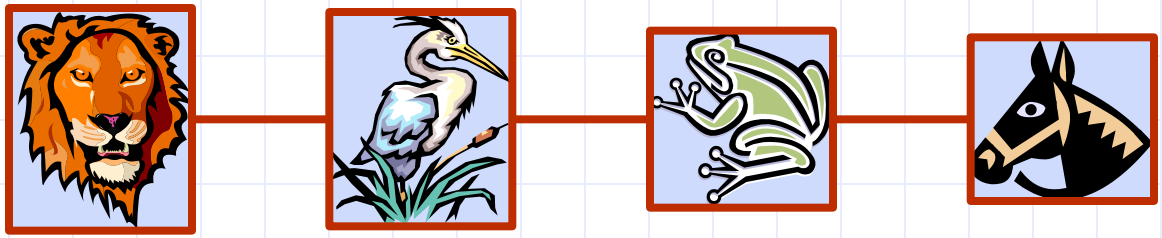
Drawbacks of Arrays

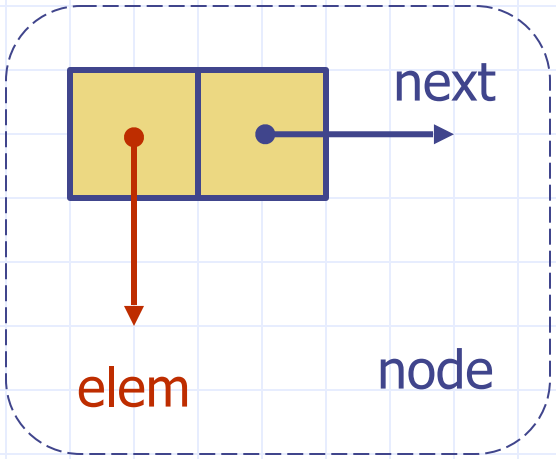
- Fixed capacity
- Insertions and Deletions involve many shifting operations

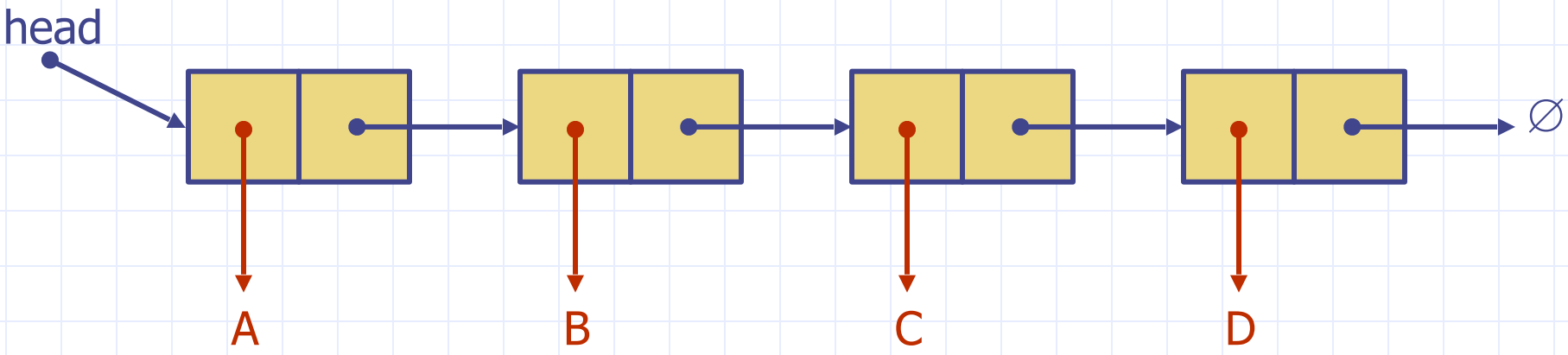
Linked Lists



Singly Linked List

- A singly linked list is a concrete data structure consisting of a sequence of nodes, starting from a head pointer
- Each node stores
  - element
  - link to the next node





