Celestial Holography : An Attempt to Understand Quantum Gravity in Asymptotically Flat Spacetime

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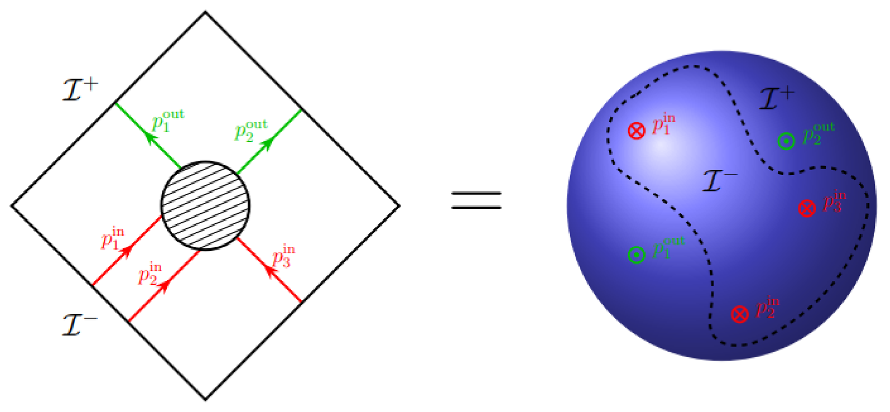
Introduction to Celestial Holography

Quantum Gravity in (3+1)D AFS $\equiv$ 2D Celestial CFT on CS^2 at Null Infinity



$$\langle out|\, S\, |in\rangle \qquad\qquad \langle \mathcal{O}^{\pm}_{\Delta_1}(z_1,\bar{z}_1)\ldots\mathcal{O}^{\pm}_{\Delta_n}(z_n,\bar{z}_n)\rangle \tag{1}$$

[Image courtesy: Strominger]

The Holographic Map

Lorentz transformations | Global conformal transformations

$$x^\mu \longrightarrow \Lambda^\mu{}_\nu x^\nu \qquad\qquad z \longrightarrow \frac{az+b}{cz+d}, \quad \bar{z} \longrightarrow \frac{\bar{a}\bar{z}+\bar{b}}{\bar{c}\bar{z}+\bar{d}}$$

states : $|\vec{p},\sigma\rangle = |\omega,\sigma,z,\bar{z}\rangle \xrightarrow[\text{Mellin transform}]{\int_0^\infty d\omega\omega^{\Delta-1}|\omega,\sigma,z,\bar{z}\rangle} |\Delta,\sigma,z,\bar{z}\rangle$

Momentum eigenstate | Boost eigenstate

Observable :

$$\langle out|\mathcal{S}|in\rangle \xrightarrow[\text{Mellin transform}]{\int_0^\infty \prod_{i=1}^n d\omega_i\omega_i^{\Delta-1}\langle out|\mathcal{S}|in\rangle} \langle \mathcal{O}_{\Delta_1}(z_1,\bar{z}_1)\ldots\mathcal{O}_{\Delta_N}(z_N,\bar{z}_N)\rangle \tag{2}$$

Scattering amplitude | Celestial amplitude

symmetry : Translation invariance manifest | conformal invariance manifest

[Review on Celestial Holography: Pasterski'21, Raclariu'21, Puhm'21]

Elements of Celestial CFT

Gluon primary :

$$\mathcal{O}^{a,\epsilon}_{h,\bar{h}}(z,\bar{z}) = \int_0^\infty d\omega\,\omega^{\Delta-1}A^a(\epsilon\omega,z,\bar{z},\sigma) \tag{3}$$

Correlation function : 3-point MHV gluon scattering amplitude

$$\boxed{\tilde{\mathcal{A}}(1^{-a_1},2^{+a_2},4^{-x}) \sim f^{a_1a_2x}\frac{z_{14}^3}{z_{12}^3 z_{24}}\delta(\bar{z}_{14})\delta(\bar{z}_{24})\prod_{i=1}^{3}\Theta(\epsilon_i\sigma_{i,1})} \tag{4}$$

