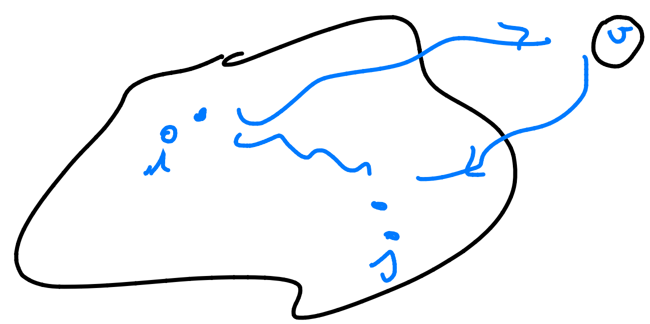
Floyd-Warshall

Idea: Think of each vertex as a resource which gets released one by one.

Everytime we ask what is the best that can do with resources at hand



$V = \{1, \ldots, n\}$

$D^{(0)} = W$

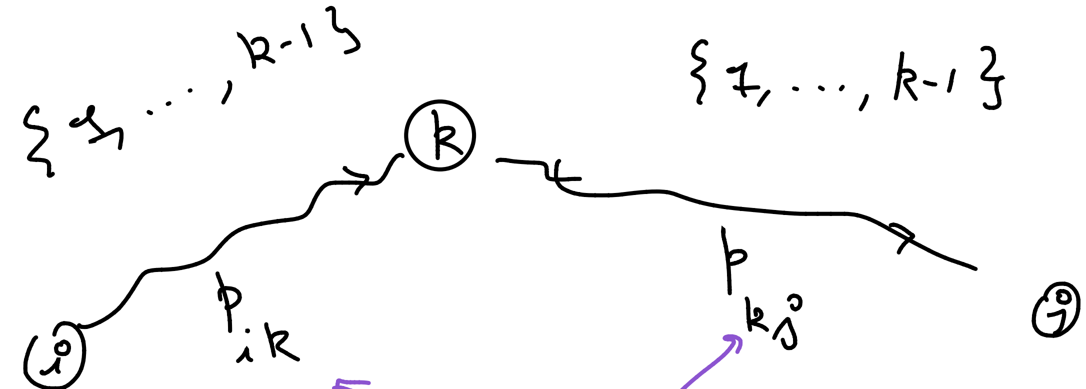
t initialisation, or $t=0$

When the vertex k is released $D^{(k)}(i,j) = \min\left(D^{(k-1)}(i,j),\ \underbrace{D^{(k-1)}(i,k)}_{}^{\text{via}} + D^{(k-1)}(k,j)\right)$

Apparant confusion: k is getting released (through / via) only now then how come we know $D(i,k)$ (to), $D(k,j)$ (from)

Say we an optimal path $p = \{ i, \{1, 2, \ldots, k-1\}, j \}$

Till $k-1$ we would not have found this path. At at time k



Already found and at k we are joining the pieces.

FLOYD-WARSHALL (W)

$D^{(0)} \leftarrow W$

via loop

(note the outer loop is stepping through the vertex that is released)

