Algebraic approach to school Geometry

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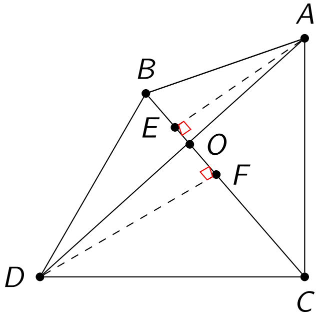
Problem Statement-Triangle Exercise

(i) ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that
$\frac{ar(ABC)}{ar(DBC)} = \frac{AO}{DO}$

Soln:

https://github.com/Rajolep/_Geometry/blob/master/figs/triexe.tex



DC=a=4 CA=b=6

D=(0,0)

C=(4,0)

O=$\frac{(A+D)}{2}$

B=(2O−C)=B(2.05,5.63)

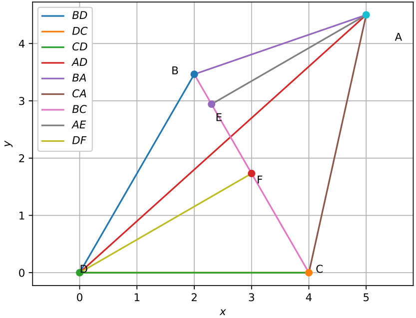
A(p,q)

p=$\frac{a^2+c^2-b^2}{2a}$ q=$\sqrt{c^2-p^2}$

A(6.05,5.63)

F=(3.57,1.23)

E=(2.47,4.40)



https://github.com/Rajolep/_Geometry/blob/master/codes/triangle/triangexer.py

$$AE \perp BC, DF \perp BC$$

$$Areaof \triangle ABC = \frac{1}{2}(BC)(AE)$$

$$Areaof \triangle DBC = \frac{1}{2}(BC)(DF)$$

$$\frac{ar\triangle ABC}{ar\triangle DBC} = \frac{\frac{1}{2}(BC)(AE)}{\frac{1}{2}(BC)(DF)}$$

$$\frac{ar\triangle ABC}{ar\triangle DBC} = \frac{AE}{DF}$$

$$\frac{AE}{DF} = \frac{AO}{DO}$$

$$\angle AEO = \angle DFO.....RA$$

$$\angle AEO = \angle DOF.....VOA$$

$$\triangle AOE \sim \triangle DOF$$

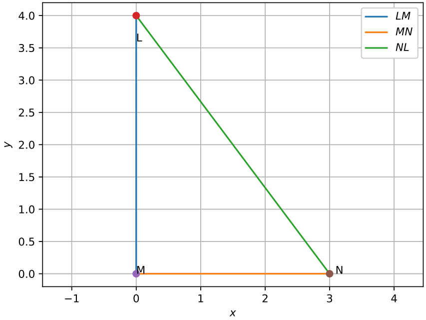
$$\frac{AE}{DF} = \frac{AO}{DO}$$

$$
\begin{aligned}
AE &= 2.407 \\
BC = e &= 9.10294 \\
DF &= 3.4756 \\
AO &= AE ...(1) \\
DO &= DF ...(2)
\end{aligned}
$$

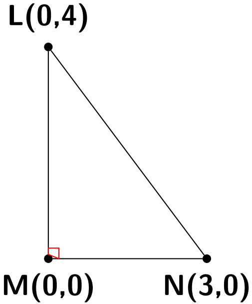
From (1) and (2)
Area of $\triangle ABC = \frac{1}{2}(BC)(AE) = 38.0739$
Area of$\triangle DBC = \frac{1}{2}(BC)(DF) = 38.0739$

Problem Statement-Triangle Construction

(i) Construct $\triangle LMN$ right angled at M such that LN $= 5$ MN $= 3$
Soln:
Given:- LN=5, MN=3, M(0,0) N(3,0) L(0,4)
https://github.com/Rajolep/_Geometry/blob/master/codes/triangle/draw_triangle.py



Applying Baudhayana's Theorem,
LN $= \sqrt{25-9}$
LN $= 5$



https://github.com/Rajolep/_Geometry/blob/master/figs/construc.tex

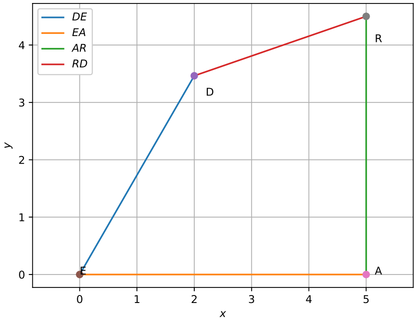
Problem Statement-Quadrilateral Construction

