An Optimal and Distributed Demand Response Strategy With Electric Vehicles in the Smart Grid

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***Abstract*—In this paper, we propose a new model of demand response management for the future smart grid that integrates plug-in electric vehicles and renewable distributed generators. A price scheme considering fluctuation cost is developed. We consider a market where users have the flexibility to sell back the energy generated from their distributed generators or the energy stored in their plug-in electric vehicles. A distributed optimization algorithm based on the alternating direction method of multipliers is developed to solve the optimization problem, in which consumers need to report their aggregated loads only to the utility company, thus ensuring their privacy. Consumers can update their loads scheduling simultaneously and locally to speed up the optimization computing. Using numerical examples, we show that the demand curve is flattened after the optimization, even though there are uncertainties in the model, thus reducing the cost paid by the utility company. The distributed algorithms are also shown to reduce the users' daily bills.**

***Index Terms*—Alternating direction method of multipliers, demand response, distributed optimization, electric vehicle, fluctuation cost, smart grid.**

SOME NOTATIONS

t	Time index with $t = \{0, 1, \ldots, T\}$.
k	User index.
$l_k(t)$	The load of user k at time t.
$l_k^{\mathrm{I}}(t)$	The type I load of user k at time t.
s_k^{S}	The sum of schedulable load of user k.
$l_k^{\max}(t)$	The max rate for schedulable load of user k.
$r_k^{\min}$	The max discharging rate of PEV k.
$r_k^{\max}$	The max charging rate of PEV k.
t_k^{ar}	The arrival time of PEV k.
t_k^{dep}	The departure time of PEV k.
s_k^{P}	The energy left in PEV k upon arrival.
s_k^{Q}	The energy required for PEV k before departure.
E_k	The battery capacity of PEV k.
c_t	The marginal unit generation cost at time t.
μ	Fluctuation cost coefficient.
$p_{\mathrm{B}}(t)$	Base price at time t.
$p_{\mathrm{F}}(t)$	Fluctuation price at time t.
C_1	Coefficient for base price.
C_2	Coefficient for fluctuation price.
c_k	Total bill of kth user.
ρ	Quadratic coefficient for augmented Lagrangian.

Manuscript received January 07, 2013; revised April 22, 2013 and September 26, 2013; accepted November 12, 2013. This work was supported in part by the International Center for Advanced Renewable Energy and Sustainability (I-CARES) in Washington University in St. Louis, MO, USA. Date of publication January 29, 2014; date of current version February 14, 2014. Paper no. TSG-00012-2013.

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Digital Object Identifier 10.1109/TSG.2013.2291330

I. INTRODUCTION

IN THE electricity market, demand response [1] is a mechanism to manage users' consumption behavior under specific supply conditions. The goal of demand response is to benefit both consumers and utilities via a more intelligent resources scheduling method. While the classical rule for operating the power system is to supply all the demand whenever it occurs, the new philosophy focuses on the concept that the system will be more efficient when the fluctuations in demand are kept as small as possible [2]. With a fixed amount of electricity generation, demand fluctuation can add significant ancillary cost to the suppliers due to the inefficient usage of thermal plants in the power grid. Therefore the goal of demand response is to flatten the demand curve by shifting the peak hour load to off-peak hours. Traditionally it is achieved by setting a time of use (TOU) price scheme [3], [4], which normally assigns high prices to the peak hours and low prices to the off-peak hours. Thus consumers will try to move some of their schedulable power usage to the off-peak hours and to reduce their electricity bill. The overall effect of this behavior will reduce the fluctuation in the power consumption level.

The TOU price scheme works well when the schedulable power usage is less than a certain amount. With the incorporation of the plug-in electric vehicles (PEV) into the power grid [5]–[7], users have more flexibility to schedule their load and tend to charge them when the electrical price is low. Therefore the effect of the TOU price scheme is simply to move the peak demand from previous peak hours into previous off-peak hours. Even so, the cost arising from load variation still remains high in this situation. With the introduction of advanced metering infrastructures (AMI) [8] and energy-management controllers (EMC) [9], [10], more effective and distributed smart demand response algorithms can be proposed to solve the peak shifting and electric vehicle charging problems [11]–[14]. In [11], an optimal distributed charging algorithm was shown but it is limited in the case where all PEVs have the same features, i.e., all PEVs have the same starting time and also the same deadline. In [10], the authors proposed a game-theoretical approach for users. A distributed algorithm is proposed and guaranteed to find the Nash equilibrium of the game. But their distributed algorithm can be applied only sequentially among the users, and the communication time and cost will reduce the effectiveness of the method. Moreover, users have to report their own usage curves to all the other users, so the privacy of each user is not protected. In [14], the authors proposed a parallel distributed optimization algorithm for the PEV charging problem and presented a convergence proof. In the numerical example, we will show that the convergence behavior of this method is sensitive to the choice of parameter in the computing algorithm.

In this paper we consider a smart grid with a certain penetration level of PEVs and also with some on-site renewable distributed generators [15]–[17], such as solar panels and wind turbines. The price model in this paper consists of two parts. The first part considers the base price, and the second part takes the fluctuation cost into account. We consider the case where users can sell back the energy they generate to the grid. The PEVs can also be used as batteries to store electricity, which can be either consumed or sold back to the grid, whichever is more advantageous. We use the alternating direction method of multipliers (ADMM) to solve the optimization [18]. Unlike in [10], our algorithm is computed in parallel. Each user needs to report their usage curve only to the utility company, therefore privacy can be guaranteed. The convergence of the ADMM requires only convexity and the saddle point condition [18], therefore the convergence of the distributed algorithm in this paper can be easily obtained.

The rest of the paper is organized as follows. In Section II, we build a model for different kinds of loads and set the pricing policy for the utility company. We also formulate the model with random prediction error in base load and distributed generation. In Section III, using the alternating direction method of multipliers, we reformulate the optimization problem into a distributed optimization problem. In Section IV, we show numerical examples to demonstrate the performance of the proposed method. In Section V, we conclude the paper and point out directions for future research.

II. System Model

We consider a smart grid model with certain number of residences provided with electricity from the same utility company. Each consumer has an energy-management controller (EMC) that controls and communicates with different appliances within the household and also has an advanced metering infrastructure (AMI) to perform two-way communication with the utility company. We also assume that there are a certain number of user-owned distributed generators and plug-in electric vehicles in the grid. Users can sell back the energy generated from their own distributed generators or store this energy in the batteries of their PEVs. Users can also sell the energy left in their PEV batteries back to the grid whenever it is profitable. The price contains two parts: the first part is based on the nonschedulable load at each time, which we will refer to as the base price; the second part is based on the fluctuation of the load throughout a day.

