Left-Right Symmetric Model

$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{\tilde{Y}=B-L}$

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References

1.Left-Right Symmetric Model Building - Eric Corrigan

2.Introduction to Left-Right Symmetric Models -W. Grimus

3.Unification and Supersymmetry - R.N.Mohapatra

4.arXiv:2001.03104v1   (for electric charge assignment)

5.arXiv:0712.4218v1

Particle Content of SM

Fermions

$$L_L^i = \begin{pmatrix} \nu_L^i \\ e_L^i \end{pmatrix}; \; e_R^i; \quad Q_L^i = \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix}; \; u_R^i, d_R^i \quad (i = 1, 2, 3)$$

Higgs Sector:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

Gauge Bosons:

W_μ^+; W_μ^-; Z_μ A_μ; G_μ^a (a=1,...,8)

Quick Review of SM

Gauge Group:

$SU(3)_C \times SU(2)_L \times U(1)_Y$

Covariant Derivatives:

For LH Quarks : $D_\mu = \partial_\mu - igW^i_\mu \frac{\sigma^i}{2} - ig' \frac{Y^q_L}{2} B_\mu - ig_{qcd} G^a_\mu \frac{T^a}{2}$

For RH Quarks : $D_\mu = \partial_\mu - ig' \frac{Y^q_R}{2} B_\mu - ig_{qcd} G^a_\mu \frac{T^a}{2}$

For LH Leptons : $D_\mu = \partial_\mu - igW^i_\mu \frac{\sigma^i}{2} - ig' \frac{Y^l_L}{2} B_\mu$

For RH Leptons : $D_\mu = \partial_\mu - ig' \frac{Y^l_R}{2} B_\mu$

For Higgs doublet : $D_\mu = \partial_\mu - igW^i_\mu \frac{\sigma^i}{2} - ig' \frac{Y_\phi}{2} B_\mu$

Spontaneous Symmetry Breaking:

$$SU(2)_L \times U(1)_Y \longrightarrow U(1)_Q$$

$$\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

$\Rightarrow$ 3 massive Gauge Bosons (W^+, W^-, Z) and one massless gauge boson, Photon (A_μ)

SM Lagrangian

$$\mathcal{L}_{SM} = \bar{L}_i^L \not{D} L_i^L + \bar{e}_i^R \not{D} e_i^R + \bar{Q}_i^L \not{D} Q_i^L + \bar{u}_i^R \not{D} u_i^R + \bar{d}_i^R \not{D} d_i^R +$$
$$(D_\mu \Phi)^\dagger (D_\mu \Phi) - \frac{1}{4}(W_{\mu\nu i} W^{\mu\nu i}) - \frac{1}{4}(G_{\mu\nu a} G^{\mu\nu a})$$
$$-\frac{1}{4}(B_{\mu\nu} B^{\mu\nu}) + \mu^2(\Phi^\dagger \Phi) - (\Phi^\dagger \Phi)^2 +$$
$$(y_i^e \bar{L}_i \Phi e_i^R + h.c) +$$
$$(y_i^u \bar{Q}_L \tilde{\Phi} u_i^R + y_i^d \bar{Q}_L \Phi d_i^R + h.c)$$

Where i runs over generation indices,i=1,2,3.

Shortcomings of SM

Though SM has been very successful in describing fundamental interactions(except gravity) in Nature very well,it cannot explain the following important things,

- No appealing Dark Matter condidates
- Huge hierarchy in particle spectra(like,why t quark is 35,000 times heavier than that of d quark)
- Why neutrinos are so extremely light
- Why are the Weak Forces not parity conserving while all other forces in Nature are or are they parity conserving at more fundamental level?
- the large number of free parameters(around 30),etc...