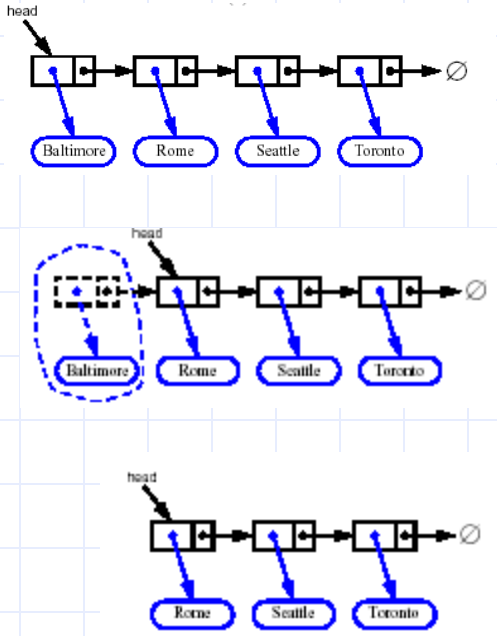
The Node Class for List Nodes

```
class _Node:
  """Lightweight, nonpublic class for storing a singly linked node."""
  __slots__ = '_element', '_next'         # streamline memory usage

  def __init__(self, element, next):      # initialize node's fields
    self._element = element               # reference to user's element
    self._next = next                     # reference to next node
```

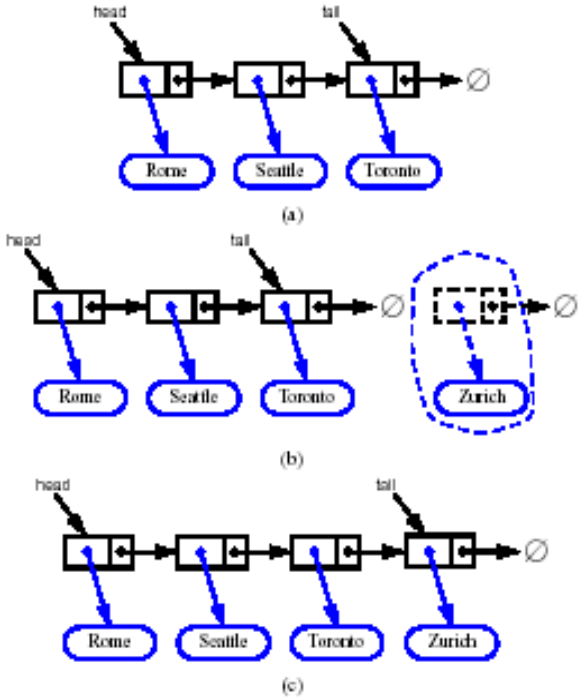
Removing at the Head

1. Update head to point to next node in the list
2. Allow garbage collector to reclaim the former first node



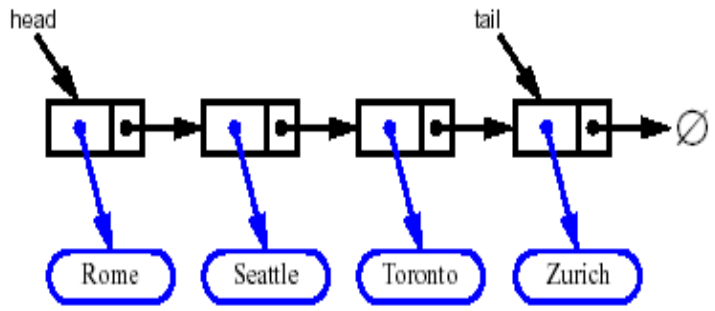
Inserting at the Tail

1. Allocate a new node
2. Insert new element
3. Have new node point to null
4. Have old last node point to new node
5. Update tail to point to new node

