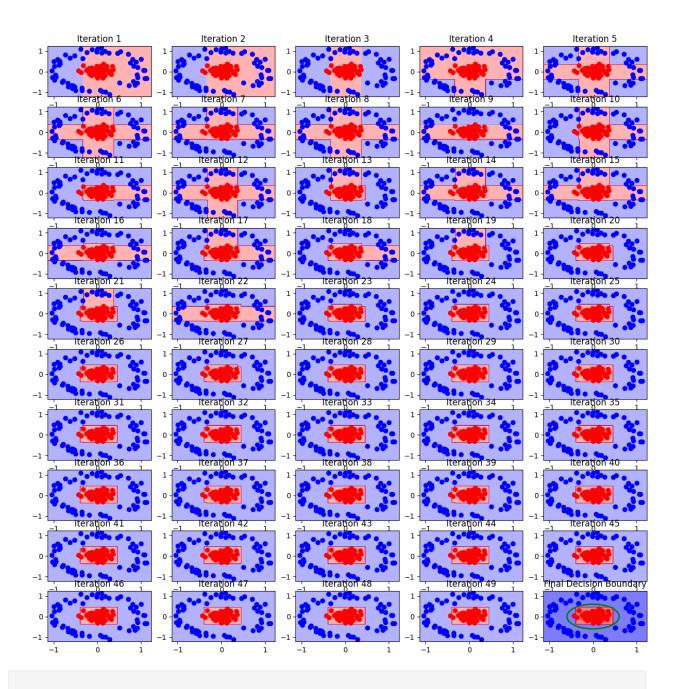
Task 1

Implement the Adaboost algorithm yourself from scratch.

```
import numpy as np
from sklearn.datasets import make circles
from sklearn.model selection import train test split
from sklearn.tree import DecisionTreeClassifier
import matplotlib.pyplot as plt
import matplotlib.colors as colors
from sklearn.metrics import accuracy score
# Step 1: Generate the circles dataset
X, y = make circles(n samples=500, noise=0.1, random state=42,
factor=0.2)
X train, X test, y train, y test = train test split(X, y,
random state=42)
# Convert labels from {0, 1} to {-1, 1} for Adaboost compatibility
y train[y train == 0] = -1
y \text{ test}[y \text{ test} == 0] = -1
# Step 2: Implement AdaBoost with deeper decision trees to capture
circular patterns
class AdaBoostCustom:
    def init (self, weak classifier=DecisionTreeClassifier,
n_estimators=50, eta=0.5, max_depth=1):
        self.weak classifier = weak classifier
        self.n estimators = n estimators
        self.eta = eta
        self.max depth = max depth
        self.alphas = []
        self.classifiers = []
    def fit(self, X, y):
        n_samples, _ = X.shape
        # Initialize weights
        w = np.ones(n samples) / n samples
        for t in range(self.n estimators):
            # Train weak classifier (now with higher max depth to
capture non-linearity)
            clf = self.weak classifier(max depth=self.max depth)
            clf.fit(X, y, sample_weight=w)
            y pred = clf.predict(X)
            # Compute weighted error
```

```
error = np.dot(w, (y_pred != y))
            if error > 0.5:
                break
            # Compute alpha
            alpha = self.eta * np.log((1 - error) / error)
            self.alphas.append(alpha)
            self.classifiers.append(clf)
            # Update weights
            w *= np.exp(alpha * (y pred != y))
            w /= np.sum(w) # Normalize weights
    def predict(self, X):
        # Initialize predictions
        final pred = np.zeros(X.shape[0])
        for alpha, clf in zip(self.alphas, self.classifiers):
            final pred += alpha * clf.predict(X)
        return np.sign(final pred)
# Step 3: Visualize decision boundaries
def plot decision boundary(clf, X, y, ax=None, title=""):
    if ax is None:
        ax = plt.gca()
    x \min, x \max = X[:, 0].\min() - 0.1, X[:, 0].\max() + 0.1
    y_{min}, y_{max} = X[:, 1].min() - 0.1, X[:, 1].max() + 0.1
    xx, yy = np.meshgrid(np.arange(x min, x max, 0.01),
                         np.arange(y_min, y_max, 0.01))
    Z = clf.predict(np.c [xx.ravel(), yy.ravel()])
    Z = Z.reshape(xx.shape)
    ax.contour(xx, yy, Z, alpha=0.3,
cmap=colors.ListedColormap(["blue", "red"]))
    ax.scatter(X[:, 0], X[:, 1], c=y,
cmap=colors.ListedColormap(["blue", "red"]))
    ax.set title(title)
# Step 4: Circle boundary visualization
def plot circle boundary(ax):
    # Add a green circle as boundary
    circle = plt.Circle((0, 0), 0.6, color='green', fill=False,
linewidth=2, label="True Circle Boundary")
    ax.add artist(circle)
# Instantiate the AdaBoost algorithm with deeper decision trees
ada boost = AdaBoostCustom(n estimators=50, eta=0.5, max depth=1)
# Train AdaBoost
ada boost.fit(X train, y train)
```

```
# Visualize the decision boundary at each iteration and final decision
boundary
fig, axes = plt.subplots(10, 5, figsize=(15, 15))
axes = axes.ravel()
for t in range(ada_boost.n estimators):
    ada boost iter = AdaBoostCustom(n estimators=t + 1, eta=0.5,
max depth=1)
    ada boost iter.fit(X train, y train)
    plot decision boundary(ada boost iter, X test, y test, ax=axes[t],
title=f"Iteration {t+1}")
    #plot_circle_boundary(axes[t]) # Add circle boundary
# Final decision boundary visualization
plot_decision_boundary(ada_boost, X_test, y_test, ax=axes[-1],
title="Final Decision Boundary")
plot circle boundary(axes[-1]) # Add circle boundary
plt.show()
# Check final accuracy
print("\n")
y pred = ada boost.predict(X test)
print("Final Accuracy:", accuracy score(y test, y pred))
```



Final Accuracy: 0.992

Task 2

Run your Adaboost implementation with several weak classifiers such as LogReg, DecisionStump, DecisionTree(depth=3), Linear SVM, and LDA. Tune the method's hyperparameters (both Adaboost and the underlying weak classifier) for maximizing the classification performance. Based on the data visualization, you can achieve >98% performance

fairly easily. Generate the decision boundary visualizations as the above figure pair for each model class.

