AN INFINITE FAMILY OF S INVARIANT THEORIES ON THE CELESTIAL SPHERE

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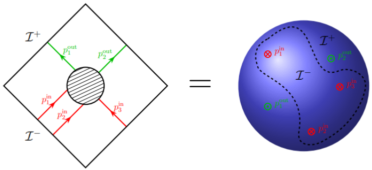
CELESTIAL CFT?

S-matrix of gravity/gauge theory in (3+1)D AFS $\equiv$ Correlators of a 2D CCFT on CS^2



$$\langle out|\, S\, |in\rangle \qquad \langle \mathcal{O}^{\pm}_{\Delta_1}(z_1,\bar{z}_1)\ldots\mathcal{O}^{\pm}_{\Delta_n}(z_n,\bar{z}_n)\rangle \tag{1}$$

☞ Parametrization of null momentum in $(-,+,+,+)$ signature:

$$p_k^\mu = \epsilon_k\omega_k(1+z_k\bar{z}_k, z_k+\bar{z}_k, -i(z_k-\bar{z}_k), 1-z_k\bar{z}_k) \tag{2}$$

CELESTIAL CORRELATION FUNCTIONS/ CELESTIAL AMPLITUDES

Primary operators: Pasterski, Shao, Strominger 2016

$$\mathcal{O}^{a,\epsilon}_{h,\bar{h}}(z,\bar{z}) = \int_0^\infty d\omega\, \omega^{\Delta-1} A^a(\epsilon\omega, z, \bar{z}, \sigma), \tag{3}$$

where, $h = \frac{\Delta+\sigma}{2}$, $\bar{h} = \frac{\Delta-\sigma}{2}$, σ: helicity, $\epsilon = \pm 1$(outgoing/incoming)

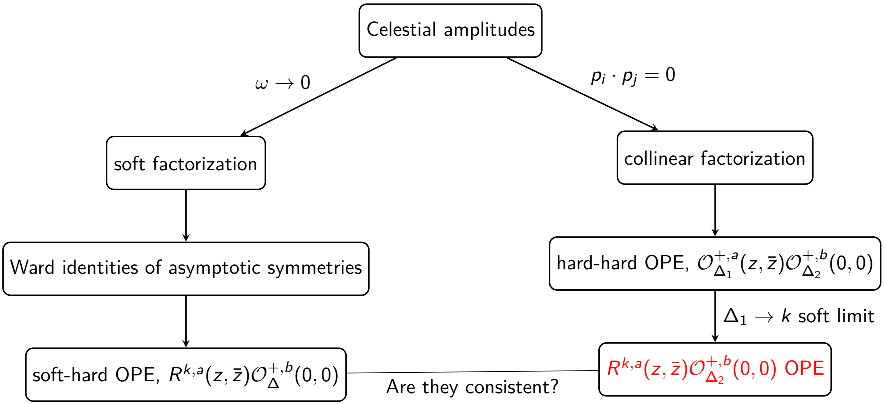
Celestial amplitude: S-matrix written in boost eigen-basis instead of momentum basis.

$$\langle \prod_{i=1}^n \mathcal{O}^{a_i,\epsilon_i}_{h_i,\bar{h}_i}(z_i,\bar{z}_i)\rangle = \prod_{i=1}^n \int_0^\infty d\omega_i\, \omega_i^{\Delta_i-1} \mathcal{S}_n(\{\omega_i, z_i, \bar{z}_i, \sigma_i, a_i, \epsilon_i\}). \tag{4}$$

Under $SL(2,\mathbb{C})$ transformations,

$$\mathcal{A}_n(\{z_i,\bar{z}_i,h_i,\bar{h}_i,a_i,\epsilon_i\}) = \prod_{i=1}^n \frac{1}{(cz_i+d)^{2h_i}} \frac{1}{(\bar{c}\bar{z}_i+\bar{d})^{2\bar{h}_i}} \mathcal{A}_n\left(\left\{\frac{az_i+b}{cz_i+d}, \frac{\bar{a}\bar{z}_i+\bar{b}}{\bar{c}\bar{z}_i+\bar{d}}, h_i, \bar{h}_i, a_i, \epsilon_i\right\}\right). \tag{5}$$

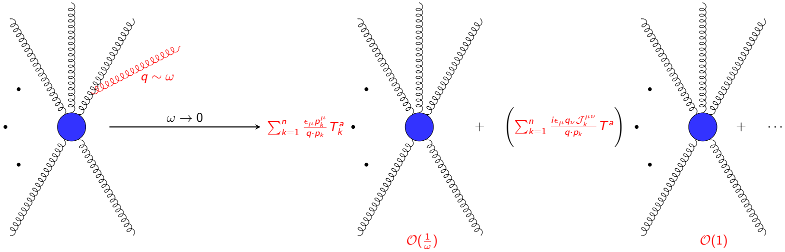
FACTORIZATION PROPERTIES OF CELESTIAL AMPLITUDE



they are same modulo a null state!

