A Quick Celestial Tour

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August 30, 2024

SPS Day, 2024

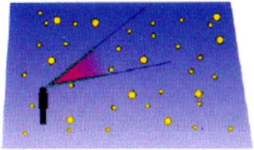
Vacuum Einstein Equation

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 0 \quad (1)$$

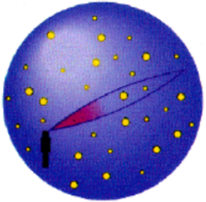
Maximally symmetric solutions:

Anti-de Sitter(AdS)	Minkowski	de Sitter(dS)
$\Lambda < 0$	$\Lambda = 0$	$\Lambda > 0$

Negatively curved universe



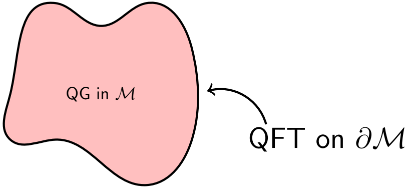
Flat universe



Positively curved universe

Holography ? Introduction

$\boxed{\text{Quantum theory of gravity in bulk}} \equiv \boxed{\text{QFT on the bdy without dynamical gravity}}$



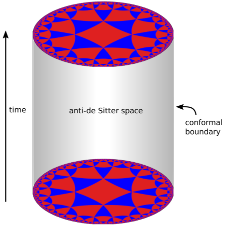
Examples? $\mathbf{\Lambda < 0}$

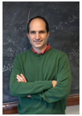
Type IIB superstring theory in $AdS_5 \times S_5$

$\equiv$

$(\mathcal{N} = 4)$ SYM in (3+1)D

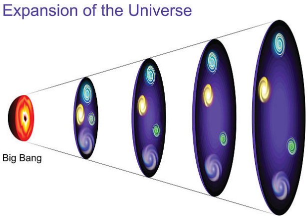
AdS-CFT duality





Maldacena'97

1. But the universe that we live in is not AdS !
2. Our universe is "asymptotically de Sitter" ! $\Lambda > 0$



3. How general is the holographic principle?
4. Can we apply it for flat($\Lambda = 0$) and expanding($\Lambda > 0$) universe also?

5. Current topics of research in string theory.

What is this tour about ?

✧ Recent advancements towards $\Lambda = 0$ holography

Why asymptotically flat spacetime ? Motivations

1. Study of isolated systems(e.g a BH) in GR, far away from the source but dist. $<$ cosmological scale.
2. Collider physics — S matrix
3. Rich asymptotic structure — BMS'60 Lorentz $\ltimes$ supertranslations
4. Observable effects — Memory effect

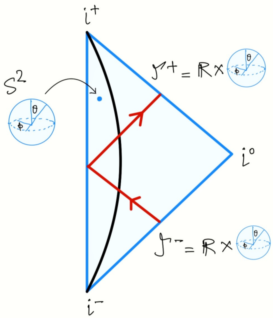
Asymptotically Flat spacetime(AFS)

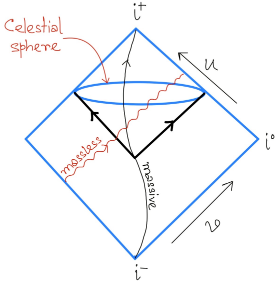
Defn :

$$g_{\mu\nu} = \eta_{\mu\nu} + \mathcal{O}\left(\frac{1}{r}\right) \qquad \text{as} \qquad r \to \infty \tag{2}$$

Minkowski :

$$ds^2 = -du^2 - 2dudr + r^2 \frac{4dzd\bar{z}}{(1+z\bar{z})^2} \tag{3}$$





Celestial holography

$\boxed{\text{Quantum Gravity in (3+1)D AFS}} \equiv \boxed{\text{2D Celestial CFTs on } CS^2 \text{ at null infinity}}$

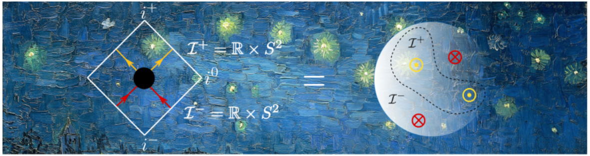
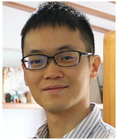


image courtesy: Andrea Puhm

Pioneers :