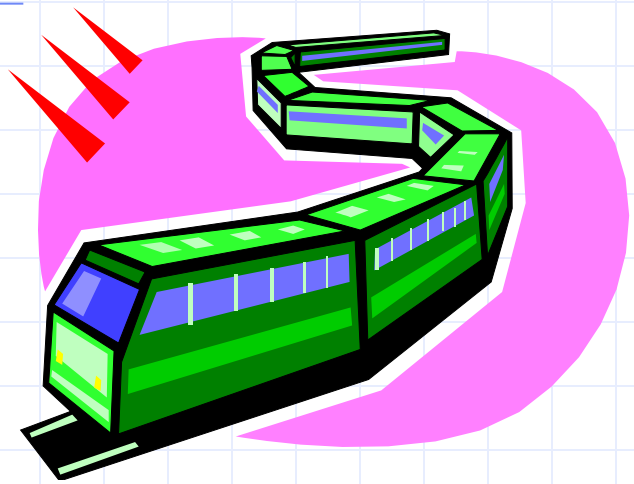


Removing at the Tail

- Removing at the tail of a singly linked list is not efficient!
- There is no constant-time way to update the tail to point to the previous node

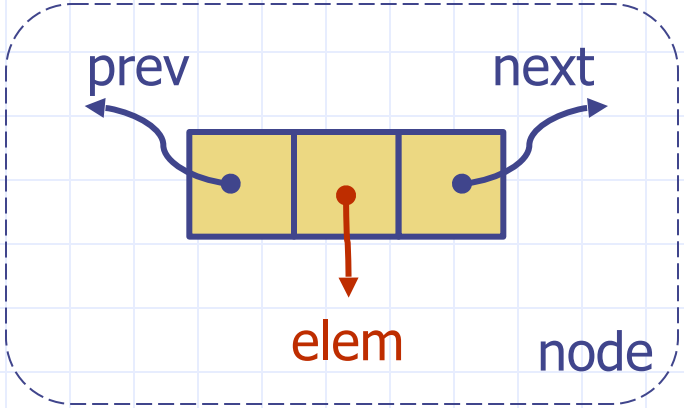


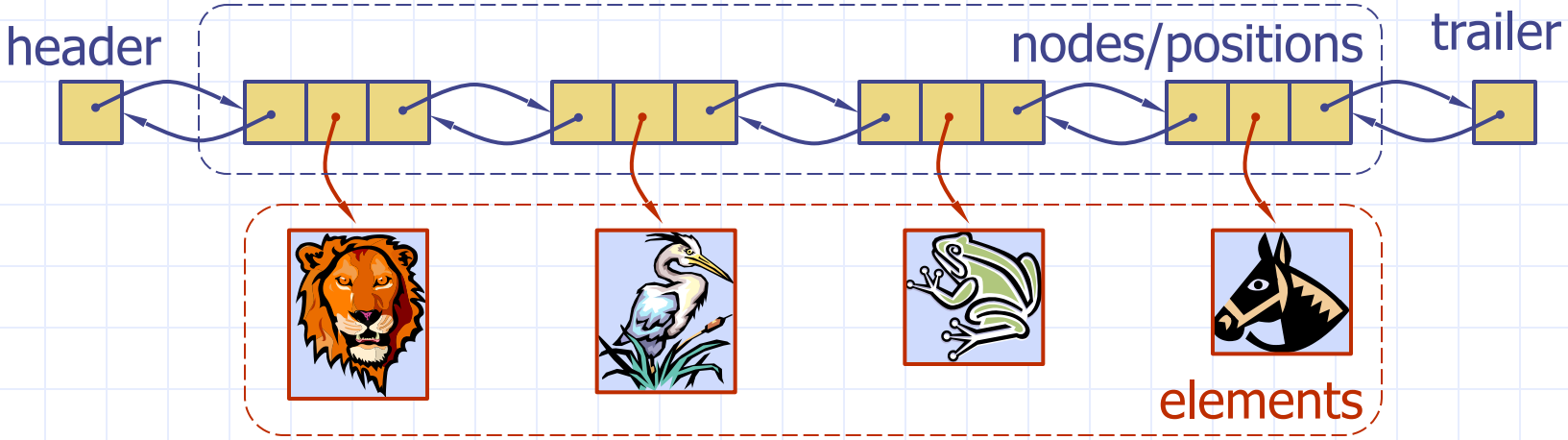