Motivation:

By this extension of SM to LRSM we would like to resolve the following unsatisfactory features of SM :

- While SM prefers one handedness over the other, LRSM models restore this symmetry
- gives explanation for small mass of LH neutrinos by introducing heavy Majorana RH neutrinos
- In SM, the hypercharge Y is an arb. quantum number,In LRSM it can be related to less arbitrary quantity $B - L$

Fermion Sector

Fermions:

$$\mathsf{L}^i_L = \begin{pmatrix} \nu^i_L \\ e^i_L \end{pmatrix} \; ; \; \mathsf{L}^i_R = \begin{pmatrix} \nu^i_R \\ e^i_R \end{pmatrix} \; ; \; \mathsf{Q}^i_L = \begin{pmatrix} u^i_L \\ d^i_L \end{pmatrix} \; ; \; \mathsf{Q}^i_R = \begin{pmatrix} u^i_R \\ d^i_R \end{pmatrix}$$

where i=1,2,3(runs over family indices).

Gauge Group:

$$G = SU(2)_L \times SU(2)_R \times U(1)_{\tilde{Y}}$$

$$\tilde{Y} = B - L$$

B=Baryon Number ; L=Lepton Number

Fermion Sector

irrep. of G is denoted by $(d_L, d_R, \tilde{Y})$

where, $\quad Q = T_{3L} + T_{3R} + \frac{1}{2}\tilde{Y}$

$$Quarks : Q_L(2, 1, \tfrac{1}{3}), \quad Q_R(1, 2, \tfrac{1}{3})$$

$$Leptons : L_L(2, 1, -1), \quad L_R(1, 2, -1)$$

$\underline{Gauge\ \ Transformations}:$

$$\Psi'_L = e^{-ig'\frac{\tilde{Y}}{2}\alpha(x)} e^{-ig_L\frac{\bar{\tau}}{2}\cdot\bar{\theta}(x)}\Psi_L$$

$$\Psi'_R = e^{-ig'\frac{\tilde{Y}}{2}\alpha(x)} e^{-ig_R\frac{\bar{\tau}}{2}\cdot\bar{\theta}(x)}\Psi_R$$

$$(\tfrac{\bar{\tau}}{2}\cdot\bar{W}_{L,R\mu}) \longrightarrow U_{L,R}(\tfrac{\bar{\tau}}{2}\cdot\bar{W}_{L,R\mu})U^\dagger_{L,R} + \frac{i}{g_{L,R}}(\partial_\mu U_{L,R})U^\dagger_{L,R}$$

$$B_\mu \longrightarrow B_\mu + \frac{1}{g'}\partial_\mu\alpha(x)$$

Fermion Sector

Covariant Derivatives:

$$\mathsf{D}_\mu \Psi_L = (\partial_\mu - i g_L \frac{\bar{\tau}}{2} \cdot \bar{W}_{L\mu} - i g' \frac{B-L}{2} B_\mu) \Psi_L$$

$$D_\mu \Psi_R = (\partial_\mu - i g_R \frac{\bar{\tau}}{2} \cdot \bar{W}_{R\mu} - i g' \frac{B-L}{2} B_\mu) \Psi_R$$

$\underline{Fermionic\ \ Gauge\ \ Lagragian}:$

$$\mathcal{L}_f = \sum_{\psi=Q,f} [\bar{\Psi}_L i \gamma^\mu (\partial_\mu - i g_L \frac{\bar{\tau}}{2} \cdot \bar{W}_{L\mu} - i g' \frac{B-L}{2} B_\mu) \Psi_L + \bar{\Psi}_R i \gamma^\mu (\partial_\mu - i g_R \frac{\bar{\tau}}{2} \cdot \bar{W}_{R\mu} - i g' \frac{B-L}{2} B_\mu) \Psi_R]$$

$\underline{SU(2)_L\ Gauge\ Bosons}:$

$W^1_{L\mu}, W^2_{L\mu}, W^3_{L\mu}$

$W^{+,-}_L, Z_L$

$\underline{SU(2)_R\ Gauge\ Bosons}:$

$W^1_{R\mu}, W^2_{R\mu}, W^3_{R\mu}$

$W^{+,-}_R, Z_R$

Fermion Sector

$$\overline{\Psi}\not{D}\Psi = \overline{\Psi}_L i\gamma^\mu(\partial_\mu - ig_L\frac{\bar{\tau}}{2}\cdot\bar{W}_{L\mu} + i\frac{g'}{2}B_\mu)\Psi_L$$

$$+$$

$$\overline{q}_L i\gamma^\mu(\partial_\mu - ig_L\frac{\bar{\tau}}{2}\cdot\bar{W}_{L\mu} - i\frac{g'}{6}B_\mu)q_L$$

$$+$$

$$\overline{\Psi}_R i\gamma^\mu(\partial_\mu - ig_R\frac{\bar{\tau}}{2}\cdot\bar{W}_{R\mu} + i\frac{g'}{2}B_\mu)\Psi_R$$

$$+$$

$$\overline{q}_R i\gamma^\mu(\partial_\mu - ig_R\frac{\bar{\tau}}{2}\cdot\bar{W}_{R\mu} - i\frac{g'}{6}B_\mu)q_R$$