Under $SL(2,\mathbb{C})/\mathbb{Z}_2$ or Lorentz transformations

$$\tilde{\mathcal{A}}_n(\{z_i,\bar{z}_i,h_i,\bar{h}_i,a_i\}) = \prod_{i=1}^n \frac{1}{(cz_i+d)^{2h_i}}\frac{1}{(\bar{c}\bar{z}_i+\bar{d})^{2\bar{h}_i}}\tilde{\mathcal{A}}_n\left(\left\{\frac{az_i+b}{cz_i+d},\frac{\bar{a}\bar{z}_i+\bar{b}}{\bar{c}\bar{z}_i+\bar{d}},h_i,\bar{h}_i,a_i\right\}\right). \tag{5}$$

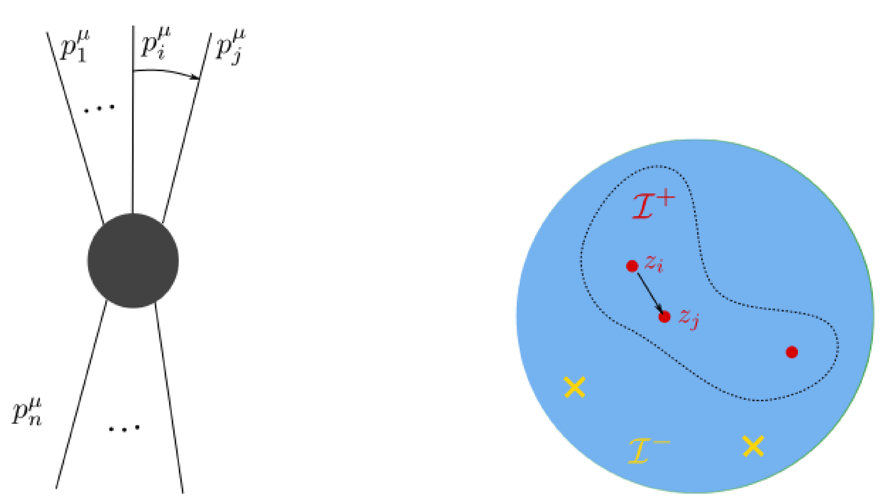
$\mathcal{O}(1)$ term in the OPE between two gluon primaries

$$\mathcal{O}^{a_4,+1}_{\Delta_4,+}(z_4,\bar{z}_4)\mathcal{O}^{a_5,+1}_{\Delta_5,+}(z_5,\bar{z}_5)\Big|_{\mathcal{O}(1)} \sim \frac{1}{2}\times B(\Delta_4-1,\Delta_5-1)\Bigg[-\frac{(\Delta_4-1)}{(\Delta_4+\Delta_5-2)}if^{xa_4a_5}L_{-1}+ \left(\frac{(\Delta_5-1)}{(\Delta_4+\Delta_5-2)}\delta^{a_4y}\delta^{a_5x}+\frac{(\Delta_4-1)}{(\Delta_4+\Delta_5-2)}\delta^{a_5y}\delta^{a_4x}\right)R^{1,y}_{-1,0}\Bigg]\mathcal{O}^{x,\epsilon_5}_{\Delta_4+\Delta_5-1}(z_5,\bar{z}_5). \tag{6}$$

How do we find the Celestial OPE ? From collinear factorization of the celestial amplitude

Celestial OPE from Collinear Limit

Celestial amplitudes get factorized upon taking collinear limit.



