# MHV Gluon Scattering in Massive Scalar Background and Celestial OPE


Raju Mandal

NISER,Bhubaneswar


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# References

- This talk is about the work done with Prof. Shamik B, Dr.Manu A.(IOPB) and Dr.P Paul(IISC) which will appear soon on arXiv(?).
- Our work is based on the work arXiv:2204.10249 done by E. Casali, W. Melton and A. Strominger.
- Other important references are arXiv:2202.08288, arXiv:2011.00017,...

# Outline of the talk

- Motivation
- Introduction to Celestial Amplitude and CCFT
- Discussion on Conformal Soft Theorems
- Objective of our work
- Results
- Summary and Discussions

# Motivation

- In attempt to understand the Flatspace Holography, A theory of QG in 4d AFS $\Leftrightarrow$ a dual QFT living on the boundary of the AFS.
- For the theory of massless particles the dual theory is called Celestial CFT which lives on the celestial sphere at null infinity.
- Hint 1 : Action of 4d Lorentz group in the bulk $\Rightarrow$ global conformal group on the celestial sphere.
- Observable in asymptotically flat spacetime is S matrix.
- Hint 2 : In this celestial holography program, S-matrix transforms as correlation function of 2D CFT under Lorentz transformations.
- To find all the non-trivial symmetries of Nature $\Rightarrow$ Gauge theories and Gravity in 4d AFS contain infinite dimensional symmetries.

# Introduction to Celestial Amplitude and CCFT

## Celestial Amplitude :

Scattering amplitude written in terms of boost eigenstates instead of momentum eigenstates of the asymptotic particle states.

## How do we obtain this?

By doing Mellin transform of the momentum space scattering amplitude, [Pasterski-Shao]

$$\mathcal{O}^{a,\epsilon}_{h,\bar{h}}(z,\bar{z}) = \int_0^\infty d\omega\, \omega^{\Delta-1} A^a(\epsilon\omega, z, \bar{z}, \sigma) \tag{1}$$

$$\mathcal{A}_n(\{z_i,\bar{z}_i,h_i,\bar{h}_i,a_i\}) = \langle \prod_{i=1}^{n} \mathcal{O}^{a_i}_{h_i,\bar{h}_i}(z_i,\bar{z}_i)\rangle = \prod_{i=1}^{n} \int_0^\infty d\omega_i\, \omega_i^{\Delta_i-1} \mathcal{S}_n(\{\omega_i, z_i, \bar{z}_i, \sigma_i, a_i\}) \tag{2}$$

where, $h_i = \frac{\Delta_i+\sigma_i}{2}$ and $\bar{h}_i = \frac{\Delta_i-\sigma_i}{2}$ with $\sigma_i = \pm1$(for gluons).

We have parametrized null momenta in split signature (-,+,-,+) as

$$p_i^\mu = \epsilon_i \omega_i (1 + z_i \bar{z}_i, z_i + \bar{z}_i, z_i - \bar{z}_i, 1 - z_i \bar{z}_i) \tag{3}$$

where, $z_i, \bar{z}_i \in \mathbb{R}$.

## Advantages that come with this basis:

- In this basis,4d scattering amplitudes transform as correlation functions of primary operators in 2d cft under $SL(2, \mathbb{C})$ transformations,i.e.

$$\mathcal{A}_n(\{z_i, \bar{z}_i, h_i, \bar{h}_i, a_i\}) \\ = \prod_{i=1}^{n} \frac{1}{(cz_i + d)^{2h_i}} \frac{1}{(\bar{c}\bar{z}_i + \bar{d})^{2\bar{h}_i}} \mathcal{A}_n\left(\left\{\frac{az_i + b}{cz_i + d}, \frac{\bar{a}\bar{z}_i + \bar{b}}{\bar{c}\bar{z}_i + \bar{d}}, h_i, \bar{h}_i, a_i\right\}\right). \tag{4}$$

- Momentum space soft theorems are interpreted as 2D Conformal Ward identities on celestial sphere at null-infinity.

- The dual 2d conformal field theory which lives on celestial sphere is dubbed as Celestial Conformal Field Theory.
- Is it same as conventional 2D CFT ? No.
- Celestial CFT is endowed with much larger (infinite dimensional) symmetry group coming from leading,subleading soft factorization theorems which have no analogs in conventional 2d CFT.
- These additional infinities of symmetry constraints help to fix the OPE coefficients.

$\Rightarrow$ For example, tree level MHV graviton and gluon scattering amplitudes in principle can be determined by solving the null equations found using these infinite symmetries.

[S.Banerjee and S. Ghosh] (2008.04330;2011.00017)

## Momentum space soft theorems:

Tree level gauge theory amplitudes obey the following soft factorization relation :

$$\mathcal{M}^a_{n+1}(p_1,...,p_n,q) \xrightarrow{\omega\to 0} (\underset{\mathcal{O}(\frac{1}{\omega})}{S^a_{(0)}} + \underset{\mathcal{O}(1)}{S^a_{(1)}})\mathcal{M}_n(p_1,...,p_n) + \mathcal{O}(q) \qquad (q\sim\omega). \tag{5}$$

Leading term,

$$S^a_{(0)} \sim \sum_{k=1}^{n} \frac{\epsilon_\mu p_k^\mu}{q\cdot p_k} T^a_k \tag{6}$$

Subleading term,

$$S^a_{(1)} = \sum_{k=1}^{n} i\frac{\epsilon_\mu q_\nu J_k^{\mu\nu}}{q\cdot p_k} T^a_k \tag{7}$$

where, $J_k^{\mu\nu}$ is the total angular momentum of the k'th particle.