Doubly-Linked Lists



Doubly Linked List

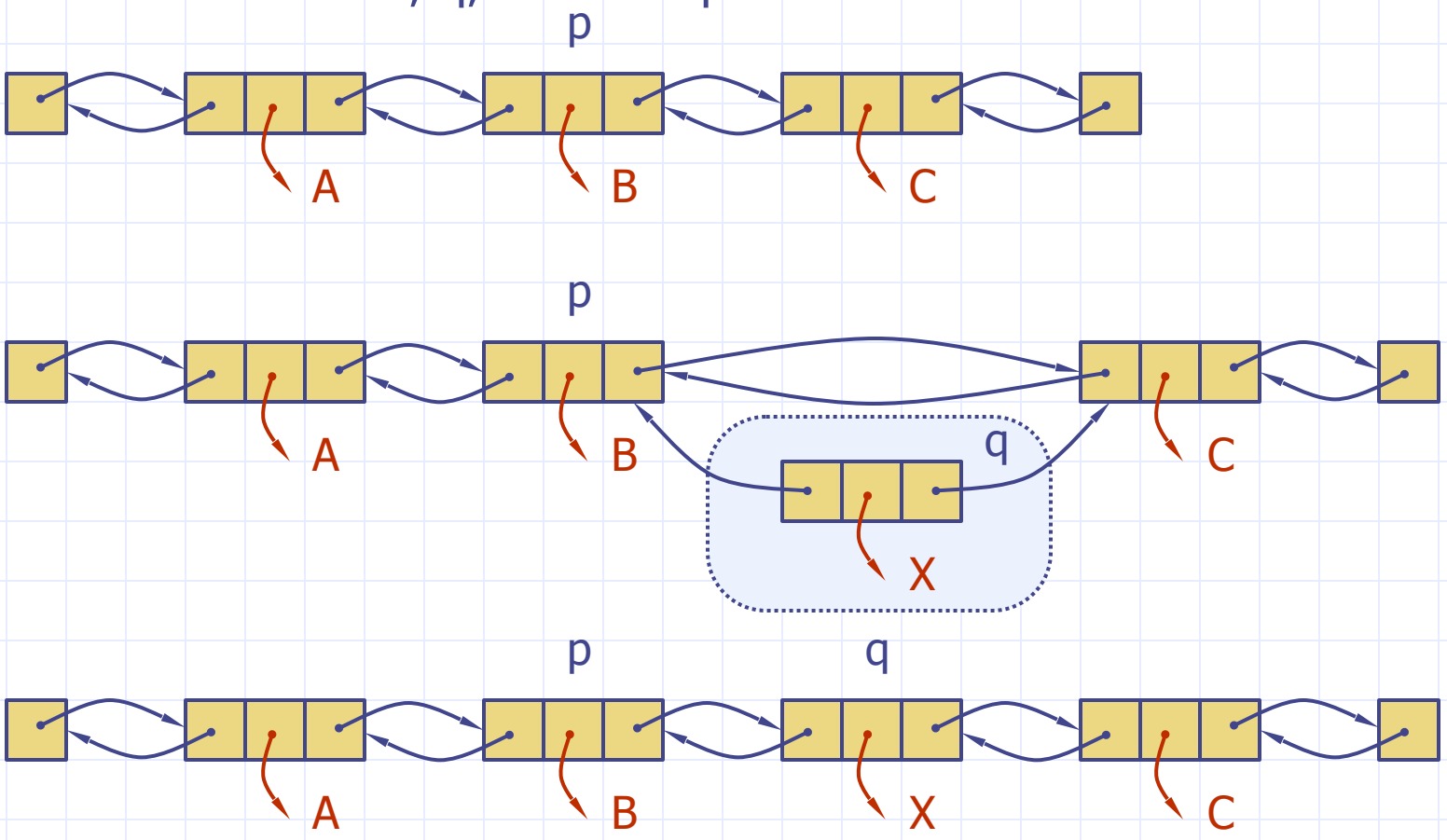
- A doubly linked list provides a natural implementation of the Node List ADT
- Nodes implement Position and store:
  - element
  - link to the previous node
  - link to the next node
- Special trailer and header nodes





