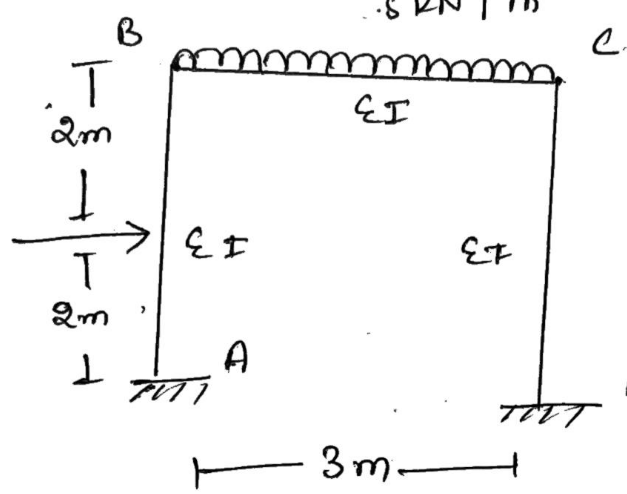
Q4
sol:

given frame is:



As there is unsymmetrical loading so, there will be sidesway also, let there is a sidesway in horizontal direction of member BC by amount Δ.

then, $\psi_{AB} = \psi_{BA} = \frac{-\Delta}{l} = \frac{-\Delta}{4}$

(as rot is clockwise)

and $\psi_{DC} = \psi_{CD} = \frac{-\Delta}{l} = -\frac{\Delta}{4}$.

Now, fixed end moments are:

$\Rightarrow M^F_{CD} = M^F_{DC} = 0$.

(as there is no loading CD member)

and $M^F_{AB} = \frac{wl}{8} = \frac{5(4)}{8} = 2.5$ kN-m.

and $M_{AB}^{F} = \frac{wl}{8} = \frac{5(4)}{8} = 2.5$ kN-m.

$\therefore M_{BA}^{F} = -2.5$ kN-m.

$M_{CB}^{F} = -3.75$ kN-m.

Now, Apply slope - deflection. method.

$$M_{AB} = M_{AB}^{F} + \frac{2EI}{l}\left(2\theta_A + \theta_B - 3\psi_{AB}\right)$$

Since, A and D are fixed supports,

so, $\theta_A = \theta_D = 0$.

$$\therefore M_{AB} = 2.5 + \frac{2EI}{4}\left(\theta_B + \frac{3\Delta}{4}\right)$$

$$\boxed{M_{AB} = 2.5 + 0.5 EI\theta_B + 0.375\Delta EI} \quad \text{— (i)}$$

$$M_{BA} = M_{BA}^{F} + \frac{2EI}{l}\left(2\theta_B + \theta_A - 3\psi_{BA}\right)$$

$$= -2.5 + \frac{2EI}{4}\left(2\theta_B - 3(-\Delta/4)\right)$$

$$\boxed{M_{BA} = -2.5 + EI\theta_B + 0.375\Delta EI} \quad \text{— (ii)}$$

$$M_{BC} = M_{BC}^F + \frac{2EI}{l}\left(2\theta_B + \theta_C - 3\psi_{BC}\right)$$

$$= 3.75 + \frac{2EI}{3}\left(2\theta_B + \theta_C\right)$$

$$\boxed{M_{BC} = 3.75 + 1.33\, EI\theta_B + 0.667\theta_C \cdot EI}$$

— (iii)

$$M_{CB} = M_{CB}^F + \frac{2EI}{l}\left(2\theta_C + \theta_B - 3\psi_{CB}\right)$$

$$\boxed{M_{CB} = -3.75 + 1.33\, EI\,\theta_C + 0.667\, EI\,\theta_B}$$

— (iv)

$$M_{CD} = M_{CD}^F + \frac{2EI}{l}\left(2\theta_C + \theta_D - 3\psi_{CD}\right)$$