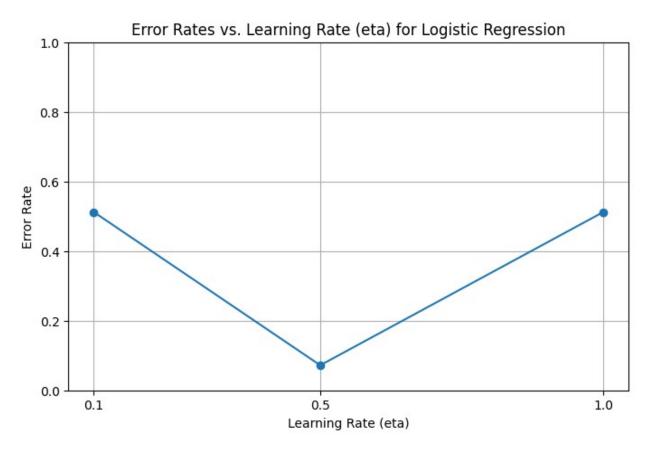
```
import numpy as np
import matplotlib.pyplot as plt
from sklearn.datasets import make circles
from sklearn.model selection import train test split
from sklearn.linear model import LogisticRegression
from sklearn.tree import DecisionTreeClassifier
from sklearn.svm import SVC
from sklearn.discriminant analysis import LinearDiscriminantAnalysis
from sklearn.metrics import accuracy score
# Generate the circles dataset
X, y = make circles(n samples=500, noise=0.1, random state=42,
factor=0.2)
y = np.where(y == 0, -1, 1) # Convert labels to {-1, 1}
X train, X test, y train, y test = train test split(X, y,
random state=42)
# Helper function to calculate weighted error
def weighted_error(y_true, y_pred, weights):
    return np.sum(weights * (y true != y pred))
# Adaboost implementation
class AdaBoost:
    def init (self, learner name, weak learner, T=50, eta=0.5):
        self.weak learner = weak learner
        self.learner name = learner name
        self.T = T
        self.eta = eta
        self.alphas = []
        self.models = []
    def fit(self, X, y):
        n \text{ samples} = X.\text{shape}[0]
        weights = np.full(n samples, 1/n samples)
        for t in range(self.T):
            model = self.weak learner()
            if self.learner name != "LDA":
                model.fit(X, y, sample weight=weights) # Always fit
with sample weights
            else:
                model.fit(X, y) # Always fit without sample weights
            y pred = model.predict(X)
            error = weighted_error(y, y_pred, weights) /
np.sum(weights)
            alpha t = self.eta * np.log((1 - error) / (error + 1e-10))
```

```
self.alphas.append(alpha t)
            self.models.append(model)
            # Update weights
            weights *= np.exp(-alpha t * y * y_pred)
            weights /= np.sum(weights) # Normalize
    def predict(self, X):
        final pred = np.zeros(X.shape[0])
        for alpha, model in zip(self.alphas, self.models):
            final pred += alpha * model.predict(X)
        return np.sign(final pred)
# Function to tune hyperparameters for a specific weak learner
def tune hyperparameters(learner name, weak learner, param grid):
    best accuracy = 0
    best params = None
    for params in param grid:
        adaboost = AdaBoost(learner_name=learner_name,
weak learner=lambda: weak learner(**params), T=50,
eta=params.get('eta', 0.5))
        adaboost.fit(X train, y train)
        y pred = adaboost.predict(X test)
        accuracy = accuracy_score(y_test, y_pred)
        if accuracy > best accuracy:
            best accuracy = accuracy
            best params = params
    print(f"Best parameters for {learner name}: {best params} with
accuracy: {best accuracy:.4f}")
    return best_params # Return best parameters
# Function to find the best eta using the best estimator
def tune eta(learner name, weak learner, best params, eta values):
    best accuracy = 0
    best eta = None
    errors = []
    for eta in eta values:
        adaboost = AdaBoost(learner name=learner name,
weak learner=lambda: weak learner(**best params), T=50, eta=eta)
        adaboost.fit(X train, y_train)
        y pred = adaboost.predict(X test)
        accuracy = accuracy_score(y_test, y_pred)
        error = 1 - accuracy
        errors.append((eta, error)) # Store eta and corresponding
```

