

```

1 # Experiment for drum m1:
2
3 for covid patient
4
5
6 total are 100 people(Universe)
7
8 Sample Space = S = { r1-7, r8-14, r15+ , d}
9                     25    20    50    5
10
11
12 events are: :
13
14 all recovered people : E1 = { r1-7, r8-14, r15+ }
15                     P[E1] = 25 + 20 +50 / 100 = 95/100
16
17 recovered in 14 days : E2 = { r1-7, r8-14 }
18                     P[E2] = 25 + 20 /100 = 45/100
19
20 recovered in 20 days : E3
21 we dont have proper data : so its not a well defined event
22 so
23                     45/100  <= P[E3] <= 95/100
24
25

```

```

In [1]: 1 # sample space : collection of all possible outcomes
2 # event : collection of any number of outcomes:
3
4 # Event is a subset of Sample space

```

## Coin :

fair coin

```

1 Coin :
2 fair coin
3
4 S = {H,T}
5
6 P[H] = 1/2
7 P[T] = 1/2
8

```

```

1 Coin toss twice:
2
3 sample space: S = { HH,HT,TH,TT }
4
5 heads : E1 = { HH,HT,TH }
6         P[E1] = 3/4
7 both heads or tail : E2 = { HH,TT }
8         P[E2] = 2/4
9
10
11

```

**Outcomes + Sample\_space + Event + Probability**

```
In [ ]: 1
```

```
In [ ]: 1
```

```
In [ ]: 1
1
2 '''
3 for Dice:
4 sample space = S = {1,2,3,4,5,6}
5
6 example of events for Dice example
7 E1 = {1,2}
8 E2 = {1}
9 E3 = {1,3,5}
10
11 '''
```

```
In [ ]: 1
1
2 dice :
3 S = {1,2,3,4,5,6}
4
5 E = {1,2,3}
6 E' = E^c = {4,5,6} E compliment, all element other than in E
```

## dice 2 times :

```
1 dice 2 times :
2
3 sample space :
4 {
5 11 12 13 14 15 16
6 21 22 23 24 25 26
7 31 32 33 34 35 36
8 41 42 43 44 45 45
9 51 52 53 54 55 56
10 61 62 63 64 65 66
11 }
12 total 36 outcomes
13
14 E : dice1 is 3 : E = {31,32,33,34,35,36}
15 P[E] = 6/36 =1/6
16 F : dice1 + +dice2 == 7
17 F = {16,25,34,43,52,61}
18 P[F] = 6/36 =1/6
19 G : dice1**2 + dice2**2 == 25
20 G = {34,43}
21 P[G] = 2/36 = 1/18
```

```
In [ ]: 1
```

```
In [ ]: 1
```

## Covid example

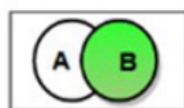
```
1 # Covid example
2
3 U : 1000 all covid patients
4 m1 : 100 given medicine
5 S : 700 survived
6
7
8 probability os patient given medicine
9 P[m1] = 100/1000 = 0.1
10
11 probability of survival
```

```

12 P[S] = 700/1000 = 0.7
13
14 people who survived and given medicine is : m1 intersection S
15                                     m1 ∩ S = 95 (given in data)
16 probability of survival who given medicine = 95/1000
17 P[ m1 ∩ S ] = 95/1000
18
19
20
21 now , if we want to know, the effectiveness of drug :
22 we need to find number of people survived who were given drug
23 we need to find probability of survival after given the drug,
24 probability wrt the people given medicine(condition)
25
26 P[S|m1] = probability of # of survival given # of people medicine
27           = P[S ∩ m1] / P[m1]
28           probability of S given m1 . (conditional probability)
29           = (95/1000) / (100/1000)
30           = 95 / 100 = 0.95
31
32
33 So , probability of Survival is 0.7
34 and probability of survival given medicine is 0.95
35 there are higher chances survival with medicine :
36

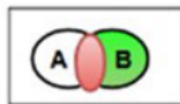
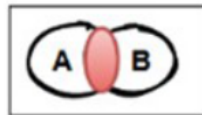
```

