Probability

- `Probability` is a measure that is associated with how certain we are of outcomes of a particular experiment or activity.
- An `experiment` is a planned operation carried out under controlled conditions.
- If the result is not predetermined, then the experiment is said to be a chance experiment.
- Flipping one fair coin twice is an example of an experiment.

- A result of an experiment is called an outcome.
- The `sample space` of an experiment is the set of all possible outcomes.
- Three ways to represent a sample space are:

1. to list the possible outcomes
2. to create a tree diagram,
3. to create a Venn diagram.

- The uppercase letter `S` is used to denote the `sample space` .
- For example, if you flip one fair coin, S = {H, T} where H = heads and T = tails are the outcomes.

- An `event` is any `combination of outcomes` . Upper case letters like A and B represent events.
- For example, if the experiment is to flip one fair coin,
- event A might be getting at most one head.
- The probability of an event A is written P(A).

- The probability of any outcome is the long-term relative frequency of that outcome.
- Probabilities are between zero and one, inclusive (that is, zero and one and all numbers between these values).
- P(A) = 0 means the event A can never happen.
- P(A) = 1 means the event A always happens.
- P(A) = 0.5 means the event A is equally likely to occur or not to occur. - For example, if you flip one fair coin repeatedly (from 20 to 2,000 to 20,000 times) the relative frequency of heads approaches 0.5 (the probability of heads).
- Equally likely means that each outcome of an experiment occurs with equal probability.
- For example, if you toss a fair, six-sided die, each face (1, 2, 3, 4, 5, or 6) is as likely to occur as any other face.
- If you toss a fair coin, a Head (H) and a Tail (T) are equally likely to occur.
- If you randomly guess the answer to a true/false question on an exam, you are equally likely to select a correct answer or an incorrect answer.

- To calculate the probability of an event A when all outcomes in the sample space are equally likely, count the number of outcomes for event A and divide by the total number of outcomes in the sample space.
- For example,
- if you toss a fair dime and a fair nickel, the sample space is {HH, TH, HT, TT}

  where T = tails and H = heads.

  The sample space has four outcomes. A = getting one head.
  - There are two outcomes that meet this condition {HT, TH},
  - so P(A) = 2/4 = 0.5.

- Suppose you roll one fair six-sided die,
- with the numbers {1, 2, 3, 4, 5, 6} on its faces.
  - Let event E = rolling a number that is at least five.
  - There are two outcomes {5, 6}.
  - P(E) = 2/6 .
  - If you were to roll the die only a few times, you would not be surprised if your observed results did not match the probability.
  - If you were to roll the die a very large number of times, you would expect that, overall, 2/6 of the rolls would result in an outcome of "at least five".
  - You would not expect exactly 2/6 . The long-term relative frequency of obtaining this result would approach the theoretical probability of 2/6 as the number of repetitions grows larger and larger.

- This important characteristic of probability experiments is known as the law of large numbers which states that `as the number of repetitions of an experiment is increased, the relative frequency obtained in the experiment tends to become closer and closer to the theoretical probability.` Even though the outcomes do not happen according to any set pattern or order, overall, the long-term observed relative frequency will approach the theoretical probability. `(The word empirical is often used instead of the word observed.)`
- It is important to realize that in many situations, the outcomes are not equally likely.
- A coin or die may be unfair, or biased.
- Two math professors in Europe had their statistics students test the Belgian one Euro coin and discovered that in 250 trials, a head was obtained 56% of the time and a tail was obtained 44% of the time.
- The data seem to show that the coin is not a fair coin; more repetitions would be helpful to draw a more accurate conclusion about such bias.
- Some dice may be biased. Look at the dice in a game you have at home; the spots on each face are usually small holes carved out and then painted to make the spots visible.
- Your dice may or may not be biased; it is possible that the outcomes may be affected by the slight weight differences due to the different numbers of holes in the faces.
- Gambling casinos make a lot of money depending on outcomes from rolling dice, so casino dice are made differently to eliminate bias.
- Casino dice have flat faces; the holes are completely filled with paint having the same density as the material that the dice are made out of so that each face is equally likely to occur.

