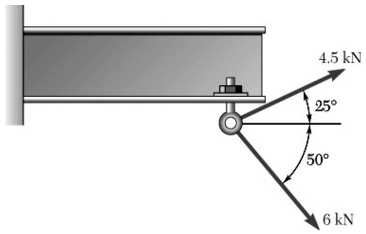
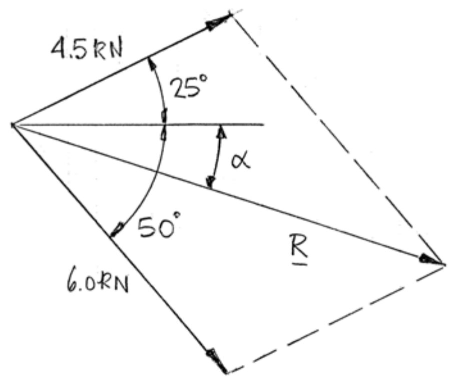


PROBLEM 2.1

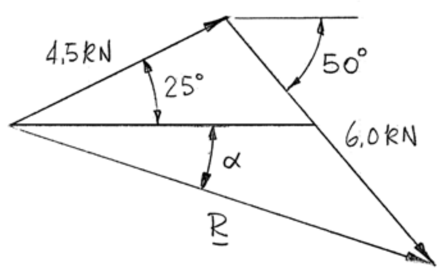
Two forces are applied to an eye bolt fastened to a beam. Determine graphically the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.

SOLUTION

(*a*)



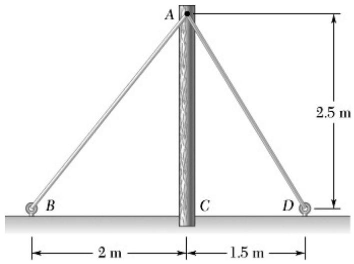
(*b*)



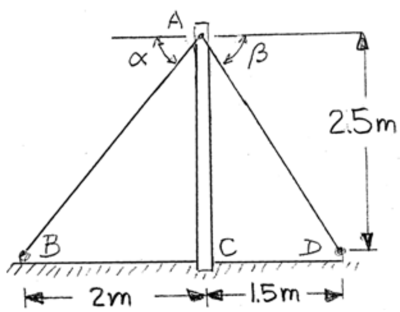
We measure: $R = 8.4$ kN

$\alpha = 19°$

$\mathbf{R} = 8.4$ kN ⦪ 19° ◀

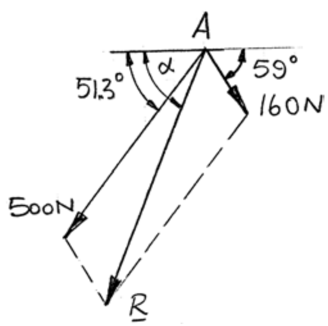
	**PROBLEM 2.2** The cable stays *AB* and *AD* help support pole *AC*. Knowing that the tension is 500 N in *AB* and 160 N in *AD*, determine graphically the magnitude and direction of the resultant of the forces exerted by the stays at *A* using (*a*) the parallelogram law, (*b*) the triangle rule.

SOLUTION

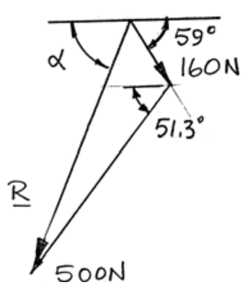


We measure: $\alpha = 51.3°, \beta = 59°$

(*a*)



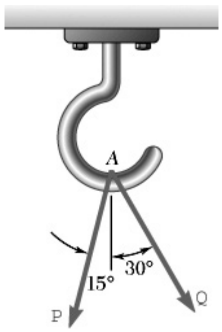
(*b*)



We measure: $R = 575 \text{ N}, \alpha = 67°$

$\mathbf{R} = 575 \text{ N} \measuredangle 67°$ ◀

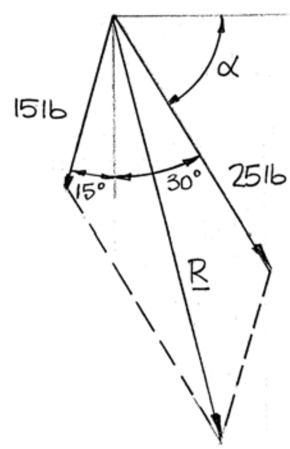
PROBLEM 2.3



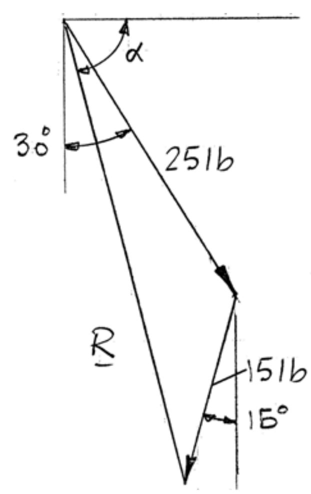
Two forces **P** and **Q** are applied as shown at point A of a hook support. Knowing that $P = 15$ lb and $Q = 25$ lb, determine graphically the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.

SOLUTION

(*a*)



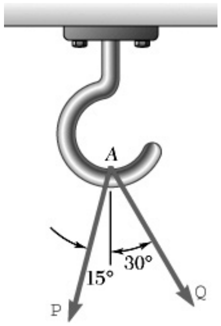
(*b*)



We measure: $R = 37$ lb, $\alpha = 76°$

$\mathbf{R} = 37$ lb ⦨ 76° ◀

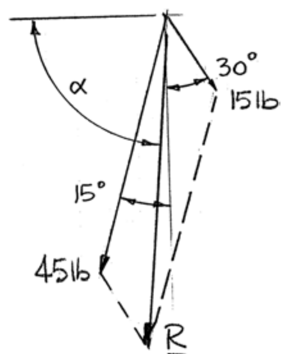
PROBLEM 2.4



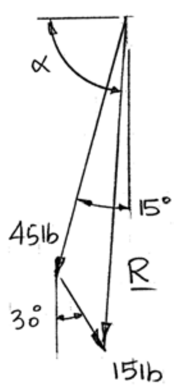
Two forces **P** and **Q** are applied as shown at point *A* of a hook support. Knowing that $P = 45$ lb and $Q = 15$ lb, determine graphically the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.

SOLUTION

(*a*)

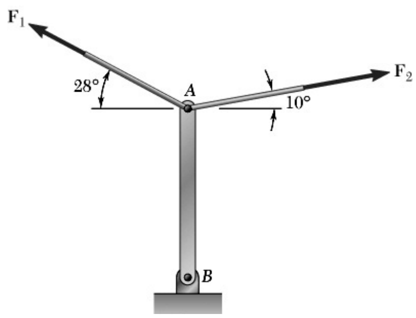


(*b*)



We measure: $R = 61.5$ lb, $\alpha = 86.5°$

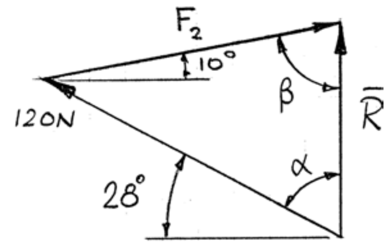
R $= 61.5$ lb ↗ 86.5° ◀



PROBLEM 2.5

Two control rods are attached at *A* to lever *AB*. Using trigonometry and knowing that the force in the left-hand rod is $F_1 = 120$ N, determine (*a*) the required force F_2 in the right-hand rod if the resultant **R** of the forces exerted by the rods on the lever is to be vertical, (*b*) the corresponding magnitude of **R**.

SOLUTION



Graphically, by the triangle law

We measure: $F_2 \cong 108$ N

$R \cong 77$ N

By trigonometry: Law of Sines

$$\frac{F_2}{\sin\alpha} = \frac{R}{\sin 38^\circ} = \frac{120}{\sin\beta}$$

$$\alpha = 90^\circ - 28^\circ = 62^\circ,\ \beta = 180^\circ - 62^\circ - 38^\circ = 80^\circ$$

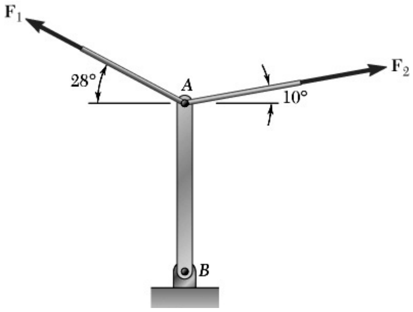
Then:

$$\frac{F_2}{\sin 62^\circ} = \frac{R}{\sin 38^\circ} = \frac{120\text{ N}}{\sin 80^\circ}$$

or (*a*) $F_2 = 107.6$ N ◀

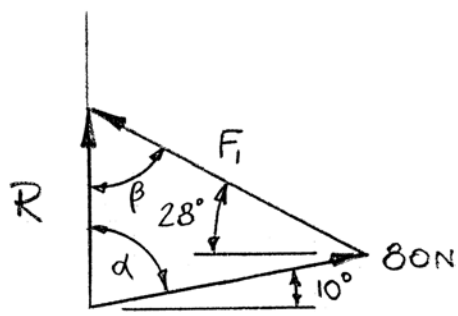
(*b*) $R = 75.0$ N ◀

PROBLEM 2.6



Two control rods are attached at *A* to lever *AB*. Using trigonometry and knowing that the force in the right-hand rod is $F_2 = 80$ N, determine (*a*) the required force F_1 in the left-hand rod if the resultant **R** of the forces exerted by the rods on the lever is to be vertical, (*b*) the corresponding magnitude of **R**.

SOLUTION



Using the Law of Sines

$$\frac{F_1}{\sin\alpha} = \frac{R}{\sin 38^\circ} = \frac{80}{\sin\beta}$$

$$\alpha = 90^\circ - 10^\circ = 80^\circ,\ \beta = 180^\circ - 80^\circ - 38^\circ = 62^\circ$$

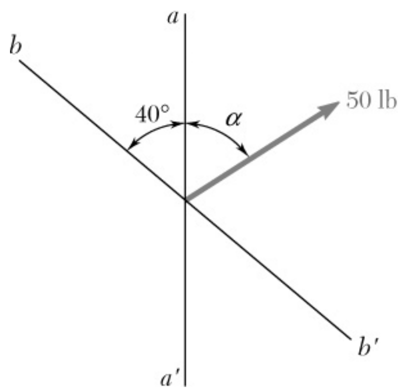
Then:

$$\frac{F_1}{\sin 80^\circ} = \frac{R}{\sin 38^\circ} = \frac{80\text{ N}}{\sin 62^\circ}$$

or (*a*) $F_1 = 89.2$ N ◀

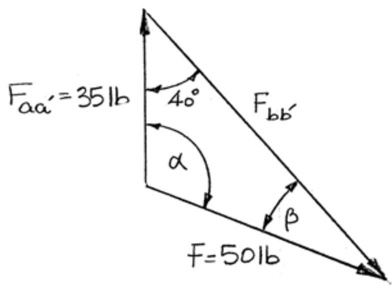
(*b*) $R = 55.8$ N ◀

PROBLEM 2.7



The 50-lb force is to be resolved into components along lines a-a' and b-b'. (a) Using trigonometry, determine the angle α knowing that the component along a-a' is 35 lb. (b) What is the corresponding value of the component along b-b'?

SOLUTION



Using the triangle rule and the Law of Sines

(a)

$$\frac{\sin\beta}{35\text{ lb}} = \frac{\sin 40^\circ}{50\text{ lb}}$$

$$\sin\beta = 0.44995$$

$$\beta = 26.74^\circ$$

Then:

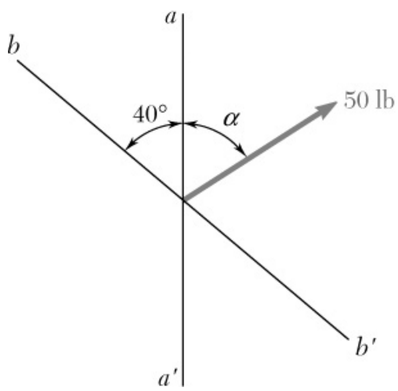
$$\alpha + \beta + 40^\circ = 180^\circ$$

$$\alpha = 113.3^\circ \blacktriangleleft$$

(b) Using the Law of Sines:

$$\frac{F_{bb'}}{\sin\alpha} = \frac{50\text{ lb}}{\sin 40^\circ}$$

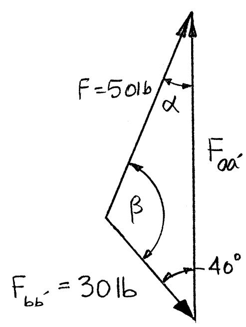
$$F_{bb'} = 71.5\text{ lb} \blacktriangleleft$$



PROBLEM 2.8

The 50-lb force is to be resolved into components along lines a-a' and b-b'. (a) Using trigonometry, determine the angle α knowing that the component along b-b' is 30 lb. (b) What is the corresponding value of the component along a-a'?

SOLUTION



Using the triangle rule and the Law of Sines

(a)

$$\frac{\sin\alpha}{30\text{ lb}} = \frac{\sin 40°}{50\text{ lb}}$$

$$\sin\alpha = 0.3857$$

$$\alpha = 22.7° \blacktriangleleft$$

(b)

$$\alpha + \beta + 40° = 180°$$

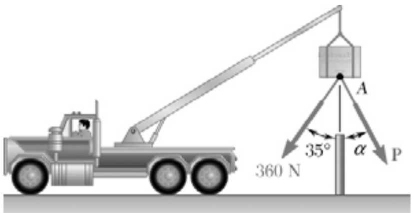
$$\beta = 117.31°$$

$$\frac{F_{aa'}}{\sin\beta} = \frac{50\text{ lb}}{\sin 40°}$$

$$F_{aa'} = 50\text{ lb}\left(\frac{\sin\beta}{\sin 40°}\right)$$

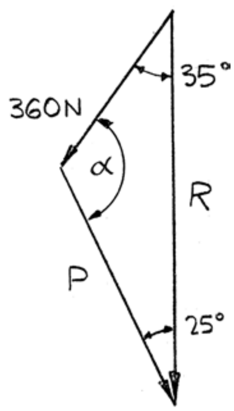
$$F_{aa'} = 69.1\text{ lb} \blacktriangleleft$$

PROBLEM 2.9



To steady a sign as it is being lowered, two cables are attached to the sign at A. Using trigonometry and knowing that $\alpha = 25°$, determine (*a*) the required magnitude of the force **P** if the resultant **R** of the two forces applied at A is to be vertical, (*b*) the corresponding magnitude of **R**.

SOLUTION



Using the triangle rule and the Law of Sines

Have:

$$\alpha = 180° - (35° + 25°)$$

$$= 120°$$

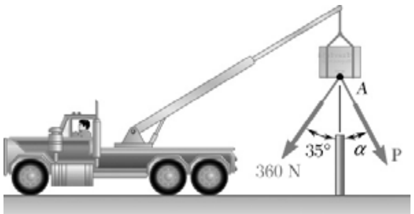
Then:

$$\frac{P}{\sin 35°} = \frac{R}{\sin 120°} = \frac{360\text{ N}}{\sin 25°}$$

or (*a*) $P = 489\text{ N}$ ◀

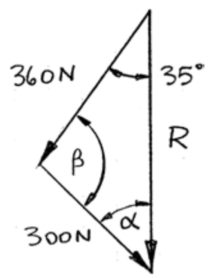
(*b*) $R = 738\text{ N}$ ◀

PROBLEM 2.10



To steady a sign as it is being lowered, two cables are attached to the sign at *A*. Using trigonometry and knowing that the magnitude of **P** is 300 N, determine (*a*) the required angle α if the resultant **R** of the two forces applied at *A* is to be vertical, (*b*) the corresponding magnitude of **R**.

SOLUTION



Using the triangle rule and the Law of Sines

(*a*) Have:

$$\frac{360 \text{ N}}{\sin\alpha} = \frac{300 \text{ N}}{\sin 35°}$$

$$\sin\alpha = 0.68829$$

$$\alpha = 43.5° ◀$$

(*b*)

$$\beta = 180 - (35° + 43.5°)$$

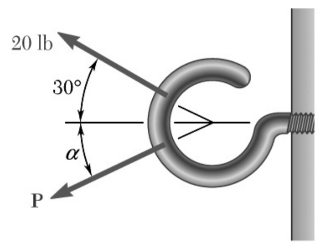
$$= 101.5°$$

Then:

$$\frac{R}{\sin 101.5°} = \frac{300 \text{ N}}{\sin 35°}$$

or $R = 513 \text{ N}$ ◀

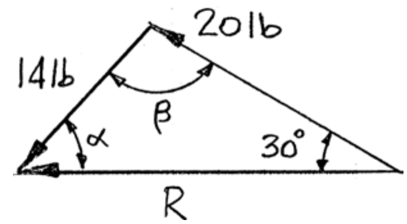
PROBLEM 2.11



Two forces are applied as shown to a hook support. Using trigonometry and knowing that the magnitude of **P** is 14 lb, determine (*a*) the required angle α if the resultant **R** of the two forces applied to the support is to be horizontal, (*b*) the corresponding magnitude of **R**.

SOLUTION

Using the triangle rule and the Law of Sines



(*a*) Have:

$$\frac{20\text{ lb}}{\sin\alpha} = \frac{14\text{ lb}}{\sin 30^\circ}$$

$$\sin\alpha = 0.71428$$

$$\alpha = 45.6^\circ \blacktriangleleft$$

(*b*)

$$\beta = 180^\circ - (30^\circ + 45.6^\circ)$$

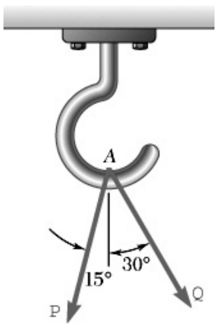
$$= 104.4^\circ$$

Then:

$$\frac{R}{\sin 104.4^\circ} = \frac{14\text{ lb}}{\sin 30^\circ}$$

$$R = 27.1\text{ lb} \blacktriangleleft$$

PROBLEM 2.12

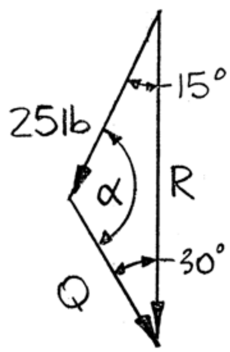


For the hook support of Problem 2.3, using trigonometry and knowing that the magnitude of **P** is 25 lb, determine (*a*) the required magnitude of the force **Q** if the resultant **R** of the two forces applied at *A* is to be vertical, (*b*) the corresponding magnitude of **R**.

Problem 2.3: Two forces **P** and **Q** are applied as shown at point *A* of a hook support. Knowing that $P = 15$ lb and $Q = 25$ lb, determine graphically the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.

SOLUTION

Using the triangle rule and the Law of Sines



(*a*) Have:

$$\frac{Q}{\sin 15°} = \frac{25 \text{ lb}}{\sin 30°}$$

$$Q = 12.94 \text{ lb} \blacktriangleleft$$

(*b*)

$$\beta = 180° - (15° + 30°)$$

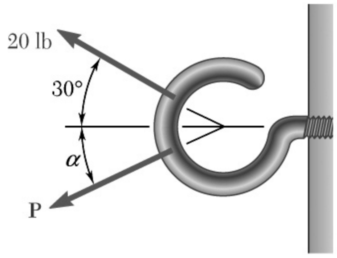
$$= 135°$$

Thus:

$$\frac{R}{\sin 135°} = \frac{25 \text{ lb}}{\sin 30°}$$

$$R = 25 \text{ lb}\left(\frac{\sin 135°}{\sin 30°}\right) = 35.36 \text{ lb}$$

$$R = 35.4 \text{ lb} \blacktriangleleft$$



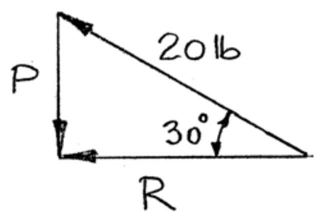
PROBLEM 2.13

For the hook support of Problem 2.11, determine, using trigonometry, (*a*) the magnitude and direction of the smallest force **P** for which the resultant **R** of the two forces applied to the support is horizontal, (*b*) the corresponding magnitude of **R**.

Problem 2.11: Two forces are applied as shown to a hook support. Using trigonometry and knowing that the magnitude of **P** is 14 lb, determine (*a*) the required angle α if the resultant **R** of the two forces applied to the support is to be horizontal, (*b*) the corresponding magnitude of **R**.

SOLUTION

(*a*) The smallest force **P** will be perpendicular to **R**, that is, vertical



$$P = (20 \text{ lb})\sin 30^\circ$$

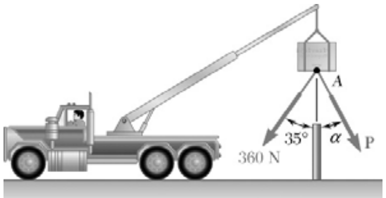
$$= 10 \text{ lb} \qquad \mathbf{P} = 10 \text{ lb} \downarrow \blacktriangleleft$$

(*b*)

$$R = (20 \text{ lb})\cos 30^\circ$$

$$= 17.32 \text{ lb} \qquad R = 17.32 \text{ lb} \blacktriangleleft$$

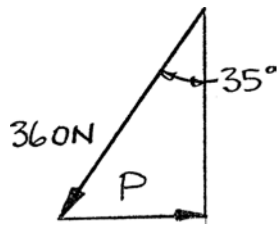
PROBLEM 2.14



As shown in Figure P2.9, two cables are attached to a sign at A to steady the sign as it is being lowered. Using trigonometry, determine (a) the magnitude and direction of the smallest force **P** for which the resultant **R** of the two forces applied at A is vertical, (b) the corresponding magnitude of **R**.

SOLUTION

We observe that force $\mathbf{P}$ is minimum when α is 90°, that is, $\mathbf{P}$ is horizontal

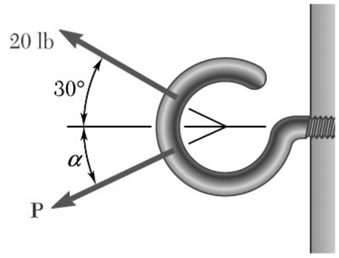


Then: (a) $P = (360 \text{ N})\sin 35°$

or $\mathbf{P} = 206 \text{ N} \longrightarrow$ ◀

And: (b) $R = (360 \text{ N})\cos 35°$

or $R = 295 \text{ N}$ ◀



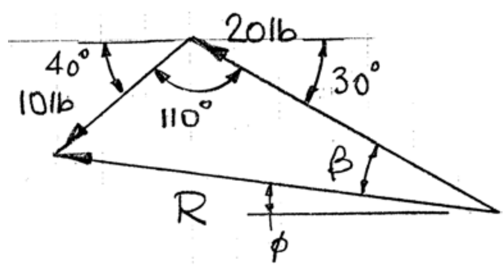
PROBLEM 2.15

For the hook support of Problem 2.11, determine, using trigonometry, the magnitude and direction of the resultant of the two forces applied to the support knowing that $P = 10$ lb and $\alpha = 40°$.

Problem 2.11: Two forces are applied as shown to a hook support. Using trigonometry and knowing that the magnitude of **P** is 14 lb, determine (*a*) the required angle α if the resultant **R** of the two forces applied to the support is to be horizontal, (*b*) the corresponding magnitude of **R**.

SOLUTION

Using the force triangle and the Law of Cosines



$$R^2 = (10 \text{ lb})^2 + (20 \text{ lb})^2 - 2(10 \text{ lb})(20 \text{ lb})\cos 110°$$

$$= \left[100 + 400 - 400(-0.342)\right] \text{lb}^2$$

$$= 636.8 \text{ lb}^2$$

$$R = 25.23 \text{ lb}$$

Using now the Law of Sines

$$\frac{10 \text{ lb}}{\sin\beta} = \frac{25.23 \text{ lb}}{\sin 110°}$$

$$\sin\beta = \left(\frac{10 \text{ lb}}{25.23 \text{ lb}}\right)\sin 110°$$

$$= 0.3724$$

So: $\beta = 21.87°$

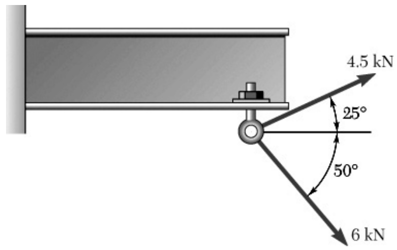
Angle of inclination of R, ϕ is then such that:

$$\phi + \beta = 30°$$

$$\phi = 8.13°$$

Hence: **R** $= 25.2$ lb ⦪ $8.13°$ ◀

PROBLEM 2.16

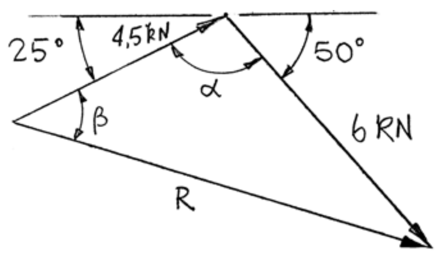


Solve Problem 2.1 using trigonometry

Problem 2.1: Two forces are applied to an eye bolt fastened to a beam. Determine graphically the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.

SOLUTION

Using the force triangle, the Law of Cosines and the Law of Sines



We have: $$\alpha = 180° - (50° + 25°)$$
$$= 105°$$

Then: $$R^2 = (4.5\text{ kN})^2 + (6\text{ kN})^2 - 2(4.5\text{ kN})(6\text{ kN})\cos 105°$$
$$= 70.226\text{ kN}^2$$

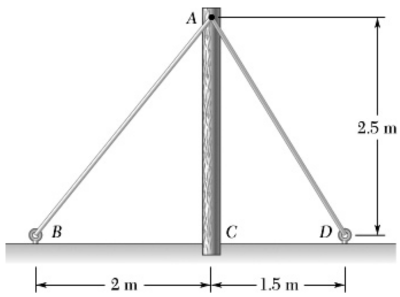
or $$R = 8.3801\text{ kN}$$

Now: $$\frac{8.3801\text{ kN}}{\sin 105°} = \frac{6\text{ kN}}{\sin\beta}$$

$$\sin\beta = \left(\frac{6\text{ kN}}{8.3801\text{ kN}}\right)\sin 105°$$
$$= 0.6916$$
$$\beta = 43.756°$$

$$\mathbf{R} = 8.38\text{ kN} \searrow 18.76° \blacktriangleleft$$

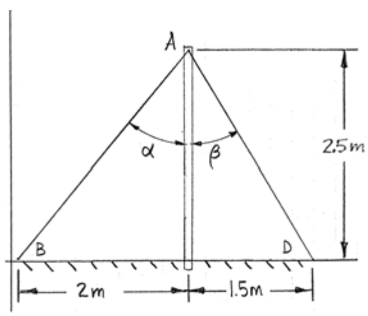
PROBLEM 2.17



Solve Problem 2.2 using trigonometry

Problem 2.2: The cable stays *AB* and *AD* help support pole *AC*. Knowing that the tension is 500 N in *AB* and 160 N in *AD*, determine graphically the magnitude and direction of the resultant of the forces exerted by the stays at *A* using (*a*) the parallelogram law, (*b*) the triangle rule.

SOLUTION



From the geometry of the problem:

$$\alpha = \tan^{-1}\frac{2}{2.5} = 38.66°$$

$$\beta = \tan^{-1}\frac{1.5}{2.5} = 30.96°$$

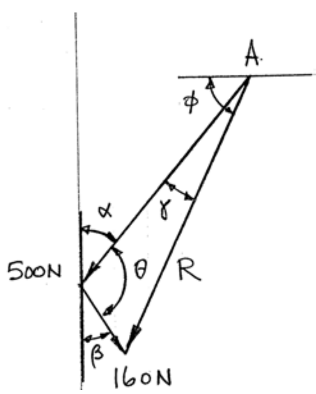
Now: $\theta = 180° - (38.66 + 30.96°) = 110.38$

And, using the Law of Cosines:

$$R^2 = (500\text{ N})^2 + (160\text{ N})^2 - 2(500\text{ N})(160\text{ N})\cos 110.38°$$

$$= 331319\text{ N}^2$$

$$R = 575.6\text{ N}$$



Using the Law of Sines:

$$\frac{160\text{ N}}{\sin\gamma} = \frac{575.6\text{ N}}{\sin 110.38°}$$

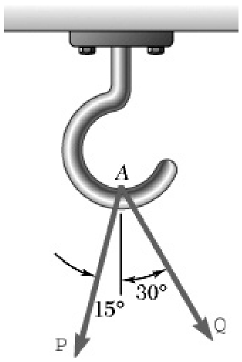
$$\sin\gamma = \left(\frac{160\text{ N}}{575.6\text{ N}}\right)\sin 110.38°$$

$$= 0.2606$$

$$\gamma = 15.1°$$

$$\phi = (90° - \alpha) + \gamma = 66.44°$$

R = 576 N ⦩ 66.4° ◀

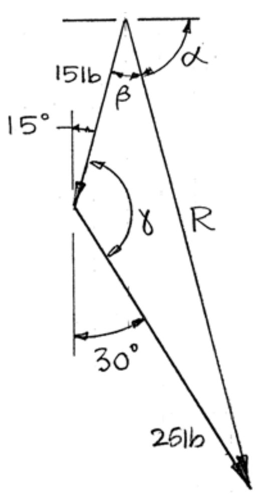


PROBLEM 2.18

Solve Problem 2.3 using trigonometry

Problem 2.3: Two forces **P** and **Q** are applied as shown at point *A* of a hook support. Knowing that $P = 15$ lb and $Q = 25$ lb, determine graphically the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.

SOLUTION



Using the force triangle and the Laws of Cosines and Sines

We have:

$$\gamma = 180° - (15° + 30°)$$

$$= 135°$$

Then: $$R^2 = (15\text{ lb})^2 + (25\text{ lb})^2 - 2(15\text{ lb})(25\text{ lb})\cos 135°$$

$$= 1380.3\text{ lb}^2$$

or $$R = 37.15\text{ lb}$$

and

$$\frac{25\text{ lb}}{\sin\beta} = \frac{37.15\text{ lb}}{\sin 135°}$$

$$\sin\beta = \left(\frac{25\text{ lb}}{37.15\text{ lb}}\right)\sin 135°$$

$$= 0.4758$$

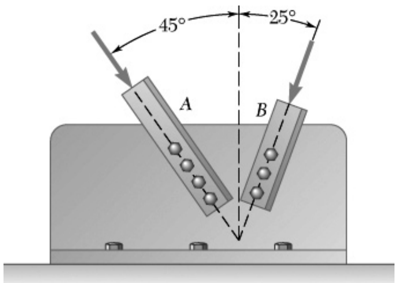
$$\beta = 28.41°$$

Then: $$\alpha + \beta + 75° = 180°$$

$$\alpha = 76.59°$$

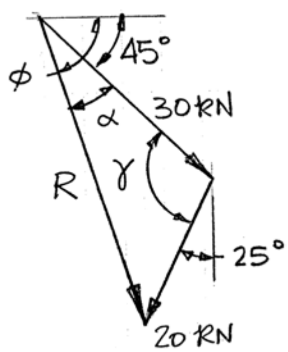
$$\mathbf{R} = 37.2\text{ lb} \;\nwarrow\; 76.6° \blacktriangleleft$$

PROBLEM 2.19



Two structural members *A* and *B* are bolted to a bracket as shown. Knowing that both members are in compression and that the force is 30 kN in member *A* and 20 kN in member *B*, determine, using trigonometry, the magnitude and direction of the resultant of the forces applied to the bracket by members *A* and *B*.

SOLUTION



Using the force triangle and the Laws of Cosines and Sines

We have: $\gamma = 180° - (45° + 25°) = 110°$

Then: $R^2 = (30\text{ kN})^2 + (20\text{ kN})^2 - 2(30\text{ kN})(20\text{ kN})\cos 110°$

$= 1710.4\text{ kN}^2$

$R = 41.357\text{ kN}$

and

$$\frac{20\text{ kN}}{\sin\alpha} = \frac{41.357\text{ kN}}{\sin 110°}$$

$$\sin\alpha = \left(\frac{20\text{ kN}}{41.357\text{ kN}}\right)\sin 110°$$

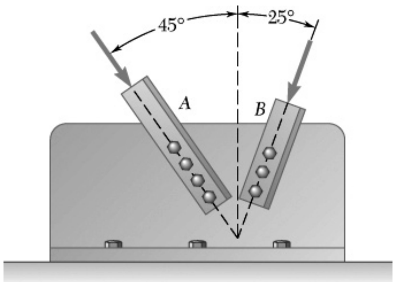
$$= 0.4544$$

$$\alpha = 27.028°$$

Hence: $\phi = \alpha + 45° = 72.028°$

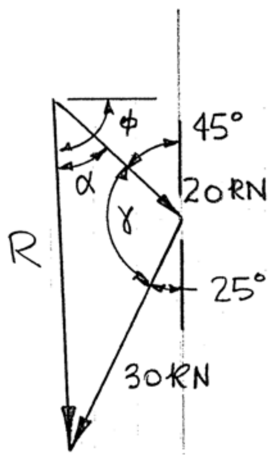
$\mathbf{R} = 41.4\text{ kN} \nwarrow 72.0°$ ◀

PROBLEM 2.20



Two structural members A and B are bolted to a bracket as shown. Knowing that both members are in compression and that the force is 20 kN in member A and 30 kN in member B, determine, using trigonometry, the magnitude and direction of the resultant of the forces applied to the bracket by members A and B.

SOLUTION



Using the force triangle and the Laws of Cosines and Sines

We have: $\gamma = 180° - (45° + 25°) = 110°$

Then: $R^2 = (30\text{ kN})^2 + (20\text{ kN})^2 - 2(30\text{ kN})(20\text{ kN})\cos 110°$

$$= 1710.4\text{ kN}^2$$

$$R = 41.357\text{ kN}$$

and

$$\frac{30\text{ kN}}{\sin\alpha} = \frac{41.357\text{ kN}}{\sin 110°}$$

$$\sin\alpha = \left(\frac{30\text{ kN}}{41.357\text{ kN}}\right)\sin 110°$$

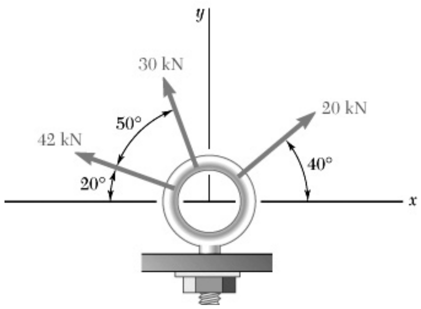
$$= 0.6816$$

$$\alpha = 42.97°$$

Finally: $\phi = \alpha + 45° = 87.97°$

$\mathbf{R} = 41.4\text{ kN} \;\nwarrow\; 88.0°$ ◀

PROBLEM 2.21



Determine the x and y components of each of the forces shown.

SOLUTION

20 kN Force:

$$F_x = +(20\text{ kN})\cos 40°, \qquad F_x = 15.32\text{ kN} ◀$$

$$F_y = +(20\text{ kN})\sin 40°, \qquad F_y = 12.86\text{ kN} ◀$$

30 kN Force:

$$F_x = -(30\text{ kN})\cos 70°, \qquad F_x = -10.26\text{ kN} ◀$$

$$F_y = +(30\text{ kN})\sin 70°, \qquad F_y = 28.2\text{ kN} ◀$$

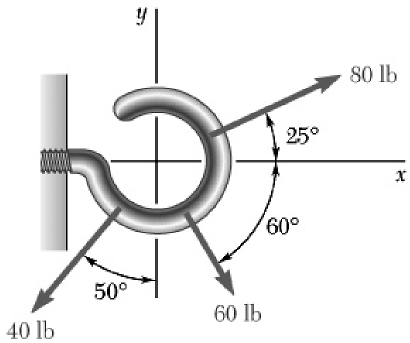
42 kN Force:

$$F_x = -(42\text{ kN})\cos 20°, \qquad F_x = -39.5\text{ kN} ◀$$

$$F_y = +(42\text{ kN})\sin 20°, \qquad F_y = 14.36\text{ kN} ◀$$

PROBLEM 2.22

Determine the x and y components of each of the forces shown.



SOLUTION

40 lb Force:

$$F_x = -(40 \text{ lb})\sin 50°, \qquad F_x = -30.6 \text{ lb} \blacktriangleleft$$

$$F_y = -(40 \text{ lb})\cos 50°, \qquad F_y = -25.7 \text{ lb} \blacktriangleleft$$

60 lb Force:

$$F_x = +(60 \text{ lb})\cos 60°, \qquad F_x = 30.0 \text{ lb} \blacktriangleleft$$

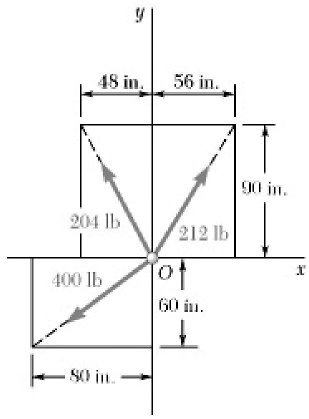
$$F_y = -(60 \text{ lb})\sin 60°, \qquad F_y = -52.0 \text{ lb} \blacktriangleleft$$

80 lb Force:

$$F_x = +(80 \text{ lb})\cos 25°, \qquad F_x = 72.5 \text{ lb} \blacktriangleleft$$

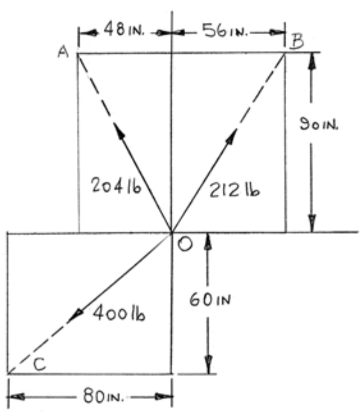
$$F_y = +(80 \text{ lb})\sin 25°, \qquad F_y = 33.8 \text{ lb} \blacktriangleleft$$

PROBLEM 2.23



Determine the x and y components of each of the forces shown.

SOLUTION



We compute the following distances:

$$OA = \sqrt{(48)^2 + (90)^2} = 102 \text{ in.}$$

$$OB = \sqrt{(56)^2 + (90)^2} = 106 \text{ in.}$$

$$OC = \sqrt{(80)^2 + (60)^2} = 100 \text{ in.}$$

Then:

204 lb Force:

$$F_x = -(102 \text{ lb})\frac{48}{102}, \qquad F_x = -48.0 \text{ lb} \blacktriangleleft$$

$$F_y = +(102 \text{ lb})\frac{90}{102}, \qquad F_y = 90.0 \text{ lb} \blacktriangleleft$$

212 lb Force:

$$F_x = +(212 \text{ lb})\frac{56}{106}, \qquad F_x = 112.0 \text{ lb} \blacktriangleleft$$

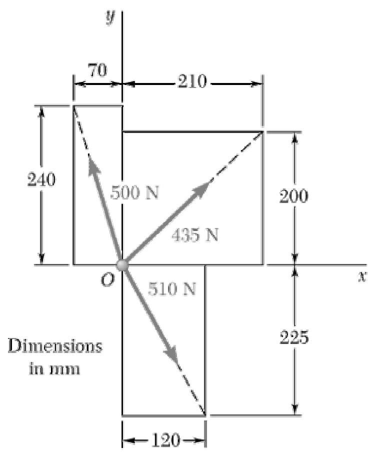
$$F_y = +(212 \text{ lb})\frac{90}{106}, \qquad F_y = 180.0 \text{ lb} \blacktriangleleft$$

400 lb Force:

$$F_x = -(400 \text{ lb})\frac{80}{100}, \qquad F_x = -320 \text{ lb} \blacktriangleleft$$

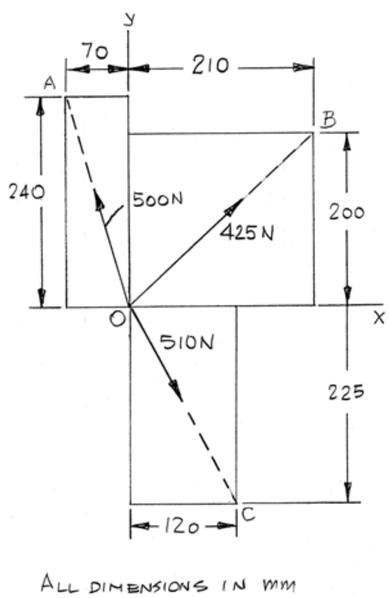
$$F_y = -(400 \text{ lb})\frac{60}{100}, \qquad F_y = -240 \text{ lb} \blacktriangleleft$$

PROBLEM 2.24



Determine the x and y components of each of the forces shown.

SOLUTION



We compute the following distances:

$$OA = \sqrt{(70)^2 + (240)^2} = 250 \text{ mm}$$

$$OB = \sqrt{(210)^2 + (200)^2} = 290 \text{ mm}$$

$$OC = \sqrt{(120)^2 + (225)^2} = 255 \text{ mm}$$

500 N Force:

$$F_x = -500 \text{ N}\left(\frac{70}{250}\right) \qquad F_x = -140.0 \text{ N} \blacktriangleleft$$

$$F_y = +500 \text{ N}\left(\frac{240}{250}\right) \qquad F_y = 480 \text{ N} \blacktriangleleft$$

435 N Force:

$$F_x = +435 \text{ N}\left(\frac{210}{290}\right) \qquad F_x = 315 \text{ N} \blacktriangleleft$$

$$F_y = +435 \text{ N}\left(\frac{200}{290}\right) \qquad F_y = 300 \text{ N} \blacktriangleleft$$

510 N Force:

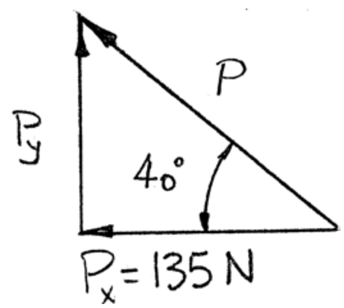
$$F_x = +510 \text{ N}\left(\frac{120}{255}\right) \qquad F_x = 240 \text{ N} \blacktriangleleft$$

$$F_y = -510 \text{ N}\left(\frac{225}{255}\right) \qquad F_y = -450 \text{ N} \blacktriangleleft$$

PROBLEM 2.25

While emptying a wheelbarrow, a gardener exerts on each handle *AB* a force **P** directed along line *CD*. Knowing that **P** must have a 135-N horizontal component, determine (*a*) the magnitude of the force **P**, (*b*) its vertical component.

SOLUTION



(*a*)

$$P = \frac{P_x}{\cos 40°}$$

$$= \frac{135 \text{ N}}{\cos 40°}$$

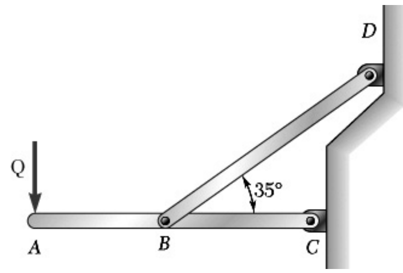
or $P = 176.2$ N ◀

(*b*)

$$P_y = P_x \tan 40° = P \sin 40°$$

$$= (135 \text{ N}) \tan 40°$$

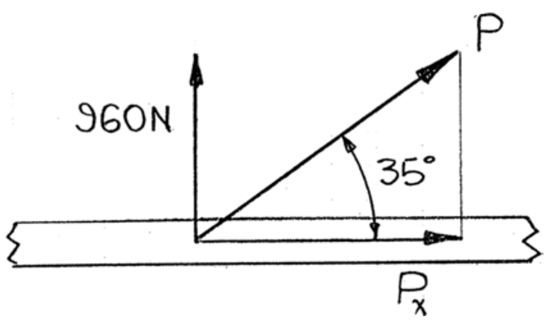
or $P_y = 113.3$ N ◀



PROBLEM 2.26

Member *BD* exerts on member *ABC* a force **P** directed along line *BD*. Knowing that **P** must have a 960-N vertical component, determine (*a*) the magnitude of the force **P**, (*b*) its horizontal component.

SOLUTION



(*a*)

$$P = \frac{P_y}{\sin 35°}$$

$$= \frac{960 \text{ N}}{\sin 35°}$$

or $P = 1674 \text{ N}$ ◀

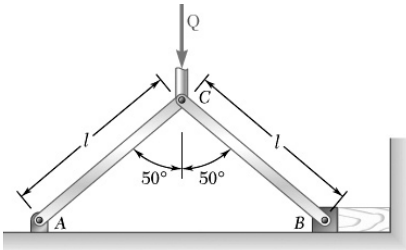
(*b*)

$$P_x = \frac{P_y}{\tan 35°}$$

$$= \frac{960 \text{ N}}{\tan 35°}$$

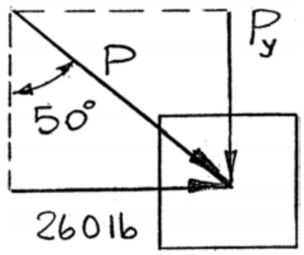
or $P_x = 1371 \text{ N}$ ◀

PROBLEM 2.27



Member *CB* of the vise shown exerts on block *B* a force **P** directed along line *CB*. Knowing that **P** must have a 260-lb horizontal component, determine (*a*) the magnitude of the force **P**, (*b*) its vertical component.

SOLUTION



We note:

CB exerts force **P** on *B* along *CB*, and the horizontal component of **P** is $P_x = 260$ lb.

Then:

(*a*)

$$P_x = P\sin 50°$$

$$P = \frac{P_x}{\sin 50°}$$

$$= \frac{260\text{ lb}}{\sin 50°}$$

$$= 339.4\text{ lb}$$

$P = 339$ lb ◀

(*b*)

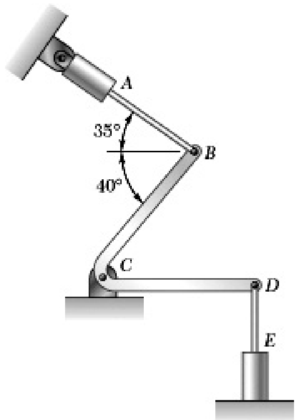
$$P_x = P_y \tan 50°$$

$$P_y = \frac{P_x}{\tan 50°}$$

$$= \frac{260\text{ lb}}{\tan 50°}$$

$$= 218.2\text{ lb}$$

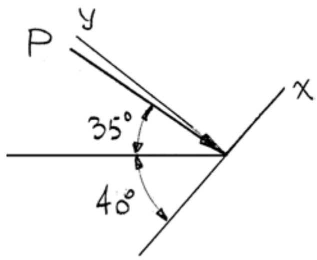
$\mathbf{P}_y = 218$ lb ↓ ◀

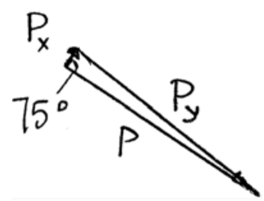


PROBLEM 2.28

Activator rod *AB* exerts on crank *BCD* a force **P** directed along line *AB*. Knowing that **P** must have a 25-lb component perpendicular to arm *BC* of the crank, determine (*a*) the magnitude of the force **P**, (*b*) its component along line *BC*.

SOLUTION





Using the *x* and *y* axes shown.

(*a*)

$$P_y = 25 \text{ lb}$$

Then:

$$P = \frac{P_y}{\sin 75°}$$

$$= \frac{25 \text{ lb}}{\sin 75°}$$

or $P = 25.9 \text{ lb}$ ◀

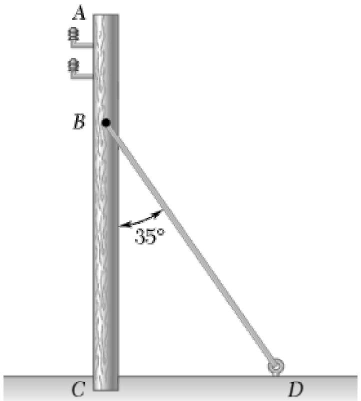
(*b*)

$$P_x = \frac{P_y}{\tan 75°}$$

$$= \frac{25 \text{ lb}}{\tan 75°}$$

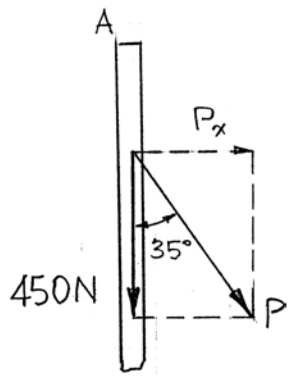
or $P_x = 6.70 \text{ lb}$ ◀

PROBLEM 2.29



The guy wire *BD* exerts on the telephone pole *AC* a force **P** directed along *BD*. Knowing that **P** has a 450-N component along line *AC*, determine (*a*) the magnitude of the force **P**, (*b*) its component in a direction perpendicular to *AC*.

SOLUTION



Note that the force exerted by *BD* on the pole is directed along *BD*, and the component of *P* along *AC* is 450 N.

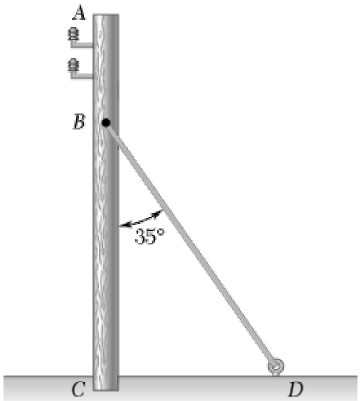
Then:

(*a*)
$$P = \frac{450\text{ N}}{\cos 35°} = 549.3\text{ N}$$

$$P = 549\text{ N} \blacktriangleleft$$

(*b*)
$$P_x = (450\text{ N})\tan 35°$$
$$= 315.1\text{ N}$$

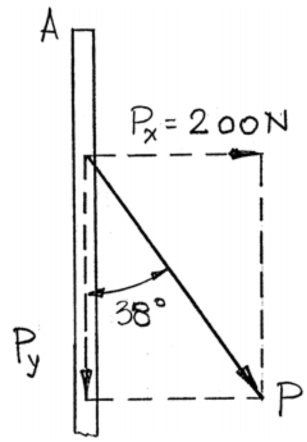
$$P_x = 315\text{ N} \blacktriangleleft$$



PROBLEM 2.30

The guy wire *BD* exerts on the telephone pole *AC* a force **P** directed along *BD*. Knowing that **P** has a 200-N perpendicular to the pole *AC*, determine (*a*) the magnitude of the force **P**, (*b*) its component along line *AC*.

SOLUTION



(*a*)

$$P = \frac{P_x}{\sin 38°}$$

$$= \frac{200 \text{ N}}{\sin 38°}$$

$$= 324.8 \text{ N}$$

or $P = 325$ N ◀

(*b*)

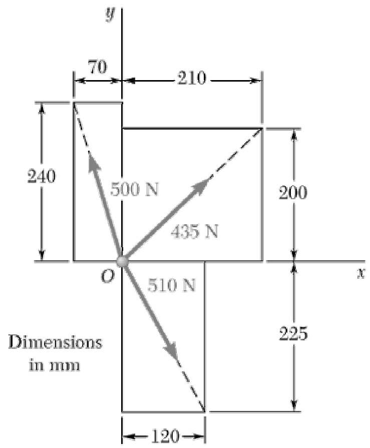
$$P_y = \frac{P_x}{\tan 38°}$$

$$= \frac{200 \text{ N}}{\tan 38°}$$

$$= 255.98 \text{ N}$$

or $P_y = 256$ N ◀

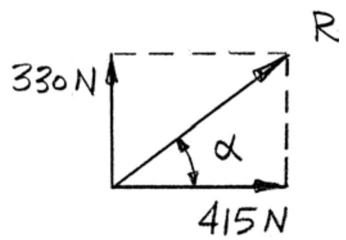
PROBLEM 2.31



Determine the resultant of the three forces of Problem 2.24.

Problem 2.24: Determine the x and y components of each of the forces shown.

SOLUTION



From Problem 2.24:

$$\mathbf{F}_{500} = -(140\text{ N})\mathbf{i} + (480\text{ N})\mathbf{j}$$

$$\mathbf{F}_{425} = (315\text{ N})\mathbf{i} + (300\text{ N})\mathbf{j}$$

$$\mathbf{F}_{510} = (240\text{ N})\mathbf{i} - (450\text{ N})\mathbf{j}$$

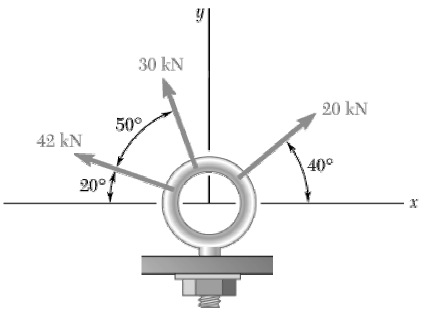
$$\mathbf{R} = \Sigma\mathbf{F} = (415\text{ N})\mathbf{i} + (330\text{ N})\mathbf{j}$$

Then:

$$\alpha = \tan^{-1}\frac{330}{415} = 38.5°$$

$$R = \sqrt{(415\text{ N})^2 + (330\text{ N})^2} = 530.2\text{ N}$$

Thus: $\mathbf{R} = 530\text{ N} \measuredangle 38.5°$ ◀

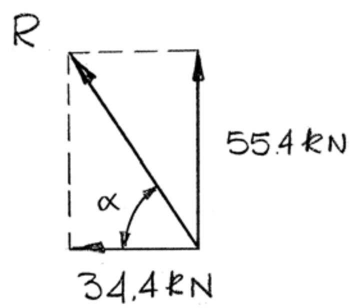


PROBLEM 2.32

Determine the resultant of the three forces of Problem 2.21.

Problem 2.21: Determine the *x* and *y* components of each of the forces shown.

SOLUTION



From Problem 2.21:

$$\mathbf{F}_{20} = (15.32 \text{ kN})\mathbf{i} + (12.86 \text{ kN})\mathbf{j}$$

$$\mathbf{F}_{30} = -(10.26 \text{ kN})\mathbf{i} + (28.2 \text{ kN})\mathbf{j}$$

$$\mathbf{F}_{42} = -(39.5 \text{ kN})\mathbf{i} + (14.36 \text{ kN})\mathbf{j}$$

$$\mathbf{R} = \Sigma\mathbf{F} = -(34.44 \text{ kN})\mathbf{i} + (55.42 \text{ kN})\mathbf{j}$$

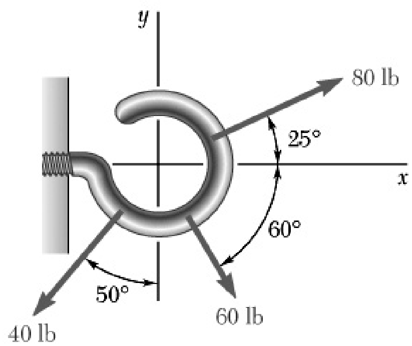
Then:

$$\alpha = \tan^{-1}\frac{55.42}{-34.44} = 58.1^\circ$$

$$R = \sqrt{(55.42 \text{ kN})^2 + (-34.44 \text{ N})^2} = 65.2 \text{ kN}$$

$R = 65.2$ kN ⦪ $58.2°$ ◀

PROBLEM 2.33



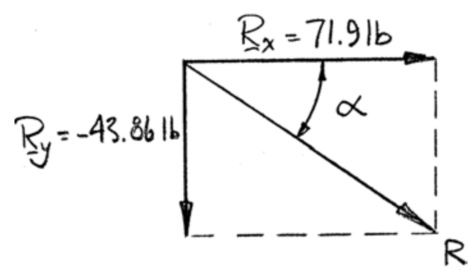
Determine the resultant of the three forces of Problem 2.22.

Problem 2.22: Determine the *x* and *y* components of each of the forces shown.

SOLUTION

The components of the forces were determined in 2.23.

Force	*x* comp. (lb)	*y* comp. (lb)
40 lb	–30.6	–25.7
60 lb	30	–51.96
80 lb	72.5	33.8
	$R_x = 71.9$	$R_y = -43.86$



$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j}$$

$$= (71.9 \text{ lb})\mathbf{i} - (43.86 \text{ lb})\mathbf{j}$$

$$\tan\alpha = \frac{43.86}{71.9}$$

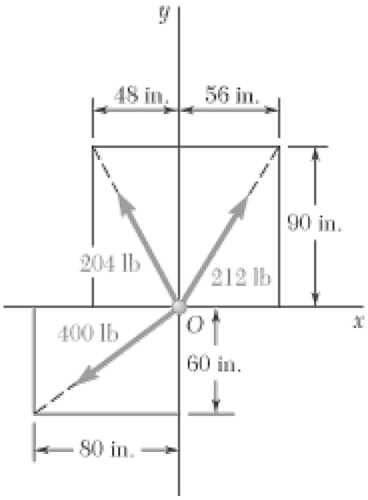
$$\alpha = 31.38°$$

$$R = \sqrt{(71.9 \text{ lb})^2 + (-43.86 \text{ lb})^2}$$

$$= 84.23 \text{ lb}$$

$\mathbf{R} = 84.2$ lb ⦫ 31.4° ◀

PROBLEM 2.34



Determine the resultant of the three forces of Problem 2.23.

Problem 2.23: Determine the x and y components of each of the forces shown.

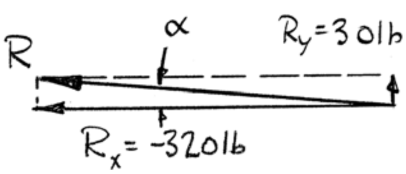
SOLUTION

The components of the forces were determined in Problem 2.23.

$$\mathbf{F}_{204} = -(48.0 \text{ lb})\mathbf{i} + (90.0 \text{ lb})\mathbf{j}$$

$$\mathbf{F}_{212} = (112.0 \text{ lb})\mathbf{i} + (180.0 \text{ lb})\mathbf{j}$$

$$\mathbf{F}_{400} = -(320 \text{ lb})\mathbf{i} - (240 \text{ lb})\mathbf{j}$$



Thus

$$\mathbf{R} = \mathbf{R}_x + \mathbf{R}_y$$

$$\mathbf{R} = -(256 \text{ lb})\mathbf{i} + (30.0 \text{ lb})\mathbf{j}$$

Now:

$$\tan\alpha = \frac{30.0}{256}$$

$$\alpha = \tan^{-1}\frac{30.0}{256} = 6.68°$$

and

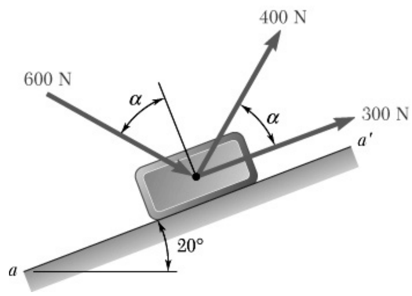
$$R = \sqrt{(-256 \text{ lb})^2 + (30.0 \text{ lb})^2}$$

$$= 257.75 \text{ lb}$$

$$\mathbf{R} = 258 \text{ lb} \quad 6.68° ◀$$

PROBLEM 2.35

Knowing that $\alpha = 35°$, determine the resultant of the three forces shown.



SOLUTION

300-N Force:

$$F_x = (300\text{ N})\cos 20° = 281.9\text{ N}$$

$$F_y = (300\text{ N})\sin 20° = 102.6\text{ N}$$

400-N Force:

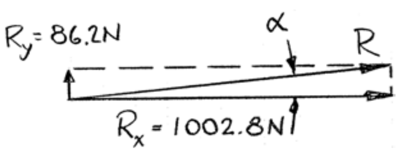
$$F_x = (400\text{ N})\cos 55° = 229.4\text{ N}$$

$$F_y = (400\text{ N})\sin 55° = 327.7\text{ N}$$

600-N Force:

$$F_x = (600\text{ N})\cos 35° = 491.5\text{ N}$$

$$F_y = -(600\text{ N})\sin 35° = -344.1\text{ N}$$



and

$$R_x = \Sigma F_x = 1002.8\text{ N}$$

$$R_y = \Sigma F_y = 86.2\text{ N}$$

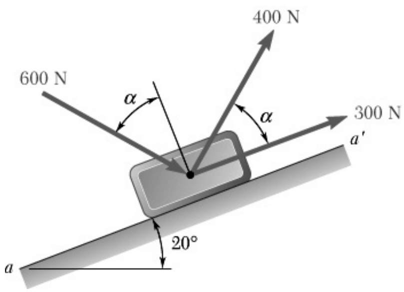
$$R = \sqrt{(1002.8\text{ N})^2 + (86.2\text{ N})^2} = 1006.5\text{ N}$$

Further:

$$\tan\alpha = \frac{86.2}{1002.8}$$

$$\alpha = \tan^{-1}\frac{86.2}{1002.8} = 4.91°$$

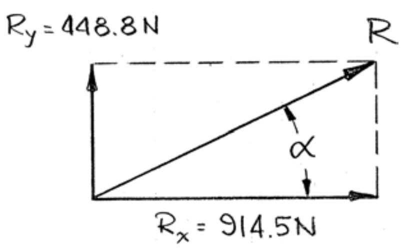
$\mathbf{R} = 1007\text{ N} \measuredangle\ 4.91°$ ◀



PROBLEM 2.36

Knowing that $\alpha = 65°$, determine the resultant of the three forces shown.

SOLUTION



300-N Force:

$$F_x = (300 \text{ N})\cos 20° = 281.9 \text{ N}$$

$$F_y = (300 \text{ N})\sin 20° = 102.6 \text{ N}$$

400-N Force:

$$F_x = (400 \text{ N})\cos 85° = 34.9 \text{ N}$$

$$F_y = (400 \text{ N})\sin 85° = 398.5 \text{ N}$$

600-N Force:

$$F_x = (600 \text{ N})\cos 5° = 597.7 \text{ N}$$

$$F_y = -(600 \text{ N})\sin 5° = -52.3 \text{ N}$$

and

$$R_x = \Sigma F_x = 914.5 \text{ N}$$

$$R_y = \Sigma F_y = 448.8 \text{ N}$$

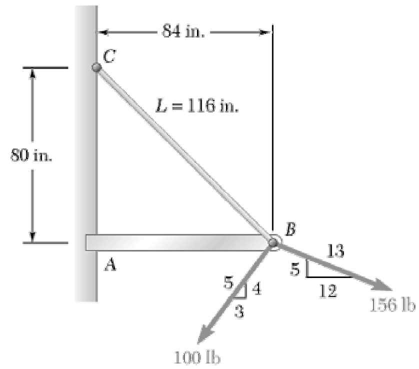
$$R = \sqrt{(914.5 \text{ N})^2 + (448.8 \text{ N})^2} = 1018.7 \text{ N}$$

Further:

$$\tan\alpha = \frac{448.8}{914.5}$$

$$\alpha = \tan^{-1}\frac{448.8}{914.5} = 26.1°$$

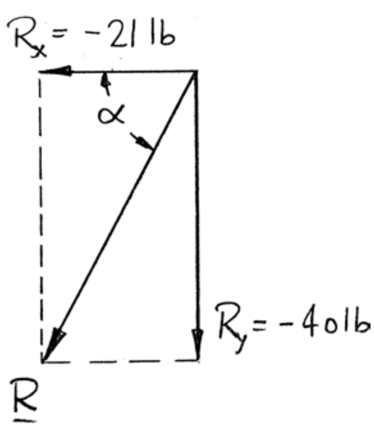
$\mathbf{R} = 1019 \text{ N} \measuredangle 26.1°$ ◀



PROBLEM 2.37

Knowing that the tension in cable BC is 145 lb, determine the resultant of the three forces exerted at point B of beam AB.

SOLUTION



Cable BC Force:

$$F_x = -(145 \text{ lb})\frac{84}{116} = -105 \text{ lb}$$

$$F_y = (145 \text{ lb})\frac{80}{116} = 100 \text{ lb}$$

100-lb Force:

$$F_x = -(100 \text{ lb})\frac{3}{5} = -60 \text{ lb}$$

$$F_y = -(100 \text{ lb})\frac{4}{5} = -80 \text{ lb}$$

156-lb Force:

$$F_x = (156 \text{ lb})\frac{12}{13} = 144 \text{ lb}$$

$$F_y = -(156 \text{ lb})\frac{5}{13} = -60 \text{ lb}$$

and

$$R_x = \Sigma F_x = -21 \text{ lb}, \qquad R_y = \Sigma F_y = -40 \text{ lb}$$

$$R = \sqrt{(-21 \text{ lb})^2 + (-40 \text{ lb})^2} = 45.177 \text{ lb}$$

Further:

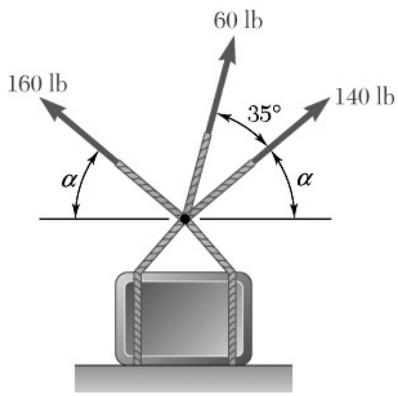
$$\tan\alpha = \frac{40}{21}$$

$$\alpha = \tan^{-1}\frac{40}{21} = 62.3°$$

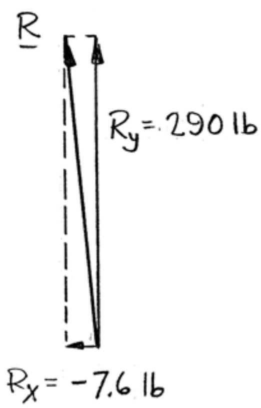
Thus: $\mathbf{R} = 45.2 \text{ lb} \measuredangle 62.3°$ ◀

PROBLEM 2.38

Knowing that $\alpha = 50°$, determine the resultant of the three forces shown.



SOLUTION



The resultant force R has the x- and y-components:

$$R_x = \Sigma F_x = (140 \text{ lb})\cos 50° + (60 \text{ lb})\cos 85° - (160 \text{ lb})\cos 50°$$

$$R_x = -7.6264 \text{ lb}$$

and

$$R_y = \Sigma F_y = (140 \text{ lb})\sin 50° + (60 \text{ lb})\sin 85° + (160 \text{ lb})\sin 50°$$

$$R_y = 289.59 \text{ lb}$$

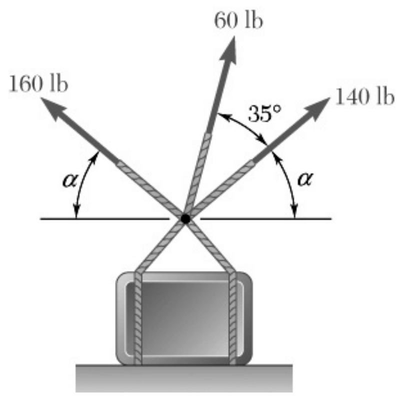
Further:

$$\tan\alpha = \frac{290}{7.6}$$

$$\alpha = \tan^{-1}\frac{290}{7.6} = 88.5°$$

Thus: $\mathbf{R} = 290 \text{ lb} \measuredangle 88.5°$ ◀

PROBLEM 2.39



Determine (*a*) the required value of α if the resultant of the three forces shown is to be vertical, (*b*) the corresponding magnitude of the resultant.

SOLUTION

For an arbitrary angle α, we have:

$$R_x = \Sigma F_x = (140 \text{ lb})\cos\alpha + (60 \text{ lb})\cos(\alpha + 35°) - (160 \text{ lb})\cos\alpha$$

(*a*) So, for *R* to be vertical:

$$R_x = \Sigma F_x = (140 \text{ lb})\cos\alpha + (60 \text{ lb})\cos(\alpha + 35°) - (160 \text{ lb})\cos\alpha = 0$$

Expanding,

$$-\cos\alpha + 3(\cos\alpha\cos 35° - \sin\alpha\sin 35°) = 0$$

Then:

$$\tan\alpha = \frac{\cos 35° - \frac{1}{3}}{\sin 35°}$$

or

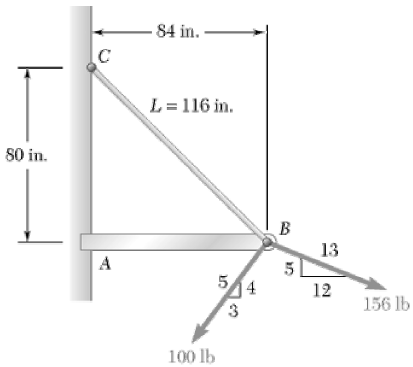
$$\alpha = \tan^{-1}\left(\frac{\cos 35° - \frac{1}{3}}{\sin 35°}\right) = 40.265° \qquad \alpha = 40.3° \blacktriangleleft$$

(*b*) Now:

$$R = R_y = \Sigma F_y = (140 \text{ lb})\sin 40.265° + (60 \text{ lb})\sin 75.265° + (160 \text{ lb})\sin 40.265°$$

$$R = |R| = 252 \text{ lb} \blacktriangleleft$$

PROBLEM 2.40



For the beam of Problem 2.37, determine (*a*) the required tension in cable *BC* if the resultant of the three forces exerted at point *B* is to be vertical, (*b*) the corresponding magnitude of the resultant.

Problem 2.37: Knowing that the tension in cable *BC* is 145 lb, determine the resultant of the three forces exerted at point *B* of beam *AB*.

SOLUTION

We have:

$$R_x = \Sigma F_x = -\frac{84}{116}T_{BC} + \frac{12}{13}(156 \text{ lb}) - \frac{3}{5}(100 \text{ lb})$$

or

$$R_x = -0.724T_{BC} + 84 \text{ lb}$$

and

$$R_y = \Sigma F_y = \frac{80}{116}T_{BC} - \frac{5}{13}(156 \text{ lb}) - \frac{4}{5}(100 \text{ lb})$$

$$R_y = 0.6897T_{BC} - 140 \text{ lb}$$

(*a*) So, for *R* to be vertical,

$$R_x = -0.724T_{BC} + 84 \text{ lb} = 0$$

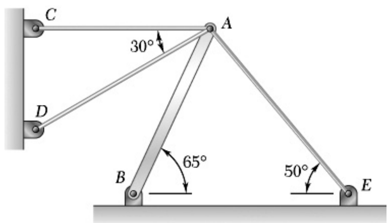
$$T_{BC} = 116.0 \text{ lb} \blacktriangleleft$$

(*b*) Using

$$T_{BC} = 116.0 \text{ lb}$$

$$R = R_y = 0.6897(116.0 \text{ lb}) - 140 \text{ lb} = -60 \text{ lb}$$

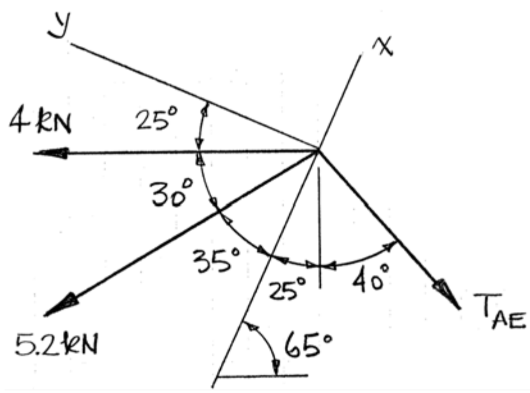
$$R = |R| = 60.0 \text{ lb} \blacktriangleleft$$



PROBLEM 2.41

Boom *AB* is held in the position shown by three cables. Knowing that the tensions in cables *AC* and *AD* are 4 kN and 5.2 kN, respectively, determine (*a*) the tension in cable *AE* if the resultant of the tensions exerted at point *A* of the boom must be directed along *AB*, (*b*) the corresponding magnitude of the resultant.

SOLUTION



Choose *x*-axis along bar *AB*.

Then

(*a*) Require

$$R_y = \Sigma F_y = 0: \quad (4 \text{ kN})\cos 25° + (5.2 \text{ kN})\sin 35° - T_{AE}\sin 65° = 0$$

or

$$T_{AE} = 7.2909 \text{ kN}$$

$$T_{AE} = 7.29 \text{ kN} \blacktriangleleft$$

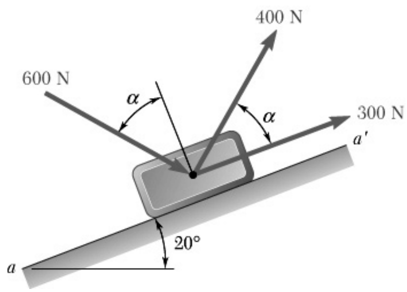
(*b*)

$$R = \Sigma F_x$$

$$= -(4 \text{ kN})\sin 25° - (5.2 \text{ kN})\cos 35° - (7.2909 \text{ kN})\cos 65°$$

$$= -9.03 \text{ kN}$$

$$|R| = 9.03 \text{ kN} \blacktriangleleft$$



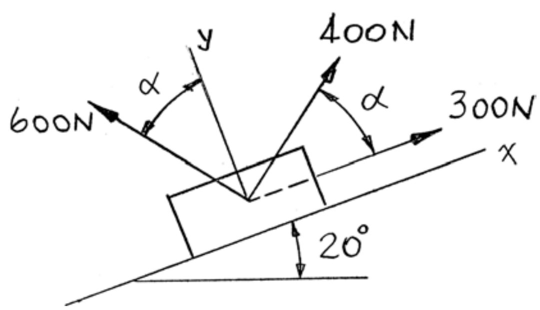
PROBLEM 2.42

For the block of Problems 2.35 and 2.36, determine (*a*) the required value of α of the resultant of the three forces shown is to be parallel to the incline, (*b*) the corresponding magnitude of the resultant.

Problem 2.35: Knowing that $\alpha = 35°$, determine the resultant of the three forces shown.

Problem 2.36: Knowing that $\alpha = 65°$, determine the resultant of the three forces shown.

SOLUTION



Selecting the *x* axis along aa', we write

$$R_x = \Sigma F_x = 300 \text{ N} + (400 \text{ N})\cos\alpha + (600 \text{ N})\sin\alpha \qquad (1)$$

$$R_y = \Sigma F_y = (400 \text{ N})\sin\alpha - (600 \text{ N})\cos\alpha \qquad (2)$$

(*a*) Setting $R_y = 0$ in Equation (2):

Thus $$\tan\alpha = \frac{600}{400} = 1.5$$

$$\alpha = 56.3° \blacktriangleleft$$

(*b*) Substituting for α in Equation (1):

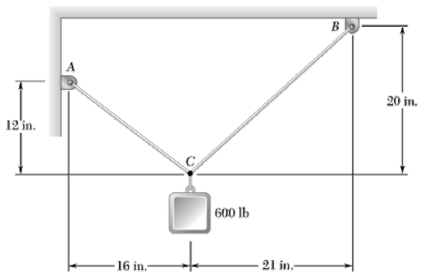
$$R_x = 300 \text{ N} + (400 \text{ N})\cos 56.3° + (600 \text{ N})\sin 56.3°$$

$$R_x = 1021.1 \text{ N}$$

$$R = R_x = 1021 \text{ N} \blacktriangleleft$$

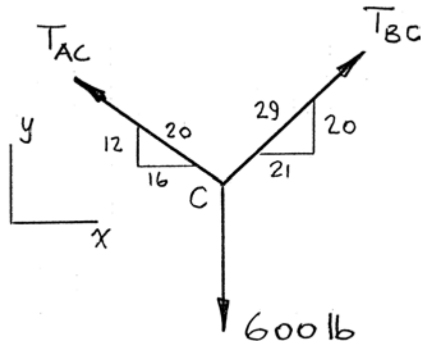
PROBLEM 2.43

Two cables are tied together at C and are loaded as shown. Determine the tension (*a*) in cable AC, (*b*) in cable BC.



SOLUTION

Free-Body Diagram



From the geometry, we calculate the distances:

$$AC = \sqrt{(16\text{ in.})^2 + (12\text{ in.})^2} = 20\text{ in.}$$

$$BC = \sqrt{(20\text{ in.})^2 + (21\text{ in.})^2} = 29\text{ in.}$$

Then, from the Free Body Diagram of point C:

$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{16}{20}T_{AC} + \frac{21}{29}T_{BC} = 0$$

or

$$T_{BC} = \frac{29}{21} \times \frac{4}{5} T_{AC}$$

and

$$+\uparrow \Sigma F_y = 0: \quad \frac{12}{20}T_{AC} + \frac{20}{29}T_{BC} - 600\text{ lb} = 0$$

or

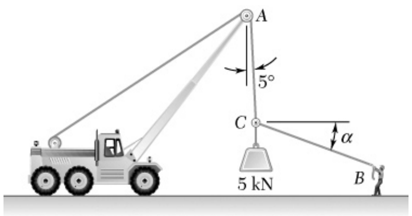
$$\frac{12}{20}T_{AC} + \frac{20}{29}\left(\frac{29}{21} \times \frac{4}{5}T_{AC}\right) - 600\text{ lb} = 0$$

Hence: $T_{AC} = 440.56$ lb

(*a*) $T_{AC} = 441$ lb ◀

(*b*) $T_{BC} = 487$ lb ◀

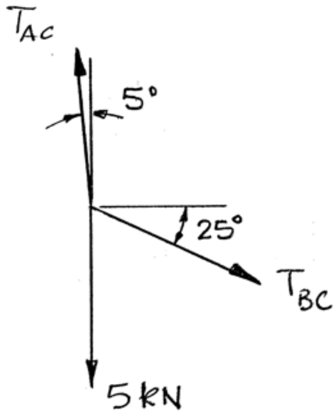
PROBLEM 2.44



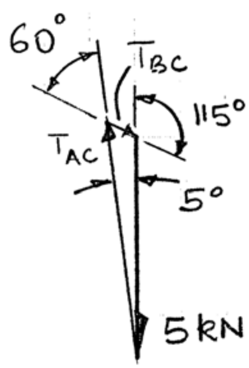
Knowing that $\alpha = 25°$, determine the tension (*a*) in cable *AC*, (*b*) in rope *BC*.

SOLUTION

Free-Body Diagram



Force Triangle



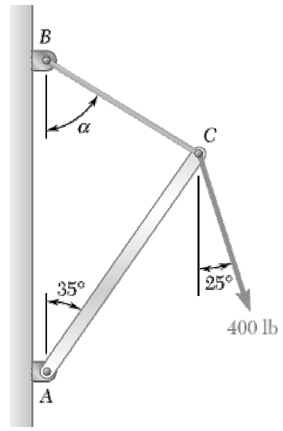
Law of Sines:

$$\frac{T_{AC}}{\sin 115°} = \frac{T_{BC}}{\sin 5°} = \frac{5\text{ kN}}{\sin 60°}$$

(*a*) $$T_{AC} = \frac{5\text{ kN}}{\sin 60°}\sin 115° = 5.23\text{ kN}$$ $T_{AC} = 5.23\text{ kN}$ ◀

(*b*) $$T_{BC} = \frac{5\text{ kN}}{\sin 60°}\sin 5° = 0.503\text{ kN}$$ $T_{BC} = 0.503\text{ kN}$ ◀

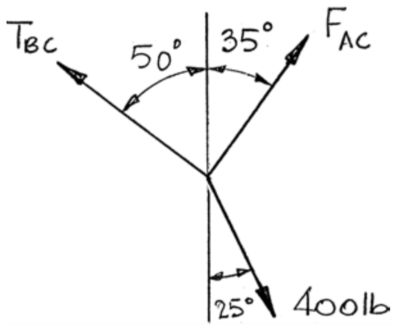
PROBLEM 2.45



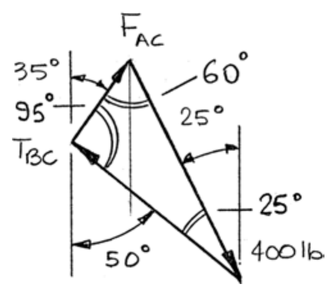
Knowing that $\alpha = 50°$ and that boom AC exerts on pin C a force directed long line AC, determine (*a*) the magnitude of that force, (*b*) the tension in cable BC.

SOLUTION

Free-Body Diagram



Force Triangle



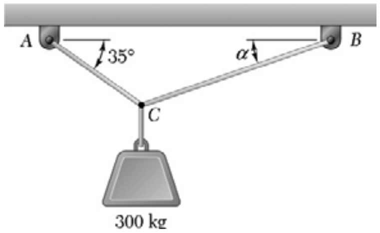
Law of Sines:

$$\frac{F_{AC}}{\sin 25°} = \frac{T_{BC}}{\sin 60°} = \frac{400 \text{ lb}}{\sin 95°}$$

(*a*) $$F_{AC} = \frac{400 \text{ lb}}{\sin 95°}\sin 25° = 169.69 \text{ lb}$$ $F_{AC} = 169.7 \text{ lb}$ ◀

(*b*) $$T_{BC} = \frac{400}{\sin 95°}\sin 60° = 347.73 \text{ lb}$$ $T_{BC} = 348 \text{ lb}$ ◀

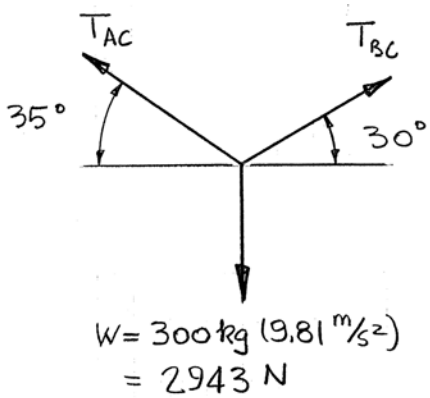
PROBLEM 2.46



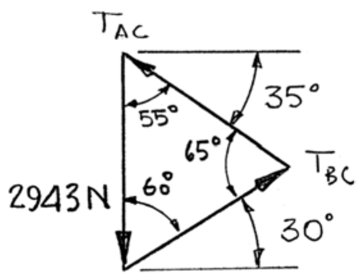
Two cables are tied together at C and are loaded as shown. Knowing that $\alpha = 30°$, determine the tension (a) in cable AC, (b) in cable BC.

SOLUTION

Free-Body Diagram



Force Triangle



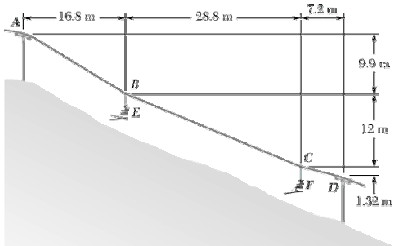
Law of Sines:

$$\frac{T_{AC}}{\sin 60°} = \frac{T_{BC}}{\sin 55°} = \frac{2943\text{ N}}{\sin 65°}$$

(a) $$T_{AC} = \frac{2943\text{ N}}{\sin 65°}\sin 60° = 2812.19\text{ N}$$ $T_{AC} = 2.81\text{ kN}$ ◀

(b) $$T_{BC} = \frac{2943\text{ N}}{\sin 65°}\sin 55° = 2659.98\text{ N}$$ $T_{BC} = 2.66\text{ kN}$ ◀

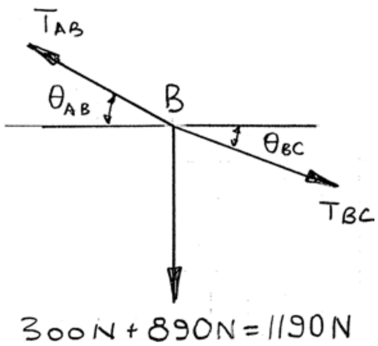
PROBLEM 2.47



A chairlift has been stopped in the position shown. Knowing that each chair weighs 300 N and that the skier in chair *E* weighs 890 N, determine that weight of the skier in chair *F*.

SOLUTION

Free-Body Diagram Point B



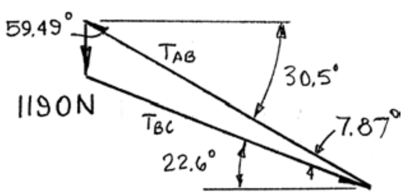
In the free-body diagram of point *B*, the geometry gives:

$$\theta_{AB} = \tan^{-1}\frac{9.9}{16.8} = 30.51°$$

$$\theta_{BC} = \tan^{-1}\frac{12}{28.8} = 22.61°$$

Thus, in the force triangle, by the Law of Sines:

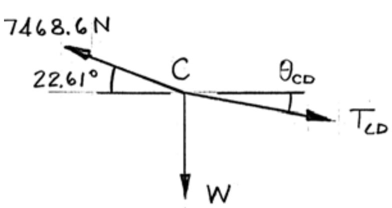
Force Triangle



$$\frac{T_{BC}}{\sin 59.49°} = \frac{1190 \text{ N}}{\sin 7.87°}$$

$$T_{BC} = 7468.6 \text{ N}$$

Free-Body Diagram Point C

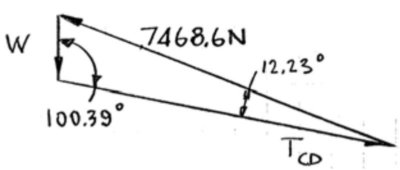


In the free-body diagram of point *C* (with *W* the sum of weights of chair and skier) the geometry gives:

$$\theta_{CD} = \tan^{-1}\frac{1.32}{7.2} = 10.39°$$

Hence, in the force triangle, by the Law of Sines:

Force Triangle



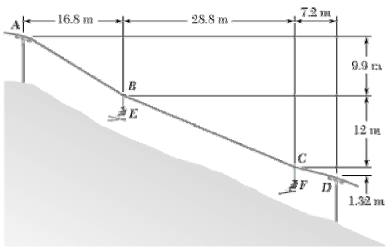
$$\frac{W}{\sin 12.23°} = \frac{7468.6 \text{ N}}{\sin 100.39°}$$

$$W = 1608.5 \text{ N}$$

Finally, the skier weight $= 1608.5 \text{ N} - 300 \text{ N} = 1308.5 \text{ N}$

skier weight $= 1309$ N ◀

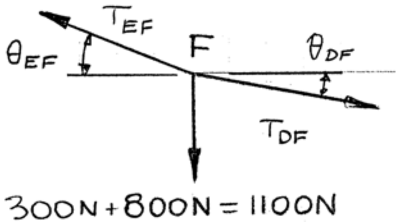
PROBLEM 2.48



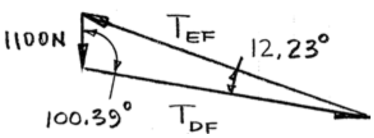
A chairlift has been stopped in the position shown. Knowing that each chair weighs 300 N and that the skier in chair *F* weighs 800 N, determine the weight of the skier in chair *E*.

SOLUTION

Free-Body Diagram Point F



Force Triangle



In the free-body diagram of point *F*, the geometry gives:

$$\theta_{EF} = \tan^{-1}\frac{12}{28.8} = 22.62°$$

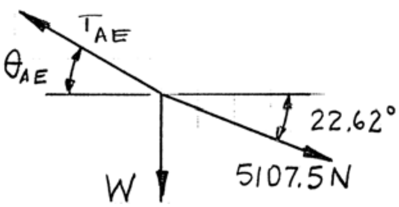
$$\theta_{DF} = \tan^{-1}\frac{1.32}{7.2} = 10.39°$$

Thus, in the force triangle, by the Law of Sines:

$$\frac{T_{EF}}{\sin 100.39°} = \frac{1100 \text{ N}}{\sin 12.23°}$$

$$T_{BC} = 5107.5 \text{ N}$$

Free-Body Diagram Point E

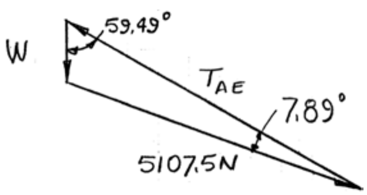


In the free-body diagram of point *E* (with *W* the sum of weights of chair and skier) the geometry gives:

$$\theta_{AE} = \tan^{-1}\frac{9.9}{16.8} = 30.51°$$

Hence, in the force triangle, by the Law of Sines:

Force Triangle



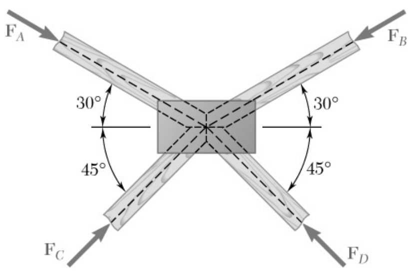
$$\frac{W}{\sin 7.89°} = \frac{5107.5 \text{ N}}{\sin 59.49°}$$

$$W = 813.8 \text{ N}$$

Finally, the skier weight $= 813.8 \text{ N} - 300 \text{ N} = 513.8 \text{ N}$

skier weight $= 514$ N ◀

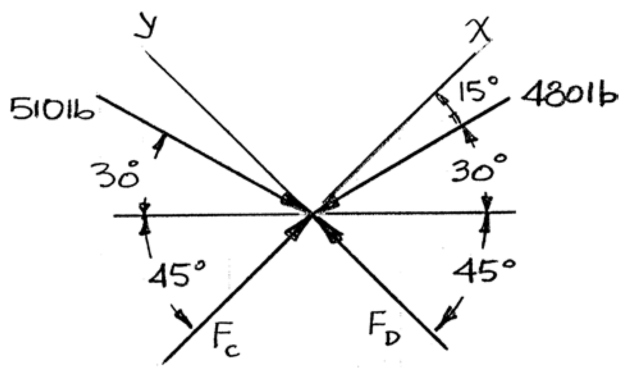
PROBLEM 2.49



Four wooden members are joined with metal plate connectors and are in equilibrium under the action of the four fences shown. Knowing that $F_A = 510$ lb and $F_B = 480$ lb, determine the magnitudes of the other two forces.

SOLUTION

Free-Body Diagram



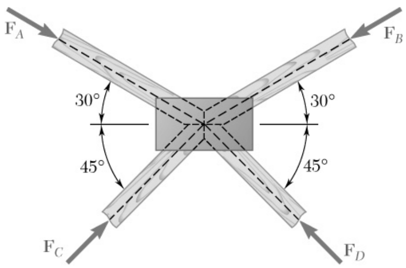
Resolving the forces into *x* and *y* components:

$$\Sigma F_x = 0: \quad F_C + (510 \text{ lb})\sin 15° - (480 \text{ lb})\cos 15° = 0$$

or $F_C = 332$ lb ◀

$$\Sigma F_y = 0: \quad F_D - (510 \text{ lb})\cos 15° + (480 \text{ lb})\sin 15° = 0$$

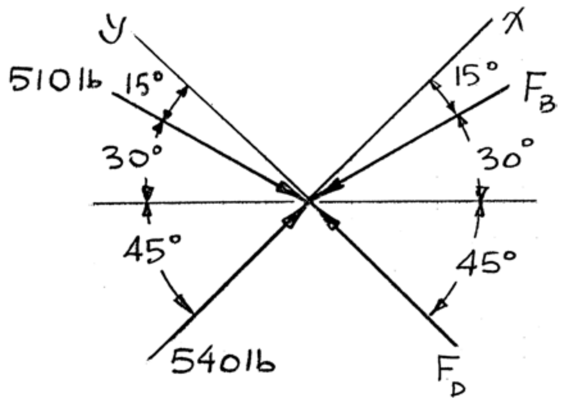
or $F_D = 368$ lb ◀



PROBLEM 2.50

Four wooden members are joined with metal plate connectors and are in equilibrium under the action of the four fences shown. Knowing that $F_A = 420$ lb and $F_C = 540$ lb, determine the magnitudes of the other two forces.

SOLUTION



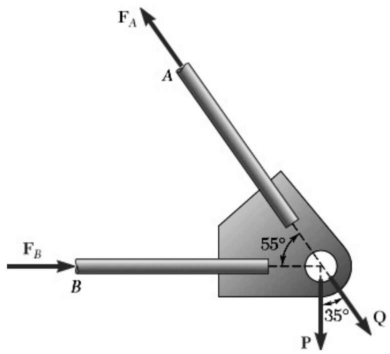
Resolving the forces into *x* and *y* components:

$$\Sigma F_x = 0: \quad -F_B \cos 15° + (540 \text{ lb}) + (420 \text{ lb})\cos 15° = 0 \qquad \text{or} \qquad F_B = 671.6 \text{ lb}$$

$$F_B = 672 \text{ lb} \blacktriangleleft$$

$$\Sigma F_y = 0: \quad F_D - (420 \text{ lb})\cos 15° + (671.6 \text{ lb})\sin 15° = 0$$

$$\text{or} \quad F_D = 232 \text{ lb} \blacktriangleleft$$

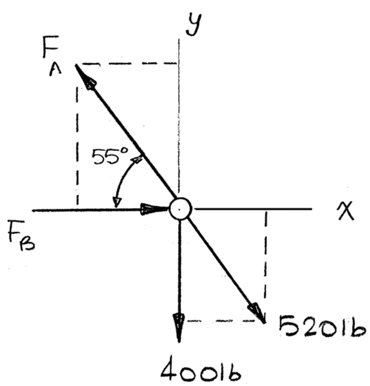


PROBLEM 2.51

Two forces **P** and **Q** are applied as shown to an aircraft connection. Knowing that the connection is in equilibrium and the $P = 400$ lb and $Q = 520$ lb, determine the magnitudes of the forces exerted on the rods A and B.

SOLUTION

Free-Body Diagram



Resolving the forces into x and y directions:

$$\mathbf{R} = \mathbf{P} + \mathbf{Q} + \mathbf{F}_A + \mathbf{F}_B = 0$$

Substituting components:

$$\mathbf{R} = -(400 \text{ lb})\mathbf{j} + \left[(520 \text{ lb})\cos 55°\right]\mathbf{i} - \left[(520 \text{ lb})\sin 55°\right]\mathbf{j}$$

$$+ F_B\mathbf{i} - (F_A \cos 55°)\mathbf{i} + (F_A \sin 55°)\mathbf{j} = 0$$

In the y-direction (one unknown force)

$$-400 \text{ lb} - (520 \text{ lb})\sin 55° + F_A \sin 55° = 0$$

Thus,

$$F_A = \frac{400 \text{ lb} + (520 \text{ lb})\sin 55°}{\sin 55°} = 1008.3 \text{ lb}$$

$$F_A = 1008 \text{ lb} \blacktriangleleft$$

In the x-direction:

$$(520 \text{ lb})\cos 55° + F_B - F_A \cos 55° = 0$$

Thus,

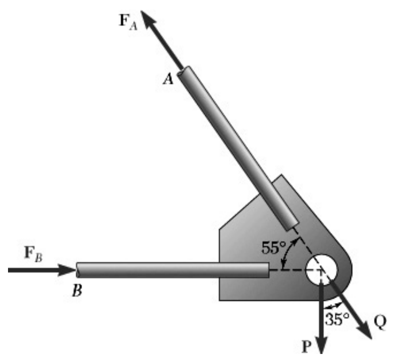
$$F_B = F_A \cos 55° - (520 \text{ lb})\cos 55°$$

$$= (1008.3 \text{ lb})\cos 55° - (520 \text{ lb})\cos 55°$$

$$= 280.08 \text{ lb}$$

$$F_B = 280 \text{ lb} \blacktriangleleft$$

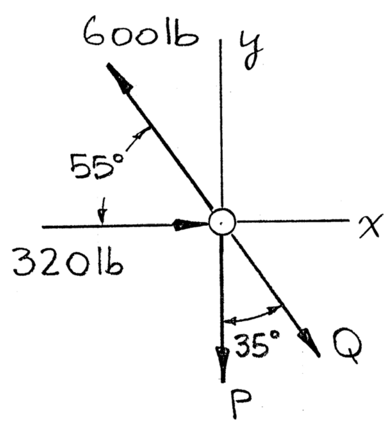
PROBLEM 2.52



Two forces **P** and **Q** are applied as shown to an aircraft connection. Knowing that the connection is in equilibrium and that the magnitudes of the forces exerted on rods A and B are $F_A = 600$ lb and $F_B = 320$ lb, determine the magnitudes of **P** and **Q**.

SOLUTION

Free-Body Diagram



Resolving the forces into x and y directions:

$$\mathbf{R} = \mathbf{P} + \mathbf{Q} + \mathbf{F}_A + \mathbf{F}_B = 0$$

Substituting components:

$$\mathbf{R} = (320\text{ lb})\mathbf{i} - \left[(600\text{ lb})\cos 55°\right]\mathbf{i} + \left[(600\text{ lb})\sin 55°\right]\mathbf{j}$$

$$+ P\mathbf{i} + (Q\cos 55°)\mathbf{i} - (Q\sin 55°)\mathbf{j} = 0$$

In the x-direction (one unknown force)

$$320\text{ lb} - (600\text{ lb})\cos 55° + Q\cos 55° = 0$$

Thus,

$$Q = \frac{-320\text{ lb} + (600\text{ lb})\cos 55°}{\cos 55°} = 42.09\text{ lb}$$

$Q = 42.1$ lb ◀

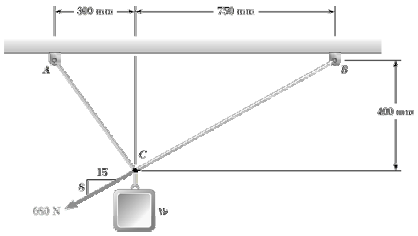
In the y-direction:

$$(600\text{ lb})\sin 55° - P - Q\sin 55° = 0$$

Thus,

$$P = (600\text{ lb})\sin 55° - Q\sin 55° = 457.01\text{ lb}$$

$P = 457$ lb ◀

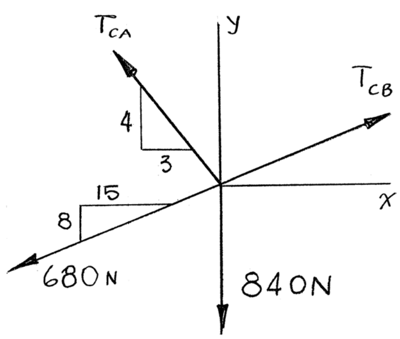


PROBLEM 2.53

Two cables tied together at C are loaded as shown. Knowing that $W = 840$ N, determine the tension (*a*) in cable AC, (*b*) in cable BC.

SOLUTION

Free-Body Diagram



From geometry:

The sides of the triangle with hypotenuse CB are in the ratio 8:15:17.

The sides of the triangle with hypotenuse CA are in the ratio 3:4:5.

Thus:

$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{3}{5}T_{CA} + \frac{15}{17}T_{CB} - \frac{15}{17}(680 \text{ N}) = 0$$

or

$$-\frac{1}{5}T_{CA} + \frac{5}{17}T_{CB} = 200 \text{ N} \tag{1}$$

and

$$+\!\uparrow \Sigma F_y = 0: \quad \frac{4}{5}T_{CA} + \frac{8}{17}T_{CB} - \frac{8}{17}(680 \text{ N}) - 840 \text{ N} = 0$$

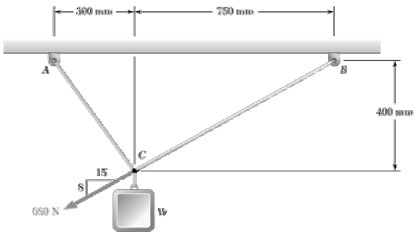
or

$$\frac{1}{5}T_{CA} + \frac{2}{17}T_{CB} = 290 \text{ N} \tag{2}$$

Solving Equations (1) and (2) simultaneously:

(*a*) $T_{CA} = 750 \text{ N}$ ◀

(*b*) $T_{CB} = 1190 \text{ N}$ ◀

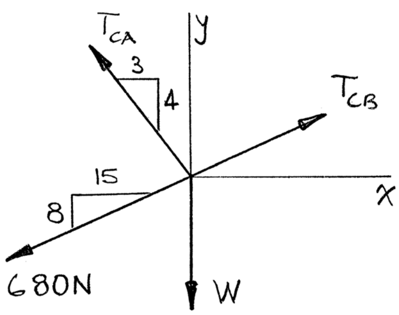


PROBLEM 2.54

Two cables tied together at C are loaded as shown. Determine the range of values of W for which the tension will not exceed 1050 N in either cable.

SOLUTION

Free-Body Diagram



From geometry:

The sides of the triangle with hypotenuse CB are in the ratio 8:15:17.

The sides of the triangle with hypotenuse CA are in the ratio 3:4:5.

Thus:

$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{3}{5}T_{CA} + \frac{15}{17}T_{CB} - \frac{15}{17}(680\text{ N}) = 0$$

or

$$-\frac{1}{5}T_{CA} + \frac{5}{17}T_{CB} = 200\text{ N} \qquad (1)$$

and

$$+\uparrow \Sigma F_y = 0: \quad \frac{4}{5}T_{CA} + \frac{8}{17}T_{CB} - \frac{8}{17}(680\text{ N}) - W = 0$$

or

$$\frac{1}{5}T_{CA} + \frac{2}{17}T_{CB} = 80\text{ N} + \frac{1}{4}W \qquad (2)$$

Then, from Equations (1) and (2)

$$T_{CB} = 680\text{ N} + \frac{17}{28}W$$

$$T_{CA} = \frac{25}{28}W$$

Now, with $T \le 1050$ N

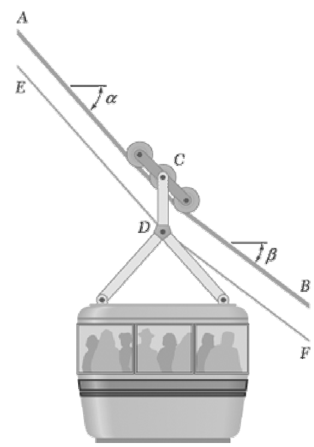
$$T_{CA}: \quad T_{CA} = 1050\text{ N} = \frac{25}{28}W$$

or $\quad W = 1176$ N

and

$$T_{CB}: \quad T_{CB} = 1050\text{ N} = 680\text{ N} + \frac{17}{28}W$$

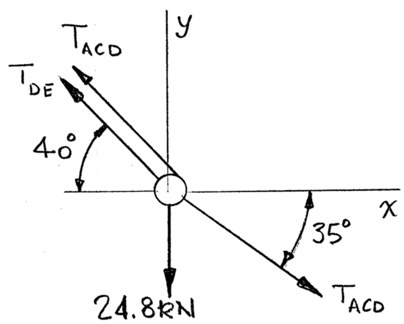
or $\quad W = 609$ N $\qquad \therefore\ 0 \le W \le 609\text{ N}$ ◀



PROBLEM 2.55

The cabin of an aerial tramway is suspended from a set of wheels that can roll freely on the support cable *ACB* and is being pulled at a constant speed by cable *DE*. Knowing that $\alpha = 40°$ and $\beta = 35°$, that the combined weight of the cabin, its support system, and its passengers is 24.8 kN, and assuming the tension in cable *DF* to be negligible, determine the tension (*a*) in the support cable *ACB*, (*b*) in the traction cable *DE*.

SOLUTION



Note: In Problems 2.55 and 2.56 the cabin is considered as a particle. If considered as a rigid body (Chapter 4) it would be found that its center of gravity should be located to the left of the centerline for the line *CD* to be vertical.

Now

$$\xrightarrow{+} \Sigma F_x = 0: \quad T_{ACB}(\cos 35° - \cos 40°) - T_{DE}\cos 40° = 0$$

or

$$0.0531T_{ACB} - 0.766T_{DE} = 0 \qquad (1)$$

and

$$+\uparrow \Sigma F_y = 0: \quad T_{ACB}(\sin 40° - \sin 35°) + T_{DE}\sin 40° - 24.8\text{ kN} = 0$$

or

$$0.0692T_{ACB} + 0.643T_{DE} = 24.8\text{ kN} \qquad (2)$$

From (1)

$$T_{ACB} = 14.426T_{DE}$$

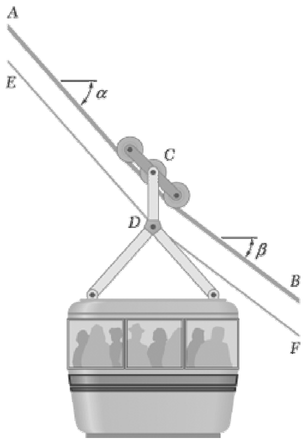
Then, from (2)

$$0.0692(14.426T_{DE}) + 0.643T_{DE} = 24.8\text{ kN}$$

and

(*b*) $T_{DE} = 15.1\text{ kN}$ ◀

(*a*) $T_{ACB} = 218\text{ kN}$ ◀

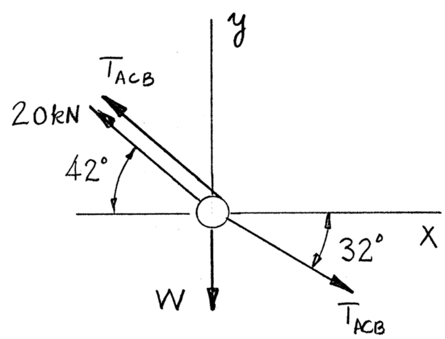


PROBLEM 2.56

The cabin of an aerial tramway is suspended from a set of wheels that can roll freely on the support cable *ACB* and is being pulled at a constant speed by cable *DE*. Knowing that $\alpha = 42°$ and $\beta = 32°$, that the tension in cable *DE* is 20 kN, and assuming the tension in cable *DF* to be negligible, determine (*a*) the combined weight of the cabin, its support system, and its passengers, (*b*) the tension in the support cable *ACB*.

SOLUTION

Free-Body Diagram



First, consider the sum of forces in the *x*-direction because there is only one unknown force:

$$\xrightarrow{+} \Sigma F_x = 0: \quad T_{ACB}(\cos 32° - \cos 42°) - (20 \text{ kN})\cos 42° = 0$$

or

$$0.1049 T_{ACB} = 14.863 \text{ kN}$$

$$(b)\quad T_{ACB} = 141.7 \text{ kN} \blacktriangleleft$$

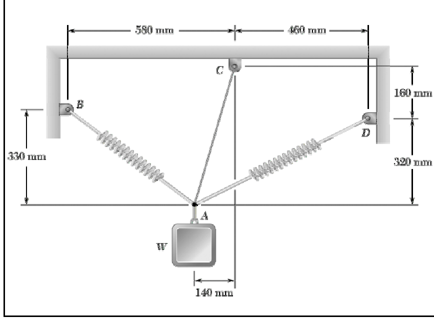
Now

$$+\uparrow \Sigma F_y = 0: \quad T_{ACB}(\sin 42° - \sin 32°) + (20 \text{ kN})\sin 42° - W = 0$$

or

$$(141.7 \text{ kN})(0.1392) + (20 \text{ kN})(0.6691) - W = 0$$

$$(a)\quad W = 33.1 \text{ kN} \blacktriangleleft$$

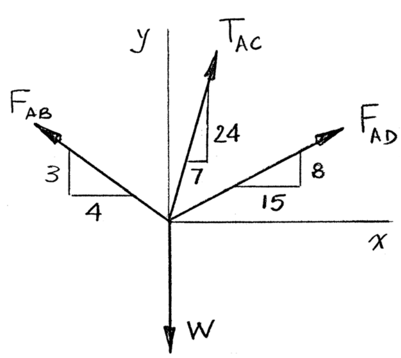


PROBLEM 2.57

A block of weight W is suspended from a 500-mm long cord and two springs of which the unstretched lengths are 450 mm. Knowing that the constants of the springs are k_{AB} = 1500 N/m and k_{AD} = 500 N/m, determine (*a*) the tension in the cord, (*b*) the weight of the block.

SOLUTION

Free-Body Diagram At A



First note from geometry:

The sides of the triangle with hypotenuse AD are in the ratio 8:15:17.

The sides of the triangle with hypotenuse AB are in the ratio 3:4:5.

The sides of the triangle with hypotenuse AC are in the ratio 7:24:25.

Then:

$$F_{AB} = k_{AB}\left(L_{AB} - L_o\right)$$

and

$$L_{AB} = \sqrt{(0.44\text{ m})^2 + (0.33\text{ m})^2} = 0.55\text{ m}$$

So:

$$F_{AB} = 1500\text{ N/m}\left(0.55\text{ m} - 0.45\text{ m}\right)$$
$$= 150\text{ N}$$

Similarly,

$$F_{AD} = k_{AD}\left(L_{AD} - L_o\right)$$

Then:

$$L_{AD} = \sqrt{(0.66\text{ m})^2 + (0.32\text{ m})^2} = 0.68\text{ m}$$

$$F_{AD} = 1500\text{ N/m}\left(0.68\text{ m} - 0.45\text{ m}\right)$$
$$= 115\text{ N}$$

(*a*)

$$\xrightarrow{+} \Sigma F_x = 0\text{:}\quad -\frac{4}{5}(150\text{ N}) + \frac{7}{25}T_{AC} - \frac{15}{17}(115\text{ N}) = 0$$

or

$$T_{AC} = 66.18\text{ N} \qquad\qquad T_{AC} = 66.2\text{ N} \blacktriangleleft$$

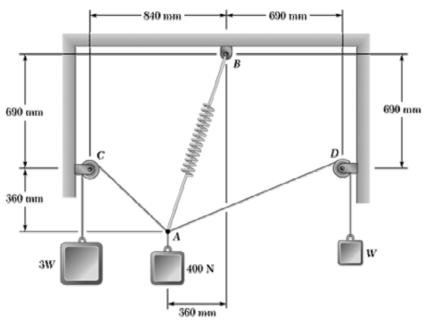
PROBLEM 2.57 CONTINUED

(*b*) and

$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}(150 \text{ N}) + \frac{24}{25}(66.18 \text{ N}) + \frac{8}{17}(115 \text{ N}) - W = 0$$

or $W = 208 \text{ N}$ ◀

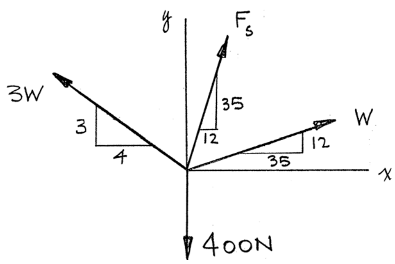
PROBLEM 2.58



A load of weight 400 N is suspended from a spring and two cords which are attached to blocks of weights $3W$ and W as shown. Knowing that the constant of the spring is 800 N/m, determine (*a*) the value of W, (*b*) the unstretched length of the spring.

SOLUTION

Free-Body Diagram At A



First note from geometry:

The sides of the triangle with hypotenuse AD are in the ratio 12:35:37.

The sides of the triangle with hypotenuse AC are in the ratio 3:4:5.

The sides of the triangle with hypotenuse AB are also in the ratio 12:35:37.

Then:

$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{4}{5}(3W) + \frac{35}{37}(W) + \frac{12}{37}F_s = 0$$

or

$$F_s = 4.4833W$$

and

$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}(3W) + \frac{12}{37}(W) + \frac{35}{37}F_s - 400\text{ N} = 0$$

Then:

$$\frac{3}{5}(3W) + \frac{12}{37}(W) + \frac{35}{37}(4.4833W) - 400\text{ N} = 0$$

or

$$W = 62.841\text{ N}$$

and

$$F_s = 281.74\text{ N}$$

or

(*a*) $W = 62.8\text{ N}$ ◀

PROBLEM 2.58 CONTINUED

(*b*) Have spring force

$$F_s = k\left(L_{AB} - L_o\right)$$

Where

$$F_{AB} = k_{AB}\left(L_{AB} - L_o\right)$$

and

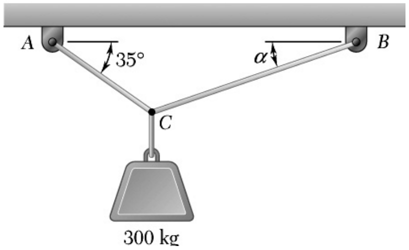
$$L_{AB} = \sqrt{\left(0.360 \text{ m}\right)^2 + \left(1.050 \text{ m}\right)^2} = 1.110 \text{ m}$$

So:

$$281.74 \text{ N} = 800 \text{ N/m}\left(1.110 - L_0\right)\text{m}$$

or $L_0 = 758 \text{ mm}$ ◀

PROBLEM 2.59

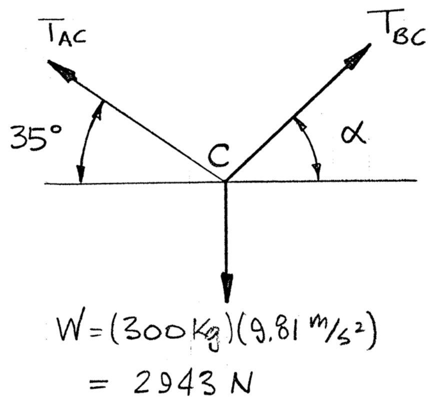


For the cables and loading of Problem 2.46, determine (*a*) the value of α for which the tension in cable *BC* is as small as possible, (*b*) the corresponding value of the tension.

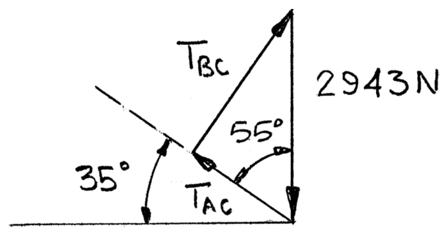
SOLUTION

The smallest T_{BC} is when T_{BC} is perpendicular to the direction of T_{AC}

Free-Body Diagram At C



Force Triangle



(*a*) $\alpha = 55.0° \blacktriangleleft$

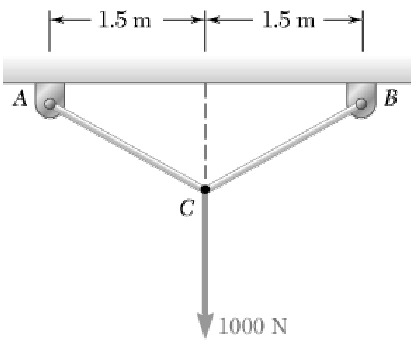
(*b*)

$$T_{BC} = (2943\text{ N})\sin 55°$$

$$= 2410.8\text{ N}$$

$$T_{BC} = 2.41\text{ kN} \blacktriangleleft$$

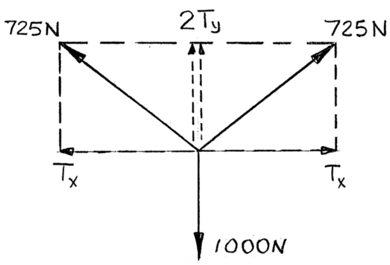
PROBLEM 2.60



Knowing that portions AC and BC of cable ACB must be equal, determine the shortest length of cable which can be used to support the load shown if the tension in the cable is not to exceed 725 N.

SOLUTION

Free-Body Diagram: C
$(\text{For } T = 725\text{ N})$



$$+\uparrow \Sigma F_y = 0\text{:} \quad 2T_y - 1000\text{ N} = 0$$

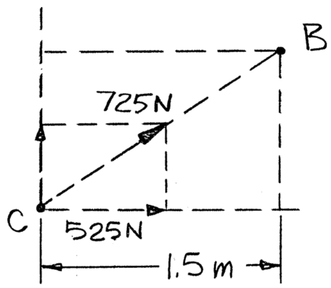
$$T_y = 500\text{ N}$$

$$T_x^2 + T_y^2 = T^2$$

$$T_x^2 + (500\text{ N})^2 = (725\text{ N})^2$$

$$T_x = 525\text{ N}$$

By similar triangles:



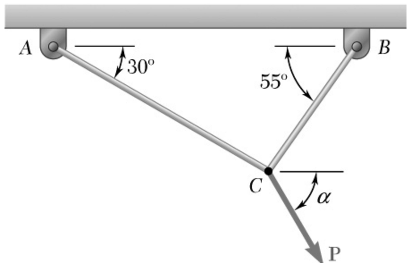
$$\frac{BC}{725} = \frac{1.5\text{ m}}{525}$$

$$\therefore\ BC = 2.07\text{ m}$$

$$L = 2(BC) = 4.14\text{ m}$$

$$L = 4.14\text{ m} \blacktriangleleft$$

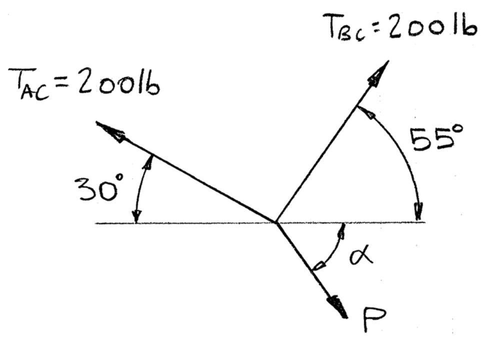
PROBLEM 2.61



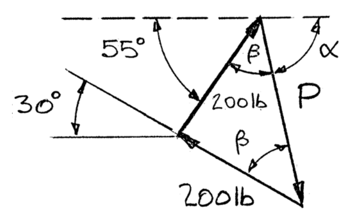
Two cables tied together at *C* are loaded as shown. Knowing that the maximum allowable tension in each cable is 200 lb, determine (*a*) the magnitude of the largest force **P** which may be applied at *C*, (*b*) the corresponding value of α.

SOLUTION

Free-Body Diagram: C



Force Triangle



Force triangle is isoceles with

$$2\beta = 180° - 85°$$

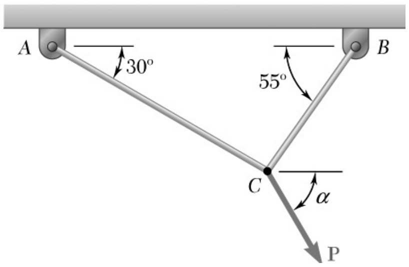
$$\beta = 47.5°$$

(*a*) $$P = 2(200 \text{ lb})\cos 47.5° = 270 \text{ lb}$$

Since $P > 0$, the solution is correct. $P = 270 \text{ lb}$ ◀

(*b*) $$\alpha = 180° - 55° - 47.5° = 77.5°$$ $\alpha = 77.5°$ ◀

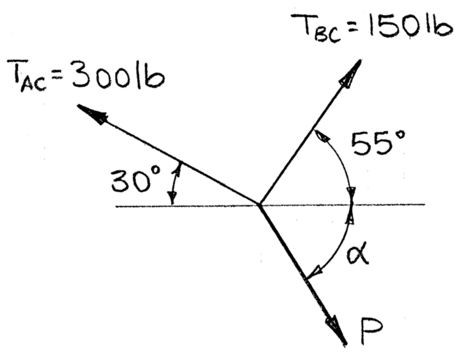
PROBLEM 2.62



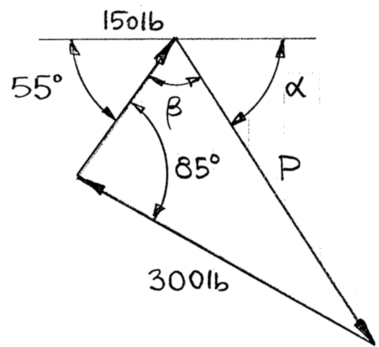
Two cables tied together at C are loaded as shown. Knowing that the maximum allowable tension is 300 lb in cable AC and 150 lb in cable BC, determine (*a*) the magnitude of the largest force **P** which may be applied at C, (*b*) the corresponding value of α.

SOLUTION

Free-Body Diagram: C



Force Triangle



(*a*) Law of Cosines:

$$P^2 = (300 \text{ lb})^2 + (150 \text{ lb})^2 - 2(300 \text{ lb})(150 \text{ lb})\cos 85°$$

$$P = 323.5 \text{ lb}$$

Since $P > 300$ lb, our solution is correct. $P = 324 \text{ lb}$ ◀

(*b*) Law of Sines:

$$\frac{\sin\beta}{300} = \frac{\sin 85°}{323.5°}$$

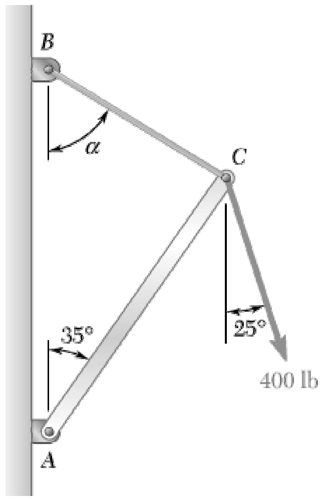
$$\sin\beta = 0.9238$$

or $$\beta = 67.49°$$

$$\alpha = 180° - 55° - 67.49° = 57.5°$$

$\alpha = 57.5°$ ◀

PROBLEM 2.63

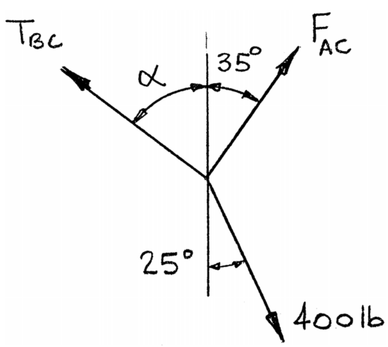


For the structure and loading of Problem 2.45, determine (*a*) the value of α for which the tension in cable *BC* is as small as possible, (*b*) the corresponding value of the tension.

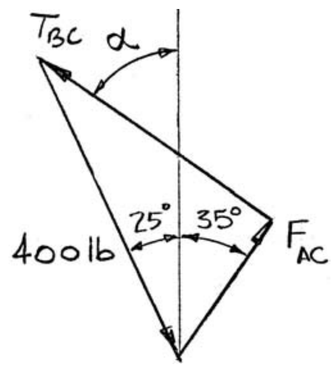
SOLUTION

T_{BC} must be perpendicular to F_{AC} to be as small as possible.

Free-Body Diagram: C



Force Triangle is a right triangle

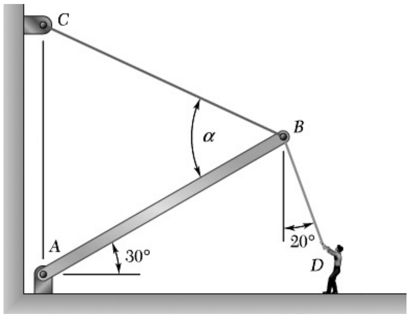


(*a*) We observe: $\alpha = 55°$ $\qquad \alpha = 55°$ ◀

(*b*) $T_{BC} = (400 \text{ lb})\sin 60°$

or $T_{BC} = 346.4 \text{ lb}$ $\qquad T_{BC} = 346 \text{ lb}$ ◀

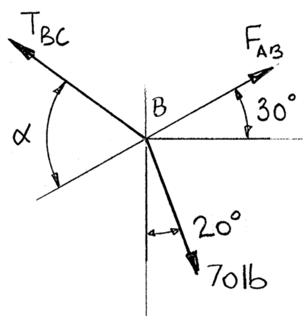
PROBLEM 2.64



Boom AB is supported by cable BC and a hinge at A. Knowing that the boom exerts on pin B a force directed along the boom and that the tension in rope BD is 70 lb, determine (a) the value of α for which the tension in cable BC is as small as possible, (b) the corresponding value of the tension.

SOLUTION

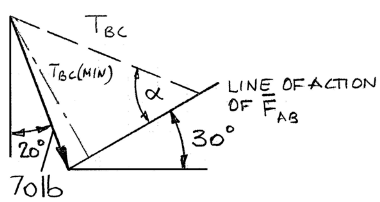
Free-Body Diagram: B



(a) Have: $\mathbf{T}_{BD} + \mathbf{F}_{AB} + \mathbf{T}_{BC} = 0$

where magnitude and direction of $\mathbf{T}_{BD}$ are known, and the direction of $\mathbf{F}_{AB}$ is known.

Then, in a force triangle:

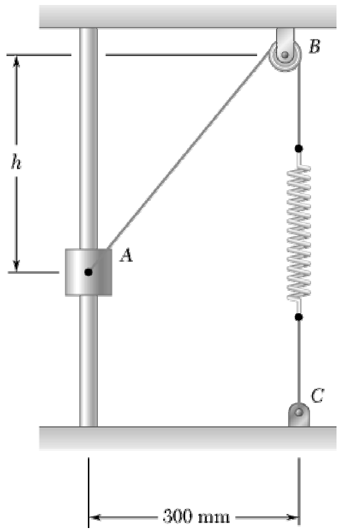


By observation, T_{BC} is minimum when $\alpha = 90.0°$ ◀

(b) Have $T_{BC} = (70 \text{ lb})\sin(180° - 70° - 30°)$

$= 68.93$ lb

$T_{BC} = 68.9$ lb ◀

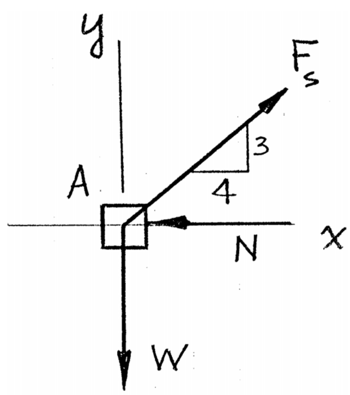


PROBLEM 2.65

Collar A shown in Figure P2.65 and P2.66 can slide on a frictionless vertical rod and is attached as shown to a spring. The constant of the spring is 660 N/m, and the spring is unstretched when $h = 300$ mm. Knowing that the system is in equilibrium when $h = 400$ mm, determine the weight of the collar.

SOLUTION

Free-Body Diagram: Collar A



Have:
$$F_s = k\left(L'_{AB} - L_{AB}\right)$$

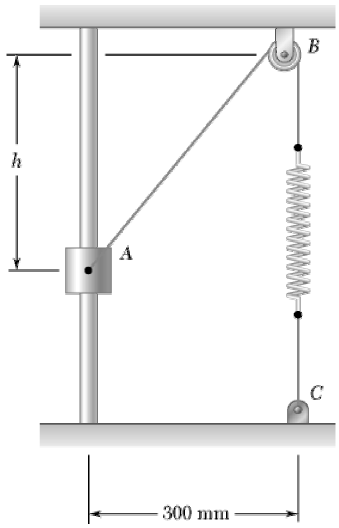
where:
$$L'_{AB} = \sqrt{(0.3\text{ m})^2 + (0.4\text{ m})^2} \qquad L_{AB} = 0.3\sqrt{2}\text{ m}$$
$$= 0.5\text{ m}$$

Then:
$$F_s = 660\text{ N/m}\left(0.5 - 0.3\sqrt{2}\right)\text{m}$$
$$= 49.986\text{ N}$$

For the collar:
$$+\uparrow \Sigma F_y = 0: \quad -W + \frac{4}{5}\left(49.986\text{ N}\right) = 0$$

or $W = 40.0$ N ◀

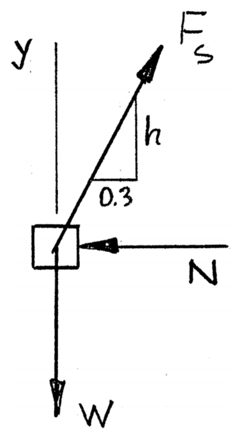
PROBLEM 2.66



The 40-N collar *A* can slide on a frictionless vertical rod and is attached as shown to a spring. The spring is unstretched when $h = 300$ mm. Knowing that the constant of the spring is 560 N/m, determine the value of *h* for which the system is in equilibrium.

SOLUTION

Free-Body Diagram: Collar A



$$+\uparrow \Sigma F_y = 0: \quad -W + \frac{h}{\sqrt{(0.3)^2 + h^2}} F_s = 0$$

or $$hF_s = 40\sqrt{0.09 + h^2}$$

Now.. $$F_s = k\left(L'_{AB} - L_{AB}\right)$$

where $$L'_{AB} = \sqrt{(0.3)^2 + h^2} \text{ m} \qquad L_{AB} = 0.3\sqrt{2} \text{ m}$$

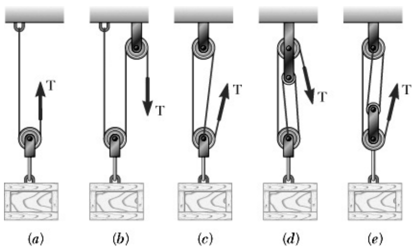
Then: $$h\left[560\left(\sqrt{0.09 + h^2} - 0.3\sqrt{2}\right)\right] = 40\sqrt{0.09 + h^2}$$

or $$(14h - 1)\sqrt{0.09 + h^2} = 4.2\sqrt{2}h \qquad h \sim \text{m}$$

Solving numerically,

$h = 415$ mm ◀

PROBLEM 2.67

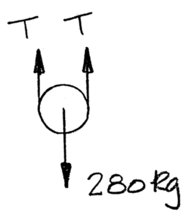


A 280-kg crate is supported by several rope-and-pulley arrangements as shown. Determine for each arrangement the tension in the rope. (*Hint:* The tension in the rope is the same on each side of a simple pulley. This can be proved by the methods of Chapter 4.)

SOLUTION

Free-Body Diagram of pulley

(*a*)

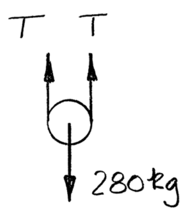


$$+\uparrow \Sigma F_y = 0: \quad 2T - (280\text{ kg})(9.81\text{ m/s}^2) = 0$$

$$T = \frac{1}{2}(2746.8\text{ N})$$

$$T = 1373\text{ N} \blacktriangleleft$$

(*b*)

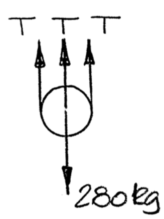


$$+\uparrow \Sigma F_y = 0: \quad 2T - (280\text{ kg})(9.81\text{ m/s}^2) = 0$$

$$T = \frac{1}{2}(2746.8\text{ N})$$

$$T = 1373\text{ N} \blacktriangleleft$$

(*c*)

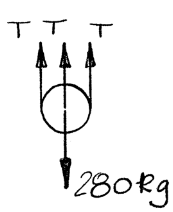


$$+\uparrow \Sigma F_y = 0: \quad 3T - (280\text{ kg})(9.81\text{ m/s}^2) = 0$$

$$T = \frac{1}{3}(2746.8\text{ N})$$

$$T = 916\text{ N} \blacktriangleleft$$

(*d*)

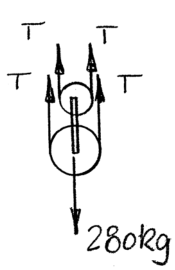


$$+\uparrow \Sigma F_y = 0: \quad 3T - (280\text{ kg})(9.81\text{ m/s}^2) = 0$$

$$T = \frac{1}{3}(2746.8\text{ N})$$

$$T = 916\text{ N} \blacktriangleleft$$

(*e*)

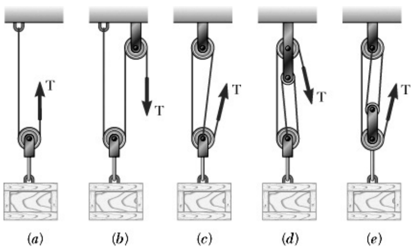


$$+\uparrow \Sigma F_y = 0: 4T - (280\text{ kg})(9.81\text{ m/s}^2) = 0$$

$$T = \frac{1}{4}(2746.8\text{ N})$$

$$T = 687\text{ N} \blacktriangleleft$$

PROBLEM 2.68



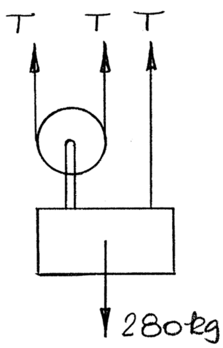
Solve parts *b* and *d* of Problem 2.67 assuming that the free end of the rope is attached to the crate.

Problem 2.67: A 280-kg crate is supported by several rope-and-pulley arrangements as shown. Determine for each arrangement the tension in the rope. (*Hint:* The tension in the rope is the same on each side of a simple pulley. This can be proved by the methods of Chapter 4.)

SOLUTION

Free-Body Diagram of pulley and crate

(*b*)

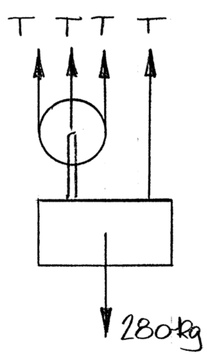


$$+\uparrow \Sigma F_y = 0: \quad 3T - (280 \text{ kg})(9.81 \text{ m/s}^2) = 0$$

$$T = \frac{1}{3}(2746.8 \text{ N})$$

$$T = 916 \text{ N} \blacktriangleleft$$

(*d*)

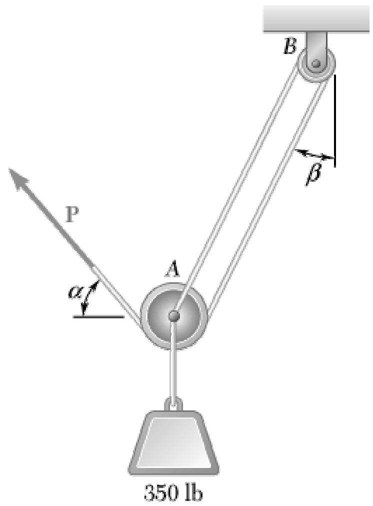


$$+\uparrow \Sigma F_y = 0: \quad 4T - (280 \text{ kg})(9.81 \text{ m/s}^2) = 0$$

$$T = \frac{1}{4}(2746.8 \text{ N})$$

$$T = 687 \text{ N} \blacktriangleleft$$

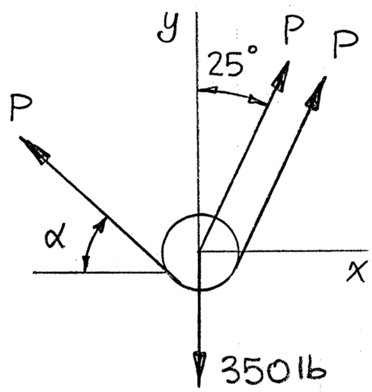
PROBLEM 2.69



A 350-lb load is supported by the rope-and-pulley arrangement shown. Knowing that $\beta = 25°$, determine the magnitude and direction of the force **P** which should be exerted on the free end of the rope to maintain equilibrium. (*Hint:* The tension in the rope is the same on each side of a simple pulley. This can be proved by the methods of Chapter 4.)

SOLUTION

Free-Body Diagram: Pulley A



$$\xrightarrow{+} \Sigma F_x = 0: \quad 2P\sin 25° - P\cos\alpha = 0$$

and

$$\cos\alpha = 0.8452 \qquad \text{or} \qquad \alpha = \pm 32.3°$$

For $\alpha = +32.3°$

$$+\uparrow \Sigma F_y = 0: \quad 2P\cos 25° + P\sin 32.3° - 350 \text{ lb} = 0$$

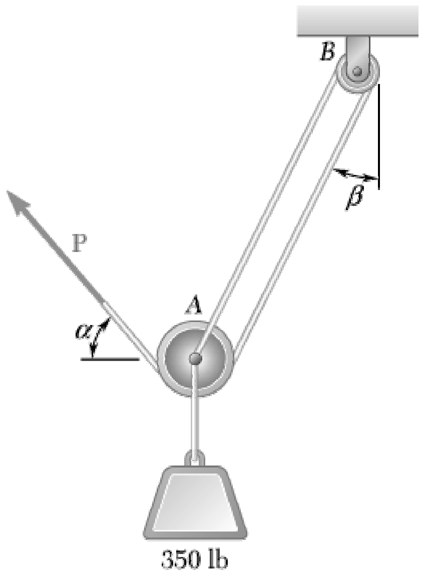
or $\mathbf{P} = 149.1 \text{ lb} \searrow 32.3°$ ◀

For $\alpha = -32.3°$

$$+\uparrow \Sigma F_y = 0: \quad 2P\cos 25° + P\sin -32.3° - 350 \text{ lb} = 0$$

or $\mathbf{P} = 274 \text{ lb} \nearrow 32.3°$ ◀

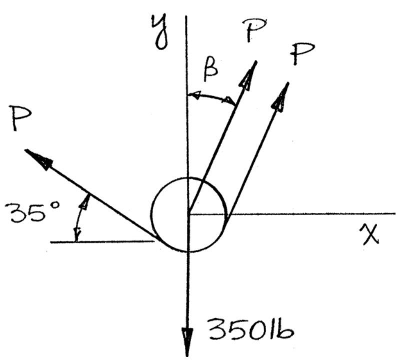
PROBLEM 2.70



A 350-lb load is supported by the rope-and-pulley arrangement shown. Knowing that $\alpha = 35°$, determine (*a*) the angle β, (*b*) the magnitude of the force **P** which should be exerted on the free end of the rope to maintain equilibrium. (*Hint:* The tension in the rope is the same on each side of a simple pulley. This can be proved by the methods of Chapter 4.)

SOLUTION

Free-Body Diagram: Pulley A



$$\xrightarrow{+} \Sigma F_x = 0: \quad 2P\sin\beta - P\cos 25° = 0$$

Hence:

(*a*) $$\sin\beta = \frac{1}{2}\cos 25° \qquad \text{or} \quad \beta = 24.2° \blacktriangleleft$$

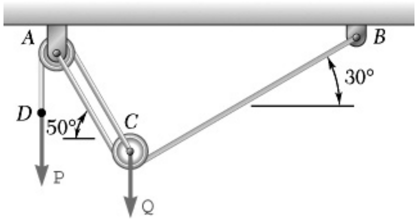
(*b*) $$+\uparrow \Sigma F_y = 0: \quad 2P\cos\beta + P\sin 35° - 350 \text{ lb} = 0$$

Hence:

$$2P\cos 24.2° + P\sin 35° - 350 \text{ lb} = 0$$

or $$P = 145.97 \text{ lb} \qquad P = 146.0 \text{ lb} \blacktriangleleft$$

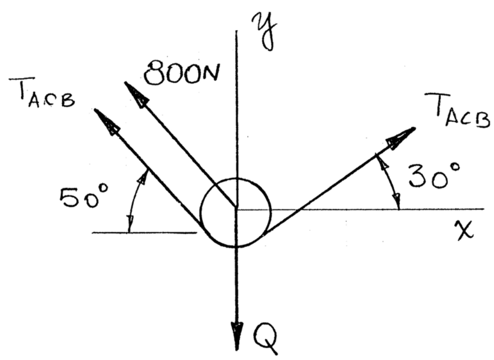
PROBLEM 2.71



A load **Q** is applied to the pulley *C*, which can roll on the cable *ACB*. The pulley is held in the position shown by a second cable *CAD*, which passes over the pulley *A* and supports a load **P**. Knowing that $P = 800$ N, determine (*a*) the tension in cable *ACB*, (*b*) the magnitude of load **Q**.

SOLUTION

Free-Body Diagram: Pulley C



(*a*) $\xrightarrow{+} \Sigma F_x = 0: \quad T_{ACB}(\cos 30° - \cos 50°) - (800 \text{ N})\cos 50° = 0$

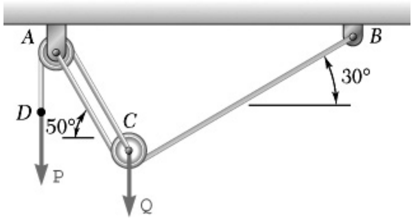
Hence $\quad T_{ACB} = 2303.5 \text{ N}$

$T_{ACB} = 2.30 \text{ kN}$ ◀

(*b*) $+\uparrow \Sigma F_y = 0: \quad T_{ACB}(\sin 30° + \sin 50°) + (800 \text{ N})\sin 50° - Q = 0$

$$(2303.5 \text{ N})(\sin 30° + \sin 50°) + (800 \text{ N})\sin 50° - Q = 0$$

or $\quad Q = 3529.2 \text{ N} \qquad Q = 3.53 \text{ kN}$ ◀

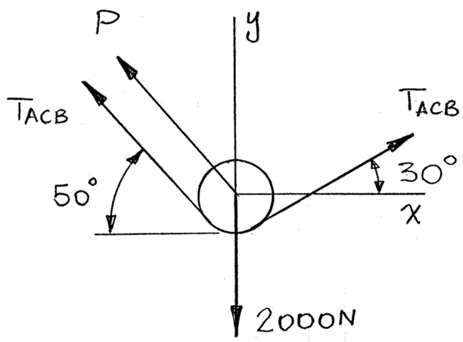


PROBLEM 2.72

A 2000-N load **Q** is applied to the pulley *C*, which can roll on the cable *ACB*. The pulley is held in the position shown by a second cable *CAD*, which passes over the pulley *A* and supports a load **P**. Determine (*a*) the tension in the cable *ACB*, (*b*) the magnitude of load **P**.

SOLUTION

Free-Body Diagram: Pulley C



$$\xrightarrow{+} \Sigma F_x = 0: \quad T_{ACB}\left(\cos 30° - \cos 50°\right) - P\cos 50° = 0$$

or $$P = 0.3473T_{ACB} \tag{1}$$

$$+\!\uparrow \Sigma F_y = 0: \quad T_{ACB}\left(\sin 30° + \sin 50°\right) + P\sin 50° - 2000 \text{ N} = 0$$

or $$1.266T_{ACB} + 0.766P = 2000 \text{ N} \tag{2}$$

(*a*) Substitute Equation (1) into Equation (2):

$$1.266T_{ACB} + 0.766\left(0.3473T_{ACB}\right) = 2000 \text{ N}$$

Hence: $$T_{ACB} = 1305.5 \text{ N}$$

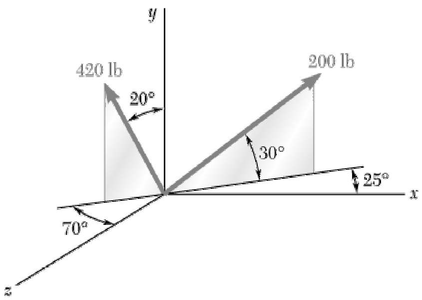
$$T_{ACB} = 1306 \text{ N} \blacktriangleleft$$

(*b*) Using (1)

$$P = 0.3473\left(1306 \text{ N}\right) = 453.57 \text{ N}$$

$$P = 454 \text{ N} \blacktriangleleft$$

PROBLEM 2.73



Determine (*a*) the *x*, *y*, and *z* components of the 200-lb force, (*b*) the angles θ_x, θ_y, and θ_z that the force forms with the coordinate axes.

SOLUTION

(*a*)

$$F_x = (200 \text{ lb})\cos 30° \cos 25° = 156.98 \text{ lb}$$

$$F_x = +157.0 \text{ lb} \blacktriangleleft$$

$$F_y = (200 \text{ lb})\sin 30° = 100.0 \text{ lb}$$

$$F_y = +100.0 \text{ lb} \blacktriangleleft$$

$$F_z = -(200 \text{ lb})\cos 30° \sin 25° = -73.1996 \text{ lb}$$

$$F_z = -73.2 \text{ lb} \blacktriangleleft$$

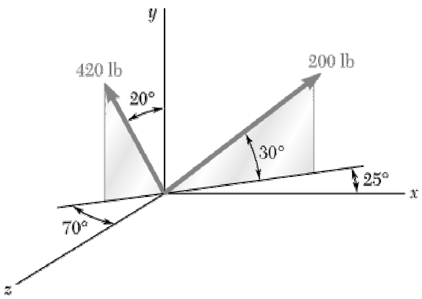
(*b*)

$$\cos\theta_x = \frac{156.98}{200} \qquad \text{or} \quad \theta_x = 38.3° \blacktriangleleft$$

$$\cos\theta_y = \frac{100.0}{200} \qquad \text{or} \quad \theta_y = 60.0° \blacktriangleleft$$

$$\cos\theta_z = \frac{-73.1996}{200} \qquad \text{or} \quad \theta_z = 111.5° \blacktriangleleft$$

PROBLEM 2.74



Determine (*a*) the *x*, *y*, and *z* components of the 420-lb force, (*b*) the angles θ_x, θ_y, and θ_z that the force forms with the coordinate axes.

SOLUTION

(*a*)

$$F_x = -(420 \text{ lb})\sin 20° \sin 70° = -134.985 \text{ lb}$$

$$F_x = -135.0 \text{ lb} \blacktriangleleft$$

$$F_y = (420 \text{ lb})\cos 20° = 394.67 \text{ lb}$$

$$F_y = +395 \text{ lb} \blacktriangleleft$$

$$F_z = (420 \text{ lb})\sin 20° \cos 70° = 49.131 \text{ lb}$$

$$F_z = +49.1 \text{ lb} \blacktriangleleft$$

(*b*)

$$\cos\theta_x = \frac{-134.985}{420}$$

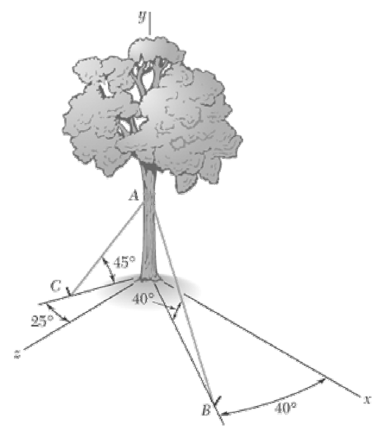
$$\theta_x = 108.7° \blacktriangleleft$$

$$\cos\theta_y = \frac{394.67}{420}$$

$$\theta_y = 20.0° \blacktriangleleft$$

$$\cos\theta_z = \frac{49.131}{420}$$

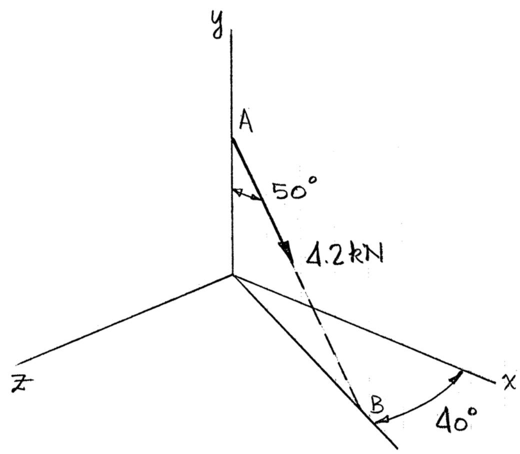
$$\theta_z = 83.3° \blacktriangleleft$$



PROBLEM 2.75

To stabilize a tree partially uprooted in a storm, cables *AB* and *AC* are attached to the upper trunk of the tree and then are fastened to steel rods anchored in the ground. Knowing that the tension in cable *AB* is 4.2 kN, determine (*a*) the components of the force exerted by this cable on the tree, (*b*) the angles θ_x, θ_y, and θ_z that the force forms with axes at *A* which are parallel to the coordinate axes.

SOLUTION



(*a*) $$F_x = (4.2 \text{ kN})\sin 50° \cos 40° = 2.4647 \text{ kN}$$

$F_x = +2.46 \text{ kN}$ ◀

$$F_y = -(4.2 \text{ kN})\cos 50° = -2.6997 \text{ kN}$$

$F_y = -2.70 \text{ kN}$ ◀

$$F_z = (4.2 \text{ kN})\sin 50° \sin 40° = 2.0681 \text{ kN}$$

$F_z = +2.07 \text{ kN}$ ◀

(*b*) $$\cos\theta_x = \frac{2.4647}{4.2}$$

$\theta_x = 54.1°$ ◀

PROBLEM 2.75 CONTINUED

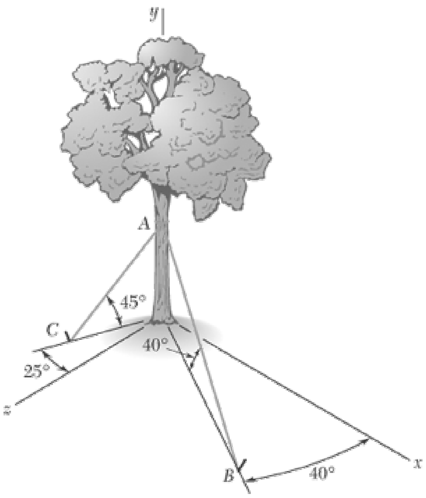
$$\cos\theta_y = \frac{-2.7}{4.2}$$

$$\theta_y = 130.0° \blacktriangleleft$$

$$\cos\theta_z = \frac{2.0681}{4.0}$$

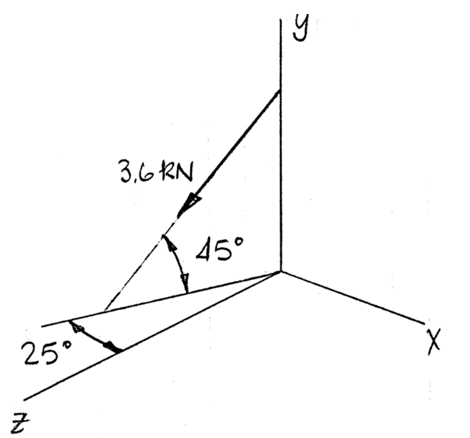
$$\theta_z = 60.5° \blacktriangleleft$$

PROBLEM 2.76



To stabilize a tree partially uprooted in a storm, cables *AB* and *AC* are attached to the upper trunk of the tree and then are fastened to steel rods anchored in the ground. Knowing that the tension in cable *AC* is 3.6 kN, determine (*a*) the components of the force exerted by this cable on the tree, (*b*) the angles θ_x, θ_y, and θ_z that the force forms with axes at *A* which are parallel to the coordinate axes.

SOLUTION



(*a*) $$F_x = -(3.6 \text{ kN})\cos 45° \sin 25° = -1.0758 \text{ kN}$$

$F_x = -1.076 \text{ kN}$ ◀

$$F_y = -(3.6 \text{ kN})\sin 45° = -2.546 \text{ kN}$$

$F_y = -2.55 \text{ kN}$ ◀

$$F_z = (3.6 \text{ kN})\cos 45° \cos 25° = 2.3071 \text{ kN}$$

$F_z = +2.31 \text{ kN}$ ◀

(*b*) $$\cos\theta_x = \frac{-1.0758}{3.6}$$

$\theta_x = 107.4°$ ◀

PROBLEM 2.76 CONTINUED

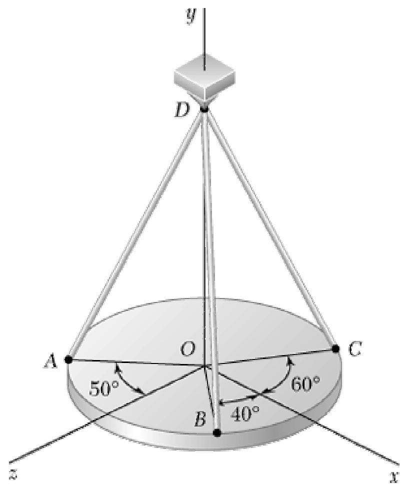
$$\cos\theta_y = \frac{-2.546}{3.6}$$

$$\theta_y = 135.0° \blacktriangleleft$$

$$\cos\theta_z = \frac{2.3071}{3.6}$$

$$\theta_z = 50.1° \blacktriangleleft$$

PROBLEM 2.77



A horizontal circular plate is suspended as shown from three wires which are attached to a support at *D* and form 30° angles with the vertical. Knowing that the *x* component of the force exerted by wire *AD* on the plate is 220.6 N, determine (*a*) the tension in wire *AD*, (*b*) the angles θ_x, θ_y, and θ_z that the force exerted at *A* forms with the coordinate axes.

SOLUTION

(*a*)

$$F_x = F \sin 30° \sin 50° = 220.6 \text{ N (Given)}$$

$$F = \frac{220.6 \text{ N}}{\sin 30° \sin 50°} = 575.95 \text{ N}$$

$$F = 576 \text{ N} \blacktriangleleft$$

(*b*)

$$\cos\theta_x = \frac{F_x}{F} = \frac{220.6}{575.95} = 0.3830$$

$$\theta_x = 67.5° \blacktriangleleft$$

$$F_y = F \cos 30° = 498.79 \text{ N}$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{498.79}{575.95} = 0.86605$$

$$\theta_y = 30.0° \blacktriangleleft$$

$$F_z = -F \sin 30° \cos 50°$$

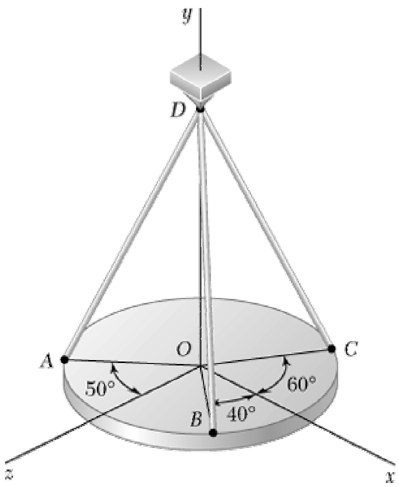
$$= -(575.95 \text{ N}) \sin 30° \cos 50°$$

$$= -185.107 \text{ N}$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{-185.107}{575.95} = -0.32139$$

$$\theta_z = 108.7° \blacktriangleleft$$

PROBLEM 2.78



A horizontal circular plate is suspended as shown from three wires which are attached to a support at *D* and form 30° angles with the vertical. Knowing that the *z* component of the force exerted by wire *BD* on the plate is –64.28 N, determine (*a*) the tension in wire *BD*, (*b*) the angles θ_x, θ_y, and θ_z that the force exerted at *B* forms with the coordinate axes.

SOLUTION

(*a*)

$$F_z = -F\sin 30°\sin 40° = -64.28 \text{ N} \ \ \text{(Given)}$$

$$F = \frac{64.28 \text{ N}}{\sin 30° \sin 40°} = 200.0 \text{ N} \qquad F = 200 \text{ N} \blacktriangleleft$$

(*b*)

$$F_x = -F\sin 30°\cos 40°$$

$$= -(200.0 \text{ N})\sin 30°\cos 40°$$

$$= -76.604 \text{ N}$$

$$\cos\theta_x = \frac{F_x}{F} = \frac{-76.604}{200.0} = -0.38302 \qquad \theta_x = 112.5° \blacktriangleleft$$

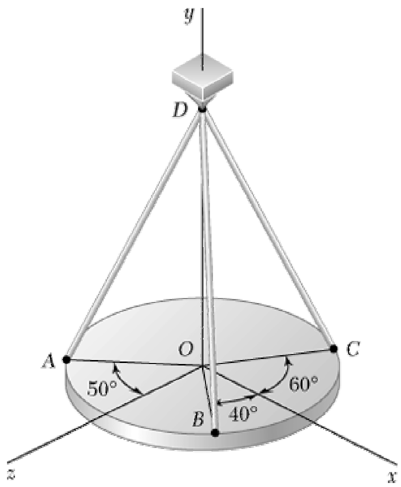
$$F_y = F\cos 30° = 173.2 \text{ N}$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{173.2}{200} = 0.866 \qquad \theta_y = 30.0° \blacktriangleleft$$

$$F_z = -64.28 \text{ N}$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{-64.28}{200} = -0.3214 \qquad \theta_z = 108.7° \blacktriangleleft$$

PROBLEM 2.79



A horizontal circular plate is suspended as shown from three wires which are attached to a support at D and form 30° angles with the vertical. Knowing that the tension in wire CD is 120 lb, determine (a) the components of the force exerted by this wire on the plate, (b) the angles θ_x, θ_y, and θ_z that the force forms with the coordinate axes.

SOLUTION

(a)

$$F_x = -(120 \text{ lb})\sin 30° \cos 60° = -30 \text{ lb}$$

$$F_x = -30.0 \text{ lb} \blacktriangleleft$$

$$F_y = (120 \text{ lb})\cos 30° = 103.92 \text{ lb}$$

$$F_y = +103.9 \text{ lb} \blacktriangleleft$$

$$F_z = (120 \text{ lb})\sin 30° \sin 60° = 51.96 \text{ lb}$$

$$F_z = +52.0 \text{ lb} \blacktriangleleft$$

(b)

$$\cos\theta_x = \frac{F_x}{F} = \frac{-30.0}{120} = -0.25$$

$$\theta_x = 104.5° \blacktriangleleft$$

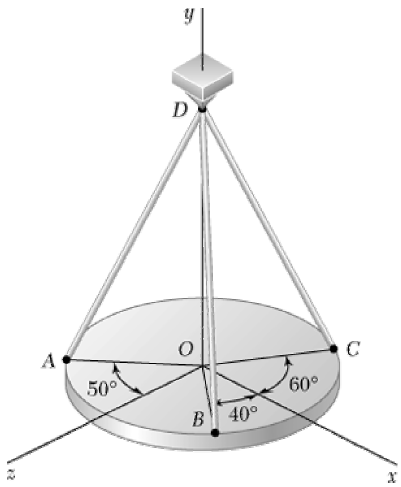
$$\cos\theta_y = \frac{F_y}{F} = \frac{103.92}{120} = 0.866$$

$$\theta_y = 30.0° \blacktriangleleft$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{51.96}{120} = 0.433$$

$$\theta_z = 64.3° \blacktriangleleft$$

PROBLEM 2.80



A horizontal circular plate is suspended as shown from three wires which are attached to a support at D and form 30° angles with the vertical. Knowing that the x component of the forces exerted by wire CD on the plate is –40 lb, determine (a) the tension in wire CD, (b) the angles θ_x, θ_y, and θ_z that the force exerted at C forms with the coordinate axes.

SOLUTION

(a)

$$F_x = -F\sin 30°\cos 60° = -40 \text{ lb (Given)}$$

$$F = \frac{40 \text{ lb}}{\sin 30° \cos 60°} = 160 \text{ lb}$$

$$F = 160.0 \text{ lb} \blacktriangleleft$$

(b)

$$\cos\theta_x = \frac{F_x}{F} = \frac{-40}{160} = -0.25$$

$$\theta_x = 104.5° \blacktriangleleft$$

$$F_y = (160 \text{ lb})\cos 30° = 103.92 \text{ lb}$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{103.92}{160} = 0.866$$

$$\theta_y = 30.0° \blacktriangleleft$$

$$F_z = (160 \text{ lb})\sin 30° \sin 60° = 69.282 \text{ lb}$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{69.282}{160} = 0.433$$

$$\theta_z = 64.3° \blacktriangleleft$$

PROBLEM 2.81

Determine the magnitude and direction of the force $\mathbf{F} = (800 \text{ lb})\mathbf{i} + (260 \text{ lb})\mathbf{j} - (320 \text{ lb})\mathbf{k}$.

SOLUTION

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2} = \sqrt{(800 \text{ lb})^2 + (260 \text{ lb})^2 + (-320 \text{ lb})^2} \qquad F = 900 \text{ lb} \blacktriangleleft$$

$$\cos\theta_x = \frac{F_x}{F} = \frac{800}{900} = 0.8889 \qquad \theta_x = 27.3° \blacktriangleleft$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{260}{900} = 0.2889 \qquad \theta_y = 73.2° \blacktriangleleft$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{-320}{900} = -0.3555 \qquad \theta_z = 110.8° \blacktriangleleft$$

PROBLEM 2.82

Determine the magnitude and direction of the force $\mathbf{F} = (400 \text{ N})\mathbf{i} - (1200 \text{ N})\mathbf{j} + (300 \text{ N})\mathbf{k}$.

SOLUTION

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2} = \sqrt{(400 \text{ N})^2 + (-1200 \text{ N})^2 + (300 \text{ N})^2} \qquad F = 1300 \text{ N} \blacktriangleleft$$

$$\cos\theta_x = \frac{F_x}{F} = \frac{400}{1300} = 0.30769 \qquad \theta_x = 72.1° \blacktriangleleft$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{-1200}{1300} = -0.92307 \qquad \theta_y = 157.4° \blacktriangleleft$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{300}{1300} = 0.23076 \qquad \theta_z = 76.7° \blacktriangleleft$$

PROBLEM 2.83

A force acts at the origin of a coordinate system in a direction defined by the angles $\theta_x = 64.5°$ and $\theta_z = 55.9°$. Knowing that the y component of the force is –200 N, determine (a) the angle θ_y, (b) the other components and the magnitude of the force.

SOLUTION

(a) We have

$$(\cos\theta_x)^2 + (\cos\theta_y)^2 + (\cos\theta_z)^2 = 1 \Rightarrow (\cos\theta_y)^2 = 1 - (\cos\theta_y)^2 - (\cos\theta_z)^2$$

Since $F_y < 0$ we must have $\cos\theta_y < 0$

Thus, taking the negative square root, from above, we have:

$$\cos\theta_y = -\sqrt{1 - (\cos 64.5°)^2 - (\cos 55.9°)^2} = -0.70735 \qquad \theta_y = 135.0° ◀$$

(b) Then:

$$F = \frac{F_y}{\cos\theta_y} = \frac{-200 \text{ N}}{-0.70735} = 282.73 \text{ N}$$

and

$$F_x = F\cos\theta_x = (282.73 \text{ N})\cos 64.5° \qquad F_x = 121.7 \text{ N} ◀$$

$$F_z = F\cos\theta_z = (282.73 \text{ N})\cos 55.9° \qquad F_y = 158.5 \text{ N} ◀$$

$$F = 283 \text{ N} ◀$$

PROBLEM 2.84

A force acts at the origin of a coordinate system in a direction defined by the angles $\theta_x = 75.4°$ and $\theta_y = 132.6°$. Knowing that the z component of the force is –60 N, determine (*a*) the angle θ_z, (*b*) the other components and the magnitude of the force.

SOLUTION

(*a*) We have

$$(\cos\theta_x)^2 + (\cos\theta_y)^2 + (\cos\theta_z)^2 = 1 \Rightarrow (\cos\theta_y)^2 = 1 - (\cos\theta_y)^2 - (\cos\theta_z)^2$$

Since $F_z < 0$ we must have $\cos\theta_z < 0$

Thus, taking the negative square root, from above, we have:

$$\cos\theta_z = -\sqrt{1 - (\cos 75.4°)^2 - (\cos 132.6°)^2} = -0.69159 \qquad \theta_z = 133.8° \blacktriangleleft$$

(*b*) Then:

$$F = \frac{F_z}{\cos\theta_z} = \frac{-60\text{ N}}{-0.69159} = 86.757\text{ N} \qquad F = 86.8\text{ N} \blacktriangleleft$$

and

$$F_x = F\cos\theta_x = (86.8\text{ N})\cos 75.4° \qquad F_x = 21.9\text{ N} \blacktriangleleft$$

$$F_y = F\cos\theta_y = (86.8\text{ N})\cos 132.6° \qquad F_y = -58.8\text{ N} \blacktriangleleft$$

PROBLEM 2.85

A force **F** of magnitude 400 N acts at the origin of a coordinate system. Knowing that $\theta_x = 28.5°$, $F_y = -80$ N, and $F_z > 0$, determine (*a*) the components F_x and F_z, (*b*) the angles θ_y and θ_z.

SOLUTION

(*a*) Have

$$F_x = F\cos\theta_x = (400\text{ N})\cos 28.5° \qquad F_x = 351.5\text{ N} \blacktriangleleft$$

Then:

$$F^2 = F_x^2 + F_y^2 + F_z^2$$

So:

$$(400\text{ N})^2 = (352.5\text{ N})^2 + (-80\text{ N})^2 + F_z^2$$

Hence:

$$F_z = +\sqrt{(400\text{ N})^2 - (351.5\text{ N})^2 - (-80\text{ N})^2} \qquad F_z = 173.3\text{ N} \blacktriangleleft$$

(*b*)

$$\cos\theta_y = \frac{F_y}{F} = \frac{-80}{400} = -0.20 \qquad \theta_y = 101.5° \blacktriangleleft$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{173.3}{400} = 0.43325 \qquad \theta_z = 64.3° \blacktriangleleft$$

PROBLEM 2.86

A force **F** of magnitude 600 lb acts at the origin of a coordinate system. Knowing that $F_x = 200$ lb, $\theta_z = 136.8°$, $F_y < 0$, determine (*a*) the components F_y and F_z, (*b*) the angles θ_x and θ_y.

SOLUTION

(*a*)

$$F_z = F\cos\theta_z = (600 \text{ lb})\cos 136.8°$$

$$= -437.4 \text{ lb} \qquad F_z = -437 \text{ lb} \blacktriangleleft$$

Then:

$$F^2 = F_x^2 + F_y^2 + F_z^2$$

So:

$$(600 \text{ lb})^2 = (200 \text{ lb})^2 + (F_y)^2 + (-437.4 \text{ lb})^2$$

Hence:

$$F_y = -\sqrt{(600 \text{ lb})^2 - (200 \text{ lb})^2 - (-437.4 \text{ lb})^2}$$

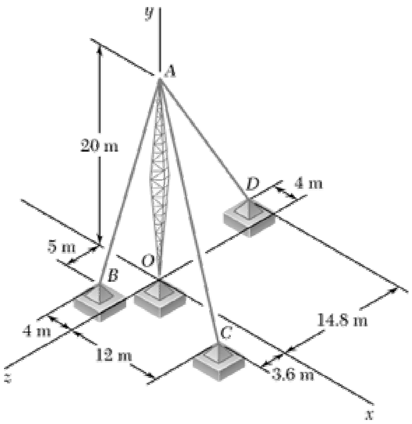
$$= -358.7 \text{ lb} \qquad F_y = -359 \text{ lb} \blacktriangleleft$$

(*b*)

$$\cos\theta_x = \frac{F_x}{F} = \frac{200}{600} = 0.333 \qquad \theta_x = 70.5° \blacktriangleleft$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{-358.7}{600} = -0.59783 \qquad \theta_y = 126.7° \blacktriangleleft$$

PROBLEM 2.87



A transmission tower is held by three guy wires anchored by bolts at *B*, *C*, and *D*. If the tension in wire *AB* is 2100 N, determine the components of the force exerted by the wire on the bolt at *B*.

SOLUTION

$$\overrightarrow{BA} = (4\text{ m})\mathbf{i} + (20\text{ m})\mathbf{j} - (5\text{ m})\mathbf{k}$$

$$BA = \sqrt{(4\text{ m})^2 + (20\text{ m})^2 + (-5\text{ m})^2} = 21\text{ m}$$

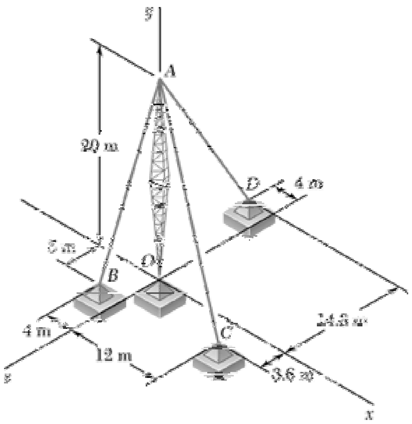
$$\mathbf{F} = F\lambda_{BA} = F\frac{\overrightarrow{BA}}{BA} = \frac{2100\text{ N}}{21\text{ m}}\left[(4\text{ m})\mathbf{i} + (20\text{ m})\mathbf{j} - (5\text{ m})\mathbf{k}\right]$$

$$\mathbf{F} = (400\text{ N})\mathbf{i} + (2000\text{ N})\mathbf{j} - (500\text{ N})\mathbf{k}$$

$F_x = +400\text{ N},\ F_y = +2000\text{ N},\ F_z = -500\text{ N}$ ◀

PROBLEM 2.88

A transmission tower is held by three guy wires anchored by bolts at *B*, *C*, and *D*. If the tension in wire *AD* is 1260 N, determine the components of the force exerted by the wire on the bolt at *D*.



SOLUTION

$$\overrightarrow{DA} = (4\text{ m})\mathbf{i} + (20\text{ m})\mathbf{j} + (14.8\text{ m})\mathbf{k}$$

$$DA = \sqrt{(4\text{ m})^2 + (20\text{ m})^2 + (14.8\text{ m})^2} = 25.2\text{ m}$$

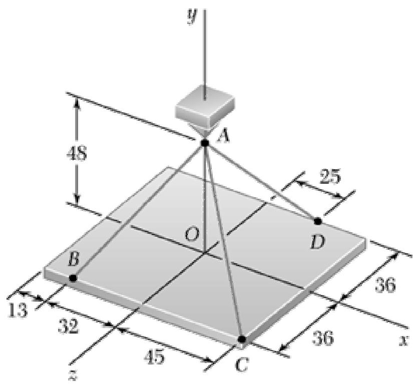
$$\mathbf{F} = F\lambda_{DA} = F\frac{\overrightarrow{DA}}{DA} = \frac{1260\text{ N}}{25.2\text{ m}}\left[(4\text{ m})\mathbf{i} + (20\text{ m})\mathbf{j} + (14.8\text{ m})\mathbf{k}\right]$$

$$\mathbf{F} = (200\text{ N})\mathbf{i} + (1000\text{ N})\mathbf{j} + (740\text{ N})\mathbf{k}$$

$$F_x = +200\text{ N},\ \ F_y = +1000\text{ N},\ \ F_z = +740\text{ N} \blacktriangleleft$$

PROBLEM 2.89

A rectangular plate is supported by three cables as shown. Knowing that the tension in cable *AB* is 204 lb, determine the components of the force exerted on the plate at *B*.



Dimensions in inches

SOLUTION

$$\overrightarrow{BA} = (32 \text{ in.})\mathbf{i} + (48 \text{ in.})\mathbf{j} - (36 \text{ in.})\mathbf{k}$$

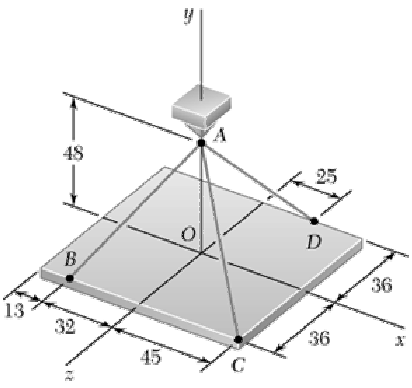
$$BA = \sqrt{(32 \text{ in.})^2 + (48 \text{ in.})^2 + (-36 \text{ in.})^2} = 68 \text{ in.}$$

$$\mathbf{F} = F\lambda_{BA} = F\frac{\overrightarrow{BA}}{BA} = \frac{204 \text{ lb}}{68 \text{ in.}}\left[(32 \text{ in.})\mathbf{i} + (48 \text{ in.})\mathbf{j} - (36 \text{ in.})\mathbf{k}\right]$$

$$\mathbf{F} = (96 \text{ lb})\mathbf{i} + (144 \text{ lb})\mathbf{j} - (108 \text{ lb})\mathbf{k}$$

$F_x = +96.0 \text{ lb}, \; F_y = +144.0 \text{ lb}, \; F_z = -108.0 \text{ lb}$ ◀

PROBLEM 2.90



Dimensions in inches

A rectangular plate is supported by three cables as shown. Knowing that the tension in cable *AD* is 195 lb, determine the components of the force exerted on the plate at *D*.

SOLUTION

$$\overrightarrow{DA} = -(25 \text{ in.})\mathbf{i} + (48 \text{ in.})\mathbf{j} + (36 \text{ in.})\mathbf{k}$$

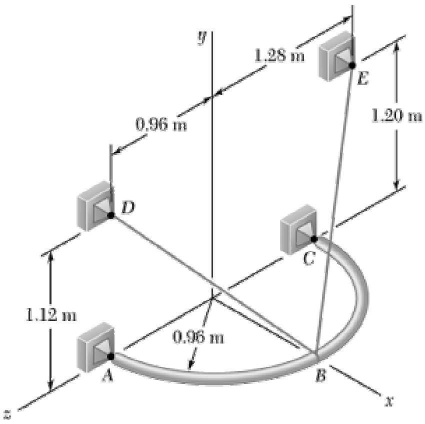
$$DA = \sqrt{(-25 \text{ in.})^2 + (48 \text{ in.})^2 + (36 \text{ in.})^2} = 65 \text{ in.}$$

$$\mathbf{F} = F\lambda_{DA} = F\frac{\overrightarrow{DA}}{DA} = \frac{195 \text{ lb}}{65 \text{ in.}}\left[(-25 \text{ in.})\mathbf{i} + (48 \text{ in.})\mathbf{j} + (36 \text{ in.})\mathbf{k}\right]$$

$$\mathbf{F} = -(75 \text{ lb})\mathbf{i} + (144 \text{ lb})\mathbf{j} + (108 \text{ lb})\mathbf{k}$$

$$F_x = -75.0 \text{ lb}, \quad F_y = +144.0 \text{ lb}, \quad F_z = +108.0 \text{ lb} \blacktriangleleft$$

PROBLEM 2.91



A steel rod is bent into a semicircular ring of radius 0.96 m and is supported in part by cables *BD* and *BE* which are attached to the ring at *B*. Knowing that the tension in cable *BD* is 220 N, determine the components of this force exerted by the cable on the support at *D*.

SOLUTION

$$\overrightarrow{DB} = (0.96 \text{ m})\mathbf{i} - (1.12 \text{ m})\mathbf{j} - (0.96 \text{ m})\mathbf{k}$$

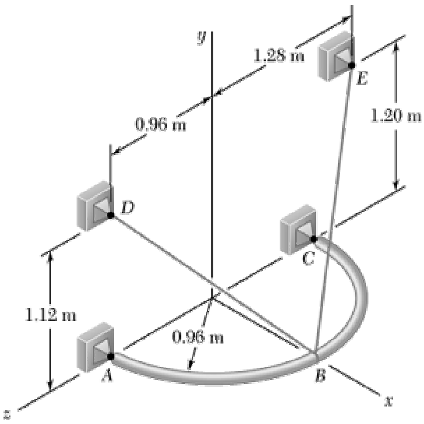
$$DB = \sqrt{(0.96 \text{ m})^2 + (-1.12 \text{ m})^2 + (-0.96 \text{ m})^2} = 1.76 \text{ m}$$

$$\mathbf{T}_{DB} = T\boldsymbol{\lambda}_{DB} = T\frac{\overrightarrow{DB}}{DB} = \frac{220 \text{ N}}{1.76 \text{ m}}\left[(0.96 \text{ m})\mathbf{i} - (1.12 \text{ m})\mathbf{j} - (0.96 \text{ m})\mathbf{k}\right]$$

$$\mathbf{T}_{DB} = (120 \text{ N})\mathbf{i} - (140 \text{ N})\mathbf{j} - (120 \text{ N})\mathbf{k}$$

$$(T_{DB})_x = +120.0 \text{ N}, \ (T_{DB})_y = -140.0 \text{ N}, \ (T_{DB})_z = -120.0 \text{ N} \blacktriangleleft$$

PROBLEM 2.92



A steel rod is bent into a semicircular ring of radius 0.96 m and is supported in part by cables *BD* and *BE* which are attached to the ring at *B*. Knowing that the tension in cable *BE* is 250 N, determine the components of this force exerted by the cable on the support at *E*.

SOLUTION

$$\overrightarrow{EB} = (0.96\text{ m})\mathbf{i} - (1.20\text{ m})\mathbf{j} + (1.28\text{ m})\mathbf{k}$$

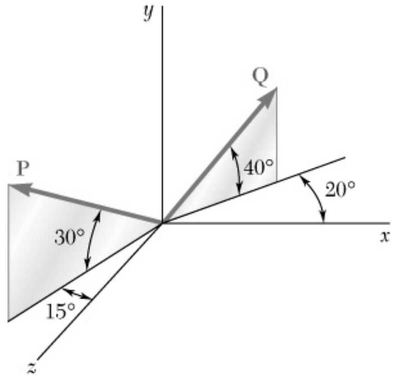
$$EB = \sqrt{(0.96\text{ m})^2 + (-1.20\text{ m})^2 + (1.28\text{ m})^2} = 2.00\text{ m}$$

$$\mathbf{T}_{EB} = T\boldsymbol{\lambda}_{EB} = T\frac{\overrightarrow{EB}}{EB} = \frac{250\text{ N}}{2.00\text{ m}}\left[(0.96\text{ m})\mathbf{i} - (1.20\text{ m})\mathbf{j} + (1.28\text{ m})\mathbf{k}\right]$$

$$\mathbf{T}_{EB} = (120\text{ N})\mathbf{i} - (150\text{ N})\mathbf{j} + (160\text{ N})\mathbf{k}$$

$$(T_{EB})_x = +120.0\text{ N},\ (T_{EB})_y = -150.0\text{ N},\ (T_{EB})_z = +160.0\text{ N} \blacktriangleleft$$

PROBLEM 2.93



Find the magnitude and direction of the resultant of the two forces shown knowing that $P = 500$ N and $Q = 600$ N.

SOLUTION

$$\mathbf{P} = (500\text{ lb})[-\cos 30°\sin 15°\mathbf{i} + \sin 30°\mathbf{j} + \cos 30°\cos 15°\mathbf{k}]$$

$$= (500\text{ lb})[-0.2241\mathbf{i} + 0.50\mathbf{j} + 0.8365\mathbf{k}]$$

$$= -(112.05\text{ lb})\mathbf{i} + (250\text{ lb})\mathbf{j} + (418.25\text{ lb})\mathbf{k}$$

$$\mathbf{Q} = (600\text{ lb})[\cos 40°\cos 20°\mathbf{i} + \sin 40°\mathbf{j} - \cos 40°\sin 20°\mathbf{k}]$$

$$= (600\text{ lb})[0.71985\mathbf{i} + 0.64278\mathbf{j} - 0.26201\mathbf{k}]$$

$$= (431.91\text{ lb})\mathbf{i} + (385.67\text{ lb})\mathbf{j} - (157.206\text{ lb})\mathbf{k}$$

$$\mathbf{R} = \mathbf{P} + \mathbf{Q} = (319.86\text{ lb})\mathbf{i} + (635.67\text{ lb})\mathbf{j} + (261.04\text{ lb})\mathbf{k}$$

$$R = \sqrt{(319.86\text{ lb})^2 + (635.67\text{ lb})^2 + (261.04\text{ lb})^2} = 757.98\text{ lb}$$

$R = 758$ lb ◀

$$\cos\theta_x = \frac{R_x}{R} = \frac{319.86\text{ lb}}{757.98\text{ lb}} = 0.42199$$

$\theta_x = 65.0°$ ◀

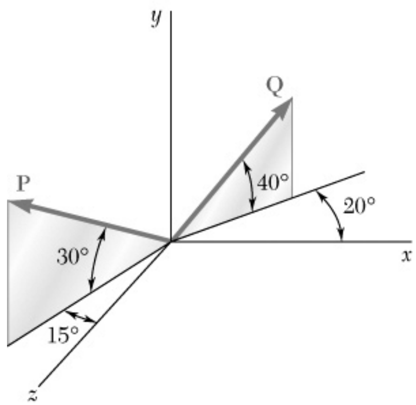
$$\cos\theta_y = \frac{R_y}{R} = \frac{635.67\text{ lb}}{757.98\text{ lb}} = 0.83864$$

$\theta_y = 33.0°$ ◀

$$\cos\theta_z = \frac{R_z}{R} = \frac{261.04\text{ lb}}{757.98\text{ lb}} = 0.34439$$

$\theta_z = 69.9°$ ◀

PROBLEM 2.94



Find the magnitude and direction of the resultant of the two forces shown knowing that $P = 600$ N and $Q = 400$ N.

SOLUTION

Using the results from 2.93:

$$\mathbf{P} = (600\text{ lb})[-0.2241\mathbf{i} + 0.50\mathbf{j} + 0.8365\mathbf{k}]$$

$$= -(134.46\text{ lb})\mathbf{i} + (300\text{ lb})\mathbf{j} + (501.9\text{ lb})\mathbf{k}$$

$$\mathbf{Q} = (400\text{ lb})[0.71985\mathbf{i} + 0.64278\mathbf{j} - 0.26201\mathbf{k}]$$

$$= (287.94\text{ lb})\mathbf{i} + (257.11\text{ lb})\mathbf{j} - (104.804\text{ lb})\mathbf{k}$$

$$\mathbf{R} = \mathbf{P} + \mathbf{Q} = (153.48\text{ lb})\mathbf{i} + (557.11\text{ lb})\mathbf{j} + (397.10\text{ lb})\mathbf{k}$$

$$R = \sqrt{(153.48\text{ lb})^2 + (557.11\text{ lb})^2 + (397.10\text{ lb})^2} = 701.15\text{ lb}$$

$R = 701$ lb ◀

$$\cos\theta_x = \frac{R_x}{R} = \frac{153.48\text{ lb}}{701.15\text{ lb}} = 0.21890$$

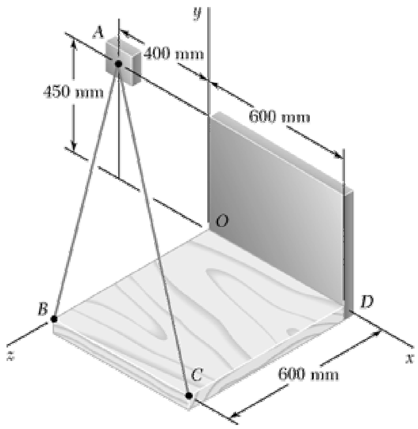
$\theta_x = 77.4°$ ◀

$$\cos\theta_y = \frac{R_y}{R} = \frac{557.11\text{ lb}}{701.15\text{ lb}} = 0.79457$$

$\theta_y = 37.4°$ ◀

$$\cos\theta_z = \frac{R_z}{R} = \frac{397.10\text{ lb}}{701.15\text{ lb}} = 0.56637$$

$\theta_z = 55.5°$ ◀



PROBLEM 2.95

Knowing that the tension is 850 N in cable *AB* and 1020 N in cable *AC*, determine the magnitude and direction of the resultant of the forces exerted at *A* by the two cables.

SOLUTION

$$\overrightarrow{AB} = (400\text{ mm})\mathbf{i} - (450\text{ mm})\mathbf{j} + (600\text{ mm})\mathbf{k}$$

$$AB = \sqrt{(400\text{ mm})^2 + (-450\text{ mm})^2 + (600\text{ mm})^2} = 850\text{ mm}$$

$$\overrightarrow{AC} = (1000\text{ mm})\mathbf{i} - (450\text{ mm})\mathbf{j} + (600\text{ mm})\mathbf{k}$$

$$AC = \sqrt{(1000\text{ mm})^2 + (-450\text{ mm})^2 + (600\text{ mm})^2} = 1250\text{ mm}$$

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = (850\text{ N})\left[\frac{(400\text{ mm})\mathbf{i} - (450\text{ mm})\mathbf{j} + (600\text{ mm})\mathbf{k}}{850\text{ mm}}\right]$$

$$\mathbf{T}_{AB} = (400\text{ N})\mathbf{i} - (450\text{ N})\mathbf{j} + (600\text{ N})\mathbf{k}$$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = (1020\text{ N})\left[\frac{(1000\text{ mm})\mathbf{i} - (450\text{ mm})\mathbf{j} + (600\text{ mm})\mathbf{k}}{1250\text{ mm}}\right]$$

$$\mathbf{T}_{AC} = (816\text{ N})\mathbf{i} - (367.2\text{ N})\mathbf{j} + (489.6\text{ N})\mathbf{k}$$

$$\mathbf{R} = \mathbf{T}_{AB} + \mathbf{T}_{AC} = (1216\text{ N})\mathbf{i} - (817.2\text{ N})\mathbf{j} + (1089.6\text{ N})\mathbf{k}$$

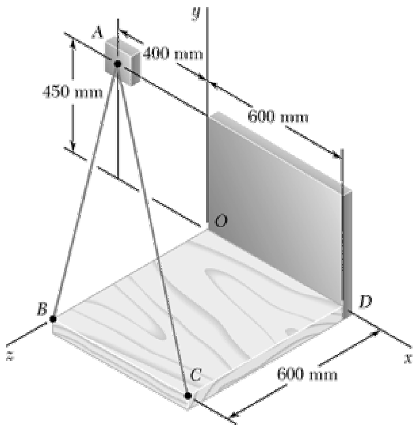
Then: $R = 1825.8\text{ N}$ $\qquad R = 1826\text{ N}$ ◀

and $\cos\theta_x = \dfrac{1216}{1825.8} = 0.66601$ $\qquad \theta_x = 48.2°$ ◀

$\cos\theta_y = \dfrac{-817.2}{1825.8} = -0.44758$ $\qquad \theta_y = 116.6°$ ◀

$\cos\theta_z = \dfrac{1089.6}{1825.8} = 0.59678$ $\qquad \theta_z = 53.4°$ ◀

PROBLEM 2.96



Assuming that in Problem 2.95 the tension is 1020 N in cable *AB* and 850 N in cable *AC*, determine the magnitude and direction of the resultant of the forces exerted at *A* by the two cables.

SOLUTION

$$\overrightarrow{AB} = (400\text{ mm})\mathbf{i} - (450\text{ mm})\mathbf{j} + (600\text{ mm})\mathbf{k}$$

$$AB = \sqrt{(400\text{ mm})^2 + (-450\text{ mm})^2 + (600\text{ mm})^2} = 850\text{ mm}$$

$$\overrightarrow{AC} = (1000\text{ mm})\mathbf{i} - (450\text{ mm})\mathbf{j} + (600\text{ mm})\mathbf{k}$$

$$AC = \sqrt{(1000\text{ mm})^2 + (-450\text{ mm})^2 + (600\text{ mm})^2} = 1250\text{ mm}$$

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = (1020\text{ N})\left[\frac{(400\text{ mm})\mathbf{i} - (450\text{ mm})\mathbf{j} + (600\text{ mm})\mathbf{k}}{850\text{ mm}}\right]$$

$$\mathbf{T}_{AB} = (480\text{ N})\mathbf{i} - (540\text{ N})\mathbf{j} + (720\text{ N})\mathbf{k}$$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = (850\text{ N})\left[\frac{(1000\text{ mm})\mathbf{i} - (450\text{ mm})\mathbf{j} + (600\text{ mm})\mathbf{k}}{1250\text{ mm}}\right]$$

$$\mathbf{T}_{AC} = (680\text{ N})\mathbf{i} - (306\text{ N})\mathbf{j} + (408\text{ N})\mathbf{k}$$

$$\mathbf{R} = \mathbf{T}_{AB} + \mathbf{T}_{AC} = (1160\text{ N})\mathbf{i} - (846\text{ N})\mathbf{j} + (1128\text{ N})\mathbf{k}$$

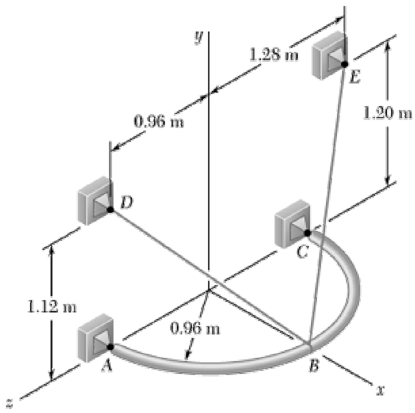
Then: $R = 1825.8\text{ N}$ $\qquad R = 1826\text{ N}$ ◀

and $\cos\theta_x = \dfrac{1160}{1825.8} = 0.6353$ $\qquad \theta_x = 50.6°$ ◀

$\cos\theta_y = \dfrac{-846}{1825.8} = -0.4634$ $\qquad \theta_y = 117.6°$ ◀

$\cos\theta_z = \dfrac{1128}{1825.8} = 0.6178$ $\qquad \theta_z = 51.8°$ ◀

PROBLEM 2.97



For the semicircular ring of Problem 2.91, determine the magnitude and direction of the resultant of the forces exerted by the cables at *B* knowing that the tensions in cables *BD* and *BE* are 220 N and 250 N, respectively.

SOLUTION

For the solutions to Problems 2.91 and 2.92, we have

$$\mathbf{T}_{BD} = -(120\text{ N})\mathbf{i} + (140\text{ N})\mathbf{j} + (120\text{ N})\mathbf{k}$$

$$\mathbf{T}_{BE} = -(120\text{ N})\mathbf{i} + (150\text{ N})\mathbf{j} - (160\text{ N})\mathbf{k}$$

Then:

$$\mathbf{R}_B = \mathbf{T}_{BD} + \mathbf{T}_{BE}$$

$$= -(240\text{ N})\mathbf{i} + (290\text{ N})\mathbf{j} - (40\text{ N})\mathbf{k}$$

and $R = 378.55\text{ N}$ $\qquad R_B = 379\text{ N}$ ◀

$$\cos\theta_x = -\frac{240}{378.55} = -0.6340$$

$\theta_x = 129.3°$ ◀

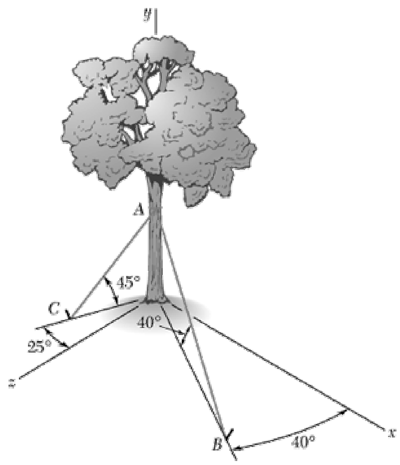
$$\cos\theta_y = \frac{290}{378.55} = -0.7661$$

$\theta_y = 40.0°$ ◀

$$\cos\theta_z = -\frac{40}{378.55} = -0.1057$$

$\theta_z = 96.1°$ ◀

PROBLEM 2.98



To stabilize a tree partially uprooted in a storm, cables *AB* and *AC* are attached to the upper trunk of the tree and then are fastened to steel rods anchored in the ground. Knowing that the tension in *AB* is 920 lb and that the resultant of the forces exerted at *A* by cables *AB* and *AC* lies in the *yz* plane, determine (*a*) the tension in *AC*, (*b*) the magnitude and direction of the resultant of the two forces.

SOLUTION

Have

$$\mathbf{T}_{AB} = (920 \text{ lb})(\sin 50° \cos 40° \mathbf{i} - \cos 50° \mathbf{j} + \sin 50° \sin 40° \mathbf{j})$$

$$\mathbf{T}_{AC} = T_{AC}(-\cos 45° \sin 25° \mathbf{i} - \sin 45° \mathbf{j} + \cos 45° \cos 25° \mathbf{j})$$

(*a*)

$$\mathbf{R}_A = \mathbf{T}_{AB} + \mathbf{T}_{AC}$$

$$(R_A)_x = 0$$

$$\therefore \quad (R_A)_x = \Sigma F_x = 0: \quad (920 \text{ lb})\sin 50° \cos 40° - T_{AC} \cos 45° \sin 25° = 0$$

or

$$T_{AC} = 1806.60 \text{ lb} \qquad\qquad T_{AC} = 1807 \text{ lb} \blacktriangleleft$$

(*b*)

$$(R_A)_y = \Sigma F_y: \quad -(920 \text{ lb})\cos 50° - (1806.60 \text{ lb})\sin 45°$$

$$(R_A)_y = -1868.82 \text{ lb}$$

$$(R_A)_z = \Sigma F_z: \quad (920 \text{ lb})\sin 50° \sin 40° + (1806.60 \text{ lb})\cos 45° \cos 25°$$

$$(R_A)_z = 1610.78 \text{ lb}$$

$$\therefore \quad R_A = -(1868.82 \text{ lb})\mathbf{j} + (1610.78 \text{ lb})\mathbf{k}$$

Then:

$$R_A = 2467.2 \text{ lb} \qquad\qquad R_A = 2.47 \text{ kips} \blacktriangleleft$$

PROBLEM 2.98 CONTINUED

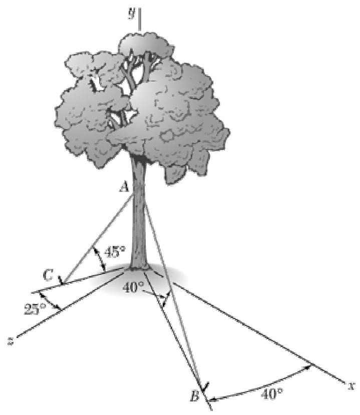
and

$$\cos\theta_x = \frac{0}{2467.2} = 0 \qquad \theta_x = 90.0° ◀$$

$$\cos\theta_y = \frac{-1868.82}{2467.2} = -0.7560 \qquad \theta_y = 139.2° ◀$$

$$\cos\theta_z = \frac{1610.78}{2467.2} = 0.65288 \qquad \theta_z = 49.2° ◀$$

PROBLEM 2.99



To stabilize a tree partially uprooted in a storm, cables *AB* and *AC* are attached to the upper trunk of the tree and then are fastened to steel rods anchored in the ground. Knowing that the tension in *AC* is 850 lb and that the resultant of the forces exerted at *A* by cables *AB* and *AC* lies in the *yz* plane, determine (*a*) the tension in *AB*, (*b*) the magnitude and direction of the resultant of the two forces.

SOLUTION

Have

$$\mathbf{T}_{AB} = T_{AB}\left(\sin 50^\circ \cos 40^\circ \mathbf{i} - \cos 50^\circ \mathbf{j} + \sin 50^\circ \sin 40^\circ \mathbf{j}\right)$$

$$\mathbf{T}_{AC} = (850 \text{ lb})\left(-\cos 45^\circ \sin 25^\circ \mathbf{i} - \sin 45^\circ \mathbf{j} + \cos 45^\circ \cos 25^\circ \mathbf{j}\right)$$

(*a*)

$$(R_A)_x = 0$$

$$\therefore \quad (R_A)_x = \Sigma F_x = 0: \quad T_{AB} \sin 50^\circ \cos 40^\circ - (850 \text{ lb}) \cos 45^\circ \sin 25^\circ = 0$$

$$T_{AB} = 432.86 \text{ lb} \qquad T_{AB} = 433 \text{ lb} \blacktriangleleft$$

(*b*)

$$(R_A)_y = \Sigma F_y: \quad -(432.86 \text{ lb}) \cos 50^\circ - (850 \text{ lb}) \sin 45^\circ$$

$$(R_A)_y = -879.28 \text{ lb}$$

$$(R_A)_z = \Sigma F_z: \quad (432.86 \text{ lb}) \sin 50^\circ \sin 40^\circ + (850 \text{ lb}) \cos 45^\circ \cos 25^\circ$$

$$(R_A)_z = 757.87 \text{ lb}$$

$$\therefore \quad \mathbf{R}_A = -(879.28 \text{ lb})\mathbf{j} + (757.87 \text{ lb})\mathbf{k}$$

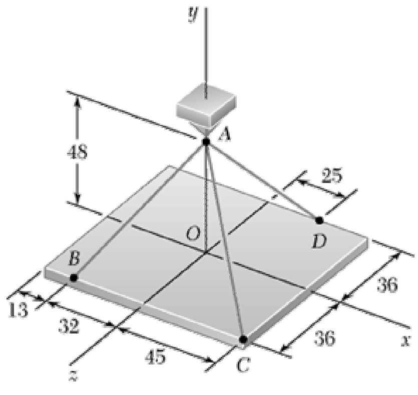
$$R_A = 1160.82 \text{ lb} \qquad R_A = 1.161 \text{ kips} \blacktriangleleft$$

$$\cos\theta_x = \frac{0}{1160.82} = 0 \qquad \theta_x = 90.0^\circ \blacktriangleleft$$

$$\cos\theta_y = \frac{-879.28}{1160.82} = -0.75746 \qquad \theta_y = 139.2^\circ \blacktriangleleft$$

$$\cos\theta_z = \frac{757.87}{1160.82} = 0.65287 \qquad \theta_z = 49.2^\circ \blacktriangleleft$$

PROBLEM 2.100



For the plate of Problem 2.89, determine the tension in cables *AB* and *AD* knowing that the tension if cable *AC* is 27 lb and that the resultant of the forces exerted by the three cables at *A* must be vertical.

SOLUTION

With:

$$\overrightarrow{AC} = (45\text{ in.})\mathbf{i} - (48\text{ in.})\mathbf{j} + (36\text{ in.})\mathbf{k}$$

$$AC = \sqrt{(45\text{ in.})^2 + (-48\text{ in.})^2 + (36\text{ in.})^2} = 75\text{ in.}$$

$$\mathbf{T}_{AC} = T_{AC}\lambda_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = \frac{27\text{ lb}}{75\text{ in.}}\left[(45\text{ in.})\mathbf{i} - (48\text{ in.})\mathbf{j} + (36\text{ in.})\mathbf{k}\right]$$

$$\mathbf{T}_{AC} = (16.2\text{ lb})\mathbf{i} - (17.28\text{ lb})\mathbf{j} + (12.96)\mathbf{k}$$

and

$$\overrightarrow{AB} = -(32\text{ in.})\mathbf{i} - (48\text{ in.})\mathbf{j} + (36\text{ in.})\mathbf{k}$$

$$AB = \sqrt{(-32\text{ in.})^2 + (-48\text{ in.})^2 + (36\text{ in.})^2} = 68\text{ in.}$$

$$\mathbf{T}_{AB} = T_{AB}\lambda_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = \frac{T_{AB}}{68\text{ in.}}\left[(-32\text{ in.})\mathbf{i} - (48\text{ in.})\mathbf{j} + (36\text{ in.})\mathbf{k}\right]$$

$$\mathbf{T}_{AB} = T_{AB}(-0.4706\mathbf{i} - 0.7059\mathbf{j} + 0.5294\mathbf{k})$$

and

$$\overrightarrow{AD} = (25\text{ in.})\mathbf{i} - (48\text{ in.})\mathbf{j} - (36\text{ in.})\mathbf{k}$$

$$AD = \sqrt{(25\text{ in.})^2 + (-48\text{ in.})^2 + (36\text{ in.})^2} = 65\text{ in.}$$

$$\mathbf{T}_{AD} = T_{AD}\lambda_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD} = \frac{T_{AD}}{65\text{ in.}}\left[(25\text{ in.})\mathbf{i} - (48\text{ in.})\mathbf{j} - (36\text{ in.})\mathbf{k}\right]$$

$$\mathbf{T}_{AD} = T_{AD}(0.3846\mathbf{i} - 0.7385\mathbf{j} - 0.5538\mathbf{k})$$

PROBLEM 2.100 CONTINUED

Now

$$\mathbf{R} = \mathbf{T}_{AB} + \mathbf{T}_{AD} + \mathbf{T}_{AD}$$

$$= T_{AB}\left(-0.4706\mathbf{i} - 0.7059\mathbf{j} + 0.5294\mathbf{k}\right) + \left[\left(16.2 \text{ lb}\right)\mathbf{i} - \left(17.28 \text{ lb}\right)\mathbf{j} + \left(12.96\right)\mathbf{k}\right]$$

$$+ T_{AD}\left(0.3846\mathbf{i} - 0.7385\mathbf{j} - 0.5538\mathbf{k}\right)$$

Since R must be vertical, the **i** and **k** components of this sum must be zero.

Hence:

$$-0.4706T_{AB} + 0.3846T_{AD} + 16.2 \text{ lb} = 0 \quad (1)$$

$$0.5294T_{AB} - 0.5538T_{AD} + 12.96 \text{ lb} = 0 \quad (2)$$

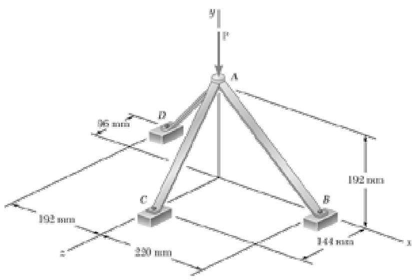
Solving (1) and (2), we obtain:

$$T_{AB} = 244.79 \text{ lb}, \qquad T_{AD} = 257.41 \text{ lb}$$

$T_{AB} = 245 \text{ lb}$ ◀

$T_{AD} = 257 \text{ lb}$ ◀

PROBLEM 2.101



The support assembly shown is bolted in place at *B*, *C*, and *D* and supports a downward force **P** at *A*. Knowing that the forces in members *AB*, *AC*, and *AD* are directed along the respective members and that the force in member *AB* is 146 N, determine the magnitude of **P**.

SOLUTION

Note that *AB*, *AC*, and *AD* are in compression.

Have

$$d_{BA} = \sqrt{(-220\text{ mm})^2 + (192\text{ mm})^2 + (0)^2} = 292\text{ mm}$$

$$d_{DA} = \sqrt{(192\text{ mm})^2 + (192\text{ mm})^2 + (96\text{ mm})^2} = 288\text{ mm}$$

$$d_{CA} = \sqrt{(0)^2 + (192\text{ mm})^2 + (-144\text{ mm})^2} = 240\text{ mm}$$

and

$$\mathbf{F}_{BA} = F_{BA}\boldsymbol{\lambda}_{BA} = \frac{146\text{ N}}{292\text{ mm}}\left[(-220\text{ mm})\mathbf{i} + (192\text{ mm})\mathbf{j}\right]$$

$$= -(110\text{ N})\mathbf{i} + (96\text{ N})\mathbf{j}$$

$$\mathbf{F}_{CA} = F_{CA}\boldsymbol{\lambda}_{CA} = \frac{F_{CA}}{240\text{ mm}}\left[(192\text{ mm})\mathbf{j} - (144\text{ mm})\mathbf{k}\right]$$

$$= F_{CA}(0.80\mathbf{j} - 0.60\mathbf{k})$$

$$\mathbf{F}_{DA} = F_{DA}\boldsymbol{\lambda}_{DA} = \frac{F_{DA}}{288\text{ mm}}\left[(192\text{ mm})\mathbf{i} + (192\text{ mm})\mathbf{j} + (96\text{ mm})\mathbf{k}\right]$$

$$= F_{DA}\left[0.66667\mathbf{i} + 0.66667\mathbf{j} + 0.33333\mathbf{k}\right]$$

With $\mathbf{P} = -P\mathbf{j}$

At *A*: $\Sigma\mathbf{F} = 0$: $\mathbf{F}_{BA} + \mathbf{F}_{CA} + \mathbf{F}_{DA} + \mathbf{P} = 0$

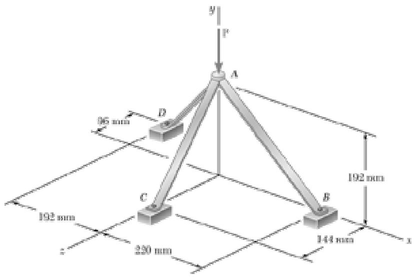
i-component: $-(110\text{ N}) + 0.66667F_{DA} = 0$ or $F_{DA} = 165\text{ N}$

j-component: $96\text{ N} + 0.80F_{CA} + 0.66667(165\text{ N}) - P = 0$ (1)

k-component: $-0.60F_{CA} + 0.33333(165\text{ N}) = 0$ (2)

Solving (2) for F_{CA} and then using that result in (1), gives $P = 279\text{ N}$ ◀

PROBLEM 2.102



The support assembly shown is bolted in place at B, C, and D and supports a downward force **P** at A. Knowing that the forces in members AB, AC, and AD are directed along the respective members and that $\mathbf{P} = 200$ N, determine the forces in the members.

SOLUTION

With the results of 2.101:

$$\mathbf{F}_{BA} = F_{BA}\boldsymbol{\lambda}_{BA} = \frac{F_{BA}}{292 \text{ mm}}\left[(-220 \text{ mm})\mathbf{i} + (192 \text{ mm})\mathbf{j}\right]$$

$$= F_{BA}\left[-0.75342\mathbf{i} + 0.65753\mathbf{j}\right]\text{N}$$

$$\mathbf{F}_{CA} = F_{CA}\boldsymbol{\lambda}_{CA} = \frac{F_{CA}}{240 \text{ mm}}\left[(192 \text{ mm})\mathbf{j} - (144 \text{ mm})\mathbf{k}\right]$$

$$= F_{CA}(0.80\mathbf{j} - 0.60\mathbf{k})$$

$$\mathbf{F}_{DA} = F_{DA}\boldsymbol{\lambda}_{DA} = \frac{F_{DA}}{288 \text{ mm}}\left[(192 \text{ mm})\mathbf{i} + (192 \text{ mm})\mathbf{j} + (96 \text{ mm})\mathbf{k}\right]$$

$$= F_{DA}\left[0.66667\mathbf{i} + 0.66667\mathbf{j} + 0.33333\mathbf{k}\right]$$

With: $\mathbf{P} = -(200 \text{ N})\mathbf{j}$

At A: $\Sigma\mathbf{F} = 0: \quad \mathbf{F}_{BA} + \mathbf{F}_{CA} + \mathbf{F}_{DA} + \mathbf{P} = 0$

Hence, equating the three (**i**, **j**, **k**) components to 0 gives three equations

i-component: $-0.75342F_{BA} + 0.66667F_{DA} = 0$ (1)

j-component: $0.65735F_{BA} + 0.80F_{CA} + 0.66667F_{DA} - 200 \text{ N} = 0$ (2)

k-component: $-0.60F_{CA} + 0.33333F_{DA} = 0$ (3)

Solving (1), (2), and (3), gives

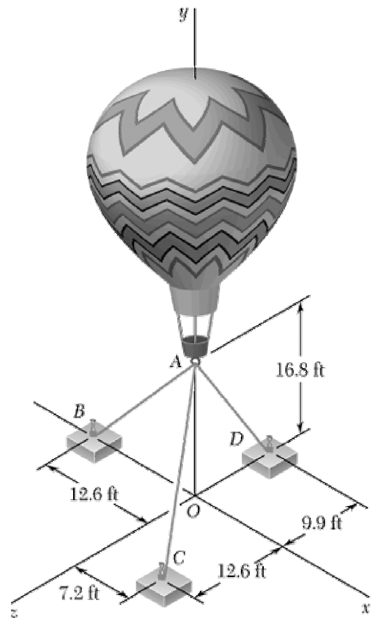
$$F_{BA} = 104.5 \text{ N}, \qquad F_{CA} = 65.6 \text{ N}, \qquad F_{\text{DA}} = 118.1 \text{ N}$$

$F_{BA} = 104.5$ N ◀

$F_{CA} = 65.6$ N ◀

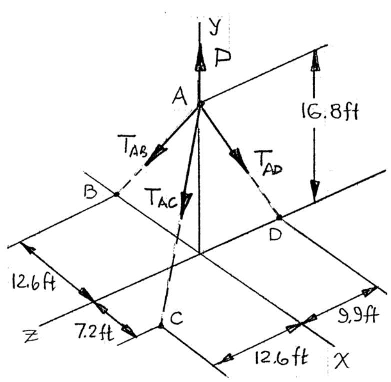
$F_{DA} = 118.1$ N ◀

PROBLEM 2.103



Three cables are used to tether a balloon as shown. Determine the vertical force **P** exerted by the balloon at A knowing that the tension in cable AB is 60 lb.

SOLUTION



The forces applied at A are:

$$\mathbf{T}_{AB},\ \mathbf{T}_{AC},\ \mathbf{T}_{AD} \text{ and } \mathbf{P}$$

where $\mathbf{P} = P\mathbf{j}$. To express the other forces in terms of the unit vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$, we write

$$\overrightarrow{AB} = -(12.6 \text{ ft})\mathbf{i} - (16.8 \text{ ft})\mathbf{j} \qquad AB = 21 \text{ ft}$$

$$\overrightarrow{AC} = (7.2 \text{ ft})\mathbf{i} - (16.8 \text{ ft})\mathbf{j} + (12.6 \text{ ft})\mathbf{k} \qquad AC = 22.2 \text{ ft}$$

$$\overrightarrow{AD} = -(16.8 \text{ ft})\mathbf{j} - (9.9 \text{ ft})\mathbf{k} \qquad AD = 19.5 \text{ ft}$$

and

$$\mathbf{T}_{AB} = T_{AB}\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = (-0.6\mathbf{i} - 0.8\mathbf{j})T_{AB}$$

$$\mathbf{T}_{AC} = T_{AC}\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = (0.3242\mathbf{i} - 0.75676\mathbf{j} + 0.56757\mathbf{k})T_{AC}$$

$$\mathbf{T}_{AD} = T_{AD}\boldsymbol{\lambda}_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD} = (-0.8615\mathbf{j} - 0.50769\mathbf{k})T_{AD}$$

PROBLEM 2.103 CONTINUED

Equilibrium Condition

$$\Sigma F = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + P\mathbf{j} = 0$$

Substituting the expressions obtained for $\mathbf{T}_{AB}$, $\mathbf{T}_{AC}$, and $\mathbf{T}_{AD}$ and factoring **i**, **j**, and **k**:

$$\left(-0.6T_{AB} + 0.3242T_{AC}\right)\mathbf{i} + \left(-0.8T_{AB} - 0.75676T_{AC} - 0.8615T_{AD} + P\right)\mathbf{j}$$

$$+ \left(0.56757T_{AC} - 0.50769T_{AD}\right)\mathbf{k} = 0$$

Equating to zero the coefficients of **i**, **j**, **k**:

$$-0.6T_{AB} + 0.3242T_{AC} = 0 \qquad (1)$$

$$-0.8T_{AB} - 0.75676T_{AC} - 0.8615T_{AD} + P = 0 \qquad (2)$$

$$0.56757T_{AC} - 0.50769T_{AD} = 0 \qquad (3)$$

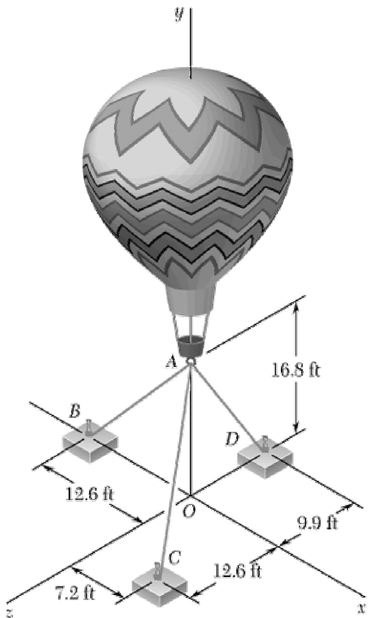
Setting $T_{AB} = 60$ lb in (1) and (2), and solving the resulting set of equations gives

$$T_{AC} = 111 \text{ lb}$$

$$T_{AD} = 124.2 \text{ lb}$$

$$\mathbf{P} = 239 \text{ lb} \uparrow \blacktriangleleft$$

PROBLEM 2.104



Three cables are used to tether a balloon as shown. Determine the vertical force **P** exerted by the balloon at *A* knowing that the tension in cable *AC* is 100 lb.

SOLUTION

See Problem 2.103 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-0.6T_{AB} + 0.3242T_{AC} = 0 \tag{1}$$

$$-0.8T_{AB} - 0.75676T_{AC} - 0.8615T_{AD} + P = 0 \tag{2}$$

$$0.56757T_{AC} - 0.50769T_{AD} = 0 \tag{3}$$

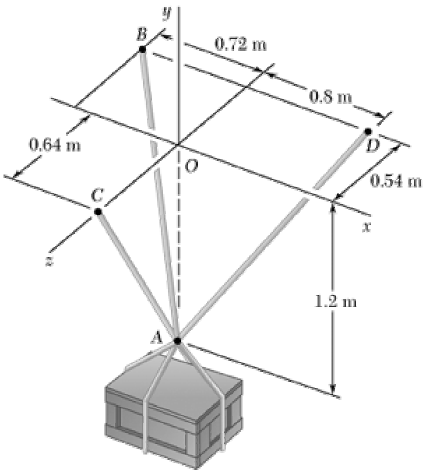
Substituting $T_{AC} = 100$ lb in Equations (1), (2), and (3) above, and solving the resulting set of equations using conventional algorithms gives

$$T_{AB} = 54 \text{ lb}$$

$$T_{AD} = 112 \text{ lb}$$

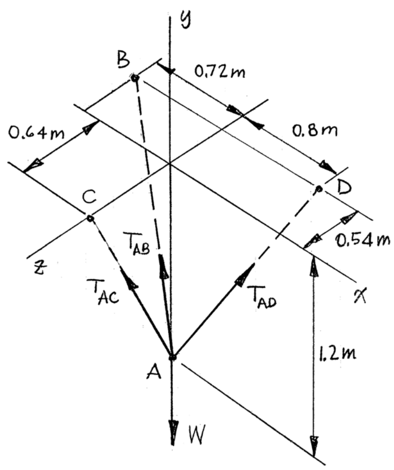
$$\mathbf{P} = 215 \text{ lb} \uparrow \blacktriangleleft$$

PROBLEM 2.105



The crate shown in Figure P2.105 and P2.108 is supported by three cables. Determine the weight of the crate knowing that the tension in cable *AB* is 3 kN.

SOLUTION



The forces applied at *A* are:

$$\mathbf{T}_{AB},\ \mathbf{T}_{AC},\ \mathbf{T}_{AD} \text{ and } \mathbf{P}$$

where $\mathbf{P} = P\mathbf{j}$. To express the other forces in terms of the unit vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$, we write

$$\overrightarrow{AB} = -(0.72\text{ m})\mathbf{i} + (1.2\text{ m})\mathbf{j} - (0.54\text{ m})\mathbf{k}, \qquad AB = 1.5\text{ m}$$

$$\overrightarrow{AC} = (1.2\text{ m})\,\mathbf{j} + (0.64\text{ m})\mathbf{k}, \qquad AC = 1.36\text{ m}$$

$$\overrightarrow{AD} = (0.8\text{ m})\mathbf{i} + (1.2\text{ m})\mathbf{j} - (0.54\text{ m})\mathbf{k}, \qquad AD = 1.54\text{ m}$$

and
$$\mathbf{T}_{AB} = T_{AB}\lambda_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = (-0.48\mathbf{i} + 0.8\mathbf{j} - 0.36\mathbf{k})T_{AB}$$

$$\mathbf{T}_{AC} = T_{AC}\lambda_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = (0.88235\mathbf{j} + 0.47059\mathbf{k})T_{AC}$$

$$\mathbf{T}_{AD} = T_{AD}\lambda_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD} = (0.51948\mathbf{i} + 0.77922\mathbf{j} - 0.35065\mathbf{k})T_{AD}$$

Equilibrium Condition with $\mathbf{W} = -W\mathbf{j}$

$$\Sigma F = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} - W\mathbf{j} = 0$$

Substituting the expressions obtained for $\mathbf{T}_{AB}$, $\mathbf{T}_{AC}$, and $\mathbf{T}_{AD}$ and factoring $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$:

$$(-0.48T_{AB} + 0.51948T_{AD})\mathbf{i} + (0.8T_{AB} + 0.88235T_{AC} + 0.77922T_{AD} - W)\mathbf{j}$$
$$+ (-0.36T_{AB} + 0.47059T_{AC} - 0.35065T_{AD})\mathbf{k} = 0$$

PROBLEM 2.105 CONTINUED

Equating to zero the coefficients of **i**, **j**, **k**:

$$-0.48T_{AB} + 0.51948T_{AD} = 0$$

$$0.8T_{AB} + 0.88235T_{AC} + 0.77922T_{AD} - W = 0$$

$$-0.36T_{AB} + 0.47059T_{AC} - 0.35065T_{AD} = 0$$

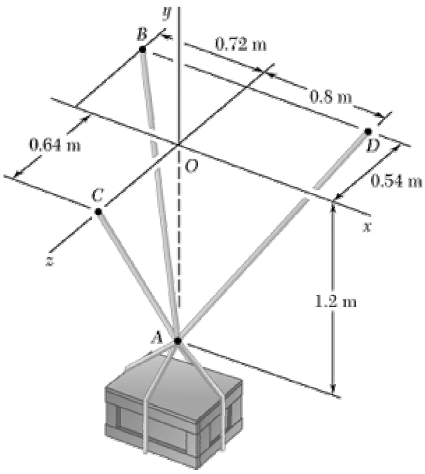
Substituting $T_{AB} = 3$ kN in Equations (1), (2) and (3) and solving the resulting set of equations, using conventional algorithms for solving linear algebraic equations, gives

$$T_{AC} = 4.3605 \text{ kN}$$

$$T_{AD} = 2.7720 \text{ kN}$$

$W = 8.41$ kN ◀

PROBLEM 2.106



For the crate of Problem 2.105, determine the weight of the crate knowing that the tension in cable *AD* is 2.8 kN.

Problem 2.105: The crate shown in Figure P2.105 and P2.108 is supported by three cables. Determine the weight of the crate knowing that the tension in cable *AB* is 3 kN.

SOLUTION

See Problem 2.105 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-0.48T_{AB} + 0.51948T_{AD} = 0$$

$$0.8T_{AB} + 0.88235T_{AC} + 0.77922T_{AD} - W = 0$$

$$-0.36T_{AB} + 0.47059T_{AC} - 0.35065T_{AD} = 0$$

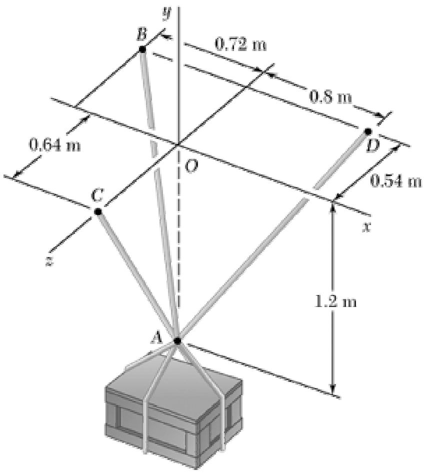
Substituting $T_{AD} = 2.8\text{ kN}$ in Equations (1), (2), and (3) above, and solving the resulting set of equations using conventional algorithms, gives

$$T_{AB} = 3.03\text{ kN}$$

$$T_{AC} = 4.40\text{ kN}$$

$$W = 8.49\text{ kN} ◀$$

PROBLEM 2.107



For the crate of Problem 2.105, determine the weight of the crate knowing that the tension in cable *AC* is 2.4 kN.

Problem 2.105: The crate shown in Figure P2.105 and P2.108 is supported by three cables. Determine the weight of the crate knowing that the tension in cable *AB* is 3 kN.

SOLUTION

See Problem 2.105 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-0.48T_{AB} + 0.51948T_{AD} = 0$$

$$0.8T_{AB} + 0.88235T_{AC} + 0.77922T_{AD} - W = 0$$

$$-0.36T_{AB} + 0.47059T_{AC} - 0.35065T_{AD} = 0$$

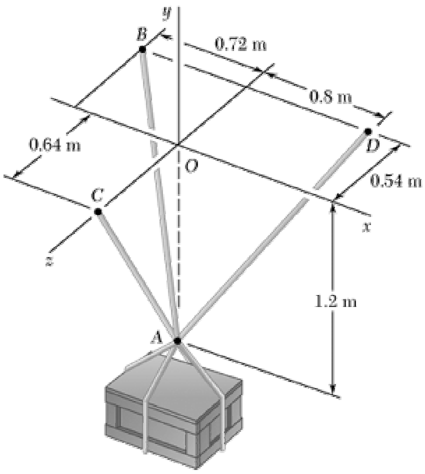
Substituting $T_{AC} = 2.4\text{ kN}$ in Equations (1), (2), and (3) above, and solving the resulting set of equations using conventional algorithms, gives

$$T_{AB} = 1.651\text{ kN}$$

$$T_{AD} = 1.526\text{ kN}$$

$W = 4.63\text{ kN}$ ◀

PROBLEM 2.108



A 750-kg crate is supported by three cables as shown. Determine the tension in each cable.

SOLUTION

See Problem 2.105 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-0.48T_{AB} + 0.51948T_{AD} = 0$$

$$0.8T_{AB} + 0.88235T_{AC} + 0.77922T_{AD} - W = 0$$

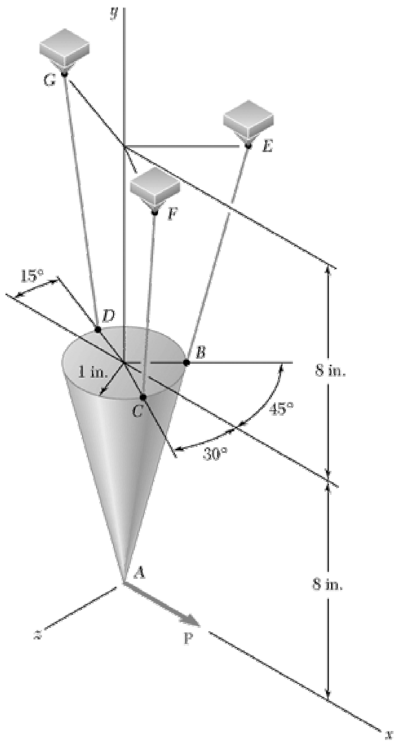
$$-0.36T_{AB} + 0.47059T_{AC} - 0.35065T_{AD} = 0$$

Substituting $W = (750 \text{ kg})(9.81 \text{ m/s}^2) = 7.36 \text{ kN}$ in Equations (1), (2), and (3) above, and solving the resulting set of equations using conventional algorithms, gives

$$T_{AB} = 2.63 \text{ kN} ◀$$

$$T_{AC} = 3.82 \text{ kN} ◀$$

$$T_{AD} = 2.43 \text{ kN} ◀$$



PROBLEM 2.109

A force **P** is applied as shown to a uniform cone which is supported by three cords, where the lines of action of the cords pass through the vertex *A* of the cone. Knowing that $P = 0$ and that the tension in cord *BE* is 0.2 lb, determine the weight *W* of the cone.

SOLUTION

Note that because the line of action of each of the cords passes through the vertex *A* of the cone, the cords all have the same length, and the unit vectors lying along the cords are parallel to the unit vectors lying along the generators of the cone.

Thus, for example, the unit vector along *BE* is identical to the unit vector along the generator *AB*.

Hence:

$$\boldsymbol{\lambda}_{AB} = \boldsymbol{\lambda}_{BE} = \frac{\cos 45°\mathbf{i} + 8\mathbf{j} - \sin 45°\mathbf{k}}{\sqrt{65}}$$

It follows that:

$$\mathbf{T}_{BE} = T_{BE}\boldsymbol{\lambda}_{BE} = T_{BE}\left(\frac{\cos 45°\mathbf{i} + 8\mathbf{j} - \sin 45°\mathbf{k}}{\sqrt{65}}\right)$$

$$\mathbf{T}_{CF} = T_{CF}\boldsymbol{\lambda}_{CF} = T_{CF}\left(\frac{\cos 30°\mathbf{i} + 8\mathbf{j} + \sin 30°\mathbf{k}}{\sqrt{65}}\right)$$

$$\mathbf{T}_{DG} = T_{DG}\boldsymbol{\lambda}_{DG} = T_{DG}\left(\frac{-\cos 15°\mathbf{i} + 8\mathbf{j} - \sin 15°\mathbf{k}}{\sqrt{65}}\right)$$

PROBLEM 2.109 CONTINUED

At A: $\Sigma\mathbf{F} = 0: \quad \mathbf{T}_{BE} + \mathbf{T}_{CF} + \mathbf{T}_{DG} + \mathbf{W} + \mathbf{P} = 0$

Then, isolating the factors of **i**, **j**, and **k**, we obtain three algebraic equations:

$$\mathbf{i}: \quad \frac{T_{BE}}{\sqrt{65}}\cos 45° + \frac{T_{CF}}{\sqrt{65}}\cos 30° - \frac{T_{DG}}{\sqrt{65}}\cos 15° + P = 0$$

or

$$T_{BE}\cos 45° + T_{CF}\cos 30° - T_{DG}\cos 15° + P\sqrt{65} = 0 \qquad (1)$$

$$\mathbf{j}: \quad T_{BE}\frac{8}{\sqrt{65}} + T_{CF}\frac{8}{\sqrt{65}} + T_{DG}\frac{8}{\sqrt{65}} - W = 0$$

or

$$T_{BE} + T_{CF} + T_{DG} - W\frac{\sqrt{65}}{8} = 0 \qquad (2)$$

$$\mathbf{k}: \quad -\frac{T_{BE}}{\sqrt{65}}\sin 45° + \frac{T_{CF}}{\sqrt{65}}\sin 30° - \frac{T_{DG}}{\sqrt{65}}\sin 15° = 0$$

or

$$-T_{BE}\sin 45° + T_{CF}\sin 30° - T_{DG}\sin 15° = 0 \qquad (3)$$

With $P = 0$ and the tension in cord $BE = 0.2$ lb:

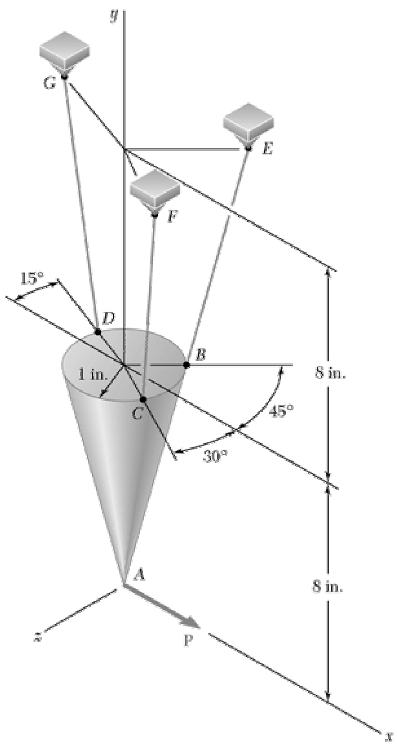
Solving the resulting Equations (1), (2), and (3) using conventional methods in Linear Algebra (elimination, matrix methods or iteration – with MATLAB or Maple, for example), we obtain:

$$T_{CF} = 0.669 \text{ lb}$$

$$T_{DG} = 0.746 \text{ lb}$$

$W = 1.603$ lb ◀

PROBLEM 2.110



A force **P** is applied as shown to a uniform cone which is supported by three cords, where the lines of action of the cords pass through the vertex *A* of the cone. Knowing that the cone weighs 1.6 lb, determine the range of values of *P* for which cord *CF* is taut.

SOLUTION

See Problem 2.109 for the Figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$\mathbf{i}: \quad T_{BE}\cos 45° + T_{CF}\cos 30° - T_{DG}\cos 15° + \sqrt{65}P = 0 \tag{1}$$

$$\mathbf{j}: \quad T_{BE} + T_{CF} + T_{DG} - W\frac{\sqrt{65}}{8} = 0 \tag{2}$$

$$\mathbf{k}: \quad -T_{BE}\sin 45° + T_{CF}\sin 30° - T_{DG}\sin 15° = 0 \tag{3}$$

With $W = 1.6$ lb, the range of values of *P* for which the cord *CF* is taut can found by solving Equations (1), (2), and (3) for the tension T_{CF} as a function of *P* and requiring it to be positive (>0).

Solving (1), (2), and (3) with unknown *P*, using conventional methods in Linear Algebra (elimination, matrix methods or iteration – with MATLAB or Maple, for example), we obtain:

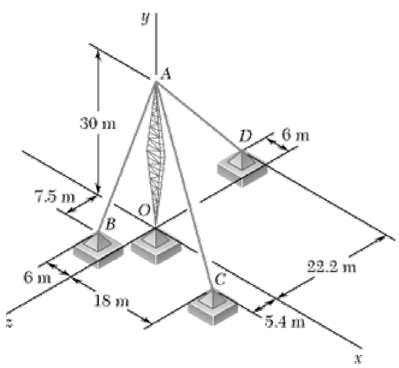
$$T_{CF} = (-1.729P + 0.668)\text{lb}$$

Hence, for $T_{CF} > 0$ $\quad -1.729P + 0.668 > 0$

or $\quad P < 0.386$ lb

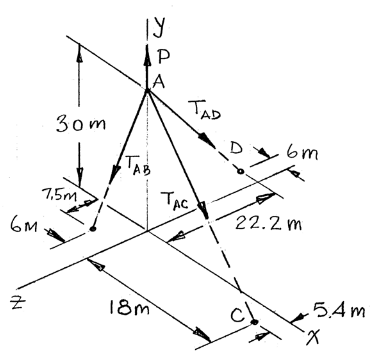
$$\therefore \quad 0 < P < 0.386 \text{ lb} \blacktriangleleft$$

PROBLEM 2.111



A transmission tower is held by three guy wires attached to a pin at *A* and anchored by bolts at *B*, *C*, and *D*. If the tension in wire *AB* is 3.6 kN, determine the vertical force **P** exerted by the tower on the pin at *A*.

SOLUTION



The force in each cable can be written as the product of the magnitude of the force and the unit vector along the cable. That is, with

$$\overrightarrow{AC} = (18\text{ m})\mathbf{i} - (30\text{ m})\mathbf{j} + (5.4\text{ m})\mathbf{k}$$

$$AC = \sqrt{(18\text{ m})^2 + (-30\text{ m})^2 + (5.4\text{ m})^2} = 35.4\text{ m}$$

$$\mathbf{T}_{AC} = T\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = \frac{T_{AC}}{35.4\text{ m}}\left[(18\text{ m})\mathbf{i} - (30\text{ m})\mathbf{j} + (5.4\text{ m})\mathbf{k}\right]$$

$$\mathbf{T}_{AC} = T_{AC}(0.5085\mathbf{i} - 0.8475\mathbf{j} + 0.1525\mathbf{k})$$

and $$\overrightarrow{AB} = -(6\text{ m})\mathbf{i} - (30\text{ m})\mathbf{j} + (7.5\text{ m})\mathbf{k}$$

$$AB = \sqrt{(-6\text{ m})^2 + (-30\text{ m})^2 + (7.5\text{ m})^2} = 31.5\text{ m}$$

$$\mathbf{T}_{AB} = T\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = \frac{T_{AB}}{31.5\text{ m}}\left[-(6\text{ m})\mathbf{i} - (30\text{ m})\mathbf{j} + (7.5\text{ m})\mathbf{k}\right]$$

$$\mathbf{T}_{AB} = T_{AB}(-0.1905\mathbf{i} - 0.9524\mathbf{j} + 0.2381\mathbf{k})$$

Finally $$\overrightarrow{AD} = -(6\text{ m})\mathbf{i} - (30\text{ m})\mathbf{j} - (22.2\text{ m})\mathbf{k}$$

$$AD = \sqrt{(-6\text{ m})^2 + (-30\text{ m})^2 + (-22.2\text{ m})^2} = 37.8\text{ m}$$

$$\mathbf{T}_{AD} = T\boldsymbol{\lambda}_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD} = \frac{T_{AD}}{37.8\text{ m}}\left[-(6\text{ m})\mathbf{i} - (30\text{ m})\mathbf{j} - (22.2\text{ m})\mathbf{k}\right]$$

$$\mathbf{T}_{AD} = T_{AD}(-0.1587\mathbf{i} - 0.7937\mathbf{j} - 0.5873\mathbf{k})$$

PROBLEM 2.111 CONTINUED

With $\mathbf{P} = P\mathbf{j}$, at A:

$$\Sigma \mathbf{F} = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + P\mathbf{j} = 0$$

Equating the factors of **i**, **j**, and **k** to zero, we obtain the linear algebraic equations:

$$\mathbf{i}: \quad -0.1905T_{AB} + 0.5085T_{AC} - 0.1587T_{AD} = 0 \tag{1}$$

$$\mathbf{j}: \quad -0.9524T_{AB} - 0.8475T_{AC} - 0.7937T_{AD} + P = 0 \tag{2}$$

$$\mathbf{k}: \quad 0.2381T_{AB} + 0.1525T_{AC} - 0.5873T_{AD} = 0 \tag{3}$$

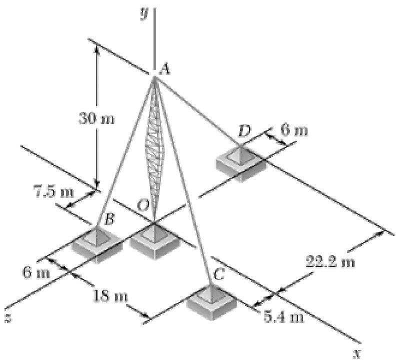
In Equations (1), (2) and (3), set $T_{AB} = 3.6$ kN, and, using conventional methods for solving Linear Algebraic Equations (MATLAB or Maple, for example), we obtain:

$$T_{AC} = 1.963 \text{ kN}$$

$$T_{AD} = 1.969 \text{ kN}$$

$\mathbf{P} = 6.66 \text{ kN} \uparrow$ ◀

PROBLEM 2.112



A transmission tower is held by three guy wires attached to a pin at *A* and anchored by bolts at *B*, *C*, and *D*. If the tension in wire *AC* is 2.6 kN, determine the vertical force **P** exerted by the tower on the pin at *A*.

SOLUTION

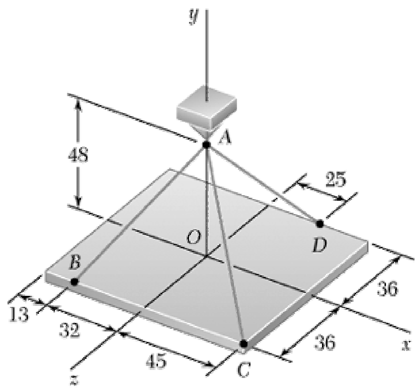
Based on the results of Problem 2.111, particularly Equations (1), (2) and (3), we substitute $T_{AC} = 2.6$ kN and solve the three resulting linear equations using conventional tools for solving Linear Algebraic Equations (MATLAB or Maple, for example), to obtain

$$T_{AB} = 4.77 \text{ kN}$$

$$T_{AD} = 2.61 \text{ kN}$$

$$\mathbf{P} = 8.81 \text{ kN} \uparrow \blacktriangleleft$$

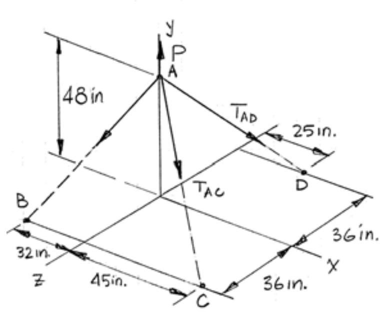
PROBLEM 2.113



Dimensions in inches

A rectangular plate is supported by three cables as shown. Knowing that the tension in cable AC is 15 lb, determine the weight of the plate.

SOLUTION



The (vector) force in each cable can be written as the product of the (scalar) force and the unit vector along the cable. That is, with

$$\overrightarrow{AB} = (32 \text{ in.})\mathbf{i} - (48 \text{ in.})\mathbf{j} + (36 \text{ in.})\mathbf{k}$$

$$AB = \sqrt{(-32 \text{ in.})^2 + (-48 \text{ in.})^2 + (36 \text{ in.})^2} = 68 \text{ in.}$$

$$\mathbf{T}_{AB} = T\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = \frac{T_{AB}}{68 \text{ in.}}\left[-(32 \text{ in.})\mathbf{i} - (48 \text{ in.})\mathbf{j} + (36 \text{ in.})\mathbf{k}\right]$$

$$\mathbf{T}_{AB} = T_{AB}(-0.4706\mathbf{i} - 0.7059\mathbf{j} + 0.5294\mathbf{k})$$

and $$\overrightarrow{AC} = (45 \text{ in.})\mathbf{i} - (48 \text{ in.})\mathbf{j} + (36 \text{ in.})\mathbf{k}$$

$$AC = \sqrt{(45 \text{ in.})^2 + (-48 \text{ in.})^2 + (36 \text{ in.})^2} = 75 \text{ in.}$$

$$\mathbf{T}_{AC} = T\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = \frac{T_{AC}}{75 \text{ in.}}\left[(45 \text{ in.})\mathbf{i} - (48 \text{ in.})\mathbf{j} + (36 \text{ in.})\mathbf{k}\right]$$

$$\mathbf{T}_{AC} = T_{AC}(0.60\mathbf{i} - 0.64\mathbf{j} + 0.48\mathbf{k})$$

Finally, $$\overrightarrow{AD} = (25 \text{ in.})\mathbf{i} - (48 \text{ in.})\mathbf{j} - (36 \text{ in.})\mathbf{k}$$

$$AD = \sqrt{(25 \text{ in.})^2 + (-48 \text{ in.})^2 + (-36 \text{ in.})^2} = 65 \text{ in.}$$

PROBLEM 2.113 CONTINUED

$$\mathbf{T}_{AD} = T\boldsymbol{\lambda}_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD} = \frac{T_{AD}}{65 \text{ in.}}\left[(25 \text{ in.})\mathbf{i} - (48 \text{ in.})\mathbf{j} - (36 \text{ in.})\mathbf{k}\right]$$

$$\mathbf{T}_{AD} = T_{AD}(0.3846\mathbf{i} - 0.7385\mathbf{j} - 0.5538\mathbf{k})$$

With $\mathbf{W} = W\mathbf{j}$, at A we have:

$$\Sigma\mathbf{F} = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + W\mathbf{j} = 0$$

Equating the factors of **i**, **j**, and **k** to zero, we obtain the linear algebraic equations:

$$\mathbf{i}: \quad -0.4706T_{AB} + 0.60T_{AC} - 0.3846T_{AD} = 0 \qquad (1)$$

$$\mathbf{j}: \quad -0.7059T_{AB} - 0.64T_{AC} - 0.7385T_{AD} + W = 0 \qquad (2)$$

$$\mathbf{k}: \quad 0.5294T_{AB} + 0.48T_{AC} - 0.5538T_{AD} = 0 \qquad (3)$$

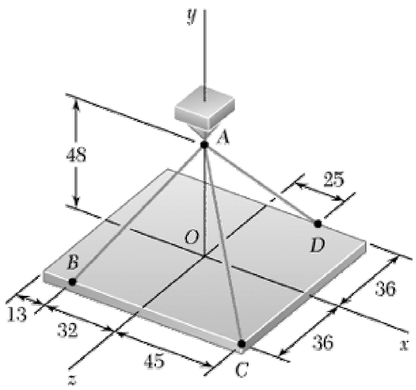
In Equations (1), (2) and (3), set $T_{AC} = 15$ lb, and, using conventional methods for solving Linear Algebraic Equations (MATLAB or Maple, for example), we obtain:

$$T_{AB} = 136.0 \text{ lb}$$

$$T_{AD} = 143.0 \text{ lb}$$

$W = 211$ lb ◀

PROBLEM 2.114



Dimensions in inches

A rectangular plate is supported by three cables as shown. Knowing that the tension in cable *AD* is 120 lb, determine the weight of the plate.

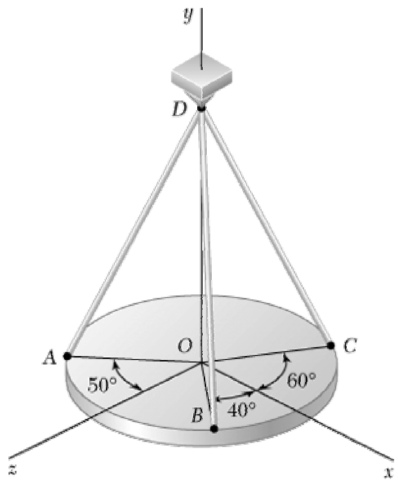
SOLUTION

Based on the results of Problem 2.111, particularly Equations (1), (2) and (3), we substitute $T_{AD} = 120$ lb and solve the three resulting linear equations using conventional tools for solving Linear Algebraic Equations (MATLAB or Maple, for example), to obtain

$$T_{AC} = 12.59 \text{ lb}$$

$$T_{AB} = 114.1 \text{ lb}$$

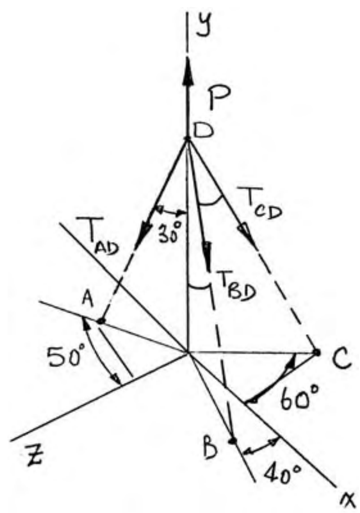
$W = 177.2$ lb ◀



PROBLEM 2.115

A horizontal circular plate having a mass of 28 kg is suspended as shown from three wires which are attached to a support D and form 30° angles with the vertical. Determine the tension in each wire.

SOLUTION



$$\Sigma F_x = 0: \quad -T_{AD} \sin 30° \sin 50° + T_{BD} \sin 30° \cos 40°$$

$$+ T_{CD} \sin 30° \cos 60° = 0$$

Dividing through by the factor $\sin 30°$ and evaluating the trigonometric functions gives

$$-0.7660 T_{AD} + 0.7660 T_{BD} + 0.50 T_{CD} = 0 \qquad (1)$$

Similarly,

$$\Sigma F_z = 0: \quad T_{AD} \sin 30° \cos 50° + T_{BD} \sin 30° \sin 40°$$

$$- T_{CD} \sin 30° \sin 60° = 0$$

or $$0.6428 T_{AD} + 0.6428 T_{BD} - 0.8660 T_{CD} = 0 \qquad (2)$$

From (1) $$T_{AD} = T_{BD} + 0.6527 T_{CD}$$

Substituting this into (2):

$$T_{BD} = 0.3573 T_{CD} \qquad (3)$$

Using T_{AD} from above:

$$T_{AD} = T_{CD} \qquad (4)$$

Now,

$$+\uparrow \Sigma F_y = 0: \quad -T_{AD} \cos 30° - T_{BD} \cos 30° - T_{CD} \cos 30°$$

$$+ (28 \text{ kg})\left(9.81 \text{ m/s}^2\right) = 0$$

or $$T_{AD} + T_{BD} + T_{CD} = 317.2 \text{ N}$$

PROBLEM 2.115 CONTINUED

Using (3) and (4), above:

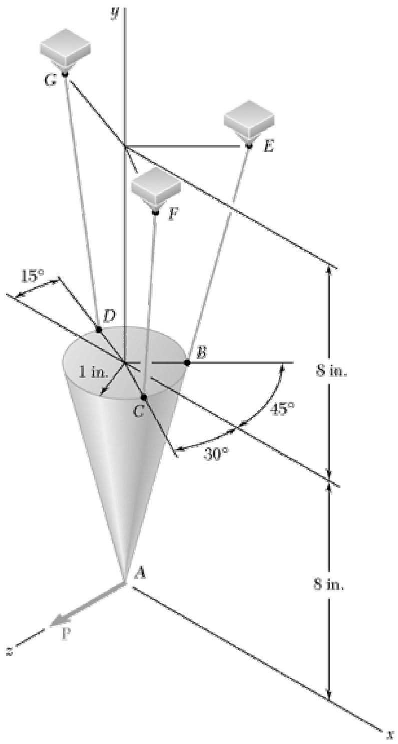
$$T_{CD} + 0.3573T_{CD} + T_{CD} = 317.2 \text{ N}$$

Then: $T_{AD} = 135.1 \text{ N}$ ◀

$T_{BD} = 46.9 \text{ N}$ ◀

$T_{CD} = 135.1 \text{ N}$ ◀

PROBLEM 2.119



A force **P** is applied as shown to a uniform cone which is supported by three cords, where the lines of action of the cords pass through the vertex A of the cone. Knowing that the cone weighs 2.4 lb and that $P = 0$, determine the tension in each cord.

SOLUTION

Note that because the line of action of each of the cords passes through the vertex A of the cone, the cords all have the same length, and the unit vectors lying along the cords are parallel to the unit vectors lying along the generators of the cone.

Thus, for example, the unit vector along BE is identical to the unit vector along the generator AB.

Hence:

$$\boldsymbol{\lambda}_{AB} = \lambda_{BE} = \frac{\cos 45°\mathbf{i} + 8\mathbf{j} - \sin 45°\mathbf{k}}{\sqrt{65}}$$

It follows that:

$$\mathbf{T}_{BE} = T_{BE}\boldsymbol{\lambda}_{BE} = T_{BE}\left(\frac{\cos 45°\mathbf{i} + 8\mathbf{j} - \sin 45°\mathbf{k}}{\sqrt{65}}\right)$$

$$\mathbf{T}_{CF} = T_{CF}\boldsymbol{\lambda}_{CF} = T_{CF}\left(\frac{\cos 30°\mathbf{i} + 8\mathbf{j} + \sin 30°\mathbf{k}}{\sqrt{65}}\right)$$

$$\mathbf{T}_{DG} = T_{DG}\boldsymbol{\lambda}_{DG} = T_{DG}\left(\frac{-\cos 15°\mathbf{i} + 8\mathbf{j} - \sin 15°\mathbf{k}}{\sqrt{65}}\right)$$

At A: $\Sigma\mathbf{F} = 0$: $\mathbf{T}_{BE} + \mathbf{T}_{CF} + \mathbf{T}_{DG} + \mathbf{W} + \mathbf{P} = 0$

PROBLEM 2.119 CONTINUED

Then, isolating the factors if **i**, **j**, and **k** we obtain three algebraic equations:

$$\mathbf{i}: \quad \frac{T_{BE}}{\sqrt{65}}\cos 45° + \frac{T_{CF}}{\sqrt{65}}\cos 30° - \frac{T_{DG}}{\sqrt{65}}\cos 15° = 0$$

or

$$T_{BE}\cos 45° + T_{CF}\cos 30° - T_{DG}\cos 15° = 0 \tag{1}$$

$$\mathbf{j}: \quad T_{BE}\frac{8}{\sqrt{65}} + T_{CF}\frac{8}{\sqrt{65}} + T_{DG}\frac{8}{\sqrt{65}} - W = 0$$

or

$$T_{BE} + T_{CF} + T_{DG} = \frac{2.4}{8}\sqrt{65} = 0.3\sqrt{65} \tag{2}$$

$$\mathbf{k}: \quad -\frac{T_{BE}}{\sqrt{65}}\sin 45° + \frac{T_{CF}}{\sqrt{65}}\sin 30° - \frac{T_{DG}}{\sqrt{65}}\sin 15° - P = 0$$

or

$$-T_{BE}\sin 45° + T_{CF}\sin 30° - T_{DG}\sin 15° = P\sqrt{65} \tag{3}$$

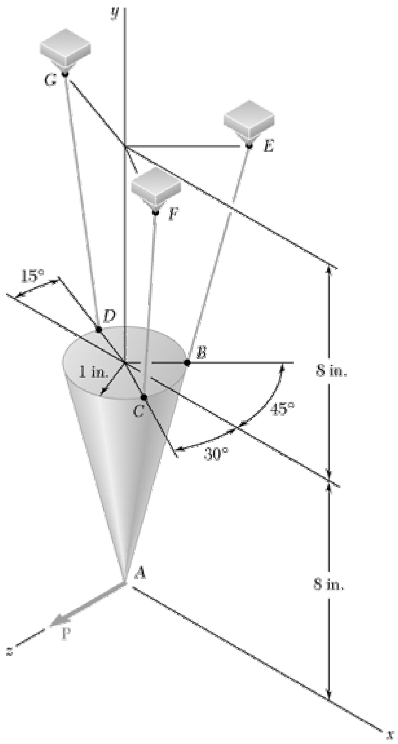
With $P = 0,$ the tension in the cords can be found by solving the resulting Equations (1), (2), and (3) using conventional methods in Linear Algebra (elimination, matrix methods or iteration–with MATLAB or Maple, for example). We obtain

$T_{BE} = 0.299$ lb ◀

$T_{CF} = 1.002$ lb ◀

$T_{DG} = 1.117$ lb ◀

PROBLEM 2.120



A force **P** is applied as shown to a uniform cone which is supported by three cords, where the lines of action of the cords pass through the vertex A of the cone. Knowing that the cone weighs 2.4 lb and that $P = 0.1$ lb, determine the tension in each cord.

SOLUTION

See Problem 2.121 for the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$T_{BE}\cos 45° + T_{CF}\cos 30° - T_{DG}\cos 15° = 0 \tag{1}$$

$$T_{BE} + T_{CF} + T_{DG} = 0.3\sqrt{65} \tag{2}$$

$$-T_{BE}\sin 45° + T_{CF}\sin 30° - T_{DG}\sin 15° = P\sqrt{65} \tag{3}$$

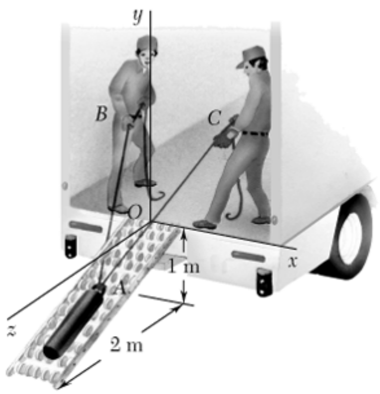
With $P = 0.1\,\text{lb}$, solving (1), (2), and (3), using conventional methods in Linear Algebra (elimination, matrix methods or iteration–with MATLAB or Maple, for example), we obtain

$T_{BE} = 1.006\text{ lb}$ ◀

$T_{CF} = 0.357\text{ lb}$ ◀

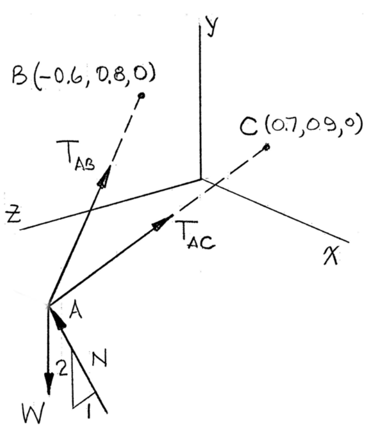
$T_{DG} = 1.056\text{ lb}$ ◀

PROBLEM 2.121



Using two ropes and a roller chute, two workers are unloading a 200-kg cast-iron counterweight from a truck. Knowing that at the instant shown the counterweight is kept from moving and that the positions of points *A*, *B*, and *C* are, respectively, *A*(0, –0.5 m, 1 m), *B*(–0.6 m, 0.8 m, 0), and *C*(0.7 m, 0.9 m, 0), and assuming that no friction exists between the counterweight and the chute, determine the tension in each rope. (*Hint:* Since there is no friction, the force exerted by the chute on the counterweight must be perpendicular to the chute.)

SOLUTION



From the geometry of the chute:

$$\mathbf{N} = \frac{N}{\sqrt{5}}(2\mathbf{j} + \mathbf{k}) = N(0.8944\mathbf{j} + 0.4472\mathbf{k})$$

As in Problem 2.11, for example, the force in each rope can be written as the product of the magnitude of the force and the unit vector along the cable. Thus, with

$$\overrightarrow{AB} = -(0.6\text{ m})\mathbf{i} + (1.3\text{ m})\mathbf{j} + (1\text{ m})\mathbf{k}$$

$$AB = \sqrt{(-0.6\text{ m})^2 + (1.3\text{ m})^2 + (1\text{ m})^2} = 1.764\text{ m}$$

$$\mathbf{T}_{AB} = T\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = \frac{T_{AB}}{1.764\text{ m}}\left[-(0.6\text{ m})\mathbf{i} + (1.3\text{ m})\mathbf{j} + (1\text{ m})\mathbf{k}\right]$$

$$\mathbf{T}_{AB} = T_{AB}(-0.3436\mathbf{i} + 0.7444\mathbf{j} + 0.5726\mathbf{k})$$

and $$\overrightarrow{AC} = (0.7\text{ m})\mathbf{i} + (1.4\text{ m})\mathbf{j} - (1\text{ m})\mathbf{k}$$

$$AC = \sqrt{(0.7\text{ m})^2 + (1.4\text{ m})^2 + (-1\text{ m})^2} = 1.8574\text{ m}$$

$$\mathbf{T}_{AC} = T\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = \frac{T_{AC}}{1.764\text{ m}}\left[(0.7\text{ m})\mathbf{i} + (1.4\text{ m})\mathbf{j} - (1\text{ m})\mathbf{k}\right]$$

$$\mathbf{T}_{AC} = T_{AC}(0.3769\mathbf{i} + 0.7537\mathbf{j} - 0.5384\mathbf{k})$$

Then: $$\Sigma\mathbf{F} = 0: \quad \mathbf{N} + \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{W} = 0$$

PROBLEM 2.121 CONTINUED

With $W = (200 \text{ kg})(9.81 \text{ m/s}) = 1962 \text{ N}$, and equating the factors of **i**, **j**, and **k** to zero, we obtain the linear algebraic equations:

$$\mathbf{i}: \quad -0.3436T_{AB} + 0.3769T_{AC} = 0 \qquad (1)$$

$$\mathbf{j}: \quad 0.7444T_{AB} + 0.7537T_{AC} + 0.8944N - 1962 = 0 \qquad (2)$$

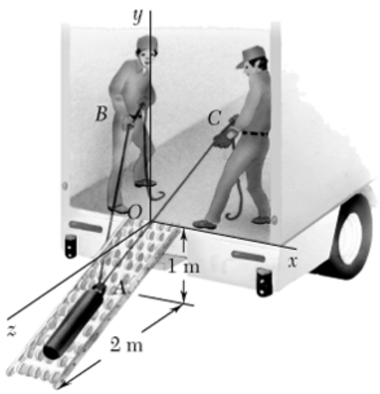
$$\mathbf{k}: \quad -0.5726T_{AB} - 0.5384T_{AC} + 0.4472N = 0 \qquad (3)$$

Using conventional methods for solving Linear Algebraic Equations (elimination, MATLAB or Maple, for example), we obtain

$$N = 1311 \text{ N}$$

$$T_{AB} = 551 \text{ N} \blacktriangleleft$$

$$T_{AC} = 503 \text{ N} \blacktriangleleft$$

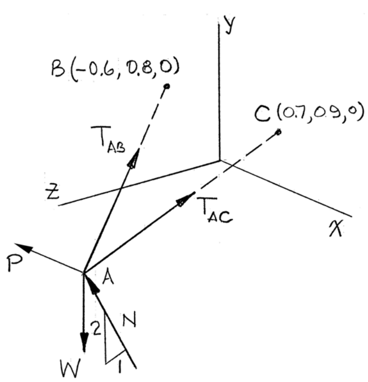


PROBLEM 2.122

Solve Problem 2.121 assuming that a third worker is exerting a force $\mathbf{P} = -(180\text{ N})\mathbf{i}$ on the counterweight.

Problem 2.121: Using two ropes and a roller chute, two workers are unloading a 200-kg cast-iron counterweight from a truck. Knowing that at the instant shown the counterweight is kept from moving and that the positions of points *A*, *B*, and *C* are, respectively, *A*(0, –0.5 m, 1 m), *B*(–0.6 m, 0.8 m, 0), and *C*(0.7 m, 0.9 m, 0), and assuming that no friction exists between the counterweight and the chute, determine the tension in each rope. (*Hint:* Since there is no friction, the force exerted by the chute on the counterweight must be perpendicular to the chute.)

SOLUTION



From the geometry of the chute:

$$\mathbf{N} = \frac{N}{\sqrt{5}}(2\mathbf{j} + \mathbf{k}) = N(0.8944\mathbf{j} + 0.4472\mathbf{k})$$

As in Problem 2.11, for example, the force in each rope can be written as the product of the magnitude of the force and the unit vector along the cable. Thus, with

$$\overrightarrow{AB} = -(0.6\text{ m})\mathbf{i} + (1.3\text{ m})\mathbf{j} + (1\text{ m})\mathbf{k}$$

$$AB = \sqrt{(-0.6\text{ m})^2 + (1.3\text{ m})^2 + (1\text{ m})^2} = 1.764\text{ m}$$

$$\mathbf{T}_{AB} = T\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = \frac{T_{AB}}{1.764\text{ m}}\left[-(0.6\text{ m})\mathbf{i} + (1.3\text{ m})\mathbf{j} + (1\text{ m})\mathbf{k}\right]$$

$$\mathbf{T}_{AB} = T_{AB}(-0.3436\mathbf{i} + 0.7444\mathbf{j} + 0.5726\mathbf{k})$$

and
$$\overrightarrow{AC} = (0.7\text{ m})\mathbf{i} + (1.4\text{ m})\mathbf{j} - (1\text{ m})\mathbf{k}$$

$$AC = \sqrt{(0.7\text{ m})^2 + (1.4\text{ m})^2 + (-1\text{ m})^2} = 1.8574\text{ m}$$

$$\mathbf{T}_{AC} = T\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = \frac{T_{AC}}{1.764\text{ m}}\left[(0.7\text{ m})\mathbf{i} + (1.4\text{ m})\mathbf{j} - (1\text{ m})\mathbf{k}\right]$$

$$\mathbf{T}_{AC} = T_{AC}(0.3769\mathbf{i} + 0.7537\mathbf{j} - 0.5384\mathbf{k})$$

Then:
$$\Sigma\mathbf{F} = 0: \quad \mathbf{N} + \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{P} + \mathbf{W} = 0$$

PROBLEM 2.122 CONTINUED

Where $\mathbf{P} = -(180\text{ N})\mathbf{i}$

and $\mathbf{W} = -\left[(200\text{ kg})(9.81\text{ m/s}^2)\right]\mathbf{j}$

$$= -(1962\text{ N})\,\mathbf{j}$$

Equating the factors of **i**, **j**, and **k** to zero, we obtain the linear equations:

$$\mathbf{i}:\quad -0.3436T_{AB} + 0.3769T_{AC} - 180 = 0$$

$$\mathbf{j}:\quad 0.8944N + 0.7444T_{AB} + 0.7537T_{AC} - 1962 = 0$$

$$\mathbf{k}:\quad 0.4472N - 0.5726T_{AB} - 0.5384T_{AC} = 0$$

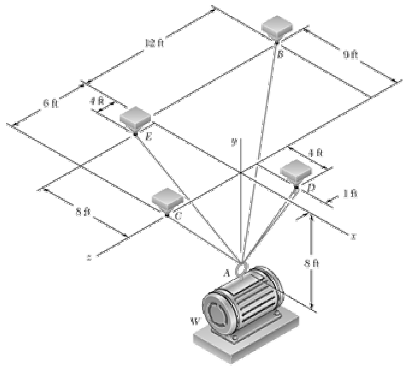
Using conventional methods for solving Linear Algebraic Equations (elimination, MATLAB or Maple, for example), we obtain

$$N = 1302\text{ N}$$

$T_{AB} = 306\text{ N}$ ◀

$T_{AC} = 756\text{ N}$ ◀

PROBLEM 2.123



A piece of machinery of weight W is temporarily supported by cables AB, AC, and ADE. Cable ADE is attached to the ring at A, passes over the pulley at D and back through the ring, and is attached to the support at E. Knowing that $W = 320$ lb, determine the tension in each cable. (*Hint:* The tension is the same in all portions of cable ADE.)

SOLUTION

The (vector) force in each cable can be written as the product of the (scalar) force and the unit vector along the cable. That is, with

$$\overrightarrow{AB} = -(9\text{ ft})\mathbf{i} + (8\text{ ft})\mathbf{j} - (12\text{ ft})\mathbf{k}$$

$$AB = \sqrt{(-9\text{ ft})^2 + (8\text{ ft})^2 + (-12\text{ ft})^2} = 17\text{ ft}$$

$$\mathbf{T}_{AB} = T\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = \frac{T_{AB}}{17\text{ ft}}\left[-(9\text{ ft})\mathbf{i} + (8\text{ ft})\mathbf{j} - (12\text{ ft})\mathbf{k}\right]$$

$$\mathbf{T}_{AB} = T_{AB}(-0.5294\mathbf{i} + 0.4706\mathbf{j} - 0.7059\mathbf{k})$$

and

$$\overrightarrow{AC} = (0)\mathbf{i} + (8\text{ ft})\mathbf{j} + (6\text{ ft})\mathbf{k}$$

$$AC = \sqrt{(0\text{ ft})^2 + (8\text{ ft})^2 + (6\text{ ft})^2} = 10\text{ ft}$$

$$\mathbf{T}_{AC} = T\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = \frac{T_{AC}}{10\text{ ft}}\left[(0\text{ ft})\mathbf{i} + (8\text{ ft})\mathbf{j} + (6\text{ ft})\mathbf{k}\right]$$

$$\mathbf{T}_{AC} = T_{AC}(0.8\mathbf{j} + 0.6\mathbf{k})$$

and

$$\overrightarrow{AD} = (4\text{ ft})\mathbf{i} + (8\text{ ft})\mathbf{j} - (1\text{ ft})\mathbf{k}$$

$$AD = \sqrt{(4\text{ ft})^2 + (8\text{ ft})^2 + (-1\text{ ft})^2} = 9\text{ ft}$$

$$\mathbf{T}_{AD} = T\boldsymbol{\lambda}_{AD} = T_{ADE}\frac{\overrightarrow{AD}}{AD} = \frac{T_{ADE}}{9\text{ ft}}\left[(4\text{ ft})\mathbf{i} + (8\text{ ft})\mathbf{j} - (1\text{ ft})\mathbf{k}\right]$$

$$\mathbf{T}_{AD} = T_{ADE}(0.4444\mathbf{i} + 0.8889\mathbf{j} - 0.1111\mathbf{k})$$

PROBLEM 2.123 CONTINUED

Finally,

$$\overrightarrow{AE} = (-8 \text{ ft})\mathbf{i} + (8 \text{ ft})\mathbf{j} + (4 \text{ ft})\mathbf{k}$$

$$AE = \sqrt{(-8 \text{ ft})^2 + (8 \text{ ft})^2 + (4 \text{ ft})^2} = 12 \text{ ft}$$

$$\mathbf{T}_{AE} = T\boldsymbol{\lambda}_{AE} = T_{ADE}\frac{\overrightarrow{AE}}{AE} = \frac{T_{ADE}}{12 \text{ ft}}\left[(-8 \text{ ft})\mathbf{i} + (8 \text{ ft})\mathbf{j} + (4 \text{ ft})\mathbf{k}\right]$$

$$\mathbf{T}_{AE} = T_{ADE}(-0.6667\mathbf{i} + 0.6667\mathbf{j} + 0.3333\mathbf{k})$$

With the weight of the machinery, $\mathbf{W} = -W\mathbf{j}$, at A, we have:

$$\Sigma\mathbf{F} = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + 2\mathbf{T}_{AD} - W\mathbf{j} = 0$$

Equating the factors of $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ to zero, we obtain the following linear algebraic equations:

$$-0.5294T_{AB} + 2(0.4444T_{ADE}) - 0.6667T_{ADE} = 0 \qquad (1)$$

$$0.4706T_{AB} + 0.8T_{AC} + 2(0.8889T_{ADE}) + 0.6667T_{ADE} - W = 0 \qquad (2)$$

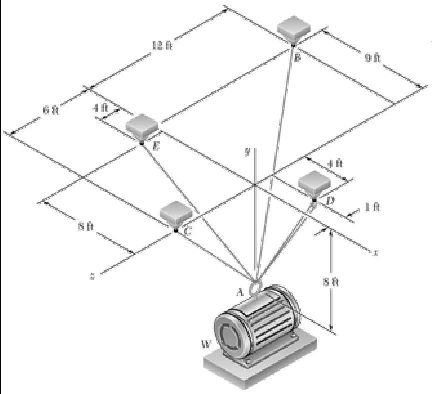
$$-0.7059T_{AB} + 0.6T_{AC} - 2(0.1111T_{ADE}) + 0.3333T_{ADE} = 0 \qquad (3)$$

Knowing that $W = 320 \text{ lb}$, we can solve Equations (1), (2) and (3) using conventional methods for solving Linear Algebraic Equations (elimination, matrix methods via MATLAB or Maple, for example) to obtain

$T_{AB} = 46.5 \text{ lb}$ ◀

$T_{AC} = 34.2 \text{ lb}$ ◀

$T_{ADE} = 110.8 \text{ lb}$ ◀



PROBLEM 2.124

A piece of machinery of weight W is temporarily supported by cables AB, AC, and ADE. Cable ADE is attached to the ring at A, passes over the pulley at D and back through the ring, and is attached to the support at E. Knowing that the tension in cable AB is 68 lb, determine (*a*) the tension in AC, (*b*) the tension in ADE, (*c*) the weight W. (*Hint:* The tension is the same in all portions of cable ADE.)

SOLUTION

See Problem 2.123 for the analysis leading to the linear algebraic Equations (1), (2), and (3), below:

$$-0.5294T_{AB} + 2\left(0.4444T_{ADE}\right) - 0.6667T_{ADE} = 0 \qquad (1)$$

$$0.4706T_{AB} + 0.8T_{AC} + 2\left(0.8889T_{ADE}\right) + 0.6667T_{ADE} - W = 0 \qquad (2)$$

$$-0.7059T_{AB} + 0.6T_{AC} - 2\left(0.1111T_{ADE}\right) + 0.3333T_{ADE} = 0 \qquad (3)$$

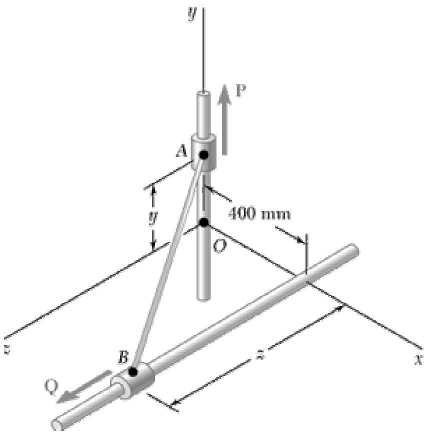
Knowing that the tension in cable AB is 68 lb, we can solve Equations (1), (2) and (3) using conventional methods for solving Linear Algebraic Equations (elimination, matrix methods via MATLAB or Maple, for example) to obtain

(*a*) $T_{AC} = 50.0\text{ lb}$ ◀

(*b*) $T_{AE} = 162.0\text{ lb}$ ◀

(*c*) $W = 468\text{ lb}$ ◀

PROBLEM 2.128



Solve Problem 2.127 assuming $y = 550$ mm.

Problem 2.127: Collars *A* and *B* are connected by a 1-m-long wire and can slide freely on frictionless rods. If a force $\mathbf{P} = (680 \text{ N})\mathbf{j}$ is applied at *A*, determine (*a*) the tension in the wire when $y = 300$ mm, (*b*) the magnitude of the force **Q** required to maintain the equilibrium of the system.

SOLUTION

From the analysis of Problem 2.127, particularly the results:

$$y^2 + z^2 = 0.84 \text{ m}^2$$

$$T_{AB} = \frac{680 \text{ N}}{y}$$

$$Q = \frac{680 \text{ N}}{y} z$$

With $y = 550 \text{ mm} = 0.55 \text{ m},$ we obtain:

$$z^2 = 0.84 \text{ m}^2 - (0.55 \text{ m})^2$$

$$\therefore \; z = 0.733 \text{ m}$$

and

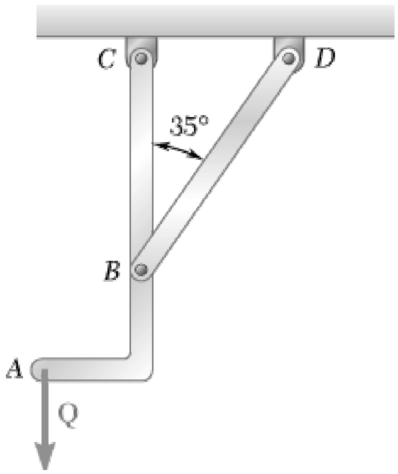
(*a*) $$T_{AB} = \frac{680 \text{ N}}{0.55} = 1236.4 \text{ N}$$

or $$T_{AB} = 1.236 \text{ kN} \blacktriangleleft$$

and

(*b*) $$Q = 1236(0.866) \text{ N} = 906 \text{ N}$$

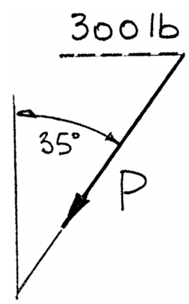
or $$Q = 0.906 \text{ kN} \blacktriangleleft$$



PROBLEM 2.129

Member *BD* exerts on member *ABC* a force **P** directed along line *BD*. Knowing that **P** must have a 300-lb horizontal component, determine (*a*) the magnitude of the force **P**, (*b*) its vertical component.

SOLUTION



(*a*)

$$P \sin 35° = 300 \text{ 1b}$$

$$P = \frac{300 \text{ lb}}{\sin 35°}$$

$P = 523 \text{ lb}$ ◀

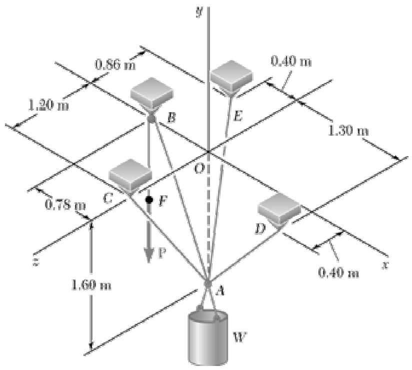
(*b*) Vertical Component

$$P_v = P \cos 35°$$

$$= (523 \text{ lb}) \cos 35°$$

$P_v = 428 \text{ lb}$ ◀

PROBLEM 2.130



A container of weight W is suspended from ring A, to which cables AC and AE are attached. A force $\mathbf{P}$ is applied to the end F of a third cable which passes over a pulley at B and through ring A and which is attached to a support at D. Knowing that $W = 1000$ N, determine the magnitude of $\mathbf{P}$. (*Hint:* The tension is the same in all portions of cable $FBAD$.)

SOLUTION

The (vector) force in each cable can be written as the product of the (scalar) force and the unit vector along the cable. That is, with

$$\overrightarrow{AB} = -(0.78\text{ m})\mathbf{i} + (1.6\text{ m})\mathbf{j} + (0\text{ m})\mathbf{k}$$

$$AB = \sqrt{(-0.78\text{ m})^2 + (1.6\text{ m})^2 + (0)^2} = 1.78\text{ m}$$

$$\mathbf{T}_{AB} = T\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = \frac{T_{AB}}{1.78\text{ m}}\left[-(0.78\text{ m})\mathbf{i} + (1.6\text{ m})\mathbf{j} + (0\text{ m})\mathbf{k}\right]$$

$$\mathbf{T}_{AB} = T_{AB}(-0.4382\mathbf{i} + 0.8989\mathbf{j} + 0\mathbf{k})$$

and

$$\overrightarrow{AC} = (0)\mathbf{i} + (1.6\text{ m})\mathbf{j} + (1.2\text{ m})\mathbf{k}$$

$$AC = \sqrt{(0\text{ m})^2 + (1.6\text{ m})^2 + (1.2\text{ m})^2} = 2\text{ m}$$

$$\mathbf{T}_{AC} = T\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = \frac{T_{AC}}{2\text{ m}}\left[(0)\mathbf{i} + (1.6\text{ m})\mathbf{j} + (1.2\text{ m})\mathbf{k}\right]$$

$$\mathbf{T}_{AC} = T_{AC}(0.8\mathbf{j} + 0.6\mathbf{k})$$

and

$$\overrightarrow{AD} = (1.3\text{ m})\mathbf{i} + (1.6\text{ m})\mathbf{j} + (0.4\text{ m})\mathbf{k}$$

$$AD = \sqrt{(1.3\text{ m})^2 + (1.6\text{ m})^2 + (0.4\text{ m})^2} = 2.1\text{ m}$$

$$\mathbf{T}_{AD} = T\boldsymbol{\lambda}_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD} = \frac{T_{AD}}{2.1\text{ m}}\left[(1.3\text{ m})\mathbf{i} + (1.6\text{ m})\mathbf{j} + (0.4\text{ m})\mathbf{k}\right]$$

$$\mathbf{T}_{AD} = T_{AD}(0.6190\mathbf{i} + 0.7619\mathbf{j} + 0.1905\mathbf{k})$$

PROBLEM 2.130 CONTINUED

Finally,

$$\overrightarrow{AE} = -(0.4\text{ m})\mathbf{i} + (1.6\text{ m})\mathbf{j} - (0.86\text{ m})\mathbf{k}$$

$$AE = \sqrt{(-0.4\text{ m})^2 + (1.6\text{ m})^2 + (-0.86\text{ m})^2} = 1.86\text{ m}$$

$$\mathbf{T}_{AE} = T\boldsymbol{\lambda}_{AE} = T_{AE}\frac{\overrightarrow{AE}}{AE} = \frac{T_{AE}}{1.86\text{ m}}\left[-(0.4\text{ m})\mathbf{i} + (1.6\text{ m})\mathbf{j} - (0.86\text{ m})\mathbf{k}\right]$$

$$\mathbf{T}_{AE} = T_{AE}(-0.2151\mathbf{i} + 0.8602\mathbf{j} - 0.4624\mathbf{k})$$

With the weight of the container $\mathbf{W} = -W\mathbf{j}$, at A we have:

$$\Sigma\mathbf{F} = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} - W\mathbf{j} = 0$$

Equating the factors of $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ to zero, we obtain the following linear algebraic equations:

$$-0.4382T_{AB} + 0.6190T_{AD} - 0.2151T_{AE} = 0 \qquad (1)$$

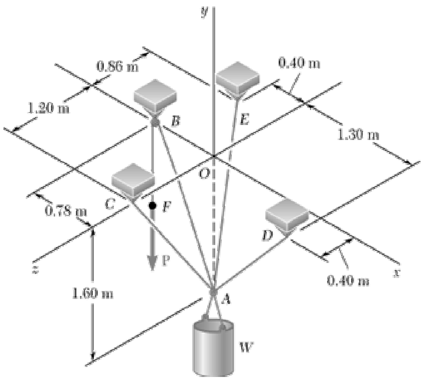
$$0.8989T_{AB} + 0.8T_{AC} + 0.7619T_{AD} + 0.8602T_{AE} - W = 0 \qquad (2)$$

$$0.6T_{AC} + 0.1905T_{AD} - 0.4624T_{AE} = 0 \qquad (3)$$

Knowing that $W = 1000\text{ N}$ and that because of the pulley system at B $T_{AB} = T_{AD} = P$, where P is the externally applied (unknown) force, we can solve the system of linear equations (1), (2) and (3) uniquely for P.

$$P = 378\text{ N} \blacktriangleleft$$

PROBLEM 2.131



A container of weight W is suspended from ring A, to which cables AC and AE are attached. A force $\mathbf{P}$ is applied to the end F of a third cable which passes over a pulley at B and through ring A and which is attached to a support at D. Knowing that the tension in cable AC is 150 N, determine (*a*) the magnitude of the force $\mathbf{P}$, (*b*) the weight W of the container. (*Hint:* The tension is the same in all portions of cable *FBAD*.)

SOLUTION

Here, as in Problem 2.130, the support of the container consists of the four cables *AE*, *AC*, *AD*, and *AB*, with the condition that the force in cables *AB* and *AD* is equal to the externally applied force P. Thus, with the condition

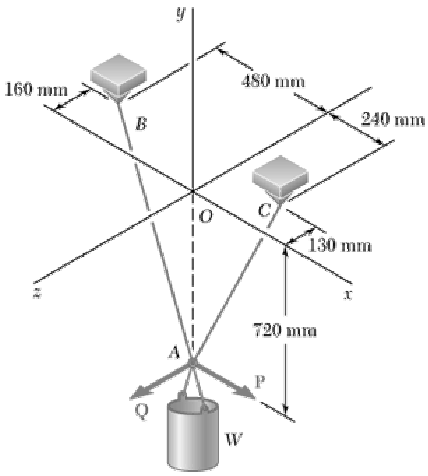
$$T_{AB} = T_{AD} = P$$

and using the linear algebraic equations of Problem 2.131 with $T_{AC} = 150$ N, we obtain

(*a*) $P = 454$ N ◀

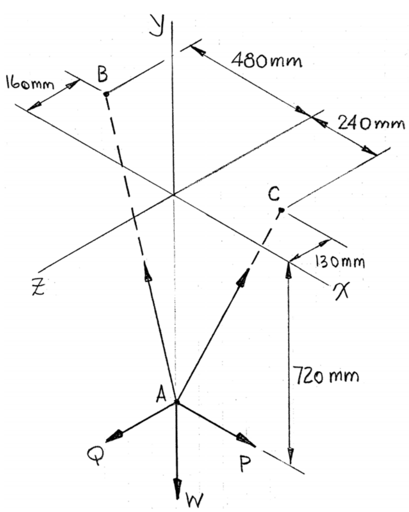
(*b*) $W = 1202$ N ◀

PROBLEM 2.125



A container of weight W is suspended from ring A. Cable BAC passes through the ring and is attached to fixed supports at B and C. Two forces $\mathbf{P} = P\mathbf{i}$ and $\mathbf{Q} = Q\mathbf{k}$ are applied to the ring to maintain the container is the position shown. Knowing that $W = 1200$ N, determine P and Q. (*Hint:* The tension is the same in both portions of cable BAC.)

SOLUTION



The (vector) force in each cable can be written as the product of the (scalar) force and the unit vector along the cable. That is, with

$$\overrightarrow{AB} = -(0.48\text{ m})\mathbf{i} + (0.72\text{ m})\mathbf{j} - (0.16\text{ m})\mathbf{k}$$

$$AB = \sqrt{(-0.48\text{ m})^2 + (0.72\text{ m})^2 + (-0.16\text{ m})^2} = 0.88\text{ m}$$

$$\mathbf{T}_{AB} = T\boldsymbol{\lambda}_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = \frac{T_{AB}}{0.88\text{ m}}\left[-(0.48\text{ m})\mathbf{i} + (0.72\text{ m})\mathbf{j} - (0.16\text{ m})\mathbf{k}\right]$$

$$\mathbf{T}_{AB} = T_{AB}(-0.5455\mathbf{i} + 0.8182\mathbf{j} - 0.1818\mathbf{k})$$

and

$$\overrightarrow{AC} = (0.24\text{ m})\mathbf{i} + (0.72\text{ m})\mathbf{j} - (0.13\text{ m})\mathbf{k}$$

$$AC = \sqrt{(0.24\text{ m})^2 + (0.72\text{ m})^2 - (0.13\text{ m})^2} = 0.77\text{ m}$$

$$\mathbf{T}_{AC} = T\boldsymbol{\lambda}_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = \frac{T_{AC}}{0.77\text{ m}}\left[(0.24\text{ m})\mathbf{i} + (0.72\text{ m})\mathbf{j} - (0.13\text{ m})\mathbf{k}\right]$$

$$\mathbf{T}_{AC} = T_{AC}(0.3177\mathbf{i} + 0.9351\mathbf{j} - 0.1688\mathbf{k})$$

At A: $\quad \Sigma\mathbf{F} = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{P} + \mathbf{Q} + \mathbf{W} = 0$

PROBLEM 2.125 CONTINUED

Noting that $T_{AB} = T_{AC}$ because of the ring A, we equate the factors of $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ to zero to obtain the linear algebraic equations:

$$\mathbf{i}: \quad (-0.5455 + 0.3177)T + P = 0$$

or $$P = 0.2338T$$

$$\mathbf{j}: \quad (0.8182 + 0.9351)T - W = 0$$

or $$W = 1.7532T$$

$$\mathbf{k}: \quad (-0.1818 - 0.1688)T + Q = 0$$

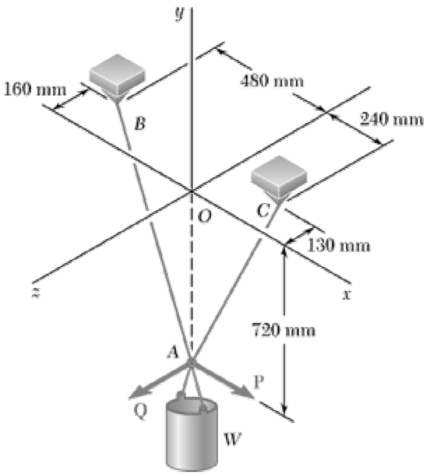
or $$Q = 0.356T$$

With $W = 1200\text{ N}$:

$$T = \frac{1200\text{ N}}{1.7532} = 684.5\text{ N}$$

$P = 160.0\text{ N}$ ◀

$Q = 240\text{ N}$ ◀



PROBLEM 2.126

For the system of Problem 2.125, determine W and P knowing that $Q = 160$ N.

Problem 2.125: A container of weight W is suspended from ring A. Cable BAC passes through the ring and is attached to fixed supports at B and C. Two forces $\mathbf{P} = P\mathbf{i}$ and $\mathbf{Q} = Q\mathbf{k}$ are applied to the ring to maintain the container is the position shown. Knowing that $W = 1200$ N, determine P and Q. (*Hint:* The tension is the same in both portions of cable BAC.)

SOLUTION

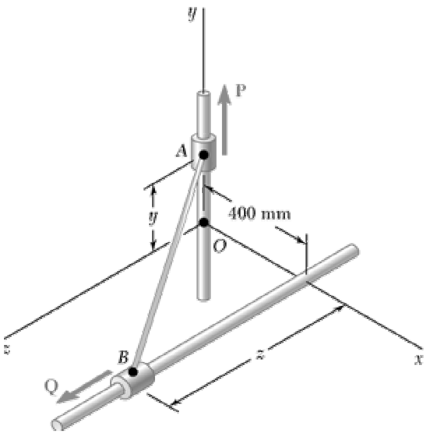
Based on the results of Problem 2.125, particularly the three equations relating P, Q, W, and T we substitute $Q = 160$ N to obtain

$$T = \frac{160 \text{ N}}{0.3506} = 456.3 \text{ N}$$

$W = 800$ N ◀

$P = 107.0$ N ◀

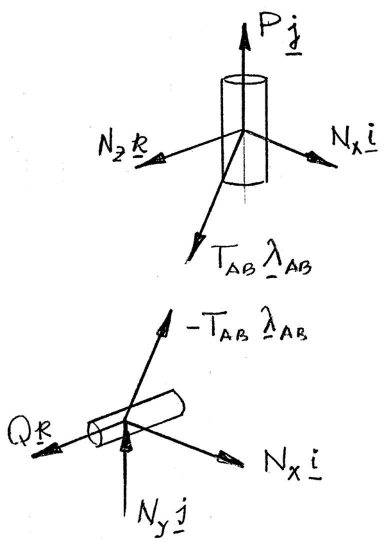
PROBLEM 2.127



Collars A and B are connected by a 1-m-long wire and can slide freely on frictionless rods. If a force $\mathbf{P} = (680\text{ N})\mathbf{j}$ is applied at A, determine (a) the tension in the wire when $y = 300$ mm, (b) the magnitude of the force $\mathbf{Q}$ required to maintain the equilibrium of the system.

SOLUTION

Free-Body Diagrams of collars



For both Problems 2.127 and 2.128:

$$(AB)^2 = x^2 + y^2 + z^2$$

Here $$(1\text{ m})^2 = (0.40\text{ m})^2 + y^2 + z^2$$

or $$y^2 + z^2 = 0.84\text{ m}^2$$

Thus, with y given, z is determined.

Now

$$\lambda_{AB} = \frac{\overrightarrow{AB}}{AB} = \frac{1}{1\text{ m}}(0.40\mathbf{i} - y\mathbf{j} + z\mathbf{k})\text{m} = 0.4\mathbf{i} - y\mathbf{k} + z\mathbf{k}$$

Where y and z are in units of meters, m.

From the F.B. Diagram of collar A:

$$\Sigma\mathbf{F} = 0: \quad N_x\mathbf{i} + N_z\mathbf{k} + P\mathbf{j} + T_{AB}\lambda_{AB} = 0$$

Setting the $\mathbf{j}$ coefficient to zero gives:

$$P - yT_{AB} = 0$$

With $P = 680$ N,

$$T_{AB} = \frac{680\text{ N}}{y}$$

Now, from the free body diagram of collar B:

$$\Sigma\mathbf{F} = 0: \quad N_x\mathbf{i} + N_y\mathbf{j} + Q\mathbf{k} - T_{AB}\lambda_{AB} = 0$$

PROBLEM 2.127 CONTINUED

Setting the **k** coefficient to zero gives:

$$Q - T_{AB}z = 0$$

And using the above result for T_{AB} we have

$$Q = T_{AB}z = \frac{680\text{ N}}{y}z$$

Then, from the specifications of the problem, $y = 300\text{ mm} = 0.3\text{ m}$

$$z^2 = 0.84\text{ m}^2 - (0.3\text{ m})^2$$

$$\therefore\ z = 0.866\text{ m}$$

and

(*a*) $$T_{AB} = \frac{680\text{ N}}{0.30} = 2266.7\text{ N}$$

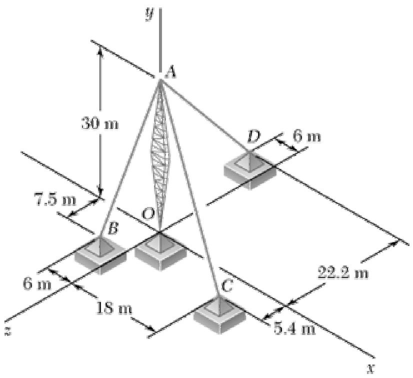
or $$T_{AB} = 2.27\text{ kN} \blacktriangleleft$$

and

(*b*) $$Q = 2266.7(0.866) = 1963.2\text{ N}$$

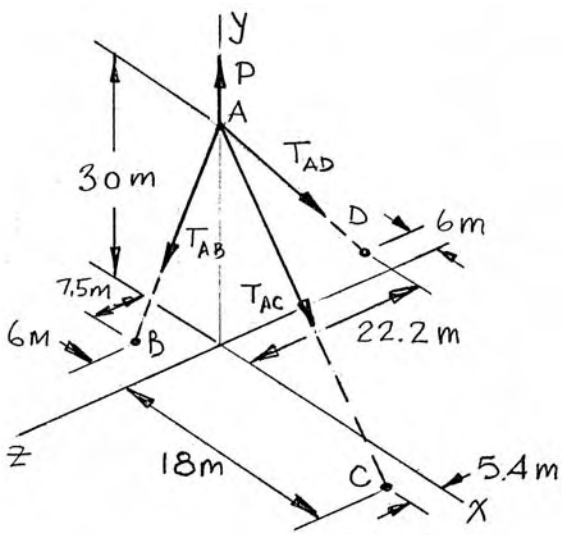
or $$Q = 1.963\text{ kN} \blacktriangleleft$$

PROBLEM 2.116



A transmission tower is held by three guy wires attached to a pin at *A* and anchored by bolts at *B*, *C*, and *D*. Knowing that the tower exerts on the pin at *A* an upward vertical force of 8 kN, determine the tension in each wire.

SOLUTION



From the solutions of 2.111 and 2.112:

$$T_{AB} = 0.5409P$$

$$T_{AC} = 0.295P$$

$$T_{AD} = 0.2959P$$

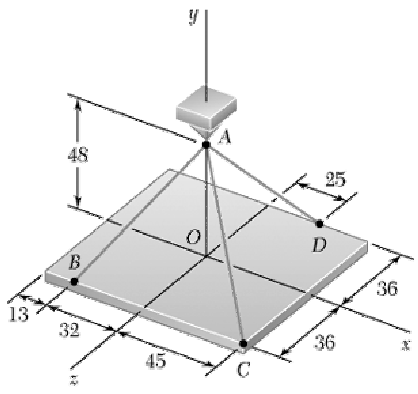
Using $P = 8$ kN:

$T_{AB} = 4.33$ kN ◀

$T_{AC} = 2.36$ kN ◀

$T_{AD} = 2.37$ kN ◀

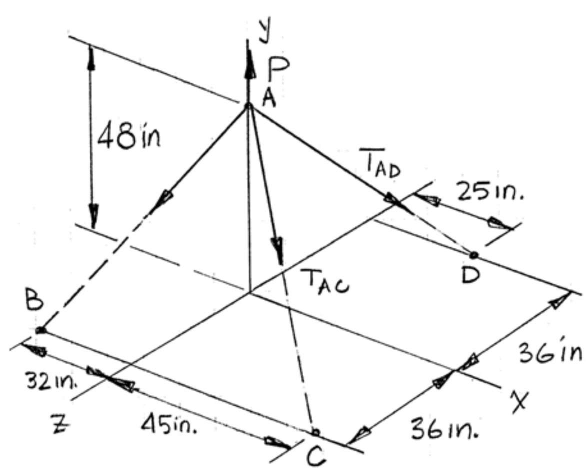
PROBLEM 2.117



Dimensions in inches

For the rectangular plate of Problems 2.113 and 2.114, determine the tension in each of the three cables knowing that the weight of the plate is 180 lb.

SOLUTION



From the solutions of 2.113 and 2.114:

$$T_{AB} = 0.6440P$$

$$T_{AC} = 0.0709P$$

$$T_{AD} = 0.6771P$$

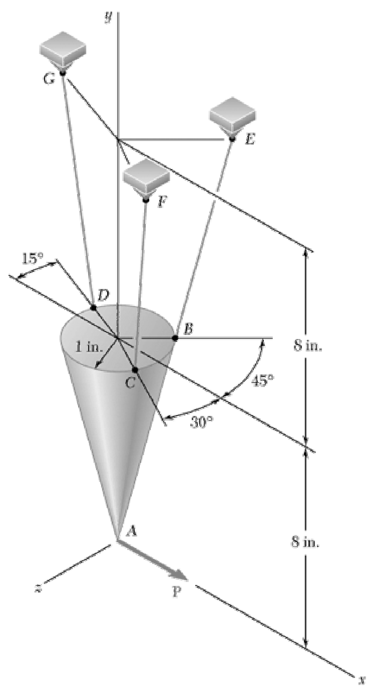
Using $P = 180$ lb:

$T_{AB} = 115.9$ lb ◀

$T_{AC} = 12.76$ lb ◀

$T_{AD} = 121.9$ lb ◀

PROBLEM 2.118



For the cone of Problem 2.110, determine the range of values of P for which cord DG is taut if **P** is directed in the $-x$ direction.

SOLUTION

From the solutions to Problems 2.109 and 2.110, have

$$T_{BE} + T_{CF} + T_{DG} = 0.2\sqrt{65} \qquad (2')$$

$$-T_{BE}\sin 45° + T_{CF}\sin 30° - T_{DG}\sin 15° = 0 \qquad (3)$$

$$T_{BE}\cos 45° + T_{CF}\cos 30° - T_{DG}\cos 15° - P\sqrt{65} = 0 \qquad (1')$$

Applying the method of elimination to obtain a desired result:

Multiplying $(2')$ by $\sin 45°$ and adding the result to (3):

$$T_{CF}\left(\sin 45° + \sin 30°\right) + T_{DG}\left(\sin 45° - \sin 15°\right) = 0.2\sqrt{65}\sin 45°$$

or $$T_{CF} = 0.9445 - 0.3714T_{DG} \qquad (4)$$

Multiplying $(2')$ by $\sin 30°$ and subtracting (3) from the result:

$$T_{BE}\left(\sin 30° + \sin 45°\right) + T_{DG}\left(\sin 30° + \sin 15°\right) = 0.2\sqrt{65}\sin 30°$$

or $$T_{BE} = 0.6679 - 0.6286T_{DG} \qquad (5)$$

PROBLEM 2.118 CONTINUED

Substituting (4) and (5) into $(1')$:

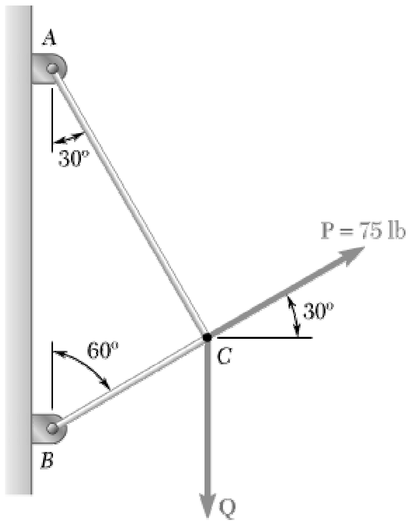
$$1.2903 - 1.7321T_{DG} - P\sqrt{65} = 0$$

$$\therefore \quad T_{DG} \text{ is taut for } P < \frac{1.2903}{\sqrt{65}} \text{ lb}$$

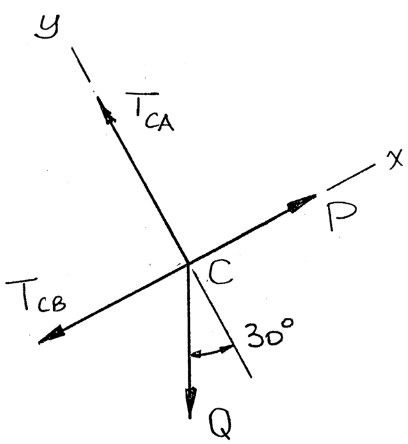
or $0 \leq P < 0.1600$ lb ◀

PROBLEM 2.132

Two cables tied together at C are loaded as shown. Knowing that $Q = 60$ lb, determine the tension (*a*) in cable AC, (*b*) in cable BC.



SOLUTION



$$\Sigma F_y = 0: \quad T_{CA} - Q\cos 30° = 0$$

With $$Q = 60 \text{ lb}$$

(*a*) $$T_{CA} = (60 \text{ lb})(0.866)$$

$$T_{CA} = 52.0 \text{ lb} \blacktriangleleft$$

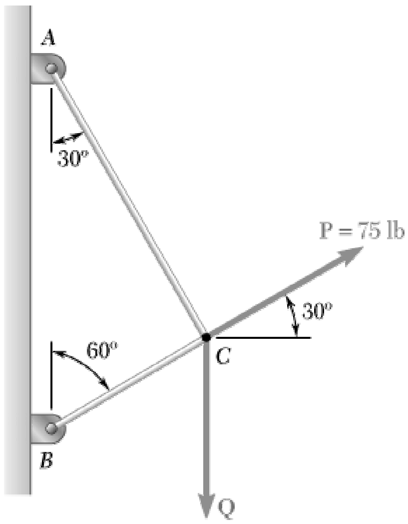
(*b*) $$\Sigma F_x = 0: \quad P - T_{CB} - Q\sin 30° = 0$$

With $$P = 75 \text{ lb}$$

$$T_{CB} = 75 \text{ lb} - (60 \text{ lb})(0.50)$$

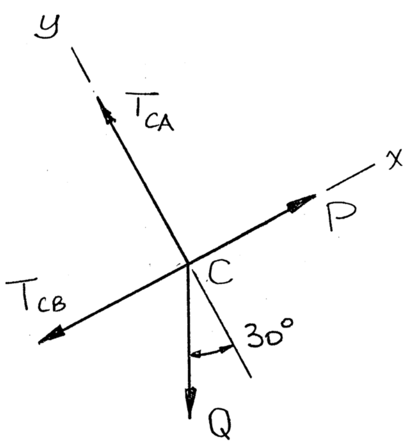
or $$T_{CB} = 45.0 \text{ lb} \blacktriangleleft$$

PROBLEM 2.133



Two cables tied together at C are loaded as shown. Determine the range of values of Q for which the tension will not exceed 60 lb in either cable.

SOLUTION



Have $\quad \Sigma F_x = 0: \quad T_{CA} - Q\cos 30° = 0$

or $\quad T_{CA} = 0.8660\text{ Q}$

Then for $\quad T_{CA} \leq 60\text{ lb}$

$$0.8660Q < 60\text{ lb}$$

or $\quad Q \leq 69.3\text{ lb}$

From $\quad \Sigma F_y = 0: \quad T_{CB} = P - Q\sin 30°$

or $\quad T_{CB} = 75\text{ lb} - 0.50Q$

For $\quad T_{CB} \leq 60\text{ lb}$

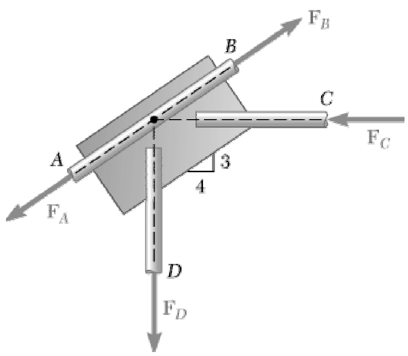
$$75\text{ lb} - 0.50Q \leq 60\text{ lb}$$

or $\quad 0.50Q \geq 15\text{ lb}$

Thus, $\quad Q \geq 30\text{ lb}$

Therefore, $\quad 30.0 \leq Q \leq 69.3\text{ lb}$ ◀

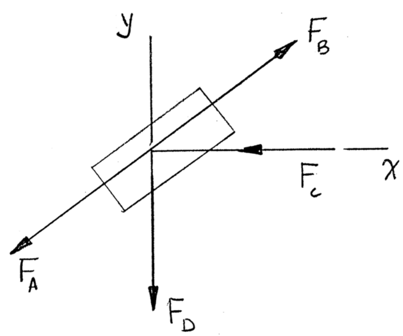
PROBLEM 2.134



A welded connection is in equilibrium under the action of the four forces shown. Knowing that $F_A = 8\text{ kN}$ and $F_B = 16\text{ kN}$, determine the magnitudes of the other two forces.

SOLUTION

Free-Body Diagram of Connection



$$\Sigma F_x = 0: \quad \frac{3}{5}F_B - F_C - \frac{3}{5}F_A = 0$$

With $$F_A = 8\text{ kN}, \; F_B = 16\text{ kN}$$

$$F_C = \frac{4}{5}(16\text{ kN}) - \frac{4}{5}(8\text{ kN})$$

$$F_C = 6.40\text{ kN} \blacktriangleleft$$

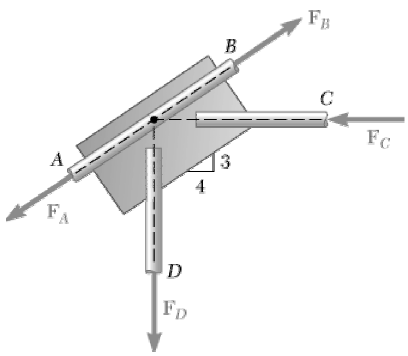
$$\Sigma F_y = 0: \quad -F_D + \frac{3}{5}F_B - \frac{3}{5}F_A = 0$$

With F_A and F_B as above:

$$F_D = \frac{3}{5}(16\text{ kN}) - \frac{3}{5}(8\text{ kN})$$

$$F_D = 4.80\text{ kN} \blacktriangleleft$$

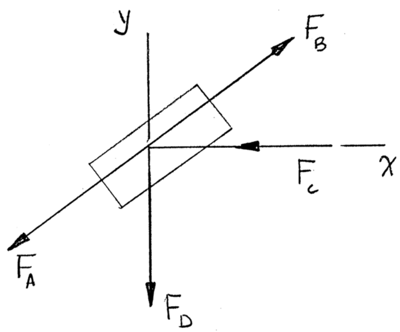
PROBLEM 2.135



A welded connection is in equilibrium under the action of the four forces shown. Knowing that $F_A = 5$ kN and $F_D = 6$ kN, determine the magnitudes of the other two forces.