## Conformal Soft theorems (for pure YM theory)

- Leading Conformal Soft Gluon Theorem, $\Delta \to 1$

$$\left\langle j^a(z) \prod_{i=1}^{n} \mathcal{O}^{a_i}_{h_i,\bar{h}_i}(z_i,\bar{z}_i) \right\rangle = -\sum_{k=1}^{n} \frac{T^a_k}{z-z_k} \left\langle \prod_{i=1}^{n} \mathcal{O}^{a_i}_{h_i,\bar{h}_i}(z_i,\bar{z}_i) \right\rangle \tag{8}$$

where leading conformal soft gluon operator is defined as,

$$j^a(z) = \lim_{\Delta\to 1} (\Delta - 1)\mathcal{O}^a_{\Delta,+}(z,\bar{z}) \tag{9}$$

and

$$T^a_k \mathcal{O}^{a_i}_{h_i,\bar{h}_i}(z_i,\bar{z}_i) = i f^{a a_i b} \mathcal{O}^b_{h_i,\bar{h}_i}(z_i,\bar{z}_i)\delta_{ik}. \tag{10}$$

$\Rightarrow$ This gives level zero Kac-Moody algebra (closed) on celestial sphere.

- Subleading Conformal Soft Gluon Theorem, $\Delta \to 0$

$$\left\langle S_1^{+a}(z,\bar{z}) \prod_{i=1}^{n} \mathcal{O}^{a_i}_{h_i,\bar{h}_i}(z_i,\bar{z}_i) \right\rangle$$
$$= -\sum_{i=1}^{n} \frac{\epsilon_k}{z-z_k}(-2\bar{h}_k + 1 + (\bar{z}-\bar{z}_k)\bar{\partial}_k) T_k^a P_k^{-1} \left\langle \prod_{i=1}^{n} \mathcal{O}^{a_i}_{h_i,\bar{h}_i}(z_i,\bar{z}_i) \right\rangle \tag{11}$$

where subleading conformal soft gluon operator is defined as,

$$S_1^{+a}(z,\bar{z}) = \lim_{\Delta\to 0} \Delta \mathcal{O}^a_{\Delta,+}(z,\bar{z}) \tag{12}$$

and

$$P_k^{-1}\mathcal{O}^{a_i}_{h_i,\bar{h}_i}(z_i,\bar{z}_i) = \mathcal{O}^{a_i}_{h_i-\frac{1}{2},\bar{h}_i-\frac{1}{2}}(z_i,\bar{z}_i)\delta_{ki}. \tag{13}$$

[Donnay,Puhm,Strominger;2018]

Where,

$$S_1^{+a}(z,\bar{z}) = J^a(z) + \bar{z}K^a(z). \tag{14}$$

$$\left\langle J^a(z) \prod_{i=1}^{n} \mathcal{O}^{a_i}_{h_i,\bar{h}_i}(z_i,\bar{z}_i) \right\rangle = -\sum_{k=1}^{n} \frac{\epsilon_k}{z-z_k}(-2\bar{h}_k+1-\bar{z}_k\bar{\partial}_k)T^a_k P^{-1}_k \left\langle \prod_{i=1}^{n} \mathcal{O}^{a_i}_{h_i,\bar{h}_i}(z_i,\bar{z}_i) \right\rangle \tag{15}$$

and

$$\left\langle K^a(z) \prod_{i=1}^{n} \mathcal{O}^{a_i}_{h_i,\bar{h}_i}(z_i,\bar{z}_i) \right\rangle = -\sum_{k=1}^{n} \frac{\epsilon_k}{z-z_k}\bar{\partial}_k T^a_k P^{-1}_k \left\langle \prod_{i=1}^{n} \mathcal{O}^{a_i}_{h_i,\bar{h}_i}(z_i,\bar{z}_i) \right\rangle \tag{16}$$

$\Rightarrow$ Current algebra of two currents $J^a(z)$ and $K^a(z)$.

# Objective of our work

- We have continued the study gluon scattering processes in chirally coupled massive scalar background.
  Lagrangian : [Casali et al.]

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi^\star - \frac{m}{2}\phi^\star \phi - \frac{1}{4} tr(F_{\mu\nu}F^{\mu\nu}) - \frac{1}{4}\phi tr(F^-_{\mu\nu}F^{-\mu\nu}) - \frac{1}{4}\phi^\star tr(F^+_{\mu\nu}F^{+\mu\nu}) \tag{17}$$

where,

$$F^{\mu\nu}_\pm = \frac{1}{2}(F^{\mu\nu} \pm \frac{i}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}) \tag{18}$$

- This breaks translation and scale invariance of the system and other conformal invariance is preserved.
- Casali,Melton and Strominger⇒ leading soft theorem and leading OPE remain same.

- Our obejective is to see if the sub-leading soft gluon theorem and the sub-leading ope structure also remain same?

- If they remain same then we get the same current algebra.

- The current algebra coming from these soft theorems have null states in it.Decoupling of these null states gives rise to PDEs of celestial amplitudes (a.k.a BG equations) which in principle can be solved to find the amplitudes. So, another goal is to check if this BG equations are also satisfied by the celestial amplitudes in this scenario.

# Celestial MHV amplitudes in massive scalar background

- It has been conjectured that n-point color-ordered MHV partial amplitude in presence of masssless scalar background is given by, [Dixon,Glover,Khoze]

$$\mathcal{A}_{n+1}(\phi, 1^{\epsilon_1,-}, 2^{\epsilon_2,-}, 3^{\epsilon_3,+}, ..., n^{\epsilon_n,+}) = \frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle...\langle n1\rangle}\delta\left(\sum_{i=1}^{n} p_i + Q\right) \tag{19}$$

- This amplitude has the same form in momentum space when coupled to massive scalar background. [Casali,Melton,...]
- The amplitude coupled to the massive background is obtained by integrating over scalar phase,

$$\begin{aligned}&\mathcal{A}_n^{\phi}(\phi, 1^{\epsilon_1,-}, 2^{\epsilon_2,-}, 3^{\epsilon_3,+}, ..., n^{\epsilon_n,+}) \\ &= \int \widetilde{d^3Q} g(Q)\mathcal{A}_{n+1}(\phi, 1^{\epsilon_1,-}, 2^{\epsilon_2,-}, 3^{\epsilon_3,+}, ..., n^{\epsilon_n,+})\end{aligned} \tag{20}$$

