CRITICAL PATH METHOD

Critical Path Method (CPM)

Critical Path Analysis.

A project network analysis technique used to predict total project duration

Critical path for a project is the series of activities that determine the earliest time by which the project can be completed.

It is the longest path through the network diagram and has the least amount of slack or float.

Critical Path Method (CPM)

Definition:

- Early start (ES) date: the earliest possible time an activity can start based on the project network logic.
- Slack (S) or float (F) : the amount of time an activity may be delayed without delaying a succeeding activity or the project finish date.
- Free slack (FS)/ free float (FF): the amount of time an activity can be delayed without delaying the early start date of any immediately following activities.
- Total slack/total float: amount of time an activity can be delayed from its early start time without delaying the planned project finish date.

Using critical path analysis

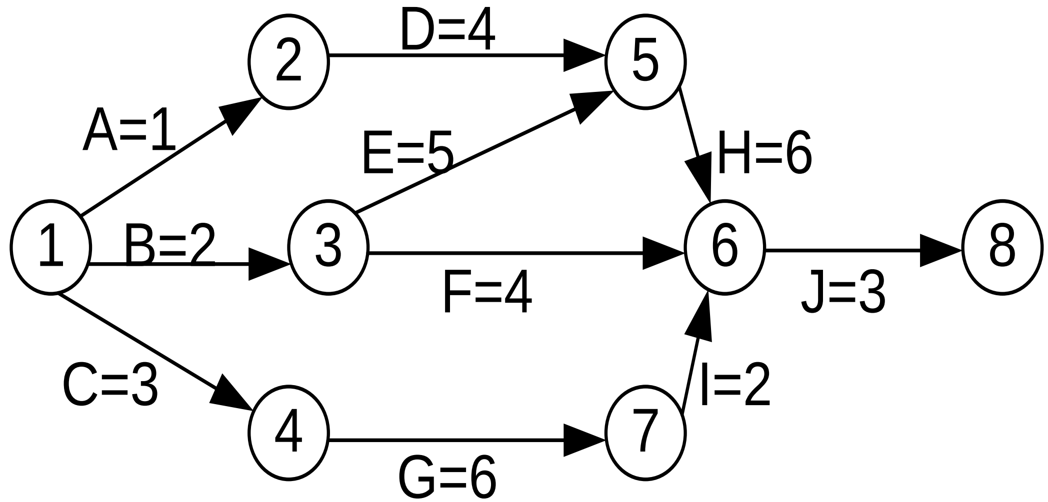
- A forward pass : determines the early start and early finish dates for each activity.
- The early finish date: the earliest possible time and activity can finish based on the project network logic.
- The project start date = the early start date for the first network diagram activity.

Using critical path analysis

- A backward pass through the network diagram: determines the late start date and the late finish date.
- The late start date: is the latest possible time an activity might begin without delaying the project finish date.
- The late finish date: is the latest possible time an activity can be complete without delaying the project finish date.

Calculating the critical path - 1

- First: develop a good network diagram → requires a good activity list based on the WBS.
- Add the duration estimates for all activities on each path through the network diagram.
- Calculating the critical path involves adding the durations for all activities on each path through the network diagram.
- The longest path is the critical path.
- If one or more of the activities on the critical path takes longer than planned, the whole project schedule will slip *unless* the project manager takes corrective action.



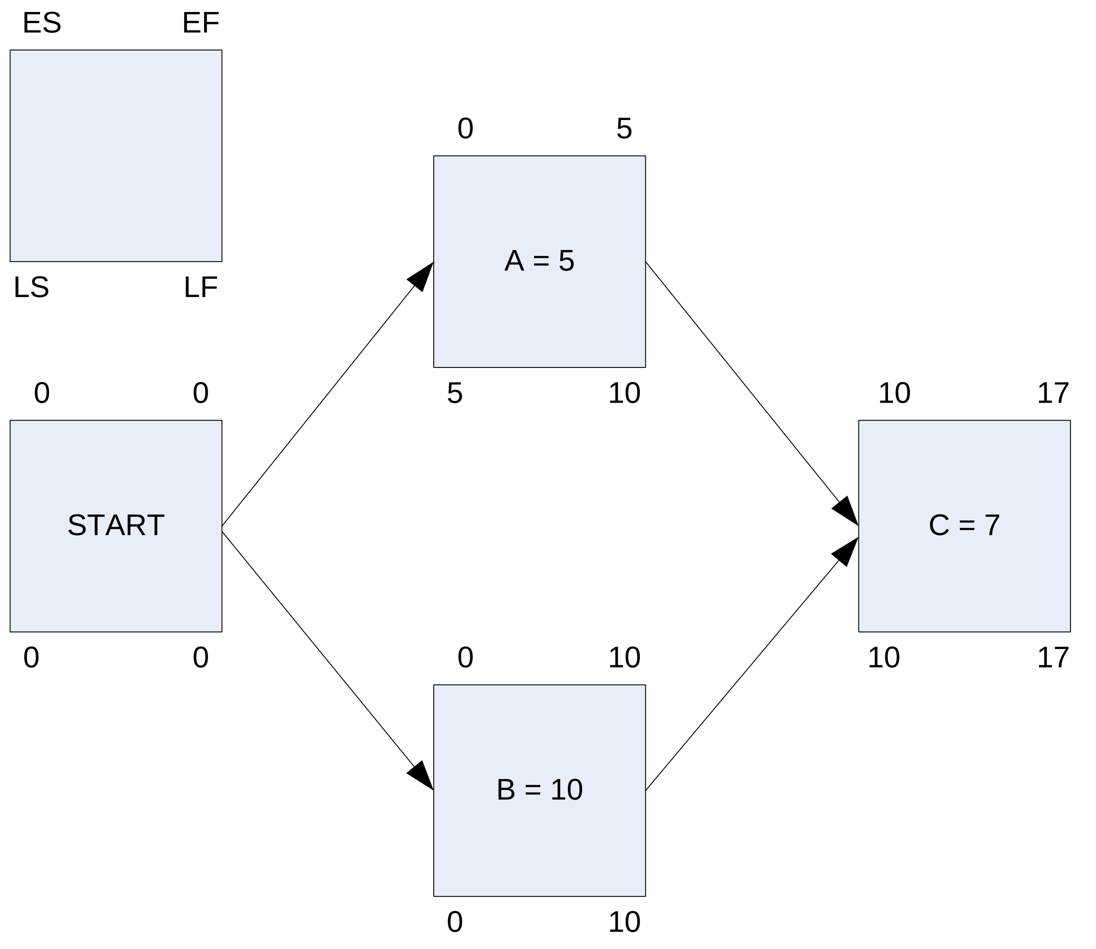
- Path 1 : A-D-H-J length= 1+4+6+3 = 14 days
- Path 2 : B-E-H-J length= 2+5+6+3 = 16 days
- Path 3 : B-F-J length= 2+4+3 = 9 days
- Path 4 : C-G-I-J length= 3+6+2+3 = 14 days

- Critical path: path 2

Using critical path analysis

- ES(A) + duration (A) = EF(A) = ES each subsequent activity.

  Early start + duration of the first activity = the earliest finish date of the first activity
- When an activity has multiple predecessor, its ES dates is the latest of the EF dates of those predecessors.


ES
EF
LS
LF
0
5
A = 5
5
10
0
0
START
0
0
10
17
C = 7
10
17
0
10
B = 10
0
10

Free and Total Float/Slack for Project X

Task	Start	Finish	Late Start	Late Finish	Free Slack	Total Slack
A	2/6/2007	2/6/2007	4/6/2007	4/6/2007	0d	2d
B	2/6/2007	3/6/2007	2/6/2007	3/6/2007	0d	0d
C	2/6/2007	4/6/2007	4/6/2007	6/6/2007	0d	2d
D	3/6/2007	6/6/2007	5/6/2007	10/6/2007	2d	2d
E	4/6/2007	10/6/2007	4/6/2007	10/6/2007		
F	4/6/2007	9/6/2007	13/6/2007	18/6/2007		
G	5/6/2007	12/6/2007	9/6/2007	16/6/2007		
H	11/6/2007	18/6/2007	11/6/2007	18/6/2007		
I	13/6/2007	16/6/2007	17/6/2007	18/6/2007		
J	19/6/2007	23/6/2007	19/6/2007	23/6/2007		

Using critical path to shorten a Project Schedule (1)

- By knowing the critical path, the project manager and his/her team can use several duration compression techniques to shorten the project schedule.
- One technique is to reduce the duration of activities on the critical path → by allocating more resources to those activities or by changing the scope.

Using critical path to shorten a Project Schedule (2)

- **Shortening** the duration of critical activities or tasks by adding more resources or changing their scope.
- **Crashing**: a technique to obtain the greatest amount of schedule compression for the least incremental cost.

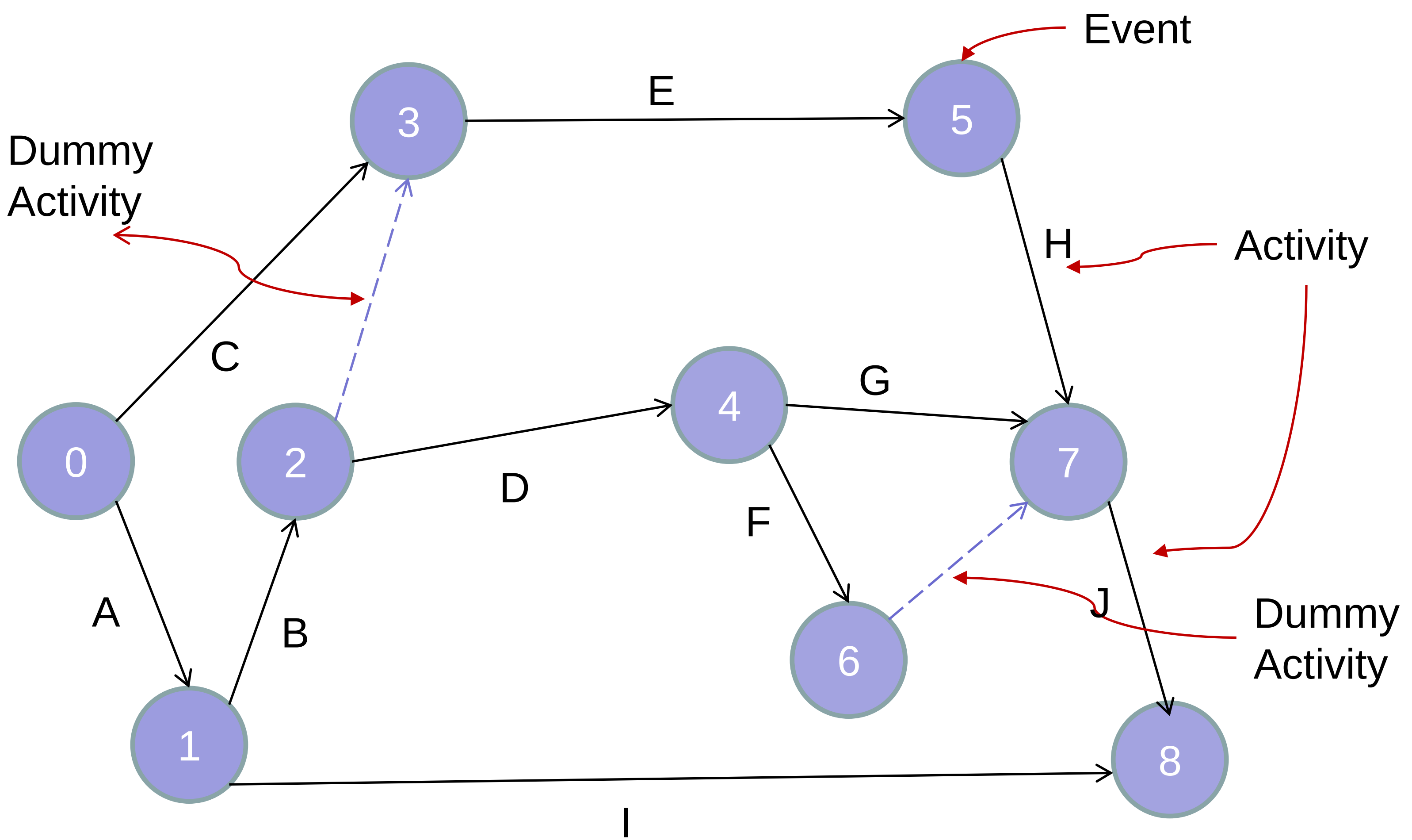
  advantage: shortening the time it takes to finish a project

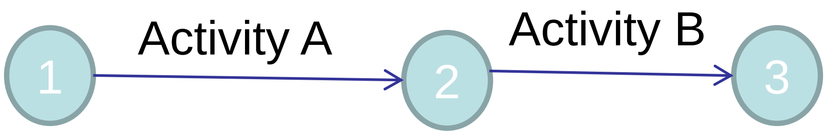
  disadvantage: increases total project costs.
- **Fast tracking**: involves doing activities in parallel that normally do in sequence.

  advantage: shorten the time it takes to finish a project.

