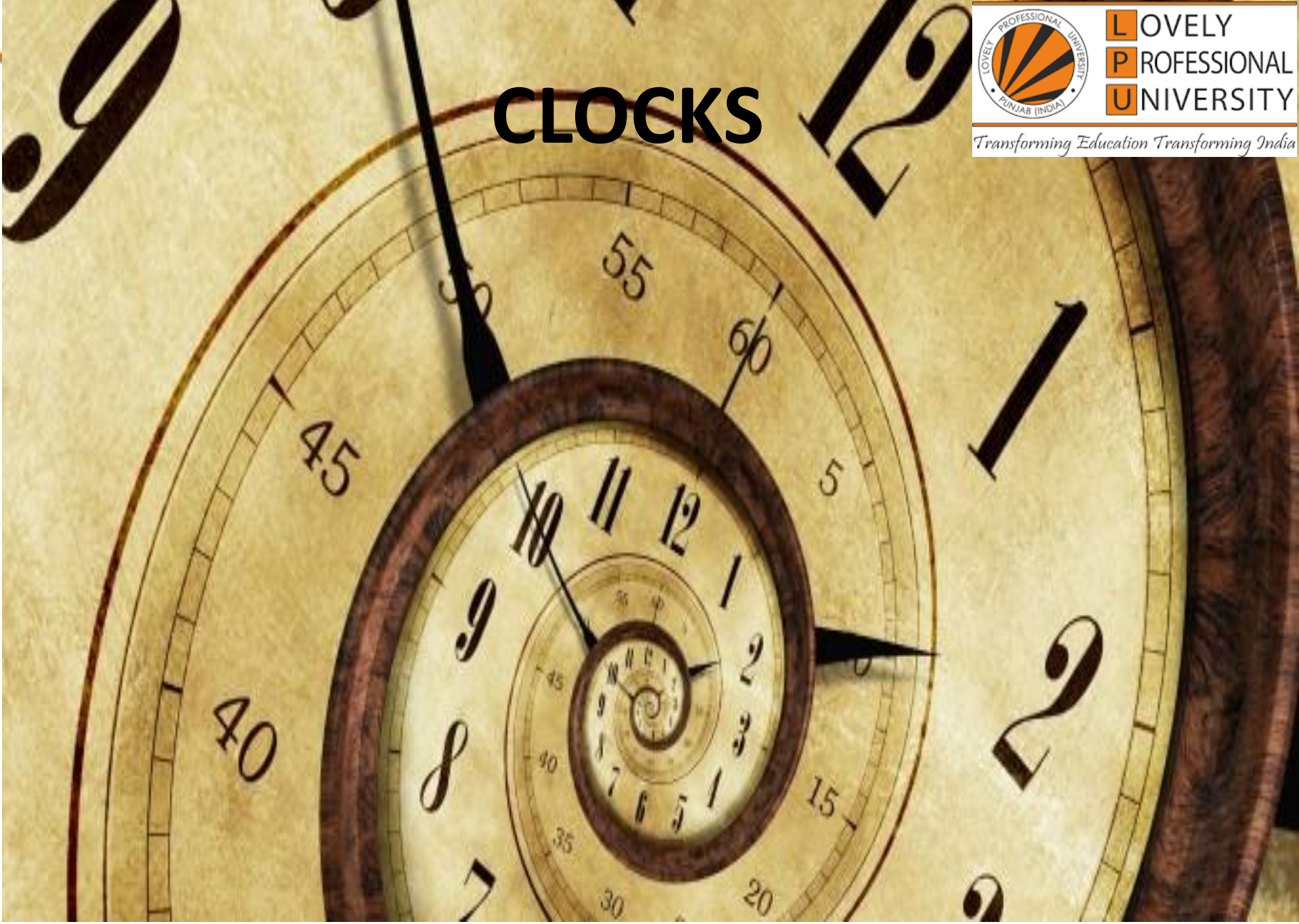
CLOCKS



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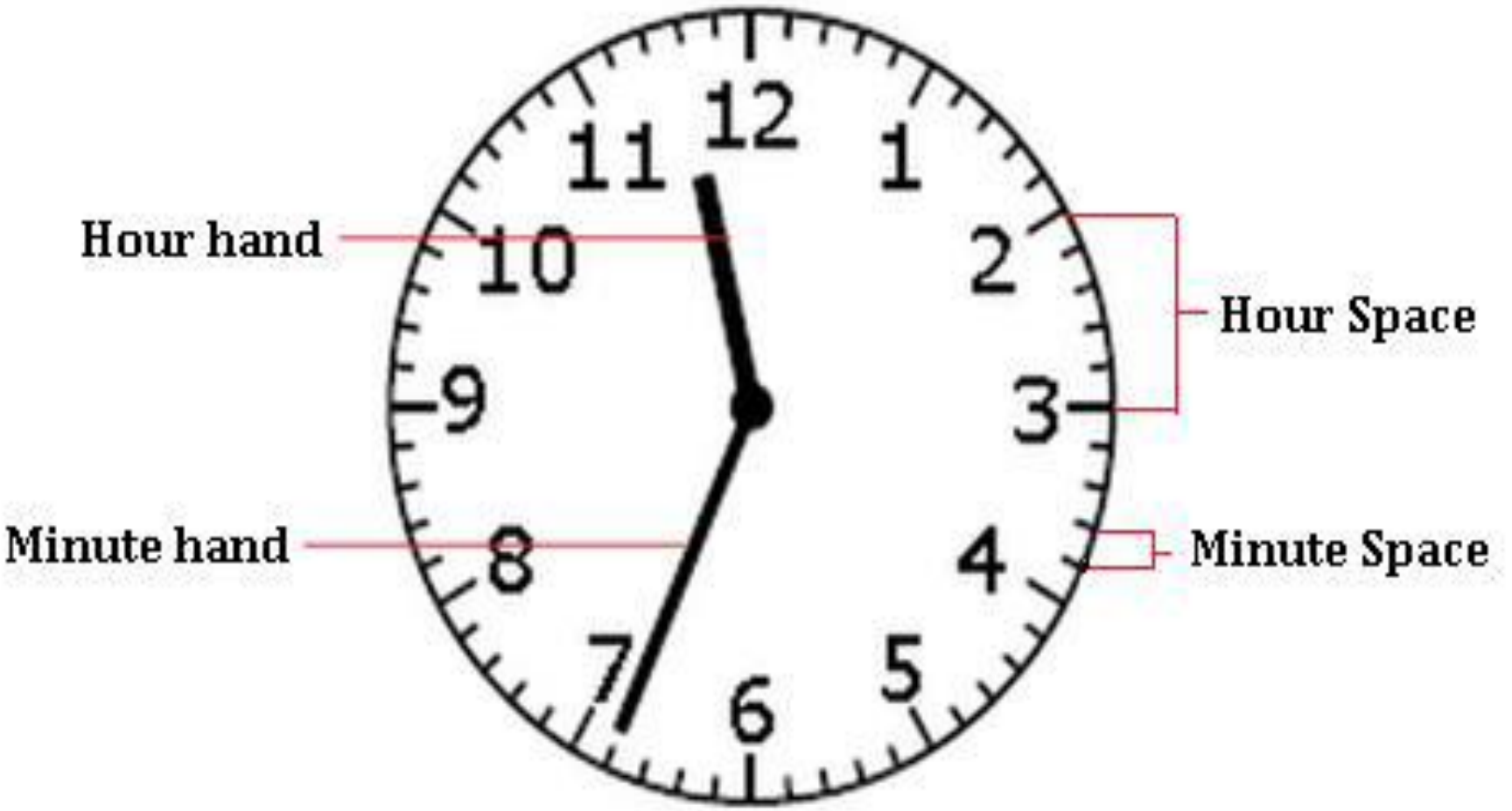
CONCEPT BASE

DIAL:

- The face or dial of a clock is a circle whose circumference is divided into 60 equal parts, called minute spaces.
- They are marked by short lines in the face of the clock. However, the end of every fifth minute space is marked longer than the others and the 60 minute spaces are represented as 12 divisions or 12 hour spaces.

They are usually numbered from 1 to 12 as shown in the figure below.



HANDS OF THE CLOCK:

- A clock has two hands; the smaller one is called the hour hand or short hand while the larger one is called the minute hand or long hand as shown in the figure above.
- The hands of the clock are the indicators of time.
- The time is read based on the positions of the hands with respect to minute or hour spaces.

