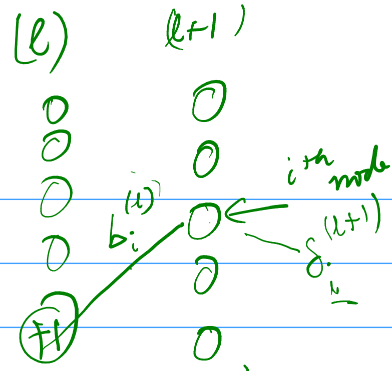


$$\frac{\partial J}{\partial b_i^{(l)}} = \underbrace{\frac{\partial J}{\partial z_i^{(l+1)}}}_{\delta_i^{(l+1)}} \underbrace{\frac{\partial z_i^{(l+1)}}{\partial b_i^{(l)}}}_{=1}$$

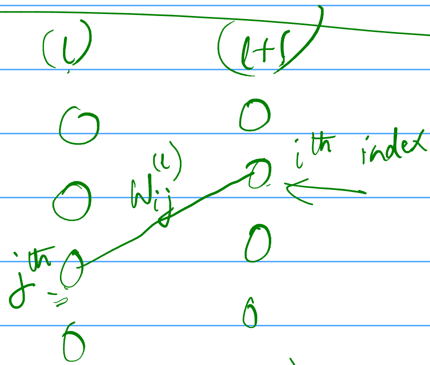
$$z^{(l+1)} = W^{(l)} a^{(l)} + b^{(l)}$$

$$z_i^{(l+1)} = \sum_j W_{ij} a_j^{(l)} + b_i^{(l)}$$

$$\frac{\partial z_i^{(l+1)}}{\partial b_i^{(l)}} = 1$$

$$\boxed{\frac{\partial J}{\partial b_i^{(l)}} = \delta_i^{(l+1)}}$$

$$\boxed{\frac{\partial J}{\partial b} = \begin{pmatrix} \frac{\partial J}{\partial b_1} \\ \frac{\partial J}{\partial b_2} \\ \vdots \end{pmatrix} = \begin{pmatrix} \delta_1^{(l+1)} \\ \delta_2^{(l+1)} \\ \vdots \end{pmatrix} = \delta^{(l+1)}}$$



$$\frac{\partial J}{\partial W_{ij}^{(l)}} = \frac{\partial J}{\partial z_i^{(l+1)}} \frac{\partial z_i^{(l+1)}}{\partial W_{ij}^{(l)}}$$

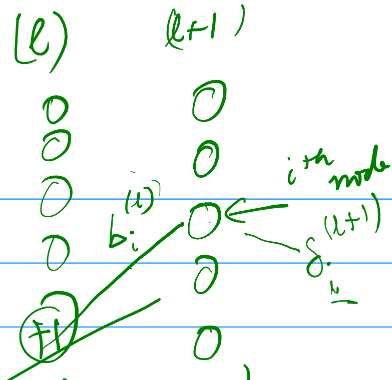
$$z^{(l+1)} = W^{(l)} a^{(l)} + b^{(l)}$$

$$z_i^{(l+1)} = \sum_{j=1} W_{ij}^{(l)} a_j^{(l)} + b_i^{(l)}$$

$$\boxed{\frac{\partial J}{\partial W_{ij}^{(l)}} = \delta_i^{(l+1)} a_j^{(l)}}$$

$$\frac{\partial J}{\partial W^{(l)}} = \begin{pmatrix} \frac{\partial J}{\partial W_{11}^{(l)}} & \frac{\partial J}{\partial W_{12}^{(l)}} & \cdots \\ \frac{\partial J}{\partial W_{21}} & \cdots \end{pmatrix} = \begin{pmatrix} \delta_1^{(l+1)} a_1^{(l)} & \delta_1^{(l+1)} a_2^{(l)} & \cdots \\ \delta_2^{(l+1)} & \cdots \end{pmatrix} = \begin{pmatrix} \delta_1^{(l+1)} \\ \delta_2^{(l+1)} \\ \vdots \end{pmatrix} \begin{pmatrix} a_1^{(l)} & a_2^{(l)} & \cdots \end{pmatrix} = \delta^{(l+1)} \left(a^{(l)}\right)^T$$

$$\boxed{\frac{\partial J}{\partial W^{(l)}} = \delta^{(l+1)} \left(a^{(l)}\right)^T}$$



$$\frac{\partial J}{\partial b_i^{(l)}} = \underbrace{\frac{\partial J}{\partial z_i^{(l+1)}}}_{\delta_i^{(l+1)}} \underbrace{\frac{\partial z_i^{(l+1)}}{\partial b_i^{(l)}}}_{=1}$$