A. Electricity Usage Model for Users

We divide a day into T time periods. We assume there are four types of loads, both positive and negative, in our model: the base load, schedulable load, plug-in electric vehicle load, and the distributed generation. The base load supplies users' basic needs, such as lighting, which can not be scheduled. The schedulable load can be scheduled, but needs to maintain a certain quantity during a day, such as refrigerators, air conditioning, laundry machines, and dishwashers. Please note that the minimum load requirement for the schedulable load can also be included in the base load. The plug-in electric vehicle load denotes the electricity usage of PEVs. It can be a negative number at time slots when users sell back the electricity stored in their PEVs. The distributed generation is considered as a negative load generated by solar or wind generators and users can sell this energy back to the grid when they have surplus. We use $l_k^{\mathrm{I}}(t)$ to represent the type I load for user k at time point t. Here, the superscript $\mathrm{I} = \mathrm{B}, \mathrm{S}, \mathrm{P}$, and D, to indicate the base load, schedulable load, PEV load, and distributed generation, respectively. Then the total load of user k at time point t can be expressed as the sum of these four types of loads:

$$l_k(t) = l_k^{\mathrm{B}}(t) + l_k^{\mathrm{S}}(t) + l_k^{\mathrm{P}}(t) + l_k^{\mathrm{D}}(t), \tag{1}$$

These equations are satisfied for all t and k.

$l_k^{\mathrm{B}}(t)$ denotes the base load of user k at time t. $l_k^{\mathrm{S}}(t)$ is the schedulable load of user k at time t. It can be scheduled during a day, but it must satisfy the following sum constraint s_k^{S} to meet the satisfaction of consumers. It should also meet the maximum physical usage rate constraints $l_k^{\max}(t)$. The constraints for schedule load are stated as follows:

$$\sum_{t=1}^{T} l_k^{\mathrm{S}}(t) = s_k^{\mathrm{S}}, \quad \text{and} \quad 0 \le l_k^{\mathrm{S}}(t) \le l_k^{\max}(t),\ \forall k. \tag{2}$$

$l_k^{\mathrm{P}}(t)$ denotes the electric vehicle load of user k. It can be decomposed into two parts, namely the charging energy and discharging energy. The equation can be written as

$$l_k^{\mathrm{P}}(t) = \frac{\tilde{l}_k^{\mathrm{P}+}(t)}{\mu_c} + \mu_d \tilde{l}_k^{\mathrm{P}-}(t), \tag{3}$$

where μ_c and μ_d denote the charging efficiency and discharging efficiency of the electric vehicle in this paper. $\tilde{l}_k^{\mathrm{P}}(t)$ represents the energy change in the battery of the electric vehicles. We have

$$\tilde{l}_k^{\mathrm{P}+}(t) = \begin{cases} \tilde{l}_k^{\mathrm{P}}(t) & \text{if } \tilde{l}_k^{\mathrm{P}}(t) \ge 0, \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

$$\tilde{l}_k^{\mathrm{P}-}(t) = \begin{cases} \tilde{l}_k^{\mathrm{P}}(t) & \text{if } \tilde{l}_k^{\mathrm{P}}(t) \le 0, \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

The term $\tilde{l}_k^{\mathrm{P}}(t)$ must also satisfy the sum constraint for the charging/discharging period. In addition, the user can use the electric vehicle as a battery to store energy when the electricity price is low and sell it back when the price is high. The charging and discharging rate also have upper bounds to meet physical constraints. The energy remaining in a battery at every time slot should also be larger than zero and less than the battery size. Let E_k denote the battery size of user k, and $r_k^{\max}$ and $r_k^{\min}$ denote the maximum charging rate and discharging rate. Let s_k^{P} denote the energy left in the battery when user k's PEV arrives home, and let s_k^{Q} denote the energy required for the next trip for the user k. Then these constraints can be stated as follows:

$$\sum_{t=t_k^{\mathrm{ar}}}^{t_k^{\mathrm{dep}}-1} \tilde{l}_k^{\mathrm{P}}(t) = s_k^{\mathrm{Q}} - s_k^{\mathrm{P}}, \quad \text{and} \quad r_k^{\min} \le \tilde{l}_k^{\mathrm{P}}(t) \le r_k^{\max}, \quad (6)$$

$$0 \le s_k^{\mathrm{P}} + \sum_{t=t_k^{\mathrm{ar}}}^{t_0} \tilde{l}_k^{\mathrm{P}}(t) \le E_k, \quad t_0 = t_{\mathrm{ar}}, t_{\mathrm{ar}}+1, \ldots, t_k^{\mathrm{dep}}-1, \quad (7)$$

where t_{ar} indicates the time of arrival and t_{dep} indicates the time of departure. If we are given a charging profile with multiple charging requests for the PEV of user k, it can be easily decomposed into several constraints which are similar to constraints (6) and (7) given above.

$l_k^{\mathrm{D}}(t)$ denotes the distributed generation of user k. It is a negative value since it is obtained from an external clean energy source and can be sold back to the grid when the user has a surplus.

B. Electricity Pricing Policy

In this section, we first describe the cost model for electricity generation and then propose the corresponding pricing scheme related to this cost model. Let c_t denote the marginal generation cost for one unit of electricity from thermal plants at time t. In an electrical power system, demand fluctuation can result in ancillary cost to the utility company since larger fluctuation will also lead to inefficient usage of the plants and the need for secondary thermal plants during the peak hours. We model this fluctuation cost as a function of the variance of the electricity load [12]. Therefore the total generation cost model for the utility company can be described as

$$\mathrm{Cost} = \sum_{t=1}^{T} c_t \sum_k l_k(t) + \mu \sum_{t=1}^{T} \left(\sum_k l_k(t) - m \right)^2, \quad (8)$$

where m is the mean usage during a day defined by $(1/T)\sum_{t=1}^{T}\sum_k l_k(t)$. μ is chosen to describe the fluctuation cost.

Electricity pricing is a complicated procedure and depends on the entire electricity market. The price discussed in this paper can be understood as a price indicator. For simplicity we will still call it "price" in the rest of the paper. The price function contains two parts, namely the base price and the price arising from the demand fluctuation. They are related to the two parts of the generation cost of electricity.