UNIT OF TIME:

- The unit of time in clocks is seconds.
- A second, the SI unit of time, is defined as the natural periodicity of the radiation of a caesium-133 atom.
- A duration of 60 seconds is defined as a minute.
- In clocks, it is also defined as the time taken by the minute hand to move across a minute space.
- A duration of 60 minutes is defined as an hour.
- In clocks, it is also defined as the time taken by the hour hand or short hand to move across five minute spaces or one hour space.
- A day is also defined as the duration of 24 hours

- The time is measured owing to the movement of the hands of a clock. A complete rotation of any of the hands would cover a measure of 360 degrees (circular displacement) .
- Degree is the unit of measurement of angles and is denoted by the symbol °.
- Hour hand covers an angle of 360° in 12 hours. So, the hour hand in one hour will cover 360 /12 = 30°.
- So for every minute, the hour hand moves through 30 /60 = 0.5 °.

- Minute hand covers an angle of 360° in 1 hour. So for every minute, the minute hand moves through 360/60 = 6 °.
- Both the hands of the clock move in same direction.
- So, their relative displacement for every minute is 5.5° or 11/2°.
- This 5.5° movement constitutes the movements of both the hands.
- So for every minute, both the hands give a relative displacement of 5.5°.

FORMULA TO FIND THE ANGLE BETWEEN THE HANDS OF THE CLOCK:

- Let us say in a race, Sam gives a head-start of 120 metres to Anand.
- Sam and Anand run at the speed of 10 m/s and 6 m/s respectively.
- Running in the same direction, the relative displacement is 10 − 6 = 4 metres for every second.
- Now, let us calculate the time taken by

  (i) Sam to catch up with Anand

  (ii) Sam to lead Anand by 20 metres

(i) Time taken by Sam to catch up with Anand

Once they begin to run, the initial gap of 120 metres starts decreasing at the rate (relative speed) of 4 m/s.

Therefore, it takes

$4 \times t = 120$ (Relative Speed × Time = Relative distance or gap)

t = 30 secs to catch up with Anand or to say that they are at the same point.

(ii) Time taken by Sam to lead Anand by 20 metres:

We know that Sam needs 30 secs to catch up with Anand and thereby it can be considered that both of them are running, starting at the same point after 30 secs.

Though Sam runs at 10 m/s, the gap between them increases at the rate of their relative speed i.e. 4 m/s.

Hence, it takes $4 \times t = 20$, $t = 5$ secs to lead

Anand by 20 metres when they start from the same point.

Thus, from the starting point Sam takes

30 + 5 = 35 secs to lead Anand by 20 metres.

However, the answer can be directly found by taking the relative distance to be

120 + 20 = 140 metres

$4 \times t = 140$

t = 35 secs.

From (i) and (ii), we can think of the hands of the clock as two runners running in a circular track of 360°. Here, the angle between the hands of the clock should be considered as the relative distance between them.

Therefore, we can formulate the following

Angle between the hands of the clock = Initial gap between the hands of the clock ~ Relative displacement for a period of time

$$\theta = 30°H - 11/2M$$

(when the minute hand is trailing the hour hand)

$$\theta = 11/2M - 30°H$$

(when the minute hand is leading the hour hand)

- θ = Angle between the hands of the clock
- $30°H$ = Initial gap; H - the division pointed by the hour hand which indicates the gap between the minute hand and hour hand initially when multiplied by 30°.
- $11/2M$ = Relative displacement for a period of time; 11/2 - Relative speed of the hands of the clock; M - no. of minute spaces moved by the minute hand.

COINCIDENCE OF THE HANDS OF THE CLOCK:

- At 12 o' clock, the hands of the clock overlap i.e. the angle formed between them is 0°.
- To find the next time they overlap, the gap between them should be 360° so that they are at the same point. Hence it takes,

  $11/2 \times t = 360$

  $t = 720/11 = 65\ 5/11$ minutes
- Thus, it can be easily concluded that the hands of the clock coincide every $65\ 5/11$ minutes.

FREQUENCY OF FORMATION OF ANGLES BETWEEN THE HANDS OF THE CLOCK:

- With the relative displacement, it is easier to find the frequency at which the hands of a clock form a particular angle.
- Let's find the no. of times the hands of the clock form 0° in a day.
- We know that the hands of the clock coincide every 65 5/11 or 720/11 minutes. So, in a day there are 24 × 60 = 1440 minutes
- Therefore, in a day the hands of the clock coincide (1440 × 11) / 720 = 22 times.

- Thus, it can be inferred that the hands of the clock coincide once in every hour except for two occasions i.e. from 11 to 1 o' clock they coincide only once at 12 o' clock.
- So, it is 11 times in 12 hours and thus 22 times in 24 hours.
- The finding can be extended to other angles by applying the formula,

$$\theta = 30°H - 11/2M \text{ or}$$

$$\theta = 11/2M - 30°H$$

for every hour in the clock.

