Cascaded LQR Control for Missile Roll Autopilot

D Viswanath

Dept of Electronics & Electrical Engg,
IIT Guwahati

TS Chandar

Dept of Electronics & Communication Engg,
PES University,
Bengaluru

Overview

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- Missile Roll Autopilot Design
  - Background
  - Design of Missile Autopilot with Cascaded LQR
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Introduction

- Modern missiles are required to be very agile and stable platforms.
- The main aim of the roll autopilot in a cruciform cartesian controlled missile is to maintain roll angle zero thus ensuring that the dynamics in the three dimensions of roll, pitch and yaw channels are decoupled.
- Decoupling of the dynamics helps in formulating simple simultaneous equations for the roll, pitch and yaw channels respectively and makes computations simpler and implementation easier.

Introduction

- Missile roll autopilots were first designed using classical control approaches.
- Classical control was effective for lower order model of roll autopilot.
- Optimal control as a modern control system design approach when used in aerospace applications can meet the requirements of fast response with minimum settling time and good stability characteristics.
- Linear Quadratic Regulator is a ubiquitous optimal controller which can be used effectively in the design of a missile roll autopilot.

Literature Survey

- Missile roll autopilots were first designed using classical control approach like proportional-integral feedback control.Instability in design due to inclusion of second order actuator dynamics was overcome using lag-lead compensation [1].
- Optimal control approach combined with classical approach for design of robust missile roll autopilots was first discussed in [2].
- [3] discusses the design of an optimal controller using a computer program called OPTSYS (Optimum System Synthesis)

[1] P. Garnell, [1980] "Guided Weapon Control Systems",2nd ed., Pergamon Press
[2] Zarchan et al [1981] "Combined Optimal/Classical Approach to Robust Missile Autopilot Design",Journal of Guidance and Control
[3] FW Nesline and Zarchan [1984] "Why Modern Controllers can go Unstable in Practice",Journal of Guidance

Literature Survey

- [4] proposes the design of a roll autopilot for the model including the rigid body dynamics and the actuator dynamics using Sliding Mode Control (SMC) technique with LQR optimization employed for design of sliding surface.
- Robust missile roll autopilot design with application of linear quadratic regulator (LQR) for solving the combined dynamics of a flexible missile is discussed in [5].

[4] Parkhi et al [2010] "Design of Roll Autopilot for a Tail Controlled Missile Using Sliding Mode Technique",IEEE Workshop on Variable Structure Systems
[5] Talole et al [2011] "Robust Autopilot Design for Tactical Missiles", Journal of Guidance, Control, and Dynamics

Literature Survey

- In [6], the authors discuss the problem of linear quadratic regulator (LQR) performance for cascade control structures of series coupled systems and obtain the necessary and sufficient condition for the linear quadratic performance of a cascade control structure to achieve the same performance as any given centralized LQR.

- Cascaded control has found applications in acceleration and displacement control of an electrodynamic shaker [7], evolutionary optimization of cascaded industrial hydraulic valves [8], and optimal control of heavy vehicle formations [9].

[6] A Gattami and A Rantzer [2005] “Linear Quadratic Performance Criteria for Cascade Control”,Proc of IEEE/ECC Conference
[7] Y Uchiyama and M Fujita,[2006] “Robust Acceleration and Displacement Control of Electrodynamic Shaker,”IEEE Conference
[8] Krettek et al [2007] “Evolutionary Hardware-in-the-loop Optimization of a Controller for Hydraulic Valves”,IEEE/ASME International Conference
[9] Gattami et al [2011] “Suboptimal Decentralized Controller Design for Chain Structures: Applications to Vehicle Formations ”, IEEE Conference

Motivation

- Advantages:
  - In cascaded LQR based approach the states can be measured as they are physical quantities.
  - Whereas when centralized LQR is used, the higher order states are obtained by differentiation of output and can only be estimated since they are not physically quantifiable.
- Disadvantages:
  - Flexible body dynamics cannot be included in this approach since they cannot be measured.

