Introduction to Language Models

Large Language Models: Introduction and Recent Advances

ELL881 · AlL821



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Mistral Large 2 drops!

Mistral AI announces the release of its 123B model.

Released yesterday July 24, 2024

https://mistral.ai/news/mistrallarge-2407/

Mistral Large 2 supports 11 languages
(French, German, Spanish, Italian,
Portuguese, Arabic, Hindi, Russian,
Chinese, Japanese, and Korean), along
with 80+ coding languages (including
Python, Java, C, C++, JavaScript, and
Bash).

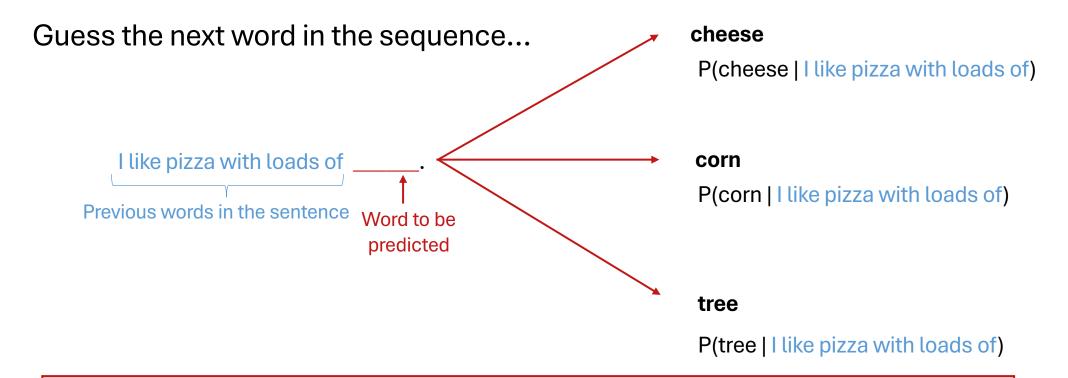


Its performance in **code generation**, **mathematics** and **reasoning** tasks is **comparable to larger LLMs** like GPT4o, Claude 3.5 Sonnet and Llama 3.1(405B).

Mistral Large 2 has a context window of **128k**!

Introduction to Statistical Language Models

Next Word Prediction



P(cheese| I like pizza with loads of) > P(corn| I like pizza with loads of) >> P(tree| I like pizza with loads of)





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Speech Recognition

- P(I bought fresh mangoes from the market) >> P(I bot fresh man goes from the mar kit)
- P(I love eating spicy samosas) >> P(eye love eat tin spy sea some o says)





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- P(Heavy rainfall) >> P(Big rainfall)
- P(The festival of lights) >> P(the festival of lamps)
- P(Family gatherings) > P(Family meetings)





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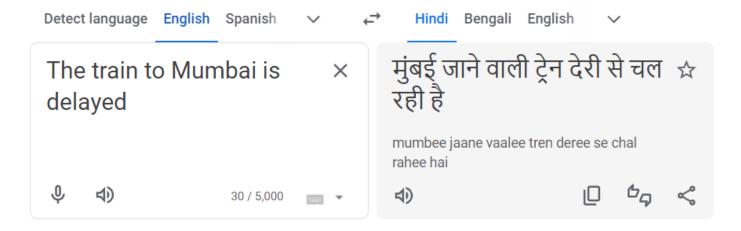
Machine Translation

- P(Heavy rainfall) >> P(Big rainfall)
- P(The festival of lights) >> P(the festival of lamps)
- P(Family gatherings) > P(Family meetings)
- Context Sensitive Spelling Correction
- Natural Language Generation
- •





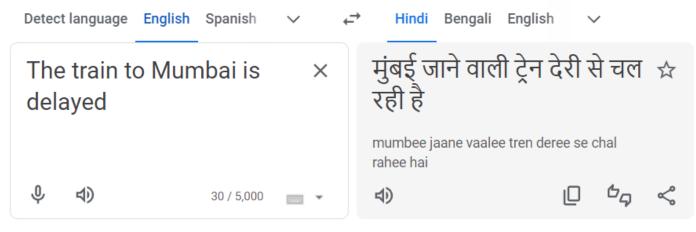
Language Models Are Everywhere





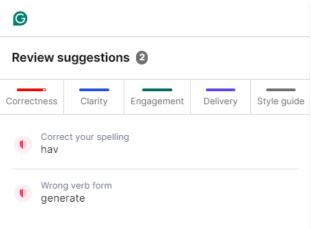


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Large Language Models (LLMs) hav revolutionized the field of natural language processing. LLMs, such as GPT-3, have demonstrated impressive capabilities in understanding and generate humanlike text across various natural language