Fermion Sector

$$\mathcal{L}_{Fermion-Gauge} = g_L[\overline{\Psi}_L\gamma^\mu\frac{\bar{\tau}}{2}\Psi_L + \overline{q}_L\gamma^\mu\frac{\bar{\tau}}{2}q_L]\cdot\bar{W}_{L\mu}$$

$$+$$

$$g_R[\overline{\Psi}_R\gamma^\mu\frac{\bar{\tau}}{2}\Psi_R + \overline{q}_R\gamma^\mu\frac{\bar{\tau}}{2}q_R]\cdot\bar{W}_{R\mu}$$

$$+$$

$$g'[\frac{1}{6}\bar{q}\gamma^\mu q - \frac{1}{2}\bar{\Psi}\gamma^\mu\Psi]B_\mu$$

under Parity operation P: $\Psi_L \leftrightarrow \Psi_R;\ \ q_L \leftrightarrow q_R;\ \ W_L \leftrightarrow W_R$

$\Rightarrow$ $$g_L = g_R = g$$

Scheme of Symmetry Breaking:

(1)We wish to break $SU(2)_L \times U(1)_Y \longrightarrow U(1)_Q$ as in SM

(2)We must first break $SU(2)_R \times U(1)_{\tilde{Y}} \longrightarrow U(1)_Y$

$\Downarrow$

an extended Higgs Sector

(3)Phenomenology $\Longrightarrow$ happens at higher scale.

$\Downarrow$

high mass for new W_R and Z_R bosons

Higgs Sector and Symmetry Breaking

Higgs Fields and Charge Assignments :

(1) $SU(2)_R \times U(1)_{\tilde{Y}} \longrightarrow U(1)_Y$

Triplets, $\Delta_{L,R} = \frac{1}{\sqrt{2}}\tau_a\delta_a = \frac{1}{\sqrt{2}}\begin{pmatrix} \delta_3 & \delta_1 - i\delta_2 \\ \delta_1 + i\delta_2 & -\delta_3 \end{pmatrix}_{L,R}$

(2) $\mathrm{SU}(2)_L \times U(1)_Y \longrightarrow U(1)_Q$

Bidoublet, $\Phi = \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix}$

Higgs Sector and Symmetry Breaking

Gauge Transformation of Higgs Fields:

$$\Phi \longrightarrow U_L \Phi U_R^\dagger \qquad (2{,}2^*, 0)$$

$$\Delta_L \longrightarrow e^{-i\alpha}\; U_L \Delta_L U_L^\dagger \qquad (3, 1, 2)$$

$$\Delta_R \longrightarrow e^{-i\alpha}\; U_R \Delta_R U_R^\dagger \qquad (1, 3, 2)$$

Charges are assigned as,

$$Q\Phi = [\tfrac{1}{2}\tau_3, \Phi] = \begin{pmatrix} 0 \cdot \phi_{11} & +1 \cdot \phi_{12} \\ -1 \cdot \phi_{21} & 0 \cdot \phi_{22} \end{pmatrix}$$

$$Q\Delta = [\tfrac{1}{2}\tau_3, \Delta] + 1 \cdot \Delta = \begin{pmatrix} \Delta_{11} & +2 \cdot \Delta_{12} \\ 0 \cdot \Delta_{21} & \Delta_{22} \end{pmatrix}$$

Higgs Sector and Symmetry Breaking

Therefore Higgs Fields can be defined as,

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}; \;\; \Delta_{L,R} = \begin{pmatrix} \frac{\delta^+}{\sqrt{2}} & \delta^{++} \\ \delta^0 & -\frac{\delta^+}{\sqrt{2}} \end{pmatrix}_{L,R}$$

