# Uncertainty and Disturbance Estimator Based Robust Pitch Autopilot

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# Outline of work

# Problem Statement

- ADD Dist in pitch plane
- ADD Provide robust solution
- ADD Analysis and Comparitive study

# Missile Model

- The missile model is a pitch axis, longitudinal, tail controlled missile with nonlinear dynamics referred from [];.
  1. $\dot{\alpha}(t) = K_\alpha M(t) C_n[\alpha(t), \delta(t), M(t)] cos(\alpha(t)) + q(t)$
  2. $\dot{q}(t) = K_q M^2(t) C_m[\alpha(t), \delta(t), M(t)]$
  3. $\dot{\delta}(t) = -\omega_a \delta(t) + \omega_a \delta_c(t)$
- where;
  1. $C_n(\alpha, \delta, M) = a_n\alpha^3 + b_n\alpha|\alpha| + c_n\left(2 - \frac{M}{3}\right)\alpha \quad + d_n\delta$
  2. $C_m(\alpha, \delta, M) = a_m\alpha^3 + b_m\alpha|\alpha| + c_m\left(-7 + \frac{8M}{3}\right)\alpha \quad + d_m\delta$
  3. $\dot{M}(t) = \frac{1}{\nu_s}[-|a_z(t)|sin|\alpha(t)| + a_x M^2(t) cos\alpha(t)]$
- Assumes constant post burnout mass, no roll rate, zero roll angle, no sideslip and no yaw rate KEEP ONLY MEANINGFUL ONES

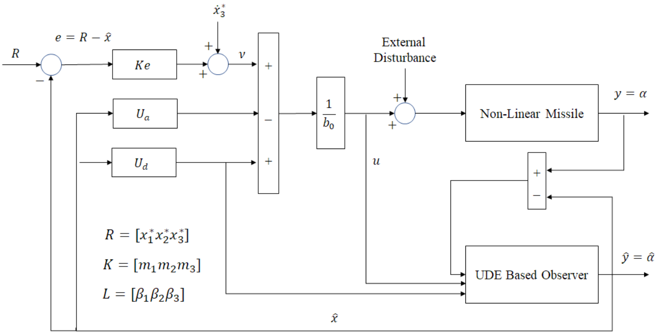
# Aerodynamic constants

- Model design constraints:
  1. $-20^\circ \leq \alpha \leq 20^\circ$
  2. $1.5 \leq M \leq 3$
  3. $\pm 25\%$ uncertainty in $C_n$ and $C_m$

Table: Performance specifications

| | | |
|---|---|---|
| $\omega_a$ | Actuator bandwidth | 150 rad/s |
| $\zeta_a$ | Drag coefficient | 9000 $1/s^2$ |
| $m$ | Mass | 204.023 kg |
| $d$ | Diameter | 0.2286 m |
| $I_y$ | Pitch moment of inertia | 247.44 $kgm^2$ |
| $C_d$ | Drag moment | 0.3 |
| $M$ | Mach | 2.25 |
| *CHECK* | IF ALL | REQD PRESENT |

# Block Diagram of UDE Controller-Observer



# Input Output Linearization

**Implementation of IOL to cancel non-linearities of missile model**

- To make relative degree = order of equation, $d_n \approx 0$
- Within $-20^\circ \leq \alpha \leq 20^\circ$, $\cos(\alpha) \approx 1$
- Now obtaining $\dddot{\alpha}$ gives

$$\begin{aligned}\dddot{\alpha} = {} & K_q M^2 (3a_m\alpha^2 + 2b_m|\alpha| + c_m\Big(-7 + \frac{8M}{3}\Big))\dot{\alpha} \\ & - K_q M^2 d_m \omega_a \delta + K_\alpha M(6a_n\alpha + 2b_n sgn(\alpha))\dot{\alpha}^2 \\ & + K_\alpha M(3a_n\alpha^2 + 2b_n|\alpha| + c_n\Big(2 - \frac{M}{3}\Big))\ddot{\alpha} \\ & + K_q M^2 d_m \omega_a \delta_c\end{aligned}$$

# Input Output Linearization

When represented in IOL form $\dddot{\alpha} = a + b\delta_c$ we get;

$$
\begin{aligned}
a = {} & K_q M^2(3a_m\alpha^2 + 2b_m|\alpha| + c_m\Big(-7 + \frac{8M}{3}\Big))\dot{\alpha} \\
& - K_q M^2 d_m \omega_a \delta + K_\alpha M(6a_n\alpha + 2b_n sgn(\alpha))\dot{\alpha}^2 \\
& + K_\alpha M(3a_n\alpha^2 + 2b_n|\alpha| + c_n\Big(2 - \frac{M}{3}\Big))\ddot{\alpha} \\
b = {} & K_q M^2 d_m \omega_a
\end{aligned}
$$

