Roll. No- DA24M015 Tutorial VII Date: 28/10/24

Name: Rajnish Maurya

Solution 1.

- Null Hypothesis (H_0): $\mu = 120$ mmHg
- Alternate Hypothesis (H_a): $\mu \neq 120$ mmHg

Sample mean $(\bar{X}) = \frac{2962}{25} = 118.48$

Z-score, $z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{118.48 - 120}{10/\sqrt{25}} = -0.76$

p-value for z-score of -0.76 in a two-tailed test

$= 2(1 - 0.77637)$

$= 0.44726$

Since the p-value (0.44726) is greater than the significance level ($\alpha = 0.05$), we fail to reject the null Hypothesis.

Conclusion:- At the 5% significance level, there is not enough evidence to conclude that the diet affects the blood pressure.

Solution 2: To test if the two populations have the same variance, we will use an F-test for equality of variances at the 5% significance level.

Given:

Sample A: 65, 66, 73, 80, 82, 84, 88, 90, 92

Sample B: 64, 66, 74, 78, 82, 85, 87, 92, 93, 95, 97.

Significance level (α) = 0.05

Null Hypothesis (H_0): $\sigma_A^2 = \sigma_B^2$

Alternate Hypothesis (H_a): $\sigma_A^2 \neq \sigma_B^2$

$\bar{X}_A = \frac{720}{9} = 80$ $\bar{X}_B = \frac{913}{11} = 83$

$\sigma_A^2 = \frac{798}{9-1} = 99.75$ $\sigma_B^2 = \frac{1297}{10} = 129.7$

$$F = \frac{\sigma_B^2}{\sigma_A^2} = \frac{129.7}{99.75} = 1.301$$

F-test with a significance value of 0.05 & degrees of freedom

$df_1 = n_B - 1 = 11 - 1 = 10$
$df_2 = n_A - 1 = 9 - 1 = 8$

It is a two-tail test

$F_{\alpha/2}, df_1, df_2$ & $F_{1-\alpha/2}, df_1, df_2$.

Lower critical value $= F_{0.025, 10, 8} = 0.259$
Upper Critical value $= F_{0.975, 10, 8} = 4.295$

As the F-test value lies between the lower critical value & upper critical value. We fail to reject the Null Hypothesis.

At 5% significance level there is not enough evidence to conclude that the variances of the two population are different

Solution 3:

Part(a): Construct a 95% confidence Interval for the difference in means

$\bar{X}_1 = 0.6097$

$\bar{X}_2 = 0.492$

$$S_1^2 = \frac{\Sigma(X - \bar{X}_1)^2}{n_1 - 1} = 0.00447, \quad S_2^2 = \frac{\Sigma(X - \bar{X}_2)^2}{n_2 - 1} = 0.00604$$

Standard Error of the difference

$$SE = \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} = \sqrt{\frac{0.00447}{6} + \frac{0.00604}{5}} = 0.0388$$

$$\text{degree of freedom} = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{\left(\frac{S_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\frac{S_2^2}{n_2^2}}{n_2 - 1}} = 8.85$$

For a 95% confidence level (two-tailed), t critical $= 2.262$

Confidence Interval $= (\bar{X}_1 - \bar{X}_2) \pm t_{\alpha/2} \cdot SE$

$= (0.6097 - 0.492) \pm 2.262 \times 0.0388$

$= 0.1177 \pm 0.0877$

Thus, the 95% confidence Interval for the difference in means is $(0.03, 0.205)$

Part (c):

Null Hypothesis (H_0): $\mu_1 = \mu_2$ (μ_1 → mean of polluted lake; μ_2 → mean of unpolluted lake.)

Alternate Hypothesis (H_a): $\mu_1 > \mu_2$

$$t\text{-statistics} = \frac{\bar{X}_1 - \bar{X}_2}{SE} = \frac{0.6097 - 0.492}{0.0388} = 3.04$$

Critical t-value for a one tailed test at $\alpha = 0.05$ with degree of freedom $= 1.833$

Since t-statistics > t-value, we reject the null Hypothesis. This provides sufficient evidence at the 5% Significance level to conclude that fish in the polluted lake have elevated levels of mercury compared to those in the unpolluted lake.

Solution 4:

a) Null Hypothesis, $\mu_D = 0$ (there is no difference in the mean shear strength b/w M_1 & M_2.

Alternate Hypothesis (H_a) $= \mu_D \neq 0$ (there is a significant difference in the mean shear strength predictions b/w M_1 & M_2)

b). $$t\text{-statistics} = \frac{\bar{D}}{S_D/\sqrt{n}}$$

$$\bar{D} = \frac{\Sigma D_i}{n} = 0.274$$

$$S_D = \sqrt{\frac{\Sigma (D_i - \bar{D})^2}{n-1}} = 0.131$$

$$t\text{-statistics} = \frac{\bar{D}}{S_D/\sqrt{n}} = \frac{0.274}{0.131/\sqrt{9}} = 6.28$$

Compare with the critical t-value & Compute p-value
Degree of freedom, $df = n-1 = 8$
Critical t-value for a two tailed test at $\alpha = 0.05$ & $df = 8$.
$= 2.306$

Since t-value is much greater than t-critical value.
we reject the null hypothesis.

c) p-value for $t = 6.28$, the p-value is very small
(less than 0.001)

Since $t = 6.28$ is much greater than $t_{critical} = 2.306$ & the p-value is less than 0.05, we reject the null hypothesis. This suggests a significant difference in the shear strength predictions between Method 1 & Method 2.