Introduction to
Statistical Language Models

Tanmoy Chakraborty
Associate Professor, IIT Delhi
https://tanmoychak.com/

Introduction to Large Language Models

Next Word Prediction

Guess the next word in the sequence...

$\underbrace{\text{I like pizza with loads of}}_{\text{Previous words in the sentence}}$ _______.

Word to be predicted

cheese

P(cheese | I like pizza with loads of)

corn

P(corn | I like pizza with loads of)

tree

P(tree | I like pizza with loads of)

P(cheese| I like pizza with loads of) > **P(corn| I like pizza with loads of)** >> **P(tree| I like pizza with loads of)**

Probabilistic Language Models: Applications

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  - P(Heavy rainfall) >> P(Big rainfall)
  - P(The festival of lights) >> P(the festival of lamps)
  - P(Family gatherings) > P(Family meetings)

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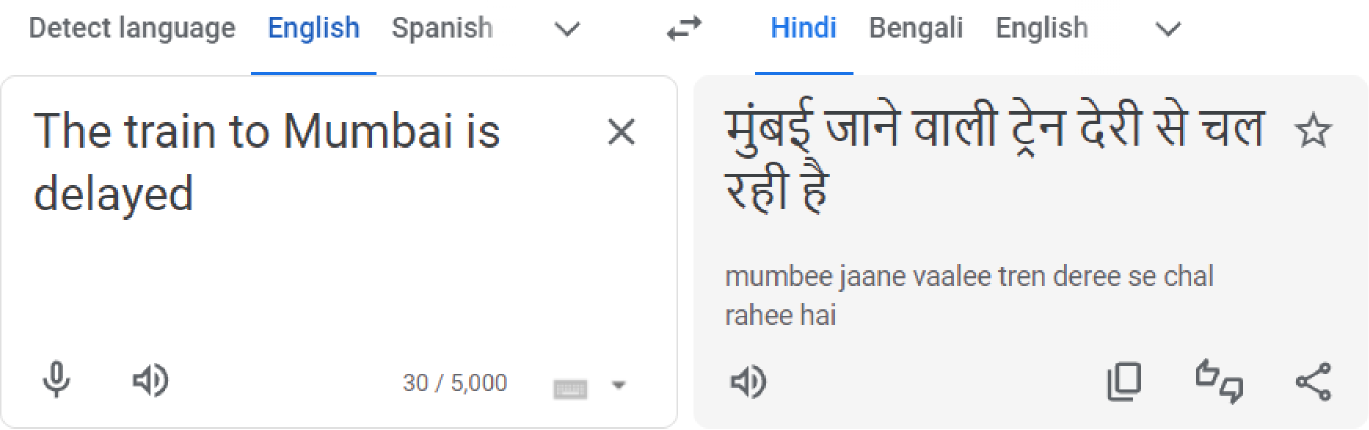
  - P(Family gatherings) > P(Family meetings)
- **Context Sensitive Spelling Correction**
- **Natural Language Generation**
- …

Language Models Are Everywhere



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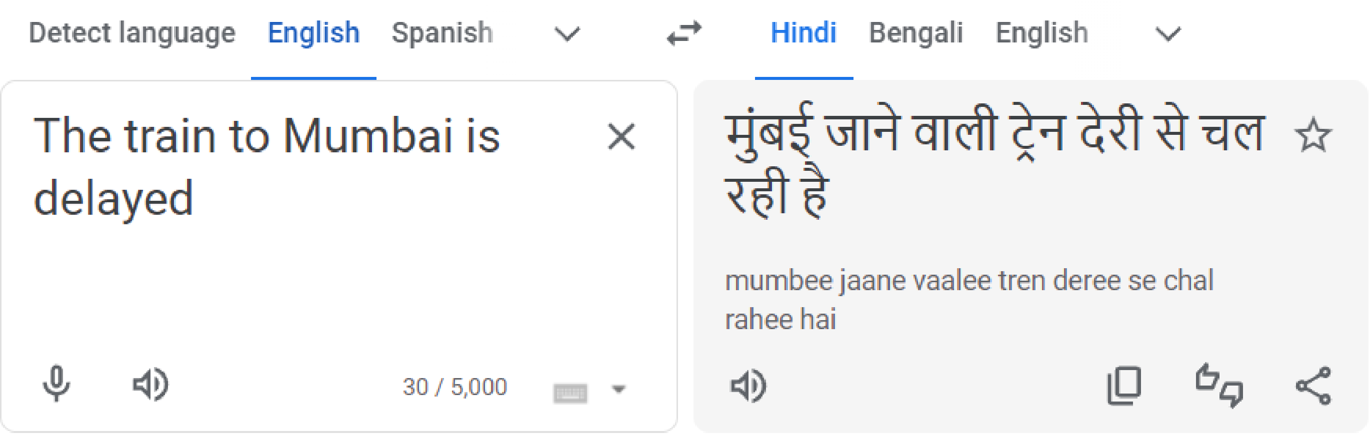

