

Algebraic approach to school Geometry

Priyanka

January 7, 2020

Problem Statement-Triangle Exercise

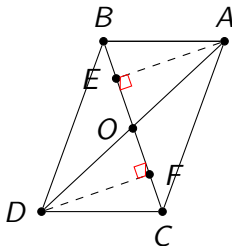
- (i) ABC and DBC are two triangles on the same base BC . If AD intersects BC at O , show that

$$\frac{ar(ABC)}{ar(DBC)} = \frac{AO}{DO}$$

Soln:

[https:](https://github.com/Rajolep/_Geometry/blob/master/figs/triexe.tex)

[//github.com/Rajolep/_Geometry/blob/master/figs/triexe.tex](https://github.com/Rajolep/_Geometry/blob/master/figs/triexe.tex)



$$DC=a=4 \quad CA=b=6$$

$$D=(0,0)$$

$$C=(4,0)$$

$$O=\frac{(A+D)}{2}$$

$$B=(2O-C)=B(2.05,5.63)$$

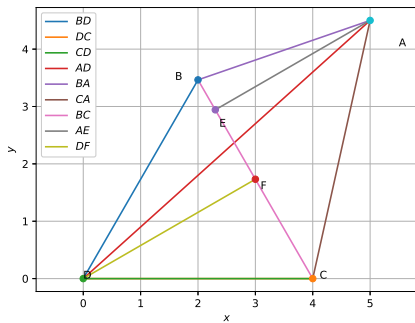
$$A(p,q)$$

$$p=\frac{a^2+c^2-b^2}{2a} \quad q=\sqrt{c^2-p^2}$$

$$A(6.05,5.63)$$

$$F=(3.57,1.23)$$

$$E=(2.47,4.40)$$



https://github.com/Rajolep/_Geometry/blob/master/codes/triangle/triangexer.py

$$AE \perp BC, DF \perp BC$$

$$\text{Area of } \triangle ABC = \frac{1}{2}(BC)(AE)$$

$$\text{Area of } \triangle DBC = \frac{1}{2}(BC)(DF)$$

$$\frac{\text{ar } \triangle ABC}{\text{ar } \triangle DBC} = \frac{\frac{1}{2}(BC)(AE)}{\frac{1}{2}(BC)(DF)}$$

$$\frac{\text{ar } \triangle ABC}{\text{ar } \triangle DBC} = \frac{AE}{DF}$$

$$\frac{AE}{DF} = \frac{AO}{DO}$$

$$\angle AEO = \angle DFO \dots \text{RA}$$

$$\angle AEO = \angle DOF \dots \text{VOA}$$

$$\triangle AOE \sim \triangle DOF$$

$$\frac{AE}{DF} = \frac{AO}{DO}$$

$$AE = 2.407$$

$$BC = e = 9.10294$$

$$DF = 3.4756$$

$$AO = AE...(1)$$

$$DO = DF...(2)$$

From (1) and (2)

$$\text{Area of } \triangle ABC = \frac{1}{2}(BC)(AE) = 38.0739$$

$$\text{Area of } \triangle DBC = \frac{1}{2}(BC)(DF) = 38.0739$$

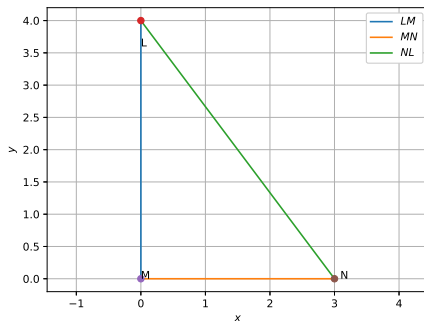
Problem Statement-Triangle Construction

- (i) Construct $\triangle LMN$ right angled at M such that $LN = 5$ $MN = 3$

Soln:

Given:- $LN=5$, $MN=3$, $M(0,0)$ $N(3,0)$ $L(0,4)$

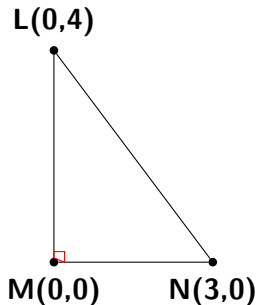
https://github.com/Rajolep/_Geometry/blob/master/codes/triangle/draw_triangle.py



Applying Baudhayana's Theorem,

$$LN = \sqrt{25 - 9}$$

$$LN = 5$$



[https:](https://github.com/Rajolep/_Geometry/blob/master/figs/construc.tex)

[//github.com/Rajolep/_Geometry/blob/master/figs/construc.tex](https://github.com/Rajolep/_Geometry/blob/master/figs/construc.tex)

