Research Statement

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Celestial holography [1–4] aims to compute the S-matrix of a quantum theory of gravity in (3+1)D asymptotically flat spacetimes in terms of the correlation functions of a 2D (Celestial) CFT which lives on the celestial sphere at null infinity. These correlation functions of the celestial CFT are obtained by performing the Mellin transform[5] of the S matrix elements of the bulk quantum theory. What lies in the heart of this duality is the fact that the Lorentz group $SO(1,3)$ acts on the celestial sphere as the global conformal group $SL(2,\mathbb{C})$ [6]. Celestial amplitudes or the correlation functions thus obtained transform like the correlators of a $2D$ CFT under global conformal transformations. Because of the bulk translation invariance the 3- and 4-point celestial amplitudes admit distributional supports on the celestial sphere which are not usually present in conventional $2D$ CFT correlators, hence the name Celestial CFT. These distributions can be removed by coupling the theory with massless or massive scalar backgrounds, few works along this direction are [7, 8]. One of the remarkable achievements of celestial holography is building the connections between seemingly different subjects like soft theorems, asymptotic symmetries and the memory effects which is known as Infrared triangle in the literature[3]. Conformally soft factorisation theorems were shown to be equivalent to the Ward identities of the asymptotic symmetries[9–11]. It was recently discovered that asymptotic symmetry algebra for gravity in AFS is the infinite dimensional wedge subalgebra of $w_{1+\infty}$ and for the gluons it is the S algebra [12, 13]. These infinite dimensional symmetry algebras admit the presence of null states in them. Null states are the primary descendants of the symmetry algebra. In celestial holography, these null states are obtained from the consistency of the conformally soft theorems and the soft limits of the celestial OPE [14–17]. Celestial amplitudes are constrained to satisfy a set of differential equations, known as Banerjee-Ghosh equations[14] which are a direct consequence of these null states. In[18], we have considered pure YM theory chirally coupled with massive scalar background, which breaks the bulk translation and scale invariance. We have shown The subleading soft gluon theorem and the OPE structure at subleading order remain unchanged in this case also. As a result, the celestial amplitudes of this theory also satisfy the same BG equations. The existence of the null states has been very powerful in classifying the boundary theories for both gluons and gravitons. In an interesting work [17], the authors have proved the existence of an infinite number of $w_{1+\infty}$ invariant theories on the celestial sphere, where they have written down all the w invariant OPEs. Following that, we have shown the same for the S algebra in [16], which will be discussed in detail in latter sections. Recently, Strominger and collaborators have shown that celestial amplitudes can be built from the more fundamental building blocks which are called leaf amplitudes[19, 20], which could be helpful to connect the flat holography with the AdS/CFT correspondence. In a recent work[21], we have shown the 4-point scalar and gluon leaf amplitudes admit simple pole type singularities in cross ratio space. Now we will briefly discuss the works [16, 18, 21], I have worked on so far.

1. **MHV gluon scattering in the massive scalar background and celestial OPE [18]:**
   As discussed earlier, 3- and 4-point celestial amplitudes of the pure Yang-Mills theory in the presence of a background are not distributional. It was shown that coupling pure YM theory to a massive scalar background does not change the leading conformally soft theorem and the leading OPE structure [8]. Later in [18], we have shown that the subleading soft gluon theorem and the subleading OPE structure also remain unchanged under such deformation of the theory. Hence it is expected that correlation functions of this theory will also satisfy the same set of differential equations. Since leading and subleading soft gluon theorems do not change and also the OPE structure at leading and subleading orders remain unchanged, it was expected that they would give rise to the same null state relations.

We have shown that the 3-point celestial amplitude in a massive scalar background indeed satisfies the same set of BG equations[14], which confirms that BG equations do not have unique solutions.

