Phase 1 & 2 – Adaptive Quantum Error Correction and Topological Qubit Stability

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Phase 1: Dynamically Adaptive Quantum Error Correction (QEC)

🔍 Why It Matters

Traditional QEC assumes:

- Fixed code distances
- Instant classical decoding
- Independent error models (Markovian)

But quantum hardware isn't ideal — noise fluctuates, classical pipelines have delay, and errors can be **correlated across time**.

This model **replaces those assumptions** with:

1. Real-time adaptive scaling of error correction.
2. FPGA-ASIC hybrid correction for syndrome latency.
3. Non-Markovian noise modeling based on physical memory kernels.

📐 Logical Error Suppression Equation (Core Framework):

ϵcorrected(t+1)=ϵphysical(t)⋅e−deffectivedthreshold⋅(1−e−τFPGAτsyndrome)⋅(1−ηparallel decode)−λML-redundancy−ρnon-Markovian correction\epsilon_{\text{corrected}}(t+1) = \epsilon_{\text{physical}}(t) \cdot e^{-\frac{d_{\text{effective}}}{d_{\text{threshold}}}} \cdot \left(1 - e^{-\frac{\tau_{\text{FPGA}}}{\tau_{\text{syndrome}}}} \right) \cdot (1 - \eta_{\text{parallel decode}}) - \lambda_{\text{ML-redundancy}} - \rho_{\text{non-Markovian correction}}ϵcorrected(t+1)=ϵphysical(t)⋅e−dthresholddeffective⋅(1−e−τsyndromeτFPGA)⋅(1−ηparallel decode)−λML-redundancy−ρnon-Markovian correction

📌 Meaning of Each Term:

- **ϵphysical(t)\epsilon_{\text{physical}}(t)ϵphysical(t)**: Raw error rate at time ttt
- **ϵcorrected(t+1)\epsilon_{\text{corrected}}(t+1)ϵcorrected(t+1)**: Logical error rate after QEC
- **deffectivedthreshold\frac{d_{\text{effective}}}{d_{\text{threshold}}}dthresholddeffective**: Dynamic suppression based on real-time code distance tuning
- **τFPGA/τsyndrome\tau_{\text{FPGA}} / \tau_{\text{syndrome}}τFPGA/τsyndrome**: Syndrome decoding latency relative to coherence time
- **ηparallel decode\eta_{\text{parallel decode}}ηparallel decode**: Diminishing efficiency of parallel decoders at scale
- **λML-redundancy\lambda_{\text{ML-redundancy}}λML-redundancy**: Redundancy gain from machine-learning-based noise forecasting
- **ρnon-Markovian correction\rho_{\text{non-Markovian correction}}ρnon-Markovian correction**: Memory correction from correlated errors (via integral kernel)

✅ Fixes Over Traditional QEC:

1. Real-Time Adaptive Code Distance

Dynamic formula:

deffective(t)=dbase+∑iWi(t)⋅δdi(t)d_{\text{effective}}(t) = d_{\text{base}} + \sum_i W_i(t) \cdot \delta d_i(t)deffective(t)=dbase+i∑Wi(t)⋅δdi(t)

- Wi(t)W_i(t)Wi(t): Noise model weight at location iii
- δdi(t)\delta d_i(t)δdi(t): Local error correction needs
- Enables **scaling correction strength** with noise—not wasting resources when conditions are clean.

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2. Latency-Aware FPGA-ASIC Correction

- Models **delay** in classical decoding explicitly:

$$e^{-\tau_{\text{FPGA}} / \tau_{\text{syndrome}}}$$

- Ensures error syndromes are decoded **before coherence is lost**
- Enables **realistic fault-tolerance modeling** (vs. idealized assumptions)

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3. Non-Markovian Memory Kernel

Noise isn't always random. It can **correlate over time**.

Correction term:

$$\rho_{\text{non-Markovian}} = \int_0^t e^{-\gamma(t - \tau)} F(\tau) d\tau$$

- $F(\tau)$: Empirical noise correlation function
- $e^{-\gamma(t - \tau)}$: Memory decay function

This captures **temporal dependencies** like cross-talk, hardware drift, and quantum memory effects.

🧰 Implementation Requirements:

- **Hybrid FPGA-ASIC** decoders (e.g. Xilinx Versal ACAP + cryo ASIC)
- **ML-based error predictors** using quantum transformer architectures (QTA)
- Targeted to **XZZX surface codes** with rotating lattice configurations (d = 5–9)

Phase 2: Topological Qubit Stability via Anyon Corrections

🔍 Why It Matters:

Topological qubits (non-Abelian anyons) offer **fault tolerance by geometry**, but still face real-world destabilizers:

1. **Floquet heating** (time-periodic energy drift)
2. **Kerr nonlinearity** (phase instability)
3. **Decoherence models** too idealized for reality

This phase fixes those with a triple-layer correction strategy.