SOLUTION

Free-Body Diagram of Connection



$$\Sigma F_y = 0: \quad -F_D - \frac{3}{5}F_A + \frac{3}{5}F_B = 0$$

or

$$F_B = F_D + \frac{3}{5}F_A$$

With

$$F_A = 5 \text{ kN}, \quad F_D = 8 \text{ kN}$$

$$F_B = \frac{5}{3}\left[6 \text{ kN} + \frac{3}{5}(5 \text{ kN})\right]$$

$$F_B = 15.00 \text{ kN} \blacktriangleleft$$

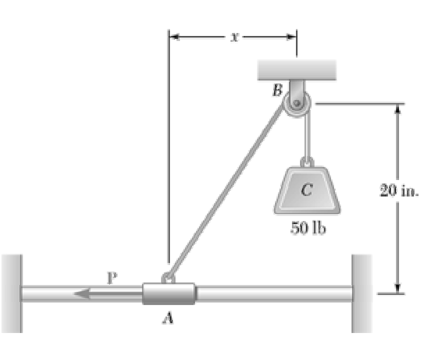
$$\Sigma F_x = 0: \quad -F_C + \frac{4}{5}F_B - \frac{4}{5}F_A = 0$$

$$F_C = \frac{4}{5}(F_B - F_A)$$

$$= \frac{4}{5}(15 \text{ kN} - 5 \text{ kN})$$

$$F_C = 8.00 \text{ kN} \blacktriangleleft$$

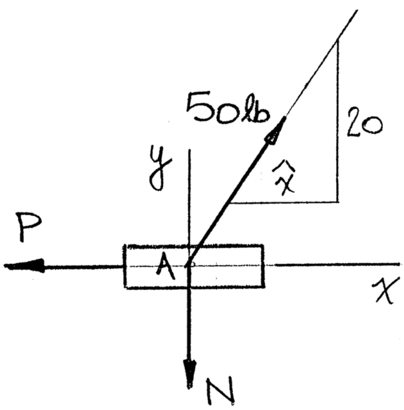
PROBLEM 2.136



Collar A is connected as shown to a 50-lb load and can slide on a frictionless horizontal rod. Determine the magnitude of the force **P** required to maintain the equilibrium of the collar when (*a*) $x = 4.5$ in., (*b*) $x = 15$ in.

SOLUTION

Free-Body Diagram of Collar



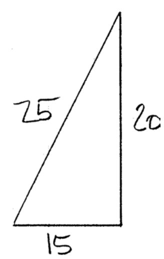
(*a*)

Triangle Proportions

$$\Sigma F_x = 0: \quad -P + \frac{4.5}{20.5}(50 \text{ lb}) = 0$$

or $P = 10.98$ lb ◀

(*b*)

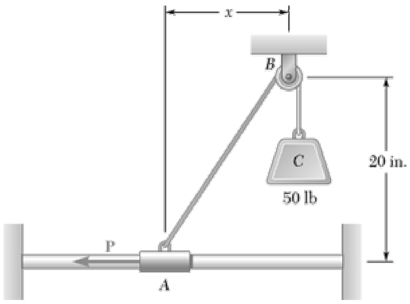


Triangle Proportions

$$\Sigma F_x = 0: \quad -P + \frac{15}{25}(50 \text{ lb}) = 0$$

or $P = 30.0$ lb ◀

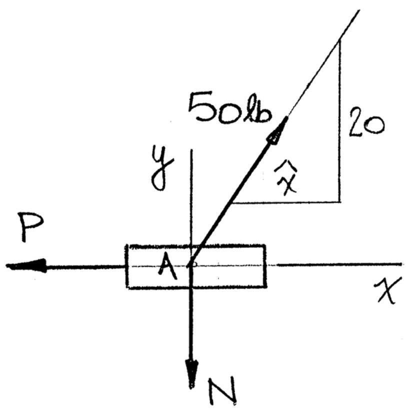
PROBLEM 2.137

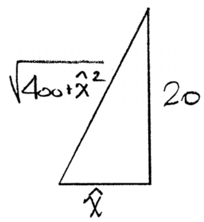


Collar A is connected as shown to a 50-lb load and can slide on a frictionless horizontal rod. Determine the distance x for which the collar is in equilibrium when $P = 48$ lb.

SOLUTION

Free-Body Diagram of Collar





Triangle Proportions

Hence: $\Sigma F_x = 0: \quad -48 + \dfrac{50\hat{x}}{\sqrt{400 + \hat{x}^2}} = 0$

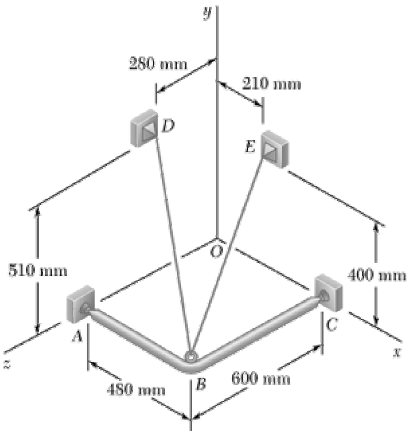
or $\hat{x} = \dfrac{48}{50}\sqrt{400 + \hat{x}^2}$

$$\hat{x}^2 = 0.92 \text{ lb}\left(400 + \hat{x}^2\right)$$

$$\hat{x}^2 = 4737.7 \text{ in}^2$$

$\hat{x} = 68.6$ in. ◀

PROBLEM 2.138



A frame *ABC* is supported in part by cable *DBE* which passes through a frictionless ring at *B*. Knowing that the tension in the cable is 385 N, determine the components of the force exerted by the cable on the support at *D*.

SOLUTION

The force in cable *DB* can be written as the product of the magnitude of the force and the unit vector along the cable. That is, with

$$\overrightarrow{DB} = (480\text{ mm})\mathbf{i} - (510\text{ mm})\mathbf{j} + (320\text{ mm})\mathbf{k}$$

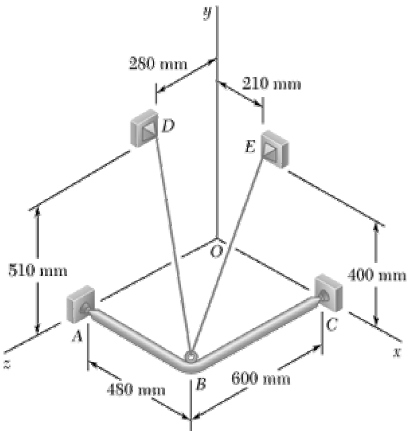
$$DB = \sqrt{(480)^2 + (510)^2 + (320)^2} = 770\text{ mm}$$

$$\mathbf{F} = F\lambda_{DB} = F\frac{\overrightarrow{DB}}{DB} = \frac{385\text{ N}}{770\text{ mm}}\left[(480\text{ mm})\mathbf{i} - (510\text{ mm})\mathbf{j} + (320\text{ mm})\mathbf{k}\right]$$

$$\mathbf{F} = (240\text{ N})\mathbf{i} - (255\text{ N})\mathbf{j} + (160\text{ N})\mathbf{k}$$

$$F_x = +240\text{ N},\ F_y = -255\text{ N},\ F_z = +160.0\text{ N} \blacktriangleleft$$

PROBLEM 2.139



A frame *ABC* is supported in part by cable *DBE* which passes through a frictionless ring at *B*. Determine the magnitude and direction of the resultant of the forces exerted by the cable at *B* knowing that the tension in the cable is 385 N.

SOLUTION

The force in each cable can be written as the product of the magnitude of the force and the unit vector along the cable. That is, with

$$\overrightarrow{BD} = -(0.48\text{ m})\mathbf{i} + (0.51\text{ m})\mathbf{j} - (0.32\text{ m})\mathbf{k}$$

$$BD = \sqrt{(-0.48\text{ m})^2 + (0.51\text{ m})^2 + (-0.32\text{ m})^2} = 0.77\text{ m}$$

$$\mathbf{T}_{BD} = T\lambda_{BD} = T_{BD}\frac{\overrightarrow{BD}}{BD} = \frac{T_{BD}}{0.77\text{ m}}\left[-(0.48\text{ m})\mathbf{i} + (0.51\text{ m})\mathbf{j} - (0.32\text{ m})\mathbf{k}\right]$$

$$\mathbf{T}_{BD} = T_{BD}(-0.6234\mathbf{i} + 0.6623\mathbf{j} - 0.4156\mathbf{k})$$

and

$$\overrightarrow{BE} = -(0.27\text{ m})\mathbf{i} + (0.40\text{ m})\mathbf{j} - (0.6\text{ m})\mathbf{k}$$

$$BE = \sqrt{(-0.27\text{ m})^2 + (0.40\text{ m})^2 + (-0.6\text{ m})^2} = 0.770\text{ m}$$

$$\mathbf{T}_{BE} = T\lambda_{BE} = T_{BE}\frac{\overrightarrow{BD}}{BD} = \frac{T_{BE}}{0.770\text{ m}}\left[-(0.26\text{ m})\mathbf{i} + (0.40\text{ m})\mathbf{j} - (0.6\text{ m})\mathbf{k}\right]$$

$$\mathbf{T}_{BE} = T_{BE}(-0.3506\mathbf{i} + 0.5195\mathbf{j} - 0.7792\mathbf{k})$$

Now, because of the frictionless ring at *B*, $T_{BE} = T_{BD} = 385\text{ N}$ and the force on the support due to the two cables is

$$\mathbf{F} = 385\text{ N}(-0.6234\mathbf{i} + 0.6623\mathbf{j} - 0.4156\mathbf{k} - 0.3506\mathbf{i} + 0.5195\mathbf{j} - 0.7792\mathbf{k})$$

$$= -(375\text{ N})\mathbf{i} + (455\text{ N})\mathbf{j} - (460\text{ N})\mathbf{k}$$

PROBLEM 2.139 CONTINUED

The magnitude of the resultant is

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2} = \sqrt{(-375 \text{ N})^2 + (455 \text{ N})^2 + (-460 \text{ N})^2} = 747.83 \text{ N}$$

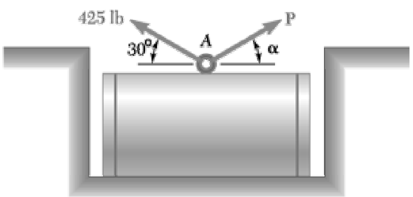
or $F = 748 \text{ N}$ ◀

The direction of this force is:

$$\theta_x = \cos^{-1}\frac{-375}{747.83}$$ or $\theta_x = 120.1°$ ◀

$$\theta_y = \cos^{-1}\frac{455}{747.83}$$ or $\theta_y = 52.5°$ ◀

$$\theta_z = \cos^{-1}\frac{-460}{747.83}$$ or $\theta_z = 128.0°$ ◀

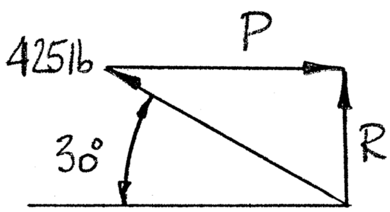


PROBLEM 2.140

A steel tank is to be positioned in an excavation. Using trigonometry, determine (*a*) the magnitude and direction of the smallest force **P** for which the resultant **R** of the two forces applied at *A* is vertical, (*b*) the corresponding magnitude of **R**.

SOLUTION

Force Triangle



(*a*) For minimum *P* it must be perpendicular to the vertical resultant *R*

$$\therefore \quad P = (425 \text{ lb})\cos 30°$$

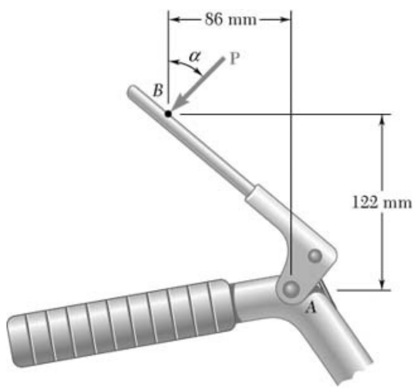
or $\mathbf{P} = 368 \text{ lb} \longrightarrow$ ◀

(*b*)

$$R = (425 \text{ lb})\sin 30°$$

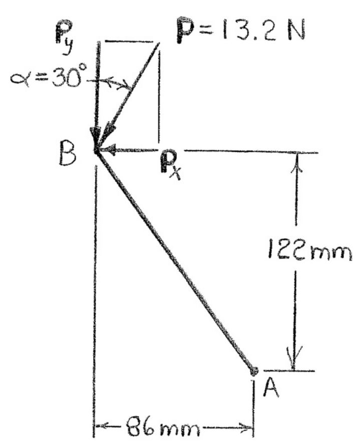
or $R = 213$ lb ◀

PROBLEM 3.1



A 13.2-N force **P** is applied to the lever which controls the auger of a snowblower. Determine the moment of **P** about A when α is equal to 30°.

SOLUTION



First note

$$P_x = P\sin\alpha = (13.2\text{ N})\sin 30° = 6.60\text{ N}$$

$$P_y = P\cos\alpha = (13.2\text{ N})\cos 30° = 11.4315\text{ N}$$

Noting that the direction of the moment of each force component about A is counterclockwise,

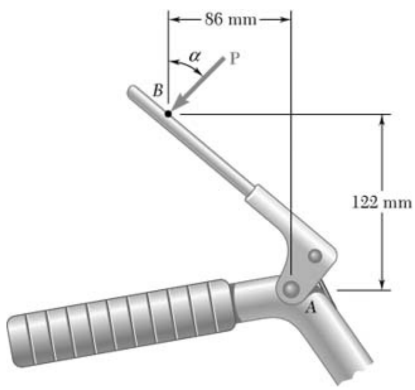
$$M_A = x_{B/A}P_y + y_{B/A}P_x$$

$$= (0.086\text{ m})(11.4315\text{ N}) + (0.122\text{ m})(6.60\text{ N})$$

$$= 1.78831\text{ N}\cdot\text{m}$$

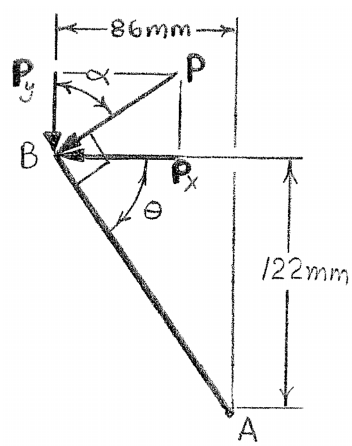
or $\mathbf{M}_A = 1.788\text{ N}\cdot\text{m}$ ↺ ◀

PROBLEM 3.2



The force **P** is applied to the lever which controls the auger of a snowblower. Determine the magnitude and the direction of the smallest force **P** which has a 2.20- N·m counterclockwise moment about *A*.

SOLUTION



For *P* to be a minimum, it must be perpendicular to the line joining points *A* and *B*.

$$r_{AB} = \sqrt{(86 \text{ mm})^2 + (122 \text{ mm})^2} = 149.265 \text{ mm}$$

$$\alpha = \theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{122 \text{ mm}}{86 \text{ mm}}\right) = 54.819°$$

Then $$M_A = r_{AB}P_{\min}$$

or $$P_{\min} = \frac{M_A}{r_{AB}}$$

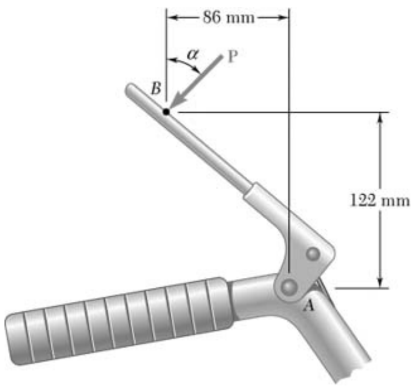
$$= \frac{2.20 \text{ N}\cdot\text{m}}{149.265 \text{ mm}}\left(\frac{1000 \text{ mm}}{1 \text{ m}}\right)$$

$$= 14.7389 \text{ N}$$

$$\therefore \ \mathbf{P}_{\min} = 14.74 \text{ N} \measuredangle \ 54.8°$$

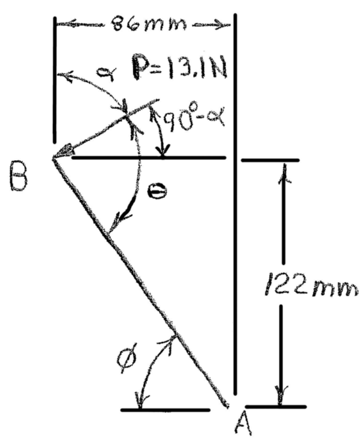
or $\mathbf{P}_{\min} = 14.74$ N $\measuredangle$ 35.2° ◀

PROBLEM 3.3



A 13.1-N force **P** is applied to the lever which controls the auger of a snowblower. Determine the value of α knowing that the moment of **P** about A is counterclockwise and has a magnitude of $1.95\ \text{N}\cdot\text{m}$.

SOLUTION



By definition $$M_A = r_{B/A} P \sin\theta$$

where $$\theta = \phi + (90° - \alpha)$$

and $$\phi = \tan^{-1}\left(\frac{122\text{ mm}}{86\text{ mm}}\right) = 54.819°$$

Also $$r_{B/A} = \sqrt{(86\text{ mm})^2 + (122\text{ mm})^2} = 149.265\text{ mm}$$

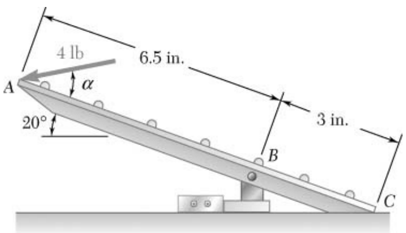
Then $$1.95\ \text{N}\cdot\text{m} = (0.149265\text{ m})(13.1\text{ N})\sin(54.819° + 90° - \alpha)$$

or $$\sin(144.819° - \alpha) = 0.99725$$

or $$144.819° - \alpha = 85.752°$$

and $$144.819° - \alpha = 94.248°$$

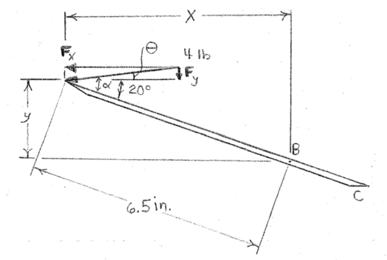
$$\therefore\ \alpha = 50.6°,\ 59.1° \blacktriangleleft$$



PROBLEM 3.4

A foot valve for a pneumatic system is hinged at B. Knowing that $\alpha = 28°$, determine the moment of the 4-lb force about point B by resolving the force into horizontal and vertical components.

SOLUTION



Note that $\theta = \alpha - 20° = 28° - 20° = 8°$

and $F_x = (4 \text{ lb})\cos 8° = 3.9611 \text{ lb}$

$F_y = (4 \text{ lb})\sin 8° = 0.55669 \text{ lb}$

Also $x = (6.5 \text{ in.})\cos 20° = 6.1080 \text{ in.}$

$y = (6.5 \text{ in.})\sin 20° = 2.2231 \text{ in.}$

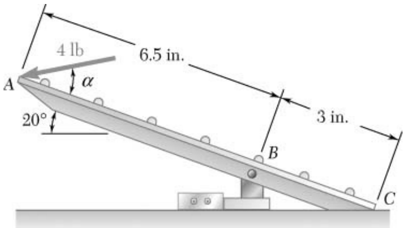
Noting that the direction of the moment of each force component about B is counterclockwise,

$$M_B = xF_y + yF_x$$

$$= (6.1080 \text{ in.})(0.55669 \text{ lb}) + (2.2231 \text{ in.})(3.9611 \text{ lb})$$

$$= 12.2062 \text{ lb}\cdot\text{in.}$$

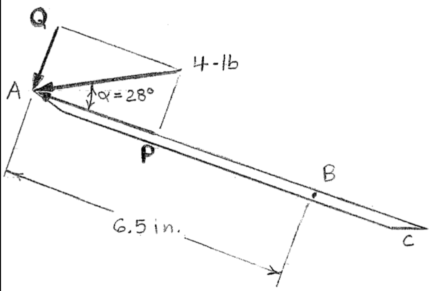
or $\mathbf{M}_B = 12.21 \text{ lb}\cdot\text{in.}$ ↺ ◀



PROBLEM 3.5

A foot valve for a pneumatic system is hinged at *B*. Knowing that $\alpha = 28°$, determine the moment of the 4-lb force about point *B* by resolving the force into components along *ABC* and in a direction perpendicular to *ABC*.

SOLUTION



First resolve the 4-lb force into components **P** and **Q**, where

$$Q = (4.0 \text{ lb})\sin 28° = 1.87787 \text{ lb}$$

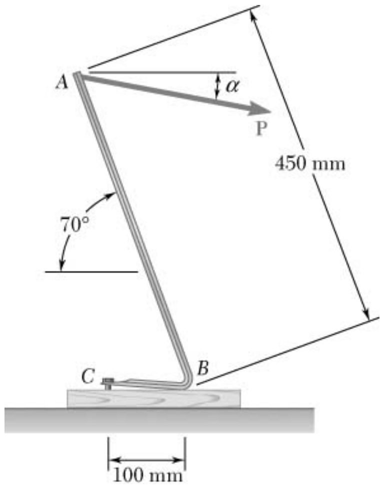
Then

$$M_B = r_{A/B}Q$$

$$= (6.5 \text{ in.})(1.87787 \text{ lb})$$

$$= 12.2063 \text{ lb}\cdot\text{in.}$$

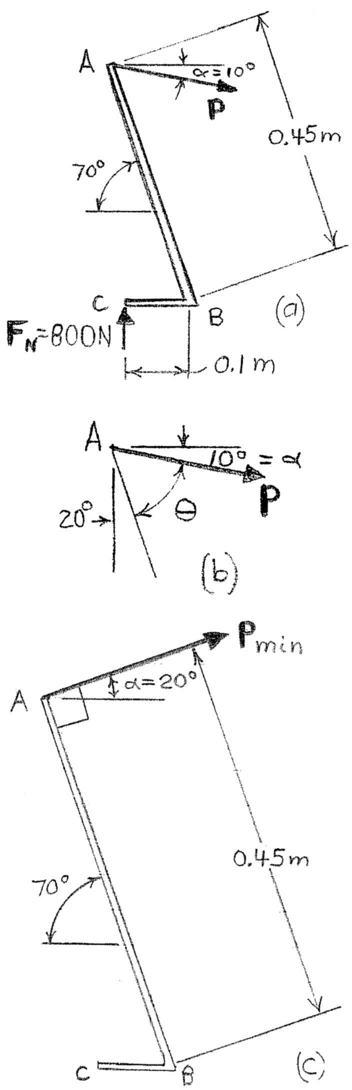
or $\mathbf{M}_B = 12.21 \text{ lb}\cdot\text{in.}$ ↻ ◀



PROBLEM 3.6

It is known that a vertical force of 800 N is required to remove the nail at C from the board. As the nail first starts moving, determine (*a*) the moment about B of the force exerted on the nail, (*b*) the magnitude of the force **P** which creates the same moment about B if $\alpha = 10°$, (*c*) the smallest force **P** which creates the same moment about B.

SOLUTION



(*a*) Have

$$M_B = r_{C/B}F_N$$

$$= (0.1\text{ m})(800\text{ N})$$

$$= 80.0\text{ N}\cdot\text{m}$$

or $\mathbf{M}_B = 80.0\text{ N}\cdot\text{m}$ ↻ ◀

(*b*) By definition

$$M_B = r_{A/B}P\sin\theta$$

where

$$\theta = 90° - (90° - 70°) - \alpha$$

$$= 90° - 20° - 10°$$

$$= 60°$$

$$\therefore \quad 80.0\text{ N}\cdot\text{m} = (0.45\text{ m})P\sin 60°$$

$$P = 205.28\text{ N}$$

or $P = 205\text{ N}$ ◀

(*c*) For **P** to be minimum, it must be perpendicular to the line joining points A and B. Thus, **P** must be directed as shown.

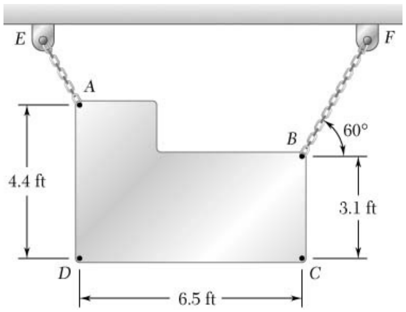
Thus
$$M_B = dP_{\min} = r_{A/B}P_{\min}$$

or
$$80.0\text{ N}\cdot\text{m} = (0.45\text{ m})P_{\min}$$

$$\therefore \quad P_{\min} = 177.778\text{ N}$$

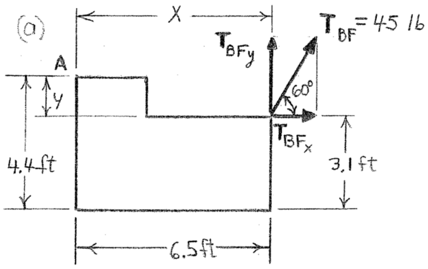
or $\mathbf{P}_{\min} = 177.8\text{ N} \measuredangle 20°$ ◀

PROBLEM 3.7



A sign is suspended from two chains *AE* and *BF*. Knowing that the tension in *BF* is 45 lb, determine (*a*) the moment about *A* of the force exert by the chain at *B*, (*b*) the smallest force applied at *C* which creates the same moment about *A*.

SOLUTION



(*a*) Have $\mathbf{M}_A = \mathbf{r}_{B/A} \times \mathbf{T}_{BF}$

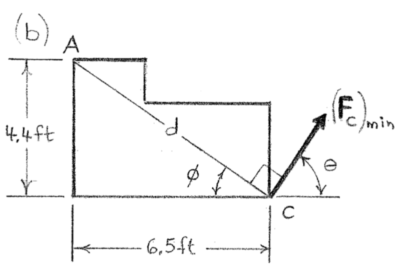
Noting that the direction of the moment of each force component about *A* is counterclockwise,

$$M_A = xT_{BFy} + yT_{BFx}$$

$$= (6.5 \text{ ft})(45 \text{ lb})\sin 60° + (4.4 \text{ ft} - 3.1 \text{ ft})(45 \text{ lb})\cos 60°$$

$$= 282.56 \text{ lb}\cdot\text{ft}$$

or $\mathbf{M}_A = 283 \text{ lb}\cdot\text{ft}$ ↺ ◀



(*b*) Have $\mathbf{M}_A = \mathbf{r}_{C/A} \times (\mathbf{F}_C)_{\text{min}}$

For $\mathbf{F}_C$ to be minimum, it must be perpendicular to the line joining points *A* and *C*.

$$\therefore\ M_A = d(F_C)_{\text{min}}$$

where $d = r_{C/A} = \sqrt{(6.5 \text{ ft})^2 + (4.4 \text{ ft})^2} = 7.8492 \text{ ft}$

$$\therefore\quad 282.56 \text{ lb}\cdot\text{ft} = (7.8492 \text{ ft})(F_C)_{\text{min}}$$

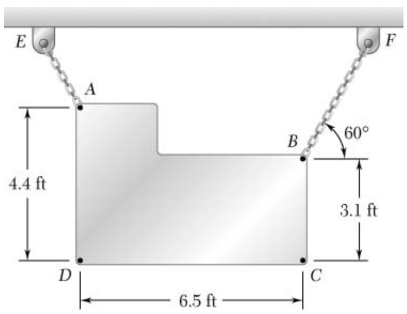
$$(F_C)_{\text{min}} = 35.999 \text{ lb}$$

$$\phi = \tan^{-1}\left(\frac{4.4 \text{ ft}}{6.5 \text{ ft}}\right) = 34.095°$$

$$\theta = 90° - \phi = 90° - 34.095° = 55.905°$$

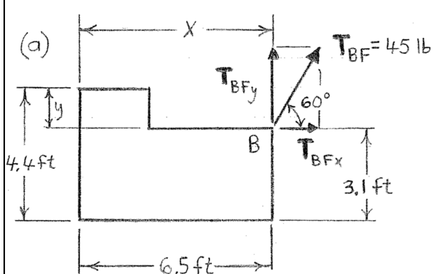
or $(\mathbf{F}_C)_{\text{min}} = 36.0 \text{ lb} \measuredangle\ 55.9°$ ◀

PROBLEM 3.8



A sign is suspended from two chains *AE* and *BF*. Knowing that the tension in *BF* is 45 lb, determine (*a*) the moment about *A* of the force exerted by the chain at *B*, (*b*) the magnitude and sense of the vertical force applied at *C* which creates the same moment about *A*, (*c*) the smallest force applied at *B* which creates the same moment about *A*.

SOLUTION



(*a*) Have $$\mathbf{M}_A = \mathbf{r}_{B/A} \times \mathbf{T}_{BF}$$

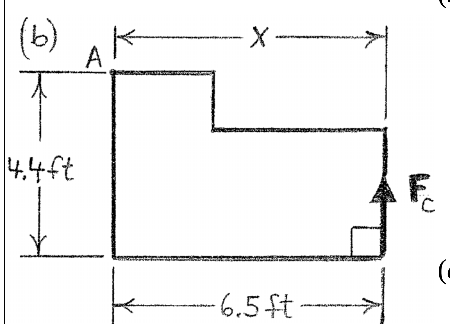
Noting that the direction of the moment of each force component about *A* is counterclockwise,

$$M_A = xT_{BFy} + yT_{BFx}$$

$$= (6.5 \text{ ft})(45 \text{ lb})\sin 60° + (4.4 \text{ ft} - 3.1 \text{ ft})(45 \text{ lb})\cos 60°$$

$$= 282.56 \text{ lb}\cdot\text{ft}$$

or $\mathbf{M}_A = 283 \text{ lb}\cdot\text{ft} \circlearrowleft$ ◀



(*b*) Have $$\mathbf{M}_A = \mathbf{r}_{C/A} \times \mathbf{F}_C$$

or $$M_A = xF_C$$

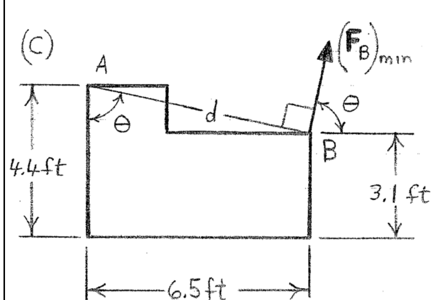
$$\therefore\ F_C = \frac{M_A}{x} = \frac{282.56 \text{ lb}\cdot\text{ft}}{6.5 \text{ ft}} = 43.471 \text{ lb}$$

or $\mathbf{F}_C = 43.5 \text{ lb}\uparrow$ ◀

(*c*) Have $$\mathbf{M}_A = \mathbf{r}_{B/A} \times (\mathbf{F}_B)_{\min}$$

For $\mathbf{F}_B$ to be minimum, it must be perpendicular to the line joining points *A* and *B*.

$$\therefore\ M_A = d(F_B)_{\min}$$



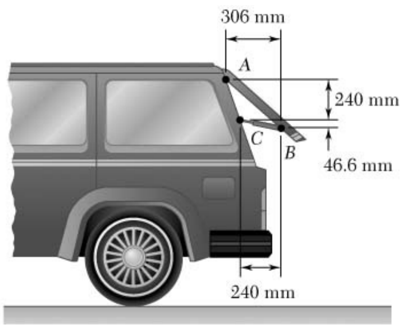
where $$d = \sqrt{(6.5 \text{ ft})^2 + (4.4 \text{ ft} - 3.1 \text{ ft})^2} = 6.6287 \text{ ft}$$

$$\therefore\ (F_B)_{\min} = \frac{M_A}{d} = \frac{282.56 \text{ lb}\cdot\text{ft}}{6.6287 \text{ ft}} = 42.627 \text{ lb}$$

and $$\theta = \tan^{-1}\left(\frac{6.5 \text{ ft}}{4.4 \text{ ft} - 3.1 \text{ ft}}\right) = 78.690°$$

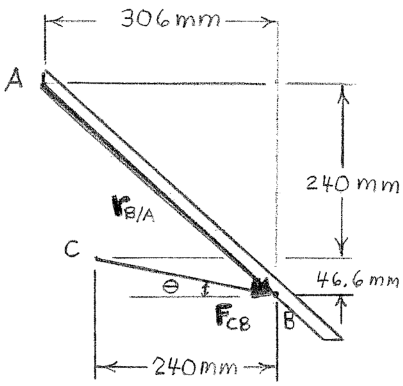
or $(\mathbf{F}_B)_{\min} = 42.6 \text{ lb} \measuredangle\ 78.7°$ ◀

PROBLEM 3.9



The tailgate of a car is supported by the hydraulic lift *BC*. If the lift exerts a 125-N force directed along its center line on the ball and socket at *B*, determine the moment of the force about *A*.

SOLUTION



First note
$$d_{CB} = \sqrt{(240 \text{ mm})^2 + (46.6 \text{ mm})^2}$$
$$= 244.48 \text{ mm}$$

Then
$$\cos\theta = \frac{240 \text{ mm}}{244.48 \text{ mm}}$$
$$\sin\theta = \frac{46.6 \text{ mm}}{244.48 \text{ mm}}$$

and
$$\mathbf{F}_{CB} = F_{CB}\cos\theta\mathbf{i} - F_{CB}\sin\theta\mathbf{j}$$
$$= \frac{125 \text{ N}}{244.48 \text{ mm}}\left[(240 \text{ mm})\mathbf{i} - (46.6 \text{ mm})\mathbf{j}\right]$$

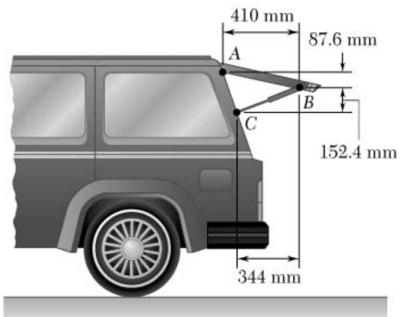
Now
$$\mathbf{M}_A = \mathbf{r}_{B/A} \times \mathbf{F}_{CB}$$

where
$$\mathbf{r}_{B/A} = (306 \text{ mm})\mathbf{i} - (240 \text{ mm} + 46.6 \text{ mm})\mathbf{j}$$
$$= (306 \text{ mm})\mathbf{i} - (286.6 \text{ mm})\mathbf{j}$$

Then
$$\mathbf{M}_A = \left[(306 \text{ mm})\mathbf{i} - (286.6 \text{ mm})\mathbf{j}\right] \times \frac{125 \text{ N}}{244.48}(240\mathbf{i} - 46.6\mathbf{j})$$
$$= (27878 \text{ N}\cdot\text{mm})\mathbf{k} = (27.878 \text{ N}\cdot\text{m})\mathbf{k}$$

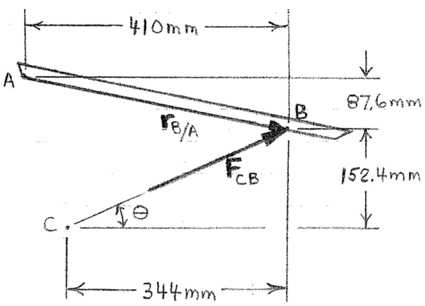
or $\mathbf{M}_A = 27.9 \text{ N}\cdot\text{m} \circlearrowleft$ ◀

PROBLEM 3.10



The tailgate of a car is supported by the hydraulic lift *BC*. If the lift exerts a 125-N force directed along its center line on the ball and socket at *B*, determine the moment of the force about *A*.

SOLUTION



First note $$d_{CB} = \sqrt{(344\text{ mm})^2 + (152.4\text{ mm})^2} = 376.25\text{ mm}$$

Then $$\cos\theta = \frac{344\text{ mm}}{376.25\text{ mm}} \qquad \sin\theta = \frac{152.4\text{ mm}}{376.25\text{ mm}}$$

and $$\mathbf{F}_{CB} = (F_{CB}\cos\theta)\mathbf{i} - (F_{CB}\sin\theta)\mathbf{j}$$

$$= \frac{125\text{ N}}{376.25\text{ mm}}\left[(344\text{ mm})\mathbf{i} + (152.4\text{ mm})\mathbf{j}\right]$$

Now $$\mathbf{M}_A = \mathbf{r}_{B/A} \times \mathbf{F}_{CB}$$

where $$\mathbf{r}_{B/A} = (410\text{ mm})\mathbf{i} - (87.6\text{ mm})\mathbf{j}$$

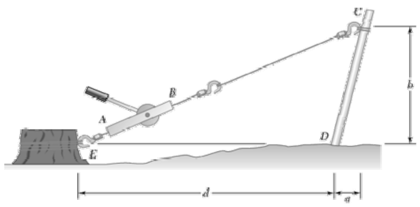
Then $$\mathbf{M}_A = \left[(410\text{ mm})\mathbf{i} - (87.6\text{ mm})\mathbf{j}\right] \times \frac{125\text{ N}}{376.25}(344\mathbf{i} - 152.4\mathbf{j})$$

$$= (30\,770\text{ N}\cdot\text{mm})\mathbf{k}$$

$$= (30.770\text{ N}\cdot\text{m})\mathbf{k}$$

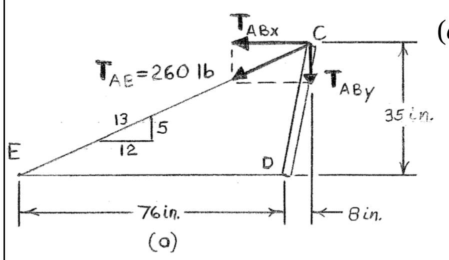
or $\mathbf{M}_A = 30.8\text{ N}\cdot\text{m}$ ⟲ ◀

PROBLEM 3.11



A winch puller AB is used to straighten a fence post. Knowing that the tension in cable BC is 260 lb, length a is 8 in., length b is 35 in., and length d is 76 in., determine the moment about D of the force exerted by the cable at C by resolving that force into horizontal and vertical components applied (*a*) at point C, (*b*) at point E.

SOLUTION



(*a*) Slope of line $EC = \dfrac{35 \text{ in.}}{76 \text{ in.} + 8 \text{ in.}} = \dfrac{5}{12}$

Then $T_{ABx} = \dfrac{12}{13}(T_{AB})$

$= \dfrac{12}{13}(260 \text{ lb}) = 240 \text{ lb}$

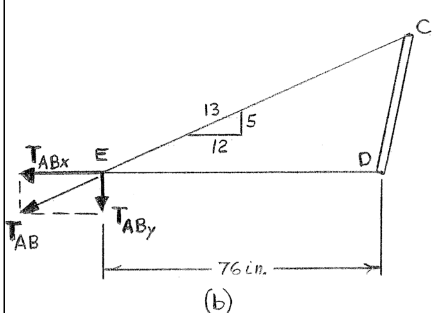
and $T_{ABy} = \dfrac{5}{13}(260 \text{ lb}) = 100 \text{ lb}$

Then $M_D = T_{ABx}(35 \text{ in.}) - T_{ABy}(8 \text{ in.})$

$= (240 \text{ lb})(35 \text{ in.}) - (100 \text{ lb})(8 \text{ in.})$

$= 7600 \text{ lb}\cdot\text{in.}$

or $\mathbf{M}_D = 7600 \text{ lb}\cdot\text{in.}$ ◀



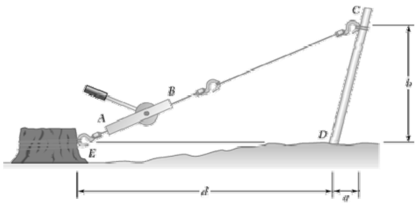
(*b*) Have $M_D = T_{ABx}(y) + T_{ABy}(x)$

$= (240 \text{ lb})(0) + (100 \text{ lb})(76 \text{ in.})$

$= 7600 \text{ lb}\cdot\text{in.}$

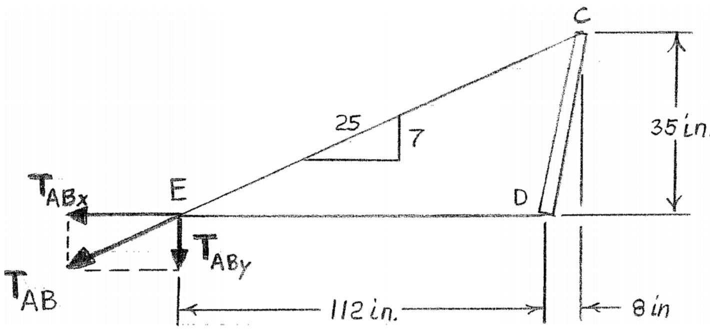
or $\mathbf{M}_D = 7600 \text{ lb}\cdot\text{in.}$ ◀

PROBLEM 3.12



It is known that a force with a moment of $7840 \text{ lb}\cdot\text{in.}$ about D is required to straighten the fence post CD. If $a = 8$ in., $b = 35$ in., and $d = 112$ in., determine the tension that must be developed in the cable of winch puller AB to create the required moment about point D.

SOLUTION



$$\text{Slope of line } EC = \frac{35 \text{ in.}}{112 \text{ in.} + 8 \text{ in.}} = \frac{7}{24}$$

Then

$$T_{ABx} = \frac{24}{25}T_{AB}$$

and

$$T_{ABy} = \frac{7}{25}T_{AB}$$

Have

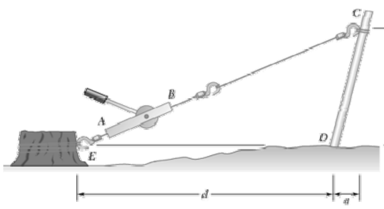
$$M_D = T_{ABx}(y) + T_{ABy}(x)$$

$$\therefore \; 7840 \text{ lb}\cdot\text{in.} = \frac{24}{25}T_{AB}(0) + \frac{7}{25}T_{AB}(112 \text{ in.})$$

$$T_{AB} = 250 \text{ lb}$$

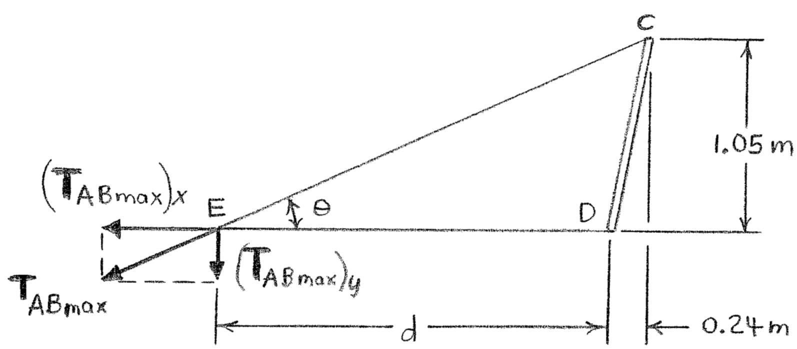
or $T_{AB} = 250$ lb ◀

PROBLEM 3.13



It is known that a force with a moment of $1152\text{ N}\cdot\text{m}$ about D is required to straighten the fence post CD. If the capacity of the winch puller AB is 2880 N, determine the minimum value of distance d to create the specified moment about point D knowing that $a = 0.24$ m and $b = 1.05$ m.

SOLUTION



The minimum value of d can be found based on the equation relating the moment of the force $\mathbf{T}_{AB}$ about D:

$$M_D = (T_{AB\max})_y(d)$$

where $$M_D = 1152\text{ N}\cdot\text{m}$$

$$(T_{AB\max})_y = T_{AB\max}\sin\theta = (2880\text{ N})\sin\theta$$

Now $$\sin\theta = \frac{1.05\text{ m}}{\sqrt{(d+0.24)^2+(1.05)^2}\text{ m}}$$

$$\therefore\ 1152\text{ N}\cdot\text{m} = 2880\text{ N}\left[\frac{1.05}{\sqrt{(d+0.24)^2+(1.05)^2}}\right](d)$$

or $$\sqrt{(d+0.24)^2+(1.05)^2} = 2.625d$$

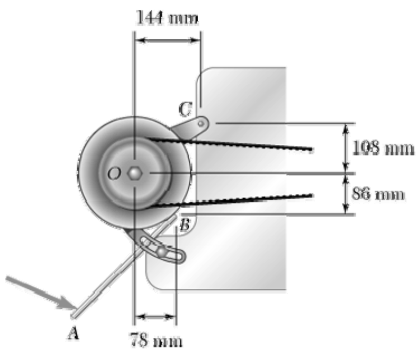
or $$(d+0.24)^2+(1.05)^2 = 6.8906d^2$$

or $$5.8906d^2 - 0.48d - 1.1601 = 0$$

Using the quadratic equation, the minimum values of d are 0.48639 m and –0.40490 m.
Since only the positive value applies here, $d = 0.48639$ m

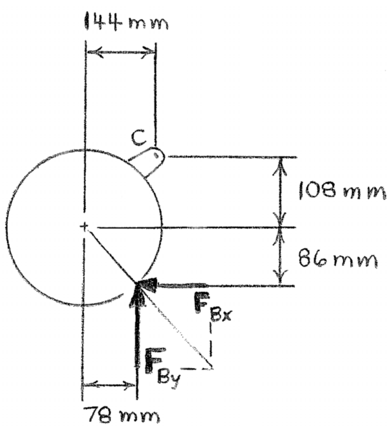
or $d = 486$ mm ◀

PROBLEM 3.14



A mechanic uses a piece of pipe AB as a lever when tightening an alternator belt. When he pushes down at A, a force of 580 N is exerted on the alternator B. Determine the moment of that force about bolt C if its line of action passes through O.

SOLUTION



Have $$\mathbf{M}_C = \mathbf{r}_{B/C} \times \mathbf{F}_B$$

Noting the direction of the moment of each force component about C is clockwise,

$$M_C = xF_{By} + yF_{Bx}$$

where $$x = 144 \text{ mm} - 78 \text{ mm} = 66 \text{ mm}$$

$$y = 86 \text{ mm} + 108 \text{ mm} = 194 \text{ mm}$$

and $$F_{Bx} = \frac{78}{\sqrt{(78)^2 + (86)^2}}(580 \text{ N}) = 389.65 \text{ N}$$

$$F_{By} = \frac{86}{\sqrt{(78)^2 + (86)^2}}(580 \text{ N}) = 429.62 \text{ N}$$

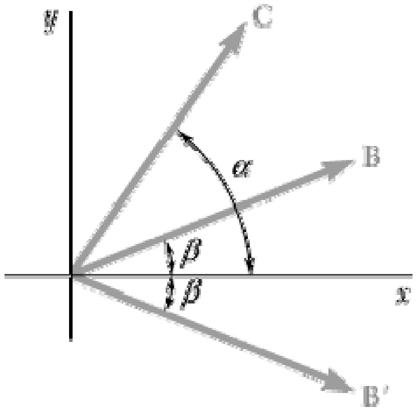
$$\therefore \quad M_C = (66 \text{ mm})(429.62 \text{ N}) + (194 \text{ mm})(389.65 \text{ N})$$

$$= 103947 \text{ N}\cdot\text{mm}$$

$$= 103.947 \text{ N}\cdot\text{m}$$

or $\mathbf{M}_C = 103.9 \text{ N}\cdot\text{m}$ ◀

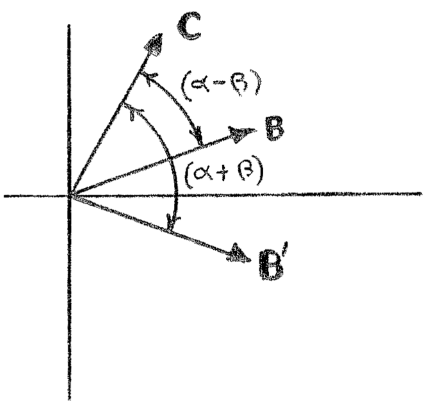
PROBLEM 3.15



Form the vector products $\mathbf{B} \times \mathbf{C}$ and $\mathbf{B}' \times \mathbf{C}$, where $B = B'$, and use the results obtained to prove the identity

$$\sin\alpha \cos\beta = \tfrac{1}{2}\sin(\alpha + \beta) + \tfrac{1}{2}\sin(\alpha - \beta).$$

SOLUTION



First note $\quad \mathbf{B} = B(\cos\beta\mathbf{i} + \sin\beta\mathbf{j})$

$$\mathbf{B}' = B(\cos\beta\mathbf{i} - \sin\beta\mathbf{j})$$

$$\mathbf{C} = C(\cos\alpha\mathbf{i} + \sin\alpha\mathbf{j})$$

By definition

$$|\mathbf{B} \times \mathbf{C}| = BC\sin(\alpha - \beta) \qquad (1)$$

$$|\mathbf{B}' \times \mathbf{C}| = BC\sin(\alpha + \beta) \qquad (2)$$

Now

$$\mathbf{B} \times \mathbf{C} = B(\cos\beta\mathbf{i} + \sin\beta\mathbf{j}) \times C(\cos\alpha\mathbf{i} + \sin\alpha\mathbf{j})$$

$$= BC(\cos\beta\sin\alpha - \sin\beta\cos\alpha)\mathbf{k} \qquad (3)$$

$$\mathbf{B} \times \mathbf{C} = B(\cos\beta\mathbf{i} - \sin\beta\mathbf{j}) \times C(\cos\alpha\mathbf{i} + \sin\alpha\mathbf{j})$$

$$= BC(\cos\beta\sin\alpha + \sin\beta\cos\alpha)\mathbf{k} \qquad (4)$$

Equating magnitudes of $\mathbf{B} \times \mathbf{C}$ from Equations (1) and (3), (5)

$$\sin(\alpha - \beta) = \cos\beta\sin\alpha - \sin\beta\cos\alpha$$

Similarly, equating magnitudes of $\mathbf{B}' \times \mathbf{C}$ from Equations (2) and (4),

$$\sin(\alpha + \beta) = \cos\beta\sin\alpha + \sin\beta\cos\alpha \qquad (6)$$

Adding Equations (5) and (6)

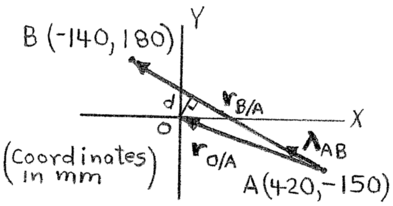
$$\sin(\alpha - \beta) + \sin(\alpha + \beta) = 2\cos\beta\sin\alpha$$

$$\therefore \quad \sin\alpha\cos\beta = \frac{1}{2}\sin(\alpha + \beta) + \frac{1}{2}\sin(\alpha - \beta) \blacktriangleleft$$

PROBLEM 3.16

A line passes through the points (420 mm, –150 mm) and (–140 mm, 180 mm). Determine the perpendicular distance d from the line to the origin O of the system of coordinates.

SOLUTION



Have $$d = \left|\lambda_{AB} \times \mathbf{r}_{O/A}\right|$$

where $$\lambda_{AB} = \frac{\mathbf{r}_{B/A}}{\left|\mathbf{r}_{B/A}\right|}$$

and $$\mathbf{r}_{B/A} = (-140\text{ mm} - 420\text{ mm})\mathbf{i} + \left[180\text{ mm} - (-150\text{ mm})\right]\mathbf{j}$$

$$= -(560\text{ mm})\mathbf{i} + (330\text{ mm})\mathbf{j}$$

$$\left|\mathbf{r}_{B/A}\right| = \sqrt{(-560)^2 + (330)^2}\text{mm} = 650\text{ mm}$$

$$\therefore\ \lambda_{AB} = \frac{-(560\text{ mm})\mathbf{i} + (330\text{ mm})\mathbf{j}}{650\text{ mm}} = \frac{1}{65}(-56\mathbf{i} + 33\mathbf{j})$$

$$\mathbf{r}_{O/A} = (0 - x_A)\mathbf{i} + (0 - y_A)\mathbf{j} = -(420\text{ mm})\mathbf{i} + (150\text{ mm})\mathbf{j}$$

$$\therefore\ d = \left|\frac{1}{65}(-56\mathbf{i} - 33\mathbf{j}) \times \left[-(420\text{ mm})\mathbf{i} + (150\text{ mm})\mathbf{j}\right]\right| = 84.0\text{ mm}$$

$d = 84.0$ mm ◀

PROBLEM 3.17

A plane contains the vectors **A** and **B**. Determine the unit vector normal to the plane when **A** and **B** are equal to, respectively, (*a*) $4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and $-2\mathbf{i} + 6\mathbf{j} - 5\mathbf{k}$, (*b*) $7\mathbf{i} + \mathbf{j} - 4\mathbf{k}$ and $-6\mathbf{i} - 3\mathbf{k} + 2\mathbf{k}$.

SOLUTION

(*a*) Have

$$\lambda = \frac{\mathbf{A} \times \mathbf{B}}{|\mathbf{A} \times \mathbf{B}|}$$

where

$$\mathbf{A} = 4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$$

$$\mathbf{B} = -2\mathbf{i} + 6\mathbf{j} - 5\mathbf{k}$$

Then

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -2 & 3 \\ -2 & 6 & -5 \end{vmatrix} = (10 - 18)\mathbf{i} + (-6 + 20)\mathbf{j} + (24 - 4)\mathbf{k} = 2(-4\mathbf{i} + 7\mathbf{j} + 10\mathbf{k})$$

and

$$|\mathbf{A} \times \mathbf{B}| = 2\sqrt{(-4)^2 + (7)^2 + (10)^2} = 2\sqrt{165}$$

$$\therefore \quad \lambda = \frac{2(-4\mathbf{i} + 7\mathbf{j} + 10\mathbf{k})}{2\sqrt{165}} \qquad \text{or} \quad \lambda = \frac{1}{\sqrt{165}}(-4\mathbf{i} + 7\mathbf{j} + 10\mathbf{k}) \blacktriangleleft$$

(*b*) Have

$$\lambda = \frac{\mathbf{A} \times \mathbf{B}}{|\mathbf{A} \times \mathbf{B}|}$$

where

$$\mathbf{A} = 7\mathbf{i} + \mathbf{j} - 4\mathbf{k}$$

$$\mathbf{B} = -6\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$$

Then

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7 & 1 & -4 \\ -6 & -3 & 2 \end{vmatrix} = (2 - 12)\mathbf{i} + (24 - 14)\mathbf{j} + (-21 + 6)\mathbf{k} = 5(-2\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$$

and

$$|\mathbf{A} \times \mathbf{B}| = 5\sqrt{(-2)^2 + (2)^2 + (-3)^2} = 5\sqrt{17}$$

$$\therefore \quad \lambda = \frac{5(-2\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})}{5\sqrt{17}} \qquad \text{or} \quad \lambda = \frac{1}{\sqrt{17}}(-2\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}) \blacktriangleleft$$

PROBLEM 3.18

The vectors **P** and **Q** are two adjacent sides of a parallelogram. Determine the area of the parallelogram when (*a*) **P** = (8 in.)**i** + (2 in.)**j** − (1 in.)**k** and **Q** = −(3 in.)**i** + (4 in.)**j** + (2 in.)**k**, (*b*) **P** = −(3 in.)**i** + (6 in.)**j** + (4 in.)**k** and **Q** = (2 in.)**i** + (5 in.)**j** − (3 in.)**k**.

SOLUTION

(*a*) Have

$$A = \left|\mathbf{P} \times \mathbf{Q}\right|$$

where

$$\mathbf{P} = (8 \text{ in.})\mathbf{i} + (2 \text{ in.})\mathbf{j} - (1 \text{ in.})\mathbf{k}$$

$$\mathbf{Q} = -(3 \text{ in.})\mathbf{i} + (4 \text{ in.})\mathbf{j} + (2 \text{ in.})\mathbf{k}$$

Then

$$\mathbf{P} \times \mathbf{Q} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & 2 & -1 \\ -3 & 4 & 2 \end{vmatrix} \text{in}^2 = \left[(4+4)\mathbf{i} + (3-16)\mathbf{j} + (32+6)\mathbf{k}\right]\text{in}^2$$

$$= \left(8 \text{ in}^2\right)\mathbf{i} - \left(13 \text{ in}^2\right)\mathbf{j} + \left(38 \text{ in}^2\right)\mathbf{k}$$

$$\therefore \quad A = \sqrt{(8)^2 + (-13)^2 + (38)^2}\,\text{in}^2 = 40.951 \text{ in}^2 \qquad \text{or} \quad A = 41.0 \text{ in}^2 \blacktriangleleft$$

(*b*) Have

$$A = \left|\mathbf{P} \times \mathbf{Q}\right|$$

where

$$\mathbf{P} = -(3 \text{ in.})\mathbf{i} + (6 \text{ in.})\mathbf{j} + (4 \text{ in.})\mathbf{k}$$

$$\mathbf{Q} = (2 \text{ in.})\mathbf{i} + (5 \text{ in.})\mathbf{j} - (3 \text{ in.})\mathbf{k}$$

Then

$$\mathbf{P} \times \mathbf{Q} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 6 & 4 \\ 2 & 5 & -3 \end{vmatrix} \text{in}^2 = \left[(-18-20)\mathbf{i} + (8-9)\mathbf{j} + (-15-12)\mathbf{k}\right]\text{in}^2$$

$$= -\left(38 \text{ in}^2\right)\mathbf{i} - \left(1 \text{ in}^2\right)\mathbf{j} - \left(27 \text{ in}^2\right)\mathbf{k}$$

$$\therefore \quad A = \sqrt{(-38)^2 + (-1)^2 + (-27)^2}\,\text{in}^2 = 46.626 \text{ in}^2 \qquad \text{or} \quad A = 46.6 \text{ in}^2 \blacktriangleleft$$

PROBLEM 3.19

Determine the moment about the origin O of the force $\mathbf{F} = -(5\text{ N})\mathbf{i} - (2\text{ N})\mathbf{j} + (3\text{ N})\mathbf{k}$ which acts at a point A. Assume that the position vector of A is (a) $\mathbf{r} = (4\text{ m})\mathbf{i} - (2\text{ m})\mathbf{j} - (1\text{ m})\mathbf{k}$, ($b$) $\mathbf{r} = -(8\text{ m})\mathbf{i} + (3\text{ m})\mathbf{j} + (4\text{ m})\mathbf{k}$, ($c$) $\mathbf{r} = (7.5\text{ m})\mathbf{i} + (3\text{ m})\mathbf{j} - (4.5\text{ m})\mathbf{k}$.

SOLUTION

(a) Have

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

where

$$\mathbf{F} = -(5\text{ N})\mathbf{i} - (2\text{ N})\mathbf{j} + (3\text{ N})\mathbf{k}$$

$$\mathbf{r} = (4\text{ m})\mathbf{i} - (2\text{ m})\mathbf{j} - (1\text{ m})\mathbf{k}$$

$$\therefore \quad \mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -2 & -1 \\ -5 & -2 & 3 \end{vmatrix} \text{N}\cdot\text{m} = \left[(-6-2)\mathbf{i} + (5-12)\mathbf{j} + (-8-10)\mathbf{k}\right] \text{N}\cdot\text{m}$$

$$= (-8\mathbf{i} - 7\mathbf{j} - 18\mathbf{k})\,\text{N}\cdot\text{m}$$

or $\mathbf{M}_O = -(8\text{ N}\cdot\text{m})\mathbf{i} - (7\text{ N}\cdot\text{m})\mathbf{j} - (18\text{ N}\cdot\text{m})\mathbf{k}$ ◀

(b) Have

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

where

$$\mathbf{F} = -(5\text{ N})\mathbf{i} - (2\text{ N})\mathbf{j} + (3\text{ N})\mathbf{k}$$

$$\mathbf{r} = -(8\text{ m})\mathbf{i} + (3\text{ m})\mathbf{j} - (4\text{ m})\mathbf{k}$$

$$\therefore \quad \mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -8 & 3 & 4 \\ -5 & -2 & 3 \end{vmatrix} \text{N}\cdot\text{m} = \left[(9+8)\mathbf{i} + (-20+24)\mathbf{j} + (16+15)\mathbf{k}\right] \text{N}\cdot\text{m}$$

$$= (17\mathbf{i} + 4\mathbf{j} + 31\mathbf{k})\,\text{N}\cdot\text{m}$$

or $\mathbf{M}_O = (17\text{ N}\cdot\text{m})\mathbf{i} + (4\text{ N}\cdot\text{m})\mathbf{j} + (31\text{ N}\cdot\text{m})\mathbf{k}$ ◀

(c) Have

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

where

$$\mathbf{F} = -(5\text{ N})\mathbf{i} - (2\text{ N})\mathbf{j} + (3\text{ N})\mathbf{k}$$

$$\mathbf{r} = (7.5\text{ m})\mathbf{i} + (3\text{ m})\mathbf{j} - (4.5\text{ m})\mathbf{k}$$

PROBLEM 3.19 CONTINUED

$$\therefore \quad \mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 7.5 & 3 & -4.5 \\ -5 & -2 & 3 \end{vmatrix} \text{N}\cdot\text{m} = \left[(9-9)\mathbf{i} + (22.5-22.5)\mathbf{j} + (-15+15)\mathbf{k}\right]\text{N}\cdot\text{m}$$

or $\mathbf{M}_O = 0$ ◀

This answer is expected since **r** and **F** are proportional $\left(\mathbf{F} = \dfrac{-2}{3}\mathbf{r}\right)$. Therefore, vector **F** has a line of action passing through the origin at *O*.

PROBLEM 3.20

Determine the moment about the origin O of the force $\mathbf{F} = -(1.5 \text{ lb})\mathbf{i} + (3 \text{ lb})\mathbf{j} - (2 \text{ lb})\mathbf{k}$ which acts at a point A. Assume that the position vector of A is (*a*) $\mathbf{r} = (2.5 \text{ ft})\mathbf{i} - (1 \text{ ft})\mathbf{j} + (2 \text{ ft})\mathbf{k}$, (*b*) $\mathbf{r} = (4.5 \text{ ft})\mathbf{i} - (9 \text{ ft})\mathbf{j} + (6 \text{ ft})\mathbf{k}$, (*c*) $\mathbf{r} = (4 \text{ ft})\mathbf{i} - (1 \text{ ft})\mathbf{j} + (7 \text{ ft})\mathbf{k}$.

SOLUTION

(*a*) Have
$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

where
$$\mathbf{F} = -(1.5 \text{ lb})\mathbf{i} + (3 \text{ lb})\mathbf{j} + (2 \text{ lb})\mathbf{k}$$
$$\mathbf{r} = (2.5 \text{ ft})\mathbf{i} - (1 \text{ ft})\mathbf{j} + (2 \text{ ft})\mathbf{k}$$

Then
$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2.5 & -1 & 2 \\ -1.5 & 3 & -2 \end{vmatrix} \text{lb}\cdot\text{ft} = \left[(2-6)\mathbf{i} + (-3+5)\mathbf{j} + (7.5-1.5)\mathbf{k}\right] \text{lb}\cdot\text{ft}$$

or $\mathbf{M}_O = -(4 \text{ lb}\cdot\text{ft})\mathbf{i} + (2 \text{ lb}\cdot\text{ft})\mathbf{j} + (6 \text{ lb}\cdot\text{ft})\mathbf{k}$ ◀

(*b*) Have
$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

where
$$\mathbf{F} = -(1.5 \text{ lb})\mathbf{i} + (3 \text{ lb})\mathbf{j} - (2 \text{ lb})\mathbf{k}$$
$$\mathbf{r} = (4.5 \text{ ft})\mathbf{i} - (9 \text{ ft})\mathbf{j} + (6 \text{ ft})\mathbf{k}$$

Then
$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4.5 & -9 & 6 \\ -1.5 & 3 & -2 \end{vmatrix} \text{lb}\cdot\text{ft} = \left[(18-18)\mathbf{i} + (-9+9)\mathbf{j} + (13.5-13.5)\mathbf{k}\right] \text{lb}\cdot\text{ft}$$

or $\mathbf{M}_O = 0$ ◀

This answer is expected since $\mathbf{r}$ and $\mathbf{F}$ are proportional $\left(\mathbf{F} = \frac{-1}{3}\mathbf{r}\right)$.

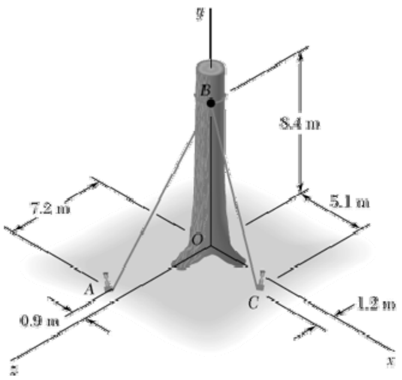
Therefore, vector $\mathbf{F}$ has a line of action passing through the origin at O.

(*c*) Have
$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

where
$$\mathbf{F} = -(1.5 \text{ lb})\mathbf{i} - (3 \text{ lb})\mathbf{j} - (2 \text{ lb})\mathbf{k}$$
$$\mathbf{r} = (4 \text{ ft})\mathbf{i} - (1 \text{ ft})\mathbf{j} + (7 \text{ ft})\mathbf{k}$$

Then
$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -1 & 7 \\ -1.5 & 3 & -2 \end{vmatrix} \text{lb}\cdot\text{ft} = \left[(2-21)\mathbf{i} + (-10.5+8)\mathbf{j} + (12-1.5)\mathbf{k}\right] \text{lb}\cdot\text{ft}$$

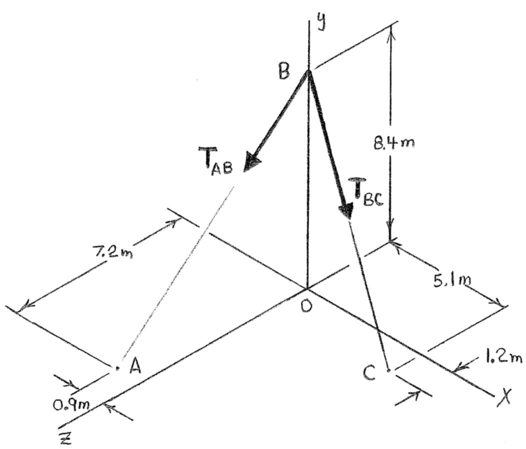
or $\mathbf{M}_O = -(19 \text{ lb}\cdot\text{ft})\mathbf{i} - (2.5 \text{ lb}\cdot\text{ft})\mathbf{j} + (10.5 \text{ lb}\cdot\text{ft})\mathbf{k}$ ◀



PROBLEM 3.21

Before the trunk of a large tree is felled, cables *AB* and *BC* are attached as shown. Knowing that the tension in cables *AB* and *BC* are 777 N and 990 N, respectively, determine the moment about *O* of the resultant force exerted on the tree by the cables at *B*.

SOLUTION



Have
$$\mathbf{M}_O = \mathbf{r}_{B/O} \times \mathbf{F}_B$$

where
$$\mathbf{r}_{B/O} = (8.4\text{ m})\,\mathbf{j}$$

$$\mathbf{F}_B = \mathbf{T}_{AB} + \mathbf{T}_{BC}$$

$$\mathbf{T}_{AB} = \lambda_{BA} T_{AB} = \frac{-(0.9\text{ m})\mathbf{i} - (8.4\text{ m})\mathbf{j} + (7.2\text{ m})\mathbf{k}}{\sqrt{(0.9)^2 + (8.4)^2 + (7.2)^2}\text{ m}}(777\text{ N})$$

$$\mathbf{T}_{BC} = \lambda_{BC} T_{BC} = \frac{(5.1\text{ m})\mathbf{i} - (8.4\text{ m})\mathbf{j} + (1.2\text{ m})\mathbf{k}}{\sqrt{(5.1)^2 + (8.4)^2 + (1.2)^2}\text{ m}}(990\text{ N})$$

PROBLEM 3.21 CONTINUED

$$\therefore \quad \mathbf{F}_B = \left[-(63.0\text{ N})\mathbf{i} - (588\text{ N})\mathbf{j} + (504\text{ N})\mathbf{k}\right] + \left[(510\text{ N})\mathbf{i} - (840\text{ N})\mathbf{j} + (120\text{ N})\mathbf{k}\right]$$

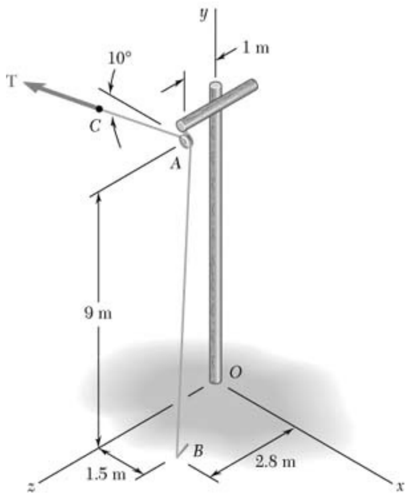
$$= (447\text{ N})\mathbf{i} - (1428\text{ N})\mathbf{j} + (624\text{ N})\mathbf{k}$$

and

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 8.4 & 0 \\ 447 & -1428 & 624 \end{vmatrix} \text{N}\cdot\text{m} = (5241.6\text{ N}\cdot\text{m})\mathbf{i} - (3754.8\text{ N}\cdot\text{m})\mathbf{k}$$

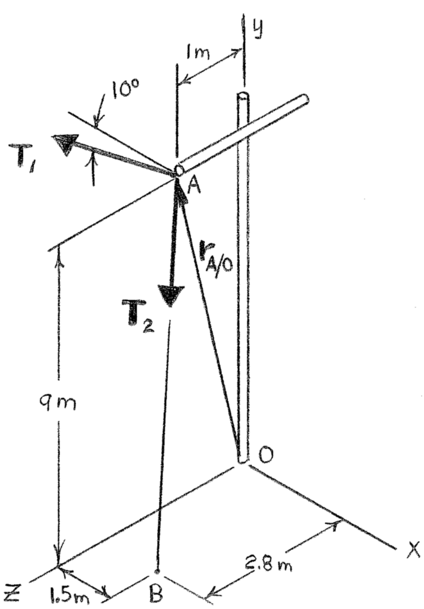
or $\mathbf{M}_O = (5.24\text{ kN}\cdot\text{m})\mathbf{i} - (3.75\text{ kN}\cdot\text{m})\mathbf{k}$ ◀

PROBLEM 3.22



Before a telephone cable is strung, rope *BAC* is tied to a stake at *B* and is passed over a pulley at *A*. Knowing that portion *AC* of the rope lies in a plane parallel to the *xy* plane and that the tension **T** in the rope is 124 N, determine the moment about *O* of the resultant force exerted on the pulley by the rope.

SOLUTION



Have $$\mathbf{M}_O = \mathbf{r}_{A/O} \times \mathbf{R}$$

where $$\mathbf{r}_{A/O} = (0\text{ m})\mathbf{i} + (9\text{ m})\mathbf{j} + (1\text{ m})\mathbf{k}$$

$$\mathbf{R} = \mathbf{T}_1 + \mathbf{T}_2$$

$$\mathbf{T}_1 = -\left[(124\text{ N})\cos 10°\right]\mathbf{i} - \left[(124\text{ N})\sin 10°\right]\mathbf{j}$$

$$= -(122.116\text{ N})\mathbf{i} - (21.532\text{ N})\mathbf{j}$$

$$\mathbf{T}_2 = \lambda T_2 = \left[\frac{(1.5\text{ m})\mathbf{i} - (9\text{ m})\mathbf{j} + (1.8\text{ m})\mathbf{k}}{\sqrt{(1.5\text{ m})^2 + (9\text{ m})^2 + (1.8\text{ m})^2}}\right](124\text{ N})$$

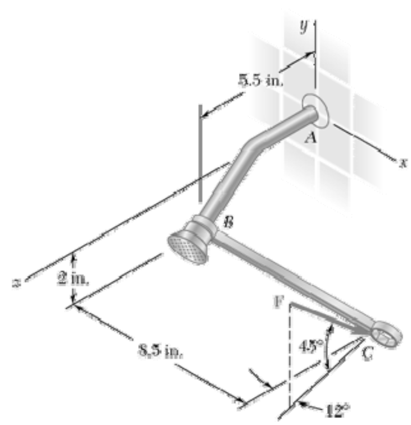
$$= (20\text{ N})\mathbf{i} - (120\text{ N})\mathbf{j} + (24\text{ N})\mathbf{k}$$

$$\therefore \quad \mathbf{R} = -(102.116\text{ N})\mathbf{i} - (141.532\text{ N})\mathbf{j} + (24\text{ N})\mathbf{k}$$

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 9 & 1 \\ -102.116 & -141.532 & 24 \end{vmatrix} \text{N}\cdot\text{m}$$

$$= (357.523\text{ N}\cdot\text{m})\mathbf{i} - (102.116\text{ N}\cdot\text{m})\mathbf{j} + (919.044\text{ N}\cdot\text{m})\mathbf{k}$$

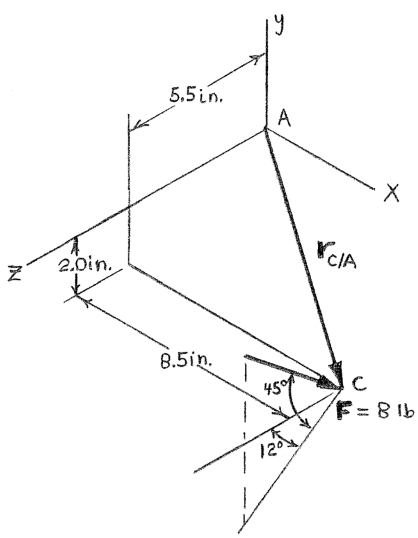
$$\text{or} \quad \mathbf{M}_O = (358\text{ N}\cdot\text{m})\mathbf{i} - (102.1\text{ N}\cdot\text{m})\mathbf{j} + (919\text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$



PROBLEM 3.23

An 8-lb force is applied to a wrench to tighten a showerhead. Knowing that the centerline of the wrench is parallel to the *x* axis, determine the moment of the force about *A*.

SOLUTION



Have $$\mathbf{M}_A = \mathbf{r}_{C/A} \times \mathbf{F}$$

where $$\mathbf{r}_{C/A} = (8.5 \text{ in.})\mathbf{i} - (2.0 \text{ in.})\mathbf{j} + (5.5 \text{ in.})\mathbf{k}$$

$$F_x = -(8\cos 45^\circ \sin 12^\circ)\text{lb}$$

$$F_y = -(8\sin 45^\circ)\text{lb}$$

$$F_z = -(8\cos 45^\circ \cos 12^\circ)\text{lb}$$

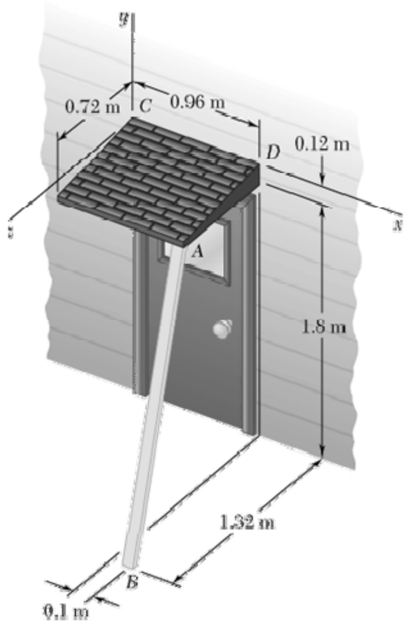
$$\therefore \quad \mathbf{F} = -(1.17613 \text{ lb})\mathbf{i} - (5.6569 \text{ lb})\mathbf{j} - (5.5332 \text{ lb})\mathbf{k}$$

and $$\mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8.5 & -2.0 & 5.5 \\ -1.17613 & -5.6569 & -5.5332 \end{vmatrix} \text{lb} \cdot \text{in.}$$

$$= (42.179 \text{ lb} \cdot \text{in.})\mathbf{i} + (40.563 \text{ lb} \cdot \text{in.})\mathbf{j} - (50.436 \text{ lb} \cdot \text{in.})\mathbf{k}$$

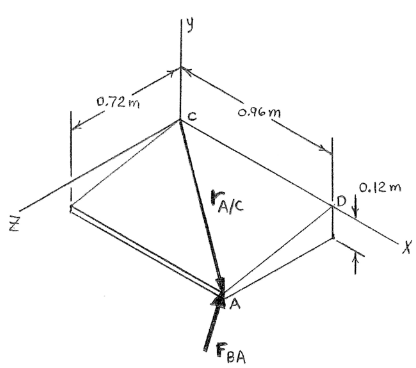
or $$\mathbf{M}_A = (42.2 \text{ lb} \cdot \text{in.})\mathbf{i} + (40.6 \text{ lb} \cdot \text{in.})\mathbf{j} - (50.4 \text{ lb} \cdot \text{in.})\mathbf{k} \blacktriangleleft$$

PROBLEM 3.24



A wooden board AB, which is used as a temporary prop to support a small roof, exerts at point A of the roof a 228 N force directed along BA. Determine the moment about C of that force.

SOLUTION



Have $$\mathbf{M}_C = \mathbf{r}_{A/C} \times \mathbf{F}_{BA}$$

where $$\mathbf{r}_{A/C} = (0.96\text{ m})\mathbf{i} - (0.12\text{ m})\mathbf{j} + (0.72\text{ m})\mathbf{k}$$

and $$\mathbf{F}_{BA} = \lambda_{BA}F_{BA}$$

$$= \left[\frac{-(0.1\text{ m})\mathbf{i} + (1.8\text{ m})\mathbf{j} - (0.6\text{ m})\mathbf{k}}{\sqrt{(0.1)^2 + (1.8)^2 + (0.6)^2}\text{ m}}\right](228\text{ N})$$

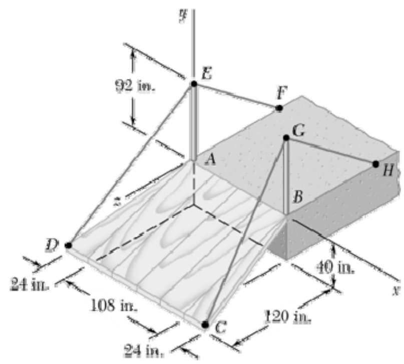
$$= -(12.0\text{ N})\mathbf{i} + (216\text{ N})\mathbf{j} - (72\text{ N})\mathbf{k}$$

$$\therefore\quad \mathbf{M}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.96 & -0.12 & 0.72 \\ -12.0 & 216 & -72 \end{vmatrix}\text{N}\cdot\text{m}$$

$$= -(146.88\text{ N}\cdot\text{m})\mathbf{i} + (60.480\text{ N}\cdot\text{m})\mathbf{j} + (205.92\text{ N}\cdot\text{m})\mathbf{k}$$

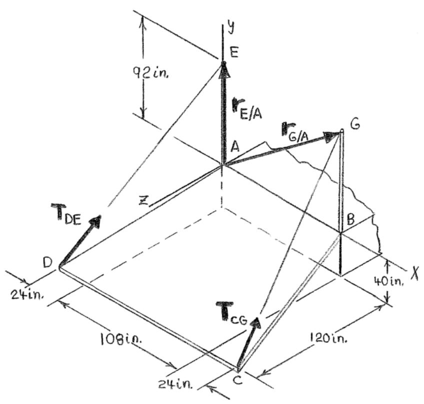
or $$\mathbf{M}_C = -(146.9\text{ N}\cdot\text{m})\mathbf{i} + (60.5\text{ N}\cdot\text{m})\mathbf{j} + (206\text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$

PROBLEM 3.25



The ramp *ABCD* is supported by cables at corners *C* and *D*. The tension in each of the cables is 360 lb. Determine the moment about *A* of the force exerted by (*a*) the cable at *D*, (*b*) the cable at *C*.

SOLUTION



(*a*) Have $$\mathbf{M}_A = \mathbf{r}_{E/A} \times \mathbf{T}_{DE}$$

where $$\mathbf{r}_{E/A} = (92 \text{ in.})\,\mathbf{j}$$

$$\mathbf{T}_{DE} = \lambda_{DE} T_{DE}$$

$$= \frac{(24 \text{ in.})\mathbf{i} + (132 \text{ in.})\mathbf{j} - (120 \text{ in.})\mathbf{k}}{\sqrt{(24)^2 + (132)^2 + (120)^2} \text{ in.}}(360 \text{ lb})$$

$$= (48 \text{ lb})\mathbf{i} + (264 \text{ lb})\mathbf{j} - (240 \text{ lb})\mathbf{k}$$

$$\therefore \quad \mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 92 & 0 \\ 48 & 264 & -240 \end{vmatrix} \text{lb}\cdot\text{in.} = -(22{,}080 \text{ lb}\cdot\text{in.})\mathbf{i} - (4416 \text{ lb}\cdot\text{in})\mathbf{k}$$

$$\text{or} \quad \mathbf{M}_A = -(1840 \text{ lb}\cdot\text{ft})\mathbf{i} - (368 \text{ lb}\cdot\text{ft})\mathbf{k} \blacktriangleleft$$

(*b*) Have $$\mathbf{M}_A = \mathbf{r}_{G/A} \times \mathbf{T}_{CG}$$

where $$\mathbf{r}_{G/A} = (108 \text{ in.})\mathbf{i} + (92 \text{ in.})\mathbf{j}$$

$$\mathbf{T}_{CG} = \lambda_{CG} T_{CG} = \frac{-(24 \text{ in.})\mathbf{i} + (132 \text{ in.})\mathbf{j} - (120 \text{ in.})\mathbf{k}}{\sqrt{(24)^2 + (132)^2 + (120)^2} \text{ in.}}(360 \text{ lb})$$

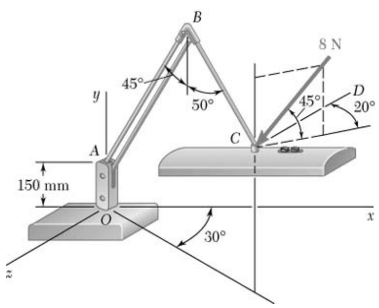
$$= -(48 \text{ lb})\mathbf{i} + (264 \text{ lb})\mathbf{j} - (240 \text{ lb})\mathbf{k}$$

$$\therefore \quad \mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 108 & 92 & 0 \\ -48 & 264 & -240 \end{vmatrix} \text{lb}\cdot\text{in.}$$

$$= -(22{,}080 \text{ lb}\cdot\text{in.})\mathbf{i} + (25{,}920 \text{ lb}\cdot\text{in.})\mathbf{j} + (32{,}928 \text{ lb}\cdot\text{in.})\mathbf{k}$$

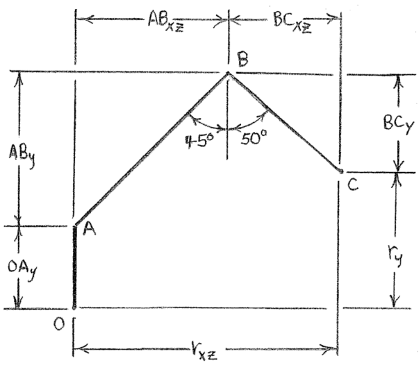
$$\text{or} \quad \mathbf{M}_A = -(1840 \text{ lb}\cdot\text{ft})\mathbf{i} + (2160 \text{ lb}\cdot\text{ft})\mathbf{j} + (2740 \text{ lb}\cdot\text{ft})\mathbf{k} \blacktriangleleft$$

PROBLEM 3.26



The arms *AB* and *BC* of a desk lamp lie in a vertical plane that forms an angle of 30° with the *xy* plane. To reposition the light, a force of magnitude 8 N is applied at *C* as shown. Determine the moment of the force about *O* knowing that $AB = 450$ mm, $BC = 325$ mm, and line *CD* is parallel to the *z* axis.

SOLUTION



Have $$\mathbf{M}_O = \mathbf{r}_{C/O} \times \mathbf{F}_C$$

where $$\left(r_{C/O}\right)_x = \left(AB_{xz} + BC_{xz}\right)\cos 30^{\circ}$$

$$AB_{xz} = (0.450 \text{ m})\sin 45^{\circ} = 0.31820 \text{ m}$$

$$BC_{xz} = (0.325 \text{ m})\sin 50^{\circ} = 0.24896 \text{ m}$$

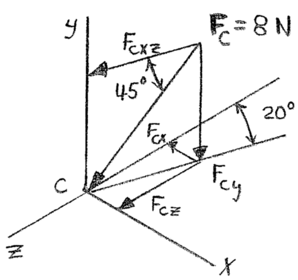
$$\left(r_{C/O}\right)_y = \left(OA_y + AB_y - BC_y\right) = 0.150 \text{ m} + (0.450 \text{ m})\cos 45^{\circ}$$

$$-(0.325 \text{ m})\cos 50^{\circ} = 0.25929 \text{ m}$$

$$\left(r_{C/O}\right)_z = \left(AB_{xz} + BC_{xz}\right)\sin 30^{\circ}$$

$$= (0.31820 \text{ m} + 0.24896 \text{ m})\sin 30^{\circ} = 0.28358 \text{ m}$$

or $$\mathbf{r}_{C/O} = (0.49118 \text{ m})\mathbf{i} + (0.25929 \text{ m})\mathbf{j} + (0.28358 \text{ m})\mathbf{k}$$



$$\left(F_C\right)_x = -(8 \text{ N})\cos 45^{\circ}\sin 20^{\circ} = -1.93476 \text{ N}$$

$$\left(F_C\right)_y = -(8 \text{ N})\sin 45^{\circ} = -5.6569 \text{ N}$$

$$\left(F_C\right)_z = (8 \text{ N})\cos 45^{\circ}\cos 20^{\circ} = 5.3157 \text{ N}$$

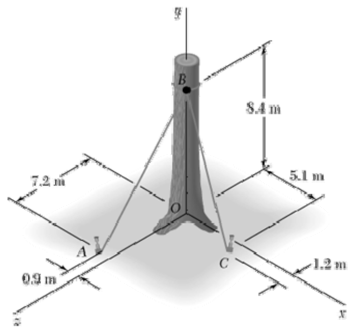
or $$\mathbf{F}_C = -(1.93476 \text{ N})\mathbf{i} - (5.6569 \text{ N})\mathbf{j} + (5.3157 \text{ N})\mathbf{k}$$

$$\therefore \quad \mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.49118 & 0.25929 & 0.28358 \\ -1.93476 & -5.6569 & 5.3157 \end{vmatrix} \text{N}\cdot\text{m}$$

$$= (2.9825 \text{ N}\cdot\text{m})\mathbf{i} - (3.1596 \text{ N}\cdot\text{m})\mathbf{j} - (2.2769 \text{ N}\cdot\text{m})\mathbf{k}$$

or $$\mathbf{M}_O = (2.98 \text{ N}\cdot\text{m})\mathbf{i} - (3.16 \text{ N}\cdot\text{m})\mathbf{j} - (2.28 \text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$

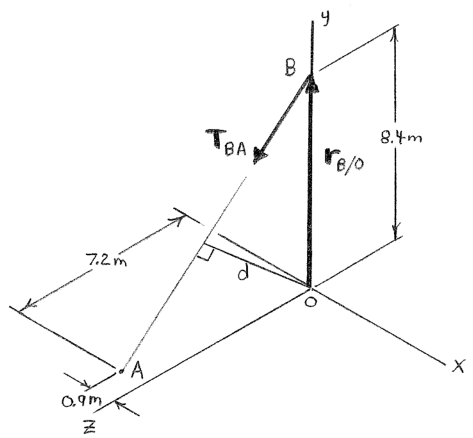
PROBLEM 3.27



In Problem 3.21, determine the perpendicular distance from point O to cable AB.

Problem 3.21: Before the trunk of a large tree is felled, cables AB and BC are attached as shown. Knowing that the tension in cables AB and BC are 777 N and 990 N, respectively, determine the moment about O of the resultant force exerted on the tree by the cables at B.

SOLUTION



Have $$|\mathbf{M}_O| = T_{BA}d$$

where $$d = \text{perpendicular distance from O to line } AB.$$

Now $$\mathbf{M}_O = \mathbf{r}_{B/O} \times \mathbf{T}_{BA}$$

and $$\mathbf{r}_{B/O} = (8.4 \text{ m})\,\mathbf{j}$$

$$\mathbf{T}_{BA} = \lambda_{BA}T_{AB} = \frac{-(0.9 \text{ m})\mathbf{i} - (8.4 \text{ m})\mathbf{j} + (7.2 \text{ m})\mathbf{k}}{\sqrt{(0.9)^2 + (8.4)^2 + (7.2)^2} \text{ m}}(777 \text{ N})$$

$$= -(63.0 \text{ N})\mathbf{i} - (588 \text{ N})\mathbf{j} + (504 \text{ N})\mathbf{k}$$

$$\therefore\ \mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 8.4 & 0 \\ -63.0 & -588 & 504 \end{vmatrix} \text{N}\cdot\text{m} = (4233.6 \text{ N}\cdot\text{m})\mathbf{i} + (529.2 \text{ N}\cdot\text{m})\mathbf{k}$$

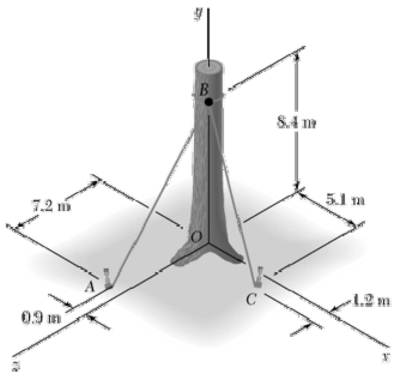
and $$|\mathbf{M}_O| = \sqrt{(4233.6)^2 + (529.2)^2} = 4266.5 \text{ N}\cdot\text{m}$$

$$\therefore\ 4266.5 \text{ N}\cdot\text{m} = (777 \text{ N})d$$

or $$d = 5.4911 \text{ m}$$

or $d = 5.49$ m ◀

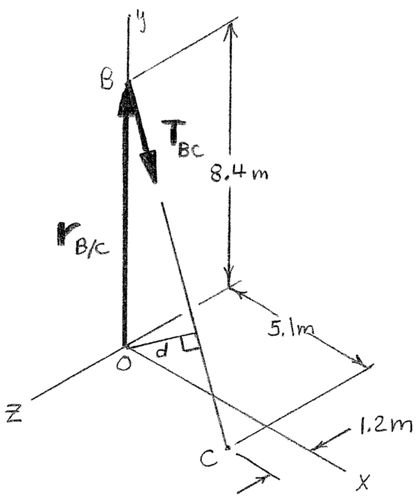
PROBLEM 3.28



In Problem 3.21, determine the perpendicular distance from point O to cable BC.

Problem 3.21: Before the trunk of a large tree is felled, cables AB and BC are attached as shown. Knowing that the tension in cables AB and BC are 777 N and 990 N, respectively, determine the moment about O of the resultant force exerted on the tree by the cables at B.

SOLUTION



Have $$|\mathbf{M}_O| = T_{BC}d$$

where d = perpendicular distance from O to line BC.

$$\mathbf{M}_O = \mathbf{r}_{B/O} \times \mathbf{T}_{BC}$$

$$\mathbf{r}_{B/O} = 8.4\text{ m}\,\mathbf{j}$$

$$\mathbf{T}_{BC} = \lambda_{BC}T_{BC} = \frac{(5.1\text{ m})\mathbf{i} - (8.4\text{ m})\mathbf{j} + (1.2\text{ m})\mathbf{k}}{\sqrt{(5.1)^2 + (8.4)^2 + (1.2)^2}\text{ m}}(990\text{ N})$$

$$= (510\text{ N})\mathbf{i} - (840\text{ N})\mathbf{j} + (120\text{ N})\mathbf{k}$$

$$\therefore\quad \mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 8.4 & 0 \\ 510 & -840 & 120 \end{vmatrix} = (1008\text{ N}\cdot\text{m})\mathbf{i} - (4284\text{ N}\cdot\text{m})\mathbf{k}$$

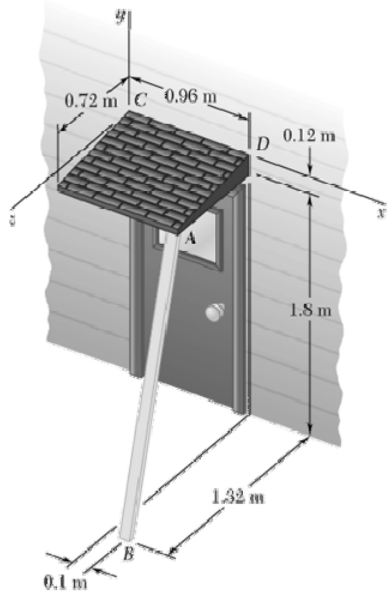
and $$|\mathbf{M}_O| = \sqrt{(1008)^2 + (4284)^2} = 4401.0\text{ N}\cdot\text{m}$$

$$\therefore\ 4401.0\text{ N}\cdot\text{m} = (990\text{ N})d$$

$$d = 4.4454\text{ m}$$

or $d = 4.45$ m ◀

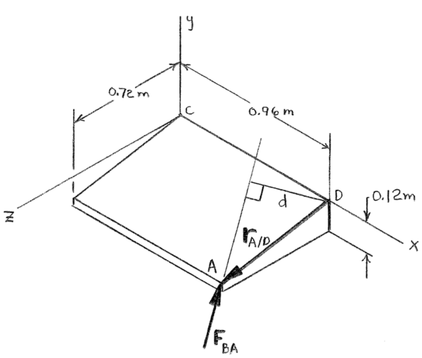
PROBLEM 3.29



In Problem 3.24, determine the perpendicular distance from point D to a line drawn through points A and B.

Problem 3.24: A wooden board AB, which is used as a temporary prop to support a small roof, exerts at point A of the roof a 228 N force directed along BA. Determine the moment about C of that force.

SOLUTION



Have $$|\mathbf{M}_D| = F_{BA}d$$

where d = perpendicular distance from D to line AB.

$$\mathbf{M}_D = \mathbf{r}_{A/D} \times \mathbf{F}_{BA}$$

$$\mathbf{r}_{A/D} = -(0.12\text{ m})\mathbf{j} + (0.72\text{ m})\mathbf{k}$$

$$\mathbf{F}_{BA} = \lambda_{BA}F_{BA} = \frac{(-(0.1\text{ m})\mathbf{i} + (1.8\text{ m})\mathbf{j} - (0.6\text{ m})\mathbf{k})}{\sqrt{(0.1)^2 + (1.8)^2 + (0.6)^2}\text{ m}}(228\text{ N})$$

$$= -(12.0\text{ N})\mathbf{i} + (216\text{ N})\mathbf{j} - (72\text{ N})\mathbf{k}$$

$$\therefore\ \mathbf{M}_D = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -0.12 & 0.72 \\ -12.0 & 216 & -72 \end{vmatrix}\text{ N}\cdot\text{m}$$

$$= -(146.88\text{ N}\cdot\text{m})\mathbf{i} - (8.64\text{ N}\cdot\text{m})\mathbf{j} - (1.44\text{ N}\cdot\text{m})\mathbf{k}$$

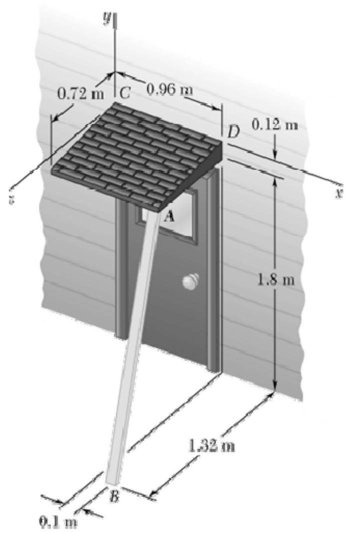
and $$|\mathbf{M}_D| = \sqrt{(146.88)^2 + (8.64)^2 + (1.44)^2} = 147.141\text{ N}\cdot\text{m}$$

$$\therefore\ 147.141\text{ N}\cdot\text{m} = (228\text{ N})d$$

$$d = 0.64536\text{ m}$$

or $d = 0.645\text{ m}$ ◀

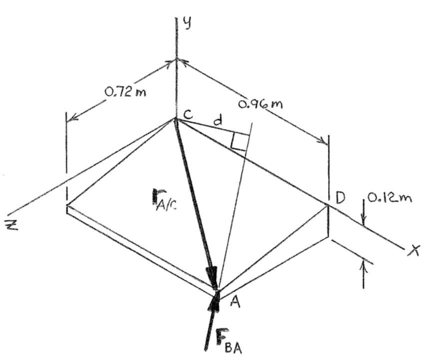
PROBLEM 3.30



In Problem 3.24, determine the perpendicular distance from point C to a line drawn through points A and B.

Problem 3.24: A wooden board AB, which is used as a temporary prop to support a small roof, exerts at point A of the roof a 228 N force directed along BA. Determine the moment about C of that force.

SOLUTION



Have $$|\mathbf{M}_C| = F_{BA}d$$

where d = perpendicular distance from C to line AB.

$$\mathbf{M}_C = \mathbf{r}_{A/C} \times \mathbf{F}_{BA}$$

$$\mathbf{r}_{A/C} = (0.96 \text{ m})\mathbf{i} - (0.12 \text{ m})\mathbf{j} + (0.72 \text{ m})\mathbf{k}$$

$$\mathbf{F}_{BA} = \lambda_{BA}F_{BA} = \frac{(-(0.1\text{ m})\mathbf{i} + (1.8\text{ m})\mathbf{j} - (0.6)\mathbf{k})}{\sqrt{(0.1)^2 + (1.8)^2 + (0.6)^2}\text{ m}}(228 \text{ N})$$

$$= -(12.0 \text{ N})\mathbf{i} + (216 \text{ N})\mathbf{j} - (72 \text{ N})\mathbf{k}$$

$$\therefore \quad \mathbf{M}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.96 & -0.12 & 0.72 \\ -12.0 & 216 & -72 \end{vmatrix} \text{N}\cdot\text{m}$$

$$= -(146.88 \text{ N}\cdot\text{m})\mathbf{i} - (60.48 \text{ N}\cdot\text{m})\mathbf{j} + (205.92 \text{ N}\cdot\text{m})\mathbf{k}$$

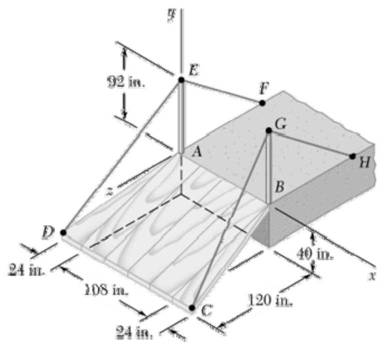
and $$|\mathbf{M}_C| = \sqrt{(146.88)^2 + (60.48)^2 + (205.92)^2} = 260.07 \text{ N}\cdot\text{m}$$

$$\therefore \; 260.07 \text{ N}\cdot\text{m} = (228 \text{ N})d$$

$$d = 1.14064 \text{ m}$$

or $d = 1.141$ m ◀

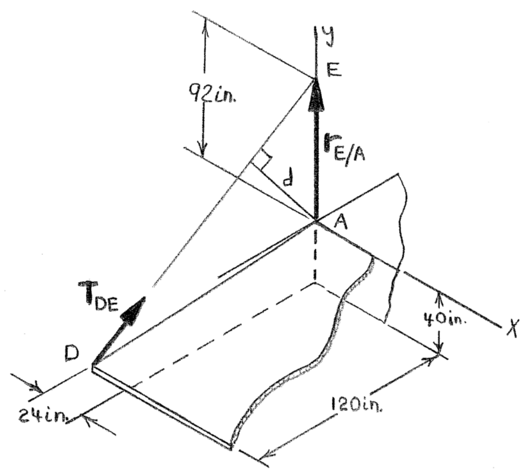
PROBLEM 3.31



In Problem 3.25, determine the perpendicular distance from point *A* to portion *DE* of cable *DEF*.

Problem 3.25: The ramp *ABCD* is supported by cables at corners *C* and *D*. The tension in each of the cables is 360 lb. Determine the moment about *A* of the force exerted by (*a*) the cable at *D*, (*b*) the cable at *C*.

SOLUTION



Have $$|\mathbf{M}_A| = T_{DE}d$$

where d = perpendicular distance from *A* to line *DE*.

$$\mathbf{M}_A = \mathbf{r}_{E/A} \times \mathbf{T}_{DE}$$

$$\mathbf{r}_{E/A} = (92\text{ in.})\,\mathbf{j}$$

$$\mathbf{T}_{DE} = \lambda_{DE}T_{DE} = \frac{(24\text{ in.})\mathbf{i} + (132\text{ in.})\mathbf{j} - (120\text{ in.})\mathbf{k}}{\sqrt{(24)^2 + (132)^2 + (120)^2}\text{ in.}}(360\text{ lb})$$

$$= (48\text{ lb})\mathbf{i} + (264\text{ lb})\mathbf{j} - (240\text{ lb})\mathbf{k}$$

$$\therefore\ \mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 92 & 0 \\ 48 & 264 & -240 \end{vmatrix} \text{N}\cdot\text{m}$$

$$= -(22{,}080\text{ lb}\cdot\text{in.})\mathbf{i} - (4416\text{ lb}\cdot\text{in.})\mathbf{k}$$

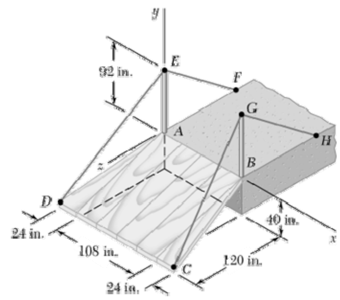
PROBLEM 3.31 CONTINUED

and $$\left|\mathbf{M}_A\right| = \sqrt{(22{,}080)^2 + (4416)^2} = 22{,}517 \text{ lb}\cdot\text{in.}$$

$$\therefore\ 22{,}517 \text{ lb}\cdot\text{in.} = (360 \text{ lb})d$$

$$d = 62.548 \text{ in.}$$

or $d = 5.21 \text{ ft}$ ◀

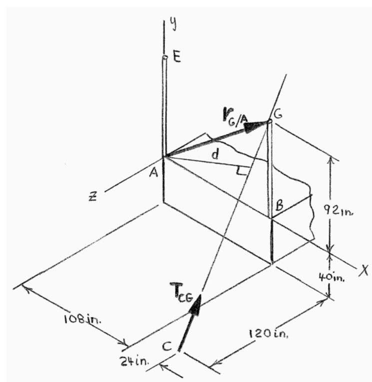


PROBLEM 3.32

In Problem 3.25, determine the perpendicular distance from point *A* to a line drawn through points *C* and *G*.

Problem 3.25: The ramp *ABCD* is supported by cables at corners *C* and *D*. The tension in each of the cables is 360 lb. Determine the moment about *A* of the force exerted by (*a*) the cable at *D*, (*b*) the cable at *C*.

SOLUTION



Have $$|\mathbf{M}_A| = T_{CG}d$$

where d = perpendicular distance from *A* to line *CG*.

$$\mathbf{M}_A = \mathbf{r}_{G/A} \times \mathbf{T}_{CG}$$

$$\mathbf{r}_{G/A} = (108 \text{ in.})\mathbf{i} + (92 \text{ in.})\mathbf{j}$$

$$\mathbf{T}_{CG} = \lambda_{CG}T_{CG}$$

$$= \frac{-(24 \text{ in.})\mathbf{i} + (132 \text{ in.})\mathbf{j} - (120 \text{ in.})\mathbf{k}}{\sqrt{(24)^2 + (132)^2 + (120)^2} \text{ in.}}(360 \text{ lb})$$

$$= -(48 \text{ lb})\mathbf{i} + (264 \text{ lb})\mathbf{j} - (240 \text{ lb})\mathbf{k}$$

$$\therefore \mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 108 & 92 & 0 \\ -48 & 264 & -240 \end{vmatrix} \text{lb}\cdot\text{in.}$$

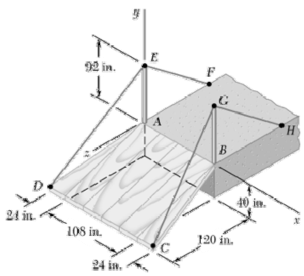
$$= -(22,080 \text{ lb}\cdot\text{in.})\mathbf{i} + (25,920 \text{ lb}\cdot\text{in.})\mathbf{j} + (32,928 \text{ lb}\cdot\text{in.})\mathbf{k}$$

and $$|\mathbf{M}_A| = \sqrt{(22,080)^2 + (25,920)^2 + (32,928)^2} = 47,367 \text{ lb}\cdot\text{in.}$$

$$\therefore 47,367 \text{ lb}\cdot\text{in.} = (360 \text{ lb})d$$

$$d = 131.575 \text{ in.}$$

or $d = 10.96$ ft ◀

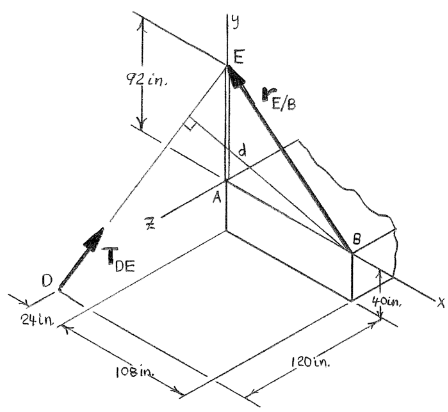


PROBLEM 3.33

In Problem 3.25, determine the perpendicular distance from point B to a line drawn through points D and E.

Problem 3.25: The ramp $ABCD$ is supported by cables at corners C and D. The tension in each of the cables is 360 lb. Determine the moment about A of the force exerted by (*a*) the cable at D, (*b*) the cable at C.

SOLUTION



Have $$|\mathbf{M}_B| = T_{DE}d$$

where $$d = \text{perpendicular distance from } B \text{ to line } DE.$$

$$\mathbf{M}_B = \mathbf{r}_{E/B} \times \mathbf{T}_{DE}$$

$$\mathbf{r}_{E/B} = -(108\text{ in.})\mathbf{i} + (92\text{ in.})\mathbf{j}$$

$$\mathbf{T}_{DE} = \lambda_{DE}T_{DE} = \frac{(24\text{ in.})\mathbf{i} + (132\text{ in.})\mathbf{j} - (120\text{ in.})\mathbf{k}}{\sqrt{(24)^2 + (132)^2 + (120)^2}\text{ in.}}(360\text{ lb})$$

$$= (48\text{ lb})\mathbf{i} + (264\text{ lb})\mathbf{j} - (240\text{ lb})\mathbf{k}$$

$$\therefore\ \mathbf{M}_B = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -108 & 92 & 0 \\ 48 & 264 & -240 \end{vmatrix} \text{lb}\cdot\text{in.}$$

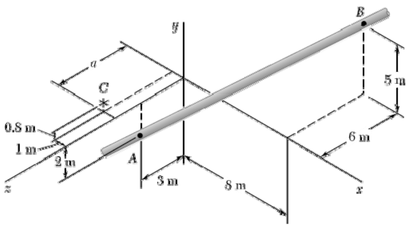
$$= -(22{,}080\text{ lb}\cdot\text{in.})\mathbf{i} - (25{,}920\text{ lb}\cdot\text{in.})\mathbf{j} - (32{,}928\text{ lb}\cdot\text{in.})\mathbf{k}$$

and $$|\mathbf{M}_B| = \sqrt{(22{,}080)^2 + (25{,}920)^2 + (32{,}928)^2} = 47{,}367\text{ lb}\cdot\text{in.}$$

$$\therefore\ 47{,}367\text{ lb}\cdot\text{in.} = (360\text{ lb})d$$

$$d = 131.575\text{ in.}$$

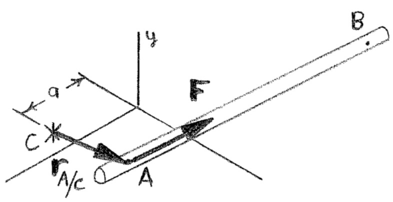
or $d = 10.96$ ft ◀



PROBLEM 3.34

Determine the value of a which minimizes the perpendicular distance from point C to a section of pipeline that passes through points A and B.

SOLUTION



Assuming a force **F** acts along AB,

$$|\mathbf{M}_C| = |\mathbf{r}_{A/C} \times \mathbf{F}| = F(d)$$

where

$$d = \text{perpendicular distance from } C \text{ to line } AB$$

$$\mathbf{F} = \lambda_{AB} F = \frac{(8\text{ m})\mathbf{i} + (7\text{ m})\mathbf{j} - (9\text{ m})\mathbf{k}}{\sqrt{(8)^2 + (7)^2 + (9)^2}\text{ m}} F$$

$$= F(0.57437)\mathbf{i} + (0.50257)\mathbf{j} - (0.64616)\mathbf{k}$$

$$\mathbf{r}_{A/C} = (1\text{ m})\mathbf{i} - (2.8\text{ m})\mathbf{j} - (a - 3\text{ m})\mathbf{k}$$

$$\therefore\ \mathbf{M}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2.8 & 3 - a \\ 0.57437 & 0.50257 & -0.64616 \end{vmatrix} F$$

$$= \big[(0.30154 + 0.50257a)\mathbf{i} + (2.3693 - 0.57437a)\mathbf{j}$$

$$+ 2.1108\mathbf{k}\big]F$$

Since $\quad |\mathbf{M}_C| = \sqrt{|\mathbf{r}_{A/C} \times \mathbf{F}^2|} \quad$ or $\quad |\mathbf{r}_{A/C} \times \mathbf{F}^2| = (dF)^2$

$$\therefore\ (0.30154 + 0.50257a)^2 + (2.3693 - 0.57437a)^2 + (2.1108)^2 = d^2$$

Setting $\dfrac{d}{da}(d^2) = 0$ to find a to minimize d

$$2(0.50257)(0.30154 + 0.50257a)$$

$$+ 2(-0.57437)(2.3693 - 0.57437a) = 0$$

Solving $\qquad a = 2.0761\text{ m}$

or $a = 2.08\text{ m}$ ◀

PROBLEM 3.35

Given the vectors $\mathbf{P} = 7\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$, $\mathbf{Q} = -3\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}$, and $\mathbf{S} = 8\mathbf{i} + \mathbf{j} - 9\mathbf{k}$, compute the scalar products $\mathbf{P}\cdot\mathbf{Q}$, $\mathbf{P}\cdot\mathbf{S}$, and $\mathbf{Q}\cdot\mathbf{S}$.

SOLUTION

$$\mathbf{P}\cdot\mathbf{Q} = (7\mathbf{i} - 2\mathbf{j} + 5\mathbf{k})\cdot(-3\mathbf{i} - 4\mathbf{j} + 6\mathbf{k})$$

$$= (7)(-3) + (-2)(-4) + (5)(6)$$

$$= 17$$

or $\mathbf{P}\cdot\mathbf{Q} = 17$ ◀

$$\mathbf{P}\cdot\mathbf{S} = (7\mathbf{i} - 2\mathbf{j} + 5\mathbf{k})\cdot(8\mathbf{i} + \mathbf{j} - 9\mathbf{k})$$

$$= (7)(8) + (-2)(1) + (5)(-9)$$

$$= 9$$

or $\mathbf{P}\cdot\mathbf{S} = 9$ ◀

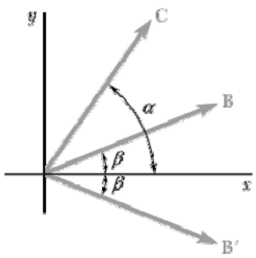
$$\mathbf{Q}\cdot\mathbf{S} = (-3\mathbf{i} - 4\mathbf{j} + 6\mathbf{k})\cdot(8\mathbf{i} + \mathbf{j} - 9\mathbf{k})$$

$$= (-3)(8) + (-4)(1) + (6)(-9)$$

$$= -82$$

or $\mathbf{Q}\cdot\mathbf{S} = -82$ ◀

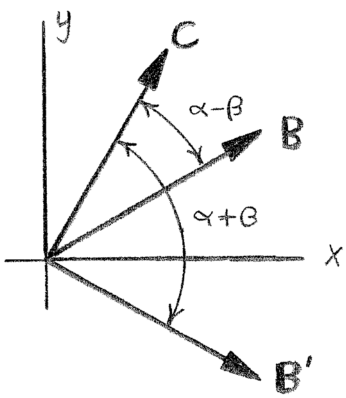
PROBLEM 3.36



Form the scalar products $\mathbf{B}\cdot\mathbf{C}$ and $\mathbf{B}'\cdot\mathbf{C}$, where $B = B'$, and use the results obtained to prove the identity

$$\cos\alpha\ \cos\beta = \tfrac{1}{2}\cos(\alpha+\beta) + \tfrac{1}{2}\cos(\alpha-\beta).$$

SOLUTION



By definition

$$\mathbf{B}\cdot\mathbf{C} = BC\cos(\alpha-\beta)$$

where

$$\mathbf{B} = B[(\cos\beta)\mathbf{i} + (\sin\beta)\mathbf{j}]$$

$$\mathbf{C} = C[(\cos\alpha)\mathbf{i} + (\sin\alpha)\mathbf{j}]$$

$$\therefore\ (B\cos\beta)(C\cos\alpha) + (B\sin\beta)(C\sin\alpha) = BC\cos(\alpha-\beta)$$

or

$$\cos\beta\cos\alpha + \sin\beta\sin\alpha = \cos(\alpha-\beta) \tag{1}$$

By definition

$$\mathbf{B}'\cdot\mathbf{C} = BC\cos(\alpha+\beta)$$

where

$$\mathbf{B}' = [(\cos\beta)\mathbf{i} - (\sin\beta)\mathbf{j}]$$

$$\therefore\ (B\cos\beta)(C\cos\alpha) + (-B\sin\beta)(C\sin\alpha) = BC\cos(\alpha+\beta)$$

or

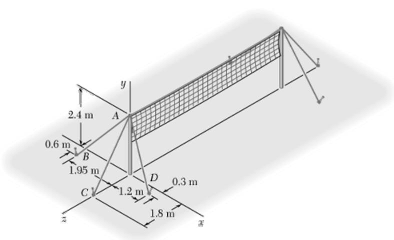
$$\cos\beta\cos\alpha - \sin\beta\sin\alpha = \cos(\alpha+\beta) \tag{2}$$

Adding Equations (1) and (2),

$$2\cos\beta\cos\alpha = \cos(\alpha-\beta) + \cos(\alpha+\beta)$$

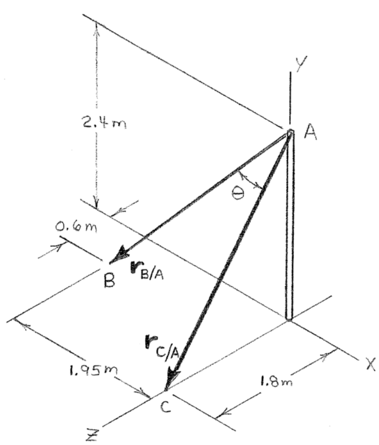
$$\text{or}\quad \cos\alpha\cos\beta = \frac{1}{2}\cos(\alpha+\beta) + \frac{1}{2}\cos(\alpha-\beta) \blacktriangleleft$$

PROBLEM 3.37



Consider the volleyball net shown. Determine the angle formed by guy wires *AB* and *AC*.

SOLUTION



First note

$$AB = |\mathbf{r}_{B/A}| = \sqrt{(-1.95 \text{ m})^2 + (-2.4 \text{ m})^2 + (0.6 \text{ m})^2}$$

$$= 3.15 \text{ m}$$

$$AC = |\mathbf{r}_{C/A}| = \sqrt{(0 \text{ m})^2 + (-2.4 \text{ m})^2 + (1.8 \text{ m})^2}$$

$$= 3.0 \text{ m}$$

and

$$\mathbf{r}_{B/A} = -(1.95 \text{ m})\mathbf{i} - (2.40 \text{ m})\mathbf{j} + (0.6 \text{ m})\mathbf{k}$$

$$\mathbf{r}_{C/A} = -(2.40 \text{ m})\mathbf{j} + (1.80 \text{ m})\mathbf{k}$$

By definition

$$\mathbf{r}_{B/A} \cdot \mathbf{r}_{C/A} = |\mathbf{r}_{B/A}||\mathbf{r}_{C/A}|\cos\theta$$

or $$(-1.95\mathbf{i} - 2.40\mathbf{j} + 0.6\mathbf{k}) \cdot (-2.40\mathbf{j} + 1.80\mathbf{k}) = (3.15)(3.0)\cos\theta$$

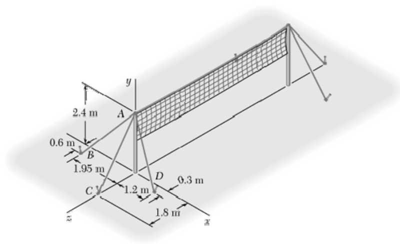
$$(-1.95)(0) + (-2.40)(-2.40) + (0.6)(1.8) = 9.45\cos\theta$$

$$\therefore \ \cos\theta = 0.72381$$

and $$\theta = 43.630°$$

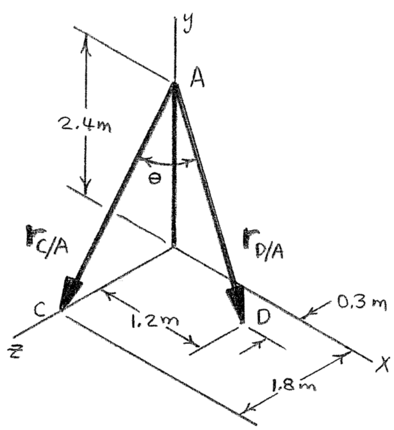
or $\theta = 43.6°$ ◀

PROBLEM 3.38



Consider the volleyball net shown. Determine the angle formed by guy wires *AC* and *AD*.

SOLUTION



First note

$$AC = |\mathbf{r}_{C/A}| = \sqrt{(-2.4)^2 + (1.8)^2} \text{ m} = 3 \text{ m}$$

$$AD = |\mathbf{r}_{D/A}| = \sqrt{(1.2)^2 + (-2.4)^2 + (0.3)^2} \text{ m} = 2.7 \text{ m}$$

and $$\mathbf{r}_{C/A} = -(2.4 \text{ m})\mathbf{j} + (1.8 \text{ m})\mathbf{k}$$

$$\mathbf{r}_{D/A} = (1.2 \text{ m})\mathbf{i} - (2.4 \text{ m})\mathbf{j} + (0.3 \text{ m})\mathbf{k}$$

By definition

$$\mathbf{r}_{C/A} \cdot \mathbf{r}_{D/A} = |\mathbf{r}_{C/A}||\mathbf{r}_{D/A}|\cos\theta$$

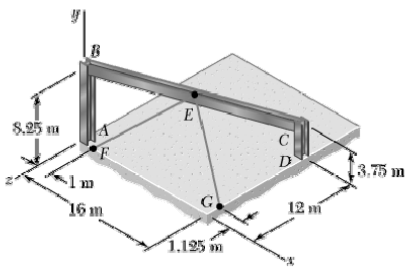
or $$(-2.4\mathbf{j} + 1.8\mathbf{k}) \cdot (1.2\mathbf{i} - 2.4\mathbf{j} + 0.3\mathbf{k}) = (3)(2.7)\cos\theta$$

$$(0)(1.2) + (-2.4)(-2.4) + (1.8)(0.3) = 8.1\cos\theta$$

and $$\cos\theta = \frac{6.3}{8.1} = 0.77778$$

$$\theta = 38.942°$$

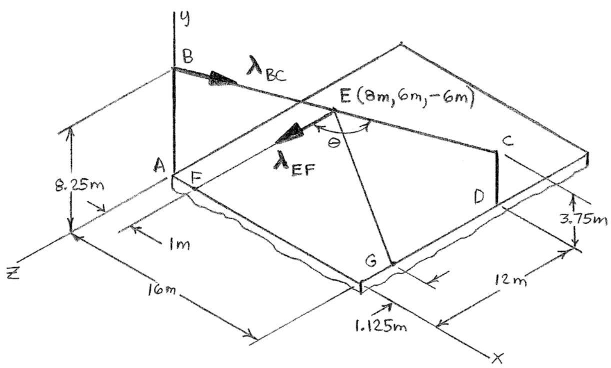
or $\theta = 38.9°$ ◀



PROBLEM 3.39

Steel framing members *AB*, *BC*, and *CD* are joined at *B* and *C* and are braced using cables *EF* and *EG*. Knowing that *E* is at the midpoint of *BC* and that the tension in cable *EF* is 330 N, determine (*a*) the angle between *EF* and member *BC*, (*b*) the projection on *BC* of the force exerted by cable *EF* at point *E*.

SOLUTION



(*a*) By definition

$$\lambda_{BC} \cdot \lambda_{EF} = (1)(1)\cos\theta$$

where

$$\lambda_{BC} = \frac{(16\text{ m})\mathbf{i} - (4.5\text{ m})\mathbf{j} - (12\text{ m})\mathbf{k}}{\sqrt{(16)^2 + (4.5)^2 + (12)^2}\text{ m}} = \frac{1}{20.5}(16\mathbf{i} - 4.5\mathbf{j} - 12\mathbf{k})$$

$$\lambda_{EF} = \frac{-(7\text{ m})\mathbf{i} - (6\text{ m})\mathbf{j} + (6\text{ m})\mathbf{k}}{\sqrt{(7)^2 + (6)^2 + (6)^2}\text{m}} = \frac{1}{11.0}(-7\mathbf{i} - 6\mathbf{j} + 6\mathbf{k})$$

$$\therefore \frac{(16\mathbf{i} - 4.5\mathbf{j} - 12\mathbf{k})}{20.5} \cdot \frac{(-7\mathbf{i} - 6\mathbf{j} + 6\mathbf{k})}{11.0} = \cos\theta$$

$$(16)(-7) + (-4.5)(-6) + (-12)(6) = (20.5)(11.0)\cos\theta$$

and

$$\theta = \cos^{-1}\left(\frac{-157}{225.5}\right) = 134.125°$$

or $\theta = 134.1°$ ◀

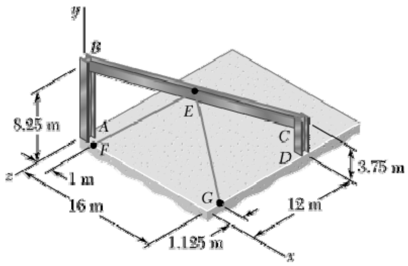
(*b*) By definition

$$(T_{EF})_{BC} = T_{EF}\cos\theta$$

$$= (330\text{ N})\cos 134.125°$$

$$= -229.26\text{ N}$$

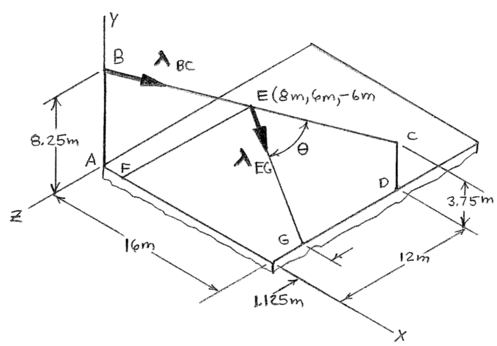
or $(T_{EF})_{BC} = -230\text{ N}$ ◀



PROBLEM 3.40

Steel framing members *AB*, *BC*, and *CD* are joined at *B* and *C* and are braced using cables *EF* and *EG*. Knowing that *E* is at the midpoint of *BC* and that the tension in cable *EG* is 445 N, determine (*a*) the angle between *EG* and member *BC*, (*b*) the projection on *BC* of the force exerted by cable *EG* at point *E*.

SOLUTION



(*a*) By definition

$$\boldsymbol{\lambda}_{BC} \cdot \boldsymbol{\lambda}_{EG} = (1)(1)\cos\theta$$

where

$$\boldsymbol{\lambda}_{BC} = \frac{(16\text{ m})\mathbf{i} - (4.5\text{ m})\mathbf{j} - (12\text{ m})\mathbf{k}}{\sqrt{(16\text{ m})^2 + (4.5)^2 + (12)^2}\text{ m}} = \frac{16\mathbf{i} - 4.5\mathbf{j} - 12\mathbf{k}}{20.5}$$

$$= 0.78049\mathbf{i} - 0.21951\mathbf{j} - 0.58537\mathbf{k}$$

$$\boldsymbol{\lambda}_{EG} = \frac{(8\text{ m})\mathbf{i} - (6\text{ m})\mathbf{j} + (4.875\text{ m})\mathbf{k}}{\sqrt{(8)^2 + (6)^2 + (4.875)^2}\text{m}} = \frac{8\mathbf{i} - 6\mathbf{j} + 4.875\mathbf{k}}{11.125}$$

$$= 0.71910\mathbf{i} - 0.53933\mathbf{j} + 0.43820\mathbf{k}$$

$$\therefore\ \boldsymbol{\lambda}_{BC} \cdot \boldsymbol{\lambda}_{EG} = \frac{16(8) + (-4.5)(-6) + (-12)(4.875)}{(20.5)(11.25)} = \cos\theta$$

and

$$\theta = \cos^{-1}\left(\frac{96.5}{228.06}\right) = 64.967°$$

or $\theta = 65.0°$ ◀

(*b*) By definition

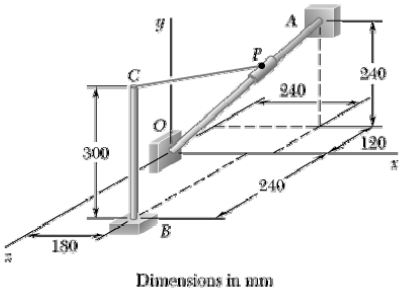
$$(T_{EG})_{BC} = T_{EG}\cos\theta$$

$$= (445\text{ N})\cos 64.967°$$

$$= 188.295\text{ N}$$

or $(T_{EG})_{BC} = 188.3\text{ N}$ ◀

PROBLEM 3.41



Slider P can move along rod OA. An elastic cord PC is attached to the slider and to the vertical member BC. Knowing that the distance from O to P is 0.12 m and the tension in the cord is 30 N, determine (*a*) the angle between the elastic cord and the rod OA, (*b*) the projection on OA of the force exerted by cord PC at point P.

SOLUTION

(*a*) By definition

$$\boldsymbol{\lambda}_{OA} \cdot \boldsymbol{\lambda}_{PC} = (1)(1)\cos\theta$$

where

$$\boldsymbol{\lambda}_{OA} = \frac{(0.24\text{ m})\mathbf{i} + (0.24\text{ m})\mathbf{j} - (0.12\text{ m})\mathbf{k}}{\sqrt{(0.24)^2 + (0.24)^2 + (0.12)^2}\text{ m}}$$

$$= \frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{1}{3}\mathbf{k}$$

Knowing that $|\mathbf{r}_{A/O}| = L_{OA} = 0.36\text{ m}$ and that P is located 0.12 m from O, it follows that the coordinates of P are $\frac{1}{3}$ the coordinates of A.

$$\therefore\; P(0.08\text{ m}, 0.08\text{ m}, -0.040\text{ m})$$

Then

$$\boldsymbol{\lambda}_{PC} = \frac{(0.10\text{ m})\mathbf{i} + (0.22\text{ m})\mathbf{j} + (0.28\text{ m})\mathbf{k}}{\sqrt{(0.10)^2 + (0.22)^2 + (0.28)^2}\text{ m}}$$

$$= 0.27037\mathbf{i} + 0.59481\mathbf{j} + 0.75703\mathbf{k}$$

$$\therefore\; \left(\frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{1}{3}\mathbf{k}\right) \cdot (0.27037\mathbf{i} + 0.59481\mathbf{j} + 0.75703\mathbf{k}) = \cos\theta$$

and

$$\theta = \cos^{-1}(0.32445) = 71.068°$$

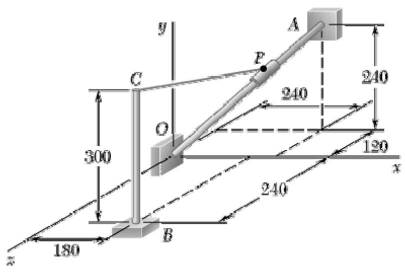
or $\theta = 71.1° \blacktriangleleft$

(*b*)

$$(T_{PC})_{OA} = T_{PC}\cos\theta = (30\text{ N})\cos 71.068°$$

$$(T_{PC})_{OA} = 9.7334\text{ N}$$

or $(T_{PC})_{OA} = 9.73\text{ N} \blacktriangleleft$



PROBLEM 3.42

Slider *P* can move along rod *OA*. An elastic cord *PC* is attached to the slider and to the vertical member *BC*. Determine the distance from *O* to *P* for which cord *PC* and rod *OA* are perpendicular.

SOLUTION

The requirement that member *OA* and the elastic cord *PC* be perpendicular implies that

$$\lambda_{OA} \cdot \lambda_{PC} = 0 \qquad \text{or} \qquad \lambda_{OA} \cdot \mathbf{r}_{C/P} = 0$$

where

$$\lambda_{OA} = \frac{(0.24\text{ m})\mathbf{i} + (0.24\text{ m})\mathbf{j} - (0.12\text{ m})\mathbf{k}}{\sqrt{(0.24)^2 + (0.24)^2 + (0.12)^2}\text{ m}}$$

$$= \frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{1}{3}\mathbf{k}$$

Letting the coordinates of *P* be $P(x, y, z)$, we have

$$\mathbf{r}_{C/P} = \left[(0.18 - x)\mathbf{i} + (0.30 - y)\mathbf{j} + (0.24 - z)\mathbf{k}\right]\text{m}$$

$$\therefore \left(\frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{1}{3}\mathbf{k}\right) \cdot \left[(0.18 - x)\mathbf{i} + (0.30 - y)\mathbf{j} + (0.24 - z)\mathbf{k}\right] = 0 \qquad (1)$$

Since

$$\mathbf{r}_{P/O} = \lambda_{OA} d_{OP} = \frac{d_{OP}}{3}(2\mathbf{i} + 2\mathbf{j} - \mathbf{k}),$$

Then

$$x = \frac{2}{3}d_{OP}, \quad y = \frac{2}{3}d_{OP}, \quad z = \frac{-1}{3}d_{OP} \qquad (2)$$

Substituting the expressions for *x*, *y*, and *z* from Equation (2) into Equation (1),

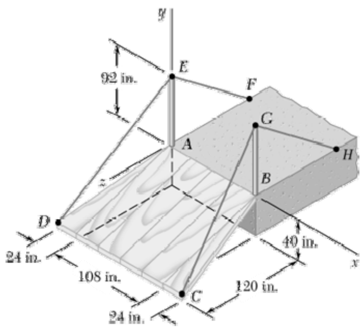
$$\frac{1}{3}(2\mathbf{i} + 2\mathbf{j} - \mathbf{k}) \cdot \left[\left(0.18 - \frac{2}{3}d_{OP}\right)\mathbf{i} + \left(0.30 - \frac{2}{3}d_{OP}\right)\mathbf{j} + \left(0.24 + \frac{1}{3}d_{OP}\right)\mathbf{k}\right] = 0$$

or

$$3d_{OP} = 0.36 + 0.60 - 0.24 = 0.72$$

$$\therefore d_{OP} = 0.24\text{ m}$$

or $d_{OP} = 240\text{ mm}$ ◀



PROBLEM 3.43

Determine the volume of the parallelepiped of Figure 3.25 when (*a*) **P** = −(7 in.)**i** − (1 in.)**j** + (2 in.)**k**, **Q** = (3 in.)**i** − (2 in.)**j** + (4 in.)**k**, and **S** = −(5 in.)**i** + (6 in.)**j** − (1 in.)**k**, (*b*) **P** = (1 in.)**i** + (2 in.)**j** − (1 in.)**k**, **Q** = −(8 in.)**i** − (1 in.)**j** + (9 in.)**k**, and **S** = (2 in.)**i** + (3 in.)**j** + (1 in.)**k**.

SOLUTION

Volume of a parallelepiped is found using the mixed triple product.

(*a*)

$$\text{Vol} = \mathbf{P}\cdot(\mathbf{Q}\times\mathbf{S})$$

$$= \begin{vmatrix} -7 & -1 & 2 \\ 3 & -2 & 4 \\ -5 & 6 & -1 \end{vmatrix} \text{in}^3 = (-14 + 168 + 20 - 3 + 36 - 20)\,\text{in}^3$$

$$= 187\ \text{in}^3$$

$$\text{or}\quad \text{Volume} = 187\ \text{in}^3 \blacktriangleleft$$

(*b*)

$$\text{Vol} = \mathbf{P}\cdot(\mathbf{Q}\times\mathbf{S})$$

$$= \begin{vmatrix} 1 & 2 & -1 \\ -8 & -1 & 9 \\ 2 & 3 & 1 \end{vmatrix} \text{in}^3 = (-1 - 27 + 36 + 16 + 24 - 2)\,\text{in}^3$$

$$= 46\ \text{in}^3$$

$$\text{or}\quad \text{Volume} = 46\ \text{in}^3 \blacktriangleleft$$

PROBLEM 3.44

Given the vectors $\mathbf{P} = 4\mathbf{i} - 2\mathbf{j} + P_z\mathbf{k}$, $\mathbf{Q} = \mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$, and $\mathbf{S} = -6\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, determine the value of P_z for which the three vectors are coplanar.

SOLUTION

For the vectors to all be in the same plane, the mixed triple product is zero.

$$\mathbf{P} \cdot (\mathbf{Q} \times \mathbf{S}) = 0$$

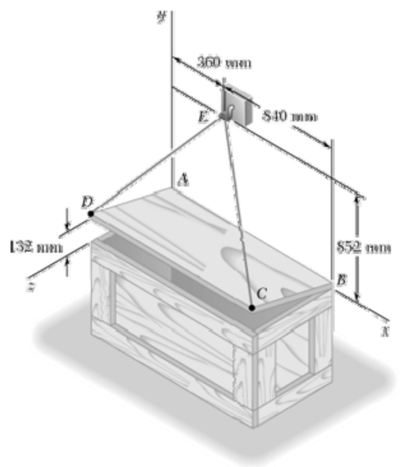
$$\therefore\ O = \begin{vmatrix} 4 & -2 & P_z \\ 1 & 3 & -5 \\ -6 & 2 & -1 \end{vmatrix} = -12 + 40 - 60 - 2 + P_z(2 + 18)$$

so that

$$P_z = \frac{34}{20} = 1.70$$

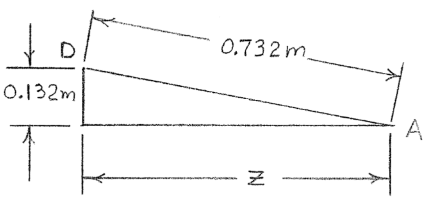
or $P_z = 1.700$ ◀

PROBLEM 3.45



The 0.732×1.2-m lid *ABCD* of a storage bin is hinged along side *AB* and is held open by looping cord *DEC* over a frictionless hook at *E*. If the tension in the cord is 54 N, determine the moment about each of the coordinate axes of the force exerted by the cord at *D*.

SOLUTION



First note $$z = \sqrt{(0.732)^2 - (0.132)^2}\ \text{m}$$

$$= 0.720\ \text{m}$$

Then $$d_{DE} = \sqrt{(0.360)^2 + (0.720)^2 + (0.720)^2}\ \text{m}$$

$$= 1.08\ \text{m}$$

and $$\mathbf{r}_{E/D} = (0.360\ \text{m})\mathbf{i} + (0.720\ \text{m})\mathbf{j} - (0.720\ \text{m})\mathbf{k}$$

Have $$\mathbf{T}_{DE} = \frac{T_{OE}}{d_{DE}}\left(\mathbf{r}_{E/D}\right)$$

$$= \frac{54\ \text{N}}{1.08}(0.360\mathbf{i} + 0.720\mathbf{j} - 0.720\mathbf{k})$$

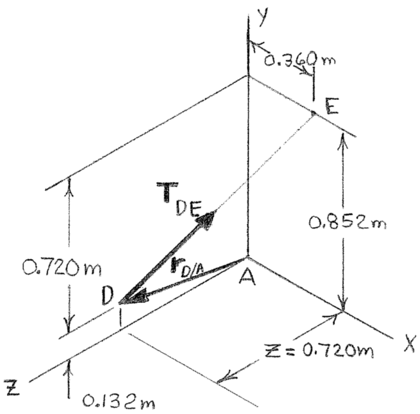
$$= (18.0\ \text{N})\mathbf{i} + (36.0\ \text{N})\mathbf{j} - (36.0\ \text{N})\mathbf{k}$$

Now $$\mathbf{M}_A = \mathbf{r}_{D/A} \times \mathbf{T}_{DE}$$

where $$\mathbf{r}_{D/A} = (0.132\ \text{m})\mathbf{j} + (0.720\ \text{m})\mathbf{k}$$

Then $$\mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.132 & 0.720 \\ 18.0 & 36.0 & -36.0 \end{vmatrix}\ \text{N}\cdot\text{m}$$

PROBLEM 3.45 CONTINUED



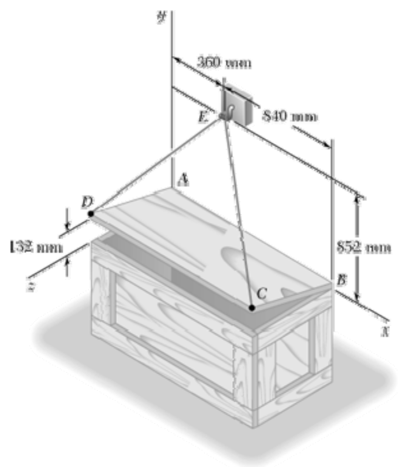
$$\therefore\ \mathbf{M}_A = \left\{\left[(0.132)(-36.0) - (0.720)(36.0)\right]\mathbf{i} + \left[(0.720)(18.0) - 0\right]\mathbf{j}\right.$$

$$\left. + \left[0 - (0.132)(18.0)\right]\mathbf{k}\right\} \text{ N}\cdot\text{m}$$

or $$\mathbf{M}_A = -(30.7 \text{ N}\cdot\text{m})\mathbf{i} + (12.96 \text{ N}\cdot\text{m})\mathbf{j} - (2.38 \text{ N}\cdot\text{m})\mathbf{k}$$

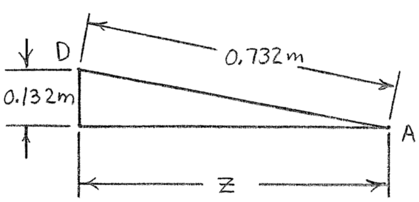
$$\therefore\ M_x = -30.7 \text{ N}\cdot\text{m},\ M_y = 12.96 \text{ N}\cdot\text{m},\ \ M_z = -2.38 \text{ N}\cdot\text{m} \blacktriangleleft$$

PROBLEM 3.46



The 0.732×1.2-m lid *ABCD* of a storage bin is hinged along side *AB* and is held open by looping cord *DEC* over a frictionless hook at *E*. If the tension in the cord is 54 N, determine the moment about each of the coordinate axes of the force exerted by the cord at *C*.

SOLUTION



First note $$z = \sqrt{(0.732)^2 - (0.132)^2}\text{ m}$$

$$= 0.720\text{ m}$$

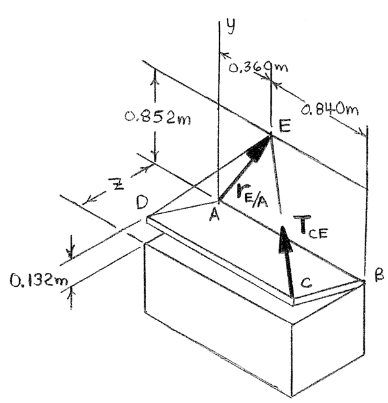
Then $$d_{CE} = \sqrt{(0.840)^2 + (0.720)^2 + (0.720)^2}\text{ m}$$

$$= 1.32\text{ m}$$

and $$\mathbf{T}_{CE} = \frac{\mathbf{r}_{E/C}}{d_{CE}}(T_{CE})$$

$$= \frac{-(0.840\text{ m})\mathbf{i} + (0.720\text{ m})\mathbf{j} - (0.720\text{ m})\mathbf{k}}{1.32\text{ m}}(54\text{ N})$$

$$= -(36.363\text{ N})\mathbf{i} + (29.454\text{ N})\mathbf{j} - (29.454\text{ N})\mathbf{k}$$



Now $$\mathbf{M}_A = \mathbf{r}_{E/A} \times \mathbf{T}_{CE}$$

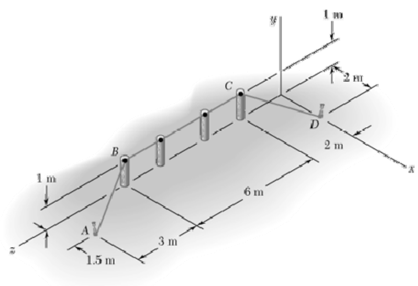
where $$\mathbf{r}_{E/A} = (0.360\text{ m})\mathbf{i} + (0.852\text{ m})\mathbf{j}$$

Then $$\mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.360 & 0.852 & 0 \\ -34.363 & 29.454 & -29.454 \end{vmatrix}\text{N}\cdot\text{m}$$

$$= -(25.095\text{ N}\cdot\text{m})\mathbf{i} + (10.6034\text{ N}\cdot\text{m})\mathbf{j} + (39.881\text{ N}\cdot\text{m})\mathbf{k}$$

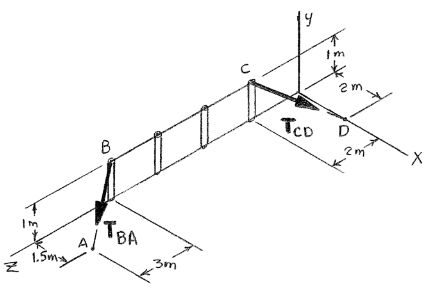
$$\therefore \quad M_x = -25.1\text{ N}\cdot\text{m}, \; M_y = 10.60\text{ N}\cdot\text{m}, \; M_z = 39.9\text{ N}\cdot\text{m} \blacktriangleleft$$

PROBLEM 3.47



A fence consists of wooden posts and a steel cable fastened to each post and anchored in the ground at A and D. Knowing that the sum of the moments about the z axis of the forces exerted by the cable on the posts at B and C is –66 N · m, determine the magnitude $\mathbf{T}_{CD}$ when $T_{BA} = 56$ N.

SOLUTION



Based on $$|\mathbf{M}_z| = \mathbf{k}\cdot\left[(\mathbf{r}_B)_y \times \mathbf{T}_{BA}\right] + \mathbf{k}\cdot\left[(\mathbf{r}_C)_y \times \mathbf{T}_{CD}\right]$$

where

$$\mathbf{M}_z = -(66\ \text{N}\cdot\text{m})\mathbf{k}$$

$$(\mathbf{r}_B)_y = (\mathbf{r}_C)_y = (1\ \text{m})\mathbf{j}$$

$$\mathbf{T}_{BA} = \lambda_{BA}T_{BA}$$

$$= \frac{(1.5\ \text{m})\mathbf{i} - (1\ \text{m})\mathbf{j} + (3\ \text{m})\mathbf{k}}{3.5\ \text{m}}(56\ \text{N})$$

$$= (24\ \text{N})\mathbf{i} - (16\ \text{N})\mathbf{j} + (48\ \text{N})\mathbf{k}$$

$$\mathbf{T}_{CD} = \lambda_{CD}T_{CD}$$

$$= \frac{(2\ \text{m})\mathbf{i} - (1\ \text{m})\mathbf{j} - (2\ \text{m})\mathbf{k}}{3.0\ \text{m}}T_{CD}$$

$$= \frac{1}{3}T_{CD}(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$$

$$\therefore\ -(66\ \text{N}\cdot\text{m}) = \mathbf{k}\cdot\left\{(1\ \text{m})\mathbf{j}\times\left[(24\ \text{N})\mathbf{i} - (16\ \text{N})\mathbf{j} + (48\ \text{N})\mathbf{k}\right]\right\}$$

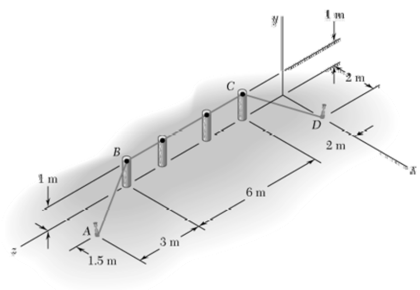
$$+\ \mathbf{k}\cdot\left\{(1\ \text{m})\mathbf{j}\times\left[\frac{1}{3}T_{CD}(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})\right]\right\}$$

or $$-66 = -24 - \frac{2}{3}T_{CD}$$

$$\therefore\ T_{CD} = \frac{3}{2}(66 - 24)\ \text{N}$$

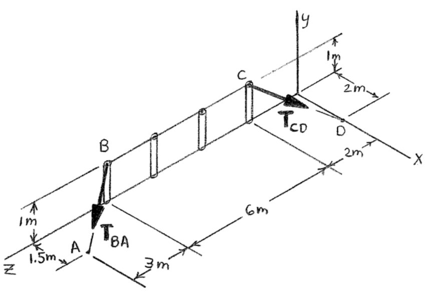
or $T_{CD} = 63.0$ N ◀

PROBLEM 3.48



A fence consists of wooden posts and a steel cable fastened to each post and anchored in the ground at A and D. Knowing that the sum of the moments about the y axis of the forces exerted by the cable on the posts at B and C is 212 N · m, determine the magnitude of $\mathbf{T}_{BA}$ when $T_{CD} = 33$ N.

SOLUTION



Based on $$|\mathbf{M}_y| = \mathbf{j}\cdot\left[(\mathbf{r}_B)_z \times \mathbf{T}_{BA} + (\mathbf{r}_C)_z \times \mathbf{T}_{CD}\right]$$

where

$$\mathbf{M}_y = (212\ \mathrm{N\cdot m})\,\mathbf{j}$$

$$(\mathbf{r}_B)_z = (8\ \mathrm{m})\mathbf{k}$$

$$(\mathbf{r}_C)_z = (2\ \mathrm{m})\mathbf{k}$$

$$\mathbf{T}_{BA} = \lambda_{BA} T_{BA}$$

$$= \frac{(1.5\ \mathrm{m})\mathbf{i} - (1\ \mathrm{m})\mathbf{j} - (3\ \mathrm{m})\mathbf{k}}{3.5\ \mathrm{m}} T_{BA}$$

$$= \frac{T_{BA}}{3.5}(1.5\mathbf{i} - \mathbf{j} + 3\mathbf{k})$$

$$\mathbf{T}_{CD} = \lambda_{CD} T_{CD}$$

$$= \frac{(2\ \mathrm{m})\mathbf{i} - (1\ \mathrm{m})\mathbf{j} - (2\ \mathrm{m})\mathbf{k}}{3.0\ \mathrm{m}}(33\ \mathrm{N})$$

$$= (22\mathbf{i} - 11\mathbf{j} - 22\mathbf{k})\ \mathrm{N}$$

$$\therefore\ (212\ \mathrm{N\cdot m}) = \mathbf{j}\cdot\left\{(8\ \mathrm{m})\mathbf{k} \times \left[\frac{T_{BA}}{3.5}(1.5\mathbf{i} - \mathbf{j} + 3\mathbf{k})\right]\right\}$$

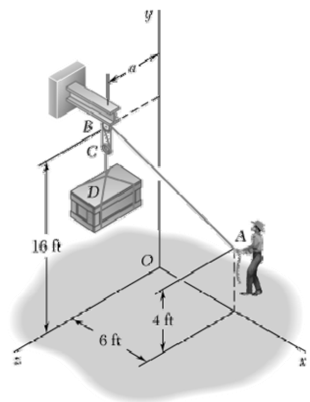
$$+\ \mathbf{j}\cdot\left[(2\ \mathrm{m})\mathbf{k} \times (22\,\mathbf{i} - 11\mathbf{j} - 22\mathbf{k})\ \mathrm{N}\right]$$

or $$212 = \frac{8(1.5)}{3.5} T_{BA} + 2(22)$$

$$\therefore\ T_{BA} = \frac{168}{18.6667}$$

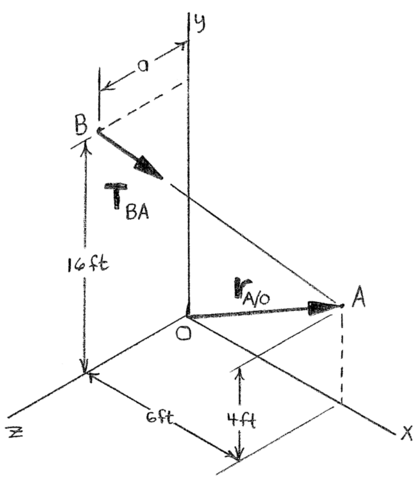
or $T_{BA} = 49.0$ N ◀

PROBLEM 3.49



To lift a heavy crate, a man uses a block and tackle attached to the bottom of an I-beam at hook B. Knowing that the moments about the y and z axes of the force exerted at B by portion AB of the rope are, respectively, $100 \text{ lb}\cdot\text{ft}$ and $-400 \text{ lb}\cdot\text{ft}$, determine the distance a.

SOLUTION



Based on $$\mathbf{M}_O = \mathbf{r}_{A/O} \times \mathbf{T}_{BA}$$

where

$$\mathbf{M}_O = M_x\mathbf{i} + M_y\mathbf{j} + M_z\mathbf{k}$$

$$= M_x\mathbf{i} + (100 \text{ lb}\cdot\text{ft})\mathbf{j} - (400 \text{ lb}\cdot\text{ft})\mathbf{k}$$

$$\mathbf{r}_{A/O} = (6 \text{ ft})\mathbf{i} + (4 \text{ ft})\mathbf{j}$$

$$\mathbf{T}_{BA} = \lambda_{BA}T_{BA}$$

$$= \frac{(6 \text{ ft})\mathbf{i} - (12 \text{ ft})\mathbf{j} - (a)\mathbf{k}}{d_{BA}}T_{BA}$$

$$\therefore\ M_x\mathbf{i} + 100\mathbf{j} - 400\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 4 & 0 \\ 6 & -12 & -a \end{vmatrix} \frac{T_{BA}}{d_{BA}}$$

$$= \frac{T_{BA}}{d_{BA}}\left[-(4a)\mathbf{i} + (6a)\mathbf{j} - (96)\mathbf{k}\right]$$

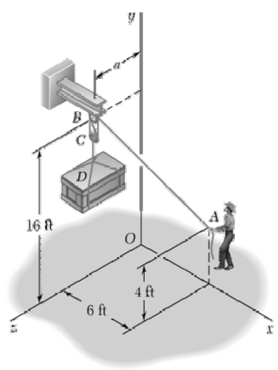
From **j**-coefficient: $$100d_{AB} = 6aT_{BA} \quad \text{or} \quad T_{BA} = \frac{100}{6a}d_{BA} \qquad (1)$$

From **k**-coefficient: $$-400d_{AB} = -96T_{BA} \quad \text{or} \quad T_{BA} = \frac{400}{96}d_{BA} \qquad (2)$$

Equating Equations (1) and (2) yields $$a = \frac{100(96)}{6(400)}$$

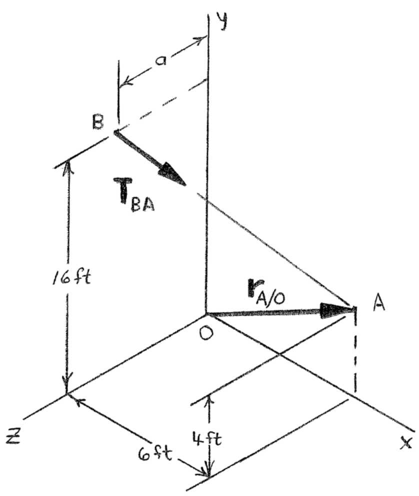
or $a = 4.00 \text{ ft}$ ◀

PROBLEM 3.50



To lift a heavy crate, a man uses a block and tackle attached to the bottom of an I-beam at hook *B*. Knowing that the man applies a 200-lb force to end *A* of the rope and that the moment of that force about the *y* axis is $175\ \text{lb}\cdot\text{ft}$, determine the distance *a*.

SOLUTION



Based on $$|\mathbf{M}_y| = \mathbf{j}\cdot\left(\mathbf{r}_{A/O}\times\mathbf{T}_{BA}\right)$$

where $$\mathbf{r}_{A/O} = (6\text{ ft})\mathbf{i} + (4\text{ ft})\mathbf{j}$$

$$\mathbf{T}_{BA} = \lambda_{BA}T_{BA} = \frac{\mathbf{r}_{A/B}}{d_{BA}}T_{BA}$$

$$= \frac{(6\text{ ft})\mathbf{i} - (12\text{ ft})\mathbf{j} - (a)\mathbf{k}}{d_{BA}}(200\text{ lb})$$

$$= \frac{200}{d_{BA}}(6\mathbf{i} - 12\mathbf{j} - a\mathbf{k})$$

$$\therefore\ 175\text{ lb}\cdot\text{ft} = \begin{vmatrix} 0 & 1 & 0 \\ 6 & 4 & 0 \\ 6 & -12 & -a \end{vmatrix}\frac{200}{d_{BA}}$$

$$175 = [0 - 6(-a)]\frac{200}{d_{BA}}$$

where $$d_{BA} = \sqrt{(6)^2 + (12)^2 + (a)^2}\text{ ft}$$

$$= \sqrt{180 + a^2}\text{ ft}$$

$$\therefore\ 175\sqrt{180 + a^2} = 1200a$$

or $$\sqrt{180 + a^2} = 6.8571a$$

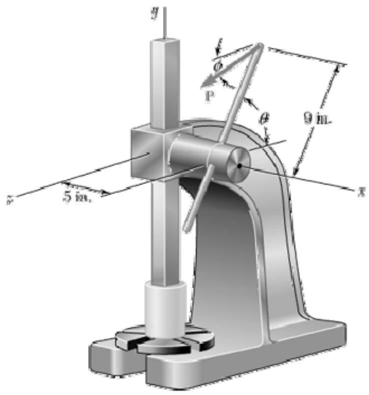
Squaring each side

$$180 + a^2 = 47.020a^2$$

Solving $$a = 1.97771\text{ ft}$$

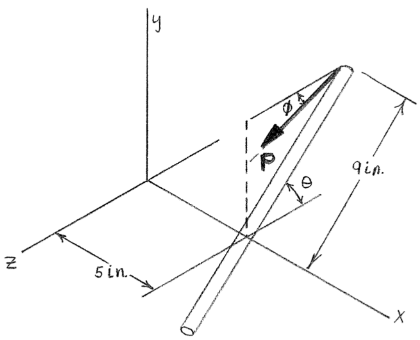
or $a = 1.978\text{ ft}$ ◀

PROBLEM 3.51



A force **P** is applied to the lever of an arbor press. Knowing that **P** lies in a plane parallel to the *yz* plane and that $M_x = 230\text{ lb}\cdot\text{in.}$, $M_y = -200\text{ lb}\cdot\text{in.}$, and $M_z = -35\text{ lb}\cdot\text{in.}$, determine the magnitude of **P** and the values of ϕ and θ.

SOLUTION



Based on $$M_x = (P\cos\phi)[(9\text{ in.})\sin\theta] - (P\sin\phi)[(9\text{ in.})\cos\theta] \quad (1)$$

$$M_y = -(P\cos\phi)(5\text{ in.}) \quad (2)$$

$$M_z = -(P\sin\phi)(5\text{ in.}) \quad (3)$$

Then $$\frac{\text{Equation (3)}}{\text{Equation (2)}}: \quad \frac{M_z}{M_y} = \frac{-(P\sin\phi)(5)}{-(P\cos\phi)(5)}$$

or $$\tan\phi = \frac{-35}{-200} = 0.175 \qquad \phi = 9.9262°$$

or $\phi = 9.93°$ ◀

Substituting ϕ into Equation (2)

$$-200\text{ lb}\cdot\text{in.} = -(P\cos 9.9262°)(5\text{ in.})$$

$$P = 40.608\text{ lb}$$

or $P = 40.6\text{ lb}$ ◀

Then, from Equation (1)

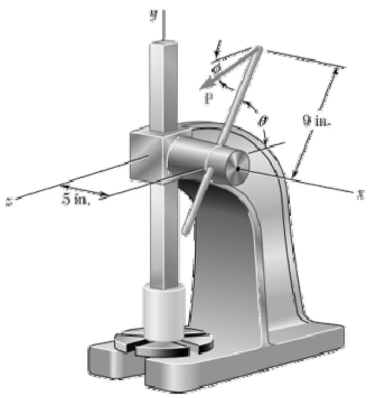
$$230\text{ lb}\cdot\text{in.} = [(40.608\text{ lb})\cos 9.9262°][(9\text{ in.})\sin\theta]$$

$$-[(40.608\text{ lb})\sin 9.9262°][(9\text{ in.})\cos\theta]$$

or $$0.98503\sin\theta - 0.172380\cos\theta = 0.62932$$

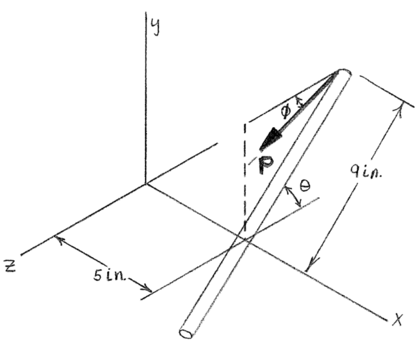
Solving numerically, $\theta = 48.9°$ ◀

PROBLEM 3.52



A force **P** is applied to the lever of an arbor press. Knowing that **P** lies in a plane parallel to the *yz* plane and that $M_y = -180 \text{ lb}\cdot\text{in.}$ and $M_z = -30 \text{ lb}\cdot\text{in.}$, determine the moment M_x of **P** about the *x* axis when $\theta = 60°$.

SOLUTION



Based on $$M_x = (P\cos\phi)[(9\text{ in.})\sin\theta] - (P\sin\phi)[(9\text{ in.})\cos\theta] \quad (1)$$

$$M_y = -(P\cos\phi)(5\text{ in.}) \quad (2)$$

$$M_z = -(P\sin\phi)(5\text{ in.}) \quad (3)$$

Then $$\frac{\text{Equation (3)}}{\text{Equation (2)}}: \quad \frac{M_z}{M_y} = \frac{-(P\sin\phi)(5)}{-(P\cos\phi)(5)}$$

or $$\frac{-30}{-180} = \tan\phi$$

$$\therefore\ \phi = 9.4623°$$

From Equation (3),

$$-30 \text{ lb}\cdot\text{in.} = -(P\sin 9.4623°)(5\text{ in.})$$

$$\therefore\ P = 36.497 \text{ lb}$$

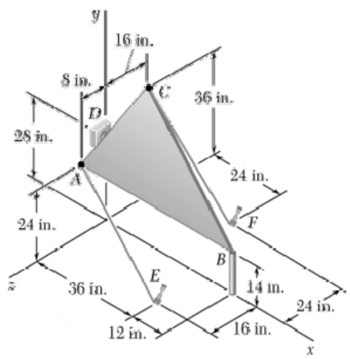
From Equation (1),

$$M_x = (36.497\text{ lb})(9\text{ in.})(\cos 9.4623°\sin 60° - \sin 9.4623°\cos 60°)$$

$$= 253.60 \text{ lb}\cdot\text{in.}$$

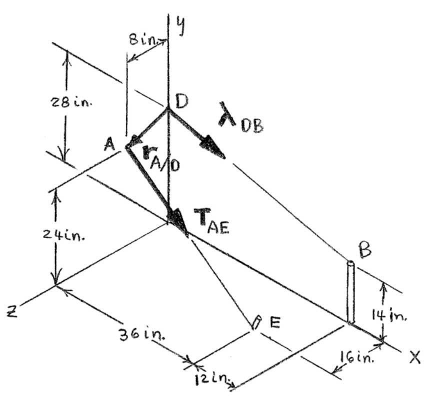
or $M_x = 254 \text{ lb}\cdot\text{in.}$ ◀

PROBLEM 3.53



The triangular plate *ABC* is supported by ball-and-socket joints at *B* and *D* and is held in the position shown by cables *AE* and *CF*. If the force exerted by cable *AE* at *A* is 220 lb, determine the moment of that force about the line joining points *D* and *B*.

SOLUTION



Have $$M_{DB} = \lambda_{DB} \cdot \left(\mathbf{r}_{A/D} \times \mathbf{T}_{AE}\right)$$

where $$\lambda_{DB} = \frac{(48 \text{ in.})\mathbf{i} - (14 \text{ in.})\mathbf{j}}{50 \text{ in.}} = 0.96\mathbf{i} - 0.28\mathbf{j}$$

$$\mathbf{r}_{A/D} = -(4 \text{ in.})\,\mathbf{j} + (8 \text{ in.})\mathbf{k}$$

$$\mathbf{T}_{AE} = \lambda_{AE} T_{AE} = \frac{\left[(36 \text{ in.})\mathbf{i} - (24 \text{ in.})\,\mathbf{j} + (8 \text{ in.})\mathbf{k}\right]}{44 \text{ in.}}(220 \text{ lb})$$

$$= (180 \text{ lb})\mathbf{i} - (120 \text{ lb})\,\mathbf{j} + (40 \text{ lb})\mathbf{k}$$

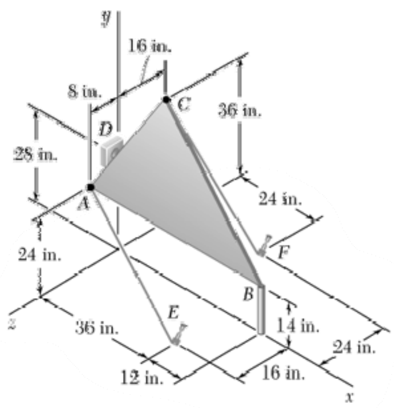
$$\therefore\ M_{DB} = \begin{vmatrix} 0.960 & -0.280 & 0 \\ 0 & -4 & 8 \\ 180 & -120 & 40 \end{vmatrix} \text{lb}\cdot\text{in.}$$

$$= (0.960)\left[(-4)(40) - (8)(-120)\right] + (-0.280)\left[8(180) - 0\right]$$

$$= 364.8 \text{ lb}\cdot\text{in.}$$

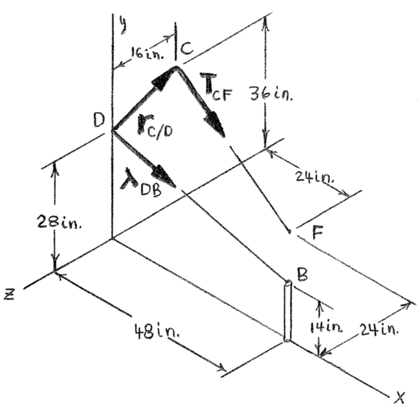
or $M_{DB} = 365 \text{ lb}\cdot\text{in.}$ ◀

PROBLEM 3.54



The triangular plate *ABC* is supported by ball-and-socket joints at *B* and *D* and is held in the position shown by cables *AE* and *CF*. If the force exerted by cable *CF* at *C* is 132 lb, determine the moment of that force about the line joining points *D* and *B*.

SOLUTION



Have $$M_{DB} = \lambda_{DB} \cdot \left(\mathbf{r}_{C/D} \times \mathbf{T}_{CF}\right)$$

where $$\lambda_{DB} = \frac{(48\text{ in.})\mathbf{i} - (14\text{ in.})\mathbf{j}}{50\text{ in.}} = 0.96\mathbf{i} - 0.28\mathbf{j}$$

$$\mathbf{r}_{C/D} = (8\text{ in.})\mathbf{j} - (16\text{ in.})\mathbf{k}$$

$$\mathbf{T}_{CF} = \lambda_{CF} T_{CF} = \frac{(24\text{ in.})\mathbf{i} - (36\text{ in.})\mathbf{j} - (8\text{ in.})\mathbf{k}}{44\text{ in.}}(132\text{ lb})$$

$$= (72\text{ lb})\mathbf{i} - (108\text{ lb})\mathbf{j} - (24\text{ lb})\mathbf{k}$$

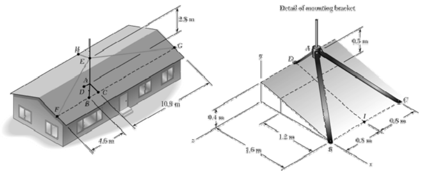
$$\therefore\ M_{DB} = \begin{vmatrix} 0.96 & -0.28 & 0 \\ 0 & 8 & -16 \\ 72 & -108 & -24 \end{vmatrix} \text{lb}\cdot\text{in.}$$

$$= 0.96\left[(8)(-24) - (-16)(-108)\right] + (-0.28)\left[(-16)(72) - 0\right]$$

$$= -1520.64\text{ lb}\cdot\text{in.}$$

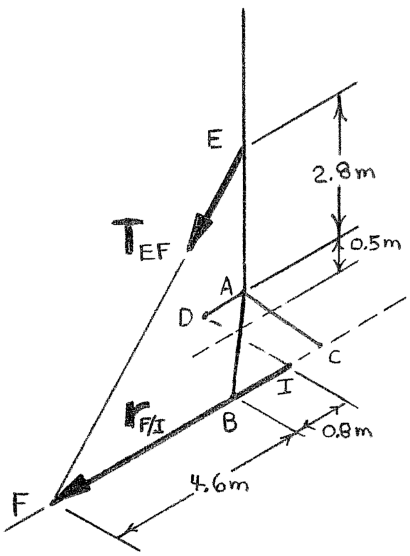
or $M_{DB} = -1521\text{ lb}\cdot\text{in.}$ ◀

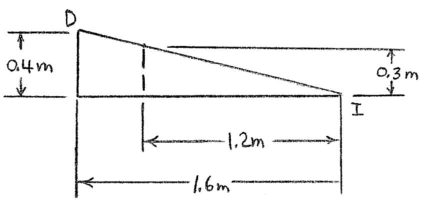
PROBLEM 3.55



A mast is mounted on the roof of a house using bracket *ABCD* and is guyed by cables *EF*, *EG*, and *EH*. Knowing that the force exerted by cable *EF* at *E* is 66 N, determine the moment of that force about the line joining points *D* and *I*.

SOLUTION





Have
$$M_{DI} = \boldsymbol{\lambda}_{DI} \cdot \left[\mathbf{r}_{F/I} \times \mathbf{T}_{EF}\right]$$

where
$$\boldsymbol{\lambda}_{DI} = \frac{(1.6\text{ m})\mathbf{i} - (0.4\text{ m})\mathbf{j}}{\sqrt{(1.6)^2 + (0.4)^2}\text{ m}} = \frac{1}{\sqrt{17}}(4\mathbf{i} - \mathbf{j})$$

$$\mathbf{r}_{F/I} = (4.6\text{ m} + 0.8\text{ m})\mathbf{k} = (5.4\text{ m})\mathbf{k}$$

$$\mathbf{T}_{EF} = \boldsymbol{\lambda}_{EF} T_{EF}$$

$$= \frac{(1.2\text{ m})\mathbf{i} - (3.6\text{ m})\mathbf{j} + (5.4\text{ m})\mathbf{k}}{6.6\text{ m}}(66\text{ N})$$

$$= (12\text{ N})\mathbf{i} - (36\text{ N})\mathbf{j} + (54\text{ N})\mathbf{k}$$

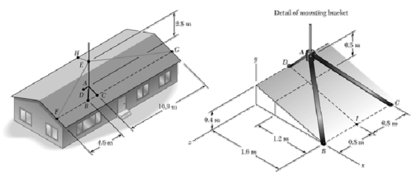
$$= 6\left[(2\text{ N})\mathbf{i} - (6\text{ N})\mathbf{j} + (9\text{ N})\mathbf{k}\right]$$

$$\therefore\ M_{DI} = \frac{(6\text{ N})(5.4\text{ m})}{\sqrt{17}}\begin{vmatrix} 4 & -1 & 0 \\ 0 & 0 & 1 \\ 2 & -6 & 9 \end{vmatrix}$$

$$= 7.8582\left[(0 + 24) + (-2 - 0)\right]$$

$$= 172.879\text{ N}\cdot\text{m}$$

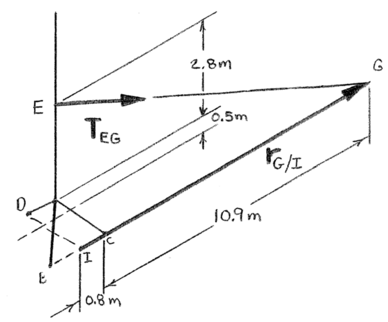
or $M_{DI} = 172.9\text{ N}\cdot\text{m}$ ◀



PROBLEM 3.56

A mast is mounted on the roof of a house using bracket *ABCD* and is guyed by cables *EF*, *EG*, and *EH*. Knowing that the force exerted by cable *EG* at *E* is 61.5 N, determine the moment of that force about the line joining points *D* and *I*.

SOLUTION



Have
$$M_{DI} = \boldsymbol{\lambda}_{DI} \cdot \left[\mathbf{r}_{G/I} \times \mathbf{T}_{EG}\right]$$

where
$$\boldsymbol{\lambda}_{DI} = \frac{(1.6 \text{ m})\mathbf{i} - (0.4 \text{ m})\mathbf{j}}{0.4\sqrt{17} \text{ m}}$$

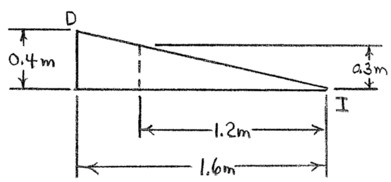
$$= \frac{1}{\sqrt{17}}(4\mathbf{i} - \mathbf{j})$$

$$\mathbf{r}_{G/I} = -(10.9 \text{ m} + 0.8 \text{ m})\mathbf{k} = -(11.7 \text{ m})\mathbf{k}$$

$$\mathbf{T}_{EG} = \boldsymbol{\lambda}_{EG} T_{EG}$$

$$= \frac{(1.2 \text{ m})\mathbf{i} - (3.6 \text{ m})\mathbf{j} - (11.7 \text{ m})\mathbf{k}}{12.3 \text{ m}}(61.5 \text{ N})$$

$$= 5\left[(1.2 \text{ N})\mathbf{i} - (3.6 \text{ N})\mathbf{j} - (11.7 \text{ N})\mathbf{k}\right]$$



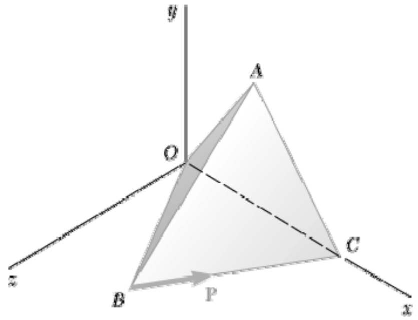
$$\therefore \quad M_{DI} = \frac{5 \text{ N}(11.7 \text{ m})}{\sqrt{17}} \begin{vmatrix} 4 & -1 & 0 \\ 0 & 0 & -1 \\ 1.2 & -3.6 & -11.7 \end{vmatrix}$$

$$= (14.1883 \text{ N}\cdot\text{m})\left\{\left[0 - (4)(-1)(-3.6)\right] + \left[(-1)(-1)(1.2) - 0\right]\right\}$$

$$= -187.286 \text{ N}\cdot\text{m}$$

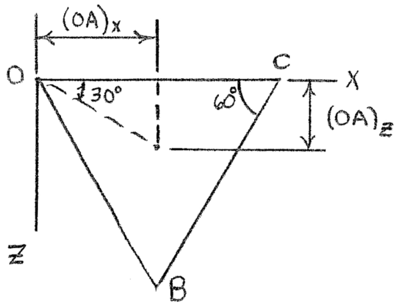
or $M_{DI} = -187.3 \text{ N}\cdot\text{m}$ ◀

PROBLEM 3.57



A rectangular tetrahedron has six edges of length a. A force $\mathbf{P}$ is directed as shown along edge BC. Determine the moment of $\mathbf{P}$ about edge OA.

SOLUTION



Have $$M_{OA} = \lambda_{OA} \cdot \left(\mathbf{r}_{C/O} \times \mathbf{P}\right)$$

where

From triangle OBC

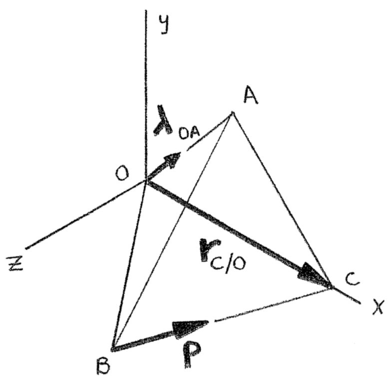
$$(OA)_x = \frac{a}{2}$$

$$(OA)_z = (OA)_x \tan 30° = \frac{a}{2}\left(\frac{1}{\sqrt{3}}\right) = \frac{a}{2\sqrt{3}}$$

Since $$(OA)^2 = (OA)_x^2 + (OA)_y^2 + (OA_z)^2$$

or $$a^2 = \left(\frac{a}{2}\right)^2 + (OA)_y^2 + \left(\frac{a}{2\sqrt{3}}\right)^2$$

$$\therefore \quad (OA)_y = \sqrt{a^2 - \frac{a^2}{4} - \frac{a^2}{12}} = a\sqrt{\frac{2}{3}}$$



Then $$\mathbf{r}_{A/O} = \frac{a}{2}\mathbf{i} + a\sqrt{\frac{2}{3}}\mathbf{j} + \frac{a}{2\sqrt{3}}\mathbf{k}$$

and $$\lambda_{OA} = \frac{1}{2}\mathbf{i} + \sqrt{\frac{2}{3}}\mathbf{j} + \frac{1}{2\sqrt{3}}\mathbf{k}$$

$$\mathbf{P} = \lambda_{BC}P$$

$$= \frac{(a\sin 30°)\mathbf{i} - (a\cos 30°)\mathbf{k}}{a}(P)$$

$$= \frac{P}{2}\left(\mathbf{i} - \sqrt{3}\mathbf{k}\right)$$

$$\mathbf{r}_{C/O} = a\mathbf{i}$$

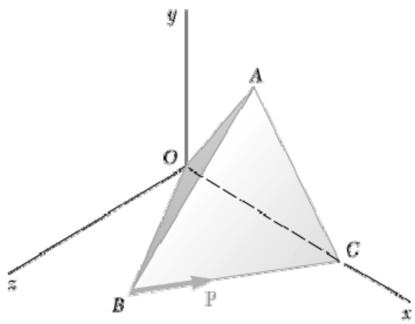
PROBLEM 3.57 CONTINUED

$$\therefore \quad M_{OA} = \begin{vmatrix} \frac{1}{2} & \sqrt{\frac{2}{3}} & \frac{1}{2\sqrt{3}} \\ 1 & 0 & 0 \\ 1 & 0 & -\sqrt{3} \end{vmatrix} (a)\left(\frac{P}{2}\right)$$

$$= \frac{aP}{2}\left(-\sqrt{\frac{2}{3}}\right)(1)\left(-\sqrt{3}\right)$$

$$= \frac{aP}{\sqrt{2}}$$

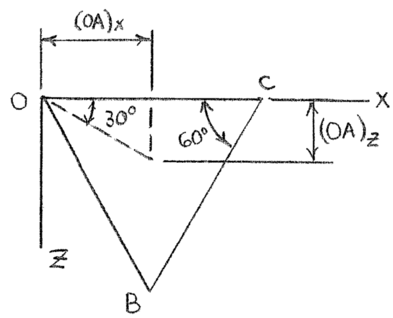
$$M_{OA} = \frac{aP}{\sqrt{2}} \blacktriangleleft$$

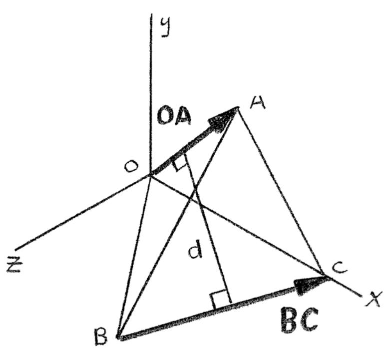


PROBLEM 3.58

A rectangular tetrahedron has six edges of length a. (a) Show that two opposite edges, such as OA and BC, are perpendicular to each other. (b) Use this property and the result obtained in Problem 3.57 to determine the perpendicular distance between edges OA and BC.

SOLUTION





(a) For edge OA to be perpendicular to edge BC,

$$\overrightarrow{OA} \cdot \overrightarrow{BC} = 0$$

where

From triangle OBC

$$(OA)_x = \frac{a}{2}$$

$$(OA)_z = (OA)_x \tan 30° = \frac{a}{2}\left(\frac{1}{\sqrt{3}}\right) = \frac{a}{2\sqrt{3}}$$

$$\therefore \quad \overrightarrow{OA} = \left(\frac{a}{2}\right)\mathbf{i} + (OA)_y\,\mathbf{j} + \left(\frac{a}{2\sqrt{3}}\right)\mathbf{k}$$

and

$$\overrightarrow{BC} = (a\sin 30°)\mathbf{i} - (a\cos 30°)\mathbf{k}$$

$$= \frac{a}{2}\mathbf{i} - \frac{a\sqrt{3}}{2}\mathbf{k}$$

$$= \frac{a}{2}\left(\mathbf{i} - \sqrt{3}\mathbf{k}\right)$$

Then

$$\left[\frac{a}{2}\mathbf{i} + (OA)_y\,\mathbf{j} + \left(\frac{a}{2\sqrt{3}}\right)\mathbf{k}\right] \cdot \left(\mathbf{i} - \sqrt{3}\mathbf{k}\right)\frac{a}{2} = 0$$

or

$$\frac{a^2}{4} + (OA)_y(0) - \frac{a^2}{4} = 0$$

$$\therefore \quad \overrightarrow{OA} \cdot \overrightarrow{BC} = 0$$

so that $\overrightarrow{OA}$ is perpendicular to $\overrightarrow{BC}$. ◀

PROBLEM 3.58 CONTINUED

(*b*) Have $M_{OA} = Pd$, with P acting along BC and d the perpendicular distance from $\overrightarrow{OA}$ to $\overrightarrow{BC}$.

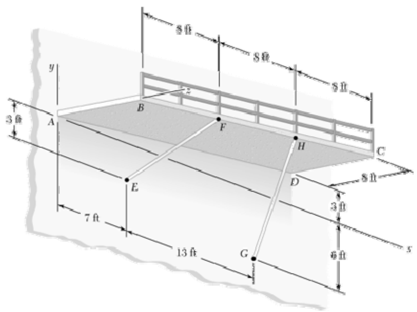
From the results of Problem 3.57,

$$M_{OA} = \frac{Pa}{\sqrt{2}}$$

$$\therefore \quad \frac{Pa}{\sqrt{2}} = Pd$$

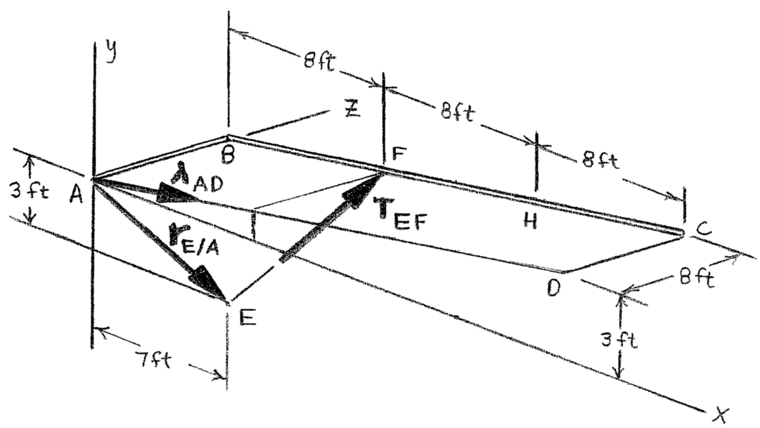
or $$d = \frac{a}{\sqrt{2}} \blacktriangleleft$$

PROBLEM 3.59



The 8-ft-wide portion *ABCD* of an inclined, cantilevered walkway is partially supported by members *EF* and *GH*. Knowing that the compressive force exerted by member *EF* on the walkway at *F* is 5400 lb, determine the moment of that force about edge *AD*.

SOLUTION



Having

$$M_{AD} = \boldsymbol{\lambda}_{AD} \cdot \left(\mathbf{r}_{E/A} \times \mathbf{T}_{EF}\right)$$

where

$$\boldsymbol{\lambda}_{AD} = \frac{(24\text{ ft})\mathbf{i} + (3\text{ ft})\mathbf{j}}{\sqrt{(24)^2 + (3)^2}\text{ ft}} = \frac{1}{\sqrt{65}}(8\mathbf{i} + \mathbf{j})$$

$$\mathbf{r}_{E/A} = (7\text{ ft})\mathbf{i} - (3\text{ ft})\mathbf{j}$$

$$\mathbf{T}_{EF} = \boldsymbol{\lambda}_{EF} T_{EF} = \frac{(8\text{ ft} - 7\text{ ft})\mathbf{i} + \left[3\text{ ft} + \left(\frac{8}{24}\right)(3\text{ ft})\right]\mathbf{j} + (8\text{ ft})\mathbf{k}}{\sqrt{(1)^2 + (4)^2 + (8)^2}\text{ ft}}(5400\text{ lb})$$

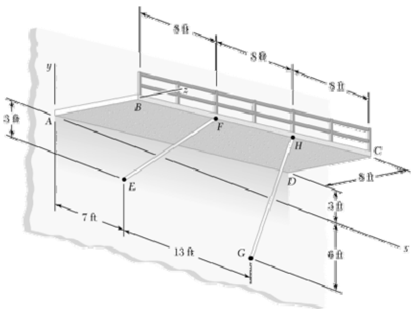
$$= 600\left[(1\text{ lb})\mathbf{i} + (4\text{ lb})\mathbf{j} + (8\text{ lb})\mathbf{k}\right]$$

$$\therefore\quad M_{AD} = \frac{600}{\sqrt{65}}\begin{vmatrix} 8 & 1 & 0 \\ 7 & -3 & 0 \\ 1 & 4 & 8 \end{vmatrix}\text{ lb}\cdot\text{ft} = \frac{600}{\sqrt{65}}(-192 - 56)\text{ lb}\cdot\text{ft}$$

$$= -18{,}456.4\text{ lb}\cdot\text{ft}$$

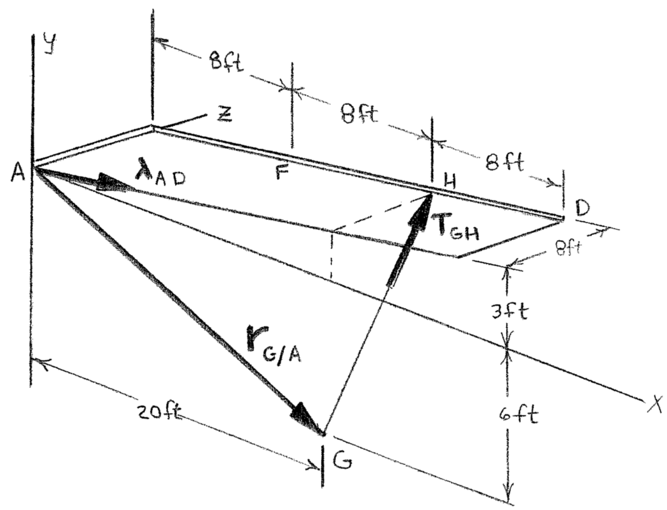
or $M_{AD} = -18.46\text{ kip}\cdot\text{ft}$ ◀

PROBLEM 3.60



The 8-ft-wide portion *ABCD* of an inclined, cantilevered walkway is partially supported by members *EF* and *GH*. Knowing that the compressive force exerted by member *GH* on the walkway at *H* is 4800 lb, determine the moment of that force about edge *AD*.

SOLUTION



Having

$$M_{AD} = \lambda_{AD} \cdot \left(\mathbf{r}_{G/A} \times \mathbf{T}_{GH}\right)$$

where

$$\lambda_{AD} = \frac{(24\text{ ft})\mathbf{i} + (3\text{ ft})\mathbf{j}}{\sqrt{(24)^2 + (3)^2}\text{ ft}} = \frac{1}{\sqrt{65}}(8\mathbf{i} + \mathbf{j})$$

$$\mathbf{r}_{G/A} = (20\text{ ft})\mathbf{i} - (6\text{ ft})\mathbf{j} = 2\left[(10\text{ ft})\mathbf{i} - (3\text{ ft})\mathbf{j}\right]$$

$$\mathbf{T}_{GH} = \lambda_{GH} T_{GH} = \frac{(16\text{ ft} - 20\text{ ft})\mathbf{i} + \left[6\text{ ft} + \left(\tfrac{16}{24}\right)(3\text{ ft})\right]\mathbf{j} + (8\text{ ft})\mathbf{k}}{\sqrt{(4)^2 + (8)^2 + (8)^2}\text{ ft}}(4800\text{ lb})$$

$$= 1600\left[-(1\text{ lb})\mathbf{i} + (2\text{ lb})\mathbf{j} + (2\text{ lb})\mathbf{k}\right]$$

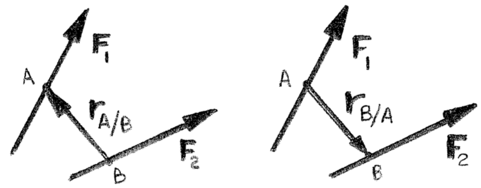
$$\therefore\quad M_{AD} = \frac{(1600\text{ lb})(2\text{ ft})}{\sqrt{65}}\begin{vmatrix} 8 & 1 & 0 \\ 10 & -3 & 0 \\ -1 & 2 & 2 \end{vmatrix} = \frac{3200\text{ lb}\cdot\text{ft}}{\sqrt{65}}(-48 - 20)$$

$$= -26{,}989\text{ lb}\cdot\text{ft} \qquad \text{or } M_{AD} = -27.0\text{ kip}\cdot\text{ft} \blacktriangleleft$$

PROBLEM 3.61

Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ in space have the same magnitude F. Prove that the moment of $\mathbf{F}_1$ about the line of action of $\mathbf{F}_2$ is equal to the moment of $\mathbf{F}_2$ about the line of action of $\mathbf{F}_1$.

SOLUTION



First note that $\mathbf{F}_1 = F_1\boldsymbol{\lambda}_1$ and $\mathbf{F}_2 = F_2\boldsymbol{\lambda}_2$

Let $M_1 =$ moment of $\mathbf{F}_2$ about the line of action of $\mathbf{M}_1$

and $M_2 =$ moment of $\mathbf{F}_1$ about the line of action of $\mathbf{M}_2$

Now, by definition

$$M_1 = \boldsymbol{\lambda}_1 \cdot \left(\mathbf{r}_{B/A} \times \mathbf{F}_2\right) = \boldsymbol{\lambda}_1 \cdot \left(\mathbf{r}_{B/A} \times \boldsymbol{\lambda}_2\right) F_2$$

$$M_2 = \boldsymbol{\lambda}_2 \cdot \left(\mathbf{r}_{A/B} \times \mathbf{F}_1\right) = \boldsymbol{\lambda}_2 \cdot \left(\mathbf{r}_{A/B} \times \boldsymbol{\lambda}_1\right) F_1$$

Since $F_1 = F_2 = F$ and $\mathbf{r}_{A/B} = -\mathbf{r}_{B/A}$

$$M_1 = \boldsymbol{\lambda}_1 \cdot \left(\mathbf{r}_{B/A} \times \boldsymbol{\lambda}_2\right) F$$

$$M_2 = \boldsymbol{\lambda}_2 \cdot \left(-\mathbf{r}_{B/A} \times \boldsymbol{\lambda}_1\right) F$$

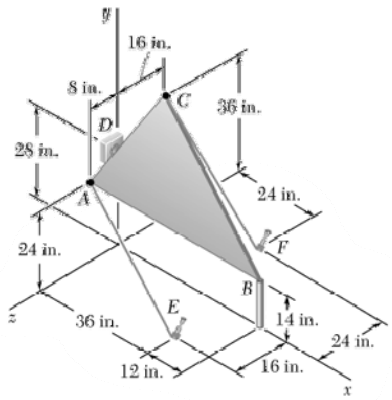
Using Equation (3.39)

$$\boldsymbol{\lambda}_1 \cdot \left(\mathbf{r}_{B/A} \times \boldsymbol{\lambda}_2\right) = \boldsymbol{\lambda}_2 \cdot \left(-\mathbf{r}_{B/A} \times \boldsymbol{\lambda}_1\right)$$

so that $M_2 = \boldsymbol{\lambda}_1 \cdot \left(\mathbf{r}_{B/A} \times \boldsymbol{\lambda}_2\right) F$

$$\therefore \quad M_{12} = M_{21} \blacktriangleleft$$

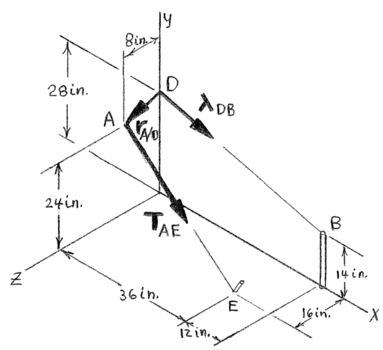
PROBLEM 3.62



In Problem 3.53, determine the perpendicular distance between cable *AE* and the line joining points *D* and *B*.

Problem 3.53: The triangular plate *ABC* is supported by ball-and-socket joints at *B* and *D* and is held in the position shown by cables *AE* and *CF*. If the force exerted by cable *AE* at *A* is 220 lb, determine the moment of that force about the line joining points *D* and *B*.

SOLUTION



Have $$M_{DB} = \lambda_{DB} \cdot \left(\mathbf{r}_{A/D} \times \mathbf{T}_{AE}\right)$$

where $$\lambda_{DB} = \frac{(48 \text{ in.})\mathbf{i} - (14 \text{ in.})\mathbf{j}}{50 \text{ in.}} = 0.96\mathbf{i} - 0.28\mathbf{j}$$

$$\mathbf{r}_{A/D} = -(4 \text{ in.})\mathbf{j} + (8 \text{ in.})\mathbf{k}$$

$$\mathbf{T}_{AE} = \lambda_{AE} T_{AE}$$

$$= \frac{(36 \text{ in.})\mathbf{i} - (24 \text{ in.})\mathbf{j} + (8 \text{ in.})\mathbf{k}}{44 \text{ in.}}(220 \text{ lb})$$

$$= (180 \text{ lb})\mathbf{i} - (120 \text{ lb})\mathbf{j} + (40 \text{ lb})\mathbf{k}$$

$$\therefore \quad M_{DB} = \begin{vmatrix} 0.96 & -0.28 & 0 \\ 0 & -4 & 8 \\ 180 & -120 & 40 \end{vmatrix} \text{lb} \cdot \text{in.}$$

$$= 364.8 \text{ lb} \cdot \text{in.}$$

Only the perpendicular component of $\mathbf{T}_{AE}$ contributes to the moment of $\mathbf{T}_{AE}$ about line *DB*. The parallel component of $\mathbf{T}_{AE}$ will be used to find the perpendicular component.

PROBLEM 3.62 CONTINUED

Have

$$(T_{AE})_{\text{parallel}} = \lambda_{DB} \cdot \mathbf{T}_{AE}$$

$$= (0.96\mathbf{i} - 0.28\mathbf{j}) \cdot \left[(180 \text{ lb})\mathbf{i} - (120 \text{ lb})\mathbf{j} + (40 \text{ lb})\mathbf{k}\right]$$

$$= \left[(0.96)(180) + (-0.28)(-120) + (0)(40)\right]\text{lb}$$

$$= (172.8 + 33.6)\text{lb}$$

$$= 206.4 \text{ lb}$$

Since $\mathbf{T}_{AE} = (\mathbf{T}_{AE})_{\text{perpendicular}} + (\mathbf{T}_{AE})_{\text{parallel}}$

$$\therefore \quad (T_{AE})_{\text{perpendicular}} = \sqrt{(T_{AE})^2 - (T_{AE})^2_{\text{parallel}}}$$

$$= \sqrt{(220)^2 - (206.41)^2}$$

$$= 76.151 \text{ lb}$$

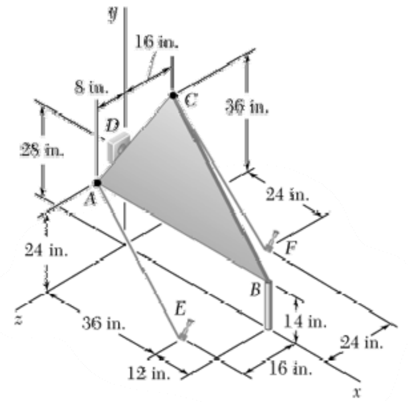
Then $\quad M_{DB} = (T_{AE})_{\text{perpendicular}}(d)$

$$364.8 \text{ lb}\cdot\text{in.} = (76.151 \text{ lb})d$$

$$d = 4.7905 \text{ in.}$$

or $d = 4.79$ in. ◀

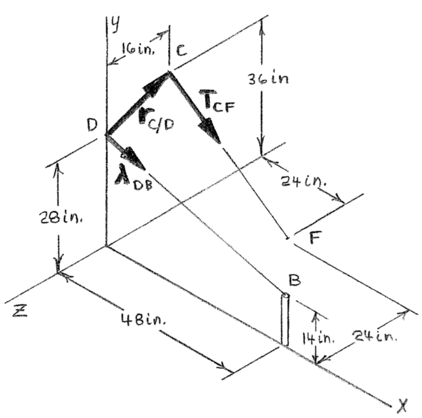
PROBLEM 3.63



In Problem 3.54, determine the perpendicular distance between cable *CF* and the line joining points *D* and *B*.

Problem 3.54: The triangular plate *ABC* is supported by ball-and-socket joints at *B* and *D* and is held in the position shown by cables *AE* and *CF*. If the force exerted by cable *CF* at *C* is 132 lb, determine the moment of that force about the line joining points *D* and *B*.

SOLUTION



Have $\left(M_{DB}\right) = \lambda_{DB} \cdot \left(\mathbf{r}_{C/D} \times \mathbf{T}_{CF}\right)$

where $$\lambda_{DB} = \frac{(48\text{ in.})\mathbf{i} - (14\text{ in.})\mathbf{j}}{50\text{ in.}}$$

$$= 0.96\mathbf{i} - 0.28\mathbf{j}$$

$$\mathbf{r}_{C/D} = (8\text{ in.})\mathbf{j} - (16\text{ in.})\mathbf{k}$$

$$\mathbf{T}_{CF} = \lambda_{CF} T_{CF}$$

$$= \frac{(24\text{ in.})\mathbf{i} - (36\text{ in.})\mathbf{j} - (8\text{ in.})\mathbf{k}}{44\text{ in.}}(132\text{ lb})$$

$$= (72\text{ lb})\mathbf{i} - (108\text{ lb})\mathbf{j} - (24\text{ lb})\mathbf{k}$$

$$\therefore \quad M_{DB} = \begin{vmatrix} 0.96 & -0.28 & 0 \\ 0 & 8 & -16 \\ 72 & -108 & -24 \end{vmatrix} \text{lb}\cdot\text{in}$$

$$= -1520.64\text{ lb}\cdot\text{in.}$$

Only the perpendicular component of $\mathbf{T}_{CF}$ contributes to the moment of $\mathbf{T}_{CF}$ about line *DB*. The parallel component of $\mathbf{T}_{CF}$ will be used to obtain the perpendicular component.

PROBLEM 3.63 CONTINUED

Have

$$(T_{CF})_{\text{parallel}} = \boldsymbol{\lambda}_{DB} \cdot \mathbf{T}_{CF}$$

$$= (0.96\mathbf{i} - 0.28\mathbf{j}) \cdot \left[(72 \text{ lb})\mathbf{i} - (108 \text{ lb})\mathbf{j} - (24 \text{ lb})\mathbf{k}\right]$$

$$= \left[(0.96)(72) + (-0.28)(-108) + (0)(-24)\right]\text{lb}$$

$$= 99.36 \text{ lb}$$

Since $\mathbf{T}_{CF} = (\mathbf{T}_{CF})_{\text{perp.}} + (\mathbf{T}_{CF})_{\text{parallel}}$

$$\therefore \quad (T_{CF})_{\text{perp.}} = \sqrt{(T_{CF})^2 - (T_{CF})^2_{\text{parallel}}}$$

$$= \sqrt{(132)^2 - (99.36)^2}$$

$$= 86.900 \text{ lb}$$

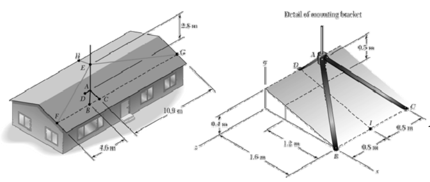
Then $$M_{DB} = (T_{CF})_{\text{perp.}}(d)$$

$$-1520.64 \text{ lb}\cdot\text{in.} = (86.900 \text{ lb})d$$

$$d = 17.4988 \text{ in.}$$

or $d = 17.50$ in. ◀

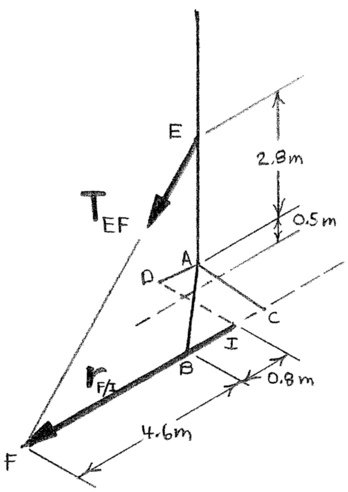
PROBLEM 3.64



In Problem 3.55, determine the perpendicular distance between cable *EF* and the line joining points *D* and *I*.

Problem 3.55: A mast is mounted on the roof of a house using bracket *ABCD* and is guyed by cables *EF*, *EG*, and *EH*. Knowing that the force exerted by cable *EF* at *E* is 66 N, determine the moment of that force about the line joining points *D* and *I*.

SOLUTION



Have $$M_{DI} = \lambda_{DI} \cdot (\mathbf{r}_{F/I} \times \mathbf{T}_{EF})$$

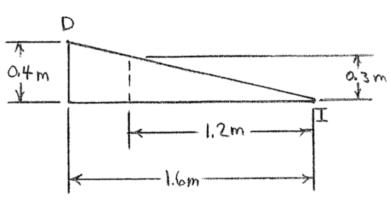
where $$\lambda_{DI} = \frac{(1.6\text{ m})\mathbf{i} - (0.4\text{ m})\mathbf{j}}{0.4\sqrt{17}\text{ m}} = \frac{1}{\sqrt{17}}(4\mathbf{i} - \mathbf{j})$$

$$\mathbf{r}_{F/I} = (5.4\text{ m})\mathbf{k}$$

$$\mathbf{T}_{EF} = \lambda_{EF} T_{EF} = \frac{(1.2\text{ m})\mathbf{i} - (3.6\text{ m})\mathbf{j} + (5.4\text{ m})\mathbf{k}}{6.6\text{ m}}(66\text{ N})$$

$$= 6[(2\text{ N})\mathbf{i} - (6\text{ N})\mathbf{j} + (9\text{ N})\mathbf{k}]$$

$$\therefore \quad M_{DI} = \frac{(6\text{ N})(5.4\text{ m})}{\sqrt{17}}\begin{vmatrix} 4 & -1 & 0 \\ 0 & 0 & 1 \\ 2 & -6 & 9 \end{vmatrix} = 172.879\text{ N}\cdot\text{m}$$



Only the perpendicular component of $\mathbf{T}_{EF}$ contributes to the moment of $\mathbf{T}_{EF}$ about line *DI*. The parallel component of $\mathbf{T}_{EF}$ will be used to find the perpendicular component.

Have

$$(T_{EF})_{\text{parallel}} = \lambda_{DI} \cdot \mathbf{T}_{EF}$$

$$= \frac{1}{\sqrt{17}}(4\mathbf{i} - \mathbf{j}) \cdot [(12\text{ N})\mathbf{i} - (36\text{ N})\mathbf{j} + (54\text{ N})\mathbf{k}]$$

$$= \frac{1}{\sqrt{17}}(48 + 36)\text{ N}$$

$$= \frac{84}{\sqrt{17}}\text{ N}$$

PROBLEM 3.64 CONTINUED

Since $\mathbf{T}_{EF} = (\mathbf{T}_{EF})_{\text{perp.}} + (\mathbf{T}_{EF})_{\text{parallel}}$

$$\therefore \quad (T_{EF})_{\text{perp.}} = \sqrt{(T_{EF})^2 - (T_{EF})^2_{\text{parallel}}}$$

$$= \sqrt{(66)^2 - \left(\frac{84}{\sqrt{17}}\right)^2}$$

$$= 62.777 \text{ N}$$

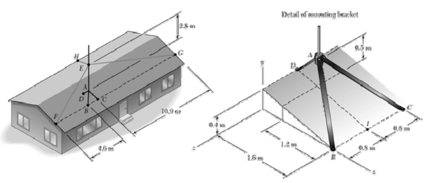
Then $$M_{DI} = (T_{EF})_{\text{perp.}}(d)$$

$$172.879 \text{ N}\cdot\text{m} = (62.777 \text{ N})(d)$$

$$d = 2.7539 \text{ m}$$

or $d = 2.75$ m ◀

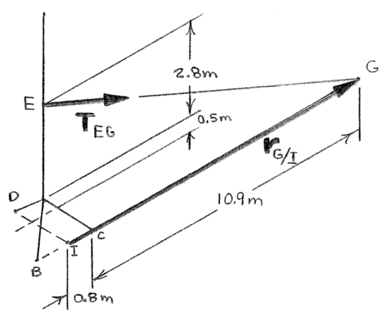
PROBLEM 3.65



In Problem 3.56, determine the perpendicular distance between cable *EG* and the line joining points *D* and *I*.

Problem 3.56: A mast is mounted on the roof of a house using bracket *ABCD* and is guyed by cables *EF*, *EG*, and *EH*. Knowing that the force exerted by cable *EG* at *E* is 61.5 N, determine the moment of that force about the line joining points *D* and *I*.

SOLUTION



Have $$M_{DI} = \lambda_{DI} \cdot \left[\mathbf{r}_{G/I} \times \mathbf{T}_{EG}\right]$$

where $$\lambda_{DI} = \frac{(1.6\text{ m})\mathbf{i} - (0.4\text{ m})\mathbf{j}}{0.4\sqrt{17}\text{ m}} = \frac{1}{\sqrt{17}}(4\mathbf{i} - \mathbf{j})$$

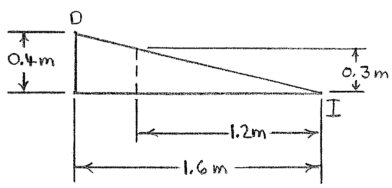
$$\mathbf{r}_{G/I} = -(10.9\text{ m} + 0.8\text{ m})\mathbf{k} = -(11.7\text{ m})\mathbf{k}$$

$$\mathbf{T}_{EG} = \lambda_{EG} T_{EG} = \frac{(1.2\text{ m})\mathbf{i} - (3.6\text{ m})\mathbf{j} - (11.7\text{ m})\mathbf{k}}{12.3\text{ m}}(61.5\text{ N})$$

$$= 5\left[(1.2\text{ N})\mathbf{i} - (3.6\text{ N})\mathbf{j} - (11.7\text{ N})\mathbf{k}\right]$$

$$\therefore\quad M_{DI} = \frac{(5\text{ N})(11.7\text{ m})}{\sqrt{17}}\begin{vmatrix} 4 & -1 & 0 \\ 0 & 0 & -1 \\ 1.2 & -3.6 & -11.7 \end{vmatrix}$$

$$= -187.286\text{ N}\cdot\text{m}$$



Only the perpendicular component of $\mathbf{T}_{EG}$ contributes to the moment of $\mathbf{T}_{EG}$ about line *DI*. The parallel component of $\mathbf{T}_{EG}$ will be used to find the perpendicular component.

Have

$$T_{EG(\text{parallel})} = \lambda_{DI} \cdot \mathbf{T}_{EG}$$

$$= \frac{1}{\sqrt{17}}(4\mathbf{i} - \mathbf{j}) \cdot 5\left[(1.2\text{ N})\mathbf{i} - (3.6\text{ N})\mathbf{j} - (11.7\text{ N})\mathbf{k}\right]$$

$$= \frac{5}{\sqrt{17}}(4.8 + 3.6)\text{N}$$

$$= \frac{42}{\sqrt{17}}\text{ N}$$

PROBLEM 3.65 CONTINUED

Since $\mathbf{T}_{EF} = (\mathbf{T}_{EG})_{\text{perp.}} + (\mathbf{T}_{EG})_{\text{parallel}}$

$$\therefore \quad (T_{EG})_{\text{perp.}} = \sqrt{(T_{EG})^2 - (T_{EG})^2_{\text{parallel}}}$$

$$= \sqrt{(61.5)^2 - \left(\frac{42}{\sqrt{17}}\right)^2}$$

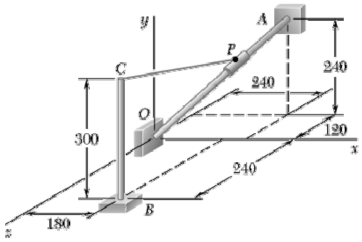
$$= 60.651 \text{ N}$$

Then $$M_{DI} = (T_{EG})_{\text{perp.}}(d)$$

$$187.286 \text{ N}\cdot\text{m} = (60.651 \text{ N})(d)$$

$$d = 3.0880 \text{ m}$$

or $d = 3.09$ m ◀



PROBLEM 3.66

In Problem 3.41, determine the perpendicular distance between post *BC* and the line connecting points *O* and *A*.

Problem 3.41: Slider *P* can move along rod *OA*. An elastic cord *PC* is attached to the slider and to the vertical member *BC*. Knowing that the distance from *O* to *P* is 0.12 m and the tension in the cord is 30 N, determine (*a*) the angle between the elastic cord and the rod *OA*, (*b*) the projection on *OA* of the force exerted by cord *PC* at point *P*.

SOLUTION

Assume post *BC* is represented by a force of magnitude F_{BC}

where
$$\mathbf{F}_{BC} = F_{BC}\mathbf{j}$$

Have
$$M_{OA} = \lambda_{OA} \cdot \left(\mathbf{r}_{B/O} \times \mathbf{F}_{BC}\right)$$

where
$$\lambda_{OA} = \frac{(0.24\text{ m})\mathbf{i} + (0.24\text{ m})\mathbf{j} - (0.12\text{ m})\mathbf{k}}{0.36\text{ m}} = \frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{1}{3}\mathbf{k}$$

$$\mathbf{r}_{B/O} = (0.18\text{ m})\mathbf{i} + (0.24\text{ m})\mathbf{k}$$

$$\therefore\quad M_{OA} = \frac{1}{3}F_{BC}\begin{vmatrix} 2 & 2 & -1 \\ 0.18 & 0 & 0.24 \\ 0 & 1 & 0 \end{vmatrix} = \frac{F_{BC}}{3}(-0.48 - 0.18) = -0.22F_{BC}$$

Only the perpendicular component of $\mathbf{F}_{BC}$ contributes to the moment of $\mathbf{F}_{BC}$ about line *OA*. The parallel component will be found first so that the perpendicular component of $\mathbf{F}_{BC}$ can be determined.

$$F_{BC(\text{parallel})} = \lambda_{OA} \cdot \mathbf{F}_{BC} = \left(\frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{1}{3}\mathbf{k}\right) \cdot F_{BC}\mathbf{j}$$

$$= \frac{2}{3}F_{BC}$$

Since
$$\mathbf{F}_{BC} = (\mathbf{F}_{BC})_{\text{parallel}} + (\mathbf{F}_{BC})_{\text{perp.}}$$

$$(F_{BC})_{\text{perp.}} = \sqrt{(F_{BC})^2 - (F_{BC})^2_{\text{parallel}}} = \sqrt{(F_{BC})^2 - \left(\frac{2F_{BC}}{3}\right)^2}$$

$$= 0.74536F_{BC}$$

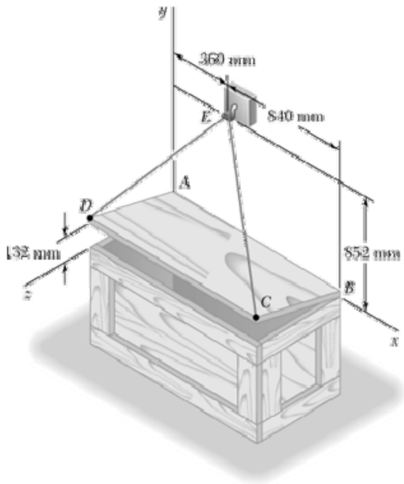
Then
$$|M_{OA}| = (F_{BC})_{\text{perp.}}(d)$$

$$0.22F_{BC} = (0.74536F_{BC})d$$

$$d = 0.29516\text{ m}$$

or $d = 295$ mm ◀

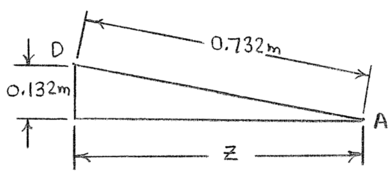
PROBLEM 3.67



In Problem 3.45, determine the perpendicular distance between cord *DE* and the *y* axis.

Problem 3.45: The 0.732×1.2-m lid *ABCD* of a storage bin is hinged along side *AB* and is held open by looping cord *DEC* over a frictionless hook at *E*. If the tension in the cord is 54 N, determine the moment about each of the coordinate axes of the force exerted by the cord at *D*.

SOLUTION



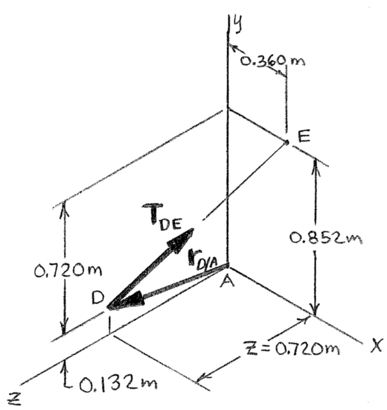
First note

$$z = \sqrt{(0.732)^2 - (0.132)^2} \text{ m}$$

$$= 0.720 \text{ m}$$

Have $$M_y = \mathbf{j} \cdot (\mathbf{r}_{D/A} \times \mathbf{T}_{DE})$$

where $$\mathbf{r}_{D/A} = (0.132\mathbf{j} + 0.720\mathbf{k}) \text{ m}$$



$$\mathbf{T}_{DE} = \lambda_{DE} T_{DE}$$

$$= \frac{(0.360 \text{ m})\mathbf{i} + (0.732 \text{ m})\mathbf{j} - (0.720 \text{ m})\mathbf{k}}{1.08 \text{ m}}(54 \text{ N})$$

$$= (18 \text{ N})\mathbf{i} + (36 \text{ N})\mathbf{j} - (36 \text{ N})\mathbf{k}$$

$$\therefore \quad M_y = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0.132 & 0.720 \\ 18 & 36 & -36 \end{vmatrix} = 12.96 \text{ N}\cdot\text{m}$$

Only the perpendicular component of $\mathbf{T}_{DE}$ contributes to the moment of $\mathbf{T}_{DE}$ about the *y*-axis. The parallel component will be found first so that the perpendicular component of $\mathbf{T}_{DE}$ can be determined.

$$T_{DE(\text{parallel})} = \mathbf{j} \cdot \mathbf{T}_{DE} = 36 \text{ N}$$

PROBLEM 3.67 CONTINUED

Since $$\left(\mathbf{T}_{DE}\right) = \left(\mathbf{T}_{DE}\right)_{\text{parallel}} + \left(\mathbf{T}_{DE}\right)_{\text{perp.}}$$

$$\left(T_{DE}\right)_{\text{perp.}} = \sqrt{\left(T_{DE}\right)^2 - \left(T_{DE}\right)^2_{\text{parallel}}}$$

$$= \sqrt{(54)^2 - (36)^2} = 40.249 \text{ N}$$

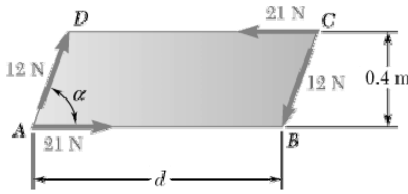
Then $$M_y = \left(T_{DE}\right)_{\text{perp.}}(d)$$

$$12.96 \text{ N}\cdot\text{m} = (40.249 \text{ N})(d)$$

$$d = 0.32199 \text{ m}$$

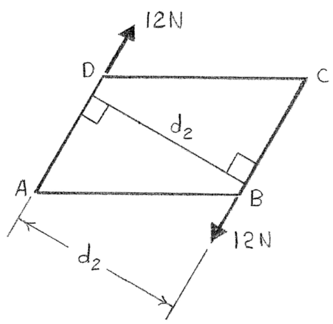
or $d = 322$ mm ◀

PROBLEM 3.68



A plate in the shape of a parallelogram is acted upon by two couples. Determine (*a*) the moment of the couple formed by the two 21-N forces, (*b*) the perpendicular distance between the 12-N forces if the resultant of the two couples is zero, (*c*) the value of α if the resultant couple is $1.8\ \text{N}\cdot\text{m}$ clockwise and *d* is 1.05 m.

SOLUTION



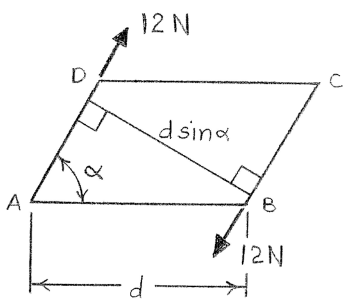
(*a*) Have $$M_1 = d_1F_1$$

where $$d_1 = 0.4\ \text{m}$$

$$F_1 = 21\ \text{N}$$

$$\therefore\quad M_1 = (0.4\ \text{m})(21\ \text{N}) = 8.4\ \text{N}\cdot\text{m}$$

$$\text{or}\quad \mathbf{M}_1 = 8.40\ \text{N}\cdot\text{m}\ \circlearrowleft \blacktriangleleft$$



(*b*) Have $$\mathbf{M}_1 + \mathbf{M}_2 = 0$$

or $$8.40\ \text{N}\cdot\text{m} - d_2(12\ \text{N}) = 0$$

$$\therefore\quad d_2 = 0.700\ \text{m} \blacktriangleleft$$

(*c*) Have $$\mathbf{M}_{\text{total}} = \mathbf{M}_1 + \mathbf{M}_2$$

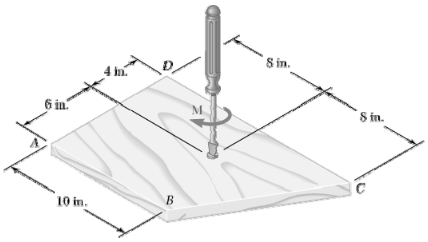
or $$1.8\ \text{N}\cdot\text{m} = 8.40\ \text{N}\cdot\text{m} - (1.05\ \text{m})(\sin\alpha)(12\ \text{N})$$

$$\therefore\quad \sin\alpha = 0.52381$$

and $$\alpha = 31.588°$$

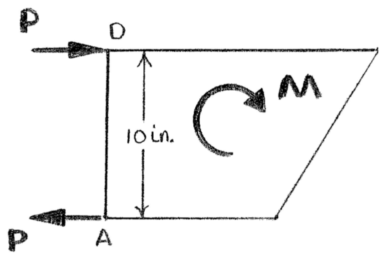
$$\text{or}\quad \alpha = 31.6° \blacktriangleleft$$

PROBLEM 3.69



A couple **M** of magnitude $10\text{ lb}\cdot\text{ft}$ is applied to the handle of a screwdriver to tighten a screw into a block of wood. Determine the magnitudes of the two smallest horizontal forces that are equivalent to **M** if they are applied (*a*) at corners *A* and *D*, (*b*) at corners *B* and *C*, (*c*) anywhere on the block.

SOLUTION

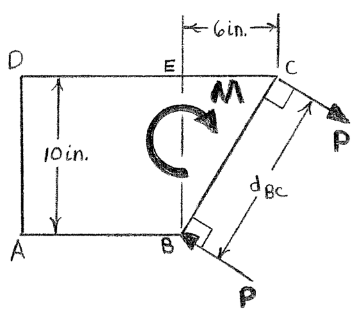


(*a*) Have $$M = Pd$$

or $$10\text{ lb}\cdot\text{ft} = P(10\text{ in.})\left(\frac{1\text{ ft}}{12\text{ in.}}\right)$$

$$\therefore \quad P = 12\text{ lb}$$

or $P_{\text{min}} = 12.00\text{ lb}$ ◀



(*b*) $$d_{BC} = \sqrt{(BE)^2 + (EC)^2}$$

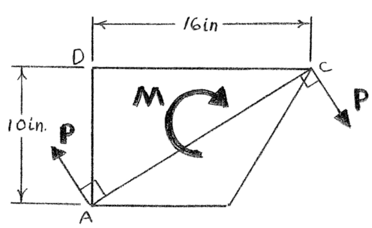
$$= \sqrt{(10\text{ in.})^2 + (6\text{ in.})^2} = 11.6619\text{ in.}$$

Have $$M = Pd$$

$$10\text{ lb}\cdot\text{ft} = P(11.6619\text{ in.})\left(\frac{1\text{ ft}}{12\text{ in.}}\right)$$

$$P = 10.2899\text{ lb}$$

or $P = 10.29\text{ lb}$ ◀



(*c*) $$d_{AC} = \sqrt{(AD)^2 + (DC)^2}$$

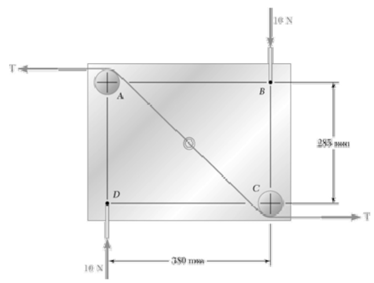
$$= \sqrt{(10\text{ in.})^2 + (16\text{ in.})^2} = 2\sqrt{89}\text{ in.}$$

Have $$M = Pd_{AC}$$

$$10\text{ lb}\cdot\text{ft} = P\left(2\sqrt{89}\text{ in.}\right)\left(\frac{1\text{ ft}}{12\text{ in.}}\right)$$

$$P = 6.3600\text{ lb}$$

or $P = 6.36\text{ lb}$ ◀



PROBLEM 3.70

Two 60-mm-diameter pegs are mounted on a steel plate at *A* and *C*, and two rods are attached to the plate at *B* and *D*. A cord is passed around the pegs and pulled as shown, while the rods exert on the plate 10-N forces as indicated. (*a*) Determine the resulting couple acting on the plate when *T* = 36 N. (*b*) If only the cord is used, in what direction should it be pulled to create the same couple with the minimum tension in the cord? (*c*) Determine the value of that minimum tension.

SOLUTION

(*a*) Have

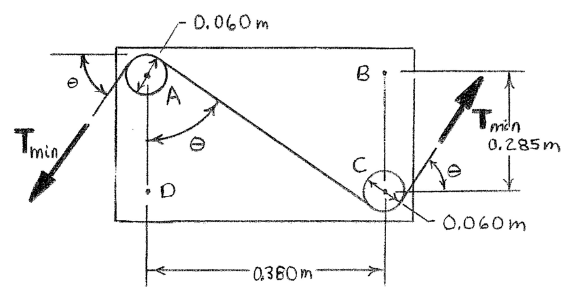
$$M = \Sigma(Fd)$$

$$= (36\text{ N})(0.345\text{ m}) - (10\text{ N})(0.380\text{ m})$$

$$= 8.62\text{ N}\cdot\text{m}$$

$$\mathbf{M} = 8.62\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

(*b*)



Have

$$M = Td = 8.62\text{ N}\cdot\text{m}$$

For *T* to be minimum, *d* must be maximum.

$$\therefore\ T_{\text{min}} \text{ must be perpendicular to line } AC$$

$$\tan\theta = \frac{0.380\text{ m}}{0.285\text{ m}} = 1.33333$$

and

$$\theta = 53.130°$$

$$\text{or } \theta = 53.1° \blacktriangleleft$$

(*c*) Have

$$M = T_{\text{min}} d_{\text{max}}$$

where

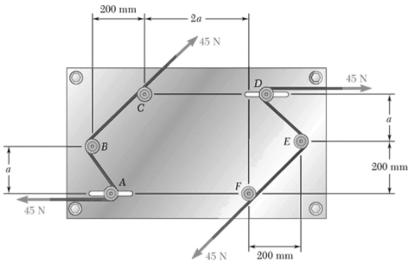
$$M = 8.62\text{ N}\cdot\text{m}$$

$$d_{\text{max}} = \left[\sqrt{(0.380)^2 + (0.285)^2} + 2(0.030)\right]\text{ m} = 0.535\text{ m}$$

$$\therefore\ 8.62\text{ N}\cdot\text{m} = T_{\text{min}}(0.535\text{ m})$$

$$T_{\text{min}} = 16.1121\text{ N}$$

$$\text{or } T_{\text{min}} = 16.11\text{ N} \blacktriangleleft$$



PROBLEM 3.71

The steel plate shown will support six 50-mm-diameter idler rollers mounted on the plate as shown. Two flat belts pass around the rollers, and rollers A and D will be adjusted so that the tension in each belt is 45 N. Determine (a) the resultant couple acting on the plate if $a = 0.2$ m, (b) the value of a so that the resultant couple acting on the plate is $54\ \text{N}\cdot\text{m}$ clockwise.

SOLUTION

(a) Note when $a = 0.2$ m, $\mathbf{r}_{C/F}$ is perpendicular to the inclined 45 N forces.

Have

$$M = \Sigma(Fd)$$

$$= -(45\ \text{N})\left[a + 0.2\ \text{m} + 2(0.025\ \text{m})\right]$$

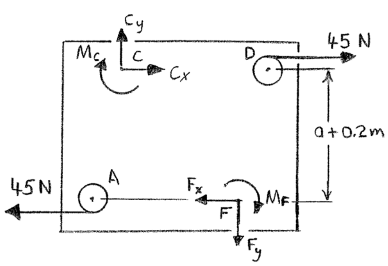
$$-(45\ \text{N})\left[2a\sqrt{2} + 2(0.025\ \text{m})\right]$$

For $a = 0.2$ m,

$$M = -(45\ \text{N})(0.450\ \text{m} + 0.61569\ \text{m})$$

$$= -47.956\ \text{N}\cdot\text{m}$$

or $\mathbf{M} = 48.0\ \text{N}\cdot\text{m}$ ◀



(b) $\mathbf{M} = 54.0\ \text{N}\cdot\text{m}$

M = Moment of couple due to horizontal forces at A and D

+ Moment of force-couple systems at C and F about C.

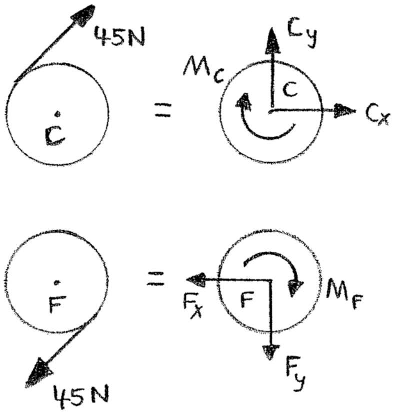
$$-54.0\ \text{N}\cdot\text{m} = -45\ \text{N}\left[a + 0.2\ \text{m} + 2(0.025\ \text{m})\right]$$

$$+\left[M_C + M_F + F_x(a + 0.2\ \text{m}) + F_y(2a)\right]$$

where $\quad M_C = -(45\ \text{N})(0.025\ \text{m}) = -1.125\ \text{N}\cdot\text{m}$

$$M_F = M_C = -1.125\ \text{N}\cdot\text{m}$$

PROBLEM 3.71 CONTINUED



$$F_x = \frac{-45}{\sqrt{2}}\ \text{N}$$

$$F_y = \frac{-45}{\sqrt{2}}\ \text{N}$$

$$\therefore \quad -54.0\ \text{N}\cdot\text{m} = -45\ \text{N}\left(a + 0.25\ \text{m}\right) - 1.125\ \text{N}\cdot\text{m} - 1.125\ \text{N}\cdot\text{m}$$

$$\frac{-45\ \text{N}}{\sqrt{2}}\left(a + 0.2\ \text{m}\right) - \frac{45\ \text{N}}{\sqrt{2}}\left(2a\right)$$

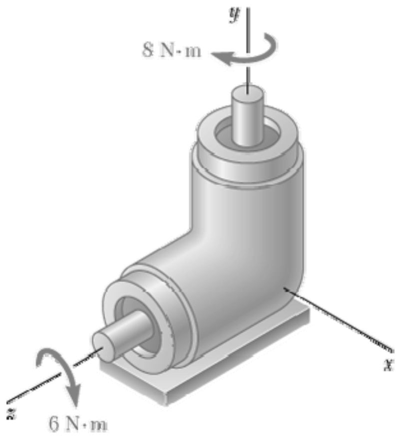
$$1.20 = a + 0.25 + 0.025 + 0.025 + \frac{a}{\sqrt{2}} + \frac{0.20}{\sqrt{2}} + \frac{2a}{\sqrt{2}}$$

$$3.1213a = 0.75858$$

$$a = 0.24303\ \text{m}$$

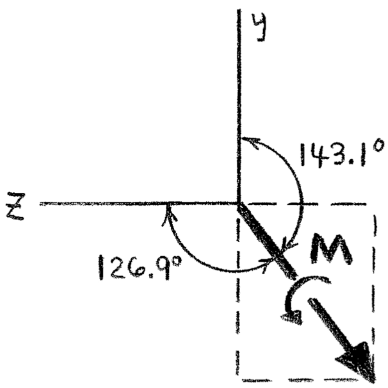
or $a = 243\ \text{mm}$ ◀

PROBLEM 3.72



The shafts of an angle drive are acted upon by the two couples shown. Replace the two couples with a single equivalent couple, specifying its magnitude and the direction of its axis.

SOLUTION



Based on
$$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2$$

where

$$\mathbf{M}_1 = -(8\ \text{N}\cdot\text{m})\,\mathbf{j}$$

$$\mathbf{M}_2 = -(6\ \text{N}\cdot\text{m})\mathbf{k}$$

$$\therefore\quad \mathbf{M} = -(8\ \text{N}\cdot\text{m})\,\mathbf{j} - (6\ \text{N}\cdot\text{m})\mathbf{k}$$

and
$$|\mathbf{M}| = \sqrt{(8)^2 + (6)^2} = 10\ \text{N}\cdot\text{m}$$

$$\text{or } M = 10.00\ \text{N}\cdot\text{m} \blacktriangleleft$$

$$\lambda = \frac{\mathbf{M}}{|\mathbf{M}|} = \frac{-(8\ \text{N}\cdot\text{m})\,\mathbf{j} - (6\ \text{N}\cdot\text{m})\mathbf{k}}{10\ \text{N}\cdot\text{m}} = -0.8\mathbf{j} - 0.6\mathbf{k}$$

or
$$\mathbf{M} = |\mathbf{M}|\lambda = (10\ \text{N}\cdot\text{m})(-0.8\mathbf{j} - 0.6\mathbf{k})$$

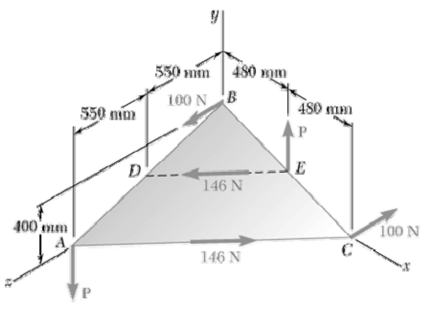
$$\cos\theta_x = 0 \qquad \therefore\quad \theta_x = 90°$$

$$\cos\theta_y = -0.8 \qquad \therefore\quad \theta_y = 143.130°$$

$$\cos\theta_z = -0.6 \qquad \therefore\quad \theta_z = 126.870°$$

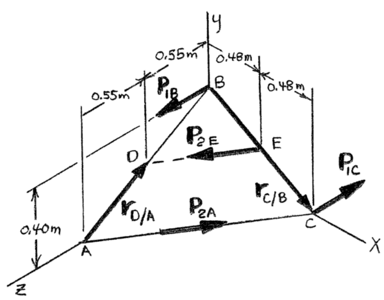
$$\text{or } \theta_x = 90.0°,\ \ \theta_y = 143.1°,\ \ \theta_z = 126.9° \blacktriangleleft$$

PROBLEM 3.73



Knowing that $P = 0$, replace the two remaining couples with a single equivalent couple, specifying its magnitude and the direction of its axis.

SOLUTION



Have $\quad \mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2$

where $\quad \mathbf{M}_1 = \mathbf{r}_{C/B} \times \mathbf{P}_{1C}$

$$\mathbf{r}_{C/B} = (0.96\text{ m})\mathbf{i} - (0.40\text{ m})\mathbf{j}$$

$$\mathbf{P}_{1C} = -(100\text{ N})\mathbf{k}$$

$$\therefore\ \mathbf{M}_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.96 & -0.40 & 0 \\ 0 & 0 & -100 \end{vmatrix} = (40\text{ N}\cdot\text{m})\mathbf{i} + (96\text{ N}\cdot\text{m})\mathbf{j}$$

Also, $\quad \mathbf{M}_2 = \mathbf{r}_{D/A} \times \mathbf{P}_{2E}$

$$\mathbf{r}_{D/A} = (0.20\text{ m})\mathbf{j} - (0.55\text{ m})\mathbf{k}$$

$$\mathbf{P}_{2E} = \lambda_{ED} P_{2E}$$

$$= \frac{-(0.48\text{ m})\mathbf{i} + (0.55\text{ m})\mathbf{k}}{\sqrt{(0.48)^2 + (0.55)^2}\text{ m}}(146\text{ N})$$

$$= -(96\text{ N})\mathbf{i} + (110\text{ N})\mathbf{k}$$

$$\therefore\ \mathbf{M}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.20 & -0.55 \\ -96 & 0 & 110 \end{vmatrix}\text{ N}\cdot\text{m}$$

$$= (22.0\text{ N}\cdot\text{m})\mathbf{i} + (52.8\text{ N}\cdot\text{m})\mathbf{j} + (19.2\text{ N}\cdot\text{m})\mathbf{k}$$

PROBLEM 3.73 CONTINUED

and $$\mathbf{M} = \left[(40\text{ N}\cdot\text{m})\mathbf{i} + (96\text{ N}\cdot\text{m})\mathbf{j}\right] + \left[(22.0\text{ N}\cdot\text{m})\mathbf{i} + (52.8\text{ N}\cdot\text{m})\mathbf{j} + (19.2\text{ N}\cdot\text{m})\mathbf{k}\right]$$

$$= (62.0\text{ N}\cdot\text{m})\mathbf{i} + (148.8\text{ N}\cdot\text{m})\mathbf{j} + (19.2\text{ N}\cdot\text{m})\mathbf{k}$$

$$|\mathbf{M}| = \sqrt{M_x^2 + M_y^2 + M_z^2} = \sqrt{(62.0)^2 + (148.8)^2 + (19.2)^2}$$

$$= 162.339\text{ N}\cdot\text{m}$$

or $M = 162.3\text{ N}\cdot\text{m}$ ◀

$$\lambda = \frac{\mathbf{M}}{|\mathbf{M}|} = \frac{62.0\mathbf{i} + 148.8\mathbf{j} + 19.2\mathbf{k}}{162.339}$$

$$= 0.38192\mathbf{i} + 0.91660\mathbf{j} + 0.118271\mathbf{k}$$

$$\cos\theta_x = 0.38192 \qquad \therefore \quad \theta_x = 67.547°$$

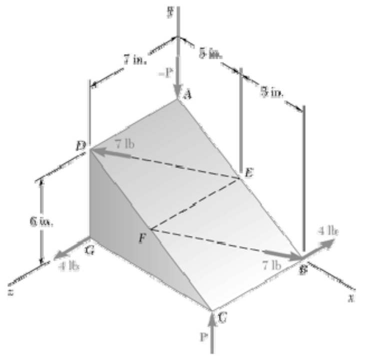
or $\theta_x = 67.5°$ ◀

$$\cos\theta_y = 0.91660 \qquad \therefore \quad \theta_y = 23.566°$$

or $\theta_y = 23.6°$ ◀

$$\cos\theta_z = 0.118271 \qquad \therefore \quad \theta_z = 83.208°$$

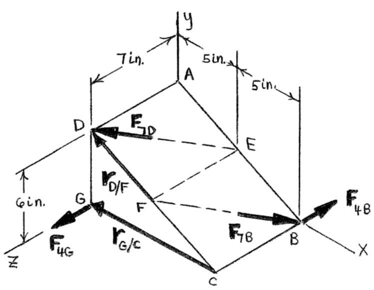
or $\theta_z = 83.2°$ ◀



PROBLEM 3.74

Knowing that $P = 0$, replace the two remaining couples with a single equivalent couple, specifying its magnitude and the direction of its axis.

SOLUTION



Have $\quad \mathbf{M} = \mathbf{M}_4 + \mathbf{M}_7$

where $\quad \mathbf{M}_4 = \mathbf{r}_{G/C} \times \mathbf{F}_{4G}$

$$\mathbf{r}_{G/C} = -(10 \text{ in.})\mathbf{i}$$

$$\mathbf{F}_{4G} = (4 \text{ lb})\mathbf{k}$$

$$\therefore \quad \mathbf{M}_4 = -(10 \text{ in.})\mathbf{i} \times (4 \text{ lb})\mathbf{k} = (40 \text{ lb}\cdot\text{in.})\mathbf{j}$$

Also, $\quad \mathbf{M}_7 = \mathbf{r}_{D/F} \times \mathbf{F}_{7D}$

$$\mathbf{r}_{D/F} = -(5 \text{ in.})\mathbf{i} + (3 \text{ in.})\mathbf{j}$$

$$\mathbf{F}_{7D} = \lambda_{ED} F_{7D}$$

$$= \frac{-(5 \text{ in.})\mathbf{i} + (3 \text{ in.})\mathbf{j} + (7 \text{ in.})\mathbf{k}}{\sqrt{(5)^2 + (3)^2 + (7)^2} \text{ in.}}(7 \text{ lb})$$

$$= \frac{7 \text{ lb}}{\sqrt{83}}(-5\mathbf{i} + 3\mathbf{j} + 7\mathbf{k})$$

$$\therefore \quad \mathbf{M}_7 = \frac{7 \text{ lb}\cdot\text{in.}}{\sqrt{83}}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -5 & 3 & 0 \\ -5 & 3 & 7 \end{vmatrix} = \frac{7 \text{ lb}\cdot\text{in.}}{\sqrt{83}}(21\mathbf{i} + 35\mathbf{j} + 0\mathbf{k})$$

$$= 0.76835(21\mathbf{i} + 35\mathbf{j}) \text{ lb}\cdot\text{in.}$$

PROBLEM 3.74 CONTINUED

and $$\mathbf{M} = \left[(40\text{ lb}\cdot\text{in.})\mathbf{j}\right] + \left[0.76835(21\mathbf{i} + 35\mathbf{j})\text{ lb}\cdot\text{in.}\right]$$

$$= (16.1353\text{ lb}\cdot\text{in.})\mathbf{i} + (66.892\text{ lb}\cdot\text{in.})\mathbf{j}$$

$$|\mathbf{M}| = \sqrt{(M_x)^2 + (M_y)^2} = \sqrt{(16.1353)^2 + (66.892)^2}$$

$$= 68.811\text{ lb}\cdot\text{in.}$$

or $M = 68.8\text{ lb}\cdot\text{in.}$ ◀

$$\lambda = \frac{\mathbf{M}}{|\mathbf{M}|} = \frac{(16.1353\text{ lb}\cdot\text{in.})\mathbf{i} + (66.892\text{ lb}\cdot\text{in.})\mathbf{j}}{68.811\text{ lb}\cdot\text{in.}}$$

$$= 0.23449\mathbf{i} + 0.97212\mathbf{j}$$

$$\cos\theta_x = 0.23449 \qquad \therefore \quad \theta_x = 76.438°$$

or $\theta_x = 76.4°$ ◀

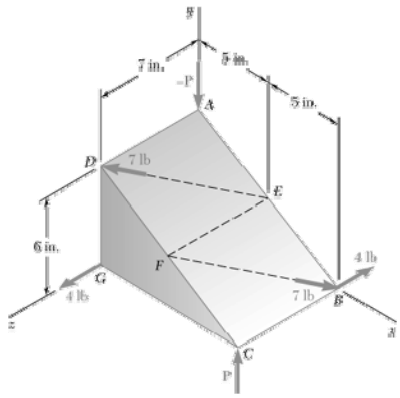
$$\cos\theta_y = 0.97212 \qquad \therefore \quad \theta_y = 13.5615°$$

or $\theta_y = 13.56°$ ◀

$$\cos\theta_z = 0.0 \qquad \therefore \quad \theta_z = 90°$$

or $\theta_z = 90.0°$ ◀

PROBLEM 3.75



Knowing that $P = 5$ lb, replace the three couples with a single equivalent couple, specifying its magnitude and the direction of its axis.

SOLUTION

Have
$$\mathbf{M} = \mathbf{M}_4 + \mathbf{M}_7 + \mathbf{M}_5$$

where

$$\mathbf{M}_4 = \mathbf{r}_{G/C} \times \mathbf{F}_{4G} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -10 & 0 & 0 \\ 0 & 0 & 4 \end{vmatrix} \text{ lb}\cdot\text{in.} = (40 \text{ lb}\cdot\text{in.})\,\mathbf{j}$$

$$\mathbf{M}_7 = \mathbf{r}_{D/F} \times \mathbf{F}_{7D} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -5 & 3 & 0 \\ -5 & 3 & 7 \end{vmatrix} \left(\frac{7}{\sqrt{83}}\right) \text{ lb}\cdot\text{in.} = 0.76835(21\mathbf{i} + 35\mathbf{j}) \text{ lb}\cdot\text{in.}$$

(See Solution to Problem 3.74.)

$$\mathbf{M}_5 = \mathbf{r}_{C/A} \times \mathbf{F}_{5C} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 10 & -6 & 7 \\ 0 & 5 & 0 \end{vmatrix} \text{ lb}\cdot\text{in.} = -(35 \text{ lb}\cdot\text{in.})\mathbf{i} + (50 \text{ lb}\cdot\text{in.})\mathbf{k}$$

$$\therefore \quad \mathbf{M} = \left[(16.1353 - 35)\mathbf{i} + (40 + 26.892)\,\mathbf{j} + (50)\mathbf{k}\right] \text{ lb}\cdot\text{in.}$$

$$= -(18.8647 \text{ lb}\cdot\text{in.})\mathbf{i} + (66.892 \text{ lb}\cdot\text{in.})\,\mathbf{j} + (50 \text{ lb}\cdot\text{in.})\mathbf{k}$$

$$|\mathbf{M}| = \sqrt{M_x^2 + M_y^2 + M_z^2} = \sqrt{(18.8647)^2 + (66.892)^2 + (50)^2} = 85.618 \text{ lb}\cdot\text{in.}$$

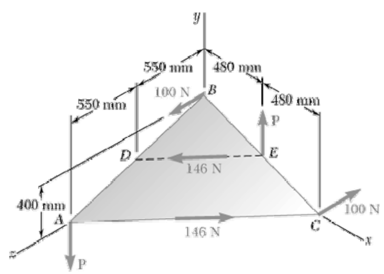
or $M = 85.6$ lb·in. ◀

$$\lambda = \frac{\mathbf{M}}{|\mathbf{M}|} = \frac{-18.8647\mathbf{i} + 66.892\mathbf{j} + 50\mathbf{k}}{85.618} = -0.22034\mathbf{i} + 0.78129\mathbf{j} + 0.58399\mathbf{k}$$

$\cos\theta_x = -0.22034 \qquad \therefore \quad \theta_x = 102.729°$ or $\theta_x = 102.7°$ ◀

$\cos\theta_y = 0.78129 \qquad \therefore \quad \theta_y = 38.621°$ or $\theta_y = 38.6°$ ◀

$\cos\theta_z = 0.58399 \qquad \therefore \quad \theta_z = 54.268°$ or $\theta_z = 54.3°$ ◀



PROBLEM 3.76

Knowing that $P = 210$ N, replace the three couples with a single equivalent couple, specifying its magnitude and the direction of its axis.

SOLUTION

Have $$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_P$$

where $$\mathbf{M}_1 = \mathbf{r}_{C/B} \times \mathbf{P}_{1C} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.96 & -0.40 & 0 \\ 0 & 0 & -100 \end{vmatrix} = (40\text{ N}\cdot\text{m})\mathbf{i} + (96\text{ N}\cdot\text{m})\mathbf{j}$$

$$\mathbf{M}_2 = \mathbf{r}_{D/A} \times \mathbf{P}_{2E} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.20 & -0.55 \\ -96 & 0 & 110 \end{vmatrix} = (22.0\text{ N}\cdot\text{m})\mathbf{i} + (52.8\text{ N}\cdot\text{m})\mathbf{j} + (19.2\text{ N}\cdot\text{m})\mathbf{k}$$

(See Solution to Problem 3.73.)

$$\mathbf{M}_P = \mathbf{r}_{E/A} \times \mathbf{P}_E = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.48 & 0.20 & -1.10 \\ 0 & 210 & 0 \end{vmatrix} = (231\text{ N}\cdot\text{m})\mathbf{i} + (100.8\text{ N}\cdot\text{m})\mathbf{k}$$

$$\therefore\ \mathbf{M} = \left[(40 + 22 + 231)\mathbf{i} + (96 + 52.8)\mathbf{j} + (19.2 + 100.8)\mathbf{k}\right]\text{ N}\cdot\text{m}$$

$$= (293\text{ N}\cdot\text{m})\mathbf{i} + (148.8\text{ N}\cdot\text{m})\mathbf{j} + (120\text{ N}\cdot\text{m})\mathbf{k}$$

$$|\mathbf{M}| = \sqrt{M_x^2 + M_y^2 + M_z^2} = \sqrt{(293)^2 + (148.8)^2 + (120)^2} = 349.84\text{ N}\cdot\text{m}$$

or $M = 350\text{ N}\cdot\text{m}$ ◀

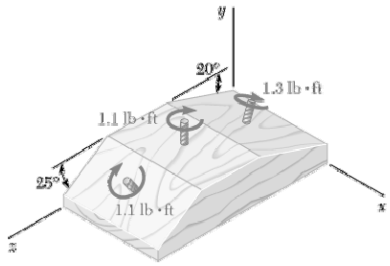
$$\lambda = \frac{\mathbf{M}}{|\mathbf{M}|} = \frac{293\mathbf{i} + 148.8\mathbf{j} + 120\mathbf{k}}{349.84} = 0.83752\mathbf{i} + 0.42533\mathbf{j} + 0.34301\mathbf{k}$$

$\cos\theta_x = 0.83752 \quad \therefore\ \theta_x = 33.121°$ or $\theta_x = 33.1°$ ◀

$\cos\theta_y = 0.42533 \quad \therefore\ \theta_y = 64.828°$ or $\theta_y = 64.8°$ ◀

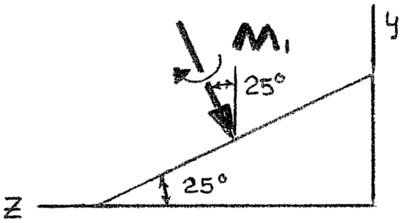
$\cos\theta_z = 0.34301 \quad \therefore\ \theta_z = 69.940°$ or $\theta_z = 69.9°$ ◀

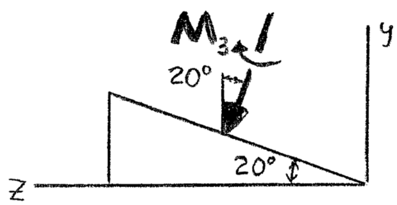
PROBLEM 3.77



In a manufacturing operation, three holes are drilled simultaneously in a workpiece. Knowing that the holes are perpendicular to the surfaces of the workpiece, replace the couples applied to the drills with a single equivalent couple, specifying its magnitude and the direction of its axis.

SOLUTION





Have $$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3$$

where $$\mathbf{M}_1 = -(1.1\text{ lb}\cdot\text{ft})(\cos 25°\mathbf{j} + \sin 25°\mathbf{k})$$

$$\mathbf{M}_2 = -(1.1\text{ lb}\cdot\text{ft})\mathbf{j}$$

$$\mathbf{M}_3 = -(1.3\text{ lb}\cdot\text{ft})(\cos 20°\mathbf{j} - \sin 20°\mathbf{k})$$

$$\therefore\quad \mathbf{M} = (-0.99694 - 1.1 - 1.22160)\mathbf{j} + (-0.46488 + 0.44463)\mathbf{k}$$

$$= -(3.3185\text{ lb}\cdot\text{ft})\mathbf{j} - (0.020254\text{ lb}\cdot\text{ft})\mathbf{k}$$

and $$|\mathbf{M}| = \sqrt{M_x^2 + M_y^2 + M_z^2} = \sqrt{(0)^2 + (3.3185)^2 + (0.020254)^2}$$

$$= 3.3186\text{ lb}\cdot\text{ft}$$

$$\text{or } M = 3.32\text{ lb}\cdot\text{ft} \blacktriangleleft$$

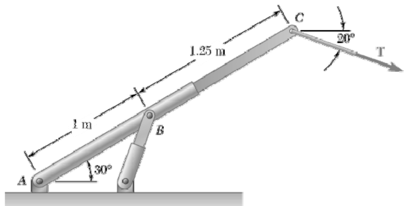
$$\lambda = \frac{\mathbf{M}}{|\mathbf{M}|} = \frac{(0)\mathbf{i} - 3.3185\mathbf{j} - 0.020254\mathbf{k}}{3.3186}$$

$$= -0.99997\mathbf{j} - 0.0061032\mathbf{k}$$

$$\cos\theta_x = 0 \qquad \therefore\ \theta_x = 90° \qquad \text{or } \theta_x = 90.0° \blacktriangleleft$$

$$\cos\theta_y = -0.99997 \qquad \therefore\ \theta_y = 179.555° \qquad \text{or } \theta_y = 179.6° \blacktriangleleft$$

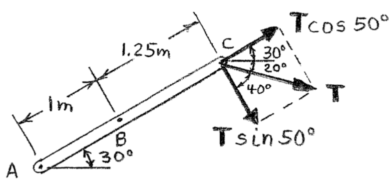
$$\cos\theta_z = -0.0061032 \qquad \therefore\ \theta_z = 90.349° \qquad \text{or } \theta_z = 90.3° \blacktriangleleft$$



PROBLEM 3.78

The tension in the cable attached to the end C of an adjustable boom ABC is 1000 N. Replace the force exerted by the cable at C with an equivalent force-couple system (a) at A, (b) at B.

SOLUTION



(a) Based on $\qquad \Sigma F: \quad F_A = T = 1000 \text{ N}$

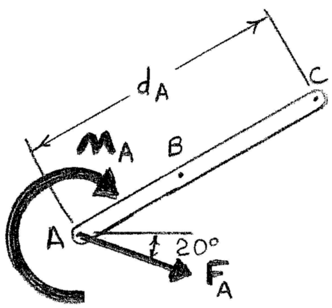
or $\mathbf{F}_A = 1000 \text{ N} \searrow 20°$ ◀

$$\Sigma M_A: \quad M_A = (T\sin 50°)(d_A)$$

$$= (1000 \text{ N})\sin 50°(2.25 \text{ m})$$

$$= 1723.60 \text{ N}\cdot\text{m}$$

or $\mathbf{M}_A = 1724 \text{ N}\cdot\text{m} \circlearrowright$ ◀



(b) Based on $\qquad \Sigma F: \quad F_B = T = 1000 \text{ N}$

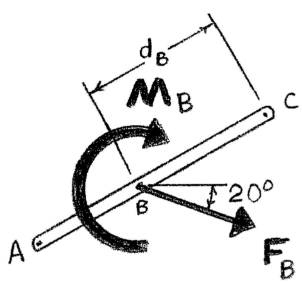
or $\mathbf{F}_B = 1000 \text{ N} \searrow 20°$ ◀

$$\Sigma M_B: \quad M_B = (T\sin 50°)(d_B)$$

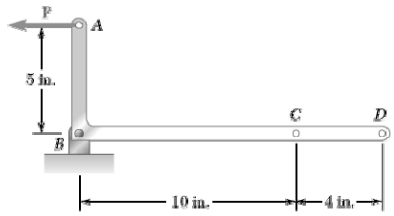
$$= (1000 \text{ N})\sin 50°(1.25 \text{ m})$$

$$= 957.56 \text{ N}\cdot\text{m}$$

or $\mathbf{M}_B = 958 \text{ N}\cdot\text{m} \circlearrowright$ ◀

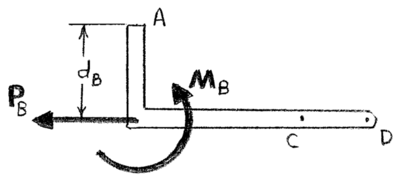


PROBLEM 3.79



The 20-lb horizontal force **P** acts on a bell crank as shown. (*a*) Replace **P** with an equivalent force-couple system at *B*. (*b*) Find the two vertical forces at *C* and *D* which are equivalent to the couple found in part *a*.

SOLUTION



(*a*) Based on $\Sigma F: \quad P_B = P = 20\text{ lb}$

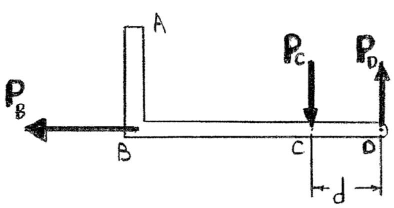
or $\mathbf{P}_B = 20\text{ lb} \longleftarrow$ ◀

$$\Sigma M: \quad M_B = Pd_B$$

$$= 20\text{ lb}(5\text{ in.})$$

$$= 100\text{ lb}\cdot\text{in.}$$

or $\mathbf{M}_B = 100\text{ lb}\cdot\text{in.}\ \circlearrowleft$ ◀



(*b*) If the two vertical forces are to be equivalent to $\mathbf{M}_B$, they must be a couple. Further, the sense of the moment of this couple must be counterclockwise.

Then, with P_C and P_D acting as shown,

$$\Sigma M: \quad M_D = P_C d$$

$$100\text{ lb}\cdot\text{in.} = P_C(4\text{ in.})$$

$$\therefore \quad P_C = 25\text{ lb}$$

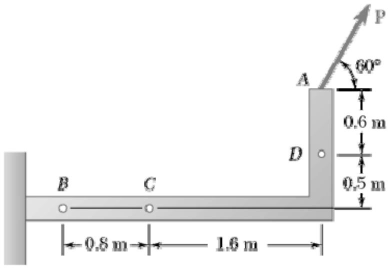
or $\mathbf{P}_C = 25\text{ lb}\downarrow$ ◀

$$\Sigma F_y: \quad 0 = P_D - P_C$$

$$\therefore \quad P_D = 25\text{ lb}$$

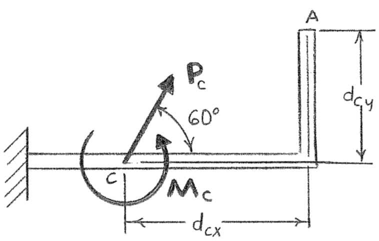
or $\mathbf{P}_D = 25\text{ lb}\uparrow$ ◀

PROBLEM 3.80



A 700-N force **P** is applied at point A of a structural member. Replace **P** with (*a*) an equivalent force-couple system at C, (*b*) an equivalent system consisting of a vertical force at B and a second force at D.

SOLUTION



(*a*) Based on ΣF: $P_C = P = 700\text{ N}$

or $\mathbf{P}_C = 700\text{ N} \measuredangle 60°$ ◀

$$\Sigma M_C: \quad M_C = -P_x d_{Cy} + P_y d_{Cx}$$

where $P_x = (700\text{ N})\cos 60° = 350\text{ N}$

$$P_y = (700\text{ N})\sin 60° = 606.22\text{ N}$$

$$d_{Cx} = 1.6\text{ m}$$

$$d_{Cy} = 1.1\text{ m}$$

$$\therefore \quad M_C = -(350\text{ N})(1.1\text{ m}) + (606.22\text{ N})(1.6\text{ m})$$

$$= -385\text{ N}\cdot\text{m} + 969.95\text{ N}\cdot\text{m}$$

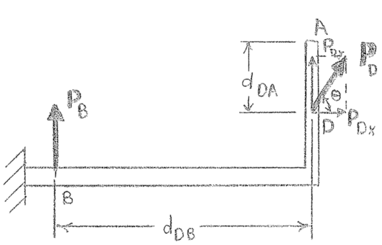
$$= 584.95\text{ N}\cdot\text{m}$$

or $\mathbf{M}_C = 585\text{ N}\cdot\text{m}$ ⤺ ◀

(*b*) Based on ΣF_x: $P_{Dx} = P\cos 60°$

$$= (700\text{ N})\cos 60°$$

$$= 350\text{ N}$$



$$\Sigma M_D: \quad (P\cos 60°)(d_{DA}) = P_B(d_{DB})$$

$$[(700\text{ N})\cos 60°](0.6\text{ m}) = P_B(2.4\text{ m})$$

$$P_B = 87.5\text{ N}$$

or $\mathbf{P}_B = 87.5\text{ N} \uparrow$ ◀

PROBLEM 3.80 CONTINUED

$$\Sigma F_y: \quad P\sin 60° = P_B + P_{Dy}$$

$$(700 \text{ N})\sin 60° = 87.5 \text{ N} + P_{Dy}$$

$$P_{Dy} = 518.72 \text{ N}$$

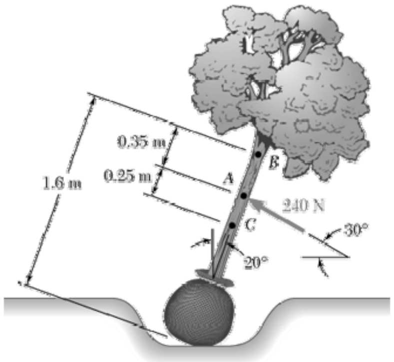
$$P_D = \sqrt{(P_{Dx})^2 + (P_{Dy})^2}$$

$$= \sqrt{(350)^2 + (518.72)^2} = 625.76 \text{ N}$$

$$\theta = \tan^{-1}\left(\frac{P_{Dy}}{P_{Dx}}\right) = \tan^{-1}\left(\frac{518.72}{350}\right) = 55.991°$$

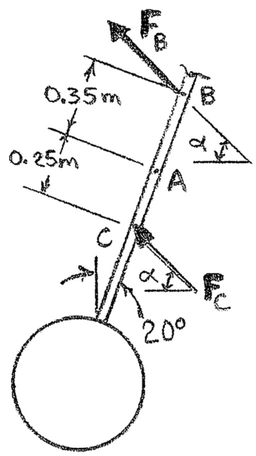
or $P_D = 626 \text{ N} \measuredangle\, 56.0°$ ◀

PROBLEM 3.81



A landscaper tries to plumb a tree by applying a 240-N force as shown. Two helpers then attempt to plumb the same tree, with one pulling at *B* and the other pushing with a parallel force at *C*. Determine these two forces so that they are equivalent to the single 240-N force shown in the figure.

SOLUTION



Based on

$$\Sigma F_x: \quad -(240\text{ N})\cos 30^\circ = -F_B\cos\alpha - F_C\cos\alpha$$

or
$$-(F_B + F_C)\cos\alpha = -(240\text{ N})\cos 30^\circ \qquad (1)$$

$$\Sigma F_y: \quad (240\text{ N})\sin 30^\circ = F_B\sin\alpha + F_C\sin\alpha$$

or
$$(F_B + F_C)\sin\alpha = (240\text{ N})\sin 30^\circ \qquad (2)$$

From

$$\frac{\text{Equation (2)}}{\text{Equation (1)}}: \quad \tan\alpha = \tan 30^\circ$$

$$\therefore \quad \alpha = 30^\circ$$

Based on

$$\Sigma M_C: \quad \left[(240\text{ N})\cos(30^\circ - 20^\circ)\right](0.25\text{ m}) = (F_B\cos 10^\circ)(0.60\text{ m})$$

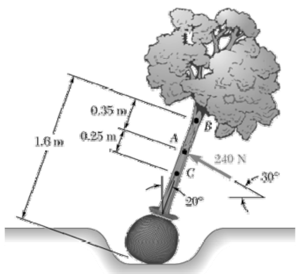
$$\therefore \quad F_B = 100\text{ N}$$

or $\mathbf{F}_B = 100.0\text{ N} \measuredangle 30^\circ$ ◀

From Equation (1), $\quad -(100\text{ N} + F_C)\cos 30^\circ = -240\cos 30^\circ$

$$F_C = 140\text{ N}$$

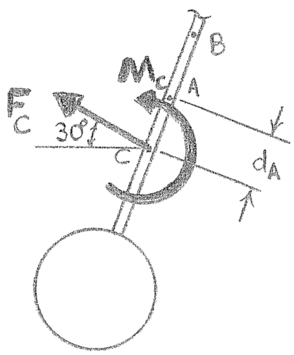
or $\mathbf{F}_C = 140.0\text{ N} \measuredangle 30^\circ$ ◀



PROBLEM 3.82

A landscaper tries to plumb a tree by applying a 240-N force as shown. (*a*) Replace that force with an equivalent force-couple system at *C*. (*b*) Two helpers attempt to plumb the same tree, with one applying a horizontal force at *C* and the other pulling at *B*. Determine these two forces if they are to be equivalent to the single force of part *a*.

SOLUTION



(*a*) Based on ΣF_x: $-(240 \text{ N})\cos 30° = -F_C \cos 30°$

$$\therefore \quad F_C = 240 \text{ N}$$

or $\mathbf{F}_C = 240 \text{ N} \searrow 30°$ ◀

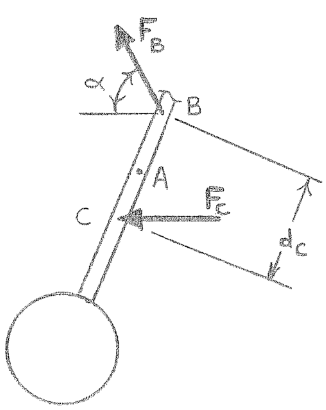
$$\Sigma M_C: \quad [(240 \text{ N})\cos 10°](d_A) = M_C \qquad d_A = 0.25 \text{ m}$$

$$\therefore \quad M_C = 59.088 \text{ N}\cdot\text{m}$$

or $\mathbf{M}_C = 59.1 \text{ N}\cdot\text{m} \circlearrowleft$ ◀

(*b*) Based on ΣF_y: $(240 \text{ N})\sin 30° = F_B \sin\alpha$

or $\qquad F_B \sin\alpha = 120 \qquad (1)$



$$\Sigma M_B: \quad 59.088 \text{ N}\cdot\text{m} - [(240 \text{ N})\cos 10°](d_C) = -F_C(d_C \cos 20°)$$

$$59.088 \text{ N}\cdot\text{m} - [(240 \text{ N})\cos 10°](0.60 \text{ m}) = -F_C[(0.60 \text{ m})\cos 20°]$$

$$0.56382 F_C = 82.724$$

$$F_C = 146.722 \text{ N}$$

or $\mathbf{F}_C = 146.7 \text{ N} \longleftarrow$ ◀

and $\quad \Sigma F_x$: $-(240 \text{ N})\cos 30° = -146.722 \text{ N} - F_B \cos\alpha$

$$F_B \cos\alpha = 61.124 \qquad (2)$$

From

$$\frac{\text{Equation (1)}}{\text{Equation (2)}}: \quad \tan\alpha = \frac{120}{61.124} = 1.96323$$

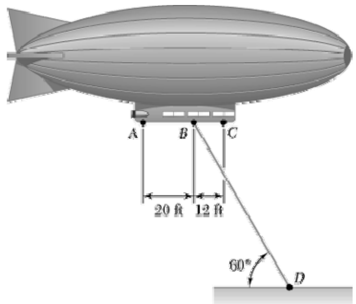
$$\alpha = 63.007°$$

or $\alpha = 63.0°$ ◀

From Equation (1), $\quad F_B = \dfrac{120}{\sin 63.007°} = 134.670 \text{ N}$

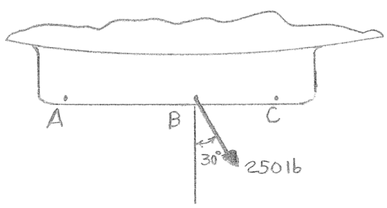
or $\mathbf{F}_B = 134.7 \text{ N} \searrow 63.0°$ ◀

PROBLEM 3.83



A dirigible is tethered by a cable attached to its cabin at *B*. If the tension in the cable is 250 lb, replace the force exerted by the cable at *B* with an equivalent system formed by two parallel forces applied at *A* and *C*.

SOLUTION

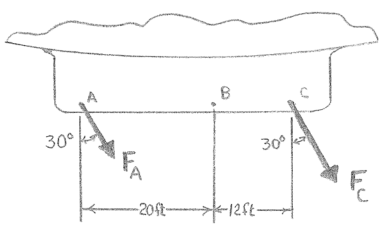


Require the equivalent forces acting at *A* and *C* be parallel and at an angle of α with the vertical.

Then for equivalence,

$$\Sigma F_x: \quad (250 \text{ lb})\sin 30° = F_A \sin\alpha + F_B \sin\alpha \qquad (1)$$

$$\Sigma F_y: \quad -(250 \text{ lb})\cos 30° = -F_A \cos\alpha - F_B \cos\alpha \qquad (2)$$



Dividing Equation (1) by Equation (2),

$$\frac{(250 \text{ lb})\sin 30°}{-(250 \text{ lb})\cos 30°} = \frac{(F_A + F_B)\sin\alpha}{-(F_A + F_B)\cos\alpha}$$

Simplifying yields $\alpha = 30°$

Based on

$$\Sigma M_C: \quad [(250 \text{ lb})\cos 30°](12 \text{ ft}) = (F_A \cos 30°)(32 \text{ ft})$$

$$\therefore \quad F_A = 93.75 \text{ lb}$$

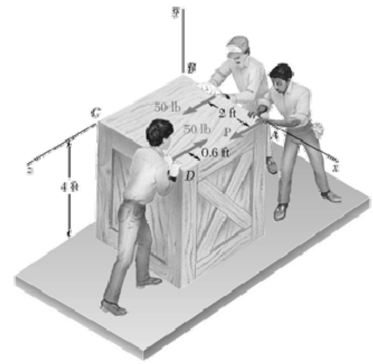
or $\mathbf{F}_A = 93.8 \text{ lb} \searrow 60°$ ◀

Based on

$$\Sigma M_A: \quad -[(250 \text{ lb})\cos 30°](20 \text{ ft}) = (F_C \cos 30°)(32 \text{ ft})$$

$$\therefore \quad F_C = 156.25 \text{ lb}$$

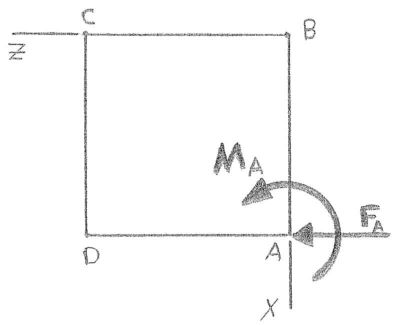
or $\mathbf{F}_C = 156.3 \text{ lb} \searrow 60°$ ◀



PROBLEM 3.84

Three workers trying to move a $3 \times 3 \times 4$-ft crate apply to the crate the three horizontal forces shown. (*a*) If $P = 60$ lb, replace the three forces with an equivalent force-couple system at *A*. (*b*) Replace the force-couple system of part *a* with a single force, and determine where it should be applied to side *AB*. (*c*) Determine the magnitude of **P** so that the three forces can be replaced with a single equivalent force applied at *B*.

SOLUTION



(*a*)

(*a*) Based on

$$\Sigma F_z: \quad -50 \text{ lb} + 50 \text{ lb} + 60 \text{ lb} = F_A$$

$$F_A = 60 \text{ lb}$$

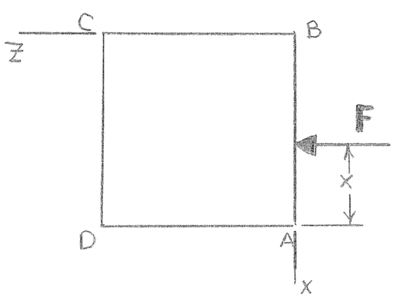
$$\text{or } \mathbf{F}_A = (60.0 \text{ lb})\mathbf{k} \blacktriangleleft$$

Based on

$$\Sigma M_A: \quad (50 \text{ lb})(2 \text{ ft}) - (50 \text{ lb})(0.6 \text{ ft}) = M_A$$

$$M_A = 70 \text{ lb}\cdot\text{ft}$$

$$\text{or } \mathbf{M}_A = (70.0 \text{ lb}\cdot\text{ft})\mathbf{j} \blacktriangleleft$$



(*b*)

(*b*) Based on

$$\Sigma F_z: \quad -50 \text{ lb} + 50 \text{ lb} + 60 \text{ lb} = F$$

$$F = 60 \text{ lb}$$

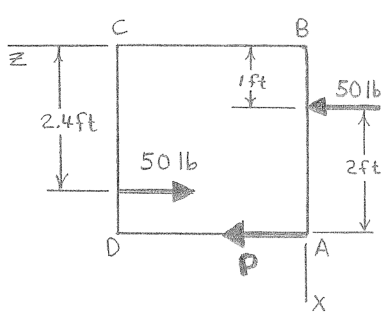
$$\text{or } \mathbf{F} = (60.0 \text{ lb})\mathbf{k} \blacktriangleleft$$

Based on

$$\Sigma M_A: \quad 70 \text{ lb}\cdot\text{ft} = 60 \text{ lb}\,(x)$$

$$x = 1.16667 \text{ ft}$$

$$\text{or } x = 1.167 \text{ ft from } A \text{ along } AB \blacktriangleleft$$



(*c*)

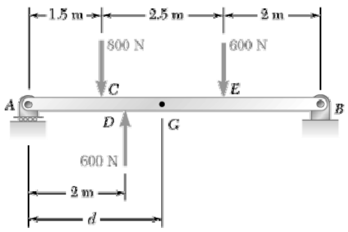
(*c*) Based on

$$\Sigma M_B: \quad -(50 \text{ lb})(1 \text{ ft}) + (50 \text{ lb})(2.4 \text{ ft}) - P(3 \text{ ft}) = R(0)$$

$$P = \frac{70}{3} = 23.333 \text{ lb}$$

$$\text{or } P = 23.3 \text{ lb} \blacktriangleleft$$

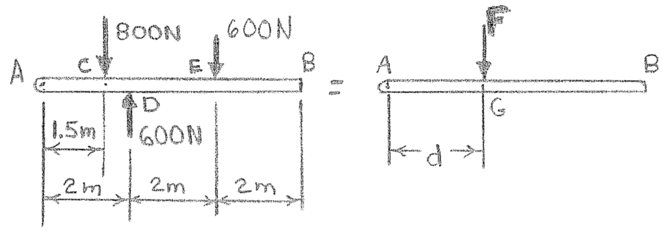
PROBLEM 3.85



A force and a couple are applied to a beam. (*a*) Replace this system with a single force **F** applied at point *G*, and determine the distance *d*. (*b*) Solve part *a* assuming that the directions of the two 600-N forces are reversed.

SOLUTION

(*a*)



Have $+\uparrow \Sigma F_y: \quad F_C + F_D + F_E = F$

$$F = -800 \text{ N} + 600 \text{ N} - 600 \text{ N}$$

$$F = -800 \text{ N}$$

or $\mathbf{F} = 800 \text{ N}\downarrow$ ◀

Have $+\circlearrowleft \Sigma M_G: \quad F_C(d - 1.5 \text{ m}) - F_D(2 \text{ m}) = 0$

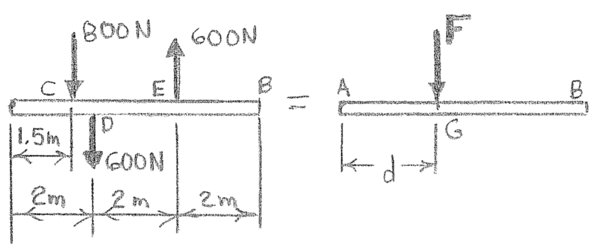
$$(800 \text{ N})(d - 1.5 \text{ m}) - (600 \text{ N})(2 \text{ m}) = 0$$

$$d = \frac{1200 + 1200}{800}$$

$$d = 3 \text{ m}$$

or $d = 3.00 \text{ m}$ ◀

(*b*)



Changing directions of the two 600 N forces only changes sign of the couple.

$$\therefore \quad F = -800 \text{ N}$$

or $\mathbf{F} = 800 \text{ N}\downarrow$ ◀

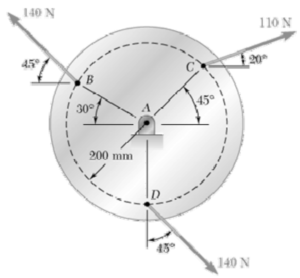
and $+\circlearrowleft \Sigma M_G: \quad F_C(d - 1.5 \text{ m}) + F_D(2 \text{ m}) = 0$

$$(800 \text{ N})(d - 1.5 \text{ m}) + (600 \text{ N})(2 \text{ m})$$

$$d = \frac{1200 - 1200}{800} = 0$$

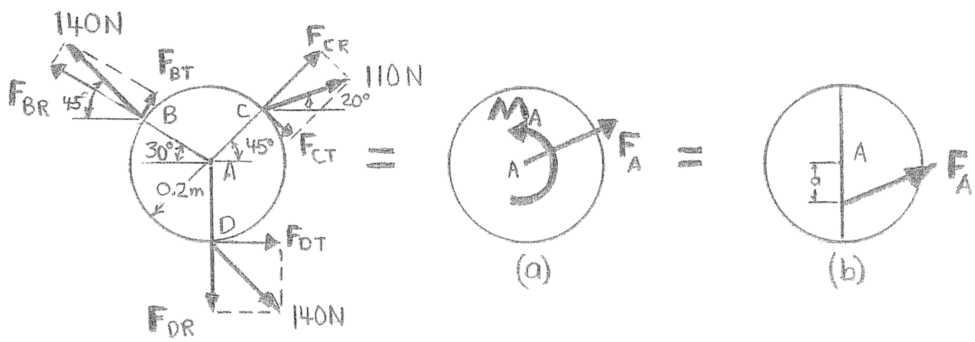
or $d = 0$ ◀

PROBLEM 3.86



Three cables attached to a disk exert on it the forces shown. (a) Replace the three forces with an equivalent force-couple system at A. (b) Determine the single force which is equivalent to the force-couple system obtained in part a, and specify its point of application on a line drawn through points A and D.

SOLUTION



(a) Have $\quad \Sigma\mathbf{F}: \quad \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D = \mathbf{F}_A$

Since $\quad \mathbf{F}_B = -\mathbf{F}_D$

$$\therefore \quad \mathbf{F}_A = \mathbf{F}_C = 110 \text{ N} \measuredangle 20°$$

or $\mathbf{F}_A = 110.0 \text{ N} \measuredangle 20.0°$ ◀

Have $\quad \Sigma M_A: \quad -F_{BT}(r) - F_{CT}(r) + F_{DT}(r) = M_A$

$$-[(140 \text{ N})\sin 15°](0.2 \text{ m}) - [(110 \text{ N})\sin 25°](0.2 \text{ m}) + [(140 \text{ N})\sin 45°](0.2 \text{ m}) = M_A$$

$$M_A = 3.2545 \text{ N}\cdot\text{m}$$

or $\mathbf{M}_A = 3.25 \text{ N}\cdot\text{m} \circlearrowright$ ◀

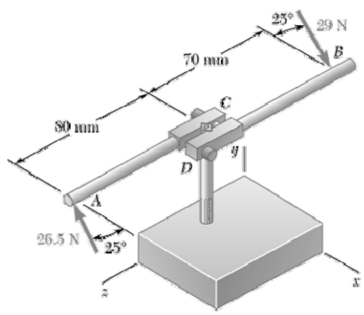
(b) Have $\quad \Sigma\mathbf{F}: \quad \mathbf{F}_A = \mathbf{F}_E \qquad$ or $\mathbf{F}_E = 110.0 \text{ N} \measuredangle 20.0°$ ◀

$$\Sigma M: \quad M_A = [F_E \cos 20°](a)$$

$$\therefore \quad 3.2545 \text{ N}\cdot\text{m} = [(110 \text{ N})\cos 20°](a)$$

$$a = 0.031485 \text{ m}$$

or $a = 31.5$ mm below A ◀



PROBLEM 3.87

While tapping a hole, a machinist applies the horizontal forces shown to the handle of the tap wrench. Show that these forces are equivalent to a single force, and specify, if possible, the point of application of the single force on the handle.

SOLUTION

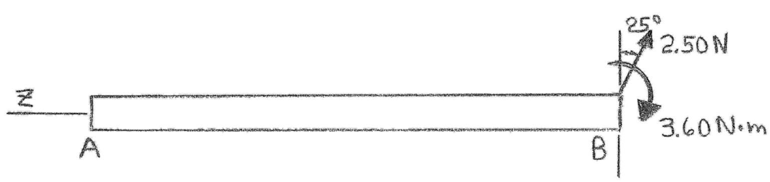
Since the forces at *A* and *B* are parallel, the force at *B* can be replaced with the sum of two forces with one of the forces equal in magnitude to the force at *A* except with an opposite sense, resulting in a force-couple.

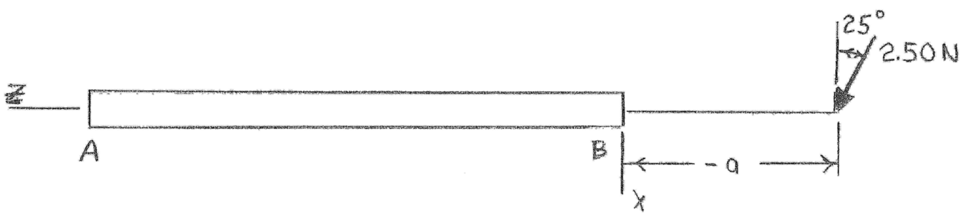
Have $F_B = 26.5\text{ N} + 2.5\text{ N},$ where the 26.5 N force be part of the couple. Combining the two parallel forces,

$$M_{\text{couple}} = (26.5\text{ N})\left[(0.080\text{ m} + 0.070\text{ m})\cos 25°\right]$$

$$= 3.60\text{ N}\cdot\text{m}$$

and, $\mathbf{M}_{\text{couple}} = 3.60\text{ N}\cdot\text{m}$ ↻





A single equivalent force will be located in the negative *z*-direction.

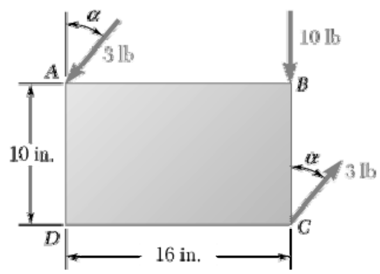
Based on $\qquad \Sigma M_B: \quad -3.60\text{ N}\cdot\text{m} = \left[(2.5\text{ N})\cos 25°\right](a)$

$$a = -1.590\text{ m}$$

$$\mathbf{F}' = (2.5\text{ N})(\cos 25°\mathbf{i} + \sin 25°\mathbf{j})$$

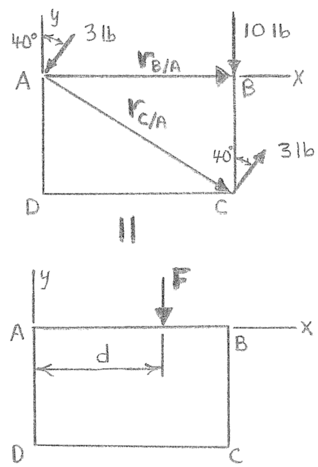
and is applied on an extension of handle *BD*
at a distance of 1.590 m to the right of *B* ◀

PROBLEM 3.88



A rectangular plate is acted upon by the force and couple shown. This system is to be replaced with a single equivalent force. (a) For $\alpha = 40°$, specify the magnitude and the line of action of the equivalent force. (b) Specify the value of α if the line of action of the equivalent force is to intersect line CD 12 in. to the right of D.

SOLUTION



(a) Have $\quad \Sigma F_x: \; -(3\text{ lb})\sin 40° + (3\text{ lb})\sin 40° = F_x$

$$\therefore \; F_x = 0$$

Have $\quad \Sigma F_y: \; -(3\text{ lb})\cos 40° - 10\text{ lb} + (3\text{ lb})\cos 40° = F_y$

$$\therefore \; F_y = -10\text{ lb}$$

or $F = 10.00$ lb ◀

Note: The two 3-lb forces form a couple

and $\quad \Sigma \mathbf{M}_A: \; \mathbf{r}_{C/A} \times \mathbf{P}_C + \mathbf{r}_{B/A} \times \mathbf{P}_B = \mathbf{r}_{X/A} \times \mathbf{F}$

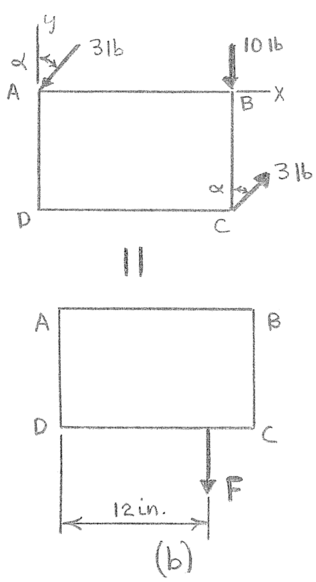
$$3\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 16 & -10 & 0 \\ \sin 40° & \cos 40° & 0 \end{vmatrix} + 160\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} = 10\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ d & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix}$$

$$\mathbf{k}: \; 3(16)\cos 40° - (-10)3\sin 40° - 160 = -10d$$

$$36.770 + 19.2836 - 160 = -10d$$

$$\therefore \; d = 10.3946\text{ in.}$$

or $\mathbf{F} = 10.00\text{ lb} \downarrow$ at 10.39 in. right of A or at 5.61 in. left of B ◀



(b) From part (a), $\quad \mathbf{F} = 10.00\text{ lb} \downarrow$

Have $\quad \Sigma \mathbf{M}_A: \; \mathbf{r}_{C/A} \times \mathbf{P}_C + \mathbf{r}_{B/A} \times \mathbf{P}_B = (12\text{ in.})\mathbf{i} \times \mathbf{F}$

$$3\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 16 & -10 & 0 \\ \sin\alpha & \cos\alpha & 0 \end{vmatrix} + 160\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} = 120\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix}$$

$$\mathbf{k}: \; 48\cos\alpha + 30\sin\alpha - 160 = -120$$

$$24\cos\alpha = 20 - 15\sin\alpha$$

PROBLEM 3.88 CONTINUED

Squaring both sides of the equation, and

using the identity $\cos^2\alpha = 1 - \sin^2\alpha$, results in

$$\sin^2\alpha - 0.74906\sin\alpha - 0.21973 = 0$$

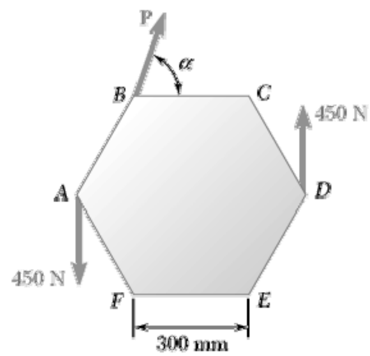
Using quadratic formula

$$\sin\alpha = 0.97453 \qquad \sin\alpha = -0.22547$$

so that

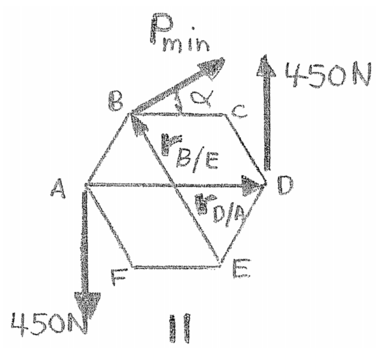
$$\alpha = 77.0° \qquad \text{and} \qquad \alpha = -13.03° \blacktriangleleft$$

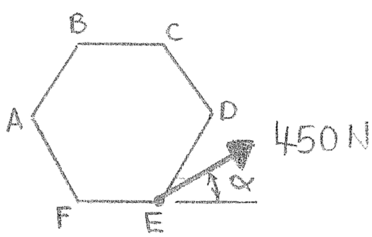
PROBLEM 3.89



A hexagonal plate is acted upon by the force **P** and the couple shown. Determine the magnitude and the direction of the smallest force **P** for which this system can be replaced with a single force at E.

SOLUTION





Since the minimum value of P acting at B is realized when P_{min} is perpendicular to a line connecting B and E, $\alpha = 30°$

Then,

$$\Sigma \mathbf{M}_E: \quad \mathbf{r}_{B/E} \times \mathbf{P}_{\text{min}} + \mathbf{r}_{D/A} \times \mathbf{P}_D = 0$$

where

$$\mathbf{r}_{B/E} = -(0.30 \text{ m})\mathbf{i} + \left[2(0.30 \text{ m})\cos 30°\right]\mathbf{j}$$

$$= -(0.30 \text{ m})\mathbf{i} + (0.51962 \text{ m})\mathbf{j}$$

$$\mathbf{r}_{D/A} = \left[0.30 \text{ m} + 2(0.3 \text{ m})\sin 30°\right]\mathbf{i}$$

$$= (0.60 \text{ m})\mathbf{i}$$

$$\mathbf{P}_D = (450 \text{ N})\mathbf{j}$$

$$\mathbf{P}_{\text{min}} = P_{\text{min}}\left[(\cos 30°)\mathbf{i} + (\sin 30°)\mathbf{j}\right]$$

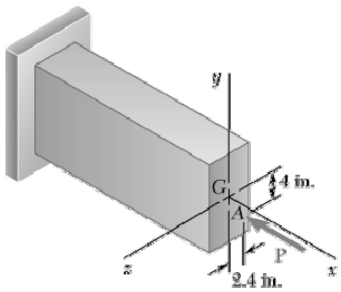
$$\therefore \quad P_{\text{min}} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.30 & 0.51962 & 0 \\ 0.86603 & 0.50 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.60 & 0 & 0 \\ 0 & 450 & 0 \end{vmatrix} \text{N} \cdot \text{m} = 0$$

$$P_{\text{min}}(-0.15 \text{ m} - 0.45 \text{ m})\mathbf{k} + (270 \text{ N} \cdot \text{m})\mathbf{k} = 0$$

$$\therefore \quad P_{\text{min}} = 450 \text{ N}$$

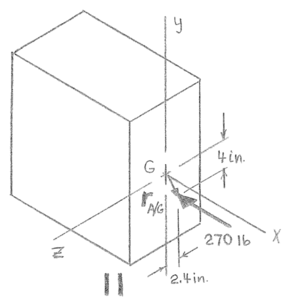
or $\mathbf{P}_{\text{min}} = 450 \text{ N} \measuredangle 30°$ ◀

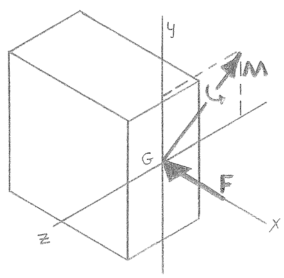
PROBLEM 3.90



An eccentric, compressive 270-lb force **P** is applied to the end of a cantilever beam. Replace **P** with an equivalent force-couple system at *G*.

SOLUTION





Have

$$\Sigma \mathbf{F}: \quad -(270 \text{ lb})\mathbf{i} = \mathbf{F}$$

$$\therefore \quad \mathbf{F} = -(270 \text{ lb})\mathbf{i} \blacktriangleleft$$

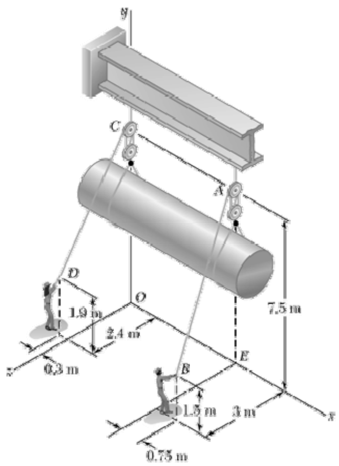
Also, have

$$\Sigma \mathbf{M}_G: \quad \mathbf{r}_{A/G} \times \mathbf{P} = \mathbf{M}$$

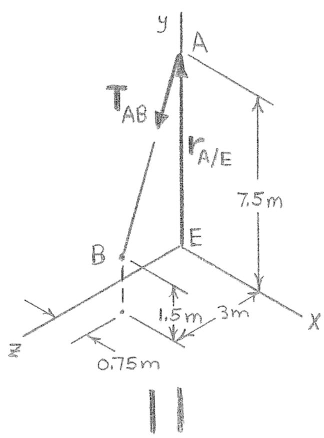
$$270 \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -4 & -2.4 \\ -1 & 0 & 0 \end{vmatrix} \text{lb} \cdot \text{in.} = \mathbf{M}$$

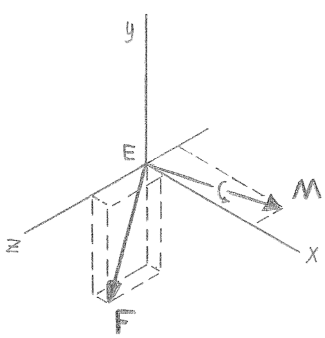
$$\therefore \quad \mathbf{M} = (270 \text{ lb} \cdot \text{in.})\left[(-2.4)(-1)\mathbf{j} - (-4)(-1)\mathbf{k}\right]$$

$$\text{or} \quad \mathbf{M} = (648 \text{ lb} \cdot \text{in.})\mathbf{j} - (1080 \text{ lb} \cdot \text{in.})\mathbf{k} \blacktriangleleft$$

	**PROBLEM 3.91** Two workers use blocks and tackles attached to the bottom of an I-beam to lift a large cylindrical tank. Knowing that the tension in rope *AB* is 324 N, replace the force exerted at *A* by rope *AB* with an equivalent force-couple system at *E*.

SOLUTION





Have $\Sigma\mathbf{F}: \quad \mathbf{T}_{AB} = \mathbf{F}$

where

$$\mathbf{T}_{AB} = \lambda_{AB} T_{AB}$$

$$= \frac{(0.75\text{ m})\mathbf{i} - (6.0\text{ m})\mathbf{j} + (3.0\text{ m})\mathbf{k}}{6.75\text{ m}}(324\text{ N})$$

$$\therefore \quad \mathbf{T}_{AB} = 36\text{ N}(\mathbf{i} - 8\mathbf{j} + 4\mathbf{k})$$

so that $\mathbf{F} = (36.0\text{ N})\mathbf{i} - (288\text{ N})\mathbf{j} + (144.0\text{ N})\mathbf{k}$ ◀

Have

$$\Sigma\mathbf{M}_E: \quad \mathbf{r}_{A/E} \times \mathbf{T}_{AB} = \mathbf{M}$$

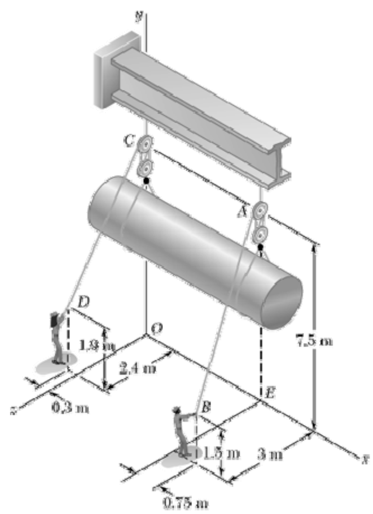
or

$$(7.5\text{ m})(36\text{ N})\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 0 \\ 1 & -8 & 4 \end{vmatrix} = \mathbf{M}$$

$$\therefore \quad \mathbf{M} = (270\text{ N}\cdot\text{m})(4\mathbf{i} - \mathbf{k})$$

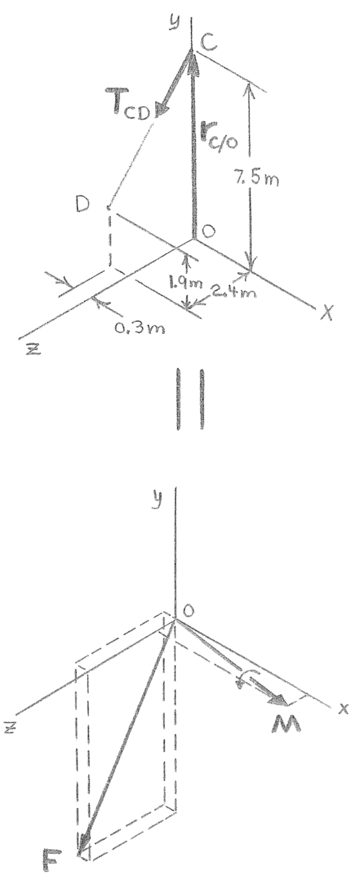
or $\mathbf{M} = (1080\text{ N}\cdot\text{m})\mathbf{i} - (270\text{ N}\cdot\text{m})\mathbf{k}$ ◀

PROBLEM 3.92



Two workers use blocks and tackles attached to the bottom of an I-beam to lift a large cylindrical tank. Knowing that the tension in rope *CD* is 366 N, replace the force exerted at *C* by rope *CD* with an equivalent force-couple system at *O*.

SOLUTION



Have $$\Sigma\mathbf{F}: \quad \mathbf{T}_{CD} = \mathbf{F}$$

where

$$\mathbf{T}_{CD} = \lambda_{CD} T_{CD}$$

$$= \frac{-(0.3\text{ m})\mathbf{i} - (5.6\text{ m})\mathbf{j} + (2.4\text{ m})\mathbf{k}}{6.1\text{ m}}(366\text{ N})$$

$$\therefore \quad \mathbf{T}_{CD} = (6.0\text{ N})(-3\mathbf{i} - 56\mathbf{j} + 24\mathbf{k})$$

so that $$\mathbf{F} = -(18.00\text{ N})\mathbf{i} - (336\text{ N})\mathbf{j} + (144.0\text{ N})\mathbf{k} \blacktriangleleft$$

Have

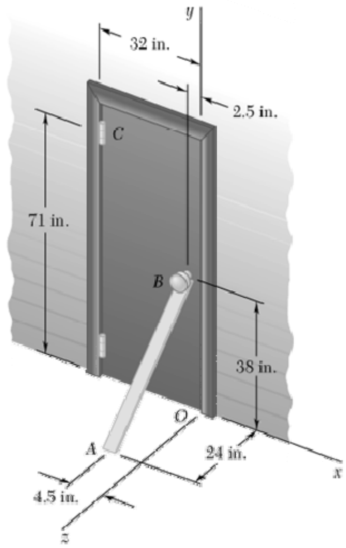
$$\Sigma\mathbf{M}_O: \quad \mathbf{r}_{C/O} \times \mathbf{T}_{CD} = \mathbf{M}$$

or $$(7.5\text{ m})(6\text{ N})\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 0 \\ -3 & -56 & 24 \end{vmatrix} = \mathbf{M}$$

$$\therefore \quad \mathbf{M} = (45\text{ N}\cdot\text{m})(24\mathbf{i} + 3\mathbf{k})$$

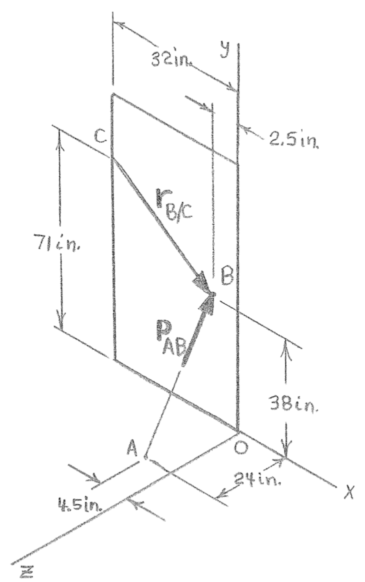
$$\text{or} \quad \mathbf{M} = (1080\text{ N}\cdot\text{m})\mathbf{i} + (135.0\text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$

PROBLEM 3.93



To keep a door closed, a wooden stick is wedged between the floor and the doorknob. The stick exerts at B a 45-lb force directed along line AB. Replace that force with an equivalent force-couple system at C.

SOLUTION



Have

$$\Sigma \mathbf{F}: \quad \mathbf{P}_{AB} = \mathbf{F}_C$$

where

$$\mathbf{P}_{AB} = \lambda_{AB} P_{AB}$$

$$= \frac{(2.0 \text{ in.})\mathbf{i} + (38 \text{ in.})\mathbf{j} - (24 \text{ in.})\mathbf{k}}{44.989 \text{ in.}}(45 \text{ lb})$$

$$\text{or } \mathbf{F}_C = (2.00 \text{ lb})\mathbf{i} + (38.0 \text{ lb})\mathbf{j} - (24.0 \text{ lb})\mathbf{k} \blacktriangleleft$$

Have

$$\Sigma \mathbf{M}_C: \quad \mathbf{r}_{B/C} \times \mathbf{P}_{AB} = \mathbf{M}_C$$

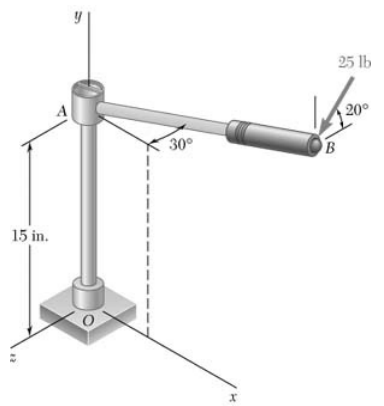
$$\mathbf{M}_C = 2\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 29.5 & -33 & 0 \\ 1 & 19 & -12 \end{vmatrix} \text{lb} \cdot \text{in.}$$

$$= (2 \text{ lb} \cdot \text{in.})\{(-33)(-12)\mathbf{i} - (29.5)(-12)\mathbf{j}$$

$$+ [(29.5)(19) - (-33)(1)]\mathbf{k}\}$$

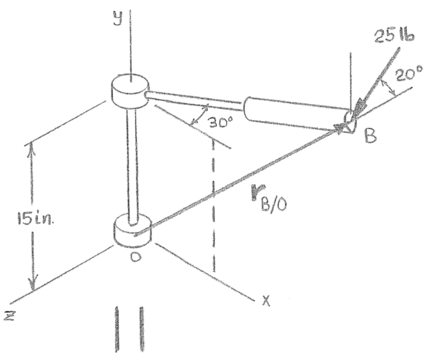
$$\text{or } \mathbf{M}_C = (792 \text{ lb} \cdot \text{in.})\mathbf{i} + (708 \text{ lb} \cdot \text{in.})\mathbf{j} + (1187 \text{ lb} \cdot \text{in.})\mathbf{k} \blacktriangleleft$$

PROBLEM 3.94



A 25-lb force acting in a vertical plane parallel to the yz plane is applied to the 8-in.-long horizontal handle AB of a socket wrench. Replace the force with an equivalent force-couple system at the origin O of the coordinate system.

SOLUTION



Have

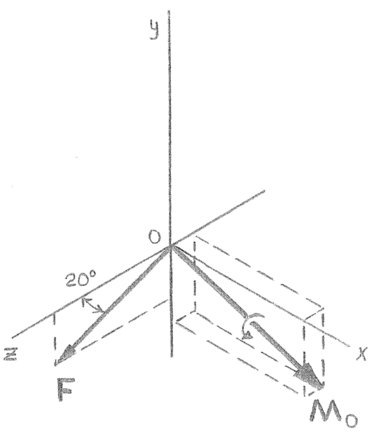
$$\Sigma \mathbf{F}: \quad \mathbf{P}_B = \mathbf{F}$$

where

$$\mathbf{P}_B = 25 \text{ lb}\left[-(\sin 20°)\,\mathbf{j} + (\cos 20°)\,\mathbf{k}\right]$$

$$= -(8.5505 \text{ lb})\,\mathbf{j} + (23.492 \text{ lb})\,\mathbf{k}$$

$$\text{or } \mathbf{F} = -(8.55 \text{ lb})\,\mathbf{j} + (23.5 \text{ lb})\,\mathbf{k} \blacktriangleleft$$



Have

$$\Sigma \mathbf{M}_O: \quad \mathbf{r}_{B/O} \times \mathbf{P}_B = \mathbf{M}_O$$

where

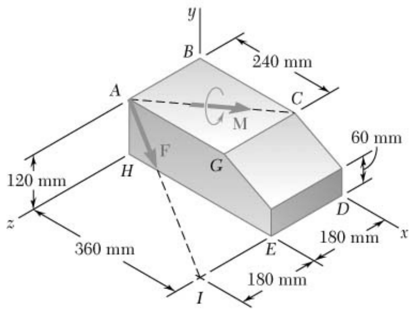
$$\mathbf{r}_{B/O} = \left[(8\cos 30°)\,\mathbf{i} + (15)\,\mathbf{j} - (8\sin 30°)\,\mathbf{k}\right] \text{ in.}$$

$$= (6.9282 \text{ in.})\,\mathbf{i} + (15 \text{ in.})\,\mathbf{j} - (4 \text{ in.})\,\mathbf{k}$$

$$\therefore \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6.9282 & 15 & -4 \\ 0 & -8.5505 & 23.492 \end{vmatrix} \text{lb}\cdot\text{in.} = \mathbf{M}_O$$

$$\mathbf{M}_O = \left[(318.18)\,\mathbf{i} - (162.757)\,\mathbf{j} - (59.240)\,\mathbf{k}\right] \text{lb}\cdot\text{in.}$$

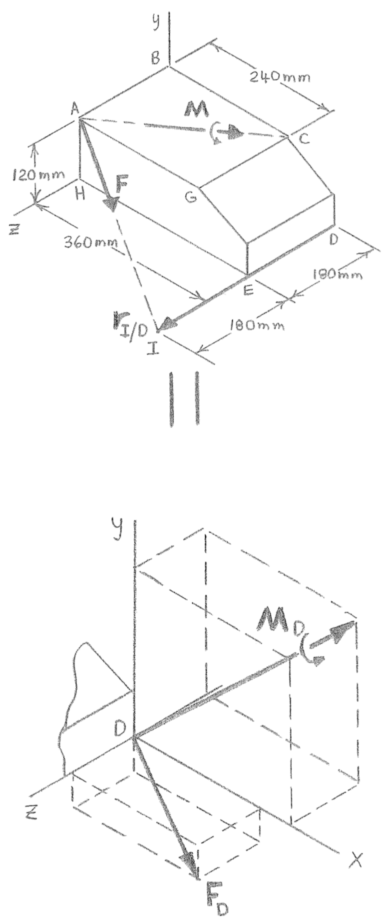
$$\text{or } \mathbf{M}_O = (318 \text{ lb}\cdot\text{in.})\,\mathbf{i} - (162.8 \text{ lb}\cdot\text{in.})\,\mathbf{j} - (59.2 \text{ lb}\cdot\text{in.})\,\mathbf{k} \blacktriangleleft$$



PROBLEM 3.95

A 315-N force **F** and 70-N · m couple **M** are applied to corner *A* of the block shown. Replace the given force-couple system with an equivalent force-couple system at corner *D*.