GAIN AND LOSS OF TIME IN AN INCORRECT CLOCK:

- A normal clock becomes incorrect when there is a change in the speeds of the hands of the clock.
- As seen already, duration of a day is measured by the time taken by the hour hand to move across the 12 divisions twice.
- So, when the speed of the hands of the clock increases they complete their rotation for a day sooner.
- Thus, they result in gain of time when actually compared with the duration for a day in a correct clock .
- Similarly, when the hands of the clock move slower than usual, they take more time to complete the rotations for a day.
- Thus, they result in losing more time for a day when compared with that in a correct clock.

KEY POINTS:

1. In 60 minutes, the minute hand gains 55 minutes on the hour hand

2. The hands of the clock coincide every 65 5/11 minutes and for every hour, both the hands coincide once

3. The hands are in the same straight line when they are coincident or opposite to each other

4. When the two hands are at right angles, they are 15 minute spaces apart

5. When the hands are in opposite directions, they are 30 minute spaces apart

6. The hands of the clock form an angle of 0° or 180° – 22 times a day

7. The hands of the clock form an angle of 1° to 179° – 44 times a day

8. If a watch or a clock indicates 8:15, when the correct time is 8, it is said to be 15 minutes faster than the correct time. On the other hand, if it indicates 7:45, when the correct time is 8, it is said to be 15 minutes slow.

PRACTICE QUESTIONS

1. Find the angle between the hands of a clock when the time is 5:40.

(a) 80^o

(b) 160^o

(c) 70^o

(d) 120^o

Ans: C

2. Find the angle between the minute hand and the hour hand of a clock when the time is 7:20.

(a) 80^o

(b) 90^o

(c) 100^o

(d) 110^o

Ans: C

3. The reflex angle between the hands of a clock at 10:25 is

(a) 180^o

(b) 162.5^o

(c) 165^o

(d) 197.5^o

Ans: D

4. Find the angle between the hour hand and the minute hand of a clock when the time is 15:25.

(a) 47.5^{o}

(b) 45.5^{o}

(c) 50^{o}

(d) None of these

Ans: A

5. At what time between 5 p.m. and 6 p.m., do the hands of a clock coincide?

(a) 8 hour 29 3/11 min

(b) 9 hour 33 8/11 min

(c) 5 hour 27 3/11 min

(d) None of these

Ans: C

6. The minute hand of a clock is found to cross the hour hand x minutes past three. Find x.

(a) 10 5/11

(b) 15 15/11

(c) 16 4/11

(d) 21 9/11

Ans: C

7. The number of minutes from midnight to now is 9 times the number of minutes from now to noon. What time is it now?

(a) 10.50 a.m.

(b) 10.48 a.m.

(c) 10.40 a.m.

(d) 10.53 a.m.

Ans: B

8. Find at what time (in minutes) past 8 o' clock but before 9 o' clock will the hands of a clock be in the same straight line but not together.

(a) 100/11

(b) 110/11

(c) 120/11

(d) None of these

Ans: C

9. How many times in a day do the hands of a clock form 60^{o}?

(a) 22

(b) 33

(c) 44

(d) 55

Ans: C

10. A few times per day, the minute hand of a clock is exactly above (or below) the hour hand. How many times per day does this occur?

(a) 11 times

(b) 24 times

(c) 22 times

(d) 44 times

Ans: C

11. At how many times between 12 o'clock and 1 o' clock are the minute hand and the hour hand of a clock at an angle of 90 degrees to each other?

(a) 4

(b) 6

(c) 3

(d) 2

Ans: D

12. How much does a watch gain per day, if its hands coincide every 64 minutes?

(a) 32 8/11 min

(b) 36 5/11 min

(c) 90 min

(d) 96 min

Ans: A

13. How many minutes is it until six o’clock if fifty minutes ago it was four times as many minutes past three o’clock?

(a) 26 min

(b) 27 min

(c) 30 min

(d) None of these

Ans: A

14. A man who went out between 3 and 4 and returned between 8 and 9, found that the hands of the watch had exactly changed places. When did he return?