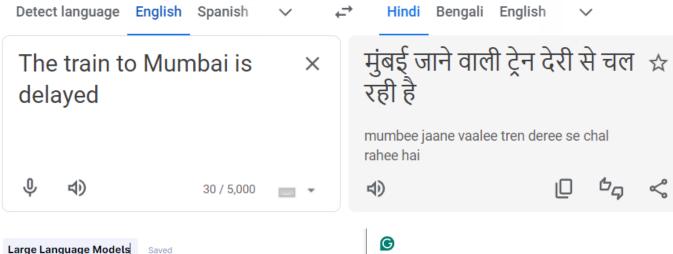




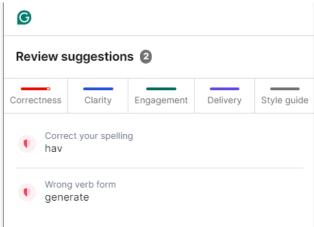
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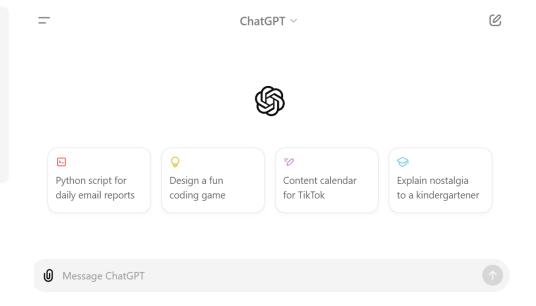


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applications.



Probabilistic Language Models

• **Goal:** Calculate the probability of a sentence or sequence consisting of *n* words

$$P(W) = P(W_1, W_2, W_3, ..., W_n)$$

or

 Related Task: Calculate the probability of the next word conditioned on the preceding words

$$P(W_6 | W_1, W_2, W_3, W_4, W_5)$$



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A model that calculates either of these is referred to as a **Language Model (LM).**





Probability of a Sentence

Let's consider the following sentence:

The monsoon season has begun

How to compute the probability of the sentence?

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P(W) = P("The monsoon season has begun")
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= P(The, monsoon, season, has, begun)





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We compute the above joint probability by using the principles of

Chain Rule of Probability.





Chain Rule of Probability

Definition of conditional probability:

$$P(A|B) = P(A,B) / P(B)$$

Rewriting: P(A, B) = P(A|B)P(B)



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• More variables: $P(A,B,C,D) = P(A) \cdot P(B \mid A) \cdot P(C \mid A,B) \cdot P(D \mid A,B,C)$





Chain Rule of Probability

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- More variables: P(A,B,C,D) = P(A) . P(B | A) . P(C | A,B) . P(D | A,B,C)
- The **Chain Rule** in general:

$$P(x_1, x_2, x_3, ..., x_n) = P(x_1) P(x_2 | x_1) P(x_3 | x_1, x_2) ... P(x_n | x_1, ..., x_{n-1})$$





Probability of a Sequence

$$P(w_1w_2...w_n) = \prod_i P(w_i | w_1w_2...w_{i-1})$$

- P(W) = P("The monsoon season has begun")
 - = P(The, monsoon, season, has, begun)
 - = P(The) x P(monsoon | The) x P(season | The monsoon) x P(has | The monsoon season) x P(begun | The monsoon season has)



Estimate Conditional Probabilities

P(begun | The monsoon season has) = $\frac{\text{Count (The monsoon season has begun)}}{\text{Count (The monsoon season has)}}$





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• **Problem:** Enough data is not available to get an accurate estimate of the above quantities.