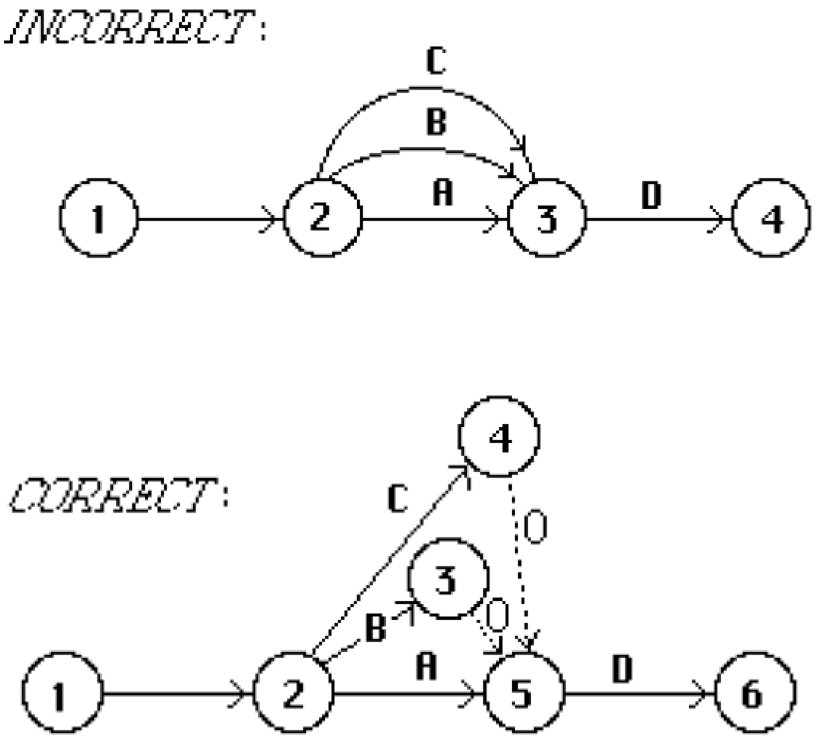
  disadvantage: can end up lengthening the project schedule since starting some task too soon often increases project risk and result in rework.

Project Network AOA

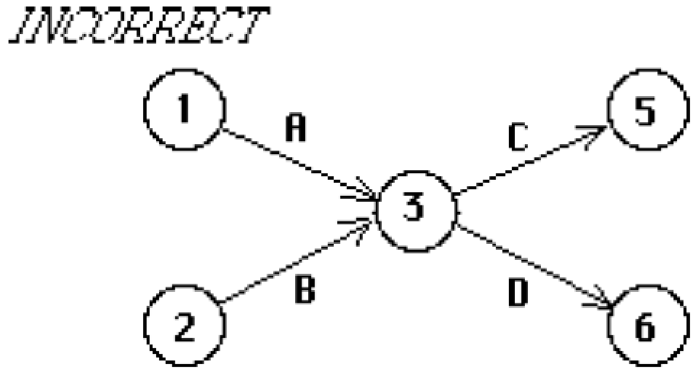


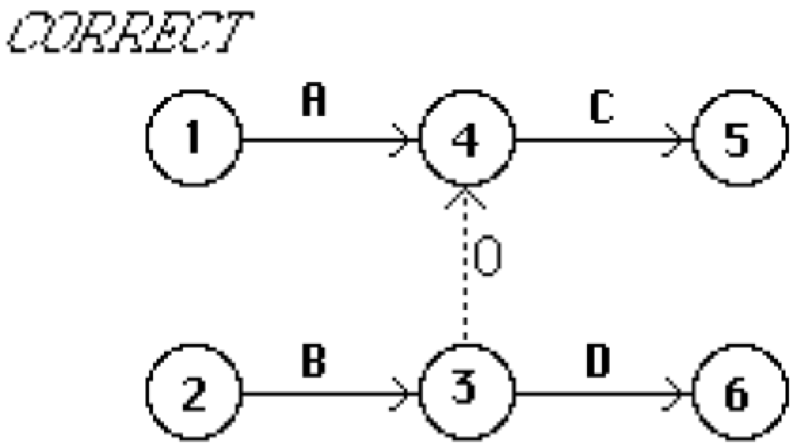


- Activity B cannot begin until activity A has been completed.
- Activity A is a predecessor of activity B
- Activity B is a successor of activity A



- Only one activity is allowed between two vertices, dummy activities may be defined if necessary (with zero duration)





- A and B are predecessors of C, but only B is a predecessor of D

- Let the beginning of the project be the event 0.
- Let the end of the project be the event n.
- ET(i) : the length of the longest path from event 0 to event i.
- If the project begin at time zero, activity (i,j) can be scheduled to start as early as (but no earlier than) time ET(i)
- ET(n): minimum project duration.

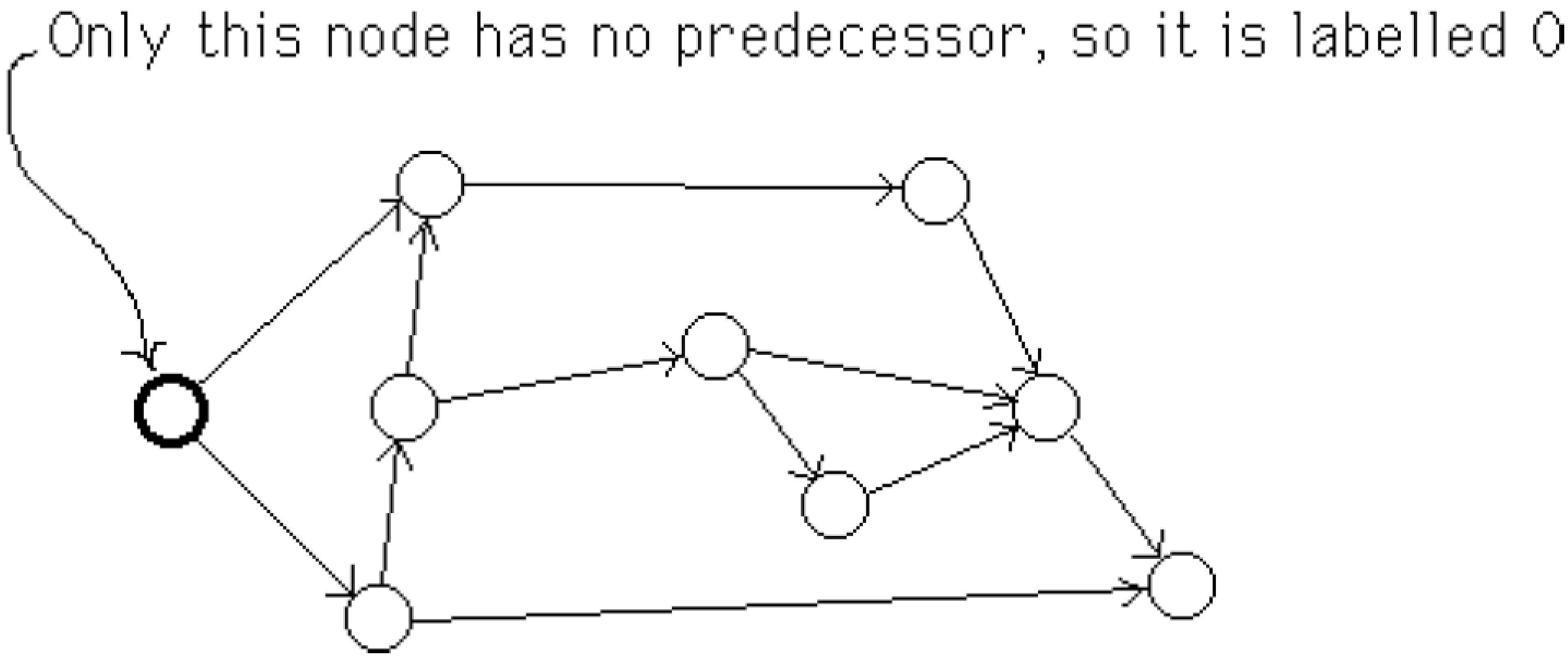
Labeling Event

- It is convenient to labeling event (vertices) of the project network so that i < j if (i , j) is an activity.
- Algorithm:

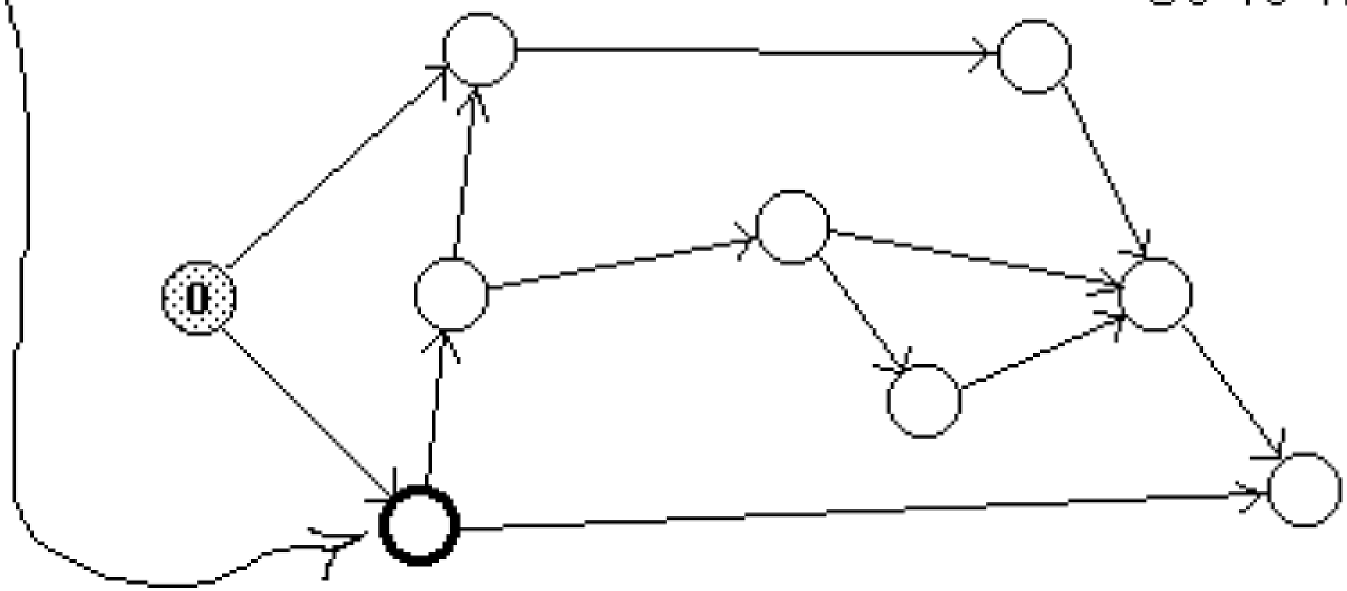
Step 0 : let j = 0

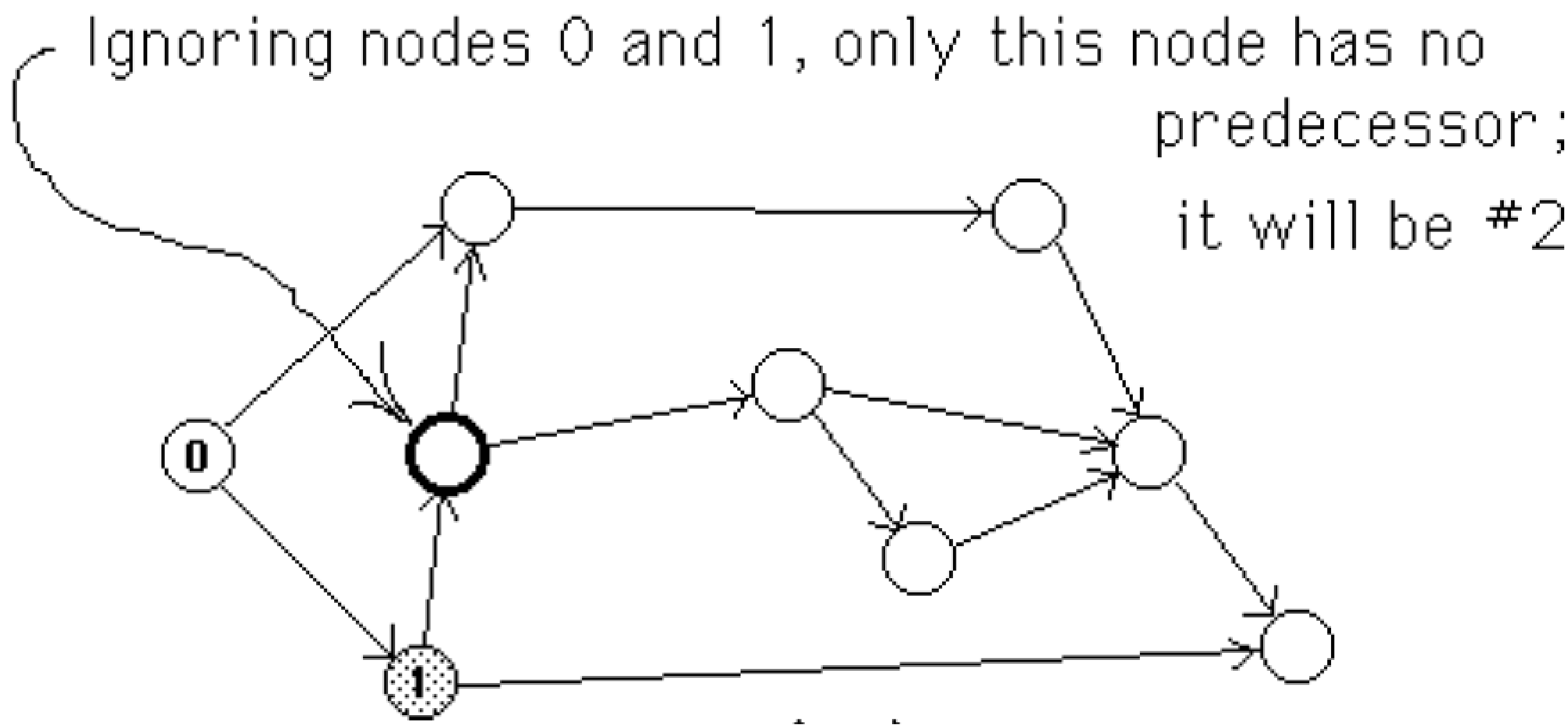
Step 1 : find a vertex without an unlabeled predecessor. If none, quit. Else label this vertex ‘j’.

Step 2 : increment j by 1 and go to step 1.

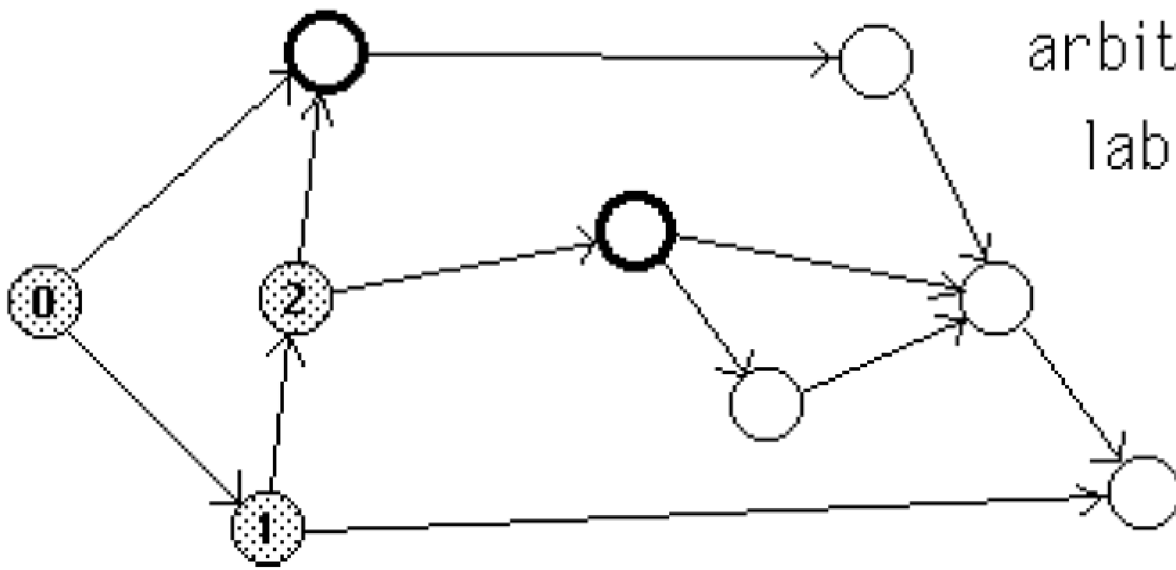

Only this node has no predecessor, so it is labelled 0

Ignoring node 0, only this node has no predecessor
so it will be #1

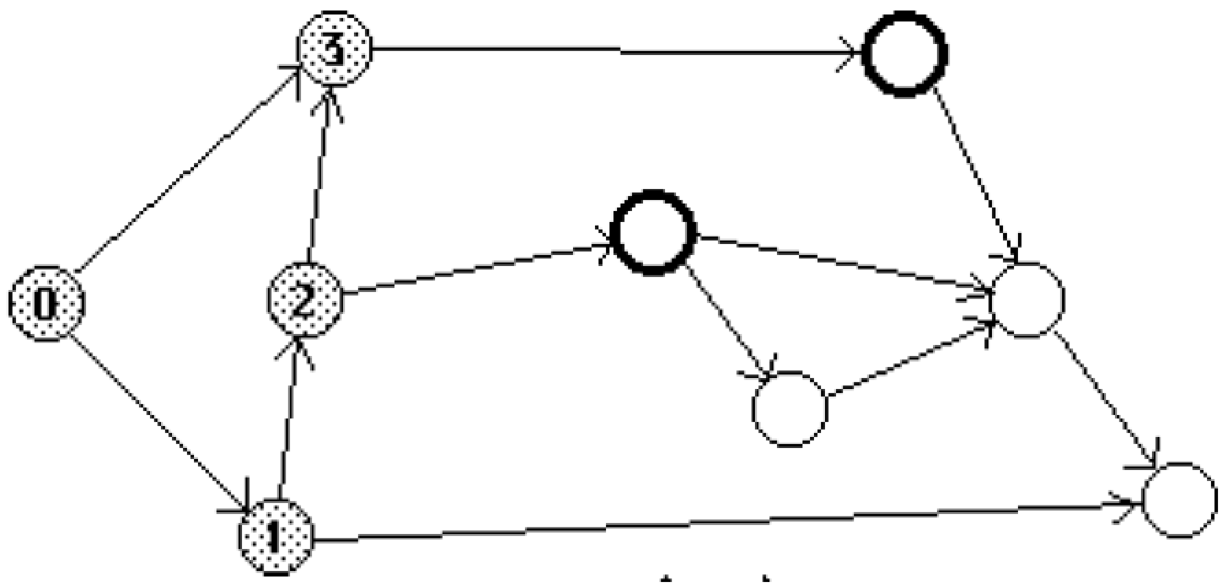



Ignoring nodes 0 and 1, only this node has no predecessor; it will be #2
0
1

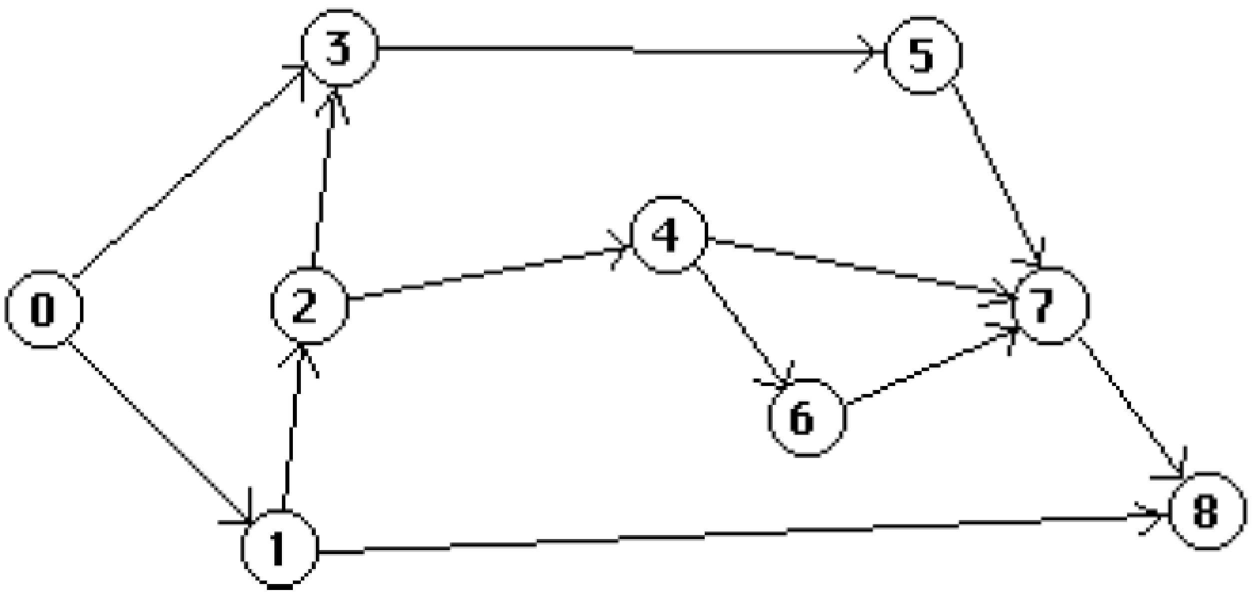
Ignoring nodes 0,1,&2, there are two nodes having no predecessor; we choose one of them arbitrarily to be labelled #3



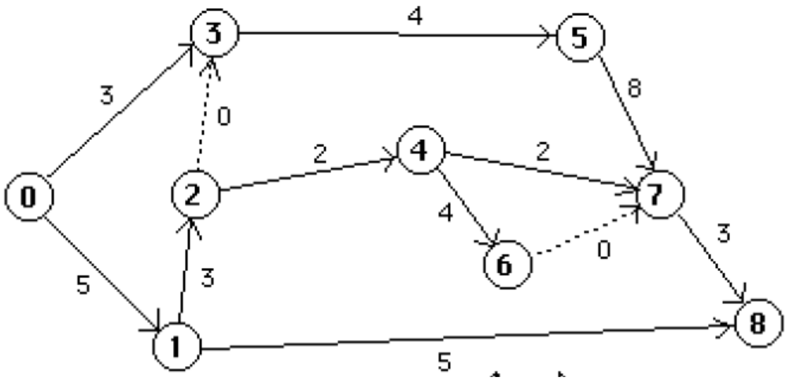
Again, there are two nodes without predecessors; we will choose one arbitrarily to be #4



The result:



Computing earliest time: Forward Pass



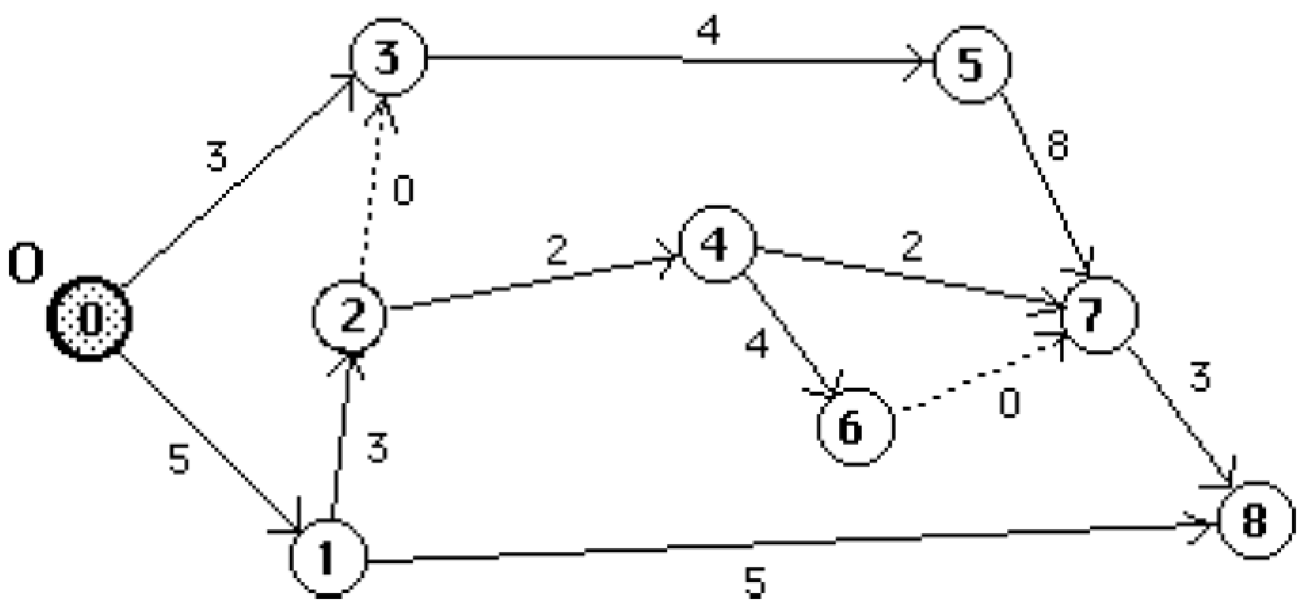
- Algorithm Forward Pass:

  ET(0) = 0

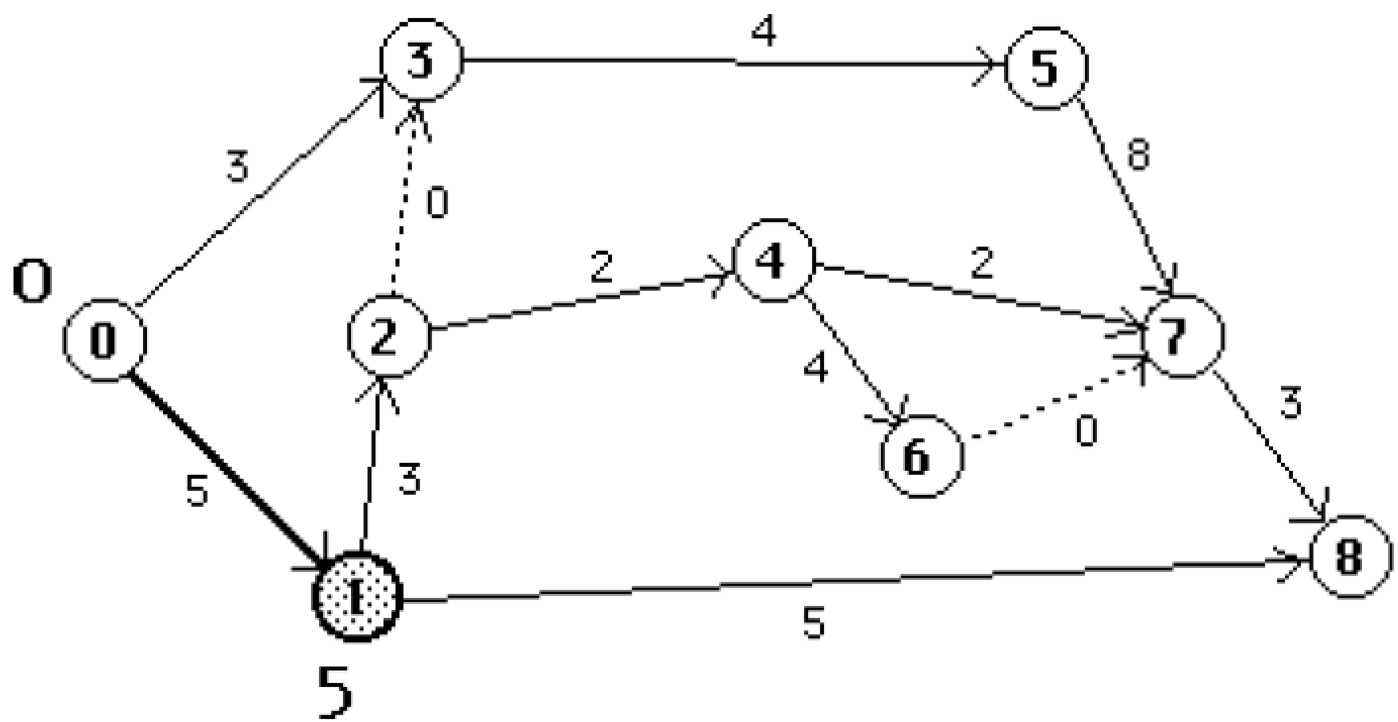
  for j = 1 to n

  ET(j) = max{ET(i)+d_{ij}}

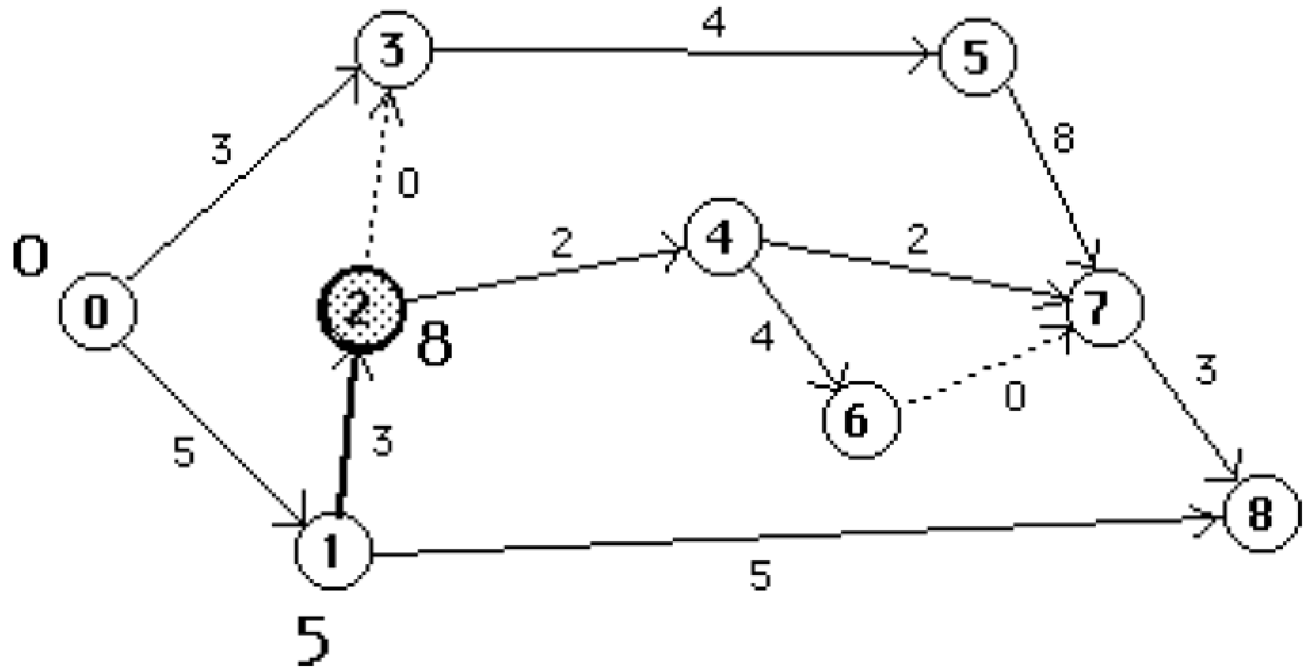
ET(0)=0



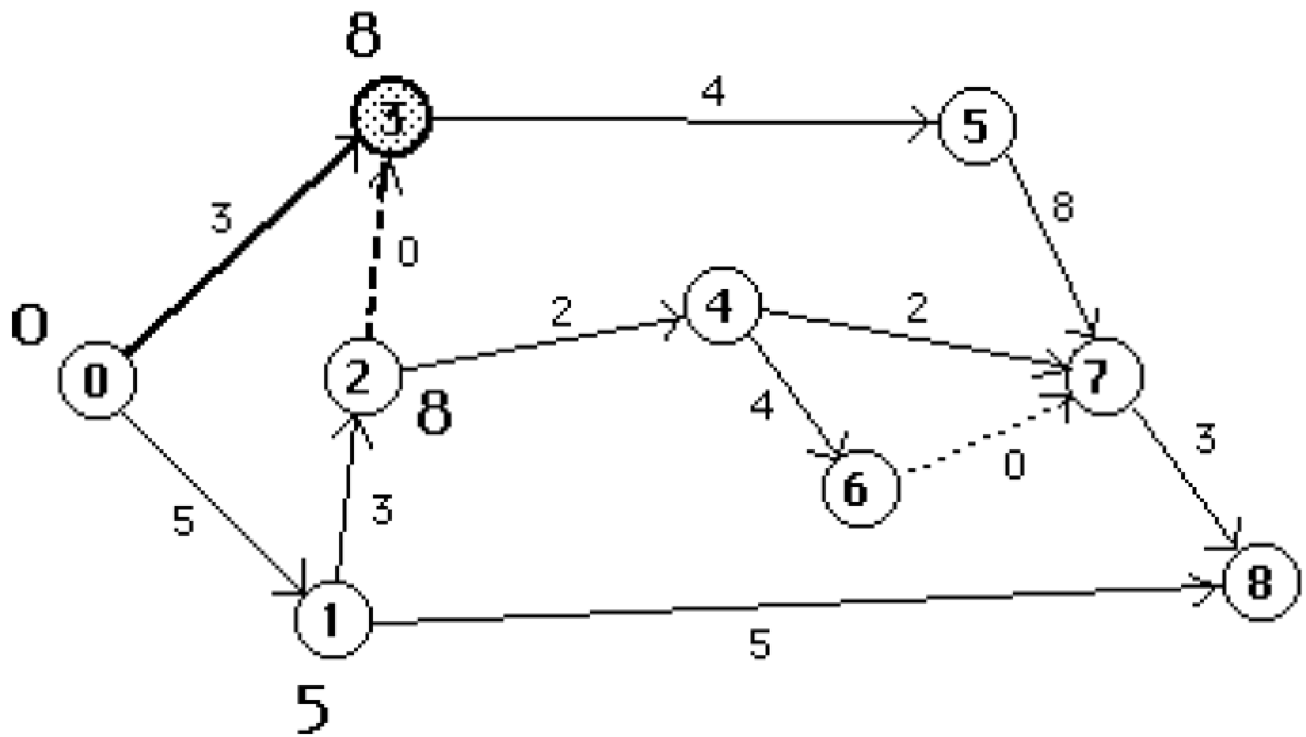
$ET(1) = ET(0) + 5 = 5$



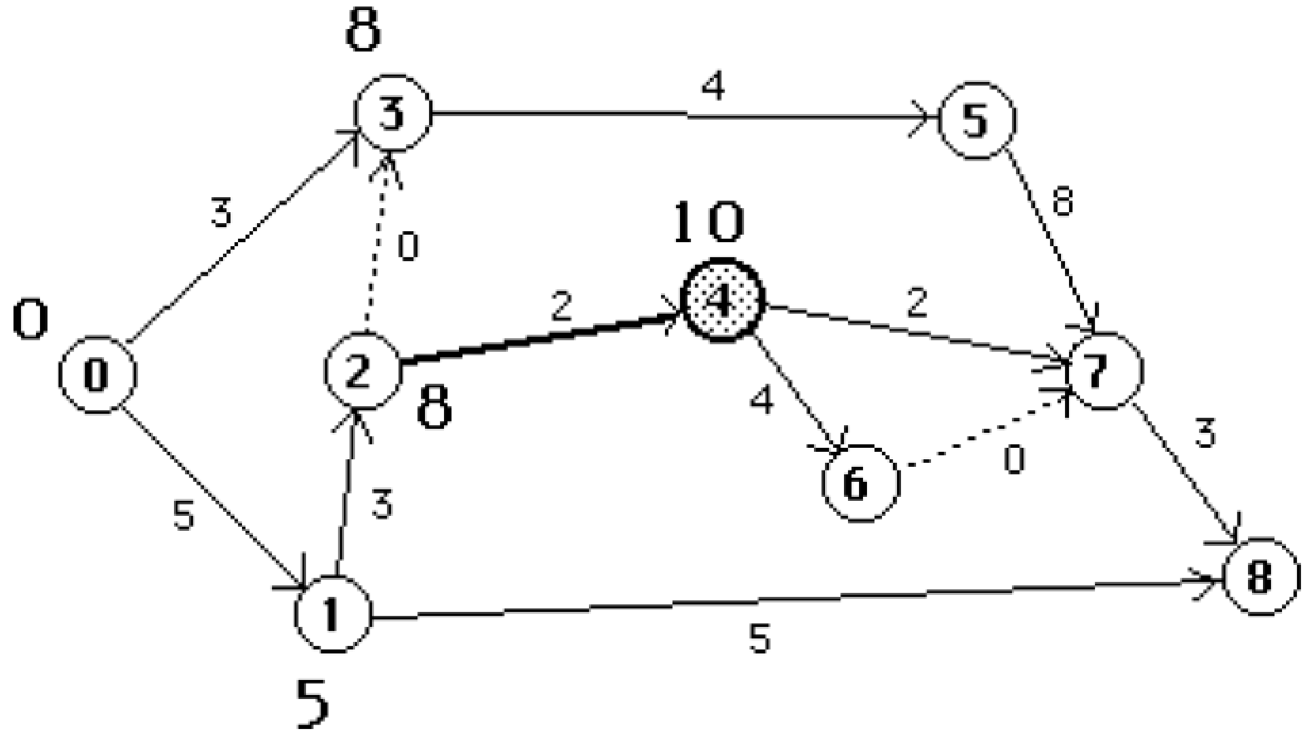
$$ET(2) = ET(1) + 3 = 8$$



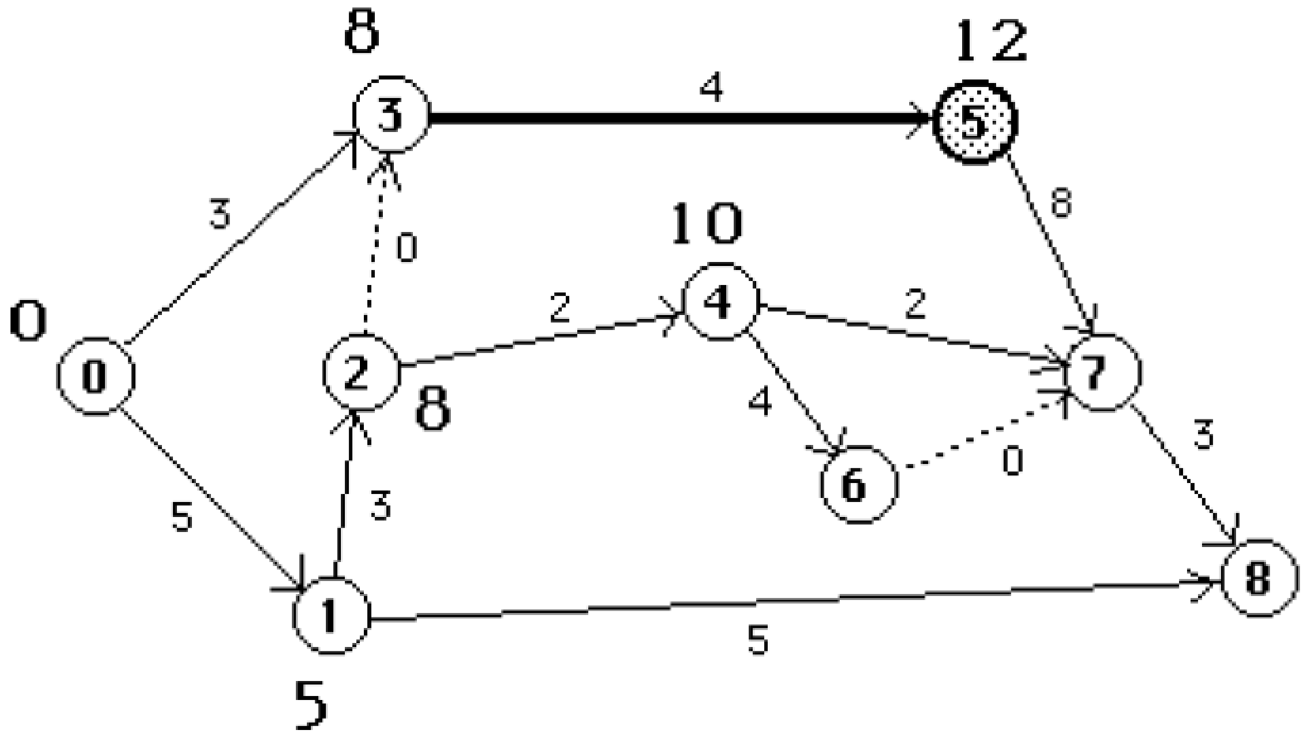
$$ET(3) = \max\{ET(0)+3,\ ET(2)+0\}$$
$$= \max\{3,8\} = 8$$



