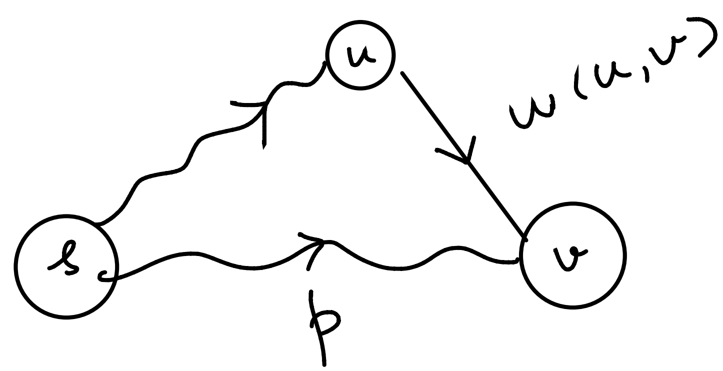
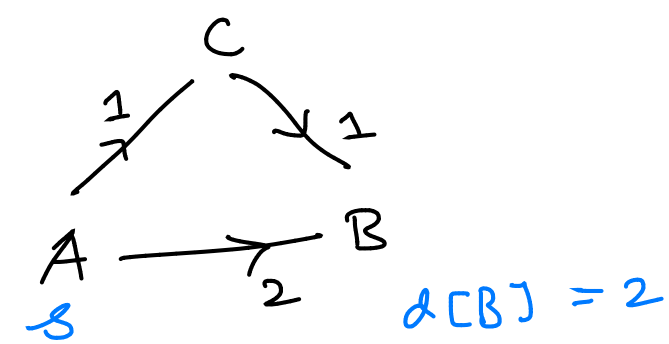
Properties

1) Triangle Inequality

$$\delta(s,v) \le \delta(s,u) + w(u,v)$$





2) Upper bound property : During the course of any algorithm for shortest path via relaxing edges

$$d[v] \ge \delta(s,v) \quad \text{and once} \quad d[v] = \delta(s,v)$$

↑ estimated

then it never changes.

3) No Path

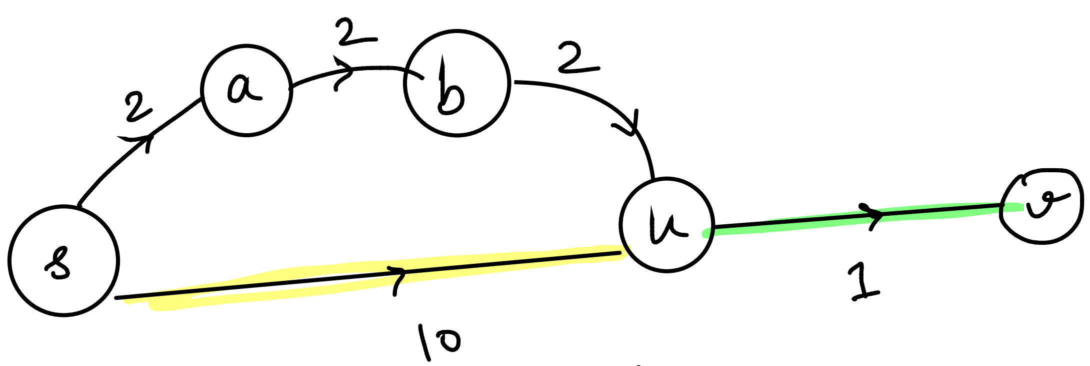
$$d[v] = \delta(s,v) = \infty$$

4) convergence Property :

$s \rightsquigarrow u \rightarrow v$

if $d[u] = \delta(s,u)$ at any time prior to relaxing (u,v), then on relaxing (u,v) you will have $d[v] = \delta(s,v)$ and it stays that way later as well.

Shortest Path to v is via u



Step 1: relax edge $(s,u) \Rightarrow d[u] = 10$

2: " $(u,v) \Rightarrow d[v] = 11$

3: " $(b,u) \Rightarrow d[u] = 10$

4: " $(a,b) \Rightarrow d[b] = \infty$

- $p = \langle v_0, v_1, \ldots, v_k \rangle$ is shortest path from s to v, and the

$v_0 = s$, $v_k = v$

edges are relaxed in the following order

$(v_0, v_1), \quad \ldots \quad , (v_1, v_2), \ldots, (v_{k-1}, v_k)$

step $\quad 10 \quad\quad 25$

(irrespective of other relaxations that happen)

$d[v_k] = \delta(s, v_k)$

- once $d[v] = \delta(s, v)$ for all $v \in V$, predecessor graph is the shortest path tree rooted at s.

