Mathematical Foundations of Data Science

Session 5 – Inferential Statistics 3

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Rejecting and failing to reject the null hypothesis

- Acceptance Matrix for hypothesis tests

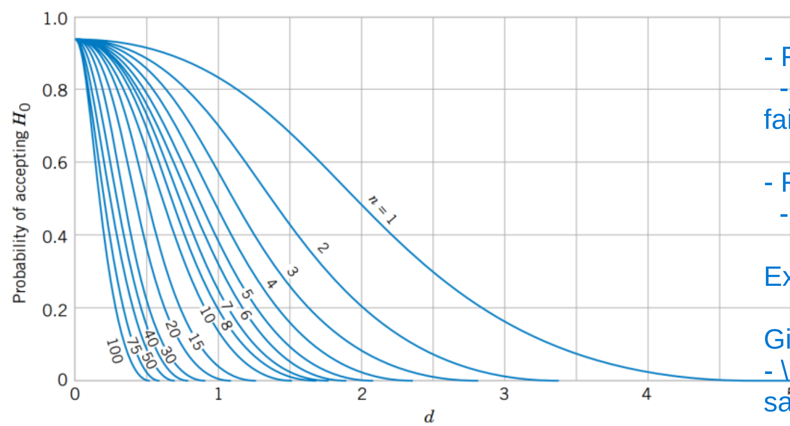
		Decision	
		Reject The Null Hypothesis	Fail to Reject the Null hypothesis
Actual	Null hypothesis is true	Type 1 Error (or Producer Risk, False Positive, alpha-risk)	Correct Decision (1-alpha)
	Alternate Hypothesis is true	Correct Decision (Power = 1-Beta)	Type 2 Error (Consumer risk, False Negative, Beta risk)

Type I and Type II Errors

- Prior to any data collection
could be as high as alpha (
analysis it is exactly equal t

- Type II error (β) is more co
  - It is a function of Delta: $\delta = |\mu - \mu_0|$
  - It is a function of standard deviation: σ
  - Often we focus on $d = {}^{\delta}/_{\sigma}$
  - It is a function of Sample Size: *n*
  - It is a function of the Type I error: α

- A graph of β versus d for a given sample size (n) is known as an OC (Operational Characteristic) curve:



(a) O.C. curves for different values of n for the two-sided normal test for a level of significance $\alpha = 0.05$.

OC Curve Interpretation:

- **X-axis:** Effect size (\(d\)) or \(|\mu - \mu_0|\)
- **Y-axis:** Probability of Type II error (\(\beta\))

- For small effect sizes:
  - \(\beta\) is large, indicating a higher chance of failing to reject a false null hypothesis.

- For large effect sizes:
  - \(\beta\) decreases, increasing the test's power.

Example:

Given:
- \(\alpha = 0.05\), \(\sigma = 10\), and various sample sizes, calculate \(\beta\) for different values of effect size (\(d\)) and plot \(\beta\) vs. \(d\).

Steps to Plot OC Curve:

1. **Choose Effect Sizes (d):**
   Pick a range of effect sizes, e.g., \(d = 0.1, 0.2, \dots, 1.0\).

2. **Compute \(\beta\) for Each Effect Size:**
   Use the z-test formula for power and Type II

Introduction

- So far statistical inference was confined to input variables that could take up two possible values (two sample tests), or there was no notion of an input variable (single sample tests).
- ANOVA
  - When there are three or more states of a single variable we can use ANOVA
- Chi-Square Test of Independence
  - Can be used when we want to compare multiple proportions

BASICS of ANOVA

- Tests the hypothesis that: $\mu_A = \mu_B$=μ_C=μ_D
- Why not multiple pairwise comparisons using t-tests?
- What do you do after a test? Tukey, Bonferroni, Scheffe tests
- Take the table:

	1	2	n
A	$y_{1,1}$	$y_{1,2}$	$y_{1,n}$
B	$y_{2,1}$	…	…
C	…	…	…
D	…	…	$y_{4,n}$

ANOVA OUTPUT

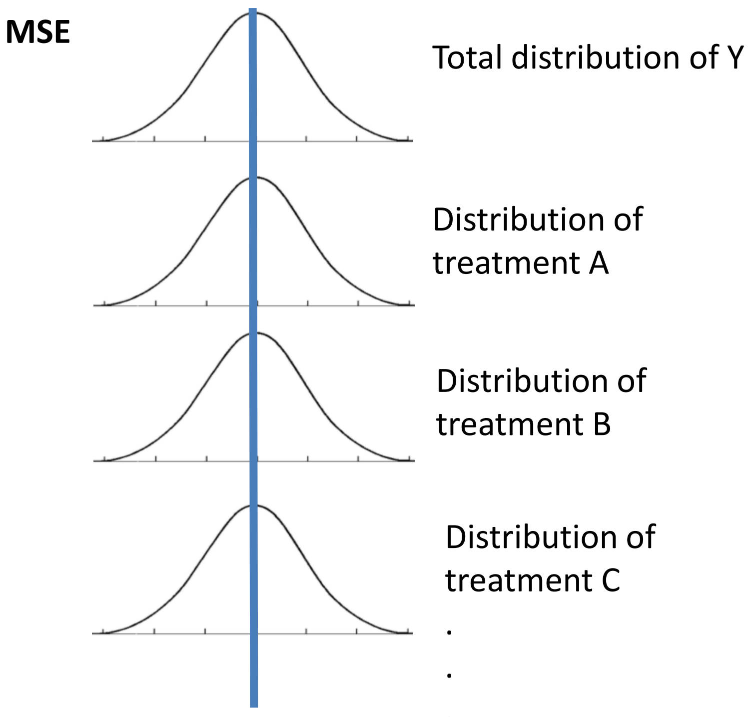
Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F-Stat
Between Treatments	$n\sum_{i=1}^{a}(\bar{y}_{i.} - \bar{\bar{y}}_{..})^2$ or SSB	a-1	MSB = SSB/DoF	F = MSB/MSE
Error within treatments	$\sum_{i=1}^{a}\sum_{j=1}^{n}(y_{i,j} - \bar{y}_{i.})^2$ or SSE	N-a	MSE = SSE/DoF	
Total	$\sum_{i=1}^{a}\sum_{j=1}^{n}(y_{i,j} - \bar{\bar{y}}_{..})^2$ or SST	N-1	MST = SST/DoF	

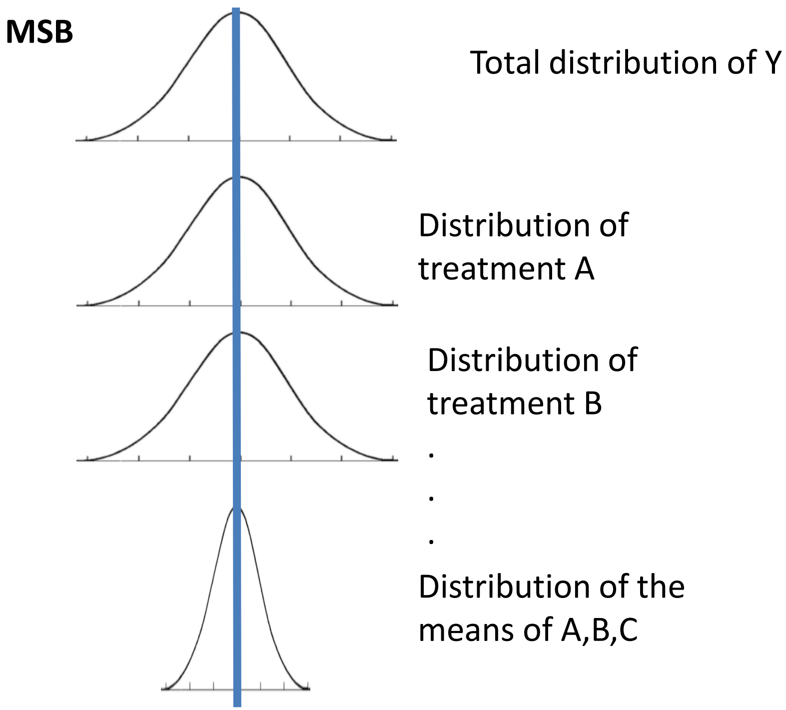
Compare F calculated against the F-distribution with a-1,N-a degrees of freedom and get a p-value

Why F for difference in means??

- The F is the ratio of two variances (where the samples come from a normal distribution and the null hypothesis is that the variances are equal)
- MSB is a way of calculating total variance
- MSE is a way of calculating total variance
- MSB, MSE and MST will be equal if the null hypothesis is true
- However is the null hypothesis is not, then MSB>MST>MSE

MSB and MSE


MSE
Total distribution of Y
Distribution of treatment A
Distribution of treatment B
Distribution of treatment C
.
.
.