**$P(A|B) = P(A \text{ given } B \text{ has occurred})$**



If B has already occurred  
then our sample space  
must be somewhere within B

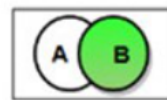
Now A can occur only  
within sample space B



$P(A|B)$  is the ratio of Red  
space divided by Green space

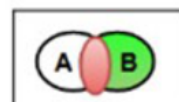
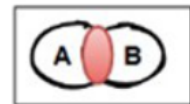
$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

**$P(B|A) = P(B \text{ given } A \text{ has occurred})$**



If A has already occurred  
then our sample space  
must be somewhere within A

Now B can occur only  
within sample space A



$P(B|A)$  is the ratio of Red  
space divided by White space

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

1 # general representation of events A and B

2

$$P[A|B] = P[A \cap B] / P[B]$$

$$P[B|A] = P[A \cap B] / P[A]$$

5

1 Conditional Probability :

2

$$P[A|B] = P[A \cap B] / P[B]$$

$$P[A \cap B] = P[B|A] * P[A]$$

5

6

$$P[A|B] = P[A \cap B] / P[B]$$

$$= P[B|A] * P[A] / P[B]$$

8

9

### **independent events**

1 what if :

2

3 probability of patient given medicine

$$P[m1] = 100/1000 = 0.1$$

5

```

6 probability of survival
7  $P[S] = 700/1000 = 0.7$ 
8
9  $P[m1 \cap S] = 70/1000$ 
10
11  $P[S|m1]$  = probability of # of survival given # of people medicine
12           =  $P[S \cap m1] / P[m1]$ 
13           =  $70 / 100 = 0.7$ 
14
15
16 So , probability of Survival is 0.7 = probability of survival given medicine is 0.7 (
     $P[S|m1] = P[S]$  )
17
18 there for no significant probability difference
19
20 That means the m1 and S are independent events :
21  $P[S|m1] = P[S]$ 

```

```

1  $P[A|B] = P[A]$  defination of indepedence
2
3 in above example :
4
5  $P[S|m1] = P[S]$ 
6
7  $P[S \cap m1] / P[m1] = P[S]$ 

```

**$P[A|B] = P[A]$  defination of indepedence**

```

1 # Independent Events :
2 A and B are independent Events if :
3
4  $P[A|B] == P[A]$ 
5  $P[B|A] == P[B]$ 
6
7  $P[A \cap B] = P[A]*P[B]$ 
8
9 """"
10 ratio of A happeneing inside B
11 is equal to
12 the ration of B happening inside the whole universe
13
14 """"
15

```

**dice 2 times : independece**

```

1 dice 2 times : independece
2
3 sample space :
4 {
5 11 12 13 14 15 16
6 21 22 23 24 25 26
7 31 32 33 34 35 36
8 41 42 43 44 45 45
9 51 52 53 54 55 56
10 61 62 63 64 65 66
11 }
12 total 36 outcomes
13
14 E : dice1 is 3 : E = {31,32,33,34,35,36}
15            $P[E] = 6/36 = 1/6$ 
16 F : dice1 + +dice2 == 7
17           F = {16,25,34,43,52,61}
18            $P[F] = 6/36 = 1/6$ 
19 G : dice1**2 + dice2**2 == 25
20           G = {34,43}
21            $P[G] = 2/36 = 1/18$ 
22 E  $\cap$  F = {34}
23  $P[E \cap F] = 1/36$ 

```

```

24
25  $P[E|F] = P[E \cap F] / P[F]$ 
26  $= (1/36) / (6/36)$ 
27  $= 1/6$ 
28  $= P[E]$ 
29
30 So , E and F are independent events
31 two dice are independent events

```

```

1       $P[E|F] = P[E \cap F] / P[F]$ 
2 conditional_prob = num_favourable_samples / num_of_conditioned_samples
3
4