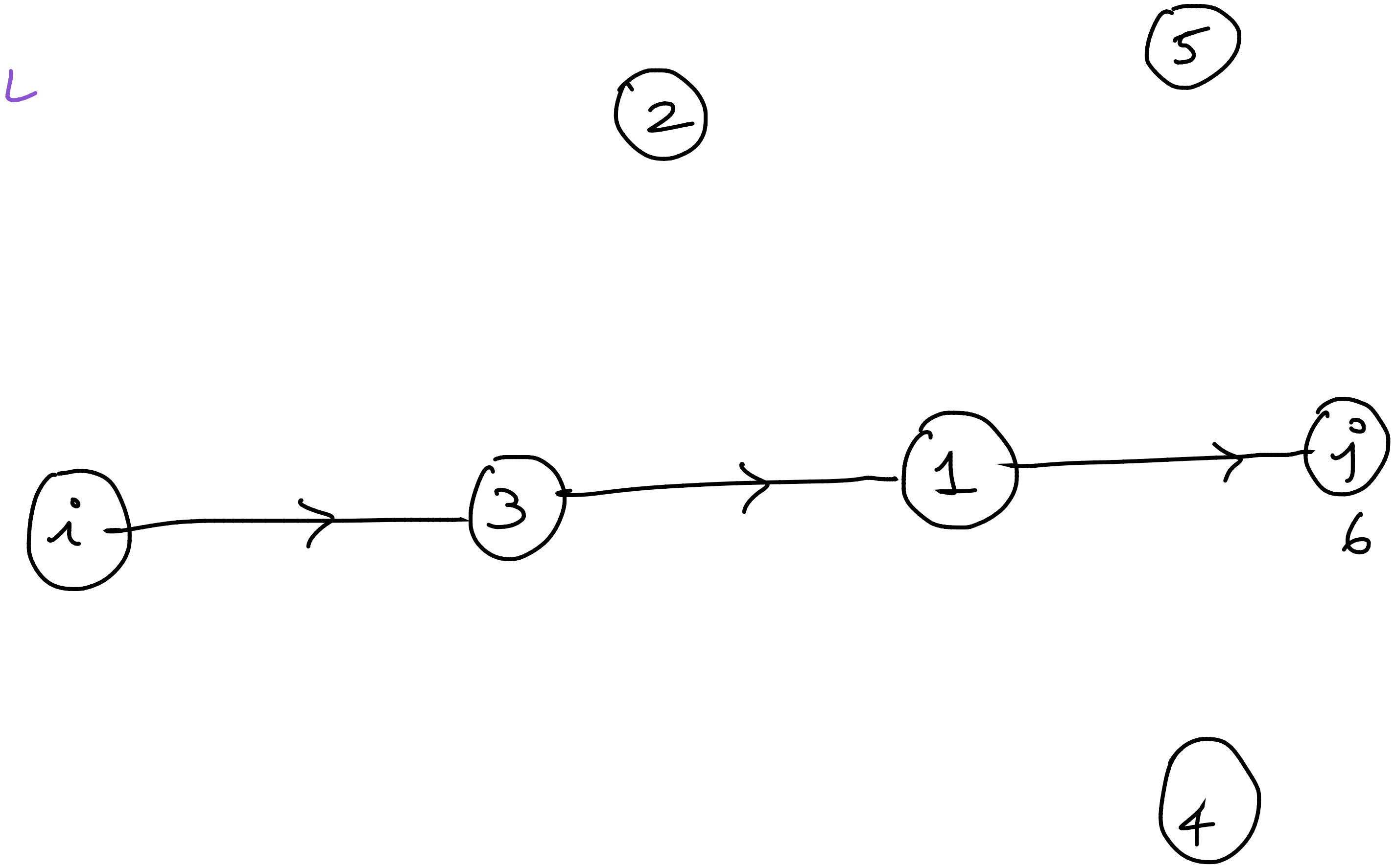
for $k \leftarrow 1$ to n

do for $i \leftarrow 1$ to n

do for $j \leftarrow 1$ to n

do $D^{(k)}(i,j) \leftarrow \min\left(D^{(k-1)}(i,j),\ D^{(k-1)}(i,k) + D^{(k-1)}(k,j) \right)$