SOLUTION



Have

$$\Sigma\mathbf{F}: \quad \mathbf{F} = \mathbf{F}_D$$

$$= \lambda_{AI}F$$

$$= \frac{(0.360\text{ m})\mathbf{i} - (0.120\text{ m})\mathbf{j} + (0.180\text{ m})\mathbf{k}}{0.420\text{ m}}(315\text{ N})$$

$$= (750\text{ N})(0.360\mathbf{i} - 0.120\mathbf{j} + 0.180\mathbf{k})$$

$$\text{or } \mathbf{F}_D = (270\text{ N})\mathbf{i} - (90.0\text{ N})\mathbf{j} + (135.0\text{ N})\mathbf{k} \blacktriangleleft$$

Have

$$\Sigma\mathbf{M}_D: \quad \mathbf{M} + \mathbf{r}_{I/D} \times \mathbf{F} = \mathbf{M}_D$$

where

$$\mathbf{M} = \lambda_{AC}M$$

$$= \frac{(0.240\text{ m})\mathbf{i} - (0.180\text{ m})\mathbf{k}}{0.300\text{ m}}(70.0\text{ N}\cdot\text{m})$$

$$= (70.0\text{ N}\cdot\text{m})(0.800\mathbf{i} - 0.600\mathbf{k})$$

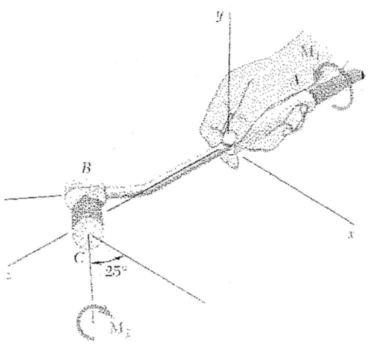
$$\mathbf{r}_{I/D} = (0.360\text{ m})\mathbf{k}$$

$$\therefore \quad \mathbf{M}_D = (70.0\text{ N}\cdot\text{m})(0.8\mathbf{i} - 0.6\mathbf{k}) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.36 \\ 0.36 & -0.12 & 0.18 \end{vmatrix}(750\text{ N}\cdot\text{m})$$

$$= (56.0\text{ N}\cdot\text{m})\mathbf{i} - (42.0\text{ N}\cdot\text{m})\mathbf{k} + \left[(32.4\text{ N}\cdot\text{m})\mathbf{i} + (97.2\text{ N}\cdot\text{m})\mathbf{j}\right]$$

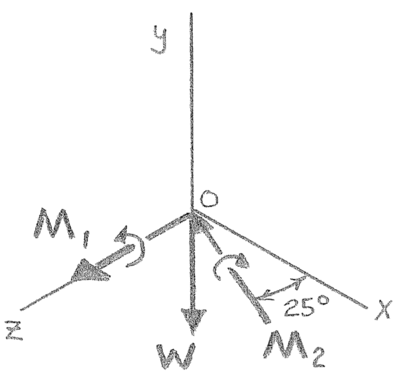
$$\text{or } \mathbf{M}_D = (88.4\text{ N}\cdot\text{m})\mathbf{i} + (97.2\text{ N}\cdot\text{m})\mathbf{j} - (42.0\text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$

PROBLEM 3.96



The handpiece of a miniature industrial grinder weighs 2.4 N, and its center of gravity is located on the *y* axis. The head of the handpiece is offset in the *xz* plane in such a way that line *BC* forms an angle of $25°$ with the *x* direction. Show that the weight of the handpiece and the two couples $\mathbf{M}_1$ and $\mathbf{M}_2$ can be replaced with a single equivalent force. Further assuming that $M_1 = 0.068\ \text{N}\cdot\text{m}$ and $M_2 = 0.065\ \text{N}\cdot\text{m}$, determine (*a*) the magnitude and the direction of the equivalent force, (*b*) the point where its line of action intersects the *xz* plane.

SOLUTION



First assume that the given force **W** and couples $\mathbf{M}_1$ and $\mathbf{M}_2$ act at the origin.

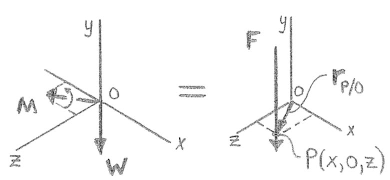
Now $$\mathbf{W} = -W_{\mathbf{j}}$$

and $$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2 = -\left(M_2 \cos 25°\right)\mathbf{i} + \left(M_1 - M_2 \sin 25°\right)\mathbf{k}$$

Note that since **W** and **M** are perpendicular, it follows that they can be replaced with a single equivalent force.

(*a*) Have $$F = \mathbf{W} \quad \text{or} \quad \mathbf{F} = -W_{\mathbf{j}} = -\left(2.4\ \text{N}\right)\mathbf{j}$$

or $\mathbf{F} = -\left(2.40\ \text{N}\right)\mathbf{j}$ ◀



(*b*) Assume that the line of action of **F** passes through point *P* (*x*, 0, *z*).

Then for equivalence

$$\mathbf{M} = \mathbf{r}_{P/O} \times \mathbf{F}$$

where $$\mathbf{r}_{P/O} = x\mathbf{i} + z\mathbf{k}$$

$$\therefore \quad -\left(M_2 \cos 25°\right)\mathbf{i} + \left(M_1 - M_2 \sin 25°\right)\mathbf{k}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & 0 & z \\ 0 & -W & 0 \end{vmatrix} = \left(Wz\right)\mathbf{i} - \left(Wx\right)\mathbf{k}$$

PROBLEM 3.96 CONTINUED

Equating the **i** and **k** coefficients,

$$z = \frac{-M_z \cos 25°}{W} \qquad \text{and} \qquad x = -\left(\frac{M_1 - M_2 \sin 25°}{W}\right)$$

(*b*) For $\quad W = 2.4 \text{ N}, M_1 = 0.068 \text{ N}\cdot\text{m}, M_2 = 0.065 \text{ N}\cdot\text{m}$

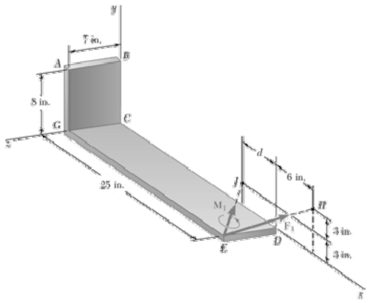
$$x = \frac{0.068 - 0.065 \sin 25°}{-2.4} = -0.0168874 \text{ m}$$

or $x = -16.89$ mm ◀

$$z = \frac{-0.065 \cos 25°}{2.4} = -0.024546 \text{ m}$$

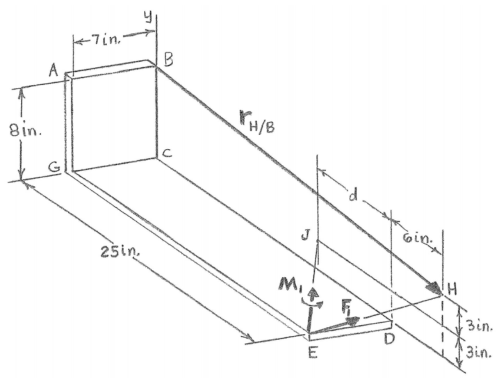
or $z = -24.5$ mm ◀

PROBLEM 3.97



A 20-lb force $\mathbf{F}_1$ and a 40-$\text{lb}\cdot\text{ft}$ couple $\mathbf{M}_1$ are applied to corner E of the bent plate shown. If $\mathbf{F}_1$ and $\mathbf{M}_1$ are to be replaced with an equivalent force-couple system $(\mathbf{F}_2, \mathbf{M}_2)$ at corner B and if $(M_2)_z = 0$, determine (a) the distance d, (b) $\mathbf{F}_2$ and $\mathbf{M}_2$.

SOLUTION



(a) Have $\qquad \Sigma M_{Bz}: \quad M_{2z} = 0$

$$\mathbf{k}\cdot(\mathbf{r}_{H/B}\times\mathbf{F}_1) + M_{1z} = 0 \qquad (1)$$

where $\qquad \mathbf{r}_{H/B} = (31\text{ in.})\mathbf{i} - (2\text{ in.})\mathbf{j}$

$$\mathbf{F}_1 = \lambda_{EH}F_1$$

$$= \frac{(6\text{ in.})\mathbf{i} + (6\text{ in.})\mathbf{j} - (7\text{ in.})\mathbf{k}}{11.0\text{ in.}}(20\text{ lb})$$

$$= \frac{20\text{ lb}}{11.0}(6\mathbf{i} + 6\mathbf{j} - 7\mathbf{k})$$

$$M_{1z} = \mathbf{k}\cdot\mathbf{M}_1$$

$$\mathbf{M}_1 = \lambda_{EJ}M_1$$

$$= \frac{-d\mathbf{i} + (3\text{ in.})\mathbf{j} - (7\text{ in.})\mathbf{k}}{\sqrt{d^2+58}\text{ in.}}(480\text{ lb}\cdot\text{in.})$$

Then from Equation (1),

$$\begin{vmatrix} 0 & 0 & 1 \\ 31 & -2 & 0 \\ 6 & 6 & -7 \end{vmatrix}\frac{20\text{ lb}\cdot\text{in.}}{11.0} + \frac{(-7)(480\text{ lb}\cdot\text{in.})}{\sqrt{d^2+58}} = 0$$

PROBLEM 3.97 CONTINUED

Solving for d, Equation (1) reduces to

$$\frac{20 \text{ lb}\cdot\text{in.}}{11.0}(186 + 12) - \frac{3360 \text{ lb}\cdot\text{in.}}{\sqrt{d^2 + 58}} = 0$$

From which $d = 5.3955$ in.

or $d = 5.40$ in. ◀

(b) $$\mathbf{F}_2 = \mathbf{F}_1 = \frac{20 \text{ lb}}{11.0}(6\mathbf{i} + 6\mathbf{j} - 7\mathbf{k})$$

$$= (10.9091\mathbf{i} + 10.9091\mathbf{j} - 12.7273\mathbf{k})\text{lb}$$

or $\mathbf{F}_2 = (10.91 \text{ lb})\mathbf{i} + (10.91 \text{ lb})\mathbf{j} - (12.73 \text{ lb})\mathbf{k}$ ◀

$$\mathbf{M}_2 = \mathbf{r}_{H/B} \times \mathbf{F}_1 + \mathbf{M}_1$$

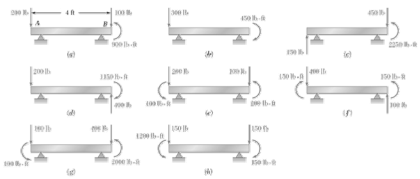
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 31 & -2 & 0 \\ 6 & 6 & -7 \end{vmatrix} \frac{20 \text{ lb}\cdot\text{in.}}{11.0} + \frac{(-5.3955)\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}}{9.3333}(480 \text{ lb}\cdot\text{in.})$$

$$= (25.455\mathbf{i} + 394.55\mathbf{j} + 360\mathbf{k})\text{lb}\cdot\text{in.}$$

$$+ (-277.48\mathbf{i} + 154.285\mathbf{j} - 360\mathbf{k})\text{lb}\cdot\text{in.}$$

$$\mathbf{M}_2 = -(252.03 \text{ lb}\cdot\text{in.})\mathbf{i} + (548.84 \text{ lb}\cdot\text{in.})\mathbf{j}$$

or $\mathbf{M}_2 = -(21.0 \text{ lb}\cdot\text{ft})\mathbf{i} + (45.7 \text{ lb}\cdot\text{ft})\mathbf{j}$ ◀

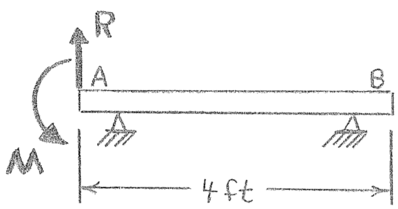


PROBLEM 3.98

A 4-ft-long beam is subjected to a variety of loadings. (*a*) Replace each loading with an equivalent force-couple system at end *A* of the beam. (*b*) Which of the loadings are equivalent?

SOLUTION

(*a*)



(*a*) Have $\Sigma F_y: \quad -200 \text{ lb} - 100 \text{ lb} = R_a$

or $\mathbf{R}_a = 300 \text{ lb}\downarrow$ ◀

and $\Sigma M_A: \quad 900 \text{ lb}\cdot\text{ft} - (100 \text{ lb})(4 \text{ ft}) = M_a$

or $\mathbf{M}_a = 500 \text{ lb}\cdot\text{ft}$ ◀

(*b*) Have $\Sigma F_y: \quad -300 \text{ lb} = R_b$

or $\mathbf{R}_b = 300 \text{ lb}\downarrow$ ◀

and $\Sigma M_A: \quad -450 \text{ lb}\cdot\text{ft} = M_b$

or $\mathbf{M}_b = 450 \text{ lb}\cdot\text{ft}$ ◀

(*c*) Have $\Sigma F_y: \quad 150 \text{ lb} - 450 \text{ lb} = R_c$

or $\mathbf{R}_c = 300 \text{ lb}\downarrow$ ◀

and $\Sigma M_A: \quad 2250 \text{ lb}\cdot\text{ft} - (450 \text{ lb})(4 \text{ ft}) = M_c$

or $\mathbf{M}_c = 450 \text{ lb}\cdot\text{ft}$ ◀

(*d*) Have $\Sigma F_y: \quad -200 \text{ lb} + 400 \text{ lb} = R_d$

or $\mathbf{R}_d = 200 \text{ lb}\uparrow$ ◀

and $\Sigma M_A: \quad (400 \text{ lb})(4 \text{ ft}) - 1150 \text{ lb}\cdot\text{ft} = M_d$

or $\mathbf{M}_d = 450 \text{ lb}\cdot\text{ft}$ ◀

(*e*) Have $\Sigma F_y: \quad -200 \text{ lb} - 100 \text{ lb} = R_e$

or $\mathbf{R}_e = 300 \text{ lb}\downarrow$ ◀

and $\Sigma M_A: \quad 100 \text{ lb}\cdot\text{ft} + 200 \text{ lb}\cdot\text{ft} - (100 \text{ lb})(4 \text{ ft}) = M_e$

or $\mathbf{M}_e = 100 \text{ lb}\cdot\text{ft}$ ◀

PROBLEM 3.98 CONTINUED

(*f*) Have $\Sigma F_y: \quad -400 \text{ lb} + 100 \text{ lb} = R_f$

or $\mathbf{R}_f = 300 \text{ lb}\downarrow$ ◀

and $\Sigma M_A: \quad -150 \text{ lb}\cdot\text{ft} + 150 \text{ lb}\cdot\text{ft} + (100 \text{ lb})(4 \text{ ft}) = M_f$

or $\mathbf{M}_f = 400 \text{ lb}\cdot\text{ft}$ ◀

(*g*) Have $\Sigma F_y: \quad -100 \text{ lb} - 400 \text{ lb} = R_g$

or $\mathbf{R}_g = 500 \text{ lb}\downarrow$ ◀

and $\Sigma M_A: \quad 100 \text{ lb}\cdot\text{ft} + 2000 \text{ lb}\cdot\text{ft} - (400 \text{ lb})(4 \text{ ft}) = M_g$

or $\mathbf{M}_g = 500 \text{ lb}\cdot\text{ft}$ ◀

(*h*) Have $\Sigma F_y: \quad -150 \text{ lb} - 150 \text{ lb} = R_h$

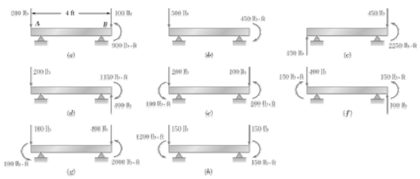
or $\mathbf{R}_h = 300 \text{ lb}\downarrow$ ◀

and $\Sigma M_A: \quad 1200 \text{ lb}\cdot\text{ft} - 150 \text{ lb}\cdot\text{ft} - (150 \text{ lb})(4 \text{ ft}) = M_h$

or $\mathbf{M}_h = 450 \text{ lb}\cdot\text{ft}$ ◀

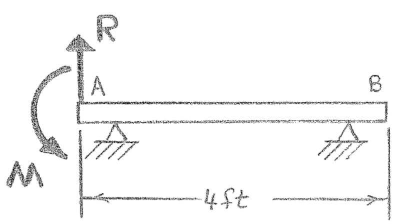
(*b*) Therefore, loadings (*c*) and (*h*) are equivalent ◀

PROBLEM 3.99



A 4-ft-long beam is loaded as shown. Determine the loading of Problem 3.98 which is equivalent to this loading.

SOLUTION



Have $\Sigma \mathbf{F}_y: \quad -100 \text{ lb} - 200 \text{ lb} = R$

or $\mathbf{R} = 300 \text{ lb} \downarrow$

and $\Sigma M_A: \quad -200 \text{ lb}\cdot\text{ft} + 1400 \text{ lb}\cdot\text{ft} - (200 \text{ lb})(4 \text{ ft}) = M$

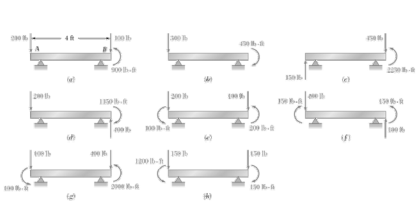
or $\mathbf{M} = 400 \text{ lb}\cdot\text{ft}$

Equivalent to case (*f*) of Problem 3.98 ◀

Problem 3.98 Equivalent force-couples at *A*

case	**R**	**M**
(*a*)	300 lb ↓	500 lb·ft
(*b*)	300 lb ↓	450 lb·ft
(*c*)	300 lb ↓	450 lb·ft
(*d*)	200 lb ↑	450 lb·ft
(*e*)	300 lb ↓	100 lb·ft
(*f*)	300 lb ↓	400 lb·ft ◀
(*g*)	500 lb ↓	500 lb·ft
(*h*)	300 lb ↓	450 lb·ft

PROBLEM 3.100



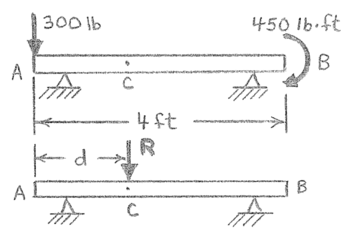
Determine the single equivalent force and the distance from point A to its line of action for the beam and loading of (a) Problem 3.98b, (b) Problem 3.98d, (c) Problem 3.98e.

Problem 3.98: A 4-ft-long beam is subjected to a variety of loadings. (a) Replace each loading with an equivalent force-couple system at end A of the beam. (b) Which of the loadings are equivalent?

SOLUTION

For equivalent single force at distance d from A

(a)



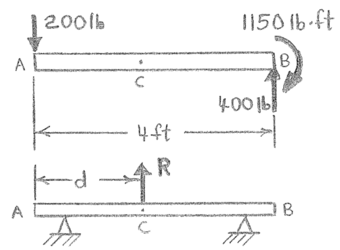
Have $\qquad \Sigma F_y: \quad -300 \text{ lb} = R$

or $\mathbf{R} = 300 \text{ lb} \downarrow$ ◀

and $\qquad \Sigma M_C: \quad (300 \text{ lb})(d) - 450 \text{ lb}\cdot\text{ft} = 0$

or $d = 1.500 \text{ ft}$ ◀

(b)



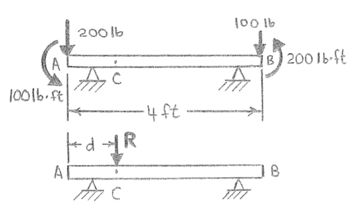
Have $\qquad \Sigma F_y: \quad -200 \text{ lb} + 400 \text{ lb} = R$

or $\mathbf{R} = 200 \text{ lb} \uparrow$ ◀

and $\qquad \Sigma M_C: \quad (200 \text{ lb})(d) + (400 \text{ lb})(4 - d) - 1150 \text{ lb}\cdot\text{ft} = 0$

or $d = 2.25 \text{ ft}$ ◀

(c)



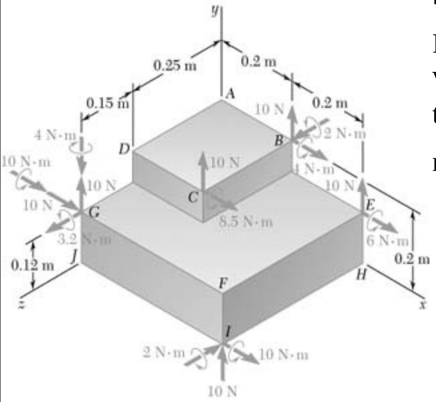
Have $\qquad \Sigma F_y: \quad -200 \text{ lb} - 100 \text{ lb} = R$

or $\mathbf{R} = 300 \text{ lb} \downarrow$ ◀

and $\qquad \Sigma M_C: \quad 100 \text{ lb}\cdot\text{ft} + (200 \text{ lb})(d) - (100 \text{ lb})(4 - d) + 200 \text{ lb}\cdot\text{ft} = 0$

or $d = 0.333 \text{ ft}$ ◀

PROBLEM 3.101

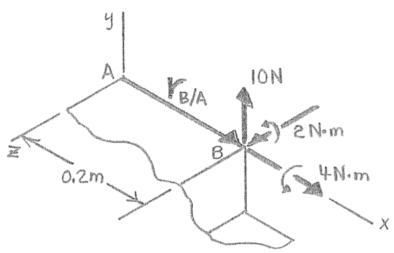


Five separate force-couple systems act at the corners of a metal block, which has been machined into the shape shown. Determine which of these systems is equivalent to a force $\mathbf{F} = (10\text{ N})\mathbf{j}$ and a couple of moment $\mathbf{M} = (6\text{ N}\cdot\text{m})\mathbf{i} + (4\text{ N}\cdot\text{m})\mathbf{k}$ located at point A.

SOLUTION

The equivalent force-couple system at A for each of the five force-couple systems will be determined. Each will then be compared to the given force-couple system to determine if they are equivalent.

Force-couple system at B



Have $$\Sigma\mathbf{F}: \quad (10\text{ N})\mathbf{j} = \mathbf{F}$$

or $$\mathbf{F} = (10\text{ N})\mathbf{j}$$

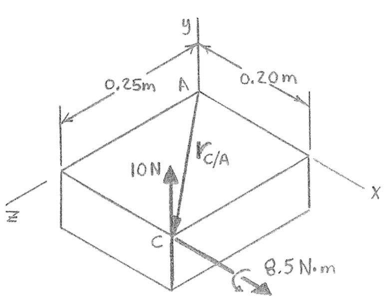
and $$\Sigma\mathbf{M}_A: \quad \Sigma\mathbf{M}_B + (\mathbf{r}_{B/A} \times \mathbf{F}) = \mathbf{M}$$

$$(4\text{ N}\cdot\text{m})\mathbf{i} + (2\text{ N}\cdot\text{m})\mathbf{k} + (0.2\text{ m})\mathbf{i} \times (10\text{ N})\mathbf{j} = \mathbf{M}$$

$$\mathbf{M} = (4\text{ N}\cdot\text{m})\mathbf{i} + (4\text{ N}\cdot\text{m})\mathbf{k}$$

Comparing to given force-couple system at A,
Is Not Equivalent ◀

Force-couple system at C



Have $$\Sigma\mathbf{F}: \quad (10\text{ N})\mathbf{j} = \mathbf{F}$$

or $$\mathbf{F} = (10\text{ N})\mathbf{j}$$

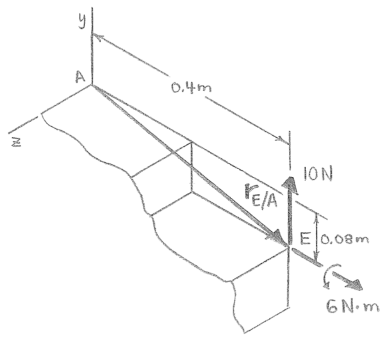
and $$\Sigma\mathbf{M}_A: \quad \mathbf{M}_C + (\mathbf{r}_{C/A} \times \mathbf{F}) = \mathbf{M}$$

$$(8.5\text{ N}\cdot\text{m})\mathbf{i} + \left[(0.2\text{ m})\mathbf{i} + (0.25\text{ m})\mathbf{k}\right] \times (10\text{ N})\mathbf{j} = \mathbf{M}$$

$$\mathbf{M} = (6\text{ N}\cdot\text{m})\mathbf{i} + (2.0\text{ N}\cdot\text{m})\mathbf{k}$$

Comparing to given force-couple system at A,
Is Not Equivalent ◀

PROBLEM 3.101 CONTINUED



Force-couple system at E

Have $\qquad \Sigma\mathbf{F}: \quad (10\text{ N})\mathbf{j} = \mathbf{F}$

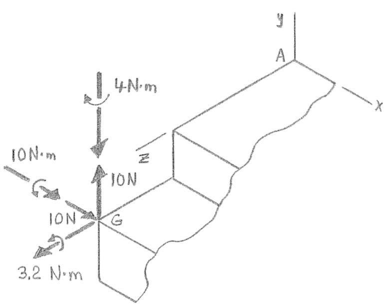
or $\qquad \mathbf{F} = (10\text{ N})\mathbf{j}$

and $\qquad \Sigma M_A: \quad \mathbf{M}_E + (\mathbf{r}_{E/A} \times \mathbf{F}) = \mathbf{M}$

$$(6\text{ N}\cdot\text{m})\mathbf{i} + \left[(0.4\text{ m})\mathbf{i} - (0.08\text{ m})\mathbf{j}\right] \times (10\text{ N})\mathbf{j} = \mathbf{M}$$

$$\mathbf{M} = (6\text{ N}\cdot\text{m})\mathbf{i} + (4\text{ N}\cdot\text{m})\mathbf{k}$$

Comparing to given force-couple system at *A*,
Is Equivalent ◀



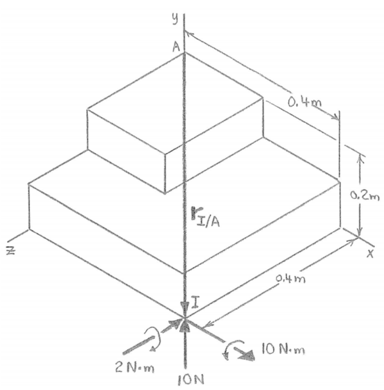
Force-couple system at G

Have $\qquad \Sigma\mathbf{F}: \quad (10\text{ N})\mathbf{i} + (10\text{ N})\mathbf{j} = \mathbf{F}$

or $\qquad \mathbf{F} = (10\text{ N})\mathbf{i} + (10\text{ N})\mathbf{j}$

$\mathbf{F}$ has two force components

$\therefore$ force-couple system at *G*
Is Not Equivalent ◀



Force-couple system at I

Have $\qquad \Sigma\mathbf{F}: \quad (10\text{ N})\mathbf{j} = \mathbf{F}$

or $\qquad \mathbf{F} = (10\text{ N})\mathbf{j}$

and $\qquad \Sigma\mathbf{M}_A: \quad \Sigma\mathbf{M}_I + (\mathbf{r}_{I/A} \times \mathbf{F}) = \mathbf{M}$

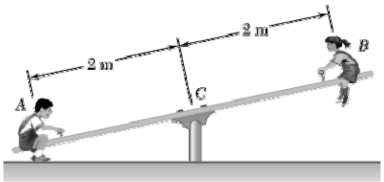
$$(10\text{ N}\cdot\text{m})\mathbf{i} - (2\text{ N}\cdot\text{m})\mathbf{k}$$

$$+\left[(0.4\text{ m})\mathbf{i} - (0.2\text{ m})\mathbf{j} + (0.4\text{ m})\mathbf{k}\right] \times (10\text{ N})\mathbf{j} = \mathbf{M}$$

or $\qquad \mathbf{M} = (6\text{ N}\cdot\text{m})\mathbf{i} + (2\text{ N}\cdot\text{m})\mathbf{k}$

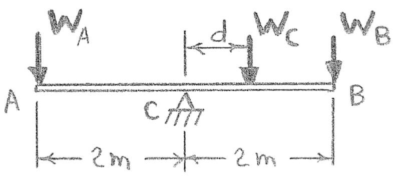
Comparing to given force-couple system at *A*,
Is Not Equivalent ◀

PROBLEM 3.102



The masses of two children sitting at ends A and B of a seesaw are 38 kg and 29 kg, respectively. Where should a third child sit so that the resultant of the weights of the three children will pass through C if she has a mass of (a) 27 kg, (b) 24 kg.

SOLUTION



First $\qquad W_A = m_A g = (38\text{ kg})g$

$$W_B = m_B g = (29\text{ kg})g$$

(a) $\qquad W_C = m_C g = (27\text{ kg})g$

For resultant weight to act at C, $\qquad \Sigma M_C = 0$

Then $\quad [(38\text{ kg})g](2\text{ m}) - [(27\text{ kg})g](d) - [(29\text{ kg})g](2\text{ m}) = 0$

$$\therefore \quad d = \frac{76-58}{27} = 0.66667\text{ m}$$

or $d = 0.667$ m ◀

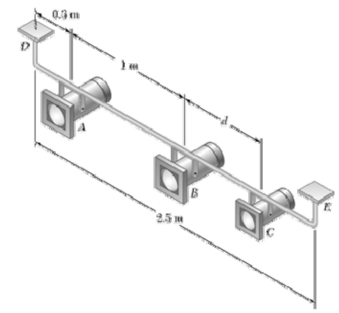
(b) $\qquad W_C = m_C g = (24\text{ kg})g$

For resultant weight to act at C, $\qquad \Sigma M_C = 0$

Then $\quad [(38\text{ kg})g](2\text{ m}) - [(24\text{ kg})g](d) - [(29\text{ kg})g](2\text{ m}) = 0$

$$\therefore \quad d = \frac{76-58}{24} = 0.75\text{ m}$$

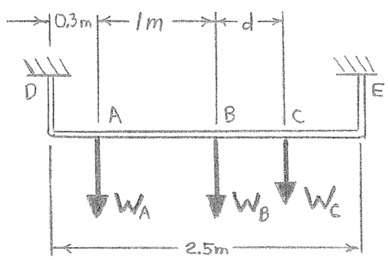
or $d = 0.750$ m ◀



PROBLEM 3.103

Three stage lights are mounted on a pipe as shown. The mass of each light is $m_A = m_B = 1.8$ kg and $m_C = 1.6$ kg. (*a*) If $d = 0.75$ m, determine the distance from *D* to the line of action of the resultant of the weights of the three lights. (*b*) Determine the value of *d* so that the resultant of the weights passes through the midpoint of the pipe.

SOLUTION



First $\quad W_A = W_B = m_A g = (1.8 \text{ kg})g$

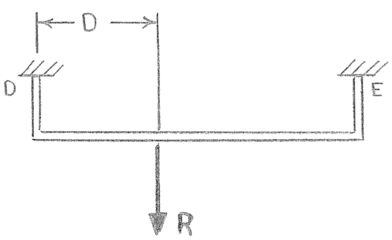
$$W_C = m_C g = (1.6 \text{ kg})g$$

(*a*) $\quad d = 0.75$ m

Have $\quad R = W_A + W_B + W_C$

$$R = [(1.8 + 1.8 + 1.6)\text{kg}]g$$

or $\quad \mathbf{R} = (5.2g) \text{ N} \downarrow$

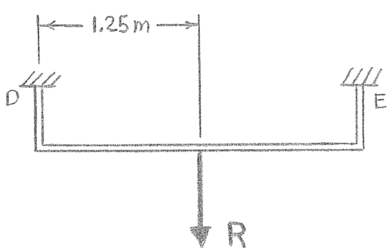


Have

$$\Sigma M_D: \quad -1.8g(0.3 \text{ m}) - 1.8g(1.3 \text{ m}) - 1.6g(2.05 \text{ m}) = -5.2g(D)$$

$$\therefore \quad D = 1.18462 \text{ m}$$

or $D = 1.185$ m ◀



(*b*) $\quad D = \dfrac{L}{2} = 1.25$ m

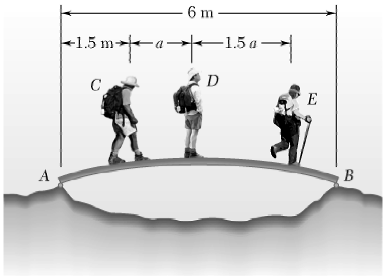
Have

$$\Sigma M_D: \quad -(1.8g)(0.3 \text{ m}) - (1.8g)(1.3 \text{ m}) - (1.6g)(1.3 \text{ m} + d)$$

$$= -(5.2g)(1.25 \text{ m})$$

$$\therefore \quad d = 0.9625 \text{ m}$$

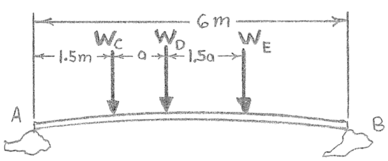
or $d = 0.963$ m ◀



PROBLEM 3.104

Three hikers are shown crossing a footbridge. Knowing that the weights of the hikers at points *C*, *D*, and *E* are 800 N, 700 N, and 540 N, respectively, determine (*a*) the horizontal distance from *A* to the line of action of the resultant of the three weights when $a = 1.1$ m, (*b*) the value of *a* so that the loads on the bridge supports at *A* and *B* are equal.

SOLUTION



(*a*) $$a = 1.1 \text{ m}$$

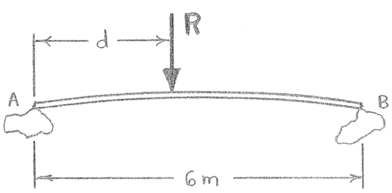
Have $$\Sigma F: \quad -W_C - W_D - W_E = R$$

$$\therefore \quad R = -800 \text{ N} - 700 \text{ N} - 540 \text{ N}$$

$$R = 2040 \text{ N}$$

or $$\mathbf{R} = 2040 \text{ N} \downarrow$$

(*a*)



Have

$$\Sigma M_A: \quad -(800 \text{ N})(1.5 \text{ m}) - (700 \text{ N})(2.6 \text{ m}) - (540 \text{ N})(4.25 \text{ m})$$

$$= -R(d)$$

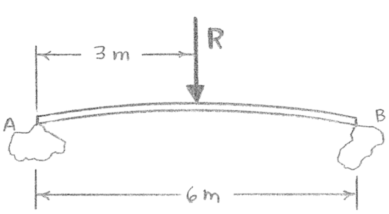
$$\therefore \quad -5315 \text{ N}\cdot\text{m} = -(2040 \text{ N})d$$

and $$d = 2.6054 \text{ m}$$

or $d = 2.61$ m to the right of *A* ◀

(*b*) For equal reaction forces at *A* and *B*, the resultant, **R**, must act at the center of the span.

(*b*)



From $$\Sigma M_A = -R\left(\frac{L}{2}\right)$$

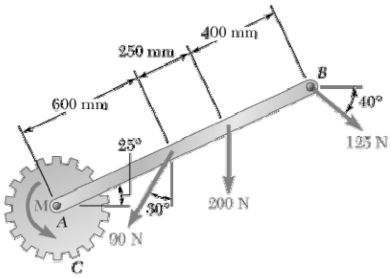
$$\therefore \quad -(800 \text{ N})(1.5 \text{ m}) - (700 \text{ N})(1.5 \text{ m} + a) - (540 \text{ N})(1.5 \text{ m} + 2.5a)$$

$$= -(2040 \text{ N})(3 \text{ m})$$

$$3060 + 2050a = 6120$$

and $$a = 1.49268 \text{ m}$$

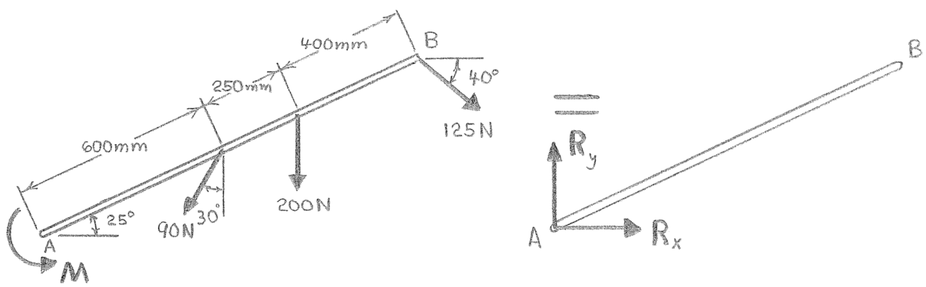
or $a = 1.493$ m ◀



PROBLEM 3.105

Gear *C* is rigidly attached to arm *AB*. If the forces and couple shown can be reduced to a single equivalent force at *A*, determine the equivalent force and the magnitude of the couple **M**.

SOLUTION



For equivalence

$$\Sigma F_x: \quad -(90\text{ N})\sin 30° + (125\text{ N})\cos 40° = R_x$$

or $R_x = 50.756$ N

$$\Sigma F_y: \quad -(90\text{ N})\cos 30° - 200\text{ N} - (125\text{ N})\sin 40° = R_y$$

or $R_y = -358.29$ N

Then $$R = \sqrt{(50.756)^2 + (-358.29)^2} = 361.87\text{ N}$$

and $$\tan\theta = \frac{R_y}{R_x} = \frac{-358.29}{50.756} = -7.0591 \qquad \therefore \quad \theta = -81.937°$$

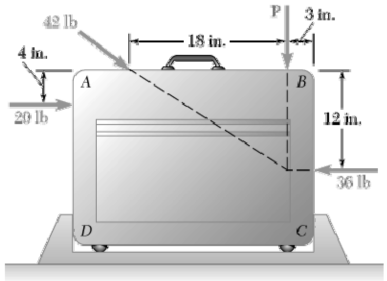
or $\mathbf{R} = 362\text{ N} \; 81.9°$ ◀

Also

$$\Sigma M_A: \quad M - [(90\text{ N})\sin 35°](0.6\text{ m}) - [(200\text{ N})\cos 25°](0.85\text{ m}) - [(125\text{ N})\sin 65°](1.25\text{ m}) = 0$$

$$\therefore \quad M = 326.66\text{ N}\cdot\text{m}$$

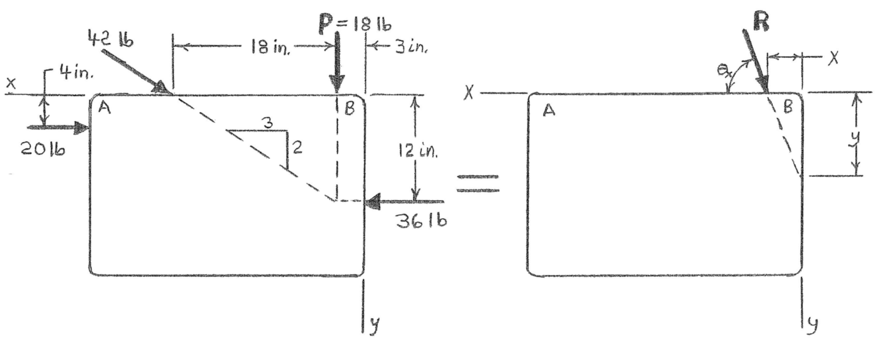
or $M = 327\text{ N}\cdot\text{m}$ ◀



PROBLEM 3.106

To test the strength of a 25×20-in. suitcase, forces are applied as shown. If $P = 18$ lb, (*a*) determine the resultant of the applied forces, (*b*) locate the two points where the line of action of the resultant intersects the edge of the suitcase.

SOLUTION



(*a*) $P = 18$ lb

Have $$\Sigma\mathbf{F}: \quad -(20\text{ lb})\mathbf{i} + \frac{42\text{ lb}}{\sqrt{13}}(-3\mathbf{i} + 2\mathbf{j}) + (18\text{ lb})\mathbf{j} + (36\text{ lb})\mathbf{i} = R_x\mathbf{i} + R_y\mathbf{j}$$

$$\therefore\ -(18.9461\text{ lb})\mathbf{i} + (41.297\text{ lb})\mathbf{j} = R_x\mathbf{i} + R_y\mathbf{j}$$

or $$\mathbf{R} = -(18.95\text{ lb})\mathbf{i} + (41.3\text{ lb})\mathbf{j}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(18.9461)^2 + (41.297)^2} = 45.436\text{ lb}$$

$$\theta_x = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{41.297}{-18.9461}\right) = -65.355°$$

or $\mathbf{R} = 45.4\text{ lb} \searrow 65.4°$ ◀

(*b*) Have $$\Sigma\mathbf{M}_B = \mathbf{M}_B$$

$$\mathbf{M}_B = (4\text{ in.})\mathbf{j} \times (-20\text{ lb})\mathbf{i} + (21\text{ in.})\mathbf{i} \times \left[\frac{42\text{ lb}}{\sqrt{13}}(-3\mathbf{i} + 2\mathbf{j})\right] + (12\text{ in.})\mathbf{j} \times (36\text{ lb})\mathbf{i} + (3\text{ in.})\mathbf{i} \times (18\text{ lb})\mathbf{j}$$

$$\therefore\ \mathbf{M}_B = (191.246\text{ lb}\cdot\text{in.})\mathbf{k}$$

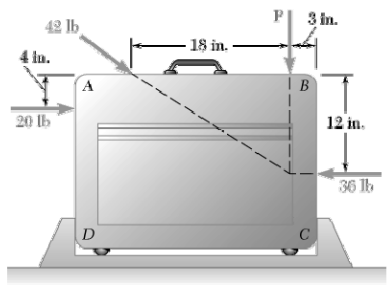
PROBLEM 3.106 CONTINUED

Since $$\mathbf{M}_B = \mathbf{r}_B \times \mathbf{R}$$

$$\therefore\ (191.246 \text{ lb}\cdot\text{in.})\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & y & 0 \\ -18.9461 & 41.297 & 0 \end{vmatrix} = (41.297x + 18.9461y)\mathbf{k}$$

For $y = 0,\quad x = \dfrac{191.246}{41.297} = 4.6310$ in. $\qquad$ or $x = 4.63$ in. ◀

For $x = 0,\quad y = \dfrac{191.246}{18.9461} = 10.0942$ in. $\qquad$ or $y = 10.09$ in. ◀

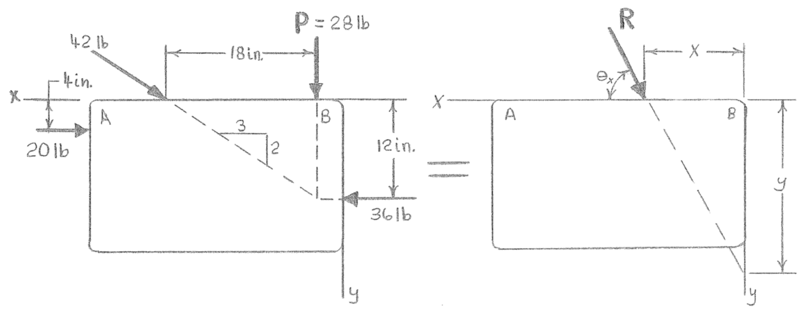


PROBLEM 3.107

Solve Problem 3.106 assuming that $P = 28$ lb.

Problem 3.106: To test the strength of a 25×20-in. suitcase, forces are applied as shown. If $P = 18$ lb, (a) determine the resultant of the applied forces, (b) locate the two points where the line of action of the resultant intersects the edge of the suitcase.

SOLUTION



(a) $P = 28$ lb

Have $\quad \Sigma\mathbf{F}: \quad -(20 \text{ lb})\mathbf{i} + \dfrac{42}{\sqrt{13}}(-3\mathbf{i} + 2\mathbf{j}) + (28 \text{ lb})\mathbf{j} + (36 \text{ lb})\mathbf{i} = R_x\mathbf{i} + R_y\mathbf{j}$

$$\therefore -(18.9461 \text{ lb})\mathbf{i} + (51.297 \text{ lb})\mathbf{j} = R_x\mathbf{i} + R_y\mathbf{j}$$

or
$$\mathbf{R} = -(18.95 \text{ lb})\mathbf{i} + (51.3 \text{ lb})\mathbf{j}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(18.9461)^2 + (51.297)^2} = 54.684 \text{ lb}$$

$$\theta_x = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{51.297}{-18.9461}\right) = -69.729°$$

or $\mathbf{R} = 54.7$ lb $\nwarrow$ 69.7° ◀

(b) Have
$$\Sigma\mathbf{M}_B = \mathbf{M}_B$$

$$\mathbf{M}_B = (4 \text{ in.})\mathbf{j} \times (-20 \text{ lb})\mathbf{i} + (21 \text{ in.})\mathbf{i} \times \left[\frac{42 \text{ lb}}{\sqrt{13}}(-3\mathbf{i} + 2\mathbf{j})\right] + (12 \text{ in.})\mathbf{j} \times (36 \text{ lb})\mathbf{i} + (3 \text{ in.})\mathbf{i} \times (28 \text{ lb})\mathbf{j}$$

$$\therefore \quad \mathbf{M}_B = (221.246 \text{ lb}\cdot\text{in.})\mathbf{k}$$

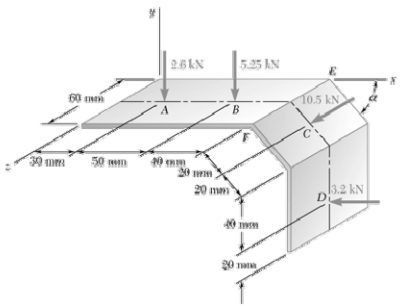
PROBLEM 3.107 CONTINUED

Since $$\mathbf{M}_B = \mathbf{r}_B \times \mathbf{R}$$

$$\therefore \; (221.246 \text{ lb} \cdot \text{in.})\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & y & 0 \\ -18.9461 & 51.297 & 0 \end{vmatrix} = (51.297x + 18.9461y)\mathbf{k}$$

For $$y = 0, \qquad x = \frac{221.246}{51.297} = 4.3130 \text{ in.}$$ $$\text{or } x = 4.31 \text{ in.} \blacktriangleleft$$

For $$x = 0, \qquad y = \frac{221.246}{18.9461} = 11.6776 \text{ in.}$$ $$\text{or } y = 11.68 \text{ in.} \blacktriangleleft$$

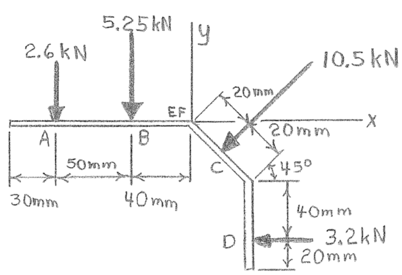


PROBLEM 3.108

As four holes are punched simultaneously in a piece of aluminum sheet metal, the punches exert on the piece the forces shown. Knowing that the forces are perpendicular to the surfaces of the piece, determine (*a*) the resultant of the applied forces when $\alpha = 45°$ and the point of intersection of the line of action of that resultant with a line drawn through points *A* and *B*, (*b*) the value of α so that the line of action of the resultant passes through fold *EF*.

SOLUTION

Position the origin for the coordinate system along the centerline of the sheet metal at the intersection with line *EF*.



(*a*) Have $\Sigma\mathbf{F} = \mathbf{R}$

$$\mathbf{R} = -\left[2.6\mathbf{j} - 5.25\mathbf{j} - 10.5(\cos 45°\mathbf{i} + \sin 45°\mathbf{j}) - 3.2\mathbf{i}\right] \text{ kN}$$

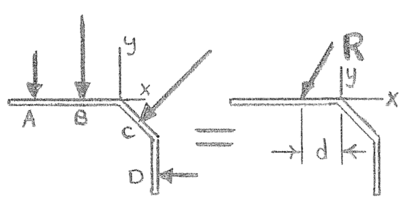
$$\therefore\ \mathbf{R} = -(10.6246 \text{ kN})\mathbf{i} - (15.2746 \text{ kN})\mathbf{j}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(10.6246)^2 + (15.2746)^2}$$

$$= 18.6064 \text{ kN}$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{-15.2746}{-10.6246}\right) = 55.179°$$

or $\mathbf{R} = 18.61 \text{ kN} \measuredangle\, 55.2°$ ◀



Have $M_{EF} = \Sigma M_{EF}$

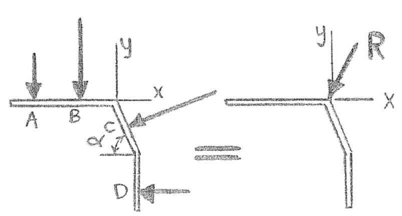
where

$$M_{EF} = (2.6 \text{ kN})(90 \text{ mm}) + (5.25 \text{ kN})(40 \text{ mm})$$

$$- (10.5 \text{ kN})(20 \text{ mm}) - (3.2 \text{ kN})\left[(40 \text{ mm})\sin 45° + 40 \text{ mm}\right]$$

$$\therefore\ M_{EF} = 15.4903 \text{ N}\cdot\text{m}$$

To obtain distance *d* left of *EF*,



Have $M_{EF} = dR_y = d(-15.2746 \text{ kN})$

$$\therefore\ d = \frac{15.4903 \text{ N}\cdot\text{m}}{-15.2746 \times 10^{-3} \text{ N}} = -1.01412 \times 10^{-3} \text{ m}$$

or $d = 1.014$ mm left of *EF* ◀

PROBLEM 3.108 CONTINUED

(*b*) Have $$M_{EF} = 0$$

$$M_{EF} = 0 = (2.6\text{ kN})(90\text{ mm}) + (5.25\text{ kN})(40\text{ mm})$$

$$-(10.5\text{ kN})(20\text{ mm})$$

$$-(3.2\text{ kN})\left[(40\text{ mm})\sin\alpha + 40\text{ mm}\right]$$

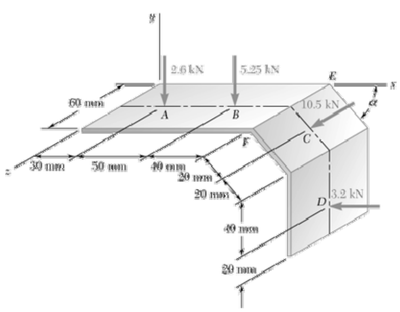
$$\therefore\ (128\text{ N}\cdot\text{m})\sin\alpha = 106\text{ N}\cdot\text{m}$$

$$\sin\alpha = 0.828125$$

$$\alpha = 55.907°$$

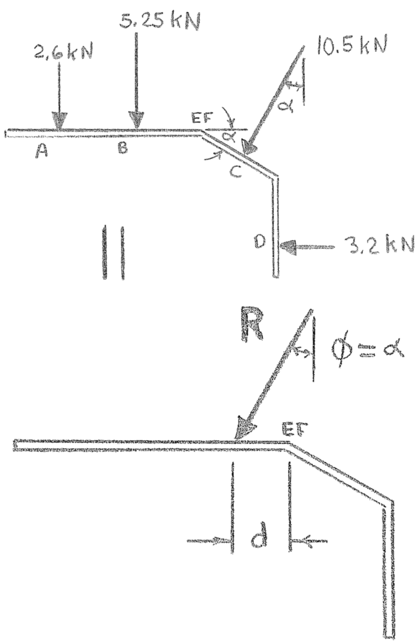
or $\alpha = 55.9°$ ◀

PROBLEM 3.109



As four holes are punched simultaneously in a piece of aluminum sheet metal, the punches exert on the piece the forces shown. Knowing that the forces are perpendicular to the surfaces of the piece, determine (*a*) the value of α so that the resultant of the applied forces is parallel to the 10.5 N force, (*b*) the corresponding resultant of the applied forces and the point of intersection of its line of action with a line drawn through points *A* and *B*.

SOLUTION



(*a*) For the resultant force, **R**, to be parallel to the 10.5 kN force,

$$\alpha = \phi$$

$$\therefore \tan\alpha = \tan\phi = \frac{R_y}{R_x}$$

where

$$R_x = -3.2 \text{ kN} - (10.5 \text{ kN})\sin\alpha$$

$$R_y = -2.6 \text{ kN} - 5.25 \text{ kN} - (10.5 \text{ kN})\cos\alpha$$

$$\therefore \tan\alpha = \frac{3.2 + 10.5\sin\alpha}{7.85 + 10.5\cos\alpha}$$

and $$\tan\alpha = \frac{3.2}{7.85} = 0.40764$$

$$\alpha = 22.178^\circ \qquad \text{or } \alpha = 22.2^\circ \blacktriangleleft$$

(*b*) From $\alpha = 22.178^\circ$

$$R_x = -3.2 \text{ kN} - (10.5 \text{ kN})\sin 22.178^\circ = -7.1636 \text{ kN}$$

$$R_y = -7.85 \text{ kN} - (10.5 \text{ kN})\cos 22.178^\circ = -17.5732 \text{ kN}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(7.1636)^2 + (17.5732)^2} = 18.9770 \text{ kN}$$

or $$\mathbf{R} = 18.98 \text{ kN} \swarrow 67.8^\circ \blacktriangleleft$$

Then

$$M_{EF} = \Sigma M_{EF}$$

where

$$M_{EF} = (2.6 \text{ kN})(90 \text{ mm}) + (5.25 \text{ kN})(40 \text{ mm}) - (10.5 \text{ kN})(20 \text{ mm})$$
$$-(3.2 \text{ kN})[(40 \text{ mm})\sin 22.178^\circ + 40 \text{ mm}]$$
$$= 57.682 \text{ N}\cdot\text{m}$$

PROBLEM 3.109 CONTINUED

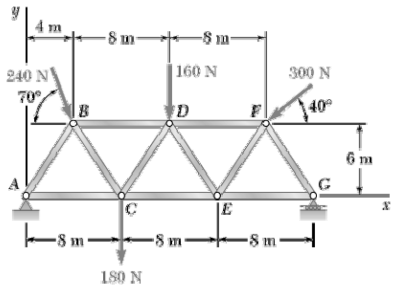
To obtain distance d left of EF,

Have $$M_{EF} = dR_y = d(-17.5732)$$

$$\therefore\ d = \frac{57.682\ \text{N}\cdot\text{m}}{-17.5732 \times 10^3\ \text{N}} = -3.2824 \times 10^{-3}\ \text{m}$$

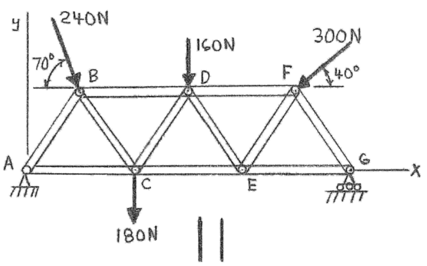
or $d = 3.28$ mm left of EF ◀

PROBLEM 3.110



A truss supports the loading shown. Determine the equivalent force acting on the truss and the point of intersection of its line of action with a line through points A and G.

SOLUTION



Have $$\mathbf{R} = \Sigma\mathbf{F}$$

$$\mathbf{R} = (240\text{ N})(\cos 70°\mathbf{i} - \sin 70°\mathbf{j}) - (160\text{ N})\mathbf{j}$$

$$+ (300\text{ N})(-\cos 40°\mathbf{i} - \sin 40°\mathbf{j}) - (180\text{ N})\mathbf{j}$$

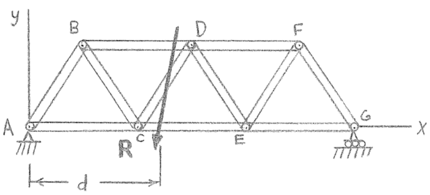
$$\therefore\ \mathbf{R} = -(147.728\text{ N})\mathbf{i} - (758.36\text{ N})\mathbf{j}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(147.728)^2 + (758.36)^2}$$

$$= 772.62\text{ N}$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{-758.36}{-147.728}\right) = 78.977°$$

or $\mathbf{R} = 773\text{ N} \swarrow 79.0°$ ◀



Have $$\Sigma M_A = dR_y$$

where

$$\Sigma M_A = -[240\text{ N}\cos 70°](6\text{ m}) - [240\text{ N}\sin 70°](4\text{ m})$$

$$-(160\text{ N})(12\text{ m}) + [300\text{ N}\cos 40°](6\text{ m})$$

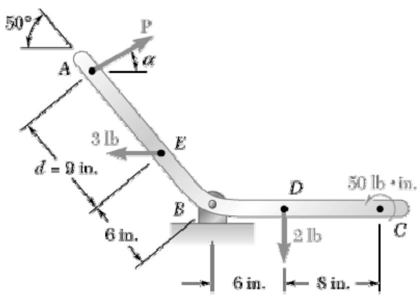
$$-[300\text{ N}\sin 40°](20\text{ m}) - (180\text{ N})(8\text{ m})$$

$$= -7232.5\text{ N}\cdot\text{m}$$

$$\therefore\ d = \frac{-7232.5\text{ N}\cdot\text{m}}{-758.36\text{ N}} = 9.5370\text{ m}$$

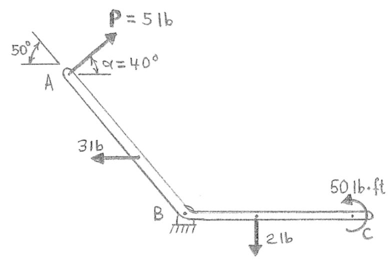
or $d = 9.54\text{ m}$ to the right of A ◀

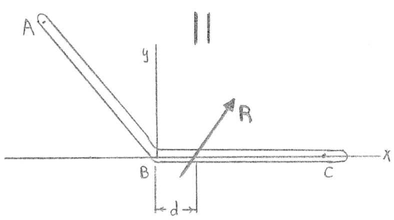
PROBLEM 3.111



Three forces and a couple act on crank ABC. For $P = 5$ lb and $\alpha = 40°$, (a) determine the resultant of the given system of forces, (b) locate the point where the line of action of the resultant intersects a line drawn through points B and C, (c) locate the point where the line of action of the resultant intersects a line drawn through points A and B.

SOLUTION





(a) $$P = 5 \text{ lb}, \qquad \alpha = 40°$$

Have $$\mathbf{R} = \Sigma\mathbf{F}$$

$$= (5 \text{ lb})(\cos 40°\mathbf{i} + \sin 40°\mathbf{j}) - (3 \text{ lb})\mathbf{i} - (2 \text{ lb})\mathbf{j}$$

$$\therefore\ \mathbf{R} = (0.83022 \text{ lb})\mathbf{i} + (1.21394 \text{ lb})\mathbf{j}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(0.83022)^2 + (1.21394)^2}$$

$$= 1.47069 \text{ lb}$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{1.21394}{0.83022}\right) = 55.632°$$

or $\mathbf{R} = 1.471 \text{ lb} \measuredangle\ 55.6°$ ◀

(b) From $$M_B = \Sigma M_B = dR_y$$

where

$$M_B = -\left[(5 \text{ lb})\cos 40°\right]\left[(15 \text{ in.})\sin 50°\right] - \left[(5 \text{ lb})\sin 40°\right]$$

$$\times\left[(15 \text{ in.})\sin 50°\right] + (3 \text{ lb})\left[(6 \text{ in.})\sin 50°\right]$$

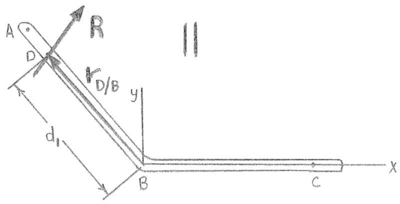
$$-(2 \text{ lb})(6 \text{ in.}) + 50 \text{ lb}\cdot\text{in.}$$

$$\therefore\ M_B = -23.211 \text{ lb}\cdot\text{in.}$$

and $$d = \frac{M_B}{R_y} = \frac{-23.211 \text{ lb}\cdot\text{in.}}{1.21394 \text{ lb}} = -19.1205 \text{ in.}$$

or $d = 19.12$ in. to the left of B ◀

PROBLEM 3.111 CONTINUED



(c) From $\mathbf{M}_B = \mathbf{r}_{D/B} \times \mathbf{R}$

$$-(23.211\text{ lb}\cdot\text{in.})\mathbf{k} = (-d_1\cos 50°\mathbf{i} + d_1\sin 50°\mathbf{j})$$

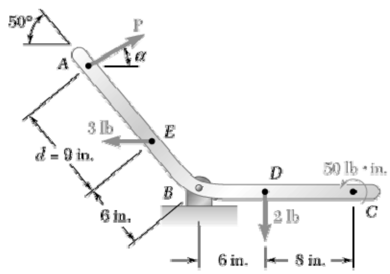
$$\times\left[(-0.83022\text{ lb})\mathbf{i} + (1.21394\text{ lb})\mathbf{j}\right]$$

$$-(23.211\text{ lb}\cdot\text{in.})\mathbf{k} = (-0.78028d_1 - 0.63599d_1)\mathbf{k}$$

$$\therefore\quad d_1 = \frac{23.211}{1.41627} = 16.3889\text{ in.}$$

or $d_1 = 16.39$ in. from B along line AB

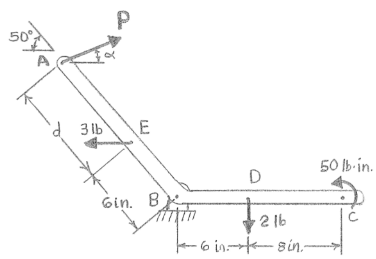
or 1.389 in. above and to the left of A ◀



PROBLEM 3.112

Three forces and a couple act on crank *ABC*. Determine the value of *d* so that the given system of forces is equivalent to zero at (*a*) point *B*, (*b*) point *D*.

SOLUTION



Based on $$\Sigma F_x = 0$$

$$P\cos\alpha - 3\text{ lb} = 0$$

$$\therefore\quad P\cos\alpha = 3\text{ lb} \qquad (1)$$

and $$\Sigma F_y = 0$$

$$P\sin\alpha - 2\text{ lb} = 0$$

$$\therefore\quad P\sin\alpha = 2\text{ lb} \qquad (2)$$

Dividing Equation (2) by Equation (1),

$$\tan\alpha = \frac{2}{3}$$

$$\therefore\quad \alpha = 33.690°$$

Substituting into Equation (1),

$$P = \frac{3\text{ lb}}{\cos 33.690°} = 3.6056\text{ lb}$$

or $$\mathbf{P} = 3.61\text{ lb} \measuredangle\ 33.7°$$

(*a*) Based on $$\Sigma M_B = 0$$

$$-\left[(3.6056\text{ lb})\cos 33.690°\right]\left[(d + 6\text{ in.})\sin 50°\right]$$

$$-\left[(3.6056\text{ lb})\sin 33.690°\right]\left[(d + 6\text{ in.})\cos 50°\right]$$

$$+(3\text{ lb})\left[(6\text{ in.})\sin 50°\right] - (2\text{ lb})(6\text{ in.}) + 50\text{ lb}\cdot\text{in.} = 0$$

$$-3.5838d = -30.286$$

$$\therefore\ d = 8.4509\text{ in.}$$

or $d = 8.45$ in. ◀

PROBLEM 3.112 CONTINUED

(*b*) Based on $\Sigma M_D = 0$

$$-\left[(3.6056 \text{ lb})\cos 33.690°\right]\left[(d + 6 \text{ in.})\sin 50°\right]$$

$$-\left[(3.6056 \text{ lb})\sin 33.690°\right]\left[(d + 6 \text{ in.})\cos 50° + 6 \text{ in.}\right]$$

$$+(3 \text{ lb})\left[(6 \text{ in.})\sin 50°\right] + 50 \text{ lb}\cdot\text{in.} = 0$$

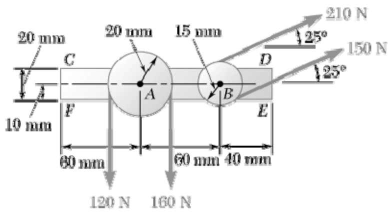
$$-3.5838d = -30.286$$

$$\therefore\ d = 8.4509 \text{ in.}$$

or $d = 8.45$ in. ◀

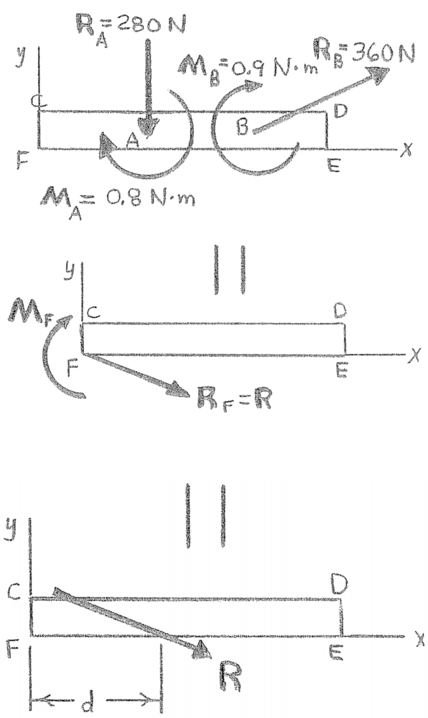
This result is expected, since $\mathbf{R} = 0$ and $\mathbf{M}_B^R = 0$ for $d = 8.45$ in. implies that $\mathbf{R} = 0$ and $\mathbf{M} = 0$ at any other point for the value of *d* found in part *a*.

PROBLEM 3.113



Pulleys *A* and *B* are mounted on bracket *CDEF*. The tension on each side of the two belts is as shown. Replace the four forces with a single equivalent force, and determine where its line of action intersects the bottom edge of the bracket.

SOLUTION



Equivalent force-couple at *A* due to belts on pulley *A*

Have $\quad \Sigma\mathbf{F}: \quad -120\text{ N} - 160\text{ N} = R_A$

$$\therefore \quad \mathbf{R}_A = 280\text{ N} \downarrow$$

Have $\quad \Sigma\mathbf{M}_A: \quad -40\text{ N}(0.02\text{ m}) = M_A$

$$\therefore \quad \mathbf{M}_A = 0.8\text{ N}\cdot\text{m} \circlearrowright$$

Equivalent force-couple at *B* due to belts on pulley *B*

Have $\quad \Sigma\mathbf{F}: \quad (210\text{ N} + 150\text{ N}) \measuredangle\, 25° = \mathbf{R}_B$

$$\therefore \quad \mathbf{R}_B = 360\text{ N} \measuredangle\, 25°$$

Have $\quad \Sigma\mathbf{M}_B: \quad -60\text{ N}(0.015\text{ m}) = M_B$

$$\therefore \quad \mathbf{M}_B = 0.9\text{ N}\cdot\text{m} \circlearrowright$$

Equivalent force-couple at *F*

Have $\quad \Sigma\mathbf{F}: \quad \mathbf{R}_F = (-280\text{ N})\mathbf{j} + (360\text{ N})(\cos 25°\mathbf{i} + \sin 25°\mathbf{j})$

$$= (326.27\text{ N})\mathbf{i} - (127.857\text{ N})\mathbf{j}$$

$$R = R_F = \sqrt{R_{Fx}^2 + R_{Fy}^2} = \sqrt{(326.27)^2 + (127.857)^2} = 350.43\text{ N}$$

$$\theta = \tan^{-1}\left(\frac{R_{Fy}}{R_{Fx}}\right) = \tan^{-1}\left(\frac{-127.857}{326.27}\right) = -21.399°$$

or $\mathbf{R}_F = \mathbf{R} = 350\text{ N} \searrow 21.4°$ ◀

PROBLEM 3.113 CONTINUED

Have

$$\Sigma \mathbf{M}_F: \quad M_F = -(280 \text{ N})(0.06 \text{ m}) - 0.80 \text{ N}\cdot\text{m}$$

$$- \left[(360 \text{ N})\cos 25°\right](0.010 \text{ m})$$

$$+ \left[(360 \text{ N})\sin 25°\right](0.120 \text{ m}) - 0.90 \text{ N}\cdot\text{m}$$

$$\mathbf{M}_F = -(3.5056 \text{ N}\cdot\text{m})\mathbf{k}$$

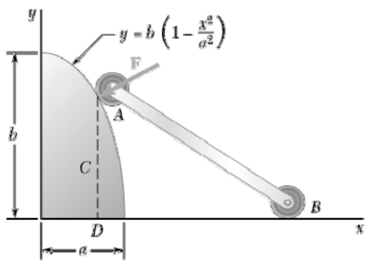
To determine where a single resultant force will intersect line *FE*,

$$M_F = dR_y$$

$$\therefore \quad d = \frac{M_F}{R_y} = \frac{-3.5056 \text{ N}\cdot\text{m}}{-127.857 \text{ N}} = 0.027418 \text{ m} = 27.418 \text{ mm}$$

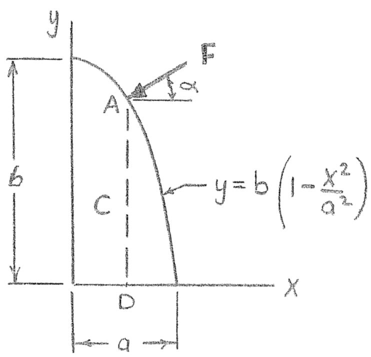
or $d = 27.4 \text{ mm}$ ◀

PROBLEM 3.114



As follower *AB* rolls along the surface of member *C*, it exerts a constant force **F** perpendicular to the surface. (*a*) Replace **F** with an equivalent force-couple system at the point *D* obtained by drawing the perpendicular from the point of contact to the *x* axis (*b*) For $a = 1\text{ m}$ and $b = 2\text{ m}$, determine the value of *x* for which the moment of the equivalent force-couple system at *D* is maximum.

SOLUTION



(*a*) The slope of any tangent to the surface of member *C* is

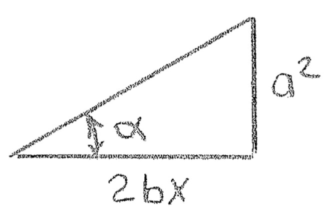
$$\frac{dy}{dx} = \frac{d}{dx}\left[b\left(1 - \frac{x^2}{a^2}\right)\right] = \frac{-2b}{a^2}x$$

Since the force **F** is perpendicular to the surface,

$$\tan\alpha = -\left(\frac{dy}{dx}\right)^{-1} = \frac{a^2}{2b}\left(\frac{1}{x}\right)$$

For equivalence

$$\Sigma F: \quad \mathbf{F} = \mathbf{R}$$



$$\Sigma M_D: \quad (F\cos\alpha)(y_A) = M_D$$

where

$$\cos\alpha = \frac{2bx}{\sqrt{(a^2)^2 + (2bx)^2}}, \qquad y_A = b\left(1 - \frac{x^2}{a^2}\right)$$

$$\therefore \quad M_D = \frac{2Fb^2\left(x - \dfrac{x^3}{a^2}\right)}{\sqrt{a^4 + 4b^2x^2}}$$

Therefore, the equivalent force-couple system at *D* is

$$\mathbf{R} = F \swarrow \tan^{-1}\left(\frac{a^2}{2bx}\right) \blacktriangleleft$$

$$\mathbf{M} = \frac{2Fb^2\left(x - \dfrac{x^3}{a^2}\right)}{\sqrt{a^4 + 4b^2x^2}} \circlearrowleft \blacktriangleleft$$

PROBLEM 3.114 CONTINUED

(*b*) To maximize *M*, the value of *x* must satisfy $\dfrac{dM}{dx} = 0$

where, for $a = 1$ m, $b = 2$ m

$$M = \frac{8F\left(x - x^3\right)}{\sqrt{1 + 16x^2}}$$

$$\therefore \quad \frac{dM}{dx} = 8F\frac{\sqrt{1+16x^2}\left(1 - 3x^2\right) - \left(x - x^3\right)\left[\frac{1}{2}\left(32x\right)\left(1+16x^2\right)^{-\frac{1}{2}}\right]}{\left(1+16x^2\right)} = 0$$

$$\left(1+16x^2\right)\left(1-3x^2\right) - 16x\left(x - x^3\right) = 0$$

or $$32x^4 + 3x^2 - 1 = 0$$

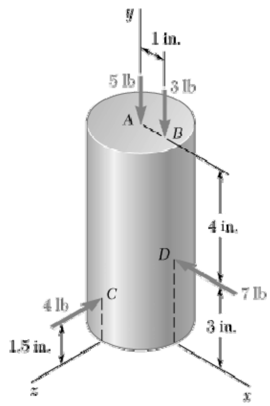
$$x^2 = \frac{-3 \pm \sqrt{9 - 4(32)(-1)}}{2(32)} = 0.136011 \text{ m}^2 \text{ and } -0.22976 \text{ m}^2$$

Using the positive value of x^2,

$$x = 0.36880 \text{ m}$$

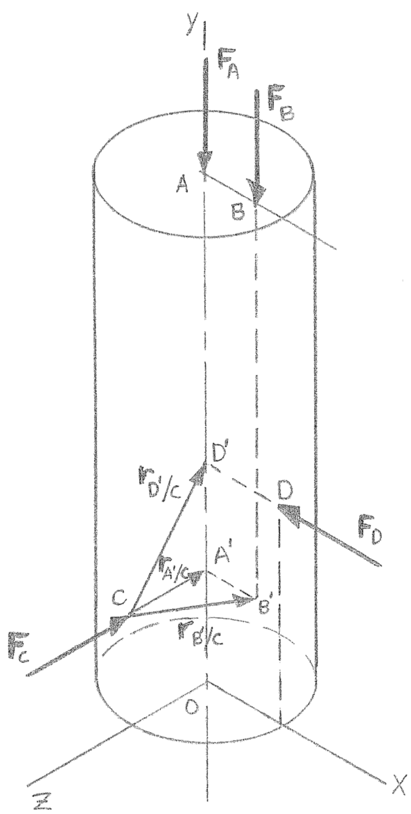
or $x = 369$ mm ◀

PROBLEM 3.115



As plastic bushings are inserted into a 3-in.-diameter cylindrical sheet metal container, the insertion tool exerts the forces shown on the enclosure. Each of the forces is parallel to one of the coordinate axes. Replace these forces with an equivalent force-couple system at *C*.

SOLUTION



For equivalence

$$\Sigma \mathbf{F}: \quad \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D = \mathbf{R}_C$$

$$\mathbf{R}_C = -(5\text{ lb})\mathbf{j} - (3\text{ lb})\mathbf{j} - (4\text{ lb})\mathbf{k} - (7\text{ lb})\mathbf{i}$$

$$\therefore \quad \mathbf{R}_C = (-7\text{ lb})\mathbf{i} - (8\text{ lb})\mathbf{j} - (4\text{ lb})\mathbf{k} \blacktriangleleft$$

Also for equivalence

$$\Sigma \mathbf{M}_C: \quad \mathbf{r}_{A'/C} \times \mathbf{F}_A + \mathbf{r}_{B'/C} \times \mathbf{F}_B + \mathbf{r}_{D'/C} \times \mathbf{F}_D = \mathbf{M}_C$$

or

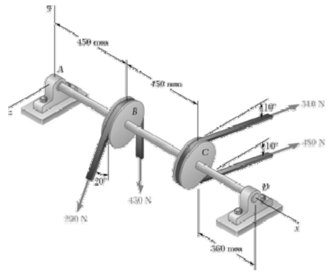
$$\mathbf{M}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -1.5\text{ in.} \\ 0 & 5\text{ lb} & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1\text{ in.} & 0 & -1.5\text{ in.} \\ 0 & -3\text{ lb} & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1.5\text{ in.} & 1.5\text{ in.} \\ -7\text{ lb} & 0 & 0 \end{vmatrix}$$

$$= \left[(-7.50\text{ lb}\cdot\text{in.} - 0)\mathbf{i}\right] + \left[(0 - 4.50\text{ lb}\cdot\text{in.})\mathbf{i} + (-3.0\text{ lb}\cdot\text{in.} - 0)\mathbf{k}\right]$$

$$+ \left[(10.5\text{ lb}\cdot\text{in.} - 0)\mathbf{j} + (0 + 10.5\text{ lb}\cdot\text{in.})\mathbf{k}\right]$$

$$\text{or } \mathbf{M}_C = -(12.0\text{ lb}\cdot\text{in.})\mathbf{i} + (10.5\text{ lb}\cdot\text{in.})\mathbf{j} + (7.5\text{ lb}\cdot\text{in.})\mathbf{k} \blacktriangleleft$$

PROBLEM 3.116

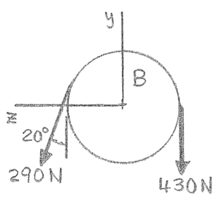


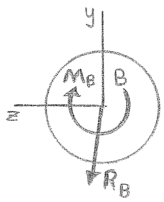
Two 300-mm-diameter pulleys are mounted on line shaft *AD*. The belts *B* and *C* lie in vertical planes parallel to the *yz* plane. Replace the belt forces shown with an equivalent force-couple system at *A*.

SOLUTION

Equivalent force-couple at each pulley

Pulley *B*





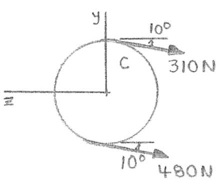
$$\mathbf{R}_B = (290\text{ N})(-\cos 20°\mathbf{j} + \sin 20°\mathbf{k}) - 430\text{ N}\mathbf{j}$$

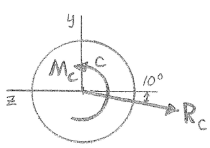
$$= -(702.51\text{ N})\mathbf{j} + (99.186\text{ N})\mathbf{k}$$

$$\mathbf{M}_B = -(430\text{ N} - 290\text{ N})(0.15\text{ m})\mathbf{i}$$

$$= -(21\text{ N}\cdot\text{m})\mathbf{i}$$

Pulley *C*





$$\mathbf{R}_C = (310\text{ N} + 480\text{ N})(-\sin 10°\mathbf{j} - \cos 10°\mathbf{k})$$

$$= -(137.182\text{ N})\mathbf{j} - (778.00\text{ N})\mathbf{k}$$

$$\mathbf{M}_C = (480\text{ N} - 310\text{ N})(0.15\text{ m})\mathbf{i}$$

$$= (25.5\text{ N}\cdot\text{m})\mathbf{i}$$

Then $\quad \mathbf{R} = \mathbf{R}_B + \mathbf{R}_C = -(839.69\text{ N})\mathbf{j} - (678.81\text{ N})\mathbf{k}$

or $\mathbf{R} = -(840\text{ N})\mathbf{j} - (679\text{ N})\mathbf{k}$ ◀

$$\mathbf{M}_A = \mathbf{M}_B + \mathbf{M}_C + \mathbf{r}_{B/A} \times \mathbf{R}_B + \mathbf{r}_{C/A} \times \mathbf{R}_C$$

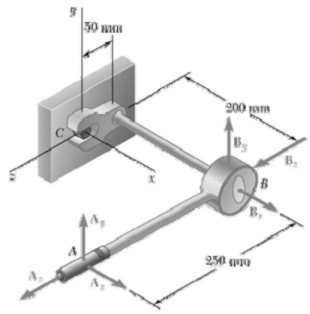
$$= -(21\text{ N}\cdot\text{m})\mathbf{i} + (25.5\text{ N}\cdot\text{m})\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.45 & 0 & 0 \\ 0 & -702.51 & 99.186 \end{vmatrix}\text{N}\cdot\text{m}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.90 & 0 & 0 \\ 0 & -137.182 & -778.00 \end{vmatrix}\text{N}\cdot\text{m}$$

$$= (4.5\text{ N}\cdot\text{m})\mathbf{i} + (655.57\text{ N}\cdot\text{m})\mathbf{j} - (439.59\text{ N}\cdot\text{m})\mathbf{k}$$

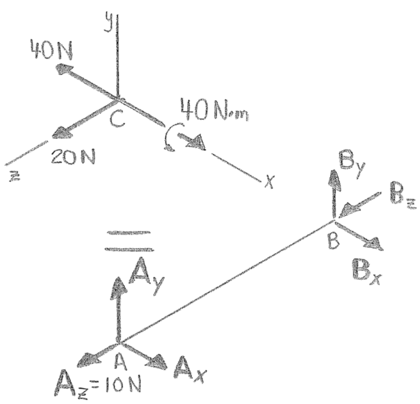
or $\mathbf{M}_A = (4.50\text{ N}\cdot\text{m})\mathbf{i} + (656\text{ N}\cdot\text{m})\mathbf{j} - (440\text{ N}\cdot\text{m})\mathbf{k}$ ◀

PROBLEM 3.117



A mechanic uses a crowfoot wrench to loosen a bolt at C. The mechanic holds the socket wrench handle at points A and B and applies forces at these points. Knowing that these forces are equivalent to a force-couple system at C consisting of the force $\mathbf{C} = -(40\text{ N})\mathbf{i} + (20\text{ N})\mathbf{k}$ and the couple $\mathbf{M}_C = (40\text{ N}\cdot\text{m})\mathbf{i}$, determine the forces applied at A and B when $A_z = 10\text{ N}$.

SOLUTION



Have $\Sigma\mathbf{F}: \quad \mathbf{A} + \mathbf{B} = \mathbf{C}$

or $F_x: \quad A_x + B_x = -40\text{ N}$

$$\therefore \quad B_x = -(A_x + 40\text{ N}) \qquad (1)$$

$$\Sigma F_y: \quad A_y + B_y = 0$$

or $A_y = -B_y \qquad (2)$

$$\Sigma F_z: \quad 10\text{ N} + B_z = 20\text{ N}$$

or $B_z = 10\text{ N} \qquad (3)$

Have $\Sigma\mathbf{M}_C: \quad \mathbf{r}_{B/C} \times \mathbf{B} + \mathbf{r}_{A/C} \times \mathbf{A} = \mathbf{M}_C$

$$\therefore \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.2 & 0 & -0.05 \\ B_x & B_y & 10 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.2 & 0 & 0.2 \\ A_x & A_y & 10 \end{vmatrix} \text{N}\cdot\text{m} = (40\text{ N}\cdot\text{m})\mathbf{i}$$

or $(0.05B_y - 0.2A_x)\mathbf{i} + (-0.05B_x - 2 + 0.2A_x - 2)\mathbf{j}$

$$+ (0.2B_y + 0.2A_y)\mathbf{k} = (40\text{ N}\cdot\text{m})\mathbf{i}$$

From $\mathbf{i}$ - coefficient $\quad 0.05B_y - 0.2A_y = 40\text{ N}\cdot\text{m} \qquad (4)$

$\mathbf{j}$ - coefficient $\quad -0.05B_x + 0.2A_x = 4\text{ N}\cdot\text{m} \qquad (5)$

$\mathbf{k}$ - coefficient $\quad 0.2B_y + 0.2A_y = 0 \qquad (6)$

PROBLEM 3.117 CONTINUED

From Equations (2) and (4): $0.05B_y - 0.2\left(-B_y\right) = 40$

$$B_y = 160 \text{ N}, A_y = -160 \text{ N}$$

From Equations (1) and (5): $-0.05\left(-A_x - 40\right) + 0.2A_x = 4$

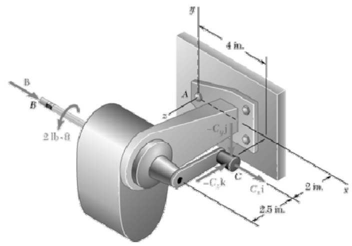
$$A_x = 8 \text{ N}$$

From Equation (1): $B_x = -\left(8 + 40\right) = -48 \text{ N}$

$$\therefore \quad \mathbf{A} = \left(8 \text{ N}\right)\mathbf{i} - \left(160 \text{ N}\right)\mathbf{j} + \left(10 \text{ N}\right)\mathbf{k} \blacktriangleleft$$

$$\mathbf{B} = -\left(48 \text{ N}\right)\mathbf{i} + \left(160 \text{ N}\right)\mathbf{j} + \left(10 \text{ N}\right)\mathbf{k} \blacktriangleleft$$

PROBLEM 3.118



While using a pencil sharpener, a student applies the forces and couple shown. (*a*) Determine the forces exerted at B and C knowing that these forces and the couple are equivalent to a force-couple system at A consisting of the force $\mathbf{R} = (3.9 \text{ lb})\mathbf{i} + R_y\mathbf{j} - (1.1 \text{ lb})\mathbf{k}$ and the couple $\mathbf{M}_A^R = M_x\mathbf{i} + (1.5 \text{ lb}\cdot\text{ft})\mathbf{j} - (1.1 \text{ lb}\cdot\text{ft})\mathbf{k}.$. (*b*) Find the corresponding values of R_y and M_x.

SOLUTION

Have $$\Sigma\mathbf{F}: \quad \mathbf{B} + \mathbf{C} = \mathbf{R}$$

$$\Sigma F_x: \quad B_x + C_x = 3.9 \text{ lb} \qquad \text{or} \qquad B_x = 3.9 \text{ lb} - C_x \tag{1}$$

$$\Sigma F_y: \quad C_y = R_y \tag{2}$$

$$\Sigma F_z: \quad C_z = -1.1 \text{ lb} \tag{3}$$

Have $$\Sigma\mathbf{M}_A: \quad \mathbf{r}_{B/A} \times \mathbf{B} + \mathbf{r}_{C/A} \times \mathbf{C} + \mathbf{M}_B = \mathbf{M}_A^R$$

$$\therefore \quad \frac{1}{12}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & 0 & 4.5 \\ B_x & 0 & 0 \end{vmatrix} + \frac{1}{12}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 0 & 2.0 \\ C_x & C_y & -1.1 \end{vmatrix} + (2 \text{ lb}\cdot\text{ft})\mathbf{i} = M_x\mathbf{i} + (1.5 \text{ lb}\cdot\text{ft})\mathbf{j} - (1.1 \text{ lb}\cdot\text{ft})\mathbf{k}$$

$$(2 - 0.166667C_y)\mathbf{i} + (0.375B_x + 0.166667C_x + 0.36667)\mathbf{j} + (0.33333C_y)\mathbf{k}$$

$$= M_x\mathbf{i} + (1.5)\mathbf{j} - (1.1)\mathbf{k}$$

From **i** - coefficient $$2 - 0.166667C_y = M_x \tag{4}$$

j - coefficient $$0.375B_x + 0.166667C_x + 0.36667 = 1.5 \tag{5}$$

k - coefficient $$0.33333C_y = -1.1 \qquad \text{or} \qquad C_y = -3.3 \text{ lb} \tag{6}$$

(*a*) From Equations (1) and (5):

$$0.375(3.9 - C_x) + 0.166667C_x = 1.13333$$

$$C_x = \frac{0.32917}{0.20833} = 1.58000 \text{ lb}$$

From Equation (1): $$B_x = 3.9 - 1.58000 = 2.32 \text{ lb}$$

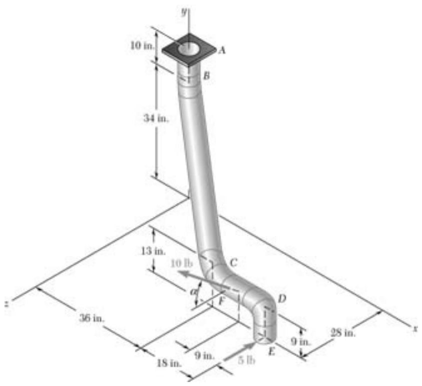
$$\therefore \quad \mathbf{B} = (2.32 \text{ lb})\mathbf{i} \blacktriangleleft$$

$$\mathbf{C} = (1.580 \text{ lb})\mathbf{i} - (3.30 \text{ lb})\mathbf{j} - (1.1 \text{ lb})\mathbf{k} \blacktriangleleft$$

(*b*) From Equation (2): $$R_y = C_y = -3.30 \text{ lb} \qquad \text{or } \mathbf{R}_y = -(3.30 \text{ lb})\mathbf{j} \blacktriangleleft$$

From Equation (4): $$M_x = -0.166667(-3.30) + 2.0 = 2.5500 \text{ lb}\cdot\text{ft}$$

$$\text{or } \mathbf{M}_x = (2.55 \text{ lb}\cdot\text{ft})\mathbf{i} \blacktriangleleft$$



PROBLEM 3.119

A portion of the flue for a furnace is attached to the ceiling at *A*. While supporting the free end of the flue at *F*, a worker pushes in at *E* and pulls out at *F* to align end *E* with the furnace. Knowing that the 10-lb force at *F* lies in a plane parallel to the *yz* plane, determine (*a*) the angle α the force at *F* should form with the horizontal if duct *AB* is not to tend to rotate about the vertical, (*b*) the force-couple system at *B* equivalent to the given force system when this condition is satisfied.

SOLUTION

(*a*) Duct *AB* will not have a tendency to rotate about the vertical or *y*-axis if:

$$M_{By}^{R} = \mathbf{j} \cdot \Sigma \mathbf{M}_B^R = \mathbf{j} \cdot \left(\mathbf{r}_{F/B} \times \mathbf{F}_F + \mathbf{r}_{E/B} \times \mathbf{F}_E \right) = 0$$

where

$$\mathbf{r}_{F/B} = (45 \text{ in.})\mathbf{i} - (23 \text{ in.})\mathbf{j} + (28 \text{ in.})\mathbf{k}$$

$$\mathbf{r}_{E/B} = (54 \text{ in.})\mathbf{i} - (34 \text{ in.})\mathbf{j} + (28 \text{ in.})\mathbf{k}$$

$$\mathbf{F}_F = 10 \text{ lb}\left[(\sin\alpha)\mathbf{j} + (\cos\alpha)\mathbf{k}\right]$$

$$\mathbf{F}_E = -(5 \text{ lb})\mathbf{k}$$

$$\therefore \quad \Sigma \mathbf{M}_B^R = (10 \text{ lb})\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 45 \text{ in.} & -23 \text{ in.} & 28 \text{ in.} \\ 0 & \sin\alpha & \cos\alpha \end{vmatrix} + (5 \text{ lb})(2 \text{ in.})\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 27 & -17 & 14 \\ 0 & 0 & -1 \end{vmatrix}$$

$$= \left[(-230\cos\alpha - 280\sin\alpha + 170)\mathbf{i} - (450\cos\alpha - 270)\mathbf{j} + (450\sin\alpha)\mathbf{k}\right] \text{lb} \cdot \text{in.}$$

Thus,

$$M_{By}^{R} = -450\cos\alpha + 270 = 0$$

$$\cos\alpha = 0.60$$

$$\alpha = 53.130°$$

or $\alpha = 53.1°$ ◀

PROBLEM 3.119 CONTINUED

(*b*) $\mathbf{R} = \mathbf{F}_E + \mathbf{F}_F$

where

$$\mathbf{F}_E = -(5\text{ lb})\mathbf{k}$$

$$\mathbf{F}_F = (10\text{ lb})(\sin 53.130°\mathbf{j} + \cos 53.130°\mathbf{k}) = (8\text{ lb})\mathbf{j} + (6\text{ lb})\mathbf{k}$$

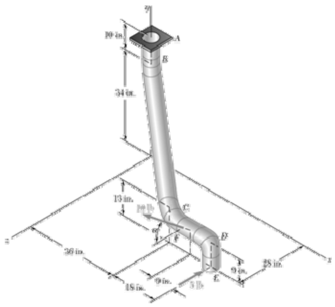
$$\therefore \quad \mathbf{R} = (8\text{ lb})\mathbf{j} + (1\text{ lb})\mathbf{k} \blacktriangleleft$$

and

$$\mathbf{M} = \Sigma\mathbf{M}_B^R = -[230(0.6) + 280(0.8) - 170]\mathbf{i} - [450(0.6) - 270]\mathbf{j} + [450(0.8)]\mathbf{k}$$

$$= -(192\text{ lb}\cdot\text{in.})\mathbf{i} - (0)\mathbf{j} + (360\text{ lb}\cdot\text{in.})\mathbf{k}$$

$$\text{or } \mathbf{M} = -(192\text{ lb}\cdot\text{in.})\mathbf{i} + (360\text{ lb}\cdot\text{in.})\mathbf{k} \blacktriangleleft$$



PROBLEM 3.120

A portion of the flue for a furnace is attached to the ceiling at *A*. While supporting the free end of the flue at *F*, a worker pushes in at *E* and pulls out at *F* to align end *E* with the furnace. Knowing that the 10-lb force at *F* lies in a plane parallel to the *yz* plane and that $\alpha = 60°$, (*a*) replace the given force system with an equivalent force-couple system at *C*, (*b*) determine whether duct *CD* will tend to rotate clockwise or counterclockwise relative to elbow *C*, as viewed from *D* to *C*.

SOLUTION

(*a*) Have

$$\mathbf{R} = \Sigma\mathbf{F} = \mathbf{F}_F + \mathbf{F}_E$$

where

$$\mathbf{F}_F = 10\text{ lb}\left[(\sin 60°)\,\mathbf{j} + (\cos 60°)\,\mathbf{k}\right] = (8.6603\text{ lb})\,\mathbf{j} + (5.0\text{ lb})\,\mathbf{k}$$

$$\mathbf{F}_E = -(5\text{ lb})\,\mathbf{k}$$

$$\therefore\quad \mathbf{R} = (8.6603\text{ lb})\,\mathbf{j}$$

or $\mathbf{R} = (8.66\text{ lb})\,\mathbf{j}$ ◀

Have

$$\mathbf{M}_C^R = \Sigma(\mathbf{r}\times\mathbf{F}) = \mathbf{r}_{F/C}\times\mathbf{F}_F + \mathbf{r}_{E/C}\times\mathbf{F}_E$$

where

$$\mathbf{r}_{F/C} = (9\text{ in.})\,\mathbf{i} - (2\text{ in.})\,\mathbf{j}$$

$$\mathbf{r}_{E/C} = (18\text{ in.})\,\mathbf{i} - (13\text{ in.})\,\mathbf{j}$$

$$\therefore\quad \mathbf{M}_C^R = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 9 & -2 & 0\\ 0 & 8.6603 & 5.0\end{vmatrix}\text{lb}\cdot\text{in.} + \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 18 & -13 & 0\\ 0 & 0 & -5\end{vmatrix}\text{lb}\cdot\text{in.}$$

$$= (55\text{ lb}\cdot\text{in.})\,\mathbf{i} + (45\text{ lb}\cdot\text{in.})\,\mathbf{j} + (77.942\text{ lb}\cdot\text{in.})\,\mathbf{k}$$

or $\mathbf{M}_C^R = (55.0\text{ lb}\cdot\text{in.})\,\mathbf{i} + (45.0\text{ lb}\cdot\text{in.})\,\mathbf{j} + (77.9\text{ lb}\cdot\text{in.})\,\mathbf{k}$ ◀

(*b*) To determine which direction duct section *CD* has a tendency to turn, have

$$M_{CD}^R = \boldsymbol{\lambda}_{DC}\cdot\mathbf{M}_C^R$$

where

$$\boldsymbol{\lambda}_{DC} = \frac{-(18\text{ in.})\,\mathbf{i} + (4\text{ in.})\,\mathbf{j}}{2\sqrt{85}\text{ in.}} = \frac{1}{\sqrt{85}}(-9\mathbf{i} + 2\mathbf{j})$$

Then

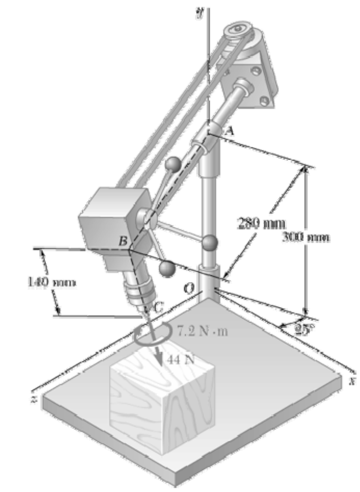
$$M_{CD}^R = \frac{1}{\sqrt{85}}(-9\mathbf{i} + 2\mathbf{j})\cdot(55\mathbf{i} + 45\mathbf{j} + 77.942\mathbf{k})\text{lb}\cdot\text{in.}$$

$$= (-53.690 + 9.7619)\text{lb}\cdot\text{in.}$$

$$= -43.928\text{ lb}\cdot\text{in.}$$

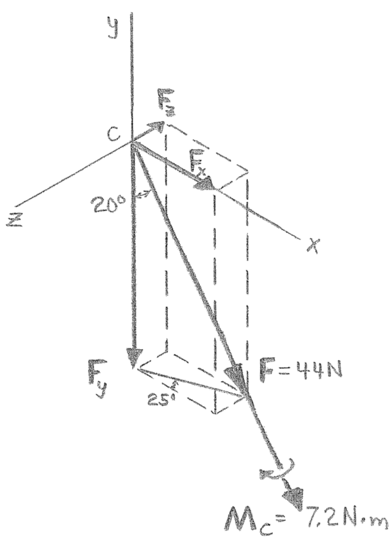
Since $\boldsymbol{\lambda}_{DC}\cdot\mathbf{M}_C^R < 0,$ duct *DC* tends to rotate *clockwise* relative to elbow *C* as viewed from *D* to *C*. ◀

PROBLEM 3.121



The head-and-motor assembly of a radial drill press was originally positioned with arm *AB* parallel to the *z* axis and the axis of the chuck and bit parallel to the *y* axis. The assembly was then rotated 25° about the *y* axis and 20° about the centerline of the horizontal arm *AB*, bringing it into the position shown. The drilling process was started by switching on the motor and rotating the handle to bring the bit into contact with the workpiece. Replace the force and couple exerted by the drill press with an equivalent force-couple system at the center *O* of the base of the vertical column.

SOLUTION



Have $\mathbf{R} = \mathbf{F}$

$$= (44\text{ N})\left[(\sin 20^{\circ}\cos 25^{\circ})\mathbf{i} - (\cos 20^{\circ})\mathbf{j} - (\sin 20^{\circ}\sin 25^{\circ})\mathbf{k}\right]$$

$$= (13.6389\text{ N})\mathbf{i} - (41.346\text{ N})\mathbf{j} - (6.3599\text{ N})\mathbf{k}$$

$$\text{or } \mathbf{R} = (13.64\text{ N})\mathbf{i} - (41.3\text{ N})\mathbf{j} - (6.36\text{ N})\mathbf{k} \blacktriangleleft$$

Have $$\mathbf{M}_O = \mathbf{r}_{B/O} \times \mathbf{F} + \mathbf{M}_C$$

where

$$\mathbf{r}_{B/O} = \left[(0.280\text{ m})\sin 25^{\circ}\right]\mathbf{i} + (0.300\text{ m})\mathbf{j} + \left[(0.280\text{ m})\cos 25^{\circ}\right]\mathbf{k}$$

$$= (0.118333\text{ m})\mathbf{i} + (0.300\text{ m})\mathbf{j} + (0.25377\text{ m})\mathbf{k}$$

$$\mathbf{M}_C = (7.2\text{ N}\cdot\text{m})\left[(\sin 20^{\circ}\cos 25^{\circ})\mathbf{i} - (\cos 20^{\circ})\mathbf{j} - (\sin 20^{\circ}\sin 25^{\circ})\mathbf{k}\right]$$

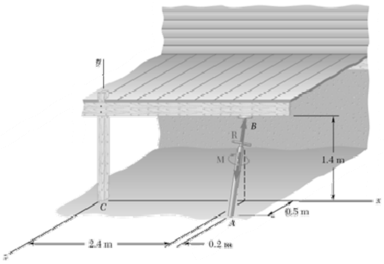
$$= (2.2318\text{ N}\cdot\text{m})\mathbf{i} - (6.7658\text{ N}\cdot\text{m})\mathbf{j} - (1.04072\text{ N}\cdot\text{m})\mathbf{k}$$

$$\therefore \quad \mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.118333 & 0.300 & 0.25377 \\ 13.6389 & -41.346 & -6.3599 \end{vmatrix} \text{N}\cdot\text{m}$$

$$+ (2.2318\mathbf{i} - 6.7658\mathbf{j} - 1.04072\mathbf{k})\text{N}\cdot\text{m}$$

$$= (10.8162\text{ N}\cdot\text{m})\mathbf{i} - (2.5521\text{ N}\cdot\text{m})\mathbf{j} - (10.0250\text{ N}\cdot\text{m})\mathbf{k}$$

$$\text{or } \mathbf{M}_O = (10.82\text{ N}\cdot\text{m})\mathbf{i} - (2.55\text{ N}\cdot\text{m})\mathbf{j} - (10.03\text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$



PROBLEM 3.122

While a sagging porch is leveled and repaired, a screw jack is used to support the front of the porch. As the jack is expanded, it exerts on the porch the force-couple system shown, where $R = 300 \text{ N}$ and $M = 37.5 \text{ N}\cdot\text{m}$. Replace this force-couple system with an equivalent force-couple system at C.

SOLUTION

From $$\mathbf{R}_C = \mathbf{R} = (300 \text{ N})\,\boldsymbol{\lambda}_{AB} = 300 \text{ N}\left[\frac{-(0.2 \text{ m})\mathbf{i} + (1.4 \text{ m})\mathbf{j} - (0.5 \text{ m})\mathbf{k}}{1.50 \text{ m}}\right]$$

$$\mathbf{R}_C = -(40.0 \text{ N})\mathbf{i} + (280 \text{ N})\mathbf{j} - (100 \text{ N})\mathbf{k} \blacktriangleleft$$

From $$\mathbf{M}_C = \mathbf{r}_{A/C} \times \mathbf{R} + \mathbf{M}$$

where

$$\mathbf{r}_{A/C} = (2.6 \text{ m})\mathbf{i} + (0.5 \text{ m})\mathbf{k}$$

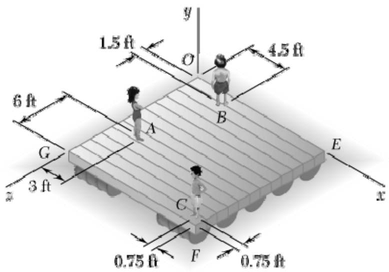
$$\mathbf{M} = (37.5 \text{ N}\cdot\text{m})\,\boldsymbol{\lambda}_{BA} = (37.5 \text{ N}\cdot\text{m})\left[\frac{(0.2 \text{ m})\mathbf{i} - (1.4 \text{ m})\mathbf{j} + (0.5 \text{ m})\mathbf{k}}{1.50 \text{ m}}\right]$$

$$= (5.0 \text{ N}\cdot\text{m})\mathbf{i} - (35.0 \text{ N}\cdot\text{m})\mathbf{j} + (12.5 \text{ N}\cdot\text{m})\mathbf{k}$$

$$\therefore \quad \mathbf{M}_C = (10 \text{ N}\cdot\text{m})\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2.6 & 0 & 0.5 \\ -4 & 28 & -10 \end{vmatrix} + (5.0 \text{ N}\cdot\text{m})\mathbf{i} - (35.0 \text{ N}\cdot\text{m})\mathbf{j} + (12.5 \text{ N}\cdot\text{m})\mathbf{k}$$

$$= \left[(-140 + 5)\text{N}\cdot\text{m}\right]\mathbf{i} + \left[(-20 + 260 - 35)\text{N}\cdot\text{m}\right]\mathbf{j} + \left[(728 + 12.5)\text{N}\cdot\text{m}\right]\mathbf{k}$$

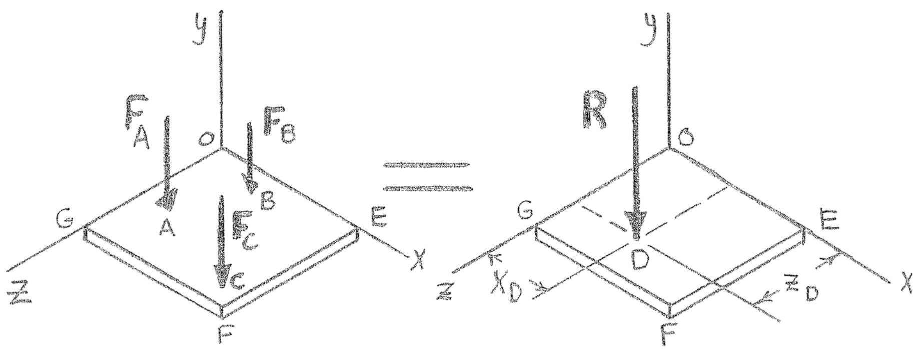
or $$\mathbf{M}_C = -(135.0 \text{ N}\cdot\text{m})\mathbf{i} + (205 \text{ N}\cdot\text{m})\mathbf{j} + (741 \text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$



PROBLEM 3.123

Three children are standing on a 15×15-ft raft. If the weights of the children at points *A*, *B*, and *C* are 85 lb, 60 lb, and 90 lb, respectively, determine the magnitude and the point of application of the resultant of the three weights.

SOLUTION



Have $$\Sigma\mathbf{F}: \quad \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C = \mathbf{R}$$

$$-(85\text{ lb})\mathbf{j} - (60\text{ lb})\mathbf{j} - (90\text{ lb})\mathbf{j} = \mathbf{R}$$

$$-(235\text{ lb})\mathbf{j} = \mathbf{R}$$

or $R = 235$ lb ◀

Have $$\Sigma M_x: \quad F_A(z_A) + F_B(z_B) + F_C(z_C) = R(z_D)$$

$$(85\text{ lb})(9\text{ ft}) + (60\text{ lb})(1.5\text{ ft}) + (90\text{ lb})(14.25\text{ ft}) = (235\text{ lb})(z_D)$$

$$\therefore \quad z_D = 9.0957\text{ ft}$$

or $z_D = 9.10$ ft ◀

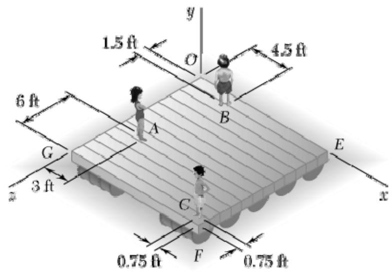
Have $$\Sigma M_z: \quad F_A(x_A) + F_B(x_B) + F_C(x_C) = R(x_D)$$

$$(85\text{ lb})(3\text{ ft}) + (60\text{ lb})(4.5\text{ ft}) + (90\text{ lb})(14.25\text{ ft}) = (235\text{ lb})(x_D)$$

$$\therefore \quad x_D = 7.6915\text{ ft}$$

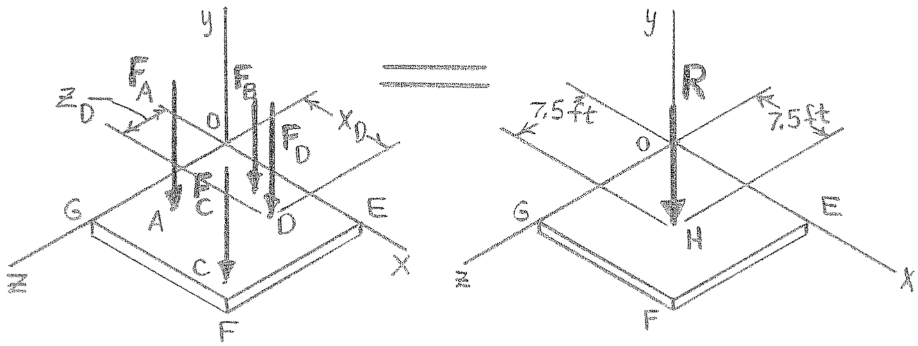
or $x_D = 7.69$ ft ◀

PROBLEM 3.124



Three children are standing on a 15×15-ft raft. The weights of the children at points *A*, *B*, and *C* are 85 lb, 60 lb, and 90 lb, respectively. If a fourth child of weight 95 lb climbs onto the raft, determine where she should stand if the other children remain in the positions shown and the line of action of the resultant of the four weights is to pass through the center of the raft.

SOLUTION



Have $\qquad \Sigma\mathbf{F}: \quad \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D = \mathbf{R}$

$$-(85 \text{ lb})\mathbf{j} - (60 \text{ lb})\mathbf{j} - (90 \text{ lb})\mathbf{j} - (95 \text{ lb})\mathbf{j} = \mathbf{R}$$

$$\therefore \quad \mathbf{R} = -(330 \text{ lb})\mathbf{j}$$

Have $\qquad \Sigma M_x: \quad F_A(z_A) + F_B(z_B) + F_C(z_C) + F_D(z_D) = R(z_H)$

$$(85 \text{ lb})(9 \text{ ft}) + (60 \text{ lb})(1.5 \text{ ft}) + (90 \text{ lb})(14.25 \text{ ft}) + (95 \text{ lb})(z_D) = (330 \text{ lb})(7.5 \text{ ft})$$

$$\therefore \quad z_D = 3.5523 \text{ ft}$$

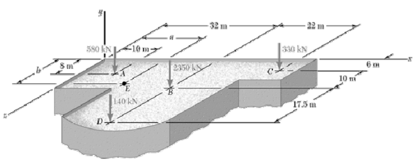
or $z_D = 3.55$ ft ◀

Have $\qquad \Sigma M_z: \quad F_A(x_A) + F_B(x_B) + F_C(x_C) + F_D(x_D) = R(x_H)$

$$(85 \text{ lb})(3 \text{ ft}) + (60 \text{ lb})(4.5 \text{ ft}) + (90 \text{ lb})(14.25 \text{ ft}) + (95 \text{ lb})(x_D) = (330 \text{ lb})(7.5 \text{ ft})$$

$$\therefore \quad x_D = 7.0263 \text{ ft}$$

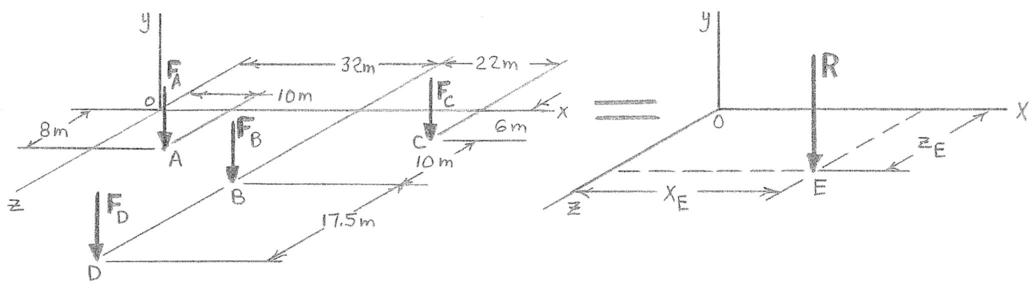
or $x_D = 7.03$ ft ◀



PROBLEM 3.125

The forces shown are the resultant downward loads on sections of the flat roof of a building because of accumulated snow. Determine the magnitude and the point of application of the resultant of these four loads.

SOLUTION



Have $$\Sigma\mathbf{F}: \quad \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D = \mathbf{R}$$

$$-(580\text{ kN})\mathbf{j} - (2350\text{ kN})\mathbf{j} - (330\text{ kN})\mathbf{j} - (140\text{ kN})\mathbf{j} = \mathbf{R}$$

$$\therefore \quad \mathbf{R} = -(3400\text{ kN})\mathbf{j}$$

$R = 3400$ kN ◀

Have $$\Sigma M_x: \quad F_A(z_A) + F_B(z_B) + F_C(z_C) + F_D(z_D) = R(z_E)$$

$$(580\text{ kN})(8\text{ m}) + (2350\text{ kN})(16\text{ m}) + (330\text{ kN})(6\text{ m}) + (140\text{ kN})(33.5\text{ m}) = (3400\text{ kN})(z_E)$$

$$\therefore \quad z_E = 14.3853\text{ m}$$

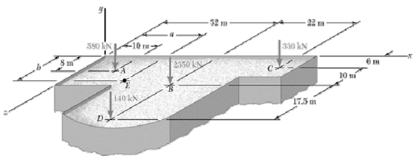
or $z_E = 14.39$ m ◀

Have $$\Sigma M_z: \quad F_A(x_A) + F_B(x_B) + F_C(x_C) + F_D(x_D) = R(x_E)$$

$$(580\text{ kN})(10\text{ m}) + (2350\text{ kN})(32\text{ m}) + (330\text{ kN})(54\text{ m}) + (140\text{ kN})(32\text{ m}) = (3400\text{ kN})(x_E)$$

$$\therefore \quad x_E = 30.382\text{ m}$$

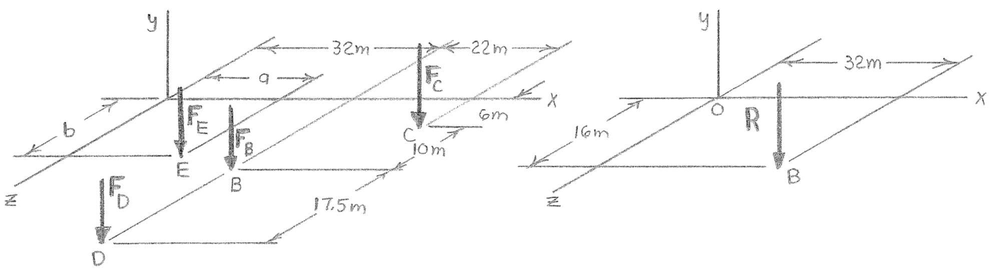
or $x_E = 30.4$ m ◀



PROBLEM 3.126

The forces shown are the resultant downward loads on sections of the flat roof of a building because of accumulated snow. If the snow represented by the 580-kN force is shoveled so that the this load acts at *E*, determine *a* and *b* knowing that the point of application of the resultant of the four loads is then at *B*.

SOLUTION



Have $$\Sigma\mathbf{F}:\quad \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D + \mathbf{F}_E = \mathbf{R}$$

$$-(2350\text{ kN})\,\mathbf{j} - (330\text{ kN})\,\mathbf{j} - (140\text{ kN})\,\mathbf{j} - (580\text{ kN})\,\mathbf{j} = \mathbf{R}$$

$$\therefore\quad \mathbf{R} = -(3400\text{ kN})\,\mathbf{j}$$

Have $$\Sigma M_x:\quad F_B(z_B) + F_C(z_C) + F_D(z_D) + F_E(z_E) = R(z_B)$$

$$(2350\text{ kN})(16\text{ m}) + (330\text{ kN})(6\text{ m}) + (140\text{ kN})(33.5\text{ m}) + (580\text{ kN})(b) = (3400\text{ kN})(16\text{ m})$$

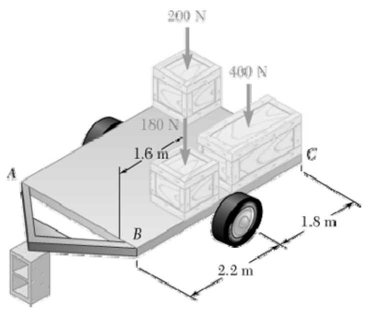
$$\therefore\quad b = 17.4655\text{ m}$$ or $b = 17.47\text{ m}$ ◀

Have $$\Sigma M_z:\quad F_B(x_B) + F_C(x_C) + F_D(x_D) + F_E(x_E) = R(x_B)$$

$$(2350\text{ kN})(32\text{ m}) + (330\text{ kN})(54\text{ m}) + (140\text{ kN})(32\text{ m}) + (580\text{ kN})(a) = (3400\text{ kN})(32\text{ m})$$

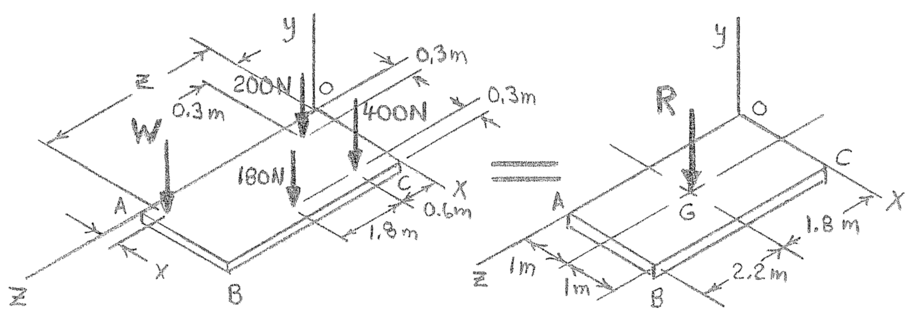
$$\therefore\quad a = 19.4828\text{ m}$$ or $a = 19.48\text{ m}$ ◀

PROBLEM 3.127



A group of students loads a 2×4-m flatbed trailer with two $0.6 \times 0.6 \times 0.6$-m boxes and one $0.6 \times 0.6 \times 1.2$-m box. Each of the boxes at the rear of the trailer is positioned so that it is aligned with both the back and a side of the trailer. Determine the smallest load the students should place in a second $0.6 \times 0.6 \times 1.2$-m box and where on the trailer they should secure it, without any part of the box overhanging the sides of the trailer, if each box is uniformly loaded and the line of action of the resultant of the weights of the four boxes is to pass through the point of intersection of the centerlines of the trailer and the axle. (*Hint:* Keep in mind that the box may be placed either on its side or on its end.)

SOLUTION



For the smallest weight on the trailer so that the resultant force of the four weights acts over the axle at the intersection with the center line of the trailer, the added $0.6 \times 0.6 \times 1.2$-m box should be placed adjacent to one of the edges of the trailer with the 0.6×0.6-m side on the bottom. The edges to be considered are based on the location of the resultant for the three given weights.

Have
$$\Sigma\mathbf{F}: \quad -(200\text{ N})\mathbf{j} - (400\text{ N})\mathbf{j} - (180\text{ N})\mathbf{j} = \mathbf{R}$$

$$\therefore \quad \mathbf{R} = -(780\text{ N})\mathbf{j}$$

Have
$$\Sigma M_z: \quad (200\text{ N})(0.3\text{ m}) + (400\text{ N})(1.7\text{ m}) + (180\text{ N})(1.7\text{ m}) = (780\text{ N})(x)$$

$$\therefore \quad x = 1.34103\text{ m}$$

Have
$$\Sigma M_x: \quad (200\text{ N})(0.3\text{ m}) + (400\text{ N})(0.6\text{ m}) + (180\text{ N})(2.4\text{ m}) = (780\text{ N})(z)$$

$$\therefore \quad z = 0.93846\text{ m}$$

From the statement of the problem, it is known that the resultant of **R** from the original loading and the lightest load **W** passes through *G*, the point of intersection of the two center lines. Thus, $\Sigma\mathbf{M}_G = 0$.

Further, since the lightest load **W** is to be as small as possible, the fourth box should be placed as far from *G* as possible without the box overhanging the trailer. These two requirements imply

$$(0.3\text{ m} \le x \le 1\text{ m}) \quad (1.8\text{ m} \le z \le 3.7\text{ m})$$

PROBLEM 3.127 CONTINUED

Let $x = 0.3$ m, $\Sigma M_{Gz}: \quad (200\text{ N})(0.7\text{ m}) - (400\text{ N})(0.7\text{ m}) - (180\text{ N})(0.7\text{ m}) + W(0.7\text{ m}) = 0$

$$\therefore \quad W = 380\text{ N}$$

$$\Sigma M_{Gx}: \quad -(200\text{ N})(1.5\text{ m}) - (400\text{ N})(1.2\text{ m}) + (180\text{ N})(0.6\text{ m}) + (380\text{ N})(z - 1.8\text{ m}) = 0$$

$$\therefore \; z = 3.5684\text{ m} < 3.7\text{ m} \qquad \therefore \quad \text{acceptable}$$

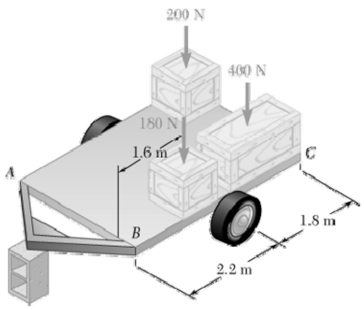
Let $z = 3.7$ m, $\Sigma M_{Gx}: \quad -(200\text{ N})(1.5\text{ m}) - (400\text{ N})(1.2\text{ m}) + (180\text{ N})(0.6\text{ m}) + W(1.7\text{ m}) = 0$

$$\therefore \quad W = 395.29\text{ N} > 380\text{ N}$$

Since the weight W found for $x = 0.3$ m is less than W found for $z = 3.7$ m, $x = 0.3$ m results in the smallest weight W.

or $W = 380$ N at $(0.3\text{ m}, 0, 3.57\text{ m})$ ◀

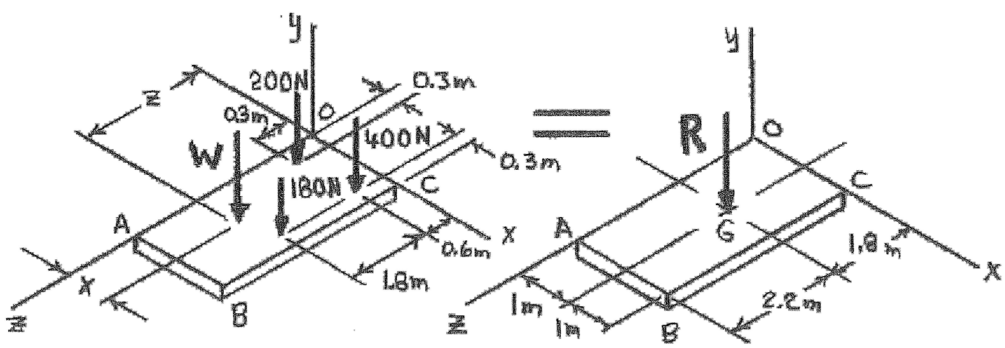
PROBLEM 3.128



Solve Problem 3.127 if the students want to place as much weight as possible in the fourth box and that at least one side of the box must coincide with a side of the trailer.

Problem 3.127: A group of students loads a 2×4-m flatbed trailer with two $0.6 \times 0.6 \times 0.6$-m boxes and one $0.6 \times 0.6 \times 1.2$-m box. Each of the boxes at the rear of the trailer is positioned so that it is aligned with both the back and a side of the trailer. Determine the smallest load the students should place in a second $0.6 \times 0.6 \times 1.2$-m box and where on the trailer they should secure it, without any part of the box overhanging the sides of the trailer, if each box is uniformly loaded and the line of action of the resultant of the weights of the four boxes is to pass through the point of intersection of the centerlines of the trailer and the axle. (*Hint:* Keep in mind that the box may be placed either on its side or on its end.)

SOLUTION



For the largest additional weight on the trailer with the box having at least one side coinsiding with the side of the trailer, the box must be as close as possible to point *G*. For $x = 0.6$ m, with a small side of the box touching the *z*-axis, satisfies this condition.

Let $x = 0.6$ m, $\quad \Sigma M_{Gz}: \quad (200 \text{ N})(0.7 \text{ m}) - (400 \text{ N})(0.7 \text{ m}) - (180 \text{ N})(0.7 \text{ m}) + W(0.4 \text{ m}) = 0$

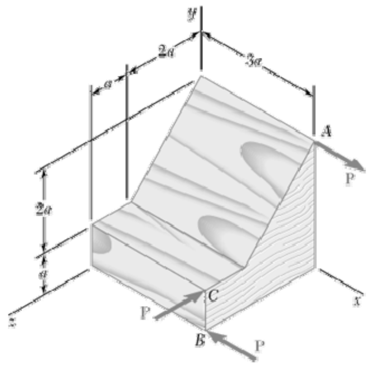
$$\therefore \quad W = 665 \text{ N}$$

and $\quad \Sigma M_{GX}: \quad -(200 \text{ N})(1.5 \text{ m}) - (400 \text{ N})(1.2 \text{ m}) + (180 \text{ N})(0.6 \text{ m}) + (665 \text{ N})(z - 1.8 \text{ m}) = 0$

$$\therefore \quad z = 2.8105 \text{ m} \qquad (2 \text{ m} < z < 4 \text{ m}) \qquad \therefore \quad \text{acceptable}$$

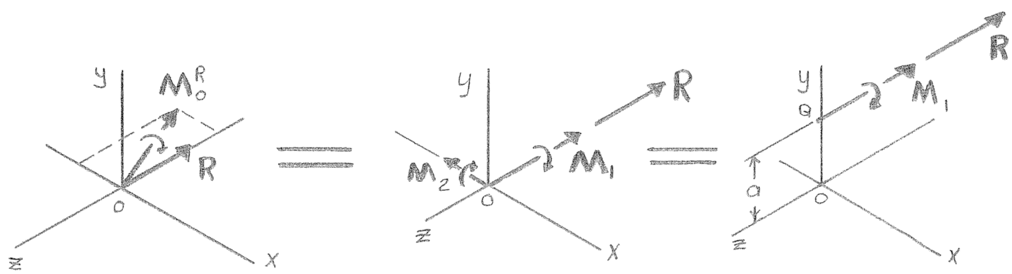
or $W = 665 \text{ N}$ at $(0.6 \text{ m}, 0, 2.81 \text{ m})$ ◀

PROBLEM 3.129



A block of wood is acted upon by three forces of the same magnitude P and having the directions shown. Replace the three forces with an equivalent wrench and determine (*a*) the magnitude and direction of the resultant **R**, (*b*) the pitch of the wrench, (*c*) the point where the axis of the wrench intersects the *xy* plane.

SOLUTION



First, reduce the given force system to a force-couple at the origin.

Have $$\Sigma\mathbf{F}: \quad P\mathbf{i} - P\mathbf{i} - P\mathbf{k} = \mathbf{R}$$

$$\therefore \quad \mathbf{R} = -P\mathbf{k}$$

Have $$\Sigma\mathbf{M}_O: \quad -P(3a)\mathbf{k} - P(3a)\mathbf{j} + P(-a\mathbf{i} + 3a\mathbf{j}) = \mathbf{M}_O^R$$

$$\therefore \quad \mathbf{M}_O^R = Pa(-\mathbf{i} - 3\mathbf{k})$$

Then let vectors $(\mathbf{R},\ \mathbf{M}_1)$ represent the components of the wrench, where their directions are the same.

(*a*) $$\mathbf{R} = -P\mathbf{k}$$ or Magnitude of $\mathbf{R} = P$ ◀

Direction of $\mathbf{R}$: $\theta_x = 90°,\ \ \theta_y = 90°,\ \ \theta_z = -180°$ ◀

(*b*) Have $$M_1 = \boldsymbol{\lambda}_R \cdot \mathbf{M}_O^R$$

$$= -\mathbf{k} \cdot \left[Pa(-\mathbf{i} - 3\mathbf{k})\right]$$

$$= 3Pa$$

and pitch $$p = \frac{M_1}{R} = \frac{3Pa}{P} = 3a$$ or $p = 3a$ ◀

PROBLEM 3.129 CONTINUED

(*c*) Have $$\mathbf{M}_O^R = \mathbf{M}_1 + \mathbf{M}_2$$

$$\therefore \quad \mathbf{M}_2 = \mathbf{M}_O^R - \mathbf{M}_1 = Pa\left(-\mathbf{i} - 3\mathbf{k}\right) - \left(-3Pa\mathbf{k}\right) = -Pa\mathbf{i}$$

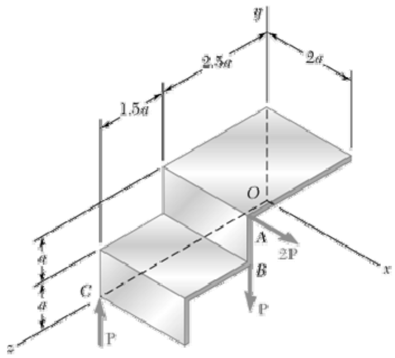
Require $$\mathbf{M}_2 = \mathbf{r}_{Q/O} \times \mathbf{R}$$

$$-Pa\mathbf{i} = \left(x\mathbf{i} + y\mathbf{j}\right) \times \left(-P\right)\mathbf{k} = Px\mathbf{j} - Py\mathbf{i}$$

From $$\mathbf{i}: \quad -Pa = -Py \qquad \text{or} \qquad y = a$$

$$\mathbf{j}: \quad x = 0$$

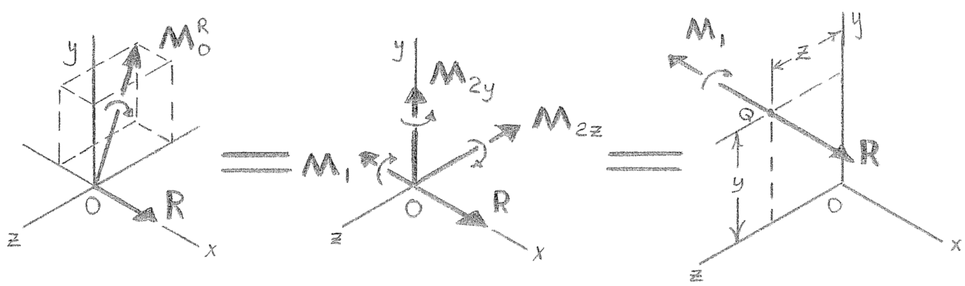
$\therefore$ The axis of the wrench is parallel to the *z*-axis and intersects the *xy* plane at $x = 0,\ y = a$ ◀



PROBLEM 3.130

A piece of sheet metal is bent into the shape shown and is acted upon by three forces. Replace the three forces with an equivalent wrench and determine (*a*) the magnitude and direction of the resultant **R**, (*b*) the pitch of the wrench, (*c*) the point where the axis of the wrench intersects the *yz* plane.

SOLUTION



First, reduce the given force system to a force-couple system at the origin.

Have $$\Sigma\mathbf{F}:\quad (2P)\mathbf{i} - (P)\mathbf{j} + (P)\mathbf{j} = \mathbf{R}$$

$$\therefore\quad \mathbf{R} = (2P)\mathbf{i}$$

Have $$\Sigma\mathbf{M}_O:\quad \Sigma(\mathbf{r}_O \times \mathbf{F}) = \mathbf{M}_O^R$$

$$\mathbf{M}_O^R = Pa\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 2 & 2.5 \\ 2 & -1 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 4 \\ 0 & 1 & 0 \end{vmatrix} = Pa(-1.5\mathbf{i} + 5\mathbf{j} - 6\mathbf{k})$$

(*a*) $$\mathbf{R} = 2P\mathbf{i}$$ or Magnitude of $\mathbf{R} = 2P$ ◀

Direction of **R**: $\theta_x = 0°,\ \theta_y = -90°,\ \theta_z = 90°$ ◀

(*b*) Have $$M_1 = \lambda_R \cdot \mathbf{M}_O^R \qquad \lambda_R = \frac{\mathbf{R}}{R}$$

$$= \mathbf{i}\cdot(-1.5Pa\mathbf{i} + 5Pa\mathbf{j} - 6Pa\mathbf{k})$$

$$= -1.5Pa$$

and pitch $$p = \frac{M_1}{R} = \frac{-1.5Pa}{2P} = -0.75a$$ or $p = -0.75a$ ◀

PROBLEM 3.130 CONTINUED

(*c*) Have

$$\mathbf{M}_O^R = \mathbf{M}_1 + \mathbf{M}_2$$

$$\therefore \quad \mathbf{M}_2 = \mathbf{M}_O^R - \mathbf{M}_1 = (5Pa)\mathbf{j} - (6Pa)\mathbf{k}$$

Require

$$\mathbf{M}_2 = \mathbf{r}_{Q/O} \times \mathbf{R}$$

$$(5Pa)\mathbf{j} - (6Pa)\mathbf{k} = (y\mathbf{j} + z\mathbf{k}) \times (2P\mathbf{i}) = -(2Py)\mathbf{k} + (2Pz)\mathbf{j}$$

From

$$\mathbf{i}: \quad 5Pa = 2Pz$$

$$\therefore \quad z = 2.5a$$

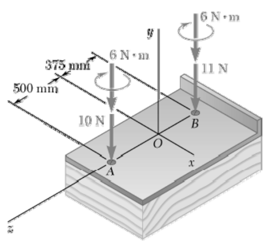
From

$$\mathbf{k}: \quad -6Pa = -2Py$$

$$\therefore \quad y = 3a$$

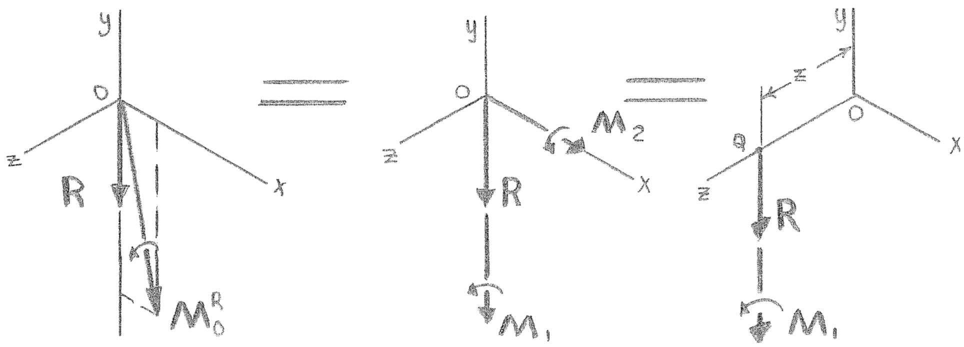
$\therefore$ The axis of the wrench is parallel to the *x*-axis and intersects the *yz*-plane at $y = 3a, \; z = 2.5a$ ◀

PROBLEM 3.131



The forces and couples shown are applied to two screws as a piece of sheet metal is fastened to a block of wood. Reduce the forces and the couples to an equivalent wrench and determine (*a*) the resultant force $\mathbf{R}$, (*b*) the pitch of the wrench, (*c*) the point where the axis of the wrench intersects the *xz* plane.

SOLUTION



First, reduce the given force system to a force-couple at the origin.

Have $$\Sigma \mathbf{F}: \quad -(10\text{ N})\mathbf{j} - (11\text{ N})\mathbf{j} = \mathbf{R}$$

$$\therefore \quad \mathbf{R} = -(21\text{ N})\mathbf{j}$$

Have $$\Sigma \mathbf{M}_O: \quad \Sigma(\mathbf{r}_O \times \mathbf{F}) + \Sigma \mathbf{M}_C = \mathbf{M}_O^R$$

$$\mathbf{M}_O^R = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.5 \\ 0 & -10 & 0 \end{vmatrix} \text{N}\cdot\text{m} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -0.375 \\ 0 & -11 & 0 \end{vmatrix} \text{N}\cdot\text{m} - (12\text{ N}\cdot\text{m})\mathbf{j}$$

$$= (0.875\text{ N}\cdot\text{m})\mathbf{i} - (12\text{ N}\cdot\text{m})\mathbf{j}$$

(*a*) $$\mathbf{R} = -(21\text{ N})\mathbf{j}$$ or $\mathbf{R} = -(21\text{ N})\mathbf{j}$ ◀

(*b*) Have $$M_1 = \lambda_R \cdot \mathbf{M}_O^R \qquad \lambda_R = \frac{\mathbf{R}}{R}$$

$$= (-\mathbf{j}) \cdot \left[(0.875\text{ N}\cdot\text{m})\mathbf{i} - (12\text{ N}\cdot\text{m})\mathbf{j}\right]$$

$$= 12\text{ N}\cdot\text{m} \qquad \text{and} \qquad \mathbf{M}_1 = -(12\text{ N}\cdot\text{m})\mathbf{j}$$

and pitch $$p = \frac{M_1}{R} = \frac{12\text{ N}\cdot\text{m}}{21\text{ N}} = 0.57143\text{ m}$$ or $p = 0.571\text{ m}$ ◀

PROBLEM 3.131 CONTINUED

(*c*) Have $$\mathbf{M}_O^R = \mathbf{M}_1 + \mathbf{M}_2$$

$$\therefore \quad \mathbf{M}_2 = \mathbf{M}_O^R - \mathbf{M}_1 = (0.875 \text{ N}\cdot\text{m})\mathbf{i}$$

Require $$\mathbf{M}_2 = \mathbf{r}_{Q/O} \times \mathbf{R}$$

$$\therefore \quad (0.875 \text{ N}\cdot\text{m})\mathbf{i} = (x\mathbf{i} + z\mathbf{k}) \times \left[-(21 \text{ N})\mathbf{j}\right]$$

$$0.875\mathbf{i} = -(21x)\mathbf{k} + (21z)\mathbf{i}$$

From **i**: $$0.875 = 21z$$

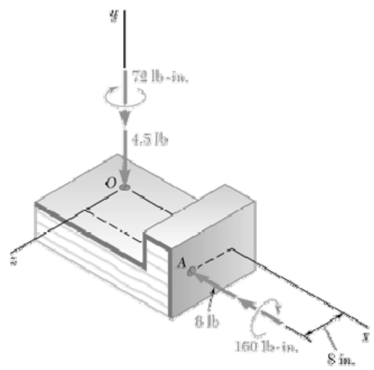
$$\therefore \quad z = 0.041667 \text{ m}$$

From **k**: $$0 = -21x$$

$$\therefore \quad z = 0$$

$\therefore$ The axis of the wrench is parallel to the *y*-axis and intersects the *xz*-plane at $x = 0$, $z = 41.7$ mm ◀

PROBLEM 3.132



The forces and couples shown are applied to two screws as a piece of sheet metal is fastened to a block of wood. Reduce the forces and the couples to an equivalent wrench and determine (*a*) the resultant force **R**, (*b*) the pitch of the wrench, (*c*) the point where the axis of the wrench intersects the *xz* plane.

SOLUTION

First, reduce the given force system to a force-couple system.

Have $\Sigma\mathbf{F}: \quad -(6\text{ lb})\mathbf{i} - (4.5\text{ lb})\mathbf{j} = \mathbf{R} \qquad R = 7.5\text{ lb}$

Have $\Sigma\mathbf{M}_O: \quad \sum(\mathbf{r}_O \times \mathbf{F}) + \sum\mathbf{M}_C = \mathbf{M}_O^R$

$$\mathbf{M}_O^R = -6\text{ lb}(8\text{ in.})\mathbf{j} - (160\text{ lb}\cdot\text{in.})\mathbf{i} - (72\text{ lb}\cdot\text{in.})\mathbf{j}$$

$$= -(160\text{ lb}\cdot\text{in.})\mathbf{i} - (120\text{ lb}\cdot\text{in.})\mathbf{j}$$

$$M_O^R = 200\text{ lb}\cdot\text{in.}$$

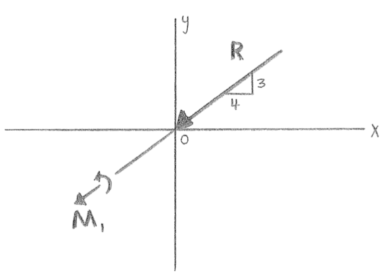
(*a*) $\mathbf{R} = -(6\text{ lb})\mathbf{i} - (4.5\text{ lb})\mathbf{j}$ ◀

(*b*) Have $M_1 = \lambda_R \cdot \mathbf{M}_O^R \qquad \lambda = \dfrac{\mathbf{R}}{R}$

$$= (-0.8\mathbf{i} - 0.6\mathbf{j}) \cdot \left[-(160\text{ lb}\cdot\text{in.})\mathbf{i} - (120\text{ lb}\cdot\text{in.})\mathbf{j}\right]$$

$$= 200\text{ lb}\cdot\text{in.}$$

and $\mathbf{M}_1 = 200\text{ lb}\cdot\text{in.}(-0.8\mathbf{i} - 0.6\mathbf{j})$



Pitch $p = \dfrac{M_1}{R} = \dfrac{200\text{ lb}\cdot\text{in.}}{7.50\text{ lb}} = 26.667\text{ in.}$

or $p = 26.7$ in. ◀

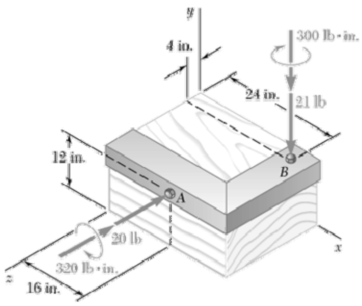
(*c*) From above note that

$$\mathbf{M}_1 = \mathbf{M}_O^R$$

Therefore, the axis of the wrench goes through the origin. The line of action of the wrench lies in the *xy* plane with a slope of

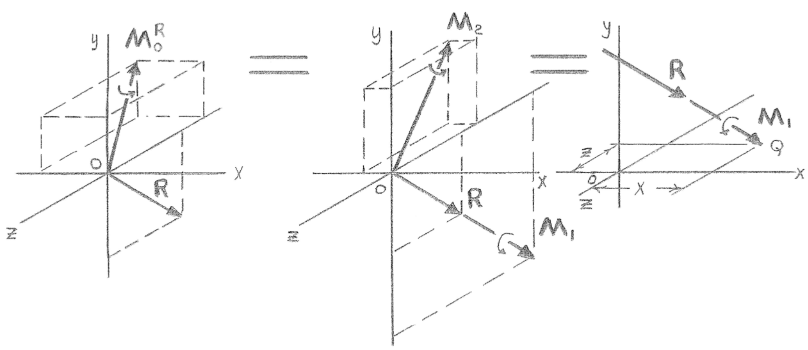
$$\frac{dy}{dx} = \frac{3}{4}$$ ◀

PROBLEM 3.133



Two bolts *A* and *B* are tightened by applying the forces and couple shown. Replace the two wrenches with a single equivalent wrench and determine (*a*) the resultant **R**, (*b*) the pitch of the single equivalent wrench, (*c*) the point where the axis of the wrench intersects the *xz* plane.

SOLUTION



First, reduce the given force system to a force-couple at the origin.

Have $\quad \Sigma\mathbf{F}: \quad -(20\text{ lb})\mathbf{k} - (21\text{ lb})\mathbf{j} = -(21\text{ lb})\mathbf{j} - (20\text{ lb})\mathbf{k} = \mathbf{R} \qquad R = 29\text{ lb}$

and $\quad \Sigma\mathbf{M}_O: \quad \sum(\mathbf{r}_O \times \mathbf{F}) + \sum\mathbf{M}_C = \mathbf{M}_O^R$

$$20\text{ lb}(4\text{ in.})\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 4 & 3 & 0\\ 0 & 0 & -1\end{vmatrix} + 21\text{ lb}(4\text{ in.})\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 6 & 0 & 1\\ 0 & -1 & 0\end{vmatrix} + (-300\mathbf{j} - 320\mathbf{k})\text{lb}\cdot\text{in.} = \mathbf{M}_O^R$$

$$\therefore \quad \mathbf{M}_O^R = -(156\text{ lb}\cdot\text{in.})\mathbf{i} + (20\text{ lb}\cdot\text{in.})\mathbf{j} - (824\text{ lb}\cdot\text{in.})\mathbf{k}$$

(*a*) $\qquad \mathbf{R} = -(21\text{ lb})\mathbf{j} - (20\text{ lb})\mathbf{k}$ ◀

(*b*) Have $\qquad M_1 = \boldsymbol{\lambda}_R \cdot \mathbf{M}_O^R \qquad \boldsymbol{\lambda}_R = \dfrac{\mathbf{R}}{R}$

$$= -\frac{-21\mathbf{j} - 20\mathbf{k}}{29}\cdot\left[-(156\text{ lb}\cdot\text{in.})\mathbf{i} + (20\text{ lb}\cdot\text{in.})\mathbf{j} - (824\text{ lb}\cdot\text{in.})\mathbf{k}\right]$$

$$= 553.80\text{ lb}\cdot\text{in.}$$

PROBLEM 3.133 CONTINUED

and $$\mathbf{M}_1 = M_1\lambda_R = -(401.03 \text{ lb}\cdot\text{in.})\mathbf{j} - (381.93 \text{ lb}\cdot\text{in.})\mathbf{k}$$

Then pitch $$p = \frac{M_1}{R} = \frac{553.80 \text{ lb}\cdot\text{in.}}{29 \text{ lb}} = 19.0964 \text{ in.}$$ or $p = 19.10$ in. ◀

(*c*) Have $$\mathbf{M}_O^R = \mathbf{M}_1 + \mathbf{M}_2$$

$$\therefore \quad \mathbf{M}_2 = \mathbf{M}_O^R - \mathbf{M}_1 = \left[(-156\mathbf{i} + 20\mathbf{j} - 824\mathbf{k}) - (-401.03\mathbf{j} - 381.93\mathbf{k})\right]\text{lb}\cdot\text{in.}$$

$$= -(156.0 \text{ lb}\cdot\text{in.})\mathbf{i} + (421.03 \text{ lb}\cdot\text{in.})\mathbf{j} - (442.07 \text{ lb}\cdot\text{in.})\mathbf{k}$$

Require $$\mathbf{M}_2 = \mathbf{r}_{Q/O} \times \mathbf{R}$$

$$(-156\mathbf{i} + 421.03\mathbf{j} - 442.07\mathbf{k}) = (x\mathbf{i} + z\mathbf{k}) \times (-21\mathbf{j} - 20\mathbf{k})$$

$$= (21z)\mathbf{i} + (20x)\mathbf{j} - (21x)\mathbf{k}$$

From **i**: $$-156 = 21z$$

$$\therefore \quad z = -7.4286 \text{ in.}$$

or $$z = -7.43 \text{ in.}$$

From **k**: $$-442.07 = -21x$$

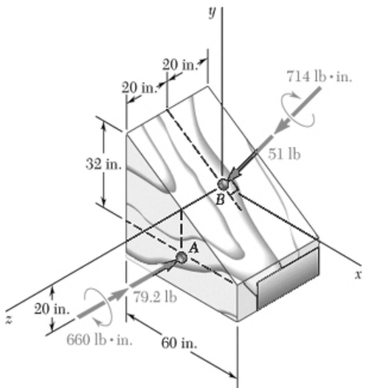
$$\therefore \quad x = 21.051 \text{ in.}$$

or $$x = 21.1 \text{ in.}$$

$\therefore$ The axis of the wrench intersects the xz-plane at

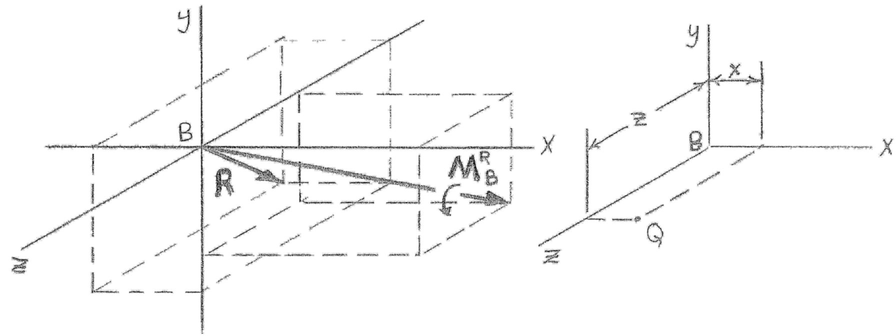
$x = 21.1$ in., $z = -7.43$ in. ◀

PROBLEM 3.134



Two bolts A and B are tightened by applying the forces and couple shown. Replace the two wrenches with a single equivalent wrench and determine (*a*) the resultant **R**, (*b*) the pitch of the single equivalent wrench, (*c*) the point where the axis of the wrench intersects the *xz* plane.

SOLUTION



First reduce the given force system to a force-couple at the origin at B.

(*a*) Have

$$\Sigma\mathbf{F}: \quad -(79.2\text{ lb})\mathbf{k} - (51\text{ lb})\left(\frac{8}{17}\mathbf{i} + \frac{15}{17}\mathbf{j}\right) = \mathbf{R}$$

$$\therefore \quad \mathbf{R} = -(24.0\text{ lb})\mathbf{i} - (45.0\text{ lb})\mathbf{j} - (79.2\text{ lb})\mathbf{k} \blacktriangleleft$$

and

$$R = 94.2\text{ lb}$$

Have

$$\Sigma\mathbf{M}_B: \quad \mathbf{r}_{A/B} \times \mathbf{F}_A + \mathbf{M}_A + \mathbf{M}_B = \mathbf{M}_B^R$$

$$\mathbf{M}_B^R = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -20 & 0 \\ 0 & 0 & -79.2 \end{vmatrix} - 660\mathbf{k} - 714\left(\frac{8}{17}\mathbf{i} + \frac{15}{17}\mathbf{j}\right) = 1584\mathbf{i} - 660\mathbf{k} - 42(8\mathbf{i} + 15\mathbf{j})$$

$$\therefore \quad \mathbf{M}_B^R = (1248\text{ lb}\cdot\text{in.})\mathbf{i} - (630\text{ lb}\cdot\text{in.})\mathbf{j} - (660\text{ lb}\cdot\text{in.})\mathbf{k}$$

(*b*) Have $\quad M_1 = \boldsymbol{\lambda}_R \cdot \mathbf{M}_O^R \qquad \boldsymbol{\lambda}_R = \dfrac{\mathbf{R}}{R}$

$$= \frac{-24.0\mathbf{i} - 45.0\mathbf{j} - 79.2\mathbf{k}}{94.2} \cdot \left[(1248\text{ lb}\cdot\text{in.})\mathbf{i} - (630\text{ lb}\cdot\text{in.})\mathbf{j} - (660\text{ lb}\cdot\text{in.})\mathbf{k}\right]$$

$$= 537.89\text{ lb}\cdot\text{in.}$$

PROBLEM 3.134 CONTINUED

and $$\mathbf{M}_1 = M_1\boldsymbol{\lambda}_R$$

$$= -(137.044 \text{ lb}\cdot\text{in.})\mathbf{i} - (256.96 \text{ lb}\cdot\text{in.})\mathbf{j} - (452.24 \text{ lb}\cdot\text{in.})\mathbf{k}$$

Then pitch $$p = \frac{M_1}{R} = \frac{537.89 \text{ lb}\cdot\text{in.}}{94.2 \text{ lb}} = 5.7101 \text{ in.}$$

or $p = 5.71$ in. ◀

(*c*) Have $$\mathbf{M}_B^R = \mathbf{M}_1 + \mathbf{M}_2$$

$$\therefore\ \mathbf{M}_2 = \mathbf{M}_B^R - \mathbf{M}_1 = (1248\mathbf{i} - 630\mathbf{j} - 660\mathbf{k}) - (-137.044\mathbf{i} - 256.96\mathbf{j} - 452.24\mathbf{k})$$

$$= (1385.04 \text{ lb}\cdot\text{in.})\mathbf{i} - (373.04 \text{ lb}\cdot\text{in.})\mathbf{j} - (207.76 \text{ lb}\cdot\text{in.})\mathbf{k}$$

Require $$\mathbf{M}_2 = \mathbf{r}_{Q/B} \times \mathbf{R}$$

$$1385.04\mathbf{i} - 373.04\mathbf{j} - 207.76\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & 0 & z \\ -24 & -45 & -79.2 \end{vmatrix}$$

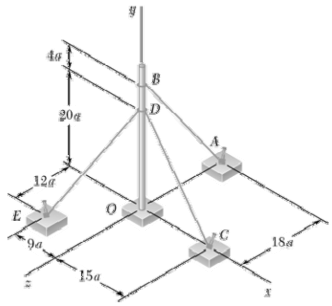
$$= (45z)\mathbf{i} - (24z)\mathbf{j} + (79.2x)\mathbf{j} - (45x)\mathbf{k}$$

From **i**: $1385.04 = 45z \qquad \therefore\ z = 30.779$ in.

From **k**: $-207.76 = -45x \qquad \therefore\ x = 4.6169$ in.

$\therefore$ The axis of the wrench intersects the *xz*-plane at

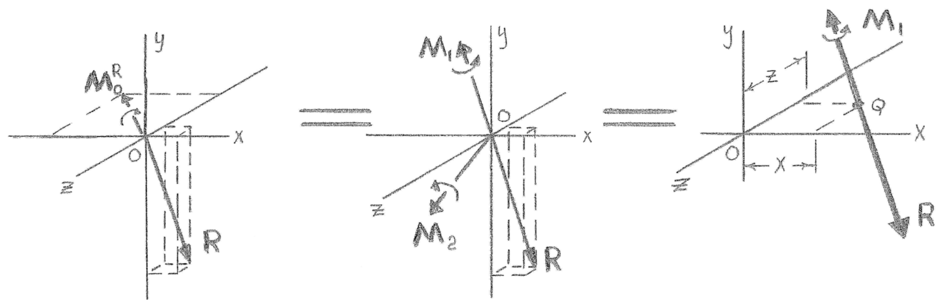
$x = 4.62$ in., $z = 30.8$ in. ◀



PROBLEM 3.135

A flagpole is guyed by three cables. If the tensions in the cables have the same magnitude P, replace the forces exerted on the pole with an equivalent wrench and determine (*a*) the resultant force **R**, (*b*) the pitch of the wrench, (*c*) the point where the axis of the wrench intersects the *xz* plane.

SOLUTION



(*a*) First reduce the given force system to a force-couple at the origin.

Have $$\Sigma\mathbf{F}:\quad P\lambda_{BA} + P\lambda_{DC} + P\lambda_{DE} = \mathbf{R}$$

$$\mathbf{R} = P\left[\left(\frac{4}{5}\mathbf{j} - \frac{3}{5}\mathbf{k}\right) + \left(\frac{3}{5}\mathbf{i} - \frac{4}{5}\mathbf{j}\right) + \left(\frac{-9}{25}\mathbf{i} - \frac{4}{5}\mathbf{j} + \frac{12}{25}\mathbf{k}\right)\right]$$

$$\therefore\quad \mathbf{R} = \frac{3P}{25}\left(2\mathbf{i} - 20\mathbf{j} - \mathbf{k}\right) \blacktriangleleft$$

$$R = \frac{3P}{25}\sqrt{(2)^2 + (20)^2 + (1)^2} = \frac{27\sqrt{5}}{25}P$$

Have $$\Sigma\mathbf{M}:\quad \Sigma\left(\mathbf{r}_O \times P\right) = \mathbf{M}_O^R$$

$$(24a)\mathbf{j} \times \left(\frac{-4P}{5}\mathbf{j} - \frac{3P}{5}\mathbf{k}\right) + (20a)\mathbf{j} \times \left(\frac{3P}{5}\mathbf{i} - \frac{4P}{5}\mathbf{j}\right) + (20a)\mathbf{j} \times \left(\frac{-9P}{25}\mathbf{i} - \frac{4P}{5}\mathbf{j} + \frac{12P}{25}\mathbf{k}\right) = \mathbf{M}_O^R$$

$$\therefore\quad \mathbf{M}_O^R = \frac{24Pa}{5}\left(-\mathbf{i} - \mathbf{k}\right)$$

(*b*) Have $$M_1 = \lambda_R \cdot \mathbf{M}_O^R$$

where $$\lambda_R = \frac{\mathbf{R}}{R} = \frac{3P}{25}\left(2\mathbf{i} - 20\mathbf{j} - \mathbf{k}\right)\frac{25}{27\sqrt{5}P} = \frac{1}{9\sqrt{5}}\left(2\mathbf{i} - 20\mathbf{j} - \mathbf{k}\right)$$

PROBLEM 3.135 CONTINUED

Then

$$M_1 = \frac{1}{9\sqrt{5}}(2\mathbf{i} - 20\mathbf{j} - \mathbf{k}) \cdot \frac{24Pa}{5}(-\mathbf{i} - \mathbf{k}) = \frac{-8Pa}{15\sqrt{5}}$$

and pitch

$$p = \frac{M_1}{R} = \frac{-8Pa}{15\sqrt{5}}\left(\frac{25}{27\sqrt{5}P}\right) = \frac{-8a}{81}$$

or $p = -0.0988a$ ◀

(*c*)

$$\mathbf{M}_1 = M_1\lambda_R = \frac{-8Pa}{15\sqrt{5}}\left(\frac{1}{9\sqrt{5}}\right)(2\mathbf{i} - 20\mathbf{j} - \mathbf{k}) = \frac{8Pa}{675}(-2\mathbf{i} + 20\mathbf{j} + \mathbf{k})$$

Then

$$\mathbf{M}_2 = \mathbf{M}_O^R - \mathbf{M}_1 = \frac{24Pa}{5}(-\mathbf{i} - \mathbf{k}) - \frac{8Pa}{675}(-2\mathbf{i} + 20\mathbf{j} + \mathbf{k}) = \frac{8Pa}{675}(-403\mathbf{i} - 20\mathbf{j} - 406\mathbf{k})$$

Require

$$\mathbf{M}_2 = \mathbf{r}_{Q/O} \times \mathbf{R}$$

$$\left(\frac{8Pa}{675}\right)(-403\mathbf{i} - 20\mathbf{j} - 406\mathbf{k}) = (x\mathbf{i} + z\mathbf{k}) \times \left(\frac{3P}{25}\right)(2\mathbf{i} - 20\mathbf{j} - \mathbf{k})$$

$$= \left(\frac{3P}{25}\right)\left[20z\mathbf{i} + (x + 2z)\mathbf{j} - 20x\mathbf{k}\right]$$

From **i**:

$$8(-403)\frac{Pa}{675} = 20z\left(\frac{3P}{25}\right) \qquad \therefore\ z = -1.99012a$$

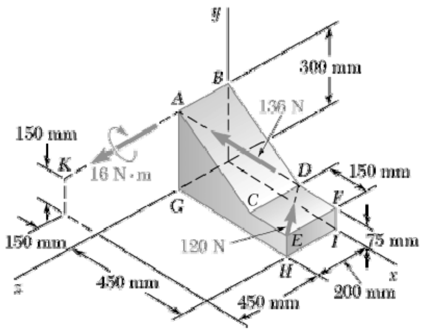
From **k**:

$$8(-406)\frac{Pa}{675} = -20x\left(\frac{3P}{25}\right) \qquad \therefore\ x = 2.0049a$$

$\therefore$ The axis of the wrench intersects the *xz*-plane at

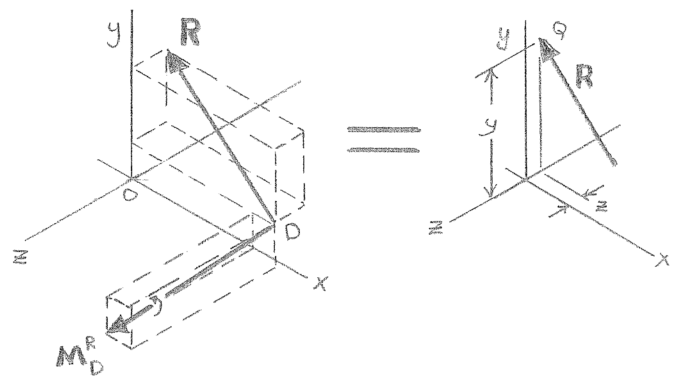
$x = 2.00a,\ z = -1.990a$ ◀

PROBLEM 3.136



Determine whether the force-and-couple system shown can be reduced to a single equivalent force **R**. If it can, determine **R** and the point where the line of action of **R** intersects the *yz* plane. If it cannot be so reduced, replace the given system with an equivalent wrench and determine its resultant, its pitch, and the point where its axis intersects the *yz* plane.

SOLUTION



First, reduce the given force system to a force-couple at *D*.

Have $$\Sigma\mathbf{F}: \quad \mathbf{F}_{DA} + \mathbf{F}_{ED} = F_{DA}\boldsymbol{\lambda}_{DA} + F_{ED}\boldsymbol{\lambda}_{ED} = \mathbf{R}$$

where $$\mathbf{F}_{DA} = 136\text{ N}\left[\frac{-(0.300\text{ m})\mathbf{i} + (0.225\text{ m})\mathbf{j} + (0.200\text{ m})\mathbf{k}}{0.425\text{ m}}\right]$$

$$= -(96\text{ N})\mathbf{i} + (72\text{ N})\mathbf{j} + (64\text{ N})\mathbf{k}$$

$$\mathbf{F}_{ED} = 120\text{ N}\left[\frac{-(0.150\text{ m})\mathbf{i} - (0.200\text{ m})\mathbf{k}}{0.250\text{ m}}\right] = -(72\text{ N})\mathbf{i} - (96\text{ N})\mathbf{k}$$

$$\therefore\ \mathbf{R} = -(168\text{ N})\mathbf{i} + (72\text{ N})\mathbf{j} - (32\text{ N})\mathbf{k} \ \blacktriangleleft$$

Have $$\Sigma\mathbf{M}_D: \quad \mathbf{M}_A = \mathbf{M}_D^R$$

or $$\mathbf{M}_D^R = (16\text{ N}\cdot\text{m})\left[\frac{-(0.150\text{ m})\mathbf{i} - (0.150\text{ m})\mathbf{j} + (0.450\text{ m})\mathbf{k}}{0.150\sqrt{11}\text{ m}}\right] = \frac{16\text{ N}\cdot\text{m}}{\sqrt{11}}(-\mathbf{i} - \mathbf{j} + 3\mathbf{k})$$

PROBLEM 3.136 CONTINUED

The force-couple at D can be replaced by a single force if $\mathbf{R}$ is perpendicular to $\mathbf{M}_D^R$. To be perpendicular, $\mathbf{R}\cdot\mathbf{M}_D^R = 0.$

Have
$$\mathbf{R}\cdot\mathbf{M}_D^R = \left(-168\mathbf{i} + 72\mathbf{j} - 32\mathbf{k}\right)\cdot\frac{16}{\sqrt{11}}\left(-\mathbf{i} - \mathbf{j} + 3\mathbf{k}\right)$$
$$= \frac{128}{\sqrt{11}}\left(21 - 9 - 12\right)$$
$$= 0$$

$\therefore$ Force-couple can be reduced to a single equivalent force.◀

To determine the coordinates where the equivalent single force intersects the *yz*-plane, $\mathbf{M}_D^R = \mathbf{r}_{Q/D}\times\mathbf{R}$

where
$$\mathbf{r}_{Q/D} = \left[(0 - 0.300)\text{m}\right]\mathbf{i} + \left[(y - 0.075)\text{m}\right]\mathbf{j} + \left[(z - 0)\text{m}\right]\mathbf{k}$$

$$\therefore\quad \frac{16\ \text{N}\cdot\text{m}}{\sqrt{11}}\left(-\mathbf{i} - \mathbf{j} + 3\mathbf{k}\right) = (8\ \text{N})\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ -0.3 & (y - 0.075) & z\\ -21 & 9 & -4\end{vmatrix}\text{m}$$

or

$$\frac{16\ \text{N}\cdot\text{m}}{\sqrt{11}}\left(-\mathbf{i} - \mathbf{j} + 3\mathbf{k}\right) = (8\ \text{N})\left\{\left[-4(y - 0.075) - 9z\right]\mathbf{i} + (-21z - 1.2)\mathbf{j} + \left[-2.7 + 21(y - 0.075)\right]\mathbf{k}\right\}\text{m}$$

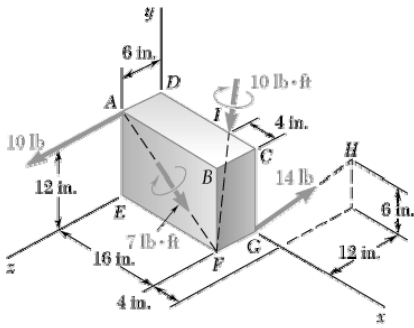
From $\mathbf{j}$:
$$\frac{-16}{\sqrt{11}} = 8(-21z - 1.2)\qquad \therefore\quad z = -0.028427\ \text{m} = -28.4\ \text{mm}$$

From $\mathbf{k}$:
$$\frac{48}{\sqrt{11}} = 8\left[-2.7 + 21(y - 0.075)\right]\qquad \therefore\quad y = 0.28972\ \text{m} = 290\ \text{mm}$$

$\therefore$ line of action of $\mathbf{R}$ intersects the *yz*-plane at

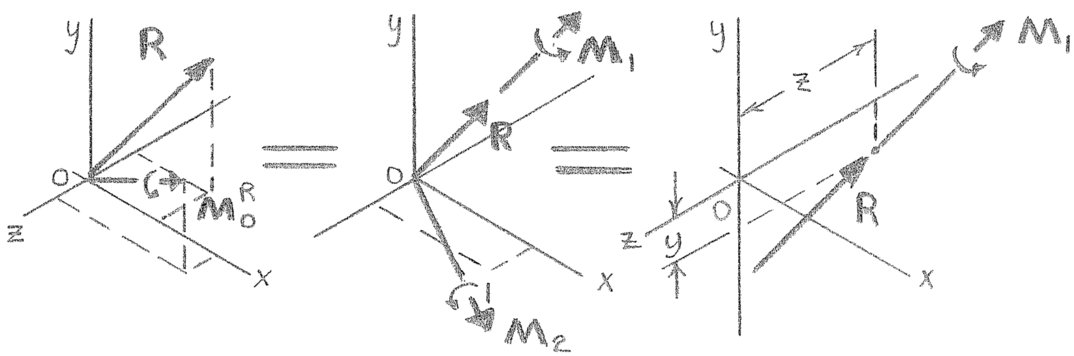
$y = 290\ \text{mm},\ z = -28.4\ \text{mm}$ ◀

PROBLEM 3.137



Determine whether the force-and-couple system shown can be reduced to a single equivalent force **R**. If it can, determine **R** and the point where the line of action of **R** intersects the *yz* plane. If it cannot be so reduced, replace the given system with an equivalent wrench and determine its resultant, its pitch, and the point where its axis intersects the *yz* plane.

SOLUTION



First, reduce the given force system to a force-couple at the origin.

Have
$$\Sigma\mathbf{F}:\quad \mathbf{F}_A + \mathbf{F}_G = \mathbf{R}$$

$$\therefore\quad \mathbf{R} = (10\text{ lb})\mathbf{k} + 14\text{ lb}\left[\frac{(4\text{ in.})\mathbf{i} + (6\text{ in.})\mathbf{j} - (12\text{ in.})\mathbf{k}}{14\text{ in.}}\right] = (4\text{ lb})\mathbf{i} + (6\text{ lb})\mathbf{j} - (2\text{ lb})\mathbf{k} \blacktriangleleft$$

and
$$R = \sqrt{56}\text{ lb}$$

Have
$$\Sigma\mathbf{M}_O:\quad \Sigma(\mathbf{r}_O \times \mathbf{F}) + \Sigma\mathbf{M}_C = \mathbf{M}_O^R$$

$$\mathbf{M}_O^R = \left[(12\text{ in.})\mathbf{j} \times (10\text{ lb})\mathbf{k}\right] + \left\{(16\text{ in.})\mathbf{i} \times \left[(4\text{ lb})\mathbf{i} + (6\text{ lb})\mathbf{j} - (12\text{ lb})\mathbf{k}\right]\right\}$$

$$+ (84\text{ lb}\cdot\text{in.})\left[\frac{(16\text{ in.})\mathbf{i} - (12\text{ in.})\mathbf{j}}{20\text{ in.}}\right] + (\ 120\text{ lb}\cdot\text{in.})\left[\frac{(4\text{ in.})\mathbf{i} - (12\text{ in.})\mathbf{j} + (6\text{ in.})\mathbf{k}}{14\text{ in.}}\right]$$

$$\therefore\quad \mathbf{M}_0^R = (221.49\text{ lb}\cdot\text{in.})\mathbf{i} + (38.743\text{ lb}\cdot\text{in.})\mathbf{j} + (147.429\text{ lb}\cdot\text{in.})\mathbf{k}$$

$$= (18.4572\text{ lb}\cdot\text{ft})\mathbf{i} + (3.2286\text{ lb}\cdot\text{ft})\mathbf{j} + (12.2858\text{ lb}\cdot\text{ft})\mathbf{k}$$

PROBLEM 3.137 CONTINUED

The force-couple at O can be replaced by a single force if the direction of $\mathbf{R}$ is perpendicular to $\mathbf{M}_O^R$.

To be perpendicular $\mathbf{R}\cdot\mathbf{M}_O^R = 0$

Have $$\mathbf{R}\cdot\mathbf{M}_O^R = (4\mathbf{i} + 6\mathbf{j} - 2\mathbf{k})\cdot(18.4572\mathbf{i} + 3.2286\mathbf{j} + 12.2858\mathbf{k}) = 0?$$

$$= 73.829 + 19.3716 - 24.572$$

$$\neq 0$$

$\therefore$ System cannot be reduced to a single equivalent force.

To reduce to an equivalent wrench, the moment component along the line of action of $\mathbf{P}$ is found.

$$M_1 = \boldsymbol{\lambda}_R\cdot\mathbf{M}_O^R \qquad \boldsymbol{\lambda}_R = \frac{\mathbf{R}}{R}$$

$$= \left[\frac{(4\mathbf{i} + 6\mathbf{j} - 2\mathbf{k})}{\sqrt{56}}\right]\cdot(18.4572\mathbf{i} + 3.2286\mathbf{j} + 12.2858\mathbf{k})$$

$$= 9.1709 \text{ lb}\cdot\text{ft}$$

and $$\mathbf{M}_1 = M_1\boldsymbol{\lambda}_R = (9.1709 \text{ lb}\cdot\text{ft})(0.53452\mathbf{i} + 0.80178\mathbf{j} - 0.26726\mathbf{k})$$

And pitch $$p = \frac{M_1}{R} = \frac{9.1709 \text{ lb}\cdot\text{ft}}{\sqrt{56}\text{ lb}} = 1.22551 \text{ ft}$$

or $p = 1.226$ ft ◀

Have

$$\mathbf{M}_2 = \mathbf{M}_O^R - \mathbf{M}_1 = (18.4572\mathbf{i} + 3.2286\mathbf{j} + 12.2858\mathbf{k}) - (9.1709)(0.53452\mathbf{i} + 0.80178\mathbf{j} - 0.26726\mathbf{k})$$

$$= (13.5552 \text{ lb}\cdot\text{ft})\mathbf{i} - (4.1244 \text{ lb}\cdot\text{ft})\mathbf{j} + (14.7368 \text{ lb}\cdot\text{ft})\mathbf{k}$$

Require $$\mathbf{M}_2 = \mathbf{r}_{Q/O}\times\mathbf{R}$$

$$(13.5552\mathbf{i} - 4.1244\mathbf{j} + 14.7368\mathbf{k}) = (y\mathbf{j} + z\mathbf{k})\times(4\mathbf{i} + 6\mathbf{j} - 2\mathbf{k})$$

$$= -(2y + 6z)\mathbf{i} + (4z)\mathbf{j} - (4y)\mathbf{k}$$

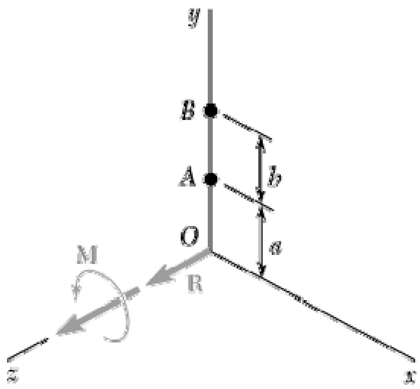
From $\mathbf{j}$: $\quad -4.1244 = 4z \quad$ or $\quad z = -1.0311$ ft

From $\mathbf{k}$: $\quad 14.7368 = -4y \quad$ or $\quad y = -3.6842$ ft

$\therefore$ line of action of the wrench intersects the yz plane at

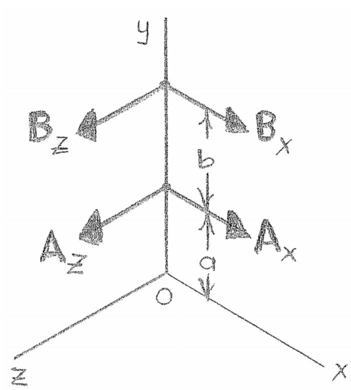
$y = -3.68$ ft, $z = 1.031$ ft ◀

PROBLEM 3.138



Replace the wrench shown with an equivalent system consisting of two forces perpendicular to the *y* axis and applied respectively at *A* and *B*.

SOLUTION



Express the forces at *A* and *B* as

$$\mathbf{A} = A_x\mathbf{i} + A_z\mathbf{k}$$

$$\mathbf{B} = B_x\mathbf{i} + B_z\mathbf{k}$$

Then, for equivalence to the given force system

$$\Sigma F_x: \quad A_x + B_x = 0 \tag{1}$$

$$\Sigma F_z: \quad A_z + B_z = R \tag{2}$$

$$\Sigma M_x: \quad A_z(a) + B_z(a+b) = 0 \tag{3}$$

$$\Sigma M_z: \quad -A_x(a) - B_x(a+b) = M \tag{4}$$

From Equation (1), $\quad B_x = -A_x$

Substitute into Equation (4)

$$-A_x(a) + A_x(a+b) = M$$

$$\therefore \quad A_x = \frac{M}{b} \qquad \text{and} \qquad B_x = -\frac{M}{b}$$

From Equation (2), $\quad B_z = R - A_z$

and Equation (3), $\quad A_z a + (R - A_z)(a+b) = 0$

$$\therefore \quad A_z = R\left(1 + \frac{a}{b}\right)$$

PROBLEM 3.138 CONTINUED

and $$B_z = R - R\left(1 + \frac{a}{b}\right)$$

$$\therefore \quad B_z = -\frac{a}{b}R$$

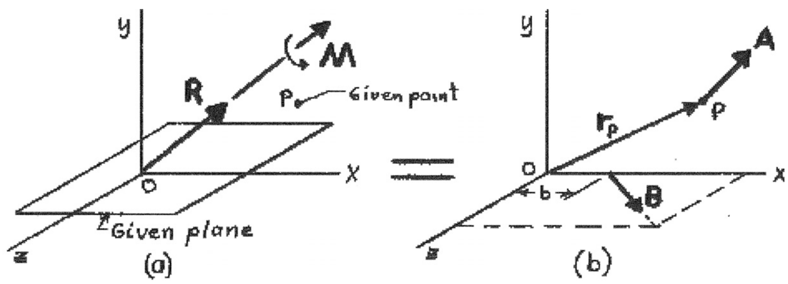
Then $$\mathbf{A} = \left(\frac{M}{b}\right)\mathbf{i} + R\left(1 + \frac{a}{b}\right)\mathbf{k} \blacktriangleleft$$

$$\mathbf{B} = -\left(\frac{M}{b}\right)\mathbf{i} - \left(\frac{a}{b}R\right)\mathbf{k} \blacktriangleleft$$

PROBLEM 3.139

Show that, in general, a wrench can be replaced with two forces chosen in such a way that one force passes through a given point while the other force lies in a given plane.

SOLUTION



First, choose a coordinate system so that the *xy* plane coincides with the given plane. Also, position the coordinate system so that the line of action of the wrench passes through the origin as shown in Figure *a*. Since the orientation of the plane and the components (**R**, **M**) of the wrench are known, it follows that the scalar components of **R** and **M** are known relative to the shown coordinate system.

A force system to be shown as equivalent is illustrated in Figure *b*. Let A be the force passing through the given point *P* and **B** be the force that lies in the given plane. Let *b* be the *x*-axis intercept of **B**.

The known components of the wrench can be expressed as

$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j} + R_z\mathbf{k} \qquad \text{and} \qquad M = M_x\mathbf{i} + M_y\mathbf{j} + M_z\mathbf{k}$$

while the unknown forces **A** and **B** can be expressed as

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k} \qquad \text{and} \qquad \mathbf{B} = B_x\mathbf{i} + B_z\mathbf{k}$$

Since the position vector of point *P* is given, it follows that the scalar components (x, y, z) of the position vector $\mathbf{r}_P$ are also known.

Then, for equivalence of the two systems

$$\Sigma F_x: \quad R_x = A_x + B_x \tag{1}$$

$$\Sigma F_y: \quad R_y = A_y \tag{2}$$

$$\Sigma F_z: \quad R_z = A_z + B_z \tag{3}$$

$$\Sigma M_x: \quad M_x = yA_z - zA_y \tag{4}$$

$$\Sigma M_y: \quad M_y = zA_x - xA_z - bB_z \tag{5}$$

$$\Sigma M_z: \quad M_z = xA_y - yA_x \tag{6}$$

PROBLEM 3.139 CONTINUED

Based on the above six independent equations for the six unknowns $\left(A_x,\ A_y,\ A_z,\ B_x,\ B_z,\ b\right)$, there exists a unique solution for **A** and **B**.

From Equation (2) $$A_y = R_y \blacktriangleleft$$

Equation (6) $$A_x = \left(\frac{1}{y}\right)\left(xR_y - M_z\right) \blacktriangleleft$$

Equation (1) $$B_x = R_x - \left(\frac{1}{y}\right)\left(xR_y - M_z\right) \blacktriangleleft$$

Equation (4) $$A_z = \left(\frac{1}{y}\right)\left(M_x + zR_y\right) \blacktriangleleft$$

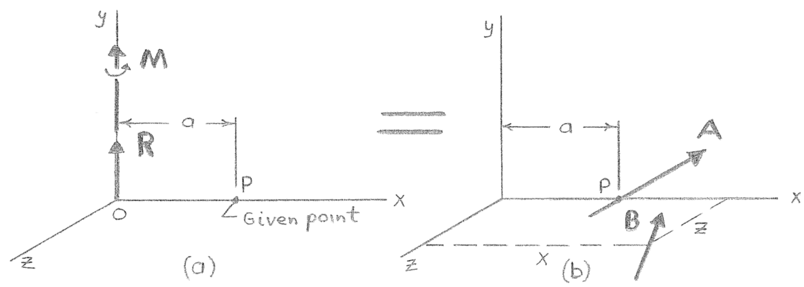
Equation (3) $$B_z = R_z - \left(\frac{1}{y}\right)\left(M_x + zR_y\right) \blacktriangleleft$$

Equation (5) $$b = \frac{\left(xM_x + yM_y + zM_z\right)}{\left(M_x - yR_z + zR_y\right)} \blacktriangleleft$$

PROBLEM 3.140

Show that a wrench can be replaced with two perpendicular forces, one of which is applied at a given point.

SOLUTION



First, observe that it is always possible to construct a line perpendicular to a given line so that the constructed line also passes through a given point. Thus, it is possible to align one of the coordinate axes of a rectangular coordinate system with the axis of the wrench while one of the other axes passes through the given point.

See Figures *a* and *b*.

Have $\mathbf{R} = R\mathbf{j}$ and $\mathbf{M} = M\mathbf{j}$ and are known.

The unknown forces **A** and **B** can be expressed as

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k} \quad \text{and} \quad \mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$$

The distance *a* is known. It is assumed that force **B** intersects the *xz* plane at $(x, 0, z)$. Then for equivalence

$$\Sigma F_x: \quad 0 = A_x + B_x \tag{1}$$

$$\Sigma F_y: \quad R = A_y + B_y \tag{2}$$

$$\Sigma F_z: \quad 0 = A_z + B_z \tag{3}$$

$$\Sigma M_x: \quad 0 = -zB_y \tag{4}$$

$$\Sigma M_y: \quad M = -aA_z - xB_z + zB_x \tag{5}$$

$$\Sigma M_z: \quad 0 = aA_y + xB_y \tag{6}$$

Since **A** and **B** are made perpendicular,

$$\mathbf{A}\cdot\mathbf{B} = 0 \quad \text{or} \quad A_xB_x + A_yB_y + A_zB_z = 0 \tag{7}$$

There are eight unknowns: $A_x, A_y, A_z, B_x, B_y, B_z, x, z$

But only seven independent equations. Therefore, *there exists an infinite number of solutions.*

PROBLEM 3.140 CONTINUED

Next consider Equation (4): $0 = -zB_y$

If $B_y = 0,$ Equation (7) becomes $A_xB_x + A_zB_z = 0$

Using Equations (1) and (3) this equation becomes $A_x^2 + A_z^2 = 0$

Since the components of **A** must be real, a nontrivial solution is not possible. Thus, it is required that $B_y \neq 0,$ so that from Equation (4), $z = 0.$

To obtain one possible solution, arbitrarily let $A_x = 0.$

(Note: Setting $A_y,\ A_z,$ or B_z equal to zero results in unacceptable solutions.)

The defining equations then become.

$$0 = B_x \tag{1$'$}$$

$$R = A_y + B_y \tag{2}$$

$$0 = A_z + B_z \tag{3}$$

$$M = -aA_z - xB_z \tag{5$'$}$$

$$0 = aA_y + xB_y \tag{6}$$

$$A_yB_y + A_zB_z = 0 \tag{7$'$}$$

Then Equation (2) can be written $A_y = R - B_y$

Equation (3) can be written $B_z = -A_z$

Equation (6) can be written $x = -\dfrac{aA_y}{B_y}$

Substituting into Equation (5)′,

$$M = -aA_z - \left(-a\frac{R - B_y}{B_y}\right)(-A_z)$$

or

$$A_z = -\frac{M}{aR}B_y \tag{8}$$

Substituting into Equation (7)′,

$$\left(R - B_y\right)B_y + \left(-\frac{M}{aR}B_y\right)\left(\frac{M}{aR}B_y\right) = 0$$

PROBLEM 3.140 CONTINUED

or
$$B_y = \frac{a^2R^3}{a^2R^2 + M^2}$$

Then from Equations (2), (8), and (3)

$$A_y = R - \frac{a^2R^3}{a^2R^2 + M^2} = \frac{RM^2}{a^2R^2 + M^2}$$

$$A_z = -\frac{M}{aR}\left(\frac{a^2R^3}{a^2R^2 + M^2}\right) = -\frac{aR^2M}{a^2R^2 + M^2}$$

$$B_z = \frac{aR^2M}{a^2R^2 + M^2}$$

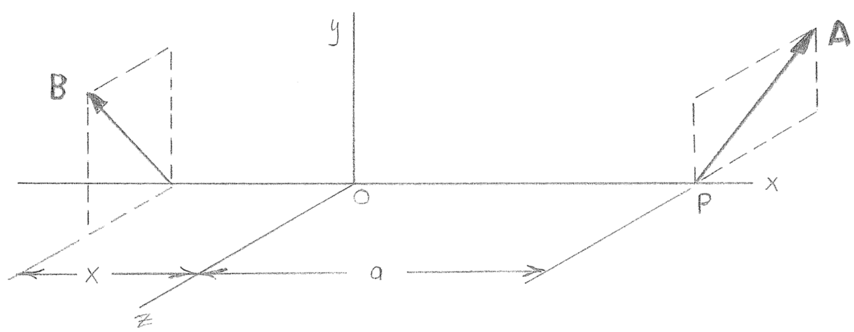
In summary

$$\mathbf{A} = \frac{RM}{a^2R^2 + M^2}\left(M\mathbf{j} - aR\mathbf{k}\right)$$

$$\mathbf{B} = \frac{aR^2}{a^2R^2 + M^2}\left(aR\mathbf{j} + M\mathbf{k}\right)$$

Which shows that it is possible to replace a wrench with two perpendicular forces, one of which is applied at a given point.

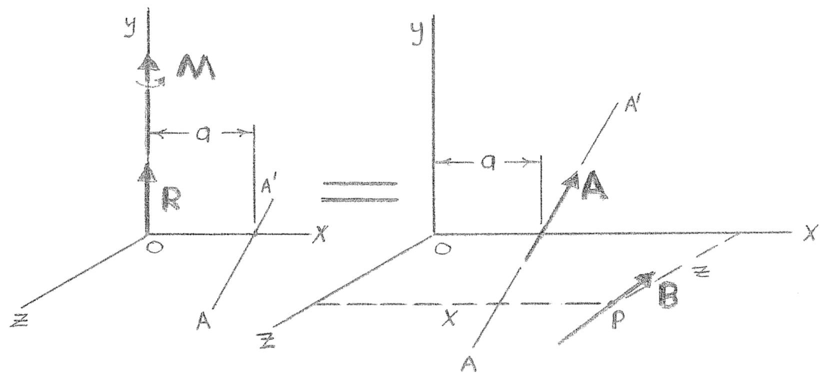
Lastly, if $R > 0$ and $M > 0$, it follows from the equations found for **A** and **B** that $A_y > 0$ and $B_y > 0$. From Equation (6), $x < 0$ (assuming $a > 0$). Then, as a consequence of letting $A_x = 0$, force **A** lies in a plane parallel to the *yz* plane and to the right of the origin, while force **B** lies in a plane parallel to the *yz* plane but to the left of the origin, as shown in the figure below.



PROBLEM 3.141

Show that a wrench can be replaced with two forces, one of which has a prescribed line of action.

SOLUTION



First, choose a rectangular coordinate system where one axis coincides with the axis of the wrench and another axis intersects the prescribed line of action (AA'). Note that it has been assumed that the line of action of force **B** intersects the *xz* plane at point $P(x,\ 0,\ z)$. Denoting the known direction of line AA' by

$$\boldsymbol{\lambda}_A = \lambda_x\mathbf{i} + \lambda_y\mathbf{j} + \lambda_z\mathbf{k}$$

it follows that force **A** can be expressed as

$$\mathbf{A} = A\boldsymbol{\lambda}_A = A\left(\lambda_x\mathbf{i} + \lambda_y\mathbf{j} + \lambda_z\mathbf{k}\right)$$

Force **B** can be expressed as

$$\mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$$

Next, observe that since the axis of the wrench and the prescribed line of action AA' are known, it follows that the distance *a* can be determined. In the following solution, it is assumed that *a* is known.

Then, for equivalence

$$\Sigma F_x: \quad 0 = A\lambda_x + B_x \tag{1}$$

$$\Sigma F_y: \quad R = A\lambda_y + B_y \tag{2}$$

$$\Sigma F_z: \quad 0 = A\lambda_z + B_z \tag{3}$$

$$\Sigma M_x: \quad 0 = -zB_y \tag{4}$$

$$\Sigma M_y: \quad M = -aA\lambda_z + zB_x - xB_z \tag{5}$$

$$\Sigma M_z: \quad 0 = aA\lambda_y + xB_y \tag{6}$$

Since there are six unknowns $\left(A,\ B_x,\ B_y,\ B_z,\ x,\ z\right)$ and six independent equations, it will be possible to obtain a solution.

PROBLEM 3.141 CONTINUED

<u>*Case 1*</u>: Let $z = 0$ to satisfy Equation (4)

Now Equation (2) $$A\lambda_y = R - B_y$$

Equation (3) $$B_z = -A\lambda_z$$

Equation (6) $$x = -\frac{aA\lambda_y}{B_y} = -\left(\frac{a}{B_y}\right)\left(R - B_y\right)$$

Substitution into Equation (5)

$$M = -aA\lambda_z - \left[-\left(\frac{a}{B_y}\right)\left(R - B_y\right)\left(-A\lambda_z\right)\right]$$

$$\therefore \quad A = -\frac{1}{\lambda_z}\left(\frac{M}{aR}\right)B_y$$

Substitution into Equation (2)

$$R = -\frac{1}{\lambda_z}\left(\frac{M}{aR}\right)B_y\lambda_y + B_y$$

$$\therefore \quad B_y = \frac{\lambda_z aR^2}{\lambda_z aR - \lambda_y M}$$

Then $$A = -\frac{MR}{\lambda_z aR - \lambda_y M} = \frac{R}{\lambda_y - \frac{aR}{M}\lambda_z}$$

$$B_x = -A\lambda_x = \frac{\lambda_x MR}{\lambda_z aR - \lambda_y M}$$

$$B_z = -A\lambda_z = \frac{\lambda_z MR}{\lambda_z aR - \lambda_y M}$$

In summary

$$\mathbf{A} = \frac{P}{\lambda_y - \frac{aR}{M}\lambda_z}\boldsymbol{\lambda}_A \blacktriangleleft$$

$$\mathbf{B} = \frac{R}{\lambda_z aR - \lambda_y M}\left(\lambda_x M\mathbf{i} + \lambda_z aR\mathbf{j} + \lambda_z M\mathbf{k}\right) \blacktriangleleft$$

and $$x = a\left(1 - \frac{R}{B_y}\right) = a\left[1 - R\left(\frac{\lambda_z aR - \lambda_y M}{\lambda_z aR^2}\right)\right]$$

$$\text{or } x = \frac{\lambda_y}{\lambda_z}\frac{M}{R} \blacktriangleleft$$

Note that for this case, the lines of action of both **A** and **B** intersect the *x* axis.

PROBLEM 3.141 CONTINUED

<u>*Case 2*</u>: Let $B_y = 0$ to satisfy Equation (4)

Now Equation (2) $$A = \frac{R}{\lambda_y}$$

Equation (1) $$B_x = -R\left(\frac{\lambda_x}{\lambda_y}\right)$$

Equation (3) $$B_z = -R\left(\frac{\lambda_z}{\lambda_y}\right)$$

Equation (6) $$aA\lambda_y = 0 \qquad \text{which requires } a = 0$$

Substitution into Equation (5)

$$M = z\left[-R\left(\frac{\lambda_x}{\lambda_y}\right)\right] - x\left[-R\left(\frac{\lambda_z}{\lambda_y}\right)\right] \qquad \text{or} \qquad \lambda_z x - \lambda_x z = \left(\frac{M}{R}\right)\lambda_y$$

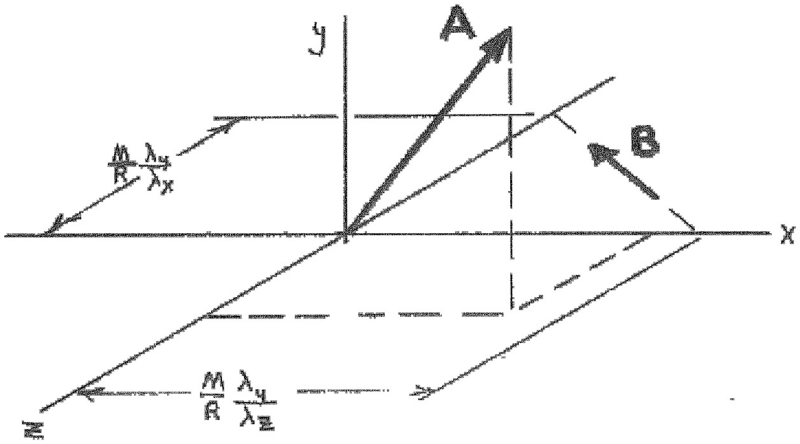
This last expression is the equation for the line of action of force **B**.

In summary

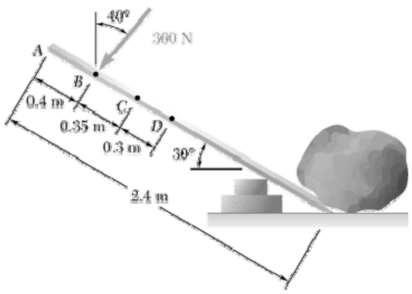
$$\mathbf{A} = \left(\frac{R}{\lambda_y}\right)\lambda_A$$

$$\mathbf{B} = \left(\frac{R}{\lambda_y}\right)(-\lambda_x\mathbf{i} - \lambda_z\mathbf{k})$$

Assuming that $\lambda_x, \lambda_y, \lambda_z > 0,$ the equivalent force system is as shown below.



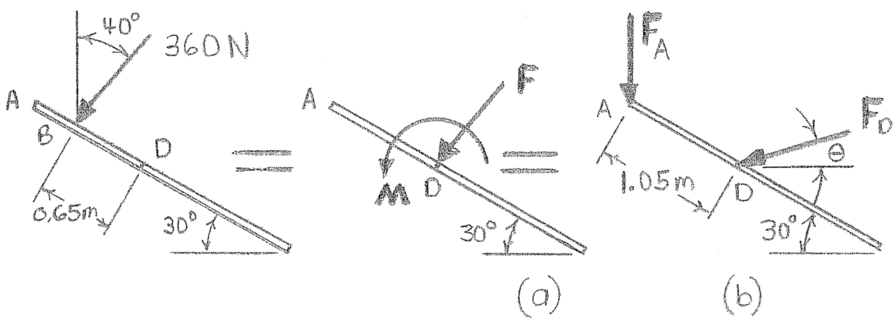
Note that the component of **A** in the *xz* plane is parallel to **B**.



PROBLEM 3.142

A worker tries to move a rock by applying a 360-N force to a steel bar as shown. (*a*) Replace that force with an equivalent force-couple system at *D*. (*b*) Two workers attempt to move the same rock by applying a vertical force at *A* and another force at *D*. Determine these two forces if they are to be equivalent to the single force of part *a*.

SOLUTION



(*a*) Have $\Sigma\mathbf{F}$: $360\text{ N}\left(-\sin 40°\mathbf{i} - \cos 40°\mathbf{j}\right) = -\left(231.40\text{ N}\right)\mathbf{i} - \left(275.78\text{ N}\right)\mathbf{j} = \mathbf{F}$

or $\mathbf{F} = 360\text{ N} \measuredangle\ 50°$ ◀

Have $\Sigma\mathbf{M}_D$: $\mathbf{r}_{B/D} \times \mathbf{R} = \mathbf{M}$

where
$$\mathbf{r}_{B/D} = -\left[\left(0.65\text{ m}\right)\cos 30°\right]\mathbf{i} + \left[\left(0.65\text{ m}\right)\sin 30°\right]\mathbf{j}$$
$$= -\left(0.56292\text{ m}\right)\mathbf{i} + \left(0.32500\text{ m}\right)\mathbf{j}$$

$$\therefore\quad \mathbf{M} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.56292 & 0.32500 & 0 \\ -231.40 & -275.78 & 0 \end{vmatrix}\text{ N}\cdot\text{m} = \left[\left(155.240 + 75.206\right)\text{N}\cdot\text{m}\right]\mathbf{k}$$

$$= \left(230.45\text{ N}\cdot\text{m}\right)\mathbf{k}$$

or $\mathbf{M} = 230\text{ N}\cdot\text{m}$ ↺ ◀

(*b*) Have $\Sigma\mathbf{M}_D$: $\mathbf{M} = \mathbf{r}_{A/D} \times \mathbf{F}_A$

where
$$\mathbf{r}_{A/D} = -\left[\left(1.05\text{ m}\right)\cos 30°\right]\mathbf{i} + \left[\left(1.05\text{ m}\right)\sin 30°\right]\mathbf{j}$$
$$= -\left(0.90933\text{ m}\right)\mathbf{i} + \left(0.52500\text{ m}\right)\mathbf{j}$$

PROBLEM 3.142 CONTINUED

$$\therefore \quad F_A \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.90933 & 0.52500 & 0 \\ 0 & -1 & 0 \end{vmatrix} \text{N}\cdot\text{m} = [230.45 \text{ N}\cdot\text{m}]\mathbf{k}$$

or
$$(0.90933F_A)\mathbf{k} = 230.45\mathbf{k}$$

$$\therefore \quad F_A = 253.42 \text{ N} \qquad\qquad \text{or } \mathbf{F}_A = 253 \text{ N} \downarrow \blacktriangleleft$$

Have
$$\Sigma\mathbf{F}: \quad \mathbf{F} = \mathbf{F}_A + \mathbf{F}_D$$

$$-(231.40 \text{ N})\mathbf{i} - (275.78 \text{ N})\mathbf{j} = -(253.42 \text{ N})\mathbf{j} + F_D(-\cos\theta\mathbf{i} - \sin\theta\mathbf{j})$$

From
$$\mathbf{i}: \quad 231.40 \text{ N} = F_D\cos\theta \qquad (1)$$

$$\mathbf{j}: \quad 22.36 \text{ N} = F_D\sin\theta \qquad (2)$$

Equation (2) divided by Equation (1)

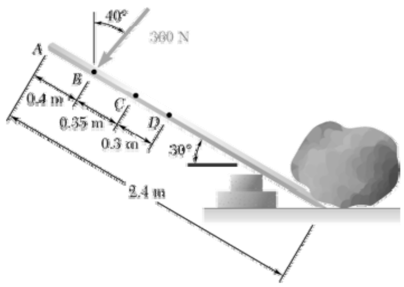
$$\tan\theta = 0.096629$$

$$\therefore \quad \theta = 5.5193° \qquad \text{or} \qquad \theta = 5.52°$$

Substitution into Equation (1)

$$F_D = \frac{231.40}{\cos 5.5193°} = 232.48 \text{ N}$$

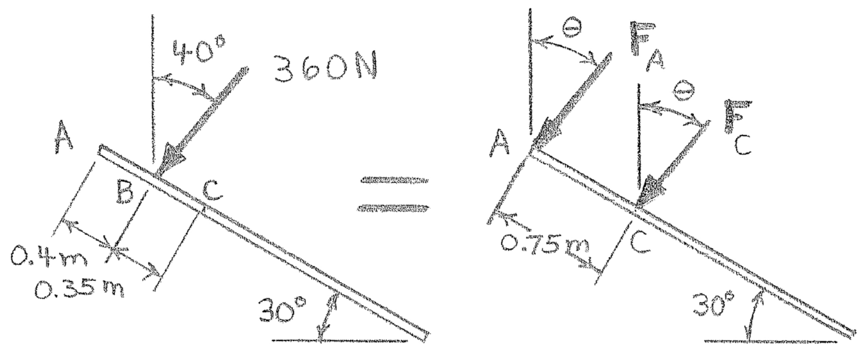
$$\text{or } \mathbf{F}_D = 232 \text{ N} \measuredangle\, 5.52° \blacktriangleleft$$



PROBLEM 3.143

A worker tries to move a rock by applying a 360-N force to a steel bar as shown. If two workers attempt to move the same rock by applying a force at A and a parallel force at C, determine these two forces so that they will be equivalent to the single 360-N force shown in the figure.

SOLUTION



Have $$\Sigma \mathbf{F}: \quad \mathbf{R} = \mathbf{F}_A + \mathbf{F}_C$$

$$-\left[(360\text{ N})\sin 40°\right]\mathbf{i} - \left[(360\text{ N})\cos 40°\right]\mathbf{j} = -\left[(F_A + F_C)\sin\theta\right]\mathbf{i} - \left[(F_A + F_C)\cos\theta\right]\mathbf{j}$$

From $$\mathbf{i}: \quad (360\text{ N})\sin 40° = (F_A + F_C)\sin\theta \qquad (1)$$

$$\mathbf{j}: \quad (360\text{ N})\cos 40° = (F_A + F_C)\cos\theta \qquad (2)$$

Dividing Equation (1) by Equation (2),

$$\tan 40° = \tan\theta$$

$$\therefore \quad \theta = 40°$$

Substituting $\theta = 40°$ into Equation (1),

$$F_A + F_C = 360\text{ N} \qquad (3)$$

Have $$\Sigma \mathbf{M}_C: \quad \mathbf{r}_{B/C} \times \mathbf{R} = \mathbf{r}_{A/C} \times \mathbf{F}_A$$

where $$\mathbf{r}_{B/C} = (0.35\text{ m})(-\cos 30°\mathbf{i} + \sin 30°\mathbf{j}) = -(0.30311\text{ m})\mathbf{i} + (0.175\text{ m})\mathbf{j}$$

PROBLEM 3.143 CONTINUED

$$\mathbf{R} = (360\text{ N})(-\sin 40°\mathbf{i} - \cos 40°\mathbf{j}) = -(231.40\text{ N})\mathbf{i} - (275.78\text{ N})\mathbf{j}$$

$$\mathbf{r}_{A/C} = (0.75\text{ m})(-\cos 30°\mathbf{i} + \sin 30\mathbf{j}) = -(0.64952\text{ m})\mathbf{i} + (0.375\text{ m})\mathbf{j}$$

$$\mathbf{F}_A = F_A(-\sin 40°\mathbf{i} - \cos 40°\mathbf{j}) = F_A(-0.64279\mathbf{i} - 0.76604\mathbf{j})$$

$$\therefore \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.30311 & 0.175 & 0 \\ -231.40 & -275.78 & 0 \end{vmatrix} \text{N}\cdot\text{m} = F_A \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.64952 & 0.375 & 0 \\ -0.64279 & -0.76604 & 0 \end{vmatrix} \text{N}\cdot\text{m}$$

$$83.592 + 40.495 = (0.49756 + 0.24105)F_A$$

$$\therefore \quad F_A = 168.002\text{ N} \qquad \text{or} \qquad F_A = 168.0\text{ N}$$

Substituting into Equation (3),

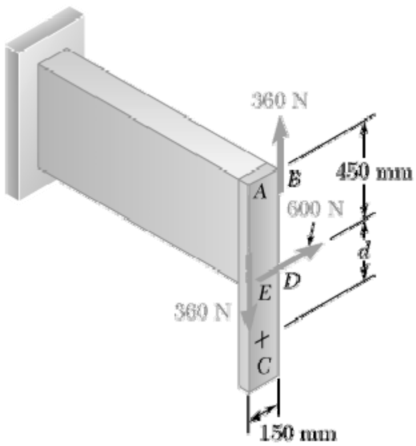
$$F_C = 360 - 168.002 = 191.998\text{ N} \qquad \text{or} \qquad F_C = 192.0\text{ N}$$

or $\mathbf{F}_A = 168.0\text{ N} \swarrow 50°$ ◀

$\mathbf{F}_C = 192.0\text{ N} \swarrow 50°$ ◀

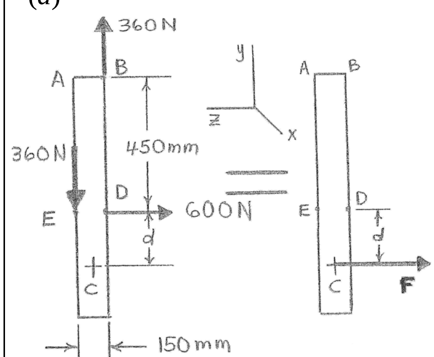
PROBLEM 3.144

A force and a couple are applied as shown to the end of a cantilever beam. (*a*) Replace this system with a single force **F** applied at point *C*, and determine the distance *d* from *C* to a line drawn through points *D* and *E*. (*b*) Solve part *a* if the directions of the two 360-N forces are reversed.



SOLUTION

(*a*)



(*a*) Have $\qquad \Sigma\mathbf{F}: \quad \mathbf{F} = (360\text{ N})\mathbf{j} - (360\text{ N})\mathbf{j} - (600\text{ N})\mathbf{k}$

or $\mathbf{F} = -(600\text{ N})\mathbf{k}$ ◀

and $\qquad \Sigma M_D: \quad (360\text{ N})(0.15\text{ m}) = (600\text{ N})(d)$

$\therefore \quad d = 0.09\text{ m}$

or $d = 90.0$ mm below *ED* ◀

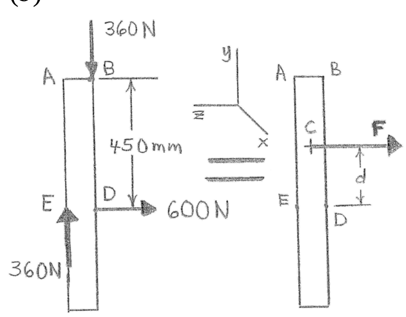
(*b*) Have from part *a* $\qquad \mathbf{F} = -(600\text{ N})\mathbf{k}$ ◀

and $\qquad \Sigma M_D: \quad -(360\text{ N})(0.15\text{ m}) = -(600\text{ N})(d)$

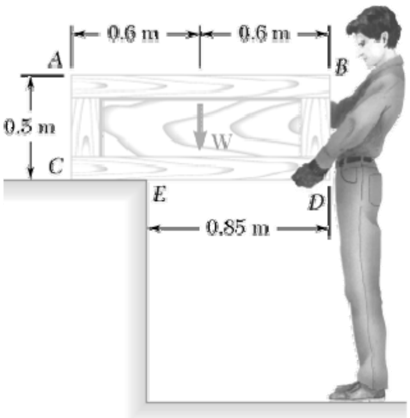
$\therefore \quad d = 0.09\text{ m}$

or $d = 90.0$ mm above *ED* ◀

(*b*)

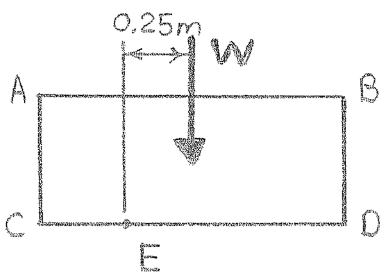


PROBLEM 3.145



A crate of mass 80 kg is held in the position shown. Determine (*a*) the moment produced by the weight **W** of the crate about *E*, (*b*) the smallest force applied at *B* which creates a moment of equal magnitude and opposite sense about *E*.

SOLUTION

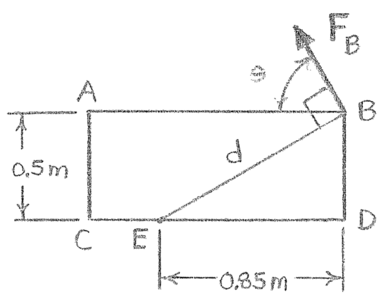


(*a*) By definition $W = mg = 80\text{ kg}\left(9.81\text{ m/s}^2\right) = 784.8\text{ N}$

Have $\Sigma M_E: \quad M_E = (784.8\text{ N})(0.25\text{ m})$

$$\therefore \quad \mathbf{M}_E = 196.2\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

(*b*) For the force at *B* to be the smallest, resulting in a moment $(\mathbf{M}_E)$ about *E*, the line of action of force $\mathbf{F}_B$ must be perpendicular to the line connecting *E* to *B*. The sense of $\mathbf{F}_B$ must be such that the force produces a counterclockwise moment about *E*.



Note: $d = \sqrt{(0.85\text{ m})^2 + (0.5\text{ m})^2} = 0.98615\text{ m}$

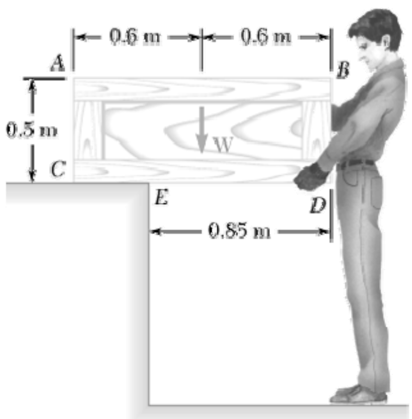
Have $\Sigma M_E: \quad 196.2\text{ N}\cdot\text{m} = F_B(0.98615\text{ m})$

$$\therefore \quad F_B = 198.954\text{ N}$$

and $\theta = \tan^{-1}\left(\dfrac{0.85\text{ m}}{0.5\text{ m}}\right) = 59.534°$

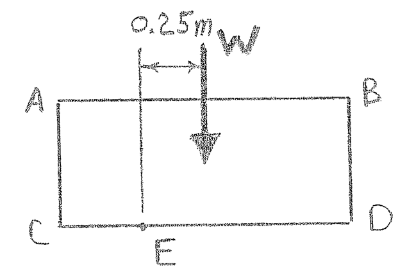
or $\mathbf{F}_B = 199.0\text{ N} \measuredangle\ 59.5° \blacktriangleleft$

PROBLEM 3.146



A crate of mass 80 kg is held in the position shown. Determine (*a*) the moment produced by the weight **W** of the crate about *E*, (*b*) the smallest force applied at *A* which creates a moment of equal magnitude and opposite sense about *E*, (*c*) the magnitude, sense, and point of application on the bottom of the crate of the smallest vertical force which creates a moment of equal magnitude and opposite sense about *E*.

SOLUTION

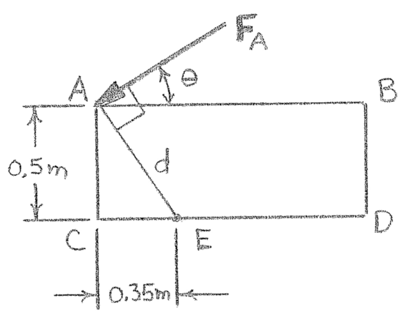


(*a*) By definition $\quad W = mg = 80\text{ kg}\left(9.81\text{ m/s}^2\right) = 784.8\text{ N}$

Have $\quad \Sigma M_E: \quad M_E = (784.8\text{ N})(0.25\text{ m})$

$\therefore \quad \mathbf{M}_E = 196.2\text{ N}\cdot\text{m}$ ◀

(*b*) For the force at *A* to be the smallest, resulting in a moment about *E*, the line of action of force $\mathbf{F}_A$ must be perpendicular to the line connecting *E* to *A*. The sense of $\mathbf{F}_A$ must be such that the force produces a counterclockwise moment about *E*.



Note: $\quad d = \sqrt{(0.35\text{ m})^2 + (0.5\text{ m})^2} = 0.61033\text{ m}$

Have $\quad \Sigma M_E: \quad 196.2\text{ N}\cdot\text{m} = F_A(0.61033\text{ m})$

$\therefore \quad F_A = 321.47\text{ N}$

and $\quad \theta = \tan^{-1}\left(\dfrac{0.35\text{ m}}{0.5\text{ m}}\right) = 34.992°$

or $\mathbf{F}_A = 321\text{ N}$ ↗ $35.0°$ ◀

(*c*) The smallest force acting on the bottom of the crate resulting in a moment about *E* will be located at the point on the bottom of the crate farthest from *E* and acting perpendicular to line *CED*. The sense of the force will be such as to produce a counterclockwise moment about *E*. A force acting vertically upward at *D* satisfies these conditions.

PROBLEM 3.146 CONTINUED

Have $\qquad \Sigma\mathbf{M}_E: \quad \mathbf{M}_E = \mathbf{r}_{D/E} \times \mathbf{F}_D$

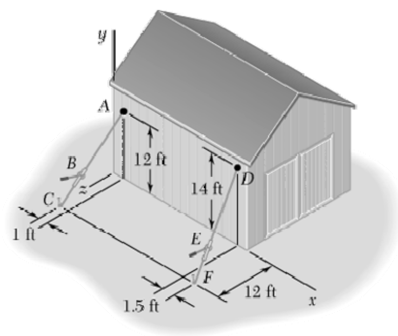
$$(196.2 \text{ N}\cdot\text{m})\mathbf{k} = (0.85 \text{ m})\mathbf{i} \times (F_D)\mathbf{j}$$

$$(196.2 \text{ N}\cdot\text{m})\mathbf{k} = (0.85F_D)\mathbf{k}$$

$$\therefore \quad F_D = 230.82 \text{ N}$$

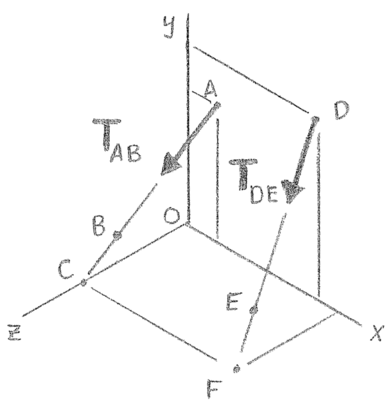
or $\mathbf{F}_D = 231 \text{ N} \uparrow$ ◀

PROBLEM 3.147



A farmer uses cables and winch pullers B and E to plumb one side of a small barn. Knowing that the sum of the moments about the x axis of the forces exerted by the cables on the barn at points A and D is equal to $4728 \text{ lb}\cdot\text{ft}$, determine the magnitude of $\mathbf{T}_{DE}$ when $\mathbf{T}_{AB} = 255$ lb.

SOLUTION



The moment about the x-axis due to the two cable forces can be found using the z-components of each force acting at their intersection with the xy-plane (A and D). The x-components of the forces are parallel to the x-axis, and the y-components of the forces intersect the x-axis. Therefore, neither the x or y components produce a moment about the x-axis.

Have $$\Sigma M_x: \quad (T_{AB})_z(y_A) + (T_{DE})_z(y_D) = M_x$$

where $$(T_{AB})_z = \mathbf{k}\cdot\mathbf{T}_{AB} = \mathbf{k}\cdot(T_{AB}\boldsymbol{\lambda}_{AB})$$

$$= \mathbf{k}\cdot\left[255 \text{ lb}\left(\frac{-\mathbf{i} - 12\mathbf{j} + 12\mathbf{k}}{17}\right)\right] = 180 \text{ lb}$$

$$(T_{DE})_z = \mathbf{k}\cdot\mathbf{T}_{DE} = \mathbf{k}\cdot(T_{DE}\boldsymbol{\lambda}_{DE})$$

$$= \mathbf{k}\cdot\left[T_{DE}\left(\frac{1.5\mathbf{i} - 14\mathbf{j} + 12\mathbf{k}}{18.5}\right)\right] = 0.64865T_{DE}$$

$$y_A = 12 \text{ ft}$$

$$y_D = 14 \text{ ft}$$

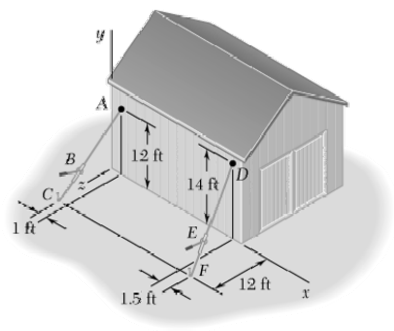
$$M_x = 4728 \text{ lb}\cdot\text{ft}$$

$$\therefore \quad (180 \text{ lb})(12 \text{ ft}) + (0.64865T_{DE})(14 \text{ ft}) = 4728 \text{ lb}\cdot\text{ft}$$

and $$T_{DE} = 282.79 \text{ lb}$$

or $T_{DE} = 283$ lb ◀

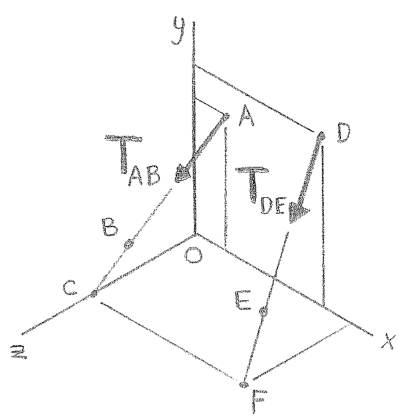
PROBLEM 3.148



Solve Problem 3.147 when the tension in cable *AB* is 306 lb.

Problem 3.147: A farmer uses cables and winch pullers *B* and *E* to plumb one side of a small barn. Knowing that the sum of the moments about the *x* axis of the forces exerted by the cables on the barn at points *A* and *D* is equal to $4728 \text{ lb} \cdot \text{ft}$, determine the magnitude of $\mathbf{T}_{DE}$ when $\mathbf{T}_{AB} = 255 \text{ lb}$.

SOLUTION



The moment about the *x*-axis due to the two cable forces can be found using the *z* components of each force acting at the intersection with the *xy* plane (*A* and *D*). The *x* components of the forces are parallel to the *x* axis, and the *y* components of the forces intersect the *x* axis. Therefore, neither the *x* or *y* components produce a moment about the *x* axis.

Have $$\Sigma M_x: \quad (T_{AB})_z (y_A) + (T_{DE})_z (y_D) = M_x$$

where $$(T_{AB})_z = \mathbf{k} \cdot \mathbf{T}_{AB} = \mathbf{k} \cdot (T_{AB} \boldsymbol{\lambda}_{AB})$$

$$= \mathbf{k} \cdot \left[306 \text{ lb} \left(\frac{-\mathbf{i} - 12\mathbf{j} + 12\mathbf{k}}{17} \right) \right] = 216 \text{ lb}$$

$$(T_{DE})_z = \mathbf{k} \cdot \mathbf{T}_{DE} = \mathbf{k} \cdot (T_{DE} \boldsymbol{\lambda}_{DE})$$

$$= \mathbf{k} \cdot \left[T_{DE} \left(\frac{1.5\mathbf{i} - 14\mathbf{j} + 12\mathbf{k}}{18.5} \right) \right] = 0.64865 T_{DE}$$

$$y_A = 12 \text{ ft}$$

$$y_D = 14 \text{ ft}$$

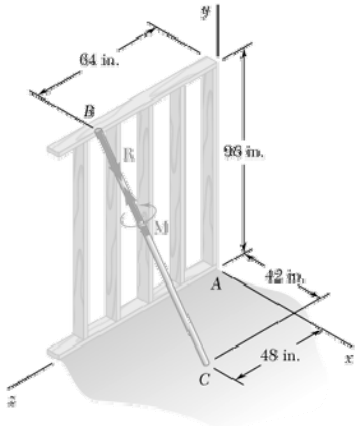
$$M_x = 4728 \text{ lb} \cdot \text{ft}$$

$$\therefore \quad (216 \text{ lb})(12 \text{ ft}) + (0.64865 T_{DE})(14 \text{ ft}) = 4728 \text{ lb} \cdot \text{ft}$$

and $$T_{DE} = 235.21 \text{ lb}$$

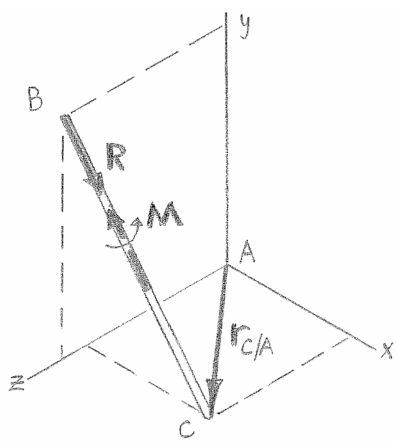
or $T_{DE} = 235 \text{ lb}$ ◀

PROBLEM 3.149



As an adjustable brace BC is used to bring a wall into plumb, the force-couple system shown is exerted on the wall. Replace this force-couple system with an equivalent force-couple system at A knowing that $R = 21.2$ lb and $M = 13.25$ lb·ft.

SOLUTION



Have $$\Sigma\mathbf{F}: \quad \mathbf{R} = \mathbf{R}_A = \mathbf{R}\boldsymbol{\lambda}_{BC}$$

where $$\lambda_{BC} = \frac{(42 \text{ in.})\mathbf{i} - (96 \text{ in.})\mathbf{j} - (16 \text{ in.})\mathbf{k}}{106 \text{ in.}}$$

$$\therefore \quad \mathbf{R}_A = \frac{21.2 \text{ lb}}{106}(42\mathbf{i} - 96\mathbf{j} - 16\mathbf{k})$$

$$\text{or } \mathbf{R}_A = (8.40 \text{ lb})\mathbf{i} - (19.20 \text{ lb})\mathbf{j} - (3.20 \text{ lb})\mathbf{k} \blacktriangleleft$$

Have $$\Sigma\mathbf{M}_A: \quad \mathbf{r}_{C/A} \times \mathbf{R} + \mathbf{M} = \mathbf{M}_A$$

where $$\mathbf{r}_{C/A} = (42 \text{ in.})\mathbf{i} + (48 \text{ in.})\mathbf{k} = \frac{1}{12}(42\mathbf{i} + 48\mathbf{k}) \text{ ft}$$

$$= (3.5 \text{ ft})\mathbf{i} + (4.0 \text{ ft})\mathbf{k}$$

$$\mathbf{R} = (8.40 \text{ lb})\mathbf{i} - (19.20 \text{ lb})\mathbf{j} - (3.20 \text{ lb})\mathbf{k}$$

$$\mathbf{M} = -\lambda_{BC}M$$

$$= \frac{-42\mathbf{i} + 96\mathbf{j} + 16\mathbf{k}}{106}(13.25 \text{ lb}\cdot\text{ft})$$

$$= -(5.25 \text{ lb}\cdot\text{ft})\mathbf{i} + (12 \text{ lb}\cdot\text{ft})\mathbf{j} + (2 \text{ lb}\cdot\text{ft})\mathbf{k}$$

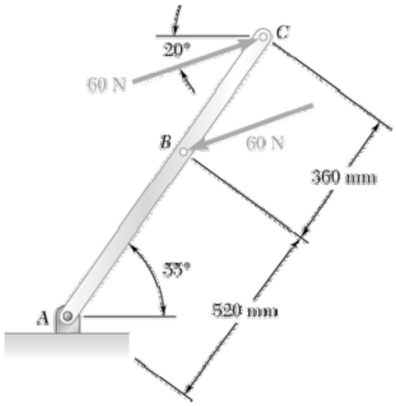
PROBLEM 3.149 CONTINUED

Then $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3.5 & 0 & 4.0 \\ 8.40 & -19.20 & -3.20 \end{vmatrix} \text{lb}\cdot\text{ft} + \left(-5.25\mathbf{i} + 12\mathbf{j} + 2\mathbf{k}\right)\text{lb}\cdot\text{ft} = \mathbf{M}_A$

$$\therefore \quad \mathbf{M}_A = \left(71.55 \text{ lb}\cdot\text{ft}\right)\mathbf{i} + \left(56.80 \text{ lb}\cdot\text{ft}\right)\mathbf{j} - \left(65.20 \text{ lb}\cdot\text{ft}\right)\mathbf{k}$$

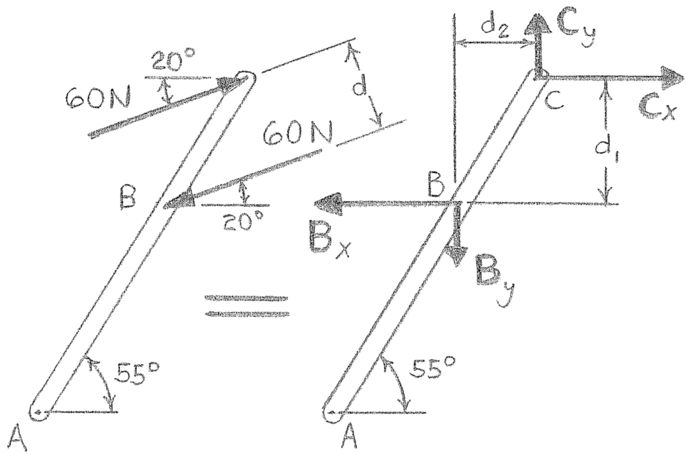
or $\mathbf{M}_A = \left(71.6 \text{ lb}\cdot\text{ft}\right)\mathbf{i} + \left(56.8 \text{ lb}\cdot\text{ft}\right)\mathbf{j} - \left(65.2 \text{ lb}\cdot\text{ft}\right)\mathbf{k}$ ◀

PROBLEM 3.150



Two parallel 60-N forces are applied to a lever as shown. Determine the moment of the couple formed by the two forces (*a*) by resolving each force into horizontal and vertical components and adding the moments of the two resulting couples, (*b*) by using the perpendicular distance between the two forces, (*c*) by summing the moments of the two forces about point *A*.

SOLUTION



(*a*) Have $$\Sigma \mathbf{M}_B: \quad -d_1 C_x + d_2 C_y = \mathbf{M}$$

where $$d_1 = (0.360 \text{ m})\sin 55° = 0.29489 \text{ m}$$

$$d_2 = (0.360 \text{ m})\cos 55° = 0.20649 \text{ m}$$

$$C_x = (60 \text{ N})\cos 20° = 56.382 \text{ N}$$

$$C_y = (60 \text{ N})\sin 20° = 20.521 \text{ N}$$

$$\therefore \quad \mathbf{M} = -(0.29489 \text{ m})(56.382 \text{ N})\mathbf{k} + (0.20649 \text{ m})(20.521 \text{ N})\mathbf{k} = -(12.3893 \text{ N}\cdot\text{m})\mathbf{k}$$

or $\mathbf{M} = 12.39 \text{ N}\cdot\text{m} \circlearrowright$ ◀

(*b*) Have $$\mathbf{M} = Fd(-\mathbf{k}) = 60 \text{ N}\left[(0.360 \text{ m})\sin(55° - 20°)\right](-\mathbf{k})$$

$$= -(12.3893 \text{ N}\cdot\text{m})\mathbf{k}$$

or $\mathbf{M} = 12.39 \text{ N}\cdot\text{m} \circlearrowright$ ◀

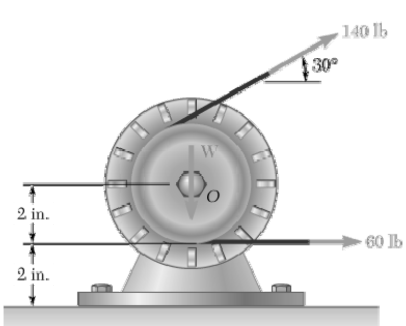
PROBLEM 3.150 CONTINUED

(*c*) Have $\Sigma \mathbf{M}_A: \quad \Sigma(\mathbf{r}_A \times \mathbf{F}) = \mathbf{r}_{B/A} \times \mathbf{F}_B + \mathbf{r}_{C/A} \times \mathbf{F}_C = \mathbf{M}$

$$\therefore \quad M = (0.520 \text{ m})(60 \text{ N})\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \cos 55^\circ & \sin 55^\circ & 0 \\ -\cos 20^\circ & -\sin 20^\circ & 0 \end{vmatrix} + (0.880 \text{ m})(60 \text{ N})\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \cos 55^\circ & \sin 55^\circ & 0 \\ \cos 20^\circ & \sin 20^\circ & 0 \end{vmatrix}$$

$$= (17.8956 \text{ N}\cdot\text{m} - 30.285 \text{ N}\cdot\text{m})\mathbf{k} = -(12.3892 \text{ N}\cdot\text{m})\mathbf{k}$$

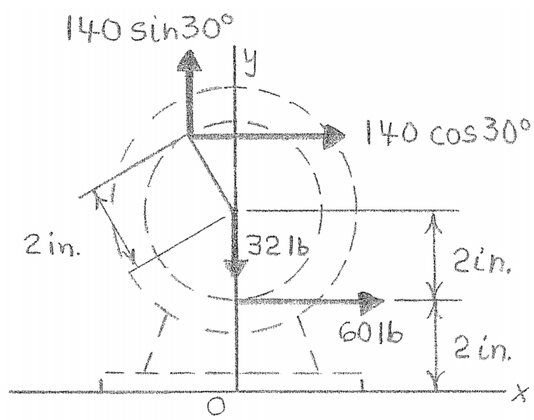
or $\mathbf{M} = 12.39 \text{ N}\cdot\text{m}$ ◀



PROBLEM 3.151

A 32-lb motor is mounted on the floor. Find the resultant of the weight and the forces exerted on the belt, and determine where the line of action of the resultant intersects the floor.

SOLUTION



Have $\Sigma\mathbf{F}: \quad (60 \text{ lb})\mathbf{i} - (32 \text{ lb})\mathbf{j} + (140 \text{ lb})(\cos 30°\mathbf{i} + \sin 30°\mathbf{j}) = \mathbf{R}$

$$\therefore \quad \mathbf{R} = (181.244 \text{ lb})\mathbf{i} + (38.0 \text{ lb})\mathbf{j}$$

or $\mathbf{R} = 185.2 \text{ lb} \measuredangle 11.84°$ ◀

Have $\Sigma M_O: \quad \Sigma M_O = xR_y$

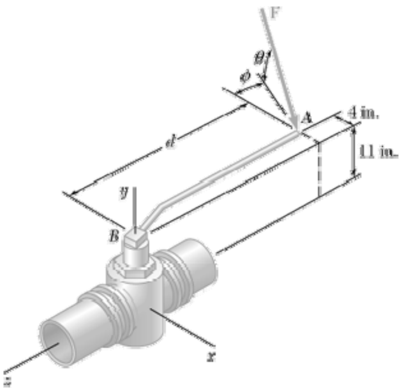
$$\therefore \quad -\left[(140 \text{ lb})\cos 30°\right]\left[(4 + 2\cos 30°)\text{ in.}\right] - \left[(140 \text{ lb})\sin 30°\right]\left[(2 \text{ in.})\sin 30°\right]$$

$$-(60 \text{ lb})(2 \text{ in.}) = x(38.0 \text{ lb})$$

$$x = \frac{1}{38.0}(-694.97 - 70.0 - 120) \text{ in.}$$

and $x = -23.289 \text{ in.}$

Or, resultant intersects the base (x axis) 23.3 in. to the left of the vertical centerline (y axis) of the motor. ◀



PROBLEM 3.152

To loosen a frozen valve, a force **F** of magnitude 70 lb is applied to the handle of the valve. Knowing that $\theta = 25°$, $M_x = -61\text{ lb}\cdot\text{ft}$, and $M_z = -43\text{ lb}\cdot\text{ft}$, determine θ and d.

SOLUTION

Have $$\Sigma\mathbf{M}_O: \quad \mathbf{r}_{A/O} \times \mathbf{F} = \mathbf{M}_O$$

where $$\mathbf{r}_{A/O} = -(4\text{ in.})\mathbf{i} + (11\text{ in.})\mathbf{j} - (d)\mathbf{k}$$

$$\mathbf{F} = F(\cos\theta\cos\phi\mathbf{i} - \sin\theta\mathbf{j} + \cos\theta\sin\phi\mathbf{k})$$

For $$F = 70\text{ lb},\ \theta = 25°$$

$$\mathbf{F} = (70\text{ lb})\left[(0.90631\cos\phi)\mathbf{i} - 0.42262\mathbf{j} + (0.90631\sin\phi)\mathbf{k}\right]$$

$$\therefore\quad \mathbf{M}_O = (70\text{ lb})\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & 11 & -d \\ -0.90631\cos\phi & -0.42262 & 0.90631\sin\phi \end{vmatrix}\text{in.}$$

$$= (70\text{ lb})\left[(9.9694\sin\phi - 0.42262d)\mathbf{i} + (-0.90631d\cos\phi + 3.6252\sin\phi)\mathbf{j}\right.$$

$$\left. + (1.69048 - 9.9694\cos\phi)\mathbf{k}\right]\text{in.}$$

and $$M_x = (70\text{ lb})(9.9694\sin\phi - 0.42262d)\text{ in.} = -(61\text{ lb}\cdot\text{ft})(12\text{ in./ft}) \quad (1)$$

$$M_y = (70\text{ lb})(-0.90631d\cos\phi + 3.6252\sin\phi)\text{in.} \quad (2)$$

$$M_z = (70\text{ lb})(1.69048 - 9.9694\cos\phi)\text{ in.} = -43\text{ lb}\cdot\text{ft}(12\text{ in./ft}) \quad (3)$$

From Equation (3)

$$\phi = \cos^{-1}\left(\frac{634.33}{697.86}\right) = 24.636°$$

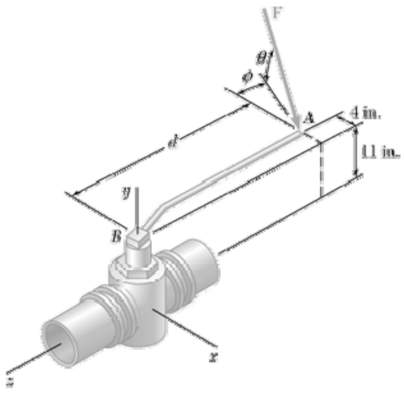
or $\phi = 24.6°$ ◀

From Equation (1)

$$d = \left(\frac{1022.90}{29.583}\right) = 34.577 \text{ in.}$$

or $d = 34.6$ in. ◀

PROBLEM 3.153



When a force **F** is applied to the handle of the valve shown, its moments about the x and z axes are, respectively, $M_x = -77\text{ lb}\cdot\text{ft}$ and $M_z = -81\text{ lb}\cdot\text{ft}$. For $d = 27$ in., determine the moment M_y of **F** about the y axis.

SOLUTION

Have
$$\Sigma\mathbf{M}_O: \quad \mathbf{r}_{A/O} \times \mathbf{F} = \mathbf{M}_O$$

where
$$\mathbf{r}_{A/O} = -(4\text{ in.})\mathbf{i} + (11\text{ in.})\mathbf{j} - (27\text{ in.})\mathbf{k}$$

$$\mathbf{F} = F(\cos\theta\cos\phi\mathbf{i} - \sin\theta\mathbf{j} + \cos\theta\sin\phi\mathbf{k})$$

$$\therefore \quad \mathbf{M}_O = F\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & 11 & -27 \\ \cos\theta\cos\phi & -\sin\theta & \cos\theta\sin\phi \end{vmatrix}\text{lb}\cdot\text{in.}$$

$$= F\big[(11\cos\theta\sin\phi - 27\sin\theta)\mathbf{i} + (-27\cos\theta\cos\phi + 4\cos\theta\sin\phi)\mathbf{j}$$

$$+ (4\sin\theta - 11\cos\theta\cos\phi)\mathbf{k}\big](\text{lb}\cdot\text{in.})$$

and
$$M_x = F(11\cos\theta\sin\phi - 27\sin\theta)(\text{lb}\cdot\text{in.}) \tag{1}$$

$$M_y = F(-27\cos\theta\cos\phi + 4\cos\theta\sin\phi)(\text{lb}\cdot\text{in.}) \tag{2}$$

$$M_z = F(4\sin\theta - 11\cos\theta\cos\phi)(\text{lb}\cdot\text{in.}) \tag{3}$$

Now, Equation (1)
$$\cos\theta\sin\phi = \frac{1}{11}\left(\frac{M_x}{F} + 27\sin\theta\right) \tag{4}$$

and Equation (3)
$$\cos\theta\cos\phi = \frac{1}{11}\left(4\sin\theta - \frac{M_z}{F}\right) \tag{5}$$

Substituting Equations (4) and (5) into Equation (2),

$$M_y = F\left\{-27\left[\frac{1}{11}\left(4\sin\theta - \frac{M_z}{F}\right)\right] + 4\left[\frac{1}{11}\left(\frac{M_x}{F} + 27\sin\theta\right)\right]\right\}$$

or
$$M_y = \frac{1}{11}(27M_z + 4M_x)$$

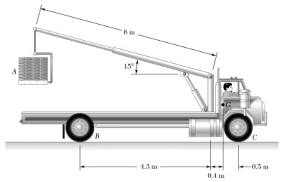
PROBLEM 3.153 CONTINUED

Noting that the ratios $\dfrac{27}{11}$ and $\dfrac{4}{11}$ are the ratios of lengths, have

$$M_y = \frac{27}{11}\left(-81 \text{ lb}\cdot\text{ft}\right) + \frac{4}{11}\left(-77 \text{ lb}\cdot\text{ft}\right) = 226.82 \text{ lb}\cdot\text{ft}$$

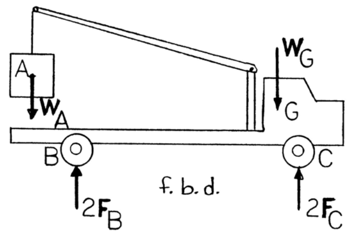
or $M_y = -227 \text{ lb}\cdot\text{ft}$ ◀

PROBLEM 4.1



The boom on a 4300-kg truck is used to unload a pallet of shingles of mass 1600 kg. Determine the reaction at each of the two (*a*) rear wheels *B*, (*b*) front wheels *C*.

SOLUTION



$$W_A = m_A g = (1600\text{ kg})(9.81\text{ m/s}^2)$$

$$= 15696\text{ N}$$

or $$\mathbf{W}_A = 15.696\text{ kN} \downarrow$$

$$W_G = m_G g = (4300\text{ kg})(9.81\text{ m/s}^2)$$

$$= 42\,183\text{ N}$$

or $$\mathbf{W}_G = 42.183\text{ kN} \downarrow$$

(*a*) From f.b.d. of truck with boom

$$+\circlearrowleft \Sigma M_C = 0\text{:}\quad (15.696\text{ kN})\left[(0.5 + 0.4 + 6\cos 15°)\text{ m}\right] - 2F_B\left[(0.5 + 0.4 + 4.3)\text{ m}\right]$$

$$+ (42.183\text{ kN})(0.5\text{ m}) = 0$$

$$\therefore\ 2F_B = \frac{126.185}{5.2} = 24.266\text{ kN}$$

or $\mathbf{F}_B = 12.13\text{ kN} \uparrow$ ◀

(*b*) From f.b.d. of truck with boom

$$+\circlearrowleft \Sigma M_B = 0\text{:}\quad (15.696\text{ kN})\left[(6\cos 15° - 4.3)\text{ m}\right] - (42.183\text{ kN})\left[(4.3 + 0.4)\text{ m}\right]$$

$$+ 2F_C\left[(4.3 + 0.9)\text{ m}\right] = 0$$

$$\therefore\ 2F_C = \frac{174.786}{5.2} = 33.613\text{ kN}$$

or $\mathbf{F}_C = 16.81\text{ kN} \uparrow$ ◀

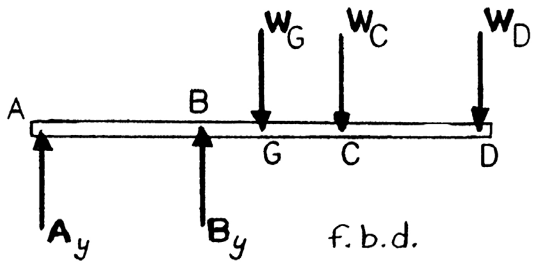
Check: $$+\uparrow \Sigma F_y = 0\text{:}\quad (33.613 - 42.183 + 24.266 - 15.696)\text{ kN} = 0?$$

$$(57.879 - 57.879)\text{ kN} = 0\ \text{ ok}$$

PROBLEM 4.2

Two children are standing on a diving board of mass 65 kg. Knowing that the masses of the children at *C* and *D* are 28 kg and 40 kg, respectively, determine (*a*) the reaction at *A*, (*b*) the reaction at *B*.

SOLUTION



$$W_G = m_G g = (65\text{ kg})(9.81\text{ m/s}^2) = 637.65\text{ N}$$

$$W_C = m_C g = (28\text{ kg})(9.81\text{ m/s}^2) = 274.68\text{ N}$$

$$W_D = m_D g = (40\text{ kg})(9.81\text{ m/s}^2) = 392.4\text{ N}$$

(*a*) From f.b.d. of diving board

$$+\circlearrowleft \Sigma M_B = 0: \quad -A_y(1.2\text{ m}) - (637.65\text{ N})(0.48\text{ m}) - (274.68\text{ N})(1.08\text{ m}) - (392.4\text{ N})(2.08\text{ m}) = 0$$

$$\therefore\ A_y = -\frac{1418.92}{1.2} = -1182.43\text{ N}$$

or $\mathbf{A}_y = 1.182\text{ kN}\downarrow$ ◀

(*b*) From f.b.d. of diving board

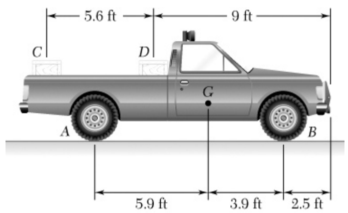
$$+\circlearrowleft \Sigma M_A = 0: \quad B_y(1.2\text{ m}) - 637.65\text{ N}(1.68\text{ m}) - 274.68\text{ N}(2.28\text{ m}) - 392.4\text{ N}(3.28\text{ m}) = 0$$

$$\therefore\ B_y = \frac{2984.6}{1.2} = 2487.2\text{ N}$$

or $\mathbf{B}_y = 2.49\text{ kN}\uparrow$ ◀

Check: $+\uparrow \Sigma F_y = 0: \quad (-1182.43 + 2487.2 - 637.65 - 274.68 - 392.4)\text{ N} = 0?$

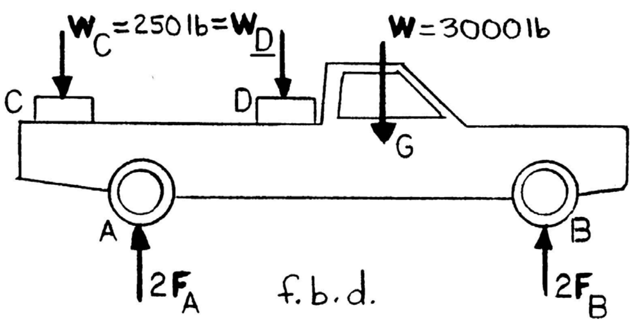
$$(2487.2 - 2487.2)\text{ N} = 0 \ \text{ ok}$$



PROBLEM 4.3

Two crates, each weighing 250 lb, are placed as shown in the bed of a 3000-lb pickup truck. Determine the reactions at each of the two (*a*) rear wheels *A*, (*b*) front wheels *B*.

SOLUTION



(*a*) From f.b.d. of truck

$$+\circlearrowleft \Sigma M_B = 0: \quad (250\text{ lb})(12.1\text{ ft}) + (250\text{ lb})(6.5\text{ ft}) + (3000\text{ lb})(3.9\text{ ft}) - (2F_A)(9.8\text{ ft}) = 0$$

$$\therefore\ 2F_A = \frac{16350}{9.8} = 1668.37\text{ lb}$$

$$\therefore\quad \mathbf{F}_A = 834\text{ lb}\uparrow \blacktriangleleft$$

(*b*) From f.b.d. of truck

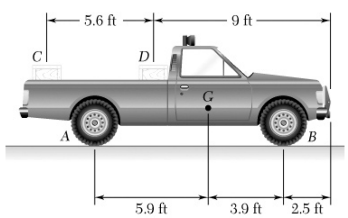
$$+\circlearrowleft \Sigma M_A = 0: \quad (2F_B)(9.8\text{ ft}) - (3000\text{ lb})(5.9\text{ ft}) - (250\text{ lb})(3.3\text{ ft}) + (250\text{ lb})(2.3\text{ ft}) = 0$$

$$\therefore\ 2F_B = \frac{17950}{9.8} = 1831.63\text{ lb}$$

$$\therefore\quad \mathbf{F}_B = 916\text{ lb}\uparrow \blacktriangleleft$$

Check: $+\uparrow \Sigma F_y = 0: \quad (-250 + 1668.37 - 250 - 3000 + 1831.63)\text{ lb} = 0?$

$$(3500 - 3500)\text{ lb} = 0 \ \text{ ok}$$

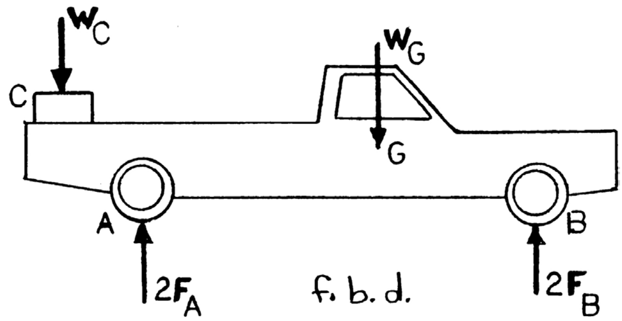


PROBLEM 4.4

Solve Problem 4.3 assuming that crate *D* is removed and that the position of crate *C* is unchanged.

P4.3 The boom on a 4300-kg truck is used to unload a pallet of shingles of mass 1600 kg. Determine the reaction at each of the two (*a*) rear wheels *B*, (*b*) front wheels *C*

SOLUTION



(*a*) From f.b.d. of truck

$$+\circlearrowleft \Sigma M_B = 0: \quad (3000\text{ lb})(3.9\text{ ft}) - (2F_A)(9.8\text{ ft}) + (250\text{ lb})(12.1\text{ ft}) = 0$$

$$\therefore \; 2F_A = \frac{14725}{9.8} = 1502.55\text{ lb}$$

or $\mathbf{F}_A = 751\text{ lb} \uparrow$ ◀

(*b*) From f.b.d. of truck

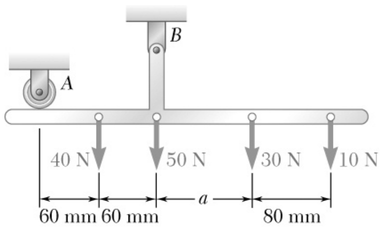
$$+\circlearrowleft \Sigma M_A = 0: \quad (2F_B)(9.8\text{ ft}) - (3000\text{ lb})(5.9\text{ ft}) + (250\text{ lb})(2.3\text{ ft}) = 0$$

$$\therefore \; 2F_B = \frac{17125}{9.8} = 1747.45\text{ lb}$$

or $\mathbf{F}_B = 874\text{ lb} \uparrow$ ◀

Check: $+\uparrow \Sigma F_y = 0: \quad [2(751 + 874) - 3000 - 250]\text{ lb} = 0?$

$$(3250 - 3250)\text{ lb} = 0 \;\; \text{ok}$$

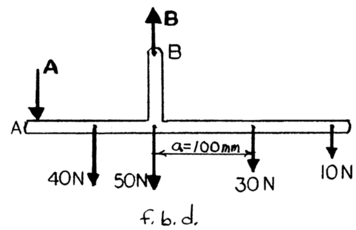


PROBLEM 4.5

A T-shaped bracket supports the four loads shown. Determine the reactions at A and B if (a) $a = 100$ mm, (b) $a = 70$ mm.

SOLUTION

(a)



From f.b.d. of bracket

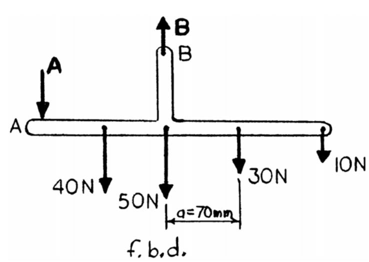
$$+\circlearrowleft\ \Sigma M_B = 0: \quad -(10\text{ N})(0.18\text{ m}) - (30\text{ N})(0.1\text{ m}) + (40\text{ N})(0.06\text{ m}) + A(0.12\text{ m}) = 0$$

$$\therefore\ A = \frac{2.400}{0.12} = 20\text{ N} \qquad \text{or } \mathbf{A} = 20.0\text{ N} \downarrow \blacktriangleleft$$

$$+\circlearrowleft\ \Sigma M_A = 0: \quad B(0.12\text{ m}) - (40\text{ N})(0.06\text{ m}) - (50\text{ N})(0.12\text{ m}) - (30\text{ N})(0.22\text{ m}) - (10\text{ N})(0.3\text{ m}) = 0$$

$$\therefore\ B = \frac{18.000}{0.12} = 150\text{ N} \qquad \text{or } \mathbf{B} = 150.0\text{ N} \uparrow \blacktriangleleft$$

(b)



From f.b.d. of bracket

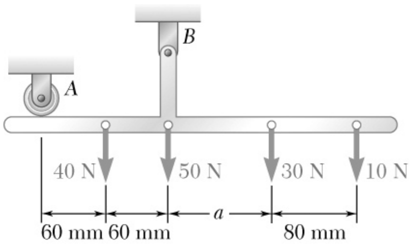
$$+\circlearrowleft\ \Sigma M_B = 0: \quad -(10\text{ N})(0.15\text{ m}) - (30\text{ N})(0.07\text{ m}) + (40\text{ N})(0.06\text{ m}) + A(0.12\text{ m}) = 0$$

$$\therefore\ A = \frac{1.200}{0.12} = 10\text{ N} \qquad \text{or } \mathbf{A} = 10.00\text{ N} \downarrow \blacktriangleleft$$

$$+\circlearrowleft\ \Sigma M_A = 0: \quad B(0.12\text{ m}) - (40\text{ N})(0.06\text{ m}) - (50\text{ N})(0.12\text{ m}) - (30\text{ N})(0.19\text{ m})$$

$$-(10\text{ N})(0.27\text{ m}) = 0$$

$$\therefore\ B = \frac{16.800}{0.12} = 140\text{ N} \qquad \text{or } \mathbf{B} = 140.0\text{ N} \uparrow \blacktriangleleft$$

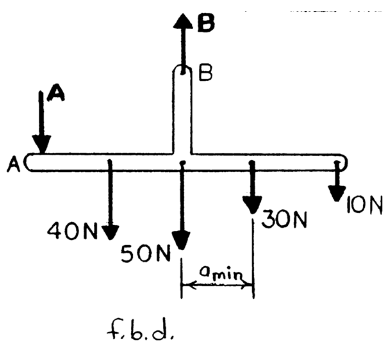


PROBLEM 4.6

For the bracket and loading of Problem 4.5, determine the smallest distance *a* if the bracket is not to move.

P4.5 A T-shaped bracket supports the four loads shown. Determine the reactions at *A* and *B* if (*a*) $a = 100$ mm, (*b*) $a = 70$ mm.

SOLUTION



The a_{min} value will be based on $\mathbf{A} = 0$

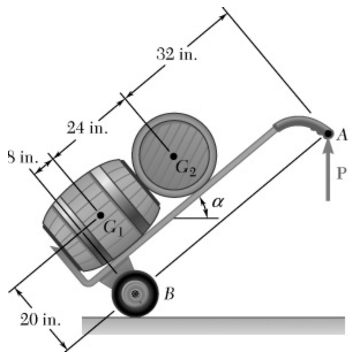
From f.b.d. of bracket

$$+\circlearrowleft \Sigma M_B = 0: \quad (40\text{ N})(60\text{ mm}) - (30\text{ N})(a) - (10\text{ N})(a + 80\text{ mm}) = 0$$

$$\therefore\ a = \frac{1600}{40} = 40\text{ mm}$$

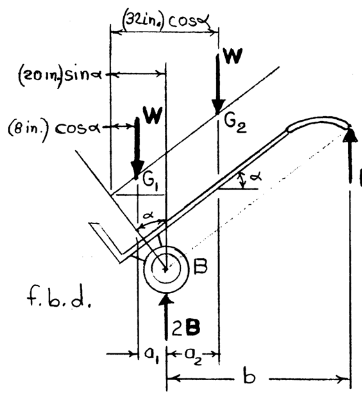
or $a_{\text{min}} = 40.0\text{ mm}$ ◀

PROBLEM 4.7



A hand truck is used to move two barrels, each weighing 80 lb. Neglecting the weight of the hand truck, determine (*a*) the vertical force **P** which should be applied to the handle to maintain equilibrium when $\alpha = 35^\circ$, (*b*) the corresponding reaction at each of the two wheels.

SOLUTION



$$a_1 = (20\text{ in.})\sin\alpha - (8\text{ in.})\cos\alpha$$

$$a_2 = (32\text{ in.})\cos\alpha - (20\text{ in.})\sin\alpha$$

$$b = (64\text{ in.})\cos\alpha$$

From f.b.d. of hand truck

$$+\circlearrowleft\ \Sigma M_B = 0:\quad P(b) - W(a_2) + W(a_1) = 0 \qquad (1)$$

$$+\uparrow\ \Sigma F_y = 0:\quad P - 2w + 2B = 0 \qquad (2)$$

For $\alpha = 35^\circ$

$$a_1 = 20\sin 35^\circ - 8\cos 35^\circ = 4.9183\text{ in.}$$

$$a_2 = 32\cos 35^\circ - 20\sin 35^\circ = 14.7413\text{ in.}$$

$$b = 64\cos 35^\circ = 52.426\text{ in.}$$

(*a*) From Equation (1)

$$P(52.426\text{ in.}) - 80\text{ lb}(14.7413\text{ in.}) + 80\text{ lb}(4.9183\text{ in.}) = 0$$

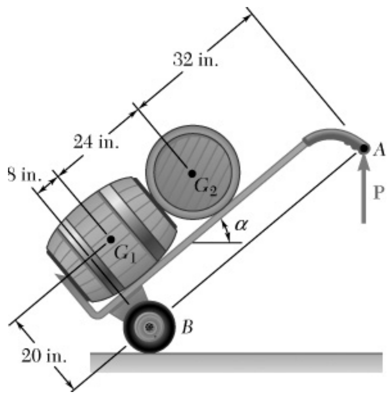
$\therefore\ P = 14.9896\text{ lb}$ or $\mathbf{P} = 14.99\text{ lb}\ \uparrow$ ◀

(*b*) From Equation (2)

$$14.9896\text{ lb} - 2(80\text{ lb}) + 2B = 0$$

$\therefore\ B = 72.505\text{ lb}$ or $\mathbf{B} = 72.5\text{ lb}\ \uparrow$ ◀

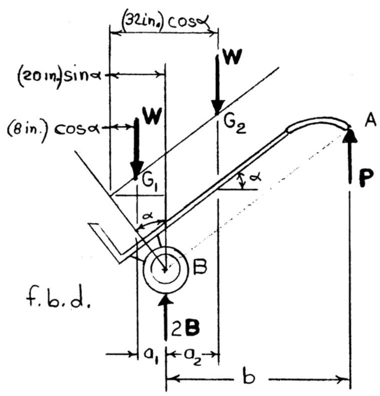
PROBLEM 4.8



Solve Problem 4.7 when $\alpha = 40^\circ$.

P4.7 A hand truck is used to move two barrels, each weighing 80 lb. Neglecting the weight of the hand truck, determine (*a*) the vertical force **P** which should be applied to the handle to maintain equilibrium when $\alpha = 35^\circ$, (*b*) the corresponding reaction at each of the two wheels.

SOLUTION



$$a_1 = (20\text{ in.})\sin\alpha - (8\text{ in.})\cos\alpha$$

$$a_2 = (32\text{ in.})\cos\alpha - (20\text{ in.})\sin\alpha$$

$$b = (64\text{ in.})\cos\alpha$$

From f.b.d. of hand truck

$$+\circlearrowleft \Sigma M_B = 0: \quad P(b) - W(a_2) + W(a_1) = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad P - 2w + 2B = 0 \qquad (2)$$

For $\qquad \alpha = 40^\circ$

$$a_1 = 20\sin 40^\circ - 8\cos 40^\circ = 6.7274\text{ in.}$$

$$a_2 = 32\cos 40^\circ - 20\sin 40^\circ = 11.6577\text{ in.}$$

$$b = 64\cos 40^\circ = 49.027\text{ in.}$$

(*a*) From Equation (1)

$$P(49.027\text{ in.}) - 80\text{ lb}(11.6577\text{ in.}) + 80\text{ lb}(6.7274\text{ in.}) = 0$$

$$\therefore\ P = 8.0450\text{ lb}$$

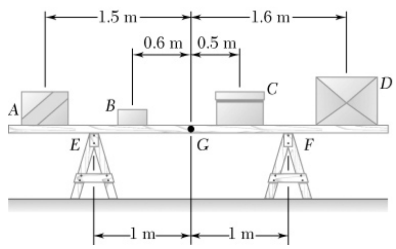
or $\mathbf{P} = 8.05\text{ lb}\uparrow$ ◀

(*b*) From Equation (2)

$$8.0450\text{ lb} - 2(80\text{ lb}) + 2B = 0$$

$$\therefore\ B = 75.9775\text{ lb}$$

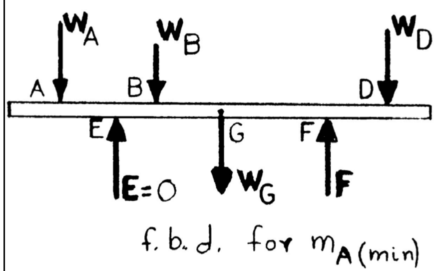
or $\mathbf{B} = 76.0\text{ lb}\uparrow$ ◀



PROBLEM 4.9

Four boxes are placed on a uniform 14-kg wooden plank which rests on two sawhorses. Knowing that the masses of boxes B and D are 4.5 kg and 45 kg, respectively, determine the range of values of the mass of box A so that the plank remains in equilibrium when box C is removed.

SOLUTION



$$W_A = m_A g \qquad W_D = m_D g = 45\,g$$

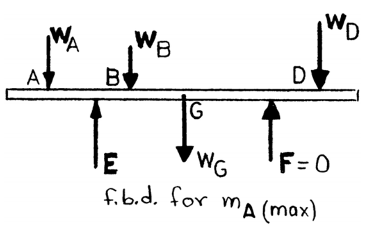
$$W_B = m_B g = 4.5g \qquad W_G = m_G g = 14g$$

For $(m_A)_{\min}$, $E = 0$

$$+\circlearrowleft \Sigma M_F = 0: \quad (m_A g)(2.5\text{ m}) + (4.5g)(1.6\text{ m})$$

$$+(14g)(1\text{ m}) - (45g)(0.6\text{ m}) = 0$$

$$\therefore\ m_A = 2.32\text{ kg}$$



For $(m_A)_{\max}$, $F = 0$:

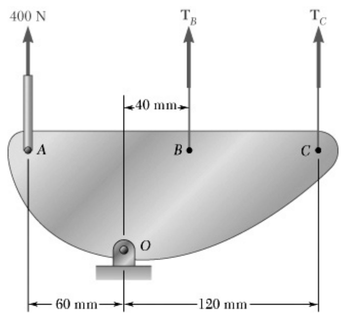
$$+\circlearrowleft \Sigma M_E = 0: \quad m_A g(0.5\text{ m}) - (4.5g)(0.4\text{ m}) - (14g)(1\text{ m})$$

$$-(45g)(2.6\text{ m}) = 0$$

$$\therefore\ m_A = 265.6\text{ kg}$$

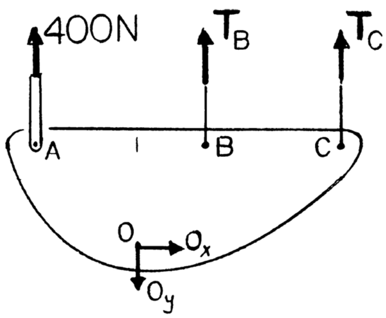
or $2.32\text{ kg} \le m_A \le 266\text{ kg}$ ◀

PROBLEM 4.10



A control rod is attached to a crank at A and cords are attached at B and C. For the given force in the rod, determine the range of values of the tension in the cord at C knowing that the cords must remain taut and that the maximum allowed tension in a cord is 180 N.

SOLUTION



f.b.d. of crank

For $\quad (T_C)_{\max}, \qquad T_B = 0$

$$+\circlearrowleft \Sigma M_O = 0: \quad (T_C)_{\max}(0.120\text{ m}) - (400\text{ N})(0.060\text{ m}) = 0$$

$$(T_C)_{\max} = 200\text{ N} > T_{\max} = 180\text{ N}$$

$$\therefore\ (T_C)_{\max} = 180.0\text{ N}$$

For $\quad (T_C)_{\min},\ T_B = T_{\max} = 180\text{ N}$

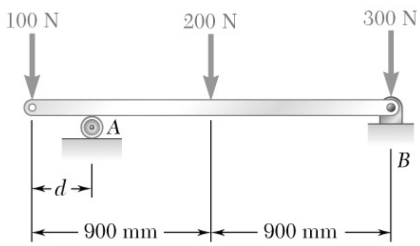
$$+\circlearrowleft \Sigma M_O = 0: \quad (T_C)_{\min}(0.120\text{ m}) + (180\text{ N})(0.040\text{ m})$$

$$-(400\text{ N})(0.060\text{ m}) = 0$$

$$\therefore\ (T_C)_{\min} = 140.0\text{ N}$$

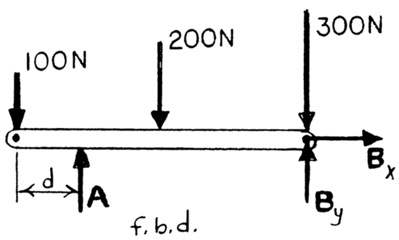
Therefore, $\quad 140.0\text{ N} \le T_C \le 180.0\text{ N}$ ◀

PROBLEM 4.11



The maximum allowable value of each of the reactions is 360 N. Neglecting the weight of the beam, determine the range of values of the distance *d* for which the beam is safe.

SOLUTION



From f.b.d. of beam

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x = 0 \qquad \text{so that} \qquad B = B_y$$

$$+\uparrow \Sigma F_y = 0: \quad A + B - (100 + 200 + 300)\text{ N} = 0$$

or $$A + B = 600 \text{ N}$$

Therefore, if either **A** or **B** has a magnitude of the maximum of 360 N, the other support reaction will be $< 360 \text{ N}(600 \text{ N} - 360 \text{ N} = 240 \text{ N})$.

$$+\circlearrowleft \Sigma M_A = 0: \quad (100 \text{ N})(d) - (200 \text{ N})(0.9 - d) - (300 \text{ N})(1.8 - d)$$

$$+ B(1.8 - d) = 0$$

or $$d = \frac{720 - 1.8B}{600 - B}$$

Since $B \le 360$ N,

$$d = \frac{720 - 1.8(360)}{600 - 360} = 0.300 \text{ m} \qquad \text{or} \qquad d \ge 300 \text{ mm}$$

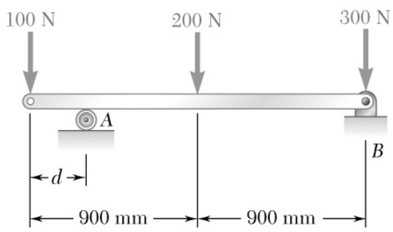
$$+\circlearrowleft \Sigma M_B = 0: \quad (100 \text{ N})(1.8) - A(1.8 - d) + (200 \text{ N})(0.9) = 0$$

or $$d = \frac{1.8A - 360}{A}$$

Since $A \le 360$ N,

$$d = \frac{1.8(360) - 360}{360} = 0.800 \text{ m} \qquad \text{or} \qquad d \le 800 \text{ mm}$$

or $300 \text{ mm} \le d \le 800 \text{ mm}$ ◀

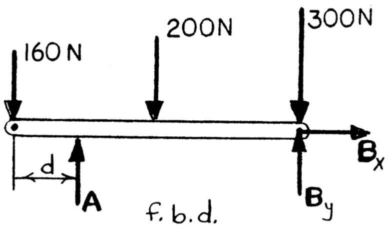


PROBLEM 4.12

Solve Problem 4.11 assuming that the 100-N load is replaced by a 160-N load.

P4.11 The maximum allowable value of each of the reactions is 360 N. Neglecting the weight of the beam, determine the range of values of the distance *d* for which the beam is safe.

SOLUTION



From f.b.d of beam

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x = 0 \qquad \text{so that} \qquad B = B_y$$

$$+\uparrow \Sigma F_y = 0: \quad A + B - (160 + 200 + 300)\text{ N} = 0$$

or $$A + B = 660\text{ N}$$

Therefore, if either **A** or **B** has a magnitude of the maximum of 360 N, the other support reaction will be $< 360\text{ N}(660 - 360 = 300\text{ N})$.

$$+\circlearrowleft \Sigma M_A = 0: \quad 160\text{ N}(d) - 200\text{ N}(0.9 - d) - 300\text{ N}(1.8 - d)$$

$$+ B(1.8 - d) = 0$$

or $$d = \frac{720 - 1.8B}{660 - B}$$

Since $B \le 360$ N,

$$d = \frac{720 - 1.8(360)}{660 - 360} = 0.240\text{ m} \qquad \text{or} \qquad d \ge 240\text{ mm}$$

$$+\circlearrowleft \Sigma M_B = 0: \quad 160\text{ N}(1.8) - A(1.8 - d) + 200\text{ N}(0.9) = 0$$

or $$d = \frac{1.8A - 468}{A}$$

Since $A \le 360$ N,

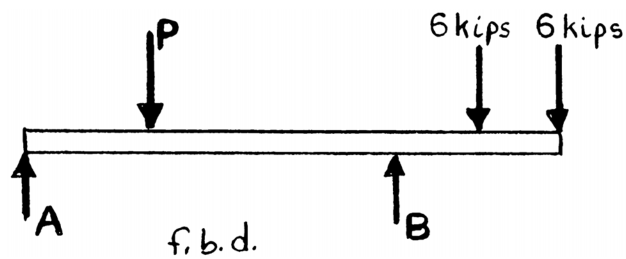
$$d = \frac{1.8(360) - 468}{360} = 0.500\text{ m} \qquad \text{or} \qquad d \ge 500\text{ mm}$$

or $240\text{ mm} \le d \le 500\text{ mm}$ ◀

PROBLEM 4.13

For the beam of Sample Problem 4.2, determine the range of values of P for which the beam will be safe knowing that the maximum allowable value of each of the reactions is 45 kips and that the reaction at A must be directed upward.

SOLUTION



For the force of **P** to be a minimum, **A** = 0.

With $A = 0$,

$$+\circlearrowleft \Sigma M_B = 0: \quad P_{\text{min}}(6\text{ ft}) - (6\text{ kips})(2\text{ ft}) - (6\text{ kips})(4\text{ ft}) = 0$$

$$\therefore\ P_{\text{min}} = 6.00\text{ kips}$$

For the force **P** to be a maximum, $\mathbf{A} = \mathbf{A}_{\text{max}} = 45\text{ kips}\uparrow$

With $A = 45$ kips,

$$+\circlearrowleft \Sigma M_B = 0: \quad -(45\text{ kips})(9\text{ ft}) + P_{\text{max}}(6\text{ ft}) - (6\text{ kips})(2\text{ ft}) - (6\text{ kips})(4\text{ ft}) = 0$$

$$\therefore\ P_{\text{max}} = 73.5\text{ kips}$$

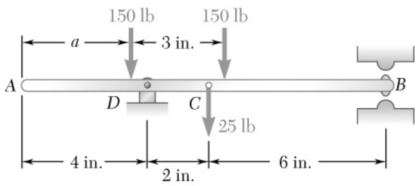
A check must be made to verify the assumption that the maximum value of **P** is based on the reaction force at A. This is done by making sure the corresponding value of B is < 45 kips.

$$+\uparrow \Sigma F_y = 0: \quad 45\text{ kips} - 73.5\text{ kips} + B - 6\text{ kips} - 6\text{ kips} = 0$$

$$\therefore\ B = 40.5\text{ kips} < 45\text{ kips} \qquad \therefore\ \text{ok} \qquad \text{or}\ \ P_{\text{max}} = 73.5\text{ kips}$$

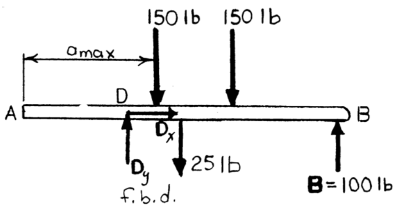
$$\text{and}\ \ 6.00\text{ kips} \le P \le 73.5\text{ kips} \blacktriangleleft$$

PROBLEM 4.14



For the beam and loading shown, determine the range of values of the distance a for which the reaction at B does not exceed 50 lb downward or 100 lb upward.

SOLUTION



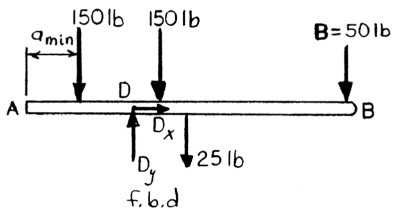
To determine a_{max} the two 150-lb forces need to be as close to B without having the vertical upward force at B exceed 100 lb.

From f.b.d. of beam with $\mathbf{B} = 100 \text{ lb} \uparrow$

$$+\circlearrowleft \Sigma M_D = 0: \quad -(150 \text{ lb})(a_{max} - 4 \text{ in.}) - (150 \text{ lb})(a_{max} - 1 \text{ in.})$$

$$-(25 \text{ lb})(2 \text{ in.}) + (100 \text{ lb})(8 \text{ in.}) = 0$$

or $\qquad a_{max} = 5.00 \text{ in.}$



To determine a_{min} the two 150-lb forces need to be as close to A without having the vertical downward force at B exceed 50 lb.

From f.b.d. of beam with $\mathbf{B} = 50 \text{ lb} \downarrow$

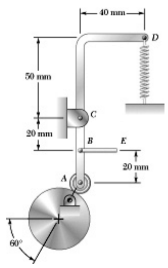
$$+\circlearrowleft \Sigma M_D = 0: \quad (150 \text{ lb})(4 \text{ in.} - a_{min}) - (150 \text{ lb})(a_{min} - 1 \text{ in.})$$

$$-(25 \text{ lb})(2 \text{ in.}) - (50 \text{ lb})(8 \text{ in.}) = 0$$

or $\qquad a_{min} = 1.00 \text{ in.}$

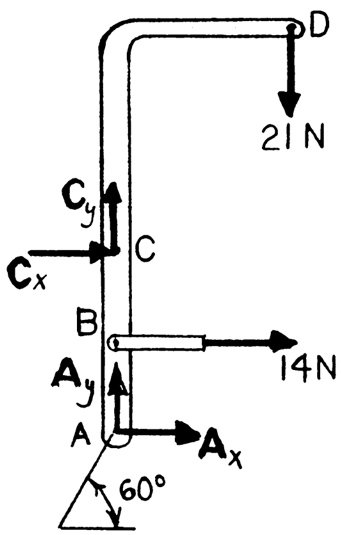
Therefore, $\qquad$ or $1.00 \text{ in.} \le a \le 5.00 \text{ in.}$ ◀

PROBLEM 4.15



A follower *ABCD* is held against a circular cam by a stretched spring, which exerts a force of 21 N for the position shown. Knowing that the tension in rod *BE* is 14 N, determine (*a*) the force exerted on the roller at *A*, (*b*) the reaction at bearing *C*.

SOLUTION



f.b.d.

Note: From f.b.d. of *ABCD*

$$A_x = A\cos 60° = \frac{A}{2}$$

$$A_y = A\sin 60° = A\frac{\sqrt{3}}{2}$$

(*a*) From f.b.d. of *ABCD*

$$+\circlearrowleft \Sigma M_C = 0: \quad \left(\frac{A}{2}\right)(40 \text{ mm}) - 21 \text{ N}(40 \text{ mm})$$

$$+ 14 \text{ N}(20 \text{ mm}) = 0$$

$$\therefore \; A = 28 \text{ N}$$

or $\mathbf{A} = 28.0 \text{ N} \measuredangle\, 60°$ ◀

(*b*) From f.b.d. of *ABCD*

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + 14 \text{ N} + (28 \text{ N})\cos 60° = 0$$

$$\therefore \; C_x = -28 \text{ N} \qquad \text{or} \qquad \mathbf{C}_x = 28.0 \text{ N} \longleftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - 21 \text{ N} + (28 \text{ N})\sin 60° = 0$$

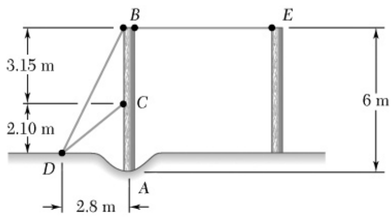
$$\therefore \; C_y = -3.2487 \text{ N} \qquad \text{or} \qquad \mathbf{C}_y = 3.25 \text{ N} \downarrow$$

Then $$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(28)^2 + (3.2487)^2} = 28.188 \text{ N}$$

and $$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{-3.2487}{-28}\right) = 6.6182°$$

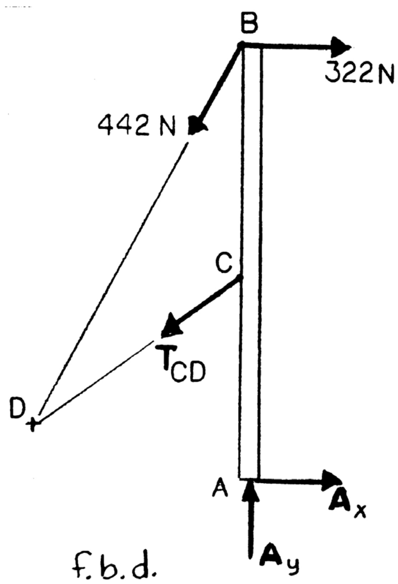
or $\mathbf{C} = 28.2 \text{ N} \measuredangle\, 6.62°$ ◀

PROBLEM 4.16



A 6-m-long pole AB is placed in a hole and is guyed by three cables. Knowing that the tensions in cables BD and BE are 442 N and 322 N, respectively, determine (*a*) the tension in cable CD, (*b*) the reaction at A.

SOLUTION



Note:

$$\overline{DB} = \sqrt{(2.8)^2 + (5.25)^2} = 5.95 \text{ m}$$

$$\overline{DC} = \sqrt{(2.8)^2 + (2.10)^2} = 3.50 \text{ m}$$

(*a*) From f.b.d. of pole

$$+\circlearrowleft \Sigma M_A = 0: \quad -(322 \text{ N})(6 \text{ m}) + \left[\left(\frac{2.8 \text{ m}}{5.95 \text{ m}}\right)(442 \text{ N})\right](6 \text{ m})$$

$$+\left[\left(\frac{2.8 \text{ m}}{3.50 \text{ m}}\right)T_{CD}\right](2.85 \text{ m}) = 0$$

$$\therefore\ T_{CD} = 300 \text{ N}$$

or $T_{CD} = 300$ N ◀

(*b*) From f.b.d. of pole

$$\xrightarrow{+} \Sigma F_x = 0: \quad 322 \text{ N} - \left(\frac{2.8 \text{ m}}{5.95 \text{ m}}\right)(442 \text{ N})$$

$$-\left(\frac{2.8 \text{ m}}{3.50 \text{ m}}\right)(300 \text{ N}) + A_x = 0$$

$$\therefore\ A_x = 126 \text{ N} \qquad \text{or} \qquad \mathbf{A}_x = 126 \text{ N} \longrightarrow$$

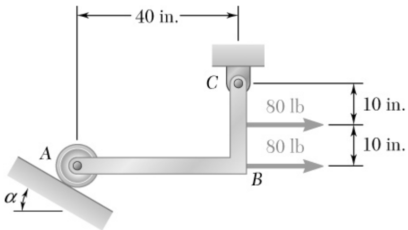
$$+\uparrow \Sigma F_y = 0: \quad A_y - \left(\frac{5.25 \text{ m}}{5.95 \text{ m}}\right)(442 \text{ N}) - \left(\frac{2.10 \text{ m}}{3.50 \text{ m}}\right)(300 \text{ N}) = 0$$

$$\therefore\ A_y = 570 \text{ N} \qquad \text{or} \qquad \mathbf{A}_y = 570 \text{ N} \uparrow$$

Then $$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(126)^2 + (570)^2} = 583.76 \text{ N}$$

and $$\theta = \tan^{-1}\left(\frac{570 \text{ N}}{126 \text{ N}}\right) = 77.535°$$

or $\mathbf{A} = 584$ N $\measuredangle$ 77.5° ◀

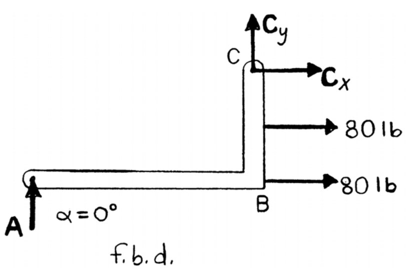


PROBLEM 4.17

Determine the reactions at A and C when (*a*) $\alpha = 0$, (*b*) $\alpha = 30^\circ$.

SOLUTION

(*a*)



(*a*) $\alpha = 0^\circ$

From f.b.d. of member *ABC*

$$+\circlearrowleft \Sigma M_C = 0: \quad (80\text{ lb})(10\text{ in.}) + (80\text{ lb})(20\text{ in.}) - A(40\text{ in.}) = 0$$

$$\therefore\ A = 60\text{ lb}$$

or $\mathbf{A} = 60.0\text{ lb}\uparrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad C_y + 60\text{ lb} = 0$$

$$\therefore\ C_y = -60\text{ lb} \qquad \text{or} \qquad \mathbf{C}_y = 60\text{ lb}\downarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad 80\text{ lb} + 80\text{ lb} + C_x = 0$$

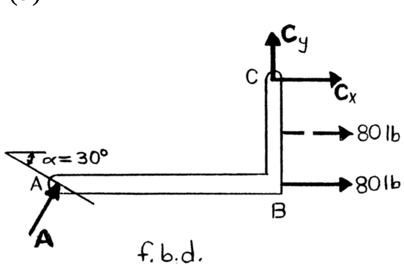
$$\therefore\ C_x = -160\text{ lb} \qquad \text{or} \qquad \mathbf{C}_x = 160\text{ lb}\longleftarrow$$

Then $$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(160)^2 + (60)^2} = 170.880\text{ lb}$$

and $$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{-60}{-160}\right) = 20.556^\circ$$

or $\mathbf{C} = 170.9\text{ lb} \measuredangle 20.6^\circ$ ◀

(*b*)



(*b*) $\alpha = 30^\circ$

From f.b.d. of member *ABC*

$$+\circlearrowleft \Sigma M_C = 0: \quad (80\text{ lb})(10\text{ in.}) + (80\text{ lb})(20\text{ in.}) - (A\cos 30^\circ)(40\text{ in.})$$

$$+ (A\sin 30^\circ)(20\text{ in.}) = 0$$

$$\therefore\ A = 97.399\text{ lb}$$

or $\mathbf{A} = 97.4\text{ lb} \measuredangle 60^\circ$ ◀

PROBLEM 4.17 CONTINUED

$$\xrightarrow{+} \Sigma F_x = 0: \quad 80 \text{ lb} + 80 \text{ lb} + (97.399 \text{ lb})\sin 30° + C_x = 0$$

$$\therefore\ C_x = -208.70 \text{ lb} \qquad \text{or} \qquad \mathbf{C}_x = 209 \text{ lb} \longleftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad C_y + (97.399 \text{ lb})\cos 30° = 0$$

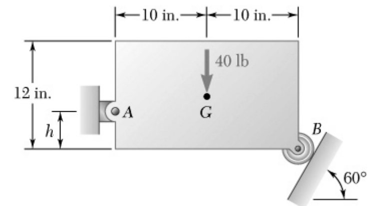
$$\therefore\ C_y = -84.350 \text{ lb} \qquad \text{or} \qquad \mathbf{C}_y = 84.4 \text{ lb} \downarrow$$

Then $$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(208.70)^2 + (84.350)^2} = 225.10 \text{ lb}$$

and $$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{-84.350}{-208.70}\right) = 22.007°$$

or $\mathbf{C} = 225 \text{ lb} \measuredangle\ 22.0°$ ◀

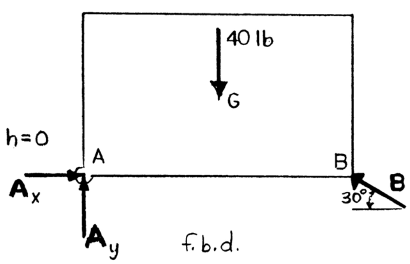
PROBLEM 4.18



Determine the reactions at A and B when (a) $h = 0$, (b) $h = 8$ in.

SOLUTION

(a)



(a) $h = 0$

From f.b.d. of plate

$$+\circlearrowleft \Sigma M_A = 0: \quad (B\sin 30°)(20 \text{ in.}) - (40 \text{ lb})(10 \text{ in.}) = 0$$

$$\therefore B = 40 \text{ lb}$$

or $\mathbf{B} = 40.0 \text{ lb} \searrow 30°$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - (40 \text{ lb})\cos 30° = 0$$

$\therefore A_x = 34.641$ lb or $\mathbf{A}_x = 34.6 \text{ lb} \longrightarrow$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 40 \text{ lb} + (40 \text{ lb})\sin 30° = 0$$

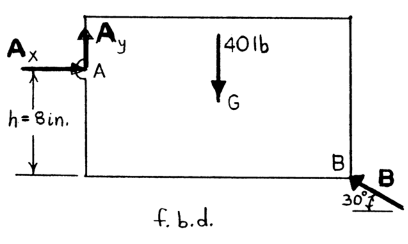
$\therefore A_y = 20$ lb or $\mathbf{A}_y = 20.0 \text{ lb} \uparrow$

Then $$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(34.641)^2 + (20)^2} = 39.999 \text{ lb}$$

and $$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{20}{34.641}\right) = 30.001°$$

or $\mathbf{A} = 40.0 \text{ lb} \measuredangle 30°$ ◀

(b)



(b) $h = 8$ in.

From f.b.d. of plate

$$+\circlearrowleft \Sigma M_A = 0: \quad (B\sin 30°)(20 \text{ in.}) - (B\cos 30°)(8 \text{ in.})$$

$$-(40 \text{ lb})(10 \text{ in.}) = 0$$

$$\therefore B = 130.217 \text{ lb}$$

or $\mathbf{B} = 130.2 \text{ lb} \searrow 30.0°$ ◀

PROBLEM 4.18 CONTINUED

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - (130.217 \text{ lb})\cos 30° = 0$$

$$\therefore\ A_x = 112.771 \text{ lb} \qquad \text{or} \qquad \mathbf{A}_x = 112.8 \text{ lb} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 40 \text{ lb} + (130.217 \text{ lb})\sin 30° = 0$$

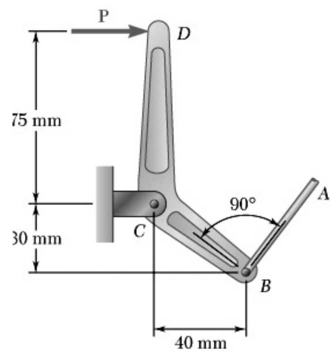
$$\therefore\ A_y = -25.108 \text{ lb} \qquad \text{or} \qquad \mathbf{A}_y = 25.1 \text{ lb} \downarrow$$

Then $$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(112.771)^2 + (25.108)^2} = 115.532 \text{ lb}$$

and $$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{-25.108}{112.771}\right) = -12.5519°$$

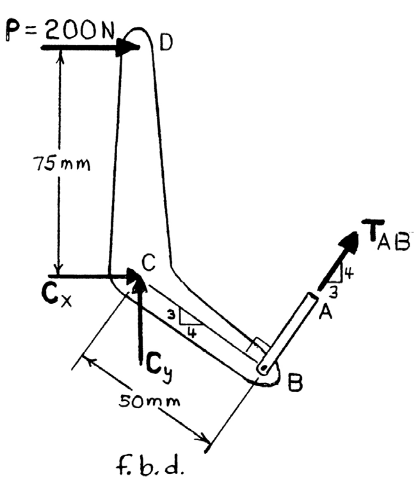
$$\text{or } \mathbf{A} = 115.5 \text{ lb} \searrow 12.55° \blacktriangleleft$$

PROBLEM 4.19



The lever *BCD* is hinged at *C* and is attached to a control rod at *B*. If $P = 200$ N, determine (*a*) the tension in rod *AB*, (*b*) the reaction at *C*.

SOLUTION



(*a*) From f.b.d. of lever *BCD*

$$+\circlearrowleft \Sigma M_C = 0: \quad T_{AB}(50 \text{ mm}) - 200 \text{ N}(75 \text{ mm}) = 0$$

$$\therefore T_{AB} = 300 \text{ N} \blacktriangleleft$$

(*b*) From f.b.d. of lever *BCD*

$$\xrightarrow{+} \Sigma F_x = 0: \quad 200 \text{ N} + C_x + 0.6(300 \text{ N}) = 0$$

$$\therefore C_x = -380 \text{ N} \qquad \text{or} \qquad \mathbf{C}_x = 380 \text{ N} \longleftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad C_y + 0.8(300 \text{ N}) = 0$$

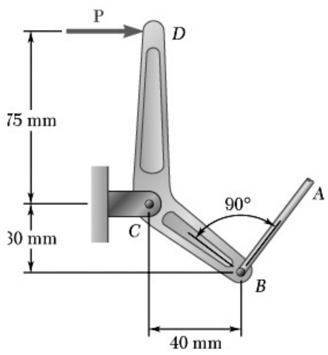
$$\therefore C_y = -240 \text{ N} \qquad \text{or} \qquad \mathbf{C}_y = 240 \text{ N} \downarrow$$

Then $\quad C = \sqrt{C_x^2 + C_y^2} = \sqrt{(380)^2 + (240)^2} = 449.44 \text{ N}$

and $\quad \theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{-240}{-380}\right) = 32.276°$

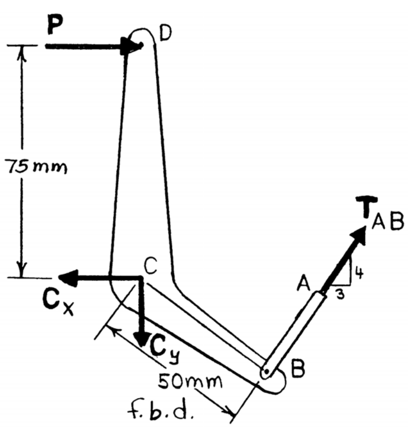
$$\text{or } \mathbf{C} = 449 \text{ N} \nearrow 32.3° \blacktriangleleft$$

PROBLEM 4.20



The lever *BCD* is hinged at *C* and is attached to a control rod at *B*. Determine the maximum force **P** which can be safely applied at *D* if the maximum allowable value of the reaction at *C* is 500 N.

SOLUTION



From f.b.d. of lever *BCD*

$$+\circlearrowleft \Sigma M_C = 0: \quad T_{AB}(50 \text{ mm}) - P(75 \text{ mm}) = 0$$

$$\therefore T_{AB} = 1.5P \qquad (1)$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad 0.6T_{AB} + P - C_x = 0$$

$$\therefore C_x = P + 0.6T_{AB} \qquad (2)$$

From Equation (1) $\quad C_x = P + 0.6(1.5P) = 1.9P$

$$+\uparrow \Sigma F_y = 0: \quad 0.8T_{AB} - C_y = 0$$

$$\therefore C_y = 0.8T_{AB} \qquad (3)$$

From Equation (1) $\quad C_y = 0.8(1.5P) = 1.2P$

From Equations (2) and (3)

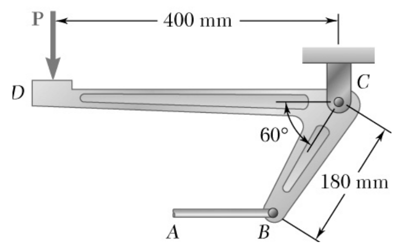
$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(1.9P)^2 + (1.2P)^2} = 2.2472P$$

Since $C_{max} = 500$ N,

$$\therefore 500 \text{ N} = 2.2472P_{max}$$

or $\quad P_{max} = 222.49$ lb

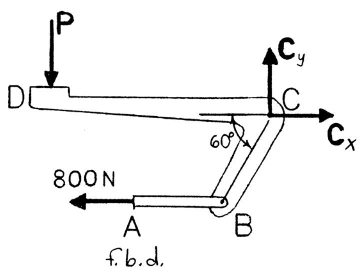
or $\mathbf{P} = 222$ lb $\longrightarrow$ ◀



PROBLEM 4.21

The required tension in cable *AB* is 800 N. Determine (*a*) the vertical force **P** which must be applied to the pedal, (*b*) the corresponding reaction at *C*.

SOLUTION



(*a*) From f.b.d. of pedal

$$+\circlearrowleft \Sigma M_C = 0: \quad P(0.4\text{ m}) - (800\text{ N})\left[(0.18\text{ m})\sin 60°\right] = 0$$

$$\therefore \quad P = 311.77\text{ N}$$

or $\mathbf{P} = 312\text{ N} \downarrow$ ◀

(*b*) From f.b.d. of pedal

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - 800\text{ N} = 0$$

$$\therefore \quad C_x = 800\text{ N}$$

or $\mathbf{C}_x = 800\text{ N} \longrightarrow$

$$+\uparrow \Sigma F_y = 0: \quad C_y - 311.77\text{ N} = 0$$

$$\therefore \quad C_y = 311.77\text{ N}$$

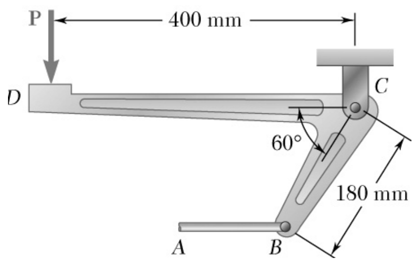
or $\mathbf{C}_y = 311.77\text{ N} \uparrow$

Then $C = \sqrt{C_x^2 + C_y^2} = \sqrt{(800)^2 + (311.77)^2} = 858.60\text{ N}$

and $\theta = \tan^{-1}\left(\dfrac{C_y}{C_x}\right) = \tan^{-1}\left(\dfrac{311.77}{800}\right) = 21.291°$

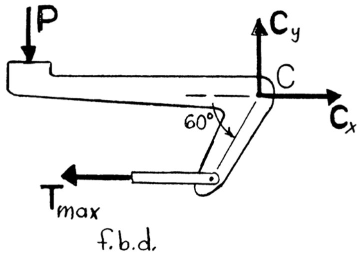
or $\mathbf{C} = 859\text{ N} \measuredangle 21.3°$ ◀

PROBLEM 4.22



Determine the maximum tension which can be developed in cable *AB* if the maximum allowable value of the reaction at *C* is 1000 N.

SOLUTION



Have $$C_{max} = 1000 \text{ N}$$

Now $$C^2 = C_x^2 + C_y^2$$

$$\therefore \quad C_y = \sqrt{(1000)^2 - C_x^2} \tag{1}$$

From f.b.d. of pedal

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - T_{max} = 0$$

$$\therefore \quad C_x = T_{max} \tag{2}$$

$$+\circlearrowleft \Sigma M_D = 0: \quad C_y(0.4 \text{ m}) - T_{max}\left[(0.18 \text{ m})\sin 60°\right] = 0$$

$$\therefore \quad C_y = 0.38971 T_{max} \tag{3}$$

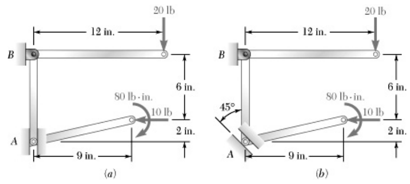
Equating the expressions for C_y in Equations (1) and (3), with $C_x = T_{max}$ from Equation (2)

$$\sqrt{(1000)^2 - T_{max}^2} = 0.389711 T_{max}$$

$$\therefore \quad T_{max}^2 = 868{,}150$$

and $$T_{max} = 931.75 \text{ N}$$

or $T_{max} = 932 \text{ N}$ ◀

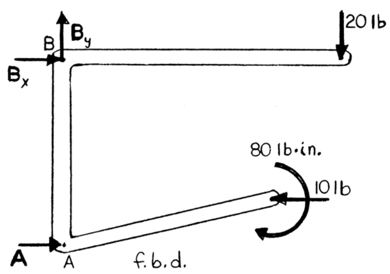


PROBLEM 4.23

A steel rod is bent to form a mounting bracket. For each of the mounting brackets and loadings shown, determine the reactions at *A* and *B*.

SOLUTION

(*a*)



(*a*) From f.b.d. of mounting bracket

$$+\circlearrowleft \Sigma M_E = 0: \quad A(8\text{ in.}) - 80\text{ lb}\cdot\text{in.} - (10\text{ lb})(6\text{ in.})$$

$$-(20\text{ lb})(12\text{ in.}) = 0$$

$$\therefore \quad A = 47.5\text{ lb}$$

$$\text{or } \mathbf{A} = 47.5\text{ lb} \longrightarrow \blacktriangleleft$$

$$\overset{+}{\longrightarrow} \Sigma F_x = 0: \quad B_x - 10\text{ lb} + 47.5\text{ lb} = 0$$

$$\therefore \quad B_x = -37.5\text{ lb}$$

or $$\mathbf{B}_x = 37.5\text{ lb} \longleftarrow$$

$$+\big\uparrow \Sigma F_y = 0: \quad B_y - 20\text{ lb} = 0$$

$$\therefore \quad B_y = 20\text{ lb}$$

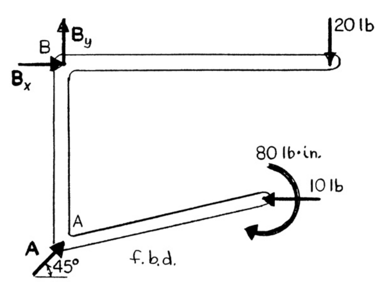
or $$\mathbf{B}_y = 20.0\text{ lb} \uparrow$$

Then $$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(37.5)^2 + (20.0)^2} = 42.5\text{ lb}$$

and $$\theta = \tan^{-1}\left(\frac{B_y}{B_x}\right) = \tan^{-1}\left(\frac{20}{-37.5}\right) = -28.072°$$

$$\text{or } \mathbf{B} = 42.5\text{ lb} \;\searrow\; 28.1° \blacktriangleleft$$

(*b*)



(*b*) From f.b.d. of mounting bracket

$$+\circlearrowleft \Sigma M_B = 0: \quad (A\cos 45°)(8\text{ in.}) - 80\text{ lb}\cdot\text{in.}$$

$$-(10\text{ lb})(6\text{ in.}) - (20\text{ lb})(12\text{ in.}) = 0$$

$$\therefore \quad A = 67.175\text{ lb}$$

$$\text{or } \mathbf{A} = 67.2\text{ lb} \;\measuredangle\; 45° \blacktriangleleft$$

$$\overset{+}{\longrightarrow} \Sigma F_x = 0: \quad B_x - 10\text{ lb} + 67.175\cos 45° = 0$$

$$\therefore \quad B_x = -37.500\text{ lb}$$

or $$\mathbf{B}_x = 37.5\text{ lb} \longleftarrow$$

PROBLEM 4.23 CONTINUED

$$+\uparrow \Sigma F_y = 0: \quad B_y - 20\text{ lb} + 67.175\sin 45° = 0$$

$$\therefore \quad B_y = -27.500\text{ lb}$$

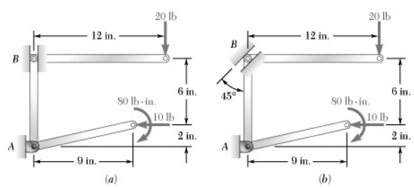
or $$\mathbf{B}_y = 27.5\text{ lb} \downarrow$$

Then $$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(37.5)^2 + (27.5)^2} = 46.503\text{ lb}$$

and $$\theta = \tan^{-1}\left(\frac{B_y}{B_x}\right) = \tan^{-1}\left(\frac{-27.5}{-37.5}\right) = 36.254°$$

or $\mathbf{B} = 46.5\text{ lb} \measuredangle 36.3° \blacktriangleleft$

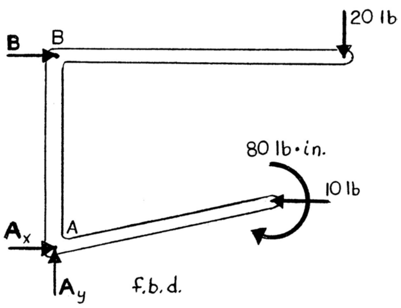
PROBLEM 4.24



A steel rod is bent to form a mounting bracket. For each of the mounting brackets and loadings shown, determine the reactions at A and B.

SOLUTION

(*a*)



(*a*) From f.b.d. of mounting bracket

$$+\circlearrowleft \Sigma M_A = 0: \quad -B(8 \text{ in.}) - (20 \text{ lb})(12 \text{ in.})$$

$$+(10 \text{ lb})(2 \text{ in.}) - 80 \text{ lb}\cdot\text{in.} = 0$$

$$\therefore \quad B = -37.5 \text{ lb}$$

or $\mathbf{B} = 37.5 \text{ lb} \longleftarrow$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad -37.5 \text{ lb} - 10 \text{ lb} + A_x = 0$$

$$\therefore \quad A_x = 47.5 \text{ lb}$$

or $\mathbf{A}_x = 47.5 \text{ lb} \longrightarrow$

$$+\uparrow \Sigma F_y = 0: \quad -20 \text{ lb} + A_y = 0$$

$$\therefore \quad A_y = 20 \text{ lb}$$

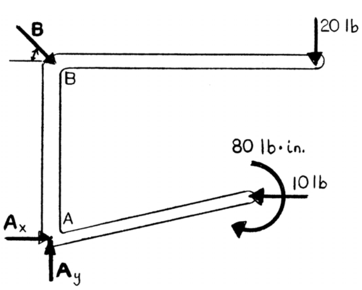
or $\mathbf{A}_y = 20.0 \text{ lb} \uparrow$

Then $A = \sqrt{A_x^2 + A_y^2} = \sqrt{(47.5)^2 + (20)^2} = 51.539 \text{ lb}$

and $\theta = \tan^{-1}\left(\dfrac{A_y}{A_x}\right) = \tan^{-1}\left(\dfrac{20}{47.5}\right) = 22.834°$

or $\mathbf{A} = 51.5 \text{ lb} \measuredangle\ 22.8°$ ◀

(*b*)



(*b*) From f.b.d. of mounting bracket

$$+\circlearrowleft \Sigma M_A = 0: \quad -(B\cos 45°)(8 \text{ in.}) - (20 \text{ lb})(2 \text{ in.})$$

$$-80 \text{ lb}\cdot\text{in.} + (10 \text{ lb})(2 \text{ in.}) = 0$$

$$\therefore \quad B = -53.033 \text{ lb}$$

or $\mathbf{B} = 53.0 \text{ lb} \searrow 45°$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + (-53.033 \text{ lb})\cos 45° - 10 = 0$$

$$\therefore \quad A_x = 47.500 \text{ lb}$$

or $\mathbf{A}_x = 47.5 \text{ lb} \longrightarrow$

PROBLEM 4.24 CONTINUED

$$+\uparrow \Sigma F_y = 0: \quad A_y - (53.033 \text{ lb})\sin 45° - 20 = 0$$

$$\therefore \quad A_y = -17.500 \text{ lb}$$

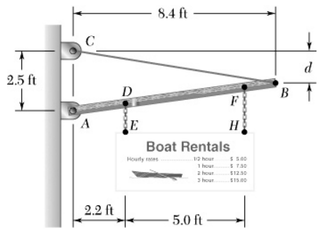
or $$\mathbf{A}_y = 17.50 \text{ lb} \downarrow$$

Then $$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(47.5)^2 + (17.5)^2} = 50.621 \text{ lb}$$

and $$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{-17.5}{47.5}\right) = -20.225°$$

or $\mathbf{A} = 50.6 \text{ lb} \searrow 20.2°$ ◀

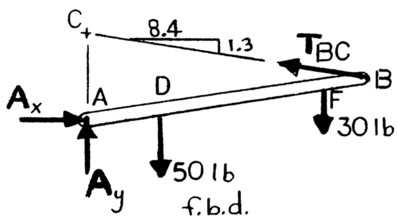
PROBLEM 4.25



A sign is hung by two chains from mast *AB*. The mast is hinged at *A* and is supported by cable *BC*. Knowing that the tensions in chains *DE* and *FH* are 50 lb and 30 lb, respectively, and that $d = 1.3$ ft, determine (*a*) the tension in cable *BC*, (*b*) the reaction at *A*.