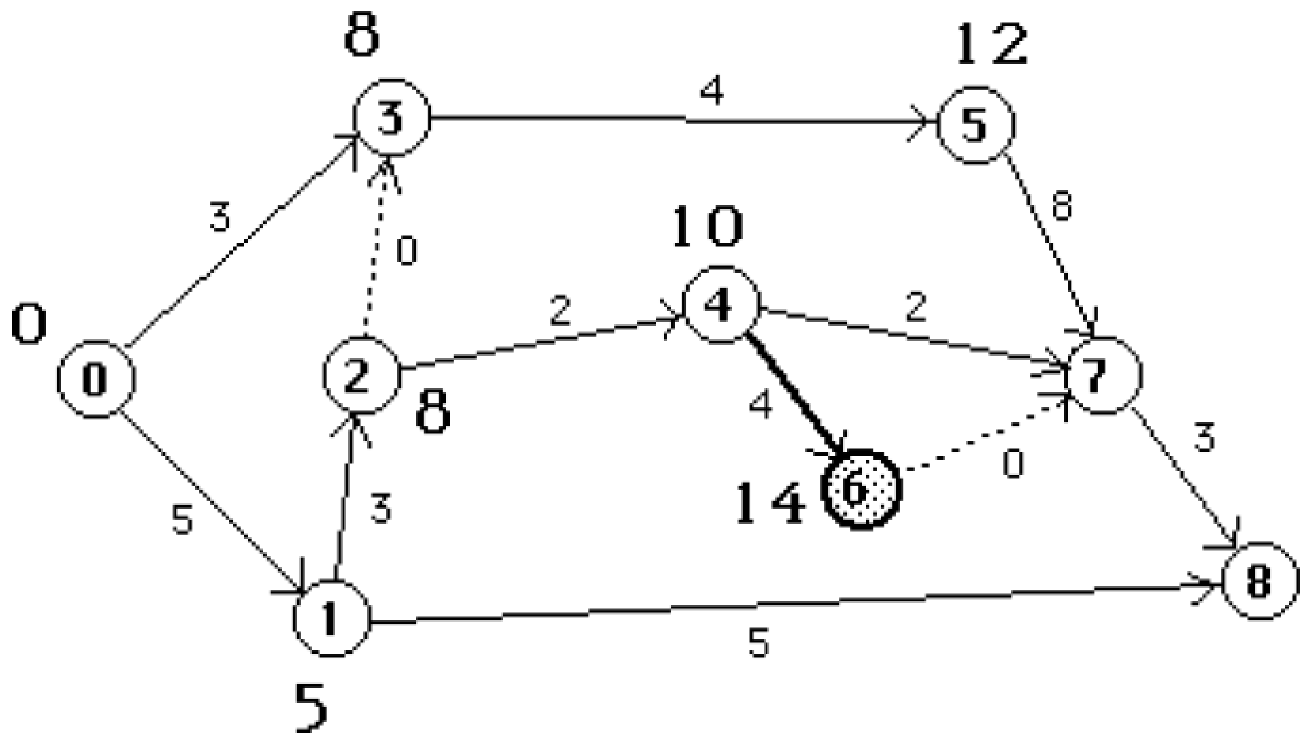
$ET(4) = ET(2) + 2 = 10$



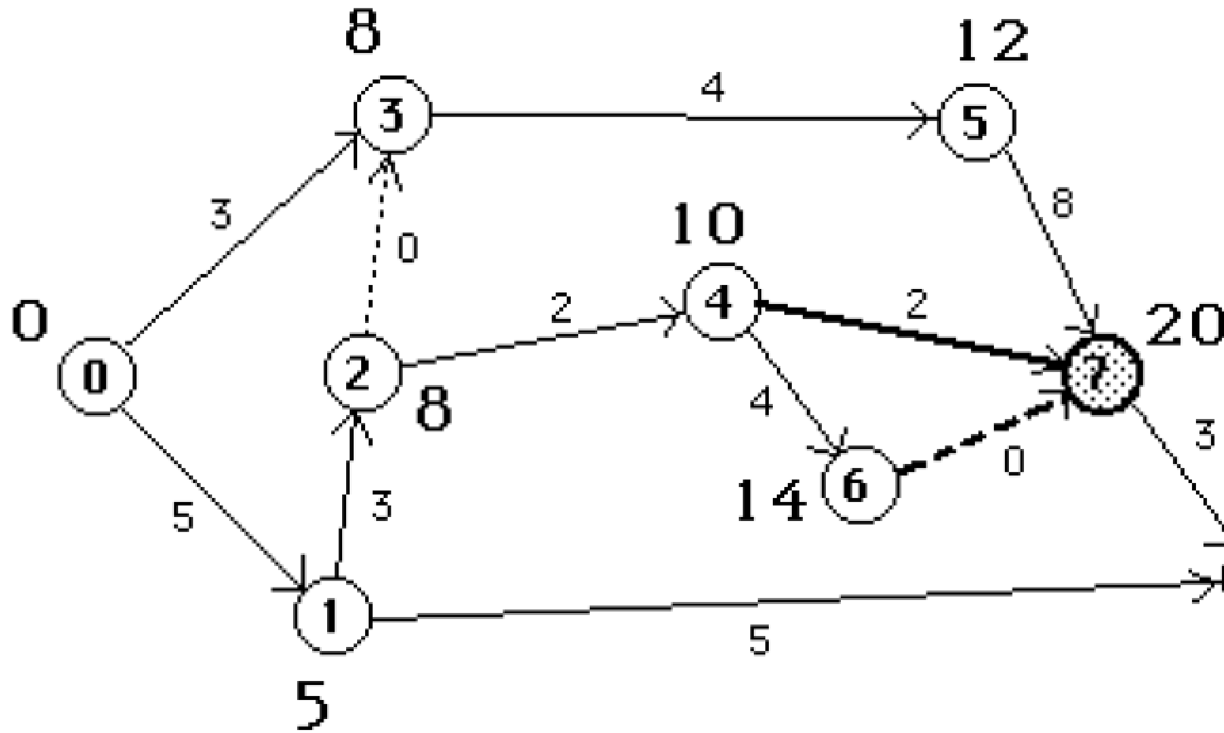
$$ET(5) = ET(3) + 4 = 12$$



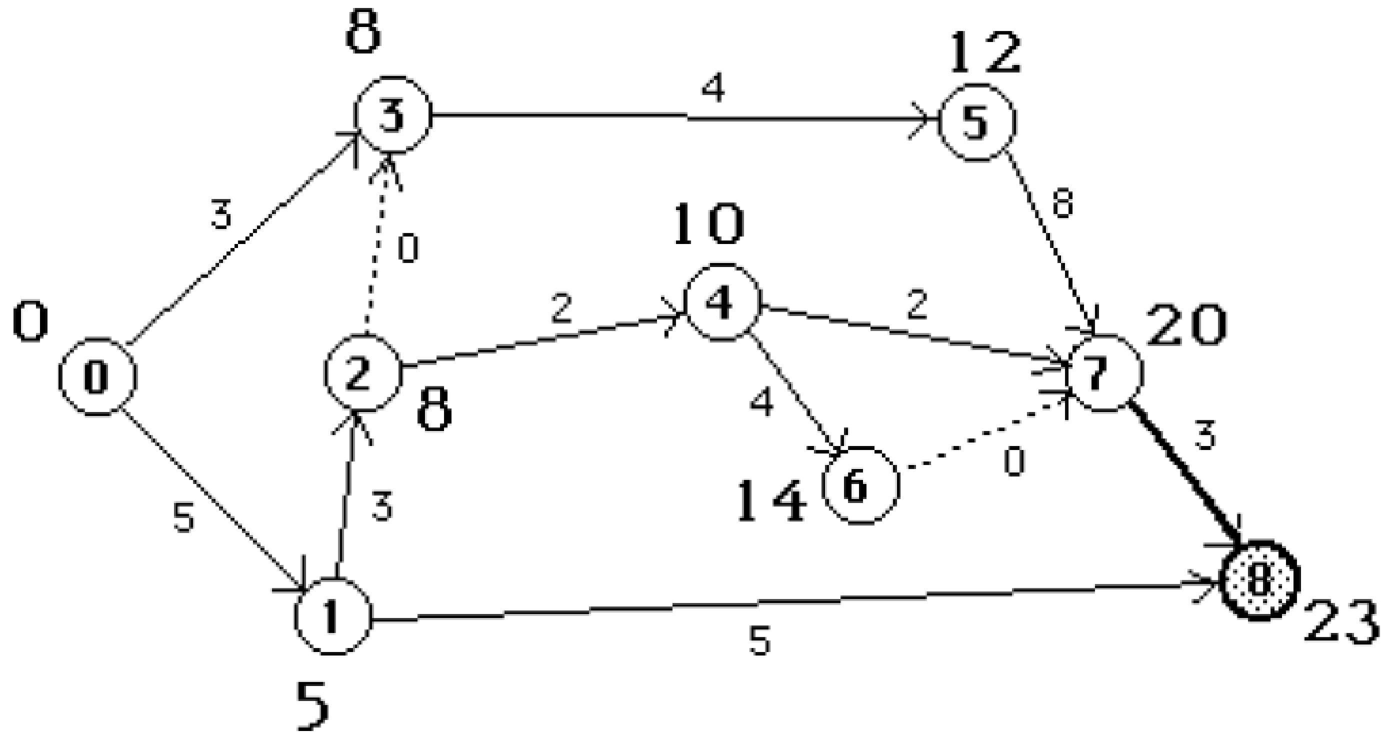
$$ET(6) = ET(4) + 4 = 14$$

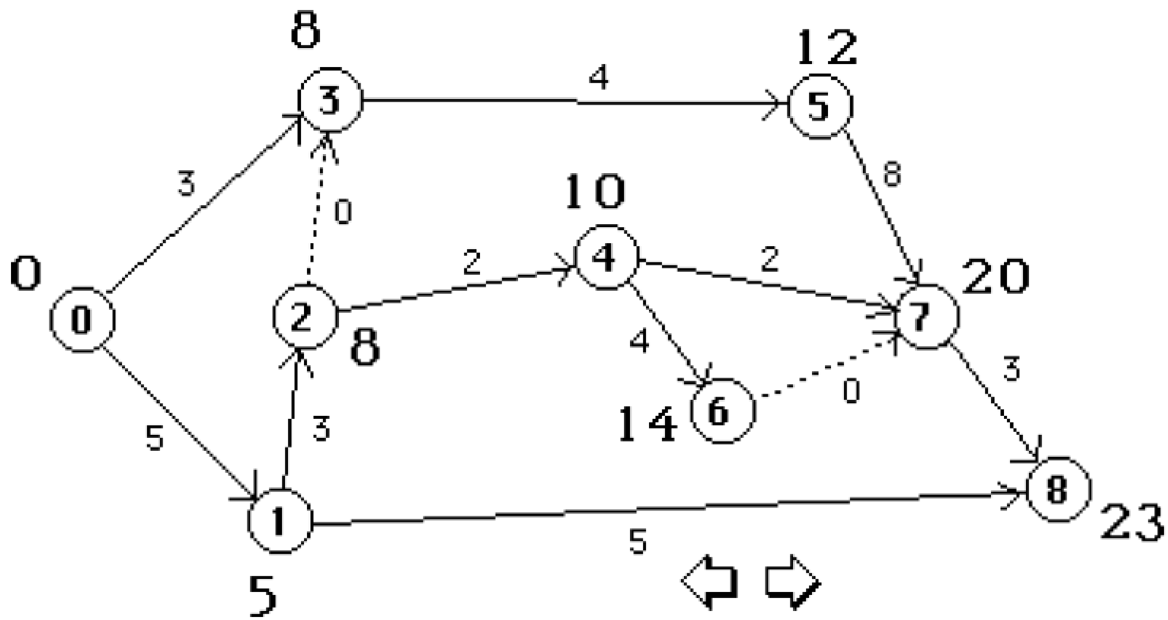


$$ET(7) = \max\{ET(4)+2,\ ET(6)+0,\ ET(5)+8\}$$
$$= \max\{12,\ 14,\ 20\} = 20$$



$$ET(8) = \max\{ET(1)+5,\ ET(7)+3\}$$
$$= \max\{10, 23\} = 23$$

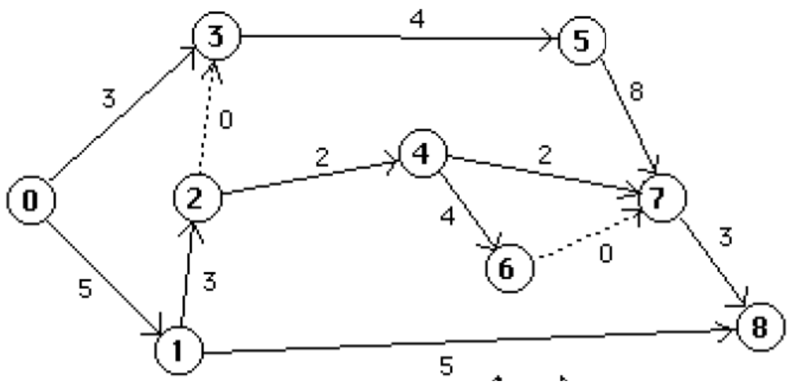




- And so the earliest time for completion of the project (event #8) is 23

Latest Time

- LT(i) : latest time at which event i can occur if the project is to be completed in minimum time.



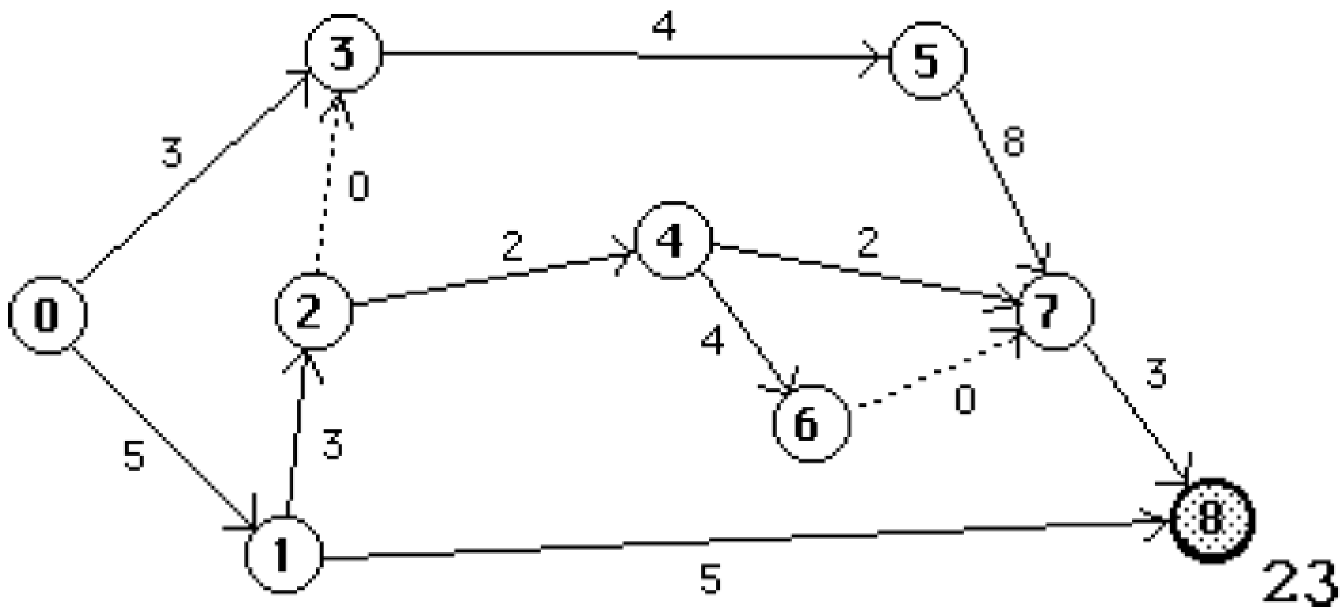
- Algorithm Backward Pass:

LT(n) = ET(n)

