Very Important

1. Find the approximate Directivity of an Antenna whose power pattern $P(\theta,\phi) = 5\sin\theta$

Sol: we know that Directivity $D = \frac{4\pi}{\Omega_A}$

$$\Omega_A = \int_0^{\pi}\int_0^{2\pi} P(\theta,\phi)\, d\Omega$$

$$d\Omega = \sin\theta\, d\theta\, d\phi$$

Beam Area

$$\Omega_A = \int_0^{\pi}\int_0^{2\pi} 5\sin\theta\, d\Omega$$

$$\Omega_A = \int_0^{\pi}\int_0^{2\pi} 5\sin^2\theta\, d\theta\, d\phi$$

$$= 5\int_0^{\pi} \sin^2\theta\, d\theta \int_0^{2\pi} d\phi \qquad \sin^2\theta = \frac{1-\cos\theta}{2}$$

$$= 5\int_0^{\pi}\left[\frac{1-\cos\theta}{2}\right] d\theta \int_0^{2\pi} d\phi$$

$$= \frac{5}{2}\left[d\theta + \sin\theta\right]_0^{\pi} \left[2\pi - 0\right]$$

$$= \frac{5}{2}\left[\pi + 0 - 0\right]\left[2\pi\right]$$

$$= \frac{5}{\cancel{2}} \times \cancel{2}\pi^2$$

$$\boxed{\Omega_A = 5\pi^2}$$

Directivity $D = \frac{4\pi}{\Omega_A} = \frac{4\cancel{\pi}}{5\pi^{\cancel{2}}} = 0.25$

$$\boxed{D = 0.25}$$

(2.) Calculate the exact "D" of an Ant whose power pattern is given by $P(\theta,\phi) = P_m \sin^2\theta \sin^2\phi$. find Directivity.

Sol

$$D = \frac{4\pi}{\Omega_A}$$

$$\Omega_A = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} P(\theta,\phi)\sin\theta\, d\theta\, d\phi$$

$$= \int_0^{\pi}\int_0^{2\pi} P_m \sin^2\theta\, \sin\theta\, d\theta\, d\phi\, \sin^2\phi = \int_0^{\pi} P_m \int_0^{2\pi} \sin^3\theta\, d\theta\, d\phi\, \sin^2\phi$$

where $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

$$\sin^3\theta = \frac{3\sin\theta - \sin 3\theta}{4}$$

$$\Omega_A = P_m \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}\left[\frac{3\sin\theta - \sin 3\theta}{4}\right] d\theta\, d\phi\, \sin^2\phi$$

Also $\sin^2\phi = \frac{1-\cos\phi}{2}$

$$\Omega_A = \frac{P_m}{4}\int_0^{\pi}(3\sin\theta - \sin 3\theta)\, d\theta \int_0^{2\pi}\frac{(1-\cos\phi)}{2}\, d\phi$$

$$\Omega_A = \frac{P_m}{4}\left[-3\cos\theta - \left(-\frac{\cos 3\theta}{3}\right)\right]_0^{\pi}\frac{1}{2}\left[d\phi + \sin\phi\right]_0^{2\pi}$$

$$= \frac{P_m}{4}\left[3 - \frac{1}{3}\right]\frac{1}{2}\left[2\pi + 0 - 0\right]$$

$$\Omega_A = \frac{P_m}{4}\left[\frac{5}{3}\right]\left(\frac{1}{2}\right)\left[2\pi\right] = \frac{4}{3}\pi P_m$$

$$D = \frac{4\pi}{\Omega_A} = \frac{4\pi}{\frac{4}{3}\pi P_m} = 1/\frac{P_m}{3} \qquad \boxed{\therefore D = \frac{3}{P_m}}$$

③ find the value of Radiation resistance of a dipol having length equal to "λ"

Sol for a dipole Radiation Resistance is

$$R_r = 80\pi^2 \left(\frac{dL}{\lambda}\right)^2$$

given length $dL = \lambda$

$$\therefore R_r = 80\pi^2 \left(\frac{\cancel{\lambda}}{\cancel{\lambda}}\right)^2 = 80\pi^2$$

$$\boxed{\therefore R_r = 789.56\,\Omega}$$

④ A dipole Antenna is radiating 1kW of Power with a gain of 2.15dB. Find the i/p power of the isotropic Antenna.

