

# RKHS APPROACH FOR ONLINE RBF UPDATE IN NEURO-ADAPTIVE CONTROL

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# INTRODUCTION

A Reproducing Kernel Hilbert Space Approach for the Online Update of Radial Bases in Neuro-Adaptive Control is a research area that focuses on developing advanced control systems for complex processes.

The relevance of this topic lies in its potential to improve the performance and reliability of complex control systems in various fields, such as robotics, aerospace, and manufacturing.

# PRELIMINARIES

- **REPRODUCING KERNEL HILBERT SPACE (RKHS):** A MATHEMATICAL SPACE WHERE THE INNER PRODUCT IS DEFINED IN TERMS OF A KERNEL FUNCTION. A GENERALIZATION OF THE EUCLIDEAN SPACE ALLOWS FOR REPRESENTING NONLINEAR FUNCTIONS AS LINEAR COMBINATIONS OF KERNEL FUNCTIONS.

- **RADIAL BASIS FUNCTION (RBF):** A MATHEMATICAL FUNCTION THAT MAPS THE DISTANCE BETWEEN DATA POINTS TO A VALUE. IT IS COMMONLY USED IN MACHINE LEARNING AND SIGNAL PROCESSING APPLICATIONS FOR INTERPOLATION, APPROXIMATION, AND CLASSIFICATION.

- **MODEL REFERENCE ADAPTIVE CONTROL (MRAC):** A CONTROL METHOD THAT USES A REFERENCE MODEL TO GUIDE THE ADAPTATION OF THE CONTROL SYSTEM.

- **ONLINE UPDATE: A TECHNIQUE USED TO ADJUST THE PARAMETERS OF A SYSTEM OR MODEL BASED ON INCOMING DATA IN REAL TIME. THE ONLINE UPDATE REFERS TO ADAPTING THE RADIAL BASIS FUNCTIONS OF A NEURAL NETWORK CONTROL SYSTEM.**

- **CONCURRENT LEARNING (CL): A ADAPTIVE CONTROL TECHNIQUE THAT SIMULTANEOUSLY INVOLVES LEARNING THE PLANT MODEL AND THE CONTROLLER PARAMETERS. CL COMBINES ONLINE AND OFFLINE LEARNING TO IMPROVE THE ADAPTATION AND CONTROL PERFORMANCE OF THE SYSTEM.**

- **NEURO-ADAPTIVE CONTROL: A CONTROL APPROACH THAT USES NEURAL NETWORKS TO ADAPT THE CONTROL LAW BASED ON THE SYSTEM'S STATE AND OUTPUT. NEURO-ADAPTIVE CONTROL IS A TYPE OF ADAPTIVE CONTROL THAT COMBINES NONLINEAR CONTROL AND ARTIFICIAL INTELLIGENCE.**

# LYAPUNOV ANALYSIS

LYAPUNOV ANALYSIS IS A MATHEMATICAL TOOL USED TO VERIFY THE STABILITY OF A DYNAMIC SYSTEM. IT INVOLVES ANALYZING THE LYAPUNOV FUNCTION TO DETERMINE WHETHER IT DECREASES OR INCREASES OVER TIME. IF THE LYAPUNOV FUNCTION DECREASES OVER TIME, THE SYSTEM IS STABLE.

LYAPUNOV ANALYSIS WAS USED TO VALIDATE THE STABILITY OF THE CONTROL SYSTEM DESIGNED USING THE REPRODUCING KERNEL HILBERT SPACE (RKHS) APPROACH FOR THE ONLINE UPDATE OF RADIAL BASES IN NEURO-ADAPTIVE CONTROL.