# Parametrizations

Split signature : $(-,+,-,+)$

$$\langle ij\rangle = 2\epsilon_i\epsilon_j\sqrt{\omega_i\omega_j}z_{ij}, \qquad [ij] = 2\sqrt{\omega_i\omega_j}\bar{z}_{ij}. \tag{21}$$

Momenta of massless particles :

$$p_i = \epsilon_i\omega_i\{1+z_i\bar{z}_i, z_i+\bar{z}_i, (z_i-\bar{z}_i), 1-z_i\bar{z}_i\} \tag{22}$$

Momenta of massive scalar of unit mass:

$$Q = \frac{1}{2y}\{1+y^2+z\bar{z}, z+\bar{z}, (z-\bar{z}), 1-y^2-z\bar{z}\} \tag{23}$$

Full 5-gluon amplitude in massive scalar background :

$$\mathcal{A}(\phi, 1^{-a_1}, 2^{-a_2}, 3^{+a_3}, 4^{+a_4}, 5^{+a_5})$$
$$= (-i) \times \Big\{ f^{a_1 a_2 x_1} f^{x_1 a_3 x_2} f^{x_2 a_4 a_5} A(\phi, 1^-, 2^-, 3^+, 4^+, 5^+) +$$
$$f^{a_1 a_2 x_1} f^{x_1 a_4 x_2} f^{x_2 a_3 a_5} A(\phi, 1^-, 2^-, 4^+, 3^+, 5^+) +$$
$$f^{a_1 a_4 x_1} f^{x_1 a_3 x_2} f^{x_2 a_2 a_5} A(\phi, 1^-, 4^+, 3^+, 2^-, 5^+) +$$
$$f^{a_1 a_3 x_1} f^{x_1 a_2 x_2} f^{x_2 a_4 a_5} A(\phi, 1^-, 3^+, 2^-, 4^+, 5^+) +$$
$$f^{a_1 a_4 x_1} f^{x_1 a_2 x_2} f^{x_2 a_3 a_5} A(\phi, 1^-, 4^+, 2^-, 3^+, 5^+) +$$
$$f^{a_1 a_3 x_1} f^{x_1 a_4 x_2} f^{x_2 a_2 a_5} A(\phi, 1^-, 3^+, 4^+, 2^-, 5^+) \Big\} \quad (24)$$

# Mellin transform of the 5-gluon amplitude coupled to massive scalar background :

$$
\begin{aligned}
&\tilde{\mathcal{A}}^{\phi}(1^{-a_1}, 2^{-a_2}, 3^{+a_3}, 4^{+a_4}, 5^{+a_5}) \\
&\quad = \langle \mathcal{O}^{a_1}_{\Delta_1,-}(z_1,\bar{z}_1)\mathcal{O}^{a_2}_{\Delta_2,-}(z_2,\bar{z}_2)\mathcal{O}^{a_3}_{\Delta_3,+}(z_3,\bar{z}_3)\mathcal{O}^{a_4}_{\Delta_4,+}(z_4,\bar{z}_4)\mathcal{O}^{a_5}_{\Delta_5,+}(z_5,\bar{z}_5)\rangle_{\phi} \\
&= (-i)\Big\{ f^{a_1a_2x_1} f^{x_1a_3x_2} f^{x_2a_4a_5} \frac{z_{12}^4}{z_{12}z_{23}z_{34}z_{45}z_{51}} + f^{a_1a_2x_1} f^{x_1a_4x_2} f^{x_2a_3a_5} \frac{z_{12}^4}{z_{12}z_{24}z_{43}z_{35}z_{51}} + \\
&f^{a_1a_4x_1} f^{x_1a_3x_2} f^{x_2a_2a_5} \frac{z_{12}^4}{z_{14}z_{43}z_{32}z_{25}z_{51}} + f^{a_1a_3x_1} f^{x_1a_2x_2} f^{x_2a_4a_5} \frac{z_{12}^4}{z_{13}z_{32}z_{24}z_{45}z_{51}} + \\
&f^{a_1a_4x_1} f^{x_1a_2x_2} f^{x_2a_3a_5} \frac{z_{12}^4}{z_{14}z_{42}z_{23}z_{35}z_{51}} + f^{a_1a_3x_1} f^{x_1a_4x_2} f^{x_2a_2a_5} \frac{z_{12}^4}{z_{13}z_{34}z_{42}z_{25}z_{51}} \Big\} \times \\
&\qquad \frac{N_5}{2(2\pi)^4} \times \Gamma(\Delta_1+1)\Gamma(\Delta_2+1)\Gamma(\Delta_3-1)\Gamma(\Delta_4-1)\Gamma(\Delta_5-1) \times f(\beta_5) \times \\
&\int \widetilde{d^3\hat{x}} \;\; (-q_1\cdot\hat{x})^{-\Delta_1-1}(-q_2\cdot\hat{x})^{-\Delta_2-1}(-q_3\cdot\hat{x})^{-\Delta_3+1}(-q_4\cdot\hat{x})^{-\Delta_4+1}(-q_5\cdot\hat{x})^{-\Delta_5+1}
\end{aligned}
\tag{25}
$$

