$$F(x) = \sum_{i=a}^{b} term(x,i)$$

$$\alpha(x,i) = \frac{term(x,i)}{term(x,i-1)}$$

k is called turnication index if $term(x,i) < \varepsilon \quad \forall \; i > k+a$ where ε is required precision.

Assuming α formula is same for all x, i

1. Term Evaluation

```
term = term(x,a) ;   result = term;
for (i= a+1  ; i<=n  ;i++) {
      term *= α(x,i) ;
      result += term;
  }
```

2. Horners method

```
    result = 1+ term(x,b) ;
for (i=b-1  ;  i>=a  ; i--) {
      result = 1+ α(x,i)* result;
 }
```

Term Evaluation	Horner's
1. Terms evaluated from 1st to last. Each term evaluated completely per step.	1. Terms evaluated from last to 1st. Terms are evaluated partially, and all terms full evaluation completed together at last step.
2. Two working variables	2. One working variables.
3. Can be turnicated at any $i < b$ during runtime based on conditionals	3. All terms must be evaluated, once a and b are set.
4. Is more prone to rounding error for small x values as small terms are added to large results.	4. Is resistant to numerical error since result is calculated and refreshed as whole without self additions.
5. Suitable for infinite series with unstable or unpridictable turnication index.	5. Suitable for finite series and infinite series with fixed, stable or reliably pridictable turnication index.
<u>Unstable</u> : K changes rapidly based on x	<u>Fixed</u> : largest value of K across full domain of $f(x)$ is small enough to be used for all values of x without compromising speed and/or efficiency of program.
<u>Unpridictable</u>: $K \neq g(x) \ \forall x \in D(f)$, K can't be reliably determine by a simple function of x for all values of x in acceptable domain of $f(x)$	

For many/most applications, performance differences may be negligible.

1. $f(r,N) = 1 + r + r^2 + r^3 - - - - + r^{N-2} + r^{N-1}$

$= 1 + r(1 + r + r^2 - - - + r^{N-3} + r^{N-2})$

$f(r,N) = 1 + r \times f(r, N-1)$

$\alpha = r$

$f(r,0) = 1$

2. $f(x,a,N) = \sum_{K=0}^{a} \frac{x^K}{K!} \left(\prod_{j=0}^{K-1} (\alpha - j) \right)$

$= 1 + xa + \frac{x^2}{2} a(a-1) + \frac{x^3}{2 \cdot 3} a(a-1)(a-2) - - - -$

$= 1 + xa \left(1 + \frac{x(a-1)}{2} + \frac{x^2 (a-1)(a-2)}{2 \cdot 3} + \frac{x^3 (a-1)(a-2)(a-3)}{2 \cdot 3 \cdot 4} \right)$

$= 1 + xa \left(1 + \frac{x}{2}(a-1) \left(1 + \frac{x}{3}(a-2) + \frac{x^2 (a-2)(a-3)}{3 \cdot 4} - - \right.\right.$

$f(x,a,T) = 1 + \frac{x(a-T)}{(1+T)} f(x,a,T+1)$ $(T_0 = 0)$

$f(x,a,N-1) = 1$

Or $\quad \alpha = \text{term}(k) \div \text{term}(k-1)$

$$\frac{x^k}{k!} \prod_{j=0}^{k-1} (\alpha - j) \div \frac{x^{k-1}}{(k-1)!} \prod_{j=0}^{k-2} (\alpha - j)$$

$$\Rightarrow \frac{x}{k}(\alpha - k + 1) \underbrace{\frac{x^{k-1}}{(k-1)!} \prod_{j=0}^{k-2} (\alpha - j) \div \frac{x^{k-1}}{(k-1)!} \prod_{j=0}^{k-2} (\alpha - j)}_{=1}$$

$$A = \frac{x}{k}(\alpha - k + 1)$$

$$k = t + 1 \Rightarrow A = \frac{x}{t+1}(\alpha - t)$$

$$3.\quad J(x) = \sum_{K=0}^{\infty} \frac{(-1)^K}{(K!)^2} \left(\frac{x}{2}\right)^{2K}$$

$$= 1 + \sum_{K=1}^{\infty} \frac{(-1)^K}{(K!)^2} \left(\frac{x}{2}\right)^{2K}$$

$$= 1 - \frac{x^2}{2^2} + \frac{x^4}{2^4}\frac{1}{(2)^2} - \frac{x^6}{2^6}\frac{1}{(3\cdot 2)^2} - - \cdot$$

$$= 1 - \frac{x^2}{2^2}\left(1 - \frac{x^2}{2^2}\frac{1}{2^2} + \frac{x^4}{2^4}\frac{1}{(3\cdot 2)^2} - \frac{x^6}{2^6 (4\cdot 3\cdot 2)^2} - - \cdot\right)$$

$$= 1 - \frac{x^2}{2^2}\left(1 - \frac{x^2}{2^2}\frac{1}{2^2}\left(1 - \frac{x^2}{2^2}\frac{1}{3^2} + \frac{x^4}{2^4}\frac{1}{(4\cdot 3)^2} \cdots - \right)\right)$$

$$J(x, t) = 1 - \frac{x^2}{2^2}\frac{1}{t^2} J(x, t+1)$$

$$J(x, N) = 1$$

$$\alpha = \frac{(-1)^K}{(K!)^2}\left(\frac{x}{2}\right)^{2K} \times \frac{((K-1)!)^2}{(-1)^{K-1}} \left(\frac{2}{x}\right)^{2(K-1)}$$

$$\alpha = \frac{-1}{K^2} \cdot \left(\frac{x}{2}\right)^2$$

4.

$$f(s) = \sum_{n=1}^{\infty} 1/n^s$$

$$f(s) = 1^{-s} + 2^{-s} + 3^{-s} \dots\dots$$

$$n^s = t_k$$

$$\log n^s = \log t_k$$

$$s \log n = \log t_k$$

$$\Rightarrow \quad t_k = e^{s \cdot \log n}$$

$$\alpha = \frac{t_k^{-1}}{t_{k-1}^{-1}} = \frac{(k-1)^s}{k^s} = \frac{e^{-s \log k}}{e^{-s \log (k-1)}}$$

$$= \left(\frac{k-1}{k}\right)^s = e^{-s \log\left(\frac{k}{k+1}\right)} \approx e^{-s \cdot \log n} = \left(\frac{1}{n}\right)^s$$

which is functionally same as evaluating t_k individually.

So, no $\alpha(s, i)$ exist for all values of s, in other words, each term is independent of each other.

5. $f(x) = \frac{2}{\sqrt{\pi}} \left(x - \frac{x^3}{3} + \frac{x^5}{10} - - - \right)$

$$= \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n \; x^{2n+1}}{(2n+1) \; n!}$$

$$\alpha = \frac{(-1)^n}{(-1)^{n-1}} \; \frac{2n-1}{2n+1} \; \frac{(n-1)!}{n!} \; \frac{x^{2n+1}}{x^{2n-1}}$$

$$\alpha = -\left(\frac{2n-1}{2n+1}\right) \frac{x^2}{n}$$