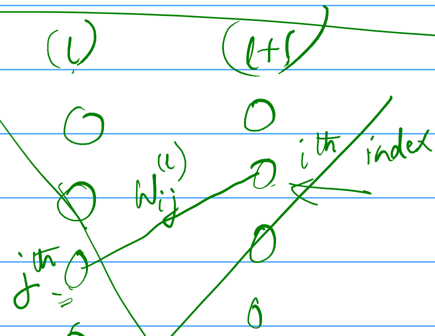
$$\boxed{\frac{\partial J}{\partial b_i^{(l)}} = \delta_i^{(l+1)}}$$

$$z^{(l+1)} = W^{(l)} a^{(l)} + b^{(l)}$$

$$z_i^{(l+1)} = \sum_j W_{ij} a_j + b_i$$

$$\frac{\partial z_i^{(n+1)}}{\partial b_i^{(l)}} = 1$$

$$\boxed{\frac{\partial J}{\partial b} = \begin{pmatrix} \frac{\partial J}{\partial b_1} \\ \frac{\partial J}{\partial b_2} \\ \vdots \end{pmatrix} = \begin{pmatrix} \delta_1^{(l+1)} \\ \delta_2^{(l+1)} \\ \vdots \end{pmatrix} = \delta^{(l+1)}}$$



$$\frac{\partial J}{\partial W_{ij}^{(l)}} = \frac{\partial J}{\partial z_i^{(l+1)}} \frac{\partial z_i^{(l+1)}}{\partial W_{ij}^{(l)}}$$

$$\boxed{\frac{\partial J}{\partial W_{ij}^{(l)}} = \delta_i^{(l+1)} \quad a_j^{(l)}}$$

$$z^{(l+1)} = W^{(l)} a^{(l)} + b^{(l)}$$

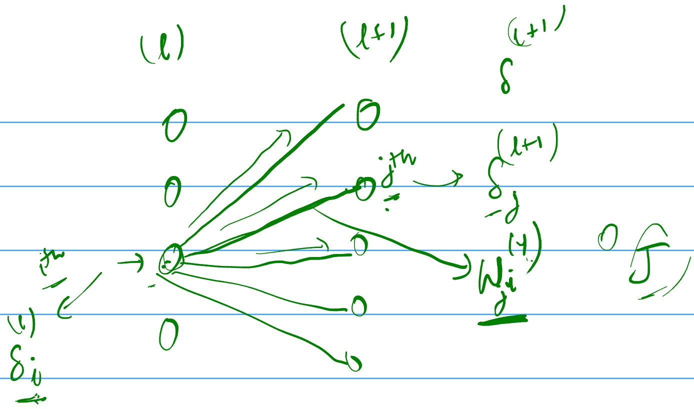
$$z_i^{(l+1)} = \sum_{j=1} W_{ij}^{(l)} a_j^{(l)} + b_i^{(l)}$$

$$\frac{\partial J}{\partial W^{(l)}} = \begin{pmatrix} \frac{\partial J}{\partial W_{11}^{(l)}} & \frac{\partial J}{\partial W_{12}^{(l)}} & \cdots \\ \frac{\partial J}{\partial W_{21}} & & \cdots \end{pmatrix} = \begin{pmatrix} \delta_1^{(l+1)} a_1^{(l)} & \delta_1^{(l+1)} a_2^{(l)} \\ \delta_2^{(l+1)} & \end{pmatrix} = \begin{pmatrix} \delta_1^{(l+1)} \\ \delta_2^{(l+1)} \end{pmatrix} \begin{pmatrix} a_1^{(l)} & a_2^{(l)} \end{pmatrix}$$

$$\delta^{(l+1)} \left(a^{(l)}\right)^T$$

$$\boxed{\frac{\partial J}{\partial W^{(l)}} = \delta^{(l+1)} \left(a^{(l)}\right)^T}$$

$$\delta_i^{(l)} = \frac{\partial J}{\partial z_i^{(l)}}$$



$$\delta_i^{(l)} = \left(\sum_j \underbrace{\frac{\partial J}{\partial z_j^{(l+1)}}}_{\delta_j^{(l+1)}} \underbrace{\frac{\partial z_j^{(l+1)}}{\partial a_i^{(l)}}}_{W_{ji}^{(l)}}\right) \underbrace{\frac{\partial a_i^{(l)}}{\partial z_i^{(l)}}}_{f'(z_i^{(l)})}$$

$$z^{(l+1)} = W^{(l)} a^{(l)} + b^{(l)}$$

$$z_j^{(l+1)} = \sum_i W_{ji}^{(l)} a_i^{(l)} + b_j^{(l)}$$

$$\boxed{\delta_i^{(l)} = \left(\sum_j W_{ji}^{(l)} \delta_j^{(l+1)}\right) f'(z_i^{(l)})}$$

$$\delta^{(l)} = \begin{pmatrix} \delta_1^{(l)} \\ \delta_2^{(l)} \\ \vdots \end{pmatrix} = \begin{pmatrix} & & \\ & W^{(l)T} & \\ & & \end{pmatrix} \begin{pmatrix} \delta_1^{(l+1)} \\ \delta_2^{(l+1)} \\ \vdots \end{pmatrix} \odot f'(z^{(l)})$$

$$\boxed{\delta^{(l)} = \left(W^{(l)T} \delta^{(l+1)}\right) \odot f'(z^{(l)})}$$