"OR" Event:

- An outcome is in the event A OR B if the outcome is in A or is in B or is in both A and B.
- For example, let
  - A = {1, 2, 3, 4, 5} and
  - B = {4, 5, 6, 7, 8}.
  - A OR B = {1, 2, 3, 4, 5, 6, 7, 8}.
  - Notice that 4 and 5 are NOT listed twice.

"AND" Event:

- An outcome is in the event A AND B if the outcome is in both A and B at the same time. For example, let
  - A and B be {1, 2, 3, 4, 5} and {4, 5, 6, 7, 8}, respectively.
  - Then A AND B = {4, 5}.

The complement of event A is denoted A′

- (read "A prime").
- A′ consists of all outcomes that are NOT in A.
- Notice that P(A) + P(A′) = 1.
  - For example, let
  - S = {1, 2, 3, 4, 5, 6} and let
  - A = {1, 2, 3, 4}. Then,
  - A′ = {5, 6}.
  - P(A) = 4/6 ,
  - P(A′) = 2/6 , and
  - P(A) + P(A′) = 4/6+2/6 = 1

`The conditional probability`

- of `A given B` is written `P(A|B)` .
- P(A|B) is the probability that event A will occur given that the event B has already occurred.
- A conditional reduces the sample space.
- We calculate the probability of A from the reduced sample space B.
- The formula to calculate P(A|B) is
  - P(A|B) = P(A AND B)/P(B)
  - where P(B) is greater than zero.

```
In [ ]: 1
```

Independent Events:

Two events are `independent if the following are true` :

`P(A|B) = P(A)`

`P(B|A) = P(B)`

`P(A AND B) = P(A)P(B)`

Two events A and B are independent if the knowledge that one occurred does not affect the chance the other occurs. For

- For example, the `outcomes of two roles of a fair die are independent` events.
- The `outcome of the first roll does not change the probability for the outcome of the second` roll.
- To show two events are independent, you must show only one of the above conditions.
- If `two events are NOT independent` , then we say that they are `dependent.`

Sampling may be done with replacement or without replacement.

With replacement:

- If each member of a population is replaced after it is picked, then that member has the possibility of being chosen more than once.
- When sampling is done with replacement, then events are considered to be `independent` , meaning the result of the first pick will not change the probabilities for the second pick.

Without replacement:

- When sampling is done without replacement, each member of a population may be chosen only once. In this case, the probabilities for the second pick are affected by the result of the first pick.
- The events are considered to be `dependent or not independent` .

If it is not known whether A and B are independent or dependent, assume they are dependent until you can show otherwise.

Example :

- You have a fair, well-shuffled deck of 52 cards.
- It consists of four suits. The suits are clubs, diamonds, hearts and spades.
- There are 13 cards in each suit consisting of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, J (jack), Q (queen), K (king) of that suit.

a. `Sampling with replacement:`

- Suppose you pick three cards with replacement.
- The first card you pick out of the 52 cards is the Q of spades.
- You put this card back, reshuffle the cards and pick a second card from the 52-card deck.
- It is the ten of clubs.
- You put this card back, reshuffle the cards and pick a third card from the 52-card deck.
- This time, the card is the Q of spades again.
- Your picks are {Q of spades, ten of clubs, Q of spades}.
- You have picked the Q of spades twice.
- You pick each card from the 52-card deck.

b. `Sampling without replacement:`

- Suppose you pick three cards without replacement.
- The first card you pick out of the 52 cards is the K of hearts.
- You put this card aside and pick the second card from the 51 cards remaining in the deck.
- It is the three of diamonds.
- You put this card aside and pick the third card from the remaining 50 cards in the deck.

- The third card is the J of spades.
- Your picks are {K of hearts, three of diamonds, J of spades}.
- Because you have picked the cards without replacement, you cannot pick the same card twice.