```

```

In [2]: 1 trials = 1100
2 num_favourable_samples = 0 # E and F
3 num_conditional_samples = 0 # F
4
5 for t in range(trials):
6     dice1 = np.random.choice([1,2,3,4,5,6])
7     dice2 = np.random.choice([1,2,3,4,5,6])
8
9     if dice1 + dice2 == 7 :
10         num_conditional_samples += 1
11         if dice1 == 3:
12             num_favourable_samples += 1
13 conditional_probability = num_favourable_samples / num_conditional_samples
14 conditional_probability
15

```

Out[2]: 0.21839080459770116

```

In [3]: 1 expected_value = 1/6
2 expected_value

```

Out[3]: 0.16666666666666666

## dice and coin together : independence:

```

1 S = { H1,H2,H3,H4,H5,H6,
2       T1,T2,T3,T4,T5,T6 }
3
4 take an experimental events :
5 E = coin is head = { H1,H2,H3,H4,H5,H6 } = 6/12 = 1/2 = P[E]
6 F = dice is 3 = { H3,T3 } = 2/12 = 1/6 = P[F]
7 E ∩ F = {H3} P[E ∩ F] = 1/12
8
9
10 probability of E given F:
11
12  $P[E|F] = P[E \cap F] / P[F]$ 
13  $= (1/12) / (1/6)$ 
14  $= 6 / 12$ 
15  $= 1 / 2$ 
16  $= P[E]$ 
17
18 probability of F given E:
19
20  $P[F|E] = P[E \cap F] / P[E]$ 
21  $= (1/12) / (1/2)$ 
22  $= 2 / 12$ 
23  $= 1 / 6$ 
24  $= P[F]$ 
25
26
27
28 So , E and F are independent events
29 So , dice and coin toss are independent events

```

```
In [4]: 1 trials = 1000
2 num_favourable_samples = 0 # E and F
3 num_conditional_samples = 0 # F
4
5 for t in range(trials):
6     coin = np.random.choice(["H","T"])
7     dice = np.random.choice([1,2,3,4,5,6])
8
9     if dice == 3 :
10         num_conditional_samples += 1
11         if coin == "H":
12             num_favourable_samples += 1
13 conditional_probability = num_favourable_samples / num_conditional_samples
14 conditional_probability
15         #  $P[E|F] = P[E \cap F] / P[F]$ 
```

Out[4]: 0.4861878453038674

```
In [5]: 1 expected_value = 1/2 #  $P[E|F] = P[E \cap F] / P[F]$ 
2 expected_value
```

Out[5]: 0.5

```
In [6]: 1 trials = 1000
2 num_favourable_samples = 0 # E and F
3 num_conditional_samples = 0 # F
4
5 for t in range(trials):
6     coin = np.random.choice(["H","T"])
7     dice = np.random.choice([1,2,3,4,5,6])
8
9     if dice == 3 and coin == "H" :
10         num_favourable_samples += 1
11     if coin == "H":
12         num_conditional_samples += 1
13 conditional_probability = num_favourable_samples / num_conditional_samples
14 conditional_probability
15         #  $P[F|E] = P[E \cap F] / P[E]$ 
```

Out[6]: 0.15415821501014199

```
In [7]: 1 expected_value = 1/6 #  $P[F|E] = P[E \cap F] / P[E]$ 
2 expected_value
```

Out[7]: 0.16666666666666666

In [ ]: 1

In [ ]: 1

In [ ]: 1

-----

```
In [8]: 1 #  $A \cap B$ 
2 #  $A \cup B$ 
3 #  $A \subset B$ 
```

In [ ]: 1

## Dice :

```

1 Dice :
2
3 S = {1,2,3,4,5,6}
4
5 E1 = {1,2}      probability of E1 : P[E1] = 2/6
6 E2 = {1,3,5}    probability of E2 : P[E2] = 3/6
7
8 probability of Event E1 and E2 :
9 E1 ∩ E2 = {1}
10 P[E1 ∩ E2] = 1/6
11
12 probability of Event E1 or E2 :
13
14 E1 ∪ E2 = {1,2,3,5}
15
16 P[ E1 ∪ E2 ] = 4/6
17
18 {1,2} ∪ {1,3,5} = {1,2} + {1,3,5} - {1} # as 1 is intersection of both events :
19 so :
20
21 P[ E1 ∪ E2 ] = P[E1] + P[E2] - P[E1 ∩ E2] = 4/6
22               {1,2}   {1,3,5}   {1}
23               = 2/6   + 3/6     - 1/6
24
25               = 4/6
26
27

