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CSE - J2

Unit - IV

Tutorial sheet - II

Part - A

1)

$\lambda = 8$ per hour

$\mu = 12$ per hour

(i) Average (waiting time in Queue) $W_q = \dfrac{\lambda}{\mu(\mu-\lambda)}$

$= \dfrac{8}{12(12-8)}$

$= \dfrac{8}{12(4)} = \dfrac{8}{48} = \dfrac{1}{6}$

$\frac{1}{6}$ hr

$1/6$ hr $= \frac{1}{6} \times 60$ min

$= 10$ min

So Customer has to wait 10 min in a Queue

2) Model $(M/M/1) : (/FIFO)$

$\lambda = 6$ per hour $\mu = 10$ per hour

P( ~~C~~ustomer has to wait more than 15 min)

$P(W_s > t)$

$t = 15$ min $= \frac{15}{60}$ hr $= \frac{1}{4}$ hr

$P(W_s > 0.25) = e^{-(\mu - \lambda)t}$

$= e^{-(10-6)0.25}$

$= e^{-((4) * 0.25)}$

$= 0.3678$

Thus the Probability that Customer has to wait more than 15 min $= 0.3678$

3) Model $(M/M/1) : (4/FIFO)$

$K = 4 \qquad \lambda = 4$ per hour $= 12$ per hour

(i) P( No Customer in System) : $P_0$ (system is idle)

$\lambda \neq \mu$

$$\text{So} \quad P_0 = \frac{1-\frac{\lambda}{\mu}}{1-\left(\frac{\lambda}{\mu}\right)^{k+1}} = \frac{1-\frac{4}{12}}{1-\left(\frac{4}{12}\right)^5}$$

$$= \frac{1-1/3}{1-(1/3)^5} = \frac{\frac{2}{3}}{1-4.1152\times10^{-3}}$$

$$= \frac{\frac{2}{3}}{0.995}$$

$$= 0.6700$$

(ii) P. If $\lambda = \mu$

$$P_0 = \frac{1}{k+1} = \frac{1}{4+1} = \frac{1}{5} = 0.2$$

4) Model $(M/M/1) : (K/FIFO)$

Little Formula Average Waiting time in system & Queue

$$L_q = L_s - \frac{\lambda}{\mu} \qquad W_s = \frac{L_s}{\lambda'}$$

$$W_q = \frac{L_q}{\lambda'}$$

$$W_q = \frac{L_s - \frac{\lambda}{\mu}}{\lambda'} \qquad \frac{L_s}{\lambda'} - \frac{\frac{\lambda}{\mu}}{\lambda'}$$

$$\boxed{W_q = W_s - \frac{\lambda/\mu}{\lambda'}}$$

Scanned with CamScanner

5) $K = 7$

$\lambda = 3$ per hour

$\mu = 4$ per hour

Model $(M/M/1) : (K/FIFO)$

Average Number of Customers in System $\lambda \neq \mu$

$$L_s = \left(\frac{\lambda}{\mu-\lambda}\right) - \frac{(k+1)\left(\frac{\lambda}{\mu}\right)^{k+1}}{1-\left(\frac{\lambda}{\mu}\right)^{k+1}}$$

$$L_s = \frac{3}{4-3} - \frac{(8)\left(\frac{3}{4}\right)^8}{1-\left(\frac{3}{4}\right)^8}$$

$$3 - \frac{8 \times \left(\frac{3}{4}\right)^8}{1-\left(\frac{3}{4}\right)^8}$$

$$\frac{8 \times 0.1001}{1-\left(\frac{3}{4}\right)^8} = \frac{0.8008}{1-0.1001}$$

$$= \frac{0.8008}{0.8999}$$

$$3 - 0.8898$$

$$2.1102$$

6) $\lambda = 4$ per hour

$\mu = \frac{1}{10} *$ $0.1 * 60 = 6$ per hour

Model $(M/M/1) : (\infty/FIFO)$

$$L_s = \frac{\lambda}{\mu - \lambda} = \frac{4}{6-4} = \frac{4}{2} = 2$$

$L_s$ = 2 per hour

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{4^2}{6(6-4)} = \frac{16}{6(2)} = \frac{\cancel{16}}{\cancel{12}} = \frac{4}{3}$$

$$= 1.33$$

$$W_s = \frac{1}{\mu - \lambda} = \frac{1}{2} = 0.5$$

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{4}{6(2)} = \frac{4}{12} = 1/3 = 0.33$$

