Property II

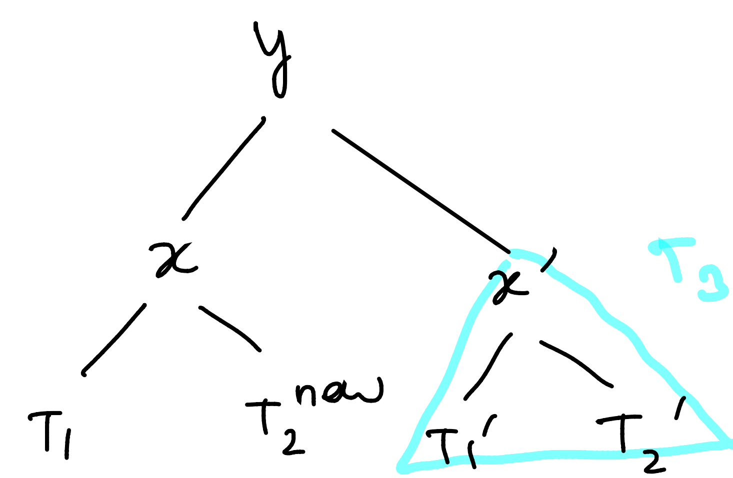
After adding, no imbalance at level y

× ★ $h(T_3) = h(T_2^{new}) + 1$ (same)

× ★ $h(T_3) = h(T_2^{new}) + 1 + 1$ (one more)

✓ ★ $h(T_3) = h(T_2^{new}) + 1 - 1$ (one less)