MSB
Total distribution of Y
Distribution of treatment A
Distribution of treatment B
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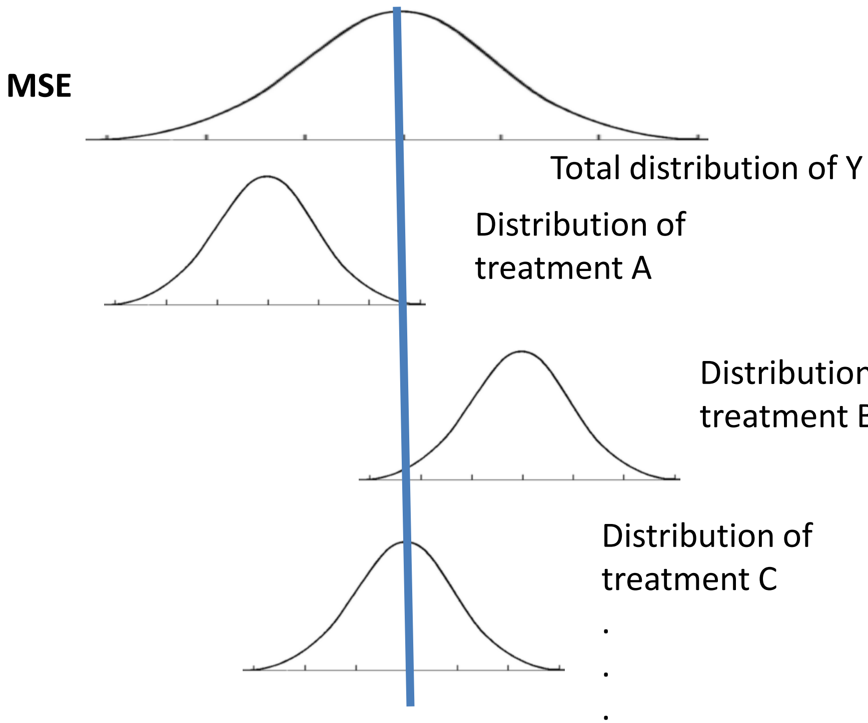
Distribution of the means of A,B,C

MSB and MSE


MSE
Total distribution of Y
Distribution of treatment A
Distribution of treatment B
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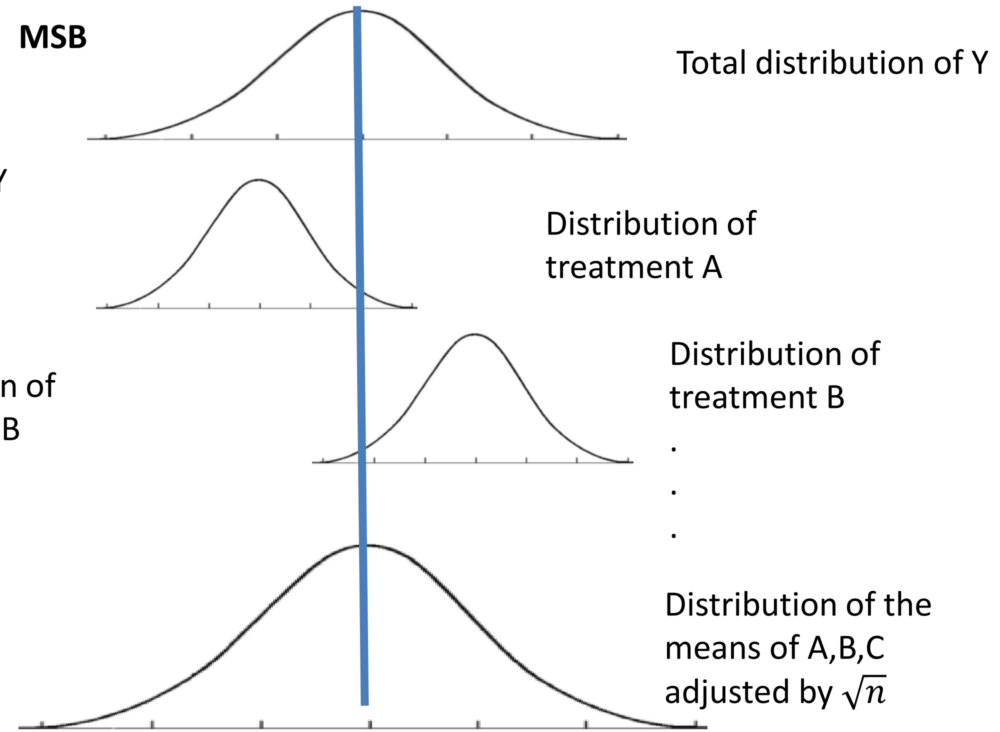
MSB
Total distribution of Y
Distribution of treatment A
Distribution of treatment B
.
.
.
Distribution of the means of A,B,C adjusted by $\sqrt{n}$

Chi-Square TOI

- When using categorical variables
- Use this to test:
  - Does the input categorical variable effect the output categorical variable (works 2 or more states of the input or output variable)
  - Independence between two variables
  - Construct a contingency table:

Exercise \ Smoking habit	Heavy	Regular	Occasional	Never
Frequent	7	9	12	87
Some	3	7	4	84
None	1	1	3	18

Chi Square TOI continued

- Create Theo values for this table in accordance to the assumption of independence
- It can be done row wise or column wise, but each cell gets an expected value
- Then if the null hypothesis is true then the test statistic is:

$$\chi^2 = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{(O_{i,j} - E_{i,j})^2}{E_{i,j}}$$

With (r-1)*(c-1) degrees of freedom (or rc-c-r+1)