Where,

$$
\beta_5 = \sum_{i=1}^{5} (\Delta_i - 1). \tag{26}
$$

Taking $\Delta_5 \to 1$ limit of the 5-point amplitude we get,

$$\lim_{\Delta_5 \to 1} (\Delta_5 - 1) \tilde{\mathcal{A}}^{\phi}(1^{-a_1}, 2^{-a_2}, 3^{+a_3}, 4^{+a_4}, 5^{+a_5})$$
$$= (-i) \Big\{ f^{a_1 a_2 x_1} f^{x_1 a_3 x_2} f^{x_2 a_4 a_5} \frac{z_{12}^4}{z_{12} z_{23} z_{34} z_{45} z_{51}} + f^{a_1 a_2 x_1} f^{x_1 a_4 x_2} f^{x_2 a_3 a_5} \frac{z_{12}^4}{z_{12} z_{24} z_{43} z_{35} z_{51}} +$$
$$f^{a_1 a_4 x_1} f^{x_1 a_3 x_2} f^{x_2 a_2 a_5} \frac{z_{12}^4}{z_{14} z_{43} z_{32} z_{25} z_{51}} + f^{a_1 a_3 x_1} f^{x_1 a_2 x_2} f^{x_2 a_4 a_5} \frac{z_{12}^4}{z_{13} z_{32} z_{24} z_{45} z_{51}} +$$
$$f^{a_1 a_4 x_1} f^{x_1 a_2 x_2} f^{x_2 a_3 a_5} \frac{z_{12}^4}{z_{14} z_{42} z_{23} z_{35} z_{51}} + f^{a_1 a_3 x_1} f^{x_1 a_4 x_2} f^{x_2 a_2 a_5} \frac{z_{12}^4}{z_{13} z_{34} z_{42} z_{25} z_{51}} \Big\} \times$$
$$\frac{N_4}{2(2\pi)^4} \times \Gamma(\Delta_1 + 1)\Gamma(\Delta_2 + 1)\Gamma(\Delta_3 - 1)\Gamma(\Delta_4 - 1) \times f(\beta_4) \times$$
$$\int \widetilde{d^3\hat{x}} \;\; (-q_1 \cdot \hat{x})^{-\Delta_1 - 1} (-q_2 \cdot \hat{x})^{-\Delta_2 - 1} (-q_3 \cdot \hat{x})^{-\Delta_3 + 1} (-q_4 \cdot \hat{x})^{-\Delta_4 + 1}. \tag{27}$$

Here,

$$\beta_5 \xrightarrow{\Delta_5 \to 1} \beta_4 = \sum_{i=1}^{4} (\Delta_i - 1). \tag{28}$$

Now, from the r.h.s of eqn.(8) we get,

$$
\begin{aligned}
&-\sum_{k=1}^{4}\frac{T_k^{a_5}}{z_5-z_k}\langle\mathcal{O}^{a_1}_{\Delta_1,-}(z_1,\bar z_1)\mathcal{O}^{a_2}_{\Delta_2,-}(z_2,\bar z_2)\mathcal{O}^{a_3}_{\Delta_3,+}(z_3,\bar z_3)\mathcal{O}^{a_4}_{\Delta_4,+}(z_4,\bar z_4)\rangle\\
&=(-i)\Big\{f^{a_5a_1x_1}f^{x_1a_2x_2}f^{x_2a_3a_4}\frac{z_{12}^4}{z_{12}z_{23}z_{34}z_{41}z_{51}}+f^{a_5a_1x_1}f^{x_1a_3x_2}f^{x_2a_2a_4}\frac{z_{12}^4}{z_{13}z_{32}z_{24}z_{41}z_{51}}+\\
&f^{a_5a_2x_1}f^{a_1x_1x_2}f^{x_2a_3a_4}\frac{z_{12}^4}{z_{12}z_{23}z_{34}z_{41}z_{52}}+f^{a_5a_2x_1}f^{a_1a_3x_2}f^{x_2x_1a_4}\frac{z_{12}^4}{z_{13}z_{32}z_{24}z_{41}z_{52}}+\\
&f^{a_5a_3x_1}f^{a_1a_2x_2}f^{x_2x_1a_4}\frac{z_{12}^4}{z_{12}z_{23}z_{34}z_{41}z_{53}}+f^{a_5a_3x_1}f^{a_1x_1x_2}f^{x_2a_2a_4}\frac{z_{12}^4}{z_{13}z_{32}z_{24}z_{41}z_{53}}\\
&f^{a_5a_4x_1}f^{a_1a_2x_2}f^{x_2a_3x_1}\frac{z_{12}^4}{z_{12}z_{23}z_{34}z_{41}z_{54}}+f^{a_5a_4x_1}f^{a_1a_3x_2}f^{x_2a_2x_1}\frac{z_{12}^4}{z_{13}z_{32}z_{24}z_{41}z_{54}}\Big\}\times\\
&\frac{N_4}{(2\pi)^4}\times\Gamma(\Delta_1+1)\Gamma(\Delta_2+1)\Gamma(\Delta_3-1)\Gamma(\Delta_4-1)\times f(\beta_4)\times\\
&\int\widetilde{d^3\hat x}\;\;(-q_1\cdot\hat x)^{-\Delta_1-1}(-q_2\cdot\hat x)^{-\Delta_2-1}(-q_3\cdot\hat x)^{-\Delta_3+1}(-q_4\cdot\hat x)^{-\Delta_4+1}.
\end{aligned}
\tag{29}
$$

