light edge $= \underset{\substack{u \in S \\ v \in V-S}}{\arg\min} \; w(u, v)$

- $A \subseteq MST$,

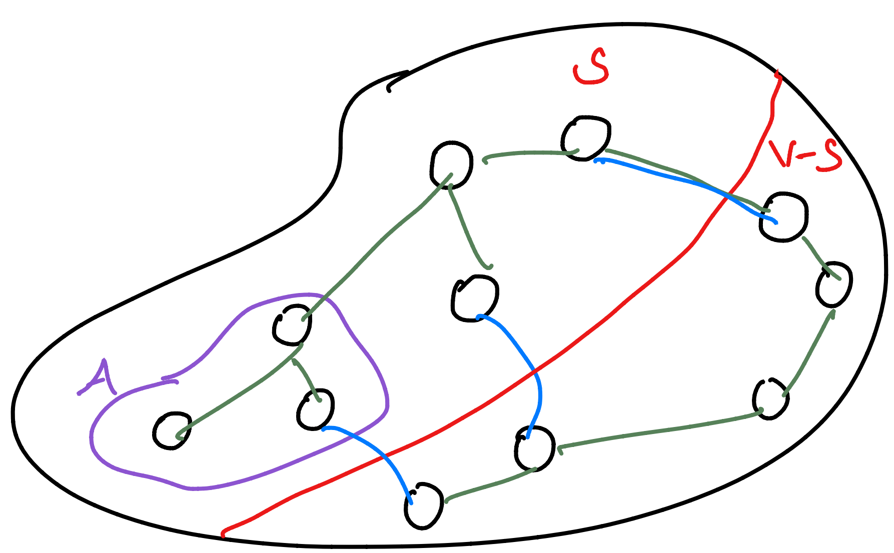
$A \Rightarrow B$

$\bar{B} \Rightarrow \bar{A}$

$(S, V-S)$ respects A,

(u, v) is a light edge crossing $(S, V-S)$

$\Rightarrow$ (u, v) is a safe edge

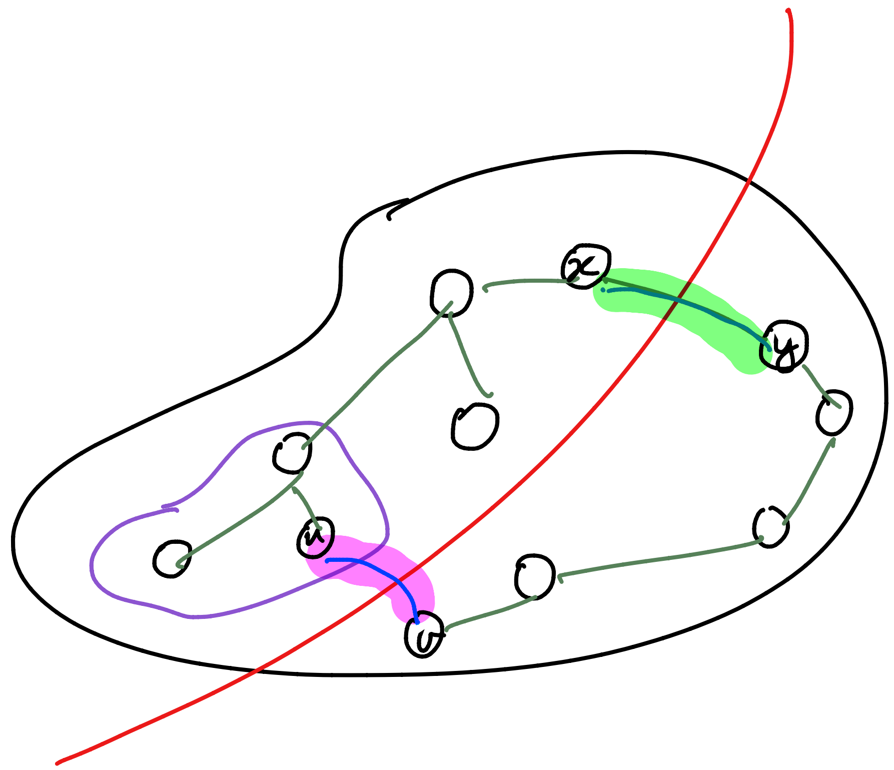


MST

(u, v) are edges crossing the cut

Case 1: (u,v) is already part of MST (Nothing to show)

Case 2: (u,v) is not part of MST



(u,v) light edge.