Thus the control law is,

$$
\begin{aligned}
\delta_c &= \frac{1}{b}(u_a + \nu) \\
u_a &= -a \\
\nu &= \dddot{\alpha}^* + m_1(\alpha^* - \alpha) + m_2(\dot{\alpha}^* - \dot{\alpha}) + m_3(\ddot{\alpha}^* - \ddot{\alpha})
\end{aligned}
$$

# UDE Augmented IOL Controller

**The UDE control law utilizes a new term $u_d$ as follows;**

- $\delta_c = \frac{1}{b}\left[u_a + u_d + \nu\right]$ where, $u_d = -\hat{d}$
- $\hat{d}$ is an estimate of the lumped disturbance and uncertainties $d$;
  1. $\dddot{\alpha} = a + b\delta_c + d$
  2. $d = \Delta a + \Delta b\delta_c + w$
  3. $\hat{d} = G_f(s)d$
  4. $G_f(s) = \frac{1}{1+s\tau}$
- Thus, we finally get $u_d = \frac{-1}{\tau}\left[\ddot{\alpha} - \int \nu dt\right]$

# UDE Observer based Control law

**A Luenberger like UDE Observer has been designed CITE EVERYWHERE**

- $\dddot{\alpha}$ is separated into linear and non-linear ($d_1$) terms;
  $\dddot{\alpha} = a_1\alpha + a_2\dot{\alpha} + a_3\ddot{\alpha} + d_1 + b\delta_c$
- Then the non-linear term $d_1$ and uncertainties are clubbed into a lumped term $d_2$, such that;
  $\dddot{\alpha} = a_{1_o}\alpha + a_{2_o}\dot{\alpha} + a_{3_o}\ddot{\alpha} + b_o\delta_c + d_2$
- This expressed in state space form can then be used to design the observer:

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= a_{1_o}x_1 + a_{2_o}x_2 + a_{3_o}x_3 + b_o\delta_c + d_2 \\
y &= x_1
\end{aligned}
$$

# UDE Observer based Control law

Now since the equations are represented in a linear manner, a Luenburger like UDE Observer is designed by introducing the observer poles $[\beta_1 \beta_2 \beta_3]$

$$
\begin{aligned}
\dot{\hat{x}}_1 &= \hat{x}_2 + \beta_1 e_o \\
\dot{\hat{x}}_2 &= \hat{x}_3 + \beta_2 e_o \\
\dot{\hat{x}}_3 &= a_{1o}\hat{x}_1 + a_{2o}\hat{x}_2 + a_{3o}\hat{x}_3 + b\delta_c + \hat{d}_2 + \beta_3 e_o \\
\hat{y} &= \hat{x}_1
\end{aligned}
$$

Here, the term $d_2$ representing the non-linearities and uncertainties is estimated by UDE

# Stability Analysis

$$\begin{bmatrix} \dot{e}_c \\ \dot{e}_o \\ \dot{\tilde{d}}_2 \end{bmatrix} = \begin{bmatrix} (A-BK) & -(BK) & -B_d \\ 0 & (A-LC) & B_d \\ 0 & 0 & -\frac{1}{\tau} \end{bmatrix} \begin{bmatrix} e_c \\ e_o \\ \tilde{d}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \dot{d}_2 \tag{1}$$

$$\left|sI-(A-BK)\right| \left|sI-(A-LC)\right| \left|s-(-\tfrac{1}{\tau})\right| = 0 \tag{2}$$

- $(A, B)$ is controllable and $(A, C)$ is observable
- $\tau$ is strictly a positive number
- Selecting appropriate controller and observer poles ensures stability of error dynamics
- Also if $\dot{d}_2 \neq 0$, then bounded-input, bounded-output stability can be assured.

# Simulations

## Parameters for simulation

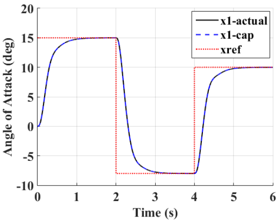
- Reference signal:

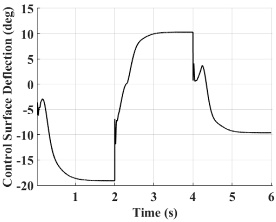
$$\alpha^* = \begin{cases} 15^\circ, & \text{if } 0 \leq t \leq 2\ s \\ -8^\circ, & \text{if } 2 < t \leq 4\ s \\ 10^\circ, & \text{if } 4 < t \leq 6\ s \end{cases}$$

