Computer Vision Assignment 3

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1.

Assignment-3

1. $E = [t]_x R$ — Rotation Matrix.
Essential Matrix ; $[t]_x$ — Translation Matrix

$$[t]_x = \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix}$$

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

$$E = \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

Null Space of E : $Ex = 0$.
$\Rightarrow [t]_x R x = 0$

The null space of E corresponds to the direction of baseline, which is the line connecting two camera centres. Since $[t]_x$ has full rank (rank 3), its null space is trivial.
$\therefore$ Null Space of E will be same as null space of R.

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Left Null Space of E :

It corresponds to the epipole in the Image plane. Its direction is ⊥ to the baseline in the Image Plane. The left null space of E will be orthogonal to T.

Left null space of E = null space of E^T

$$E^T = R^T([t]_x)^T$$

$$= R^T \begin{bmatrix} 0 & t_z & -t_y \\ -t_z & 0 & t_x \\ t_y & -t_x & 0 \end{bmatrix}$$

Find y, such that $yE = 0$.

2. $R = I$
$t = [t_x, 0, 0]^T$.

$$E = [t]_x R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix}$$

Let p_1 & p_2 be image coordinates of P in 2 cameras

$$p_2^T E p_1 = 0.$$

$$p_2^T \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix} p_1 = 0.$$

Expanding : $t_x(p_{1y} p_{2z} - p_{1z} p_{2y}) = 0.$

Since $t_x \neq 0$.

$$P_{1y} \cdot P_{2z} = P_{1z} \cdot P_{2y}.$$

It would be wise to assume that they are on same z-axis because of the stereo camera setup.

$$\boxed{\therefore \quad P_{1y} = P_{2y}}$$

Hence they are on same y-coordinate.

3. Epipole Transformation: First we will transform the epipoles of the two cameras to points at infinity along x-axis.

$$T_1 = \begin{bmatrix} 1 & 0 & -e_{1x} \\ 0 & 1 & -e_{1y} \\ 0 & 0 & 1 \end{bmatrix}; \quad T_2 = \begin{bmatrix} 1 & 0 & -e_{2x} \\ 0 & 1 & -e_{2y} \\ 0 & 0 & 1 \end{bmatrix}$$

Secondly, we should make the epipolar line || & horizontal. Let R_{rect} be the horizontal matrix to achieve this.

$$R_{rect} = \begin{bmatrix} r_1^T \\ r_2^T \\ r_3^T \end{bmatrix}$$

Given: epipole e.
(using SVD in E)
(translation from E)

$$r_1 = e_1 = \frac{T}{\|T\|}$$

$$r_2 = \frac{1}{\sqrt{T_x^2 + T_y^2}} \begin{bmatrix} -T_y & T_x & 0 \end{bmatrix}$$

$$r_3 = r_1 \times r_2.$$

If $r_1 = e_1 = \frac{T}{\|T\|}$ & r_2 r_3 orthogonal

then $R_{rect} e_1 = \begin{bmatrix} r_1^T e_1 \\ r_2^T e_1 \\ r_3^T e_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

The point is located at x - infinity on image plane.

2.

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des1 :
[[ 9.  3.  4. ...  0.  0.  2.]
 [27.  9.  3. ...  0.  0.  0.]
 [13.  0.  0. ...  5.  2.  2.]
 ...
 [11.  8.  1. ...  2.  0.  0.]
 [68.  8.  0. ...  0.  0.  0.]
 [ 0.  1.  0. ...  0.  3.  7.]]

