MHV Gluon Scattering in the Massive Scalar Background and Celestial OPE

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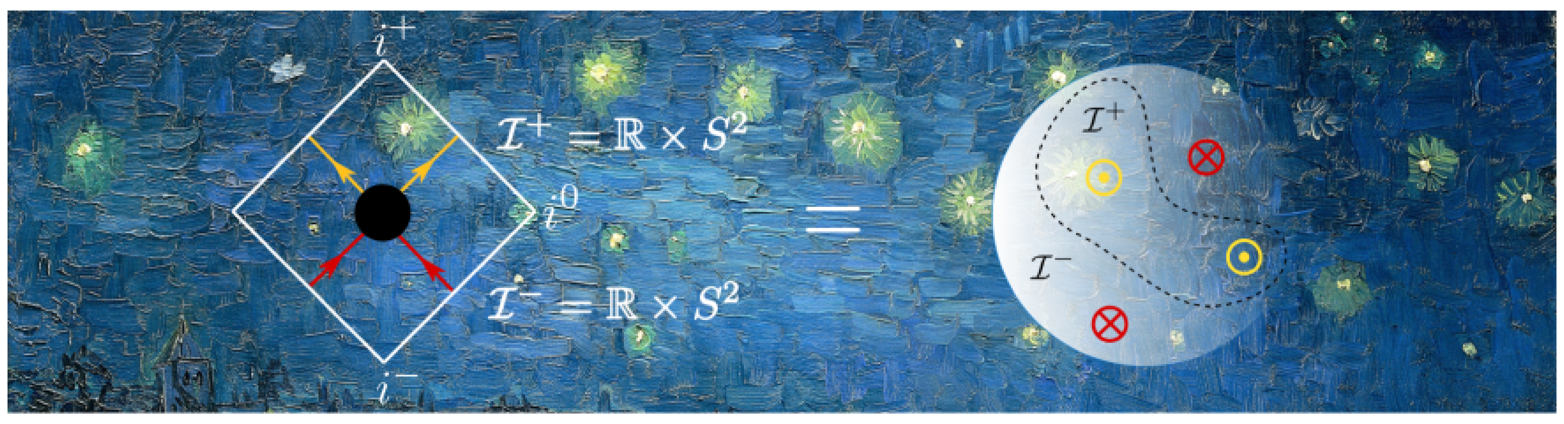
Introduction to Celestial Holography

Conjecture :

Quantum theories of Gravity in (3+1)-D AFS $\Longleftrightarrow$ 2D CCFTs on the Celestial sphere at null infinity.



[Picture Courtesy: Andrea Puhm]

Asymptotic Symmetries of Gravity and Gauge Theories in AFS

Gravity

- '60s : Poincaré$\Longrightarrow$ BMS = Lorentz $\times$ Supertranslations (Bondi,Burg,Metzner and Sachs)
- '10s : BMS $\longrightarrow$ Virasoro $\ltimes$ Supertranslations Extended BMS
  (using asymptotic symmetry analysis and soft theorems.)
  Lorentz group $\longrightarrow$ Virasoro Kapec,Mitra,Strominger,...
  Another extension :
  Lorentz group $\longrightarrow$ $\overline{SL(2,\mathbb{C})}$ current algebra Banerjee,Ghosh,Paul '20
  $\downarrow$ Guevara-Himwich-Pate-Strominger '21
  wedge subalgebra of $w_{1+\infty}$ algebra

Gauge Theories

- Asymptotic symmetry algbera is HSA $\times$Poincaré. Guevara-Himwich-Pate-Strominger '21

Mellin Transform and the Celestial Amplitude

Scattering amplitudes in boost eigenbasis.

$$\tilde{\mathcal{A}}_n(\{z_i,\bar{z}_i,h_i,\bar{h}_i,a_i\}) = \langle\prod_{i=1}^{n}\mathcal{O}^{a_i}_{h_i,\bar{h}_i}(z_i,\bar{z}_i)\rangle = \prod_{i=1}^{n}\int_0^\infty d\omega_i\omega_i^{\Delta_i-1}\mathcal{S}_n(\{\omega_i,z_i,\bar{z}_i,\sigma_i,a_i\}) \quad (1)$$

Example : [S. Banerjee and S. Ghosh ; arXiv:2011.00017]

$$\boxed{\tilde{\mathcal{A}}(1^{-a_1},2^{+a_2},4^{-x}) \sim f^{a_1a_2x}\frac{z_{14}^3}{z_{12}^3z_{24}}\underbrace{\delta(\bar{z}_{14})\delta(\bar{z}_{24})}\prod_{i=1}^{3}\Theta(\epsilon_i\sigma_{i,1})} \quad (2)$$

$\nearrow$
delta distributions ??