Vacuum Expectation Values:

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} ke^{i\alpha_k} & 0 \\ 0 & k'e^{i\alpha_{k'}} \end{pmatrix}; \;\; \langle\Delta_L\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_L e^{i\beta_L} & 0 \end{pmatrix}$$

$$\langle\Delta_R\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_R e^{i\beta_R} & 0 \end{pmatrix}$$

Two of the phases can be gauge-transformed away to give,

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} k & 0 \\ 0 & k' e^{i\alpha_{k'}} \end{pmatrix}; \;\; \langle \Delta_L \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_L e^{i\beta_L} & 0 \end{pmatrix}$$

$$\langle \Delta_R \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}$$

<u>The Gauged Higgs Lagrangian</u>:

$$\mathcal{L}_{Higgs} = Tr[(D_\mu \Delta_L)^\dagger (D^\mu \Delta_L)] + Tr[(D_\mu \Delta_R)^\dagger (D^\mu \Delta_R)] + Tr[(D_\mu \Phi)^\dagger (D^\mu \Phi)]$$

$with,$ $D_\mu \Delta_{L,R} = \partial_\mu \Delta_{L,R} - \frac{ig}{2} [W^i_{L,R_\mu} \sigma^i, \Delta_{L,R}] - igB_\mu \Delta_{L,R}$

and $D_\mu \Phi = \partial_\mu \Phi - \frac{ig}{2} (W^i_{L,R_\mu} \sigma^i \Phi - \Phi W^i_{L,R_\mu} \sigma^i)$

Masses of W bosons:

Evaluating the above Lagrangian with Higgs VEVs and collectting the bilinear terms with $W^{+-}_{L,R}$ we get,

$$\mathcal{L}^{W^{+-}}_{mass} = \begin{pmatrix} W^-_L & W^-_R \end{pmatrix} \begin{pmatrix} \frac{1}{4}g^2(k^2+k'^2+2v_L^2) & -\frac{1}{2}g^2kk'e^{-i\alpha_{k'}} \\ -\frac{1}{2}g^2kk'e^{i\alpha_{k'}} & \frac{1}{4}g^2(k^2+k'^2+2v_R^2) \end{pmatrix} \begin{pmatrix} W^+_L \\ W^+_R \end{pmatrix}$$

Where, $W^{+-}_{L,R} = \frac{1}{\sqrt{2}}(W^1_{L,R} \mp iW^2_{L,R})$

Diagonalizing this matrix and defining $W^{+-}_{1,2}$ as mass eigenstates we get,

$$M^2_{W_{1,2}} = \frac{g^2}{4}\left[k^2+k'^2+v_L^2+v_R^2 \mp \sqrt{(v_L^2-v_R^2)^2+4k^2k'^2}\right]$$

Invoking the hierarchy $v_R >> k$, $k' >> v_L$ we get

$$M_{W_1}^2 \approx \frac{g^2}{4} k_+^2 (1 - \frac{2k^2 k'^2}{k_+^2 v_R^2})$$

$$M_{W_2}^2 \approx \frac{g^2}{4} v_R^2$$

Mixing:

$$\begin{pmatrix} W_L^{+-} \\ W_R^{+-} \end{pmatrix} \begin{pmatrix} cos\zeta & -sin\zeta e^{i\lambda} \\ sin\zeta e^{-i\lambda} & cos\zeta \end{pmatrix} \begin{pmatrix} W_1^{+-} \\ W_2^{+-} \end{pmatrix}$$

$where \;\; \lambda = -\alpha_{k'}$ and,

$$tan\zeta = -\frac{kk'}{v_R^2}$$

Gauge Boson Masses

Masses of Z bosons :

$$\mathcal{L}^{Z}_{mass}=\frac{1}{2}\begin{pmatrix} W^3_{L\mu} & W^3_{R\mu} & B_\mu \end{pmatrix} \mathsf{M}_0^2 \begin{pmatrix} W_L^{3\mu} \\ W_R^{3\mu} \\ B^\mu \end{pmatrix}$$

