Uncertainty and Disturbance Estimator Based Robust Pitch Autopilot Design

*Note: Sub-titles are not captured in Xplore and should not be used

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***Abstract*—This document is a model and instructions for LaTeX. This and the IEEEtran.cls file define the components of your paper [title, text, heads, etc.]. *CRITICAL: Do Not Use Symbols, Special Characters, Footnotes, or Math in Paper Title or Abstract.**

***Index Terms*—component, formatting, style, styling, insert**

I. Introduction

Missile controls primarily use classical linear control theory...done so by linearizing the system in finite number of operating points in the flight envelop. H-inf and mu controllers. Linearization of systems and gain scheduling not sufficient when non-linearities are dominant portion of the system, uncertainties arise, and/or external disturbance exists. Thus this has lead to non-linear control theory for missiles...SMC, PC, IOL.

IOL in brief...drawbacks in an 'explanation' of IOL manner and not as strict drawbacks. SMC and PC remarks? None in Godbole. UDE has the capability of ... counter points to IOL, SMC, PC and MOST IMPORTANTLY any magnitude of disturbance (only bandwidth matters, which can be adjusted in Tau).

This study goes over the missile model used, UDE control law and how it has been adapted to the given missile auto pilot, and has comparitive simulations of performance between smc, iol, pc in varying conditions of mach, aerodynamic constants, external disturbances. CAN WE ALSO BREAK ESO SOMEHOW?? MAYBE LARGE MAGNITUDES WILL BREAK IT.

II. Missile Model

The missile model considered/employed in this study is a pitch axis, longitudinal missile model with non-linear dynamics [legacy papers]. The model used in this analysis is based on the hypothetical TAIL-CONTROLLED missile used as a baseline for previous nonlinear control research [7, 8]. This model has been modified in this study to include gravitational forces and a pitch rate damping term (Cmq)- The missile model assumes constant mass, that is, post burnout, no roll rate, zero roll angle, no sideslip, and no yaw rate. Under these assumptions, the longitudinal nonlinear equations of motion for a rigid airframe reduce to two force, one moment and three kinematic equations. FROM mrakle/imech paper...rephrase and cite

The model seems to be well established/well noted/quite distinguished/quite eminent as a number of studies/papers [cite them here] have used this model for the simulation of their corresponding control techniques xxxx, yyyy, zzzz. The non-linear dynamics are described by,

$$\begin{aligned}
\dot{\alpha}(t) &= K_\alpha M(t) C_n[\alpha(t), \delta(t), M(t)] cos(\alpha(t) + q(t) \\
\dot{q}(t) &= K_q M^2(t) C_m[\alpha(t), \delta(t), M(t)] \\
\dot{\delta}(t) &= -\omega_a \delta(t) + \omega_a \delta_c(t)
\end{aligned} \tag{1}$$

where δ_c is the commanded input, α is the output being tracked, δ is the tail-fin deflection and M is the mach dynamics of the missile. The terms C_n and C_m seen in the **??** equations are aerodynamic constants which are defined by

$$\begin{aligned}
C_n(\alpha, \delta, M) &= a_n \alpha^3 + b_n \alpha|\alpha| + c_n \Big(2 - \frac{M}{3}\Big)\alpha \\
&\quad + d_n \delta \\
C_m(\alpha, \delta, M) &= a_m \alpha^3 + b_\alpha |\alpha| + c_m \Big(-7 + \frac{8M}{3}\Big)\alpha \\
&\quad + d_m \delta
\end{aligned} \tag{2}$$

The normal acceleration of the missile can be controlled as desired using the angle of attack α, which in turn is controlled by the missile's tail-fin deflection. Thus the input to the system becomes commanded tail-fin deflection δ_c and the output to be tracked is alpha. The model is works/functions within the constraints of $-20 \leq \alpha \leq 20$, $1.5 \leq M \leq 3$ and $\pm 25\%$ uncertainty in C_n and C_m CITE paper again for these constraints. Various other constants seen in equations **??**, **??** and the remainder of the study are provided in table **??**.

TABLE I
A TEST CAPTION

$P_0 = 46660.185\ N/m^2$	Static pressure at 6096 m
$S = 0.040877\ m^2$	Surface area
$m = 204.023\ kg$	Mass
$v_s = 315.89\ m/s$	Speed of sound
$I_y = 247.44\ kgm^2$	Pitch moment of inertia
$\omega_a = 150\ rad/s$	Actuator bandwidth
$\zeta_a = 0.7$	Actuator damping
$K_q = 0.7 P_0 S d / I_y$	
$K_\alpha = 0.7 P_0 S / m V_s$	
$K_z = 0.7 P_0 S / m$	
$a_n = 19.373\ rad^{-3}$	$a_m = 40.440\ rad^{-3}$
$b_n = -31.023\ rad^{-2}$	$b_m = -64.015\ rad^{-2}$
$c_n = -9.717\ rad^{-1}$	$c_m = 2.922\ rad^{-1}$
$d_n = -1.948\ rad^{-1}$	$d_m = -11.803\ rad^{-1}$

III. Uncertainty and Disturbance Estimator

Short intro paragraph please! ... just two sentences describing what is explained in this section

A. Input Output Linearization (IOL)

Input Output Linearization (IOL) is a control technique applied to non-linear systems in order to linearize them. An inner loop control helps achieve model linearization while an outer control loop works on the linearized system to bring about the desired dynamics. Once the system is linearized, any standard/straightforward/simple linear control law can be applied to the system. As seen in **??**, u_a is the inner control loop used to deal with/handle/linearize the system non-linearlities and ν helps achieve desired tracking performance.

