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Calculating standard deviation step by step

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Introduction

In this article, we'll learn how to calculate standard deviation "by hand".

Interestingly, in the real world no statistician would ever calculate standard deviation by hand. The calculations involved are somewhat complex, and the risk of making a mistake is high. Also, calculating by hand is slow. Very slow. This is why statisticians rely on spreadsheets and computer programs to crunch their numbers.

So what's the point of this article? Why are we taking time to learn a process statisticians don't actually use? The answer is that learning to do the calculations by hand will give us insight into how standard deviation really works. This insight is valuable. Instead of viewing standard deviation as some magical number our spreadsheet or computer program gives us, we'll be able to explain where that number comes from.

Overview of how to calculate standard deviation

The formula for standard deviation (SD) is

$$\text{SD} = \sqrt{\frac{\sum |x - \mu|^2}{N}}$$

where $\sum$ means "sum of", x is a value in the data set, μ is the mean of the data set, and N is the number of data points in the population.

The standard deviation formula may look confusing, but it will make sense after we break it down. In the coming sections, we'll walk through a step-by-step interactive example. Here's a quick preview of the steps we're about to follow:

Step 1: Find the mean.

Step 2: For each data point, find the square of its distance to the mean.

Step 3: Sum the values from Step 2.

Step 4: Divide by the number of data points.

deviation of a population

Next tutorial

Step 5: Take the square root.

An important note

The formula above is for finding the standard deviation of a population. If you're dealing with a sample, you'll want to use a slightly different formula (below), which uses $n-1$ instead of N. The point of this article, however, is to familiarize you with the the process of computing standard deviation, which is basically the same no matter which formula you use.

$$\text{SD}_{\text{sample}} = \sqrt{\frac{\sum |x - \bar{x}|^2}{n-1}}$$

[Why are there two formulas?]

Step-by-step interactive example for calculating standard deviation

First, we need a data set to work with. Let's pick something small so we don't get overwhelmed by the number of data points. Here's a good one:

$$6, 2, 3, 1$$

Step 1: Finding μ in $\sqrt{\dfrac{\sum |x-\mu|^2}{N}}$

In this step, we find the mean of the data set, which is represented by the variable μ.

Fill in the blank.

$\mu =$ 3

Check

[Explain]

Step 2: Finding $|x-\mu|^2$ in $\sqrt{\dfrac{\sum |x-\mu|^2}{N}}$

In this step, we find the distance from each data point to the mean (i.e., the deviations) and square each of those distances.

For example, the first data point is 6 and the mean is 3, so the distance between them is 3. Squaring this distance gives us 9.

Complete the table below.

Data point x	Square of the distance from the mean $\|x-\mu\|^2$
6	9
2	1
3	0
1	4

Check

[Explain]

Step 3: Finding $\sum|x-\mu|^2$ in $\sqrt{\dfrac{\sum|x-\mu|^2}{N}}$

The symbol $\sum$ means "sum", so in this step we add up the four values we found in Step 2.

Fill in the blank.

$\sum|x-\mu|^2 =$ 14

Check

[Explain]

Step 4: Finding $\dfrac{\sum|x-\mu|^2}{N}$ in $\sqrt{\dfrac{\sum|x-\mu|^2}{N}}$

In this step, we divide our result from Step 3 by the variable N, which is the number of data points.

Fill in the blank.

$$\frac{\sum|x-\mu|^2}{N} = $$ 3.5

Check

[Explain]

Step 5: Finding the standard deviation $\sqrt{\dfrac{\sum|x-\mu|^2}{N}}$

We're almost finished! Just take the square root of the answer from Step 4 and we're done.

Fill in the blank.

Round your answer to the nearest hundredth.

$$\text{SD} = \sqrt{\dfrac{\sum |x - \mu|^2}{N}} \approx \text{1.87}$$

Check

[Explain]

Yes! We did it! We successfully calculated the standard deviation of a small data set.

Summary of what we did

We broke down the formula into five steps:

Step 1: Find the mean μ.

$$\mu = \frac{6+2+3+1}{4} = \frac{12}{4} = 3$$

Step 2: Find the square of the distance from each data point to the mean $|x - \mu|^2$.

x	$\lvert x-\mu\rvert^2$
6	$\lvert 6-3\rvert^2 = 3^2 = 9$
2	$\lvert 2-3\rvert^2 = 1^2 = 1$
3	$\lvert 3-3\rvert^2 = 0^2 = 0$
1	$\lvert 1-3\rvert^2 = 2^2 = 4$

Steps 3, 4, and 5:

$$\text{SD} = \sqrt{\frac{\sum |x - \mu|^2}{N}}$$

$$= \sqrt{\frac{9+1+0+4}{4}}$$

$$= \sqrt{\frac{14}{4}} \qquad \text{Sum the squares of the}$$

$$= \sqrt{3.5} \qquad \text{Divide by the number o}$$

$$\approx 1.87 \qquad \text{Take the square root (St}$$

Try it yourself

Here's a reminder of the formula:

$$\text{SD} = \sqrt{\frac{\sum |x-\mu|^2}{N}}$$

And here's a data set:

$$1, 4, 7, 2, 6$$

Find the standard deviation of the data set.