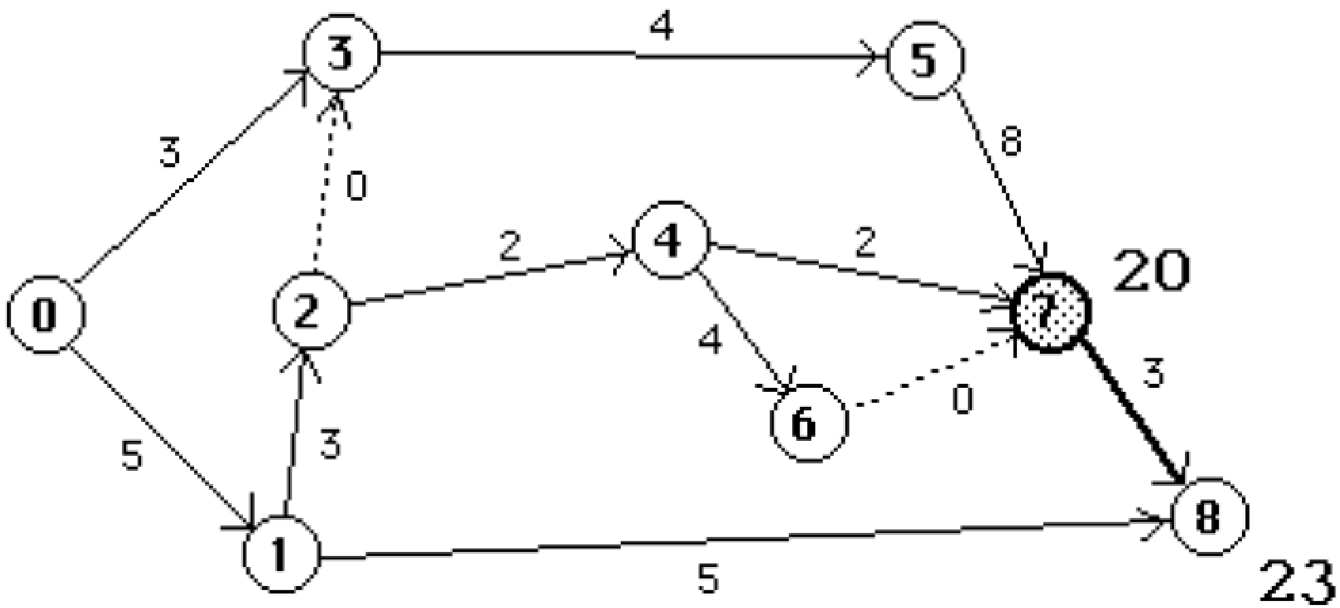
For i = n-1, n-2, …, 0

LT(i) = min{LT(j) – d}

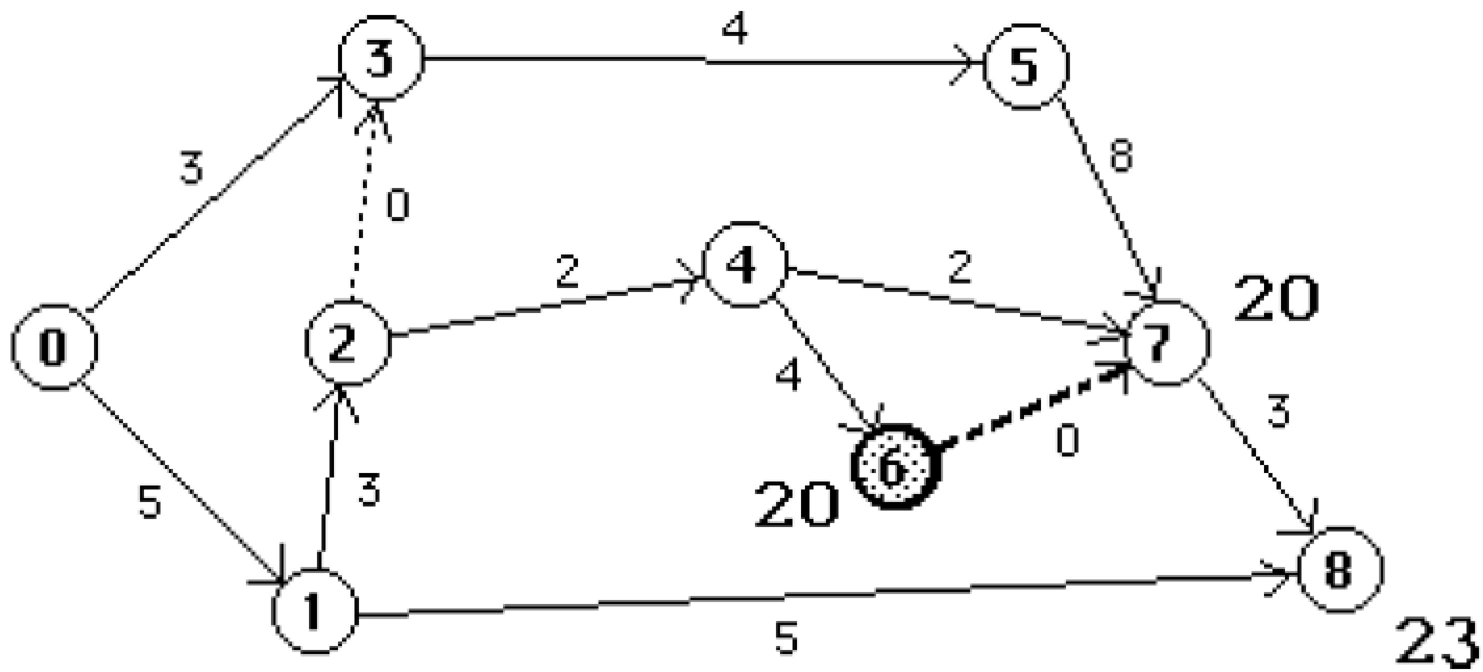
$LT(8) = ET(8) = 23$



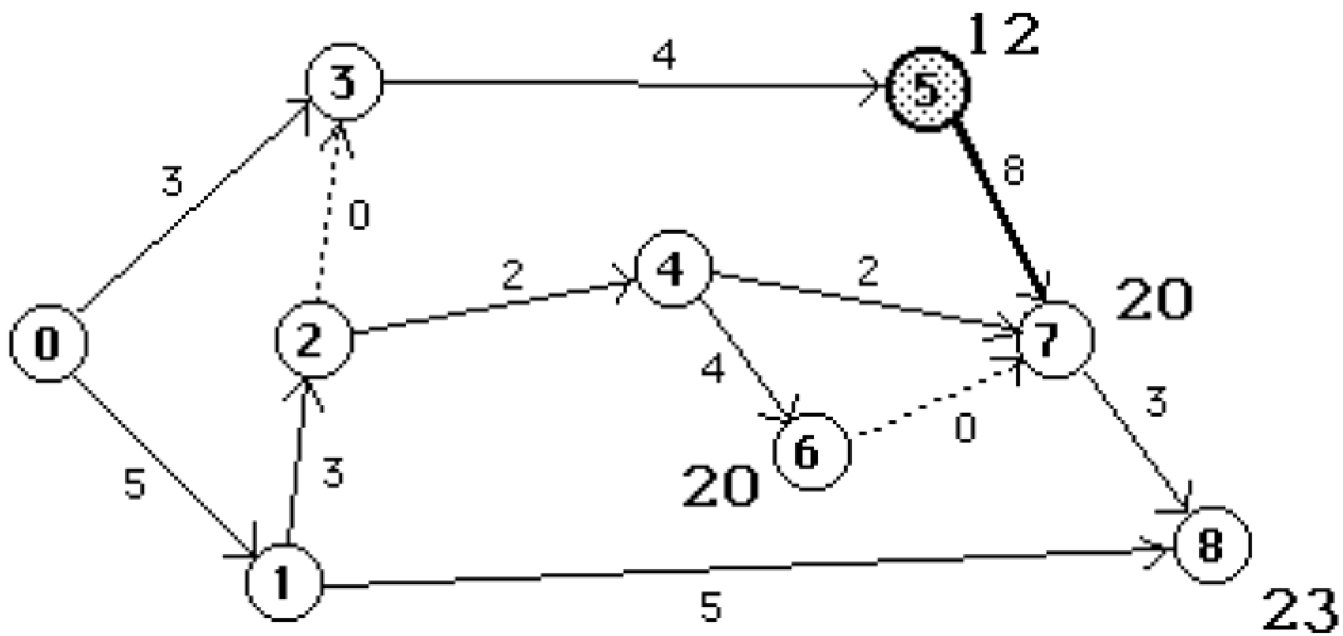
LT(7) = LT(8) - 3 = 20