Cascaded LQR

The realistic model of the roll autopilot considering the missile airframe, actuator,and gyro dynamics is as shown in block diagram below

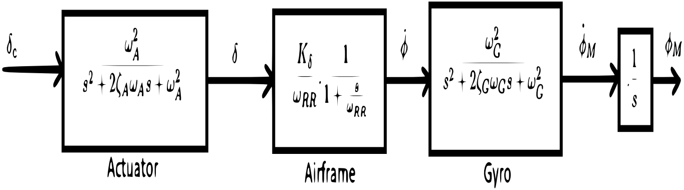

Figure: **Missile Roll Autopilot [2][3]**

[2] Zarchan et al [1981] "Combined Optimal/Classical Approach to Robust Missile Autopilot Design",Journal of Guidance and Control
[3] FW Nesline and Zarchan [1984] "Why Modern Controllers can go Unstable in Practice",Journal of Guidance

Background

The transfer function for this model is given by

$$\begin{aligned}\frac{\phi_M}{\delta_c} &= \frac{\omega_A^2}{s^2 + 2\zeta_A\omega_A s + \omega_A^2} \\ &\times \left[\frac{K_\delta}{\omega_{RR}}\cdot\frac{1}{(1+\frac{s}{\omega_{RR}})}\right] \\ &\times \frac{\omega_G^2}{s^2 + 2\zeta_G\omega_G s + \omega_G^2}\cdot\frac{1}{s}\end{aligned} \tag{1}$$

where

- ϕ_M is measured roll angle and δ_c is the fin deflection command,
- K_δ is the fin effectiveness and ω_{RR} is the roll rate bandwidth,
- ω_A is the actuator bandwidth and ζ_A is the actuator damping,
- ω_G is the rate gyro bandwidth and ζ_G is the rate gyro damping,

Background

- Most classical control approaches [1] omit the higher order dynamics of the second order gyro.
- In recent times, with advent of modern state space approach, strategies like optimal control based on LQR [2],[3] and nonlinear control laws like Extended State Observer (ESO) [5] which include these higher order dynamics have been proposed.

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[1] P. Garnell, [1980] “Guided Weapon Control Systems”,2nd ed., Pergamon Press
[2] Zarchan et al [1981] “Combined Optimal/Classical Approach to Robust Missile Autopilot Design”,Journal of Guidance and Control
[3] FW Nesline and Zarchan [1984] “Why Modern Controllers can go Unstable in Practice”,Journal of Guidance
[5] Talole et al [2011] “Robust Autopilot Design for Tactical Missiles”, Journal of Guidance, Control, and Dynamics

Cascaded LQR

Let us consider the block diagram for the realistic model of the roll autopilot again

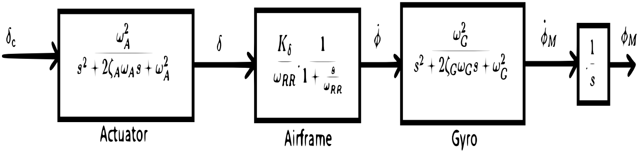

Figure: **Missile Roll Autopilot**

In the cascaded form

- The first block is represented by system Σ_4 with input $U_4 = \delta_C$ and output $Y_4 = \delta$,
- The second block is represented by system Σ_3 with input $U_3 = Y_4 = \delta$ and output $Y_3 = \dot{\phi}$,
- The third block is represented by system Σ_2 with input $U_2 = Y_3 = \dot{\phi}$ and output $Y_2 = \dot{\phi}_M$, and
- The fourth block is represented by system Σ_1 with input $U_1 = Y_4 = \dot{\phi}_M$ and output $Y_1 = \phi_M$.

Cascaded LQR

Let the state space form for each block be given by

- $$\begin{aligned} \dot{X}_4 &= A_4 X_4 + B_4 U_4 \\ Y_4 &= C_4 X_4 \end{aligned} \tag{2}$$

- $$\begin{aligned} \dot{X}_3 &= A_3 X_3 + B_3 U_3 \\ Y_3 &= C_3 X_3 \end{aligned} \tag{3}$$

- $$\begin{aligned} \dot{X}_2 &= A_2 X_2 + B_2 U_2 \\ Y_2 &= C_2 X_2 \end{aligned} \tag{4}$$

- $$\begin{aligned} \dot{X}_1 &= A_1 X_1 + B_1 U_1 \\ Y_1 &= C_1 X_1 \end{aligned} \tag{5}$$