Sol for isotropic Antenna

$$U_0 = \frac{P_{rad}}{4\pi} = \frac{1000}{4\pi} = 79.5\ W/Sr$$

$$G = 2.15dB \Rightarrow 10 \log G = 2.15$$

$$G = (10)^{0.215} = 1.64$$

$$P_{rad} = 1000W$$

$$Gain = \frac{4\pi U_0}{P_{in}} =$$

$$1.64 = \frac{4\pi(79.5)}{P_{in}}$$

$$P_{in} = \frac{999}{1.64} = 609.1\ watt$$

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(5) An Antenna has a loss Resistance of 8 Ω and a power gain of 12dB, exhibiting resistance of 72 Ω find the Antenna efficiency and Directivity ?

Sol :-

Given $G = 12dB$ $R_L = 8\Omega$ $R_r = 72\Omega$

$\eta = ?$ $D = ?$

W.K.T. $G = \eta D$

~~12dB × ...~~

$$\eta = \frac{R_r}{R_r + R_L} = \frac{72}{80} = 0.9$$

$D = ?$

$G = 12dB \Rightarrow (Gain)_{dB} = 10 \log G$

$$\log G = \frac{(G)_{dB}}{10} = \frac{12}{10} = 1.2$$

$$G = (10)^{1.2} = 15.84$$

$G = \eta D$

$15.84 = 0.9 D$

$D = 17.6$

(D) in dB $= 10 \log D = 10 \log (17.6)$

$$\boxed{(\text{Directivity})_{dB} = 12.45 dB}$$

(6) Find the directivity of an Antenna if it radiates on one plane is 30° & other 45°

Sol

Given $\theta_{HP} = 30°$ $\phi_{HP} = 45°$

$$D = \frac{41253}{\theta_{HP} * \phi_{HP}} = \frac{41253}{30 \times 45} = 30.55$$

(D) in dB $= 10 \log (30.55) = 14.85 dB$

⑦ Calculate Maximum Effective Area of an Antenna having directivity of 900.

Sol

$$D = \frac{4\pi A_e}{\lambda^2}$$

$$A_e = \frac{D\lambda^2}{4\pi} = \frac{900}{4\pi}\lambda^2$$

$$\boxed{A_e = 71.6\lambda^2 \text{ meter}^2}$$

① An antenna has a radiation resistance of 72Ω, a loss resistance of 8Ω and a power gain of 12db. Determine the antenna efficiency and its directivity.

Sol:- Given $R_r = 72\Omega$

$R_L = 8\Omega$.

$G_p = 12\,db$

$\eta = ?$

$D = ?$

$$\eta = \left(\frac{R_r}{R_r + R_L}\right) \times 100 = 0.90 \times 100 = 90\%$$

$$\eta = \frac{G_p}{G_d}, \quad (G_p)_{dB} = 10\log_{10} G_p$$

$$12 = 10\log_{10} G_p$$

$$G_p = \text{Antilog}(1.2)$$

$$= 15.84.$$

$$\therefore \eta = \frac{15.84}{G_d}, \quad 0.9 = \frac{15.84}{G_d}$$

$$G_d = 17.611$$

$$(G_d)_{dB} = 10\log_{10} 17.611 = 12.458.$$

2) In a microwave communication link, two identical antenna operating at 10GHz are used with power gain of 40dB. If the transmitter power is 1W, find the received power, if the range of the link is 30km.

Sol:- Two identical antenna's so transmitter gain and receiver gain $G_t = G_r = 40dB$

$$(G_t)_{dB} = 10\log_{10} G_t$$

$$40 = 10\log_{10} G_t, \quad G_t = \text{Antilog}(4) = 10^4.$$

$$\frac{W_r}{W_t} = G_t \cdot G_r \left(\frac{\lambda}{4\pi r}\right)^2 \quad , \lambda = \frac{c}{f} = \frac{3\times10^8}{10\times10^9} = 0.03\,\text{mts}$$

$$W_r = \frac{1\times(10^4)^2 \times (0.03)^2}{(4\pi)^2\times(30\times10^3)^2}$$

$$= 0.634\,\mu W.$$

3) Find total radiated power for the given $U = A_0 \sin\theta$

Sol:-

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi} U \sin\theta\, d\theta\, d\phi$$

$$= \int_0^{2\pi}\int_0^{\pi} A_0 \sin\theta \sin\theta\, d\theta\, d\phi$$

$$= \int_0^{2\pi}\int_0^{\pi} A_0 \sin^2\theta\, d\theta\, d\phi.$$

$$= A_0 \int_0^{2\pi}\int_0^{\pi} \left(\frac{1-\cos 2\theta}{2}\right) d\theta\, d\phi.$$