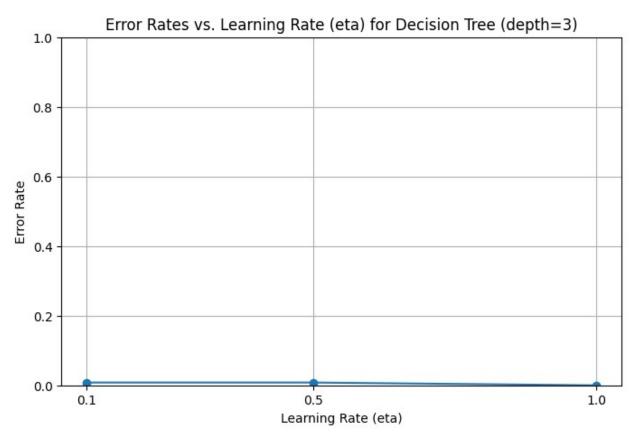
```
error
        print(f"Parameters: {best_params}, eta: {eta}, Accuracy:
{accuracy:.4f}, Error: {error:.4f}")
        if accuracy > best accuracy:
            best accuracy = accuracy
            best eta = eta
    print(f"Best eta for {learner name}: {best eta} with accuracy:
{best accuracy:.4f}")
    print("\n")
    return best_eta, errors # Return best eta and errors for plotting
# Define weak learners with hyperparameter grids
weak learners params = {
    "Logistic Regression": lambda **params:
LogisticRegression(solver='lbfgs', max iter=1000, C=params.get('C',
1.0)),
    "Decision Stump": lambda **params:
DecisionTreeClassifier(max depth=1),
    "Decision Tree (depth=3)": lambda **params:
DecisionTreeClassifier(max depth=3),
    "Linear SVM": lambda **params: SVC(kernel='rbf', probability=True,
C=params.qet('C', 1)),
    "LDA": lambda **params: LinearDiscriminantAnalysis(),
}
# Hyperparameter grids
param grids = {
    "Logistic Regression": [\{'C': c\} \text{ for } c \text{ in } [0.1, 1, 10]],
    "Decision Stump": [{}],
    "Decision Tree (depth=3)": [{}],
    "Linear SVM": [\{'C': c\} \text{ for } c \text{ in } [0.1, 1, 10]],
    "LDA": [{}],
}
# Eta values for tuning
eta values = [0.1, 0.5, 1]
# Dictionary to hold best parameters and errors for plotting
all best params = {}
all errors = {}
# Run tuning for each weak learner to find best hyperparameters and
then best eta
for learner name, weak learner in weak learners params.items():
    print(f"Tuning hyperparameters for {learner name}...")
    best params = tune hyperparameters(learner name, weak learner,
```

```
param grids[learner name])
    print(f"Tuning eta for {learner name} with best params:
{best params}...")
    best eta, errors = tune eta(learner name, weak learner,
best params, eta values)
    all best params[learner name] = (best params, best eta) # Store
best parameters and eta
    all errors[learner name] = errors # Store errors for plotting
# Plotting separate error plots for each algorithm
for learner name, errors in all errors.items():
    plt.figure(figsize=(8, 5))
    eta values = [error[0] for error in errors]
    error rates = [error[1] for error in errors]
    plt.plot(eta_values, error_rates, marker='o')
    plt.title(f'Error Rates vs. Learning Rate (eta) for
{learner name}')
    plt.xlabel('Learning Rate (eta)')
    plt.ylabel('Error Rate')
    plt.xticks(np.unique(eta values)) # Unique eta values
    plt.grid()
    plt.ylim(0, 1)
    plt.show()
for i, (learner_name, (best_params, best_eta)) in
enumerate(all best params.items()):
    # Create the model with the best parameters and best eta
    if learner name == "LDA":
        model = weak learners params[learner name]()
    else:
        model = weak learners params[learner name](**best params)
    model.fit(X train, y train) # Fit the model to the training data
plt.tight_layout()
plt.show()
Tuning hyperparameters for Logistic Regression...
Best parameters for Logistic Regression: {'C': 1} with accuracy:
0.9280
Tuning eta for Logistic Regression with best params: {'C': 1}...
Parameters: {'C': 1}, eta: 0.1, Accuracy: 0.4880, Error: 0.5120
Parameters: {'C': 1}, eta: 0.5, Accuracy: 0.9280, Error: 0.0720
Parameters: {'C': 1}, eta: 1, Accuracy: 0.4880, Error: 0.5120
Best eta for Logistic Regression: 0.5 with accuracy: 0.9280
```

```
Tuning hyperparameters for Decision Stump...
Best parameters for Decision Stump: {} with accuracy: 0.9920
Tuning eta for Decision Stump with best params: {}...
Parameters: {}, eta: 0.1, Accuracy: 0.9520, Error: 0.0480
Parameters: {}, eta: 0.5, Accuracy: 0.9920, Error: 0.0080
Parameters: {}, eta: 1, Accuracy: 0.7600, Error: 0.2400
Best eta for Decision Stump: 0.5 with accuracy: 0.9920
Tuning hyperparameters for Decision Tree (depth=3)...
Best parameters for Decision Tree (depth=3): {} with accuracy: 0.9920
Tuning eta for Decision Tree (depth=3) with best params: {}...
Parameters: {}, eta: 0.1, Accuracy: 0.9920, Error: 0.0080
Parameters: {}, eta: 0.5, Accuracy: 0.9920, Error: 0.0080
Parameters: {}, eta: 1, Accuracy: 1.0000, Error: 0.0000
Best eta for Decision Tree (depth=3): 1 with accuracy: 1.0000
Tuning hyperparameters for Linear SVM...
Best parameters for Linear SVM: {'C': 10} with accuracy: 1.0000
Tuning eta for Linear SVM with best params: {'C': 10}...
Parameters: {'C': 10}, eta: 0.1, Accuracy: 1.0000, Error: 0.0000
Parameters: {'C': 10}, eta: 0.5, Accuracy: 1.0000, Error: 0.0000
Parameters: {'C': 10}, eta: 1, Accuracy: 1.0000, Error: 0.0000
Best eta for Linear SVM: 0.1 with accuracy: 1.0000
Tuning hyperparameters for LDA...
Best parameters for LDA: {} with accuracy: 0.7360
Tuning eta for LDA with best params: {}...
Parameters: {}, eta: 0.1, Accuracy: 0.7360, Error: 0.2640
Parameters: {}, eta: 0.5, Accuracy: 0.7360, Error: 0.2640
Parameters: {}, eta: 1, Accuracy: 0.7360, Error: 0.2640
Best eta for LDA: 0.1 with accuracy: 0.7360
```