- Tracking Constraints:
  1. To be tracked with a time constant of less than $0.25s$
  2. Less than 10% overshoot
  3. Less than 1% steady-state error
- Pole placement to meet this target:
  1. Controller gains $[m_1 \ \ m_2 \ \ m_3]$ placed at $s_{1,2,3} = -12$
  2. Observer gains $[\beta_1 \ \ \beta_2 \ \ \beta_3]^T$ placed at $s_{1,2,3} = -360$
- Controller and observer designed at $M = 2.25$ (mid-point of Mach envelope)

# Case I: UDE with Mach Dynamics & External Disturbance

- UDE simulated with varying mach and external disturbance
- Mach i.c is $M(0) = 2.5$ and follows $M$ equation reference?? till $M = 2$
- Extrenal disturbance modeled as sinusoidal amplitude $8^\circ$ and frequency $0.25Hz$



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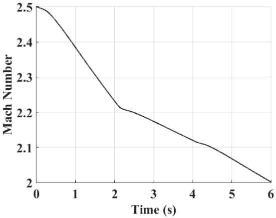
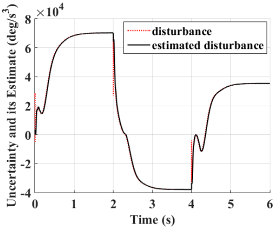
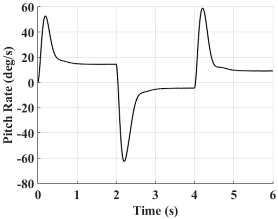


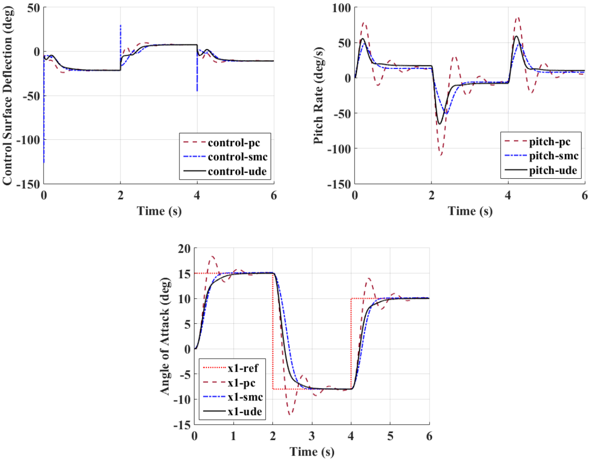



# Case II: Comparative Study With Aerodynamic Uncertainties

- Comparative analysis of UDE has been done against Predictive Control (PC) and Sliding Mode Control (SMC) ADD Citation
- Uncertainty of $+30\%$ in aerodynamic force coefficient $C_n$ and $-30\%$ in aerodynamic moment coefficient $C_m$
- Mach has been maintained at the nominal constant of $M = 2.25$
- No external disturbances added to the system.

# Case II: Comparative Study With Aerodynamic Uncertainties



# Results and conclusions

- Results of CASE I
  1. Tracking performance is as desired and control effort stays smooth and within the practical bounds of $\pm 30^\circ$
  2. UDE observer is able to estimate states quickly and accurately
  3. Estimation of disturbance by UDE is minimally delayed and follows closely to the actual value
- Results of CASE II
  1. Due to aerodynamic uncertainty, PC has overshoots in tracking. UDE and SMC are smooth
  2. Control effort of PC is oscillatory. SMC has high overshoots at transition points. UDE is able to provide smooth control within $\pm 30^\circ$
  3. Pitch graph of PC is oscillatory while SMC is slightly delayed. Pitch graph of UDE is on point.

# Novelty

- Despite uncertainties such as varying mach (case I) and varying aerodynamic constants (case II) and even the presence of external disturbance (case I), UDE is able to provide robust tracking control.
- Only the frequency bound (captured through $\tau$) of the external disturbance and uncertainty is required to provide robust tracking with UDE. There is no dependency on the magnitude bounds of the disturbance/uncertainty. include in UDE slide also...relate to $\tau$
- Unlike PC and SMC simulations which have used the actual states while implementing the control law, UDE simulation has utilized the estimated states obtained from the UDE observer. As is well known, use of estimated states might result in degraded performance; in contrast the proposed strategy utilizes the estimated states and still proves its worthiness.

# Future Work

- Design of an integrated pith-yaw-roll autopilot
- Application of UDE to a nonlinear missile model with and without prior linearization of the model

# References

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- K. Ogata, Modern Control Engineering, 5th ed. PHI, New Delhi, 2010, pp. 743-746.

# Thank You!