprise: with input -2147483648, the program prints false!

We want to understand the problem

- ▶ Another test:
`System.out.println(Math.abs(-2147483648))`
prints
-2147483648
- ▶ We cannot come any closer to the problem by testing/debugging.
- ▶ So how can we?

Specification is the Answer!

From the Java API Specification, class Math:

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From the Java API Specification, class Math:

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Returns the absolute value of an int value. If the argument is not negative, the argument is returned. If the argument is negative, the negation of the argument is returned.

Note that **if the argument is** equal to the value of `Integer.MIN_VALUE`, the most negative representable int value, **the result is that same value**, which is negative.

Caller and Callee disagree

The problem was:

Caller (here `alwaysTrue()`)
had `unfulfilled expectations` about
Callee (here `Math.abs()`).

Example 2: equal Objects in Sets

```
public class Book {  
  
    private String title;  
    private String author;  
    private long isbn;  
  
    public Book(...) { ... }  
}  
  
    public boolean equals(Object obj) {  
        if (obj instanceof Book) {  
            Book other = (Book) obj;  
            return (isbn == other.isbn);  
        }  
        return false;  
    }  
  
    public String toString() { ... }  
}
```

Example 2: equal Objects in Sets

From the Java API Specification, Interface Set:

```
public interface Set  
extends Collection
```

Sets contain **no pair** of elements e_1, e_2 such that
 $e_1.equals(e_2)$...

...

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Sets contain **no pair** of elements e_1, e_2 such that
 $e_1.equals(e_2)$...

...

```
boolean add(E e)
```

Adds e to this set **if** the set contains **no** element **e_2** such that
 $e.equals(e_2)$...

Example 2: equal Objects in Sets

Adding two equal books to a set:

```
Set<Book> catalogue = new HashSet<Book>();
```

```
Book b1 = new Book("Effective_Java",  
                    "Joshua_Bloch",  
                    201310058);
```

```
Book b2 = new Book("Effective_Java",  
                    "J._Bloch",  
                    201310058);
```

```
catalogue.add(b1);  
catalogue.add(b2);
```

How many elements has catalogue now?

Demo: AddTwoBooks.java

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two!(?)

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- ▶ Here, specification of Set or HashSet does not reveal problem
- ▶ Instead: check the specification of Book!
- ▶ Is there any?

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- ▶ Here, specification of Set or HashSet does not reveal problem
- ▶ Instead: check the specification of Book!
- ▶ Is there any?
- ▶ Yes, because Book extends Object, and inherits the specifications from there!

Checking the API of Object

```
public int hashCode()
```

...

If two objects are equal according to the `equals(Object)` method, then calling the `hashCode` method on each of the two objects must produce the same integer result.

...

Checking the API of Object

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...

If two objects are equal according to the `equals(Object)` method, then calling the `hashCode` method on each of the two objects must produce the same integer result.

...

By overriding `equals` only, and not `hashCode`, we broke the specification of `Book.hashCode()`.

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Caller (here `HashSet.add()`)
had **unfulfilled expectations** about
Callee (here `Book.hashCode()`).

Specifications as Contracts

To stress the different roles – obligations – responsibilities in a specification:

Widely used analogy of the *specification as a contract*

“Design by Contract” methodology

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“Design by Contract” methodology

Contract between *caller* and *callee* of method

callee guarantees certain outcome *provided* *caller guarantees prerequisites*

Formal Specifications

Natural language specs are very important (see the examples above).

Still:
we focus on

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Motivation:

- ▶ higher degree of precision.
- ▶ **Automation** of program analysis of various kinds:
 - ▶ formal verification
 - ▶ test case generation

A first glance at Dafny

- ▶ Object oriented, like Java.
- ▶ Designed to make it easy to write correct code.
- ▶ Write specification in formal language (**annotations specifying program behaviour**).
- ▶ **Automatically proves** that the code **matches annotations**.
- ▶ We will look at Dafny briefly in the coming lectures.

Knowledge about formal specification/verification is useful (enables precise thinking), even if you will not regularly use Isabelle/Dafny/etc.

