

Indian Institute of Technology Madras
Department of Data Science and Artificial Intelligence

DA5000: Mathematical Foundations of Data Science

Tutorial V

Problem

1. Prove that the binomial distribution converges to the Poisson distribution as $n \rightarrow \infty$ and $p \rightarrow 0$, subject to the constraint that the product $np = \lambda$, where λ is a constant.
2. Given that in a factory, roughly 1.5 % of all mobile phones manufactured are defective. Find the probability that in a sample of 700 units, less than 3 mobiles are defective.
Suppose the company improves its manufacturing techniques and reduces the defective % to 0.5 %, find the probability of less than 3 mobiles being defective for the same sample size.

3. Consider a random variable X and Y that follows an Exponential distribution with a rate parameter $\lambda > 0$ and Bernoulli distribution respectively. Show that the expectation $\mathbb{E}[\cdot]$ and the cumulative distribution function $F[\cdot]$ have closed-form solutions.

Hint: A closed-form solution is an expression that can be computed in a finite number of standard operations (addition, subtraction, multiplication, division, and root extraction). It typically does not involve infinite series or integrals. If a solution requires infinite series, integrals, or limits that can't be resolved in a finite number of steps, it is not considered closed form.

4. A random variable X is uniformly distributed over the interval $[5, 15]$. Calculate the probability that a randomly chosen value of X is closer to the mean than to either endpoint of the interval.
5. The memoryless property states that for all $s, t \geq 0$:

$$P(X > s + t \mid X > s) = P(X > t).$$

Show that the exponential distribution satisfies this property.