SOLUTION

First note $\overline{BC} = \sqrt{(8.4)^2 + (1.3)^2} = 8.5$ ft



(*a*) From f.b.d. of mast *AB*

$$+\circlearrowleft \Sigma M_A = 0: \quad \left[\left(\frac{8.4}{8.5}\right)T_{BC}\right](2.5\text{ ft}) - (30\text{ lb})(7.2\text{ ft})$$

$$-50\text{ lb}(2.2\text{ ft}) = 0$$

$$\therefore \quad T_{BC} = 131.952\text{ lb}$$

or $T_{BC} = 132.0$ lb ◀

(*b*) From f.b.d. of mast *AB*

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - \left(\frac{8.4}{8.5}\right)(131.952\text{ lb}) = 0$$

$$\therefore \quad A_x = 130.400\text{ lb}$$

or $\mathbf{A}_x = 130.4\text{ lb} \longrightarrow$

$$+\uparrow \Sigma F_y = 0: \quad A_y + \left(\frac{1.3}{8.5}\right)(131.952\text{ lb}) - 30\text{ lb} - 50\text{ lb} = 0$$

$$\therefore \quad A_y = 59.819\text{ lb}$$

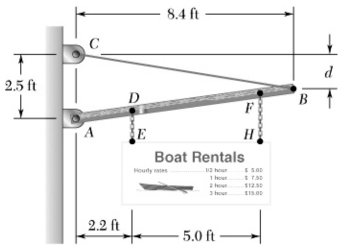
or $\mathbf{A}_y = 59.819\text{ lb} \uparrow$

Then $A = \sqrt{A_x^2 + A_y^2} = \sqrt{(130.4)^2 + (59.819)^2} = 143.466$ lb

and $\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{59.819}{130.4}\right) = 24.643°$

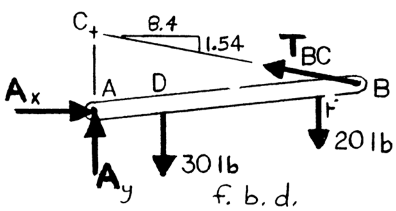
or $\mathbf{A} = 143.5$ lb $\measuredangle$ 24.6° ◀

PROBLEM 4.26



A sign is hung by two chains from mast *AB*. The mast is hinged at *A* and is supported by cable *BC*. Knowing that the tensions in chains *DE* and *FH* are 30 lb and 20 lb, respectively, and that $d = 1.54$ ft, determine (*a*) the tension in cable *BC*, (*b*) the reaction at *A*.

SOLUTION



First note $\qquad \overline{BC} = \sqrt{(8.4)^2 + (1.54)^2} = 8.54 \text{ ft}$

(*a*) From f.b.d. of mast *AB*

$$+\circlearrowleft\ \Sigma M_A = 0: \quad \left[\left(\frac{8.4}{8.54}\right)T_{BC}\right](2.5 \text{ ft}) - 20 \text{ lb}(7.2 \text{ ft})$$

$$-\ 30 \text{ lb}(2.2 \text{ ft}) = 0$$

$$\therefore \quad T_{BC} = 85.401 \text{ lb}$$

or $T_{BC} = 85.4$ lb ◀

(*b*) From f.b.d. of mast *AB*

$$\xrightarrow{+}\ \Sigma F_x = 0: \quad A_x - \left(\frac{8.4}{8.54}\right)(85.401 \text{ lb}) = 0$$

$$\therefore \quad A_x = 84.001 \text{ lb}$$

or $\qquad \mathbf{A}_x = 84.001 \text{ lb} \longrightarrow$

$$+\uparrow\ \Sigma F_y = 0: \quad A_y + \left(\frac{1.54}{8.54}\right)(85.401 \text{ lb}) - 20 \text{ lb} - 30 \text{ lb} = 0$$

$$\therefore \quad A_y = 34.600 \text{ lb}$$

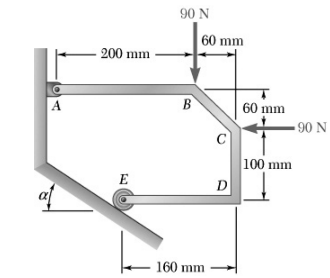
or $\qquad \mathbf{A}_y = 34.600 \text{ lb} \uparrow$

Then $\quad A = \sqrt{A_x^2 + A_y^2} = \sqrt{(84.001)^2 + (34.600)^2} = 90.848 \text{ lb}$

and $\qquad \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{34.6}{84.001}\right) = 22.387°$

or $\mathbf{A} = 90.8 \text{ lb} \measuredangle 22.4°$ ◀

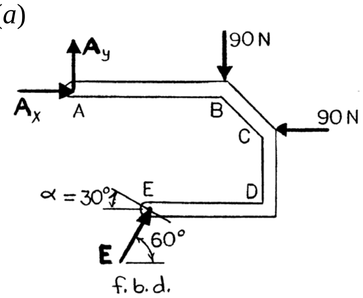
PROBLEM 4.27



For the frame and loading shown, determine the reactions at A and E when (a) $\alpha = 30^{\text{o}}$, (b) $\alpha = 45^{\text{o}}$.

SOLUTION

(a)



(a) Given $\alpha = 30°$

From f.b.d. of frame

$$+\circlearrowleft \Sigma M_A = 0: \quad -(90 \text{ N})(0.2 \text{ m}) - (90 \text{ N})(0.06 \text{ m})$$

$$+(E\cos 60°)(0.160 \text{ m}) + (E\sin 60°)(0.100 \text{ m}) = 0$$

$$\therefore \quad E = 140.454 \text{ N}$$

or $\mathbf{E} = 140.5 \text{ N} \measuredangle 60°$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - 90 \text{ N} + (140.454 \text{ N})\cos 60° = 0$$

$$\therefore \quad A_x = 19.7730 \text{ N}$$

or $\qquad \mathbf{A}_x = 19.7730 \text{ N} \longrightarrow$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 90 \text{ N} + (140.454 \text{ N})\sin 60° = 0$$

$$\therefore \quad A_y = -31.637 \text{ N}$$

or $\qquad \mathbf{A}_y = 31.6 \text{ N} \downarrow$

Then $\qquad A = \sqrt{A_x^2 + A_y^2} = \sqrt{(19.7730)^2 + (31.637)^2}$

$$= 37.308 \text{ lb}$$

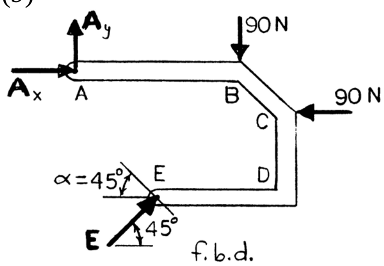
and $\qquad \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{-31.637}{19.7730}\right)$

$$= -57.995°$$

or $\mathbf{A} = 37.3 \text{ N} \searrow 58.0°$ ◀

PROBLEM 4.27 CONTINUED

(*b*)



(*b*) Given $\alpha = 45°$

From f.b.d. of frame

$$+\circlearrowleft \Sigma M_A = 0: \quad -(90\text{ N})(0.2\text{ m}) - (90\text{ N})(0.06\text{ m})$$

$$+(E\cos 45°)(0.160\text{ m}) + (E\sin 45°)(0.100\text{ m}) = 0$$

$$\therefore\ E = 127.279\text{ N}$$

or $\mathbf{E} = 127.3\text{ N} \measuredangle 45°$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - 90 + (127.279\text{ N})\cos 45° = 0$$

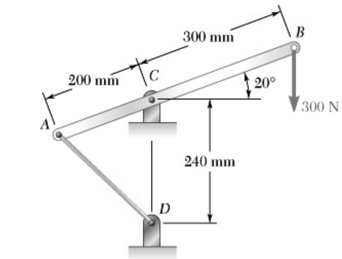
$$\therefore\ A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 90 + (127.279\text{ N})\sin 45° = 0$$

$$\therefore\ A_y = 0$$

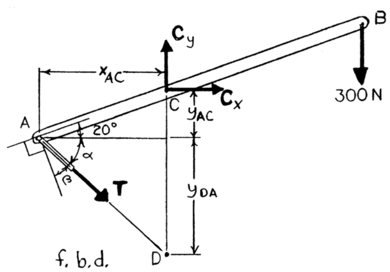
or $\mathbf{A} = 0$ ◀

PROBLEM 4.28



A lever AB is hinged at C and is attached to a control cable at A. If the lever is subjected to a 300-N vertical force at B, determine (a) the tension in the cable, (b) the reaction at C.

SOLUTION



First

$$x_{AC} = (0.200 \text{ m})\cos 20° = 0.187\ 939 \text{ m}$$

$$y_{AC} = (0.200 \text{ m})\sin 20° = 0.068\ 404 \text{ m}$$

Then

$$y_{DA} = 0.240 \text{ m} - y_{AC}$$

$$= 0.240 \text{ m} - 0.068404 \text{ m}$$

$$= 0.171596 \text{ m}$$

and

$$\tan\alpha = \frac{y_{DA}}{x_{AC}} = \frac{0.171\ 596}{0.187\ 939}$$

$$\therefore \quad \alpha = 42.397°$$

and

$$\beta = 90° - 20° - 42.397° = 27.603°$$

(*a*) From f.b.d. of lever AB

$$+\circlearrowleft \Sigma M_C = 0: \quad T\cos 27.603°(0.2 \text{ m})$$

$$- 300 \text{ N}\left[(0.3 \text{ m})\cos 20°\right] = 0$$

$$\therefore \quad T = 477.17 \text{ N} \qquad \text{or } T = 477 \text{ N} ◀$$

(*b*) From f.b.d. of lever AB

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + (477.17 \text{ N})\cos 42.397° = 0$$

$$\therefore \quad C_x = -352.39 \text{ N}$$

or

$$\mathbf{C}_x = 352.39 \text{ N} \longleftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - 300 \text{ N} - (477.17 \text{ N})\sin 42.397° = 0$$

$$\therefore \quad C_y = 621.74 \text{ N}$$

or

$$\mathbf{C}_y = 621.74 \text{ N} \uparrow$$

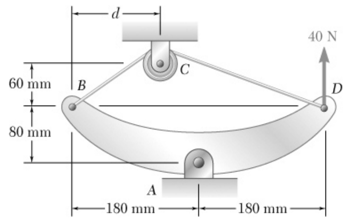
PROBLEM 4.28 CONTINUED

Then $\quad C = \sqrt{C_x^2 + C_y^2} = \sqrt{(352.39)^2 + (621.74)^2} = 714.66 \text{ N}$

and $\quad \theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{621.74}{-352.39}\right) = -60.456°$

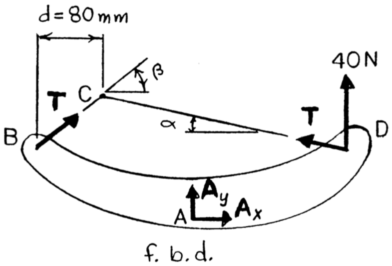
or $\mathbf{C} = 715 \text{ N} \measuredangle 60.5° \blacktriangleleft$

PROBLEM 4.29



Neglecting friction and the radius of the pulley, determine the tension in cable BCD and the reaction at support A when $d = 80$ mm.

SOLUTION



First

$$\alpha = \tan^{-1}\left(\frac{60}{280}\right) = 12.0948°$$

$$\beta = \tan^{-1}\left(\frac{60}{80}\right) = 36.870°$$

From f.b.d. of object BAD

$$+\circlearrowleft \Sigma M_A = 0: \quad (40\text{ N})(0.18\text{ m}) + (T\cos\alpha)(0.08\text{ m})$$

$$+(T\sin\alpha)(0.18\text{ m}) - (T\cos\beta)(0.08\text{ m})$$

$$-(T\sin\beta)(0.18\text{ m}) = 0$$

$$\therefore \quad T = \left(\frac{7.2\text{ N}\cdot\text{m}}{0.056061}\right) = 128.433\text{ N}$$

or $T = 128.4$ N ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad (128.433\text{ N})(\cos\beta - \cos\alpha) + A_x = 0$$

$$\therefore \quad A_x = 22.836\text{ N}$$

or $\qquad \mathbf{A}_x = 22.836\text{ N} \longrightarrow$

$$+\uparrow \Sigma F_y = 0: \quad A_y + (128.433\text{ N})(\sin\beta + \sin\alpha) + 40\text{ N} = 0$$

$$\therefore \quad A_y = -143.970\text{ N}$$

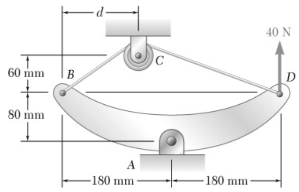
or $\qquad \mathbf{A}_y = 143.970\text{ N} \downarrow$

Then $\qquad A = \sqrt{A_x^2 + A_y^2} = \sqrt{(22.836)^2 + (143.970)^2} = 145.770\text{ N}$

and $\qquad \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{-143.970}{22.836}\right) = -80.987°$

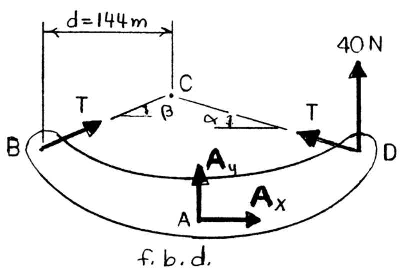
or $\mathbf{A} = 145.8$ N $\searrow$ 81.0° ◀

PROBLEM 4.30



Neglecting friction and the radius of the pulley, determine the tension in cable *BCD* and the reaction at support *A* when $d = 144$ mm.

SOLUTION



First note

$$\alpha = \tan^{-1}\left(\frac{60}{216}\right) = 15.5241°$$

$$\beta = \tan^{-1}\left(\frac{60}{144}\right) = 22.620°$$

From f.b.d. of member *BAD*

$$+\circlearrowleft \Sigma M_A = 0: \quad (40 \text{ N})(0.18 \text{ m}) + (T\cos\alpha)(0.08 \text{ m})$$

$$+(T\sin\alpha)(0.18 \text{ m}) - (T\cos\beta)(0.08 \text{ m})$$

$$-(T\sin\beta)(0.18 \text{ m}) = 0$$

$$\therefore \quad T = \left(\frac{7.2 \text{ N}\cdot\text{m}}{0.0178199 \text{ m}}\right) = 404.04 \text{ N}$$

or $T = 404$ N ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + (404.04 \text{ N})(\cos\beta - \cos\alpha) = 0$$

$$\therefore \quad A_x = 16.3402 \text{ N}$$

or $\mathbf{A}_x = 16.3402 \text{ N} \longrightarrow$

$$+\uparrow \Sigma F_y = 0: \quad A_y + (404.04 \text{ N})(\sin\beta + \sin\alpha) + 40 \text{ N} = 0$$

$$\therefore \quad A_y = -303.54 \text{ N}$$

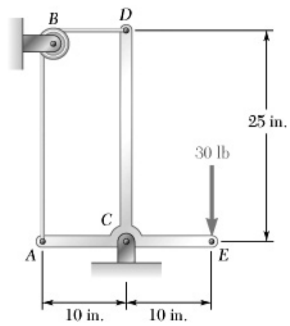
or $\mathbf{A}_y = 303.54 \text{ N} \downarrow$

Then $A = \sqrt{A_x^2 + A_y^2} = \sqrt{(16.3402)^2 + (303.54)^2} = 303.98 \text{ N}$

and $\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{-303.54}{16.3402}\right) = -86.919°$

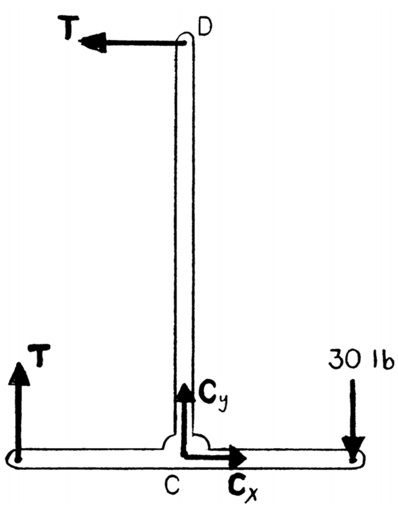
or $\mathbf{A} = 304 \text{ N} \searrow 86.9°$ ◀

PROBLEM 4.31



Neglecting friction, determine the tension in cable *ABD* and the reaction at support *C*.

SOLUTION



From f.b.d. of inverted T-member

$$+\circlearrowleft \Sigma M_C = 0: \quad T(25 \text{ in.}) - T(10 \text{ in.}) - (30 \text{ lb})(10 \text{ in.}) = 0$$

$$\therefore \; T = 20 \text{ lb}$$

or $T = 20.0 \text{ lb}$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - 20 \text{ lb} = 0$$

$$\therefore \; C_x = 20 \text{ lb}$$

or $\mathbf{C}_x = 20.0 \text{ lb} \longrightarrow$

$$+\uparrow \Sigma F_y = 0: \quad C_y + 20 \text{ lb} - 30 \text{ lb} = 0$$

$$\therefore \; C_y = 10 \text{ lb}$$

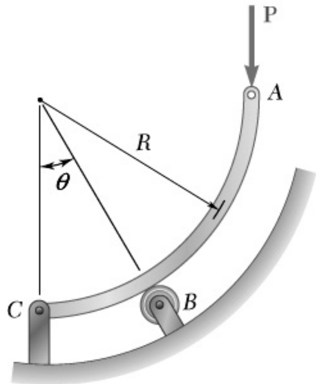
or $\mathbf{C}_y = 10.00 \text{ lb} \uparrow$

Then $C = \sqrt{C_x^2 + C_y^2} = \sqrt{(20)^2 + (10)^2} = 22.361 \text{ lb}$

and $\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{10}{20}\right) = 26.565°$

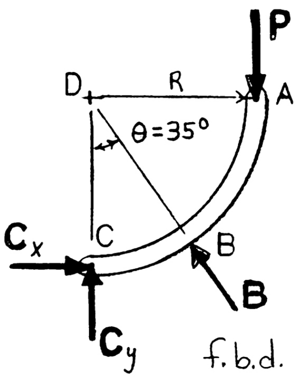
or $\mathbf{C} = 22.4 \text{ lb} \measuredangle 26.6°$ ◀

PROBLEM 4.32



Rod *ABC* is bent in the shape of a circular arc of radius *R*. Knowing that $\theta = 35^{\circ}$, determine the reaction (*a*) at *B*, (*b*) at *C*.

SOLUTION



For $\theta = 35°$

(*a*) From the f.b.d. of rod *ABC*

$$+\circlearrowleft \Sigma M_D = 0: \quad C_x(R) - P(R) = 0$$

$$\therefore \quad C_x = P$$

or $$\mathbf{C}_x = P \longrightarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad P - B\sin 35° = 0$$

$$\therefore \quad B = \frac{P}{\sin 35°} = 1.74345P$$

or $\mathbf{B} = 1.743P \searrow 55.0°$ ◀

(*b*) From the f.b.d. of rod *ABC*

$$+\uparrow \Sigma F_y = 0: \quad C_y + (1.74345P)\cos 35° - P = 0$$

$$\therefore \quad C_y = -0.42815P$$

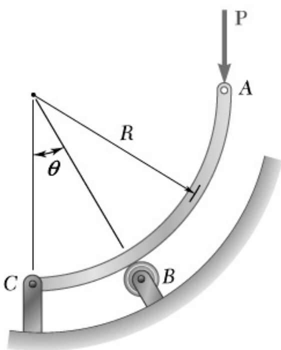
or $$\mathbf{C}_y = 0.42815P \downarrow$$

Then $$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(P)^2 + (0.42815P)^2} = 1.08780P$$

and $$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{-0.42815P}{P}\right) = -23.178°$$

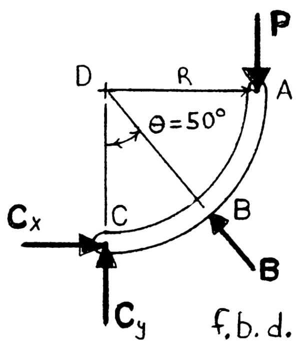
or $\mathbf{C} = 1.088P \nwarrow 23.2°$ ◀

PROBLEM 4.33



Rod ABC is bent in the shape of a circular arc of radius R. Knowing that $\theta = 50^\circ$, determine the reaction (*a*) at B, (*b*) at C.

SOLUTION



For $\theta = 50^\circ$

(*a*) From the f.b.d. of rod ABC

$$+\circlearrowleft \Sigma M_D = 0: \quad C_x(R) - P(R) = 0$$

$$\therefore \quad C_x = P$$

or $$\mathbf{C}_x = P \longrightarrow$$

$$\overset{+}{\longrightarrow} \Sigma F_x = 0: \quad P - B\sin 50^\circ = 0$$

$$\therefore \quad B = \frac{P}{\sin 50^\circ} = 1.30541P$$

or $\mathbf{B} = 1.305P \searrow 40.0^\circ$ ◀

(*b*) From the f.b.d. of rod ABC

$$+\uparrow \Sigma F_y = 0: \quad C_y - P + (1.30541P)\cos 50^\circ = 0$$

$$\therefore \quad C_y = 0.160900P$$

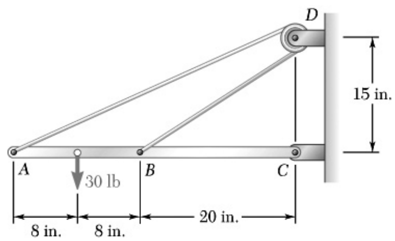
or $$\mathbf{C}_y = 0.1609P \uparrow$$

Then $$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(P)^2 + (0.1609P)^2} = 1.01286P$$

and $$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{0.1609P}{P}\right) = 9.1405^\circ$$

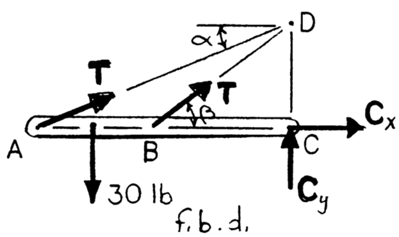
or $\mathbf{C} = 1.013P \measuredangle 9.14^\circ$ ◀

PROBLEM 4.34



Neglecting friction and the radius of the pulley, determine (*a*) the tension in cable *ABD*, (*b*) the reaction at *C*.

SOLUTION



First note

$$\alpha = \tan^{-1}\left(\frac{15}{36}\right) = 22.620°$$

$$\beta = \tan^{-1}\left(\frac{15}{20}\right) = 36.870°$$

(*a*) From f.b.d. of member *ABC*

$$+\circlearrowleft \Sigma M_C = 0: \quad (30\text{ lb})(28\text{ in.}) - (T\sin 22.620°)(36\text{ in.})$$

$$-(T\sin 36.870°)(20\text{ in.}) = 0$$

$$\therefore \quad T = 32.500\text{ lb}$$

or $T = 32.5\text{ lb}$ ◀

(*b*) From f.b.d. of member *ABC*

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + (32.500\text{ lb})(\cos 22.620° + \cos 36.870°) = 0$$

$$\therefore \quad C_x = -56.000\text{ lb}$$

or $\qquad \mathbf{C}_x = 56.000\text{ lb} \longleftarrow$

$$+\uparrow \Sigma F_y = 0: \quad C_y - 30\text{ lb} + (32.500\text{ lb})(\sin 22.620° + \sin 36.870°) = 0$$

$$\therefore \quad C_y = -2.0001\text{ lb}$$

or $\qquad \mathbf{C}_y = 2.0001\text{ lb} \downarrow$

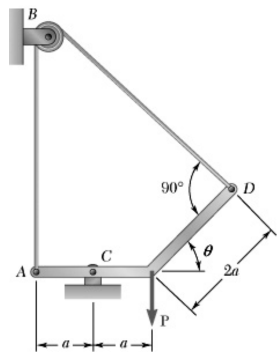
Then $\qquad C = \sqrt{C_x^2 + C_y^2} = \sqrt{(56.0)^2 + (2.001)^2} = 56.036\text{ lb}$

and $\qquad \theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{-2.0}{-56.0}\right) = 2.0454°$

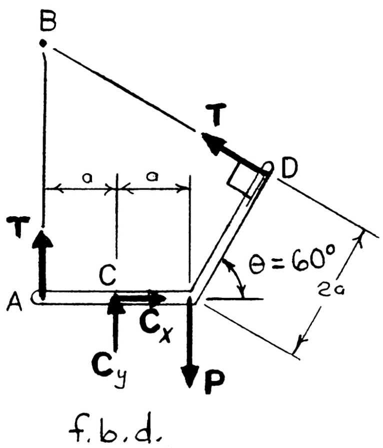
or $\mathbf{C} = 56.0\text{ lb} \measuredangle\ 2.05°$ ◀

PROBLEM 4.35

Neglecting friction, determine the tension in cable *ABD* and the reaction at *C* when $\theta = 60^\circ$.



SOLUTION



From f.b.d. of bent *ACD*

$$+\circlearrowleft \Sigma M_C = 0: \quad (T\cos 30^\circ)(2a\sin 60^\circ) + (T\sin 30^\circ)(a + 2a\cos 60^\circ)$$

$$-T(a) - P(a) = 0$$

$$\therefore \quad T = \frac{P}{1.5}$$

$$\text{or } T = \frac{2P}{3} \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - \left(\frac{2P}{3}\right)\cos 30^\circ = 0$$

$$\therefore \quad C_x = \frac{\sqrt{3}}{3}P = 0.57735P$$

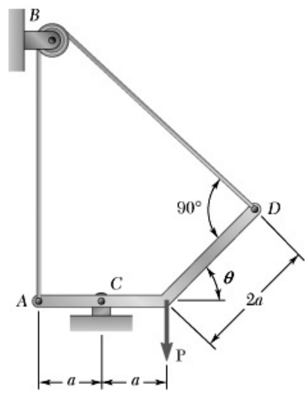
or $\qquad \mathbf{C}_x = 0.577P \longrightarrow$

$$+\uparrow \Sigma F_y = 0: \quad C_y + \frac{2}{3}P - P + \left(\frac{2P}{3}\right)\cos 60^\circ = 0$$

$$\therefore \quad C_y = 0$$

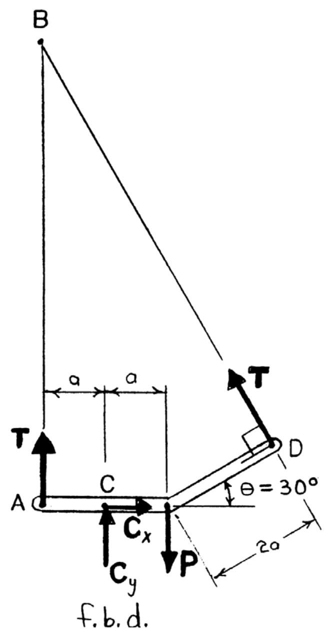
$$\text{or } \mathbf{C} = 0.577P \longrightarrow \blacktriangleleft$$

PROBLEM 4.36



Neglecting friction, determine the tension in cable *ABD* and the reaction at *C* when $\theta = 30^{\circ}$.

SOLUTION



From f.b.d. of bent *ACD*

$$+\circlearrowleft \Sigma M_C = 0: \quad (T\cos 60^{\circ})(2a\sin 30^{\circ}) + T\sin 60^{\circ}(a + 2a\cos 30^{\circ})$$

$$-P(a) - T(a) = 0$$

$$\therefore \quad T = \frac{P}{1.86603} = 0.53590P$$

or $T = 0.536P$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - (0.53590P)\cos 60^{\circ} = 0$$

$$\therefore \quad C_x = 0.26795P$$

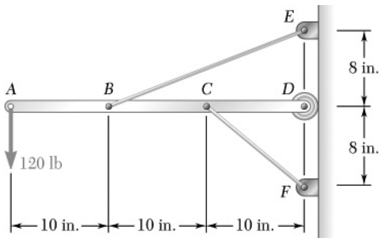
or $\mathbf{C}_x = 0.268P \longrightarrow$

$$+\uparrow \Sigma F_y = 0: \quad C_y + 0.53590P - P + (0.53590P)\sin 60^{\circ} = 0$$

$$\therefore \quad C_y = 0$$

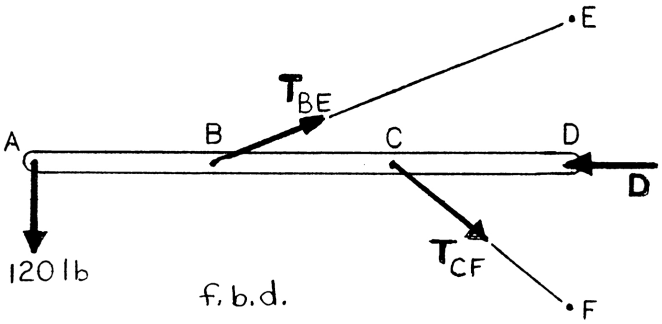
or $\mathbf{C} = 0.268P \longrightarrow$ ◀

PROBLEM 4.37



Determine the tension in each cable and the reaction at D.

SOLUTION



First note

$$\overline{BE} = \sqrt{(20)^2 + (8)^2} \text{ in.} = 21.541 \text{ in.}$$

$$\overline{CF} = \sqrt{(10)^2 + (8)^2} \text{ in.} = 12.8062 \text{ in.}$$

From f.b.d. of member $ABCD$

$$+\circlearrowleft \Sigma M_C = 0: \quad (120 \text{ lb})(20 \text{ in.}) - \left[\left(\frac{8}{21.541}\right)T_{BE}\right](10 \text{ in.}) = 0$$

$$\therefore \quad T_{BE} = 646.24 \text{ lb}$$

or $T_{BE} = 646$ lb ◀

$$+\uparrow \Sigma F_y = 0: \quad -120 \text{ lb} + \left(\frac{8}{21.541}\right)(646.24 \text{ lb}) - \left(\frac{8}{12.8062}\right)T_{CF} = 0$$

$$\therefore \quad T_{CF} = 192.099 \text{ lb}$$

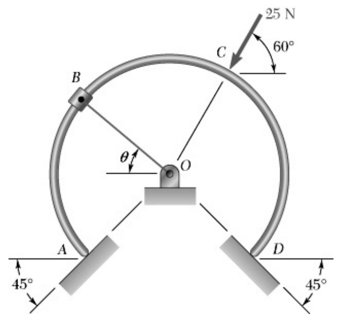
or $T_{CF} = 192.1$ lb ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad \left(\frac{20}{21.541}\right)(646.24 \text{ lb}) + \left(\frac{10}{12.8062}\right)(192.099 \text{ lb}) - D = 0$$

$$\therefore \quad D = 750.01 \text{ lb}$$

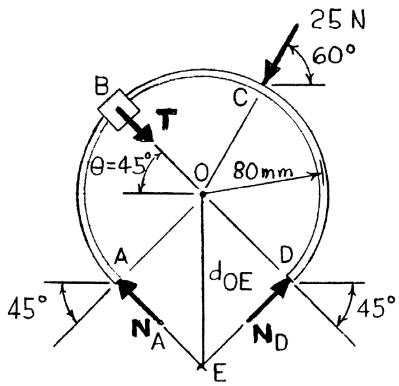
or $\mathbf{D} = 750$ lb $\longleftarrow$ ◀

PROBLEM 4.38



Rod *ABCD* is bent in the shape of a circular arc of radius 80 mm and rests against frictionless surfaces at *A* and *D*. Knowing that the collar at *B* can move freely on the rod and that $\theta = 45^{\circ}$. determine (*a*) the tension in cord *OB*, (*b*) the reactions at *A* and *D*.

SOLUTION



f. b. d.

(*a*) From f.b.d. of rod *ABCD*

$$+\circlearrowleft \Sigma M_E = 0: \quad (25\text{ N})\cos 60^{\circ}(d_{OE}) - (T\cos 45^{\circ})(d_{OE}) = 0$$

$$\therefore \quad T = 17.6777\text{ N}$$

or $T = 17.68\text{ N}$ ◀

(*b*) From f.b.d. of rod *ABCD*

$$\xrightarrow{+} \Sigma F_x = 0: \quad -(17.6777\text{ N})\cos 45^{\circ} + (25\text{ N})\cos 60^{\circ}$$

$$+ N_D\cos 45^{\circ} - N_A\cos 45^{\circ} = 0$$

$$\therefore \quad N_A - N_D = 0$$

or $$N_D = N_A \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad N_A\sin 45^{\circ} + N_D\sin 45^{\circ} - (17.6777\text{ N})\sin 45^{\circ}$$

$$-(25\text{ N})\sin 60^{\circ} = 0$$

$$\therefore \quad N_A + N_D = 48.296\text{ N} \qquad (2)$$

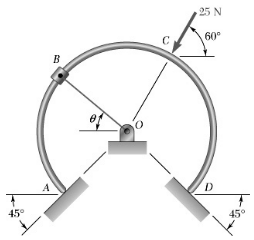
Substituting Equation (1) into Equation (2),

$$2N_A = 48.296\text{ N}$$

$$N_A = 24.148\text{ N}$$

or $\mathbf{N}_A = 24.1\text{ N} \measuredangle\, 45.0^{\circ}$ ◀

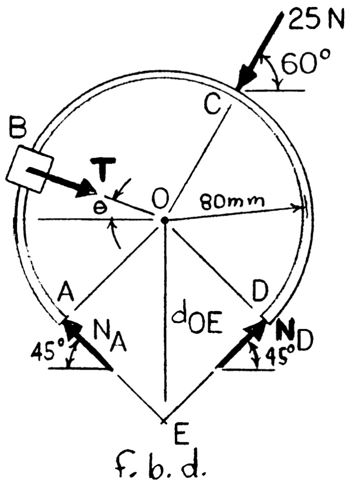
and $\mathbf{N}_D = 24.1\text{ N} \measuredangle\, 45.0^{\circ}$ ◀



PROBLEM 4.39

Rod *ABCD* is bent in the shape of a circular arc of radius 80 mm and rests against frictionless surfaces at *A* and *D*. Knowing that the collar at *B* can move freely on the rod, determine (*a*) the value of θ for which the tension in cord *OB* is as small as possible, (*b*) the corresponding value of the tension, (*c*) the reactions at *A* and *D*.

SOLUTION



(*a*) From f.b.d. of rod *ABCD*

$$+\circlearrowleft \Sigma M_E = 0: \quad (25\text{ N})\cos 60^\circ (d_{OE}) - (T\cos\theta)(d_{OE}) = 0$$

or

$$T = \frac{12.5\text{ N}}{\cos\theta} \tag{1}$$

$\therefore$ T is minimum when $\cos\theta$ is maximum,

or $\theta = 0^\circ$ ◀

(*b*) From Equation (1)

$$T = \frac{12.5\text{ N}}{\cos 0} = 12.5\text{ N}$$

or $T_{\min} = 12.50\text{ N}$ ◀

(*c*) $\xrightarrow{+} \Sigma F_x = 0: \quad -N_A\cos 45^\circ + N_D\cos 45^\circ + 12.5\text{ N}$

$$-(25\text{ N})\cos 60^\circ = 0$$

$$\therefore \quad N_D - N_A = 0$$

or

$$N_D = N_A \tag{2}$$

$$+\uparrow \Sigma F_y = 0: \quad N_A\sin 45^\circ + N_D\sin 45^\circ - (25\text{ N})\sin 60^\circ = 0$$

$$\therefore \quad N_D + N_A = 30.619\text{ N} \tag{3}$$

Substituting Equation (2) into Equation (3),

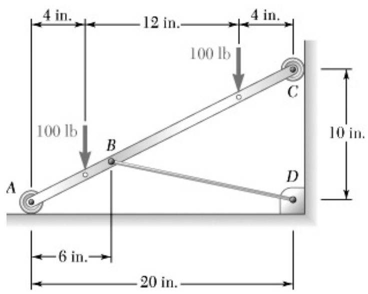
$$2N_A = 30.619$$

$$N_A = 15.3095\text{ N}$$

or $\mathbf{N}_A = 15.31\text{ N} \measuredangle\, 45.0^\circ$ ◀

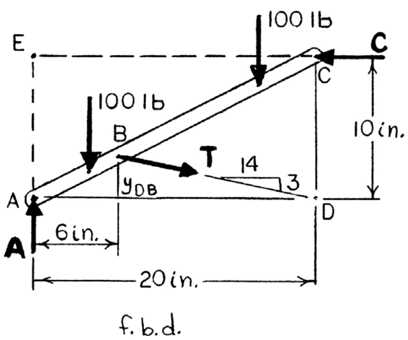
and $\mathbf{N}_D = 15.31\text{ N} \measuredangle\, 45.0^\circ$ ◀

PROBLEM 4.40



Bar AC supports two 100-lb loads as shown. Rollers A and C rest against frictionless surfaces and a cable BD is attached at B. Determine (*a*) the tension in cable BD, (*b*) the reaction at A, (*c*) the reaction at C.

SOLUTION



First note that from similar triangles

$$\frac{y_{DB}}{6} = \frac{10}{20} \qquad \therefore \quad y_{DB} = 3 \text{ in.}$$

and

$$\overline{BD} = \sqrt{(3)^2 + (14)^2} \text{ in.} = 14.3178 \text{ in.}$$

$$T_x = \frac{14}{14.3178}T = 0.97780T$$

$$T_y = \frac{3}{14.3178}T = 0.20953T$$

(*a*) From f.b.d. of bar AC

$$+\circlearrowleft \Sigma M_E = 0: \quad (0.97780T)(7 \text{ in.}) - (0.20953T)(6 \text{ in.})$$

$$-(100 \text{ lb})(16 \text{ in.}) - (100 \text{ lb})(4 \text{ in.}) = 0$$

$$\therefore \quad T = 357.95 \text{ lb}$$

or $T = 358$ lb ◀

(*b*) From f.b.d. of bar AC

$$+\uparrow \Sigma F_y = 0: \quad A - 100 - 0.20953(357.95) - 100 = 0$$

$$\therefore \quad A = 275.00 \text{ lb}$$

or $\mathbf{A} = 275$ lb $\uparrow$ ◀

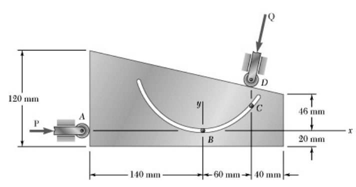
(*c*) From f.b.d of bar AC

$$\xrightarrow{+} \Sigma F_x = 0: \quad 0.97780(357.95) - C = 0$$

$$\therefore \quad C = 350.00 \text{ lb}$$

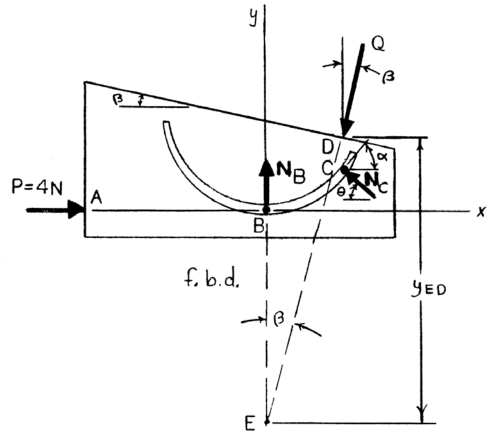
or $\mathbf{C} = 350$ lb $\longleftarrow$ ◀

PROBLEM 4.41



A parabolic slot has been cut in plate *AD*, and the plate has been placed so that the slot fits two fixed, frictionless pins *B* and *C*. The equation of the slot is $y = x^2/100$, where *x* and *y* are expressed in mm. Knowing that the input force $P = 4$ N, determine (*a*) the force each pin exerts on the plate, (*b*) the output force **Q**.

SOLUTION



The equation of the slot is $$y = \frac{x^2}{100}$$

Now $$\left(\frac{dy}{dx}\right)_C = \text{slope of the slot at } C$$

$$= \left[\frac{2x}{100}\right]_{x=60\text{ mm}} = 1.200$$

$$\therefore \quad \alpha = \tan^{-1}(1.200) = 50.194°$$

and $$\theta = 90° - \alpha = 90° - 50.194° = 39.806°$$

Coordinates of *C* are

$$x_C = 60 \text{ mm}, \qquad y_C = \frac{(60)^2}{100} = 36 \text{ mm}$$

Also, the coordinates of *D* are $$x_D = 60 \text{ mm}$$

$$y_D = 46 \text{ mm} + (40 \text{ mm})\sin\beta$$

where $$\beta = \tan^{-1}\left(\frac{120 - 66}{240}\right) = 12.6804°$$

$$\therefore \quad y_D = 46 \text{ mm} + (40 \text{ mm})\tan 12.6804°$$

$$= 55.000 \text{ mm}$$

PROBLEM 4.41 CONTINUED

Also, $$y_{ED} = \frac{60 \text{ mm}}{\tan\beta} = \frac{60 \text{ mm}}{\tan 12.6804^\circ}$$

$$= 266.67 \text{ mm}$$

From f.b.d. of plate *AD*

$$+\circlearrowright \Sigma M_E = 0: \quad (N_C\cos\theta)\left[y_{ED} - (y_D - y_C)\right] + (N_C\sin\theta)(x_C) - (4 \text{ N})(y_{ED} - y_D) = 0$$

$$(N_C\cos 39.806^\circ)\left[266.67 - (55.0 - 36.0)\right] \text{ mm} + N_C\sin(39.806^\circ)(60 \text{ mm}) - (4 \text{ N})(266.67 - 55.0) \text{ mm} = 0$$

$$\therefore\ N_C = 3.7025 \text{ N}$$

or $$\mathbf{N}_C = 3.70 \text{ N} \searrow 39.8^\circ$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -4 \text{ N} + N_C\cos\theta + Q\sin\beta = 0$$

$$-4 \text{ N} + (3.7025 \text{ N})\cos 39.806^\circ + Q\sin 12.6804^\circ = 0$$

$$\therefore\ Q = 5.2649 \text{ N}$$

or $$\mathbf{Q} = 5.26 \text{ N} \swarrow 77.3^\circ$$

$$+\uparrow \Sigma F_y = 0: \quad N_B + N_C\sin\theta - Q\cos\beta = 0$$

$$N_B + (3.7025 \text{ N})\sin 39.806^\circ - (5.2649 \text{ N})\cos 12.6804^\circ = 0$$

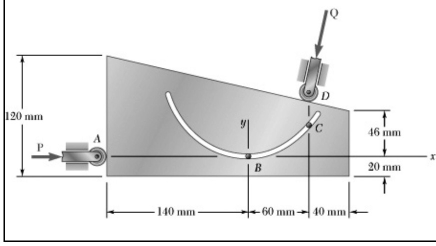
$$\therefore\ N_B = 2.7662 \text{ N}$$

or $$\mathbf{N}_B = 2.77 \text{ N} \uparrow$$

(*a*) $$\mathbf{N}_B = 2.77 \text{ N} \uparrow, \quad \mathbf{N}_C = 3.70 \text{ N} \searrow 39.8^\circ \blacktriangleleft$$

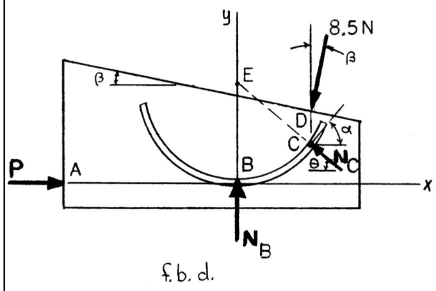
(*b*) $$\mathbf{Q} = 5.26 \text{ N} \nearrow 77.3^\circ (\text{output}) \blacktriangleleft$$

PROBLEM 4.42



A parabolic slot has been cut in plate *AD*, and the plate has been placed so that the slot fits two fixed, frictionless pins *B* and *C*. The equation of the slot is $y = x^2/100$, where *x* and *y* are expressed in mm. Knowing that the maximum allowable force exerted on the roller at *D* is 8.5 N, determine (*a*) the corresponding magnitude of the input force **P**, (*b*) the force each pin exerts on the plate.

SOLUTION



The equation of the slot is, $$y = \frac{x^2}{100}$$

Now $$\left(\frac{dy}{dx}\right)_C = \text{slope of slot at } C$$

$$= \left[\frac{2x}{100}\right]_{x=60\text{ mm}} = 1.200$$

$$\therefore \quad \alpha = \tan^{-1}(1.200) = 50.194°$$

and $$\theta = 90° - \alpha = 90° - 50.194° = 39.806°$$

Coordinates of *C* are

$$x_C = 60\text{ mm}, \quad y_C = \frac{(60)^2}{100} = 36\text{ mm}$$

Also, the coordinates of *D* are

$$x_D = 60\text{ mm}$$

$$y_D = 46\text{ mm} + (40\text{ mm})\sin\beta$$

where $$\beta = \tan^{-1}\left(\frac{120 - 66}{240}\right) = 12.6804°$$

$$\therefore \quad y_D = 46\text{ mm} + (40\text{ mm})\tan 12.6804° = 55.000\text{ mm}$$

Note: $$x_E = 0$$

$$y_E = y_C + (60\text{ mm})\tan\theta$$

$$= 36\text{ mm} + (60\text{ mm})\tan 39.806°$$

$$= 86.001\text{ mm}$$

(*a*) From f.b.d. of plate *AD*

$$+\circlearrowleft \Sigma M_E = 0: \quad P(y_E) - [(8.5\text{ N})\sin\beta](y_E - y_D)$$

$$-[(8.5\text{ N})\cos\beta](60\text{ mm}) = 0$$

PROBLEM 4.42 CONITNIUED

$$P(86.001\text{ mm}) - [(8.5\text{ N})\sin 12.6804°](31.001\text{ mm})$$

$$-[(8.5\text{ N})\cos 12.6804°](60\text{ mm}) = 0$$

$$\therefore \quad P = 6.4581\text{ N}$$

or $P = 6.46\text{ N}$ ◀

(*b*) $\xrightarrow{+} \Sigma F_x = 0: \quad P - (8.5\text{ N})\sin\beta - N_C\cos\theta = 0$

$$6.458\text{ N} - (8.5\text{ N})(\sin 12.6804°) - N_C(\cos 39.806°) = 0$$

$$\therefore \quad N_C = 5.9778\text{ N}$$

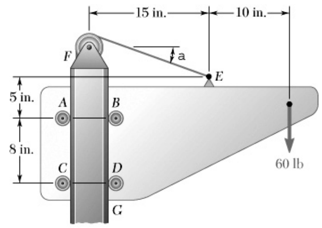
or $\mathbf{N}_C = 5.98\text{ N} \searrow 39.8°$ ◀

$$+\uparrow \Sigma F_y = 0: \quad N_B + N_C\sin\theta - (8.5\text{ N})\cos\beta = 0$$

$$N_B + (5.9778\text{ N})\sin 39.806° - (8.5\text{ N})\cos 12.6804° = 0$$

$$\therefore \quad N_B = 4.4657\text{ N}$$

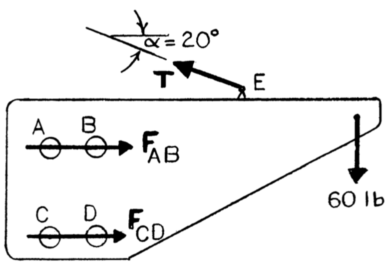
or $\mathbf{N}_B = 4.47\text{ N} \uparrow$ ◀



PROBLEM 4.43

A movable bracket is held at rest by a cable attached at *E* and by frictionless rollers. Knowing that the width of post *FG* is slightly less than the distance between the rollers, determine the force exerted on the post by each roller when $\alpha = 20^{\circ}$.

SOLUTION



f.b.d.

From f.b.d. of bracket

$$+\uparrow \Sigma F_y = 0: \quad T \sin 20^{\circ} - 60 \text{ lb} = 0$$

$$\therefore \quad T = 175.428 \text{ lb}$$

$$T_x = (175.428 \text{ lb}) \cos 20^{\circ} = 164.849 \text{ lb}$$

$$T_y = (175.428 \text{ lb}) \sin 20^{\circ} = 60 \text{ lb}$$

Note: T_y and 60 lb force form a couple of

$$60 \text{ lb}(10 \text{ in.}) = 600 \text{ lb} \cdot \text{in.} \circlearrowleft$$

$$+\circlearrowleft \Sigma M_B = 0: \quad 164.849 \text{ lb}(5 \text{ in.}) - 600 \text{ lb} \cdot \text{in.} + F_{CD}(8 \text{ in.}) = 0$$

$$\therefore \quad F_{CD} = -28.030 \text{ lb}$$

or $$\mathbf{F}_{CD} = 28.0 \text{ lb} \longleftarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{CD} + F_{AB} - T_x = 0$$

$$-28.030 \text{ lb} + F_{AB} - 164.849 \text{ lb} = 0$$

$$\therefore \quad F_{AB} = 192.879 \text{ lb}$$

or $$\mathbf{F}_{AB} = 192.9 \text{ lb} \longrightarrow$$

Rollers *A* and *C* can only apply a horizontal force to the right onto the vertical post corresponding to the equal and opposite force to the left on the bracket. Since $\mathbf{F}_{AB}$ is directed to the right onto the bracket, roller *B* will react $\mathbf{F}_{AB}$. Also, since $\mathbf{F}_{CD}$ is acting to the left on the bracket, it will act to the right on the post at roller *C*.

PROBLEM 4.43 CONTINUED

$$\therefore \quad \mathbf{A} = \mathbf{D} = 0$$

$$\mathbf{B} = 192.9 \text{ lb} \longrightarrow$$

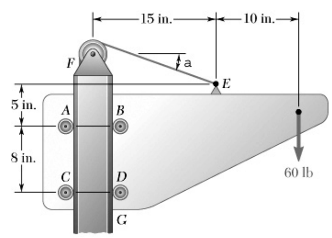
$$\mathbf{C} = 28.0 \text{ lb} \longleftarrow$$

Forces exerted on the post are

$$\mathbf{A} = \mathbf{D} = 0 \blacktriangleleft$$

$$\mathbf{B} = 192.9 \text{ lb} \longleftarrow \blacktriangleleft$$

$$\mathbf{C} = 28.0 \text{ lb} \longrightarrow \blacktriangleleft$$

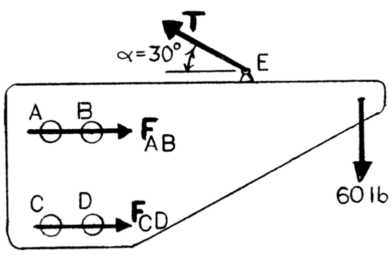


PROBLEM 4.44

Solve Problem 4.43 when $\alpha = 30^{\circ}$.

P4.43 A movable bracket is held at rest by a cable attached at *E* and by frictionless rollers. Knowing that the width of post *FG* is slightly less than the distance between the rollers, determine the force exerted on the post by each roller when $\alpha = 20^{\circ}$.

SOLUTION



f.b.d.

From f.b.d. of bracket

$$+\uparrow\ \Sigma F_y = 0: \quad T\sin 30^{\circ} - 60\text{ lb} = 0$$

$$\therefore \quad T = 120\text{ lb}$$

$$T_x = (120\text{ lb})\cos 30^{\circ} = 103.923\text{ lb}$$

$$T_y = (120\text{ lb})\sin 30^{\circ} = 60\text{ lb}$$

Note: T_y and 60 lb force form a couple of

$$(60\text{ lb})(10\text{ in.}) = 600\text{ lb}\cdot\text{in.} \circlearrowleft$$

$$+\circlearrowright\ \Sigma M_B = 0: \quad (103.923\text{ lb})(5\text{ in.}) - 600\text{ lb}\cdot\text{in.} + F_{CD}(8\text{ in.}) = 0$$

$$\therefore \quad F_{CD} = 10.0481\text{ lb}$$

or

$$\mathbf{F}_{CD} = 10.05\text{ lb} \longrightarrow$$

$$\xrightarrow{+}\ \Sigma F_x = 0: \quad F_{CD} + F_{AB} - T_x = 0$$

$$10.0481\text{ lb} + F_{AB} - 103.923\text{ lb} = 0$$

$$\therefore \quad F_{AB} = 93.875\text{ lb}$$

or

$$\mathbf{F}_{AB} = 93.9\text{ lb} \longrightarrow$$

Rollers *A* and *C* can only apply a horizontal force to the right on the vertical post corresponding to the equal and opposite force to the left on the bracket. The opposite direction apply to roller *B* and *D*. Since both $\mathbf{F}_{AB}$ and $\mathbf{F}_{CD}$ act to the right on the bracket, rollers *B* and *D* will react these forces.

$$\therefore \quad \mathbf{A} = \mathbf{C} = 0$$

$$\mathbf{B} = 93.9\text{ lb} \longrightarrow$$

$$\mathbf{D} = 10.05\text{ lb} \longrightarrow$$

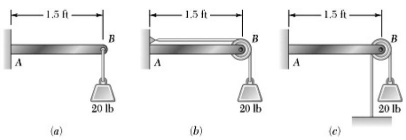
Forces exerted on the post are

$$\mathbf{A} = \mathbf{C} = 0 \blacktriangleleft$$

$$\mathbf{B} = 93.9\text{ lb} \longleftarrow \blacktriangleleft$$

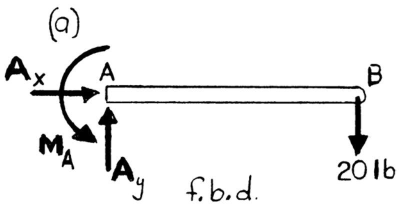
$$\mathbf{D} = 10.05\text{ lb} \longleftarrow \blacktriangleleft$$

PROBLEM 4.45



A 20-lb weight can be supported in the three different ways shown. Knowing that the pulleys have a 4-in. radius, determine the reaction at A in each case.

SOLUTION



(a) From f.b.d. of AB

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 20 \text{ lb} = 0$$

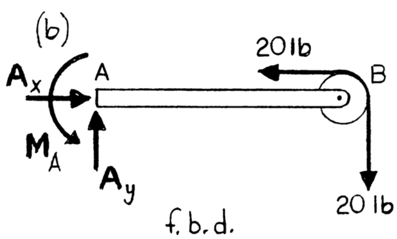
or $$A_y = 20.0 \text{ lb}$$

and $\mathbf{A} = 20.0 \text{ lb}\uparrow$ ◀

$$+\circlearrowleft \Sigma M_A = 0: \quad M_A - (20 \text{ lb})(1.5 \text{ ft}) = 0$$

$$\therefore\ M_A = 30.0 \text{ lb}\cdot\text{ft}$$

or $\mathbf{M}_A = 30.0 \text{ lb}\cdot\text{ft} \circlearrowleft$ ◀



(b) Note: $$4 \text{ in.}\left(\frac{1 \text{ ft}}{12 \text{ in.}}\right) = 0.33333 \text{ ft}$$

From f.b.d. of AB

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - 20 \text{ lb} = 0$$

or $$A_x = 20.0 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 20 \text{ lb} = 0$$

or $$A_y = 20.0 \text{ lb}$$

Then $$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(20.0)^2 + (20.0)^2} = 28.284 \text{ lb}$$

$\therefore\ \mathbf{A} = 28.3 \text{ lb} \measuredangle 45°$ ◀

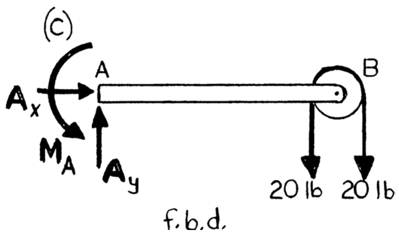
$$+\circlearrowleft \Sigma M_A = 0: \quad M_A + (20 \text{ lb})(0.33333 \text{ ft})$$

$$-(20 \text{ lb})(1.5 \text{ ft} + 0.33333 \text{ ft}) = 0$$

$$\therefore\ M_A = 30.0 \text{ lb}\cdot\text{ft}$$

or $\mathbf{M}_A = 30.0 \text{ lb}\cdot\text{ft} \circlearrowleft$ ◀

PROBLEM 4.45 CONTINUED



(*c*) From f.b.d. of *AB*

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 20 \text{ lb} - 20 \text{ lb} = 0$$

or $$A_y = 40.0 \text{ lb}$$

and $\mathbf{A} = 40.0 \text{ lb} \uparrow$ ◀

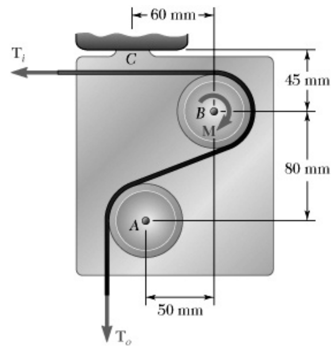
$$+\circlearrowleft \Sigma M_A = 0: \quad M_A - (20 \text{ lb})(1.5 \text{ ft} - 0.33333 \text{ ft})$$

$$-(20 \text{ lb})(1.5 \text{ ft} + 0.33333 \text{ ft}) = 0$$

$$\therefore \quad M_A = 60.0 \text{ lb}\cdot\text{ft}$$

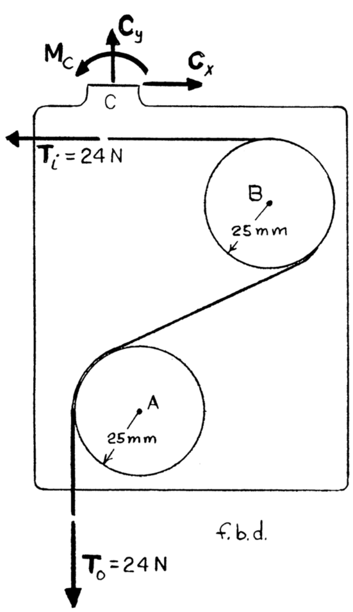
or $\mathbf{M}_A = 60.0 \text{ lb}\cdot\text{ft} \circlearrowleft$ ◀

PROBLEM 4.46



A belt passes over two 50-mm-diameter pulleys which are mounted on a bracket as shown. Knowing that $M = 0$ and $T_i = T_O = 24\text{ N}$, determine the reaction at *C*.

SOLUTION



From f.b.d. of bracket

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - 24\text{ N} = 0$$

$$\therefore\ C_x = 24\text{ N}$$

$$+\!\uparrow \Sigma F_y = 0: \quad C_y - 24\text{ N} = 0$$

$$\therefore\ C_y = 24\text{ N}$$

Then $$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(24)^2 + (24)^2} = 33.941\text{ N}$$

$$\therefore\ \mathbf{C} = 33.9\text{ N} \measuredangle 45.0° \blacktriangleleft$$

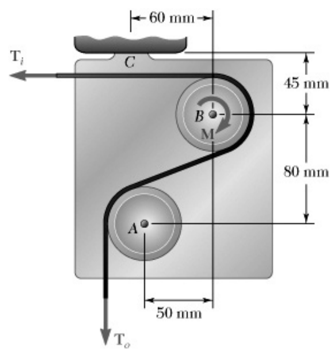
$$+\circlearrowleft \Sigma M_C = 0: \quad M_C - (24\text{ N})\big[(45 - 25)\text{ mm}\big]$$

$$+ (24\text{ N})\big[(25 + 50 - 60)\text{ mm}\big] = 0$$

$$\therefore\ M_C = 120\text{ N}\cdot\text{mm}$$

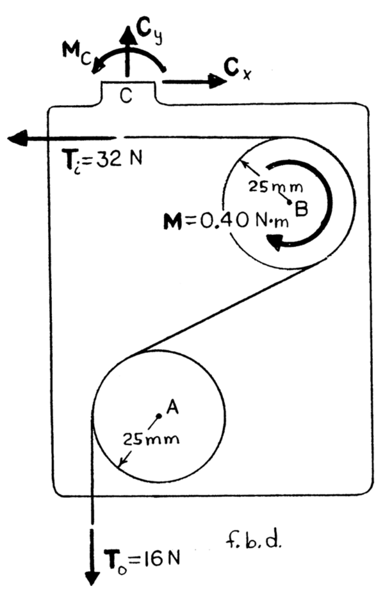
$$\text{or}\quad \mathbf{M}_C = 0.120\text{ N}\cdot\text{m} \circlearrowleft \blacktriangleleft$$

PROBLEM 4.47



A belt passes over two 50-mm-diameter pulleys which are mounted on a bracket as shown. Knowing that $M = 0.40\text{ N}\cdot\text{m}$ m and that T_i and T_O are equal to 32 N and 16 N, respectively, determine the reaction at C.

SOLUTION



From f.b.d. of bracket

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - 32\text{ N} = 0$$

$$\therefore \quad C_x = 32\text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - 16\text{ N} = 0$$

$$\therefore \quad C_y = 16\text{ N}$$

Then $\qquad C = \sqrt{C_x^2 + C_y^2} = \sqrt{(32)^2 + (16)^2} = 35.777\text{ N}$

and $\qquad \theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{16}{32}\right) = 26.565°$

or $\mathbf{C} = 35.8\text{ N} \measuredangle 26.6°$ ◀

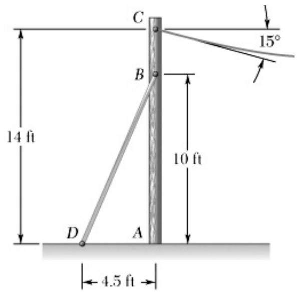
$$+\circlearrowleft \Sigma M_C = 0: \quad M_C - (32\text{ N})(45\text{ mm} - 25\text{ mm})$$

$$+ (16\text{ N})(25\text{ mm} + 50\text{ mm} - 60\text{ mm}) - 400\text{ N}\cdot\text{mm} = 0$$

$$\therefore \quad M_C = 800\text{ N}\cdot\text{mm}$$

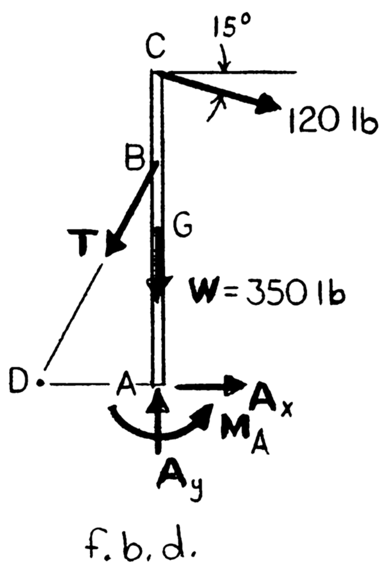
or $\mathbf{M}_C = 0.800\text{ N}\cdot\text{m} \circlearrowleft$ ◀

PROBLEM 4.48



A 350-lb utility pole is used to support at C the end of an electric wire. The tension in the wire is 120 lb, and the wire forms an angle of $15°$ with the horizontal at C. Determine the largest and smallest allowable tensions in the guy cable BD if the magnitude of the couple at A may not exceed $200\ \text{lb}\cdot\text{ft}$.

SOLUTION



First note

$$L_{BD} = \sqrt{(4.5)^2 + (10)^2} = 10.9659 \text{ ft}$$

T_{max}: From f.b.d. of utility pole with $\mathbf{M}_A = 200 \text{ lb}\cdot\text{ft}$ ↻

$$+\circlearrowleft \Sigma M_A = 0: \quad -200 \text{ lb}\cdot\text{ft} - \left[(120 \text{ lb})\cos 15°\right](14 \text{ ft})$$

$$+\left[\left(\frac{4.5}{10.9659}\right)T_{max}\right](10 \text{ ft}) = 0$$

$$\therefore \quad T_{max} = 444.19 \text{ lb}$$

or $T_{max} = 444 \text{ lb}$ ◀

T_{min}: From f.b.d. of utility pole with $\mathbf{M}_A = 200 \text{ lb}\cdot\text{ft}$ ↺

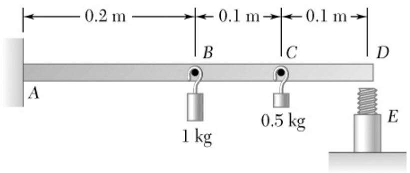
$$+\circlearrowleft \Sigma M_A = 0: \quad 200 \text{ lb}\cdot\text{ft} - \left[(120 \text{ lb})\cos 15°\right](14 \text{ ft})$$

$$+\left[\left(\frac{4.5}{10.9659}\right)T_{min}\right](10 \text{ ft}) = 0$$

$$\therefore \quad T_{min} = 346.71 \text{ lb}$$

or $T_{min} = 347 \text{ lb}$ ◀

PROBLEM 4.49

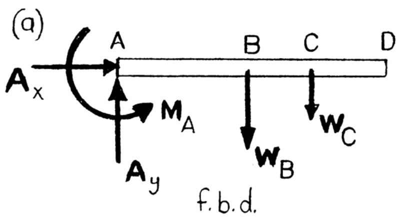


In a laboratory experiment, students hang the masses shown from a beam of negligible mass. (*a*) Determine the reaction at the fixed support *A* knowing that end *D* of the beam does not touch support *E*. (*b*) Determine the reaction at the fixed support *A* knowing that the adjustable support *E* exerts an upward force of 6 N on the beam.

SOLUTION

$$W_B = m_B g = (1\text{ kg})(9.81\text{ m/s}^2) = 9.81\text{ N}$$

$$W_C = m_C g = (0.5\text{ kg})(9.81\text{ m/s}^2) = 4.905\text{ N}$$



(*a*) From f.b.d. of beam *ABCD*

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - W_B - W_C = 0$$

$$A_y - 9.81\text{ N} - 4.905\text{ N} = 0$$

$$\therefore \quad A_y = 14.715\text{ N}$$

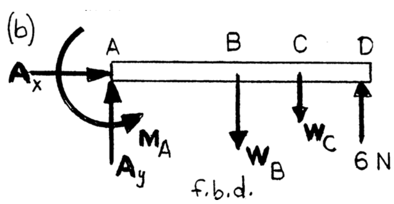
or $\mathbf{A} = 14.72\text{ N} \uparrow$ ◀

$$+\circlearrowleft \Sigma M_A = 0: \quad M_A - W_B(0.2\text{ m}) - W_C(0.3\text{ m}) = 0$$

$$M_A - (9.81\text{ N})(0.2\text{ m}) - (4.905\text{ N})(0.3\text{ m}) = 0$$

$$\therefore \quad M_A = 3.4335\text{ N}\cdot\text{m}$$

or $\mathbf{M}_A = 3.43\text{ N}\cdot\text{m} \circlearrowleft$ ◀



(*b*) From f.b.d. of beam *ABCD*

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - W_B - W_C + 6\text{ N} = 0$$

$$A_y - 9.81\text{ N} - 4.905\text{ N} + 6\text{ N} = 0$$

$$\therefore \quad A_y = 8.715\text{ N}$$

or $\mathbf{A} = 8.72\text{ N} \uparrow$ ◀

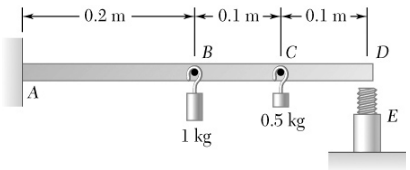
$$+\circlearrowleft \Sigma M_A = 0: \quad M_A - W_B(0.2\text{ m}) - W_C(0.3\text{ m}) + (6\text{ N})(0.4\text{ m}) = 0$$

$$M_A - (9.81\text{ N})(0.2\text{ m}) - (4.905\text{ N})(0.3\text{ m}) + (6\text{ N})(0.4\text{ m}) = 0$$

$$\therefore \quad M_A = 1.03350\text{ N}\cdot\text{m}$$

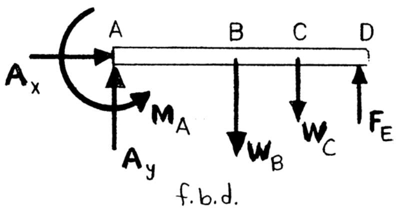
or $\mathbf{M}_A = 1.034\text{ N}\cdot\text{m} \circlearrowleft$ ◀

PROBLEM 4.50



In a laboratory experiment, students hang the masses shown from a beam of negligible mass. Determine the range of values of the force exerted on the beam by the adjustable support E for which the magnitude of the couple at A does not exceed $2.5\text{ N}\cdot\text{m}$.

SOLUTION



$$W_B = m_B g = 1\text{ kg}\left(9.81\text{ m/s}^2\right) = 9.81\text{ N}$$

$$W_C = m_C g = 0.5\text{ kg}\left(9.81\text{ m/s}^2\right) = 4.905\text{ N}$$

Maximum M_A value is $2.5\text{ N}\cdot\text{m}$

F_{min}: From f.b.d. of beam $ABCD$ with $\mathbf{M}_A = 2.5\text{ N}\cdot\text{m}$

$$+\circlearrowleft\ \Sigma M_A = 0:\quad 2.5\text{ N}\cdot\text{m} - W_B(0.2\text{ m}) - W_C(0.3\text{ m})$$

$$+ F_{\text{min}}(0.4\text{ m}) = 0$$

$$2.5\text{ N}\cdot\text{m} - (9.81\text{ N})(0.2\text{ m}) - (4.905\text{ N})(0.3\text{ m}) + F_{\text{min}}(0.4\text{ m}) = 0$$

$$\therefore\quad F_{\text{min}} = 2.3338\text{ N}$$

or $$F_{\text{min}} = 2.33\text{ N}$$

F_{max}: From f.b.d. of beam $ABCD$ with $\mathbf{M}_A = 2.5\text{ N}\cdot\text{m}$

$$+\circlearrowleft\ \Sigma M_A = 0:\quad -2.5\text{ N}\cdot\text{m} - W_B(0.2\text{ m}) - W_C(0.3\text{ m})$$

$$+ F_{\text{max}}(0.4\text{ m}) = 0$$

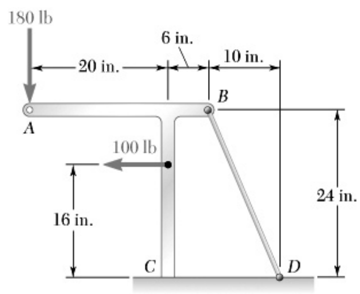
$$-2.5\text{ N}\cdot\text{m} - (9.81\text{ N})(0.2\text{ m}) - (4.905\text{ N})(0.3\text{ m}) + F_{\text{max}}(0.4\text{ m}) = 0$$

$$\therefore\quad F_{\text{max}} = 14.8338\text{ N}$$

or $$F_{\text{max}} = 14.83\text{ N}$$

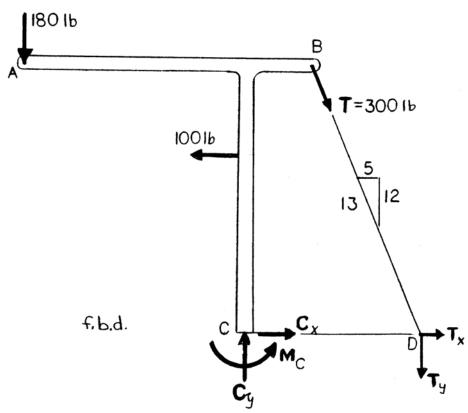
or $2.33\text{ N} \le F_E \le 14.83\text{ N}$ ◀

PROBLEM 4.51



Knowing that the tension in wire *BD* is 300 lb, determine the reaction at fixed support *C* for the frame shown.

SOLUTION



From f.b.d. of frame with $T = 300$ lb

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - 100 \text{ lb} + \left(\frac{5}{13}\right)300 \text{ lb} = 0$$

$$\therefore \quad C_x = -15.3846 \text{ lb} \qquad \text{or} \qquad \mathbf{C}_x = 15.3846 \text{ lb} \longleftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - 180 \text{ lb} - \left(\frac{12}{13}\right)300 \text{ lb} = 0$$

$$\therefore \quad C_y = 456.92 \text{ lb} \qquad \text{or} \qquad \mathbf{C}_y = 456.92 \text{ lb} \uparrow$$

Then $$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(15.3846)^2 + (456.92)^2} = 457.18 \text{ lb}$$

and $$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{456.92}{-15.3846}\right) = -88.072°$$

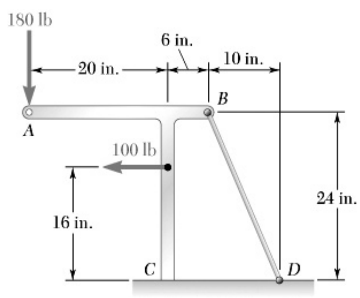
or $\mathbf{C} = 457$ lb ⦣ 88.1° ◀

$$+\circlearrowleft \Sigma M_C = 0: \quad M_C + (180 \text{ lb})(20 \text{ in.}) + (100 \text{ lb})(16 \text{ in.}) - \left[\left(\frac{12}{13}\right)300 \text{ lb}\right](16 \text{ in.}) = 0$$

$$\therefore \quad M_C = -769.23 \text{ lb}\cdot\text{in.}$$

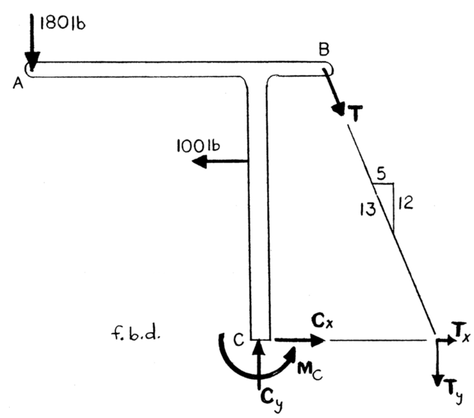
or $\mathbf{M}_C = 769$ lb·in. $\circlearrowright$ ◀

PROBLEM 4.52



Determine the range of allowable values of the tension in wire *BD* if the magnitude of the couple at the fixed support *C* is not to exceed $75\ \text{lb}\cdot\text{ft}$.

SOLUTION



T_{max} From f.b.d. of frame with $\mathbf{M}_C = 75\ \text{lb}\cdot\text{ft} \circlearrowleft = 900\ \text{lb}\cdot\text{in.} \circlearrowleft$

$$+\circlearrowleft \Sigma M_C = 0: \quad 900\ \text{lb}\cdot\text{in.} + (180\ \text{lb})(20\ \text{in.}) + (100\ \text{lb})(16\ \text{in.}) - \left[\left(\frac{12}{13}\right)T_{max}\right](16\ \text{in.}) = 0$$

$$\therefore \quad T_{max} = 413.02\ \text{lb}$$

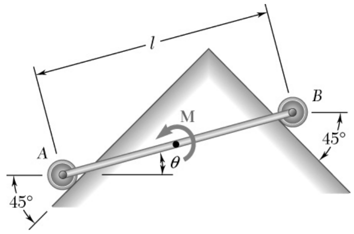
T_{min} From f.b.d. of frame with $\mathbf{M}_C = 75\ \text{lb}\cdot\text{ft} \circlearrowright = 900\ \text{lb}\cdot\text{in.} \circlearrowright$

$$+\circlearrowleft \Sigma M_C = 0: \quad -900\ \text{lb}\cdot\text{in.} + (180\ \text{lb})(20\ \text{in.}) + (100\ \text{lb})(16\ \text{in.}) - \left[\left(\frac{12}{13}\right)T_{min}\right](16\ \text{in.}) = 0$$

$$\therefore \quad T_{min} = 291.15\ \text{lb}$$

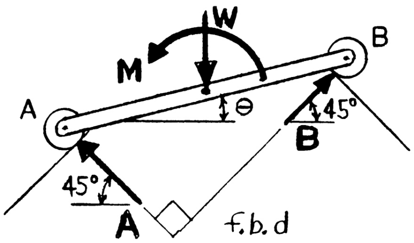
$$\therefore \quad 291\ \text{lb} \le T \le 413\ \text{lb} \blacktriangleleft$$

PROBLEM 4.53



Uniform rod AB of length l and weight W lies in a vertical plane and is acted upon by a couple **M**. The ends of the rod are connected to small rollers which rest against frictionless surfaces. (a) Express the angle θ corresponding to equilibrium in terms of M, W, and l. (b) Determine the value of θ corresponding to equilibrium when $M = 1.5\text{ lb}\cdot\text{ft}$, $W = 4\text{ lb}$, and $l = 2\text{ ft}$.

SOLUTION



(a) From f.b.d. of uniform rod AB

$$\xrightarrow{+} \Sigma F_x = 0: \quad -A\cos 45° + B\cos 45° = 0$$

$$\therefore\ -A + B = 0 \qquad \text{or} \qquad B = A \tag{1}$$

$$+\!\uparrow \Sigma F_y = 0: \quad A\sin 45° + B\sin 45° - W = 0$$

$$\therefore\ A + B = \sqrt{2}W \tag{2}$$

From Equations (1) and (2)

$$2A = \sqrt{2}W$$

$$\therefore\ A = \frac{1}{\sqrt{2}}W$$

From f.b.d. of uniform rod AB

$$+\circlearrowleft \Sigma M_B = 0: \quad W\left[\left(\frac{l}{2}\right)\cos\theta\right] + M$$

$$-\left(\frac{1}{\sqrt{2}}W\right)\left[l\cos\left(45° - \theta\right)\right] = 0 \tag{3}$$

From trigonometric identity

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

Equation (3) becomes

$$\left(\frac{Wl}{2}\right)\cos\theta + M - \left(\frac{Wl}{2}\right)\left(\cos\theta + \sin\theta\right) = 0$$

PROBLEM 4.53 CONTINUED

or $$\left(\frac{Wl}{2}\right)\cos\theta + M - \left(\frac{Wl}{2}\right)\cos\theta - \left(\frac{Wl}{2}\right)\sin\theta = 0$$

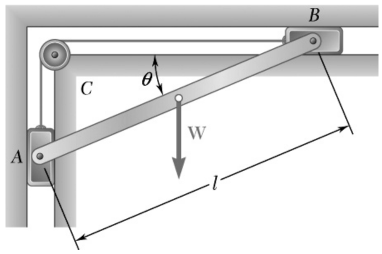
$$\therefore \quad \sin\theta = \frac{2M}{Wl}$$

$$\text{or} \quad \theta = \sin^{-1}\left(\frac{2M}{Wl}\right) \blacktriangleleft$$

(*b*) $$\theta = \sin^{-1}\left[\frac{2(1.5\text{ lb}\cdot\text{ft})}{(4\text{ lb})(2\text{ ft})}\right] = 22.024°$$

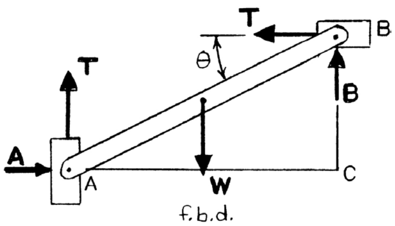
$$\text{or} \quad \theta = 22.0° \blacktriangleleft$$

PROBLEM 4.54



A slender rod *AB*, of weight *W*, is attached to blocks *A* and *B*, which move freely in the guides shown. The blocks are connected by an elastic cord which passes over a pulley at *C*. (*a*) Express the tension in the cord in terms of *W* and θ. (*b*) Determine the value of θ for which the tension in the cord is equal to 3*W*.

SOLUTION



(*a*) From f.b.d. of rod *AB*

$$+\circlearrowleft \Sigma M_C = 0: \quad T(l\sin\theta) + W\left[\left(\frac{l}{2}\right)\cos\theta\right] - T(l\cos\theta) = 0$$

$$\therefore\ T = \frac{W\cos\theta}{2(\cos\theta - \sin\theta)}$$

Dividing both numerator and denominator by $\cos\theta$,

$$T = \frac{W}{2}\left(\frac{1}{1-\tan\theta}\right)$$

$$\text{or } T = \frac{\left(\frac{W}{2}\right)}{(1-\tan\theta)} \blacktriangleleft$$

(*b*) For $T = 3W$,

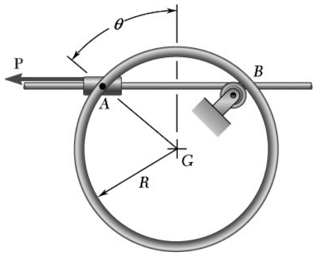
$$3W = \frac{\left(\frac{W}{2}\right)}{(1-\tan\theta)}$$

$$\therefore\ 1 - \tan\theta = \frac{1}{6}$$

or $$\theta = \tan^{-1}\left(\frac{5}{6}\right) = 39.806°$$

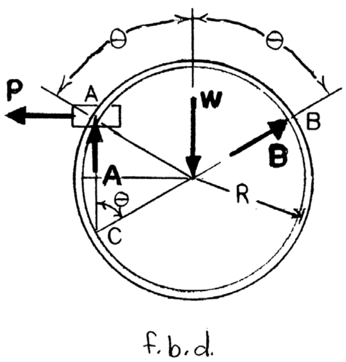
$$\text{or } \theta = 39.8° \blacktriangleleft$$

PROBLEM 4.55



A thin, uniform ring of mass m and radius R is attached by a frictionless pin to a collar at A and rests against a small roller at B. The ring lies in a vertical plane, and the collar can move freely on a horizontal rod and is acted upon by a horizontal force **P**. (a) Express the angle θ corresponding to equilibrium in terms of m and P. (b) Determine the value of θ corresponding to equilibrium when $m = 500$ g and $P = 5$ N.

SOLUTION



(a) From f.b.d. of ring

$$+\circlearrowleft \Sigma M_C = 0: \quad P(R\cos\theta + R\cos\theta) - W(R\sin\theta) = 0$$

$$2P = W\tan\theta \quad \text{where} \quad W = mg$$

$$\therefore \quad \tan\theta = \frac{2P}{mg}$$

$$\text{or} \quad \theta = \tan^{-1}\left(\frac{2P}{mg}\right) \blacktriangleleft$$

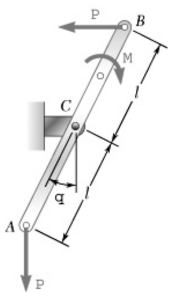
(b) Have $\quad m = 500\text{ g} = 0.500\text{ kg} \quad \text{and} \quad P = 5\text{ N}$

$$\therefore \quad \theta = \tan^{-1}\left[\frac{2(5\text{ N})}{(0.500\text{ kg})(9.81\text{ m/s}^2)}\right]$$

$$= 63.872°$$

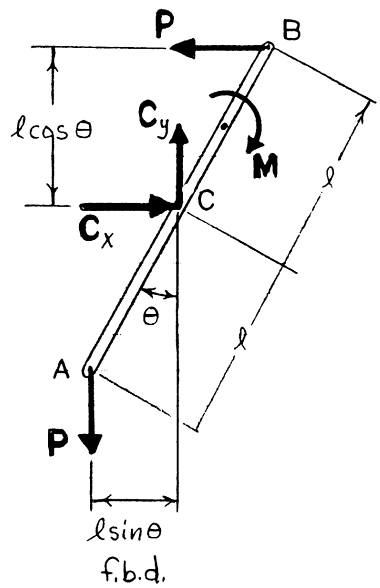
$$\text{or} \quad \theta = 63.9° \blacktriangleleft$$

PROBLEM 4.56



Rod AB is acted upon by a couple **M** and two forces, each of magnitude P. (a) Derive an equation in θ, P, M, and l which must be satisfied when the rod is in equilibrium. (b) Determine the value of θ corresponding to equilibrium when $M = 150$ lb · in., $P = 20$ lb, and $l = 6$ in.

SOLUTION



(a) From f.b.d. of rod AB

$$+\circlearrowright \Sigma M_C = 0: \quad P(l\cos\theta) + P(l\sin\theta) - M = 0$$

$$\text{or} \quad \sin\theta + \cos\theta = \frac{M}{Pl} \blacktriangleleft$$

(b) For $\quad M = 150\text{ lb}\cdot\text{in.}, \; P = 20\text{ lb}, \text{ and } l = 6\text{ in.}$

$$\sin\theta + \cos\theta = \frac{150\text{ lb}\cdot\text{in.}}{(20\text{ lb})(6\text{ in.})} = \frac{5}{4} = 1.25$$

Using identity $\quad \sin^2\theta + \cos^2\theta = 1$

$$\sin\theta + \left(1 - \sin^2\theta\right)^{\frac{1}{2}} = 1.25$$

$$\left(1 - \sin^2\theta\right)^{\frac{1}{2}} = 1.25 - \sin\theta$$

$$1 - \sin^2\theta = 1.5625 - 2.5\sin\theta + \sin^2\theta$$

$$2\sin^2\theta - 2.5\sin\theta + 0.5625 = 0$$

Using quadratic formula

$$\sin\theta = \frac{-(-2.5) \pm \sqrt{(6.25) - 4(2)(0.5625)}}{2(2)}$$

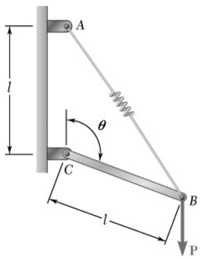
$$= \frac{2.5 \pm \sqrt{1.75}}{4}$$

or $\quad \sin\theta = 0.95572 \quad$ and $\quad \sin\theta = 0.29428$

$$\therefore \quad \theta = 72.886° \quad \text{and} \quad \theta = 17.1144°$$

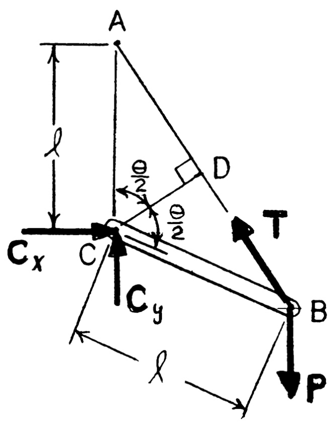
$$\text{or} \quad \theta = 17.11° \text{ and } \theta = 72.9° \blacktriangleleft$$

PROBLEM 4.57



A vertical load **P** is applied at end *B* of rod *BC*. The constant of the spring is *k*, and the spring is unstretched when $\theta = 90°$. (*a*) Neglecting the weight of the rod, express the angle θ corresponding to equilibrium in terms of *P*, *k*, and *l*. (*b*) Determine the value of θ corresponding to equilibrium when $P = \frac{1}{4}kl$.

SOLUTION



First note

$$T = \text{tension in spring} = ks$$

where $\quad s = \text{elongation of spring}$

$$= \left(\overline{AB}\right)_\theta - \left(\overline{AB}\right)_{\theta=90°}$$

$$= 2l\sin\left(\frac{\theta}{2}\right) - 2l\sin\left(\frac{90°}{2}\right)$$

$$= 2l\left[\sin\left(\frac{\theta}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\right]$$

$$\therefore \quad T = 2kl\left[\sin\left(\frac{\theta}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\right] \qquad (1)$$

(*a*) From f.b.d. of rod *BC*

$$+\circlearrowleft \Sigma M_C = 0: \quad T\left[l\cos\left(\frac{\theta}{2}\right)\right] - P(l\sin\theta) = 0$$

Substituting *T* From Equation (1)

$$2kl\left[\sin\left(\frac{\theta}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\right]\left[l\cos\left(\frac{\theta}{2}\right)\right] - P(l\sin\theta) = 0$$

$$2kl^2\left[\sin\left(\frac{\theta}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\right]\cos\left(\frac{\theta}{2}\right) - Pl\left[2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\right] = 0$$

Factoring out $\quad 2l\cos\left(\frac{\theta}{2}\right)$, leaves

PROBLEM 4.57 CONTINUED

$$kl\left[\sin\left(\frac{\theta}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\right] - P\sin\left(\frac{\theta}{2}\right) = 0$$

or

$$\sin\left(\frac{\theta}{2}\right) = \frac{1}{\sqrt{2}}\left(\frac{kl}{kl - P}\right)$$

$$\therefore \quad \theta = 2\sin^{-1}\left[\frac{kl}{\sqrt{2}(kl - P)}\right] \blacktriangleleft$$

$(b)\quad P = \dfrac{kl}{4}$

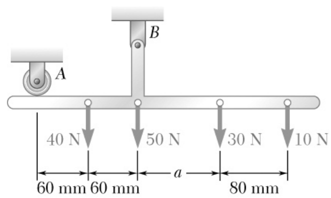
$$\theta = 2\sin^{-1}\left[\frac{kl}{\sqrt{2}\left(kl - \frac{kl}{4}\right)}\right] = 2\sin^{-1}\left[\frac{kl}{\sqrt{2}}\left(\frac{4}{3\,\text{kl}}\right)\right] = 2\sin^{-1}\left(\frac{4}{3\sqrt{2}}\right)$$

$$= 2\sin^{-1}(0.94281)$$

$$= 141.058°$$

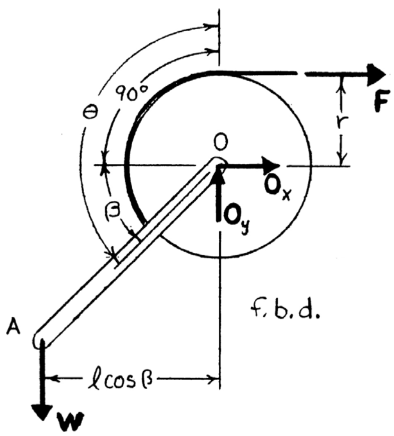
$$\text{or}\quad \theta = 141.1° \blacktriangleleft$$

PROBLEM 4.58



Solve Sample Problem 4.5 assuming that the spring is unstretched when $\theta = 90^{\circ}$.

SOLUTION



First note

$$T = \text{tension in spring} = ks$$

where $$s = \text{deformation of spring}$$

$$= r\beta$$

$$\therefore \quad F = kr\beta$$

From f.b.d. of assembly

$$+\circlearrowleft \Sigma M_0 = 0: \quad W(l\cos\beta) - F(r) = 0$$

or $$Wl\cos\beta - kr^2\beta = 0$$

$$\therefore \quad \cos\beta = \frac{kr^2}{Wl}\beta$$

For $$k = 250 \text{ lb/in.}, \ r = 3 \text{ in.}, \ l = 8 \text{ in.}, \ W = 400 \text{ lb}$$

$$\cos\beta = \frac{(250 \text{ lb/in.})(3 \text{ in.})^2}{(400 \text{ lb})(8 \text{ in.})}\beta$$

or $$\cos\beta = 0.703125\beta$$

Solving numerically,

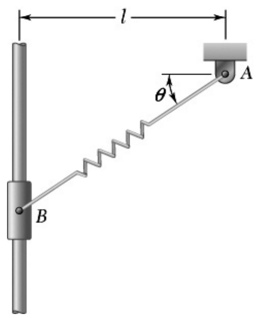
$$\beta = 0.89245 \text{ rad}$$

or $$\beta = 51.134^{\circ}$$

Then $$\theta = 90^{\circ} + 51.134^{\circ} = 141.134^{\circ}$$

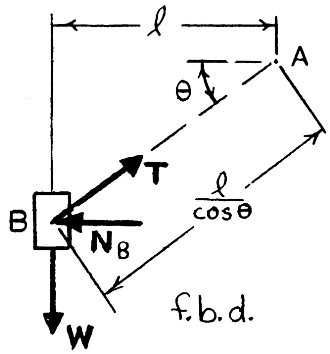
or $\theta = 141.1^{\circ}$ ◀

PROBLEM 4.59



A collar B of weight W can move freely along the vertical rod shown. The constant of the spring is k, and the spring is unstretched when $\theta = 0$. (a) Derive an equation in θ, W, k, and l which must be satisfied when the collar is in equilibrium. (b) Knowing that $W = 3\text{ lb}$, $l = 6\text{ in.}$, and $k = 8\text{ lb/ft}$, determine the value of θ corresponding to equilibrium.

SOLUTION



First note $\quad T = ks$

where $\quad k = \text{spring constant}$

$$s = \text{elongation of spring}$$

$$= \frac{l}{\cos\theta} - l = \frac{l}{\cos\theta}(1 - \cos\theta)$$

$$\therefore \quad T = \frac{kl}{\cos\theta}(1 - \cos\theta)$$

(a) From f.b.d. of collar B

$$+\uparrow \Sigma F_y = 0: \quad T\sin\theta - W = 0$$

or
$$\frac{kl}{\cos\theta}(1 - \cos\theta)\sin\theta - W = 0$$

$$\text{or} \quad \tan\theta - \sin\theta = \frac{W}{kl} \blacktriangleleft$$

(b) For $W = 3\text{ lb}$, $l = 6\text{ in.}$, $k = 8\text{ lb/ft}$

$$l = \frac{6\text{ in.}}{12\text{ in./ft}} = 0.5\text{ ft}$$

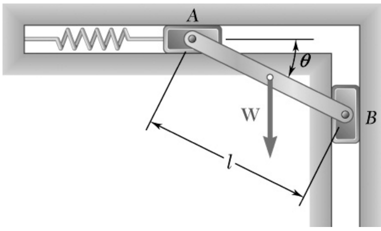
$$\tan\theta - \sin\theta = \frac{3\text{ lb}}{(8\text{ lb/ft})(0.5\text{ ft})} = 0.75$$

Solving Numerically,

$$\theta = 57.957°$$

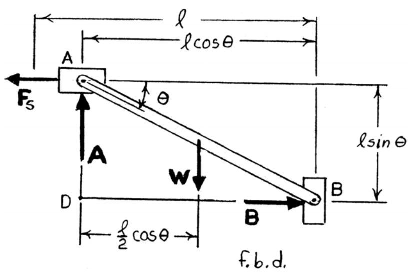
$$\text{or} \quad \theta = 58.0° \blacktriangleleft$$

PROBLEM 4.60



A slender rod *AB*, of mass *m*, is attached to blocks *A* and *B* which move freely in the guides shown. The constant of the spring is *k*, and the spring is unstretched when $\theta = 0$. (*a*) Neglecting the mass of the blocks, derive an equation in *m*, *g*, *k*, *l*, and θ which must be satisfied when the rod is in equilibrium. (*b*) Determine the value of θ when $m = 2$ kg, $l = 750$ mm, and $k = 30$ N/m.

SOLUTION



First note

$$F_s = \text{spring force} = ks$$

where

$$k = \text{spring constant}$$

$$s = \text{spring deformation}$$

$$= l - l\cos\theta$$

$$= l(1 - \cos\theta)$$

$$\therefore \quad F_s = kl(1 - \cos\theta)$$

(*a*) From f.b.d. of assembly

$$+\circlearrowleft \Sigma M_D = 0: \quad F_s(l\sin\theta) - W\left(\frac{l}{2}\cos\theta\right) = 0$$

$$kl(1 - \cos\theta)(l\sin\theta) - W\left(\frac{l}{2}\cos\theta\right) = 0$$

$$kl(\sin\theta - \cos\theta\sin\theta) - \left(\frac{W}{2}\right)\cos\theta = 0$$

Dividing by $\cos\theta$

$$kl(\tan\theta - \sin\theta) = \frac{W}{2}$$

$$\therefore \quad \tan\theta - \sin\theta = \frac{W}{2kl}$$

$$\text{or} \quad \tan\theta - \sin\theta = \frac{mg}{2kl} \blacktriangleleft$$

(*b*) For $m = 2$ kg, $l = 750$ mm, $k = 30$ N/m

$$l = 750 \text{ mm} = 0.750 \text{ m}$$

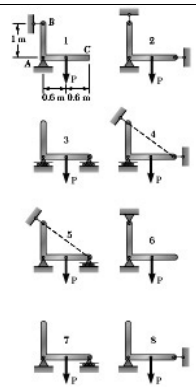
PROBLEM 4.60 CONTINUED

Then $$\tan\theta - \sin\theta = \frac{(2\text{ kg})(9.81\text{ m/s}^2)}{2(30\text{ N/m})(0.750\text{ m})} = 0.436$$

Solving Numerically,

$$\theta = 50.328°$$

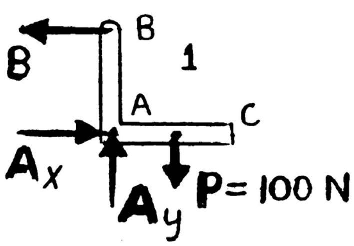
or $\theta = 50.3°$ ◀



PROBLEM 4.61

The bracket *ABC* can be supported in the eight different ways shown. All connections consist of smooth pins, rollers, or short links. In each case, determine whether (*a*) the plate is completely, partially, or improperly constrained, (*b*) the reactions are statically determinate or indeterminate, (*c*) the equilibrium of the plate is maintained in the position shown. Also, wherever possible, compute the reactions assuming that the magnitude of the force **P** is 100 N.

SOLUTION



1. Three non-concurrent, non-parallel reactions

(*a*) Completely constrained ◀

(*b*) Determinate ◀

(*c*) Equilibrium ◀

From f.b.d. of bracket:

$$+\circlearrowleft\ \Sigma M_A = 0: \quad B(1\text{ m}) - (100\text{ N})(0.6\text{ m}) = 0$$

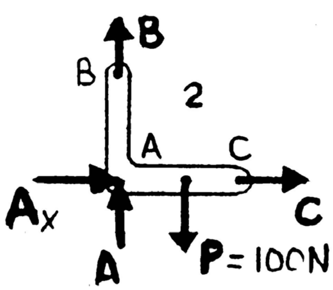
$$\therefore \quad \mathbf{B} = 60.0\text{ N} \longleftarrow \blacktriangleleft$$

$$\xrightarrow{+}\ \Sigma F_x = 0: \quad A_x - 60\text{ N} = 0$$

$$\therefore \quad \mathbf{A}_x = 60.0\text{ N} \longrightarrow$$

$$+\uparrow\ \Sigma F_y = 0: \quad A_y - 100\text{ N} = 0$$

$$\therefore \quad \mathbf{A}_y = 100\text{ N} \uparrow$$



Then $$A = \sqrt{(60.0)^2 + (100)^2} = 116.619\text{ N}$$

and $$\theta = \tan^{-1}\left(\frac{100}{60.0}\right) = 59.036^\circ$$

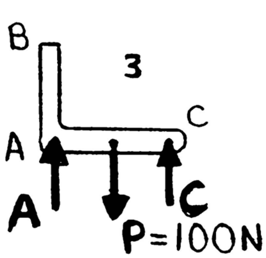
$$\therefore \quad \mathbf{A} = 116.6\text{ N} \measuredangle 59.0^\circ \blacktriangleleft$$

2. Four concurrent reactions through *A*

(*a*) Improperly constrained ◀

(*b*) Indeterminate ◀

(*c*) No equilibrium ◀



3. Two reactions

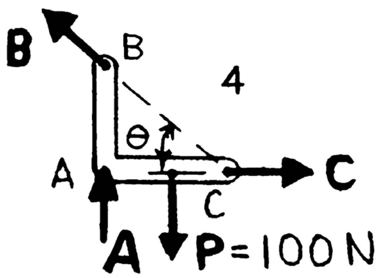
(*a*) Partially constrained ◀

(*b*) Determinate ◀

(*c*) Equilibrium ◀

PROBLEM 4.61 CONTINUED

From f.b.d. of bracket



$$+\circlearrowleft \Sigma M_A = 0: \quad C(1.2 \text{ m}) - (100 \text{ N})(0.6 \text{ m}) = 0$$

$\therefore$ **C** $= 50.0$ N $\uparrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad A - 100 \text{ N} + 50 \text{ N} = 0$$

$\therefore$ **A** $= 50.0$ N $\uparrow$ ◀

4. Three non-concurrent, non-parallel reactions

(*a*) Completely constrained ◀

(*b*) Determinate ◀

(*c*) Equilibrium ◀

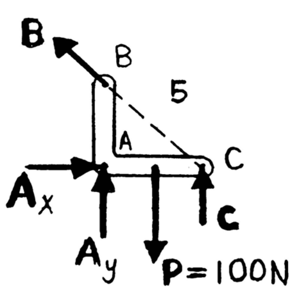
From f.b.d. of bracket

$$\theta = \tan^{-1}\left(\frac{1.0}{1.2}\right) = 39.8°$$

$$\overline{BC} = \sqrt{(1.2)^2 + (1.0)^2} = 1.56205 \text{ m}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad \left[\left(\frac{1.2}{1.56205}\right)B\right](1 \text{ m}) - (100 \text{ N})(0.6 \text{ m}) = 0$$

$\therefore$ **B** $= 78.1$ N $\searrow$ 39.8° ◀



$$\xrightarrow{+} \Sigma F_x = 0: \quad C - (78.102 \text{ N})\cos 39.806° = 0$$

$\therefore$ **C** $= 60.0$ N $\longrightarrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad A + (78.102 \text{ N})\sin 39.806° - 100 \text{ N} = 0$$

$\therefore$ **A** $= 50.0$ N $\uparrow$ ◀

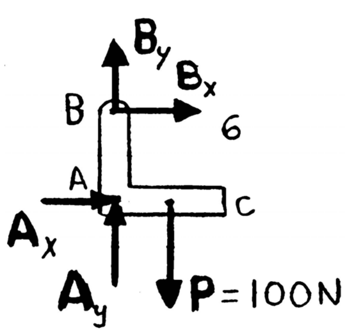
5. Four non-concurrent, non-parallel reactions

(*a*) Completely constrained ◀

(*b*) Indeterminate ◀

(*c*) Equilibrium ◀

From f.b.d. of bracket



$$+\circlearrowleft \Sigma M_C = 0: \quad (100 \text{ N})(0.6 \text{ m}) - A_y(1.2 \text{ m}) = 0$$

$\therefore$ $A_y = 50$ N or $\mathbf{A}_y = 50.0$ N $\uparrow$ ◀

6. Four non-concurrent non-parallel reactions

(*a*) Completely constrained ◀

(*b*) Indeterminate ◀

(*c*) Equilibrium ◀

PROBLEM 4.61 CONTINUED

From f.b.d. of bracket

$$+\circlearrowleft \Sigma M_A = 0: \quad -B_x(1\text{ m}) - (100\text{ N})(0.6\text{ m}) = 0$$

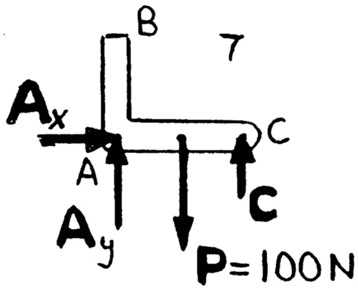
$$\therefore \quad B_x = -60.0\text{ N}$$

or $\mathbf{B}_x = 60.0\text{ N} \longleftarrow$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad -60 + A_x = 0$$

$$\therefore \quad A_x = 60.0\text{ N}$$

or $\mathbf{A}_x = 60.0\text{ N} \longrightarrow$ ◀



7. Three non-concurrent, non-parallel reactions

(*a*) Completely constrained ◀

(*b*) Determinate ◀

(*c*) Equilibrium ◀

From f.b.d. of bracket

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$

$$+\circlearrowleft \Sigma M_A = 0: \quad C(1.2\text{ m}) - (100\text{ N})(0.6\text{ m}) = 0$$

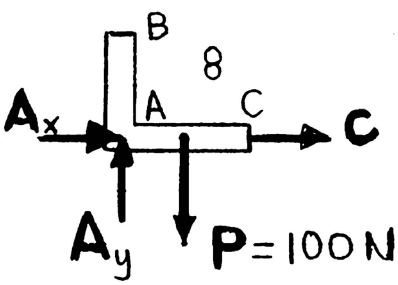
$$\therefore \quad C = 50.0\text{ N}$$

or $\mathbf{C} = 50.0\text{ N} \uparrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad A_y - 100\text{ N} + 50.0\text{ N} = 0$$

$$\therefore \quad A_y = 50.0\text{ N}$$

$\therefore \quad \mathbf{A} = 50.0\text{ N} \uparrow$ ◀



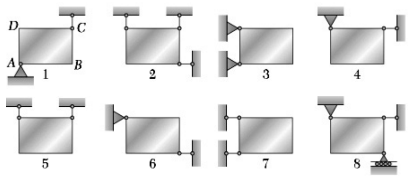
8. Three concurrent, non-parallel reactions

(*a*) Improperly constrained ◀

(*b*) Indeterminate ◀

(*c*) No equilibrium ◀

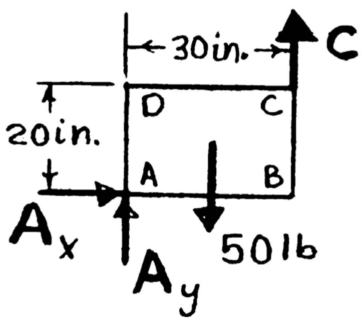
PROBLEM 4.62



Eight identical 20×30-in. rectangular plates, each weighing 50 lb, are held in a vertical plane as shown. All connections consist of frictionless pins, rollers, or short links. For each case, answer the questions listed in Problem 4.61, and, wherever possible, compute the reactions.

P6.1 The bracket *ABC* can be supported in the eight different ways shown. All connections consist of smooth pins, rollers, or short links. In each case, determine whether (*a*) the plate is completely, partially, or improperly constrained, (*b*) the reactions are statically determinate or indeterminate, (*c*) the equilibrium of the plate is maintained in the position shown. Also, wherever possible, compute the reactions assuming that the magnitude of the force **P** is 100 N.

SOLUTION



1. Three non-concurrent, non-parallel reactions

(*a*) Completely constrained ◀

(*b*) Determinate ◀

(*c*) Equilibrium ◀

From f.b.d. of plate

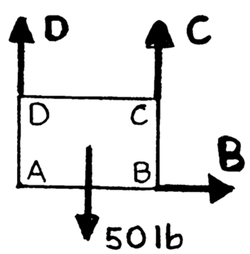
$$+\circlearrowleft \Sigma M_A = 0: \quad C(30 \text{ in.}) - 50 \text{ lb}(15 \text{ in.}) = 0$$

$\mathbf{C} = 25.0 \text{ lb} \uparrow$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 50 \text{ lb} + 25 \text{ lb} = 0$$

$A_y = 25 \text{ lb}$ $\quad\quad$ $\mathbf{A} = 25.0 \text{ lb} \uparrow$ ◀



2. Three non-current, non-parallel reactions

(*a*) Completely constrained ◀

(*b*) Determinate ◀

(*c*) Equilibrium ◀

From f.b.d. of plate

$\xrightarrow{+} \Sigma F_x = 0:$ $\quad\quad$ $\mathbf{B} = 0$ ◀

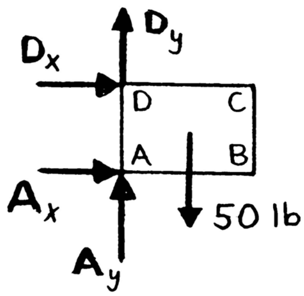
$$+\circlearrowleft \Sigma M_B = 0: \quad (50 \text{ lb})(15 \text{ in.}) - D(30 \text{ in.}) = 0$$

$\mathbf{D} = 25.0 \text{ lb} \uparrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad 25.0 \text{ lb} - 50 \text{ lb} + C = 0$$

$\mathbf{C} = 25.0 \text{ lb} \uparrow$ ◀

PROBLEM 4.62 CONTINUED



3. Four non-concurrent, non-parallel reactions

(*a*) Completely constrained ◀

(*b*) Indeterminate ◀

(*c*) Equilibrium ◀

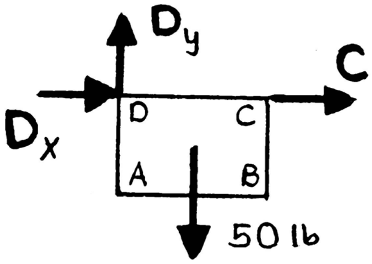
From f.b.d. of plate

$$+\circlearrowleft \Sigma M_D = 0: \quad A_x(20 \text{ in.}) - (50 \text{ lb})(15 \text{ in.})$$

$$\therefore \quad \mathbf{A}_x = 37.5 \text{ lb} \longrightarrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad D_x + 37.5 \text{ lb} = 0$$

$$\therefore \quad \mathbf{D}_x = 37.5 \text{ lb} \longleftarrow \blacktriangleleft$$

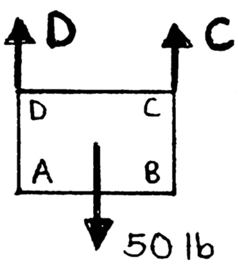


4. Three concurrent reactions

(*a*) Improperly constrained ◀

(*b*) Indeterminate ◀

(*c*) No equilibrium ◀



5. Two parallel reactions

(*a*) Partial constraint ◀

(*b*) Determinate ◀

(*c*) Equilibrium ◀

From f.b.d. of plate

$$+\circlearrowleft \Sigma M_D = 0: \quad C(30 \text{ in.}) - (50 \text{ lb})(15 \text{ in.}) = 0$$

$$\mathbf{C} = 25.0 \text{ lb} \uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad D - 50 \text{ lb} + 25 \text{ lb} = 0$$

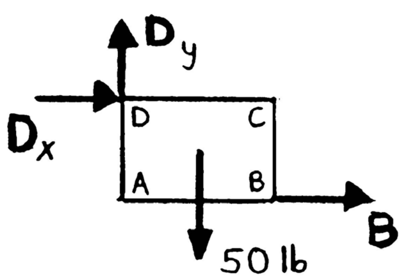
$$\mathbf{D} = 25.0 \text{ lb} \uparrow \blacktriangleleft$$

6. Three non-concurrent, non-parallel reactions

(*a*) Completely constrained ◀

(*b*) Determinate ◀

(*c*) Equilibrium ◀



From f.b.d. of plate

$$+\circlearrowleft \Sigma M_D = 0: \quad B(20 \text{ in.}) - (50 \text{ lb})(15 \text{ in.}) = 0$$

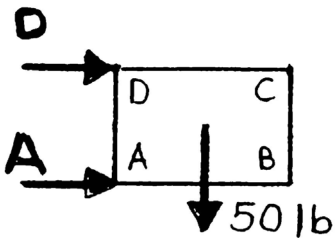
$$\mathbf{B} = 37.5 \text{ lb} \longrightarrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad D_x + 37.5 \text{ lb} = 0 \qquad \mathbf{D}_x = 37.5 \text{ lb} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad D_y - 50 \text{ lb} = 0 \qquad \mathbf{D}_y = 50.0 \text{ lb} \uparrow$$

$$\text{or } \mathbf{D} = 62.5 \text{ lb} \searrow 53.1^\circ \blacktriangleleft$$

PROBLEM 4.62 CONTINUED



7. Two parallel reactions

(*a*) Improperly constrained ◀

(*b*) Reactions determined by dynamics ◀

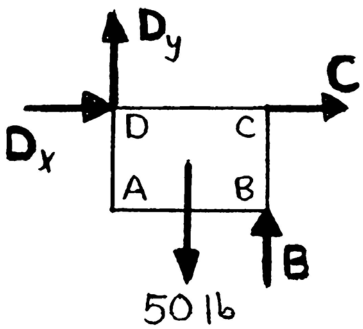
(*c*) No equilibrium ◀

8. Four non-concurrent, non-parallel reactions

(*a*) Completely constrained ◀

(*b*) Indeterminate ◀

(*c*) Equilibrium ◀



From f.b.d. of plate

$$+\circlearrowright \Sigma M_D = 0: \quad B(30 \text{ in.}) - (50 \text{ lb})(15 \text{ in.}) = 0$$

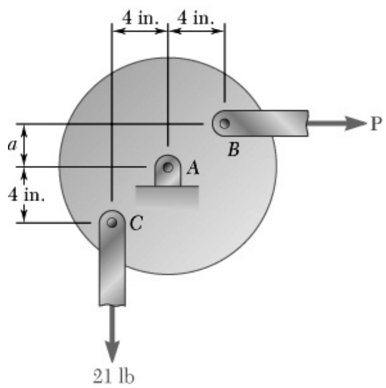
$$\mathbf{B} = 25.0 \text{ lb} \uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad D_y - 50 \text{ lb} + 25.0 \text{ lb} = 0$$

$$\mathbf{D}_y = 25.0 \text{ lb} \uparrow \blacktriangleleft$$

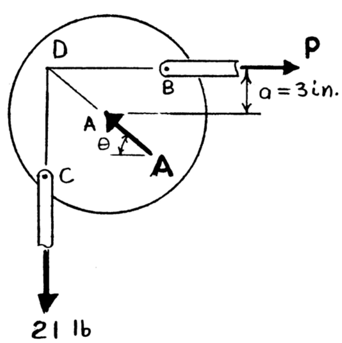
$$\xrightarrow{+} \Sigma F_x = 0: \quad D_x + C = 0$$

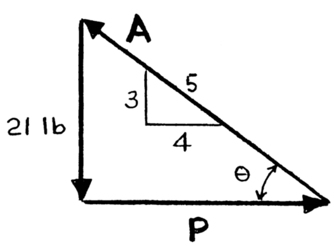
PROBLEM 4.63



Horizontal and vertical links are hinged to a wheel, and forces are applied to the links as shown. Knowing that $a = 3.0$ in., determine the value of P and the reaction at A.

SOLUTION





As shown on the f.b.d., the wheel is a three-force body. Let point D be the intersection of the three forces.

From force triangle

$$\frac{A}{5} = \frac{P}{4} = \frac{21\text{ lb}}{3}$$

$$\therefore \quad P = \frac{4}{3}(21\text{ lb}) = 28\text{ lb}$$

or $P = 28.0$ lb ◀

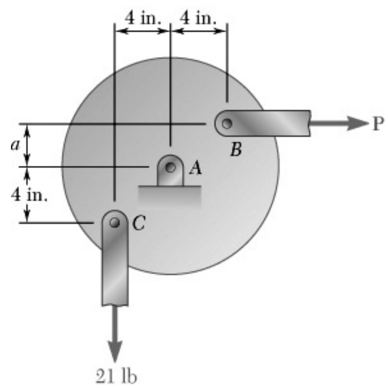
and

$$A = \frac{5}{3}(21\text{ lb}) = 35\text{ lb}$$

$$\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.870°$$

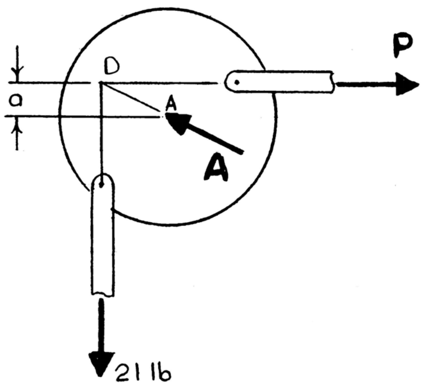
$\therefore \quad \mathbf{A} = 35.0\text{ lb} \measuredangle 36.9°$ ◀

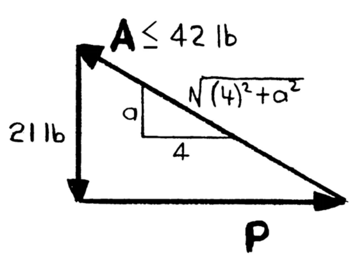
PROBLEM 4.64



Horizontal and vertical links are hinged to a wheel, and forces are applied to the links as shown. Determine the range of values of the distance a for which the magnitude of the reaction at A does not exceed 42 lb.

SOLUTION





Let D be the intersection of the three forces acting on the wheel.

From the force triangle

$$\frac{21\text{ lb}}{a} = \frac{A}{\sqrt{16 + a^2}}$$

or $$A = 21\sqrt{\frac{16}{a^2} + 1}$$

For $$A = 42 \text{ lb}$$

$$\frac{21\text{ lb}}{a} = \frac{42\text{ lb}}{\sqrt{16 + a^2}}$$

or $$a^2 = \frac{16 + a^2}{4}$$

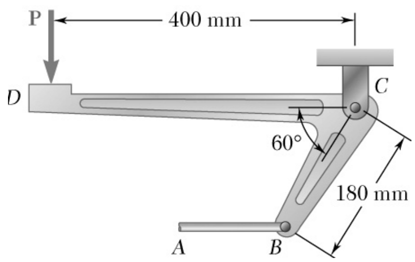
or $$a = \sqrt{\frac{16}{3}} = 2.3094\text{ in.}$$

or $a \geq 2.31$ in. ◀

Since $$A = 21\sqrt{\frac{16}{a^2} + 1}$$

as a increases, A decreases

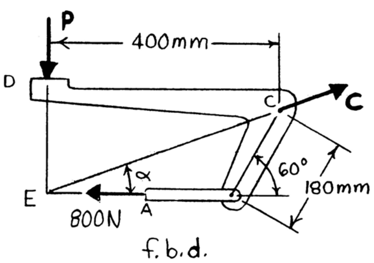
PROBLEM 4.65

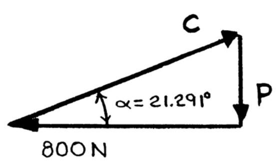


Using the method of Section 4.7, solve Problem 4.21.

P4.21 The required tension in cable *AB* is 800 N. Determine (*a*) the vertical force **P** which must be applied to the pedal, (*b*) the corresponding reaction at *C*.

SOLUTION





Let *E* be the intersection of the three forces acting on the pedal device.

First note

$$\alpha = \tan^{-1}\left[\frac{(180 \text{ mm})\sin 60°}{400 \text{ mm}}\right] = 21.291°$$

From force triangle

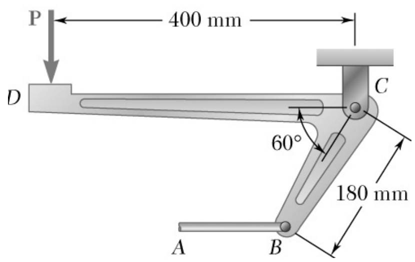
(*a*)
$$P = (800 \text{ N})\tan 21.291°$$
$$= 311.76 \text{ N}$$

or $\mathbf{P} = 312 \text{ N} \downarrow$ ◀

(*b*)
$$C = \frac{800 \text{ N}}{\cos 21.291°}$$
$$= 858.60 \text{ N}$$

or $\mathbf{C} = 859 \text{ N} \measuredangle 21.3°$ ◀

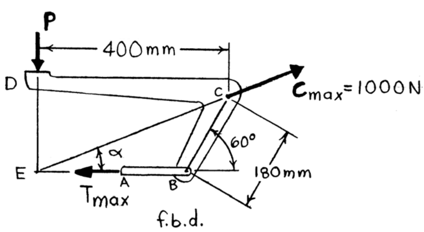
PROBLEM 4.66

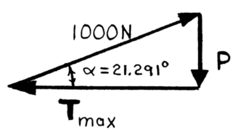


Using the method of Section 4.7, solve Problem 4.22.

P4.22 Determine the maximum tension which can be developed in cable *AB* if the maximum allowable value of the reaction at *C* is 1000 N.

SOLUTION





Let *E* be the intersection of the three forces acting on the pedal device.

First note

$$\alpha = \tan^{-1}\left[\frac{(180\text{ mm})\sin 60°}{400\text{ mm}}\right] = 21.291°$$

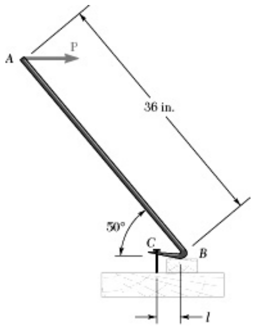
From force triangle

$$T_{max} = (1000\text{ N})\cos 21.291°$$

$$= 931.75\text{ N}$$

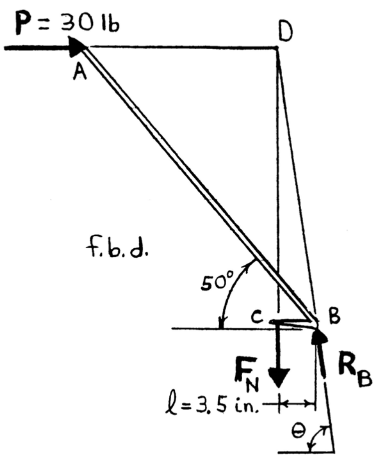
or $T_{max} = 932\text{ N}$ ◀

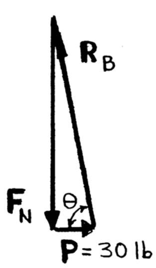
PROBLEM 4.67



To remove a nail, a small block of wood is placed under a crowbar, and a horizontal force **P** is applied as shown. Knowing that $l = 3.5\text{ in.}$ and $P = 30\text{ lb}$, determine the vertical force exerted on the nail and the reaction at *B*.

SOLUTION





Let *D* be the intersection of the three forces acting on the crowbar.

First note

$$\theta = \tan^{-1}\left[\frac{(36\text{ in.})\sin 50°}{3.5\text{ in.}}\right] = 82.767°$$

From force triangle

$$F_N = P\tan\theta = (30\text{ lb})\tan 82.767°$$

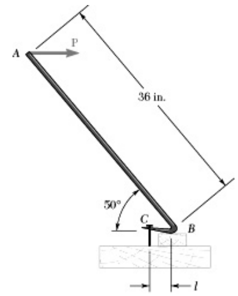
$$= 236.381\text{ lb}$$

$$\therefore \quad \text{on nail } \mathbf{F}_N = 236\text{ lb} \uparrow \blacktriangleleft$$

$$R_B = \frac{P}{\cos\theta} = \frac{30\text{ lb}}{\cos 82.767°} = 238.28\text{ lb}$$

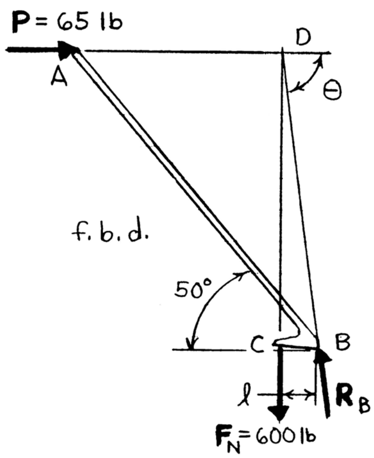
$$\text{or } \mathbf{R}_B = 238\text{ lb} \quad 82.8° \blacktriangleleft$$

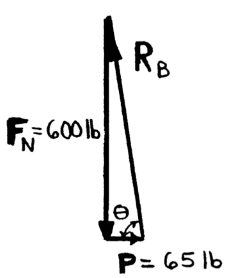
PROBLEM 4.68



To remove a nail, a small block of wood is placed under a crowbar, and a horizontal force **P** is applied as shown. Knowing that the maximum vertical force needed to extract the nail is 600 lb and that the horizontal force **P** is not to exceed 65 lb, determine the largest acceptable value of distance l.

SOLUTION





Let D be the intersection of the three forces acting on the crowbar.

From force diagram

$$\tan\theta = \frac{F_N}{P} = \frac{600\text{ lb}}{65\text{ lb}} = 9.2308$$

$$\therefore \quad \theta = 83.817°$$

From f.b.d.

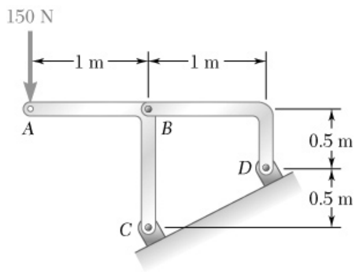
$$\tan\theta = \frac{(36\text{ in.})\sin 50°}{l}$$

$$\therefore \quad l = \frac{(36\text{ in.})\sin 50°}{\tan 83.817°} = 2.9876\text{ in.}$$

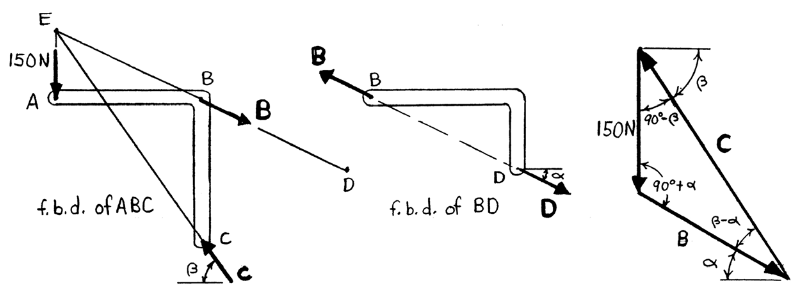
or $l = 2.99$ in. ◀

PROBLEM 4.69

For the frame and loading shown, determine the reactions at *C* and *D*.



SOLUTION



Since member *BD* is acted upon by two forces, **B** and **D**, they must be colinear, have the same magnitude, and be opposite in direction for *BD* to be in equilibrium. The force **B** acting at *B* of member *ABC* will be equal in magnitude but opposite in direction to force **B** acting on member *BD*. Member *ABC* is a three-force body with member forces intersecting at *E*. The f.b.d.'s of members *ABC* and *BD* illustrate the above conditions. The force triangle for member *ABC* is also shown. The angles α and β are found from the member dimensions:

$$\alpha = \tan^{-1}\left(\frac{0.5 \text{ m}}{1.0 \text{ m}}\right) = 26.565°$$

$$\beta = \tan^{-1}\left(\frac{1.5 \text{ m}}{1.0 \text{ m}}\right) = 56.310°$$

Applying the law of sines to the force triangle for member *ABC*,

$$\frac{150 \text{ N}}{\sin(\beta - \alpha)} = \frac{C}{\sin(90° + \alpha)} = \frac{B}{\sin(90° - \beta)}$$

or

$$\frac{150 \text{ N}}{\sin 29.745°} = \frac{C}{\sin 116.565°} = \frac{B}{\sin 33.690°}$$

$$\therefore \quad C = \frac{(150 \text{ N})\sin 116.565°}{\sin 29.745°} = 270.42 \text{ N}$$

or $\mathbf{C} = 270$ N ↘ 56.3° ◀

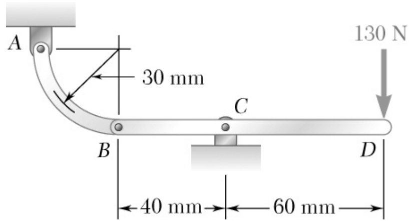
and

$$D = B = \frac{(150 \text{ N})\sin 33.690°}{\sin 29.745°} = 167.704 \text{ N}$$

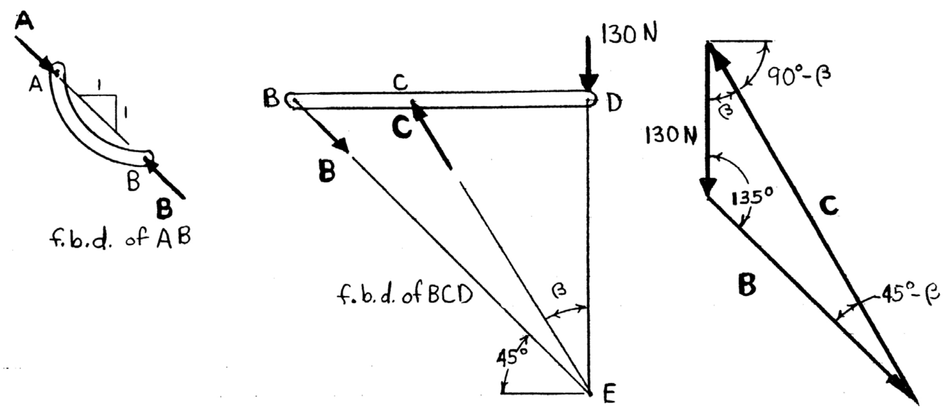
or $\mathbf{D} = 167.7$ N ↘ 26.6° ◀

PROBLEM 4.70

For the frame and loading shown, determine the reactions at *A* and *C*.



SOLUTION



Since member *AB* is acted upon by two forces, **A** and **B**, they must be colinear, have the same magnitude, and be opposite in direction for *AB* to be in equilibrium. The force **B** acting at *B* of member *BCD* will be equal in magnitude but opposite in direction to force **B** acting on member *AB*. Member *BCD* is a three-force body with member forces intersecting at *E*. The f.b.d.'s of members *AB* and *BCD* illustrate the above conditions. The force triangle for member *BCD* is also shown. The angle β is found from the member dimensions:

$$\beta = \tan^{-1}\left(\frac{60 \text{ m}}{100 \text{ m}}\right) = 30.964°$$

Applying of the law of sines to the force triangle for member *BCD*,

$$\frac{130 \text{ N}}{\sin(45° - \beta)} = \frac{B}{\sin\beta} = \frac{C}{\sin 135°}$$

or

$$\frac{130 \text{ N}}{\sin 14.036°} = \frac{B}{\sin 30.964°} = \frac{C}{\sin 135°}$$

$$\therefore \quad A = B = \frac{(130 \text{ N})\sin 30.964°}{\sin 14.036°} = 275.78 \text{ N}$$

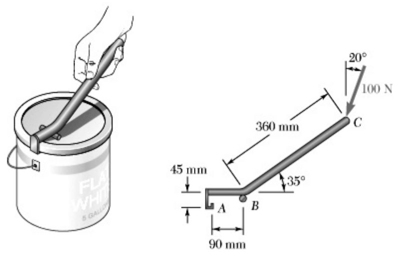
or **A** = 276 N ⦪ 45.0° ◀

and

$$C = \frac{(130 \text{ N})\sin 135°}{\sin 14.036°} = 379.02 \text{ N}$$

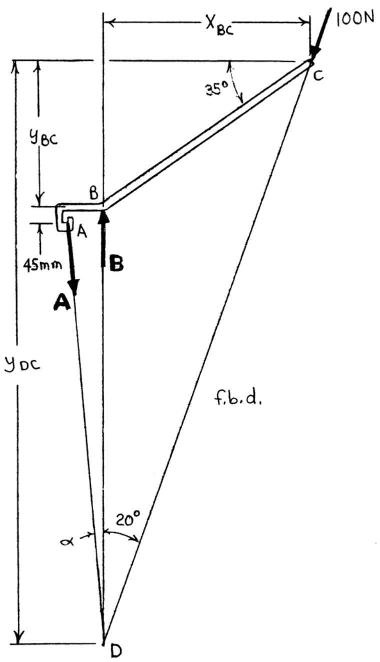
or **C** = 379 N ⦪ 59.0° ◀

PROBLEM 4.71



To remove the lid from a 5-gallon pail, the tool shown is used to apply an upward and radially outward force to the bottom inside rim of the lid. Assuming that the rim rests against the tool at A and that a 100-N force is applied as indicated to the handle, determine the force acting on the rim.

SOLUTION



The three-force member *ABC* has forces that intersect at *D*, where

$$\alpha = \tan^{-1}\left(\frac{90 \text{ mm}}{y_{DC} - y_{BC} - 45 \text{ mm}}\right)$$

and

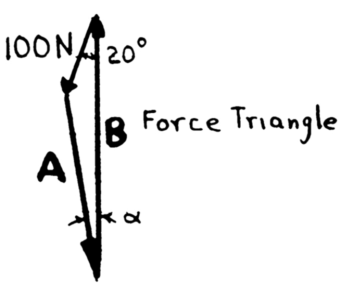
$$y_{DC} = \frac{x_{BC}}{\tan 20°} = \frac{(360 \text{ mm})\cos 35°}{\tan 20°}$$

$$= 810.22 \text{ mm}$$

$$y_{BC} = (360 \text{ mm})\sin 35°$$

$$= 206.49 \text{ mm}$$

$$\therefore \alpha = \tan^{-1}\left(\frac{90}{558.73}\right) = 9.1506°$$



Based on the force triangle, the law of sines gives

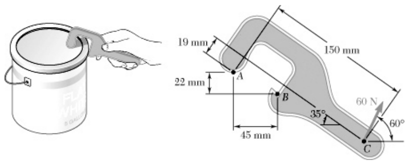
$$\frac{100 \text{ N}}{\sin \alpha} = \frac{A}{\sin 20°}$$

$$\therefore A = \frac{(100 \text{ N})\sin 20°}{\sin 9.1506°} = 215.07 \text{ N}$$

or $\mathbf{A} = 215 \text{ N} \nwarrow 80.8°$ on tool

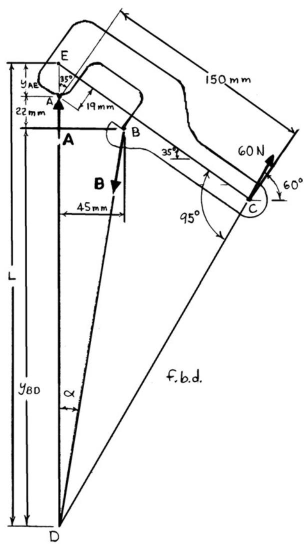
and $\mathbf{A} = 215 \text{ N} \searrow 80.8°$ on rim of can ◀

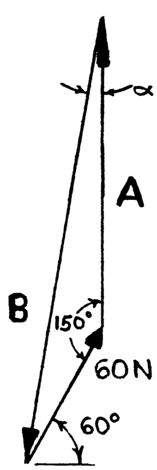
PROBLEM 4.72



To remove the lid from a 5-gallon pail, the tool shown is used to apply an upward and radially outward force to the bottom inside rim of the lid. Assuming that the top and the rim of the lid rest against the tool at *A* and *B*, respectively, and that a 60-N force is applied as indicated to the handle, determine the force acting on the rim.

SOLUTION





The three-force member *ABC* has forces that intersect at point *D*, where, from the law of sines (ΔCDE)

$$\frac{L}{\sin 95^\circ} = \frac{150 \text{ mm} + (19 \text{ mm})\tan 35^\circ}{\sin 30^\circ}$$

$$\therefore L = 325.37 \text{ mm}$$

Then

$$\alpha = \tan^{-1}\left(\frac{45 \text{ mm}}{y_{BD}}\right)$$

where

$$\begin{aligned} y_{BD} &= L - y_{AE} - 22 \text{ mm} \\ &= 325.37 \text{ mm} - \frac{19 \text{ mm}}{\cos 35^\circ} - 22 \text{ mm} \\ &= 280.18 \text{ mm} \end{aligned}$$

$$\therefore \alpha = \tan^{-1}\left(\frac{45 \text{ mm}}{280.18 \text{ mm}}\right) = 9.1246^\circ$$

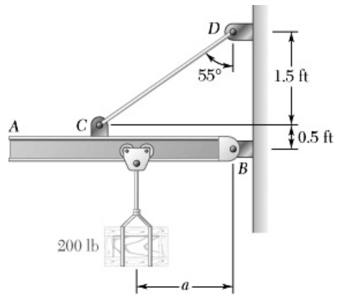
Applying the law of sines to the force triangle,

$$\frac{B}{\sin 150^\circ} = \frac{60 \text{ N}}{\sin 9.1246^\circ}$$

$$\therefore B = 189.177 \text{ N}$$

Or, on member $\quad \mathbf{B} = 189.2 \text{ N} \; \swarrow \; 80.9^\circ$

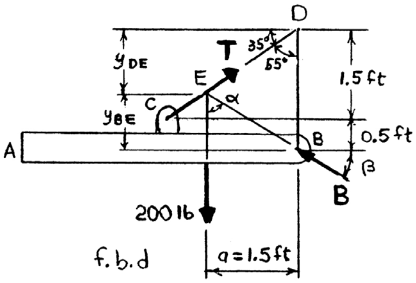
and, on lid $\quad \mathbf{B} = 189.2 \text{ N} \; \nearrow \; 80.9^\circ$ ◀



PROBLEM 4.73

A 200-lb crate is attached to the trolley-beam system shown. Knowing that $a = 1.5$ ft, determine (*a*) the tension in cable *CD*, (*b*) the reaction at *B*.

SOLUTION



From geometry of forces

$$\beta = \tan^{-1}\left(\frac{y_{BE}}{1.5\text{ ft}}\right)$$

where

$$y_{BE} = 2.0 - y_{DE}$$

$$= 2.0 - 1.5\tan 35°$$

$$= 0.94969\text{ ft}$$

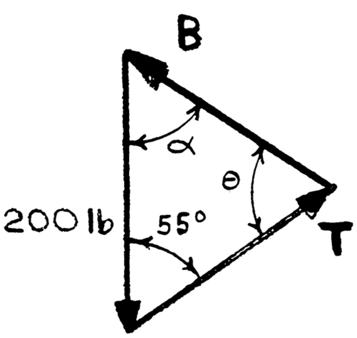
$$\therefore\ \beta = \tan^{-1}\left(\frac{0.94969}{1.5}\right) = 32.339°$$

and

$$\alpha = 90° - \beta = 90° - 32.339° = 57.661°$$

$$\theta = \beta + 35° = 32.339° + 35° = 67.339°$$

Applying the law of sines to the force triangle,



$$\frac{200\text{ lb}}{\sin\theta} = \frac{T}{\sin\alpha} = \frac{B}{\sin 55°}$$

or

$$\frac{(200\text{ lb})}{\sin 67.339°} = \frac{T}{\sin 57.661°} = \frac{B}{\sin 55°}$$

(*a*)

$$T = \frac{(200\text{ lb})(\sin 57.661°)}{\sin 67.339°} = 183.116\text{ lb}$$

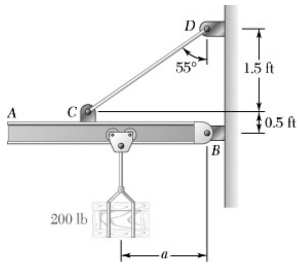
or $T = 183.1$ lb ◀

(*b*)

$$B = \frac{(200\text{ lb})(\sin 55°)}{\sin 67.339°} = 177.536\text{ lb}$$

or **B** = 177.5 lb ↖ 32.3° ◀

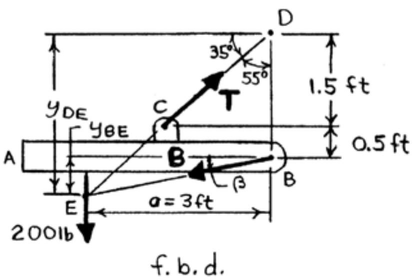
PROBLEM 4.74



Solve Problem 4.73 assuming that $a = 3$ ft.

P4.73 A 200-lb crate is attached to the trolley-beam system shown. Knowing that $a = 1.5$ ft, determine (*a*) the tension in cable *CD*, (*b*) the reaction
at *B*.

SOLUTION



From geometry of forces

$$\beta = \tan^{-1}\left(\frac{y_{BE}}{3 \text{ ft}}\right)$$

where

$$y_{BE} = y_{DE} - 2.0 \text{ ft}$$

$$= 3\tan 35° - 2.0$$

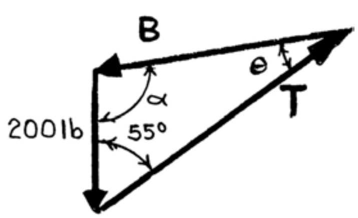
$$= 0.100623 \text{ ft}$$

$$\therefore \beta = \tan^{-1}\left(\frac{0.100623}{3}\right) = 1.92103°$$

and $$\alpha = 90° + \beta = 90° + 1.92103° = 91.921°$$

$$\theta = 35° - \beta = 35° - 1.92103° = 33.079°$$

Applying the law of sines to the force triangle,



$$\frac{200 \text{ lb}}{\sin\theta} = \frac{T}{\sin\alpha} = \frac{B}{\sin 55°}$$

or $$\frac{200 \text{ lb}}{\sin 33.079°} = \frac{T}{\sin 91.921°} = \frac{\text{B}}{\sin 55°}$$

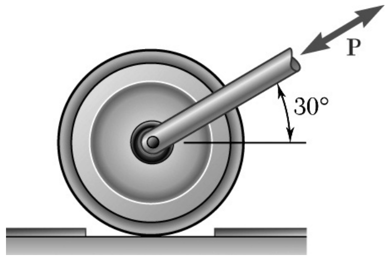
(*a*) $$T = \frac{(200 \text{ lb})(\sin 91.921°)}{\sin 33.079°} = 366.23 \text{ lb}$$

or $T = 366$ lb ◀

(*b*) $$B = \frac{(200 \text{ lb})(\sin 55°)}{\sin 33.079°} = 300.17 \text{ lb}$$

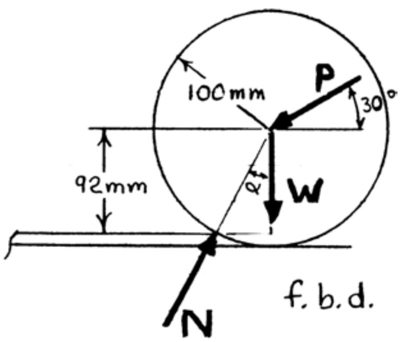
or **B** = 300 lb $\nearrow$ 1.921° ◀

PROBLEM 4.75



A 20-kg roller, of diameter 200 mm, which is to be used on a tile floor, is resting directly on the subflooring as shown. Knowing that the thickness of each tile is 8 mm, determine the force **P** required to move the roller onto the tiles if the roller is pushed to the left.

SOLUTION



Based on the roller having impending motion to the left, the only contact between the roller and floor will be at the edge of the tile.

First note $$W = mg = (20\text{ kg})(9.81\text{ m/s}^2) = 196.2\text{ N}$$

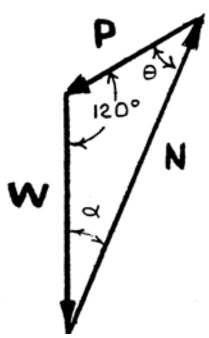
From the geometry of the three forces acting on the roller

$$\alpha = \cos^{-1}\left(\frac{92\text{ mm}}{100\text{ mm}}\right) = 23.074°$$

and $$\theta = 90° - 30° - \alpha$$

$$= 60° - 23.074$$

$$= 36.926°$$



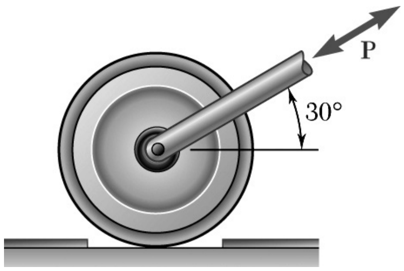
Applying the law of sines to the force triangle,

$$\frac{W}{\sin\theta} = \frac{P}{\sin\alpha}$$

or $$\frac{196.2\text{ N}}{\sin 36.926°} = \frac{P}{\sin 23.074°}$$

$$\therefore P = 127.991\text{ N}$$

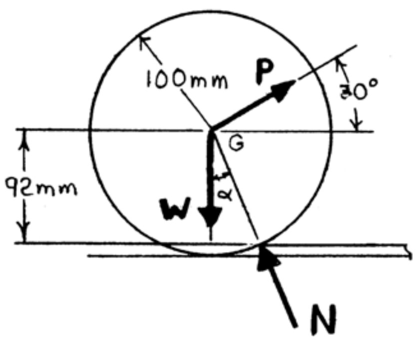
or $\mathbf{P} = 128.0\text{ N} \nearrow 30°$ ◀



PROBLEM 4.76

A 20-kg roller, of diameter 200 mm, which is to be used on a tile floor, is resting directly on the subflooring as shown. Knowing that the thickness of each tile is 8 mm, determine the force **P** required to move the roller onto the tiles if the roller is pulled to the right.

SOLUTION



Based on the roller having impending motion to the right, the only contact between the roller and floor will be at the edge of the tile.

First note $$W = mg = (20\text{ kg})(9.81\text{ m/s}^2)$$

$$= 196.2\text{ N}$$

From the geometry of the three forces acting on the roller

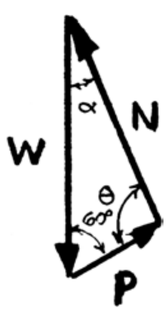
$$\alpha = \cos^{-1}\left(\frac{92\text{ mm}}{100\text{ mm}}\right) = 23.074°$$

and $$\theta = 90° + 30° - \alpha$$

$$= 120° - 23.074°$$

$$= 96.926°$$

Applying the law of sines to the force triangle,



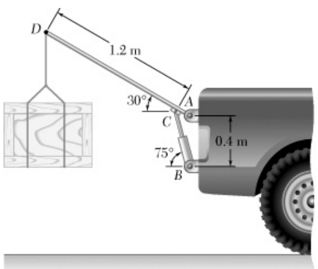
$$\frac{W}{\sin\theta} = \frac{P}{\sin\alpha}$$

or $$\frac{196.2\text{ N}}{\sin 96.926°} = \frac{P}{\sin 23.074}$$

$$\therefore P = 77.460\text{ N}$$

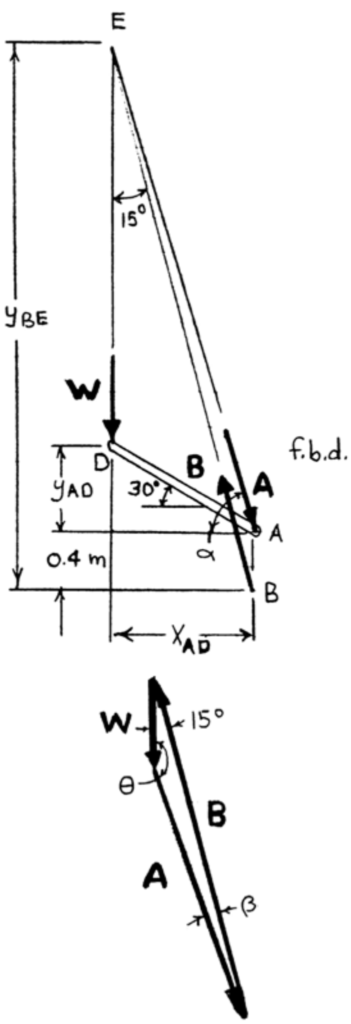
or $\mathbf{P} = 77.5\text{ N} \measuredangle 30°$ ◀

PROBLEM 4.77



A small hoist is mounted on the back of a pickup truck and is used to lift a 120-kg crate. Determine (*a*) the force exerted on the hoist by the hydraulic cylinder *BC*, (*b*) the reaction at *A*.

SOLUTION



First note $W = mg = (120\text{ kg})(9.81\text{ m/s}^2) = 1177.2\text{ N}$

From the geometry of the three forces acting on the small hoist

$$x_{AD} = (1.2\text{ m})\cos 30° = 1.03923\text{ m}$$

$$y_{AD} = (1.2\text{ m})\sin 30° = 0.6\text{ m}$$

and $$y_{BE} = x_{AD}\tan 75° = (1.03923\text{ m})\tan 75° = 3.8785\text{ m}$$

Then $$\alpha = \tan^{-1}\left(\frac{y_{BE} - 0.4\text{ m}}{x_{AD}}\right) = \tan^{-1}\left(\frac{3.4785}{1.03923}\right) = 73.366°$$

$$\beta = 75° - \alpha = 75° - 73.366° = 1.63412°$$

$$\theta = 180° - 15° - \beta = 165° - 1.63412° = 163.366°$$

Applying the law of sines to the force triangle,

$$\frac{W}{\sin\beta} = \frac{B}{\sin\theta} = \frac{A}{\sin 15°}$$

or $$\frac{1177.2\text{ N}}{\sin 1.63412°} = \frac{B}{\sin 163.366°} = \frac{A}{\sin 15°}$$

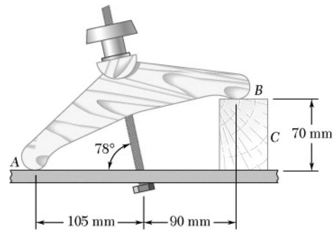
(*a*) $B = 11\,816.9\text{ N}$

or $\mathbf{B} = 11.82\text{ kN}$ ⦨ $75.0°$ ◀

(*b*) $A = 10\,684.2\text{ N}$

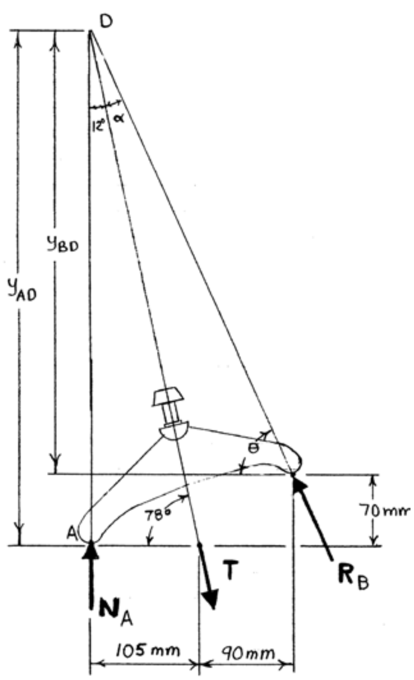
or $\mathbf{A} = 10.68\text{ kN}$ ⦨ $73.4°$ ◀

PROBLEM 4.78



The clamp shown is used to hold the rough workpiece *C*. Knowing that the maximum allowable compressive force on the workpiece is 200 N and neglecting the effect of friction at *A*, determine the corresponding (*a*) reaction at *B*, (*b*) reaction at *A*, (*c*) tension in the bolt.

SOLUTION



From the geometry of the three forces acting on the clamp

$$y_{AD} = (105\text{ mm})\tan 78° = 493.99\text{ mm}$$

$$y_{BD} = y_{AD} - 70\text{ mm} = (493.99 - 70)\text{ mm} = 423.99\text{ mm}$$

Then $$\theta = \tan^{-1}\left(\frac{y_{BD}}{195\text{ mm}}\right) = \tan^{-1}\left(\frac{423.99}{195}\right) = 65.301°$$

$$\alpha = 90° - \theta - 12° = 78° - 65.301° = 12.6987°$$

(*a*) Based on the maximum allowable compressive force on the workpiece of 200 N,

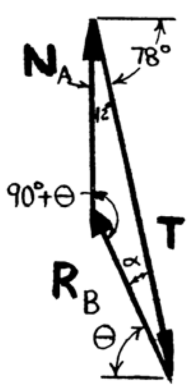
$$(R_B)_y = 200\text{ N}$$

or $$R_B \sin\theta = 200\text{ N}$$

$$\therefore R_B = \frac{200\text{ N}}{\sin 65.301°} = 220.14\text{ N}$$

or $\mathbf{R}_B = 220\text{ N} \searrow 65.3°$ ◀

Applying the law of sines to the force triangle,



$$\frac{R_B}{\sin 12°} = \frac{N_A}{\sin\alpha} = \frac{T}{\sin(90° + \theta)}$$

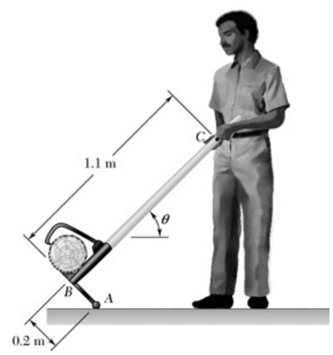
or $$\frac{220.14\text{ N}}{\sin 12°} = \frac{N_A}{\sin 12.6987°} = \frac{T}{\sin 155.301°}$$

(*b*) $$N_A = 232.75\text{ N}$$

or $\mathbf{N}_A = 233\text{ N} \uparrow$ ◀

(*c*) $$T = 442.43\text{ N}$$

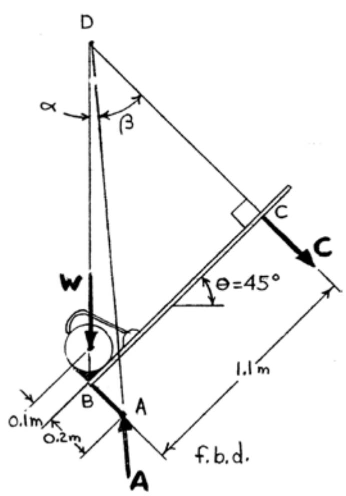
or $T = 442\text{ N}$ ◀

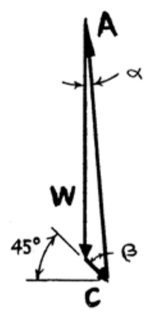


PROBLEM 4.79

A modified peavey is used to lift a 0.2-m-diameter log of mass 36 kg. Knowing that $\theta = 45°$ and that the force exerted at C by the worker is perpendicular to the handle of the peavey, determine (a) the force exerted at C, (b) the reaction at A.

SOLUTION





First note $W = mg = (36 \text{ kg})(9.81 \text{ m/s}^2) = 353.16 \text{ N}$

From the geometry of the three forces acting on the modified peavey

$$\beta = \tan^{-1}\left(\frac{1.1 \text{ m}}{1.1 \text{ m} + 0.2 \text{ m}}\right) = 40.236°$$

$$\alpha = 45° - \beta = 45° - 40.236° = 4.7636°$$

Applying the law of sines to the force triangle,

$$\frac{W}{\sin\beta} = \frac{C}{\sin\alpha} = \frac{A}{\sin 135°}$$

or

$$\frac{353.16 \text{ N}}{\sin 40.236°} = \frac{C}{\sin 4.7636} = \frac{A}{\sin 135°}$$

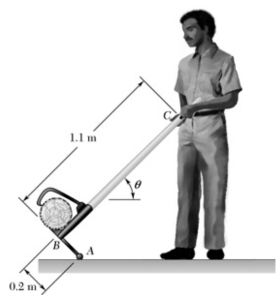
(a) $C = 45.404 \text{ N}$

or $\mathbf{C} = 45.4 \text{ N}$ ⦫ $45.0°$ ◀

(b) $A = 386.60 \text{ N}$

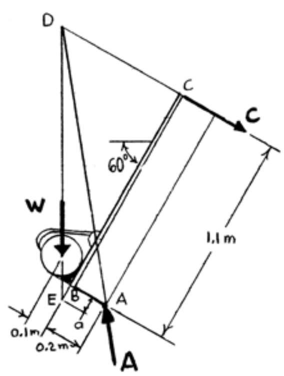
or $\mathbf{A} = 387 \text{ N}$ ⦫ $85.2°$ ◀

PROBLEM 4.80



A modified peavey is used to lift a 0.2-m-diameter log of mass 36 kg. Knowing that $\theta = 60^\circ$ and that the force exerted at *C* by the worker is perpendicular to the handle of the peavey, determine (*a*) the force exerted at *C*, (*b*) the reaction at *A*.

SOLUTION



First note $\qquad W = mg = (36 \text{ kg})(9.81 \text{ m/s}^2) = 353.16 \text{ N}$

From the geometry of the three forces acting on the modified peavey

$$\beta = \tan^{-1}\left(\frac{1.1 \text{ m}}{DC + 0.2 \text{ m}}\right)$$

where $\qquad DC = (1.1 \text{ m} + a)\tan 30^\circ$

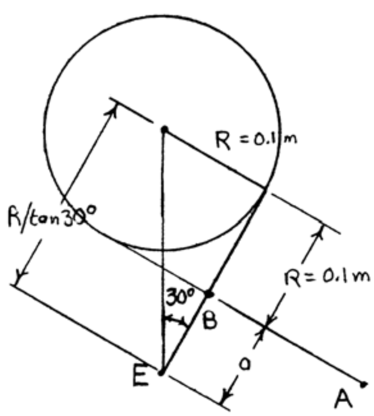
$$a = \left(\frac{R}{\tan 30^\circ}\right) - R$$

$$= \left(\frac{0.1 \text{ m}}{\tan 30^\circ}\right) - 0.1 \text{ m}$$

$$= 0.073205 \text{ m}$$

$$\therefore\ DC = (1.173205)\tan 30^\circ$$

$$= 0.67735 \text{ m}$$



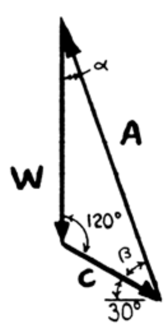
and $\qquad \beta = \tan^{-1}\left(\frac{1.1}{0.87735}\right) = 51.424^\circ$

$$\alpha = 60^\circ - \beta = 60^\circ - 51.424^\circ = 8.5756^\circ$$

Applying the law of sines to the force triangle,

$$\frac{W}{\sin\beta} = \frac{C}{\sin\alpha} = \frac{A}{\sin 120^\circ}$$

or $\qquad \dfrac{353.16 \text{ N}}{\sin 51.424^\circ} = \dfrac{C}{\sin 8.5756^\circ} = \dfrac{A}{\sin 120^\circ}$



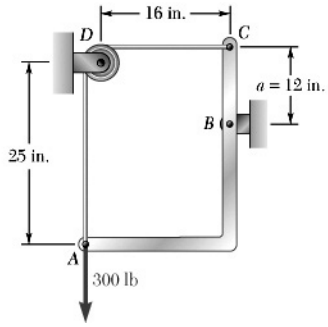
(*a*) $\qquad C = 67.360 \text{ N}$

or $\mathbf{C} = 67.4 \text{ N}$ 30° ◀

(*b*) $\qquad A = 391.22 \text{ N}$

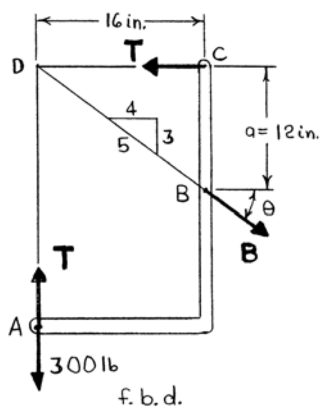
or $\mathbf{A} = 391 \text{ N}$ 81.4° ◀

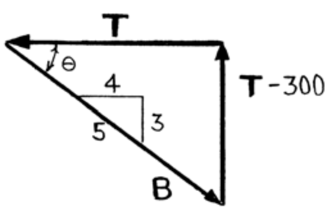
PROBLEM 4.81



Member *ABC* is supported by a pin and bracket at *B* and by an inextensible cord at *A* and *C* and passing over a frictionless pulley at *D*. The tension may be assumed to be the same in portion *AD* and *CD* of the cord. For the loading shown and neglecting the size of the pulley, determine the tension in the cord and the reaction at *B*.

SOLUTION





From the f.b.d. of member *ABC*, it is seen that the member can be treated as a three-force body.

From the force triangle

$$\frac{T - 300}{T} = \frac{3}{4}$$

$$3T = 4T - 1200$$

$$\therefore \quad T = 1200 \text{ lb} \blacktriangleleft$$

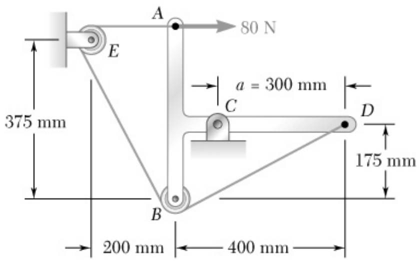
Also, $$\frac{B}{T} = \frac{5}{4}$$

$$\therefore \quad B = \frac{5}{4}T = \frac{5}{4}(1200 \text{ lb}) = 1500 \text{ lb}$$

$$\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.870°$$

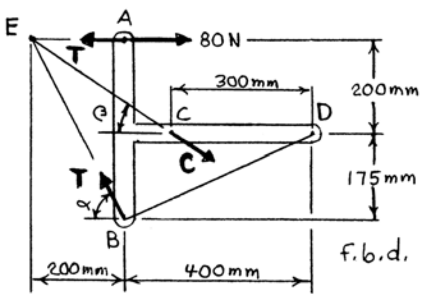
and $\mathbf{B} = 1500$ lb $\searrow$ 36.9° ◀

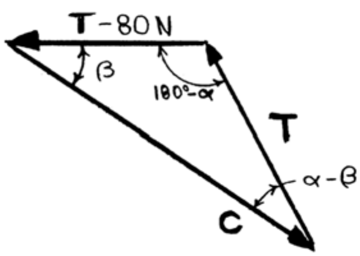
PROBLEM 4.82



Member *ABCD* is supported by a pin and bracket at *C* and by an inextensible cord attached at *A* and *D* and passing over frictionless pulleys at *B* and *E*. Neglecting the size of the pulleys, determine the tension in the cord and the reaction at *C*.

SOLUTION





From the geometry of the forces acting on member *ABCD*

$$\beta = \tan^{-1}\left(\frac{200}{300}\right) = 33.690°$$

$$\alpha = \tan^{-1}\left(\frac{375}{200}\right) = 61.928°$$

$$\alpha - \beta = 61.928° - 33.690° = 28.237°$$

$$180° - \alpha = 180° - 61.928° = 118.072°$$

Applying the law of sines to the force triangle,

$$\frac{T - 80\text{ N}}{\sin(\alpha - \beta)} = \frac{T}{\sin\beta} = \frac{C}{\sin(180° - \alpha)}$$

or $$\frac{T - 80\text{ N}}{\sin 28.237°} = \frac{T}{\sin 33.690°} = \frac{C}{\sin 118.072°}$$

Then $$(T - 80\text{ N})\sin 33.690° = T\sin 28.237°$$

$$\therefore\quad T = 543.96\text{ N}$$

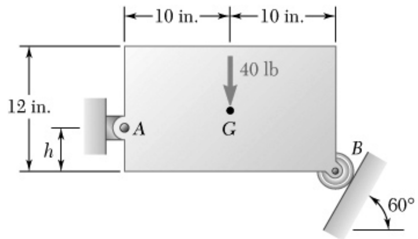
or $T = 544$ N ◀

and $$(543.96\text{ N})\sin 118.072 = C\sin 33.690°$$

$$\therefore\quad C = 865.27\text{ N}$$

or $\mathbf{C} = 865$ N ⦪ 33.7° ◀

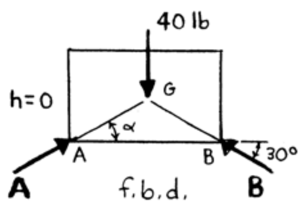
PROBLEM 4.83

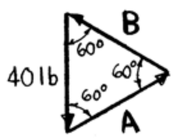


Using the method of Section 4.7, solve Problem 4.18.

P4.18 Determine the reactions at A and B when (a) $h = 0$, (b) $h = 8$ in.

SOLUTION





(a) Based on symmetry

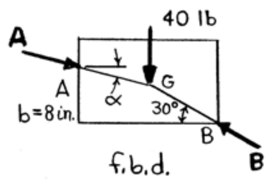
$$\alpha = 30°$$

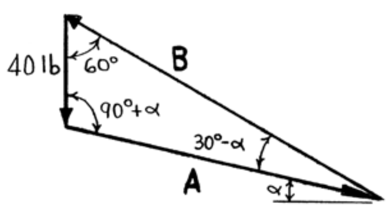
From force triangle

$$A = B = 40 \text{ lb}$$

or $\mathbf{A} = 40.0$ lb $\measuredangle$ 30° ◀

and $\mathbf{B} = 40.0$ lb $\searrow$ 30° ◀





(b) From geometry of forces

$$\alpha = \tan^{-1}\left(\frac{8 \text{ in.} - (10 \text{ in.})\tan 30°}{10 \text{ in.}}\right) = 12.5521°$$

Also,

$$30° - \alpha = 30° - 12.5521° = 17.4479°$$

$$90° + \alpha = 90° + 12.5521° = 102.5521°$$

Applying law of sines to the force triangle,

$$\frac{40 \text{ lb}}{\sin(30° - \alpha)} = \frac{A}{\sin 60°} = \frac{B}{\sin(90° + \alpha)}$$

or $$\frac{40 \text{ lb}}{\sin 17.4479°} = \frac{A}{\sin 60°} = \frac{B}{\sin 102.5521}$$

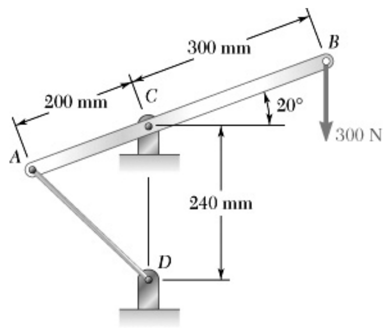
$$A = 115.533 \text{ lb}$$

or $\mathbf{A} = 115.5$ lb $\nwarrow$ 12.55° ◀

$$B = 130.217 \text{ lb}$$

or $\mathbf{B} = 130.2$ lb $\searrow$ 30.0° ◀

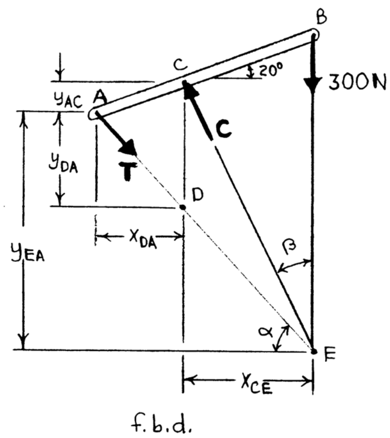
PROBLEM 4.84



Using the method of Section 4.7, solve Problem 4.28.

P4.28 A lever is hinged at *C* and is attached to a control cable at *A*. If the lever is subjected to a 300-N vertical force at *B*, determine (a) the tension in the cable, (b) the reaction at *C*.

SOLUTION



From geometry of forces acting on lever

$$\alpha = \tan^{-1}\left(\frac{y_{DA}}{x_{DA}}\right)$$

where

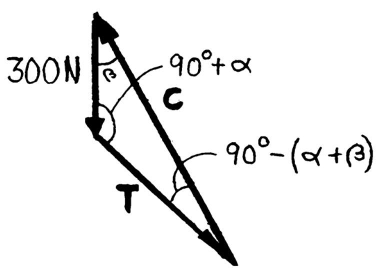
$$y_{DA} = 0.24\text{ m} - y_{AC} = 0.24\text{ m} - (0.2\text{ m})\sin 20°$$

$$= 0.171596\text{ m}$$

$$x_{DA} = (0.2\text{ m})\cos 20°$$

$$= 0.187939\text{ m}$$

$$\therefore\quad \alpha = \tan^{-1}\left(\frac{0.171596}{0.187939}\right) = 42.397°$$



$$\beta = 90° - \tan^{-1}\left(\frac{y_{AC} + y_{EA}}{x_{CE}}\right)$$

where $\quad x_{CE} = (0.3\text{ m})\cos 20° = 0.28191\text{ m}$

$$y_{AC} = (0.2\text{ m})\sin 20° = 0.068404\text{ m}$$

$$y_{EA} = (x_{DA} + x_{CE})\tan\alpha$$

$$= (0.187939 + 0.28191)\tan 42.397°$$

$$= 0.42898\text{ m}$$

$$\therefore\quad \beta = 90° - \tan^{-1}\left(\frac{0.49739}{0.28191}\right) = 29.544°$$

Also, $\quad 90° - (\alpha + \beta) = 90° - 71.941° = 18.0593°$

$$90° + \alpha = 90° + 42.397° = 132.397°$$

PROBLEM 4.84 CONTINUED

Applying the law of sines to the force triangle,

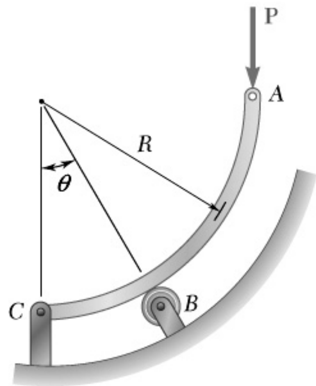
$$\frac{300\text{ N}}{\sin\left[90^\circ - (\alpha + \beta)\right]} = \frac{T}{\sin\beta} = \frac{C}{\sin(90^\circ + \alpha)}$$

or

$$\frac{300\text{ N}}{\sin 18.0593^\circ} = \frac{T}{\sin 29.544^\circ} = \frac{C}{\sin 132.397^\circ}$$

(*a*) $T = 477.18\text{ N}$ or $T = 477\text{ N}$ ◀

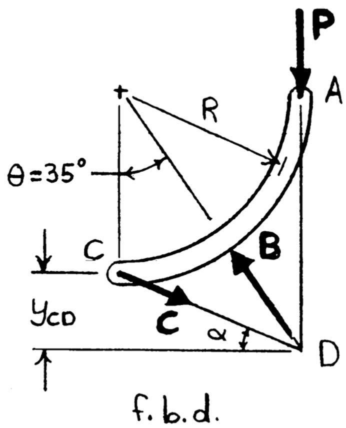
(*b*) $C = 714.67\text{ N}$ or $\mathbf{C} = 715\text{ N} \searrow 60.5^\circ$ ◀

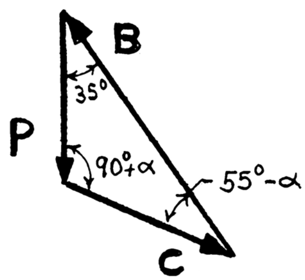


PROBLEM 4.85

Knowing that $\theta = 35^\text{o}$, determine the reaction (a) at B, (b) at C.

SOLUTION





From the geometry of the three forces applied to the member ABC

$$\alpha = \tan^{-1}\left(\frac{y_{CD}}{R}\right)$$

where

$$y_{CD} = R\tan 55° - R = 0.42815R$$

$$\therefore \quad \alpha = \tan^{-1}(0.42815) = 23.178°$$

Then $\quad 55° - \alpha = 55° - 23.178° = 31.822°$

$$90° + \alpha = 90° + 23.178° = 113.178°$$

Applying the law of sines to the force triangle,

$$\frac{P}{\sin(55° - \alpha)} = \frac{B}{\sin(90° + \alpha)} = \frac{C}{\sin 35°}$$

or $\quad \dfrac{P}{\sin 31.822°} = \dfrac{B}{\sin 113.178°} = \dfrac{C}{\sin 35°}$

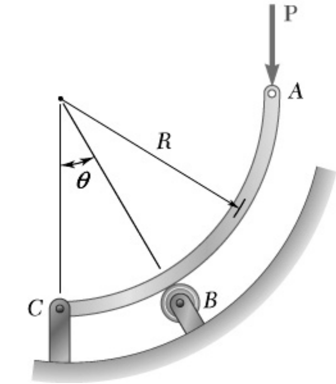
(a) $\quad B = 1.74344P$

or $\mathbf{B} = 1.743P \searrow 55.0°$ ◀

(b) $\quad C = 1.08780P$

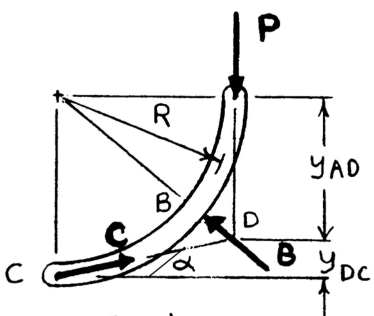
or $\mathbf{C} = 1.088P \nwarrow 23.2°$ ◀

PROBLEM 4.86

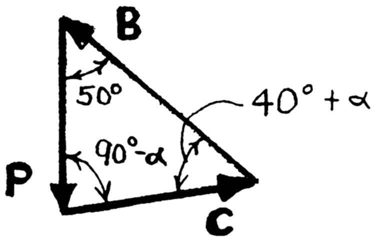


Knowing that $\theta = 50^\circ$, determine the reaction (*a*) at *B*, (*b*) at *C*.

SOLUTION



f.b.d.



From the geometry of the three forces acting on member *ABC*

$$\alpha = \tan^{-1}\left(\frac{y_{DC}}{R}\right)$$

where

$$y_{DC} = R - y_{AD} = R\left[1 - \tan\left(90^\circ - 50^\circ\right)\right]$$

$$= 0.160900R$$

$$\therefore \quad \alpha = \tan^{-1}\left(0.160900\right) = 9.1406^\circ$$

Then $$90^\circ - \alpha = 90^\circ - 9.1406^\circ = 80.859^\circ$$

$$40^\circ + \alpha = 40^\circ + 9.1406^\circ = 49.141^\circ$$

Applying the law of sines to the force triangle,

$$\frac{P}{\sin\left(40^\circ + \alpha\right)} = \frac{B}{\sin\left(90^\circ - \alpha\right)} = \frac{C}{\sin 50^\circ}$$

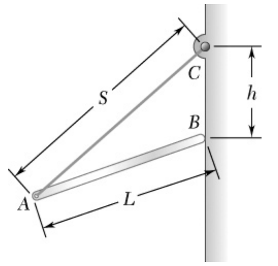
or $$\frac{P}{\sin 49.141^\circ} = \frac{B}{\sin\left(80.859^\circ\right)} = \frac{C}{\sin 50^\circ}$$

(*a*) $$B = 1.30540P$$

or $\mathbf{B} = 1.305P$ ⦪ 40.0° ◀

(*b*) $$C = 1.01286P$$

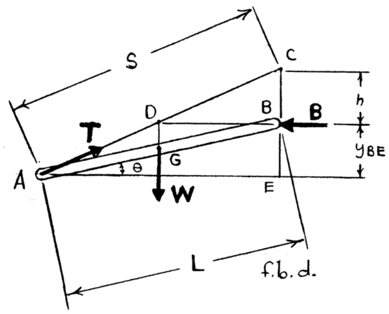
or $\mathbf{C} = 1.013P$ ⦩ 9.14° ◀



PROBLEM 4.87

A slender rod of length L and weight W is held in equilibrium as shown, with one end against a frictionless wall and the other end attached to a cord of length S. Derive an expression for the distance h in terms of L and S. Show that this position of equilibrium does not exist if $S > 2L$.

SOLUTION



From the f.b.d of the three-force member AB, forces must intersect at D. Since the force T intersects point D, directly above G,

$$y_{BE} = h$$

For triangle ACE:

$$S^2 = (AE)^2 + (2h)^2 \qquad (1)$$

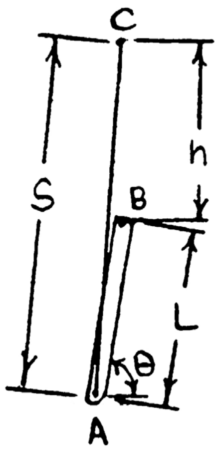
For triangle ABE:

$$L^2 = (AE)^2 + (h)^2 \qquad (2)$$

Subtracting Equation (2) from Equation (1)

$$S^2 - L^2 = 3h^2 \qquad (3)$$

$$\text{or} \quad h = \sqrt{\frac{S^2 - L^2}{3}} \blacktriangleleft$$



As length S increases relative to length L, angle θ increases until rod AB is vertical. At this vertical position:

$$h + L = S \qquad \text{or} \qquad h = S - L$$

Therefore, for all positions of AB $\qquad h \geq S - L \qquad (4)$

or $$\sqrt{\frac{S^2 - L^2}{3}} \geq S - L$$

or $$S^2 - L^2 \geq 3(S - L)^2 = 3(S^2 - 2SL + L^2) = 3S^2 - 6SL + 3L^2$$

or $$0 \geq 2S^2 - 6SL + 4L^2$$

and $$0 \geq S^2 - 3SL + 2L^2 = (S - L)(S - 2L)$$

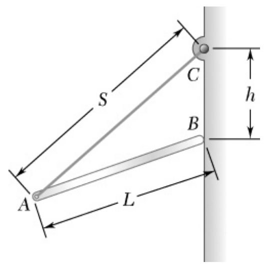
For $$S - L = 0 \qquad S = L$$

$\therefore$ Minimum value of S is L

For $$S - 2L = 0 \qquad S = 2L$$

$\therefore$ Maximum value of S is $2L$

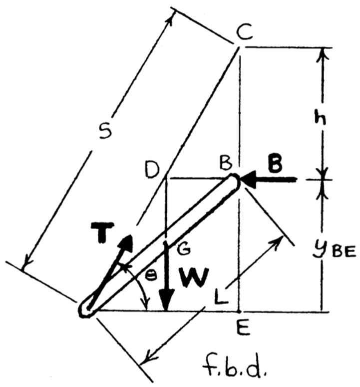
Therefore, equilibrium does not exist if $S > 2L$ ◀



PROBLEM 4.88

A slender rod of length $L = 200$ mm is held in equilibrium as shown, with one end against a frictionless wall and the other end attached to a cord of length $S = 300$ mm. Knowing that the mass of the rod is 1.5 kg, determine (*a*) the distance *h*, (*b*) the tension in the cord, (*c*) the reaction at *B*.

SOLUTION



From the f.b.d of the three-force member *AB*, forces must intersect at *D*. Since the force *T* intersects point *D*, directly above *G*,

$$y_{BE} = h$$

For triangle *ACE*:

$$S^2 = (AE)^2 + (2h)^2 \tag{1}$$

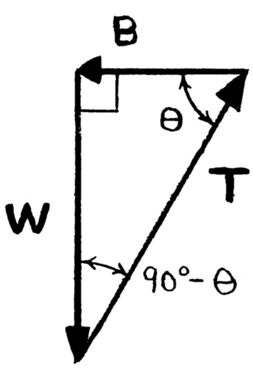
For triangle *ABE*:

$$L^2 = (AE)^2 + (h)^2 \tag{2}$$

Subtracting Equation (2) from Equation (1)

$$S^2 - L^2 = 3h^2$$

$$\text{or} \quad h = \sqrt{\frac{S^2 - L^2}{3}}$$



(*a*) For $L = 200$ mm and $S = 300$ mm

$$h = \sqrt{\frac{(300)^2 - (200)^2}{3}} = 129.099 \text{ mm}$$

or $h = 129.1$ mm ◀

(*b*) Have $W = mg = (1.5 \text{ kg})(9.81 \text{ m/s}^2) = 14.715$ N

and $$\theta = \sin^{-1}\left(\frac{2h}{s}\right) = \sin^{-1}\left[\frac{2(129.099)}{300}\right]$$

$$\theta = 59.391°$$

From the force triangle

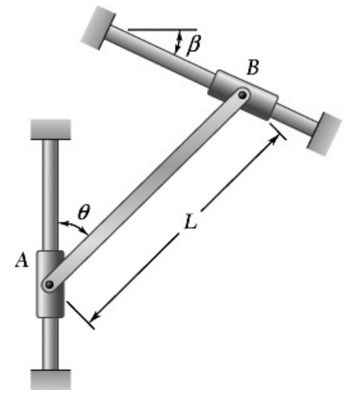
$$T = \frac{W}{\sin\theta} = \frac{14.715 \text{ N}}{\sin 59.391°} = 17.0973 \text{ N}$$

or $T = 17.10$ N ◀

(*c*) $$B = \frac{W}{\tan\theta} = \frac{14.715 \text{ N}}{\tan 59.391°} = 8.7055 \text{ N}$$

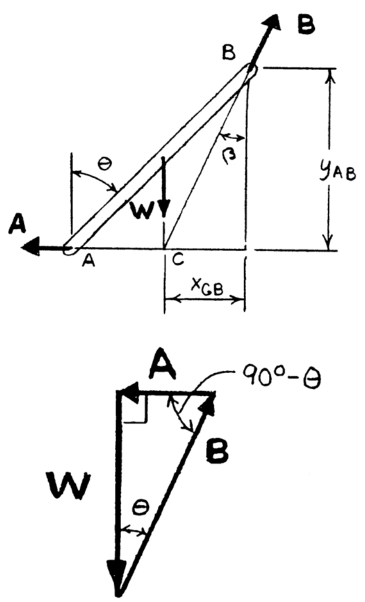
or $\mathbf{B} = 8.71 \text{ N} \longleftarrow$ ◀

PROBLEM 4.89



A slender rod of length L and weight W is attached to collars which can slide freely along the guides shown. Knowing that the rod is in equilibrium, derive an expression for the angle θ in terms of the angle β.

SOLUTION



As shown in the f.b.d of the slender rod AB, the three forces intersect at C. From the force geometry

$$\tan\beta = \frac{x_{GB}}{y_{AB}}$$

where

$$y_{AB} = L\cos\theta$$

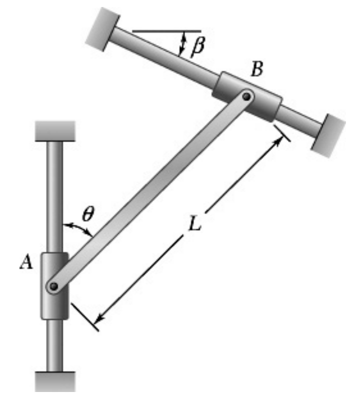
and

$$x_{GB} = \frac{1}{2}L\sin\theta$$

$$\therefore \quad \tan\beta = \frac{\frac{1}{2}L\sin\theta}{L\cos\theta} = \frac{1}{2}\tan\theta$$

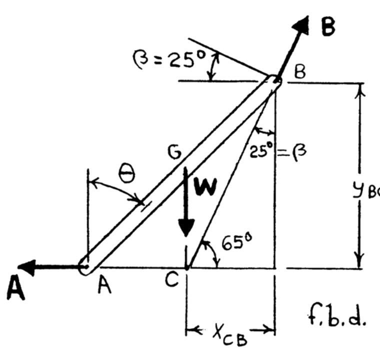
$$\text{or} \quad \tan\theta = 2\tan\beta \blacktriangleleft$$

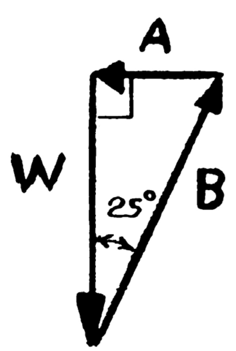
PROBLEM 4.90



A 10-kg slender rod of length L is attached to collars which can slide freely along the guides shown. Knowing that the rod is in equilibrium and that $\beta = 25°$, determine (*a*) the angle θ that the rod forms with the vertical, (*b*) the reactions at A and B.

SOLUTION





(*a*) As shown in the f.b.d. of the slender rod AB, the three forces intersect at C. From the geometry of the forces

$$\tan\beta = \frac{x_{CB}}{y_{BC}}$$

where

$$x_{CB} = \frac{1}{2}L\sin\theta$$

and

$$y_{BC} = L\cos\theta$$

$$\therefore \quad \tan\beta = \frac{1}{2}\tan\theta$$

or

$$\tan\theta = 2\tan\beta$$

For

$$\beta = 25°$$

$$\tan\theta = 2\tan 25° = 0.93262$$

$$\therefore \quad \theta = 43.003°$$

or $\theta = 43.0°$ ◀

(*b*)

$$W = mg = (10\text{ kg})(9.81\text{ m/s}^2) = 98.1\text{ N}$$

From force triangle

$$A = W\tan\beta$$

$$= (98.1\text{ N})\tan 25°$$

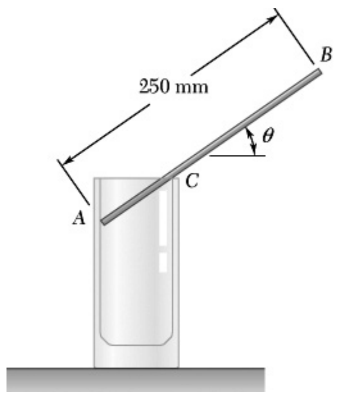
$$= 45.745\text{ N}$$

or $\mathbf{A} = 45.7\text{ N} \longleftarrow$ ◀

and

$$B = \frac{W}{\cos\beta} = \frac{98.1\text{ N}}{\cos 25°} = 108.241\text{ N}$$

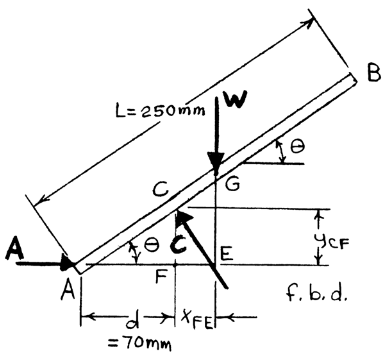
or $\mathbf{B} = 108.2\text{ N} \measuredangle 65.0°$ ◀



PROBLEM 4.91

A uniform slender rod of mass 5 g and length 250 mm is balanced on a glass of inner diameter 70 mm. Neglecting friction, determine the angle θ corresponding to equilibrium.

SOLUTION



From the geometry of the forces acting on the three-force member *AB*

Triangle *ACF*

$$y_{CF} = d\tan\theta$$

Triangle *CEF* $\quad x_{FE} = y_{CF}\tan\theta = d\tan^2\theta$

Triangle *AGE*

$$\cos\theta = \frac{d + x_{FE}}{\left(\frac{L}{2}\right)} = \frac{d + d\tan^2\theta}{\left(\frac{L}{2}\right)}$$

$$= \frac{2d}{L}\left(1 + \tan^2\theta\right)$$

Now $\quad \left(1 + \tan^2\theta\right) = \sec^2\theta \quad$ and $\quad \sec\theta = \dfrac{1}{\cos\theta}$

Then $\quad \cos\theta = \dfrac{2d}{L}\sec^2\theta = \dfrac{2d}{L}\left(\dfrac{1}{\cos^2\theta}\right)$

$$\therefore \quad \cos^3\theta = \frac{2d}{L}$$

For $\quad d = 70 \text{ mm} \quad$ and $\quad L = 250 \text{ mm}$

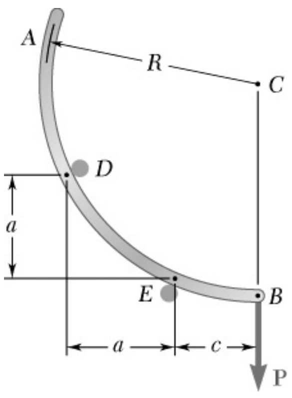
$$\cos^3\theta = \frac{2(70)}{250} = 0.56$$

$$\therefore \quad \cos\theta = 0.82426$$

and $\quad \theta = 34.487°$

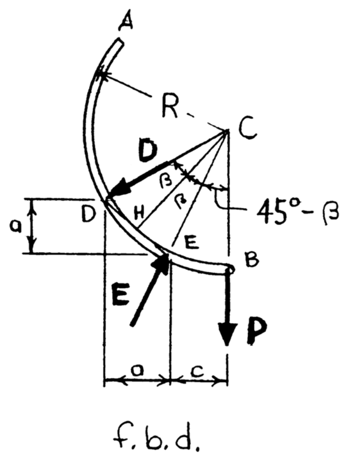
or $\theta = 34.5°$ ◀

PROBLEM 4.92



Rod AB is bent into the shape of a circular arc and is lodged between two pegs D and E. It supports a load $\mathbf{P}$ at end B. Neglecting friction and the weight of the rod, determine the distance c corresponding to equilibrium when $a = 1$ in. and $R = 5$ in.

SOLUTION



Since $$y_{ED} = x_{ED} = a,$$

Slope of ED is $\measuredangle$ 45°

$\therefore$ slope of HC is $\measuredangle$ 45°

Also $$DE = \sqrt{2}a$$

and $$DH = HE = \left(\frac{1}{2}\right)DE = \frac{a}{\sqrt{2}}$$

For triangles DHC and EHC

$$\sin\beta = \frac{a/\sqrt{2}}{R} = \frac{a}{\sqrt{2}R}$$

Now $$c = R\sin(45° - \beta)$$

For $$a = 1\text{ in.}\quad\text{and}\quad R = 5\text{ in.}$$

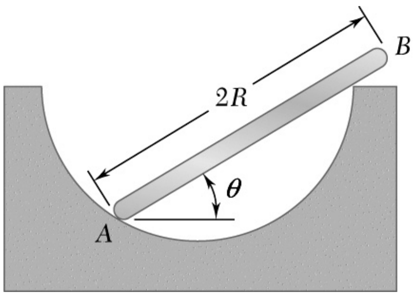
$$\sin\beta = \frac{1\text{ in.}}{\sqrt{2}(5\text{ in.})} = 0.141421$$

$$\therefore\ \beta = 8.1301°$$ or $\beta = 8.13°$ ◀

and $$c = (5\text{ in.})\sin(45° - 8.1301°) = 3.00\text{ in.}$$

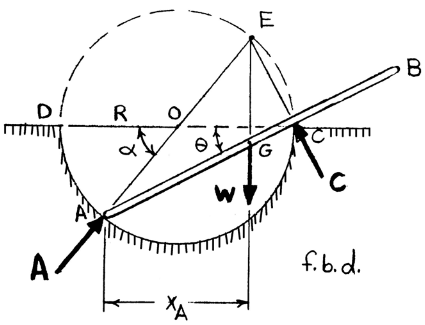
or $c = 3.00$ in. ◀

PROBLEM 4.93



A uniform rod AB of weight W and length $2R$ rests inside a hemispherical bowl of radius R as shown. Neglecting friction determine the angle θ corresponding to equilibrium.

SOLUTION



Based on the f.b.d., the uniform rod AB is a three-force body. Point E is the point of intersection of the three forces. Since force A passes through O, the center of the circle, and since force **C** is perpendicular to the rod, triangle ACE is a right triangle inscribed in the circle. Thus, E is a point on the circle.

Note that the angle α of triangle DOA is the central angle corresponding to the inscribed angle θ of triangle DCA.

$$\therefore \ \alpha = 2\theta$$

The horizontal projections of $AE, (x_{AE})$, and $AG, (x_{AG})$, are equal.

$$\therefore \ x_{AE} = x_{AG} = x_A$$

or $$(AE)\cos 2\theta = (AG)\cos\theta$$

and $$(2R)\cos 2\theta = R\cos\theta$$

Now $$\cos 2\theta = 2\cos^2\theta - 1$$

then $$4\cos^2\theta - 2 = \cos\theta$$

or $$4\cos^2\theta - \cos\theta - 2 = 0$$

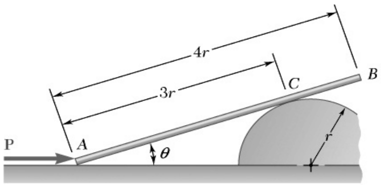
Applying the quadratic equation

$$\cos\theta = 0.84307 \quad \text{and} \quad \cos\theta = -0.59307$$

$$\therefore \ \theta = 32.534° \quad \text{and} \quad \theta = 126.375°\text{(Discard)}$$

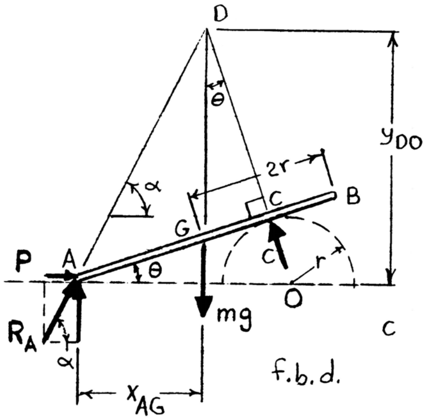
or $\theta = 32.5°$ ◀

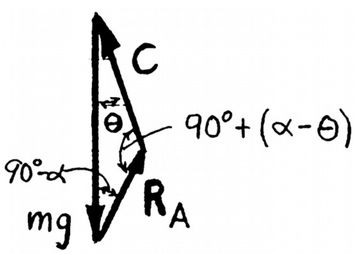
PROBLEM 4.94



A uniform slender rod of mass m and length $4r$ rests on the surface shown and is held in the given equilibrium position by the force **P**. Neglecting the effect of friction at A and C, (a) determine the angle θ, (b) derive an expression for P in terms of m.

SOLUTION





The forces acting on the three-force member intersect at D.

(a) From triangle ACO

$$\theta = \tan^{-1}\left(\frac{r}{3r}\right) = \tan^{-1}\left(\frac{1}{3}\right) = 18.4349^\circ \qquad \text{or} \quad \theta = 18.43^\circ \blacktriangleleft$$

(b) From triangle DCG

$$\tan\theta = \frac{r}{DC}$$

$$\therefore \quad DC = \frac{r}{\tan\theta} = \frac{r}{\tan 18.4349^\circ} = 3r$$

and

$$DO = DC + r = 3r + r = 4r$$

$$\alpha = \tan^{-1}\left(\frac{y_{DO}}{x_{AG}}\right)$$

where

$$y_{DO} = (DO)\cos\theta = (4r)\cos 18.4349^\circ$$

$$= 3.4947r$$

and

$$x_{AG} = (2r)\cos\theta = (2r)\cos 18.4349^\circ$$

$$= 1.89737r$$

$$\therefore \quad \alpha = \tan^{-1}\left(\frac{3.4947r}{1.89737r}\right) = 63.435^\circ$$

where

$$90^\circ + (\alpha - \theta) = 90^\circ + 45^\circ = 135.00^\circ$$

Applying the law of sines to the force triangle,

$$\frac{mg}{\sin\left[90^\circ + (\alpha - \theta)\right]} = \frac{R_A}{\sin\theta}$$

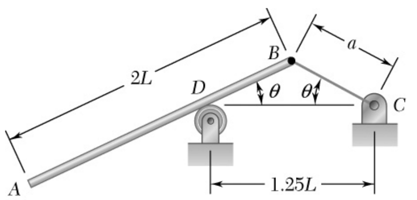
$$\therefore \quad R_A = (0.44721)mg$$

Finally,

$$P = R_A \cos\alpha$$

$$= (0.44721mg)\cos 63.435^\circ$$

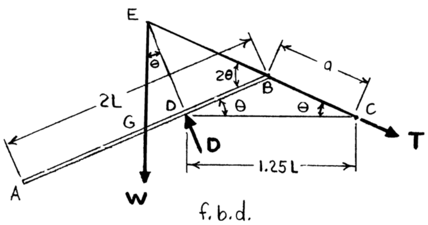
$$= 0.20000mg \qquad \text{or} \quad P = \frac{mg}{5} \blacktriangleleft$$



PROBLEM 4.95

A uniform slender rod of length $2L$ and mass m rests against a roller at D and is held in the equilibrium position shown by a cord of length a. Knowing that $L = 200$ mm, determine (*a*) the angle θ, (*b*) the length a.

SOLUTION



(*a*) The forces acting on the three-force member *AB* intersect at *E*. Since triangle *DBC* is isosceles, $DB = a$.

From triangle *BDE*

$$ED = DB\tan 2\theta = a\tan 2\theta$$

From triangle *GED*

$$ED = \frac{(L-a)}{\tan\theta}$$

$$\therefore \quad a\tan 2\theta = \frac{L-a}{\tan\theta} \qquad \text{or} \qquad a(\tan\theta\tan 2\theta + 1) = L \qquad (1)$$

From triangle *BCD* $\quad a = \dfrac{\frac{1}{2}(1.25L)}{\cos\theta} \quad$ or $\quad \dfrac{L}{a} = 1.6\cos\theta \qquad (2)$

Substituting Equation (2) into Equation (1) yields

$$1.6\cos\theta = 1 + \tan\theta\tan 2\theta$$

Now

$$\tan\theta\tan 2\theta = \frac{\sin\theta}{\cos\theta}\frac{\sin 2\theta}{\cos 2\theta}$$

$$= \frac{\sin\theta}{\cos\theta}\frac{2\sin\theta\cos\theta}{2\cos^2\theta - 1}$$

$$= \frac{2(1-\cos^2\theta)}{2\cos^2\theta - 1}$$

Then

$$1.6\cos\theta = 1 + \frac{2(1-\cos^2\theta)}{2\cos^2\theta - 1}$$

or

$$3.2\cos^3\theta - 1.6\cos\theta - 1 = 0$$

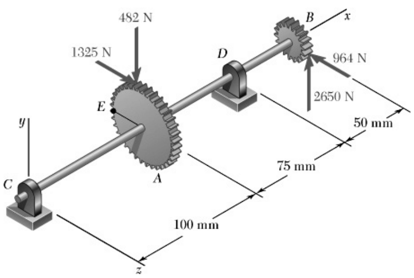
Solving numerically $\qquad \theta = 23.515° \quad$ or $\quad \theta = 23.5°$ ◀

(*b*) From Equation (2) for $L = 200$ mm and $\theta = 23.5°$

$$a = \frac{5}{8}\frac{(200\text{ mm})}{\cos 23.515°} = 136.321\text{ mm}$$

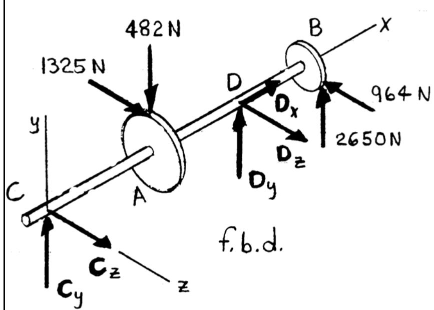
or $a = 136.3$ mm ◀

PROBLEM 4.96



Gears A and B are attached to a shaft supported by bearings at C and D. The diameters of gears A and B are 150 mm and 75 mm, respectively, and the tangential and radial forces acting on the gears are as shown. Knowing that the system rotates at a constant rate, determine the reactions at C and D. Assume that the bearing at C does not exert any axial force, and neglect the weights of the gears and the shaft.

SOLUTION



Assume moment reactions at the bearing supports are zero. From f.b.d. of shaft

$$\Sigma F_x = 0: \quad \therefore \quad D_x = 0$$

$$\Sigma M_{D(z\text{-axis})} = 0: \quad -C_y(175\text{ mm}) + (482\text{ N})(75\text{ mm})$$

$$+(2650\text{ N})(50\text{ mm}) = 0$$

$$\therefore \quad C_y = 963.71\text{ N}$$

or $$\mathbf{C}_y = (964\text{ N})\mathbf{j}$$

$$\Sigma M_{D(y\text{-axis})} = 0: \quad C_z(175\text{ mm}) + (1325\text{ N})(75\text{ mm})$$

$$+(964\text{ N})(50\text{ mm}) = 0$$

$$\therefore \quad C_z = -843.29\text{ N}$$

or $$\mathbf{C}_z = (843\text{ N})\mathbf{k}$$

$$\text{and} \quad \mathbf{C} = (964\text{ N})\mathbf{j} - (843\text{ N})\mathbf{k} \blacktriangleleft$$

$$\Sigma M_{C(z\text{-axis})} = 0: \quad -(482\text{ N})(100\text{ mm}) + D_y(175\text{ mm})$$

$$+(2650\text{ N})(225\text{ mm}) = 0$$

$$\therefore \quad D_y = -3131.7\text{ N}$$

or $$\mathbf{D}_y = -(3130\text{ N})\mathbf{j}$$

$$\Sigma M_{C(y\text{-axis})} = 0: \quad -(1325\text{ N})(100\text{ mm}) - D_z(175\text{ mm})$$

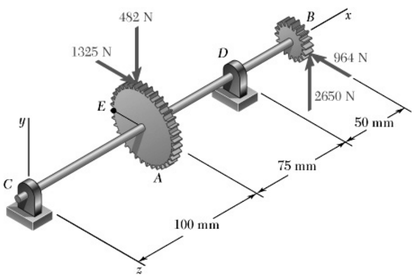
$$+(964\text{ N})(225\text{ mm}) = 0$$

$$\therefore \quad D_z = 482.29\text{ N}$$

or $$\mathbf{D}_z = (482\text{ N})\mathbf{k}$$

$$\text{and} \quad \mathbf{D} = -(3130\text{ N})\mathbf{j} + (482\text{ N})\mathbf{k} \blacktriangleleft$$

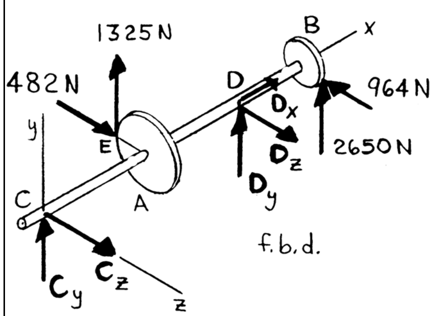
PROBLEM 4.97



Solve Problem 4.96 assuming that for gear *A* the tangential and radial forces are acting at *E*, so that $\mathbf{F}_A = (1325\text{ N})\mathbf{j} + (482\text{ N})\mathbf{k}$.

P4.96 Gears *A* and *B* are attached to a shaft supported by bearings at *C* and *D*. The diameters of gears *A* and *B* are 150 mm and 75 mm, respectively, and the tangential and radial forces acting on the gears are as shown. Knowing that the system rotates at a constant rate, determine the reactions at *C* and *D*. Assume that the bearing at *C* does not exert any axial force, and neglect the weights of the gears and the shaft.

SOLUTION



Assume moment reactions at the bearing supports are zero. From f.b.d. of shaft

$$\Sigma F_x = 0: \quad \therefore \quad D_x = 0$$

$$\Sigma M_{D(z\text{-}axis)} = 0: \quad -C_y(175\text{ mm}) - (1325\text{ N})(75\text{ mm})$$

$$+ (2650\text{ N})(50\text{ mm}) = 0$$

$$\therefore \quad C_y = 189.286\text{ N}$$

or $$\mathbf{C}_y = (189.3\text{ N})\mathbf{j}$$

$$\Sigma M_{D(y\text{-axis})} = 0: \quad C_z(175\text{ mm}) + (482\text{ N})(75\text{ mm})$$

$$+ (964\text{ N})(50\text{ mm}) = 0$$

$$\therefore \quad C_z = -482.00\text{ N}$$

or $$\mathbf{C}_z = -(482\text{ N})\mathbf{k}$$

$$\text{and} \quad \mathbf{C} = (189.3\text{ N})\mathbf{j} - (482\text{ N})\mathbf{k} \blacktriangleleft$$

$$\Sigma M_{C(z\text{-axis})} = 0: \quad (1325\text{ N})(100\text{ mm}) + D_y(175\text{ mm})$$

$$+ (2650\text{ N})(225\text{ mm}) = 0$$

$$\therefore \quad D_y = -4164.3\text{ N}$$

or $$\mathbf{D}_y = -(4160\text{ N})\mathbf{j}$$

$$\Sigma M_{C(y\text{-axis})} = 0: \quad -(482\text{ N})(100\text{ mm}) - D_z(175\text{ mm})$$

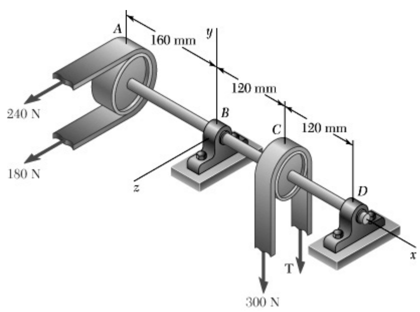
$$+(964\text{ N})(225\text{ mm}) = 0$$

$$\therefore \quad D_z = 964.00\text{ N}$$

or $$\mathbf{D}_z = (964\text{ N})\mathbf{k}$$

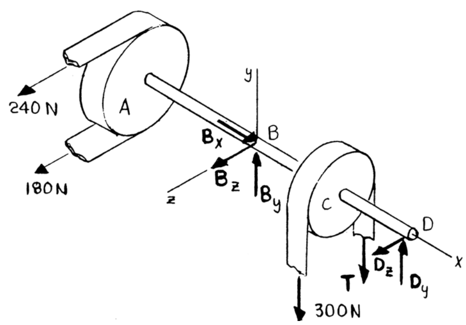
$$\text{and} \quad \mathbf{D} = -(4160\text{ N})\mathbf{j} + (964\text{ N})\mathbf{k} \blacktriangleleft$$

PROBLEM 4.98



Two transmission belts pass over sheaves welded to an axle supported by bearings at *B* and *D*. The sheave at *A* has a radius of 50 mm, and the sheave at *C* has a radius of 40 mm. Knowing that the system rotates with a constant rate, determine (*a*) the tension *T*, (*b*) the reactions at *B* and *D*. Assume that the bearing at *D* does not exert any axial thrust and neglect the weights of the sheaves and the axle.

SOLUTION



Assume moment reactions at the bearing supports are zero. From f.b.d. of shaft

(*a*) $$\Sigma M_{x\text{-axis}} = 0: \quad (240\text{ N} - 180\text{ N})(50\text{ mm}) + (300\text{ N} - T)(40\text{ mm}) = 0$$

$$\therefore\ T = 375\text{ N} ◀$$

(*b*) $$\Sigma F_x = 0: \quad B_x = 0$$

$$\Sigma M_{D(z\text{-axis})} = 0: \quad (300\text{ N} + 375\text{ N})(120\text{ mm}) - B_y(240\text{ mm}) = 0$$

$$\therefore\ B_y = 337.5\text{ N}$$

$$\Sigma M_{D(y\text{-axis})} = 0: \quad (240\text{ N} + 180\text{ N})(400\text{ mm}) + B_z(240\text{ mm}) = 0$$

$$\therefore\ B_z = -700\text{ N}$$

$$\text{or}\quad \mathbf{B} = (338\text{ N})\mathbf{j} - (700\text{ N})\mathbf{k} ◀$$

$$\Sigma M_{B(z\text{-axis})} = 0: \quad -(300\text{ N} + 375\text{ N})(120\text{ mm}) + D_y(240\text{ mm}) = 0$$

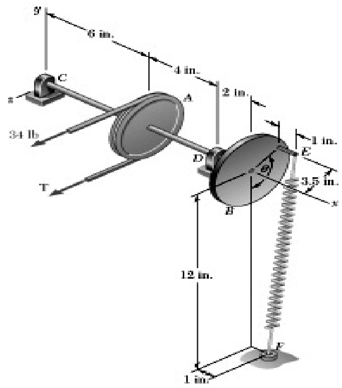
$$\therefore\ D_y = 337.5\text{ N}$$

$$\Sigma M_{B(y\text{-axis})} = 0: \quad (240\text{ N} + 180\text{ N})(160\text{ mm}) + D_z(240\text{ mm}) = 0$$

$$\therefore\ D_z = -280\text{ N}$$

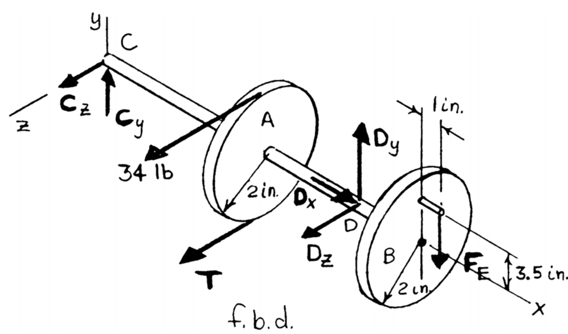
$$\text{or}\quad \mathbf{D} = (338\text{ N})\mathbf{j} - (280\text{ N})\mathbf{k} ◀$$

PROBLEM 4.99



For the portion of a machine shown, the 4-in.-diameter pulley *A* and wheel *B* are fixed to a shaft supported by bearings at *C* and *D*. The spring of constant 2 lb/in. is unstretched when $\theta = 0$, and the bearing at *C* does not exert any axial force. Knowing that $\theta = 180°$ and that the machine is at rest and in equilibrium, determine (*a*) the tension *T*, (*b*) the reactions at *C* and *D*. Neglect the weights of the shaft, pulley, and wheel.

SOLUTION



First, determine the spring force, $\mathbf{F}_E$, at $\theta = 180°$.

$$F_E = k_s x$$

where $$k_s = 2 \text{ lb/in.}$$

$$x = (y_E)_{\text{final}} - (y_E)_{\text{initial}} = (12 \text{ in.} + 3.5 \text{ in.}) - (12 \text{ in.} - 3.5 \text{ in.}) = 7.0 \text{ in.}$$

$$\therefore \quad F_E = (2 \text{ lb/in.})(7.0 \text{ in.}) = 14.0 \text{ lb}$$

(*a*) From f.b.d. of machine part

$$\Sigma M_x = 0: \quad (34 \text{ lb})(2 \text{ in.}) - T(2 \text{ in.}) = 0$$

$$\therefore \quad T = 34 \text{ lb} \qquad \text{or} \quad T = 34.0 \text{ lb} \blacktriangleleft$$

(*b*) $$\Sigma M_{D(z\text{-axis})} = 0: \quad -C_y(10 \text{ in.}) - F_E(2 \text{ in.} + 1 \text{ in.}) = 0$$

$$-C_y(10 \text{ in.}) - 14.0 \text{ lb}(3 \text{ in.}) = 0$$

$$\therefore \quad C_y = -4.2 \text{ lb} \qquad \text{or} \qquad \mathbf{C}_y = -(4.20 \text{ lb})\mathbf{j}$$

$$\Sigma M_{D(y\text{-axis})} = 0: \quad C_z(10 \text{ in.}) + 34 \text{ lb}(4 \text{ in.}) + 34 \text{ lb}(4 \text{ in.}) = 0$$

$$\therefore \quad C_z = -27.2 \text{ lb} \qquad \text{or} \qquad \mathbf{C}_z = -(27.2 \text{ lb})\mathbf{k}$$

and $\mathbf{C} = -(4.20 \text{ lb})\mathbf{j} - (27.2 \text{ lb})\mathbf{k} \blacktriangleleft$

PROBLEM 4.99 CONTINUED

$$\Sigma F_x = 0: \quad D_x = 0$$

$$\Sigma M_{C(z\text{-axis})} = 0: \quad D_y(10\text{ in.}) - F_E(12\text{ in.} + 1\text{ in.}) = 0$$

or

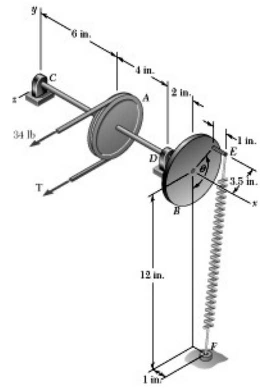
$$D_y(10\text{ in.}) - 14.0(13\text{ in.}) = 0$$

$$\therefore\ D_y = 18.2\text{ lb} \qquad \text{or} \qquad \mathbf{D}_y = (18.20\text{ lb})\mathbf{j}$$

$$\Sigma M_{C(y\text{-axis})} = 0: \quad -2(34\text{ lb})(6\text{ in.}) - D_z(10\text{ in.}) = 0$$

$$\therefore\ D_z = -40.8\text{ lb} \qquad \text{or} \qquad \mathbf{D}_z = -(40.8\text{ lb})\mathbf{k}$$

and $\mathbf{D} = (18.20\text{ lb})\mathbf{j} - (40.8\text{ lb})\mathbf{k}$ ◀

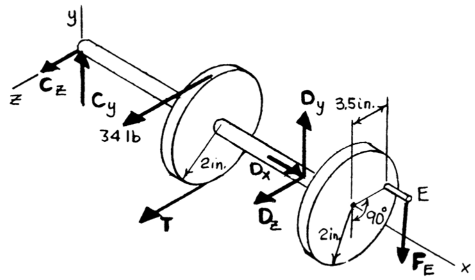


PROBLEM 4.100

Solve Problem 4.99 for $\theta = 90°$.

P4.99 For the portion of a machine shown, the 4-in.-diameter pulley *A* and wheel *B* are fixed to a shaft supported by bearings at *C* and *D*. The spring of constant 2 lb/in. is unstretched when $\theta = 0$, and the bearing at *C* does not exert any axial force. Knowing that $\theta = 180°$ and that the machine is at rest and in equilibrium, determine (*a*) the tension *T*, (*b*) the reactions at *C* and *D*. Neglect the weights of the shaft, pulley, and wheel.

SOLUTION



First, determine the spring force, $\mathbf{F}_E$, at $\theta = 90°$.

$$F_E = k_s x$$

where $$k_s = 2 \text{ lb/in.}$$

and $$x = L_{\text{final}} - L_{\text{initial}} = \left(\sqrt{(3.5)^2 + (12)^2}\right) - (12 - 3.5) = 12.5 - 8.5 = 4.0 \text{ in.}$$

$$\therefore\ F_E = (2 \text{ lb/in.})(4.0 \text{ in.}) = 8.0 \text{ lb}$$

Then $$\mathbf{F}_E = \frac{-12.0}{12.5}(8.0 \text{ lb})\mathbf{j} + \frac{3.5}{12.5}(8.0 \text{ lb})\mathbf{k} = -(7.68 \text{ lb})\mathbf{j} + (2.24 \text{ lb})\mathbf{k}$$

(*a*) From f.b.d. of machine part

$$\Sigma M_x = 0: \quad (34 \text{ lb})(2 \text{ in.}) - T(2 \text{ in.}) - (7.68 \text{ lb})(3.5 \text{ in.}) = 0$$

$$\therefore\ T = 20.56 \text{ lb} \qquad \text{or } T = 20.6 \text{ lb} \blacktriangleleft$$

(*b*) $$\Sigma M_{D(z\text{-axis})} = 0: \quad -C_y(10 \text{ in.}) - (7.68 \text{ lb})(3.0 \text{ in.}) = 0$$

$$\therefore\ C_y = -2.304 \text{ lb} \qquad \text{or} \qquad \mathbf{C}_y = -(2.30 \text{ lb})\mathbf{j}$$

$$\Sigma M_{D(y\text{-axis})} = 0: \quad C_z(10 \text{ in.}) + (34 \text{ lb})(4.0 \text{ in.}) + (20.56 \text{ lb})(4.0 \text{ in.}) - (2.24 \text{ lb})(3 \text{ in.}) = 0$$

$$\therefore\ C_z = -21.152 \text{ lb} \qquad \text{or} \qquad \mathbf{C}_z = -(21.2 \text{ lb})\mathbf{k}$$

$$\text{and } \mathbf{C} = -(2.30 \text{ lb})\mathbf{j} - (21.2 \text{ lb})\mathbf{k} \blacktriangleleft$$

PROBLEM 4.100 CONTINUED

$$\Sigma F_x = 0: \quad D_x = 0$$

$$\Sigma M_{C(z\text{-axis})} = 0: \quad D_y(10 \text{ in.}) - (7.68 \text{ lb})(13 \text{ in.}) = 0$$

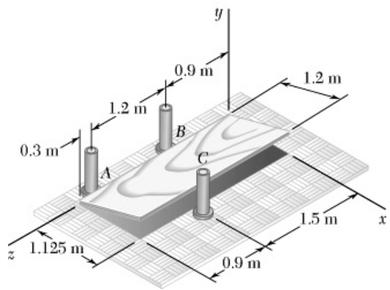
$$\therefore \quad D_y = 9.984 \text{ lb} \qquad \text{or} \qquad \mathbf{D}_y = (9.98 \text{ lb})\mathbf{j}$$

$$\Sigma M_{C(y\text{-axis})} = 0: \quad -(34 \text{ lb})(6 \text{ in.}) - (20.56 \text{ lb})(6 \text{ in.}) - D_z(10 \text{ in.}) - (2.24 \text{ lb})(13 \text{ in.}) = 0$$

$$\therefore \quad D_z = -35.648 \text{ lb} \qquad \text{or} \qquad \mathbf{D}_z = -(35.6 \text{ lb})\mathbf{k}$$

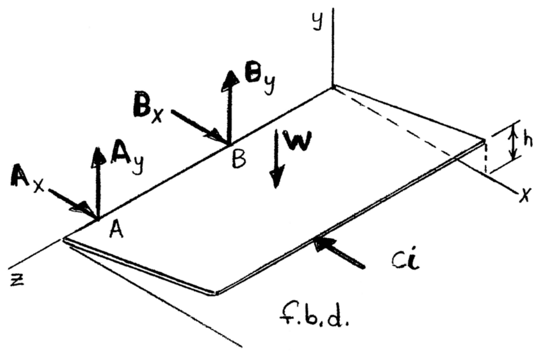
and $\mathbf{D} = (9.98 \text{ lb})\mathbf{j} - (35.6 \text{ lb})\mathbf{k}$ ◀

PROBLEM 4.101



A 1.2×2.4-m sheet of plywood having a mass of 17 kg has been temporarily placed among three pipe supports. The lower edge of the sheet rests on small collars *A* and *B* and its upper edge leans against pipe *C*. Neglecting friction at all surfaces, determine the reactions at *A*, *B*, and *C*.

SOLUTION



First note $$W = mg = (17 \text{ kg})(9.81 \text{ m/s}^2) = 166.77 \text{ N}$$

$$h = \sqrt{(1.2)^2 - (1.125)^2} = 0.41758 \text{ m}$$

From f.b.d. of plywood sheet

$$\Sigma M_z = 0: \quad C(h) - W\left[\frac{(1.125 \text{ m})}{2}\right] = 0$$

$$C(0.41758 \text{ m}) - (166.77 \text{ N})(0.5625 \text{ m}) = 0$$

$$\therefore \quad C = 224.65 \text{ N} \qquad \text{or} \qquad \mathbf{C} = -(225 \text{ N})\mathbf{i}$$

$$\Sigma M_{B(y\text{-axis})} = 0: \quad -(224.65 \text{ N})(0.6 \text{ m}) + A_x(1.2 \text{ m}) = 0$$

$$\therefore \quad A_x = 112.324 \text{ N} \qquad \text{or} \qquad \mathbf{A}_x = (112.3 \text{ N})\mathbf{i}$$

$$\Sigma M_{B(x\text{-axis})} = 0: \quad (166.77 \text{ N})(0.3 \text{ m}) - A_y(1.2 \text{ m}) = 0$$

$$\therefore \quad A_y = 41.693 \text{ N} \qquad \text{or} \qquad \mathbf{A}_y = (41.7 \text{ N})\mathbf{j}$$

$$\Sigma M_{A(y\text{-axis})} = 0: \quad (224.65 \text{ N})(0.6 \text{ m}) - B_x(1.2 \text{ m}) = 0$$

$$\therefore \quad B_x = 112.325 \text{ N} \qquad \text{or} \qquad \mathbf{B}_x = (112.3 \text{ N})\mathbf{i}$$

PROBLEM 4.101 CONTINUED

$$\Sigma M_{A(x\text{-axis})} = 0: \quad B_y(1.2 \text{ m}) - (166.77 \text{ N})(0.9 \text{ m}) = 0$$

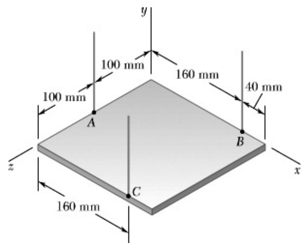
$$\therefore \quad B_y = 125.078 \text{ N} \qquad \text{or} \qquad \mathbf{B}_y = (125.1 \text{ N})\mathbf{j}$$

$$\therefore \quad \mathbf{A} = (112.3 \text{ N})\mathbf{i} + (41.7 \text{ N})\mathbf{j} \blacktriangleleft$$

$$\mathbf{B} = (112.3 \text{ N})\mathbf{i} + (125.1 \text{ N})\mathbf{j} \blacktriangleleft$$

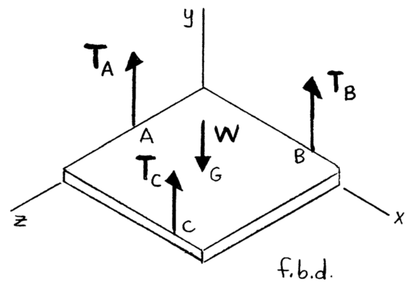
$$\mathbf{C} = -(225 \text{ N})\mathbf{i} \blacktriangleleft$$

PROBLEM 4.102



The 200×200-mm square plate shown has a mass of 25 kg and is supported by three vertical wires. Determine the tension in each wire.

SOLUTION



First note $$W = mg = (25\text{ kg})(9.81\text{ m/s}^2) = 245.25\text{ N}$$

From f.b.d. of plate

$$\Sigma M_x = 0: \quad (245.25\text{ N})(100\text{ mm}) - T_A(100\text{ mm}) - T_C(200\text{ mm}) = 0$$

$$\therefore \quad T_A + 2T_C = 245.25\text{ N} \qquad (1)$$

$$\Sigma M_z = 0: \quad T_B(160\text{ mm}) + T_C(160\text{ mm}) - (245.25\text{ N})(100\text{ mm}) = 0$$

$$\therefore \quad T_B + T_C = 153.281\text{ N} \qquad (2)$$

$$\Sigma F_y = 0: \quad T_A + T_B + T_C - 245.25\text{ N} = 0$$

$$\therefore \quad T_B + T_C = 245.25 - T_A \qquad (3)$$

Equating Equations (2) and (3) yields

$$T_A = 245.25\text{ N} - 153.281\text{ N} = 91.969\text{ N} \qquad (4)$$

or $$T_A = 92.0\text{ N}$$

Substituting the value of T_A into Equation (1)

$$T_C = \frac{(245.25\text{ N} - 91.969\text{ N})}{2} = 76.641\text{ N} \qquad (5)$$

or $$T_C = 76.6\text{ N}$$

Substituting the value of T_C into Equation (2)

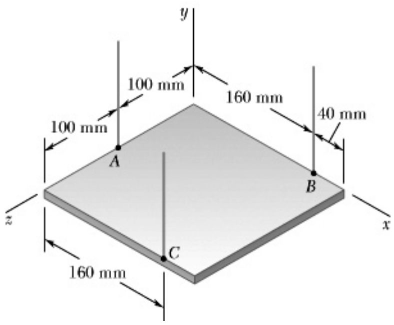
$$T_B = 153.281\text{ N} - 76.641\text{ N} = 76.639\text{ N} \qquad \text{or} \qquad T_B = 76.6\text{ N}$$

$T_A = 92.0\text{ N}$ ◀

$T_B = 76.6\text{ N}$ ◀

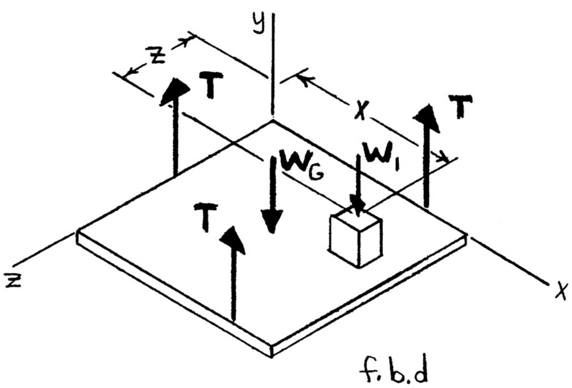
$T_C = 76.6\text{ N}$ ◀

PROBLEM 4.103



The 200×200-mm square plate shown has a mass of 25 kg and is supported by three vertical wires. Determine the mass and location of the lightest block which should be placed on the plate if the tensions in the three cables are to be equal.

SOLUTION



First note $$W_G = m_{p1}g = (25 \text{ kg})\left(9.81 \text{ m/s}^2\right) = 245.25 \text{ N}$$

$$W_1 = mg = m\left(9.81 \text{ m/s}^2\right) = (9.81m)\,\text{N}$$

From f.b.d. of plate

$$\Sigma F_y = 0: \quad 3T - W_G - W_1 = 0\,(1)$$

$$\Sigma M_x = 0: \quad W_G(100 \text{ mm}) + W_1(z) - T(100 \text{ mm}) - T(200 \text{ mm}) = 0$$

$$\text{or} \quad -300T + 100W_G + W_1 z = 0 \qquad (2)$$

$$\Sigma M_z = 0: \quad 2T(160 \text{ mm}) - W_G(100 \text{ mm}) - W_1(x) = 0$$

$$\text{or} \quad 320T - 100W_G - W_1 x = 0 \qquad (3)$$

Eliminate T by forming $100 \times [\text{Eq. }(1) + \text{Eq. }(2)]$

$$-100W_1 + W_1 z = 0$$

$$\therefore \quad z = 100 \text{ mm} \qquad 0 \le z \le 200 \text{ mm}, \quad \therefore \quad \text{okay}$$

Now, $3 \times [\text{Eq. }(3)] - 320 \times [\text{Eq. }(1)]$ yields

$$3(320T) - 3(100)W_G - 3W_1 x - 320(3T) + 320W_G + 320W_1 = 0$$

PROBLEM 4.103 CONTINUED

or $$20W_G + (320 - 3x)W_1 = 0$$

or $$\frac{W_1}{W_G} = \frac{20}{(3x - 320)}$$

The smallest value of $\frac{W_1}{W_G}$ will result in the smallest value of W_1 since W_G is given.

$$\therefore \quad \text{Use } x = x_{\text{max}} = 200 \text{ mm}$$

and then $$\frac{W_1}{W_G} = \frac{20}{3(200) - 320} = \frac{1}{14}$$

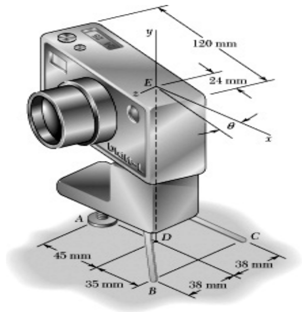
$$\therefore \quad W_1 = \frac{W_G}{14} = \frac{245.25 \text{ N}}{14} = 17.5179 \text{ N}(\text{minimum})$$

and $$m = \frac{W_1}{g} = \frac{17.5179 \text{ N}}{9.81 \text{ m/s}^2} = 1.78571 \text{ kg}$$

or $m = 1.786$ kg ◀

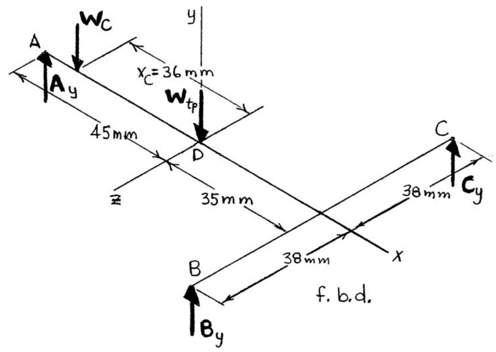
at $x = 200$ mm, $z = 100$ mm ◀

PROBLEM 4.104



A camera of mass 240 g is mounted on a small tripod of mass 200 g. Assuming that the mass of the camera is uniformly distributed and that the line of action of the weight of the tripod passes through *D*, determine (*a*) the vertical components of the reactions at *A*, *B*, and *C* when $\theta = 0$, (*b*) the maximum value of θ if the tripod is not to tip over.

SOLUTION



First note $$W_C = m_C g = (0.24 \text{ kg})(9.81 \text{ m/s}^2) = 2.3544 \text{ N}$$

$$W_{\text{tp}} = m_{\text{tp}} g = (0.20 \text{ kg})(9.81 \text{ m/s}^2) = 1.9620 \text{ N}$$

For $\theta = 0$ $$x_C = -(60 \text{ mm} - 24 \text{ mm}) = -36 \text{ mm}$$

$$z_C = 0$$

(*a*) From f.b.d. of camera and tripod as projected onto plane *ABCD*

$$\Sigma F_y = 0: \quad A_y + B_y + C_y - W_C - W_{\text{tp}} = 0$$

$$\therefore \quad A_y + B_y + C_y = 2.3544 \text{ N} + 1.9620 \text{ N} = 4.3164 \text{ N} \tag{1}$$

$$\Sigma M_x = 0: \quad C_y(38 \text{ mm}) - B_y(38 \text{ mm}) = 0 \qquad \therefore \quad C_y = B_y \tag{2}$$

$$\Sigma M_z = 0: \quad B_y(35 \text{ mm}) + C_y(35 \text{ mm}) + (2.3544 \text{ N})(36 \text{ mm}) - A_y(45 \text{ mm}) = 0$$

$$\therefore \quad 9A_y - 7B_y - 7C_y = 16.9517 \tag{3}$$

Substitute C_y with B_y from Equation (2) into Equations (1) and (3), and solve by elimination

$$7\left(A_y + 2B_y = 4.3164\right)$$

$$\underline{9A_y - 14B_y = 16.9517}$$

$$16A_y \qquad = 47.166$$

PROBLEM 4.104 CONTINUED

$$\therefore \quad A_y = 2.9479 \text{ N}$$

or $\mathbf{A}_y = 2.95 \text{ N} \uparrow$ ◀

Substituting $A_y = 2.9479$ N into Equation (1)

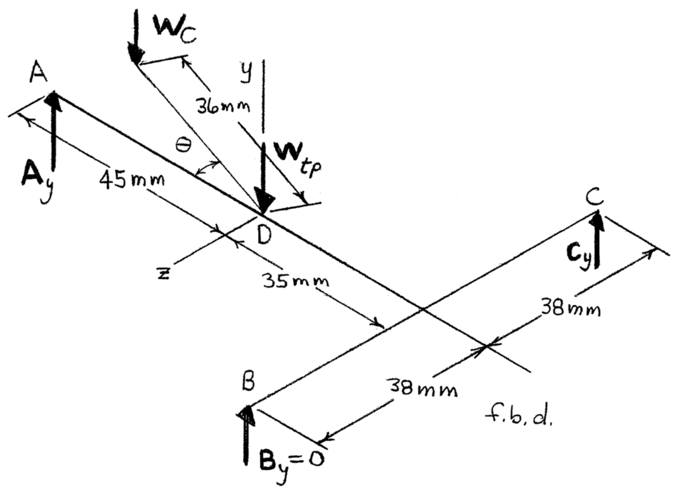
$$2.9479 \text{ N} + 2B_y = 4.3164$$

$$\therefore \quad B_y = 0.68425 \text{ N}$$

$$C_y = 0.68425 \text{ N}$$

or $\mathbf{B}_y = \mathbf{C}_y = 0.684 \text{ N} \uparrow$ ◀

(*b*) $B_y = 0$ for impending tipping



From f.b.d. of camera and tripod as projected onto plane *ABCD*

$$\Sigma F_y = 0: \quad A_y + C_y - W_C - W_{tp} = 0$$

$$\therefore \quad A_y + C_y = 4.3164 \text{ N} \qquad (1)$$

$$\Sigma M_x = 0: \quad C_y(38 \text{ mm}) - (2.3544 \text{ N})\left[(36 \text{ mm})\sin\theta\right] = 0$$

$$\therefore \quad C_y = 2.2305\sin\theta \qquad (2)$$

$$\Sigma M_z = 0: \quad C_y(35 \text{ mm}) - A_y(45 \text{ mm}) + (2.3544 \text{ N})\left[(36 \text{ mm})\cos\theta\right] = 0$$

$$\therefore \quad 9A_y - 7C_y = (16.9517 \text{ N})\cos\theta \qquad (3)$$

Forming $7 \times [\text{Eq. } (1)] + [\text{Eq. } (3)]$ yields

$$16A_y = 30.215 \text{ N} + (16.9517 \text{ N})\cos\theta \qquad (4)$$

PROBLEM 4.104 CONTINUED

Substituting Equation (2) into Equation (3)

$$9A_y - (15.6134\text{ N})\sin\theta = (16.9517\text{ N})\cos\theta \qquad (5)$$

Forming $9 \times [\text{Eq. }(4)] - 16 \times [\text{Eq. }(5)]$ yields

$$(249.81\text{ N})\sin\theta = 271.93\text{ N} - (118.662\text{ N})\cos\theta$$

or

$$\cos^2\theta = [2.2916\text{ N} - (2.1053\text{ N})\sin\theta]^2$$

Now

$$\cos^2\theta = 1 - \sin^2\theta$$

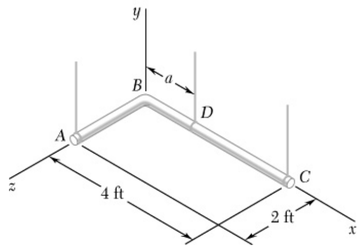
$$\therefore \quad 5.4323\sin^2\theta - 9.6490\sin\theta + 4.2514 = 0$$

Using quadratic formula to solve,

$$\sin\theta = 0.80981 \quad \text{and} \quad \sin\theta = 0.96641$$

$$\therefore \quad \theta = 54.078° \quad \text{and} \quad \theta = 75.108°$$

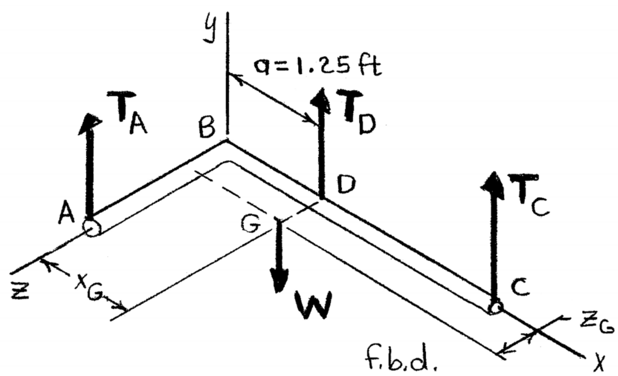
or $\theta_{max} = 54.1°$ before tipping ◀



PROBLEM 4.105

Two steel pipes *AB* and *BC*, each having a weight per unit length of 5 lb/ft, are welded together at *B* and are supported by three wires. Knowing that $a = 1.25$ ft, determine the tension in each wire.

SOLUTION



First note

$$W_{AB} = (5 \text{ lb/ft})(2 \text{ ft}) = 10 \text{ lb}$$

$$W_{BC} = (5 \text{ lb/ft})(4 \text{ ft}) = 20 \text{ lb}$$

$$W = W_{AB} + W_{BC} = 30 \text{ lb}$$

To locate the equivalent force of the pipe assembly weight

$$\mathbf{r}_{G/B} \times \mathbf{W} = \Sigma(\mathbf{r_i} \times \mathbf{W_i}) = \mathbf{r}_{G(AB)} \times \mathbf{W}_{AB} + \mathbf{r}_{G(BC)} \times \mathbf{W}_{BC}$$

or

$$(x_G\mathbf{i} + z_G\mathbf{k}) \times (-30 \text{ lb})\mathbf{j} = (1 \text{ ft})\mathbf{k} \times (-10 \text{ lb})\mathbf{j} + (2 \text{ ft})\mathbf{i} \times (-20 \text{ lb})\mathbf{j}$$

$$\therefore \quad -(30 \text{ lb})x_G\mathbf{k} + (30 \text{ lb})z_G\mathbf{i} = (10 \text{ lb}\cdot\text{ft})\mathbf{i} - (40 \text{ lb}\cdot\text{ft})\mathbf{k}$$

From **i**-coefficient

$$z_G = \frac{10 \text{ lb}\cdot\text{ft}}{30 \text{ lb}} = \frac{1}{3}\text{ft}$$

k-coefficient

$$x_G = \frac{40 \text{ lb}\cdot\text{ft}}{30 \text{ lb}} = 1\frac{1}{3}\text{ft}$$

From f.b.d. of piping

$$\Sigma M_x = 0: \quad W(z_G) - T_A(2 \text{ ft}) = 0$$

$$\therefore \quad T_A = \left(\frac{1}{2} \text{ ft}\right)30 \text{ lb}\left(\frac{1}{3}\text{ft}\right) = 5 \text{ lb} \qquad \text{or} \qquad T_A = 5.00 \text{ lb}$$

$$\Sigma F_y = 0: \quad 5 \text{ lb} + T_D + T_C - 30 \text{ lb} = 0$$

$$\therefore \quad T_D + T_C = 25 \text{ lb} \tag{1}$$

PROBLEM 4.105 CONTINUED

$$\Sigma M_z = 0: \quad T_D(1.25\text{ ft}) + T_C(4\text{ ft}) - 30\text{ lb}\left(\frac{4}{3}\text{ft}\right) = 0$$

$$\therefore \quad 1.25T_D + 4T_C = 40\text{ lb}\cdot\text{ft} \qquad (2)$$

$-4\left[\text{Equation }(1)\right]$ $$-4T_D - 4T_C = -100 \qquad (3)$$

Equation (2) + Equation (3) $$-2.75T_D = -60$$

$$\therefore \quad T_D = 21.818\text{ lb} \qquad \text{or} \qquad T_D = 21.8\text{ lb}$$

From Equation (1) $$T_C = 25 - 21.818 = 3.1818\text{ lb} \qquad \text{or} \qquad T_C = 3.18\text{ lb}$$

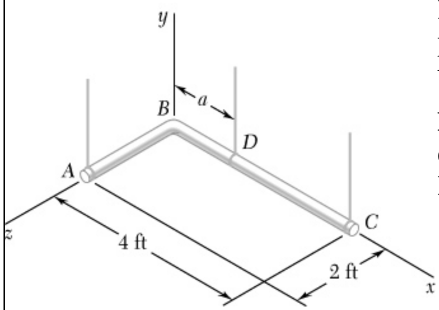
Results:

$T_A = 5.00$ lb ◀

$T_C = 3.18$ lb ◀

$T_D = 21.8$ lb ◀

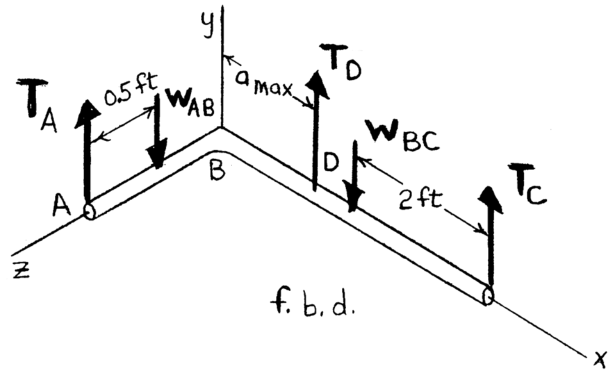
PROBLEM 4.106



For the pile assembly of Problem 4.105, determine (*a*) the largest permissible value of *a* if the assembly is not to tip, (*b*) the corresponding tension in each wire.

P4.105 Two steel pipes *AB* and *BC*, each having a weight per unit length of 5 lb/ft, are welded together at *B* and are supported by three wires. Knowing that $a = 1.25$ ft, determine the tension in each wire.

SOLUTION



First note $$W_{AB} = (5\text{ lb/ft})(2\text{ ft}) = 10\text{ lb}$$

$$W_{BC} = (5\text{ lb/ft})(4\text{ ft}) = 20\text{ lb}$$

From f.b.d. of pipe assembly

$$\Sigma F_y = 0: \quad T_A + T_C + T_D - 10\text{ lb} - 20\text{ lb} = 0$$

$$\therefore \quad T_A + T_C + T_D = 30\text{ lb} \qquad (1)$$

$$\Sigma M_x = 0: \quad (10\text{ lb})(1\text{ ft}) - T_A(2\text{ ft}) = 0$$

or $$T_A = 5.00\text{ lb} \qquad (2)$$

From Equations (1) and (2) $$T_C + T_D = 25\text{ lb} \qquad (3)$$

$$\Sigma M_z = 0: \quad T_C(4\text{ ft}) + T_D(a_{\max}) - 20\text{ lb}(2\text{ ft}) = 0$$

or $$(4\text{ ft})T_C + T_D a_{\max} = 40\text{ lb}\cdot\text{ft} \qquad (4)$$

PROBLEM 4.106 CONTINUED

Using Equation (3) to eliminate T_C

$$4\left(25 - T_D\right) + T_D a_{max} = 40$$

or
$$a_{max} = 4 - \frac{60}{T_D}$$

By observation, a is maximum when T_D is maximum. From Equation (3), $\left(T_D\right)_{max}$ occurs when $T_C = 0$. Therefore, $\left(T_D\right)_{max} = 25 \text{ lb}$ and

$$a_{max} = 4 - \frac{60}{25}$$

$$= 1.600 \text{ ft}$$

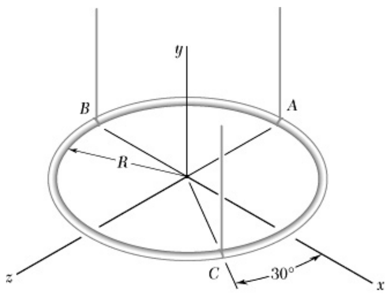
Results: (*a*) $a_{max} = 1.600 \text{ ft}$ ◀

(*b*) $T_A = 5.00 \text{ lb}$ ◀

$T_C = 0$ ◀

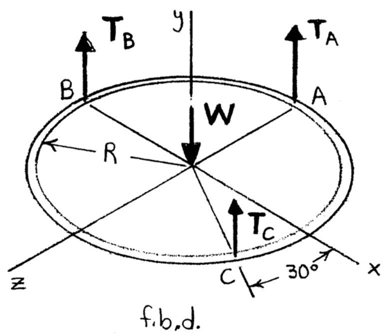
$T_D = 25.0 \text{ lb}$ ◀

PROBLEM 4.107



A uniform aluminum rod of weight W is bent into a circular ring of radius R and is supported by three wires as shown. Determine the tension in each wire.

SOLUTION



From f.b.d. of ring

$$\Sigma F_y = 0: \quad T_A + T_B + T_C - W = 0$$

$$\therefore \quad T_A + T_B + T_C = W \tag{1}$$

$$\Sigma M_x = 0: \quad T_A(R) - T_C(R\sin 30°) = 0$$

$$\therefore \quad T_A = 0.5T_C \tag{2}$$

$$\Sigma M_z = 0: \quad T_C(R\cos 30°) - T_B(R) = 0$$

$$\therefore \quad T_B = 0.86603T_C \tag{3}$$

Substituting T_A and T_B from Equations (2) and (3) into Equation (1)

$$0.5T_C + 0.86603T_C + T_C = W$$

$$\therefore \quad T_C = 0.42265W$$

From Equation (2)

$$T_A = 0.5(0.42265W) = 0.21132W$$

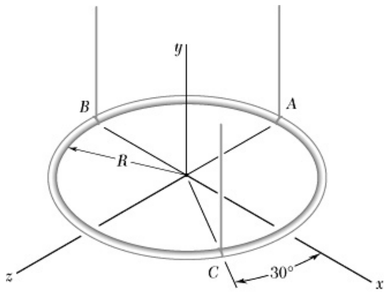
From Equation (3)

$$T_B = 0.86603(0.42265W) = 0.36603W$$

or $T_A = 0.211W$ ◀

$T_B = 0.366W$ ◀

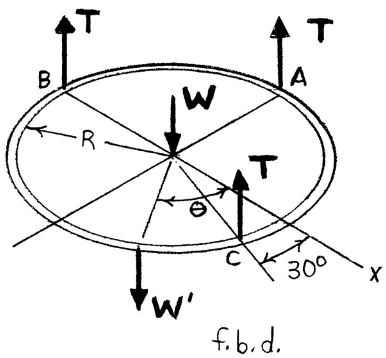
$T_C = 0.423W$ ◀



PROBLEM 4.108

A uniform aluminum rod of weight W is bent into a circular ring of radius R and is supported by three wires as shown. A small collar of weight W' is then placed on the ring and positioned so that the tensions in the three wires are equal. Determine (*a*) the position of the collar, (*b*) the value of W', (*c*) the tension in the wires.

SOLUTION



Let θ = angle from x-axis to small collar of weight W'

From f.b.d. of ring

$$\Sigma F_y = 0: \quad 3T - W - W' = 0 \qquad (1)$$

$$\Sigma M_x = 0: \quad T(R) - T(R\sin 30°) + W'(R\sin\theta) = 0$$

or

$$W'\sin\theta = -\frac{1}{2}T \qquad (2)$$

$$\Sigma M_z = 0: \quad T(R\cos 30°) - W'(R\cos\theta) - T(R) = 0$$

or

$$W'\cos\theta = -\left(1 - \frac{\sqrt{3}}{2}\right)T \qquad (3)$$

Dividing Equation (2) by Equation (3)

$$\tan\theta = \left(\frac{1}{2}\right)\left[1 - \left(\frac{\sqrt{3}}{2}\right)\right]^{-1} = 3.7321$$

$$\therefore \quad \theta = 75.000° \qquad \text{and} \qquad \theta = 255.00°$$

Based on Equations (2) and (3), $\theta = 75.000°$ will give a negative value for W', which is not acceptable.

(*a*) $\therefore$ W' is located at $\theta = 255°$ from the x-axis or $15°$ from A towards B. ◀

(*b*) From Equation (1) and Equation (2)

$$W' = 3(-2W')(\sin 255°) - W$$

$$\therefore \quad W' = 0.20853W$$

or $W' = 0.209W$ ◀

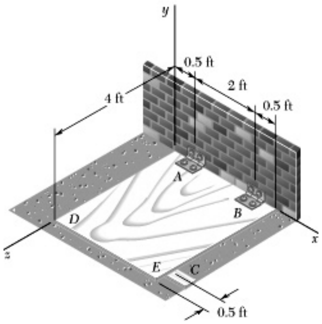
(*c*) From Equation (1)

$$T = -2(0.20853W)\sin 255°$$

$$= 0.40285W$$

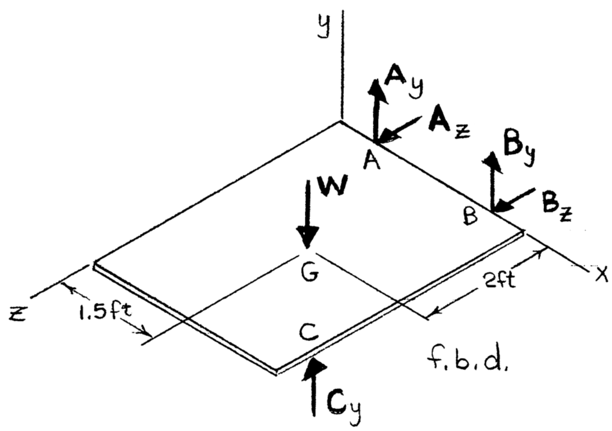
or $T = 0.403W$ ◀

PROBLEM 4.109



An opening in a floor is covered by a 3×4-ft sheet of plywood weighing 12 lb. The sheet is hinged at *A* and *B* and is maintained in a position slightly above the floor by a small block *C*. Determine the vertical component of the reaction (*a*) at *A*, (*b*) at *B*, (*c*) at *C*.

SOLUTION



From f.b.d. of plywood sheet

$$\Sigma M_x = 0: \quad (12 \text{ lb})(2 \text{ ft}) - C_y(3.5 \text{ ft}) = 0$$

$$\therefore \quad C_y = 6.8571 \text{ lb} \qquad \text{or} \qquad C_y = 6.86 \text{ lb}$$

$$\Sigma M_{B(z\text{-axis})} = 0: \quad (12 \text{ lb})(1 \text{ ft}) + (6.8571 \text{ lb})(0.5 \text{ ft}) - A_y(2 \text{ ft}) = 0$$

$$\therefore \quad A_y = 7.7143 \text{ lb} \qquad \text{or} \qquad A_y = 7.71 \text{ lb}$$

$$\Sigma M_{A(z\text{-axis})} = 0: \quad -(12 \text{ lb})(1 \text{ ft}) + B_y(2 \text{ ft}) + (6.8571 \text{ lb})(2.5 \text{ ft}) = 0$$

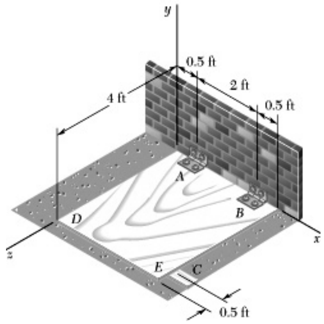
$$\therefore \quad B_y = 2.5714 \text{ lb} \qquad \text{or} \qquad B_y = 2.57 \text{ lb}$$

(*a*) $A_y = 7.71 \text{ lb}$ ◀

(*b*) $B_y = 2.57 \text{ lb}$ ◀

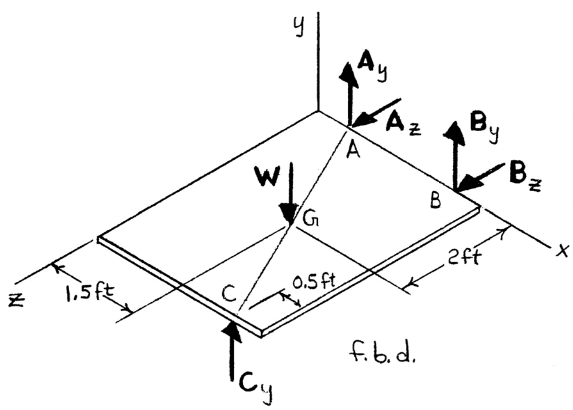
(*c*) $C_y = 6.86 \text{ lb}$ ◀

PROBLEM 4.110



Solve Problem 4.109 assuming that the small block C is moved and placed under edge DE at a point 0.5 ft from corner E.

SOLUTION



First, $\mathbf{r}_{B/A} = (2\text{ ft})\mathbf{i}$

$$\mathbf{r}_{C/A} = (2\text{ ft})\mathbf{i} + (4\text{ ft})\mathbf{k}$$

$$\mathbf{r}_{G/A} = (1\text{ ft})\mathbf{i} + (2\text{ ft})\mathbf{k}$$

From f.b.d. of plywood sheet

$$\Sigma\mathbf{M}_A = 0: \quad \mathbf{r}_{B/A} \times \left(B_y\mathbf{j} + B_z\mathbf{k}\right) + \mathbf{r}_{C/A} \times C_y\mathbf{j} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) = 0$$

$$(2\text{ ft})\mathbf{i} \times B_y\mathbf{j} + (2\text{ ft})\mathbf{i} \times B_z\mathbf{k} + [(2\text{ ft})\mathbf{i} + (4\text{ ft})\mathbf{k}] \times C_y\mathbf{j}$$

$$+ [(1\text{ ft})\mathbf{i} + (2\text{ ft})\mathbf{k}] \times (-12\text{ lb})\mathbf{j} = 0$$

$$2B_y\mathbf{k} - 2B_z\mathbf{j} + 2C_y\mathbf{k} - 4C_y\mathbf{i} - 12\mathbf{k} + 24\mathbf{i} = 0$$

i-coeff. $\quad -4C_y + 24 = 0 \qquad \therefore\ C_y = 6.00\text{ lb}$

j-coeff. $\quad -2B_z = 0 \qquad \therefore\ B_z = 0$

k-coeff. $\quad 2B_y + 2C_y - 12 = 0$

or $\quad 2B_y + 2(6) - 12 = 0 \qquad \therefore\ B_y = 0$

PROBLEM 4.110 CONTINUED

$$\Sigma\mathbf{F} = 0: \quad A_y\mathbf{j} + A_z\mathbf{k} + B_y\mathbf{j} + B_z\mathbf{k} + C_y\mathbf{j} - W\mathbf{j} = 0$$

$$A_y\mathbf{j} + A_z\mathbf{k} + 0\mathbf{j} + 0\mathbf{k} + 6\mathbf{j} - 12\mathbf{j} = 0$$

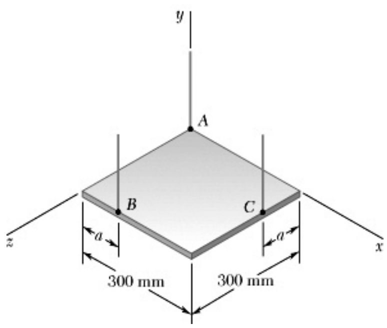
j-coeff. $\quad A_y + 6 - 12 = 0 \qquad \therefore \; A_y = 6.00 \text{ lb}$

k-coeff. $\quad A_z = 0 \qquad A_z = 0$

$\therefore$ *a*) $A_y = 6.00 \text{ lb}$ ◀

b) $B_y = 0$ ◀

c) $C_y = 6.00 \text{ lb}$ ◀



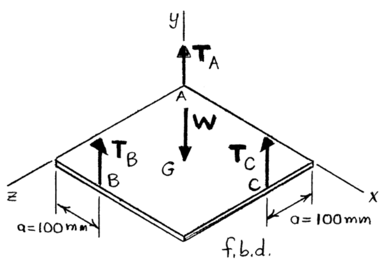
PROBLEM 4.111

The 10-kg square plate shown is supported by three vertical wires. Determine (*a*) the tension in each wire when $a = 100$ mm, (*b*) the value of *a* for which tensions in the three wires are equal.

SOLUTION

First note $\qquad W = mg = (10\text{ kg})(9.81\text{ m/s}^2) = 98.1\text{ N}$

(*a*)



(*a*) From f.b.d. of plate

$$\Sigma F_y = 0: \quad T_A + T_B + T_C - W = 0$$

$$\therefore \quad T_A + T_B + T_C = 98.1\text{ N} \qquad (1)$$

$$\Sigma M_x = 0: \quad W(150\text{ mm}) - T_B(300\text{ mm}) - T_C(100\text{ mm}) = 0$$

$$\therefore \quad 6T_B + 2T_C = 294.3 \qquad (2)$$

$$\Sigma M_z = 0: \quad T_B(100\text{ mm}) + T_C(300\text{ mm}) - (98.1\text{ N})(150\text{ mm}) = 0$$

$$\therefore \quad -6T_B - 18T_C = -882.9 \qquad (3)$$

Equation (2) + Equation (3)

$$-16T_C = -588.6$$

$$\therefore \quad T_C = 36.788\text{ N}$$

or $\qquad T_C = 36.8\text{ N} \blacktriangleleft$

Substitution into Equation (2)

$$6T_B + 2(36.788\text{ N}) = 294.3\text{ N}$$

$$\therefore \quad T_B = 36.788\text{ N} \qquad \text{or} \qquad T_B = 36.8\text{ N} \blacktriangleleft$$

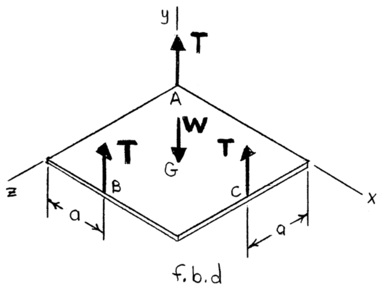
From Equation (1)

$$T_A + 36.788 + 36.788 = 98.1\text{ N}$$

$$\therefore \quad T_A = 24.525\text{ N} \qquad \text{or} \qquad T_A = 24.5\text{ N} \blacktriangleleft$$

PROBLEM 4.111 CONTINUED

(*b*)



(*b*) From f.b.d. of plate

$$\Sigma F_y = 0: \quad 3T - W = 0$$

$$\therefore \quad T = \frac{1}{3}W \tag{1}$$

$$\Sigma M_x = 0: \quad W(150 \text{ mm}) - T(a) - T(300 \text{ mm}) = 0$$

$$\therefore \quad T = \frac{150W}{a + 300} \tag{2}$$

Equating Equation (1) to Equation (2)

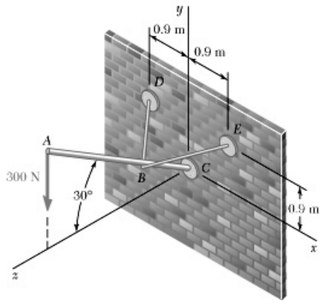
$$\frac{1}{3}W = \frac{150W}{a + 300}$$

or

$$a + 300 = 3(150)$$

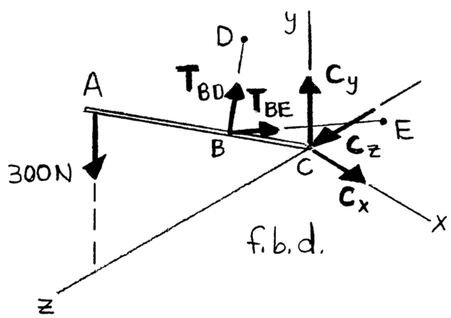
or $a = 150.0$ mm ◀

PROBLEM 4.112



The 3-m flagpole AC forms an angle of 30^{o} with the z axis. It is held by a ball-and-socket joint at C and by two thin braces BD and BE. Knowing that the distance BC is 0.9 m, determine the tension in each brace and the reaction at C.

SOLUTION



T_{BE} can be found from ΣM about line CE

From f.b.d. of flagpole

$$\Sigma M_{CE} = 0: \quad \lambda_{CE} \cdot \left(\mathbf{r}_{B/C} \times \mathbf{T}_{BD}\right) + \lambda_{CE} \cdot \left(\mathbf{r}_{A/C} \times \mathbf{F}_A\right) = 0$$

where $$\lambda_{CE} = \frac{(0.9\text{ m})\mathbf{i} + (0.9\text{ m})\mathbf{j}}{\sqrt{(0.9)^2 + (0.9)^2}\text{ m}} = \frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j})$$

$$\mathbf{r}_{B/C} = \left[(0.9\text{ m})\sin 30°\right]\mathbf{j} + \left[(0.9\text{ m})\cos 30°\right]\mathbf{k}$$

$$= (0.45\text{ m})\mathbf{j} + (0.77942\text{ m})\mathbf{k}$$

$$\mathbf{T}_{BD} = \lambda_{BD}T_{BD} = \left\{\frac{-(0.9\text{ m})\mathbf{i} + \left[0.9\text{ m} - (0.9\text{ m})\sin 30°\right]\mathbf{j} - \left[(0.9\text{ m})\cos 30°\right]\mathbf{k}}{\sqrt{(0.9)^2 + (0.45)^2 + (0.77942)^2}\text{ m}}\right\}T_{BD}$$

$$= \left[-(0.9\text{ m})\mathbf{i} + (0.45\text{ m})\mathbf{j} - (0.77942\text{ m})\mathbf{k}\right]\frac{T_{BD}}{\sqrt{1.62}}$$

$$= (-0.70711\mathbf{i} + 0.35355\mathbf{j} - 0.61237\mathbf{k})T_{BD}$$

$$\mathbf{r}_{A/C} = (3\text{ m})\sin 30°\mathbf{j} + (3\text{ m})\cos 30°\mathbf{k} = (1.5\text{ m})\mathbf{j} + (2.5981\text{ m})\mathbf{k}$$

$$\mathbf{F}_A = -(300\text{ N})\mathbf{j}$$

$$\therefore \begin{vmatrix} 1 & 1 & 0 \\ 0 & 0.45 & 0.77942 \\ -0.70711 & 0.35355 & -0.61237 \end{vmatrix}\left(\frac{T_{BD}}{\sqrt{2}}\right) + \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1.5 & 2.5981 \\ 0 & -300 & 0 \end{vmatrix}\left(\frac{1}{\sqrt{2}}\right) = 0$$

PROBLEM 4.112 CONTINUED

or $$-1.10227T_{BD} + 779.43 = 0$$

$$\therefore \quad T_{BD} = 707.12 \text{ N} \qquad \text{or } T_{BD} = 707 \text{ N} \blacktriangleleft$$

Based on symmetry with *yz*-plane, $$T_{BE} = T_{BD} = 707.12 \text{ N} \qquad \text{or } T_{BE} = 707 \text{ N} \blacktriangleleft$$

The reaction forces at C are found from $\Sigma\mathbf{F} = 0$

$$\Sigma F_x = 0: \quad -(T_{BD})_x + (T_{BE})_x + C_x = 0 \qquad \text{or} \qquad C_x = 0$$

$$\Sigma F_y = 0: \quad (T_{BD})_y + (T_{BE})_y + C_y - 300 \text{ N} = 0$$

$$C_y = 300 \text{ N} - 2(0.35355)(707.12 \text{ N})$$

$$\therefore \quad C_y = -200.00 \text{ N}$$

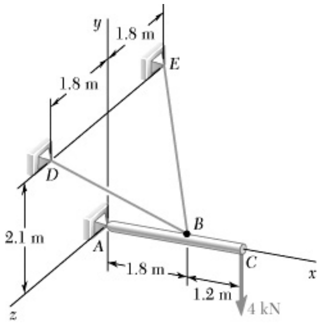
$$\Sigma F_z = 0: \quad C_z - (T_{BD})_z - (T_{BE})_z = 0$$

$$C_z = 2(0.61237)(707.12 \text{ N})$$

$$\therefore \quad C_z = 866.04 \text{ N}$$

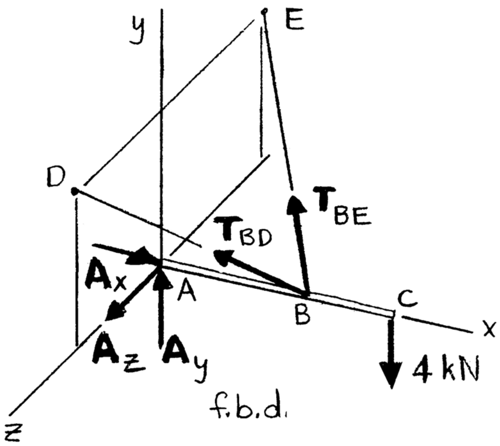
$$\text{or } \mathbf{C} = -(200 \text{ N})\mathbf{j} + (866 \text{ N})\mathbf{k} \blacktriangleleft$$

PROBLEM 4.113



A 3-m boom is acted upon by the 4-kN force shown. Determine the tension in each cable and the reaction at the ball-and-socket joint at *A*.

SOLUTION



From f.b.d. of boom

$$\Sigma M_{AE} = 0: \quad \lambda_{AE} \cdot \left(\mathbf{r}_{B/A} \times \mathbf{T}_{BD}\right) + \lambda_{AE} \cdot \left(\mathbf{r}_{C/A} \times \mathbf{F}_C\right) = 0$$

where
$$\lambda_{AE} = \frac{(2.1 \text{ m})\mathbf{j} - (1.8 \text{ m})\mathbf{k}}{\sqrt{(2.1)^2 + (1.8)^2} \text{ m}}$$

$$= 0.27451\mathbf{j} - 0.23529\mathbf{k}$$

$$\mathbf{r}_{B/A} = (1.8 \text{ m})\mathbf{i}$$

$$\mathbf{T}_{BD} = \lambda_{BD} T_{BD} = \frac{(-1.8 \text{ m})\mathbf{i} + (2.1 \text{ m})\mathbf{j} + (1.8 \text{ m})\mathbf{k}}{\sqrt{(1.8)^2 + (2.1)^2 + (1.8)^2} \text{ m}} T_{BD}$$

$$= (-0.54545\mathbf{i} + 0.63636\mathbf{j} + 0.54545\mathbf{k}) T_{BD}$$

$$\mathbf{r}_{C/A} = (3.0 \text{ m})\mathbf{i}$$

$$\mathbf{F}_C = -(4 \text{ kN})\mathbf{j}$$

PROBLEM 4.113 CONTINUED

$$\therefore \begin{vmatrix} 0 & 0.27451 & -0.23529 \\ 1.8 & 0 & 0 \\ -0.54545 & 0.63636 & 0.54545 \end{vmatrix} T_{BD} + \begin{vmatrix} 0 & 0.27451 & -0.23529 \\ 3 & 0 & 0 \\ 0 & -4 & 0 \end{vmatrix} = 0$$

$$(-0.149731 - 0.149729)1.8T_{BD} + 2.82348 = 0$$

$$\therefore \quad T_{BD} = 5.2381 \text{ kN} \qquad \text{or } T_{BD} = 5.24 \text{ kN} \blacktriangleleft$$

Based on symmetry, $T_{BE} = T_{BD} = 5.2381 \text{ kN}$ or $T_{BE} = 5.24 \text{ kN} \blacktriangleleft$

$$\Sigma F_z = 0: \quad A_z + (T_{BD})_z - (T_{BE})_z = 0 \qquad A_z = 0$$

$$\Sigma F_y = 0: \quad A_y + (T_{BD})_y + (T_{BD})_y - 4 \text{ kN} = 0$$

$$A_y + 2(0.63636)(5.2381 \text{ kN}) - 4 \text{ kN} = 0$$

$$\therefore \quad A_y = -2.6666 \text{ kN}$$

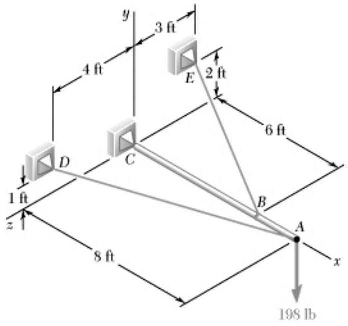
$$\Sigma F_x = 0: \quad A_x - (T_{BD})_x - (T_{BE})_x = 0$$

$$A_x - 2(0.54545)(5.2381 \text{ kN}) = 0$$

$$\therefore \quad A_x = 5.7142 \text{ kN}$$

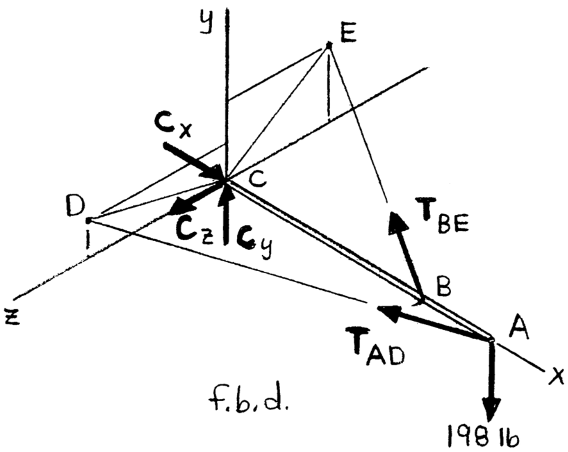
$$\text{and } \mathbf{A} = (5.71 \text{ N})\mathbf{i} - (2.67 \text{ N})\mathbf{j} \blacktriangleleft$$

PROBLEM 4.114



An 8-ft-long boom is held by a ball-and-socket joint at C and by two cables AD and BE. Determine the tension in each cable and the reaction at C.

SOLUTION



From f.b.d. of boom

$$\Sigma M_{CE} = 0: \quad \lambda_{CE} \cdot \left(\mathbf{r}_{A/C} \times \mathbf{T}_{AD}\right) + \lambda_{CE} \cdot \left(\mathbf{r}_{A/C} \times \mathbf{F}_A\right) = 0$$

where

$$\lambda_{CE} = \frac{(2\text{ ft})\mathbf{j} - (3\text{ ft})\mathbf{k}}{\sqrt{(2)^2 + (3)^2}\text{ ft}} = \frac{1}{\sqrt{13}}(2\mathbf{j} - 3\mathbf{k})$$

$$\mathbf{r}_{A/C} = (8\text{ ft})\mathbf{i}$$

$$\mathbf{T}_{AD} = \lambda_{AD}T_{AD} = \frac{-(8\text{ ft})\mathbf{i} + (1\text{ ft})\mathbf{j} + (4\text{ ft})\mathbf{k}}{\sqrt{(8)^2 + (1)^2 + (4)^2}\text{ ft}}T_{AD}$$

$$= \left(\frac{1}{9}\right)T_{AD}(-8\mathbf{i} + \mathbf{j} + 4\mathbf{k})$$

$$\mathbf{F}_A = -(198\text{ lb})\mathbf{j}$$

$$\therefore \quad \begin{vmatrix} 0 & 2 & -3 \\ 8 & 0 & 0 \\ -8 & 1 & 4 \end{vmatrix}\left(\frac{T_{AD}}{9\sqrt{13}}\right) + \begin{vmatrix} 0 & 2 & -3 \\ 8 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix}\left(\frac{198}{\sqrt{13}}\right) = 0$$

PROBLEM 4.114 CONTINUED

$$(-64 - 24)\frac{T_{AD}}{9\sqrt{13}} + (24)\frac{198}{\sqrt{13}} = 0$$

$$\therefore \quad T_{AD} = 486.00 \text{ lb}$$

or $T_{AD} = 486 \text{ lb}$ ◀

$$\Sigma M_{CD} = 0: \quad \boldsymbol{\lambda}_{CD} \cdot \left(\mathbf{r}_{B/C} \times \mathbf{T}_{BE}\right) + \boldsymbol{\lambda}_{CD} \cdot \left(\mathbf{r}_{A/C} \times \mathbf{F}_A\right)$$

where $\quad \boldsymbol{\lambda}_{CD} = \dfrac{(1 \text{ ft})\mathbf{j} + (4 \text{ ft})\mathbf{k}}{\sqrt{17} \text{ ft}} = \dfrac{1}{\sqrt{17}}(1\mathbf{j} + 4\mathbf{k})$

$$\mathbf{r}_{B/C} = (6 \text{ ft})\mathbf{i}$$

$$\mathbf{T}_{BE} = \boldsymbol{\lambda}_{BE} T_{BE} = \frac{-(6 \text{ ft})\mathbf{i} + (2 \text{ ft})\mathbf{j} - (3 \text{ ft})\mathbf{k}}{\sqrt{(6)^2 + (2)^2 + (3)^2} \text{ ft}} T_{BE} = \left(\frac{1}{7}\right) T_{BE} (-6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k})$$

$$\therefore \quad \begin{vmatrix} 0 & 1 & 4 \\ 6 & 0 & 0 \\ -6 & 2 & -3 \end{vmatrix} \frac{T_{BE}}{7\sqrt{17}} + \begin{vmatrix} 0 & 1 & 4 \\ 8 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} \frac{198}{\sqrt{17}} = 0$$

$$(18 + 48)\frac{T_{BE}}{7} + (-32)198 = 0$$

$$\therefore \quad T_{BE} = 672.00 \text{ lb}$$

or $T_{BE} = 672 \text{ lb}$ ◀

$$\Sigma F_x = 0: \quad C_x - (T_{AD})_x - (T_{BE})_x = 0$$

$$C_x - \left(\frac{8}{9}\right)486 - \left(\frac{6}{7}\right)672 = 0$$

$$\therefore \quad C_x = 1008 \text{ lb}$$

$$\Sigma F_y = 0: \quad C_y + (T_{AD})_y + (T_{BE})_y - 198 \text{ lb} = 0$$

$$C_y + \left(\frac{1}{9}\right)486 + \left(\frac{2}{7}\right)672 - 198 \text{ lb} = 0$$

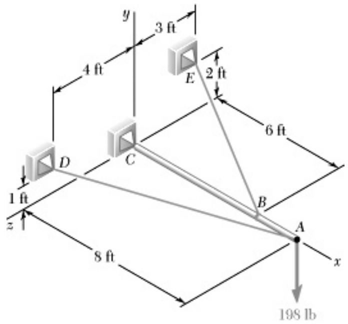
$$\therefore \quad C_y = -48.0 \text{ lb}$$

$$\Sigma F_z = 0: \quad C_z + (T_{AD})_z - (T_{BE})_z = 0$$

$$C_z + \left(\frac{4}{9}\right)486 - \left(\frac{3}{7}\right)(672) = 0$$

$$\therefore \quad C_z = 72.0 \text{ lb}$$

or $\mathbf{C} = (1008 \text{ lb})\mathbf{i} - (48.0 \text{ lb})\mathbf{j} + (72.0 \text{ lb})\mathbf{k}$ ◀

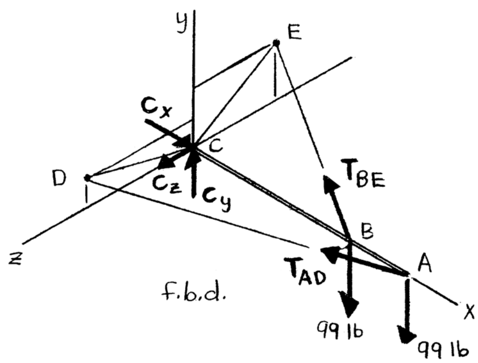


PROBLEM 4.115

Solve Problem 4.114 assuming that the given 198-lb load is replaced with two 99-lb loads applied at *A* and *B*.

P4.114 An 8-ft-long boom is held by a ball-and-socket joint at *C* and by two cables *AD* and *BE*. Determine the tension in each cable and the reaction at *C*.

SOLUTION



From f.b.d. of boom

$$\Sigma M_{CE} = 0: \quad \boldsymbol{\lambda}_{CE} \cdot \left(\mathbf{r}_{A/C} \times \mathbf{T}_{AD}\right) + \boldsymbol{\lambda}_{CE} \cdot \left(\mathbf{r}_{A/C} \times \mathbf{F}_A\right) + \boldsymbol{\lambda}_{CE} \cdot \left(\mathbf{r}_{B/C} \times \mathbf{F}_B\right) = 0$$

where

$$\boldsymbol{\lambda}_{CE} = \frac{(2\text{ ft})\mathbf{j} - (3\text{ ft})\mathbf{k}}{\sqrt{(2)^2 + (3)^2}\text{ ft}} = \frac{1}{\sqrt{13}}(2\mathbf{j} - 3\mathbf{k})$$

$$\mathbf{r}_{A/C} = (8\text{ ft})\mathbf{i}$$

$$\mathbf{r}_{B/C} = (6\text{ ft})\mathbf{i}$$

$$\mathbf{T}_{AD} = \boldsymbol{\lambda}_{AD} T_{AD} = \frac{-(8\text{ ft})\mathbf{i} + (1\text{ ft})\mathbf{j} + (4\text{ ft})\mathbf{k}}{\sqrt{(8)^2 + (1)^2 + (4)^2}\text{ ft}} T_{AD}$$

$$= \left(\frac{1}{9}\right) T_{AD}(-8\mathbf{i} + \mathbf{j} + 4\mathbf{k})$$

$$\mathbf{F}_A = -(99\text{ lb})\mathbf{j}$$

$$\mathbf{F}_B = -(99\text{ lb})\mathbf{j}$$

$$\therefore \quad \begin{vmatrix} 0 & 2 & -3 \\ 8 & 0 & 0 \\ -8 & 1 & 4 \end{vmatrix} \frac{T_{AD}}{9\sqrt{13}} + \begin{vmatrix} 0 & 2 & -3 \\ 8 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} \frac{99}{\sqrt{13}} + \begin{vmatrix} 0 & 2 & -3 \\ 6 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} \frac{99}{\sqrt{13}} = 0$$

PROBLEM 4.115 CONTINUED

$$\left(-64 - 24\right)\frac{T_{AD}}{9\sqrt{13}} + \left(24 + 18\right)\frac{99}{\sqrt{13}} = 0$$

or
$$T_{AD} = 425.25 \text{ lb}$$

or $T_{AD} = 425 \text{ lb}$ ◀

$$\Sigma M_{CD} = 0: \quad \boldsymbol{\lambda}_{CD} \cdot \left(\mathbf{r}_{B/C} \times \mathbf{T}_{BE}\right) + \boldsymbol{\lambda}_{CD} \cdot \left(\mathbf{r}_{A/C} \times \mathbf{F}_A\right) + \boldsymbol{\lambda}_{CD} \cdot \left(\mathbf{r}_{B/C} \times \mathbf{F}_B\right) = 0$$

where
$$\boldsymbol{\lambda}_{CD} = \frac{(1 \text{ ft})\mathbf{j} + (4 \text{ ft})\mathbf{k}}{\sqrt{17}} = \frac{1}{\sqrt{17}}\left(\mathbf{j} + 4\mathbf{k}\right)$$

$$\mathbf{r}_{B/C} = (6 \text{ ft})\mathbf{i}$$

$$\mathbf{r}_{A/C} = (8 \text{ ft})\mathbf{j}$$

$$\mathbf{T}_{BE} = \boldsymbol{\lambda}_{BE} T_{BE} = \frac{-(6 \text{ ft})\mathbf{i} + (2 \text{ ft})\mathbf{j} - (3 \text{ ft})\mathbf{k}}{\sqrt{(6)^2 + (2)^2 + (3)^2} \text{ ft}} T_{BE} = \frac{T_{BE}}{7}\left(-6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}\right)$$

$$\therefore \quad \begin{vmatrix} 0 & 1 & 4 \\ 6 & 0 & 0 \\ -6 & 2 & -3 \end{vmatrix} \left(\frac{T_{BE}}{7\sqrt{17}}\right) + \begin{vmatrix} 0 & 1 & 4 \\ 8 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} \left(\frac{99}{\sqrt{17}}\right) + \begin{vmatrix} 0 & 1 & 4 \\ 6 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} \left(\frac{99}{\sqrt{17}}\right) = 0$$

$$\left(18 + 48\right)\left(\frac{T_{BE}}{7\sqrt{17}}\right) + \left(-32 - 24\right)\left(\frac{99}{\sqrt{17}}\right) = 0$$

or
$$T_{BE} = 588.00 \text{ lb}$$

or $T_{BE} = 588 \text{ lb}$ ◀

$$\Sigma F_x = 0: \quad C_x - \left(T_{AD}\right)_x - \left(T_{BE}\right)_x = 0$$

$$C_x - \left(\frac{8}{9}\right)425.25 - \left(\frac{6}{7}\right)588.00 = 0$$

$$\therefore \quad C_x = 882 \text{ lb}$$

$$\Sigma F_y = 0: \quad C_y + \left(T_{AD}\right)_y + \left(T_{BE}\right)_y - 99 - 99 = 0$$

$$C_y + \left(\frac{1}{9}\right)425.25 + \left(\frac{2}{7}\right)588.00 - 198 = 0$$

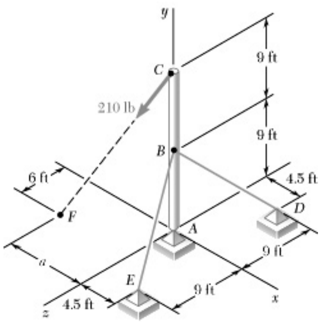
$$\therefore \quad C_y = -17.25 \text{ lb}$$

$$\Sigma F_z = 0: \quad C_z + \left(T_{AD}\right)_z - \left(T_{BE}\right)_z = 0$$

$$C_z + \left(\frac{4}{9}\right)425.25 - \left(\frac{3}{7}\right)588.00 = 0$$

$$\therefore \quad C_z = 63.0 \text{ lb}$$

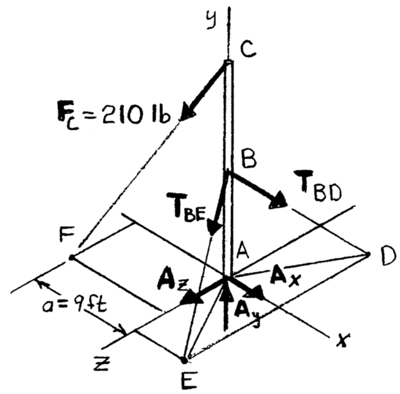
or $\mathbf{C} = (882 \text{ lb})\mathbf{i} - (17.25 \text{ lb})\mathbf{j} + (63.0 \text{ lb})\mathbf{k}$ ◀



PROBLEM 4.116

The 18-ft pole *ABC* is acted upon by a 210-lb force as shown. The pole is held by a ball-and-socket joint at *A* and by two cables *BD* and *BE*. For $a = 9$ ft, determine the tension in each cable and the reaction at *A*.

SOLUTION



f.b.d.

From f.b.d. of pole *ABC*

$$\Sigma M_{AE} = 0: \quad \lambda_{AE} \cdot \left(\mathbf{r}_{B/A} \times \mathbf{T}_{BD}\right) + \lambda_{AE} \cdot \left(\mathbf{r}_{C/A} \times \mathbf{F}_C\right) = 0$$

where
$$\lambda_{AE} = \frac{(4.5\text{ ft})\mathbf{i} + (9\text{ ft})\mathbf{k}}{\sqrt{(4.5)^2 + (9)^2}\text{ ft}} = \frac{1}{\sqrt{101.25}}(4.5\mathbf{i} + 9\mathbf{k})$$

$$\mathbf{r}_{B/A} = (9\text{ ft})\,\mathbf{j}$$

$$\mathbf{r}_{C/A} = (18\text{ ft})\,\mathbf{j}$$

$$\mathbf{T}_{BD} = \lambda_{BD}T_{BD} = \frac{(4.5\text{ ft})\mathbf{i} - (9\text{ ft})\,\mathbf{j} - (9\text{ ft})\mathbf{k}}{\sqrt{(4.5)^2 + (9)^2 + (9)^2}\text{ ft}}T_{BD}$$

$$= \left(\frac{T_{BD}}{13.5}\right)(4.5\mathbf{i} - 9\mathbf{j} - 9\mathbf{k})$$

$$\mathbf{F}_C = \lambda_{CF}(210\text{ lb}) = \frac{-9\mathbf{i} - 18\mathbf{j} + 6\mathbf{k}}{\sqrt{(9)^2 + (18)^2 + (6)^2}}(210\text{ lb}) = 10\text{ lb}(-9\mathbf{i} - 18\mathbf{j} + 6\mathbf{k})$$

$$\therefore \quad \begin{vmatrix} 4.5 & 0 & 9 \\ 0 & 9 & 0 \\ 4.5 & -9 & -9 \end{vmatrix}\left(\frac{T_{BD}}{13.5\sqrt{101.25}}\right) + \begin{vmatrix} 4.5 & 0 & 9 \\ 0 & 18 & 0 \\ -9 & -18 & 6 \end{vmatrix}\left(\frac{10\text{ lb}}{\sqrt{101.25}}\right) = 0$$

PROBLEM 4.116 CONTINUED

$$\frac{(-364.5 - 364.5)}{13.5\sqrt{101.25}}T_{BD} + \frac{(486 + 1458)}{\sqrt{101.25}}(10 \text{ lb}) = 0$$

and
$$T_{BD} = 360.00 \text{ lb}$$

or $T_{BD} = 360$ lb ◀

$$\Sigma M_{AD} = 0: \quad \lambda_{AD} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}_{BE}) + \lambda_{AD} \cdot (\mathbf{r}_{C/A} \times \mathbf{F}_C) = 0$$

where
$$\lambda_{AD} = \frac{(4.5 \text{ ft})\mathbf{i} - (9 \text{ ft})\mathbf{k}}{\sqrt{(4.5)^2 + (9)^2} \text{ ft}} = \frac{1}{\sqrt{101.25}}(4.5\mathbf{i} - 9\mathbf{k})$$

$$\mathbf{r}_{B/A} = (9 \text{ ft})\mathbf{j}$$

$$\mathbf{r}_{C/A} = (18 \text{ ft})\mathbf{j}$$

$$\mathbf{T}_{BE} = \lambda_{BE}T_{BE} = \frac{(4.5 \text{ ft})\mathbf{i} - (9 \text{ ft})\mathbf{j} + (9 \text{ ft})\mathbf{k}}{\sqrt{(4.5)^2 + (9)^2 + (9)^2} \text{ ft}}T_{BE} = \frac{T_{BE}}{13.5}(4.5\mathbf{i} - 9\mathbf{j} + 9\mathbf{k})$$

$$\therefore \quad \begin{vmatrix} 4.5 & 0 & -9 \\ 0 & 9 & 0 \\ 4.5 & -9 & 9 \end{vmatrix}\left(\frac{T_{BE}}{13.5\sqrt{101.25}}\right) + \begin{vmatrix} 4.5 & 0 & -9 \\ 0 & 18 & 0 \\ -9 & -18 & 6 \end{vmatrix}\left(\frac{10 \text{ lb}}{\sqrt{101.25}}\right) = 0$$

$$\frac{(364.5 + 364.5)}{13.5\sqrt{101.25}}T_{BE} + \frac{(486 - 1458)10 \text{ lb}}{\sqrt{101.25}} = 0$$

or
$$T_{BE} = 180.0 \text{ lb}$$

or $T_{BE} = 180.0$ lb ◀

$$\Sigma F_x = 0: \quad A_x + (T_{BD})_x + (T_{BE})_x - (F_C)_x = 0$$

$$A_x + \left(\frac{4.5}{13.5}\right)360 + \left(\frac{4.5}{13.5}\right)180 - \left(\frac{9}{21}\right)210 = 0$$

$$\therefore \quad A_x = -90.0 \text{ lb}$$

$$\Sigma F_y = 0: \quad A_y - (T_{BD})_y - (T_{BE})_y - (F_C)_y = 0$$

$$A_y - \left(\frac{9}{13.5}\right)360 - \left(\frac{9}{13.5}\right)180 - \left(\frac{18}{21}\right)210 = 0$$

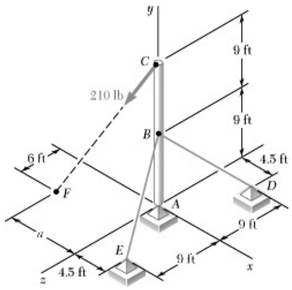
$$\therefore \quad A_y = 540 \text{ lb}$$

$$\Sigma F_z = 0: \quad A_z - (T_{BD})_z + (T_{BE})_z + (F_C)_z = 0$$

$$A_z - \left(\frac{9}{13.5}\right)360 + \left(\frac{9}{13.5}\right)180 + \left(\frac{6}{21}\right)210 = 0$$

$$\therefore \quad A_z = 60.0 \text{ lb}$$

or $\mathbf{A} = -(90.0 \text{ lb})\mathbf{i} + (540 \text{ lb})\mathbf{j} + (60.0 \text{ lb})\mathbf{k}$ ◀

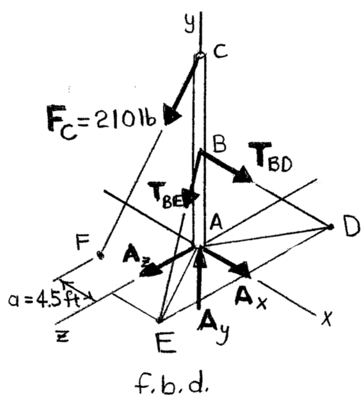


PROBLEM 4.117

Solve Problem 4.116 for $a = 4.5$ ft.

P4.116 The 18-ft pole *ABC* is acted upon by a 210-lb force as shown. The pole is held by a ball-and-socket joint at *A* and by two cables *BD* and *BE*. For $a = 9$ ft, determine the tension in each cable and the reaction at *A*.

SOLUTION



From f.b.d. of pole *ABC*

$$\Sigma M_{AE} = 0: \quad \lambda_{AE} \cdot \left(\mathbf{r}_{B/A} \times \mathbf{T}_{BD}\right) + \lambda_{AE} \cdot \left(\mathbf{r}_{C/A} \times \mathbf{F}_C\right) = 0$$

where

$$\lambda_{AE} = \frac{(4.5 \text{ ft})\mathbf{i} + (9 \text{ ft})\mathbf{k}}{\sqrt{(4.5)^2 + (9)^2} \text{ ft}} = \frac{1}{\sqrt{101.25}}(4.5\mathbf{i} + 9\mathbf{k})$$

$$\mathbf{r}_{B/A} = (9 \text{ ft})\mathbf{j}$$

$$\mathbf{r}_{C/A} = (18 \text{ ft})\mathbf{j}$$

$$\mathbf{T}_{BD} = \lambda_{BD}T_{BD} = \frac{(4.5 \text{ ft})\mathbf{i} - (9 \text{ ft})\mathbf{j} - (9 \text{ ft})\mathbf{k}}{\sqrt{(4.5)^2 + (9)^2 + (9)^2} \text{ ft}}T_{BD}$$

$$= \left(\frac{T_{BD}}{13.5}\right)(4.5\mathbf{i} - 9\mathbf{j} - 9\mathbf{k})$$

$$\mathbf{F}_C = \lambda_{CF}(210 \text{ lb}) = \frac{-4.5\mathbf{i} - 18\mathbf{j} + 6\mathbf{k}}{\sqrt{(4.5)^2 + (18)^2 + (6)^2}}(210 \text{ lb})$$

$$= \left(\frac{210 \text{ lb}}{19.5}\right)(-4.5\mathbf{i} - 18\mathbf{j} + 6\mathbf{k})$$

$$\therefore \quad \begin{vmatrix} 4.5 & 0 & 9 \\ 0 & 9 & 0 \\ 4.5 & -9 & -9 \end{vmatrix}\left(\frac{T_{BD}}{13.5\sqrt{101.25}}\right) + \begin{vmatrix} 4.5 & 0 & 9 \\ 0 & 18 & 0 \\ -4.5 & -18 & 6 \end{vmatrix}\left(\frac{210 \text{ lb}}{19.5\sqrt{101.25}}\right) = 0$$

PROBLEM 4.117 CONTINUED

$$\frac{(-364.5 - 364.5)}{13.5\sqrt{101.25}}T_{BD} + \frac{(486 + 729)}{19.5\sqrt{101.25}}(210 \text{ lb}) = 0$$

or $\qquad T_{BD} = 242.31 \text{ lb} \qquad$ or $T_{BD} = 242 \text{ lb}$ ◀

$$\Sigma M_{AD} = 0: \quad \lambda_{AD} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}_{BE}) + \lambda_{AD} \cdot (\mathbf{r}_{C/A} \times \mathbf{F}_C) = 0$$

where $\qquad \lambda_{AD} = \dfrac{(4.5 \text{ ft})\mathbf{i} - (9 \text{ ft})\mathbf{k}}{\sqrt{(4.5)^2 + (9)^2} \text{ ft}} = \dfrac{1}{\sqrt{101.25}}(4.5\mathbf{i} - 9\mathbf{k}),$

$$\mathbf{r}_{B/A} = (9 \text{ ft})\mathbf{j}$$

$$\mathbf{r}_{C/A} = (18 \text{ ft})\mathbf{j}$$

$$\mathbf{T}_{BE} = \lambda_{BE}T_{BE} = \frac{(4.5 \text{ ft})\mathbf{i} - (9 \text{ ft})\mathbf{j} + (9 \text{ ft})\mathbf{k}}{\sqrt{(4.5)^2 + (9)^2 + (9)^2} \text{ ft}}T_{BE} = \frac{T_{BE}}{13.5}(4.5\mathbf{i} - 9\mathbf{j} + 9\mathbf{k})$$

$$\therefore \quad \begin{vmatrix} 4.5 & 0 & -9 \\ 0 & 9 & 0 \\ 4.5 & -9 & 9 \end{vmatrix}\left(\frac{T_{BE}}{13.5\sqrt{101.25}}\right) + \begin{vmatrix} 4.5 & 0 & -9 \\ 0 & 18 & 0 \\ -4.5 & -18 & 6 \end{vmatrix}\left(\frac{210 \text{ lb}}{19.5\sqrt{101.25}}\right) = 0$$

$$\frac{(364.5 + 364.5)}{13.5\sqrt{101.25}}T_{BE} + \frac{(486 - 729)(210 \text{ lb})}{19.5\sqrt{101.25}} = 0$$

or $\qquad T_{BE} = 48.462 \text{ lb}$

or $T_{BE} = 48.5 \text{ lb}$ ◀

$$\Sigma F_x = 0: \quad A_x + (T_{BD})_x + (T_{BE})_x - (F_C)_x = 0$$

$$A_x + \left(\frac{4.5}{13.5}\right)242.31 + \left(\frac{4.5}{13.5}\right)48.462 - \left(\frac{4.5}{19.5}\right)210 = 0$$

$$\therefore \quad A_x = -48.459 \text{ lb}$$

$$\Sigma F_y = 0: \quad A_y - (T_{BD})_y - (T_{BE})_y - (F_C)_y = 0$$

$$A_y - \left(\frac{9}{13.5}\right)242.31 - \left(\frac{9}{13.5}\right)48.462 - \left(\frac{18}{19.5}\right)210 =$$

$$\therefore \quad A_y = 387.69 \text{ lb}$$

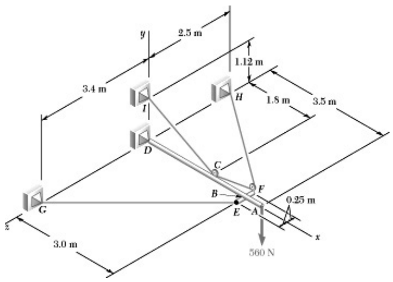
$$\Sigma F_z = 0: \quad A_z - (T_{BD})_z + (T_{BE})_z + (F_C)_z = 0$$

$$A_z - \left(\frac{9}{13.5}\right)242.31 + \left(\frac{9}{13.5}\right)48.462 + \left(\frac{6}{19.5}\right)2$$

$$\therefore \quad A_z = 64.591 \text{ lb}$$

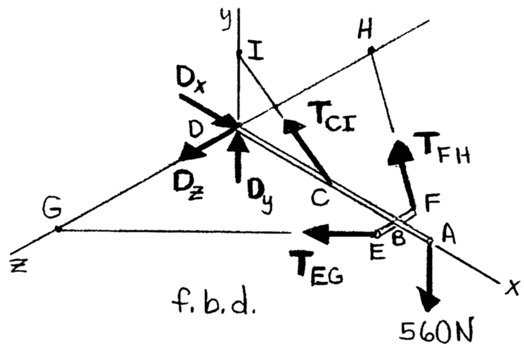
or $\mathbf{A} = -(48.5 \text{ lb})\mathbf{i} + (388 \text{ lb})\mathbf{j} + (64.6 \text{ lb})\mathbf{k}$ ◀

PROBLEM 4.118



Two steel pipes *ABCD* and *EBF* are welded together at *B* to form the boom shown. The boom is held by a ball-and-socket joint at *D* and by two cables *EG* and *ICFH*; cable *ICFH* passes around frictionless pulleys at *C* and *F*. For the loading shown, determine the tension in each cable and the reaction at *D*.

SOLUTION



From f.b.d. of boom

$$\Sigma M_z = 0: \quad \mathbf{k}\cdot\left(\mathbf{r}_{C/D}\times\mathbf{T}_{CI}\right)+\mathbf{k}\cdot\left(\mathbf{r}_{A/D}\times\mathbf{F}_A\right)=0$$

where

$$\mathbf{r}_{C/D} = (1.8\text{ m})\mathbf{i}$$

$$\mathbf{T}_{CI} = \lambda_{CI}T_{CI} = \frac{-(1.8\text{ m})\mathbf{i}+(1.12\text{ m})\mathbf{j}}{\sqrt{(1.8)^2+(1.12)^2}\text{ m}}T_{CI}$$

$$= \left(\frac{T_{CI}}{2.12}\right)(-1.8\mathbf{i}+1.12\mathbf{j})$$

$$\mathbf{r}_{A/D} = (3.5\text{ m})\mathbf{i}$$

$$\mathbf{F}_A = -(560\text{ N})\mathbf{j}$$

$$\therefore \quad \Sigma M_z = \begin{vmatrix} 0 & 0 & 1 \\ 1.8 & 0 & 0 \\ -1.8 & 1.12 & 0 \end{vmatrix}\left(\frac{T_{CI}}{2.12}\right)+\begin{vmatrix} 0 & 0 & 1 \\ 3.5 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix}(560\text{ N})=0$$

$$(2.016)\frac{T_{CI}}{2.12}+(-3.5)560=0$$

or

$$T_{CI} = T_{FH} = 2061.1\text{ N}$$

$$T_{ICFH} = 2.06\text{ kN} \blacktriangleleft$$

PROBLEM 4.118 CONTINUED

$$\Sigma M_y = 0: \quad \mathbf{j}\cdot\left(\mathbf{r}_{G/D}\times\mathbf{T}_{EG}\right) + \mathbf{j}\cdot\left(\mathbf{r}_{H/D}\times\mathbf{T}_{FH}\right) = 0$$

where $\mathbf{r}_{G/D} = (3.4\text{ m})\mathbf{k}$

$$\mathbf{r}_{H/D} = -(2.5\text{ m})\mathbf{k}$$

$$\mathbf{T}_{EG} = \frac{-(3.0\text{ m})\mathbf{i} + (3.15\text{ m})\mathbf{k}}{\sqrt{(3)^2 + (3.15)^2}\text{ m}}T_{EG} = \left(\frac{T_{EG}}{4.35}\right)(-3\mathbf{i} + 3.15\mathbf{k})$$

$$\mathbf{T}_{FH} = \lambda_{FH}T_{FH} = \frac{-(3.0\text{ m})\mathbf{i} - (2.25\text{ m})\mathbf{k}}{\sqrt{(3)^2 + (2.25)^2}\text{ m}}(2061.1\text{ N}) = \frac{2061.1\text{ N}}{3.75}(-3\mathbf{i} - 2.25\mathbf{k})$$

$$\therefore \quad \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 3.4 \\ -3 & 0 & 3.15 \end{vmatrix}\left(\frac{T_{EG}}{4.35}\right) + \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & -2.5 \\ -3 & 0 & -2.25 \end{vmatrix}\left(\frac{2061.1\text{ N}}{3.75}\right) = 0$$

$$-(10.2)\frac{T_{EG}}{4.35} + (7.5)\frac{2061.1\text{ N}}{3.75} = 0$$

or $$T_{EG} = 1758.00\text{ N}$$

$$T_{EG} = 1.758\text{ kN} \blacktriangleleft$$

$$\Sigma F_x = 0: \quad D_x - (T_{CI})_x - (T_{FH})_x - (T_{EG})_x = 0$$

$$D_x - \left(\frac{1.8}{2.12}\right)(2061.1\text{ N}) - \left(\frac{3.0}{3.75}\right)(2061.1\text{ N}) - \left(\frac{3}{4.35}\right)(1758\text{ N}) = 0$$

$$\therefore \quad D_x = 4611.3\text{ N}$$

$$\Sigma F_y = 0: \quad D_y + (T_{CI})_y - 560\text{ N} = 0$$

$$D_y + \left(\frac{1.12}{2.12}\right)(2061.1\text{ N}) - 560\text{ N} = 0$$

$$\therefore \quad D_y = -528.88\text{ N}$$

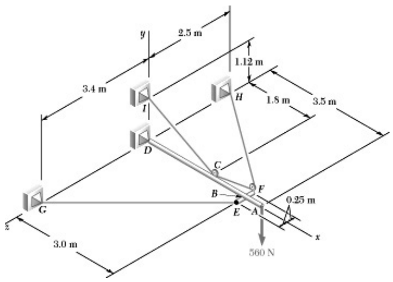
$$\Sigma F_z = 0: \quad D_z + (T_{EG})_z - (T_{FH})_z = 0$$

$$D_z + \left(\frac{3.15}{4.35}\right)(1758\text{ N}) - \left(\frac{2.25}{3.75}\right)(2061.1\text{ N}) = 0$$

$$\therefore \quad D_z = -36.374\text{ N}$$

and $\mathbf{D} = (4610\text{ N})\mathbf{i} - (529\text{ N})\mathbf{j} - (36.4\text{ N})\mathbf{k} \blacktriangleleft$

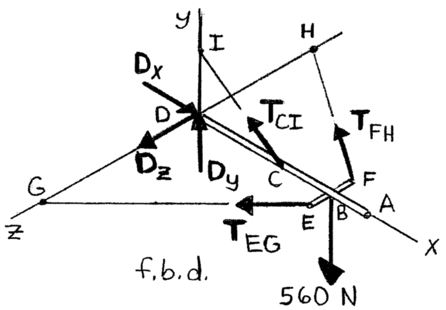
PROBLEM 4.119



Solve Problem 4.118 assuming that the 560-N load is applied at *B*.

P4.118 Two steel pipes *ABCD* and *EBF* are welded together at *B* to form the boom shown. The boom is held by a ball-and-socket joint at *D* and by two cables *EG* and *ICFH*; cable *ICFH* passes around frictionless pulleys at *C* and *F*. For the loading shown, determine the tension in each cable and the reaction at *D*.

