

# Introduction to Transformer

## Part-1

Tanmoy Chakraborty  
Associate Professor, IIT Delhi  
<https://tanmoychak.com/>

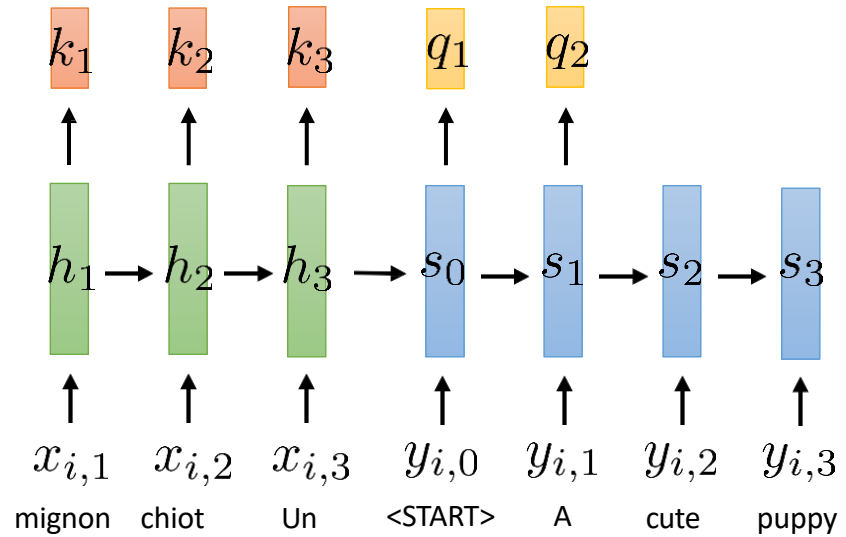


**Introduction to Large Language Models**

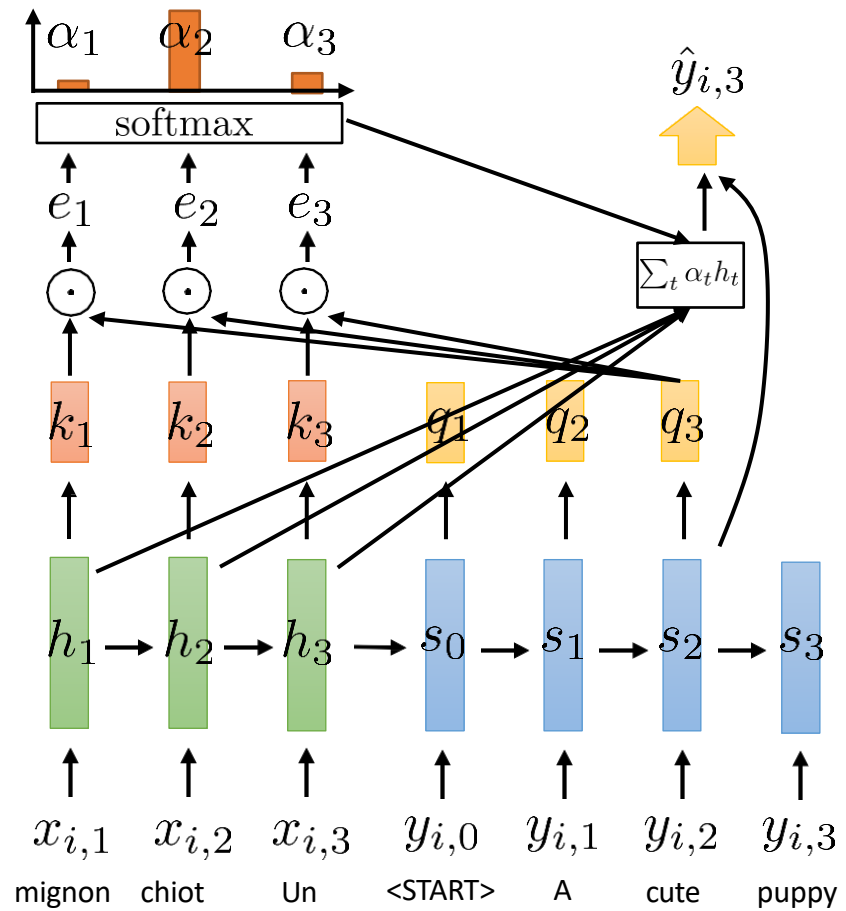


# Is Attention All We Need?

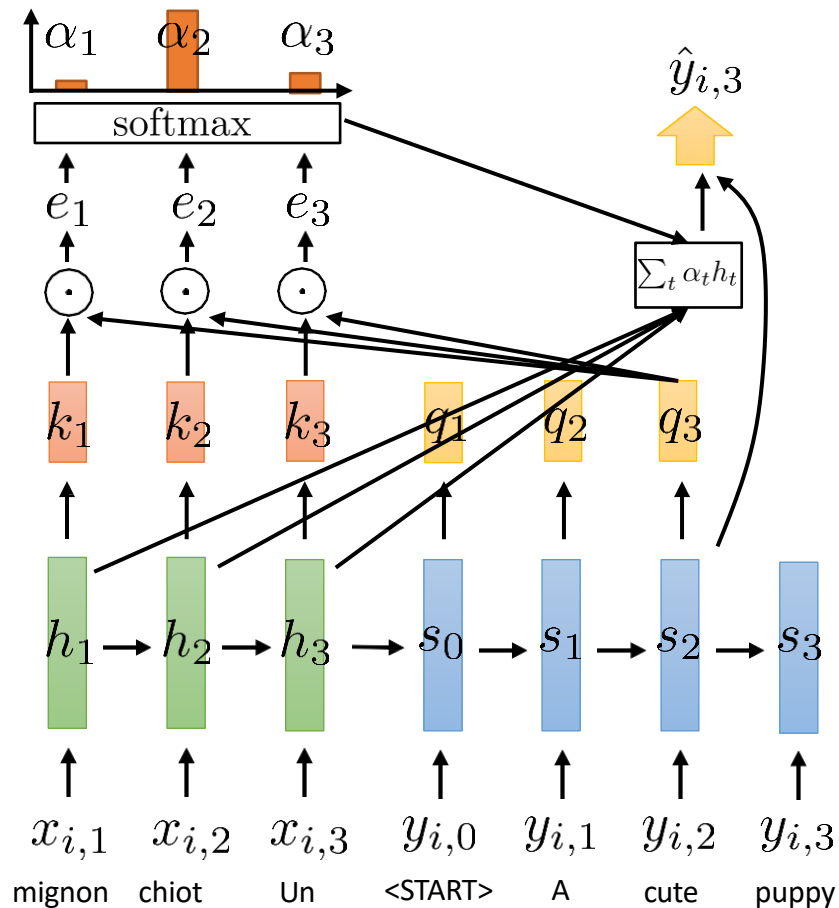
# Recap: Attention



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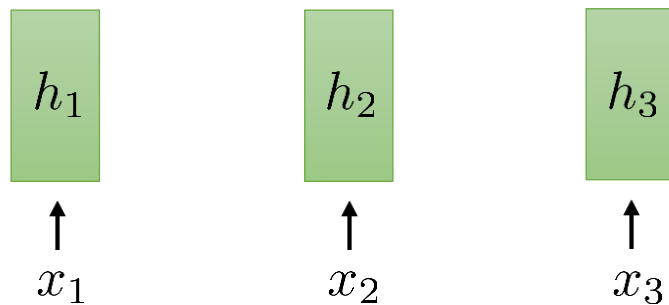
# Recap: Attention



- If we have **attention**, do we even need recurrent connections?
- Can we transform our RNN into a **purely attention-based model**?
- Attention can access all time steps simultaneously, potentially doing everything that recurrence can, and even more. However, this approach presents some challenges:

The encoder lacks temporal dependencies at all!

# Self-Attention



this is *not* a recurrent model!

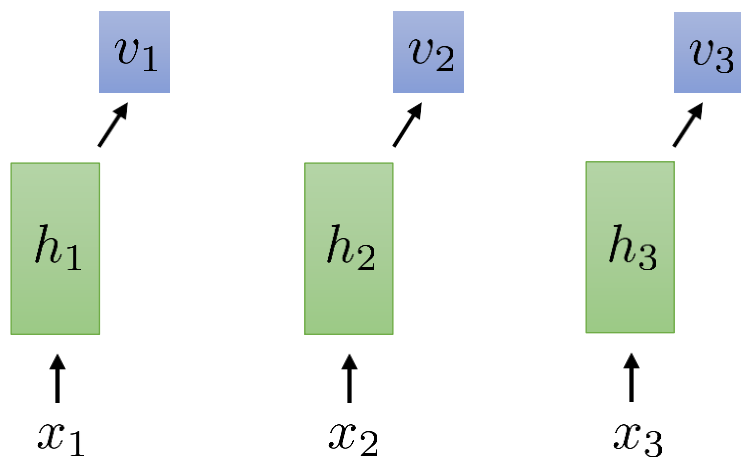
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$$h_t = \sigma(Wx_t + b)$$

shared weights at all time steps

(or any other nonlinear function)