Where,

$$\mathsf{M}_0^2 = \begin{pmatrix} \frac{g^2}{4}(k_+^2 + 4v_L^2) & -\frac{g^2}{4}k_+^2 & -gg'v_L^2 \\ -\frac{g^2}{4}k_+^2 & \frac{g^2}{4}(k_+^2 + 4v_R^2) & -gg'v_R^2 \\ -gg'v_L^2 & -gg'v_R^2 & g'^2(v_L^2 + v_R^2) \end{pmatrix}$$

Gauge Boson Masses

Diagonalizing we get, $M_A = 0$ and

$$M^2_{Z_{1,2}} = \frac{1}{2}\Bigg[g^2(k_+^2 + 2v_R^2) + 2g'^2 v^2_{R+}{}^{-}$$

$$\sqrt{g^4 k_+^4 + 4v_R^4(g^2+g'^2)^2 - 4g^2 g'^2 v^2 R k_+^2}\Bigg]$$

$$\mathrm{M}^2_{Z_1} \approx \frac{k_+^2 g^2}{4cos^2\theta_w}\left(1 - \frac{k_+^2}{4v_R^2 cos^4\theta_Y}\right)$$

$$\mathrm{M}^2_{Z_2} \approx g^2 v_R^2$$

Gauge Boson Masses

Mixing:

$$\begin{pmatrix} A \\ Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} s_W & c_W s_Y & c_W c_Y \\ -c_W & s_W s_Y & s_W c_Y \\ 0 & -c_Y & s_Y \end{pmatrix} \begin{pmatrix} W_L^3 \\ W_R^3 \\ B \end{pmatrix}$$

Mixing Angles:

$$s_W = \frac{g'}{\sqrt{g^2+2g'2}}$$

$$c_W = \sqrt{\frac{g^2+g'2}{g^2+2g'2}}$$

$$s_Y = \frac{g'}{\sqrt{g^2+g'2}}$$

$$c_Y = \frac{g}{\sqrt{g^2+g'2}}$$

Currents in Terms of Weak Eigenstates

Lepton Fields :

Charged Current Lagrangian:

$$\mathcal{L}^{c.c}_{gauge-lepton}= \frac{g}{\sqrt{2}}(\bar{\nu}'_L\gamma^{\mu}W^{+}_{L\mu}l'_L + \bar{l}'_L\gamma^{\mu}W^{-}_{L\mu}\nu'_L) + (L \rightarrow R)$$

Neutral Current Lagrangian:

$$\mathsf{L}^{n.c}_{gauge-lepton}=\bar{\nu}'_L\gamma^{\mu}\left[\frac{g}{2}W^3_{L\mu} - \frac{g'}{2}B_{\mu}\right]\nu'_L - \bar{l}'_L\gamma^{\mu}\left[\frac{g}{2}W^3_{L\mu} + \frac{g'}{2}B_{\mu}\right]l'_L + (L \rightarrow R)$$

Currents in Terms of Weak Eigenstates

Quark Fields :

Charged Current Lagrangian:

$$\mathcal{L}^{c.c}_{gauge-quark} = \frac{g}{\sqrt{2}}(\bar{U}\,'_L \gamma^\mu W^+_{L\mu} D'_L + \bar{D}'_L \gamma^\mu W^-_{L\mu} U'_L) + (L \to R)$$

Neutral Current Lagrangian:

$$\mathsf{L}^{n.c}_{gauge-quark} = \bar{U}'_L \gamma^\mu \left[\frac{g}{2} W^3_{L\mu} - \frac{g'}{6} B_\mu\right] \nu'_L + \bar{D}'_L \gamma^\mu \left[-\frac{g}{2} W^3_{L\mu} + \frac{g'}{6} B_\mu\right] D'_L + (L \to R)$$