When α is chosen as the output and δ_c is used as the input, it can be seen easily that the relative degree of the system becomes 2. **??** shows that without loss of generality small terms/negligible terms can be made zero/ignored to improve the relative degree of the system. Thus, as per **??**, $d_n \approx 0$ allows for the relative degree to equal the order of the system and and ensure no zero dynamics. Further, for the constraints $-20^\circ \leq \alpha \leq 20^\circ$, $\cos(\alpha) \approx 1$. Successive differentiation of α with the above considerations gives/yields/results in

$$\begin{aligned}\dddot{\alpha} &= K_q M^2 (3a_m\alpha^2 + 2b_m|\alpha| + c_m\Big(-7 + \frac{8M}{3}\Big))\dot{\alpha} \\ &\quad - K_q M^2 d_m \omega_a \delta + K_\alpha M(6a_n\alpha + 2b_n sgn(\alpha))\dot{\alpha}^2 \\ &\quad + K_\alpha M(3a_n\alpha^2 + 2b_n|\alpha| + c_n\Big(2 - \frac{M}{3}\Big))\ddot{\alpha} \\ &\quad + K_q M^2 d_m \omega_a \delta_c\end{aligned} \tag{3}$$

To make it more convenient to apply IOL, the above can be written as

$$\dddot{\alpha} \quad = a + b\delta_c \tag{4}$$

where

$$\begin{aligned}a \triangleq & K_q M^2 (3a_m\alpha^2 + 2b_m|\alpha| + c_m\Big(-7 + \frac{8M}{3}\Big))\dot{\alpha} \\ & - K_q M^2 d_m \omega_a \delta + K_\alpha M(6a_n\alpha + 2b_n sgn(\alpha))\dot{\alpha}^2 \\ & + K_\alpha M(3a_n\alpha^2 + 2b_n|\alpha| + c_n\Big(2 - \frac{M}{3}\Big))\ddot{\alpha} \\ b \triangleq & K_q M^2 d_m \omega_a\end{aligned} \tag{5}$$

Finally, IOL theory can be applied by setting the input δ_c of the system to

$$\delta_c = \frac{1}{b}(u_a + \nu) \tag{6}$$

where

$$\begin{aligned}u_a &= -a \\ \nu &= \dddot{\alpha}^* + m_1(\alpha^* - \alpha) + m_2(\dot{\alpha}^* - \dot{\alpha}) + m_3(\ddot{\alpha}^* - \ddot{\alpha})\end{aligned} \tag{7}$$

In **??**, α^* represents the reference signal/angle of attack/desired angle attack that the system is to track, α is the system's actual/present angle of attack and m_1, m_2, and m_3 are the outer loop controller gains.

B. Uncertainty and Disturbance Estimation (UDE)

The Uncertainty and Disturbance Estimator/tion (UDE) technique is used to address the issue of robustness that is associated with IOL controller. Firstly, in order to use IOL, the system needs to be linearizable. This can be a barrier for the use of IOL in many systems SURE???. Secondly, the IOL controller requires the exact knowledge of the system and can work only within that frame/narrow bound. Any form of deviation from the nominal functional state of the system leads to deterioration in remove:tracking: performance. Thus, parametric uncertainties and external disturbance is very poorly handled by the IOL control law however, UDE has the capability of being robust/providing robustness in such situations. It has the ability to estimate uncertainties and disturbance remove:that may arise: by passing them through a filter. Finally, an important advantage/feature that UDE provides is that it only requires the bandwidth of the uncertainties and disturbance to be bounded and known, and can perform irrespective of the value of the uncertainty/disturbance's magnitude.

ADD MORE FROM GODBOLE (UNDERLINED PARTS) FOR ADVANTAGES OF UDE ??