Take the case of $h(T_3) = h(T_2^{new}) + 1$

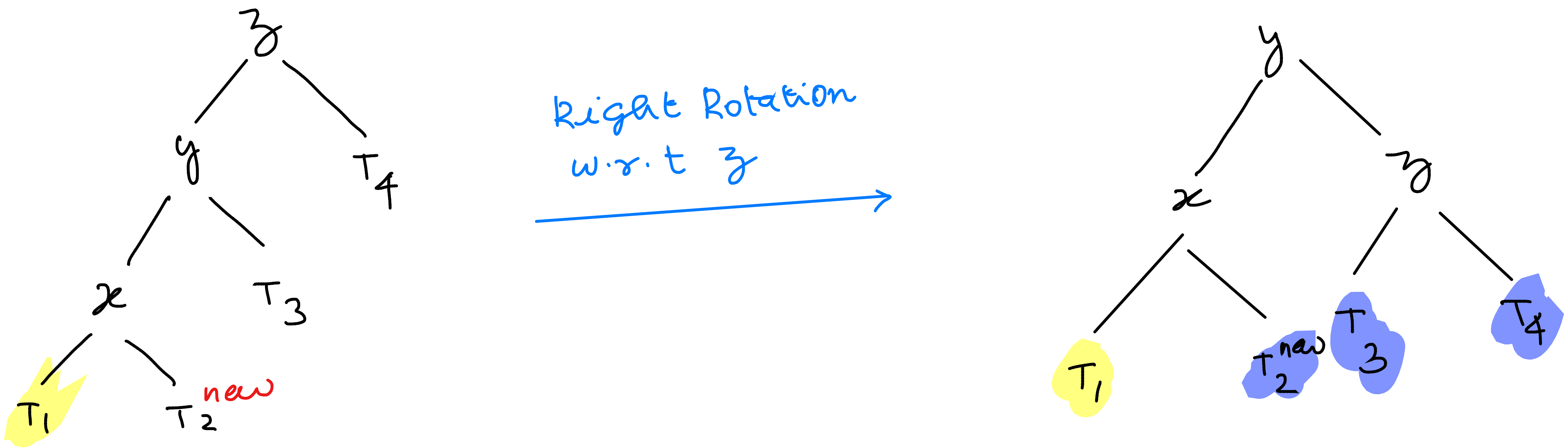



either $h(T_1')$ or $h(T_2')$ should be equal to $h(T_2^{new})$

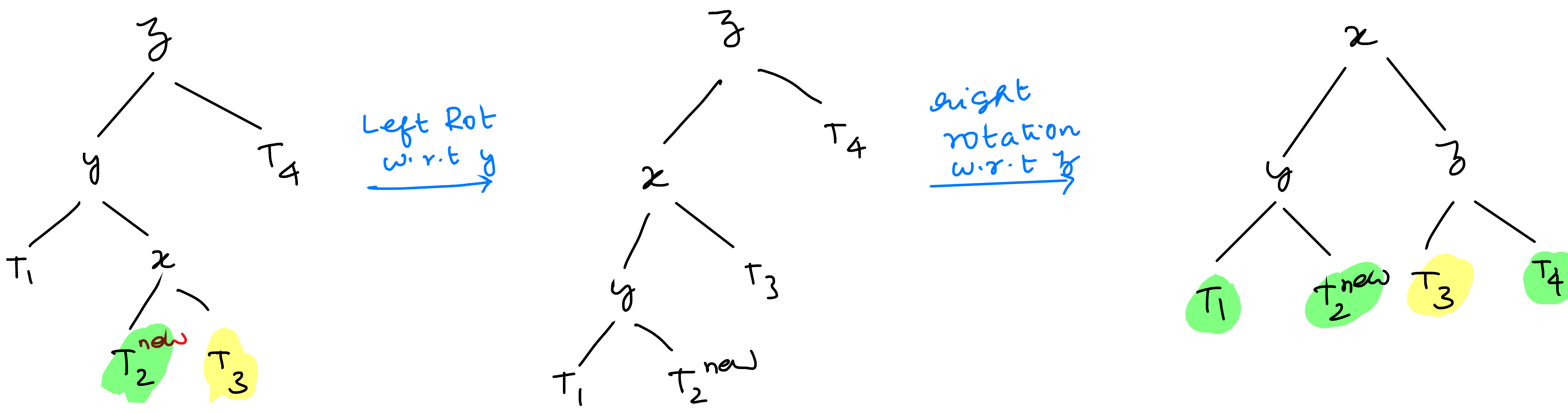
$\Rightarrow$ already imbalance was there in $T_3$ side itself before adding

$h(T_3) = h(T_2^{new})$

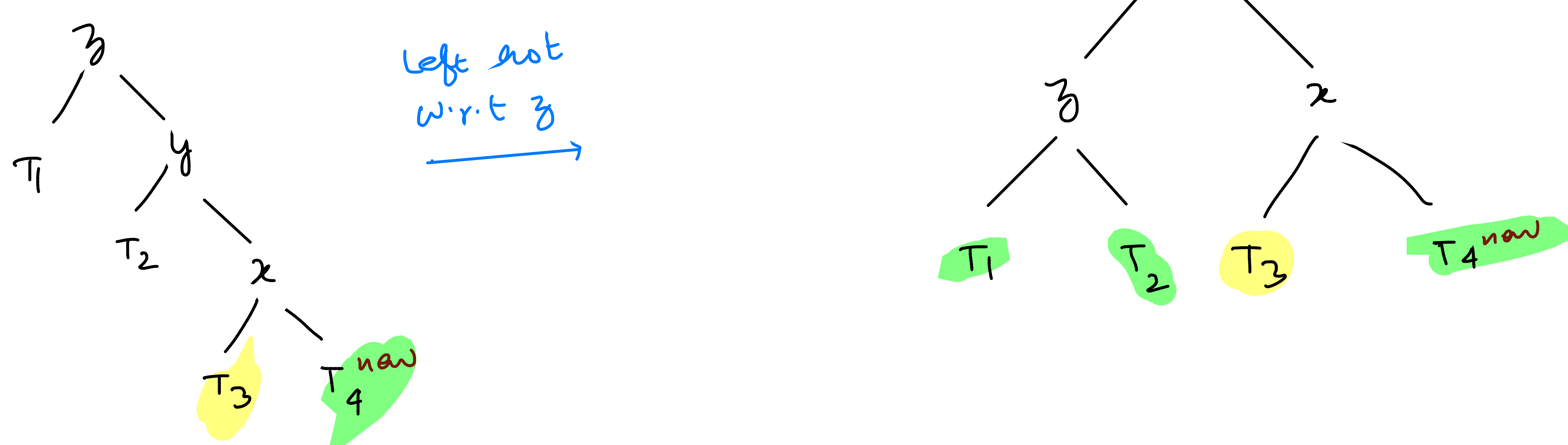
$h(T_2^{new}) = h(T_1) + 1$ , $h(T_2^{new}) = h(T_4)$ , $h(T_3) = h(T_2^{new})$