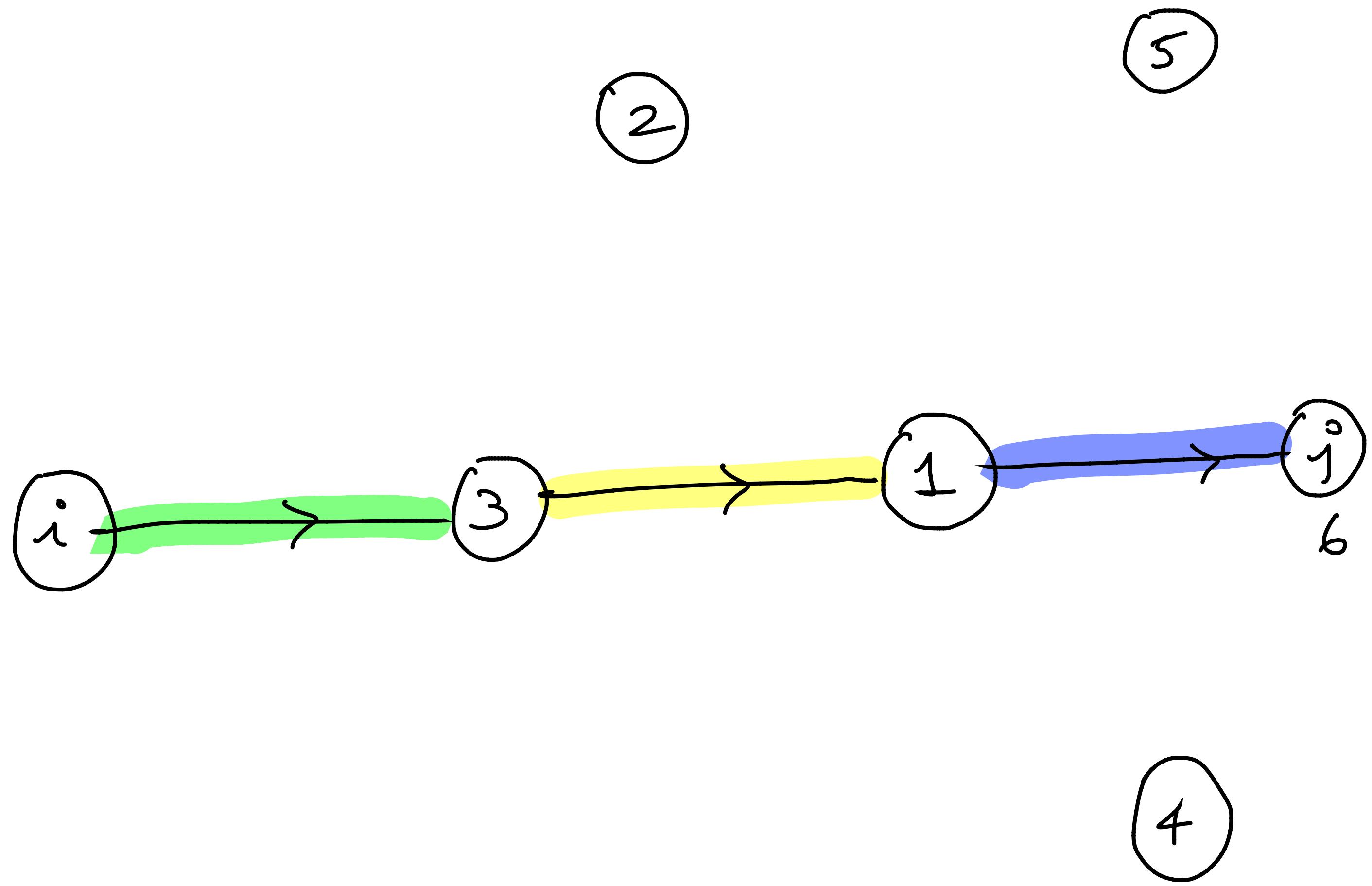
NOT THE ACTUAL GRAPH



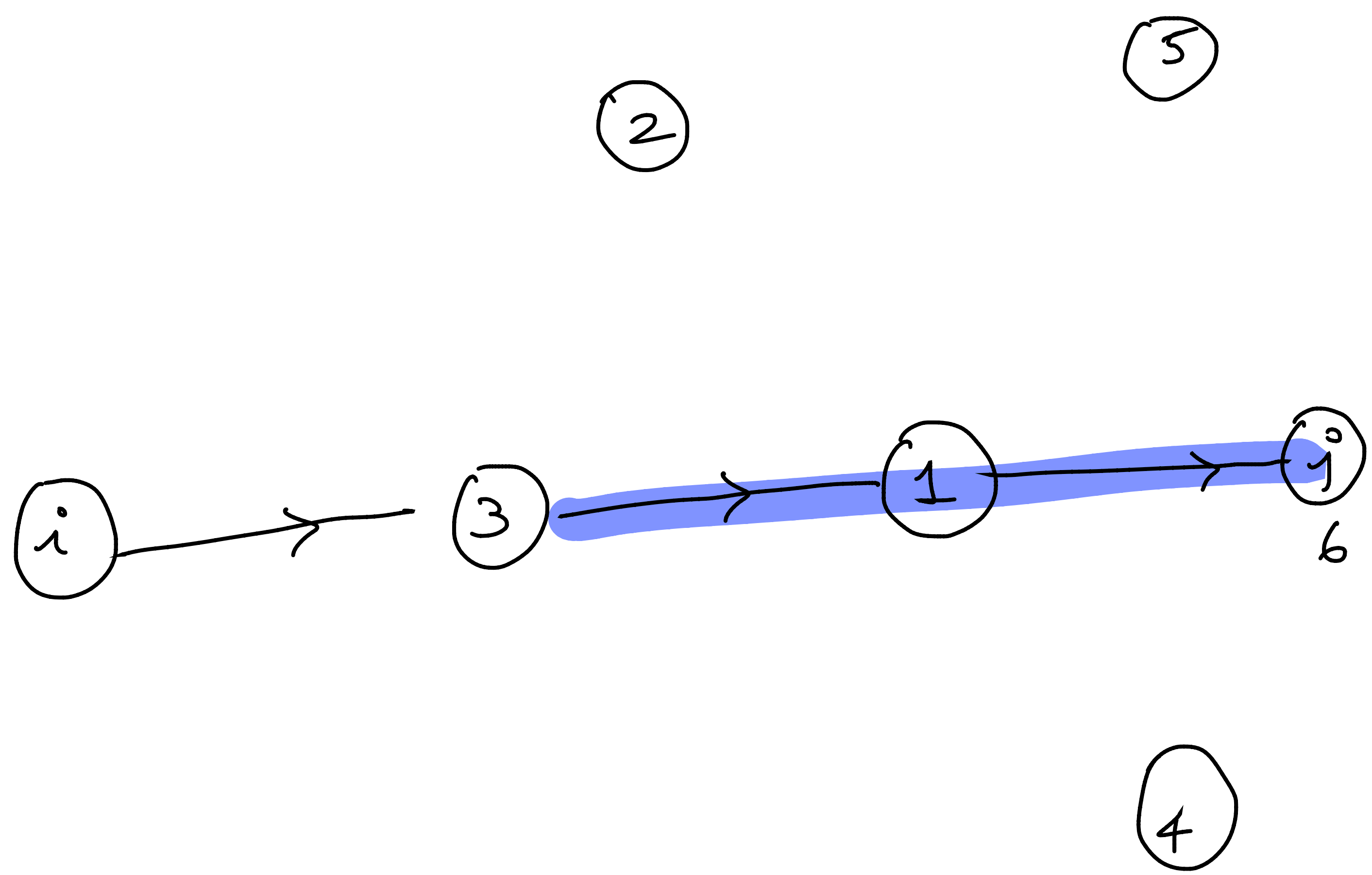
We are only showing a snap short: how path from i to j gets formed given this release order of vertices.