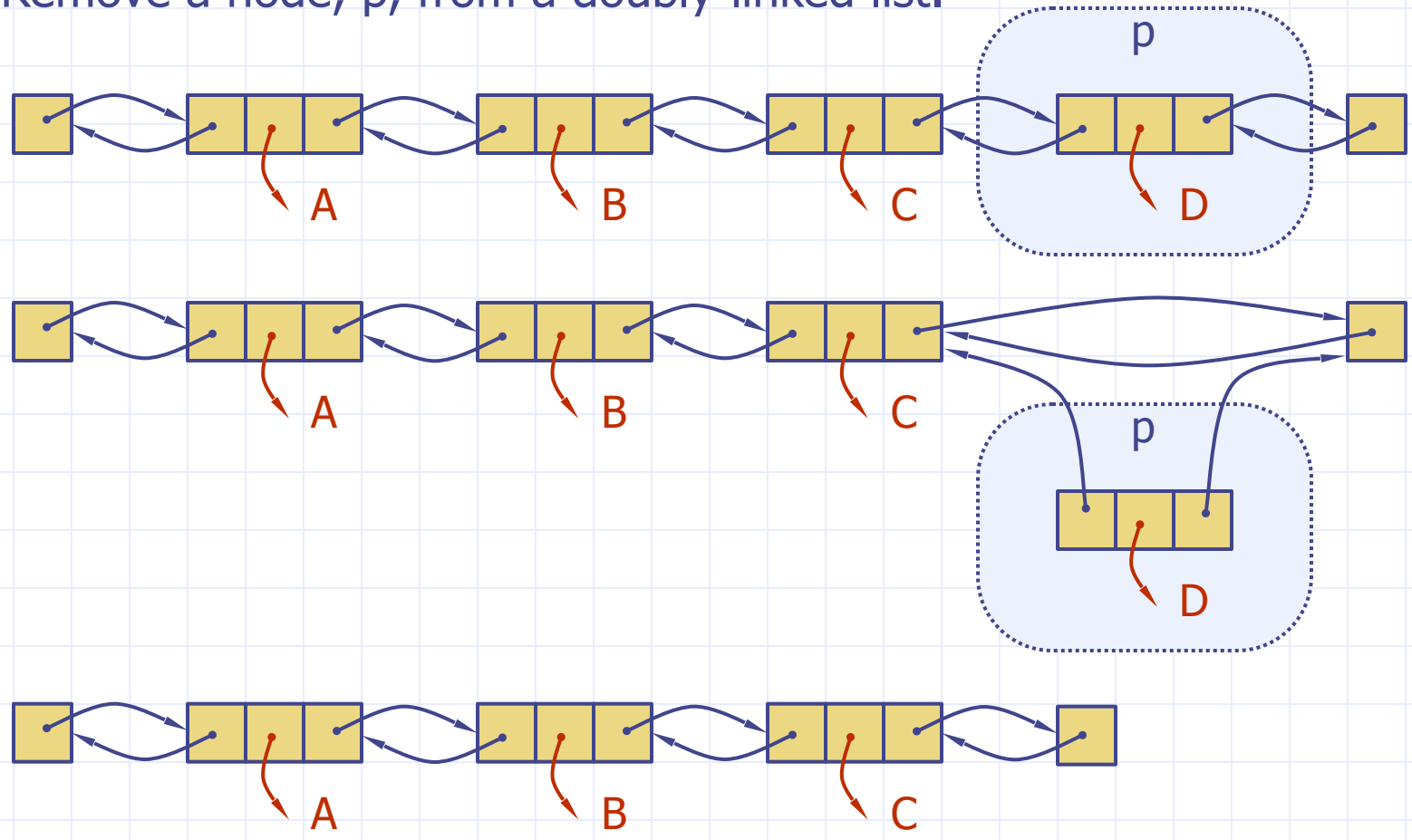
Insertion

- Insert a new node, q, between p and its successor.