$$= 0 + \frac{2EI}{4}\left[2\theta_C - 3\left(-\Delta/4\right)\right]$$

$$\boxed{M_{CD} = EI\,\theta_C + 0.375\, EI\,\Delta}$$

— (v)

$$M_{DC} = -M_{CD}^F + \frac{2EI}{l}\left(2\theta_D + \theta_C - 3\psi_{DC}\right)$$

$$= 0 + \frac{2EI}{4}\left(\theta_C - 3\left(-\Delta/4\right)\right)$$

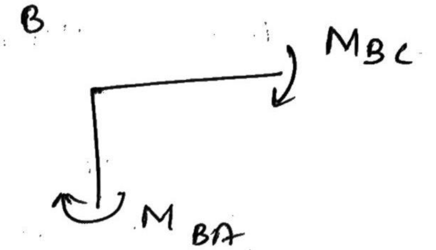
$$\boxed{M_{DC} = 0.5\, EI\,\theta_C + 0.375\, EI\,\Delta}$$

— (vi)

Consider joint B,

for eq^m of joint B,

$$M_{BA} + M_{BC} = 0$$



$\Rightarrow$

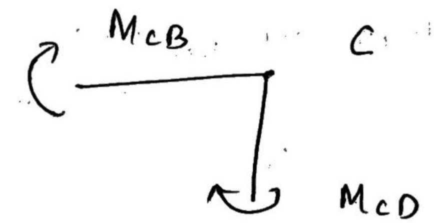
$$-2.5 + EI\,\theta_B + 0.375\Delta\,EI + 3.75 + 1.33EI\,\theta_B + 0.667\,EI\theta_C = 0$$

$$\Rightarrow \quad 2.33\,EI\,\theta_B + 0.66\,EI\theta_C + 0.375\,EI\,\Delta + 1.25 = 0. \quad \text{— (A)}$$

Consider joint C,

for eq^m at joint C,

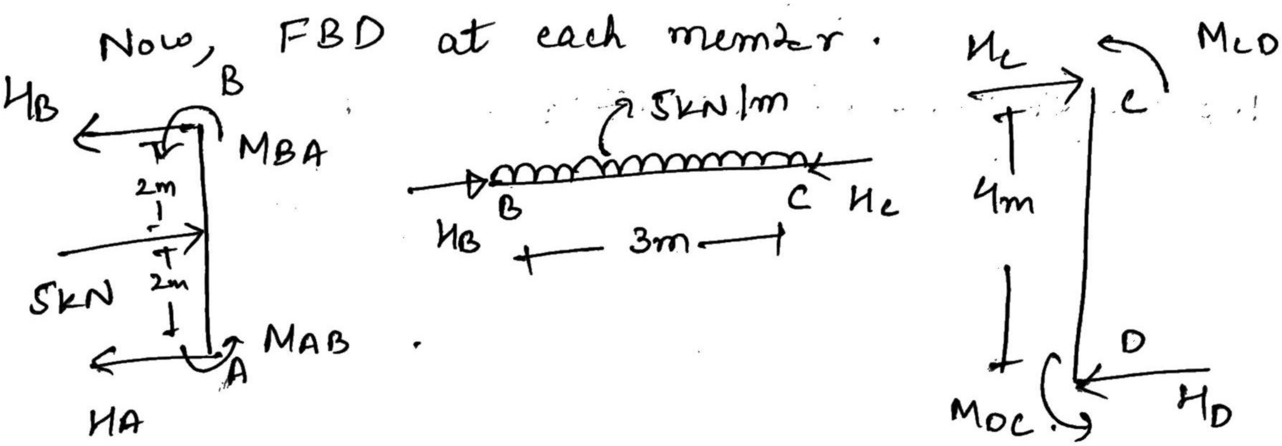
$$M_{CB} + M_{CD} = 0.$$



$$-3.75 + 0.667\,EI\,\theta_B + 1.33\,EI\,\theta_C + EI\theta_C + 0.315\,EI\Delta = 0.$$

$$\Rightarrow 0.667\,EI\,\theta_B + 2.33\,EI\,\theta_C + 0.375\,EI\,\Delta - 3.75 = 0. \quad \text{— (B)}$$