Mutually Exclusive Events

A and B are mutually exclusive events `if they cannot occur at the same time.`

This means that `A and B do not share any outcomes` and `P(A AND B) = 0`.

- For example,
- suppose the sample space S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. Let A = {1, 2, 3, 4, 5},

  B = {4, 5, 6, 7, 8}, and

  C = {7, 9}.

  A AND B = {4, 5}.

  P(A AND B) = 210

  and is not equal to zero. Therefore, A and B are not mutually exclusive.

  A and C do not have any numbers in common so

  P(A AND C) = 0. Therefore, A and C are mutually exclusive.

If it is not known whether A and B are mutually exclusive, assume they are not until you can show otherwise. The following examples illustrate these definitions and terms.

Flip two fair coins.

- The sample space is {HH, HT, TH, TT} where T = tails and H = heads.
- The outcomes are
  - HH, HT, TH, and TT.
  - The outcomes HT and TH are different.
  - The HT means that the first coin showed heads and the second coin showed tails.
  - The TH means that the first coin showed tails and the second coin showed heads.

- Let A = the event of getting at most one tail.
  - (At most one tail means zero or one tail.)
  - Then A can be written as
    - {HH, HT, TH}.
    - The outcome HH shows zero tails.
    - HT and TH each show one tail.

- Let B = the event of getting all tails.
  - B can be written as
  - {TT}.
  - B is the complement of A,
  - so B = A′. Also,
    - P(A) + P(B) = P(A) + P(A′) = 1.

- Let C = the event of getting all heads.
  - C = {HH}.
  - Since B = {TT},
  - P(B AND C) = 0.
  - B and C are mutually exclusive. (B and C have no members in common because you cannot have all tails and all heads at the same time.)

- Let D = event of getting more than one tail.
  - D = {TT}.
    - P(D) = 1/4

- Let E = event of getting a head on the first roll.

- (This implies you can get either a head or tail on the second roll.)
  - E = {HT, HH}.
  - P(E) = 2/4

- Find the probability of getting at least one (one or two) tail in two flips.
- Let F = event of getting at least one tail in two flips.
  - F = {HT, TH, TT}.
  - P(F) = 3/4

In [ ]:
```
1
```

When calculating probability, there are two rules to consider when determining if two events are independent or dependent and if they are mutually exclusive or not.

The Multiplication Rule

- If A and B are two events defined on a sample space,
  - `P(A AND B) = P(B) * P(A|B)`

- This rule may also be written as:
  - `P(A|B) =  P(A AND B) / P(B)`

- (The probability of A given B equals the probability of A and B divided by the probability of B.)

- If A and B are independent, then
  - `P(A|B) = P(A).`
  - Then `P(A AND B) = P(A|B) * P(B)`

    becomes `P(A AND B) = P(A)*P(B).`

The Addition Rule

- If A and B are defined on a sample space, then:
  - `P(A OR B) = P(A) + P(B) - P(A AND B).`
- If `A and B are mutually exclusive` , then
  - `P(A AND B) = 0.`

    Then
    - P(A OR B) = P(A) + P(B) - P(A AND B)

      becomes P(A OR B) = P(A) + P(B).

In [ ]:
```
1
```

In [ ]:
```
1
```

Conditional Probability

- the likelihood that an event will occur given that another event has already occurred

contingency table

- the method of displaying a frequency distribution as a table with rows and columns to show how two variables may be dependent (contingent) upon each other; the table provides an easy way to calculate conditional probabilities.

Dependent Events

- If two events are NOT independent, then we say that they are dependent.

Equally Likely

- Each outcome of an experiment has the same probability.

Event

- a subset of the set of all outcomes of an experiment; the set of all outcomes of an experiment is called a sample space and is usually denoted by S.
- An event is an arbitrary subset in S.
- It can contain one outcome, two outcomes, no outcomes (empty subset), the entire sample space, and the like.
- Standard notations for events are capital letters such as A, B, C, and so on.