[image: Andrea Puhm]

$$\mathcal{A}(1^{-a_1},2^{+a_2},3^{+a_3},4^{-a_4}) \xrightarrow[p_3\cdot p_4=0]{z_{34}\to 0} -\frac{f^{a_3a_4x}}{z_{34}}B(\Delta_3-1,\Delta_4+1)\mathcal{A}(1^{-a_1},2^{+a_2},4^{-x}) + \text{subleading in}\, z_{34}+\ldots \tag{7}$$

OPE at leading order :

$$\mathcal{O}^{a_3}_{\Delta_3,+}(z_3,\bar{z}_3)\mathcal{O}^{a_3}_{\Delta_4,-}(z_4,\bar{z}_4) \sim -\frac{f^{a_3a_4x}}{z_{34}}B(\Delta_3-1,\Delta_4+1)\mathcal{O}^x_{\Delta_3+\Delta_4-1,-}(z_4,\bar{z}_4) \tag{8}$$

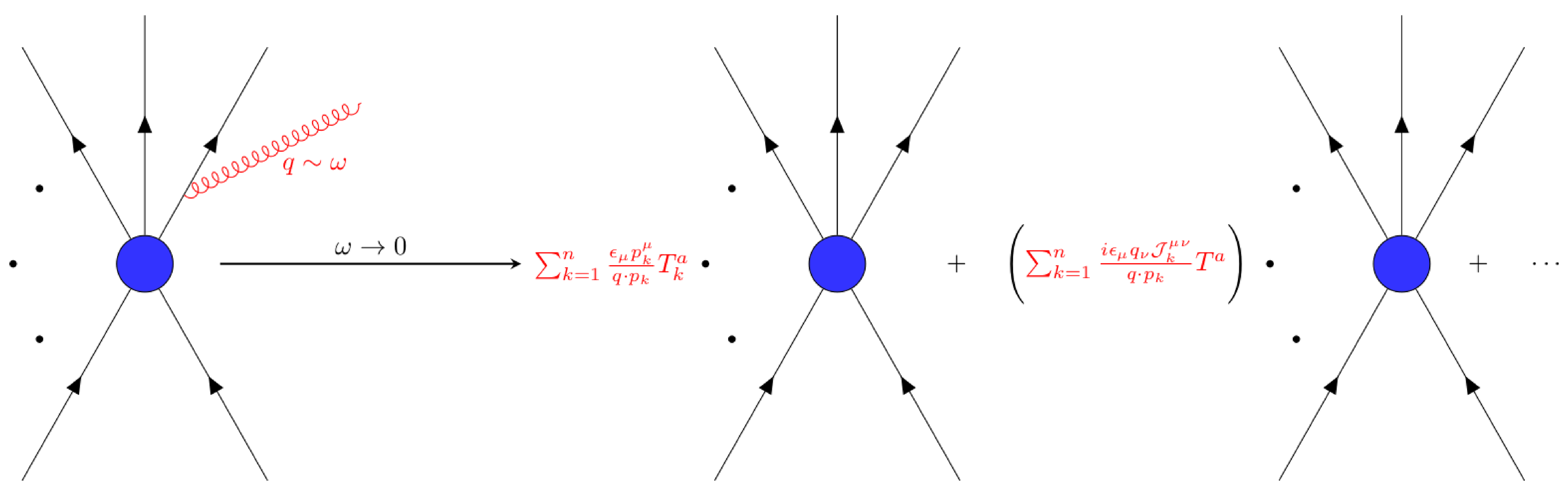
More generally,

$$\mathcal{O}^{a,+}_{\Delta_1}(z_1,\bar{z}_1)\mathcal{O}^{b,+}_{\Delta_2}(z_2,\bar{z}_2) \sim -\frac{if^{ab}{}_c}{z_{12}}\sum_{n=0}^\infty B(\Delta_1+n-1,\Delta_2-1)\frac{\bar{z}_{12}^n}{n!}\bar{\partial}_2^n\mathcal{O}^{c,+}_{\Delta_1+\Delta_2-1}(z_2,\bar{z}_2). \tag{9}$$

(Guevara, Himwich, Pate, Strominger '21)