The base price $p_{\mathrm{B}}(t)$ is determined by the sum of the base loads of all individual users at time t. Since the sum of the base loads can be well predicted in a day-ahead market, the base price is well defined. We assume that the base price $p_B(t)$ is proportional to the sum of base loads at time t:

$$p_{\mathrm{B}}(t) = C_1 \left(\sum_k l_k^{\mathrm{B}}(t) \right)^{\alpha}, \quad (9)$$

where C_1 is chosen so that $c_t \le p_{\mathrm{B}}(t)$ for all time points t. This choice will guarantee that revenue can cover the regular electricity generation cost as long as there is more energy demand than the generation from the on-site renewable generators when fluctuation cost is not considered. The parameter α is within the range [0, 1], which can influence changes in the base electricity price between the different time slots and therefore influence the users' usage pattern. We will show this influence in the numerical examples.

If only base price is used, consumers have limited motivation to reschedule their demand response to lower the fluctuation in the demand curve. When the percentage of schedulable load and the penetration level of PEVs are high, another peak will be created in the time periods with low base price. In order to align the incentives of consumers to lower the fluctuation cost in (8), an extra price based on how much they contribute to this demand curve fluctuation needs to be introduced. Let f_0 denote the variance of the aggregated demand load:

$$f_0 = \sum_{t=1}^{T} \left(\sum_k l_k(t) - m \right)^2. \quad (10)$$

An extra price term $p_{\mathrm{F}}(t)$ related to this variance is introduced in our price model. This price term is added to the base price only in the time periods when the total load of all users is larger than the mean usage m. Let $\mathcal{T}_0$ denote the set containing these time points. Then the fluctuation price $p_{\mathrm{F}}(t)$ can be written as

$$p_{\mathrm{F}}(t) = \begin{cases} C_2 f_0 \dfrac{\sum_{k'} l_{k'}(t) - m}{\sum_{t' \in \mathcal{T}_0} \left(\sum_{k'} l_{k'}(t') - m \right)} \quad \dfrac{l_k(t)}{\sum_{k'} l_{k'}(t)} & \text{if } t \in \mathcal{T}_0, \\ 0, & \text{otherwise.} \end{cases} \quad (11)$$

Therefore the actual price charged to the consumers is $p(t) = p_{\mathrm{B}}(t) + p_{\mathrm{F}}(t)$. The electricity bill to each individual user during a day can be summarized as

$$c_k = \sum_{t=1}^{T} \left((p_{\mathrm{B}}(t) + p_{\mathrm{F}}(t) \right) l_k(t). \quad (12)$$

By summing c_k over all the consumers, we have the total revenue for the utility company as

$$\mathrm{Revenue} = \sum_{t=1}^{T} p_{\mathrm{B}}(t) \sum_k l_k(t) + C_2 \sum_{t=1}^{T} \left(\sum_k l_k(t) - m \right)^2,$$

in which C_2 is chosen to be larger than μ from (8) to cover the generation cost.

With the introduction of the fluctuation price, consumers will cooperate with each other to reduce the variance of the overall load curve, and therefore lower the generation cost to the utility company and their own bills. The savings result from a more efficient utilization of the generation infrastructures, which can be shared between the utility company and consumers. The saving for consumers is directly reflected in a reduction of their daily bills, which will be shown in the numerical example. The extension to the unbundling situation, i.e., when the generation companies and retailers are separated entities in the electricity market, is interesting, but more complicated. Discussion on the unbundling model is beyond the scope of this preliminary paper.

C. Prediction and Uncertainty of Base Load and Distributed Generation

In order to perform the optimization in the next section, we need to predict the base load, the distributed generation, and also parameters of schedulable load and PEV usage. The parameters related to PEVs and schedulable loads can be inputed by users directly through their own EMC device, and there is no need to report them to the utility company if the distributed algorithm is used. The predication of the base load has been investigated in the literature, using artificial neural networks [19] and pattern analysis [20]. The prediction can be quite precise. Therefore in our model, we assume the predicted base load for each individual user is a Gaussian random variable as follows:

$$l_k^{\mathrm{B}}(t) = l_k^{\mathrm{B0}}(t) + \epsilon_k^{\mathrm{B}}(t), \tag{13}$$

where $l_k^{\mathrm{B0}}(t)$ denotes the actual base load of user k, and ϵ_k^{B} is a random Gaussian noise with distribution $\mathcal{N}(0, \sigma_b^2)$, which shows the prediction error in the mathematical model. We assume that the noise term ϵ_k^{B} is uncorrelated among all the users.

The renewable energy can be predicated using a short-term prediction method [21]. By assuming the prediction error as a addictive Gaussian noise [22], we will have a linear model as,

$$l_k^{\mathrm{D}}(t) = l_k^{\mathrm{D0}}(t) + \epsilon_k^{\mathrm{D}}(t), \tag{14}$$

where $l_k^{\mathrm{D0}}(t)$ represents the actual distributed generation of user k, and ϵ_k^{D} is the prediction error, which can be regarded as random noise in the model. Unlike the users' base load, we assume that the noise term ϵ_k^{D} is highly correlated among all the users since they are all affected by the same weather conditions if they are located in the same geographic area. Then we can write the noise term as

$$\epsilon_k^{\mathrm{D}}(t) = e^{\mathrm{D}}(t) + e_k^{\mathrm{D}}(t). \tag{15}$$

The term $e^{\mathrm{D}}(t)$ follows a Gaussian distribution $\mathcal{N}(0, \sigma_0^2)$, and it shows that the prediction errors of distribution generators in different households are correlated. The term $e_k^{\mathrm{D}}(t)$ follows a Gaussian distribution $\mathcal{N}(0, \sigma_d^2)$. The impact of the uncertainties in the base load and also the distributed generation are analyzed in the numerical example section.

III. Distributed Optimization Algorithm

A. Centralized Optimization of Loads

The goal of users is to minimize their bills. The utility company also has the incentive to minimize this total bill. Since minimizing the total bills of all the users will lead to a more flattened load curve, this will lower the fluctuation cost for the utility company. In other words, the utility company should minimize the total electricity bill under the condition that its revenue can cover the its own cost, which can be guaranteed with a proper choice of C_1 and C_2. Then, with fixed C_1 and C_2, the goal of the utility company is to minimize the total bill to all the users. Therefore the centralized optimization problem can be formulated as

$$\min_{\{\boldsymbol{l}_k\}} \quad \sum_k \boldsymbol{p}_{\mathrm{B}}^{\mathrm{T}} \boldsymbol{l}_k + C_2 f_0 \left(\sum_k \boldsymbol{l}_k \right), \tag{16}$$

$$\text{subject to} \quad \boldsymbol{l}_k \in \mathcal{F}_k, \ \forall k. \tag{17}$$

Here $\boldsymbol{p}_{\mathrm{B}} = [p_{\mathrm{B}}(1), p_{\mathrm{B}}(2), \ldots, p_{\mathrm{B}}(T)]^{\mathrm{T}}$ and $\boldsymbol{l}_k = [l_k(1), l_k(2), \ldots, l_k(T)]^{\mathrm{T}}$. $\mathcal{F}_k$ denotes the feasible set for load $\boldsymbol{l}_k$, which means that $\mathcal{F}_k = \{\boldsymbol{l}_k : \boldsymbol{l}_k \text{ satisfies conditions } (1) - (7)\}$.

