Probability
Basic Definitions

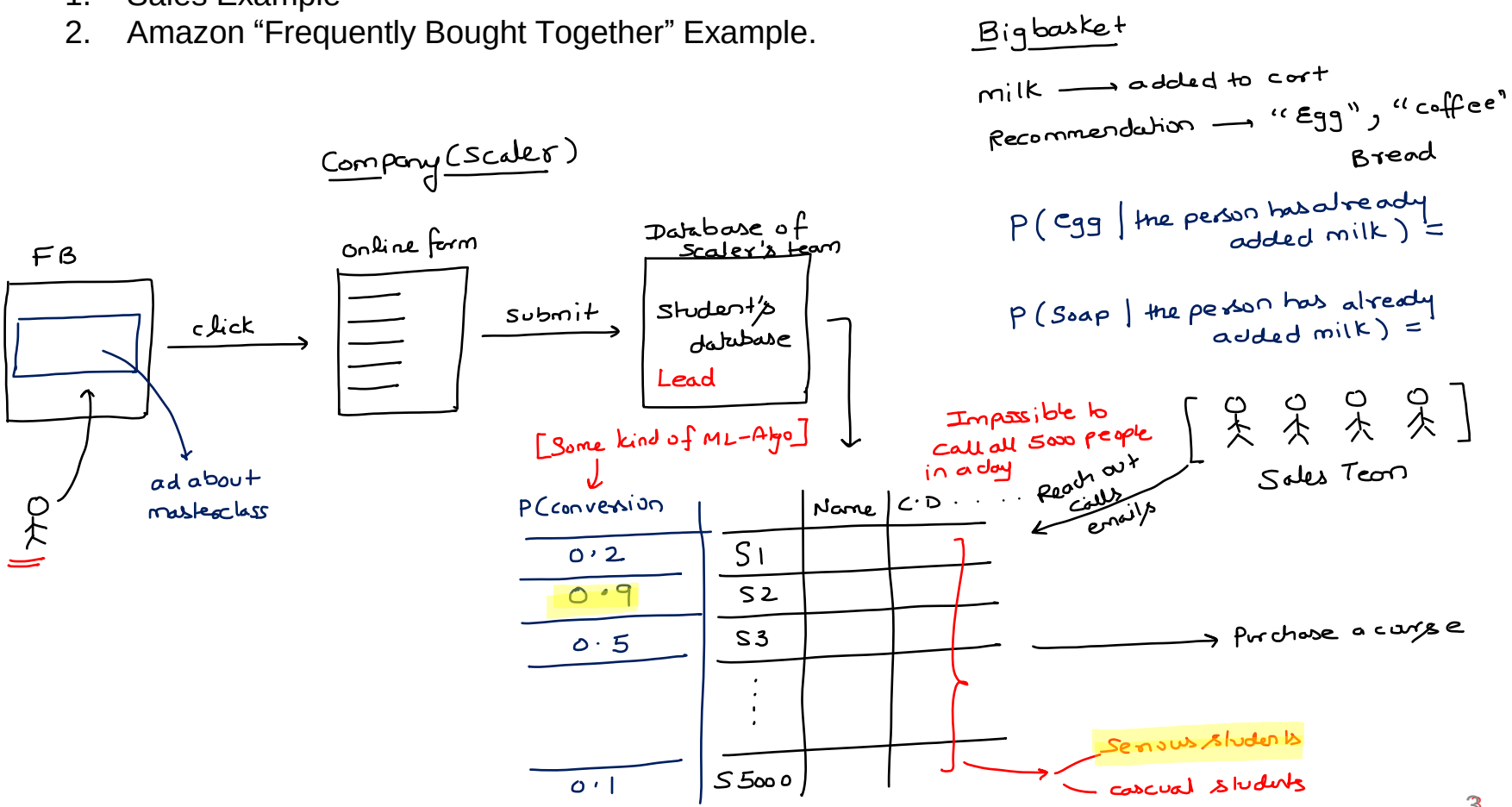
Agenda:

- Why we need to learn Probability
- Basic Terminologies
- Set Operations
- Probability Formula
- Rules of Probability
- Cross Tab
- Case Study