BELLMAN-FORD ALGORITHM (Negative Edge weights and even $\underbrace{\text{negative cycles}}_{\text{Can detect}}$)

INIT-SINGLE-SOURCE (G, s)

for $i \leftarrow 1$ to $|V| - 1$

do for each edge $(u, v) \in E$

do Relax (u, v, w)

Shortest Path Computation

for each edge $(u, v) \in E$

do if $d[v] > d[u] + w(u, v)$

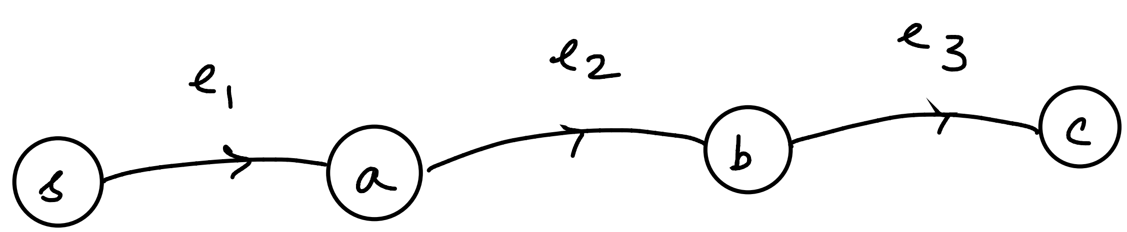
then return $\underbrace{\text{FALSE}}_{\text{graph has negative cycles}}$

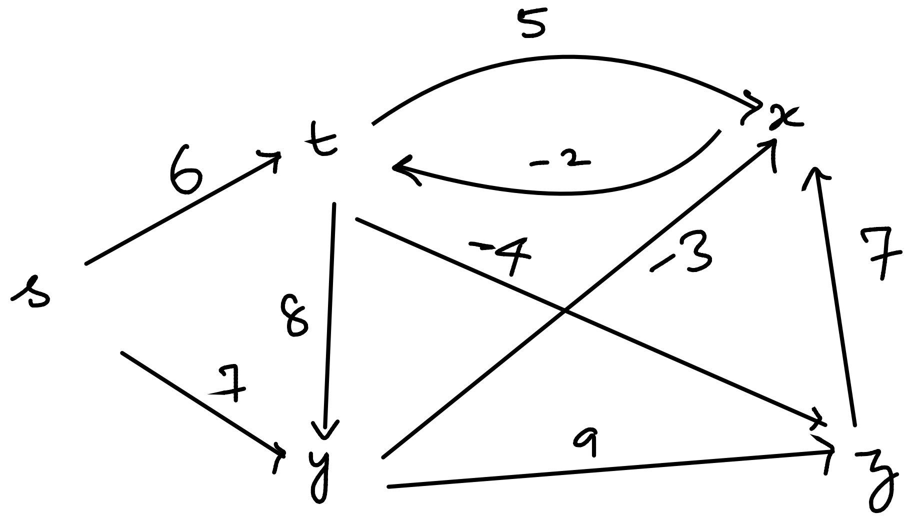
edge detection

return $\underbrace{\text{TRUE}}_{\text{graph has no negative cycles}}$ $\Rightarrow d[v] = \delta(s, v)$

