Algebraic approach to school Geometry

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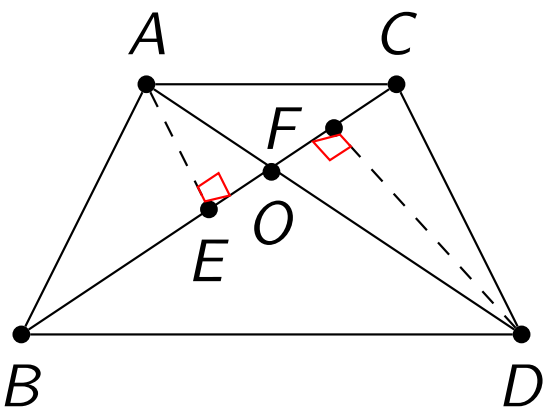
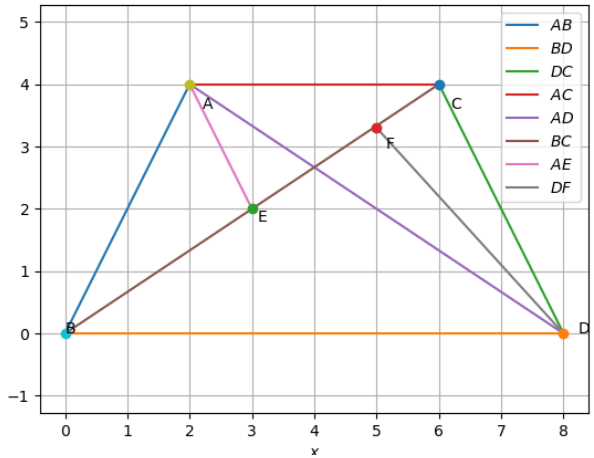
January 2, 2020

Problem Statement-Triangle Exercise

(i) ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that $\frac{ar(ABC)}{ar(DBC)} = \frac{AO}{DO}$

Soln:





$$AE \perp BC, DF \perp BC$$

$$Areaof \triangle ABC = \frac{1}{2} BC * AE$$

$$Areaof \triangle DBC = \frac{1}{2} BC * DF$$

$$\frac{ar\triangle ABC}{ar\triangle DBC} = \frac{\frac{1}{2} BC * AE}{\frac{1}{2} BC * DF}$$

$$\frac{ar\triangle ABC}{ar\triangle DBC} = \frac{AE}{DF}$$

$$\frac{AE}{DF} = \frac{AO}{DO}$$

$$\angle AEO = \angle DFO .....RA$$

$$\angle AEO = \angle DOF .....VOA$$

$$\triangle AOE \sim \triangle DOF$$

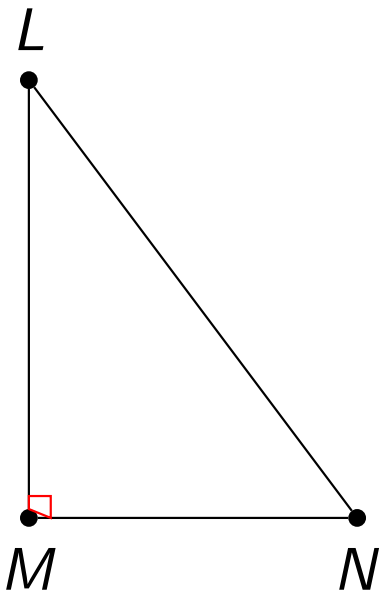
$$\frac{AE}{DF} = \frac{AO}{DO}$$

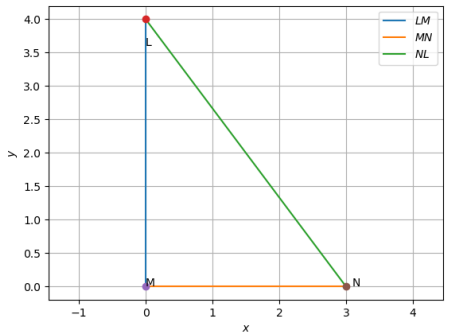
Problem Statement-Triangle Construction

(i) Construct $\triangle LMN$ right angled at M such that LN $= 5$ MN $= 3$

Soln:

https://github.com/Rajolep/_Geometry/blob/master/codes/triangle/draw_triangle.py https://github.com/Rajolep/_Geometry/blob/master/figs/construc.tex