Now, FBD at each member.



for eqm consideration of member AB, we have.

$$H_A = \frac{M_{AB} + M_{BA} + 10}{4} \quad - (vii)$$

and from eqm consideration of member CD, we have.

$$H_D = \frac{M_{CD} + M_{DC}}{4} \quad - (viii)$$

Consider Horizontal eqm of whole frame structure,

we have $H_A + H_D = 5$

$$\Rightarrow \frac{M_{AB} + M_{BA} + 10}{4} + \frac{M_{CD} + M_{DC}}{4} = 5$$

(using (vii) and viii)

$$\Rightarrow M_{AB} + M_{BA} + M_{CD} + M_{DC} = 10$$

$$\Rightarrow \cancel{2.5} + 0.5 EI\theta_B + 0.375 EI\Delta - \cancel{2.5} + EI\theta_B + 0.375 EI\Delta$$

$$+ EI\theta_C + 0.375 EI\Delta + 0.5 EI\theta_C + 0.375 EI\Delta = 0$$

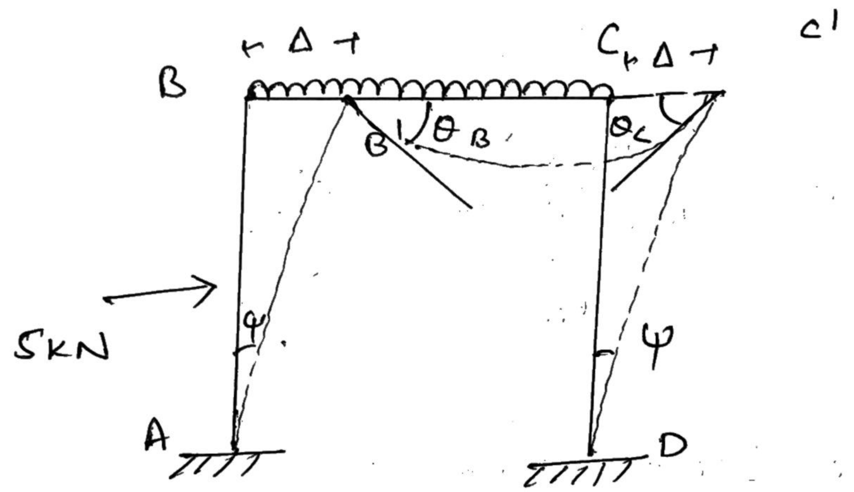
$$\Rightarrow 1.5 EI\theta_B + 1.5 EI\theta_C + 1.5 EI\Delta = 10.$$

$$- (c)$$

on solving (A), (B) and (C), we get.

$$\Delta = \frac{7.7}{EI}$$

$$\theta_B = \frac{-2.055}{EI} \qquad \theta_C = \frac{0.945}{EI}$$



Putting the values of Δ, θ_B and θ_C in eq. (i), (ii), (iii), (iv), (v) and (vi) we get,

$M_{AB} = +4.39$ kN m

$M_{BA} = -1.64$ kNm.