Experiment

- a planned activity carried out under controlled conditions

Independent Events

- The occurrence of one event has no effect on the probability of the occurrence of another event. Events A and B are independent if one of the following is true:

```
P(A|B) = P(A)
P(B|A) = P(B)
P(A AND B) = P(A)P(B)
```

Mutually Exclusive

- Two events are mutually exclusive if the probability that they both happen at the same time is zero. If events A and B are mutually exclusive, then P(A AND B) = 0.

Outcome

- a particular result of an experiment

Probability

- a number between zero and one, inclusive, that gives the likelihood that a specific event will occur; the foundation of statistics is given by the following 3 axioms (by A.N. Kolmogorov, 1930's): Let S denote the sample space and A and B are two events in S. Then:

```
- 0 ≤ P(A) ≤ 1
- If A and B are any two mutually exclusive events, then P(A OR B) = P(A) + P(B).
- P(S) = 1
```

Sample Space

- the set of all possible outcomes of an experiment

Sampling with Replacement

- If each member of a population is replaced after it is picked, then that member has the possibility of being chosen more than once.

Sampling without Replacement

- When sampling is done without replacement, each member of a population may be chosen only once.

The AND Event

- An outcome is in the event A AND B if the outcome is in both A AND B at the same time.

The Complement Event

- The complement of event A consists of all outcomes that are NOT in A.

The Conditional Probability of A GIVEN B

- P(A|B) is the probability that event A will occur given that the event B has already occurred.

• P(A|B) is the probability that event A will occur given that the event B has already occurred.

The Conditional Probability of One Event Given Another Event

- P(A|B) is the probability that event A will occur given that the event B has already occurred.

The Or Event

- An outcome is in the event A OR B if the outcome is in A or is in B or is in both A and B.

The OR of Two Events

- An outcome is in the event A OR B if the outcome is in A, is in B, or is in both A and B.

Tree Diagram

- the useful visual representation of a sample space and events in the form of a “tree” with branches marked by possible outcomes together with associated probabilities (frequencies, relative frequencies)

Venn Diagram

- the visual representation of a sample space and events in the form of circles or ovals showing their intersections

In [ ]:

```

```

3.1 Terminology

A and B are events

$P(S) = 1$ where S is the sample space

$0 \leq P(A) \leq 1$

$P(A|B) = \frac{P(A AND B)}{P(B)}$

3.2 Independent and Mutually Exclusive Events

If A and B are independent, $P(A \text{ AND } B) = P(A)P(B)$, $P(A|B) = P(A)$ and $P(B|A) = P(B)$.

If A and B are mutually exclusive, $P(A \text{ OR } B) = P(A) + P(B)$ and $P(A \text{ AND } B) = 0$.

3.3 Two Basic Rules of Probability

The multiplication rule: $P(A \text{ AND } B) = P(A|B)P(B)$

The addition rule: $P(A \text{ OR } B) = P(A) + P(B) - P(A \text{ AND } B)$

In [ ]:

```

```

In [ ]:

```

```

In [ ]:

```

```

In [11]:

```
from collections import Counter
import numpy as np
```

n trials of tosing fair coin : # number of heads and tails

In [7]:

```
ans = []
for i in range(1000000):
    ans.append(np.random.choice( a = ["H","T"] , p=[ 1- 0.5 , 0.5] ))
# print(ans)
Counter(ans)
```

Out[7]:

```
Counter({'T': 500682, 'H': 499318})
```

In [8]:
```
ans = []
for i in range(100000):
    ans.append(np.random.choice( a = ["H","T"] ))
# print(ans)
Counter(ans)
```

Out[8]: Counter({'H': 50139, 'T': 49861})

cumulative probability of heads and tails (1000 tosses | fair coin)