(x,y) already part of MST (Not a light edge)

$$MST' = \{MST - \{(x,y)\}\} \cup \{(u,v)\}$$

$$W(MST') = W(MST) - \underbrace{w(x,y) + w(u,v)}_{-ve}$$

$$w(u,v) \leq w(x,y) \implies W(MST') \leq W(MST)$$

MST-KRUSKAL

$A \leftarrow \phi$

for each vertex $v \in V$

do MAKE-SET(v)

careful

sort edges in non-decreasing order by W

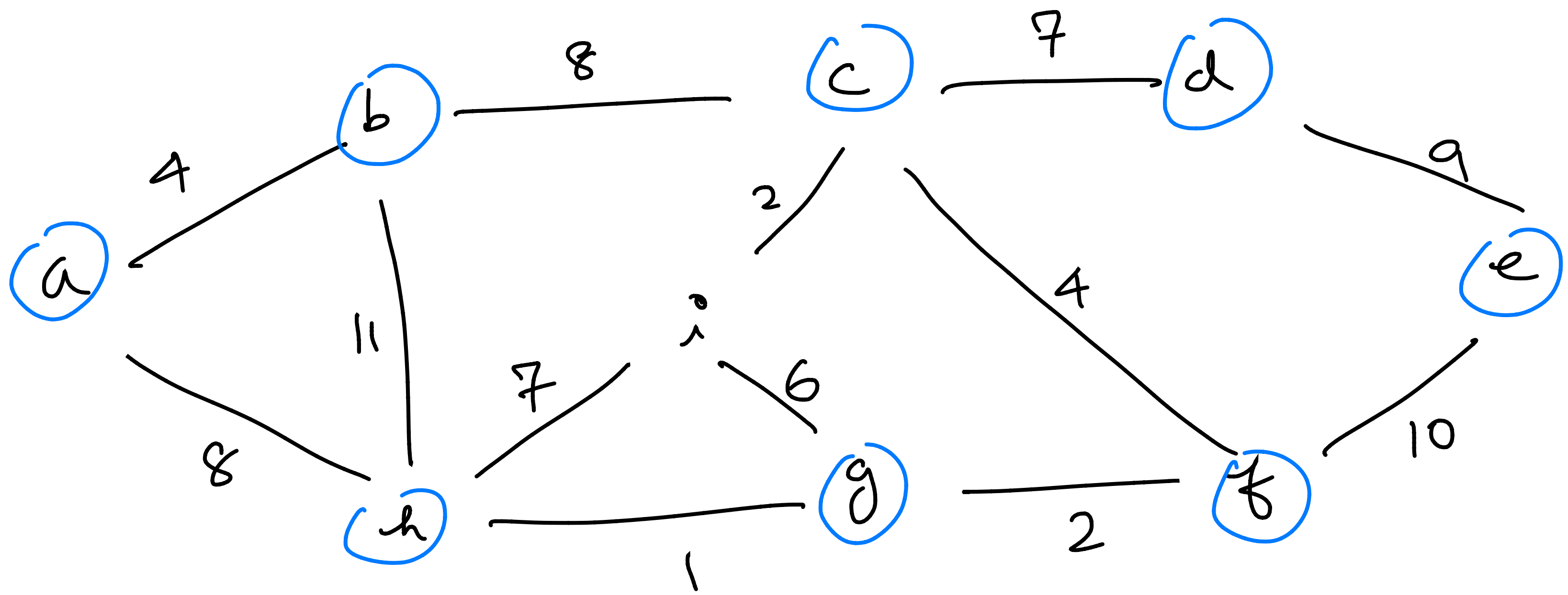
for each edge $(u, v) \in E$ in non-decreasing order

do if FIND-SET$(u) \neq$ FIND-SET(v)

check if (u, v) is not connecting things that are already spanned.