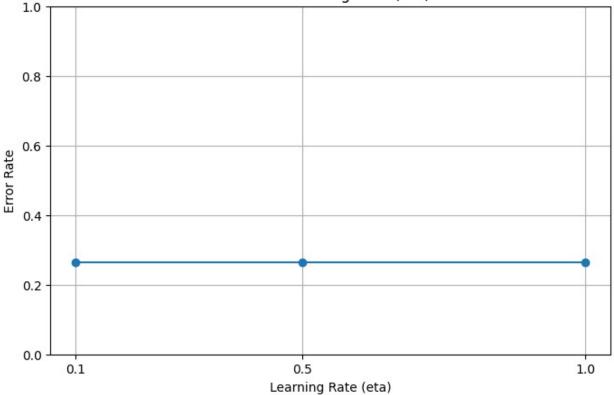












<Figure size 640x480 with 0 Axes>

Code for Deicsion boundary and plot for error rate vs eta for different algorithm

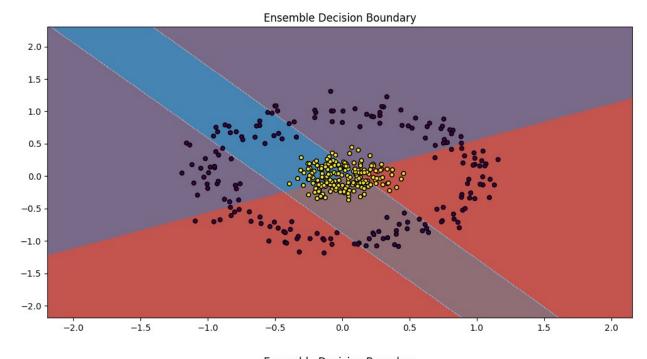
```
import numpy as np
import matplotlib.pyplot as plt
from sklearn.datasets import make circles
from sklearn.model selection import train test split
from sklearn.linear model import LogisticRegression
from sklearn.tree import DecisionTreeClassifier
from sklearn.svm import SVC
from sklearn.discriminant analysis import LinearDiscriminantAnalysis
from sklearn.metrics import accuracy score
# Generate the circles dataset
X, y = make circles(n samples=500, noise=0.1, random state=42,
factor=0.2)
y = np.where(y == 0, -1, 1) # Convert labels to {-1, 1}
X_train, X_test, y_train, y_test = train_test_split(X, y,
random state=42)
# Helper function to calculate weighted error
def weighted_error(y_true, y_pred, weights):
    return np.sum(weights * (y_true != y_pred))
```

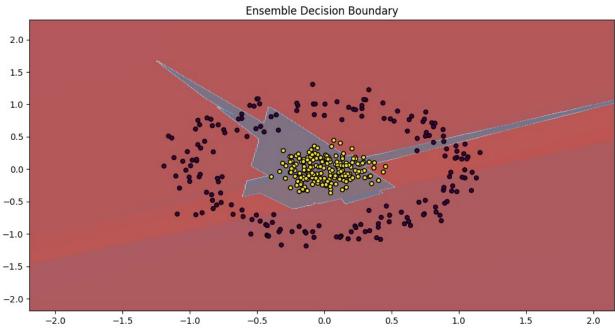
```
# Adaboost implementation
class AdaBoost:
    def init (self, learner name, weak learner, T=50, eta=0.5):
        self.weak learner = weak learner
        self.learner name = learner name
        self.T = T
        self.eta = eta
        self.alphas = []
        self.models = []
    def fit(self, X, y):
        n \text{ samples} = X.shape[0]
        weights = np.full(n_samples, 1/n_samples)
        plt.figure(figsize=(12, 6))
        for t in range(self.T):
            model = self.weak learner()
            if self.learner_name != "LDA":
                model.fit(X, y, sample weight=weights) # Always fit
with sample weights
            else:
                model.fit(X, y) # Always fit without sample weights
            y pred = model.predict(X)
            # Compute the weighted error
            error = weighted_error(y, y_pred, weights) /
np.sum(weights)
            alpha_t = self.eta * np.log((1 - error) / (error + 1e-10))
            self.alphas.append(alpha t)
            self.models.append(model)
            # Update weights
            weights *= np.exp(-alpha t * y * y pred)
            weights /= np.sum(weights) # Normalize
            # Plot the decision boundary after each iteration
            self.plot classifier fit(X, y, model, t + 1)
        self.plot final decision boundary(X, y)
    def predict(self, X):
        final pred = np.zeros(X.shape[0])
        for alpha, model in zip(self.alphas, self.models):
            final pred += alpha * model.predict(X)
        return np.sign(final pred)
    def plot classifier fit(self, X, y, model, iteration):
        """Plots classifier fit at every iteration without clearing
previous iterations."""
```

```
x_{min}, x_{max} = X[:, 0].min() - 1, X[:, 0].max() + 1
        y \min, y \max = X[:, 1].\min() - 1, X[:, 1].\max() + 1
        xx, yy = np.meshgrid(np.arange(x_min, x_max, 0.01),
                             np.arange(y min, y max, 0.01))
        Z = model.predict(np.c [xx.ravel(), yy.ravel()])
        Z = Z.reshape(xx.shape)
        # Plot decision boundary of current iteration
        plt.contourf(xx, yy, Z, alpha=0.3, cmap=plt.cm.RdBu, levels=[-
1, 0, 1]
        plt.scatter(X[:, 0], X[:, 1], c=y, edgecolors='k', marker='o',
s=20)
        plt.title(f'Classifier fit after {iteration} iterations')
    def plot final decision boundary(self, X, y):
        """Plots the final ensembled decision boundary."""
        x \min, x \max = X[:, 0].\min() - 1, X[:, 0].\max() + 1
        y_{min}, y_{max} = X[:, 1].min() - 1, X[:, 1].max() + 1
        xx, yy = np.meshgrid(np.arange(x_min, x_max, 0.01),
                             np.arange(y min, y max, 0.01))
        Z = self.predict(np.c [xx.ravel(), yy.ravel()])
        Z = Z.reshape(xx.shape)
        plt.contourf(xx, yy, Z, alpha=0.4, cmap=plt.cm.RdBu)
        plt.scatter(X[:, 0], X[:, 1], c=y, edgecolors='k', marker='o',
s=20)
        plt.title('Ensemble Decision Boundary')
        plt.show()
# Function to tune hyperparameters for a specific weak learner
def tune hyperparameters(learner name, weak learner, param grid):
    best accuracy = 0
    best params = None
    for params in param grid:
        adaboost = AdaBoost(learner name=learner name,
weak_learner=lambda: weak_learner(**params), T=50,
eta=params.get('eta', 0.5))
        adaboost.fit(X train, y_train)
        y pred = adaboost.predict(X test)
        accuracy = accuracy score(y test, y pred)
        if accuracy > best accuracy:
            best accuracy = accuracy
            best params = params
    print(f"Best parameters for {learner_name}: {best params} with
accuracy: {best accuracy:.4f}")
    return best_params # Return best parameters
```