We could possibly be relaxing in the worst possible order

e_3, e_2, e_1



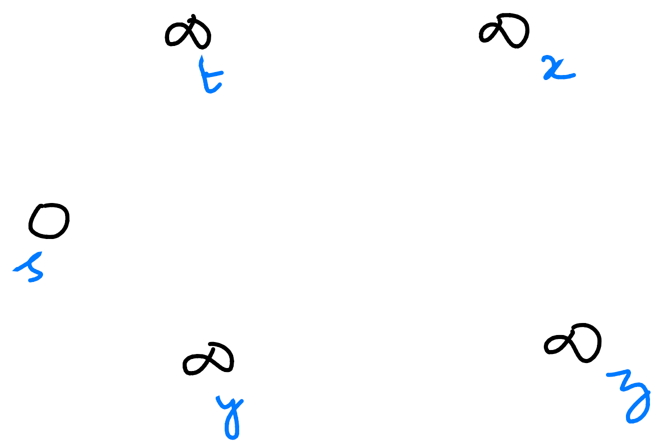


relaxation

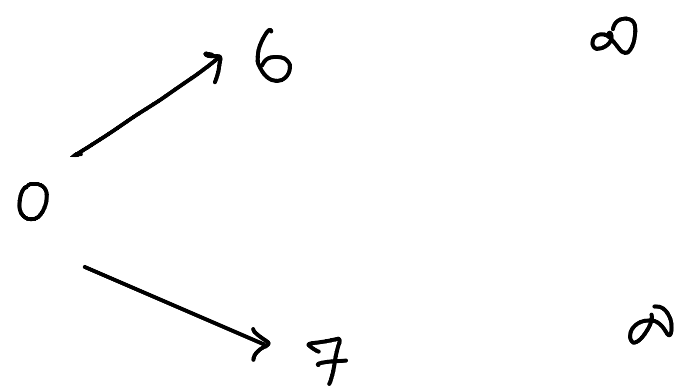
Predecessor Graph

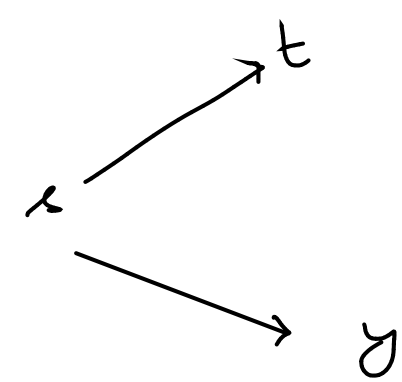
Cost

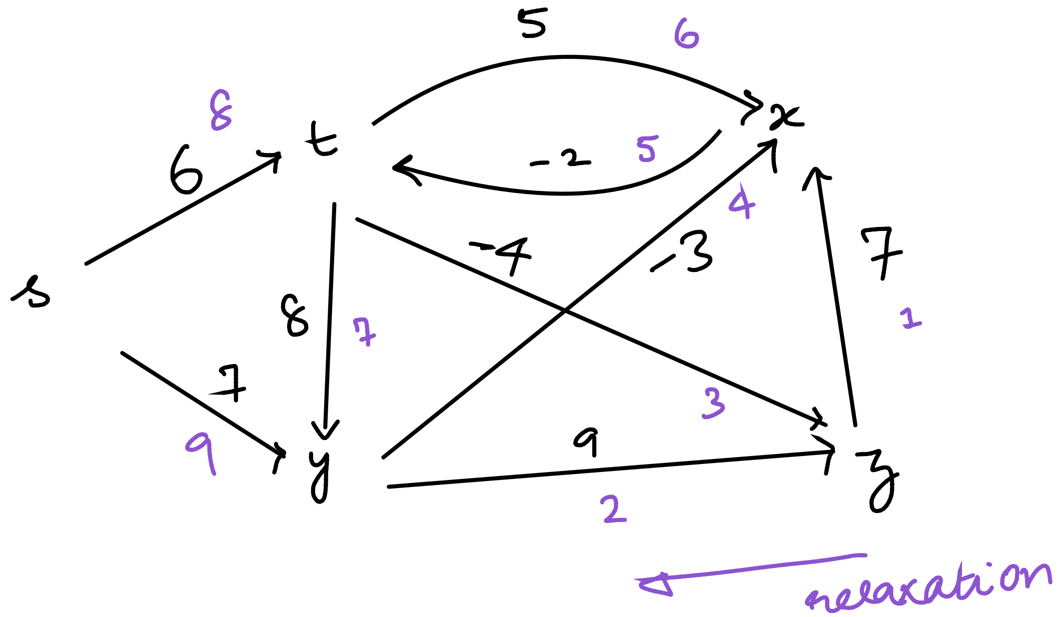
Initialisation



Step 1