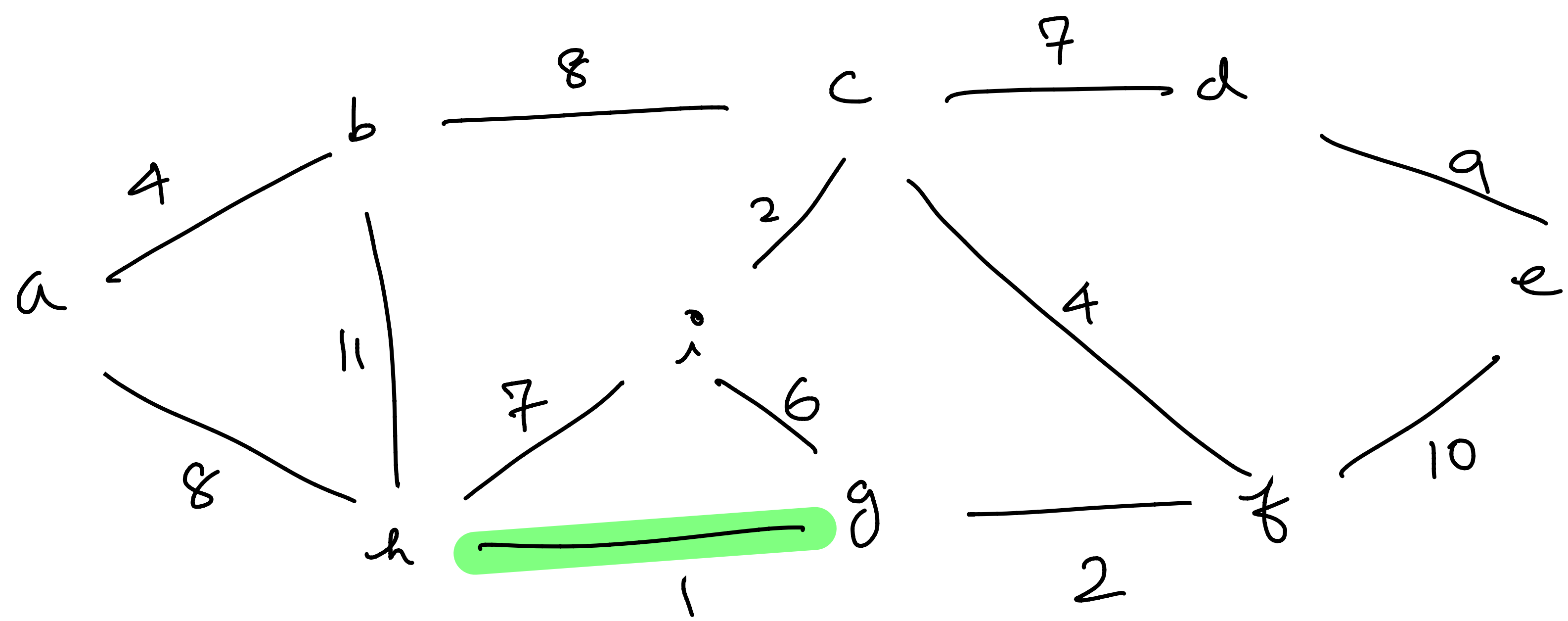
then $A \leftarrow A \cup \{(u, v)\}$

UNION(u, v)



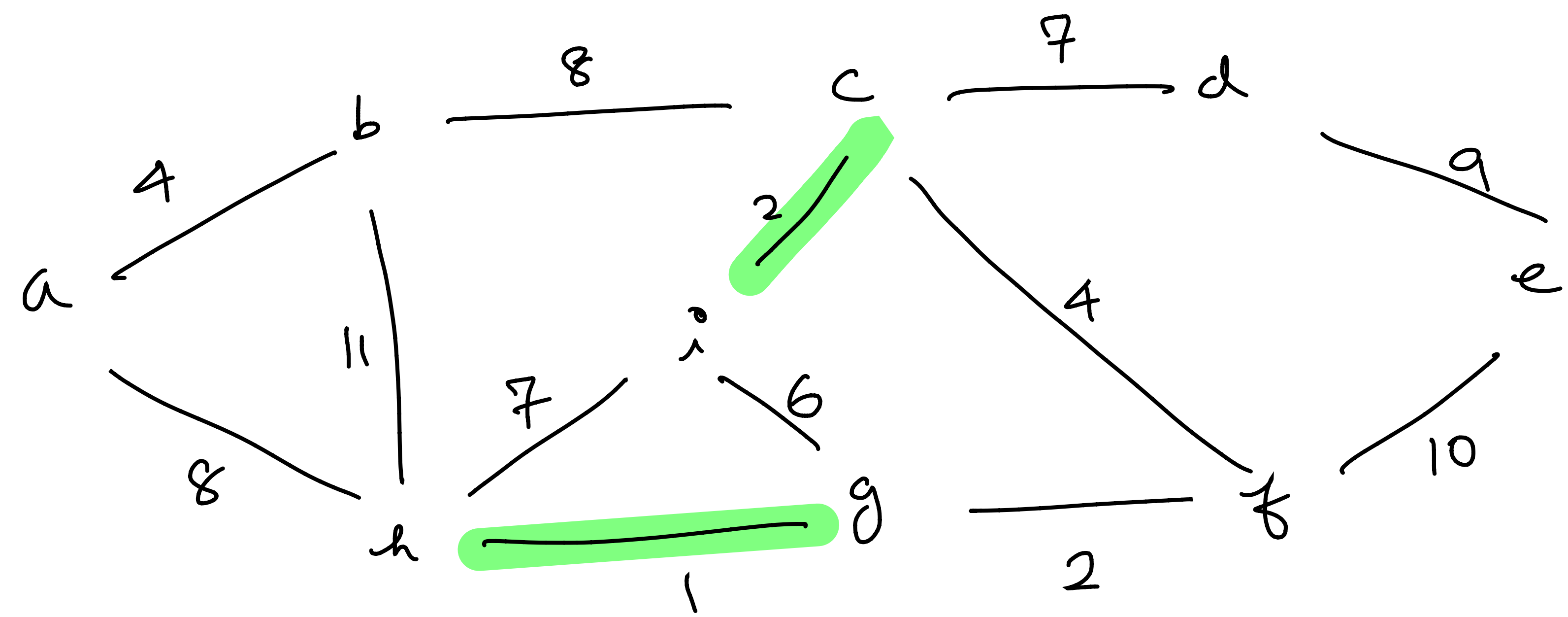
{a} . . . {g} (All vertices are their separate sets)