# STABILITY ANALYSIS

- **THE NN APPROXIMATION ERROR:**  $\Delta(\bar{x}) = W^{k*T} \sigma^k(\bar{x}) + \tilde{\epsilon}^k(\bar{x}).$

- **TRACKING ERROR:**  $\dot{e} = Ae + [W^T \sigma^k(\bar{x}) - \Delta(\bar{x})].$

- **WEIGHT UPDATE LAW:**  $\dot{W} = -\Gamma_W \sigma^k(\bar{x}) e^T P B - \Gamma_W \sum_{j=1}^p \sigma^k(\bar{x}_j) \epsilon_j^{kT}.$

- **LYAPUNOV FUNCTION:**

$$\begin{aligned} \dot{V}^k(e, \tilde{W}^k) &= -\frac{1}{2} e^T Q e + e^T P B \tilde{\epsilon}^k \\ &\quad - \text{Tr} \left( \tilde{W}^{kT} \left[ \sum_j \sigma_j^k \sigma_j^{kT} \tilde{W}^k - \sum_j \sigma_j^k \tilde{\epsilon}_j^k \right] \right) \\ &\leq -\frac{1}{2} \lambda_{\min}(Q) e^T e + \|e^T P B \tilde{\epsilon}^k\| \\ &\quad - \lambda_{\min}(\Omega^k) \tilde{W}^{kT} \tilde{W}^k - \|\tilde{W}^{kT} \sum_i \sigma_i^k \tilde{\epsilon}^k\| \\ &\leq \|e\| \left( C_1^k - \frac{1}{2} \lambda_{\min}(Q) \|e\| \right) \\ &\quad + \|\tilde{W}^k\| (C_2^k - \lambda_{\min}(\Omega^k) \|\tilde{W}^k\|) \end{aligned}$$

# VARIOUS FUNCTIONS

**ERROR** : Manually calculated the values of  $x$ ,  $\dot{x}$ ,  $u$ ,  $\sigma_{\text{bar}}$  from 6 time instances each of 10 sec.

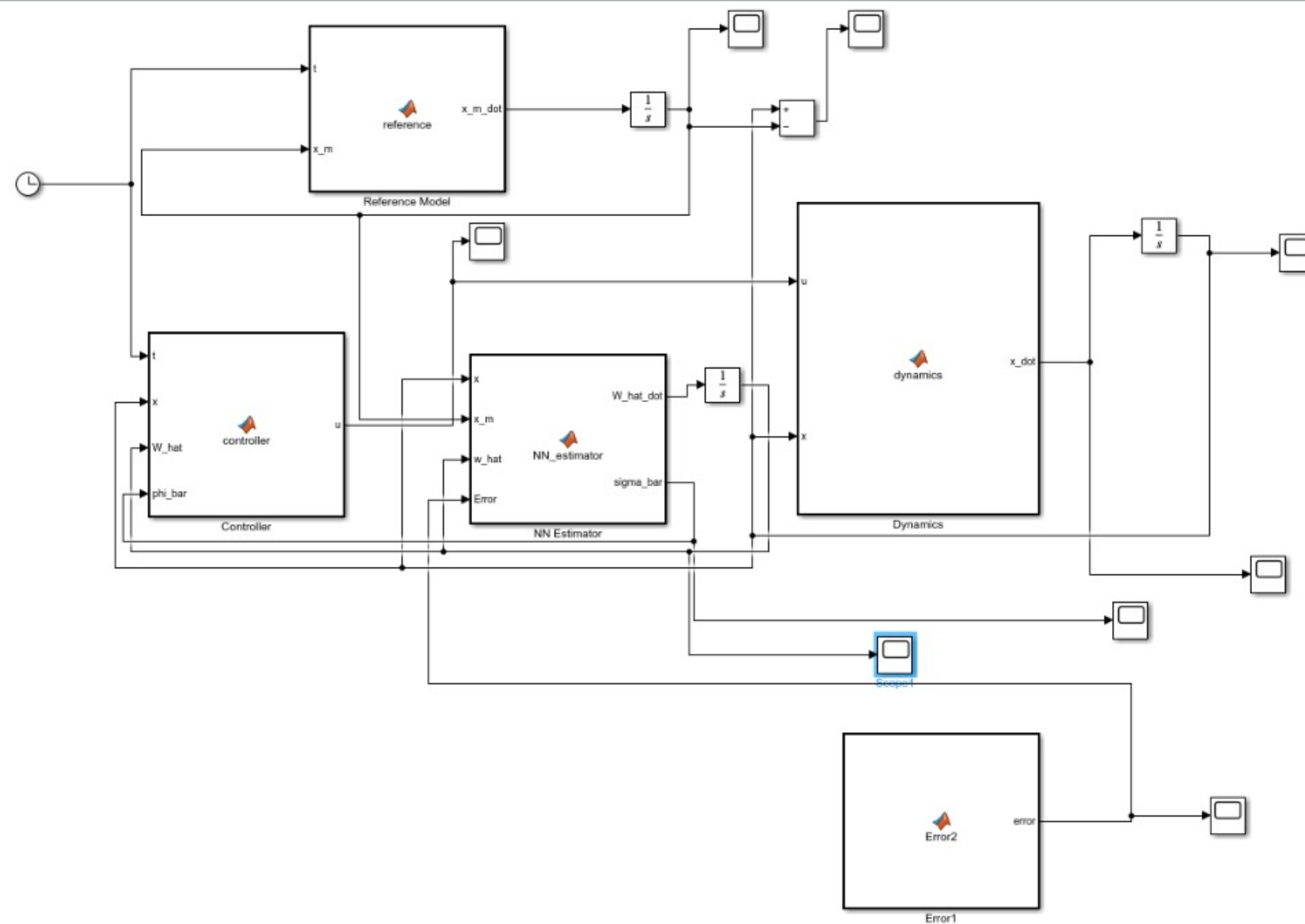
$$\epsilon_j^k(\bar{x}) = W^T \sigma^k(\bar{x}) - W^{k*^T} \sigma^k(\bar{x}) - \tilde{\epsilon}^k(\bar{x})$$

