Uncertainty and Disturbance Estimator Based Robust Pitch Autopilot

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Outline of work

Problem Statement

- ADD Dist in pitch plane
- ADD Provide robust solution
- ADD Analysis and Comparitive study

Missile Model

As referred from [];

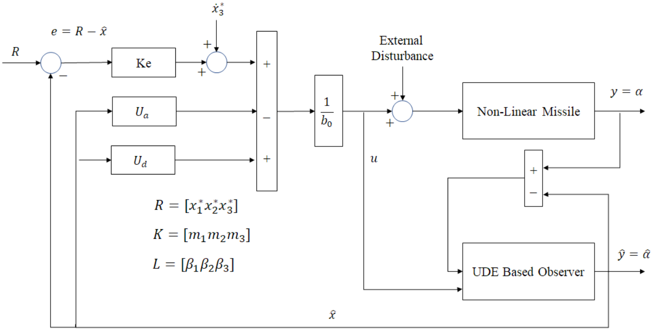
- Transfer Function of linearized roll dynamics
  1. $\dot{\alpha}(t) = K_\alpha M(t) C_n[\alpha(t), \delta(t), M(t)] cos(\alpha(t)) + q(t)$
  2. $\dot{q}(t) = K_q M^2(t) C_m[\alpha(t), \delta(t), M(t)]$
  3. $\dot{\delta}(t) = -\omega_a \delta(t) + \omega_a \delta_c(t)$
  4. $C_n(\alpha, \delta, M) = a_n \alpha^3 + b_n \alpha|\alpha| + c_n \left(2 - \frac{M}{3}\right)\alpha \quad + d_n \delta$
  5. $C_m(\alpha, \delta, M) = a_m \alpha^3 + b_m \alpha|\alpha| + c_m \left(-7 + \frac{8M}{3}\right)\alpha \quad + d_m \delta$
- Transfer Function of second order actuator
  1. $\dot{M}(t) = \frac{1}{\nu_s}[-|a_z(t)| sin|\alpha(t)| + a_x M^2(t) cos\alpha(t)]$
  2. $a_z = K_z M^2(t) C_n[\alpha(t), \delta(t), M(t)]$
  3. $a_x = \frac{0.7 P_0 S C_d}{m}$

Aerodynamic constants

Table: Performance specifications

ω_a	Actuator bandwidth	150 rad/s
ζ_a	Drag coefficient	9000 $1/s^2$
m	Mass	204.023 kg
d	Diameter	0.2286 m
I_y	Pitch moment of inertia	247.44 kgm^2
C_d	Drag moment	0.3
M	Mach	2.25

Block Diagram of UDE Controller-Observer



Input Output Linearization

Plant dynamics in state space form

- Trip Diff
- $d_n \approx 0$ – relative degree
- alpha constraints

$$\dddot{\alpha} = K_q M^2 (3a_m\alpha^2 + 2b_m|\alpha| + c_m\Big(-7 + \frac{8M}{3}\Big))\dot{\alpha} \\ - K_q M^2 d_m \omega_a \delta + K_\alpha M(6a_n\alpha + 2b_n sgn(\alpha))\dot{\alpha}^2 \\ + K_\alpha M(3a_n\alpha^2 + 2b_n|\alpha| + c_n\Big(2 - \frac{M}{3}\Big))\ddot{\alpha} \\ + K_q M^2 d_m \omega_a \delta_c$$

where to get the form al-trip-dot = a + bu,

$$
\begin{aligned}
a &= K_q M^2 (3a_m\alpha^2 + 2b_m|\alpha| + c_m\Big(-7 + \frac{8M}{3}\Big))\dot{\alpha} \\
&\quad - K_q M^2 d_m \omega_a \delta + K_\alpha M(6a_n\alpha + 2b_n sgn(\alpha))\dot{\alpha}^2 \\
&\quad + K_\alpha M(3a_n\alpha^2 + 2b_n|\alpha| + c_n\Big(2 - \frac{M}{3}\Big))\ddot{\alpha} \\
b &= K_q M^2 d_m \omega_a
\end{aligned}
$$

Thus,

$$
\begin{aligned}
\delta_c &= \frac{1}{b}(u_a + \nu) \\
u_a &= -a \\
\nu &= \dddot{\alpha}^* + m_1(\alpha^* - \alpha) + m_2(\dot{\alpha}^* - \dot{\alpha}) + m_3(\ddot{\alpha}^* - \ddot{\alpha})
\end{aligned}
$$

UDE Augmented IOL Controller

Defining u_a, ν and u_d

- $\delta_c = \frac{1}{b}\left[u_a + u_d + \nu\right]$
- $\dddot{\alpha} = (a + \Delta a) + (b + \Delta b)\delta_c + w$
  1. $d = \Delta a + \Delta b \delta_c + w$
  2. $\dddot{\alpha} = a + b\delta_c + d$
- To define u_d
  1. $\hat{d} = G_f(s)d$
  2. $G_f(s) = \frac{1}{1+s\tau}$
     we finally get
  3. $u_d = \frac{-1}{\tau}\left[\ddot{\alpha} - \int \nu dt\right]$