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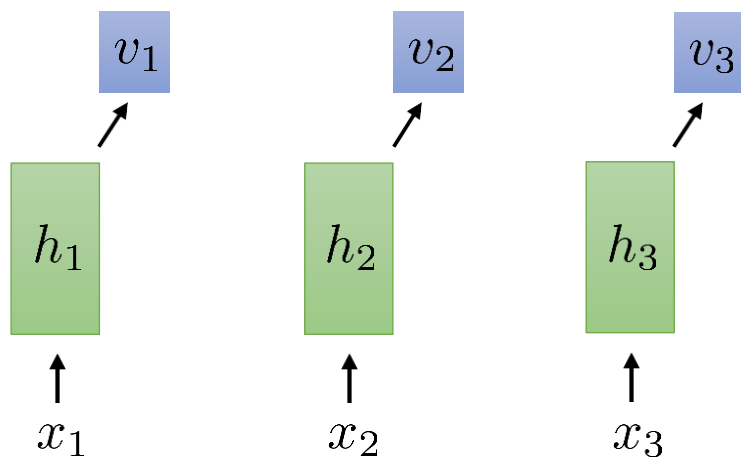
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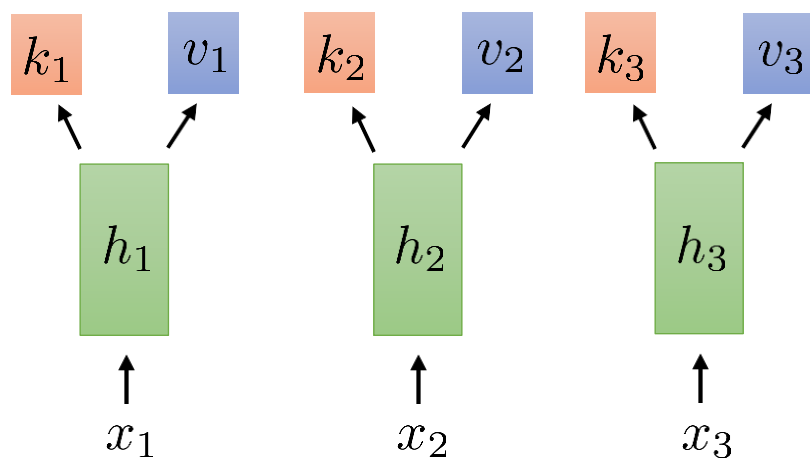
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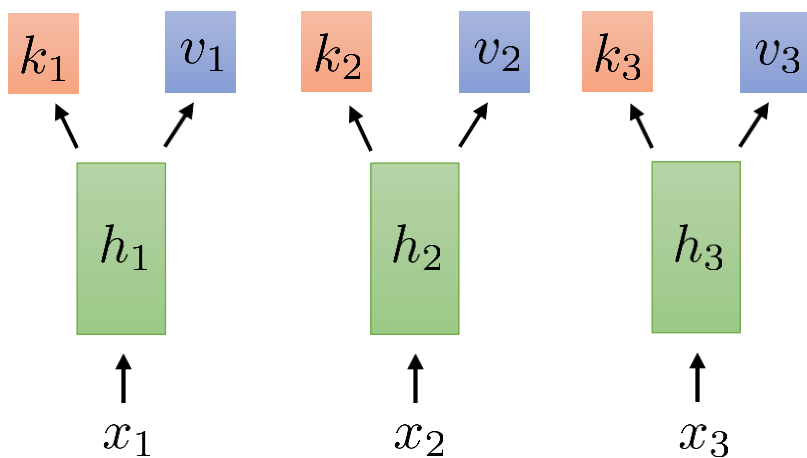
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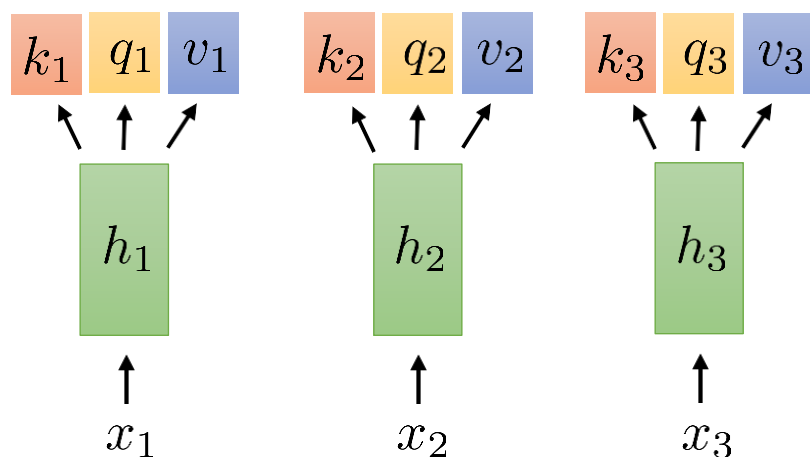
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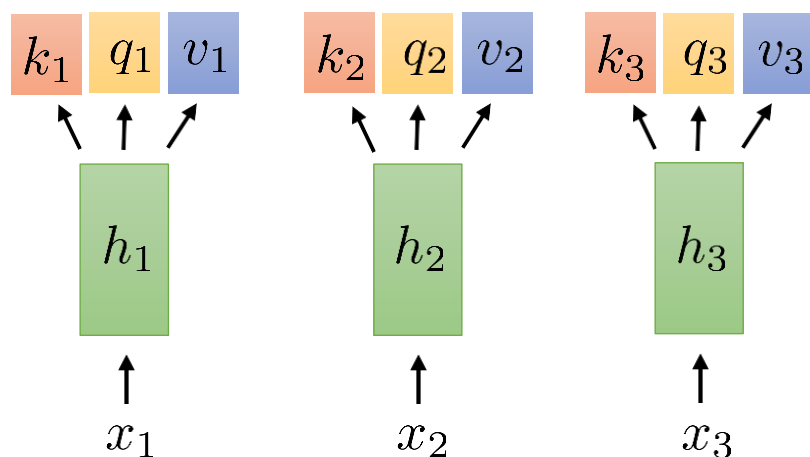
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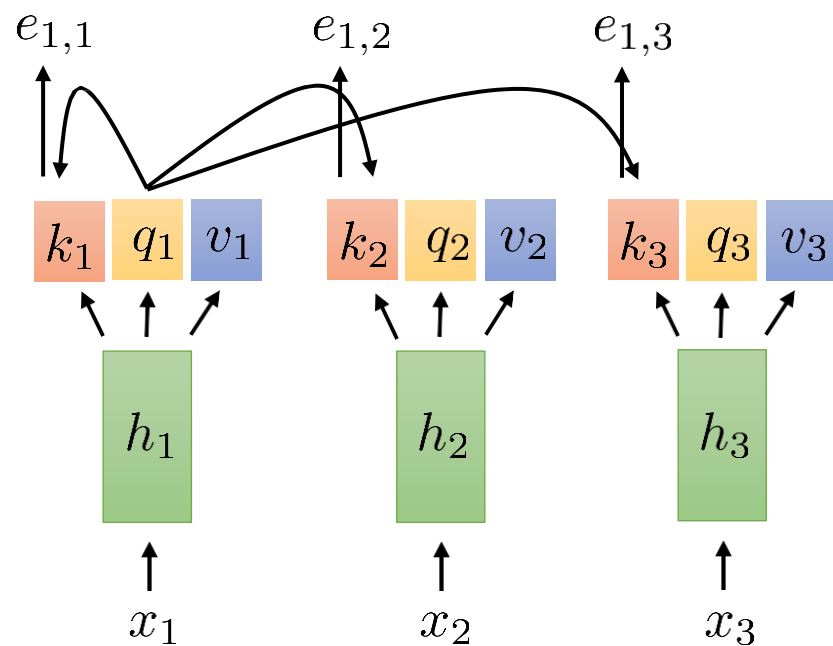
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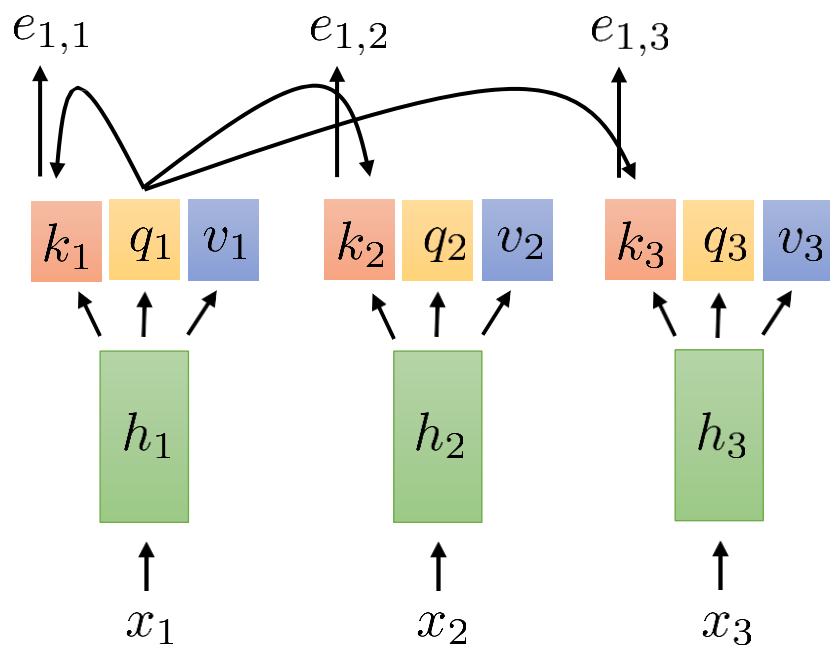
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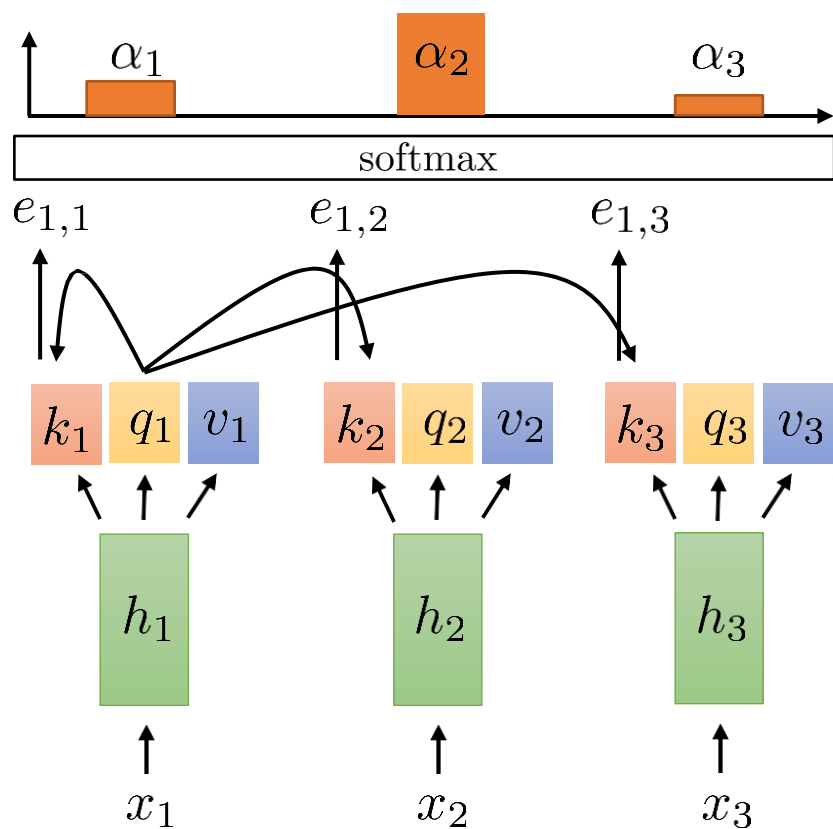
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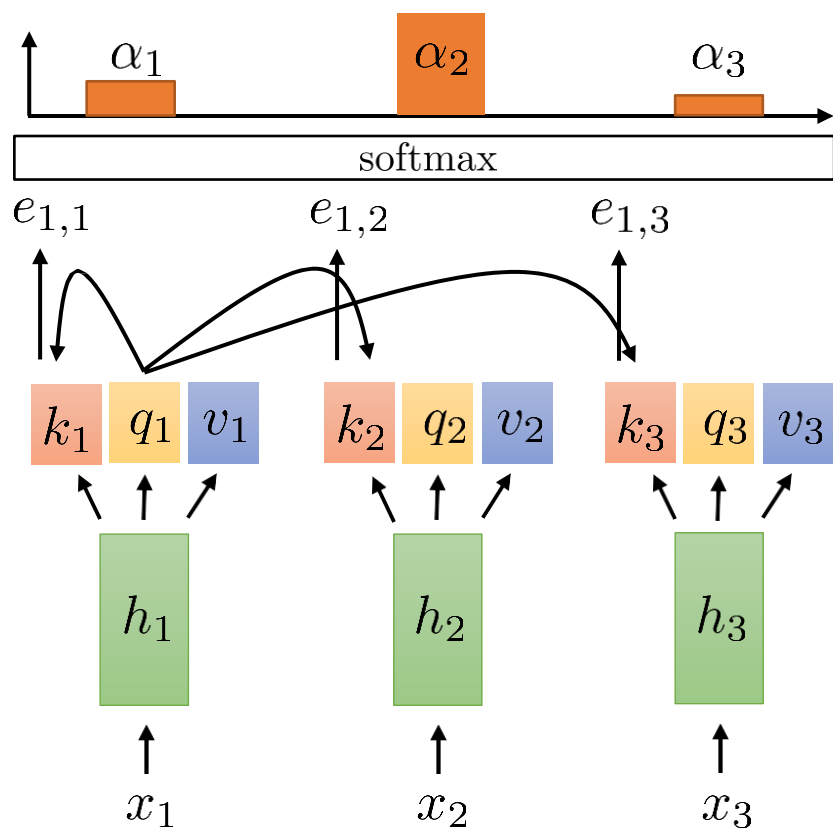
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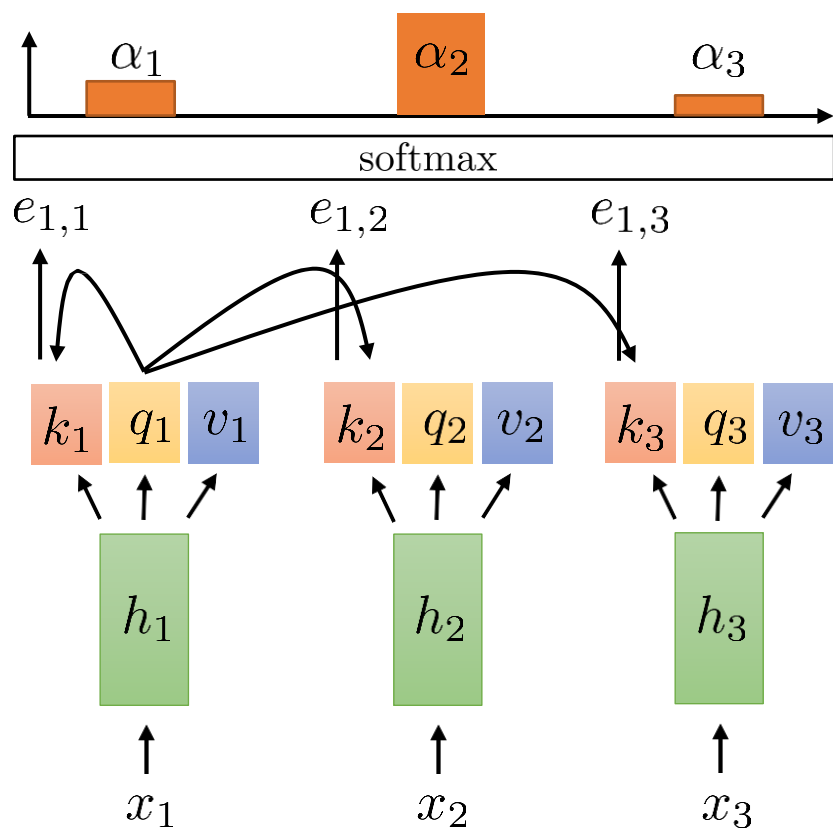
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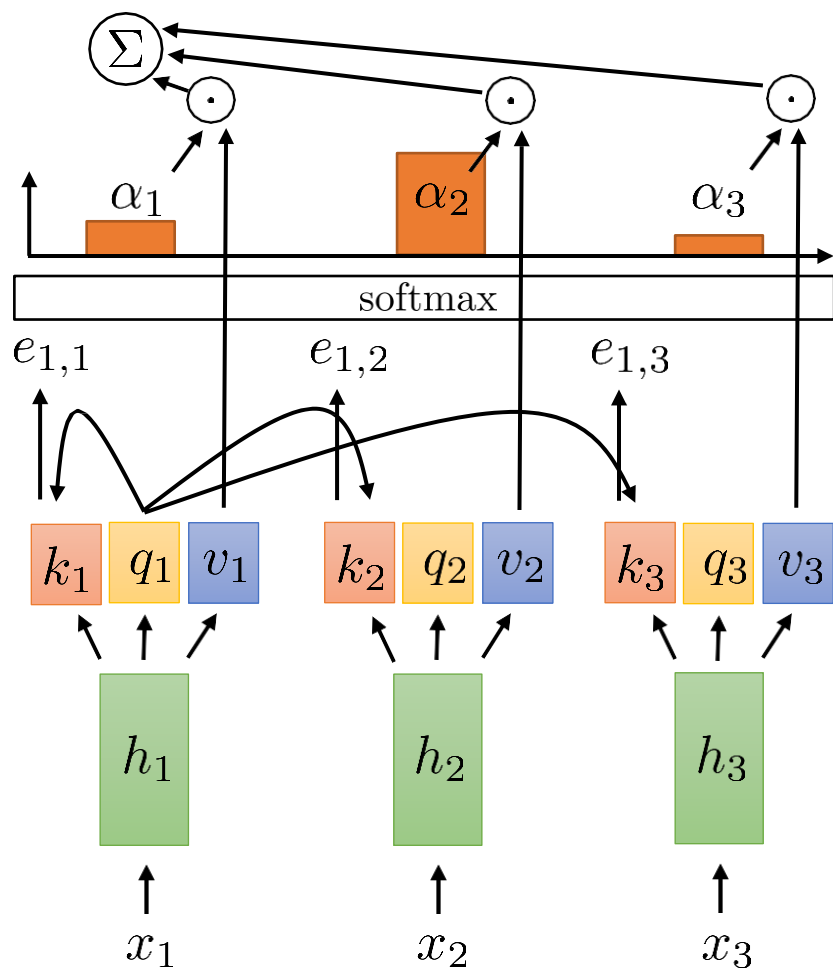
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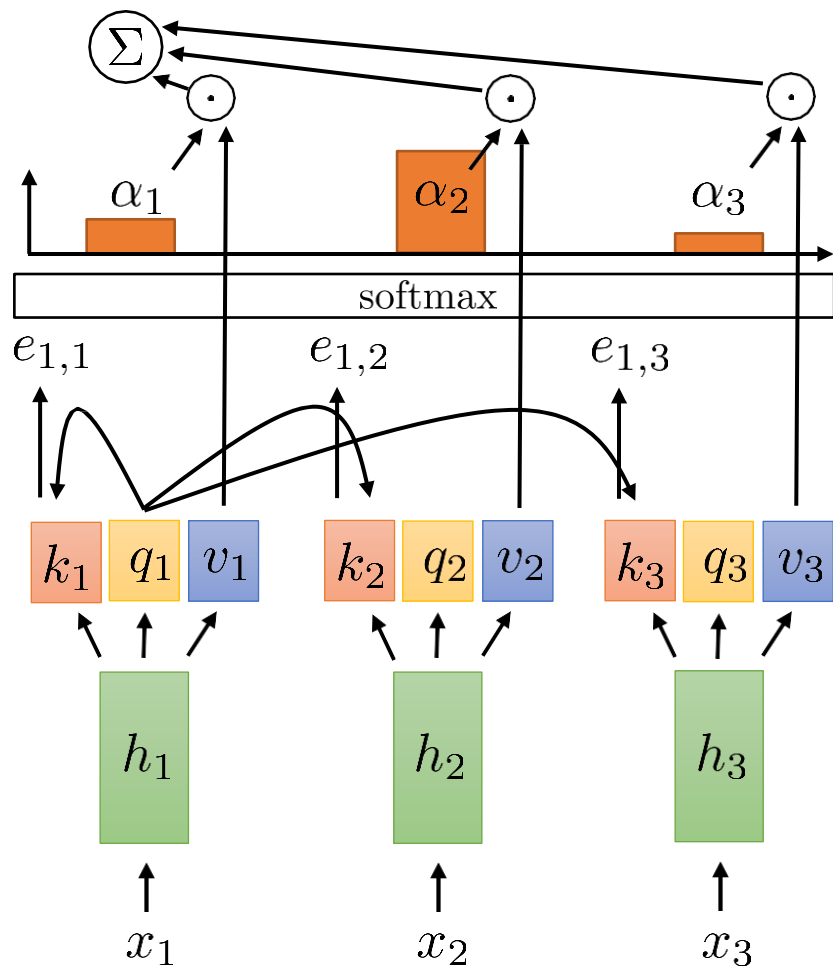
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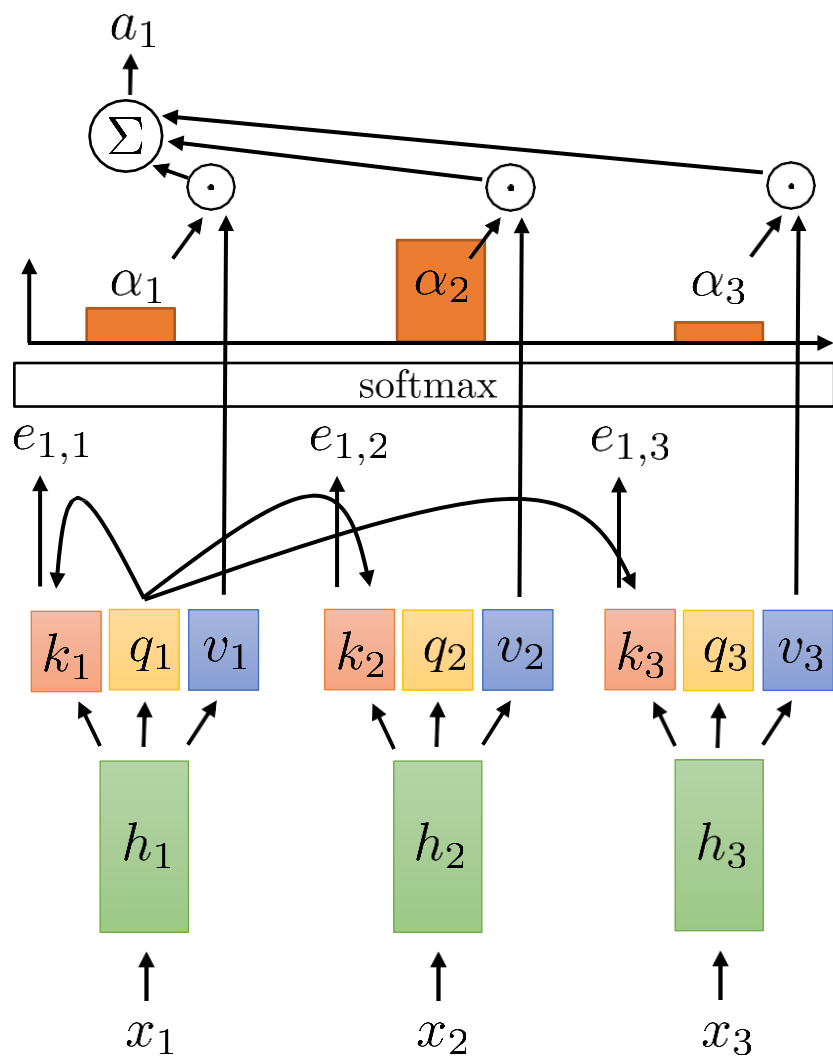
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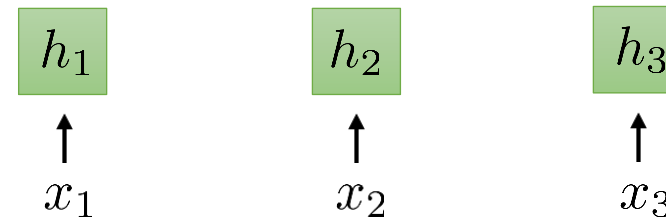
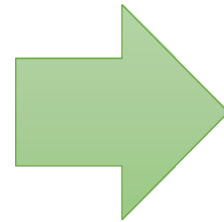
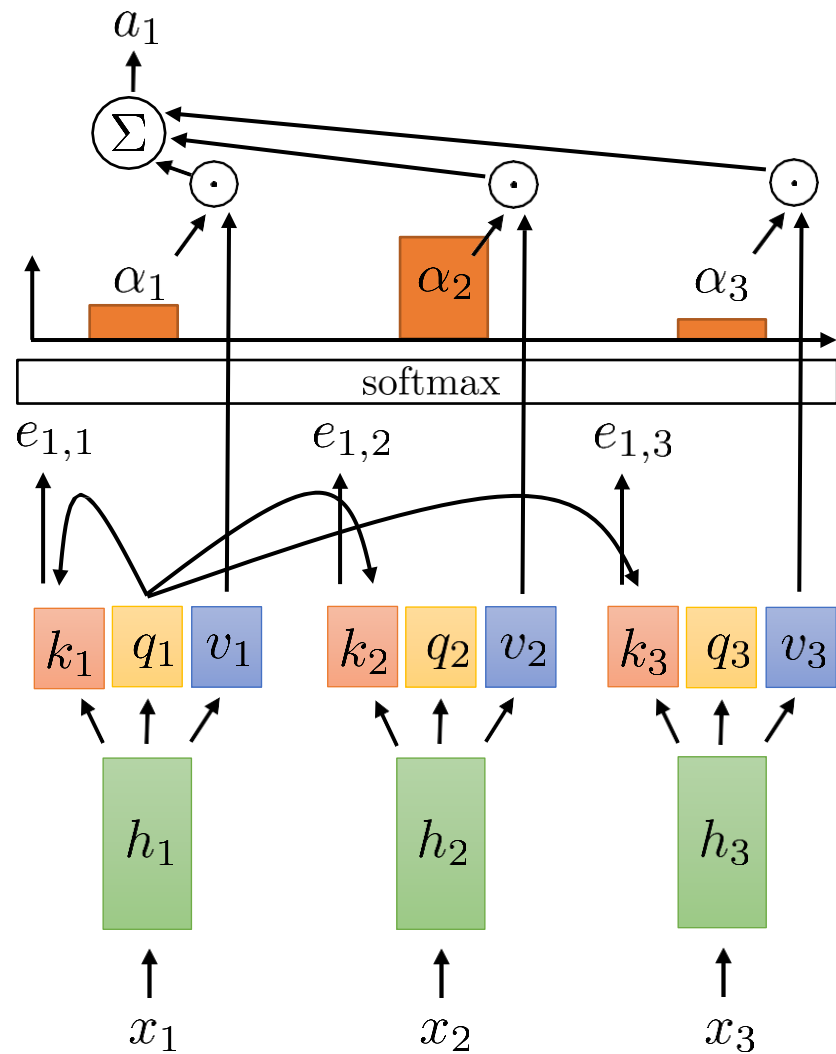
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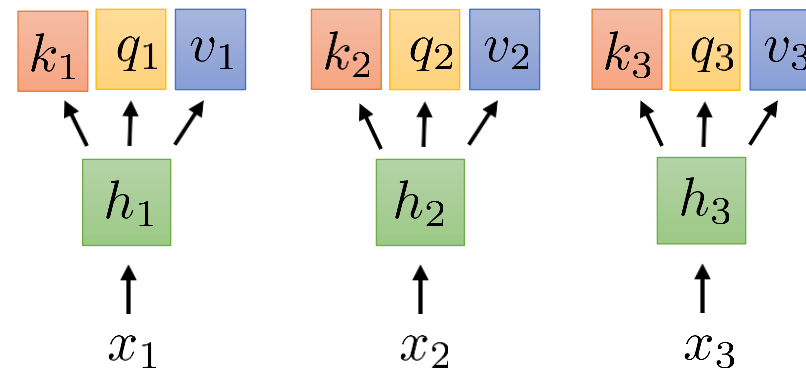
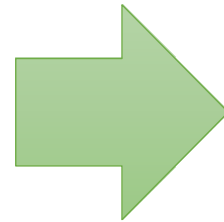
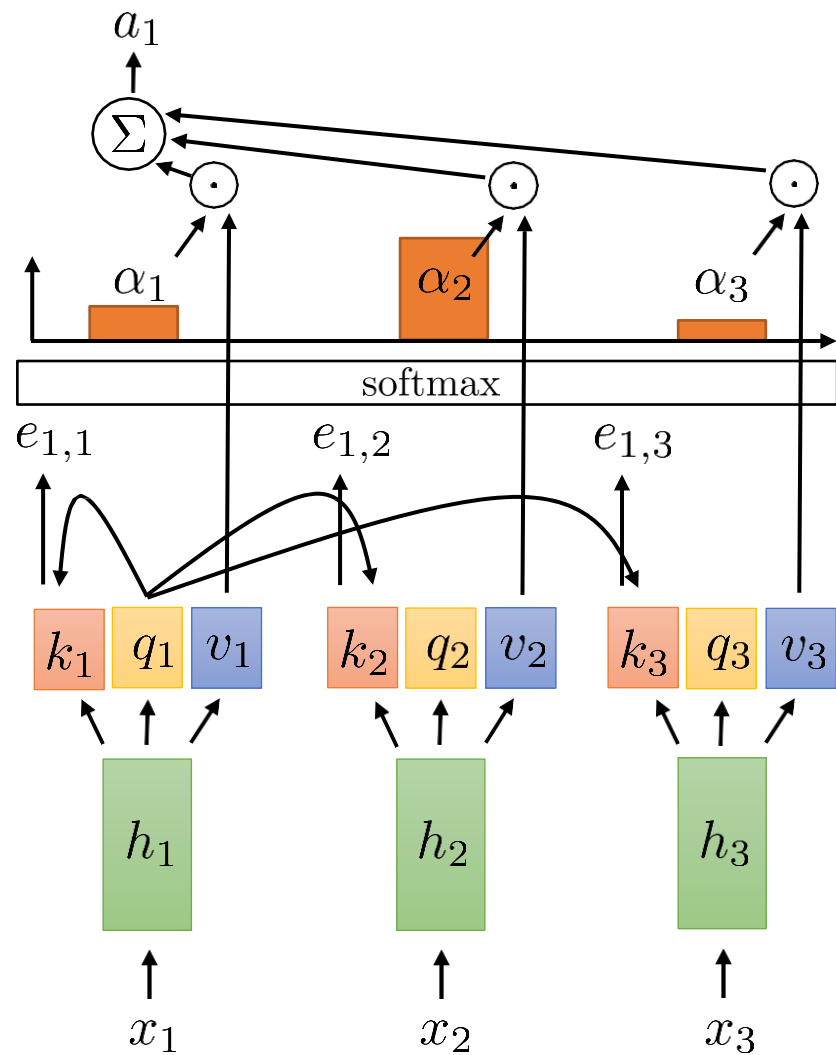
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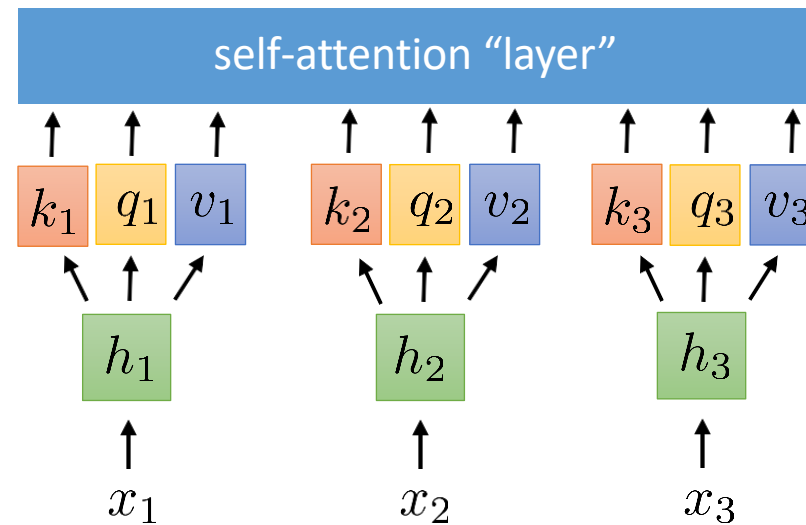
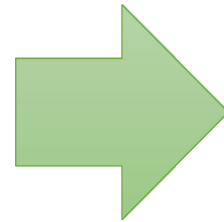
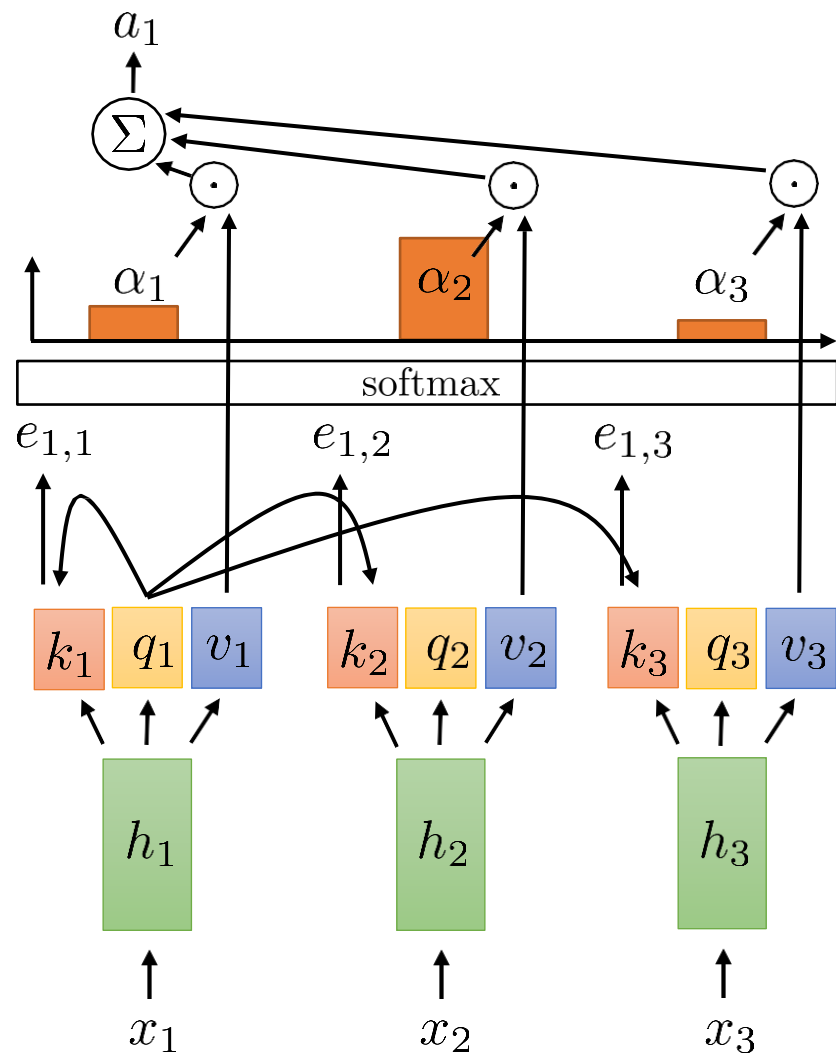
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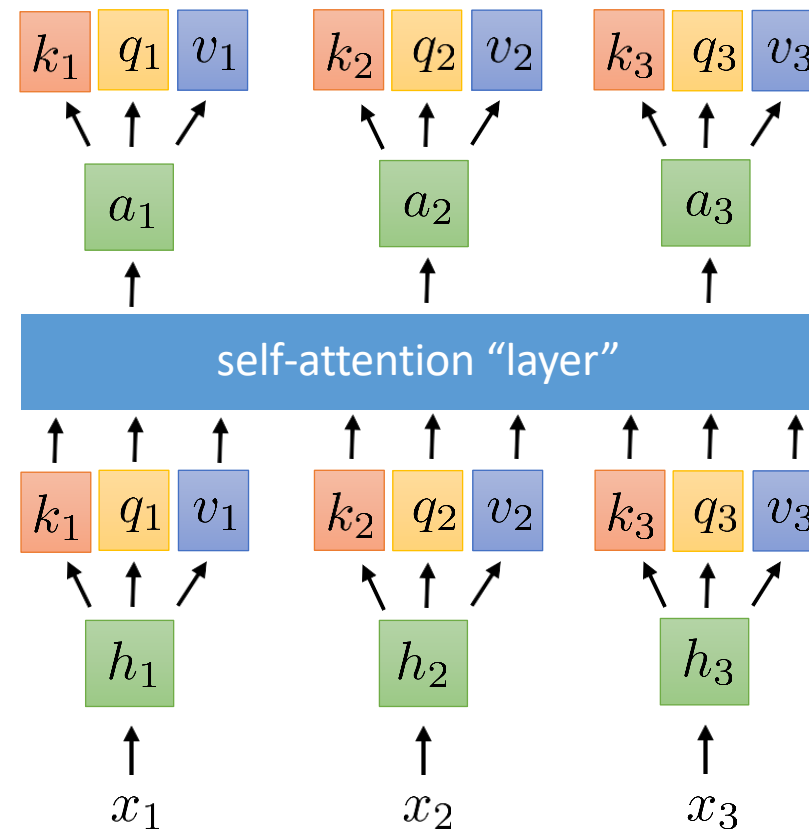
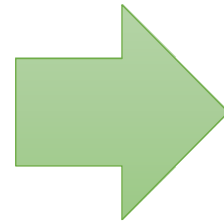
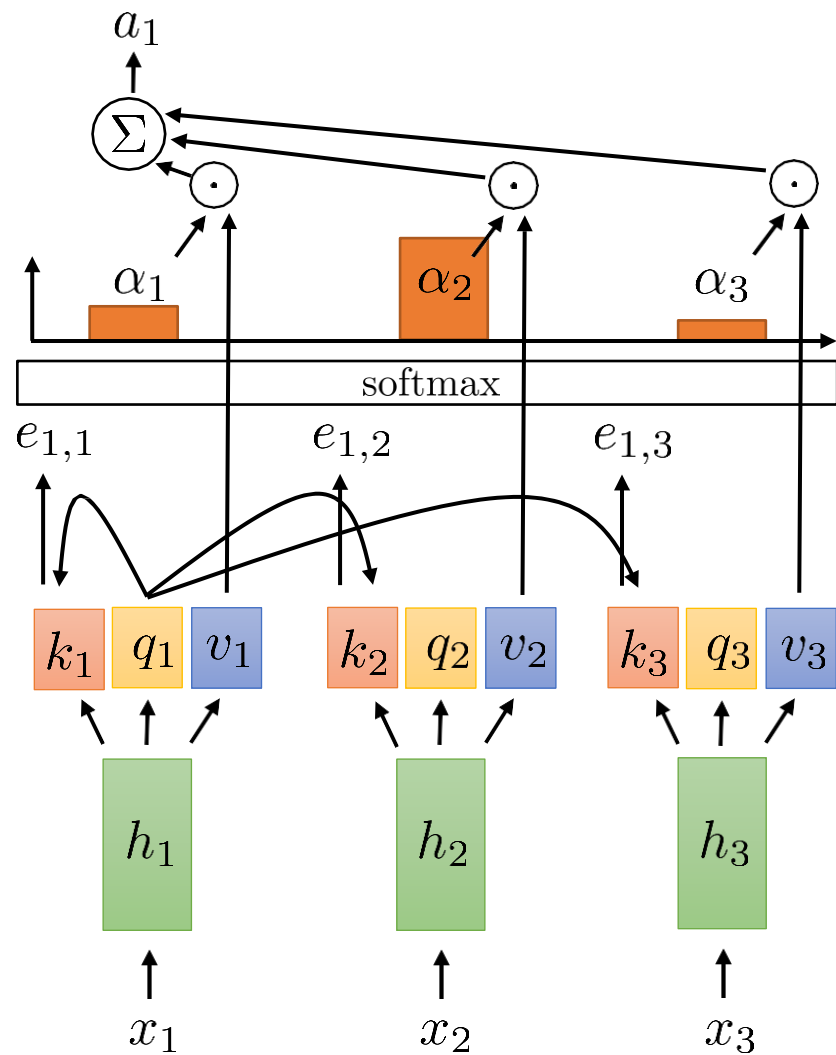
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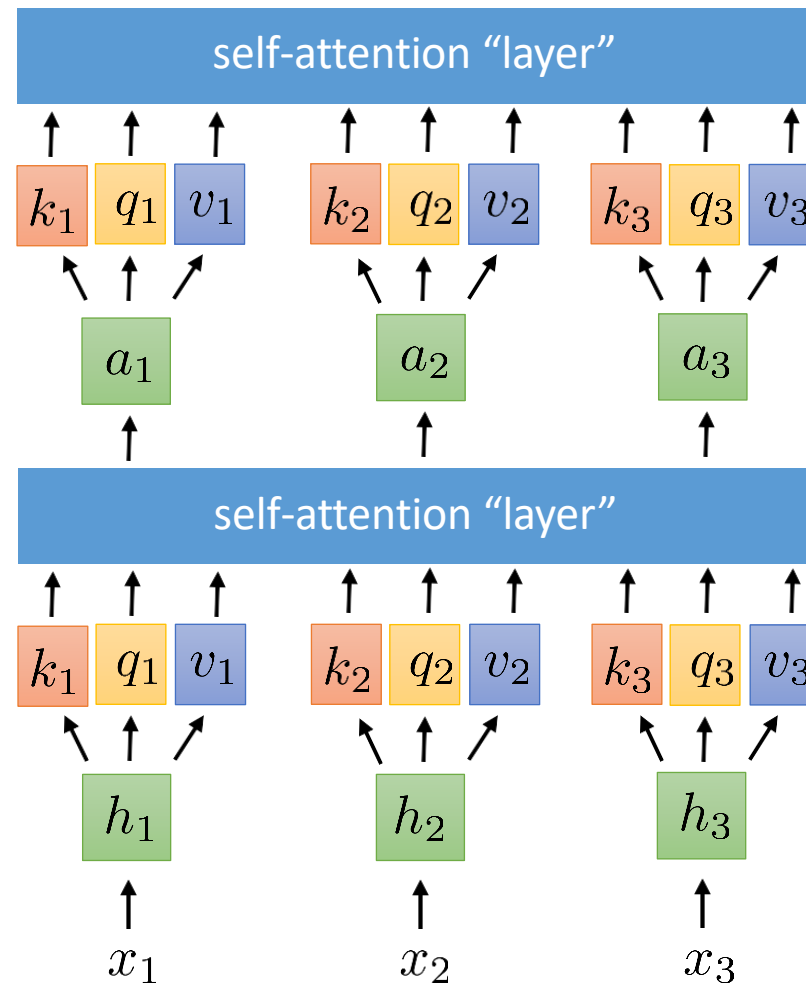
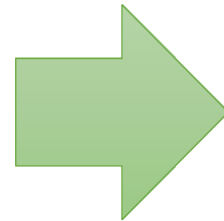
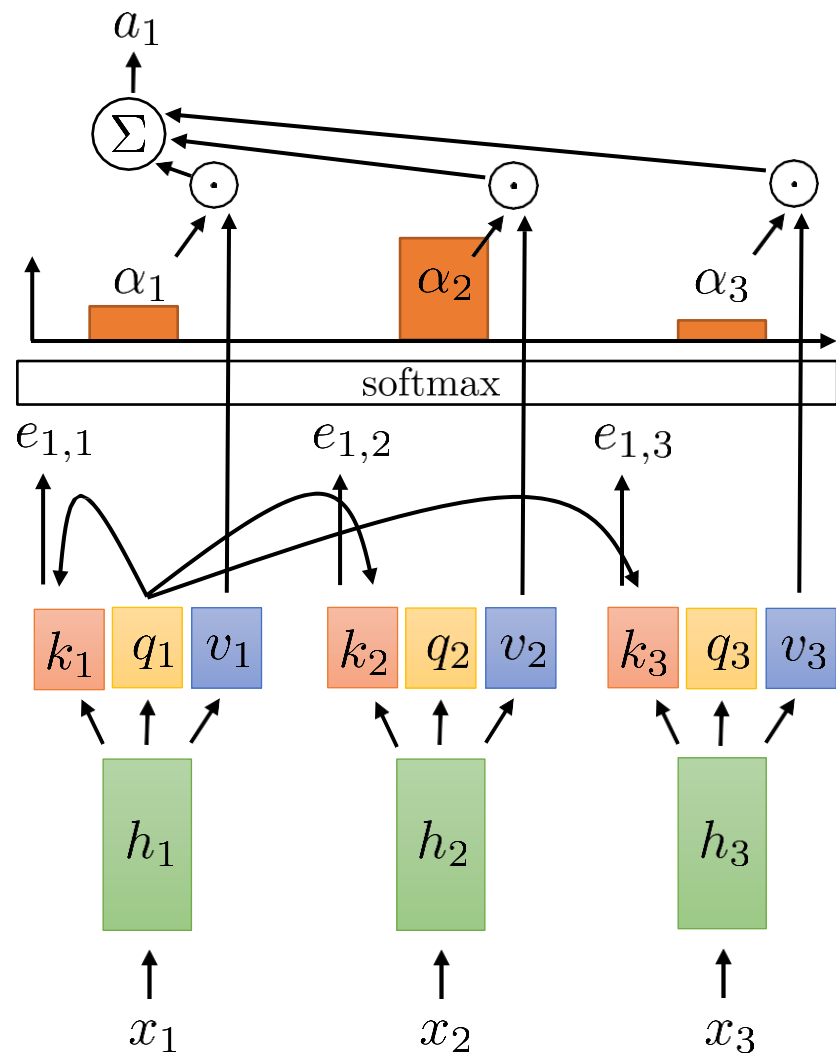