Step 1



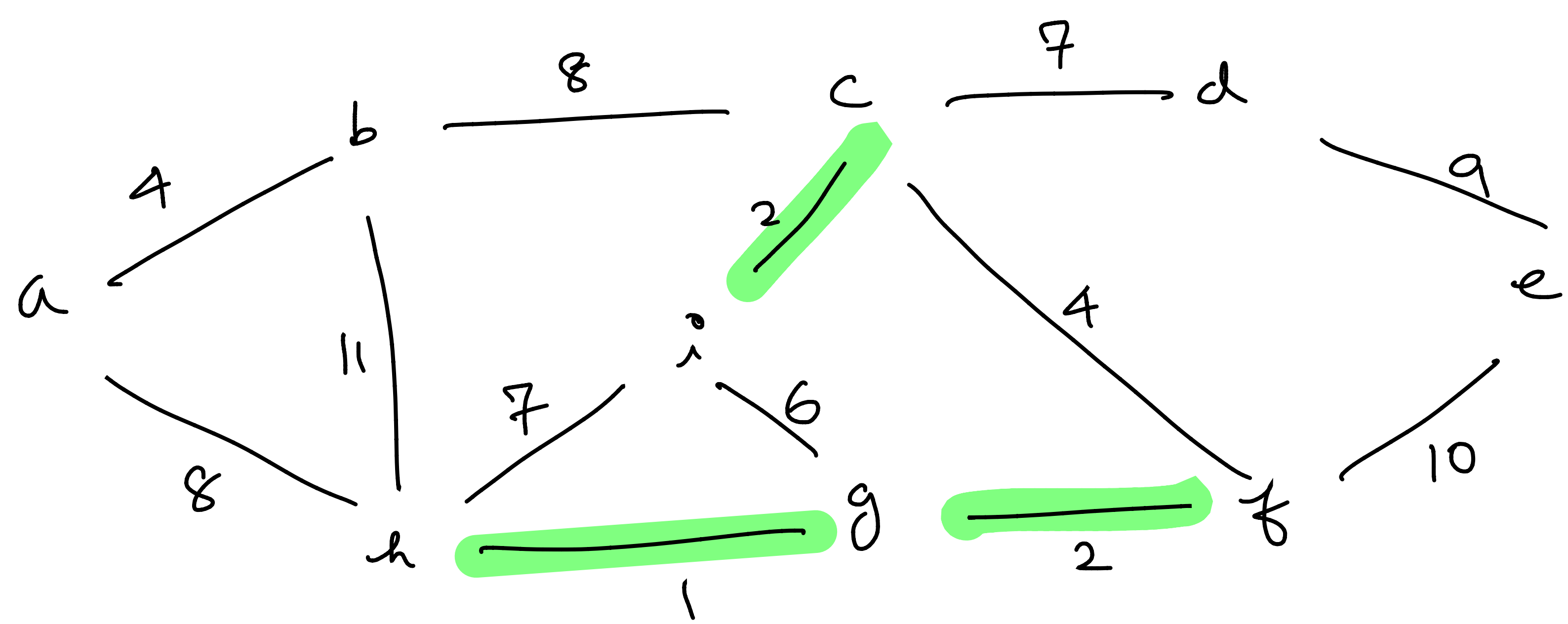
{a} {b} . . . {h, g}

Step 2



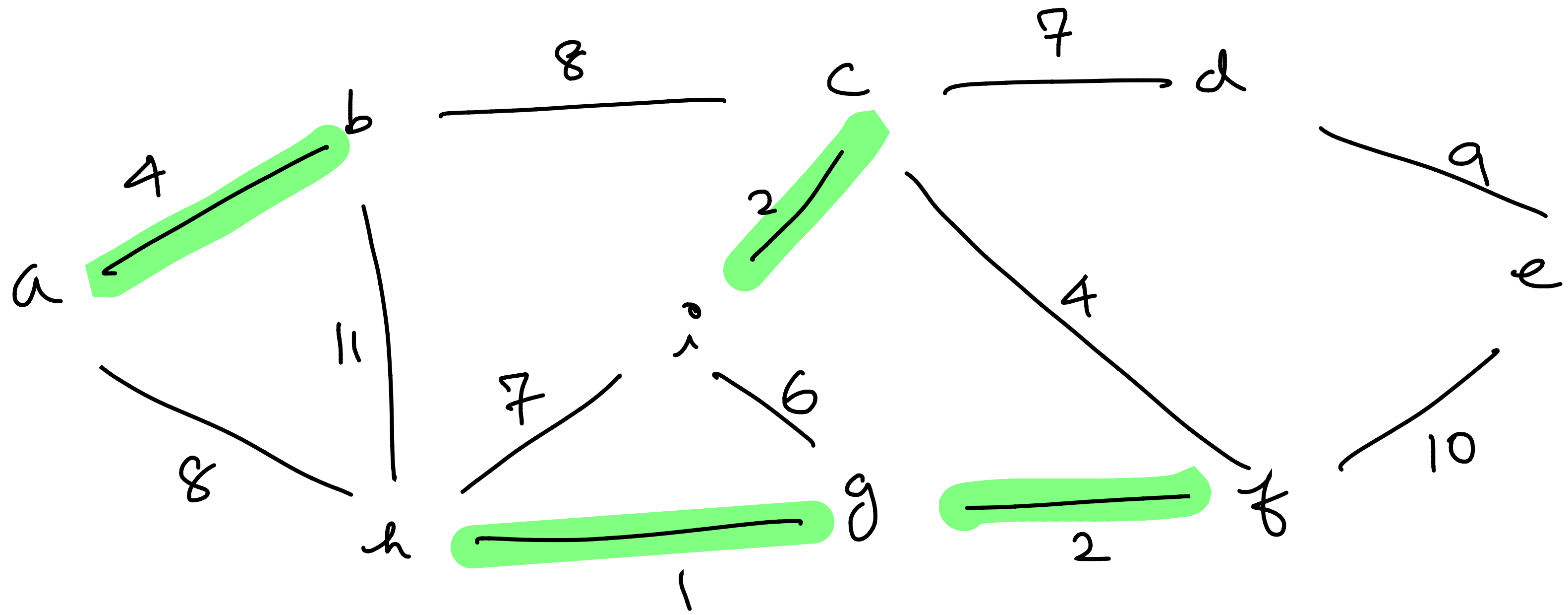
{a} {b} {c, i} . . . {h, g}

Step 3