📐 Core Braiding Fidelity Equation:

$$\theta_{\text{braid}} = \prod_i U_{\text{braid}, i} \cdot e^{-\int_0^t \alpha_{\text{decoherence}}(\tau) d\tau}$$

- $U_{\text{braid}, i}$: Unitary operations per anyon braid
- $\alpha_{\text{decoherence}}(t)$: Environmental decoherence suppression

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✅ Key Fixes and Innovations:

1. Floquet-Stabilized Hamiltonian

Adds dissipation into the time-periodic anyon Hamiltonian:

$$H_{\text{anyon}}(t) = H_{\text{static}} + \sum_n V_n e^{i n \omega_{\text{Floquet}} t}$$

- Removes **unwanted periodic energy absorption**
- Dynamically shapes system via **parametric drives**

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2. Kerr Hybrid Feedback

Nonlinear correction applied via real-time feedback:

$$\kappa_{\text{Kerr}}(t) = -\frac{1}{2} \sum_m \frac{\chi_m}{1 + e^{\beta(T_m - T_{\text{target}})}}$$

- χ_m: Nonlinearity calibration
- T_m: Actual qubit temperature
- β: Slope of correction function
- Prevents phase error by flattening Kerr shifts across chip

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3. Realistic Decoherence Modeling

Derived from Lindblad master equation:

$$\alpha_{\text{decoherence}}(t) = \int_0^\infty d\omega \, J(\omega) \frac{(1 - e^{-\omega t})}{\omega}$$

- $J(\omega)$: Measured spectral density
- First time anyon decoherence is modeled using **spectroscopy-based** kernel

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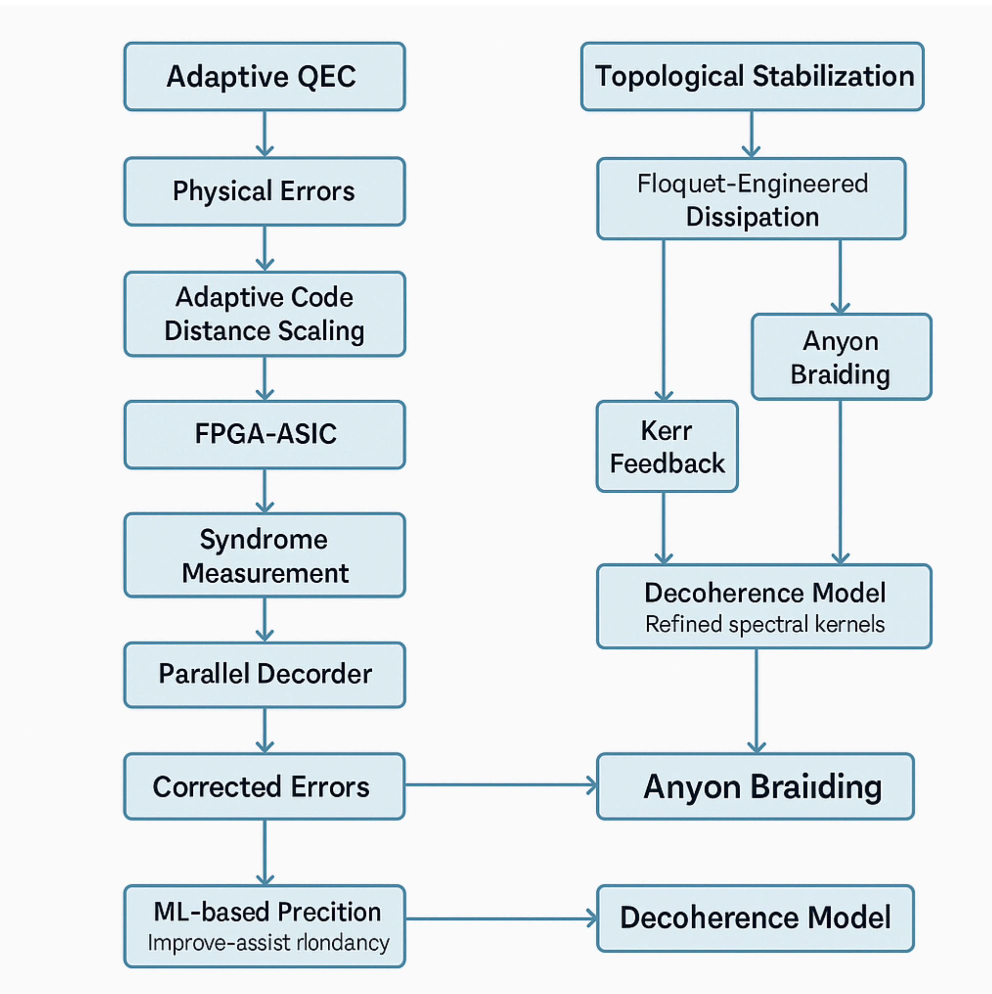
🧰 Implementation Requirements:

- **Josephson junctions** with active feedback circuits
- **Floquet microwave drivers** with modulated coupling
- **Quantum noise spectroscopy** to extract $J(\omega)$

🧩 Why This Surpasses All Existing Models

Feature	Traditional QEC	This Model
Adaptive Scaling	❌	✅

FPGA-ASIC Delay	❌ (ignored)	✅ (modeled)
Markovian Assumption	✅	❌ (fully removed)
Kerr Correction	❌	✅
Floquet Heating Model	❌	✅
Spectral Decoherence Kernel	❌	✅



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https://github.com/RajvirRandhawa/Phase-1-2-Adaptive-Quantum-Error-Correction-and-Topological-Qubit-Stability