$$
\begin{aligned}
&-\sum_{k=1}^{4}\frac{T_k^{a_5}}{z_5-z_k}\langle \mathcal{O}^{a_1}_{\Delta_1,-}(z_1,\bar z_1)\mathcal{O}^{a_2}_{\Delta_2,-}(z_2,\bar z_2)\mathcal{O}^{a_3}_{\Delta_3,+}(z_3,\bar z_3)\mathcal{O}^{a_4}_{\Delta_4,+}(z_4,\bar z_4)\rangle_\phi \\
&= (-i)\Bigg\{ f^{a_1a_2x_1}f^{x_1a_3x_2}f^{x_2a_4a_5}\frac{z_{12}^4}{z_{12}z_{23}z_{34}z_{45}z_{51}} + f^{a_1a_2x_1}f^{x_1a_4x_2}f^{x_2a_3a_5}\frac{z_{12}^4}{z_{12}z_{24}z_{43}z_{35}z_{51}} + \\
&f^{a_1a_4x_1}f^{x_1a_3x_2}f^{x_2a_2a_5}\frac{z_{12}^4}{z_{14}z_{43}z_{32}z_{25}z_{51}} + f^{a_1a_3x_1}f^{x_1a_2x_2}f^{x_2a_4a_5}\frac{z_{12}^4}{z_{13}z_{32}z_{24}z_{45}z_{51}} + \\
&f^{a_1a_4x_1}f^{x_1a_2x_2}f^{x_2a_3a_5}\frac{z_{12}^4}{z_{14}z_{42}z_{23}z_{35}z_{51}} + f^{a_1a_3x_1}f^{x_1a_4x_2}f^{x_2a_2a_5}\frac{z_{12}^4}{z_{13}z_{34}z_{42}z_{25}z_{51}} \Bigg\}\times \\
&\frac{N_4}{(2\pi)^4}\times\Gamma(\Delta_1+1)\Gamma(\Delta_2+1)\Gamma(\Delta_3-1)\Gamma(\Delta_4-1)\times f(\beta_4)\times \\
&\int \widetilde{d^3\hat x}\;\; (-q_1\cdot\hat x)^{-\Delta_1-1}(-q_2\cdot\hat x)^{-\Delta_2-1}(-q_3\cdot\hat x)^{-\Delta_3+1}(-q_4\cdot\hat x)^{-\Delta_4+1}
\end{aligned}
\tag{30}
$$

$\Rightarrow$ Leading soft theorem (8) is satisfied upto an overall $\frac{1}{2}$ factor on the LHS.(This $\frac{1}{2}$ factor comes due to the choice of coupling constant.)

# Subleading conformal soft limit($\Delta \to 0$) :

Taking the subleading soft conformal limit i.e. $\Delta_5 \to 0$ of the 5 point amplitude we get,

$$
\begin{aligned}
&\lim_{\Delta_5 \to 0} \Delta_5 \langle \mathcal{O}^{a_1}_{\Delta_1,-}(z_1,\bar{z}_1)\mathcal{O}^{a_2}_{\Delta_2,-}(z_2,\bar{z}_2)\mathcal{O}^{a_3}_{\Delta_3,+}(z_3,\bar{z}_3)\mathcal{O}^{a_4}_{\Delta_4,+}(z_4,\bar{z}_4)\mathcal{O}^{a_5}_{\Delta_5,+}(z_5,\bar{z}_5)\rangle_\phi \\
&= \lim_{\Delta_5 \to 0} \Delta_5 \tilde{\mathcal{A}}^\phi(1^{-a_1}, 2^{-a_2}, 3^{+a_3}, 4^{+a_4}, 5^{+a_5}) \\
&= -\Bigg\{ f^{a_1 a_2 x_1} f^{x_1 a_3 x_2} f^{x_2 a_4 a_5} \frac{z_{12}^4}{z_{12} z_{23} z_{34} z_{45} z_{51}} + f^{a_1 a_2 x_1} f^{x_1 a_4 x_2} f^{x_2 a_3 a_5} \frac{z_{12}^4}{z_{12} z_{24} z_{43} z_{35} z_{51}} + \\
&f^{a_1 a_4 x_1} f^{x_1 a_3 x_2} f^{x_2 a_2 a_5} \frac{z_{12}^4}{z_{14} z_{43} z_{32} z_{25} z_{51}} + f^{a_1 a_3 x_1} f^{x_1 a_2 x_2} f^{x_2 a_4 a_5} \frac{z_{12}^4}{z_{13} z_{32} z_{24} z_{45} z_{51}} + \\
&f^{a_1 a_4 x_1} f^{x_1 a_2 x_2} f^{x_2 a_3 a_5} \frac{z_{12}^4}{z_{14} z_{42} z_{23} z_{35} z_{51}} + f^{a_1 a_3 x_1} f^{x_1 a_4 x_2} f^{x_2 a_2 a_5} \frac{z_{12}^4}{z_{13} z_{34} z_{42} z_{25} z_{51}} \Bigg\} \times \\
&\frac{N_4}{2(2\pi)^4} \times \Gamma(\Delta_1 + 1)\Gamma(\Delta_2 + 1)\Gamma(\Delta_3 - 1)\Gamma(\Delta_4 - 1) \times f(\tilde{\beta}_4) \times \\
&\int \widetilde{d^3\hat{x}} \;\; (-q_1 \cdot \hat{x})^{-\Delta_1 - 1}(-q_2 \cdot \hat{x})^{-\Delta_2 - 1}(-q_3 \cdot \hat{x})^{-\Delta_3 + 1}(-q_4 \cdot \hat{x})^{-\Delta_4 + 1}(-q_5 \cdot \hat{x}) \qquad (31)
\end{aligned}
$$