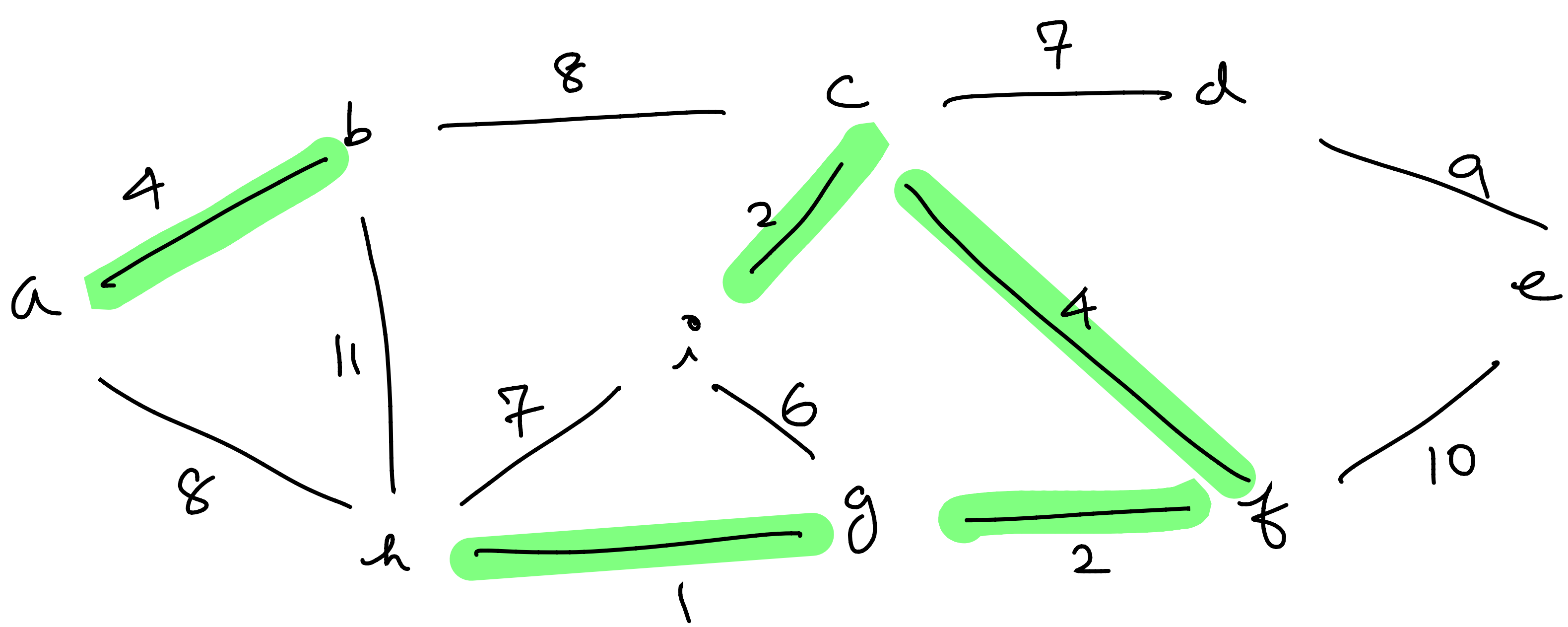
{a} {b} {c, i} . . . {f, g, h}

Step 4



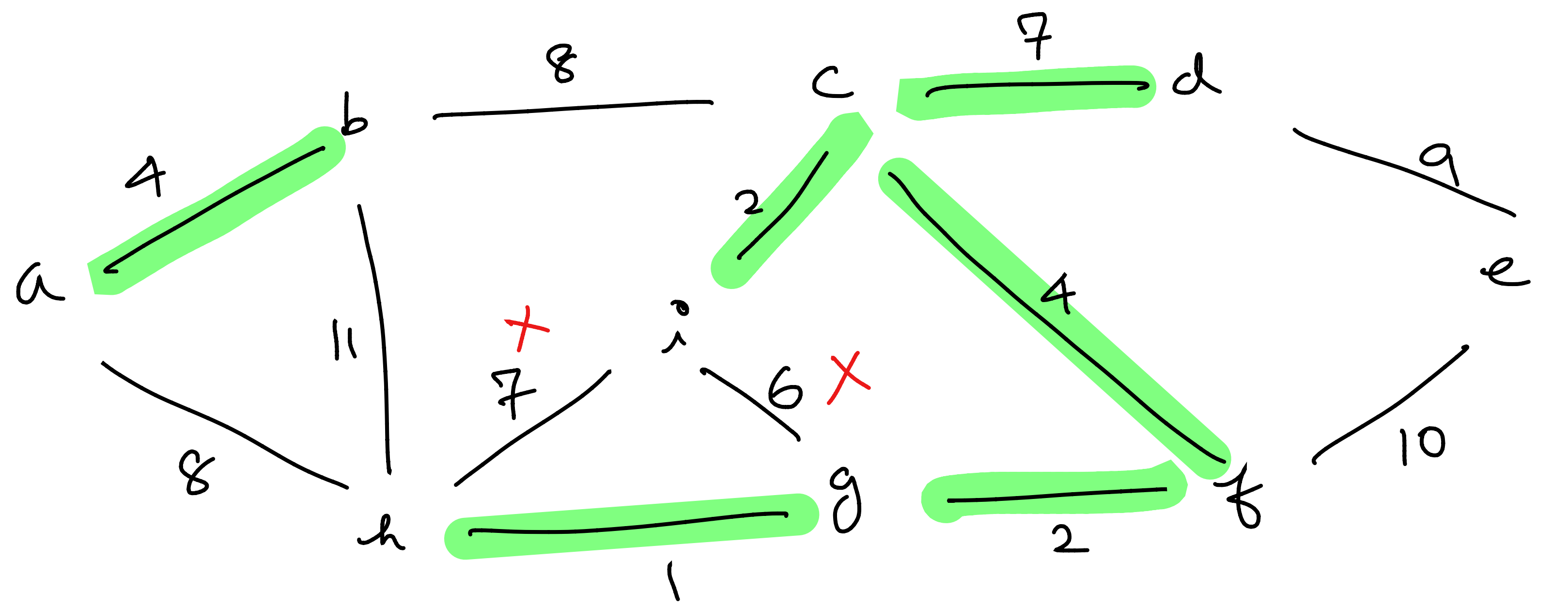
$\{a, b\}$ $\{c, i\}$ . . . $\{f, g, h\}$