7) $\lambda$ = 12 per hour

$\mu$ = 30 per hour

(i) P(No customer at Counter) : $P_0$(Idle) $= 1 - \frac{\lambda}{\mu}$

$$1 - \frac{12}{30}$$

$$1 - \frac{\cancel{4}}{10} \; \frac{2}{5}$$

$$\frac{3}{5} = 0.6$$

(ii) P(There are more than two customers) $= \frac{2}{5} = P(n>2) = \left(\frac{\lambda}{\mu}\right)^{k+1}$

$$= \left(\frac{12}{30}\right)^3 = \left(\frac{4}{10}\right) = \left(\frac{2}{5}\right)^3$$

$$= 0.064$$

(iii) P( No customer waiting to be served) = $e^{-(}$

$1 - \frac{\lambda}{\mu} = 1/3 = 0.333$

(IV)

(V) $P(W_q \le t)$

$1 - P(W_q > t)$

$= 1 - \frac{\lambda}{\mu} e^{-(\mu-\lambda)t}$

$1 - \frac{12}{30} e^{-(30-12) \times \frac{\cancel{4}}{\cancel{60}}}$

$1 - \frac{2}{5} e^{-\cancel{18} \times \cancel{2} \, \cancel{30} \, \frac{1}{\cancel{15}}}$ 6 5

$1 - \frac{2}{5} e^{-6/5}$

$1 - \frac{2}{5} e$

$0.30 \times 0.4$

$= 1 - 0.12$

$0.88$

(iv) P

9) K

(i) P(

(ii)

(iii)

8) $\lambda = 1/10$ min

$\mu = 1/8$ min

(ii)

(i) $L_s$ (ii) $W_s$ (iii) $L_q$ (iv) $P_0$

(i) $L_s = \frac{\lambda}{\mu - \lambda} = \frac{1/10}{1/8 - 1/10} = \frac{1/10}{\frac{10-8}{80}} = \frac{1/10}{2/80} = \frac{80}{20} = 4$

(ii) $W_s = \frac{1}{\mu - \lambda} = \frac{1}{\frac{2}{80}} = 40$ min

(iii) Lq : $\frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{1/100}{1/8(1/8-1/10)} = \frac{1/100}{1/8(\frac{2}{80})}$

$= \frac{64\cancel{0}}{20\cancel{0}} = \frac{64}{20} = 3.2$

$= 3.2$

(iv) $P_0 = 1-P = 1-\frac{\lambda}{\mu} = 1-\frac{1/10}{1/8} = \frac{8}{10}$

$1-4/5$

$= 1/5$

9) $K = 3$ (2 wait + one handled) $= 3$

$\lambda = 6$ per hour

$\mu = 6$ per hour

$P_0 = \frac{1}{k+1} = 1/4$

(i) P( No of trains in System) Ls :

(ii) P( Wq

(iii) If $\lambda$ = doubled $= \cancel{1}2$ per hour

$Ls = \frac{k}{2} = \frac{3}{2} = 1.5$

$\lambda' = \mu(1-P_0)$

$\lambda' = 6(1-1/4)$

$6 \times 3/4$

$\lambda' = \frac{18}{4} = \frac{9}{2}$

$\boxed{\lambda' = 9/2}$

(ii) Wq $= \frac{Lq}{\lambda'} = \frac{Lq}{}$

Wq' $= Lq = Ls - \frac{\lambda}{\mu}$

$1.5 - 1$

$= 0.5$

$= \frac{0.5 \times 2}{9} = 1/9$

$\boxed{Wq = 1/9}$

10) $\lambda = 30$ per hour

$k = 14+1 = 15$ Mode $(M|M|1) : (K|FIFO)$

$\mu = 20$ per hour

(i) $\lambda' = \mu(1 - P_0)$

$$P_0 = \frac{1 - \frac{\lambda}{\mu}}{1 - \left(\frac{\lambda}{\mu}\right)^{k+1}}$$ when $\lambda \neq \mu$

$$\frac{1 - \frac{30}{20}}{1 - 3/2} \qquad \frac{-1/2}{1 - \left(\frac{3}{2}\right)^{16}} = \frac{-1/2}{1 - 656.84} \qquad \frac{-0.5}{-655.84}$$

~~Question wrong~~;

$P_0 = = 0.00076238$

$\lambda = 20 \times 0.999 = 19.998$

(ii) $\cdot P_0 = 0.00076238$

(iii) $W_s = \dfrac{L_s}{\lambda}$

$$L_s = \frac{\lambda}{\mu - \lambda} - \frac{(k+1)\left(\frac{\lambda}{\mu}\right)^{k+1}}{1 - \left(\frac{\lambda}{\mu}\right)^{k+1}} = \frac{30}{10} - $$

$$3 - \frac{(16)\left(\frac{3}{2}\right)^{16}}{1 - \left(\frac{3}{2}\right)^{16}}$$

$$3 + \frac{10509.44}{655.84}$$

$L_s = 19$ patients $= 19.02$ $\quad 16.02 + 3$

$W_s = \dfrac{19}{19.998} \simeq 1$ patient hour