An Infinite Family of S Invariant Theories on the Celestial Sphere

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Based on :
"All OPEs invariant under the infinite symmetry algebra for gluons on the celestial sphere"
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The S algebra is obtained from the singular part of the OPE between two **positive helicity outgoing gluons** on the celestial sphere :

Guevara, Himwich, Pate and Strominger '21

$$\mathcal{O}^{a,+}_{\Delta_1}(z_1,\bar{z}_1)\mathcal{O}^{b,+}_{\Delta_2}(z_2,\bar{z}_2) \sim -\frac{if^{ab}{}_c}{z_{12}}\sum_{n=0}^{\infty} B(\Delta_1+n-1,\Delta_2-1)\frac{\bar{z}_{12}^n}{n!}\bar{\partial}_2^n\mathcal{O}^{c,+}_{\Delta_1+\Delta_2-1}(z_2,\bar{z}_2). \quad (1)$$

Soft gluons :

$$R^{k,a}(z,\bar{z}) = \lim_{\Delta\to k}(\Delta-k)O^{a,+}_{\Delta}(z,\bar{z}), \qquad k=1,0,-1,\ldots \quad (2)$$

Holomorphic soft gluon currents :

$$R^{k,a}(z,\bar{z}) = \sum_{n=\frac{k-1}{2}}^{\frac{1-k}{2}} \frac{R^{k,a}_n(z)}{\bar{z}^{n+\frac{k-1}{2}}} \quad (3)$$

Modes of the Holomorphic currents :

$$R_n^{k,a}(z) = \sum_{\alpha \in \mathbb{Z} - \frac{k+1}{2}} \frac{R_{\alpha,n}^{k,a}}{z^{\alpha + \frac{k+1}{2}}} \tag{4}$$

Algebra :

$$[R_{\alpha,m}^{k,a}, R_{\beta,n}^{l,b}] = -if^{ab}{}_{c} \frac{(\frac{1-k}{2} - m + \frac{1-l}{2} - n)!}{(\frac{1-k}{2} - m)!(\frac{1-l}{2} - n)!} \frac{(\frac{1-k}{2} + m + \frac{1-l}{2} + n)!}{(\frac{1-k}{2} + m)!(\frac{1-l}{2} + n)!} R_{\alpha+\beta,m+n}^{k+l-1,c} \tag{5}$$

Redefinition :

$$S_{\alpha,m}^{q,a} = (q - m - 1)!(q + m - 1)! R_{\alpha,m}^{3-2q,a} \tag{6}$$

S Algebra :

$$[S_{\alpha,m}^{p,a}, S_{\beta,n}^{q,b}] = -if^{abc} S_{\alpha+\beta,m+n}^{p+q-1,c} \tag{7}$$

Guevara, Himwich, Pate and Strominger '21

General structure of the OPE is

$$\begin{aligned}
&\mathcal{O}^{a,+}_{\Delta_1}(z_1,\bar{z}_1)\mathcal{O}^{b,+}_{\Delta_2}(z_2,\bar{z}_2) \\
&= -\frac{if^{ab}{}_c}{z_{12}}\sum_{n=0}^{\infty} B(\Delta_1+n-1,\Delta_2-1)\frac{\bar{z}_{12}^n}{n!}\bar{\partial}_2^n\mathcal{O}^{c,+}_{\Delta_1+\Delta_2-1}(z_2,\bar{z}_2) \\
&\qquad + \sum_{p,q=0}^{\infty}\sum_{k=1}^{\tilde{n}_{p,q}} z_{12}^p\bar{z}_{12}^q C^k_{p,q}(\Delta_1,\Delta_2)\tilde{\mathcal{O}}^{ab}_{k,p,q}(z_2,\bar{z}_2).
\end{aligned} \tag{8}$$

Our goal is to find :

- ✍ The OPE coefficients $C^k_{p,q}$
- ✍ the S-algebra descendants $\tilde{\mathcal{O}}^{ab}_{k,p,q}$ of a positive helicity soft gluon at $\mathcal{O}(1)$
- ✍ to argue that there exists a discrete infinite number of S invariant OPEs which correspond to infinite number of theories
- ✍ Also to find the KZ-type null states of such theories.