Step 5



$\{a, b\}$, . . . , $\{c, i, f, g, h\}$

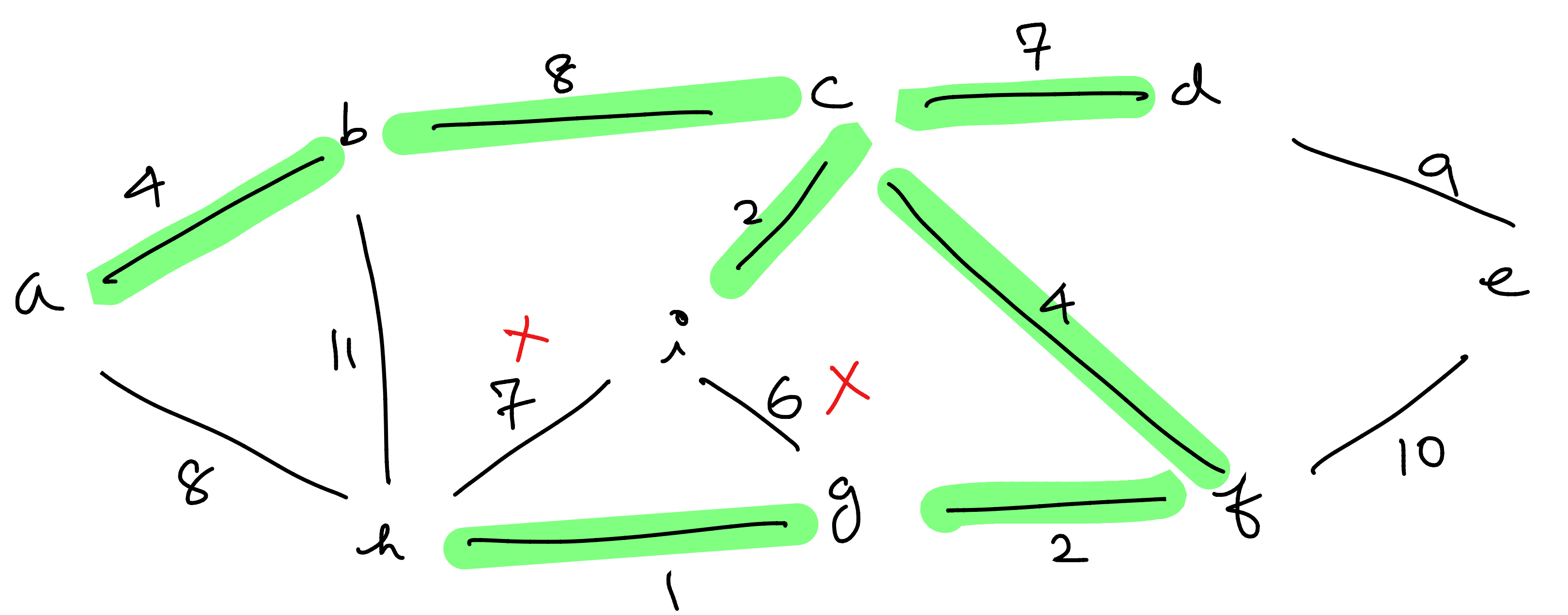
Step 5



× No use of adding

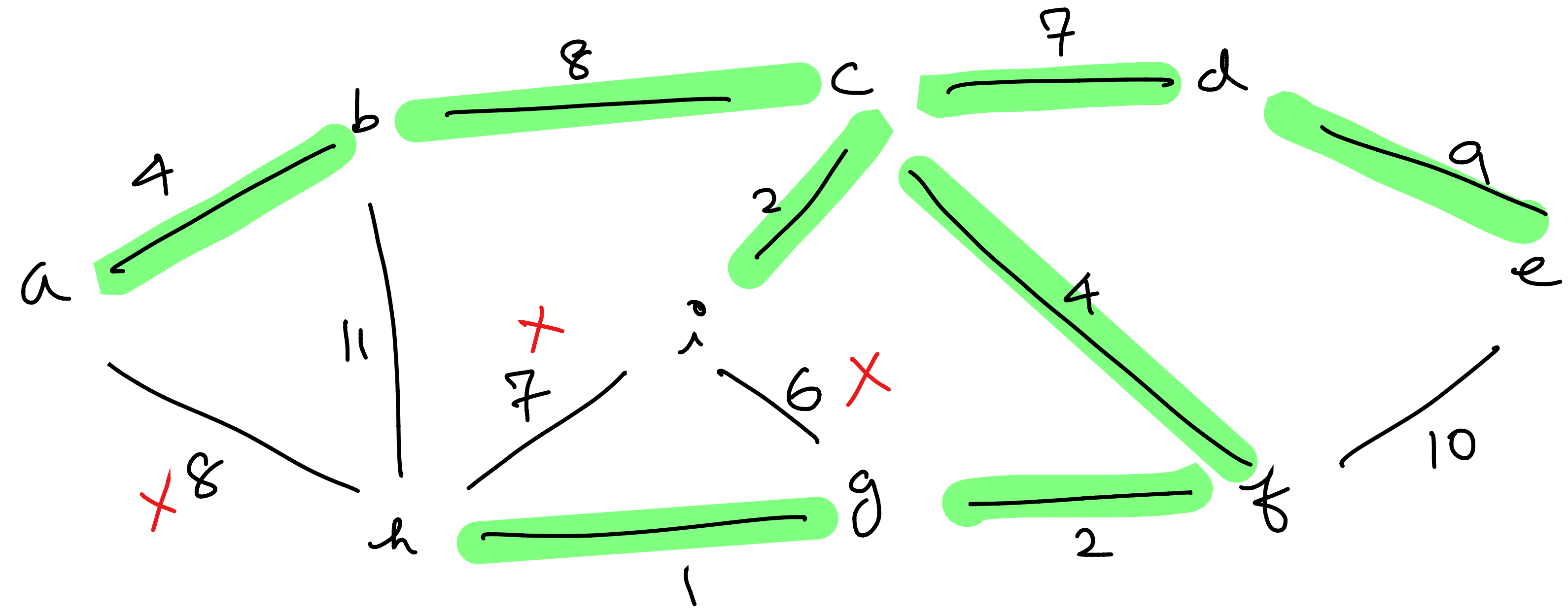
$\{a, b\}, \; . \; . \; . \; , \{c, d, i, f, g, h\}$

Step 6



$\{a, b\; c, d, i, f, g, h\}, \; . \; . \; .$

Step 6



