Uncertainty and Disturbance Estimator Based Robust Pitch Autopilot

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Outline of work

Problem Statement

- ADD Dist in pitch plane
- ADD Provide robust solution
- ADD Analysis and Comparitive study

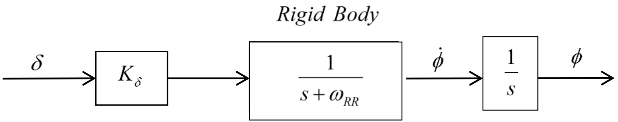
Missile Model

As referred from [];

- Transfer Function of linearized roll dynamics
  1. $\dot{\alpha}(t) = K_\alpha M(t) C_n[\alpha(t), \delta(t), M(t)] cos(\alpha(t)) + q(t)$
  2. $\dot{q}(t) = K_q M^2(t) C_m[\alpha(t), \delta(t), M(t)]$
  3. $\dot{\delta}(t) = -\omega_a \delta(t) + \omega_a \delta_c(t)$
- Transfer Function of second order actuator
  1. $\dot{M}(t) = \frac{1}{\nu_s}[-|a_z(t)| sin|\alpha(t)| + a_x M^2(t) cos\alpha(t)]$
  2. $a_z = K_z M^2(t) C_n[\alpha(t), \delta(t), M(t)]$
  3. $a_x = \frac{0.7 P_0 S C_d}{m}$

Block Diagram of UDE Controller-Observer

Assuming ideal actuator ($\delta = \delta_c$), the block diagram schematic of roll dynamics of roll autopilot is shown below;



- ADD
- ADD
- ADD TABLE ALSO AFTER OR BEFORE THIS...WHERE EVER TO INDICATE WHAT THE TERMS MEAN

Input Output Linearization

Plant dynamics in state space form

- Trip Diff
- $d_n \approx 0$ – relative degree
- alpha constraints

$$\dddot{\alpha} = K_q M^2 (3a_m\alpha^2 + 2b_m|\alpha| + c_m\Big(-7 + \frac{8M}{3}\Big))\dot{\alpha}$$
$$- K_q M^2 d_m \omega_a \delta + K_\alpha M(6a_n\alpha + 2b_n sgn(\alpha))\dot{\alpha}^2$$
$$+ K_\alpha M(3a_n\alpha^2 + 2b_n|\alpha| + c_n\Big(2 - \frac{M}{3}\Big))\ddot{\alpha}$$
$$+ K_q M^2 d_m \omega_a \delta_c$$

where to get the form al-trip-dot = a + bu,

$$
\begin{aligned}
a = {} & K_q M^2(3a_m\alpha^2 + 2b_m|\alpha| + c_m\Big(-7 + \frac{8M}{3}\Big))\dot{\alpha} \\
& - K_q M^2 d_m \omega_a \delta + K_\alpha M(6a_n\alpha + 2b_n sgn(\alpha))\dot{\alpha}^2 \\
& + K_\alpha M(3a_n\alpha^2 + 2b_n|\alpha| + c_n\Big(2 - \frac{M}{3}\Big))\ddot{\alpha} \\
b = {} & K_q M^2 d_m \omega_a
\end{aligned}
$$

Thus,

$$
\begin{aligned}
\delta_c &= \frac{1}{b}(u_a + \nu) \\
u_a &= -a \\
\nu &= \dddot{\alpha}^* + m_1(\alpha^* - \alpha) + m_2(\dot{\alpha}^* - \dot{\alpha}) + m_3(\ddot{\alpha}^* - \ddot{\alpha})
\end{aligned}
$$

UDE Augmented IOL Controller

Defining u_a, ν and u_d

- $\delta_c = \frac{1}{b}\left[u_a + u_d + \nu\right]$
- $\dddot{\alpha} = (a + \Delta a) + (b + \Delta b)\delta_c + w$
  1. $d = \Delta a + \Delta b \delta_c + w$
  2. $\dddot{\alpha} = a + b\delta_c + d$
- To define u_d
  1. $\hat{d} = G_f(s)d$
  2. $G_f(s) = \frac{1}{1+s\tau}$

     we finally get
  3. $u_d = \frac{-1}{\tau}\left[\ddot{\alpha} - \int \nu dt\right]$