STRATEGY :

- ✍ We consider S invariant theories (OPEs) for all of which S-algebra is universal $\Rightarrow$ Existence of a Master OPE .
- ✍ We know that tree-level MHV sector of the pure YM theory is an example of such S invariant theories.
- ✍ This Master OPE inserted in a MHV gluon scattering amplitude $\Rightarrow$ known MHV OPE.
- ✍ $\boxed{\textbf{Master OPE} = \textbf{MHV-sector OPE} + \mathbf{R}}$
- ✍ R should vanish inside MHV scattering amplitude $\Rightarrow$ R is a lin. combination of MHV null states.
- ✍ R consists only non-singular terms.

Null states are important !

Using the above arguments we can rewrite (8) as,

$$\boxed{\begin{aligned}\mathcal{O}^{a,+}_{\Delta_1}(z_1,\bar{z}_1)\mathcal{O}^{b,+}_{\Delta_2}(z_2,\bar{z}_2)\big|_{\text{Any Theory}} &= \mathcal{O}^{a,+}_{\Delta_1}(z_1,\bar{z}_1)\mathcal{O}^{b,+}_{\Delta_2}(z_2,\bar{z}_2)\big|_{\text{MHV}} \\ &+ \sum_{p,q=0}^{\infty} z_{12}^{p}\bar{z}_{12}^{q}\sum_{i=1}^{\tilde{n}_{p,q}} \tilde{C}^{k}_{p,q}(\Delta_1,\Delta_2)M^{a,b}_{k,p,q}(\Delta_1,\Delta_2,z_2,\bar{z}_2).\end{aligned}} \tag{9}$$

- ☆ $M^{a,b}_{k,p,q}$ are the MHV null states at $\mathcal{O}(z_{12}^{p}\bar{z}_{12}^{q})$.
- ☆ We perform the analysis at $\mathcal{O}(z_{12}^{0}\bar{z}_{12}^{0})$.

MHV NULL STATES AT $\mathcal{O}(1)$

The general null state at $\mathcal{O}(1)$ in the MHV-sector is given by

$$\Psi_j^{ab}(\Delta) = R^{-j,a}_{\frac{j-1}{2},\frac{j+1}{2}} \mathcal{O}^b_{\Delta+j,+} - \frac{(-1)^j j}{\Gamma(j+2)} \frac{\Gamma(\Delta+j-1)}{\Gamma(\Delta-2)} R^{1,a}_{-1,0} \mathcal{O}^b_{\Delta-1,+} - \frac{(-1)^j}{\Gamma(j+1)} \frac{\Gamma(\Delta+j-1)}{\Gamma(\Delta-1)} R^{0,a}_{-1/2,1/2} \mathcal{O}^b_{\Delta,+} \tag{10}$$

where $j = 1, 2, 3, ...$

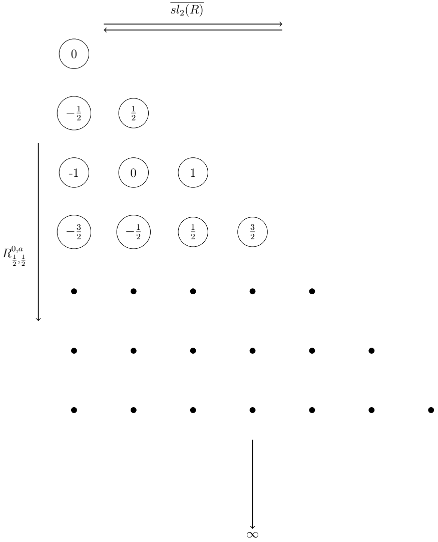
Let's consider the following basis :

$$M_k^{ab}(\Delta) = \sum_{i=1}^{k} \frac{1}{\Gamma(k-i+1)} \frac{\Gamma(\Delta+k-1)}{\Gamma(\Delta+i-1)} \Psi_i^{ab}(\Delta). \tag{11}$$

Also define,

$$M_k^a(\Delta) = f^{abc} M_k^{bc}. \tag{12}$$

✧ Focus only on the generators $(R^{1,a}_{n,0}, R^{0,a}_{\frac{1}{2},\frac{1}{2}}, H^0_{0,0}, H^0_{0,\pm 1})$ to study the action of S-algebra on the null states.