$$= \frac{A_0}{2}\int_0^{2\pi} \theta\Big]_0^{\pi} - \left[\frac{\sin 2\theta}{2}\right]_0^{\pi} d\phi$$

$$= \frac{A_0}{2}\times\int_0^{2\pi} \left[(\pi) - \frac{1}{2}\left[\sin 2\pi - \sin 0\right]\right] d\phi$$

$$= \frac{A_0}{2}\int_0^{2\pi} [\pi - 0]\, d\phi$$

$$= \frac{A_0}{2}\times\pi\left[\phi\right]_0^{2\pi}$$

$$= \frac{A_0}{\cancel{2}}\times\pi\times\cancel{2}\pi$$

$$P_{rad} = A_0\pi^2$$

Problems:-

1. Calculate the cutoff frequency and cutoff wavelength of R.W.G with TE_{10} mode having dimensions 3×2 cm
$a = 3$, $b = 2$

A:- Cutoff frequency $(f_c) = \frac{c}{2a} = \frac{c}{2 \times 3} = \frac{c}{6}$

$$= \frac{3 \times 10^{10}}{\cancel{6}_5} = 0.5 \times 10^{10}$$

where $C = 3 \times 10^{10}$

$$= 5 \times 10^{9} = 5\,GHz$$

WKT $f = \frac{c}{\lambda}$

$$\therefore \lambda_c = \frac{c}{f_c} = \frac{\overset{0.6}{\cancel{3}} \times 10^{10}}{\cancel{5} \times 10^{9}} = 0.6 \times 10$$

$$= 6\,cm$$

2. The cutoff frequency of a Rwg in the dominent mode is 10GHz. find the width of wave guide.

A:- $TE_{10} = \frac{c}{2a}$

$$10 \times 10^{9} = \frac{\overset{1.5}{\cancel{3}} \times 10^{10}}{\cancel{2}a}$$

$a = 1.5$cm

3. Consider a air field rectangular w.g width dimensions a = 2.286 cm , b = 1.016 cm give the Increasing order of the cutoff frequency for different modes. TE_{01}, TE_{10}, TE_{11}, TE_{20}

A:- TE

$$\therefore f_c = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$$

$TE_{01} \Rightarrow m=0 \quad n=0 \quad a=2.286 \quad b=1.016$

$$f_c = \frac{3\times10^{10}}{2b} \text{ i.e } \frac{3\times10^{10}}{2\times1.016} = 14.7\,GHz$$

$TE_{10} \Rightarrow m=1 \quad n=0 \quad a=2.286 \quad b=1.016$

$$f_c = \frac{3\times10^{10}}{2a} \text{ i.e, } = \frac{3\times10^{8}}{2\times2.286} = 6.56\,HZ$$

$TE_{11} \Rightarrow m=1, n=1 \quad a=2.286 \quad b=1.016$

$$f_c = \frac{c\sqrt{a^2+b^2}}{2ab} = \frac{3\times10^{10}\sqrt{(2.286)^2+(1.016)^2}}{2\times2.286\times1.016}$$

$$f_c = \frac{3\times10^{10}\sqrt{5.22+1.0321}}{4.645}$$

$$f_c = \frac{3\times10^{10}\sqrt{6.2521}}{4.645}$$

$$f_c = \frac{3\times10^{10}\times2.5}{4.645}$$

$$f_c = 16.14\,GHz$$

TE_{20} $$f_c = \frac{c}{a} = \frac{3\times10^{10}}{2.286} = 1.312\times10^{10}$$

$$= 13.1\,GHz$$

$\therefore TE_{10} < TE_{20} < TE_{01} < TE_{11}$

4. An air field Raw of dimensions i.e $7\times3.5\ cm^2$ operates in the dominent mode of the waveguide the value of phase velocity at a frequency of $3.5\,GHz$

TE_{10} $$V_p = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$f_c = \frac{c}{2a}$$

$c = 3\times10^{10}$

$a = 7$

$b = 3.5$

$f = 3.5\,GHz$

$$f_c = \frac{3\times10^{10}}{14}$$

$$f_c = 0.214\times10^{10}$$

$$\boxed{f_c = 2.14\,GHz}$$

$$V_p = \frac{3 \times 10^{10}}{\sqrt{1 - \left(\frac{2.14 \times \cancel{10^9}}{3.5 \times \cancel{10^9}}\right)^2}}$$

$$= \frac{3 \times 10^{10}}{\sqrt{1 - 0.373}} = \frac{3 \times 10^{10}}{\sqrt{0.627}}$$

$$V_p = \frac{3 \times 10^{10}}{0.791} = 37.92\ cm/sec^2$$