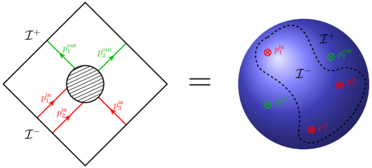
Sabrina Pasterski



Shu-Heng Shao

Andrew Strominger

Duality



Lorentz transformations | **Global conformal transfns.**

$$x^\mu \longrightarrow \Lambda^\mu{}_\nu x^\nu \qquad\qquad z \longrightarrow \frac{az+b}{cz+d}, \text{ c.c}$$

states :

$$|\vec{p},\sigma\rangle = |\omega,\sigma,z,\bar{z}\rangle \xrightarrow[\text{Mellin}]{\int_0^\infty d\omega\omega^{\Delta-1}\ldots} |\Delta,\sigma,z,\bar{z}\rangle$$

momentum | Boost

Observable :

$$\langle out|\mathcal{S}|in\rangle \xrightarrow[\text{Mellin transform}]{\int_0^\infty d\omega\omega^{\Delta-1}\ldots} \langle \mathcal{O}_1(z_1,\bar{z}_1)\ldots\mathcal{O}_N(z_N,\bar{z}_N)\rangle \tag{4}$$

Scattering amplitude | Celestial amplitude

symmetry : translation invariance manifest | conformal invariance manifest

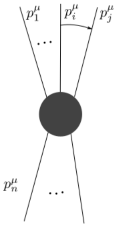
What makes us happy ? Finding a CFT maybe

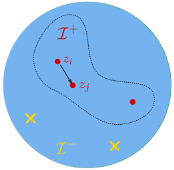
1. States $|\Delta, \sigma, z, \bar{z}\rangle$ transform as the primary states of a 2D CFT under Lorentz transformations.
2. Celestial amplitudes transform nicely under Lorentz transformations/ $SL(2,\mathbb{C})$ transformations.
3. We have found a 2D CFT on the Celestial sphere !!
4. CFT data ?
   Given $\{\Delta_i, \sigma_i\}$ and C_{ijk}, one can build higher point correlation functions from the OPE(Operator product expansion),

$$\mathcal{O}_i(z_1,\bar{z}_1)\mathcal{O}_j(z_2,\bar{z}_2) = \sum_k c_{ij}^k(z_{12},\bar{z}_{12})\mathcal{O}_k(z_2,\bar{z}_2) \tag{5}$$

5. Then how do you find the OPE on celestial sphere?

Celestial amplitudes get factorized upon taking collinear limit.





$$\mathcal{A}(1^{-a_1},2^{+a_2},3^{+a_3},4^{-a_4}) \xrightarrow[p_3\cdot p_4=0]{z_{34}\to 0} -\frac{f^{a_3a_4x}}{z_{34}}B(\Delta_3-1,\Delta_4+1)\mathcal{A}(1^{-a_1},2^{+a_2},4^{-x}) + \text{subleading in } z_{34} + \dots \tag{6}$$

OPE at leading order :

$$\mathcal{O}^{a_3}_{\Delta_3,+}(z_3,\bar{z}_3)\mathcal{O}^{a_3}_{\Delta_4,-}(z_4,\bar{z}_4) \sim -\frac{f^{a_3a_4x}}{z_{34}}B(\Delta_3-1,\Delta_4+1)\mathcal{O}^{x}_{\Delta_3+\Delta_4-1,-}(z_4,\bar{z}_4) \tag{7}$$