Under $SL(2,\mathbb{C})/\mathbb{Z}_2$ or Lorentz transformations,

$$\tilde{\mathcal{A}}_n(\{z_i,\bar{z}_i,h_i,\bar{h}_i,a_i\}) = \prod_{i=1}^{n}\frac{1}{(cz_i+d)^{2h_i}}\frac{1}{(\bar{c}\bar{z}_i+\bar{d})^{2\bar{h}_i}}\tilde{\mathcal{A}}_n\left(\left\{\frac{az_i+b}{cz_i+d},\frac{\bar{a}\bar{z}_i+\bar{b}}{\bar{c}\bar{z}_i+\bar{d}},h_i,\bar{h}_i,a_i\right\}\right). \quad (3)$$

Momentum Space Soft Theorems

Gauge theories at tree level

$$\mathcal{M}^a_{n+1}(p_1,...,p_n,q) \xrightarrow{\omega\to 0} (S^a_{(0)} + S^a_{(1)})\mathcal{M}_n(p_1,...,p_n) + \mathcal{O}(q)$$

$\nearrow$ $\nwarrow$ no further factorisation

$\mathcal{O}(\frac{1}{\omega})$ $\mathcal{O}(1)$ $(q\sim\omega)$. (4)

where,

$$S^a_{(0)} \sim \sum_{k=1}^{n}\frac{\epsilon_\mu p^\mu_k}{q\cdot p_k}T^a_k \qquad S^a_{(1)} = \sum_{k=1}^{n}i\frac{\epsilon_\mu q_\nu J^{\mu\nu}_k}{q\cdot p_k}T^a_k \qquad \text{(universal)} \quad (5)$$

and $J^{\mu\nu}_k$ is the total angular momentum of the k'th particle.

Conformal Soft Theorems in Mellin Basis

CONFORMAL SOFT THEOREMS $\equiv$ WARD IDENTITIES FOR ASYMPTOTIC SYMMETRIES

Leading Conformal Soft Theorem $\Longrightarrow$ level-zero Kac-Moody Algebra

$$\left\langle R^{1,a}(z)\prod_{i=1}^{n}\mathcal{O}^{a_i}_{h_i,\bar{h}_i}(z_i,\bar{z}_i)\right\rangle = -\sum_{k=1}^{n}\frac{T^a_k}{z-z_k}\left\langle\prod_{i=1}^{n}\mathcal{O}^{a_i}_{h_i,\bar{h}_i}(z_i,\bar{z}_i)\right\rangle \quad (6)$$

where leading conformal soft gluon operator is defined as,

$$R^{1,a}(z) = \lim_{\Delta\to 1}(\Delta-1)\mathcal{O}^a_{\Delta,+}(z,\bar{z}) \quad (7)$$

Sub-leading Conformal Soft Theorem $\Longrightarrow$ Current Algebra

$$\left\langle R^{0,a}(z,\bar{z})\prod_{i=1}^{n}\mathcal{O}^{a_i}_{h_i,\bar{h}_i}(z_i,\bar{z}_i)\right\rangle = -\sum_{i=1}^{n}\frac{\epsilon_k}{z-z_k}(-2\bar{h}_k+1+(\bar{z}-\bar{z}_k)\bar{\partial}_k)T^a_kP^{-1}_k\left\langle\prod_{i=1}^{n}\mathcal{O}^{a_i}_{h_i,\bar{h}_i}(z_i,\bar{z}_i)\right\rangle \quad (8)$$

where sub-leading conformal soft gluon operator is defined as,

$$R^{0,a}(z,\bar{z}) = \lim_{\Delta\to 0}\Delta\mathcal{O}^a_{\Delta,+}(z,\bar{z}) \quad (9)$$