SOFT FACTORIZATION THEOREM

momentum space tree level



(6)

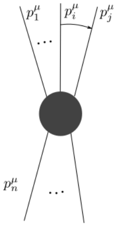
Leading conformally soft gluon theorem: Mellin space

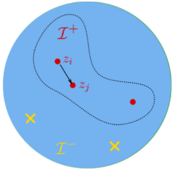
$$\left\langle R^{1,a}(z) \prod_{i=1}^{n} \mathcal{O}^{a_i}_{h_i,\bar{h}_i}(z_i,\bar{z}_i) \right\rangle = -\sum_{k=1}^{n} \frac{T_k^a}{z - z_k} \left\langle \prod_{i=1}^{n} \mathcal{O}^{a_i}_{h_i,\bar{h}_i}(z_i,\bar{z}_i) \right\rangle \tag{7}$$

$$\text{where,} \quad R^{1,a}(z,\bar{z}) = \lim_{\Delta\to 1} (\Delta - 1)\mathcal{O}^a_{\Delta,+}(z,\bar{z}) = R^{1,a}(z) = \sum_{\alpha\in\mathbb{Z}-1} \frac{R^{1,a}_\alpha}{z^{\alpha+1}} \tag{8}$$

➥ Level zero Kac-Moody algebra, $[R^{1,a}_\alpha, R^{1,b}_{\alpha'}] = -if^{abc} R^{1,c}_{\alpha+\alpha'}$

COLLINEAR FACTORIZATION AND CELESTIAL OPE





[image courtesy: Andrea Puhm]

$$\mathcal{A}(1^{-a_1}, 2^{+a_2}, 3^{+a_3}, 4^{-a_4}) \xrightarrow[p_3 \cdot p_4 = 0]{z_{34} \to 0} -\frac{f^{a_3 a_4 x}}{z_{34}} B(\Delta_3 - 1, \Delta_4 + 1) \mathcal{A}(1^{-a_1}, 2^{+a_2}, 4^{-x}) \qquad (9)$$

$$+ \text{subleading in } z_{34} + \ldots$$

Leading OPE structure $\downarrow$ between two outgoing gluons

$$\mathcal{O}^{a_3}_{\Delta_3,+}(z_3, \bar{z}_3)\mathcal{O}^{a_4}_{\Delta_4,-}(z_4, \bar{z}_4) \sim -\frac{f^{a_3 a_4 x}}{z_{34}} B(\Delta_3 - 1, \Delta_4 + 1) \mathcal{O}^{x}_{\Delta_3+\Delta_4-1,-}(z_4, \bar{z}_4) \qquad (10)$$

The S algebra is obtained from the singular part of the OPE between two **positive helicity outgoing gluons** on the celestial sphere :

Guevara, Himwich, Pate and Strominger '21

$$\boxed{\mathcal{O}^{a,+}_{\Delta_1}(z_1,\bar{z}_1)\mathcal{O}^{b,+}_{\Delta_2}(z_2,\bar{z}_2) \sim -\frac{if^{ab}{}_c}{z_{12}}\sum_{n=0}^{\infty} B(\Delta_1+n-1,\Delta_2-1)\frac{\bar{z}^n_{12}}{n!}\bar{\partial}^n_2\mathcal{O}^{c,+}_{\Delta_1+\Delta_2-1}(z_2,\bar{z}_2)} \tag{11}$$

Infinite tower of soft gluons:

$$R^{k,a}(z,\bar{z}) = \lim_{\Delta\to k}(\Delta-k)O^{a,+}_{\Delta}(z,\bar{z}), \qquad k=1,0,-1,\ldots \tag{12}$$

Holomorphic soft gluon currents:

$$R^{k,a}(z,\bar{z}) = \sum_{n=\frac{k-1}{2}}^{\frac{1-k}{2}} \frac{R^{k,a}_n(z)}{\bar{z}^{n+\frac{k-1}{2}}} \tag{13}$$