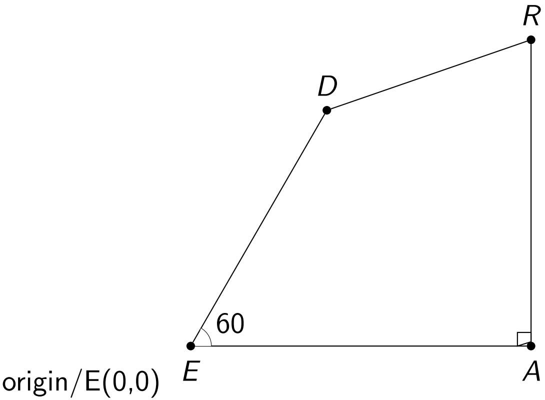
(i) Construct DEAR with DE $= 4$, EA $= 5$, AR $= 4.5$, $\angle E = 60^\circ$ and $\angle A = 90^\circ$.

Soln:

Given:- DE $= 4$, EA $= 5$, AR $= 4.5$, $\angle E = 60^\circ$ and $\angle A = 90^\circ$

https://github.com/Rajolep/_Geometry/blob/master/codes/Quad/drawquad.py





https://github.com/Rajolep/_Geometry/blob/master/figs/quadccon.tex

Construction steps:

E(0,0) A(4,0) R(4,5)

step1 : From given draw the line segments

step2 : draw a line segment/diagnol and calculate its value
p=$\sqrt{(EA)^2(AR)^2 - 2(EA)(AR)\cos 60}$
by using cosine formula,

step3 : coordinates of D(x,y)
x=$\frac{(c)^2+(b)^2-(p)^2}{2c}$ y=$\sqrt{(b)^2 - (x)^2}$
D(2,3.46)

Problem Statement-Circle Construction

(i) Draw a line segment AB of length 8 units. Taking A as centre, draw a circle of radius 4 units and taking B as centre, draw another circle of radius 3 units. Construct tangents to each circle from the centre of the other circle.

Soln:

Given:- AB=8, r1=4, r2=3, $A=\begin{pmatrix}0\\0\end{pmatrix}$ $B=\begin{pmatrix}8\\0\end{pmatrix}$

https://github.com/Rajolep/_Geometry/blob/master/codes/circle/circon.py

Construction steps:

step1 : Draw a tangent AP=$(AB)^2-(PB)^2$=7.416198

MB=$(AB)^2-(AM)^2$6.928203

M(p1,q1)

M(2 , 3.46410162)

r1=4 r2=3

a=$\sqrt{(8)^2-(r2)^2)}$

c=r2 b=8 d=r1

e=$\sqrt{(8)^2-(r1)^2)}$

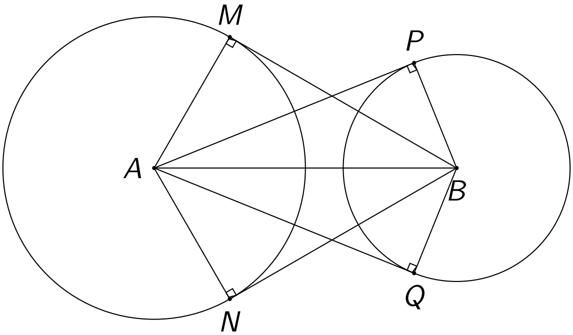
p1 $=\frac{(b)^2+(d)^2-(e)^2)}{(2b)}$

q1 $=\sqrt{(d)^2-(p1)^2}$

N(2 , -3.46410162)

P(6.875 , 2.78107443)

Q(6.875 , -2.78107443)



https://github.com/Rajolep/_Geometry/blob/master/figs/circon.tex

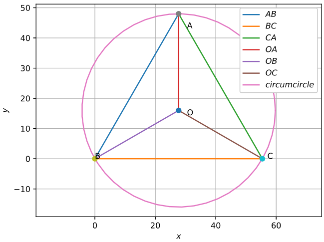
Problem Statement-Miscellenous

(i) In a circular table cover of radius 32 cm, a design is formed leaving an equilateral $\triangle ABC$ in the middle. Find the area of the design.