des2 :
[[  0.   0.  44. ...   0.  16.   6.]
 [  3.   0.   0. ...  39.  14.  15.]
 [ 15.   8.   2. ...   0.   0.   0.]
 ...
 [138.  24.   6. ...   0.   0.   0.]
 [  0.   0.   0. ...  26.   2.  10.]
 [  0.   0.   0. ...   1.   0.   3.]]
Homography Matrix:
[[-5.08764786e+01  1.01329378e+00  1.84180938e+04]
 [-1.54419885e+01 -3.14571037e+01  9.60978388e+03]
 [-5.28756889e-02 -1.58679848e-03  1.00000000e+00]]
```

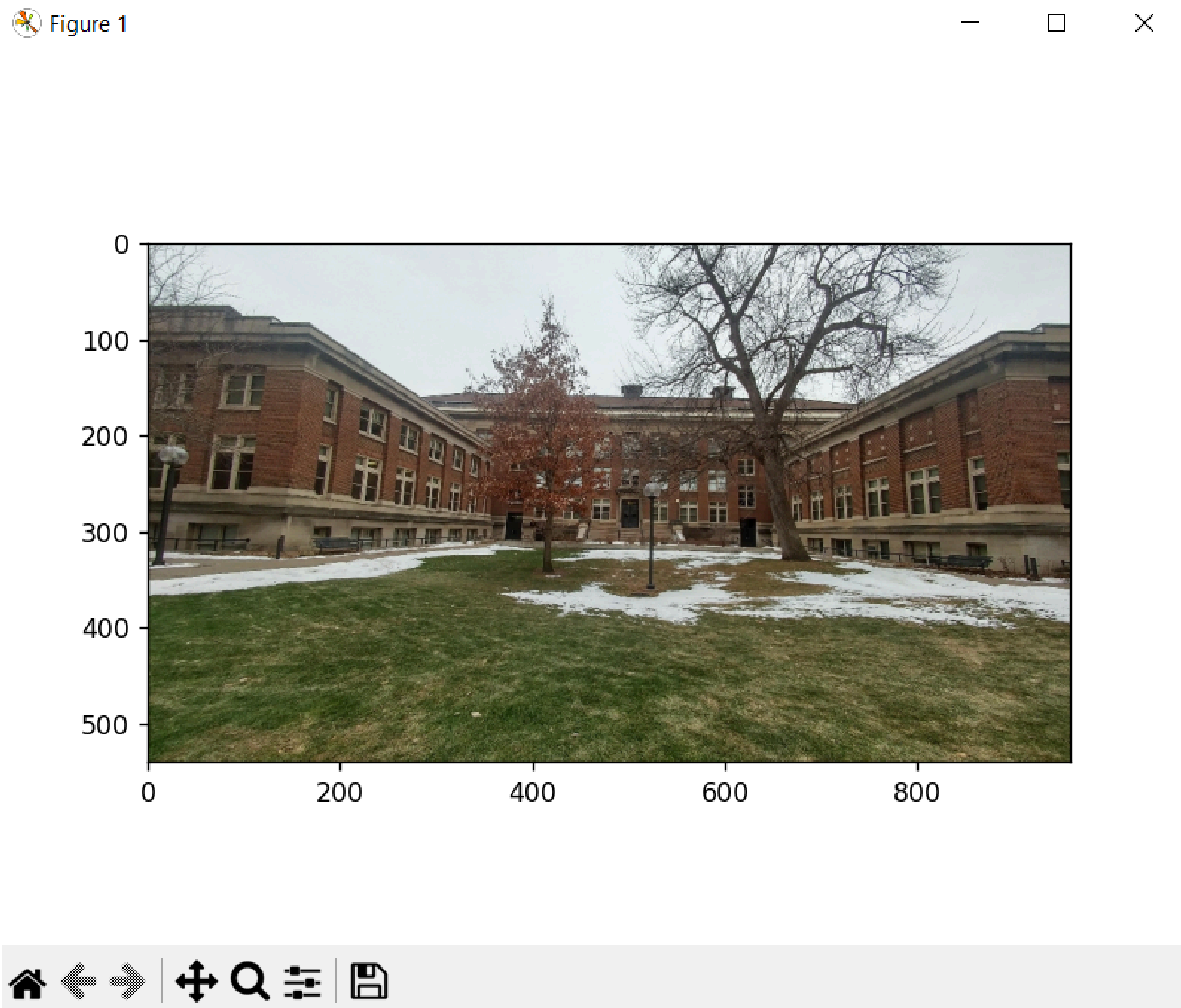