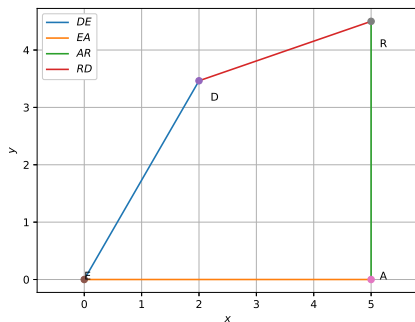
Problem Statement-Quadrilateral Construction

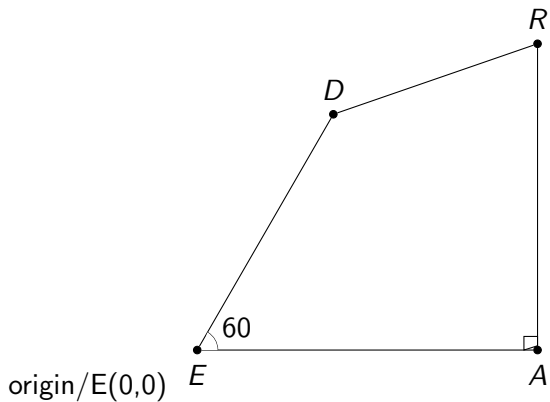
- (i) Construct DEAR with $DE = 4$, $EA = 5$, $AR = 4.5$, $\angle E = 60^\circ$ and $\angle A = 90^\circ$.

Soln:

Given:- $DE = 4$, $EA = 5$, $AR = 4.5$, $\angle E = 60^\circ$ and $\angle A = 90^\circ$

https://github.com/Rajolep/_Geometry/blob/master/codes/Quad/drawquad.py





https://github.com/Rajolep/_Geometry/blob/master/figs/quadccon.tex

Construction steps:

E(0,0) A(4,0) R(4,5)

step1 : From given draw the line segments

step2 : draw a line segment/diagonal and calculate its value

$$p = \sqrt{(EA)^2 + (AR)^2 - 2(EA)(AR) \cos 60}$$

by using cosine formula,

step3 : coordinates of D(x,y)

$$x = \frac{(c)^2 + (b)^2 - (p)^2}{2c} \quad y = \sqrt{(b)^2 - (x)^2}$$

D(2,3.46)

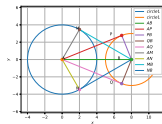
Problem Statement-Circle Construction

- (i) Draw a line segment AB of length 8 units. Taking A as centre, draw a circle of radius 4 units and taking B as centre, draw another circle of radius 3 units. Construct tangents to each circle from the centre of the other circle.

Soln:

Given:- $AB=8$, $r_1=4$, $r_2=3$, $A=\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $B=\begin{pmatrix} 8 \\ 0 \end{pmatrix}$

https://github.com/Rajolep/_Geometry/blob/master/codes/circle/circon.py



Construction steps:

step1 : Draw a tangent $AP = \sqrt{(AB)^2 - (PB)^2} = 7.416198$

$MB = \sqrt{(AB)^2 - (AM)^2} = 6.928203$

M(p1,q1)

M(2 , 3.46410162)

r1=4 r2=3

$a = \sqrt{(8)^2 - (r2)^2}$

c=r2 b=8 d=r1

$e = \sqrt{(8)^2 - (r1)^2}$

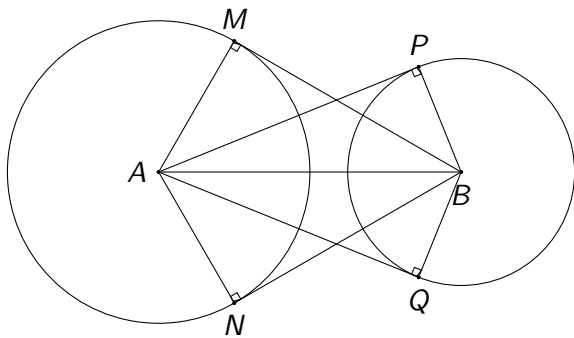
$p1 = \frac{(b)^2 + (d)^2 - (e)^2}{(2b)}$

$q1 = \sqrt{(d)^2 - (p1)^2}$

N(2 , -3.46410162)

P(6.875 , 2.78107443)

Q(6.875 , -2.78107443)



[https:](https://github.com/Rajolep/_Geometry/blob/master/figs/circon.tex)

[//github.com/Rajolep/_Geometry/blob/master/figs/circon.tex](https://github.com/Rajolep/_Geometry/blob/master/figs/circon.tex)

Problem Statement-Miscellaneous

- (i) In a circular table cover of radius 32 cm, a design is formed leaving an equilateral $\triangle ABC$ in the middle. Find the area of the design.

