Rajnish Maurya Tutorial VIII 04/11/24

DA24M015

Solution 2.

a) Probability of committing a type I error is $\alpha = 0.05$

b) Power of test $= 1-\beta$

$1-\beta = 0.8$

$\beta = 0.2$

c) Effects of increasing sample size on Type I & II error

- Type I error (α) remains unchanged, as the significance level unchanged.
- Type II error (β) decreases, because larger sample provide more information about the population, making easier to detect a true effect if it exists. Increasing sample size, increases power of test, hence decreases β.

d) Reducing significance level α, makes the confidence interval wider. The confidence Interval (CI) increases on decreasing α.

e) effect on precision & recall

$$\text{Precision} = \frac{TP}{TP+FP} = \frac{1-\beta}{1-\beta+\alpha}, \quad \text{Recall} = \frac{TP}{TP+FN} = \frac{1-\beta}{1-\beta+\beta} = 1-\beta$$

(i) α increases keeping β constant
- Precision decreases due to increase of denominator.
- Recall does not change as it does not depend on α.

(ii) β increases keeping α constant
- Precision remains unchanges
- Recall decreases, as on increasing β.

Solution 1.

$H_0 \geq 28$

$H_a < 28 \longrightarrow$ left tail test

Given: $\sigma^2 = 10000$, $n = 100$, $\alpha = 0.05$

Critical value for rejection of H_0

$$z_c = \frac{\bar{x}_c - \mu_0}{\sigma/\sqrt{n}} =$$

for $\alpha = 0.05$, $z_c = -1.645$ (from z-table)

$$-1.645 = \frac{x_c - \mu_0}{\sqrt{10000}/\sqrt{100}} = \frac{x_c - 28}{10}$$

$$x_c = 11.55$$

So, we reject the H_0 if $X_0 < 11.55$

a). Probability of type 2 error assuming $\mu = 25$

Type 2 error (β) is probability of failing to reject H_0, when H_1 is true, i.e. $\mu = 25$. This corresponds to probability of $\bar{X}$ is greater than equal the critical value of 11.55. (assuming $\mu = 25$)

$$z = \frac{\bar{x}_c - \mu}{\sigma/\sqrt{n}} = \frac{11.55 - 25}{10} = -1.345$$

Probability corresponding to $z = -1.345$ is 0.0894

type 2 error $= 0.0894$ Ans

b) Type 2 error assuming $\mu = 30$

as $\mu = 30$ & the null hypothesis states that $\mu \geq 28$.

So the null Hypothesis true.

$\boxed{\beta = \text{Type 2 error} = 0}$. Ans

c) probability of type 1 error

Probability of type 1 error is simple the significance level α of the test.

$\alpha = 0.05$ (Given)

$\boxed{\text{Type I error} = 0.05}$ Ans

Solution 3:

a) $X \sim N(\mu, \sigma^2)$.

$\sigma^2 = (0.06)^2$.

$\bar{X} \sim N(\mu, \sigma^2/36)$

$H_0, \mu = 32$

$H_A, \mu \neq 32$

$$\alpha = 0.05 = P(\bar{X} \geq 32 + a) + P(\bar{X} \leq 32 - a)$$

$$= 2P\left(\frac{\bar{X} - 32}{\sigma/6} > \frac{a}{\sigma/6}\right)$$

$$= 2P\left(Z \geq \frac{6a}{\sigma}\right)$$

therefore $\frac{6a}{\sigma} = Z_{0.025} = 0.0196$

The rejects ~~regions~~ regions are $32 \pm a = 32.0196, 31.9804$

Ans

b) Power$_1$ $= P(\bar{X} > 32.0196 \mid \mu = 31.97) + P(\bar{X} < 31.9804 \mid \mu = 31.97)$

$= 0.8508$

Power 2 $= P(Z > 2.96) + P(Z < -0.96) = 0.1700$

Power 3 $= P(Z > 1.96) + P(Z < -1.96) \geq 0.05$

Power 4 $= P(Z > 0.96) + P(Z < -2.96) = 0.1700$

Power 5 $= P(Z > -1.04) + P(Z < -4.96) = 0.8508$

c) $\beta = 1 - \text{Power}_5 = 1 - 0.8508$
$= 0.1492$ Ans

Solution 4:

a) Type I Error (False Positive): This occurs when the test indicates that a patient has the disease when they actually do not.

Impact: A false positive can cause significant emotional distress and lead to unnecessary further testing, treatments. This also incurs extra medical costs and may cause the patient & their family to worry unnecessarily.

Type II Error (False Negative): - This occurs when the test indicates that a patient does not have the disease when they actually do.

Impact A false negative can delay the patient's diagnosis, potentially allowing the disease to progress untreated. For rare disease, timely treatment is often critical, so missing the diagnosis can have severe health consequences.

b) Minimizing Type-I Error: - Since the disease is rare, a positive result is likely to be a false +ve simply due to the low base rate of true disease cases. Reducing the type-I Error rate helps prevent a large no. of false positives. which reduces unnecessary stress for patients, as well as the costs & potential risks associated with further diagnostic procedure or treatment

Implication for Type-II Error: If the test is designed to minimize type I Error, it becomes more conservative in identifying positives. This conservatism generally increases the probability of type-II errors; it means that there is a greater chance of missing true cases (false -ve). This trade off reflects the balance between reducing false alarms & ensuring true cases are detected.

c) Power $= 1-\beta = 1-0.2 = 0.8$.
So, the test's power is 0.8 (80%), it means that if a patient does have the disease, the test will correctly identify it 80% of the time.

d) Impact on Reliability: With very low disease prevalence, even a test with a low Type-I error will produce more False +ve than true +ve.

- Type-I Errors: Given the low prevalence, the majority of positive results will be false positives. This can reduce confidence in the test's accuracy for identifying actual cases.
- Type II error: True cases may be missed, resulting in untreated patients.

Interpreting a positive result:- Positive test results should be interpreted carefully, often requiring confirmatory tests.

Solution 5:

a) $H_0 : u = 0.7$ (Null Hypothesis)
$H_A : u > 0.7$ (Alternate Hypothesis

b) $\alpha = = P(X \geq 375 / H_0)$

$$= P\left(\left(\frac{X - 500 \times 0.7}{\sqrt{500 \times 0.7 \times 0.3}}\right) > \left(\frac{375 - 500 \times 0.7}{\sqrt{500 \times 0.7 \times 0.3}}\right) \Big| H_0\right)$$

$= P(Z > 2.4398)$

$= 0.073$

c) Power $= P(X > 375 / C = 0.8)$

$$= P\left(Z \geq \frac{375 - 500 \times 0.8}{\sqrt{500 \times 0.8 \times 0.2}}\right)$$

$= P(Z \geq -2.8)$

$= 0.9974$

d) $P = P(X > 395 / H_0)$

$$= P\left(Z \geq \frac{395 - 500 \times 0.7}{\sqrt{500 \times 0.7 \times 0.3}}\right)$$

$= P(Z \geq 4.39)$

$=$ less than 0.0001