Gauge Boson Interactions

$$\mathcal{L}_{gauge-gauge} = -\frac{1}{4}W_L^{i\mu\nu}W^i_{L\mu\nu} - \frac{1}{4}W_R^{i\mu\nu}W^i_{R\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu}$$

where,

$$W^{1\mu\nu}_{L,R} = \partial^\mu W^{1\nu}_{L,R} - \partial^\nu W^{1\mu}_{L,R} + g(W^{2\mu}_{L,R}W^{3\nu}_{L,R} - W^{3\mu}_{L,R}W^{2\nu}_{L,R}),$$

$$W^{2\mu\nu}_{L,R} = \partial^\mu W^{2\nu}_{L,R} - \partial^\nu W^{2\mu}_{L,R} + g(W^{3\mu}_{L,R}W^{1\nu}_{L,R} - W^{1\mu}_{L,R}W^{3\nu}_{L,R}),$$

$$W^{3\mu\nu}_{L,R} = \partial^\mu W^{3\nu}_{L,R} - \partial^\nu W^{3\mu}_{L,R} + g(W^{1\mu}_{L,R}W^{2\nu}_{L,R} - W^{2\mu}_{L,R}W^{1\nu}_{L,R}),$$

$$B^{\mu\nu} = \partial^\mu B^\nu - \partial^\nu B^\mu$$

Scalar Potential

$$
\begin{aligned}
&V(\Delta_L,\Delta_R,\Phi) = \\
&-\mu_1^2 Tr\Phi^\dagger\Phi - \mu_2^2\left[Tr\tilde{\Phi}\Phi^\dagger + Tr\tilde{\Phi^\dagger}\Phi\right] - \mu_3^2\left[Tr\Delta_L\Delta_L^\dagger + Tr\Delta_R\Delta_R^\dagger\right] \\
&\lambda_1(Tr\Phi^\dagger\Phi)^2 + \lambda_2\left[(Tr\tilde{\Phi}\Phi^\dagger)^2 + (Tr\tilde{\Phi^\dagger}\Phi)^2\right] + \lambda_3 Tr\tilde{\Phi}\Phi^\dagger Tr\tilde{\Phi}^\dagger\Phi \\
&\lambda_4 Tr\Phi^\dagger\Phi\left[Tr\tilde{\Phi}\Phi^\dagger + Tr\tilde{\Phi}^\dagger\Phi\right] + \rho_1\left[(Tr\Delta_L\Delta_L^\dagger)^2 + (Tr\Delta_R\Delta_R^\dagger)^2\right] \\
&\rho_2\left[Tr\Delta_L\Delta_L Tr\Delta_L^\dagger\Delta_L^\dagger + Tr\Delta_R\Delta_R Tr\Delta_R^\dagger\Delta_R^\dagger\right] + \\
&\rho_3 Tr\Delta_L\Delta_L^\dagger Tr\Delta_R\Delta_R^\dagger \\
&\rho_4\left[Tr\Delta_L\Delta_L Tr\Delta_R^\dagger\Delta_L^\dagger + Tr\Delta_L^\dagger\Delta_L^\dagger Tr\Delta_R\Delta_R\right] + \\
&\alpha_1 Tr\Phi^\dagger\Phi\left[Tr\Delta_L^\dagger\Delta_L + Tr\Delta_R^\dagger\Delta_R\right] \\
&+ \left[\alpha_2 e^{i\delta}\left[Tr\tilde{\Phi}\Phi^\dagger Tr\Delta_L\Delta_L^\dagger + Tr\tilde{\Phi}^\dagger\Phi Tr\Delta_R\Delta_R^\dagger\right] + h.c\right] \\
&+ \alpha_3(Tr\Phi\Phi^\dagger\Delta_L\Delta_L^\dagger + Tr\Phi^\dagger\Phi\Delta_R\Delta_R^\dagger) \\
&+ \beta_1(Tr\Phi\Delta_R\Phi^\dagger\Delta_L^\dagger + Tr\Phi^\dagger\Delta_L\Phi\Delta_R^\dagger) \\
&+ \beta_2(Tr\tilde{\Phi}\Delta_R\Phi^\dagger\Delta_L^\dagger + Tr\tilde{\Phi}^\dagger\Delta_L\Phi\Delta_R^\dagger) \\
&+ \beta_3(Tr\Phi\Delta_R\tilde{\Phi}^\dagger\Delta_L^\dagger + Tr\Phi^\dagger\Delta_L\tilde{\Phi}\Delta_R^\dagger)
\end{aligned}
$$

Where, $\tilde{\Phi} = \sigma_2\Phi^\star\sigma_2$

Scalar Potential

the parameters $\mu^2_{1,2,3}$, $\lambda_{1,2,3,4}$, $\rho_{1,2,3,4}$,$\alpha_{1,2,3}$ and $\beta_{1,2,3}$ are all real except α_2 which may be complex,indicated explicitly by inclusion of the cp violating phase δ.