Now from the RHS of eqn.(11) we get,

$$
\begin{aligned}
&-\sum_{k=1}^{4}\frac{\epsilon_k}{z_5-z_k}\left(-2\bar{h}_k+1+(\bar{z}_5-\bar{z}_k)\bar{\partial}_k\right)T_k^{a_5}P_k^{-1}\langle\mathcal{O}^{a_1}_{\Delta_1,-}(z_1,\bar{z}_1)\mathcal{O}^{a_2}_{\Delta_2,-}(z_2,\bar{z}_2)\mathcal{O}^{a_3}_{\Delta_3,+}(z_3,\bar{z}_3)\mathcal{O}^{a_4}_{\Delta_4,+}(z_4,\bar{z}_4)\rangle_\phi \\
&= (i)\Bigg\{f^{a_1a_2x_1}f^{x_1a_3x_2}f^{x_2a_4a_5}\frac{z_{12}^4}{z_{12}z_{23}z_{34}z_{45}z_{51}}+f^{a_1a_2x_1}f^{x_1a_4x_2}f^{x_2a_3a_5}\frac{z_{12}^4}{z_{12}z_{24}z_{43}z_{35}z_{51}}+ \\
&f^{a_1a_4x_1}f^{x_1a_3x_2}f^{x_2a_2a_5}\frac{z_{12}^4}{z_{14}z_{43}z_{32}z_{25}z_{51}}+f^{a_1a_3x_1}f^{x_1a_2x_2}f^{x_2a_4a_5}\frac{z_{12}^4}{z_{13}z_{32}z_{24}z_{45}z_{51}}+ \\
&f^{a_1a_4x_1}f^{x_1a_2x_2}f^{x_2a_3a_5}\frac{z_{12}^4}{z_{14}z_{42}z_{23}z_{35}z_{51}}+f^{a_1a_3x_1}f^{x_1a_4x_2}f^{x_2a_2a_5}\frac{z_{12}^4}{z_{13}z_{34}z_{42}z_{25}z_{51}}\Bigg\}\times \\
&\frac{N_4}{(2\pi)^4}\times\Gamma(\Delta_1+1)\Gamma(\Delta_2+1)\Gamma(\Delta_3-1)\Gamma(\Delta_4-1)\times f(\tilde{\beta}_4)\times \\
&\int\widetilde{d^3\hat{x}}\ \ (-q_1\cdot\hat{x})^{-\Delta_1-1}(-q_2\cdot\hat{x})^{-\Delta_2-1}(-q_3\cdot\hat{x})^{-\Delta_3+1}(-q_4\cdot\hat{x})^{-\Delta_4+1}(-q_5\cdot\hat{x}) \qquad (32)
\end{aligned}
$$

$\Rightarrow$ Hence subleading soft theorem is also satisfied (upto a factor of $\frac{1}{2}$).

# OPE factorization from collinear limit

In the OPE limit i.e. $z_4 \to z_5$, $\bar{z}_4 \to \bar{z}_5$ we will write the following expansion,

$$(-q_4 \cdot \hat{x})^{-\Delta_4+1} = \left[\frac{y^2 + (z - z_4)(\bar{z} - \bar{z}_4)}{y}\right]^{-\Delta_4+1}$$

$$= (-q_5 \cdot \hat{x})^{-\Delta_4+1}\left[1 + \frac{\Delta_4 - 1}{y^2 + (z - z_5)(\bar{z} - \bar{z}_5)}\left\{(\bar{z} - \bar{z}_5)z_{45} + (z - z_5)\bar{z}_{45} - z_{45}\bar{z}_{45}\right\}\right] + \dots \tag{33}$$

Substituting this expression in 5-point mellin amplitude we get,

$$
\begin{aligned}
= (-i)\Bigg\{ & f^{a_1 a_2 x_1} f^{x_1 a_3 x_2} f^{x_2 a_4 a_5}\left(\frac{1}{z_{34}} + \frac{1}{z_{45}}\right) \times \frac{z_{12}^4}{z_{12} z_{23} z_{35} z_{51}} + \\
& f^{a_1 a_2 x_1} f^{x_1 a_4 x_2} f^{x_2 a_3 a_5}\left(\frac{1}{z_{24}} + \frac{1}{z_{43}}\right) \times \frac{z_{12}^4}{z_{12} z_{23} z_{35} z_{51}} + \\
& f^{a_1 a_4 x_1} f^{x_1 a_3 x_2} f^{x_2 a_2 a_5}\left(\frac{1}{z_{14}} + \frac{1}{z_{43}}\right) \times \frac{z_{12}^4}{z_{13} z_{32} z_{25} z_{51}} + \\
& f^{a_1 a_3 x_1} f^{x_1 a_2 x_2} f^{x_2 a_4 a_5}\left(\frac{1}{z_{24}} + \frac{1}{z_{45}}\right) \times \frac{z_{12}^4}{z_{13} z_{32} z_{25} z_{51}} + \\
& f^{a_1 a_4 x_1} f^{x_1 a_2 x_2} f^{x_2 a_3 a_5}\left(\frac{1}{z_{14}} + \frac{1}{z_{42}}\right) \times \frac{z_{12}^4}{z_{12} z_{23} z_{35} z_{51}} + \\
& f^{a_1 a_3 x_1} f^{x_1 a_4 x_2} f^{x_2 a_2 a_5}\left(\frac{1}{z_{34}} + \frac{1}{z_{42}}\right) \times \frac{z_{12}^4}{z_{13} z_{32} z_{25} z_{51}} \Bigg]\Bigg\} \times \\
& \frac{N_5}{2(2\pi)^4} \times \Gamma(\Delta_1 + 1)\Gamma(\Delta_2 + 1)\Gamma(\Delta_3 - 1)\Gamma(\Delta_4 - 1)\Gamma(\Delta_5 - 1) \times f(\beta_5) \times \\
& \int \widetilde{d^3\hat{x}} \quad (-q_1 \cdot \hat{x})^{-\Delta_1 - 1}(-q_2 \cdot \hat{x})^{-\Delta_2 - 1}(-q_3 \cdot \hat{x})^{-\Delta_3 + 1}(-q_5 \cdot \hat{x})^{-\Delta_4 - \Delta_5 + 2} \times \\
& \left[1 + \frac{\Delta_4 - 1}{y^2 + (z - z_5)(\bar{z} - \bar{z}_5)}\left\{(\bar{z} - \bar{z}_5) z_{45} + (z - z_5)\bar{z}_{45} - z_{45}\bar{z}_{45}\right\}\right] + \ldots \qquad (34)
\end{aligned}
$$