2. **All OPEs invariant under the infinite symmetry algebra for gluons on the celestial sphere[16]:** In this work, we have shown there exists an infinite family of S invariant theories on the celestial sphere. We compute the gluon-gluon OPEs of all such theories. Our strategy was similar to the gravity case done in [17]. Since the MHV sector of pure YM theory is a S-invariant theory, we use the MHV sector as the basis of our classification. Here we first find the $\mathcal{O}(1)$ null states of the MHV sector of pure Yang-Mills theory. We then write the $\mathcal{O}(1)$ OPEs of all other S invariant theories in terms of the MHV OPE and the null states of the MHV sector (1). We show that these OPEs are invariant under the S algebra. The number n of MHV null states appearing in the OPE (1) is not fixed by S invariance which hints at the existence of the infinite number of S invariant theories on the celestial sphere for each value of n [16]

$$\begin{aligned}&\mathcal{O}^b_{\Delta_1,+}(z,\bar{z})\mathcal{O}^c_{\Delta_2,+}(0,0)\Big|^{\text{Any Theory}}_{\mathcal{O}(1)}\\ &=\mathcal{O}^b_{\Delta_1,+}(z,\bar{z})\mathcal{O}^c_{\Delta_2,+}(0,0)\Bigg|^{MHV}_{\mathcal{O}(1)}+\sum_{k=1}^{n}B(\Delta_1+k,\Delta_2-1)M^{bc}_k(\Delta_1+\Delta_2)\end{aligned} \tag{1}$$

where M^{ab}_k are the MHV null states. The interesting point is that bulk descriptions of these theories are not known except for the MHV sector($n=0$) and the self-dual Yang-Mills(SDYM) theory($n=1$). We have explicitly checked that the celestial OPE of the SDYM theory can be organised according to our prescription. To find the other bulk theories corresponding to the celestial OPEs (1) with $n\geq 2$ would be an interesting task to explore in future. We also find the KZ-type null states (2) of all such theories which could be useful to explore these theories

$$K^a(\Delta)=\xi^a(\Delta)-i\sum_{k=1}^{n}M^a_k(\Delta+1), \tag{2}$$

with

$$\xi^a(\Delta)=C_A L_{-1}\mathcal{O}^{a,+}_{\Delta}-(\Delta+1)R^{1,b}_{-1,0}R^{1,b}_{0,0}\mathcal{O}^{a,+}_{\Delta}-R^{0,b}_{-\frac{1}{2},\frac{1}{2}}R^{1,b}_{0,0}\mathcal{O}^{a,+}_{\Delta+1} \tag{3}$$

where $\xi^a(\Delta)$s are the KZ-type null states of the MHV sector. These null states put strong constraints on the form of celestial amplitudes.

3. **Singularity Structure of the Four Point Celestial Leaf Amplitudes :** As mentioned before, celestial amplitudes can be obtained from more fundamental building blocks, known as celestial leaf amplitudes. These leaf amplitudes are translationally non-invariant and non-distributional in nature. Translationally invariant celestial amplitudes are obtained by adding the time-like and spacelike leaf amplitudes[19]. It was also shown that the leaf amplitudes admit the same infinite dimensional S algebra[20]. In our work [21], we have studied the singularity structure of the four point celestial leaf amplitudes for massless scalars and MHV gluons. Bulk scale invariance puts constraints on the total conformal weight of the scalars and gluons under considerations. On the support of these constraints, we show that 4-point scalar and gluon (4) leaf amplitudes have a simple pole singularity at $z=\bar{z}$, where z and $\bar{z}$ are two independent cross ratios in klein space.

$$\begin{aligned}&\text{Timelike gluon leaf,}\qquad \delta(\lambda)\mathcal{G}(z,\bar{z})|_{z\to\bar{z}}=\delta(\lambda)\frac{i\pi}{2}\frac{z}{z-1}\frac{4\pi^2}{\bar{z}-z+i\epsilon},z>1\\ &\text{Spacelike gluon leaf,}\qquad \delta(\lambda)\bar{\mathcal{G}}(z,\bar{z})|_{z\to\bar{z}}=-\delta(\lambda)\frac{i\pi}{2}\frac{z}{z-1}\frac{4\pi^2}{\bar{z}-z-i\epsilon},z>1\end{aligned} \tag{4}$$

where $\lambda=\sum_{i=1}^{4}\lambda_i$. We also argue that MHV gluon leaf amplitudes satisfy the same set of differential equations, known as BG equations.

In the next section, we will discuss the possible extensions of our works that can be done in future.