Listen Understand and React ✓
Power Learning ✓

-**Class will start sharp at 07: 05 AM**
- **No Break**
- **Class will sharply end at 09:05 AM**

1. Sales Example
2. Amazon “Frequently Bought Together” Example.



(Basic Building Blocks Prob)

Random Experiment and Sample Space

- Is a process by which we observe something that is uncertain
- outcome is the result of the random experiment
- The set of all possible outcomes out of a random experiment is called a sample space

Since I don't know the actual outcome before I roll the dice, this is a random exp. for me

{ 1, 2, 3, 4, 5, 6 }

Sample-space

outcomes of the random experiment

Random Experiment and Sample Space

for my mother my birthday is not a random exp.

1. Number of runs scored in a 50 overs cricket match (RE)
2. Number of coffee orders in a cafe (RE)
3. My birthday (may) → Random exp. {1, 2, 3, 4 . . . . 31} — sample space
4. Time taken by an employee to travel from home to office (commute time)
5. Height of a random person you will meet next

{0, 1, 2, 3 . . . 600, . . .} → sample space

{0, 1, 2 . . . . . 1000 . . . } → sample space

Toss of a single fair coin

Ω = Sample space
S = Sample space

Event ⟶ Subset of a sample space

Toss of a coin ⟶ Random Exp.

$\Omega = \{H, T\}$

$A = \{H\}$
$B = \{T\}$
$C = \{H, T\}$
$\phi = \{\}$

all events

Rule

1. It should have outcome from sample space
2. No outcome should be outside the sample space

$A \cup D = \{HH, TT\}$

$A \cap D = \{\}$

Toss of two fair coins

$\Omega = \{HH, HT, TH, TT\}$

A: I will win if both coins landing with some outcome — $P(A) = 2/4 = 1/2$ — $\{HH, TT\}$

B: I will win if both coins lands with different outcome — $P(B) = 2/4 = 1/2$ — $\{HT, TH\}$

C: I will win in any possible outcome — $P(C) = 4/4 = 1$ — $\{HH, HT, TH, TT\} = \Omega$

D: I will not put money / I will not play — $P(D) = 0/4 = 0$ — $\{\} = \phi$

E: I will win if atleast on head appears in the outcome — $P(E) = 3/4$ — $\{HH, HT, TH\}$

Q: when A and B will win together?

$A \cap B = \{\} = \phi$

Q: when A and E will win together?

$A \cap E = \{HH\}$

Q: when A or E both win ~~to~~?

$A \cup E = \{HH, HT, TH, TT\} = \Omega$

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(1) when the union of two or more events results in Ω (Superset), then we call those events to be collectively exhaustive

(2) when the intersection of two events results in ϕ, null set we call those events to be mutually exclusive

→ Ind vs pak → {0}

→ Ind vs pak — was suspended → {}
wer

$\phi = \{\} =$ null

Event with no outcome

$A \cap E = \{HH\}$ — not mutually exclusive

$A \cup E = \{\Omega\}$ — But collectively exhaustive

→ $(A \cap B)$ → mutually exclusive

→ $(A \cup B)$ → $\{HH, TT, HT, TH\}$ → collectively exhaustive

Set Operations

$\cap$ = And = Intersection

$\cup$ = or = union

RE = Toss of two coins

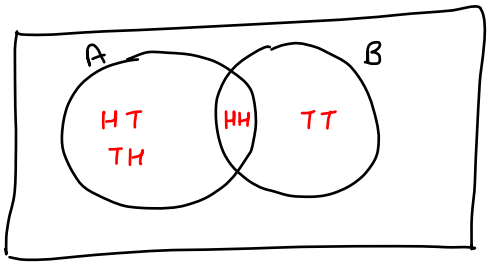
A = I will win if at least one head appear

$\{HH, HT, TH\}$

B = I will win if both outcome are same

$\{HH, TT\}$

Both A and B will win if the outcome is HH



→ $A \cap B = \{HH\}$

→ $A \cup B = \{HH, HT, TH, TT\}$

Roll of a dice → R.E

$\Omega = \{1, 2, 3, 4, 5, 6\}$

A: I will win if the outcome is an even number $\{2, 4, 6\}$
B: I will win if the outcome is an odd number $\{1, 3, 5\}$
C: I will win if the outcome is 1, 5, 6 $\{1, 5, 6\}$

