Research Statement

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One approach to understanding quantum gravity is the holographic principle, which posits an equivalence between a quantum theory of gravity and a lower dimensional non-gravitational theory. The precise nature of the dual theory depends on the boundary of the spacetime under consideration. The best-established example of this principle is the AdS/CFT correspondence. While the holography in anti-de Sitter spacetime is well understood, a good understanding of duality in the case of asymptotically flat spacetime is still lacking. Two leading approaches to understanding quantum gravity in asymptotically flat spacetimes(AFS) are: Celstial holography and Carrollian holography. Celestial holography [1–4] aims to compute the S-matrix of a quantum theory of gravity in (3+1)D asymptotically flat spacetimes in terms of the correlation functions of a 2D (Celestial) CFT which lives on the celestial sphere at null infinity. These correlation functions of the celestial CFT are obtained by performing the Mellin transform[5] of the S matrix elements of the bulk quantum theory. The fact that the Lorentz group $SO(1,3)$ acts on the celestial sphere as the global conformal group $SL(2,\mathbb{C})$ [6] plays as a hint for this correspondence. Celestial amplitudes or the correlation functions thus obtained transform like the correlators of a $2D$ CFT under global conformal transformations. Because of the bulk translation invariance the 3- and 4-point celestial amplitudes admit distributional supports on the celestial sphere which are not usually present in conventional $2D$ CFT correlators, hence the name Celestial CFT. One of the remarkable achievements of Celestial holography is building the connections between seemingly different subjects like soft theorems, asymptotic symmetries and the memory effects which is known as Infrared triangle in the literature[3]. It was discovered that Conformally soft factorisation theorems are equivalent to the Ward identities of the asymptotic symmetries[7–9]. It was recently discovered that asymptotic symmetry algebra arising out of confomrally soft positive helicity gravitons in AFS is the infinite dimensional wedge subalgebra of $w_{1+\infty}$ and for the positive helicity conformally soft gluons it is the S algebra [10, 11]. This rich symmetry structure has enabled the computation of certain class of scattering amplitudes known as MHV gluon and graviton scattering amplitudes. For example, the Celestial amplitudes are constrained to satisfy a set of differential equations, known as Banerjee-Ghosh equations[12] which are a direct consequence of the presence of null states in the infinite dimensional symmetry algebra. We plan to explore more in this direction by developing new tools to calculate various other flat space scattering amplitudes. We also plan to advance our understanding of the scattering amplitudes involving massive external particles which is a relatively less explored area under Celestial framework. Another direction we want to explore is the study of black holes within the celestial framework, hoping to gain some new insights into their properties. The project will explore the mathematical structures of asymptotic symmetry algebras, helping us better understand the symmetries of scattering amplitudes. Furthermore, we seek to extend the tools and techniques of celestial holography beyond the self-dual and MHV scattering amplitudes.

Carrollian holography[13] offers another promising approach to flat space holography. This framework posits that the dual theory resides on the codimension-one boundary (null infinity) of spacetime. Notably, Carrollian holography has yielded significant results in the three dimensional bulk and two dimensional boundary theories, including computations of entanglement entropy, stress tensor calculations, and entropy matching [14–16]. A key objective of this project is to extend these successes by tackling the currently unexplored domain of scattering problems within the Carrollian holography framework. We also plan to investigate the large radius limit of AdS spacetime, with a focus on recovering the celestial and carrollian correlators and their infinite dimensional symmetry algebra. Now I briefly discuss the work that I have done and the tools that I have used and the possible extensions of the work which can be done in future.

1. **MHV gluon scattering in the massive scalar background and celestial OPE [17]:**
The 3- and 4-point celestial amplitudes have distributional supports on the celestial sphere which is a consequence of the bulk translation invariances. For the gluons, it was shown that coupling pure YM theory to a massive scalar background does not change the leading conformally soft theorem and the leading OPE structure [18]. Later in [17], we have shown that the subleading soft gluon theorem and the subleading OPE structure also remain unchanged under such deformation of the theory. Hence it is expected that correlation functions of this theory will also satisfy the same set of differential equations. Since leading and subleading soft gluon theorems do not change and also the OPE structure at leading and subleading orders remain unchanged, it was expected that they would give rise to the same null state relations. We have shown that the 3-point celestial amplitude in a massive scalar background indeed satisfies the same set of BG equations[12], which confirms that BG equations do not have unique solutions.
It would be interesting to look into the consequences of breaking some of these bulk symmetries in graviton scattering amplitudes. For example, if we break the the translation invariance of the bulk, the leading soft graviton theorem would change since it is the consequences of the supertransltion invariance. Then it would be interesting to see how the decoupling equations in graviton scattering amplitudes change. We also plan to classify the theories on the celestial sphere for the case of deformed soft algebras[19] by doing the similar analysis.