SOLUTION



From f.b.d. of boom

$$\Sigma M_z = 0: \quad \mathbf{k}\cdot\left(\mathbf{r}_{C/D}\times\mathbf{T}_{CI}\right)+\mathbf{k}\cdot\left(\mathbf{r}_{B/D}\times\mathbf{F}_B\right)=0$$

where $\mathbf{r}_{C/D} = (1.8\text{ m})\mathbf{i}$

$$\mathbf{T}_{CI} = \lambda_{CI}T_{CI} = \frac{-(1.8\text{ m})\mathbf{i}+(1.12\text{ m})\mathbf{j}}{\sqrt{(1.8)^2+(1.12)^2}\text{ m}}T_{CI}$$

$$= \left(\frac{T_{CI}}{2.12}\right)(-1.8\mathbf{i}+1.12\mathbf{j})$$

$$\mathbf{r}_{B/D} = (3.0\text{ m})\mathbf{i}$$

$$\mathbf{F}_B = -(560\text{ N})\mathbf{j}$$

$$\therefore \begin{vmatrix} 0 & 0 & 1 \\ 1.8 & 0 & 0 \\ -1.8 & 1.12 & 0 \end{vmatrix}\left(\frac{T_{CI}}{2.12}\right)+\begin{vmatrix} 0 & 0 & 1 \\ 3 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix}(560\text{ N}) = 0$$

$$(2.016)\frac{T_{CI}}{2.12}+(-3)560 = 0$$

or $T_{CI} = T_{FH} = 1766.67\text{ N}$

$T_{ICFH} = 1.767\text{ kN}$ ◀

PROBLEM 4.119 CONTINUED

$$\Sigma M_y = 0: \quad \mathbf{j} \cdot \left(\mathbf{r}_{G/D} \times \mathbf{T}_{EG}\right) + \mathbf{j} \cdot \left(\mathbf{r}_{H/D} \times \mathbf{T}_{FH}\right) = 0$$

where $\mathbf{r}_{G/D} = (3.4 \text{ m})\mathbf{k}$

$$\mathbf{r}_{H/D} = -(2.5 \text{ m})\mathbf{k}$$

$$\mathbf{T}_{EG} = \lambda_{EG} T_{EG} = \frac{-(3.0 \text{ m})\mathbf{i} + (3.15 \text{ m})\mathbf{k}}{\sqrt{(3)^2 + (3.15)^2} \text{ m}} T_{EG} = \frac{T_{EG}}{4.35}(-3\mathbf{i} + 3.15\mathbf{k})$$

$$\mathbf{T}_{FH} = \lambda_{FH} T_{FH} = \frac{-(3.0 \text{ m})\mathbf{i} - (2.25 \text{ m})\mathbf{k}}{\sqrt{(3)^2 + (2.25)^2} \text{ m}} T_{FH} = \frac{1766.67 \text{ N}}{3.75}(-3\mathbf{i} - 2.25\mathbf{k})$$

$$\therefore \quad \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 3.4 \\ -3 & 0 & 3.15 \end{vmatrix} \left(\frac{T_{EG}}{4.35}\right) + \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & -2.5 \\ -3 & 0 & -2.25 \end{vmatrix} \left(\frac{1766.67}{3.75}\right) = 0$$

$$-(10.2)\frac{T_{EG}}{4.35} + (7.5)\frac{1766.67}{3.75} = 0$$

or $$T_{EG} = 1506.86 \text{ N}$$

$T_{EG} = 1.507 \text{ kN}$ ◀

$$\Sigma F_x = 0: \quad D_x - (T_{CI})_x - (T_{FH})_x - (T_{EG})_x = 0$$

$$D_x - \left(\frac{1.8}{2.12}\right)(1766.67 \text{ N}) - \left(\frac{3}{3.75}\right)(1766.67 \text{ N}) - \left(\frac{3}{4.35}\right)(1506.86 \text{ N}) = 0$$

$$\therefore \quad D_x = 3952.5 \text{ N}$$

$$\Sigma F_y = 0: \quad D_y + (T_{CI})_y - 560 \text{ N} = 0$$

$$D_y + \left(\frac{1.12}{2.12}\right)(1766.67 \text{ N}) - 560 \text{ N} = 0$$

$$\therefore \quad D_y = -373.34 \text{ N}$$

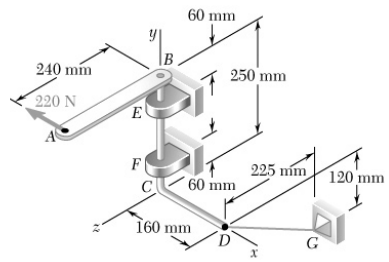
$$\Sigma F_z = 0: \quad D_z + (T_{EG})_z - (T_{FH})_z = 0$$

$$D_z + \left(\frac{3.15}{4.35}\right)(1506.86 \text{ N}) - \left(\frac{2.25}{3.75}\right)(1766.67 \text{ N}) = 0$$

$$\therefore \quad D_z = -31.172 \text{ N}$$

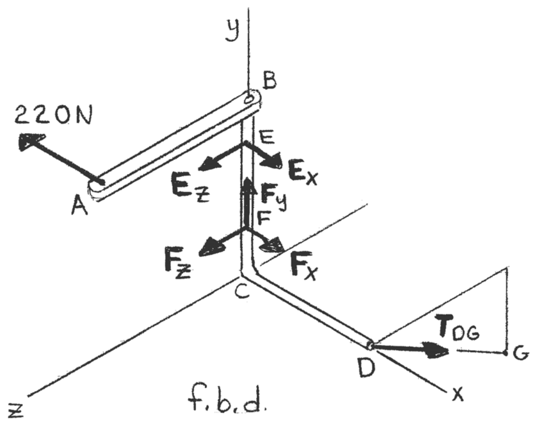
$\mathbf{D} = (3950 \text{ N})\mathbf{i} - (373 \text{ N})\mathbf{j} - (31.2 \text{ N})\mathbf{k}$ ◀

PROBLEM 4.120



The lever AB is welded to the bent rod BCD which is supported by bearings at E and F and by cable DG. Knowing that the bearing at E does not exert any axial thrust, determine (*a*) the tension in cable DG, (*b*) the reactions at E and F.

SOLUTION



(*a*) From f.b.d. of assembly

$$\mathbf{T}_{DG} = \lambda_{DG} T_{DG} = \left[\frac{-(0.12\text{ m})\mathbf{j} - (0.225\text{ m})\mathbf{k}}{\sqrt{(0.12)^2 + (0.225)^2}\text{ m}}\right] = \frac{T_{DG}}{0.255}\left[-(0.12)\mathbf{j} - (0.225)\mathbf{k}\right]$$

$$\Sigma M_y = 0: \quad -(220\text{ N})(0.24\text{ m}) + \left[T_{DG}\left(\frac{0.225}{0.255}\right)\right](0.16\text{ m}) = 0$$

$$\therefore \quad T_{DG} = 374.00\text{ N}$$

or $T_{DG} = 374\text{ N}$ ◀

(*b*) From f.b.d. of assembly

$$\Sigma M_{F(z\text{-axis})} = 0: \quad (220\text{ N})(0.19\text{ m}) - E_x(0.13\text{ m}) - \left[374\text{ N}\left(\frac{0.120}{0.255}\right)\right](0.16\text{ m}) = 0$$

$$\therefore \quad E_x = 104.923\text{ N}$$

$$\Sigma F_x = 0: \quad F_x + 104.923\text{ N} - 220\text{ N} = 0$$

$$\therefore \quad F_x = 115.077\text{ N}$$

$$\Sigma M_{F(x\text{-axis})} = 0: \quad E_z(0.13\text{ m}) + \left[374\text{ N}\left(\frac{0.225}{0.255}\right)\right](0.06\text{ m}) = 0$$

$$\therefore \quad E_z = -152.308\text{ N}$$

PROBLEM 4.120 CONTINUED

$$\Sigma F_z = 0: \quad F_z - 152.308\text{ N} - (374\text{ N})\left(\frac{0.225}{0.255}\right) = 0$$

$$\therefore \quad F_z = 482.31\text{ N}$$

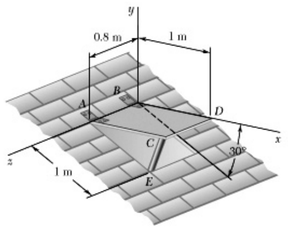
$$\Sigma F_y = 0: \quad F_y - (374\text{ N})\left(\frac{0.12}{0.255}\right) = 0$$

$$\therefore \quad F_y = 176.0\text{ N}$$

$$\mathbf{E} = (104.9\text{ N})\mathbf{i} - (152.3\text{ N})\mathbf{k} \blacktriangleleft$$

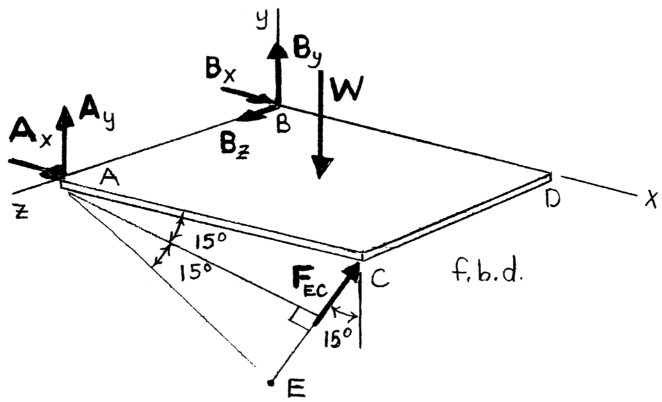
$$\mathbf{F} = (115.1\text{ N})\mathbf{i} + (176.0\text{ N})\mathbf{j} + (482\text{ N})\mathbf{k} \blacktriangleleft$$

PROBLEM 4.121



A 30-kg cover for a roof opening is hinged at corners *A* and *B*. The roof forms an angle of 30° with the horizontal, and the cover is maintained in a horizontal position by the brace *CE*. Determine (*a*) the magnitude of the force exerted by the brace, (*b*) the reactions at the hinges. Assume that the hinge at *A* does not exert any axial thrust.

SOLUTION



First note $$W = mg = (30 \text{ kg})\left(9.81 \text{ m/s}^2\right) = 294.3 \text{ N}$$

$$\mathbf{F}_{EC} = \lambda_{EC}F_{EC} = \left[(\sin 15^{\circ})\mathbf{i} + (\cos 15^{\circ})\mathbf{j}\right]F_{EC}$$

From f.b.d. of cover

(*a*) $$\Sigma M_z = 0: \quad \left(F_{EC}\cos 15^{\circ}\right)(1.0 \text{ m}) - W(0.5 \text{ m}) = 0$$

or $$F_{EC}\cos 15^{\circ}(1.0 \text{ m}) - (294.3 \text{ N})(0.5 \text{ m}) = 0$$

$$\therefore \quad F_{EC} = 152.341 \text{ N} \qquad \text{or } F_{EC} = 152.3 \text{ N} ◀$$

(*b*) $$\Sigma M_x = 0: \quad W(0.4 \text{ m}) - A_y(0.8 \text{ m}) - \left(F_{EC}\cos 15^{\circ}\right)(0.8 \text{ m}) = 0$$

or $$(294.3 \text{ N})(0.4 \text{ m}) - A_y(0.8 \text{ m}) - \left[(152.341 \text{ N})\cos 15^{\circ}\right](0.8 \text{ m}) = 0$$

$$\therefore \quad A_y = 0$$

$$\Sigma M_y = 0: \quad A_x(0.8 \text{ m}) + \left(F_{EC}\sin 15^{\circ}\right)(0.8 \text{ m}) = 0$$

or $$A_x(0.8 \text{ m}) + \left[(152.341 \text{ N})\sin 15^{\circ}\right](0.8 \text{ m}) = 0$$

$$\therefore \quad A_x = -39.429 \text{ N}$$

$$\Sigma F_x = 0: \quad A_x + B_x + F_{EC}\sin 15^{\circ} = 0$$

$$-39.429 \text{ N} + B_x + (152.341 \text{ N})\sin 15^{\circ} = 0$$

$$\therefore \quad B_x = 0$$

PROBLEM 4.121 CONTINUED

$$\Sigma F_y = 0: \quad F_{EC}\cos 15° - W + B_y = 0$$

or

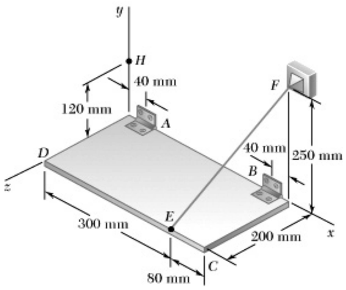
$$(152.341 \text{ N})\cos 15° - 294.3 \text{ N} + B_y = 0$$

$$\therefore \quad B_y = 147.180 \text{ N}$$

or $\mathbf{A} = -(39.4 \text{ N})\mathbf{i}$ ◀

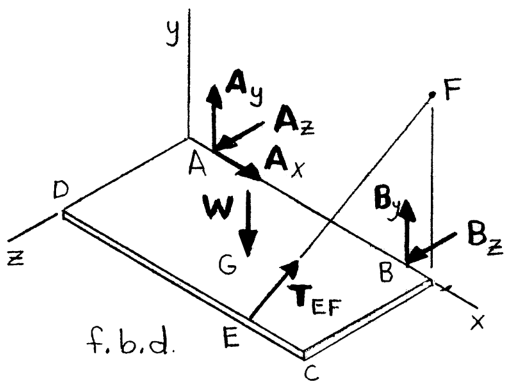
$\mathbf{B} = (147.2 \text{ N})\mathbf{j}$ ◀

PROBLEM 4.122



The rectangular plate shown has a mass of 15 kg and is held in the position shown by hinges A and B and cable EF. Assuming that the hinge at B does not exert any axial thrust, determine (a) the tension in the cable, (b) the reactions at A and B.

SOLUTION



First note $$W = mg = (15\text{ kg})(9.81\text{ m/s}^2) = 147.15\text{ N}$$

$$\mathbf{T}_{EF} = \lambda_{EF} T_{EF} = \left[\frac{(0.08\text{ m})\mathbf{i} + (0.25\text{ m})\mathbf{j} - (0.2\text{ m})\mathbf{k}}{\sqrt{(0.08)^2 + (0.25)^2 + (0.2)^2}\text{ m}}\right] T_{EF} = \frac{T_{EF}}{0.33}(0.08\mathbf{i} + 0.25\mathbf{j} - 0.2\mathbf{k})$$

From f.b.d. of rectangular plate

$$\Sigma M_x = 0: \quad (147.15\text{ N})(0.1\text{ m}) - (T_{EF})_y(0.2\text{ m}) = 0$$

or $$14.715\text{ N}\cdot\text{m} - \left[\left(\frac{0.25}{0.33}\right)T_{EF}\right](0.2\text{ m}) = 0$$

or $$T_{EF} = 97.119\text{ N}$$

or $T_{EF} = 97.1\text{ N}$ ◀

$$\Sigma F_x = 0: \quad A_x + (T_{EF})_x = 0$$

$$A_x + \left(\frac{0.08}{0.33}\right)(97.119\text{ N}) = 0$$

$$\therefore \quad A_x = -23.544\text{ N}$$

PROBLEM 4.122 CONTINUED

$$\Sigma M_{B(z\text{-axis})} = 0: \quad -A_y(0.3\text{ m}) - (T_{EF})_y(0.04\text{ m}) + W(0.15\text{ m}) = 0$$

or

$$-A_y(0.3\text{ m}) - \left[\left(\frac{0.25}{0.33}\right)97.119\text{ N}\right](0.04\text{ m}) + 147.15\text{ N}(0.15\text{ m}) = 0$$

$$\therefore \quad A_y = 63.765\text{ N}$$

$$\Sigma M_{B(y\text{-axis})} = 0: \quad A_z(0.3\text{ m}) + (T_{EF})_x(0.2\text{ m}) + (T_{EF})_z(0.04\text{ m}) = 0$$

$$A_z(0.3\text{ m}) + \left[\left(\frac{0.08}{0.33}\right)T_{EF}\right](0.2\text{ m}) - \left[\left(\frac{0.2}{0.33}\right)T_{EF}\right](0.04\text{ m}) = 0$$

$$\therefore \quad A_z = -7.848\text{ N}$$

and $\mathbf{A} = -(23.5\text{ N})\mathbf{i} + (63.8\text{ N})\mathbf{j} - (7.85\text{ N})\mathbf{k}$ ◀

$$\Sigma F_y = 0: \quad A_y - W + (T_{EF})_y + B_y = 0$$

$$63.765\text{ N} - 147.15\text{ N} + \left(\frac{0.25}{0.33}\right)(97.119\text{ N}) + B_y = 0$$

$$\therefore \quad B_y = 9.81\text{ N}$$

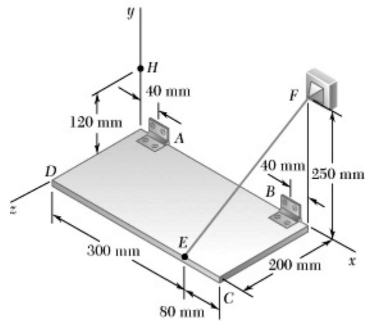
$$\Sigma F_z = 0: \quad A_z - (T_{EF})_z + B_z = 0$$

$$-7.848\text{ N} - \left(\frac{0.2}{0.33}\right)(97.119\text{ N}) + B_z = 0$$

$$\therefore \quad B_z = 66.708\text{ N}$$

and $\mathbf{B} = (9.81\text{ N})\mathbf{j} + (66.7\text{ N})\mathbf{k}$ ◀

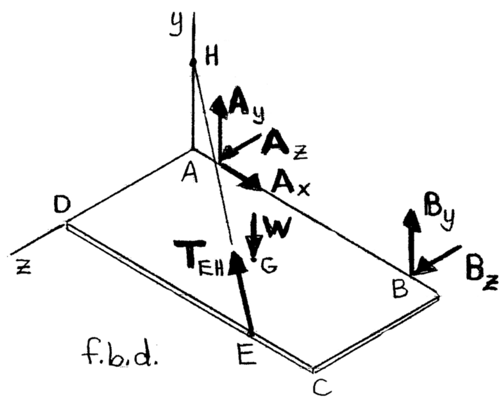
PROBLEM 4.123



Solve Problem 4.122 assuming that cable *EF* is replaced by a cable attached at points *E* and *H*.

P4.122 The rectangular plate shown has a mass of 15 kg and is held in the position shown by hinges *A* and *B* and cable *EF*. Assuming that the hinge at *B* does not exert any axial thrust, determine (*a*) the tension in the cable, (*b*) the reactions at *A* and *B*.

SOLUTION



First note $$W = mg = (15\text{ kg})(9.81\text{ m/s}^2) = 147.15\text{ N}$$

$$\mathbf{T}_{EH} = \lambda_{EH}T_{EH} = \left[\frac{-(0.3\text{ m})\mathbf{i} + (0.12\text{ m})\mathbf{j} - (0.2\text{ m})\mathbf{k}}{\sqrt{(0.3)^2 + (0.12)^2 + (0.2)^2}\text{ m}}\right]T_{EH} = \frac{T_{EH}}{0.38}\left[-(0.3)\mathbf{i} + (0.12)\mathbf{j} - (0.2)\mathbf{k}\right]$$

From f.b.d. of rectangular plate

$$\Sigma M_x = 0: \quad (147.15\text{ N})(0.1\text{ m}) - (T_{EH})_y(0.2\text{ m}) = 0$$

or $$(147.15\text{ N})(0.1\text{ m}) - \left[\left(\frac{0.12}{0.38}\right)T_{EH}\right](0.2\text{ m}) = 0$$

or $$T_{EH} = 232.99\text{ N}$$

or $T_{EH} = 233\text{ N}$ ◀

$$\Sigma F_x = 0: \quad A_x + (T_{EH})_x = 0$$

$$A_x - \left(\frac{0.3}{0.38}\right)(232.99\text{ N}) = 0$$

$$\therefore \quad A_x = 183.938\text{ N}$$

PROBLEM 4.123 CONTINUED

$$\Sigma M_{B(z\text{-axis})} = 0: \quad -A_y(0.3\text{ m}) - (T_{EH})_y(0.04\text{ m}) + W(0.15\text{ m}) = 0$$

or

$$-A_y(0.3\text{ m}) - \left[\frac{0.12}{0.38}(232.99\text{ N})\right](0.04\text{ m}) + (147.15\text{ N})(0.15\text{ m}) = 0$$

$$\therefore \quad A_y = 63.765\text{ N}$$

$$\Sigma M_{B(y\text{-axis})} = 0: \quad A_z(0.3\text{ m}) + (T_{EH})_x(0.2\text{ m}) + (T_{EH})_z(0.04\text{ m}) = 0$$

or

$$A_z(0.3\text{ m}) - \left[\left(\frac{0.3}{0.38}\right)(232.99\text{ N})\right](0.2\text{ m}) - \left[\left(\frac{0.2}{0.38}\right)(232.99)\right](0.04\text{ m}) = 0$$

$$\therefore \quad A_z = 138.976\text{ N}$$

and $\mathbf{A} = (183.9\text{ N})\mathbf{i} + (63.8\text{ N})\mathbf{j} + (139.0\text{ N})\mathbf{k}$ ◀

$$\Sigma F_y = 0: \quad A_y + B_y - W + (T_{EH})_y = 0$$

$$63.765\text{ N} + B_y - 147.15\text{ N} + \left(\frac{0.12}{0.38}\right)(232.99\text{ N}) = 0$$

$$\therefore \quad B_y = 9.8092\text{ N}$$

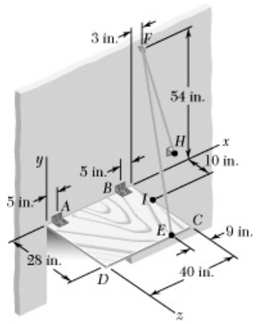
$$\Sigma F_z = 0: \quad A_z + B_z - (T_{EH})_z = 0$$

$$138.976\text{ N} + B_z - \left(\frac{0.2}{0.38}\right)(232.99\text{ N}) = 0$$

$$\therefore \quad B_z = -16.3497\text{ N}$$

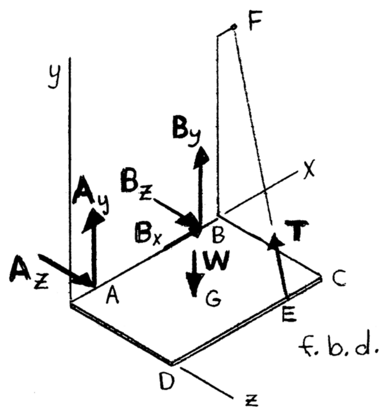
and $\mathbf{B} = (9.81\text{ N})\mathbf{j} - (16.35\text{ N})\mathbf{k}$ ◀

PROBLEM 4.124



A small door weighing 16 lb is attached by hinges A and B to a wall and is held in the horizontal position shown by rope EFH. The rope passes around a small, frictionless pulley at F and is tied to a fixed cleat at H. Assuming that the hinge at A does not exert any axial thrust, determine (a) the tension in the rope, (b) the reactions at A and B.

SOLUTION



First note

$$\mathbf{T} = \lambda_{EF}T = \frac{(12 \text{ in.})\mathbf{i} + (54 \text{ in.})\mathbf{j} - (28 \text{ in.})\mathbf{k}}{\sqrt{(12)^2 + (54)^2 + (28)^2} \text{ in.}}T$$

$$= \frac{T}{62}(12\mathbf{i} + 54\mathbf{j} - 28\mathbf{k}) = \frac{T}{31}(6\mathbf{i} + 27\mathbf{j} - 14\mathbf{k})$$

$$\mathbf{W} = -(16 \text{ lb})\mathbf{j} \quad \text{at } G$$

From f.b.d. of door $ABCD$

(a) $\quad \Sigma M_x = 0: \quad T_y(28 \text{ in.}) - W(14 \text{ in.}) = 0$

$$\left[T\left(\frac{27}{31}\right)\right](28 \text{ in.}) - (16 \text{ lb})(14 \text{ in.}) = 0$$

$$\therefore \quad T = 9.1852 \text{ lb}$$

or $T = 9.19$ lb ◀

(b) $\quad \Sigma M_{B(z\text{-axis})} = 0: \quad -A_y(30 \text{ in.}) + W(15 \text{ in.}) - T_y(4 \text{ in.}) = 0$

$$-A_y(30 \text{ in.}) + (16 \text{ lb})(15 \text{ in.}) - \left[(9.1852 \text{ lb})\left(\frac{27}{31}\right)\right](4 \text{ in.}) = 0$$

$$\therefore \quad A_y = 6.9333 \text{ lb}$$

PROBLEM 4.124 CONTINUED

$$\Sigma M_{B(y\text{-axis})} = 0: \quad A_z(30 \text{ in.}) + T_x(28 \text{ in.}) - T_z(4 \text{ in.}) = 0$$

$$A_z(30 \text{ in.}) + \left[(9.1852 \text{ lb})\left(\frac{6}{31}\right)\right](28 \text{ in.}) - \left[(9.1852 \text{ lb})\left(\frac{14}{31}\right)\right](4 \text{ in.}) = 0$$

$$\therefore \quad A_z = -1.10617 \text{ lb}$$

$$\text{or } \mathbf{A} = (6.93 \text{ lb})\mathbf{j} - (1.106 \text{ lb})\mathbf{k} \blacktriangleleft$$

$$\Sigma F_x = 0: \quad B_x + T_x = B_x + (9.1852 \text{ lb})\left(\frac{6}{31}\right) = 0$$

$$\therefore \quad B_x = -1.77778 \text{ lb}$$

$$\Sigma F_y = 0: \quad B_y + T_y - W + A_y = 0$$

$$B_y + (9.1852 \text{ lb})\left(\frac{27}{31}\right) - 16 \text{ lb} + 6.9333 \text{ lb} = 0$$

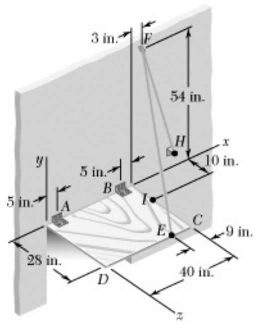
$$\therefore \quad B_y = 1.06666 \text{ lb}$$

$$\Sigma F_z = 0: \quad A_z - T_z + B_z = 0$$

$$-1.10617 \text{ lb} - (9.1852 \text{ lb})\left(\frac{14}{31}\right) + B_z = 0$$

$$\therefore \quad B_z = 5.2543 \text{ lb}$$

$$\text{or } \mathbf{B} = -(1.778 \text{ lb})\mathbf{i} + (1.067 \text{ lb})\mathbf{j} + (5.25 \text{ lb})\mathbf{k} \blacktriangleleft$$

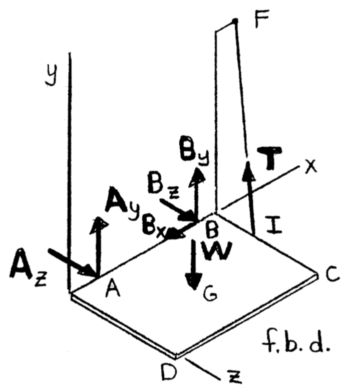


PROBLEM 4.125

Solve Problem 4.124 assuming that the rope is attached to the door at *I*.

P4.124 A small door weighing 16 lb is attached by hinges *A* and *B* to a wall and is held in the horizontal position shown by rope *EFH*. The rope passes around a small, frictionless pulley at *F* and is tied to a fixed cleat at *H*. Assuming that the hinge at *A* does not exert any axial thrust, determine (*a*) the tension in the rope, (*b*) the reactions at *A* and *B*.

SOLUTION



First note

$$\mathbf{T} = \lambda_{IF} T = \frac{(3 \text{ in.})\mathbf{i} + (54 \text{ in.})\mathbf{j} - (10 \text{ in.})\mathbf{k}}{\sqrt{(3)^2 + (54)^2 + (10)^2} \text{ in.}} T$$

$$= \frac{T}{55}(3\mathbf{i} + 54\mathbf{j} - 10\mathbf{k})$$

$$\mathbf{W} = -(16 \text{ lb})\mathbf{j}$$

From f.b.d. of door *ABCD*

(*a*) $\quad \Sigma M_x = 0: \quad W(14 \text{ in.}) - T_y(10 \text{ in.}) = 0$

$$(16 \text{ lb})(14 \text{ in.}) - \left(\frac{54}{55}\right) T(10 \text{ in.}) = 0$$

$$\therefore \quad T = 22.815 \text{ lb}$$

or $T = 22.8$ lb ◀

(*b*) $\quad \Sigma M_{B(z\text{-axis})} = 0: \quad -A_y(30 \text{ in.}) + W(15 \text{ in.}) + T_y(5 \text{ in.}) = 0$

$$-A_y(30 \text{ in.}) + (16 \text{ lb})(15 \text{ in.}) + (22.815 \text{ lb})\left(\frac{54}{55}\right)(5 \text{ in.}) = 0$$

$$\therefore \; A_y = 11.7334 \text{ lb}$$

PROBLEM 4.125 CONTINUED

$$\Sigma M_{B(y\text{-axis})} = 0: \quad A_z(30 \text{ in.}) + T_x(10 \text{ in.}) + T_z(5 \text{ in.}) = 0$$

$$A_z(30 \text{ in.}) + \left[(22.815 \text{ lb})\left(\frac{3}{55}\right)\right](10 \text{ in.}) + \left[(22.815 \text{ lb})\left(\frac{10}{55}\right)\right](5 \text{ in.}) = 0$$

$$\therefore A_z = -1.10618 \text{ lb}$$

$$\text{or } \mathbf{A} = (11.73 \text{ lb})\mathbf{j} - (1.106 \text{ lb})\mathbf{k} \blacktriangleleft$$

$$\Sigma F_x = 0: \quad B_x + T_x = 0$$

$$B_x + \left(\frac{3}{55}\right)(22.815 \text{ lb}) = 0$$

$$\therefore \quad B_x = -1.24444 \text{ lb}$$

$$\Sigma F_y = 0: \quad A_y - W + T_y + B_y = 0$$

$$11.7334 \text{ lb} - 16 \text{ lb} + (22.815 \text{ lb})\left(\frac{54}{55}\right) + B_y = 0$$

$$\therefore \quad B_y = -18.1336 \text{ lb}$$

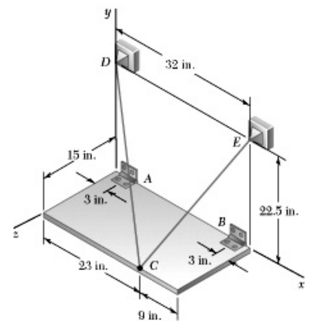
$$\Sigma F_z = 0: \quad A_z - T_z + B_z = 0$$

$$-1.10618 \text{ lb} - (22.815 \text{ lb})\left(\frac{10}{55}\right) + B_z = 0$$

$$\therefore \quad B_z = 5.2544 \text{ lb}$$

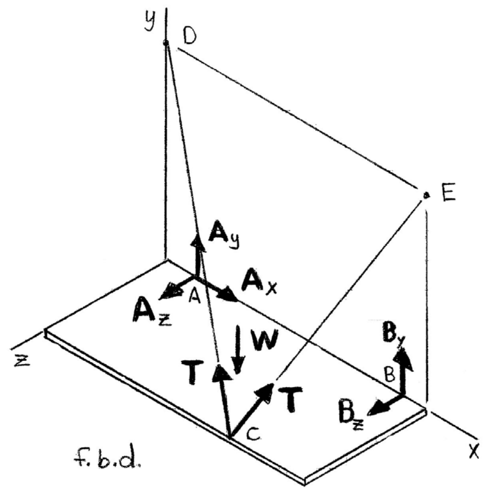
$$\text{or } \mathbf{B} = -(1.244 \text{ lb})\mathbf{i} - (18.13 \text{ lb})\mathbf{j} + (5.25 \text{ lb})\mathbf{k} \blacktriangleleft$$

PROBLEM 4.126



A 285-lb uniform rectangular plate is supported in the position shown by hinges *A* and *B* and by cable *DCE*, which passes over a frictionless hook at *C*. Assuming that the tension is the same in both parts of the cable, determine (*a*) the tension in the cable, (*b*) the reactions at *A* and *B*. Assume that the hinge at *B* does not exert any axial thrust.

SOLUTION



First note

$$\lambda_{CD} = \frac{-(23\text{ in.})\mathbf{i} + (22.5\text{ in.})\mathbf{j} - (15\text{ in.})\mathbf{k}}{35.5\text{ in.}}$$

$$= \frac{1}{35.5}(-23\mathbf{i} + 22.5\mathbf{j} - 15\mathbf{k})$$

$$\lambda_{CE} = \frac{(9\text{ in.})\mathbf{i} + (22.5\text{ in.})\mathbf{j} - (15\text{ in.})\mathbf{k}}{28.5\text{ in.}}$$

$$= \frac{1}{28.5}(9\mathbf{i} + 22.5\mathbf{j} - 15\mathbf{k})$$

$$\mathbf{W} = -(285\text{ lb})\mathbf{j}$$

From f.b.d. of plate

(*a*) $\Sigma M_x = 0$: $(285\text{ lb})(7.5\text{ in.}) - \left[\left(\frac{22.5}{35.5}\right)T\right](15\text{ in.}) - \left[\left(\frac{22.5}{28.5}\right)T\right](15\text{ in.}) = 0$

$$\therefore\quad T = 100.121\text{ lb}$$

or $T = 100.1$ lb ◀

PROBLEM 4.126 CONTINUED

(*b*)

$$\Sigma F_x = 0: \quad A_x - T\left(\frac{23}{35.5}\right) + T\left(\frac{9}{28.5}\right) = 0$$

$$A_x - (100.121\text{ lb})\left(\frac{23}{35.5}\right) + (100.121\text{ lb})\left(\frac{9}{28.5}\right) = 0$$

$$\therefore \quad A_x = 33.250\text{ lb}$$

$$\Sigma M_{B(z\text{-axis})} = 0: \quad -A_y(26\text{ in.}) + W(13\text{ in.}) - \left[T\left(\frac{22.5}{35.5}\right)\right](6\text{ in.}) - \left[T\left(\frac{22.5}{28.5}\right)\right](6\text{ in.}) = 0$$

or

$$-A_y(26\text{ in.}) + (285\text{ lb})(13\text{ in.}) - \left[(100.121\text{ lb})\left(\frac{22.5}{35.5}\right)\right](6\text{ in.})$$

$$-\left[(100.121\text{ lb})\left(\frac{22.5}{28.5}\right)\right](6\text{ in.}) = 0$$

$$\therefore \quad A_y = 109.615\text{ lb}$$

$$\Sigma M_{B(y\text{-axis})} = 0: \quad A_z(26\text{ in.}) - \left[T\left(\frac{15}{35.5}\right)\right](6\text{ in.}) - \left[T\left(\frac{23}{35.5}\right)\right](15\text{ in.})$$

$$-\left[T\left(\frac{15}{28.5}\right)\right](6\text{ in.}) + \left[T\left(\frac{9}{28.5}\right)\right](15\text{ in.}) = 0$$

or

$$A_z(26\text{ in.}) + \left[\frac{-1}{35.5}(90 + 345) - \frac{1}{28.5}(90 - 135)\right](100.121\text{ lb}) = 0$$

$$\therefore \quad A_z = 41.106\text{ lb}$$

or $\mathbf{A} = (33.3\text{ lb})\mathbf{i} + (109.6\text{ lb})\mathbf{j} + (41.1\text{ lb})\mathbf{k}$ ◀

$$\Sigma F_y = 0: \quad B_y - W + T\left(\frac{22.5}{35.5}\right) + T\left(\frac{22.5}{28.5}\right) + A_y = 0$$

$$B_y - 285\text{ lb} + (100.121\text{ lb})\left(\frac{22.5}{35.5} + \frac{22.5}{28.5}\right) + 109.615\text{ lb} = 0$$

$$\therefore \quad B_y = 32.885\text{ lb}$$

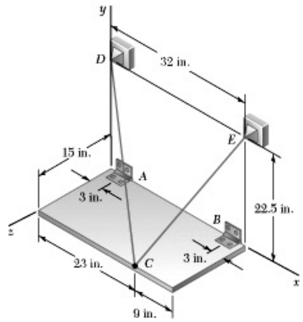
$$\Sigma F_z = 0: \quad B_z + A_z - T\left(\frac{15}{35.5}\right) - T\left(\frac{15}{28.5}\right) = 0$$

$$B_z + 41.106\text{ lb} - (100.121\text{ lb})\left(\frac{15}{35.5} + \frac{15}{28.5}\right) = 0$$

$$\therefore \quad B_z = 53.894\text{ lb}$$

or $\mathbf{B} = (32.9\text{ lb})\mathbf{j} + (53.9\text{ lb})\mathbf{k}$ ◀

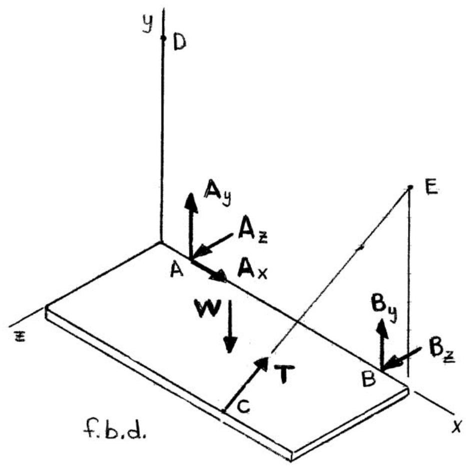
PROBLEM 4.127



Solve Problem 4.126 assuming that cable *DCE* is replaced by a cable attached to point *E* and hook *C*.

P4.126 A 285-lb uniform rectangular plate is supported in the position shown by hinges *A* and *B* and by cable *DCE*, which passes over a frictionless hook at *C*. Assuming that the tension is the same in both parts of the cable, determine (*a*) the tension in the cable, (*b*) the reactions at *A* and *B*. Assume that the hinge at *B* does not exert any axial thrust.

SOLUTION



First note

$$\lambda_{CE} = \frac{(9\text{ in.})\mathbf{i} + (22.5\text{ in.})\mathbf{j} - (15\text{ in.})\mathbf{k}}{28.5\text{ in.}}$$

$$= \frac{1}{28.5}(9\mathbf{i} + 22.5\mathbf{j} - 15\mathbf{k})$$

$$\mathbf{W} = -(285\text{ lb})\mathbf{j}$$

From f.b.d. of plate

(*a*)

$$\Sigma M_x = 0: \quad (285\text{ lb})(7.5\text{ in.}) - \left[\left(\frac{22.5}{28.5}\right)T\right](15\text{ in.}) = 0$$

$$\therefore \quad T = 180.500\text{ lb}$$

or $T = 180.5$ lb ◀

(*b*)

$$\Sigma F_x = 0: \quad A_x + T\left(\frac{9}{28.5}\right) = 0$$

$$A_x + 180.5\text{ lb}\left(\frac{9}{28.5}\right) = 0$$

$$\therefore \quad A_x = -57.000\text{ lb}$$

PROBLEM 4.127 CONTINUED

$$\Sigma M_{B(z\text{-axis})} = 0: \quad -A_y(26\text{ in.}) + W(13\text{ in.}) - \left[T\left(\frac{22.5}{28.5}\right)\right](6\text{ in.}) = 0$$

$$-A_y(26\text{ in.}) + (285\text{ lb})(13\text{ in.}) - \left[(180.5\text{ lb})\left(\frac{22.5}{28.5}\right)\right](6\text{ in.}) = 0$$

$$\therefore \quad A_y = 109.615\text{ lb}$$

$$\Sigma M_{B(y\text{-axis})} = 0: \quad A_z(26\text{ in.}) - \left[T\left(\frac{15}{28.5}\right)\right](6\text{ in.}) + \left[T\left(\frac{9}{28.5}\right)\right](15\text{ in.}) = 0$$

$$A_z(26\text{ in.}) + (180.5\text{ lb})\left(\frac{45}{28.5}\right) = 0$$

$$\therefore \quad A_z = -10.9615\text{ lb}$$

$$\text{or } \mathbf{A} = -(57.0\text{ lb})\mathbf{i} + (109.6\text{ lb})\mathbf{j} - (10.96\text{ lb})\mathbf{k} \blacktriangleleft$$

$$\Sigma F_y = 0: \quad B_y - W + T\left(\frac{22.5}{28.5}\right) + A_y = 0$$

$$B_y - 285\text{ lb} + (180.5\text{ lb})\left(\frac{22.5}{28.5}\right) - 109.615\text{ lb} = 0$$

$$\therefore \quad B_y = 32.885\text{ lb}$$

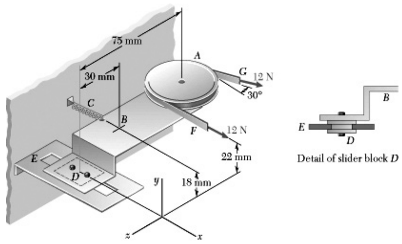
$$\Sigma F_z = 0: \quad B_z + A_z - T\left(\frac{15}{28.5}\right) = 0$$

$$B_z - 10.9615\text{ lb} - 180.5\text{ lb}\left(\frac{15}{28.5}\right) = 0$$

$$\therefore \quad B_z = 105.962\text{ lb}$$

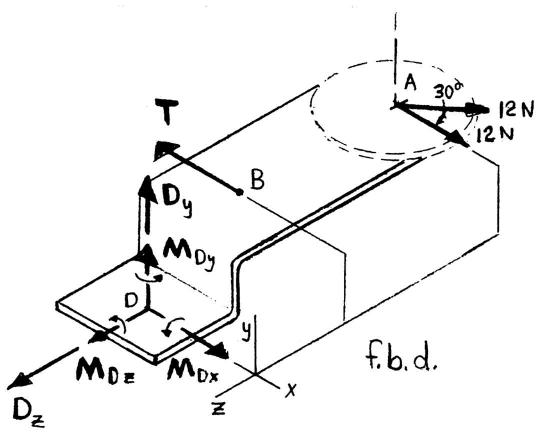
$$\text{or } \mathbf{B} = (32.9\text{ lb})\mathbf{j} + (106.0\text{ lb})\mathbf{k} \blacktriangleleft$$

PROBLEM 4.128



The tensioning mechanism of a belt drive consists of frictionless pulley *A*, mounting plate *B*, and spring *C*. Attached below the mounting plate is slider block *D* which is free to move in the frictionless slot of bracket *E*. Knowing that the pulley and the belt lie in a horizontal plane, with portion *F* of the belt parallel to the *x* axis and portion *G* forming an angle of 30° with the *x* axis, determine (*a*) the force in the spring, (*b*) the reaction at *D*.

SOLUTION



From f.b.d. of plate *B*

(*a*) $\Sigma F_x = 0\colon\ \ 12\text{ N} + (12\text{ N})\cos 30° - T = 0$

$$\therefore\ \ T = 22.392\text{ N}$$

or $T = 22.4\text{ N}$ ◀

(*b*) $\Sigma F_y = 0\colon\ \ D_y = 0$

$$\Sigma F_z = 0\colon\ \ D_z - (12\text{ N})\sin 30° = 0$$

$$\therefore\ \ D_z = 6\text{ N}$$

or $\mathbf{D} = (6.00\text{ N})\mathbf{k}$ ◀

$$\Sigma M_x = 0\colon\ \ M_{D_x} - \left[(12\text{ N})\sin 30°\right](22\text{ mm}) = 0$$

$$\therefore\ \ M_{D_x} = 132.0\text{ N}\cdot\text{mm}$$

$$\Sigma M_{D(y\text{-axis})} = 0\colon\ \ M_{D_y} + (22.392\text{ N})(30\text{ mm}) - (12\text{ N})(75\text{ mm}) - \left[(12\text{ N})\cos 30°\right](75\text{ mm}) = 0$$

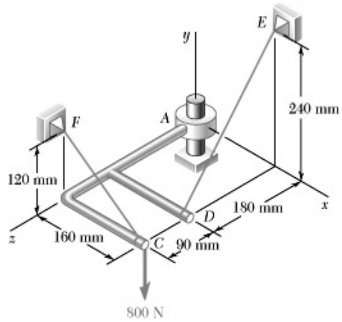
$$\therefore\ \ M_{D_y} = 1007.66\text{ N}\cdot\text{mm}$$

$$\Sigma M_{D(z\text{-axis})} = 0\colon\ \ M_{D_z} + (22.392\text{ N})(18\text{ mm}) - (12\text{ N})(22\text{ mm}) - \left[(12\text{ N})\cos 30°\right](22\text{ mm}) = 0$$

$$\therefore\ \ M_{D_z} = 89.575\text{ N}\cdot\text{mm}$$

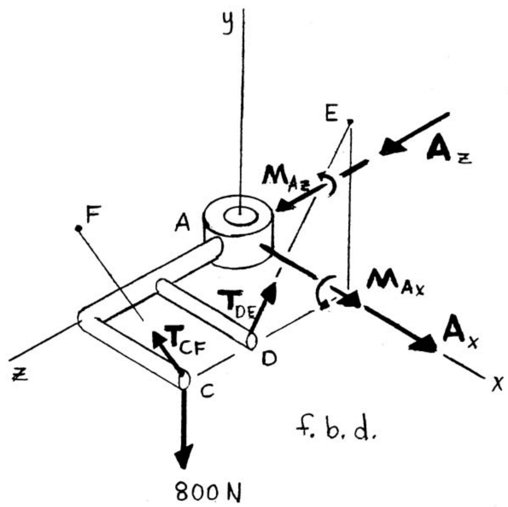
or $\mathbf{M}_D = (0.1320\text{ N}\cdot\text{m})\mathbf{i} + (1.008\text{ N}\cdot\text{m})\mathbf{j} + (0.0896\text{ N}\cdot\text{m})\mathbf{k}$ ◀

PROBLEM 4.129



The assembly shown is welded to collar A which fits on the vertical pin shown. The pin can exert couples about the x and z axes but does not prevent motion about or along the y axis. For the loading shown, determine the tension in each cable and the reaction at A.

SOLUTION



First note
$$\mathbf{T}_{CF} = \lambda_{CF}T_{CF} = \frac{-(0.16\text{ m})\mathbf{i} + (0.12\text{ m})\mathbf{j}}{\sqrt{(0.16)^2 + (0.12)^2}\text{ m}}T_{CF}$$

$$= T_{CF}(-0.8\mathbf{i} + 0.6\mathbf{j})$$

$$\mathbf{T}_{DE} = \lambda_{DE}T_{DE} = \frac{(0.24\text{ m})\mathbf{j} - (0.18\text{ m})\mathbf{k}}{\sqrt{(0.24)^2 + (0.18)^2}\text{ m}}T_{DE}$$

$$= T_{DE}(0.8\mathbf{j} - 0.6\mathbf{k})$$

(a) From f.b.d. of assembly

$$\Sigma F_y = 0\text{:}\quad 0.6T_{CF} + 0.8T_{DE} - 800\text{ N} = 0$$

or
$$0.6T_{CF} + 0.8T_{DE} = 800\text{ N} \qquad (1)$$

$$\Sigma M_y = 0\text{:}\quad -(0.8T_{CF})(0.27\text{ m}) + (0.6T_{DE})(0.16\text{ m}) = 0$$

or
$$T_{DE} = 2.25T_{CF} \qquad (2)$$

PROBLEM 4.129 CONTINUED

Substituting Equation (2) into Equation (1)

$$0.6T_{CF} + 0.8\left[(2.25)T_{CF}\right] = 800 \text{ N}$$

$\therefore \quad T_{CF} = 333.33 \text{ N}$ or $T_{CF} = 333 \text{ N}$ ◀

and from Equation (2) $\quad T_{DE} = 2.25(333.33 \text{ N}) = 750.00 \text{ N}$ or $T_{DE} = 750 \text{ N}$ ◀

(*b*) From f.b.d. of assembly

$\Sigma F_z = 0\text{:} \quad A_z - (0.6)(750.00 \text{ N}) = 0 \qquad \therefore A_z = 450.00 \text{ N}$

$\Sigma F_x = 0\text{:} \quad A_x - (0.8)(333.33 \text{ N}) = 0 \qquad \therefore A_x = 266.67 \text{ N}$

or $\mathbf{A} = (267 \text{ N})\mathbf{i} + (450 \text{ N})\mathbf{k}$ ◀

$$\Sigma M_x = 0\text{:} \quad M_{A_x} + (800 \text{ N})(0.27 \text{ m}) - \left[(333.33 \text{ N})(0.6)\right](0.27 \text{ m}) - \left[(750 \text{ N})(0.8)\right](0.18 \text{ m}) = 0$$

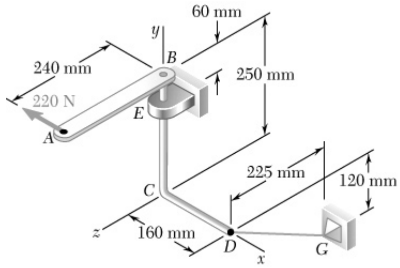
$$\therefore \quad M_{A_x} = -54.001 \text{ N}\cdot\text{m}$$

$$\Sigma M_z = 0\text{:} \quad M_{A_z} - (800 \text{ N})(0.16 \text{ m}) + \left[(333.33 \text{ N})(0.6)\right](0.16 \text{ m}) + \left[(750 \text{ N})(0.8)\right](0.16 \text{ m}) = 0$$

$$\therefore \quad M_{A_z} = 0$$

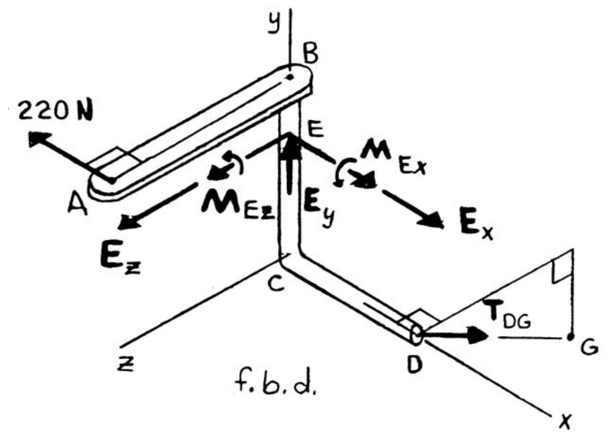
or $\mathbf{M}_A = -(54.0 \text{ N}\cdot\text{m})\mathbf{i}$ ◀

PROBLEM 4.130



The lever *AB* is welded to the bent rod *BCD* which is supported by bearing *E* and by cable *DG*. Assuming that the bearing can exert an axial thrust and couples about axes parallel to the *x* and *z* axes, determine (*a*) the tension in cable *DG*, (*b*) the reaction at *E*.

SOLUTION



First note

$$\mathbf{T}_{DG} = \lambda_{DG}T_{DG} = \frac{-(0.12\text{ m})\mathbf{j} - (0.225\text{ m})\mathbf{k}}{\sqrt{(0.12)^2 + (0.225)^2}\text{ m}}T_{DG}$$

$$= \frac{T_{DG}}{0.255}(-0.12\mathbf{j} - 0.225\mathbf{k})$$

(*a*) From f.b.d. of weldment

$$\Sigma M_y = 0: \quad \left[\left(\frac{0.225}{0.255}\right)T_{DG}\right](0.16\text{ m}) - (220\text{ N})(0.24\text{ m}) = 0$$

$$\therefore \quad T_{DG} = 374.00\text{ N} \qquad \text{or } T_{DG} = 374\text{ N} \blacktriangleleft$$

(*b*) From f.b.d. of weldment

$$\Sigma F_x = 0: \quad E_x - 220\text{ N} = 0$$

$$\therefore \quad E_x = 220.00\text{ N}$$

$$\Sigma F_y = 0: \quad E_y - (374.00\text{ N})\left(\frac{0.12}{0.255}\right) = 0$$

$$\therefore \quad E_y = 176.000\text{ N}$$

PROBLEM 4.130 CONTINUED

$$\Sigma F_z = 0: \quad E_z - (374.00 \text{ N})\left(\frac{0.225}{0.255}\right) = 0$$

$$\therefore \quad E_z = 330.00 \text{ N}$$

$$\text{or } \mathbf{E} = (220 \text{ N})\mathbf{i} + (176.0 \text{ N})\mathbf{j} + (330 \text{ N})\mathbf{k} \blacktriangleleft$$

$$\Sigma M_x = 0: \quad M_{E_x} + (330.00 \text{ N})(0.19 \text{ m}) = 0$$

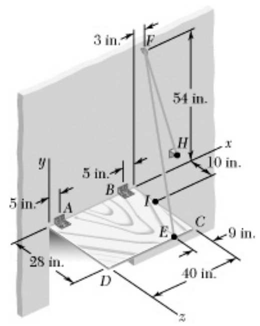
$$\therefore \quad M_{E_x} = -62.700 \text{ N}\cdot\text{m}$$

$$\Sigma M_z = 0: \quad (220 \text{ N})(0.06 \text{ m}) + M_{E_z} - \left[(374.00 \text{ N})\left(\frac{0.12}{0.255}\right)\right](0.16 \text{ m}) = 0$$

$$\therefore \quad M_{E_z} = -14.9600 \text{ N}\cdot\text{m}$$

$$\text{or } \mathbf{M}_E = -(62.7 \text{ N}\cdot\text{m})\mathbf{i} - (14.96 \text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$

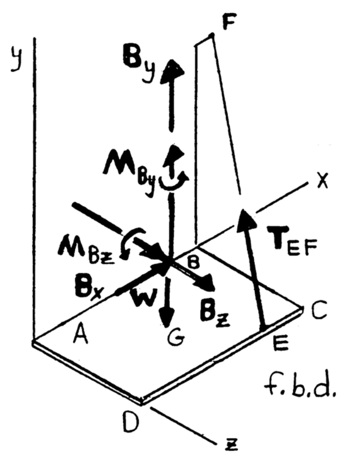
PROBLEM 4.131



Solve Problem 4.124 assuming that the hinge at *A* is removed and that the hinge at *B* can exert couples about the *y* and *z* axes.

P4.124 A small door weighing 16 lb is attached by hinges *A* and *B* to a wall and is held in the horizontal position shown by rope *EFH*. The rope passes around a small, frictionless pulley at *F* and is tied to a fixed cleat at *H*. Assuming that the hinge at *A* does not exert any axial thrust, determine (*a*) the tension in the rope, (*b*) the reactions at *A* and *B*.

SOLUTION



From f.b.d. of door

(*a*) $$\Sigma \mathbf{M}_B = 0: \quad \mathbf{r}_{G/B} \times \mathbf{W} + \mathbf{r}_{E/B} \times \mathbf{T}_{EF} + \mathbf{M}_B = 0$$

where

$$\mathbf{W} = -(16 \text{ lb})\mathbf{j}$$

$$\mathbf{M}_B = M_{B_y}\mathbf{j} + M_{B_z}\mathbf{k}$$

$$\mathbf{T}_{EF} = \lambda_{EF} T_{EF} = \frac{(12 \text{ in.})\mathbf{i} + (54 \text{ in.})\mathbf{j} - (28 \text{ in.})\mathbf{k}}{\sqrt{(12)^2 + (54)^2 + (28)^2} \text{ in.}} T_{EF}$$

$$= \frac{T_{EF}}{31}(6\mathbf{i} + 27\mathbf{j} - 14\mathbf{k})$$

$$\mathbf{r}_{G/B} = -(15 \text{ in.})\mathbf{i} + (14 \text{ in.})\mathbf{k}$$

$$\mathbf{r}_{E/B} = -(4 \text{ in.})\mathbf{i} + (28 \text{ in.})\mathbf{k}$$

$$\therefore \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -15 & 0 & 14 \\ 0 & -1 & 0 \end{vmatrix} (16 \text{ lb}) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & 0 & 28 \\ 6 & 27 & -14 \end{vmatrix} \left(\frac{T_{EF}}{31}\right) + \left(M_{B_y}\mathbf{j} + M_{B_z}\mathbf{k}\right) = 0$$

or $$(224 - 24.387T_{EF})\mathbf{i} + \left(3.6129T_{EF} + M_{B_y}\right)\mathbf{j}$$

$$+ \left(240 - 3.4839T_{EF} + M_{B_z}\right)\mathbf{k} = 0$$

From **i**-coefficient $$224 - 24.387T_{EF} = 0$$

$$\therefore \quad T_{EF} = 9.1852 \text{ lb}$$

or $T_{EF} = 9.19$ lb ◀

(*b*) From **j**-coefficient $$3.6129(9.1852) + M_{B_y} = 0$$

$$\therefore \quad M_{B_y} = -33.185 \text{ lb}\cdot\text{in.}$$

PROBLEM 4.131 CONTINUED

From **k**-coefficient $\quad 240 - 3.4839(9.1852) + M_{B_z} = 0$

$$\therefore \quad M_{B_z} = -208.00 \text{ lb}\cdot\text{in.}$$

$$\text{or } \mathbf{M}_B = -(33.2 \text{ lb}\cdot\text{in.})\mathbf{j} - (208 \text{ lb}\cdot\text{in.})\mathbf{k} \blacktriangleleft$$

$$\Sigma F_x = 0: \quad B_x + \frac{6}{31}(9.1852 \text{ lb}) = 0$$

$$\therefore \quad B_x = -1.77778 \text{ lb}$$

$$\Sigma F_y = 0: \quad B_y - 16 \text{ lb} + \frac{27}{31}(9.1852 \text{ lb}) = 0$$

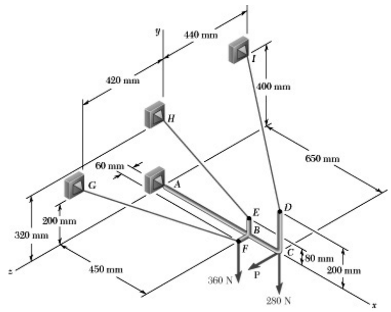
$$\therefore \quad B_y = 8.0000 \text{ lb}$$

$$\Sigma F_z = 0: \quad B_z - \frac{14}{31}(9.1852 \text{ lb}) = 0$$

$$\therefore \quad B_z = 4.1482 \text{ lb}$$

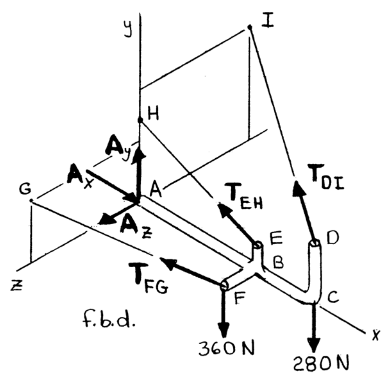
$$\text{or } \mathbf{B} = -(1.778 \text{ lb})\mathbf{i} + (8.00 \text{ lb})\mathbf{j} + (4.15 \text{ lb})\mathbf{k} \blacktriangleleft$$

PROBLEM 4.132



The frame shown is supported by three cables and a ball-and-socket joint at *A*. For $\mathbf{P} = 0$, determine the tension in each cable and the reaction at *A*.

SOLUTION



First note

$$\mathbf{T}_{DI} = \lambda_{DI} T_{DI} = \frac{-(0.65\text{ m})\mathbf{i} + (0.2\text{ m})\mathbf{j} - (0.44\text{ m})\mathbf{k}}{\sqrt{(0.65)^2 + (0.2)^2 + (0.44)^2}\text{ m}} T_{DI}$$

$$= \frac{T_{DI}}{0.81}(-0.65\mathbf{i} + 0.2\mathbf{j} - 0.44\mathbf{k})$$

$$\mathbf{T}_{EH} = \lambda_{EH} T_{EH} = \frac{-(0.45\text{ m})\mathbf{i} + (0.24\text{ m})\mathbf{j}}{\sqrt{(0.45)^2 + (0.24)^2}\text{ m}} T_{EH}$$

$$= \frac{T_{EH}}{0.51}(-0.45\mathbf{i}) + (0.24\,\mathbf{j})$$

$$\mathbf{T}_{FG} = \lambda_{FG} T_{FG} = \frac{-(0.45\text{ m})\mathbf{i} + (0.2\text{ m})\mathbf{j} + (0.36\text{ m})\mathbf{k}}{\sqrt{(0.45\,)^2 + (0.2)^2 + (0.36)^2}\text{ m}} T_{FG}$$

$$= \frac{T_{FG}}{0.61}(-0.45\mathbf{i} + 0.2\mathbf{j} + 0.36\mathbf{k})$$

From f.b.d. of frame

$$\Sigma \mathbf{M}_A = 0: \quad \mathbf{r}_{D/A} \times \mathbf{T}_{DI} + \mathbf{r}_{C/A} \times (-280\text{ N})\mathbf{j} + \mathbf{r}_{H/A} \times \mathbf{T}_{EH} + \mathbf{r}_{F/A} \times \mathbf{T}_{FG} + \mathbf{r}_{F/A} \times (-360\text{ N})\mathbf{j} = 0$$

or
$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.65 & 0.2 & 0 \\ -0.65 & 0.2 & -0.44 \end{vmatrix} \left(\frac{T_{DI}}{0.81}\right) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.65 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix} (280\text{ N}) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.32 & 0 \\ -0.45 & 0.24 & 0 \end{vmatrix} \left(\frac{T_{EH}}{0.51}\right) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.45 & 0 & 0.06 \\ -0.45 & 0.2 & 0.36 \end{vmatrix} \left(\frac{T_{FG}}{0.61}\right)$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.45 & 0 & 0.06 \\ 0 & -1 & 0 \end{vmatrix} (360\text{ N}) = 0$$

or
$$(-0.088\mathbf{i} + 0.286\mathbf{j} + 0.26\mathbf{k})\frac{T_{DI}}{0.81} + (-0.65\mathbf{k})280\text{ N} + (0.144\mathbf{k})\frac{T_{EH}}{0.51}$$

$$+ (-0.012\mathbf{i} - 0.189\mathbf{j} + 0.09\mathbf{k})\frac{T_{FG}}{0.61} + (0.06\mathbf{i} - 0.45\mathbf{k})(360\text{ N}) = 0$$

PROBLEM 4.132 CONTINUED

From **i**-coefficient $$-0.088\left(\frac{T_{DI}}{0.81}\right) - 0.012\left(\frac{T_{FG}}{0.61}\right) + 0.06(360\text{ N}) = 0$$

$$\therefore\quad 0.108642T_{DI} + 0.0196721T_{FG} = 21.6 \qquad (1)$$

From **j**-coefficient $$0.286\left(\frac{T_{DI}}{0.81}\right) - 0.189\left(\frac{T_{FG}}{0.61}\right) = 0$$

$$\therefore\quad T_{FG} = 1.13959T_{DI} \qquad (2)$$

From **k**-coefficient

$$0.26\left(\frac{T_{DI}}{0.81}\right) - 0.65(280\text{ N}) + 0.144\left(\frac{T_{EH}}{0.51}\right) + 0.09\left(\frac{T_{FG}}{0.61}\right)$$

$$-0.45(360\text{ N}) = 0$$

$$\therefore\quad 0.32099T_{DI} + 0.28235T_{EH} + 0.147541T_{FG} = 344\text{ N} \qquad (3)$$

Substitution of Equation (2) into Equation (1)

$$0.108642T_{DI} + 0.0196721(1.13959T_{DI}) = 21.6$$

$$\therefore\quad T_{DI} = 164.810\text{ N}$$

or $$T_{DI} = 164.8\text{ N} \blacktriangleleft$$

Then from Equation (2)

$$T_{FG} = 1.13959(164.810\text{ N}) = 187.816\text{ N}$$

or $$T_{FG} = 187.8\text{ N} \blacktriangleleft$$

And from Equation (3)

$$0.32099(164.810\text{ N}) + 0.28235T_{EH} + 0.147541(187.816\text{ N}) = 344\text{ N}$$

$$\therefore\quad T_{EH} = 932.84\text{ N}$$

or $$T_{EH} = 933\text{ N} \blacktriangleleft$$

The vector forms of the cable forces are:

$$\mathbf{T}_{DI} = \frac{164.810\text{ N}}{0.81}(-0.65\mathbf{i} + 0.2\mathbf{j} - 0.44\mathbf{k})$$

$$= -(132.25\text{ N})\mathbf{i} + (40.694\text{ N})\mathbf{j} - (89.526\text{ N})\mathbf{k}$$

$$\mathbf{T}_{EH} = \frac{932.84\text{ N}}{0.51}(-0.45\mathbf{i} + 0.24\mathbf{j}) = -(823.09\text{ N})\mathbf{i} + (438.98\text{ N})\mathbf{j}$$

$$\mathbf{T}_{FG} = \frac{187.816\text{ N}}{0.61}(-0.45\mathbf{i} + 0.2\mathbf{j} + 0.36\mathbf{k})$$

$$= -(138.553\text{ N})\mathbf{i} + (61.579\text{ N})\mathbf{j} + (110.842\text{ N})\mathbf{k}$$

PROBLEM 4.132 CONTINUED

Then, from f.b.d. of frame

$$\Sigma F_x = 0: \quad A_x - 132.25 - 823.09 - 138.553 = 0$$

$$\therefore \quad A_x = 1093.89 \text{ N}$$

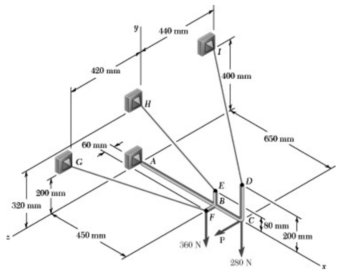
$$\Sigma F_y = 0: \quad A_y + 40.694 + 438.98 + 61.579 - 360 - 280 = 0$$

$$\therefore \quad A_y = 98.747 \text{ N}$$

$$\Sigma F_z = 0: \quad A_z - 89.526 + 110.842 = 0$$

$$\therefore \quad A_z = -21.316 \text{ N}$$

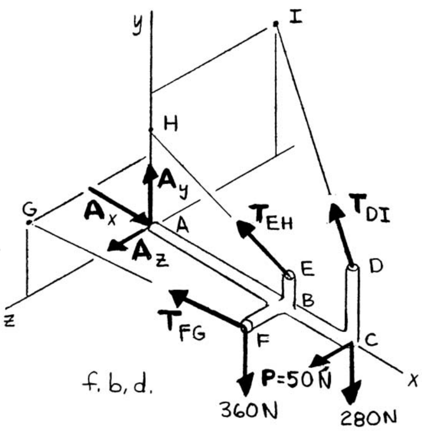
or $$\mathbf{A} = (1094 \text{ N})\mathbf{i} + (98.7 \text{ N})\mathbf{j} - (21.3 \text{ N})\mathbf{k} \blacktriangleleft$$



PROBLEM 4.133

The frame shown is supported by three cables and a ball-and-socket joint at A. For $P = 50$ N, determine the tension in each cable and the reaction at A.

SOLUTION



First note

$$\mathbf{T}_{DI} = \lambda_{DI}T_{DI} = \frac{-(0.65\text{ m})\mathbf{i} + (0.2\text{ m})\mathbf{j} - (0.44\text{ m})\mathbf{k}}{\sqrt{(0.65)^2 + (0.2)^2 + (0.44)^2}\text{ m}}T_{DI}$$

$$= \frac{T_{DI}}{81}(-65\mathbf{i} + 20\mathbf{j} - 44\mathbf{k})$$

$$\mathbf{T}_{EH} = \lambda_{EH}T_{EH} = \frac{-(0.45\text{ m})\mathbf{i} + (0.24\text{ m})\mathbf{j}}{\sqrt{(0.45)^2 + (0.24)^2}\text{ m}}T_{EH}$$

$$= \frac{T_{EH}}{17}(-15\mathbf{i} + 8\mathbf{j})$$

$$\mathbf{T}_{FG} = \lambda_{FG}T_{FG} = \frac{-(0.45\text{ m})\mathbf{i} + (0.2\text{ m})\mathbf{j} + (0.36\text{ m})\mathbf{k}}{\sqrt{(0.45)^2 + (0.2)^2 + (0.36)^2}\text{ m}}T_{FG}$$

$$= \frac{T_{FG}}{61}(-45\mathbf{i} + 20\mathbf{j} + 36\mathbf{k})$$

From f.b.d. of frame

$$\Sigma\mathbf{M}_A = 0: \quad \mathbf{r}_{D/A} \times \mathbf{T}_{DI} + \mathbf{r}_{C/A} \times \left[-(280\text{ N})\mathbf{j} + (50\text{ N})\mathbf{k}\right]$$

$$+ \mathbf{r}_{H/A} \times \mathbf{T}_{EH} + \mathbf{r}_{F/A} \times \mathbf{T}_{FG} + \mathbf{r}_{F/A} \times (-360\text{ N})\mathbf{j}$$

or
$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.65 & 0.2 & 0 \\ -65 & 20 & -44 \end{vmatrix}\left(\frac{T_{DI}}{81}\right) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.65 & 0 & 0 \\ 0 & -280 & 50 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.32 & 0 \\ -15 & 8 & 0 \end{vmatrix}\left(\frac{T_{EH}}{17}\right)$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.45 & 0 & 0.06 \\ -45 & 20 & 36 \end{vmatrix}\left(\frac{T_{FG}}{61}\right) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.45 & 0 & 0.06 \\ 0 & -1 & 0 \end{vmatrix}(360\text{ N}) = 0$$

and
$$(-8.8\mathbf{i} + 28.6\mathbf{j} + 26\mathbf{k})\left(\frac{T_{DI}}{81}\right) + (-32.5\mathbf{j} - 182\mathbf{k}) + (4.8\mathbf{k})\left(\frac{T_{EH}}{17}\right)$$

$$+(-1.2\mathbf{i} - 18.9\mathbf{j} + 9.0\mathbf{k})\left(\frac{T_{FG}}{61}\right) + (0.06\mathbf{i} - 0.45\mathbf{k})(360) = 0$$

PROBLEM 4.133 CONTINUED

From **i**-coefficient $-8.8\left(\frac{T_{DI}}{81}\right) - 1.2\left(\frac{T_{FG}}{61}\right) + 0.06(360) = 0$

$$\therefore \quad 0.108642 T_{DI} + 0.0196721 T_{FG} = 21.6 \qquad (1)$$

From **j**-coefficient $28.6\left(\frac{T_{DI}}{81}\right) - 32.5 - 18.9\left(\frac{T_{FG}}{61}\right) = 0$

$$\therefore \quad 0.35309 T_{DI} - 0.30984 T_{FG} = 32.5 \qquad (2)$$

From **k**-coefficient

$$26\left(\frac{T_{DI}}{81}\right) - 182 + 4.8\left(\frac{T_{EH}}{17}\right) + 9.0\left(\frac{T_{FG}}{61}\right) - 0.45(360) = 0$$

$$\therefore \quad 0.32099 T_{DI} + 0.28235 T_{EH} + 0.147541 T_{FG} = 344 \qquad (3)$$

$-3.25 \times$ Equation (1) $\qquad -0.35309 T_{DI} - 0.063935 T_{FG} = -70.201$

Add Equation (2) $\qquad \underline{0.35309 T_{DI} - 0.30984 T_{FG} = 32.5}$

$$-0.37378 T_{FG} = -37.701$$

$$\therefore \quad T_{FG} = 100.864 \text{ N}$$

or $\qquad T_{FG} = 100.9 \text{ N}$ ◀

Then from Equation (1)

$$0.108642 T_{DI} + 0.0196721(100.864) = 21.6$$

$$\therefore \quad T_{DI} = 180.554 \text{ N}$$

or $\qquad T_{DI} = 180.6 \text{ N}$ ◀

and from Equation (3)

$$0.32099(180.554) + 0.28235 T_{EH} + 0.147541(100.864) = 344$$

$$\therefore \quad T_{EH} = 960.38 \text{ N}$$

or $\qquad T_{EH} = 960 \text{ N}$ ◀

The vector forms of the cable forces are:

$$\mathbf{T}_{DI} = \frac{180.554 \text{ N}}{81}(-65\mathbf{i} + 20\mathbf{j} - 44\mathbf{k})$$

$$= -(144.889 \text{ N})\mathbf{i} + (44.581 \text{ N})\mathbf{j} - (98.079 \text{ N})\mathbf{k}$$

$$\mathbf{T}_{EH} = \frac{960.38 \text{ N}}{17}(-15\mathbf{i} + 8\mathbf{j}) = -(847.39 \text{ N})\mathbf{i} + (451.94 \text{ N})\mathbf{j}$$

$$\mathbf{T}_{FG} = \frac{100.864 \text{ N}}{61}(-45\mathbf{i} + 20\mathbf{j} + 36\mathbf{k})$$

$$= -(74.409 \text{ N})\mathbf{i} + (33.070 \text{ N})\mathbf{j} + (59.527 \text{ N})\mathbf{k}$$

PROBLEM 4.133 CONTINUED

Then from f.b.d. of frame

$$\Sigma F_x = 0: \quad A_x - 144.889 - 847.39 - 74.409 = 0$$

$$\therefore \quad A_x = 1066.69 \text{ N}$$

$$\Sigma F_y = 0: \quad A_y + 44.581 + 451.94 + 33.070 - 360 - 280 = 0$$

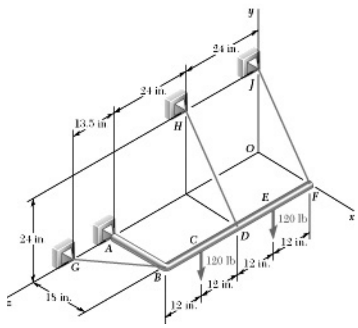
$$\therefore \quad A_y = 110.409 \text{ N}$$

$$\Sigma F_z = 0: \quad A_z - 98.079 + 59.527 + 50 = 0$$

$$\therefore \quad A_z = -11.448 \text{ N}$$

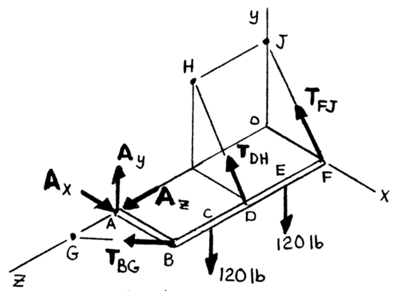
Therefore, $\mathbf{A} = (1067 \text{ N})\mathbf{i} + (110.4 \text{ N})\mathbf{j} - (11.45 \text{ N})\mathbf{k}$ ◀

PROBLEM 4.134



The rigid L-shaped member *ABF* is supported by a ball-and-socket joint at *A* and by three cables. For the loading shown, determine the tension in each cable and the reaction at *A*.

SOLUTION



f.b.d.

First note

$$\mathbf{T}_{BG} = \lambda_{BG} T_{BG} = \frac{-(18 \text{ in.})\mathbf{i} + (13.5 \text{ in.})\mathbf{k}}{\sqrt{(18)^2 + (13.5)^2} \text{ in.}} T_{BG}$$

$$= T_{BG}(-0.8\mathbf{i} + 0.6\mathbf{k})$$

$$\mathbf{T}_{DH} = \lambda_{DH} T_{DH} = \frac{-(18 \text{ in.})\mathbf{i} + (24 \text{ in.})\mathbf{j}}{\sqrt{(18)^2 + (24)^2} \text{ in.}} T_{DH}$$

$$= T_{DH}(-0.6\mathbf{i} + 0.8\mathbf{j})$$

Since $\lambda_{FJ} = \lambda_{DH}$,

$$\mathbf{T}_{FJ} = T_{FJ}(-0.6\mathbf{i} + 0.8\mathbf{j})$$

From f.b.d. of member *ABF*

$$\Sigma M_{A(x\text{-}axis)} = 0: \quad (0.8T_{FJ})(48 \text{ in.}) + (0.8T_{DH})(24 \text{ in.}) - (120 \text{ lb})(36 \text{ in.}) - (120 \text{ lb})(12 \text{ in.}) = 0$$

$$\therefore \quad 3.2T_{FJ} + 1.6T_{DH} = 480 \qquad (1)$$

$$\Sigma M_{A(z\text{-}axis)} = 0: \quad (0.8T_{FJ})(18 \text{ in.}) + (0.8T_{DH})(18 \text{ in.}) - (120 \text{ lb})(18 \text{ in.}) - (120 \text{ lb})(18 \text{ in.}) = 0$$

$$\therefore \quad -3.2T_{FJ} - 3.2T_{DH} = -960 \qquad (2)$$

Equation (1) + Equation (2) $\qquad T_{DH} = 300 \text{ lb}$ ◀

Substituting in Equation (1) $\qquad T_{FJ} = 0$ ◀

$$\Sigma M_{A(y\text{-}axis)} = 0: \quad (0.6T_{FJ})(48 \text{ in.}) + [0.6(300 \text{ lb})](24 \text{ in.}) - (0.6T_{BG})(18 \text{ in.}) = 0$$

$$\therefore \quad T_{BG} = 400 \text{ lb}$$ ◀

PROBLEM 4.134 CONTINUED

$$\Sigma F_x = 0: \quad -0.6T_{FJ} - 0.6T_{DH} - 0.8T_{BG} + A_x = 0$$

$$-0.6(300\text{ lb}) - 0.8(400\text{ lb}) + A_x = 0$$

$$\therefore \quad A_x = 500\text{ lb}$$

$$\Sigma F_y = 0: \quad 0.8T_{FJ} + 0.8T_{DH} - 240\text{ lb} + A_y = 0$$

$$0.8(300\text{ lb}) - 240 + A_y = 0$$

$$\therefore \quad A_y = 0$$

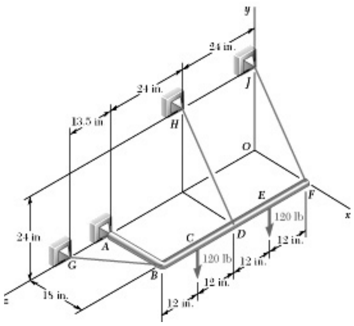
$$\Sigma F_z = 0: \quad 0.6T_{BG} + A_z = 0$$

$$0.6(400\text{ lb}) + A_z = 0$$

$$\therefore \quad A_z = -240\text{ lb}$$

Therefore, $\mathbf{A} = (500\text{ lb})\mathbf{i} - (240\text{ lb})\mathbf{k}$ ◀

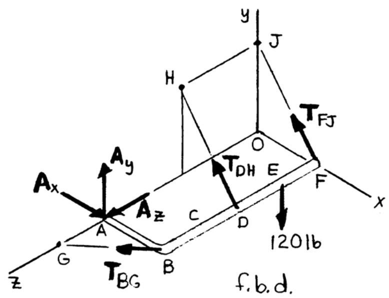
PROBLEM 4.135



Solve Problem 4.134 assuming that the load at C has been removed.

P4.134 The rigid L-shaped member ABF is supported by a ball-and-socket joint at A and by three cables. For the loading shown, determine the tension in each cable and the reaction at A.

SOLUTION



First

$$\mathbf{T}_{BG} = \lambda_{BG}T_{BG} = \frac{-(18\text{ in.})\mathbf{i} + (13.5\text{ in.})\mathbf{k}}{\sqrt{(18)^2 + (13.5)^2}\text{ in.}}T_{BG}$$

$$= T_{BG}(-0.8\mathbf{i} + 0.6\mathbf{k})$$

$$\mathbf{T}_{DH} = \lambda_{DH}T_{DH} = \frac{-(18\text{ in.})\mathbf{i} + (24\text{ in.})\mathbf{j}}{\sqrt{(18)^2 + (24)^2}\text{ in.}}T_{DH}$$

$$= T_{DH}(-0.6\mathbf{i} + 0.8\mathbf{j})$$

Since $\lambda_{FJ} = \lambda_{DH}$

$$\mathbf{T}_{FJ} = T_{FJ}(-0.6\mathbf{i} + 0.8\mathbf{j})$$

From f.b.d. of member ABF

$$\Sigma M_{A(x\text{-}axis)} = 0: \quad (0.8T_{FJ})(48\text{ in.}) + (0.8T_{DH})(24\text{ in.}) - (120\text{ lb})(36\text{ in.}) = 0$$

$$\therefore \quad 3.2T_{FJ} + 1.6T_{DH} = 360 \tag{1}$$

$$\Sigma M_{A(z\text{-}axis)} = 0: \quad (0.8T_{FJ})(18\text{ in.}) + (0.8T_{DH})(18\text{ in.}) - (120\text{ lb})(18\text{ in.}) = 0$$

$$\therefore \quad -3.2T_{FJ} - 3.2T_{DH} = -480 \tag{2}$$

Equation (1) + Equation (2) $T_{DH} = 75.0\text{ lb}$ ◀

Substituting into Equation (2) $T_{FJ} = 75.0\text{ lb}$ ◀

$$\Sigma M_{A(y\text{-}axis)} = 0: \quad (0.6T_{FJ})(48\text{ in.}) + (0.6T_{DH})(24\text{ in.}) - (0.6T_{BG})(18\text{ in.}) = 0$$

or

$$(75.0\text{ lb})(48\text{ in.}) + (75.0\text{ lb})(24\text{ in.}) = T_{BG}(18\text{ in.})$$

$T_{BG} = 300\text{ lb}$ ◀

PROBLEM 4.135 CONTINUED

$$\Sigma F_x = 0: \quad -0.6T_{FJ} - 0.6T_{DH} - 0.8T_{BG} + A_x = 0$$

$$-0.6(75.0 + 75.0) - 0.8(300) + A_x = 0$$

$$\therefore \quad A_x = 330 \text{ lb}$$

$$\Sigma F_y = 0: \quad 0.8T_{FJ} + 0.8T_{DH} - 120 \text{ lb} + A_y = 0$$

$$0.8(150 \text{ lb}) - 120 \text{ lb} + A_y = 0$$

$$\therefore \quad A_y = 0$$

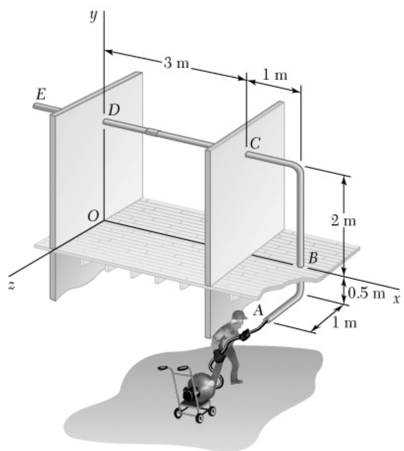
$$\Sigma F_z = 0: \quad 0.6T_{BG} + A_z = 0$$

$$0.6(300 \text{ lb}) + A_z = 0$$

$$\therefore \quad A_z = -180 \text{ lb}$$

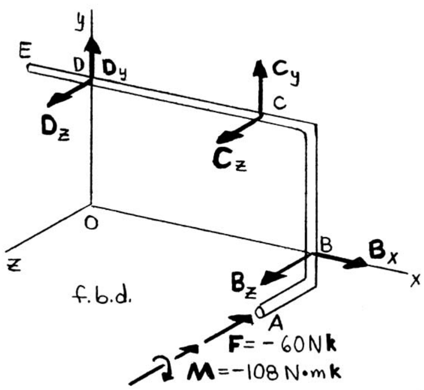
Therefore $\mathbf{A} = (330 \text{ lb})\mathbf{i} - (180 \text{ lb})\mathbf{k}$ ◀

PROBLEM 4.136



In order to clean the clogged drainpipe *AE*, a plumber has disconnected both ends of the pipe and inserted a power snake through the opening at *A*. The cutting head of the snake is connected by a heavy cable to an electric motor which rotates at a constant speed as the plumber forces the cable into the pipe. The forces exerted by the plumber and the motor on the end of the cable can be represented by the wrench $\mathbf{F} = -(60\ \text{N})\mathbf{k}$, $\mathbf{M} = -(108\ \text{N}\cdot\text{m})\mathbf{k}$. Determine the additional reactions at *B*, *C*, and *D* caused by the cleaning operation. Assume that the reaction at each support consists of two force components perpendicular to the pipe.

SOLUTION



From f.b.d. of pipe assembly *ABCD*

$$\Sigma F_x = 0: \quad B_x = 0$$

$$\Sigma M_{D(x\text{-axis})} = 0: \quad (60\ \text{N})(2.5\ \text{m}) - B_z(2\ \text{m}) = 0$$

$$\therefore \quad B_z = 75.0\ \text{N}$$

$$\text{and} \quad \mathbf{B} = (75.0\ \text{N})\mathbf{k} \blacktriangleleft$$

$$\Sigma M_{D(z\text{-axis})} = 0: \quad C_y(3\ \text{m}) - 108\ \text{N}\cdot\text{m} = 0$$

$$\therefore \quad C_y = 36.0\ \text{N}$$

$$\Sigma M_{D(y\text{-axis})} = 0: \quad -C_z(3\ \text{m}) - (75\ \text{N})(4\ \text{m}) + (60\ \text{N})(4\ \text{m}) = 0$$

$$\therefore \quad C_z = -20.0\ \text{N}$$

$$\text{and} \quad \mathbf{C} = (36.0\ \text{N})\mathbf{j} - (20.0\ \text{N})\mathbf{k} \blacktriangleleft$$

$$\Sigma F_y = 0: \quad D_y + 36.0 = 0$$

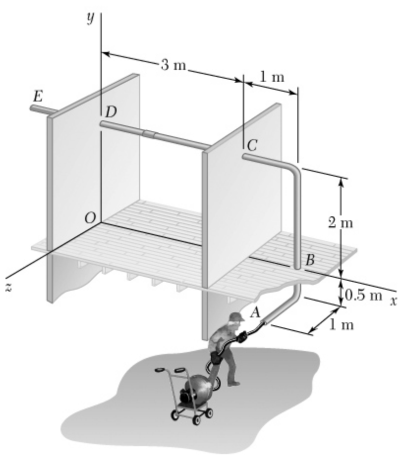
$$\therefore \quad D_y = -36.0\ \text{N}$$

$$\Sigma F_z = 0: \quad D_z - 20.0\ \text{N} + 75.0\ \text{N} - 60\ \text{N} = 0$$

$$\therefore \quad D_z = 5.00\ \text{N}$$

$$\text{and} \quad \mathbf{D} = -(36.0\ \text{N})\mathbf{j} + (5.00\ \text{N})\mathbf{k} \blacktriangleleft$$

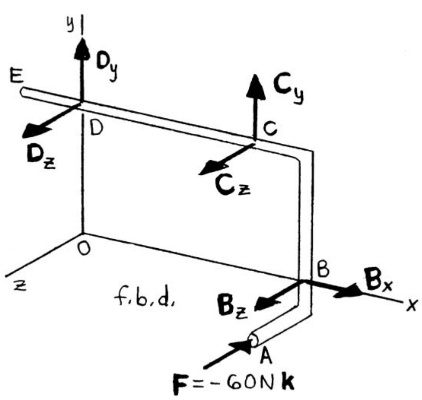
PROBLEM 4.137



Solve Problem 4.136 assuming that the plumber exerts a force $\mathbf{F} = -(60\ \text{N})\mathbf{k}$ and that the motor is turned off $(\mathbf{M} = 0)$.

P4.136 In order to clean the clogged drainpipe *AE*, a plumber has disconnected both ends of the pipe and inserted a power snake through the opening at *A*. The cutting head of the snake is connected by a heavy cable to an electric motor which rotates at a constant speed as the plumber forces the cable into the pipe. The forces exerted by the plumber and the motor on the end of the cable can be represented by the wrench $\mathbf{F} = -(60\ \text{N})\mathbf{k}$, $\mathbf{M} = -(108\ \text{N}\cdot\text{m})\mathbf{k}$. Determine the additional reactions at *B*, *C*, and *D* caused by the cleaning operation. Assume that the reaction at each support consists of two force components perpendicular to the pipe.

SOLUTION



From f.b.d. of pipe assembly *ABCD*

$$\Sigma F_x = 0: \quad B_x = 0$$

$$\Sigma M_{D(x\text{-axis})} = 0: \quad (60\ \text{N})(2.5\ \text{m}) - B_z(2\ \text{m}) = 0$$

$$\therefore \quad B_z = 75.0\ \text{N}$$

$$\text{and} \quad \mathbf{B} = (75.0\ \text{N})\mathbf{k} \blacktriangleleft$$

$$\Sigma M_{D(z\text{-axis})} = 0: \quad C_y(3\ \text{m}) - B_x(2\ \text{m}) = 0$$

$$\therefore \quad C_y = 0$$

$$\Sigma M_{D(y\text{-axis})} = 0: \quad C_z(3\ \text{m}) - (75.0\ \text{N})(4\ \text{m}) + (60\ \text{N})(4\ \text{m}) = 0$$

$$\therefore \quad C_z = -20\ \text{N}$$

$$\text{and} \quad \mathbf{C} = -(20.0\ \text{N})\mathbf{k} \blacktriangleleft$$

$$\Sigma F_y = 0: \quad D_y + C_y = 0$$

$$\therefore \quad D_y = 0$$

$$\Sigma F_z = 0: \quad D_z + B_z + C_z - F = 0$$

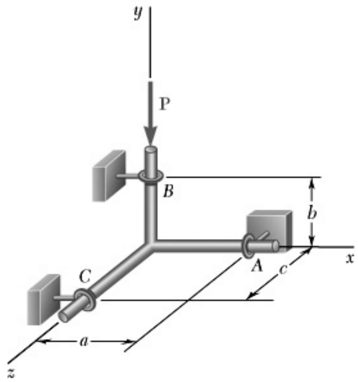
$$D_z + 75\ \text{N} - 20\ \text{N} - 60\ \text{N} = 0$$

$$\therefore \quad D_z = 5.00\ \text{N}$$

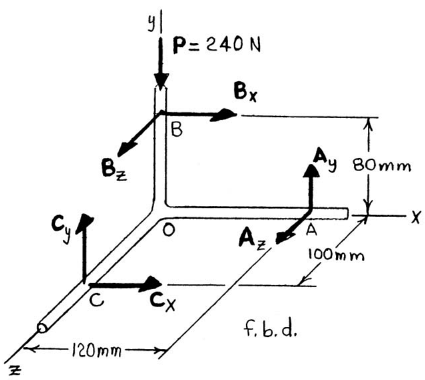
$$\text{and} \quad \mathbf{D} = (5.00\ \text{N})\mathbf{k} \blacktriangleleft$$

PROBLEM 4.138

Three rods are welded together to form a "corner" which is supported by three eyebolts. Neglecting friction, determine the reactions at A, B, and C when $P = 240$ N, $a = 120$ mm, $b = 80$ mm, and $c = 100$ mm.



SOLUTION



From f.b.d. of weldment

$$\Sigma \mathbf{M}_O = 0: \quad \mathbf{r}_{A/O} \times \mathbf{A} + \mathbf{r}_{B/O} \times \mathbf{B} + \mathbf{r}_{C/O} \times \mathbf{C} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 120 & 0 & 0 \\ 0 & A_y & A_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 80 & 0 \\ B_x & 0 & B_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 100 \\ C_x & C_y & 0 \end{vmatrix} = 0$$

$$\left(-120A_z\mathbf{j} + 120A_y\mathbf{k}\right) + \left(80B_z\mathbf{i} - 80B_x\mathbf{k}\right) + \left(-100C_y\mathbf{i} + 100C_x\mathbf{j}\right) = 0$$

From **i**-coefficient $\qquad 80B_z - 100C_y = 0$

or $\qquad B_z = 1.25C_y \qquad (1)$

j-coefficient $\qquad -120A_z + 100C_x = 0$

or $\qquad C_x = 1.2A_z \qquad (2)$

k-coefficient $\qquad 120A_y - 80B_x = 0$

or $\qquad B_x = 1.5A_y \qquad (3)$

$$\Sigma \mathbf{F} = 0: \quad \mathbf{A} + \mathbf{B} + \mathbf{C} - \mathbf{P} = 0$$

or $\qquad \left(B_x + C_x\right)\mathbf{i} + \left(A_y + C_y - 240 \text{ N}\right)\mathbf{j} + \left(A_z + B_z\right)\mathbf{k} = 0$

From **i**-coefficient $\qquad B_x + C_x = 0$

or $\qquad C_x = -B_x \qquad (4)$

j-coefficient $\qquad A_y + C_y - 240 \text{ N} = 0$

or $\qquad A_y + C_y = 240 \text{ N} \qquad (5)$

k-coefficient $\qquad A_z + B_z = 0$

or $\qquad A_z = -B_z \qquad (6)$

PROBLEM 4.138 CONTINUED

Substituting C_x from Equation (4) into Equation (2)

$$-B_z = 1.2A_z \tag{7}$$

Using Equations (1), (6), and (7)

$$C_y = \frac{B_z}{1.25} = \frac{-A_z}{1.25} = \frac{1}{1.25}\left(\frac{B_x}{1.2}\right) = \frac{B_x}{1.5} \tag{8}$$

From Equations (3) and (8)

$$C_y = \frac{1.5A_y}{1.5} \qquad \text{or} \qquad C_y = A_y$$

and substituting into Equation (5)

$$2A_y = 240 \text{ N}$$

$$\therefore \quad A_y = C_y = 120 \text{ N} \tag{9}$$

Using Equation (1) and Equation (9)

$$B_z = 1.25(120 \text{ N}) = 150.0 \text{ N}$$

Using Equation (3) and Equation (9)

$$B_x = 1.5(120 \text{ N}) = 180.0 \text{ N}$$

From Equation (4) $\qquad C_x = -180.0 \text{ N}$

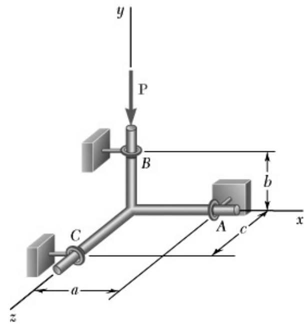
From Equation (6) $\qquad A_z = -150.0 \text{ N}$

Therefore $\qquad \mathbf{A} = (120.0 \text{ N})\mathbf{j} - (150.0 \text{ N})\mathbf{k}$ ◀

$\mathbf{B} = (180.0 \text{ N})\mathbf{i} + (150.0 \text{ N})\mathbf{k}$ ◀

$\mathbf{C} = -(180.0 \text{ N})\mathbf{i} + (120.0 \text{ N})\mathbf{j}$ ◀

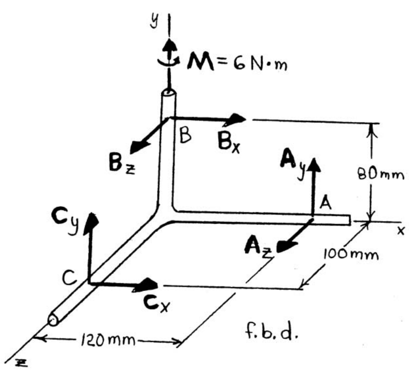
PROBLEM 4.139



Solve Problem 4.138 assuming that the force **P** is removed and is replaced by a couple $\mathbf{M} = +(6\ \text{N}\cdot\text{m})\mathbf{j}$ acting at *B*.

P4.138 Three rods are welded together to form a "corner" which is supported by three eyebolts. Neglecting friction, determine the reactions at *A*, *B*, and *C* when $P = 240$ N, $a = 120$ mm, $b = 80$ mm, and $c = 100$ mm.

SOLUTION



From f.b.d. of weldment

$$\Sigma \mathbf{M}_O = 0: \quad \mathbf{r}_{A/O} \times \mathbf{A} + \mathbf{r}_{B/O} \times \mathbf{B} + \mathbf{r}_{C/O} \times \mathbf{C} + \mathbf{M} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.12 & 0 & 0 \\ 0 & A_y & A_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.08 & 0 \\ B_x & 0 & B_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.1 \\ C_x & C_y & 0 \end{vmatrix} + (6\ \text{N}\cdot\text{m})\mathbf{j} = 0$$

$$\left(-0.12A_z\mathbf{j} + 0.12A_y\mathbf{k}\right) + \left(0.08B_z\mathbf{j} - 0.08B_x\mathbf{k}\right)$$

$$+\left(-0.1C_y\mathbf{i} + 0.1C_x\mathbf{j}\right) + (6\ \text{N}\cdot\text{m})\mathbf{j} = 0$$

From **i**-coefficient $\qquad 0.08B_z - 0.1C_y = 0$

or $\qquad C_y = 0.8B_z \qquad (1)$

j-coefficient $\qquad -0.12A_z + 0.1C_x + 6 = 0$

or $\qquad C_x = 1.2A_z - 60 \qquad (2)$

k-coefficient $\qquad 0.12A_y - 0.08B_x = 0$

or $\qquad B_x = 1.5A_y \qquad (3)$

$$\Sigma\mathbf{F} = 0: \quad \mathbf{A} + \mathbf{B} + \mathbf{C} = 0$$

$$\left(B_x + C_x\right)\mathbf{i} + \left(A_y + C_y\right)\mathbf{j} + \left(A_z + B_z\right)\mathbf{k} = 0$$

From **i**-coefficient $\qquad C_x = -B_x \qquad (4)$

j-coefficient $\qquad C_y = -A_y \qquad (5)$

k-coefficient $\qquad A_z = -B_z \qquad (6)$

Substituting C_x from Equation (4) into Equation (2)

$$A_z = 50 - \left(\frac{B_x}{1.2}\right) \qquad (7)$$

PROBLEM 4.139 CONTINUED

Using Equations (1), (6), and (7)

$$C_y = 0.8B_z = -0.8A_z = \left(\frac{2}{3}\right)B_x - 40 \qquad (8)$$

From Equations (3) and (8)

$$C_y = A_y - 40$$

Substituting into Equation (5) $\qquad 2A_y = 40$

$$\therefore \quad A_y = 20.0 \text{ N}$$

From Equation (5) $\qquad C_y = -20.0 \text{ N}$

Equation (1) $\qquad B_z = -25.0 \text{ N}$

Equation (3) $\qquad B_x = 30.0 \text{ N}$

Equation (4) $\qquad C_x = -30.0 \text{ N}$

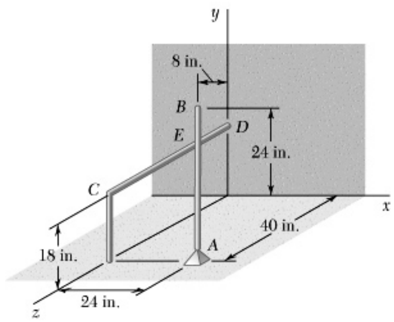
Equation (6) $\qquad A_z = 25.0 \text{ N}$

Therefore $\qquad \mathbf{A} = (20.0 \text{ N})\mathbf{j} + (25.0 \text{ N})\mathbf{k}$ ◀

$\mathbf{B} = (30.0 \text{ N})\mathbf{i} - (25.0 \text{ N})\mathbf{k}$ ◀

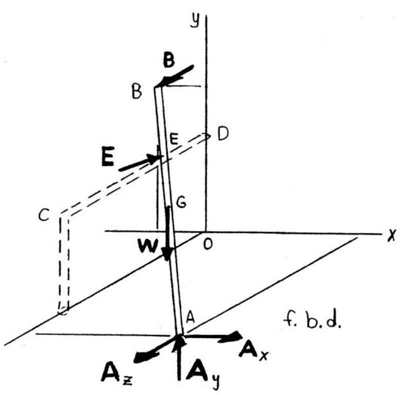
$\mathbf{C} = -(30.0 \text{ N})\mathbf{i} - (20.0 \text{ N})\mathbf{j}$ ◀

PROBLEM 4.140



The uniform 10-lb rod *AB* is supported by a ball-and-socket joint at *A* and leans against both the rod *CD* and the vertical wall. Neglecting the effects of friction, determine (*a*) the force which rod *CD* exerts on *AB*, (*b*) the reactions at *A* and *B*. (*Hint:* The force exerted by *CD* on *AB* must be perpendicular to both rods.)

SOLUTION



(*a*) The force acting at *E* on the f.b.d. of rod *AB* is perpendicular to *AB* and *CD*. Letting $\lambda_E =$ direction cosines for force **E**,

$$\lambda_E = \frac{\mathbf{r}_{B/A} \times \mathbf{k}}{|\mathbf{r}_{B/A} \times \mathbf{k}|}$$

$$= \frac{\left[-(32 \text{ in.})\mathbf{i} + (24 \text{ in.})\mathbf{j} - (40 \text{ in.})\mathbf{k}\right] \times \mathbf{k}}{\sqrt{(32)^2 + (24)^2} \text{ in.}}$$

$$= 0.6\mathbf{i} + 0.8\mathbf{j}$$

Also, $\mathbf{W} = -(10 \text{ lb})\mathbf{j}$

$\mathbf{B} = B\mathbf{k}$

$\mathbf{E} = E(0.6\mathbf{i} + 0.8\mathbf{j})$

From f.b.d. of rod *AB*

$$\Sigma \mathbf{M}_A = 0: \quad \mathbf{r}_{G/A} \times \mathbf{W} + \mathbf{r}_{E/A} \times \mathbf{E} + \mathbf{r}_{B/A} \times \mathbf{B} = 0$$

$$\therefore \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -16 & 12 & -20 \\ 0 & -1 & 0 \end{vmatrix} (10 \text{ lb}) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -24 & 18 & -30 \\ 0.6 & 0.8 & 0 \end{vmatrix} E + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -32 & 24 & -40 \\ 0 & 0 & 1 \end{vmatrix} B = 0$$

$$(-20\mathbf{i} + 16\mathbf{k})(10 \text{ lb}) + (24\mathbf{i} - 18\mathbf{j} - 30\mathbf{k})E + (24\mathbf{i} + 32\mathbf{j})B = 0$$

From **k**-coefficient $\qquad 160 - 30E = 0$

$$\therefore \quad E = 5.3333 \text{ lb}$$

and $\qquad \mathbf{E} = 5.3333 \text{ lb}(0.6\mathbf{i} + 0.8\mathbf{j})$

or $\qquad \mathbf{E} = (3.20 \text{ lb})\mathbf{i} + (4.27 \text{ lb})\mathbf{j}$ ◀

(*b*) From **j**-coefficient $\qquad -18(5.3333 \text{ lb}) + 32B = 0$

$$\therefore \quad B = 3.00 \text{ lb}$$

or $\qquad \mathbf{B} = (3.00 \text{ lb})\mathbf{k}$ ◀

PROBLEM 4.140 CONTINUED

From f.b.d. of rod AB

$$\Sigma \mathbf{F} = 0: \quad \mathbf{A} + \mathbf{W} + \mathbf{E} + \mathbf{B} = 0$$

$$A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k} - (10\text{ lb})\mathbf{j} + \left[(3.20\text{ lb})\mathbf{i} + (4.27\text{ lb})\mathbf{j}\right] + (3.00\text{ lb})\mathbf{k} = 0$$

From **i**-coefficient $\quad A_x + 3.20\text{ lb} = 0$

$$\therefore \quad A_x = -3.20\text{ lb}$$

j-coefficient $\quad A_y - 10\text{ lb} + 4.27\text{ lb} = 0$

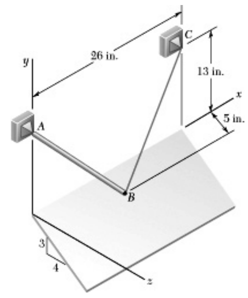
$$\therefore \quad A_y = 5.73\text{ lb}$$

k-coefficient $\quad A_z + 3.00\text{ lb} = 0$

$$\therefore \quad A_z = -3.00\text{ lb}$$

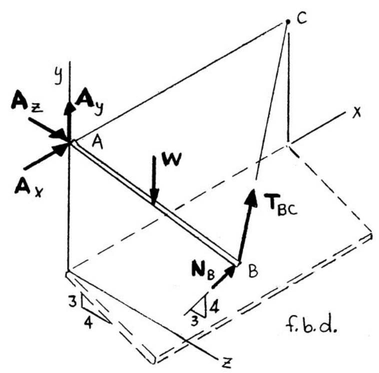
Therefore $\quad \mathbf{A} = -(3.20\text{ lb})\mathbf{i} + (5.73\text{ lb})\mathbf{j} - (3.00\text{ lb})\mathbf{k}$ ◀

PROBLEM 4.141



A 21-in.-long uniform rod AB weighs 6.4 lb and is attached to a ball-and-socket joint at A. The rod rests against an inclined frictionless surface and is held in the position shown by cord BC. Knowing that the cord is 21 in. long, determine (a) the tension in the cord, (b) the reactions at A and B.

SOLUTION



First note

$$\mathbf{W} = -(6.4 \text{ lb})\mathbf{j}$$

$$\mathbf{N}_B = N_B(0.8\mathbf{j} + 0.6\mathbf{k})$$

$$L_{AB} = 21 \text{ in.}$$

$$= \sqrt{(x_B)^2 + (13+3)^2 + (4)^2} = \sqrt{x_B^2 + (16)^2 + (4)^2}$$

$$\therefore\ x_B = 13 \text{ in.}$$

$$\mathbf{T}_{BC} = \lambda_{BC}T_{BC} = \frac{(13 \text{ in.})\mathbf{i} + (16 \text{ in.})\mathbf{j} - (4 \text{ in.})\mathbf{k}}{21 \text{ in.}}T_{BC}$$

$$= \frac{T_{BC}}{21}(13\mathbf{i} + 16\mathbf{j} - 4\mathbf{k})$$

From f.b.d. of rod AB

$$\Sigma\mathbf{M}_A = 0\text{:}\quad \mathbf{r}_{G/A}\times\mathbf{W} + \mathbf{r}_{B/A}\times\mathbf{N}_B + \mathbf{r}_{C/A}\times\mathbf{T}_{BC} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6.5 & -8 & 2 \\ 0 & -6.4 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 13 & -16 & 4 \\ 0 & 0.8 & 0.6 \end{vmatrix} N_B + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 0 \\ 13 & 16 & -4 \end{vmatrix} \frac{26T_{BC}}{21} = 0$$

$$(12.8\mathbf{i} - 41.6\mathbf{k}) + (-12.8\mathbf{i} - 7.8\mathbf{j} + 10.4\mathbf{k})N_B + (4\mathbf{j} + 16\mathbf{k})\frac{26T_{BC}}{21} = 0$$

PROBLEM 4.141 CONTINUED

From **i**-coeff. $12.8 - 12.8N_B = 0 \qquad \therefore \quad N_B = 1.00 \text{ lb}$

or $\mathbf{N}_B = (0.800 \text{ lb})\mathbf{j} + (0.600 \text{ lb})\mathbf{k}$

From **j**-coeff. $-7.8N_B + 4\left(\frac{26}{21}\right)T_{BC} = 0 \qquad \therefore \quad T_{BC} = 1.575 \text{ lb}$

From f.b.d. of rod *AB*

$$\Sigma\mathbf{F} = 0: \quad \mathbf{A} + \mathbf{W} + \mathbf{N}_B + \mathbf{T}_{BC} = 0$$

$$\left(A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}\right) - (6.4 \text{ lb})\mathbf{j} + (0.800 \text{ lb})\mathbf{j} + (0.600 \text{ lb})\mathbf{k} + \left(\frac{1.575}{21}\right)(13\mathbf{i} + 16\mathbf{j} - 4\mathbf{k}) = 0$$

From **i**-coefficient $A_x = -0.975 \text{ lb}$

j-coefficient $A_y = 4.40 \text{ lb}$

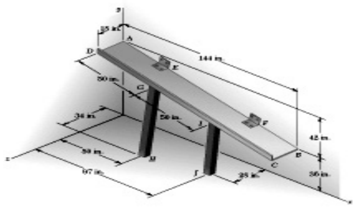
k-coefficient $A_z = -0.3 \text{ lb}$

$\therefore$ (*a*) $T_{BC} = 1.575 \text{ lb}$ ◀

(*b*) $\mathbf{A} = -(0.975 \text{ lb})\mathbf{i} + (4.40 \text{ lb})\mathbf{j} - (0.300 \text{ lb})\mathbf{k}$ ◀

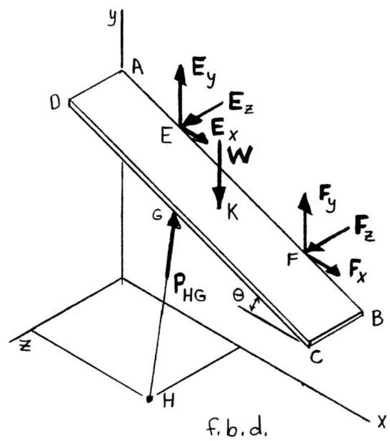
$\mathbf{N}_B = (0.800 \text{ lb})\mathbf{j} + (0.600 \text{ lb})\mathbf{k}$ ◀

PROBLEM 4.142



While being installed, the 56-lb chute *ABCD* is attached to a wall with brackets *E* and *F* and is braced with props *GH* and *IJ*. Assuming that the weight of the chute is uniformly distributed, determine the magnitude of the force exerted on the chute by prop *GH* if prop *IJ* is removed.

SOLUTION



First note

$$\theta = \tan^{-1}\left(\frac{42 \text{ in.}}{144 \text{ in.}}\right) = 16.2602°$$

$$x_G = (50 \text{ in.})\cos 16.2602° = 48 \text{ in.}$$

$$y_G = 78 \text{ in.} - (50 \text{ in.})\sin 16.2602° = 64 \text{ in.}$$

$$\boldsymbol{\lambda}_{BA} = \frac{-(144 \text{ in.})\mathbf{i} + (42 \text{ in.})\mathbf{j}}{\sqrt{(144)^2 + (42)^2} \text{ in.}} = \frac{1}{25}(-24\mathbf{i} + 7\mathbf{j})$$

$$\mathbf{r}_{K/A} = (72 \text{ in.})\mathbf{i} - (21 \text{ in.})\mathbf{j} + (9 \text{ in.})\mathbf{k}$$

$$\mathbf{r}_{G/A} = (48 \text{ in.})\mathbf{i} - (78 \text{ in.} - 64 \text{ in.})\mathbf{j} + (18 \text{ in.})\mathbf{k} = (48 \text{ in.})\mathbf{i} - (14 \text{ in.})\mathbf{j} + (18 \text{ in.})\mathbf{k}$$

$$\mathbf{W} = -(56 \text{ lb})\mathbf{j}$$

$$\mathbf{P}_{HG} = \boldsymbol{\lambda}_{HG} P_{HG}$$

$$= \frac{-(2 \text{ in.})\mathbf{i} + (64 \text{ in.})\mathbf{j} - (16 \text{ in.})\mathbf{k}}{\sqrt{(2)^2 + (64)^2 + (16)^2} \text{ in.}} P_{HG}$$

$$= \frac{P_{HG}}{33}(-\mathbf{i} + 32\mathbf{j} - 8\mathbf{k})$$

PROBLEM 4.142 CONTINUED

From the f.b.d. of the chute

$$\Sigma M_{BA} = 0: \quad \boldsymbol{\lambda}_{BA} \cdot \left(\mathbf{r}_{K/A} \times \mathbf{W}\right) + \boldsymbol{\lambda}_{BA} \cdot \left(\mathbf{r}_{G/A} \times \mathbf{P}_{HG}\right) = 0$$

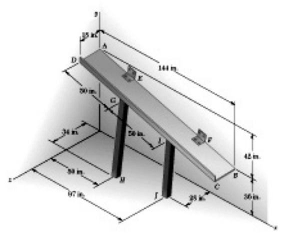
$$\begin{vmatrix} -24 & 7 & 0 \\ 72 & -21 & 9 \\ 0 & -1 & 0 \end{vmatrix} \left(\frac{56}{25}\right) + \begin{vmatrix} -24 & 7 & 0 \\ 48 & -14 & 18 \\ -1 & 32 & -8 \end{vmatrix} \left[\frac{P_{HG}}{33(25)}\right] = 0$$

$$\frac{-216(56)}{25} + \left[-24(-14)(-8) - (-24)(18)(32) + 7(18)(-1) - (7)(48)(-8)\right] \frac{P_{HG}}{33(25)} = 0$$

$$\therefore \quad P_{HG} = 29.141 \text{ lb}$$

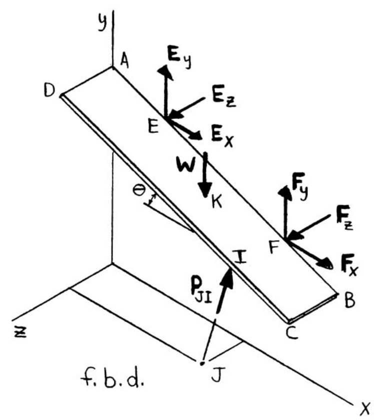
or $P_{HG} = 29.1 \text{ lb}$ ◀

PROBLEM 4.143



While being installed, the 56-lb chute *ABCD* is attached to a wall with brackets *E* and *F* and is braced with props *GH* and *IJ*. Assuming that the weight of the chute is uniformly distributed, determine the magnitude of the force exerted on the chute by prop *IJ* if prop *GH* is removed.

SOLUTION



First note

$$\theta = \tan^{-1}\left(\frac{42\text{ in.}}{144\text{ in.}}\right) = 16.2602°$$

$$x_I = (100\text{ in.})\cos 16.2602° = 96\text{ in.}$$

$$y_I = 78\text{ in.} - (100\text{ in.})\sin 16.2602° = 50\text{ in.}$$

$$\lambda_{BA} = \frac{-(144\text{ in.})\mathbf{i} + (42\text{ in.})\mathbf{j}}{\sqrt{(144)^2 + (42)^2}\text{ in.}} = \frac{1}{25}(-24\mathbf{i} + 7\mathbf{j})$$

$$\mathbf{r}_{K/A} = (72\text{ in.})\mathbf{i} - (21\text{ in.})\mathbf{j} + (9\text{ in.})\mathbf{k}$$

$$\mathbf{r}_{I/A} = (96\text{ in.})\mathbf{i} - (78\text{ in.} - 50\text{ in.})\mathbf{j} + (18\text{ in.})\mathbf{k} = (96\text{ in.})\mathbf{i} - (28\text{ in.})\mathbf{j} + (18\text{ in.})\mathbf{k}$$

$$\mathbf{W} = -(56\text{ lb})\mathbf{j}$$

$$\mathbf{P}_{JI} = \lambda_{JI}P_{JI}$$

$$= \frac{-(1\text{ in.})\mathbf{i} + (50\text{ in.})\mathbf{j} - (10\text{ in.})\mathbf{k}}{\sqrt{(1)^2 + (50)^2 + (10)^2}\text{ in.}}P_{JI}$$

$$= \frac{P_{JI}}{51}(-\mathbf{i} + 50\mathbf{j} - 10\mathbf{k})$$

PROBLEM 4.143 CONTINUED

From the f.b.d. of the chute

$$\Sigma M_{BA} = 0: \quad \boldsymbol{\lambda}_{BA} \cdot \left(\mathbf{r}_{K/A} \times \mathbf{W}\right) + \boldsymbol{\lambda}_{BA} \cdot \left(\mathbf{r}_{I/A} \times \mathbf{P}_{JI}\right) = 0$$

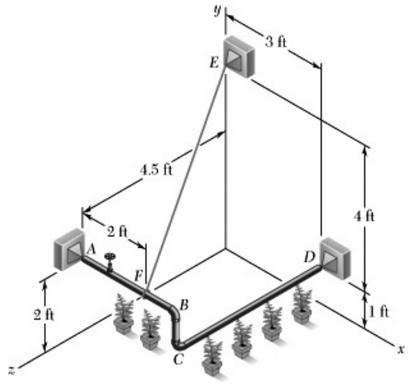
$$\begin{vmatrix} -24 & 7 & 0 \\ 72 & -21 & 9 \\ 0 & -1 & 0 \end{vmatrix}\left(\frac{56}{25}\right) + \begin{vmatrix} -24 & 7 & 0 \\ 96 & -28 & 18 \\ -1 & 50 & -10 \end{vmatrix}\left[\frac{P_{JI}}{51(25)}\right] = 0$$

$$\frac{-216(56)}{25} + \left[-24(-28)(-10) - (-24)(18)(50) + 7(18)(-1) - (7)(96)(-10)\right]\frac{P_{JI}}{51(25)} = 0$$

$$\therefore \quad P_{JI} = 28.728 \text{ lb}$$

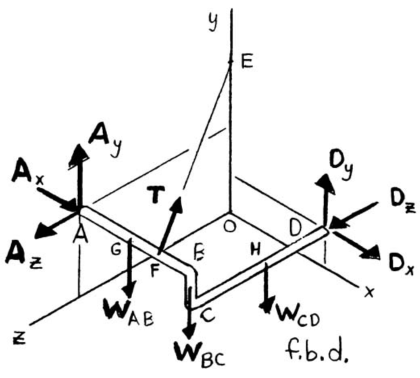
or $P_{JI} = 28.7$ lb ◀

PROBLEM 4.144



To water seedlings, a gardener joins three lengths of pipe, *AB*, *BC*, and *CD*, fitted with spray nozzles and suspends the assembly using hinged supports at *A* and *D* and cable *EF*. Knowing that the pipe weighs 0.85 lb/ft, determine the tension in the cable.

SOLUTION



First note $\mathbf{r}_{G/A} = (1.5\text{ ft})\mathbf{i}$

$$\mathbf{W}_{AB} = -(0.85\text{ lb/ft})(3\text{ ft})\,\mathbf{j} = -(2.55\text{ lb})\,\mathbf{j}$$

$$\mathbf{r}_{F/A} = (2\text{ ft})\mathbf{i}$$

$$\mathbf{T} = \lambda_{FE}T = \frac{-(2\text{ ft})\mathbf{i} + (3\text{ ft})\mathbf{j} - (4.5\text{ ft})\mathbf{k}}{\sqrt{(2)^2 + (3)^2 + (4.5)^2}\text{ ft}}T$$

$$= \left(\frac{T}{\sqrt{33.25}}\right)(-2\mathbf{i} + 3\mathbf{j} - 4.5\mathbf{k})$$

$$\mathbf{r}_{B/A} = (3\text{ ft})\mathbf{i}$$

$$\mathbf{W}_{BC} = -(0.85\text{ lb/ft})(1\text{ ft})\,\mathbf{j} = -(0.85\text{ lb})\,\mathbf{j}$$

$$\mathbf{r}_{H/A} = (3\text{ ft})\mathbf{i} - (2.25\text{ ft})\mathbf{k}$$

$$\mathbf{W}_{CD} = -(0.85\text{ lb/ft})(4.5\text{ ft})\,\mathbf{j} = -(3.825\text{ lb})\,\mathbf{j}$$

$$\lambda_{AD} = \frac{(3\text{ ft})\mathbf{i} - (1\text{ ft})\mathbf{j} - (4.5\text{ ft})\mathbf{k}}{\sqrt{(3)^2 + (1)^2 + (4.5)^2}\text{ ft}} = \frac{1}{5.5}(3\mathbf{i} - \mathbf{j} - 4.5\mathbf{k})$$

PROBLEM 4.144 CONTINUED

From f.b.d. of the pipe assembly

$$\Sigma M_{AD} = 0: \quad \boldsymbol{\lambda}_{AD} \cdot \left(\mathbf{r}_{G/A} \times \mathbf{W}_{AB}\right) + \boldsymbol{\lambda}_{AD} \cdot \left(\mathbf{r}_{F/A} \times \mathbf{T}\right)$$

$$+ \ \boldsymbol{\lambda}_{AD} \cdot \left(\mathbf{r}_{B/A} \times \mathbf{W}_{BC}\right) + \boldsymbol{\lambda}_{AD} \cdot \left(\mathbf{r}_{H/A} \times \mathbf{W}_{CD}\right) = 0$$

$$\therefore \quad \begin{vmatrix} 3 & -1 & -4.5 \\ 1.5 & 0 & 0 \\ 0 & -2.55 & 0 \end{vmatrix} \left(\frac{1}{5.5}\right) + \begin{vmatrix} 3 & -1 & -4.5 \\ 2 & 0 & 0 \\ -2 & 3 & -4.5 \end{vmatrix} \left(\frac{T}{5.5\sqrt{33.25}}\right)$$

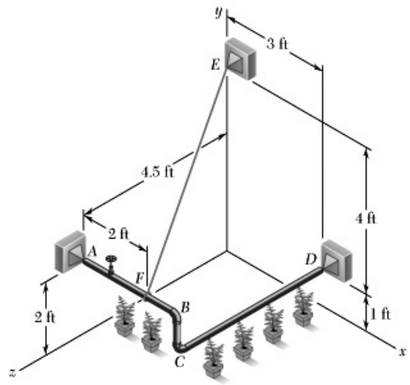
$$+ \begin{vmatrix} 3 & -1 & -4.5 \\ 3 & 0 & 0 \\ 0 & -0.85 & 0 \end{vmatrix} \left(\frac{1}{5.5}\right) + \begin{vmatrix} 3 & -1 & -4.5 \\ 3 & 0 & -2.25 \\ 0 & -3.825 & 0 \end{vmatrix} \left(\frac{1}{5.5}\right) = 0$$

$$(17.2125) + (-36)\left(\frac{T}{\sqrt{33.25}}\right) + (11.475) + (25.819) = 0$$

$$\therefore \quad T = 8.7306 \text{ lb}$$

or $T = 8.73$ lb ◀

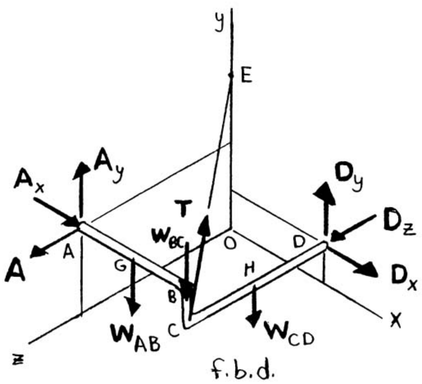
PROBLEM 4.145



Solve Problem 4.144 assuming that cable *EF* is replaced by a cable connecting *E* and *C*.

P4.144 To water seedlings, a gardener joins three lengths of pipe, *AB*, *BC*, and *CD*, fitted with spray nozzles and suspends the assembly using hinged supports at *A* and *D* and cable *EF*. Knowing that the pipe weighs 0.85 lb/ft, determine the tension in the cable.

SOLUTION



First note $\mathbf{r}_{G/A} = (1.5 \text{ ft})\mathbf{i}$

$$\mathbf{W}_{AB} = -(0.85 \text{ lb/ft})(3 \text{ ft})\mathbf{j} = -(2.55 \text{ lb})\mathbf{j}$$

$$\mathbf{r}_{C/A} = (3 \text{ ft})\mathbf{i} - (1 \text{ ft})\mathbf{j}$$

$$\mathbf{T} = \boldsymbol{\lambda}_{CE}T = \frac{-(3 \text{ ft})\mathbf{i} + (4 \text{ ft})\mathbf{j} - (4.5 \text{ ft})\mathbf{k}}{\sqrt{(3)^2 + (4)^2 + (4.5)^2} \text{ ft}}T$$

$$= \left(\frac{T}{\sqrt{45.25}}\right)(-3\mathbf{i} + 4\mathbf{j} - 4.5\mathbf{k})$$

$$\mathbf{r}_{B/A} = (3 \text{ ft})\mathbf{i}$$

$$\mathbf{W}_{BC} = -(0.85 \text{ lb/ft})(1 \text{ ft})\mathbf{j} = -(0.85 \text{ lb})\mathbf{j}$$

$$\mathbf{r}_{H/A} = (3 \text{ ft})\mathbf{i} - (2.25 \text{ ft})\mathbf{k}$$

$$\mathbf{W}_{CD} = -(0.85 \text{ lb/ft})(1 \text{ ft})\mathbf{j} = -(3.825 \text{ lb})\mathbf{j}$$

$$\boldsymbol{\lambda}_{AD} = \frac{(3 \text{ ft})\mathbf{i} - (1 \text{ ft})\mathbf{j} - (4.5 \text{ ft})\mathbf{k}}{\sqrt{(3)^2 + (1)^2 + (4.5)^2} \text{ ft}} = \frac{1}{5.5}(3\mathbf{i} - \mathbf{j} - 4.5\mathbf{k})$$

PROBLEM 4.145 CONTINUED

From f.b.d. of the pipe assembly

$$\Sigma M_{AD} = 0: \quad \boldsymbol{\lambda}_{AD} \cdot \left(\mathbf{r}_{G/A} \times \mathbf{W}_{AB}\right) + \boldsymbol{\lambda}_{AD} \cdot \left(\mathbf{r}_{C/A} \times \mathbf{T}\right)$$

$$+ \ \boldsymbol{\lambda}_{AD} \cdot \left(\mathbf{r}_{B/A} \times \mathbf{W}_{BC}\right) + \boldsymbol{\lambda}_{AD} \cdot \left(\mathbf{r}_{H/A} \times \mathbf{W}_{CD}\right) = 0$$

$$\therefore \quad \begin{vmatrix} 3 & -1 & -4.5 \\ 1.5 & 0 & 0 \\ 0 & -2.55 & 0 \end{vmatrix} \left(\frac{1}{5.5}\right) + \begin{vmatrix} 3 & -1 & -4.5 \\ 3 & -1 & 0 \\ -3 & 4 & -4.5 \end{vmatrix} \left(\frac{T}{5.5\sqrt{45.25}}\right)$$

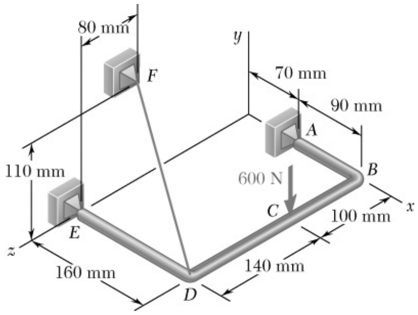
$$+ \begin{vmatrix} 3 & -1 & -4.5 \\ 3 & 0 & 0 \\ 0 & -0.85 & 0 \end{vmatrix} \left(\frac{1}{5.5}\right) + \begin{vmatrix} 3 & -1 & -4.5 \\ 3 & 0 & -2.25 \\ 0 & -3.825 & 0 \end{vmatrix} \left(\frac{1}{5.5}\right) = 0$$

$$(17.2125) + (-40.5)\left(\frac{T}{\sqrt{45.25}}\right) + (11.475) + (25.819) = 0$$

$$\therefore \quad T = 9.0536 \text{ lb}$$

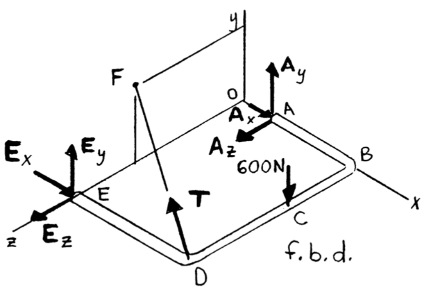
or $T = 9.05$ lb ◀

PROBLEM 4.146



The bent rod *ABDE* is supported by ball-and-socket joints at *A* and *E* and by the cable *DF*. If a 600-N load is applied at *C* as shown, determine the tension in the cable.

SOLUTION



First note

$$\lambda_{AE} = \frac{-(70\text{ mm})\mathbf{i} + (240\text{ mm})\mathbf{k}}{\sqrt{(70)^2 + (240)^2}\text{ mm}} = \frac{1}{25}(-7\mathbf{i} + 24\mathbf{k})$$

$$\mathbf{r}_{C/A} = (90\text{ mm})\mathbf{i} + (100\text{ mm})\mathbf{k}$$

$$\mathbf{F}_C = -(600\text{ N})\mathbf{j}$$

$$\mathbf{r}_{D/A} = (90\text{ mm})\mathbf{i} + (240\text{ mm})\mathbf{k}$$

$$\mathbf{T} = \lambda_{DF}T = \frac{-(160\text{ mm})\mathbf{i} + (110\text{ mm})\mathbf{j} - (80\text{ mm})\mathbf{k}}{\sqrt{(160)^2 + (110)^2 + (80)^2}\text{ mm}}T$$

$$= \frac{T}{21}(-16\,\mathbf{i} + 11\mathbf{j} - 8\mathbf{k})$$

From the f.b.d. of the bend rod

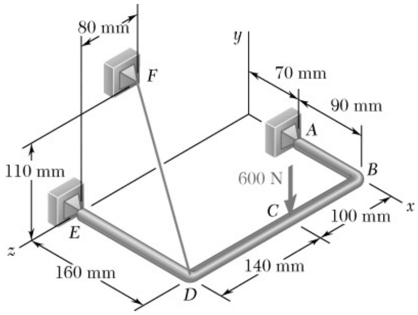
$$\Sigma M_{AE} = 0: \quad \lambda_{AE} \cdot (\mathbf{r}_{C/A} \times \mathbf{F}_C) + \lambda_{AE} \cdot (\mathbf{r}_{D/A} \times \mathbf{T}) = 0$$

$$\therefore \quad \begin{vmatrix} -7 & 0 & 24 \\ 90 & 0 & 100 \\ 0 & -1 & 0 \end{vmatrix}\left(\frac{600}{25}\right) + \begin{vmatrix} -7 & 0 & 24 \\ 90 & 0 & 240 \\ -16 & 11 & -8 \end{vmatrix}\left[\frac{T}{25(21)}\right] = 0$$

$$(-700 - 2160)\left(\frac{600}{25}\right) + (18\,480 + 23\,760)\left[\frac{T}{25(21)}\right] = 0$$

$$\therefore \quad T = 853.13\text{ N}$$

or $T = 853\text{ N}$ ◀

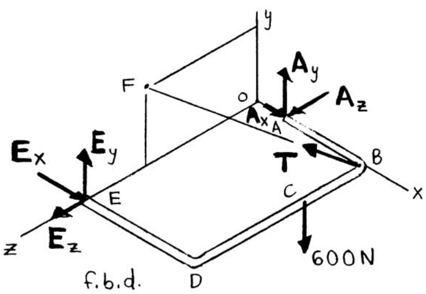


PROBLEM 4.147

Solve Problem 4.146 assuming that cable *DF* is replaced by a cable connecting *B* and *F*.

P4.146 The bent rod *ABDE* is supported by ball-and-socket joints at *A* and *E* and by the cable *DF*. If a 600-N load is applied at *C* as shown, determine the tension in the cable.

SOLUTION



First note

$$\boldsymbol{\lambda}_{AE} = \frac{-(70\text{ mm})\mathbf{i} + (240\text{ mm})\mathbf{k}}{\sqrt{(70)^2 + (240)^2}\text{ mm}} = \frac{1}{25}(-7\mathbf{i} + 24\mathbf{k})$$

$$\mathbf{r}_{C/A} = (90\text{ mm})\mathbf{i} + (100\text{ mm})\mathbf{k}$$

$$\mathbf{F}_C = -(600\text{ N})\mathbf{j}$$

$$\mathbf{r}_{B/A} = (90\text{ mm})\mathbf{i}$$

$$\mathbf{T} = \boldsymbol{\lambda}_{BF}T = \frac{-(160\text{ mm})\mathbf{i} + (110\text{ mm})\mathbf{j} + (160\text{ mm})\mathbf{k}}{\sqrt{(160)^2 + (110)^2 + (160)^2}\text{ mm}}T$$

$$= \frac{1}{251.59}(-160\,\mathbf{i} + 110\mathbf{j} + 160\mathbf{k})$$

From the f.b.d. of the bend rod

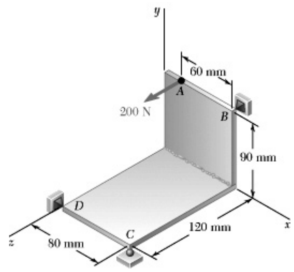
$$\Sigma M_{AE} = 0: \quad \boldsymbol{\lambda}_{AE} \cdot (\mathbf{r}_{C/A} \times \mathbf{F}_C) + \boldsymbol{\lambda}_{AE} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}) = 0$$

$$\therefore \begin{vmatrix} -7 & 0 & 24 \\ 90 & 0 & 100 \\ 0 & -1 & 0 \end{vmatrix}\left(\frac{600}{25}\right) + \begin{vmatrix} -7 & 0 & 24 \\ 90 & 0 & 0 \\ -160 & 110 & 160 \end{vmatrix}\left[\frac{T}{25(251.59)}\right] = 0$$

$$(-700 - 2160)\left(\frac{600}{25}\right) + (237\ 600)\left[\frac{T}{25(251.59)}\right] = 0$$

$$\therefore \quad T = 1817.04\text{ N}$$

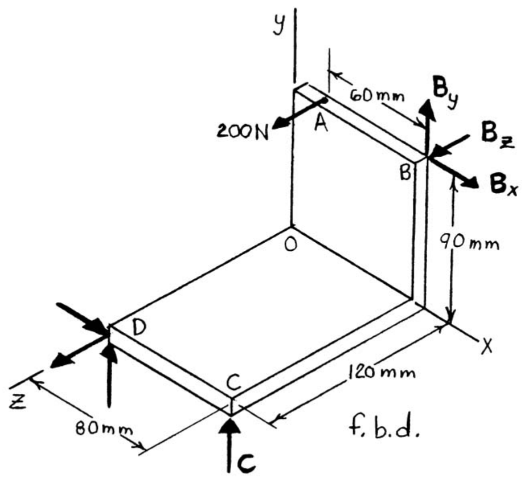
or $T = 1817\text{ N}$ ◀



PROBLEM 4.148

Two rectangular plates are welded together to form the assembly shown. The assembly is supported by ball-and-socket joints at B and D and by a ball on a horizontal surface at C. For the loading shown, determine the reaction at C.

SOLUTION



First note

$$\lambda_{BD} = \frac{-(80\text{ mm})\mathbf{i} - (90\text{ mm})\mathbf{j} + (120\text{ mm})\mathbf{k}}{\sqrt{(80)^2 + (90)^2 + (120)^2}\text{ mm}}$$

$$= \frac{1}{17}(-8\mathbf{i} - 9\mathbf{j} + 12\mathbf{k})$$

$$\mathbf{r}_{A/B} = -(60\text{ mm})\mathbf{i}$$

$$\mathbf{P} = (200\text{ N})\mathbf{k}$$

$$\mathbf{r}_{C/D} = (80\text{ mm})\mathbf{i}$$

$$\mathbf{C} = (C)\mathbf{j}$$

From the f.b.d. of the plates

$$\Sigma M_{BD} = 0: \quad \lambda_{BD} \cdot (\mathbf{r}_{A/B} \times \mathbf{P}) + \lambda_{BD} \cdot (\mathbf{r}_{C/D} \times \mathbf{C}) = 0$$

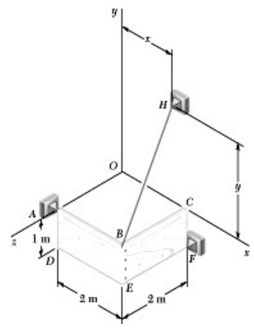
$$\therefore \quad \begin{vmatrix} -8 & -9 & 12 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} \left[\frac{60(200)}{17}\right] + \begin{vmatrix} -8 & -9 & 12 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} \left[\frac{C(80)}{17}\right] = 0$$

$$(-9)(60)(200) + (12)(80)C = 0$$

$$\therefore \quad C = 112.5\text{ N}$$

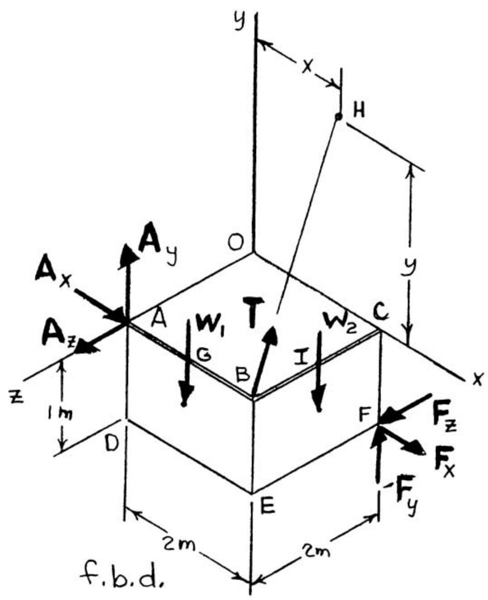
or $\mathbf{C} = (112.5\text{ N})\mathbf{j}$ ◀

PROBLEM 4.149



Two $1 \times 2\text{-m}$ plywood panels, each of mass 15 kg, are nailed together as shown. The panels are supported by ball-and-socket joints at *A* and *F* and by the wire *BH*. Determine (*a*) the location of *H* in the *xy* plane if the tension in the wire is to be minimum, (*b*) the corresponding minimum tension.

SOLUTION



Let $$\mathbf{W}_1 = \mathbf{W}_2 = -(mg)\,\mathbf{j} = -(15\text{ kg})\left(9.81\text{ m/s}^2\right)\mathbf{j}$$

$$= -(147.15\text{ N})\,\mathbf{j}$$

From the f.b.d. of the panels

$$\Sigma M_{AF} = 0\text{:}\quad \boldsymbol{\lambda}_{AF}\cdot\left(\mathbf{r}_{G/A}\times\mathbf{W}_1\right) + \boldsymbol{\lambda}_{AF}\cdot\left(\mathbf{r}_{B/A}\times\mathbf{T}\right) + \boldsymbol{\lambda}_{AF}\cdot\left(\mathbf{r}_{T/A}\times\mathbf{W}_2\right) = 0$$

where $$\boldsymbol{\lambda}_{AF} = \frac{(2\text{ m})\mathbf{i} - (1\text{ m})\mathbf{j} - (2\text{ m})\mathbf{k}}{\sqrt{(2)^2 + (1)^2 + (2)^2}\text{ m}} = \frac{1}{3}(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$$

$$\mathbf{r}_{G/A} = (1\text{ m})\mathbf{i}$$

$$\mathbf{r}_{B/A} = (2\text{ m})\mathbf{i}$$

$$\mathbf{r}_{I/A} = (2\text{ m})\mathbf{i} - (1\text{ m})\mathbf{k}$$

PROBLEM 4.149 CONTINUED

$$\lambda_{BH} = \frac{(x-2)\mathbf{i} + (y)\mathbf{j} - (2)\mathbf{k}}{\sqrt{(x-2)^2 + y^2 + (2)^2}}$$

$$\mathbf{T} = \lambda_{BH}T = \frac{(x-2)\mathbf{i} + (y)\mathbf{j} - (2)\mathbf{k}}{\sqrt{(x-2)^2 + y^2 + (2)^2}}$$

$$\therefore \quad \begin{vmatrix} 2 & -1 & -2 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix}\left(\frac{147.15}{3}\right) + \begin{vmatrix} 2 & -1 & -2 \\ 2 & 0 & 0 \\ x-2 & y & -2 \end{vmatrix}\left(\frac{T}{3\sqrt{(x-2)^2 + y^2 + (2)^2}}\right) + \begin{vmatrix} 2 & -1 & -2 \\ 2 & 0 & -1 \\ 0 & -1 & 0 \end{vmatrix}\left(\frac{147.15}{3}\right) = 0$$

$$\frac{2(147.15)}{3} + (-4 - 4y)\frac{T}{3\sqrt{(x-2)^2 + y^2 + (2)^2}} + (-2 + 4)\frac{147.15}{3} = 0$$

or $$T = \frac{147.15}{1+y}\sqrt{(x-2)^2 + y^2 + (2)^2}$$

For $x - 2$ m, $T = T_{\text{min}}$ $$\therefore \quad T_{\text{min}} = \frac{147.15}{(1+y)}\left(y^2 + 4\right)^{\frac{1}{2}}$$

The y-value for T_{min} is found from $$\left(\frac{dT}{dy}\right) = 0: \quad \frac{(1+y)\frac{1}{2}\left(y^2+4\right)^{-\frac{1}{2}}(2y) - \left(y^2+4\right)^{\frac{1}{2}}(1)}{(1+y)^2} = 0$$

Setting the numerator equal to zero, $$(1+y)y = y^2 + 4$$

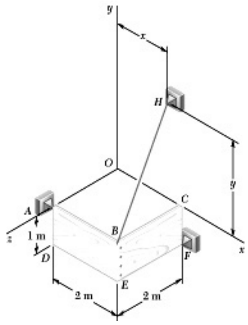
$$y = 4 \text{ m}$$

Then $$T\,\text{min} = \frac{147.15}{(1+4)}\sqrt{(2-2)^2 + (4)^2 + (2)^2} = 131.615 \text{ N}$$

$\therefore$ (*a*) $x = 2.00$ m, $y = 4.00$ m ◀

(*b*) $T_{\text{min}} = 131.6$ N ◀

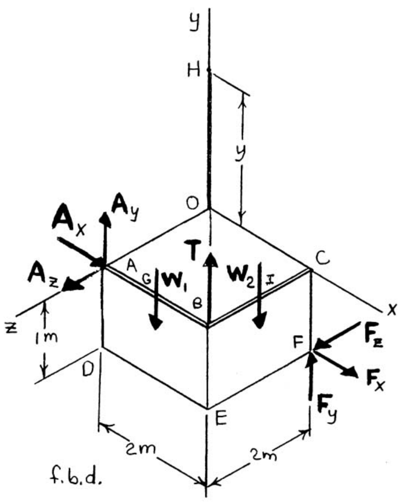
PROBLEM 4.150



Solve Problem 4.149 subject to the restriction that H must lie on the y axis.

P4.149 Two 1×2-m plywood panels, each of mass 15 kg, are nailed together as shown. The panels are supported by ball-and-socket joints at A and F and by the wire BH. Determine (a) the location of H in the xy plane if the tension in the wire is to be minimum, (b) the corresponding minimum tension.

SOLUTION



Let $\qquad \mathbf{W}_1 = \mathbf{W}_2 = -(mg)\mathbf{j} = -(15\text{ kg})(9.81\text{ m/s}^2)\mathbf{j} = -(147.15\text{ N})\mathbf{j}$

From the f.b.d. of the panels

$$\Sigma M_{AF} = 0: \quad \lambda_{AF} \cdot (\mathbf{r}_{G/A} \times \mathbf{W}_1) + \lambda_{AF} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}) + \lambda_{AF} \cdot (\mathbf{r}_{I/A} \times \mathbf{W}_2) = 0$$

where $$\lambda_{AF} = \frac{(2\text{ m})\mathbf{i} - (1\text{ m})\mathbf{j} - (2\text{ m})\mathbf{k}}{\sqrt{(2)^2 + (1)^2 + (2)^2}\text{ m}} = \frac{1}{3}(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$$

$$\mathbf{r}_{G/A} = (1\text{ m})\mathbf{i}$$

$$\mathbf{r}_{B/A} = (2\text{ m})\mathbf{i}$$

$$\mathbf{r}_{I/A} = (2\text{ m})\mathbf{i} - (1\text{ m})\mathbf{k}$$

$$\mathbf{T} = \lambda_{BH}T = \frac{-(2\text{ m})\mathbf{i} + (y)\mathbf{j} - (2\text{ m})\mathbf{k}}{\sqrt{(2)^2 + (y)^2 + (2)^2}\text{ m}}T$$

$$= \frac{T}{\sqrt{8 + y^2}}(-2\mathbf{i} + y\mathbf{j} - 2\mathbf{k})$$

PROBLEM 4.150 CONTINUED

$$\therefore \quad \begin{vmatrix} 2 & -1 & -2 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{vmatrix}\left(\frac{147.15}{3}\right) + \begin{vmatrix} 2 & -1 & -2 \\ 2 & 0 & 0 \\ -2 & y & -2 \end{vmatrix}\left(\frac{T}{3\sqrt{8 + y^2}}\right) + \begin{vmatrix} 2 & -1 & -2 \\ 2 & 0 & -1 \\ 0 & -1 & 0 \end{vmatrix}\left(\frac{147.15}{3}\right) = 0$$

$$2(147.15) + (-4 - 4y)\left(T\sqrt{8 + y^2}\right) + (2)147.15 = 0$$

$$\therefore \quad T = \frac{(147.15)\sqrt{8 + y^2}}{(1 + y)}$$

For $T_{\min}$, $\quad \left(\frac{dT}{dy}\right) = 0 \qquad \therefore \quad \frac{(1 + y)\frac{1}{2}\left(8 + y^2\right)^{-\frac{1}{2}}(2y) - \left(8 + y^2\right)^{\frac{1}{2}}(1)}{(1 + y)^2} = 0$

Setting the numerator equal to zero,

$$(1 + y)y = 8 + y^2$$

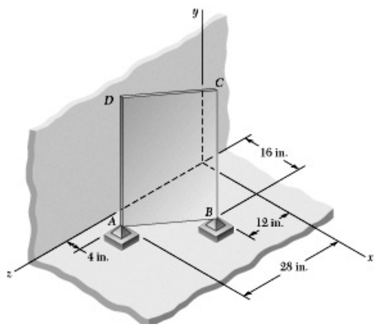
$$\therefore \quad y = 8.00 \text{ m}$$

and $\quad T_{\min} = \frac{(147.15)\sqrt{8 + (8)^2}}{(1 + 8)} = 138.734 \text{ N}$

$\therefore$ (*a*) $\quad x = 0, \; y = 8.00 \text{ m}$ ◀

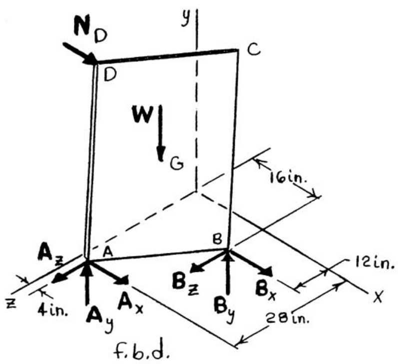
(*b*) $\quad T_{\min} = 138.7 \text{ N}$ ◀

PROBLEM 4.151



A uniform 20 × 30-in. steel plate *ABCD* weighs 85 lb and is attached to ball-and-socket joints at *A* and *B*. Knowing that the plate leans against a frictionless vertical wall at *D*, determine (*a*) the location of *D*, (*b*) the reaction at *D*.

SOLUTION



(*a*) Since $\mathbf{r}_{D/A}$ is perpendicular to $\mathbf{r}_{B/A}$,

$$\mathbf{r}_{D/A} \cdot \mathbf{r}_{B/A} = 0$$

where coordinates of *D* are $(0, y, z)$, and

$$\mathbf{r}_{D/A} = -(4\text{ in.})\mathbf{i} + (y)\mathbf{j} + (z - 28\text{ in.})\mathbf{k}$$

$$\mathbf{r}_{B/A} = (12\text{ in.})\mathbf{i} - (16\text{ in.})\mathbf{k}$$

$$\therefore\ \mathbf{r}_{D/A} \cdot \mathbf{r}_{B/A} = -48 - 16z + 448 = 0$$

or $\qquad z = 25\text{ in.}$

Since $\qquad L_{AD} = 30\text{ in.}$

$$30 = \sqrt{(4)^2 + (y)^2 + (25 - 28)^2}$$

$$900 = 16 + y^2 + 9$$

or $\qquad y = \sqrt{875}\text{ in.} = 29.580\text{ in.}$

$\therefore$ Coordinates of *D*: $\qquad x = 0,\ y = 29.6\text{ in.},\ z = 25.0\text{ in.}$ ◀

(*b*) From f.b.d. of steel plate *ABCD*

$$\Sigma M_{AB} = 0: \quad \boldsymbol{\lambda}_{AB} \cdot (\mathbf{r}_{D/A} \times \mathbf{N}_D) + \boldsymbol{\lambda}_{AB} \cdot (\mathbf{r}_{G/B} \times \mathbf{W}) = 0$$

where $\qquad \boldsymbol{\lambda}_{AB} = \dfrac{(12\text{ in.})\mathbf{i} - (16\text{ in.})\mathbf{k}}{\sqrt{(12)^2 + (16)^2}\text{ in.}} = \dfrac{1}{5}(3\mathbf{i} - 4\mathbf{k})$

$$\mathbf{r}_{D/A} = -(4\text{ in.})\mathbf{i} + (29.580\text{ in.})\mathbf{j} - (3\text{ in.})\mathbf{k}$$

$$\mathbf{N}_D = N_D\mathbf{i}$$

PROBLEM 4.151 CONTINUED

$$\mathbf{r}_{G/B} = \frac{1}{2}\mathbf{r}_{D/B} = \frac{1}{2}\left[-(16\text{ in.})\mathbf{i} + (29.580\text{ in.})\mathbf{j} + (25\text{ in.} - 12\text{ in.})\mathbf{k}\right]$$

$$\mathbf{W} = -(85\text{ lb})\mathbf{j}$$

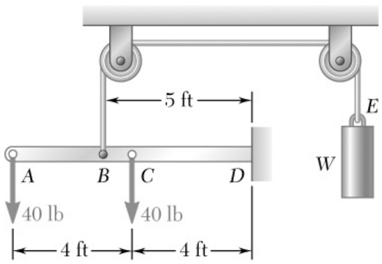
$$\therefore \begin{vmatrix} 3 & 0 & -4 \\ -4 & 29.580 & -3 \\ 1 & 0 & 0 \end{vmatrix}\left(\frac{N_D}{5}\right) + \begin{vmatrix} 3 & 0 & -4 \\ -16 & 29.580 & 13 \\ 0 & -1 & 0 \end{vmatrix}\left[\frac{85}{2(5)}\right] = 0$$

$$118.32N_D + (39 - 64)42.5 = 0$$

$$\therefore N_D = 8.9799\text{ lb}$$

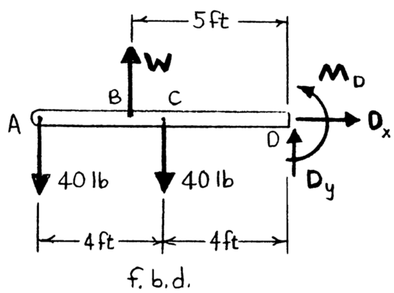
$$\text{or} \quad \mathbf{N}_D = (8.98\text{ lb})\mathbf{i} \blacktriangleleft$$

PROBLEM 4.152



Beam AD carries the two 40-lb loads shown. The beam is held by a fixed support at D and by the cable BE which is attached to the counter-weight W. Determine the reaction at D when (a) $W = 100$ lb, (b) $W = 90$ lb.

SOLUTION



(a) $W = 100$ lb

From f.b.d. of beam AD

$$\xrightarrow{+} \Sigma F_x = 0: \quad D_x = 0$$

$$+\!\uparrow \Sigma F_y = 0: \quad D_y - 40 \text{ lb} - 40 \text{ lb} + 100 \text{ lb} = 0$$

$$\therefore \; D_y = -20.0 \text{ lb}$$

$$\text{or} \quad \mathbf{D} = 20.0 \text{ lb} \downarrow \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_D = 0: \quad M_D - (100 \text{ lb})(5 \text{ ft}) + (40 \text{ lb})(8 \text{ ft}) + (40 \text{ lb})(4 \text{ ft}) = 0$$

$$\therefore \; M_D = 20.0 \text{ lb}\cdot\text{ft}$$

$$\text{or} \quad \mathbf{M}_D = 20.0 \text{ lb}\cdot\text{ft} \circlearrowleft \blacktriangleleft$$

(b) $W = 90$ lb

From f.b.d. of beam AD

$$\xrightarrow{+} \Sigma F_x = 0: \quad D_x = 0$$

$$+\!\uparrow \Sigma F_y = 0: \quad D_y + 90 \text{ lb} - 40 \text{ lb} - 40 \text{ lb} = 0$$

$$\therefore \; D_y = -10.00 \text{ lb}$$

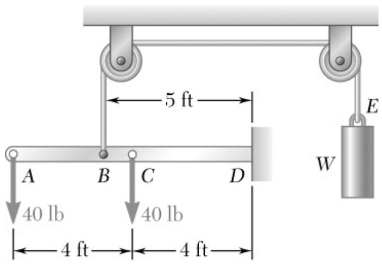
$$\text{or} \quad \mathbf{D} = 10.00 \text{ lb} \downarrow \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_D = 0: \quad M_D - (90 \text{ lb})(5 \text{ ft}) + (40 \text{ lb})(8 \text{ ft}) + (40 \text{ lb})(4 \text{ ft}) = 0$$

$$\therefore \; M_D = -30.0 \text{ lb}\cdot\text{ft}$$

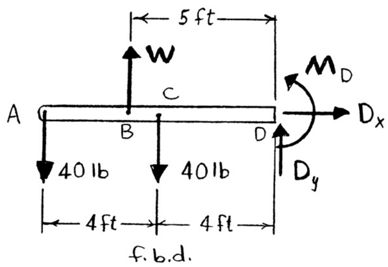
$$\text{or} \quad \mathbf{M}_D = 30.0 \text{ lb}\cdot\text{ft} \circlearrowright \blacktriangleleft$$

PROBLEM 4.153



For the beam and loading shown, determine the range of values of W for which the magnitude of the couple at D does not exceed $40\text{ lb}\cdot\text{ft}$.

SOLUTION



For W_{min}, $\quad M_D = -40\text{ lb}\cdot\text{ft}$

From f.b.d. of beam AD

$$+\circlearrowleft\ \Sigma M_D = 0: \quad (40\text{ lb})(8\text{ ft}) - W_{\text{min}}(5\text{ ft}) + (40\text{ lb})(4\text{ ft}) - 40\text{ lb}\cdot\text{ft} = 0$$

$$\therefore\ W_{\text{min}} = 88.0\text{ lb}$$

For W_{max}, $\quad M_D = 40\text{ lb}\cdot\text{ft}$

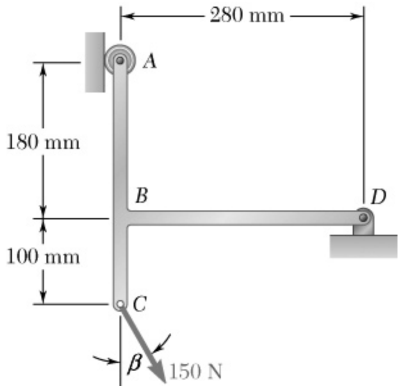
From f.b.d. of beam AD

$$+\circlearrowleft\ \Sigma M_D = 0: \quad (40\text{ lb})(8\text{ ft}) - W_{\text{max}}(5\text{ ft}) + (40\text{ lb})(4\text{ ft}) + 40\text{ lb}\cdot\text{ft} = 0$$

$$\therefore\ W_{\text{max}} = 104.0\text{ lb}$$

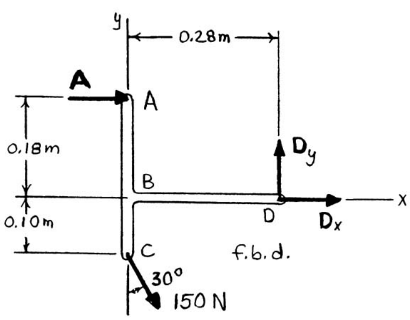
or $88.0\text{ lb} \le W \le 104.0\text{ lb}$ ◀

PROBLEM 4.154



Determine the reactions at A and D when $\beta = 30°$.

SOLUTION



From f.b.d. of frame $ABCD$

$$+\circlearrowleft \Sigma M_D = 0: \quad -A(0.18 \text{ m}) + [(150 \text{ N})\sin 30°](0.10 \text{ m})$$

$$+[(150 \text{ N})\cos 30°](0.28 \text{ m}) = 0$$

$$\therefore \; A = 243.74 \text{ N}$$

or $\mathbf{A} = 244 \text{ N} \longrightarrow$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad (243.74 \text{ N}) + (150 \text{ N})\sin 30° + D_x = 0$$

$$\therefore \; D_x = -318.74 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad D_y - (150 \text{ N})\cos 30° = 0$$

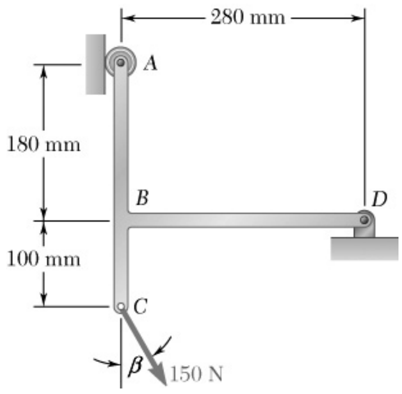
$$\therefore \; D_y = 129.904 \text{ N}$$

Then $$D = \sqrt{(D_x)^2 + D_x^2} = \sqrt{(318.74)^2 + (129.904)^2} = 344.19 \text{ N}$$

and $$\theta = \tan^{-1}\left(\frac{D_y}{D_x}\right) = \tan^{-1}\left(\frac{129.904}{-318.74}\right) = -22.174°$$

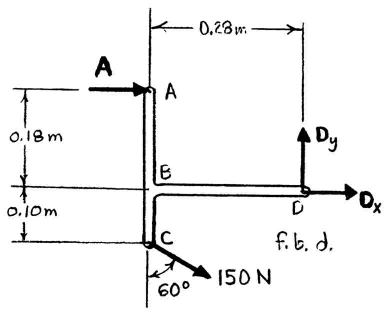
or $\mathbf{D} = 344 \text{ N} \searrow 22.2°$ ◀

PROBLEM 4.155



Determine the reactions at A and D when $\beta = 60°$.

SOLUTION



From f.b.d. of frame $ABCD$

$$+\circlearrowleft \Sigma M_D = 0: \quad -A(0.18\text{ m}) + \left[(150\text{ N})\sin 60°\right](0.10\text{ m})$$

$$+\left[(150\text{ N})\cos 60°\right](0.28\text{ m}) = 0$$

$$\therefore\ A = 188.835\text{ N}$$

or $\mathbf{A} = 188.8\text{ N} \longrightarrow$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad (188.835\text{ N}) + (150\text{ N})\sin 60° + D_x = 0$$

$$\therefore\ D_x = -318.74\text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad D_y - (150\text{ N})\cos 60° = 0$$

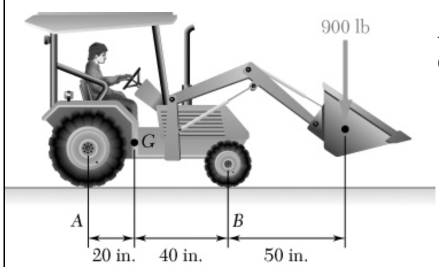
$$\therefore\ D_y = 75.0\text{ N}$$

Then $\quad D = \sqrt{(D_x)^2 + (D_y)^2} = \sqrt{(318.74)^2 + (75.0)^2} = 327.44\text{ N}$

and $\quad \theta = \tan^{-1}\left(\dfrac{D_y}{D_x}\right) = \tan^{-1}\left(\dfrac{75.0}{-318.74}\right) = -13.2409°$

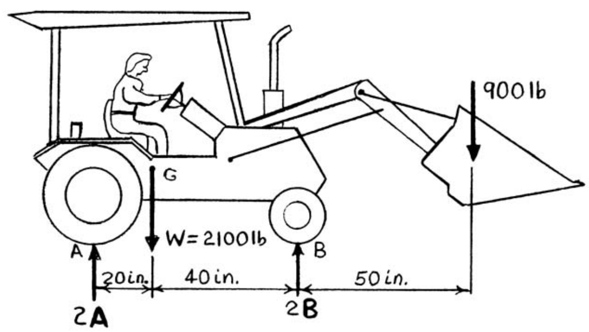
or $\mathbf{D} = 327\text{ N} \measuredangle\ 13.24°$ ◀

PROBLEM 4.156



A 2100-lb tractor is used to lift 900 lb of gravel. Determine the reaction at each of the two (*a*) rear wheels *A*, (*b*) front wheels *B*.

SOLUTION



(*a*) From f.b.d. of tractor

$$+\circlearrowleft \Sigma M_B = 0: \quad (2100\text{ lb})(40\text{ in.}) - (2A)(60\text{ in.}) - (900\text{ lb})(50\text{ in.}) = 0$$

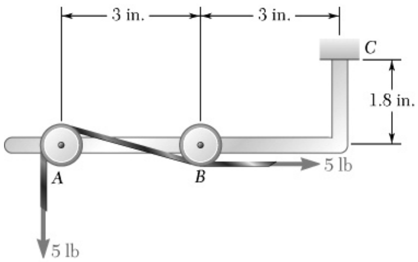
$\therefore\ A = 325\text{ lb}$ or $\mathbf{A} = 325\text{ lb}\uparrow$ ◀

(*b*) From f.b.d. of tractor

$$+\circlearrowleft \Sigma M_A = 0: \quad (2B)(60\text{ in.}) - (2100\text{ lb})(20\text{ in.}) - (900\text{ lb})(110\text{ in.}) = 0$$

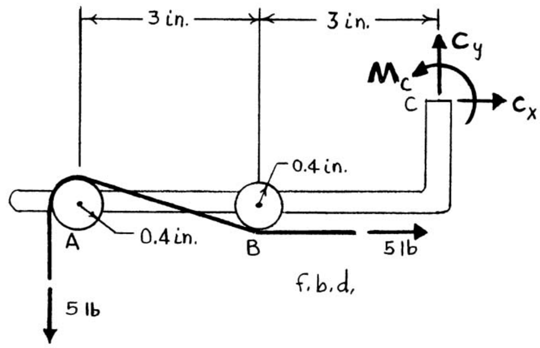
$\therefore\ B = 1175\text{ lb}$ or $\mathbf{B} = 1175\text{ lb}\uparrow$ ◀

PROBLEM 4.157



A tension of 5 lb is maintained in a tape as it passes the support system shown. Knowing that the radius of each pulley is 0.4 in., determine the reaction at C.

SOLUTION



From f.b.d. of system

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + (5\text{ lb}) = 0$$

$$\therefore\ C_x = -5\text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - (5\text{ lb}) = 0$$

$$\therefore\ C_y = 5\text{ lb}$$

Then $$C = \sqrt{(C_x)^2 + (C_y)^2} = \sqrt{(5)^2 + (5)^2} = 7.0711\text{ lb}$$

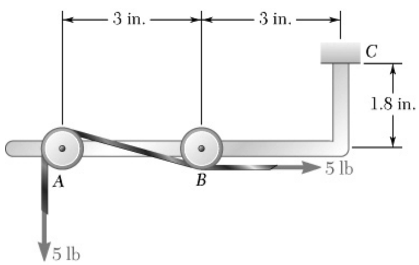
and $$\theta = \tan^{-1}\left(\frac{+5}{-5}\right) = -45°$$

or $\mathbf{C} = 7.07\text{ lb} \searrow 45.0°$ ◀

$$+\circlearrowleft \Sigma M_C = 0: \quad M_C + (5\text{ lb})(6.4\text{ in.}) + (5\text{ lb})(2.2\text{ in.}) = 0$$

$$\therefore\ M_C = -43.0\text{ lb}\cdot\text{in}$$

or $\mathbf{M}_C = 43.0\text{ lb}\cdot\text{in.} \circlearrowright$ ◀

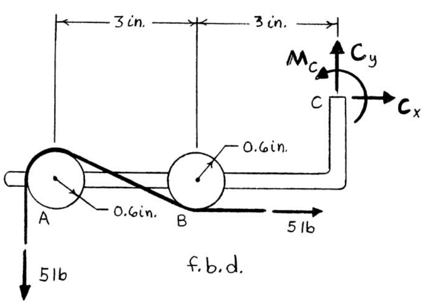


PROBLEM 4.158

Solve Problem 4.157 assuming that 0.6-in.-radius pulleys are used.

P4.157 A tension of 5 lb is maintained in a tape as it passes the support system shown. Knowing that the radius of each pulley is 0.4 in., determine the reaction at *C*.

SOLUTION



From f.b.d of system

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + (5\text{ lb}) = 0$$

$$\therefore \; C_x = -5\text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - (5\text{ lb}) = 0$$

$$\therefore \; C_y = 5\text{ lb}$$

Then $$C = \sqrt{(C_x)^2 + (C_y)^2} = \sqrt{(5)^2 + (5)^2} = 7.0711\text{ lb}$$

and $$\theta = \tan^{-1}\left(\frac{5}{-5}\right) = -45.0°$$

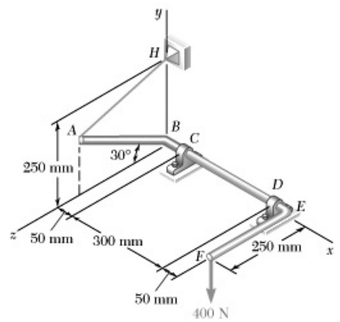
or $\mathbf{C} = 7.07\text{ lb} \searrow 45.0°$ ◀

$$+\circlearrowleft \Sigma M_C = 0: \quad M_C + (5\text{ lb})(6.6\text{ in.}) + (5\text{ lb})(2.4\text{ in.}) = 0$$

$$\therefore \; M_C = -45.0\text{ lb}\cdot\text{in.}$$

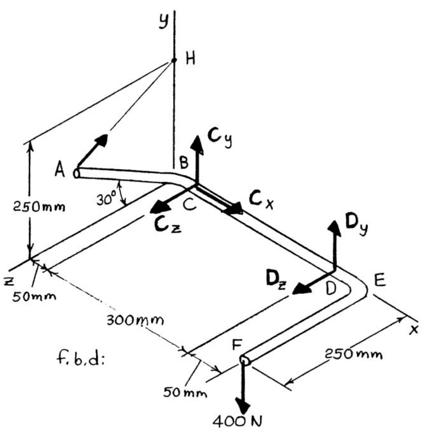
or $\mathbf{M}_C = 45.0\text{ lb}\cdot\text{in.} \circlearrowright$ ◀

PROBLEM 4.159



The bent rod *ABEF* is supported by bearings at *C* and *D* and by wire *AH*. Knowing that portion *AB* of the rod is 250 mm long, determine (*a*) the tension in wire *AH*, (*b*) the reactions at *C* and *D*. Assume that the bearing at *D* does not exert any axial thrust.

SOLUTION



(*a*) From f.b.d. of bent rod

$$\Sigma M_{CD} = 0: \quad \lambda_{CD} \cdot \left(\mathbf{r}_{H/B} \times \mathbf{T}\right) + \lambda_{CD} \cdot \left(\mathbf{r}_{F/E} \times \mathbf{F}\right) = 0$$

where $\lambda_{CD} = \mathbf{i}$

$$\mathbf{r}_{H/B} = (0.25\text{ m})\mathbf{j}$$

$$\mathbf{T} = \lambda_{AH} T$$

$$= \frac{(y_{AH})\mathbf{j} - (z_{AH})\mathbf{k}}{\sqrt{(y_{AH})^2 + (z_{AH})^2}} T$$

$$y_{AH} = (0.25\text{ m}) - (0.25\text{ m})\sin 30°$$

$$= 0.125\text{ m}$$

$$z_{AH} = (0.25\text{ m})\cos 30°$$

$$= 0.21651\text{ m}$$

$$\therefore\ \mathbf{T} = \frac{T}{0.25}(0.125\mathbf{j} - 0.21651\mathbf{k})$$

$$\mathbf{r}_{F/E} = (0.25\text{ m})\mathbf{k}$$

$$\mathbf{F} = -400\text{ N }\mathbf{j}$$

$$\therefore \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0.125 & -0.21651 \end{vmatrix} (0.25)\left(\frac{T}{0.25}\right) + \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{vmatrix} (0.25)(400\text{ N}) = 0$$

$$-0.21651T + 0.25(400\text{ N}) = 0$$

$$\therefore\ T = 461.88\text{ N}$$

or $T = 462\text{ N}$ ◀

PROBLEM 4.159 CONTINUED

(*b*) From f.b.d. of bent rod

$$\Sigma F_x = 0: \quad C_x = 0$$

$$\Sigma M_{D(z\text{-axis})} = 0: \quad -\left[(461.88\text{ N})\sin 30^\circ\right](0.35\text{ m}) - C_y(0.3\text{ m})$$

$$-(400\text{ N})(0.05\text{ m}) = 0$$

$$\therefore\ C_y = -336.10\text{ N}$$

$$\Sigma M_{D(y\text{-axis})} = 0: \quad C_z(0.3\text{ m}) - \left[(461.88\text{ N})\cos 30^\circ\right](0.35\text{ m}) = 0$$

$$\therefore\ C_z = 466.67\text{ N}$$

or $\mathbf{C} = -(336\text{ N})\mathbf{j} + (467\text{ N})\mathbf{k}$ ◀

$$\Sigma F_y = 0: \quad D_y - 336.10\text{ N} + (461.88\text{ N})\sin 30^\circ - 400\text{ N} = 0$$

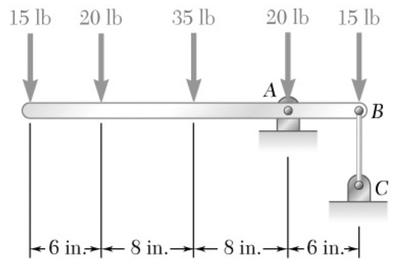
$$\therefore\ D_y = 505.16\text{ N}$$

$$\Sigma F_z = 0: \quad D_z + 466.67\text{ N} - (461.88\text{ N})\cos 30^\circ = 0$$

$$\therefore\ D_z = -66.670\text{ N}$$

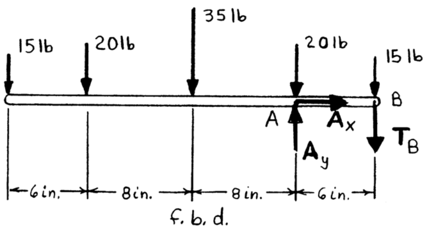
or $\mathbf{D} = (505\text{ N})\mathbf{j} - (66.7\text{ N})\mathbf{k}$ ◀

PROBLEM 4.160



For the beam and loading shown, determine (*a*) the reaction at *A*, (*b*) the tension in cable *BC*.

SOLUTION



(*a*) From f.b.d of beam

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (15 \text{ lb})(28 \text{ in.}) + (20 \text{ lb})(22 \text{ in.}) + (35 \text{ lb})(14 \text{ in.})$$

$$+ (20 \text{ lb})(6 \text{ in.}) - A_y(6 \text{ in.}) = 0$$

$$\therefore A_y = 245 \text{ lb}$$

or $\mathbf{A} = 245 \text{ lb} \uparrow$ ◀

(*b*) From f.b.d of beam

$$+\circlearrowleft \Sigma M_A = 0: \quad (15 \text{ lb})(22 \text{ in.}) + (20 \text{ lb})(16 \text{ in.}) + (35 \text{ lb})(8 \text{ in.})$$

$$- (15 \text{ lb})(6 \text{ in.}) - T_B(6 \text{ in.}) = 0$$

$$\therefore T_B = 140.0 \text{ lb}$$

or $T_B = 140.0 \text{ lb}$ ◀

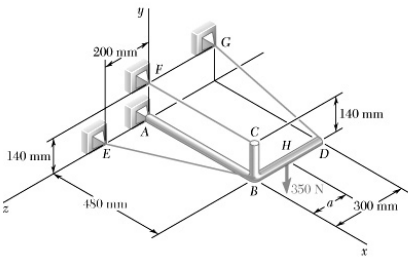
Check:

$$+\uparrow \Sigma F_y = 0: \quad -15 \text{ lb} - 20 \text{ lb} - 35 \text{ lb} - 20 \text{ lb}$$

$$- 15 \text{ lb} - 140 \text{ lb} + 245 \text{ lb} = 0?$$

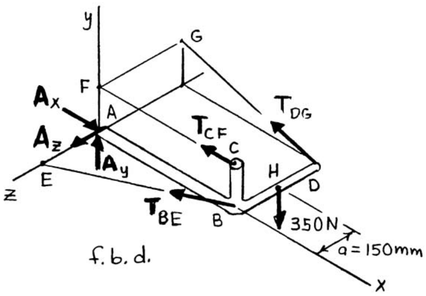
$$245 \text{ lb} - 245 \text{ lb} = 0 \quad \text{ok}$$

PROBLEM 4.161



Frame *ABCD* is supported by a ball-and-socket joint at *A* and by three cables. For $a = 150$ mm, determine the tension in each cable and the reaction at *A*.

SOLUTION



First note

$$\mathbf{T}_{DG} = \lambda_{DG}T_{DG} = \frac{-(0.48\text{ m})\mathbf{i} + (0.14\text{ m})\mathbf{j}}{\sqrt{(0.48)^2 + (0.14)^2}\text{ m}}T_{DG}$$

$$= \frac{-0.48\mathbf{i} + 0.14\mathbf{j}}{0.50}T_{DG}$$

$$= \frac{T_{DG}}{25}(24\mathbf{i} + 7\mathbf{j})$$

$$\mathbf{T}_{BE} = \lambda_{BE}T_{BE} = \frac{-(0.48\text{ m})\mathbf{i} + (0.2\text{ m})\mathbf{k}}{\sqrt{(0.48)^2 + (0.2)^2}\text{ m}}T_{BE}$$

$$= \frac{-0.48\mathbf{i} + 0.2\mathbf{k}}{0.52}T_{BE}$$

$$= \frac{T_{BE}}{13}(-12\mathbf{j} + 5\mathbf{k})$$

From f.b.d. of frame *ABCD*

$$\Sigma M_x = 0: \quad \left(\frac{7}{25}T_{DG}\right)(0.3\text{ m}) - (350\text{ N})(0.15\text{ m}) = 0$$

or $T_{DG} = 625\text{ N}$ ◀

$$\Sigma M_y = 0: \quad \left(\frac{24}{25}\times 625\text{ N}\right)(0.3\text{ m}) - \left(\frac{5}{13}T_{BE}\right)(0.48\text{ m}) = 0$$

or $T_{BE} = 975\text{ N}$ ◀

$$\Sigma M_z = 0: \quad T_{CF}(0.14\text{ m}) + \left(\frac{7}{25}\times 625\text{ N}\right)(0.48\text{ m})$$

$$-(350\text{ N})(0.48\text{ m}) = 0$$

or $T_{CF} = 600\text{ N}$ ◀

PROBLEM 4.161 CONTINUED

$$\Sigma F_x = 0: \quad A_x + T_{CF} + (T_{BE})_x + (T_{DG})_x = 0$$

$$A_x - 600\text{ N} - \left(\frac{12}{13} \times 975\text{ N}\right) - \left(\frac{24}{25} \times 625\text{ N}\right) = 0$$

$$\therefore\ A_x = 2100\text{ N}$$

$$\Sigma F_y = 0: \quad A_y + (T_{DG})_y - 350\text{ N} = 0$$

$$A_y + \left(\frac{7}{25} \times 625\text{ N}\right) - 350\text{ N} = 0$$

$$\therefore\ A_y = 175.0\text{ N}$$

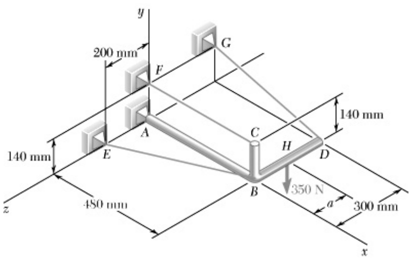
$$\Sigma F_z = 0: \quad A_z + (T_{BE})_z = 0$$

$$A_z + \left(\frac{5}{13} \times 975\text{ N}\right) = 0$$

$$\therefore\ A_z = -375\text{ N}$$

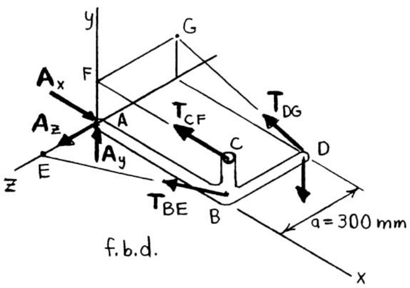
Therefore $\mathbf{A} = (2100\text{ N})\mathbf{i} + (175.0\text{ N})\mathbf{j} - (375\text{ N})\mathbf{k}$ ◀

PROBLEM 4.162



Frame *ABCD* is supported by a ball-and-socket joint at *A* and by three cables. Knowing that the 350-N load is applied at *D* (a = 300 mm), determine the tension in each cable and the reaction at *A*.

SOLUTION



First note

$$\mathbf{T}_{DG} = \lambda_{DG} T_{DG} = \frac{-(0.48\text{ m})\mathbf{i} + (0.14\text{ m})\mathbf{j}}{\sqrt{(0.48)^2 + (0.14)^2}\text{ m}} T_{DG}$$

$$= \frac{-0.48\mathbf{i} + 0.14\mathbf{j}}{0.50} T_{DG}$$

$$= \frac{T_{DG}}{25}(24\mathbf{i} + 7\mathbf{j})$$

$$\mathbf{T}_{BE} = \lambda_{BE} T_{BE} = \frac{-(0.48\text{ m})\mathbf{i} + (0.2\text{ m})\mathbf{k}}{\sqrt{(0.48)^2 + (0.2)^2}\text{ m}} T_{BE}$$

$$= \frac{-0.48\mathbf{i} + 0.2\mathbf{k}}{0.52} T_{BE}$$

$$= \frac{T_{BE}}{13}(-12\mathbf{i} + 5\mathbf{k})$$

From f.b.d of frame *ABCD*

$$\Sigma M_x = 0: \quad \left(\frac{7}{25}T_{DG}\right)(0.3\text{ m}) - (350\text{ N})(0.3\text{ m}) = 0$$

or $T_{DG} = 1250\text{ N}$ ◀

$$\Sigma M_y = 0: \quad \left(\frac{24}{25} \times 1250\text{ N}\right)(0.3\text{ m}) - \left(\frac{5}{13}T_{BE}\right)(0.48\text{ m}) = 0$$

or $T_{BE} = 1950\text{ N}$ ◀

$$\Sigma M_z = 0: \quad T_{CF}(0.14\text{ m}) + \left(\frac{7}{25} \times 1250\text{ N}\right)(0.48\text{ m})$$

$$-(350\text{ N})(0.48\text{ m}) = 0$$

or $T_{CF} = 0$ ◀

PROBLEM 4.162 CONTINUED

$$\Sigma F_x = 0: \quad A_x + T_{CF} + \left(T_{BE}\right)_x + \left(T_{DG}\right)_x = 0$$

$$A_x + 0 - \left(\frac{12}{13} \times 1950 \text{ N}\right) - \left(\frac{24}{25} \times 1250 \text{ N}\right) = 0$$

$$\therefore \; A_x = 3000 \text{ N}$$

$$\Sigma F_y = 0: \quad A_y + \left(T_{DG}\right)_y - 350 \text{ N} = 0$$

$$A_y + \left(\frac{7}{25} \times 1250 \text{ N}\right) - 350 \text{ N} = 0$$

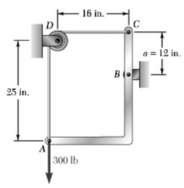
$$\therefore \; A_y = 0$$

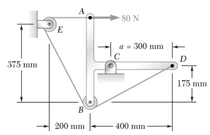
$$\Sigma F_z = 0: \quad A_z + \left(T_{BE}\right)_z = 0$$

$$A_z + \left(\frac{5}{13} \times 1950 \text{ N}\right) = 0$$

$$\therefore \; A_z = -750 \text{ N}$$

Therefore $\quad \mathbf{A} = \left(3000 \text{ N}\right)\mathbf{i} - \left(750 \text{ N}\right)\mathbf{k}$ ◀



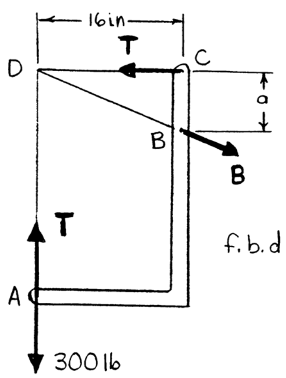


PROBLEM 4.163

In the problems listed below, the rigid bodies considered were completely constrained and the reactions were statically determinate. For each of these rigid bodies it is possible to create an improper set of constraints by changing a dimension of the body. In each of the following problems determine the value of *a* which results in improper constraints. (*a*) Problem 4.81, (*b*) Problem 4.82.

SOLUTION

(*a*)



(*a*) $+\circlearrowleft \Sigma M_B = 0: \quad (300\text{ lb})(16\text{ in.}) - T(16\text{ in.}) + T(a) = 0$

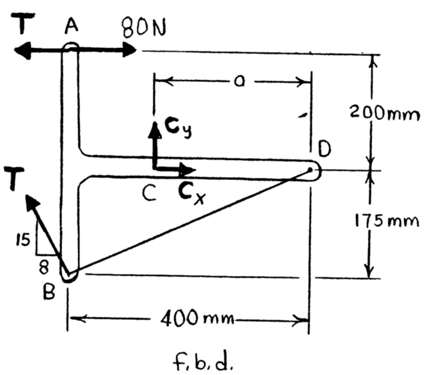
or $$T = \frac{(300\text{ lb})(16\text{ in.})}{(16 - a)\text{in.}}$$

$\therefore$ T becomes infinite when

$$16 - a = 0$$

or $a = 16.00$ in. ◀

(*b*)



(*b*) $+\circlearrowleft \Sigma M_C = 0: \quad (T - 80\text{ N})(0.2\text{ m}) - \left(\frac{8}{17}T\right)(0.175\text{ m})$

$$-\left(\frac{15}{17}T\right)(0.4\text{ m} - a) = 0$$

$$0.2T - 16.0 - 0.82353T - 0.35294T + 0.88235Ta = 0$$

or $$T = \frac{16.0}{0.88235a - 0.23529}$$

$\therefore$ T becomes infinite when

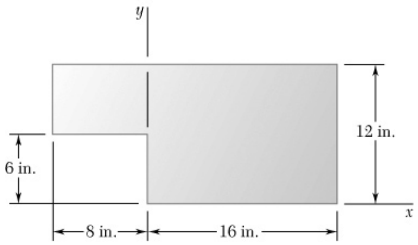
$$0.88235a - 0.23529 = 0$$

$$a = 0.26666\text{ m}$$

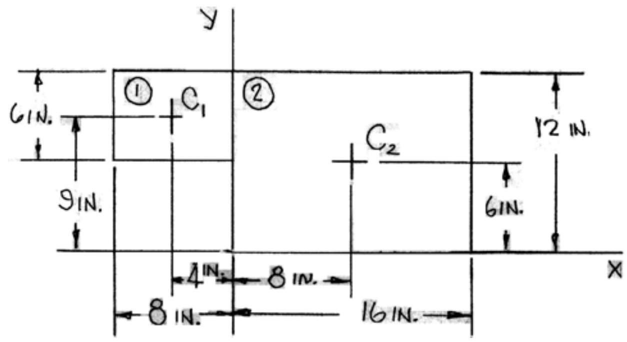
or $a = 267$ mm ◀

PROBLEM 5.1

Locate the centroid of the plane area shown.



SOLUTION



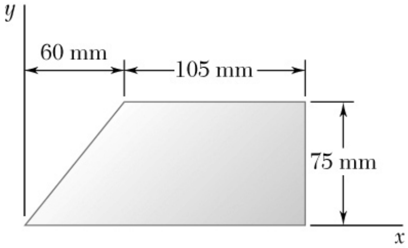
	A, in^2	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}A$, in^3	$\bar{y}A$, in^3
1	$8 \times 6 = 48$	–4	9	–192	432
2	$16 \times 12 = 192$	8	6	1536	1152
Σ	240			1344	1584

Then $$\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{1344 \text{ in}^3}{240 \text{ in}^2}$$ or $\bar{X} = 5.60$ in. ◀

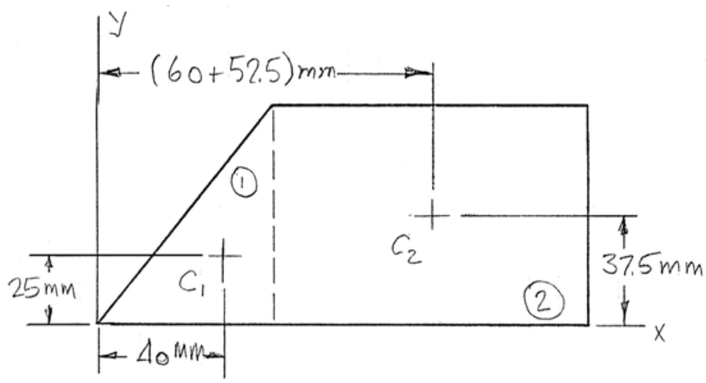
and $$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{1584 \text{ in}^3}{240 \text{ in}^2}$$ or $\bar{Y} = 6.60$ in. ◀

PROBLEM 5.2

Locate the centroid of the plane area shown.



SOLUTION

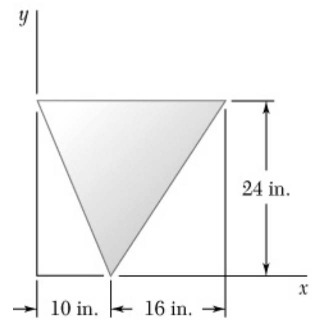


	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	$\frac{1}{2} \times 60 \times 75 = 2250$	40	25	90 000	56 250
2	$105 \times 75 = 7875$	112.5	37.5	885 900	295 300
Σ	10 125			975 900	351 600

Then $$\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{975\ 900 \text{ mm}^3}{10\ 125 \text{ mm}^2}$$ or $\bar{X} = 96.4$ mm ◀

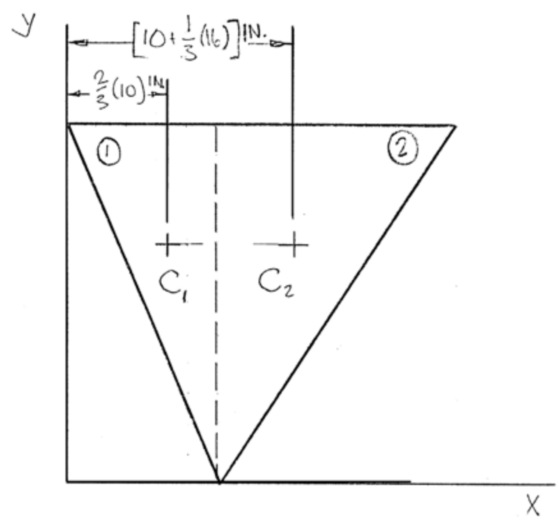
and $$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{351\ 600 \text{ mm}^3}{10\ 125 \text{ mm}^2}$$ or $\bar{Y} = 34.7$ mm ◀

PROBLEM 5.3



Locate the centroid of the plane area shown.

SOLUTION



For the area as a whole, it can be concluded by observation that

$$\bar{Y} = \frac{2}{3}(24 \text{ in.}) \qquad \text{or } \bar{Y} = 16.00 \text{ in.} \blacktriangleleft$$

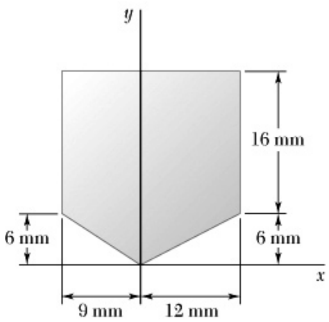
	A, in^2	$\bar{x}$, in.	$\bar{x}A$, in^3
1	$\frac{1}{2} \times 24 \times 10 = 120$	$\frac{2}{3}(10) = 6.667$	800
2	$\frac{1}{2} \times 24 \times 16 = 192$	$10 + \frac{1}{3}(16) = 15.333$	2944
Σ	312		3744

Then

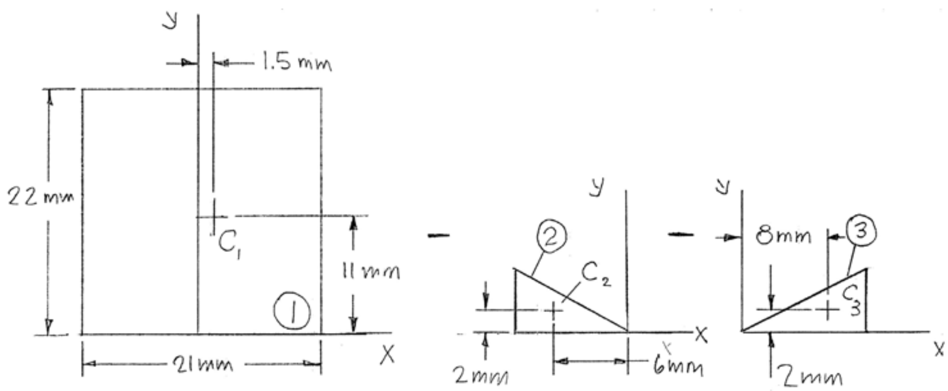
$$\bar{X} = \frac{\Sigma \bar{x} A}{\Sigma A} = \frac{3744 \text{ in}^3}{312 \text{ in}^2} \qquad \text{or } \bar{X} = 12.00 \text{ in.} \blacktriangleleft$$

PROBLEM 5.4

Locate the centroid of the plane area shown.



SOLUTION



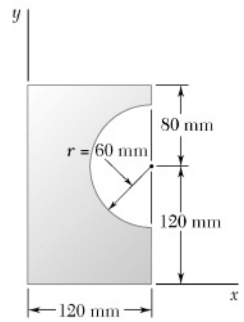
	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	$21 \times 22 = 462$	1.5	11	693	5082
2	$-\frac{1}{2}(6)(9) = -27$	-6	2	162	-54
3	$-\frac{1}{2}(6)(12) = -36$	8	2	-288	-72
Σ	399			567	4956

Then $\quad \bar{X} = \dfrac{\Sigma \bar{x}A}{\Sigma A} = \dfrac{567 \text{ mm}^3}{399 \text{ mm}^2}$ $\qquad$ or $\bar{X} = 1.421$ mm ◀

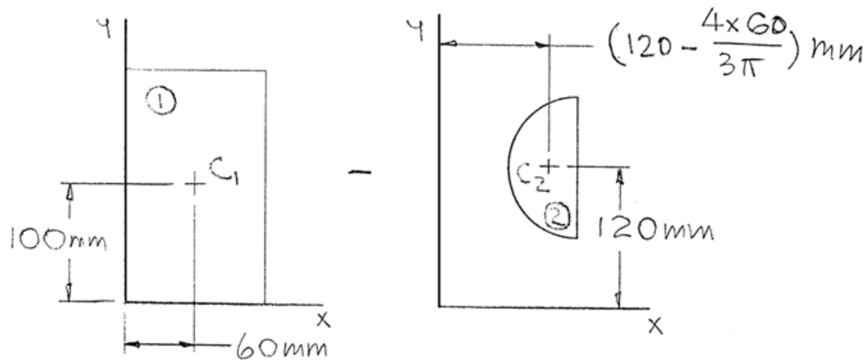
and $\quad \bar{Y} = \dfrac{\Sigma \bar{y}A}{\Sigma A} = \dfrac{4956 \text{ mm}^3}{399 \text{ mm}^2}$ $\qquad$ or $\bar{Y} = 12.42$ mm ◀

PROBLEM 5.5

Locate the centroid of the plane area shown.



SOLUTION

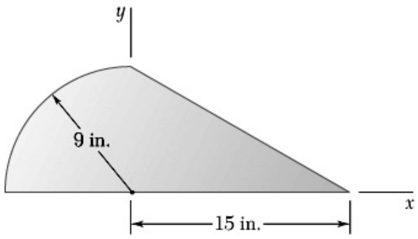


	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	$120 \times 200 = 24\ 000$	60	120	1 440 000	2 880 000
2	$-\dfrac{\pi(60)^2}{2} = -5654.9$	94.5	120	−534 600	−678 600
Σ	18 345			905 400	2 201 400

Then $$\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{905\ 400 \text{ mm}^3}{18\ 345 \text{ mm}^2}$$ or $\bar{X} = 49.4$ mm ◀

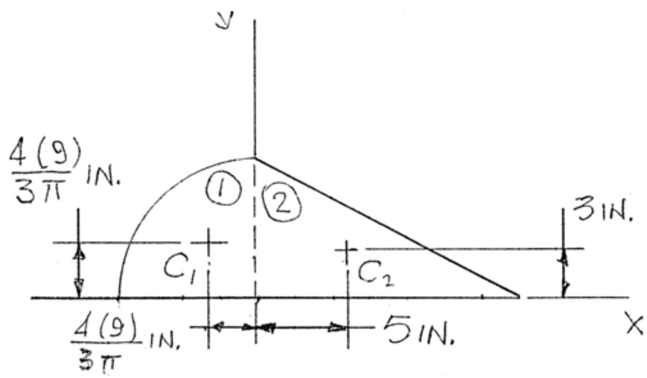
and $$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{2\ 201\ 400 \text{ mm}^3}{18\ 345 \text{ mm}^2}$$ or $\bar{Y} = 93.8$ mm ◀

PROBLEM 5.6



Locate the centroid of the plane area shown.

SOLUTION

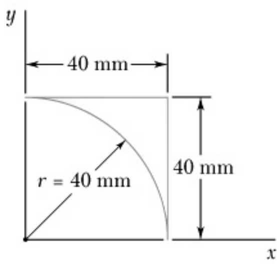


	A, in^2	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}A$, in^3	$\bar{y}A$, in^3
1	$\dfrac{\pi(9)^2}{4} = 63.617$	$\dfrac{-4(9)}{(3\pi)} = -3.8917$	3.8917	–243	243
2	$\dfrac{1}{2}(15)(9) = 67.5$	5	3	337.5	202.5
Σ	131.1			94.5	445.5

Then $$\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{94.5 \text{ in}^3}{131.1 \text{ in}^2}$$ or $\bar{X} = 0.721$ in. ◀

and $$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{445.5 \text{ in}^3}{131.1 \text{ in}^2}$$ or $\bar{Y} = 3.40$ in. ◀

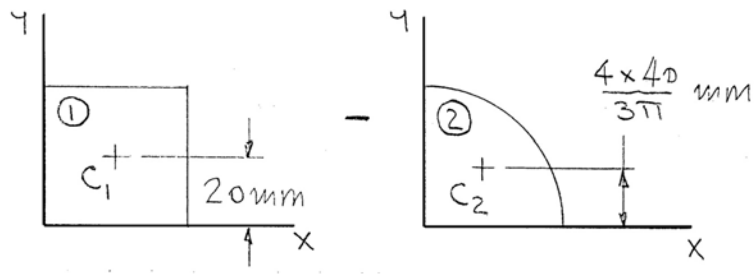
PROBLEM 5.7



Locate the centroid of the plane area shown.

SOLUTION

First note that symmetry implies $\overline{X} = \overline{Y}$



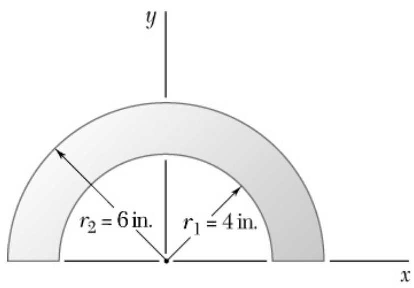
	A, mm^2	$\bar{x}$, mm	$\bar{x}A$, mm^3
1	$40 \times 40 = 1600$	20	32 000
2	$-\dfrac{\pi(40)^2}{4} = -1257$	16.98	−21 330
Σ	343		10 667

Then $$\overline{X} = \frac{\Sigma\bar{x}A}{\Sigma A} = \frac{10\ 667\ \text{mm}^3}{343\ \text{mm}^2}$$

or $\overline{X} = 31.1$ mm ◀

and $\overline{Y} = \overline{X} = 31.1$ mm ◀

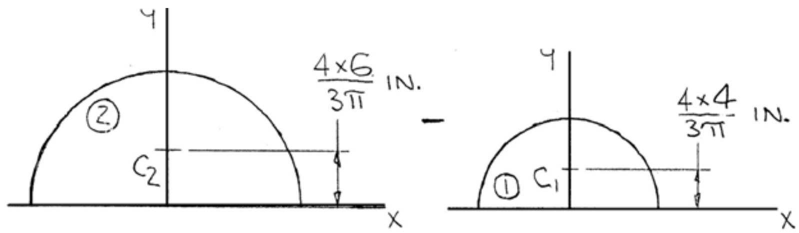
PROBLEM 5.8



Locate the centroid of the plane area shown.

SOLUTION

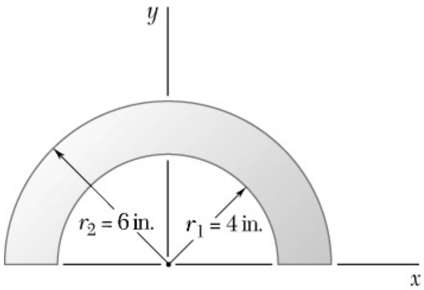
First note that symmetry implies $\bar{X} = 0$ ◀



	A, in^2	$\bar{y}$, in.	$\bar{y}A$, in^3
1	$-\dfrac{\pi(4)^2}{2} = -25.13$	1.6977	–42.67
2	$\dfrac{\pi(6)^2}{2} = 56.55$	2.546	144
Σ	31.42		101.33

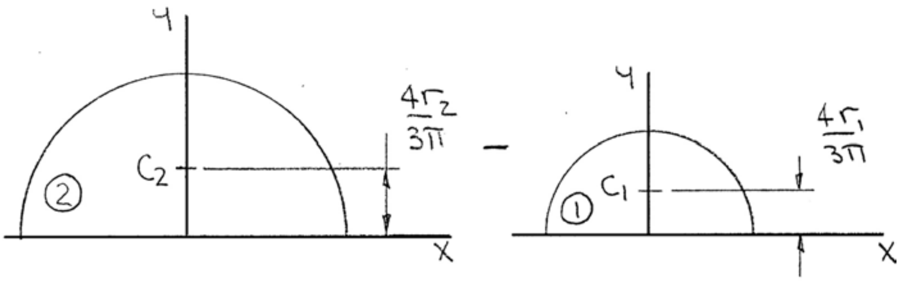
Then $\bar{Y} = \dfrac{\Sigma \bar{y}A}{\Sigma A} = \dfrac{101.33 \text{ in}^3}{31.42 \text{ in}^2}$ or $\bar{Y} = 3.23$ in. ◀

PROBLEM 5.9



For the area of Problem 5.8, determine the ratio r_2/r_1 so that $\bar{y} = 3r_1/4$.

SOLUTION



	A	$\bar{y}$	$\bar{y}A$
1	$-\frac{\pi}{2}r_1^2$	$\frac{4r_1}{3\pi}$	$-\frac{2}{3}r_1^3$
2	$\frac{\pi}{2}r_2^2$	$\frac{4r_2}{3\pi}$	$\frac{2}{3}r_2^3$
Σ	$\frac{\pi}{2}\left(r_2^2 - r_1^2\right)$		$\frac{2}{3}\left(r_2^3 - r_1^3\right)$

Then
$$\bar{Y}\Sigma A = \Sigma\bar{y}A$$

or
$$\frac{3}{4}r_1 \times \frac{\pi}{2}\left(r_2^2 - r_1^2\right) = \frac{2}{3}\left(r_2^3 - r_1^3\right)$$

$$\frac{9\pi}{16}\left[\left(\frac{r_2}{r_1}\right)^2 - 1\right] = \left(\frac{r_2}{r_1}\right)^3 - 1$$

Let
$$p = \frac{r_2}{r_1}$$

$$\frac{9\pi}{16}\left[(p+1)(p-1)\right] = (p-1)(p^2 + p + 1)$$

or
$$16p^2 + (16 - 9\pi)p + (16 - 9\pi) = 0$$

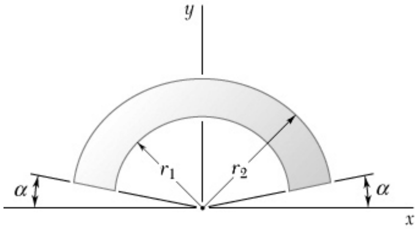
PROBLEM 5.9 CONTINUED

Then
$$p = \frac{-(16 - 9\pi) \pm \sqrt{(16 - 9\pi)^2 - 4(16)(16 - 9\pi)}}{2(16)}$$

or
$$p = -0.5726 \qquad p = 1.3397$$

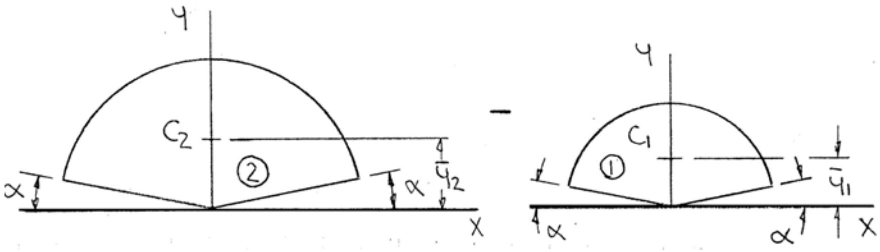
Taking the positive root
$$\frac{r_2}{r_1} = 1.340 \blacktriangleleft$$

PROBLEM 5.10



Show that as r_1 approaches r_2, the location of the centroid approaches that of a circular arc of radius $(r_1 + r_2)/2$.

SOLUTION



First, determine the location of the centroid.

From Fig. 5.8A:

$$\bar{y}_2 = \frac{2}{3}r_2\frac{\sin\left(\frac{\pi}{2}-\alpha\right)}{\left(\frac{\pi}{2}-\alpha\right)} \qquad A_2 = \left(\frac{\pi}{2}-\alpha\right)r_2^2$$

$$= \frac{2}{3}r_2\frac{\cos\alpha}{\left(\frac{\pi}{2}-\alpha\right)}$$

Similarly

$$\bar{y}_1 = \frac{2}{3}r_1\frac{\cos\alpha}{\left(\frac{\pi}{2}-\alpha\right)} \qquad A_1 = \left(\frac{\pi}{2}-\alpha\right)r_1^2$$

Then

$$\Sigma\bar{y}A = \frac{2}{3}r_2\frac{\cos\alpha}{\left(\frac{\pi}{2}-\alpha\right)}\left[\left(\frac{\pi}{2}-\alpha\right)r_2^2\right] - \frac{2}{3}r_1\frac{\cos\alpha}{\left(\frac{\pi}{2}-\alpha\right)}\left[\left(\frac{\pi}{2}-\alpha\right)r_1^2\right]$$

$$= \frac{2}{3}\left(r_2^3 - r_1^3\right)\cos\alpha$$

and

$$\Sigma A = \left(\frac{\pi}{2}-\alpha\right)r_2^2 - \left(\frac{\pi}{2}-\alpha\right)r_1^2$$

$$= \left(\frac{\pi}{2}-\alpha\right)\left(r_2^2 - r_1^2\right)$$

Now

$$\bar{Y}\Sigma A = \Sigma\bar{y}A$$

$$\bar{Y}\left[\left(\frac{\pi}{2}-\alpha\right)\left(r_2^2 - r_1^2\right)\right] = \frac{2}{3}\left(r_2^3 - r_1^3\right)\cos\alpha$$

$$\bar{Y} = \frac{2}{3}\,\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}\,\frac{\cos\alpha}{\frac{\pi}{2}-\alpha}$$

PROBLEM 5.10 CONTINUED

Using Figure 5.8B, $\bar{Y}$ of an arc of radius $\frac{1}{2}(r_1 + r_2)$ is

$$\bar{Y} = \frac{1}{2}(r_1 + r_2)\frac{\sin\left(\frac{\pi}{2} - \alpha\right)}{\left(\frac{\pi}{2} - \alpha\right)}$$

$$= \frac{1}{2}(r_1 + r_2)\frac{\cos\alpha}{\left(\frac{\pi}{2} - \alpha\right)} \qquad (1)$$

Now

$$\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2} = \frac{(r_2 - r_1)\left(r_2^2 + r_1 r_2 + r_1^2\right)}{(r_2 - r_1)(r_2 + r_1)}$$

$$= \frac{r_2^2 + r_1 r_2 + r_1^2}{r_2 + r_1}$$

Let

$$r_2 = r + \Delta$$
$$r_1 = r - \Delta$$

Then

$$r = \frac{1}{2}(r_1 + r_2)$$

and

$$\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2} = \frac{(r + \Delta)^2 + (r + \Delta)(r - \Delta) + (r - \Delta)^2}{(r + \Delta) + (r - \Delta)}$$

$$= \frac{3r^2 + \Delta^2}{2r}$$

In the limit as $\Delta \to 0$ (i.e., $r_1 = r_2$), then

$$\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2} = \frac{3}{2}r$$

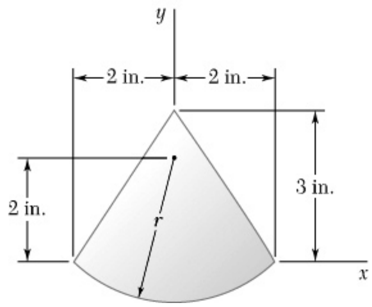
$$= \frac{3}{2} \times \frac{1}{2}(r_1 + r_2)$$

so that

$$\bar{Y} = \frac{2}{3} \times \frac{3}{4}(r_1 + r_2)\frac{\cos\alpha}{\frac{\pi}{2} - \alpha} \qquad \text{or} \quad \bar{Y} = \frac{1}{2}(r_1 + r_2)\frac{\cos\alpha}{\frac{\pi}{2} - \alpha} \blacktriangleleft$$

Which agrees with Eq. (1).

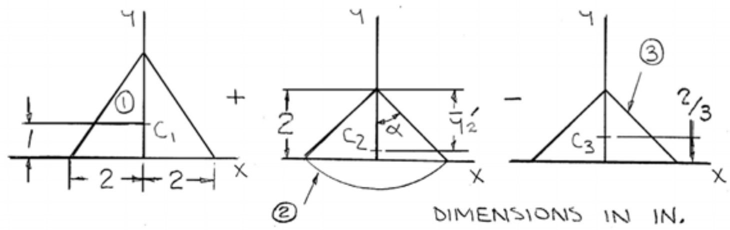
PROBLEM 5.11



Locate the centroid of the plane area shown.

SOLUTION

First note that symmetry implies $\bar{X} = 0$ ◀



$$r_2 = 2\sqrt{2} \text{ in.}, \ \alpha = 45°$$

$$\bar{y}_2' = \frac{2r\sin\alpha}{3\alpha} = \frac{2\left(2\sqrt{2}\right)\sin\left(\frac{\pi}{4}\right)}{3\left(\frac{\pi}{4}\right)} = 1.6977 \text{ in.}$$

	A, in^2	$\bar{y}$, in.	$\bar{y}A$, in^3
1	$\frac{1}{2}(4)(3) = 6$	1	6
2	$\frac{\pi}{4}\left(2\sqrt{2}\right)^2 = 6.283$	$2 - \bar{y}' = 0.3024$	1.8997
3	$-\frac{1}{2}(4)(2) = -4$	0.6667	−2.667
Σ	8.283		5.2330

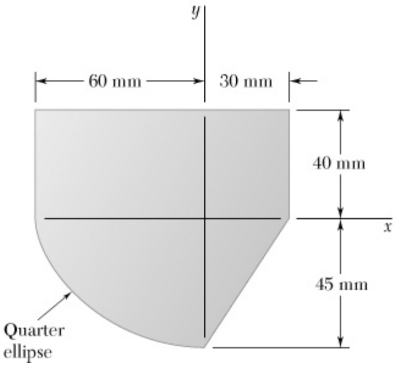
Then $$\bar{Y}\Sigma A = \Sigma\,\bar{y}A$$

$$\bar{Y}\left(8.283 \text{ in}^2\right) = 5.2330 \text{ in}^3$$

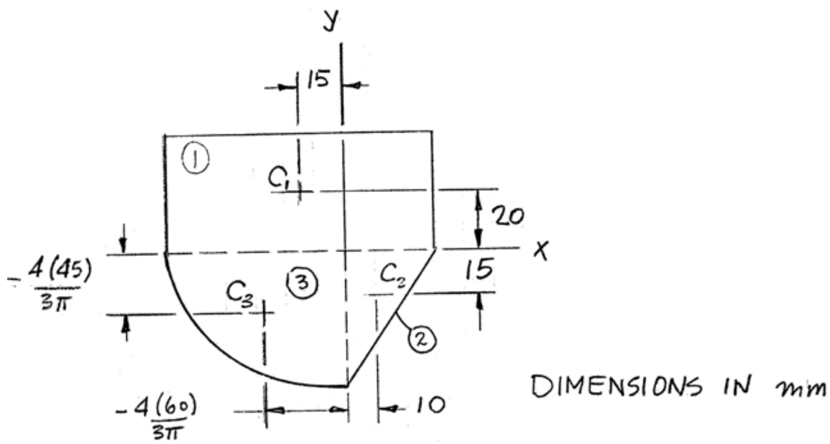
or $\bar{Y} = 0.632$ in. ◀

PROBLEM 5.12

Locate the centroid of the plane area shown.



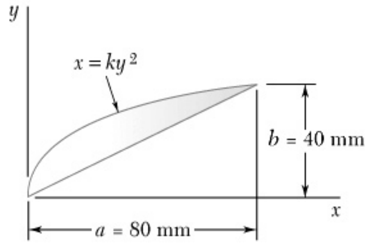
SOLUTION



	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	$(40)(90) = 3600$	–15	20	–54 000	72 000
2	$\frac{\pi(40)(60)}{4} = 2121$	10	–15	6750	–10 125
3	$\frac{1}{2}(30)(45) = 675$	–25.47	–19.099	–54 000	–40 500
Σ	6396			–101 250	21 375

Then
$$\bar{X}A = \Sigma\bar{x}A$$
$$\bar{X}\left(6396 \text{ mm}^2\right) = -101\,250 \text{ mm}^3 \qquad \text{or} \quad \bar{X} = -15.83 \text{ mm} \blacktriangleleft$$

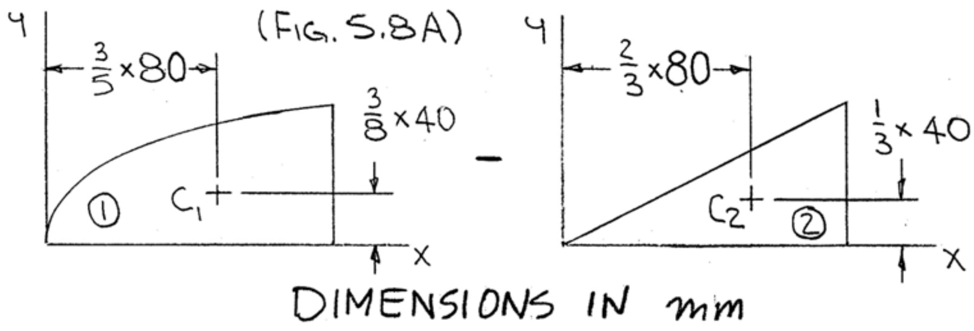
and
$$\bar{Y}A = \Sigma\,\bar{y}A$$
$$\bar{Y}\left(6396 \text{ mm}^2\right) = 21\,375 \text{ mm}^3 \qquad \text{or} \quad \bar{Y} = 3.34 \text{ mm} \blacktriangleleft$$



PROBLEM 5.13

Locate the centroid of the plane area shown.

SOLUTION



	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	$\frac{2}{3}(40)(80) = 2133$	48	15	102 400	32 000
2	$-\frac{1}{2}(40)(80) = -1600$	53.33	13.333	−85 330	−21 330
Σ	533.3			17 067	10 667

Then
$$\bar{X}\Sigma A = \Sigma \bar{x}A$$

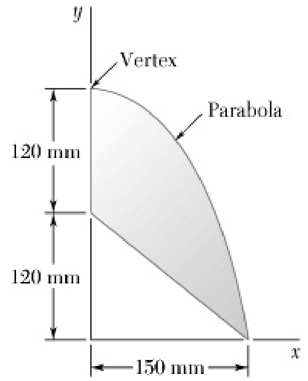
$$\bar{X}\left(533.3 \text{ mm}^2\right) = 17\ 067 \text{ mm}^3 \qquad \text{or} \quad \bar{X} = 32.0 \text{ mm} \blacktriangleleft$$

and
$$\bar{Y}\Sigma A = \Sigma \bar{y}A$$

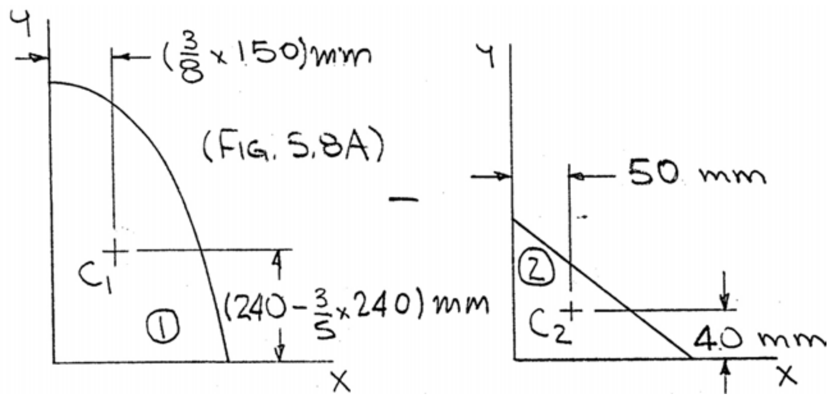
$$\bar{Y}\left(533.3 \text{ mm}^2\right) = 10\ 667 \text{ mm}^3 \qquad \text{or} \quad \bar{Y} = 20.0 \text{ mm} \blacktriangleleft$$

PROBLEM 5.14

Locate the centroid of the plane area shown.



SOLUTION



	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	$\frac{2}{3}(150)(240) = 24\ 000$	56.25	96	1 350 000	2 304 000
2	$-\frac{1}{2}(150)(120) = -9000$	50	40	–450 000	–360 000
Σ	15 000			900 000	1 944 000

Then $$\bar{X}\Sigma A = \Sigma \bar{x}A$$

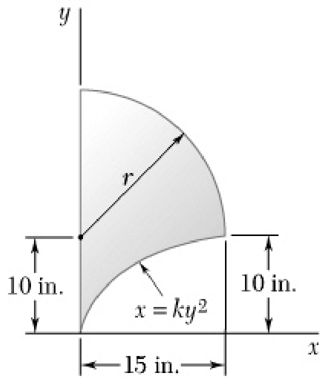
$$\bar{X}\left(15\ 000 \text{ mm}^2\right) = 900\ 000 \text{ mm}^3$$ or $\bar{X} = 60.0$ mm ◀

and $$\bar{Y}\Sigma A = \Sigma \bar{y}A$$

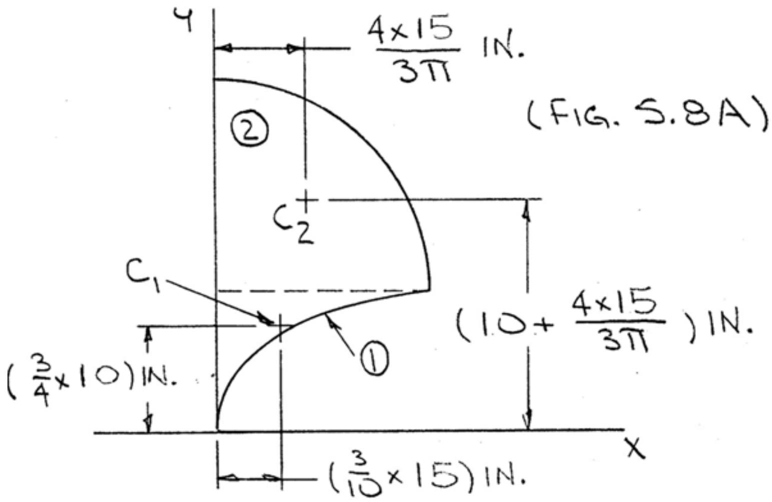
$$\bar{Y}\left(15\ 000 \text{ mm}^2\right) = 1\ 944\ 000$$ or $\bar{Y} = 129.6$ mm ◀

PROBLEM 5.15

Locate the centroid of the plane area shown.



SOLUTION



	A, in^2	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}A$, in^3	$\bar{y}A$, in^3
1	$\frac{1}{3}(10)(15) = 50$	4.5	7.5	225	375
2	$\frac{\pi}{4}(15)^2 = 176.71$	6.366	16.366	1125	2892
Σ	226.71			1350	3267

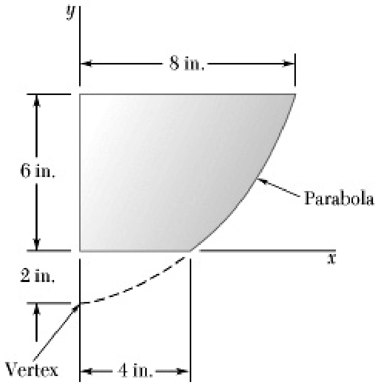
Then $$\bar{X}\Sigma A = \Sigma \bar{x} A$$

$$\bar{X}\left(226.71 \text{ in}^2\right) = 1350 \text{ in}^3$$ or $\bar{X} = 5.95$ in. ◀

and $$\bar{Y}\Sigma A = \Sigma \bar{y} A$$

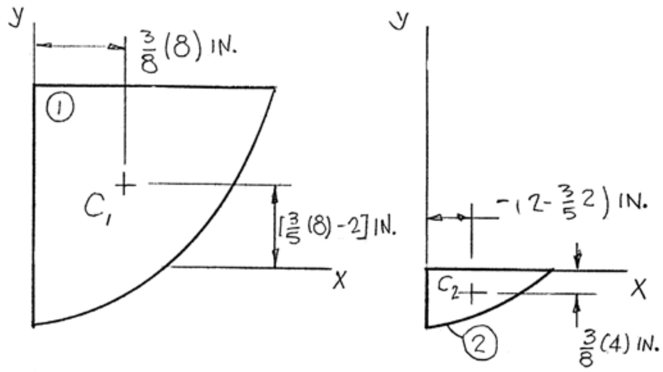
$$\bar{Y}\left(226.71 \text{ in}^2\right) = 3267 \text{ in}^3$$ or $\bar{Y} = 14.41$ in. ◀

PROBLEM 5.16



Locate the centroid of the plane area shown.

SOLUTION



	A, in^2	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}A$, in^3	$\bar{y}A$, in^3
1	$\frac{2}{3}(8)(8) = 42.67$	3	2.8	128	119.47
2	$-\frac{2}{3}(4)(2) = -5.333$	1.5	–0.8	–8	4.267
Σ	37.33			120	123.73

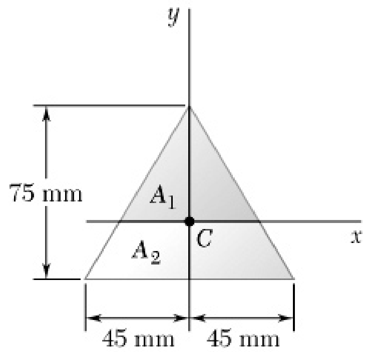
Then $$\bar{X}\Sigma A = \Sigma \bar{x} A$$

$$\bar{X}\left(37.33 \text{ in}^2\right) = 120 \text{ in}^3$$ or $\bar{X} = 3.21$ in. ◀

and $$\bar{Y}\Sigma A = \Sigma \bar{y} A$$

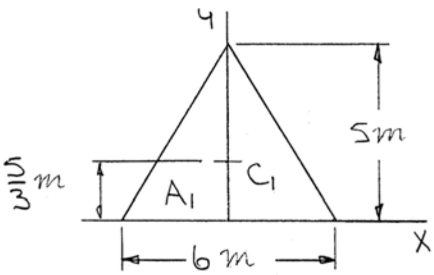
$$\bar{Y}\left(37.33 \text{ in}^2\right) = 123.73 \text{ in}^3$$ or $\bar{Y} = 3.31$ in. ◀

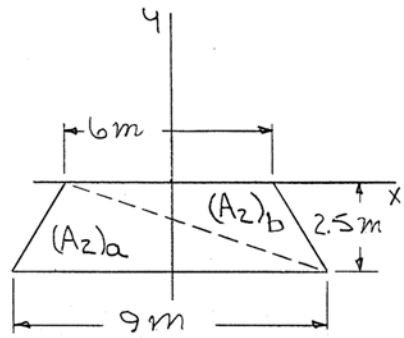
PROBLEM 5.17



The horizontal x axis is drawn through the centroid C of the area shown and divides the area into two component areas A_1 and A_2. Determine the first moment of each component area with respect to the x axis, and explain the results obtained.

SOLUTION





Note that $Q_x = \Sigma \overline{y} A$

Then $(Q_x)_1 = \left(\frac{5}{3}\text{ m}\right)\left(\frac{1}{2} \times 6 \times 5\right)\text{m}^2$ or $(Q_x)_1 = 25.0 \times 10^3\text{ mm}^3$ ◀

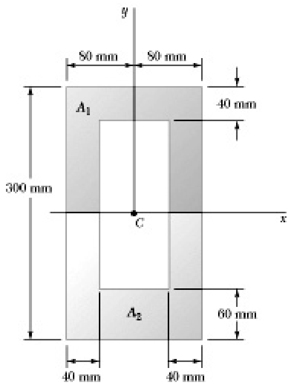
and $(Q_x)_2 = \left(-\frac{2}{3} \times 2.5\text{ m}\right)\left(\frac{1}{2} \times 9 \times 2.5\right)\text{m}^2 + \left(-\frac{1}{3} \times 2.5\text{ m}\right)\left(\frac{1}{2} \times 6 \times 2.5\right)\text{m}^2$

or $(Q_x)_2 = -25.0 \times 10^3\text{ mm}^3$ ◀

Now $Q_x = (Q_x)_1 + (Q_x)_2 = 0$

This result is expected since x is a centroidal axis $(\text{thus } \overline{y} = 0)$

and $Q_x = \Sigma \overline{y} A = \overline{Y} \Sigma A \quad (\overline{y} = 0 \Rightarrow Q_x = 0)$

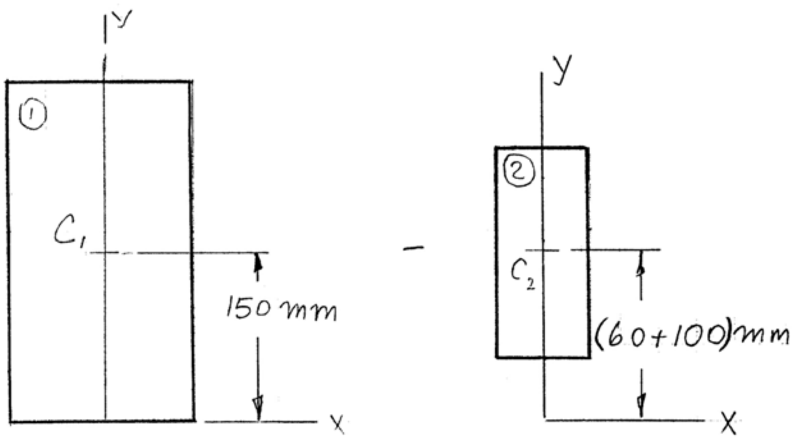


PROBLEM 5.18

The horizontal x axis is drawn through the centroid C of the area shown and divides the area into two component areas A_1 and A_2. Determine the first moment of each component area with respect to the x axis, and explain the results obtained.

SOLUTION

First, locate the position $\bar{y}$ of the figure.



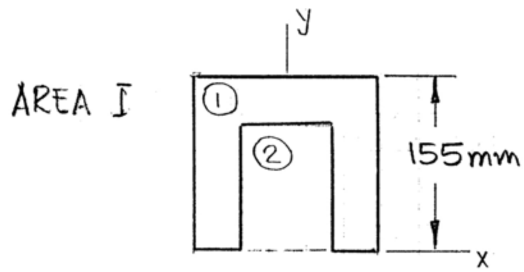
	A, mm^2	$\bar{y}$, mm	$\bar{y}A$, mm^3
1	$160 \times 300 = 48\ 000$	150	7 200 000
2	$-150 \times 80 = -16\ 000$	160	−2 560 000
Σ	32 000		4 640 000

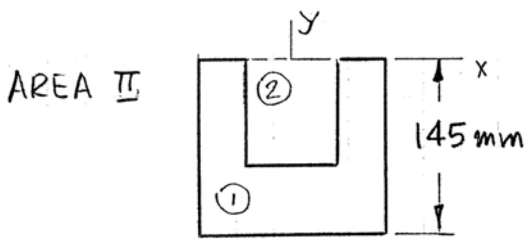
Then
$$\bar{Y}\Sigma A = \Sigma\bar{y}A$$

$$\bar{Y}\left(32\ 000\ \text{mm}^2\right) = 4\ 640\ 000\ \text{mm}^3$$

or
$$\bar{Y} = 145.0\ \text{mm}$$

PROBLEM 5.18 CONTINUED





$$A_{\mathrm{I}}: \quad Q_{\mathrm{I}} = \Sigma \bar{y} A$$

$$= \frac{155}{2}(160 \times 155) + \frac{115}{2}\left[-\left(80 \times 115\right)\right]$$

$$= 1.393 \times 10^{6}\ \mathrm{mm}^{3} \quad \blacktriangleleft$$

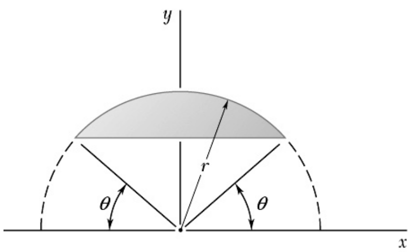
$$A_{\mathrm{II}}: \quad Q_{\mathrm{II}} = \Sigma \bar{y} A$$

$$= -\frac{145}{2}\left(160 \times 145\right) - \left[-\frac{85}{2}\left(80 \times 85\right)\right]$$

$$= -1.393 \times 10^{6}\ \mathrm{mm}^{3} \quad \blacktriangleleft$$

$$\therefore \quad \left(\mathrm{Q}_{\mathrm{area}}\right)_{x} = Q_{\mathrm{I}} + Q_{\mathrm{II}} = 0$$

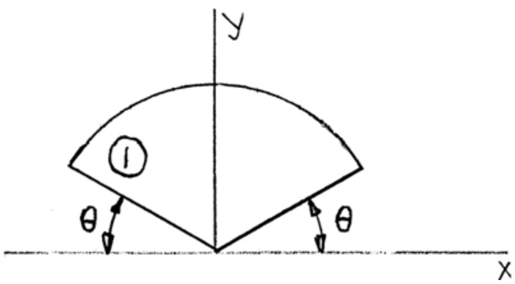
Which is expected since $Q_x = \Sigma \bar{y} A = \bar{y} A$ and $\bar{y} = 0$, since x is a centroidal axis.

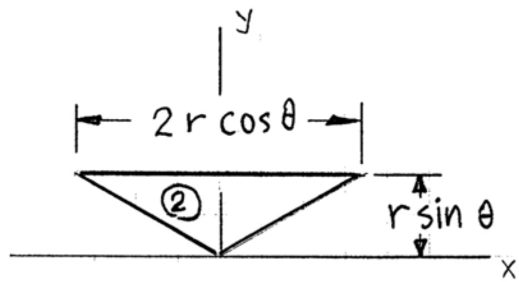


PROBLEM 5.19

The first moment of the shaded area with respect to the x axis is denoted by Q_x. (a) Express Q_x in terms of r and θ. (b) For what value of θ is Q_x maximum, and what is the maximum value?

SOLUTION





(a) With $Q_x = \Sigma \bar{y} A$ and using Fig. 5.8 A,

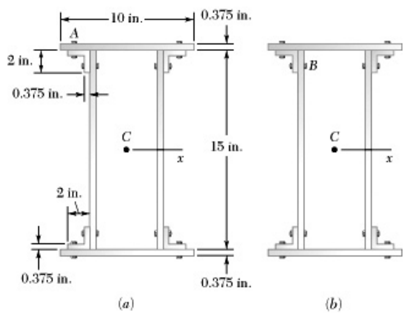
$$Q_x = \left[\frac{\frac{2}{3} r \sin\left(\frac{\pi}{2} - \theta\right)}{\frac{\pi}{2} - \theta}\right]\left[r^2\left(\frac{\pi}{2} - \theta\right)\right] - \left(\frac{2}{3} r \sin\theta\right)\left(\frac{1}{2} \times 2r\cos\theta \times r\sin\theta\right)$$

$$= \frac{2}{3} r^3 \left(\cos\theta - \cos\theta \sin^2\theta\right)$$

or $Q_x = \dfrac{2}{3} r^3 \cos^3\theta$ ◀

(b) By observation, Q_x is maximum when $\theta = 0$ ◀

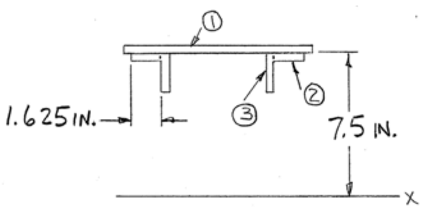
and then $Q_x = \dfrac{2}{3} r^3$ ◀



PROBLEM 5.20

A composite beam is constructed by bolting four plates to four $2 \times 2 \times 3/8$-in. angles as shown. The bolts are equally spaced along the beam, and the beam supports a vertical load. As proved in mechanics of materials, the shearing forces exerted on the bolts at A and B are proportional to the first moments with respect to the centroidal x axis of the red shaded areas shown, respectively, in parts *a* and *b* of the figure. Knowing that the force exerted on the bolt at A is 70 lb, determine the force exerted on the bolt at B.

SOLUTION



From the problem statement: $F \propto Q_x$

so that $$\frac{F_A}{(Q_x)_A} = \frac{F_B}{(Q_x)_B}$$

and $$F_B = \frac{(Q_x)_B}{(Q_x)_A} F_A$$

Now $$Q_x = \sum \bar{y} A$$

So $$(Q_x)_A = \left(7.5 \text{ in.} + \frac{0.375}{2} \text{ in.}\right)\left[10 \text{ in.} \times (0.375 \text{ in.})\right] = 28.82 \text{ in}^3$$

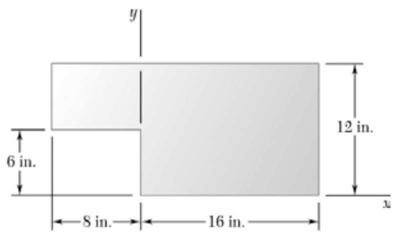
and $$(Q_x)_B = (Q_x)_A + 2\left(7.5 \text{ in.} - \frac{0.375}{2} \text{ in.}\right)\left[(1.625 \text{ in.})(0.375 \text{ in.})\right]$$
$$+ 2(7.5 \text{ in.} - 1 \text{ in.})\left[(2 \text{ in.})(0.375 \text{ in.})\right]$$

$$= 28.82 \text{ in}^3 + 8.921 \text{ in}^3 + 9.75 \text{ in}^3$$

$$= 47.49 \text{ in}^3$$

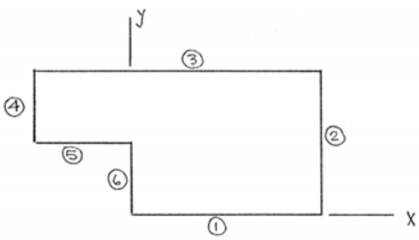
Then $$F_B = \frac{47.49 \text{ in}^3}{28.82 \text{ in}^3}(70 \text{ lb}) = 115.3 \text{ lb} \blacktriangleleft$$

PROBLEM 5.21



A thin, homogeneous wire is bent to form the perimeter of the figure indicated. Locate the center of gravity of the wire figure thus formed.

SOLUTION



First note that because the wire is homogeneous, its center of gravity will coincide with the centroid of the corresponding line.

	L, in.	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}L$, in^2	$\bar{y}L$, in^2
1	16	8	0	128	0
2	12	16	6	102	72
3	24	4	12	96	288
4	6	–8	9	–48	54
5	8	–4	6	–32	48
6	6	0	3	0	18
Σ	72			336	480

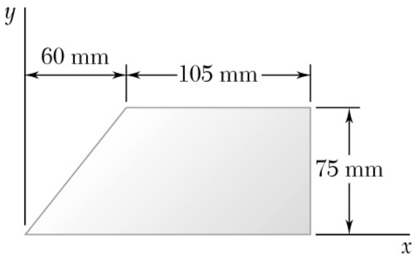
Then $$\bar{X}\Sigma L = \Sigma \bar{x} L$$

$$\bar{X}(72 \text{ in.}) = 336 \text{ in}^2 \qquad \text{or} \quad \bar{X} = 4.67 \text{ in.} \blacktriangleleft$$

and $$\bar{Y}\Sigma L = \Sigma \bar{y} L$$

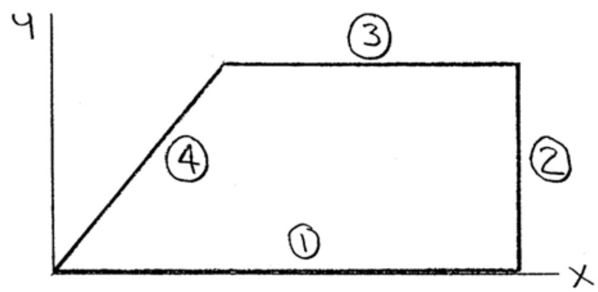
$$\bar{Y}(72 \text{ in.}) = 480 \text{ in}^2 \qquad \text{or} \quad \bar{Y} = 6.67 \text{ in.} \blacktriangleleft$$

PROBLEM 5.22



A thin, homogeneous wire is bent to form the perimeter of the figure indicated. Locate the center of gravity of the wire figure thus formed.

SOLUTION



First note that because the wire is homogeneous, its center of gravity will coincide with the centroid of the corresponding line.

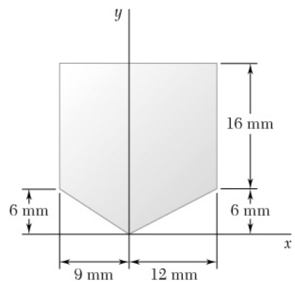
	L, mm	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}L$, mm^2	$\bar{y}L$, mm^2
1	165	82.5	0	13 612	0
2	75	165	37.5	12 375	2812
3	105	112.5	75	11 812	7875
4	$\sqrt{60^2 + 75^2} = 96.05$	30	37.5	2881	3602
Σ	441.05			40 680	14 289

Then $$\bar{X}\Sigma L = \Sigma \bar{x} L$$

$$\bar{X}(441.05 \text{ mm}) = 40\,680 \text{ mm}^2$$ or $\bar{X} = 92.2$ mm ◀

and $$\bar{Y}\Sigma L = \Sigma \bar{y} L$$

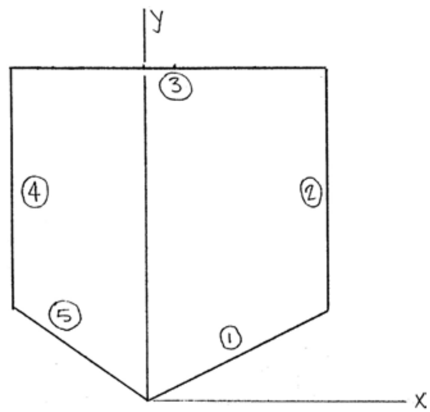
$$\bar{Y}(441.05 \text{ mm}) = 14\,289 \text{ mm}^2$$ $\bar{Y} = 32.4$ mm ◀



PROBLEM 5.23

A thin, homogeneous wire is bent to form the perimeter of the figure indicated. Locate the center of gravity of the wire figure thus formed.

SOLUTION



First note that because the wire is homogeneous, its center of gravity will coincide with the centroid of the corresponding line.

	L, mm	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}L$, mm^2	$\bar{y}L$, mm^2
1	$\sqrt{12^2 + 6^2} = 13.416$	6	3	80.50	40.25
2	16	12	14	192	224
3	21	1.5	22	31.50	462
4	16	–9	14	–144	224
5	$\sqrt{6^2 + 9^2} = 10.817$	–4.5	3	–48.67	32.45
Σ	77.233			111.32	982.7

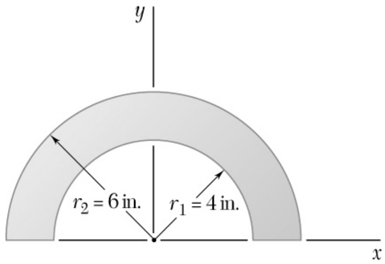
Then
$$\bar{X}\Sigma L = \Sigma \bar{x} L$$

$$\bar{X}(77.233 \text{ mm}) = 111.32 \text{ mm}^2$$
or $\bar{X} = 1.441$ mm ◀

and
$$\bar{Y}\Sigma L = \Sigma \bar{y} L$$

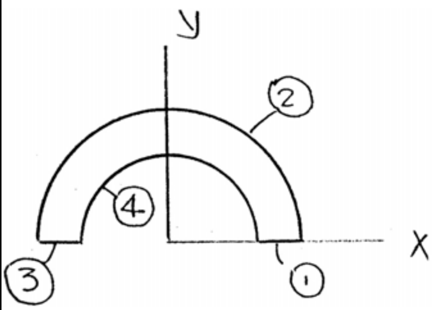
$$\bar{Y}(77.233 \text{ mm}) = 982.7 \text{ mm}^2$$
or $\bar{Y} = 12.72$ mm ◀

PROBLEM 5.24



A thin, homogeneous wire is bent to form the perimeter of the figure indicated. Locate the center of gravity of the wire figure thus formed.

SOLUTION



First note that because the wire is homogeneous, its center of gravity will coincide with the centroid of the corresponding line.

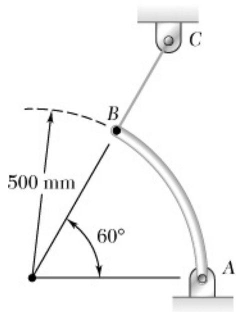
By symmetry $\bar{X} = 0$ ◀

	L, in.	$\bar{y}$, in.	$\bar{y}L$, in^2
1	2	0	0
2	$\pi(6)$	$\frac{2(6)}{\pi} = 3.820$	72
3	2	0	0
4	$\pi(4)$	$\frac{2(4)}{\pi} = 2.546$	32
Σ	35.416		104

Then $$\bar{Y}\Sigma L = \Sigma\bar{y}L$$

$$\bar{Y}(35.416 \text{ in.}) = 104 \text{ in}^2$$

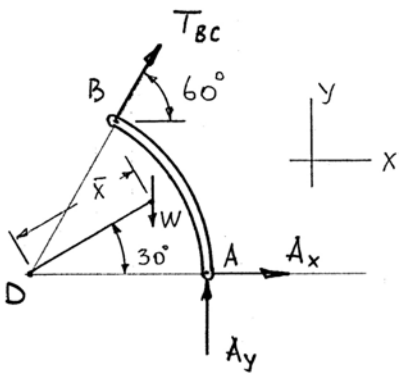
or $\bar{Y} = 2.94$ in. ◀



PROBLEM 5.25

A 750 = g uniform steel rod is bent into a circular arc of radius 500 mm as shown. The rod is supported by a pin at *A* and the cord *BC*. Determine (*a*) the tension in the cord, (*b*) the reaction at *A*.

SOLUTION



First note, from Figure 5.8B: $\bar{X} = \dfrac{(0.5\text{ m})\sin 30°}{\pi/6}$

$$= \frac{1.5}{\pi}\text{ m}$$

Then
$$W = mg$$
$$= (0.75\text{ kg})(9.81\text{ m/s}^2)$$
$$= 7.358\text{ N}$$

Also note that ΔABD is an equilateral triangle.
Equilibrium then requires

(*a*) $\Sigma M_A = 0$:

$$\left[0.5\text{ m} - \left(\frac{1.5}{\pi}\text{ m}\right)\cos 30°\right](7.358\text{ N}) - \left[(0.5\text{ m})\sin 60°\right]T_{BC} = 0$$

or $T_{BC} = 1.4698\text{ N}$ or $T_{BC} = 1.470\text{ N}$ ◀

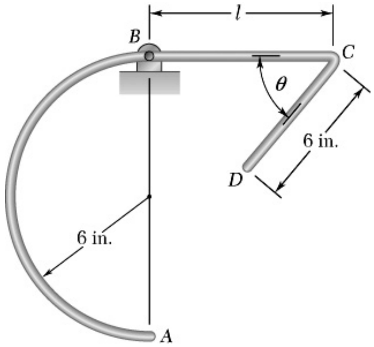
(*b*) $\Sigma F_x = 0$: $A_x + (1.4698\text{ N})\cos 60° = 0$

or $A_x = -0.7349\text{ N}$

$\Sigma F_y = 0$: $A_y - 7.358\text{ N} + (1.4698\text{ N})\sin 60° = 0$

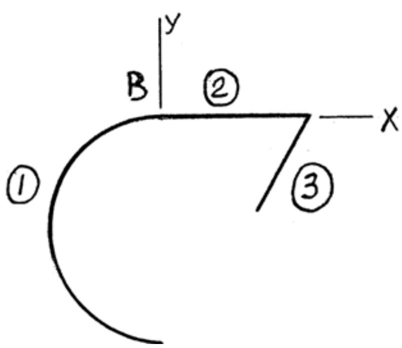
or $A_y = 6.085\text{ N}$ thus $\mathbf{A} = 6.13\text{ N} \measuredangle 83.1°$ ◀

PROBLEM 5.26



The homogeneous wire *ABCD* is bent as shown and is supported by a pin at *B*. Knowing that $l = 8$ in., determine the angle θ for which portion *BC* of the wire is horizontal.

SOLUTION



First note that for equilibrium, the center of gravity of the wire must lie on a vertical line through *B*. Further, because the wire is homogeneous, its center of gravity will coincide with the centroid of the corresponding line.

Thus $\Sigma M_B = 0$, which implies that $\bar{x} = 0$ or $\Sigma xL = 0$

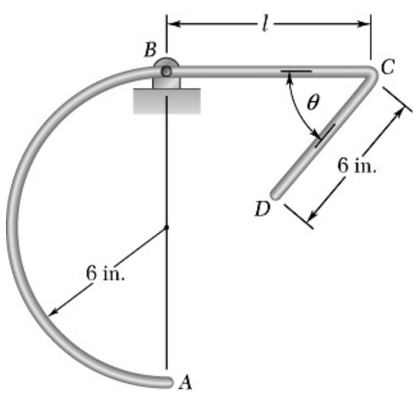
Hence

$$-\frac{2(6\text{ in.})}{\pi}(\pi \times 6\text{ in.}) + \left(\frac{8\text{ in.}}{2}\right)(8\text{ in.})$$

$$+\left(8\text{ in.} - \frac{6\text{ in.}}{2}\cos\theta\right)(6\text{ in.}) = 0$$

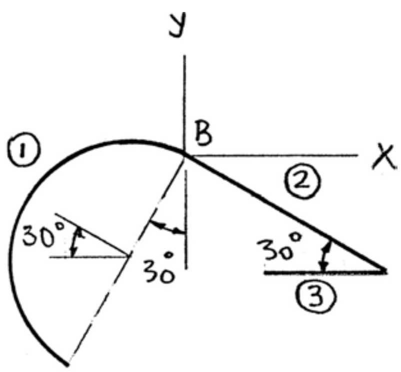
Then $\quad \cos\theta = \dfrac{4}{9} \quad$ or $\theta = 63.6°$ ◀

PROBLEM 5.27



The homogeneous wire *ABCD* is bent as shown and is supported by a pin at *B*. Knowing that $\theta = 30°$, determine the length l for which portion *CD* of the wire is horizontal.

SOLUTION



First note that for equilibrium, the center of gravity of the wire must lie on a vertical line through *B*. Further, because the wire is homogeneous, its center of gravity will coincide with the centroid of the corresponding line.

Thus $\Sigma M_B = 0$, which implies that $\bar{x} = 0$ or $\Sigma x_i L_i = 0$

Hence
$$-\left[\frac{2(6\text{ in.})}{\pi}\cos 30° + (6\text{ in.})\sin 30°\right](\pi \times 6\text{ in.})$$

$$+\left[\frac{(l\text{ in.})}{2}\cos 30°\right](l\text{ in.})$$

$$+\left[(l\text{ in.})\cos 30° - \frac{6\text{ in.}}{2}\right](6\text{ in.}) = 0$$

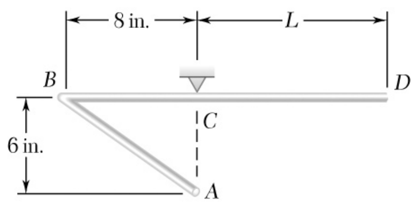
or
$$l^2 + 12.0l - 316.16 = 0$$

with roots $l_1 = 12.77$ and -24.77.

Taking the positive root

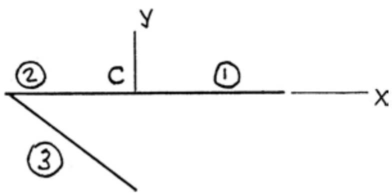
$$l = 12.77\text{ in.} \blacktriangleleft$$

PROBLEM 5.28



The homogeneous wire *ABCD* is bent as shown and is attached to a hinge at *C*. Determine the length *L* for which the portion *BCD* of the wire is horizontal.

SOLUTION



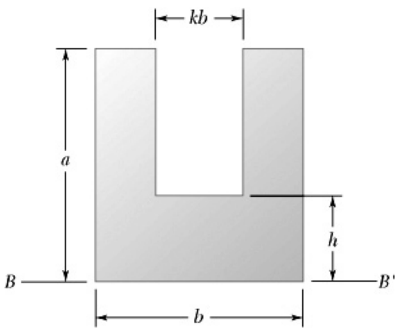
First note that for equilibrium, the center of gravity of the wire must lie on a vertical line through *C*. Further, because the wire is homogeneous, its center of gravity will coincide with the centroid of the corresponding line.

Thus $\Sigma M_C = 0$, which implies that $\overline{x} = 0$

or $$\Sigma \overline{x}_i L_i = 0$$

Hence $$\frac{L}{2}(L) + (-4 \text{ in.})(8 \text{ in.}) + (-4 \text{ in.})(10 \text{ in.}) = 0$$

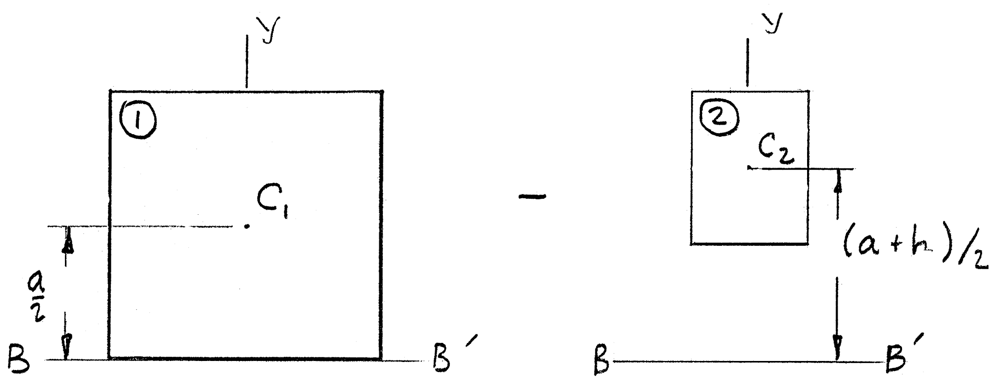
or $$L^2 = 144 \text{ in}^2$$ or $L = 12.00$ in. ◀



PROBLEM 5.29

Determine the distance h so that the centroid of the shaded area is as close to line BB' as possible when (a) $k = 0.2$, (b) $k = 0.6$.

SOLUTION



Then
$$\bar{y} = \frac{\Sigma yA}{\Sigma A}$$

or
$$\bar{y} = \frac{\frac{a}{2}(ab) - \left[\frac{(a+h)}{2}\right]\left[kb(a-h)\right]}{ba - kb(a-h)}$$

$$= \frac{1}{2}\frac{a^2(1-k) + kh^2}{a(1-k) + kh}$$

Let
$$c = 1 - k \qquad \text{and} \qquad \zeta = \frac{h}{a}$$

Then
$$\bar{y} = \frac{a}{2}\frac{c + k\zeta^2}{c + k\zeta} \tag{1}$$

Now find a value of ζ (or h) for which $\bar{y}$ is minimum:

$$\frac{d\bar{y}}{d\zeta} = \frac{a}{2}\frac{2k\zeta(c + k\zeta) - k\left(c + k\zeta^2\right)}{(c + k\zeta)^2} = 0 \qquad \text{or} \qquad 2\zeta(c + k\zeta) - \left(c + k\zeta^2\right) = 0 \tag{2}$$

PROBLEM 5.29 CONTINUED

Expanding (2) $\quad 2c\zeta + 2\zeta^2 - c - k\zeta^2 = 0 \quad$ or $\quad k\zeta^2 + 2c\zeta - c = 0$

Then

$$\zeta = \frac{-2c \pm \sqrt{(2c)^2 - 4(k)(c)}}{2k}$$

Taking the positive root, since $h > 0$ (hence $\zeta > 0$)

$$h = a\frac{-2(1-k) + \sqrt{4(1-k)^2 + 4k(1-k)}}{2k}$$

(*a*) $k = 0.2$:

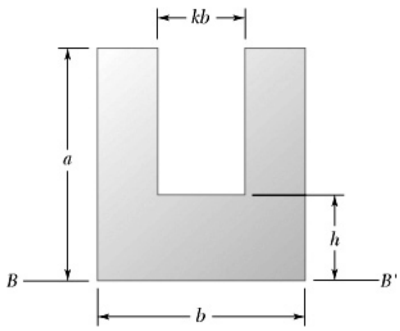
$$h = a\frac{-2(1-0.2) + \sqrt{4(1-0.2)^2 + 4(0.2)(1-0.2)}}{2(0.2)}$$

or $h = 0.472a$ ◀

(*b*) $k = 0.6$:

$$h = a\frac{-2(1-0.6) + \sqrt{4(1-0.6)^2 + 4(0.6)(1-0.6)}}{2(0.6)}$$

or $h = 0.387a$ ◀



PROBLEM 5.30

Show when the distance h is selected to minimize the distance $\bar{y}$ from line BB' to the centroid of the shaded area that $\bar{y} = h$.

SOLUTION

From Problem 5.29, note that Eq. (2) yields the value of ζ that minimizes h.

Then from Eq. (2)

We see
$$2\zeta = \frac{c + k\zeta^2}{c + k\zeta} \tag{3}$$

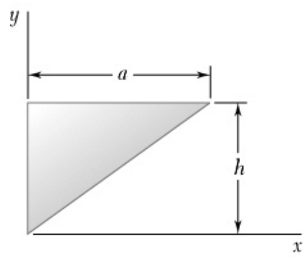
Then, replacing the right-hand side of (1) by 2ζ, from Eq. (3)

We obtain
$$\bar{y} = \frac{a}{2}(2\zeta)$$

But
$$\zeta = \frac{h}{a}$$

So
$$\bar{y} = h$$

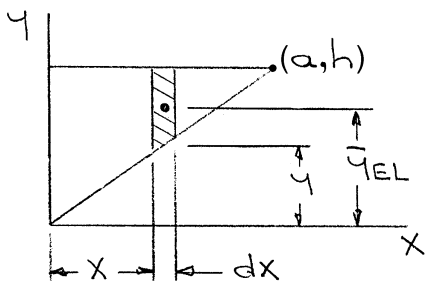
Q.E.D. ◀



PROBLEM 5.31

Determine by direct integration the centroid of the area shown. Express your answer in terms of *a* and *h*.

SOLUTION



For the element of area (EL) shown $$y = \frac{h}{a}x$$

and
$$dA = (h - y)dx$$
$$= h\left(1 - \frac{x}{a}\right)dx$$

Then
$$x_{EL} = x$$
$$y_{EL} = \frac{1}{2}(h + y)$$
$$= \frac{h}{2}\left(1 + \frac{x}{a}\right)$$

Then area $$A = \int dA = \int_0^a h\left(1 - \frac{x}{a}\right)dx = h\left(x - \frac{x^2}{2a}\right)\Big|_0^a = \frac{1}{2}ah$$

and
$$\int \bar{x}_{EL}dA = \int_0^a x\left[h\left(1 - \frac{x}{a}\right)dx\right] = h\left(\frac{x^2}{2} - \frac{x^3}{3a}\right)\Big|_0^a = \frac{1}{6}a^2h$$
$$\int \bar{y}_{EL}dA = \int_0^a \frac{h}{2}\left(1 + \frac{x}{a}\right)\left[h\left(1 - \frac{x}{a}\right)dx\right] = \frac{h^2}{2}\int_0^a\left(1 - \frac{x^2}{a^2}\right)dx$$
$$= \frac{h^2}{2}\left(x - \frac{x^3}{3a^2}\right)\Big|_0^a = \frac{1}{3}ah^2$$

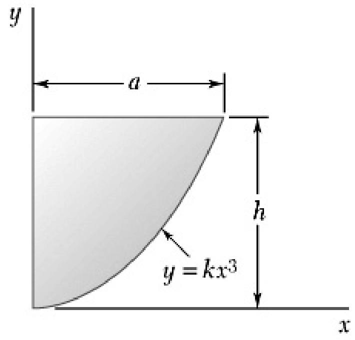
Hence
$$\bar{x}A = \int \bar{x}_{EL}dA$$
$$\bar{x}\left(\frac{1}{2}ah\right) = \frac{1}{6}a^2h$$

$$\bar{x} = \frac{1}{3}a \blacktriangleleft$$

$$\bar{y}A = \int \bar{y}_{EL}dA$$
$$\bar{y}\left(\frac{1}{2}ah\right) = \frac{1}{3}ah^2$$

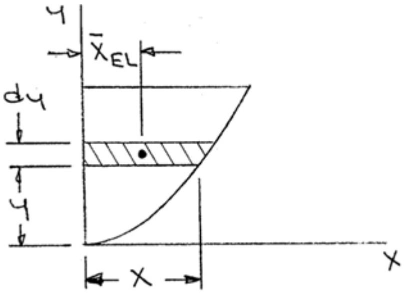
$$\bar{y} = \frac{2}{3}h \blacktriangleleft$$

PROBLEM 5.32



Determine by direct integration the centroid of the area shown. Express your answer in terms of a and h.

SOLUTION



For the element (EL) shown

At $\qquad x = a, y = h\colon h = ka^3 \qquad$ or $\qquad k = \dfrac{h}{a^3}$

Then $$x = \frac{a}{h^{1/3}} y^{1/3}$$

Now $$dA = xdy$$
$$= \frac{a}{h^{1/3}} y^{1/3}\, dy$$

$$\bar{x}_{EL} = \frac{1}{2}x = \frac{1}{2}\frac{a}{h^{1/3}} y^{1/3}, \quad \bar{y}_{EL} = y$$

Then $$A = \int dA = \int_0^h \frac{a}{h^{1/3}} y^{1/3} dy = \frac{3}{4}\frac{a}{h^{1/3}}\left(y^{4/3}\right)\Big|_0^h = \frac{3}{4}ah$$

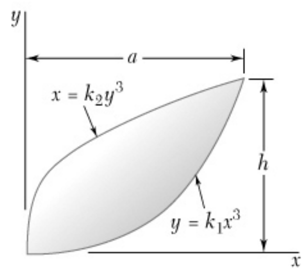
and $$\int \bar{x}_{EL}dA = \int_0^h \frac{1}{2}\frac{a}{h^{1/3}} y^{1/3}\left(\frac{a}{h^{1/3}} y^{1/3} dy\right) = \frac{1}{2}\frac{a}{h^{2/3}}\left(\frac{3}{5}y^{5/3}\right)\Big|_0^h = \frac{3}{10}a^2h$$

$$\int \bar{y}_{EL}dA = \int_0^h y\left(\frac{a}{h^{1/3}} y^{1/3} dy\right) = \frac{a}{h^{1/3}}\left(\frac{3}{7}y^{7/3}\right)\Big|_0^h = \frac{3}{7}ah^2$$

Hence $$\bar{x}A = \int \bar{x}_{EL}dA\colon \bar{x}\left(\frac{3}{4}ah\right) = \frac{3}{10}a^2h \qquad \bar{x} = \frac{2}{5}a \blacktriangleleft$$

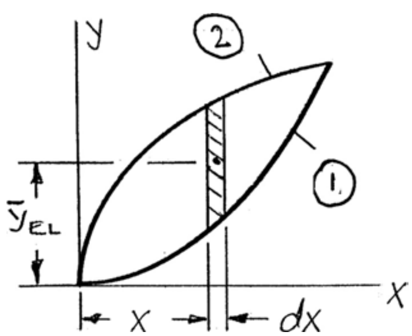
$$\bar{y}A = \int \bar{y}_{EL}dA\colon \bar{y}\left(\frac{3}{4}ah\right) = \frac{3}{7}ah^2 \qquad \bar{y} = \frac{4}{7}h \blacktriangleleft$$

PROBLEM 5.33



Determine by direct integration the centroid of the area shown. Express your answer in terms of a and h.

SOLUTION



For the element (EL) shown

At $\quad x = a, y = h: \quad h = k_1a^3 \quad$ or $\quad k_1 = \dfrac{h}{a^3}$

$$a = k_2h^3 \quad \text{or} \quad k_2 = \frac{a}{h^3}$$

Hence, on line 1

$$y = \frac{h}{a^3}x^3$$

and on line 2

$$y = \frac{h}{a^{1/3}}x^{1/3}$$

Then

$$dA = \left(\frac{h}{a^{1/3}}x^{1/3} - \frac{h}{a^3}x^3\right)dx \quad \text{and} \quad \bar{y}_{EL} = \frac{1}{2}\left(\frac{h}{a^{1/3}}x^{1/3} + \frac{h}{a^3}x^3\right)$$

$$\therefore \; A = \int dA = \int_0^a\left(\frac{h}{a^{1/3}}x^{1/3} - \frac{h}{a^3}x^3\right)dx = h\left(\frac{3}{4a^{1/3}}x^{4/3} - \frac{1}{4a^3}x^4\right)\Bigg|_0^a = \frac{1}{2}ah$$

$$\int \bar{x}_{EL}dA = \int_0^a x\left(\frac{h}{a^{1/3}}x^{1/3} - \frac{h}{a^3}x^3\right)dx = h\left(\frac{3}{7a^{1/3}}x^{7/3} - \frac{1}{5a^3}x^5\right)\Bigg|_0^a = \frac{8}{35}a^2h$$

$$\int \bar{y}_{EL}dA = \int_0^a \frac{1}{2}\left(\frac{h}{a^{1/3}}x^{1/3} + \frac{h}{a^3}x^3\right)\left(\frac{h}{a^{1/3}}x^{1/3} - \frac{h}{a^3}x^3\right)dx$$

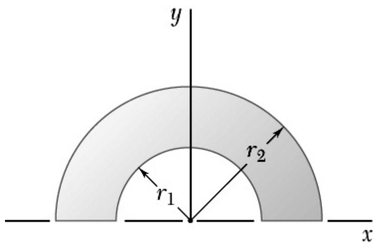
$$= \frac{h^2}{2}\int_0^a\left(\frac{x^{2/3}}{a^{2/3}} - \frac{x^6}{a^6}\right)dx = \frac{h^2}{2}\left(\frac{3}{5}\frac{x^{5/3}}{a^{5/3}} - \frac{1}{7}\frac{x^6}{a^6}\right)\Bigg|_0^a = \frac{8}{35}ah^2$$

From $\quad \bar{x}A = \int \bar{x}_{EL}dA: \quad \bar{x}\left(\dfrac{ah}{2}\right) = \dfrac{8}{35}a^2h \quad$ or $\quad \bar{x} = \dfrac{16}{35}a$ ◀

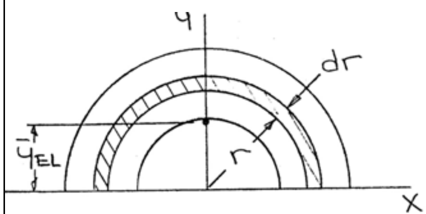
and $\quad \bar{y}A = \int \bar{y}_{EL}dA: \quad \bar{y}\left(\dfrac{ah}{2}\right) = \dfrac{8}{35}ah^2 \quad$ or $\quad \bar{y} = \dfrac{16}{35}h$ ◀

PROBLEM 5.34

Determine by direct integration the centroid of the area shown.



SOLUTION



First note that symmetry implies $\qquad \bar{x} = 0$ ◀

For the element (EL) shown

$$\bar{y}_{EL} = \frac{2r}{\pi} \quad \text{(Figure 5.8B)}$$

$$dA = \pi r\, dr$$

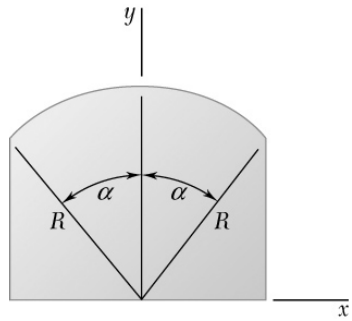
Then $\qquad A = \int dA = \int_{r_1}^{r_2} \pi r\, dr = \pi\left(\frac{r^2}{2}\right)\Bigg|_{r_1}^{r_2} = \frac{\pi}{2}\left(r_2^2 - r_1^2\right)$

and $\qquad \int \bar{y}_{EL}\, dA = \int_{r_1}^{r_2} \frac{2r}{\pi}\left(\pi r\, dr\right) = 2\left(\frac{1}{3}r^3\right)\Bigg|_{r_1}^{r_2} = \frac{2}{3}\left(r_2^3 - r_1^3\right)$

So $\qquad \bar{y}A = \int \bar{y}_{EL}\, dA: \quad \bar{y}\left[\frac{\pi}{2}\left(r_2^2 - r_1^2\right)\right] = \frac{2}{3}\left(r_2^3 - r_1^3\right)$

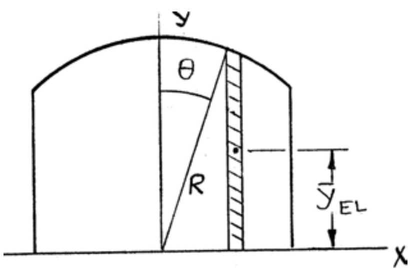
or $\quad \bar{y} = \dfrac{4}{3\pi}\dfrac{r_2^3 - r_1^3}{r_2^2 - r_1^2}$ ◀

PROBLEM 5.35



Determine by direct integration the centroid of the area shown.

SOLUTION



First note that symmetry implies $\bar{x} = 0$ ◀

For the element (EL) shown

$$y = R\cos\theta, \quad x = R\sin\theta$$

$$dx = R\cos\theta\, d\theta$$

$$dA = ydx = R^2\cos^2\theta\, d\theta$$

Hence

$$A = \int dA = 2\int_0^\alpha R^2\cos^2\theta d\theta = 2R^2\left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right)\Bigg|_0^\alpha = \frac{1}{2}R^2(2\alpha\sin 2\alpha)$$

$$\int \bar{y}_{EL}dA = 2\int_0^\alpha \frac{R}{2}\cos\theta\left(R^2\cos^2\theta d\theta\right) = R^3\left(\frac{1}{3}\cos^2\theta\sin\theta + \frac{2}{3}\sin\theta\right)\Bigg|_0^\alpha$$

$$= \frac{R^3}{3}\left(\cos^2\alpha\sin\alpha + 2\sin\alpha\right)$$

But $\bar{y}A = \int \bar{y}_{EL}dA$ so

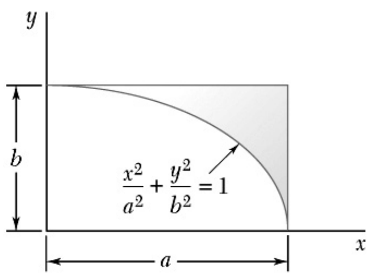
$$\bar{y} = \frac{\frac{R^3}{3}\left(\cos^2\alpha\sin\alpha + 2\sin\alpha\right)}{\frac{R^2}{2}(2\alpha + \sin 2\alpha)}$$

or

$$\bar{y} = \frac{2}{3}R\sin\alpha\,\frac{\left(\cos^2\alpha + 2\right)}{(2\alpha + \sin 2\alpha)}$$

Alternatively,

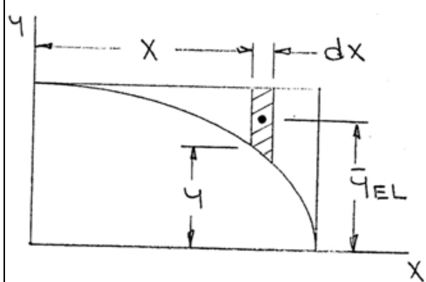
$$\bar{y} = \frac{2}{3}R\sin\alpha\,\frac{3 - \sin^2\alpha}{2\alpha + \sin 2\alpha}$$ ◀



PROBLEM 5.36

Determine by direct integration the centroid of the area shown.

SOLUTION



For the element (EL) shown

$$y = \frac{b}{a}\sqrt{a^2 - x^2}$$

and $$dA = (b - y)\,dx$$

$$= \frac{b}{a}\left(a - \sqrt{a^2 - x^2}\right)dx$$

$$\bar{x}_{EL} = x;\ \bar{y}_{EL} = \frac{1}{2}(y + b) = \frac{b}{2a}\left(a + \sqrt{a^2 - x^2}\right)$$

Then $$A = \int dA = \int_0^a \frac{b}{a}\left(a - \sqrt{a^2 - x^2}\right)dx$$

To integrate, let $x = a\sin\theta$: $\sqrt{a^2 - x^2} = a\cos\theta,\ dx = a\cos\theta\, d\theta$

Then $$A = \int_0^{\pi/2} \frac{b}{a}(a - a\cos\theta)(a\cos\theta d\theta)$$

$$= \frac{b}{a}\left[a^2\sin\theta - a^2\left(\frac{\theta}{2} + \sin\frac{2\theta}{4}\right)\right]\Bigg|_0^{\pi/2} = ab\left(1 - \frac{\pi}{4}\right)$$

and $$\int \bar{x}_{EL}\,dA = \int_0^a x\left[\frac{b}{a}\left(a - \sqrt{a^2 - x^2}\right)dx\right] = \frac{b}{a}\left[\left(\frac{a}{2}x^2 + \frac{1}{3}\left(a^2 - x^2\right)^{3/2}\right)\right]\Bigg|_0^{\pi/2}$$

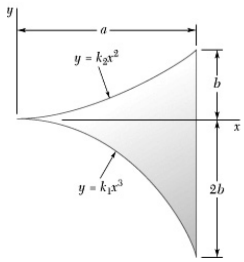
$$= \frac{1}{6}a^3 b$$

$$\int \bar{y}_{EL}\,dA = \int_0^a \frac{b}{2a}\left(a + \sqrt{a^2 - x^2}\right)\left[\frac{b}{a}\left(a - \sqrt{a^2 - x^2}\right)dx\right]$$

$$= \frac{b^2}{2a^2}\int_0^a \left(x^2\right)dx = \frac{b^2}{2a^2}\left(\frac{x^3}{3}\right)\Bigg|_0^a = \frac{1}{6}ab^2$$

$$\bar{x}A = \int \bar{x}_{EL}\,dA:\quad \bar{x}\left[ab\left(1 - \frac{\pi}{4}\right)\right] = \frac{1}{6}a^2 b \qquad \text{or } \bar{x} = \frac{2a}{3(4 - \pi)} \blacktriangleleft$$

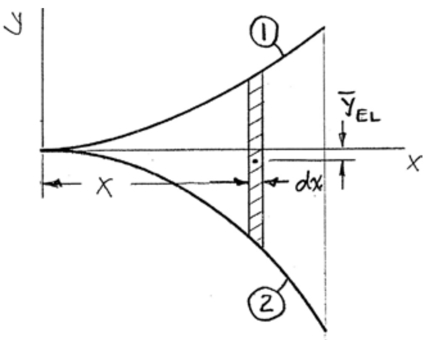
$$\bar{y}A = \int \bar{y}_{EL}\,dA:\quad \bar{y}\left[ab\left(1 - \frac{\pi}{4}\right)\right] = \frac{1}{6}ab^2 \qquad \text{or } \bar{y} = \frac{2b}{3(4 - \pi)} \blacktriangleleft$$



PROBLEM 5.37

Determine by direct integration the centroid of the area shown. Express your answer in terms of a and b.

SOLUTION



For the element (EL) shown on line 1 at

$$x = a,\ b = k_2 a^2 \qquad \text{or} \qquad k_2 = \frac{b}{a^2}$$

$$\therefore \quad y = \frac{b}{a^2}x^2$$

On line 2 at

$$x = a,\ \ -2b = k_1 a^3 \qquad \text{or} \qquad k_2 = \frac{-2b}{a^3}$$

$$\therefore \quad y = \frac{-2b}{a^3}x^3$$

$$dA = \left(\frac{b}{a^2}x^2 + \frac{2b}{a^3}x^3\right)dx$$

Then

$$A = \int dA = \int_0^a \frac{b}{a^2}\left(x^2 + \frac{2x^3}{x}\right)dx = \frac{b}{a^2}\left(\frac{x^3}{3} + \frac{2x^4}{4a}\right)\Bigg|_0^a$$

$$= ab\left(\frac{1}{3} + \frac{1}{2}\right) = \frac{5}{6}ab$$

and

$$\int \bar{x}_{EL}\,dA = \int_0^a x\left(\frac{b}{a^2}x^2 + \frac{2b}{a^3}x^3\right)dx = \frac{b}{a^2}\left(\frac{x^4}{4} + \frac{2x^5}{5a}\right)\Bigg|_0^a = a^2b\left(\frac{1}{4} + \frac{2}{5}\right)$$

$$= \frac{13}{20}a^2b$$

$$\int \bar{y}_{EL}\,dA = \int_0^a \frac{1}{2}\left(\frac{b}{a^2}x^2 - \frac{2b}{a^3}x^3\right)\left[\left(\frac{b}{a^2}x^2 + \frac{2b}{a^3}x^3\right)dx\right]$$

$$= \int_0^a \frac{1}{2}\left[\left(\frac{b}{a^2}x^2\right)^2 - \left(\frac{2b}{a^3}x^3\right)^2\right]dx = \frac{b^2}{2a^4}\left(\frac{x^5}{5} - \frac{2}{7a^2}x^7\right)\Bigg|_0^a$$

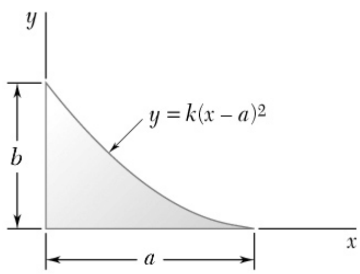
$$= b^2a^5\left(\frac{1}{10} - \frac{2}{7}\right) = -\frac{13}{70}ab^2$$

Then

$$\bar{x}A = \int \bar{x}_{EL}\,dA: \quad \bar{x}\left(\frac{5}{6}ab\right) = \frac{13}{20}a^2b \qquad \text{or} \quad \bar{x} = \frac{39}{50}a \ \blacktriangleleft$$

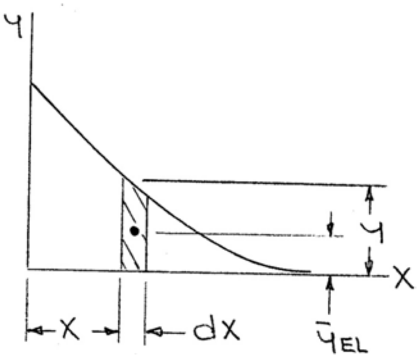
$$\bar{y}A = \int \bar{y}_{EL}\,dA: \quad \bar{y}\left(\frac{5}{6}ab\right) - \frac{13}{70}ab^2 \qquad \text{or} \quad \bar{y} = -\frac{39}{175}b \ \blacktriangleleft$$

PROBLEM 5.38



Determine by direct integration the centroid of the area shown. Express your answer in terms of *a* and *b*.

SOLUTION



At $$x = 0, y = b$$

$$b = k(0-a)^2 \qquad \text{or} \qquad k = \frac{b}{a^2}$$

Then $$y = \frac{b}{a^2}(x-a)^2$$

Now $$\bar{x}_{EL} = x,\ \ \bar{y}_{EL} = \frac{y}{2} = \frac{b}{2a^2}(x-a)^2$$

and $$dA = y\,dx = \frac{b}{a^2}(x-a)^2\,dx$$

Then $$A = \int dA = \int_0^a \frac{b}{a^2}(x-a)^2\,dx = \frac{b}{3a^2}\left[(x-a)^3\right]_0^a = \frac{1}{3}ab$$

and $$\int \bar{x}_{EL}\,dA = \int_0^a x\left[\frac{b}{a^2}(x-a)^2\,dx\right] = \frac{b}{a^2}\int_0^a \left(x^3 - 2ax^2 + a^2x\right)dx$$

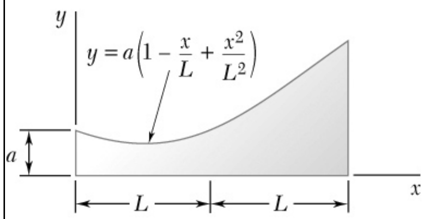
$$= \frac{b}{a^2}\left(\frac{x^4}{4} - \frac{2}{3}ax^3 + \frac{a^2}{2}x^2\right) = \frac{1}{12}a^2b$$

$$\int \bar{y}_{EL}\,dA = \int_0^a \frac{b}{2a^2}(x-a)^2\left[\frac{b}{a^2}(x-a)^2\,dx\right] = \frac{b^2}{2a^4}\left[\frac{1}{5}(x-a)^5\right]_0^a$$

$$= \frac{1}{10}ab^2$$

Hence $$\bar{x}A = \int \bar{x}_{EL}\,dA: \quad \bar{x}\left(\frac{1}{3}ab\right) = \frac{1}{12}a^2b \qquad \bar{x} = \frac{1}{4}a \ \blacktriangleleft$$

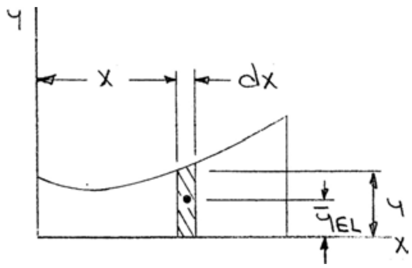
$$\bar{y}A = \int \bar{y}_{EL}\,dA: \quad \bar{y}\left(\frac{1}{3}ab\right) = \frac{1}{10}ab^2 \qquad \bar{y} = \frac{3}{10}b \ \blacktriangleleft$$



PROBLEM 5.39

Determine by direct integration the centroid of the area shown.

SOLUTION



Have $\bar{x}_{EL} = x$

$$\bar{y}_{EL} = \frac{1}{2}y = \frac{a}{2}\left(1 - \frac{x}{L} + \frac{x^2}{L^2}\right)$$

$$dA = ydx = a\left(1 - \frac{x}{L} + \frac{x^2}{L^2}\right)dx$$

Then $A = \int dA = \int_0^{2L} a\left(1 - \frac{x}{L} + \frac{x^2}{L^2}\right)dx = a\left[x - \frac{x^2}{2L} + \frac{x^3}{3L^2}\right]_0^{2L} = \frac{8}{3}aL$

and $\int \bar{x}_{EL}dA = \int_0^{2L} x\left[a\left(1 - \frac{x}{L} + \frac{x^2}{L^2}\right)dx\right] = a\left[\frac{x^2}{2} - \frac{x^3}{3L} + \frac{x^4}{4L^2}\right]_0^{2L}$

$$= \frac{10}{3}aL^2$$

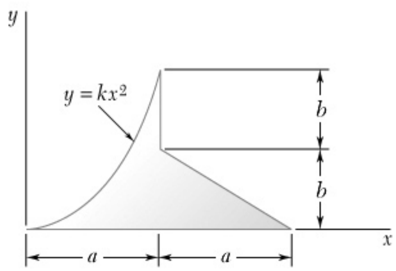
$$\int \bar{y}_{EL}dA = \int_0^{2L} \frac{a}{2}\left(1 - \frac{x}{L} + \frac{x^2}{L^2}\right)\left[a\left(1 - \frac{x}{L} + \frac{x^2}{L^2}\right)dx\right]$$

$$= \frac{a^2}{2}\int_0^{EL}\left(1 - 2\frac{x}{L} + 3\frac{x^2}{L^2} - 2\frac{x^3}{L^3} + \frac{x^4}{L^4}\right)dx$$

$$= \frac{a^2}{2}\left[x - \frac{x^2}{L} + \frac{x^3}{L^2} - \frac{x^4}{2L^3} + \frac{x^5}{5L^4}\right]_0^{2L} = \frac{11}{5}a^2L$$

Hence $\bar{x}A = \int \bar{x}_{EL}dA: \quad \bar{x}\left(\frac{8}{3}aL\right) = \frac{10}{3}aL^2$ $\quad \bar{x} = \frac{5}{4}L$ ◀

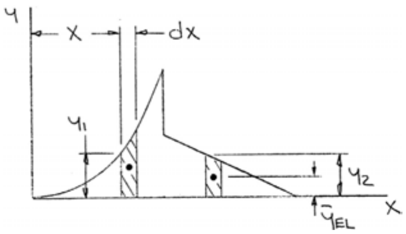
$\bar{y}A = \int \bar{y}_{EL}dA: \quad \bar{y}\left(\frac{1}{8}a\right) = \frac{11}{5}a^2$ $\quad \bar{y} = \frac{33}{40}a$ ◀



PROBLEM 5.40

Determine by direct integration the centroid of the area shown. Express your answer in terms of *a* and *b*.

SOLUTION



For y_1 at $x = a,\ y = 2b$ $\quad 2b = ka^2 \quad$ or $\quad k = \dfrac{2b}{a^2}$

Then
$$y_1 = \frac{2b}{a^2}x^2$$

By observation
$$y_2 = -\frac{b}{a}(x + 2b) = b\left(2 - \frac{x}{a}\right)$$

Now
$$\overline{x}_{EL} = x$$

and for $0 \le x \le a$:

$$\overline{y}_{EL} = \frac{1}{2}y_1 = \frac{b}{a^2}x^2 \quad \text{and} \quad dA = y_1 dx = \frac{2b}{a^2}x^2 dx$$

For $a \le x \le 2a$:

$$\overline{y}_{EL} = \frac{1}{2}y_2 = \frac{b}{2}\left(2 - \frac{x}{a}\right) \quad \text{and} \quad dA = y_2 dx = b\left(2 - \frac{x}{a}\right)dx$$

Then
$$A = \int dA = \int_0^a \frac{2b}{a^2}x^2 dx + \int_a^{2a} b\left(2 - \frac{x}{a}\right)dx$$

$$= \frac{2b}{a^2}\left[\frac{x^3}{3}\right]_0^a + b\left[-\frac{a}{2}\left(2 - \frac{x}{a}\right)^2\right]_0^{2a} = \frac{7}{6}ab$$

and
$$\int \overline{x}_{EL} dA = \int_0^a x\left(\frac{2b}{a^2}x^2 dx\right) + \int_a^{2a} x\left[b\left(2 - \frac{x}{a}\right)dx\right]$$

$$= \frac{2b}{a^2}\left[\frac{x^4}{4}\right]_0^a + b\left[x^2 - \frac{x^3}{3a}\right]_0^{2a}$$

$$= \frac{1}{2}a^2 b + b\left\{\left[(2a)^2 - (a)^2\right] + \frac{1}{3a}\left[\left(2a^2\right) - (a)^3\right]\right\}$$

$$= \frac{7}{6}a^2 b$$

PROBLEM 5.40 CONTINUED

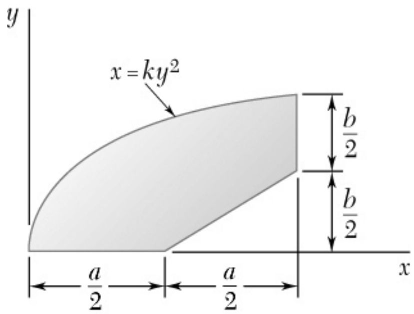
$$\int \bar{y}_{EL} dA = \int_0^a \frac{b}{a^2}x^2\left[\frac{2b}{a^2}x^2 dx\right] + \int_0^{2a}\frac{b}{2}\left(2-\frac{x}{a}\right)\left[b\left(2-\frac{x}{a}\right)dx\right]$$

$$= \frac{2b^2}{a^4}\left[\frac{x^5}{5}\right]_0^a + \frac{b^2}{2}\left[-\frac{a}{3}\left(2-\frac{x}{a}\right)^3\right]_a^{2a}$$

$$= \frac{17}{30}ab^2$$

Hence $\qquad \bar{x}A = \int \bar{x}_{EL} dA: \quad \bar{x}\left(\frac{7}{6}ab\right) = \frac{7}{6}a^2b \qquad \bar{x} = a$ ◀

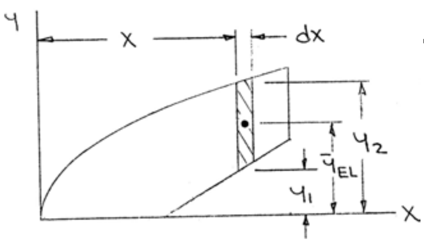
$$\bar{y}A = \int \bar{y}_{EL} dA: \quad \bar{y}\left(\frac{7}{6}ab\right) = \frac{17}{30}ab^2 \qquad \bar{y} = \frac{17}{35}b \; ◀$$



PROBLEM 5.41

Determine by direct integration the centroid of the area shown. Express your answer in terms of *a* and *b*.

SOLUTION



For y_2 at $x = a,\ y = b:\ a = kb^2$ or $k = \dfrac{a}{b^2}$

Then $$y_2 = \frac{b}{\sqrt{a}}x^{1/2}$$

Now $$\overline{x}_{EL} = x$$

and for $0 \le x \le \dfrac{a}{2}$: $\overline{y}_{EL} = \dfrac{y_2}{2} = \dfrac{b}{2}\dfrac{x^{1/2}}{\sqrt{a}},\ dA = y_2 dx = b\dfrac{x^{1/2}}{\sqrt{a}}dx$

For $\dfrac{a}{2} \le x \le a$: $\overline{y}_{EL} = \dfrac{1}{2}(y_1 + y_2) = \dfrac{b}{2}\left(\dfrac{x}{a} - \dfrac{1}{2} + \dfrac{x^{1/2}}{\sqrt{a}}\right)$

$$dA = (y_2 - y_1)dx = b\left(\frac{x^{1/2}}{\sqrt{a}} - \frac{x}{a} + \frac{1}{2}\right)dx$$

Then $$A = \int dA = \int_0^{a/2} b\frac{x^{1/2}}{\sqrt{a}}dx + \int_{a/2}^{a} b\left(\frac{x^{1/2}}{\sqrt{a}} - \frac{x}{a} + \frac{1}{2}\right)dx$$

$$= \frac{b}{\sqrt{a}}\left[\frac{2}{3}x^{3/2}\right]_0^{a/2} + b\left[\frac{2}{3}\frac{x^{3/2}}{\sqrt{a}} - \frac{x^2}{2a} + \frac{1}{2}x\right]_{a/2}^{a}$$

$$= \frac{2}{3}\frac{b}{\sqrt{a}}\left[\left(\frac{a}{2}\right)^{3/2} + (a)^{3/2} - \left(\frac{a}{2}\right)^{3/2}\right]$$

$$+ b\left\{-\frac{1}{2a}\left[(a^2) - \left(\frac{a}{2}\right)^2\right] + \frac{1}{2}\left[(a) - \left(\frac{a}{2}\right)\right]\right\}$$

$$= \frac{13}{24}ab$$

PROBLEM 5.41 CONTINUED

and $$\int \overline{x}_{EL}\,dA = \int_0^{a/2} x\left(b\frac{x^{1/2}}{\sqrt{a}}dx\right) + \int_{a/2}^{a} x\left[b\left(\frac{x^{1/2}}{\sqrt{a}} - \frac{x}{a} + \frac{1}{2}\right)\right]dx$$

$$= \frac{b}{\sqrt{a}}\left[\frac{2}{5}x^{5/2}\right]_0^{a/2} + b\left[\frac{2}{5}\frac{x^{5/2}}{\sqrt{a}} - \frac{x^3}{3a} + \frac{x^4}{4}\right]_{a/2}^{a}$$

$$= \frac{2}{5}\frac{b}{\sqrt{a}}\left[\left(\frac{a}{2}\right)^{5/2} + (a)^{5/2} - \left(\frac{a}{2}\right)^{5/2}\right]$$

$$+ b\left\{-\frac{1}{3a}\left[(a)^3 - \left(\frac{a}{2}\right)^3\right] + \frac{1}{4}\left[(a)^2 - \left(\frac{a}{2}\right)^2\right]\right\}$$

$$= \frac{71}{240}a^2 b$$

$$\int \overline{y}_{EL}\,dA = \int_0^{a/2} \frac{b}{2}\frac{x^{1/2}}{\sqrt{a}}\left[b\frac{x^{1/2}}{\sqrt{a}}dx\right]$$

$$+ \int_{a/2}^{a} \frac{b}{2}\left(\frac{x}{a} - \frac{1}{2} + \frac{x^{1/2}}{\sqrt{a}}\right)\left[b\left(\frac{x^{1/2}}{\sqrt{a}} - \frac{x}{a} + \frac{1}{2}\right)dx\right]$$

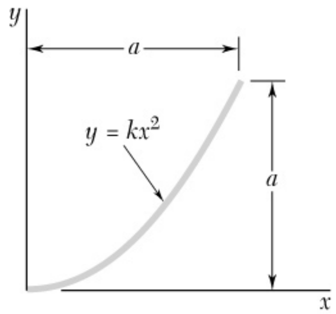
$$= \frac{b^2}{2a}\left[\frac{1}{2}x^2\right]_0^{a/2} + \frac{b^2}{2}\left[\left(\frac{x^2}{2a} - \frac{1}{3a}\left(\frac{x}{a} - \frac{1}{2}\right)^3\right)\right]_{a/2}^{a}$$

$$= \frac{b}{4a}\left[\left(\frac{a}{2}\right)^2 + (a)^2 - \left(\frac{a}{2}\right)^2\right] - \frac{b^2}{6a}\left(\frac{a}{2} - \frac{1}{2}\right)^3$$

$$= \frac{11}{48}ab^2$$

Hence $\overline{x}A = \int \overline{x}_{EL}\,dA$: $\overline{x}\left(\frac{13}{24}ab\right) = \frac{71}{240}a^2 b$ $\qquad \overline{x} = \frac{17}{130}a = 0.546a$ ◀

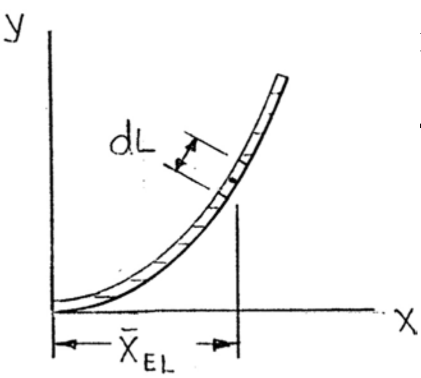
$\overline{y}A = \int \overline{y}_{EL}\,dA$: $\overline{y}\left(\frac{13}{24}ab\right) = \frac{11}{48}ab^2$ $\qquad \overline{y} = \frac{11}{26}b = 0.423b$ ◀



PROBLEM 5.42

A homogeneous wire is bent into the shape shown. Determine by direct integration the x coordinate of its centroid. Express your answer in terms of a.

SOLUTION



First note that because the wire is homogeneous, its center of gravity coincides with the centroid of the corresponding line

Have at $\quad x = a,\ y = a:\ a = ka^2 \qquad \text{or} \qquad k = \dfrac{1}{a}$

Thus $\quad y = \dfrac{1}{a}x^2 \qquad \text{and} \qquad dy = \dfrac{2}{a}xdx$

Then $\quad dL = \sqrt{1 + \left(\dfrac{dy}{dx}\right)^2}dx = \sqrt{1 + \left(\dfrac{2}{a}x\right)^2}dx$

$$\therefore \quad L = \int dL = \int_0^a \sqrt{1 + \frac{4}{a^2}x^2}dx = \left[\frac{x}{2}\sqrt{1 + \frac{4x^2}{a^2}} + \frac{a}{4}\ln\left(\frac{2}{a}x + \sqrt{1 + \frac{4x^2}{a^2}}\right)\right]_0^a$$

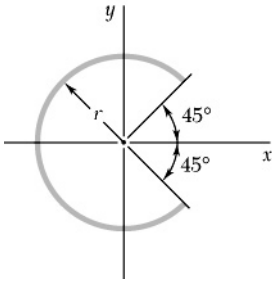
$$= \frac{a}{2}\sqrt{5} + \frac{a}{4}\ln\left(2 + \sqrt{5}\right) = 1.4789a$$

$$\int \overline{x}_{EL}dL = \int_0^a x\left(\sqrt{1 + \frac{4x^2}{a^2}}dx\right) = \left[\frac{2}{3}\left(\frac{a^2}{8}\right)\left(1 + \frac{4}{a^2}x^2\right)^{3/2}\right]_0^a$$

$$= \frac{a^2}{12}\left(5^{3/2} - 1\right) = 0.8484a^2$$

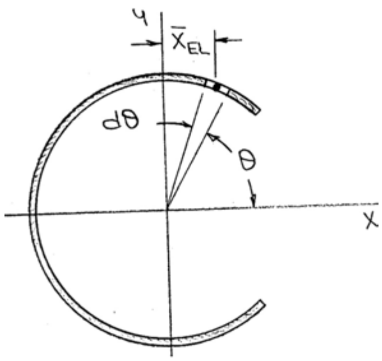
Then $\quad \overline{x}L = \int \overline{x}_{EL}dL: \quad \overline{x}\left(1.4789a\right) = 0.8484a^2 \qquad \overline{x} = 0.574a$ ◀

PROBLEM 5.43



A homogeneous wire is bent into the shape shown. Determine by direct integration the x coordinate of its centroid.

SOLUTION



First note that because the wire is homogeneous, its center of gravity coincides with the centroid of the corresponding line

Now $\qquad \overline{x}_{EL} = r\cos\theta \qquad$ and $\qquad dL = r\,d\theta$

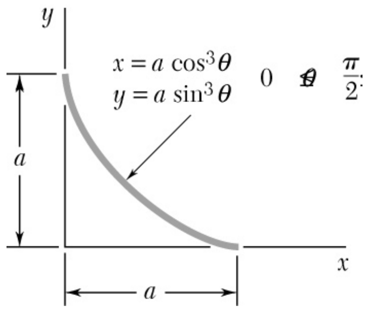
Then $\qquad L = \int dL = \int_{\pi/4}^{7\pi/4} r\,d\theta = r\left[\theta\right]_{\pi/4}^{7\pi/4} = \dfrac{3}{2}\pi r$

and $\qquad \int \overline{x}_{EL}\,dL = \int_{\pi/4}^{7\pi/4} r\cos\theta\left(r\,d\theta\right)$

$$= r^2\left[\sin\theta\right]_{\pi/4}^{7\pi/4} = r^2\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) = -r^2\sqrt{2}$$

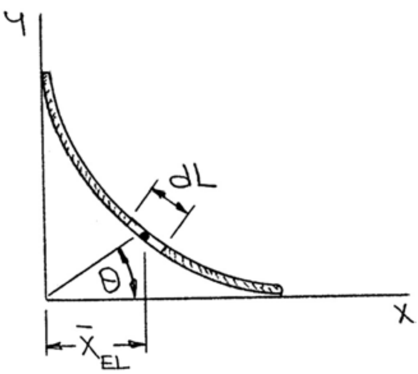
Thus $\qquad \overline{x}L = \int \overline{x}\,dL: \quad \overline{x}\left(\dfrac{3}{2}\pi r\right) = -r^2\sqrt{2} \qquad\qquad \overline{x} = -\dfrac{2\sqrt{2}}{3\pi}r$ ◀

PROBLEM 5.44



A homogeneous wire is bent into the shape shown. Determine by direct integration the *x* coordinate of its centroid.

SOLUTION



First note that because the wire is homogeneous, its center of gravity coincides with the centroid of the corresponding line

Now $\quad \overline{x}_{EL} = a\cos^3\theta \quad$ and $\quad dL = \sqrt{dx^2 + dy^2}$

Where $\quad x = a\cos^3\theta: \quad dx = -3a\cos^2\theta\sin\theta d\theta$

$$y = a\sin^3\theta: \quad dy = 3a\sin^2\theta\cos\theta d\theta$$

Then
$$dL = \left[\left(-3a\cos^2\theta\sin\theta d\theta\right)^2 + \left(3a\sin^2\theta\cos\theta d\theta\right)^2\right]^{1/2}$$

$$= 3a\cos\theta\sin\theta\left(\cos^2\theta + \sin^2\theta\right)^{1/2} d\theta$$

$$= 3a\cos\theta\sin\theta d\theta$$

$$\therefore \quad L = \int dL = \int_0^{\pi/2} 3a\cos\theta\sin\theta d\theta = 3a\left[\frac{1}{2}\sin^2\theta\right]_0^{\pi/2}$$

$$= \frac{3}{2}a$$

and
$$\int \overline{x}_{EL} dL = \int_0^{\pi/2} a\cos^3\theta\left(3a\cos\theta\sin\theta d\theta\right)$$

$$= 3a^2\left[-\frac{1}{5}\cos^5\theta\right]_0^{\pi/2} = \frac{3}{5}a^2$$

Hence $\quad \overline{x}L = \int \overline{x}_{EL} dL: \quad \overline{x}\left(\frac{3}{2}a\right) = \frac{3}{5}a^2 \qquad \overline{x} = \frac{2}{5}a$ ◀

PROBLEM 5.44 CONTINUED

Alternative solution

$$x = a\cos^3\theta \;\Rightarrow\; \cos^2\theta = \left(\frac{x}{a}\right)^{2/3}$$

$$y = a\sin^3\theta \;\Rightarrow\; \sin^2\theta = \left(\frac{y}{a}\right)^{2/3}$$

$$\therefore \quad \left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{a}\right)^{2/3} = 1 \qquad \text{or} \qquad y = \left(a^{2/3} - x^{2/3}\right)^{3/2}$$

Then $$\frac{dy}{dx} = \left(a^{2/3} - x^{2/3}\right)^{1/2}\left(-x^{-1/3}\right)$$

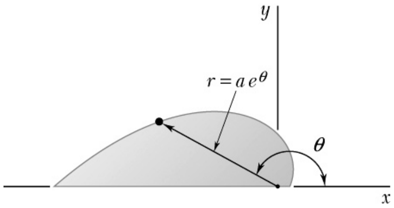
Now $$\overline{x}_{EL} = x$$

and $$dL = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx = \left\{1 + \left[\left(a^{2/3} - x^{2/3}\right)^{1/2}\left(-x^{-1/3}\right)\right]^2\right\}^{1/2} dx$$

Then $$L = \int dL = \int_0^a \frac{a^{1/3}}{x^{1/3}}dx = a^{1/3}\left[\frac{3}{2}x^{2/3}\right]_0^a = \frac{3}{2}a$$

and $$\int \overline{x}_{EL}dL = \int_0^a x\left(\frac{a^{1/3}}{x^{1/3}}dx\right) = a^{1/3}\left[\frac{3}{5}x^{5/3}\right]_0^a = \frac{3}{5}a^2$$

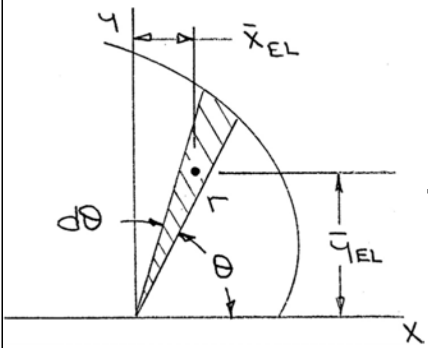
Hence $$\overline{x}L = \int \overline{x}_{EL}dL: \quad \overline{x}\left(\frac{3}{2}a\right) = \frac{3}{5}a^2 \qquad \overline{x} = \frac{2}{5}a \;\blacktriangleleft$$



PROBLEM 5.45

Determine by direct integration the centroid of the area shown.

SOLUTION



Have $\quad \bar{x}_{EL} = \dfrac{2}{3} r\cos\theta = \dfrac{2}{3} ae^{\theta}\cos\theta$

$$\bar{y}_{EL} = \frac{2}{3} r\sin\theta = \frac{2}{3} ae^{\theta}\sin\theta$$

and $\quad dA = \dfrac{1}{2}(r)(rd\theta) = \dfrac{1}{2}a^2e^{2\theta}d\theta$

Then

$$A = \int dA = \int_0^{\pi} \frac{1}{2}a^2e^{2\theta}d\theta = \frac{1}{2}a^2\left[\frac{1}{2}e^{2\theta}\right]_0^{\pi} = \frac{1}{4}a^2\left(e^{2\pi} - 1\right) = 133.623a^2$$

and $\quad \displaystyle\int \bar{x}_{EL}dA = \int_0^{\pi} \frac{2}{3}ae^{\theta}\cos\theta\left(\frac{1}{2}a^2e^{2\theta}d\theta\right) = \frac{1}{3}a^3\int_0^{\pi} e^{3\theta}\cos\theta\, d\theta$

To proceed, use integration by parts, with

$$u = e^{3\theta} \quad \text{and} \quad du = 3e^{3\theta}d\theta$$

$$dv = \cos\theta d\theta \quad \text{and} \quad v = \sin\theta$$

Then $\quad \displaystyle\int e^{3\theta}\cos\theta d\theta = e^{3\theta}\sin\theta - \int \sin\theta\left(3e^{3\theta}d\theta\right)$

Now let $\quad u = e^{3\theta} \quad \text{then} \quad du = 3e^{3\theta}d\theta$

$$dv = \sin\theta d\theta, \quad \text{then} \quad v = -\cos\theta$$

Then $\quad \displaystyle\int e^{3\theta}\sin\theta d\theta = e^{3\theta}\sin\theta - 3\left[-e^{-3\theta}\cos\theta - \int(-\cos\theta)\left(3e^{3\theta}d\theta\right)\right]$

So that $\quad \displaystyle\int e^{3\theta}\cos\theta d\theta = \frac{e^{3\theta}}{10}(\sin\theta + 3\cos\theta)$

$$\therefore \quad \int \bar{x}_{EL}dA = \frac{1}{3}a^3\left[\frac{e^{3\theta}}{10}(\sin\theta + 3\cos\theta)\right]_0^{\pi} = \frac{a^3}{30}\left(-3e^{3\pi} - 3\right) = -1239.26a^3$$

Also $\quad \displaystyle\int \bar{y}_{EL}dA = \int_0^{\pi} \frac{2}{3}ae^{\theta}\sin\theta\left(\frac{1}{2}a^2e^{2\theta}d\theta\right) = \frac{1}{3}a^3\int_0^{\pi} e^{3\theta}\sin\theta\, d\theta$

PROBLEM 5.45 CONTINUED

Using integration by parts, as above, with

$$u = e^{3\theta} \quad \text{and} \quad du = 3e^{3\theta}d\theta$$

$$dv = \int \sin\theta d\theta \quad \text{and} \quad v = -\cos\theta$$

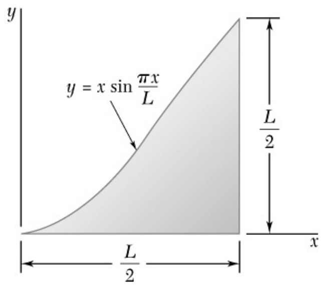
Then $\int e^{3\theta}\sin\theta d\theta = -e^{3\theta}\cos\theta - \int(-\cos\theta)\left(3e^{3\theta}d\theta\right)$

So that $\int e^{3\theta}\sin\theta d\theta = \dfrac{e^{3\theta}}{10}(-\cos\theta + 3\sin\theta)$

$$\therefore \quad \int \overline{y}_{EL}dA = \frac{1}{3}a^3\left[\frac{e^{3\theta}}{10}(-\cos\theta + 3\sin\theta)\right]_0^{\pi} = \frac{a^3}{30}\left(e^{3\pi} + 1\right) = 413.09a^3$$

Hence $\overline{x}A = \int \overline{x}_{EL}dA$: $\quad \overline{x}\left(133.623a^2\right) = -1239.26a^3 \quad$ or $\quad \overline{x} = -9.27a$ ◀

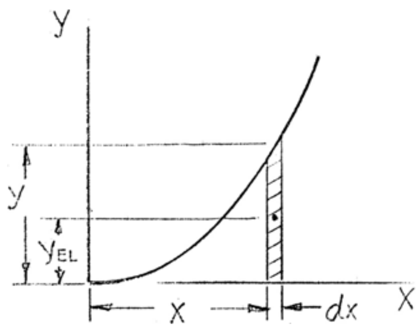
$\overline{y}A = \int \overline{y}_{EL}dA$: $\quad \overline{y}\left(133.623a^2\right) = 413.09a^3 \quad$ or $\quad \overline{y} = 3.09a$ ◀



PROBLEM 5.46

Determine by direct integration the centroid of the area shown.