In [9]:
```
p_h = 0.5 # probability of heads
num_tosses = 1000


def toss_coin():
    all_tosses = []
    cummulative =[0]

    for toss_id in range(num_tosses):
        toss_value = np.random.choice( a = [0,1] , p=[ 1-p_h , p_h ] )    # 0 : heads   |  1: tail
        all_tosses.append(toss_value)
        cummulative.append(cummulative[toss_id] + toss_value)

    cummulative = cummulative[1:]
    cummulative = [val / (i+1) for i , val in enumerate(cummulative)]
#     print(np.array(all_tosses))
    return  np.array(cummulative)[-1]

```

In [10]:
```
toss_coin()  # cumulative probability  of heads and tails
```

Out[10]: 0.516

n trials of throwing dice : # number of outcomes per throw

In [12]:
```
ans = []
for i in range(1000000):
    ans.append(np.random.choice( a = [1,2,3,4,5,6] , p=[0.16666666666666666,0.16666666666666666,0
# print(ans)
Counter(ans)
```

Out[12]: Counter({2: 166977, 4: 166806, 3: 166798, 6: 165815, 1: 166894, 5: 166710})

In [14]:
```
ans = []
for i in range(1000000):
    ans.append(np.random.choice( a = [1,2,3,4,5,6]  ))
# print(ans)
dice_throw_100000_times = Counter(ans)
dice_throw_100000_times

```

Out[14]: Counter({6: 166224, 3: 167031, 5: 166728, 4: 166288, 1: 167356, 2: 166373})

In [15]:
```
dice_throw_100000_times.values()
```

Out[15]: dict_values([166224, 167031, 166728, 166288, 167356, 166373])

In [21]:
```
Expected_value_of_100000_tosses = ((1*166224)/1000000) + ((2*167031)/1000000) + ((3*166728)/10000
Expected_value_of_100000_tosses
```

Out[21]: 3.5006399999999998

In [23]:
```
expected_value_of_one_toss = (1*1/6)+(2*1/6)+(3*1/6)+(4*1/6)+(5*1/6)+(6*1/6)
expected_value_of_one_toss
```

Out[23]: 3.5

In [ ]:
```

```

In [24]:
```
num_throws=10000



def throw_dice():
    all_tosses = []
    for toss_id in range(num_throws):
        toss_value = np.random.choice( a = [1,2,3,4,5,6]  )
        all_tosses.append(toss_value)
        ans = Counter(all_tosses)
    return ans
throw_dice()
```

Out[24]: Counter({4: 1669, 1: 1628, 2: 1681, 5: 1736, 3: 1660, 6: 1626})

In [ ]:
```

```

In [ ]:
```

```

Two dice example : compute_conditional_prob

conditional probability

E = (dice 1 = 3)

F = (dice 1 + dice2 = 7)

find P[E|F]

In [25]:

```
class TwoDiceConditional:
    """
    Find the probability that sum of two dice is 7 given Dice 1 = 3
    Home Work: Find one flaw in this current code
    """

    def __init__(self, num_trials=100000):
        self.num_trials = num_trials
        self.conditional_prob = self.compute_conditional_prob()

    def set_num_trials(self, num_trials):
        self.num_trials = num_trials

    def compute_conditional_prob(self):
        all_tosses = self.get_all_tosses()
        num_conditioned_samples = 0
        num_favourable_samples = 0
        for dice1, dice2 in all_tosses:
            if dice1 == 3:
                num_conditioned_samples += 1
                if dice1 + dice2 == 7:
                    num_favourable_samples += 1
        conditional_prob = num_favourable_samples / num_conditioned_samples
        print(
            "The conditional probability is {}".format(
                np.round(conditional_prob, 4)
            )
        )
        return conditional_prob

    def get_all_tosses(self):
        all_tosses = []
        for trial in range(self.num_trials):
            dice1 = np.random.choice([1, 2, 3, 4, 5, 6])
            dice2 = np.random.choice([1, 2, 3, 4, 5, 6])
            all_tosses.append([dice1, dice2])
        return all_tosses


if __name__ == "__main__":
    tdc = TwoDiceConditional()
```

```
The conditional probability is 0.1671
```

