Exercise : 1 :-

1) $T(n) = \begin{cases} 1, & \text{if } n=1. \\ T(n-1)+\frac{1}{n}. & \text{if } n>1 \end{cases}$

Sol :-

$T(n) = T(n-1)+\frac{1}{n}$

$T(n-1) = T(n-2)+\frac{1}{n-1}+\frac{1}{n}$

$T(n-2) = T(n-3)+\frac{1}{n-2}+\frac{1}{n-1}+\frac{1}{n}$

$T(n-k) = T(n-(k+1))+\frac{1}{n-k}+\frac{1}{n-(k-1)}+\frac{1}{n-(k-2)}$

So,

$T(1) = 1$

$T(n-k) = 1$

$\boxed{n-1=k}$

$= T(n-(n-1+1))+\frac{1}{n-(n-1)}+\frac{1}{n-(n-1-1)}$

$= T(0)+\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{n-(n-1)}$

$= 0+\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+$

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2) $T(n) = \begin{cases} 1, & \text{if } n=0. \\ T(n-2)+n^2, & \text{if } n>0 \end{cases}$

$T(n) = T(n-2)+n^2$

$T(n-2) = T(n-4)+(n-2)^2$

$T(n-4) = T(n-6)+(n-4)^2+(n-2)^2+n^2$

$= T(n-k)+(n-k+2)^2+(n-k+4)^2+n^2$

$\boxed{n-k=0}$

$= T(0)+(2)^2+(4)^2+n^2$

$= 1+2^2+4^2+6^2+n^2$

Sum of squares

3) $T(n) = \begin{cases} 10, & \text{if } n = 0 \\ T(n-2) + \log(n) & \text{if } n > 0 \end{cases}$

$T(n) = T(n-2) + \log(n)$

$T(n-2) = T(n-4) + (n-2) + \log(n)$

$T(n-4) = T(n-6) + n-4 + n-2 + \log(n)$

$\boxed{n - k = 10}$

$= T(n-k) + \log(n - \{k+2\}) + \log(n - \{k+4\}) + \log(n)$

$= \cancel{10 + 12 + 14 + \ldots \log(n)}$

$= 2(5 + 6 + 7 + \ldots$

$T(n-n) = T(n-n) + \log(n-n+2) + \log(n-n+4) + \ldots + \log n$

$= T(0) + \log 2 + \log 4 + \log 6 + \ldots \log n$

$= 1 + \log(2 * 4 * 6 * \ldots n)$

$= 1 + \log(2 \times 1 \times (2 \times 2) + 2 \times 3 + \ldots (2 \times \frac{n}{2}))$

$= 1 + \log(2^n)(1 \times 2 \times 3 \times \ldots \times n/2)$

$= 1 + \log(2^n \times (\frac{n}{2})!)$

Exercise - 2 :-

Ex - 2 :-

$f(n) = n$ & $g(n) = n^{(1+\sin n)}$

[$\because$ value of sin varies from -1 to 1.

$f(n) <= c \cdot g(n)$

$c = ?$

$n = c \cdot n^{(1+\sin n)}$

$\log(n) = c \cdot \log(n^{1+\sin n})$

$= (1 + \sin n)\, c \cdot \log(n)$

if n; is an positive integer.

if $n = 1$;

$\log(1) = 1 + 1$

$1 < = 2$

$\boxed{\therefore f(n) = O\, g(n)}$

DO

$f_3(n), f_2(n), f_4(n), f_1(n)$

Increasing of asymptotic complexity.

Practice:

$f_1(n) = 2^n$

$f_2(n) = n^{3/2}$

$f_3(n) = n \cdot \log_2 n$

$f_4(n) = n^{\log_2 n}$

2^n

$n^{\log_2 n} > n \cdot \log_2 n$

$2^n > n^{3/2}$

f_3, f_2, f_4, f_1