Uncertainty and Disturbance Estimator Based Robust Pitch Autopilot

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Outline of work

Problem Statement

- Missile control systems have traditionally faced many challenges in predicting the uncertainty and disturbance acting on the missile at various points in its flight envelope.
- Few robust control techniques (SMC, PC) can tackle non-linear system, but it requires prior knowledge of the magnitude bounds of the disturbance acting on the system
- Uncertainty and Disturbance Estimator (UDE) has been employed to address these concerns

Missile Model

- The missile model is a pitch axis, longitudinal, tail controlled missile with nonlinear dynamics [1];
  1. $\dot{\alpha}(t) = K_\alpha M(t) C_n[\alpha(t), \delta(t), M(t)] cos(\alpha(t)) + q(t)$
  2. $\dot{q}(t) = K_q M^2(t) C_m[\alpha(t), \delta(t), M(t)]$
  3. $\dot{\delta}(t) = -\omega_a \delta(t) + \omega_a \delta_c(t)$
- where;
  1. $C_n(\alpha, \delta, M) = a_n \alpha^3 + b_n \alpha|\alpha| + c_n \left(2 - \frac{M}{3}\right)\alpha \quad + d_n \delta$
  2. $C_m(\alpha, \delta, M) = a_m \alpha^3 + b_m \alpha|\alpha| + c_m \left(-7 + \frac{8M}{3}\right)\alpha \quad + d_m \delta$
  3. $\dot{M}(t) = \frac{1}{\nu_s}[-|a_z(t)| sin|\alpha(t)| + a_x M^2(t) cos\alpha(t)]$
- Assumes no roll rate, no sideslip and no yaw rate
- In this model δ_c is the commanded input to the tail fins and α is the angle of attack

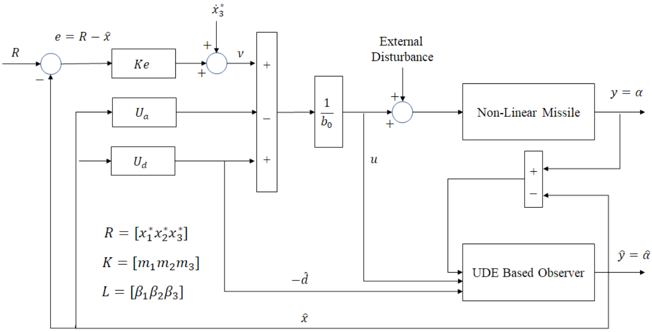
Aerodynamic constants

- Model design constraints:
  1. $-20^\circ \leq \alpha \leq 20^\circ$
  2. $1.5 \leq M \leq 3$
  3. $\pm 25\%$ uncertainty in C_n and C_m

Table: Performance specifications

ω_a	Actuator bandwidth	150 rad/s
ζ_a	Actuator damping	0.7
m	Mass	204.023 kg
d	Diameter	0.2286 m
I_y	Pitch moment of inertia	247.44 kgm^2
C_d	Drag coefficient	0.3
v_s	Speed of sound	315.89 m/s
M	Mach	2.25

Overview of UDE Controller-Observer



Input Output Linearization

Implementation of IOL to cancel non-linearities in model [2]

- To make relative degree = order of equation, $d_n \approx 0$
- Within $-20^\circ \leq \alpha \leq 20^\circ$, $\cos(\alpha) \approx 1$
- Now obtaining $\dddot{\alpha}$ gives

$$
\begin{aligned}
\dddot{\alpha} = {} & K_q M^2 (3a_m\alpha^2 + 2b_m|\alpha| + c_m\Big(-7 + \frac{8M}{3}\Big))\dot{\alpha} \\
& - K_q M^2 d_m \omega_a \delta + K_\alpha M(6a_n\alpha + 2b_n sgn(\alpha))\dot{\alpha}^2 \\
& + K_\alpha M(3a_n\alpha^2 + 2b_n|\alpha| + c_n\Big(2 - \frac{M}{3}\Big))\ddot{\alpha} \\
& + K_q M^2 d_m \omega_a \delta_c
\end{aligned}
$$

Input Output Linearization

When represented in IOL form $\dddot{\alpha} = a + b\delta_c$ we get;

$$
\begin{aligned}
a = {} & K_q M^2 (3a_m\alpha^2 + 2b_m|\alpha| + c_m\Big(-7 + \frac{8M}{3}\Big))\dot{\alpha} \\
& - K_q M^2 d_m \omega_a \delta + K_\alpha M(6a_n\alpha + 2b_n sgn(\alpha))\dot{\alpha}^2 \\
& + K_\alpha M(3a_n\alpha^2 + 2b_n|\alpha| + c_n\Big(2 - \frac{M}{3}\Big))\ddot{\alpha} \\
b = {} & K_q M^2 d_m \omega_a
\end{aligned}
$$