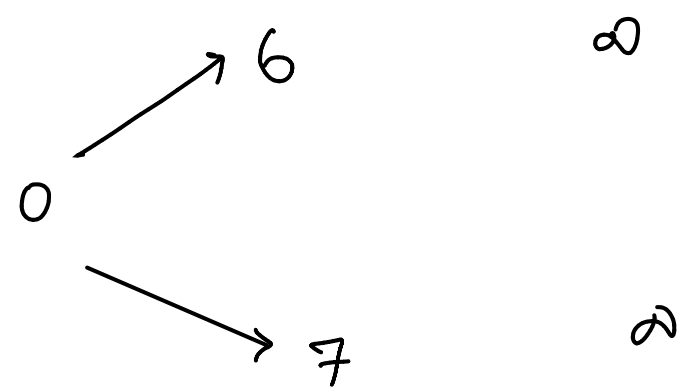


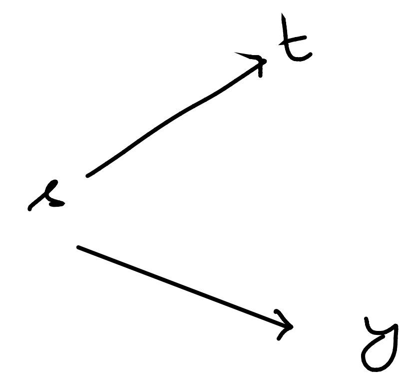


Cost

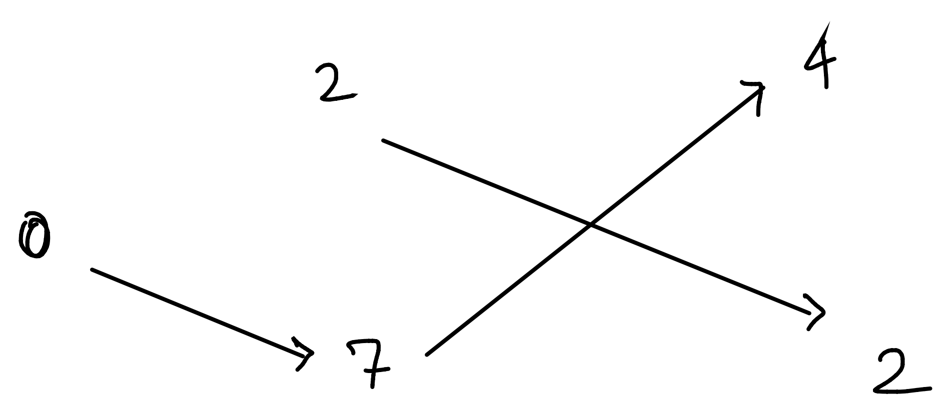
Predecessor Graph

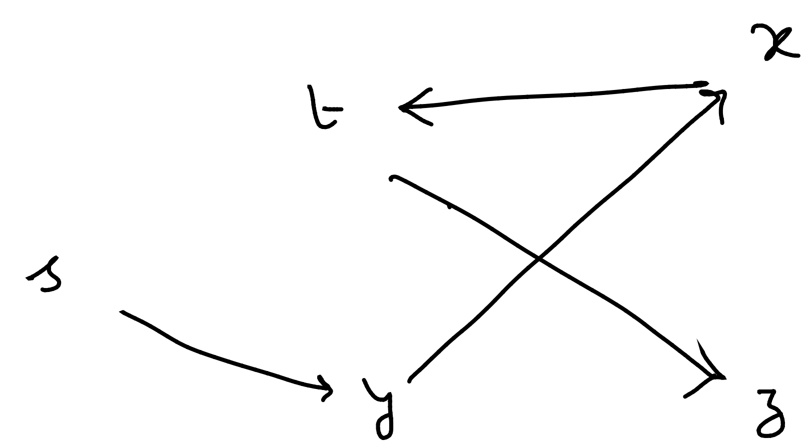
Step 1



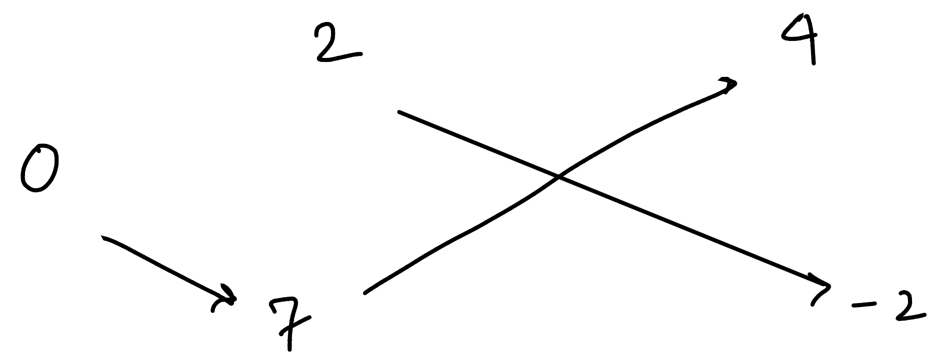


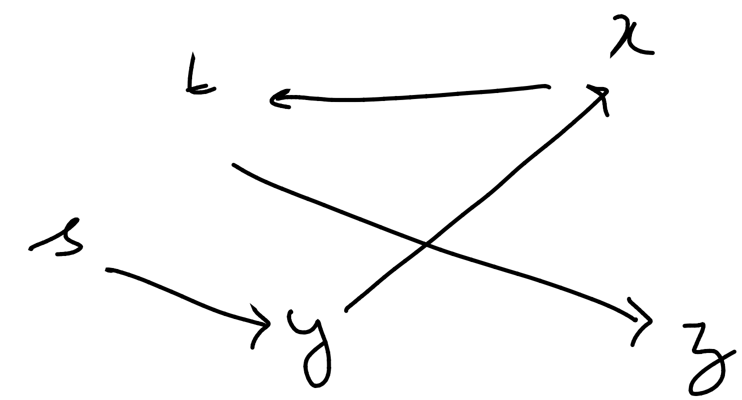
step 2





Step3





Time: $VE \rightarrow V^2$