```

In [9]: 1 (2/6) + (3/6) - (1/6)

Out[9]: 0.6666666666666666

In [10]: 1 4/6

Out[10]: 0.6666666666666666

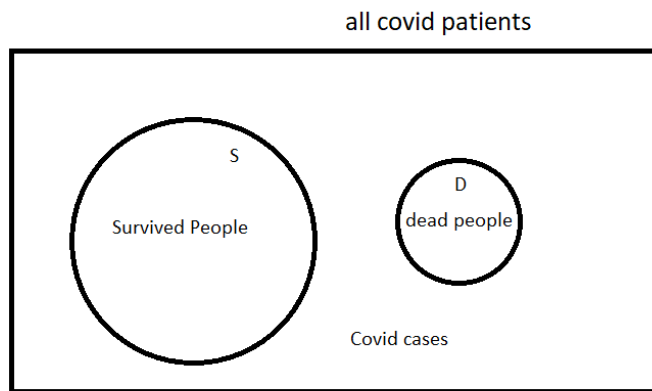
## Covid Example

∩ ∪

```

1 Covid Example :
2
3 U : all patients
4 S : survived (recovered)
5 C : cases (symptomatic )
6 d : died
7
8 S ∩ d = {} :
9 S ∩ c = {} : Mutually Exclusive Events
10 c ∩ d = {} :
11
12 S ∪ c ∪ d = Universe Mutually Exhausted Events
13
14
15

```



```

1
2  A ∩ A' = {}
3  A ∪ A' = S
4  so , A and A' are mutually exclusive and exhaustive
5
6

```

In [ ]:

1

In [ ]:

1

$$P[A \text{ or } B \text{ or } C] = P[A] + P[B] + P[C] - P[A \text{ and } B] + P[B \text{ and } C] + P[A \text{ and } C] + P[A \text{ and } B \text{ and } C]$$

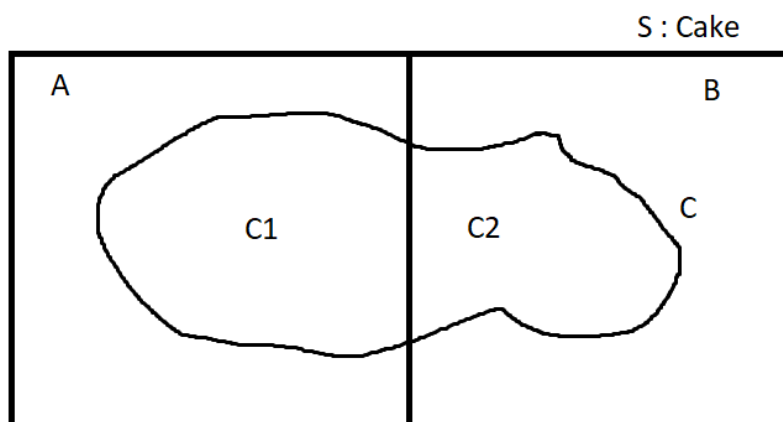
$\cap \cup$

## Cake example :

```

1  Universe U = SampleSpace S = Cake
2
3  cut the cake in two pieces A and B:
4
5
6  there's a cream on cake S : called C
7
8  both pieces of cream C1 and C2
9
10 A ∩ B = {} # mutually exclusive
11 A ∪ B = whole cake : S : # mutually exhaustive
12
13 A and B are mutually exhaustive and exclusive events
14