At $t=0$

$D^{(0)}$

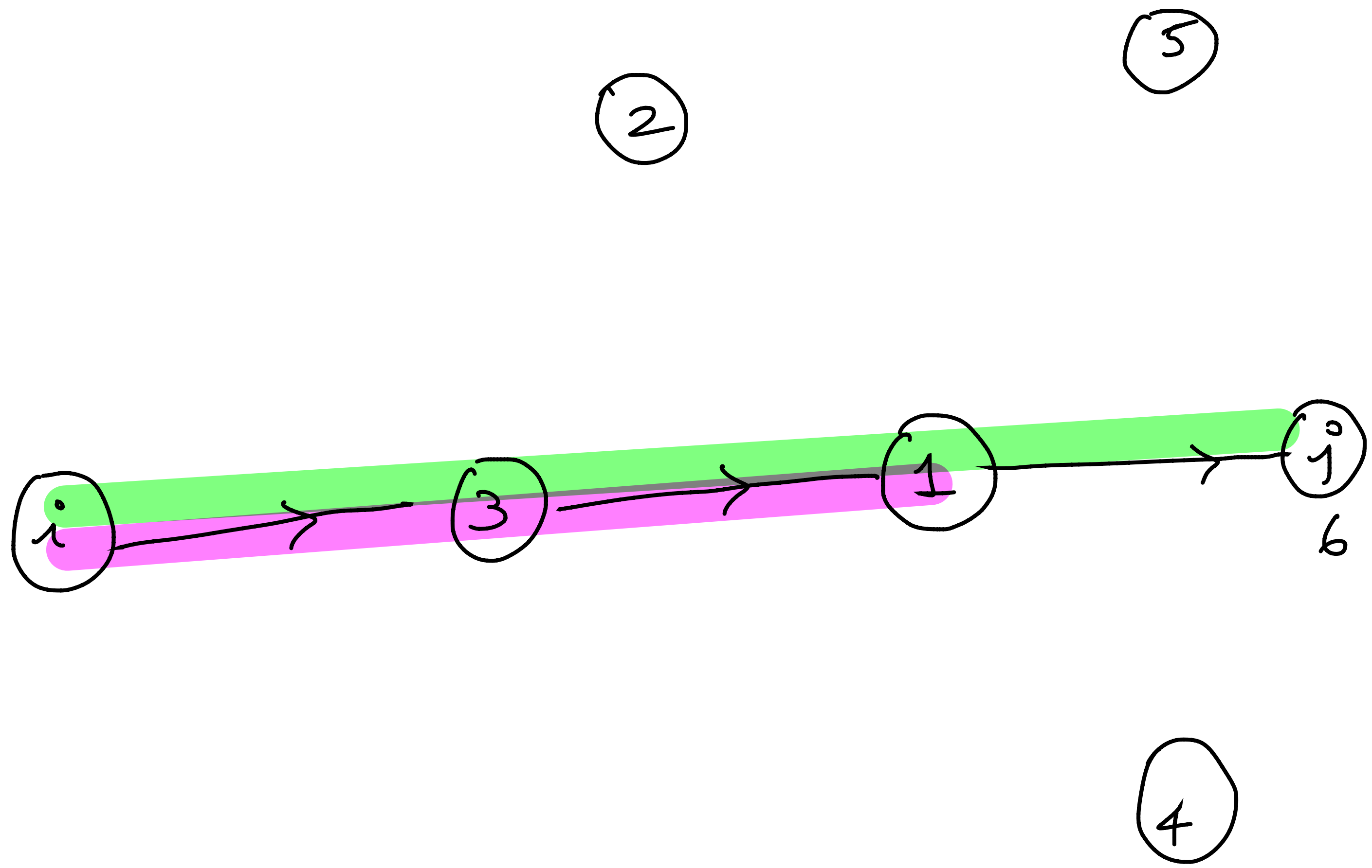


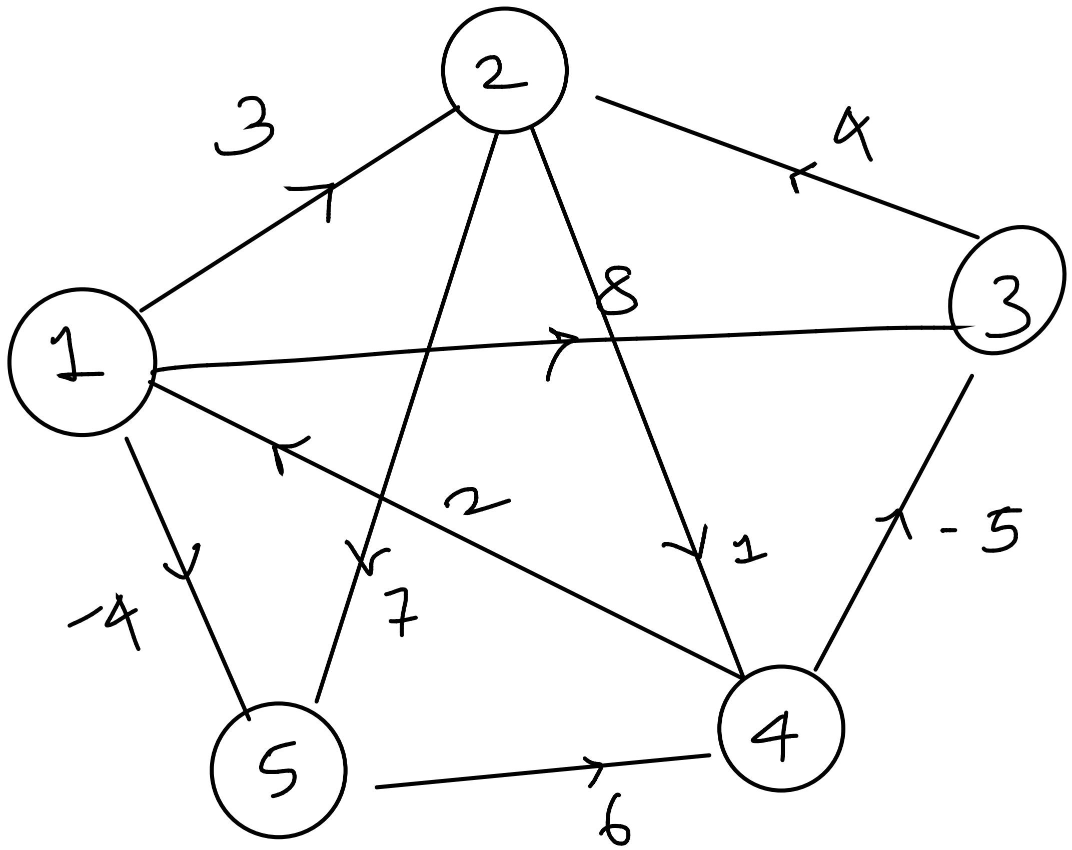
via 1



via 2 , no activity

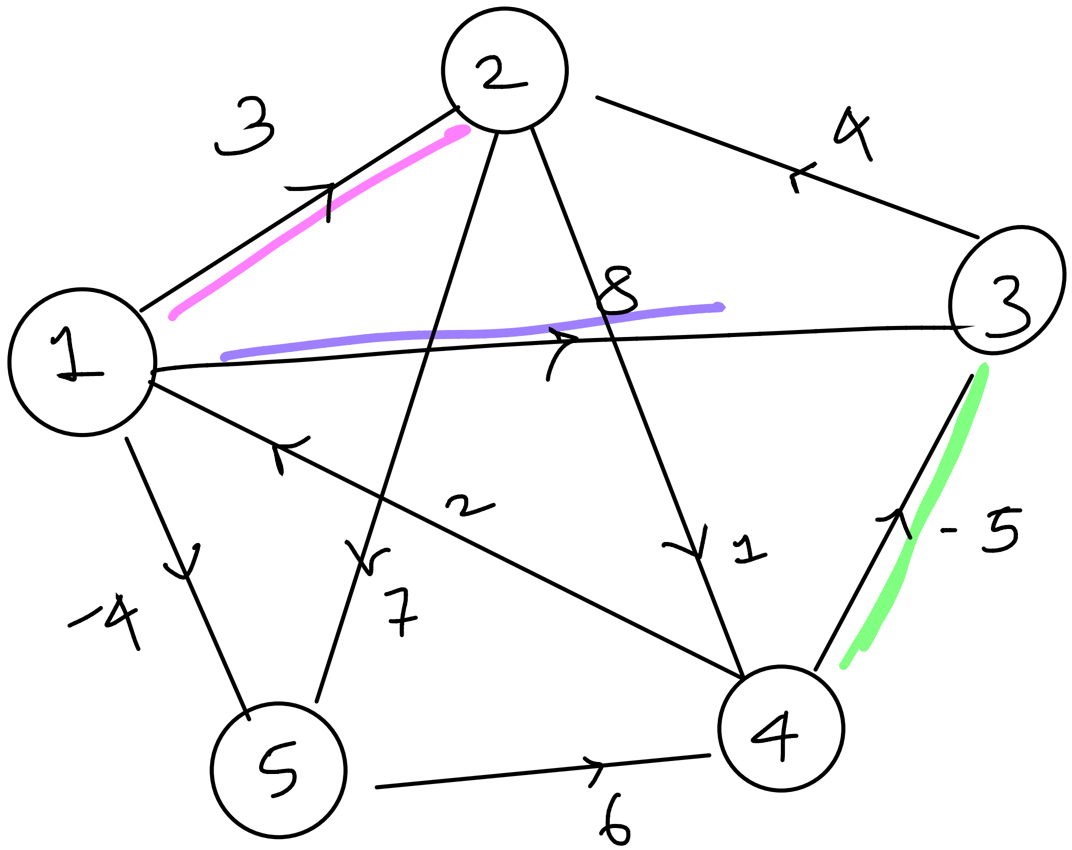
via 3





	1	2	3	4	5
1					
2					
3					
4					
5					

	1	2	3	4	5
1					
2					
3					
4					
5					



$D^{(0)}$

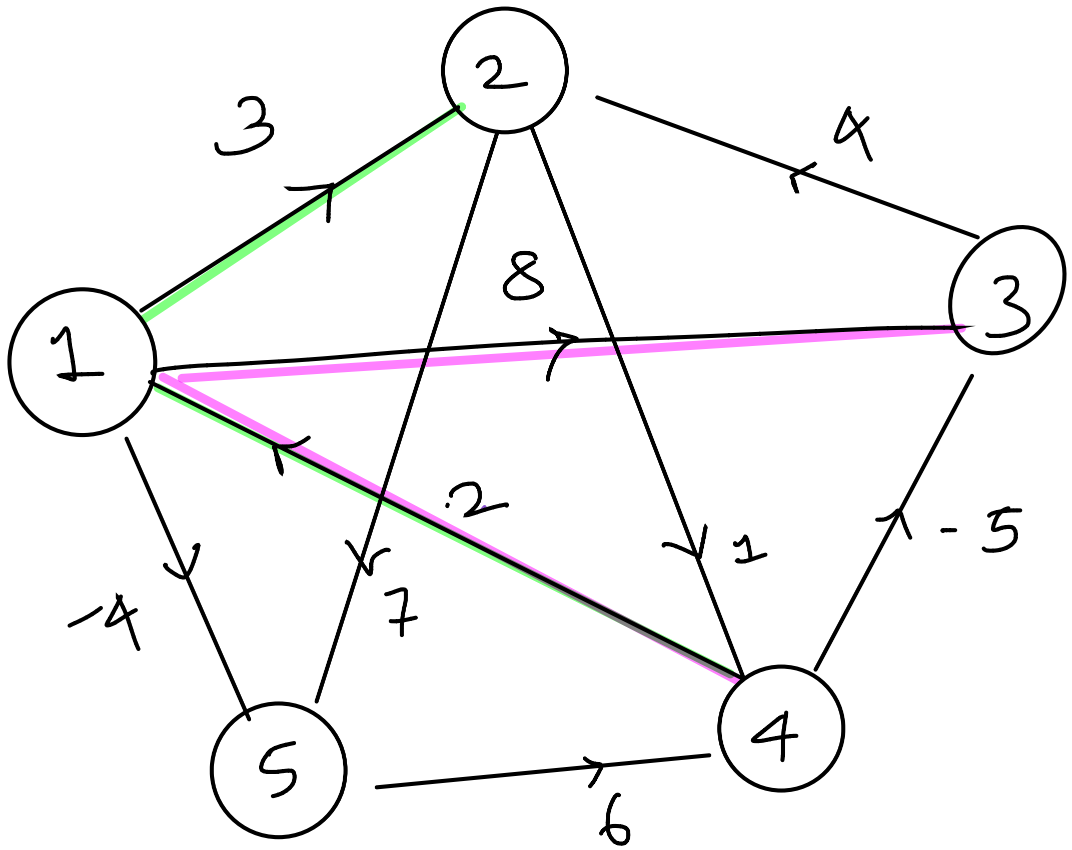
Destination

Source	1	2	3	4	5
1	0	3	8	∞	-4
2	∞	0	∞	1	7
3	∞	4	0	∞	∞
4	2	∞	-5	0	∞
5	∞	∞	∞	6	0

$\Pi^{(0)}$ Predecessor (how

Destination

Source	1	2	3	4	5