Consider the system **??** when affected by uncertainties Δa and Δb in the aerodynamic constants/terms a and b and when exposed to external disturbances w. (OR in cases on parametric uncertainties). In such a case/situation, it can be represented as

$$\dddot{\alpha} = (a + \Delta a) + (b + \Delta b)\delta_c + w \tag{8}$$

Now, clubbing the uncertainty terms and external disturbance into a lumped uncertainty $d = \Delta a + \Delta b + w$, we get

$$\dddot{\alpha} = a + b\delta_c + d \tag{9}$$

This can be re-written as,

$$d = \dddot{\alpha} - a - b\delta_c \tag{10}$$

The estimated disturbance $\hat{d}$ can be obtained by passing the disturbance d through a first order filter as follows

$$\hat{d} = G_f(s)d \tag{11}$$

where

$$G_f(s) = \frac{1}{1+s\tau} \tag{12}$$

The term τ is the filter constant which determines the bandwidth of the UDE estimator. It must be selected such that it encompasses the bandwidth of the entire lumped uncertainty d. The input δ_c can now be made to compensate for effects of d by setting $u_d = -d$ and taking it as

$$\delta_c = \frac{1}{b}\Big[u_a + u_d + \nu\Big] \tag{13}$$

On substituting **??** 7, **??** 8, **??** 9 in**??** 4 (rewrite this), we get

$$d = \dddot{\alpha} - \nu - u_d \tag{14}$$

Since $\hat{d} = G_f(s)d$ we get,

$$\hat{d} = G_f(s)\Big[\dddot{\alpha} - \nu - u_d\Big] \tag{15}$$

Using (8)(rewrite this)

$$u_d = \frac{-G_f(s)}{1-G_f(s)}\Big[\dddot{\alpha} - \nu\Big] \tag{16}$$

Rearrangement results in the uncertainty and disturbance estimation u_d to be

$$u_d = \frac{-1}{\tau}\Big[\ddot{\alpha} - \int \nu dt\Big] \tag{17}$$

In this way, robustness of the missile dynamics in the presence of uncertainty and disturbance is ensured. u Eqn is needed again right?: Finally, the UDE control law is obtained as additional layer/loop over IOL as,

$$u = \frac{1}{b}\Big[u_a + u_d + \nu\Big] \tag{18}$$

C. UDE State Observer

A UDE Observer has been constructed to obtain the output derivatives and thus system states. In order to represent the system in state space form, equation **??** can be rewritten as

$$\dddot{\alpha} = a_1\alpha + a_2\dot{\alpha} + a_3\ddot{\alpha} + d' + bu + d \tag{19}$$

where $a_1 = 0$, $a_2 = K_q M^2 c_m(-7 + 8M/3)$, $a_3 = K_\alpha M c_n(2 + M/3)$ and d' represents all the non-linear terms of a. By further clubbing the terms d and d' as $\bar{d}$, we get

$$\dddot{\alpha} = a_1\alpha + a_2\dot{\alpha} + a_3\ddot{\alpha} + bu + \bar{d} \tag{20}$$

Note that the clubbed uncertainty and disturbance term $\bar{d}$ is not strictly bounded to only account for the uncertainties $(\Delta a, \Delta b)$ and external disturbances w. It can also encompass/deal with/handle/account for unaccounted and unpredicted dynamics, disturbances and uncertainties such as cross-coupling effects, effects of linearization approximations, un-modeled dynamics, and also the effect of neglecting the contribution of control surface deflection in the force equation.

ALSO add this in results/conclusion section ... But how have we proved this in the simulations? Do we need to?

Rewriting **??** in state space form we get

$$\begin{aligned}
\dot{x}_1 &= x_2\\
\dot{x}_2 &= x_3\\
\dot{x}_3 &= a_{1o}x_1 + a_{2o}x_2 + a_{3o}x_3 + b\delta_c + \bar{d}\\
y &= x_1
\end{aligned} \tag{21}$$

where x_1=α, x_2=$\dot{\alpha}$ and x_3=$\ddot{\alpha}$. Now, (21) can be represented as,

$$\begin{aligned}
\dot{x}_p &= A_p x_p + B_p u + B_d\bar{d}\\
y_p &= C_p x_p
\end{aligned} \tag{22}$$

where

$$A_p = \begin{bmatrix} 0 & 1 & 0\\ 0 & 0 & 1\\ a_{1o} & a_{2o} & a_{3o}\end{bmatrix} B_p = \begin{bmatrix}0\\0\\b_0\end{bmatrix} B_d = \begin{bmatrix}0\\0\\1\end{bmatrix} \tag{23}$$

and

$$C_p = \begin{bmatrix}1 & 0 & 0\end{bmatrix} \tag{24}$$

The observer for the above system can be described as

$$\begin{aligned}
\dot{\hat{x}}_1 &= \hat{x_2} + \beta_1 e_o\\
\dot{\hat{x}}_2 &= \hat{x_3} + \beta_2 e_o\\
\dot{\hat{x}}_3 &= a_{1o}\hat{x}_1 + a_{2o}\hat{x}_2 + a_{3o}\hat{x}_3 + b\delta_c + \hat{\bar{d}} + \beta_3 e_o\\
\hat{y} &= \hat{x_1}
\end{aligned} \tag{25}$$

where $e_o = y - \hat{y}$ and $L = [\beta_1\beta_2\beta_3]^T$ are the observer gains. As per **??** $\bar{d} = d' + d$, thus the estimated terms can be represented as $\hat{\bar{d}} = \hat{d}' + \hat{d}$, where the $\hat{d}$ is obtained from the UDE estimator and $\hat{d}'$ is the value of the non-linear terms of a when fed with the estimated state variables $[\hat{x}_1\hat{x}_2\hat{x}_3]$ from the observer.