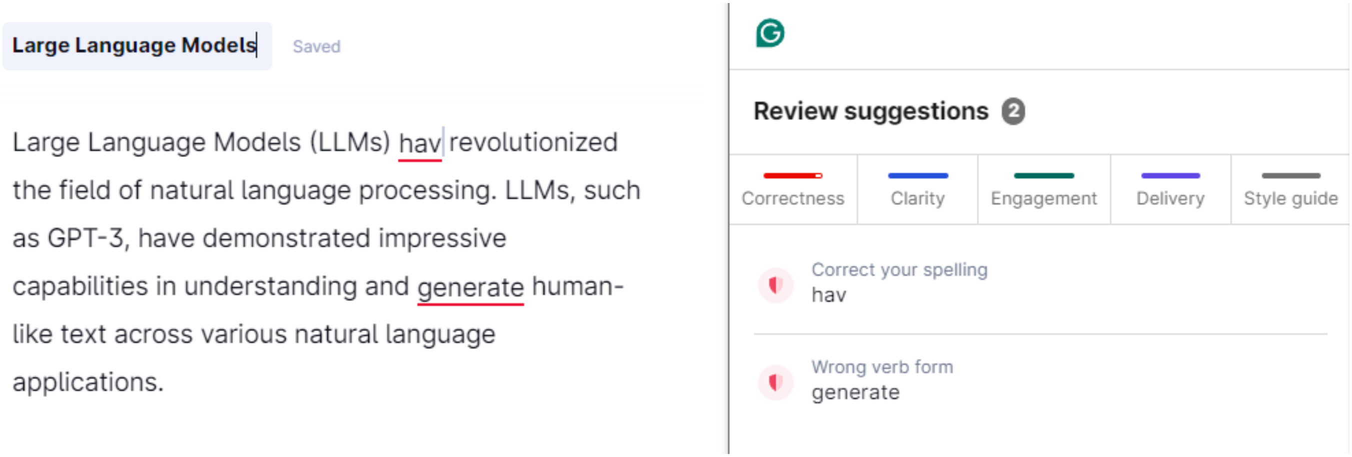

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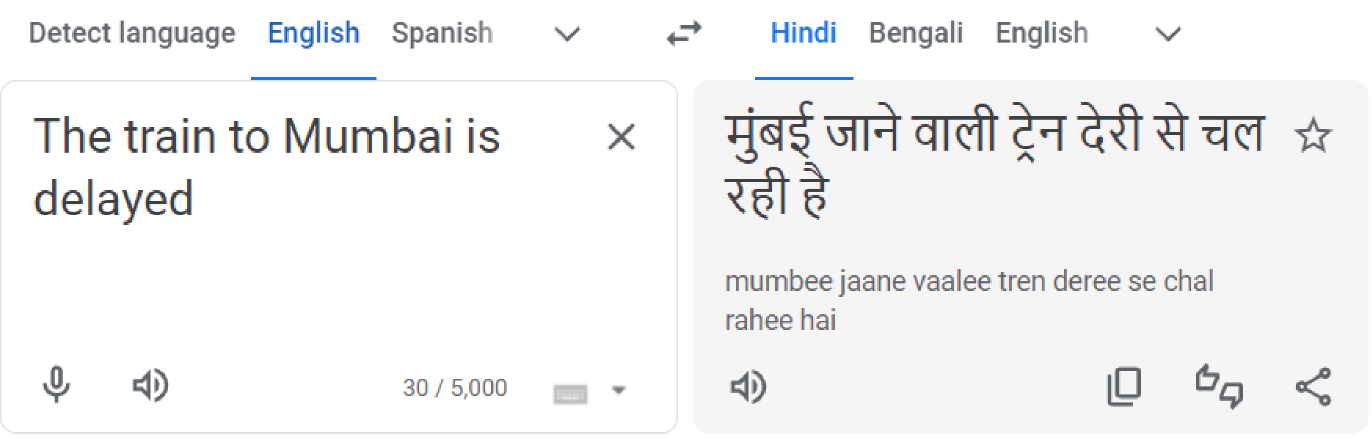