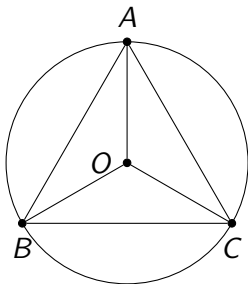
Soln:

Given: $R=32\text{cm}$

https://github.com/Rajolep/_Geometry/blob/master/codes/triangle/draw_triangle.py

[https:](https://github.com/Rajolep/_Geometry/blob/master/figs/miscell.tex)

[//github.com/Rajolep/_Geometry/blob/master/figs/miscell.tex](https://github.com/Rajolep/_Geometry/blob/master/figs/miscell.tex)



Problem Statement-Circle Exercise

- (i) In any $\triangle ABC$, if the angle bisector of $\angle A$ and perpendicular bisector of BC intersect, prove that they intersect on the circumcircle of the $\triangle ABC$

Soln:

Given: https://github.com/Rajolep/_Geometry/blob/master/codes/circle/5ques.py

$a=5$ $b=6$ $c=7$

$R=3.57$

$O=(2.50, 2.55)$

$$\angle BOC = 2\angle BAC = 2\angle A...(1)$$

$$OB = OC$$

$$\angle OEB = \angle OEC$$

$$\triangle BOE \cong \triangle COE...(2)$$

$$\angle BOE + \angle COE = \angle BOC$$

therefore,

$$\angle BOE + \angle BOE = 2\angle A$$

$$\angle BOD = \angle BOE = \angle A$$

$$\angle BAD = \frac{\angle A}{2}$$

$$2\angle BAD = \angle A$$

$$\angle BOD = 2\angle BAD$$

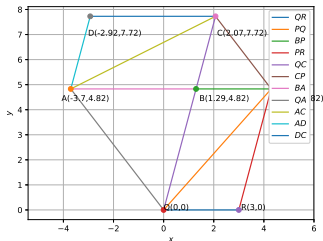
Problem Statement-Quadrilateral Exercise

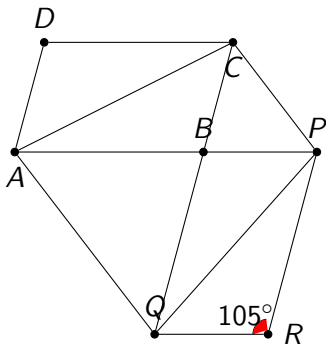
- (i) The side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed. Show that $\text{ar}(\text{ABCD}) = \text{ar}(\text{PBQR})$.

Soln:

Given:- $CP \parallel AQ$

https://github.com/Rajolep/_Geometry/blob/master/codes/quadr/quadexer.py





https://github.com/Rajolep/_Geometry/blob/master/figs/quadexe.tex

$$Q = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad R = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \quad \angle QRP = (105)^\circ$$

$$QP = \sqrt{a^2 + b^2 - 2ab \cos 105}$$

$$P = \begin{pmatrix} p \\ q \end{pmatrix} \quad a=3, \quad b=5 \quad c=QP$$

$$p = \frac{a^2 + c^2 - b^2}{2a} \quad q = \sqrt{c^2 - p^2}$$

$$P(4.29, 4.82)$$

$$C = \begin{pmatrix} p1 \\ q1 \end{pmatrix} \quad f = b + a$$

$$CP = g = \sqrt{f^2 + c^2 - 2fc \cos 75}$$

$$p1 = \frac{c^2 + f^2 - g^2}{2c} \quad q1 = \sqrt{f^2 - p1^2}$$

$$C(2.07, 7.72)$$

$$CP \parallel AQ$$

area of $\triangle ACQ$ = area of $\triangle AQP$

subtract area $\triangle ABQ$ in above Eqn

$$\text{ar}\triangle ACQ - \text{ar}\triangle ABQ = \text{ar}\triangle APQ - \text{ar}\triangle ACQ$$

$$\text{ar}\triangle ABC = \text{ar}\triangle PBQ \dots (1)$$

$$\triangle ABC \cong \triangle ADC$$

$$\text{ar}\triangle ABC = \text{ar}\triangle ADC$$

$$\text{ar}\triangle ABC = \text{ar}\triangle ADC = \frac{1}{2}(ABCD) \dots (2)$$

similarly in PBQR,

$$\triangle PBQ \cong \triangle PRQ$$

$$\text{ar}\triangle PBQ = \text{ar}\triangle PRQ$$

$$\text{ar}\triangle PBQ = \text{ar}\triangle PRQ = \frac{1}{2}(ABCD)....(3)$$

frm (1)

$$\triangle ABC = \triangle PBQ$$

frm (2) (3)

$$\frac{1}{2}(ABCD) = \frac{1}{2}(ABCD)$$

$$\text{Ar}(ABCD) = \text{Ar}(PBQR)$$