## OPE factorization at leading order

$$\tilde{\mathcal{A}}^{\phi}(1^{-a_1}_{\Delta_1}, 2^{-a_2}_{\Delta_2}, 3^{a_3}_{\Delta_3}, 4^{a_4}_{\Delta_4}, 5^{a_5}_{\Delta_5})\Big|_{\mathcal{O}(\frac{1}{z_{45}})}$$
$$= \frac{-if^{xa_4a_5}}{2z_{45}}\left(\frac{\epsilon_4}{\epsilon_5}\right)^{\Delta_4-1} \times B(\Delta_4-1, \Delta_5-1) \times \tilde{\mathcal{A}}^{\phi}(1^{-a_1}_{\Delta_1}, 2^{-a_2}_{\Delta_2}, 3^{a_3}_{\Delta_3}, 5^{x}_{\Delta_4+\Delta_5-1}). \quad (35)$$

## Leading order OPE of two gluon primaries :

$$\mathcal{O}^{a_4,\epsilon_4}_{\Delta_4,+}(z_4,\bar{z}_4)\mathcal{O}^{a_5,\epsilon_5}_{\Delta_5,+}(z_5,\bar{z}_5) \sim \left(-\frac{1}{2z_{45}}\right)\left(\frac{\epsilon_4}{\epsilon_5}\right)^{\Delta_4-1} B\left(\Delta_4-1, \Delta_5-1\right)\left(if^{xa_4a_5}\right)\mathcal{O}^{x,\epsilon_5}_{\Delta_4+\Delta_5-1,+}(z_5,\bar{z}_5). \quad (36)$$

## OPE factorization at subleading order, $\mathcal{O}(1)$

$$\tilde{\mathcal{A}}^{\phi}(1^{-a_1}_{\Delta_1}, 2^{-a_2}_{\Delta_2}, 3^{+a_3}_{\Delta_3}, 4^{+a_4}_{\Delta_4}, 5^{+a_5}_{\Delta_5})\Big|_{\mathcal{O}(1)}$$
$$= \frac{1}{2} \times \left(\frac{\epsilon_4}{\epsilon_5}\right)^{\Delta_4-1} B(\Delta_4-1, \Delta_5-1)\Bigg[-\frac{(\Delta_4-1)}{(\Delta_4+\Delta_5-2)}(if^{xa_4a_5})\mathcal{L}_{-1}(5)\tilde{A}^{\phi}(1^{-a_1}_{\Delta_1}, 2^{-a_2}_{\Delta_2}, 3^{+a_3}_{\Delta_3}, 5^{+x}_{\Delta_4+\Delta_5-1})$$
$$+ \frac{(\Delta_5-1)}{(\Delta_4+\Delta_5-2)}\mathscr{J}^{a_4}_{-1}(5)\tilde{A}^{\phi}(1^{-a_1}_{\Delta_1}, 2^{-a_2}_{\Delta_2}, 3^{+a_3}_{\Delta_3}, 5^{+a_5}_{\Delta_4+\Delta_5-1})$$
$$+ \frac{(\Delta_4-1)}{(\Delta_4+\Delta_5-2)}\mathscr{J}^{a_5}_{-1}(5)\tilde{A}^{\phi}(1^{-a_1}_{\Delta_1}, 2^{-a_2}_{\Delta_2}, 3^{+a_3}_{\Delta_3}, 5^{+a_4}_{\Delta_4+\Delta_5-1})\Bigg]. \qquad (37)$$

## $\mathcal{O}(1)$ term in the OPE of two gluon primaries

$$\mathcal{O}^{a_4\epsilon_4}_{\Delta_4,+}(z_4,\bar{z}_4)\mathcal{O}^{a_5\epsilon_5}_{\Delta_5,+}(z_5,\bar{z}_5)\Big|_{\mathcal{O}(1)} \sim \frac{1}{2}\times\left(\frac{\epsilon_4}{\epsilon_5}\right)^{\Delta_4-1}\times B(\Delta_4-1,\Delta_5-1)\Bigg[-\frac{(\Delta_4-1)}{(\Delta_4+\Delta_5-2)}if^{xa_4a_5}L_{-1}+$$
$$\left(\frac{(\Delta_5-1)}{(\Delta_4+\Delta_5-2)}\delta^{a_4y}\delta^{a_5x} + \frac{(\Delta_4-1)}{(\Delta_4+\Delta_5-2)}\delta^{a_5y}\delta^{a_4x}\right)j^{y}_{-1}\Bigg]\mathcal{O}^{x,\epsilon_5}_{\Delta_4+\Delta_5-1}(z_5,\bar{z}_5). \qquad (38)$$

Where the actions of the above operators are given by,

$$\mathscr{J}^{a_4}_{-1}(5)\tilde{A}^{\phi}(1^{-a_1}_{\Delta_1}, 2^{-a_2}_{\Delta_2}, 3^{+a_3}_{\Delta_3}, 5^{+a_5}_{\Delta_4+\Delta_5-1})$$

$$= \sum_{k=1}^{3} \frac{T^{a_4}_k}{z_k - z_5} \langle \mathcal{O}^{a_1}_{\Delta_1,-}(z_1,\bar{z}_1)\mathcal{O}^{a_2}_{\Delta_2,-}(z_2,\bar{z}_2)\mathcal{O}^{a_3}_{\Delta_3,+}(z_3,\bar{z}_3)\mathcal{O}^{a_5}_{\Delta_4+\Delta_5-1,+}(z_5,\bar{z}_5)\rangle_{\phi} \tag{39}$$

$$\mathscr{J}^{a_5}_{-1}(5)\tilde{A}^{\phi}(1^{-a_1}_{\Delta_1}, 2^{-a_2}_{\Delta_2}, 3^{+a_3}_{\Delta_3}, 5^{+a_4}_{\Delta_4+\Delta_5-1})$$