B. Distributed Optimization of Loads

Solving the optimization problem (16) in a centralized way is inefficient due to the huge dimensionality and thousands of constraints. The size of this optimization problem increases with the number of households. In addition consumers need to report their specific load usage to the utility company, which will lead to privacy issues. The objective function in problem (16) is a sharing problem and therefore can be decentralized into parallel programming using the alternating direction method of multipliers (ADMM). In order to formulate the problem to use ADMM, the optimization problem (16) is reformulated by introducing auxiliary variables $\{\boldsymbol{z}_k\}$ as follows:

$$\min_{\{\boldsymbol{l}_k\},\{\boldsymbol{z}_k\}} \quad \sum_k \boldsymbol{p}_{\mathrm{B}}^{\mathrm{T}} \boldsymbol{l}_k + C_2 f_0 \left(\sum_k \boldsymbol{z}_k \right) \tag{18}$$

$$\text{subject to} \quad \boldsymbol{l}_k = \boldsymbol{z}_k, \ \boldsymbol{l}_k \in \mathcal{F}_k, \ \forall k. \tag{19}$$

Due to the constraint above, it leads to the same optimal solutions as (16). By Introducing auxiliary l_2 norm penalty terms in the objective function, we will have an equivalent optimization problem as

$$\min_{\{\boldsymbol{l}_k\},\{\boldsymbol{z}_k\}} \quad \sum_k \boldsymbol{p}_{\mathrm{B}}^{\mathrm{T}} \boldsymbol{l}_k + \frac{\rho}{2} \|\boldsymbol{l}_k - \boldsymbol{z}_k\|_2^2 \tag{20}$$

$$+ C_2 f_0 \left(\sum_k \boldsymbol{z}_k \right) \tag{21}$$

$$\text{subject to} \quad \boldsymbol{l}_k = \boldsymbol{z}_k, \ \boldsymbol{l}_k \in \mathcal{F}_k, \ \forall k. \tag{22}$$

Introducing the Lagrange multipliers $\boldsymbol{v}_k \in \mathrm{R}^T$ for each $\boldsymbol{l}_k = \boldsymbol{z}_k$ constraint in the above optimization, we can obtain the augmented Lagrangian function as

$$L(\{\boldsymbol{l}_k\}, \{\boldsymbol{z}_k\}, \{\boldsymbol{v}_k\}) = \sum_k \left(\boldsymbol{p}_{\mathrm{B}}^{\mathrm{T}} \boldsymbol{l}_k + \boldsymbol{v}_k^{\mathrm{T}} (\boldsymbol{l}_k - \boldsymbol{z}_k) + \frac{\rho}{2} \|\boldsymbol{l}_k - \boldsymbol{z}_k\|_2^2 \right) + C_2 f_0 \left(\sum_k \boldsymbol{z}_k \right), \tag{23}$$

where ρ is a pre-defined constant. The original optimal problem can be solved using a Gauss-Seidel algorithm on the augmented Lagrangian function $L(\{\boldsymbol{l}_k\}, \{\boldsymbol{z}_k\}, \{\boldsymbol{v}_k\})$ [18]. Basically, the

ADMM cycles through the following steps until some kind of convergence is reached:

$$\boldsymbol{l}_k^{i+1} = \underset{\boldsymbol{l}_k \in \mathcal{F}_k}{\arg\min} \quad \boldsymbol{p}_{\mathrm{B}}^{\mathrm{T}} \boldsymbol{l}_k + \boldsymbol{v}_k^{i\mathrm{T}} \left(\boldsymbol{l}_k - \boldsymbol{z}_k^i\right) + \frac{\rho}{2} \left\|\boldsymbol{l}_k - \boldsymbol{z}_k^i\right\|_2^2, \tag{24}$$

$$\{\boldsymbol{z}_k^{i+1}\} = \underset{\boldsymbol{z}_k}{\arg\min} \sum_k \left(\boldsymbol{v}_k^{i\mathrm{T}} \left(\boldsymbol{l}_k^{i+1} - \boldsymbol{z}_k\right) + \frac{\rho}{2} \left\|\boldsymbol{l}_k^{i+1} - \boldsymbol{z}_k\right\|_2^2\right) + C_2 f_0 \left(\sum_k \boldsymbol{z}_k\right), \tag{25}$$

$$\boldsymbol{v}_k^{i+1} = \boldsymbol{v}_k^i + \rho \left(\boldsymbol{l}_k^{i+1} - \boldsymbol{z}_k^{i+1}\right), \ \forall k. \tag{26}$$

In (24), $\boldsymbol{l}_k$ is updated by solving a convex optimization problem while keeping $\boldsymbol{z}_k$ and $\boldsymbol{v}_k$ fixed to the values from the previous iteration. Likewise, we solve for $\boldsymbol{z}_k$. Equation (26) is a gradient descent of the augmented Lagrangian multiplier with step size ρ. The optimization problems (24) and (26) can be solved locally and also in parallel. Each user needs to report his/her total usage during each time slot only to the utility company, which then solves the optimization problem (25). As an intuitive interpretation of the ADMM procedure, we can regard $\boldsymbol{z}_k$ as the load which is suggested by the utility company to minimize the fluctuation in the demand response, and we regard $\boldsymbol{l}_k$ as the load according to the user's own benefit. The whole algorithm is a process of negotiation between each user and the utility company. $\boldsymbol{v}_k$ and $\rho/2$ are the penalty coefficients for the first and second order terms of disagreement. This optimization problem (24) and (25) can be reformulated as constrained convex optimization.

Since in real world application, the charging and discharging efficiencies are less than 1, we can set μ_c and μ_d to be 1 in the ADMM iterations to ensure convexity of the problem. After convergence of the ADMM algorithm, we post-process the load of the electric vehicles using (3) to get a suboptimal solution of the distributed optimization algorithm. In the remainder of this paper, we refer to this method simply as the ADMM scheduling method. This ADMM scheduling method can be extended to other forms of price scheme with more complicated generation cost functions as long as the problem is convex.