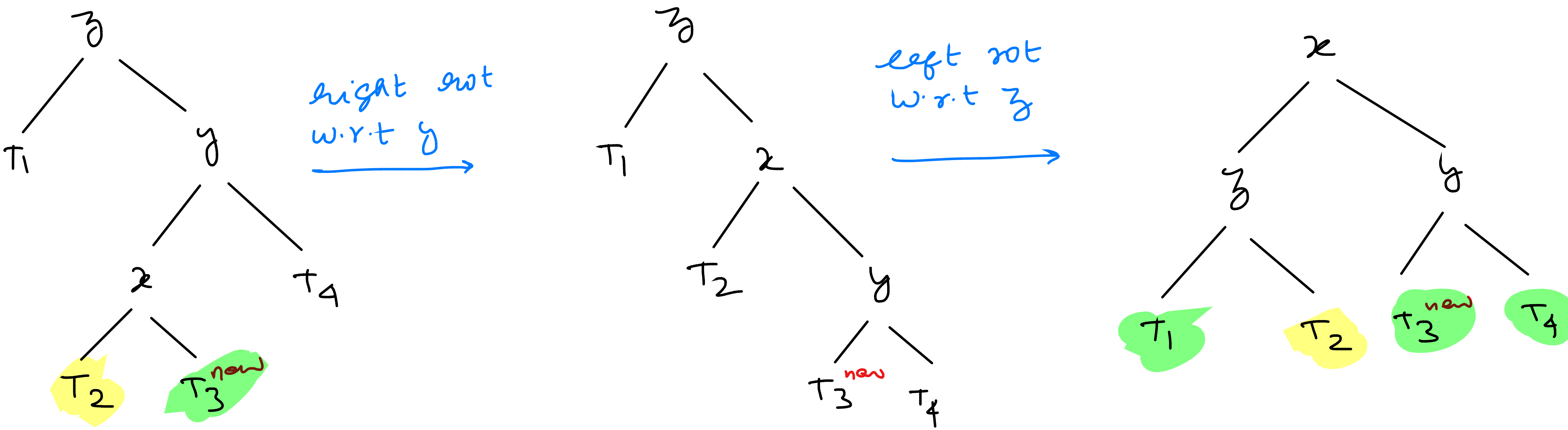
Left Right Imbalance



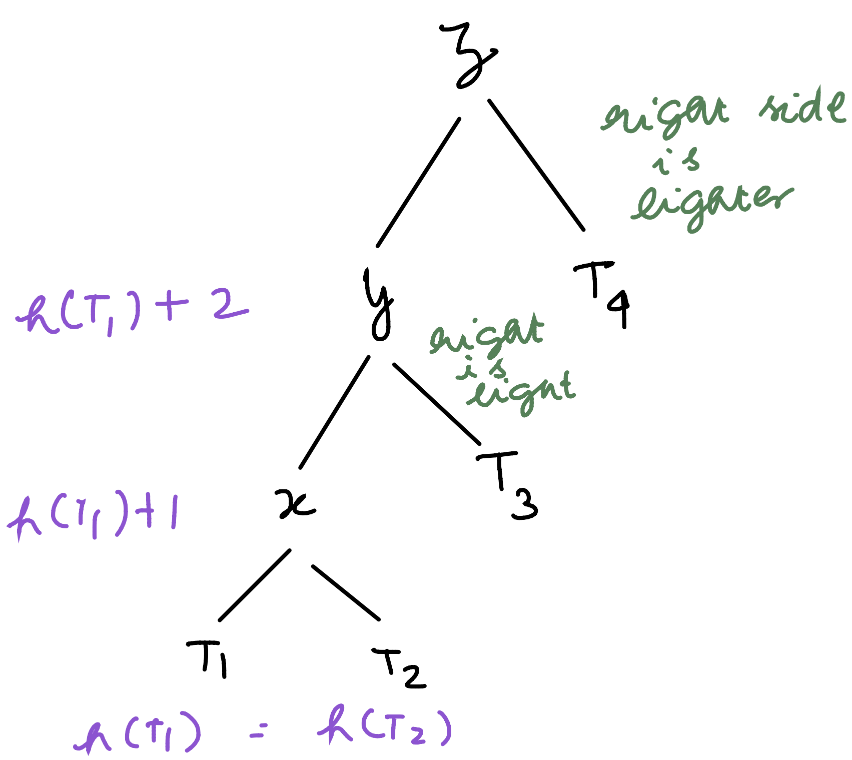

Right Right Imbalance



# Right Left Imbalance



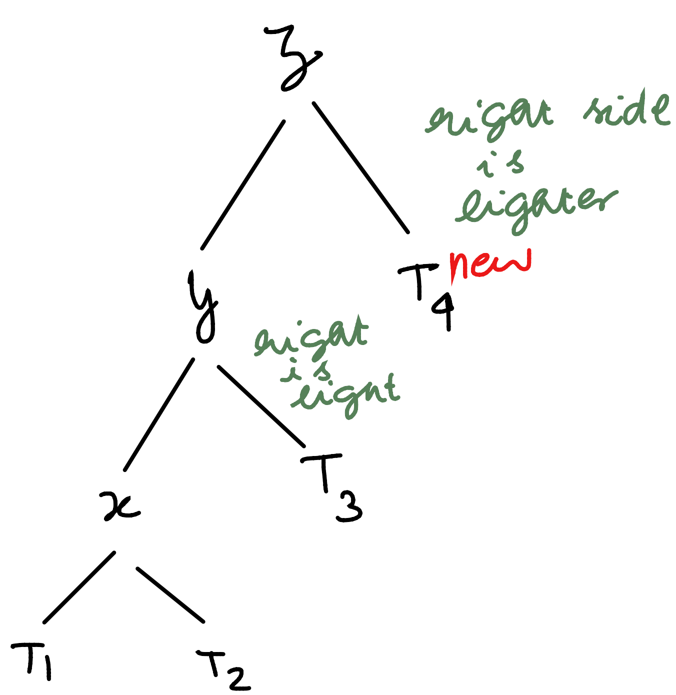
Before Deletion




- All nodes are balanced
- $h(T_3) = h(T_1)$
- $T_4$ is lighter side at level x