SOLUTION



Have $$\bar{x}_{EL} = x, \quad \bar{y}_{EL} = \frac{1}{2} x \sin \frac{\pi x}{L}$$

and $$dA = ydx$$

$$A = \int dA = \int_0^{L/2} x \sin \frac{\pi x}{L} dx = \left[\frac{L^2}{\pi^2} \sin \frac{\pi x}{L} - \frac{L}{\pi} x \cos \frac{\pi x}{L} \right]_0^{L/2} = \frac{L^2}{\pi^2}$$

and $$\bar{x} = \int \bar{x}_{EL} dA = \int_0^{L/2} x \left(x \sin \frac{\pi x}{L} dx \right)$$

$$= \left[\frac{2L^2}{\pi^2} x \sin \left(\frac{\pi x}{L} \right) + \frac{2L^3}{\pi^3} \cos \left(\frac{\pi x}{L} \right) - \frac{L}{\pi} x^2 \sin \left(\frac{\pi x}{L} \right) \right]_0^{L/2} = \frac{L^3}{\pi^2} - 2 \frac{L^3}{\pi^3}$$

Also $$\bar{y} = \int \bar{y}_{EL} dA = \int_0^{L/2} \frac{1}{2} x \sin \frac{\pi x}{L} \left(x \sin \frac{\pi x}{L} dx \right)$$

$$= \frac{1}{2} \left[\frac{2L^2}{\pi^2} x \sin \frac{\pi x}{L} - \left(\frac{L}{\pi} x - \frac{2L^3}{\pi^3} \right) \cos \frac{\pi x}{L} \right]_0^{L/2}$$

$$= \frac{1}{2} \left[\frac{1}{6} \left(\frac{L^3}{8} \right) - \frac{L^2}{4\pi^2} \left(\frac{L}{2} \right) (-1) \right] = \frac{L^3}{96\pi^2} \left(6 + \pi^2 \right)$$

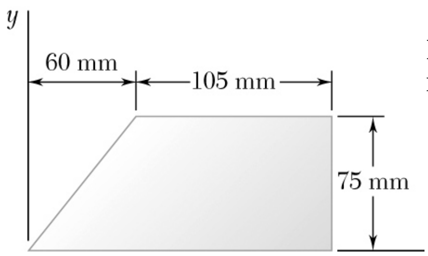
PROBLEM 5.46 CONTINUED

Hence $$\bar{x}A = \int \bar{x}_{EL}dA: \quad \bar{x}\left(\frac{L^2}{\pi^2}\right) = L^3\left(\frac{1}{\pi^2} - \frac{z}{\pi^3}\right)$$

or $\bar{x} = 0.363L$ ◀

$$\bar{y}A = \int \bar{y}_{EL}dA: \quad \bar{y}\left(\frac{L^2}{\pi^2}\right) = \frac{L^3}{96\pi^2}\left(\frac{1}{\pi^2} - \frac{2}{\pi^3}\right)$$

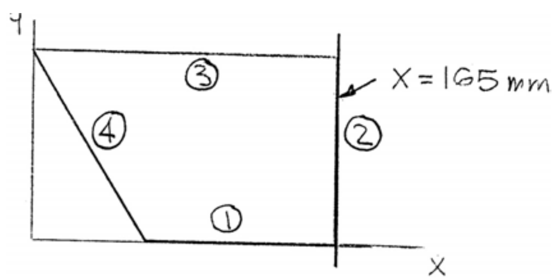
or $\bar{y} = 0.1653L$ ◀



PROBLEM 5.47

Determine the volume and the surface area of the solid obtained by rotating the area of Problem. 5.2 about (*a*) the *x* axis, (*b*) the line $x = 165$ mm.

SOLUTION



From the solution to Problem 5.2:

$$A = 10\,125\text{ mm}^2,\ \bar{X}_{\text{area}} = 96.4\text{ mm},\ \bar{Y}_{\text{area}} = 34.7\text{ mm} \qquad (\text{Area})$$

From the solution to Problem 5.22:

$$L = 441.05\text{ mm}\ \bar{X}_{\text{line}} = 92.2\text{ mm},\ \bar{Y}_{\text{line}} = 32.4\text{ mm} \qquad (\text{Line})$$

Applying the theorems of Pappus-Guldinus, we have

(*a*) Rotation about the *x* axis:

$$\text{Area} = 2\pi\,\bar{Y}_{\text{line}}\,L = 2\pi(32.4\text{ mm})(441.05\text{ mm}) = 89.786\times10^3\text{ mm}^2$$

$$A = 89.8\times10^3\text{ mm}^2 \blacktriangleleft$$

$$\text{Volume} = 2\pi\bar{Y}_{\text{area}}A = 2\pi(34.7\text{ mm})(10\,125\text{ mm}) = 2.2075\times10^6\text{ mm}^3$$

$$V = 2.21\times10^6\text{ mm}^3 \blacktriangleleft$$

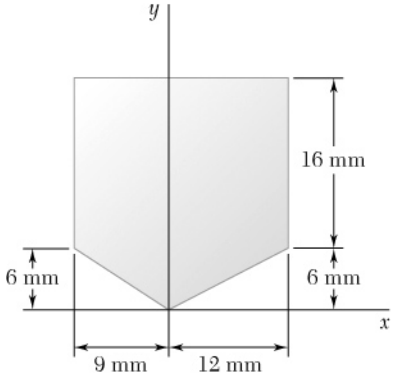
(*b*) Rotation about $x = 165$ mm:

$$\text{Area} = 2\pi(165 - \bar{X}_{\text{line}})L = 2\pi[(165 - 92.2)\text{ mm}](441.05\text{ mm}) = 2.01774\times10^5\text{ mm}^2$$

$$A = 0.202\times10^6\text{ mm}^2 \blacktriangleleft$$

$$\text{Volume} = 2\pi(165 - \bar{X}_{\text{area}})A = 2\pi[(165 - 96.4)\text{ mm}](10\,125\text{ mm}) = 4.3641\times10^6\text{ mm}^3$$

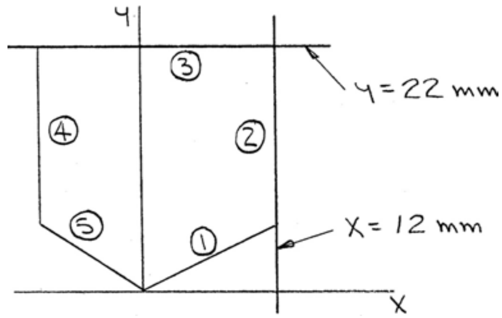
$$V = 4.36\times10^6\text{ mm}^3 \blacktriangleleft$$



PROBLEM 5.48

Determine the volume and the surface area of the solid obtained by rotating the area of Problem 5.4 about (*a*) the line $y = 22$ mm, (*b*) the line $x = 12$ mm.

SOLUTION



From the solution to Problem 5.4:

$$A = 399\text{ mm}^2,\ \overline{X}_{\text{area}} = 1.421\text{ mm},\ \overline{Y}_{\text{area}} = 12.42\text{ mm} \qquad \text{(Area)}$$

From the solution to Problem 5.23:

$$L = 77.233\text{ mm},\ \overline{X}_{\text{line}} = 1.441\text{ mm},\ \overline{Y}_{\text{line}} = 12.72\text{ mm} \qquad \text{(Line)}$$

Applying the theorems of Pappus-Guldinus, we have

(*a*) Rotation about the line $y = 22$ mm:

$$\text{Area} = 2\pi\left(22 - \overline{Y}_{\text{line}}\right)L = 2\pi\left[(22 - 12.72)\text{ mm}\right](77.233\text{ mm}) = 4503\text{ mm}^2$$

$$A = 4.50 \times 10^3\text{ mm}^2 \blacktriangleleft$$

$$\text{Volume} = 2\pi\left(22 - \overline{Y}_{\text{area}}\right)A = 2\pi\left[(22 - 12.42)\text{ mm}\right]\left(399\text{ mm}^2\right) = 24\ 016.97\text{ mm}^3$$

$$V = 24.0 \times 10^3\text{ mm}^3 \blacktriangleleft$$

(*b*) Rotation about line $x = 12$ mm:

$$\text{Area} = 2\pi\left(12 - \overline{X}_{\text{line}}\right)L = 2\pi\left[(12 - 1.441)\text{ mm}\right](77.233\text{ mm}) = 5124.45\text{ mm}^2$$

$$A = 5.12 \times 10^3\text{ mm}^2 \blacktriangleleft$$

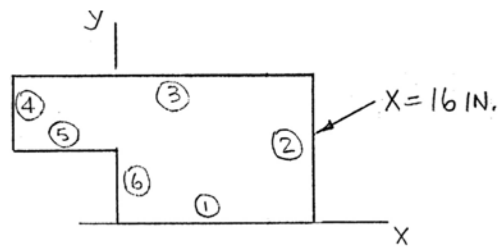
$$\text{Volume} = 2\pi(12 - 1.421)A = 2\pi\left[(12 - 1.421)\text{ mm}\right]\left(399\text{ mm}^2\right) = 26\ 521.46\text{ mm}^3$$

$$V = 26.5 \times 10^3\text{ mm}^3 \blacktriangleleft$$

PROBLEM 5.49

Determine the volume and the surface area of the solid obtained by rotating the area of Problem 5.1 about (*a*) the *x* axis, (*b*) the line $x = 16$ in.

SOLUTION



From the solution to Problem 5.1:

$$A = 240 \text{ in}^2, \bar{X}_{\text{area}} = 5.60 \text{ in.}, \bar{Y}_{\text{area}} = 6.60 \text{ in.} \qquad \text{(Area)}$$

From the solution to Problem 5.21:

$$L = 72 \text{ in.}, \bar{X}_{\text{line}} = 4.67 \text{ in.}, \bar{Y}_{\text{line}} = 6.67 \text{ in.}$$

Applying the theorems of Pappus-Guldinus, we have

(*a*) Rotation about the *x* axis:

$$A_x = 2\pi Y_{\text{line}} L = 2\pi(6.67 \text{ in.})(72 \text{ in.}) = 3017.4 \text{ in}^2$$

$$A = 3020 \text{ in}^2 \blacktriangleleft$$

$$V_x = 2\pi Y_{\text{area}} A = 2\pi(6.60 \text{ in.})(240 \text{ in}^2) = 9952.6 \text{ in}^3$$

$$V = 9950 \text{ in}^3 \blacktriangleleft$$

(*b*) Rotation about $x = 16$ in.:

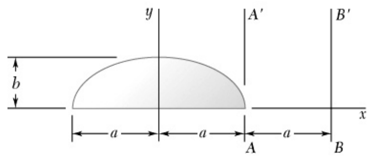
$$A_{x=16} = 2\pi(16 - \bar{X}_{\text{line}})L = 2\pi[(16 - 4.67) \text{ in.}](72 \text{ in.}) = 5125.6 \text{ in}^2$$

$$A_{x=16} = 5130 \text{ in}^2 \blacktriangleleft$$

$$V_{x=16} = 2\pi(16 - \bar{X}_{\text{area}})A = 2\pi[(16 - 5.60) \text{ in.}](240 \text{ in}^2) = 15\,682.8 \text{ in}^3$$

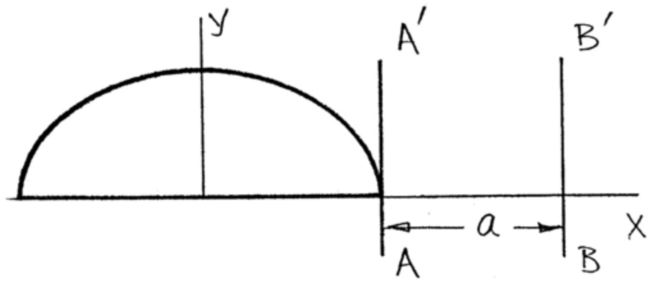
$$V_{x=16} = 15.68 \times 10^3 \text{ in}^3 \blacktriangleleft$$

PROBLEM 5.50



Determine the volume of the solid generated by rotating the semielliptical area shown about (*a*) the axis AA', (*b*) the axis BB', (*c*) the *y* axis.

SOLUTION



Applying the second theorem of Pappus-Guldinus, we have

(*a*) Rotation about axis AA':

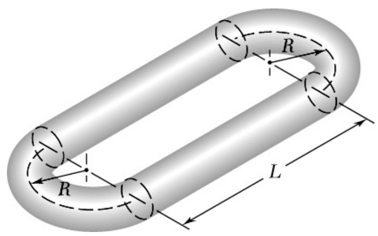
$$\text{Volume} = 2\pi\bar{y}A = 2\pi(a)\left(\frac{\pi ab}{2}\right) = \pi^2a^2b \qquad V = \pi^2a^2b \blacktriangleleft$$

(*b*) Rotation about axis BB':

$$\text{Volume} = 2\pi\bar{y}A = 2\pi(2a)\left(\frac{\pi ab}{2}\right) = 2\pi^2a^2b \qquad V = 2\pi^2a^2b \blacktriangleleft$$

(*c*) Rotation about *y*-axis:

$$\text{Volume} = 2\pi\bar{y}A = 2\pi\left(\frac{4a}{3\pi}\right)\left(\frac{\pi ab}{2}\right) = \frac{2}{3}\pi a^2b \qquad V = \frac{2}{3}\pi a^2b \blacktriangleleft$$



PROBLEM 5.51

Determine the volume and the surface area of the chain link shown, which is made from a 2-in.-diameter bar, if $R = 3$ in. and $L = 10$ in.

SOLUTION

First note that the area A and the circumference C of the cross section of the bar are

$$A = \frac{\pi}{4}d^2 \qquad \text{and} \qquad C = \pi d$$

Observe that the semicircular ends of the link can be obtained by rotating the cross section through a horizontal semicircular arc of radius R. Then, applying the theorems of Pappus-Guldinus, we have

$$\text{Volume} = 2(V_{\text{side}}) + 2(V_{\text{end}}) = 2(AL) + 2(\pi RA) = 2(L + \pi R)A$$

$$= 2[10\text{ in.} + \pi(3\text{ in.})]\left[\frac{\pi}{4}(2\text{ in.})^2\right]$$

$$= 122.049\text{ in}^3$$

$$V = 122.0\text{ in}^3 \blacktriangleleft$$

$$\text{Area} = 2(A_{\text{side}}) + 2(A_{\text{end}}) = 2(CL) + 2(\pi RC) = 2(L + \pi R)C$$

$$= 2[10\text{ in.} + \pi(3\text{ in.})][\pi(4\text{ in.})]$$

$$= 488.198\text{ in}^2$$

$$A = 488\text{ in}^2 \blacktriangleleft$$

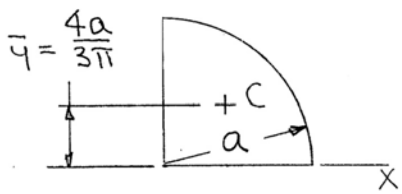
PROBLEM 5.52

Verify that the expressions for the volumes of the first four shapes in Figure 5.21 on page 261 are correct.

SOLUTION

Following the second theorem of Pappus-Guldinus, in each case a specific generating area A will be rotated about the x axis to produce the given shape. Values of $\bar{y}$ are from Fig. 5.8A.

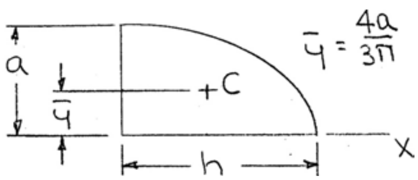
(1) Hemisphere: the generating area is a quarter circle



Have $$V = 2\pi\bar{y}A = 2\pi\left(\frac{4a}{3\pi}\right)\left(\frac{\pi}{4}a^2\right)$$

$$\text{or } V = \frac{2}{3}\pi a^3 \blacktriangleleft$$

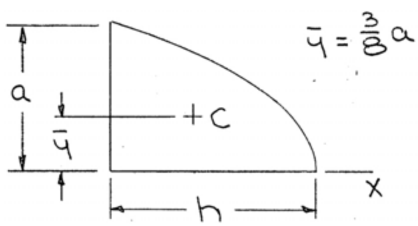
(2) Semiellipsoid of revolution: the generating area is a quarter ellipse



Have $$V = 2\pi\bar{y}A = 2\pi\left(\frac{4a}{3\pi}\right)\left(\frac{\pi}{4}ha\right)$$

$$\text{or } V = \frac{2}{3}\pi a^2 h \blacktriangleleft$$

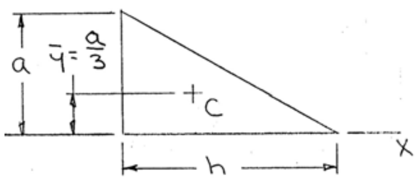
(3) Paraboloid of revolution: the generating area is a quarter parabola



Have $$V = 2\pi\bar{y}A = 2\pi\left(\frac{3}{8}a\right)\left(\frac{2}{3}ah\right)$$

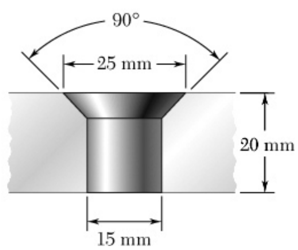
$$\text{or } V = \frac{1}{2}\pi a^2 h \blacktriangleleft$$

(4) Cone: the generating area is a triangle



Have $$V = 2\pi\bar{y}A = 2\pi\left(\frac{a}{3}\right)\left(\frac{1}{2}ha\right)$$

$$\text{or } V = \frac{1}{3}\pi a^2 h \blacktriangleleft$$

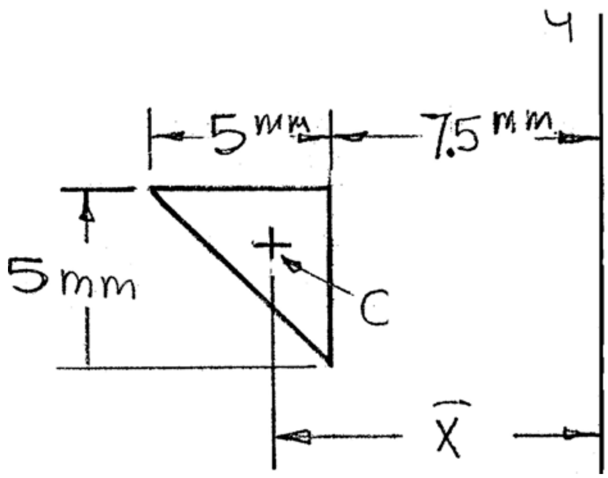


PROBLEM 5.53

A 15-mm-diameter hole is drilled in a piece of 20-mm-thick steel; the hole is then countersunk as shown. Determine the volume of steel removed during the countersinking process.

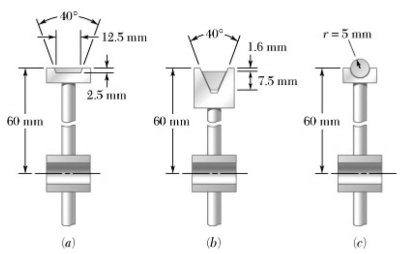
SOLUTION

The required volume can be generated by rotating the area shown about the *y* axis. Applying the second theorem of Pappus-Guldinus, we have



$$V = 2\pi\bar{x}A = 2\pi\left[\left(\frac{5}{3} + 7.5\right)\text{mm}\right] \times \left[\frac{1}{2} \times 5\text{ mm} \times 5\text{ mm}\right]$$

or $V = 720\text{ mm}^3$ ◀



PROBLEM 5.54

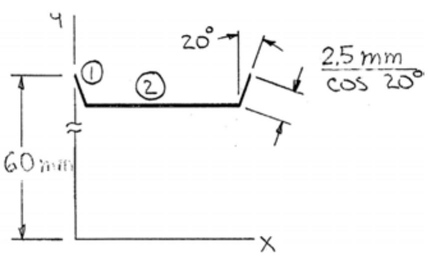
Three different drive belt profiles are to be studied. If at any given time each belt makes contact with one-half of the circumference of its pulley, determine the *contact area* between the belt and the pulley for each design.

SOLUTION

Applying the first theorem of Pappus-Guldinus, the contact area A_C of a belt is given by

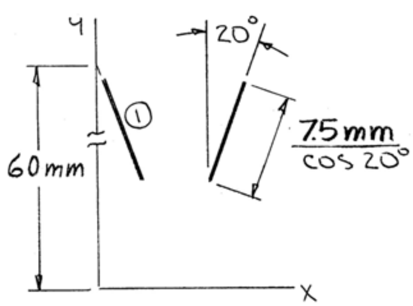
$$A_C = \pi \bar{y} L = \pi \Sigma \bar{y} L$$

Where the individual lengths are the "Lengths" of the belt cross section that are in contact with the pulley



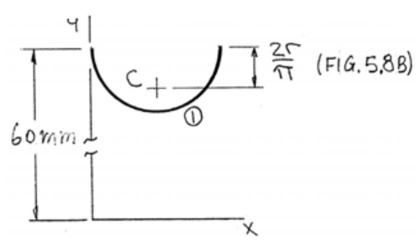
Have
$$A_C = \pi\left[2\left(\bar{y}_1 L_1\right) + \bar{y}_2 L_2\right]$$
$$= \pi\left\{2\left[\left(60 - \frac{2.5}{2}\right)\text{mm}\right]\left[\frac{2.5\text{ mm}}{\cos 20°}\right] + \left[\left(60 - 2.5\right)\text{mm}\right]\left(12.5\text{ mm}\right)\right\}$$

or $A_C = 3.24 \times 10^3\text{ mm}^2$ ◀



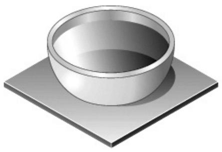
Have
$$A_C = \pi\left[2\left(\bar{y}_1 L_1\right)\right]$$
$$= 2\pi\left[\left(60 - 1.6 - \frac{7.5}{2}\right)\text{mm}\right] \times \left(\frac{7.5\text{ mm}}{\cos 20^\circ}\right)$$

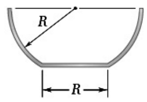
or $A_C = 2.74 \times 10^3\text{ mm}^2$ ◀



Have
$$A_C = \pi\left(\bar{y}_1 L_1\right) = \pi\left[\left(60 - \frac{2 \times 5}{\pi}\right)\text{mm}\right]\left(\pi \times 5\text{ mm}\right)$$

or $A_C = 2.80 \times 10^3\text{ mm}^2$ ◀



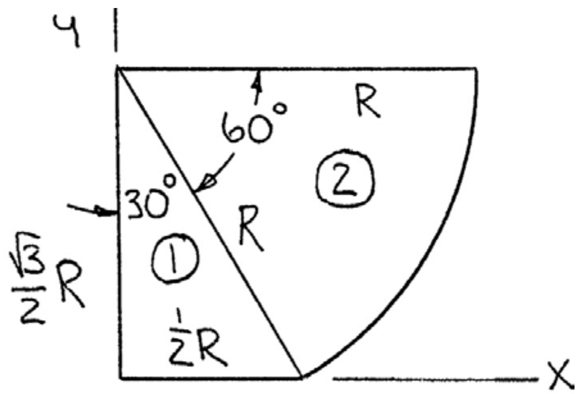


PROBLEM 5.55

Determine the capacity, in gallons, of the punch bowl shown if $R = 12$ in.

SOLUTION

The volume can be generated by rotating the triangle and circular sector shown about the *y* axis. Applying the second theorem of Pappus-Guldinus and using Fig. 5.8A, we have



$$V = 2\pi\bar{x}A = 2\pi\Sigma\bar{x}A = 2\pi\left(\bar{x}_1A_1 + \bar{x}_2A_2\right)$$

$$= 2\pi\left[\left(\frac{1}{3}\times\frac{1}{2}R\right)\left(\frac{1}{2}\times\frac{1}{2}R\times\frac{\sqrt{3}}{2}R\right) + \left(\frac{2R\sin 30^\circ}{3\times\frac{\pi}{6}}\cos 30^\circ\right)\left(\frac{\pi}{6}R^2\right)\right]$$

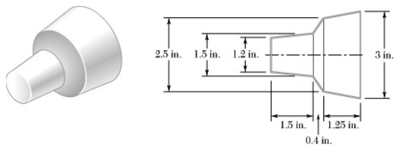
$$= 2\pi\left(\frac{R^3}{16\sqrt{3}} + \frac{R^3}{2\sqrt{3}}\right) = \frac{3\sqrt{3}}{8}\pi R^3$$

$$= \frac{3\sqrt{3}}{8}\pi\left(12\text{ in.}\right)^3 = 3526.03\text{ in}^3$$

Since $$1\text{ gal} = 231\text{ in}^3$$

$$V = \frac{3526.03\text{ in}^3}{231\text{ in}^3/\text{gal}} = 15.26\text{ gal}$$

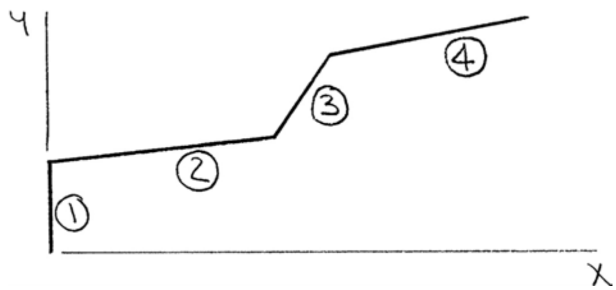
$V = 15.26$ gal ◀



PROBLEM 5.56

The aluminum shade for a small high-intensity lamp has a uniform thickness of 3/32 in. Knowing that the specific weight of aluminum is $0.101\ \text{lb/in}^3$, determine the weight of the shade.

SOLUTION



The weight of the lamp shade is given by

$$W = \gamma V = \gamma A t$$

where A is the surface area of the shade. This area can be generated by rotating the line shown about the x axis. Applying the first theorem of Pappus-Guldinus, we have

$$A = 2\pi \overline{y} L = 2\pi \Sigma \overline{y} L = 2\pi \left(\overline{y}_1 L_1 + \overline{y}_2 L_2 + \overline{y}_3 L_3 + \overline{y}_4 L_4 \right)$$

$$= 2\pi \left[\frac{0.6\ \text{mm}}{2} (0.6\ \text{mm}) + \left(\frac{0.60 + 0.75}{2} \right) \text{mm} \times \sqrt{(0.15\ \text{mm})^2 + (1.5\ \text{mm})^2} \right.$$

$$+ \left(\frac{0.75 + 1.25}{2} \right) \text{mm} \times \sqrt{(0.50\ \text{mm})^2 + (0.40\ \text{mm})^2}$$

$$\left. + \left(\frac{1.25 + 1.5}{2} \right) \text{mm} \times \sqrt{(0.25\ \text{mm})^2 + (1.25\ \text{mm})^2} \right]$$

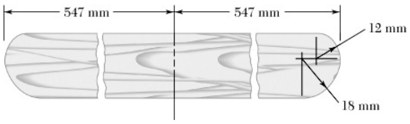
$$= 22.5607\ \text{in}^2$$

Then

$$W = 0.101\ \text{lb/in}^3 \times 22.5607\ \text{in}^2 \times \frac{3}{32}\ \text{in.} = 0.21362\ \text{lb}$$

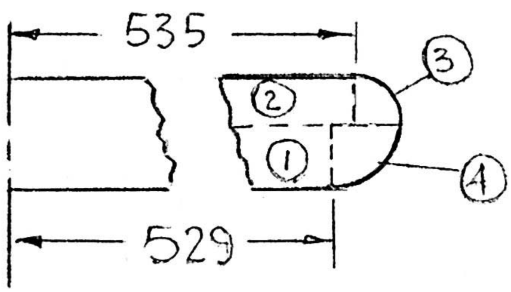
$$W = 0.214\ \text{lb} \blacktriangleleft$$

PROBLEM 5.57



The top of a round wooden table has the edge profile shown. Knowing that the diameter of the top is 1100 mm before shaping and that the density of the wood is 690 kg/m^3, determine the weight of the waste wood resulting from the production of 5000 tops.

SOLUTION



All dimensions are in mm

Have $$V_{\text{waste}} = V_{\text{blank}} - V_{\text{top}}$$

$$V_{\text{blank}} = \pi(550\text{ mm})^2 \times (30\text{ mm}) = 9.075\pi \times 10^6\text{ mm}^3$$

$$V_{\text{top}} = V_1 + V_2 + V_3 + V_4$$

Applying the second theorem of Pappus-Guldinus to parts 3 and 4

$$\begin{aligned} V_{\text{top}} &= \left[\pi(529\text{ mm})^2 \times (18\text{ mm})\right] + \left[\pi(535\text{ mm})^2 \times (12\text{ mm})\right] \\ &\quad + 2\pi\left\{\left[\left(535 + \frac{4\times 12}{3\pi}\right)\text{mm}\right] \times \frac{\pi}{4}(12\text{ mm})^2\right\} \\ &\quad + 2\pi\left\{\left[\left(529 + \frac{4\times 18}{3\pi}\right)\text{mm}\right] \times \frac{\pi}{4}(18\text{ mm})^2\right\} \\ &= \pi(5.0371 + 3.347 + 0.1222 + 0.2731) \times 10^6\text{ mm}^3 \\ &= 8.8671\pi \times 10^6\text{ mm}^3 \end{aligned}$$

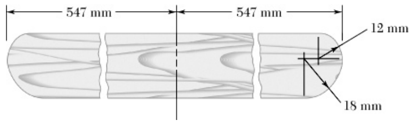
$$\therefore \quad V_{\text{waste}} = (9.0750 - 8.8671)\pi \times 10^6\text{ mm}^3 = 0.2079\pi \times 10^{-3}\text{ m}^3$$

Finally $$W_{\text{waste}} = \rho_{\text{wood}} V_{\text{waste}}\, g\, N_{\text{tops}}$$

$$= 690\text{ kg/m}^3 \times \left(0.2079\pi \times 10^{-3}\text{ m}^3\right) \times 9.81\text{ m/s}^2 \times 5000\text{ (tops)}$$

or $W_{\text{waste}} = 2.21\text{ kN}$ ◀

PROBLEM 5.58



The top of a round wooden table has the shape shown. Determine how many liters of lacquer are required to finish 5000 tops knowing that each top is given three coats of lacquer and that 1 liter of lacquer covers 12 m^2.

SOLUTION

Referring to the figure in solution of Problem 5.57 and using the first theorem of Pappus-Guldinus, we have

$$A_{\text{surface}} = A_{\text{top circle}} + A_{\text{bottom circle}} + A_{\text{edge}}$$

$$= \left[\pi(535\text{ mm})^2\right] + \left[\pi(529\text{ mm})^2\right]$$

$$+ \left\{2\pi\left[\left(535 + \frac{2\times 12}{\pi}\right)\text{mm}\right] \times \frac{\pi}{2}(12\text{ mm})\right\}$$

$$+ \left\{2\pi\left[\left(529 + \frac{2\times 18}{\pi}\right)\text{mm}\right] \times \frac{\pi}{2}(18\text{ mm})\right\}$$

$$= 617.115\pi \times 10^3\text{ mm}^2$$

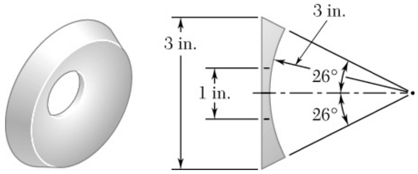
Then

$$\#\text{ liters} = A_{\text{surface}} \times \text{Coverage} \times N_{\text{tops}} \times N_{\text{coats}}$$

$$= 617.115\pi \times 10^{-3}\text{ m}^2 \times \frac{1\text{ liter}}{12\text{ m}^2} \times 5000 \times 3$$

or $\#\text{ liters} = 2424\text{ L}$ ◀

PROBLEM 5.59

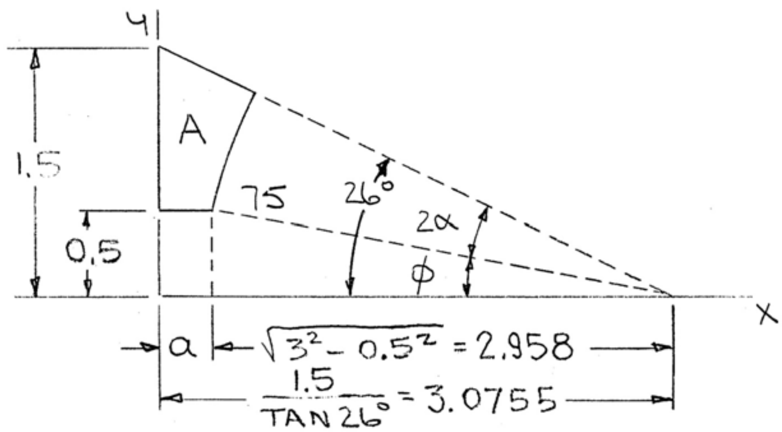


The escutcheon (a decorative plate placed on a pipe where the pipe exits from a wall) shown is cast from yellow brass. Knowing that the specific weight of yellow brass is 0.306 lb/in^3. determine the weight of the escutcheon.

SOLUTION

The weight of the escutcheon is given by $\qquad W = (\text{specific weight})V$

where V is the volume of the plate. V can be generated by rotating the area A about the x axis.

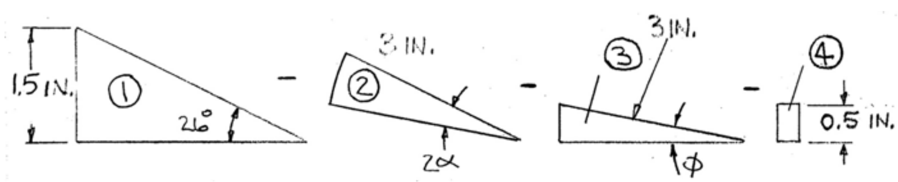


Have $\qquad a = 3.0755\text{ in.} - 2.958\text{ in.} = 0.1175\text{ in.}$

and $\qquad \sin\phi = \dfrac{0.5}{3} \Rightarrow \phi = 0.16745\text{ R} = 9.5941°$

Then $\qquad 2\alpha = 26° - 9.5941° = 16.4059° \qquad$ or $\qquad \alpha = 8.20295° = 0.143169\text{ rad}$

The area A can be obtained by combining the following four areas, as indicated.



Applying the second theorem of Pappus-Guldinus and then using Figure 5.8A, we have

$$V = 2\pi\bar{y}A = 2\pi\Sigma\bar{y}A$$

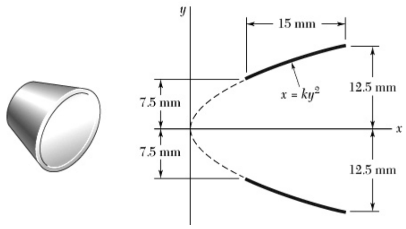
PROBLEM 5.59 CONTINUED

	A, in^2	$\bar{y}$, in.	$\bar{y}A$, in^3
1	$\frac{1}{2}(3.0755)(1.5) = 2.3066$	$\frac{1}{3}(1.5) = 0.5$	1.1533
2	$-\alpha(3)^2 = -1.28851$	$\frac{2(3)\sin\alpha}{3\alpha} \times \sin(\alpha + \phi) = 0.60921$	–0.78497
3	$-\frac{1}{2}(2.958)(0.5) = -0.7395$	$\frac{1}{3}(0.5) = 0.16667$	–0.12325
4	$-(0.1755)(0.5) = -0.05875$	$\frac{1}{2}(0.5) = 0.25$	–0.14688
			$\Sigma\bar{y}A = 0.44296 \text{ in}^3$

Then $$V = 2\pi\left(0.44296 \text{ in}^3\right) = 1.4476 \text{ in}^3$$

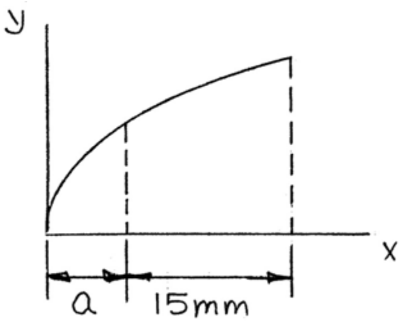
so that $$W = 1.4476 \text{ in}^3\left(0.306 \text{ lb/in}^3\right) = 0.44296 \text{ lb}$$ $W = 0.443 \text{ lb}$ ◀

PROBLEM 5.60



The reflector of a small flashlight has the parabolic shape shown. Determine the surface area of the inside of the reflector.

SOLUTION



First note that the required surface area A can be generated by rotating the parabolic cross section through 2π radians about the x axis. Applying the first theorem of Pappus-Guldinus, we have

$$A = 2\pi \overline{y} L$$

Now, since $\quad x = ky^2, \quad$ at $\quad x = a: a = k(7.5)^2$

or $\quad a = 56.25\,k \qquad (1)$

At $\quad x = (a + 15)\,\text{mm}: a + 15 = k(12.5)^2$

or $\quad a + 15 = 156.25k \qquad (2)$

Then $\quad \dfrac{\text{Eq. (2)}}{\text{Eq. (1)}}: \quad \dfrac{a + 15}{a} = \dfrac{156.25k}{56.25k} \quad$ or $\quad a = 8.4375\text{ mm}$

$$\text{Eq. (1)} \Rightarrow k = 0.15\frac{1}{\text{mm}}$$

$$\therefore\ x = 0.15\,y^2 \qquad \text{and} \qquad \frac{dx}{dy} = 0.3y$$

Now $\quad dL = \sqrt{1 + \left(\dfrac{dx}{dy}\right)^2}\,dy = \sqrt{1 + 0.09y^2}\,dy$

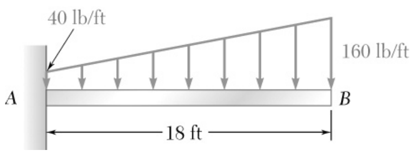
So $\quad A = 2\pi\overline{y}L \qquad \text{and} \qquad \overline{y}L = \int y\,dL$

$$\therefore\ A = 2\pi \int_{7.5}^{12.5} y\sqrt{1 + 0.09y^2}\,dy$$

$$= 2\pi\left[\frac{2}{3}\left(\frac{1}{0.18}\right)\left(1 + 0.09y^2\right)^{3/2}\right]_{7.5}^{12.5}$$

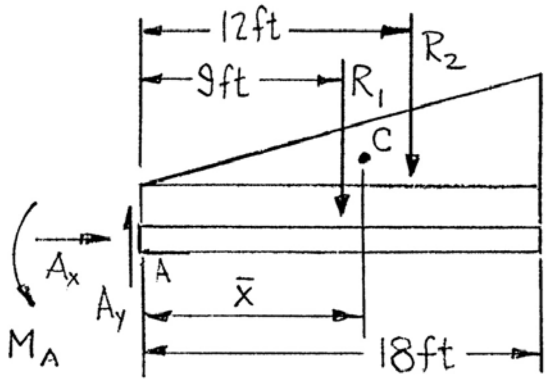
$$= 1013\text{ mm}^2 \qquad \text{or } A = 1013\text{ mm}^2 \blacktriangleleft$$

PROBLEM 5.61



For the beam and loading shown, determine (*a*) the magnitude and location of the resultant of the distributed load, (*b*) the reactions at the beam supports.

SOLUTION



Resultant $\quad R = R_1 + R_2$

(*a*) Have $\quad R_1 = (40 \text{ lb/ft})(18 \text{ ft}) = 720 \text{ lb}$

$$R_2 = \frac{1}{2}(120 \text{ lb/ft})(18 \text{ ft}) = 1080 \text{ lb}$$

or $\quad R = 1800 \text{ lb}$

The resultant is located at the centroid C of the distributed load $\bar{x}$

Have $\quad +\circlearrowleft \Sigma M_A: \quad (1800 \text{ lb})\bar{x} = (40 \text{ lb/ft})(18 \text{ ft})(9 \text{ ft}) + \frac{1}{2}(120 \text{ lb/ft})(18 \text{ ft})(12 \text{ ft})$

or $\quad \bar{x} = 10.80 \text{ ft}$

$R = 1800 \text{ lb}$ ◀

$\bar{x} = 10.80 \text{ ft}$

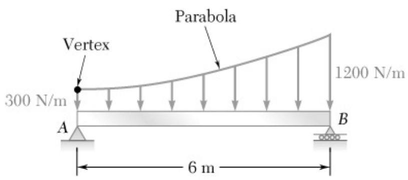
(*b*) $\quad \xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$

$+\uparrow \Sigma F_y = 0: \quad A_y - 1800 \text{ lb} = 0, \; A_y = 1800 \text{ lb} \qquad \therefore \; \mathbf{A} = 1800 \text{ lb} \uparrow$ ◀

$+\circlearrowleft \Sigma M_A = 0: \quad M_A - (1800 \text{ lb})(10.8 \text{ ft}) = 0$

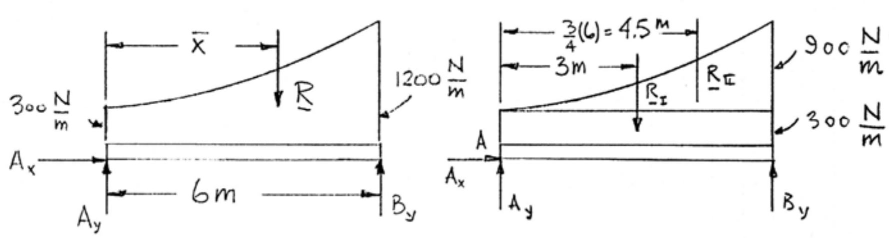
$M_A = 19.444 \text{ lb} \cdot \text{ft} \qquad$ or $\; \mathbf{M}_A = 19.44 \text{ kip} \cdot \text{ft} \circlearrowleft$ ◀

PROBLEM 5.62



For the beam and loading shown, determine (*a*) the magnitude and location of the resultant of the distributed load, (*b*) the reactions at the beam supports.

SOLUTION



(*a*) Have $$R_{\text{I}} = (300\text{ N/m})(6\text{ m}) = 1800\text{ N}$$

$$R_{\text{II}} = \frac{1}{3}(6\text{ m})(900\text{ N/m}) = 1800\text{ N}$$

Then $$+\uparrow \Sigma F_y: \quad -R = -R_{\text{I}} - R_{\text{II}}$$

or $$R = 1800\text{ N} + 1800\text{ N} = 3600\text{ N}$$

$$+\circlearrowleft \Sigma M_A: \quad -\bar{x}(3600\text{ N}) = -(3\text{ m})(1800\text{ N}) - (4.5\text{ m})(1800\text{ N})$$

or $$\bar{x} = 3.75\text{ m}$$

$R = 3600\text{ N}$ ◀
$\bar{x} = 3.75\text{ m}$

(*b*) Reactions

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$

$$+\circlearrowleft \Sigma M_A = 0: \quad (6\text{ m})B_y - (3600\text{ N})(3.75\text{ m}) = 0$$

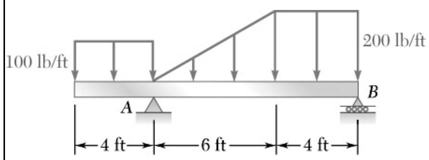
or $$B_y = 2250\text{ N}$$ $\mathbf{B} = 2250\text{ N}\uparrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad A_y + 2250\text{ N} = 3600\text{ N}$$

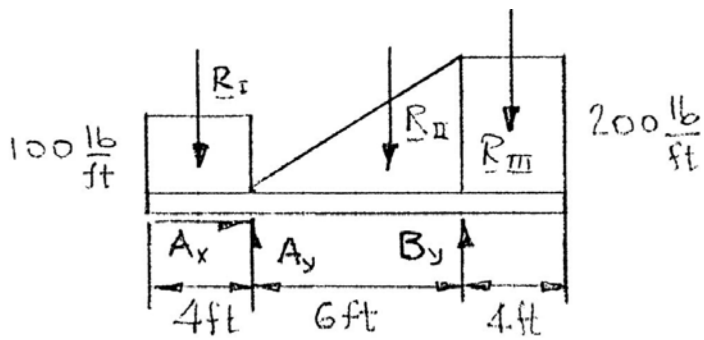
or $$A_y = 1350\text{ N}$$ $\mathbf{A} = 1350\text{ N}\uparrow$ ◀

PROBLEM 5.63

Determine the reactions at the beam supports for the given loading.



SOLUTION



Have $$R_{\text{I}} = (100\text{ lb/ft})(4\text{ ft}) = 400\text{ lb}$$

$$R_{\text{II}} = \frac{1}{2}(200\text{ lb/ft})(6\text{ ft}) = 600\text{ lb}$$

$$R_{\text{III}} = (200\text{ lb/ft})(4\text{ ft}) = 800\text{ lb}$$

Then $$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$

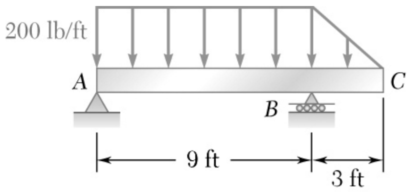
$$+\circlearrowleft \Sigma M_A = 0: \quad (2\text{ ft})(400\text{ lb}) - (4\text{ ft})(600\text{ lb}) - (12\text{ ft})(800\text{ lb}) + (10\text{ ft})B_y = 0$$

or $$B_y = 800\text{ lb}$$ $\mathbf{B} = 800\text{ lb} \uparrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad A_y + 800\text{ lb} - 400\text{ lb} - 600\text{ lb} - 800 = 0$$

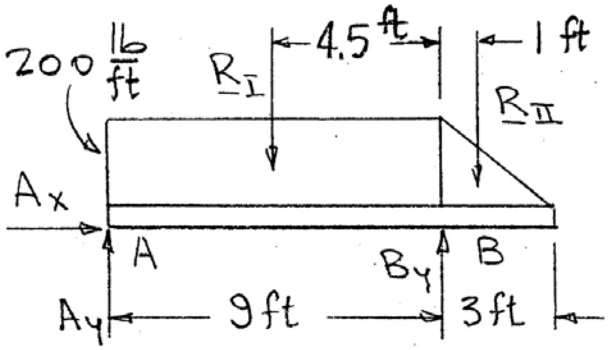
or $$A_y = 1000\text{ lb}$$ $\mathbf{A} = 1000\text{ lb} \uparrow$ ◀

PROBLEM 5.64



Determine the reactions at the beam supports for the given loading.

SOLUTION



Have
$$R_{\mathrm{I}} = (9\text{ ft})(200\text{ lb/ft}) = 1800\text{ lb}$$

$$R_{\mathrm{II}} = \frac{1}{2}(3\text{ ft})(200\text{ lb/ft}) = 300\text{ lb}$$

Then
$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -(4.5\text{ ft})(1800\text{ lb}) - (10\text{ ft})(300\text{ lb}) + (9\text{ ft})B_y = 0$$

or
$$B_y = 1233.3\text{ lb}$$

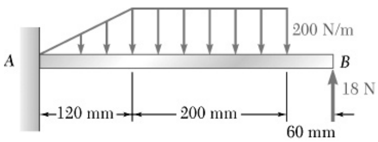
$\mathbf{B} = 1233\text{ lb} \uparrow$ ◀

$$\uparrow \Sigma F_y = 0: \quad A_y - 1800\text{ lb} - 300\text{ lb} + 1233.3\text{ lb} = 0$$

or
$$A_y = 866.7\text{ lb}$$

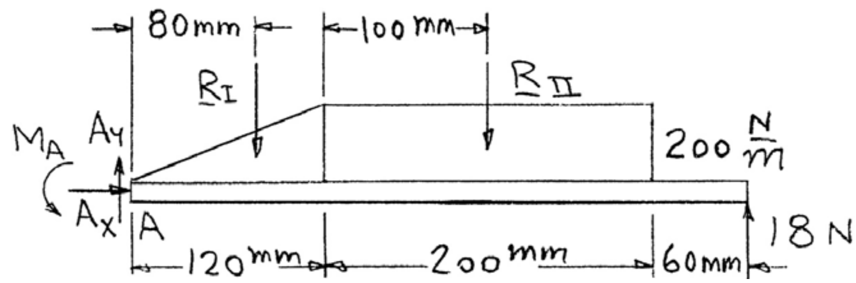
$\mathbf{A} = 867\text{ lb} \uparrow$ ◀

PROBLEM 5.65



Determine the reactions at the beam supports for the given loading.

SOLUTION



Have $$R_{\text{I}} = \frac{1}{2}(200 \text{ N/m})(0.12 \text{ m}) = 12 \text{ N}$$

$$R_{\text{II}} = (200 \text{ N/m})(0.2 \text{ m}) = 40 \text{ N}$$

Then $$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$

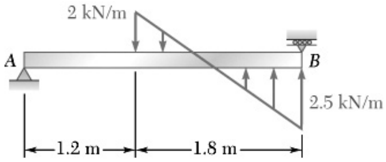
$$+\uparrow \Sigma F_y = 0: \quad A_y + 18 \text{ N} - 12 \text{ N} - 40 \text{ N} = 0$$

or $$A_y = 34 \text{ N}$$ $$\mathbf{A} = 34.0 \text{ N} \uparrow \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_A = 0: \quad M_A - (0.8 \text{ m})(12 \text{ N}) - (0.22 \text{ m})(40 \text{ N}) + (0.38 \text{ m})(18 \text{ N})$$

or $$M_A = 2.92 \text{ N}\cdot\text{m}$$ $$\mathbf{M}_A = 2.92 \text{ N}\cdot\text{m} \circlearrowleft \blacktriangleleft$$

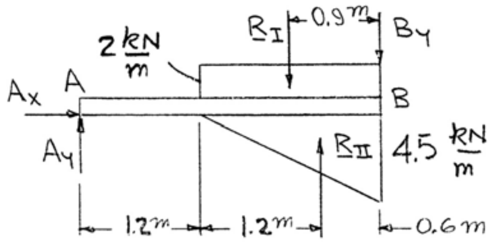
PROBLEM 5.66



Determine the reactions at the beam supports for the given loading.

SOLUTION

First, replace the given loading with the loading shown below. The two loadings are equivalent because both are defined by a linear relation between load and distance, and the values at the end points are the same.



Have $$R_{\mathrm{I}} = (1.8\text{ m})(2000\text{ N/m}) = 3600\text{ N}$$

$$R_{\mathrm{II}} = \frac{1}{2}(1.8\text{ m})(4500\text{ N/m}) = 4050\text{ N}$$

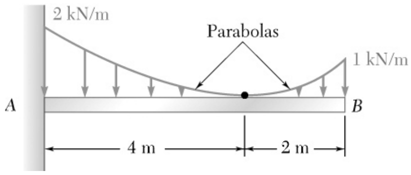
Then $$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$

$$+\circlearrowleft \Sigma M_B = 0: \quad -(3\text{ m})A_y - (2.1\text{ m})(3600\text{ N}) + (2.4\text{ m})(4050\text{ N})$$

or $$A_y = 270\text{ N}$$ $\mathbf{A} = 270\text{ N} \uparrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad 270\text{ N} - 3600\text{ N} + 4050\text{ N} - B_y = 0$$

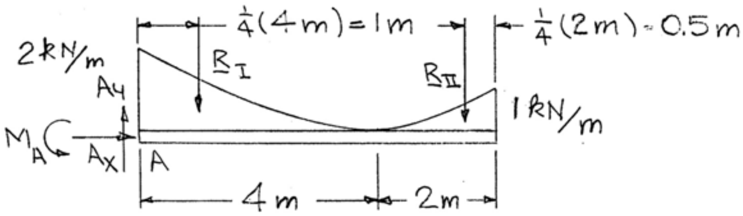
or $$B_y = 720\text{ N}$$ $\mathbf{B} = 720\text{ N} \downarrow$ ◀



PROBLEM 5.67

Determine the reactions at the beam supports for the given loading.

SOLUTION



Have $$R_{\text{I}} = \frac{1}{3}(4\text{ m})(2000\text{ kN/m}) = 2667\text{ N}$$

$$R_{\text{II}} = \frac{1}{3}(2\text{ m})(1000\text{ kN/m}) = 666.7\text{ N}$$

Then $$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$

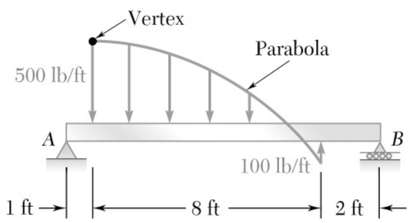
$$+\uparrow \Sigma F_y = 0: \quad A_y - 2667\text{ N} - 666.7\text{ N} = 0$$

or $$A_y = 3334\text{ N}$$ $\mathbf{A} = 3.33\text{ kN}\uparrow$ ◀

$$+\circlearrowleft \Sigma M_A = 0: \quad M_A - (1\text{ m})(2667\text{ N}) - (5.5\text{ m})(666.7\text{ N})$$

or $$M_A = 6334\text{ N}\cdot\text{m}$$ $\mathbf{M}_A = 6.33\text{ kN}\cdot\text{m} \circlearrowleft$ ◀

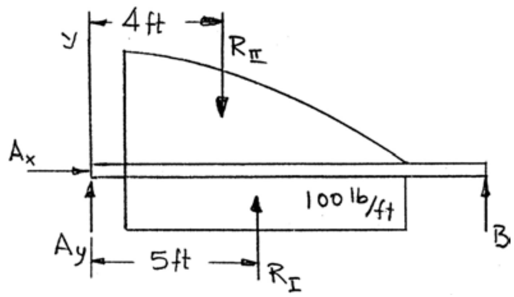
PROBLEM 5.68



Determine the reactions at the beam supports for the given loading.

SOLUTION

First, replace the given loading with the loading shown below. The two loadings are equivalent because both are defined by a parabolic relation between load and distance, and the values at end points are the same.



Have

$$R_I = (8 \text{ ft})(100 \text{ lb/ft}) = 800 \text{ lb}$$

$$R_{II} = \frac{2}{3}(8 \text{ ft})(600 \text{ lb/ft}) = 3200 \text{ lb}$$

Then

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$

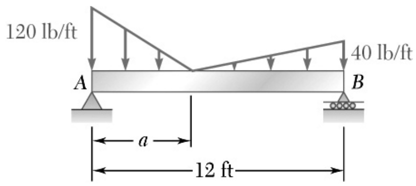
$$+\circlearrowleft \Sigma M_A = 0: \quad 11B + (5 \text{ ft})(800 \text{ lb}) - (4 \text{ ft})(3200) \text{lb} = 0$$

or $\mathbf{B} = 800 \text{ lb} \uparrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad A_y - 3200 \text{ lb} + 800 \text{ lb} + 800 \text{ lb} = 0$$

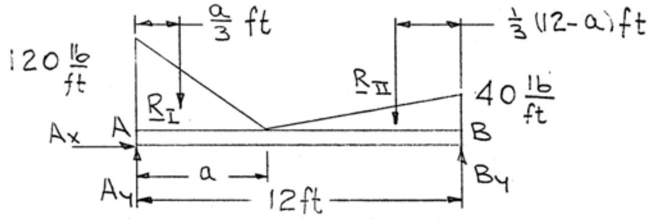
or $\mathbf{A} = 1600 \text{ lb} \uparrow$ ◀

PROBLEM 5.69



Determine (*a*) the distance *a* so that the vertical reactions at supports *A* and *B* are equal, (*b*) the corresponding reactions at the supports.

SOLUTION



(*a*) Have $$R_{\mathrm{I}} = \frac{1}{2}(a \text{ ft})(120 \text{ lb/ft}) = (60a) \text{ lb}$$

$$R_{\mathrm{II}} = \frac{1}{2}(12 - a)(40 \text{ lb/ft}) = (240 - 20a) \text{ lb}$$

Then $$+\uparrow \Sigma F_y = 0: \quad A_y - 60a - (240 - 2a) + B_y = 0$$

or $$A_y + B_y = 240 + 40a$$

Now $$A_y = B_y \Rightarrow A_y = B_y = 120 + 20a \qquad (1)$$

Also $$+\circlearrowleft \Sigma M_B = 0: \quad -(12 \text{ m})A_y + [(60a) \text{ lb}]\left[\left(12 - \frac{a}{3}\right) \text{ft}\right] + \left[\left(\frac{1}{3}(12 - a) \text{ ft}\right)\right][(240 - 20a) \text{ lb}] = 0$$

or $$A_y = 80 - \frac{140}{3}a - \frac{10}{9}a^2 \qquad (2)$$

Equating Eqs. (1) and (2)

$$120 + 20a = 80 - \frac{140}{3}a - \frac{10}{9}a^2$$

or $$\frac{40}{3}a^2 - 320a + 480 = 0$$

Then $$a = 1.6077 \text{ ft}, \qquad a = 22.392$$

Now $$a \le 12 \text{ ft}$$

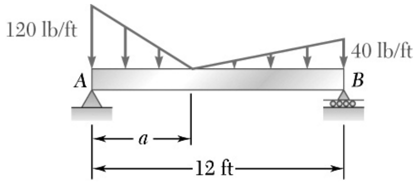
$a = 1.608$ ft ◀

(*b*) Have $$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$

Eq. (1) $$A_y = B_y = 120 + 20(1.61) = 152.2 \text{ lb}$$

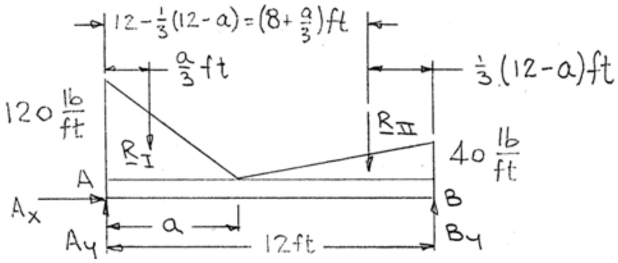
$\mathbf{A} = \mathbf{B} = 152.2 \text{ lb} \uparrow$ ◀

PROBLEM 5.70



Determine (*a*) the distance *a* so that the vertical reaction at support *B* is minimum, (*b*) the corresponding reactions at the supports.

SOLUTION



(*a*) Have

$$R_{\text{I}} = \frac{1}{2}(a\text{ ft})(120\text{ lb/ft}) = 60a\text{ lb}$$

$$R_{\text{II}} = \frac{1}{2}\left[(12-a)\text{ ft}\right](40\text{ lb/ft}) = (240-20a)\text{lb}$$

Then $\quad +\circlearrowleft \Sigma M_A = 0\text{:} \quad -\left(\frac{a}{3}\text{ft}\right)(60a\text{ lb}) - \left[(240-20a)\text{ lb}\right]\left[\left(8+\frac{a}{3}\right)\text{ft}\right] + (12\text{ ft})B_y = 0$

or

$$B_y = \frac{10}{9}a^2 - \frac{20}{3}a + 160 \tag{1}$$

Then

$$\frac{dB_y}{da} = \frac{20}{9}a - \frac{20}{3} = 0$$

or $a = 3.00$ ft ◀

(*b*) Eq. (1)

$$B_y = \frac{10}{9}(3.00)^2 - \frac{20}{3}(3.00) + 160$$

$$= 150\text{ lb}$$

$\mathbf{B} = 150.0\text{ lb}\uparrow$ ◀

and $\quad \xrightarrow{+} \Sigma F_x = 0\text{:} \quad A_x = 0$

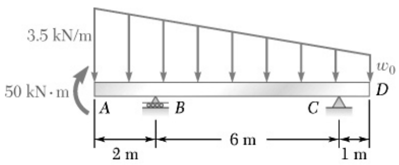
$$+\uparrow \Sigma F_y = 0\text{:} \quad A_y - \left[60(3.00)\right]\text{lb} - \left[240-20(3.00)\right]\text{lb} + 150\text{ lb} = 0$$

or

$$A_y = 210\text{ lb}$$

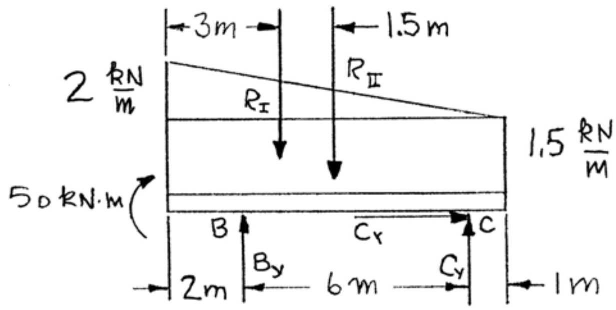
$\mathbf{A} = 210\text{ lb}\uparrow$ ◀

PROBLEM 5.71



Determine the reactions at the beam supports for the given loading when $w_0 = 1.5$ kN/m.

SOLUTION



Have
$$R_I = \frac{1}{2}(9\text{ m})(2\text{ kN/m}) = 9\text{ kN}$$

$$R_{II} = (9\text{ m})(1.5\text{ kN/m}) = 13.5\text{ kN}$$

Then
$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x = 0$$

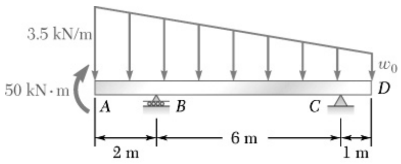
$$+\circlearrowleft \Sigma M_B = 0: \quad -50\text{ kN}\cdot\text{m} - (1\text{ m})(9\text{ kN}) - (2.5\text{ m})(13.5\text{ kN}) + (6\text{ m})C_y = 0$$

or
$$C_y = 15.4583\text{ kN}$$
$\mathbf{C} = 15.46$ kN $\uparrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad B_y - 9\text{ kN} - 13.5\text{ kN} + 15.4583 = 0$$

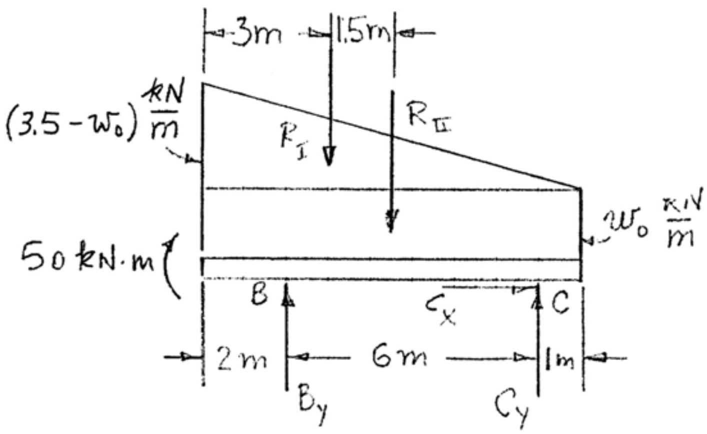
or
$$B_y = 7.0417\text{ kN}$$
$\mathbf{B} = 7.04$ kN $\uparrow$ ◀

PROBLEM 5.72



Determine (*a*) the distributed load w_0 at the end *D* of the beam *ABCD* for which the reaction at *B* is zero, (*b*) the corresponding reactions at *C*.

SOLUTION



Have $$R_{\text{I}} = \frac{1}{2}(9\text{ m})\big[(3.5 - w_0)\text{ kN/m}\big] = 4.5(3.5 - w_0)\text{ kN}$$

$$R_{\text{II}} = (9\text{ m})(w_0\text{ kN/m}) = 9w_0\text{ kN}$$

(*a*) Then $$+\circlearrowleft\ \Sigma M_C = 0:\quad -50\text{ kN}\cdot\text{m} + (5\text{ m})\big[4.5(3.5 - w_0)\text{ kN}\big] + (3.5\text{ m})(9w_0\text{ kN}) = 0$$

or $$9w_0 + 28.75 = 0$$

so $$w_0 = -3.1944\text{ kN/m}$$ $w_0 = 3.19\text{ kN/m}\uparrow$ ◀

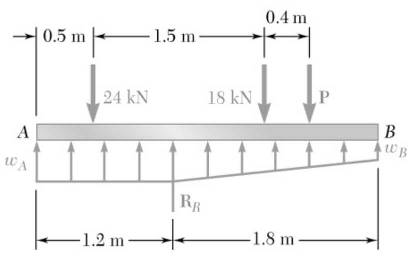
Note: the negative sign means that the distributed force w_0 is upward.

(*b*) $$\xrightarrow{+}\ \Sigma F_x = 0:\quad C_x = 0$$

$$+\uparrow\ \Sigma F_y = 0:\quad -4.5(3.5 + 3.19)\text{ kN} + 9(3.19)\text{ kN} + C_y = 0$$

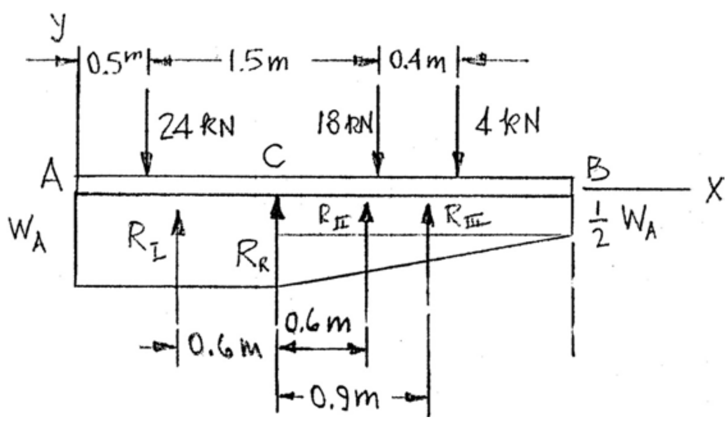
or $$C_y = 1.375\text{ kN}$$ $\mathbf{C} = 1.375\text{ kN}\uparrow$ ◀

PROBLEM 5.73



A grade beam *AB* supports three concentrated loads and rests on soil and the top of a large rock. The soil exerts an upward distributed load, and the rock exerts a concentrated load $\mathbf{R}_R$ as shown. Knowing that $P = 4$ kN and $w_B = \frac{1}{2} w_A$, determine the values of w_A and R_R corresponding to equilibrium.

SOLUTION



Have $$R_{\text{I}} = (1.2\text{ m})(w_A\text{ kN/m}) = 1.2\, w_A\text{ kN}$$

$$R_{\text{II}} = \frac{1}{2}(1.8\text{ m})\left(\frac{1}{2}w_A\text{ kN/m}\right) = 0.45\, w_A\text{ kN}$$

$$R_{\text{III}} = (1.8\text{ m})\left(\frac{1}{2}w_A\text{ kN/m}\right) = 0.9\, w_A\text{ kN}$$

Then $$+\circlearrowleft \Sigma M_C = 0:\quad -(0.6\text{ m})\big[(1.2\, w_A)\text{ kN}\big] + (0.6\text{ m})\big[(0.45\, w_A)\text{kN/m}\big]$$

$$+(0.9\text{ m})\big[(0.9\, w_A)\text{kN/m}\big] - (1.2\text{ m})(4\text{ kN/m})$$

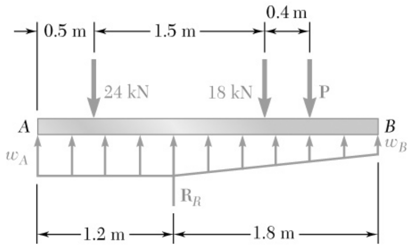
$$-(0.8\text{ m})(18\text{ kN/m}) + (0.7\text{ m})(24\text{ kN/m}) = 0$$

or $$w_A = 6.667\text{ kN/m}$$ $w_A = 6.67$ kN/m ◀

and $$+\uparrow \Sigma F_y = 0:\quad R_R + (1.2\text{ m})(6.67\text{ kN/m}) + (0.45\text{ m})(6.67\text{ kN/m})$$

$$+(0.9\text{ m})(6.67\text{ kN/m}) - 24\text{ kN} - 18\text{ kN} - 4\text{ kN}$$

or $$R_R = 29.0\text{ kN}$$ $R_R = 29.0$ kN ◀



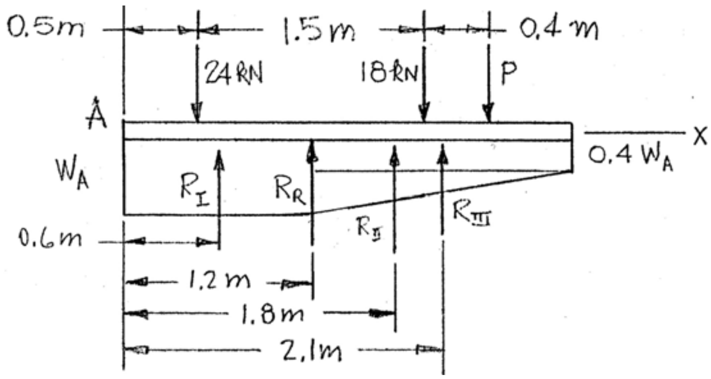
PROBLEM 5.74

A grade beam *AB* supports three concentrated loads and rests on soil and the top of a large rock. The soil exerts an upward distributed load, and the rock exerts a concentrated load $\mathbf{R}_R$ as shown. Knowing that $w_B = 0.4w_A$, determine (*a*) the largest value of **P** for which the beam is in equilibrium, (*b*) the corresponding value of w_A.

In the following problems, use $\gamma = 62.4$ lb/ft^3 for the specific weight of fresh water and $\gamma_C = 150$ lb/ft^3 for the specific weight of concrete if U.S. customary units are used. With SI units, use $\rho = 10^3$ kg/m^3 for the density of fresh water and $\rho_C = 2.40 \times 10^3$ kg/m^3 for the density of concrete. (See the footnote on page 222 for how to determine the specific weight of a material given its density.)

j

SOLUTION



Have $$R_{\mathrm{I}} = (1.2\text{ m})(w_A\text{ kN/m}) = 1.2\, w_A\text{ kN}$$

$$R_{\mathrm{II}} = \frac{1}{2}(1.8\text{ m})(0.6\, w_A\text{ kN/m}) = 0.54\, w_A\text{ kN}$$

$$R_{\mathrm{III}} = (1.8\text{ m})(0.4\, w_A\text{ kN/m}) = 0.72\, w_A\text{ kN}$$

(*a*) Then $$+\circlearrowleft\ \Sigma M_A = 0\!:\quad (0.6\text{ m})\left[(1.2\, w_A)\text{ kN}\right] + (1.2\text{ m})R_R + (1.8\text{ m})\left[(0.54\, w_A)\text{ kN}\right]$$

$$+ (2.1\text{ m})\left[(0.72\, w_A)\text{ kN}\right] - (0.5\text{ m})(24\text{ kN})$$

$$- (2.0\text{ m})(18\text{ kN}) + (2.4\text{ m})P = 0$$

or $$3.204\, w_A + 1.2\, R_R - 2.4\, P = 48 \tag{1}$$

and $$+\uparrow\ \Sigma F_y = 0\!:\quad R_R + 1.2\, W_A + 0.54\, W_A + 0.72\, W_A - 24 - 18 - P = 0$$

or $$R_R + 2.46\, W_A - P = 42 \tag{2}$$

Now combine Eqs. (1) and (2) to eliminate w_A:

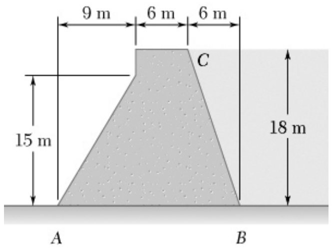
$$(3.204)\text{Eq. } 2 - (2.46)\text{Eq. } 1 \Rightarrow\ 0.252\, R_R = 16.488 - 2.7\, P$$

Since R_R must be ≥ 0, the maximum acceptable value of P is that for which $R = 0$,

or $$P = 6.1067\text{ kN}$$ $P = 6.11\text{ kN}$ ◀

(*b*) Then, from Eq. (2): $$2.46\, W_A - 6.1067 = 42$$ or $W_A = 19.56\text{ kN/m}$ ◀

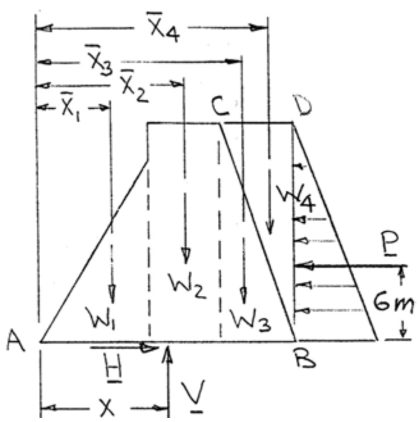
PROBLEM 5.75



The cross section of a concrete dam is as shown. For a dam section of unit width, determine (*a*) the reaction forces exerted by the ground on the base *AB* of the dam, (*b*) the point of application of the resultant of the reaction forces of part *a*, (*c*) the resultant of the pressure forces exerted by the water on the face *BC* of the dam.

In the following problems, use $\gamma = 62.4\ \text{lb/ft}^3$ for the specific weight of fresh water and $\gamma_C = 150\ \text{lb/ft}^3$ for the specific weight of concrete if U.S. customary units are used. With SI units, use $\rho = 10^3\ \text{kg/m}^3$ for the density of fresh water and $\rho_C = 2.40 \times 10^3\ \text{kg/m}^3$ for the density of concrete. (See the footnote on page 222 for how to determine the specific weight of a material given its density.)

SOLUTION



The free body shown consists of a 1-m thick section of the dam and the triangular section *BCD* of the water behind the dam.

Note: $\bar{X}_1 = 6\text{ m}$

$$\bar{X}_2 = (9+3)\text{ m} = 12\text{ m}$$

$$\bar{X}_3 = (15+2)\text{ m} = 17\text{ m}$$

$$\bar{X}_4 = (15+4)\text{ m} = 19\text{ m}$$

(*a*) Now $\quad W = \rho g V \quad$ so that

$$W_1 = (2400\text{ kg/m}^3)(9.81\text{ m/s}^2)\left[\frac{1}{2}(9\text{ m})(15\text{ m})(1\text{ m})\right] = 1589\text{ kN}$$

$$W_2 = (2400\text{ kg/m}^3)(9.81\text{ m/s}^2)\left[(6\text{ m})(18\text{ m})(1\text{ m})\right] = 2543\text{ kN}$$

$$W_3 = (2400\text{ kg/m}^3)(9.81\text{ m/s}^2)\left[\frac{1}{2}(6\text{ m})(18\text{ m})(1\text{ m})\right] = 1271\text{ kN}$$

$$W_4 = (2400\text{ kg/m}^3)(9.81\text{ m/s}^2)\left[\frac{1}{2}(6\text{ m})(18\text{ m})(1\text{ m})\right] = 529.7\text{ kN}$$

Also $\quad P = \frac{1}{2}Ap = \frac{1}{2}\left[(18\text{ m})(1\text{ m})\right]\left[(10^3\text{ kg/m}^3)(9.81\text{ m/s}^2)(18\text{ m})\right]$

$$= 1589\text{ kN}$$

Then $\quad \xrightarrow{+} \Sigma F_x = 0: \quad H - 1589\text{ kN} = 0$

or $\quad H = 1589\text{ kN} \qquad \mathbf{H} = 1589\text{ kN} \longrightarrow$ ◀

$+\uparrow \Sigma F_y = 0: \quad V - 1589\text{ kN} - 2543\text{ kN} - 1271\text{ kN} - 529.7\text{ kN}$

or $\quad V = 5933\text{ kN} \qquad \mathbf{V} = 5.93\text{ MN} \uparrow$ ◀

PROBLEM 5.75 CONTINUED

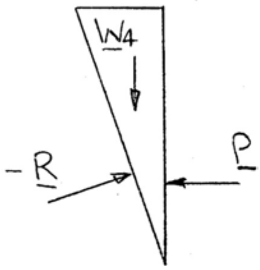
(*b*) Have $\circlearrowleft + \; \Sigma M_A = 0\!: \quad X\left(5933\text{ kN}\right) + \left(6\text{ m}\right)\left(1589\text{ kN}\right)$

$$- \left(6\text{ m}\right)\left(1589\text{ kN}\right) - \left(12\text{ m}\right)\left(2543\text{ kN}\right)$$

$$- \left(17\text{ m}\right)\left(1271\text{ kN}\right) - \left(19\text{ m}\right)\left(529.7\right) = 0$$

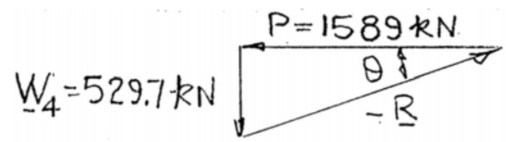
or $\quad X = 10.48\text{ m} \qquad\qquad X = 10.48\text{ m}$ ◀

to the right of *A*



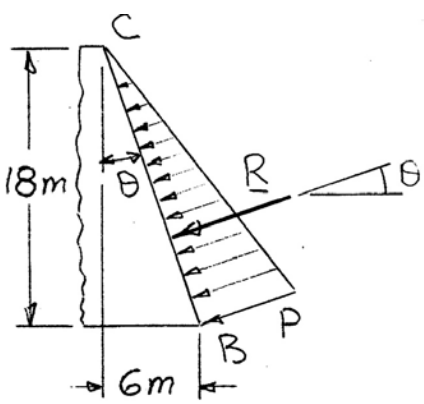
(*c*) Consider water section *BCD* as the free body.

Have $\quad \Sigma\mathbf{F} = 0$



Then $\quad -\mathbf{R} = 1675\text{ kN} \angle 18.43°$

or $\mathbf{R} = 1675\text{ kN} \angle 18.43°$ ◀



Alternative solution to part (c)

Consider the face *BC* of the dam.

Have $\quad BC = \sqrt{6^2 + 18^2} = 18.9737\text{ m}$

$$\tan\theta = \frac{6}{18} \qquad \theta = 18.43°$$

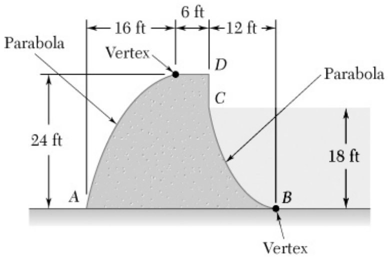
and $\quad p = \left(\rho g\right)h = \left(1000\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)\left(18\text{ m}\right)$

$$= 176.6\text{ kN/m}^2$$

Then $\quad R = \frac{1}{2}Ap = \frac{1}{2}\left[\left(18.97\text{ m}\right)\left(1\text{ m}\right)\right]\left(176.6\text{ kN/m}^2\right)$

$$= 1675\text{ kN}$$

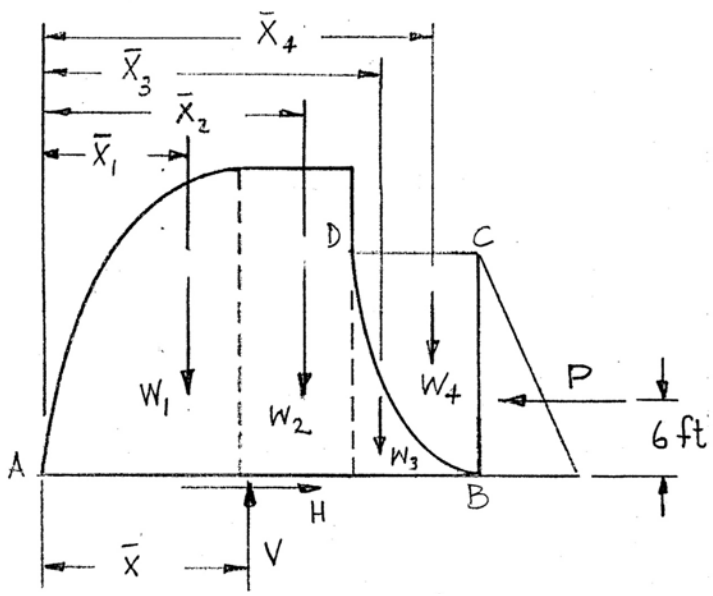
$$\therefore \quad \mathbf{R} = 1675\text{ kN} \angle 18.43°$$



PROBLEM 5.76

The cross section of a concrete dam is as shown. For a dam section of unit width, determine (*a*) the reaction forces exerted by the ground on the base *AB* of the dam, (*b*) the point of application of the resultant of the reaction forces of part *a*, (*c*) the resultant of the pressure forces exerted by the water on the face *BC* of the dam.

SOLUTION



The free body shown consists of a 1-ft thick section of the dam and the parabolic section of water above (and behind) the dam.

Note

$$\bar{x}_1 = \frac{5}{8}(16\text{ ft}) = 10\text{ ft}$$

$$\bar{x}_2 = \left[16 + \frac{1}{2}(6)\right]\text{ft} = 19\text{ ft}$$

$$\bar{x}_3 = \left[22 + \frac{1}{4}(12)\right]\text{ft} = 25\text{ ft}$$

$$\bar{x}_4 = \left[22 + \frac{5}{8}(12)\right]\text{ft} = 29.5\text{ ft}$$

PROBLEM 5.76 CONTINUED

Now $$W = \gamma V$$

$$W_1 = \left(150\ \text{lb/ft}^3\right)\left[\frac{2}{3}\left(16\ \text{ft}\right)\left(24\ \text{ft}\right)\times\left(1\ \text{ft}\right)\right] = 38,400\ \text{lb}$$

$$W_2 = \left(150\ \text{lb/ft}^3\right)\left[\left(6\ \text{ft}\right)\left(24\ \text{ft}\right)\times\left(1\ \text{ft}\right)\right] = 21,600\ \text{lb}$$

$$W_3 = \left(150\ \text{lb/ft}^3\right)\left[\frac{1}{3}\left(12\ \text{ft}\right)\left(18\ \text{ft}\right)\times\left(1\ \text{ft}\right)\right] = 10,800\ \text{lb}$$

$$W_4 = \left(62.4\ \text{lb/ft}^3\right)\left[\frac{2}{3}\left(12\ \text{ft}\right)\left(18\ \text{ft}\right)\times\left(1\ \text{ft}\right)\right] = 8985.6\ \text{lb}$$

Also $$P = \frac{1}{2}Ap = \frac{1}{2}\left[\left(18\times 1\right)\text{ft}^2\right]\times\left(62.4\ \text{lb/ft}^3\times 18\ \text{ft}\right) = 10,108.8\ \text{lb}$$

(a) Then $$\xrightarrow{+}\ \Sigma F_x = 0:\quad H - 10,108.8\ \text{lb} = 0$$

or $\mathbf{H} = 10.11\ \text{kips} \longrightarrow$ ◀

$$+\uparrow \Sigma F_y = 0:\quad V - 38,400\ \text{lb} - 21,600\ \text{lb} - 10,800\ \text{lb} - 8995.6\ \text{lb} = 0$$

or $$V = 79,785.6$$ $\mathbf{V} = 79.8\ \text{kips} \uparrow$ ◀

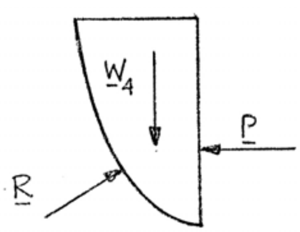
(b) $$+\circlearrowleft \Sigma M_A = 0:\quad \overline{X}\left(79,785.6\ \text{lb}\right) - \left(6\ \text{ft}\right)\left(38,400\ \text{lb}\right) - \left(19\ \text{ft}\right)\left(21,600\ \text{lb}\right) - \left(25\ \text{ft}\right)\left(10,800\ \text{lb}\right)$$
$$- \left(29.5\ \text{ft}\right)\left(8985.6\ \text{lb}\right) + \left(6\ \text{ft}\right)\left(10,108.8\ \text{lb}\right) = 0$$

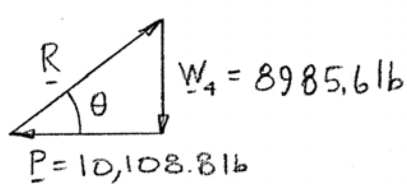
or $$\overline{X} = 15.90\ \text{ft}$$

The point of application of the resultant is 15.90 ft to the right of A ◀

(c) Consider the water section BCD as the free body.

Have $$\Sigma\mathbf{F} = 0$$



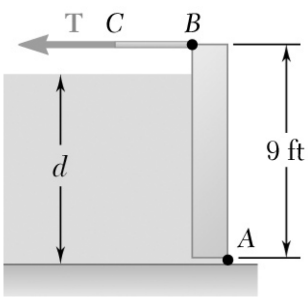


$$\therefore\quad R = 13.53\ \text{kips}$$
$$\theta = 41.6°$$

On the face BD of the dam

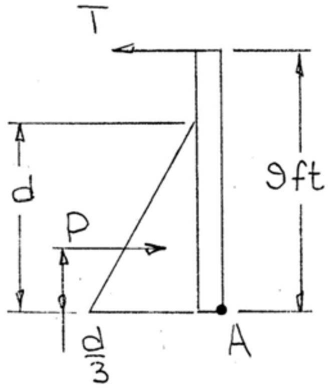
$\mathbf{R} = 13.53\ \text{kips} \nearrow 41.6°$ ◀

PROBLEM 5.77



The $9 \times 12\text{-ft}$ side *AB* of a tank is hinged at its bottom *A* and is held in place by a thin rod *BC*. The maximum tensile force the rod can withstand without breaking is 40 kips, and the design specifications require the force in the rod not exceed 20 percent of this value. If the tank is slowly filled with water, determine the maximum allowable depth of water *d* in the tank.

SOLUTION



Consider the free-body diagram of the side.

Have
$$P = \frac{1}{2}Ap = \frac{1}{2}A(\gamma d)$$

Now
$$+\circlearrowleft \Sigma M_A = 0: \quad (9\text{ ft})T - \frac{d}{3}P = 0$$

Then, for $d_{\max}$:

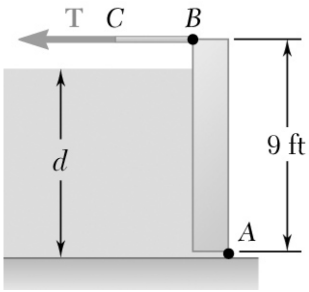
$$(9\text{ ft})\left[(0.2)\left(40\times10^3\text{ lb}\right)\right] - \frac{d_{\max}}{3}\left\{\frac{1}{2}\left[(12\text{ ft})(d_{\max})\right]\left(62.4\text{ lb/ft}^3\right)d_{\max}\right\} = 0$$

or
$$216\times10^3\text{ ft}^3 = 374.4\,d_{\max}^3$$

or
$$d_{\max}^3 = 576.92\text{ ft}^3$$

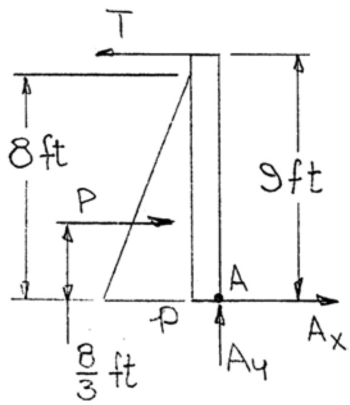
$d_{\max} = 8.32\text{ ft}$ ◀

PROBLEM 5.78



The 9×12-ft side of an open tank is hinged at its bottom A and is held in place by a thin rod. The tank is filled with glycerine, whose specific weight is $80\ \text{lb/ft}^3$. Determine the force **T** in the rod and the reactions at the hinge after the tank is filled to a depth of 8 ft.

SOLUTION



Consider the free-body diagram of the side.

Have $$P = \frac{1}{2}Ap = \frac{1}{2}A(\gamma d)$$

$$= \frac{1}{2}\left[(8\text{ ft})(12\text{ ft})\right]\left(80\text{ lb/ft}^3\right)(8\text{ ft}) = 30{,}720\text{ lb}$$

Then $$+\uparrow \Sigma F_y = 0: \quad A_y = 0$$

$$+\circlearrowright \Sigma M_A = 0: \quad (9\text{ ft})T - \left(\frac{8}{3}\text{ ft}\right)(30{,}720\text{ lb}) = 0$$

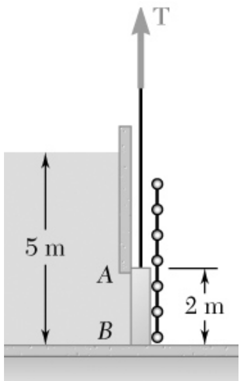
or $$T = 9102.22\text{ lb}$$

$$\mathbf{T} = 9.10\text{ kips} \longleftarrow \blacktriangleleft$$

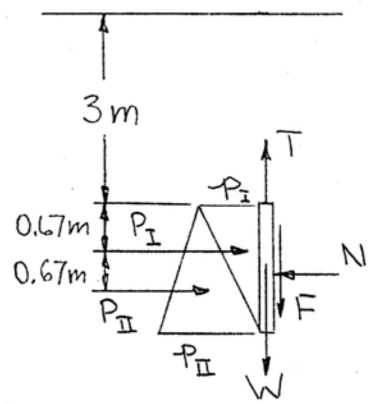
$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + 30{,}720\text{ lb} - 9102.22\text{ lb} = 0$$

or $$A = -21{,}618\text{ lb}$$

$$\mathbf{A} = 21.6\text{ kips} \longleftarrow \blacktriangleleft$$

	**PROBLEM 5.79** The friction force between a $2 \times 2\text{-m}$ square sluice gate *AB* and its guides is equal to 10 percent of the resultant of the pressure forces exerted by the water on the face of the gate. Determine the initial force needed to lift the gate that its mass is 500 kg.

SOLUTION



Consider the free-body diagram of the gate.

Now $$P_{\text{I}} = \frac{1}{2}Ap_{\text{I}} = \frac{1}{2}\left[(2 \times 2)\,\text{m}^2\right]\left[\left(10^3\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(3\ \text{m})\right]$$

$$= 58.86\ \text{kN}$$

$$P_{\text{II}} = \frac{1}{2}Ap_{\text{II}} = \frac{1}{2}\left[(2 \times 2)\,\text{m}^2\right]\left[\left(10^3\ \text{kg/m}^3\right)\left(9.81\ \text{m/s}^2\right)(5\ \text{m})\right]$$

$$= 98.10\ \text{kN}$$

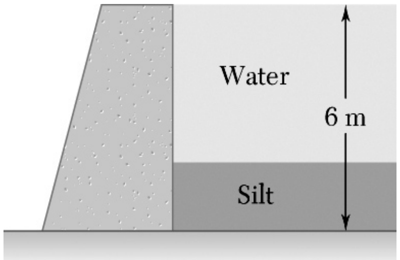
Then $$F = 0.1P = 0.1\left(P_{\text{I}} + P_{\text{II}}\right)$$

$$= 0.1(58.86 + 98.10)\ \text{kN}$$

$$= 15.696\ \text{kN}$$

Finally $$+\uparrow \Sigma F_y = 0: \quad T - 15.696\ \text{kN} - (500\ \text{kg})\left(9.81\ \text{m/s}^2\right) = 0$$

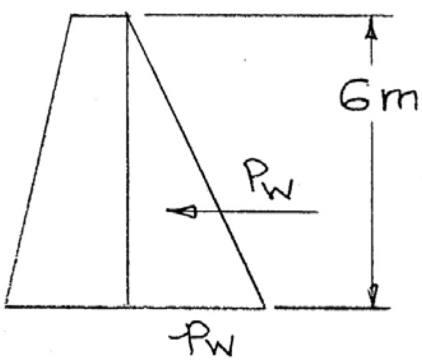
or $\mathbf{T} = 20.6\ \text{kN}\uparrow$ ◀



PROBLEM 5.80

The dam for a lake is designed to withstand the additional force caused by silt which has settled on the lake bottom. Assuming that silt is equivalent to a liquid of density $\rho_s = 1.76 \times 10^3$ kg/m^3 and considering a 1-m-wide section of dam, determine the percentage increase in the force acting on the dam face for a silt accumulation of depth 1.5 m.

SOLUTION

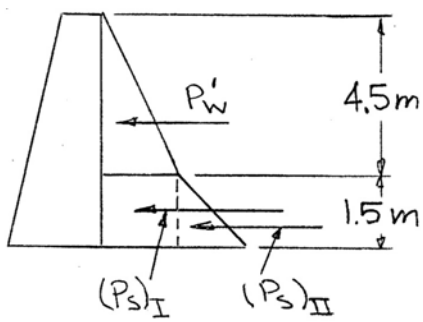


First, determine the force on the dam face without the silt.

Have $$P_w = \frac{1}{2}Ap_w = \frac{1}{2}A(\rho g h)$$

$$= \frac{1}{2}\left[(6\text{ m})(1\text{ m})\right]\left[\left(10^3\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)(6\text{ m})\right]$$

$$= 176.58\text{ kN}$$



Next, determine the force on the dam face with silt.

Have $$P'_w = \frac{1}{2}\left[(4.5\text{ m})(1\text{m})\right]\left[\left(10^3\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)(4.5\text{ m})\right]$$

$$= 99.326\text{ kN}$$

$$(P_s)_{\text{I}} = \left[(1.5\text{ m})(1\text{ m})\right]\left[\left(10^3\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)(4.5\text{ m})\right]$$

$$= 66.218\text{ kN}$$

$$(P_s)_{\text{II}} = \frac{1}{2}\left[(1.5\text{ m})(1\text{ m})\right]\left[\left(1.76\times 10^3\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)(1.5\text{ m})\right]$$

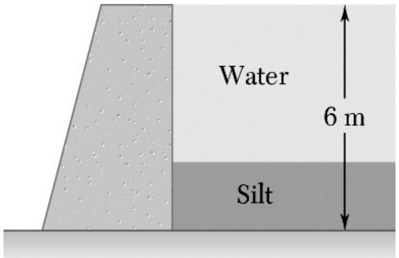
$$= 19.424\text{ kN}$$

Then $$P' = P'_w + (P_s)_{\text{I}} + (P_s)_{\text{II}} = 184.97\text{ kN}$$

The percentage increase, % inc., is then given by

$$\%\text{ inc.} = \frac{P' - P_w}{P_w}\times 100\% = \frac{(184.97 - 176.58)}{176.58}\times 100\% = 4.7503\%$$

$$\%\text{ inc.} = 4.75\% \blacktriangleleft$$



PROBLEM 5.81

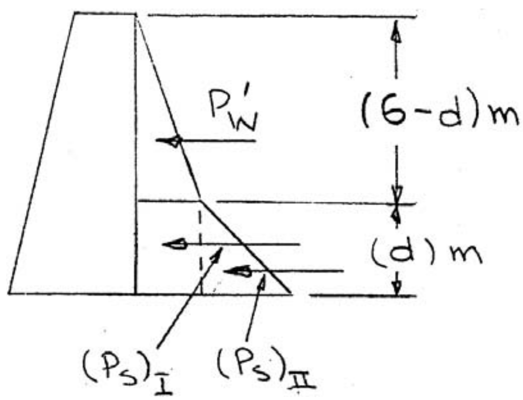
The base of a dam for a lake is designed to resist up to 150 percent of the horizontal force of the water. After construction, it is found that silt (which is equivalent to a liquid of density $\rho_s = 1.76 \times 10^3\ \text{kg/m}^3$) is settling on the lake bottom at a rate of 20 mm/y. Considering a 1-m-wide section of dam, determine the number of years until the dam becomes unsafe.

SOLUTION

From Problem 5.80, the force on the dam face before the silt is deposited, is $P_w = 176.58\ \text{kN}$. The maximum allowable force P_{allow} on the dam is then:

$$P_{\text{allow}} = 1.5P_w = (1.5)(176.58\ \text{kN}) = 264.87\ \text{kN}$$

Next determine the force P' on the dam face after *a* depth *d* of silt has settled.



Have

$$P_w' = \frac{1}{2}\left[(6-d)\text{m} \times (1\ \text{m})\right]\left[(10^3\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(6-d)\text{m}\right]$$

$$= 4.905(6-d)^2\ \text{kN}$$

$$(P_s)_{\text{I}} = \left[d(1\ \text{m})\right]\left[(10^3\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(6-d)\text{m}\right]$$

$$= 9.81(6d - d^2)\ \text{kN}$$

$$(P_s)_{\text{II}} = \frac{1}{2}\left[d(1\ \text{m})\right]\left[(1.76 \times 10^3\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(d)\text{m}\right]$$

$$= 8.6328d^2\ \text{kN}$$

$$P' = P_w' + (P_s)_{\text{I}} + (P_s)_{\text{II}} = \left[4.905(36 - 12d + d^2) + 9.81(6d - d^2) + 8.6328d^2\right]\text{kN}$$

$$= \left[3.7278d^2 + 176.58\right]\text{kN}$$

PROBLEM 5.81 CONTINUED

Now required that $P' = P_{\text{allow}}$ to determine the maximum value of d.

$$\therefore \quad \left(3.7278d^2 + 176.58\right)\text{kN} = 264.87\ \text{kN}$$

or

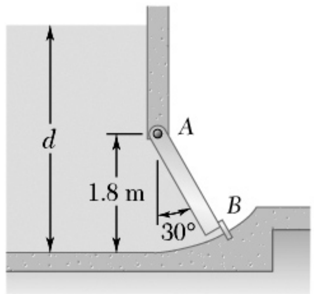
$$d = 4.8667\ \text{m}$$

Finally

$$4.8667\ \text{m} = 20 \times 10^{-3}\ \frac{\text{m}}{\text{year}} \times N$$

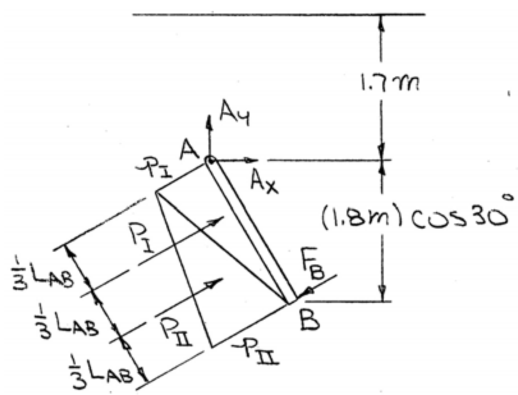
or $N = 243$ years ◀

PROBLEM 5.82



The square gate *AB* is held in the position shown by hinges along its top edge *A* and by a shear pin at *B*. For a depth of water $d = 3.5$ m, determine the force exerted on the gate by the shear pin.

SOLUTION



First consider the force of the water on the gate.

Have $$P = \frac{1}{2}Ap = \frac{1}{2}A(\rho g h)$$

Then $$P_{\text{I}} = \frac{1}{2}(18\text{ m})^2(10^3\text{ kg/m}^3)(9.81\text{ m/s}^2)(1.7\text{ m})$$

$$= 26.99\text{ kN}$$

$$P_{\text{II}} = \frac{1}{2}(18\text{ m})^2(10^3\text{ kg/m}^3)(9.81\text{ m/s}^2)(1.7 \times 1.8\cos 30°)\text{m}$$

$$= 51.74\text{ kN}$$

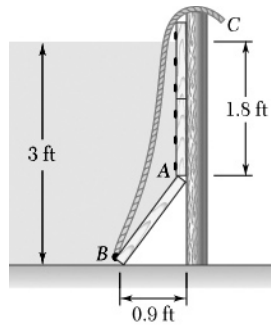
Now $$+\circlearrowleft \Sigma M_A = 0\text{:}\quad \frac{1}{3}(L_{AB})P_{\text{I}} + \frac{2}{3}(L_{AB})P_{\text{II}} - L_{AB}F_B = 0$$

or $$\frac{1}{3}(26.99\text{ kN}) + \frac{2}{3}(51.74\text{ kN}) - F_B = 0$$

or $$F_B = 43.49\text{ kN}$$

$$\mathbf{F}_B = 4.35\text{ kN} \measuredangle 30.0° \blacktriangleleft$$

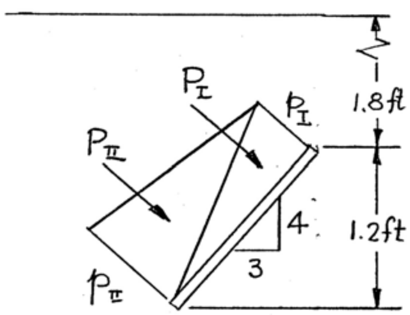
PROBLEMS 5.83 AND 5.84



Problem 5.83: A temporary dam is constructed in a 5-ft-wide fresh water channel by nailing two boards to pilings located at the sides of the channel and propping a third board *AB* against the pilings and the floor of the channel. Neglecting friction, determine the reactions at *A* and *B* when rope *BC* is slack.

Problem 5.84: A temporary dam is constructed in a 5-ft-wide fresh water channel by nailing two boards to pilings located at the sides of the channel and propping a third board *AB* against the pilings and the floor of the channel. Neglecting friction, determine the magnitude and direction of the minimum tension required in rope *BC* to move board *AB*.

SOLUTION



First, consider the force of the water on the gate.

Have $$P = \frac{1}{2}Ap = \frac{1}{2}A(\gamma h)$$

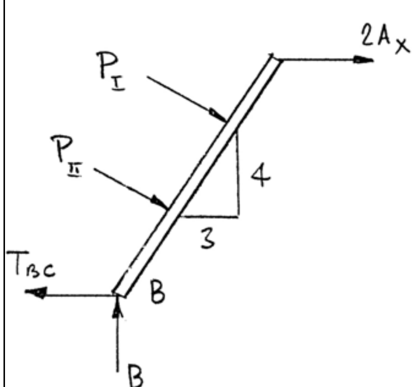
So that $$P_{\text{I}} = \frac{1}{2}\left[(1.5\text{ ft})(5\text{ ft})\right]\left[\left(62.4\text{ lb/ft}^3\right)(1.8\text{ ft})\right]$$

$$= 421.2\text{ lb}$$

$$P_{\text{II}} = \frac{1}{2}\left[(1.5\text{ ft})(5\text{ ft})\right]\left[\left(62.4\text{ lb/ft}^3\right)(3\text{ ft})\right]$$

$$= 702\text{ lb}$$

5.83 Find the reactions at *A* and *B* when rope is slack.



$$+\circlearrowleft \Sigma M_A = 0: \quad -(0.9\text{ ft})B + (0.5\text{ ft})(421.2\text{ lb}) + (1.0\text{ ft})(702\text{ lb}) = 0$$

or $$B = 1014\text{ lb}$$

$$\mathbf{B} = 1014\text{ lb} \uparrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad 2A_x + \frac{4}{5}(421.2\text{ lb}) + \frac{4}{5}(702\text{ lb}) = 0$$

or $$A_x = -449.28\text{ lb}$$

Note that the factor 2 $(2A_x)$ is included since A_x is the horizontal force exerted by the board on each piling.

$$+\uparrow \Sigma F_y = 0: \quad 1014\text{ lb} - \frac{3}{5}(421.2\text{ lb}) - \frac{3}{5}(702\text{ lb}) + A_y = 0$$

or $$A_y = -340.08\text{ lb}$$

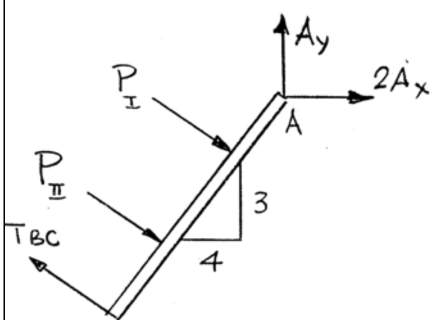
$$\therefore \quad \mathbf{A} = 563\text{ lb} \measuredangle 37.1° \blacktriangleleft$$

PROBLEMS 5.83 AND 5.84 CONTINUED

5.84 Note that there are two ways to move the board:

1. Pull upward on the rope fastened at B so that the board rotates about A. For this case $\mathbf{B} \to 0$ and $\mathbf{T}_{BC}$ is perpendicular to AB for minimum tension.

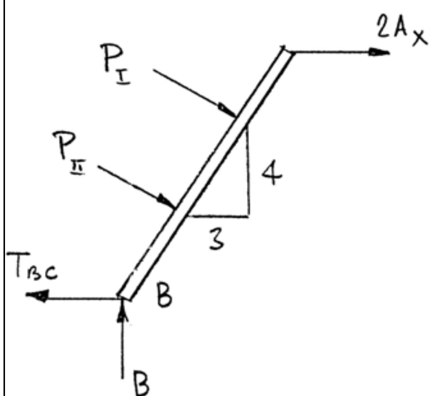
2. Pull horizontally at B so that the edge B of the board moves to the left. For this case $A_y \to 0$ and the board remains against the pilings because of the force of the water.



Case (1) $\quad +\circlearrowleft \Sigma M_A = 0$: $\quad -1.5T_{BC} + (0.5 \text{ ft})(421.2 \text{ lb})$

$$+ (1.0 \text{ ft})(702 \text{ lb}) = 0$$

or $\quad T_{BC} = 608.4 \text{ lb}$



Case (2) $\quad +\circlearrowleft \Sigma M_B = 0$: $\quad -(1.2 \text{ ft})(2A_x) - (0.5 \text{ ft})(702 \text{ lb})$

$$-(1.0 \text{ ft})(421.2 \text{ lb}) = 0$$

or $\quad 2A_x = -643.5 \text{ lb}$

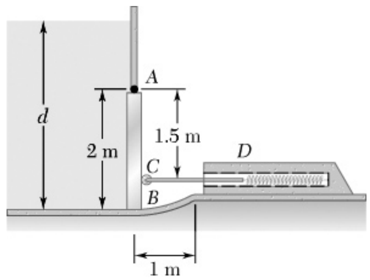
$$\xrightarrow{+} \Sigma F_x = 0: \quad -T_{BC} - 643.5 \text{ lb} + \frac{4}{5}(421.2 \text{ lb})$$

$$+ \frac{4}{5}(702 \text{ lb}) = 0$$

or $\quad T_{BC} = 255.06 \text{ lb}$

$$\therefore \quad (\mathbf{T}_{BC})_{\min} = 255 \text{ lb} \longleftarrow \blacktriangleleft$$

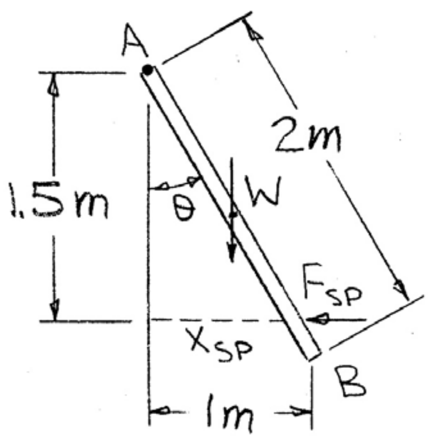
PROBLEMS 5.85 AND 5.86

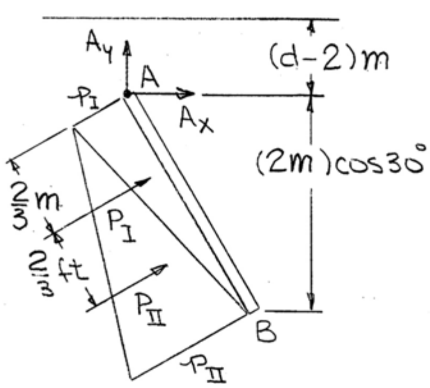


Problem 5.85: A 2×3-m gate is hinged at *A* and is held in position by rod *CD*. End *D* rests against a spring whose constant is 12 kN/m. The spring is undeformed when the gate is vertical. Assuming that the force exerted by rod *CD* on the gate remains horizontal, determine the minimum depth of water *d* for which the bottom *B* of the gate will move to the end of the cylindrical portion of the floor.

Problem 5.86: Solve Problem 5.85 if the mass of the gate is 500 kg.

SOLUTION





First, determine the forces exerted on the gate by the spring and the water when *B* is at the end of the cylindrical portion of the floor.

Have $\quad \sin\theta = \dfrac{1}{2} \qquad \therefore \quad \theta = 30°$

Then $\quad x_{sp} = (1.5\text{ m})\tan 30°$

and $\quad F_{sp} = kx_{sp}$

$$= (12\text{ kN/m})(1.5\text{ m})\tan 30°$$

$$= 10.39\text{ kN}$$

Assume $\quad d \geq 2\text{ m}$

Have $\quad P = \dfrac{1}{2}Ap = \dfrac{1}{2}A(\rho g)h$

Then $\quad P_{\text{I}} = \dfrac{1}{2}\left[(2\text{ m})(3\text{ m})\right]\left[\left(10^3\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)(d-2)\text{m}\right]$

$$= 29.43(d-2)\text{ kN}$$

$$P_{\text{II}} = \frac{1}{2}\left[(2\text{ m})(3\text{ m})\right]\left[\left(10^3\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)(d-2+2\cos 30°)\text{m}\right]$$

$$= 29.43(d-0.2679)\text{ kN}$$

PROBLEMS 5.85 AND 5.86 CONTINUED

5.85 Find $d_{\min}$ so that gate opens, $W = 0$.

Using the above free-body diagrams of the gate, we have

$$+\circlearrowright \Sigma M_A = 0: \quad \left(\frac{2}{3}\text{m}\right)\left[29.43(d-2)\text{kN}\right]$$

$$+\left(\frac{4}{3}\text{m}\right)\left[29.43(d-0.2679)\text{kN}\right]$$

$$-(1.5\text{ m})(10.39\text{ kN}) = 0$$

or $$19.62(d-2) + 39.24(d-0.2679) = 15.585$$

$$58.86d = 65.3374$$

or $$d = 1.1105\text{ m} \qquad d = 1.110\text{ m} \blacktriangleleft$$

5.86 Find $d_{\min}$ so that the gate opens.

$$W = \left(9.81\text{ m/s}^2\right)(500\text{ kg}) = 4.905\text{ kN}$$

Using the above free-body diagrams of the gate, we have

$$+\circlearrowright \Sigma M_A = 0: \quad \left(\frac{2}{3}\text{m}\right)\left[29.43(d-2)\text{kN}\right]$$

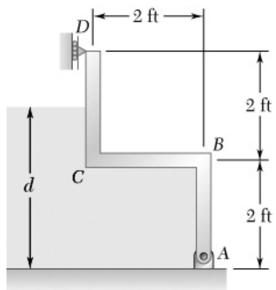
$$+\left(\frac{4}{3}\text{m}\right)\left[29.43(d-0.2679)\text{kN}\right]$$

$$-(1.5\text{ m})(10.39\text{ kN}) +$$

$$-(0.5\text{ m})(4.905\text{ kN}) = 0$$

or $$19.62(d-2) + 39.24(d-0.2679) = 18.0375$$

or $$d = 1.15171\text{ m} \qquad d = 1.152\text{ m} \blacktriangleleft$$

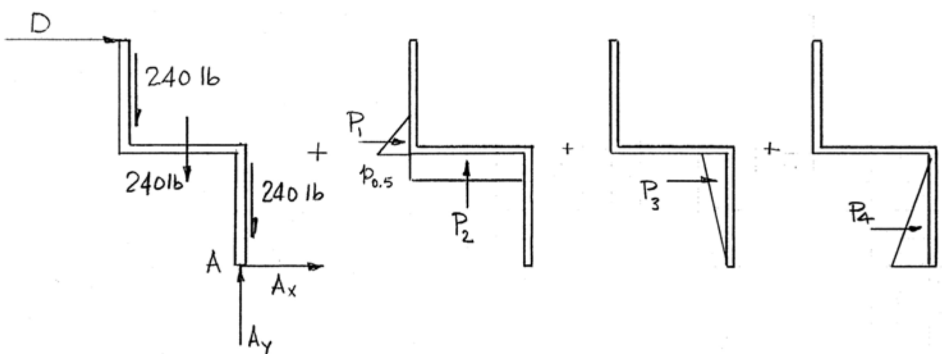


PROBLEMS 5.87 AND 5.88

Problem 5.87: The gate at the end of a 3-ft-wide fresh water channel is fabricated from three 240-lb, rectangular steel plates. The gate is hinged at *A* and rests against a frictionless support at *D*. Knowing that $d = 2.5$ ft, determine the reactions at *A* and *D*.

Problem 5.88: The gate at the end of a 3-ft-wide fresh water channel is fabricated from three 240-lb, rectangular steel plates. The gate is hinged at *A* and rests against a frictionless support at *D*. Determine the depth of water *d* for which the gate will open.

SOLUTION



5.87 Note that in addition to the weights of the gate segments, the water exerts pressure on all submerged surfaces $(p = \gamma h)$.

Thus, at

$$h = 0.5\text{ ft},\ \ p_{0.5} = \left(62.4\text{ lb/ft}^3\right)(0.5)\text{ ft} = 31.2\text{ lb/ft}^2$$

$$h = 2.5\text{ ft},\ \ p_{2.5} = \left(6.24\text{ lb/ft}^3\right)(2.5)\text{ ft} = 156.0\text{ lb/ft}^2$$

Then

$$P_1 = \frac{1}{2}\big[(0.5\text{ ft})(3\text{ ft})\big]\left(31.2\text{ lb/ft}^2\right) = 23.4\text{ lb}$$

$$P_2 = \big[(2\text{ ft})(3\text{ ft})\big]\left(31.2\text{ lb/ft}^2\right) = 187.2\text{ lb}$$

$$P_3 = \frac{1}{2}\big[(2\text{ ft})(3\text{ ft})\big]\left(31.2\text{ lb/ft}^2\right) = 93.6\text{ lb}$$

$$P_4 = \frac{1}{2}\big[(2\text{ ft})(3\text{ ft})\big]\left(156\text{ lb/ft}^2\right) = 468\text{ lb}$$

and

$$+\circlearrowleft\ \Sigma M_A = 0\!:\ \ -4D + (2\text{ ft})(240\text{ lb}) + (1\text{ ft})(240\text{ lb}) - \left[\left(2 + \frac{1}{3}\times 0.5\right)\text{ft}\,(23.4\text{ lb})\right] - (1\text{ ft})(187.2\text{ lb})$$

$$-\frac{2}{3}(2\text{ ft})(93.6\text{ lb}) - \frac{1}{3}(2\text{ ft})(468\text{ lb}) = 0$$

or

$$D = 11.325\text{ lb}$$

$\therefore$ $\mathbf{D} = 11.33\text{ lb} \longrightarrow$ ◀

PROBLEMS 5.87 AND 5.88 CONTINUED

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + 11.32 + 23.4 + 93.6 + 468 = 0$$

or $$A_x = -596.32 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 240 - 240 - 240 + 187.2 = 0$$

or $$A_y = 532.8 \text{ lb}$$ $\therefore$ $\mathbf{A} = 800 \text{ lb} \searrow 41.8°$ ◀

5.88 At $h = (d-2)\text{ft}, \quad p_{d-2} = \gamma(d-2) \text{ lb/ft}^2$ where $\gamma = 62.4 \text{ lb/ft}^3$

$$h = d \text{ ft}, \quad p_d = (\gamma d) \text{ lb/ft}^2$$

Then $$P_1 = \frac{1}{2}A_1 p_{d-2} = \frac{1}{2}\left[(d-2)\text{ft} \times (3\text{ ft})\right]\left[\gamma \text{ lb/ft}^3 (d-2)\text{ft}\right] = \frac{3}{2}\gamma(d-2)^2 \text{ lb}$$

(Note: For simplicity, the numerical value of the density γ will be substituted into the equilibrium equations below, rather than at this level of the calculations.)

$$P_2 = A_2 p_{d-2} = \left[(2\text{ ft})(3\text{ ft})\right]\gamma\left[(d-2)\text{ft}\right] = 6\gamma(d-2) \text{ lb}$$

$$P_3 = \frac{1}{2}A_3 p_{d-2} = \frac{1}{2}\left[(2\text{ ft})(3\text{ ft})\right]\gamma\left[(d-2)\text{ft}\right] = 3\gamma(d-2) \text{ lb}$$

$$P_4 = \frac{1}{2}A_4 p_d = \frac{1}{2}\left[(2\text{ ft})(3\text{ ft})\right]\gamma(d \text{ ft}) = (3\gamma d) \text{ lb} = \left[3\gamma(d-2) + 6\gamma\right] \text{ lb}$$

As the gate begins to open, $\mathbf{D} \to 0$

$$\therefore \; +\circlearrowright \Sigma M_A = 0: \quad (2\text{ ft})(240\text{ lb}) + (1\text{ ft})(240\text{ lb}) - \left[2\text{ ft} + \frac{1}{3}(d-2)\text{ft}\right]\left[\frac{3}{2}\gamma(d-2)^2 \text{ lb}\right] +$$

$$-(1\text{ ft})\left[6\gamma(d-2)\text{ lb}\right] - \left[\frac{2}{3}(2\text{ ft})\right]\left[3\gamma(d-2)\text{ lb}\right]$$

$$-\left[\frac{1}{3}(2\text{ ft})\right]\left[3\gamma(d-2)\text{ lb} + 6\gamma \text{ lb}\right] = 0$$

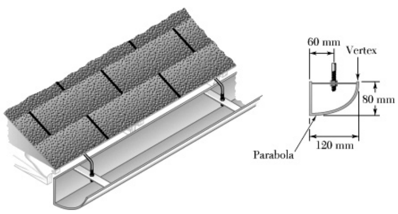
or $$\frac{1}{2}(d-2)^3 + 3(d-2)^2 + 12(d-2) = \frac{720}{\gamma} - 4$$

$$= \frac{720}{62.4} - 4$$

$$= 7.53846$$

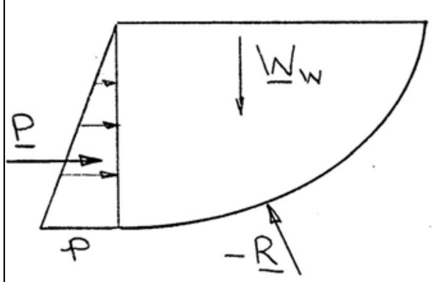
Solving numerically yields $d = 2.55 \text{ ft}$ ◀

PROBLEM 5.89



A rain gutter is supported from the roof of a house by hangers that are spaced 0.6 m apart. After leaves clog the gutter's drain, the gutter slowly fills with rainwater. When the gutter is completely filled with water, determine (*a*) the resultant of the pressure force exerted by the water on the 0.6-m section of the curved surface of the gutter, (*b*) the force-couple system exerted on a hanger where it is attached to the gutter.

SOLUTION



(*a*) Consider a 0.6 m long parabolic section of water.

Then $P = \frac{1}{2}Ap = \frac{1}{2}A(\rho gh)$

$$= \frac{1}{2}(0.08\text{ m})(0.6\text{ m})\left[(10^3\text{ kg/m}^3)(9.81\text{ m/s}^2)(0.08\text{ m})\right]$$

$$= 18.84\text{ N}$$

$$W_w = \rho g V$$

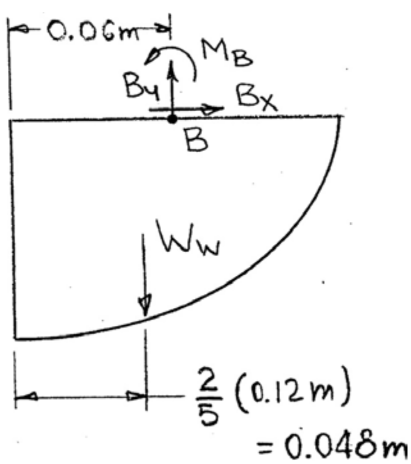
$$= (10^3\text{ kg/m}^3)(9.81\text{ m/s}^2)\left[\frac{2}{3}(0.12\text{ m})(0.08\text{ m})(0.6\text{ m})\right]$$

$$= 37.67\text{ N}$$

Now $\Sigma\mathbf{F} = 0\text{:}\quad (-\mathbf{R}) + \mathbf{P} + \mathbf{W}_w = 0$

So that $R = \sqrt{P^2 + W_w^{\,2}}, \qquad \tan\theta = \frac{W_w}{P}$

$= 42.12\text{ N},\ \theta = 63.4°$ $\qquad \mathbf{R} = 42.1\text{ N} \nwarrow 63.4°$ ◀



(*b*) Consider the free-body diagram of a 0.6 m long section of water and gutter.

Then $\xrightarrow{+} \Sigma F_x = 0\text{:}\quad B_x = 0$

$+\uparrow \Sigma F_y = 0\text{:}\quad B_y - 37.67\text{ N} = 0$

or $B_y = 37.67\text{ N}$

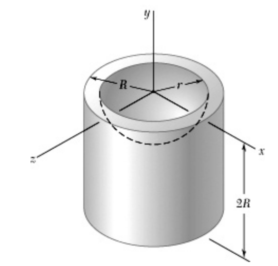
$+\circlearrowright \Sigma M_B = 0\text{:}\quad M_B + [(0.06 - 0.048)\text{ m}](37.67\text{ N}) = 0$

or $M_B = -0.4520\text{ N}\cdot\text{m}$

The force-couple system exerted on the hanger is then

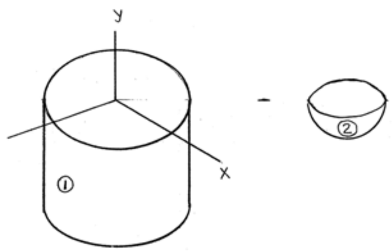
$37.7\text{ N}\downarrow,\ 0.452\text{ N}\cdot\text{m} \circlearrowright$ ◀

PROBLEM 5.90



The composite body shown is formed by removing a hemisphere of radius r from a cylinder of radius R and height $2R$. Determine (a) the y coordinate of the centroid when $r = 3R/4$, (b) the ratio r/R for which $\bar{y} = -1.2R$.

SOLUTION



Note, for the axes shown

	V	$\bar{y}$	$\bar{y}V$
1	$(\pi R^2)(2R) = 2\pi R^3$	$-R$	$-2\pi R^4$
2	$-\frac{2}{3}\pi r^3$	$-\frac{3}{8}r$	$\frac{1}{4}\pi r^4$
Σ	$2\pi\left(R^3 - \frac{r^3}{3}\right)$		$-2\pi\left(R^4 - \frac{r^4}{8}\right)$

Then
$$\bar{Y} = \frac{\Sigma\bar{y}V}{\Sigma V} = -\frac{R^4 - \frac{1}{8}r^4}{R^3 - \frac{1}{3}r^3}$$

$$= \frac{1 - \frac{1}{8}\left(\frac{r}{R}\right)^4}{1 - \frac{1}{3}\left(\frac{r}{R}\right)^3}$$

(a)
$$r = \frac{3}{4}R: \quad \bar{y} = -\frac{1 - \frac{1}{3}\left(\frac{3}{4}\right)^4}{1 - \frac{1}{3}\left(\frac{3}{4}\right)^3}R$$

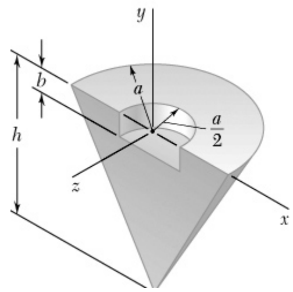
or $\bar{y} = -1.118R$ ◀

(b)
$$\bar{y} = -1.2R: \quad -1.2R = -\frac{1 - \frac{1}{8}\left(\frac{r}{R}\right)^4}{1 - \frac{1}{3}\left(\frac{r}{R}\right)^3}R$$

or
$$\left(\frac{r}{R}\right)^4 - 3.2\left(\frac{r}{R}\right)^3 + 1.6 = 0$$

Solving numerically $\frac{r}{R} = 0.884$ ◀

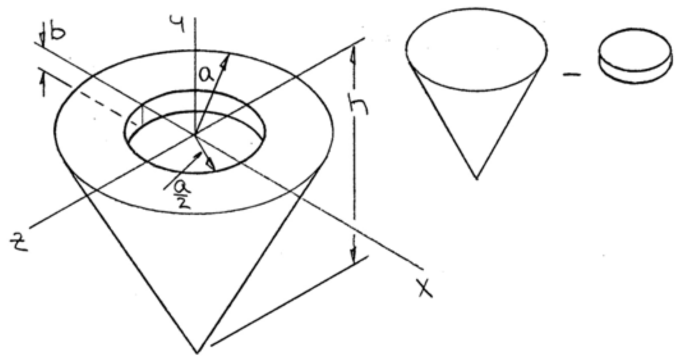
PROBLEM 5.91



Determine the y coordinate of the centroid of the body shown.

SOLUTION

First note that the values of $\bar{Y}$ will be the same for the given body and the body shown below. Then



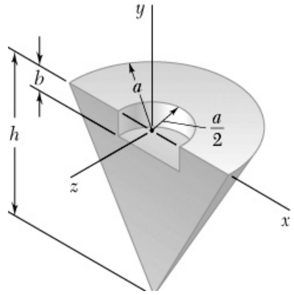
	V	$\bar{y}$	$\bar{y}V$
Cone	$\frac{1}{3}\pi a^2 h$	$-\frac{1}{4}h$	$-\frac{1}{12}\pi a^2 h^2$
Cylinder	$-\pi\left(\frac{a}{2}\right)^2 b = -\frac{1}{4}\pi a^2 b$	$-\frac{1}{2}b$	$\frac{1}{8}\pi a^2 b^2$
Σ	$\frac{\pi}{12}a^2(4h - 3b)$		$-\frac{\pi}{24}a^2\left(2h^2 - 3b^2\right)$

Have $$\bar{Y}\Sigma V = \Sigma\bar{y}V$$

Then $$\bar{Y}\left[\frac{\pi}{12}a^2(4h - 3b)\right] = -\frac{\pi}{24}a^2\left(2h^2 - 3b^2\right)$$

or $$\bar{Y} = -\frac{2h^2 - 3b^2}{2(4h - 3b)} \blacktriangleleft$$

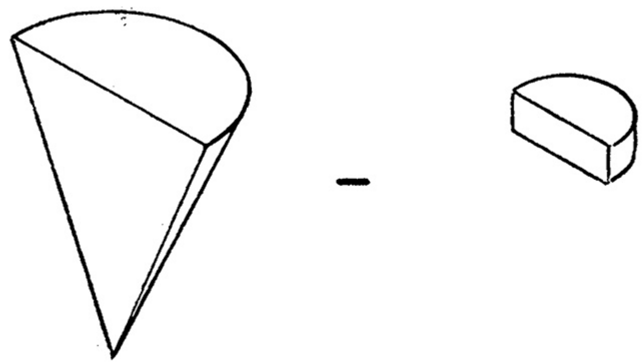
PROBLEM 5.92



Determine the z coordinate of the centroid of the body shown. (*Hint:* Use the result of Sample Problem 5.13.)

SOLUTION

First note that the body can be formed by removing a "half-cylinder" from a "half-cone," as shown.



	V	$\bar{z}$	$\bar{z}V$
Half-Cone	$\frac{1}{6}\pi a^2 h$	$-\frac{a}{\pi}$ *	$-\frac{1}{6}a^3 h$
Half-Cylinder	$-\frac{\pi}{2}\left(\frac{a}{2}\right)^2 b = -\frac{\pi}{8}a^2 b$	$-\frac{4}{3\pi}\left(\frac{a}{2}\right) = -\frac{2a}{3\pi}$	$\frac{1}{12}a^3 b$
Σ	$\frac{\pi}{24}a^2(4h-3b)$		$-\frac{1}{12}a^3(2h-b)$

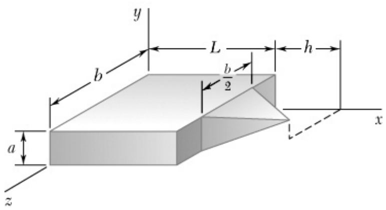
From Sample Problem 5.13

Have $$\bar{Z}\Sigma V = \Sigma \bar{z} V$$

Then $$\bar{Z}\left[\frac{\pi}{24}a^2(4h-3b)\right] = -\frac{1}{12}a^3(2h-b)$$

$$\text{or } \bar{Z} = -\frac{2a}{\pi}\frac{2h-b}{4h-3b} \blacktriangleleft$$

PROBLEM 5.93



Consider the composite body shown. Determine (*a*) the value of $\bar{x}$ when $h = L/2$, (*b*) the ratio h/L for which $\bar{x} = L$.

SOLUTION

	V	$\bar{x}$	$\bar{x}V$
Rectangular prism	Lab	$\frac{1}{2}L$	$\frac{1}{2}L^2ab$
Pyramid	$\frac{1}{3}a\left(\frac{b}{2}\right)h$	$L + \frac{1}{4}h$	$\frac{1}{6}abh\left(L + \frac{1}{4}h\right)$

Then $$\Sigma V = ab\left(L + \frac{1}{6}h\right) \qquad \Sigma\bar{x}V = \frac{1}{6}ab\left[3L^2 + h\left(L + \frac{1}{4}h\right)\right]$$

Now $$\bar{X}\Sigma V = \Sigma\bar{x}V \qquad \text{so that}$$

$$\bar{X}\left[ab\left(L + \frac{1}{6}h\right)\right] = \frac{1}{6}ab\left(3L^2 + hL + \frac{1}{4}h^2\right)$$

or $$\bar{X}\left(1 + \frac{1}{6}\frac{h}{L}\right) = \frac{1}{6}L\left(3 + \frac{h}{L} + \frac{1}{4}\frac{h^2}{L^2}\right) \qquad (1)$$

(*a*) $\bar{X} = ?$ when $h = \frac{1}{2}L$

Substituting $\frac{h}{L} = \frac{1}{2}$ into Eq. (1)

$$\bar{X}\left[1 + \frac{1}{6}\left(\frac{1}{2}\right)\right] = \frac{1}{6}L\left[3 + \left(\frac{1}{2}\right) + \frac{1}{4}\left(\frac{1}{2}\right)^2\right]$$

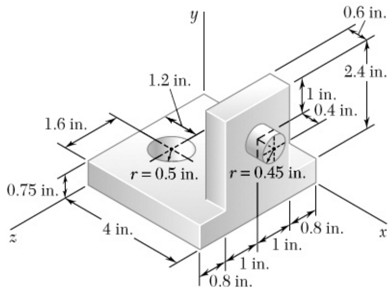
or $$\bar{X} = \frac{57}{104}L$$ $\bar{X} = 0.548L$ ◀

(*b*) $\frac{h}{L} = ?$ when $\bar{X} = L$

Substituting into Eq. (1) $$L\left(1 + \frac{1}{6}\frac{h}{L}\right) = \frac{1}{6}L\left(3 + \frac{h}{L} + \frac{1}{4}\frac{h^2}{L^2}\right)$$

or $$1 + \frac{1}{6}\frac{h}{L} = \frac{1}{2} + \frac{1}{6}\frac{h}{L} + \frac{1}{24}\frac{h^2}{L^2}$$

or $$\frac{h^2}{L^2} = 12$$ $\therefore\ \frac{h}{L} = 2\sqrt{3}$ ◀



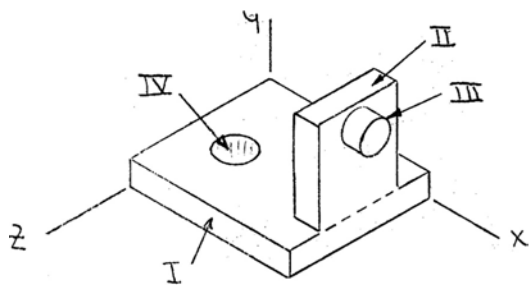
PROBLEMS 5.94 AND 5.95

Problem 5.94: For the machine element shown, determine the *x* coordinate of the center of gravity.

Problem 5.95: For the machine element shown, determine the *y* coordinate of the center of gravity.

SOLUTIONS

First, assume that the machine element is homogeneous so that its center of gravity coincides with the centroid of the corresponding volume.



	V, in^3	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}V$, in^4	$\bar{y}V$, in^4
I	$(4)(3.6)(0.75) = 10.8$	2.0	0.375	21.6	4.05
II	$(2.4)(2.0)(0.6) = 2.88$	3.7	1.95	10.656	5.616
III	$\pi(0.45)^2(0.4) = 0.2545$	4.2	2.15	1.0688	0.54711
IV	$-\pi(0.5)^2(0.75) = -0.5890$	1.2	0.375	–0.7068	–0.22089
Σ	13.3454			32.618	9.9922

5.94

Have
$$\bar{X}\Sigma V = \Sigma \bar{x} V$$

$$\bar{X}\left(13.3454 \text{ in}^3\right) = 32.618 \text{ in}^4$$

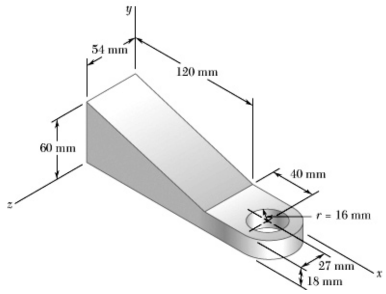
or $\bar{X} = 2.44$ in. ◀

5.95

Have
$$\bar{Y}\Sigma V = \Sigma \bar{y} V$$

$$\bar{Y}\left(13.3454 \text{ in}^3\right) = 9.9922 \text{ in}^4$$

or $\bar{Y} = 0.749$ in. ◀



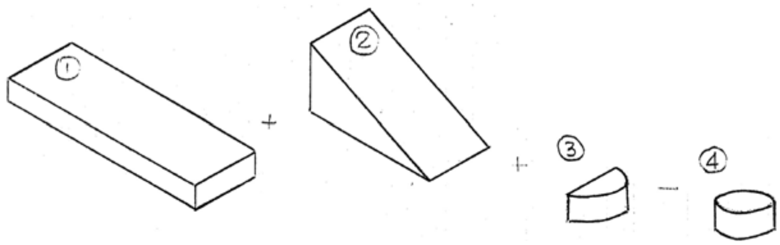
PROBLEMS 5.96 AND 5.97

Problem 5.96: For the machine element shown, locate the x coordinate of the center of gravity.

Problem 5.97: For the machine element shown, locate the y coordinate of the center of gravity.

SOLUTIONS

First, assume that the machine element is homogeneous so that its center of gravity coincides with the centroid of the corresponding volume.



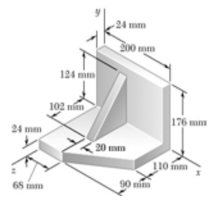
	V, mm^3	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}V$, mm^4	$\bar{y}V$, mm^4
1	$(160)(54)(18) = 155\ 520$	80	9	12 441 600	1 399 680
2	$\frac{1}{2}(120)(42)(54) = 136\ 080$	40	32	5 443 200	4 354 560
3	$\frac{\pi}{2}(27)^2(18) = 6561\pi$	$160 + \frac{36}{\pi}$	9	3 534 114	185 508
4	$-\pi(16)^2(18) = -4608\pi$	160	9	–2 316 233	–130 288
Σ	297 736			19 102 681	5 809 460

5.96

Have
$$\bar{X}\Sigma V = \Sigma \bar{x} V$$
$$\bar{X}\left(297\ 736\ \text{mm}^3\right) = 19\ 102\ 681\ \text{mm}^4$$
or $\bar{X} = 64.2$ mm ◀

5.97

Have
$$\bar{Y}\Sigma V = \Sigma \bar{y} V$$
$$\bar{Y}\left(297\ 736\ \text{mm}^3\right) = 5\ 809\ 460\ \text{mm}^4$$
or $\bar{Y} = 19.51$ mm ◀



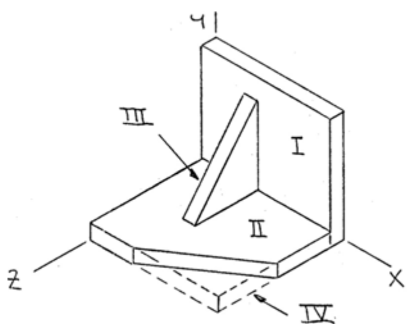
PROBLEMS 5.98 AND 5.99

Problem 5.98: For the stop bracket shown, locate the *x* coordinate of the center of gravity.

Problem 5.99: For the stop bracket shown, locate the *z* coordinate of the center of gravity.

SOLUTIONS

First, assume that the bracket is homogeneous so that its center of gravity coincides with the centroid of the corresponding volume.



Have..

$$\bar{Z}_{\text{II}} = 24 \text{ mm} + \frac{1}{2}(90 + 86)\text{mm} = 112 \text{ mm}$$

$$\bar{Z}_{\text{III}} = 24 \text{ mm} + \frac{1}{3}(102)\text{mm} = 58 \text{ mm}$$

$$\bar{X}_{\text{III}} = 68 \text{ mm} + \frac{1}{2}(20)\text{mm} = 78 \text{ mm}$$

$$\bar{Z}_{\text{IV}} = 110 \text{ mm} + \frac{2}{3}(90)\text{mm} = 170 \text{ mm}$$

$$\bar{X}_{\text{IV}} = 60 \text{ mm} + \frac{2}{3}(132)\text{mm} = 156 \text{ mm}$$

	V, mm^3	$\bar{x}$, mm	$\bar{z}$, mm	$\bar{x}V$, mm^4	$\bar{z}V$, mm^4
I	$(200)(176)(24) = 844\ 800$	100	12	84 480 000	1 013 760
II	$(200)(24)(176) = 844\ 800$	100	112	84 480 000	94 617 600
III	$\frac{1}{2}(20)(124)(102) = 126\ 480$	78	58	9 865 440	733 840
IV	$-\frac{1}{2}(90)(132)(24) = -142\ 560$	156	170	–22 239 360	–24 235 200
Σ	1 673 520			156 586 080	8 785 584

5.98

Have $\bar{X}\Sigma V = \Sigma \bar{x} V$

$$\bar{X}\left(1\ 673\ 520 \text{ mm}^3\right) = 156\ 586\ 080 \text{ mm}^4$$

or $\bar{X} = 93.6$ mm ◀

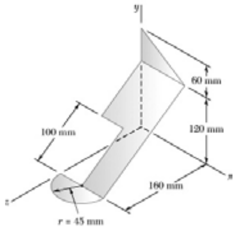
5.99

Have $\bar{Z}\Sigma V = \Sigma \bar{z} V$

$$\bar{Z}\left(1\ 673\ 520 \text{ mm}^3\right) = 8\ 785\ 584 \text{ mm}^4$$

or $\bar{Z} = 52.5$ mm ◀

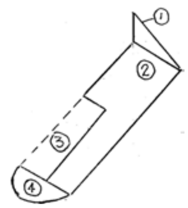
PROBLEM 5.100



Locate the center of gravity of the sheet-metal form shown.

SOLUTION

First, assume that the sheet metal is homogeneous so that the center of gravity coincides with the centroid of the corresponding area.



	A, mm^2	$\overline{x}$, mm	$\overline{y}$, mm	$\overline{z}$, mm	$\overline{x}A$, mm^3	$\overline{y}A$, mm^3	$\overline{z}A$, mm^3
1	$\frac{1}{2}(90)(60) = 2700$	30	$120 + 20 = 140$	0	81 000	378 000	0
2	$(90)(200) = 18\,000$	45	60	80	810 000	1 080 000	1 440 000
3	$-(45)(100) = -4500$	22.5	30	120	–101 250	–135 000	–540 000
4	$\frac{\pi}{2}(45)^2 = 1012.5\pi$	45	0	$160 + \frac{(4)(45)}{3\pi} = 179.1$	143 139	0	569 688
Σ	19 380.9				932 889	1 323 000	1 469 688

Have $\overline{X}\Sigma A = \Sigma\overline{x}A$:

$$\overline{X}\left(19\,380.9 \text{ mm}^2\right) = 932\,889 \text{ mm}^3$$

or $= 48.1 \text{ mm}$ $\quad\overline{X} = 48.1 \text{ mm}$ ◀

$$\overline{Y}\Sigma A = \Sigma\overline{y}A$$

$$\overline{Y}\left(19\,380.9 \text{ mm}^2\right) = 1\,323\,000 \text{ mm}^3$$

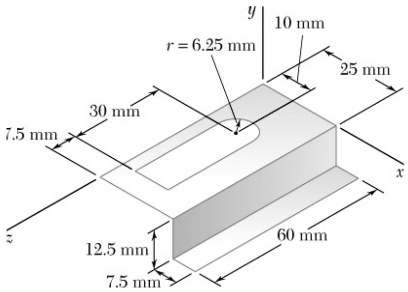
or $\overline{Y} = 68.3 \text{ mm}$ $\quad\overline{Y} = 68.3 \text{ mm}$ ◀

$$\overline{Z}\Sigma A = \Sigma\overline{z}A$$

$$\overline{Z}\left(19\,380.9 \text{ mm}^2\right) = 1\,469\,688 \text{ mm}^3$$

or $\overline{Z} = 75.8 \text{ mm}$ $\quad\overline{Z} = 75.8 \text{ mm}$ ◀

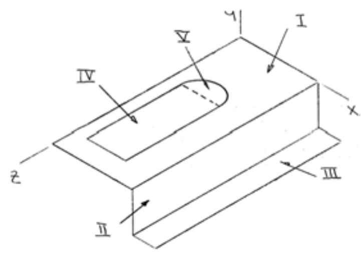
PROBLEM 5.101



A mounting bracket for electronic components is formed from sheet metal of uniform thickness. Locate the center of gravity of the bracket.

SOLUTION

First, assume that the sheet metal is homogeneous so that the center of gravity of the bracket coincides with the centroid of the corresponding area. Then (see diagram)



$$\bar{z}_V = 22.5 - \frac{4(6.25)}{3\pi}$$

$$= 19.85 \text{ mm}$$

$$A_V = -\frac{\pi}{2}(6.25)^2$$

$$= -61.36 \text{ mm}^2$$

	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{z}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3	$\bar{z}A$, mm^3
I	$(25)(60) = 1500$	12.5	0	30	18 750	0	45 000
II	$(12.5)(60) = 750$	25	–6.25	30	18 750	–4687.5	22 500
III	$(7.5)(60) = 450$	28.75	–12.5	30	12 937.5	–5625	13 500
IV	$-(12.5)(30) = -375$	10	0	37.5	–3750	0	–14 062.5
V	–61.36	10	0	19.85	–613.6	0	–1218.0
Σ	2263.64				46 074	–10 313	65 720

Have

$$\bar{X}\Sigma A = \Sigma \bar{x}A$$

$$\bar{X}(2263.64 \text{ mm}^2) = 46\ 074 \text{ mm}^3 \qquad \text{or } \bar{X} = 20.4 \text{ mm} ◀$$

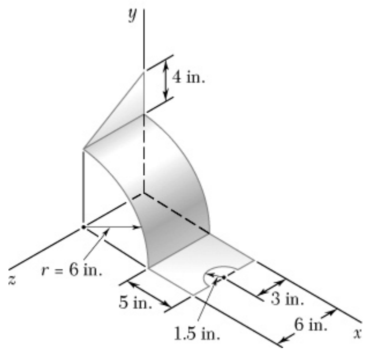
$$\bar{Y}\Sigma A = \Sigma \bar{y}A$$

$$\bar{Y}(2263.64 \text{ mm}^2) = -10\ 313 \text{ mm}^3 \qquad \text{or } \bar{Y} = -4.55 \text{ mm} ◀$$

$$\bar{Z}\Sigma A = \Sigma \bar{z}A$$

$$\bar{Z}(2263.64 \text{ mm}^2) = 65\ 720 \text{ mm}^3 \qquad \text{or } \bar{Z} = 29.0 \text{ mm} ◀$$

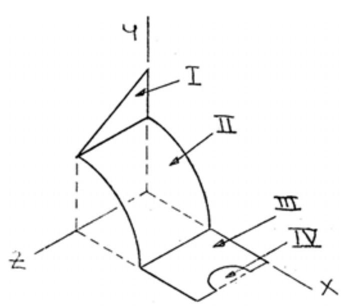
PROBLEM 5.102



Locate the center of gravity of the sheet-metal form shown.

SOLUTION

First, assume that the sheet metal is homogeneous so that the center of gravity of the form coincides with the centroid of the corresponding area.



$$\bar{y}_I = 6 + \frac{4}{3} = 7.333 \text{ in.}$$

$$\bar{z}_I = \frac{1}{3}(6) = 2 \text{ in.}$$

$$\bar{x}_{II} = \bar{y}_{II} = \frac{1}{\pi}(2)(6) = 3.8197 \text{ in.}$$

$$\bar{x}_{IV} = 11 - \frac{1}{3\pi}(4)(1.5) = 10.363 \text{ in.}$$

	A, in^2	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{z}$, in.	$\bar{x}A$, in^3	$\bar{y}A$, in^3	$\bar{z}A$, in^3
I	12	0	7.333	2	0	88	24
II	56.55	3.8197	3.8197	3	216	216	169.65
III	30	8.5	0	3	255	0	90
IV	–3.534	10.363	0	3	–36.62	0	–10.603
Σ	95.01				434.4	304	273.0

Have $\bar{X}\Sigma A = \Sigma \bar{x} A$

$$\bar{X}\left(95.01 \text{ in}^2\right) = 434 \text{ in}^3$$

or $\bar{X} = 4.57$ in. ◀

$$\bar{Y}\Sigma A = \Sigma \bar{y} A$$

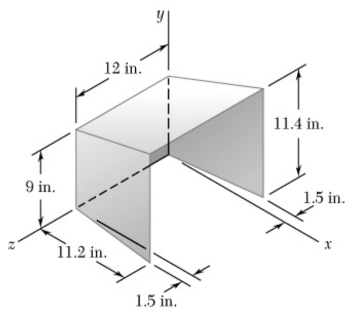
$$\bar{Y}\left(95.01 \text{ in}^2\right) = 304.0 \text{ in}^3$$

or $\bar{Y} = 3.20$ in. ◀

$$\bar{Z}\Sigma A = \Sigma \bar{z} A$$

$$Z\left(95.01 \text{ in}^2\right) = 273.0 \text{ in}^3$$

or $\bar{Z} = 2.87$ in. ◀



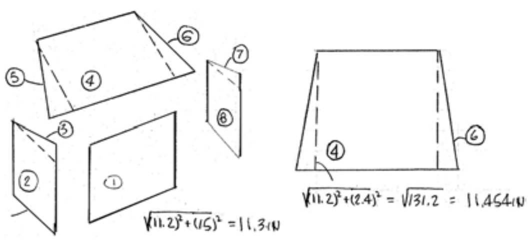
PROBLEM 5.103

An enclosure for an electronic device is formed from sheet metal of uniform thickness. Locate the center of gravity of the enclosure.

SOLUTION

First, assume that the sheet metal is homogeneous so that the center of gravity of the form coincides with the centroid of the corresponding area.

Consider the division of the back, sides, and top into eight segments according to the sketch.



Note that symmetry implies $\bar{Z} = 6.00 \text{ in.}$ ◀

and

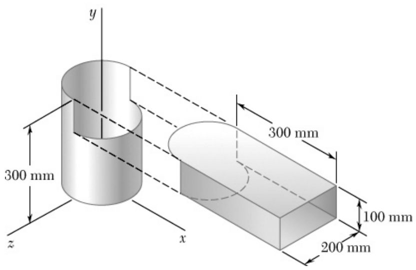
$$A_8 = A_2$$
$$A_7 = A_3$$
$$A_6 = A_5$$

Thus

	A, in^2	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}A$, in^3	$\bar{y}A$, in^3
1	$(12)(9) = 108$	0	4.5	0	486
2	$(11.3)(9) = 101.7$	5.6	4.5	569.5	457.6
3	$\frac{1}{2}(11.3)(2.4) = 13.56$	7.467	9.8	101.25	132.89
4	$(12)(11.454) = 137.45$	5.6	10.2	769.72	1402.0
5	$\frac{1}{2}(1.5)(11.454) = 8.591$	7.467	10.6	64.15	91.06
6	8.591	7.467	10.6	64.15	91.06
7	13.56	7.467	9.8	101.25	132.89
8	101.7	5.6	4.5	569.5	457.6
Σ	493.2			2239.5	3251.1

Have $\bar{X}\Sigma A = \Sigma\bar{x}A$: $\bar{X}(493.2 \text{ in}^2) = 2239.5 \text{ in}^3$ or $\bar{X} = 4.54 \text{ in.}$ ◀

$\bar{Y}\Sigma A = \Sigma\bar{y}A$: $\bar{Y}(493.2 \text{ in}^2) = 3251.1 \text{ in}^3$ or $\bar{Y} = 6.59 \text{ in.}$ ◀

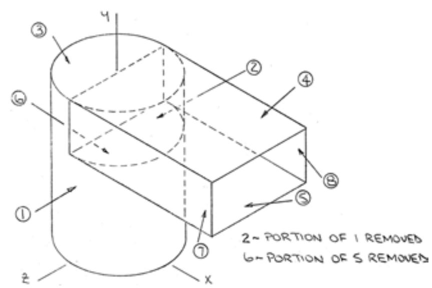


PROBLEM 5.104

A 200-mm-diameter cylindrical duct and a 100×200-mm rectangular duct are to be joined as indicated. Knowing that the ducts are fabricated from the same sheet metal, which is of uniform thickness, locate the center of gravity of the assembly.

SOLUTION

First, assume that the sheet metal is homogeneous so that the center of gravity of the duct coincides with the centroid of the corresponding area. Also note that symmetry implies $\bar{Z} = 0$ ◀



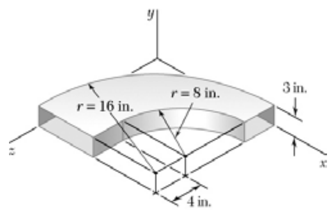
	A, m^2	$\bar{x}$, m	$\bar{y}$, m	$\bar{x}A$, m^3	$\bar{y}A$, m^3
1	$\pi(0.2)(0.3) = 0.1885$	0	0.15	0	0.028274
2	$-\frac{\pi}{2}(0.2)(0.1) = -0.0314$	$\frac{2(0.1)}{\pi} = 0.06366$	0.25	–0.02000	–0.007854
3	$\frac{\pi}{2}(0.1)^2 = 0.01571$	$\frac{-4(0.1)}{3\pi} = -0.04244$	0.30	–0.000667	0.004712
4	$(0.3)(0.2) = 0.060$	0.15	0.30	0.00900	0.001800
5	$(0.3)(0.2) = 0.060$	0.15	0.20	0.00900	0.001200
6	$-\frac{\pi}{2}(0.1)^2 = -0.1571$	$\frac{4(0.1)}{3\pi}$	0.20	–0.000667	–0.003142
7	$(0.3)(0.1) = 0.030$	0.15	0.25	0.004500	0.007500
8	$(0.3)(0.1) = 0.030$	0.15	0.25	0.004500	0.007500
Σ	0.337080			0.023667	0.066991

Have $\bar{X}\Sigma A = \Sigma \bar{x}A$: $\bar{X}(0.337080 \text{ mm}^2) = 0.023667 \text{ mm}^3$

or $\bar{X} = 0.0702$ m $\qquad \bar{X} = 70.2$ mm ◀

$\bar{Y}\Sigma A = \Sigma \bar{y}A$: $\bar{Y}\left(0.337080 \text{ mm}^2\right) = 0.066991 \text{ mm}^3$

or $\bar{Y} = 0.19874$ m $\qquad \bar{Y} = 198.7$ mm ◀

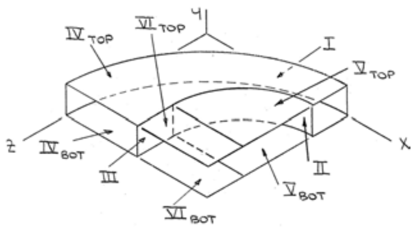


PROBLEM 5.105

An elbow for the duct of a ventilating system is made of sheet metal of uniform thickness. Locate the center of gravity of the elbow.

SOLUTION

First, assume that the sheet metal is homogeneous so that the center of gravity of the duct coincides with the centroid of the corresponding area. Also, note that the shape of the duct implies $\overline{Y} = 1.5 \text{ in.}$ ◀



Note that $\bar{x}_{\text{I}} = \bar{z}_{\text{I}} = 16 \text{ in.} - \frac{2}{\pi}(16 \text{ in.}) = 5.81408 \text{ in.}$

$$\bar{x}_{\text{II}} = 16 \text{ in.} - \frac{2}{\pi}(8 \text{ in.}) = 10.9070 \text{ in.}$$

$$\bar{z}_{\text{II}} = 12 \text{ in.} - \frac{2}{\pi}(8 \text{ in.}) = 6.9070 \text{ in.}$$

$$\bar{x}_{\text{IV}} = \bar{z}_{\text{IV}} = 16 \text{ in.} - \frac{4}{3\pi}(16 \text{ in.}) = 9.2094 \text{ in.}$$

$$\bar{x}_{\text{V}} = 16 \text{ in.} - \frac{4}{3\pi}(8 \text{ in.}) = 12.6047 \text{ in.}$$

$$\bar{z}_{\text{V}} = 12 \text{ in.} - \frac{4}{3\pi}(8 \text{ in.}) = 8.6047 \text{ in.}$$

Also note that the corresponding top and bottom areas will contribute equally when determining $\bar{x}$ and $\bar{z}$.

Thus

	A, in^2	$\bar{x}$, in.	$\bar{z}$, in.	$\bar{x}A$, in^3	$\bar{z}A$, in^3
I	$\frac{\pi}{2}(16)(3) = 75.3982$	5.81408	5.81408	438.37	438.37
II	$\frac{\pi}{2}(8)(3) = 37.6991$	10.9070	6.9070	411.18	260.39
III	$4(3) = 12$	8	14	96.0	168.0
IV	$2\left(\frac{\pi}{4}\right)(16)^2 = 402.1239$	9.2094	9.2094	3703.32	3703.32
V	$-2\left(\frac{\pi}{4}\right)(8)^2 = -100.5309$	12.6047	8.6047	–1267.16	–865.04
VI	$-2(4)(8) = -64$	12	14	–768.0	–896.0
Σ	362.69			2613.71	2809.04

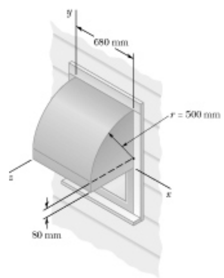
Have $\overline{X}\Sigma A = \Sigma \bar{x}A$: $\overline{X}\left(362.69 \text{ in}^2\right) = 2613.71 \text{ in}^3$

or $\overline{X} = 7.21 \text{ in.}$ ◀

$\overline{Z}\Sigma A = \Sigma \bar{z}A$: $\overline{Z}\left(362.69 \text{ in}^2\right) = 2809.04 \text{ in}^3$

or $\overline{Z} = 7.74 \text{ in.}$ ◀

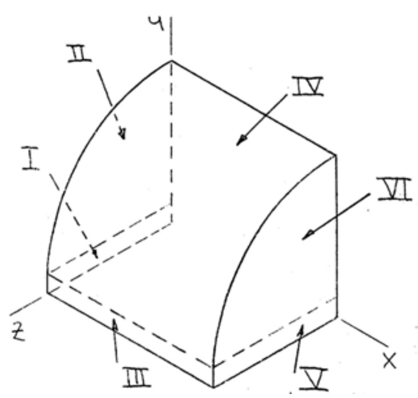
PROBLEM 5.106



A window awning is fabricated from sheet metal of uniform thickness. Locate the center of gravity of the awning.

SOLUTION

First, assume that the sheet metal is homogeneous so that the center of gravity of the awning coincides with the centroid of the corresponding area.



$$\bar{y}_{II} = \bar{y}_{VI} = 80 + \frac{(4)(500)}{3\pi} = 292.2 \text{ mm}$$

$$\bar{z}_{II} = \bar{z}_{VI} = \frac{(4)(500)}{3\pi} = 212.2 \text{ mm}$$

$$\bar{y}_{IV} = 80 + \frac{(2)(500)}{\pi} = 398.3 \text{ mm}$$

$$\bar{z}_{IV} = \frac{(2)(500)}{\pi} = 318.3 \text{ mm}$$

$$A_{II} = A_{VI} = \frac{\pi}{4}(500)^2 = 196\ 350 \text{ mm}^2$$

$$A_{IV} = \frac{\pi}{2}(500)(680) = 534\ 071 \text{ mm}^2$$

	A, mm^2	$\bar{y}$, mm	$\bar{z}$, mm	$\bar{y}A$, mm^3	$\bar{z}A$, mm^3
I	$(80)(500) = 40\ 000$	40	250	1.6×10^6	10×10^6
II	196 350	292.2	212.2	57.4×10^6	41.67×10^6
III	$(80)(680) = 54\ 400$	40	500	0.2176×10^6	27.2×10^6
IV	534 071	398.3	318.3	212.7×10^6	170×10^6
V	$(80)(500) = 40\ 000$	40	250	1.6×10^6	10×10^6
VI	196 350	292.2	212.2	57.4×10^6	41.67×10^6
Σ	1.061×10^6			332.9×10^6	300.5×10^6

Now, symmetry implies $\bar{X} = 340 \text{ mm}$ ◀

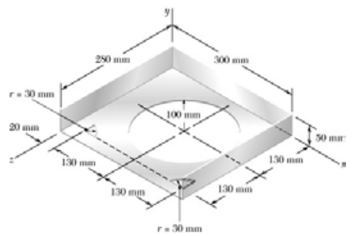
and $\bar{Y}\Sigma A = \Sigma\bar{y}A$: $\bar{Y}(1.061 \times 10^6 \text{ mm}^2) = 332.9 \times 10^6 \text{ mm}^3$

or $\bar{Y} = 314 \text{ mm}$ ◀

$\bar{Z}\Sigma A = \Sigma\bar{z}A$: $\bar{Z}(1.061 \times 10^6 \text{ mm}^2) = 300.5 \times 10^6 \text{ mm}^3$

or $\bar{Z} = 283 \text{ mm}$ ◀

PROBLEM 5.107

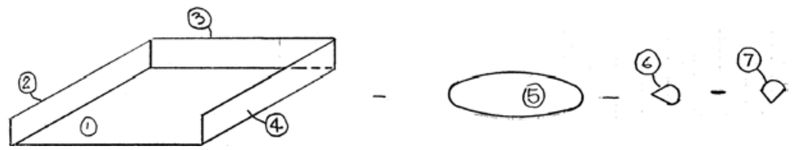


The thin, plastic front cover of a wall clock is of uniform thickness. Locate the center of gravity of the cover.

SOLUTION

First, assume that the plastic is homogeneous so that the center of gravity of the cover coincides with the centroid of the corresponding area.

Next, note that symmetry implies $\bar{X} = 150.0$ mm



	A, mm^2	$\bar{y}$, mm	$\bar{z}$, mm	$\bar{y}A$, mm^3	$\bar{z}$, mm^3
1	$(300)(280) = 84\,000$	0	140	0	11 760 000
2	$(280)(50) = 14\,000$	25	140	350 000	1 960 000
3	$(300)(50) = 15\,000$	25	0	375 000	0
4	$(280)(50) = 14\,000$	25	140	350 000	1 960 000
5	$-\pi(100)^2 = -31\,416$	0	130	0	–4 084 070
6	$\frac{-\pi}{4}(30)^2 = -706.86$	0	$260 - \frac{(4)(30)}{3\pi} = 247.29$	0	–174 783
7	$\frac{-\pi}{4}(30)^2 = -706.86$	0	$260 - \frac{(4)(30)}{3\pi} = 247.29$	0	–174 783
Σ	94 170			1 075 000	11 246 363

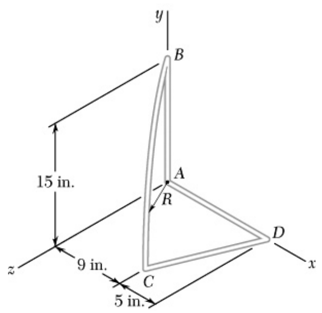
Have $\bar{Y}\Sigma A = \Sigma\bar{y}A$: $\quad \bar{Y}\left(94\,170 \text{ mm}^2\right) = 1\,075\,000 \text{ mm}^3$

or $\bar{Y} = 11.42$ mm ◀

$\bar{Z}\Sigma A = \Sigma\bar{z}A$: $\quad \bar{Z}\left(94\,170 \text{ mm}^2\right) = 11\,246\,363 \text{ mm}^3$

or $\bar{Z} = 119.4$ mm ◀

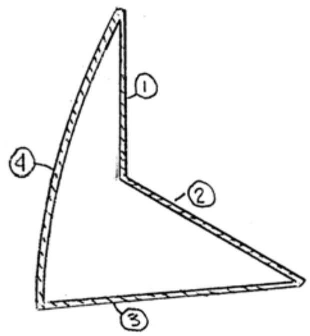
PROBLEM 5.108



A thin steel wire of uniform cross section is bent into the shape shown, where arc BC is a quarter circle of radius R. Locate its center of gravity.

SOLUTION

First, assume that the wire is homogeneous so that its center of gravity coincides with the centroid of the corresponding line.



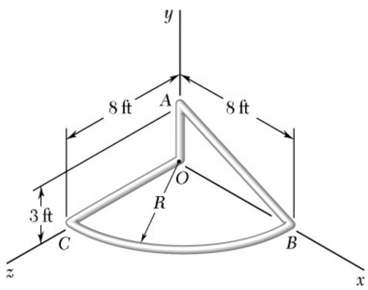
	L, in^2	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{z}$, in.	$\bar{x}L$, in^2	$\bar{y}L$, in^2	$\bar{z}L$, in^2
1	15	0	7.5	0	0	112.5	0
2	14	7	0	0	98	0	0
3	13	$9\left(\frac{5}{12}\right) = 11.5$	0	6	149.5	0	78
4	$\frac{\pi}{2}(15) = 23.56$	$\frac{3}{5}\left(\frac{2\times 15}{\pi}\right) = 5.73$	$\frac{30}{\pi} = 9.549$	$\frac{24}{\pi} = 7.639$	135.0	225.0	180.0
Σ	65.56				382.5	337.5	258.0

Have $\quad \bar{X}\Sigma L = \Sigma\bar{x}L: \quad \bar{X}(65.56\text{ in.}) = 382.5\text{ in}^2 \quad$ or $\bar{X} = 5.83\text{ in.}$ ◀

$\bar{Y}\Sigma L = \Sigma\bar{y}L: \quad \bar{Y}(65.56\text{ in.}) = 337.5\text{ in}^2 \quad$ or $\bar{Y} = 5.15\text{ in.}$ ◀

$\bar{Z}\Sigma L = \Sigma\bar{z}L: \quad \bar{Z}(65.56\text{ in.}) = 258.0\text{ in}^2 \quad$ or $\bar{Z} = 3.94\text{ in.}$ ◀

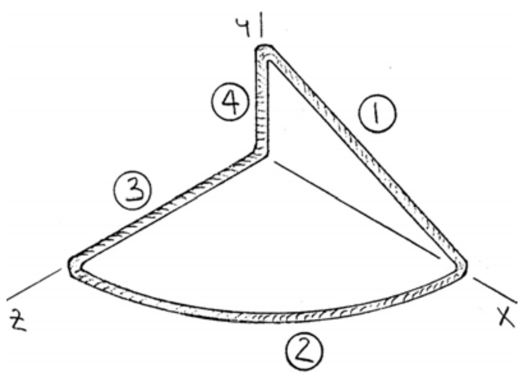
PROBLEM 5.109



A thin steel wire of uniform cross section is bent into the shape shown, where arc *BC* is a quarter circle of radius *R*. Locate its center of gravity.

SOLUTION

First, assume that the wire is homogeneous so that its center of gravity coincides with the centroid of the corresponding line



Have $\bar{x}_2 = \bar{z}_2 = \dfrac{(2)(8)}{\pi} = \dfrac{16}{\pi}\text{ ft}$

$$L_1 = \sqrt{8^2 + 3^2} = 8.5440\text{ ft}$$

$$L_2 = \frac{8\pi}{2} = 4\pi\text{ ft}$$

4

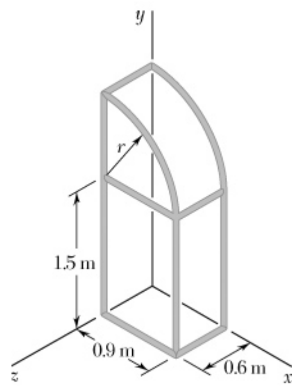
	L, ft	$\bar{x}$, ft	$\bar{y}$, ft	$\bar{z}$, ft	$\bar{x}L$, ft^2	$\bar{y}L$, ft^2	$\bar{z}L$, ft^2
1	8.5440	4	1.5	0	34.176	12.816	0
2	4π	16π	0	$\dfrac{16}{\pi}$	64.0	0	64.0
3	8	0	0	4	0	0	32
4	3	0	1.5	0	0	4.5	0
	32.110				98.176	17.316	96.0

Have $\bar{X}\Sigma L = \Sigma\bar{x}L$: $\bar{X}(32.110\text{ ft}) = 98.176\text{ ft}^2$ or $\bar{X} = 3.06\text{ ft}$ ◀

$\bar{Y}\Sigma L = \Sigma\bar{y}L$: $\bar{Y}(32.110\text{ ft}) = 17.316\text{ ft}^2$ or $\bar{Y} = 0.539\text{ ft}$ ◀

$\bar{Z}\Sigma L = \Sigma\bar{z}L$: $\bar{Z}(32.110\text{ ft}) = 96.0\text{ ft}^2$ or $\bar{Z} = 2.99\text{ ft}$ ◀

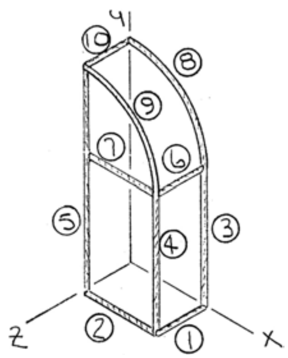
PROBLEM 5.110



The frame of a greenhouse is constructed from uniform aluminum channels. Locate the center of gravity of the portion of the frame shown.

SOLUTION

First, assume that the channels are homogeneous so that the center of gravity of the frame coincides with the centroid of the corresponding line.



Note $$\bar{x}_8 = \bar{x}_9 = \frac{(2)(0.9)}{\pi} = 0.57296 \text{ m}$$

$$\bar{y}_8 = \bar{y}_9 = 1.5 + \frac{(2)(0.9)}{\pi} = 2.073 \text{ m}$$

$$L_7 = L_8 = \frac{\pi}{2}(0.9) = 1.4137 \text{ m}$$

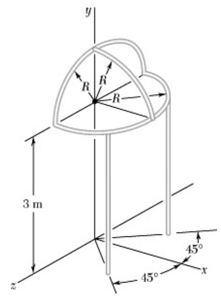
	L, m	$\bar{x}$, m	$\bar{y}$, m	$\bar{z}$, m	$\bar{x}L$, m^2	$\bar{y}L$, m^2	$\bar{z}L$, m^2
1	0.6	0.9	0	0.3	0.540	0	0.18
2	0.9	0.45	0	0.6	0.4050	0	0.54
3	1.5	0.9	0.75	0	1.350	1.125	0
4	1.5	0.9	0.75	0.6	1.350	1.125	0.9
5	2.4	0	1.2	0.6	0	2.880	1.44
6	0.6	0.9	1.5	0.3	0.540	0.9	0.18
7	0.9	0.45	1.5	0.6	0.4050	1.350	0.54
8	1.4137	0.573	2.073	0	0.8100	2.9306	0
9	1.4137	0.573	2.073	0.6	0.8100	2.9306	0.8482
10	0.6	0	2.4	0.3	0	1.440	0.18
Σ	11.827				6.210	14.681	4.8082

Have $\bar{X}\Sigma L = \Sigma\bar{x}L$: $\bar{X}(11.827 \text{ m}) = 6.210 \text{ m}^2$ or $\bar{X} = 0.525 \text{ m}$ ◀

$\bar{Y}\Sigma L = \Sigma\bar{y}L$: $\bar{Y}(11.827 \text{ m}) = 14.681 \text{ m}^2$ or $\bar{Y} = 1.241 \text{ m}$ ◀

$\bar{Z}\Sigma L = \Sigma\bar{z}L$: $\bar{Z}(11.827 \text{ m}) = 4.8082 \text{ m}^2$ or $\bar{Z} = 0.406 \text{ m}$ ◀

*

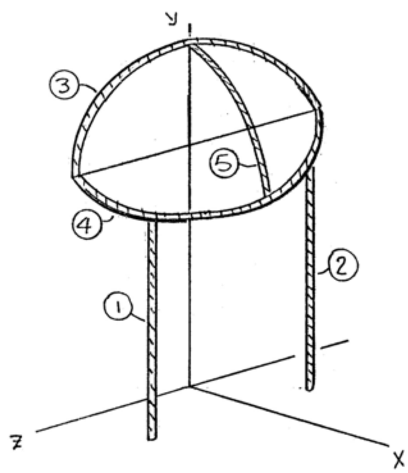


PROBLEM 5.111

The decorative metalwork at the entrance of a store is fabricated from uniform steel structural tubing. Knowing that $R = 1.2$ m, locate the center of gravity of the metalwork.

SOLUTION

First, assume that the tubes are homogeneous so that the center of gravity of the metalwork coincides with the centroid of the corresponding line.



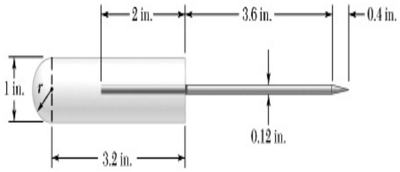
Note that symmetry implies $\bar{Z} = 0$ ◀

	L, m	$\bar{x}$, m	$\bar{y}$, m	$\bar{x}L$, m^2	$\bar{y}L$, m^2
1	3	$(1.2)\cos 45° = 0.8485$	1.5	2.5456	4.5
2	3	$(1.2)\cos 45° = 0.8485$	1.5	2.5456	4.5
3	1.2π	0	3.7639	0	14.1897
4	1.2π	$\dfrac{(2)(1.2)}{\pi} = 0.7639$	3	2.88	11.3097
5	0.6π	$\dfrac{(2)(1.2)}{\pi} = 0.7639$	3.7639	1.44	7.0949
Σ	15.425			9.4112	41.594

Have $\quad \bar{X}\Sigma L = \Sigma \bar{x}L: \quad \bar{X}(15.425 \text{ m}) = 9.4112 \text{ m}^2 \quad$ or $\bar{X} = 0.610$ m ◀

$\bar{Y}\Sigma L = \Sigma \bar{y}L: \quad \bar{Y}(15.425 \text{ m}) = 41.594 \text{ m}^2 \quad$ or $\bar{Y} = 2.70$ m ◀

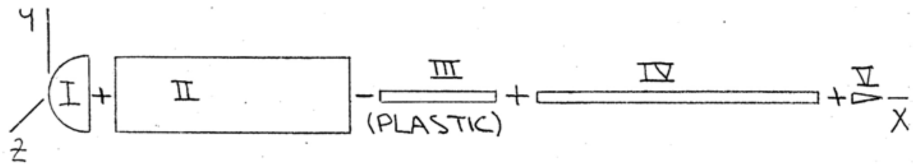
PROBLEM 5.112



A scratch awl has a plastic handle and a steel blade and shank. Knowing that the specific weight of plastic is $0.0374\ \text{lb/in}^3$ and of steel is $0.284\ \text{lb/in}^3$, locate the center of gravity of the awl.

SOLUTION

First, note that symmetry implies $\qquad \overline{Y} = \overline{Z} = 0$ ◀



$$\overline{x}_{\text{I}} = \frac{5}{8}(0.5\ \text{in.}) = 0.3125\ \text{in.},\ W_{\text{I}} = \left(0.0374\ \text{lb/in}^3\right)\left(\frac{2\pi}{3}\right)(0.5\ \text{in.})^3 = 0.009791\ \text{lb}$$

$$\overline{x}_{\text{II}} = 1.6\ \text{in.} + 0.5\ \text{in.} = 2.1\ \text{in.}\ W_{\text{II}} = \left(0.0374\ \text{lb/in}^3\right)(\pi)(0.5\ \text{in.})^2(3.2\ \text{in.}) = 0.093996\ \text{lb}$$

$$\overline{x}_{\text{III}} = 3.7\ \text{in.} - 1\ \text{in.} = 2.7\ \text{in.},\ W_{\text{III}} = -\left(0.0374\ \text{lb/in}^3\right)\left(\frac{\pi}{4}\right)(0.12\ \text{in.})^2(2\ \text{in.}) = -0.000846\ \text{lb}$$

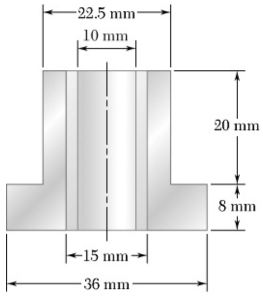
$$\overline{x}_{\text{IV}} = 7.3\ \text{in.} - 2.8\ \text{in.} = 4.5\ \text{in.},\ W_{\text{IV}} = \left(0.284\ \text{lb/in}^3\right)\left(\frac{\pi}{4}\right)(0.12\ \text{in.})^2(5.6\ \text{in.})^2 = 0.017987\ \text{lb}$$

$$\overline{x}_{\text{V}} = 7.3\ \text{in.} + \frac{1}{4}(0.4\ \text{in.}) = 7.4\ \text{in.},\ W_{\text{V}} = \left(0.284\ \text{lb/in}^3\right)\left(\frac{\pi}{3}\right)(0.06\ \text{in.})^2(0.4\ \text{in.}) = 0.000428\ \text{lb}$$

	W, lb	$\overline{x}$, in.	$\overline{x}W$, in·lb
I	0.009791	0.3125	0.003060
II	0.093996	2.1	0.197393
III	–0.000846	2.7	–0.002284
IV	0.017987	4.5	0.080942
V	0.000428	7.4	0.003169
Σ	0.12136		0.28228

Have $\qquad \overline{X}\Sigma W = \Sigma\overline{x}W: \quad \overline{X}(0.12136\ \text{lb}) = 0.28228\ \text{in.}\cdot\text{lb}$ $\qquad$ or $\overline{X} = 2.33\ \text{in.}$ ◀

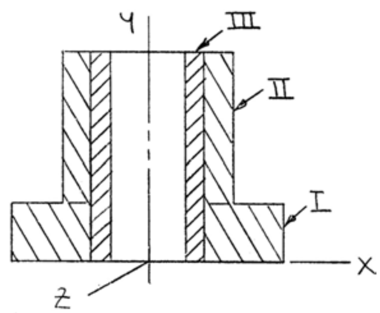
PROBLEM 5.113



A bronze bushing is mounted inside a steel sleeve. Knowing that the density of bronze is 8800 kg/m^3 and of steel is 7860 kg/m^3, determine the center of gravity of the assembly.

SOLUTION

First, note that symmetry implies $\bar{X} = \bar{Z} = 0$ ◀



Now $W = (\rho g)V$

$$\bar{y}_{\text{I}} = 4\text{ mm}, \quad W_{\text{I}} = (7860\text{ kg/m}^3)(9.81\text{ m/s}^2)\left\{\left(\frac{\pi}{4}\right)\left[(0.036^2 - 0.015^2)\text{m}^2\right](0.008\text{ m})\right\}$$

$$= 0.51887\text{ N}$$

$$\bar{y}_{\text{II}} = 18\text{ mm}, \quad W_{\text{II}} = (7860\text{ kg/m}^3)(9.81\text{ m/s}^2)\left\{\left(\frac{\pi}{4}\right)\left[(0.0225^2 - 0.05^2)\text{m}^2\right](0.02\text{ m})\right\}$$

$$= 0.34065\text{ N}$$

$$\bar{y}_{\text{III}} = 14\text{ mm}, \quad W_{\text{III}} = (8800\text{ kg/m}^3)(9.81\text{ m/s}^2)\left\{\left(\frac{\pi}{4}\right)\left[(0.15^2 - 0.10^2)\text{m}^2\right](0.028\text{ m})\right\}$$

$$= 0.23731\text{ N}$$

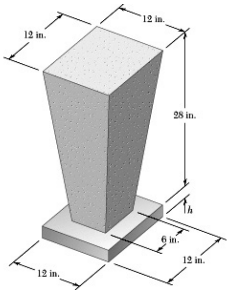
Have $\bar{Y}\Sigma W = \Sigma \bar{y} W$

$$\bar{Y} = \frac{(4\text{ mm})(0.5189\text{ N}) + (18\text{ mm})(0.3406\text{ N}) + (14\text{ mm})(0.2373\text{ N})}{0.5189\text{ N} + 0.3406\text{ N} + 0.2373\text{ N}}$$

or $\bar{Y} = 10.51\text{ mm}$ ◀

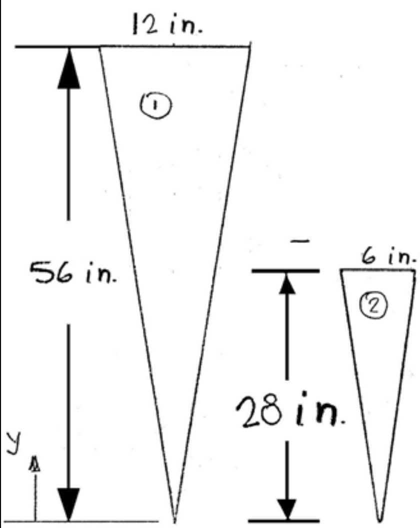
(above base)

PROBLEM 5.114



A marker for a garden path consists of a truncated regular pyramid carved from stone of specific weight 160 lb/ft^3. The pyramid is mounted on a steel base of thickness h. Knowing that the specific weight of steel is 490 lb/ft^3 and that steel plate is available in $\frac{1}{4}$ in. increments, specify the minimum thickness h for which the center of gravity of the marker is approximately 12 in. above the top of the base.

SOLUTION



First, locate the center of gravity of the stone. Assume that the stone is homogeneous so that the center of gravity coincides with the centroid of the corresponding volume.

Have $\quad \bar{y}_1 = \frac{3}{4}(56\text{ in.}) = 42\text{ in.}, \qquad V_1 = \frac{1}{3}(12\text{ in.})(12\text{ in.})(56\text{ in.})$

$$= 2688\text{ in}^3$$

$$\bar{y}_2 = \frac{3}{4}(28\text{ in.}) = 21\text{ in.}, \qquad V_2 = -\frac{1}{3}(6\text{ in.})(6\text{ in.})(28\text{ in.})$$

$$= -366\text{ in}^3$$

Then $\quad V_{\text{stone}} = 2688\text{ in}^3 - 366\text{ in}^3$

$$= 2352\text{ in}^3$$

and $\quad \bar{Y} = \frac{\Sigma \bar{y}V}{\Sigma V}$

$$= \frac{(42\text{ in.})(2688\text{ in}^3) + (21\text{ in.})(-366\text{ in}^3)}{2352\text{ in}^3}$$

$$= 45\text{ in.}$$

Therefore, the center of gravity of the stone is $(45 - 28)$ in. = 17 in. above the base.

Now $\quad W_{\text{stone}} = \gamma_{\text{stone}} V_{\text{stone}} = (160\text{ lb/ft}^3)(2352\text{ in}^3)\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^3$

$$= 217.78\text{ lb}$$

$$W_{\text{steel}} = \gamma_{\text{steel}} V_{\text{steel}}$$

$$= (490\text{ lb/ft}^3)\left[(12\text{ in.})(12\text{ in.})h\right]\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^3$$

$$= (40.833h)\text{ 1b}$$

PROBLEM 5.114 CONTINUED

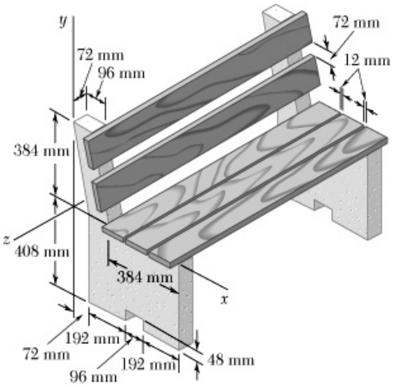
Then $\qquad \bar{Y}_{\text{marker}} = \dfrac{\Sigma yW}{\Sigma W} = 12 \text{ in.}$

$$= \frac{(17 \text{ in.})(217.78 \text{ lb}) + \left(-\frac{h}{2} \text{ in.}\right)(40.833 \text{ h}) \text{ lb}}{(217.78 + 40.833h)\text{lb}}$$

or $\qquad h^2 + 24h - 53.334 = 0$

With positive solution $\ h = 2.0476 \text{ in.}$

$\therefore \quad$ specify $h = 2$ in. ◀



PROBLEM 5.115

The ends of the park bench shown are made of concrete, while the seat and back are wooden boards. Each piece of wood is $36 \times 120 \times 1180$ mm. Knowing that the density of concrete is 2320 kg/m^3 and of wood is 470 kg/m^3, determine the x and y coordinates of the center of gravity of the bench.

SOLUTION

First, note that we will account for the two concrete ends by counting twice the weights of components 1, 2, and 3.

$$W_1 = (\rho_c g)V_1 = \left(2320 \text{ kg/m}^3\right)\left(9.81 \text{ m/s}^2\right)\left[(0.480 \text{ m})(0.408 \text{ m})(0.072 \text{ m})\right]$$

$$= 320.9 \text{ N}$$

$$W_2 = -(\rho_c g)V_2 = -\left(2320 \text{ kg/m}^3\right)\left(9.81 \text{ m/s}^2\right)\left[(0.096 \text{ m})(0.048 \text{ m})(0.072 \text{ m})\right]$$

$$= -7.551 \text{ N}$$

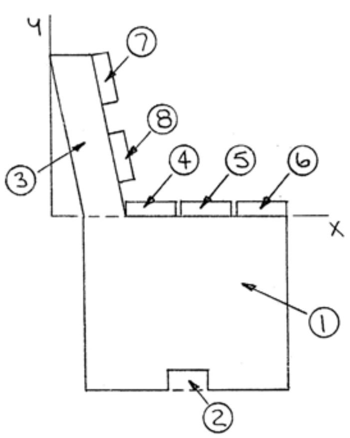
$$W_3 = (\rho_c g)V_3 = \left(2320 \text{ kg/m}^3\right)\left(9.81 \text{ m/s}^2\right)\left[(0.096 \text{ m})(0.384 \text{ m})(0.072 \text{ m})\right]$$

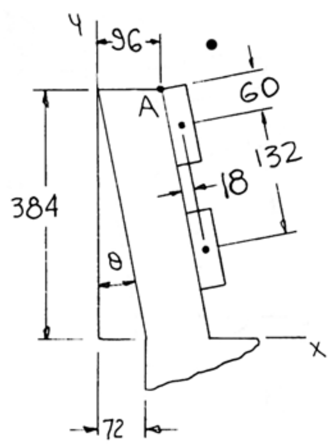
$$= 60.41 \text{ N}$$

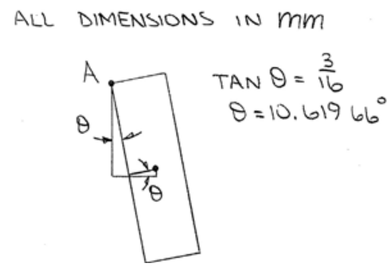
$$W_4 = W_5 = W_6 = W_7 = \rho_w V_{\text{board}}$$

$$= \left(470 \text{ kg/m}^3\right)\left(9.81 \text{ m/s}^2\right)\left[(0.120 \text{ m})(0.036 \text{ m})(1.180 \text{ m})\right]$$

$$= 23.504 \text{ N}$$







PROBLEM 5.115 CONTINUED

	W, N	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}W$, mm·N	$\bar{y}W$, mm·N
1	$2(320.4) = 641.83$	312	–204	200 251.4	–130 933.6
2	$2(-7.551) = -15.10$	312	–384	–4711.8	5799.1
3	$2(60.41) = 120.82$	84	192	10 148.5	23 196.5
4	23.504	228	18	5358.8	423.1
5	23.504	360	18	8461.3	423.1
6	23.504	442	18	10 388.5	423.1
7	23.504	124.7	328.3	2930.9	7716.2
8	23.504	160.1	139.6	3762.9	3281.1
Σ	865.06			236 590	–89 671

Have $$\bar{X}\Sigma W = \Sigma\bar{x}W: \quad \bar{X}(865.06 \text{ N}) = 236\,590 \text{ mm}\cdot\text{N}$$

or $\bar{X} = 274 \text{ mm}$ ◀

$$\bar{Y}\Sigma W = \Sigma\bar{y}W: \quad \bar{Y}(865.06 \text{ N}) = -89\,671 \text{ mm}\cdot\text{N}$$

or $\bar{Y} = -103.6 \text{ mm}$ ◀

PROBLEM 5.116

Determine by direct integration the values of $\bar{x}$ for the two volumes obtained by passing a vertical cutting plane through the given shape of Fig. 5.21. The cutting plane is parallel to the base of the given shape and divides the shape into two volumes of equal height.

A hemisphere.

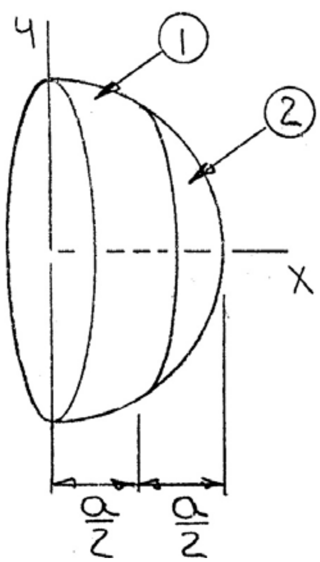
SOLUTION

Choose as the element of volume a disk of radius r and thickness dx. Then

$$dV = \pi r^2 dx, \quad \bar{x}_{EL} = x$$

The equation of the generating curve is $x^2 + y^2 = a^2$ so that $r^2 = a^2 - x^2$ and then

$$dV = \pi\left(a^2 - x^2\right)dx$$



Component 1

$$V_1 = \int_0^{a/2} \pi\left(a^2 - x^2\right)dx = \pi\left[a^2x - \frac{x^3}{3}\right]_0^{a/2}$$

$$= \frac{11}{24}\pi a^3$$

and

$$\int_1 \bar{x}_{EL}dV = \int_0^{a/2} x\left[\pi\left(a^2 - x^2\right)dx\right]$$

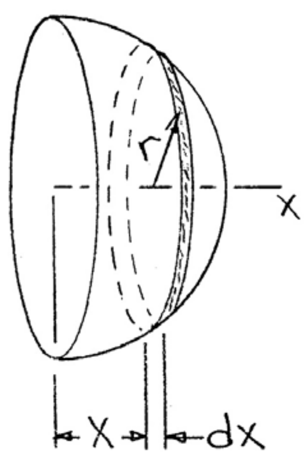
$$= \pi\left[a^2\frac{x^2}{2} - \frac{x^4}{4}\right]_0^{a/2}$$

$$= \frac{7}{64}\pi a^4$$

Now

$$\bar{x}_1 V_1 = \int_1 \bar{x}_{EL}dV: \quad \bar{x}_1\left(\frac{11}{24}\pi a^3\right) = \frac{7}{64}\pi a^4$$

$$\text{or } \bar{x}_1 = \frac{21}{88}a \blacktriangleleft$$



Component 2

$$V_2 = \int_{a/2}^{a} \pi\left(a^2 - x^2\right)dx = \pi\left[a^2x - \frac{x^3}{3}\right]_{a/2}^{a}$$

$$= \pi\left\{\left[a^2(a) - \frac{a^3}{3}\right] - \left[a^2\left(\frac{a}{2}\right) - \frac{\left(\frac{a}{2}\right)^3}{3}\right]\right\}$$

$$= \frac{5}{24}\pi a^3$$

PROBLEM 5.116 CONTINUED

and $\int_2 \overline{x}_{EL} dV = \int_{a/2}^{a} x\left[\pi\left(a^2 - x^2\right)dx\right] = \pi\left[a^2\frac{x^2}{2} - \frac{x^4}{4}\right]_{a/2}^{a}$

$$= \pi\left\{\left[a^2\frac{(a)^2}{2} - \frac{(a)^4}{4}\right] - \left[a^2\frac{\left(\frac{a}{2}\right)^2}{2} - \frac{\left(\frac{a}{2}\right)^4}{4}\right]\right\}$$

$$= \frac{9}{64}\pi a^4$$

Now $\overline{x}_2 V_2 = \int_2 \overline{x}_{EL} dV$: $\quad \overline{x}_2\left(\frac{5}{24}\pi a^3\right) = \frac{9}{64}\pi a^4$

or $\overline{x}_2 = \frac{27}{40}a$ ◀

PROBLEM 5.117

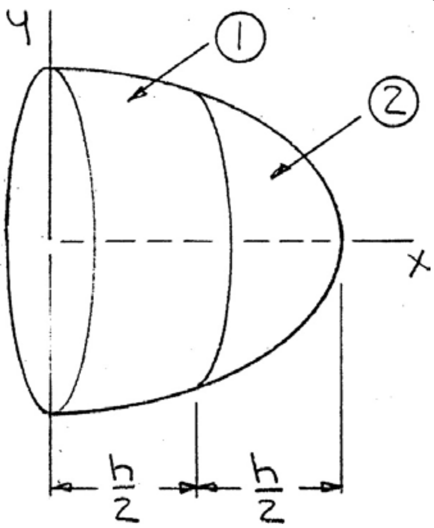
Determine by direct integration the values of $\bar{x}$ for the two volumes obtained by passing a vertical cutting plane through the given shape of Fig. 5.21. The cutting plane is parallel to the base of the given shape and divides the shape into two volumes of equal height.

A semiellipsoid of revolution.

SOLUTION

Choose as the element of volume a disk of radius r and thickness dx. Then

$$dV = \pi r^2 dx, \quad \bar{x}_{\text{EL}} = x$$



The equation of the generating curve is $\dfrac{x^2}{h^2} + \dfrac{y^2}{a^2} = 1$ so that

$r^2 = \dfrac{a^2}{h^2}\left(h^2 - x^2\right)$ and then

$$dV = \pi \frac{a^2}{h^2}\left(h^2 - x^2\right)dx$$

Component 1

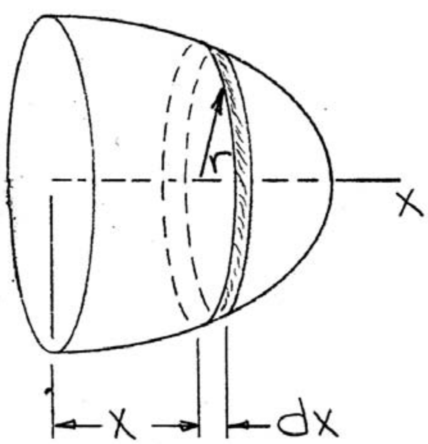
$$V_1 = \int_0^{h/2} \pi \frac{a^2}{h^2}\left(h^2 - x^2\right)dx = \pi \frac{a^2}{h^2}\left[h^2 x - \frac{x^3}{3}\right]_0^{h/2}$$

$$= \frac{11}{24}\pi a^2 h$$

and $$\int_1 \bar{x}_{EL}\,dV = \int_0^{h/2} x\left[\pi \frac{a^2}{h^2}\left(h^2 - x^2\right)dx\right]$$

$$= \pi \frac{a^2}{h^2}\left[h^2 \frac{x^2}{2} - \frac{x^4}{4}\right]_0^{h/2}$$

$$= \frac{7}{64}\pi a^2 h^2$$



Now $$\bar{x}_1 V_1 = \int_1 \bar{x}_{EL}\,dV: \quad \bar{x}_1\left(\frac{11}{24}\pi a^2 h\right) = \frac{7}{64}\pi a^2 h^2$$

or $\bar{x}_1 = \dfrac{21}{88}h$ ◀

PROBLEM 5.117 CONTINUED

Component 2

$$V_2 = \int_{h/2}^{h} \pi \frac{a^2}{h^2}\left(h^2 - x^2\right)dx = \pi\frac{a^2}{h^2}\left[h^2 x - \frac{x^3}{3}\right]_{h/2}^{h}$$

$$= \pi\frac{a^2}{h^2}\left\{\left[h^2(h) - \frac{(h)^3}{3}\right] - \left[h^2\left(\frac{h}{2}\right) - \frac{\left(\frac{h}{2}\right)^3}{3}\right]\right\}$$

$$= \frac{5}{24}\pi a^2 h$$

and $$\int_2 \overline{x}_{EL} dV = \int_{h/2}^{h} x\left[\pi \frac{a^2}{h^2}\left(h^2 - x^2\right)dx\right]$$

$$= \pi\frac{a^2}{h^2}\left[h^2\frac{x^2}{2} - \frac{x^4}{4}\right]_{h/2}^{h}$$

$$= \pi\frac{a^2}{h^2}\left\{\left[h^2\frac{(h)^2}{2} - \frac{(h)^4}{4}\right] - \left[h^2\frac{\left(\frac{h}{2}\right)^2}{2} - \frac{\left(\frac{h}{2}\right)^4}{4}\right]\right\}$$

$$= \frac{9}{64}\pi a^2 h^2$$

Now $$\overline{x}_2 V_2 = \int_2 \overline{x}_{EL} dV: \quad \overline{x}_2\left(\frac{5}{24}\pi a^2 h\right) = \frac{9}{64}\pi a^2 h^2$$

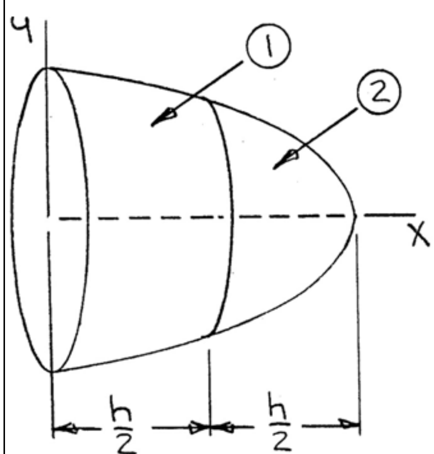
or $$\overline{x}_2 = \frac{27}{40}h \blacktriangleleft$$

PROBLEM 5.118

Determine by direct integration the values of $\bar{x}$ for the two volumes obtained by passing a vertical cutting plane through the given shape of Fig. 5.21. The cutting plane is parallel to the base of the given shape and divides the shape into two volumes of equal height.

A paraboloid of revolution.

SOLUTION



Choose as the element of volume a disk of radius r and thickness dx. Then

$$dV = \pi r^2 dx, \quad \bar{x}_{\text{EL}} = x$$

The equation of the generating curve is $x = h - \frac{h}{a^2}y^2$ so that

$r^2 = \frac{a^2}{h}(h-x)$ and then

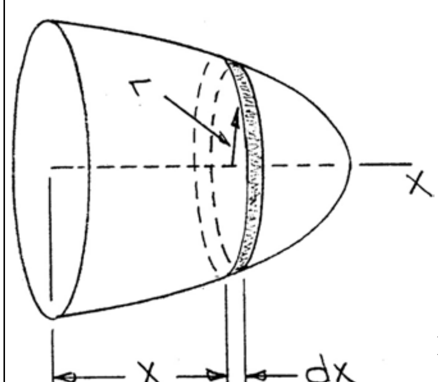
$$dV = \pi\frac{a^2}{h}(h-x)dx$$

Component 1

$$V_1 = \int_0^{h/2} \pi\frac{a^2}{h}(h-x)dx$$

$$= \pi\frac{a^2}{h}\left[hx - \frac{x^2}{2}\right]_0^{h/2}$$

$$= \frac{3}{8}\pi a^2 h$$



and $$\int_1 \bar{x}_{EL}dV = \int_0^{h/2} x\left[\pi\frac{a^2}{h}(h-x)dx\right]$$

$$= \pi\frac{a^2}{h}\left[h\frac{x^2}{2} - \frac{x^3}{3}\right]_0^{h/2}$$

$$= \frac{1}{12}\pi a^2 h^2$$

Now $$\bar{x}_1 V_1 = \int_1 \bar{x}_{EL}dV: \quad \bar{x}_1\left(\frac{3}{8}\pi a^2 h\right) = \frac{1}{12}\pi a^2 h^2$$

$$\text{or } \bar{x}_1 = \frac{2}{9}h \blacktriangleleft$$

PROBLEM 5.118 CONTINUED

Component 2

$$V_2 = \int_{h/2}^{h} \pi \frac{a^2}{h}(h - x)dx = \pi \frac{a^2}{h}\left[hx - \frac{x^2}{2}\right]_{h/2}^{h}$$

$$= \pi \frac{a^2}{h}\left\{\left[h(h) - \frac{(h)^2}{2}\right] - \left[h\left(\frac{h}{2}\right) - \frac{\left(\frac{h}{2}\right)^2}{2}\right]\right\}$$

$$= \frac{1}{8}\pi a^2 h$$

and
$$\int_2 \overline{x}_{EL} dV = \int_{h/2}^{h} x\left[\pi \frac{a^2}{h}(h - x)dx\right] = \pi \frac{a^2}{h}\left[h\frac{x^2}{2} - \frac{x^3}{3}\right]_{h/2}^{h}$$

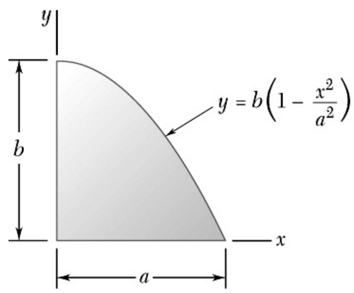
$$= \pi \frac{a^2}{h}\left\{\left[h\frac{(h)^2}{2} - \frac{(h)^3}{3}\right] - \left[h\frac{\left(\frac{h}{2}\right)^2}{2} - \frac{\left(\frac{h}{2}\right)^3}{3}\right]\right\}$$

$$= \frac{1}{12}\pi a^2 h^2$$

Now
$$\overline{x}_2 V_2 = \int_2 \overline{x}_{EL} dV: \quad \overline{x}_2\left(\frac{1}{8}\pi a^2 h\right) = \frac{1}{12}\pi a^2 h^2$$

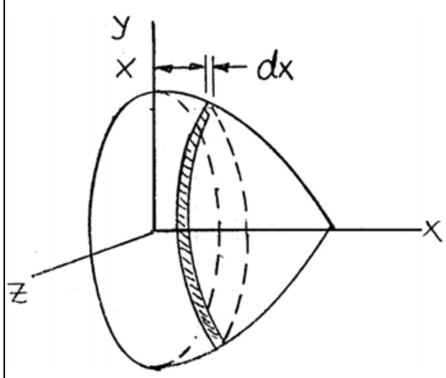
$$\text{or } \overline{x}_2 = \frac{2}{3}h \blacktriangleleft$$

PROBLEM 5.119



Locate the centroid of the volume obtained by rotating the shaded area about the x axis.

SOLUTION



First note that symmetry implies $\bar{y} = 0$ ◀

$\bar{z} = 0$ ◀

Choose as the element of volume a disk of radius r and thickness dx. Then

$$dV = \pi r^2 dx, \quad \bar{x}_{EL} = x$$

Now $r = b\left(1 - \frac{x^2}{a^2}\right)$ so that

$$dV = \pi b^2 \left(1 - \frac{x^2}{a^2}\right)^2 dx$$

Then
$$V = \int_0^a \pi b^2 \left(1 - \frac{x^2}{a^2}\right)^2 dx = \int_0^a \pi b^2 \left(1 - \frac{2x^2}{a^2} + \frac{x^4}{a^4}\right) dx$$

$$= \pi b^2 \left(x - \frac{2x^3}{3a^2} + \frac{x^5}{5a^4}\right)\Bigg|_0^a$$

$$= \pi a b^2 \left(1 - \frac{2}{3} + \frac{1}{5}\right)$$

$$= \frac{8}{15}\pi a b^2$$

and
$$\int \bar{x}_{EL} dV = \int_0^a \pi b^2 x \left(1 - \frac{2x^2}{a^2} + \frac{x^4}{a^4}\right) dx$$

$$= \pi b^2 \left(\frac{x^2}{2} - \frac{2x^4}{4a^2} + \frac{x^6}{6a^4}\right)\Bigg|_0^a$$

$$= \pi a^2 b^2 \left(\frac{1}{2} - \frac{1}{2} + \frac{1}{6}\right)$$

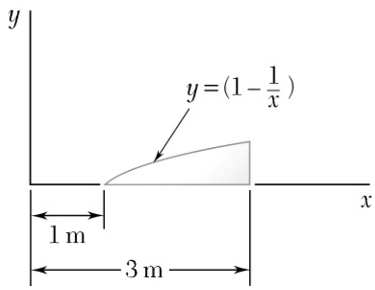
$$= \frac{1}{6}\pi a^2 b^2$$

PROBLEM 5.119 CONTINUED

Then $\qquad \bar{x}V = \int x_{EL}dV: \quad \bar{x}\left(\frac{8}{15}\pi ab^2\right) = \frac{1}{16}\pi a^2b^2$

$$\text{or } \bar{x} = \frac{15}{6}a \blacktriangleleft$$

PROBLEM 5.120



Locate the centroid of the volume obtained by rotating the shaded area about the x axis.

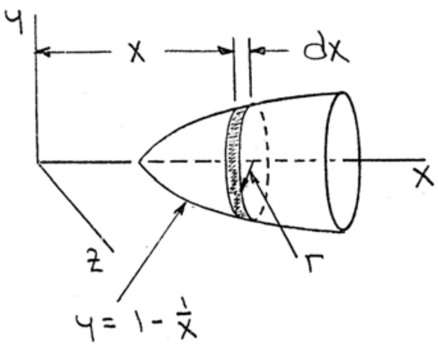
SOLUTION

First, note that symmetry implies $\bar{y} = 0$ ◀

$\bar{z} = 0$ ◀

Choose as the element of volume a disk of radius r and thickness dx. Then

$$dV = \pi r^2 dx, \quad \bar{x}_{EL} = x$$



Now $r = 1 - \dfrac{1}{x}$ so that

$$dV = \pi\left(1 - \frac{1}{x}\right)^2 dx$$

$$= \pi\left(1 - \frac{2}{x} + \frac{1}{x^2}\right)dx$$

Then $$V = \int_1^3 \pi\left(1 - \frac{2}{x} + \frac{1}{x^2}\right)dx = \pi\left[x - 2\ln x - \frac{1}{x}\right]_1^3$$

$$= \pi\left[\left(3 - 2\ln 3 - \frac{1}{3}\right) - \left(1 - 2\ln 1 - \frac{1}{1}\right)\right]$$

$$= (0.46944\pi)\text{ m}^3$$

and $$\int \bar{x}_{EL} dV = \int_1^3 x\left[\pi\left(1 - \frac{2}{x} + \frac{1}{x^2}\right)dx\right] = \pi\left[\frac{x^2}{2} - 2x + \ln x\right]_1^3$$

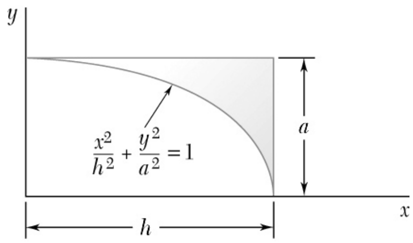
$$= \pi\left\{\left[\frac{3^2}{2} - 2(3) + \ln 3\right] - \left[\frac{1^3}{2} - 2(1) + \ln 1\right]\right\}$$

$$= (1.09861\pi)\text{ m}$$

Now $$\bar{x}V = \int \bar{x}_{EL} dV: \quad \bar{X}\left(0.46944\pi\text{ m}^3\right) = 1.09861\pi\text{ m}^4$$

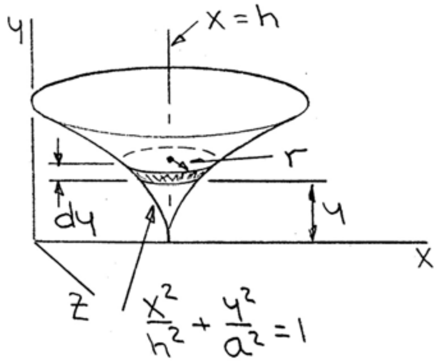
or $\bar{x} = 2.34$ m ◀

PROBLEM 5.121



Locate the centroid of the volume obtained by rotating the shaded area about the line $x = h$.

SOLUTION



First, note that symmetry implies $\bar{x} = h$ ◀

$\bar{z} = 0$ ◀

Choose as the element of volume a disk of radius r and thickness dx. Then

$$dV = \pi r^2 dy, \quad \bar{y}_{EL} = y$$

Now $x^2 = \frac{h^2}{a^2}\left(a^2 - y^2\right)$ so that $r = h - \frac{h}{a}\sqrt{a^2 - y^2}$

Then $$dV = \pi\frac{h^2}{a^2}\left(a - \sqrt{a^2 - y^2}\right)^2 dy$$

and $$V = \int_0^a \pi\frac{h^2}{a^2}\left(a - \sqrt{a^2 - y^2}\right)^2 dy$$

Let $$y = a\sin\theta \Rightarrow dy = a\cos\theta\, d\theta$$

Then
$$\begin{aligned}
V &= \pi\frac{h^2}{a^2}\int_0^{\pi/2}\left(a - \sqrt{a^2 - a^2\sin^2\theta}\right)^2 a\cos\theta\, d\theta \\
&= \pi\frac{h^2}{a^2}\int_0^{\pi/2}\left[a^2 - 2a\left(a\cos\theta\right) + \left(a^2 - a^2\sin^2\theta\right)\right]a\cos\theta\, d\theta \\
&= \pi a h^2\int_0^{\pi/2}\left(2\cos\theta - 2\cos^2\theta - \sin^2\theta\cos\theta\right)d\theta \\
&= \pi a h^2\left[2\sin\theta - 2\left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right) - \frac{1}{3}\sin^3\theta\right]_0^{\pi/2} \\
&= \pi a h^2\left[2 - 2\left(\frac{\frac{\pi}{2}}{2}\right) - \frac{1}{3}\right] \\
&= 0.095870\pi a h^2
\end{aligned}$$

PROBLEM 5.121 CONTINUED

and $$\int \bar{y}_{EL}\,dV = \int_0^a y\left[\pi\frac{h^2}{a^2}\left(a - \sqrt{a^2 - y^2}\right)^2 dy\right]$$

$$= \pi\frac{h^2}{a^2}\int_0^a \left(2a^2y - 2ay\sqrt{a^2 - y^2} - y^3\right)dy$$

$$= \pi\frac{h^2}{a^2}\left[a^2y^2 + \frac{2}{3}a\left(a^2 - y^2\right)^{3/2} - \frac{1}{4}y^4\right]_0^a$$

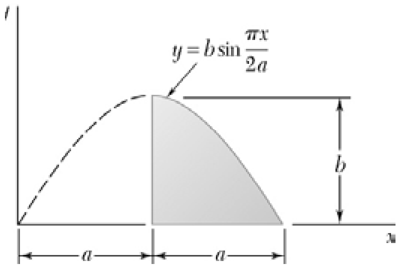
$$= \pi\frac{h^2}{a^2}\left\{\left[a^2(a)^2 - \frac{1}{4}a^4\right] - \left[\frac{2}{3}a\left(a^2\right)^{3/2}\right]\right\}$$

$$= \frac{1}{12}\pi a^2h^2$$

Now $$\bar{y}V = \int \bar{y}_{EL}\,dV: \quad \bar{y}\left(0.095870\pi ah^2\right) = \frac{1}{12}\pi a^2h^2$$

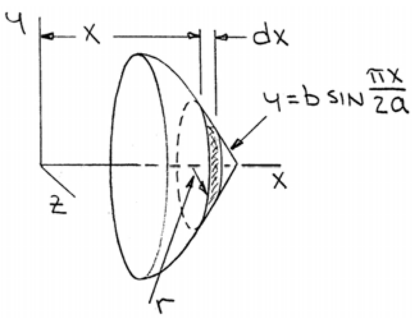
or $\bar{y} = 0.869a$ ◀

PROBLEM 5.122



Locate the centroid of the volume generated by revolving the portion of the sine curve shown about the x axis.

SOLUTION



First, note that symmetry implies $\bar{y} = 0$ ◀

$\bar{z} = 0$ ◀

Choose as the element of volume a disk of radius r and thickness dx. Then

$$dV = \pi r^2 dx, \quad \bar{x}_{\mathrm{EL}} = x$$

Now $$r = b\sin\frac{\pi x}{2a}$$

so that $$dV = \pi b^2 \sin^2 \frac{\pi x}{2a} dx$$

Then $$V = \int_a^{2a} \pi b^2 \sin^2 \frac{\pi x}{2a} dx$$

$$= \pi b^2 \left[\frac{x}{2} - \frac{\sin \frac{\pi x}{2a}}{2\frac{\pi}{a}} \right]_a^{2a}$$

$$= \pi b^2 \left[\left(\tfrac{2a}{2}\right) - \left(\tfrac{a}{2}\right) \right]$$

$$= \frac{1}{2}\pi a b^2$$

and $$\int \bar{x}_{EL} dV = \int_a^{2a} x \left(\pi b^2 \sin^2 \frac{\pi x}{2a} dx \right)$$

Use integration by parts with

$$u = x \qquad dV = \sin^2 \frac{\pi x}{2a}$$

$$du = dx \qquad V = \frac{x}{2} - \frac{\sin \frac{\pi x}{a}}{\frac{2\pi}{a}}$$

PROBLEM 5.122 CONTINUED

Then $\int \overline{x}_{EL}dV = \pi b^2\left\{\left[x\left(\frac{x}{2} - \frac{\sin\frac{\pi x}{a}}{\frac{2\pi}{a}}\right)\right]_a^{2a} - \int_a^{2a}\left(\frac{x}{2} - \frac{\sin\frac{\pi x}{a}}{\frac{2\pi}{a}}\right)dx\right\}$

$$= \pi b^2\left\{\left[2a\left(\frac{2a}{2}\right) - a\left(\frac{a}{2}\right)\right] - \left[\frac{1}{4}x^2 + \frac{a^2}{2\pi^2}\cos\frac{\pi x}{a}\right]_a^{2a}\right\}$$

$$= \pi b^2\left\{\left(\frac{3}{2}a^2\right) - \left[\frac{1}{4}(2a)^2 + \frac{a^2}{2\pi^2} - \frac{1}{4}(a)^2 + \frac{a^2}{2\pi^2}\right]\right\}$$

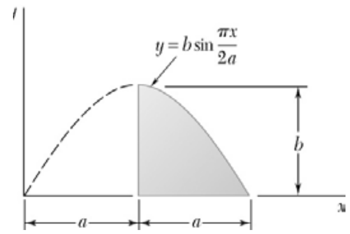
$$= \pi a^2 b^2\left(\frac{3}{4} - \frac{1}{\pi^2}\right)$$

$$= 0.64868\pi a^2 b^2$$

Now $\overline{x}V = \int \overline{x}_{EL}dV$: $\overline{x}\left(\frac{1}{2}\pi ab^2\right) = 0.64868\pi a^2 b^2$

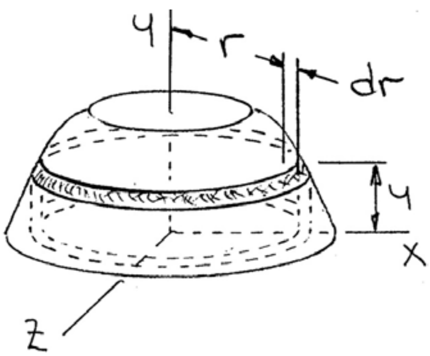
or $\overline{x} = 1.297a$ ◀

PROBLEM 5.123



Locate the centroid of the volume generated by revolving the portion of the sine curve shown about the y axis. (*Hint:* Use a thin cylindrical shell of radius r and thickness dr as the element of volume.)

SOLUTION



First note that symmetry implies $\bar{x} = 0$ ◀

$\bar{z} = 0$ ◀

Choose as the element of volume a cylindrical shell of radius r and thickness dr.

Then $\quad dV = (2\pi r)(y)(dr), \ \bar{y}_{EL} = \frac{1}{2}y$

Now $\quad y = b\sin\frac{\pi r}{2a}$

so that $\quad dV = 2\pi br\sin\frac{\pi r}{2a}dr$

Then $\quad V = \int_a^{2a} 2\pi br\sin\frac{\pi r}{2a}dr$

Use integration by parts with

$$u = r \qquad dv = \sin\frac{\pi r}{2a}dr$$

$$du = dr \qquad v = -\frac{2a}{\pi}\cos\frac{\pi r}{2a}$$

Then
$$V = 2\pi b\left\{\left[(r)\left(-\frac{2a}{\pi}\cos\frac{\pi r}{2a}\right)\right]_a^{2a} - \int_a^{2a}\left(\frac{2a}{\pi}\cos\frac{\pi r}{2a}\right)dr\right\}$$

$$= 2\pi b\left\{-\frac{2a}{\pi}\left[(2a)(-1)\right] + \left[\frac{4a^2}{\pi^2}\sin\frac{\pi r}{2a}\right]_a^{2a}\right\}$$

$$V = 2\pi b\left(\frac{4a^2}{\pi} - \frac{4a^2}{\pi^2}\right)$$

$$= 8a^2b\left(1 - \frac{1}{\pi}\right)$$

$$= 5.4535\, a^2b$$

Also
$$\int \bar{y}_{EL}dV = \int_a^{2a}\left(\tfrac{1}{2}b\sin\frac{\pi r}{2a}\right)\left(2\pi br\sin\frac{\pi r}{2a}dr\right)$$

$$= \pi b^2\int_a^{2a} r\sin^2\frac{\pi r}{2a}dr$$

PROBLEM 5.123 CONTINUED

Use integration by parts with

$$u = r \qquad dv = \sin^2 \frac{\pi r}{2a} dr$$

$$du = dr \qquad v = \frac{r}{2} - \frac{\sin \frac{\pi r}{a}}{\frac{2\pi}{a}}$$

Then $$\int \overline{y}_{EL} dV = \pi b^2 \left\{ \left[(r)\left(\frac{r}{2} - \frac{\sin \frac{\pi r}{a}}{\frac{2\pi}{a}} \right) \right]_a^{2a} - \int_a^{2a} \left(\frac{r}{2} - \frac{\sin \frac{\pi r}{a}}{\frac{2\pi}{a}} \right) dr \right\}$$

$$= \pi b^2 \left\{ \left[(2a)\left(\frac{2a}{2} \right) - (a)\left(\frac{a}{2} \right) \right] - \left[\frac{r^2}{4} + \frac{a^2}{2\pi^2} \cos \frac{\pi r}{a} \right]_a^{2a} \right\}$$

$$= \pi b^2 \left\{ \frac{3}{2} a^2 - \left[\frac{(2a)^2}{4} + \frac{a^2}{2\pi^2} - \frac{(a)^2}{4} + \frac{a^2}{2\pi^2} \right] \right\}$$

$$= \pi a^2 b^2 \left(\frac{3}{4} - \frac{1}{\pi^2} \right)$$

$$= 2.0379 a^2 b^2$$

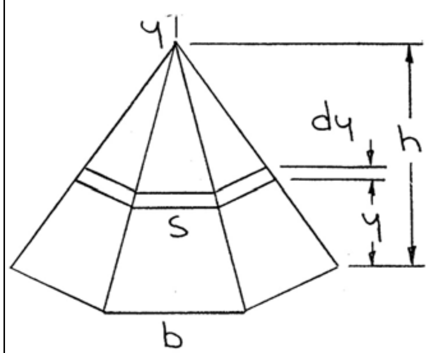
Now $$\overline{y}V = \int \overline{y}_{EL} dV: \quad \overline{y}\left(5.4535 a^2 b\right) = 2.0379 a^2 b^2$$

or $\overline{y} = 0.374b$ ◀

PROBLEM 5.124

Show that for a regular pyramid of height h and n sides ($n = 3, 4, \ldots$) the centroid of the volume of the pyramid is located at a distance $h/4$ above the base.

SOLUTION



Choose as the element of a horizontal slice of thickness dy. For any number N of sides, the area of the base of the pyramid is given by

$$A_{\text{base}} = kb^2$$

where $k = k(N)$; see note below. Using similar triangles, have

$$\frac{s}{b} = \frac{h-y}{h}$$

or

$$s = \frac{b}{h}(h-y)$$

Then

$$dV = A_{\text{slice}}dy = ks^2 dy = k\frac{b^2}{h^2}(h-y)^2\,dy$$

and

$$V = \int_0^h k\frac{b^2}{h^2}(h-y)^2\,dy = k\frac{b^2}{h^2}\left[-\frac{1}{3}(h-y)^3\right]_0^h$$

$$= \frac{1}{3}kb^2h$$

Also $\qquad \bar{y}_{EL} = y$

so then

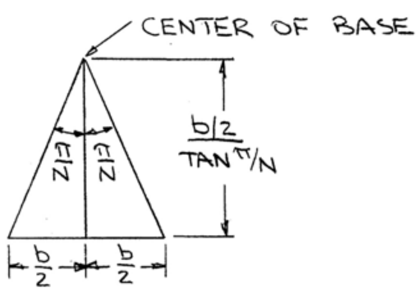
$$\int \bar{y}_{EL}dV = \int_0^h y\left[k\frac{b^2}{h^2}(h-y)^2\,dy\right] = k\frac{b^2}{h^2}\int_0^h \left(h^2y - 2hy^2 + y^3\right)dy$$

$$= k\frac{b^2}{h^2}\left[\frac{1}{2}h^2y^2 - \frac{2}{3}hy^3 + \frac{1}{4}y^4\right]_0^h = \frac{1}{12}kb^2h^2$$

Now

$$\bar{y}V = \int \bar{y}_{EL}dV: \quad \bar{y}\left(\frac{1}{3}kb^2h\right) = \frac{1}{12}kb^2h^2$$

or $y = \frac{1}{4}h$ Q.E.D. ◀



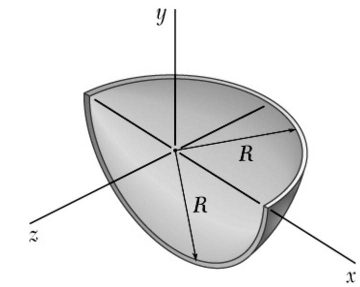
Note

$$A_{\text{base}} = N\left(\frac{1}{2} \times b \times \frac{\frac{b}{2}}{\tan\frac{\pi}{N}}\right)$$

$$= \frac{N}{4\tan\frac{\pi}{N}}b^2$$

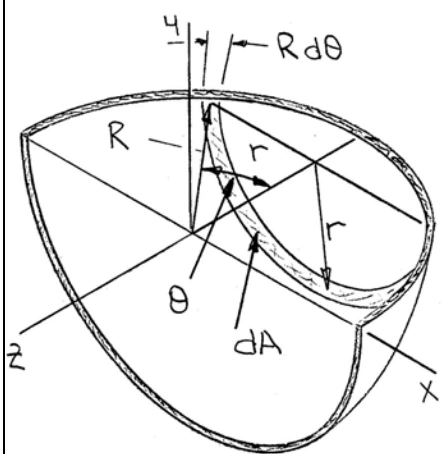
$$= k(N)\,b^2$$

PROBLEM 5.125



Determine by direct integration the location of the centroid of one-half of a thin, uniform hemispherical shell of radius *R*.

SOLUTION



First note that symmetry implies $\bar{x} = 0$ ◀

The element of area *dA* of the shell shown is obtained by cutting the shell with two planes parallel to the *xy* plane. Now

$$dA = (\pi r)(R d\theta)$$

$$\bar{y}_{EL} = -\frac{2r}{\pi}$$

where $r = R\sin\theta$

so that $dA = \pi R^2 \sin\theta d\theta$

$$\bar{y}_{\mathrm{EL}} = -\frac{2R}{\pi}\sin\theta$$

Then $A = \int_0^{\frac{\pi}{2}} \pi R^2 \sin\theta d\theta = \pi R^2 \left[-\cos\theta\right]_0^{\frac{\pi}{2}}$

$$= \pi R^2$$

and $\int \bar{y}_{EL} dA = \int_0^{\frac{\pi}{2}} \left(-\frac{2R}{\pi}\sin\theta\right)\left(\pi R^2 \sin\theta d\theta\right)$

$$= -2R^3\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\frac{\pi}{2}}$$

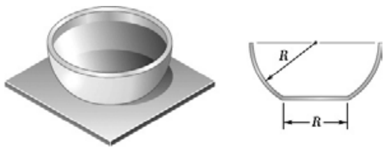
$$= -\frac{\pi}{2}R^3$$

Now $\bar{y}A = \int \bar{y}_{EL} dA\!: \; \bar{y}\left(\pi R^2\right) = -\frac{\pi}{2}R^3$

or $\bar{y} = -\frac{1}{2}R$ ◀

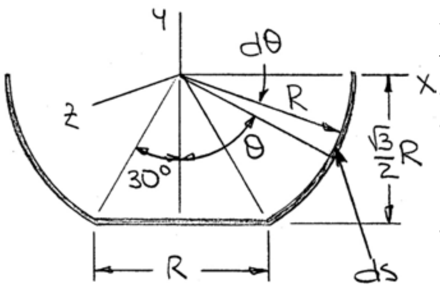
Symmetry implies $\bar{z} = \bar{y} \; \therefore \; \bar{z} = -\frac{1}{2}R$ ◀

PROBLEM 5.126



The sides and the base of a punch bowl are of uniform thickness t. If $t << R$ and $R = 350$ mm, determine the location of the center of gravity of (a) the bowl, (b) the punch.

SOLUTION



(a) Bowl

First note that symmetry implies $\qquad \bar{x} = 0$ ◀

$\bar{z} = 0$ ◀

for the coordinate axes shown below. Now assume that the bowl may be treated as a shell; the center of gravity of the bowl will coincide with the centroid of the shell. For the walls of the bowl, an element of area is obtained by rotating the arc ds about the y axis. Then

$$dA_{\text{wall}} = (2\pi R \sin\theta)(R d\theta)$$

and $(\bar{y}_{EL})_{\text{wall}} = -R\cos\theta$

Then $\qquad A_{\text{wall}} = \int_{\pi/6}^{\pi/2} 2\pi R^2 \sin\theta d\theta = 2\pi R^2 [-\cos\theta]_{\pi/6}^{\pi/2}$

$$= \pi\sqrt{3}R^2$$

and $\qquad \bar{y}_{\text{wall}} A_{\text{wall}} = \int (\bar{y}_{EL})_{\text{wall}} dA$

$$= \int_{\pi/6}^{\pi/2} (-R\cos\theta)(2\pi R^2 \sin\theta d\theta)$$

$$= \pi R^3 \left[\cos^2\theta\right]_{\pi/6}^{\pi/2}$$

$$= -\frac{3}{4}\pi R^3$$

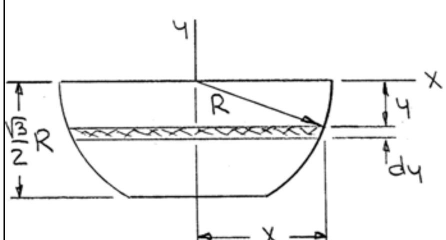
By observation $\qquad A_{\text{base}} = \frac{\pi}{4}R^2, \qquad \bar{y}_{\text{base}} = -\frac{\sqrt{3}}{2}R$

Now $\qquad \bar{y}\Sigma A = \Sigma \bar{y} A$

or $\qquad \bar{y}\left(\pi\sqrt{3}R^2 + \frac{\pi}{4}R^2\right) = -\frac{3}{4}\pi R^3 + \frac{\pi}{4}R^2\left(-\frac{\sqrt{3}}{2}R\right)$

or $\qquad \bar{y} = -0.48763R \qquad R = 350 \text{ mm}$

$\therefore\ \bar{y} = -170.7 \text{ mm}$ ◀



(b) Punch

First note that symmetry implies $\qquad \bar{x} = 0$ ◀

$\bar{z} = 0$ ◀

and that because the punch is homogeneous, its center of gravity will coincide with the centroid of the corresponding volume. Choose as the element of volume a disk of radius x and thickness dy. Then

$$dV = \pi x^2 dy, \quad \bar{y}_{EL} = y$$

PROBLEM 5.126 CONTINUED

Now $\quad x^2 + y^2 = R^2 \quad$ so that $\quad dV = \pi\left(R^2 - y^2\right)dy$

Then
$$V = \int_{-\sqrt{3}/2R}^{0} \pi\left(R^2 - y^2\right)dy = \pi\left[R^2 y - \frac{1}{3}y^3\right]_{-\sqrt{3}/2R}^{0}$$

$$= -\pi\left[R^2\left(-\frac{\sqrt{3}}{2}R\right) - \frac{1}{3}\left(-\frac{\sqrt{3}}{2}R\right)^3\right] = \frac{3}{8}\pi\sqrt{3}R^3$$

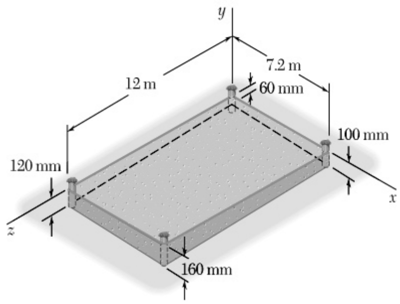
and
$$\int \overline{y}_{EL} dV = \int_{-\sqrt{3}/2R}^{0} (y)\left[\pi\left(R^2 - y^2\right)dy\right] = \pi\left[\frac{1}{2}R^2 y^2 - \frac{1}{4}y^4\right]_{-\sqrt{3}/2R}^{0}$$

$$= -\pi\left[\frac{1}{2}R^2\left(-\frac{\sqrt{3}}{2}R\right)^2 - \frac{1}{4}\left(-\frac{\sqrt{3}}{2}R\right)^4\right] = -\frac{15}{64}\pi R^4$$

Now
$$\overline{y}V = \int \overline{y}_{EL} dV: \quad \overline{y}\left(\frac{3}{8}\pi\sqrt{3}R^3\right) = -\frac{15}{64}\pi R^4$$

or
$$\overline{y} = -\frac{5}{8\sqrt{3}}R \qquad\qquad R = 350 \text{ mm}$$

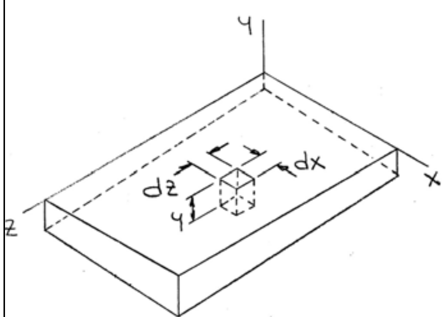
$$\therefore\ \overline{y} = -126.3 \text{ mm} \blacktriangleleft$$



PROBLEM 5.127

After grading a lot, a builder places four stakes to designate the corners of the slab for a house. To provide a firm, level base for the slab, the builder places a minimum of 60 mm of gravel beneath the slab. Determine the volume of gravel needed and the *x* coordinate of the centroid of the volume of the gravel. (*Hint:* The bottom of the gravel is an oblique plane, which can be represented by the equation $y = a + bx + cz$.)

SOLUTION



First, determine the constants *a*, *b*, and *c*.

At $\quad x = 0, z = 0: \; y = -60 \text{ mm}$

$$\therefore -60 \text{ mm} = a; \; a = -60 \text{ mm}$$

At $\quad x = 7200 \text{ mm}, z = 0: \; y = -100 \text{ mm}$

$$\therefore -100 \text{ mm} = -60 + b(7200)$$

$$b = -\frac{1}{180}$$

At $\quad x = 0, z = 12\,000 \text{ mm}: \; y = -120 \text{ mm}$

$$\therefore -120 \text{ mm} = -60 \text{ mm} + c(12000)$$

$$c = -\frac{1}{200}$$

It follows that $y = -60 - \frac{1}{180}x - \frac{1}{120}z$ where all dimensions are in mm.

Choose as the element of volume a filament of base $dx \times dz$ and height $|y|$. Then

$$dV = |y|dxdz, \quad \overline{x}_{EL} = x$$

or

$$dV = \left|-60 - \frac{1}{180}x - \frac{1}{200}z\right| dxdz$$

Then

$$V = \int_0^{12000}\int_0^{7200}\left(60 + \frac{1}{180}x + \frac{1}{200}z\right)dxdz$$

$$= \int_0^{12000}\left[60x + \frac{1}{360}x^2 + \frac{1}{200}xz\right]_0^{7200} dz$$

$$= \int_0^{12000}\left[(60)(7200) + \frac{(7200)^2}{360} + \frac{(7200)}{200}z\right]dz$$

$$= \left[576000z + \frac{36}{2}z^2\right]_0^{12000}$$

$$= 9.504 \times 10^9 \text{ mm}^3 = 9.50 \text{ m}^3$$

$$V = 9.50 \text{ m}^3 \blacktriangleleft$$

PROBLEM 5.127 CONTINUED

and $$\int \bar{x}_{EL}dV = \int_0^{12000}\int_0^{7200} x\left(60 + \frac{1}{180}x + \frac{1}{200}z\right)dxdz$$

$$= \int_0^{12000}\left[\frac{60}{2}x^2 + \frac{1}{540}x^3 + \frac{1}{400}x^2z\right]_0^{7200} dz$$

$$= \int_0^{12000}\left(2246.4\times 10^6 + 129\ 600z\right)dz$$

$$= \left[2246.4\times 10^6 z - \frac{129\ 600}{2}z^2\right]_0^{12000}$$

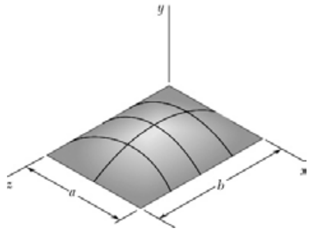
$$= 2.695\times 10^{13} + 0.933\times 10^{13}$$

$$= 3.63\times 10^{13}\text{mm}^4 = 36.3\text{ m}^4$$

Now $$\bar{x}V = \int \bar{x}_{EL}dV:\ \bar{x}\left(9.50\text{ m}^3\right) = 36.3\text{ m}^4$$

or $\bar{x} = 3.82$ m ◀

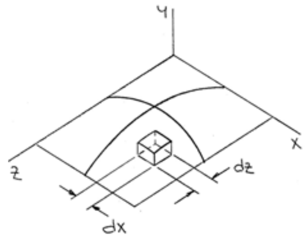
PROBLEM 5.128



Determine by direct integration the location of the centroid of the volume between the *xz* plane and the portion shown of the surface

$$y = \frac{16h\left(ax - x^2\right)\left(bz - z^2\right)}{a^2b^2}.$$

SOLUTION



First note that symmetry implies

$$\overline{x} = \frac{a}{2} \blacktriangleleft$$

$$\overline{z} = \frac{b}{2} \blacktriangleleft$$

Choose as the element of volume a filament of base $dx \times dz$ and height y. Then

$$dV = ydxdz, \overline{y}_{\text{EL}} = \frac{1}{2}y$$

or

$$dV = \frac{16h}{a^2b^2}\left(ax - x^2\right)\left(bz - z^2\right)dxdz$$

Then

$$V = \int_0^b\int_0^a \frac{16h}{a^2b^2}\left(ax - x^2\right)\left(bz - z^2\right)dxdz$$

$$V = \frac{16h}{a^2b^2}\int_0^b\left(bz - z^2\right)\left[\frac{a}{z}x^2 - \frac{1}{3}x^3\right]_0^a dz$$

$$= \frac{16h}{a^2b^2}\left[\frac{a}{2}\left(a^2\right) - \frac{1}{3}\left(a\right)^3\right]\left[\frac{b}{2}z^2 - \frac{1}{3}z^3\right]_0^b = \frac{8ah}{3b^2}\left[\frac{b}{2}\left(b\right)^2 - \frac{1}{3}\left(b\right)^3\right] = \frac{4}{9}abh$$

and

$$\int \overline{y}_{EL}dV = \int_0^b\int_0^a \frac{1}{2}\left[\frac{16h}{a^2b^2}\left(ax - x^2\right)\left(bz - z^2\right)\right]\left[\frac{16h}{a^2b^2}\left(ax - x^2\right)\left(bz - z^2\right)dxdz\right]$$

$$= \frac{128h^2}{a^4b^4}\int_0^b\int_0^a\left(a^2x^2 - 2ax^3 + x^4\right)\left(b^2z^2 - 2bz^3 + z^4\right)dxdz$$

$$= \frac{128h^2}{a^2b^4}\int_0^b\left(b^2z^2 - 2bz^3 + z^4\right)\left[\frac{a^2}{3}x^3 - \frac{a}{2}x^4 + \frac{1}{5}x^5\right]_0^a dz$$

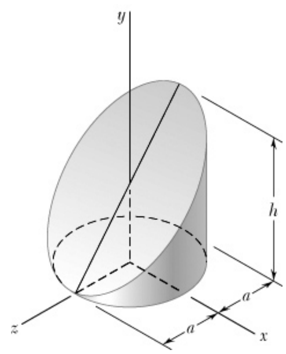
$$= \frac{128h^2}{a^4b^4}\left[\frac{a^2}{3}\left(a\right)^3 - \frac{a}{2}\left(a\right)^4 + \frac{1}{5}\left(a\right)^5\right]\left[\frac{b^2}{3}Z^3 - \frac{b}{Z}Z^4 + \frac{1}{5}Z^5\right]_0^b$$

$$= \frac{64ah^2}{15b^4}\left[\frac{b^3}{3}\left(b\right)^3 - \frac{b}{2}\left(b\right)^4 + \frac{1}{5}\left(b\right)^5\right] = \frac{32}{225}abh^2$$

PROBLEM 5.128 CONTINUED

Now $$\bar{y}V = \int \bar{y}_{EL}dV: \quad \bar{y}\left(\frac{4}{9}abh\right) = \frac{32}{225}abh^2$$

$$\text{or } \bar{y} = \frac{8}{25}h \blacktriangleleft$$

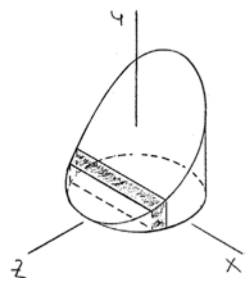


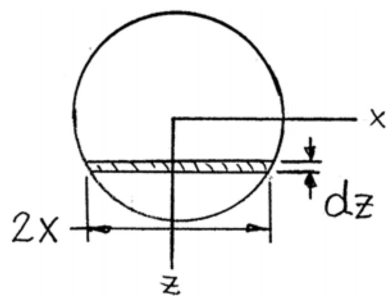
PROBLEM 5.129

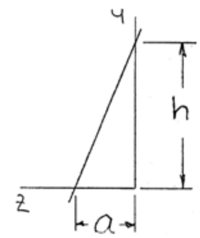
Locate the centroid of the section shown, which was cut from a circular cylinder by an inclined plane.

SOLUTION

First note that symmetry implies $\bar{x} = 0$ ◀







Choose as the element of volume a vertical slice of width $2x$, thickness dz, and height y. Then

$$dV = 2xy\,dz, \quad \bar{y}_{EL} = \frac{1}{2}y, \quad \bar{z}_{EL} = z$$

Now $\quad x = \sqrt{a^2 - z^2} \quad$ and $\quad y = \frac{h}{2} - \frac{h}{2a}z = \frac{h}{2}\left(1 - \frac{z}{a}\right)$

So $\quad dV = h\sqrt{a^2 - z^2}\left(1 - \frac{z}{a}\right)dz$

Then $\quad V = \int_0^a h\sqrt{a^2 - z^2}\left(1 - \frac{z}{a}\right)dz = h\left\{\frac{1}{2}\left[z\sqrt{a^2 - z^2} + a^2\sin^{-1}\left(\frac{z}{a}\right)\right] + \frac{1}{3a}\left(a^2 - z^2\right)^{3/2}\right\}\Bigg|_{-a}^{a}$

$$= \frac{1}{2}a^2h\left[\sin^{-1}(1) - \sin^{-1}(-1)\right]$$

$$= \frac{\pi}{2}a^2h$$

PROBLEM 5.129 CONTINUED

Then
$$\int \bar{y}_{EL} dV = \int_{-a}^{a} \left[\frac{1}{2} \times \frac{h}{2}\left(1 - \frac{z}{a}\right)\right]\left[h\sqrt{a^2 - z^2}\left(1 - \frac{z}{a}\right)dz\right]$$

$$= \frac{h^2}{4}\int_{-a}^{a} \sqrt{a^2 - z^2}\left(1 - 2\frac{z}{a} + \frac{z^2}{a^2}\right)dz$$

$$= \frac{h^2}{4}\left\{\frac{1}{2}\left[z\sqrt{a^2 - z^2} + a^2 \sin^{-1}\left(\frac{z}{a}\right)\right] + \left[\frac{2}{3a}\left(a^2 - z^2\right)^{\frac{3}{2}}\right]\right.$$

$$\left. + \frac{1}{a^2}\left[-\frac{z}{4}\left(a^2 - z^2\right)^{\frac{3}{2}} + \frac{a^2 z}{8}\sqrt{a^2 - z^2} + \frac{a^4}{8}\sin^{-1}\left(\frac{z}{a}\right)\right]\right\}\Bigg|_{-a}^{a}$$

$$= \frac{5h^2a^2}{32}\left[\sin^{-1}(1) - \sin^{-1}(-1)\right]$$

Then
$$\bar{y}V = \int \bar{y}_{EL} dV: \quad \bar{y}\left(\frac{\pi a^2}{2}h\right) = \frac{5h^2a^2}{32}(\pi)$$

$$\text{or } \bar{y} = \frac{5}{16}h \blacktriangleleft$$

and
$$\int z_{EL} dV = \int_{-a}^{a} z\left[h\sqrt{a^2 - z^2}\left(1 - \frac{z}{a}\right)dz\right]$$

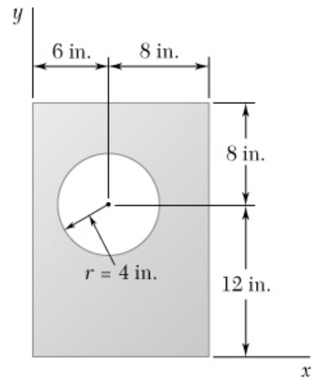
$$= h\left\{-\frac{1}{3}\left(a^2 - z^2\right)^{\frac{3}{2}} - \frac{1}{a}\left[-\frac{z}{4}\left(a^2 - z^2\right)^{\frac{3}{2}} + \frac{a^2 z}{8}\sqrt{a^2 - z^2} + \frac{a^4}{8}\sin^{-1}\left(\frac{z}{a}\right)\right]\right\}\Bigg|_{-a}^{a}$$

$$= -\frac{a^3h}{8}\left[\sin^{-1}(1) - \sin^{-1}(-1)\right]$$

$$\bar{z}V = \int \bar{z}_{EL} dV: \quad \bar{z}\left(\frac{\pi a^2 h}{2}\right) = -\frac{\pi a^3 h}{8}$$

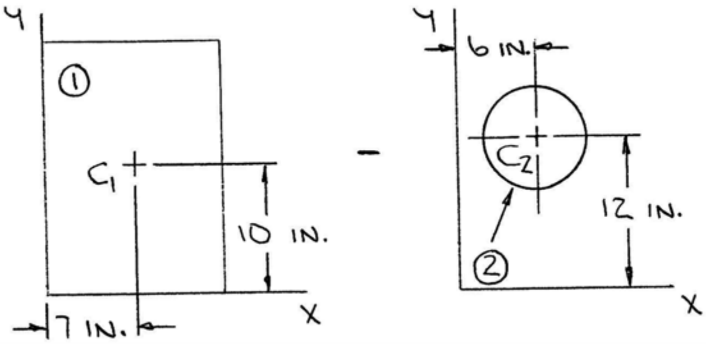
$$\text{or } \bar{z} = -\frac{a}{4} \blacktriangleleft$$

PROBLEM 5.130



Locate the centroid of the plane area shown.

SOLUTION



	A, in^2	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}A$, in^3	$\bar{y}A$, in^3
1	$14 \times 20 = 280$	7	10	1960	2800
2	$-\pi(4)^2 = -16\pi$	6	12	–301.59	–603.19
Σ	229.73			1658.41	2196.8

Then

$$\bar{X}\Sigma A = \Sigma\bar{x}A$$

$$\bar{X}\left(229.73\,\text{in}^2\right) = 1658.41\,\text{in}^3$$

or $\bar{X} = 7.22$ in. ◀

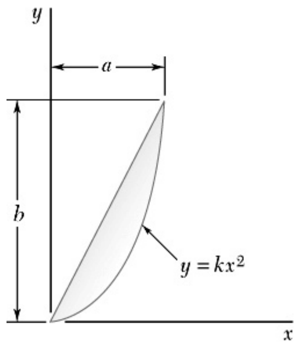
and

$$\bar{Y}\Sigma A = \Sigma\bar{y}A$$

$$\bar{Y}\left(229.73\,\text{in}^2\right) = 2196.8\,\text{in}^3$$

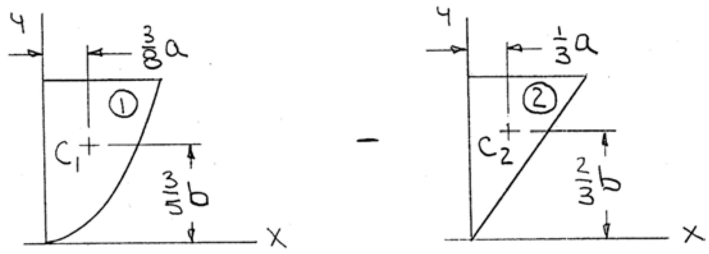
or $\bar{Y} = 9.56$ in. ◀

PROBLEM 5.131



For the area shown, determine the ratio a/b for which $\bar{x} = \bar{y}$.

SOLUTION



	A	$\bar{x}$	$\bar{y}$	$\bar{x}A$	$\bar{y}A$
1	$\frac{2}{3}ab$	$\frac{3}{8}a$	$\frac{3}{5}b$	$\frac{a^2b}{4}$	$\frac{2ab^2}{5}$
2	$-\frac{1}{2}ab$	$\frac{1}{3}a$	$\frac{2}{3}b$	$-\frac{a^2b}{6}$	$-\frac{ab^2}{3}$
Σ	$\frac{1}{6}ab$			$\frac{a^2b}{12}$	$\frac{ab^2}{15}$

Then
$$\bar{X}\Sigma A = \Sigma\bar{x}A$$
$$\bar{X}\left(\frac{1}{6}ab\right) = \frac{a^2b}{12}$$

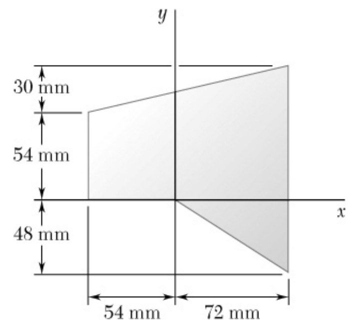
or
$$\bar{X} = \frac{1}{2}a$$
$$\bar{Y}\Sigma A = \Sigma\bar{y}A$$
$$\bar{Y}\left(\frac{1}{6}ab\right) = \frac{ab^2}{15}$$

or
$$\bar{Y} = \frac{2}{5}b$$

Now
$$\bar{X} = \bar{Y} \Rightarrow \quad \frac{1}{2}a = \frac{2}{5}b$$

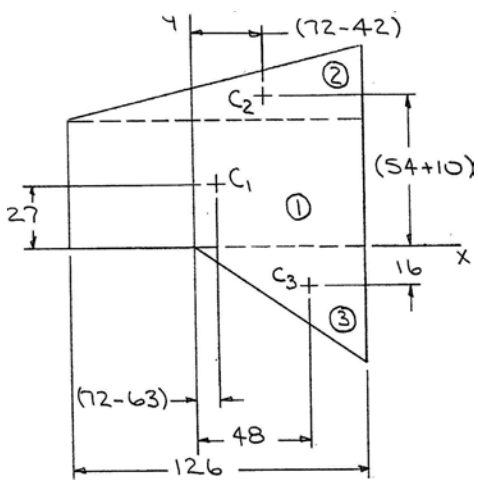
or $\frac{a}{b} = \frac{4}{5}$ ◀

PROBLEM 5.132



Locate the centroid of the plane area shown.

SOLUTION



Dimensions in mm

	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
1	$126 \times 54 = 6804$	9	27	61 236	183 708
2	$\frac{1}{2} \times 126 \times 30 = 1890$	30	64	56 700	120 960
3	$\frac{1}{2} \times 72 \times = 1728$	48	–16	82 944	–27 648
Σ	10 422			200 880	277 020

Then

$$\bar{X}\Sigma A = \Sigma \bar{x}A$$

$$\bar{X}\left(10\ 422\ \text{m}^2\right) = 200\ 880\ \text{mm}^2$$

or $\bar{X} = 19.27$ mm ◀

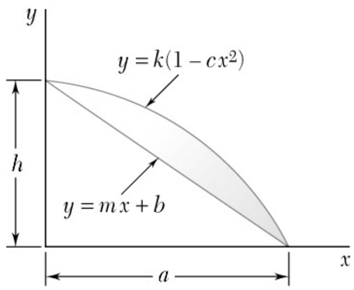
and

$$\bar{Y}\Sigma A = \Sigma \bar{y}A$$

$$\bar{Y}\left(10\ 422\ \text{m}^2\right) = 270\ 020\ \text{mm}^3$$

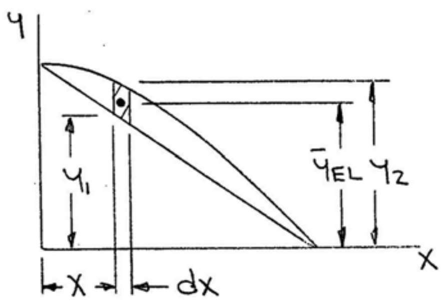
or $\bar{Y} = 26.6$ mm ◀

PROBLEM 5.133



Determine by direct integration the centroid of the area shown. Express your answer in terms of *a* and *h*.

SOLUTION



By observation

$$y_1 = -\frac{h}{a}x + h$$

$$= h\left(1 - \frac{x}{a}\right)$$

For y_2: At $x = 0, y = h$: $h = k(1-0)$ or $k = h$

At $x = a, y = 0$: $0 = h(1 - ca^2)$ or $C = \dfrac{1}{a^2}$

Then $$y_2 = h\left(1 - \frac{x^2}{a^2}\right)$$

Now $$dA = (y_2 - y_1)dx = h\left[\left(1 - \frac{x^2}{a^2}\right) - \left(1 - \frac{x}{a}\right)\right]dx$$

$$= h\left(\frac{x}{a} - \frac{x^2}{a^2}\right)dx$$

$\bar{x}_{EL} = x$ $\qquad \bar{y}_{EL} = \dfrac{1}{2}(y_1 - y_2) = \dfrac{h}{2}\left[\left(1 - \dfrac{x}{a}\right) + \left(1 - \dfrac{x^2}{a^2}\right)\right]$

$$= \frac{h}{2}\left(2 - \frac{x}{a} - \frac{x^2}{a^2}\right)$$

Then $$A = \int dA = \int_0^a h\left(\frac{x}{a} - \frac{x^2}{a^2}\right)dx = h\left[\frac{x^2}{2a} - \frac{x^3}{3a^2}\right]_0^a$$

$$= \frac{1}{6}ah$$

and $$\int \bar{x}_{EL}dA = \int_0^a x\left[h\left(\frac{x}{a} - \frac{x^2}{a^2}\right)dx\right] = h\left[\left(\frac{x^3}{3a} - \frac{x^4}{4a^2}\right)\right]_0^a$$

$$= \frac{1}{12}a^2h$$

PROBLEM 5.133 CONTINUED

$$\int \overline{y}_{EL} dA = \int_0^a \frac{h}{2}\left(2 - \frac{x}{a} - \frac{x^2}{a^2}\right)\left[h\left(\frac{x}{a} - \frac{x^2}{a^2}\right)dx\right]$$

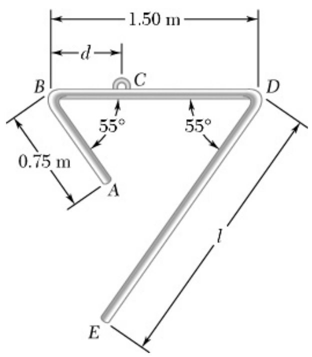
$$= \frac{h^2}{2}\int_0^a \left(2\frac{x}{a} - 3\frac{x^2}{a^2} + \frac{x^4}{a^4}\right)dx$$

$$= \frac{h^2}{2}\left[\frac{x^2}{a} - \frac{x^3}{a^2} + \frac{x^5}{5a^4}\right]_0^a = \frac{1}{10}ah^2$$

$$\overline{x}A = \int \overline{x}_{EL} dA\text{:} \quad x\left(\frac{1}{6}ah\right) = \frac{1}{12}a^2h \qquad \overline{x} = \frac{1}{2}a \blacktriangleleft$$

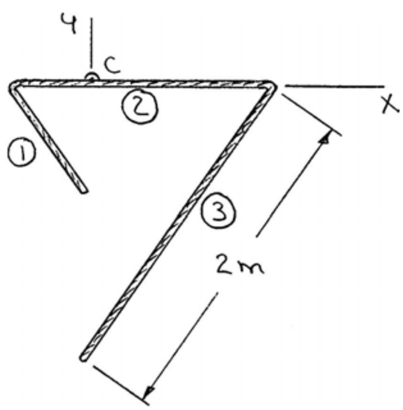
$$\overline{y}A = \int \overline{y}_{EL} dA\text{:} \quad y\left(\frac{1}{6}ah\right) = \frac{1}{10}a^2h \qquad \overline{y} = \frac{3}{5}h \blacktriangleleft$$

PROBLEM 5.134



Member *ABCDE* is a component of a mobile and is formed from a single piece of aluminum tubing. Knowing that the member is supported at *C* and that $l = 2$ m, determine the distance *d* so that portion *BCD* of the member is horizontal.

SOLUTION



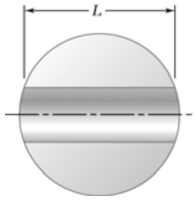
First note that for equilibrium, the center of gravity of the component must lie on a vertical line through C. Further, because the tubing is uniform, the center of gravity of the component will coincide with the centroid of the corresponding line. Thus, $\bar{X} = 0$

So that $\Sigma \bar{x}L = 0$

Then
$$-\left(d - \frac{0.75}{2}\cos 55°\right)\text{m} \times (0.75\text{ m})$$
$$+ (0.75 - d)\text{m} \times (1.5\text{ m})$$
$$+ \left[(1.5 - d)\text{m} - \left(\frac{1}{2} \times 2\text{ m} \times \cos 55°\right)\right] \times (2\text{ m}) = 0$$

or
$$(0.75 + 1.5 + 2)d = \left[\frac{1}{2}(0.75)^2 - 2\right]\cos 55° + (0.75)(1.5) + 3$$

or $d = 0.739$ m ◀

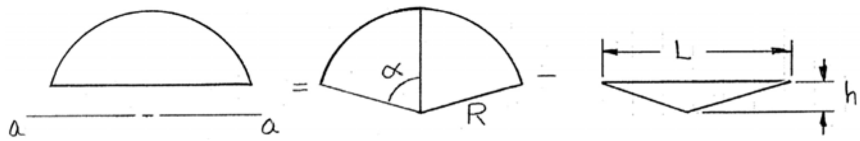


PROBLEM 5.135

A cylindrical hole is drilled through the center of a steel ball bearing shown here in cross section. Denoting the length of the *hole* by L, show that the volume of the steel remaining is equal to the volume of a sphere of diameter L.

SOLUTION

Calculate volumes by rotating cross sections about a line and using Theorem II of Pappus-Guldinus



For the sector: $\bar{y}_{AA} = \dfrac{2R\sin\alpha}{3\alpha}$ $\qquad A = \alpha R^2$

For the triangle: $h = \sqrt{R^2 - \left(\dfrac{L}{2}\right)^2} = \dfrac{1}{2}\sqrt{4R^2 - L^2}$ $\qquad \bar{y}_{AA} = \dfrac{2}{3}h = \dfrac{1}{3}\sqrt{4R^2 - L^2},$

$$A = \frac{1}{2}(L)(h)$$

$$= \frac{1}{4}L\sqrt{4\text{R}^2 - L^2}$$

Using Theorem II of Pappus-Guldinus

$$V_{\text{ball}} = 2\pi(\bar{y}_{AA})_1 A_1 - 2\pi(\bar{y}_{AA})_2 A_2$$

$$= 2\pi\left[\frac{2R\sin\alpha}{3\alpha}(\alpha R^2) - \left(\frac{1}{3}\sqrt{4R^2 - L^2}\right)\left(\frac{1}{4}L\sqrt{4R^2 - L^2}\right)\right]$$

$$= 2\pi\left[\frac{2}{3}R^3\sin\alpha - \frac{L}{12}(4R^2 - L^2)\right]$$

Now $\qquad R\sin\alpha = \dfrac{L}{2}$

Then $\qquad V = 2\pi\left[\dfrac{2}{3}\left(\dfrac{L}{2}\right)R^2 - \dfrac{1}{3}LR^2 + \dfrac{1}{12}L^3\right]$

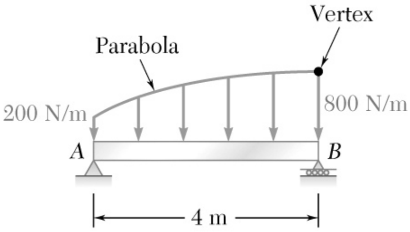
$$= \frac{\pi}{6}L^3$$

Note $V_{\text{sphere}} = \dfrac{4}{3}\pi r$ where r is the radius

If $r = \dfrac{L}{2}$, then $\qquad V_{\text{sphere}} = \dfrac{4}{3}\pi\left(\dfrac{L}{2}\right)^3 = \dfrac{\pi}{6}L^3$

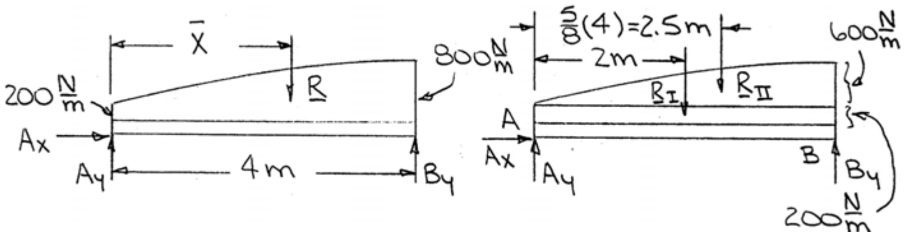
Therefore, $\qquad V_{\text{ball}} = V_{\text{sphere}} = \dfrac{\pi}{6}L^3$ Q.E.D. ◀

PROBLEM 5.136



For the beam and loading shown, determine (*a*) the magnitude and location of the resultant of the distributed load, (*b*) the reactions at the beam supports.

SOLUTION



(*a*) Have
$$R_I = (4\text{ m})(200\text{ N/m}) = 800\text{ N}$$

$$R_{II} = \frac{2}{3}(4\text{ m})(600\text{ N/m}) = 1600\text{ N}$$

Then
$$\Sigma F_y: \quad -R = -R_I - R_{II}$$

or
$$R = 800 + 1600 = 2400\text{ N}$$

and
$$\Sigma M_A: \quad -\bar{X}(2400) = -2(800) - 2.5(1600)$$

or
$$\bar{X} = \frac{7}{3}\text{m}$$

$$\therefore\ \mathbf{R} = 2400\text{ N}\downarrow,\ \bar{X} = 2.33\text{ m} \blacktriangleleft$$

(*b*) Reactions

$$\xrightarrow{+} \Sigma F_x = 0: \ A_x = 0$$

$$+\circlearrowleft \Sigma M_A = 0: \ (4\text{ m})B_y - \left(\frac{7}{3}\text{ m}\right)(2400\text{ N}) = 0$$

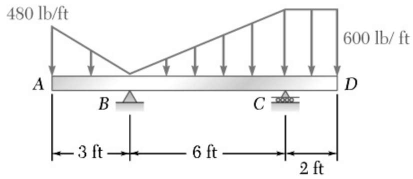
or
$$B_y = 1400\text{ N}$$

$$+\uparrow \Sigma F_y = 0: \ A_y + 1400\text{ N} - 2400\text{ N} = 0$$

or
$$A_y = 1000\text{ N}$$

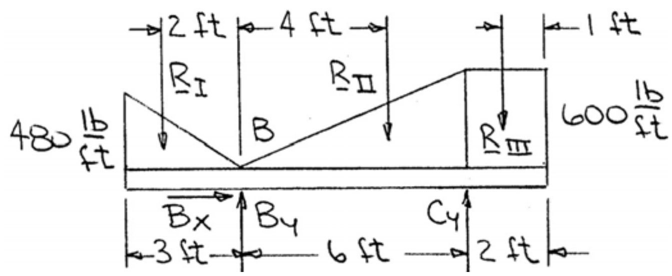
$$\therefore\ \mathbf{A} = 1000\text{ N}\uparrow,\ \mathbf{B} = 1400\text{ N}\uparrow \blacktriangleleft$$

PROBLEM 5.137



Determine the reactions at the beam supports for the given loading.

SOLUTION



Have
$$R_{\text{I}} = \frac{1}{2}(3\text{ ft})(480\text{ lb/ft}) = 720\text{ lb}$$

$$R_{\text{II}} = \frac{1}{2}(6\text{ ft})(600\text{ lb/ft}) = 1800\text{ lb}$$

$$R_{\text{III}} = (2\text{ ft})(600\text{ lb/ft}) = 1200\text{ lb}$$

Then
$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x = 0$$

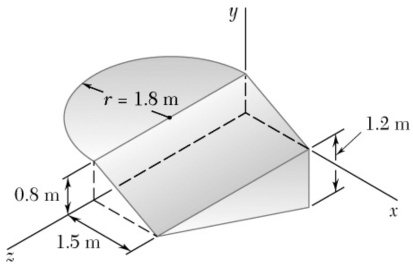
$$+\circlearrowleft \Sigma M_B = 0: \quad (2\text{ ft})(720\text{ lb}) - (4\text{ ft})(1800\text{ lb})$$
$$+(6\text{ ft})Cy - (7\text{ ft})(1200\text{ lb}) = 0$$

or $\quad \text{C}_y = 2360\text{ lb}$ $\qquad$ $\mathbf{C} = 2360\text{ lb} \uparrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad -720\text{ lb} + B_y - 1800\text{ lb} + 2360\text{ lb} - 1200\text{ lb} = 0$$

or $\quad B_y = 1360\text{ lb}$ $\qquad$ $\mathbf{B} = 1360\text{ lb} \uparrow$ ◀

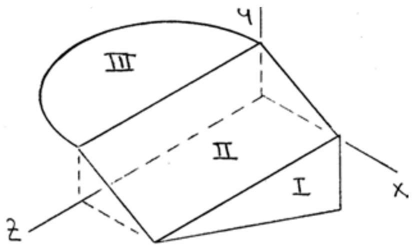
PROBLEM 5.138



Locate the center of gravity of the sheet-metal form shown.

SOLUTION

First, assume that the sheet metal is homogeneous so that the center of gravity of the form will coincide with the centroid of the corresponding area.



$$\bar{y}_{\mathrm{I}} = -\frac{1}{3}(1.2) = -0.4\text{ m}$$

$$\bar{z}_{\mathrm{I}} = \frac{1}{3}(3.6) = 1.2\text{ m}$$

$$\bar{x}_{\mathrm{III}} = -\frac{4(1.8)}{3\pi} = -\frac{2.4}{\pi}\text{ m}$$

	A, m^2	$\bar{x}$, m	$\bar{y}$, m	$\bar{z}$, m	$\bar{x}A$, m^3	$\bar{y}A$, m^3	$\bar{z}A$, m^3
I	$\frac{1}{2}(3.6)(1.2) = 2.16$	1.5	–0.4	1.2	3.24	–0.864	2.592
II	$(3.6)(1.7) = 6.12$	0.75	0.4	1.8	4.59	2.448	11.016
III	$\frac{\pi}{2}(1.8)^2 = 5.0894$	$-\frac{2.4}{\pi}$	0.8	1.8	–3.888	4.0715	9.1609
Σ	13.3694				3.942	5.6555	22.769

Have $\quad \bar{X}\Sigma V = \Sigma\bar{x}V: \quad \bar{X}\left(13.3694\text{ m}^2\right) = 3.942\text{ m}^3$

or $\bar{X} = 0.295\text{ m}$ ◀

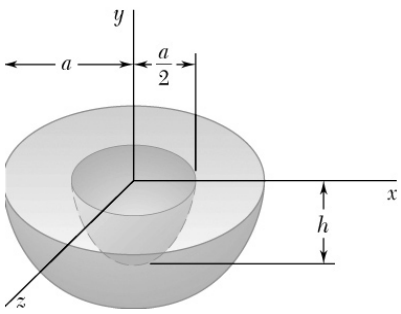
$$\bar{Y}\Sigma V = \Sigma\bar{y}V: \quad \bar{Y}\left(13.3694\text{ m}^2\right) = 5.6555\text{ m}^3$$

or $\bar{Y} = 0.423\text{ m}$ ◀

$$\bar{Z}\Sigma V = \Sigma\bar{z}V: \quad \bar{Z}\left(13.3694\text{ m}^2\right) = 22.769\text{ m}^3$$

or $\bar{Z} = 1.703\text{ m}$ ◀

PROBLEM 5.139



The composite body shown is formed by removing a semiellipsoid of revolution of semimajor axis h and semiminor axis $\frac{a}{2}$ from a hemisphere of radius a. Determine (*a*) the y coordinate of the centroid when $h = a/2$, (*b*) the ratio h/a for which $\bar{y} = -0.4a$.

SOLUTION

	V	$\bar{y}$	$\bar{y}V$
Hemisphere	$\frac{2}{3}\pi a^3$	$-\frac{3}{8}a$	$-\frac{1}{4}\pi a^4$
Semiellipsoid	$-\frac{2}{3}\pi\left(\frac{a}{2}\right)^2 h = -\frac{1}{6}\pi a^2 h$	$-\frac{3}{8}h$	$+\frac{1}{16}\pi a^2 h^2$

Then $$\Sigma V = \frac{\pi}{6}a^2(4a - h) \qquad \Sigma\bar{y}V = -\frac{\pi}{16}a^2(4a^2 - h^2)$$

Now $$\bar{Y}\Sigma V = \Sigma\bar{y}V$$

so that $$\bar{Y}\left[\frac{\pi}{6}a^2(4a - h)\right] = -\frac{\pi}{16}a^2(4a^2 - h^2)$$

or $$\bar{Y}\left(4 - \frac{h}{a}\right) = -\frac{3}{8}a\left[4 - \left(\frac{h}{a}\right)^2\right] \qquad (1)$$

(*a*) $\bar{Y} = ?$ when $h = \frac{a}{2}$

Substituting $\frac{h}{a} = \frac{1}{2}$ into Eq. (1)

$$\bar{Y}\left(4 - \frac{1}{2}\right) = -\frac{3}{8}a\left[4 - \left(\frac{1}{2}\right)^2\right]$$

or $$\bar{Y} = -\frac{45}{112}a$$ $\bar{Y} = -0.402a$ ◀

PROBLEM 5.139 CONTINUED

(*b*) $\dfrac{h}{a} = ?$ when $\bar{Y} = -0.4a$

Substituting into Eq. (1)

$$(-0.4a)\left(4 - \frac{h}{a}\right) = -\frac{3}{8}a\left[4 - \left(\frac{h}{a}\right)^2\right]$$

or

$$3\left(\frac{h}{a}\right)^2 - 3.2\left(\frac{h}{a}\right) + 0.8 = 0$$

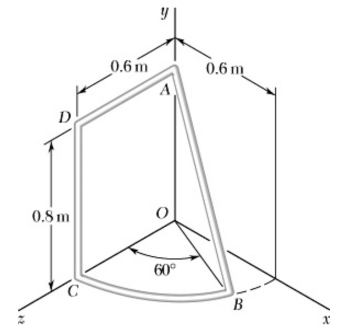
Then

$$\frac{h}{a} = \frac{3.2 \pm \sqrt{(-3.2)^2 - 4(3)(0.8)}}{2(3)}$$

$$= \frac{3.2 \pm 0.8}{6}$$

or $\dfrac{h}{a} = \dfrac{2}{5}$ and $\dfrac{h}{a} = \dfrac{2}{3}$ ◀

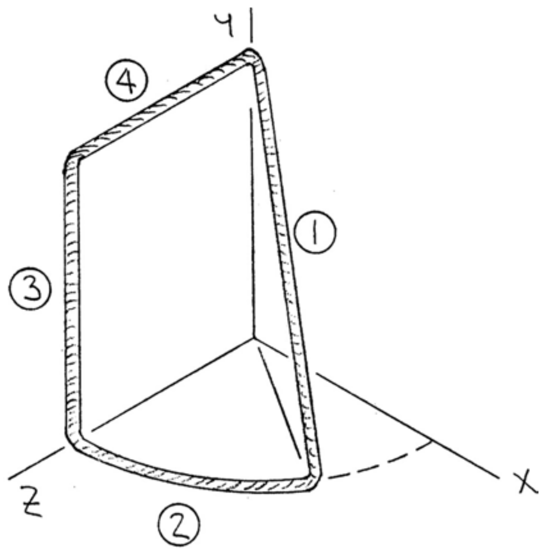
PROBLEM 5.140



A thin steel wire of uniform cross section is bent into the shape shown. Locate its center of gravity.

SOLUTION

First assume that the wire is homogeneous so that its center of gravity will coincide with the centroid of the corresponding line.



$$\bar{x}_1 = 0.3\sin 60° = 0.15\sqrt{3}\text{ m}$$

$$\bar{z}_1 = 0.3\cos 60° = 0.15\text{ m}$$

$$\bar{x}_2 = \left(\frac{0.6\sin 30°}{\frac{\pi}{6}}\right)\sin 30° = \frac{0.9}{\pi}\text{ m}$$

$$\bar{z}_2 = \left(\frac{0.6\sin 30°}{\frac{\pi}{6}}\right)\cos 30° = \frac{0.9}{\pi}\sqrt{3}\text{ m}$$

$$L_2 = \left(\frac{\pi}{3}\right)(0.6) = (0.2\pi)\text{ m}$$

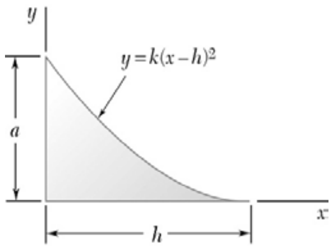
	L, m	$\bar{x}$, m	$\bar{y}$, m	$\bar{z}$, m	$\bar{x}L$, m^2	$\bar{y}L$, m^2	$\bar{z}L$, m^2
1	1.0	$0.15\sqrt{3}$	0.4	0.15	0.25981	0.4	0.15
2	0.2π	$\frac{0.9}{\pi}$	0	$\frac{0.9\sqrt{3}}{\pi}$	0.18	0	0.31177
3	0.8	0	0.4	0.6	0	0.32	0.48
4	0.6	0	0.8	0.3	0	0.48	0.18
Σ	3.0283				0.43981	1.20	1.12177

Have $\bar{X}\Sigma L = \Sigma \bar{x}L$: $\quad \bar{X}(3.0283\text{ m}) = 0.43981\text{ m}^2$ or $\bar{X} = 0.1452\text{ m}$ ◀

$\bar{Y}\Sigma L = \Sigma \bar{y}L$: $\quad \bar{Y}(3.0283\text{ m}) = 1.20\text{ m}^2$ or $\bar{Y} = 0.396\text{ m}$ ◀

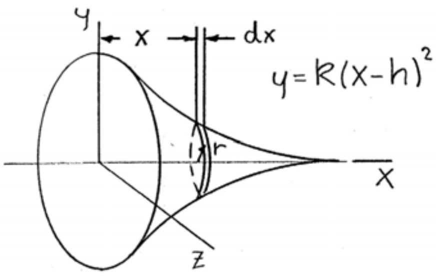
$\bar{Z}\Sigma L = \Sigma \bar{z}L$: $\quad \bar{Z}(3.0283\text{ m}) = 1.12177\text{ m}^2$ or $\bar{Z} = 0.370\text{ m}$ ◀

PROBLEM 5.141



Locate the centroid of the volume obtained by rotating the shaded area about the x axis.

SOLUTION



First note that symmetry implies $\bar{y} = 0$ ◀

and $\bar{z} = 0$ ◀

Have $y = k(X - h)^2$

At $x = 0, y = a$: $a = k(-h)^2$

or $k = \dfrac{a}{h^2}$

Choose as the element of volume a disk of radius r and thickness dx. Then

$$dV = \pi r^2 dx, \ \ \bar{X}_{EL} = x$$

Now $r = \dfrac{a}{h^2}(x - h)^2$

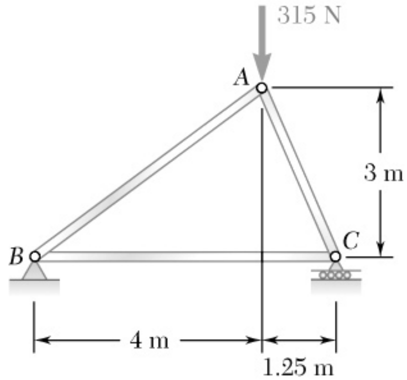
so that $dV = \pi\dfrac{a^2}{h^4}(x - h)^4\,dx$

Then
$$V = \int_0^h \pi\frac{a^2}{h^4}(x - h)^4\,dx = \frac{\pi}{5}\frac{a^2}{h^4}\Big[(x - h)^5\Big]_0^h$$
$$= \frac{1}{5}\pi a^2 h$$

and
$$\int \bar{x}_{EL}\,dV = \int_0^h x\left[\pi\frac{a^2}{h^4}(x - h)^4\,dx\right]$$
$$= \pi\frac{a^2}{h^4}\int_0^h \left(x^5 - 4hx^4 + 6h^2x^3 - 4h^3x^2 + h^4x\right)dx$$
$$= \pi\frac{a^2}{h^4}\left[\frac{1}{6}x^6 - \frac{4}{5}hx^5 + \frac{3}{2}h^2x^4 - \frac{4}{3}h^3x^3 + \frac{1}{2}h^4x^2\right]_0^h$$
$$= \frac{1}{30}\pi a^2h^2$$

Now $\bar{x}V = \int \bar{x}_{EL}\,dV$: $\bar{x}\left(\dfrac{\pi}{5}a^2h\right) = \dfrac{\pi}{30}a^2h^2$

or $\bar{x} = \dfrac{1}{6}h$ ◀

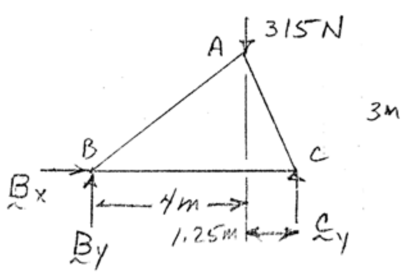


PROBLEM 6.1

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

FBD Truss:



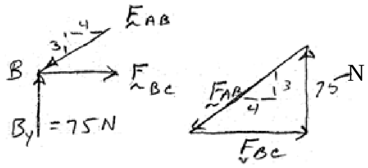
$$\Sigma M_B = 0: (6.25\text{ m})C_y - (4\text{ m})(315\text{ N}) = 0 \qquad \mathbf{C}_y = 240\text{ N}\uparrow$$

$$\uparrow \Sigma F_y = 0: B_y - 315\text{ N} + C_y = 0 \qquad \mathbf{B}_y = 75\text{ N}\uparrow$$

$$\rightarrow \Sigma F_x = 0: \qquad \mathbf{B}_x = 0$$

Joint FBDs:

Joint *B*:

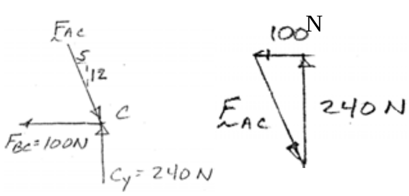


$$\frac{F_{AB}}{5} = \frac{F_{BC}}{4} = \frac{75\text{ N}}{3}$$

$F_{AB} = 125.0\text{ N}$ C ◀

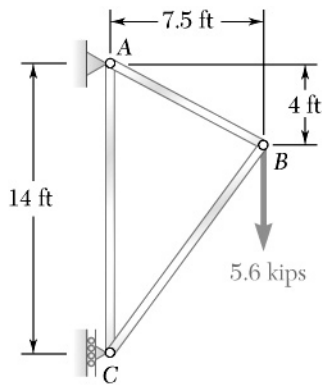
$F_{BC} = 100.0\text{ N}$ T ◀

Joint *C*:



By inspection: $F_{AC} = 260\text{ N}$ C ◀

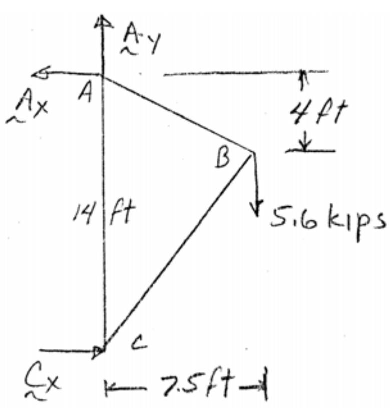
PROBLEM 6.2



Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

FBD Truss:



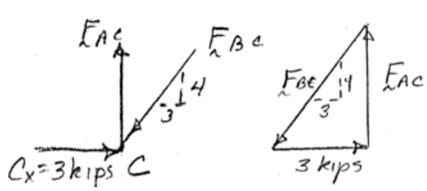
$$\circlearrowleft \Sigma M_A = 0: (14 \text{ ft})C_x - (7.5 \text{ ft})(5.6 \text{ kips}) = 0 \qquad \mathbf{C}_x = 3 \text{ kips} \longrightarrow$$

$$\longrightarrow \Sigma F_x = 0: -A_x + C_x = 0 \qquad \mathbf{A}_x = 3 \text{ kips} \longleftarrow$$

$$\uparrow \Sigma F_y = 0: A_y - 5.6 \text{ kips} = 0 \qquad \mathbf{A}_y = 5.6 \text{ kips} \uparrow$$

Joint FBDs:

Joint *C*:

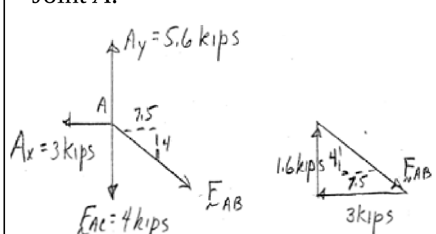


$$\frac{F_{BC}}{5} = \frac{F_{AC}}{4} = \frac{3 \text{ kips}}{3}$$

$F_{BC} = 5.00$ kips C ◀

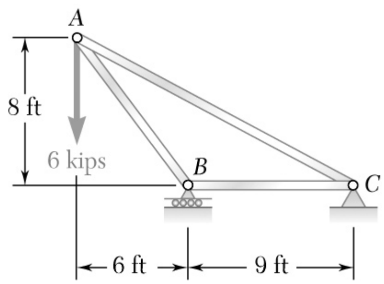
$F_{AC} = 4.00$ kips T ◀

Joint *A*:



$$\frac{F_{AB}}{8.5} = \frac{1.6 \text{ kips}}{4}$$

$F_{AB} = 3.40$ kips T ◀

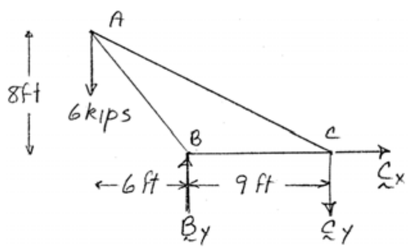


PROBLEM 6.3

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

FBD Truss:



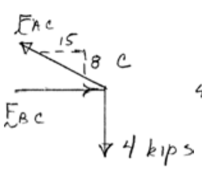
$$\circlearrowleft \Sigma M_B = 0\text{: } (6\text{ ft})(6\text{ kips}) - (9\text{ ft})C_y = 0 \qquad \mathbf{C}_y = 4\text{ kips} \downarrow$$

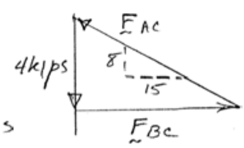
$$\uparrow \Sigma F_y = 0\text{: } B_y - 6\text{ kips} - C_y = 0 \qquad \mathbf{B}_y = 10\text{ kips} \uparrow$$

$$\longrightarrow \Sigma F_x = 0\text{: } \mathbf{C}_x = 0$$

Joint FBDs:

Joint C:



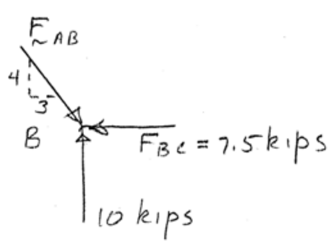


$$\frac{F_{AC}}{17} = \frac{F_{BC}}{15} = \frac{4\text{ kips}}{8}$$

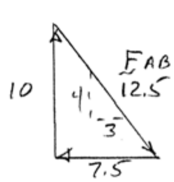
$F_{AC} = 8.50$ kips T ◀

$F_{BC} = 7.50$ kips C ◀

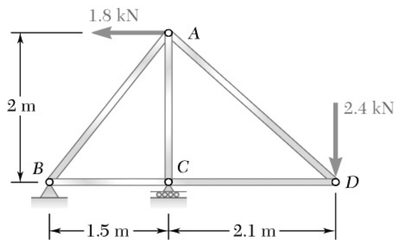
Joint B:



By inspection: $F_{AB} = 12.50$ kips C ◀



$$\frac{F_{AB}}{5} = \frac{10\text{ kips}}{4}$$

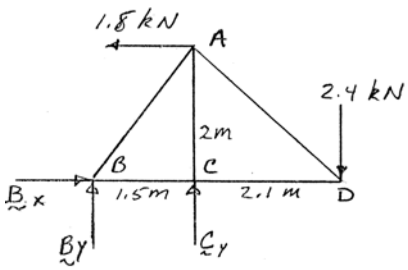


PROBLEM 6.4

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

FBD Truss:



$$\circlearrowleft \Sigma M_B = 0: (1.5\text{ m})C_y + (2\text{ m})(1.8\text{ kN}) - 3.6\text{ m}(2.4\text{ kN}) = 0$$

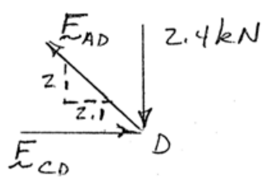
$$\mathbf{C}_y = 3.36\text{ kN} \uparrow$$

$$\uparrow \Sigma F_y = 0: B_y + 3.36\text{ kN} - 2.4\text{ kN} = 0$$

$$\mathbf{B}_y = 0.96\text{ kN} \downarrow$$

Joint FBDs:

Joint *D*:

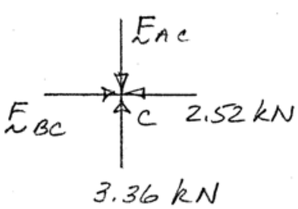


$$\uparrow \Sigma F_y = 0: \frac{2}{2.9}F_{AD} - 2.4\text{ kN} = 0 \qquad F_{AD} = 3.48\text{ kN T} \blacktriangleleft$$

$$\rightarrow \Sigma F_x = 0: F_{CD} - \frac{2.1}{2.9}F_{AD} = 0$$

$$F_{CD} = \frac{2.1}{2.9}(3.48\text{ kN}) \qquad F_{CD} = 2.52\text{ kN C} \blacktriangleleft$$

Joint *C*:

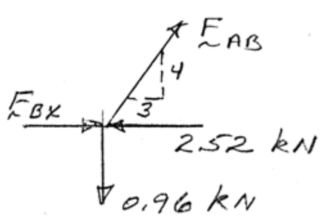


By inspection:

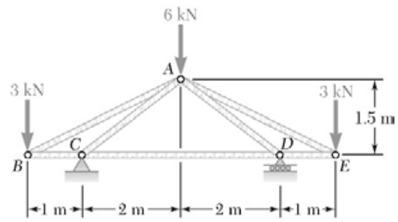
$$F_{AC} = 3.36\text{ kN C} \blacktriangleleft$$

$$F_{BC} = 2.52\text{ kN C} \blacktriangleleft$$

Joint *B*:



$$\uparrow \Sigma F_y = 0: \frac{4}{5}F_{AB} - 0.9\text{ kN} = 0 \qquad F_{AB} = 1.200\text{ kN T} \blacktriangleleft$$

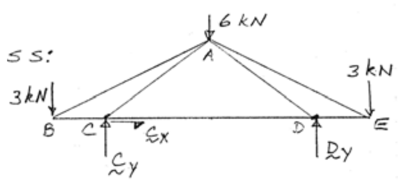


PROBLEM 6.5

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

FBD Truss:

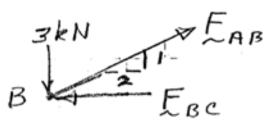


$\longrightarrow \Sigma F_x = 0: \ \mathbf{C}_x = 0$

By symmetry: $\mathbf{C}_y = \mathbf{D}_y = 6 \text{ kN} \uparrow$

Joint FBDs:

Joint *B*:

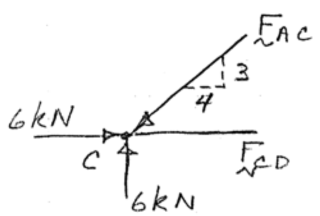


$$\uparrow \Sigma F_y = 0: -3 \text{ kN} + \frac{1}{\sqrt{5}} F_{AB} = 0$$

$$F_{AB} = 3\sqrt{5} = 6.71 \text{ kN T} \blacktriangleleft$$

$$\longrightarrow \Sigma F_x = 0: \frac{2}{\sqrt{5}} F_{AB} - F_{BC} = 0 \qquad F_{BC} = 6.00 \text{ kN C} \blacktriangleleft$$

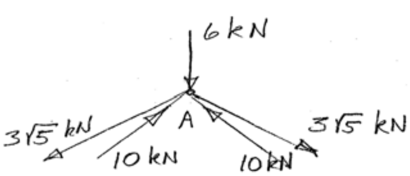
Joint *C*:



$$\uparrow \Sigma F_y = 0: 6 \text{ kN} - \frac{3}{5} F_{AC} = 0 \qquad F_{AC} = 10.00 \text{ kN C} \blacktriangleleft$$

$$\longrightarrow \Sigma F_x = 0: 6 \text{ kN} - \frac{4}{5} F_{AC} + F_{CD} = 0 \qquad F_{CD} = 2.00 \text{ kN T} \blacktriangleleft$$

Joint *A*:



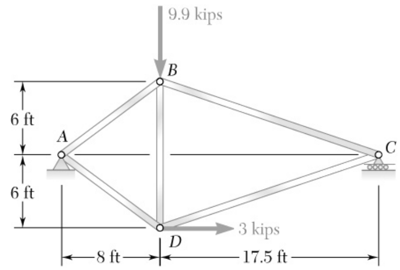
$$\uparrow \Sigma F_y = 0: -2\left(\frac{1}{\sqrt{5}} 3\sqrt{5} \text{ kN}\right) + 2\left(\frac{3}{5} 10 \text{ kN}\right) - 6 \text{ kN} = 0 \text{ check}$$

By symmetry:

$$F_{AE} = F_{AB} = 6.71 \text{ kN T} \blacktriangleleft$$

$$F_{AD} = F_{AC} = 10.00 \text{ kN C} \blacktriangleleft$$

$$F_{DE} = F_{BC} = 6.00 \text{ kN C} \blacktriangleleft$$

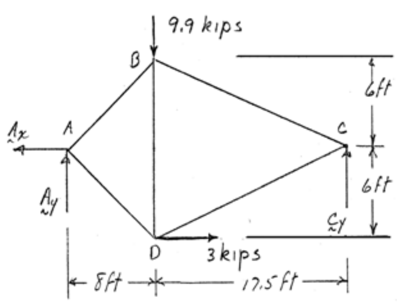


PROBLEM 6.6

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

FBD Truss:



$$\circlearrowleft \Sigma M_A = 0: (25.5 \text{ ft})C_y + (6 \text{ ft})(3 \text{ kips}) - (8 \text{ ft})(9.9 \text{ kips}) = 0$$

$$\mathbf{C}_y = 2.4 \text{ kips} \uparrow$$

$$\uparrow \Sigma F_y = 0: A_y + 2.4 \text{ kips} - 9.9 \text{ kips} = 0$$

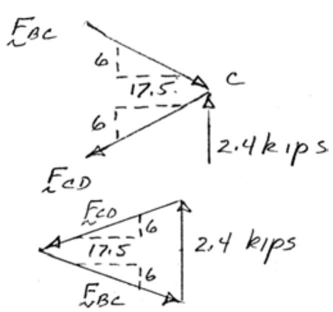
$$\mathbf{A}_y = 7.4 \text{ kips} \uparrow$$

$$\longrightarrow \Sigma F_x = 0: -A_x + 3 \text{ kips} = 0$$

$$\mathbf{A}_x = 3 \text{ kips} \longleftarrow$$

Joint FBDs:

Joint C:



$$\frac{2.4 \text{ kips}}{12} = \frac{F_{CD}}{18.5} = \frac{F_{BC}}{18.5}$$

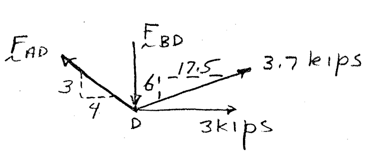
$F_{CD} = 3.70$ kips T ◀

$F_{BC} = 3.70$ kips C ◀

or: $\Sigma F_x = 0: F_{BC} = F_{CD}$ $\quad$ $\uparrow \Sigma F_y = 0: 2.4 \text{ kips} - 2\dfrac{6}{18.5}F_{BC} = 0$

same answers

Joint D:



$$\longrightarrow \Sigma F_x = 0: 3 \text{ kips} + \frac{17.5}{18.5}(3.70 \text{ kips}) - \frac{4}{5}F_{AD} = 0$$

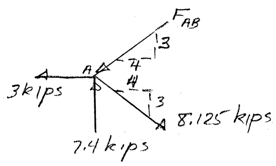
$F_{AD} = 8.125$ kips $\qquad$ $F_{AD} = 8.13$ kips T ◀

$$\uparrow \Sigma F_y = 0: \frac{6}{18.5}(3.7 \text{ kips}) + \frac{3}{5}(8.125 \text{ kips}) - F_{BD} = 0$$

$F_{BD} = 6.075$ kips $\qquad$ $F_{BD} = 6.08$ kips C ◀

PROBLEM 6.6 CONTINUED

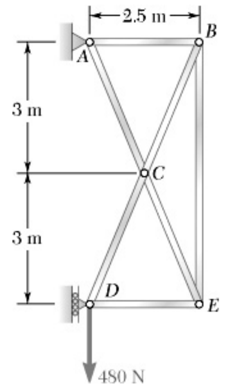
Joint A:



$$\xrightarrow{} \Sigma F_x = 0: -3 \text{ kips} + \frac{4}{5}(8.125 \text{ kips}) - \frac{4}{5}F_{AB} = 0$$

$F_{AB} = 4.375$ kips

$F_{AB} = 4.38$ kips C ◀

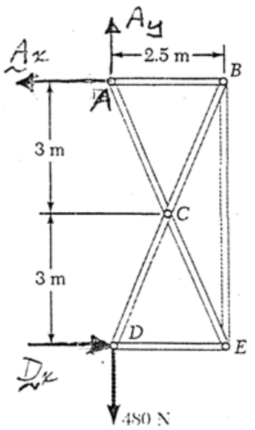


PROBLEM 6.7

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

FBD Truss:



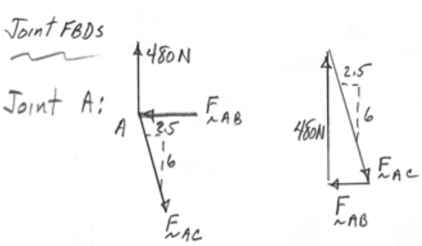
$$\uparrow \Sigma F_y = 0: A_y - 480 \text{ N} = 0 \qquad \mathbf{A}_y = 480 \text{ N} \uparrow$$

$$\circlearrowright \Sigma M_A = 0: (6 \text{ m}) D_x = 0 \qquad \mathbf{D}_x = 0$$

$$\longrightarrow \Sigma F_x = 0: -A_x = 0 \qquad \mathbf{A}_x = 0$$

Joint FBDs:

Joint A:

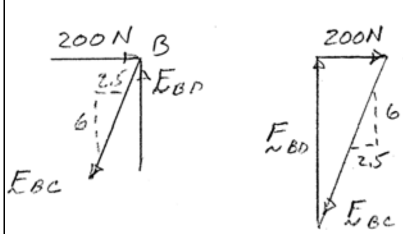


$$\frac{480 \text{ N}}{6} = \frac{F_{AB}}{2.5} = \frac{F_{AC}}{6.5}$$

$F_{AB} = 200 \text{ N C}$ ◀

$F_{AC} = 520 \text{ N T}$ ◀

Joint B:



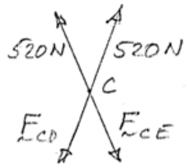
$$\frac{200 \text{ N}}{2.5} = \frac{F_{BE}}{6} = \frac{F_{BC}}{6.5}$$

$F_{BE} = 480 \text{ N C}$ ◀

$F_{BC} = 520 \text{ N T}$ ◀

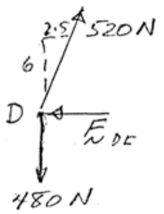
PROBLEM 6.7 CONTINUED

Joint C:



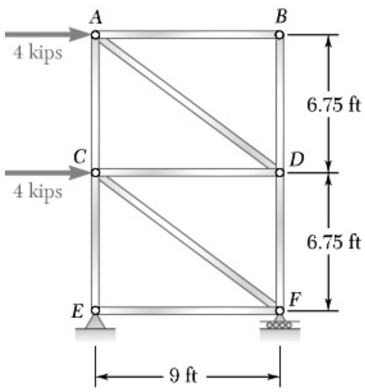
By inspection: $F_{CD} = F_{CE} = 520\text{ N}$ T ◀

Joint D:



$\longrightarrow \Sigma F_x = 0: \dfrac{2.5}{6.5}(520\text{ N}) - F_{DE} = 0$

$F_{DE} = 200\text{ N}$ C ◀

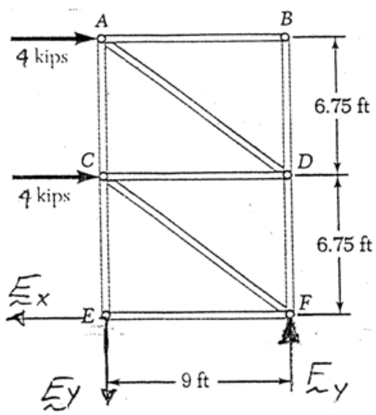


PROBLEM 6.8

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

FBD Truss:



$$\circlearrowleft \Sigma M_E = 0: (9 \text{ ft})F_y - (6.75 \text{ ft})(4 \text{ kips}) - (13.5 \text{ ft})(4 \text{ kips}) = 0$$

$$\mathbf{F}_y = 9 \text{ kips} \uparrow$$

$$\uparrow \Sigma F_y = 0: -E_y + 9 \text{ kips} = 0 \qquad \mathbf{E}_y = 9 \text{ kips} \downarrow$$

$$\longrightarrow \Sigma F_x = 0: -E_x + 4 \text{ kips} + 4 \text{ kips} = 0 \qquad \mathbf{E}_x = 8 \text{ kips} \longleftarrow$$

By inspection of joint E: $F_{EC} = 9.00 \text{ kips T}$ ◀

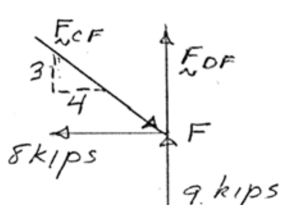
$F_{EF} = 8.00 \text{ kips T}$ ◀

By inspection of joint B: $F_{AB} = 0$ ◀

$F_{BD} = 0$ ◀

Joint FBDs:

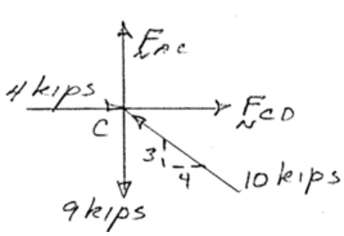
Joint F:



$$\longrightarrow \Sigma F_x = 0: \frac{4}{5}F_{CF} - 8 \text{ kips} = 0 \qquad F_{CF} = 10.00 \text{ kips C} ◀$$

$$\uparrow \Sigma F_y = 0: F_{DF} - \frac{3}{5}(10 \text{ kips}) = 0 \qquad F_{DF} = 6.00 \text{ kips T} ◀$$

Joint C:



$$\longrightarrow \Sigma F_x = 0: 4 \text{ kips} - \frac{4}{5}(10 \text{ kips}) + F_{CD} = 0$$

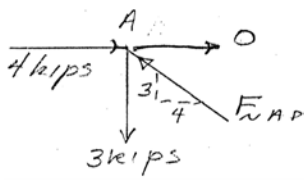
$F_{CD} = 4.00 \text{ kips T}$ ◀

$$\uparrow \Sigma F_y = 0: F_{AC} - 9 \text{ kips} + \frac{3}{5}(10 \text{ kips}) = 0$$

$F_{AC} = 3.00 \text{ kips T}$ ◀

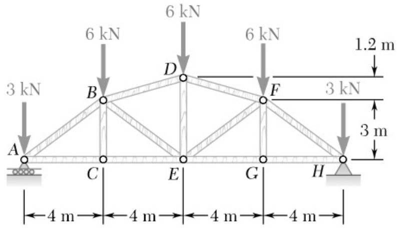
PROBLEM 6.8 CONTINUED

Joint A:



$\xrightarrow{} \Sigma F_x = 0\text{: } 4 \text{ kips} - \frac{4}{5}F_{AD} = 0$ $\qquad F_{AD} = 5.00 \text{ kips C}$ ◀

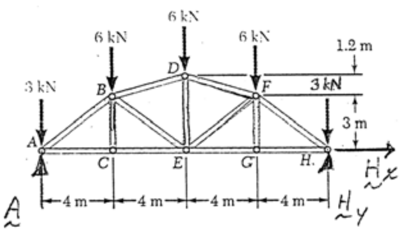
PROBLEM 6.9



Determine the force in each member of the Gambrel roof truss shown. State whether each member is in tension or compression.

SOLUTION

FBD Truss:



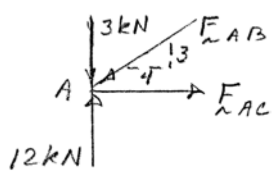
$\xrightarrow{} \Sigma F_x = 0: \mathbf{H}_x = 0$

By symmetry: $\mathbf{A} = \mathbf{H}_y = 12 \text{ kN} \uparrow$

By inspection of joints C and G, $F_{CE} = F_{AC}$ and $F_{BC} = 0$ ◀

$F_{EG} = F_{GH}$ and $F_{FG} = 0$ ◀

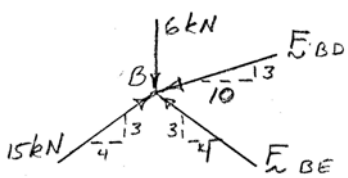
Joint A:



$\uparrow \Sigma F_y = 0: 12 \text{ kN} - 3 \text{ kN} - \frac{3}{5}F_{AB} = 0$ $\qquad F_{AB} = 15.00 \text{ kN C}$ ◀

$\xrightarrow{} \Sigma F_x = 0: F_{AC} - \frac{4}{5}(15 \text{ kN}) = 0$ $\qquad F_{AC} = 12.00 \text{ kN T}$ ◀

Joint B:



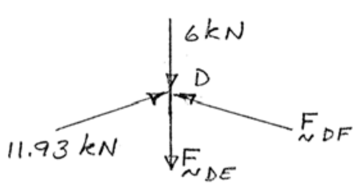
$\xrightarrow{} \Sigma F_x = 0: \frac{4}{5}(15 \text{ kN}) - \frac{10}{10.44}F_{BD} - \frac{4}{5}F_{BE} = 0$

$\uparrow \Sigma F_y = 0: \frac{3}{5}(15 \text{ kN}) - 6 \text{ kN} - \frac{3}{10.44}F_{BD} + \frac{3}{5}F_{BE} = 0$

Solving yields $\qquad F_{BD} = 11.93 \text{ kN C}$ ◀

$F_{BE} = 0.714 \text{ kN C}$ ◀

Joint D:



$\xrightarrow{} \Sigma F_x = 0:$ by symmetry $\qquad F_{DF} = 11.93 \text{ kN C}$ ◀

$\uparrow \Sigma F_y = 0: -F_{DE} - 6 \text{ kN} + 2\frac{3}{10.44}(11.93 \text{ kN}) = 0$

$F_{DE} = 0.856 \text{ kN T}$ ◀

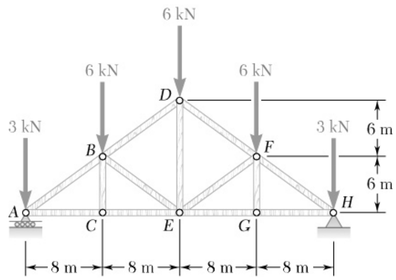
By symmetry: $\qquad F_{EF} = F_{BE}$ so $\qquad F_{EF} = 0.714 \text{ kN C}$ ◀

$F_{FH} = F_{AB}$ $\qquad F_{FH} = 15.00 \text{ kN C}$ ◀

$F_{GH} = F_{AC}$ $\qquad F_{GH} = 12.00 \text{ kN T}$ ◀

From above $\qquad F_{CE} = F_{AC}$ $\qquad F_{CE} = 12.00 \text{ kN T}$ ◀

$F_{EG} = F_{GH}$ $\qquad F_{EG} = 12.00 \text{ kN T}$ ◀

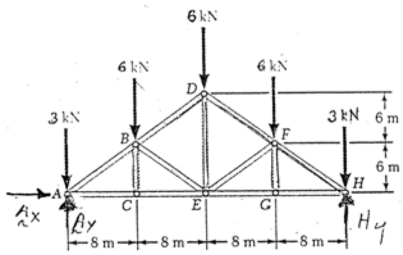


PROBLEM 6.10

Determine the force in each member of the Howe roof truss shown. State whether each member is in tension or compression.