$P(A) = 3/6 = 1/2$
$P(B) = 3/6 = 1/2$
$P(C) = 3/6 = 1/2$

A and B
Both are mutually ex. and collectively exhaustive

$A \cup B = \{1, 2, 3, 4, 5, 6\} = \Omega$ = collectively exhaustive
$A \cap B = \{\}$ $= \phi$ = mutually exclusive

A and C are neither mutually exclusive nor collectively exhaustive

$A \cup C = \{1, 2, 4, 5, 6\} \neq \Omega$ = not collectively exhaustive
$A \cap C = \{6\} \neq \phi$ = not mutually exclusive

Probability

$$P(A) = \frac{\#\text{ of outcomes in event } A}{\text{Total no. of outcomes in the Sample space } \Omega}$$

Axioms Of Probability

1. $0 \leq P(x) \leq 1$
2. $P(\Omega) = 1$
3. $P(\phi) = 0$

$\equiv$

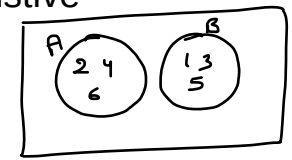
$P(A \cup B) = P(A) + P(B)$

Mutually Exclusive and Collectively Exhaustive

When A and B are mutually exclusive.

$P(A \cap B) = 0$

$P(A \cup B) = P(A) + P(B)$



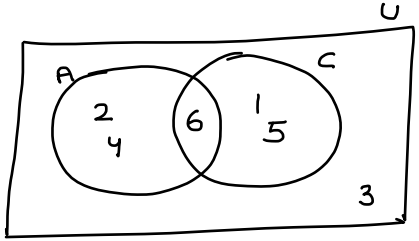
Roll of a dice

$\Omega = \{1, 2, 3, 4, 5, 6\}$

A: Even no. outcome : $\{2, 4, 6\}$

B: odd no. outcome : $\{1, 3, 5\}$

C: Random no. $\{1, 5, 6\}$



$P(A) = \frac{3}{6}$ $\{2, 4, 6\}$

$P(C) = \frac{3}{6}$ $\{1, 5, 6\}$

$P(A \cup C) = P(A) + P(B)$

$A \cup C = \{2, 4, 6, 1, 5\} = 5/6$

When two events are not mutually exclusive (Something in common) =

$P(A \cup C) = P(A) + P(C) - P(A \cap C)$

$= \frac{3}{6} + \frac{3}{6} - \frac{1}{6}$

$= 5/6$

$=$

Quiz-1. We are tossing a dice, where the sample space is {1, 2, 3, 4, 5, 6}. Which of following is not an event?

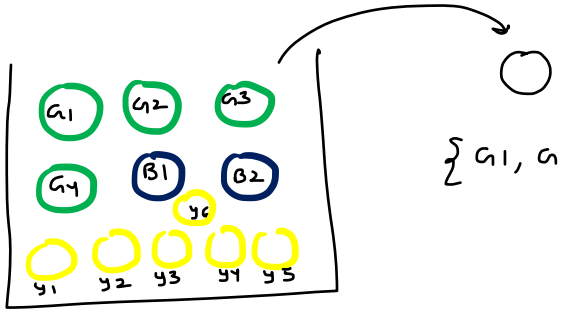
Quiz-2: We are tossing a coin followed by a Dice, how many elements in the sample space?

{ H1, H2, H3, H4, H5, H6,
T1, T2, T3, T4, T5, T6 }

# of outcome = 2
Coin toss
X
# of outcomes = 6
Roll dice
12

{ H1, H2

Quiz-3. There are 4 green balls, 6 yellow balls, and 2 blue balls in a bag. A random ball is chosen. Find the probability that a yellow or blue ball is chosen.