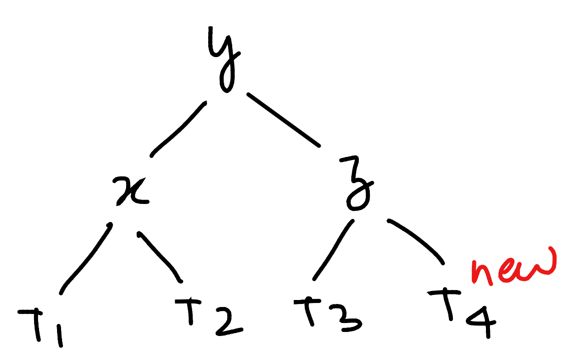
$$h(T_4) = h(T_1) + 2 - 1$$
$$= h(T_1) + 1$$

After Deletion




- $h(T_4^{new}) = h(T_4) - 1$

$$= h(T_1) + 1 - 1$$
$$= h(T_1)$$



Exercise: Finish Left Right, Right Right, Right Left Cases.

Property: $h = O(\log n)$

$\leq C \log n$

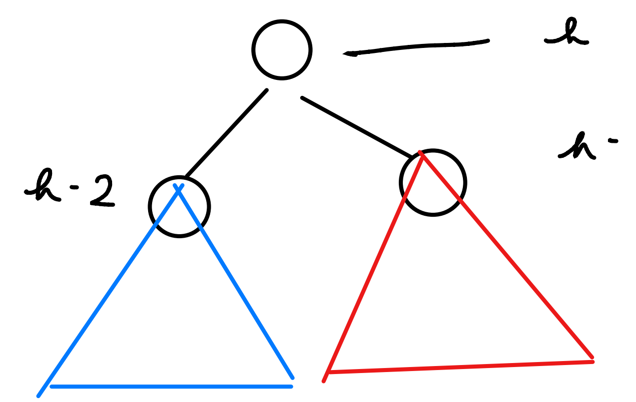
Proof:

$m(h)$ = minimum number of nodes in a AVL tree of height $h$

$m(1) = 1$

$m(2) = 2$

$m(h) = 1 + m(h-1) + m(h-2)$



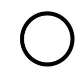

$m(h-1) \geq m(h-2)$

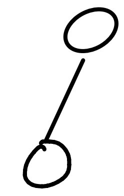
$m(h) \geq 1 + 2m(h-2)$

$> 2m(h-2) \Rightarrow m(h) > 2^2 m(h-2^2)$

$\vdots$

$m(h) > 2^i m(h-2^i)$, $i = 1, \ldots, \left\lfloor \frac{h-1}{2} \right\rfloor$

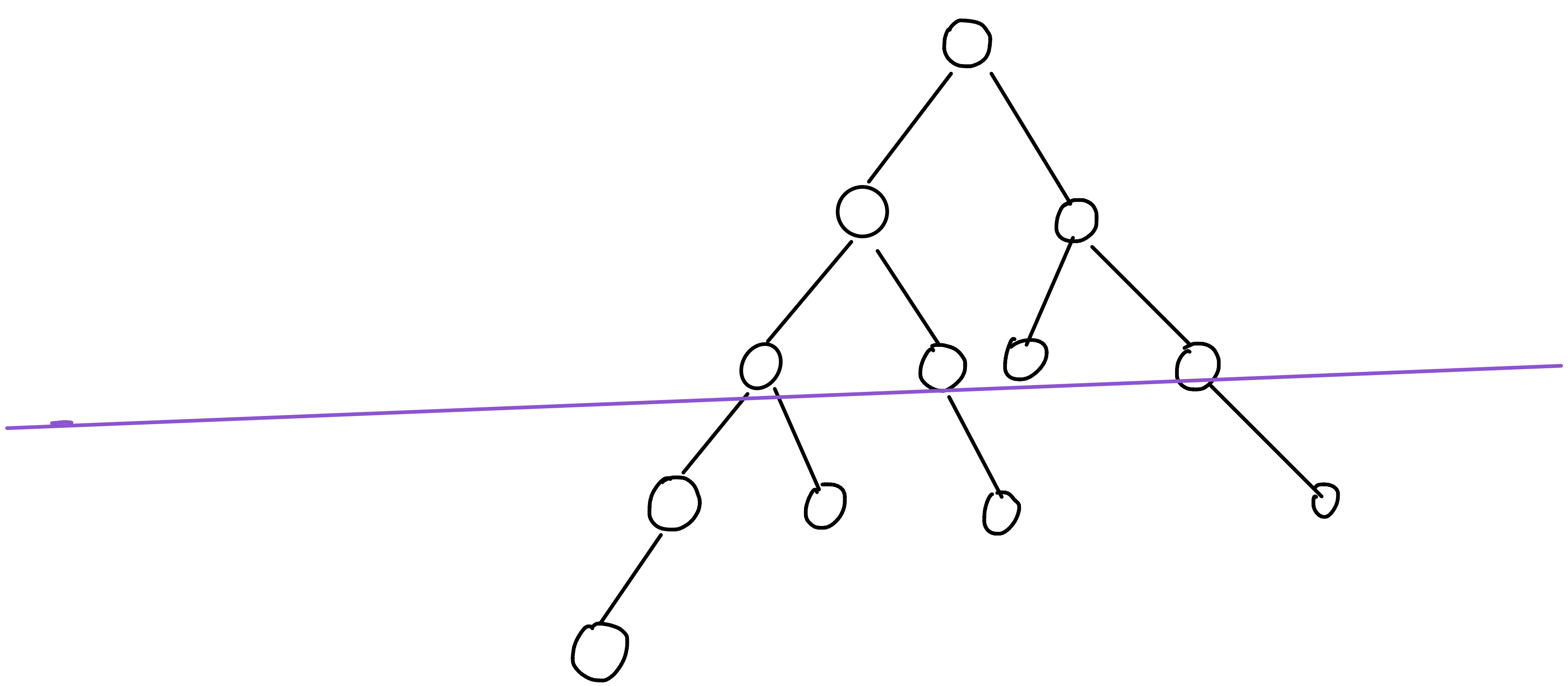
$n(1) =$ 

$n(2) =$ 