Comment about beta values here also?? CHECK IF THE OVERALL FLOW IS MAINTAINED IN HERE

The conventional Luenberger observer fails to provide accurate state estimation for this system due to the presence of uncertainty and disturbance terms. However, a Luenberger type observer can be described as, OR Now, constructing a Luenberger based UDE observer for the above controller gives/results in

$$\begin{aligned}
\dot{\hat{x}}_p &= A_p\hat{x}_p + B_p u + B_d\hat{\bar{d}} + L(y - \hat{y}_p)\\
\hat{y} &= C_p\hat{x}_p
\end{aligned} \tag{26}$$

Change notation of d to d-bar so that state space form and UDE d is much clearer.

D. Stability Analysis

In this section, stability analysis of the entire missile autopilot system with UDE controller and observer is presented. The final UDE control law was presented in **??**. By substituting **??**, **??** in **??** we get

ADD xp-dot equation here??? Need to leave them as x itself in UDE observer section

$$u = \frac{1}{b_o}\Big[- a_{10}\alpha - a_{20}\dot{\alpha} - a_{30}\ddot{\alpha} - \hat{d} + \dddot{\alpha} \\ - m_1(\alpha - \alpha^*) - m_2(\dot{\alpha} - \dot{\alpha}^*) - m_3(\ddot{\alpha} - \ddot{\alpha}^*)\Big] \tag{27}$$

The reference state vector can be represented as $R = [\alpha, \dot{\alpha}, \ddot{\alpha}]^T$ while the feedback gain vector can be denoted as $K_p = [k_1, k_2, k_3]$ where $k_1 = \frac{m_1 + a_{10}}{b_0}$, $k_2 = \frac{m_2 + a_{20}}{b_0}$ and $k_3 = \frac{m_3 + a_{30}}{b_0}$. Using the these notations, the UDE control law (27) can be expressed more compactly as

$$u = -K_p\hat{x}_p + K_pR - K_RR + \frac{\dddot{\alpha}^*}{b_0} - \frac{\hat{d}}{b_0} \tag{28}$$

where $K_R = [-\frac{\hat{a}_{10}}{\hat{b}_0}, -\frac{\hat{a}_{20}}{\hat{b}_0}, -\frac{\hat{a}_{30}}{\hat{b}_0}]$. It can be shown that dynamics of reference state vector R is

$$\dot{R} = A_pR + B_d\dddot{\alpha}^* + A_0R \tag{29}$$

where

$$A_0 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ a_{1o} & a_{2o} & a_{3o} \end{bmatrix} \tag{30}$$

Tracking error, e_c is described as $e_c = R - x_p$. By subracting **??** from 29 and substituting 28, we get

$$\dot{R} - \dot{x}_p = A_pe_c + B_d\dddot{\alpha}^* - B_d\bar{d} + B_pK_p\hat{x}_p - K_pB_pR \\ + B_pK_RR - \frac{B_p}{b_0}\dddot{\alpha}^* + \frac{B_p}{b_0}\hat{d} + A_0R \tag{31}$$

which on simplifying further, we obtain ?? what are these question marks for?

$$\dot{e}_c = (A_p - B_pK_p)e_c - (B_pK_p)e_0 - B_d\tilde{d} \tag{32}$$

where $\tilde{d} = \bar{d} - \hat{d}$ is the uncertainty estimation error and $e_0 = \alpha - \hat{\alpha}$ is the observer state estimation error.

To obtain the observer error dynamics equation, a similar subtraction of **??** from **??** can be performed to yield

$$\dot{e}_0 = A_pe_0 + B_d\tilde{d} - L(y_p - \hat{y}_p) \tag{33}$$

Now, to derive/arrive at the dynamcis of the disturbnace and uncertainty terms, we consider the UDE filter equation

$$\hat{d} = G_f(s)d \tag{34}$$

where the first order filter G_f can be represented as

$$G_f = (1 + s\tau) \tag{35}$$

Substituting 35 in uncertainty estimation equation 34, we obtain

$$d = (1 + s\tau)[d - \tilde{d}] \tag{36}$$

which on rearranging gives

$$\dot{\tilde{d}} = \frac{-1}{\tau}\tilde{d} + \dot{d} \tag{37}$$

By combining (32), (33) and (37), the following error dynamics for the UDE missile autopilot controller-observer pair is obtained.