Example: ATM.dfy

```
class ATM {  
  
  // fields:  
  var insertedCard : BankCard;  
  var wrongPINCounter : int;  
  var customerAuthenticated : bool;  
  
  // methods:  
  method insertCard (card : BankCard) { ... }  
  method enterPIN (pin : int) { ... }  
  ...  
  
}
```

Informal Specification

Very informal specification of 'enterPIN (pin:int)':

Enter the PIN that belongs to the currently inserted bank card into the ATM. If a wrong PIN is entered three times in a row, the card is invalidated and confiscated. After having entered the correct PIN, the customer is regarded as authenticated.

Getting More Precise: Specification as Contract

Contract states **what is guaranteed** **under which conditions**.

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Implicit preconditions in natural language spec: inserted card is not null, the card is valid. Should also be formalised!

Question: Identifying pre- and postconditions

The method `insertCard(card:BankCard)` has the following informal specification:

Inserts a bank card into the ATM if the card slot is free and provided the card is valid.

- ▶ Identify at least two **preconditions** and at least one **postcondition**.
- ▶ Optional: think of sensible additional ones, not mentioned explicitly by the informal specification.

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class ATM {  
    var insertedCard : BankCard;  
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class ATM {  
    var insertedCard : BankCard;  
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```

preconditions:

ATM card slot is free.

The card is valid.

(The card is not null)

postconditions:

The ATM card slot is occupied.

(The user is not authenticated.)

(wrongPINCounter is 0.)

How do we express pre- and postconditions formally?

Need a formal language to express:

- ▶ Set of
 - ▶ preconditions
 - ▶ postconditions
- ▶ A language to express these conditions, capturing:
 - ▶ relations, equality, logical connectives
 - ▶ *quantification*

Before diving in to Dafny:

Pause and learn a bit about **First Order Logic**.

Recall: Propositional Logic

A propositional logic formula is built from

- ▶ **Constants:** true, false
- ▶ **Boolean variables:** P, Q, R... (atomic propositions)
- ▶ **Connectives:** \wedge , \vee , \neg , \rightarrow , \leftrightarrow

Connective	Meaning	Dafny
$\neg P$	not P	!P
$P \wedge Q$	P and Q	P && Q
$P \vee Q$	P or Q	P Q
$P \rightarrow Q$	P implies Q	P ==> Q
$P \leftrightarrow Q$	P is equivalent to Q	P <==> Q

Example: *"If you are a bunny, then you eat carrots."*

P: You are a bunny.

Q: You eat carrots.

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Truth table:

P	Q	$P \rightarrow Q$
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A formula F is:

- ▶ **Satisfiable** if F can be true.
- ▶ **Valid** if F is always true.

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Exercise:

Draw the truth-table for $\neg P \vee Q$. Do you notice anything interesting?

Propositional Satisfiability Problem (SAT Solver)

Given propositional logic formula, check whether it is satisfiable, and return a solution if it is.



- ▶ Program that solves whether a formula F satisfiable.
- ▶ can be also used to check for validity of F (if $\neg F$ is not satisfiable).

First-Order Logic (FOL)

Extends propositional logic by:

- ▶ **Types**, other than boolean
e.g. int, real, BankCard,
- ▶ **Functions** (mathematical)
e.g. $+$, max, abs, fibonacci, ...
 - ▶ **Constants** are functions with **no arguments**
e.g. 0, 1, ...
- ▶ **Predicates** (functions returning a boolean)
e.g. isEven, $>$, isPrime...
- ▶ **Quantifiers**
for all (\forall), there exists (\exists)

First Order Logic: Syntax

Terms

$$t ::= x | c | f(t_1, \dots, t_n)$$

x is any variable symbol, c is any constant, f is any function symbol of some arity n .

Formulas

$$\begin{aligned} \phi ::= & P(t_1, \dots, t_n) \\ & | (\phi \wedge \phi) | (\phi \vee \phi) | (\neg \phi) | \dots \\ & | (\forall x : \phi) | (\exists x : \phi) \end{aligned}$$

P is any predicate symbol of some arity n and t_i are terms.

First Order Logic: Terms and Formulas

Terms are built from

- ▶ Functions
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- ▶ Variables
- ▶ E.g. $x + 2$, -5

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FO Formulas are built recursively from atomic formulas and boolean connectives. E.g.