$$n(h) > 2^{\lfloor \frac{h-1}{2} \rfloor} \, n(1)$$

$$\left\lfloor \frac{h-1}{2} \right\rfloor < \log_2 n(h) < \log_2 n$$

$$\Rightarrow h = O(\log(n))$$

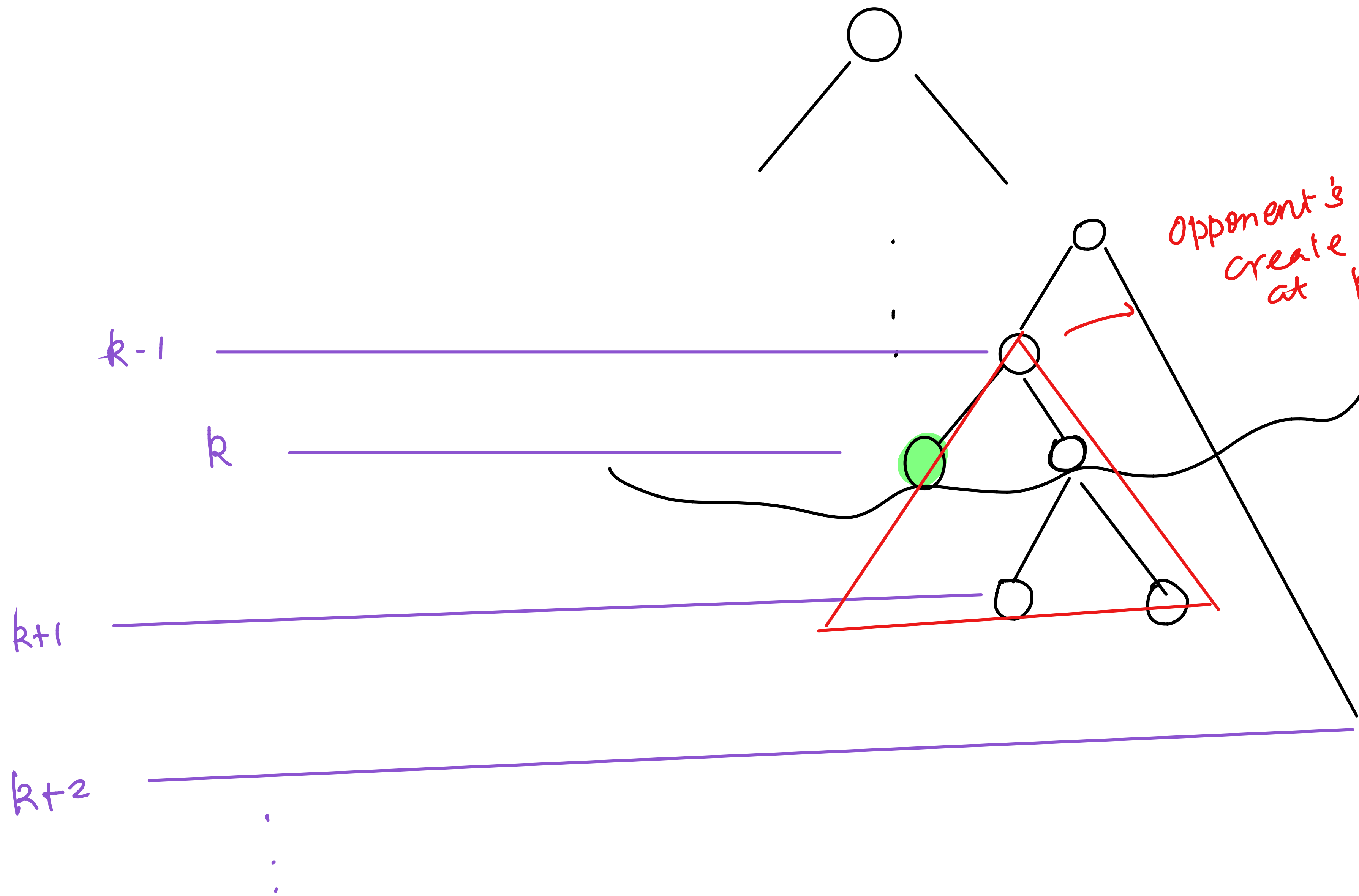


Property : Leaf closest to root is at level $k$

(leaf ht vs max ht)

$\Downarrow$

Ht of the tree is at most $2k-1$




Opponent's idea create imbalance at parent

Aim is to play the adversary or the devil or opponent who tries to break the statement

$k + k - 1 = 2k - 1$