Modes of the Holomorphic currents :

$$R_n^{k,a}(z) = \sum_{\alpha \in \mathbb{Z} - \frac{k+1}{2}} \frac{R_{\alpha,n}^{k,a}}{z^{\alpha + \frac{k+1}{2}}} \tag{14}$$

New symmetry algebra :

$$\boxed{[R_{\alpha,m}^{k,a}, R_{\beta,n}^{l,b}] = -if^{ab}{}_c \frac{(\frac{1-k}{2} - m + \frac{1-l}{2} - n)!}{(\frac{1-k}{2} - m)!(\frac{1-l}{2} - n)!} \frac{(\frac{1-k}{2} + m + \frac{1-l}{2} + n)!}{(\frac{1-k}{2} + m)!(\frac{1-l}{2} + n)!} R_{\alpha+\beta,m+n}^{k+l-1,c}} \tag{15}$$

Redefinition :

$$S_{\alpha,m}^{q,a} = (q - m - 1)!(q + m - 1)! R_{\alpha,m}^{3-2q,a} \tag{16}$$

S Algebra :

$$[S_{\alpha,m}^{p,a}, S_{\beta,n}^{q,b}] = -if^{abc} S_{\alpha+\beta,m+n}^{p+q-1,c} \tag{17}$$

Guevara, Himwich, Pate and Strominger '21

General structure of celestial OPE between two positive helicity outgoing gluons

$$\boxed{\begin{aligned}\mathcal{O}^{a,+}_{\Delta_1}(z_1,\bar{z}_1)\mathcal{O}^{b,+}_{\Delta_2}(z_2,\bar{z}_2) = -\frac{if^{ab}{}_c}{z_{12}}\sum_{n=0}^{\infty} B(\Delta_1+n-1,\Delta_2-1)\frac{\bar{z}_{12}^n}{n!}\bar{\partial}_2^n\mathcal{O}^{c,+}_{\Delta_1+\Delta_2-1}(z_2,\bar{z}_2)\\ + \sum_{p,q=0}^{\infty}\sum_{k=1}^{\tilde{n}_{p,q}} z_{12}^p\bar{z}_{12}^q C^k_{p,q}(\Delta_1,\Delta_2)\tilde{\mathcal{O}}^{ab}_{k,p,q}(z_2,\bar{z}_2)\end{aligned}} \tag{18}$$

MOTIVATION

✧ If we can classify all the S invariant theories on the celestial sphere, it could in principle help us explore more interesting sectors of Yang-Mills theory.

GOAL

❶ To determine all the OPEs which are invariant under S algebra.

❷ Also to find the KZ-type null states of such theories.

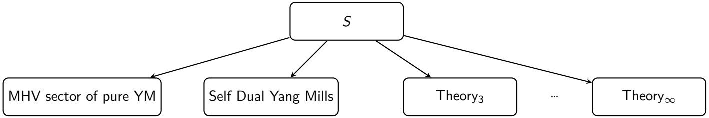
Results:

S invariant OPEs at $\mathcal{O}(1)$:

$$\boxed{\mathcal{O}^a_{\Delta_1,+}(z,\bar{z})\mathcal{O}^b_{\Delta_2,+}(0,0)|_{\mathcal{O}(1)} = \mathcal{O}^a_{\Delta_1,+}(z,\bar{z})\mathcal{O}^b_{\Delta_2,+}(0,0)\Big|^{MHV}_{\mathcal{O}(1)} + \sum_{k=1}^{n} B(\Delta_1+k,\Delta_2-1)M^{ab}_k(\Delta_1+\Delta_2)} \tag{19}$$

- S invariance cannot fix the the value of integer n.
- Notice, we are distinguishing different theories by the null states and not by the operator spectrum.

Bulk theories?



KZ type null states: $$K^a(\Delta) = \xi^a(\Delta) - i\sum_{k=1}^{n} M^a_k(\Delta+1), \tag{20}$$

where, $$\xi^a(\Delta) = C_A L_{-1}\mathcal{O}^{a,+}_{\Delta} - (\Delta+1)R^{1,b}_{-1,0}R^{1,b}_{0,0}\mathcal{O}^{a,+}_{\Delta} - R^{0,b}_{-\frac{1}{2},\frac{1}{2}}R^{1,b}_{0,0}\mathcal{O}^{a,+}_{\Delta+1}. \tag{21}$$

Thank you for your attention !!