- ▶ $(x < y \wedge x = 4) \rightarrow 0 < (y - 4)$
- ▶ $\forall i : int. isEven(i) \rightarrow isOdd(i + 1)$

Quantifiers

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Example: All entries in the array a are greater than 0

$\forall i : int. 0 \leq i < a.Length \rightarrow a[i] > 0$

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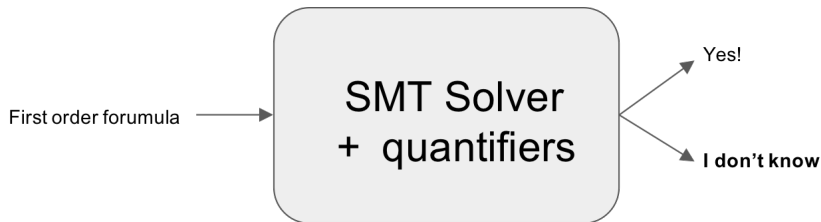
Example: All entries in the array a are greater than 0

$\forall i : int. 0 \leq i < a.Length \rightarrow a[i] > 0$

Example: There is at least one prime number in the array a

$\exists i : int. 0 \leq i < a.Length \wedge isPrime(a[i])$

Satisfiability modulo theories + Quantifiers



- ▶ Semi-decidable problem (often gives good results).

Validity

A first order logic formula is valid if it is true in every interpretation (however we interpret the functions and constants)

Valid Formulas

The following formulas are valid:

1. $\neg(\exists x : t. \phi) \leftrightarrow \forall x : t. \neg\phi$
2. $\neg(\forall x : t. \phi) \leftrightarrow \exists x : t. \neg\phi$
3. $(\forall x : t. \phi \wedge \psi) \leftrightarrow (\forall x : t. \phi) \wedge (\forall x : t. \psi)$
4. $(\exists x : t. \phi \vee \psi) \leftrightarrow (\exists x : t. \phi) \vee (\exists x : t. \psi)$

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Are the following formulas also valid?

- ▶ $(\forall x : t. \phi \vee \psi) \leftrightarrow (\forall x : t. \phi) \vee (\forall x : t. \psi)$
- ▶ $(\exists x : t. \phi \wedge \psi) \leftrightarrow (\exists x : t. \phi) \wedge (\exists x : t. \psi)$

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Are the following formulas also valid?

- ▶ $(\forall x : t. \phi \vee \psi) \leftrightarrow (\forall x : t. \phi) \vee (\forall x : t. \psi)$
 - ▶ No! On the left, each x must make either ϕ or ψ true. On the right, one of ϕ or ψ must hold for every x .
- ▶ $(\exists x : t. \phi \wedge \psi) \leftrightarrow (\exists x : t. \phi) \wedge (\exists x : t. \psi)$

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 - ▶ No! On the left, each x must make either ϕ or ψ true. On the right, one of ϕ or ψ must hold for every x .
- ▶ $(\exists x : t. \phi \wedge \psi) \leftrightarrow (\exists x : t. \phi) \wedge (\exists x : t. \psi)$
 - ▶ No! On the left, must pick same x for ϕ and ψ . On the right, might pick different x for ϕ and ψ .

Formal Specification Examples

```
int[] sort(int[] a)
```

- ▶ **requires:** $a \neq \text{null}$
- ▶ **ensures:** $\text{isSorted}(\text{sort}(a)) \wedge \text{isPermutationOf}(\text{sort}(a), a)$

Formal Specification Examples

`int[] sort(int[] a)`

- ▶ **requires:** $a \neq \text{null}$
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`int binarySearch(int[] a, int elem)`

- ▶ **requires:** $a \neq \text{null} \wedge \text{isSorted}(a)$
- ▶ **ensures:**
 $(\text{result} = -1 \wedge \forall i : \text{int}, 0 \leq i < a.\text{length} \rightarrow a[i] \neq \text{elem})$
 \vee
 $(a[\text{result}] = \text{elem} \wedge \forall i : \text{int}, 0 \leq i < \text{result} \rightarrow a[i] \neq \text{elem})$

We learned...

- ▶ What design by contract is.
- ▶ Pre-conditions and post-conditions.
- ▶ Formal specification: what and why.
- ▶ First-order logic.