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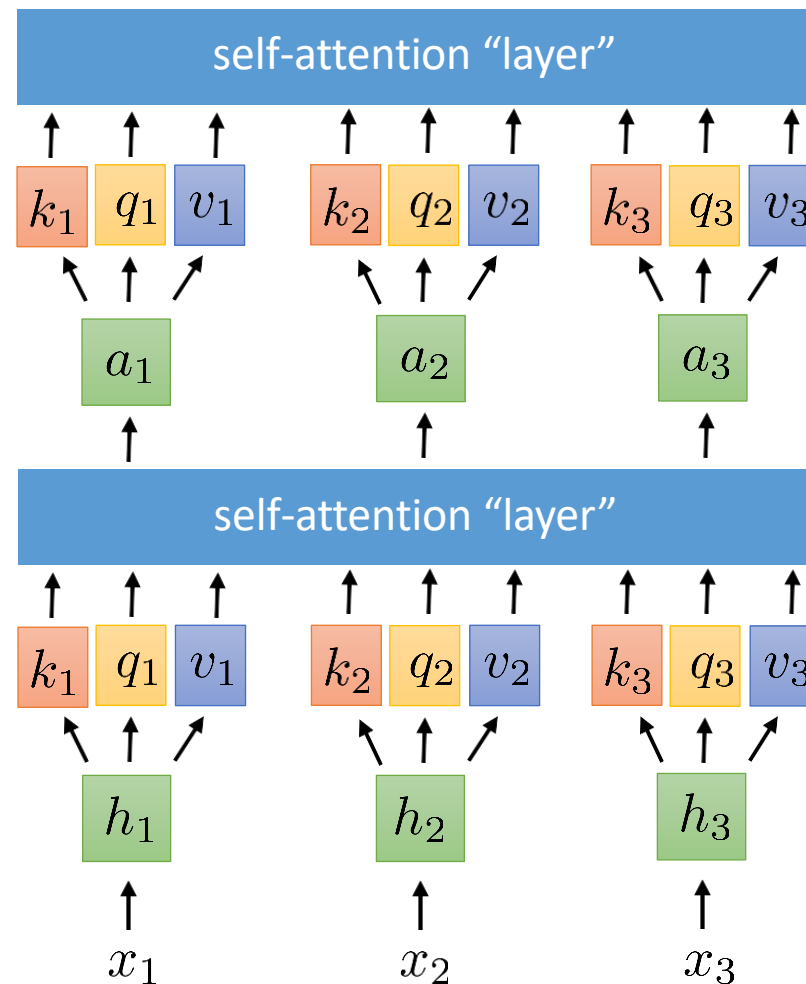
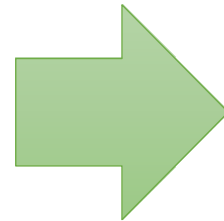
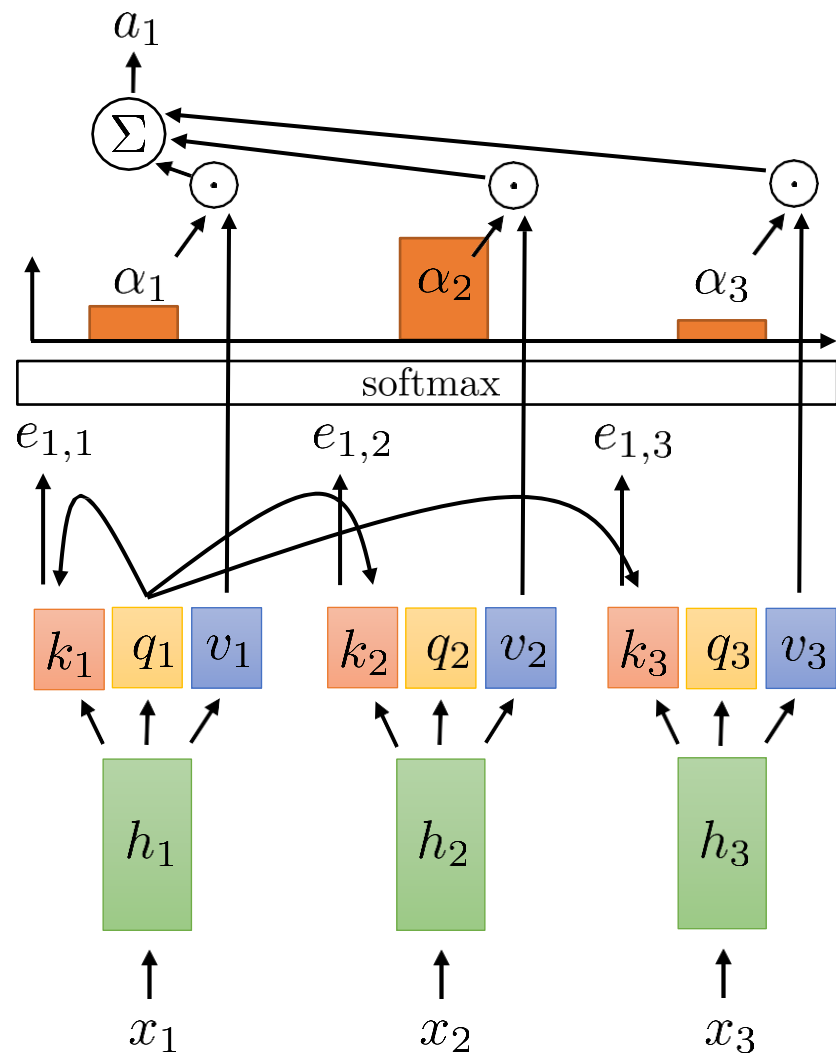