Thus the control law is,

$$
\begin{aligned}
\delta_c &= \frac{1}{b}(u_a + \nu) \\
u_a &= -a \\
\nu &= \dddot{\alpha}^* + m_1(\alpha^* - \alpha) + m_2(\dot{\alpha}^* - \dot{\alpha}) + m_3(\ddot{\alpha}^* - \ddot{\alpha})
\end{aligned}
$$

UDE Augmented IOL Controller

The UDE control law utilizes a new term u_d as follows [3];

- $\delta_c = \frac{1}{b}\left[u_a + u_d + \nu\right]$ where, $u_d = -\hat{d}$
- $\hat{d}$ is an estimate of the lumped disturbance and uncertainties d;
    1. $\ddot{\alpha} = a + b\delta_c + d$
    2. $d = \Delta a + \Delta b\delta_c + w$
    3. $\hat{d} = G_f(s)d$
    4. $G_f(s) = \frac{1}{1+s\tau}$
- Thus, we finally get $u_d = \frac{-1}{\tau}\left[\ddot{\alpha} - \int \nu dt\right]$
- UDE does not require knowledge of magnitude bounds and only requires the frequency bounds (tuned using τ) of the uncertainty or disturbance

UDE Observer based Control law

A Luenberger like UDE Observer has been designed [4]

- $\dddot{\alpha}$ is separated into linear and non-linear (d_1) terms;
  $\dddot{\alpha} = a_1\alpha + a_2\dot{\alpha} + a_3\ddot{\alpha} + d_1 + b\delta_c$
- Then the non-linear term d_1 and uncertainties are clubbed into a lumped term d_2, such that;
  $\dddot{\alpha} = a_{1_o}\alpha + a_{2_o}\dot{\alpha} + a_{3_o}\ddot{\alpha} + b_o\delta_c + d_2$
- This expressed in state space form can then be used to design the observer:

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= a_{1_o}x_1 + a_{2_o}x_2 + a_{3_o}x_3 + b_o\delta_c + d_2 \\
y &= x_1
\end{aligned}$$

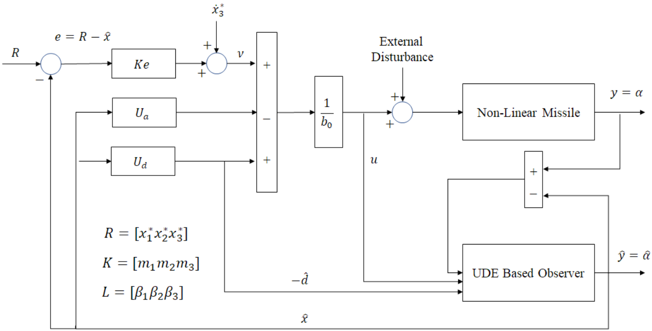
UDE Observer based Control law

Now since the equations are represented in a linear manner, a Luenburger like UDE Observer is designed by introducing the observer poles $[\beta_1 \beta_2 \beta_3]$

$$\dot{\hat{x}}_1 = \hat{x}_2 + \beta_1 e_o$$
$$\dot{\hat{x}}_2 = \hat{x}_3 + \beta_2 e_o$$
$$\dot{\hat{x}}_3 = a_{1o}\hat{x}_1 + a_{2o}\hat{x}_2 + a_{3o}\hat{x}_3 + b\delta_c + \hat{d}_2 + \beta_3 e_o$$
$$\hat{y} = \hat{x}_1$$

Here, the term d_2 representing the non-linearities and uncertainties is estimated by UDE as $\hat{d}_2$

Overview of UDE Controller-Observer



Stability Analysis

Error dynamics for UDE missile autopilot controller-observer structure

$$\begin{bmatrix} \dot{e}_c \\ \dot{e}_o \\ \dot{\tilde{d}}_2 \end{bmatrix} = \begin{bmatrix} (A-BK) & -(BK) & -B_d \\ 0 & (A-LC) & B_d \\ 0 & 0 & -\frac{1}{\tau} \end{bmatrix} \begin{bmatrix} e_c \\ e_o \\ \tilde{d}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \dot{d}_2 \tag{1}$$

Eigen values of the system matrix can be computed from

$$\left|sI-(A-BK)\right|\left|sI-(A-LC)\right|\left|s-(-\tfrac{1}{\tau})\right| = 0 \tag{2}$$

- (A, B) is controllable and (A, C) is observable
- τ is strictly a positive number
- Selecting appropriate controller and observer poles ensures stability of error dynamics
- Also if $\dot{d}_2 \neq 0$, then bounded-input, bounded-output stability can be assured.