UDE Observer based Control law

The final UDE based control law using the expressions for u_a, ν and u_d

$$\dddot{\alpha} = a_1\alpha + a_2\dot{\alpha} + a_3\ddot{\alpha} + d_1 + b\delta_c$$

d1 = non-linear terms, a1 is coeffs alpha, a2 is coeffs alpha-dot, a3 is coeffs alpha-ddot
Now,

$$\dddot{\alpha} = a_{1o}\alpha + a_{2o}\dot{\alpha} + a_{3o}\ddot{\alpha} + b_o\delta_c + d_2$$

UDE Observer based Control law

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= a_{1o}x_1 + a_{2o}x_2 + a_{3o}x_3 + b_o\delta_c + d_2 \\
y &= x_1
\end{aligned}
$$

Intro with observer poles we get

$$
\begin{aligned}
\dot{\hat{x}}_1 &= \hat{x}_2 + \beta_1 e_o \\
\dot{\hat{x}}_2 &= \hat{x}_3 + \beta_2 e_o \\
\dot{\hat{x}}_3 &= a_{1o}\hat{x}_1 + a_{2o}\hat{x}_2 + a_{3o}\hat{x}_3 + b\delta_c + \hat{d}_2 + \beta_3 e_o \\
\hat{y} &= \hat{x}_1
\end{aligned}
$$

Stability Analysis

quick points

Parameters used in roll dynamics

Simulation Parameters as referred from [1]

Table: Performance specifications

ω_{RRn}	roll rate bandwidth	2 rad/s
$K_{\delta} n$	fin effectiveness	9000 $1/s^2$
ω_A	actuator bandwidth	100 rad/s
ζ_A	actuator damping	0.65
ϕ_{max}	maximum desired roll angle	10 deg
$\dot{\phi}_{max}$	maximum desired roll rate	300 deg/s
d_{ext}	external disturbance	200 rad/s^2
δ_{cmax}	maximum desired fin deflection	30 deg

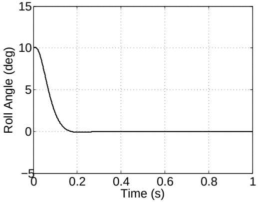
Performance of UDE based controller

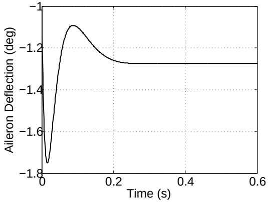
Parameters for simulation

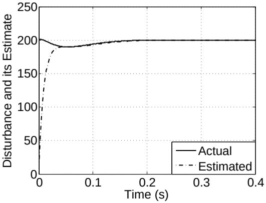
- filter constant $\tau = 0.01$
- desired settling time $t_s = 180$ ms
- damping factor $\zeta = 0.8$
    1. Using these values, feedback gains m_1 and m_2 were evaluated to be $m_1 = 42.45$, $m_2 = 771.13$
- external disturbance $d_{ext} = 200$ rad/s^2
- taking ω_{RR} to be -3 rad/s against the nominal value of 2 rad/s
- desired roll orientation = 0 deg
- initial condition in $\phi = 10$ deg
- all other values as referred from Table.1

For this controller and plant system, the phase margin was found to be 69 deg, validating the proposed control law

Performance of UDE based controller

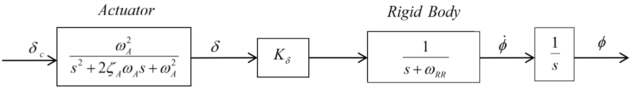






Cascading second order actuator Second order actuator as referred in [1] of the form $\frac{\delta(s)}{\delta_c(s)} = \frac{\omega_A^2}{s^2+2\zeta_A\omega_A s+\omega_A^2}$ is introduced

- ω_A is actuator bandwidth in rad/s
- ζ_A is actuator damping ratio

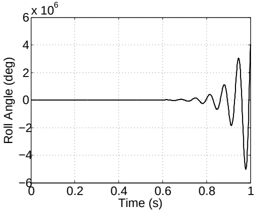


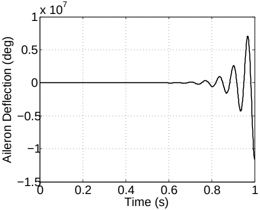
Performance of UDE based controller with second order actuator in the loop