Figure 1
0
100
200
300
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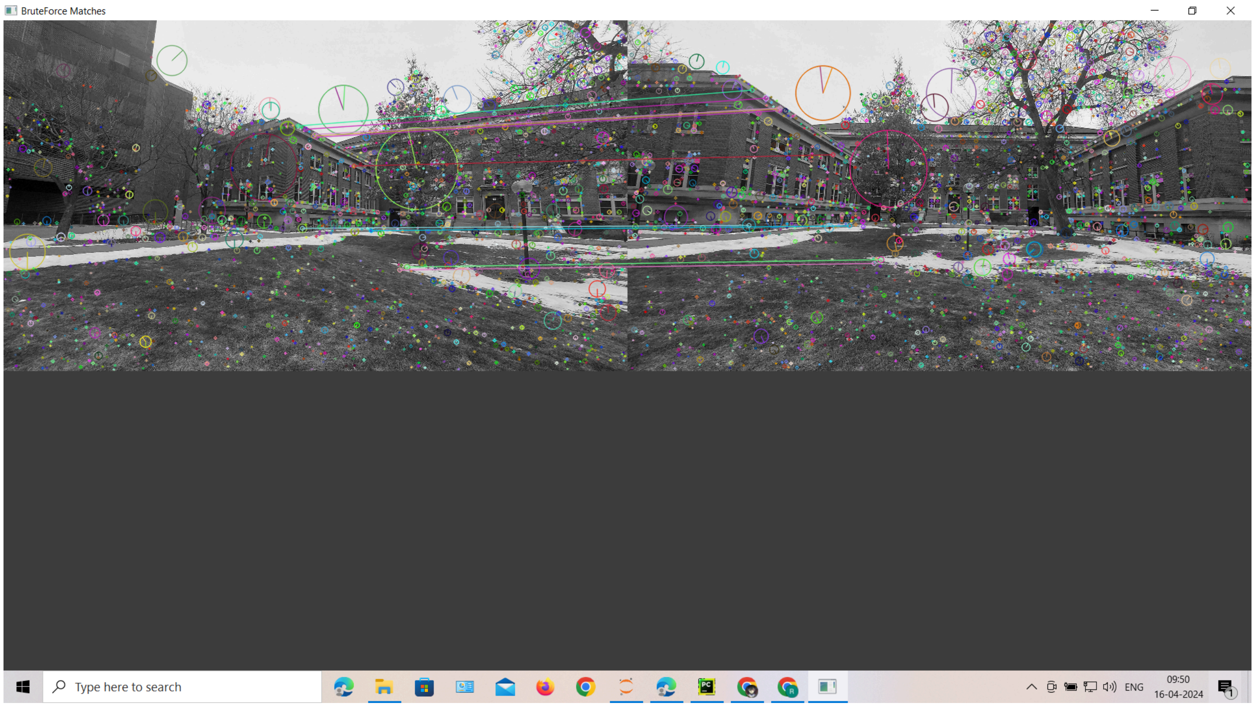
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ENG
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