Deletion

- Remove a node, p, from a doubly-linked list.



The Node Class for List Nodes

```
class _Node:
  """Lightweight, nonpublic class for storing a doubly linked node."""
  __slots__ = '_element', `_next', `_prev'          # streamline memory us

  def __init__(self, element, next, prev):       # initialize node's fields
    self._element = element                  # reference to user's element
    self._next = next                      # reference to next node
     self._prev = prev
```

Doubly-Linked List in Python

```
1  class _DoublyLinkedBase:
2    """A base class providing a doubly linked list representation."""
3
4    class _Node:
5      """Lightweight, nonpublic class for storing a doubly linked node."""
6      (omitted here; see previous code fragment)
7
8    def __init__(self):
9      """Create an empty list."""
10     self._header = self._Node(None, None, None)
11     self._trailer = self._Node(None, None, None)
12     self._header._next = self._trailer                # trailer is after header
13     self._trailer._prev = self._header                # header is before trailer
14     self._size = 0                                    # number of elements
15
16   def __len__(self):
17     """Return the number of elements in the list."""
18     return self._size
19
20   def is_empty(self):
21     """Return True if list is empty."""
22     return self._size == 0
23
```

```
24   def _insert_between(self, e, predecessor, successor):
25     """Add element e between two existing nodes and return new node."""
26     newest = self._Node(e, predecessor, successor)   # linked to neighbors
27     predecessor._next = newest
28     successor._prev = newest
29     self._size += 1
30     return newest
31
32   def _delete_node(self, node):
33     """Delete nonsentinel node from the list and return its element."""
34     predecessor = node._prev
35     successor = node._next
36     predecessor._next = successor
37     successor._prev = predecessor
38     self._size -= 1
39     element = node._element                          # record deleted element
40     node._prev = node._next = node._element = None   # deprecate node
41     return element                                   # return deleted element
```

Performance

- In a doubly linked list
  - The space used by a list with n elements is $O(n)$
  - The space used by each position of the list is $O(1)$
  - All the standard operations of a list run in $O(1)$ time

Positional List

- To provide for a general abstraction of a sequence of elements with the ability to identify the location of an element, we define a **positional list** ADT.
- A position acts as a marker or token within the broader positional list.
- A position p is unaffected by changes elsewhere in a list; the only way in which a position becomes invalid is if an explicit command is issued to delete it.
- A position instance is a simple object, supporting only the following method:
  - p.element( ): Return the element stored at position p.

Positional Accessor Operations

L.first( ):	Return the position of the first element of L, or None if L is empty.
L.last( ):	Return the position of the last element of L, or None if L is empty.
L.before(p):	Return the position of L immediately before position p, or None if p is the first position.
L.after(p):	Return the position of L immediately after position p, or None if p is the last position.
L.is_empty( ):	Return True if list L does not contain any elements.
len(L):	Return the number of elements in the list.
iter(L):	Return a forward iterator for the *elements* of the list. See Section 1.8 for discussion of iterators in Python.

Positional Update Operations

L.add_first(e):	Insert a new element e at the front of L, returning the position of the new element.
L.add_last(e):	Insert a new element e at the back of L, returning the position of the new element.
L.add_before(p, e):	Insert a new element e just before position p in L, returning the position of the new element.
L.add_after(p, e):	Insert a new element e just after position p in L, returning the position of the new element.
L.replace(p, e):	Replace the element at position p with element e, returning the element formerly at position p.
L.delete(p):	Remove and return the element at position p in L, invalidating the position.