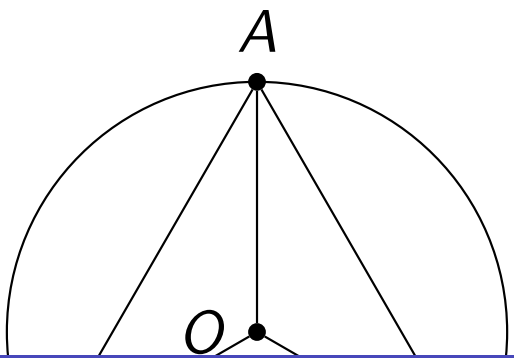
Problem Statement-Miscellenous

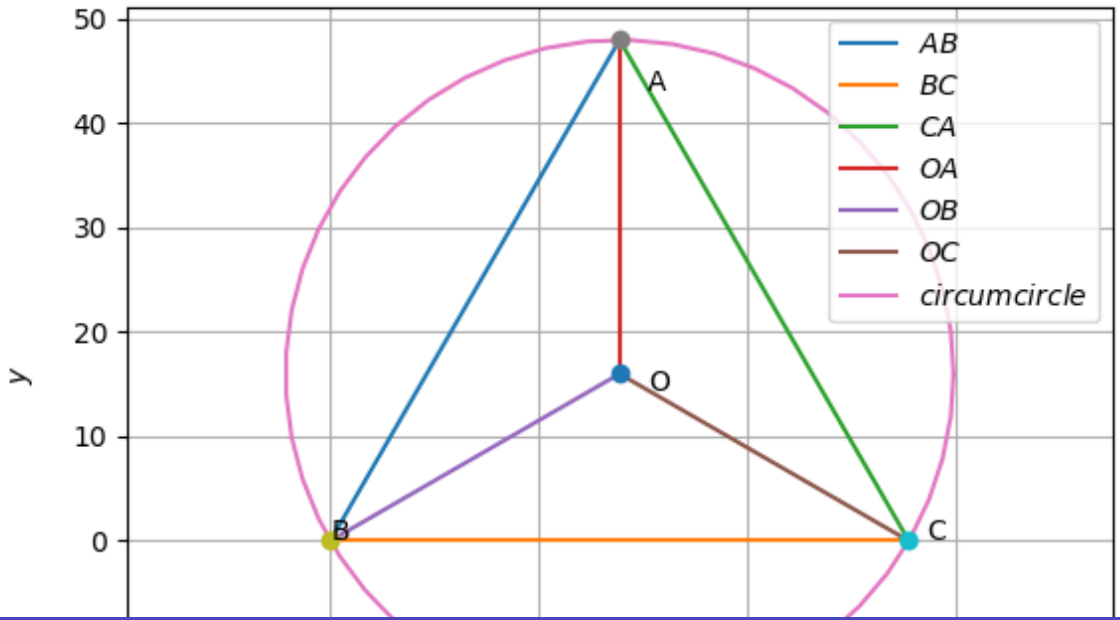
(i) In a circular table cover of radius 32 cm, a design is formed leaving an equilateral $\triangle ABC$ in the middle. Find the area of the design.

Soln:

Given: R=32cm

https://github.com/Rajolep/_Geometry/blob/master/codes/triangle/draw_triangle.py https://github.com/Rajolep/_Geometry/blob/master/figs/miscell.tex





$$\triangle BOC = 120^\circ$$

$$BO = OC = 32$$

$$BC = \sqrt{(BO)^2 + (OC)^2} - 2 * BO * OC * \cos(120) = 55.425$$

Area of design $= \pi * R * R$ - $\frac{\sqrt{3}4}{*}(BC) * 2$

Area $= 1886.81$

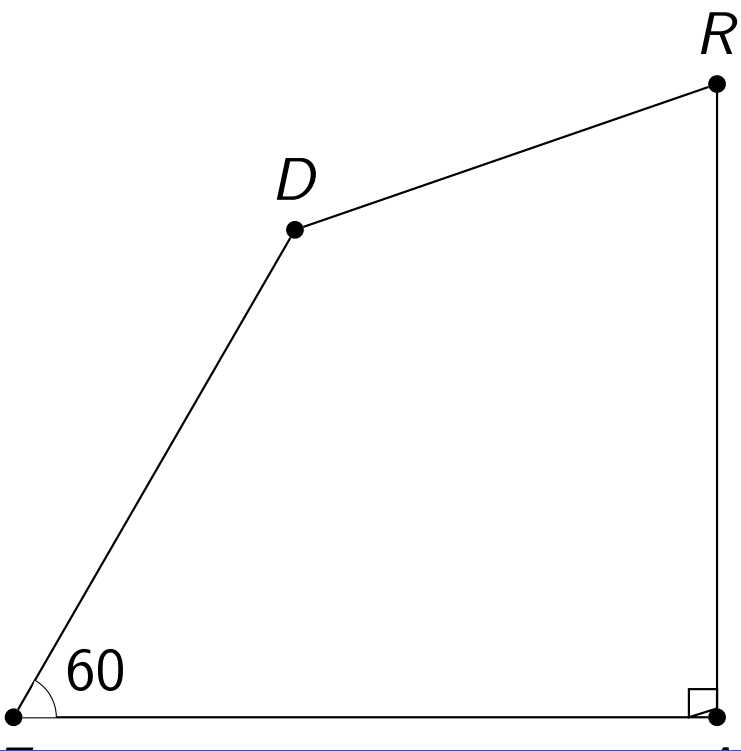
Problem Statement-Quadrilateral Construction

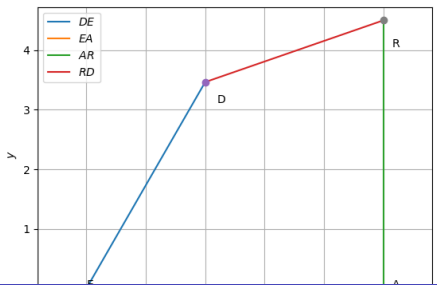
(i) Construct DEAR with DE $= 4$, EA $= 5$, AR $= 4.5$, $\angle E = 60^\circ$ and $\angle A = 90^\circ$.