IV. Numerical Example

In the simulation we consider the case where there is only one electricity supplier and 120 households in the smart grid. There are 30 households with both wind distributed generators and plug-in electric vehicles, 20 households with only distributed generators, 30 households with only plug-in electric vehicles, and 40 households with none of these. The sum of the base load demand and schedulable demand from each household is generated randomly according to the MISO daily report by the U.S. Federal Regulatory Commission (FERC) [23]. The distributed wind generation values are taken from the Ontario Power Authority [24]. We set the battery size of the plug-in electric vehicle as either 10 kW or 20 kW, to reflect different kinds of vehicles. The maximum charging rate is assumed to be 3.3 kW/h, and the maximum discharging rate is 1.5 kW/h. We also assume the charging and discharging rates can change continuously between the maximum discharging rate and maximum charging rate. The statistical mean of arrival time and departure are 18:00 and 8:00, respectively. The specific time slot is generated according to Gaussian distributions.

In our simulation, we first show three different types of distributed scheduling algorithms. The first one uses no optimization which is widely used when the EMC and AMI devices are not applied. Users randomly select time slots to put their schedulable loads and charge their PEVs as soon as their vehicles are in the garages. The second algorithm is a greedy algorithm, in which everyone tries to lower their total electricity bill according to the pre-determined base price. This one simulates the scenario when there is no fluctuation fee in the cost (12) and users are not cooperating with each other when making decisions. We compare them with the ADMM scheduling method proposed earlier. Then we show the impact of α in pricing (10) for reducing the daily bill of each individual user. The effect of the proportion of schedulable power is also discussed in this section. The motivation for consumers to participate in this algorithm is given by showing the reduction of the daily bill for each individual user compared with the greedy method. We also show how the prediction error in base load and distributed generation will affect the performance of ADMM scheduling method. At last, we compare our method with another distributed optimization algorithm proposed in [14]. We show that the convergence behavior of ADMM is less sensitive to the choice of parameter in the algorithm compared with the method in [14].

A. Valley Filling Properies of the ADMM Scheduling Method

In the first part of the results, we present the experiment when 20% of the load (except PEV and load supplied by distributed generators) of an individual user is schedulable. Thus the ratio of the base load and schedulable load is 4:1. The α in (10) is set to be 1. Both charging efficiency and discharging efficiency are set to be 0.8. ρ in the ADMM iteration is chosen to be 0.006. In the pricing model, we have $C_1 = 7.8 \times 10^{-5}$ and $\mathrm{C}_2 = 5 \times 10^{-4}$. We show the valley filling results of the ADMM scheduling algorithm in Fig. 2. We can see that the load without optimization creates a peak when most PEVs arrive home. The greedy algorithm simply moves the peak to another time period, which has the lowest base electricity price. The ADMM-based distributed optimization model proposed in this paper can fill the valley of the original base load and thus lead to a lower fluctuation of the load curve.

In Fig. 3, we show the specific load arrangement with our proposed method, the method without optimization and also the greedy algorithm of one user with distributed generator and PEV. We can see that the greedy user moves all his schedulable load into the periods with cheaper electricity prices. He/She also tends to charge his/her PEV at times when the electricity prices are low and discharge his/her PEV at times when electricity prices are high. Comparing part (b) with Fig. 2, we can see that actually this decision pattern will lead to a high fluctuation price for this user. Therefore when the proportion of the fluctuation price is high, it is not worth doing this greedy algorithm. In part (c) this figure, we can see that the user under ADMM strategy

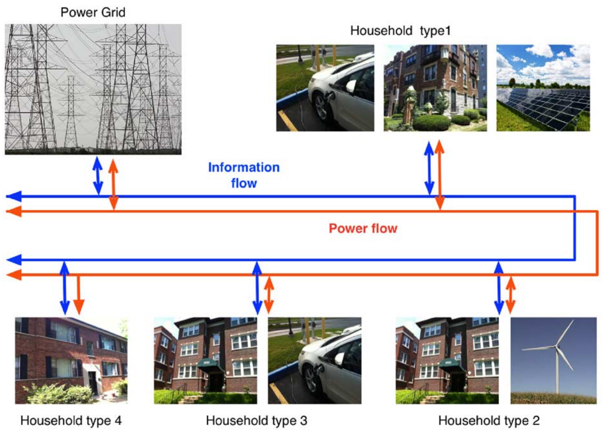

Fig. 1. The smart grid model considered in this paper.

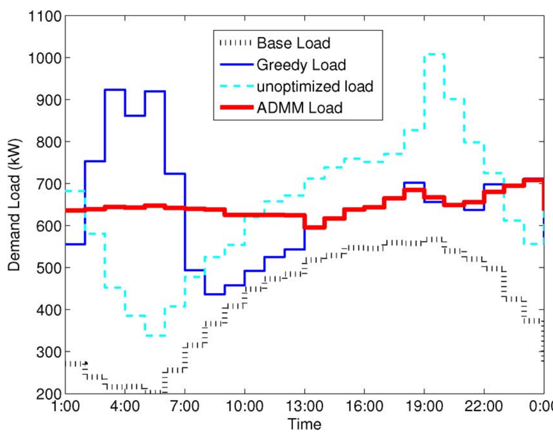

Fig. 2. Total load of four scheduling methods.

is more rational and cooperative. With this kind of cooperation among different users, the total fluctuation cost will be lowered and therefore reduce the bill every user pays for their utility usage.

The total daily bill of all users is also compared among these three methods. The unoptimized method has a total bill as high as \$689, and the greedy method has a bill amount as \$592. While the ADMM scheduling method leads to a total bill as low as \$521.

B. Impact of Pricing Parameter α

As we mentioned earlier in the pricing policy section, the parameter α in (10) has the ability to affect users' behavior. When α is set to be 1, the base price is proportional to the base load; therefore, the difference between different time slots is relatively large, and then user will become more sensitive to this base price. When α is set to be 1/2, the base price is proportional to the square root of the base price, and the user will become less sensitive to which time slot is the cheapest for their schedulable energy. When α is set to be zero, the user will not care about which time slots to allocate their energy and their goal becomes lowering the load variation as much as possible.