$$\begin{bmatrix} \dot{e}_c \\ \dot{e}_0 \\ \dot{\tilde{d}} \end{bmatrix} = \begin{bmatrix} (A_p - B_pK_p) & -(B_pK_p) & -B_d \\ 0 & (A_p - LC_p) & B_d \\ 0 & 0 & -\frac{1}{\tau} \end{bmatrix} \begin{bmatrix} e_c \\ e_0 \\ \tilde{d} \end{bmatrix} \\ + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \dot{d} \tag{38}$$

The eigen values of the system matrix 38 are easily seen to be

$$\left|sI - (A_p - B_pK_p)\right| \left|sI - (A_p - LC_p)\right| \left|s - (-\tfrac{1}{\tau})\right| = 0 \tag{39}$$

Since, τ is strictly a positive number, (A_p, B_p) is controllable and (A_p, C_p) is observable, the stability of the error dynamics (39) can be ensured by selecting appropriate controller and observer poles. ELABORATE ?

IV. Simulations and Results

A. Uncertainty and Disturbance Estimator Simulations

Emphasize how UDE simulation in first case and final uncert, and ext dist case is important to showing its advantage. In this section, various simulations are carried out to evaluate the performance of the missile autopilot/system with UDE control law. The UDE controller has been designed to track the reference signal given by

$$\alpha^* = \begin{cases} 15^\circ, & \text{if } 0 \le t \le 2 \ s \\ -8^\circ, & \text{if } 2 < t \le 4 \ s \\ 10^\circ, & \text{if } 4 < t \le 6 \ s \end{cases} \tag{40}$$

Controller gains $[m_1, m_2, m_3]$ have been obtained by placing the closed loop poles at $(1 + \frac{\tau_c s}{3})^3$ with a time constant $\tau_c = 0.25$ OR just say As per godbole they were placed at s = -12**??** while the observer gains $[\beta_1, \beta_2, \beta_3]$ have been obtained by placed the observer poles at at $s = -120$, which is 10 times the distance of the controller poles.

The UDE's disturbance filter bandwidth was taken as $\frac{1}{0.01}$. The nominal values of parameters/aerodynamic constants appearing in the controller $u = \frac{1}{b}[u_a + u_d + \nu]$ are taken from Table **??**. The controller is designed for $M = 2.25$ as it represents midpoint of the considered Mach dynamics. The initial conditions for the plant has been taken as zero.

In reality, as Mach is not constant due to the presence of external disturbances and turbulence, simulations have been performed for varying mach dynamics as well. The plant's varying mach dynamics has been simulated according to **??** with initial condition $M(0) = 3$;

$$\dot{M}(t) = \frac{1}{\nu_s}[-|a_z(t)|sin|\alpha(t)| + a_xM^2(t)cos\alpha(t)] \tag{41}$$

where a_z is the normal acceleration and a_x is the longitudinal acceleration given by

$$\begin{aligned} a_z &= K_z M^2(t) C_n[\alpha(t), \delta(t), M(t)] \\ a_x &= \frac{0.7 P_O S C_a}{m} \end{aligned} \tag{42}$$

Need to include the x vs x-cap graph along with observer error...Just to prove that the error is actually very small...or else they may question how the observer error is so high

B. Comparison with other control techniques

The performance of UDE is compared against the performance of other non-linear control techniques like Predictive Control (PC) and Sliding Mode Control (SMC). All controllers have been designed for $M = 2.25$ and results have been evaluated with and without the presence of uncertainty. WHAT ABOUT THE THIRD CASE WHERE UNCERT AND DISTURBANCE EXIST??

1) Predictive Control: This controller is based on non-linear continuous time predictive control technique, where future response of the states or the output of a nonlinear dynamic system is predicted using functional expansion. A quadratic cost function is utilized on the errors between the predicted and desired states to minimize the control expenditure/effort at each point and obtain a continuous time-optimal, non-linear feedback control law. **??** provides the design of an angle of attack tracking controller, and the same has been used in this study;

$$u = -\frac{6}{b_p \Delta t^3}\left[e + \Delta t \dot{e} + \frac{\Delta t^2}{2}\ddot{e} + \frac{\Delta t^3}{6}(a_p - \dddot{\alpha}^*)\right] \tag{43}$$

where Δt represents the predictive horizon, $e = \alpha - \alpha^*$ is the output tracking error, and the functions ap and bp are given as

$$\begin{aligned} a_p =& K_q M^2(3a_m\alpha^2 + 2b_m|\alpha| + c_m\big(-7 + \frac{8M}{3}\big)\dot{\alpha} \\ & - K_q M^2 d_m \omega_a \delta + K_\alpha M(6a_n\alpha + 2b_n sgn(\alpha))\dot{\alpha}^2 \\ & + K_\alpha M(3a_n\alpha^2 + 2b_n|\alpha| + c_n\Big(2 - \frac{M}{3}\Big)\ddot{} \\ b_p =& K_q M^2 d_m \omega_a \end{aligned} \tag{44}$$

In simulations, the value of prediction horizon, $\Delta t = 0.18$ has been used **??**.