Round your answer to the nearest hundredth.

$\text{SD} =$ [ ]

Check

[Explain]

Tags

Variance and standard deviation

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Questions Tips & Thanks

Question

Ask a question...

Ben Fullstop 3 years ago

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Why in the formula of the second paragraph they used the absolute value and not simple parenthesis? Squaring the algebraic sum in the parenthesis will return a positive value and quite frankly I never saw this formula with the absolute value.

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Surf 3 years ago

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Mathematically there is no difference.

One of the advantages of writing it that way is to explicitly show what the operation being done is.

*In this step, **we find the distance from each data point to the mean** (i.e., the deviations) and square each of those distances.*

By writing it as the absolute value instead of just parentheses, it emphasises the bit in bold.

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kbranch451 3 years ago

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Why isn't "n" upper case (N) since this is a population and not a sample? Also, why use the sample mean symbol vs mu? I didn't know whether to divide by N or (n-1). Guessed and got it right. Thank you.

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Mr. Stuckey 9 months ago more

For every parameter, like mean or standard deviation, and the number of data points, there is a different notation for population or sample. Population mean is μ (greek letter mu), sample mean is x̄ (x-bar). Population standard deviation is σ, sample standard deviation is s. So for the number of data points, population is N and sample is n.

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Tais Price 2 years ago more

What are the steps to finding the square root of 3.5? I can't figure out how to get to 1.87 with out knowing the answer before hand.

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Jordan Cooper 2 years ago more

While it is possible to calculate decimal places of an irrational square root by hand, every method I've seen is a LOT of work. I use a calculator.

7 comments (6 votes) Upvote Downvote Flag

akanksha.rph 3 years ago more

I want to understand the significance of squaring the values, like it is done at step 2.

Why actually we square the number values?

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Matthew Daly 3 years ago more

The important thing is that we want to be sure that the deviations from the mean are always given as positive, so that a sample value one greater than the mean doesn't cancel out a sample value one less than the mean. There are two strategies for doing that, squaring the values (which gives you the variance) and taking the absolute value (which gives you a thing called the Mean Absolute Deviation). Even though taking the absolute value is being done by hand, it's easier to prove that the variance has a lot of pleasant properties that make a difference by the time you get to the end of the statistics playlist.

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jkcrain12 3 years ago more

From the class that I am in, my Professor has labeled this equation of finding standard deviation as the population standard deviation, which uses a different formula from the sample standard deviation. Is there a way to differentiate when to use the population and when to use the sample? Or would such a thing be more based on context or directly asking for a giving one? Why do we use two different types of standard deviation in the first place when the goal of both is the same?

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Dr C 3 years ago

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If we have data on the entire population, then we use the formula shown to compute the population SD. Usually, that will not be the case, so we will *need* to use the sample SD.

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origamidc17 2 years ago

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If I have a set of data with repeating values, say 2,3,4,6,6,6,9, would you take the sum of the squared distance for all 7 points or would you only add the 5 different values?

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Muazzam Ali 2 years ago

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Yes, all 7 points

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See 1 more reply

Shannon a year ago

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But what actually is standard deviation? I understand how to get it and all but what does it actually tell us about the data?

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ZeroFK a year ago

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The standard deviation is a measure of how close the numbers are to the mean. If the standard deviation is big, then the data is more "dispersed" or "diverse".

As an example let's take two small sets of numbers:
4.9, 5.1, 6.2, 7.8
and
1.6, 3.9, 7.7, 10.8
The average (mean) of both these sets is 6. But the second set is more dispersed: the numbers are further away from the mean.
This is reflected in the standard deviation: if I calculated correctly (please check!) the first set has a standard deviation of 2.3, the second has 7.05.

Comment (6 votes) Upvote Downvote F

Brian Hall 3 years ago more

Thank you Sal for all the help you have provided me thus far. I came across a problem that gave me quite a bit of trouble as it did not fit any of the practice problems I have done so far.
A cell phone company inspects 80% of their cell phone keypads produced. A random sample is taken of 116 keypads to analyze. Find the standard deviation?
Any help you can give would be greatly appreciated.

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Matt Wilson 3 years ago more

I believe you would need to provide more information to answer this, right? There is no sample data set.

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Madradubh 2 years ago

more

Hi,
How do I calculate the standard deviation of bivariate data by hand?
Thanks
Sean

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chung.k2 a year ago

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In the formula for the SD of a population, they use mu for the mean. Is there a difference from the x with a line over it in the SD for a sample?

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bysctusa 9 months ago

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No, μ and x̄ mean the same thing (no pun intended). At least when it comes to standard deviation.

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