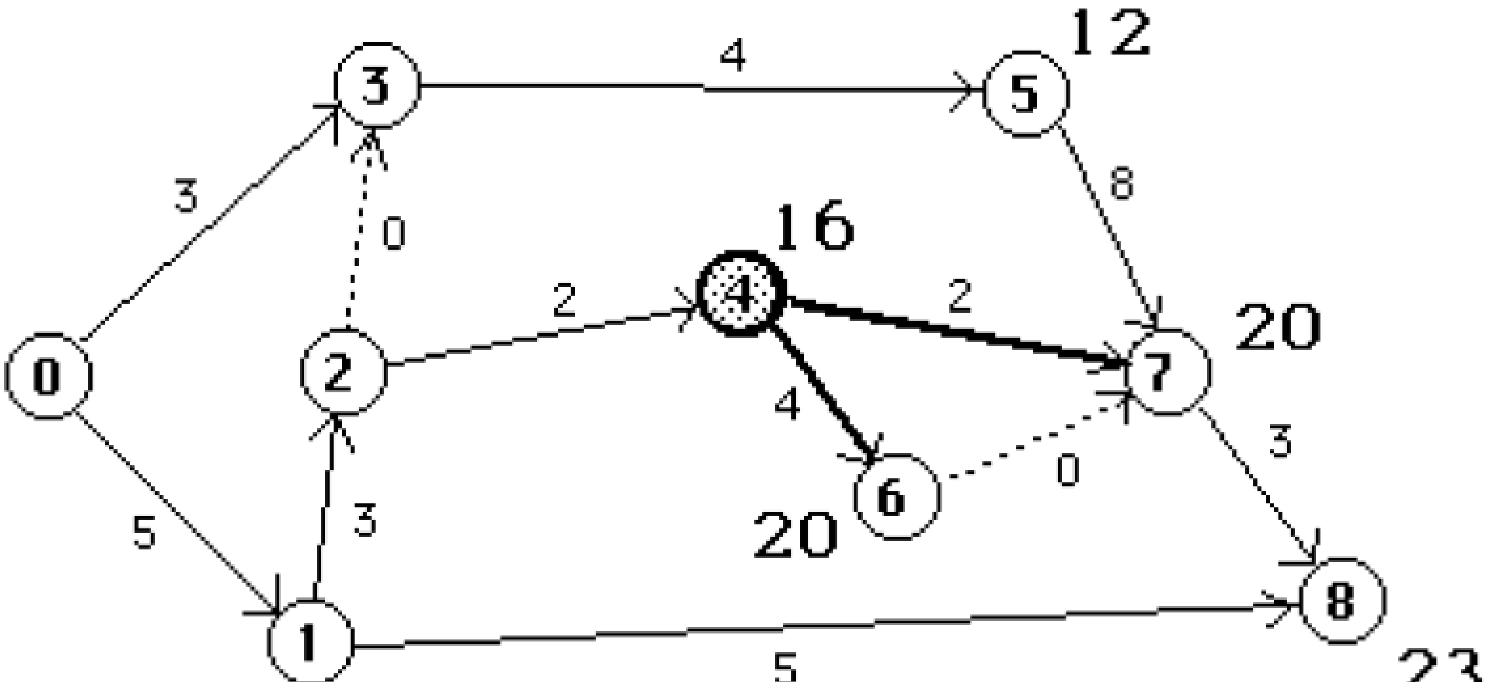
LT(6) = LT(7) - 0 = 20



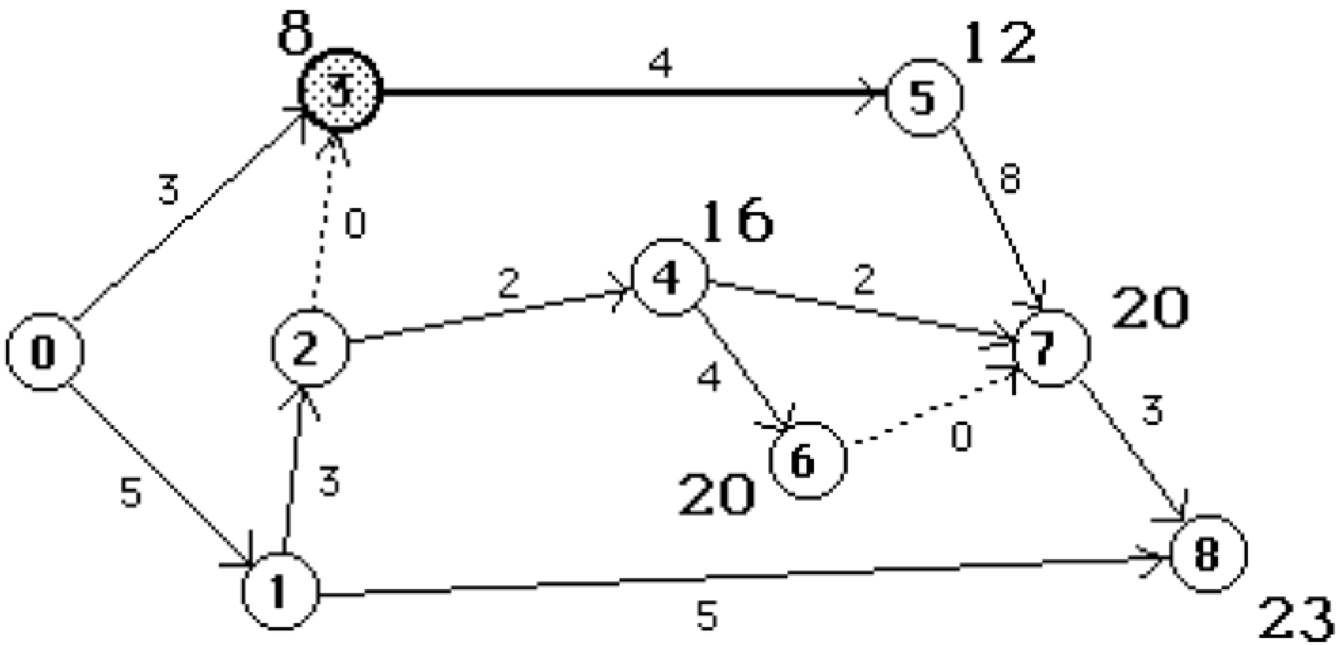
LT(5) = LT(7) - 8 = 12



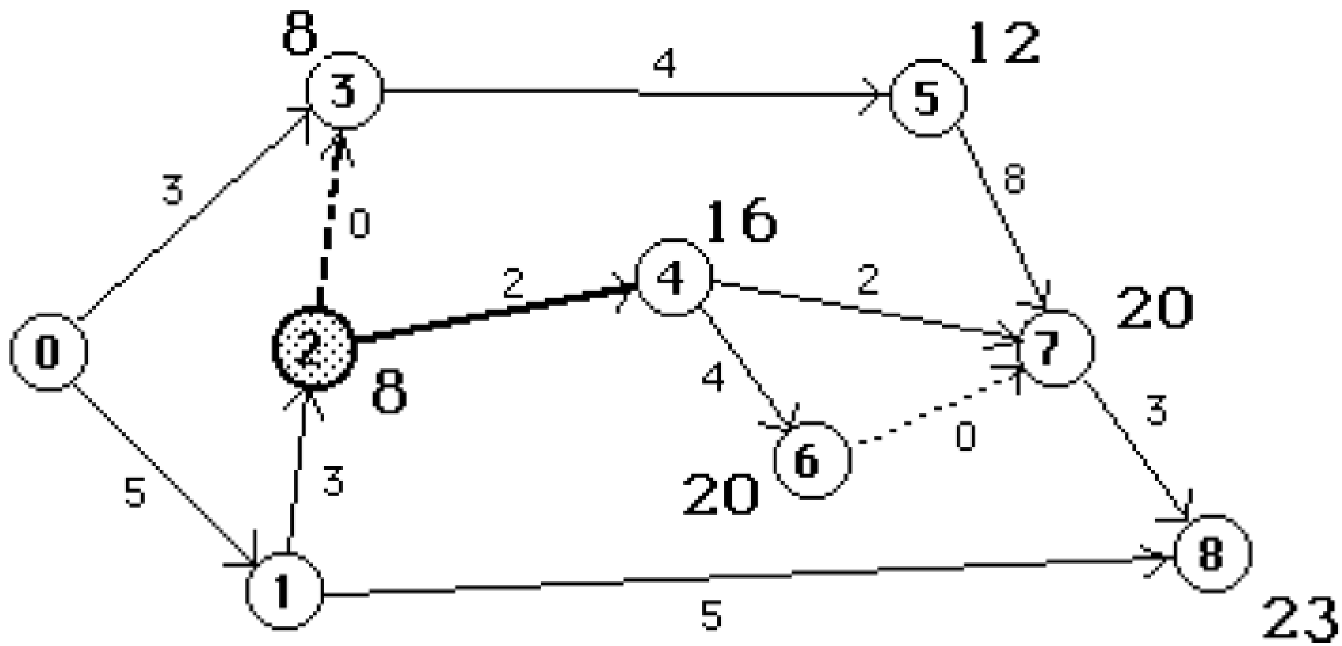
$$LT(4) = \min\{LT(6)-4, LT(7)-2\}$$
$$= \min\{16, 18\} = 16$$