$$= \sum_{k=1}^{3} \frac{T^{a_5}_k}{z_k - z_5} \langle \mathcal{O}^{a_1}_{\Delta_1,-}(z_1,\bar{z}_1)\mathcal{O}^{a_2}_{\Delta_2,-}(z_2,\bar{z}_2)\mathcal{O}^{a_3}_{\Delta_3,+}(z_3,\bar{z}_3)\mathcal{O}^{a_4}_{\Delta_4+\Delta_5-1,+}(z_5,\bar{z}_5)\rangle_{\phi} \tag{40}$$

and

$$\mathcal{L}_{-1}(5)\tilde{A}^{\phi}(1^{-a_1}_{\Delta_1}, 2^{-a_2}_{\Delta_2}, 3^{+a_3}_{\Delta_3}, 5^{+x}_{\Delta_4+\Delta_5-1}) = \partial_{z_5}\tilde{A}^{\phi}(1^{-a_1}_{\Delta_1}, 2^{-a_2}_{\Delta_2}, 3^{+a_3}_{\Delta_3}, 5^{+x}_{\Delta_4+\Delta_5-1}). \tag{41}$$

# BG equations in massive scalar background

- BG equations for MHV gluon amplitude in pure Yang-Mills theory was first found by Banerjee-Ghosh in [2011.00017].
- In presence of massive sacalar background,we have seen that subleading soft gluon theorem and ope factorization at $\mathcal{O}(1)$ also do not change.
  $\Rightarrow$We expect that scattering amplitude in the present theory will also satisfy the same set of BG equations.
- The presence of massive scalar background breaks the translational and scale invarinace ,none of which are required for leading and sub-leading soft gluon theorem.

## Solutions of BG equations :

We have checked explicitly that 3-point MHV amplitude in massive scalar background satisfies BG equations.

The most general form of the color-ordered $SL(2,\mathbb{C})$ covariant 3-point amplitude is,

$$\widetilde{\mathcal{M}}_3(1^-_{\Delta_1}, 2^-_{\Delta_2}, 3^+_{\Delta_3})$$
$$= C(\Delta_1,\Delta_2,\Delta_3) z_{12}^{h_3-h_1-h_2} z_{13}^{h_2-h_1-h_3} z_{23}^{h_1-h_2-h_3} \bar{z}_{12}^{\bar{h}_3-\bar{h}_1-\bar{h}_2} \bar{z}_{13}^{\bar{h}_2-\bar{h}_1-\bar{h}_3} \bar{z}_{23}^{\bar{h}_1-\bar{h}_2-\bar{h}_3} \tag{42}$$

## Decoupling equations :

$$\left(\partial_3 - \frac{\Delta_3}{z_{13}} - \frac{1}{z_{23}}\right)\widetilde{\mathcal{M}}_3(1^-_{\Delta_1}, 2^-_{\Delta_2}, 3^+_{\Delta_3}) + \epsilon_1\epsilon_3 \frac{\Delta_1 - \sigma_1 - 1 + \bar{z}_{13}\bar{\partial}_1}{z_{13}} \widetilde{\mathcal{M}}_3(1^-_{\Delta_1-1}, 2^-_{\Delta_2}, 3^+_{\Delta_3+1}) = 0$$
$$\left(\partial_3 - \frac{\Delta_3}{z_{23}} - \frac{1}{z_{13}}\right)\widetilde{\mathcal{M}}_3(1^-_{\Delta_1}, 2^-_{\Delta_2}, 3^+_{\Delta_3}) + \epsilon_2\epsilon_3 \frac{\Delta_2 - \sigma_2 - 1 + \bar{z}_{23}\bar{\partial}_2}{z_{23}} \widetilde{\mathcal{M}}_3(1^-_{\Delta_1}, 2^-_{\Delta_2-1}, 3^+_{\Delta_3+1}) = 0 \tag{43}$$

Recursion relations of the coefficients:

$$C(\Delta_1-1,\Delta_2,\Delta_3+1)=\epsilon_1\epsilon_3\frac{(\Delta_1-\Delta_2-\Delta_3+1)}{(\Delta_3-\Delta_1-\Delta_2-1)}C(\Delta_1,\Delta_2,\Delta_3)$$
$$C(\Delta_1,\Delta_2-1,\Delta_3+1)=\epsilon_2\epsilon_3\frac{(\Delta_2-\Delta_1-\Delta_3+1)}{(\Delta_3-\Delta_1-\Delta_2-1)}C(\Delta_1,\Delta_2,\Delta_3) \tag{44}$$

solution:

$$\begin{aligned}&C(\Delta_1,\Delta_2,\Delta_3)\\&=\mathcal{N}_3\Gamma\left(\frac{\Delta_1+\Delta_2-\Delta_3+3}{2}\right)\Gamma\left(\frac{\Delta_1-\Delta_2+\Delta_3-3}{2}\right)\Gamma\left(-\frac{\Delta_1+\Delta_2+\Delta_3-1}{2}\right)f(\beta)\end{aligned} \tag{45}$$

where, $\beta=\sum_{i=1}^{3}\Delta_i$ and $\mathcal{N}_3=\prod_{j=1}^{3}(-i\epsilon_j)^{\Delta_j-\sigma_j}$.

# Summary and Discussions

Summary :

- We have shown that sub-leading soft gluon theorem doesn't change for this chirally coupled YM theory.
- We have also extracted the ope of two outgoing positive helicity gluon primaries at subleading order.
- OPE factorization $\mathcal{O}(1)$ is completely determined in terms of the descendants of $SL(2,\mathbb{C})$ and the leading soft symetry algebra.
- We have checked that this theory is also a solution of BG equation $\Rightarrow$ BG equations don't have unique solutions.

Discussion for future exploration:

- In graviton scattering amplitude if someone breaks few of these symmetries (e.g translation and scaling) then situation will change completely.
- It would be interesting to ask what happpens to the null equations for MHV graviton scattering in this scenario.

Thank You !!