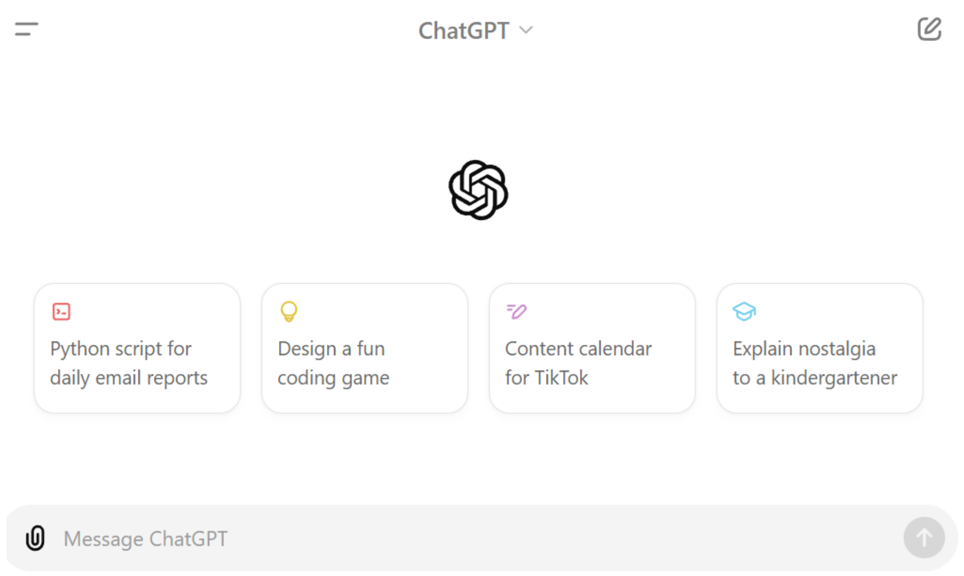

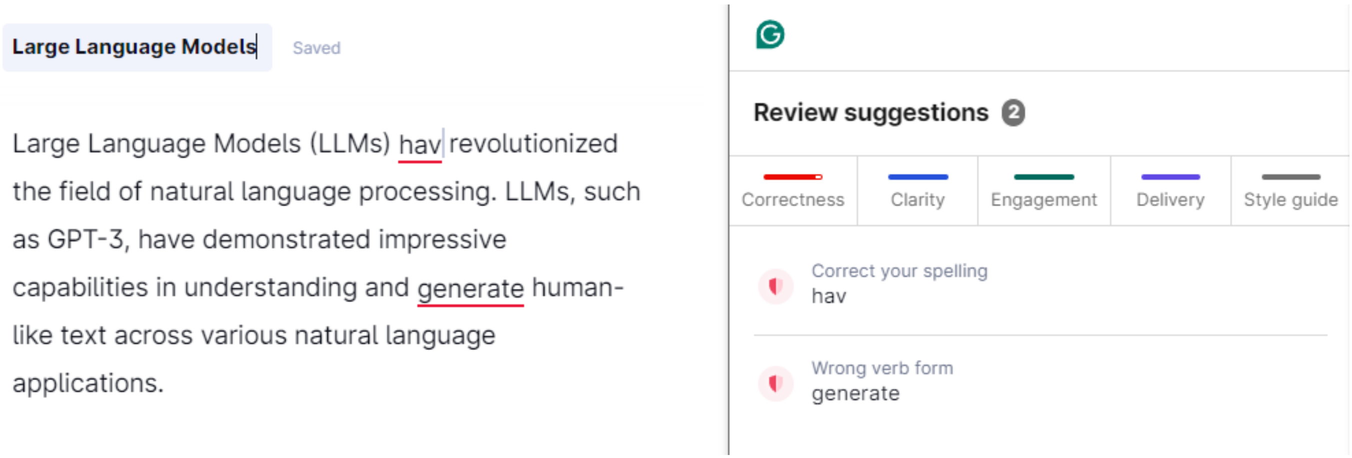

Probabilistic Language Models

- **Goal:** Calculate the probability of a sentence or sequence consisting of *n* words