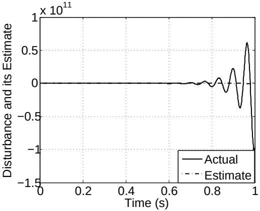
For simulation;

- τ continues to be 0.01
- All other simulation parameters are as per Table.1

Performance of UDE based controller with second order actuator in the loop







Actuator compensated UDE based control law

Need for actuator compensation

- It has been observed that addition of a second order actuator in the plant, makes the system unstable
- There is a need to compensate the effects of the actuator by proper tuning the UDE based robust control law or by designing a compensator for the actuator

Method 1: Tuning of τ

Assuming uncertainties and disturbances in the original plant and expressing δ in terms of δ_c, we re-write the dynamics as

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\omega_{RRn} x_2 + K_{\delta n} \left[\delta_c - \frac{1}{\omega_A^2} \left(\ddot{\delta} + 2\zeta_A \omega_A \dot{\delta} \right) \right] \\
&\quad + d
\end{aligned}
$$

Method 1: Tuning of τ

Since $\frac{-K_{\delta n}}{\omega_A^2}\left(\ddot{\delta}+2\zeta_A\omega_A\dot{\delta}\right)$ is unknown, it can be treated as an unknown disturbance and clubbed with d and denoted as d_1

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= -\omega_{RRn}x_2 + K_{\delta n}\delta_c + d_1\end{aligned}$$

- $d_1 = \frac{-K_{\delta n}}{\omega_A^2}\left(\ddot{\delta}+2\zeta_A\omega_A\dot{\delta}\right) + d$
- $u_a = \omega_{RRn}x_2$
- $\nu = \ddot{\phi}_{ref} - m_1(\dot{\phi}-\dot{\phi}_{ref}) - m_2(\phi-\phi_{ref})$
- $u_d = -\frac{1}{\tau}\left[x_2 - \int \nu dt\right]$

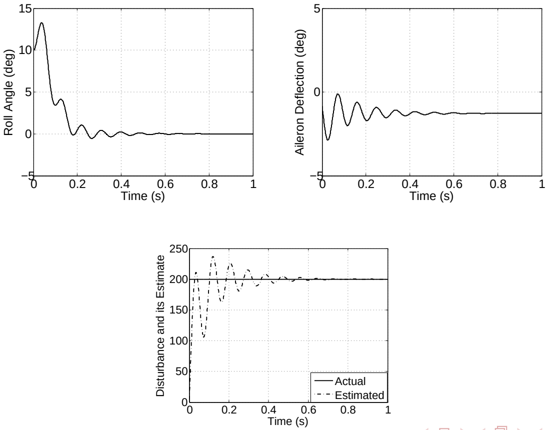
Performance of UDE based control law with actuator compensation using Method 1

Simulations were carried out with the data in Table 1, with the value of filter constant τ chosen as follows;

- τ when chosen to be 0.01 rendered the system unstable
- It was thus varied to achieve the desired performance
- As the value of τ increases from 0.01 till 0.025, the system remained unstable
- On further increasing the value of τ, the system gradually began to stabilize
- The value of τ was thereby tuned to achieve the best possible performance

The value of τ which gave the best possible performance was 0.04

Performance of UDE based control law with actuator compensation using Method 1



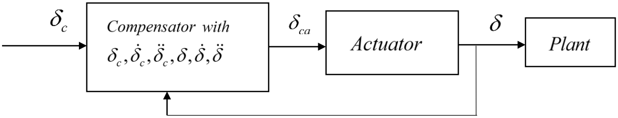
Performance of UDE based control law with actuator compensation using Method 1

Inference

- Control effort, though oscillatory was found to be within the desired limits
- Disturbance Estimation was found to be satisfactory after 500 ms
- The overshoot in output response was dependent on magnitude of disturbance

Method 2: Using Compensator Form 1

The block diagram schematic for this proposed method



- In this approach, δ_c is taken as a reference signal which δ is required to track before it is fed to the plant
- δ_{ca} is taken as the compensated control to the actuator