$\{G1, G2, G3, G4, B1, B2, Y1, Y2, Y3, Y4, Y5, Y6\}$

Yellow $A = \{Y1, Y2, Y3, Y4, Y5, Y6\}$ > mutually exclusive

Blue $B = \{B1, B2\}$

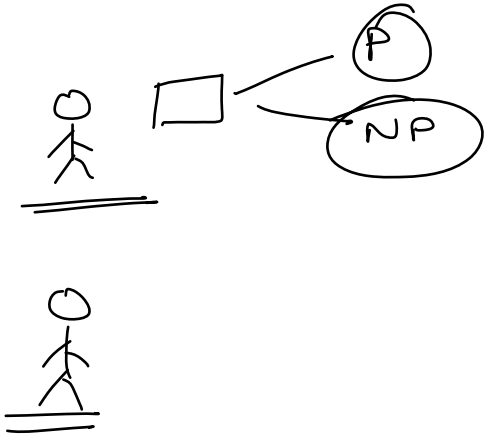
$$P(A \cup B) = P(A) + P(B)$$

$$= \frac{6}{12} + \frac{2}{12}$$

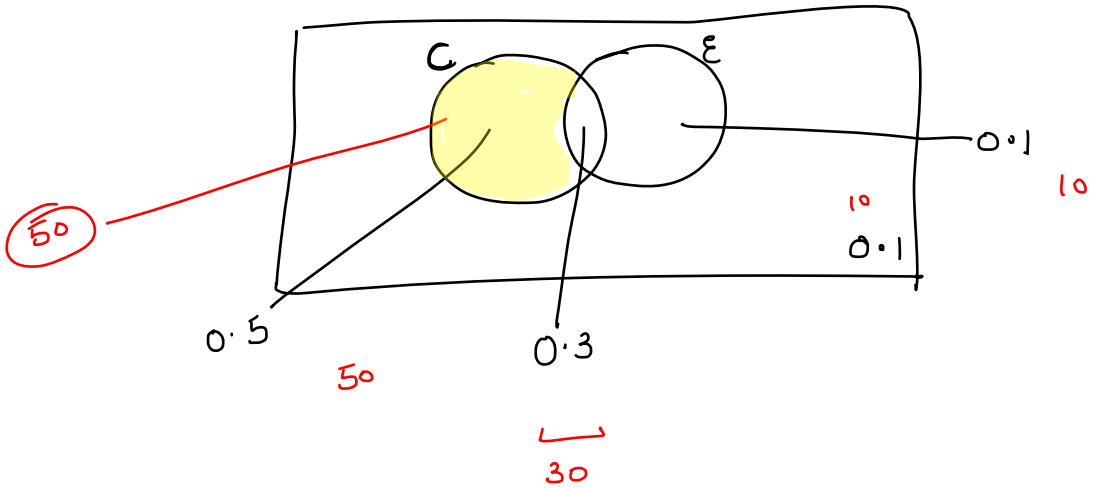
$$= \frac{8}{12}$$

Quiz-4: Which of the following represent mutually exclusive sets?

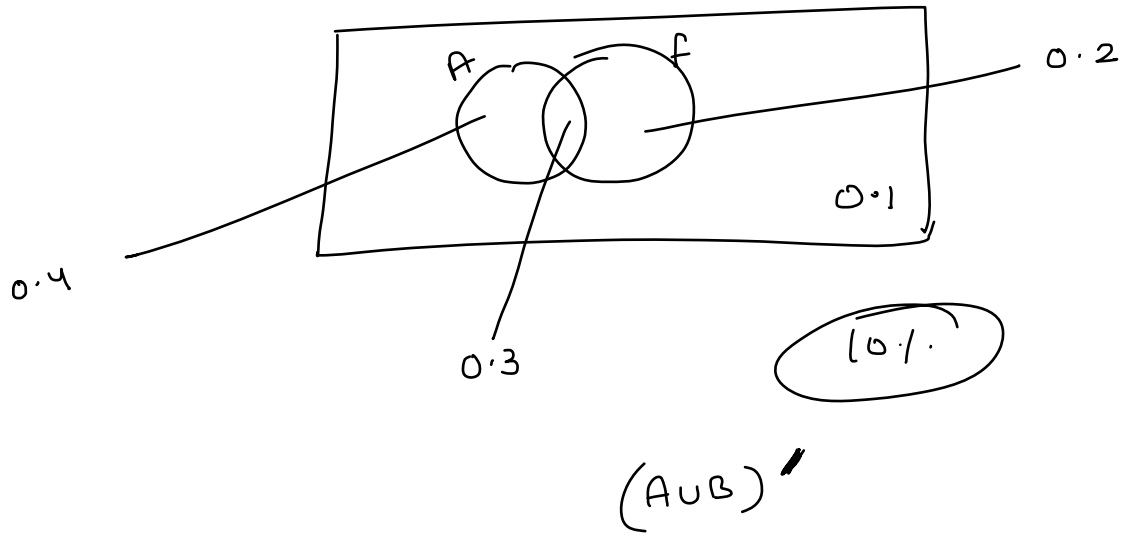
A. Youtube premium Vs Non-premium users
B. People who like Cappuccino Vs Espresso
C. Users of Swiggy Vs Zomato
D. Users of Amazon Vs Flipkart



