18MAB204T-Assignment#1

Register No.	**:**	
Name	**:**	
Subject	**:**	**18MAB204T-Probability and Queuing Theory**
Class/Room No.	**:**	**TP202**
Sem./Branch	**:**	**IV/CSE**

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Instructions:

Answer all the questions. Do the assignment neatly in the A4 sheets and file it then submit to me. Print these questions and keep it in the front of your Assignment file. Draw the diagrams wherever needed. To follow the uniformity you can print the above details in the front page of your assignment.

Questions to solve:

1. Two random samples gave the following data. Test whether the population have come from the same population.

Sample 1	20	16	26	27	23	22	18	24	25	19		
Sample 2	17	23	32	25	22	24	28	18	31	33	20	27

2. Fit normal distribution for the following distribution and also test the goodness of fit.

x	125	135	145	155	165	175	185	195	205
f	1	1	14	22	25	19	13	3	2

3. Fit Poisson distribution for the following distribution and also test the goodness of fit.

x	0	1	2	3	4
f	123	59	14	3	1

4. The table given below gives the results of survey in which 250 respondents were classified according to levels of education and attitude towards students agitation in a certain town. Test whether the two criteria of classification are independent.

		Attitude		
		Against	Neutral	For
Education	Middle School	40	25	5
	High School	40	20	5
	College	30	15	30
	Post Graduate	15	15	10

5. In an epidemic of certain disease, 92 children contacted the disease. Of these 41 received no treatment and of these 10 showed after effects. Of the remainder who did receive the treatment, 17 showed after effects. Test hypothesis that the treatment was not effective.

6. People arrive at a telephone booth according to a Poisson process at an average rate of 12 per hour, and the average time for each call is an exponential r.v. with mean 2 minutes. **(a)** What is the probability that an arriving customer will find the telephone booth occupied? **(b)** It is the policy of the telephone company to install additional booths if customers wait an average of 3 or more minutes for the phone. Find the average arrival rate needed to justify a second booth.

7. At a railway station, only one train is handled at a time. The railway yard is sufficient only for 2 trains to wait while other is given signal to leave the station. Trains arrive at the station at an average rate of 6 per hour and the railway station can handle them on a average of 12 per hour. Assuming Poisson arrival and exponential service distribution, find the

   (a) steady state probabilities of the various no. of trains in the system

   (b) average no. of trains in the system and

   (c) average waiting time

8. a) Customers arrive at a one man barber shop according to a Poisson process with a mean inter-arrival time of 20 minutes. Customers spend an average of 15 minutes in the barber's chair. If an hour is used as a unit of time, then
   i) What is the probability that a customer need not wait for a haircut?
   ii) What is the expected number of customers in the barber shop and in the queue?
   iii) How much time can a customer expect to spend in the barber shop?
   iv) Find the average time that the customer spends in the queue
   v) The owner of the shop will provide another chair and hire another

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barber when a customers average time in the shop exceeds 1.25 hr. By how much should the average rate of arrivals increase in order to justify a second barber?
vi) Estimate the fraction of the day that the customer will be idle.
vii) What is the probability that there will be more than 6 customers waiting for service?
viii) Estimate the percentage of customers who have to wait prior to getting into the barbers chair.
ix) What is the probability that the waiting time (a) in the system (b) in the queue, is greater than 12 minutes?

9. a) Customers arrive at a one man barber shop according to a Poisson process with a mean inter-arrival time of 20 minutes. Customers spend an average of 15 minutes in the barber's chair. Assume that the shop has only 6 chairs for waiting and a customer who arrive will leave the shop if all the 6 chairs are full. If an hour is used as a unit of time, then
i) What is the probability that a customer need not wait for a haircut?
ii) What is the expected number of customers in the barber shop and in the queue?
iii) How much time can a customer expect to spend in the barber shop?
iv) Find the average time that the customer spends in the queue
v) The owner of the shop will provide another chair and hire another barber when a customers average time in the shop exceeds 1.25 hr. By how much should the average rate of arrivals increase in order to justify a second barber?
vi) Estimate the fraction of the day that the customer will be idle.
vii) What is the probability that there will be more than 6 customers waiting for service?
viii) Estimate the percentage of customers who have to wait prior to getting into the barbers chair.
ix) What is the probability that the waiting time (a) in the system (b) in the queue, is greater than 12 minutes?

10. A gambler has $Rs.2$. He bets $Re.1$ at a time and wins $Re.1$ with probability $\frac{1}{2}$. He stops playing if he loses $Rs.2$ or wins $Rs.4$. What is the

    (a) TPM of the related Markov chain?
    (b) probability that he has lost his money at the end of 5 plays?
    (c) probability that the game last more than 9 plays?

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11. Assume that people in a particular society can be classified as belonging to the upper class (U), middle class (M), and lower class (L). Membership in any class is inherited in the following probabilistic manner. Given that a person is raised in an upper-class family, he or she will have an upper-class family with probability 0.7, a middle-class family with probability 0.2, and a lower class family with probability 0.1. Similarly, given that a person is raised in a middle-class family, he or she will have an upper-class family with probability 0.1, a middle-class family with probability 0.6, and a lower-class family with probability 0.3. Finally, given that a person is raised in a lower-class family, he or she will have a middle-class family with probability 0.3 and a lower-class family with probability 0.7. Determine **(a)** the transition probability matrix and **(b)** the state transition diagram for this problem.

12. Find the nature of the states (Classify the states) of the Markov chain

$$\begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{pmatrix} \end{array}.$$

13. The tpm of a Markov Process $\{X_n\}$, $n = 1, 2, 3$ having 3 states 0,1,2 is

$$\begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{pmatrix} \end{array}$$

and the initial distribution $P^{(0)} = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$.

Find

(a) $P(X_3 = 2/X_2 = 1)$

(b) $P(X_2 = 2, X_1 = 1, X_0 = 2)$

(c) $P(X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2)$

(d) $P(X_2 = 2)$

14. Two boys B_1 and B_2 and two girls G_1 and G_2 are throwing a ball from one to the other. Each boy throws the to the other boy with the

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probability 0.5 and to each girl with probability 0.25. On the other hand each girl throws the ball to each boy with probability 0.5 and never throw to other girl. In the long run, how often does each receive the ball?

****************All The Best****************

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