SOLUTION

FBD Truss:



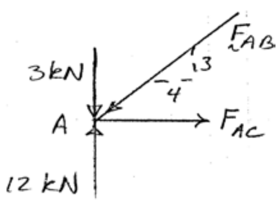
$\xrightarrow{} \Sigma F_x = 0\colon\ \mathbf{A}_x = 0$

By symmetry: $\mathbf{A}_y = \mathbf{H}_y = 12\text{ kN} \uparrow$

and $F_{FH} = F_{AB};\ F_{GH} = F_{AC};\ F_{FG} = F_{BC}$

$F_{DF} = F_{BD};\ F_{EF} = F_{BE};\ F_{EG} = F_{CE}$

Joint FBDs:

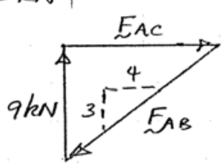


$$\frac{9\text{ kN}}{3} = \frac{F_{AC}}{4} = \frac{F_{AB}}{5}$$

$F_{AC} = 12.00\text{ kN T}$ ◀

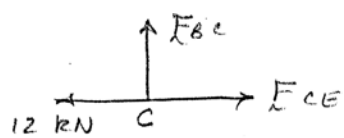
$F_{AB} = 15.00\text{ kN C}$ ◀

so $F_{FH} = 15.00\text{ kN C}$ ◀



By inspection: $F_{GH} = 12.00\text{ kN T}$ ◀

Joint C:



$F_{BC} = 0;\ F_{CE} = 12\text{ kN}$ $\qquad F_{BC} = 0 = F_{FG}$ ◀

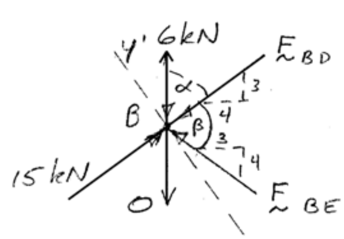
$F_{CE} = 12.00\text{ kN T}$ ◀

$F_{EG} = 12.00\text{ kN T}$ ◀

Note: $\alpha = \tan^{-1}\dfrac{4}{3}$ so $\sin\alpha = 0.8$

$\beta = 2\tan^{-1}\dfrac{3}{4}$ so $\sin\beta = 0.96$

Joint B:



$\nwarrow \Sigma F_{y1} = 0\colon -6\text{ kN}\sin\alpha + F_{BE}\sin\beta = 0$

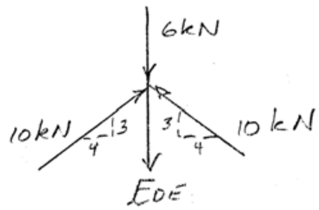
$F_{BE} = 5.00\text{ kN C}$ ◀

so $F_{EF} = 5.00\text{ kN C}$ ◀

$$\uparrow \Sigma F_y = 0\colon -6\text{ kN} - \frac{3}{5}F_{BD} + \frac{3}{5}F_{BE} + \frac{3}{5}(15\text{ kN}) = 0$$

PROBLEM 6.10 CONTINUED

Joint D:

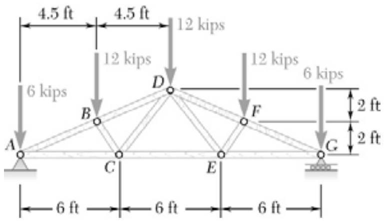


$$\frac{3}{5}F_{BD} = \frac{3}{5}(5\text{ kN}) + \frac{3}{5}(15\text{ kN}) - 6\text{ kN} \qquad F_{BD} = 10.00\text{ kN C} \blacktriangleleft$$

$$\text{so } F_{DF} = 10.00\text{ kN C} \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0\text{: } -6\text{ kN} + 2\left(\frac{3}{5}10\text{ kN}\right) - F_{DE} = 0 \qquad F_{DE} = 6.00\text{ kN T} \blacktriangleleft$$

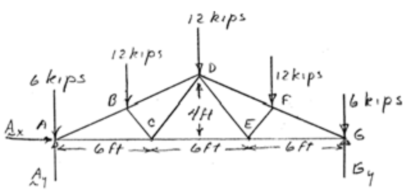
PROBLEM 6.11



Determine the force in each member of the Fink roof truss shown. State whether each member is in tension or compression.

SOLUTION

FBD Truss:



$\rightarrow \Sigma F_x = 0: \quad \mathbf{A}_x = 0$

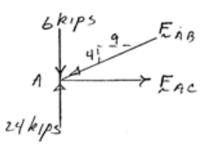
By symmetry: $\mathbf{A}_y = \mathbf{G}_y = 24 \text{ kips} \uparrow$

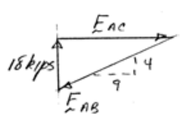
also; $F_{FG} = F_{AB};\ F_{EG} = F_{AC};\ F_{EF} = F_{BC}$

$F_{DF} = F_{BD};\ F_{DE} = F_{CD}$

Joint FBDs:

Joint A:





$$\frac{18 \text{ kips}}{4} = \frac{F_{AC}}{9} = \frac{F_{AB}}{\sqrt{97}};$$

$F_{AC} = 40.5 \text{ kips}$ T ◀

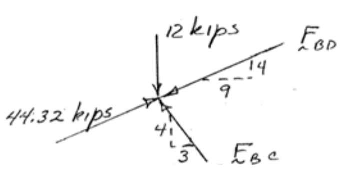
$F_{AB} = 44.32 \text{ kips}$

$F_{AB} = 44.3 \text{ kips}$ C ◀

so $F_{FG} = 44.3 \text{ kips}$ C ◀

$F_{EG} = 40.5 \text{ kips}$ T ◀

Joint B:



$$\rightarrow \Sigma F_x = 0: \frac{9}{\sqrt{97}}(44.32 \text{ kips} - F_{BD}) - \frac{3}{5}F_{BC} = 0$$

$$\uparrow \Sigma F_y = 0: \frac{4}{\sqrt{97}}(44.32 \text{ kips} - F_{BD}) + \frac{4}{5}F_{BC} - 12 = 0$$

Solving:

$F_{BC} = 11.25 \text{ kips}$ C ◀

$F_{BD} = 36.9 \text{ kips}$ C ◀

so $F_{EF} = 11.25 \text{ kips}$ C ◀

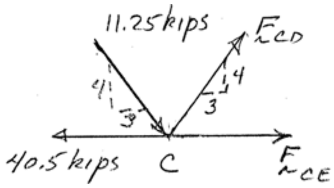
$F_{DF} = 36.9 \text{ kips}$ C ◀

PROBLEM 6.11 CONTINUED

$$\uparrow \Sigma F_y = 0: \frac{4}{5}(11.25 \text{ kips}) - \frac{4}{5}F_{CD} = 0 \qquad F_{CD} = 11.25 \text{ kips T} \blacktriangleleft$$

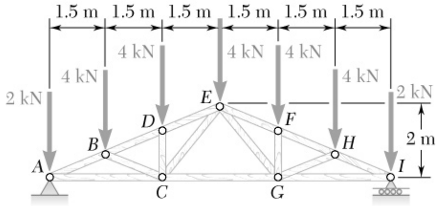
Joint *C*:

$$F_{DE} = 11.25 \text{ kips T} \blacktriangleleft$$



$$\rightarrow \Sigma F_x = 0: F_{CE} + 2\left[\frac{3}{5}(11.25 \text{ kips})\right] - 40.5 \text{ kips} = 0$$

$$F_{CE} = 27.0 \text{ kips T} \blacktriangleleft$$

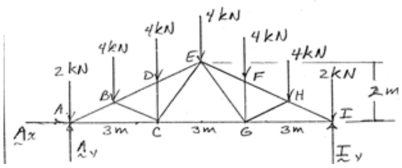


PROBLEM 6.12

Determine the force in each member of the fan roof truss shown. State whether each member is in tension or compression.

SOLUTION

FBD Truss:



$\xrightarrow{+} \Sigma F_x = 0: \quad \mathbf{A}_x = 0$

By symmetry: $\mathbf{A}_y = \mathbf{I}_y = 12 \text{ kN} \uparrow$

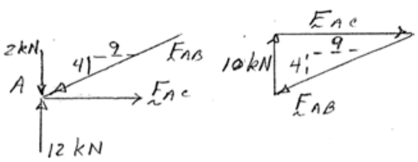
and $F_{AB} = F_{HI};\ F_{AC} = F_{GI};\ F_{BC} = F_{GH}$

$F_{BD} = F_{FH};\ F_{DC} = F_{FG};\ F_{DE} = F_{EF}$

$F_{CE} = F_{EG}$

Joint FBDs:

Joint *A*:



$$\frac{10 \text{ kN}}{4} = \frac{F_{AC}}{9} = \frac{F_{AB}}{\sqrt{97}}$$

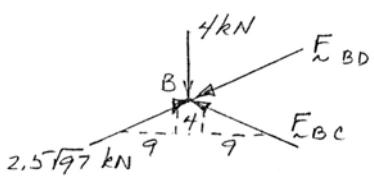
$F_{AC} = 22.5 \text{ kN T}$ ◀

so $F_{GI} = 22.5 \text{ kN T}$ ◀

$F_{AB} = 2.5\sqrt{97} \text{ kN}$ $\qquad F_{AB} = 24.6 \text{ kN C}$ ◀

so $F_{HI} = 24.6 \text{ kN C}$ ◀

Joint *B*:



$$\xrightarrow{+} \Sigma F_x = 0: 22.5 \text{ kN} - \frac{9}{\sqrt{97}}\left(F_{BD} + F_{BC}\right) = 0$$

$$+\uparrow \Sigma F_y = 0: 10 \text{ kN} - 4 \text{ kN} + \frac{4}{\sqrt{97}}\left(F_{BC} - F_{BD}\right) = 0$$

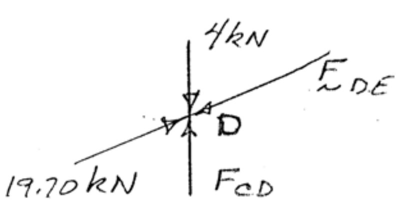
Solving: $F_{BD} = 19.70 \text{ kN C}$ ◀

so $F_{FH} = 19.70 \text{ kN C}$ ◀

and $F_{BC} = 4.92 \text{ kN C}$ ◀

so $F_{GH} = 4.92 \text{ kN C}$ ◀

Joint *D*:



By inspection: $F_{DE} = 19.70 \text{ kN C}$ ◀

so $F_{EF} = 19.70 \text{ kN C}$ ◀

and $F_{CD} = 4.00 \text{ kN C}$ ◀

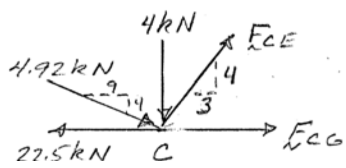
so $F_{FG} = 4.00 \text{ kN C}$ ◀

PROBLEM 6.12 CONTINUED

Joint C:

$$\xrightarrow{} \Sigma F_x = 0: -22.5\text{ kN} + \frac{9}{\sqrt{97}}(4.92\text{ kN}) + \frac{3}{5}F_{CE} + F_{CG} = 0$$

$$\uparrow \Sigma F_y = 0: -4\text{ kN} - \frac{4}{\sqrt{97}}(4.92\text{ kN}) + \frac{4}{5}F_{CE} = 0$$



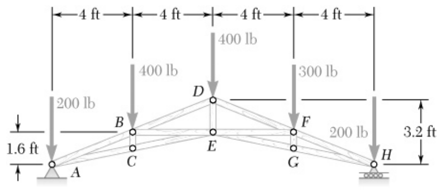
Solving:

$F_{CE} = 7.50\text{ kN T}$ ◀

so $F_{EG} = 7.50\text{ kN T}$ ◀

and $F_{CG} = 13.50\text{ kN T}$ ◀

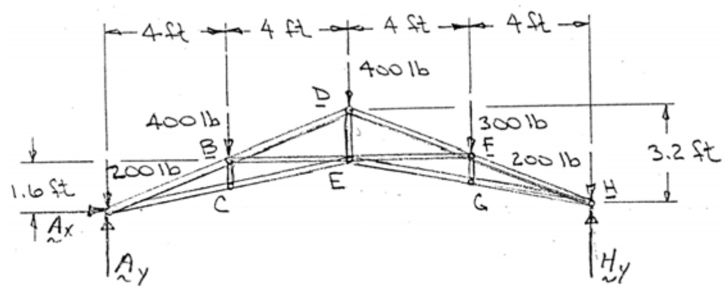
PROBLEM 6.13



Determine the force in each member of the roof truss shown. State whether each member is in tension or compression.

SOLUTION

FBD Truss:



$$\circlearrowright \Sigma M_A = 0: (16\text{ ft})H_y - (16\text{ ft})(200\text{ lb}) - (12\text{ ft})(300\text{ lb}) - (8\text{ ft})(400\text{ lb}) - (4\text{ ft})(400\text{ lb}) = 0$$

$$\mathbf{H}_y = 725\text{ lb} \uparrow$$

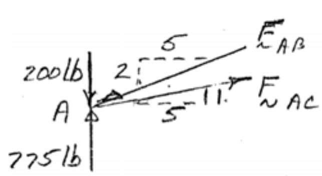
$$\uparrow \Sigma F_y = 0: A_y - 200\text{ lb} - 400\text{ lb} - 400\text{ lb} - 300\text{ lb} - 200\text{ lb} + 725\text{ lb} = 0$$

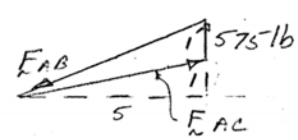
$$\mathbf{A}_y = 775\text{ lb} \uparrow$$

$$\rightarrow \Sigma F_x = 0: \qquad \mathbf{A}_x = 0$$

Joint FBDs:

Joint *A*:





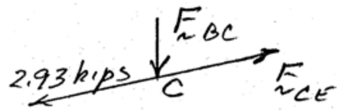
$$\frac{575\text{ lb}}{1} = \frac{F_{AC}}{\sqrt{26}} = \frac{F_{AB}}{\sqrt{29}}$$

$F_{AB} = 3096.5$ lb; $F_{AB} = 3.10$ kips C ◀

$F_{AC} = 2931.9$ lb; $F_{AC} = 2.93$ kips T ◀

PROBLEM 6.13 CONTINUED

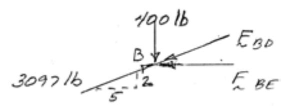
Joint C:



By inspection: $F_{BC} = 0$ ◀

$F_{CE} = 2.93\text{ kips}\ \ \text{T}$ ◀

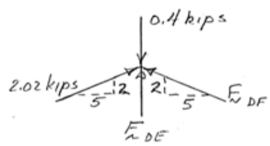
Joint B:



$+\uparrow \Sigma F_y = 0{:}\ \dfrac{2}{\sqrt{29}}(3097\text{ lb}) - 400\text{ lb} - \dfrac{2}{\sqrt{29}}F_{BD} = 0$ $\quad F_{BD} = 2020.0\text{ lb};\ F_{BD} = 2.02\text{ kips}\ \ \text{C}$ ◀

$\xrightarrow{+} \Sigma F_x = 0{:}\ \dfrac{5}{\sqrt{29}}(3097 - 2020)\text{lb} - F_{BE} = 0$ $\quad F_{BE} = 1000.0\text{ lb};\ F_{BE} = 1.000\text{ kip}\ \ \text{C}$ ◀

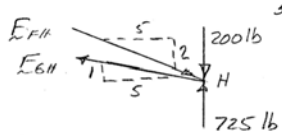
Joint D:

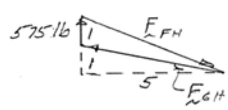


$\xrightarrow{+} \Sigma F_x = 0{:}\ \dfrac{5}{\sqrt{29}}(2.02\text{ kips} - F_{DF}) = 0$ $\quad F_{DF} = 2.02\text{ kips}\ \ \text{C}$ ◀

$+\uparrow \Sigma F_y = 0{:}\ F_{DE} + 2\dfrac{2}{\sqrt{29}}(2.02\text{ kips}) - 0.4\text{ kips} = 0$ $\quad F_{DE} = 1.100\text{ kips}\ \ \text{C}$ ◀

Joint H:



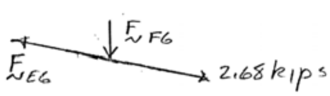


$$\frac{525\text{ lb}}{1} = \frac{F_{FH}}{\sqrt{29}} = \frac{F_{GH}}{\sqrt{26}}$$

$F_{FH} = 2827\text{ lb};\ F_{FH} = 2.83\text{ kips}\ \ \text{C}$ ◀

$F_{GH} = 2677\text{ lb};\ F_{GH} = 2.68\text{ kips}\ \ \text{T}$ ◀

Joint G:

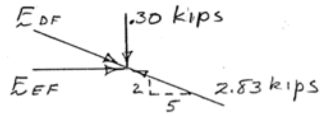


By inspection: $F_{EG} = 2.68\text{ kips}\ \ \text{T}$ ◀

$F_{FG} = 0$ ◀

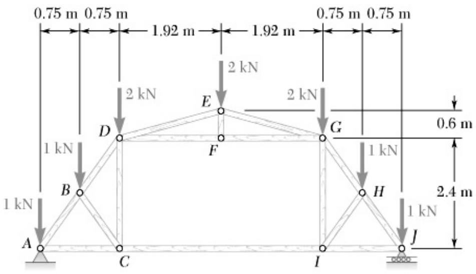
PROBLEM 6.13 CONTINUED

Joint *F*:



$$+\uparrow \Sigma F_y = 0: \frac{2}{\sqrt{29}}\left(2.83 \text{ kips} - F_{DF}\right) - 0.3 \text{ kips} = 0 \qquad F_{DF} = 2.02 \text{ kips}$$

$$\xrightarrow{+} \Sigma F_x = 0: F_{EF} + \frac{5}{\sqrt{29}}\left(2.02 \text{ kips} - 2.827 \text{ kips}\right) = 0 \qquad F_{EF} = 0.750 \text{ kips C} \blacktriangleleft$$

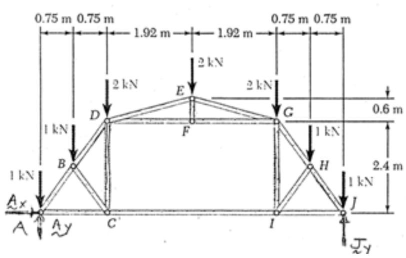


PROBLEM 6.14

Determine the force in each member of the Gambrel roof truss shown. State whether each member is in tension or compression.

SOLUTION

FBD Truss:



$$\rightarrow \Sigma F_x = 0 \qquad \mathbf{A}_x = 0$$

By symmetry: $\mathbf{A}_y = \mathbf{J}_y = 5 \text{ kN} \uparrow$

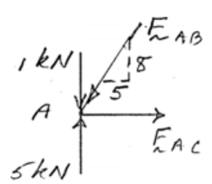
and

$$F_{AB} = F_{HJ};\ F_{AC} = F_{IJ};\ F_{BD} = F_{GH}$$
$$F_{CD} = F_{GI};\ F_{DE} = F_{EG};\ F_{DF} = F_{FG}$$
$$F_{BC} = F_{HI}$$

By inspection of joint F: $F_{EF} = 0$ ◀

Joint FBDs:

Joint A:



$$\uparrow \Sigma F_y = 0:\ 5 \text{ kN} - 1 \text{ kN} - \frac{8}{\sqrt{89}} F_{AB} = 0 \qquad F_{AB} = \frac{\sqrt{89}}{2} \text{ kN}$$

$F_{AB} = 4.72 \text{ kN C}$ ◀

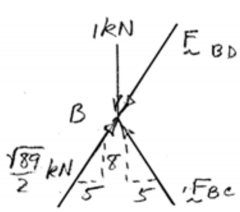
$$\rightarrow \Sigma F_x = 0:\ F_{AC} - \frac{5}{\sqrt{89}} \frac{\sqrt{89}}{2} \text{ kN} = 0$$

$F_{AC} = 2.50 \text{ kN T}$ ◀

so $F_{HJ} = 4.72 \text{ kN C}$ ◀

$F_{IJ} = 2.50 \text{ kN T}$ ◀

Joint B:



$$\rightarrow \Sigma F_x = 0:\ \frac{5}{\sqrt{89}} \left(\frac{\sqrt{89}}{2} \text{ kN} - F_{BD} - F_{BC} \right) = 0$$

$$\uparrow \Sigma F_y = 0:\ \frac{8}{\sqrt{89}} \left(\frac{\sqrt{89}}{2} \text{ kN} - F_{BD} + F_{BC} \right) - 1 \text{ kN} = 0$$

Solving: $F_{BD} = 4.127 \text{ kN}$ so $F_{BD} = 4.13 \text{ kN C}$ ◀

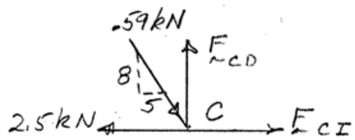
$F_{AB} = 0.5896 \text{ kN}$ and $F_{BC} = 0.590 \text{ kN C}$ ◀

so $F_{GH} = 4.13 \text{ kN C}$ ◀

and $F_{HI} = 0.590 \text{ kN C}$ ◀

PROBLEM 6.14 CONTINUED

Joint *C*:



$$\xrightarrow{} \Sigma F_x = 0:\ F_{CI} + \frac{5}{\sqrt{89}}(.59\text{ kN}) - 2.5\text{ kN} = 0; \qquad F_{CI} = 2.187\text{ kN}$$

$F_{CI} = 2.19\text{ kN T}$ ◀

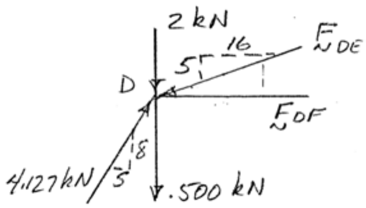
$$\uparrow \Sigma F_y = 0:\ F_{CD} - \frac{8}{\sqrt{89}}(.59\text{ kN}) = 0 \qquad F_{CD} = 0.500\text{ kN T}$$ ◀

so $F_{GI} = 0.500\text{ kN T}$ ◀

$$\uparrow \Sigma F_y = 0:\ \frac{8}{\sqrt{89}}(4.127\text{ kN}) - 2.5\text{ kN} - \frac{5}{\sqrt{281}}F_{DE} = 0$$

$$F_{DE} = 3.352\text{ kN}$$

Joint *D*:



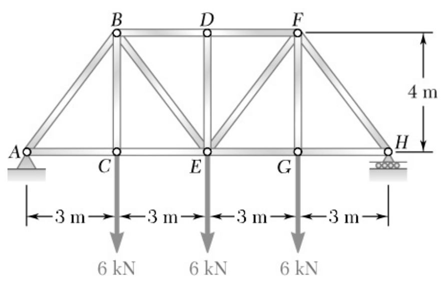
so $F_{DE} = 3.35\text{ kN C}$ ◀

and $F_{EG} = 3.35\text{ kN C}$ ◀

$$\xrightarrow{} \Sigma F_x = \frac{5}{\sqrt{89}}(4.127\text{ kN}) - \frac{16}{\sqrt{281}}(3.352\text{ kN}) + F_{DF} = 0$$

$F_{DF} = 1.012\text{ kN T}$ ◀

$F_{FG} = 1.012\text{ kN T}$ ◀

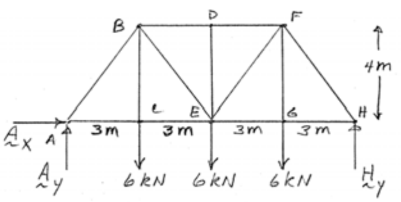


PROBLEM 6.15

Determine the force in each member of the Pratt bridge truss shown. State whether each member is in tension or compression.

SOLUTION

FBD Truss:



$\xrightarrow{} \Sigma F_x = 0: \quad \mathbf{A}_x = 0$

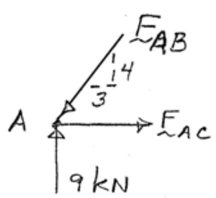
By symmetry: $\mathbf{A}_y = \mathbf{H}_y = 9 \text{ kN} \uparrow$

and

$$F_{AB} = F_{FH};\ F_{AC} = F_{GH}$$
$$F_{BC} = F_{FG};\ F_{BD} = F_{DF}$$
$$F_{BE} = F_{EF};\ F_{CE} = F_{EG}$$

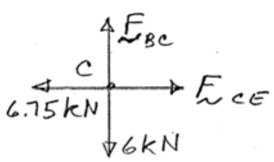
By inspection of joint D: $\qquad F_{DE} = 0$ ◀

FBDs Joints:



$+\uparrow \Sigma F_y = 0: 9 \text{ kN} - \frac{4}{5}F_{AB} = 0 \qquad F_{AB} = 11.25 \text{ kN C}$ ◀

$\xrightarrow{} \Sigma F_x = 0: F_{AC} - \frac{3}{5}F_{AB} = 0 \qquad F_{AC} = 6.75 \text{ kN T}$ ◀

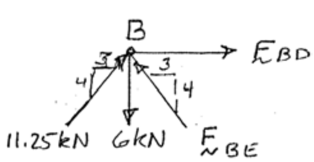


$\xrightarrow{} \Sigma F_x = 0: F_{CE} - 6.75 \text{ kN} = 0 \qquad F_{CE} = 6.75 \text{ kN T}$ ◀

$+\uparrow \Sigma F_y = 0: F_{BC} - 6 \text{ kN} = 0 \qquad F_{BC} = 6.00 \text{ kN T}$ ◀

$+\uparrow \Sigma F_y = 0: \frac{4}{5}(11.25 \text{ kN}) - 6 \text{ kN} + \frac{4}{5}F_{BE} = 0$

$F_{BE} = 3.75 \text{ kN C}$ ◀



$\xrightarrow{} \Sigma F_x = 0: F_{BD} - \frac{3}{5}(11.25 \text{ kN}) - \frac{3}{5}(3.75 \text{ kN}) = 0$

$F_{BD} = 9.00 \text{ kN T}$ ◀

From symmetry conditions above $\qquad F_{FH} = 11.25 \text{ kN C}$ ◀

$F_{GH} = 6.75 \text{ kN T}$ ◀

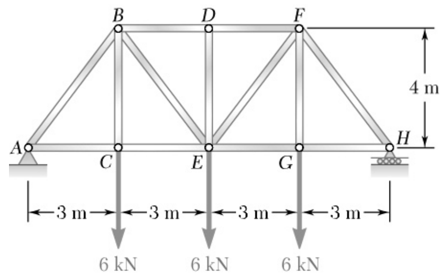
$F_{EG} = 6.75 \text{ kN T}$ ◀

$F_{FG} = 6.00 \text{ kN T}$ ◀

$F_{EF} = 3.75 \text{ kN C}$ ◀

$F_{DF} = 9.00 \text{ kN T}$ ◀

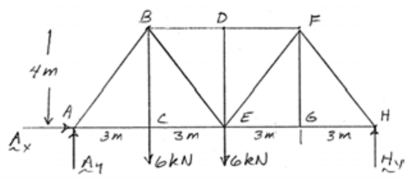
PROBLEM 6.16



Determine the force in each member of the Pratt bridge truss shown. State whether each member is in tension or compression. Assume that the load at G has been removed.

SOLUTION

FBD Truss:



$$\rightarrow \Sigma F_x = 0: \mathbf{A}_x = 0$$

$$\Sigma M_A = 0: (12\text{ m})H_y - (6\text{ m})(6\text{ kN}) - (3\text{ m})(6\text{ kN}) = 0$$

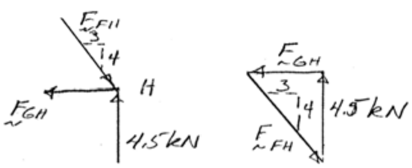
$$\mathbf{H}_y = 4.5\text{ kN} \uparrow$$

$$\uparrow \Sigma F_y = 0: A_y - 6\text{ kN} - 6\text{ kN} + 4.5\text{ kN} = 0$$

$$\mathbf{A}_y = 7.5\text{ kN} \uparrow$$

Joint FBDs:

Joint H:



$$\frac{4.5\text{ kN}}{4} = \frac{F_{GH}}{3} = \frac{F_{FH}}{5}$$

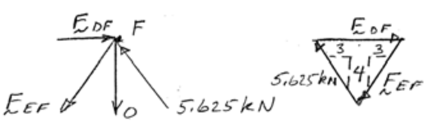
$F_{GH} = 3.375\text{ kN}$ $\qquad F_{GH} = 3.38\text{ kN T}$ ◀

$F_{FH} = 5.625\text{ kN}$ $\qquad F_{FH} = 5.63\text{ kN C}$ ◀

By inspection of joint G: $\qquad F_{FG} = 0$ ◀

$F_{EG} = F_{GH} = 3.38\text{ kN T}$ ◀

Joint F:



$$\frac{5.625\text{ kN}}{5} = \frac{F_{EF}}{5} = \frac{F_{DF}}{6}$$

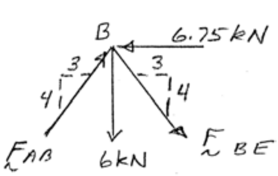
$F_{EF} = 5.63\text{ kN T}$ ◀

$F_{DF} = 6.75\text{ kN C}$ ◀

By inspection of joint D: $\qquad F_{DE} = 0$ ◀

$F_{BD} = F_{DF} = 6.75\text{ kN C}$ ◀

Joint B:



By inspection of joint C: $F_{AC} = F_{CE}$ $\qquad$ and $F_{BC} = 6.00\text{ kN T}$ ◀

$$\rightarrow \Sigma F_x = 0: \frac{3}{5}(F_{AB} + F_{BE}) - 6.75\text{ kN} = 0$$

$$\uparrow \Sigma F_y = 0: \frac{4}{5}(F_{AB} - F_{BE}) - 6\text{ kN} = 0$$

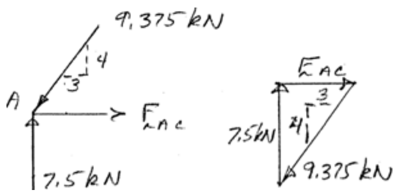
$$\Sigma F_y = 0: \frac{4}{5}(F_{AB} - F_{BE}) - 6\text{ kN} = 0$$

PROBLEM 6.16 CONTINUED

Solving: $F_{AB} = 9.375 \text{ kN}$ so $F_{AB} = 9.38 \text{ kN}$ C ◀

$F_{BE} = 1.875 \text{ kN}$ $F_{BE} = 1.875 \text{ kN}$ T ◀

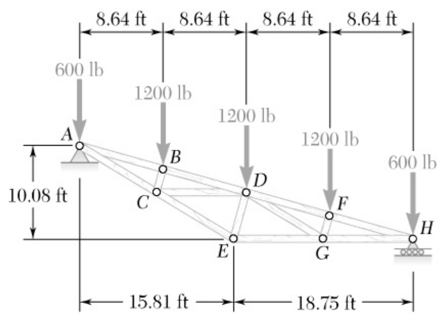
Joint A:



$$\frac{F_{AC}}{3} = \frac{7.5 \text{ kN}}{4} = \frac{9.375 \text{ kN}}{5} \qquad F_{AC} = 5.625 \text{ kN}$$

$F_{AC} = 5.63 \text{ kN}$ T ◀

From above $F_{CE} = 5.63 \text{ kN}$ T ◀

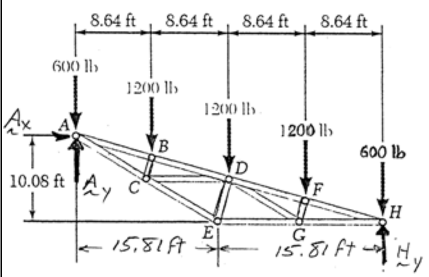


PROBLEM 6.17

Determine the force in member *DE* and in each of the members located to the left of *DE* for the inverted Howe roof truss shown. State whether each member is in tension or compression.

SOLUTION

FBD Truss:



$$\rightarrow \Sigma F_x = 0: \mathbf{A}_x = 0$$

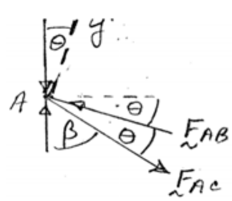
By load symmetry $\mathbf{A}_y = \mathbf{H}_y = 2400 \text{ lb} \uparrow$

Note: $$\theta = \tan^{-1}\frac{10.08}{15.81 + 18.75} = 16.26°$$

$$\beta = 90 - 2\theta = 57.48°; \ \alpha = 180 - \beta = 32.52°$$

Joint FBDs:

Joint *A*:



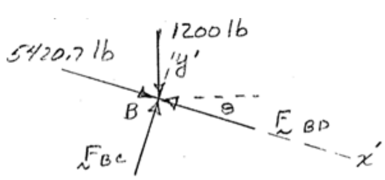
$$\nearrow \Sigma F_{y'} = 0: (2400 \text{ lb} - 600 \text{ lb})\cos\theta - F_{AC}\sin\theta = 0$$

$$F_{AC} = \frac{(1800 \text{ lb})}{\tan 16.26°} = 6171.5 \text{ lb}; \qquad F_{AC} = 6.17 \text{ kips T} \blacktriangleleft$$

$$\searrow \Sigma F_x = 0: (6171.5 \text{ lb})\cos 2\theta - F_{AB}\cos\theta = 0$$

$$F_{AB} = 6171.5\frac{\cos 32.52°}{\cos 16.26°} = 5420.7 \text{ lb}; \qquad F_{AB} = 5.42 \text{ kips C} \blacktriangleleft$$

Joint *B*:



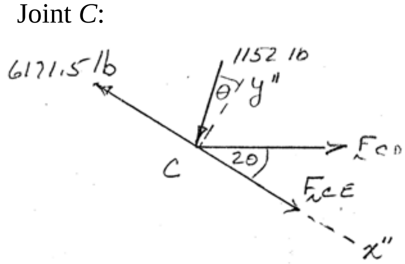
$$\searrow \Sigma F_{x'} = 0: 5420.7 \text{ lb} + (1200 \text{ lb})\sin\theta - F_{BD} = 0$$

$$F_{BD} = 5420.7 + 1200\sin 16.26° = 5756.7 \text{ lb} \qquad F_{BD} = 5.76 \text{ kips C} \blacktriangleleft$$

$$\nearrow \Sigma F_{y'} = 0: F_{BC} - (1200 \text{ lb})\cos\theta = 0 \qquad F_{BC} = 1152 \text{ lb}$$

$$F_{BC} = 1.152 \text{ kips C} \blacktriangleleft$$

Joint *C*:



$$\nearrow \Sigma F_{y''} = 0: F_{CD}\sin 2\theta - (1152 \text{ lb})\cos\theta = 0$$

$$F_{CD} = 1152\frac{\cos 16.26°}{\sin 32.52°} = 2057.2 \text{ lb} \qquad F_{CD} = 2.06 \text{ kips T} \blacktriangleleft$$

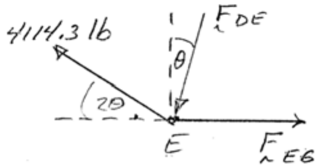
$$\searrow \Sigma F_{x''} = 0: F_{CE} + (1152 \text{ lb})\sin\theta + (2057.2 \text{ lb})\cos 2\theta - 6171.5 \text{ lb} = 0$$

PROBLEM 6.17 CONTINUED

$$F_{CE} = 6171.5 - 1152\sin 16.26° - 2057.2\cos 32.52°$$

$$= 4114.3 \text{ lb} \qquad\qquad F_{CE} = 4.11 \text{ kips T} ◀$$

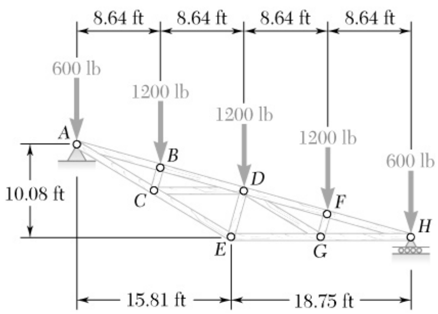
Joint E:



$$+\uparrow \Sigma F_y = 0: (4114.3 \text{ lb})\sin 2\theta - F_{DE}\cos\theta = 0$$

$$F_{DE} = 4114.3\frac{\sin 32.52°}{\cos 16.26°} = 2304.0 \text{ lb} \qquad F_{DE} = 2.30 \text{ kips C} ◀$$

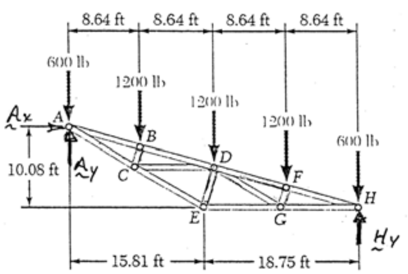
PROBLEM 6.18



Determine the force in each of the members located to the right of *DE* for the inverted Howe roof truss shown. State whether each member is in tension or compression.

SOLUTION

FBD Truss:



$$\xrightarrow{+} \Sigma F_x = 0: \mathbf{A}_x = 0$$

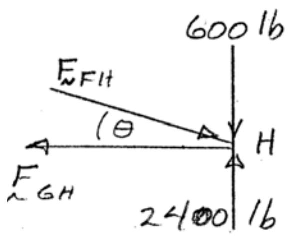
By symmetry of loads $\mathbf{A}_y = \mathbf{H}_y = 2400 \text{ lb} \uparrow$

Note: $$\theta = \tan^{-1} \frac{10.08}{15.81 + 18.75} = 16.26°$$

$$\beta = 90 - 2\theta = 57.48°$$

Joint FBDs:

Joint *H*:



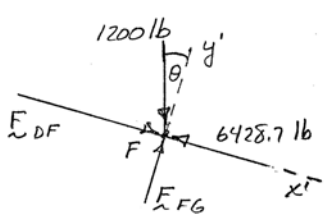
$$+\uparrow \Sigma F_y = 0: 2400 \text{ lb} - 600 \text{ lb} - F_{FH} \sin\theta = 0$$

$$F_{FH} = \frac{1800 \text{ lb}}{\sin 16.26°} = 6428.7 \text{ lb} \qquad F_{FH} = 6.43 \text{ kips C} \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: (6428.7 \text{ lb})\cos\theta - F_{GH} = 0$$

$$F_{GH} = 6428.7 \cos 16.32° = 6171.5 \text{ lb} \qquad F_{GH} = 6.17 \text{ kips T} \blacktriangleleft$$

Joint *F*:



$$\nearrow \Sigma F_{y'} = 0: F_{FG} - (1200 \text{ lb})\cos\theta = 0 \qquad F_{FG} = 1152.0 \text{ lb}$$

$$F_{FG} = 1.152 \text{ kips C} \blacktriangleleft$$

$$\searrow \Sigma F_{x'} = 0: F_{DF} + (1200 \text{ lb})\sin\theta - 6428.7 \text{ lb} = 0$$

$$F_{DF} = 6428.7 - 1200 \sin 16.26° = 6092.7 \text{ lb} \qquad F_{DF} = 6.09 \text{ kips C} \blacktriangleleft$$

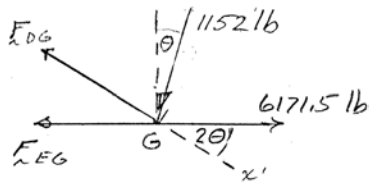
PROBLEM 6.18 CONTINUED

Joint G:

$$\uparrow \Sigma F_y = 0: F_{DG} \sin 2\theta - (1152 \text{ lb}) \cos\theta = 0$$

$$F_{DG} = 1152 \frac{\cos 16.26°}{\sin 32.52°} = 2057.2 \text{ lb}$$

$F_{DG} = 2.06$ kips T ◀

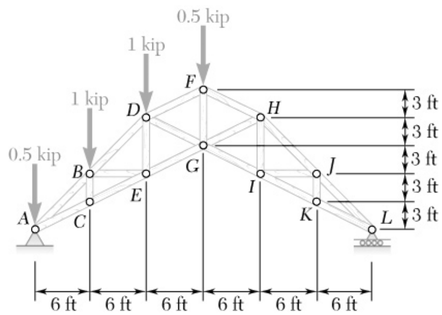


$$\rightarrow \Sigma F_x = 0: 6171.5 \text{ lb} - 2057.2 \cos 2\theta - F_{EG} - 1152 \text{ lb} \sin\theta = 0$$

$$F_{EG} = 6171.5 - 2057.2 \cos 32.52° - (1152 \text{ lb}) \sin 16.26° = 4114.3 \text{ lb}$$

$F_{EG} = 4.11$ kips T ◀

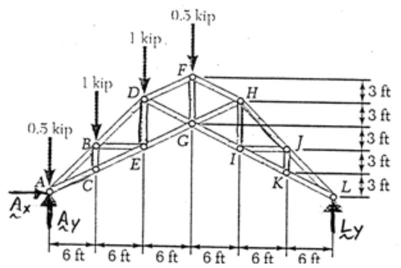
PROBLEM 6.19



Determine the force in each of the members located to the left of member *FG* for the scissor roof truss shown. State whether each member is in tension or compression.

SOLUTION

FBD Truss:



$$\circlearrowleft \Sigma M_A = 0\text{: } (6\text{ ft})\left[6L_y - 3(.5\text{ kip}) - 2(1\text{ kip}) - 1(1\text{ kip})\right] = 0$$

$$\mathbf{L}_y = 0.75\text{ kip} \uparrow$$

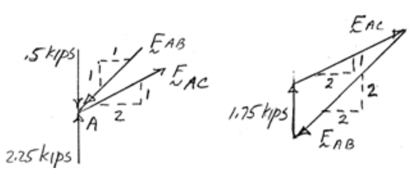
$$\uparrow \Sigma F_y = 0\text{: } A_y - 0.5\text{ kip} - 1\text{ kip} - 1\text{ kip} - 0.5\text{ kip} + 0.75\text{ kip} = 0$$

$$\mathbf{A}_y = 2.25\text{ kips} \uparrow$$

$$\rightarrow \Sigma F_x = 0\text{: } \mathbf{A}_x = 0$$

Joint FBDs:

Joint *A*:



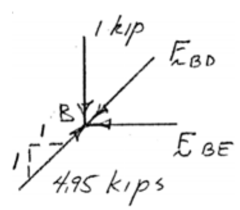
$$\frac{1.75\text{ kips}}{1} = \frac{F_{AC}}{\sqrt{5}} = \frac{F_{AB}}{\sqrt{8}} \qquad F_{AB} = 4.95\text{ kips C} \blacktriangleleft$$

$$F_{AC} = 3.913\text{ kips} \qquad F_{AC} = 3.91\text{ kips T} \blacktriangleleft$$

By inspection of joint *C*: $F_{BC} = 0 \blacktriangleleft$

and $F_{CE} = F_{AC}$ so $F_{CE} = 3.91\text{ kips T} \blacktriangleleft$

Joint *B*:



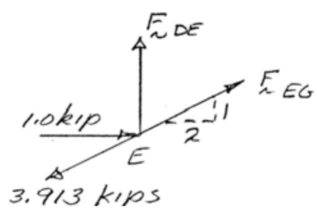
$$\uparrow \Sigma F_y = 0\text{: } \frac{4.95\text{ kips}}{\sqrt{2}} - 1\text{ kip} - \frac{F_{BD}}{\sqrt{2}} = 0$$

$$F_{BD} = 3.536\text{ kips} \qquad F_{BD} = 3.54\text{ kips C} \blacktriangleleft$$

$$\rightarrow \Sigma F_x = 0\text{: } (4.95\text{ kips} - 3.536\text{ kips})\frac{1}{\sqrt{2}} - F_{BE} = 0$$

$$F_{BE} = 1.000\text{ kip} \qquad F_{BE} = 1.000\text{ kip C} \blacktriangleleft$$

Joint *E*:



$$\rightarrow \Sigma F_x = 0\text{: } \frac{2}{\sqrt{5}}\left[F_{EG} - 3.913\text{ kips}\right] + 1\text{ kip} = 0$$

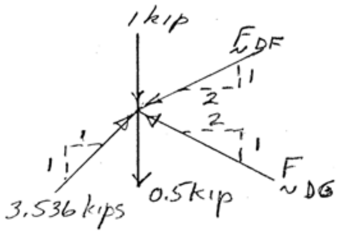
$$F_{EG} = 2.795\text{ kips} \qquad F_{EG} = 2.80\text{ kips T} \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0\text{: } \frac{1}{\sqrt{5}}(2.795\text{ kips} - 3.913\text{ kips}) + F_{DE} = 0$$

$$F_{DE} = 0.500\text{ kip T} \blacktriangleleft$$

PROBLEM 6.19 CONTINUED

Joint D:

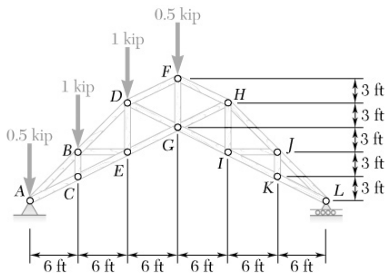


$$\xrightarrow{} \Sigma F_x = 0: \frac{1}{\sqrt{2}}(3.536\text{ kips}) - \frac{2}{\sqrt{5}}(F_{DF} + F_{DG}) = 0$$

$$\uparrow \Sigma F_y = 0: \frac{1}{\sqrt{2}}(3.536\text{ kips}) - 1.5\text{ kips} + \frac{1}{\sqrt{5}}(F_{DG} - F_{DF}) = 0$$

Solving: $F_{DF} = 2.516\text{ kips}$ so $F_{DF} = 2.52\text{ kips C}$ ◀

$F_{DG} = 0.280\text{ kip}$ $F_{DG} = 0.280\text{ kip C}$ ◀

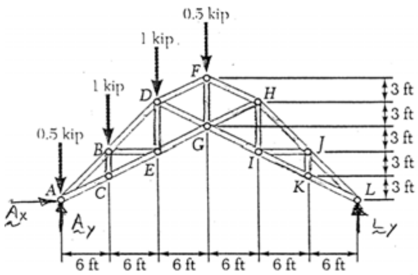


PROBLEM 6.20

Determine the force in member *FG* and in each of the members located to the right of member *FG* for the scissor roof truss shown. State whether each member is in tension or compression.

SOLUTION

FBD Truss:



$$\circlearrowleft \Sigma M_A = 0\text{: } (6\text{ ft})\left[6L_y - 3(0.5\text{ kip}) - 2(1\text{ kip}) - 1(1\text{ kip})\right] = 0$$

$$\mathbf{L}_y = 0.75\text{ kip} \uparrow$$

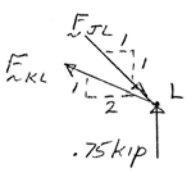
Inspection of joints K, J, and I, in order, shows that $F_{JK} = 0$ ◀

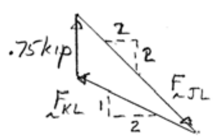
$F_{IJ} = 0$ ◀

$F_{HI} = 0$ ◀

and that $F_{IK} = F_{KL}$; $F_{HJ} = F_{JL}$ and $F_{GI} = F_{IK}$

Joint FBDs:



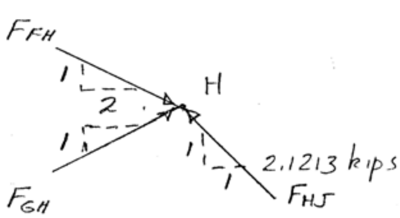


$$\frac{0.75}{1} = \frac{F_{JL}}{\sqrt{8}} = \frac{F_{KL}}{\sqrt{5}} \qquad F_{JL} = 2.1213\text{ kips} \qquad F_{JL} = 2.12\text{ kips C} ◀$$

$$F_{KL} = 1.6771\text{ kips} \qquad F_{KL} = 1.677\text{ kips T} ◀$$

and, from above: $F_{HJ} = 2.12$ kips C ◀

$F_{GI} = F_{IK} = 1.677$ kips T ◀



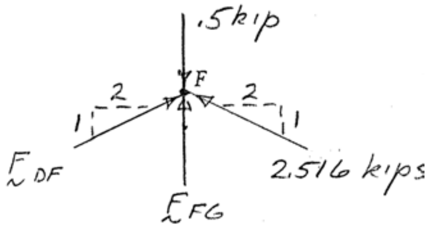
$$\xrightarrow{} \Sigma F_x = 0\text{: } \frac{2}{\sqrt{5}}\left(F_{FH} + F_{GH}\right) - \frac{1}{\sqrt{2}}(2.1213\text{ kips}) = 0$$

$$\uparrow \Sigma F_y = 0\text{: } \frac{1}{\sqrt{5}}\left(F_{GH} + F_{FH}\right) + \frac{1}{\sqrt{2}}(2.1213\text{ kips}) = 0$$

Solving: $F_{FH} = 2.516$ kips $\qquad F_{FH} = 2.52$ kips C ◀

$F_{GH} = -0.8383$ kips $\qquad F_{GH} = 0.838$ kips T ◀

PROBLEM 6.20 CONTINUED

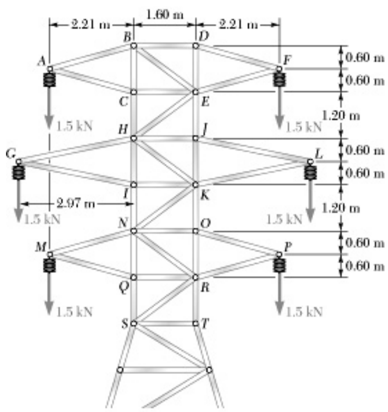


$\xrightarrow{} \Sigma F_x = 0: \frac{2}{\sqrt{5}}\left(F_{DF} - 2.516 \text{ kips}\right) = 0 \qquad F_{DF} = 2.52 \text{ kips C}$

$\uparrow \Sigma F_y = 0: F_{FG} - 0.5 \text{ kip} + \frac{1}{\sqrt{5}}(2)(2.516 \text{ kips}) = 0$

$F_{FG} = 1.750 \text{ kips T}$ ◀

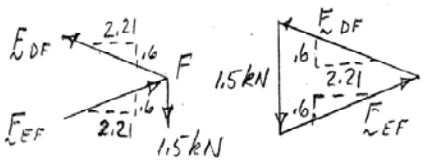
PROBLEM 6.21



The portion of truss shown represents the upper part of a power transmission line tower. For the given loading, determine the force in each of the members located above *HJ*. State whether each member is in tension or compression.

SOLUTION

Joint FBDs:



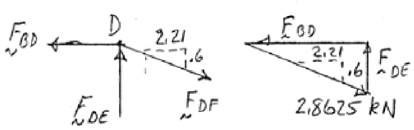
$$\frac{F_{DF}}{2.29} = \frac{F_{EF}}{2.29} = \frac{1.5 \text{ kN}}{1.2}$$

$F_{DF} = F_{EF} = 2.8625 \text{ kN}$ $\quad$ $F_{DF} = 2.86 \text{ kN T}$ ◀

$F_{EF} = 2.86 \text{ kN C}$ ◀

$$\frac{F_{BD}}{2.21} = \frac{F_{DE}}{0.6} = \frac{2.8625 \text{ kN}}{2.29} = 1.25 \text{ kN}$$

$F_{BD} = 2.7625 \text{ kN}$ $\quad$ $F_{BD} = 2.76 \text{ kN T}$ ◀

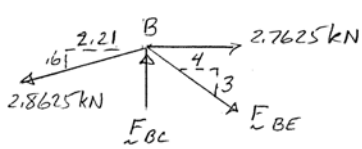


$F_{DE} = 0.750 \text{ kN C}$ ◀

By symmetry of joint *A* vs. joint *F* $\quad$ $F_{AB} = 2.86 \text{ kN T}$ ◀

$F_{AC} = 2.86 \text{ kN T}$ ◀

$$\rightarrow \Sigma F_x = 0: 2.7625 \text{ kN} - \frac{2.21}{2.29}(2.8625 \text{ kN}) + \frac{4}{5}F_{BE} = 0$$



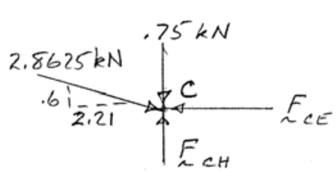
$F_{BE} = 0$ ◀

$$\uparrow \Sigma F_y = 0: F_{BC} - \frac{0.6}{2.29}(2.8625 \text{ kN}) = 0;$$

$F_{BC} = 0.750 \text{ kN C}$ ◀

$$\rightarrow \Sigma F_x = 0: \frac{2.21}{2.29}(2.8625 \text{ kN}) - F_{CE} = 0 \qquad F_{CE} = 2.7625 \text{ kN}$$

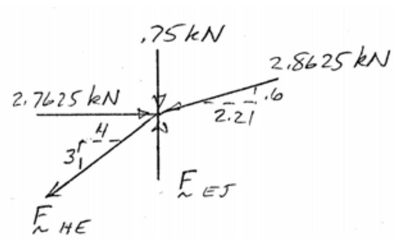
$F_{CE} = 2.76 \text{ kN C}$ ◀



$$\uparrow \Sigma F_y = 0: F_{CH} - 0.75 \text{ kN} - \frac{0.6}{2.21}(2.8625 \text{ kN}) = 0$$

$F_{CH} = 1.500 \text{ kN C}$ ◀

PROBLEM 6.21 CONTINUED



$$\xrightarrow{} \Sigma F_x = 0\text{: } 2.7625\text{ kN} - \frac{2.21}{2.29}(2.8625\text{ kN}) - \frac{4}{5}F_{HE} = 0$$

$$F_{HE} = 0 \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0\text{: } F_{EJ} - 0.75\text{ kN} - \frac{0.6}{2.29}(2.8625\text{ kN}) = 0$$

$$F_{EJ} = 1.500\text{ kN} \blacktriangleleft$$

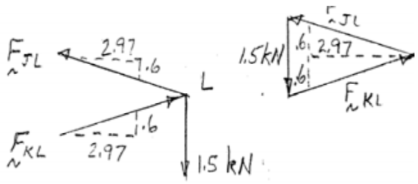
PROBLEM 6.22

For the tower and loading of Prob. 6.21 and knowing that $F_{CH} = F_{EJ} = 1.5$ kN C and $F_{EH} = 0$, determine the force in member *HJ* and in each of the members located between *HJ* and *NO*. State whether each member is in tension or compression.

SOLUTION

$$\frac{1.5 \text{ kN}}{1.2} = \frac{F_{JL}}{3.03} = \frac{F_{KL}}{3.03}$$

Joint FBDs:



$F_{JL} = F_{KL} = 3.7875$ kN $\qquad F_{JL} = 3.79$ kN T ◀

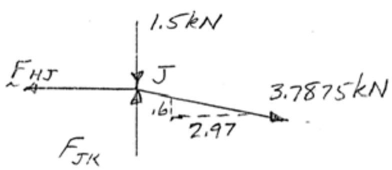
$F_{KL} = 3.79$ kN C ◀

By symmetry: $\qquad F_{GH} = 3.79$ kN T ◀

$F_{GI} = 3.79$ kN C ◀

$$\xrightarrow{} \Sigma F_x = 0: \frac{2.97}{3.03}(3.7875 \text{ kN}) - F_{HJ} = 0$$

$F_{HJ} = 3.7125$ kN $\qquad F_{HJ} = 3.71$ kN T ◀



$$\uparrow \Sigma F_y = 0: F_{JK} - \frac{0.6}{3.03}(3.7875 \text{ kN}) - 1.5 \text{ kN} = 0$$

$F_{JK} = 2.25$ kN C ◀

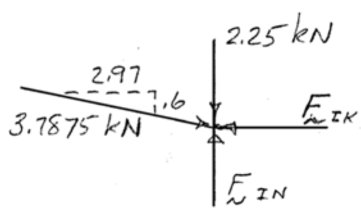
Knowing $F_{HE} = 0$; by symmetry $\qquad F_{HK} = 0$ ◀

$F_{HI} = 2.25$ kN C ◀

$$\xrightarrow{} \Sigma F_x = 0: \frac{2.97}{3.03}(3.7875 \text{ kN}) - F_{IK} = 0 \qquad F_{IK} = 3.7125$$

$F_{IK} = 3.71$ kN C ◀

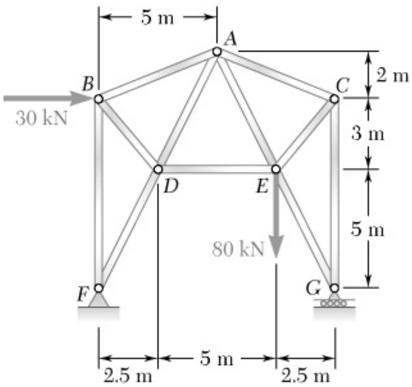
$$\uparrow \Sigma F_y = 0: F_{IN} - 2.25 \text{ kN} - \frac{0.6}{3.03}(3.7875 \text{ kN}) = 0$$



$F_{IN} = 3.00$ kN C ◀

Knowing that $F_{HK} = 0$, by symmetry $\qquad F_{KO} = 3.00$ kN C ◀

$F_{KN} = 0$ ◀

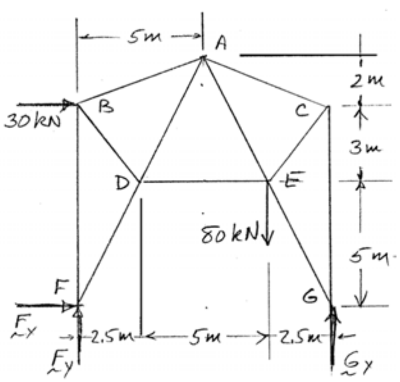


PROBLEM 6.23

Determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

FBD Truss:



$$\circlearrowleft \Sigma M_F = 0: (10 \text{ m})G_y - (7.5 \text{ m})(80 \text{ kN}) - (8 \text{ m})(30 \text{ kN}) = 0$$

$$\mathbf{G}_y = 84 \text{ kN} \uparrow$$

$$\rightarrow \Sigma F_x = 0: -F_x + 30 \text{ kN} = 0 \qquad \mathbf{F}_x = 30 \text{ kN} \leftarrow$$

$$\uparrow \Sigma F_y = 0: F_y + 84 \text{ kN} - 80 \text{ kN} = 0 \qquad \mathbf{F}_y = 4 \text{ kN} \downarrow$$

By inspection of joint G: $F_{EG} = 0$ ◀

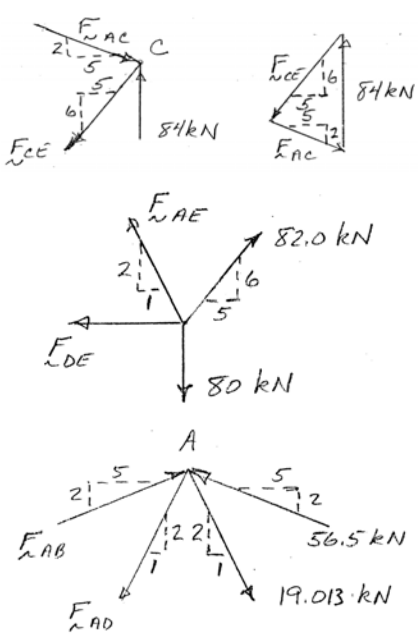
$F_{CG} = 84 \text{ kN}$ C ◀

$$\frac{84 \text{ kN}}{8} = \frac{F_{CE}}{\sqrt{61}} = \frac{F_{AC}}{\sqrt{29}} = 10.5 \text{ kN}$$

$F_{CE} = 82.0 \text{ kN}$ T ◀

$F_{AC} = 56.5 \text{ kN}$ C ◀

Joint FBDs:



$$\uparrow \Sigma F_y = 0: \frac{2}{\sqrt{5}} F_{AE} + \frac{6}{\sqrt{61}}(82.0 \text{ kN}) - 80 \text{ kN} = 0$$

$F_{AE} = 19.01312$ $\qquad F_{AE} = 19.01 \text{ kN}$ T ◀

$$\rightarrow \Sigma F_x = 0: -F_{DE} - \frac{1}{\sqrt{5}}(19.013 \text{ kN}) + \frac{5}{\sqrt{61}}(82.0 \text{ kN}) = 0$$

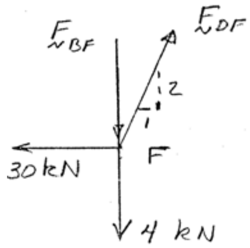
$F_{DE} = 43.99 \text{ kN}$ $\qquad F_{DE} = 44.0 \text{ kN}$ T ◀

$$\rightarrow \Sigma F_x = 0: \frac{1}{\sqrt{5}} F_{DF} - 30 \text{ kN} = 0$$

$F_{DF} = 67.082 \text{ kN}$ $\qquad F_{DF} = 67.1 \text{ kN}$ T ◀

PROBLEM 6.23 CONTINUED

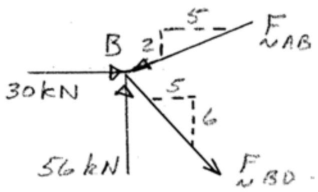
$$+\uparrow \Sigma F_y = 0: \frac{2}{\sqrt{5}}(67.082\text{ kN}) - F_{BF} - 4\text{ kN} = 0$$



$F_{BF} = 56.00\text{ kN}$ $\qquad F_{BF} = 56.0\text{ kN C}$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: 30\text{ kN} + \frac{5}{\sqrt{61}}F_{BD} - \frac{5}{\sqrt{29}}F_{AB} = 0$$

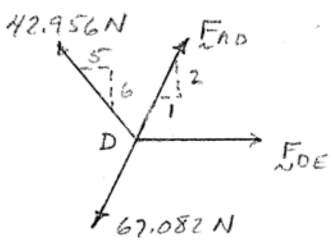
$$+\uparrow \Sigma F_y = 0: 56\text{ kN} - \frac{6}{\sqrt{61}}F_{BD} - \frac{3}{\sqrt{29}}F_{AB} = 0$$



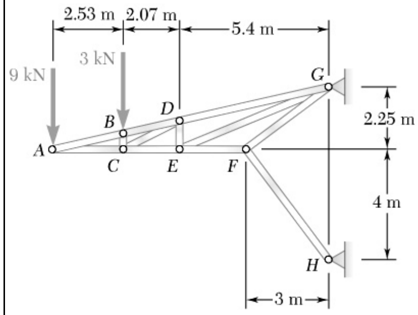
Solving: $F_{BD} = 42.956\text{ kN}$ $\qquad F_{BD} = 43.0\text{ kN T}$ ◀

$F_{AB} = 61.929\text{ kN}$ $\qquad F_{AB} = 61.9\text{ kN C}$ ◀

$$+\uparrow \Sigma F_y = 0: \frac{6}{\sqrt{61}}(42.956\text{ N}) + \frac{2}{\sqrt{5}}(F_{AD} - 67.082\text{ N}) = 0$$



$F_{AD} = 30.157\text{ kN}$ $\qquad F_{AD} = 30.2\text{ N T}$ ◀

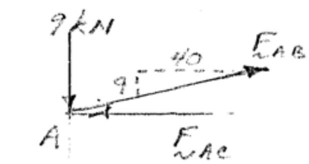


PROBLEM 6.24

Determine the force in each member of the truss shown. State whether each member is in tension or compression.

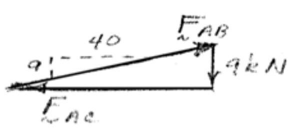
SOLUTION

Joint FBDs:



$$\frac{9\text{ kN}}{9} = \frac{F_{AC}}{40} = \frac{F_{AB}}{41}$$

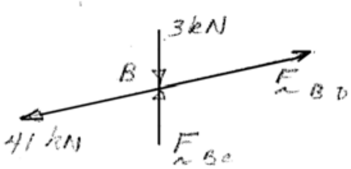
$F_{AB} = 41.0\text{ kN}$ T ◀



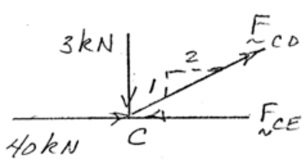
$F_{AC} = 40.0\text{ kN}$ C ◀

By inspection of joint B: $F_{BD} = 41.0\text{ kN}$ T ◀

$F_{BC} = 3.00\text{ kN}$ C ◀

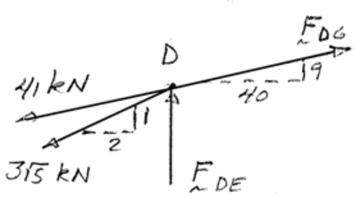


Note: to determine slope of CD: $DE = \frac{4.6}{10.0}(2.25\text{ m}) = 1.035\text{ m}$



$$\uparrow \Sigma F_y = 0{:}\quad \frac{1}{\sqrt{5}}F_{CD} - 3\text{ kN} = 0 \qquad F_{CD} = 3\sqrt{5}\text{ kN} = 6.71\text{ kN T} ◀$$

$$\longrightarrow \Sigma F_x = 0{:}\quad \frac{2}{\sqrt{5}}\left(3\sqrt{5}\text{ kN}\right) + 40\text{ kN} - F_{CE} = 0 \qquad F_{CE} = 46.0\text{ kN C} ◀$$

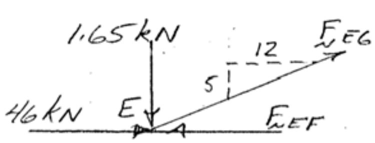


$$\longrightarrow \Sigma F_x = 0{:}\quad \frac{40}{41}\left(F_{DG} - 41\text{ kN}\right) - \frac{2}{\sqrt{5}}\left(3\sqrt{5}\text{ kN}\right) = 0$$

$F_{DG} = 47.15\text{ kN}$ $\qquad$ $F_{DG} = 47.2\text{ kN}$ T ◀

$$\uparrow \Sigma F_y = 0{:}\quad F_{DE} + \frac{9}{41}\left(47.15\text{ kN} - 41\text{ kN}\right) - \frac{1}{\sqrt{5}}\left(3\sqrt{5}\text{ kN}\right) = 0$$

$F_{DE} = 1.650\text{ kN}$ C ◀

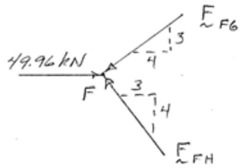


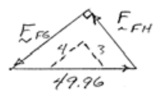
$$\uparrow \Sigma F_y = 0{:}\quad \frac{5}{13}F_{EG} - 1.65\text{ kN} = 0 \qquad F_{EG} = 4.29\text{ kN T} ◀$$

$$\longrightarrow \Sigma F_x = 0{:}\quad 46\text{ kN} + \frac{12}{13}\left(4.29\text{ kN}\right) - F_{EF} = 0 \qquad F_{EF} = 49.96\text{ kN}$$

$F_{EF} = 50.0\text{ kN}$ C ◀

PROBLEM 6.24 CONTINUED



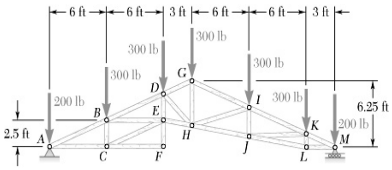


$$\frac{49.96\text{ kN}}{5} = \frac{F_{FG}}{4} = \frac{F_{FH}}{3}$$

$F_{FG} = 40.0\text{ kN}$ C ◀

$F_{FH} = 30.0\text{ kN}$ C ◀

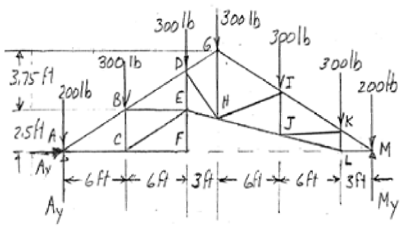
PROBLEM 6.25



For the roof truss shown in Fig. *P6.25* and *P6.26*, determine the force in each of the members located to the left of member *GH*. State whether each member is in tension or compression.

SOLUTION

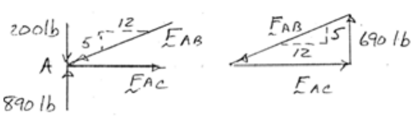
FBD Truss:



$$\circlearrowleft \Sigma M_M = 0: (3\text{ ft})(300\text{ lb}) + (9\text{ ft})(300\text{ lb}) + (15\text{ ft})(300\text{ lb})$$
$$+ (18\text{ ft})(300\text{ lb}) + (24\text{ ft})(300\text{ lb}) + (30\text{ ft})(200\text{ lb})$$
$$-(30\text{ ft})(A_y) = 0 \qquad \mathbf{A}_y = 890\text{ lb} \uparrow$$

$$\rightarrow \Sigma F_x = 0: \ \mathbf{A}_x = 0$$

Joint FBDs:

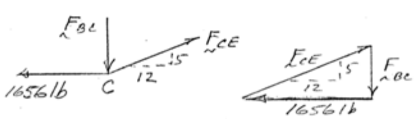


$$\frac{690\text{ lb}}{5} = \frac{F_{AC}}{12} = \frac{F_{AB}}{13} \qquad F_{AB} = 1794\text{ lb C} \blacktriangleleft$$

$$F_{AC} = 1656\text{ lb T} \blacktriangleleft$$

By inspection of joint F: $F_{CF} = 0 \blacktriangleleft$

$$F_{EF} = 0 \blacktriangleleft$$

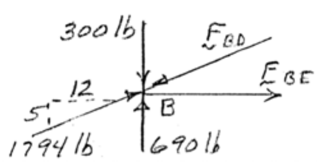


$$\frac{1656\text{ lb}}{12} = \frac{F_{CE}}{13} = \frac{F_{BC}}{5} \qquad F_{CE} = 1794\text{ lb T} \blacktriangleleft$$

$$F_{BC} = 890\text{ lb C} \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \frac{5}{13}(1794\text{ lb} - F_{BD}) + 690\text{ lb} - 300\text{ lb} = 0$$

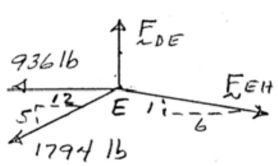
$$F_{BD} = 2808\text{ lb} \qquad F_{BD} = 2.81\text{ kips C} \blacktriangleleft$$



$$\rightarrow \Sigma F_x = 0: F_{BE} + \frac{12}{13}(1794\text{ lb} - 2808\text{ lb}) = 0 \qquad F_{BE} = 936\text{ lb T} \blacktriangleleft$$

$$\rightarrow \Sigma F_x = 0: \frac{6}{\sqrt{37}}F_{EH} - 936\text{ lb} - \frac{12}{13}1794\text{ lb} = 0$$

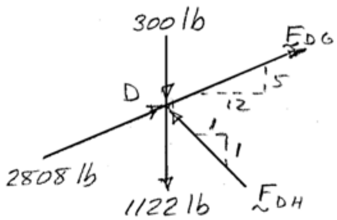
$$F_{EH} = 432\sqrt{37}\text{ lb} \qquad F_{EH} = 2.63\text{ kips T} \blacktriangleleft$$



$$\uparrow \Sigma F_y = 0: F_{DE} - \frac{5}{13}(1794\text{ lb}) - \frac{1}{\sqrt{37}}\left(432\sqrt{37}\text{ lb}\right) = 0$$

$$F_{DE} = 1122\text{ lb T} \blacktriangleleft$$

PROBLEM 6.25 CONTINUED



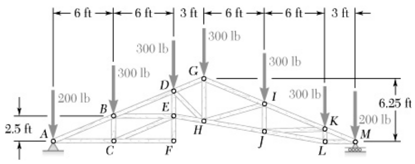
$$\xrightarrow{} \Sigma F_x = 0: \frac{12}{13}\left(2808 \text{ lb} + F_{DG}\right) - \frac{1}{\sqrt{2}}F_{DH} = 0$$

$$+\uparrow \Sigma F_y = 0: \frac{5}{13}\left(2808 \text{ lb} + F_{DG}\right) + \frac{1}{\sqrt{2}}F_{DH} - 300 \text{ lb} - 1122 \text{ lb} = 0$$

Solving:

$F_{DG} = 1721 \text{ lb}$ T ◀

$F_{DH} = 1419 \text{ lb}$ C ◀

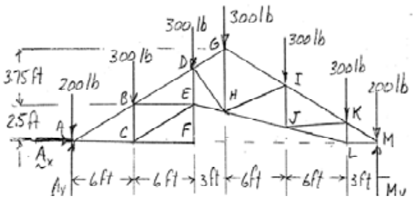


PROBLEM 6.26

Determine the force in member *GH* and in each of the members located to the right of *GH* for the roof truss shown. State whether each member is in tension or compression.

SOLUTION

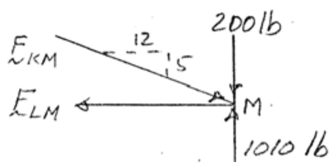
FBD Truss:

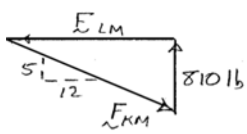


$$\circlearrowleft \Sigma M_A = 0: (30\text{ ft})M_y - (30\text{ ft})(200\text{ lb}) - (27\text{ ft})(300\text{ lb})$$
$$- (21\text{ ft})(300\text{ lb}) - (15\text{ ft})(300\text{ lb})$$
$$- (12\text{ ft})(300\text{ lb}) - (6\text{ ft})(300\text{ lb}) = 0$$

$$\mathbf{M}_y = 1010\text{ lb} \uparrow$$

Joint FBDs:

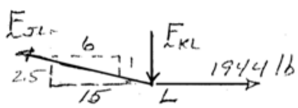


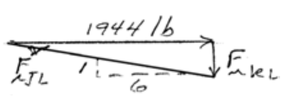


$$\frac{810\text{ lb}}{5} = \frac{F_{LM}}{12} = \frac{F_{KM}}{13}$$

$F_{KM} = 2106$ lb $\qquad F_{KM} = 2.11$ kips C ◀

$F_{LM} = 1944$ lb $\qquad F_{LM} = 1.944$ kips T ◀





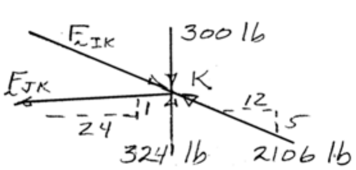
$$\frac{F_{JL}}{\sqrt{37}} = \frac{F_{KL}}{1} = \frac{1944\text{ lb}}{6}$$

$F_{KL} = 324$ lb C ◀

$F_{JL} = 1970.8$ lb $\qquad F_{JL} = 1.971$ kips T ◀

$$\longrightarrow \Sigma F_x = 0: \frac{12}{13}(F_{IK} - 2106\text{ lb}) + \frac{24}{\sqrt{577}}F_{JK} = 0$$

$$\uparrow \Sigma F_y = 0: \frac{5}{13}(-F_{IK} + 2106\text{ lb}) - \frac{1}{\sqrt{577}}F_{JK} + 24\text{ lb} = 0$$

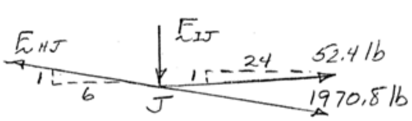


Solving: $F_{IK} = 2162.7$ lb $\qquad F_{IK} = 2.16$ kips C ◀

$F_{JK} = 52.4$ lb T ◀

$$\longrightarrow \Sigma F_x = 0: \frac{6}{\sqrt{37}}(1970.8\text{ lb} - F_{HJ}) + \frac{24}{\sqrt{577}}(52.4\text{ lb}) = 0$$

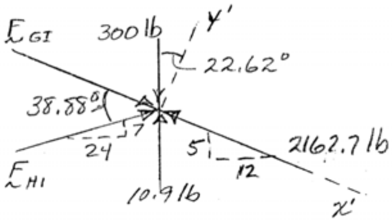
$F_{HJ} = 2024$ lb $\qquad F_{HJ} = 2.02$ kips T ◀



$$\uparrow \Sigma F_y = 0: \frac{1}{\sqrt{37}}(2024\text{ lb} - 1970.8\text{ lb}) + \frac{1}{\sqrt{577}}(52.4\text{ lb}) - F_{IJ} = 0$$

$F_{IJ} = 10.90$ lb C ◀

PROBLEM 6.26 CONTINUED

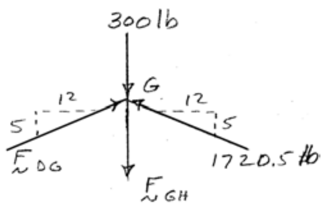


$$\nearrow \Sigma F_{y'} = 0\colon\ F_{HI} \sin 38.88° + (10.9 \text{ lb} - 300 \text{ lb}) \cos 22.62° = 0$$

$$F_{HJ} = 425.1 \text{ lb} \qquad F_{HI} = 425 \text{ lb C} \blacktriangleleft$$

$$\searrow \Sigma F_{x'} = 0\colon\ F_{GI} + F_{HI} \cos 38.88° + (300 \text{ lb} - 10.9 \text{ lb}) \sin 22.62°$$

$$-2162.7 \text{ lb} = 0 \qquad F_{GI} = 1720.5 \text{ lb} = 1.721 \text{ kips C} \blacktriangleleft$$



By symmetry $F_{DG} = 1720.5$ lb

$$\uparrow \Sigma F_y = 0\colon\ \frac{5}{13} 2(1720.5 \text{ lb}) - 300 \text{ lb} - F_{GH} = 0$$

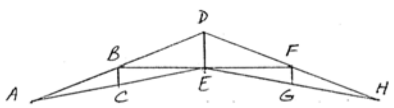
$$F_{GH} = 1023.46 \text{ lb} \qquad F_{GH} = 1.023 \text{ kips T} \blacktriangleleft$$

PROBLEM 6.27

Determine whether the trusses of Probs. 6.13, 6.14, and 6.25 are simple trusses.

SOLUTION

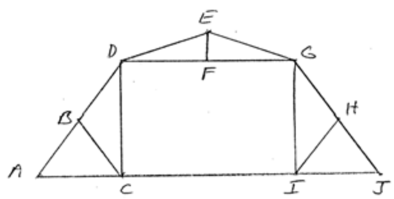
Truss of 6.13:



Start with ΔABC and add, in order, joints E, D, F, G, H

This is a simple truss. ◀

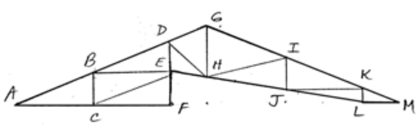
Truss of 6.14:



ABDC, *DEGF*, and *GHJI* are all individually simple trusses, but no simple extension (one joint, two members at a time) will produce the given truss:

$\therefore$ Not a simple truss. ◀

Truss of 6.25:



Starting with ΔABC, add, in order, joints $E, F, D, H, G, I, J, K, L, M$

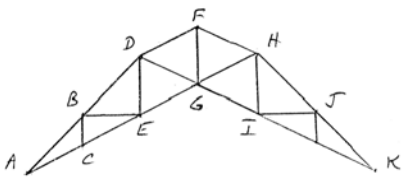
This is a simple truss. ◀

PROBLEM 6.28

Determine whether the trusses of Probs. 6.19, 6.21, and 6.23 are simple trusses.

SOLUTION

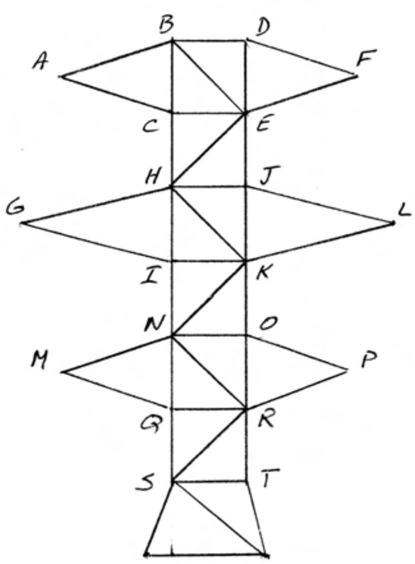
Truss of 6.19:



Start with ΔABC, and add, in order, *E*, *D*, *G*, *F*, *H*, *I*, *J*, *K*

$\therefore$ Simple truss ◀

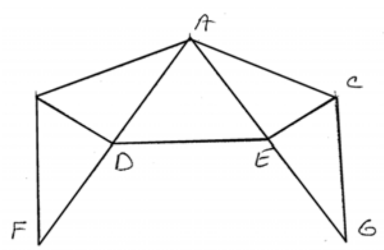
Truss of 6.21:



Start with ΔABC, and add, in order, *E*, *D*, *F*, *H*, *J*, *K*, *I*, *G*, *L*, *N*, *O*, *R*, *Q*, *M*, *P*, *S*, *T*, etc.

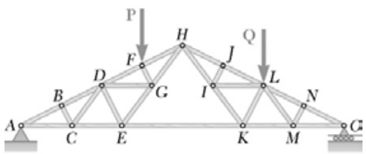
$\therefore$ Simple truss ◀

Truss of 6.23:



Start with ΔBDF, and add, in order, *A*, *E*, *C*, *G*.

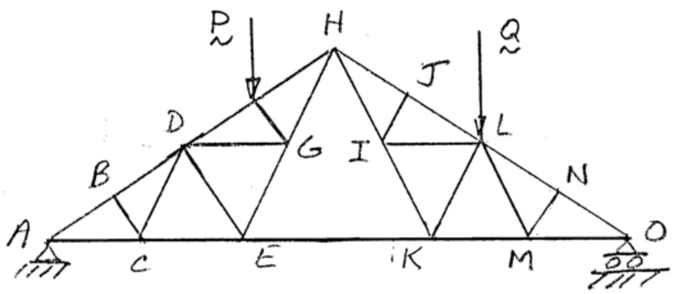
$\therefore$ Simple truss ◀



PROBLEM 6.29

For the given loading, determine the zero-force members in the truss shown.

SOLUTION



By inspection of joint B: $F_{BC} = 0$ ◀

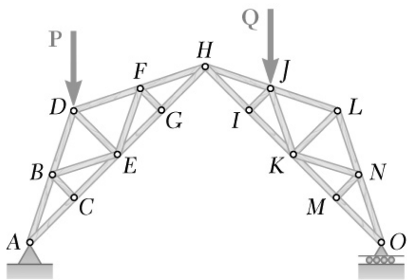
Then by inspection of joint C: $F_{CD} = 0$ ◀

By inspection of joint J: $F_{IJ} = 0$ ◀

Then by inspection of joint I: $F_{IL} = 0$ ◀

By inspection of joint N: $F_{MN} = 0$ ◀

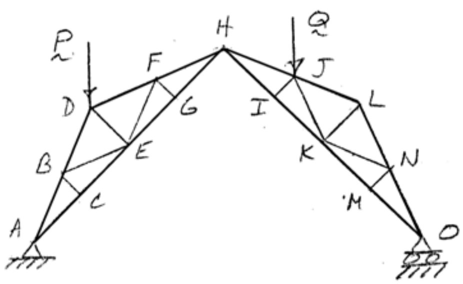
Then by inspection of joint M: $F_{LM} = 0$ ◀



PROBLEM 6.30

For the given loading, determine the zero-force members in the truss shown.

SOLUTION



By inspection of joint C: $F_{BC} = 0$ ◀

Then by inspection of joint B: $F_{BE} = 0$ ◀

By inspection of joint G: $F_{FG} = 0$ ◀

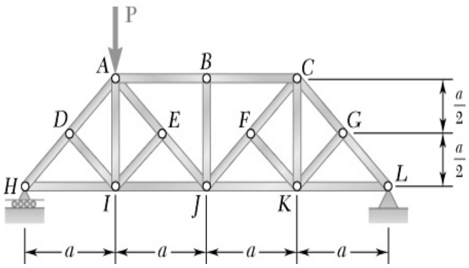
Then by inspection of joint F: $F_{EF} = 0$ ◀

Then by inspection of joint E: $F_{DE} = 0$ ◀

By inspection of joint M: $F_{MN} = 0$ ◀

Then by inspection of joint N: $F_{KN} = 0$ ◀

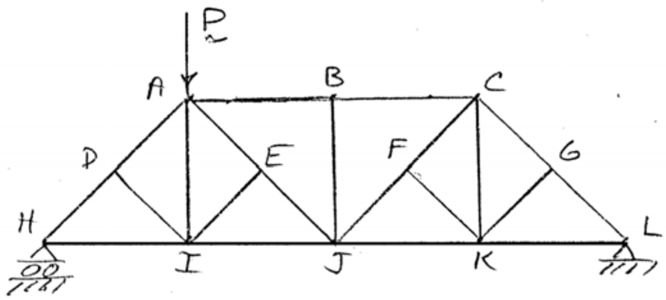
By inspection of joint I: $F_{IJ} = 0$ ◀



PROBLEM 6.31

For the given loading, determine the zero-force members in the truss shown.

SOLUTION



By inspection of joint D: $F_{DI} = 0$ ◀

By inspection of joint E: $F_{EI} = 0$ ◀

Then by inspection of joint I: $F_{AI} = 0$ ◀

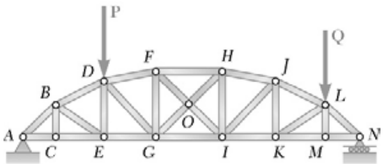
By inspection of joint B: $F_{BJ} = 0$ ◀

By inspection of joint F: $F_{FK} = 0$ ◀

By inspection of joint G: $F_{GK} = 0$ ◀

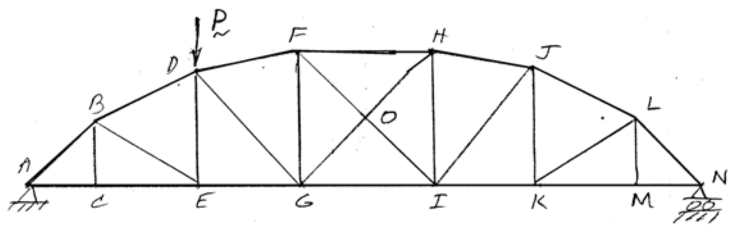
Then by inspection of joint K: $F_{CK} = 0$ ◀

PROBLEM 6.32



For the given loading, determine the zero-force members in the truss shown.

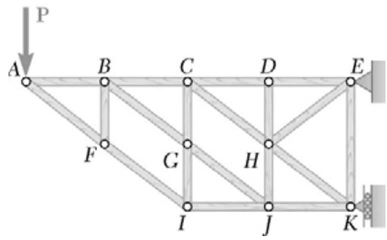
SOLUTION



By inspection of joint C: $F_{BC} = 0$ ◀

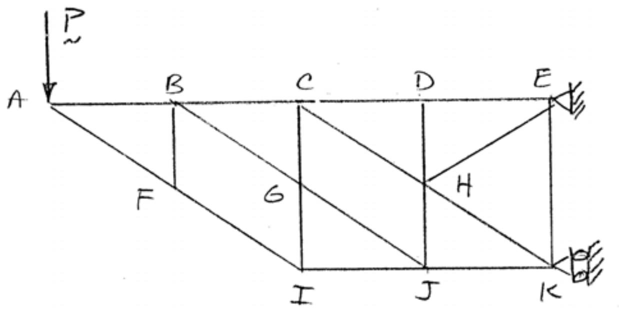
By inspection of joint M: $F_{LM} = 0$ ◀

PROBLEM 6.33



For the given loading, determine the zero-force members in the truss shown.

SOLUTION



By inspection of joint F: $F_{BF} = 0$ ◀

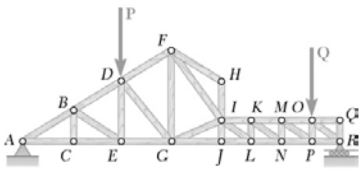
Then by inspection of joint B: $F_{BG} = 0$ ◀

Then by inspection of joint G: $F_{GJ} = 0$ ◀

Then by inspection of joint J: $F_{HJ} = 0$ ◀

By inspection of joint D: $F_{DH} = 0$ ◀

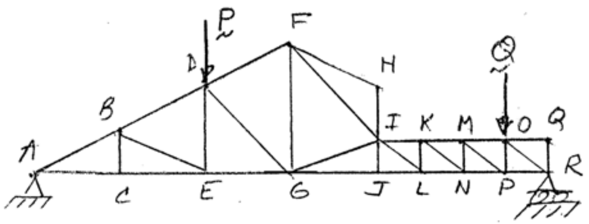
Then by inspection of joint H: $F_{HE} = 0$ ◀



PROBLEM 6.34

For the given loading, determine the zero-force members in the truss shown.

SOLUTION



By inspection of joint C: $F_{BC} = 0$ ◀

Then by inspection of joint B: $F_{BE} = 0$ ◀

Then by inspection of joint E: $F_{DE} = 0$ ◀

By inspection of joint H: $F_{FH} = 0$ ◀

and $F_{HI} = 0$ ◀

By inspection of joint Q: $F_{OQ} = 0$ ◀

and $F_{QR} = 0$ ◀

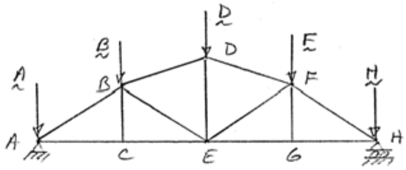
By inspection of joint J: $F_{IJ} = 0$ ◀

PROBLEM 6.35

Determine the zero-force members in the truss of (*a*) Prob. 6.9, (*b*) Prob. 6.19.

SOLUTION

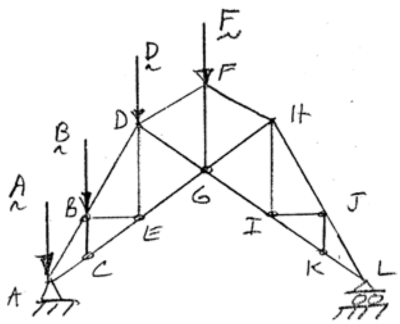
Truss of 6.9:



By inspection of joint C: $F_{BC} = 0$ ◀

By inspection of joint G: $F_{FG} = 0$ ◀

Truss of 6.19:



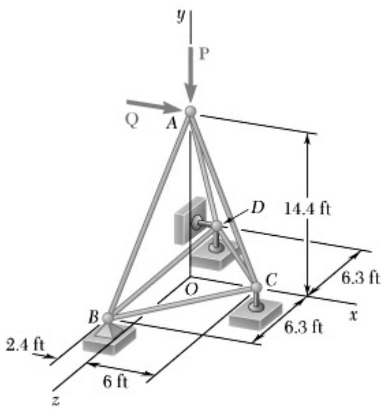
By inspection of joint C: $F_{BC} = 0$ ◀

By inspection of joint K: $F_{JK} = 0$ ◀

Then by inspection of joint J: $F_{IJ} = 0$ ◀

Then by inspection of joint I: $F_{HI} = 0$ ◀

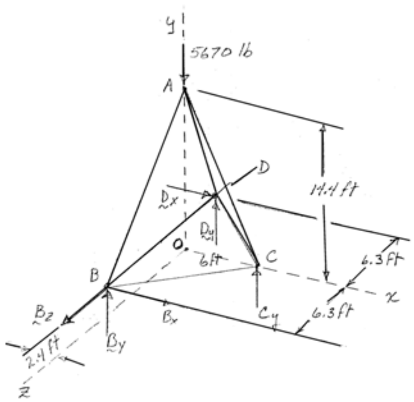
PROBLEM 6.36



The truss shown consists of six members and is supported by a ball and socket at B, a short link at C, and two short links at D. Determine the force in each of the members for $\mathbf{P} = (-5670\ \text{lb})\,\mathbf{j}$ and $\mathbf{Q} = 0$.

SOLUTION

FBD Truss:



$$\Sigma F_z = 0:\ \mathbf{B}_z = 0$$

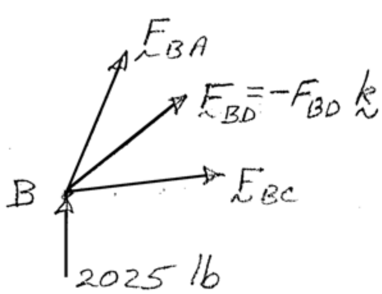
$$\Sigma M_{BD} = 0:\ (2.4\text{ ft})(5670\text{ lb}) - (8.4\text{ ft})C_y = 0 \qquad \mathbf{C}_y = (1620\text{ lb})\,\mathbf{j}$$

$$\Sigma M_x = 0:\ (6.3\text{ ft})D_y - (6.3\text{ ft})B_y = 0 \qquad B_y = D_y$$

$$\Sigma F_y = 0:\ B_y + D_y - 5670\text{ lb} + 1620\text{ lb} = 0 \qquad \mathbf{B}_y = \mathbf{D}_y = (2025\text{ lb})\,\mathbf{j}$$

$$\left.\begin{array}{l}\Sigma M_y = 0:\ (6.3\text{ ft})B_x - (6.3\text{ ft})D_x = 0\\ \Sigma F_x = 0:\ B_x + D_x = 0\end{array}\right\} \quad \mathbf{B}_x = \mathbf{D}_x = 0$$

Joint B:



Where $\mathbf{F}_{BA} = F_{BA}\dfrac{2.4\mathbf{i} + 14.4\mathbf{j} - 6.3\mathbf{k}}{15.9}$

$$= F_{BA}(0.1509\mathbf{i} + 0.9057\mathbf{j} - 0.3962\mathbf{k})$$

$$\mathbf{F}_{BC} = F_{BC}\frac{8.4\mathbf{i} - 6.3\mathbf{k}}{10.5} = F_{BC}(0.8\mathbf{i} - 0.6\mathbf{k})$$

$$\Sigma F_y = 0:\ 0.9057F_{BA} + 2025\text{ lb} = 0$$

$F_{BA} = -2236$ lb

$F_{BA} = 2.24$ kips C ◀

By symmetry $F_{AD} = 2.24$ kips C ◀

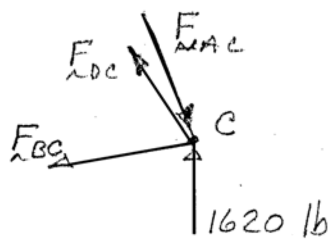
$$\Sigma F_x = 0:\ 0.1509(-2236\text{ lb}) + 0.8F_{BC} = 0$$

$F_{BC} = 422$ lb T ◀

By symmetry $F_{DC} = 422$ lb T ◀

PROBLEM 6.36 CONTINUED

Joint C:

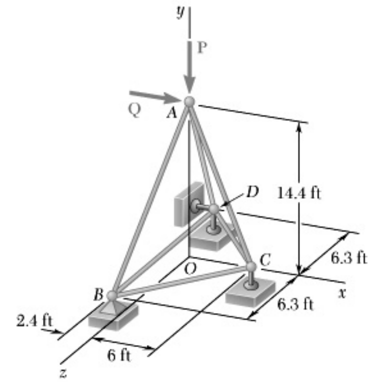


$$\swarrow \Sigma F_z = 0: -0.3962(-2236 \text{ lb}) - F_{BD} - 0.6(422 \text{ lb}) = 0$$

$$F_{BD} = 633 \text{ lb T} \blacktriangleleft$$

$$\mathbf{F}_{AC} = F_{AC}\frac{6\mathbf{i} - 14.4\mathbf{j}}{15.6} = F_{AC}(0.3846\mathbf{i} - 0.9231\mathbf{j})$$

$$\uparrow \Sigma F_y = 0: 1620 \text{ lb} - (0.9231)F_{AC} = 0 \qquad F_{AC} = 1755 \text{ lb C} \blacktriangleleft$$

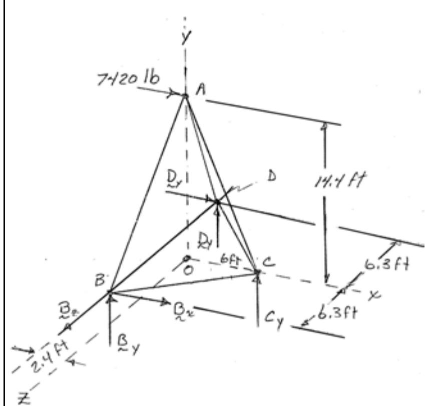


PROBLEM 6.37

The truss shown consists of six members and is supported by a ball and socket at B, a short link at C, and two short links at D. Determine the force in each of the members for $\mathbf{P} = 0$ and $\mathbf{Q} = (7420\ \text{lb})\mathbf{i}$.

SOLUTION

FBD Truss:



$$\Sigma F_z = 0: \mathbf{B}_z = 0$$

$$\Sigma M_{BD} = 0: (8.4\text{ ft})C_y - (14.4\text{ ft})(7420\text{ lb}) = 0$$

$$\mathbf{C}_y = (12720\text{ lb})\mathbf{j}$$

$$\Sigma M_x = 0: (6.3\text{ ft})(D_y - B_y) = 0 \qquad D_y = B_y$$

$$\Sigma F_y = 0: B_y + D_y + C_y = 0; \qquad 2D_y + 12720\text{ lb} = 0$$

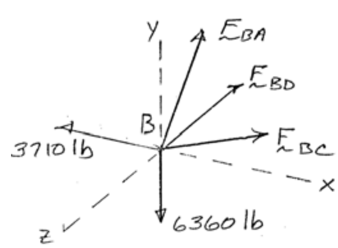
so

$$\mathbf{B}_y = \mathbf{D}_y = -(6360\text{ lb})\mathbf{j}$$

$$\Sigma M_y = 0: (6.3\text{ ft})(B_x - D_x) = 0; \qquad B_x = D_x$$

$$\Sigma F_x = 0: B_x + D_x + 7420\text{ lb} = 0; \qquad \mathbf{B}_x = \mathbf{D}_x = -(3710\text{ lb})\mathbf{i}$$

Joint FBDs:



$$\mathbf{F}_{BA} = F_{BA}\frac{(2.4\text{ ft }\mathbf{i} + 14.4\text{ ft }\mathbf{j} - 6.3\text{ ft }\mathbf{k})}{15.9\text{ ft}}$$

$$= F_{BA}(0.1509\mathbf{i} + 0.9057\mathbf{j} - 0.3962\mathbf{k}) \qquad \mathbf{F}_{BD} = F_{BD}\mathbf{k}$$

$$\mathbf{F}_{BC} = F_{BC}\frac{(8.4\text{ ft }\mathbf{i} - 6.3\text{ ft }\mathbf{j})}{10.5\text{ ft}} = F_{BC}(0.8\mathbf{i} - 0.6\mathbf{j})$$

$$\Sigma F_y = 0: 0.9057F_{BA} - 6360\text{ lb} = 0 \qquad F_{BA} = 7022\text{ lb}$$

$$F_{BA} = 7.02\text{ kips T} \blacktriangleleft$$

By symmetry $\qquad F_{DA} = 7.02\text{ kips T} \blacktriangleleft$

$$\Sigma F_x = 0: 0.1509(7022\text{ lb}) + 0.8F_{BC} - 3710\text{ lb} = 0 \qquad F_{BC} = 3313\text{ lb}$$

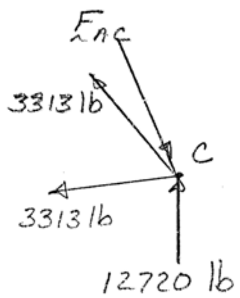
so $F_{BC} = 3.31\text{ kips T} \blacktriangleleft$

By symmetry $\qquad F_{DC} = 3.31\text{ kips T} \blacktriangleleft$

$$\Sigma F_z = 0: -0.3962(7022\text{ lb}) + 0.6(3313\text{ lb}) - F_{BD} = 0$$

$$F_{BD} = -4770\text{ lb} \qquad F_{BD} = 4.77\text{ kips C} \blacktriangleleft$$

PROBLEM 6.37 CONTINUED

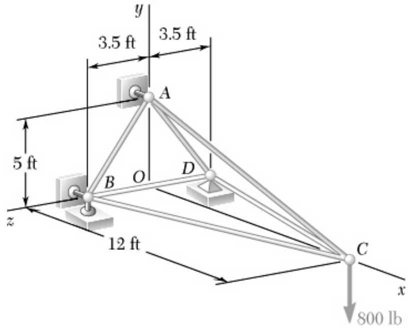


$$\mathbf{F}_{AC} = F_{AC}\frac{(6\text{ ft }\mathbf{i} - 14.4\text{ ft }\mathbf{j})}{15.6\text{ ft}}$$

$$= F_{AC}(0.3846\mathbf{i} - 0.9231\mathbf{j})$$

$$\uparrow \Sigma F_y = 12720\text{ lb} - 0.9231F_{AC} = 0; \qquad F_{AC} = 13780\text{ lb}$$

$$F_{AC} = 1.378\text{ kips C} \blacktriangleleft$$

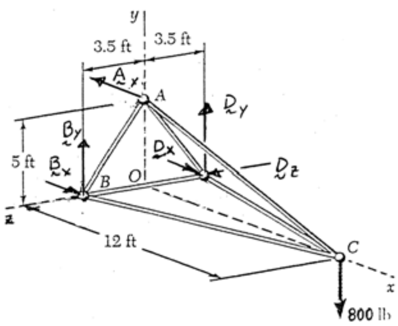


PROBLEM 6.38

The truss shown consists of six members and is supported by a short link at A, two short links at B, and a ball and socket at D. Determine the force in each of the members for the given loading.

SOLUTION

FBD Truss:



$$\Sigma F_z = 0: \mathbf{D}_z = 0$$

$$\Sigma M_z = 0: (5 \text{ ft})A_x - (12 \text{ ft})(800 \text{ lb}) = 0 \qquad \mathbf{A}_x = (1920 \text{ lb})\mathbf{i}$$

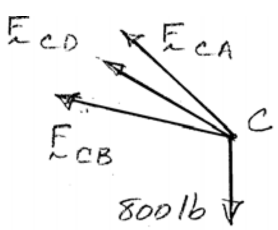
$$\Sigma M_y = 0: (3.5 \text{ ft})(B_x - D_x) = 0; \quad B_x = D_x$$

$$\Sigma F_x = 0: B_x + D_x - 1920 \text{ lb} = 0 \qquad \text{so} \qquad \mathbf{B}_x = \mathbf{D}_x = (960 \text{ lb})\mathbf{i}$$

$$\Sigma M_x = 0: (3.5 \text{ ft})(D_y - B_y) = 0; \quad D_y = B_y$$

$$\Sigma F_y = 0: B_y + D_y - 800 \text{ lb} = 0 \qquad \text{so} \qquad \mathbf{B}_y = \mathbf{D}_y = (400 \text{ lb})\mathbf{j}$$

Joint FBDs:



$$\mathbf{F}_{CA} = F_{AC}\frac{(-12 \text{ ft } \mathbf{i} + 5 \text{ ft } \mathbf{j})}{13 \text{ ft}} = \frac{F_{AC}}{13}(-12\mathbf{i} + 5\mathbf{j})$$

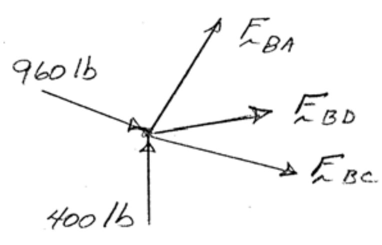
$$\mathbf{F}_{CD} = F_{CD}\frac{(-12 \text{ ft } \mathbf{i} - 3.5 \text{ ft } \mathbf{k})}{12.5 \text{ ft}} = \frac{F_{CD}}{12.5}(-12\mathbf{i} - 3.5\mathbf{k})$$

Similarly $$F_{CB} = \frac{F_{CB}}{12.5}(-12\mathbf{i} + 3.5\mathbf{k})$$

$$\Sigma F_z = 0: \frac{3.5}{12.5}(F_{CB} - F_{CD}) = 0; \qquad F_{CB} = F_{CD}$$

$$\Sigma F_y = 0: F_{AC}\left(\frac{5}{13}\right) - 800 = 0 \qquad F_{AC} = 2080 \text{ lb}$$

$F_{AC} = 2.08$ kips T ◀



$$\mathbf{F}_{BA} = F_{BA}\frac{(5 \text{ ft } \mathbf{j} - 3.5 \text{ ft } \mathbf{k})}{6.1033 \text{ ft}} = \frac{F_{BA}}{6.1033}(5\mathbf{j} - 3.5\mathbf{k})$$

$$\mathbf{F}_{BD} = -F_{BD}\,\mathbf{k}$$

$$\mathbf{F}_{BC} = -\mathbf{F}_{CB} = \frac{F_{CB}}{12.5}(+12\mathbf{i} - 3.5\mathbf{k})$$

PROBLEM 6.38 CONTINUED

$$\uparrow \Sigma F_y = 0: \frac{5F_{BA}}{6.1033} + 400 \text{ lb} = 0 \qquad F_{BA} = -488 \text{ lb}$$

so $F_{BA} = 488 \text{ lb} \ \ C$ ◀

By symmetry: $F_{AD} = 488 \text{ lb} \ \ C$ ◀

$$\longrightarrow \Sigma F_x = 0: F_{BC}\left(\frac{12}{12.5}\right) + 960 \text{ lb} = 0 \qquad F_{BC} = -1000 \text{ lb}$$

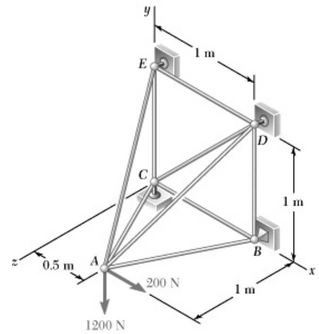
$F_{BC} = 1.000 \text{ kip} \ \ C$ ◀

By symmetry: $F_{CD} = 1.000 \text{ kip} \ \ C$ ◀

$$\swarrow \Sigma F_z = 0: -F_{BD} - 488 \text{ lb}\left(\frac{3.5}{6.1033}\right) + (1000 \text{ lb})\frac{3.5}{12.5} = 0$$

$F_{BD} = -559.9 \text{ lb}$ $\qquad F_{BD} = 560 \text{ lb} \ \ C$ ◀

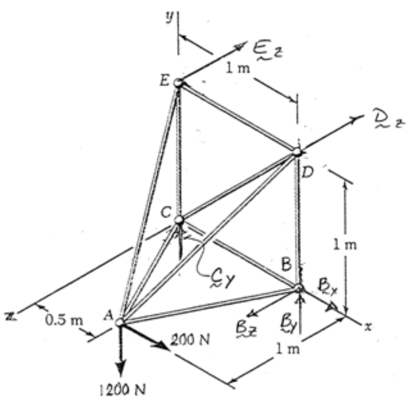
PROBLEM 6.39



The portion of a power line transmission tower shown consists of nine members and is supported by a ball and socket at *B* and short links at *C*, *D*, and *E*. Determine the force in each of the members for the given loading.

SOLUTION

FBD Truss:



$$\Sigma M_{BD} = 0: (1\text{ m})(200\text{ N} - E_z) = 0 \qquad \mathbf{E}_z = (200\text{ N})\mathbf{k}$$

$$\Sigma M_x = 0: (1\text{ m})(1200\text{ N} - E_z - D_z) = 0$$

$$D_z = 1200\text{ N} - 200\text{ N} = 1000\text{ N} \quad \mathbf{D}_Z = (1000\text{ N})\mathbf{k}$$

$$\Sigma F_z = 0: B_z - 1000\text{ N} - 200\text{ N} = 0 \qquad \mathbf{B}_z = (1200\text{ N})\mathbf{k}$$

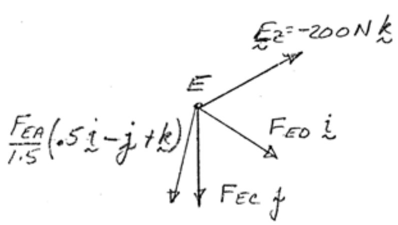
$$\Sigma M_{Bz} = 0: (.5\text{ m})(1200\text{ N}) - (1\text{ m})C_y = 0 \qquad \mathbf{C}_y = (600\text{ N})\mathbf{j}$$

$$\Sigma F_x = 0: B_x - 200\text{ N} = 0 \qquad \mathbf{B}_x = (200\text{ N})\mathbf{i}$$

$$\Sigma F_y = 0: B_y + C_y - 1200\text{ N} = 0$$

$$B_y = 1200\text{ N} - 600\text{ N} \qquad \mathbf{B}_y = (600\text{ N})\mathbf{j}$$

Joint FBDs:



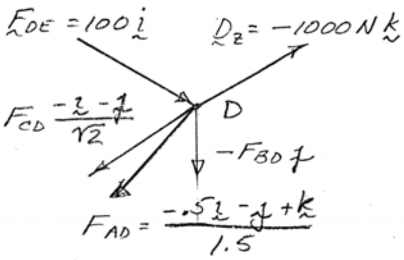
$$\Sigma F_z = 0: \frac{F_{EA}}{1.5} - 200\text{ N} = 0 \qquad F_{EA} = 300\text{ N T} \blacktriangleleft$$

$$\Sigma F_x = 0: F_{ED} + \frac{.5}{1.5}F_{EA} = 0 \qquad F_{ED} = -100\text{ N}$$

$$F_{ED} = 100\text{ N C} \blacktriangleleft$$

$$\Sigma F_y = 0: \quad -\frac{F_{EA}}{1.5} - F_{EC} = 0 \qquad F_{EC} = -200\text{ N}$$

$$F_{EC} = 200\text{ N C} \blacktriangleleft$$



$$\Sigma F_z = 0: \frac{F_{AD}}{1.5} - 1000\text{ N} = 0 \qquad F_{AD} = 1500\text{ N T} \blacktriangleleft$$

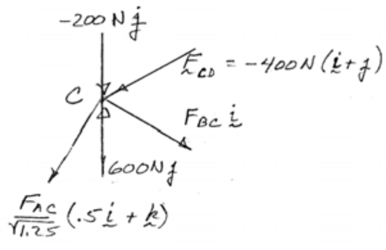
$$\Sigma F_x = 0: 100\text{ N} - \frac{0.5}{1.5}F_{AD} - \frac{1}{\sqrt{2}}F_{CD} = 0$$

$$F_{CD} = \sqrt{2}\left(100\text{ N} - \frac{1500\text{ N}}{3}\right) = -400\sqrt{2} \qquad F_{CD} = 566\text{ N C} \blacktriangleleft$$

$$\Sigma F_y = 0: \qquad -\frac{F_{CD}}{\sqrt{2}} - \frac{F_{AD}}{1.5} - F_{BD} = 0$$

$$F_{BD} = +400 - 1000 = -600\text{ N} \qquad F_{BD} = 600\text{ N C} \blacktriangleleft$$

PROBLEM 6.39 CONTINUED

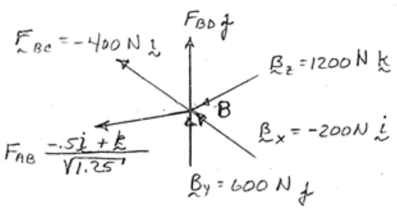


$$\Sigma F_y = 0\text{: } 600 \text{ N} - 200 \text{ N} - 400 \text{ N} = 0$$

$$\Sigma F_z = 0\text{: } \frac{F_{AC}}{\sqrt{1.25}} = 0 \qquad F_{AC} = 0 \blacktriangleleft$$

$$\Sigma F_x = 0\text{: } F_{BC} - 400 \text{ N} + \frac{.5}{\sqrt{1.25}}\cancel{F_{AC}}^{0} = 0$$

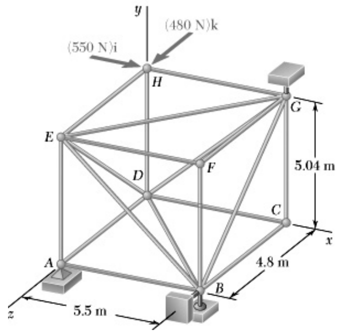
$$F_{BC} = 400 \text{ N T} \blacktriangleleft$$



$$\Sigma F_x = 0\text{: } -200 \text{ N} - 400 \text{ N} - F_{AB}\frac{.5}{\sqrt{1.25}} = 0$$

$$F_{AB} = 1200\sqrt{1.25} \text{ N} \qquad F_{AB} = 1342 \text{ N C} \blacktriangleleft$$

PROBLEM 6.40



The truss shown consists of 18 members and is supported by a ball and socket at *A*, two short links at *B*, and one short link at *G*. (*a*) Check that this truss is a simple truss, that it is completely constrained, and that the reactions at its supports are statically determinate. (*b*) For the given loading, determine the force in each of the six members joined at *E*.

SOLUTION

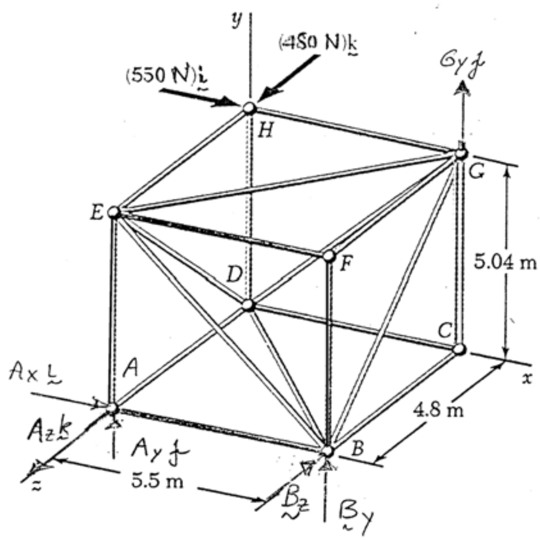
(a) To check for simple truss, start with *ABDE* and add three members at a time which meet at a single joint, thus successively adding joints *F*, *G*, *H*, and *C*, to complete the truss.

This is, therefore, a simple truss. ◀

There are six reaction force components, none of which are in-line, so they are determined by the six equilibrium equations. Constraints prevent motion. Truss is completely constrained and ◀

statically determinate ◀

(b) FBD Truss:



$\Sigma M_{BC} = 0: (5.04 \text{ m})(550 \text{ N}) + (5.5 \text{ m})(A_y) = 0 \qquad \mathbf{A}_y = -(504 \text{ N})\,\mathbf{j}$

$\Sigma M_{BF} = 0: (5.5 \text{ m})(A_z + 480 \text{ N}) - (4.8 \text{ m})(550 \text{ N}) = 0 \qquad \mathbf{A}_z = 0$

By inspection of joint *C*: $F_{DC} = F_{BC} = F_{GC} = 0$

By inspection of joint *A*: $F_{AE} = -A_y = 504 \text{ N}$ T ◀

$F_{AD} = A_z = 0$

By inspection of joint *H*: $F_{DH} = 0$

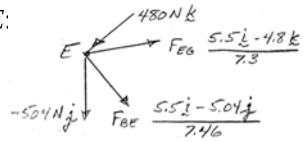
$F_{EH} = 480 \text{ N}$ C ◀

By inspection of joint *F*: $F_{EF} = 0$ ◀

PROBLEM 6.40 CONTINUED

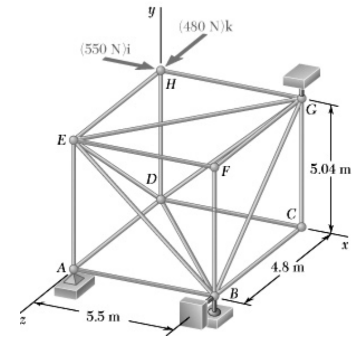
Then, since ED is the only non-zero member at D, not in the plane BDG, $F_{DE} = 0$ ◀

Joint E:



$\Sigma F_z = 0$: $480\text{ N} - \dfrac{4.8}{7.3}F_{EG} = 0$ $F_{EG} = 730\text{ N}$ T ◀

$\Sigma F_y = 0$: $-504\text{ N} - \dfrac{5.04}{7.46}F_{BE} = 0$ $F_{BE} = -746\text{ N}$ $F_{BE} = 746\text{ N}$ C ◀



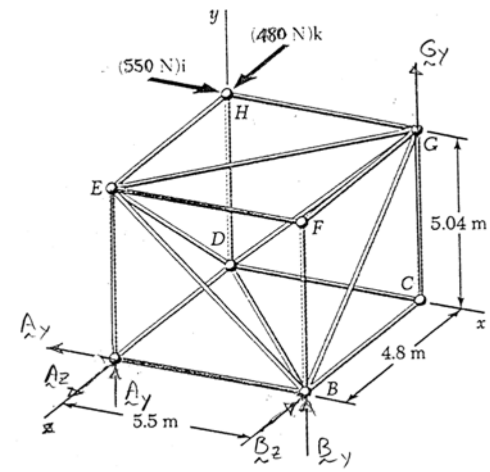
PROBLEM 6.41

The truss shown consists of 18 members and is supported by a ball and socket at *A*, two short links at *B*, and one short link at *G*. (*a*) Check that this truss is a simple truss, that it is completely constrained, and that the reactions at its supports are statically determinate. (*b*) For the given loading, determine the force in each of the six members joined at *G*.

SOLUTION

(a) See part (a) solution 6.40 above

(b) FBD Truss:



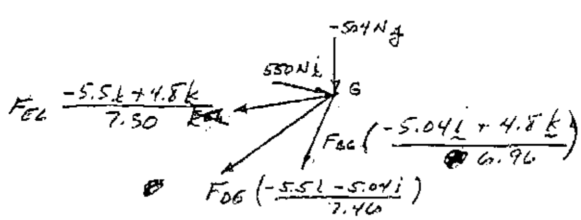
$$\Sigma M_{AB} = 0: \ (4.8 \text{ m})G_y + (5.04 \text{ m})(480 \text{ N}) = 0 \qquad \mathbf{G}_y = -504 \text{ N}$$

By inspection of joint *C*: $F_{CG} = 0$ ◀

By inspection of joint *H*: $F_{HG} = 550 \text{ N C}$ ◀

By inspection of joint *F*: $F_{FG} = 0$ ◀

Joint *G*:



$$\xrightarrow{} \Sigma F_x = 0: 550 \text{ N} - \frac{5.5}{7.46}F_{DG} - \frac{5.5}{7.30}F_{EG} = 0$$

PROBLEM 6.41 CONTINUED

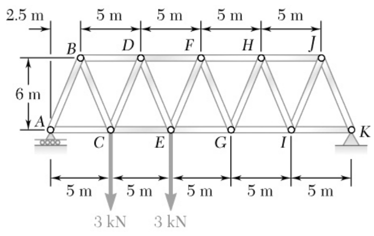
$$\uparrow \Sigma F_y = 0: -504\text{ N} - \frac{5.04}{7.46}F_{DG} - \frac{5.04}{6.96}F_{BG} = 0$$

$$\swarrow \Sigma F_z = 0: \frac{4.8}{7.30}F_{EG} + \frac{4.8}{6.96}F_{BG} = 0$$

Solving: $F_{BG} = -696\text{ N}$ $\qquad F_{BG} = 696\text{ N C} \blacktriangleleft$

$F_{DG} = 0$ $\qquad F_{DG} = 0 \blacktriangleleft$

$F_{EG} = 730\text{ N}$ $\qquad F_{EG} = 730\text{ N T} \blacktriangleleft$

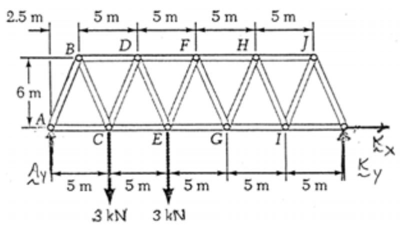


PROBLEM 6.42

A Warren bridge truss is loaded as shown. Determine the force in members *CE*, *DE*, and *DF*.

SOLUTION

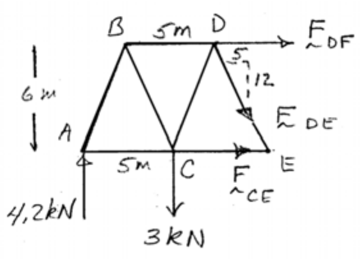
FBD Truss:



$$\circlearrowleft \Sigma M_K = 0: (15\text{ m})(3\text{ kN}) + (20\text{ m})(3\text{ kN}) - (25\text{ m})A_y = 0$$

$$\mathbf{A}_y = 4.2\text{ kN} \uparrow$$

Section ABDC:



$$\circlearrowleft \Sigma M_D = 0: (6\text{ m})F_{CE} + (2.5\text{ m})(3\text{ kN}) - (7.5\text{ m})(4.2\text{ kN}) = 0$$

$$F_{CE} = 4\text{ kN} \qquad F_{CE} = 4.00\text{ kN T} \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: 4.2\text{ kN} - 3\text{ kN} - \frac{12}{13}F_{DE} = 0$$

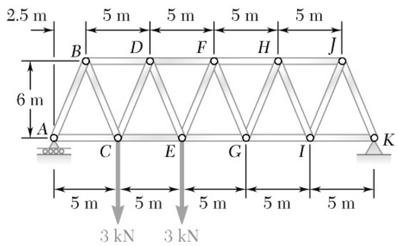
$$F_{DE} = 1.3\text{ kN} \qquad F_{DE} = 1.300\text{ kN T} \blacktriangleleft$$

$$\rightarrow \Sigma F_x = 0: F_{DF} + \frac{5}{13}F_{DE} + F_{CE} = 0$$

$$F_{DF} = -\frac{5}{13}(1.3\text{ kN}) - (4\text{ kN}) = -4.5\text{ kN}$$

$$F_{DF} = 4.50\text{ kN C} \blacktriangleleft$$

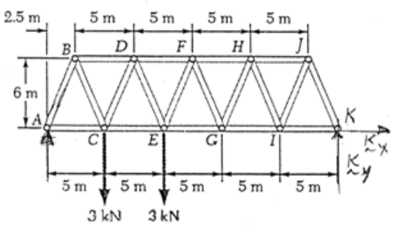
PROBLEM 6.43



A Warren bridge truss is loaded as shown. Determine the force in members *EG*, *FG*, and *FH*.

SOLUTION

FBD Truss:



$$\rightarrow \Sigma F_x = 0: \mathbf{K}_x = 0$$

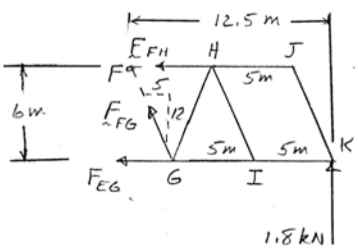
$$\circlearrowleft \Sigma M_A = 0: (25\text{ m})K_y - (10\text{ m})(3\text{ kN}) - (5\text{ m})(3\text{ kN}) = 0$$

$$\mathbf{K}_y = 1.8\text{ kN} \uparrow$$

$$\circlearrowleft \Sigma M_G = 0: (10\text{ m})(1.8\text{ kN}) + (6\text{ m})F_{FH} = 0$$

$F_{FH} = -3\text{ kN}$ $\qquad$ $F_{FH} = 3.00\text{ kN C}$ ◀

Section FBD:



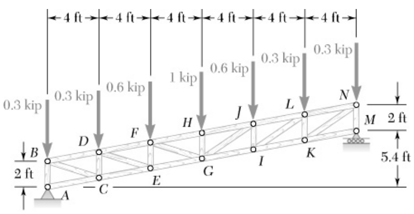
$$\circlearrowleft \Sigma M_F = 0: (12.5\text{ m})(1.8\text{ kN}) - (6\text{ m})(F_{EG}) = 0$$

$F_{EG} = 3.75\text{ kN}$ $\qquad$ $F_{EG} = 3.75\text{ kN T}$ ◀

$$\uparrow \Sigma F_y = 0: \frac{12}{13}F_{FG} + 1.8\text{ kN} = 0$$

$F_{FG} = -1.95\text{ kN}$ $\qquad$ $F_{FG} = 1.950\text{ kN C}$ ◀

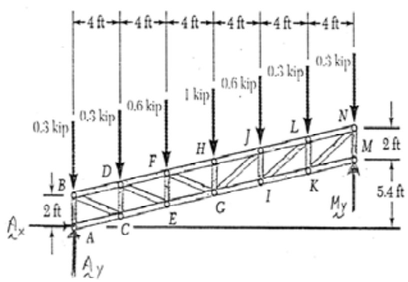
PROBLEM 6.44



A parallel chord Howe truss is loaded as shown. Determine the force in members *CE*, *DE*, and *DF*.

SOLUTION

FBD Truss:



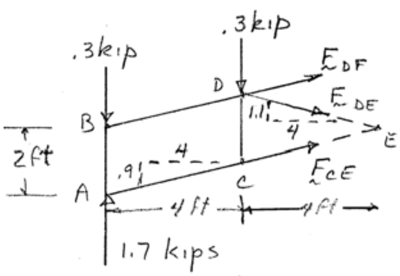
$$\rightarrow \Sigma F_x = 0: \mathbf{A}_x = 0$$

$$\Sigma M_M = 0: (4\text{ ft})\big[1(0.3\text{ kip}) + 2(0.6\text{ kip}) + 3(1\text{ kip}) + 4(0.6\text{ kip}) + 5(0.3\text{ kip}) + 6(0.3\text{ kip}) - 6A_y\big] = 0$$

$$\mathbf{A}_y = 1.7\text{ kips} \uparrow$$

$$\Sigma M_D = 0: (4\text{ ft})(3\text{ kips} - 1.7\text{ kips}) + (2\text{ ft})\left(\frac{4}{4.1}F_{CE} = 0\right)$$

FBD Section:



$$F_{CE} = 2.87\text{ kips} \qquad F_{CE} = 2.87\text{ kips T} \blacktriangleleft$$

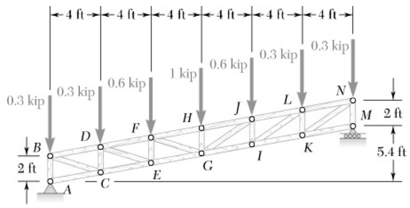
$$\Sigma M_E = 0: (8\text{ ft})(3\text{ kips} - 1.7\text{ kips}) + (4\text{ ft})(0.3\text{ kip}) - (2\text{ ft})\left(\frac{4}{4.1}F_{DF}\right) = 0$$

$$F_{DF} = -5.125\text{ kips} \qquad F_{DF} = 5.13\text{ kips C} \blacktriangleleft$$

$$\rightarrow \Sigma F_x = 0: \frac{4}{4.1}(F_{DF} + F_{CE}) + \frac{4}{\sqrt{17.21}}F_{DE} = 0$$

$$F_{DE} = -\frac{\sqrt{17.21}}{4.1}(-5.125 + 2.87)\text{kips} \qquad F_{DE} = 2.28\text{ kips}$$

$$F_{DE} = 2.28\text{ kips T} \blacktriangleleft$$

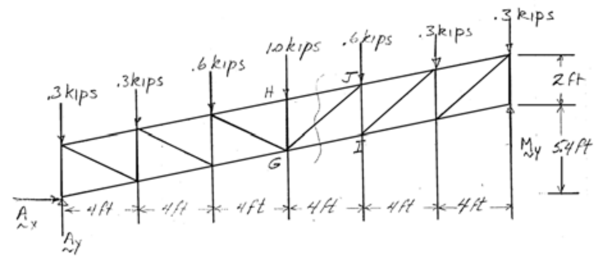


PROBLEM 6.45

A parallel chord Howe truss is loaded as shown. Determine the force in members *GI*, *GJ*, and *HJ*.

SOLUTION

FBD Truss:

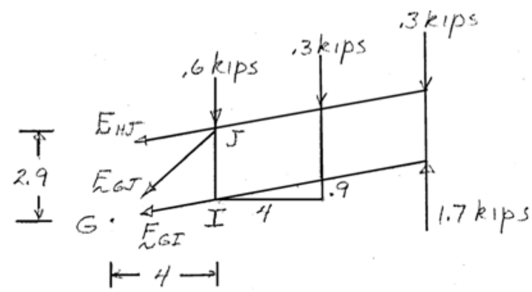


$$\circlearrowleft \Sigma M_A = (24\text{ ft})M_y - 4\text{ ft}(0.3\text{ kip}) - 8\text{ ft}(0.6\text{ kip}) - 12\text{ ft}(1\text{ kip})$$

$$- 16\text{ ft}(0.6\text{ kip}) - 20\text{ ft}(0.3\text{ kip}) - 24\text{ ft}(0.3\text{ kip}) = 0$$

$$\mathbf{M}_y = 1.7\text{ kips}$$

FBD Section:



$$\circlearrowleft \Sigma M_J = 8\text{ ft}(1.7 - 0.3)\text{ kips} - 4\text{ ft}(0.3\text{ kip}) - 2\text{ ft}\left(\frac{4}{4.1}F_{GI}\right) = 0$$

$$F_{GI} = 5.125\text{ kips} \qquad F_{GI} = 5.3\text{ kips T} \blacktriangleleft$$

$$\circlearrowleft \Sigma M_G = 12\text{ ft}(1.7 - 0.3)\text{ kips} - 8\text{ ft}(0.3\text{ kip})$$

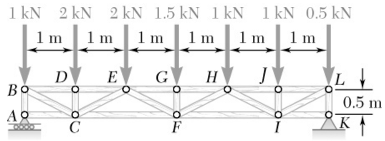
$$- 4\text{ ft}(0.6\text{ kip}) + 2\text{ ft}\left(\frac{4}{4.1}F_{HJ}\right) = 0$$

$$F_{HJ} = -6.15\text{ kips} = 6.15\text{ kips C} \blacktriangleleft$$

$$\xrightarrow{} \Sigma F_x = \frac{4}{4.1}(6.15 - 5.125)\text{ kips} - \frac{4}{4.94}F_{GJ}$$

$$F_{GJ} = 1.235\text{ kips T} \blacktriangleleft$$

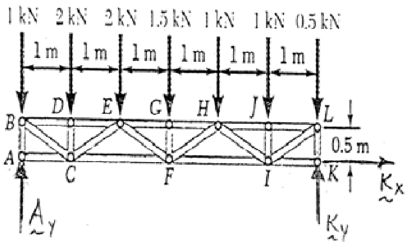
PROBLEM 6.46



A floor truss is loaded as shown. Determine the force in members *CF*, *EF*, and *EG*.

SOLUTION

FBD Truss:



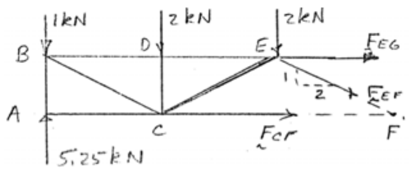
$$\Sigma M_K = 0: (1\text{ m})\big[1\text{ kN} + 2(1\text{ kN}) + 3(1.5\text{ kN})$$
$$+4(2\text{ kN}) + 5(2\text{ kN}) + 6(1\text{ kN}) - 6A_y\big] = 0$$

$$\mathbf{A}_y = 5.25\text{ kN} \uparrow$$

$$\Sigma M_E = 0: (1\text{ m})\big[1(2\text{ kN}) + 2(1\text{ kN} - 5.25\text{ kN})\big] + (0.5\text{ m})F_{CE} = 0$$

$$F_{CF} = 13.0\text{ kN} \qquad F_{CF} = 13.00\text{ kN T} \blacktriangleleft$$

FBD Section:

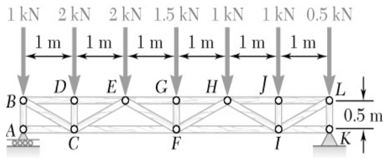


$$\Sigma M_F = 0: (1\text{ m})\big[1(2\text{ kN}) + 2(2\text{ kN}) + 3(1\text{ kN} - 5.25\text{ kN})\big]$$
$$-(0.5\text{ m})F_{EG} = 0$$

$$F_{EG} = -13.5\text{ kN} \qquad F_{EG} = 13.50\text{ kN C} \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: 5.25\text{ kN} - 1\text{ kN} - 2\text{ kN} - 2\text{ kN} - \frac{1}{\sqrt{5}}F_{EF} = 0$$

$$F_{EF} = \frac{\sqrt{5}}{4} = 0.5590\text{ kN} \qquad F_{EF} = 559\text{ N T} \blacktriangleleft$$

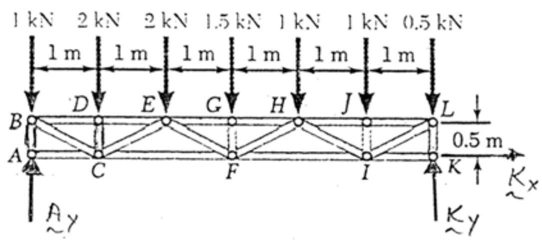


PROBLEM 6.47

A floor truss is loaded as shown. Determine the force in members *FI*, *HI*, and *HJ*.

SOLUTION

FBD Truss:

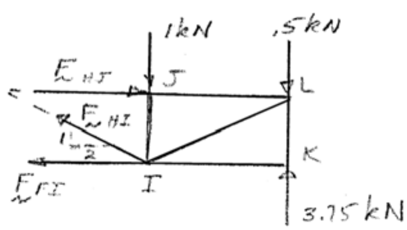


$$\rightarrow \Sigma F_x = 0: \mathbf{K}_x = 0$$

$$\circlearrowleft \Sigma M_A = 0: (1\text{m})\Big[6\big(K_y - 0.5\text{ kN}\big) - 5\big(1\text{ kN}\big) - 4\big(1\text{ kN}\big) - 3\big(1.5\text{ kN}\big) - 2\big(2\text{ kN}\big) - 1\big(2\text{ kN}\big)\Big] = 0$$

$$\mathbf{K}_y = 3.75\text{ kN} \uparrow$$

FBD Section:



$$\circlearrowleft \Sigma M_I = 0: \; (1\text{ m})(3.75\text{ kN} - .5\text{ kN}) - (.5\text{ m})F_{HJ} = 0$$

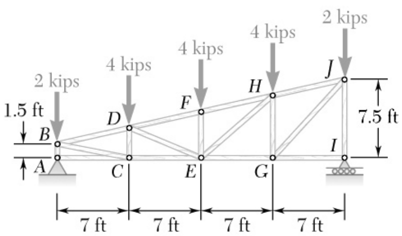
$F_{HJ} = 6.5\text{ kN}$ $\qquad$ $F_{HJ} = 6.50\text{ kN C}$ ◀

$$\uparrow \Sigma F_y = 0: \frac{1}{\sqrt{5}}F_{HI} - 1\text{ kN} - 0.5\text{ kN} + 3.75\text{ kN} = 0$$

$F_{HI} = -2.25\sqrt{5}\text{ kN}$ $\qquad$ $F_{HI} = 5.03\text{ kN C}$ ◀

$$\rightarrow \Sigma F_y = 0: -\frac{2}{\sqrt{5}}F_{HI} - F_{FI} + F_{HJ} = 0$$

$F_{FI} = 2\big(2.25\text{ kN}\big) + 6.50\text{ kN}$ $\qquad$ $F_{FI} = 11.00\text{ kN T}$ ◀

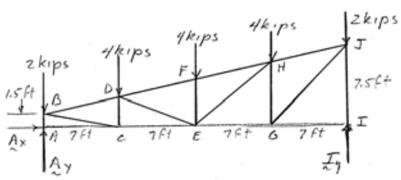


PROBLEM 6.48

A pitched flat roof truss is loaded as shown. Determine the force in members *CE*, *DE*, and *DF*.

SOLUTION

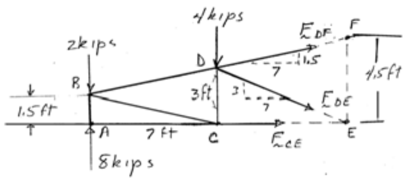
FBD Truss:



$$\rightarrow \Sigma F_x = 0: \mathbf{A}_x = 0$$

By load symmetry: $\mathbf{A}_y = \mathbf{I}_y = 8 \text{ kips} \uparrow$

FBD Section:



$$\circlearrowleft \Sigma M_D = 0: (7 \text{ ft})(2 \text{ kips} - 8 \text{ kips}) + (3 \text{ ft})(F_{CE}) = 0$$

$$F_{CE} = 14 \text{ kips} \qquad F_{CE} = 14.00 \text{ kips T} \blacktriangleleft$$

$$\circlearrowleft \Sigma M_E = 0: (7 \text{ ft})\left[1(4 \text{ kips}) + 2(2 \text{ kips} - 8 \text{ kips})\right]$$

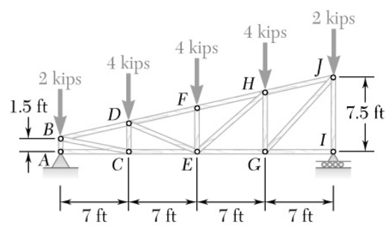
$$(4.5 \text{ ft})\frac{7}{\sqrt{51.25}} F_{DF} = 0$$

$$F_{DF} = \frac{8\sqrt{51.25}}{4.5} \text{ kips} \qquad F_{DF} = 12.73 \text{ kips C} \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: 8 \text{ kips} - 2 \text{ kips} - 4 \text{ kips} + \frac{1.5}{\sqrt{51.25}} \frac{8\sqrt{51.25}}{4.5} \text{ kips} - \frac{3}{\sqrt{58}} F_{DE} = 0$$

$$F_{DE} = -1.692 \text{ kips} \qquad F_{DE} = 1.692 \text{ kips C} \blacktriangleleft$$

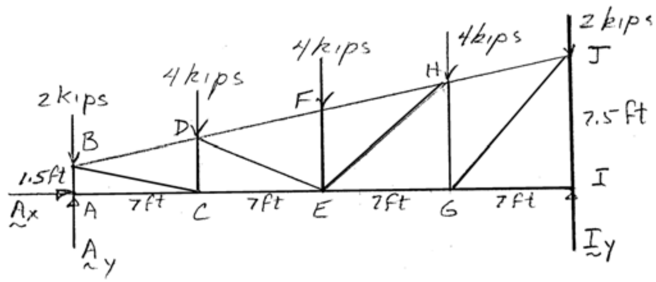
PROBLEM 6.49



A pitched flat roof truss is loaded as shown. Determine the force in members *EG*, *GH*, and *HJ*.

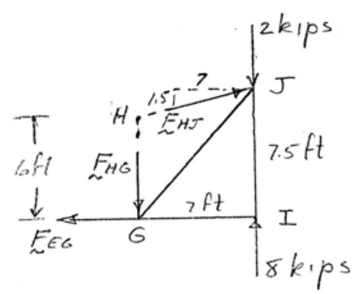
SOLUTION

FBD Truss:



By load symmetry: $\mathbf{A}_y = \mathbf{I}_y = 8 \text{ kips} \uparrow$

FBD Section:



$$\circlearrowright \Sigma M = 0: (7 \text{ ft})(8 \text{ kips} - 2 \text{ kips}) - (6 \text{ ft})F_{EG} = 0$$

$F_{EG} = 7.00 \text{ kips T}$ ◀

$$\longrightarrow \Sigma F_x = 0: \frac{7}{\sqrt{51.25}} F_{HJ} - 7.00 \text{ kips} = 0$$

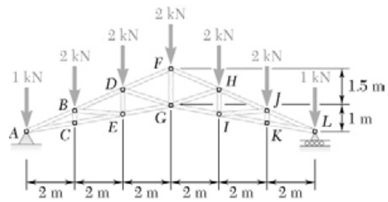
$F_{HJ} = \sqrt{51.25}$ kips

$F_{HJ} = 7.16 \text{ kips C}$ ◀

$$\uparrow \Sigma F_y = 0: \frac{1.5}{\sqrt{51.25}}\left(\sqrt{51.25} \text{ kips}\right) - F_{HG} + (8 - 2) \text{ kips} = 0$$

$F_{HG} = 7.50 \text{ kips C}$ ◀

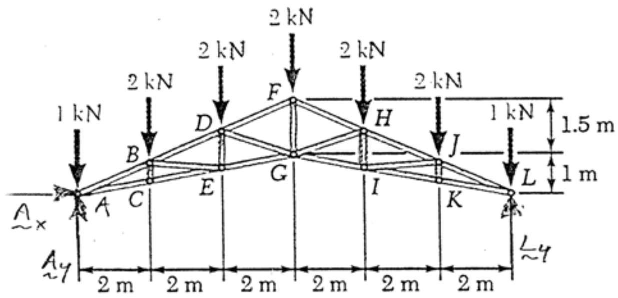
PROBLEM 6.50



A Howe scissors roof truss is loaded as shown. Determine the force in members *DF*, *DG*, and *EG*.

SOLUTION

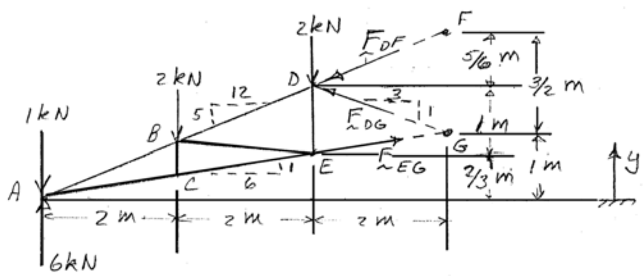
FBD Truss:



$\longrightarrow \Sigma F_X = 0: \quad \mathbf{A}_x = 0$

By symmetry: $\mathbf{A}_y = \mathbf{L}_y = 6 \text{ kN} \uparrow$

FBD Section:



Notes:

$$y_F = \frac{15}{6} \text{ m}$$

$$y_D = \frac{2}{3} \cdot \frac{5}{2} = \frac{5}{3} \text{ m}$$

$$y_E = \frac{2}{3} \cdot 1 = \frac{2}{3} \text{ m}$$

$$y_F - y_D = \frac{5}{6} \text{ m}$$

$$y_G = 1 \text{ m}$$

$$y_D - y_G = \frac{2}{3} \text{ m}$$

PROBLEM 6.50 CONTINUED

$$\circlearrowleft \Sigma M_D = 0: (1\text{ m})\frac{6}{\sqrt{37}}F_{EG} + (2\text{ m})(2\text{ kN}) + (4\text{ m})(1\text{ kN} - 6\text{ kN}) = 0$$

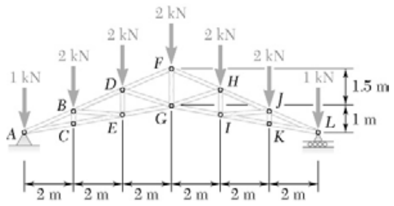
$$F_{EG} = \frac{8}{3}\sqrt{37}\text{ kN} \qquad F_{EG} = 16.22\text{ kN T} \blacktriangleleft$$

$$\circlearrowleft \Sigma M_A = 0: (2\text{ m})(2\text{ kN}) + (4\text{ m})(2\text{ kN}) - (6\text{ m})\left(\frac{1}{\sqrt{10}}F_{DG}\right) - (1\text{ m})\left(\frac{3}{\sqrt{10}}F_{DG}\right) = 0$$

$$F_{DG} = \frac{4}{3}\sqrt{10}\text{ kN} \qquad F_{DG} = 4.22\text{ kN C} \blacktriangleleft$$

$$\xrightarrow{} \Sigma F_x = 0: \frac{6}{\sqrt{37}}F_{EG} - \frac{3}{\sqrt{10}}F_{DG} - \frac{12}{13}F_{DF} = 0 \qquad 16 - 4 - \frac{12}{13}F_{DF} = 0$$

$$F_{DF} = 13\text{ kN} \qquad F_{DF} = 13.00\text{ kN C} \blacktriangleleft$$

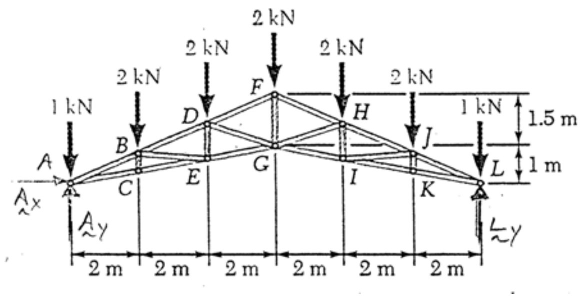


PROBLEM 6.51

A Howe scissors roof truss is loaded as shown. Determine the force in members *GI*, *HI*, and *HJ*.

SOLUTION

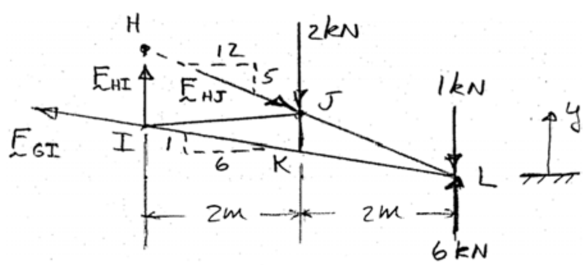
FBD Truss:



$$\xrightarrow{} \Sigma F_x = 0: \quad \mathbf{A}_x = 0$$

By symmetry: $\mathbf{A}_y = \mathbf{L}_y = 6 \text{ kN} \uparrow$

FBD Section:



Notes: $y_I = \dfrac{2}{3}$ m

$$y_H = \frac{2}{3} \cdot \frac{5}{2} = \frac{5}{3} \text{ m}$$

so $y_H - y_I = 1 \text{ m}$

PROBLEM 6.51 CONTINUED

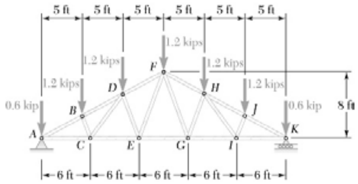
$$\circlearrowleft \Sigma M_I = 0: (4\text{ m})(6\text{ kN} - 1\text{ kN}) - (2\text{ m})(2\text{ kN}) - (1\text{ m})\left(\frac{12}{13}F_{HJ}\right) = 0$$

$$F_{HJ} = \frac{52}{3}\text{ kN} \qquad F_{HJ} = 17.33\text{ kN C} \blacktriangleleft$$

$$\circlearrowleft \Sigma M_H = 0: (4\text{ m})(6\text{ kN} - 1\text{ kN}) - (2\text{ m})(2\text{ kN}) - (1\text{ m})\left(\frac{6}{\sqrt{37}}F_{GI}\right) = 0$$

$$F_{GI} = \frac{8}{3}\sqrt{37}\text{ kN} \qquad F_{GI} = 16.22\text{ kN T} \blacktriangleleft$$

$$\circlearrowleft \Sigma M_L = 0: (2\text{ m})(2\text{ kN}) - (4\text{ m})F_{HI} = 0 \qquad F_{HI} = 1.000\text{ kN T} \blacktriangleleft$$

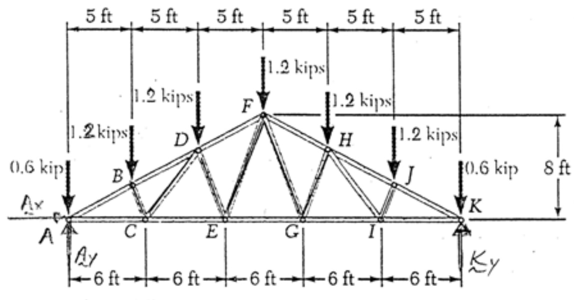


PROBLEM 6.52

A Fink roof truss is loaded as shown. Determine the force in members *BD*, *CD*, and *CE*.

SOLUTION

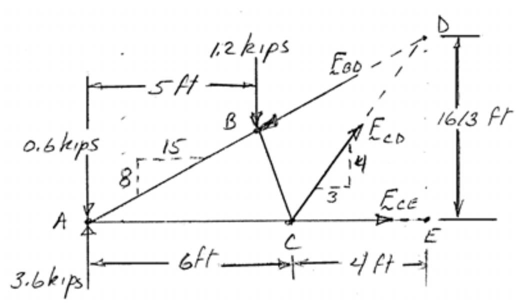
FBD Truss:



$$\xrightarrow{} \Sigma F_x = 0: \quad \mathbf{A}_x = 0$$

$$\text{By symmetry: } \mathbf{A}_y = \mathbf{K}_y = 3.6 \text{ kips} \uparrow$$

FBD Section:



$$\circlearrowleft \Sigma M_D = 0: \left(\frac{16}{3}\text{ ft}\right)F_{CE} + (5\text{ ft})(1.2\text{ kips}) + (10\text{ ft})(0.6\text{ kips}) - (10\text{ ft})(3.6\text{ kips}) = 0$$

$$F_{CE} = 4.50 \text{ kips T} \blacktriangleleft$$

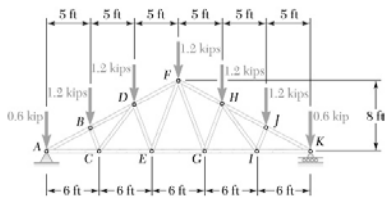
$$\circlearrowleft \Sigma M_A = 0: (6\text{ ft})\left(\frac{4}{5}F_{CD}\right) - (5\text{ ft})(1.2\text{ kips}) = 0$$

$$F_{CD} = 1.250 \text{ kips T} \blacktriangleleft$$

$$\circlearrowleft \Sigma F_y = 0: (3.6 - 0.6)\text{ kips} - 1.2\text{ kips} + \frac{4}{5}(1.25\text{ kips}) - \frac{8}{17}F_{BD} = 0$$

$$F_{BD} = 5.95 \text{ kips}$$

$$F_{BD} = 5.95 \text{ kips C} \blacktriangleleft$$

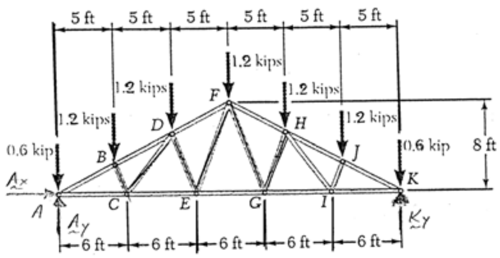


PROBLEM 6.53

A Fink roof truss is loaded as shown. Determine the force in members *FH*, *FG*, and *EG*.

SOLUTION

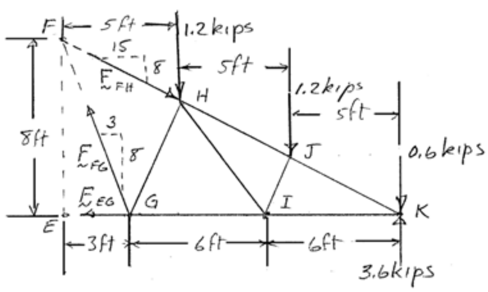
FBD Truss:



$$\xrightarrow{+} \Sigma F_x = 0: \quad \mathbf{A}_x = 0$$

By symmetry: $\mathbf{A}_y = \mathbf{K}_y = 3.6 \text{ kips} \uparrow$

FBD Section:



$$\circlearrowleft \Sigma M_F = 0: (15 \text{ ft})(3.6 - .6) \text{ kips} - (10 \text{ ft})(1.2 \text{ kips}) - (5 \text{ ft})(1.2 \text{ kips}) - (8\text{ft})F_{EG} = 0$$

$F_{EG} = 3.375 \text{ kips}$ $\qquad$ $F_{EG} = 3.38 \text{ kips T}$ ◀

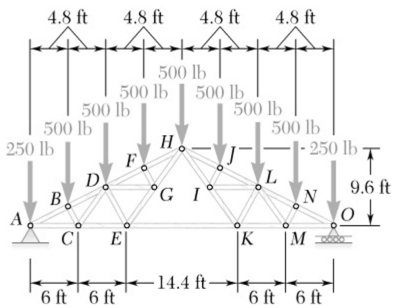
$$\circlearrowleft \Sigma M_K = 0: (5 \text{ ft})(1.2 \text{ kips}) + (10 \text{ ft})(1.2 \text{ kips}) - (12 \text{ ft})\left(\frac{8}{\sqrt{73}} F_{FG}\right) = 0$$

$F_{FG} = \dfrac{3}{16}\sqrt{73} \text{ kips}$ $\qquad$ $F_{FG} = 1.602 \text{ kips T}$ ◀

$$+\uparrow \Sigma F_y = 0 \qquad \frac{8}{\sqrt{73}}\left(\frac{3}{16}\sqrt{73} \text{ kips}\right) - \frac{8}{17}F_{FH} - 1.2 \text{ kips} - 1.2 \text{ kips} - 0.6 \text{ kip} + 3.6 \text{ kips} = 0$$

$F_{FH} = 4.4625 \text{ kips}$ $\qquad$ $F_{FH} = 4.46 \text{ kips C}$ ◀

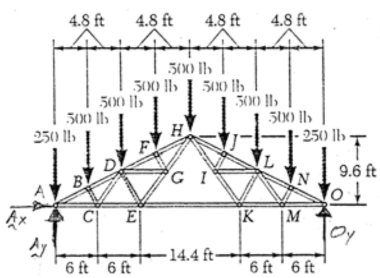
PROBLEM 6.54



A Fink roof truss is loaded as shown. Determine the force in members *DF*, *DG*, and *EG*. (*Hint:* First determine the force in member *EK*.)

SOLUTION

FBD Truss:

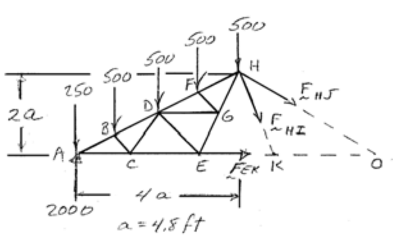


$\longrightarrow \Sigma F_x = 0: \quad \mathbf{A}_x = 0$

By symmetry: $\mathbf{A}_y = \mathbf{O}_y = 2000 \text{ lb} \uparrow$

FBD Sections:

Forces in 1b



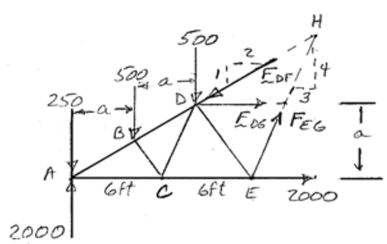
$$\circlearrowleft \Sigma M_H = 0: (a + 2a + 3a)(500 \text{ lb}) + 4a(250 \text{ lb})$$

$$- 4a(2000 \text{ lb}) + 2aF_{EK} = 0$$

$$F_{EK} = 2000 \text{ lb T}$$

$$\circlearrowleft \Sigma M_D = 0: a(500 \text{ lb}) + 2a(250 \text{ lb}) - 2a(2000 \text{ lb}) + a(2000 \text{ lb})$$

$$+ a\left(\frac{3}{5}F_{EG}\right) + (12 \text{ ft} - 2a)\left(\frac{4}{5}F_{EG}\right) = 0$$



$$F_{EG} = 1000 \text{ lb} \qquad F_{EG} = 1.000 \text{ kip T} \blacktriangleleft$$

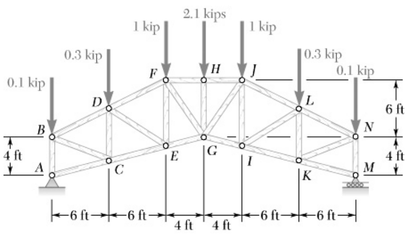
$$\uparrow \Sigma F_y = 0: \frac{4}{5}(1000 \text{ lb}) - \frac{1}{\sqrt{5}}F_{DF} + (2000 - 250 - 500 - 500) \text{ lb} = 0$$

$$F_{DF} = 1550\sqrt{5} \text{ lb} \qquad F_{DF} = 3.47 \text{ kips C} \blacktriangleleft$$

$$\longrightarrow \Sigma F_x = 0: 2000 \text{ lb} + \frac{3}{5}(1000 \text{ lb}) - \frac{2}{\sqrt{5}}(1550\sqrt{5} \text{ lb}) + F_{DG} = 0$$

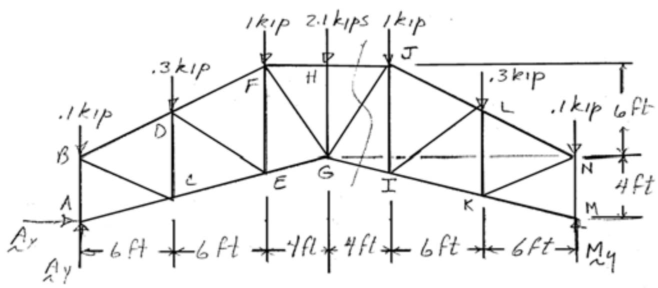
$$F_{DG} = 500 \text{ lb T} \blacktriangleleft$$

PROBLEM 6.55



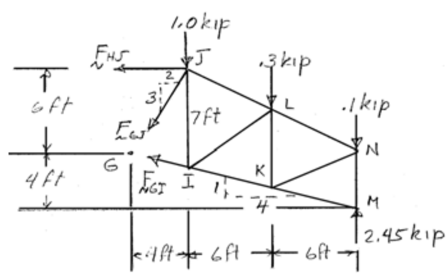
A roof truss is loaded as shown. Determine the force in members *FH*, *GJ*, and *GI*.

SOLUTION



By symmetry: $\mathbf{A}_y = \mathbf{M}_y = 2.45 \text{ kips} \uparrow$

$\xrightarrow{} \Sigma F_x = 0: \quad \mathbf{A}_x = 0$



$$\circlearrowleft \Sigma M_J = 6 \text{ ft}(0.3 \text{ kip}) - 12 \text{ ft}(2.35 \text{ kips}) + 7 \text{ ft}\left(\frac{4}{\sqrt{17}} F_{GI}\right) = 0$$

$$F_{GI} = 3.887 \text{ kips} \qquad\qquad F_{GI} = 3.89 \text{ kips T} \blacktriangleleft$$

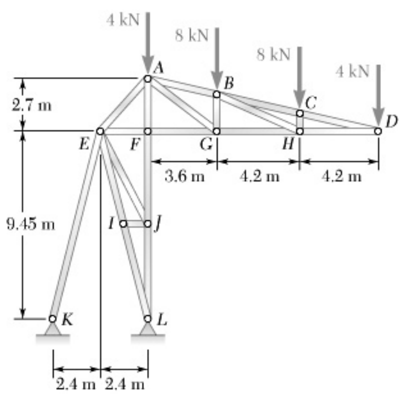
$$\circlearrowleft \Sigma M_G = (6 \text{ ft})F_{HJ} - (4 \text{ ft})(1.0 \text{ kips}) - (10 \text{ ft})(0.3 \text{ kips}) - (16 \text{ ft})(0.1 \text{ kips})$$

$$+ \ (16 \text{ ft})(2.45 \text{ kips}) = 0 \qquad\qquad F_{HJ} = -5.10 = 5.10 \text{ kips C}$$

By inspection: $F_{FH} = F_{HJ} = 5.10 \text{ kips C} \blacktriangleleft$

$$\uparrow \Sigma F_y = 0: \frac{1}{\sqrt{17}}(3.887 \text{ kips}) + 2.45 \text{ kips} - 1.4 \text{ kips} - \frac{3}{\sqrt{13}} F_{GJ} = 0$$

$$F_{GJ} = 2.40 \text{ kips T} \blacktriangleleft$$

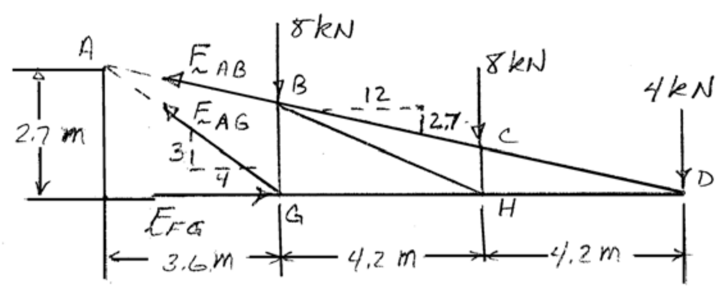


PROBLEM 6.56

A stadium roof truss is loaded as shown. Determine the force in members *AB*, *AG*, and *FG*.

SOLUTION

FBD Section:



Note: $\widehat{BG} = \dfrac{8.4}{12.0}(2.7\text{ m}) = 1.89\text{ m}$

$$\circlearrowleft \Sigma M_A = 0: (2.7\text{ m})F_{FG} - (3.6\text{ m})(8\text{ kN}) - (7.8\text{ m})(8\text{ kN}) - (12\text{ m})(4\text{ kN}) = 0$$

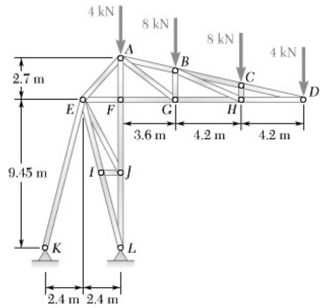
$F_{FG} = 51.56\text{ kN}$ $\qquad F_{FG} = 51.6\text{ kN C}$ ◀

$$\circlearrowleft \Sigma M_G = 0: (1.89\text{ m})\left(\frac{12}{12.3}F_{AB}\right) - (4.2\text{ m})(8\text{ kN}) - (8.4\text{ m})(4\text{ kN}) = 0$$

$F_{AB} = 36.44\text{ kN}$ $\qquad F_{AB} = 36.4\text{ kN T}$ ◀

$$\circlearrowleft \Sigma M_D = 0: (4.2\text{ m})(8\text{ kN}) + (8.4\text{ m})(8\text{ kN}) - (8.4\text{ m})\left(\frac{3}{5}F_{AG}\right) = 0$$

$F_{AG} = 20.0\text{ kN T}$ ◀

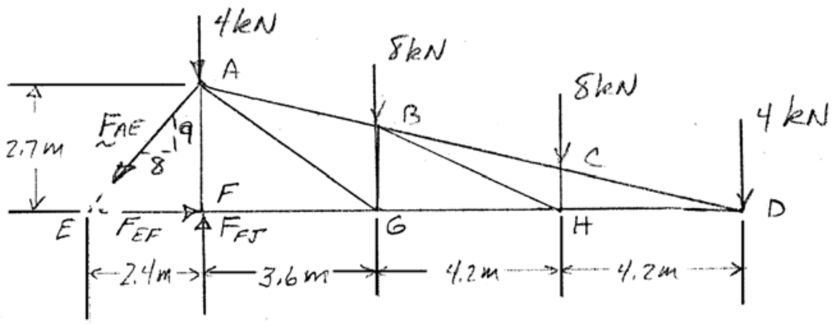


PROBLEM 6.57

A stadium roof truss is loaded as shown. Determine the force in members *AE*, *EF*, and *FJ*.

SOLUTION

FBD Section:



$$\circlearrowleft \Sigma M_F = 0: (2.7\text{ m})\left(\frac{8}{\sqrt{145}}F_{AE}\right) - (3.6\text{ m})(8\text{ kN}) - (7.8\text{ m})(8\text{ kN}) - (12\text{ m})(4\text{ kN}) = 0$$

$$F_{AE} = \frac{17.4}{2.7}\sqrt{145}\text{ kN}$$

$F_{AE} = 77.6\text{ kN T}$ ◀

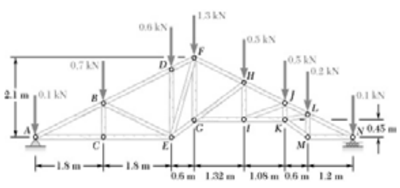
$$\xrightarrow{} \Sigma F_x = 0: F_{EF} - \frac{8}{\sqrt{145}}\left(\frac{17.4}{2.7}\sqrt{145}\text{ kN}\right) = 0$$

$$F_{EF} = 51.555\text{ kN}$$

$F_{EF} = 51.6\text{ kN C}$ ◀

$$\uparrow \Sigma F_y = 0: F_{FJ} - \frac{9}{\sqrt{145}}\left(\frac{17.4}{2.7}\sqrt{145}\text{ kN}\right) - (4+8+8+4)\text{ kN} = 0$$

$F_{FJ} = 82.0\text{ kN C}$ ◀

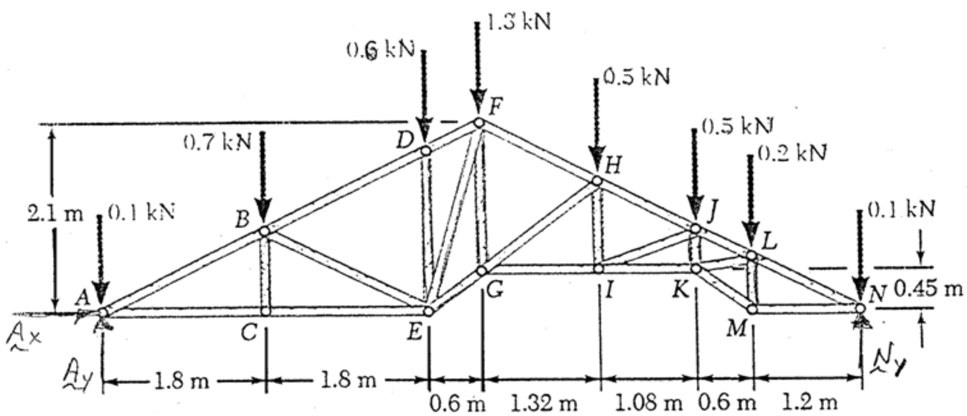


PROBLEM 6.58

A vaulted roof truss is loaded as shown. Determine the force in members *BE*, *CE*, and *DF*.

SOLUTION

FBD Truss:



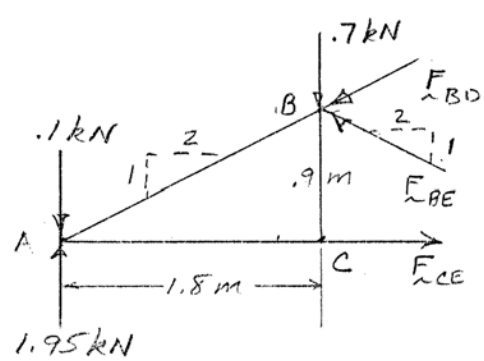
$$\circlearrowleft \Sigma M_N = 0: (1.2\text{ m})(0.2\text{ kN}) + (1.8\text{ m})(0.5\text{ kN}) + (2.88\text{ m})(0.5\text{ kN}) + (4.2\text{ m})(1.3\text{ kN})$$

$$+ (4.8\text{ m})(0.6\text{ kN}) + (6.6\text{ m})(0.7\text{ kN}) + (8.4\text{ m})(0.1\text{ kN}) - (8.4\text{ m})A_y = 0$$

$$\mathbf{A}_y = 1.95\text{ kN} \uparrow$$

$$\longrightarrow \Sigma F_x = 0: \mathbf{A}_x = 0$$

FBD Section:



$$\circlearrowleft \Sigma M_B = 0: (0.9\text{ m})F_{CE} - (1.8\text{ m})(1.95\text{ kN} - 0.1\text{ kN}) = 0$$

$F_{CE} = 3.70\text{ kN T}$ ◀

PROBLEM 6.58 CONTINUED

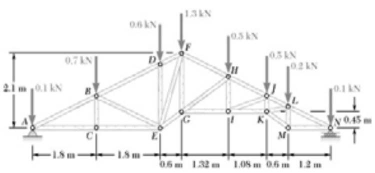
$$\circlearrowright \Sigma M_A = 0: (1.8 \text{ m})\left[\left(\frac{1}{\sqrt{5}}F_{BE}\right) - .7 \text{ kN}\right] + (.9 \text{ m})\left(\frac{2}{\sqrt{5}}F_{BE}\right) = 0$$

$$F_{BE} = 0.35\sqrt{5} \text{ kN} \qquad\qquad F_{BE} = 783 \text{ N C} \blacktriangleleft$$

$$\xrightarrow{} \Sigma F_x = 0: 3.70 \text{ kN} - \frac{2}{\sqrt{5}}\left(0.35\sqrt{5} \text{ kN}\right) - \frac{2}{\sqrt{5}}F_{BD} = 0$$

$$F_{BD} = 1.5\sqrt{5} \text{ kN} = 3.35 \text{ kN C}$$

Then by inspection of joint D: $F_{DF} = F_{BD}$ so $F_{DF} = 3.35 \text{ kN C} \blacktriangleleft$

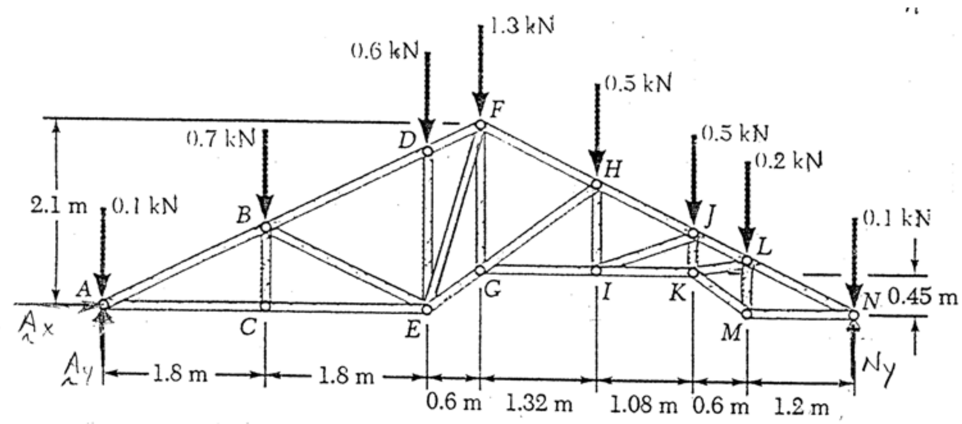


PROBLEM 6.59

A vaulted roof truss is loaded as shown. Determine the force in members *HJ*, *IJ*, and *GI*.

SOLUTION

FBD Truss:

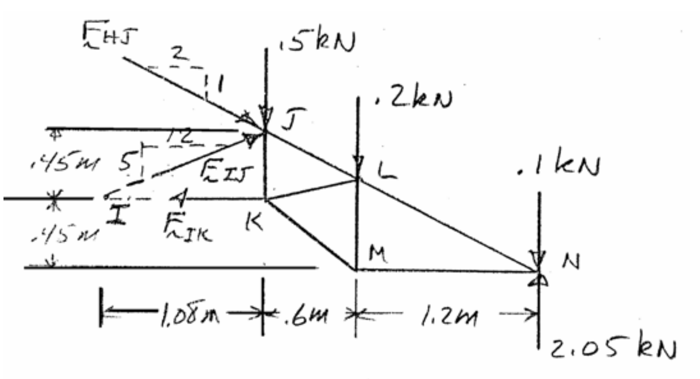


$$\circlearrowleft \Sigma M_A = 0\text{: } (1.8\text{ m})(0.7\text{ kN}) + (3.6\text{ m})(0.6\text{ kN}) + (4.2\text{ m})(1.3\text{ kN}) + (5.52\text{ m})(0.5\text{ kN})$$

$$+ (6.6\text{ m})(0.5\text{ kN}) + (7.2\text{ m})(0.2\text{ kN}) + (8.4\text{ m})(0.1\text{ kN}) - (8.4\text{ m})(N_y) = 0$$

$$\mathbf{N}_y = 2.05\text{ kN} \uparrow$$

FBD Section:



$$\circlearrowleft \Sigma M_J = 0\text{: } (1.8\text{ m})(2.05 - 0.1)\text{ kN} - (0.6\text{ m})(0.2\text{ kN}) + (0.45\text{ m})F_{IK} = 0$$

$$F_{IK} = 7.533\text{ kN} \qquad\qquad F_{IK} = 7.53\text{ kN T}$$

PROBLEM 6.59 CONTINUED

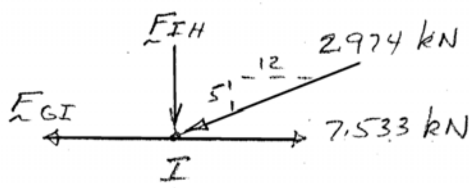
$$\circlearrowleft \Sigma M_I = 0: (2.88 \text{ m})(2.05 - 0.1) \text{ kN} - (1.68 \text{ m})(0.2 \text{ kN}) - (0.45 \text{ m})\left(\frac{2}{\sqrt{5}} F_{HJ}\right) - (1.08 \text{ m})\left(\frac{1}{\sqrt{5}} F_{HJ}\right) = 0$$

$F_{HJ} = 2.3939\sqrt{5}$ kN $\qquad$ $F_{HJ} = 5.35$ kN C ◀

$$\uparrow \Sigma F_y = 0: -\frac{1}{\sqrt{5}}\left(2.3939\sqrt{5} \text{ kN}\right) + \frac{5}{13} F_{IJ} + 2.05 \text{ kN} - 0.5 \text{ kN} - 0.2 \text{ kN} - 0.1 \text{ kN} = 0$$

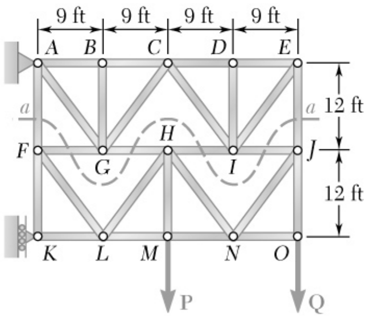
$F_{IJ} = 2.974$ kN $\qquad$ $F_{IJ} = 2.97$ kN C ◀

FBD Joint:



$$\longrightarrow \Sigma F_x = 0: -F_{GI} - \frac{12}{13}(2.974 \text{ kN}) + 7.533 \text{ kN} = 0$$

$F_{GI} = 4.788$ kN $\qquad$ $F_{GI} = 4.79$ kN T ◀

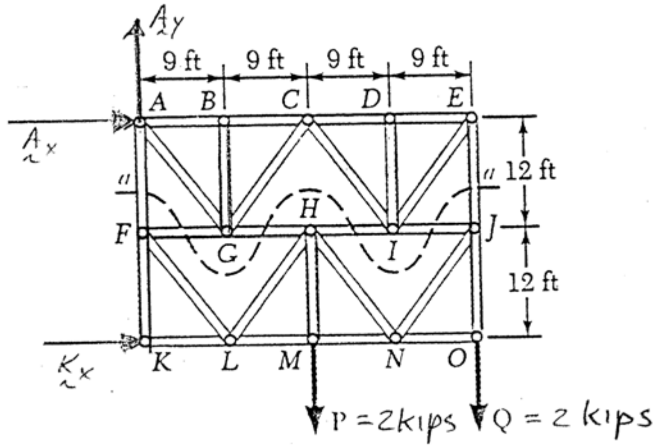


PROBLEM 6.60

Determine the force in members AF and EJ of the truss shown when $P = Q = 2$ kips. (*Hint:* Use section aa.)

SOLUTION

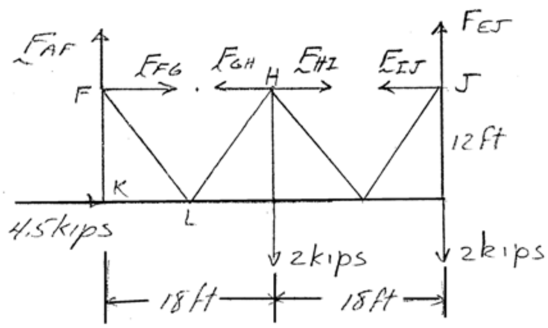
FBD Truss:



$$\circlearrowleft \Sigma M_A = 0: (24 \text{ ft})K_x - (18 \text{ ft})(2 \text{ kips}) - (36 \text{ ft})(2 \text{ kips}) = 0$$

$$\mathbf{K}_x = 4.5 \text{ kips} \longrightarrow$$

FBD Section:

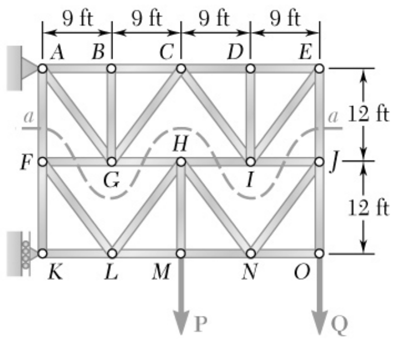


$$\circlearrowleft \Sigma M_F = 0: (12 \text{ ft})(4.5 \text{ kips}) - (18 \text{ ft})(2 \text{ kips}) - (36 \text{ ft})(2 \text{ kips}) + (36 \text{ ft})F_{EJ} = 0$$

$$F_{EJ} = 1.500 \text{ kips T} \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: (18 \text{ ft})(2 \text{ kips}) + (12 \text{ ft})(4.5 \text{ kips}) - (36 \text{ ft})F_{AF} = 0$$

$$F_{AF} = 2.50 \text{ kips T} \blacktriangleleft$$

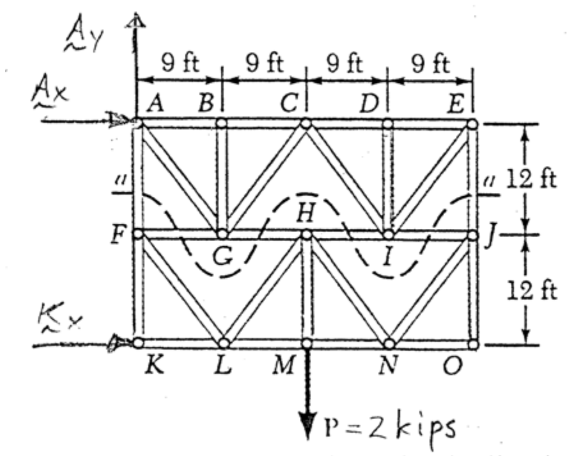


PROBLEM 6.61

Determine the force in members AF and EJ of the truss shown when $P = 2$ kips and $Q = 0$. (*Hint:* Use section aa.)

SOLUTION

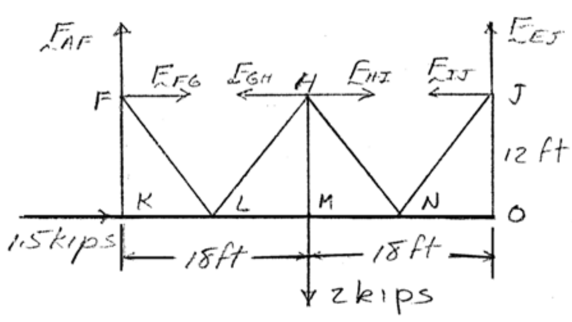
FBD Truss:



$$\circlearrowleft \Sigma M_A = 0: (24 \text{ ft})K_x - (18 \text{ ft})(2 \text{ kips}) = 0$$

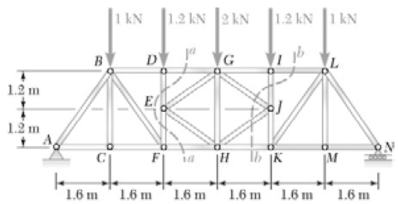
$$\mathbf{K}_x = 1.5 \text{ kips} \longrightarrow$$

FBD Section:



$$\circlearrowleft \Sigma M_F = 0: (12 \text{ ft})(1.5 \text{ kips}) - (18 \text{ ft})(2 \text{ kips}) + (36 \text{ ft})F_{EJ} = 0 \qquad F_{EJ} = 0.500 \text{ kip T} \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: (18 \text{ ft})(2 \text{ kips}) + (12 \text{ ft})(1.5 \text{ kips}) - (36 \text{ ft})F_{AF} = 0 \qquad F_{AF} = 1.500 \text{ kips T} \blacktriangleleft$$

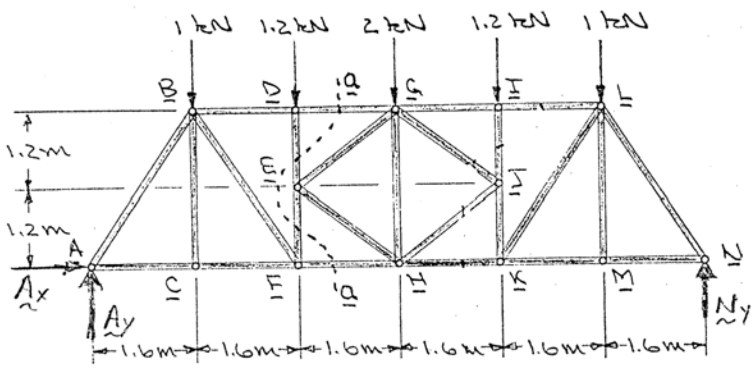


PROBLEM 6.62

Determine the force in members *DG* and *FH* of the truss shown. (*Hint:* Use section *aa*.)

SOLUTION

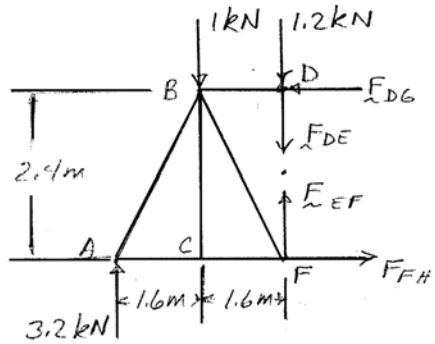
FBD Truss:



$$\rightarrow \Sigma F_x = 0: \quad \mathbf{A}_x = 0$$

By symmetry: $\mathbf{A}_y = \mathbf{N}_y = 3.2 \text{ kN} \uparrow$

FBD Section:

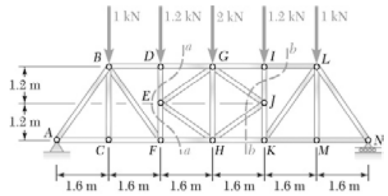


$$\circlearrowleft \Sigma M_F = 0: (2.4 \text{ m})F_{DG} - (3.2 \text{ m})(3.2 \text{ kN}) + (1.6 \text{ m})(1 \text{ kN}) = 0$$

$$F_{DG} = 3.60 \text{ kN C} \blacktriangleleft$$

$$\circlearrowleft \Sigma M_D = 0: (2.4 \text{ m})F_{FH} + (1.6 \text{ m})(1 \text{ kN}) - (3.2 \text{ m})(3.2 \text{ kN}) = 0$$

$$F_{FH} = 3.60 \text{ kN T} \blacktriangleleft$$

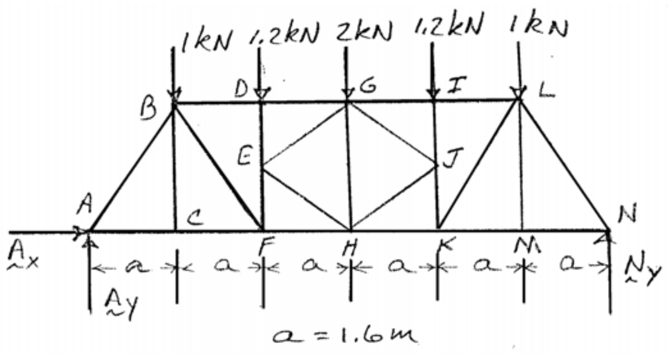


PROBLEM 6.63

Determine the force in members *IL*, *GJ*, and *HK* of the truss shown. (*Hint:* Begin with pins *I* and *J* and then use section *bb*.)

SOLUTION

FBD Truss:

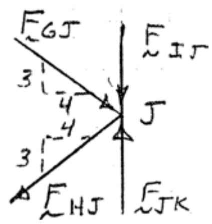


$\longrightarrow \Sigma F_x = 0$: $\quad \mathbf{A}_x = 0$

By symmetry: $A_x = 0$; $A_y = N_y = 3.2$ kN $\uparrow$

Joints: By inspection of joint *I*: $F_{GI} = F_{IL}$ and $F_{IJ} = 1.2$ kN C

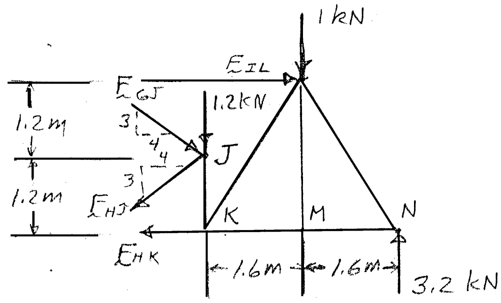
By inspection of joint *J*:



$F_{GJ}\,(\mathrm{C}) = F_{HJ}\,(\mathrm{T})$

PROBLEM 6.63 CONTINUED

FBD Section:



$$\xrightarrow{} \Sigma F_x = 0; F_{IL} - F_{HK} + \frac{4}{5}\left(\cancel{F_{GJ} - F_{HJ}}^{\,0}\right) = 0 \qquad \text{so} \qquad F_{IL} - F_{HK} = 0$$

$$\circlearrowleft \Sigma M_J = 0: (3.2\text{ m})(3.2\text{ kN}) - (1.2\text{ m})(F_{IL} + F_{HK}) - (1.6\text{ m})(1\text{ kN}) = 0 \qquad \text{so} \qquad F_{IL} + F_{HK} = 7.2\text{ kN}$$

Solving: $F_{IL} = F_{HK} = 3.6\text{ kN}$

$F_{IL} = 3.60\text{ kN C}$ ◀

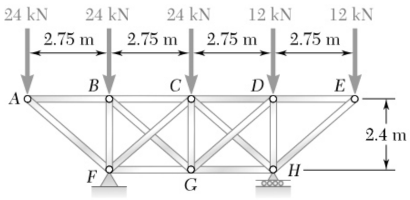
$F_{HK} = 3.60\text{ kN T}$ ◀

$$\uparrow \Sigma F_y = 0: 3.2\text{ kN} - 1\text{ kN} - 1.2\text{ kN} - \frac{3}{5}(F_{GJ} + F_{HJ}) = 0$$

$F_{GJ} + F_{HJ} = \frac{5}{3}\text{ kN}$ but $F_{GJ} = F_{HJ} = \frac{5}{6}\text{ kN}$

$F_{GJ} = .833\text{ kN C}$ ◀

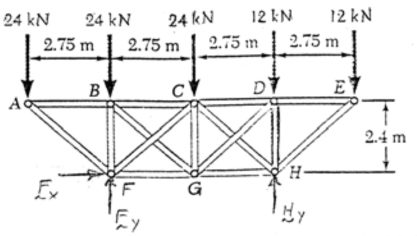
PROBLEM 6.64



The diagonal members in the center panels of the truss shown are very slender and can act only in tension; such members are known as *counters*. Determine the forces in the counters which are acting under the given loading.

SOLUTION

FBD Truss:



$$\rightarrow \Sigma F_x = 0: \mathbf{F}_x = 0$$

$$\Sigma M_F = 0: (5.5 \text{ m})H_y + (2.75 \text{ m})(24 \text{ kN}) - (2.75 \text{ m})(24 \text{ kN})$$

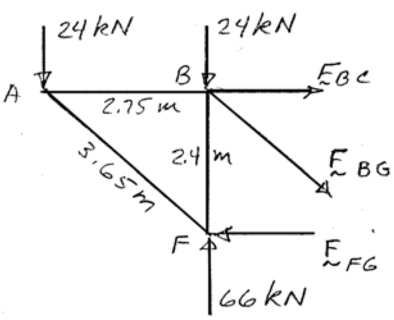
$$- (5.5 \text{ m})(12 \text{ kN}) - (8.25 \text{ m})(12 \text{ kN}) = 0$$

$$\mathbf{H}_y = 30 \text{ kN} \uparrow$$

$$\uparrow \Sigma F_y = 0: F_y + 30 \text{ kN} - 3(24 \text{ kN}) - 2(12 \text{ kN}) = 0$$

$$\mathbf{F}_y = 66 \text{ kN} \uparrow$$

FBD Section:



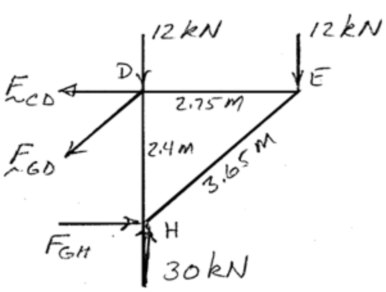
Assume there is no pretension in any counter so that, as the truss is loaded, one of each pair becomes taut while the other becomes slack.

Here tension is required in *BG* to provide downward force, so *CF* is slack.

$$\uparrow \Sigma F_y = 0: 66 \text{ kN} - 24 \text{ kN} - 24 \text{ kN} - \frac{2.4}{3.65}F_{BG} = 0$$

$F_{BG} = 27.375$ kN $\qquad F_{BG} = 27.4$ kN T ◀

$F_{CF} = 0$ ◀



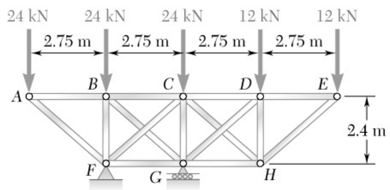
Here tension is required in *GD* to provide downward force, so *CH* is slack.

$F_{CH} = 0$ ◀

$$\uparrow \Sigma F_y = 0: 30 \text{ kN} - 2(12 \text{ kN}) - \frac{2.4}{3.65}F_{GD} = 0$$

$F_{GD} = 9.125$ kN $\qquad F_{GD} = 9.13$ kN T ◀

PROBLEM 6.65

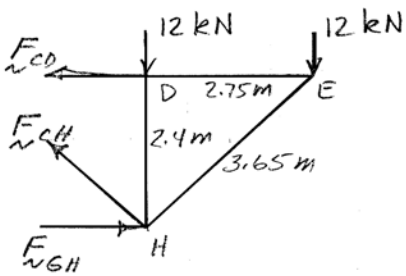


The diagonal members in the center panels of the truss shown are very slender and can act only in tension; such members are known as *counters*. Determine the forces in the counters which are acting under the given loading.

SOLUTION

Assume that there is no pretension in any counter. So that, as the truss is loaded, one of each crossing pair becomes taut while the other becomes slack.

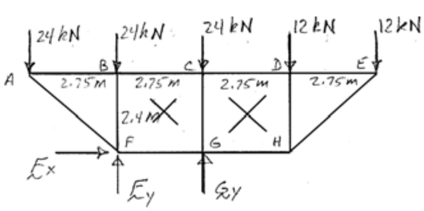
FBD Section:



Note: Tension is required in *CH* to provide upward force; thus *GD* is slack

$$\uparrow \Sigma F_y = 0: \frac{2.4}{3.65} F_{CH} - 24 \text{ kN} = 0 \qquad F_{CH} = 36.5 \text{ kN T} \blacktriangleleft$$

FBD Truss:



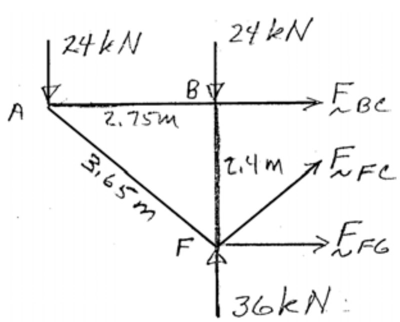
$$\circlearrowleft \Sigma M_G = 0: (5.5 \text{ m})(24 \text{ kN}) + (2.75 \text{ m})(24 \text{ kN}) - (2.75 \text{ m})(F_y)$$

$$- (2.75 \text{ m})(12 \text{ kN}) - (5.5 \text{ m})(12 \text{ kN}) = 0$$

$$\mathbf{F}_y = 36 \text{ kN} \uparrow$$

$$\Sigma F_x = 0: \mathbf{F}_x = 0$$

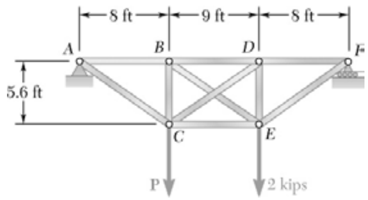
FBD Section:



Note: Tension is required in *FC* to provide upward force; so *BG* is slack.

$$\circlearrowleft \Sigma F_y = 0: (36 - 24 - 24) \text{ kN} + \frac{2.4}{3.65} F_{FC} = 0$$

$$F_{FC} = 18.25 \text{ kN T} \blacktriangleleft$$

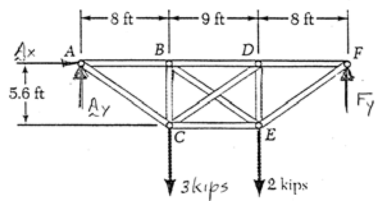


PROBLEM 6.66

The diagonal members in the center panel of the truss shown are very slender and can act only in tension; such members are known as *counters*. Determine the force in members *BD* and *CE* and in the counter which is acting when $P = 3$ kips.

SOLUTION

FBD Truss:



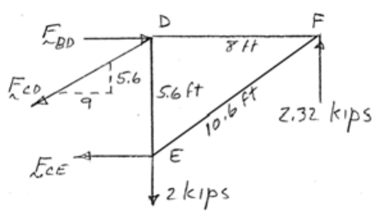
$$\xrightarrow{} \Sigma F_x = 0: \mathbf{A}_x = 0$$

$$\Sigma M_A = 0: (25 \text{ ft})F_y - (17 \text{ ft})(2 \text{ kips}) - (8 \text{ ft})(3 \text{ kips}) = 0$$

$$\mathbf{F}_y = 2.32 \text{ kips} \uparrow$$

Assume there is no pretension in either counter so that, as the truss is loaded, one becomes taut while the other becomes slack.

FBD Section:



Here tension is required in *CD* to provide downward force, so

$$F_{BE} = 0$$

$$\uparrow \Sigma F_y = 0: 2.32 \text{ kips} - 2 \text{ kips} - \frac{5.6}{10.6} F_{CD} = 0$$

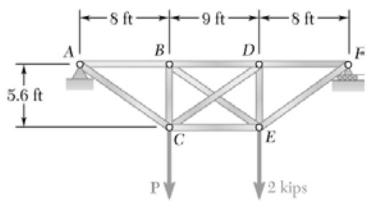
$$F_{CD} = 0.6057 \text{ kip} \qquad F_{CD} = 606 \text{ lb T} \blacktriangleleft$$

$$\Sigma M_D = 0: (8 \text{ ft})(2.32 \text{ kips}) - (5.6 \text{ ft})F_{CE} = 0$$

$$F_{CE} = 3.314 \text{ kips} \qquad F_{CE} = 3.31 \text{ kips T} \blacktriangleleft$$

$$\xrightarrow{} \Sigma F_x = 0: F_{BD} - \frac{9}{10.6}(0.6057 \text{ kip}) - 3.314 \text{ kips} = 0$$

$$F_{BD} = 3.829 \text{ kips} \qquad F_{BD} = 3.83 \text{ kips C} \blacktriangleleft$$

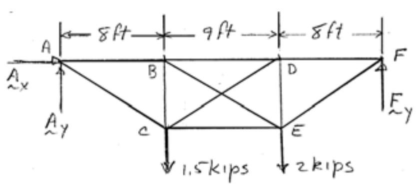


PROBLEM 6.67

Solve Prob. 6.66 when $P = 1.5$ kips.

SOLUTION

FBD Truss:



$$\circlearrowleft \Sigma M_A = 0: (25\text{ ft})F_y - (17\text{ ft})(2\text{ kips}) - (8\text{ ft})(1.5\text{ kips}) = 0$$

$$\mathbf{F}_y = 1.84\text{ kips} \uparrow$$

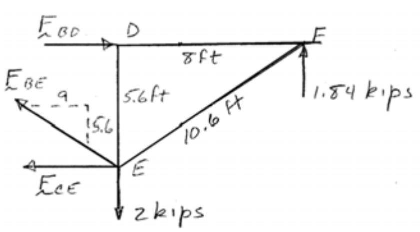
Here tension is needed in *BE* to provide upward force, so $F_{CD} = 0$

FBD Section:

$$\uparrow \Sigma F_y = 0: \frac{5.6}{10.6}F_{BE} + 1.84\text{ kips} - 2\text{ kips} = 0$$

$$F_{BE} = 0.03029\text{ kip}$$

$$F_{BE} = 303\text{ lb T} \blacktriangleleft$$



$$\circlearrowleft \Sigma M_E = 0: (8\text{ ft})(1.84\text{ kips}) - (5.6\text{ ft})F_{BD} = 0$$

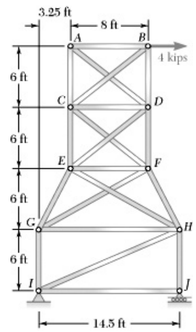
$$F_{BD} = 2.629\text{ kips}$$

$$F_{BD} = 2.63\text{ kips C} \blacktriangleleft$$

$$\longrightarrow \Sigma F_x = 0: 2.629\text{ kips} - \frac{9}{10.6}(0.3029\text{ kip}) - F_{CE} = 0$$

$$F_{CE} = 2.371\text{ kips}$$

$$F_{CE} = 2.37\text{ kips T} \blacktriangleleft$$

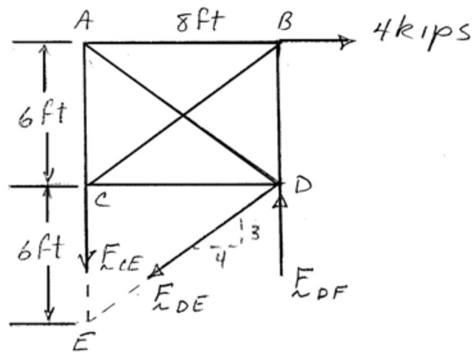


PROBLEM 6.68

The diagonal members *CF* and *DE* of the truss shown are very slender and can act only in tension; such members are known as *counters*. Determine the force in members *CE* and *DF* and in the counter which is acting.

SOLUTION

FBD Section:



DE must be in tension to provide leftward force, so *CF* is slack.

$$\xrightarrow{} \Sigma F_x = 0: 4\text{ kips} - \frac{4}{5}F_{DE} = 0$$

$$F_{DE} = 5.00\text{ kips T} \blacktriangleleft$$

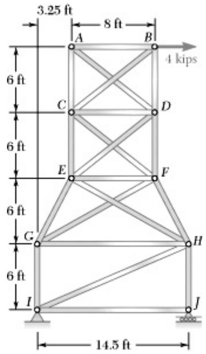
$$\circlearrowleft \Sigma M_D = 0: (8\text{ ft})F_{CE} - (6\text{ ft})(4\text{ kips}) = 0$$

$$F_{CE} = 3.00\text{ kips T} \blacktriangleleft$$

$$\circlearrowleft \Sigma M_E = 0: (8\text{ ft})F_{DF} - (12\text{ ft})(4\text{ kips}) = 0$$

$$F_{DF} = 6.00\text{ kips C} \blacktriangleleft$$

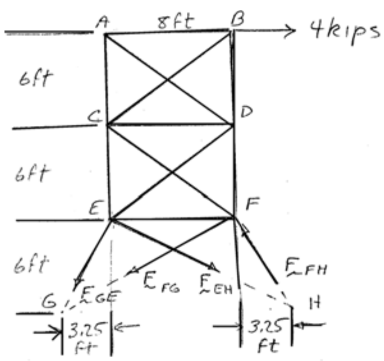
PROBLEM 6.69



The diagonal members *EH* and *FG* of the truss shown are very slender and can act only in tension; such members are known as *counters*. Determine the force in members *EG* and *FH* and in the counter which is acting.

SOLUTION

FBD Section:



It is not obvious which counter is active, so assume *FG* is (and thus *EH* is slack)

$$\circlearrowleft \Sigma M_F = 0: (8\text{ ft})\left(\frac{6}{6.824}F_{GE}\right) - (12\text{ ft})(4\text{ kips}) = 0$$

$$F_{GE} = 6.824\text{ kips T}$$

$$\circlearrowleft \Sigma M_G = 0: (14.5\text{ ft})\left(\frac{6}{6.824}F_{FH}\right) - (18\text{ ft})(4\text{ kips}) = 0$$

$$F_{FH} = 5.647\text{ kips C}$$

$$\uparrow \Sigma F_y = 0: \left(\frac{6}{6.824}\right)(5.647\text{ kips} - 6.824\text{ kips}) - \frac{6}{12.75}F_{FG} = 0$$

This gives $F_{FG} < 0$ which is impossible, so the assumption is wrong, *FG* is slack, and *EH* is in tension.

Then $$\circlearrowleft \Sigma M_E = 0: (8\text{ ft})\left(\frac{6}{6.824}F_{FH}\right) - (12\text{ ft})(4\text{ kips}) = 0$$

$$F_{FH} = 6.824\text{ kips}$$

$$F_{FH} = 6.83\text{ kips C} \blacktriangleleft$$

$$\circlearrowleft \Sigma M_H = 0: (14.5\text{ ft})\left(\frac{6}{6.824}F_{GE}\right) - (18\text{ ft})(4\text{ kips}) = 0$$

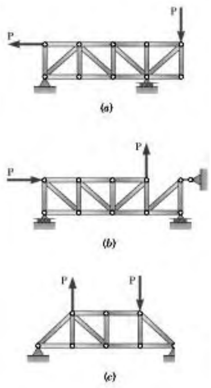
$$F_{GE} = 5.647\text{ kips}$$

$$F_{GE} = 5.65\text{ kips T} \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \frac{6}{6.824}(6.824\text{ kips} - 5.647\text{ kips}) - \frac{6}{12.75}F_{EH} = 0$$

$$F_{EH} = 2.198\text{ kips}$$

$$F_{EH} = 2.20\text{ kips T} \blacktriangleleft$$



PROBLEM 6.70

Classify each of the structures shown as completely, partially, or improperly constrained; if completely constrained, further classify it as determinate or indeterminate. (All members can act both in tension and in compression.)

SOLUTION

Structure (a):

Simple truss with $r = 4$, $m = 16$, $n = 10$

So $m + r = 20 = 2n$ so completely constrained and determinate ◀

Structure (b):

Compound truss with $r = 3$, $m = 16$, $n = 10$

So $m + r = 19 < 2n = 20$ so partially constrained ◀

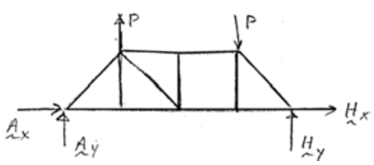
Structure (c):

Non-simple truss with $r = 4$, $m = 12$, $n = 8$

So $m + r = 16 = 2n$ but must examine further, note that reaction forces

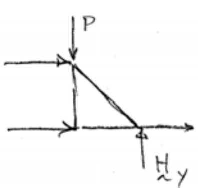
$\mathbf{A}_x$ and $\mathbf{H}_x$ are aligned, so no equilibrium equation will resolve them.

$\therefore$ Statically indeterminate ◀



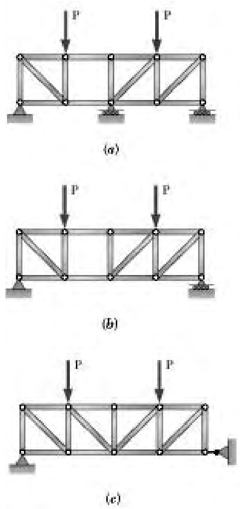
For $\Sigma F_y = 0$: $H_y = 0$, but then $\Sigma M_A \neq 0$ in FBD Truss,

Also consider



$\therefore$ Improperly constrained ◀

PROBLEM 6.71



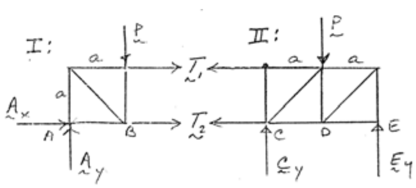
Classify each of the structures shown as completely, partially, or improperly constrained; if completely constrained, further classify it as determinate or indeterminate. (All members can act both in tension and in compression.)

SOLUTION

Structure (a): Non-simple truss with $r = 4, \; m = 16, \; n = 10$

so $m + r = 20 = 2n,$ but must examine further.

FBD Sections:



FBD I: $\Sigma M_A = 0 \;\Rightarrow\; T_1$

II: $\Sigma F_x = 0 \;\Rightarrow\; T_2$

I: $\Sigma F_x = 0 \;\Rightarrow\; A_x$

I: $\Sigma F_y = 0 \;\Rightarrow\; A_y$

II: $\Sigma M_E = 0 \;\Rightarrow\; C_y$

II: $\Sigma F_y = 0 \;\Rightarrow\; E_y$

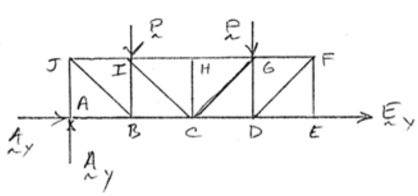
Since each section is a simple truss with reactions determined,

structure is completely constrained and determinate. ◀

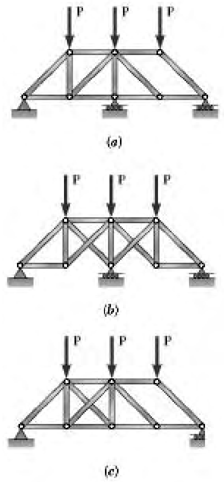
Structure (b): Non-simple truss with $r = 3, \; m = 16, \; n = 10$

so $m + r = 19 < 2n = 20 \quad \therefore$ structure is partially constrained ◀

Structure (c):



Simple truss with $r = 3, \; m = 17, \; n = 10$
$m + r = 20 = 2n,$ but the horizontal reaction forces A_x and E_x are collinear and no equilibrium equation will resolve them, so the structure is improperly constrained and indeterminate ◀

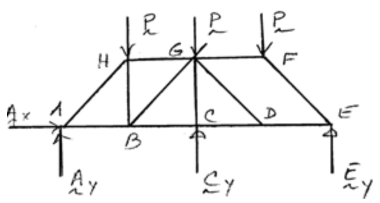


PROBLEM 6.72

Classify each of the structures shown as completely, partially, or improperly constrained; if completely constrained, further classify it as determinate or indeterminate. (All members can act both in tension and in compression.)

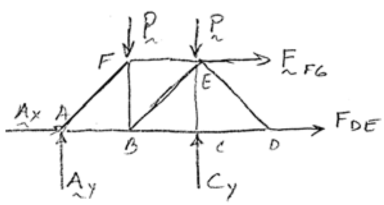
SOLUTION

Structure (a):



Non-simple truss with $r = 4,\ \ m = 12,\ \ n = 8$ so $r + m = 16 = 2n,$ check for determinacy:

One can solve joint F for forces in EF, FG and then solve joint E for $\mathbf{E}_y$ and force in DE.

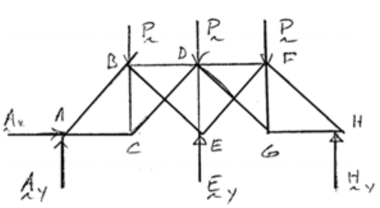


This leaves a simple truss $ABCDGH$ with

$$r = 3,\ \ m = 9,\ \ n = 6 \qquad \text{so} \qquad r + m = 12 = 2n$$

Structure is completely constrained and determinate ◀

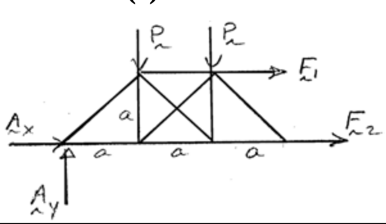
Structure (b):



Simple truss (start with ABC and add joints alphabetically to complete truss) with $r = 4,\ \ m = 13,\ \ n = 8$

so $\qquad r + m = 17 > 2n = 16 \qquad$ Constrained but indeterminate ◀

Structure (c):

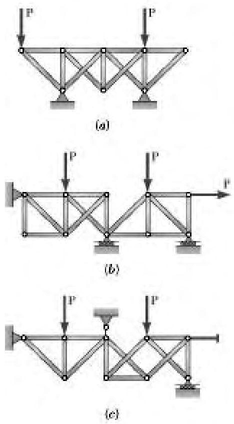


Non-simple truss with $r = 3,\ \ m = 13,\ \ n = 8$ so $r + m = 16 = 2n$. To further examine, follow procedure in part (a) above to get truss at left.

Since $\mathbf{F}_1 \neq 0$ (from solution of joint F),

$\Sigma M_A = aF_1 \neq 0$ and there is no equilibrium.

Structure is improperly constrained ◀

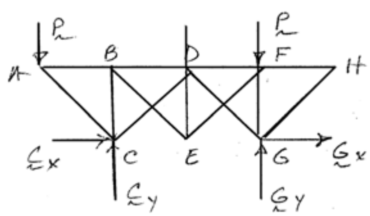


PROBLEM 6.73

Classify each of the structures shown as completely, partially, or improperly constrained; if completely constrained, further classify it as determinate or indeterminate. (All members can act both in tension and in compression.)

SOLUTION

Structure (a):

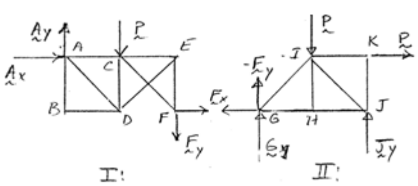


Simple truss (start with *ABC* and add joints alphabetical to complete truss), with

$$r = 4, \quad m = 13, \quad n = 8 \qquad \text{so} \qquad r + m = 17 > 2n = 16$$

Structure is completely constrained but indeterminate. ◀

Structure (b):



From FBD II: $\Sigma M_G = 0 \Rightarrow J_y$

$\Sigma F_x = 0 \Rightarrow F_x$

FBD I: $\Sigma M_A = 0 \Rightarrow F_y$

$\Sigma F_y = 0 \Rightarrow A_y$

$\Sigma F_x = 0 \Rightarrow A_x$

FBD II: $\Sigma F_y = 0 \Rightarrow G_y$

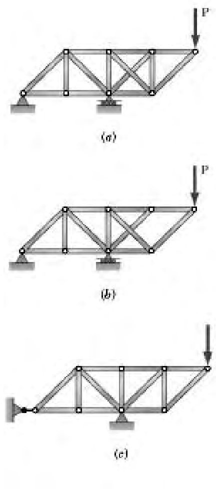
Thus have two simple trusses with all reactions known,

so structure is completely constrained and determinate. ◀

Structure (c):

Structure has $r = 4, \quad m = 13, \quad n = 9$

so $\quad r + m = 17 < 2n = 18,\quad$ structure is partially constrained ◀



PROBLEM 6.74

Classify each of the structures shown as completely, partially, or improperly constrained; if completely constrained, further classify it as determinate or indeterminate. (All members can act both in tension and in compression.)

SOLUTION

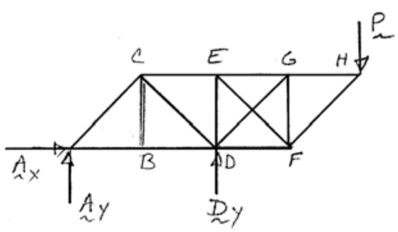
Structure (a):

Rigid truss with $r = 3, \;\; m = 14, \;\; n = 8$

so $$r + m = 17 > 2n = 16$$

so completely constrained but indeterminate ◀

Structure (b):

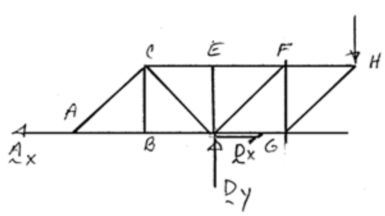


Simple truss (start with *ABC* and add joints alphabetically), with

$$r = 3, \;\; m = 13, \;\; n = 8 \qquad \text{so} \qquad r + m = 16 = 2n$$

so completely constrained and determinate ◀

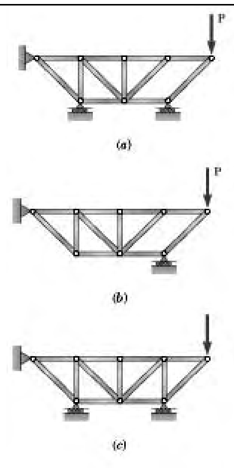
Structure (c):



Simple truss with $r = 3, \;\; m = 13, \;\; n = 8$ so $r + m = 16 = 2n,$ but horizontal reactions $(A_x$ and $D_x)$ are collinear so cannot be resolved by any equilibrium equation.

$\therefore$ structure is improperly constrained ◀

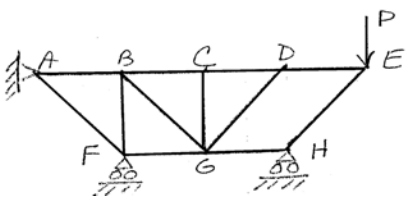
PROBLEM 6.75



Classify each of the structures shown as completely, partially, or improperly constrained; if completely constrained, further classify it as determinate or indeterminate. (All members can act both in tension and in compression.)

SOLUTION

Structure (a):

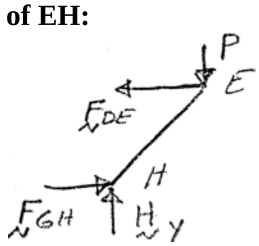


No. of members $m = 12$

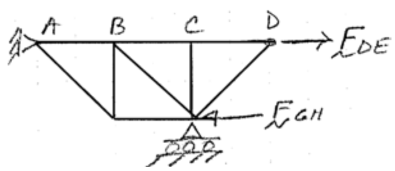
No. of joints $n = 8$ $\qquad m + r = 16 = 2n$

No. of react. comps. $r = 4$ $\qquad$ unks = eqns

FBD of EH:



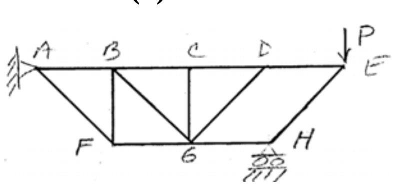
$$\Sigma M_H = 0 \;\rightarrow\; F_{DE}; \;\; \Sigma F_x = 0 \;\rightarrow\; F_{GH}; \;\; \Sigma F_y = 0 \;\rightarrow\; H_y$$



Then *ABCDGF* is a simple truss and all forces can be determined.

This example is completely constrained and determinate. ◀

Structure (b):



No. of members $m = 12$

No. of joints $n = 8$ $\qquad m + r = 15 < 2n = 16$

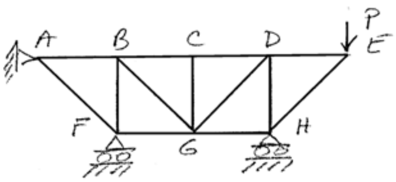
No. of react. comps. $r = 3$ $\qquad$ unks < eqns

partially constrained ◀

Note: Quadrilateral *DEHG* can collapse with joint *D* moving downward: in (a) the roller at *F* prevents this action.

PROBLEM 6.75 CONTINUED

Structure (c):

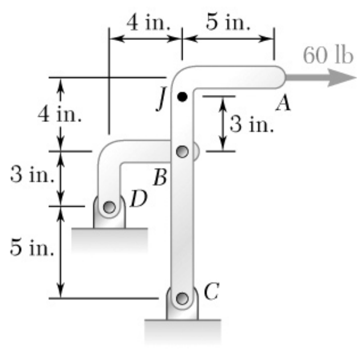


No. of members $m = 13$

No. of joints $n = 8$ $\qquad m + r = 17 > 2n = 16$

No. of react. comps. $r = 4$ $\qquad$ unks $>$ eqns

completely constrained but indeterminate ◀

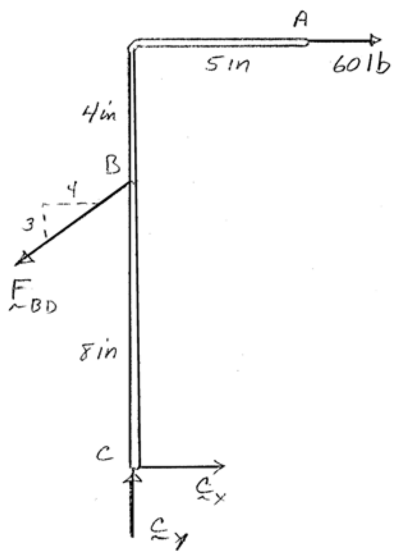


PROBLEM 6.76

For the frame and loading shown, determine the force acting on member *ABC* (*a*) at *B*, (*b*) at *C*.

SOLUTION

FBD member ABC:



Note: *BD* is a two-force member so $\mathbf{F}_{BD}$ is through *D*.

$$(a)\quad \Sigma M_C = 0:\ (8\text{ in.})\left(\frac{4}{5}F_{BD}\right) - (12\text{ in.})(60\text{ lb}) = 0$$

$$\mathbf{F}_{BD} = 112.5\text{ lb} \measuredangle 36.9° \blacktriangleleft$$

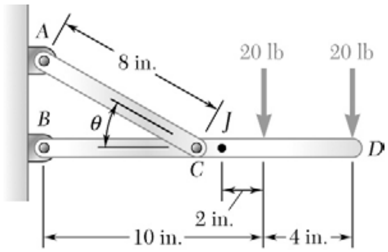
$$(b)\quad \longrightarrow \Sigma F_x = 0:\ 60\text{ lb} + C_x - \frac{4}{5}(112.5\text{ lb}) = 0$$

$$C_x = 30\text{ lb}$$

$$\uparrow \Sigma F_y = 0:\ C_y - \frac{3}{5}(112.5\text{ lb}) = 0$$

$$C_y = 67.5\text{ lb}$$

$$\text{so } \mathbf{C} = 73.9\text{ lb} \measuredangle 66.0° \blacktriangleleft$$

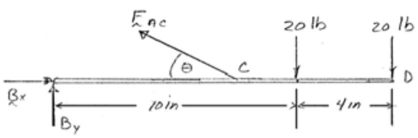


PROBLEM 6.77

Determine the force in member AC and the reaction at B when (*a*) $\theta = 30^\circ$, (*b*) $\theta = 60^\circ$.

SOLUTION

FBD member BCD:



Note: AC is two-force member so $\mathbf{F}_{AC}$ is through A.

$$\widehat{BC} = (8 \text{ in.})\cos\theta$$

$$\circlearrowleft \Sigma M_B = 0: (8 \text{ in.})\cos\theta\left(F_{AC}\sin\theta\right) - (10 \text{ in.})(20 \text{ lb})$$

$$-(14 \text{ in.})(20 \text{ lb}) = 0$$

$$F_{AC} = \frac{60 \text{ lb}}{\sin\theta\,\cos\theta}$$

$$\longrightarrow \Sigma F_x = 0: \; B_x - F_{AC}\cos\theta = 0 \qquad B_x = \frac{60 \text{ lb}}{\sin\theta}$$

$$\uparrow \Sigma F_y = 0: B_y + F_{AC}\sin\theta - 20 \text{ lb} - 20 \text{ lb} = 0$$

$$B_y = 40\,lb - \frac{60 \text{ lb}}{\cos\theta}$$

(*a*) $\theta = 30^\circ$ $\qquad F_{AC} = 138.56 \text{ lb}$

$F_{AC} = 138.6 \text{ lb T}$ ◀

$B_x = 120.0 \text{ lb} \qquad B_y = -29.28 \text{ lb}$

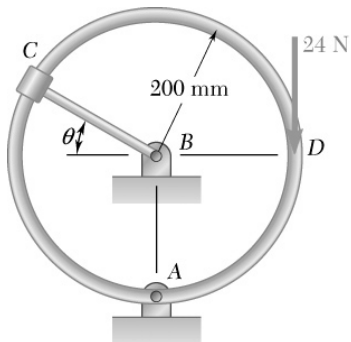
$\mathbf{B} = 123.5 \text{ lb} \;\searrow 13.71^\circ$ ◀

(*b*) $\theta = 60^\circ$ $\qquad F_{AC} = 138.56 \text{ lb}$

$F_{AC} = 138.6 \text{ lb T}$ ◀

$B_x = 69.28 \text{ lb} \qquad B_y = -80 \text{ lb}$

$\mathbf{B} = 105.8 \text{ lb} \;\searrow 49.1^\circ$ ◀

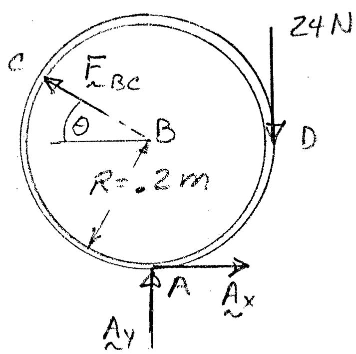


PROBLEM 6.78

A circular ring of radius 200 mm is pinned at *A* and is supported by rod *BC*, which is fitted with a collar at *C* that can be moved along the ring. For the position when $\theta = 35°$, determine (*a*) the force in rod *BC*, (*b*) the reaction at *A*.

SOLUTION

FBD ring:



$(a)\ \theta = 35°$

$$\circlearrowleft \Sigma M_A = 0: (0.2\text{ m})F_{BC}\cos 35° - (0.2\text{ m})(24\text{ N}) = 0$$

$$F_{BC} = \frac{24\text{ N}}{\cos 35°} = 29.298\text{ N}$$

$$F_{BC} = 29.3\text{ N C} \blacktriangleleft$$

(b)

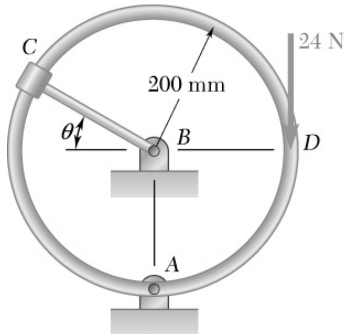
$$\rightarrow \Sigma F_x = 0: A_x - \frac{24\text{ N}}{\cos 35°}\cos 35° = 0$$

$$A_x = 24\text{ N}$$

$$\uparrow \Sigma F_y = 0: A_y + \frac{24\text{ N}}{\cos 35°}\sin 35° - 24\text{ N} = 0$$

$$A_y = 7.195\text{ N}$$

so $\mathbf{A} = 25.1\text{ N} \measuredangle 16.69° \blacktriangleleft$

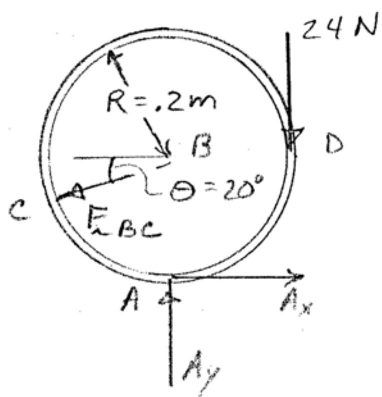


PROBLEM 6.79

Solve Prob. 6.78 when $\theta = -20°$.

SOLUTION

FBD ring:



$$(a)\quad \theta = 20° \quad \circlearrowright \Sigma M_A = 0: (0.2\text{ m})(F_{BC}\ \cos 20°) - (0.2\text{ m})(24\text{ N}) = 0$$

$$F_{BC} = \frac{24\text{ N}}{\cos 20°} = 25.54\text{ N}$$

$$F_{BC} = 25.5\text{ N C} \blacktriangleleft$$

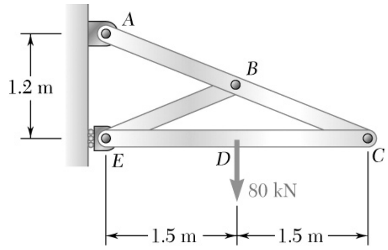
$$(b)\quad \uparrow \Sigma F_y = 0: A_y - \frac{24\text{ N}}{\cos 20°}\sin 20° - 24\text{ N} = 0$$

$$A_y = 32.735\text{ lb}$$

$$\circlearrowright \Sigma M_B = 0: (0.2\text{ m})A_x - (0.2\text{ m})(24\text{ N}) = 0$$

$$A_x = 24\text{ N}$$

$$\text{so}\quad \mathbf{A} = 40.6\text{ N} \measuredangle 53.8° \blacktriangleleft$$

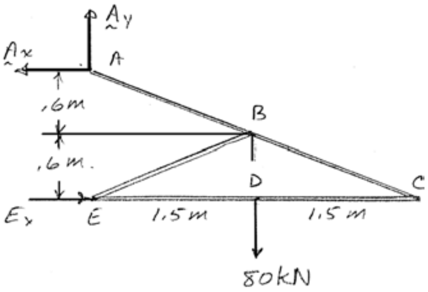


PROBLEM 6.80

For the frame and loading shown, determine the components of all forces acting on member *ABC*.

SOLUTION

FBD Frame:

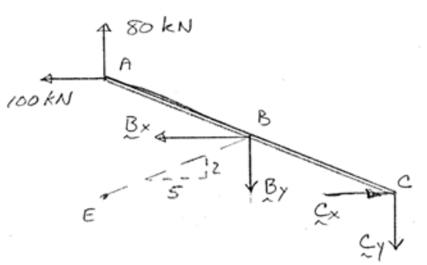


$$\circlearrowleft \Sigma M_E = 0{:}\ (1.2\text{ m})A_x - (1.5\text{ m})(80\text{ kN}) = 0$$

$$\mathbf{A}_x = 100.0\text{ kN} \longleftarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0{:}\ A_y - 80\text{ kN} = 0 \qquad \mathbf{A}_y = 80.0\text{ kN} \uparrow \blacktriangleleft$$

FBD member ABC:



Note: *BE* is two-force member so

$$B_y = \frac{2}{5}B_x = 0.4B_x$$

$$\circlearrowleft \Sigma M_C = 0{:}\ (1.2\text{ m})(100\text{ kN}) - (3.0\text{ m})(80\text{ kN})$$

$$+ (0.6\text{ m})(B_x) + (1.5\text{ m})(0.4B_x) = 0$$

$$\mathbf{B}_x = 100.0\text{ kN} \longleftarrow \blacktriangleleft$$

$$\text{so } \mathbf{B}_y = 40.0\text{ kN} \downarrow \blacktriangleleft$$

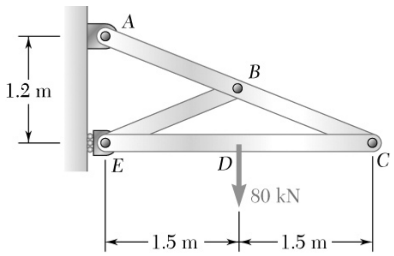
$$\longrightarrow \Sigma F_x = 0{:}\ -100\text{ kN} - 100\text{ kN} + C_x = 0$$

$$\mathbf{C}_x = 200\text{ kN} \longrightarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0{:}\ 80\text{ kN} - 40\text{ kN} - C_y = 0$$

$$\mathbf{C}_y = 40.0\text{ kN} \downarrow \blacktriangleleft$$

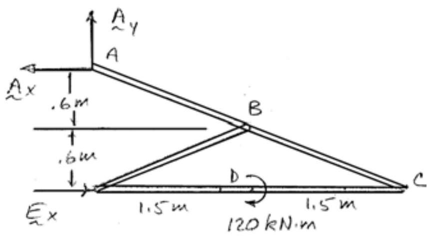
PROBLEM 6.81



Solve Prob. 6.80 assuming that the 80-kN load is replaced by a clockwise couple of magnitude 120 kN·m applied to member *EDC* at point *D*.

SOLUTION

FBD Frame:

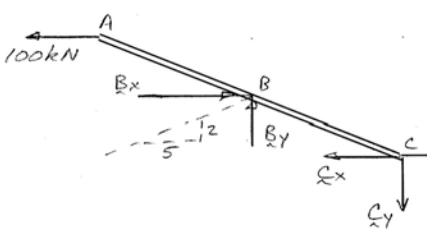


$\uparrow \Sigma F_y = 0$: $\mathbf{A}_y = 0$ ◀

$\circlearrowright \Sigma M_E = 0: (1.2 \text{ m})A_x - 120 \text{ kN} \cdot \text{m} = 0$

$\mathbf{A}_x = 100.0 \text{ kN} \longleftarrow$ ◀

FBD member ABC:



Note: *BE* is two-force member, so

$$B_y = \frac{2}{5}B_x = 0.4B_x$$

$$\circlearrowright \Sigma M_C = 0: (1.2 \text{ m})100 \text{ kN} - (0.6 \text{ m})B_x - (1.5 \text{ m})(0.4B_x) = 0$$

$\mathbf{B}_x = 100.0 \text{ kN} \longrightarrow$ ◀

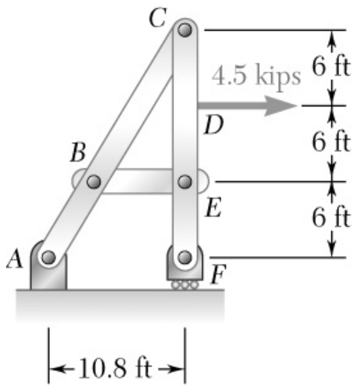
so $\mathbf{B}_y = 40.0 \text{ kN} \uparrow$ ◀

$\longrightarrow \Sigma F_x = 0: -100 \text{ kN} + 100 \text{ kN} - C_x = 0$

$\mathbf{C}_x = 0$ ◀

$\uparrow \Sigma F_y = 0: 40 \text{ kN} - C_y = 0$ $\mathbf{C}_y = 40.0 \text{ kN} \downarrow$ ◀

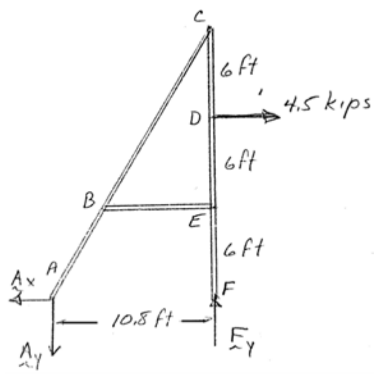
PROBLEM 6.82



For the frame and loading shown, determine the components of all forces acting on member *ABC*.

SOLUTION

FBD Frame:



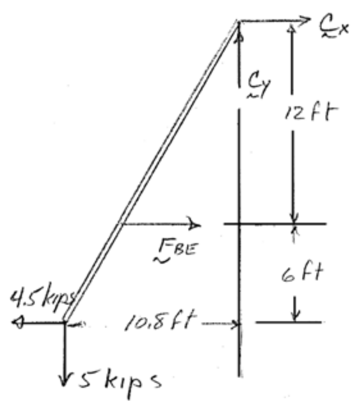
$$\circlearrowleft \Sigma M_F = 0: (10.8 \text{ ft})A_y - (12 \text{ ft})(4.5 \text{ kips}) = 0$$

$$\mathbf{A}_y = 5.00 \text{ kips} \downarrow \blacktriangleleft$$

$$\xrightarrow{} \Sigma F_x = 0: -A_x + 4.5 \text{ kips} = 0$$

$$\mathbf{A}_x = 4.50 \text{ kips} \longleftarrow \blacktriangleleft$$

FBD member ABC:



Note: *BE* is a two-force member

$$\circlearrowleft \Sigma M_C = 0: (12 \text{ ft})F_{BE} + (10.8 \text{ ft})(5 \text{ kips}) - (18 \text{ ft})(4.5 \text{ kips}) = 0$$

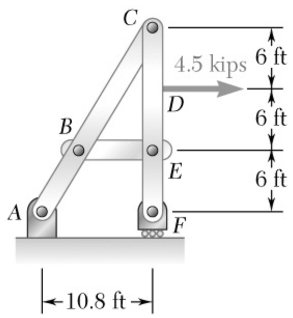
$$\mathbf{F}_{BE} = 2.25 \text{ kips} \longrightarrow \blacktriangleleft$$

$$\xrightarrow{} \Sigma F_x = 0: C_x + 2.25 \text{ kips} - 4.5 \text{ kips} = 0$$

$$\mathbf{C}_x = 2.25 \text{ kips} \longrightarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: C_y - 5 \text{ kips} = 0$$

$$\mathbf{C}_y = 5.00 \text{ kips} \uparrow \blacktriangleleft$$

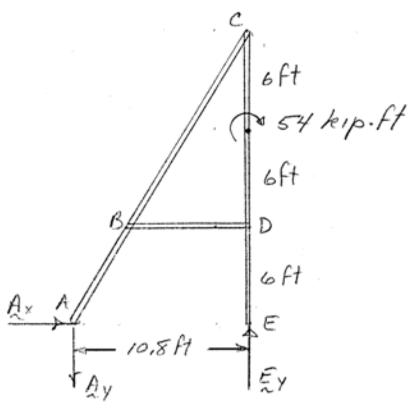


PROBLEM 6.83

Solve Prob. 6.82 assuming that the 4.5-kip load is replaced by a clockwise couple of magnitude $54\text{ kip}\cdot\text{ft}$ applied to member *CDEF* at point *D*.

SOLUTION

FBD Frame:

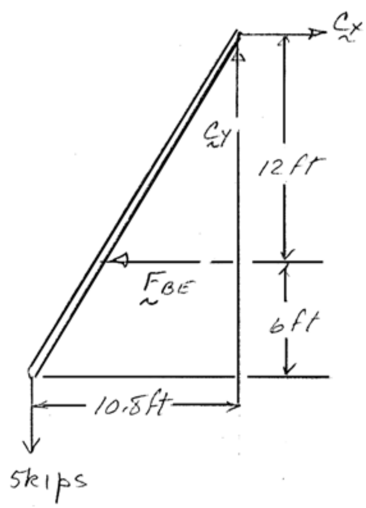


$$\longrightarrow \Sigma F_x = 0: \qquad \mathbf{A}_x = 0 \blacktriangleleft$$

$$\circlearrowleft \Sigma M_E = 0: (10.8\text{ ft})A_y - 54\text{ kip}\cdot\text{ft} = 0$$

$$\mathbf{A}_y = 5.00\text{ kips} \downarrow \blacktriangleleft$$

FBD member ABC:



$$\circlearrowleft \Sigma M_C = 0: (-12\text{ ft})F_{BE} + (10.8\text{ ft})(5\text{ kips}) = 0$$

$$\mathbf{F}_{BE} = 4.50\text{ kips} \longleftarrow \blacktriangleleft$$

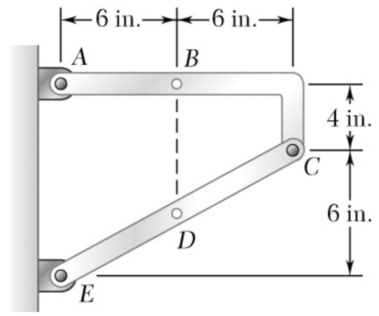
$$\longrightarrow \Sigma F_x = 0: C_x - 4.5\text{ kips} = 0$$

$$\mathbf{C}_x = 4.50\text{ kips} \longrightarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: C_y - 5\text{ kips} = 0$$

$$\mathbf{C}_y = 5.00\text{ kips} \uparrow \blacktriangleleft$$

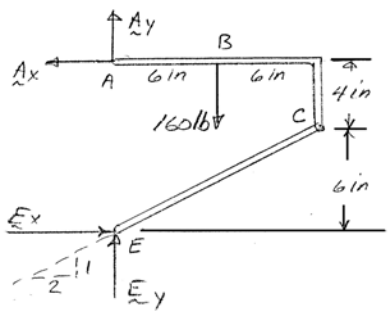
PROBLEM 6.84



Determine the components of the reactions at A and E when a 160-lb force directed vertically downward is applied (a) at B, (b) at D.

SOLUTION

FBD Frame (part a):



Note: EC is a two-force member, so

$$E_y = \frac{1}{2} E_x$$

$$\circlearrowleft \Sigma M_A = 0: (10 \text{ in.}) E_x - (6 \text{ in.})(160 \text{ lb}) = 0$$

$$\mathbf{E}_x = 96.0 \text{ lb} \longrightarrow \blacktriangleleft$$

$$\text{so } \mathbf{E}_y = 48.0 \text{ lb} \uparrow \blacktriangleleft$$

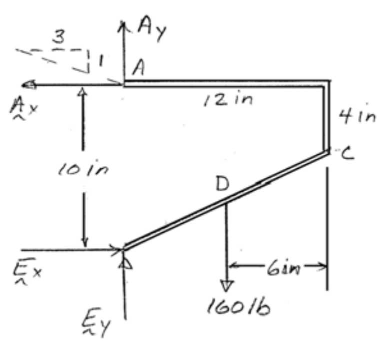
$$\longrightarrow \Sigma F_x = 0: -A_x + 96 \text{ lb} = 0$$

$$\mathbf{A}_x = 96.0 \text{ lb} \longleftarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: A_y - 160 \text{ lb} + 48 \text{ lb} = 0$$

$$\mathbf{A}_y = 112.0 \text{ lb} \uparrow \blacktriangleleft$$

FBD member (part b):



Note: AC is a two-force member, so

$$A_x = 3A_y$$

$$\circlearrowleft \Sigma M_A = 0: \text{same as part (a)}$$

$$\mathbf{E}_x = 96.0 \text{ lb} \longrightarrow \blacktriangleleft$$

$$\longrightarrow \Sigma F_x = 0: \text{same as part (a)}$$

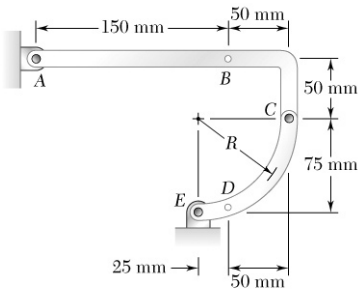
$$\mathbf{A}_x = 96.0 \text{ lb} \longleftarrow \blacktriangleleft$$

Here $\quad A_y = \frac{1}{3} A_x \quad$ so $\mathbf{A}_y = 32.0 \text{ lb} \uparrow \blacktriangleleft$

$$\uparrow \Sigma F_y = 0: 32 \text{ lb} + E_y - 160 \text{ lb} = 0$$

$$\mathbf{E}_y = 128.0 \text{ lb} \uparrow \blacktriangleleft$$

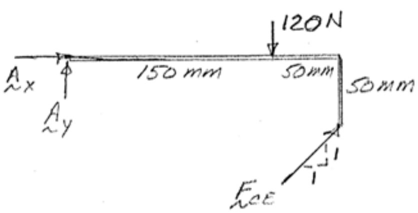
PROBLEM 6.85



Determine the components of the reactions at A and E when a 120-N force directed vertically downward is applied (a) at B, (b) at D.

SOLUTION

FBD ABC:



(a) CE is a two-force member

$$\circlearrowleft \Sigma M_A = 0: (200 \text{ mm})\frac{1}{\sqrt{2}}F_{CE} + (50 \text{ mm})\frac{1}{\sqrt{2}}F_{CE}$$

$$-150 \text{ mm}(120 \text{ N}) = 0$$

$$F_{CE} = 72\sqrt{2} \text{ N} \qquad \text{so } \mathbf{E}_x = 72.0 \text{ N} \longrightarrow \blacktriangleleft$$

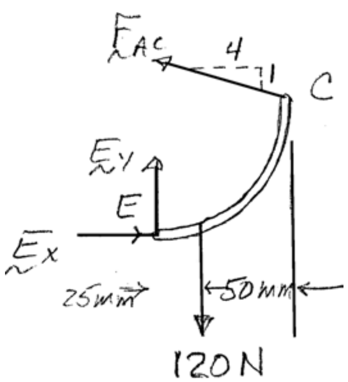
$$\mathbf{E}_y = 72.0 \text{ N} \uparrow \blacktriangleleft$$

$$\longrightarrow \Sigma F_x = 0: A_x + \frac{1}{\sqrt{2}}F_{CE} = 0 \qquad A_x = -72 \text{ N} \blacktriangleleft$$

$$\mathbf{A}_x = 72.0 \text{ N} \longleftarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: A_y - 120 \text{ N} + \frac{1}{\sqrt{2}}F_{CE} = 0 \qquad \mathbf{A}_y = 48.0 \text{ N} \uparrow \blacktriangleleft$$

FBD CE:



(b) AC is a two-force member

$$\circlearrowleft \Sigma M_E = 0: (75 \text{ mm})\left(\frac{4}{\sqrt{17}}F_{AC}\right) + (75 \text{ mm})\left(\frac{1}{\sqrt{17}}F_{AC}\right)$$

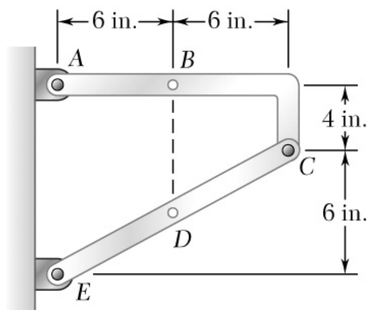
$$-(25 \text{ mm})(120 \text{ N}) = 0 \qquad F_{AC} = 8\sqrt{17} \text{ N}$$

$$\longrightarrow \Sigma F_x = 0: E_x - \frac{4}{\sqrt{17}}F_{AC} = 0 \qquad \mathbf{E}_x = 32.0 \text{ N} \longrightarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: E_y + \frac{1}{\sqrt{17}}F_{AC} - 120 = 0 \qquad \mathbf{E}_y = 112.0 \text{ N} \uparrow \blacktriangleleft$$

$$\text{and } \mathbf{A}_x = 32.0 \text{ N} \longleftarrow \blacktriangleleft$$

$$\mathbf{A}_y = 8.00 \text{ N} \uparrow \blacktriangleleft$$



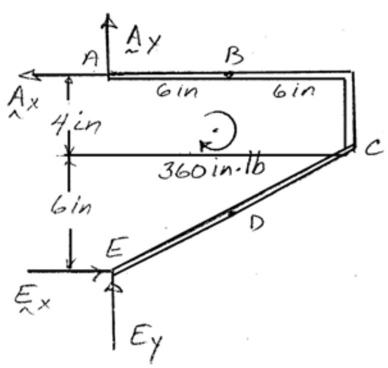
PROBLEM 6.86

Determine the components of the reactions at *A* and *E* when the frame is loaded by a clockwise couple of magnitude $360\ \text{lb}\cdot\text{in.}$ applied

(*a*) at *B*, (*b*) at *D*.

SOLUTION

FBD Frame:



Note for analysis of the frame FBD, the location of the applied couple is immaterial.

$$\circlearrowleft \Sigma M_A = 0: (10\text{ in.})E_x - 360\text{ in}\cdot\text{lb} = 0$$

$$\mathbf{E}_x = 36.0\text{ lb} \longrightarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_E = 0: (10\text{ in.})A_x - 360\text{ in}\cdot\text{lb} = 0$$

$$\mathbf{A}_x = 36.0\text{ lb} \longleftarrow \blacktriangleleft$$

Part (a): If couple acts at *B*, *EC* is a two-force member, so

$$E_y = \frac{1}{2}E_x \qquad \mathbf{E}_y = 18.0\text{ lb} \uparrow \blacktriangleleft$$

and then $\uparrow \Sigma F_y = 0: A_y + 18\text{ lb} = 0$

$$\mathbf{A}_y = 18.00\text{ lb} \downarrow \blacktriangleleft$$

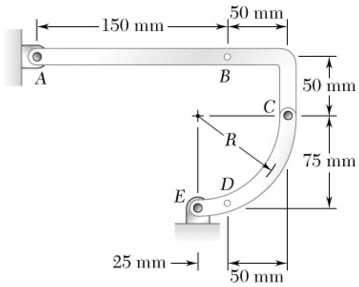
Part (b): If couple acts at *D*, *AC* is a two-force member, so

$$A_y = \frac{1}{3}A_x \qquad \mathbf{A}_y = 12.00\text{ lb} \uparrow \blacktriangleleft$$

Then $\uparrow \Sigma F_y = 0: 12\text{ lb} + E_y = 0 \qquad E_y = -12\text{ lb}$

$$\mathbf{E}_y = 12.00\text{ lb} \downarrow \blacktriangleleft$$

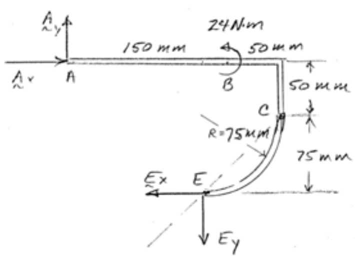
PROBLEM 6.87



Determine the components of the reactions at A and E when the frame is loaded by a counterclockwise couple of magnitude 24 N·m applied (a) at B, (b) at D.

SOLUTION

(a) FBD Frame:



Note: CE is a two-force member, so $E_x = E_y$

$$\circlearrowleft \Sigma M_A = 0: 24 \text{ N} \cdot \text{m} - (0.125 \text{ m})E_x - (0.125 \text{ m})E_y = 0$$

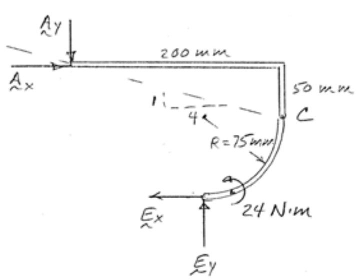
$$E_x = E_y = 96 \text{ N} \qquad \mathbf{E}_x = 96.0 \text{ N} \longleftarrow \blacktriangleleft$$

$$\mathbf{E}_y = 96.0 \text{ N} \downarrow \blacktriangleleft$$

$$\longrightarrow \Sigma F_x = 0: A_x - 96 \text{ N} = 0 \qquad \mathbf{A}_x = 96.0 \text{ N} \longrightarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: A_y - 96 \text{ N} = 0 \qquad \mathbf{A}_y = 96.0 \text{ N} \uparrow \blacktriangleleft$$

(b) FBD Frame:



Note: AC is a two-force member, so $A_x = 4A_y$

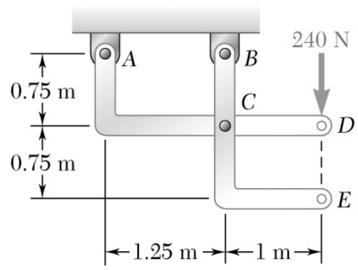
$$\circlearrowleft \Sigma M_E = 0: 24 \text{ N} \cdot \text{m} + (0.125 \text{ m})A_y - (0.125 \text{ m})(4A_y) = 0$$

$$A_y = 64 \text{ N} \qquad \mathbf{A}_y = 64.0 \text{ N} \downarrow \blacktriangleleft$$

$$\mathbf{A}_x = 256 \text{ N} \longrightarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: E_y - 64 \text{ N} = 0 \qquad \mathbf{E}_y = 64.0 \text{ N} \uparrow \blacktriangleleft$$

$$\longrightarrow \Sigma F_x = 0: -E_x + 256 \text{ N} = 0 \qquad \mathbf{E}_x = 256 \text{ N} \longleftarrow \blacktriangleleft$$

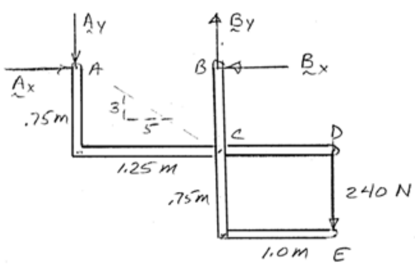


PROBLEM 6.88

Determine the components of the reactions at *A* and *B* if (*a*) the 240-N load is applied as shown, (*b*) the 240-N load is moved along its line of action and is applied at *E*.

SOLUTION

FBD Frame:



Regardless of the point of application of the 240 N load;

$$\Sigma M_A = 0: (1.25 \text{ m})B_y - (2.25 \text{ m})(240 \text{ N}) = 0$$

$$\mathbf{B}_y = 432 \text{ N} \uparrow \blacktriangleleft$$

$$\Sigma M_B = 0: (1.25 \text{ m})A_y - (1.0 \text{ m})(240 \text{ N}) = 0$$

$$\mathbf{A}_y = 192.0 \text{ N} \downarrow \blacktriangleleft$$

Part (a): If load at *D*, *BCE* is a two-force member, so $\mathbf{B}_x = 0 \blacktriangleleft$

Then $\quad \rightarrow \Sigma F_x = 0: \quad A_x - B_x = 0 \qquad A_x = B_x = 0$

$$\mathbf{A}_x = 0 \blacktriangleleft$$

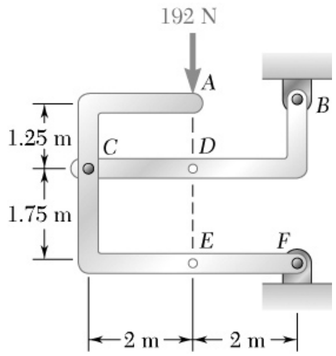
Part (b): If load at *E*, *ACD* is a two-force member, so $A_x = \dfrac{5}{3}A_y$

then $\mathbf{A}_x = 320 \text{ N} \longrightarrow \blacktriangleleft$

and $\quad \rightarrow \Sigma F_x = 0: A_x - B_x = 0$

$$\mathbf{B}_x = 320 \text{ N} \longleftarrow \blacktriangleleft$$

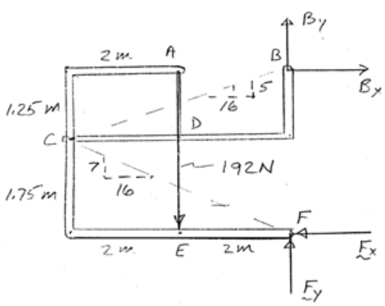
PROBLEM 6.89



The 192-N load can be moved along the line of action shown and applied at A, D, or E. Determine the components of the reactions at B and F when the 192-N load is applied (a) at A, (b) at D, (c) at E.

SOLUTION

FBD Frame:



Note, regardless of the point of application of the 192 N load,

$$\circlearrowleft \Sigma M_B = 0: (2 \text{ m})(192 \text{ N}) - (3 \text{ m})F_x = 0$$

$$\mathbf{F}_x = 128.0 \text{ N} \longleftarrow \blacktriangleleft$$

$$\longrightarrow \Sigma F_x = 0: B_x - 128 \text{ N} = 0$$

$$\mathbf{B}_x = 128.0 \text{ N} \longrightarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: B_y + F_y - 192 \text{ N} = 0$$

(a) and (c): If load applied at either A or E, BC is a two-force member

so $\quad B_y = \dfrac{5}{16}B_x \qquad \mathbf{B}_y = 40.0 \text{ N} \uparrow \blacktriangleleft$

Then $\quad \uparrow \Sigma F_y = 0: 40 \text{ N} + F_y - 192 \text{ N} = 0$

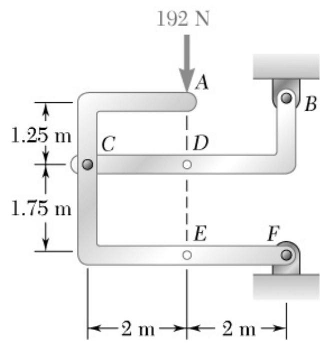
$$\mathbf{F}_y = 152.0 \text{ N} \uparrow \blacktriangleleft$$

(b): If load applied at D, $ACEF$ is a two-force member, so

$$F_y = \frac{7}{16}F_y \qquad \mathbf{F}_y = 56.0 \text{ N} \uparrow \blacktriangleleft$$

Then $\quad \uparrow \Sigma F_y = 0: B_y + 56 \text{ N} - 192 \text{ N} = 0$

$$\mathbf{B}_y = 136.0 \text{ N} \uparrow \blacktriangleleft$$

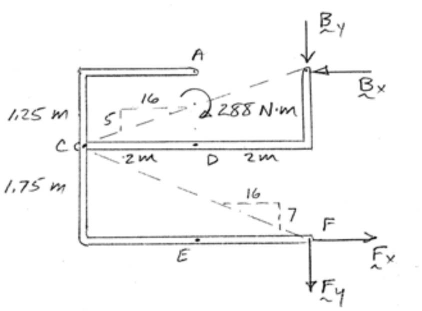


PROBLEM 6.90

The 192-N load is removed and a $288\ \text{N}\cdot\text{m}$ clockwise couple is applied successively at *A*, *D*, and *E*. Determine the components of the reactions at *B* and *F* when the couple is applied (*a*) at *A*, (*b*) at *B*, (*c*) at *E*.

SOLUTION

FBD Frame:



Regardless of the location of applied couple,

$$\circlearrowright \Sigma M_B = 0: (3\text{ m})F_x - 288\text{ N}\cdot\text{m} = 0 \qquad \mathbf{F}_x = 96.0\text{ N} \longrightarrow \blacktriangleleft$$

$$\longrightarrow \Sigma F_x = 0: -B_x + 96\text{ N} = 0 \qquad \mathbf{B}_x = 96.0\text{ N} \longleftarrow \blacktriangleleft$$

$$\downarrow \Sigma F_y = 0: B_y + F_y = 0$$

(*a*) and (*c*): If couple applied *anywhere* on *ACEF*, *BC* is a two-force member,

so $$B_y = \frac{5}{16}B_x \qquad \mathbf{B}_y = 30.0\text{ N} \downarrow \blacktriangleleft$$

Then $$\downarrow \Sigma F_y = 0: 30\text{ N} + F_y = 0 \qquad F_y = -30\text{ N}$$

$$\mathbf{F}_y = 30.0\text{ N} \uparrow \blacktriangleleft$$

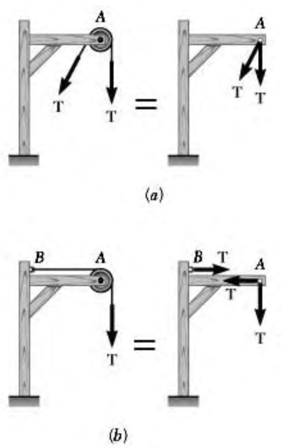
(*b*): If couple is applied *anywhere* on *BC*, *ACEF* is a two-force member,

so $$F_y = \frac{7}{16}F_x \qquad \mathbf{F}_y = 42.0\text{ N} \downarrow \blacktriangleleft$$

$$\downarrow \Sigma F_y = 0: B_y + 42\text{ N} = 0 \qquad B_y = -42\text{ N}$$

$$\mathbf{B}_y = 42.0\text{ N} \uparrow \blacktriangleleft$$

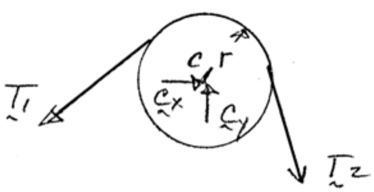
PROBLEM 6.91



(*a*) Show that when a frame supports a pulley at *A*, an equivalent loading of the frame and of each of its component parts can be obtained by removing the pulley and applying at *A* two forces equal and parallel to the forces that the cable exerted on the pulley.
(*b*) Show that if one end of the cable is attached to the frame at point *B*, a force of magnitude equal to the tension in the cable should also be applied at *B*.

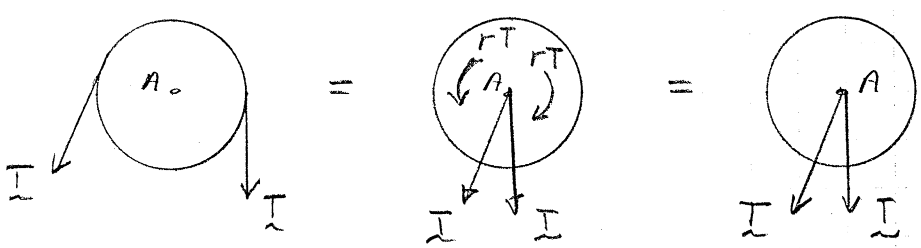
SOLUTION

First note that, when a cable or cord passes over a *frictionless*, *motionless* pulley, the tension is unchanged.

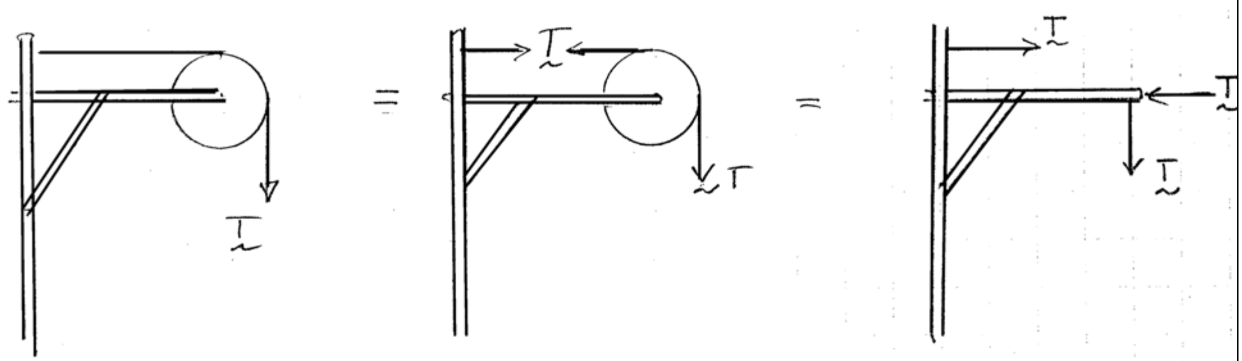


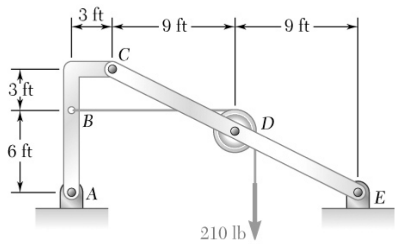
$$\circlearrowleft \Sigma M_C = 0: rT_1 - rT_2 = 0 \qquad T_1 = T_2$$

(*a*) Replace each force with an equivalent force-couple.



(*b*) Cut cable and replace forces on pulley with equivalent pair of forces at *A* as above.



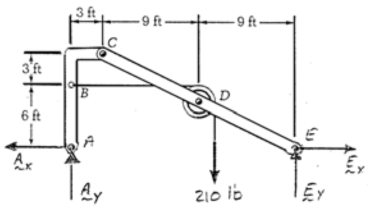


PROBLEM 6.92

Knowing that the pulley has a radius of 1.5 ft, determine the components of the reactions at A and E.

SOLUTION

FBD Frame:



$$\circlearrowleft \Sigma M_A = 0: (21 \text{ ft})E_y - (13.5 \text{ ft})(210 \text{ lb}) = 0$$

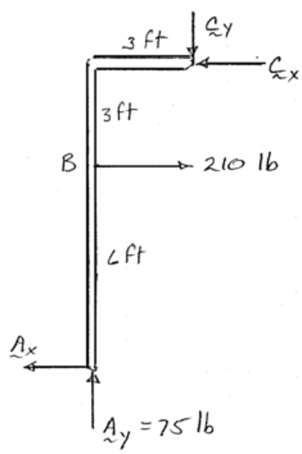
$$\mathbf{E}_y = 135.0 \text{ lb} \uparrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: A_y - 210 \text{ lb} + 135 \text{ lb} = 0$$

$$\mathbf{A}_y = 75.0 \text{ lb} \uparrow \blacktriangleleft$$

$$\longrightarrow \Sigma F_x = 0: A_x - E_x = 0 \qquad A_x = E_x$$

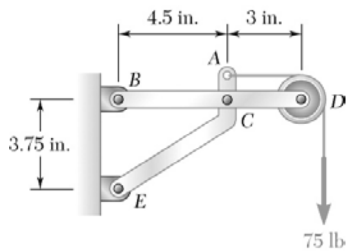
FBD member ABC:



$$\circlearrowleft \Sigma M_C = 0: (3 \text{ ft})(210 \text{ lb}) - (3 \text{ ft})(75 \text{ lb}) - (9 \text{ ft})A_x = 0$$

$$\mathbf{A}_x = 45.0 \text{ lb} \longleftarrow \blacktriangleleft$$

$$\text{so } \mathbf{E}_x = 45.0 \text{ lb} \longrightarrow \blacktriangleleft$$

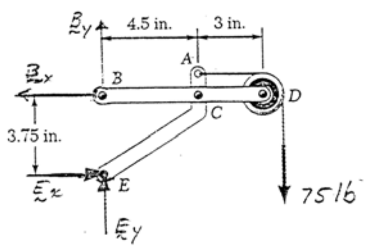


PROBLEM 6.93

Knowing that the pulley has a radius of 1.25 in., determine the components of the reactions at B and E.

SOLUTION

FBD Frame:



$$\circlearrowleft \Sigma M_E = 0: (3.75 \text{ in.})B_x + (8.75 \text{ in.})(75 \text{ lb}) = 0$$

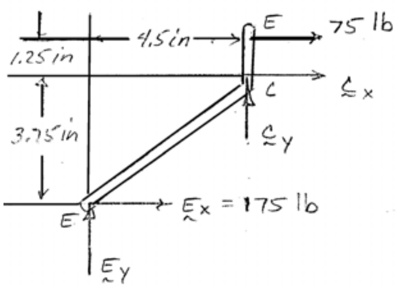
$$B_x = 175 \text{ lb} \qquad \mathbf{B}_x = 175.0 \text{ lb} \longleftarrow \blacktriangleleft$$

$$\longrightarrow \Sigma F_x = 0: E_x - B_x = 0 \qquad \mathbf{E}_x = 175.0 \text{ lb} \longrightarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: E_y + B_y - 75 \text{ lb} = 0$$

$$B_y = 75 \text{ lb} - E_y$$

FBD member ACE:

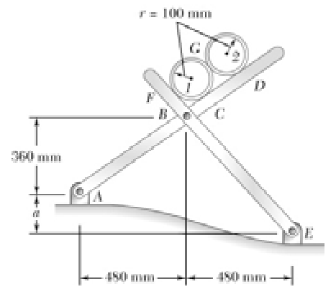


$$\circlearrowleft \Sigma M_C = 0: -(1.25 \text{ in.})(75 \text{ lb}) + (3.75 \text{ in.})(175 \text{ lb}) - (4.5 \text{ in.})E_y = 0$$

$$\mathbf{E}_y = 125.0 \text{ lb} \uparrow \blacktriangleleft$$

Thus $$B_y = 75 \text{ lb} - 125 \text{ lb} = -50 \text{ lb}$$

$$\mathbf{B}_y = 50.0 \text{ lb} \downarrow \blacktriangleleft$$



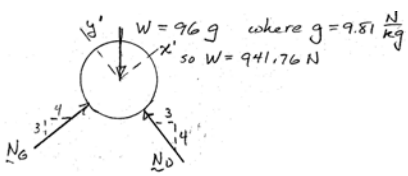
PROBLEM 6.94

Two 200-mm-diameter pipes (pipe *1* and pipe *2*) are supported every 3 m by a small frame like the one shown. Knowing that the combined mass per unit length of each pipe and its contents is 32 kg/m and assuming frictionless surfaces, determine the components of the reactions at A and E when $a = 0$.

SOLUTION

FBDs:

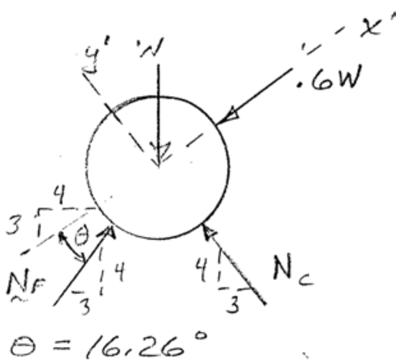
pipe 2:



$$\nearrow \Sigma F_{x'} = 0: N_G - \frac{3}{5}W = 0 \qquad N_G = 0.6W$$

$$\nwarrow \Sigma F_{y'} = 0: N_D - \frac{4}{5}W = 0 \qquad N_D = 0.8W$$

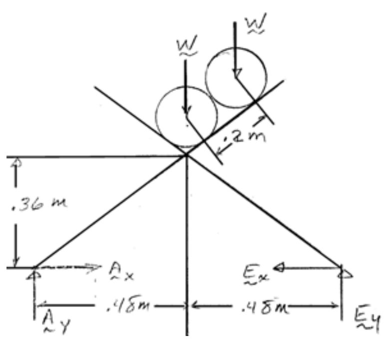
pipe 1:



$$\nearrow \Sigma F_{x'} = 0: N_F \cos 16.26° - \frac{3}{5}W - 0.6W = 0 \qquad N_F = 1.25W$$

$$\nwarrow \Sigma F_{y'} = 0: N_C + 1.25W \sin 16.26° - \frac{4}{5}W = 0 \qquad N_C = 0.45W$$

Frame:



$$\circlearrowleft \Sigma M_A = 0: (0.96 \text{ m})E_y - (0.48 \text{ m})W - \left\{\left[0.48 + \frac{4}{5}(0.2)\right]\text{m}\right\}W = 0$$

$$E_y = 1.16667W \qquad \mathbf{E}_y = 1099 \text{ N} \uparrow \blacktriangleleft$$

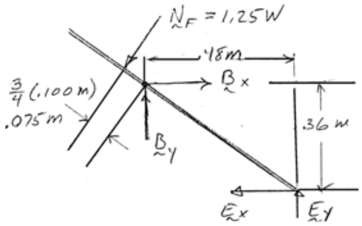
$$\uparrow \Sigma F_y = 0: A_y + 1.16667W - W - W = 0$$

$$A_y = 0.83333W \qquad \mathbf{A}_y = 785 \text{ N} \uparrow \blacktriangleleft$$

$$\rightarrow \Sigma F_x = 0: A_x - E_x = 0 \qquad \text{so} \qquad A_x = E_x$$

PROBLEM 6.94 CONTINUED

member BE:



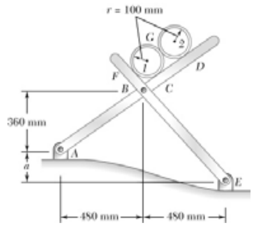
$$\circlearrowright \Sigma M_B = 0\text{: } (0.075 \text{ m})(1.25W) + (0.48 \text{ m})(1.16667W)$$

$$-(0.36 \text{ m})E_x = 0$$

$$E_x = 1.816W$$

$$\mathbf{E}_x = 1710 \text{ N} \longleftarrow \blacktriangleleft$$

$$\text{thus } \mathbf{A}_x = 1710 \text{ N} \longrightarrow \blacktriangleleft$$



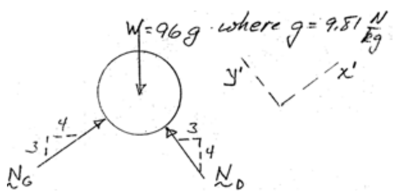
PROBLEM 6.95

Solve Prob. 6.94 when $a = 280$ mm.

SOLUTION

FBDs

pipe 2

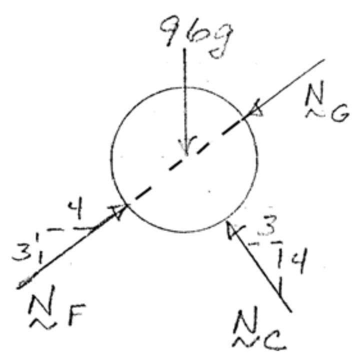


$$W = 941.76\,N$$

$$\Sigma F_{y'} = 0{:}\; N_D - \frac{4}{5}W = 0 \qquad N_D = 0.8W$$

$$\Sigma F_{x'} = 0{:}\; N_G - \frac{3}{5}W = 0 \qquad N_G = 0.6W$$

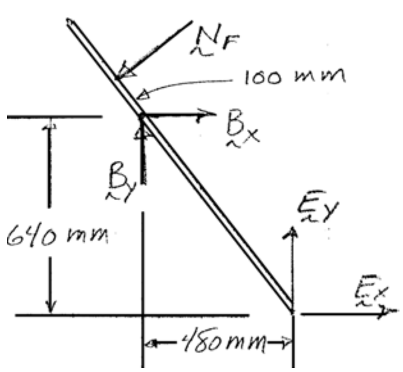
pipe 1:



$$\Sigma F_{y'} = 0{:}\; N_C - \frac{4}{5}W = 0 \qquad N_C = 0.8W$$

$$\Sigma F_{x'} = 0{:}\; N_F - \frac{3}{5}\,96\text{ g} - N_G = 0 \qquad N_F = 1.2W$$

member BE:



$$\Sigma M_B = 0{:}\; (640\text{ mm})E_x + (480\text{ mm})E_y + (100\text{ mm})N_F = 0$$

$$\Sigma M_A = 0{:}\; (280\text{ mm})E_x + (960\text{ mm})E_y + (100\text{ mm})N_F$$

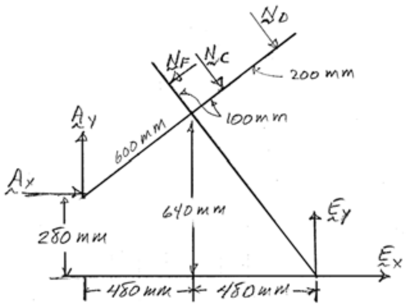
$$-(700\text{ mm})N_C - (900\text{ mm})N_D = 0$$

$$E_x = -1.400W \qquad \mathbf{E}_x = 1318\text{ N} \longleftarrow \blacktriangleleft$$

$$E_y = 1.617W \qquad \mathbf{E}_y = 1523\text{ N} \uparrow \blacktriangleleft$$

PROBLEM 6.95 CONTINUED

FBD Frame:



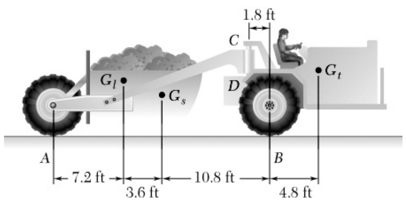
$$\xrightarrow{} \Sigma F_x = 0: A_x + E_x + \frac{3}{5}(N_C + N_D) - \frac{4}{5}N_F = 0$$

$$A_x = 1.400W \qquad \mathbf{A}_x = 1318 \text{ N} \longrightarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: A_y + E_y - \frac{4}{5}(N_C + N_D) - \frac{3}{5}N_F = 0$$

$$A_y = 0.3833W \qquad \mathbf{A}_y = 361 \text{ N} \uparrow \blacktriangleleft$$

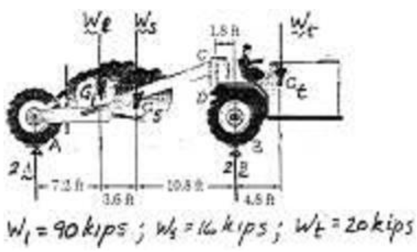
PROBLEM 6.96



The tractor and scraper units shown are connected by a vertical pin located 1.8 ft behind the tractor wheels. The distance from C to D is 2.25 ft. The center of gravity of the 20-kip tractor unit is located at G_t, while the centers of gravity of the 16-kip scraper unit and the 90-kip load are located at G_s and G_l, respectively. Knowing that the tractor is at rest with its brakes released, determine (*a*) the reactions at each of the four wheels, (*b*) the forces exerted on the tractor unit at C and D.

SOLUTION

FBD Entire machine:



(*a*)

$$\circlearrowleft \Sigma M_A = 0: (21.6 \text{ ft})2B - (7.2 \text{ ft})(90 \text{ kips}) - (10.8 \text{ ft})(16 \text{ kips})$$

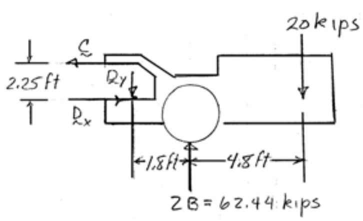
$$- (26.4 \text{ ft})(20 \text{ kips}) = 0$$

$$B = 31.22 \text{ kips} \qquad \mathbf{B} = 31.2 \text{ kips} \uparrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: 2A + 2(31.22 \text{ kips}) - (90 + 16 + 20)\text{kips} = 0$$

$$A = 31.78 \text{ kips} \qquad \mathbf{A} = 31.8 \text{ kips} \uparrow \blacktriangleleft$$

FBD Tractor:



(*b*)

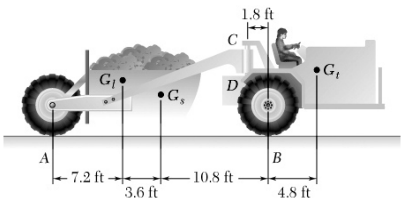
$$\circlearrowleft \Sigma M_D = 0: (2.25 \text{ ft})C + (1.8 \text{ ft})(62.44 \text{ kips}) - (6.6 \text{ ft})(20 \text{ kips}) = 0$$

$$C = 8.7146 \text{ kips} \qquad \mathbf{C} = 8.71 \text{ kips} \longleftarrow \blacktriangleleft$$

$$\longrightarrow \Sigma F_x = 0: D_x - C = 0 \qquad D_x = 8.715 \text{ kips} \longrightarrow$$

$$\uparrow \Sigma F_y = 0: 62.44 \text{ kips} - D_y - 20 \text{ kips} = 0 \qquad D_y = 42.44 \text{ kips} \downarrow$$

$$\text{so } \mathbf{D} = 43.3 \text{ kips} \searrow 78.4^\circ \blacktriangleleft$$

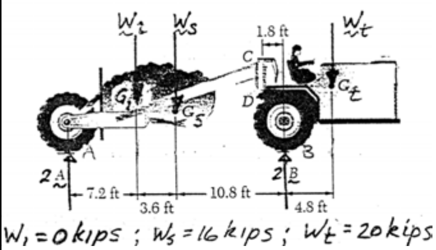


PROBLEM 6.97

Solve Prob. 6.96 assuming that the 90-kip load has been removed.

SOLUTION

FBD Entire machine:



(*a*)

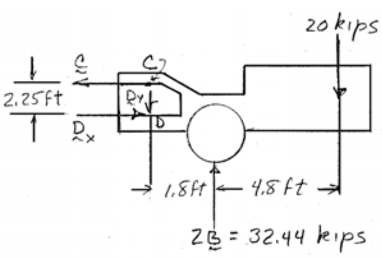
$$\circlearrowleft \Sigma M_A = 0: (21.6\text{ ft})(2B) - (10.8\text{ ft})(16\text{ kips}) - (26.4\text{ ft})(20\text{ kips}) = 0$$

$$B = 16.222\text{ kips} \qquad \mathbf{B} = 16.22\text{ kips}\uparrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: 2A + 2(16.222\text{ kips}) - (16+20)\text{kips} = 0$$

$$A = 1.778\text{ kips} \qquad \mathbf{A} = 1.778\text{ kips}\uparrow \blacktriangleleft$$

FBD Tractor:



(*b*)

$$\circlearrowleft \Sigma M_D = 0: (2.25\text{ ft})C + (1.8\text{ ft})(32.44\text{ kips}) - (6.6\text{ ft})(20\text{ kips}) = 0$$

$$C = 32.71\text{ kips} \qquad \mathbf{C} = 32.7\text{ kips}\leftarrow \blacktriangleleft$$

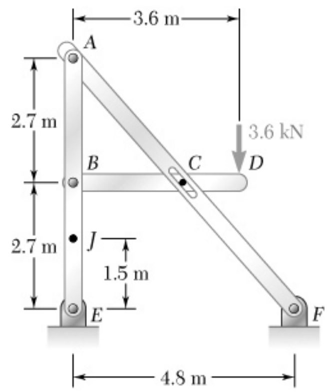
$$\rightarrow \Sigma F_x = 0: D_x - C = 0 \qquad D_x = 32.71\text{ kips}\rightarrow$$

$$\uparrow \Sigma F_y = 0: -D_y + 2(32.44\text{ kips}) - 20\text{ kips} = 0$$

$$D_y = 12.44\text{ kips}\downarrow$$

$$\text{so } \mathbf{D} = 35.0\text{ kips} \searrow 20.8^\circ \blacktriangleleft$$

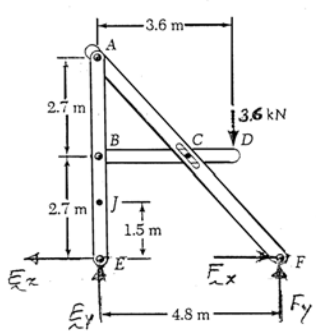
PROBLEM 6.98



For the frame and loading shown, determine the components of all forces acting on member *ABE*.

SOLUTION

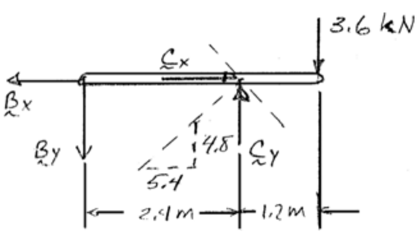
FBD Frame:



$$\circlearrowleft \Sigma M_F = 0: (1.2 \text{ m})(3.6 \text{ kN}) - (4.8 \text{ m})E_y = 0$$

$$\mathbf{E}_y = 0.9 \text{ kN} \uparrow \blacktriangleleft$$

FBD member BC:



$$C_y = \frac{4.8}{5.4}C_x = \frac{8}{9}C_x$$

$$\circlearrowleft \Sigma M_C = 0: (2.4 \text{ m})B_y - (1.2 \text{ m})(3.6 \text{ kN}) = 0 \quad B_y = 1.8 \text{ kN} \downarrow$$

on *ABE*: $\mathbf{B}_y = 1.800 \text{ kN} \uparrow \blacktriangleleft$

$$\uparrow \Sigma F_y = 0: -1.8 \text{ kN} + C_y - 3.6 \text{ kN} = 0 \qquad C_y = 5.40 \text{ kN} \uparrow$$

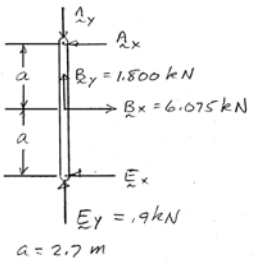
so $C_x = \frac{9}{8}C_y \qquad C_x = 6.075 \text{ kN} \longrightarrow$

$$\longrightarrow \Sigma F_x = 0: -B_x + C_x = 0 \qquad B_x = 6.075 \text{ kN} \longleftarrow \text{ on } BC$$

on *ABE*: $\mathbf{B}_x = 6.08 \text{ kN} \longrightarrow \blacktriangleleft$

PROBLEM 6.98 CONTINUED

FBD member AB0E:



$$\circlearrowleft \Sigma M_A = 0: a(6.075 \text{ kN}) - 2aE_x = 0$$

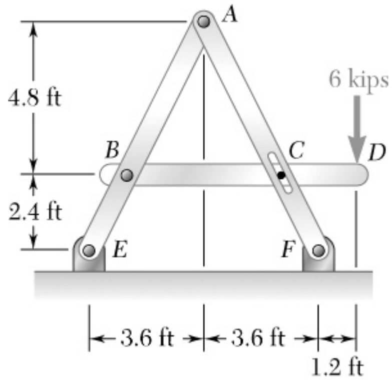
$$E_x = 3.038 \text{ kN} \qquad \mathbf{E}_x = 3.04 \text{ kN} \longleftarrow \blacktriangleleft$$

$$\longrightarrow \Sigma F_x = 0: -A_x + (6.075 - 3.038)\text{kN} = 0$$

$$\mathbf{A}_x = 3.04 \text{ kN} \longleftarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: 0.9 \text{ kN} + 1.8 \text{ kN} - A_y = 0 \qquad \mathbf{A}_y = 2.70 \text{ kN} \downarrow \blacktriangleleft$$

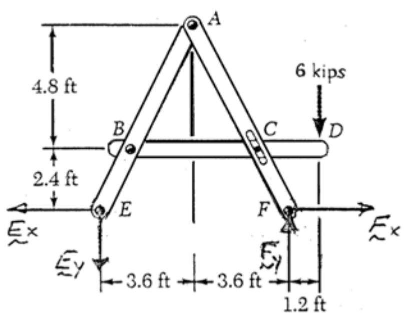
PROBLEM 6.99



For the frame and loading shown, determine the components of all forces acting on member *ABE*.

SOLUTION

FBD Frame:

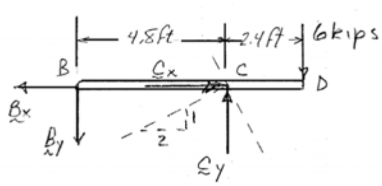


$$\circlearrowleft \Sigma M_F = 0: (7.2\text{ ft})F_y - (1.2\text{ ft})(6\text{ kips}) = 0$$

$$\mathbf{E}_y = 1.000\text{ kip} \downarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_B = 0: (4.8\text{ ft})C_y - (7.2\text{ ft})(6\text{ kips}) = 0 \qquad C_y = 9\text{ kips}$$

FBD member BCD:



But **C** is $\perp$ *ACF*, so $C_x = 2C_y$; $C_x = 18\text{ kips} \longrightarrow$

$$\longrightarrow \Sigma F_x = 0: -B_x + C_x = 0 \qquad B_x = C_x = 18\text{ kips}$$

$$B_x = 18.00\text{ kips} \longleftarrow \text{ on } BCD$$

$$\uparrow \Sigma F_y = 0: -B_y + 9\text{ kips} - 6\text{ kips} = 0 \qquad B_y = 3\text{ kips} \downarrow \text{ on } BCD$$

On *ABE*: $\mathbf{B}_x = 18.00\text{ kips} \longrightarrow \blacktriangleleft$

$$\mathbf{B}_y = 3.00\text{ kips} \uparrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_A = 0: (4.8\text{ ft})(18\text{ kips}) - (2.4\text{ ft})(3\text{ kips})$$

$$+ (3.6\text{ ft})(1\text{ kip}) - (7.2\text{ ft})(E_x) = 0$$

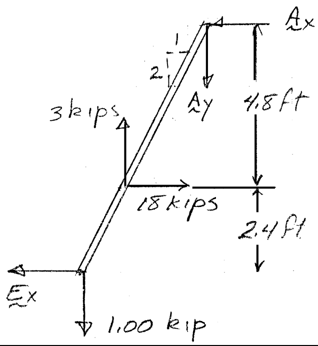
$$\mathbf{E}_x = 11.50\text{ kips} \longleftarrow \blacktriangleleft$$

$$\longrightarrow \Sigma F_x = 0: -11.50\text{ kips} + 18\text{ kips} - A_x = 0$$

$$\mathbf{A}_x = 6.50\text{ kips} \longleftarrow \blacktriangleleft$$

PROBLEM 6.99 CONTINUED

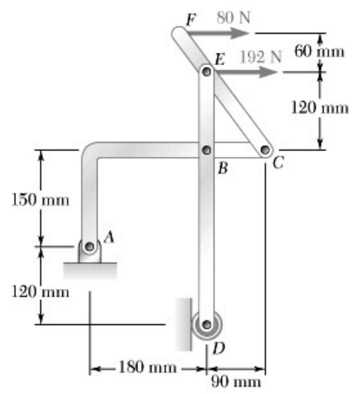
FBD member ABE:



$$+\uparrow \Sigma F_y = 0: -1.00 \text{ kip} + 3.00 \text{ kips} - A_y = 0$$

$$\mathbf{A}_y = 2.00 \text{ kips} \downarrow \blacktriangleleft$$

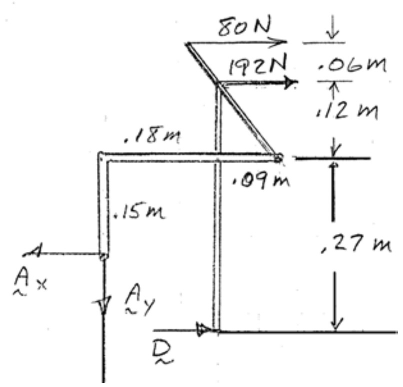
PROBLEM 6.100



For the frame and loading shown, determine the components of the forces acting on member *ABC* at *B* and *C*.

SOLUTION

FBD Frame:

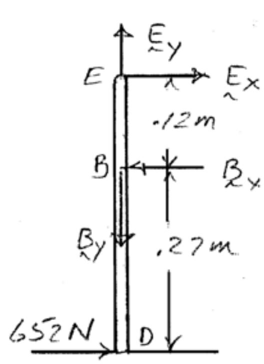


$$\circlearrowleft \Sigma M_A = 0: (0.12\text{ m})D - (0.27\text{ m})(192\text{ N}) - (0.33\text{ m})(80\text{ N}) = 0$$

$$\mathbf{D} = 652\text{ N} \longrightarrow$$

$$\uparrow \Sigma F_y = 0: \mathbf{A}_y = 0$$

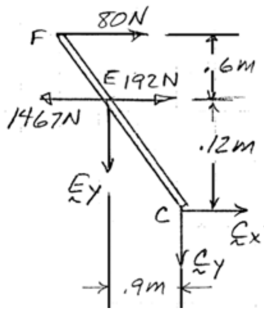
FBD members:



$$\circlearrowleft \Sigma M_E = 0: (0.39\text{ m})(652\text{ N}) - (0.12\text{ m})B_x = 0 \qquad B_x = 2119\text{ N} \longleftarrow$$

$$\longrightarrow \Sigma F_x = 0: E_x - 2119\text{ N} + 652\text{ N} = 0 \qquad E_x = 1467\text{ N} \longrightarrow$$

PROBLEM 6.100 CONTINUED



$\rightarrow \Sigma F_x = 0\text{: } 80\text{ N} + 192\text{ N} - 1467\text{ N} + C_x = 0 \qquad C_x = 1195\text{ N} \rightarrow$

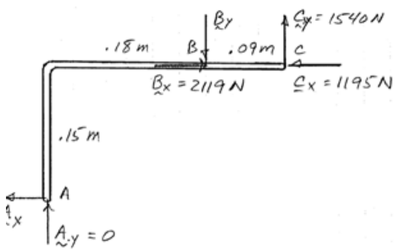
$\circlearrowright \Sigma M_E = 0\text{: } -(0.9\text{ m})C_y + (0.12\text{ m})(1195\text{ N}) - (0.6\text{ m})(80\text{ N}) = 0$

$C_y = 1540\text{ N} \downarrow$

From above, on *ABC* $\qquad \mathbf{C}_x = 1.195\text{ kN} \leftarrow$ ◀

$\mathbf{C}_y = 1.540\text{ kN} \uparrow$ ◀

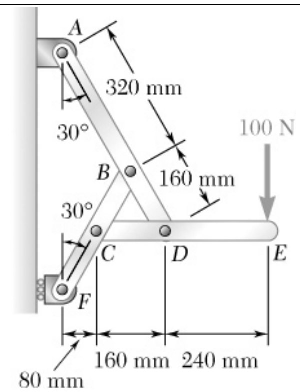
$\mathbf{B}_x = 2.12\text{ kN} \rightarrow$ ◀



$\uparrow \Sigma F_y = 0\text{: } -B_y + 1540\text{ N} = 0 \qquad B_y = 1540\text{ N}$

$\mathbf{B}_y = 1.540\text{ kN} \downarrow$ ◀

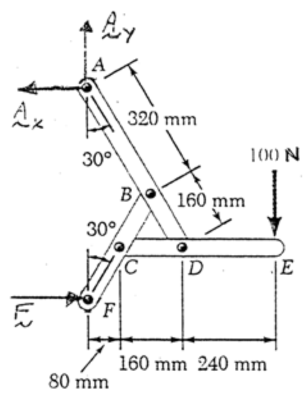
PROBLEM 6.101



For the frame and loading shown, determine the components of the forces acting on member *CDE* at *C* and *D*.

SOLUTION

FBD Frame:



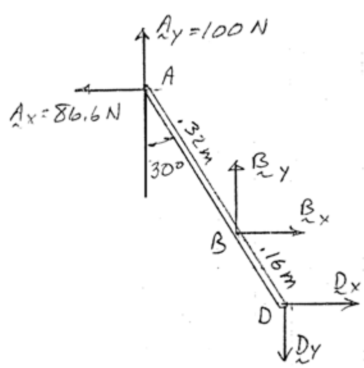
Note: $\overset{\frown}{AF} = 2(0.32\text{ m})\cos 30° = 0.5543\text{ m}$

$$\circlearrowleft \Sigma M_F = 0: (0.5543\text{ m})A_x - (0.48\text{ m})(100\text{ N}) = 0$$

$$\mathbf{A}_x = 86.603\text{ N} \longleftarrow$$

$$\uparrow \Sigma F_y = 0: A_y - 100\text{ N} = 0 \qquad \mathbf{A}_y = 100\text{ N} \uparrow$$

FBD members:



$$\circlearrowleft \Sigma M_B = 0: (0.32\text{ m})(\cos 30°)(86.603\text{ N}) + (0.16\text{ m})(\cos 30°)D_x$$

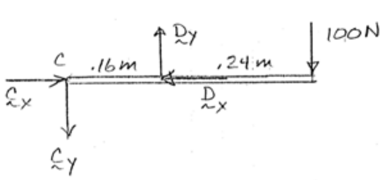
$$-(0.32\text{ m})(\sin 30°)(100\text{ N}) - (0.16\text{ m})(\sin 30°)D_y = 0$$

$$D_x = D_y \tan 30° - 57.736\text{ N}$$

$$\circlearrowleft \Sigma M_C = 0: (0.16\text{ m})D_y - (0.40\text{ m})(100\text{ N}) = 0 \qquad D_y = 250\text{ N}$$

$$\mathbf{D}_y = 250\text{ N} \uparrow \blacktriangleleft$$

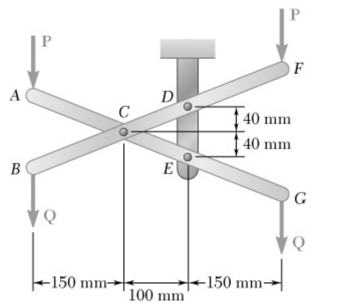
Then, from above $\qquad D_x = 86.6\text{ N} \qquad \mathbf{D}_x = 86.6\text{ N} \longleftarrow \blacktriangleleft$



$$\longrightarrow \Sigma F_x = 0: C_x - 86.6\text{ N} = 0 \qquad \mathbf{C}_x = 86.6\text{ N} \longrightarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: -C_y + 250\text{ N} - 100\text{ N} = 0 \qquad \mathbf{C}_y = 150.0\text{ N} \downarrow \blacktriangleleft$$

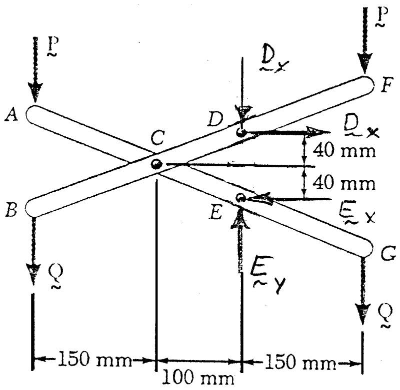
PROBLEM 6.102



Knowing that $P = 15$ N and $Q = 65$ N, determine the components of the forces exerted (*a*) on member *BCDF* at *C* and *D*, (*b*) on member *ACEG* at *E*.

SOLUTION

FBD Frame:



$$P = 15\text{ N}\downarrow \qquad Q = 65\text{ N}\downarrow$$

$$\circlearrowleft \Sigma M_D = 0\text{: } (0.25\text{ m})(P+Q) - (.15\text{ m})(P+Q) - (0.08\text{ m})E_x = 0$$

$$E_x = 1.2(P+Q) = 100\text{ N} \longleftarrow \qquad \mathbf{E}_x = 100.0\text{ N} \longleftarrow \blacktriangleleft$$

$$\longrightarrow \Sigma F_x = 0\text{: } D_x - E_x = 0 = D_x - 100\text{ N} \qquad D_x = 100\text{ N} \longrightarrow$$

$$\mathbf{D}_x = 100.0\text{ N} \longrightarrow \blacktriangleleft$$

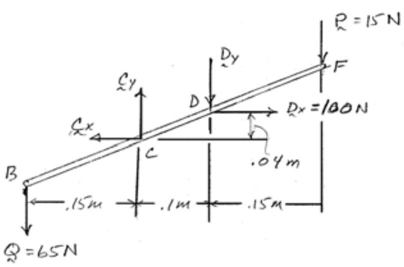
$$\uparrow \Sigma F_y = 0\text{: } E_y - D_y - 2P - 2Q = 0$$

$$E_y = D_y + 2(P+Q) = D_y + 160\text{ N}$$

FBD member BF:

$$\circlearrowleft \Sigma M_C = 0\text{: } (0.15\text{ m})(65\text{ N}) - (0.1\text{ m})D_y - (0.04\text{ m})(100\text{ N})$$

$$- (0.25\text{ m})(15\text{ N}) = 0$$



$$D_y = 20\text{ N} \qquad \mathbf{D}_y = 20.0\text{ N}\downarrow \blacktriangleleft$$

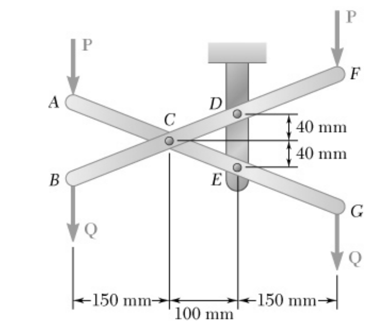
From above $\quad E_y = 20\text{ N} + 160\text{ N} = 180\text{ N} \qquad \mathbf{E}_y = 180.0\text{ N}\uparrow \blacktriangleleft$

$$\longrightarrow \Sigma F_x = 0\text{: } -C_x + 100\text{ N} = 0$$

$$\mathbf{C}_x = 100.0\text{ N} \longleftarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0\text{: } -65\text{ N} + C_y - 20\text{ N} - 15\text{ N} = 0$$

$$\mathbf{C}_y = 100.0\text{ N}\uparrow \blacktriangleleft$$

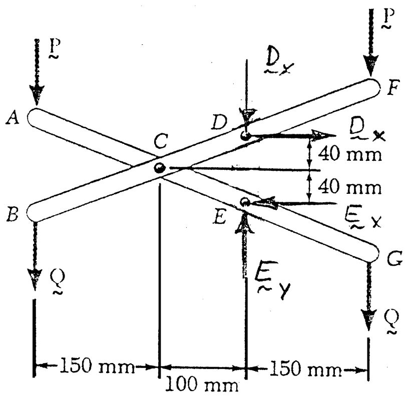


PROBLEM 6.103

Knowing that $P = 25$ N and $Q = 55$ N, determine the components of the forces exerted (*a*) on member *BCDF* at *C* and *D*, (*b*) on member *ACEG* at *E*.

SOLUTION

FBD Frame:



$$P = 25\text{ N}\downarrow \qquad Q = 55\text{ N}\downarrow$$

$$\Sigma M_D = 0: (0.25\text{ m})(P+Q) - (0.15\text{ m})(P+Q) - (0.08\text{m})E_x = 0$$

$$E_x = 1.20(P+Q) = 100\text{ N}$$

$$\mathbf{E}_x = 100.0\text{ N} \longleftarrow \blacktriangleleft$$

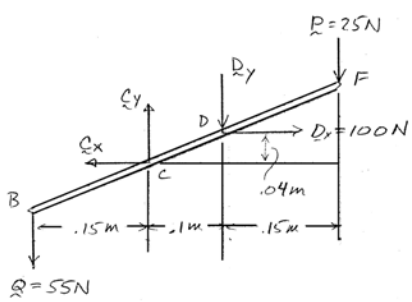
$$\Sigma F_x = D_x - 100\text{ N} = 0$$

$$\mathbf{D}_x = 100.0\text{ N} \longrightarrow \blacktriangleleft$$

$$\Sigma F_y = E_y - D_y - 2P - 2Q = 0$$

$$E_y = D_y + 2(P+Q) = D_y + 160\text{ N}$$

FBD member BF:



$$\Sigma M_C = 0: (0.15\text{ m})(55\text{ N}) - (0.1\text{ m})D_y - (0.04)(100\text{ N})$$

$$-(0.25\text{ m})(25\text{ N}) = 0$$

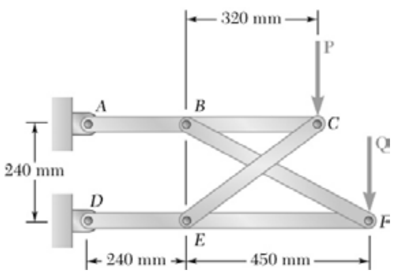
$$D_y = -20\text{ N} \qquad \mathbf{D}_y = 20.0\text{ N}\uparrow \blacktriangleleft$$

From above $\quad E_y = -20\text{ N} + 160\text{ N} = 140\text{ N} \qquad \mathbf{E}_y = 140.0\text{ N}\uparrow \blacktriangleleft$

$$\longrightarrow \Sigma F_x = 0: -C_x + 100\text{ N} = 0 \qquad \mathbf{C}_x = 100.0\text{ N} \longleftarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: -55\text{ N} + C_y - (-20\text{ N}) - 25\text{ N} = 0$$

$$\mathbf{C}_y = 60.0\text{ N}\uparrow \blacktriangleleft$$

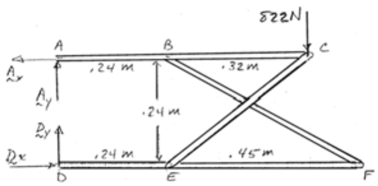


PROBLEM 6.104

Knowing that $P = 822$ N and $Q = 0$, determine for the frame and loading shown (*a*) the reaction at *D*, (*b*) the force in member *BF*.

SOLUTION

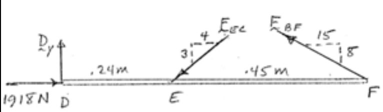
FBD Frame:



$$\circlearrowleft \Sigma M = 0: (0.24 \text{ m})D_x - (0.56 \text{ m})(822 \text{ N}) = 0 \qquad D_x = 1918 \text{ N}$$

$$\mathbf{D}_x = 1.918 \text{ kN} \longrightarrow$$

FBD member DF:



$$\circlearrowleft \Sigma M_D = 0: (0.69 \text{ m})\frac{8}{17}F_{BF} - (0.24 \text{ m})\frac{3}{5}F_{EC} = 0$$

$$0.3247F_{BF} - 0.144F_{EC} = 0$$

$$\longrightarrow \Sigma F_x = 0: 1918 \text{ N} - \frac{15}{17}F_{BF} - \frac{4}{5}F_{EC} = 0$$

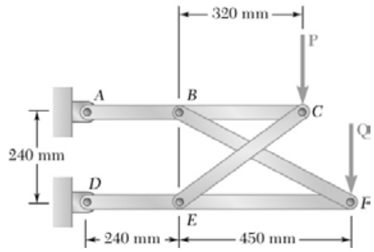
$$0.8824F_{BF} + 0.800F_{EC} = 1918 \text{ N}$$

Solving: $F_{BF} = 714 \text{ N T}$ ◀

$$\circlearrowleft \Sigma M_E = 0: (0.45 \text{ m})\frac{8}{17}(714 \text{ N}) - (0.24 \text{ m})D_y = 0$$

$$D_y = 630 \text{ N} \uparrow$$

so $\mathbf{D} = 2.02 \text{ kN} \measuredangle 18.18°$ ◀

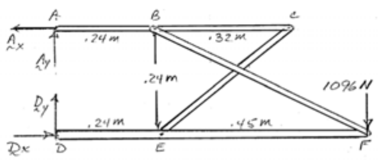


PROBLEM 6.105

Knowing that $P = 0$ and $Q = 1096$ N, determine for the frame and loading shown (*a*) the reaction at *D*, (*b*) the force in member *BF*.