In Fig. 4, we range the value of α from 0.2 to 1, and plot the money saved for each individual user when consumers use the ADMM method compared with the greedy scheduling method. The portion of schedulable load is 20%, and ρ is chosen to be 0.006. We can see that even when α is high, which makes the

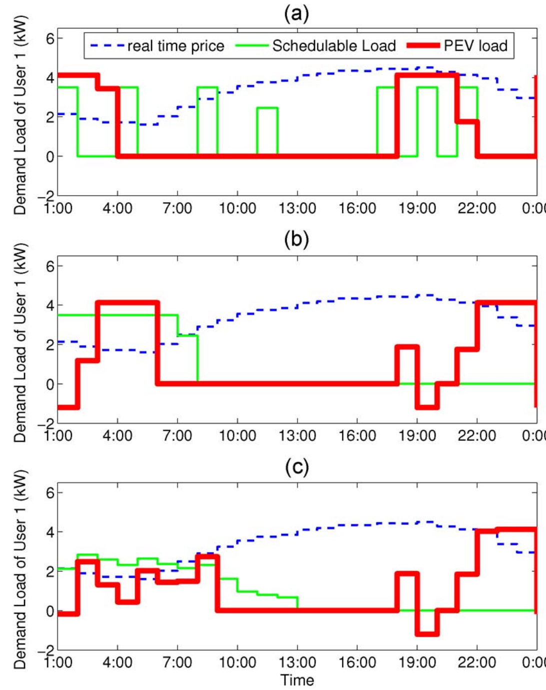

Fig. 3. Comparsion of user's load scheduling strategy under three different scheduling methods. (a) Unoptimized user; (b) greedy user; (c) ADMM user.

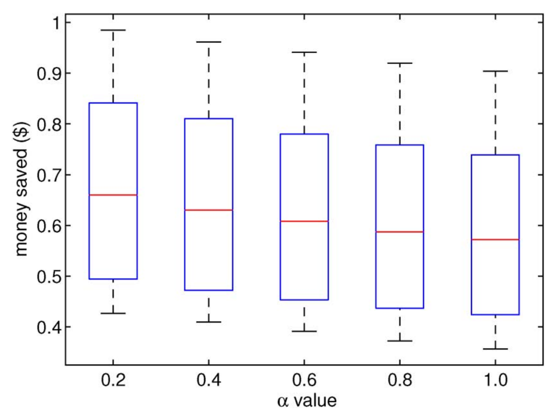

Fig. 4. Money saved with changing α $(\text{percentage of schedulable load} = 20\%)$.

greedy algorithm perform well, the consumer will still save money using the ADMM method. Overall every consumer will save their money every day under the ADMM scheduling method.

C. Results With Different Proportions of Schedulable Energy

In Fig. 5, we show the daily bill reduction achieved by using the proposed distributed ADMM optimization algorithm, compared with the greedy algorithm. The percentage of schedulable load of each individual user changes from 10% to 50%. α is set to be 1, and ρ equals 0.006 in the algorithm. Every user will gain under the proposed distributed algorithm, and thus everyone has the incentive to be cooperative. In fact, if some users have perfect information about others' usage, they can gain even more by deviating from the ADMM solution for themselves, but it is impossible to get such information in real applications. In Fig. 5 we notice that when percentage of schedulable load is 20% and 30%, the difference between ADMM scheduling method and

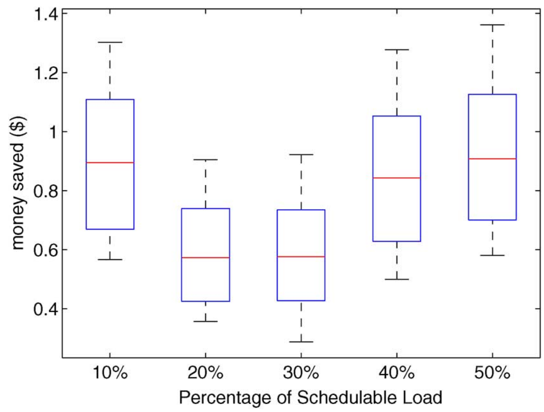

Fig. 5. Money saved with changing proportion of schedulable load. ($\alpha = 1$).

greedy method is less than other cases. This is due to the fact that with this percentage of schedulable energy, the valley in the base load can be filled without creating a new peak at the same time therefore greedy method is more efficient than greedy algorithm in other cases.

D. Results With Uncertainties in the Model

In order to set the base price for the next day, the utility company needs to predict the sum of the base load for all the households. The prediction will suffer from some random noise. We will see how much the prediction error will deteriorate the performance of ADMM. We will also show the impact of randomness in the distributed generation on the performance of the ADMM scheduling method. The models describing these randomness are shown in Section II-C. The percentage of schedulable energy is 20%. We let $\alpha = 1$ and $\rho = 0.006$ in this simulation. We range the standard deviation σ_b for the base load prediction error from 0.5 kW to 2.5 kW. The scale for the base load for a household at a certain time point ranges from 0 kW to 7 kW. We set σ_d to be 0.01 kW, and change σ_0 from 0.2 kW to 1 kW to see the impact of randomness in the distributed generation.

From Fig. 6, we can see that the ADMM scheduling method still outperform the greedy method and also the method without optimization. The total bill will increase with more uncertainties in the model. The ADMM method is more sensitive to the randomness in the distributed generation than that in the base load since the prediction error terms in the distributed generation are correlated among different households.

E. Comparsion With Another Distributed Load Scheduling Method

In this section, we compare our distributed scheduling method with the optimal decentralized charging method (ODC) proposed in [14]. The ODC algorithm basically cycles between the following two operations until some kind of convergence criteria is satisfied:

$$\boldsymbol{l}_k^{i+1} = \arg\min_{\boldsymbol{l}_k} \quad \boldsymbol{p}^{i\mathrm{T}}\boldsymbol{l}_k + \beta \left\| \boldsymbol{l}_k - \boldsymbol{l}_k^i \right\|_2^2, \quad \boldsymbol{l}_k \in \mathcal{F}_k, \ \forall k, \tag{27}$$

$$\boldsymbol{p}^{i+1} = \gamma \sum_k \boldsymbol{l}_k^{i+1}. \tag{28}$$

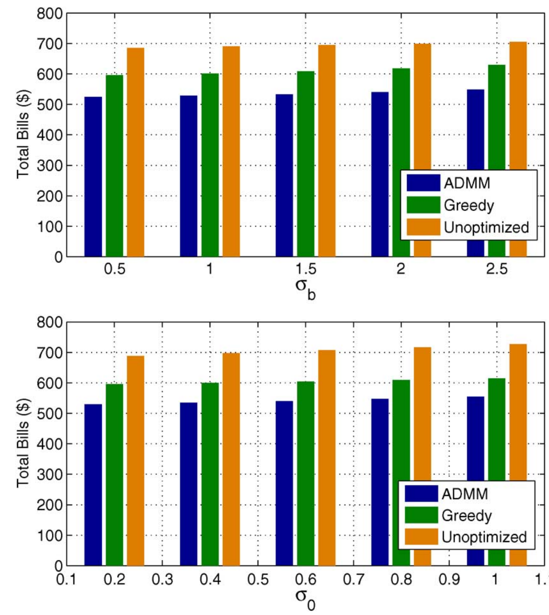

Fig. 6. Total daily bills for three scheduling methods with uncertainty in the model. The upper figure shows the impact of uncertainty in the base load, and the lower figure shows the impact of uncertainty in the distributed generation.