Conformally Soft Theorems

CONFORMALLY SOFT THEOREMS $\equiv$ WARD IDENTITIES FOR ASYMPTOTIC SYMMETRIES



Leading Conformally Soft Theorem $\Longrightarrow$ level-zero Kac-Moody Algebra

$$\left\langle R^{1,a}(z)\prod_{i=1}^n\mathcal{O}^{a_i}_{h_i,\bar{h}_i}(z_i,\bar{z}_i)\right\rangle = -\sum_{k=1}^n\frac{T_k^a}{z-z_k}\left\langle\prod_{i=1}^n\mathcal{O}^{a_i}_{h_i,\bar{h}_i}(z_i,\bar{z}_i)\right\rangle \tag{10}$$

where leading conformally soft gluon operator is defined as,

$$R^{1,a}(z) = \lim_{\Delta\to 1}(\Delta-1)\mathcal{O}^a_{\Delta,+}(z,\bar{z}) \tag{11}$$

Sub-leading Conformally Soft Theorem $\Longrightarrow$ Current Algebra

$$\left\langle R^{0,a}(z,\bar{z})\prod_{i=1}^n\mathcal{O}^{a_i}_{h_i,\bar{h}_i}(z_i,\bar{z}_i)\right\rangle = -\sum_{i=1}^n\frac{\epsilon_k}{z-z_k}(-2\bar{h}_k+1+(\bar{z}-\bar{z}_k)\bar{\partial}_k)T_k^aP_k^{-1}\left\langle\prod_{i=1}^n\mathcal{O}^{a_i}_{h_i,\bar{h}_i}(z_i,\bar{z}_i)\right\rangle \tag{12}$$

where sub-leading conformally soft gluon operator is defined as,

$$R^{0,a}(z,\bar{z}) = \lim_{\Delta\to 0}\Delta\mathcal{O}^a_{\Delta,+}(z,\bar{z}) \tag{13}$$

Holographic Symmetry Algebras

gluon-gluon OPE (9) $\xrightarrow[\text{mode expansions}]{\text{soft limits}}$ S algebra (Strominger and collaborators '21)

$$[S^{p,a}_{\alpha,m},S^{q,b}_{\beta,n}] = -if^{abc}S^{p+q-1,c}_{\alpha+\beta,m+n} \tag{14}$$

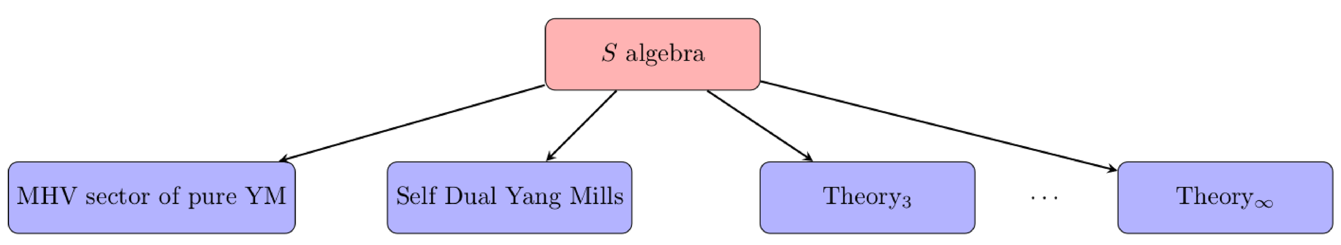
graviton-graviton OPE $\xrightarrow[\text{mode expansions}]{\text{soft limits}}$ (wedge) $w_{1+\infty}$ algebra

$$[w^p_{\alpha,m},w^q_{\beta,n}] = [m(q-1)-n(p-1)]w^{p+q-2}_{\alpha+\beta,m+n} \tag{15}$$