$$P(W) = P(w_1, w_2, w_3, \dots, w_n)$$

or

- **Related Task:** Calculate the probability of the next word conditioned on the preceding words

$$P(w_6 \mid w_1, w_2, w_3, w_4, w_5)$$

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A model that calculates either of these is referred to as a **Language Model (LM).**

Probability of a Sentence

Let's consider the following sentence:

The monsoon season has begun

- How to compute the probability of the sentence?

$$
\begin{aligned}
P(W) &= P(\text{“The monsoon season has begun”}) \\
&= P(\text{The, monsoon, season, has, begun})
\end{aligned}
$$

Probability of a Sentence

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- How to compute the probability of the sentence?

$$P(W) = P(\text{"The monsoon season has begun"})$$
$$= P(\text{The, monsoon, season, has, begun})$$

We compute the above joint probability by using the principles of

Chain Rule of Probability.

Chain Rule of Probability

- Definition of **conditional probability**:

$$P(A \mid B) = P(A, B) / P(B)$$

Rewriting: $P(A, B) = P(A \mid B)\, P(B)$

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- More variables: $P(A,B,C,D) = P(A) \cdot P(B \mid A) \cdot P(C \mid A,B) \cdot P(D \mid A,B,C)$

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- The **Chain Rule** in general:

$$P(x_1, x_2, x_3, \ldots, x_n) = P(x_1)\, P(x_2 \mid x_1)\, P(x_3 \mid x_1,x_2) \ldots P(x_n \mid x_1,\ldots,x_{n-1})$$

Probability of a Sequence

$$P(w_1 w_2 \ldots w_n) = \prod_i P(w_i \mid w_1 w_2 \ldots w_{i-1})$$

P(W) = P("The monsoon season has begun")

= P(The, monsoon, season, has, begun)

= P(The) x P(monsoon | The) x P(season | The monsoon) x P(has | The monsoon season) x P(begun | The monsoon season has)

Estimate Conditional Probabilities

$$P(\text{begun} \mid \text{The monsoon season has}) = \frac{\text{Count (The monsoon season has begun)}}{\text{Count (The monsoon season has)}}$$

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- **Solution:** Markov Assumption

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Every next state depends only the previous k states

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- Applying Markov Assumption we condition on only the preceding k words:

$$P(w_1 w_2 \ldots w_n) = \prod_i P(w_i | w_{i-k} \ldots w_{i-1})$$

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- Probabilistic Language Models exploit the **Chain Rule of Probability** and **Markov Assumption** to build a probability distribution over sequences of words.

N-gram Language Models

- Let's consider the following conditional probability:

$$P(\text{begun} \mid \text{the monsoon season has})$$

- An **N-gram model** considers only the preceding **N −1 words**.

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P(begun | the monsoon season has)

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  - Unigram: P(begun)
  - Bigram: P(begun | has)
  - Trigram: P(begun | season has)

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Relation between Markov model and Language Model:

An N-gram Language Model ≡ (N −1) order Markov Model

Raw bigram counts (absolute measure)

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

Raw unigram counts (absolute measure)

i	want	to	eat	chinese	food	lunch	spend
2533	927	2417	746	158	1093	341	278

Unigram and bigram counts for eight of the words (out of V = 1446) in the Berkeley Restaurant Project corpus of 9332 sentences. Previously-zero counts are in gray.