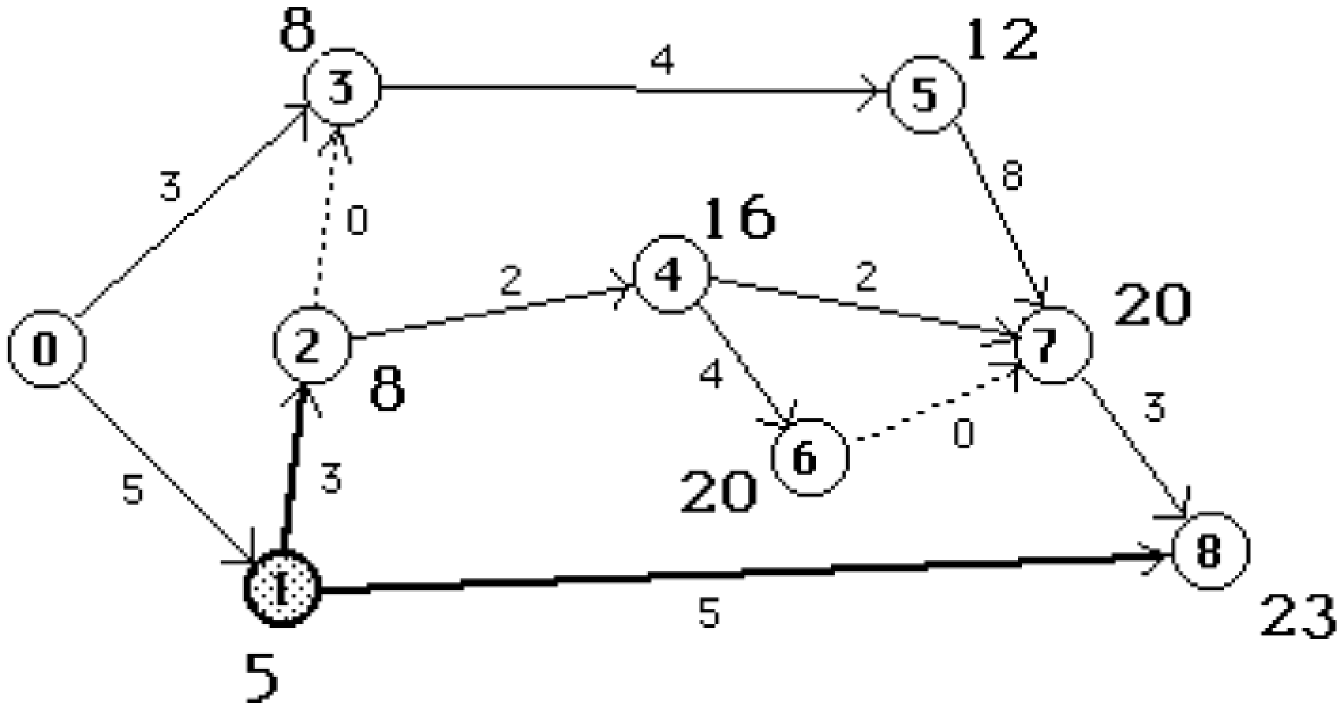
LT(3) = LT(5) - 4 = 8



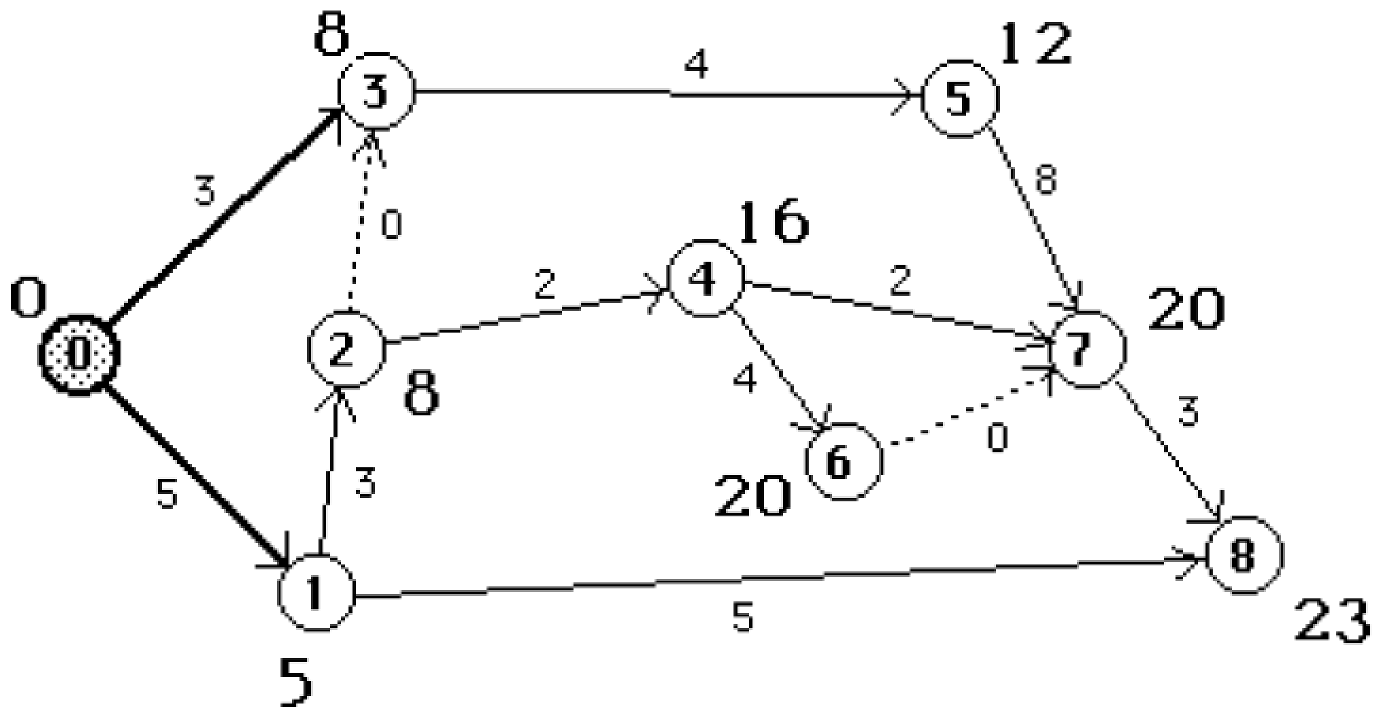
$$LT(2) = \min\{LT(3)-0,\ LT(4)-2\}$$
$$= \min\{8, 14\} = 8$$



$$LT(1) = \min\{LT(2)-3,\ LT(8)-5\}$$
$$= \min\{5, 18\} = 5$$



$$LT(0) = \min\{LT(1)-5,\ LT(3)-3\}$$
$$= \min\{0, 5\} = 0$$



If LT(0) ≠ 0, then an error was made!!!

For each activity define:

Earliest start time	$ES(i,j) = ET(i)$
Earliest finish time	$EF(i,j) = ET(i) + d_{ij}$
Latest finish time	$LF(i,j) = LT(j)$
Latest start time	$LS(i,j) = LT(j) - d_{ij}$

For each activity, define;

Total float $TF(i,j) = LS(i,j) - ES(i,j)$

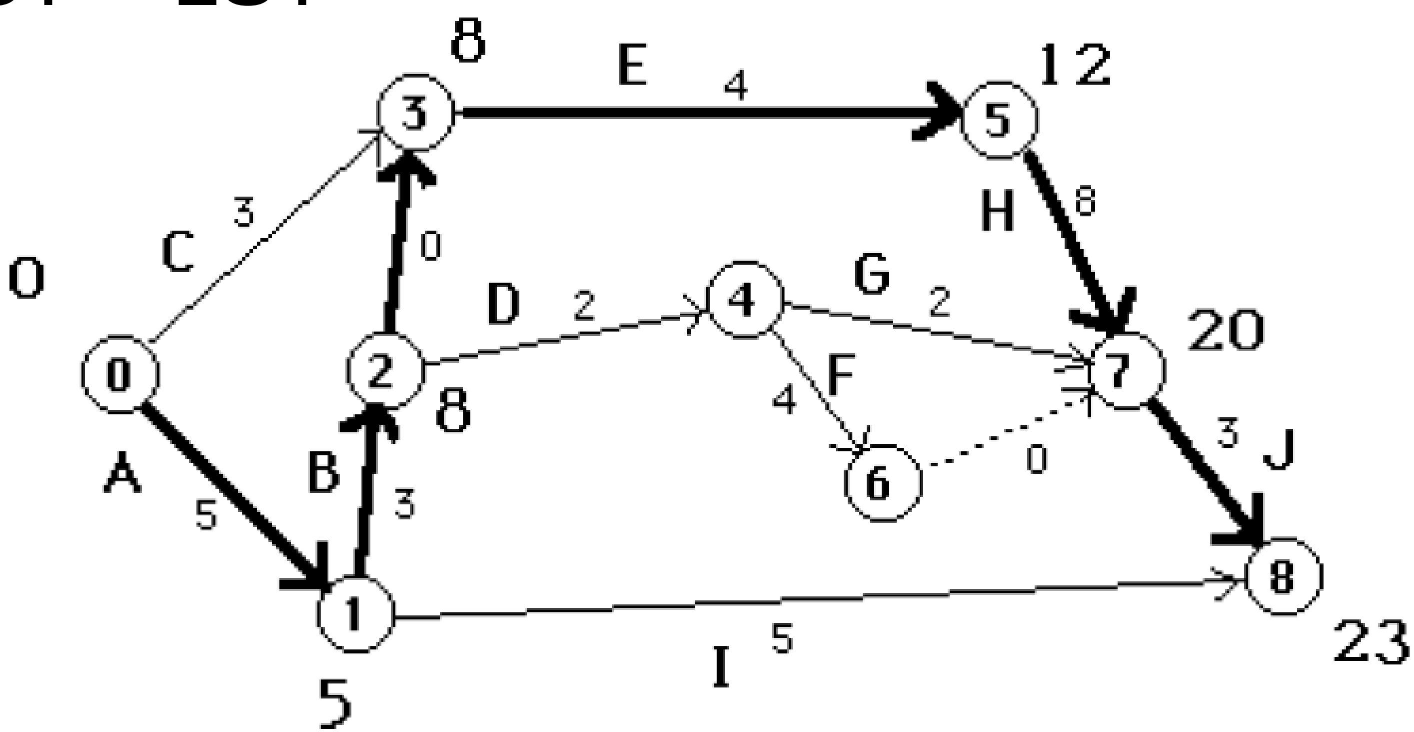
Free float $FF(i,j) = [ET(j) - d_{ij}] - ET(i)$

Critical path

	TASK	I	D	ES	EF	LS	LF	TS	FS
**	Start	1	0	0	0	0	0	0	0
**	A	2	5	0	5	0	5	0	0
**	B	3	3	5	8	5	8	0	0
	C	4	3	0	3	5	8	5	5
	D	5	2	8	10	14	16	6	0
**	E	6	4	8	12	8	12	0	0
	F	7	4	10	14	16	20	6	6
	G	8	2	10	12	18	20	8	8
**	H	9	8	12	20	12	20	0	0
	I	10	5	5	10	18	23	13	13
**	J	11	3	20	23	20	23	0	0
**	End	12	0	23	23	23	23	0	0

Critical Path

- If the total float of an activity is zero,
- EST = LST



Program Evaluation and Review Technique (PERT)

- Another project time management technique.
- A network analysis technique used to estimate project duration when there is a high degree of uncertainty about the individual activity duration estimate.
- PERT applies the critical path method to weighted average duration estimate.
- PERT uses probabilistic time estimate : duration estimate based on using optimistic, most likely, pessimistic estimates of activities duration.
- PERT based on network diagram.

PERT Formula and Example

- PERT weighted average**=**

$$\frac{\text{optimistic time} + 4X \text{ most likely time} + \text{pessimistic time}}{6}$$

- Example:
  - PERT weighted average =

$$\frac{8 \text{ workdays} + 4 \text{ X } 10 \text{ workdays} + 24 \text{ workdays}}{6} = \mathbf{12\ days}$$

where:
  - optimistic time= 8 days
  - most likely time = **10 days**
  - pessimistic time = 24 days
- Therefore, you'd use **12 days** on the network diagram instead of 10 when using PERT for the above example.

Schedule control

- Ensure that the project schedule is realistic.
- Use discipline and leadership to emphasize the importance of following and meeting project schedules.
- Review the draft schedule.
- Prepare a more detail schedule and get stakeholders' approval
- Don't plan for everyone to work at 100 percent capacity all the time.

Schedule Control

- Goals are to know the status of the schedule, influence factors that cause schedule changes, determine that the schedule has changed, and manage changes when they occur.
- Tools and techniques include:
  - Progress reports.
  - A schedule change control system.
  - Project management software, including schedule comparison charts, such as the tracking Gantt chart.
  - Variance analysis, such as analyzing float or slack.
  - Performance management, such as earned value

Working with People Issues

- Strong leadership helps projects succeed more than good PERT charts do.
- Project managers should use:
  - Empowerment
  - Incentives
  - Discipline
  - Negotiation