Cascaded LQR

The state space representation of the cascaded systems Σ_1, Σ_2, Σ_3 and Σ_4 with $U_c = U_4$ can be written as a larger system Σ_c given by

$$\dot{X}_c = \mathbf{A}_c X + \mathbf{B}_c U_c \quad (6)$$

$$Y_c = \mathbf{C}_c X_c \quad (7)$$

where

$$X_c = [\delta \;\; \dot{\delta} \;\; \dot{\phi} \;\; \dot{\phi}_M \;\; \ddot{\phi}_M \;\; \phi_M]^T$$

$$\mathbf{A}_c = \begin{bmatrix} \mathbf{A}_1 & \mathbf{B}_1\mathbf{C}_2 & 0 & 0 & 0 \\ 0 & \mathbf{A}_2 & \mathbf{B}_2\mathbf{C}_3 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{A}_3 & \mathbf{B}_3\mathbf{C}_4 \\ 0 & 0 & 0 & 0 & \mathbf{A}_4 \end{bmatrix} \quad (8)$$

and

$$\mathbf{B}_c = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \mathbf{B}_4 \end{bmatrix} \; ; \; \mathbf{C}_c = \begin{bmatrix} \mathbf{C}_1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (9)$$

Cascaded LQR

The linear quadratic regulator for Σ_c is designed to minimize the performance index given by

$$J_c = \frac{1}{2}\int_0^\infty \left[X_c^T \mathbf{Q}_c X_c + U_c^T \mathbf{R}_c U_c \right] dt \tag{10}$$

The control law is given by

$$U_c = -\mathbf{L}_c X_c \tag{11}$$

where $\mathbf{L}_c = \mathbf{R}_c^{-1}\mathbf{B}_c^T\mathbf{P}_c$ and $\mathbf{P}_c$ is the solution to the algebraic matrix Riccati equation given by

$$\mathbf{P}_c\mathbf{A}_c + \mathbf{A}_c^T\mathbf{P}_c - \mathbf{P}_c\mathbf{B}_c\mathbf{R}_c^{-1}\mathbf{B}_c^T\mathbf{P}_c + \mathbf{Q}_c = 0 \tag{12}$$

The matrices $\mathbf{Q}_c$ and $\mathbf{R}_c$ are chosen to be positive definite.

Simulation Parameters

The design values in [2] for the gains and bandwidths which were used in the simulations are given in table below.

Table: Simulation Parameters

Parameter	Name	Value
ω_{RR}	Roll Rate Bandwidth	2 rad/s
K_δ	Fin effectiveness	$9000\ \frac{1}{s^2}$
ω_A	Actuator Bandwidth	100 rad/s
ζ_A	Actuator Damping	0.65
ω_G	Gyro Bandwidth	200 rad/s
ζ_G	Gyro Damping	0.5

[2] Zarchan et al [1981] "Combined Optimal/Classical Approach to Robust Missile Autopilot Design",Journal of Guidance and Control

Simulation Parameters

The maximum values for a safe and controlled launch used in [3] are given in table below.

Table: Maximum Values for Desired LQR Performance

Parameter	Name	Value
ϕ_{MX}	Maximum Desired Roll Angle	0.174 rad
$\dot{\phi}_{MX}$	Maximum Desired Roll Rate	5.23 rad/sec
δ_{CMX}	Maximum Desired Fin Deflection	0.524 rad

[3] FW Nesline and Zarchan [1984] "Why Modern Controllers can go Unstable in Practice",Journal of Guidance

Simulation Parameters

The matrices $\mathbf{Q}_c$ and $\mathbf{R}_c$ are chosen as

$$\mathbf{Q}_c = \begin{bmatrix} \frac{1}{\phi_{MX}^2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\dot{\phi}_{MX}^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{13}$$

and

$$\mathbf{R}_c = \frac{1}{\delta_{CMX}^2} \tag{14}$$

Solving equation (12), the controller gains are computed as

$$\mathbf{L}_c = [3.0115 \quad 0.0102 \quad 0.000013 \quad 0.1348 \quad 8.985 \quad 0.0313]$$

Simulation Results

For a comparison with the sliding mode control based roll autopilot in [4] with nominal parameters,

- The initial values in measured roll angle and roll rate are assumed to be $-0.7\ rad$ and $+0.7\ rad/s$
- All other initial conditions are assumed to be zero, giving $X_c(0) = [0 \ \ 0 \ \ 0 \ \ 0.7 \ \ 0 \ \ -0.7]^T$.
- The time domain specifications from [4] are rise time not more than $50\ ms$, settling time less than $200\ ms$ and peak overshoot(M_p) below 15%.