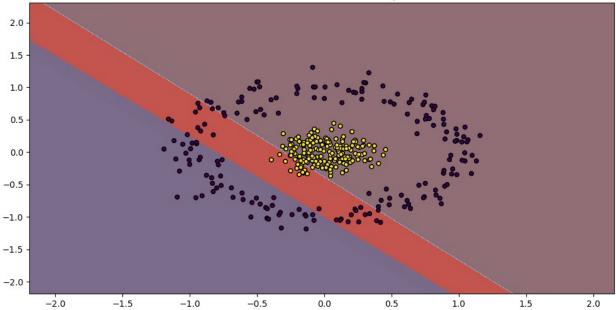
```
# Function to find the best eta using the best estimator
def tune eta(learner name, weak learner, best params, eta values):
    best accuracy = 0
    best eta = None
    errors = []
    for eta in eta values:
        adaboost = AdaBoost(learner name=learner name,
weak learner=lambda: weak learner(**best params), T=50, eta=eta)
        adaboost.fit(X train, y train)
        y_pred = adaboost.predict(X test)
        accuracy = accuracy_score(y_test, y_pred)
        error = 1 - accuracy
        errors.append((eta, error)) # Store eta and corresponding
error
        print(f"Parameters: {best params}, eta: {eta}, Accuracy:
{accuracy:.4f}, Error: {error:.4f}")
        if accuracy > best accuracy:
            best accuracy = accuracy
            best eta = eta
    print(f"Best eta for {learner name}: {best eta} with accuracy:
{best accuracy:.4f}")
    print("\n")
    return best eta, errors # Return best eta and errors for plotting
# Define weak learners with hyperparameter grids
weak learners params = {
    "Logistic Regression": lambda **params:
LogisticRegression(solver='lbfgs', max iter=1000, C=params.get('C',
1.0)),
    "Decision Stump": lambda **params:
DecisionTreeClassifier(max depth=1),
    "Decision Tree (depth=3)": lambda **params:
DecisionTreeClassifier(max depth=3),
    "Linear SVM": lambda **params: SVC(kernel='rbf', probability=True,
C=params.get('C', 1)),
    "LDA": lambda **params: LinearDiscriminantAnalysis(),
}
# Hyperparameter grids
param grids = {
    "Logistic Regression": [\{'C': c\} \text{ for } c \text{ in } [0.1, 1, 10]],
    "Decision Stump": [{}],
    "Decision Tree (depth=3)": [{}],
```

```
"Linear SVM": [{'C': c} for c in [0.1, 1, 10]],
    "LDA": [{}],
}
# Eta values for tuning
eta values = [0.1, 0.5, 1]
# Dictionary to hold best parameters and errors for plotting
all best params = {}
all errors = {}
# Run tuning for each weak learner to find best hyperparameters and
then best eta
for learner name, weak learner in weak learners params.items():
    print(f"Tuning hyperparameters for {learner name}...")
    best params = tune hyperparameters(learner name, weak learner,
param grids[learner name])
    print(f"Tuning eta for {learner name} with best params:
{best params}...")
    best eta, errors = tune eta(learner name, weak learner,
best params, eta values)
    all best params[learner name] = (best params, best eta) # Store
best parameters and eta
    all errors[learner name] = errors # Store errors for plotting
# Plotting separate error plots for each algorithm
for learner name, errors in all errors.items():
    plt.figure(figsize=(8, 5))
    eta values = [error[0] for error in errors]
    error rates = [error[1] for error in errors]
    plt.plot(eta values, error rates, marker='o')
    plt.title(f'Error Rates vs. Learning Rate (eta) for
{learner name}')
    plt.xlabel('Learning Rate (eta)')
    plt.ylabel('Error Rate')
    plt.xticks(np.unique(eta values)) # Unique eta values
    plt.grid()
    plt.ylim(0, 1)
    plt.show()
Tuning hyperparameters for Logistic Regression...
```

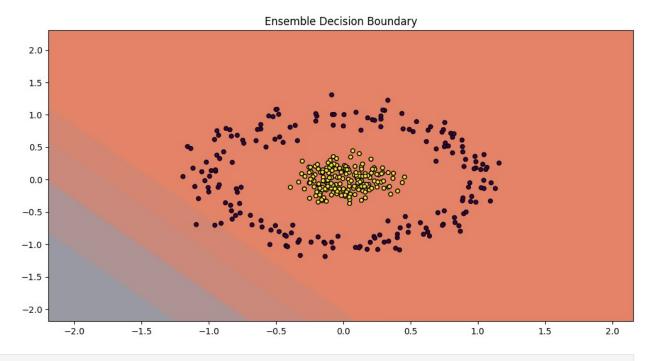






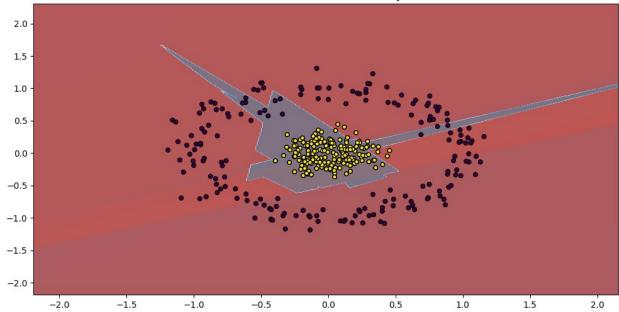


Best parameters for Logistic Regression: {'C': 1} with accuracy: 0.9280
Tuning eta for Logistic Regression with best params: {'C': 1}...