Null states and classification of S and w-invariant theories

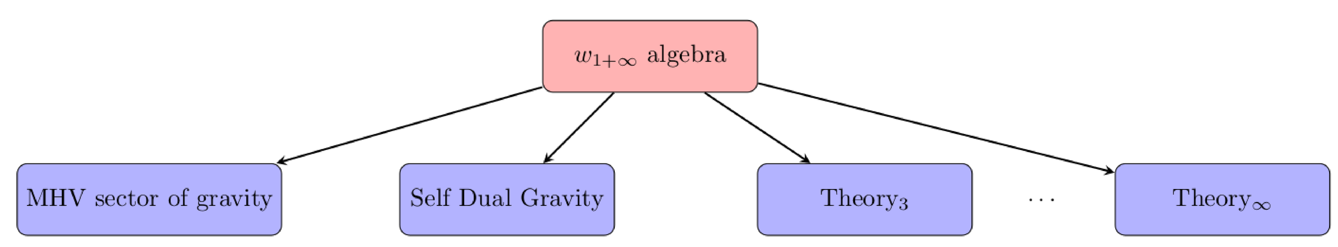
Gauge theories :

$$\boxed{\mathcal{O}^b_{\Delta_1,+}(z,\bar{z})\mathcal{O}^c_{\Delta_2,+}(0,0)|_{\mathcal{O}(1)} = \mathcal{O}^b_{\Delta_1,+}(z,\bar{z})\mathcal{O}^c_{\Delta_2,+}(0,0)\Big|^{MHV}_{\mathcal{O}(1)} + \sum_{k=1}^n B(\Delta_1+k,\Delta_2-1)M_k^{bc}(\Delta_1+\Delta_2)} \tag{16}$$



(Banerjee,Paul,Panda,Misra,RM'23)

Gravity :



(Banerjee,Kulkarni and Paul '23)

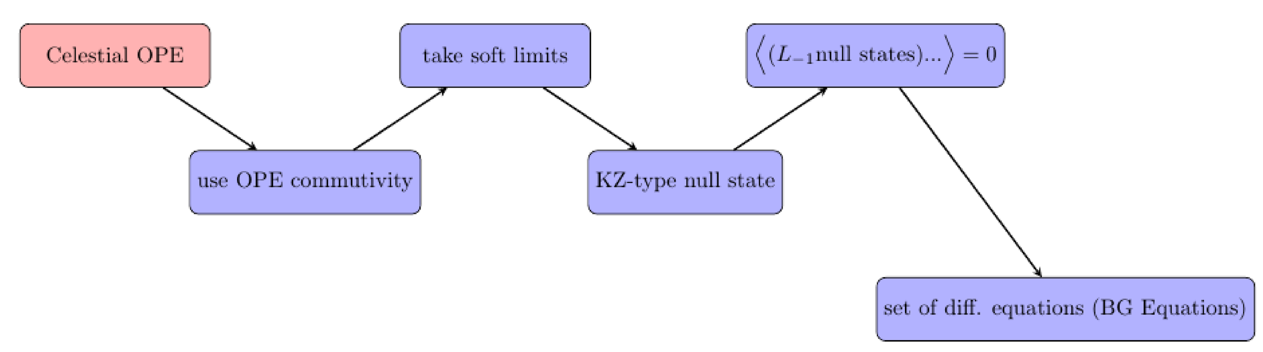
KZ-type Null states and the BG(Banerjee-Ghosh) equations

Gluons :

$$K^a(\Delta) = \xi^a(\Delta) - i\sum_{k=1}^n M_k^a(\Delta+1), \tag{17}$$

where

$$\xi^a(\Delta) = C_AL_{-1}\mathcal{O}^{a,+}_\Delta - (\Delta+1)R^{1,b}_{-1,0}R^{1,b}_{0,0}\mathcal{O}^{a,+}_\Delta - R^{0,b}_{-\frac{1}{2},\frac{1}{2}}R^{1,b}_{0,0}\mathcal{O}^{a,+}_{\Delta+1}. \tag{18}$$



References

1. S. Banerjee, R. Mandal, S. Misra, S. Panda and P. Paul, "All OPEs invariant under the infinite symmetry algebra for gluons on the celestial sphere," Phys. Rev. D 110 (2024) no.2, 026020

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