[4] Parkhi et al [2010] "Design of Roll Autopilot for a Tail Controlled Missile Using Sliding Mode Technique",IEEE Workshop on Variable Structure Systems

Simulation Results

Figure below shows the measured roll angle (ϕ_M). The response obtained is similar to the SMC Roll Autopilot i.e., dead-beat type with no overshoot and settling time approximately $180\ ms$.

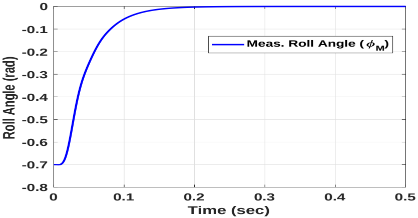

Figure: **Measured Roll Angle.**

Simulation Results

Figure below shows the actual roll rate ($\dot{\phi}$) and measured roll rate ($\dot{\phi}_M$) respectively. The settling time is approximately $180\ ms$; rise time is less than $50\ ms$. The initial lag in the actual and measured roll rates is due to the second order gyro dynamics. The lag is negligible after $100\ ms$.

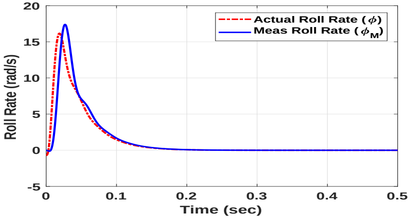

Figure: **Roll Rate.**

Simulation Results

Figure below shows the commanded fin deflection (δ_C) and the actual fin deflection (δ). The observed maximum fin deflection in both cases is well within the limits of the desired maximum fin deflection given in Table II.

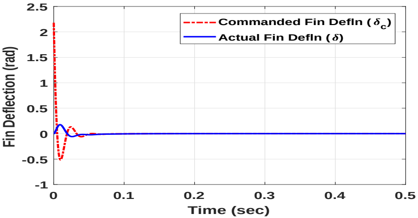

Figure: **Fin Deflection**

Simulation Results

A comparison of the time response of SMC Roll Autopilot from [4] and Cascaded LQR (CLQR) is given in Table below.

Table: COMPARATIVE TIME RESPONSE

SMC Autopilot			**CLQR Autopilot**		
M_p	$\|\|\dot{\phi}_M\|\|_\infty$	$\|\|\delta\|\|_\infty$	M_p	$\|\|\dot{\phi}_M\|\|_\infty$	$\|\|\delta\|\|_\infty$
(%)	(deg/s)	(deg)	(%)	(deg/s)	(deg)
1.00	922.14	10.10	1.00	996.775	10.07

[4] Parkhi et al [2010] "Design of Roll Autopilot for a Tail Controlled Missile Using Sliding Mode Technique",IEEE Workshop on Variable Structure Systems

Conclusion

- Cascaded linear quadratic regulator is employed in the design of feedback controller for the missile roll autopilot considering the dynamics of second order actuator and second order gyro also in a realistic scenario.
- Simulations were carried out to verify the effectiveness of the controller and the time response was compared with SMC based roll autopilot.
- The proposed cascaded LQR is not in a position to handle unmodeled flexible body dynamics which can be extended to parametric uncertainties and disturbances.

Future Scope

- The proposed cascaded LQR controller would be augmented with another controller for robustness in the presence of flexible body dynamics and model uncertainties.
- Performance of this augmented controller will be compared with existing approaches.

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[1] P. Garnell, " *Guided Weapon Control Systems,"2nd ed., Pergamon Press*, Oxford, England, 1980.

[2] F. William Nesline, Brian H. Wells, and P. Zarchan, "Combined Optimal/Classical Approach to Robust Missile Autopilot Design," *Journal of Guidance and Control*, Vol. 4, No. 3, May-Jun 1981, pp.495-500.

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References III

[9] Assad Al Alam, Ather Gattami and Karl H. Johansson, “Suboptimal Decentralized Controller Design for Chain Structures: Applications to Vehicle Formations ,” *2011 50th IEEE Conference on Decision and Control and European Control Conference (CDC-ECC)*, Orlando, FL, USA, December 12-15, 2011, pp.6894-6900.