Special case, $\delta = 0$ and $\alpha_{k'} = 0$ (known as manifest leftright symmetric limit).

Define, $\phi^0_{1,2} = \frac{1}{\sqrt{2}}(\phi^{0r}_{1,2} + i\phi^{0i}_{1,2})$ and similarly for $\delta^0_{L,R}$.

Scalar Potential

The potential after SSB is extremum at the VEVs,

$$\frac{\partial V}{\partial \phi_1^{0r}} = \frac{\partial V}{\partial \phi_2^{0r}} = \frac{\partial V}{\partial \delta_R^{0r}} = \frac{\partial V}{\partial \delta_L^{0r}} = \frac{\partial V}{\partial \phi_2^{0i}} = \frac{\partial V}{\partial \delta_L^{0i}} = 0$$

If we evaluate these equations at VEVs,one of the equations we get is,

$$\beta_2 = (-\beta_1 k k' - \beta_3 k'^2 + v_L v_R(2\rho_1 - \rho_3))/k^2$$

now in the case when,$\beta_1 = \beta_2 = \beta_3 = 0$,we get

$$v_L v_R(\rho_3 - 2\rho_1) = 0 \quad \text{(VEV see-saw relation)}$$

Scalar Potential

- Either v_L, v_R or $(\rho_3 - 2\rho_1)$ must be zero.
- v_R must not be zero because we have to break $SU(2)_R$ symmetry
- $(\rho_3 - 2\rho_1)$ is nonzero due to phenomenology because several new Higgs Bosons will have masses proportional to $(\rho_3 - 2\rho_1)$ upto first oder.
- only posibility left is $v_L = 0$ in this case.

Yukawa Sector

Quarks(Gauge Eigenstates):

$$\mathcal{L}_{Yukawa}=\bar{Q}_{Li}(h_{ij}\Phi + \tilde{h}_{ij}\tilde{\Phi})Q_{Rj} + h.c$$

where,
$\tilde{\Phi} \equiv \sigma_2 \Phi^\star \sigma_2$, i,j=1,2,3(generation indices)
under Parity operation P : $Q_L \longleftrightarrow Q_R, \quad \Phi \longleftrightarrow \Phi^\dagger$

Parity invariance of $\mathcal{L}_{Yukawa}$ $\Rightarrow$ h and $\tilde{h}$ are Hermitian matrices.

Mass Matrices:

$$M_U = \frac{1}{\sqrt{2}}(kh + k' e^{-i\alpha}\tilde{h})$$

$$M_D = \frac{1}{\sqrt{2}}(k' e^{i\alpha} h + k\tilde{h})$$

with hierarchy:

$$M_U \approx \frac{1}{\sqrt{2}} kh$$

$$M_D = \frac{1}{\sqrt{2}}(k' e^{i\alpha} h + k\tilde{h})$$

Yukawa Sector

Diagonalization and CKM matrices :

we can choose a basis such that $M_U = S_U \hat{M}_U$

where, $\hat{M}_U$=diag(m_u, m_c, m_t) and S_U=diag(s_u, s_c, s_t)

where, $s_q = \pm 1$

- M_D in general is not diagonal in this basis.
- Define CKM matrices to make M_D diagonal.

$$M_D = V_R^{CKM} \hat{M}_D V_L^{CKM^\dagger} S_U$$

$$V_R \equiv V_R^{CKM} \neq V_L^{CKM} \equiv V_L$$

CKM matrices :

$$V_L = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

$$\hat{V}_R = P_U \tilde{V}_L P_D$$

Where, $P_U = \text{diag}(s_u, s_c e^{2i\theta_2}, s_t e^{2i\theta_3})$,
$P_D = \text{diag}(s_d e^{i\theta_1}, s_s e^{-i\theta_2}, s_b e^{-i\theta_3})$

$$\tilde{V}_L = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 e^{-2i\theta_2} \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 e^{2i\theta_2} & 1 \end{pmatrix}$$

[arXiv:0712.4218v1]