We can see from optimization problem (27) that the ODC algorithm is similar to the optimization problem (24) in the ADMM scheduling method. Rather than being given a reference usage pattern $\boldsymbol{z}_k$ by the utility company, the users in ODC are required to choose their usage pattern only within the neighborhood of the usage pattern in previous iteration. The pricing parameter γ is chosen to be 10^{-4}. In the pricing model of ADMM, we let $C_1 = 7.8 \times 10^{-5}$ and $C_2 = 5 \times 10^{-4}$. We test the performance of ADMM and ODC algorithms when β and ρ are set to be 0.0006, 0.006, and 0.06 respectively. In this simulation, the percentage of schedulable load is 20%, and α is set to 1 in the pricing model.

From Fig. 7, we can see that when both ρ and β are set to be 0.006, the ADMM and ODC will have almost the same optimal convergence behavior after three iterations. When we increase the value of the parameters to 0.06, the ODC converges much slower than the ADMM scheduling method. When we decrease the value of these two parameter to 0.0006, which is not shown in the figure, the ODC algorithm will diverge, while the ADMM gives a total bill of \$537 after ten iterations. From this numerical example, we can see that the convergence of the ADMM scheduling method is less sensitive than ODC method to the parameter used.

V. Conclusion

In this paper, we first built an electricity usage model for four types of loads, namely the base load, schedulable load, PEV load, and distributed generation. A price scheme considering both base price and demand fluctuation in the demand response was proposed. By applying the alternating direction method of multipliers, we decomposed the centralized optimization problem into distributed and parallel optimization problems. Theoretical convergence proof was given for the ADMM case. By showing numerical examples, we demonstrated that by

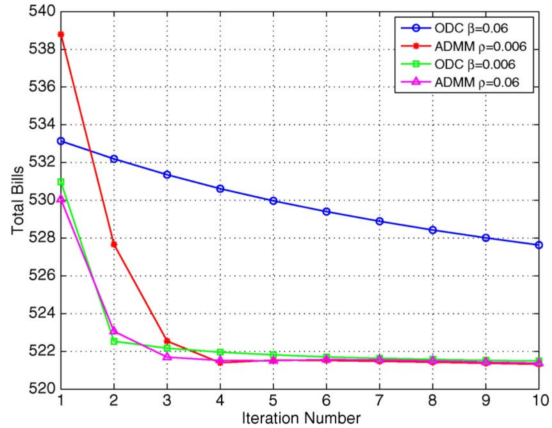

Fig. 7. Total bill with respect to number of iterations for ADMM and ODC.

using the ADMM based distributed scheduling method the demand response was flattened and the electrical bill was reduced for each individual user.

In our future work, we will employ a more detailed cost function for the utility company. We will also develop more advanced machine learning models to predict the users' future electricity usage behavior and will propose detailed strategies for choosing α in real-world applications. Extension of the price scheme proposed in this paper to an unbundling market situation will also be included in our future work.

Appendix A

The conditions for the convergence of the ADMM scheduling method in this paper are given by the following lemma and theorem:

Lemma 1: Suppose set $\mathcal{D}$ is a closed nonempty convex set and $f(\mathcal{D})$ is a indicator function, then the epigraph of indicator function $f(\mathcal{D})$ is also a closed nonempty convex set.

Proof: Since set $\mathcal{D}$ is nonempty, there exists an element $\boldsymbol{x} \in \mathcal{D}$, so that we have $f(\boldsymbol{x}) = 0 \leq 1$. Therefore $(\boldsymbol{x}, 1) \in \boldsymbol{epi} f$. The epigraph is nonempty. Convexity can be easily obtained by using the convexity of set $\mathcal{D}$. Suppose we have a sequence $(\boldsymbol{x}_k, t_k) \in \boldsymbol{epi} f$ satisfying $f(\boldsymbol{x}_k) \leq t_k$ that converges to point $(\boldsymbol{x}^*, t^*)$. $\boldsymbol{x}_k \in \mathcal{D}$ for all k. Since the sequence $\{\boldsymbol{x}_k\}$ also converges to $\boldsymbol{x}^*$, and $\mathcal{D}$ is closed, then $\boldsymbol{x}^* \in \mathcal{D}$. Then $f(\boldsymbol{x}^*) = 0 \leq \lim_{k\to\infty} t_k = t^*$. Therefore point $(\boldsymbol{x}^*, t^*)$ is also in the $\boldsymbol{epi} f$, and the epigraph is closed.

Theorem 1: If the charging and discharging efficiency μ_c and μ_d are both equal to one and the charging and scheduling constraints are feasible, which means there is at least one $\{\boldsymbol{l}_k\}$ satisfies conditions (1)–(7); Then by iterating between (24), (25) and (26), we have the residual convergence behavior. Also, the objective function converges to the optimal value p^*, and the Lagrangian multipliers $\boldsymbol{v}_k$ converge to the optimal solution $\boldsymbol{v}_k^*$:

$$\lim_{i\to\infty} \left(\boldsymbol{l}_k^i - \boldsymbol{z}_k^i\right) = 0,\ \forall k. \tag{29}$$

$$\lim_{i\to\infty} \left(\sum_k g_k(\boldsymbol{l}_k) + f_0\left(\sum_k \boldsymbol{z}_k^i\right)\right) = p^*, \tag{30}$$

$$\lim_{i\to\infty} \boldsymbol{v}_k^i = \boldsymbol{v}_k^*,\ \forall k, \tag{31}$$

where $g_k(\boldsymbol{l}_k) = \boldsymbol{p}_b^{\mathrm{T}} \boldsymbol{l}_k + f(\mathcal{F}_k)$.