Method 2: Using Compensator Form 1

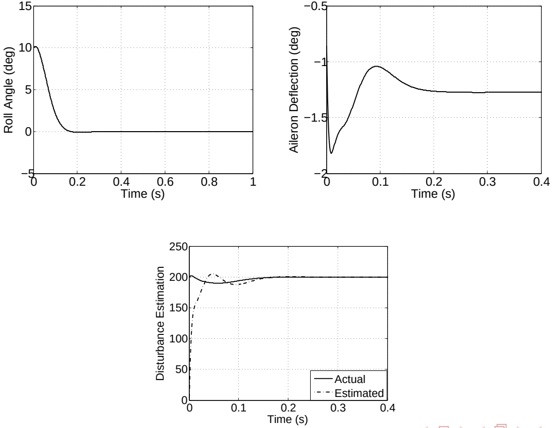
- $\delta_{ca} = \frac{1}{\omega_A^2}(\nu_a + u_{na})$
- $u_{na} = 2\zeta_A \omega_A \dot{\delta} + \omega_A^2 \delta$
- $\nu_a = \ddot{\delta}_c - k_{a1}\dot{\delta}_e - k_{a2}\delta_e$
  1. where $\delta_e = \delta_c - \delta$
  2. k_{a1} and k_{a2} are the feedback gains, whose proper choice will ensure stability
- This results in tracking error dynamics between δ and δ_c of the form $\ddot{\delta}_e + k_{a1}\dot{\delta}_e + k_{a2}\delta_e = 0$

Performance of UDE based control law with actuator compensation using Method 2

Simulation parameters

- Using desired settling time to be 50ms, damping ratio to be 0.8, k_{a1} and k_{a2} are evaluated to be 160 and 10000
- τ is taken as 0.01
- Other simulation parameters are as per Table 1.
- Simulations are done with d_{ext} and uncertainty in ω_{RR}

Performance of UDE based control law with actuator compensation using Method 2



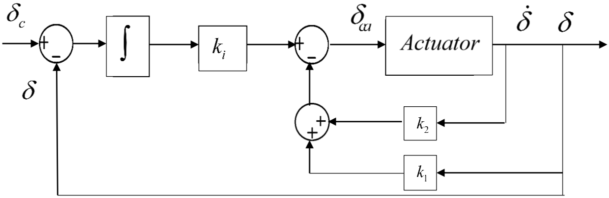
Performance of UDE based control law with actuator compensation using Method 2

Inference

- The compensator proposed in Method 2 has been successful in ensuring that δ tracks δ_c
- However, the drawback of this approach is that it requires first and second derivatives of δ and δ_c
- While these derivatives can be assumed to be available for simulation purposes, their availability cannot be assured in real-life situations

Method 3: Using Compensator Form 2

Block Diagram Schematic for this proposed method



The proposed compensator in this strategy not only reduces the number of derivatives of δ and δ_c required, it also ensures that δ tracks δ_c.
In this proposed strategy, δ_{ca} is the compensated actuator input.

Method 3: Using Compensator Form 2

Deducing expression for δ_{ca} As referred from [3], The transfer function of the second order actuator can be written as;

$$\frac{\delta(s)}{\delta_c(s)} = \frac{b}{s^2 + as + b}$$

with $b = \omega_A^2$ and $a = 2\zeta_A\omega_A$. The actuator dynamics can be re-written in the state space model as follows;

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}$$

A, B and C are the matrices of the system and are given by

$$A = \begin{bmatrix} 0 & 1 \\ -b & -a \end{bmatrix}; B = \begin{bmatrix} 0 \\ b \end{bmatrix}; C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Method 3: Using Compensator Form 2

The compensated actuator control input δ_{ca} is thus defined as follows;

$$u = \delta_{ca} = -k_1\delta - k_2\dot{\delta} + k_i \int (\delta_c - \delta)dt$$

Method 3: Using Compensator Form 2

Determination of feedback gains k_1, k_2 and k_i

- The desired characteristic equation for this system is of the form $(s^2 + 2\zeta_d\omega_d s + \omega_d^2)(s + \zeta_d\omega_d) = 0$
  1. damping factor 0.8
  2. to ensure that the compensator is faster than the actuator, its desired settling time should be less than actuator settling time
- To derive the actual characteristic equation for this system, following [3], we first need to define matrices $\hat{A}$, $\hat{B}$, $\hat{K}$;
- Computing $sI - (\hat{A} - \hat{B}\hat{K}) = 0$ results in the actual characteristic equation of the form $s^3 + s^2(a + bk_2) + s(b + bk_1) + bk_i = 0$
- Comparing the desired and actual characteristic equations, gives us the value of k_1, k_2 and k_i