Positional List in Python

```
1  class PositionalList(_DoublyLinkedBase):
2    """A sequential container of elements allowing positional access."""
3
4    #------------------------- nested Position class -------------------------
5    class Position:
6      """An abstraction representing the location of a single element."""
7
8      def __init__(self, container, node):
9        """Constructor should not be invoked by user."""
10       self._container = container
11       self._node = node
12
13     def element(self):
14       """Return the element stored at this Position."""
15       return self._node._element
16
17     def __eq__(self, other):
18       """Return True if other is a Position representing the same location."""
19       return type(other) is type(self) and other._node is self._node
20
21     def __ne__(self, other):
22       """Return True if other does not represent the same location."""
23       return not (self == other)           # opposite of __eq__
24
25   #------------------------------ utility method ------------------------------
26   def _validate(self, p):
27     """Return position's node, or raise appropriate error if invalid."""
28     if not isinstance(p, self.Position):
29       raise TypeError('p must be proper Position type')
30     if p._container is not self:
31       raise ValueError('p does not belong to this container')
32     if p._node._next is None:              # convention for deprecated nodes
33       raise ValueError('p is no longer valid')
34     return p._node
```

Positional List in Python, Part 2

```
35    #------------------------------ utility method ------------------------------
36    def _make_position(self, node):
37      """Return Position instance for given node (or None if sentinel)."""
38      if node is self._header or node is self._trailer:
39        return None                                  # boundary violation
40      else:
41        return self.Position(self, node)             # legitimate position
42
43    #------------------------------ accessors ------------------------------
44    def first(self):
45      """Return the first Position in the list (or None if list is empty)."""
46      return self._make_position(self._header._next)
47
48    def last(self):
49      """Return the last Position in the list (or None if list is empty)."""
50      return self._make_position(self._trailer._prev)
51
52    def before(self, p):
53      """Return the Position just before Position p (or None if p is first)."""
54      node = self._validate(p)
55      return self._make_position(node._prev)
56
57    def after(self, p):
58      """Return the Position just after Position p (or None if p is last)."""
59      node = self._validate(p)
60      return self._make_position(node._next)
61
62    def __iter__(self):
63      """Generate a forward iteration of the elements of the list."""
64      cursor = self.first()
65      while cursor is not None:
66        yield cursor.element()
67        cursor = self.after(cursor)
```

Positional List in Python, Part 3

```
68    #------------------------------ mutators ------------------------------
69    # override inherited version to return Position, rather than Node
70    def _insert_between(self, e, predecessor, successor):
71      """Add element between existing nodes and return new Position."""
72      node = super()._insert_between(e, predecessor, successor)
73      return self._make_position(node)
74
75    def add_first(self, e):
76      """Insert element e at the front of the list and return new Position."""
77      return self._insert_between(e, self._header, self._header._next)
78
79    def add_last(self, e):
80      """Insert element e at the back of the list and return new Position."""
81      return self._insert_between(e, self._trailer._prev, self._trailer)
82
83    def add_before(self, p, e):
84      """Insert element e into list before Position p and return new Position."""
85      original = self._validate(p)
86      return self._insert_between(e, original._prev, original)
87
88    def add_after(self, p, e):
89      """Insert element e into list after Position p and return new Position."""
90      original = self._validate(p)
91      return self._insert_between(e, original, original._next)
92
93    def delete(self, p):
94      """Remove and return the element at Position p."""
95      original = self._validate(p)
96      return self._delete_node(original)      # inherited method returns element
97
98    def replace(self, p, e):
99      """Replace the element at Position p with e.
100
101     Return the element formerly at Position p.
102     """
103     original = self._validate(p)
104     old_value = original._element           # temporarily store old element
105     original._element = e                   # replace with new element
106     return old_value                        # return the old element value
```