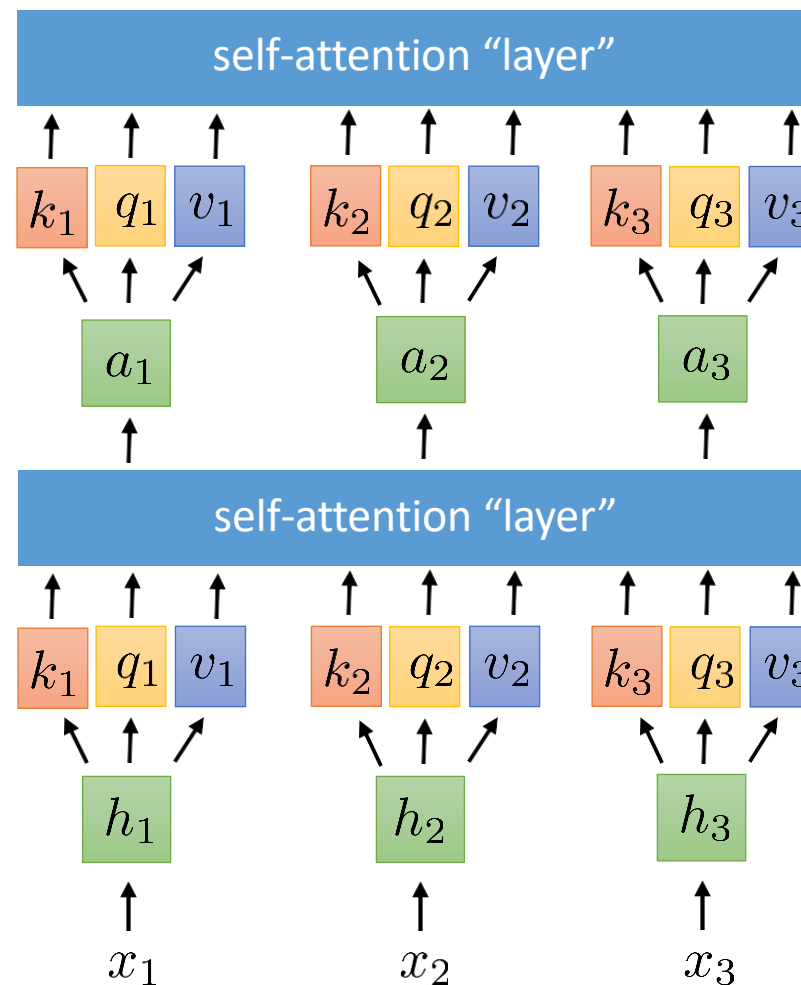
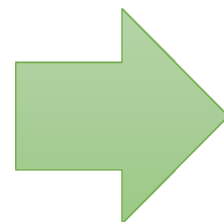
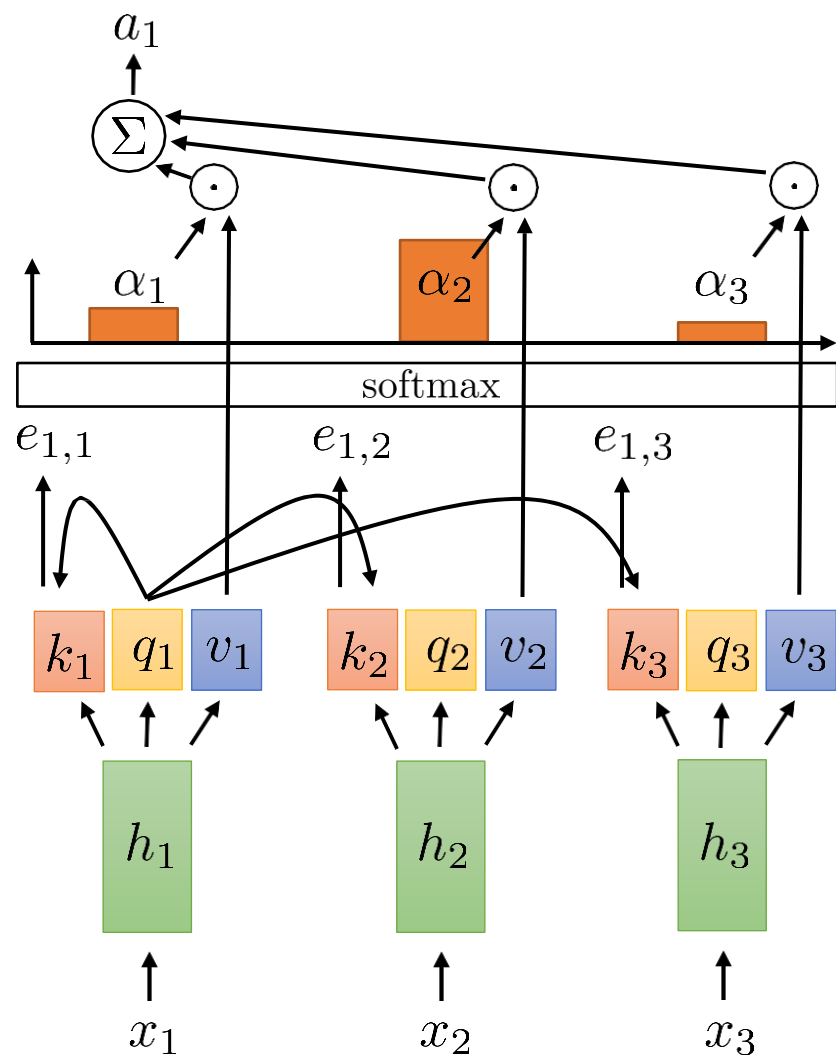
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# From Self-Attention to Transformers

- We will talk about a class of models for processing sequences that does not use recurrent connections but instead relies entirely on attention and will build up towards a class of models called **Transformers**.
- To address a few key limitations, we need to add certain elements:
  1. Positional encoding addresses lack of sequence information
  2. Multi-headed attention allows querying multiple positions at each layer
  3. Adding nonlinearities so far, each successive layer is *linear* in the previous one
  4. Masked decoding how to prevent attention lookups into the future?