Where,

$$R^{0,a}(z,\bar{z}) = \sum_{n=-\frac{1}{2}}^{\frac{1}{2}}\frac{R^{0,a}_n(z)}{\bar{z}^{n-\frac{1}{2}}} \quad (10)$$

YM Theory Coupled with Massive Complex Scalar Background

- Tree level celestial amplitudes in trivial background have kinematic singularities due to scale and translation invariances.
- The YM theory chirally coupled with massive complex scalar background breaks translation as well as scale invariance.
- This theory indeed removes those singularities by replacing the delta distributions with some smooth functions.
- Leading conformal soft theorem and leading OPE structure were shown to remain unmodified. [Casali,Melton,Strominger'22]
- We have shown that the sub-leading soft gluon theorem and the OPE structure of two outgoing positive helicity gluons at $\mathcal{O}(1)$ also remain same.
- As a consequence of this, the scattering amplitudes of this theory also satisfy the same BG equations.
- Calculation done in $(2,2)$ signature. Celestial torus

Tree Level 5-point Amplitude

In momentum basis

$$\begin{aligned}\mathcal{A}(\phi,1^{-a_1},2^{-a_2},3^{+a_3},4^{+a_4},5^{+a_5}) = (-i)\times\Big\{&f^{a_1a_2x_1}f^{x_1a_3x_2}f^{x_2a_4a_5}A(\phi,1^-,2^-,3^+,4^+,5^+)+f^{a_1a_2x_1}f^{x_1a_4x_2}f^{x_2a_3a_5}A(\phi,1^-,2^-,4^+,3^+,5^+)\\&+f^{a_1a_4x_1}f^{x_1a_3x_2}f^{x_2a_2a_5}A(\phi,1^-,4^+,3^+,2^-,5^+)+f^{a_1a_3x_1}f^{x_1a_2x_2}f^{x_2a_4a_5}A(\phi,1^-,3^+,2^-,4^+,5^+)\\&+f^{a_1a_4x_1}f^{x_1a_2x_2}f^{x_2a_3a_5}A(\phi,1^-,4^+,2^-,3^+,5^+)+f^{a_1a_3x_1}f^{x_1a_4x_2}f^{x_2a_2a_5}A(\phi,1^-,3^+,4^+,2^-,5^+)\Big\}\end{aligned} \quad (11)$$

In boost eigenbasis

$$\begin{aligned}\tilde{\mathcal{A}}^\phi(1^{-a_1},2^{-a_2},3^{+a_3},4^{+a_4},5^{+a_5}) &= \langle\mathcal{O}^{a_1}_{\Delta_1,-}(z_1,\bar{z}_1)\mathcal{O}^{a_2}_{\Delta_2,-}(z_2,\bar{z}_2)\mathcal{O}^{a_3}_{\Delta_3,+}(z_3,\bar{z}_3)\mathcal{O}^{a_4}_{\Delta_4,+}(z_4,\bar{z}_4)\mathcal{O}^{a_5}_{\Delta_5,+}(z_5,\bar{z}_5)\rangle_\phi\\&= (-i)\Big\{f^{a_1a_2x_1}f^{x_1a_3x_2}f^{x_2a_4a_5}\frac{z_{12}^4}{z_{12}z_{23}z_{34}z_{45}z_{51}}+f^{a_1a_2x_1}f^{x_1a_4x_2}f^{x_2a_3a_5}\frac{z_{12}^4}{z_{12}z_{24}z_{43}z_{35}z_{51}}+f^{a_1a_4x_1}f^{x_1a_3x_2}f^{x_2a_2a_5}\frac{z_{12}^4}{z_{14}z_{43}z_{32}z_{25}z_{51}}+\\&f^{a_1a_3x_1}f^{x_1a_2x_2}f^{x_2a_4a_5}\frac{z_{12}^4}{z_{13}z_{32}z_{24}z_{45}z_{51}}+f^{a_1a_4x_1}f^{x_1a_2x_2}f^{x_2a_3a_5}\frac{z_{12}^4}{z_{14}z_{42}z_{23}z_{35}z_{51}}+f^{a_1a_3x_1}f^{x_1a_4x_2}f^{x_2a_2a_5}\frac{z_{12}^4}{z_{13}z_{34}z_{42}z_{25}z_{51}}\Big\}\times\\&\frac{N_5}{2(2\pi)^4}\times\Gamma(\Delta_1+1)\Gamma(\Delta_2+1)\Gamma(\Delta_3-1)\Gamma(\Delta_4-1)\Gamma(\Delta_5-1)\times f(\beta_5)\times\\&\int\widetilde{d^3\hat{x}}\,(-q_1\cdot\hat{x})^{-\Delta_1-1}(-q_2\cdot\hat{x})^{-\Delta_2-1}(-q_3\cdot\hat{x})^{-\Delta_3+1}(-q_4\cdot\hat{x})^{-\Delta_4+1}(-q_5\cdot\hat{x})^{-\Delta_5+1}\end{aligned} \quad (12)$$