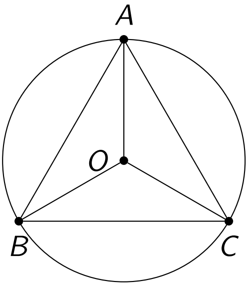
Soln:

Given: R=32cm

https://github.com/Rajolep/_Geometry/blob/master/codes/miscel.py



https://github.com/Rajolep/_Geometry/blob/master/figs/miscell.tex

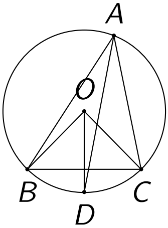


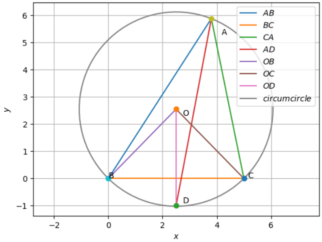
Problem Statement-Circle Exercise

(i) In any $\triangle ABC$, if the angle bisector of $\angle A$ and perpendicular bisector of BC intersect, prove that they intersect on the circumcircle of the $\triangle ABC$

Soln:

Given: https://github.com/Rajolep/_Geometry/blob/master/codes/circle/5ques.py




AB
BC
CA
AD
OB
OC
OD
circumcircle
A
B
C
D
O
x
y
−2
0
2
4
6
−1
1
3
5

a=5 b=6 c=7

R=3.57

O=(2.50,2.55)

$$\angle BOC = 2\angle BAC = 2\angle A ...(1)$$
$$OB = OC$$
$$\angle OEB = \angle OEC$$
$$\triangle BOE \cong \triangle COE ...(2)$$
$$\angle BOE + \angle COE = \angle BOC$$

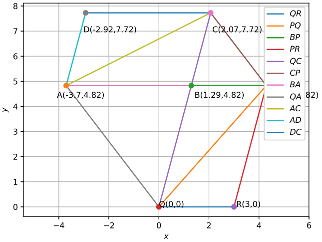
therefore,

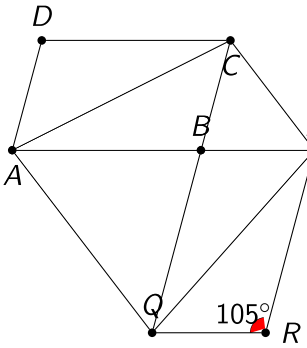
$$\angle BOE + \angle BOE = 2\angle A$$
$$\angle BOD = \angle BOE = \angle A$$
$$\angle BAD = \frac{\angle A}{2}$$
$$2\angle BAD = \angle A$$
$$\angle BOD = 2\angle BAD$$

Problem Statement-Quadrilateral Exercise

(i) The side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed. Show that ar(ABCD) = ar(PBQR). **Soln:**
Given:- CP $\parallel$ AQ

https://github.com/Rajolep/_Geometry/blob/master/codes/quadr/quadexer.py





https:
//github.com/Rajolep/_Geometry/blob/master/figs/quadexe.tex

Q=$\begin{pmatrix}0\\0\end{pmatrix}$ R=$\begin{pmatrix}3\\0\end{pmatrix}$ $\angle QRP = (105)^\circ$

QP=$\sqrt{a^2 + b^2 - 2ab\cos 105}$

P=$\begin{pmatrix}p\\q\end{pmatrix}$ a=3, b=5 c=QP

p=$\frac{a^2+c^2-b^2}{2a}$ q=$\sqrt{c^2 - p^2}$

P(4.29,4.82)

C=$\begin{pmatrix}p1\\q1\end{pmatrix}$ f=b+a

CP=g=$\sqrt{f^2 + c^2 - 2fc\cos 75}$

p1=$\frac{c^2+f^2-g^2}{2c}$ q1=$\sqrt{f^2 - p1^2}$

C(2.07,7.72)

$$CP \parallel AQ$$

area of $\triangle ACQ$ = area of $\triangle AQP$

subtract area$\triangle ABQ$ in above Eqn

ar$\triangle ACQ$-ar$\triangle ABQ$ = ar$\triangle APQ$-ar$\triangle ACQ$

ar$\triangle ABC$ = ar$\triangle PBQ$....(1)

$\triangle ABC \cong \triangle ADC$

ar$\triangle ABC$ = ar$\triangle ADC$

ar$\triangle ABC$ = ar$\triangle ADC$ = $\frac{1}{2}(ABCD)$...(2)

similarly in PBQR,
$\triangle PBQ \cong \triangle PRQ$
ar$\triangle PBQ =$ ar$\triangle PRQ$
ar$\triangle PBQ =$ ar$\triangle PRQ = \frac{1}{2}(ABCD)$....(3)
frm (1)
$\triangle ABC = \triangle PBQ$
frm (2) (3)
$\frac{1}{2}(ABCD) = \frac{1}{2}(ABCD)$
Ar(ABCD) $=$ Ar(PBQR)