**NN\_ESTIMATOR** :  $L$  is taken to be 5. The eqn of  $\dot{W}_{\text{hat}}$  used is :

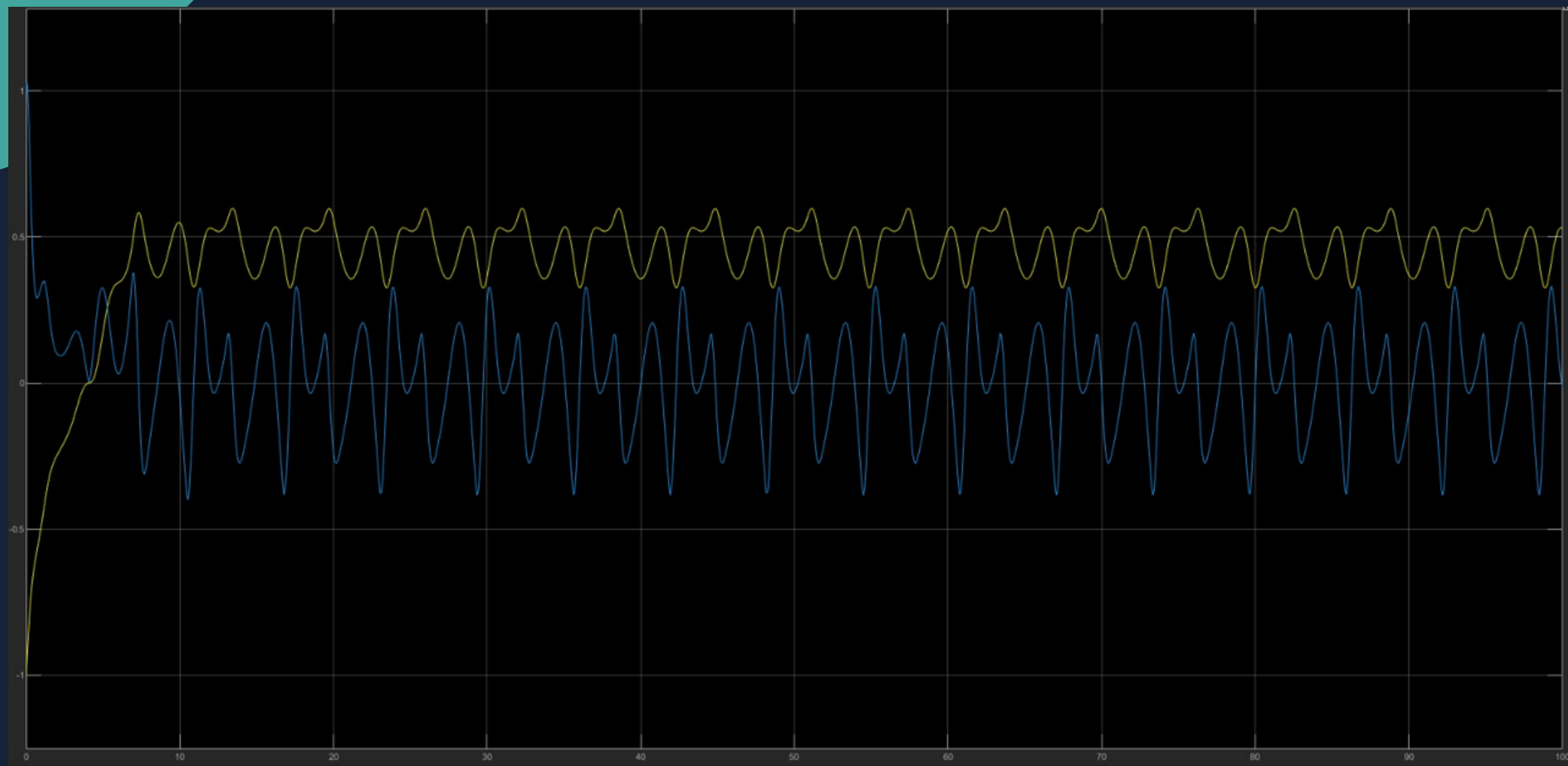
$$\dot{W} = -\Gamma_W \sigma^k(\bar{x}) e^T P B - \Gamma_W \sum_{j=1}^p \sigma^k(\bar{x}_j) \epsilon_j^{k^T}.$$