Sub-leading Conformal Soft Theorem and the Structure of OPE

Check for sub-leading conformal soft theorem

$$\begin{aligned}&\lim_{\Delta_5\to 0}\Delta_5\langle\mathcal{O}^{a_1}_{\Delta_1,-}(z_1,\bar{z}_1)\mathcal{O}^{a_2}_{\Delta_2,-}(z_2,\bar{z}_2)\mathcal{O}^{a_3}_{\Delta_3,+}(z_3,\bar{z}_3)\mathcal{O}^{a_4}_{\Delta_4,+}(z_4,\bar{z}_4)\mathcal{O}^{a_5}_{\Delta_5,+}(z_5,\bar{z}_5)\rangle_\phi\\&= -\sum_{k=1}^{4}\frac{\epsilon_k}{z_5-z_k}\left(-2\bar{h}_k+1+(\bar{z}_5-\bar{z}_k)\bar{\partial}_k\right)T^{a_5}_kP^{-1}_k\langle\mathcal{O}^{a_1}_{\Delta_1,-}(z_1,\bar{z}_1)\mathcal{O}^{a_2}_{\Delta_2,-}(z_2,\bar{z}_2)\mathcal{O}^{a_3}_{\Delta_3,+}(z_3,\bar{z}_3)\mathcal{O}^{a_4}_{\Delta_4,+}(z_4,\bar{z}_4)\rangle_\phi\end{aligned} \quad (13)$$

Extraction of OPE at $\mathcal{O}(1)$ from collinear limit for two positive-helicity outgoing gluons

Collinear limit $z_4\to z_5,\ \ \bar{z}_4\to\bar{z}_5$

OPE factorization at sub-leading order,$\mathcal{O}(1)$

$$\begin{aligned}\tilde{\mathcal{A}}^\phi(1^{-a_1}_{\Delta_1},2^{-a_2}_{\Delta_2},3^{+a_3}_{\Delta_3},4^{+a_4}_{\Delta_4},5^{+a_5}_{\Delta_5})\Big|_{\mathcal{O}(1)} &= \frac{1}{2}\times B(\Delta_4-1,\Delta_5-1)\Big[-\frac{(\Delta_4-1)}{(\Delta_4+\Delta_5-2)}(if^{xa_4a_5})\mathcal{L}_{-1}(5)\tilde{\mathcal{A}}^\phi(1^{-a_1}_{\Delta_1},2^{-a_2}_{\Delta_2},3^{+a_3}_{\Delta_3},5^{+x}_{\Delta_4+\Delta_5-1})\\&+\frac{(\Delta_5-1)}{(\Delta_4+\Delta_5-2)}\mathscr{J}^{a_4}_{-1}(5)\tilde{\mathcal{A}}^\phi(1^{-a_1}_{\Delta_1},2^{-a_2}_{\Delta_2},3^{+a_3}_{\Delta_3},5^{+a_5}_{\Delta_4+\Delta_5-1})\\&+\frac{(\Delta_4-1)}{(\Delta_4+\Delta_5-2)}\mathscr{J}^{a_5}_{-1}(5)\tilde{\mathcal{A}}^\phi(1^{-a_1}_{\Delta_1},2^{-a_2}_{\Delta_2},3^{+a_3}_{\Delta_3},5^{+a_4}_{\Delta_4+\Delta_5-1})\Big].\end{aligned} \quad (14)$$