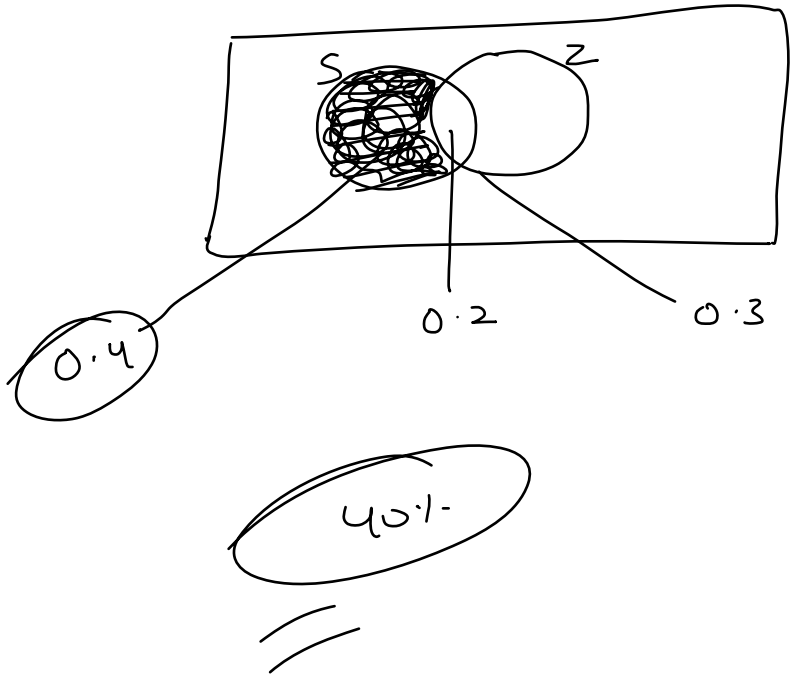
It is known that 80% people like cappuccino, 40% people like espresso, and 30% like both. What percentage of the people like cappuccino, but do not like espresso?



Quiz: It is known that 70% people use Amazon, 50% use Flipkart. 30% people use both. What percentage of people use neither Amazon, nor Flipkart?



Quiz-5: It is known that 60% people use Swiggy, 50% use Zomato. 20% people use both. What percentage use Swiggy, but do not use Zomato?



century	False	True	All
False	160	154	314
True	16	30	46
All	176	184	360

160 matches in which century not made and India lost

all matches in which India lost

century	False	True	All
False	0.444444	0.427778	0.872222
True	0.044444	0.083333	0.127778
All	0.488889	0.511111	1.000000

Joint prob

marginal

marginal

S = Sachine Century

S' = Sachine not century

I = India win

I' = India lost

$S' \cap I' = 160$

$S' \cap I = 154$

$S \cap I' = 16$

$S \cap I = 30$

$P(S' \cap I') = \frac{160}{360} = 0.444$

$P(S' \cap I) = \frac{154}{360} = 0.427$

$P(S \cap I') = \frac{16}{360} = 0.04$

$P(S \cap I) = \frac{30}{360} = 0.08$

Joint

$P(I') = \frac{176}{360}$

$P(I) = \frac{184}{360}$

$P(S') = \frac{314}{360}$

$P(S) = \frac{46}{360}$

marginal

divide each no. with grand total

normalize = True