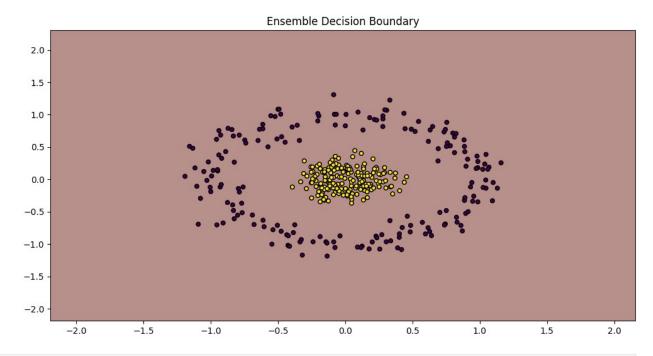


Parameters: {'C': 1}, eta: 0.1, Accuracy: 0.4880, Error: 0.5120





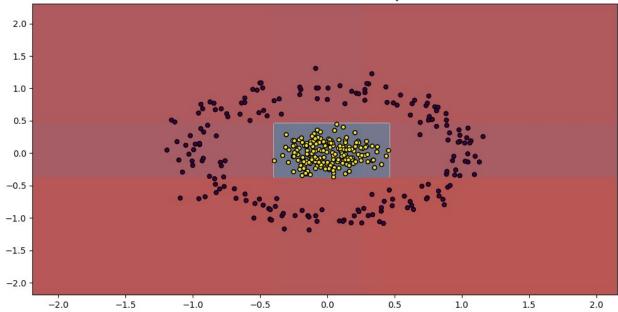
Parameters: {'C': 1}, eta: 0.5, Accuracy: 0.9280, Error: 0.0720



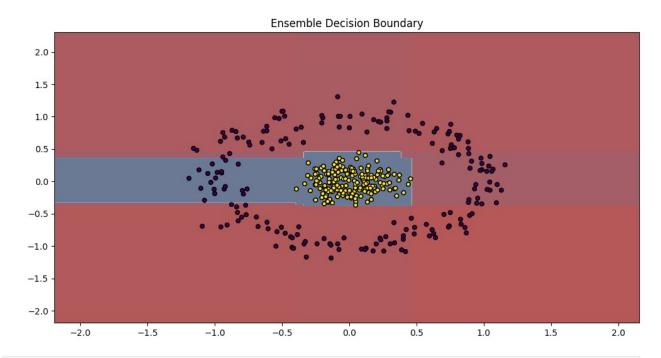
Parameters: {'C': 1}, eta: 1, Accuracy: 0.4880, Error: 0.5120 Best eta for Logistic Regression: 0.5 with accuracy: 0.9280

Tuning hyperparameters for Decision Stump...



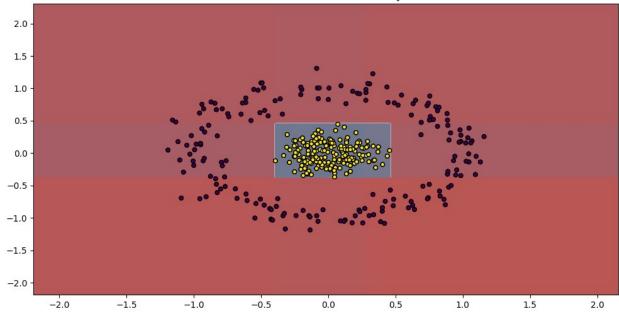


Best parameters for Decision Stump: {} with accuracy: 0.9920 Tuning eta for Decision Stump with best params: {}...

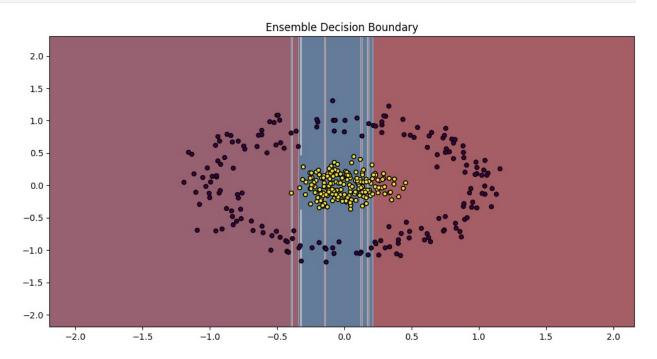


Parameters: {}, eta: 0.1, Accuracy: 0.9520, Error: 0.0480



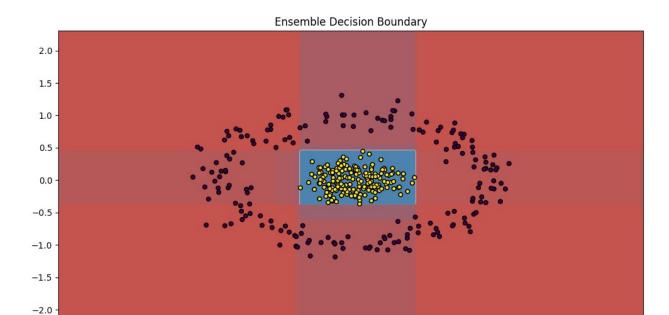


Parameters: {}, eta: 0.5, Accuracy: 0.9920, Error: 0.0080



Parameters: {}, eta: 1, Accuracy: 0.7600, Error: 0.2400 Best eta for Decision Stump: 0.5 with accuracy: 0.9920

Tuning hyperparameters for Decision Tree (depth=3)...