$\mathcal{O}(1)$ term in the OPE of two gluon primaries

$$\begin{aligned}\mathcal{O}^{a_4,+1}_{\Delta_4,+}(z_4,\bar{z}_4)\mathcal{O}^{a_5,+1}_{\Delta_5,+}(z_5,\bar{z}_5)\Big|_{\mathcal{O}(1)} &\sim \frac{1}{2}\times B(\Delta_4-1,\Delta_5-1)\Big[-\frac{(\Delta_4-1)}{(\Delta_4+\Delta_5-2)}if^{xa_4a_5}L_{-1}+\\&\left(\frac{(\Delta_5-1)}{(\Delta_4+\Delta_5-2)}\delta^{a_4y}\delta^{a_5x}+\frac{(\Delta_4-1)}{(\Delta_4+\Delta_5-2)}\delta^{a_5y}\delta^{a_4x}\right)R^{1,y}_{-1,0}\Big]\mathcal{O}^{x,\epsilon_5}_{\Delta_4+\Delta_5-1}(z_5,\bar{z}_5).\end{aligned} \quad (15)$$

BG Equations and 3-point Amplitude

The general form of the color-ordered $SL(2,\mathbb{C})$ covariant 3-point amplitude is,

$$\widetilde{\mathcal{M}}_3(1^-_{\Delta_1},2^-_{\Delta_2},3^+_{\Delta_3}) = C(\Delta_1,\Delta_2,\Delta_3)z_{12}^{h_3-h_1-h_2}z_{13}^{h_2-h_1-h_3}z_{23}^{h_1-h_2-h_3}\bar{z}_{12}^{\bar{h}_3-\bar{h}_1-\bar{h}_2}\bar{z}_{13}^{\bar{h}_2-\bar{h}_1-\bar{h}_3}\bar{z}_{23}^{\bar{h}_1-\bar{h}_2-\bar{h}_3} \quad (16)$$

BG equations

$$\begin{aligned}&\left(\partial_3-\frac{\Delta_3}{z_{13}}-\frac{1}{z_{23}}\right)\widetilde{\mathcal{M}}_3(1^-_{\Delta_1},2^-_{\Delta_2},3^+_{\Delta_3})+\epsilon_1\epsilon_3\frac{\Delta_1-\sigma_1-1+\bar{z}_{13}\bar{\partial}_1}{z_{13}}\widetilde{\mathcal{M}}_3(1^-_{\Delta_1-1},2^-_{\Delta_2},3^+_{\Delta_3+1})=0\\&\left(\partial_3-\frac{\Delta_3}{z_{23}}-\frac{1}{z_{13}}\right)\widetilde{\mathcal{M}}_3(1^-_{\Delta_1},2^-_{\Delta_2},3^+_{\Delta_3})+\epsilon_2\epsilon_3\frac{\Delta_2-\sigma_2-1+\bar{z}_{23}\bar{\partial}_2}{z_{23}}\widetilde{\mathcal{M}}_3(1^-_{\Delta_1},2^-_{\Delta_2-1},3^+_{\Delta_3+1})=0\end{aligned} \quad (17)$$

$\Longrightarrow$ Recursion relation for the coefficients $C(\Delta_1,\Delta_2,\Delta_3)$.

solution :

$$C(\Delta_1,\Delta_2,\Delta_3) = \mathcal{N}_3\Gamma\left(\frac{\Delta_1+\Delta_2-\Delta_3+3}{2}\right)\Gamma\left(\frac{\Delta_1-\Delta_2+\Delta_3-3}{2}\right)\Gamma\left(-\frac{\Delta_1+\Delta_2+\Delta_3-1}{2}\right)f(\beta) \quad (18)$$

where, $\beta = \sum_{i=1}^{3}\Delta_i$ and $\mathcal{N}_3 = \prod_{j=1}^{3}(-i\epsilon_j)^{\Delta_j-\sigma_j}$.

References

1. S. Banerjee, R. Mandal, A. Manu and P. Paul, "MHV Gluon Scattering in the Massive Scalar Background and Celestial OPE," [arXiv:2302.10245 [hep-th]]

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