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## 2. Multi-headed attention

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## 3. Adding nonlinearities

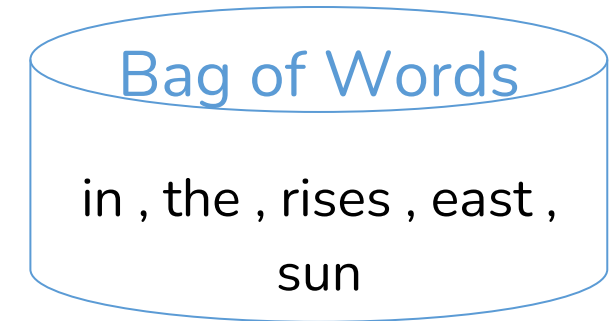
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## 4. Masked decoding

how to prevent attention lookups into the future?

# Positional Encoding - Motivation

- **Problem** : Self-attention processes all the elements of a sequence in parallel without any regard for their order.
  - Example : the sun rises in the east
  - Permuted version : rises in the sun the east  
the east rises in the sun
  - Self-attention is permutation invariant.
  - In natural language, it is important to take into account the order of words in a sentence.
- **Solution** : Explicitly add positional information to indicate where a word appears in a sequence

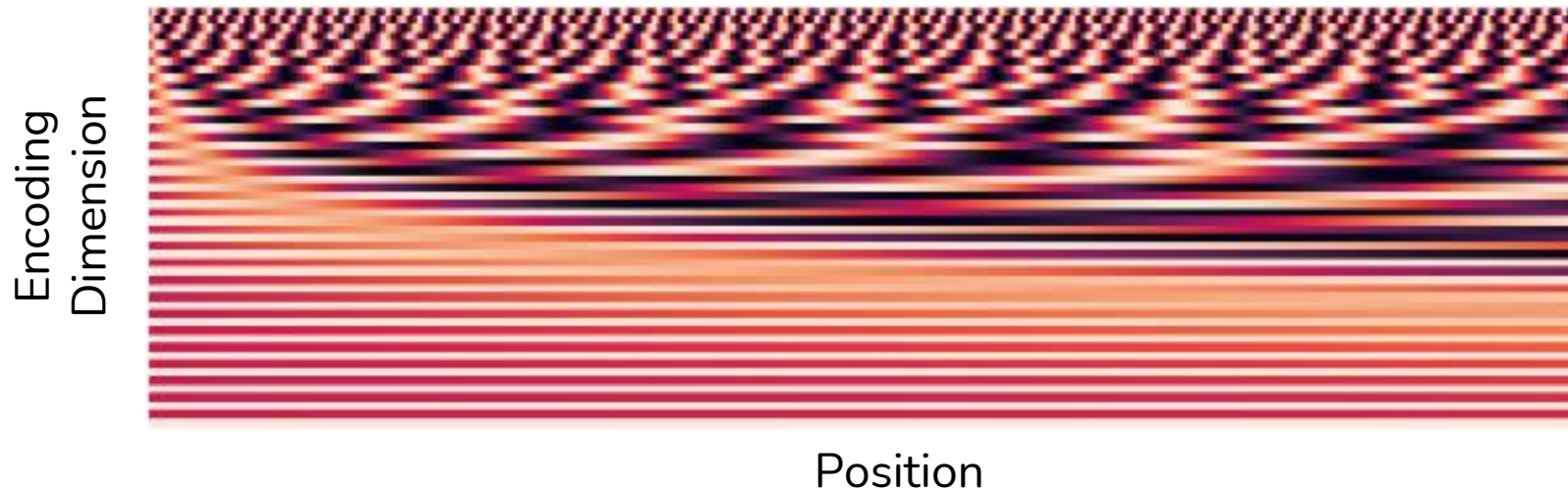


# Sinusoidal Positional Encoding

- Helps it determine the position of each word (absolute positional information), or the distance between different words in the sequence (relative positional information)
- The frequency decreases along the encoding dimension.

$$PE_{(pos, 2i)} = \sin(pos/10000^{2i/d_{\text{model}}})$$

$$PE_{(pos, 2i+1)} = \cos(pos/10000^{2i/d_{\text{model}}})$$



We will see this in more detail later !

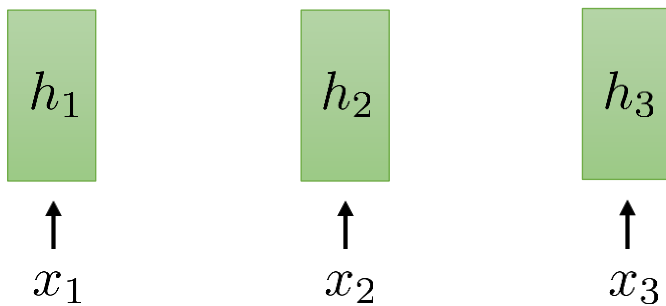
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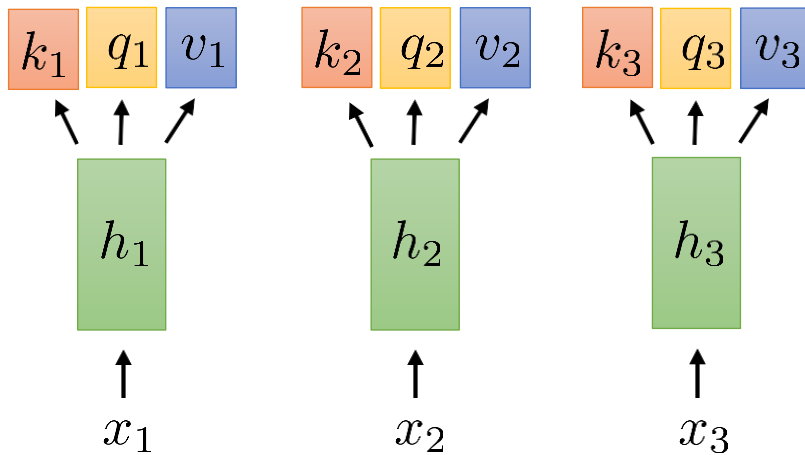
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Given that we're fully depending on attention now, it could be beneficial to include more than one time step.



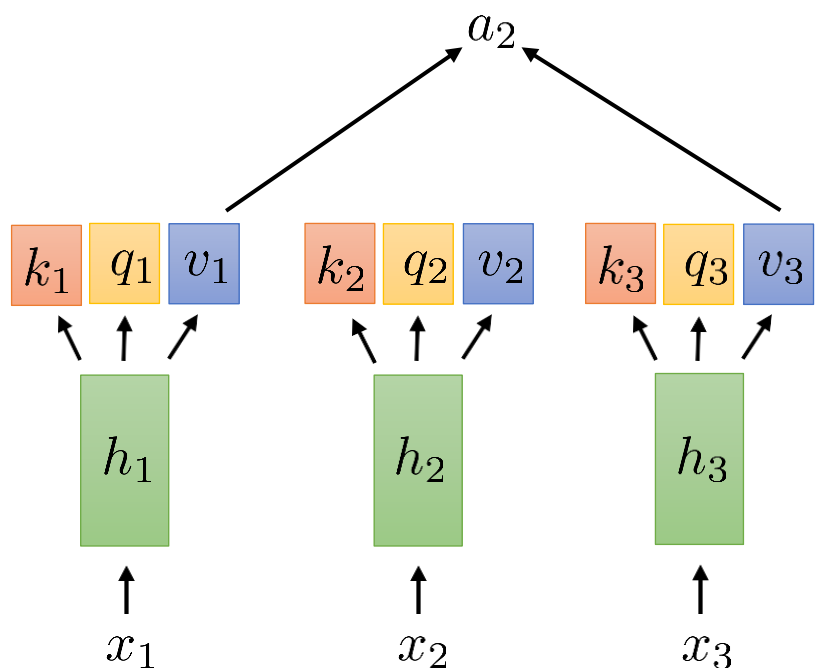
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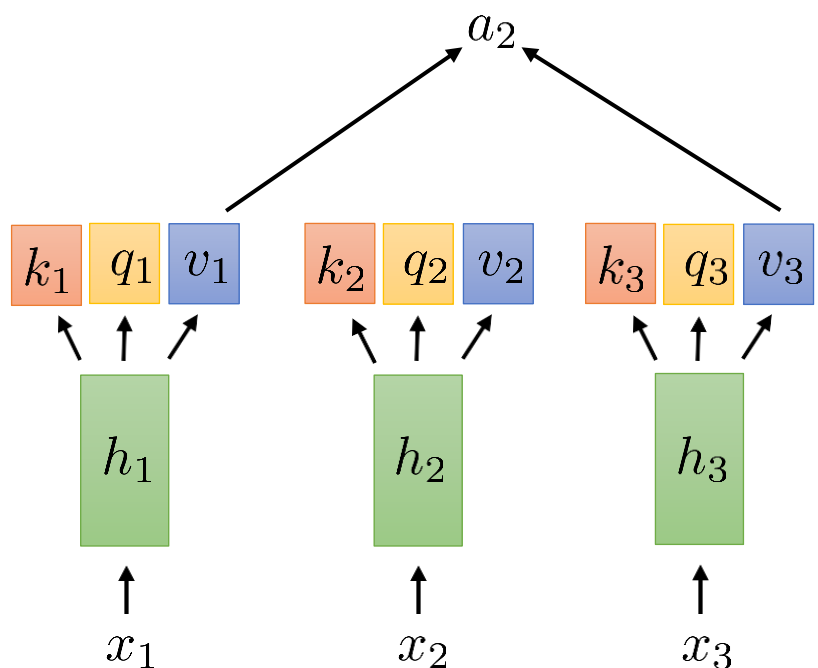


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Due to the softmax function, this will be heavily influenced by a single value.

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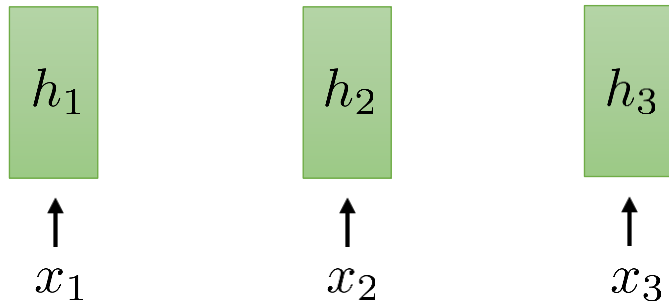
Due to the softmax function, this will be heavily influenced by a single value.

$$e_{l,t} = q_l \cdot k_t$$

It's challenging to clearly specify that you want two distinct elements, like the subject and object in a sentence.

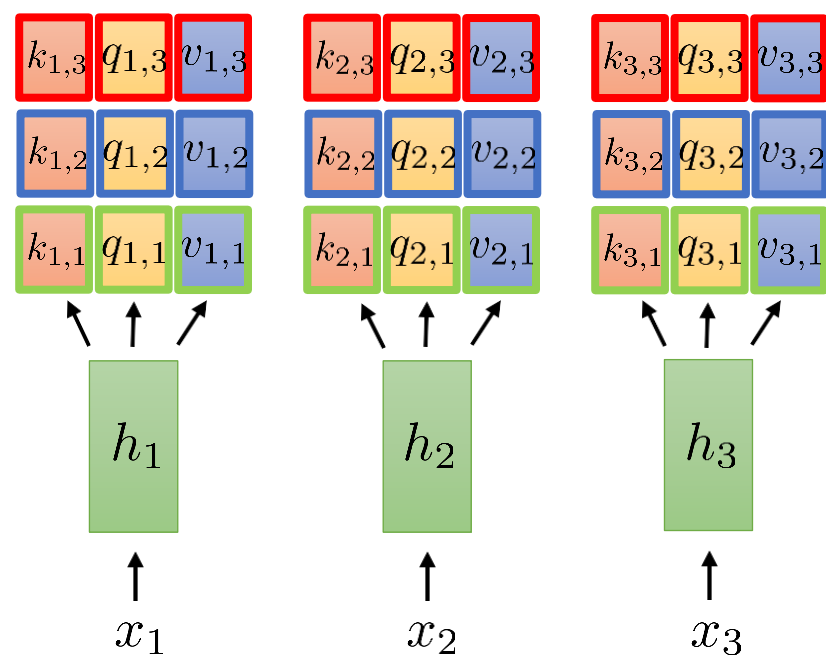
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Solution: Use multiple keys, queries, and values for each time step



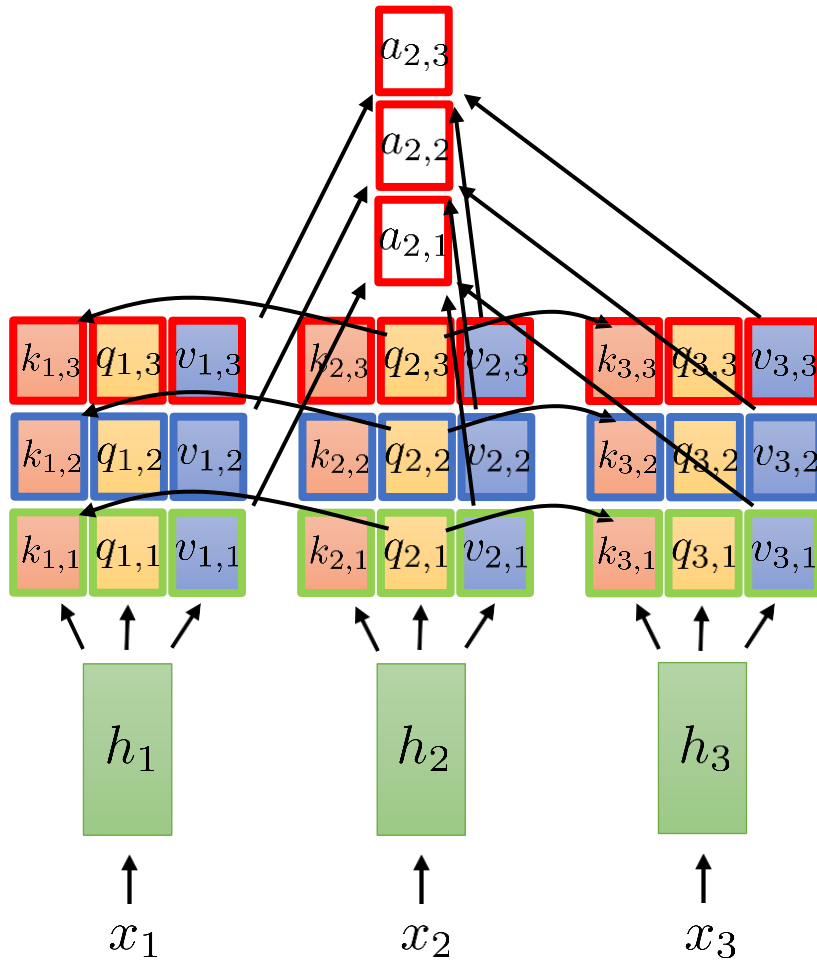
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full attention vector formed by concatenation:

$$a_2 = \begin{bmatrix} a_{2,1} \\ a_{2,2} \\ a_{2,3} \end{bmatrix}$$

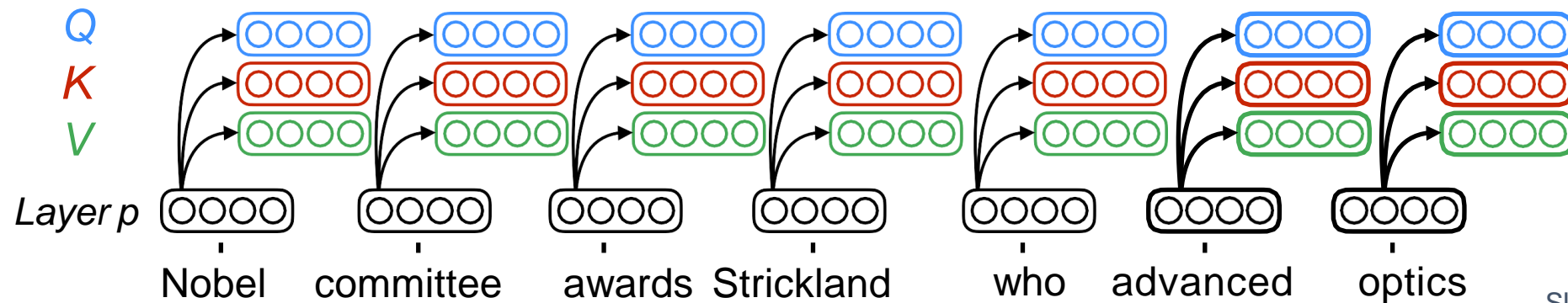
compute weights **independently** for each head

$$e_{l,t,i} = q_{l,i} \cdot k_{l,i}$$

$$\alpha_{l,t,i} = \exp(e_{l,t,i}) / \sum_{t'} \exp(e_{l,t',i})$$

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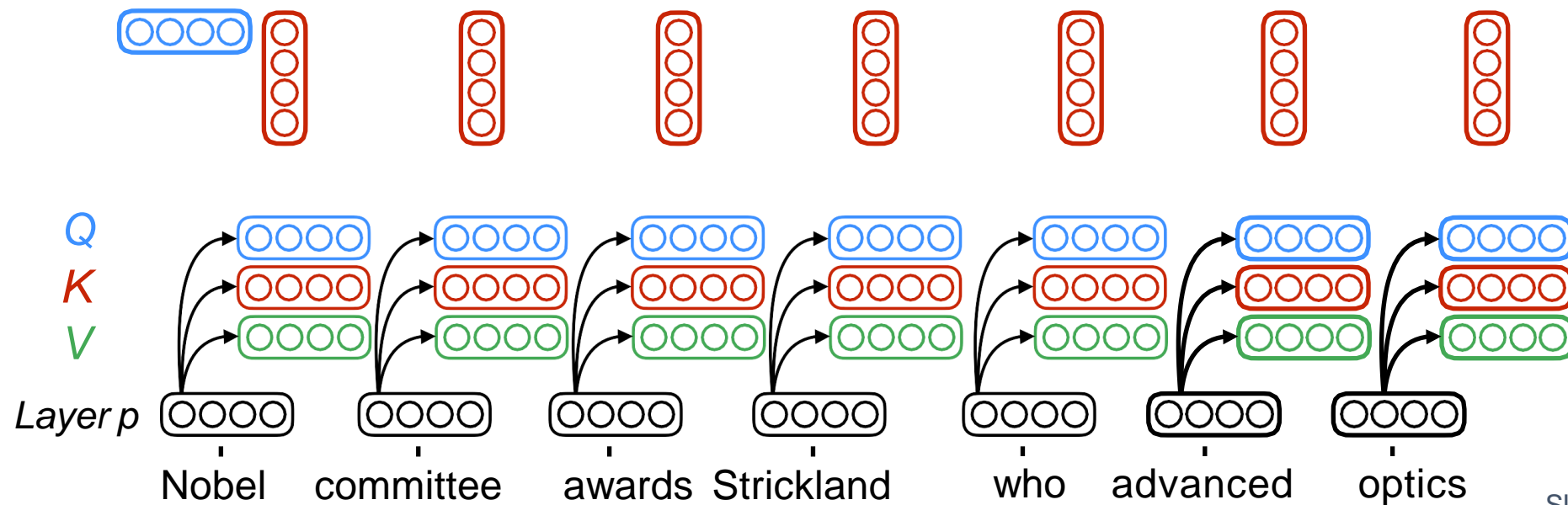
# Self-Attention (In Encoder)