In [28]:

```
1/6 # expected outcome
```

Out[28]:

```
0.16666666666666666
```

In [ ]:

```
### conditional probability E =  ( dice 1 = 3)
#                               F = (dice 1 + dice2 = 7)

# find P[E|F]
```

In [27]:
```
num_trials = 1000000
num_favorable_samples = 0   # E intersection F
num_conditioned_samples = 0  # denominator F

for trial in range(num_trials):

    dice1 = np.random.choice([1,2,3,4,5,6])
    dice2 = np.random.choice([1,2,3,4,5,6])

    if dice1 + dice2 == 7:
        num_conditioned_samples += 1   #F
        if dice1 == 3 :
            num_favorable_samples += 1  #E and F

conditional_probability = num_favorable_samples / num_conditioned_samples
conditional_probability
```

Out[27]: 0.16543652530533687

In [ ]:
```

```

Covid example of Condtional Probability:

```
# Covid example

U : 1000 all covid patients
m1 : 100 given medicine
S : 700 survived


probability os patient given medicine
P[m1] = 100/1000 = 0.1

probability of survival
P[S] = 700/1000 = 0.7

people who servived and given medicine is : m1 intersection S
                                            m1 ∩ S = 95 (given in data)
          probability of survival who given  medicine = 95/1000
          P[ m1 ∩ S ] = 95/1000



now  , if we want to know, the effectiveness of drug :
we need to find number of people survived who were given drug
we need to find probability of survival after given the drug,
probability wrt the people given medicine(condition)

P[S|m1] = probability of # of survival given # of people medicine
        = P[S ∩ m1] / P[m1]
        probability  of S given m1 . (conditional probability)
        = (95/1000) / (100/1000)
        = 95 / 100 = 0.95


So , probability of Survival is 0.7
and probability of survival given medicine  is 0.95
there are higher chances survival with medicine :

```

Conditional Probability :

```
Conditional Probability :

P[A|B] = P[A ∩ B] / P[B]
P[A ∩ B] = P[B|A]*P[A]


```

```
P[A|B] = P[A ∩ B] / P[B]
       = P[B|A]*P[A] / P[B]

```

```
what if :

probability os patient given medicine
P[m1] = 100/1000 = 0.1

probability of survival
P[S] = 700/1000 = 0.7

P[ m1 ∩ S ] = 70/1000

P[S|m1] = probability of # of survival given # of people medicine
        = P[S ∩ m1] / P[m1]
        = 70 / 100 = 0.7


So , probability of Survival is 0.7  = probability of survival given medicine  is 0.7  ( P[S|m1] = P[S] )

there for no significant probability difference

That means the m1 and S are independent events :
P[S|m1] = P[S]
```

Defination of indepedence

```
P[A|B] = P[A] defination of indepedence

in above example :

P[S|m1] = P[S]

P[S ∩ m1] / P[m1] = P[S]
```

Independent Events :

```
# Independent Events :
A and B are independent Events if :

P[A|B] == P[A]
P[B|A] == P[B]

P[A ∩ B] = P[A]*P[B]

"""
ratio of A happeneing inside B
is equal to
the ration of B happening inside the whole universe

"""

```

In [ ]:

```

```

2 Dice throw example for independecе

```
dice 2 times : independece

sample space :
{
11 12 13 14 15 16
21 22 23 24 25 26
31 32 33 34 35 36
41 42 43 44 45 45
```

```
51 52 53 54 55 56
61 62 63 64 65 66
}
total 36 outcomes

E : dice1 is 3 : E = {31,32,33,34,35,36}
                 P[E] = 6/36 =1/6
F : dice1 + +dice2 == 7
                 F = {16,25,34,43,52,61}
                 P[F] = 6/36 =1/6
G : dice1**2 + dice2**2 == 25
                 G = {34,43}
                 P[G] = 2/36 = 1/18
E ∩ F = {34}
P[E ∩ F] = 1/36

P[E|F] = P[E ∩ F]/ P[F]
       = (1/36)  / (6/36)
       = 1/6
       = P[E]

So , E and F are independent events
two dice are independent events
```

```
     P[E|F]       = P[E ∩ F]                /     P[F]
conditional_prob  = num_favourable_samples  / num_of_conditioned_samples