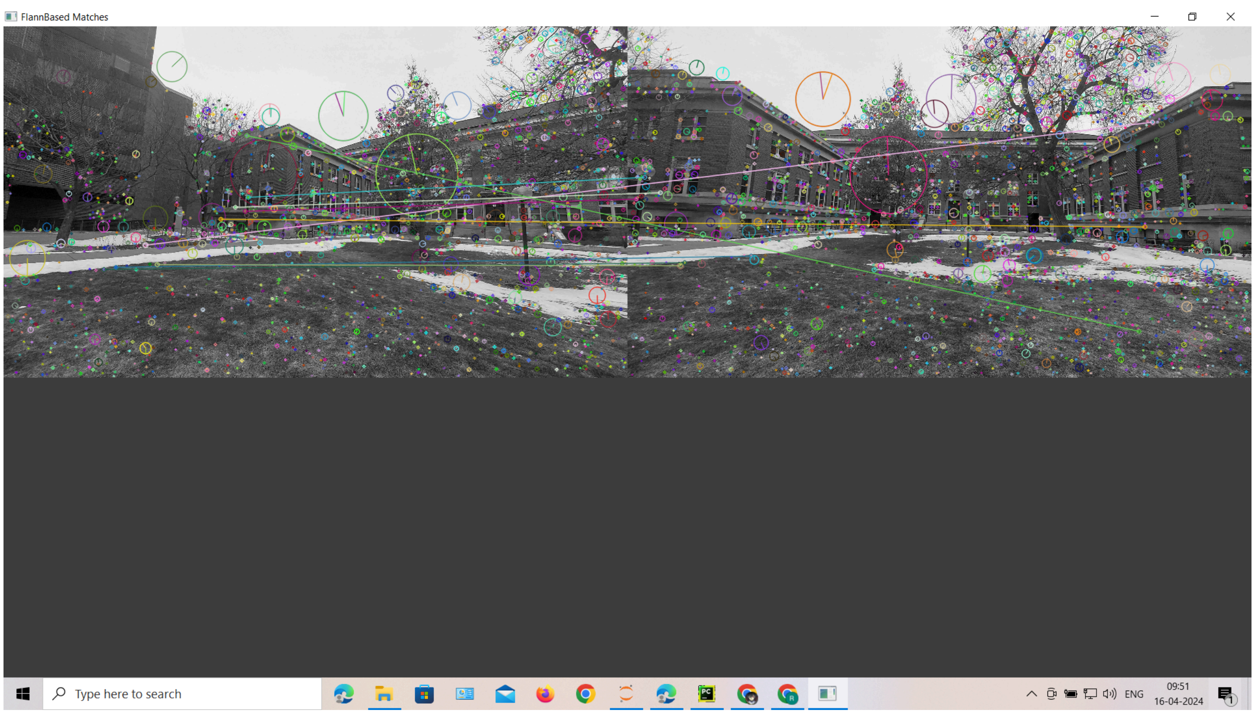
16-04-2024


FlannBased Matches
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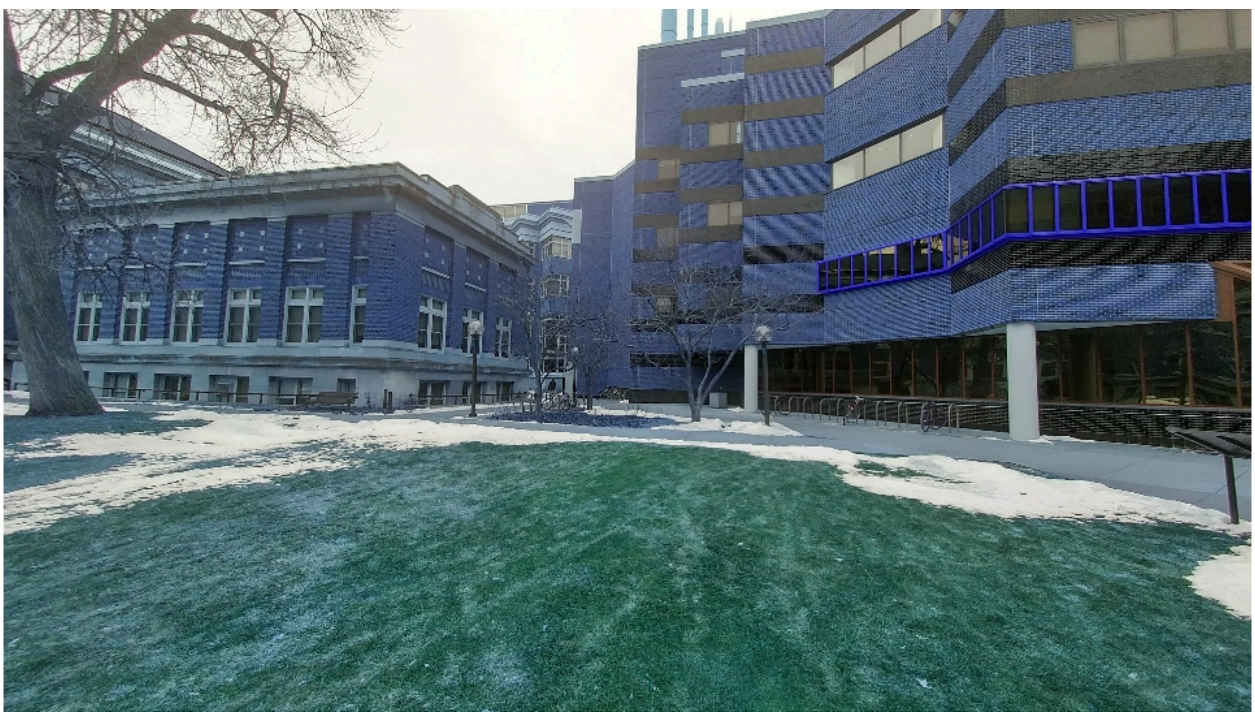

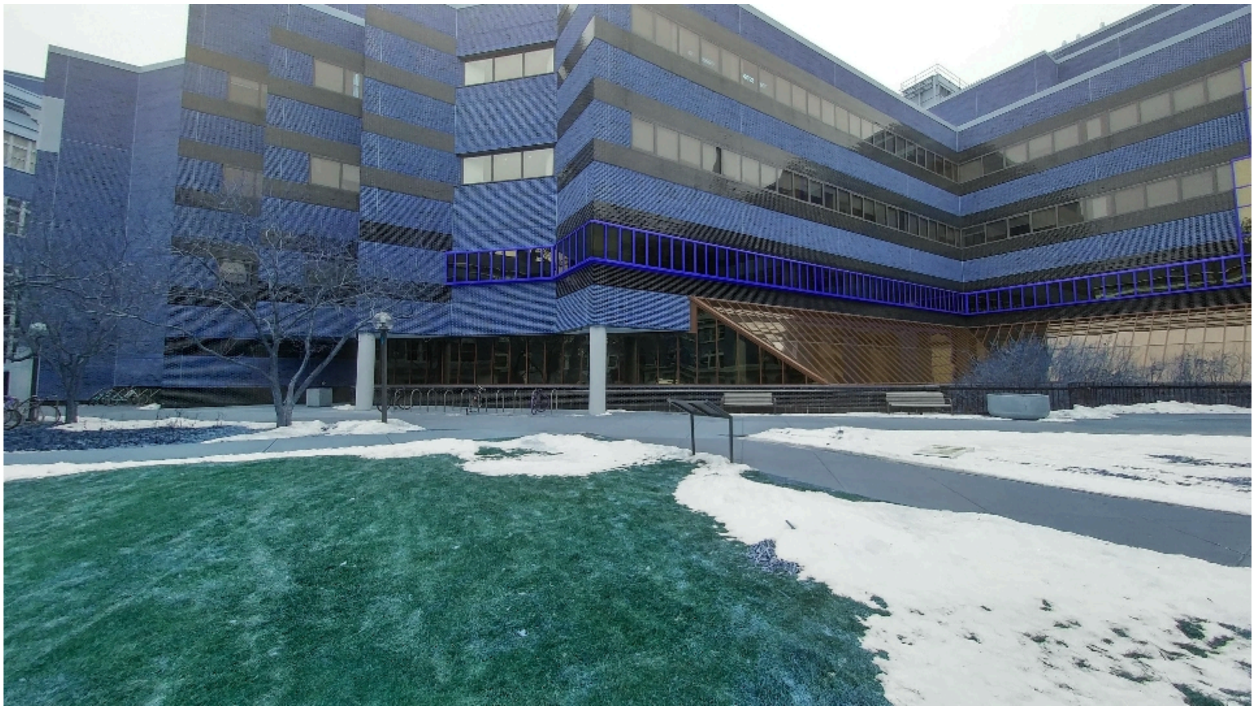

Final Panorama