Best parameters for Decision Tree (depth=3): {} with accuracy: 0.9920 Tuning eta for Decision Tree (depth=3) with best params: {}...

0.5

1.0

1.5

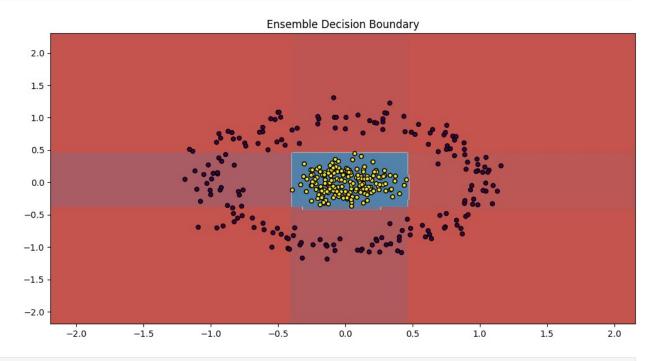
2.0

-0.5

-2.0

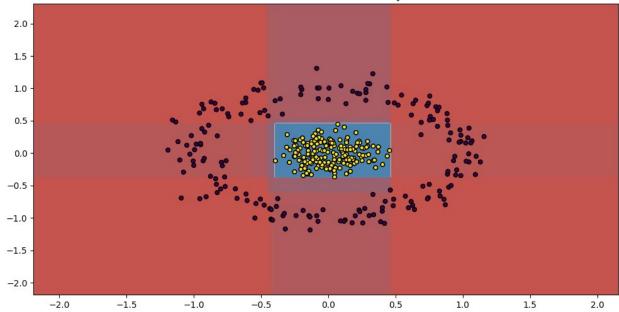
-1.5

-1.0

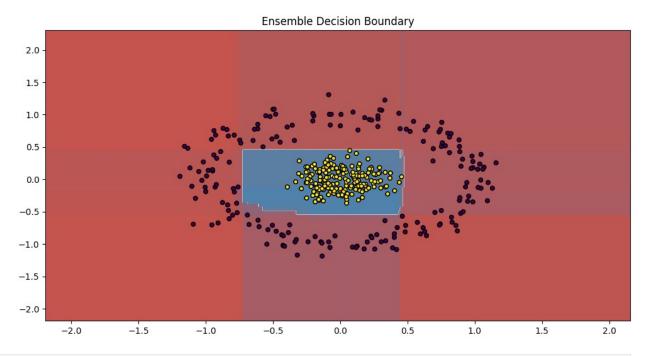


Parameters: {}, eta: 0.1, Accuracy: 0.9920, Error: 0.0080



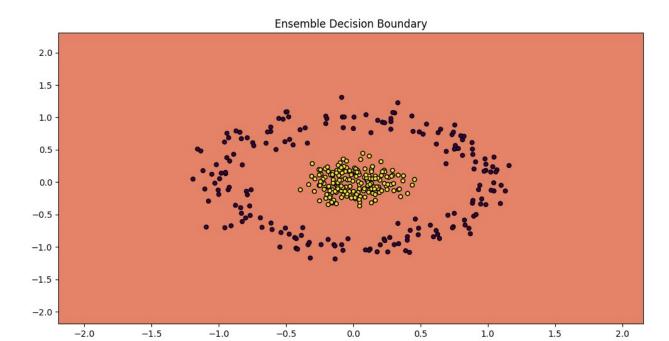


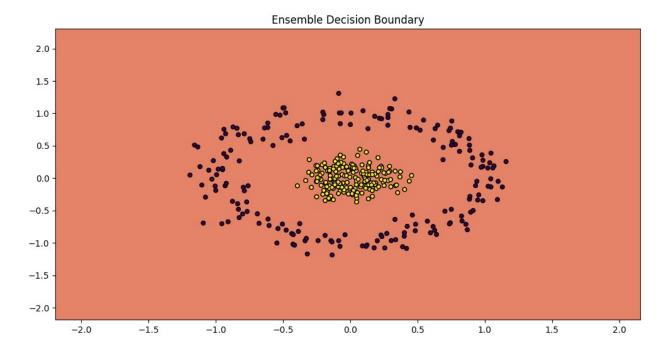
Parameters: {}, eta: 0.5, Accuracy: 0.9920, Error: 0.0080



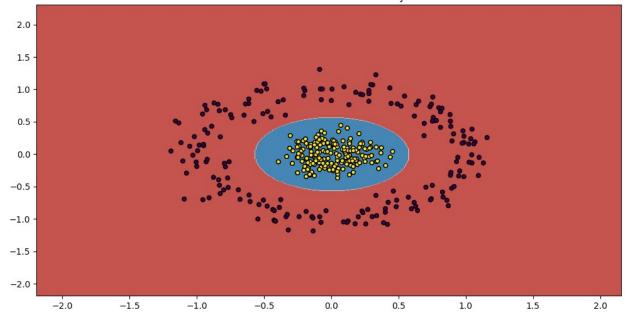
Parameters: {}, eta: 1, Accuracy: 1.0000, Error: 0.0000 Best eta for Decision Tree (depth=3): 1 with accuracy: 1.0000

Tuning hyperparameters for Linear SVM...