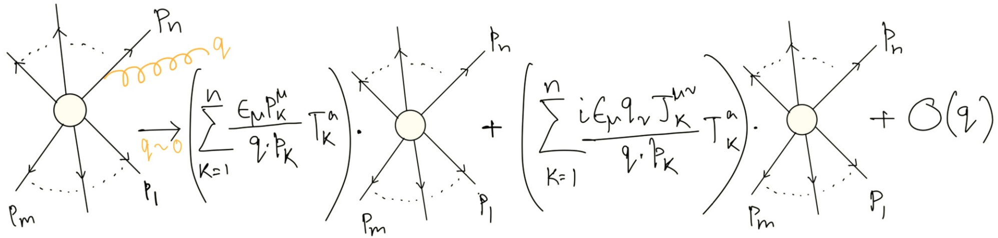
More generally,

$$\mathcal{O}^{a,+}_{\Delta_1}(z_1,\bar{z}_1)\mathcal{O}^{b,+}_{\Delta_2}(z_2,\bar{z}_2) \sim -\frac{if^{ab}{}_c}{z_{12}}\sum_{n=0}^{\infty} B(\Delta_1+n-1,\Delta_2-1)\frac{\bar{z}_{12}^n}{n!}\bar{\partial}_2^n\mathcal{O}^{c,+}_{\Delta_1+\Delta_2-1}(z_2,\bar{z}_2). \quad (8)$$

(Guevara, Himwich, Pate, Strominger '21)

Soft factorization theorem

Energetic soft theorems for gauge theory amplitudes



1. Celestial analog :
   $\boxed{\text{conformally soft theorem}} \equiv \boxed{\text{WIs of asymptotic symmetries}}$
   a surprising connection
2. Infite tower of soft gluons :

$$R^{k,a}(z,\bar{z}) = \lim_{\Delta \to k} (\Delta - k) O^{a,+}_{\Delta}(z,\bar{z}), \qquad k = 1, 0, -1, \dots \tag{9}$$

3. $\boxed{\text{gluon-gluon OPE (8)}} \xrightarrow[\text{mode expansions}]{\text{soft limits}} \boxed{\text{current algebra, S}}$ (Strominger and collaborators '21)

$$[S^{p,a}_{\alpha,m}, S^{q,b}_{\beta,n}] = -if^{abc} S^{p+q-1,c}_{\alpha+\beta,m+n} \tag{10}$$

3. $\boxed{\text{graviton-graviton OPE}} \xrightarrow[\text{mode expansions}]{\text{soft limits}} \boxed{\text{current algebra, } w_{1+\infty}}$

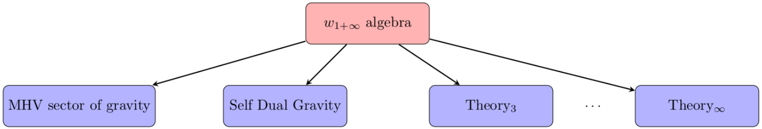
$$[w^p_{\alpha,m}, w^q_{\beta,n}] = [m(q-1) - n(p-1)] w^{p+q-2}_{\alpha+\beta,m+n} \tag{11}$$

4. These are new infinite global symmetries of gauge theories and gravity in AFS.

more happiness

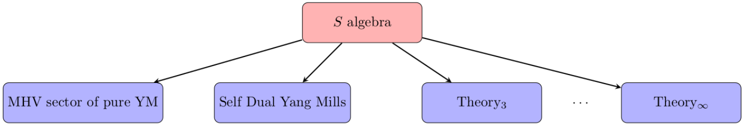
NISER's contributions to Celestial Holography ?

1. OPE = (singular part of the OPE)+ (what else ?) (Banerjee, Paul, Panda, Misra, RM'23)
2. Find the "null states" of these algebras. But why ?
3. They put constraints on the celestial amplitudes $\longrightarrow$ $\boxed{\text{Banerjee-Ghosh eqns.}}$ 2020
4.



(Banerjee,Kulkarni and Paul '23)

5.

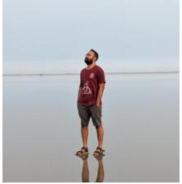


(Banerjee, Paul, Panda, Misra, RM'23)

6. Looking beyond MHV theories (e.g. NMHV gravitons),...

Celestial Holography group in NISER

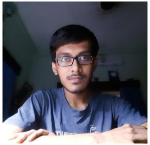
Shamik Banerjee



Nishant Gupta

Mousumi Maitra

Sagnik Misra



Suman Guchait

RM

Thank you for your attention !!

Keywords : celestial holography celestial cft asymptotic symmetries

asymptotically flat spacetime conformally soft theorems celestial ope

Flatspace holography gauge-gravity duality MHV gluons MHV gravitons

$w_{1+\infty}$ algebra S algebra celestial ope BG equations