```



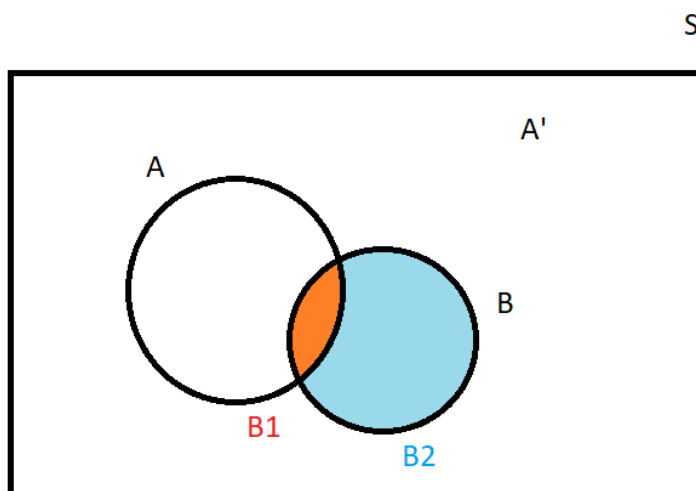


```

1 C1 is intersection of whole cream and part A cake
2 C2 is intersection of whole cream and part B cake
3
4  $C1 = C \cap A$ 
5  $C2 = C \cap B$ 
6
7 whole cream :  $C = C1 \cup C2$ 
8 and  $C1 \cap C2 = \{\}$ 
9
10  $P[C] = P[C1] + P[C2] - P[C1 \cap C2]$ 
11  $= P[C1] + P[C2] - \emptyset$ 
12  $= P[C1] + P[C2]$ 
13  $= P[C \cap A] + P[C \cap B]$ 
14
15 as per conditional probability :
16 we can replace :
17  $P[C \cap A] = P[C|A]*P[A] = P[A|C]*P[C] / P[A]$ 
18  $P[C \cap B] = P[C|B]*P[B] = P[B|C]*P[B] / P[B]$ 
19
20
21
22
23  $P[C] = P[C1] + P[C2] - P[C1 \cap C2]$ 
24  $= P[C1] + P[C2] - \emptyset$ 
25  $= P[C1] + P[C2]$ 
26  $= P[C \cap A] + P[C \cap B]$ 
27  $= P[C|A]*P[A] + P[C|B]*P[B]$ 

```

1



```

1  $B = B1 \cup B2$ 
2
3  $P[B] = P[B1] + P[B2]$ 
4  $= P[B \cap A] + P[B \cap A']$ 
5  $= P[B|A]*P[A] + P[B|A']*P[A']$ 
6
7
8

```

```

1
2  $A \cap A' = \{\}$ 
3  $A \cup A' = S$ 
4
5 so , A and A' are mutually exclusive and exhaustive
6
7  $P[A|B] = P[A \cap B]/P[B] = P[B|A]*P[A] / P[B]$ 
8
9  $= P[B|A]*P[A] / (P[B|A]*P[A] + P[B|A']*P[A'])$ 
10

```

```
In [ ]: 1
```

```
In [ ]: 1
```

```
In [ ]: 1
```

```
1 P[C] = P[C1]+P[C2] - P[C1 n C2]
2     = P[C1]+P[C2] - 0
3     = P[C1] + P[C2]
4     = P[C n A] + P[C n B]
5     = P[C|A]*P[A] + P[C|B]*P[B]
```

```
In [ ]: 1
```

```
In [ ]: 1
```

## Covid case

```
In [ ]: 1
```

```
1 people : heathy or discesed
2     test : +ve or -ve
```

```
1 P[H] = 0.9 #(90% population is heathy)
2 that means P[D] = 1- P[H] = 0.1 (disceased)
3
4
5
6 P[-ve|H] = 0.75    P[-ve|D] = 0.25
7
8 P[+ve|H] = 0.25    P[+ve|D] = 0.75
9
```

```
1 # if we get a +ve , what is the probability that i m still healthy
2
3 P[H|+ve]
```

```
In [11]: 1 # P[A|B] = P[A n B]/P[B] = P[B|A]*P[A] / P[B]
2
3 #           = P[B|A]*P[A] / ( P[B|A]*P[A] + P[B|A']*P[A'])
4
```

```
1 P[H|+ve] = P[+ve|H]*P[H] / ( P[+ve|H]*P[H] + P[+ve|D]*P[D])
2           = (0.25 * 0.9) / (0.25*0.9 + 0.75*0.1 )
3           = 0.75
```

```
In [12]: 1 (0.25 * 0.9) / ((0.25*0.9) + ( 0.75*0.1 ))
```

```
Out[12]: 0.7499999999999999
```

```
In [ ]: 1
```

```
In [ ]: 1
```

```
1
2 P[D] = 0.6    p
3 P[H] = 0.4    1-p
```

```

4 P[+ve|D] = 0.9 d
5 P[+ve|H] = 0.1 h
6 P[D|+ve] = ?
7
8 P[D|+ve] = P[+ve|D]*P[D] / ( P[+ve|D]*P[D] + P[+ve|H]*P[H])
9           ( 0.9 * 0.6) /      ( ( 0.9 * 0.6) + (0.1 * 0.4))
10
11           (d*p) / ((d*p)+(h*(1-p)))
12