Future directions :

1. In [18], we considered pure YM theory chirally coupled with a massive scalar background. This theory breaks the bulk translation and the bulk scale invariances. It was shown that in this situation also, the leading [8] and subleading [18] soft gluon theorems and the OPE structures at leading and subleading order do not change. As a consequence of this, the celestial amplitudes of this theory also satisfy the BG equations. Now it would be interesting to look into the consequences of breaking some of these bulk symmetries in graviton scattering amplitude. For example, if we break the the translation invariance of the bulk, the leading soft graviton theorem would change since it is the consequences of the supertransltion invariance. Then it would be interesting to see how the decoupling equations in graviton scattering amplitudes change. It would also be an interesting task to check how it changes in case of deformed soft algebras[22].

2. As mentioned before, our work [16] shows there exists an infinite family of S invariant OPEs on the celestial sphere, among which only the bulk descriptions of MHV gluons and the self-dual YM theory are known. An interesting task to explore in future would be to find the bulk descriptions of other theories. We have found the KZ-type null states of all the boundary theories which could be useful in finding these theories. Another important point to notice is that we have classified the boundary theories using the null states (primary descendants) and only finite number of null states appeared in the OPE of a particular theory although the operator spectrum ($\Delta = 1 + i\mathbb{R}$) of CCFT is not bounded from below. This can motivate us to think about the reformulation of CCFT in which the operator spectrum is discrete and bounded from below. There are other interesting questions that stem from this work. For example, in our analysis we assumed the bulk theories are Lorentz invariant because the s algebra does not have the $sl(2,c)$ in it. So, one can now seek for the physical interpretations of the theories which are S-invariant but lack the bulk Lorentz invariance. Another interesting work would be to think about Lagrangian formulations of the S-invariant CFTs and to check how the existence of null states constrains the form of the Lagrangian.

3. As already discussed, authors in [19, 20] have shown that translationally invariant celestial gluon amplitudes can be written as a sum of AdS_3-Witten diagrams for each leaf of the hyperbolic foliation of Klein space. They also showed that these leaf amplitudes are governed by the same infinite dimensional S algebra. The important point is that these leaf amplitudes are not translationally invariant and the S algebra also does not have the Poincaré algebra in it. Translation invariance was retrieved by adding the two leaf amplitudes. So, it might be very interesting to think about the leaf amplitudes for gravitons and also to find the algebra that will govern the graviton leaf amplitudes. The gravity case is interesting because the $w_{1+\infty}$ includes translational symmetries, so it is expected that graviton leaf amplitudes will not be governed by the $w_{1+\infty}$ algebra. To verify this would be an interesting future work. In [21], we have shown that on the support of the constraints on the total conformal weights, 4-point scalar and gluon leaf amplitudes show a simple pole type singularity in the cross ratio space. As an extension of this work, it would also be interesting to check if our conclusion holds in the absence of this constraint. Another interesting work would be to check the singularity structures of the 4-point amplitudes for other bulk scattering processes.

4. Though celestial holography has shown many promising developments towards the flat holography and has been very successful in producing a plethora of interesting results in the last decade, it still lacks the intrinsic formulation of the boundary CCFT. I am interested to work further in this area and to contribute to the understanding of flat holography. There are many aspects in CCFT that need to be well understood. For example, how the above analysis would change in case of deformed soft algebras and for the loop corrected soft theorems. Some progresses along this direction are [22, 23]. I am also interested in understanding black holes in the context of celestial holography. There are a few works on self-dual black holes in this context [24, 25]. My interest also lies in AdS/CFT correspondence in general because the celestial leaf amplitudes that we have discussed can be interpreted by AdS_3/CFT_2 correspondence. Other topics that I am interested in are the study of edge modes and entanglement entropy in celestial holography, there are some recent works [26, 27] in this direction. Since $2D$ celestial CFTs have various infinite dimensional symmetries and as there are no Lagrangian descriptions of these theories, one can think about formulating them using Bootstrap technique. This is one of the future

research directions I would like to work on. Last but not least, since the universe that we live in is asymptotically de Sitter and there are some recent steps towards the formulation of dS holography [28, 29], I also have interest in exploring this area in future.

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