```

In [31]:

```python
trials = 1100
num_favourable_samples = 0    # E and F
num_conditional_samples = 0   # F

for t in range(trials):
    dice1 = np.random.choice([1,2,3,4,5,6])
    dice2 = np.random.choice([1,2,3,4,5,6])

    if dice1 + dice2 == 7 :
        num_conditional_samples += 1
        if  dice1 == 3:
            num_favourable_samples+= 1
conditional_probability = num_favourable_samples / num_conditional_samples
conditional_probability

```

Out[31]: 0.1656441717791411

In [32]:

```python
expected_value = 1/6
expected_value
```

Out[32]: 0.16666666666666666

```

E : dice1 is 3 : E = {31,32,33,34,35,36}
                 P[E] = 6/36 =1/6

F : dice1 + +dice2 == 7
                 F = {16,25,34,43,52,61}
                 P[F] = 6/36 =1/6

P[E|F] = P[E ∩ F]/ P[F]
       = (1/36)  / (6/36)
       = 1/6                        = 0.1656441717791411
       = P[E]                       = 0.16
```

In [ ]:

Dice and Coin | independence

```
S = { H1,H2,H3,H4,H5,H6,
      T1,T2,T3,T4,T5,T6 }

take an experimental events  :
E = coin is head = { H1,H2,H3,H4,H5,H6 }   = 6/12 = 1/2 = P[E]
F = dice is 3 = { H3,T3 }                  = 2/12 = 1/6 = P[F]
E ∩ F = {H3}              P[E ∩ F] = 1/12


probability of E given F:

P[E|F] = P[E ∩ F]/ P[F]
       = (1/12)  / (1/6)
       = 6 / 12
       = 1 / 2
       = P[E]

probability of F given E:

P[F|E] = P[E ∩ F]/ P[E]
       = (1/12)  / (1/2)
       = 2 / 12
       = 1 / 6
       = P[F]



So , E and F are independent events
       So , dice and coin toss are independent events
```

P[E|F] = P[E ∩ F]/ P[F]

In [35]:
```python
trials = 1000
num_favourable_samples = 0    # E and F
num_conditional_samples = 0   # F

for t in range(trials):
    coin = np.random.choice(["H","T"])
    dice = np.random.choice([1,2,3,4,5,6])

    if dice == 3 :
        num_conditional_samples += 1
        if  coin == "H":
            num_favourable_samples+= 1
conditional_probability = num_favourable_samples / num_conditional_samples
conditional_probability
            # P[E|F] = P[E ∩ F]/ P[F]
```

Out[35]: 0.4945652173913043

In [34]:
```python
expected_value = 1/2   # P[E|F] = P[E ∩ F]/ P[F]
expected_value
```

Out[34]: 0.5

P[F|E] = P[E ∩ F]/ P[E]

In [38]:
```
trials = 1000
num_favourable_samples = 0    # E and F
num_conditional_samples = 0   # F

for t in range(trials):
    coin = np.random.choice(["H","T"])
    dice = np.random.choice([1,2,3,4,5,6])

    if dice == 3 and coin == "H" :
        num_favourable_samples += 1
    if coin == "H":
        num_conditional_samples += 1
conditional_probability = num_favourable_samples / num_conditional_samples
conditional_probability
            # P[F|E] = P[E ∩ F]/ P[E]
```