1	N	1	1	N	1
2	N	N	N	2	2
3	N	3	N	N	N
4	4	N	4	N	N
5	N	N	N	5	N



Via 1

Can something better happen.

$-5 = \min(-5,\ 2+8)$

Predecessor (how

$D^{(1)}$

Source \ Destination	1	2	3	4	5	
1	0	3	8	∞	-4	→ Does not make sense
2	∞	0	∞	1	7	No path to 1
3	∞	4	0	∞	∞	No path to 1
4	2	5	-5	0	-2	Has a path
5	∞	∞	∞	6	0	No path to 1

No change because 1 is destination

No change because, already a better path

$\Pi^{(1)}$

Source \ Destination	1	2	3	4	5
1	N	1	1	N	1
2	N	N	N	2	2
3	N	3	N	N	N
4	4	1	4	N	1
5	N	N	N	5	N

Exercise: complete the rest.

Transitive closure:

$G^* = (V, E^*)$

$E^* = \{(i,j) :$ there is a path from i to j in $G\}$

j is reachable from i then put a 1

Solution I: Floyd-Warshall with edges as 1

$D_{ij}^{(n)} < n \rightarrow E^*(i,j) = 1$

$D_{ij}^{(n)} = \infty \rightarrow E^*(i,j) = 0$

Solution II: use and / + and OR / min

$$D^{(k)}(i,j) = D^{(k-1)}(i,j) \text{ OR } \left(D^{(k-1)}(i,k) \text{ AND } D^{(k-1)}(k,j)\right)$$

bit 0/1

bit operations.