# SIMULATION RESULTS

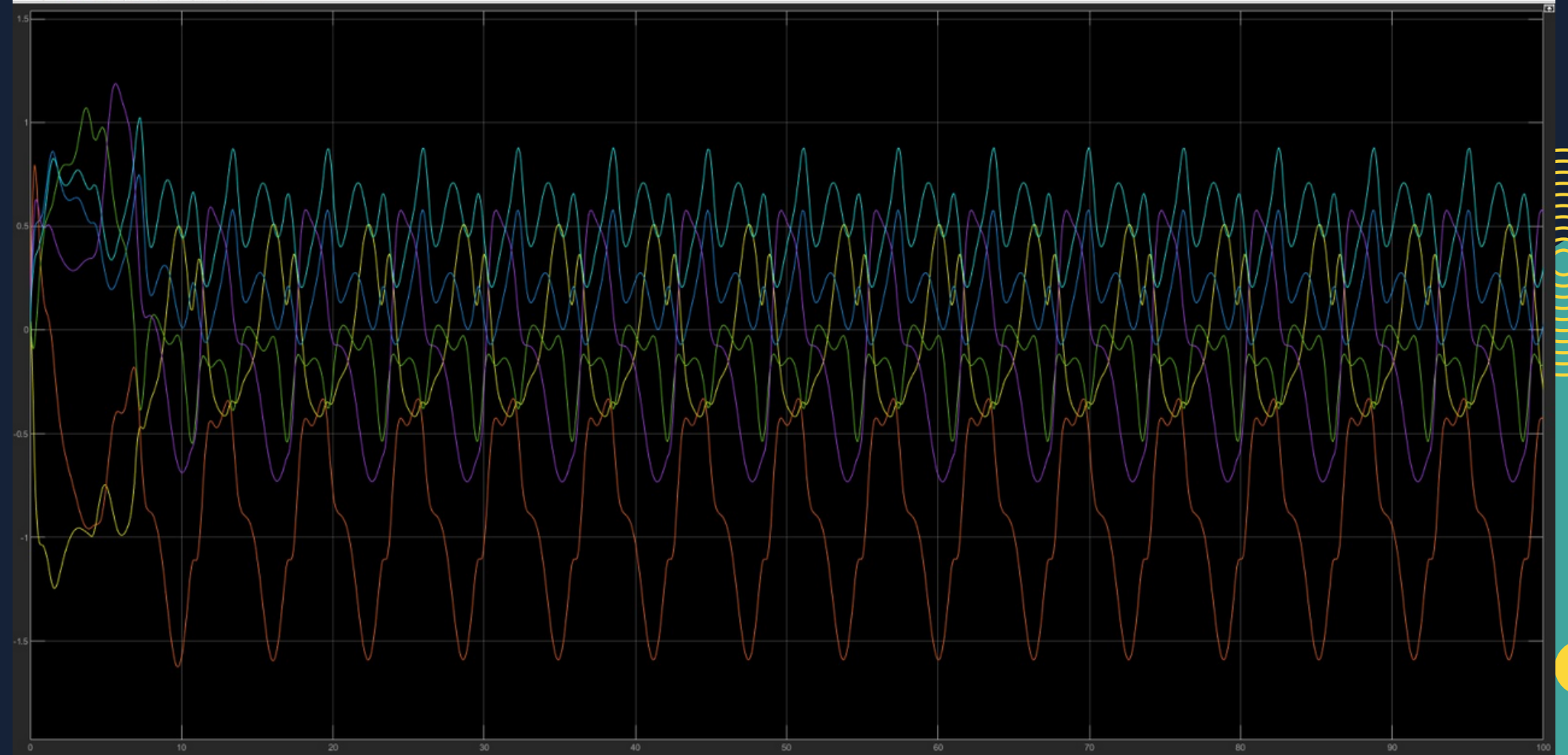






Error vs Time

$\hat{W}$  vs Time



# APPLICATION TO CONTROL OF WING ROCK DYNAMICS

THE CONTROL OF WING ROCK DYNAMICS IS AN IMPORTANT AREA OF RESEARCH IN AERONAUTICS. WING ROCK IS A PHENOMENON THAT OCCURS WHEN AN AIRCRAFT EXPERIENCES A LATERAL OSCILLATION OF THE WING DURING FLIGHT. THIS OSCILLATION CAN BE CAUSED BY A VARIETY OF FACTORS, INCLUDING GUSTS OF WIND OR TURBULENCE, AND CAN BE DIFFICULT TO CONTROL. HOWEVER, THERE ARE SEVERAL APPROACHES THAT CAN BE TAKEN TO MINIMIZE THE EFFECTS OF WING ROCK AND MAINTAIN THE STABILITY OF THE AIRCRAFT.

BKR-CL IS A CONTROL TECHNIQUE USED TO IMPROVE THE CONTROL PERFORMANCE OF WING ROCK DYNAMICS. IT COMBINES BACKSTEPPING CONTROL, KALMAN FILTERING, AND REINFORCEMENT LEARNING ALGORITHMS TO ACHIEVE A LONG-TERM LEARNING EFFECT, ALLOWING IT TO ADAPT TO SYSTEM