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2) Sliding Mode Control: SMC is a robust control algorithm which can handle significant uncertainties and unmeasurable disturbances. It uses discontinuous control based on the bounds of uncertainties. As long as the magnitude of the uncertainty lies within the bound assumed for SMC, satisfactory performance is obtained, however, in the case where the magnitude of the uncertainty is very low compared to the bound, an necessarily high control effort is applied. Also, in cases where the uncertainty is larger than the bound, tracking performance deteriorates. Reference **??** has been used for simulations performed in this study;

$$u = \frac{1}{b}(-\nu(x,t) - \rho sat(s/\phi)) \tag{45}$$

where s is the switching surface given by $s(t) = \dot{e}(t) + \lambda e(t)$, $e(t) = \alpha - \alpha^*$ and $sat(x)$ is defined as

$$sat(x) = \begin{cases} x, & \text{if } \mid x \mid \leq 1 \\ sgn(x), & \text{otherwise} \end{cases} \tag{46}$$

where

$$sgn(x) = \begin{cases} -1, & \text{if } x \leq 0 \\ 0, & \text{if } x = 0 \\ 1, & \text{if } x \geq 0 \end{cases} \tag{47}$$

The quantities bs and ν are given as

$$\begin{aligned} b_s =& K_\alpha M d_n \omega_a \\ \nu =& \lambda\dot{e} - \ddot{\alpha}^* + K_\alpha M[(3a_n\alpha^2 + 2b_n \mid \alpha \mid \\ & + c_n(2 - M/3)\dot{\alpha}cos(\alpha) - C_n sin(\alpha)\dot{\alpha})] \\ & - K_\alpha M d_n \omega_a \delta + \dot{q} \end{aligned} \tag{48}$$

In simulations provided, SMC was used while setting design parameters to $\lambda = 7$, $\rho = 10$, and $\phi = 0.5$ as per **??**. Check values and ensure they match with that given in reference

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V. Results and Discussion

Describe Simulations in detail. Where all uncert was there. where all what all failed. How UDE was better.

From the UDE Observer section: ALSO add this in results/conclusion section ... But how have we proved this in the simulations? Do we need to?

SMC — However, the exact value of the bound is rarely known. In addition, the controller needs plant states for its implementation.

Acknowledgment

The preferred spelling of the word "acknowledgment" in America is without an "e" after the "g". Avoid the stilted expression "one of us (R. B. G.) thanks ...". Instead, try "R. B. G. thanks...". Put sponsor acknowledgments in the unnumbered footnote on the first page.

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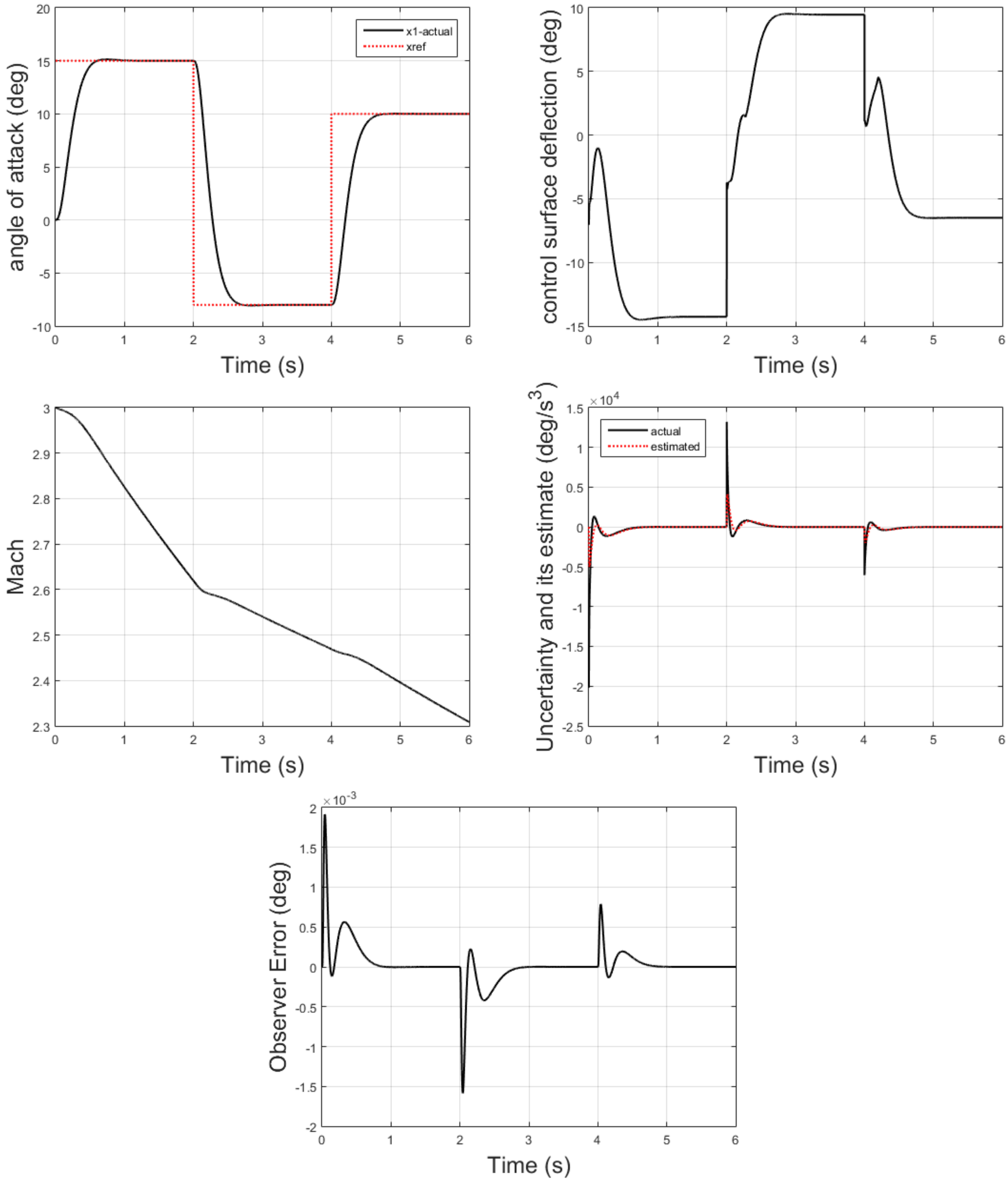