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	i	want	to	eat	chinese	food	lunch	spend
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want								
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	i	**want**	**to**	**eat**	**chinese**	**food**	**lunch**	**spend**
i	0.002	0.33	0	0.0036	0	0	0	0.00079
want	0.0022	0	0.66	0.0011	0.0065	0.0065	0.0054	0.0011
to	0.00083	0	0.0017	0.28	0.00083	0	0.0025	0.087
eat	0	0	0.0027	0	0.021	0.0027	0.056	0
chinese	0.0063	0	0	0	0	0.52	0.0063	0
food	0.014	0	0.014	0	0.00092	0.0037	0	0
lunch	0.0059	0	0	0	0	0.0029	0	0
spend	0.0036	0	0.0036	0	0	0	0	0

Raw unigram counts (

i	want	to	eat	chinese	food	lunch	spend
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Limitation of N-gram Language Models

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  - Example:

    The **project**, which he had been working on for months, was finally **approved** by the committee.

  The above example highlights the long-distance dependency between “project” and “approved”, where the context provided by earlier words affects the interpretation of later parts of the sentence.

Estimate N-gram Probabilities

- Maximum Likelihood Estimate (MLE):
  - Used to estimate the parameters of a statistical model
  - Determine the most likely values of the parameters that would make the observed data most probable

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- For example, bigram probabilities can be estimated as follows:

$$P(w_i \mid w_{i-1}) = \frac{count(w_{i-1}, w_i)}{count(w_{i-1})} = \frac{c(w_{i-1}, w_i)}{c\,(w_{i-1})}$$

Limitations with MLE Estimation

Problem: N-grams only work well for word prediction if the test corpus looks like the training corpus. It is often not the case in real scenarios (data sparsity problem).

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Training set:

- ... enjoyed the movie
- ... enjoyed the food
- ... enjoyed the game
- ... enjoyed the vacation

Test set:

- ... enjoyed the concert
- ... enjoyed the festival
- ... enjoyed the walk

Zero probability n-grams:

- P(concert | enjoyed the) = P(festival | enjoyed the) = P(walk | enjoyed the) = 0
- As a result, the probability of the test set will be 0.
- Perplexity cannot be computed (Cannot divide by 0).

Limitations with MLE Estimation

	i	want	to	eat	chinese	food	lunch	spend
i	5	827	0	9	0	0	0	2
want	2	0	608	1	6	6	5	1
to	2	0	4	686	2	0	6	211
eat	0	0	2	0	16	2	42	0
chinese	1	0	0	0	0	82	1	0
food	15	0	15	0	1	4	0	0
lunch	2	0	0	0	0	1	0	0
spend	1	0	1	0	0	0	0	0

Solution: Various smoothing techniques

Laplace Smoothing (Add-One Estimation)

- Imagine that we encountered each word (N-gram) one more time than its actual occurrence.
- Simply increase all the counts by one!

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- Simply increase all the counts by one!
- MLE estimate (in case of bigram model)

$$P_{MLE}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c\,(w_{i-1})}$$

- Add-1 estimate:

$$P_{Add\text{-}1}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) + 1}{c\,(w_{i-1}) + |V|}$$

Laplace Smoothing (Add-One Estimation)

- Imagine that we encountered each word (N-gram) one more time than its actual occurrence.
- Simply increase all the counts by one!
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$$P_{MLE}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i)}{c\,(w_{i-1})}$$

- Add-1 estimate:

$$P_{Add\text{-}1}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) + 1}{c\,(w_{i-1}) + |V|}$$

- Effective bigram count ($c^*(w_{n-1}w_n)$):

$$\frac{c^*(w_{n-1}w_n)}{c(w_{n-1})} = \frac{c(w_{n-1}, w_n) + 1}{c\,(w_{n-1}) + |V|}$$

Comparing with Bigrams: Before and After Smoothing

	i	want	to	eat	chinese	food	lunch	spend
i	6	828	1	10	1	1	1	3
want	3	1	609	2	7	7	6	2
to	3	1	5	687	3	1	7	212
eat	1	1	3	1	17	3	43	1
chinese	2	1	1	1	1	83	2	1
food	16	1	16	1	2	5	1	1
lunch	3	1	1	1	1	2	1	1
spend	2	1	2	1	1	1	1	1

Add-one smoothed bigram counts for eight of the words (out of V = 1446) in the Berkeley Restaurant Project corpus of 9332 sentences. Previously-zero counts are in gray.