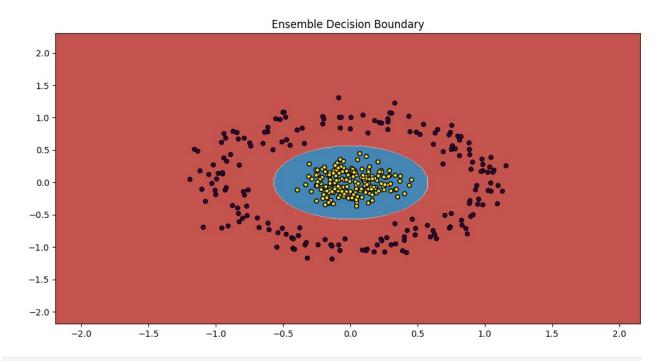






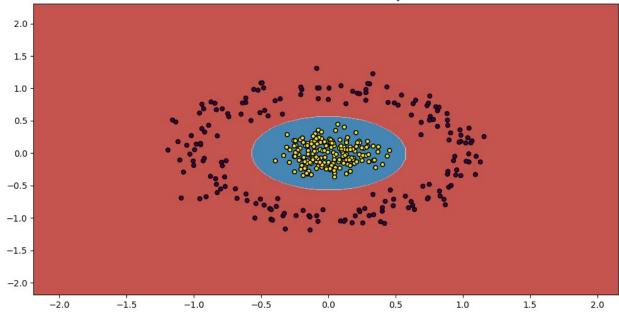


Best parameters for Linear SVM: {'C': 10} with accuracy: 1.0000 Tuning eta for Linear SVM with best params: {'C': 10}...

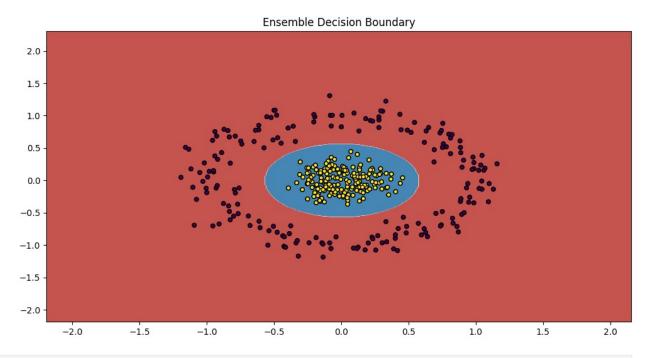


Parameters: {'C': 10}, eta: 0.1, Accuracy: 1.0000, Error: 0.0000





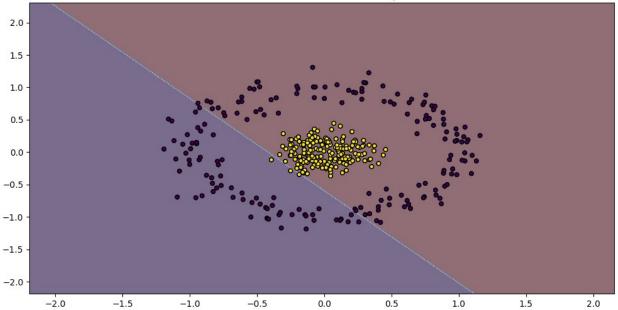
Parameters: {'C': 10}, eta: 0.5, Accuracy: 1.0000, Error: 0.0000



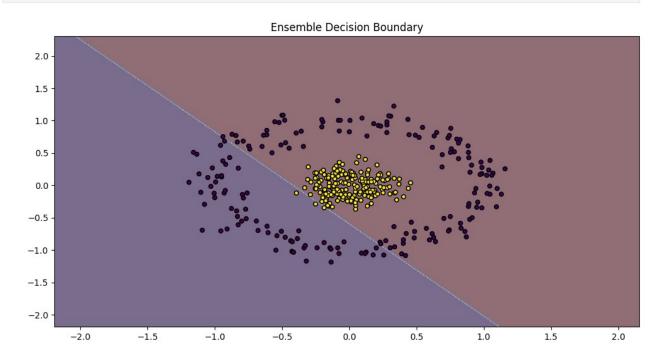
Parameters: {'C': 10}, eta: 1, Accuracy: 1.0000, Error: 0.0000 Best eta for Linear SVM: 0.1 with accuracy: 1.0000

Tuning hyperparameters for LDA...



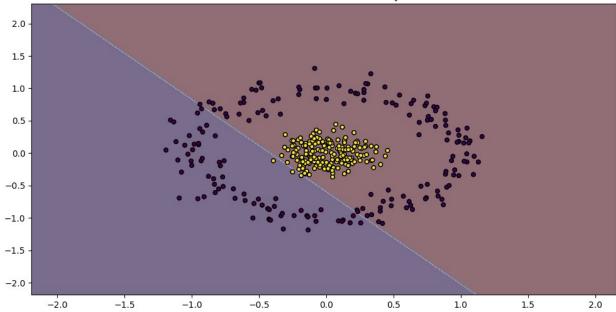


Best parameters for LDA: {} with accuracy: 0.7360 Tuning eta for LDA with best params: {}...

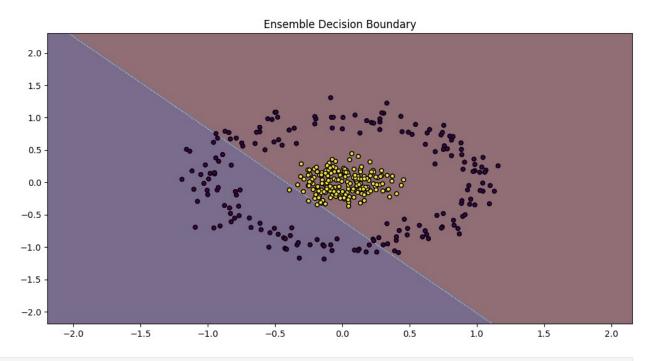


Parameters: {}, eta: 0.1, Accuracy: 0.7360, Error: 0.2640





Parameters: {}, eta: 0.5, Accuracy: 0.7360, Error: 0.2640



Parameters: {}, eta: 1, Accuracy: 0.7360, Error: 0.2640 Best eta for LDA: 0.1 with accuracy: 0.7360