$M_{BC} = +1.64$ kN m

$M_{CB} = -3.86$ kN m

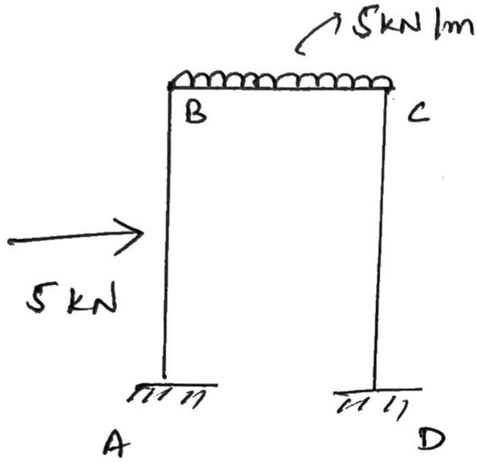
$M_{CD} = +3.86$ kNm

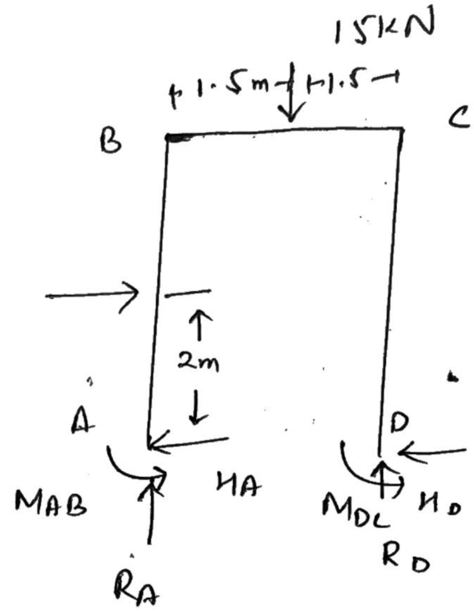
$M_{DC} = +3.39$ kN

from eq. (vii) and (viii)

$$H_A = 3.1875 \text{ kN}.$$

and $$H_D = 1.8125 \text{ kN}$$





$$\Sigma \overline{M_D} = 0$$

$$\Rightarrow M_{AB} + M_{DC} - 3R_A - 10 + 22.5 = 0.$$

$$\Rightarrow 4.39 + 3.39 - 3R_A + 12.5 = 0.$$

$$\Rightarrow 3R_A = 20.28.$$

$$\therefore \boxed{R_A = 6.76 \text{ kN}}$$

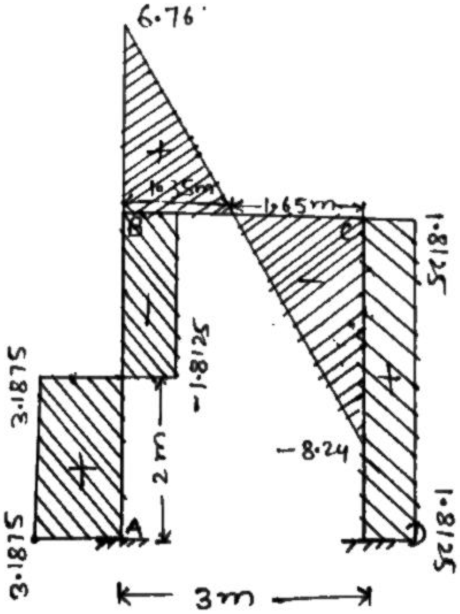
$$\Sigma \overline{F_y} = 0$$

$$\Rightarrow R_A + R_D - 15 = 0.$$

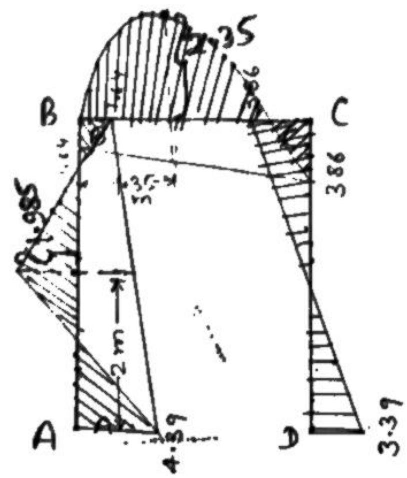
$$\Rightarrow R_D = 15 - R_A = 15 - 6.76.$$

$$\therefore \boxed{R_D = 8.24 \text{ kN}}$$

SFD



BMD



Deflected Shape