Simulations

Parameters for simulation

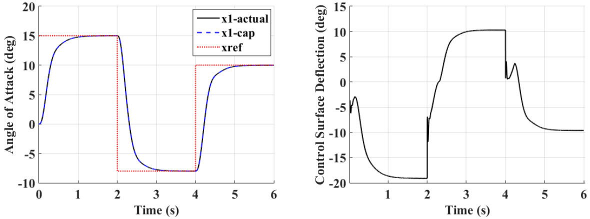
- Reference signal:

$$\alpha^* = \begin{cases} 15^\circ, & \text{if } 0 \leq t \leq 2\ s \\ -8^\circ, & \text{if } 2 < t \leq 4\ s \\ 10^\circ, & \text{if } 4 < t \leq 6\ s \end{cases}$$

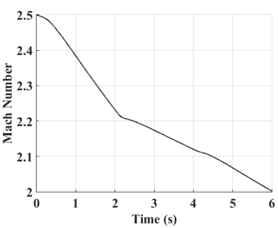
- Tracking Constraints:
  1. To be tracked with a time constant of less than $0.25s$
  2. Less than 10% overshoot
  3. Less than 1% steady-state error
- Pole placement to meet this target:
  1. Controller gains $[m_1\ \ m_2\ \ m_3]$ placed at $s_{1,2,3} = -12$
  2. Observer gains $[\beta_1\ \ \beta_2\ \ \beta_3]^T$ placed at $s_{1,2,3} = -360$
- Controller and observer designed at M = 2.25 (mid-point of Mach envelope)

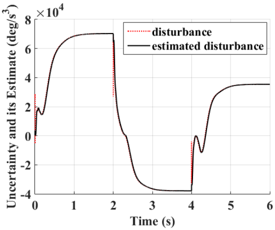
Case I: UDE with Mach Dynamics & External Disturbance

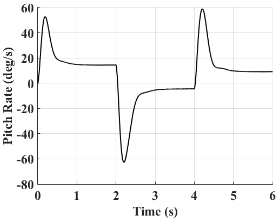
- UDE simulated with varying mach and external disturbance
- Mach i.c is $M(0) = 2.5$ and follows M equation [1] till $M = 2$
- External disturbance modeled as sinusoidal amplitude 8° and frequency $0.25Hz$



Case I: UDE with Mach Dynamics & External Disturbance



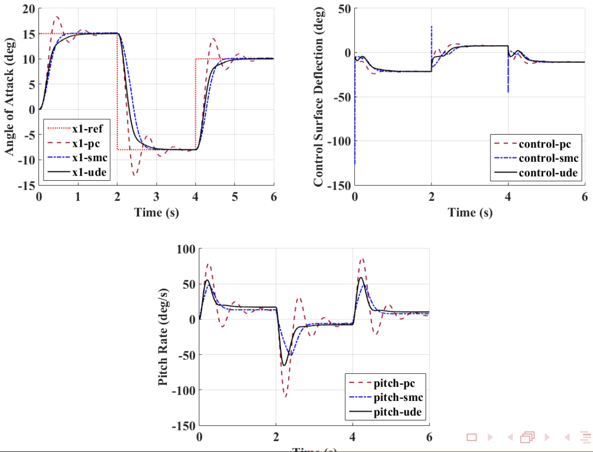




Case II: Comparative Study With Aerodynamic Uncertainties

- Comparative analysis of UDE has been done against Predictive Control (PC) [4] and Sliding Mode Control (SMC) [5]
- Uncertainty of $+30\%$ in aerodynamic force coefficient C_n and -30% in aerodynamic moment coefficient C_m
- Mach has been maintained at the nominal constant of $M = 2.25$
- No external disturbances added to the system.

Case II: Comparative Study With Aerodynamic Uncertainties



Results and conclusions

- Results of CASE I
  1. Tracking performance is as desired and control effort stays smooth and within the practical bounds of $\pm 30^\circ$
  2. UDE observer is able to estimate states quickly and accurately
  3. Estimation of disturbance by UDE is minimally delayed and follows closely to the actual value
- Results of CASE II
  1. Due to aerodynamic uncertainty, PC has overshoots in tracking. UDE and SMC are smooth
  2. Control effort of PC is oscillatory. SMC has high overshoots at transition points. UDE is able to provide smooth control within $\pm 30^\circ$
  3. Pitch graph of PC is oscillatory while SMC is slightly delayed. Pitch graph of UDE is on point.

Novelty

- Despite uncertainties such as varying Mach (case I) and varying aerodynamic constants (case II) and even the presence of external disturbance (case I), UDE is able to provide robust tracking control.
- Only the frequency bound (captured through τ) of the external disturbance and uncertainty is required to provide robust tracking with UDE. There is no dependency on the magnitude bounds of the disturbance/uncertainty.
- Unlike PC and SMC simulations which have used the actual states while implementing the control law, UDE simulation has utilized the estimated states obtained from the UDE observer. As is well known, use of estimated states might result in degraded performance; in contrast the proposed strategy utilizes the estimated states and still proves its worthiness.

Future Work

Our future work would include the design of an integrated pith-yaw-roll autopilot for a nonlinear missile model with and without linearization, using UDE theory.

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Thank You!