2. **All OPEs invariant under the infinite symmetry algebra for gluons on the celestial sphere[20]:**
In this work, we compute all S-invariant gluon-gluon OPEs on the celestial sphere. Our strategy was similar to the gravity case done in [21]. Since the MHV sector of pure YM theory is a S-invariant theory, we use the MHV sector as the basis of our classification of theories. Here we first find the $\mathcal{O}(1)$ null states of the MHV sector of pure Yang-Mills theory. We then write the $\mathcal{O}(1)$ OPEs of all other S invariant theories in terms of the MHV OPE and the null states of the MHV sector (1). We show that these OPEs are invariant under the S algebra. The number n of MHV null states appearing in the OPE (1) is not fixed by S invariance which hints at the existence of the infinite number of S invariant theories on the celestial sphere for each value of n [20]

$$\mathcal{O}^b_{\Delta_1,+}(z,\bar{z})\mathcal{O}^c_{\Delta_2,+}(0,0)\Big|^{\text{Any Theory}}_{\mathcal{O}(1)} = \mathcal{O}^b_{\Delta_1,+}(z,\bar{z})\mathcal{O}^c_{\Delta_2,+}(0,0)\Bigg|^{MHV}_{\mathcal{O}(1)} + \sum_{k=1}^{n} B(\Delta_1+k,\Delta_2-1)M^{bc}_k(\Delta_1+\Delta_2) \tag{1}$$

where M^{ab}_k are the MHV null states. The interesting point is that bulk descriptions of these theories are not known except for the MHV sector($n=0$) and the self-dual Yang-Mills(SDYM) theory($n=1$). We have explicitly checked that the celestial OPE of the SDYM theory can be organised according to our prescription. To find the other bulk theories corresponding to the celestial OPEs (1) with $n \geq 2$ would be an interesting task to explore in future. We also find the KZ-type null states (2) of all such theories which could be useful to explore these theories

$$K^a(\Delta) = \xi^a(\Delta) - i\sum_{k=1}^{n} M^a_k(\Delta+1), \tag{2}$$

with

$$\xi^a(\Delta) = C_A L_{-1}\mathcal{O}^{a,+}_{\Delta} - (\Delta+1)R^{1,b}_{-1,0}R^{1,b}_{0,0}\mathcal{O}^{a,+}_{\Delta} - R^{0,b}_{-\frac{1}{2},\frac{1}{2}}R^{1,b}_{0,0}\mathcal{O}^{a,+}_{\Delta+1} \tag{3}$$

where $\xi^a(\Delta)$s are the KZ-type null states of the MHV sector. These null states put strong constraints on the form of celestial amplitudes.
As mentioned, our work [20] shows there exists an infinite family of S invariant OPEs on the celestial sphere, among which only the bulk descriptions of MHV gluons and the self-dual YM theory are known. We hope to find other theories which fit into our proposed prescription. We have found the KZ-type null states of all the boundary theories which could be useful in finding these theories. Another important point to notice is that we have classified the boundary theories using the null states (primary descendants) and only a finite number of null states appeared in the OPE of a particular theory although the operator spectrum ($\Delta = 1 + i\mathbb{R}$) of CCFT is not bounded from below. Hence we can think about the

reformulation of CCFT in which the operator spectrum is discrete and bounded from below. There are other interesting questions that we can explore in this project. For example, in our analysis we assumed the bulk theories are Lorentz invariant because the s algebra does not have the $sl(2,c)$ in it. So, we can now plan to seek for the physical interpretations of the theories which are S-invariant but lack the bulk Lorentz invariance. We also hope to explore the Lagrangian description of S-invariant theories and plan to investigate the constraints on the form of the Lagrangians that comes form the existence of the null states.