Example from Speech and Language Processing book by Daniel Jurafsky and James H. Martin

Comparing with Bigrams: Before and After Smoothing

	i	want	to	eat	chinese	food	lunch	spend
i	0.0015	0.21	0.00025	0.0025	0.00025	0.00025	0.00025	0.00075
want	0.0013	0.00042	0.26	0.00084	0.0029	0.0029	0.0025	0.00084
to	0.00078	0.00026	0.0013	0.18	0.00078	0.00026	0.0018	0.055
eat	0.00046	0.00046	0.0014	0.00046	0.0078	0.0014	0.02	0.00046
chinese	0.0012	0.00062	0.00062	0.00062	0.00062	0.052	0.0012	0.00062
food	0.0063	0.00039	0.0063	0.00039	0.00079	0.002	0.00039	0.00039
lunch	0.0017	0.00056	0.00056	0.00056	0.00056	0.0011	0.00056	0.00056
spend	0.0012	0.00058	0.0012	0.00058	0.00058	0.00058	0.00058	0.00058

Add-one smoothed bigram probabilities for eight of the words (out of V = 1446) in the BeRP corpus of 9332 sentences. Previously-zero probabilities are in gray.

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Comparing with Bigrams: Before and After Smoothing

	i	want	to	eat	chinese	food	lunch	spend
i	3.8	527	0.64	6.4	0.64	0.64	0.64	1.9
want	1.2	0.39	238	0.78	2.7	2.7	2.3	0.78
to	1.9	0.63	3.1	430	1.9	0.63	4.4	133
eat	0.34	0.34	1	0.34	5.8	1	15	0.34
chinese	0.2	0.098	0.098	0.098	0.098	8.2	0.2	0.098
food	6.9	0.43	6.9	0.43	0.86	2.2	0.43	0.43
lunch	0.57	0.19	0.19	0.19	0.19	0.38	0.19	0.19
spend	0.32	0.16	0.32	0.16	0.16	0.16	0.16	0.16

Add-one reconstituted counts for eight words (of V = 1446) in the BeRP corpus of 9332 sentences. Previously-zero counts are in gray.

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Comparing with Bigrams: Before and After Smoothing

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lunch	0.57	0.19	0.19	0.19	0.19	0.38	0.19	0.19
spend	0.32	0.16	0.32	0.16	0.16	0.16	0.16	0.16

More General Smoothing Techniques

- **Add-k smoothing:**

$$P_{Add\text{-}k}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) + k}{c(w_{i-1}) + k|V|}$$

$$P_{Add\text{-}k}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) + m(\frac{1}{|V|})}{c(w_{i-1}) + m}$$

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$$P_{\text{Add-k}}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) + k}{c\,(w_{i-1}) + k|V|}$$

$$P_{\text{Add-k}}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) + m(\frac{1}{|V|})}{c\,(w_{i-1}) + m}$$

- **Unigram prior smoothing:**

$$P_{\text{UnigramPrior}}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}, w_i) + m\,P(w_i)}{c\,(w_{i-1}) + m}$$

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An optimal value for *k* or *m* can be determined using a held-out dataset.

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- When we have limited knowledge about larger contexts, it can be helpful to consider less context.
- **Back-off:**
  - Opt for a trigram when there is sufficient evidence, otherwise use bigram, otherwise unigram
- **Interpolation:**
  - Mix unigram, bigram, trigram
  - Interpolation generally results in improved performance

Interpolation

Linear interpolation

$$\hat{P}(w_n|w_{n-2}w_{n-1}) = \lambda_1 P(w_n|w_{n-2}w_{n-1}) + \lambda_2 P(w_n|w_{n-1}) + \lambda_3 P(w_n)$$

$$\sum_i \lambda_i = 1$$

Context-dependent interpolation

$$\hat{P}(w_n|w_{n-2}w_{n-1}) = \lambda_1(w_{n-2}^{n-1})P(w_n|w_{n-2}w_{n-1}) + \lambda_2(w_{n-2}^{n-1})P(w_n|w_{n-1}) + \lambda_3(w_{n-2}^{n-1})P(w_n)$$

Advanced Smoothing Algorithms

- Naïve smoothing algorithms have limited usage and are not very effective. Not frequently used for N-grams.

- However, they can be used in domains where the number of zeros isn't so huge.