Action of the Leading Soft Gluon mode:

$$R^{1,a}_{0,0} M^{bc}_{k}(\Delta) = -if^{abd} M^{dc}_{k}(\Delta) - if^{acd} M^{bd}_{k} \tag{13}$$

$$R^{1,a}_{n,0} M^{bc}_{k}(\Delta) = 0, n > 0 \tag{14}$$

Action of the Subleading Soft Gluon mode :

$$\begin{aligned} &[R^{0,a}_{1/2,1/2}, M^{bc}_{k}(\Delta)] \\ &= -if^{abd}(k+2) M^{dc}_{k+1}(\Delta - 1) \\ &\qquad + (\Delta + k - 2)\Big\{ if^{acd} M^{bd}_{k}(\Delta - 1) + if^{abd} M^{dc}_{k}(\Delta - 1) \Big\} . \end{aligned} \tag{15}$$

Building up S invariant OPE

Observation :

- Consider the following set of null states,

$$M_k^{bc}(\Delta), k = 1, 2, 3, ..., n. \tag{16}$$

- Action of $R^{0,a}_{1/2,1/2}$ on the null states is closed if we set

$$M_{k+1}^{ab}(\Delta) = 0, k \geq n \geq 0. \tag{17}$$

Inference :

- We can get an S invariant OPE if we consider the finite set of null states (16).

S invariant OPEs at $\mathcal{O}(1)$:

$$\boxed{\begin{aligned}&\mathcal{O}^a_{\Delta_1,+}(z,\bar{z})\mathcal{O}^b_{\Delta_2,+}(0,0)\big|_{\mathcal{O}(1)}\\ &= \mathcal{O}^a_{\Delta_1,+}(z,\bar{z})\mathcal{O}^b_{\Delta_2,+}(0,0)\Big|^{MHV}_{\mathcal{O}(1)} + \sum_{k=1}^{n} B(\Delta_1 + k, \Delta_2 - 1) M_k^{ab}(\Delta_1 + \Delta_2)\end{aligned}} \tag{18}$$

S invariance of the OPE

Action of $R^{0,a}_{1/2,1/2}$:

$$R^{0,x}_{\frac{1}{2},\frac{1}{2}}\left(\mathcal{O}^{a}_{\Delta_1,+}(z,\bar{z})\mathcal{O}^{b,+}_{\Delta_2,+}(0,0)\right)\Big|_{\mathcal{O}(1)} - R^{0,x}_{\frac{1}{2},\frac{1}{2}}\Bigg[\mathcal{O}^{a}_{\Delta_1,+}(z,\bar{z})\mathcal{O}^{b}_{\Delta_2,+}(0,0)\Big|^{MHV}_{\mathcal{O}(1)} + \sum_{k=1}^{n} B(\Delta_1+k,\Delta_2-1)M^{ab}_{k}(\Delta_1+\Delta_2)\Bigg] = if^{xay}(n+2)B(\Delta_1+n,\Delta_2-1)M^{yb}_{n+1}(\Delta_1+\Delta_2-1) = 0. \tag{19}$$

- One can also verify that OPE (18) is invariant under the action of $R^{1,a}_{n,0}$ and $H^{0}_{0,1}$.
- Truncated OPE (18) is invariant under the S algebra.

Infinite family of S invariant theories :

- We have shown that the following set of equations are S invariant.

$$M^{ab}_{k+1}(\Delta) = 0, k \geq n \geq 0. \tag{20}$$

- We can truncate the OPE at $\mathcal{O}(1)$ at an arbitrary n in S invariant way.
- But the S invariance does not fix the value of integer n.
- Hence, different choices of the integer n give rise to a discrete infinite family of S-invariant OPEs.
- Each of these consistent OPEs correspond to a S invariant theory.
- We do not know the Lagrangian description of these theories except for for the MHV YM and the self-dual Yang-Mills theory.

- KZ-type null states involve the L_{-1} descendants on the CS^2.
- We obtain KZ-type null states by using OPE commutivity and taking different soft limits.

$$\mathcal{O}^{a,+}_{\Delta_1}(z_1,\bar{z}_1)\mathcal{O}^{b,+}_{\Delta_2}(z_2,\bar{z}_2)=\mathcal{O}^{b,+}_{\Delta_2}(z_2,\bar{z}_2)\mathcal{O}^{a,+}_{\Delta_1}(z_1,\bar{z}_1). \tag{21}$$

- **KZ-type null states :**

$$K^a(\Delta)=\xi^a(\Delta)-i\sum_{k=1}^{n}M^a_k(\Delta+1), \tag{22}$$

where

$$\xi^a(\Delta)=C_A L_{-1}\mathcal{O}^{a,+}_{\Delta}-(\Delta+1)R^{1,b}_{-1,0}R^{1,b}_{0,0}\mathcal{O}^{a,+}_{\Delta}-R^{0,b}_{-\frac{1}{2},\frac{1}{2}}R^{1,b}_{0,0}\mathcal{O}^{a,+}_{\Delta+1}. \tag{23}$$

Thank you for your attention !!