Algebraic approach to school Geometry

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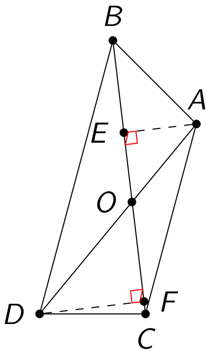
Problem Statement-Triangle Exercise

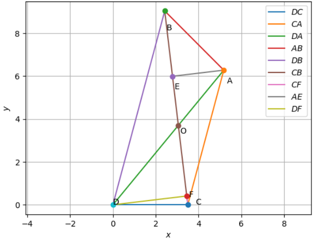
(i) ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that
$\frac{ar(ABC)}{ar(DBC)} = \frac{AO}{DO}$

Soln:

https://github.com/Rajolep/_Geometry/blob/master/figs/construc.tex



D(0,0)
B(2.4219,9.0388)
A(5.1823,6.27881)



https://github.com/Rajolep/_Geometry/blob/master/codes/triangle/draw_triangle.py

$$AE \perp BC, DF \perp BC$$

$$Areaof \triangle ABC = \frac{1}{2}(BC)(AE)$$

$$Areaof \triangle DBC = \frac{1}{2}(BC)(DF)$$

$$\frac{ar\triangle ABC}{ar\triangle DBC} = \frac{\frac{1}{2}(BC)(AE)}{\frac{1}{2}(BC)(DF)}$$

$$\frac{ar\triangle ABC}{ar\triangle DBC} = \frac{AE}{DF}$$

$$\frac{AE}{DF} = \frac{AO}{DO}$$

$$\angle AEO = \angle DFO.....RA$$

$$\angle AEO = \angle DOF.....VOA$$

$$\triangle AOE \sim \triangle DOF$$

$$\frac{AE}{DF} = \frac{AO}{DO}$$

$$
\begin{aligned}
AE &= 2.407 \\
BC = e &= 9.10294 \\
DF &= 3.4756 \\
AO &= AE...(1) \\
DO &= DF...(2)
\end{aligned}
$$

From (1) and (2)
Area of $\triangle ABC = \frac{1}{2}(BC)(AE) = 38.0739$
Area of$\triangle DBC = \frac{1}{2}(BC)(DF) = 38.0739$

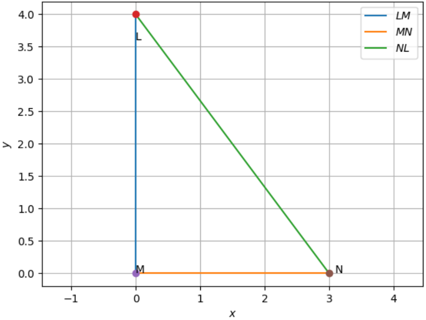
Problem Statement-Triangle Construction

(i) Construct $\triangle LMN$ right angled at M such that LN $= 5$ MN $= 3$

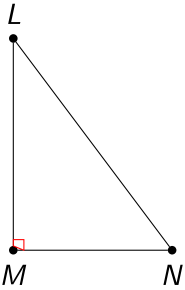
Soln:

Given:- LN=5, MN=3, M(0,0)

https://github.com/Rajolep/_Geometry/blob/master/codes/triangle/draw_triangle.py



$\mathrm{LM} = \sqrt{25 - 9}$
$\mathrm{LM} = 5$



https://github.com/Rajolep/_Geometry/blob/master/figs/construc.tex

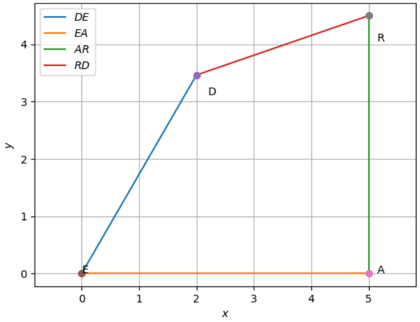
Problem Statement-Quadrilateral Construction

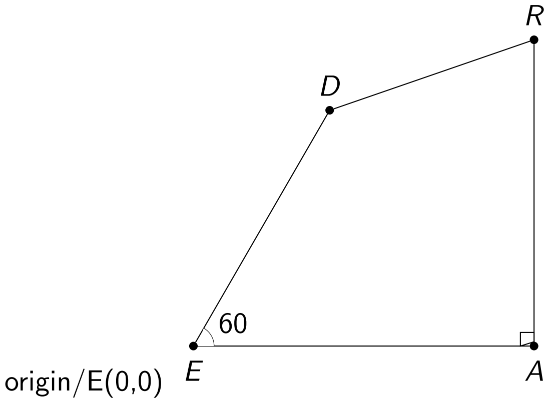
(i) Construct DEAR with DE $= 4$, EA $= 5$, AR $= 4.5$, $\angle E = 60^\circ$ and $\angle A = 90^\circ$.

Soln:

Given:- DE $= 4$, EA $= 5$, AR $= 4.5$, $\angle E = 60^\circ$ and $\angle A = 90^\circ$

https://github.com/Rajolep/_Geometry/blob/master/codes/Quad/drawquad.py





https://github.com/Rajolep/_Geometry/blob/master/figs/quadccon.tex

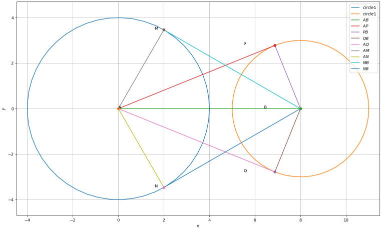
Problem Statement-Circle Construction

(i) Draw a line segment AB of length 8 units. Taking A as centre, draw a circle of radius 4 units and taking B as centre, draw another circle of radius 3 units. Construct tangents to each circle from the centre of the other circle.