Slides by Emma Strubell

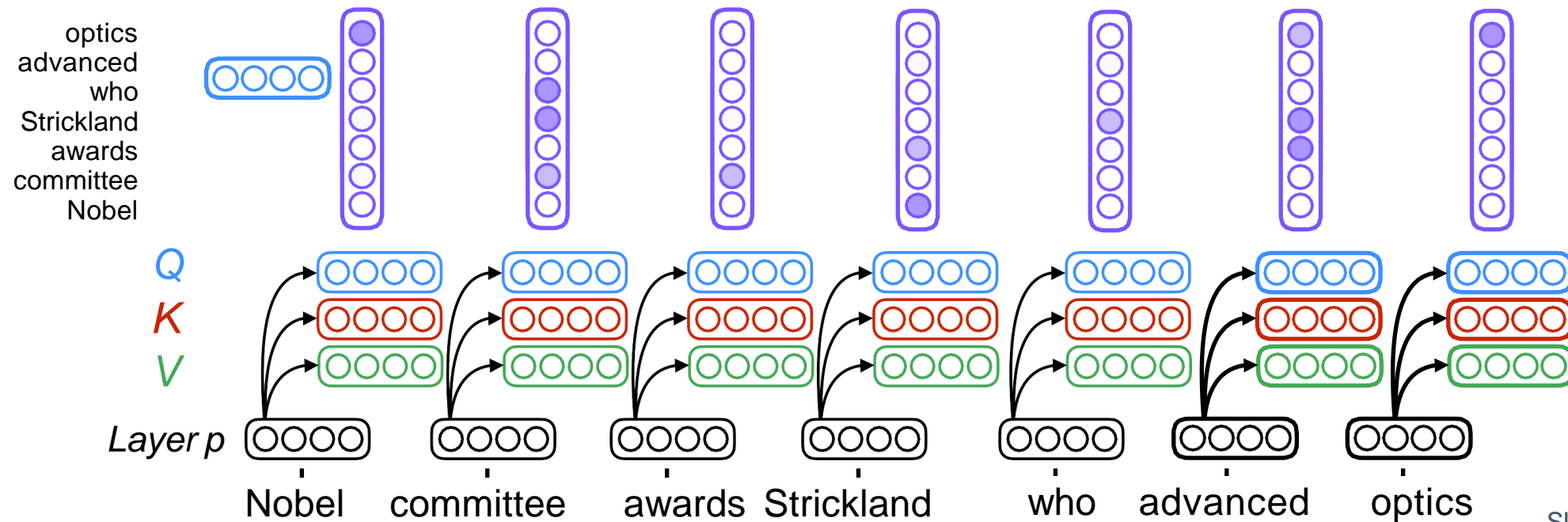


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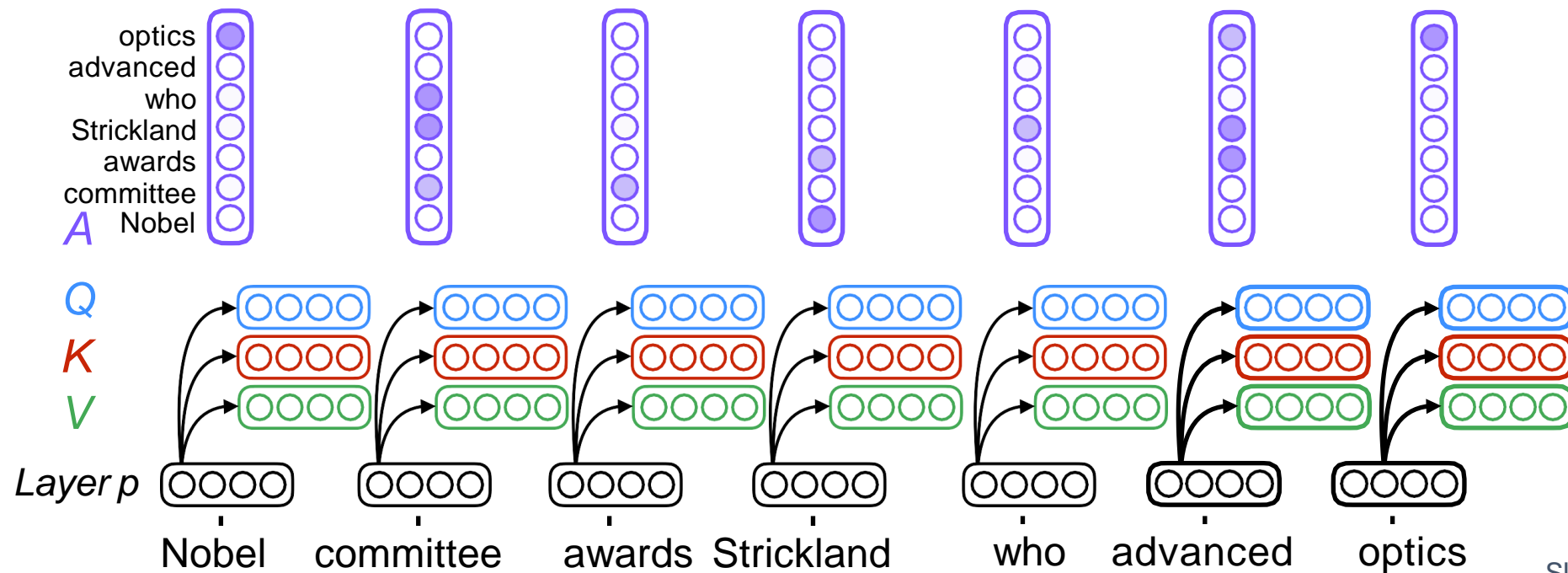
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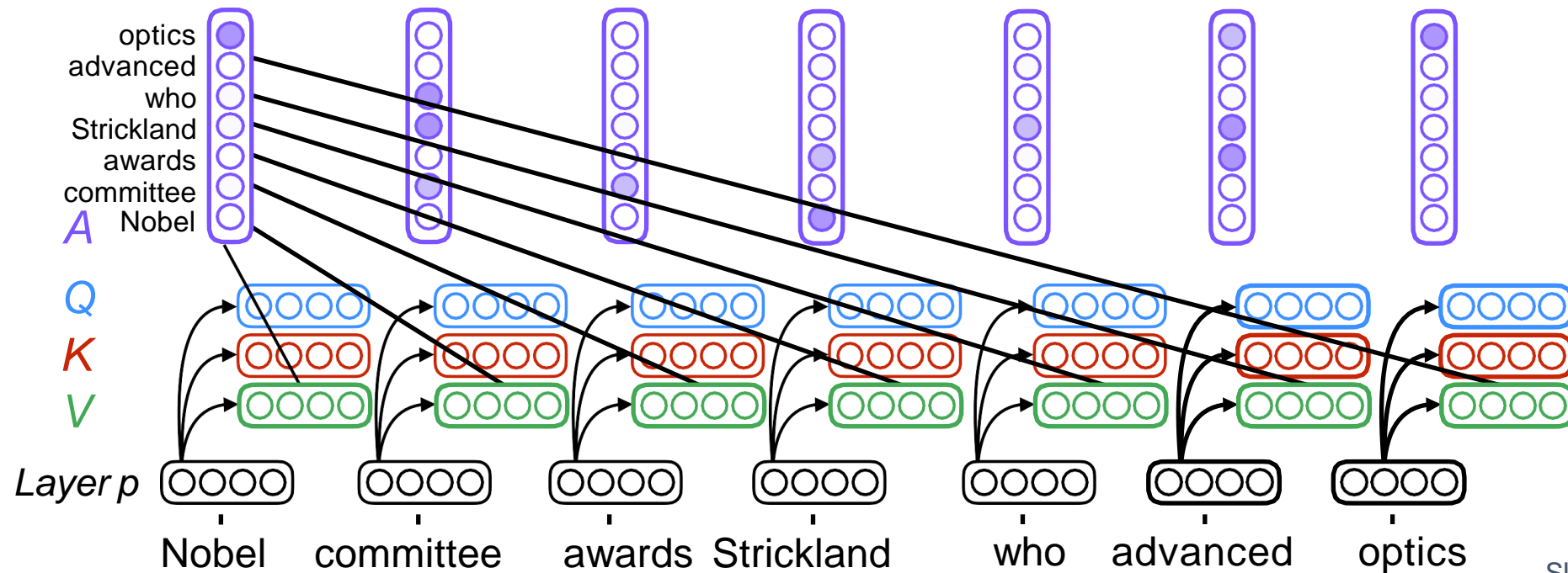
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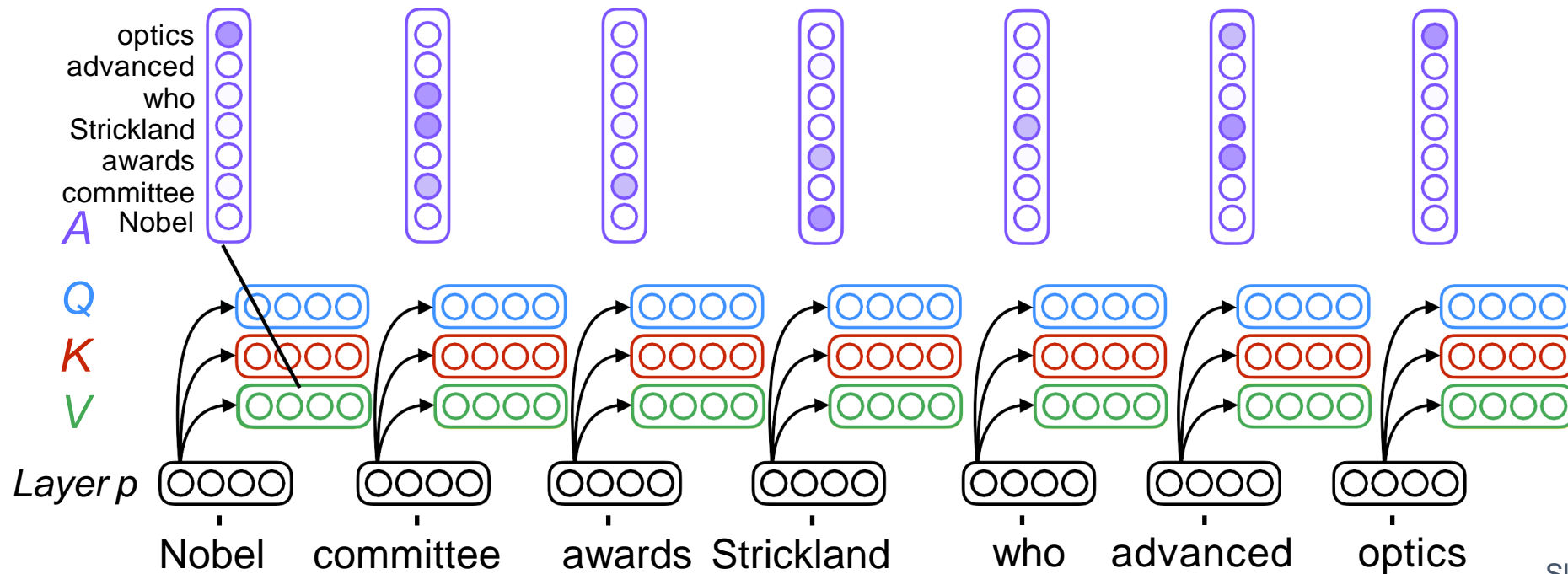
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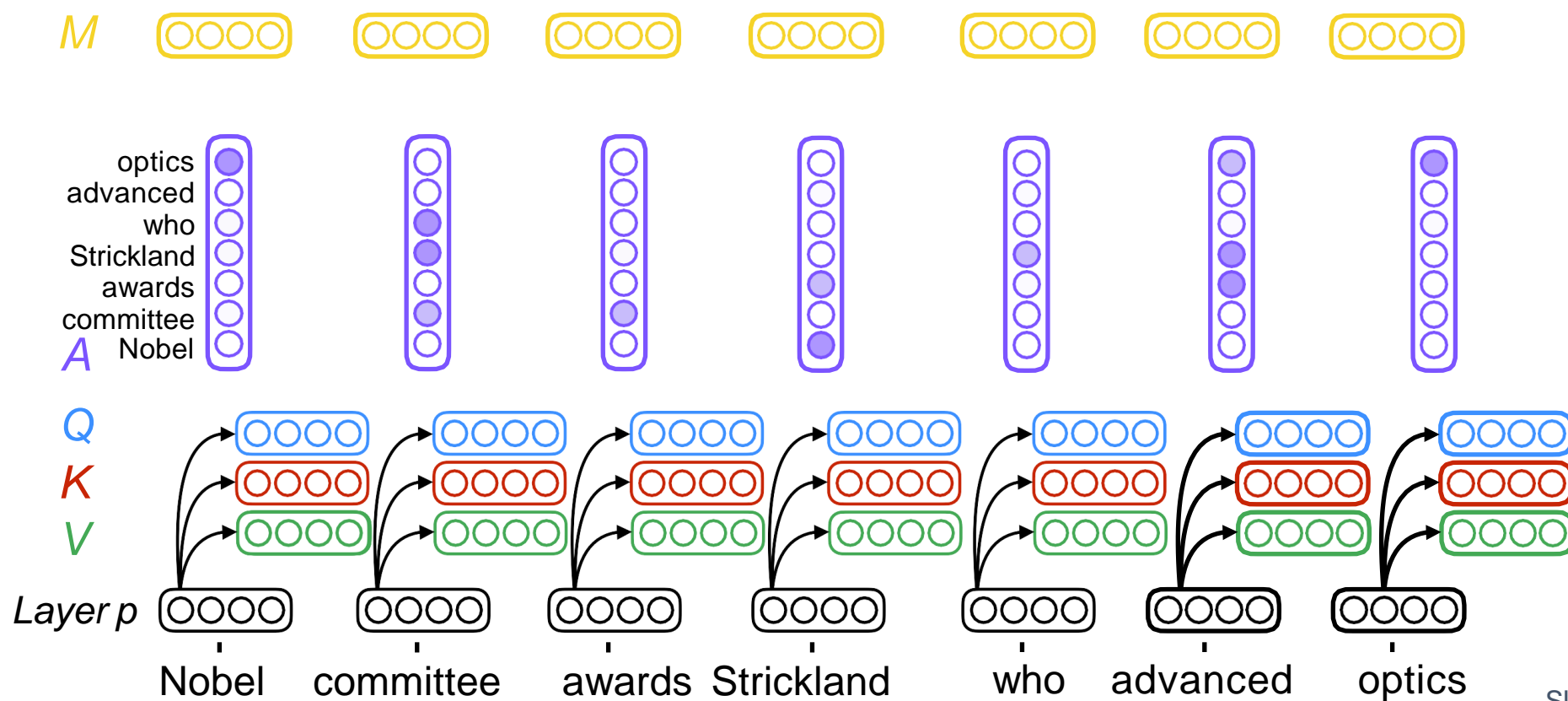
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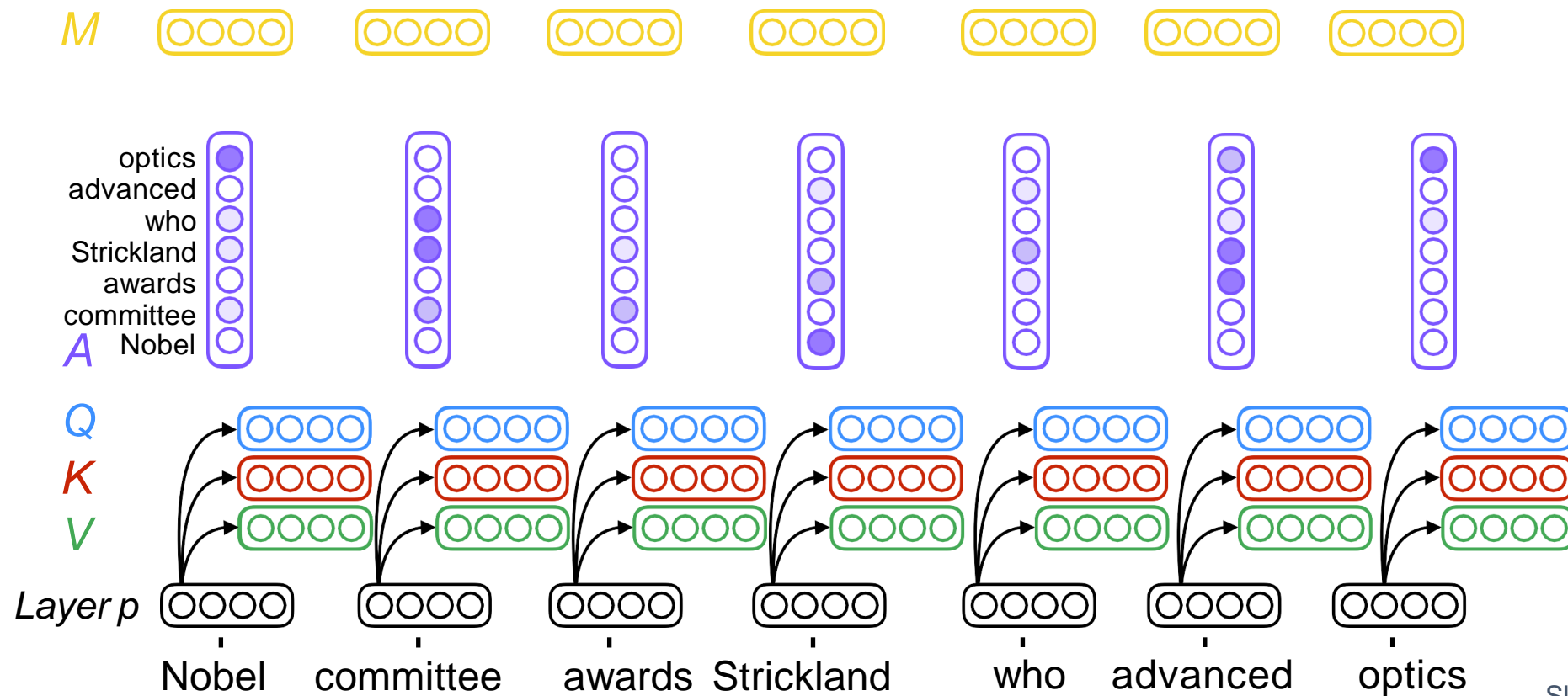
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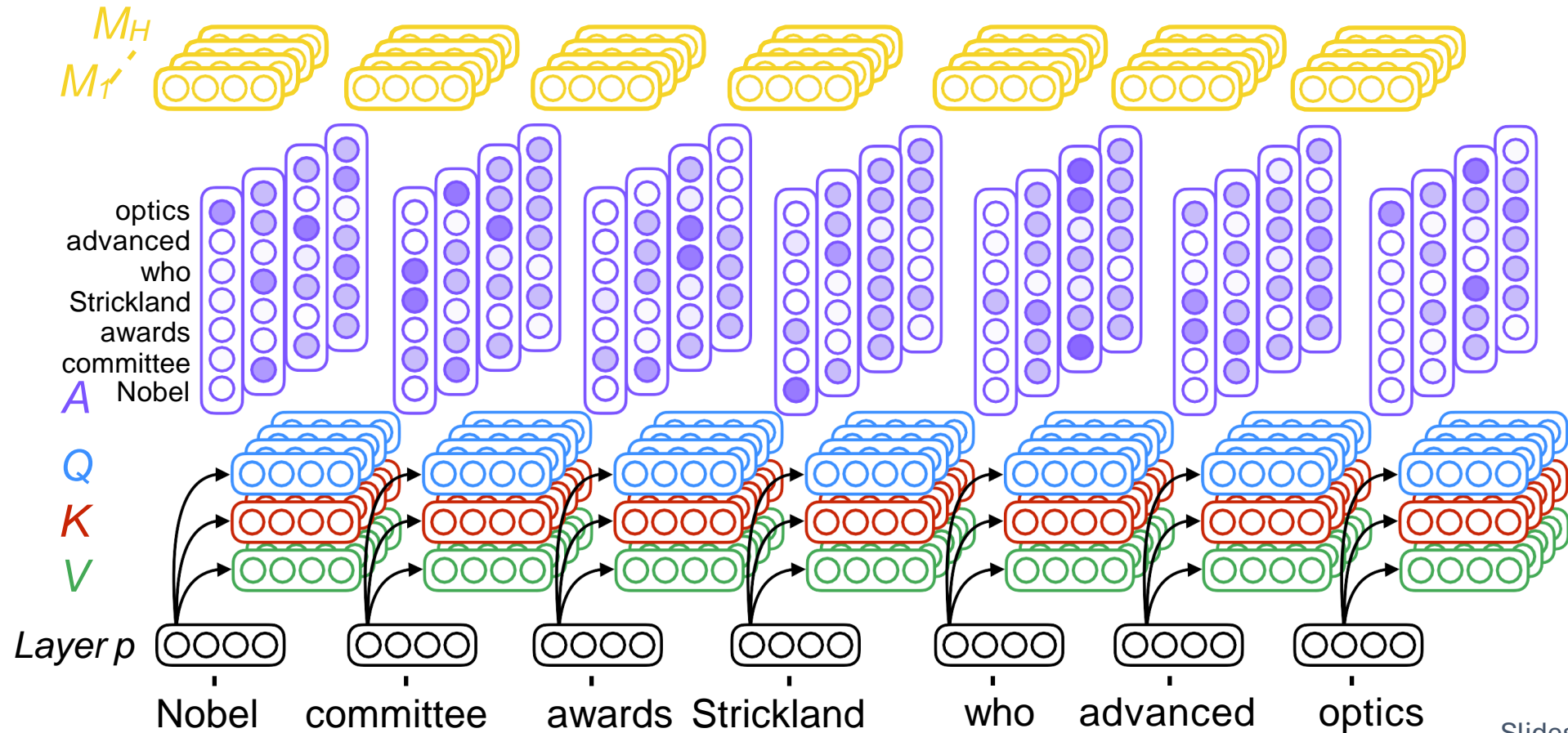
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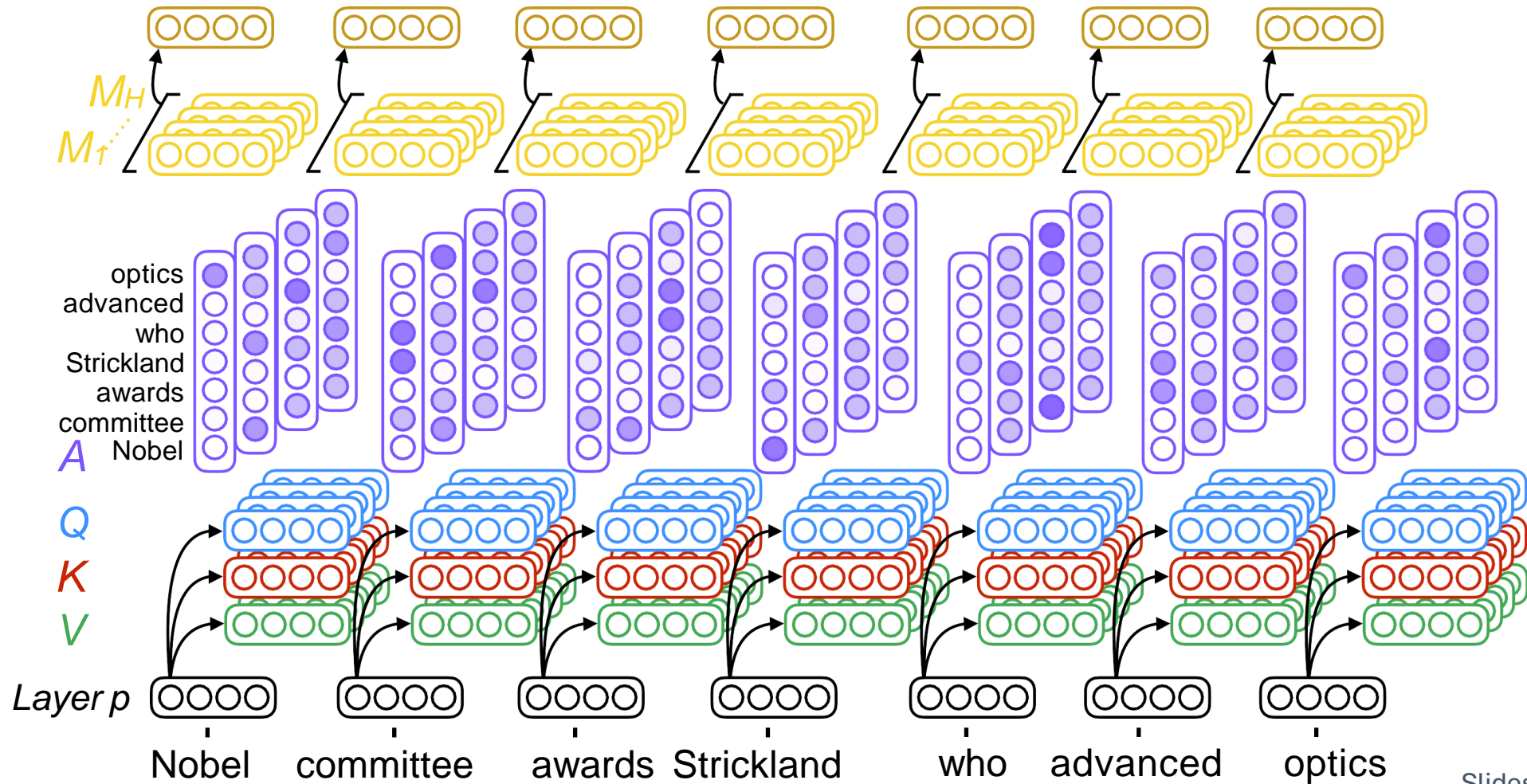
# Multi-Head Self-Attention



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# Multi-Head Self-Attention

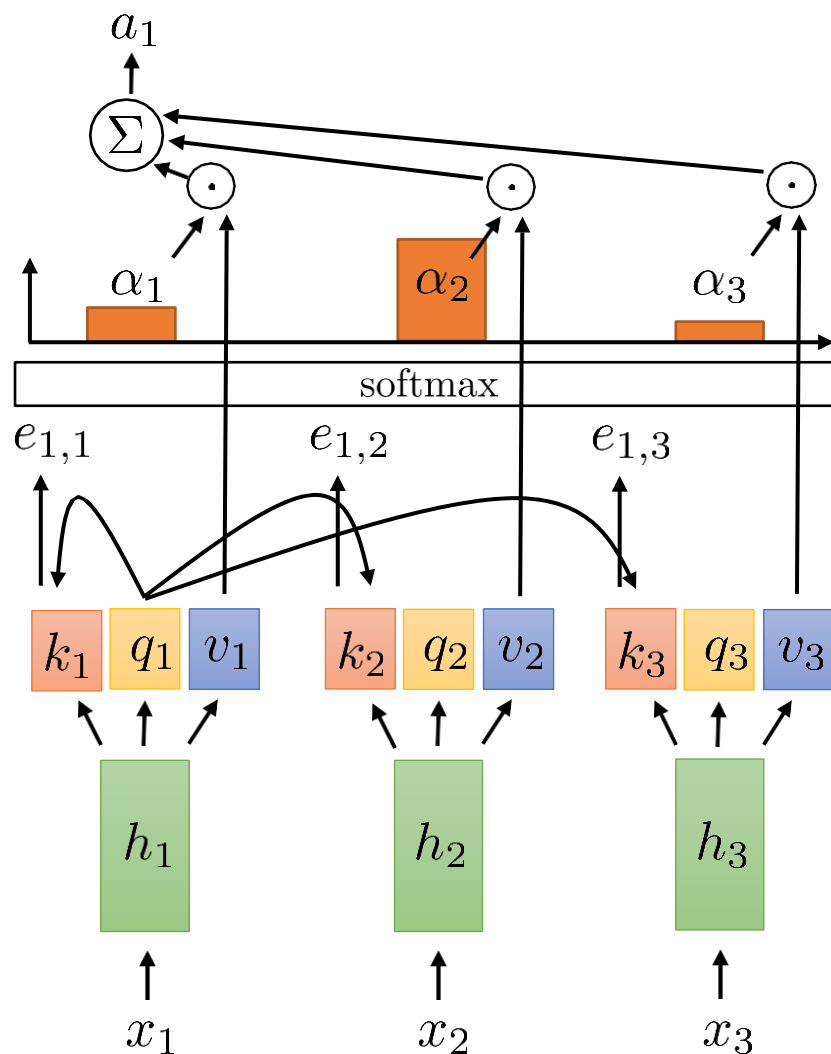


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# From Self-Attention to Transformers

- We will talk about a class of models for processing sequences that does not use recurrent connections but instead relies entirely on attention and will build up towards a class of models called transformers.
- To address a few key limitations, we need to add certain elements:
  1. Positional encoding addresses lack of sequence information
  2. Multi-headed attention allows querying multiple positions at each layer
  3. Adding nonlinearities so far, each successive layer is *linear* in the previous one
  4. Masked decoding how to prevent attention lookups into the future?