Soln:

given:- DE $= 4$, EA $= 5$, AR $= 4.5$, $\angle E = 60^\circ$ and $\angle A = 90^\circ$

https://github.com/Rajolep/_Geometry/blob/master/codes/Quad/drawquad.py https://github.com/Rajolep/_Geometry/blob/master/figs/quadccon.tex

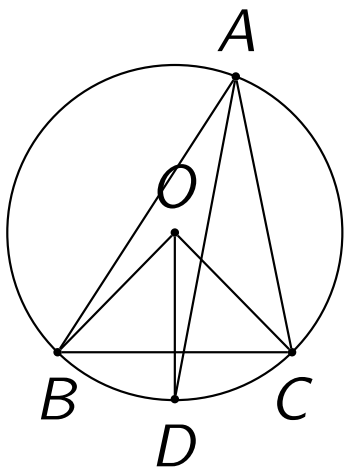


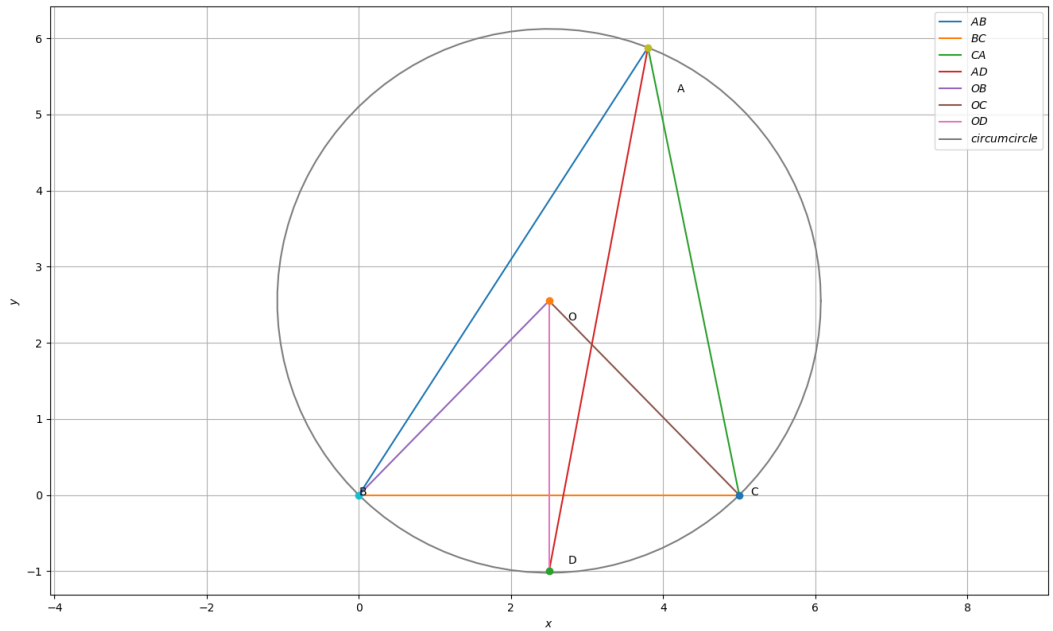


Problem Statement-Circle Exercise

(i) In any $\triangle ABC$, if the angle bisector of $\angle A$ and perpendicular bisector of BC intersect, prove that they intersect on the circumcircle of the $\triangle ABC$

Soln:





$$\angle BOC = 2\angle BAC = 2\angle A ...(1)$$
$$OB = OC$$
$$\angle OEB = \angle OEC$$
$$\triangle BOE \cong \triangle COE ...(2)$$
$$\angle BOE + \angle COE = \angle BOC$$

therefore,

$$\angle BOE + \angle BOE = 2\angle A \tag{0.1}$$
$$\angle BOD = \angle BOE = \angle A \tag{0.2}$$
$$\angle BAD = \frac{\angle A}{2} \tag{0.3}$$
$$2\angle BAD = \angle A \tag{0.4}$$
$$\angle BOD = 2\angle BAD \tag{0.5}$$
$$\tag{0.6}$$

Problem Statement-Circle Construction

(i) Draw a line segment AB of length 8 units. Taking A as centre, draw a circle of radius 4 units and taking B as centre, draw another circle of radius 3 units. Construct tangents to each circle from the centre of the other circle.
Soln:

https://github.com/Rajolep/_Geometry/blob/master/codes/circle/circon.py