```

```
In [13]: 1 ( 0.9 * 0.6) /      ( ( 0.9 * 0.6) + (0.1 * 0.4))
```

```
Out[13]: 0.9310344827586207
```

```
In [14]: 1 # # h/+VE = [+VE|h] * [h] / [+VE|h] * [h] + [+VE|D] * [D]
2 #           (0.25 * 0.9) / (( 0.25*0.9) + (0.75 * 0.1))
```

```
In [15]: 1 def solve(p,d,h):
2         # YOUR CODE GOES HERE
3
4         return np.round((d*p) / ((d*p)+(h*(1-p))),3)
5

```

```
In [16]: 1 solve(0.6,0.9,0.1)
```

```
Out[16]: 0.931
```

```
In [ ]: 1
```

```
In [17]: 1 (0.25 * 0.9) /      (( 0.25*0.9) + (0.75 * 0.1))
```

```
Out[17]: 0.7499999999999999
```

```
In [ ]: 1
```

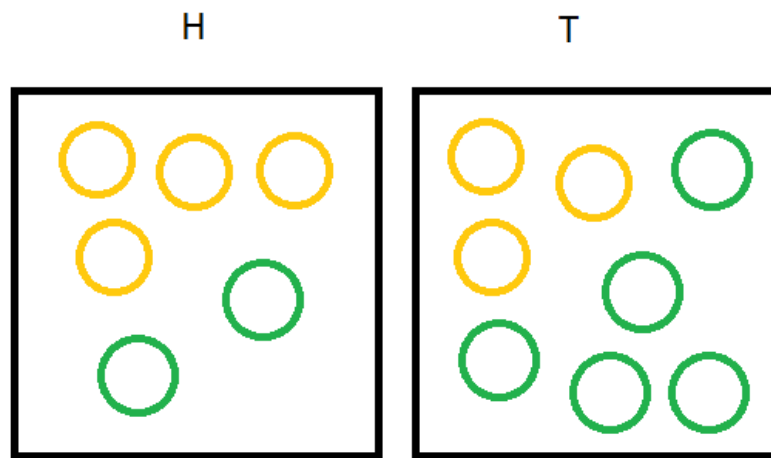
```
In [ ]: 1
```

```
In [ ]: 1
```

```
In [ ]: 1
```

```
In [ ]: 1
```

```
In [ ]: 1
```



```

1 two boxes :
2 1st contains 4 yellow and 2 green balls
3 2nd contains 5 green and 3 yellow balls.
4
5 we toss a coin
6 if H : we select 1st box
7 if T : we select 2nd box
8
9 if the ball so chosen is yello , what is the probability that coin toss was heads.
10
11  $P[H|Y] = ?$ 
12

```

```

1  $P[H|Y] = \frac{P[Y|H]*P[H]}{P[Y|H]*P[H] + P[Y|T]*P[T]}$ 
2
3
4  $= \frac{(4/6)*(1/2)}{((4/6)*(1/2)) + ((3/8)*(1/2))}$ 
5

```

```
In [18]: 1 ((4/6)*(1/2))/(((4/6)*(1/2))+((3/8)*(1/2)))
```

Out[18]: 0.64

In [ ]: 1

In [ ]: 1

In [ ]: 1

In [ ]: 1

In [ ]: 1

In [ ]: 1

In [ ]: 1