UDE Observer based Control law

The final UDE based control law using the expressions for u_a, ν and u_d

$$\dddot{\alpha} = a_1\alpha + a_2\dot{\alpha} + a_3\ddot{\alpha} + d_1 + b\delta_c$$

d1 = non-linear terms, a1 is coeffs alpha, a2 is coeffs alpha-dot, a3 is coeffs alpha-ddot
Now,

$$\dddot{\alpha} = a_{1o}\alpha + a_{2o}\dot{\alpha} + a_{3o}\ddot{\alpha} + b_o\delta_c + d_2$$

UDE Observer based Control law

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= a_{1o}x_1 + a_{2o}x_2 + a_{3o}x_3 + b_o\delta_c + d_2 \\
y &= x_1
\end{aligned}
$$

Intro with observer poles we get

$$
\begin{aligned}
\dot{\hat{x}}_1 &= \hat{x}_2 + \beta_1 e_o \\
\dot{\hat{x}}_2 &= \hat{x}_3 + \beta_2 e_o \\
\dot{\hat{x}}_3 &= a_{1o}\hat{x}_1 + a_{2o}\hat{x}_2 + a_{3o}\hat{x}_3 + b\delta_c + \hat{d}_2 + \beta_3 e_o \\
\hat{y} &= \hat{x}_1
\end{aligned}
$$

Stability Analysis

$$\begin{bmatrix} \dot{e}_c \\ \dot{e}_o \\ \dot{\tilde{d}}_2 \end{bmatrix} = \begin{bmatrix} (A-BK) & -(BK) & -B_d \\ 0 & (A-LC) & B_d \\ 0 & 0 & -\frac{1}{\tau} \end{bmatrix} \begin{bmatrix} e_c \\ e_o \\ \tilde{d}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \dot{d}_2 \qquad (1)$$

$$\left|sI-(A-BK)\right| \left|sI-(A-LC)\right| \left|s-(-\tfrac{1}{\tau})\right| = 0 \qquad (2)$$

- (A, B) is controllable and (A, C) is observable
- τ is strictly a positive number
- Selecting appropriate controller and observer poles ensures stability of error dynamics
- If $\dot{d}_2 \neq 0$, then bounded-input, bounded-output stability can be assured.

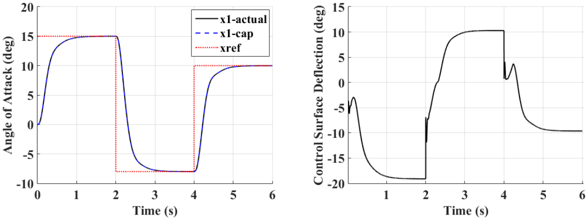
Performance of UDE based controller

Parameters for simulation

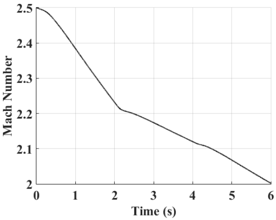
- filter constant $\tau = 0.01$
- desired settling time $t_s = 180$ ms
- damping factor $\zeta = 0.8$
  1. Using these values, feedback gains m_1 and m_2 were evaluated to be $m_1 = 42.45$, $m_2 = 771.13$
- external disturbance $d_{ext} = 200$ rad$/s^2$
- taking ω_{RR} to be -3 rad/s against the nominal value of 2 rad/s
- desired roll orientation $= 0$ deg
- initial condition in $\phi = 10$ deg
- all other values as referred from Table.1

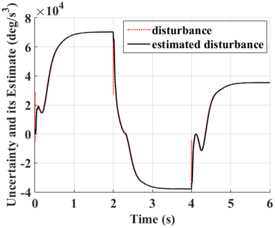
For this controller and plant system, the phase margin was found to be 69 deg, validating the proposed control law

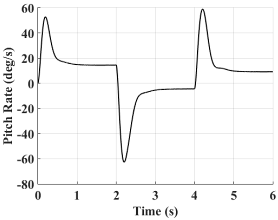
Performance of UDE (Case 1)



Performance of UDE (Case 1)

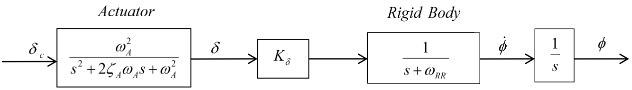






Cascading second order actuator Second order actuator as referred in [1] of the form $\frac{\delta(s)}{\delta_c(s)} = \frac{\omega_A^2}{s^2+2\zeta_A\omega_A s+\omega_A^2}$ is introduced

- ω_A is actuator bandwidth in rad/s
- ζ_A is actuator damping ratio

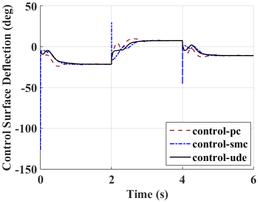


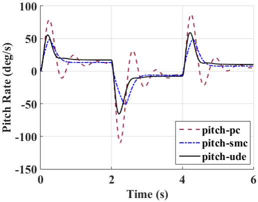
Performance of UDE based controller with second order actuator in the loop

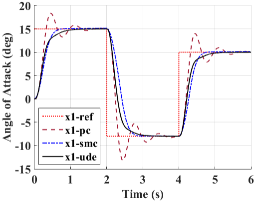
For simulation;

- τ continues to be 0.01
- All other simulation parameters are as per Table.1

Comparative analysis (Case 2)







Comparative analysis (Case 2)

ADD SOME CONTENT HERE FOR CASE 2

Results and conclusions

Novelty and future work

References

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Thank You!