3. **Singularity Structure of the Four Point Celestial Leaf Amplitudes :**
As mentioned before, celestial amplitudes can be obtained from more fundamental building blocks, known as celestial leaf amplitudes. These leaf amplitudes are translationally non-invariant and non-distributional in nature. Translationally invariant celestial amplitudes are obtained by adding the timelike and spacelike leaf amplitudes[22]. It was also shown that the leaf amplitudes admit the same infinite dimensional S algebra[23]. In our work [24], we have studied the singularity structure of the four point celestial leaf amplitudes for massless scalars and MHV gluons. Bulk scale invariance puts constraints on the total conformal weight of the scalars and gluons under considerations. On the support of these constraints, we show that 4-point scalar and gluon (4) leaf amplitudes have a simple pole singularity at $z = \bar{z}$, where z and $\bar{z}$ are two independent cross ratios in klein space.
$$\begin{aligned} &\text{Timelike gluon leaf,} \qquad \delta(\lambda)\mathcal{G}(z,\bar{z})|_{z\to\bar{z}} = \delta(\lambda)\frac{i\pi}{2}\frac{z}{z-1}\frac{4\pi^2}{\bar{z}-z+i\epsilon}, z>1 \\ &\text{Spacelike gluon leaf,} \qquad \delta(\lambda)\bar{\mathcal{G}}(z,\bar{z})|_{z\to\bar{z}} = -\delta(\lambda)\frac{i\pi}{2}\frac{z}{z-1}\frac{4\pi^2}{\bar{z}-z-i\epsilon}, z>1 \end{aligned} \tag{4}$$
where $\lambda = \sum_{i=1}^{4}\lambda_i$. We also argue that MHV gluon leaf amplitudes satisfy the same set of differential equations, known as BG equations.
Translationally invariant celestial amplitude can be recovered by adding the two leaf amplitudes which are not invariant under translation individually. We plan to study the leaf amplitudes for gravitons and also to find the algebra that governs them. The gravity case is interesting because the $w_{1+\infty}$ includes super translation symmetries, so it is expected that each graviton leaf amplitude will not be governed by the $w_{1+\infty}$ algebra. We hope to investigate this by obtaining the graviton-graviton ope from the leaf amplitudes. In [24], we have shown that on the support of the constraints on the total conformal weights, 4-point scalar and gluon leaf amplitudes show a simple pole type singularity in the cross ratio space. As an extension of this work, we also plan to check if our conclusion holds in the absence of this constraint. Another interesting aspect we would like to investigate is the singularity structures of the 4-point amplitudes for other bulk scattering processes.

4. In this section, we discuss other plans and methods for implementing them. While Celestial holography has shown many promising developments towards the flat holography and has been very successful in producing a plethora of interesting results in the last decade, it still lacks the intrinsic formulation of the boundary CCFTs. This project aims to advance our understanding of CCFTs, and the tools and techniques developed herein may pave the way for constructing a self-contained formulation of these boundary theories. We plan to develop tools and techniques for computing scattering amplitudes involving massive external particles following[4], extending beyond the current understanding of massless scattering amplitudes. Additionally, we seek to explore black holes within the celestial framework, hoping to gain some new insights into their properties. There are a few work on self-dual black holes in this context [25, 26]. One of the major discoveries of the celestial holography is finding $w_{1+\infty}$ as the asymptotic symmetry algebra for gravity and the infinite dimensional S algebra for gluon scattering amplitudes. The existence of these infinite-dimensional symmetries suggests the potential for bootstrap techniques to calculate various flat spacetime amplitudes. We plan to build those tools and find more efficient ways to compute various amplitudes in asymptotically flat spacetime. We also plan to explore the rich mathematical structure of the asymptotic symmetries of the S matrix in flat space by studying the representation theories of these symmetry algebras. We also aim to extend the application of aforementioned tools and techniques of celestial holography beyond the MHV and self-dual scattering amplitudes. Another important aspect we hope to explore is recovering the celestial and carrollian correlators and

their infinite dimensional symmetry algebra by investigating the large radius limit of the AdS spacetime.

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