DYNAMICS AND DISTURBANCES CHANGES. HOWEVER, THE COMPUTATIONAL COMPLEXITY OF BKR-CL CAN BE SIGNIFICANT DUE TO THE INTENSIVE COMPUTATION REQUIRED BY THE ALGORITHMS, BUT IT CAN BE OPTIMIZED BY USING PARALLEL COMPUTING TECHNIQUES.

# CONCLUSION

THIS PAPER INTRODUCES AN ALGORITHM CALLED BUDGETED KERNEL RESTRUCTURING (BKR) FOR DESIGNING ADAPTIVE CONTROLLERS THAT RETAIN ANY INSERTED EXCITATION. THE ALGORITHM UPDATES THE RADIAL BASIS FUNCTION (RBF) NETWORK TO ENSURE PERSISTENTLY EXCITING (PE) SIGNALS. THE METHOD IS BETTER THAN EVENLY SPACED CENTERS AS THE CENTERS ARE SELECTED ALONG THE PATH OF THE SYSTEM THROUGH THE STATE SPACE. BKR-CL, AN AUGMENTED VERSION OF BKR WITH CONCURRENT LEARNING (CL), IS ALSO PRESENTED AND SHOWN TO IMPROVE TRACKING PERFORMANCE THROUGH SIMULATIONS

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**THANK YOU &  
HAVE A NICE DAY!**