Out[38]: 0.16247582205029013

In [37]:
```
expected_value = 1/6   # P[F|E] = P[E ∩ F]/ P[E]
expected_value
```

Out[37]: 0.16666666666666666

In [ ]:
```

```

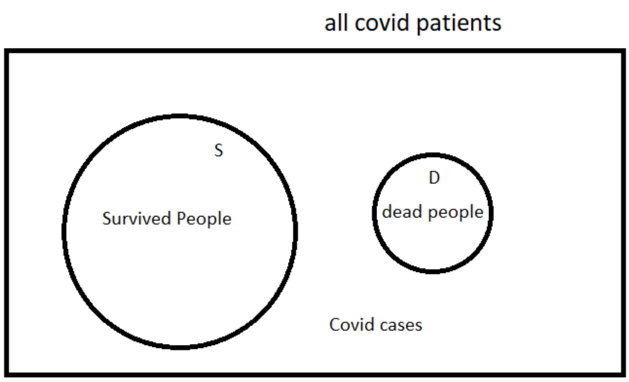
Mututally exclusive and mutually exhaustive :

```
Covid Example :

U : all patients
S : survived (recovered)
C : cases  (symptomatic )
d : died

S ∩ d = {}   :
S ∩ c = {}   :    Mutually Exclusive Events
c ∩ d = {}   :

S ∪ c ∪ d = Universe    Mutually Exhausted Events
```



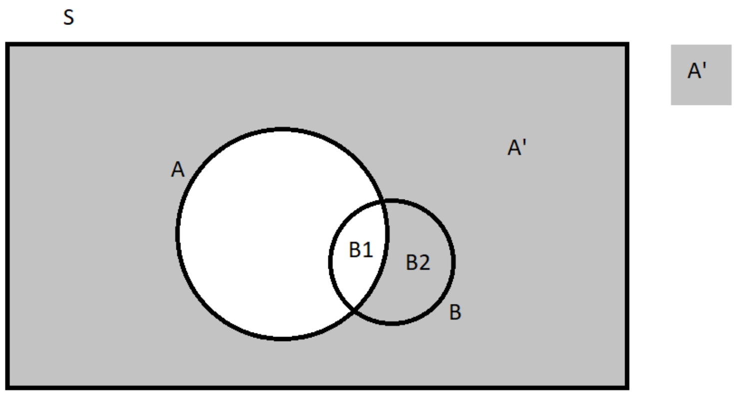
In [ ]:
```

```

In [ ]:
```

```

Bayes' Theorem




```
B = B1 ∪ B2

P[B1] = P[A ∩ B]
P[B2] = P[A'∩ B]


P[B] = P[B1]        +  P[B2]
     = P[B ∩ A]     +  P[ B ∩ A']
     = P[B|A]*P[A]  +  P[B|A']*P[A']



A ∩ A' = {}
A ∪ A' = S

so , A and A' are mtually exclusive and exhaustive

P[A|B] = P[A ∩ B]
         /P[B]

       = P[B|A]*P[A]
         / P[B]

       = P[B|A]*P[A]
         / P[B1] +  P[B2]

       = P[B|A]*P[A]
         / P[B ∩ A] + P[ B ∩ A']

       =        P[B|A]*P[A]
         /( P[B|A]*P[A] + P[B|A']*P[A'])
```

Covid example | Bayes' Theorem:

```
people : heathy or discesed
    test : +ve or -ve

P[H] = 0.9 #(90% population is heathy)
that means P[D] = 1- P[H] = 0.1 (disceased)



P[-ve|H] = 0.75    P[-ve|D] = 0.25

P[+ve|H] = 0.25    P[+ve|D] = 0.75


# if we get a +ve , what is the probability that i m still healthy

P[H|+ve]


P[H|+ve] = P[+ve|H]*P[H]   /  ( P[+ve|H]*P[H] + P[+ve|D]*P[D])
```

```
        = (0.25 * 0.9)  /  (0.25*0.9 + 0.75*0.1 )
        = 0.75

```

In [39]:
```python
(0.25 * 0.9)  /  ((0.25*0.9) + ( 0.75*0.1 ))
```

Out[39]: 0.7499999999999999

In [ ]:
```python

```

In [ ]:
```python

```

In [ ]:
```python

```

In [ ]:
```python

```