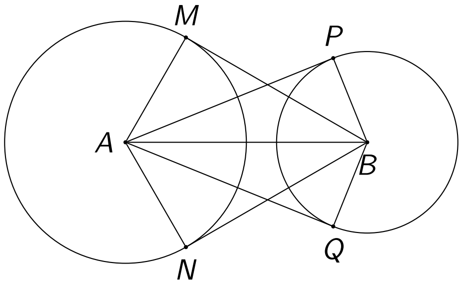
Soln:

Given:- AB=8, r1=4, r2=3, A(0,0)

https://github.com/Rajolep/_Geometry/blob/master/codes/circle/circon.py



AP=7.416198
MB=6.928203
M(2 , 3.46410162)
N(2 , -3.46410162)
P(6.875 , 2.78107443)
Q(6.875 , -2.78107443)



https://github.com/Rajolep/_Geometry/blob/master/figs/circon.tex

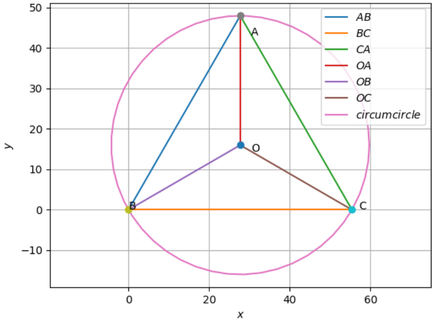
Problem Statement-Miscellenous

(i) In a circular table cover of radius 32 cm, a design is formed leaving an equilateral $\triangle ABC$ in the middle. Find the area of the design.

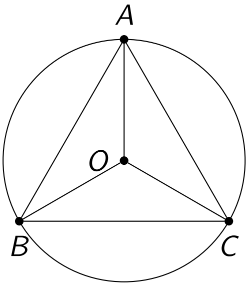
Soln:

Given: R=32cm

https://github.com/Rajolep/_Geometry/blob/master/codes/triangle/draw_triangle.py



https://github.com/Rajolep/_Geometry/blob/master/figs/miscell.tex



$$\triangle BOC = 120^\circ$$

$$BO = OC = 32$$

$$BC = \sqrt{(BO)^2 + (OC)^2} - 2(BO)(OC)\cos(120) = 55.425$$

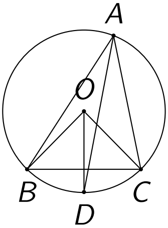
Area of design $= \pi(R)(R)$ - $\frac{\sqrt{3}}{4}(BC)(BC)$
Area $= 1886.81$

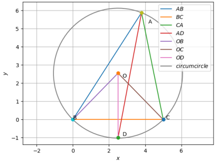
Problem Statement-Circle Exercise

(i) In any △ABC, if the angle bisector of ∠A and perpendicular bisector of BC intersect, prove that they intersect on the circumcircle of the △ABC

Soln:

https://github.com/Rajolep/_Geometry/blob/master/codes/circle/5ques.py





$$\angle BOC = 2\angle BAC = 2\angle A ...(1)$$
$$OB = OC$$
$$\angle OEB = \angle OEC$$
$$\triangle BOE \cong \triangle COE ...(2)$$
$$\angle BOE + \angle COE = \angle BOC$$

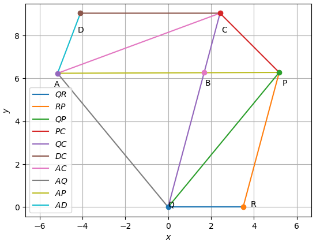
therefore,

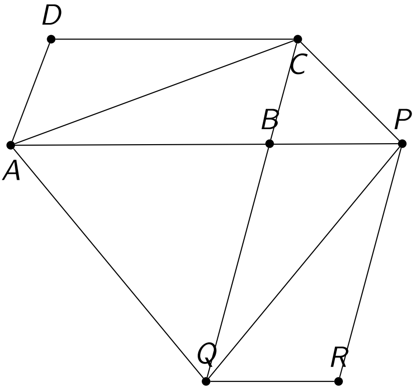
$$\angle BOE + \angle BOE = 2\angle A$$
$$\angle BOD = \angle BOE = \angle A$$
$$\angle BAD = \frac{\angle A}{2}$$
$$2\angle BAD = \angle A$$
$$\angle BOD = 2\angle BAD$$

Problem Statement-Quadrilateral Exercise

(i) The side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed. Show that ar(ABCD) = ar(PBQR). **Soln:**
Given:- CP $\parallel$ AQ

https://github.com/Rajolep/_Geometry/blob/master/codes/quadr/quadexer.py





https://github.com/Rajolep/_Geometry/blob/master/figs/quadccon.tex

$$CP \parallel AQ$$

area of $\triangle ACQ$ = area of $\triangle AQP$
subtract area$\triangle ABQ$ in above Eqn
ar$\triangle ACQ$-ar$\triangle ABQ$ = ar$\triangle APQ$-ar$\triangle ACQ$
ar$\triangle ABC$ = ar$\triangle PBQ$....(1)
$\triangle ABC \cong \triangle ADC$
ar$\triangle ABC$ = ar$\triangle ADC$
ar$\triangle ABC$ = ar$\triangle ADC$ = $\frac{1}{2}(ABCD)$...(2)

similarly in PBQR,
$\triangle PBQ \cong \triangle PRQ$
ar$\triangle PBQ =$ ar$\triangle PRQ$
ar$\triangle PBQ =$ ar$\triangle PRQ = \frac{1}{2}(ABCD)$....(3)
frm (1)
$\triangle ABC = \triangle PBQ$
frm (2) (3)
$\frac{1}{2}(ABCD) = \frac{1}{2}(ABCD)$
Ar(ABCD) $=$ Ar(PBQR)