$$f(x) = \prod_{j=a}^{b} term(x, j)$$

1. Term Evaluation

$$\alpha(x,j) = \frac{term(x,j)}{term(x,j-1)} \quad ;$$

if α holds true for all j

- term = term(x,a); f = term;

```
for (i=a+1; i<=j ; i++) {
    term *= α(x,i);
    f += term;
}
```

2. Horner's Metod

$$f(x,b) = 1 + g(x,b) * f(x, b-1) \qquad (f(a) = value)$$

```
f = f(a);
for ( i=a+1; i<=b ; i++){
    f = 1+ g(x,i)*f ;
}
```

- Horner's Method Accumulates less approximation, round off errors and is more resistant to overflow & underflow errors.

- Term Evaluation can be halted anytime without going to n^{th} term, while Horner's method HAS to go to n^{th} term.

1. $$f(r,N) = 1 + r + r^2 + r^3 \cdots + r^{N-2} + r^{N-1}$$

$$= 1 + r\left(1 + r + r^2 \cdots + r^{N-3} + r^{N-2}\right)$$

$$f(r,N) = 1 + r \times f(r, N-1)$$

$$\alpha = r$$

$$f(r,0) = 1$$

2. $$f(x,a,N) = \sum_{K=0}^{a} \frac{x^K}{K!}\left(\prod_{j=0}^{K-1}(\alpha - j)\right)$$

$$= 1 + xa + \frac{x^2}{2}a(a-1) + \frac{x^3}{2\cdot 3}a(a-1)(a-2) \cdots$$

$$= 1 + xa\left(1 + \frac{x(a-1)}{2} + \frac{x^2(a-1)(a-2)}{2\cdot 3} + \frac{x^3(a-1)(a-2)(a-3)}{2\cdot 3\cdot 4}\right)$$

$$= 1 + xa\left(1 + \frac{x}{2}(a-1)\left(1 + \frac{x}{3}(a-2) + \frac{x^2(a-2)(a-3)}{3\cdot 4} \cdots\right.\right.$$

$$f(x,a,T) = 1 + \frac{x(a-T)}{(1+T)} f(x,a,T+1) \qquad (T_0 = 0)$$

$$f(x,a,N-1) = 1 \qquad (\text{To stop at } N^{th} \text{ term})$$

Or $\quad \alpha = \text{term}(k) \div \text{term}(k-1)$

$$\frac{x^k}{k!}\prod_{j=0}^{k-1}(\alpha-j) \div \frac{x^{k-1}}{(k-1)!}\prod_{j=0}^{k-2}(\alpha-j)$$

$$\Rightarrow \frac{x}{k}(\alpha-k+1)\,\frac{x^{k-1}}{(k-1)!}\prod_{j=0}^{k-2}(\alpha-j) \div \frac{x^{k-1}}{(k-1)!}\prod_{j=0}^{k-2}(\alpha-j) \quad = 1$$

$$\alpha = \frac{x}{k}(\alpha-k+1)$$

3. $$J(x) = \sum_{K=0}^{\infty} \frac{(-1)^K}{(K!)^2} \left(\frac{x}{2}\right)^{2K}$$

$$= 1 + \sum_{K=1}^{\infty} \frac{(-1)^K}{(K!)^2} \left(\frac{x}{2}\right)^{2K}$$

$$= 1 - \frac{x^2}{2^2} + \frac{x^4}{2^4}\frac{1}{(2)^2} - \frac{x^6}{2^6}\frac{1}{(3\cdot 2)^2} - - \cdot$$

$$= 1 - \frac{x^2}{2^2}\left(1 - \frac{x^2}{2^2}\frac{1}{2^2} + \frac{x^4}{2^4}\frac{1}{(3\cdot 2)^2} - \frac{x^6}{2^6 (4\cdot 3\cdot 2)^2} - - \cdot\right)$$

$$= 1 - \frac{x^2}{2^2}\left(1 - \frac{x^2}{2^2}\frac{1}{2^2}\left(1 - \frac{x^2}{2^2}\frac{1}{3^2} + \frac{x^4}{2^4}\frac{1}{(4\cdot 3)^2} - \cdot\cdot - \right)\right.$$

$$J(x, t, N) = 1 - \frac{x^2}{2^2}\frac{1}{t^2} J(x, t+1, N)$$

$$J(x, N, N) = 1$$

$$\alpha = \frac{(-1)^K}{(K!)^2}\left(\frac{x}{2}\right)^{2K} \times \frac{((K-1)!)^2}{(-1)^{K-1}}\left(\frac{2}{K}\right)^{2(K-1)}$$

$$\alpha = \frac{-1}{K^2}\cdot\left(\frac{x}{2}\right)^2$$

4.

$$f(s) = \sum_{n=1}^{\infty} 1/n^s$$

$$f(s) = 1^{-s} + 2^{-s} + 3^{-s} \ldots\ldots$$

$$n^s = t$$

$$\log n^s = \log t$$

$$s \log n = \log t$$

$$\Rightarrow \quad t = e^{s \cdot \log n}$$

5. $f(x) = \frac{2}{\sqrt{\pi}} \left(x - \frac{x^3}{3} + \frac{x^5}{10} - - - \right)$

$$= \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n \quad x^{2n+1}}{(2n+1) \quad n!}$$

$$\alpha = \frac{(-1)^n}{(-1)^{n-1}} \quad \frac{2n-1}{2n+1} \quad \frac{(n-1)!}{n!} \quad \frac{x^{2n+1}}{x^{2n-1}}$$

$$\alpha = -\left(\frac{2n-1}{2n+1}\right) \frac{x^2}{n}$$

$$F(x,a,N,T) = 1 + \frac{x(a-T)}{(1+T)} F(x,a,N,T+1) \qquad T \leq N$$

$$T = N = 1$$

$$F(x,a,N) = \sum_{k=0}^{a} \frac{x^k}{k!} \left(\prod_{j=0}^{k-1} (\alpha - j) \right)$$

$$t(k) = \frac{x^k}{k!} \prod_{j=0}^{k-1} (\alpha - j)$$

$$\frac{t(k+1)}{t(k)} = \frac{x}{k+1} \quad \frac{1}{\alpha - k}$$

$$\text{term} = \frac{x}{(k+1)(\alpha - k)}$$