{a, b c, d, e, i, f, g, h}

MST - PRIM (G, W, r) root (any node)

for each vertex $u \in V$

do $key[u] \leftarrow \infty$

$\pi[u] \leftarrow NIL$

$key[r] \leftarrow 0$

$Q \leftarrow V$

while $Q \neq \phi$

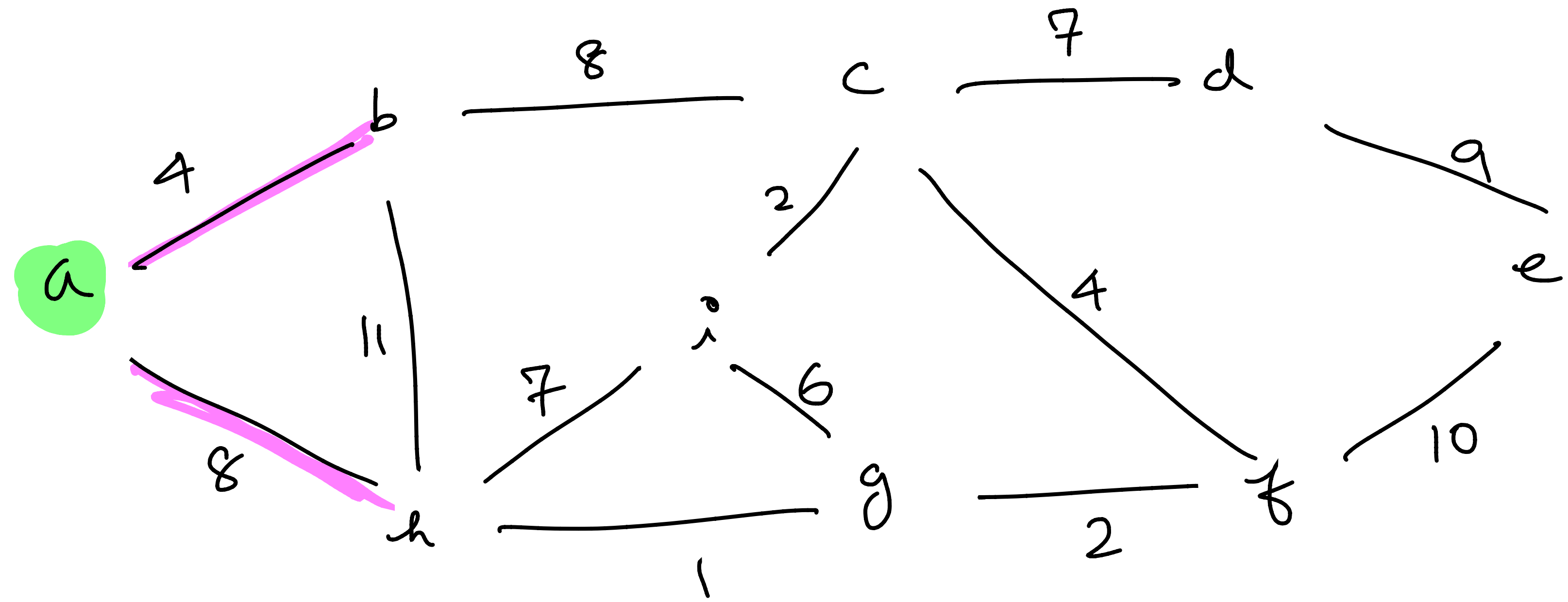
do $u \leftarrow$ EXTRACT - MIN (Q)