(a) 14 min past 8

(b) 21 1/3min past 8

(c) 19 2/3 min past 8

(d) 18 6/13 min past 8

Ans: D

15. My watch was 8 minutes behind at 8 p.m. on Sunday but it was 7 minutes ahead of time at 8 p.m. on Wednesday. During this period, at which time has this watch shown the correct time?

(a) Tuesday 10.24 a.m.

(b) Wednesday 9.16 p.m.

(c) Tuesday 10.24 p.m.

(d) Wednesday 9.16 a.m.

Ans: A

16. A friend of mine invented a new clock. It times an entire day in 10 hours, each hour in 100 minutes and each minute in 100 seconds. What normal time is it when it reads 5:41?

(a) 12:59:02

(b) 12:59:01

(c) 11:59:02

(d) None of these

Ans: A

17. The minute hand of a clock overtakes the hour hand at intervals of 65 minutes of the correct time. How much a day does the clock gain or lose in minutes?

(a) 1440/143

(b) 1444/143

(c) 1400/143

(d) 4440/143

Ans: A

18. Between 11 o'clock and 12 o'clock, how often will the difference between the positions of the minute and hour hands of a clock have an integral number of minutes?

(a) 4

(b) 6

(c) 3

(d) 5

Ans: D

19. A clock loses 1% time during the first week and then gains 2% time during the next one week. If the clock was set right at 12 noon on a Sunday, what will be the time shown by the clock exactly after 14 days from the time it was set right?

(a) 1 : 40 : 48 p.m.

(b) 1 : 40 : 28 p.m.

(c) 1 : 40 : 46 p.m.

(d) 1 : 40 : 26 p.m.

Ans: A

20. A clock is set right at 8 a.m. The clock gains 10 minutes in 24 hours. What will be the true time when the clock indicates 1 p.m. on the following day?

(a) 48 min past 12

(b) 48 min past 11

(c) 45 min past 12

(d) 45 min past 11

Ans: A

21. At what time between 4 and 5 o'clock, will the hands of a watch point in opposite directions?

(a) 54 min past 4

(b) 53 7/11 min past 4

(c) 54 7/11 min past 4

(d) 54 6/11 min past 4

Ans: D

22. A watch which gains 5 seconds in 3 minutes was set right at 7 a.m. In the afternoon of the same day, when the watch indicated quarter past 4 o'clock, the true time is

(a) 4 p.m.

(b) 6 p.m.

(c) 5 p.m.

(d) 7 p.m.

Ans: A

23. At what time between 5.30 and 6 o' clock, will the hands of a clock be at right angles?

(a) 43 5/11 min past 5

(b) 43 7/11 min past 5

(c) 5 p.m.

(d) 7 p.m.

Ans: B

24. Find the reflex angle between the hour hand and the minute hand of a clock when the time is 11:25.

(a) 192.5 ^{o}

(b) 230 ^{o}

(c) 45 ^{o}

(d) 72 ^{o}

Ans: A

25. Three clocks were set to true time. First runs with exact time. Second slows one minute per day. Third gains one minute per day. After how many days will they show the same time?

(a) 150 days

(b) 550 days

(c) 1440 days

(d) 2220 days

Ans: C

26. A clock showing 6 o'clock takes 30 secs to strike 6 times. How long will it take to strike 12 at midnight?

(a) 45 secs

(b) 40 secs

(c) 66 secs

(d) 60 secs

Ans: C

27. Light glows for every 12 seconds. How many times did it glow between 2:58:59 a.m. and 4:10:23 a.m.?

(a) 384

(b) 357

(c) 383

(d) 380

Ans: B

28. Three cuckoo clocks are such that the cuckoos chime after every 10 minutes, 20 minutes and 35 minutes respectively. If the 3 clocks chime simultaneously at 2pm, at what time will they chime together again?

(a) 3 p.m.

(b) 1:40 p.m.

(c) 3.20 p.m.

(d) 4:20 p.m.

Ans: D

29. A certain clock marks every hour by striking a number of times equal to the hour and the time required for a stroke is exactly equal to the time interval between the two consecutive strokes. At 6.00, the time lapse between the beginning of the first stroke and the end of the last stroke is 22 seconds. At 12.00, how many seconds do elapse between the beginning of the first stroke and the end of the last stroke?

(a) 44

(b) 43

(c) 46

(d) 54

Ans: C

30. The quarter of the time from midnight to present time added to the half of the time from the present to midnight gives the present time. What is the present time?

(a) 36 min past 9

(b) 12 min past 9

(c) 36 min past 8

(d) 12 min past 8

Ans: A