# Self-Attention Is “Linear”



$$k_t = W_k h_t \quad q_t = W_q h_t \quad v_t = W_v h_t$$

$$\alpha_{l,t} = \exp(e_{l,t}) / \sum_{t'} \exp(e_{l,t'})$$

$$e_{l,t} = q_l \cdot k_t$$

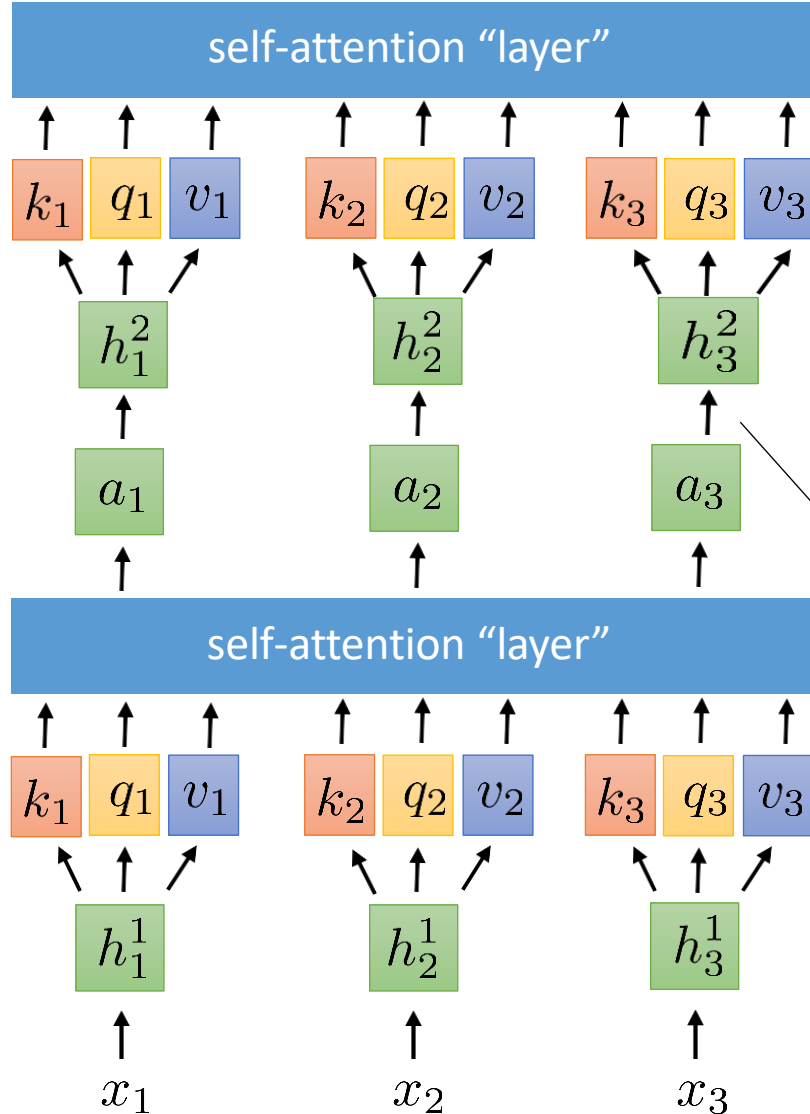
$$a_l = \sum_t \alpha_{l,t} v_t = \sum_t \alpha_{l,t} W_v h_t = W_v \sum_t \alpha_{l,t} h_t$$

linear transformation

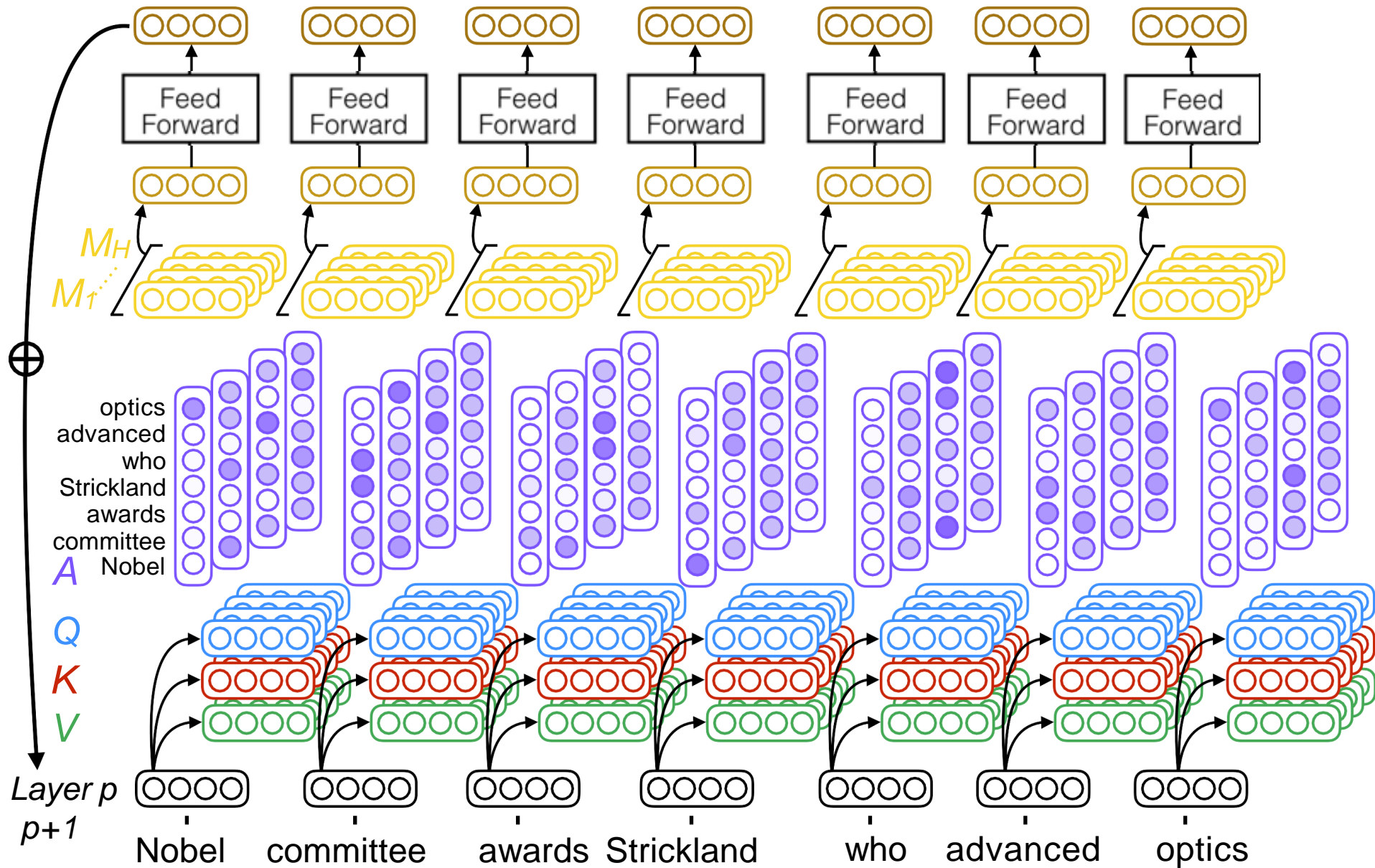
non-linear weights

Problem: Every self-attention layer is a linear transformation of the previous layer with non-linear weights.

# Position-wise Feed-Forward Networks



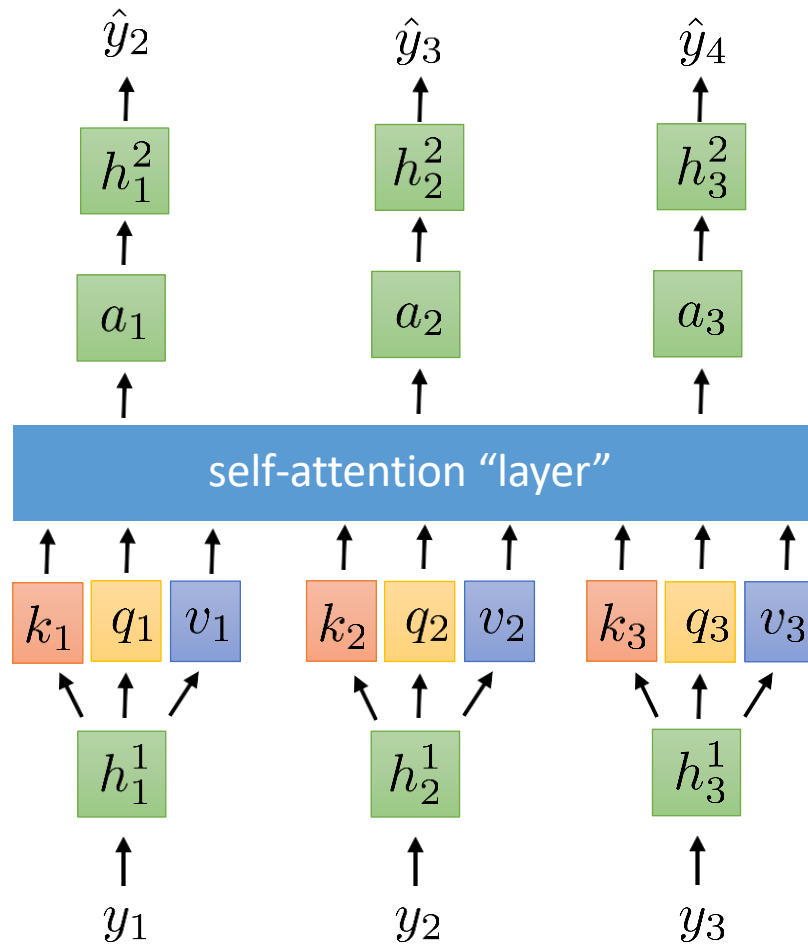
- **Solution** : Make the model more expressive is by alternating use of self-attention and non-linearity.
- Non-linearity is incorporated by means of a feed-forward network which consists of two linear transformations with a ReLU activation in between.  
$$\text{FFN}(x) = \max(0, xW_1 + b_1)W_2 + b_2$$
- The same non-linearity is utilized across various positions but they differ from layer to layer.



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# Self-attention can see the future!

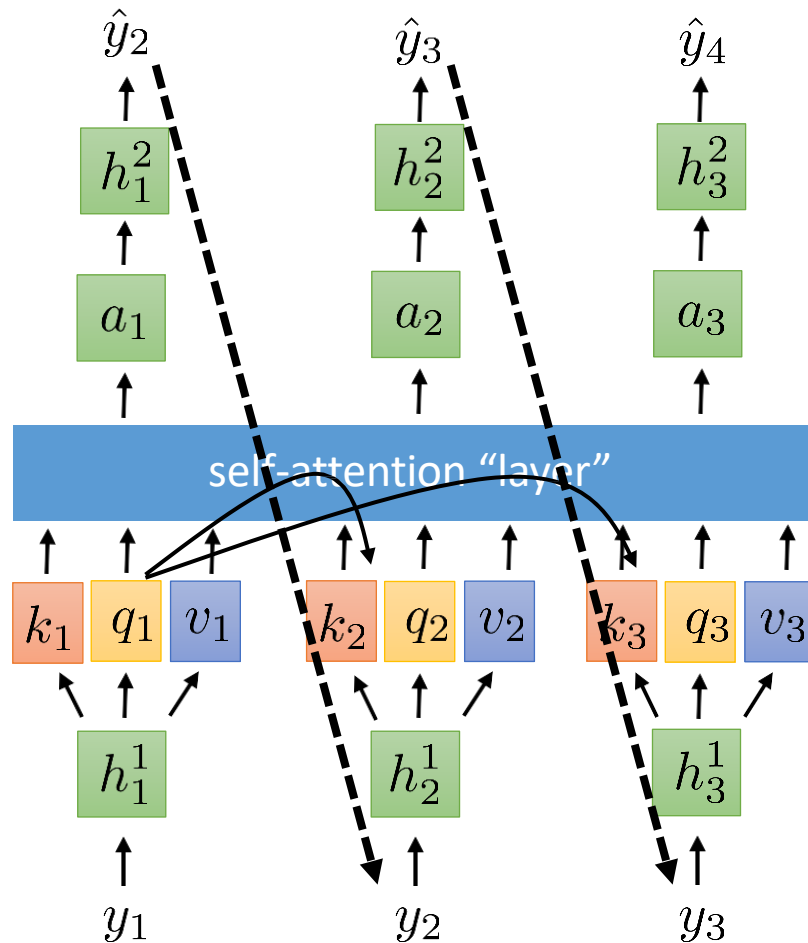


A **crude** self-attention “language model”:

In practice, there would be several alternating self-attention layers and position-wise feedforward networks



# Self-attention can see the future!



A **crude** self-attention “language model”:

In practice, there would be several alternating self-attention layers and position-wise feedforward networks

**Big problem:** self-attention at step 1 can look at the value at steps 2 & 3, which is based on the **inputs** at steps 2 & 3

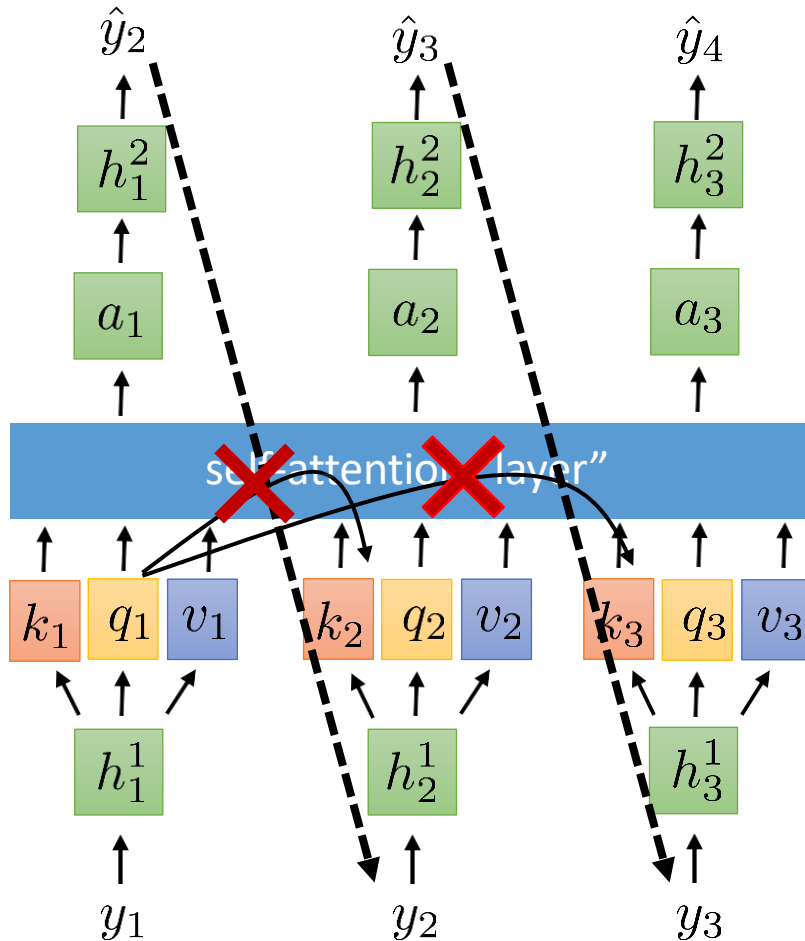
**At test time** (when decoding), the **inputs** at steps 2 & 3 will be based on the output at step 1...

...which requires knowing the **input** at steps 2 & 3



# Masked Attention

A **crude** self-attention “language model”:



**At test time** (when decoding), the **inputs** at steps 2 & 3 will be based on the output at step 1...

...which requires knowing the **input** at steps 2 & 3

Must allow self-attention into the **past**...

...but not into the **future**

Easy solution:

~~$$e_{l,t} = q_l \cdot k_t$$~~

$$e_{l,t} = \begin{cases} q_l \cdot k_t & \text{if } l \geq t \\ -\infty & \text{otherwise} \end{cases}$$

in practice:

just replace  $\exp(e_{l,t})$  with 0 if  $l < t$

inside the softmax