for each $v \in Adj[u]$

do if $v \in Q$ & $w(u, v) < key[v]$

then $\pi[v] \leftarrow u$

$key[v] \leftarrow w(u, v)$



Q = a, ...

key $\overset{\downarrow}{0}$, ∞ , ∞ , ∞ , . . .

Pick a to be the root

Step 1 : EXTRACT

Adj [a] are h, b.

a is inside S , and you want to reach anything in the outside

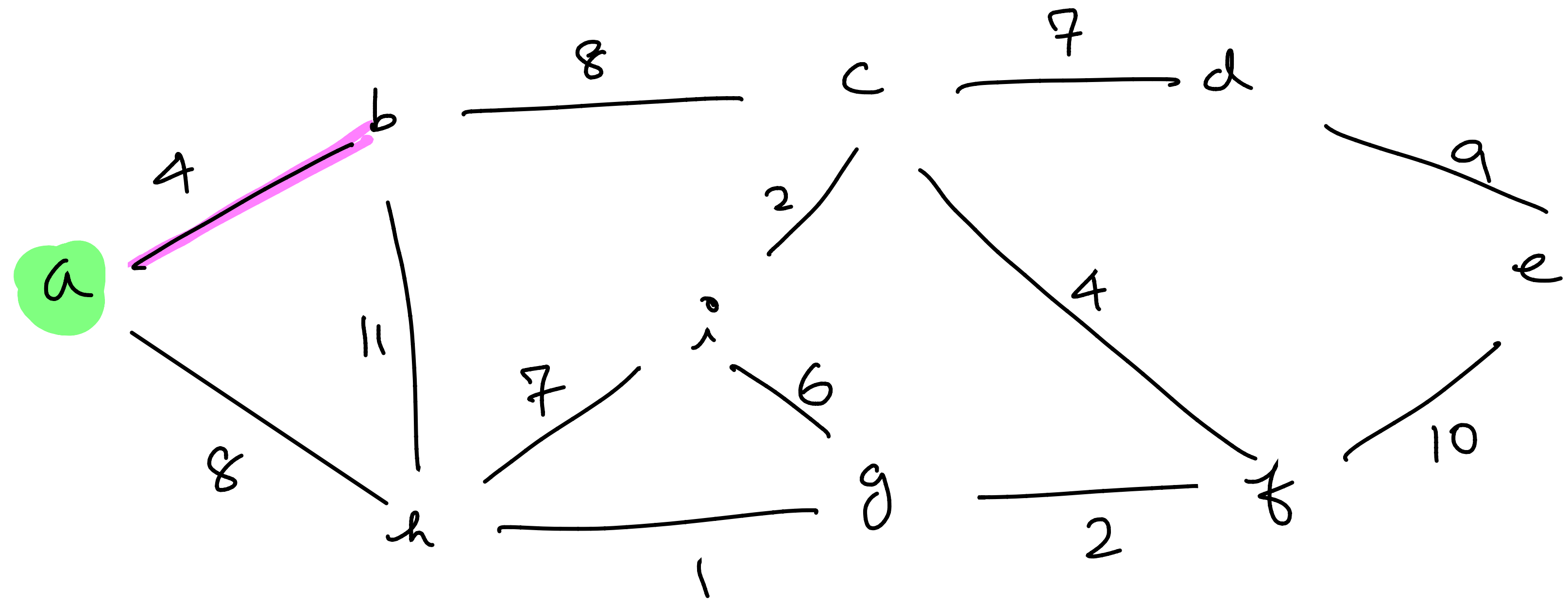
$\pi[b] = a$

$key[b] = 4$

$\pi[h] = a$

$key[h] = 8$

Step 2:



b get dequeued.

Adj [b] = h, c

but h will not be considered because its key = 8 < 11

π [c] = b

key [c] = 8