Fig. 1. Image A.

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1. Should we add block diagram of entire, ua, IOL, UDE, Observer system?
2. observer Graphs

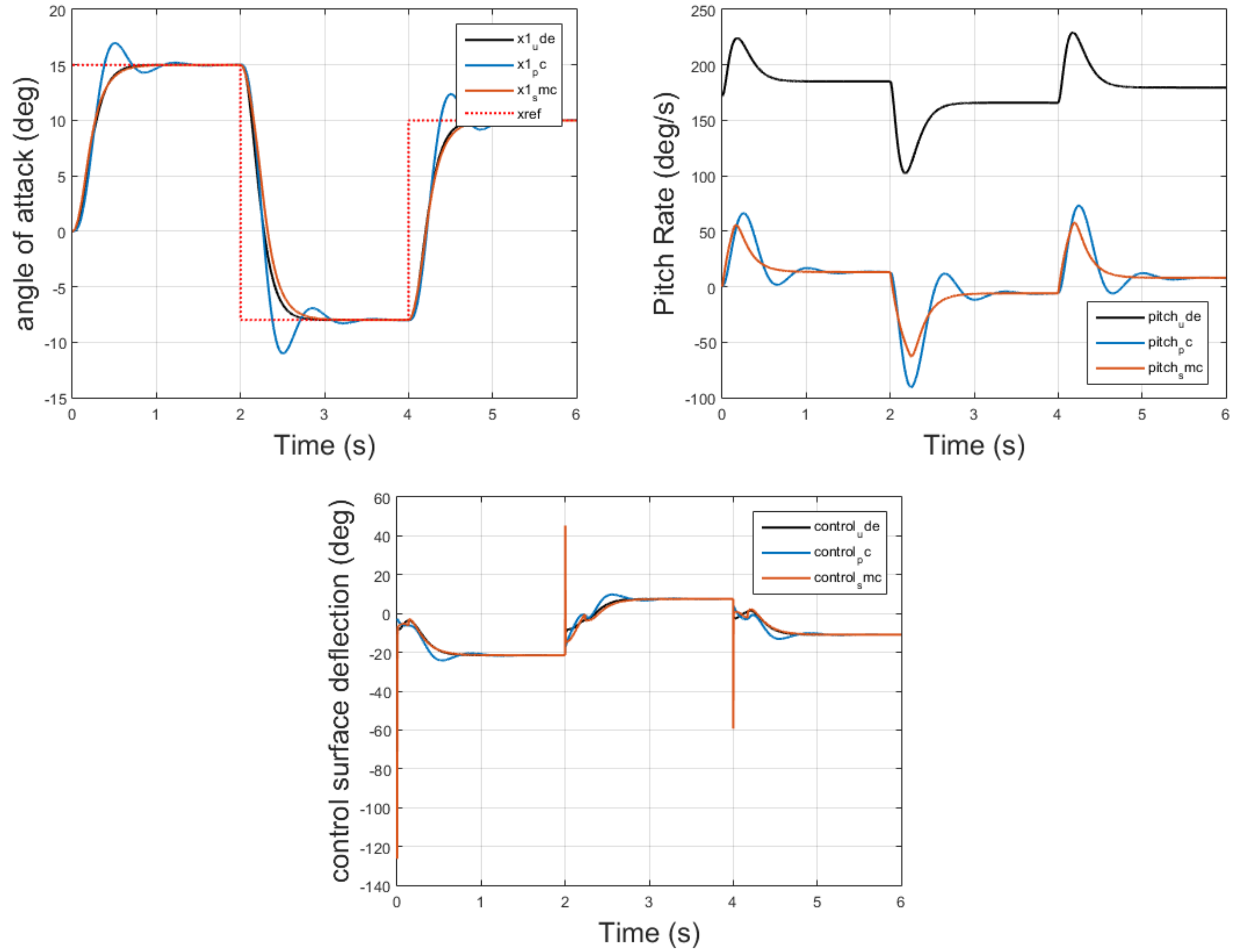

Fig. 2. No Uncertainty in Cn Cm

3. IOL simulation results
4. Tense through out the paper (Keep it as past tense)