Proof: Function $f_0(\sum_I \sum_k \boldsymbol{z}_{I,k}^i))$ is a quadratic function in this paper, therefore it is easy to show the epigraph of this function is a nonempty closed convex set. When μ_c and μ_d are equal to 1, the constraints in the optimization problem are all convex. Combining the solution feasibility and Lemma 1, we can easily show that the epigraph of $\sum_I \sum_k g_{I,k}(\boldsymbol{l}_{I,k})$ is also a nonempty closed convex set. Since strong duality holds for the original optimization problem (18), the optimal solution is the saddle point for the unaugmented saddle points, satisfying $L(\{\boldsymbol{l}_{I,k}^*\}, \{\boldsymbol{z}_{I,k}^*\}, \{\boldsymbol{v}_{I,k}\}) \leq L(\{\boldsymbol{l}_{I,k}^*\}, \{\boldsymbol{z}_{I,k}^*\}, \{\boldsymbol{v}_{I,k}^*\}) \leq L(\{\boldsymbol{l}_{I,k}\}, \{\boldsymbol{z}_{I,k}\}, \{\boldsymbol{v}_{I,k}^*\})$ for $\rho = 0$, where $\{\boldsymbol{l}_{I,k}^*\}, \{\boldsymbol{z}_{I,k}^*\}$ are the optimal solution for the primal problem and $\{\boldsymbol{v}_{I,k}^*\})$ are the optimal dual variables. Combining this property with the properties of the epigraphs we get the convergence results [18].

References

[1] C. Gellings, "The concept of demand-side management for electric utilities," *Proc. IEEE*, vol. 73, no. 10, pp. 1468–1470, Oct. 1985.

[2] M. H. Albadi and E. El-Saadany, "Demand response in electricity markets: An overview," in *Proc. IEEE Power Eng. Soc. Gen. Meet. 2007*, pp. 1–5.

[3] S. P. R. Hartway and C. K. Woo, "Smart meter, customer choice and profitable time-of-use rate option," *Energy*, vol. 24, pp. 895–903, 1999.

[4] Q. Wu, L. Wang, and H. Cheng, "Research of TOU power price based on multi-objective optimization of DSM and costs of power consumers," in *Proc. 2004 IEEE Int. Conf. Elect. Util. Deregulation, Restructuring, Power Technol. (DRPT 2004)*, Apr. 2004.

[5] A. R. L. Kelly and P. Wild, "Analyzing the impacts of plug-in electric vehicles on distribution networks in British Columbia," in *Proc. IEEE Electr. Power Energy Conf. (EPEC)*, Oct. 2009, pp. 1–6.

[6] C. Roe, J. Meisel, A. Meliopoulos, F. Evangelos, and T. Overbye, "Power system level impacts of PHEVs," in *Proc. 42nd Hawaii Int. Conf. Syst. Sci. (HICSS '09)*, Jan. 2009, pp. 1–10.

[7] J. Lopes, F. Soares, and P. Almeida, "Integration of electric vehicles in the electric power system," *Proc. IEEE*, vol. 99, no. 1, pp. 168–183, Jan. 2011.

[8] O. T. S. Karnouskos and P. Karnouskos, "An advanced metering infrastructure for future energy networks," in *Proc. NTMS Conf.*, Paris, France, May 2007.

[9] C. Chen, S. Kishore, and L. Snyder, "An innovative RTP-based residential power scheduling scheme for smart grids," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Process. (ICASSP)*, May 2011, pp. 5956–5959.

[10] A.-H. Mohsenian-Rad, V. Wong, J. Jatskevich, R. Schober, and A. Leon-Garcia, "Autonomous demand-side management based on game-theoretic energy consumption scheduling for the future smart grid," *IEEE Trans. Smart Grid*, vol. 1, no. 3, pp. 320–331, 2010.

[11] Z. Ma, D. Callaway, and I. Hiskens, "Decentralized charging control for large populations of plug-in electric vehicles: Application of the nash certainty equivalence principle," in *Proc. IEEE Int. Conf. Control Appl. (CCA)*, 2010, pp. 191–195.

[12] P. Yang, G. Tang, and A. Nehorai, "A game-theoretic approach for optimal time-of-use electricity pricing," *IEEE Trans. Power Syst.*, vol. 28, no. 2, pp. 884–892, May 2013.

[13] M. Caramanis and J. Foster, "Coupling of day ahead and real-time power markets for energy and reserves incorporating local distribution network costs and congestion," in *Proc. 2010 48th Annu. Allerton Conf. Commun., Control, Comput. (Allerton)*, pp. 42–49.

[14] L. Gan, U. Topcu, and S. Low, "Optimal decentralized protocol for electric vehicle charging," *IEEE Trans. Power Syst.*, vol. 28, no. 2, pp. 940–951, 2013.

[15] J. Brey, A. Castro, E. Moreno, and C. Garcia, "Integration of renewable energy sources as an optimised solution for distributed generation," in *Proc. 28th Annu. Conf. IEEE Ind. Electron. Soc. (IECON)*, Nov. 2002, vol. 4, pp. 3355–3359.

[16] J. Carrasco, L. Franquelo, J. Bialasiewicz, E. Galvan, R. Guisado, M. Prats, J. Leon, and N. Moreno-Alfonso, "Power-electronic systems for the grid integration of renewable energy sources: A survey," *IEEE Trans. Ind. Electron.*, vol. 53, no. 4, pp. 1002–1016, Jun. 2006.

[17] J. P. Lopes, N. Hatziargyriou, J. Mutale, P. Djapic, and N. Jenkins, "Integrating distributed generation into electric power systems: A review of drivers, challenges and opportunities," *Elect. Power Syst. Res.*, vol. 77, no. 9, pp. 1189–1203, 2007.
[18] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, "Distributed optimization and statistical learning via alternating direction method of multipliers," *Found. Trends Mach. Learn.*, vol. 3, pp. 1–122, 2010.
[19] J. Taylor and R. Buizza, "Neural network load forecasting with weather ensemble predictions," *IEEE Trans. Power Syst.*, vol. 17, no. 3, pp. 626–632, 2002.
[20] Y. Xia, Y. Yang, F. Ge, J. Su, and H. Yu, "Pattern analysis for load forecasting," in *Proc. 2012 8th Int. Conf. Comput. Technol. Inf. Manage. (ICCM)*, vol. 1, pp. 339–343.
[21] S. Soman, H. Zareipour, O. Malik, and P. Mandal, "A review of wind power and wind speed forecasting methods with different time horizons," in *Proc. North Amer. Power Symp. (NAPS) 2010*, pp. 1–8.
[22] M. He, S. Murugesan, and J. Zhang, "Multiple timescale dispatch and scheduling for stochastic reliability in smart grids with wind generation integration," in *Proc. IEEE INFOCOM 2011*, pp. 461–465.
[23] FERC, "MISO daily report: Electric power markets: Midwest (MISO) 2011 [Online]. Available: http://www.ferc.gov/market-oversight/mkt-electric/midwest/miso-archives.asp
[24] OPA, "Simulated wind generation data," Ontario Power Authority, 2007 [Online]. Available: http://www.powerauthority.on.ca/integrated-power-system-plan/simulated-wind-generation-data

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