Performance of UDE based control law with actuator compensation using Method 2

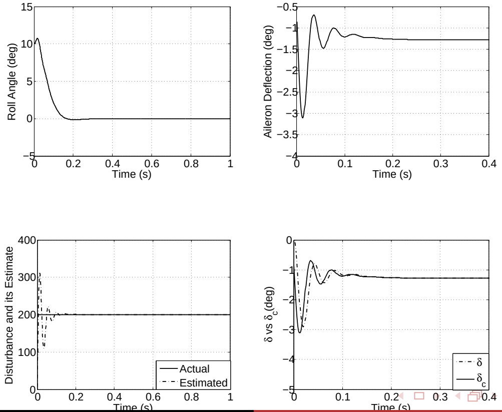
Simulations were carried with data from Table 1. with an external disturbance d_{ext} present.

Desired compensator settling time : 12.5ms

- First a desired settling time of 1/4th that of actuator settling time, i.e. 12.5ms is chosen.
- for this value, with damping ratio of 0.8, k_1, k_2 and k_i were calculated to be 35.48, 0.083, 5120, respectively

Performance of UDE based control law with actuator compensation using Method 2

Simulation results for desired compensator settling time : 12.5ms



Performance of UDE based control law with actuator compensation using Method 2

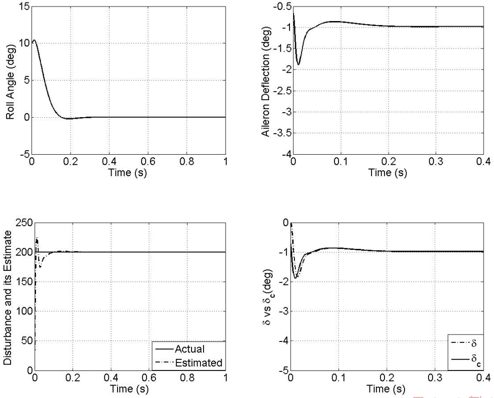
Simulations were carried with data from Table 1. with an external disturbance d_{ext} present.

Desired compensator settling time : 7.96ms

- It was observed that for a given damping ratio, say 0.8, the relation between τ and settling time for the error dynamics (t_s) can be expressed as $\tau = 1.25t_s$.
- For $\tau = 0.01$, the desired settling time was computed to be 7.96 ms.
- for this value, with damping ratio of 0.8, k_1, k_2 and k_i were calculated to be 88.89, 0.137 and 19800, respectively
- an uncertainty of -30 % in ω_{RR} and +30 % in K_δ along with d_{ext} were introduced

Performance of UDE based control law with actuator compensation using Method 2

Simulation results for desired settling time : 7.96ms



Performance of UDE based control law with actuator compensation using Method 3

Inference

- The compensator proposed in Method 3 has been successful in ensuring that δ tracks δ_c
- As compared to the previous approach, the number of derivatives of δ and δ_c required has been reduced
- We now require only the control input δ_c, actuator output δ and its first derivative $\dot{\delta}$

Conclusion and future work

Our work has proposed three strategies for compensating actuator lag;

- The strategy of tuning filter constant τ, though promising, requires a formula linking τ with the parameters of the plant or the performance indices
- Compensator Form 1, though satisfactory in ensuring tracking of δ_c by δ, required a number of their derivatives
- Compensator Form 2, which gave the desired results at the same time reducing the number of derivatives required

Our future work would include the following

- Further investigation into the role of filter time constant τ in actuator compensation
- Design of UDE based control law to cater to time varying uncertainties and disturbances along with actuator compensation
- Applicability of the proposed methods for non-linear roll dynamics

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Q. C. Zhong, and D. Rees, “Control of LTI systems based on an uncertainty and disturbance estimator,” ASME Trans. J. of Dynamic systems, Measurement and Control, 126(4), 2004, pp. 905-910.

K. Ogata, Modern Control Engineering, 5th ed. PHI, New Delhi, 2010, pp. 743-746.

Thank You!