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- Solution: Markov Assumption





Markov Assumption

Every next state depends only the previous k states





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Applying Markov Assumption we condition on only the preceding k words:

$$P(w_1w_2...w_n) = \prod_i P(w_i|w_{i-k}...w_{i-1})$$



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Chain Rule:

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 Probabilistic Language Models exploit the Chain Rule of Probability and Markov Assumption to build a probability distribution over sequences of words.



N-gram Language Models

• Let's consider the following conditional probability:

P(begun | the monsoon season has)

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 - Trigram: P(begun | the monsoon)



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Relation between Markov model and Language Model:

An N-gram Language Model ≡ (N -1) order Markov Model







Raw bigram counts (absolute measure)

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

Raw unigram counts (absolute measure)

i	want	to	eat	chinese	food	lunch	spend
2533	927	2417	746	158	1093	341	278

Unigram and bigram counts for eight of the words (out of V = 1446) in the Berkeley Restaurant Project corpus of 9332 sentences. Previously-zero counts are in gray.







Raw bigram counts (absolute measure)

		i	want	to	eat		chin	nese f		od	lunch	sp	end		
	i	5	827	0	9		0	0			0 2				
	want	i			want	to		eat chine		se food		lunc	h	spend	
	to	i	0.0	02	0.33	0.66		0.0020				0			0.00079
	eat	want	0.0	022	0							.0065	0.0054		0.0011
	chinese	to	0.0	0083	0	0.0	0.0017			0.00083			0.00	25	0.087
	food	eat	0		0	0.00		0	0.021		0	.0027	0.05	6	0
	lunch	chine	ese 0.0	063	0	0		0		0	0	.52	0.0063		0
	spend	food 0.0		14	0	0.0	14	0	0.0009		92 0	0.0037 0			0
Raw unigram counts		luncl	o .0	059	0	0		0		0	0	.0029	0		0
		spen	d 0.0	036	0	0.0	036	0		0	0		0		0
i	want	to	e	at	chine	se	fc	ood 1		lunch		end			
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Limitation of N-gram Language Models

• An insufficient model of language since they are **not effective in capturing long-range dependencies present in language**.



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Example:

The **project**, which he had been working on for months, was finally **approved** by the committee.

The above example highlights the long-distance dependency between "project" and "approved", where the context provided by earlier words affects the interpretation of later parts of the sentence.





Estimate N-gram Probabilities

- Maximum Likelihood Estimate (MLE):
 - Used to estimate the parameters of a statistical model
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- Maximum Likelihood Estimate (MLE):
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- For example, bigram probabilities can be estimated as follows:

$$P(w_{i} | w_{i-1}) = \frac{count(w_{i-1}, w_{i})}{count(w_{i-1})} = \frac{c(w_{i-1}, w_{i})}{c(w_{i-1})}$$



Limitations with MLE Estimation

Problem: N-grams only work well for word prediction if the test corpus looks like the training corpus. It is often not the case in real scenarios (data sparsity problem).





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Training set:

- ... enjoyed the movie
- ... enjoyed the food
- ... enjoyed the game
- ... enjoyed the vacation

Test set:

- ... enjoyed the concert
- ... enjoyed the festival
- ... enjoyed the walk

Zero probability n-grams:

- P(concert | enjoyed the) = P(festival | enjoyed the) = P(walk | enjoyed the) = 0
- As a result, the probability of the test set will be 0.
- Perplexity cannot be computed (Cannot divide by 0).





Limitations with MLE Estimation

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

Solution: Various smoothing techniques







Laplace Smoothing (Add-One Estimation)

- Imagine that we encountered each word (N-gram) one more time than its actual occurrence.
- Simply increase all the counts by one!



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- MLE estimate (in case of bigram model)

$$P_{MLE}(W_i \mid W_{i-1}) = \frac{C(W_{i-1}, W_i)}{C(W_{i-1})}$$

Add-1 estimate:

$$P_{Add-1}(W_i \mid W_{i-1}) = \frac{C(W_{i-1}, W_i) + 1}{C(W_{i-1}) + |V|}$$



Laplace Smoothing (Add-One Estimation)

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- Simply increase all the counts by one!
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Add-1 estimate:

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Effective bigram count (c*(w_{n-1}w_n)):

$$\frac{C^*(W_{n-1}W_n)}{C(W_{n-1})} = \frac{C(W_{n-1},W_n) + 1}{C(W_{n-1}) + |V|}$$





	i	want	to	eat	chinese	food	lunch	spend
i	6	828	1	10	1	1	1	3
want	3	1	609	2	7	7	6	2
to	3	1	5	687	3	1	7	212
eat	1	1	3	1	17	3	43	1
chinese	2	1	1	1	1	83	2	1
food	16	1	16	1	2	5	1	1
lunch	3	1	1	1	1	2	1	1
spend	2	1	2	1	1	1	1	1

Add-one smoothed bigram counts for eight of the words (out of V = 1446) in the Berkeley Restaurant Project corpus of 9332 sentences. Previously-zero counts are in gray.

Example from Speech and Language Processing book by Daniel Jurafsky and James H. Martin







	i	want	to	eat	chinese	food	lunch	spend
i	0.0015	0.21	0.00025	0.0025	0.00025	0.00025	0.00025	0.00075
want	0.0013	0.00042	0.26	0.00084	0.0029	0.0029	0.0025	0.00084
to	0.00078	0.00026	0.0013	0.18	0.00078	0.00026	0.0018	0.055
eat	0.00046	0.00046	0.0014	0.00046	0.0078	0.0014	0.02	0.00046
chinese	0.0012	0.00062	0.00062	0.00062	0.00062	0.052	0.0012	0.00062
food	0.0063	0.00039	0.0063	0.00039	0.00079	0.002	0.00039	0.00039
lunch	0.0017	0.00056	0.00056	0.00056	0.00056	0.0011	0.00056	0.00056
spend	0.0012	0.00058	0.0012	0.00058	0.00058	0.00058	0.00058	0.00058

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	i	want	to	eat	chinese	food	lunch	spend
i	3.8	527	0.64	6.4	0.64	0.64	0.64	1.9
want	1.2	0.39	238	0.78	2.7	2.7	2.3	0.78
to	1.9	0.63	3.1	430	1.9	0.63	4.4	133
eat	0.34	0.34	1	0.34	5.8	1	15	0.34
chinese	0.2	0.098	0.098	0.098	0.098	8.2	0.2	0.098
food	6.9	0.43	6.9	0.43	0.86	2.2	0.43	0.43
lunch	0.57	0.19	0.19	0.19	0.19	0.38	0.19	0.19
spend	0.32	0.16	0.32	0.16	0.16	0.16	0.16	0.16

Add-one reconstituted counts for eight words (of V = 1446) in the BeRP corpus of 9332 sentences. Previously-zero counts are in gray.

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food	6.9	0.43	6.9	0.43	0.86	2.2	0.43	0.43
lunch	0.57	0.19	0.19	0.19	0.19	0.38	0.19	0.19
spend	0.32	0.16	0.32	0.16	0.16	0.16	0.16	0.16





More General Smoothing Techniques

Add-k smoothing:

$$P_{Add-k}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) + k}{c(w_{i-1}) + k|V|}$$

$$P_{Add-k}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) + m(\frac{1}{|V|})}{c(w_{i-1}) + m}$$



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Unigram prior smoothing:

$$P_{\text{UnigramPrior}}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) + m P(w_i)}{c(w_{i-1}) + m}$$



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An optimal value for k or m can be determined using a held-out dataset.



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 - Opt for a trigram when there is sufficient evidence, otherwise use bigram, otherwise unigram



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Interpolation:

- Mix unigram, bigram, trigram
- Interpolation generally results in improved performance





Interpolation

Linear interpolation

$$\hat{P}(w_n|w_{n-2}w_{n-1}) = \lambda_1 P(w_n|w_{n-2}w_{n-1}) + \lambda_2 P(w_n|w_{n-1}) + \lambda_3 P(w_n)$$

$$\sum_{i} \lambda_i = 1$$

Context-dependent interpolation

$$\hat{P}(w_n|w_{n-2}w_{n-1}) = \lambda_1(w_{n-2}^{n-1})P(w_n|w_{n-2}w_{n-1})
+ \lambda_2(w_{n-2}^{n-1})P(w_n|w_{n-1})
+ \lambda_3(w_{n-2}^{n-1})P(w_n)$$