5. CHECK IF THEORY IN PC AND SMC MAKES SENSE and if it matches stuff given in references
6. Fix line numbers – label only those needed, and club only those eqns that must be clubbed

Shortforms:

1. IOL
2. UDE
3. SMC
4. PC
5.

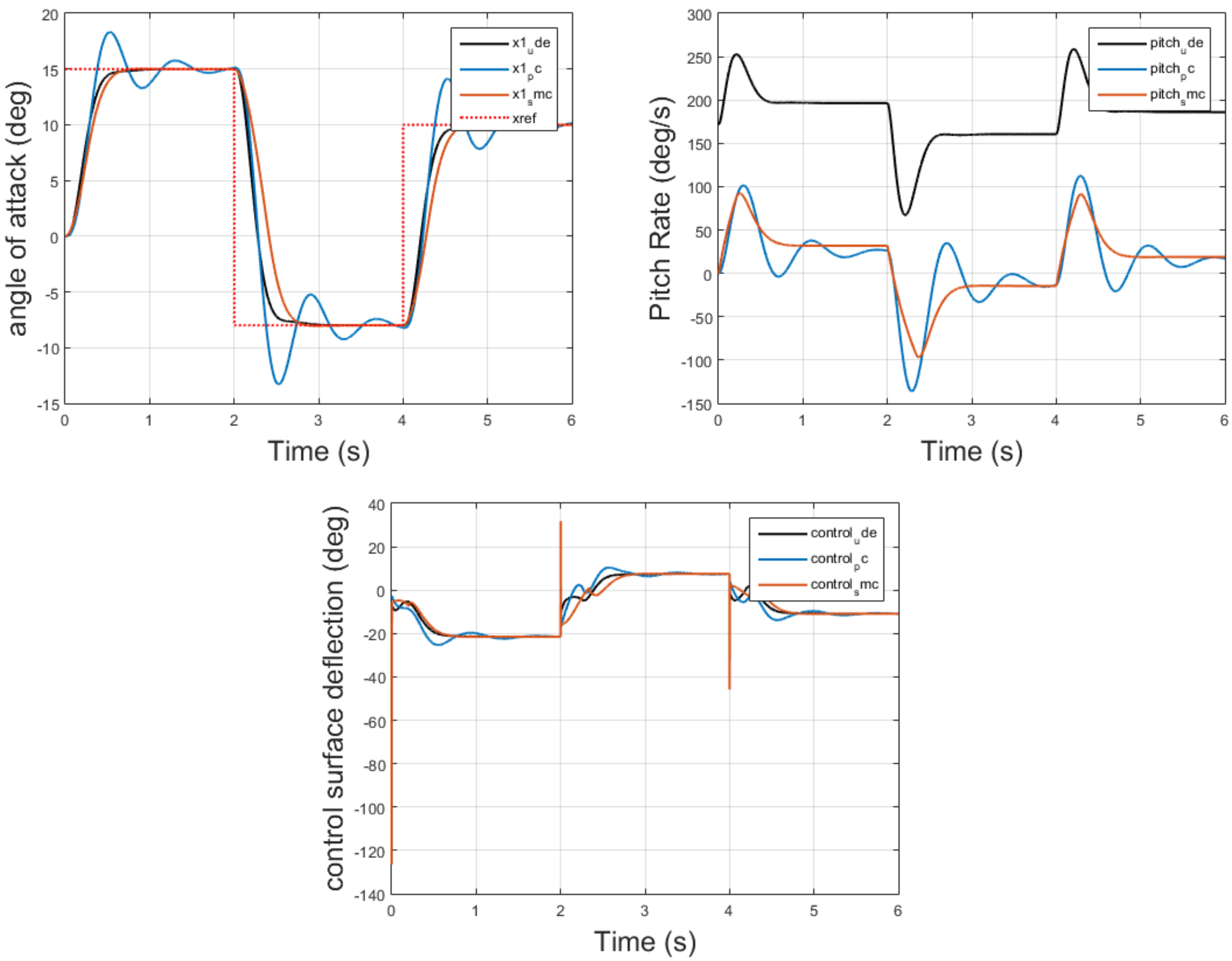

Fig. 3. +30% uncertainty in Cn, -30% uncertainty in Cm