Yukawa Sector

$$\hat{V}_R = \begin{pmatrix} s_d s_u (1-\frac{1}{2}\lambda^2)e^{i\theta_1} & s_d s_u \lambda e^{-i\theta_2} & s_b s_u A\lambda^3(\rho - i\eta)e^{-i\theta_3} \\ -s_d s_c \lambda e^{i(\theta_1+2\theta_2)} & s_s s_c (1-\frac{1}{2}\lambda^2)e^{i\theta_2} & s_b s_c A\lambda^2 e^{-i\theta_3} \\ s_d s_t A\lambda^3(1-\rho-i\eta)e^{i(\theta_1+2\theta_3)} & -s_s s_t A\lambda^2 e^{i(\theta_2+2\theta_3)} & s_b s_t e^{i\theta_3} \end{pmatrix}$$

Where, $\lambda \equiv sin\theta_{Cabibbo}$ and η is CP violating parameter.

Quark mass terms:

$$\mathcal{L}^q_{mass} = \bar{U}_{Li}(M_U)_{ij}\bar{U'}_{Rj} + \bar{D}_{Li}(M_D)_{ij}\bar{D'}_{Rj} + h.c$$

Yukawa Sector

Leptons(Gauge Eigenstate)

$$\mathsf{L}^{L}_{Yukawa} = [\overline{L}_{Li}(h_{ij}\phi + \tilde{h}_{ij}\tilde{\Phi})L_{Rj} + h.c] + \\ [\overline{L}^{c}_{Ri}(h_M)_{ij}\Sigma_L L_{Lj} + \overline{L}^{c}_{Li}(h_M)_{ij}\Sigma_R L_{Rj} + h.c]$$

where, $\Sigma_{L,R} = i\sigma_2\Delta_{L,R}$

h_M is the Majorana mass matrix.

Yukawa Sector

Lepton Mass Lagrangian:

$$\mathcal{L}^{l}_{mass} = \bar{l}'_{Li}(M_l)_{ij} l'_R + \bar{l}'_{Ri}(M_l^\dagger)_{ij} l'_L$$

Where, $M_l = M_l^\dagger = \frac{1}{\sqrt{2}}(k\tilde{h}_l + k' h_l)$

Neutrino Mass Lagrangian:

$$\mathcal{L}^{\nu}_{mass} = (\bar{n}'^c_L M_\nu n'_R + \bar{n}'^c_R M^\star_\nu n'_L)$$

with fields $n'_R = \begin{pmatrix} \nu'^c_R \\ \nu'_R \end{pmatrix}$ and $n'_L = \begin{pmatrix} \nu'_L \\ \nu'^c_L \end{pmatrix}$

The neutrino mass matrix is, $M_\nu = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}$

with $M_D = \frac{1}{\sqrt{2}}(kh + k'\tilde{h}_L)$, $M_R = \sqrt{2} h_M v_R$ and (1,1) element comes from $v_L = 0$

Neutrino Mass and See-Sawing

- M_D in the neutrino mass matrix above are Dirac Mass Matrices which depend on SM scale VEVs.
- And M_R depends on the large VEV v_R
- for simplicity, if we consider the case where the dimensionality is reduced, so that M_R and M_D are numbers,the eigenvalues of M_ν are

$$\lambda_{1,2} = \frac{1}{2}\left(M_R \pm \sqrt{M_R^2 + 4M_D^2}\right)$$

now since $M_R >> M_D$, the eigenvalues are,

$\lambda_1 \approx M_R$,

$\lambda_2 \approx \frac{M_D^2}{M_R}$

Neutrino Mass and See-Sawing

- neutrino mass is proportional to the large scale introduced in v_R and the other is suppressed by it.
- This see-sawing gives an explanation for the light LH neutrinos from only the hierarchy $v_R >> k, k'$,where SM cannot.
- Measured upper limit of SM neutrino puts the lower limits on the scale v_R,which is at least 10^{10} GeV. [arXiv:hep-ph/0107121v3]

Unresolved Problems

The following problems still remain unresolved in this model

- The arbitraryness of large mass hierarchies
- large number of free parameters,etc...

These issues are addressed in more symmetric Grand Unified Theories(GUTs).

Thank You!