SOLUTION

FBD Frame:



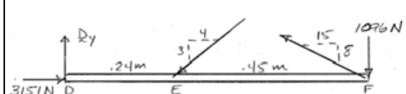
$$\circlearrowleft \Sigma M_A = 0\colon (0.24\text{ m})D_x - (0.69\text{ m})(1096\text{ N}) = 0 \qquad \mathbf{D}_x = 3151\text{ N} \longrightarrow$$

$$\circlearrowleft \Sigma M_D = 0\colon (0.69\text{ m})\frac{8}{17}F_{BF} - (0.69\text{ m})(1096\text{ N}) - (0.24\text{ m})\frac{3}{5}F_{EC} = 0$$

FBD member DF:

$$0.3247F_{BF} - 0.144F_{EC} = 756.24\text{ N}$$

$$\circlearrowleft \Sigma F_x = 0\colon 3151\text{ N} - \frac{4}{5}F_{EC} - \frac{15}{17}F_{BF} = 0$$

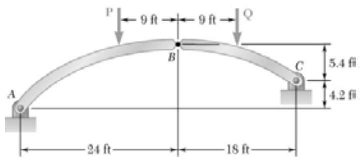


$$0.8824F_{BF} + 0.800F_{EC} = 3151\text{ N}$$

Solving: $\quad F_{BF} = 2737$ N $\qquad F_{BF} = 2.74$ kN T ◀

$$\circlearrowleft \Sigma M_E = 0\colon (0.45\text{ m})\left[\frac{8}{17}(2737\text{ N}) - 1096\text{ N}\right] - (0.24\text{ m})D_y = 0$$

$\mathbf{D}_y = 360.06$ N $\uparrow \qquad \mathbf{D} = 3.17$ kN $\angle 6.52°$ ◀

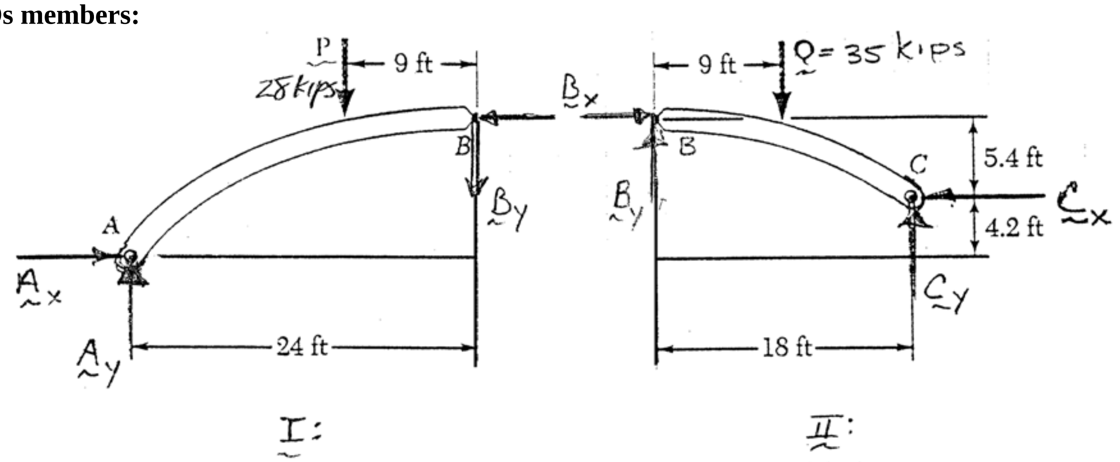


PROBLEM 6.106

The axis of the three-hinge arch ABC is a parabola with vertex at B. Knowing that $P = 28$ kips and $Q = 35$ kips, determine (a) the components of the reaction at A, (b) the components of the force exerted at B on segment AB.

SOLUTION

FBDs members:



From FBD I: $\Sigma M_A = 0$: $(9.6 \text{ ft})B_x - (24 \text{ ft})B_y - (15 \text{ ft})(28 \text{ kips}) = 0$

$$1.2B_x - 3.0B_y = 52.5 \text{ kips}$$

FBD II: $\Sigma M_C = 0$: $(5.4 \text{ ft})B_x + (18 \text{ ft})B_y - (9 \text{ ft})(35 \text{ kips}) = 0$

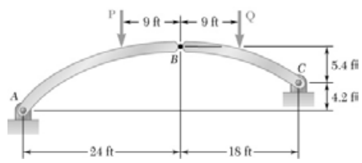
$$0.6B_x - 2B_y = 35 \text{ kips}$$

Solving: $B_x = 50$ kips; $B_y = 2.5$ kips as drawn, so on AB: $\mathbf{B}_x = 50.0 \text{ kips} \longleftarrow$ ◀

$\mathbf{B}_y = 2.50 \text{ kips} \downarrow$ ◀

FBD I: $\longrightarrow \Sigma F_x = 0$: $A_x - 50 \text{ kips} = 0$ $\qquad \mathbf{A}_x = 50.0 \text{ kips} \longrightarrow$ ◀

$\uparrow \Sigma F_x = 0$: $A_y - 28 \text{ kips} - 2.5 \text{ kips} = 0$ $\qquad \mathbf{A}_y = 30.5 \text{ kips} \uparrow$ ◀

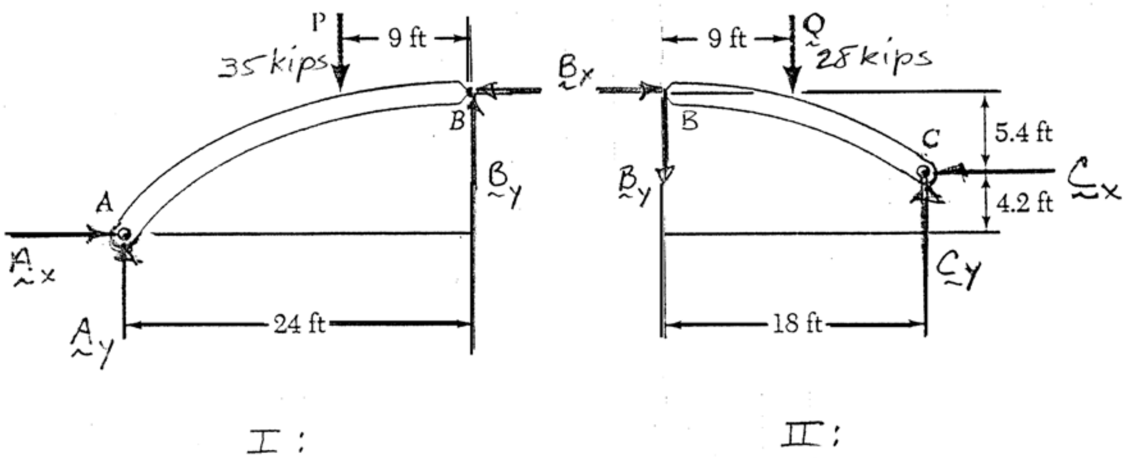


PROBLEM 6.107

The axis of the three-hinge arch *ABC* is a parabola with vertex at *B*. Knowing that $P = 35$ kips and $Q = 28$ kips, determine (*a*) the components of the reaction at *A*, (*b*) the components of the force exerted at *B* on segment *AB*.

SOLUTION

member FBDs:



From FBD I: $\Sigma M_A = 0$: $(9.6\text{ ft})B_x + (24\text{ ft})B_y - (15\text{ ft})(35\text{ kips}) = 0$

$$3.2B_x + 8B_y = 175\text{ kips}$$

FBD I: $\Sigma M_C = 0$: $(5.4\text{ ft})B_x - (18\text{ ft})B_y - (9\text{ ft})(28\text{ kips}) = 0$

$$0.6B_x - 2B_y = 28\text{ kips}$$

Solving: $B_x = 51.25$ kips; $B_x = 1.375$ kips as drawn, so on *AB*: $\mathbf{B}_x = 51.3$ kips ⟵ ◀

$\mathbf{B}_y = 1.375$ kips ↑ ◀

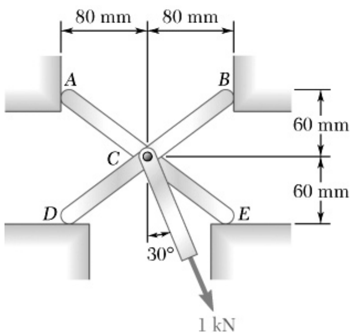
FBD I: ⟶ $\Sigma F_x = 0$: $A_x - 51.25$ kips

$\mathbf{A}_x = 51.3$ kips ⟶ ◀

↑ $\Sigma F_y = 0$: $A_y - 35\text{ kips} + 1.375\text{ kips}$

$\mathbf{A}_y = 33.6$ kips ↑ ◀

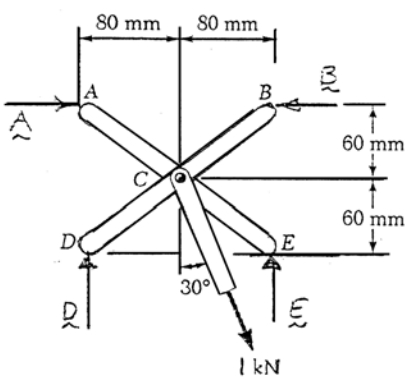
PROBLEM 6.108



For the frame and loading shown, determine the reactions at *A*, *B*, *D*, and *E*. Assume that the surface at each support is frictionless.

SOLUTION

FBD Frame:



$$\circlearrowleft \Sigma M_A = 0: (0.16\text{ m})E - (0.08\text{ m})(1\text{ kN})\cos 30°$$

$$-(0.06\text{ m})(1\text{ kN})\ \sin 30° = 0$$

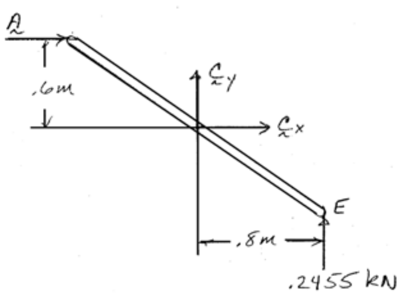
$$E = 0.2455\text{ kN}$$ $\mathbf{E} = 246\text{ N}\uparrow$ ◀

$\uparrow \Sigma F_y = 0: D + 0.2455\text{ kN} - (1\text{ kN})\cos 30° = 0$ $D = 0.6205\text{ kN}$

$\mathbf{D} = 621\text{ N}\uparrow$ ◀

$\longrightarrow \Sigma F_x = 0: A - B + (1\text{ kN})\sin 30° = 0$ $B = A + 0.5\text{ kN}$

FBD member ACE:

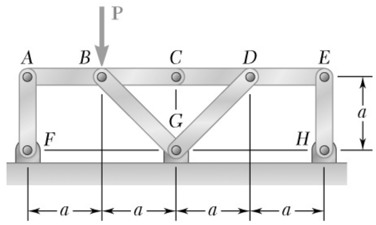


$\circlearrowleft \Sigma M_C = 0: (0.8\text{ m})(0.2455\text{ kN}) - (0.6\text{ m})(A) = 0$ $A = 0.3274\text{ kN}$

$\mathbf{A} = 327\text{ N}\longrightarrow$ ◀

From above $B = A + 0.05\text{ kN}$

$B = (0.327 + 0.50)\text{kN} = 0.827\text{ kN}$ $\mathbf{B} = 827\text{ N}\longleftarrow$ ◀

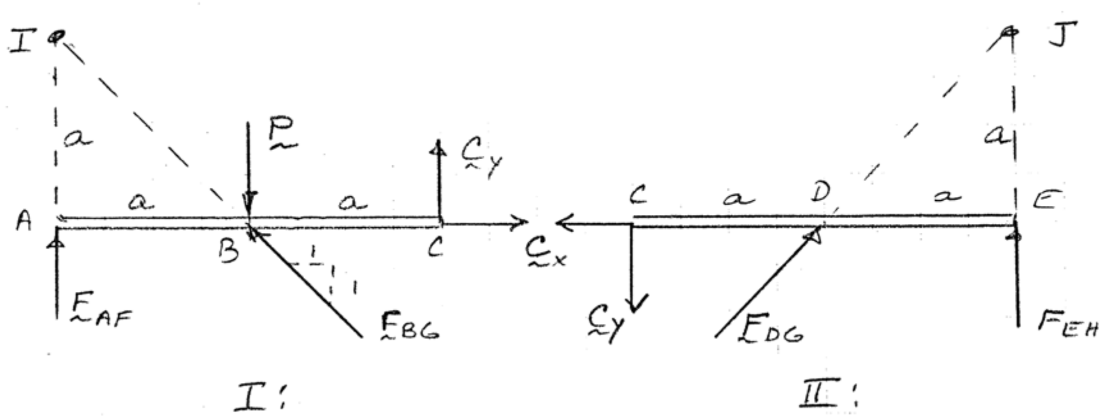


PROBLEM 6.109

Members *ABC* and *CDE* are pin-connected at *C* and are supported by four links. For the loading shown, determine the force in each link.

SOLUTION

member FBDs:



FBD I: $\Sigma M_I = 0\colon 2aC_y + aC_x - aP = 0 \qquad 2C_y + C_x = P$

FBD II: $\Sigma M_J = 0\colon 2aC_y - aC_x = 0 \qquad 2C_y - C_x = 0$

Solving: $C_x = \dfrac{P}{2};\ C_y = \dfrac{P}{4}$ as shown

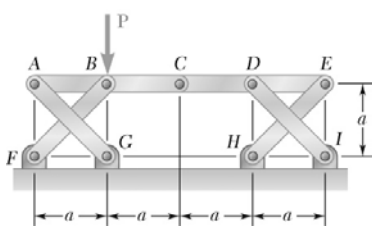
FBD I: $\longrightarrow \Sigma F_x = 0\colon -\dfrac{1}{\sqrt{2}}F_{BG} + C_x = 0 \qquad F_{BG} = C_x\sqrt{2}$ $\qquad F_{BG} = \dfrac{\sqrt{2}}{2}P$ C ◀

$\uparrow \Sigma F_y = 0\colon F_{AF} - P + \dfrac{1}{\sqrt{2}}\left(\dfrac{\sqrt{2}}{2}P\right) + \dfrac{P}{4} = 0$ $\qquad F_{AF} = \dfrac{P}{4}$ C ◀

FBD II: $\longrightarrow \Sigma F_x = 0\colon -C_x + \dfrac{1}{\sqrt{2}}F_{DG} = 0 \qquad F_{DG} = C_x\sqrt{2}$ $\qquad F_{DG} = \dfrac{\sqrt{2}}{2}P$ C ◀

$\uparrow \Sigma F_y = 0\colon -C_y + \dfrac{1}{\sqrt{2}}\left(\dfrac{\sqrt{2}}{2}P\right) + F_{EH} = 0 \qquad F_{EH} = \dfrac{P}{4} - \dfrac{P}{2} = -\dfrac{P}{4}$ $\qquad F_{EH} = \dfrac{P}{4}$ T ◀

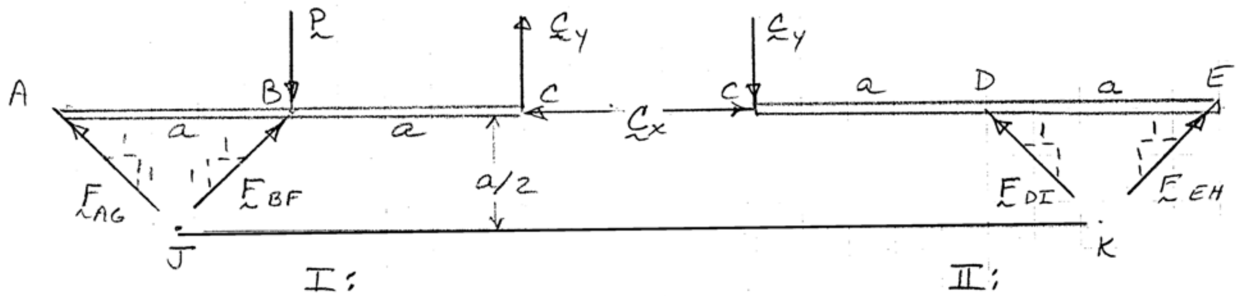
PROBLEM 6.110



Members *ABC* and *CDE* are pin-connected at *C* and are supported by four links. For the loading shown, determine the force in each link.

SOLUTION

member FBDs:



From FBD I: $\circlearrowleft \Sigma M_J = 0$: $\frac{a}{2}C_x + \frac{3a}{2}C_y - \frac{a}{2}P = 0 \qquad C_x + 3C_y = P$

FBD II: $\circlearrowleft \Sigma M_K = 0$: $\frac{a}{2}C_x - \frac{3a}{2}C_y = 0 \qquad C_x - 3C_y = 0$

Solving : $C_x = \frac{P}{2}$; $C_y = \frac{P}{6}$ as drawn

FBD I: $\circlearrowleft \Sigma M_B = 0$: $aC_y - a\frac{1}{\sqrt{2}}F_{AG} = 0 \qquad F_{AG} = \sqrt{2}C_y = \frac{\sqrt{2}}{6}P$

$F_{AG} = \frac{\sqrt{2}}{6}P$ C ◀

$\longrightarrow \Sigma F_x = 0$: $-\frac{1}{\sqrt{2}}F_{AG} + \frac{1}{\sqrt{2}}F_{BF} - C_x = 0 \qquad F_{BF} = F_{AG} + C_x\sqrt{2} = \frac{\sqrt{2}}{6}P + \frac{\sqrt{2}}{2}P$

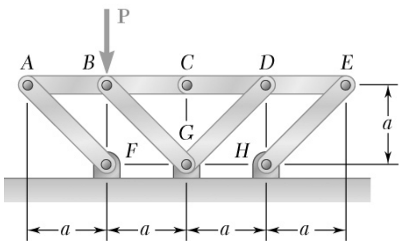
$F_{BF} = \frac{2\sqrt{2}}{3}P$ C ◀

FBD II: $\circlearrowleft \Sigma M_D = 0$: $a\frac{1}{\sqrt{2}}F_{EH} + aC_y = 0 \qquad F_{EH} = -\sqrt{2}C_y = -\frac{\sqrt{2}}{6}P$

$F_{EH} = \frac{\sqrt{2}}{6}P$ T ◀

$\longrightarrow \Sigma F_x = 0$: $C_x - \frac{1}{\sqrt{2}}F_{DI} + \frac{1}{\sqrt{2}}F_{EH} = 0 \qquad F_{DI} = F_{EH} + C_x\sqrt{2} = -\frac{\sqrt{2}}{6}P + \frac{\sqrt{2}}{2}P$

$F_{DI} = \frac{\sqrt{2}}{3}P$ C ◀

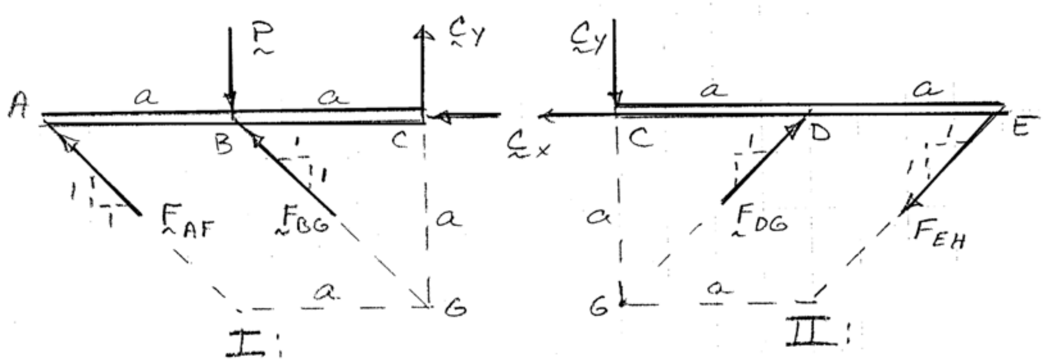


PROBLEM 6.111

Members ABC and CDE are pin-connected at C and are supported by four links. For the loading shown, determine the force in each link.

SOLUTION

member FBDs:



FBD I: $\Sigma M_B = 0: aC_y - a\frac{1}{\sqrt{2}}F_{AF} = 0 \qquad F_{AF} = \sqrt{2}C_y$

FBD II: $M_D = 0: aC_y - a\frac{1}{\sqrt{2}}F_{EH} = 0 \qquad F_{EH} = \sqrt{2}C_y$

FBDs combined: $\Sigma M_G = 0: aP - a\frac{1}{\sqrt{2}}F_{AF} - a\frac{1}{\sqrt{2}}F_{EH} = 0 \qquad P = \frac{1}{\sqrt{2}}\sqrt{2}C_y + \frac{1}{\sqrt{2}}\sqrt{2}C_y$

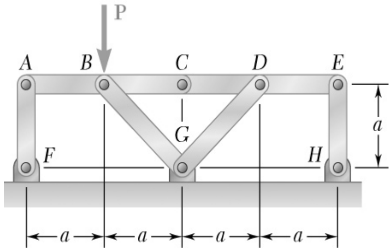
$C_y = \frac{P}{2}$

so $F_{AF} = \frac{\sqrt{2}}{2}P \quad \text{C}$ ◀

$F_{EH} = \frac{\sqrt{2}}{2}P \quad \text{T}$ ◀

FBD I: $\Sigma F_y = 0: \frac{1}{\sqrt{2}}F_{AF} + \frac{1}{\sqrt{2}}F_{BG} - P + C_y = 0 \qquad \frac{P}{2} + \frac{1}{\sqrt{2}}F_{BG} - P + \frac{P}{2} = 0 \qquad F_{BG} = 0$ ◀

FBD II: $\Sigma F_y = 0: -C_y + \frac{1}{\sqrt{2}}F_{DG} - \frac{1}{\sqrt{2}}F_{EH} = 0 \qquad -\frac{P}{2} + \frac{1}{\sqrt{2}}F_{DG} - \frac{P}{2} = 0 \qquad F_{DG} = \sqrt{2}P \quad \text{C}$ ◀

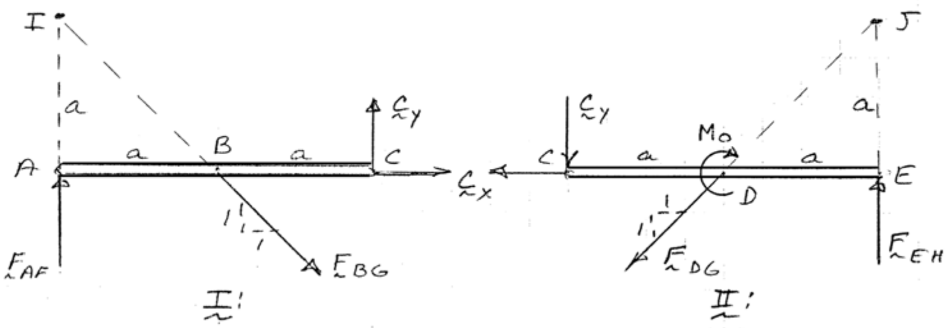


PROBLEM 6.112

Solve Prob. 6.109 assuming that the force **P** is replaced by a clockwise couple of moment $\mathbf{M}_0$ applied to member *CDE* at *D*.

SOLUTION

FBDs members:



FBD I: $\quad \Sigma M_A = 0\text{: } 2aC_y - a\dfrac{1}{\sqrt{2}}F_{BG} = 0 \qquad F_{BG} = 2\sqrt{2}C_y$

FBD II: $\quad \Sigma M_E = 0\text{: } 2aC_y - M_0 + a\dfrac{1}{\sqrt{2}}F_{DG} = 0 \qquad F_{DG} = -2\sqrt{2}C_y + \dfrac{\sqrt{2}}{a}M_0$

FBDs combined: $\quad \rightarrow \Sigma F_x = 0\text{: } \dfrac{1}{\sqrt{2}}F_{BG} + C_x - C_x - \dfrac{1}{\sqrt{2}}F_{DG} = 0$

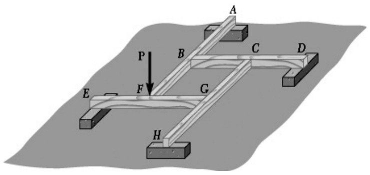
$$F_{BG} = F_{DG}\text{: } 2\sqrt{2}C_y = -2\sqrt{2}C_y + \frac{\sqrt{2}}{a}M_0 \qquad C_y = \frac{M_0}{4a}$$

$$F_{BG} = 2\sqrt{2}\frac{M_0}{4a} \qquad\qquad F_{BG} = \frac{\sqrt{2}}{2}\frac{M_0}{a} \quad \text{T} \blacktriangleleft$$

$$F_{DG} = \frac{\sqrt{2}}{a}M_0 - 2\sqrt{2}\frac{M_0}{4a} \qquad\qquad F_{DG} = \frac{\sqrt{2}}{2}\frac{M_0}{a} \quad \text{T} \blacktriangleleft$$

FBD I: $\uparrow \Sigma F_y = 0\text{: } F_{AF} - \dfrac{1}{\sqrt{2}}F_{BG} + C_y = 0 \qquad F_{AF} = \dfrac{1}{\sqrt{2}}\dfrac{\sqrt{2}}{2}\dfrac{M_0}{a} - \dfrac{M_0}{4a} \qquad\qquad F_{AF} = \dfrac{M_0}{4a} \quad \text{C} \blacktriangleleft$

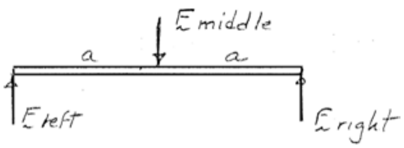
FBD II: $\Sigma H_D = 0\text{: } aC_y - M_0 + aF_{EH} = 0 \qquad aF_{EH} = M_0 - a\dfrac{M_0}{4a} \qquad\qquad F_{EH} = \dfrac{3}{4}\dfrac{M_0}{a} \quad \text{C} \blacktriangleleft$



PROBLEM 6.113

Four wooden beams, each of length $2a$, are nailed together at their midpoints to form the support system shown. Assuming that only vertical forces are exerted at the connections, determine the vertical reactions at A, D, E, and H.

SOLUTION



Note that, if we assume P is applied to EG, each individual member FBD looks like

so $$2F_{\text{left}} = 2F_{\text{right}} = F_{\text{middle}}$$

Labeling each interaction force with the letter corresponding to the joint of its application, we see that

$$B = 2A = 2F$$

$$C = 2B = 2D$$

$$G = 2C = 2H$$

$$P + F = 2G(=4C = 8B = 16F) = 2E$$

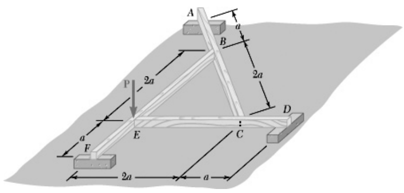
From $$P + F = 16F, \quad F = \frac{P}{15}$$ so $\mathbf{A} = \frac{P}{15}\uparrow$ ◀

$\mathbf{D} = \frac{2P}{15}\uparrow$ ◀

$\mathbf{H} = \frac{4P}{15}\uparrow$ ◀

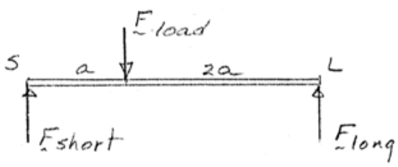
$\mathbf{E} = \frac{8P}{15}\uparrow$ ◀

PROBLEM 6.114



Three wooden beams, each of length of $3a$, are nailed together to form the support system shown. Assuming that only vertical forces are exerted at the connections, determine the vertical reactions at A, D, and F.

SOLUTION



Note that, if we assume P is applied to BF, each individual member FBD looks like:

so
$$F_{\text{short}} = 2F_{\text{long}} = \frac{2}{3}F_{\text{load}}$$

(by moment equations about S and L).

Labeling each interaction force with the letter corresponding to the joint of application, we have:

$$F = \frac{2(P+E)}{3} = 2B$$

$$E = \frac{C}{3} = \frac{D}{2}$$

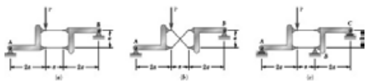
$$C = \frac{B}{3} = \frac{A}{2}$$

so
$$\frac{2(P+E)}{3} = 2B = 6C = 18E \qquad P + E = 27E \qquad \mathbf{E} = \frac{P}{26}\uparrow$$

so $\mathbf{D} = 2\mathbf{E} = \dfrac{P}{13}\uparrow$ ◀

$$A = 2C = 3E \qquad \mathbf{A} = \frac{3P}{13}\uparrow \quad ◀$$

$$F = \frac{2}{3}\left(P + \frac{P}{26}\right) \qquad \mathbf{F} = \frac{9P}{13}\uparrow \quad ◀$$

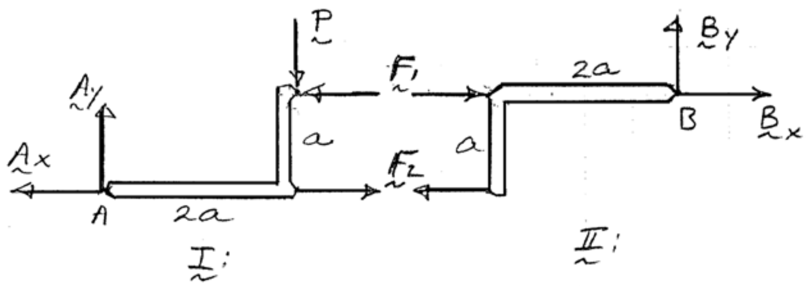


PROBLEM 6.115

Each of the frames shown consists of two L-shaped members connected by two rigid links. For each frame, determine the reactions at the supports and indicate whether the frame is rigid.

SOLUTION

(*a*) **member FBDs:**



FBD I: $\circlearrowleft \Sigma M_A = 0: aF_1 - 2aP = 0 \qquad F_1 = 2P; \uparrow \Sigma F_y = 0: A_y - P = 0 \qquad \mathbf{A}_y = P \uparrow$

FBD II: $\circlearrowleft \Sigma M_B = 0: -aF_2 = 0 \qquad F_2 = 0$

$\longrightarrow \Sigma F_x = 0: B_x + F_1 = 0, \; B_x = -F_1 = -2P \qquad \mathbf{B}_x = 2P \longrightarrow$

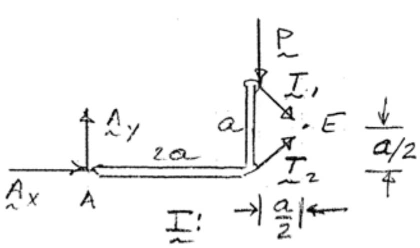
$\uparrow \Sigma F_y = 0: \; B_y = 0$ so $\mathbf{B} = 2P \longrightarrow$ ◀

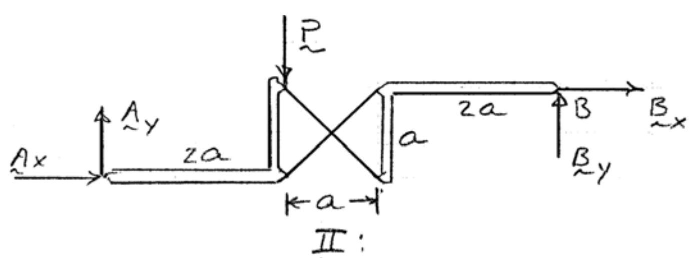
FBD I: $\longrightarrow \Sigma F_x = 0: -A_x - F_1 + F_2 = 0 \qquad A_x = F_2 - F_1 = 0 - 2P \qquad \mathbf{A}_x = 2P \longrightarrow$

so $\mathbf{A} = 2.24P \measuredangle 26.6°$ ◀

frame is rigid ◀

(*b*) **FBD left:** **FBD whole:**





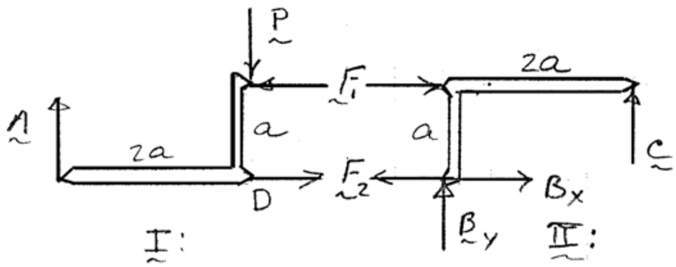
FBD I: $\circlearrowleft \Sigma M_E = 0: \frac{a}{2}P + \frac{a}{2}A_x - \frac{5a}{2}A_y = 0 \qquad A_x - 5A_y = -P$

FBD II: $\circlearrowleft \Sigma M_B = 0: 3aP + aA_x - 5aA_y = 0 \qquad A_x - 5A_y = -3P$

This is impossible unless $P = 0 \quad \therefore$ not rigid ◀

PROBLEM 6.115 CONTINUED

(*c*) **member FBDs:**



FBD I: $\Sigma F_y = 0: A - P = 0$ $\qquad$ $\mathbf{A} = P \uparrow$ ◀

$\circlearrowright \Sigma M_D = 0: aF_1 - 2aA = 0 \qquad F_1 = 2P$

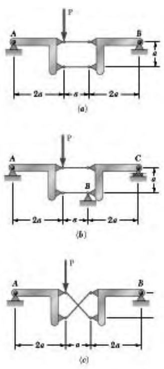
$\rightarrow \Sigma F_x = 0: F_2 - F_1 = 0 \qquad F_2 = 2P$

FBD II: $\circlearrowright \Sigma M_B = 0: 2aC - aF_1 = 0 \qquad C = \frac{F_1}{2} = P$ $\qquad$ $\mathbf{C} = P \uparrow$ ◀

$\rightarrow \Sigma F_x = 0: F_1 - F_2 + B_x = 0 \qquad B_x = P - P = 0$

$\uparrow \Sigma F_x = 0: B_y + C = 0 \qquad B_y = -C = -P$ $\qquad$ $\mathbf{B} = P$ ◀

Frame is rigid ◀

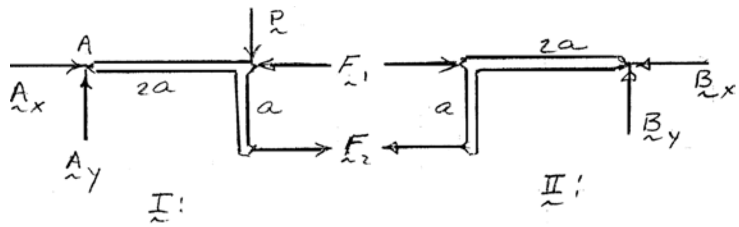


PROBLEM 6.116

Each of the frames shown consists of two L-shaped members connected by two rigid links. For each frame, determine the reactions at the supports and indicate whether the frame is rigid.

SOLUTION

(*a*) **member FBDs:**

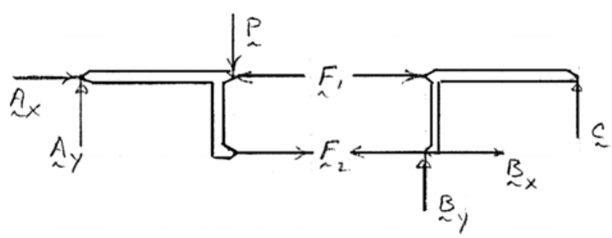


FBD II: $\uparrow \Sigma F_y = 0: B_y = 0$ $\quad \circlearrowleft \Sigma M_B = 0: aF_2 = 0$ $\quad F_2 = 0$

FBD I: $\circlearrowleft \Sigma M_A = 0: aF_2 - 2aP = 0$ $\quad$ but $F_2 = 0$

so $P = 0$ $\quad$ not rigid for $P \neq 0$ ◀

(*b*) **member FBDs:**

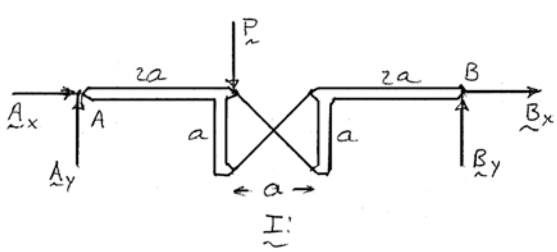


Note: 7 unknowns $\left(A_x, A_y, B_x, B_y, F_1, F_2, C\right)$ but only 6 independent equations.

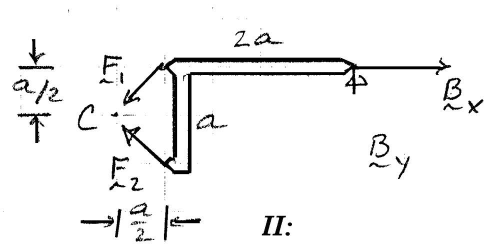
System is statically indeterminate ◀

System is, however, rigid ◀

(*c*) **FBD whole:**



FBD right:



II:

PROBLEM 6.116 CONTINUED

FBD I: $\circlearrowleft \Sigma M_A = 0: 5aB_y - 2aP = 0$ $\qquad$ $\mathbf{B}_y = \frac{2}{5}P \uparrow$

$\uparrow \Sigma F_y = 0: A_y - P + \frac{2}{5}P = 0$ $\qquad$ $\mathbf{A}_y = \frac{3}{5}P \uparrow$

FBD II: $\circlearrowright \Sigma M_c = 0: \frac{a}{2}B_x - \frac{5a}{2}B_y = 0$ $\qquad$ $B_x = 5B_y$ $\qquad$ $\mathbf{B}_x = 2P \longrightarrow$

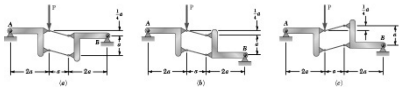
FBD I: $\longrightarrow \Sigma F_x = 0: A_x + B_x = 0$ $\qquad$ $A_x = -B_x$ $\qquad$ $\mathbf{A}_x = 2P \longleftarrow$

$\mathbf{A} = 2.09P \measuredangle 16.70° \blacktriangleleft$

$\mathbf{B} = 2.04P \measuredangle 11.31° \blacktriangleleft$

System is rigid ◀

PROBLEM 6.117

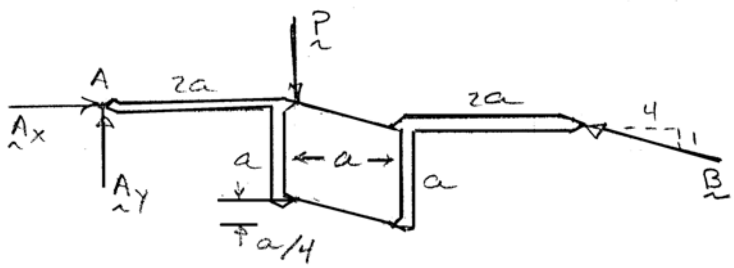


Each of the frames shown consists of two L-shaped members connected by two rigid links. For each frame, determine the reactions at the supports and indicate whether the frame is rigid.

SOLUTION

Note: In all three cases, the right member has only three forces acting, two of which are parallel. Thus the third force, at B, must be parallel to the link forces.

(a) **FBD whole:**



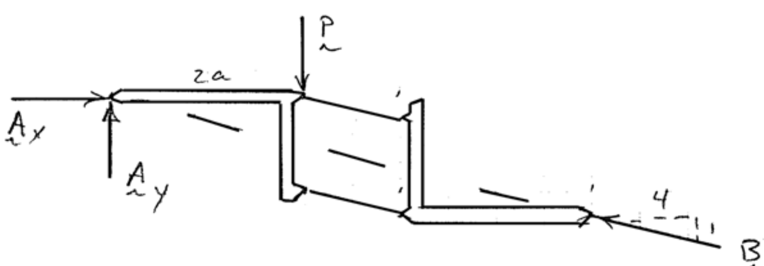
$$\circlearrowleft \Sigma M_A = 0\text{:}\; -2aP - \frac{a}{4}\frac{4}{\sqrt{17}}B + 5a\frac{1}{\sqrt{17}}B = 0 \qquad B = 2.06P \qquad \mathbf{B} = 2.06P \;\searrow 14.04° \blacktriangleleft$$

$$\rightarrow \Sigma F_x = 0\text{:}\; A_x - \frac{4}{\sqrt{17}}B = 0 \qquad \mathbf{A}_x = 2P \longleftarrow$$

$$\uparrow \Sigma F_y = 0\text{:}\; A_y - P + \frac{1}{\sqrt{17}}B = 0 \qquad \mathbf{A}_y = \frac{P}{2} \uparrow \qquad \mathbf{A} = 2.06P \;\measuredangle\, 14.04° \blacktriangleleft$$

rigid ◀

(b) **FBD whole:**



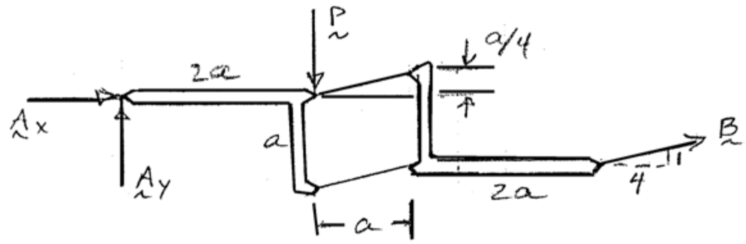
Since **B** passes through A, $\circlearrowleft \Sigma M_A = 2aP = 0$ only if $P = 0$

$\therefore$ no equilibrium if $P \neq 0$

not rigid ◀

PROBLEM 6.117 CONTINUED

(*c*) **FBD whole:**



$$\circlearrowleft \Sigma M_A = 0\text{:}\ \ 5a\frac{1}{\sqrt{17}}B + \frac{3a}{4}\frac{4}{\sqrt{17}}B - 2aP = 0 \qquad B = \frac{\sqrt{17}}{4}P$$

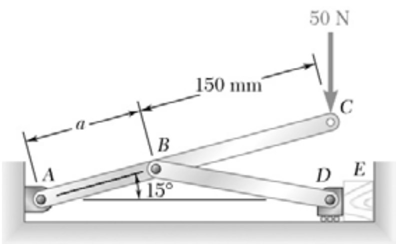
$\mathbf{B} = 1.031P \measuredangle 14.04°$ ◀

$$\rightarrow \Sigma F_x = 0\text{:}\ \ A_x + \frac{4}{\sqrt{17}}B = 0 \qquad A_x = -P$$

$$\uparrow \Sigma F_y = 0\text{:}\ \ A_y - P + \frac{1}{\sqrt{17}}B = 0 \qquad A_y = P - \frac{P}{4} = \frac{3P}{4}$$

$\mathbf{A} = 1.250P$ ◸ $36.9°$ ◀

System is rigid ◀

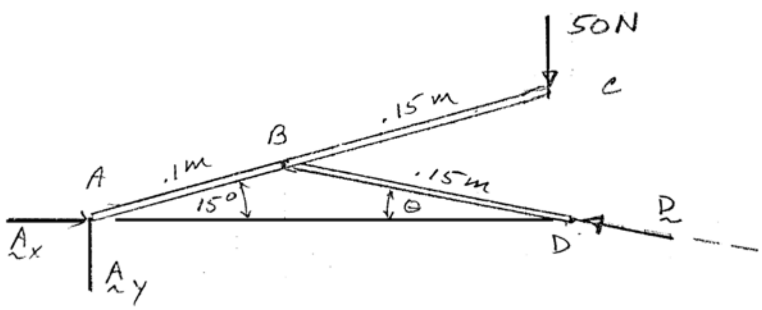


PROBLEM 6.118

A 50-N force directed vertically downward is applied to the toggle vise at *C*. Knowing that link *BD* is 150 mm long and that $a = 100$ mm, determine the horizontal force exerted on block *E*.

SOLUTION

FBD machine:



Note: $(0.1\text{ m})\sin 15° = (0.15\text{ m})\sin\theta$

$\theta = \sin^{-1}(0.17255) = 9.9359°$

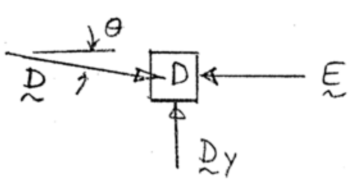
$$\overline{AD} = (0.1\text{ m})\cos 15° + (0.15\text{ m})\cos\theta$$

$$= 0.24434\text{ m}$$

$$\circlearrowleft \Sigma M_A = 0\text{: } (0.24434\text{ m})D\sin\theta - (0.25\text{ m})(\cos 15°)(50\text{ N}) = 0$$

$$D = \frac{0.25\cos 15°}{(0.24434)(0.17255)}50\text{ N} = 286.38\text{ N}$$

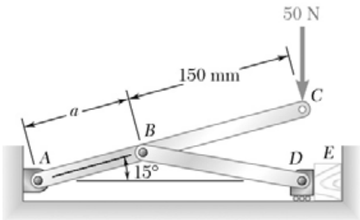
FBD part D :



$$\xrightarrow{} \Sigma F_x = 0\text{: } D\cos\theta - E = 0$$

$$E = D\cos\theta = 282.1\text{ N}$$

$$\mathbf{E}_{\text{block}} = 282\text{ N} \longrightarrow \blacktriangleleft$$

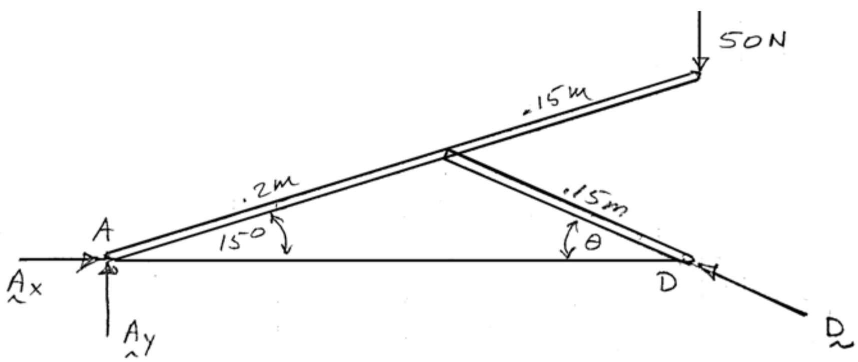


PROBLEM 6.119

A 50-N force directed vertically downward is applied to the toggle vise at C. Knowing that link BD is 150 mm long and that $a = 200$ mm, determine the horizontal force exerted on block E.

SOLUTION

FBD machine:



Note: $(0.2 \text{ m})\sin 15° = (0.15 \text{ m})\sin\theta$

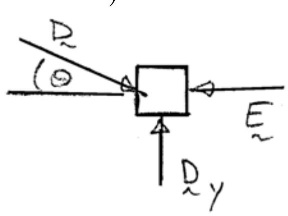
$$\theta = \sin^{-1}(0.3451) = 20.187°$$

$$\overline{AD} = (0.2 \text{ m})\cos 15° + (0.15\text{m})\cos\theta$$

$$= 0.33397 \text{ m}$$

$$\circlearrowleft \Sigma M_A = 0{:}\ (0.33397 \text{ m})D \sin 20.187° - (0.35\text{m})(\cos 15°)(50 \text{ N}) = 0 \qquad D = 146.67 \text{ N}$$

FBD part D:



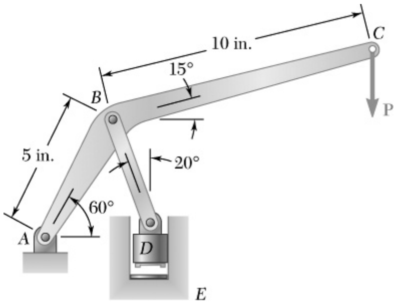
$$\xrightarrow{} \Sigma F_x = 0{:}\ D\cos\theta - E = 0$$

$$E = (146.67 \text{ N})\cos 20.187°$$

$$= 137.66 \text{ N}$$

$$\mathbf{E}_{\text{on block}} = 137.7 \text{ N} \longrightarrow \blacktriangleleft$$

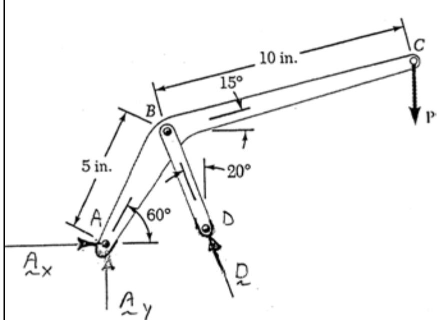
PROBLEM 6.120



The press shown is used to emboss a small seal at E. Knowing that $P = 60$ lb, determine (a) the vertical component of the force exerted on the seal, (b) the reaction at A.

SOLUTION

FBD machine:



$$\circlearrowleft \Sigma M_A = 0: \left[(5\text{ in.})\cos 60°\right]D\cos 20° + \left[(5\text{ in.})\sin 60°\right]D\sin 20°$$

$$-\left[(5\text{ in.})\cos 60° + (10\text{ in.})\cos 15°\right](60\text{ lb}) = 0$$

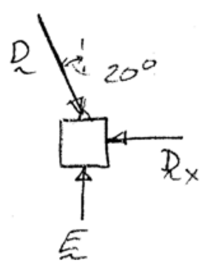
$$D = 190.473\text{ lb}$$

$$\xrightarrow{} \Sigma F_x = 0: A_x - D\sin 20° = 0 \qquad A_x = 65.146\text{ lb}$$

$$\uparrow \Sigma F_y = 0: A_y + D\cos 20° - 60\text{ lb} = 0 \qquad A_x = -118.99\text{ lb}$$

$$\text{so } \mathbf{A} = 135.7\text{ lb} \searrow 61.3° \blacktriangleleft$$

FBD part D:

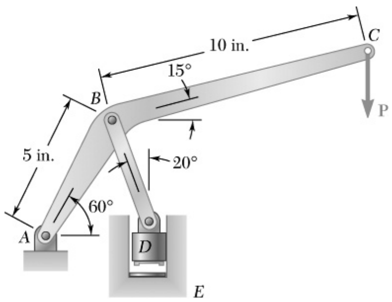


$$\uparrow \Sigma F_y = 0: E - D\cos 20° = 0 \qquad E = (190.47\text{ lb})\cos 20°$$

$$= 178.98\text{ lb}$$

$$\mathbf{E}_{\text{on seal}} = 179.0\text{ lb} \downarrow$$

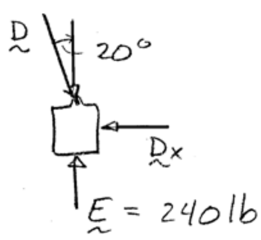
PROBLEM 6.121



The press shown is used to emboss a small seal at E. Knowing that the vertical component of the force exerted on the seal must be 240 lb, determine (a) the required vertical force **P**, (b) the corresponding reaction at A.

SOLUTION

FBD part D:

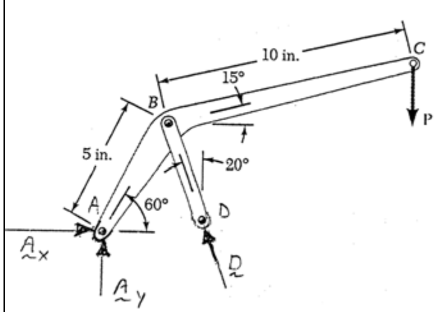


(a)

$$\uparrow \Sigma F_y = 0: E - D\cos 20° = 0$$

$$D = \frac{240 \text{ lb}}{\cos 20°} = 255.40 \text{ lb}$$

FBD machine:



$$\circlearrowright \Sigma M_A = 0: [(5\text{in.})\cos 60°]D\cos 20° + [(5 \text{ in.})\cos 60°]D\sin 20°$$

$$-[(5\text{in.})\cos 60° + 10\text{in.})\cos 15°]P = 0$$

$$P = 80.453 \text{ lb}$$

$\mathbf{P} = 80.5 \text{ lb} \downarrow$ ◀

(b)

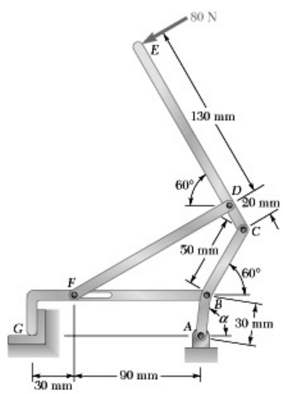
$$\rightarrow \Sigma F_x = 0: A_x - D\sin 20° = 0$$

$$A_x = 87.35 \text{ lb}$$

$$\uparrow \Sigma F_y = 0: A_y + 240 \text{ lb} - 80.5 \text{ lb} = 0$$

$$A_y = 159.5 \text{ lb}$$

$\mathbf{A} = 181.9 \text{ lb} \searrow 61.3°$ ◀

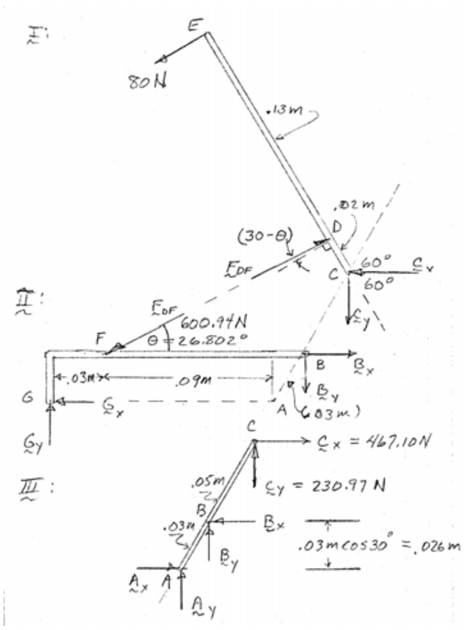


PROBLEM 6.122

The double toggle latching mechanism shown is used to hold member G against the support. Knowing that $\alpha = 60°$, determine the force exerted on G.

SOLUTION

member FBDs:



Note:
$$\tan\theta = \frac{(0.05\text{ m} + 0.02\text{ m})\sin 60°}{(0.09\text{ m}) + \left[(0.03 + 0.05 - 0.02)\text{m}\right]\cos 60°}$$

$$= 0.50518$$

$$\theta = 26.802°$$

FBD I:

$$\Sigma M_C = 0:\ (0.15\text{ m})80\text{ N} - (0.02\text{ m})F_{DF}\cos(30° - 26.802°) = 0$$

$$F_{DF} = 600.94\text{ N}$$

$$\xrightarrow{} \Sigma F_x = 0:\ (600.94\text{ N})\cos 26.802° - (80\text{ N})\sin 60° - C_x = 0$$

$$C_x = 467.10\text{ N}$$

$$\uparrow \Sigma F_y = 0:\ -C_y + (600.94\text{ N})\sin 26.802° - (80\text{ N})\cos 60° = 0$$

$$C_y = 230.97\text{ N}$$

FBD II:

$$\Sigma M_G = 0:\ \left[(0.03\text{ m})(\cos 30°)\right]B_x$$

$$+\left[0.12\text{ m} + (0.03\text{m})\cos 60°\right]B_y$$

$$+(0.03\text{ m})\left[(600.94\text{ N})\sin 26.802°\right]$$

$$-(.026\text{ m})\left[(600.94\text{ N})\cos 26.802°\right] = 0$$

$$0.015\sqrt{3}B_x + 0.135B_y = 5.8065\text{ N} \qquad (1)$$

PROBLEM 6.122 CONTINUED

FBD III:

$$\circlearrowleft \Sigma M_A = 0: \left[(0.03\,\text{m})(\sin 60^\circ)\right]B_x + \left[(0.03\text{ m})(\cos 60^\circ)\right]B_y$$

$$-\left[(0.08\text{ m})(\sin 60^\circ)\right]467.10\text{ N}$$

$$+\left[(0.08\text{ m})(\cos 60^\circ)\right]230.97\text{ N} = 0$$

$$0.015\sqrt{3}B_x + 0.015B_y = 23.123\text{ N} \qquad (2)$$

Solving (1) and (2) $\quad B_x = 973.31\text{ N}$

$$B_y = -144.303\text{ N}$$

FBD II:

$$\xrightarrow{} \Sigma F_x = 0: \; -G_x - (600.94\text{ N})\cos 26.802^\circ + 973.31\text{ N} = 0$$

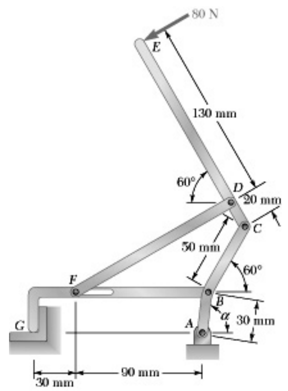
$$\mathbf{G}_x = 436.93\text{ N} \longleftarrow$$

$$\uparrow \Sigma F_y = 0: \; -(-144.303\text{ N}) + G_y - (600.94\text{ N})\sin 26.802^\circ = 0$$

$$\mathbf{G}_y = 126.67\text{ N} \uparrow$$

Therefore, the force acting on member G is $\quad \mathbf{G} = 455\text{ N} \;\searrow\; 16.17^\circ$ ◀

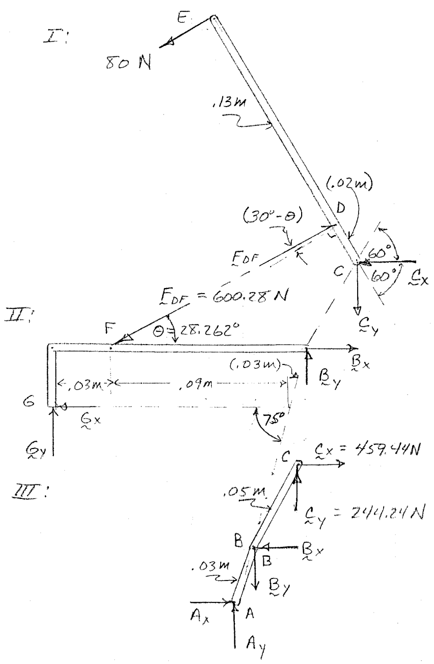
PROBLEM 6.123



The double toggle latching mechanism shown is used to hold member G against the support. Knowing that $\alpha = 75°$, determine the force exerted on G.

SOLUTION

FBDs:



Note: $$\tan\theta = \frac{(0.07\text{ m})\cos 30°}{0.09\text{ m} + (0.03\text{ m})\cos 75° + (0.03\text{ m})\sin 30°}$$

$$\theta = 28.262°$$

FBD I:

$$\circlearrowleft \Sigma M_C = 0\text{: } (0.15\text{ m})(80\text{ N}) - (0.02\text{ m})\ F_{DF}\ \cos(30° - 28.262°) = 0$$

$$F_{DF} = 600.28\text{ N}$$

$$\xrightarrow{} \Sigma F_x = 0\text{: } -C_x - (80\text{ N})\cos 30° + (600.28\text{ N})\cos 28.262° = 0$$

$$C_x = 459.44\text{ N}$$

$$\uparrow \Sigma F_y = 0\text{: } -C_y - (80\text{ N})\sin 30° - (600.28\text{ N})\sin 28.262° = 0$$

$$C_y = 244.24\text{ N}$$

FBD II:

$$\circlearrowleft \Sigma M_G = 0\text{: } -\left[(0.03\text{m})\sin 75°\right]B_x + \left[0.120\text{ m} + (0.03\text{m})\cos 75°\right]B_y$$

$$+\left[(0.03\text{ m})\sin 75°\right]\left[(600.28\text{ N})\cos 28.262°\right]$$

$$-(0.03\text{ m})\left[(600.28\text{ N})\sin 28.262°\right] = 0$$

$$0.9659B_x - 4.2588B_y = 226.47\text{ N} \qquad (1)$$

PROBLEM 6.123 CONTINUED

FBD III:

$$\circlearrowleft \Sigma M_A = 0: \left[(0.03\text{ m})\sin 75°\right]B_x - \left[(0.03\text{ m})\cos 75°\right]B_y$$

$$-\left[(0.03m)\sin 75° + (0.05\text{ m})\sin 60°\right](459.44\text{ N})$$

$$+\left[(0.03\text{ m})\cos 75° + (0.05\text{ m})\cos 60°\right](244.24\text{ N}) = 0$$

$$0.9659B_x - 0.2588B_y = 840.18\text{ N} \qquad (2)$$

Solving (1) and (2): $B_x = 910.93\text{ N}$

$B_y = 153.428\text{ N}$

FBD II:

$$\xrightarrow{} \Sigma F_x = 0: \; -G_x - (600.28\text{ N})\cos 28.262° + 910.93\text{ N} = 0$$

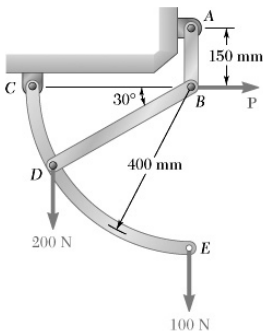
$$\mathbf{G}_x = 382.21\text{ N} \longleftarrow$$

$$\uparrow \Sigma F_y = 0: G_y - (600.28\text{ N})\sin 28.262° + 153.428\text{ N} = 0$$

$$\mathbf{G}_y = 130.81\text{ N} \uparrow$$

Therefore, the force acting on member G is: $\mathbf{G} = 404\text{ N} \searrow 18.89°$ ◀

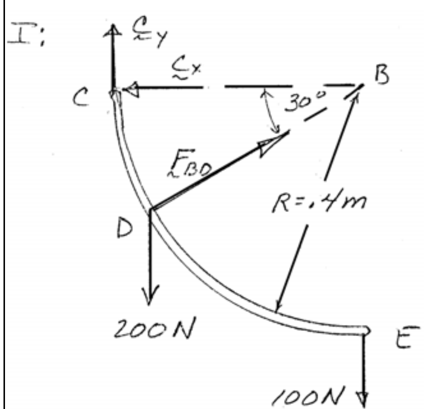
PROBLEM 6.124

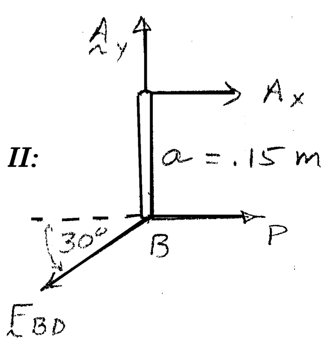


For the system and loading shown, determine (*a*) the force **P** required for equilibrium, (*b*) the corresponding force in member *BD*, (*c*) the corresponding reaction at *C*.

SOLUTION

member FBDs:





FBD I :

$$\Sigma M_C = 0: R\left(F_{BD}\sin 30°\right)$$

$$-\left[R\left(1-\cos 30°\right)\right]\left(200\text{ N}\right) - R\left(100\text{ N}\right) = 0$$

$F_{BD} = 253.6$ N $\qquad F_{BD} = 254$ N T ◀

$$\longrightarrow \Sigma F_x = 0: \ -C_x + \left(253.6\text{ N}\right)\cos 30° = 0$$

$\mathbf{C}_x = 219.6$ N $\longleftarrow$

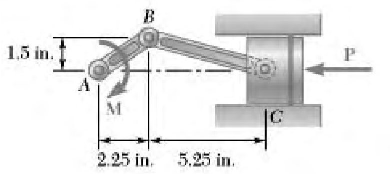
$$\uparrow \Sigma F_y = 0: C_y + \left(253.6\text{ N}\right)\sin 30° - 200\text{ N} - 100\text{ N} = 0$$

$\mathbf{C}_y = 173.2$ N $\uparrow$ $\qquad$ so $\mathbf{C} = 280$ N $\searrow$ 38.3° ◀

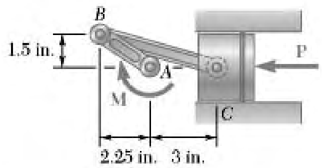
FBD II : $\Sigma M_A = 0: aP - a\left[\left(253.6\text{ N}\right)\cos 30°\right] = 0$

$\mathbf{P} = 220$ N $\longrightarrow$ ◀

PROBLEM 6.125



(a)

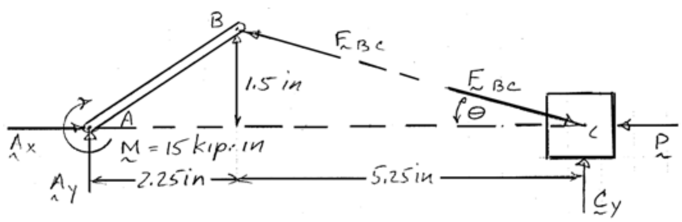


(b)

A couple **M** of magnitude $15 \text{ kip} \cdot \text{in.}$ is applied to the crank of the engine system shown. For each of the two positions shown, determine the force **P** required to hold the system in equilibrium.

SOLUTION

(*a*) **FBDs:**



Note: $\tan\theta = \dfrac{1.5 \text{ in.}}{5.25 \text{ in.}}$

$$= \frac{2}{7}$$

FBD whole: $\circlearrowleft \Sigma M_A = 0\colon (7.50 \text{ in.})C_y - 15 \text{ kip} \cdot \text{in.} = 0 \qquad C_y = 2.00 \text{ kips}$

FBD piston: $\uparrow \Sigma F_y = 0\colon C_y - F_{BC}\sin\theta = 0 \qquad F_{BC} = \dfrac{C_y}{\sin\theta} = \dfrac{2 \text{ kips}}{\sin\theta}$

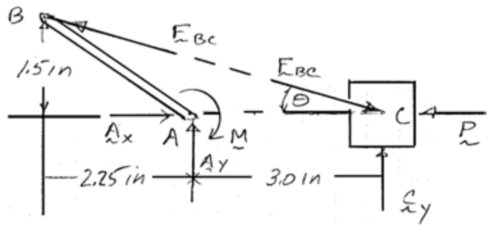
$$\longrightarrow \Sigma F_x = 0\colon F_{BC}\cos\theta - P = 0$$

$$P = F_{BC}\cos\theta = \frac{2 \text{ kips}}{\tan\theta} = 7 \text{ kips}$$

$\mathbf{P} = 7.00 \text{ kips} \longleftarrow$ ◀

PROBLEM 6.125 CONTINUED

(*b*) **FBDs:**



Note: $\tan\theta = \dfrac{2}{7}$ as above

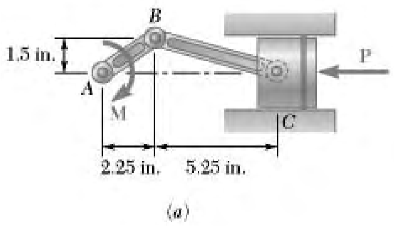
FBD whole: $\circlearrowleft \Sigma M_A = 0\text{: } (3\text{ in.})C_y - 15\text{ kip}\cdot\text{in.} = 0 \qquad C_y = 5\text{ kips}$

$$\Sigma F_y = 0\text{: } C_y - F_{BC}\sin\theta = 0 \qquad F_{BC} = \frac{C_y}{\sin\theta}$$

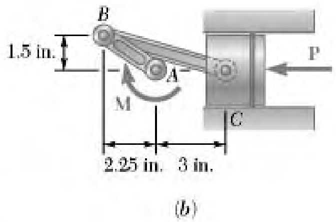
$$\longrightarrow \Sigma F_x = 0\text{: } F_{BC}\cos\theta - P = 0$$

$$P = F_{BC}\cos\theta = \frac{C_y}{\tan\theta} = \frac{5\text{ kips}}{2/7}$$

$\mathbf{P} = 17.50\text{ kips} \longleftarrow$ ◀



(*a*)



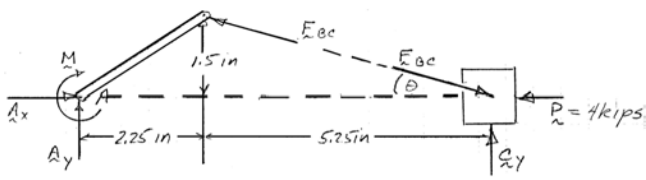
(*b*)

PROBLEM 6.126

A force **P** of magnitude 4 kips is applied to the piston of the engine system shown. For each of the two positions shown, determine the couple **M** required to hold the system in equilibrium.

SOLUTION

(*a*) **FBDs:**



Note: $\tan\theta = \dfrac{1.5 \text{ in.}}{5.25 \text{ in.}}$

$= \dfrac{2}{7}$

FBD piston: $\longrightarrow \Sigma F_x = 0: F_{BC}\cos\theta - P = 0 \qquad F_{BC} = \dfrac{P}{\cos\theta}$

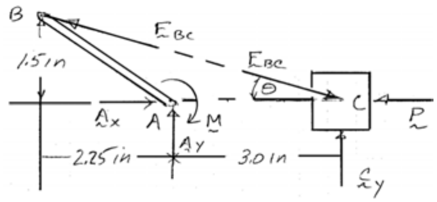
$\uparrow \Sigma F_y = 0: C_y - F_{BC}\sin\theta = 0 \qquad C_y = F_{BC}\sin\theta = P\tan\theta = \dfrac{2}{7}P$

FBD whole: $\circlearrowleft \Sigma M_A = 0: (7.50 \text{ in.})C_y - M = 0 \qquad M = 7.5 \text{ in.} \qquad C_y = \dfrac{15 \text{ in.}}{7}P$

$\mathbf{M} = 8.57 \text{ kip}\cdot\text{in.} \circlearrowright$ ◀

PROBLEM 6.126 CONTINUED

(*b*) **FBDs:**



Note: $\tan\theta = \dfrac{2}{7}$ as above

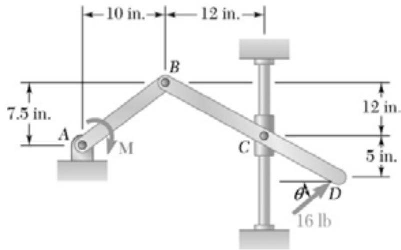
FBD piston: as above $C_y = P\tan\theta = \dfrac{2}{7}P$

FBD whole: $\circlearrowleft \Sigma M_A = 0\text{: } (3.0 \text{ in.})C_y - M = 0 \qquad M = (3.0 \text{ in.})\dfrac{2}{7}P$

$$M = \frac{24}{7} \text{ kip} \cdot \text{in.}$$

$\mathbf{M} = 3.43 \text{ kip} \cdot \text{in.}$ ◀

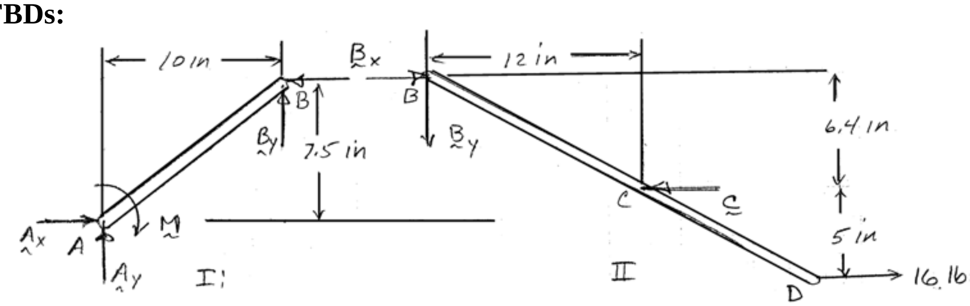
PROBLEM 6.127



Arm *BCD* is connected by pins to crank *AB* at *B* and to a collar at *C*. Neglecting the effect of friction, determine the couple **M** required to hold the system in equilibrium when $\theta = 0$.

SOLUTION

member FBDs:

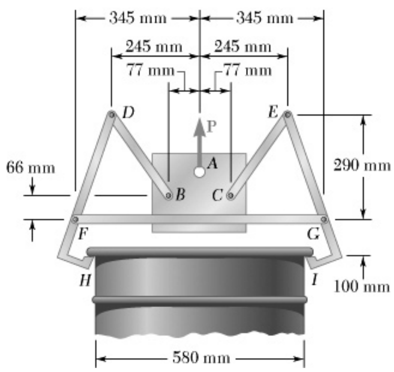


FBD II: $\uparrow \Sigma F_y = 0\text{: } B_y = 0$

$$\circlearrowleft \Sigma M_C = 0\text{: } (6.4 \text{ in.})B_x - (5 \text{ in.})16 \text{ lb} = 0 \qquad B_x = 12.5 \text{ lb}$$

FBD I: $\circlearrowleft \Sigma M_A = 0\text{: } (7.5 \text{ in.})B_x - M = 0 \qquad M = (7.5 \text{ in.})(12.5 \text{ lb}) = 93.8 \text{ lb}\cdot\text{in.}$

$$\mathbf{M} = 93.8 \text{ lb}\cdot\text{in.} \circlearrowright \blacktriangleleft$$

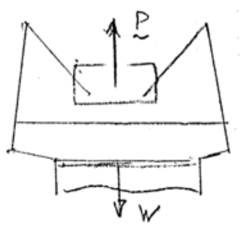


PROBLEM 6.137

The drum lifter shown is used to lift a steel drum. Knowing that the mass of the drum and its contents is 240 kg, determine the forces exerted at *F* and *H* on member *DFH*.

SOLUTION

FBD System:



$$\uparrow \Sigma F_y = 0;$$

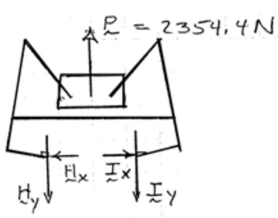
$$P - W = 0$$

$$P = W$$

$$= (240 \text{ kg})(9.81 \text{ m/s}^2)$$

$$P = 2354.4 \text{ N}$$

FBD machine:



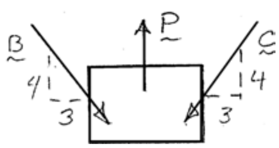
Symmetry: $H_x = I_x$

$$H_y = I_y$$

$$\uparrow \Sigma F_y = 0: P - 2H_y = 0$$

$$H_y = \frac{P}{2} = 1177.2 \text{ N}$$

FBD ABC:



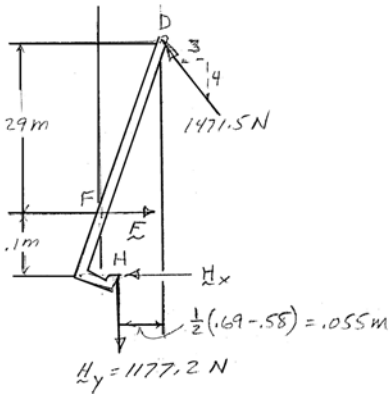
Symmetry: $C = B$

$$\Sigma F_y = 0: P - 2\frac{4}{5}B = 0$$

$$B = \frac{5}{8}P = 1471.5 \text{ N}$$

PROBLEM 6.137 CONTINUED

FBD DFH:



$$\circlearrowleft \Sigma M_H = 0: \; -(0.1 \text{ m})F + (0.39 \text{ m})\left(\frac{3}{5}1471.5 \text{ N}\right)$$

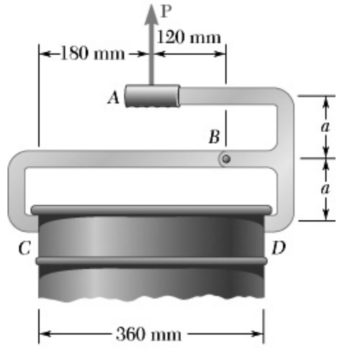
$$-(0.055 \text{ m})\left(\frac{4}{5}1471.5 \text{ N}\right) = 0$$

$$F = 3973.05 \text{ N} \qquad \mathbf{F} = 3.97 \text{ kN} \longrightarrow \blacktriangleleft$$

$$\longrightarrow \Sigma F_x = 0: 3973.05 \text{ N} - \frac{3}{5}1471.5 \text{ N} - H_x = 0$$

$$H_x = 3090.15 \text{ N}$$

$$\mathbf{H} = 3.31 \text{ kN} \;\swarrow\; 20.9° \blacktriangleleft$$

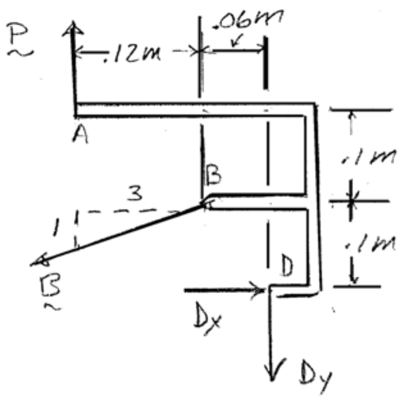


PROBLEM 6.138

A small barrel having a mass of 72 kg is lifted by a pair of tongs as shown. Knowing that $a = 100$ mm, determine the forces exerted at B and D on tong ABD.

SOLUTION

FBB ABD:



Notes: From FBD whole, by inspection,

$$P = W = mg = (72\text{ kg})(9.81\text{ m/s}^2)$$

$$P = 706.32\text{ N}$$

BC is a two-force member: $B_x = 3B_y$

$$\circlearrowleft \Sigma M_D = 0: (0.1\text{ m})B_x + (0.06\text{ m})B_y - (0.18\text{ m})P = 0$$

$$3B_y + 0.6B_y = 1.8P$$

$$B_y = \frac{P}{2} = 353.16\text{ N}$$

so

$$B_x = \frac{3P}{2} = 1059.48\text{ N}$$

and $\mathbf{B} = 1.117$ kN $\swarrow$ 18.43° ◀

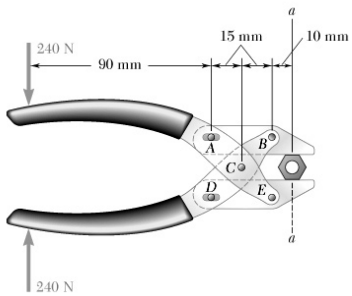
$$\longrightarrow \Sigma F_x = 0: -B_x + D_x = 0$$

$$D_x = \frac{3P}{2} = 1059.48\text{ N}$$

$$\uparrow \Sigma F_y = 0: P - B_y - D_y = 0$$

$$D_y = P - \frac{P}{2} = 353.16\text{ N}$$

so $\mathbf{D} = 1.117$ kN $\searrow$ 18.43° ◀

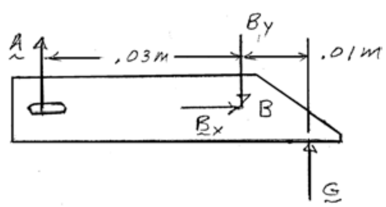


PROBLEM 6.139

Determine the magnitude of the ripping forces exerted along line *aa* on the nut when two 240-N forces are applied to the handles as shown. Assume that pins *A* and *D* slide freely in slots cut in the jaws.

SOLUTION

FBD jaw AB:



$$\rightarrow \Sigma F_x = 0: B_x = 0$$

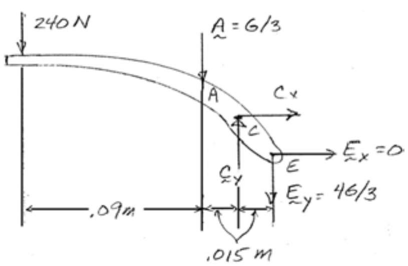
$$\circlearrowleft \Sigma M_B = 0: (0.01\text{ m})G - (0.03\text{ m})A = 0$$

$$A = \frac{G}{3}$$

$$\uparrow \Sigma F_y = 0: A + G - B_y = 0$$

$$B_y = A + G = \frac{4G}{3}$$

FBD handle ACE:

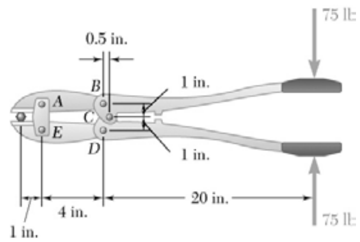


By symmetry and FBD jaw *DE*: $D = A = \frac{G}{3}, E_x = B_x = 0,$

$$E_y = B_y = \frac{4G}{3}$$

$$\circlearrowleft \Sigma M_C = 0: (0.105\text{ m})(240\text{ N}) + (0.015\text{ m})\frac{G}{3} - (0.015\text{ m})\frac{4G}{3} = 0$$

$$G = 1680\text{ N} \blacktriangleleft$$

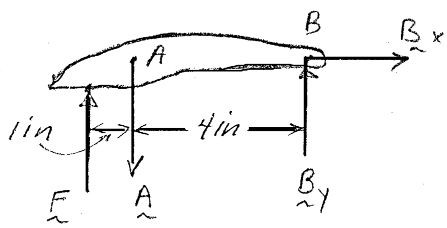


PROBLEM 6.140

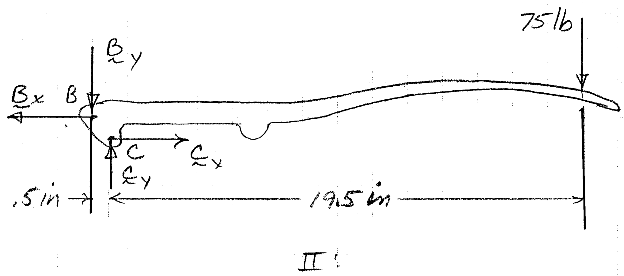
In using the bolt cutter shown, a worker applies two 75-lb forces to the handles. Determine the magnitude of the forces exerted by the cutter on the bolt.

SOLUTION

FBD Cutter AB:



FBD handle BC:



FBD I: $\longrightarrow \Sigma F_x = 0: B_x = 0$

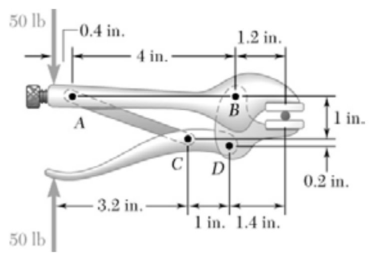
FBD II: $\circlearrowleft\ \Sigma M_C = 0: (0.5 \text{ in.})B_y - (19.5 \text{ in.})75 \text{ lb} = 0$

$$B_y = 2925 \text{ lb}$$

Then FBD I: $\circlearrowleft\ \Sigma M_A = 0: (4 \text{ in.})B_y - (1 \text{ in.})F = 0 \qquad F = 4B_y$

$$F = 11700 \text{ lb} = 11.70 \text{ kips} \blacktriangleleft$$

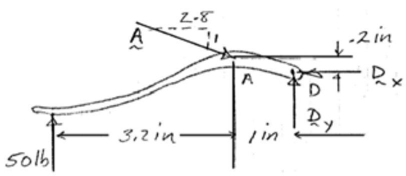
PROBLEM 6.141



Determine the magnitude of the gripping forces produced when two 50-lb forces are applied as shown.

SOLUTION

FBD handle CD:

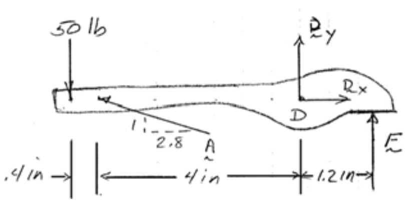


$$\circlearrowright \Sigma M_D = 0: -(4.2 \text{ in.})(50 \text{ lb}) - (0.2 \text{ in.})\frac{2.8}{\sqrt{8.84}}A$$

$$+(1 \text{ in.})\left(\frac{1}{\sqrt{8.84}}A\right) = 0$$

$$A = 477.27\sqrt{8.84} \text{ lb}$$

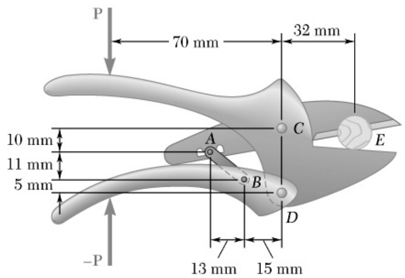
FBD handle AD:



$$\circlearrowright \Sigma M_D = 0: (4.4 \text{ in.})(50 \text{ lb}) - (4 \text{ in.})\frac{1}{\sqrt{8.84}}\left(477.27\sqrt{8.84} \text{ lb}\right)$$

$$+(1.2 \text{ in.})F = 0$$

$$F = 1.408 \text{ kips} \blacktriangleleft$$

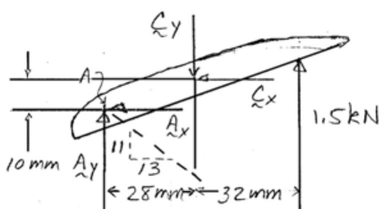


PROBLEM 6.142

The compound-lever pruning shears shown can be adjusted by placing pin A at various ratchet positions on blade ACE. Knowing that 1.5-kN vertical forces are required to complete the pruning of a small branch, determine the magnitude P of the forces that must be applied to the handles when the shears are adjusted as shown.

SOLUTION

FBD cutter AC:



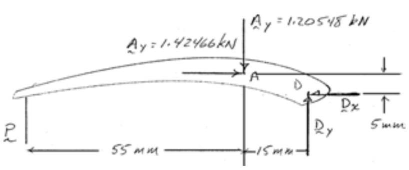
$$\Sigma M_C = 0: (32 \text{ mm})1.5 \text{ KN} - (28 \text{ mm})A_y - (10 \text{ mm})A_x = 0$$

$$10A_x + 28\left(\frac{11}{13}A_x\right) = 48 \text{ kN}$$

$$A_x = 1.42466 \text{ kN}$$

$$A_y = 1.20548 \text{ kN}$$

FBD handle AD:

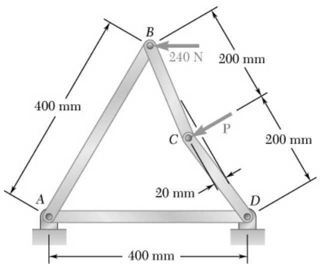


$$\Sigma M_D = 0: (15 \text{ mm})(1.20548 \text{ kN}) - (5 \text{ mm})(1.42466 \text{ kN})$$

$$-(70 \text{ mm})P = 0$$

$$P = 0.1566 \text{ kN} = 156.6 \text{ N} \blacktriangleleft$$

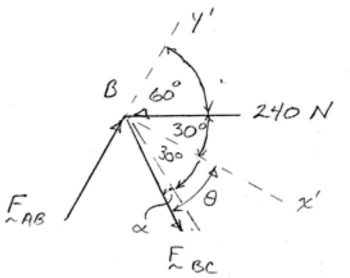
PROBLEM 6.143



Determine the force **P** which must be applied to the toggle *BCD*
To maintain equilibrium in the position shown.

SOLUTION

FBD joint B:



Note: $\theta = 30° + \alpha$

$$= 30° + \tan^{-1}\frac{20}{200}$$

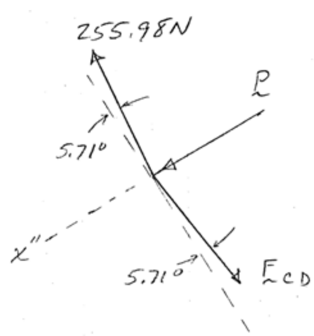
$$= 30° + 5.711°$$

$$= 35.711°$$

$$\searrow \Sigma F_{x'} = 0: F_{BC}\cos 35.711° - (240 \text{ N})\cos 30° = 0$$

$$F_{BC} = 255.98 \text{ N T}$$

FBD joint C:

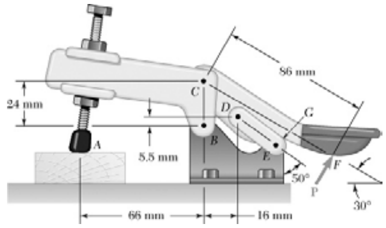


By symmetry: $F_{CD} = 255.98$ N

$$\swarrow \Sigma F_{x''} = 0: P - 2(255.98 \text{ N})\sin 5.711° = 0$$

$$\mathbf{P} = 50.9 \text{ N} \measuredangle 30.0° \blacktriangleleft$$

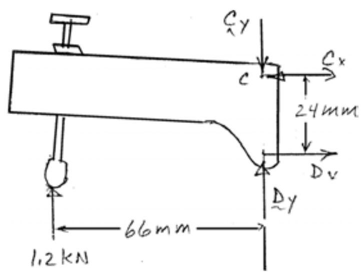
PROBLEM 6.144



In the locked position shown, the toggle clamp exerts at A a vertical 1.2-kN force on the wooden block, and handle CF rests against the stop at G. Determine the force **P** required to release the clamp. (*Hint:* To release the clamp, the forces of contact at G must be zero.)

SOLUTION

FBD BC:

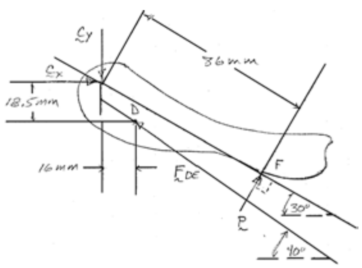


$$\circlearrowleft \Sigma M_B = 0\text{: } (24\text{ mm})C_x - (66\text{ mm})(1.2\text{ kN}) = 0$$

$$C_x = 3.3\text{ kN}$$

$$\longrightarrow \Sigma F_x = 0\text{: } D_x - C_x = 0 \qquad D_x = 3.3\text{ kN}$$

FBD CDF:



$$\circlearrowleft \Sigma M_C = 0\text{: } (86\text{ mm})P$$

$$-\left[18.5\ mm - (16\ mm)\tan 40°\right]\left(F_{DE}\cos 40°\right) = 0$$

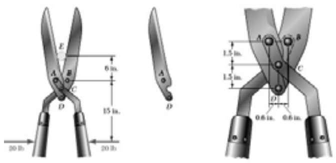
$$F_{DE} = 22.124\,P$$

$$\longrightarrow \Sigma F_x = 0\text{: } C_x - F_{DE}\cos 40° + P\sin 30° = 0$$

$$3.3\text{ kN} - (22.124\,P)\cos 40° + P\sin 30° = 0$$

$$\mathbf{P} = 201\text{ N} \measuredangle\ 60° \blacktriangleleft$$

PROBLEM 6.145

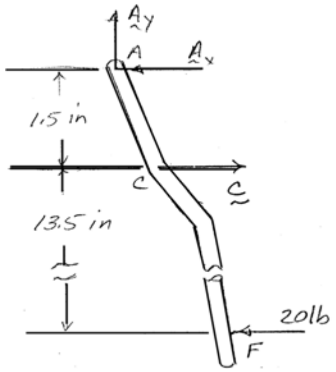


The garden shears shown consist of two blades and two handles. The two handles are connected by pin *C* and the two blades are connected by pin *D*. The left blade and the right handle are connected by pin *A*; the right blade and the left handle are connected by pin *B*. Determine the magnitude of the forces exerted on the small branch *E* when two 20-lb forces are applied to the handles as shown.

SOLUTION

Note: By symmetry the vertical components of pin forces **C** and **D** are zero.

FBD handle ACF: (not to scale)

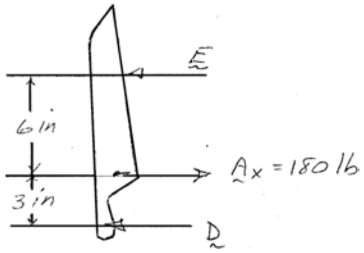


$$\uparrow \Sigma F_y = 0: A_y = 0$$

$$\circlearrowleft \Sigma M_C = 0: (13.5 \text{ in.})(20 \text{ lb}) - (1.5 \text{ in.})A_x = 0 \qquad A_x = 180 \text{ lb}$$

$$\longrightarrow \Sigma F_x = 0: C - A_x - 20 \text{ lb} = 0 \qquad C = (180 + 20) \text{ lb} = 200 \text{ lb}$$

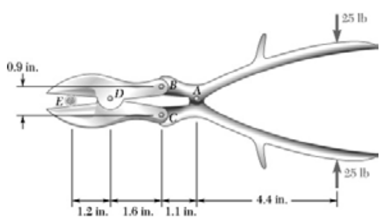
FBD blade DE:



$$\circlearrowright \Sigma M_D = 0: (9 \text{ in.})E - (3 \text{ in.})(180 \text{ lb}) = 0$$

$$E = 60.0 \text{ lb} \blacktriangleleft$$

PROBLEM 6.146

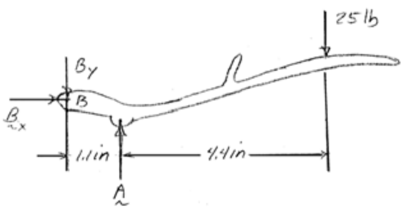


The bone rongeur shown is used in surgical procedures to cut small bones. Determine the magnitude of the forces exerted on the bone at E when two 25-lb forces are applied as shown.

SOLUTION

Note: By symmetry the horizontal components of pin forces at A and D are zero.

FBD handle AB:

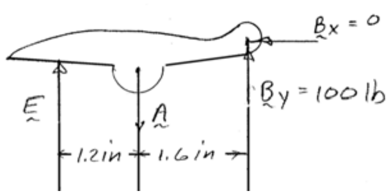


$$\rightarrow \Sigma F_x = 0: \; B_x = 0$$

$$\circlearrowleft \Sigma M_A = 0: \; (1.1 \text{ in.})B_y - (4.4 \text{ in.})(25 \text{ lb})$$

$$B_y = 100 \text{ lb}$$

FBD Blade BD:



$$\circlearrowleft \Sigma M_A = 0: \; (1.6 \text{ in.})(100 \text{ lb}) - (1.2 \text{ in.})(E) = 0$$

$$E = 133.3 \text{ lb} \blacktriangleleft$$

PROBLEM 6.147

The telescoping arm *ABC* is used to provide an elevated platform for construction workers. The workers and the platform together have a mass of 240 kg and have a combined center of gravity located directly above *C*. For the position when $\theta = 24^\circ$, determine (*a*) the force exerted at *B* by the single hydraulic cylinder *BD*, (*b*) the force exerted on the supporting carriage at *A*.

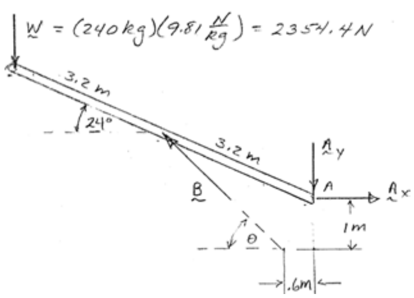
SOLUTION

Note:
$$\theta = \tan^{-1}\frac{(3.2\sin 24^\circ - 1)\text{ m}}{(3.2\cos 24^\circ - 0.6)\text{ m}}$$

FBD boom:

$$\theta = 44.73^\circ$$

(*a*)



$$\circlearrowleft \Sigma M_A = 0: [(6.4\text{ m})\cos 24^\circ](2.3544\text{ kN}) - [(3.2\text{ m})\cos 24^\circ]B\sin 44.73^\circ$$

$$+[(3.2\text{ m})\sin 24^\circ]B\cos 44.73^\circ = 0$$

$B = 12.153$ kN $\qquad$ $\mathbf{B} = 12.15\text{ kN} \;\nwarrow\; 44.7^\circ$ ◀

$$\xrightarrow{} \Sigma F_x = 0: A_x - (12.153\text{ kN})\cos 44.73^\circ = 0$$

$$\mathbf{A}_x = 8.633\text{ kN} \longrightarrow$$

(*b*)

$$\uparrow \Sigma F_y = 0: -2.3544\text{ kN} + (12.153\text{ kN})\sin 44.73^\circ - A_y = 0$$

$$\mathbf{A}_y = 6.198\text{ kN} \downarrow$$

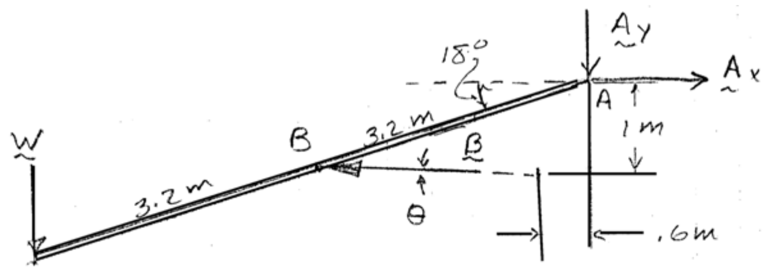
On boom: $\mathbf{A} = 10.63\text{ kN} \;\searrow\; 35.7^\circ$

On carriage: $\mathbf{A} = 10.63\text{ kN} \;\nwarrow\; 35.7^\circ$ ◀

PROBLEM 6.148

The telescoping arm *ABC* can be lowered until end *C* is close to the ground, so that workers can easily board the platform. For the position when $\theta = -18^\circ$, determine (*a*) the force exerted at *B* by the single hydraulic cylinder *BD*, (*b*) the force exerted on the supporting carriage at *A*.

SOLUTION



FBD boom:

$$\theta = \tan^{-1}\frac{1\text{ m} - 3.2\text{ m}\sin 18^\circ}{3.2\text{ m}\cos 18^\circ - 0.6\text{ m}}$$

$$\theta = 0.2614^\circ$$

$$\mathbf{W} = (240\text{ kg})(9.81\text{ N/kg}) = 2354.4\text{ N}$$

(*a*)

$$\circlearrowleft \Sigma M_A = 0: \left[(6.4\text{ m})\cos 18^\circ\right]2.3544\text{ kN} - \left[(3.2\text{ m})\cos 18^\circ\right]B\sin(0.2614^\circ)$$

$$-\left[(3.2\text{ m})\sin 18^\circ\right]B\cos(0.2614^\circ) = 0$$

$$B = 14.292\text{ kN} \qquad \mathbf{B} = 14.29\text{ kN} \searrow .261^\circ \blacktriangleleft$$

(*b*)

$$\xrightarrow{} \Sigma F_x = 0: A_x - B\cos(0.2614^\circ) = 0$$

$$A_x = (14.292\text{ kN})\cos(0.2614^\circ) = 14.292\text{ kN}$$

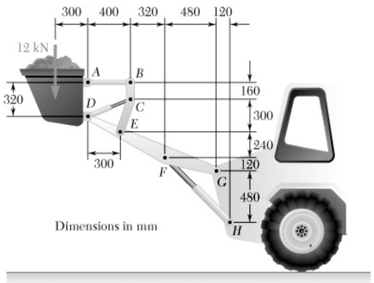
$$\uparrow \Sigma F_y = 0: A_y + B\sin(0.2614^\circ) - 2.3544\text{ kN} = 0$$

$$A_y = 2.3544\text{ kN} - (14.292\text{ kN})\sin(0.2614^\circ)$$

$$A_y = 2.2892\text{ kN}$$

On boom: $\mathbf{A} = 14.47\text{ kN} \searrow 9.10^\circ$

On carriage: $\mathbf{A} = 14.47\text{ kN} \searrow 9.10^\circ \blacktriangleleft$

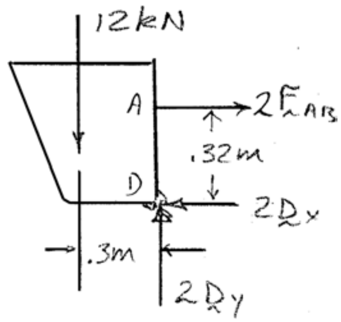


PROBLEM 6.149

The bucket of the front-end loader shown carries a 12-kN load. The motion of the bucket is controlled by two identical mechanisms, only one of which is shown. Knowing that the mechanism shown supports one-half of the 12-kN load, determine the force exerted (*a*) by cylinder *CD*, (*b*) by cylinder *FH*.

SOLUTION

FBD bucket:



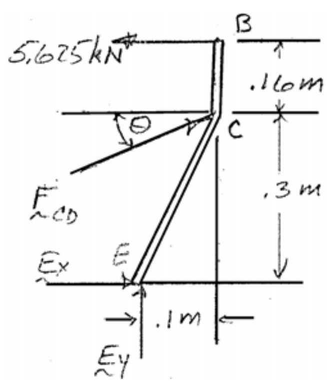
(*a*)

$$\circlearrowleft \Sigma M_D = 0: (0.3\text{ m})(12\text{ kN}) - (0.32\text{ m})(2F_{AB}) = 0 \quad F_{AB} = 5.625\text{ kN}$$

$$\xrightarrow{\quad} \Sigma F_x = 0: 2F_{AB} - 2D_x = 0 \qquad D_x = F_{AB} = 5.625\text{ kN}$$

$$\uparrow \Sigma F_y = 0: D_y - 12\text{ kN} = 0 \qquad D_y = 12\text{ kN}$$

FBD link BCE:



$$\theta = \tan^{-1}\frac{160}{400} = 21.801°$$

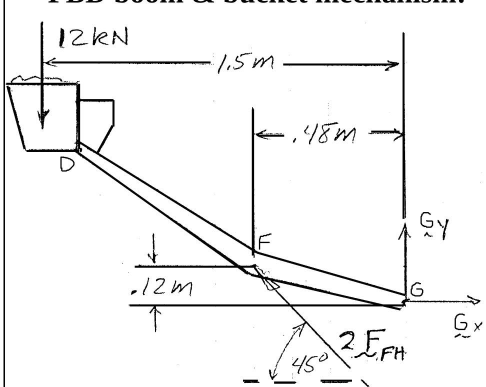
$$\circlearrowleft \Sigma M_E = 0: (0.46\text{ m})(5.625\text{ kN}) + (0.1\text{ m})(F_{CD}\sin 21.801°)$$

$$-(0.3\text{ m})(F_{CD}\cos 21.801°) = 0$$

$$F_{CD} = 10.7185\text{ kN} \quad (\text{C})$$

On *BCE*: $\mathbf{F}_{CD} = 10.72\text{ kN} \measuredangle\, 21.8°$ ◀

FBD boom & bucket mechanism:



(*b*)

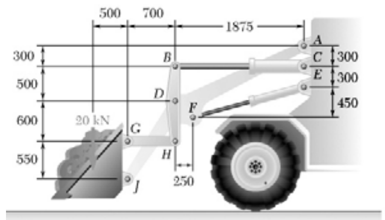
$$\circlearrowleft \Sigma M_G = 0: (1.5\text{ m})(12\text{ kN}) + (0.12\text{ m})(2F_{FH}\cos 45°)$$

$$-(0.48\text{ m})(2F_{FH}\sin 45°) = 0$$

$$F_{FH} = 35.4\text{ kN C}$$

On *DFG*: $\mathbf{F}_{FH} = 35.4\text{ kN} \searrow 45°$ ◀

PROBLEM 6.150

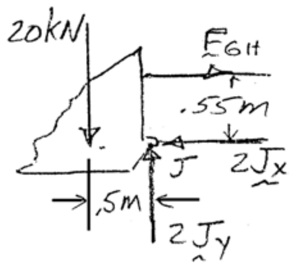


Dimensions in mm

The motion of the bucket of the front-end loader shown is controlled by two arms and a linkage which are pin-connected at *D*. The arms are located symmetrically with respect to the central, vertical, and longitudinal plane of the loader; one arm *AFJ* and its control cylinder *EF* are shown. The single linkage *GHBD* and its control cylinder *BC* are located in the plane of symmetry. For the position shown, determine the force exerted (*a*) by cylinder *BC*, (*b*) by cylinder *EF*.

SOLUTION

FBD bucket:

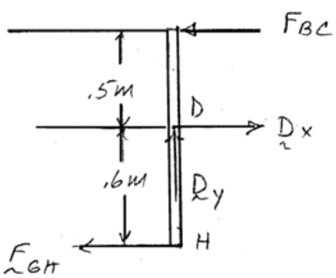


(*a*)

$$\circlearrowleft \Sigma M_J = 0:\ (0.5\text{ m})(20\text{ kN}) - (0.55\text{ m})F_{GH} = 0$$

$$F_{GH} = 18.1818\text{ kN}$$

FBD link BH:

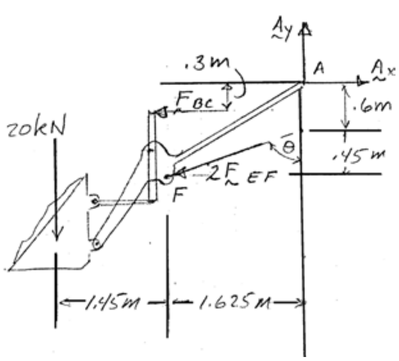


$$\circlearrowleft \Sigma M_D = 0:\ (0.5\text{ m})F_{BC} - (0.6\text{ m})F_{GH} = 0$$

$$F_{BC} = \frac{6}{5}F_{GH} = \frac{6}{5}18.1818\text{ kN} = 21.818\text{ kN}$$

On *BH*: $\mathbf{F}_{BC} = 21.8\text{ kN} \longleftarrow$ ◀

FBD mechanism with bucket:



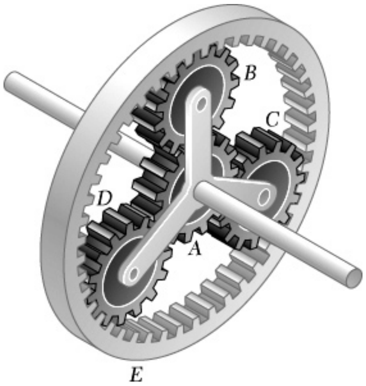
(*b*)

$$\theta = \tan^{-1}\frac{1.625\text{ m}}{0.45\text{ m}} = 74.521°$$

$$\Sigma M_A = (3.075\text{ m})(20\text{ kN}) - (0.3\text{ m})(28.818\text{ kN})$$

$$- (0.6\text{ m})(2F_{EF}\sin 74.521°) = 0$$

On *AF*: $\mathbf{F}_{EF} = 47.5\text{ kN} \nearrow 15.48°$ ◀

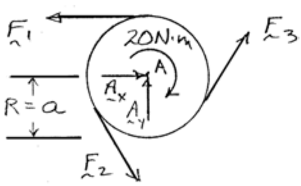


PROBLEM 6.151

In the planetary gear system shown, the radius of the central gear A is $a = 20$ mm, the radius of the planetary gear is b, and the radius of the outer gear E is $(a + 2b)$. A clockwise couple of magnitude $M_A = 20\ \text{N}\cdot\text{m}$ is applied to the central gear A, and a counter-clockwise couple of magnitude $M_S = 100\ \text{N}\cdot\text{m}$ is applied to the spider BCD. If the system is to be in equilibrium, determine (*a*) the required radius b of the planetary gears, (*b*) the couple $\mathbf{M}_E$ that must be applied to the outer gear E.

SOLUTION

FBD Gear A:

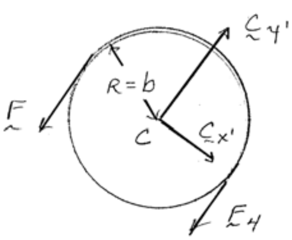


(*a*) By symmetry $F_1 = F_2 = F_3 = F$

$$\circlearrowleft \Sigma M_A = 0: 3aF - 20\ \text{N}\cdot\text{m} = 0$$

$$F = \frac{20}{3a}\ \text{N}\cdot\text{m}$$

FBD Gear C:



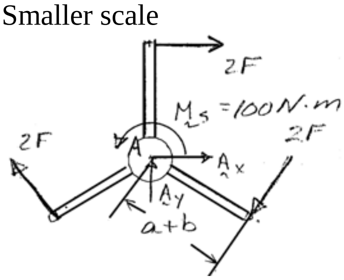
$$\searrow \Sigma F_{x'} = 0: C_{x'} = 0$$

$$\circlearrowleft \Sigma M_C = 0: bF - bF_4 = 0 \qquad F_4 = F = \frac{20\ \text{N}\cdot\text{m}}{3a}$$

$$\nearrow \Sigma F_{y'} = 0: C_{y'} - F - F_4 = 0 \qquad C_{y'} = 2F = \frac{40\ \text{N}\cdot\text{m}}{3a}$$

By symmetry central forces on gears B and D are the same

FBD Spider:
Smaller scale



$$\circlearrowleft \Sigma M_A = 0: M_S - (a + b)2F = 0$$

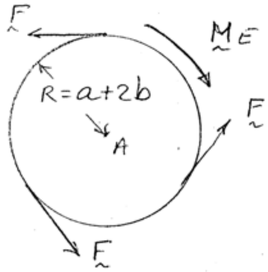
$$100\ \text{N}\cdot\text{m} = 6(a + b)F = (a + b)\frac{40}{a}\ \text{N}\cdot\text{m}$$

$$\frac{100}{40} = 1 + \frac{b}{a} \qquad \frac{b}{a} = \frac{3}{2}$$

$$a = 20\ \text{mm} \text{ so that } b = 30.0\ \text{mm} \blacktriangleleft$$

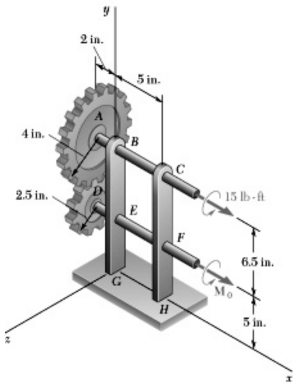
PROBLEM 6.151 CONTINUED

FBD Outer gear:



(*b*) $\curvearrowleft \Sigma M_A = 0: 3(a + 2b)F - M_E = 0$

$$M_E = 3(20\text{ mm} + 60\text{ mm})\frac{20\text{ N}\cdot\text{m}}{3(20\text{ mm})} = 80.0\text{ N}\cdot\text{m} \blacktriangleleft$$

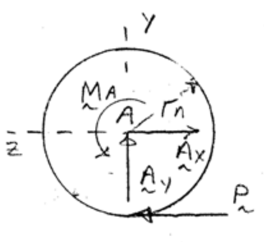


PROBLEM 6.152

Gears A and D are rigidly attached to horizontal shafts that are held by frictionless bearings. Determine (a) the couple $\mathbf{M}_0$ that must be applied to shaft DEF to maintain equilibrium, (b) the reactions at G and H.

SOLUTION

FBD Gear A: looking from C



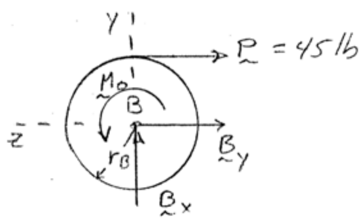
(a)

$$M_A = 15\text{ lb}\cdot\text{ft} \qquad r_A = 4\text{ in.}$$

$$\circlearrowleft \Sigma M_A = 0\colon M_A - P r_A = 0 \qquad P = \frac{M_A}{r_A} = \frac{180\text{ lb}\cdot\text{in.}}{4\text{ in.}}$$

$$P = 45\text{ lb}$$

FBD Gear B: looking from F

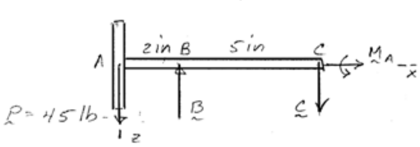


$$\circlearrowleft \Sigma M_B = 0\colon M_0 - r_B P = 0$$

$$M_0 = r_B P = (2.5\text{ in.})(45\text{ lb}) = 112.5\text{ lb}\cdot\text{in.} \circlearrowright$$

$$\mathbf{M}_0 = 112.5\text{ lb}\cdot\text{in. } \mathbf{i} \blacktriangleleft$$

FBD ABC: looking down



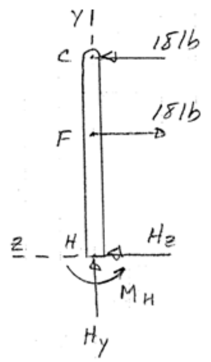
(b)

$$\circlearrowleft \Sigma M_B = 0\colon (2\text{ in.})(45\text{ lb}) - (5\text{ in.})C = 0 \qquad \mathbf{C} = 18\text{ lb } \mathbf{k}$$

$$\downarrow \Sigma F_z = 0\colon 45\text{ lb} - B + 18\text{ lb} = 0 \qquad \mathbf{B} = -63\text{ lb } \mathbf{k}$$

PROBLEM 6.152 CONTINUED

FBD BEG:



By analogy, using FBD *DEF* $\quad \mathbf{E} = 63 \text{ lb } \mathbf{k} \quad \mathbf{F} = 18 \text{ lb } \mathbf{k}$

$$\leftarrow \Sigma F_z = 0: G_z + 63 \text{ lb} - 63 \text{ lb} = 0$$

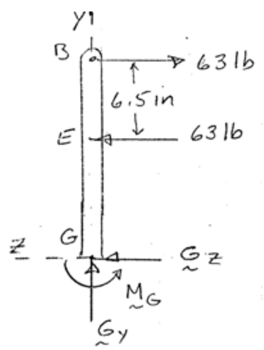
$$G_z = 0$$

$$\uparrow \Sigma F_y = 0 \qquad G_y = 0$$

$$\Sigma M_G = 0 \qquad M_G - (6.5 \text{ in.})(63 \text{ lb}) = 0$$

$$\mathbf{M}_G = (410 \text{ lb}\cdot\text{in.})\mathbf{i} \blacktriangleleft$$

FBD CFH:



$$\Sigma \mathbf{F} = 0: H_z = H_y = 0$$

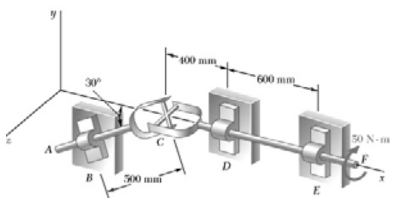
$$\Sigma M_H = 0$$

$$M_H = -(6.5 \text{ in.})(18 \text{ lb})$$

$$= -117 \text{ lb}\cdot\text{in.}$$

$$\mathbf{M}_G = -(117.0 \text{ lb}\cdot\text{in.})\mathbf{i} \blacktriangleleft$$

PROBLEM 6.153



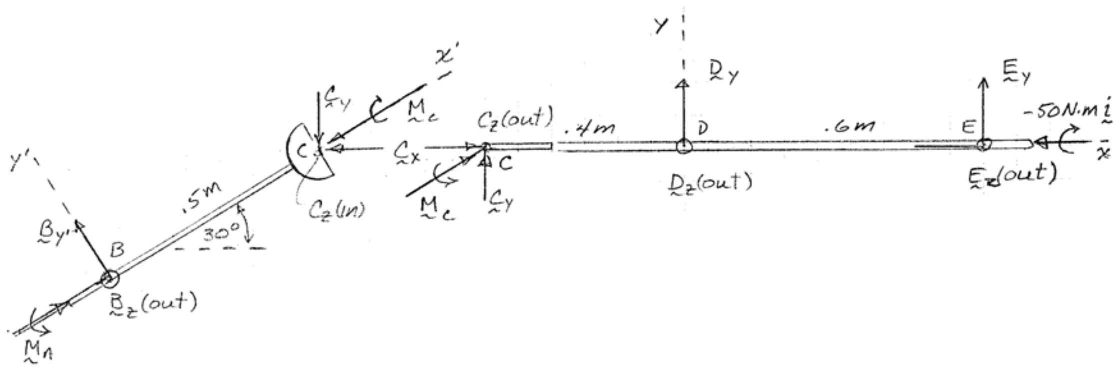
Two shafts *AC* and *CF*, which lie in the vertical *xy* plane, are connected by a universal joint at *C*. The bearings at *B* and *D* do ot exert any axial force. A couple of magnitude $50\ \text{N}\cdot\text{m}$ (clockwise when viewed from the positive *x* axis) is applied to shaft *CF* at *F*. At a time when the arm of the crosspiece attached to shaft *CF* is horizontal, determine (*a*) the magnitude of the couple which must be applied to shaft *AC* at *A* to maintain equilibrium, (*b*) the reactions at *B*, *D*, and *E*. (*Hint:* The sum of the couples exerted on the crosspiece must be zero).

SOLUTION

Note: The couples exerted by the two yokes on the crosspiece must be equal and opposite. Since neither yoke can exert a couple along the arm of the crosspiece it contacts, these equal and opposite couples must be normal to the plane of the crosspiece.

If the crosspiece arm attached to shaft *CF* is horizontal, the plane of the crosspiece is normal to shaft *AC*, so couple $\mathbf{M}_C$ is along *AC*.

FBDs shafts with yokes:



(*a*) FBD *CDE*: $\Sigma M_x = 0$: $\quad M_C \cos 30° - 50\ \text{N}\cdot\text{m} = 0 \qquad M_C = 57.735\ \text{N}\cdot\text{m}$

FBD *BC*: $\Sigma M_{x'} = 0$: $M_A - M_C = 0 \qquad M_A = 57.7\ \text{N}\cdot\text{m}$ ◀

(*b*) $\Sigma \mathbf{M}_C = 0$: $M_A \mathbf{i}' + (0.5\ \text{m})B_z\,\mathbf{j}' - (0.5\ \text{m})B_{y'}\,\mathbf{k} = 0 \qquad \mathbf{B} = 0$ ◀

$\Sigma \mathbf{F} = 0$: $\mathbf{B} + \mathbf{C} = 0 \quad$ so $\quad \mathbf{C} = 0$

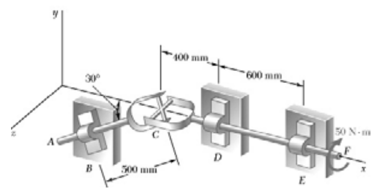
FBD *CDF*: $\Sigma M_{Dy} = 0$: $-(0.6\ \text{m})E_z + (57.735\ \text{N}\cdot\text{m})\sin 30° = 0$

$$\mathbf{E}_z = 48.1\ \text{N}\ \mathbf{k}$$

$\Sigma F_x = 0$: $E_x = 0$

$\Sigma M_{Dz} = 0$: $(0.6\ \text{m})E_y = 0 \qquad E_y = 0$ so $\mathbf{E} = (48.1\ \text{N})\mathbf{k}$ ◀

$\Sigma \mathbf{F} = 0$: $\cancel{\mathbf{C}}^{\,0} + \mathbf{D} + \mathbf{E} = 0 \qquad \mathbf{D} = -\mathbf{E} = -(48.1\ \text{N})\mathbf{k}$ ◀



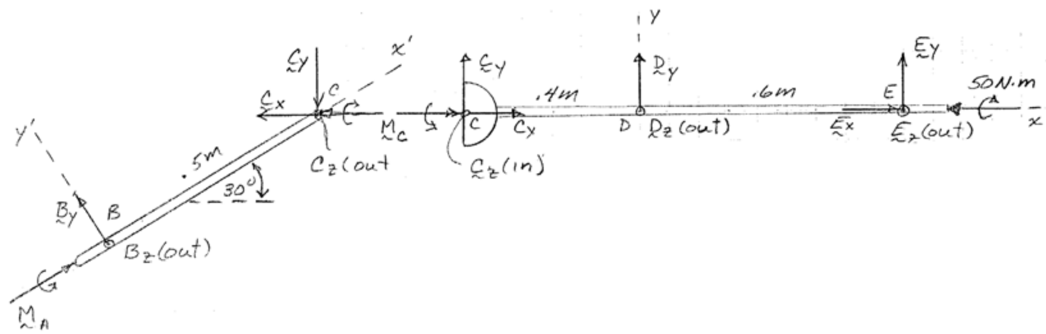
PROBLEM 6.154

Solve Prob. 6.153 assuming that the arm of the crosspiece attached to shaft *CF* is vertical.

SOLUTION

Note: The couples exerted by the two yokes on the crosspiece must be equal and opposite. Since neither yoke can exert a couple along the arm of the crosspiece it contacts, these equal and opposite couples must be normal to the plane of the crosspiece.

If the crosspiece arm attached to *CF* is vertical, the plane of the crosspiece is normal to *CF*, so the couple $\mathbf{M}_C$ is along *CF*.



(*a*) FBD *CDE*: $\Sigma M_x = 0$: $M_C - 50\ \text{N}\cdot\text{m} = 0 \qquad M_C = 50\ \text{N}\cdot\text{m}$

FBD *BC*: $\Sigma M_{x'} = 0$: $M_A - M_C \cos 30° = 0 \qquad M_A = (50\ \text{N}\cdot\text{m})\cos 30°$

$M_A = 43.3\ \text{N}\cdot\text{m}$ ◀

(*b*) $\Sigma M_{Cy'} = 0$: $M_C \sin 30° + (0.5\ \text{m})B_z = 0 \qquad B_z = -\dfrac{(50\ \text{N}\cdot\text{m})(0.5)}{0.5\ \text{m}} = -50\ \text{N}$

$\Sigma M_{Cz} = 0$: $-(0.5\ \text{m})B_y = 0 \qquad B_y = 0 \qquad$ so $\mathbf{B} = -(50.0\ \text{N})\mathbf{k}$ ◀

$\Sigma \mathbf{F} = 0$: $\mathbf{B} + \mathbf{C} = 0 \qquad \mathbf{C} = -\mathbf{B} \qquad$ so $\qquad C = (50\ \text{N})\mathbf{k}$ on BC

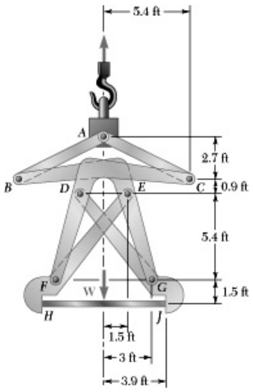
FBD *CDE*: $\Sigma M_{Dy} = 0$: $-(0.4\ \text{m})C_z - (0.6\ \text{m})E_z = 0 \qquad E_z = -(50\ \text{N})\left(\dfrac{4}{6}\right) = -33.3\ \text{N}$

$\Sigma M_{Dz} = 0$: $E_y = 0$

$\Sigma F_x = 0$: $E_x = 0 \qquad$ so $\mathbf{E} = -(33.3\ \text{N})\ \mathbf{k}$ ◀

$\Sigma \mathbf{F} = 0$: $\mathbf{C} + \mathbf{D} + \mathbf{E} = 0 \qquad -(50\ \text{N})\ \mathbf{k} + \mathbf{D} - (33.3\ \text{N})\ \mathbf{k} = 0$

$\mathbf{D} = (83.3\ \text{N})\ \mathbf{k}$ ◀

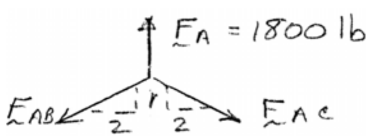


PROBLEM 6.155

The large mechanical tongs shown are used to grab and lift a thick 1800-lb steel slab *HJ*. Knowing that slipping does not occur between the tong grips and the slab at *H* and *J*, determine the components of all forces acting on member *EFH*. (*Hint:* Consider the symmetry of the tongs to establish relationships between the components of the force acting at *E* on *EFH* and the components of the force acting at *D* on *CDF*.)

SOLUTION

FBD A:



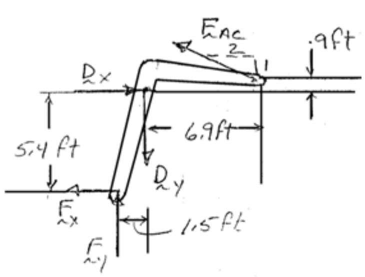
By inspection of FBD whole: $F_A = W = 1800$ lb

By symmetry: $F_{AB} = F_{AC} = T$ (say)

$$\uparrow \Sigma F_y = 0: 1800 \text{ lb} - 2\frac{1}{\sqrt{5}}T = 0 \qquad T = 900\sqrt{5} \text{ lb} = F_{AC} = F_{AB}$$

$$\longrightarrow \Sigma F_x = 0: D_x - F_x - \frac{2}{\sqrt{5}}\left(900\sqrt{5} \text{ lb}\right) = 0$$

FBD CDF:



$$D_x - F_x = 1800 \text{ lb} \qquad (1)$$

$$\uparrow \Sigma F_y = 0: -D_y + F_y + \frac{1}{\sqrt{5}}\left(900\sqrt{5} \text{ lb}\right) = 0$$

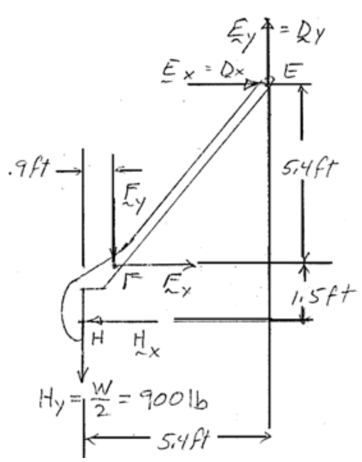
$$D_y - F_y = 900 \text{ lb} \qquad (2)$$

$$\circlearrowleft \Sigma M_D = 0: (6.9 \text{ ft})\left[\frac{1}{\sqrt{5}}\left(900\sqrt{5}\right) \text{ lb}\right] + (0.9 \text{ ft})\left[\frac{2}{\sqrt{5}}\left(900\sqrt{5}\right) \text{ lb}\right]$$

$$-(1.5 \text{ ft})F_y - (5.4 \text{ ft})F_x = 0$$

$$5.4F_x + 1.5F_y = 7830 \text{ lb} \qquad (3)$$

FBD EFH:



Note: By symmetry $E_x = D_x$; $E_y = D_y$

$$\circlearrowleft \Sigma M_F = 0: (4.5 \text{ ft})D_y - (5.4 \text{ ft})D_y - (1.5 \text{ ft})H_x$$

$$+ (0.9 \text{ ft})900 \text{ lb} = 0$$

$$5.4D_x - 4.5D_y + 1.5H_x = 810 \text{ lb} \qquad (4)$$

$$\longrightarrow \Sigma F_x = 0: D_x + F_x - H_x = 0 \qquad (5)$$

$\mathbf{F}_x = 648$ lb $\longrightarrow$ ◀

PROBLEM 6.155 CONTINUED

Solving equations (1) through (5):

$\mathbf{F}_x = 648 \text{ lb} \longrightarrow$ ◀

$\mathbf{F}_y = 2.89 \text{ kips} \downarrow$ ◀

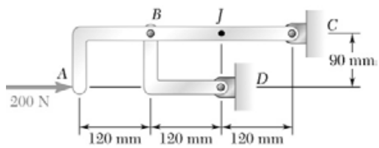
$\mathbf{E}_x = \mathbf{D}_x = 2.45 \text{ kips} \longrightarrow$ ◀

$\mathbf{E}_y = \mathbf{D}_y = 3.79 \text{ kips} \uparrow$ ◀

$\mathbf{H}_x = 3.10 \text{ kip} \longleftarrow$ ◀

and, as noted $\mathbf{H}_y = 900 \text{ lb} \downarrow$ ◀

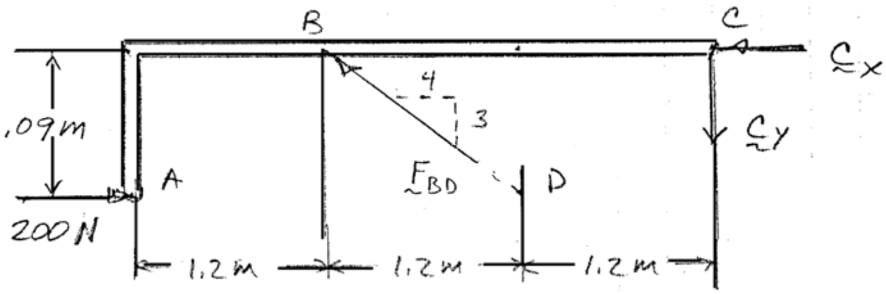
PROBLEM 6.156



For the frame and loading shown, determine the force acting on member *ABC* (*a*) at *B*, (*b*) at *C*.

SOLUTION

FBD ABC:



Note: *BD* is two-force member

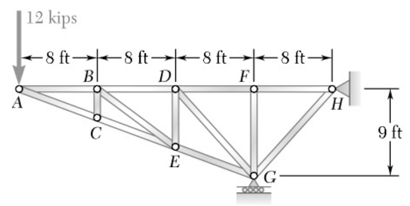
(*a*) $$\circlearrowleft \Sigma M_C = 0: (0.09 \text{ m})(200 \text{ N}) - (2.4 \text{ m})\left(\frac{3}{5}F_{BD}\right) = 0$$

$$\mathbf{F}_{BD} = 125.0 \text{ N} \searrow 36.9° \blacktriangleleft$$

(*b*) $$\xrightarrow{} \Sigma F_x = 0: 200 \text{ N} - \frac{4}{5}(125 \text{ N}) - C_x = 0 \qquad \mathbf{C}_x = 100 \text{ N} \longleftarrow$$

$$\uparrow \Sigma F_y = 0: \frac{3}{5}F_{BD} - C_y = 0 \qquad \mathbf{C}_y = \frac{3}{5}(125 \text{ N}) = 75 \text{ N} \downarrow$$

$$\mathbf{C} = 125.0 \text{ N} \swarrow 36.9° \blacktriangleleft$$



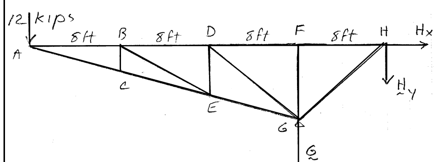
PROBLEM 6.157

Determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

$$\xrightarrow{+} \Sigma F_x = 0\text{: } H_x = 0$$

FBD Truss:

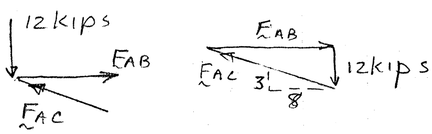


$$\circlearrowleft \Sigma M_H = 0\text{: } (32 \text{ ft})(12 \text{ kips}) - (8 \text{ ft})G = 0 \qquad \mathbf{G} = 48 \text{ kips} \uparrow$$

$$+\uparrow \Sigma F_y = 0\text{: } -12 \text{ kips} + G - H_y = 0$$

$$H_y = 48 \text{ kips} - 12 \text{ kips} = 36 \text{ kips} \qquad \mathbf{H}_y = 36 \text{ kips} \downarrow$$

FBD joint A:



$$\frac{12 \text{ kips}}{3} = \frac{F_{AB}}{8} = \frac{F_{AC}}{\sqrt{73}}$$

so $F_{AB} = 32.0$ kips T ◀

$F_{AC} = 4\sqrt{73}$ kips; $F_{AC} = 34.2$ kips C ◀

By inspection of joint C: $F_{BC} = 0$ ◀

$F_{CE} = 34.2$ kips C ◀

Then by inspection of joint B: $F_{BE} = 0$ ◀

$F_{BD} = 32.0$ kips T ◀

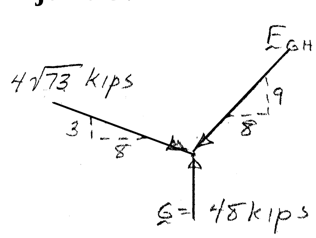
Then by inspection of joint E: $F_{DE} = 0$ ◀

$F_{EG} = 34.2$ kips C ◀

Then by inspection of joint D: $F_{DG} = 0$ ◀

$F_{DF} = 32.0$ kips T ◀

FBD joint G:



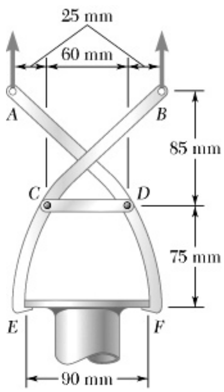
By inspection of joint F: $F_{FG} = 0$ ◀

$F_{FH} = 32.0$ kips T ◀

$$\xrightarrow{+} \Sigma F_x = 0\text{: } \frac{8}{\sqrt{73}}\left(4\sqrt{73} \text{ kips}\right) - \frac{8}{\sqrt{145}} F_{GH} = 0$$

$F_{GH} = 4\sqrt{145}$ kips $\qquad F_{GH} = 48.2$ kips C ◀

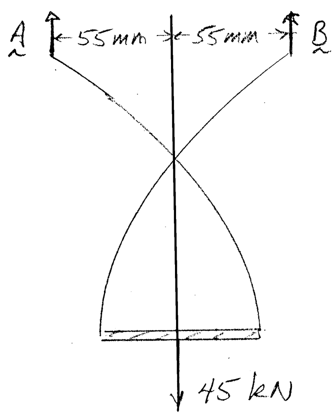
PROBLEM 6.158



The tongs shown are used to apply a total upward force of 45 kN on a pipe cap. Determine the forces exerted at D and F on tong ADF.

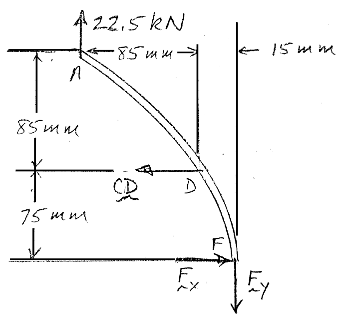
SOLUTION

FBD whole:



By symmetry $\mathbf{A} = \mathbf{B} = 22.5 \text{ kN} \uparrow$

FBD ADF:



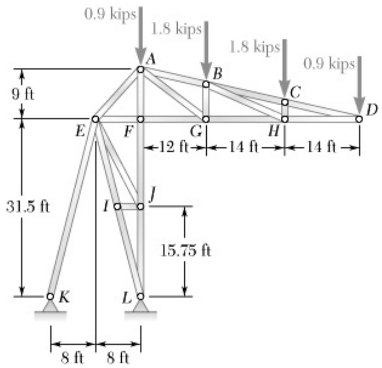
$$\circlearrowleft \Sigma M_F = 0\text{: } (75 \text{ mm})CD - (100 \text{ mm})(22.5 \text{ kN}) = 0$$

$$\mathbf{CD} = 30 \text{ kN} \longleftarrow \blacktriangleleft$$

$$\longrightarrow \Sigma F_x = 0\text{: } F_x - CD = 0 \qquad F_x = CD = 30 \text{ kN}$$

$$\uparrow \Sigma F_y = 0\text{: } 22.5 \text{ kN} - F_y = 0 \qquad F_y = 22.5 \text{ kN}$$

$$\text{so } \mathbf{F} = 37.5 \text{ kN} \searrow 36.9° \blacktriangleleft$$

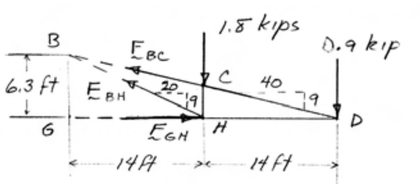


PROBLEM 6.159

A stadium roof truss is loaded as shown. Determine the force in members *BC*, *BH*, and *GH*.

SOLUTION

FBD Section:



$$\circlearrowleft \Sigma M_B = 0:\ (6.3\text{ ft})F_{GH} - (14\text{ ft})(1.8\text{ kips}) - (28\text{ ft})(0.9\text{ kip}) = 0$$

$$F_{GH} = 8.00\text{ kips C} \blacktriangleleft$$

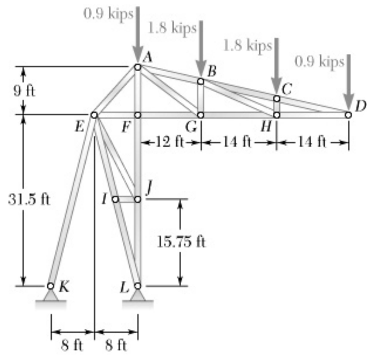
$$\circlearrowleft \Sigma M_H = 0:\ (3.15\text{ ft})\left(\frac{40}{41}F_{BC}\right) - (14\text{ ft})(0.9\text{ kip}) = 0$$

$$F_{BC} = 4.10\text{ kips T} \blacktriangleleft$$

$$\uparrow \Sigma F_Y = 0:\ \frac{9}{41}F_{BC} - 1.8\text{ kips} - 0.9\text{ kip} + \frac{9}{21.93}F_{BH} = 0$$

$$F_{BH} = \frac{21.93}{9}\left[2.7\text{ kips} - \frac{9}{41}(4.10\text{ kips})\right] = 4.386\text{ kips}$$

$$F_{BH} = 4.39\text{ kips T} \blacktriangleleft$$

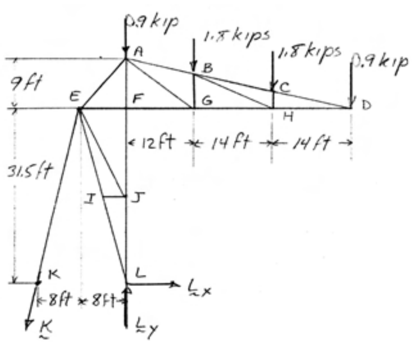


PROBLEM 6.160

A stadium roof truss is loaded as shown. Determine the force in members *EJ*, *FJ*, and *EI*.

SOLUTION

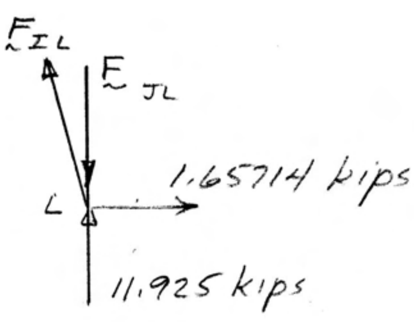
FBD Truss:



$$\circlearrowleft \Sigma M_K = 0: (16\text{ ft})(L_y - 0.9\text{ kip})$$
$$-(28\text{ ft})(1.8\text{ kips})$$
$$-(42\text{ ft})(1.8\text{ kips})$$
$$-(56\text{ ft})(0.9\text{ kip}) = 0$$
$$L_y = 11.925\text{ kips}$$

$$\circlearrowleft \Sigma M_E = 0: (8\text{ ft})(11.925\text{ kips} - 0.9\text{ kip})$$
$$-(20\text{ ft})(1.8\text{ kips}) - (34\text{ ft})(1.8\text{ kips})$$
$$-(48\text{ ft})(0.9\text{ kip}) + (31.5\text{ ft})L_x = 0$$
$$L_x = 1.65714\text{ kips}$$

Joint L:



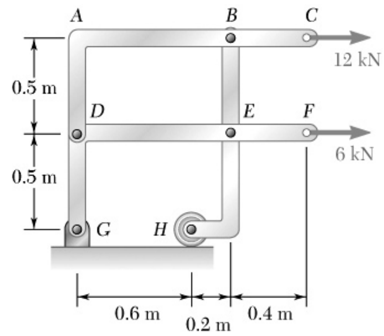
$$\xrightarrow{} \Sigma F_x = 0: -\frac{8}{32.5}F_{IL} + 1.65714\text{ kips} = 0$$
$$F_{IL} = 6.7321\text{ kips}$$

$$\uparrow \Sigma F_y = 0: \frac{31.5}{32.5}(6.7321\text{ kips}) + 11.925\text{ kips}$$
$$-F_{JL} = 0$$
$$F_{JL} = 18.4500\text{ kips}$$

By inspection of joint I, $F_{IJ} = 0$ *and* $F_{EI} = F_{IL} = 6.73\text{ kips}$ T ◀

Then by inspection of joint J, $F_{EJ} = 0$ ◀

and $F_{FJ} = F_{JL} = 18.45\text{ kips}$ C ◀

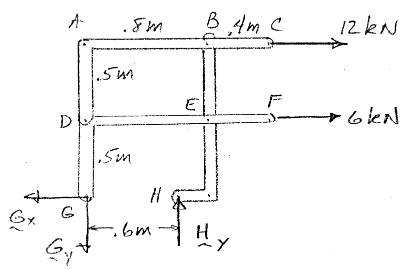


PROBLEM 6.161

For the frame and loading shown, determine the components of the forces acting on member *DABC* at *B* and at *D*.

SOLUTION

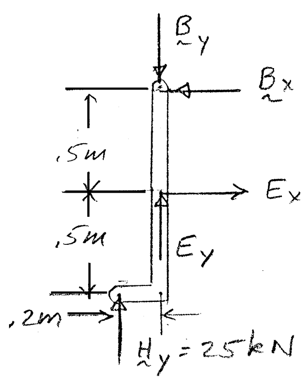
FBD Frame:



$$\circlearrowleft \Sigma M_G = 0\text{: } (0.6\text{ m})H_y - (0.5\text{ m})6\text{ kN} - (1.0\text{ m})(12\text{ kN}) = 0$$

$$\mathbf{H}_y = 25\text{ kN} \uparrow$$

FBD BEH:



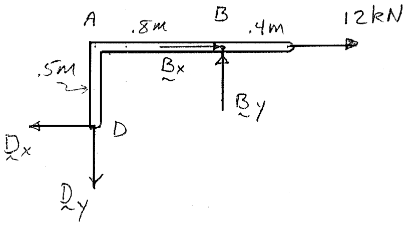
$$\circlearrowleft \Sigma M_E = 0\text{: } (0.5\text{ m})B_x - (0.2\text{ m})(25\text{ kN}) = 0$$

$$B_x = 10\text{ kN}$$

$$\text{on } DABC \quad \mathbf{B}_x = 10.00\text{ kN} \longrightarrow \blacktriangleleft$$

$$\longrightarrow \Sigma F_x = 0\text{: } -D_x + B_x + 12\text{ kN} = 0$$

FBD DABC:



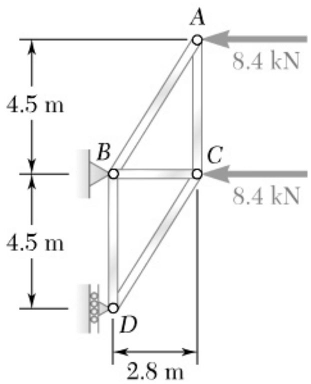
$$D_x = (10\text{ kN} + 12\text{ kN}) = 22\text{ kN} \qquad \mathbf{D}_x = 22.0\text{ kN} \longleftarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_B = 0\text{: } (0.8\text{ m})D_y - (0.5\text{ m})D_x = 0$$

$$D_y = 13.75\text{ kN} \qquad \mathbf{D}_y = 13.75\text{ kN} \downarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0\text{: } B_y - D_y = 0$$

$$B_y = 13.75\text{ kN} \qquad \mathbf{B}_y = 13.75\text{ kN} \uparrow \blacktriangleleft$$



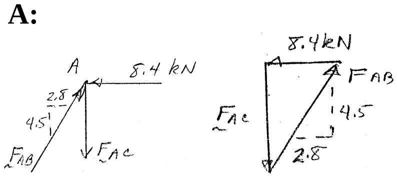
PROBLEM 6.162

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

Joint FBDs:

A:

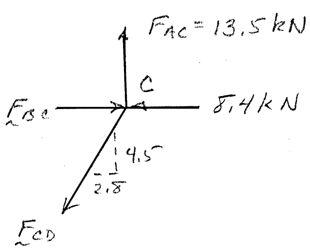


$$\frac{8.4\text{ kN}}{2.8} = \frac{F_{AC}}{4.5} = \frac{F_{AB}}{5.3}$$

$F_{AB} = 15.90\text{ kN C}$ ◀

$F_{AC} = 13.50\text{ kN T}$ ◀

C:



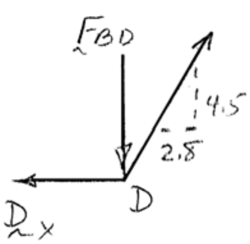
$$\uparrow \Sigma F_y = 0: 13.5\text{ kN} - \frac{4.5}{5.3}F_{CD} = 0$$

$F_{CD} = 15.90\text{ kN T}$ ◀

$$\longrightarrow \Sigma F_y = 0: F_{BC} - \frac{2.8}{5.3}(15.9\text{ kN}) - 8.4\text{ kN} = 0$$

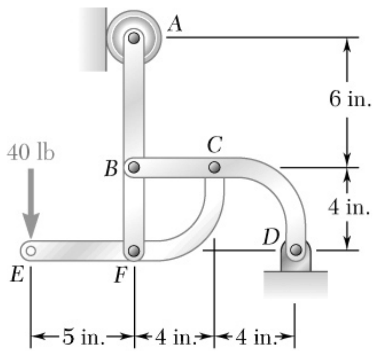
$F_{BC} = 16.80\text{ kN C}$ ◀

D:



$$\uparrow \Sigma F_y = 0: \frac{4.5}{5.3}(15.9\text{ kN}) - F_{BD} = 0$$

$F_{BD} = 13.50\text{ kN C}$ ◀

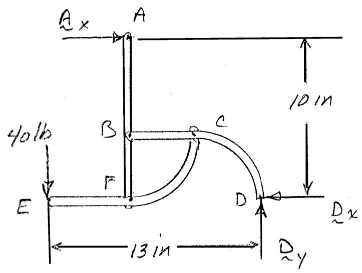


PROBLEM 6.163

For the frame and loading shown, determine the components of the forces acting on member *CFE* at *C* and at *F*.

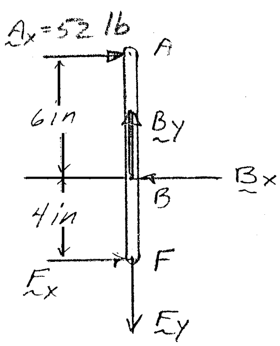
SOLUTION

FBD Frame:



$$\circlearrowleft \Sigma M_D = 0: (13 \text{ in.})(40 \text{ lb}) - (10 \text{ in.})A_x = 0 \qquad \mathbf{A}_x = 52 \text{ lb} \longrightarrow$$

FBD ABF:

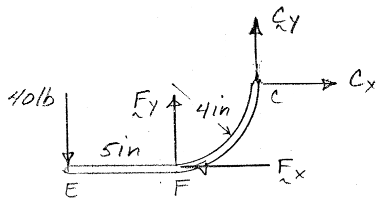


$$\circlearrowleft \Sigma M_B = 0: (4 \text{ in.})F_x - (6 \text{ in.})(52 \text{ lb}) = 0$$

$F_x = 78 \text{ lb} \longrightarrow$ on *ABF*

on *CFE* from above $\qquad \mathbf{F}_x = 78.0 \text{ lb} \longleftarrow$ ◀

FBD CFE:



$$\circlearrowleft \Sigma M_c = 0: (9 \text{ in.})(40 \text{ lb}) - (4 \text{ in.})F_y - (4 \text{ in.})(78 \text{ lb}) = 0$$

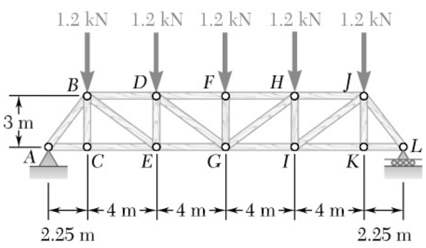
$\mathbf{F}_y = 12.00 \text{ lb} \uparrow$ ◀

$$\circlearrowleft \Sigma F_x = 0: C_x - F_x = 0 \qquad C_x = 78 \text{ lb}$$

$\mathbf{C}_x = 78.0 \text{ lb} \longrightarrow$ ◀

$$\uparrow \Sigma F_y = 0: -40 \text{ lb} + F_y + C_y = 0$$

$C_y = 40 \text{ lb} - 12 \text{ lb} = 28 \text{ lb} \qquad \mathbf{C}_y = 28.0 \text{ lb} \uparrow$ ◀

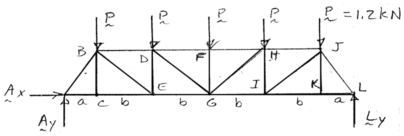


PROBLEM 6.164

A Mansard roof truss is loaded as shown. Determine the force in members *DF*, *DG*, and *EG*.

SOLUTION

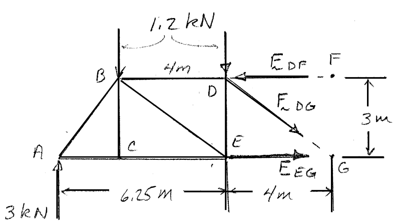
FBD Truss:



$$\xrightarrow{} \Sigma F_x = 0: A_x = 0$$

By symmetry: $A_y = L_y = \dfrac{5P}{2}$ or $A_y = L_y = 3\text{ kN} \uparrow$

FBD Section:



$$\circlearrowleft \Sigma M_D = 0: (3\text{ m})F_{EG} + (4\text{ m})(1.2\text{ kN}) - (6.25\text{ m})(3\text{ kN}) = 0$$

$$F_{EG} = 4.65\text{ kN T} \blacktriangleleft$$

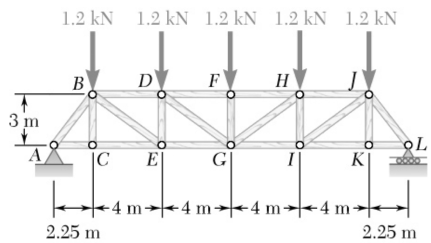
$$\uparrow \Sigma F_y = 0: 3\text{ kN} - 2(1.2\text{ kN}) - \frac{3}{5}F_{DG} = 0$$

$$F_{DG} = 1.000\text{ kN T} \blacktriangleleft$$

$$\xrightarrow{} \Sigma F_x = 0: F_{EG} + \frac{4}{5}F_{DG} - F_{DF} = 0$$

$$F_{DF} = 4.65\text{ kN} + \frac{4}{5}(1\text{ kN}) = 5.45\text{ kN}$$

$$F_{DF} = 5.45\text{ kN C} \blacktriangleleft$$

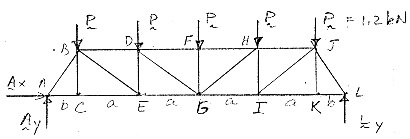


PROBLEM 6.165

A Mansard roof truss is loaded as shown. Determine the force in members *GI*, *HI*, and *HJ*.

SOLUTION

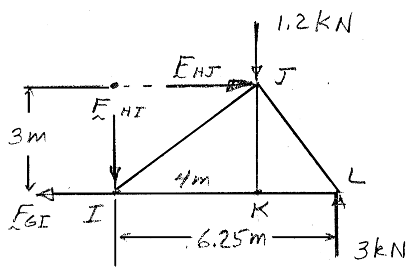
FBD Truss:



$\longrightarrow \Sigma F_x = 0: A_x = 0$

By symmetry: $A_y = L_y = \dfrac{5P}{2}$ or $A_y = L_y = 3\text{ kN} \uparrow$

FBD Section:



$\circlearrowleft \Sigma M_I = 0: (6.25\text{ m})(3\text{ kN}) - (4\text{ m})(1.2\text{ kN}) - (3\text{ m})F_{HJ} = 0$

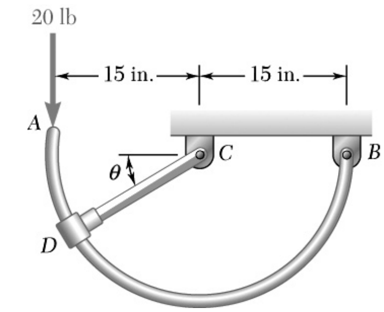
$F_{HJ} = 4.65\text{ kN C}$ ◀

$\longrightarrow \Sigma F_x = 0: F_{HJ} - F_{GI} = 0 \qquad F_{GI} = F_{HJ}$

$F_{GI} = 4.65\text{ kN T}$ ◀

$\uparrow \Sigma F_y = 0: -F_{HI} - 1.2\text{ kN} + 3\text{ kN} = 0$

$F_{HI} = 1.800\text{ kN C}$ ◀

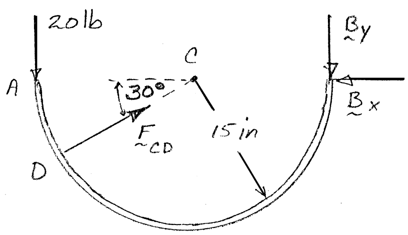


PROBLEM 6.166

Rod *CD* is fitted with a collar at *D* that can be moved along rod *AB*, which is bent in the shape of a circular arc. For the position when $\theta = 30°$, determine (*a*) the force in rod *CD*, (*b*) the reaction at *B*.

SOLUTION

FBD:



(*a*)

$$\circlearrowleft \Sigma M_C = 0{:}\ (15 \text{ in.})\left(20 \text{ lb} - B_y\right) = 0$$

$$\mathbf{B}_y = 20 \text{ lb} \downarrow$$

$$\uparrow \Sigma F_y = 0{:}\ -20 \text{ lb} + F_{CD} \sin 30° - 20 \text{ lb} = 0$$

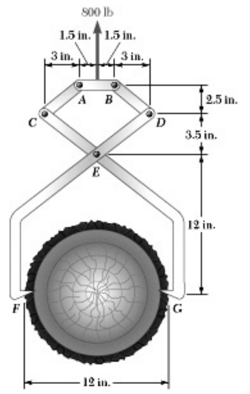
$$F_{CD} = 80.0 \text{ lb T} \blacktriangleleft$$

(*b*)

$$\longrightarrow \Sigma F_x = 0{:}\ (80 \text{ lb})\cos 30° - B_x = 0$$

$$\mathbf{B}_x = 69.282 \text{ lb} \longleftarrow$$

$$\text{so } \mathbf{B} = 72.1 \text{ lb} \swarrow 16.10° \blacktriangleleft$$

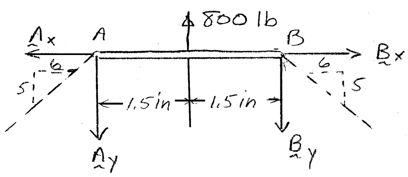


PROBLEM 6.167

A log weighing 800 lb is lifted by a pair of tongs as shown. Determine the forces exerted at E and at F on tong DEF.

SOLUTION

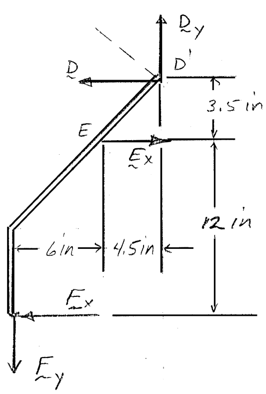
FBD AB:



By symmetry: $A_y = B_y = 400$ lb

and $$A_x = B_x = \frac{6}{5}(400 \text{ lb}) = 480 \text{ lb}$$

FBD DEF:



Note: $\mathbf{D} = -\mathbf{B}$ so $D_x = 480$ lb

$D_y = 400$ lb

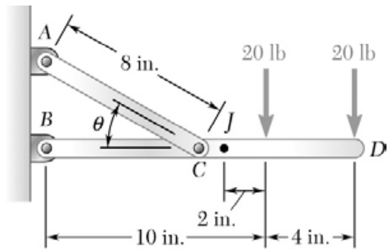
$$\circlearrowleft \Sigma M_F = (10.5 \text{ in.})(400 \text{ lb}) + (15.5 \text{ in.})(480 \text{ lb}) - (12 \text{ in.})E_x = 0$$

$E_x = 970$ lb $\qquad$ $\mathbf{E} = 970$ lb $\longrightarrow$ ◀

$$\xrightarrow{} \Sigma F_x = 0: -480 \text{ lb} + 970 \text{ lb} - F_x = 0 \qquad F_x = 490 \text{ lb}$$

$$\uparrow \Sigma F_y = 0: 400 \text{ lb} - F_y = 0 \qquad F_y = 400 \text{ lb}$$

$\mathbf{F} = 633$ lb $\measuredangle$ 39.2° ◀



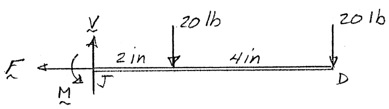
PROBLEM 7.1

Determine the internal forces (axial force, shearing force, and bending moment) at point J of the structure indicated:
Frame and loading of Prob. 6.77.

SOLUTION

$$\xrightarrow{} \Sigma F_x = 0: -F = 0$$

FBD JD:



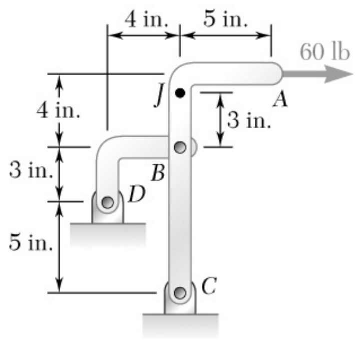
$$\mathbf{F} = 0 \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: V - 20 \text{ lb} - 20 \text{ lb} = 0$$

$$\mathbf{V} = 40.0 \text{ lb} \uparrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: M - (2 \text{ in.})(20 \text{ lb}) - (6 \text{ in.})(20 \text{ lb}) = 0$$

$$\mathbf{M} = 160.0 \text{ lb} \cdot \text{in.} \circlearrowright \blacktriangleleft$$

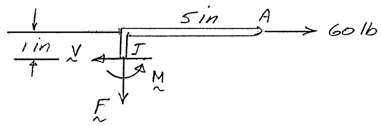


PROBLEM 7.2

Determine the internal forces (axial force, shearing force, and bending moment) at point J of the structure indicated:
Frame and loading of Prob. 6.76.

SOLUTION

FBD AJ:



$$\rightarrow \Sigma F_x = 0: 60 \text{ lb} - V = 0$$

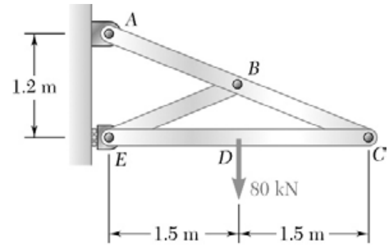
$$\mathbf{V} = 60.0 \text{ lb} \longleftarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: -F = 0$$

$$\mathbf{F} = 0 \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: M - (1 \text{ in.})(60 \text{ lb}) = 0$$

$$\mathbf{M} = 60.0 \text{ lb}\cdot\text{in.} \circlearrowright \blacktriangleleft$$

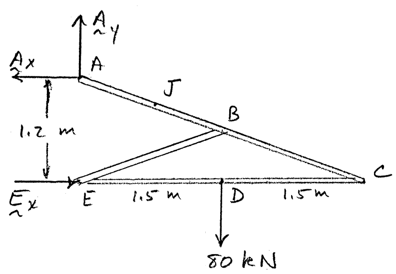


PROBLEM 7.3

For the frame and loading of Prob. 6.80, determine the internal forces at a point J located halfway between points A and B.

SOLUTION

FBD Frame:



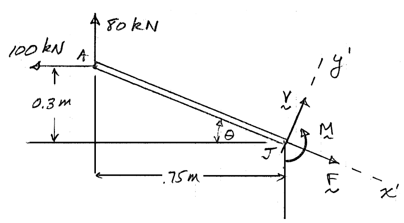
$$\Sigma F_y = 0:\ A_y - 80\text{ kN} = 0 \qquad \mathbf{A}_y = 80\text{ kN}$$

$$\Sigma M_E = 0:\ (1.2\text{ m})A_x - (1.5\text{ m})(80\text{ kN}) = 0$$

$$\mathbf{A}_x = 100\text{ kN}$$

$$\theta = \tan^{-1}\left(\frac{0.3\text{ m}}{0.75\text{ m}}\right) = 21.801°$$

FBD AJ:



$$\Sigma F_{x'} = 0:\ F - (80\text{ kN})\sin 21.801° - (100\text{ kN})\cos 21.801° = 0$$

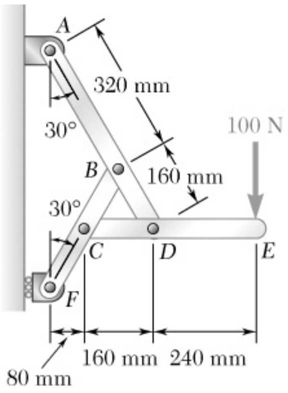
$$\mathbf{F} = 122.6\text{ kN} \blacktriangleleft$$

$$\Sigma F_{y'} = 0:\ V + (80\text{ kN})\cos 21.801° - (100\text{ kN})\sin 21.801° = 0$$

$$\mathbf{V} = 37.1\text{ kN} \blacktriangleleft$$

$$\Sigma M_J = 0:\ M + (.3\text{ m})(100\text{ kN}) - (.75\text{ m})(80\text{ kN}) = 0$$

$$\mathbf{M} = 30.0\text{ kN}\cdot\text{m} \blacktriangleleft$$

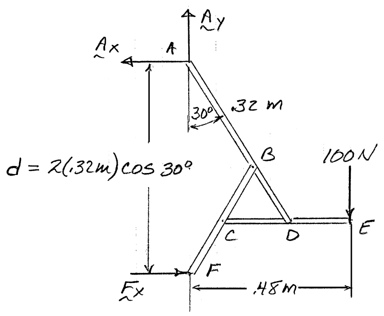


PROBLEM 7.4

For the frame and loading of Prob. 6.101, determine the internal forces at a point J located halfway between points A and B.

SOLUTION

FBD Frame:

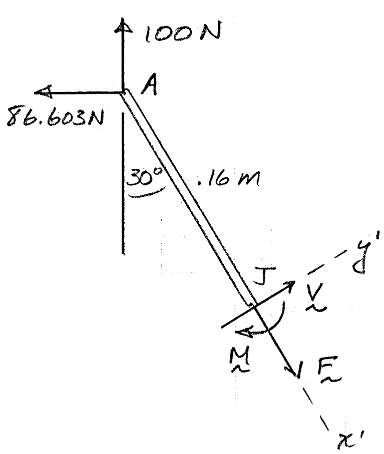


$$\uparrow \Sigma F_y = 0: \; A_y - 100\text{ N} = 0 \qquad \mathbf{A}_y = 100\text{ N} \uparrow$$

$$\circlearrowleft \Sigma M_F = 0: \; \left[2(0.32\text{ m})\cos 30°\right]A_x - (0.48\text{ m})(100\text{ N}) = 0$$

$$\mathbf{A}_x = 86.603\text{ N} \longleftarrow$$

FBD AJ:



$$\searrow \Sigma F_{x'} = 0: \; F - (100\text{ N})\cos 30° - (86.603\text{ N})\sin 30° = 0$$

$$\mathbf{F} = 129.9\text{ N} \searrow \blacktriangleleft$$

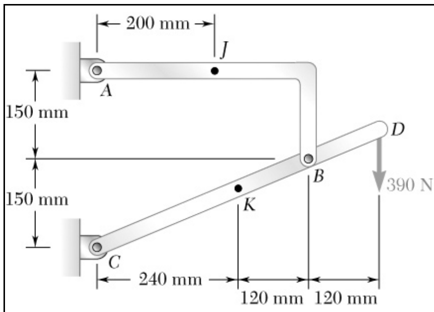
$$\nearrow \Sigma F_{y'} = 0: \; V + (100\text{ N})\sin 30° - (86.603\text{ N})\cos 30° = 0$$

$$\mathbf{V} = 25.0\text{ N} \nearrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: \; \left[(0.16\text{ m})\cos 30°\right](86.603\text{ N})$$

$$- \left[(0.16\text{ m})\sin 30°\right](100\text{ N}) - M = 0$$

$$\mathbf{M} = 4.00\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

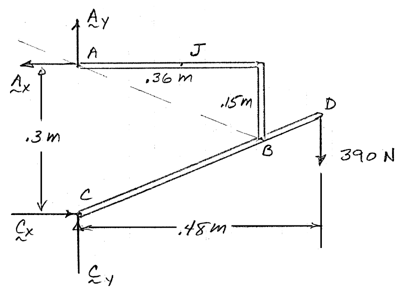


PROBLEM 7.5

Determine the internal forces at point J of the structure shown.

SOLUTION

FBD Frame:



AB is two-force member, so

$$\frac{A_x}{0.36 \text{ m}} = \frac{A_y}{0.15 \text{ m}} \qquad A_y = \frac{5}{12} A_x$$

$$\circlearrowleft \Sigma M_C = 0: (0.3 \text{ m}) A_x - (0.48 \text{ m})(390 \text{ N}) = 0$$

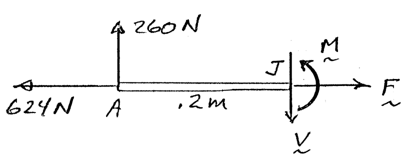
$$\mathbf{A}_x = 624 \text{ N} \longleftarrow$$

$$A_y = \frac{5}{12} A_x = 260 \text{ N} \text{ or } \mathbf{A}_y = 260 \text{ N} \uparrow$$

$$\longrightarrow \Sigma F_x = 0: F - 624 \text{ N} = 0$$

$$\mathbf{F} = 624 \text{ N} \longrightarrow \blacktriangleleft$$

FBD AJ:

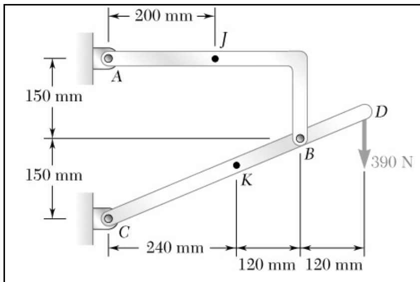


$$\uparrow \Sigma F_y = 0: 260 \text{ N} - V = 0$$

$$\mathbf{V} = 260 \text{ N} \downarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: M - (0.2 \text{ m})(260 \text{ N}) = 0$$

$$\mathbf{M} = 52.0 \text{ N} \cdot \text{m} \circlearrowright \blacktriangleleft$$

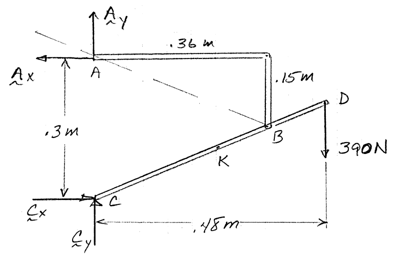


PROBLEM 7.6

Determine the internal forces at point K of the structure shown.

SOLUTION

FBD Frame:



$$\circlearrowleft \Sigma M_C = 0: (0.3\text{ m})A_x - (0.48\text{ m})(390\text{ N}) = 0$$

$$\mathbf{A}_x = 624\text{ N} \longleftarrow$$

AB is two-force member, so

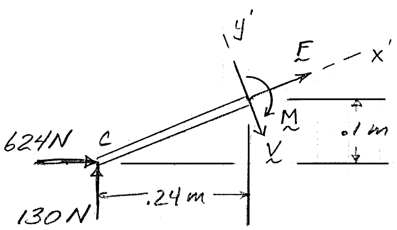
$$\frac{A_x}{0.36\text{ m}} = \frac{A_y}{0.15\text{ m}} \rightarrow A_y = \frac{5}{12}A_x \qquad \mathbf{A}_y = 260\text{ N} \uparrow$$

$$\longrightarrow \Sigma F_x = 0: -A_x + C_x = 0 \qquad \mathbf{C}_x = \mathbf{A}_x = 624\text{ N} \longrightarrow$$

$$\uparrow \Sigma F_y = 0: A_y + C_y - 390\text{ N} = 0$$

$$C_y = 390\text{ N} - 260\text{ N} = 130\text{ N or } \mathbf{C}_y = 130\text{ N} \uparrow$$

FBD CK:



$$\nearrow \Sigma F_{x'} = 0: F + \frac{12}{13}(624\text{ N}) + \frac{5}{13}(130\text{ N}) = 0$$

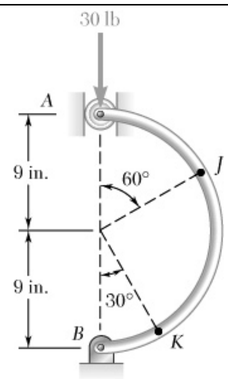
$$F = -626\text{ N} \qquad \mathbf{F} = 626\text{ N} \swarrow \blacktriangleleft$$

$$\nwarrow \Sigma F_{y'} = 0: \frac{12}{13}(130\text{ N}) - \frac{5}{13}(624\text{ N}) - V = 0$$

$$V = -120\text{ N} \qquad \mathbf{V} = 120.0\text{ N} \nwarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_K = 0: (0.1\text{ m})(624\text{ N}) - (0.24\text{ m})(130\text{ N}) - M = 0$$

$$\mathbf{M} = 31.2\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

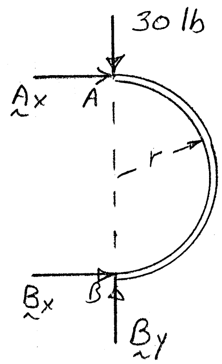


PROBLEM 7.7

A semicircular rod is loaded as shown. Determine the internal forces at point *J*.

SOLUTION

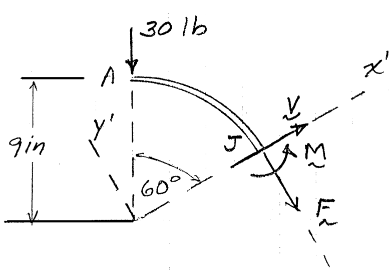
FBD Rod:



$$\circlearrowleft \Sigma M_B = 0: A_x(2r) = 0$$

$$\mathbf{A}_x = 0$$

FBD AJ:



$$\nearrow \Sigma F_{x'} = 0: V - (30 \text{ lb})\cos 60° = 0$$

$$\mathbf{V} = 15.00 \text{ lb} \nearrow \blacktriangleleft$$

$$\searrow \Sigma F_{y'} = 0: F + (30 \text{ lb})\sin 60° = 0$$

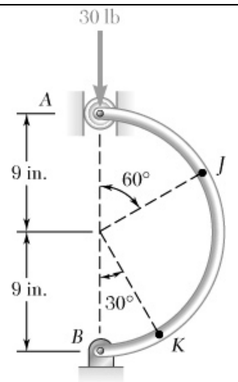
$$F = -25.98 \text{ lb}$$

$$\mathbf{F} = 26.0 \text{ lb} \nwarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: M - [(9 \text{ in.})\sin 60°](30 \text{ lb}) = 0$$

$$M = -233.8 \text{ lb}\cdot\text{in.}$$

$$\mathbf{M} = 234 \text{ lb}\cdot\text{in.} \circlearrowright \blacktriangleleft$$

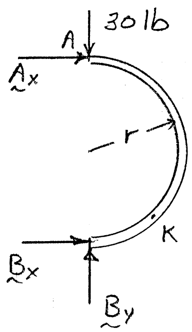


PROBLEM 7.8

A semicircular rod is loaded as shown. Determine the internal forces at point *K*.

SOLUTION

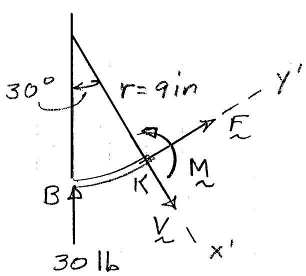
FBD Rod:



$$\uparrow \Sigma F_y = 0: B_y - 30 \text{ lb} = 0 \qquad \mathbf{B}_y = 30 \text{ lb} \uparrow$$

$$\circlearrowleft \Sigma M_A = 0: 2rB_x = 0 \qquad \mathbf{B}_x = 0$$

FBD BK:



$$\searrow \Sigma F_{x'} = 0: V - (30 \text{ lb})\cos 30° = 0$$

$$V = 25.98 \text{ lb}$$

$$\mathbf{V} = 26.0 \text{ lb} \searrow \blacktriangleleft$$

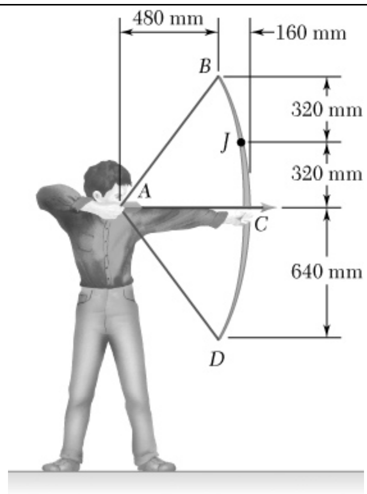
$$\nearrow \Sigma F_{y'} = 0: F + (30 \text{ lb})\sin 30° = 0$$

$$F = -15 \text{ lb}$$

$$\mathbf{F} = 15.00 \text{ lb} \swarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_K = 0: M - [(9 \text{ in.})\sin 30°](30 \text{ lb}) = 0$$

$$\mathbf{M} = 135.0 \text{ lb}\cdot\text{in.} \circlearrowright \blacktriangleleft$$

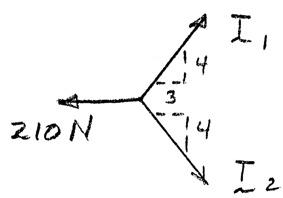


PROBLEM 7.9

An archer aiming at a target is pulling with a 210-N force on the bowstring. Assuming that the shape of the bow can be approximated by a parabola, determine the internal forces at point J.

SOLUTION

FBD Point A:

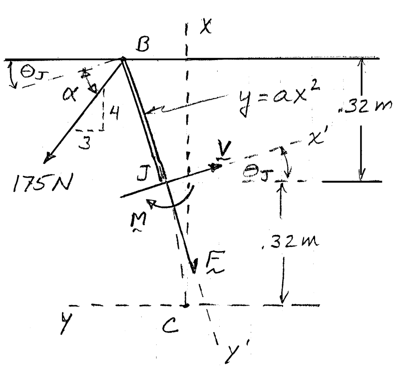


By symmetry $T_1 = T_2$

$$\rightarrow \Sigma F_x = 0: 2\left(\frac{3}{5}T_1\right) - 210 \text{ N} = 0 \qquad T_1 = T_2 = 175 \text{ N}$$

Curve CJB is parabolic: $y = ax^2$

FBD BJ:



At B: $\quad x = 0.64 \text{ m}, \qquad y = 0.16 \text{ m} \qquad a = \dfrac{0.16 \text{ m}}{(0.64 \text{ m})^2} = \dfrac{1}{2.56 \text{ m}}$

So, at J: $\quad y_J = \dfrac{1}{2.56 \text{ m}}(0.32 \text{ m})^2 = 0.04 \text{ m}$

$$\text{Slope of parabola} = \tan\theta = \frac{dy}{dx} = 2ax$$

At J: $\quad \theta_J = \tan^{-1}\left[\dfrac{2}{2.56 \text{ m}}(0.32 \text{ m})\right] = 14.036°$

So $\quad \alpha = \tan^{-1}\dfrac{4}{3} - 14.036° = 39.094°$

$$\nearrow \Sigma F_{x'} = 0: V - (175 \text{ N})\cos(39.094°) = 0$$

$$\mathbf{V} = 135.8 \text{ N} \nearrow \blacktriangleleft$$

$$\searrow \Sigma F_{y'} = 0: F + (175 \text{ N})\sin(39.094°) = 0$$

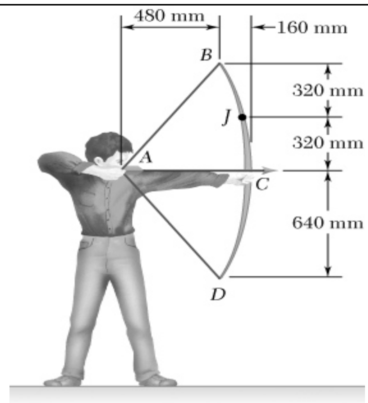
$$F = -110.35 \text{ N} \qquad \mathbf{F} = 110.4 \text{ N} \nwarrow \blacktriangleleft$$

PROBLEM 7.9 CONTINUED

$$\circlearrowleft \Sigma M_J = 0: M + (0.32 \text{ m})\left[\frac{3}{5}(175 \text{ N})\right]$$

$$+\left[(0.16 - 0.04)\text{m}\right]\left[\frac{4}{5}(175 \text{ N})\right] = 0$$

$$\mathbf{M} = 50.4 \text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$



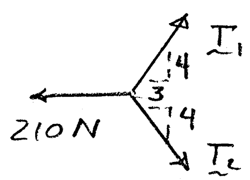
PROBLEM 7.10

For the bow of Prob. 7.9, determine the magnitude and location of the maximum (*a*) axial force, (*b*) shearing force, (*c*) bending moment.

SOLUTION

By symmetry $T_1 = T_2 = T$

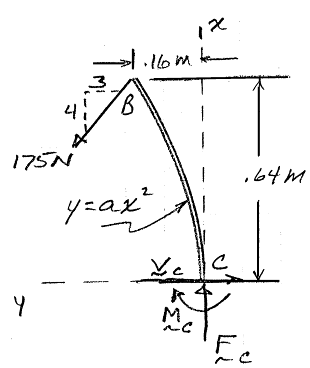
FBD Point A:



$$\longrightarrow \Sigma F_x = 0: 2T_1\left(\frac{3}{5}\right) - 210\text{ N} = 0 \qquad T_1 = 175\text{ N}$$

$$\uparrow \Sigma F_y = 0: F_C - \frac{4}{5}(175\text{ N}) = 0 \qquad F_C = 140\text{ N} \uparrow$$

FBD BC:



$$\longleftarrow \Sigma F_x = 0: \frac{3}{5}(175\text{ N}) - V_C = 0 \qquad V_C = 105\text{ N} \longrightarrow$$

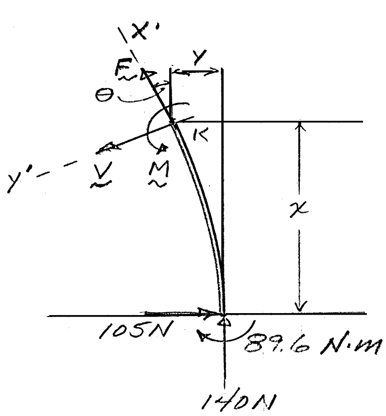
$$\circlearrowleft \Sigma M_C = 0: M_C - (0.64\text{ m})\left[\frac{3}{5}(175\text{ N})\right] - (0.16\text{ m})\left[\frac{4}{5}(175\text{ N})\right] = 0$$

$$M_C = 89.6\text{ N}\cdot\text{m} \circlearrowright$$

Also: if $y = ax^2$ and, at B, $y = 0.16$ m, $x = 0.64$ m

Then $$a = \frac{0.16\text{ m}}{(0.64\text{ m})^2} = \frac{1}{2.56\text{ m}};$$

FBD CK:



And $$\theta = \tan^{-1}\frac{dy}{dx} = \tan^{-1}2ax$$

$$\nwarrow \Sigma F_{x'} = 0: (140\text{ N})\cos\theta - (105\text{ N})\sin\theta + F = 0$$

So $$F = (105\text{ N})\sin\theta - (140\text{ N})\cos\theta$$

$$\frac{dF}{d\theta} = (105\text{ N})\cos\theta + (140\text{ N})\sin\theta$$

$$\swarrow \Sigma F_{y'} = 0: V - (105\text{ N})\cos\theta - (140\text{ N})\sin\theta = 0$$

So $$V = (105\text{ N})\cos\theta + (140\text{ N})\sin\theta$$

PROBLEM 7.10 CONTINUED

And $\dfrac{dV}{d\theta} = -(105 \text{ N})\sin\theta + (140 \text{ N})\cos\theta$

$$\Sigma M_K = 0: M + x(105 \text{ N}) + y(140 \text{ N}) - 89.6 \text{ N}\cdot\text{m} = 0$$

$$M = -(105 \text{ N})x - \frac{(140 \text{ N})x^2}{(2.56 \text{ m})} + 89.6 \text{ N}\cdot\text{m}$$

$$\frac{dM}{dx} = -(105 \text{ N}) - (109.4 \text{ N/m})x + 89.6 \text{ N}\cdot\text{m}$$

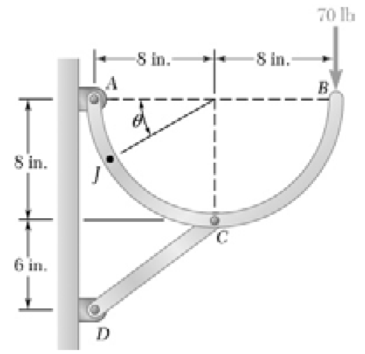
Since none of the functions, F, V, or M has a vanishing derivative in the valid range of $0 \le x \le 0.64 \text{ m}(0 \le \theta \le 26.6°)$, the maxima are at the limits $(x = 0, \text{ or } x = 0.64 \text{ m})$.

Therefore, (*a*) $\mathbf{F}_{\text{max}} = 140.0 \text{ N}$ $\uparrow$ at C ◀

(*b*) $\mathbf{V}_{\text{max}} = 156.5 \text{ N}$ at B ◀

(*c*) $\mathbf{M}_{\text{max}} = 89.6 \text{ N}\cdot\text{m}$ at C ◀

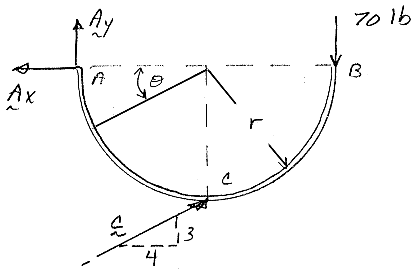
PROBLEM 7.11



A semicircular rod is loaded as shown. Determine the internal forces at point J knowing that $\theta = 30^{\circ}$.

SOLUTION

FBD AB:



$$\circlearrowleft \Sigma M_A = 0: r\left(\frac{4}{5}C\right) + r\left(\frac{3}{5}C\right) - 2r(70\text{ lb}) = 0$$

$$\mathbf{C} = 100\text{ lb} \nearrow$$

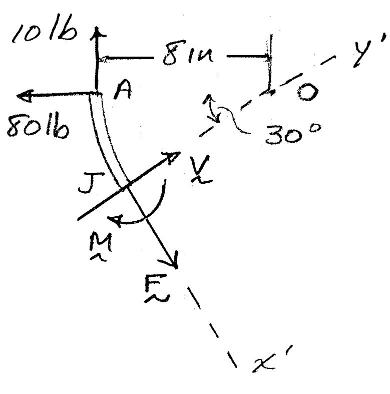
$$\rightarrow \Sigma F_x = 0: -A_x + \frac{4}{5}(100\text{ lb}) = 0$$

$$\mathbf{A}_x = 80\text{ lb} \longleftarrow$$

$$\uparrow \Sigma F_y = 0: A_y + \frac{3}{5}(100\text{ lb}) - 70\text{ lb} = 0$$

$$\mathbf{A}_y = 10\text{ lb} \uparrow$$

FBD AJ:



$$\searrow \Sigma F_{x'} = 0: F - (80\text{ lb})\sin 30° - (10\text{ lb})\cos 30° = 0$$

$$F = 48.66\text{ lb}$$

$$\mathbf{F} = 48.7\text{ lb} \nwarrow 60° \blacktriangleleft$$

$$\nearrow \Sigma F_{y'} = 0: V - (80\text{ lb})\cos 30° + (10\text{ lb})\sin 30° = 0$$

$$V = 64.28\text{ lb}$$

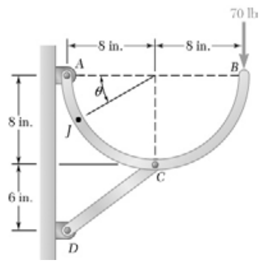
$$\mathbf{V} = 64.3\text{ lb} \measuredangle 30° \blacktriangleleft$$

$$\circlearrowleft \Sigma M_0 = 0: (8\text{ in.})(48.66\text{ lb}) - (8\text{ in.})(10\text{ lb}) - M = 0$$

$$M = 309.28\text{ lb}\cdot\text{in.}$$

$$\mathbf{M} = 309\text{ lb}\cdot\text{in.} \circlearrowright \blacktriangleleft$$

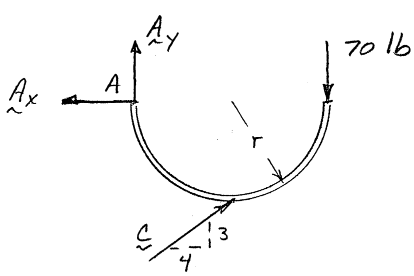
PROBLEM 7.12



A semicircular rod is loaded as shown. Determine the magnitude and location of the maximum bending moment in the rod.

SOLUTION

FBD AB:



$$\circlearrowleft \Sigma M_A = 0: r\left(\frac{4}{5}C\right) + r\left(\frac{3}{5}C\right) - 2r(70 \text{ lb}) = 0$$

$$\mathbf{C} = 100 \text{ lb} \nearrow$$

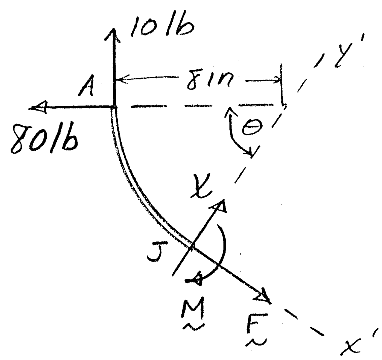
$$\longrightarrow \Sigma F_x = 0: -A_x + \frac{4}{5}(100 \text{ lb}) = 0$$

$$\mathbf{A}_x = 80 \text{ lb} \longleftarrow$$

$$\uparrow \Sigma F_y = 0: A_y + \frac{3}{5}(100 \text{ lb}) - 70 \text{ lb} = 0$$

$$\mathbf{A}_y = 10 \text{ lb} \uparrow$$

FBD AJ:



$$\circlearrowleft \Sigma M_J = 0: M - (8 \text{ in.})(1 - \cos\theta)(10 \text{ lb}) - (8 \text{ in.})(\sin\theta)(80 \text{ lb}) = 0$$

$$M = (640 \text{ lb}\cdot\text{in.})\sin\theta + (80 \text{ lb}\cdot\text{in.})(\cos\theta - 1)$$

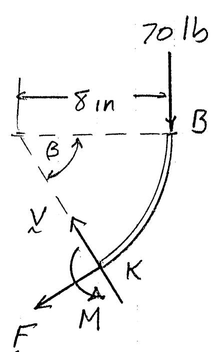
$$\frac{dM}{d\theta} = (640 \text{ lb}\cdot\text{in.})\cos\theta - (80 \text{ lb}\cdot\text{in.})\sin\theta = 0$$

for $\theta = \tan^{-1} 8 = 82.87°$,

where $\dfrac{d^2M}{d\theta^2} = -(640 \text{ lb}\cdot\text{in.})\sin\theta - (80 \text{ lb}\cdot\text{in.})\cos\theta < 0$

So $M = 565 \text{ lb}\cdot\text{in.}$ at $\theta = 82.9°$ is a *max* for *AC*

FBD BK:



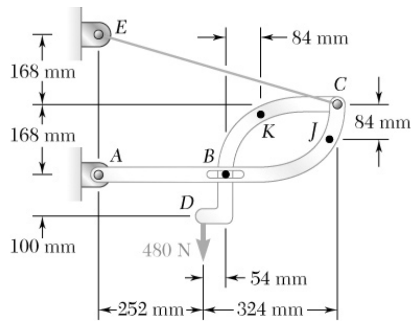
$$\circlearrowleft \Sigma M_K = 0: M - (8 \text{ in.})(1 - \cos\beta)(70 \text{ lb}) = 0$$

$$M = (560 \text{ lb}\cdot\text{in.})(1 - \cos\beta)$$

$$\frac{dM}{d\beta} = (560 \text{ lb}\cdot\text{in.})\sin\beta = 0 \quad \text{for } \beta = 0, \text{ where } M = 0$$

So, for $\beta = \dfrac{\pi}{2}$, $M = 560 \text{ lb}\cdot\text{in.}$ is max for *BC*

$$\therefore \ \mathbf{M}_{\max} = 565 \text{ lb}\cdot\text{in. at } \theta = 82.9° \blacktriangleleft$$

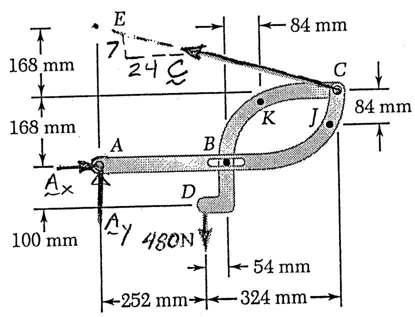


PROBLEM 7.13

Two members, each consisting of straight and 168-mm-radius quarter-circle portions, are connected as shown and support a 480-N load at *D*. Determine the internal forces at point *J*.

SOLUTION

FBD Frame:



$$\circlearrowleft \Sigma M_A = 0: (0.336 \text{ m})\left(\frac{24}{25}C\right) - (0.252 \text{ m})(480 \text{ N}) = 0$$

$$C = 375 \text{ N}$$

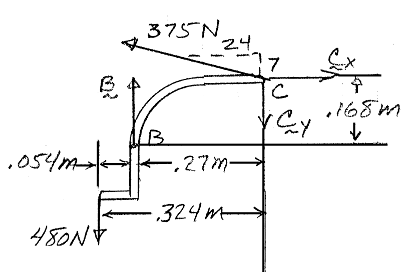
$$\longrightarrow \Sigma F_y = 0: A_x - \frac{24}{25}C = 0 \qquad A_x = \frac{24}{25}(375 \text{ N}) = 360 \text{ N}$$

$$\mathbf{A}_x = 360 \text{ N} \longrightarrow$$

$$\uparrow \Sigma F_y = 0: A_y - 480 \text{ N} + \frac{7}{24}(375 \text{ N}) = 0$$

$$\mathbf{A}_y = 375 \text{ N} \uparrow$$

FBD CD:



$$\circlearrowleft \Sigma M_C = 0: (0.324 \text{ m})(480 \text{ N}) - (0.27 \text{ m})B = 0$$

$$B = 576 \text{ N}$$

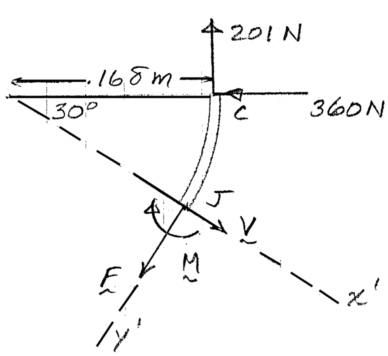
$$\longrightarrow \Sigma F_x = 0: C_x - \frac{24}{25}(375 \text{ N}) = 0$$

$$\mathbf{C}_x = 360 \text{ N} \longrightarrow$$

$$\uparrow \Sigma F_y = 0: -480 \text{ N} + \frac{7}{25}(375 \text{ N}) + (576 \text{ N}) - C_y = 0$$

$$\mathbf{C}_y = 201 \text{ N} \downarrow$$

FBD CJ:



$$\searrow \Sigma F_{x'} = 0: V - (360 \text{ N})\cos 30° - (201 \text{ N})\sin 30° = 0$$

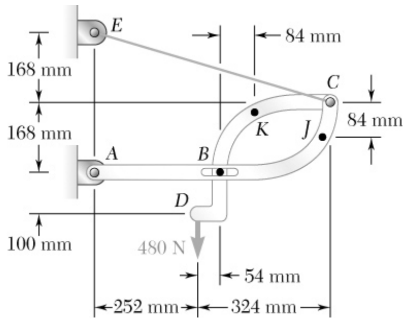
$$\mathbf{V} = 412 \text{ N} \searrow \blacktriangleleft$$

$$\swarrow \Sigma F_{y'} = 0: F + (360 \text{ N})\sin 30° - (201 \text{ N})\cos 30° = 0$$

$$F = -5.93 \text{ N} \qquad \mathbf{F} = 5.93 \text{ N} \nearrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_0 = 0: (0.168 \text{ m})(201 \text{ N} + 5.93 \text{ N}) - M = 0$$

$$M = 34.76 \text{ N}\cdot\text{m} \qquad \mathbf{M} = 34.8 \text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

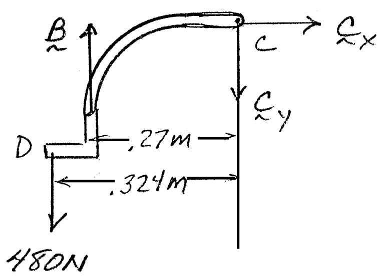


PROBLEM 7.14

Two members, each consisting of straight and 168-mm-radius quarter-circle portions, are connected as shown and support a 480-N load at *D*. Determine the internal forces at point *K*.

SOLUTION

FBD CD:



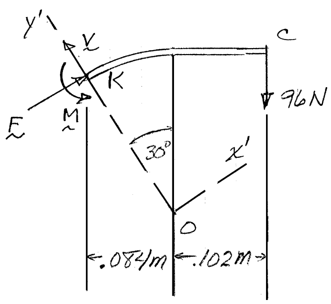
$$\rightarrow \Sigma F_x = 0: \quad \mathbf{C}_x = 0$$

$$\circlearrowleft \Sigma M_B = 0: (0.054 \text{ m})(480 \text{ N}) - (0.27 \text{ m})C_y = 0$$

$$\mathbf{C}_y = 96 \text{ N} \downarrow$$

$$\uparrow \Sigma F_y = 0: B - C_y = 0 \qquad \mathbf{B} = 96 \text{ N} \uparrow$$

FBD CK:



$$\nwarrow \Sigma F_{y'} = 0: V - (96 \text{ N})\cos 30° = 0$$

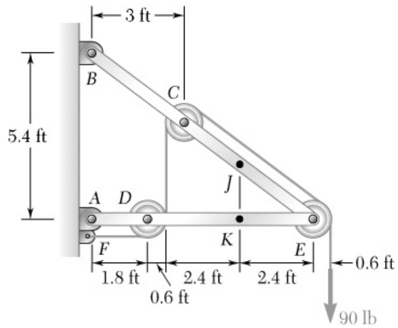
$$\mathbf{V} = 83.1 \text{ N} \nwarrow \blacktriangleleft$$

$$\nearrow \Sigma F_{x'} = 0: F - (96 \text{ N})\sin 30° = 0$$

$$\mathbf{F} = 48.0 \text{ N} \nearrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_K = 0: M - (0.186 \text{ m})(96 \text{ N}) = 0$$

$$\mathbf{M} = 17.86 \text{ N} \cdot \text{m} \circlearrowright \blacktriangleleft$$

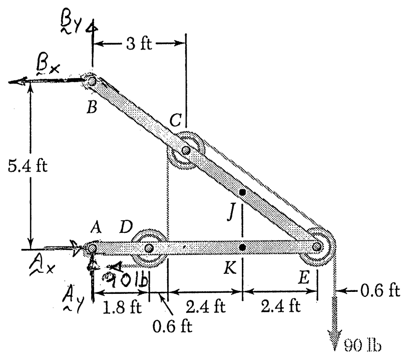


PROBLEM 7.15

Knowing that the radius of each pulley is 7.2 in. and neglecting friction, determine the internal forces at point J of the frame shown.

SOLUTION

FBD Frame:

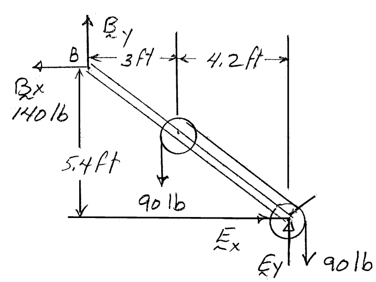


Note: Tension T in cord is 90 lb at any cut. All radii = 0.6 ft

$$\circlearrowleft \Sigma M_A = 0: (5.4 \text{ ft})B_x - (7.8 \text{ ft})(90 \text{ lb}) - (0.6 \text{ ft})(90 \text{ lb}) = 0$$

$$\mathbf{B}_x = 140 \text{ lb} \longleftarrow$$

FBD BCE with pulleys and cord:



$$\circlearrowleft \Sigma M_E = 0: (5.4 \text{ ft})(140 \text{ lb}) - (7.2 \text{ ft})B_y$$

$$+ (4.8 \text{ ft})90 \text{ lb} - (0.6 \text{ ft})90 \text{ lb} = 0$$

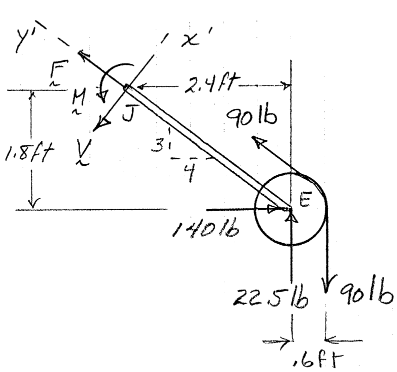
$$\mathbf{B}_y = 157.5 \text{ lb} \uparrow$$

$$\longrightarrow \Sigma F_x = 0: E_x - 140 \text{ lb} = 0 \qquad \mathbf{E}_x = 140 \text{ lb} \longrightarrow$$

$$\uparrow \Sigma F_y = 0: 157.5 \text{ lb} - 90 \text{ lb} - 90 \text{ lb} + E_y = 0$$

$$\mathbf{E}_y = 22.5 \text{ lb} \uparrow$$

FBD EJ:



$$\nearrow \Sigma F_{x'} = 0: -V + \frac{3}{5}(140 \text{ lb}) + \frac{4}{5}(22.5 \text{ lb} - 90 \text{ lb}) = 0$$

$$V = 30 \text{ lb} \qquad \mathbf{V} = 30.0 \text{ lb} \swarrow \blacktriangleleft$$

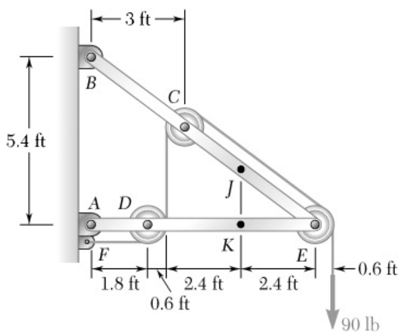
$$\nwarrow \Sigma F_{y'} = 0: F + 90 \text{ lb} - \frac{4}{5}(140 \text{ lb}) - \frac{3}{5}(90 \text{ lb} - 22.5 \text{ lb}) = 0$$

$$F = 62.5 \text{ lb} \qquad \mathbf{F} = 62.5 \text{ lb} \nwarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: M + (1.8 \text{ ft})(140 \text{ lb}) + (0.6 \text{ ft})(90 \text{ lb})$$

$$+ (2.4 \text{ ft})(22.5 \text{ lb}) - (3.0 \text{ ft})(90 \text{ lb}) = 0$$

$$M = -90 \text{ lb}\cdot\text{ft} \qquad \mathbf{M} = 90.0 \text{ lb}\cdot\text{ft} \circlearrowright \blacktriangleleft$$

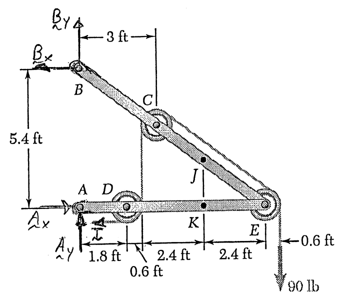


PROBLEM 7.16

Knowing that the radius of each pulley is 7.2 in. and neglecting friction, determine the internal forces at point K of the frame shown.

SOLUTION

FBD Whole:

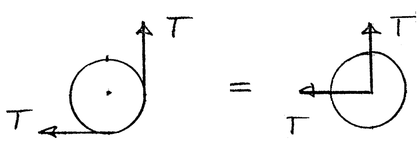


Note: $T = 90$ lb

$$\circlearrowleft \Sigma M_B = 0: (5.4 \text{ ft})A_x - (6 \text{ ft})(90 \text{ lb}) - (7.8 \text{ ft})(90 \text{ lb}) = 0$$

$$\mathbf{A}_x = 2.30 \text{ lb} \longrightarrow$$

FBD AE:



Note: Cord tensions moved to point D as per Problem 6.91

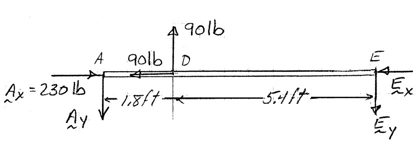
$$\longrightarrow \Sigma F_x = 0: 230 \text{ lb} - 90 \text{ lb} - E_x = 0$$

$$\mathbf{E}_x = 140 \text{ lb} \longleftarrow$$

$$\circlearrowleft \Sigma M_A = 0: (1.8 \text{ ft})(90 \text{ lb}) - (7.2 \text{ ft})E_y = 0$$

$$\mathbf{E}_y = 22.5 \text{ lb} \downarrow$$

FBD KE:



$$\longrightarrow \Sigma F_x = 0: F - 140 \text{ lb} = 0$$

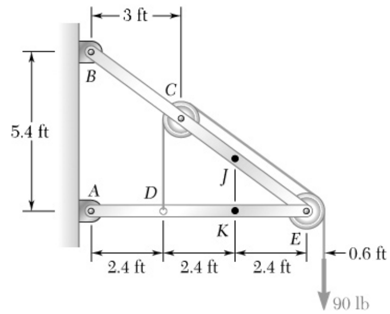
$$\mathbf{F} = 140.0 \text{ lb} \longrightarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: V - 22.5 \text{ lb} = 0$$

$$\mathbf{V} = 22.5 \text{ lb} \uparrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_K = 0: M - (2.4 \text{ ft})(22.5 \text{ lb}) = 0$$

$$\mathbf{M} = 54.0 \text{ lb}\cdot\text{ft} \circlearrowright \blacktriangleleft$$

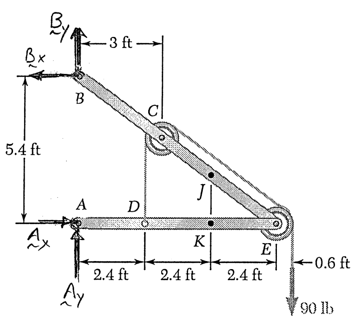


PROBLEM 7.17

Knowing that the radius of each pulley is 7.2 in. and neglecting friction, determine the internal forces at point J of the frame shown.

SOLUTION

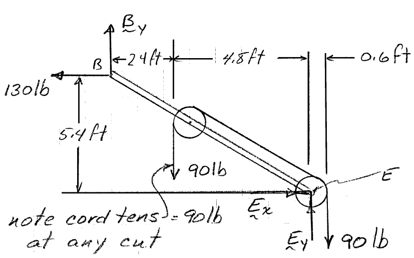
FBD Whole:



$$\circlearrowleft \Sigma M_A = 0: (5.4 \text{ ft})B_x - (7.8 \text{ ft})(90 \text{ lb}) = 0$$

$$\mathbf{B}_x = 130 \text{ lb} \longleftarrow$$

FBD BE with pulleys and cord:



$$\circlearrowleft \Sigma M_E = 0: (5.4 \text{ ft})(130 \text{ lb}) - (7.2 \text{ ft})B_y$$
$$+ (4.8 \text{ ft})(90 \text{ lb}) - (0.6 \text{ ft})(90 \text{ lb}) = 0$$

$$\mathbf{B}_y = 150 \text{ lb} \uparrow$$

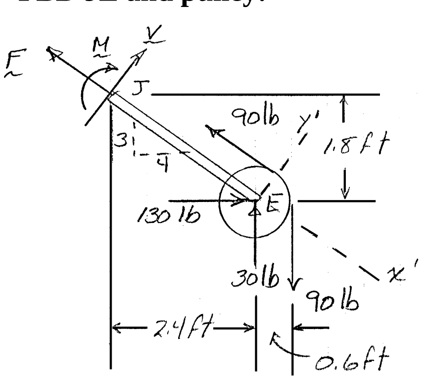
$$\longrightarrow \Sigma F_x = 0: E_x - 130 \text{ lb} = 0$$

$$\mathbf{E}_x = 130 \text{ lb} \longrightarrow$$

$$\uparrow \Sigma F_y = 0: E_y + 150 \text{ lb} - 90 \text{ lb} - 90 \text{ lb} = 0$$

$$\mathbf{E}_y = 30 \text{ lb} \uparrow$$

FBD JE and pulley:



$$\searrow \Sigma F_{x'} = 0: -F - 90 \text{ lb} + \frac{4}{5}(130 \text{ lb}) + \frac{3}{5}(90 \text{ lb} - 30 \text{ lb}) = 0$$

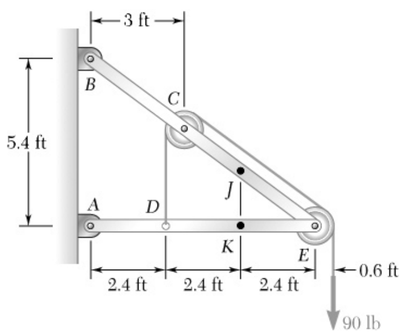
$$\mathbf{F} = 50.0 \text{ lb} \nwarrow \blacktriangleleft$$

$$\nearrow \Sigma F_{y'} = 0: V + \frac{3}{5}(130 \text{ lb}) + \frac{4}{5}(30 \text{ lb} - 90 \text{ lb}) = 0$$

$$V = -30 \text{ lb} \qquad \mathbf{V} = 30.0 \text{ lb} \swarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: -M + (1.8 \text{ ft})(130 \text{ lb}) + (2.4 \text{ ft})(30 \text{ lb}) + (0.6 \text{ ft})(90 \text{ lb})$$
$$- (3.0 \text{ ft})(90 \text{ lb}) = 0$$

$$\mathbf{M} = 90.0 \text{ lb}\cdot\text{ft} \circlearrowright \blacktriangleleft$$

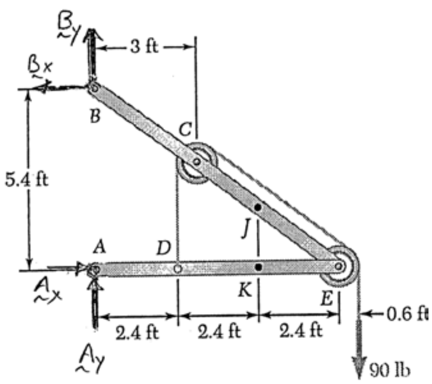


PROBLEM 7.18

Knowing that the radius of each pulley is 7.2 in. and neglecting friction, determine the internal forces at point *K* of the frame shown.

SOLUTION

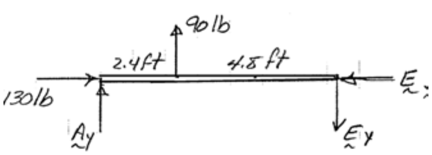
FBD Whole:



$$\circlearrowleft \Sigma M_B = 0: (5.4\text{ ft})A_x - (7.8\text{ ft})(90\text{ lb}) = 0$$

$$\mathbf{A}_x = 130\text{ lb} \longrightarrow$$

FBD AE:

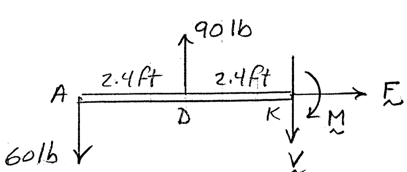


$$\circlearrowleft \Sigma M_E = 0: -(7.2\text{ ft})A_y - (4.8\text{ ft})(90\text{ lb}) = 0$$

$$A_y = -60\text{ lb} \qquad \mathbf{A}_y = 60\text{ lb} \downarrow$$

$$\longrightarrow \Sigma F_x = 0: \qquad \mathbf{F} = 0 \blacktriangleleft$$

FBD AK:

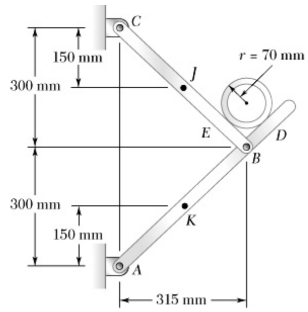


$$\uparrow \Sigma F_y = 0: -60\text{ lb} + 90\text{ lb} - \text{V} = 0$$

$$\mathbf{V} = 30.0\text{ lb} \downarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_K = 0: (4.8\text{ ft})(60\text{ lb}) - (2.4\text{ ft})(90\text{ lb}) - M = 0$$

$$\mathbf{M} = 72.0\text{ lb}\cdot\text{ft} \circlearrowright \blacktriangleleft$$

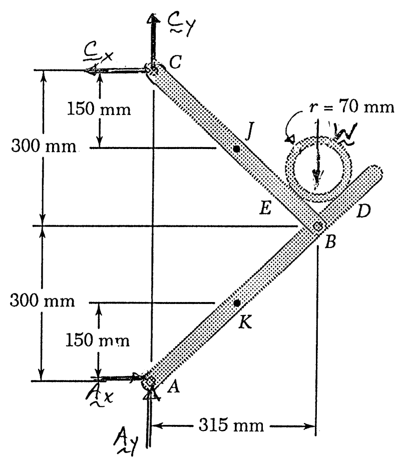


PROBLEM 7.19

A 140-mm-diameter pipe is supported every 3 m by a small frame consisting of two members as shown. Knowing that the combined mass per unit length of the pipe and its contents is 28 kg/m and neglecting the effect of friction, determine the internal forces at point *J*.

SOLUTION

FBD Whole:

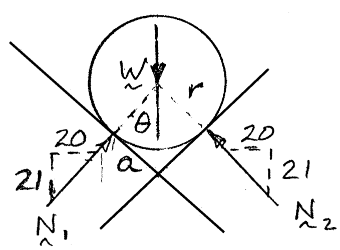


$$W = (3\text{ m})(28\text{ kg/m})\left(9.81\text{ m/s}^2\right) = 824.04\text{ N}$$

$$\circlearrowleft \Sigma M_A = (0.6\text{ m})C_x - (0.315\text{ m})(824.04\text{ N}) = 0$$

$$\mathbf{C}_x = 432.62\text{ N} \longleftarrow$$

FBD pipe:



By symmetry: $N_1 = N_2$

$$\uparrow \Sigma F_y = 0{:}\ 2\frac{21}{29}N_1 - W = 0$$

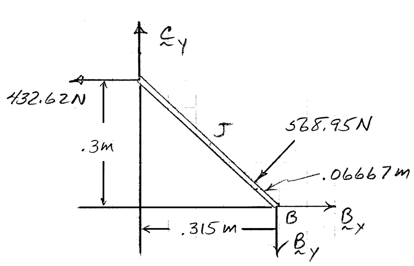
$$N_1 = \frac{29}{42}(824.04\text{ N})$$

$$= 568.98\text{ N}$$

Also note: $a = r\tan\theta = 70\text{ mm}\left(\frac{20}{21}\right)$

$$a = 66.67\text{ mm}$$

FBD BC:



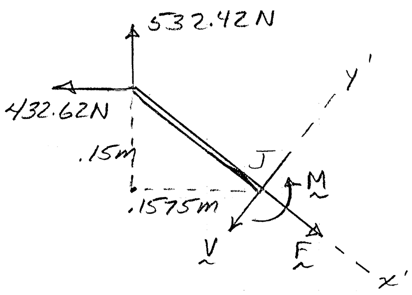
$$\circlearrowleft \Sigma M_B = 0{:}\ (0.3\text{ m})(432.62\text{ N}) - (0.315\text{ m})C_y$$

$$+ (0.06667\text{ m})(568.98\text{ N}) = 0$$

$$\mathbf{C}_y = 532.42\text{ N} \uparrow$$

PROBLEM 7.19 CONTINUED

FBD CJ:



$$\searrow \Sigma F_{x'} = 0: F - \frac{21}{29}(432.62\text{ N}) - \frac{20}{29}(532.42\text{ N}) = 0$$

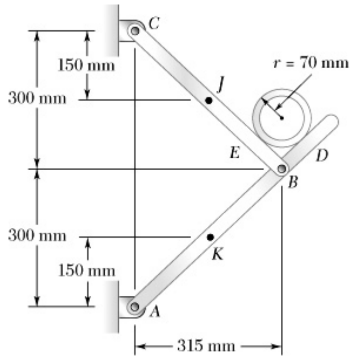
$\mathbf{F} = 680\text{ N}$ ◀

$$\nearrow \Sigma F_{y'} = 0: \frac{21}{29}(532.42\text{ N}) - \frac{20}{29}(432.62\text{ N}) - V = 0$$

$\mathbf{V} = 87.2\text{ N}$ ◀

$$\Sigma M_J = 0: (0.15\text{ m})(432.62\text{ N}) - (0.1575\text{ m})(532.42\text{ N}) + M = 0$$

$\mathbf{M} = 18.96\text{ N}\cdot\text{m}$ ◀

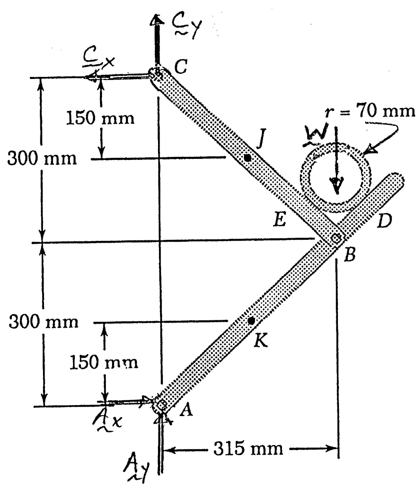


PROBLEM 7.20

A 140-mm-diameter pipe is supported every 3 m by a small frame consisting of two members as shown. Knowing that the combined mass per unit length of the pipe and its contents is 28 kg/m and neglecting the effect of friction, determine the internal forces at point *K*.

SOLUTION

FBD Whole:

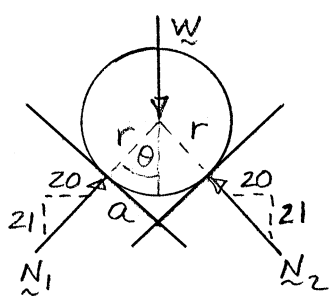


$$W = (3\text{ m})(28\text{ kg/m})(9.81\text{ m/s}^2) = 824.04\text{ N}$$

$$\Sigma M_C = 0: (.6\text{ m})A_x - (.315\text{ m})(824.04\text{ N}) = 0$$

$$\mathbf{A}_x = 432.62\text{ N} \longrightarrow$$

FBD pipe



By symmetry: $N_1 = N_2$

$$\uparrow \Sigma F_y = 0: 2\frac{21}{29}N_1 - W = 0$$

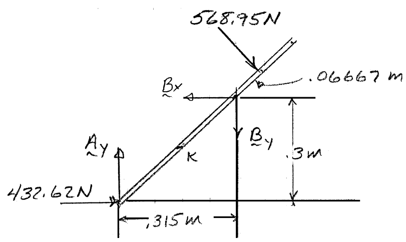
$$N_2 = \frac{29}{42}\,824.04\text{ N}$$

$$= 568.98\text{ N}$$

Also note: $a = r\tan\theta = (70\text{ mm})\dfrac{20}{21}$

$$a = 66.67\text{ mm}$$

FBD AD:



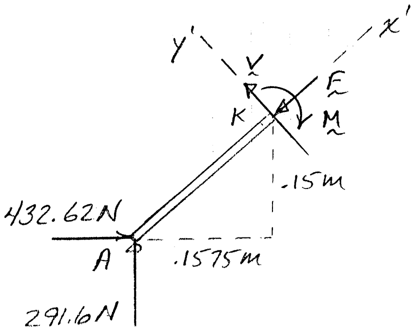
$$\Sigma M_B = 0: (0.3\text{ m})(432.62\text{ N}) - (0.315\text{ m})A_y$$

$$- (0.06667\text{ m})(568.98\text{ N}) = 0$$

$$\mathbf{A}_y = 291.6\text{ N} \uparrow$$

PROBLEM 7.20 CONTINUED

FBD AK:



$$\nearrow \Sigma F_{x'} = 0: \frac{21}{29}(432.62\text{ N}) + \frac{20}{29}(291.6\text{ N}) - F = 0$$

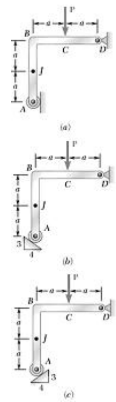
$$\mathbf{F} = 514\text{ N} \swarrow \blacktriangleleft$$

$$\nwarrow \Sigma F_{y'} = 0: \frac{21}{29}(291.6\text{ N}) - \frac{20}{29}(432.62\text{ N}) + V = 0$$

$$\mathbf{V} = 87.2\text{ N} \nwarrow \blacktriangleleft$$

$$\Sigma M_K = 0: (0.15\text{ m})(432.62\text{ N}) - (0.1575\text{ m})(291.6\text{ N}) - M = 0$$

$$\mathbf{M} = 18.97\text{ N}\cdot\text{m} \blacktriangleleft$$

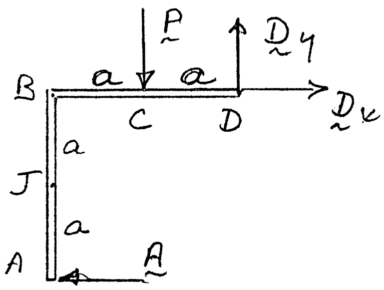


PROBLEM 7.21

A force **P** is applied to a bent rod which is supported by a roller and a pin and bracket. For each of the three cases shown, determine the internal forces at point *J*.

SOLUTION

(*a*) **FBD Rod:**



$$\circlearrowleft \Sigma M_D = 0: aP - 2aA = 0$$

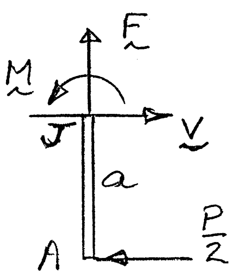
$$\mathbf{A} = \frac{P}{2} \longleftarrow$$

$$\longrightarrow \Sigma F_x = 0: V - \frac{P}{2} = 0$$

$$\mathbf{V} = \frac{P}{2} \longrightarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \qquad \mathbf{F} = 0 \blacktriangleleft$$

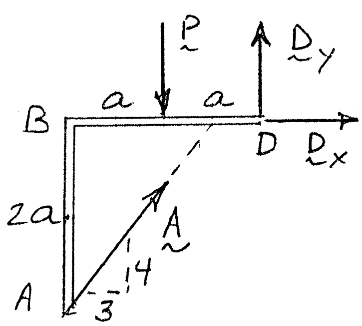
FBD AJ:



$$\circlearrowleft \Sigma M_J = 0: M - a\frac{P}{2} = 0$$

$$\mathbf{M} = \frac{aP}{2} \circlearrowright \blacktriangleleft$$

(*b*) **FBD Rod:**

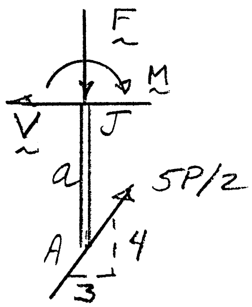


$$\circlearrowleft \Sigma M_D = 0: aP - \frac{a}{2}\left(\frac{4}{5}A\right) = 0$$

$$\mathbf{A} = \frac{5P}{2} \nearrow$$

PROBLEM 7.21 CONTINUED

FBD AJ:



$$\longrightarrow \Sigma F_x = 0: \frac{3}{5}\frac{5P}{2} - V = 0$$

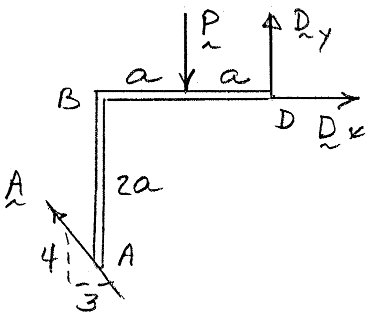
$$\mathbf{V} = \frac{3P}{2} \longleftarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \frac{4}{5}\frac{5P}{2} - F = 0$$

$$\mathbf{F} = 2P \downarrow \blacktriangleleft$$

$$\mathbf{M} = \frac{3}{2}aP \curvearrowright \blacktriangleleft$$

(*c*) **FBD Rod:**



$$\curvearrowleft \Sigma M_D = 0: aP - 2a\left(\frac{3}{5}A\right) - 2a\left(\frac{4}{5}A\right) = 0$$

$$A = \frac{5P}{14}$$

$$\longrightarrow \Sigma F_x = 0: V - \left(\frac{3}{5}\frac{5P}{14}\right) = 0$$

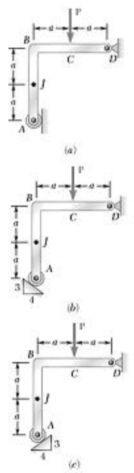
$$\mathbf{V} = \frac{3P}{14} \longrightarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \frac{4}{5}\frac{5P}{14} - F = 0$$

$$\mathbf{F} = \frac{2P}{7} \downarrow \blacktriangleleft$$

$$\curvearrowleft \Sigma M_J = 0: M - a\left(\frac{3}{5}\frac{5P}{14}\right) = 0$$

$$\mathbf{M} = \frac{3}{14}aP \curvearrowright \blacktriangleleft$$

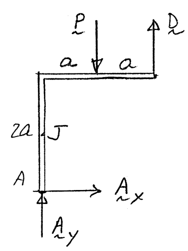


PROBLEM 7.22

A force **P** is applied to a bent rod which is supported by a roller and a pin and bracket. For each of the three cases shown, determine the internal forces at point *J*.

SOLUTION

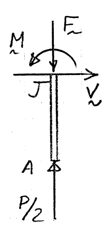
(*a*) **FBD Rod:**



$$\xrightarrow{} \Sigma F_x = 0: A_x = 0$$

$$\circlearrowleft \Sigma M_D = 0: aP - 2aA_y = 0 \qquad A_y = \frac{P}{2}$$

FBD AJ:



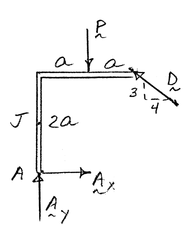
$$\xrightarrow{} \Sigma F_x = 0: \mathbf{V} = 0 \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \frac{P}{2} - F = 0$$

$$\mathbf{F} = \frac{P}{2} \downarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: \mathbf{M} = 0 \blacktriangleleft$$

(*b*) **FBD Rod:**



$$\circlearrowleft \Sigma M_A = 0$$

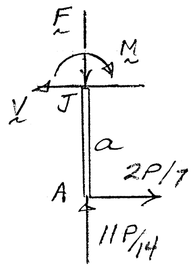
$$2a\left(\frac{4}{5}D\right) + 2a\left(\frac{3}{5}D\right) - aP = 0 \qquad D = \frac{5P}{14}$$

$$\xrightarrow{} \Sigma F_x = 0: A_x - \frac{4}{5}\frac{5}{14}P = 0 \qquad A_x = \frac{2P}{7}$$

$$\uparrow \Sigma F_y = 0: A_y - P + \frac{3}{5}\frac{5}{14}P = 0 \qquad A_y = \frac{11P}{14}$$

PROBLEM 7.22 CONTINUED

FBD AJ:



$$\rightarrow \Sigma F_x = 0: \frac{2}{7}P - V = 0$$

$$\mathbf{V} = \frac{2P}{7} \leftarrow \blacktriangleleft$$

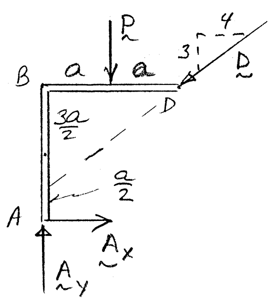
$$\uparrow \Sigma F_y = 0: \frac{11P}{14} - F = 0$$

$$\mathbf{F} = \frac{11P}{14} \downarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: a\frac{2P}{7} - M = 0$$

$$\mathbf{M} = \frac{2}{7}aP \circlearrowright \blacktriangleleft$$

(c) **FBD Rod:**

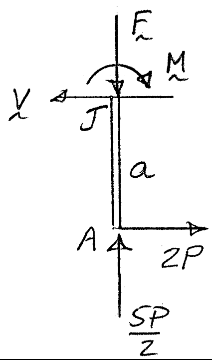


$$\circlearrowleft \Sigma M_A = 0: \frac{a}{2}\left(\frac{4D}{5}\right) - aP = 0 \qquad D = \frac{5P}{2}$$

$$\rightarrow \Sigma F_x = 0: A_x - \frac{4}{5}\frac{5P}{2} = 0 \qquad A_x = 2P$$

$$\uparrow \Sigma F_y = 0: A_y - P - \frac{3}{5}\frac{5P}{2} = 0 \qquad A_y = \frac{5P}{2}$$

FBD AJ:



$$\rightarrow \Sigma F_x = 0: 2P - V = 0$$

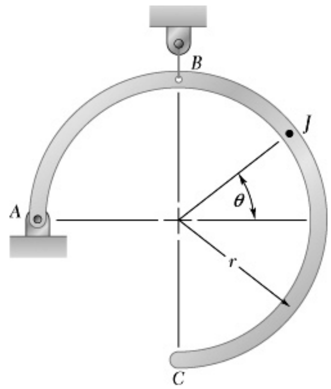
$$\mathbf{V} = 2P \leftarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \frac{5P}{2} - F = 0$$

$$\mathbf{F} = \frac{5P}{2} \downarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: a(2P) - M = 0$$

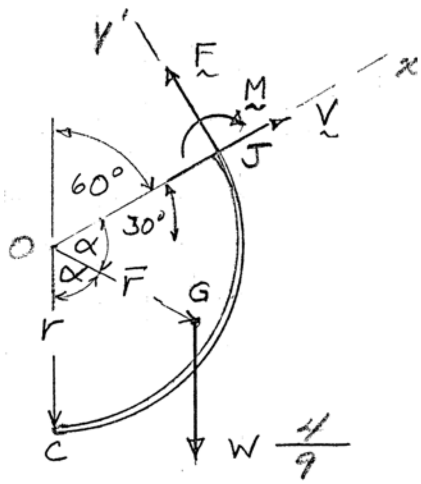
$$\mathbf{M} = 2aP \circlearrowright \blacktriangleleft$$



PROBLEM 7.23

A rod of weight W and uniform cross section is bent into the circular arc of radius r shown. Determine the bending moment at point J when $\theta = 30°$.

SOLUTION



Note $\alpha = \dfrac{180° - 60°}{2} = 60° = \dfrac{\pi}{3}$

$$\bar{r} = \frac{r}{\alpha}\sin\alpha = \frac{3r}{\pi}\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2\pi}r$$

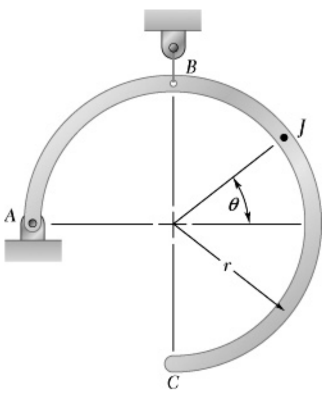
$$\text{Weight of section} = W\frac{120}{270} = \frac{4}{9}W$$

$$\nwarrow \Sigma F_{y'} = 0\colon\ F - \frac{4}{9}W\cos 30° = 0 \qquad F = \frac{2\sqrt{3}}{9}W$$

$$\circlearrowleft \Sigma M_0 = 0\colon\ rF - (\bar{r}\sin 60°)\frac{4W}{9} - M = 0$$

$$M = r\left[\frac{2\sqrt{3}}{9} - \frac{3\sqrt{3}}{2\pi}\frac{\sqrt{3}}{2}\frac{4}{9}\right]W = \left[\frac{2\sqrt{3}}{9} - \frac{1}{\pi}\right]Wr$$

$$\mathbf{M} = 0.0666Wr \circlearrowright \blacktriangleleft$$

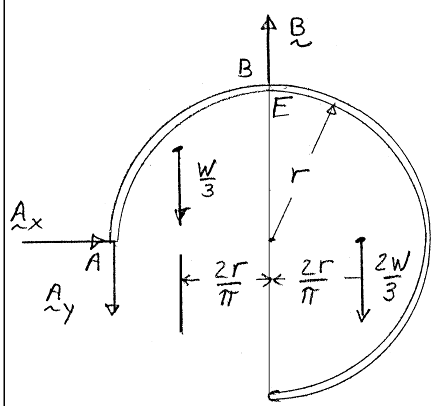


PROBLEM 7.24

A rod of weight W and uniform cross section is bent into the circular arc of radius r shown. Determine the bending moment at point J when $\theta = 120°$.

SOLUTION

(a) **FBD Rod:**

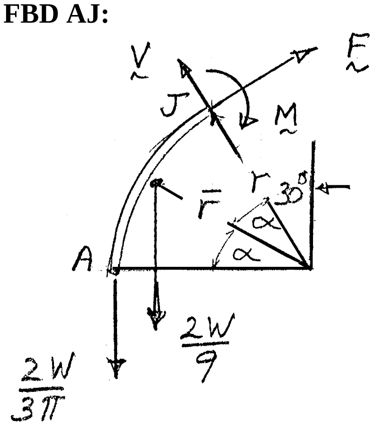


$$\xrightarrow{} \Sigma F_x = 0:\ A_x = 0$$

$$\circlearrowleft \Sigma M_B = 0:\ rA_y + \frac{2r}{\pi}\frac{W}{3} - \frac{2r}{\pi}\frac{2W}{3} = 0$$

$$A_y = \frac{2W}{3\pi}$$

FBD AJ:



Note: $$\alpha = \frac{60°}{2} = 30° = \frac{\pi}{6}$$

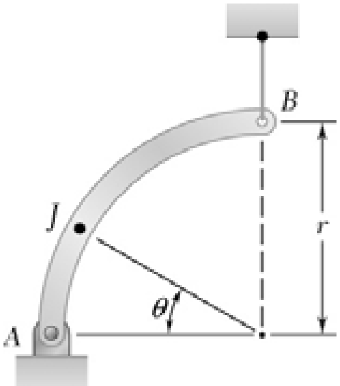
$$\text{Weight of segment } = W\frac{60}{270} = \frac{2W}{9}$$

$$F = \frac{r}{\alpha}\sin\alpha = \frac{r}{\pi/6}\sin 30° = \frac{3r}{\pi}$$

$$\circlearrowleft \Sigma M_J = 0:\ (\bar{r}\cos\alpha - r\sin 30°)\frac{2W}{9} + (r - r\sin 30°)\frac{2W}{3\pi} - M = 0$$

$$M = \frac{2W}{9}\left(\frac{3r}{\pi}\frac{\sqrt{3}}{2} - \frac{r}{2} + \frac{3r}{2\pi}\right) = Wr\left(\frac{\sqrt{3}}{3\pi} - \frac{1}{9} + \frac{1}{3\pi}\right)$$

$$\mathbf{M} = 0.1788Wr \circlearrowright \blacktriangleleft$$

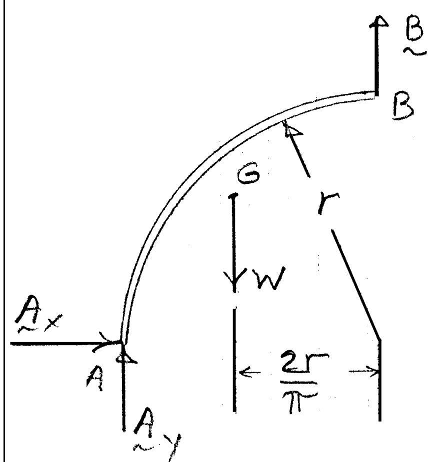


PROBLEM 7.25

A quarter-circular rod of weight W and uniform cross section is supported as shown. Determine the bending moment at point J when $\theta = 30^{\circ}$.

SOLUTION

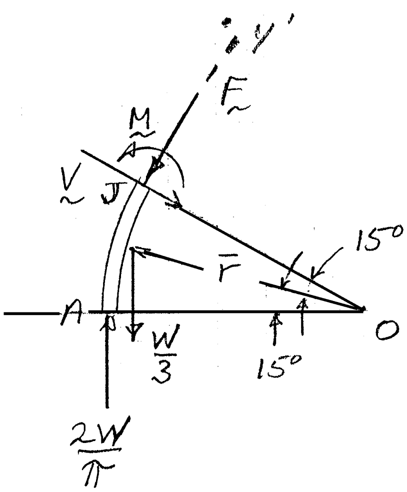
FBD Rod:



$$\rightarrow \Sigma F_x = 0: \mathbf{A}_x = 0$$

$$\circlearrowleft \Sigma M_B = 0: \frac{2r}{\pi}W - rA_y = 0 \qquad \mathbf{A}_y = \frac{2W}{\pi} \uparrow$$

FBD AJ:



$$\alpha = 15^{\circ}, \text{ weight of segment } = W\frac{30^{\circ}}{90^{\circ}} = \frac{W}{3}$$

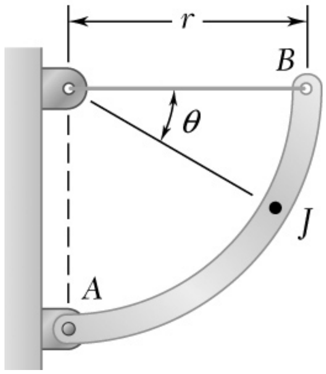
$$\bar{r} = \frac{r}{\alpha}\sin\alpha = \frac{r}{\pi/12}\sin 15^{\circ} = 0.9886r$$

$$\nearrow \Sigma F_{y'} = 0: \frac{2W}{\pi}\cos 30^{\circ} - \frac{W}{3}\cos 30^{\circ} - F = 0$$

$$\mathbf{F} = \frac{W\sqrt{3}}{2}\left(\frac{2}{\pi} - \frac{1}{3}\right) \swarrow$$

$$\circlearrowleft \Sigma M_0 = M + r\left(F - \frac{2W}{\pi}\right) + \bar{r}\cos 15^{\circ}\frac{W}{3} = 0$$

$$\mathbf{M} = 0.0557Wr \circlearrowright \blacktriangleleft$$

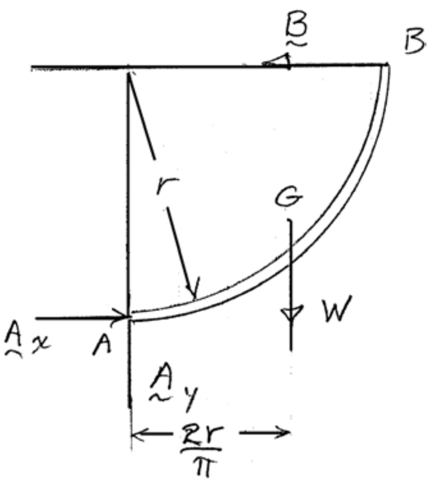


PROBLEM 7.26

A quarter-circular rod of weight W and uniform cross section is supported as shown. Determine the bending moment at point J when $\theta = 30^{\circ}$.

SOLUTION

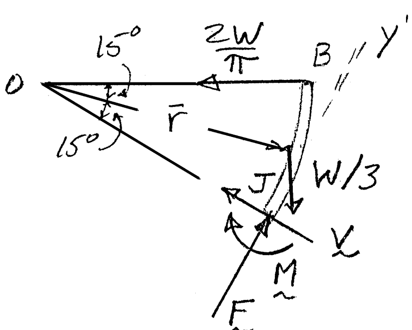
FBD Rod:



$$\Sigma M_A = 0: rB - \frac{2r}{\pi}W = 0$$

$$\mathbf{B} = \frac{2W}{\pi} \longleftarrow$$

FBD BJ:



$$\alpha = 15^{\circ} = \frac{\pi}{12}$$

$$\bar{r} = \frac{r}{\pi/12}\sin 15^{\circ} = 0.98862r$$

$$\text{Weight of segment} = W\frac{30^{\circ}}{90^{\circ}} = \frac{W}{3}$$

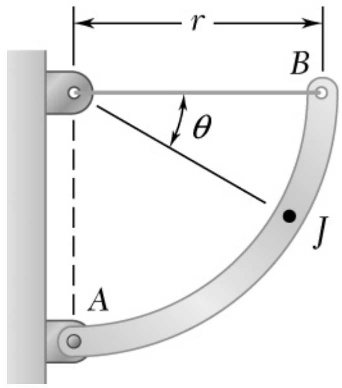
$$\Sigma F_{y'} = 0: F - \frac{W}{3}\cos 30^{\circ} - \frac{2W}{\pi}\sin 30^{\circ} = 0$$

$$\mathbf{F} = \left(\frac{\sqrt{3}}{6} + \frac{1}{\pi}\right)W$$

$$\Sigma M_0 = 0: rF - (\bar{r}\cos 15^{\circ})\frac{W}{3} - M = 0$$

$$M = rW\left(\frac{\sqrt{3}}{6} + \frac{1}{\pi}\right) - \left(0.98862\frac{\cos 15^{\circ}}{3}\right)Wr$$

$$\mathbf{M} = 0.289Wr \blacktriangleleft$$

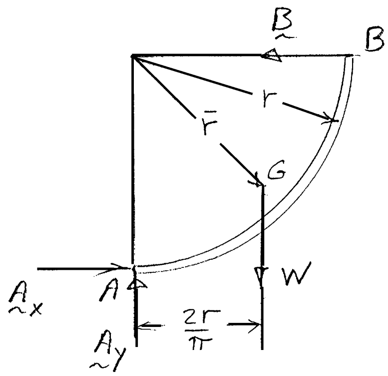


PROBLEM 7.27

For the rod of Prob.7.26, determine the magnitude and location of the maximum bending moment.

SOLUTION

FBD Bar:



$$\circlearrowleft \Sigma M_A = 0: rB - \frac{2r}{\pi}W = 0 \qquad \mathbf{B} = \frac{2W}{\pi} \longleftarrow$$

$$\alpha = \frac{\theta}{2} \qquad \text{so} \qquad 0 \le \alpha \le \frac{\pi}{4}$$

$$\bar{r} = \frac{r}{\alpha}\sin\alpha,$$

$$\text{Weight of segment } = W\frac{2\alpha}{\pi/2}$$

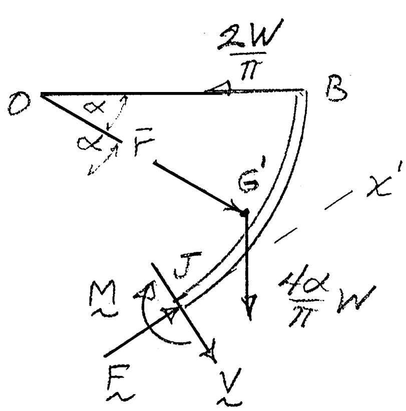
$$= \frac{4\alpha}{\pi}W$$

$$\nearrow \Sigma F_{x'} = 0: F - \frac{4\alpha}{\pi}W\cos 2\alpha - \frac{2W}{\pi}\sin 2\alpha = 0$$

$$F = \frac{2W}{\pi}\left(\sin 2\alpha + 2\alpha\cos 2\alpha\right)$$

$$= \frac{2W}{\pi}\left(\sin\theta + \theta\cos\theta\right)$$

FBD BJ:



$$\circlearrowleft \Sigma M_0 = 0: rF - \left(\bar{r}\cos\alpha\right)\frac{4\alpha}{\pi}W - M = 0$$

$$M = \frac{2}{\pi}Wr\left(\sin\theta + \theta\cos\theta\right) - \left(\frac{r}{\alpha}\sin\alpha\cos\alpha\right)\frac{4\alpha}{\pi}W$$

$$\text{But,} \qquad \sin\alpha\cos\alpha = \frac{1}{2}\sin 2\alpha = \frac{1}{2}\sin\theta$$

$$\text{so} \qquad M = \frac{2Wr}{\pi}\left(\sin\theta + \theta\cos\theta - \sin\theta\right)$$

$$\text{or} \qquad M = \frac{2}{\pi}Wr\theta\cos\theta$$

$$\frac{dM}{d\theta} = \frac{2}{\pi}Wr\left(\cos\theta - \theta\sin\theta\right) = 0 \quad \text{at } \theta\tan\theta = 1$$

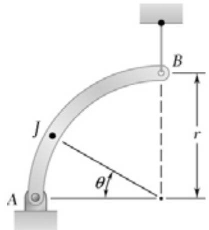
PROBLEM 7.27 CONTINUED

Solving numerically $\theta = 0.8603\text{ rad}$ and $\mathbf{M} = 0.357Wr$ ◀

at $\theta = 49.3°$ ◀

(Since $M = 0$ at both limits, this is the maximum)

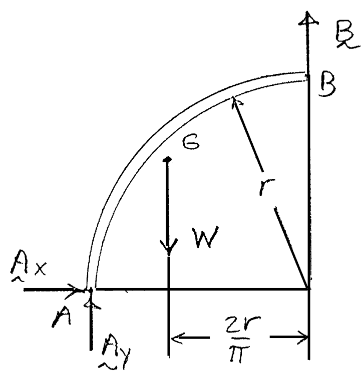
PROBLEM 7.28



For the rod of Prob.7.25, determine the magnitude and location of the maximum bending moment.

SOLUTION

FBD Rod:



$$\rightarrow \Sigma F_x = 0: A_x = 0$$

$$\Sigma M_B = 0: \frac{2r}{\pi}W - rA_y = 0 \qquad A_y = \frac{2W}{\pi}$$

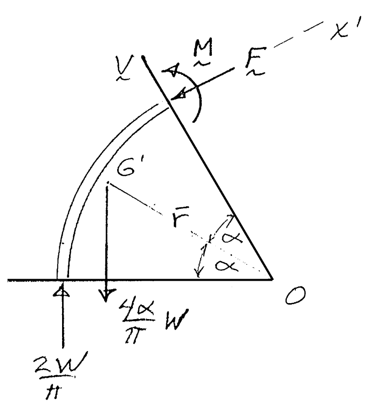
$$\alpha = \frac{\theta}{2}, \qquad \bar{r} = \frac{r}{\alpha}\sin\alpha$$

$$\text{Weight of segment } = W\frac{2\alpha}{\pi/2} = \frac{4\alpha}{\pi}W$$

$$\Sigma F_{x'} = 0: -F - \frac{4\alpha}{\pi}W\cos 2\alpha + \frac{2W}{\pi}\cos 2\alpha = 0$$

$$F = \frac{2W}{\pi}(1-2\alpha)\cos 2\alpha = \frac{2W}{\pi}(1-\theta)\cos\theta$$

FBD AJ:



$$\Sigma M_0 = 0: M + \left(F - \frac{2W}{\pi}\right)r + (\bar{r}\cos\alpha)\frac{4\alpha}{\pi}W = 0$$

$$M = \frac{2W}{\pi}(1 + \theta\cos\theta - \cos\theta)r - \frac{4\alpha W}{\pi}\frac{r}{\alpha}\sin\alpha\cos\alpha$$

But, $\quad \sin\alpha\cos\alpha = \frac{1}{2}\sin 2\alpha = \frac{1}{2}\sin\theta$

so $\quad M = \frac{2r}{\pi}W(1 - \cos\theta + \theta\cos\theta - \sin\theta)$

$$\frac{dM}{d\theta} = \frac{2rW}{\pi}(\sin\theta - \theta\sin\theta + \cos\theta - \cos\theta) = 0$$

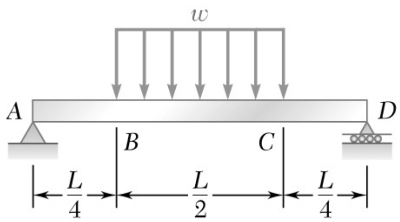
for $\quad (1-\theta)\sin\theta = 0$

$$\frac{dM}{d\theta} = 0 \qquad \text{for} \qquad \theta = 0, 1, n\pi \ (n = 1, 2, \cdots)$$

Only 0 and 1 in valid range

At $\quad \theta = 0 \quad M = 0, \quad$ at $\theta = 1$ rad

at $\quad \theta = 57.3° \qquad M = M_{max} = 0.1009\,Wr$ ◀

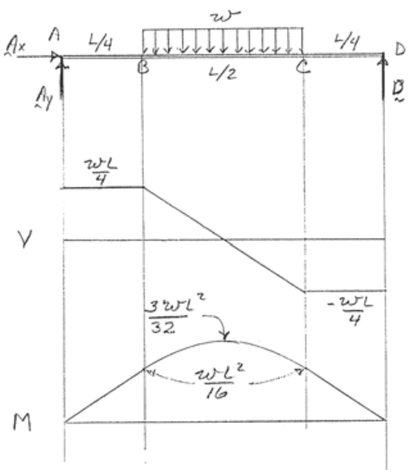


PROBLEM 7.29

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

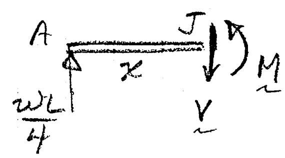
SOLUTION

FBD beam:



(*a*) By symmetry: $A_y = D = \frac{1}{2}(w)\frac{L}{2} \qquad \mathbf{A}_y = \mathbf{D} = \frac{wL}{4} \uparrow$

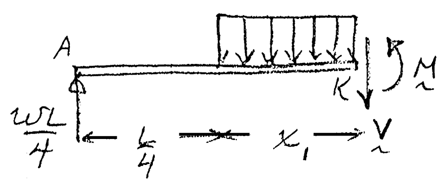
Along AB:



$$\uparrow \Sigma F_y = 0: \frac{wL}{4} - V = 0 \qquad V = \frac{wL}{4}$$

$$\circlearrowleft \Sigma M_J = 0: M - x\frac{wL}{4} = 0 \qquad M = \frac{wL}{4}x \text{ (straight)}$$

Along BC:



$$\uparrow \Sigma F_y = 0: \frac{wL}{4} - wx_1 - V = 0$$

$$V = \frac{wL}{4} - wx_1$$

straight with $\quad V = 0 \quad$ at $\quad x_1 = \frac{L}{4}$

$$\circlearrowleft \Sigma M_k = 0: M + \frac{x_1}{2}wx_1 - \left(\frac{L}{4} + x_1\right)\frac{wL}{4} = 0$$

$$M = \frac{w}{2}\left(\frac{L^2}{8} + \frac{L}{2}x_1 - x_1^2\right)$$

PROBLEM 7.29 CONTINUED

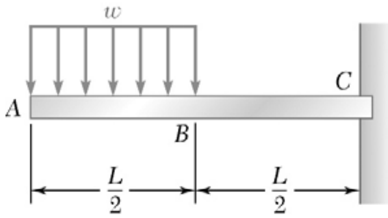
Parabola with $M = \frac{3}{32}wL^2$ at $x_1 = \frac{L}{4}$

Section *CD* by symmetry

(*b*) From diagrams:

$$|V|_{max} = \frac{wL}{4} \text{ on } AB \text{ and } CD \blacktriangleleft$$

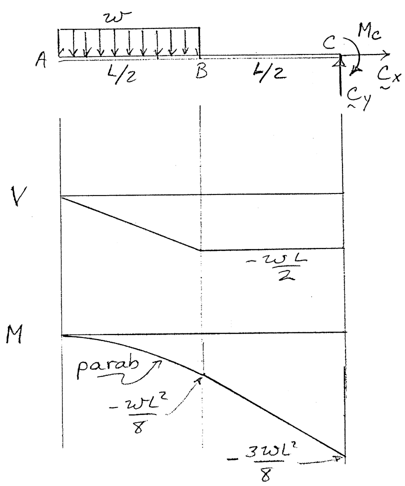
$$|M|_{max} = \frac{3wL^2}{32} \text{ at center} \blacktriangleleft$$



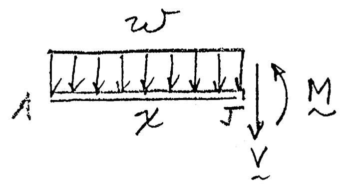
PROBLEM 7.30

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



(*a*) **Along AB:**



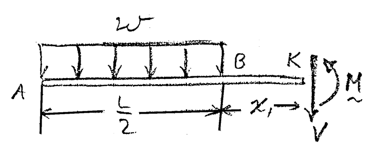
$$+\uparrow \Sigma F_y = 0: -wx - V = 0 \qquad V = -wx$$

straight with $V = -\dfrac{wL}{2}$ at $x = \dfrac{L}{2}$

$$+\circlearrowleft \Sigma M_J = 0: M + \frac{x}{2}wx = 0 \qquad M = -\frac{1}{2}wx^2$$

parabola with $M = -\dfrac{wL^2}{8}$ at $x = \dfrac{L}{2}$

Along BC:



$$+\uparrow \Sigma F_y = 0: -w\frac{L}{2} - V = 0 \qquad V = -\frac{1}{2}wL$$

$$+\circlearrowleft \Sigma M_k = 0: M + \left(x_1 + \frac{L}{4}\right)w\frac{L}{2} = 0$$

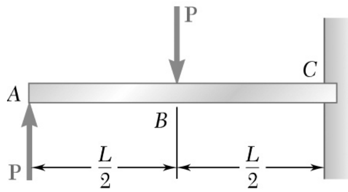
$$M = -\frac{wL}{2}\left(\frac{L}{4} + x_1\right)$$

straight with $M = -\dfrac{3}{8}wL^2$ at $x_1 = \dfrac{L}{2}$

(*b*) From diagrams:

$$|V|_{max} = \frac{wL}{2} \text{ on } BC \blacktriangleleft$$

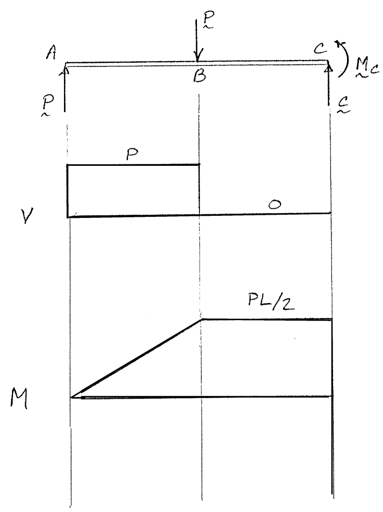
$$|M|_{max} = \frac{3wL^2}{8} \text{ at } C \blacktriangleleft$$



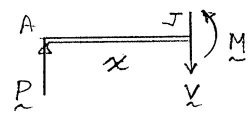
PROBLEM 7.31

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



(*a*) **Along AB:**

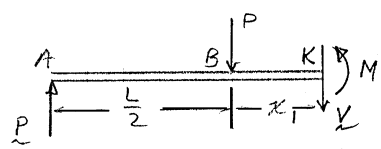


$$\uparrow \Sigma F_y = 0:\ P - V = 0 \qquad V = P$$

$$\Sigma M_J = 0:\ M - Px = 0 \qquad M = Px$$

straight with $M = \dfrac{PL}{2}$ at B

Along BC:



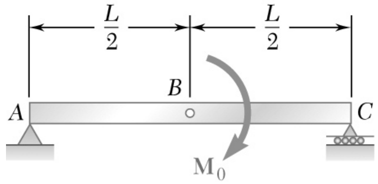
$$\uparrow \Sigma F_y = 0: P - P - V = 0 \qquad V = 0$$

$$\Sigma M_K = 0: M + Px_1 - P\left(\frac{L}{2} + x_1\right) = 0$$

$$M = \frac{PL}{2} \quad \text{(constant)}$$

(*b*) From diagrams: $|V|_{\max} = P$ along AB ◀

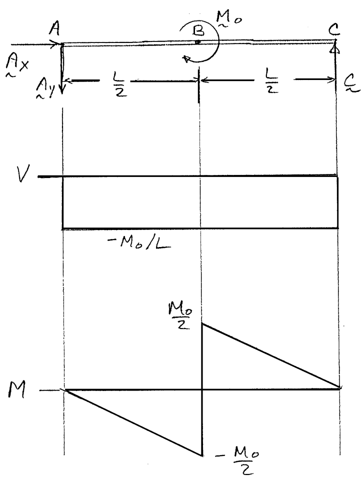
$|M|_{\max} = \dfrac{PL}{2}$ along BC ◀



PROBLEM 7.32

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION



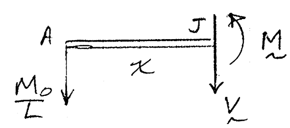
(a) **FBD Beam:**

$$\circlearrowleft \Sigma M_C = 0:\ LA_y - M_0 = 0$$

$$\mathbf{A}_y = \frac{M_0}{L} \downarrow$$

$$\uparrow \Sigma F_y = 0: -A_y + C = 0 \qquad \mathbf{C} = \frac{M_0}{L} \uparrow$$

Along AB:

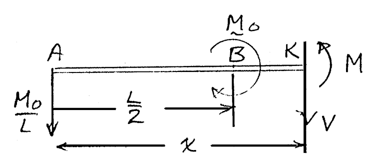


$$\uparrow \Sigma F_y = 0: -\frac{M_0}{L} - V = 0 \qquad V = -\frac{M_0}{L}$$

$$\circlearrowleft \Sigma M_J = 0:\ x\frac{M_0}{L} + M = 0 \qquad M = -\frac{M_0}{L}x$$

$$\text{straight with } M = -\frac{M_0}{2} \text{ at } B$$

Along BC:



$$\uparrow \Sigma F_y = 0: \quad -\frac{M_0}{L} - V = 0 \qquad V = -\frac{M_0}{L}$$

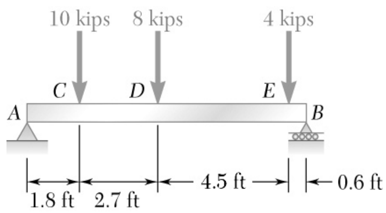
$$\circlearrowleft \Sigma M_K = 0:\ M + x\frac{M_0}{L} - M_0 = 0 \qquad M = M_0\left(1 - \frac{x}{L}\right)$$

$$\text{straight with } M = \frac{M_0}{2} \text{ at } B \qquad M = 0 \text{ at } C$$

(b) From diagrams:

$$|V|_{\max} = P \text{ everywhere} \blacktriangleleft$$

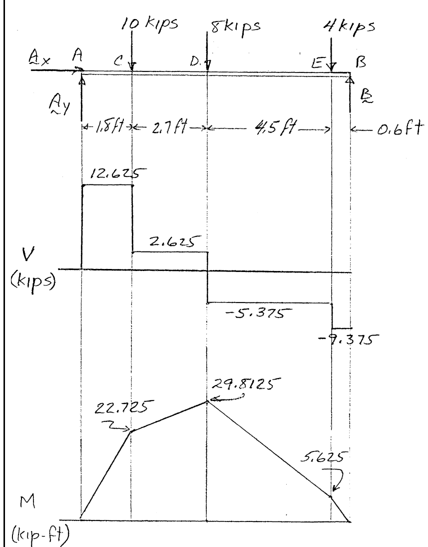
$$|M|_{\max} = \frac{M_0}{2} \text{ at } B \blacktriangleleft$$



PROBLEM 7.33

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION



(a) **FBD Beam:**

$$\circlearrowleft \Sigma M_B = 0:$$

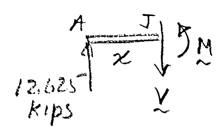
$$(.6\text{ ft})(4\text{ kips}) + (5.1\text{ ft})(8\text{ kips}) + (7.8\text{ ft})(10\text{ kips}) - (9.6\text{ ft})A_y = 0$$

$$\mathbf{A}_y = 12.625\text{ kips} \uparrow$$

$$\uparrow \Sigma F_y = 0:\ \ 12.625\text{ kips} - 10\text{ kips} - 8\text{ kips} - 4\text{ kips} + B = 0$$

$$\mathbf{B} = 9.375\text{ kips} \uparrow$$

Along AC:



$$\uparrow \Sigma F_y = 0:\ \ 12.625\text{ kips} - V = 0$$

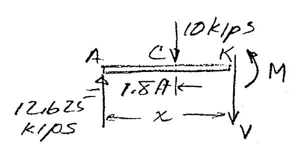
$$V = 12.625\text{ kips}$$

$$\circlearrowleft \Sigma M_J = 0:\ \ M - x(12.625\text{ kips}) = 0$$

$$M = (12.625\text{ kips})x$$

$$M = 22.725\text{ kip}\cdot\text{ft at } C$$

Along CD:



$$\uparrow \Sigma F_y = 0:\ \ 12.625\text{ kips} - 10\text{ kips} - V = 0$$

$$V = 2.625\text{ kips}$$

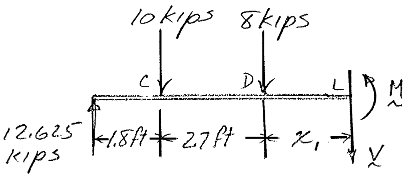
$$\circlearrowleft \Sigma M_K = 0:\ \ M + (x - 1.8\text{ ft})(10\text{ kips}) - x(12.625\text{ kips}) = 0$$

$$M = 18\text{ kip}\cdot\text{ft} + (2.625\text{ kips})\,x$$

$$M = 29.8125\text{ kip}\cdot\text{ft at } D\ (x = 4.5\text{ ft})$$

PROBLEM 7.33 CONTINUED

Along DE:



Along DE:

$$\uparrow \Sigma F_y = 0: (12.625 - 10 - 8) \text{ kips} - V = 0 \qquad V = -5.375 \text{ kips}$$

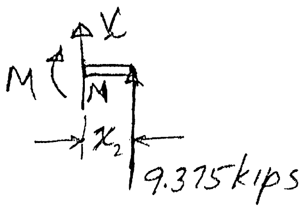
$$\circlearrowleft \Sigma M_L = 0: \; M + x_1(8 \text{ kips}) + (2.7 \text{ ft} + x_1)(10 \text{ kips})$$

$$- (4.5 \text{ ft} + x_1)(12.625 \text{ kips}) = 0$$

$$M = 29.8125 \text{ kip}\cdot\text{ft} - (5.375 \text{ kips})\, x_1$$

$$M = 5.625 \text{ kip}\cdot\text{ft at } E \quad (x_1 = 4.5 \text{ ft})$$

Along EB:



Along EB:

$$\uparrow \Sigma F_y = 0: V + 9.375 \text{ kips} = 0 \qquad V = 9.375 \text{ kips}$$

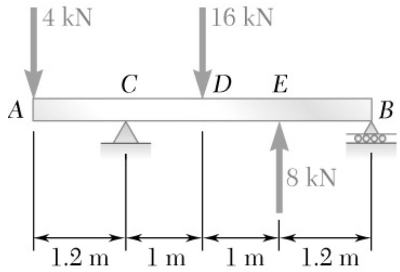
$$\circlearrowleft \Sigma M_N = 0: \; x_2(9.375 \text{ kip}) - M = 0$$

$$M = (9.375 \text{ kips})\, x_2$$

$$M = 5.625 \text{ kip}\cdot\text{ft at } E$$

(*b*) From diagrams: $|V|_{\max} = 12.63 \text{ kips on } AC$ ◀

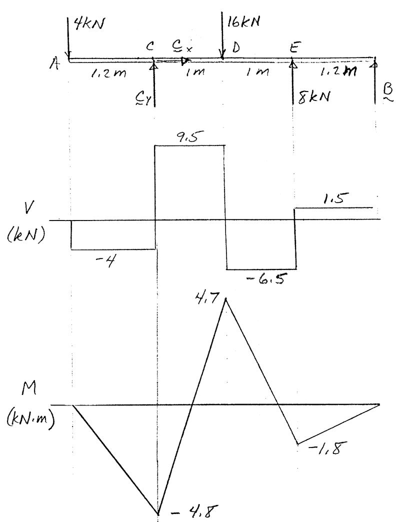
$|M|_{\max} = 29.8 \text{ kip}\cdot\text{ft at } D$ ◀



PROBLEM 7.34

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



(*a*) **FBD Beam:**

$\circlearrowleft \Sigma M_C = 0$:

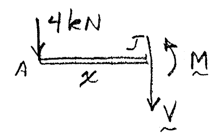
$$(1.2 \text{ m})(4 \text{ kN}) - (1 \text{ m})(16 \text{ kN}) + (2 \text{ m})(8 \text{ kN}) + (3.2 \text{ m})B = 0$$

$$\mathbf{B} = -1.5 \text{ kN} \uparrow$$

$$\uparrow \Sigma F_y = 0: -4 \text{ kN} + C_y - 16 \text{ kN} + 8 \text{ kN} - 1.5 \text{ kN} = 0$$

$$\mathbf{C}_y = 13.5 \text{ kN} \uparrow$$

Along AC:



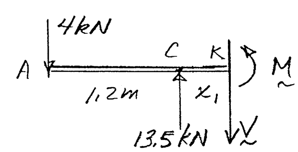
$$\uparrow \Sigma F_y = 0: -4 \text{ kN} - V = 0$$

$$V = -4 \text{ kN}$$

$$\circlearrowleft \Sigma M_J = 0: M + x(4 \text{ kN}) = 0 \qquad M = -4 \text{ kN}\, x$$

$$M = -4.8 \text{ kN}\cdot\text{m at } C$$

Along CD:



$$\uparrow \Sigma F_y = 0: -4 \text{ kN} + 13.5 \text{ kN} - V = 0$$

$$V = 9.5 \text{ kN}$$

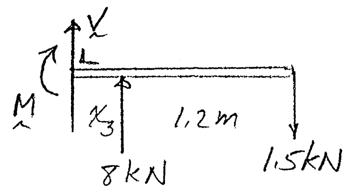
$$\circlearrowleft \Sigma M_K = 0: \; M + (1.2 \text{ m} + x_1)(4 \text{ kN}) - x_1(13.5 \text{ kN}) = 0$$

$$M = -4.8 \text{ kN}\cdot\text{m} + (9.5 \text{ kN})x_1$$

$$M = 4.7 \text{ kN}\cdot\text{m at } D \; (x_1 = 1 \text{ m})$$

PROBLEM 7.34 CONTINUED

Along DE:



$$\uparrow \Sigma F_y = 0: V + 8 \text{ kN} - 1.5 \text{ kN} = 0$$

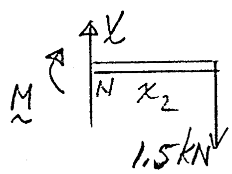
$$V = -6.5 \text{ kN}$$

$$\circlearrowleft \Sigma M_L = 0: M - x_3(8 \text{ kN}) + (x_3 + 1.2 \text{ m})(1.5 \text{ kN}) = 0$$

$$M = -1.8 \text{ kN}\cdot\text{m} + (6.5 \text{ kN})x_3$$

$$M = 4.7 \text{ kN}\cdot\text{m at } D \ (x_3 = 1 \text{ m})$$

Along EB:



$$\uparrow \Sigma F_y = 0: V - 1.5 \text{ kN} = 0$$

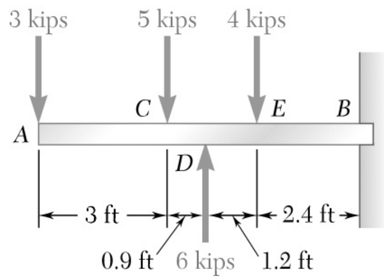
$$V = 1.5 \text{ kN}$$

$$\circlearrowleft \Sigma M_N = 0: \ x_2(1.5 \text{ kN}) + M = 0$$

$$M = -(1.5 \text{ kN})x_2 \qquad M = -1.8 \text{ kN}\cdot\text{m at } E$$

(*b*) From diagrams: $|V|_{\max} = 9.50 \text{ kN}\cdot on \ CD$ ◀

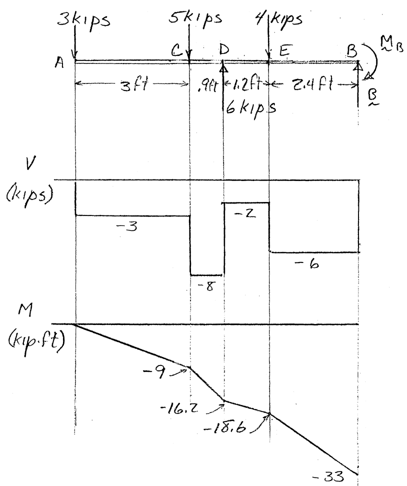
$|M|_{\max} = 4.80 \text{ kN}\cdot\text{m at } C$ ◀



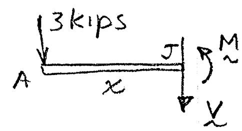
PROBLEM 7.35

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



(*a*) **Along AC:**

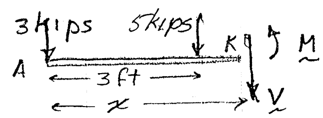


$$+\uparrow \Sigma F_y = 0: -3 \text{ kip} - V = 0 \qquad V = -3 \text{ kips}$$

$$\circlearrowleft \Sigma M_J = 0: \; M + x(3 \text{ kips}) = 0 \qquad M = (3 \text{ kips})x$$

$$M = -9 \text{ kip}\cdot\text{ft at } C$$

Along CD:



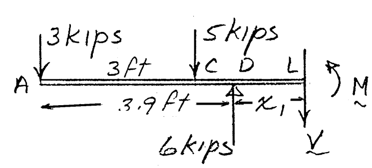
$$+\uparrow \Sigma F_y = 0: -3 \text{ kips} - 5 \text{ kips} - V = 0 \qquad V = -8 \text{ kips}$$

$$\circlearrowleft \Sigma M_K = 0: \; M + (x - 3 \text{ ft})(5 \text{ kips}) + x(3 \text{ kips}) = 0$$

$$M = +15 \text{ kip}\cdot\text{ft} - (8 \text{ kips})x$$

$$M = -16.2 \text{ kip}\cdot\text{ft at } D \; (x = 3.9 \text{ ft})$$

Along DE:



$$+\uparrow \Sigma F_y = 0: -3 \text{ kips} - 5 \text{ kips} + 6 \text{ kips} - V = 0$$

$$V = -2 \text{ kips}$$

$$\circlearrowleft \Sigma M_L = 0: \; M - x_1(6 \text{ kips}) + (.9 \text{ ft} + x_1)(5 \text{ kips})$$

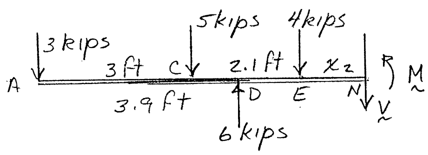
$$+ (3.9 \text{ ft} + x_1)(3 \text{ kips}) = 0$$

$$M = -16.2 \text{ kip}\cdot\text{ft} - (2 \text{ kips})x_1$$

$$M = -18.6 \text{ kip}\cdot\text{ft at } E \; (x_1 = 1.2 \text{ ft})$$

PROBLEM 7.35 CONTINUED

Along EB:



$+\uparrow \Sigma F_y = 0$: $-3\text{ kips} - 5\text{ kips} + 6\text{ kips} - 4\text{ kips} - V = 0 \qquad V = -6\text{ kips}$

$$+\circlearrowleft \Sigma M_N = 0:\ M + (4\text{ kips})x_2 + (2.1\text{ ft} + x_2)(5\text{ kips})$$

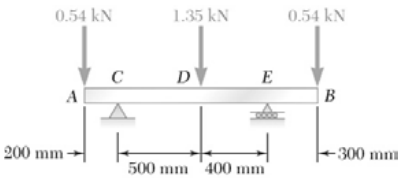
$$+(5.1\text{ ft} + x_2)(3\text{ kips}) - (1.2\text{ ft} + x_2)(6\text{ kips}) = 0$$

$$M = -18.6\text{ kip}\cdot\text{ft} - (6\text{ kips})x_2$$

$$M = -33\text{ kip}\cdot\text{ft at } B \quad (x_2 = 2.4\text{ ft})$$

(*b*) From diagrams: $|V|_{\max} = 8.00\text{ kips on } CD$ ◀

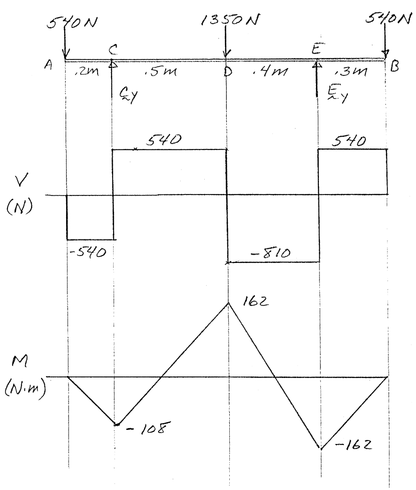
$|M|_{\max} = 33.0\text{ kip}\cdot\text{ft at } B$ ◀



PROBLEM 7.36

For the beam and loading shown, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION



(a) **FBD Beam:**

$$\circlearrowleft \Sigma M_E = 0:$$

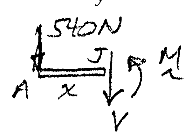
$$(1.1\text{ m})(540\text{ N}) - (0.9\text{ m})C_y + (0.4\text{ m})(1350\text{ N}) - (0.3\text{ m})(540\text{ N}) = 0$$

$$\mathbf{C}_y = 1080\text{ N}\uparrow$$

$$\uparrow \Sigma F_y = 0: -540\text{ N} + 1080\text{ N} - 1350\text{ N}$$

$$-540\text{ N} + E_y = 0 \qquad \mathbf{E}_y = 1350\text{ N}\uparrow$$

Along AC:

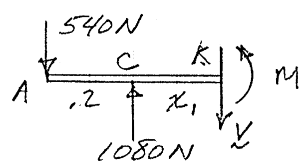


$$\uparrow \Sigma F_y = 0: -540\text{ N} - V = 0$$

$$V = -540\text{ N}$$

$$\circlearrowleft \Sigma M_J = 0: \; x(540\text{ N}) + M = 0 \qquad M = -(540\text{ N})x$$

Along CD:



$$\uparrow \Sigma F_y = 0: -540\text{ N} + 1080\text{ N} - V = 0 \qquad V = 540\text{ N}$$

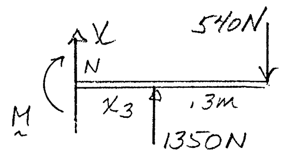
$$\circlearrowleft \Sigma M_K = 0: \; M + (0.2\text{ m} + x_1)(540\text{ N}) - x_1(1080\text{ N}) = 0$$

$$M = -108\text{ N}\cdot\text{m} + (540\text{ N})x_1$$

$$M = 162\text{ N}\cdot\text{m at } D \; (x_1 = 0.5\text{ m})$$

PROBLEM 7.36 CONTINUED

Along DE:



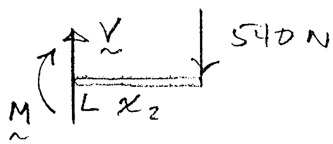
$\uparrow \Sigma F_y = 0: \quad V + 1350\text{ N} - 540\text{ N} = 0 \qquad V = -810\text{ N}$

$\circlearrowleft \Sigma M_N = 0: \ M + (x_3 + 0.3\text{ m})(540\text{ N}) - x_3(1350\text{ N}) = 0$

$M = -162\text{ N}\cdot\text{m} + (810\text{ N})x_3$

$M = 162\text{ N}\cdot\text{m} \ \text{at}\ D\ (x_3 = 0.4)$

Along EB:



$\uparrow \Sigma F_y = 0: V - 540\text{ N} = 0 \qquad V = 540\text{ N}$

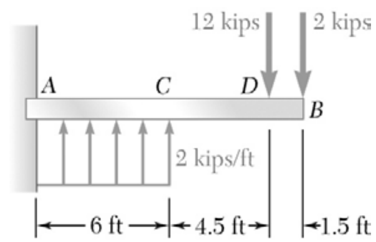
$\circlearrowleft \Sigma M_L = 0: M + x_2(540\text{ N}) = 0 \qquad M = -540\text{ N}\,x_2$

$M = -162\text{ N}\cdot\text{m at } E \quad (x_2 = 0.3\text{ m})$

(*b*) From diagrams $|V|_{max} = 810\text{ N on } DE$ ◀

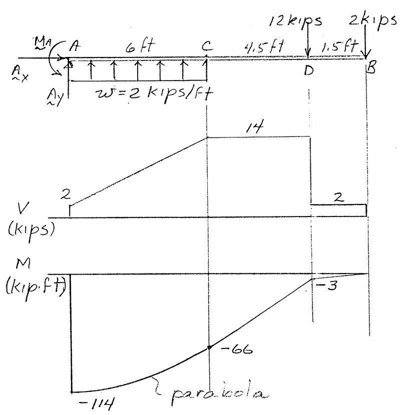
$|M|_{max} = 162.0\text{ N}\cdot\text{m at } D \text{ and } E$ ◀

PROBLEM 7.37



For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



(*a*) **FBD Beam:**

$$+\uparrow \Sigma F_y = 0: A_y + (6 \text{ ft})(2 \text{ kips/ft}) - 12 \text{ kips} - 2 \text{ kips} = 0$$

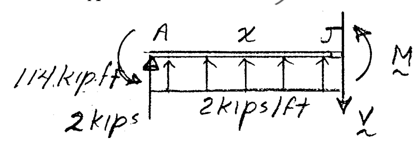
$$\mathbf{A}_y = 2 \text{ kips} \uparrow$$

$$\circlearrowleft \Sigma M_A = 0: M_A + (3 \text{ ft})(6 \text{ ft})(2 \text{ kips/ft})$$

$$- (10.5 \text{ ft})(12 \text{ kips}) - (12 \text{ ft})(2 \text{ kips}) = 0$$

$$\mathbf{M}_A = 114 \text{ kip}\cdot\text{ft} \circlearrowleft$$

Along AC:



$$+\uparrow \Sigma F_y = 0: 2 \text{ kips} + x(2 \text{ kips/ft}) - V = 0$$

$$V = 2 \text{ kips} + (2 \text{ kips/ft})\, x$$

$$V = 14 \text{ kips at } C \;(x = 6 \text{ ft})$$

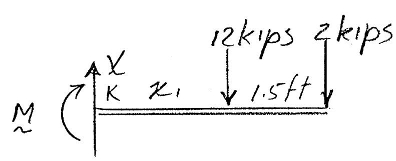
$$\circlearrowleft \Sigma M_J = 0: 114 \text{ kip}\cdot\text{ft} - x(2 \text{ kips})$$

$$- \frac{x}{2} x(2 \text{ kips/ft}) + M = 0$$

$$M = (1 \text{ kip/ft})x^2 + (2 \text{ kips})x - 114 \text{ kip}\cdot\text{ft}$$

$$M = -66 \text{ kip}\cdot\text{ft at } C \;(x = 6 \text{ ft})$$

Along CD:



$$+\uparrow \Sigma F_y = 0: V - 12 \text{ kips} - 2 \text{ kips} = 0 \qquad V = 14 \text{ kips}$$

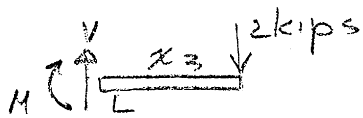
$$\circlearrowleft \Sigma M_k = 0: -M - x_1(12 \text{ kips}) - (1.5 \text{ ft} + x_1)(2 \text{ kips}) = 0$$

PROBLEM 7.37 CONTINUED

$$M = -3 \text{ kip}\cdot\text{ft} - (14 \text{ kips})x_1$$

$$M = -66 \text{ kip}\cdot\text{ft at } C \quad (x_1 = 4.5 \text{ ft})$$

Along DB:



$$\uparrow \Sigma F_y = 0: \quad V - 2 \text{ kips} = 0 \qquad V = +2 \text{ kips}$$

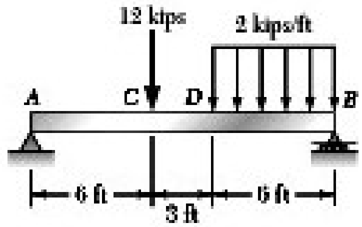
$$\circlearrowleft \Sigma M_L = 0: -M - 2 \text{ kip } x_3 = 0$$

$$M = -(2 \text{ kips})x_3$$

$$M = -3 \text{ kip}\cdot\text{ft at } D \quad (x = 1.5 \text{ ft})$$

(b) From diagrams: $|V|_{max} = 14.00$ kips on CD ◀

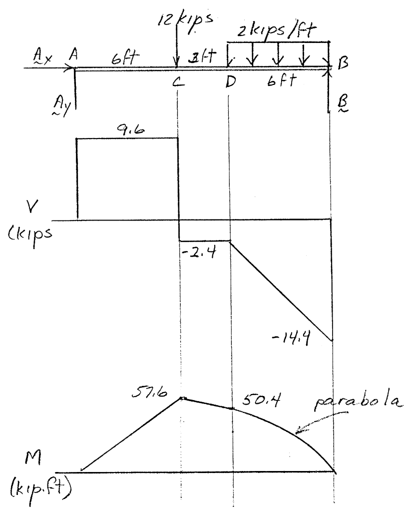
$|M|_{max} = 114.0$ kip·ft at A ◀



PROBLEM 7.38

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



(*a*) **FBD Beam:**

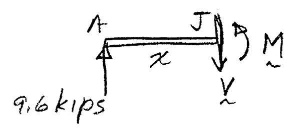
$$\Sigma M_A = (15 \text{ ft})B - (12 \text{ ft})(2 \text{ kips/ft})(6 \text{ ft}) - (6 \text{ ft})(12 \text{ kips}) = 0$$

$$\mathbf{B} = 14.4 \text{ kips} \uparrow$$

$$\uparrow \Sigma F_y = 0\text{: } A_y - 12 \text{ kips} - (2 \text{ kips/ft})(6 \text{ ft}) + 14.4 \text{ kips}$$

$$\mathbf{A}_y = 9.6 \text{ kips} \uparrow$$

Along AC:



$$\uparrow \Sigma F_y = 0\text{: } 9.6 \text{ kips} - V = 0$$

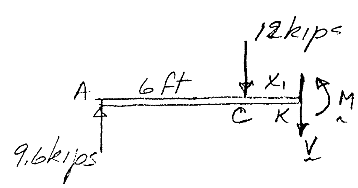
$$V = 9.6 \text{ kips}$$

$$\Sigma M_J = 0\text{: } M - x(9.6 \text{ kips}) = 0$$

$$M = (9.6 \text{ kips})x$$

$$M = 57.6 \text{ kip}\cdot\text{ft at } C \ (x = 6 \text{ ft})$$

Along CD:



$$\uparrow \Sigma F_y = 0\text{: } 9.6 \text{ kips} - 12 \text{ kips} - V = 0$$

$$V = -2.4 \text{ kips}$$

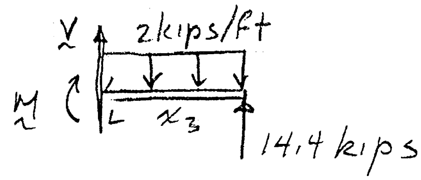
$$\Sigma M_K = 0\text{: } M + x_1(12 \text{ kips}) - (6 \text{ ft} + x_1)(9.6 \text{ kips}) = 0$$

$$M = 57.6 \text{ kip}\cdot\text{ft} - (2.4 \text{ kips})x_1$$

$$M = 50.4 \text{ kip}\cdot\text{ft at } D$$

PROBLEM 7.38 CONTINUED

Along DB:



$$\Sigma F_y = 0: \; V - x_3(2 \text{ kips/ft}) + 14.4 \text{ kips} = 0$$

$$V = -14.4 \text{ kips} + (2 \text{ kips/ft})x_3$$

$$V = -2.4 \text{ kips at } D$$

$$\Sigma M_L = 0: M + \frac{x_3}{2}(2 \text{ kips/ft})(x_3) - x_3(14.4 \text{ kips}) = 0$$

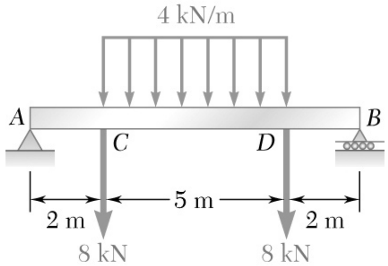
$$M = (14.4 \text{ kips})x_3 - (1 \text{ kip/ft})x_3^2$$

$$M = 50.4 \text{ kip}\cdot\text{ft at } D \; (x_3 = 6 \text{ ft})$$

(b) From diagrams: $|V|_{max} = 14.40$ kips at B ◀

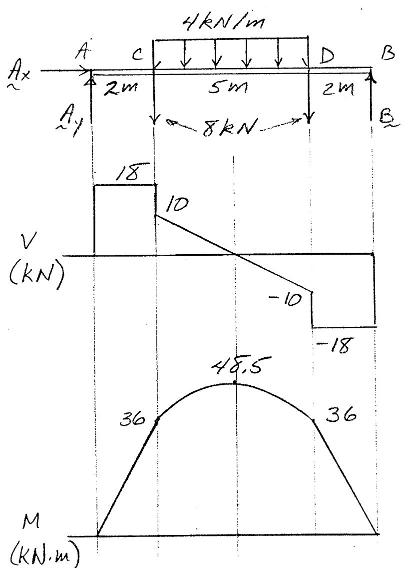
$|M|_{max} = 57.6$ kip·ft at C ◀

PROBLEM 7.39



For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

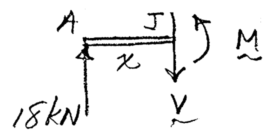
SOLUTION



(*a*) By symmetry:

$$A_y = B = 8\text{ kN} + \frac{1}{2}(4\text{ kN/m})(5\text{ m}) \qquad \mathbf{A}_y = \mathbf{B} = 18\text{ kN}\uparrow$$

Along AC:

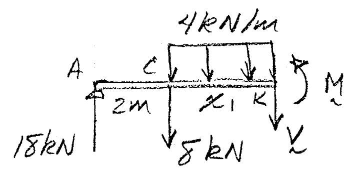


$$+\uparrow \Sigma F_y = 0{:}\ 18\text{ kN} - V = 0 \qquad V = 18\text{ kN}$$

$$\circlearrowleft \Sigma M_J = 0{:}\ M - x(18\text{ kN}) \qquad M = (18\text{ kN})x$$

$$M = 36\text{ kN}\cdot\text{m at } C\ (x = 2\text{ m})$$

Along CD:



$$+\uparrow \Sigma F_y = 0{:}\ 18\text{ kN} - 8\text{ kN} - (4\text{ kN/m})x_1 - V = 0$$

$$V = 10\ \text{kN} - (4\text{ kN/m})x_1$$

$$V = 0 \text{ at } x_1 = 2.5\text{ m}(\text{at center})$$

$$\circlearrowleft \Sigma M_K = 0{:}\ M + \frac{x_1}{2}(4\text{ kN/m})x_1 + (8\text{ kN})x_1 - (2\text{ m} + x_1)(18\text{ kN}) = 0$$

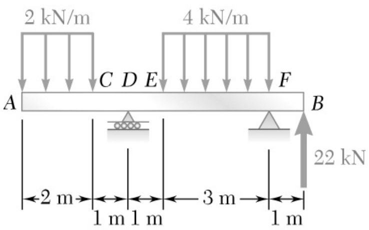
$$M = 36\text{ kN}\cdot\text{m} + (10\text{ kN/m})x_1 - (2\text{ kN/m})x_1^2$$

$$M = 48.5\text{ kN}\cdot\text{m at } x_1 = 2.5\text{ m}$$

Complete diagram by symmetry

(*b*) From diagrams: $|V|_{\max} = 18.00\text{ kN on } AC \text{ and } DB$ ◀

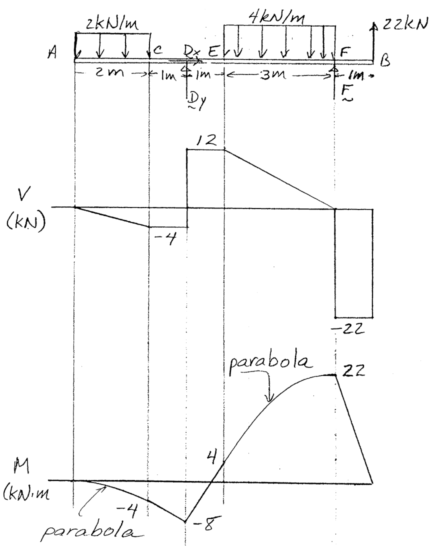
$|M|_{\max} = 48.5\text{ kN}\cdot\text{m at center}$ ◀



PROBLEM 7.40

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



(*a*) $\circlearrowleft \Sigma M_D = 0\text{: } (2\text{ m})(2\text{ kN/m})(2\text{ m}) - (2.5\text{ m})(4\text{ kN/m})(3\text{ m})$

$$-(4\text{ m})F - (5\text{ m})(22\text{ kN}) = 0$$

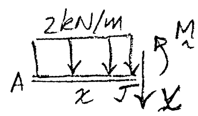
$$\mathbf{F} = 22\text{ kN} \downarrow$$

$$\uparrow \Sigma F_y = 0\text{: } -(2\text{ m})(2\text{ kN/m}) + D_y$$

$$-(3\text{ m})(4\text{ kN/m}) - 22\text{ kN} + 22\text{ kN} = 0$$

$$\mathbf{D}_y = 16\text{ kN} \uparrow$$

Along AC:



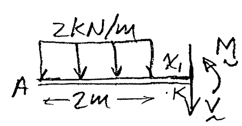
$$\uparrow \Sigma F_y = 0\text{: } -x(2\text{ kN/m}) - V = 0$$

$$V = -(2\text{ kN/m})x \qquad V = -4\text{ kN at } C$$

$$\circlearrowleft \Sigma M_J = 0\text{: } M + \frac{x}{2}(2\text{ kN/m})(x) \neq 0$$

$$M = -(1\text{ kN/m})x^2 \qquad M = -4\text{ kN}\cdot\text{m at } C$$

Along CD:



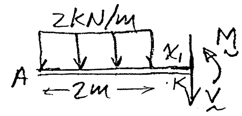
$$\uparrow \Sigma F_y = 0\text{: } -(2\text{ m})(2\text{ kN/m}) - V = 0 \qquad V = -4\text{ kN}$$

$$\circlearrowleft \Sigma M_K = 0\text{: } (1\text{ m} + x_1)(2\text{ kN/m})(2\text{ m}) = 0$$

$$M = -4\text{ kN}\cdot\text{m} - (4\text{ kN/m})x_1 \qquad M = -8\text{ kN}\cdot\text{m at } D$$

PROBLEM 7.40 CONTINUED

Along DE:

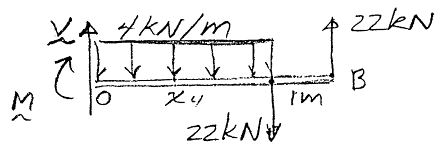


$$\uparrow \Sigma F_y = 0: -(2\ \text{kN/m})(2\ \text{m}) + 16\ \text{kN} - V = 0 \qquad V = 12\ \text{kN}$$

$$\circlearrowleft \Sigma M_L = 0: M - x_2(16\ \text{kN}) + (x_2 + 2\ \text{m})(2\ \text{kN/m})(2\ \text{m}) = 0$$

$$M = -8\ \text{kN}\cdot\text{m} + (12\ \text{kN})x_2 \qquad M = 4\ \text{kN}\cdot\text{m at } E$$

Along EF:



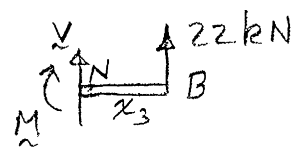
$$\uparrow \Sigma F_y = 0: V - x_4(4\ \text{kN/m}) - 22\ \text{kN} + 22\ \text{kN} = 0$$

$$V = (4\ \text{kN/m})x_4 \qquad V = 12\ \text{kN at } E$$

$$\circlearrowleft \Sigma M_0 = 0: M + \frac{x_4}{2}(4\ \text{kN/m})x_4 - (1\ \text{m})(22\ \text{kN}) = 0$$

$$M = 22\ \text{kN}\cdot\text{m} - (2\ \text{kN/m})x_4^2 \qquad M = 4\ \text{kN}\cdot\text{m at } E$$

Along FB:



$$\uparrow \Sigma F_y = 0: V + 22\ \text{kN} = 0 \qquad V = 22\ \text{kN}$$

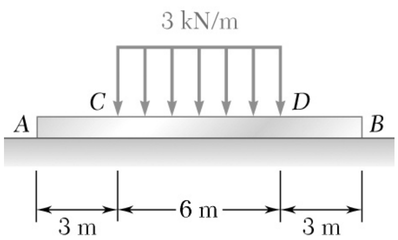
$$\circlearrowleft \Sigma M_N = 0: M - x_3(22\ \text{kN}) = 0$$

$$M = (22\ \text{kN})x_3$$

$$M = 22\ \text{kN}\cdot\text{m at } F$$

(*b*) From diagrams: $|V|_{max} = 22.0\ \text{kN on } FB$ ◀

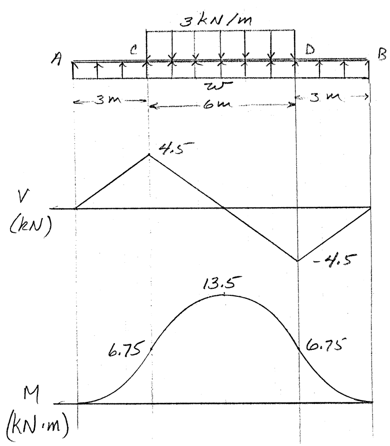
$|M|_{max} = 22.0\ \text{kN}\cdot\text{m at } F$ ◀



PROBLEM 7.41

Assuming the upward reaction of the ground on beam *AB* to be uniformly distributed, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

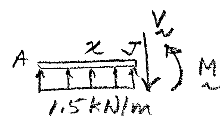
SOLUTION



(*a*) $\uparrow \Sigma F_y = 0: (12 \text{ m})w - (6 \text{ m})(3 \text{ kN/m}) = 0$

$$w = 1.5 \text{ kN/m}$$

Along AC:



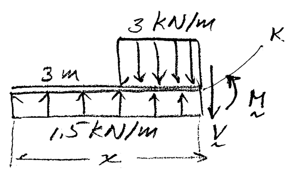
$$\uparrow \Sigma F_y = 0: x(1.5 \text{ kN/m}) - V = 0 \qquad V = (1.5 \text{ kN/m})x$$

$$V = 4.5 \text{ kN at } C$$

$$\circlearrowleft \Sigma M_J = 0: M - \frac{x}{2}(1.5 \text{ kN/m})(x) = 0$$

$$M = (0.75 \text{ kN/m})x^2 \qquad M = 6.75 \text{ N}\cdot\text{m at } C$$

Along CD:



$$\uparrow \Sigma F_y = 0: x(1.5 \text{ kN/m}) - (x - 3 \text{ m})(3 \text{ kN/m}) - V = 0$$

$$V = 9 \text{ kN} - (1.5 \text{ kN/m})x \qquad V = 0 \text{ at } x = 6 \text{ m}$$

$$\circlearrowleft \Sigma M_K = 0: M + \left(\frac{x - 3 \text{ m}}{2}\right)(3 \text{ kN/m})(x - 3 \text{ m}) - \frac{x}{2}(1.5 \text{ kN/m})x = 0$$

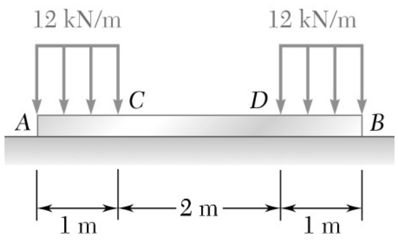
$$M = -13.5 \text{ kN}\cdot\text{m} + (9 \text{ kN})x - (0.75 \text{ kN/m})x^2$$

$$M = 13.5 \text{ kN}\cdot\text{m at center } (x = 6 \text{ m})$$

Finish by symmetry

(*b*) From diagrams: $|V|_{\max} = 4.50 \text{ kN at } C \text{ and } D$ ◀

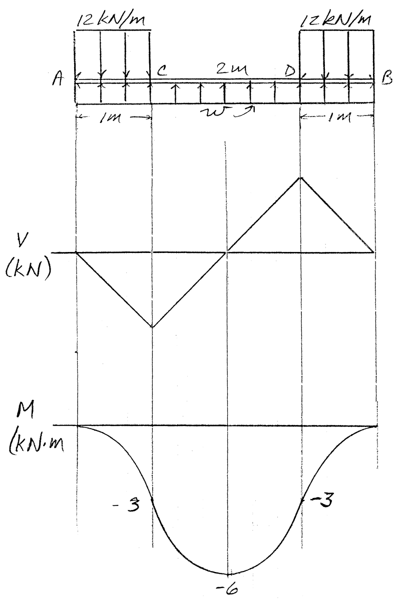
$|M|_{\max} = 13.50 \text{ kN}\cdot\text{m at center}$ ◀



PROBLEM 7.42

Assuming the upward reaction of the ground on beam AB to be uniformly distributed, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

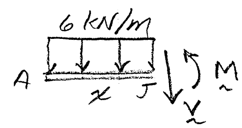


(a) **FBD Beam:**

$$\uparrow \Sigma F_y = 0: (4 \text{ m})(w) - (2 \text{ m})(12 \text{ kN/m}) = 0$$

$$w = 6 \text{ kN/m}$$

Along AC:



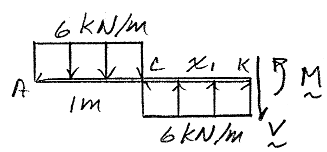
$$\uparrow \Sigma F_y = 0: -x(6 \text{ kN/m}) - V = 0 \qquad V = -(6 \text{ kN/m})x$$

$$V = -6 \text{ kN at } C(x = 1 \text{ m})$$

$$\circlearrowleft \Sigma M_J = 0: M + \frac{x}{2}(6 \text{ kN/m})(x) = 0$$

$$M = -(3 \text{ kN/m})x^2 \qquad M = -3 \text{ kN}\cdot\text{m at } C$$

Along CD:



$$\uparrow \Sigma F_y = 0: -(1 \text{ m})(6 \text{ kN/m}) + x_1(6 \text{ kN/m}) - v = 0$$

$$V = (6 \text{ kN/m})(1 \text{ m} - x_1) \qquad V = 0 \text{ at } x_1 = 1 \text{ m}$$

$$\circlearrowleft \Sigma M_K = 0: M + (0.5 \text{ m} + x_1)(6 \text{ kN/m})(1 \text{ m}) - \frac{x_1}{2}(6 \text{ kN/m})x_1 = 0$$

$$M = -3 \text{ kN}\cdot\text{m} - (6 \text{ kN})x_1 + (3 \text{ kN/m})x_1^2$$

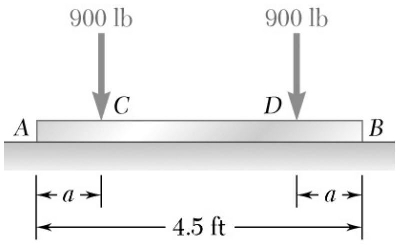
$$M = -6 \text{ kN}\cdot\text{m at center } (x_1 = 1 \text{ m})$$

Finish by symmetry

(b) From diagrams:

$$|V|_{\max} = 6.00 \text{ kN at } C \text{ and } D \blacktriangleleft$$

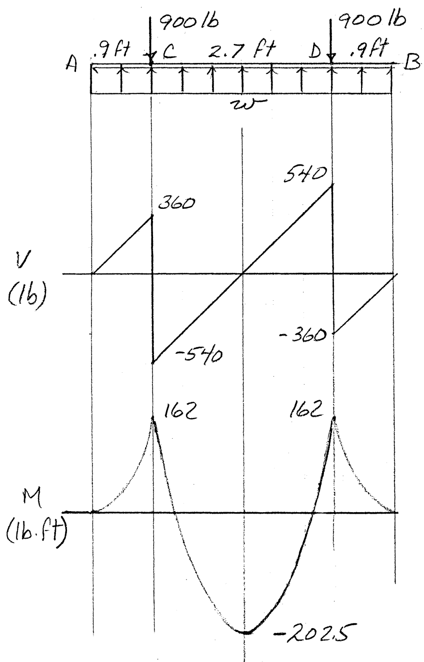
$$|M|_{\max} = 6.00 \text{ kN at center} \blacktriangleleft$$



PROBLEM 7.43

Assuming the upward reaction of the ground on beam AB to be uniformly distributed and knowing that $a = 0.9$ ft, (a) draw the shear and bending-moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

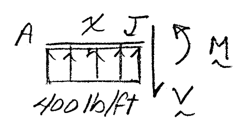


(a) **FBD Beam:**

$$+\uparrow \Sigma F_y = 0: (4.5\text{ ft})w - 900\text{ lb} - 900\text{ lb} = 0$$

$$w = 400\text{ lb/ft}$$

Along AC:



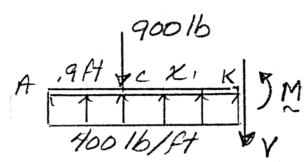
$$+\uparrow \Sigma F_y = 0: x(400\text{ lb}) - V = 0 \qquad V = (400\text{ lb})x$$

$$V = 360\text{ lb at } C \quad (x = 0.9\text{ ft})$$

$$\Sigma M_J = 0: M - \frac{x}{2}(400\text{ lb/ft})x = 0$$

$$M = (200\text{ lb/ft})x^2 \qquad M = 162\text{ lb}\cdot\text{ft at } C$$

Along CD:



$$+\uparrow \Sigma F_y = 0: (0.9\text{ ft} + x_1)(400\text{ lb/ft}) - 900\text{ lb} - V = 0$$

$$V = -540\text{ lb} + (400\text{ lb/ft})x_1 \qquad V = 0 \text{ at } x_1 = 1.35\text{ ft}$$

$$\Sigma M_K = 0: M + x_1(900\text{ lb}) - \frac{0.9\text{ ft} + x_1}{2}(400\text{ lb/ft})(0.9\text{ ft} + x_1) = 0$$

$$M = 162\text{ lb}\cdot\text{ft} - (540\text{ lb})x_1 + (200\text{ lb/ft})x_1^2$$

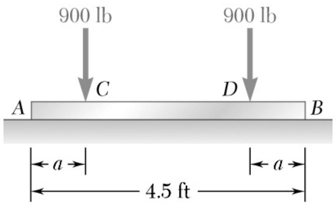
$$M = -202.5\text{ lb}\cdot\text{ft at center} \quad (x_1 = 1.35\text{ ft})$$

Finish by symmetry

(b) From diagrams:

$$|V|_{max} = 540\text{ lb at } C^+ \text{ and } D^- \blacktriangleleft$$

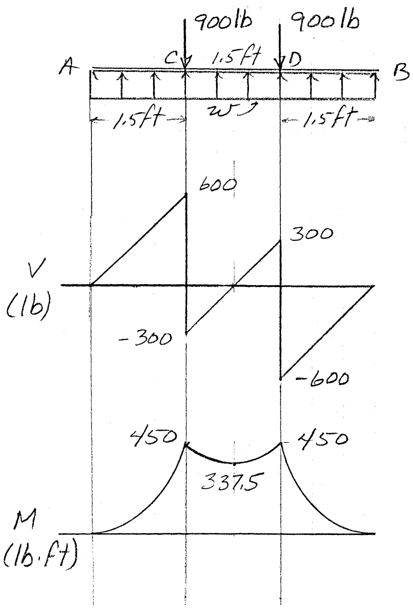
$$|M|_{max} = 203\text{ lb}\cdot\text{ft at center} \blacktriangleleft$$



PROBLEM 7.44

Solve Prob. 7.43 assuming that $a = 1.5$ ft.

SOLUTION

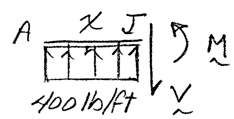


(*a*) **FBD Beam:**

$$+\uparrow \Sigma F_y = 0: (4.5\text{ ft})w - 900\text{ lb} - 900\text{ lb} = 0$$

$$w = 400\text{ lb/ft}$$

Along AC:



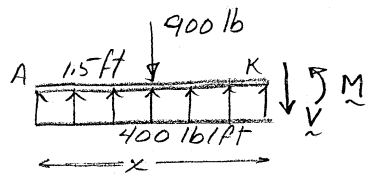
$$+\uparrow \Sigma F_y = 0: \qquad x(400\text{ lb/ft}) - V = 0$$

$$V = (400\text{ lb/ft})x \qquad V = 600\text{ lb at } C \quad (x = 1.5\text{ ft})$$

$$\circlearrowleft \Sigma M_J = 0: M - \frac{x}{2}(400\text{ lb/ft})x = 0$$

$$M = (200\text{ lb/ft})x^2 \qquad M = 450\text{ lb}\cdot\text{ft at } C$$

Along CD:



$$+\uparrow \Sigma F_y = 0: x(400\text{ lb/ft}) - 900\text{ lb} - V = 0$$

$$V = -900\text{ lb} + (400\text{ lb/ft})x \qquad V = -300 \text{ at } x = 1.5\text{ ft}$$

$$V = 0 \text{ at } x = 2.25\text{ ft}$$

$$\circlearrowleft \Sigma M_K = 0: M + (x - 1.5\text{ ft})(900\text{ lb}) - \frac{x}{2}(400\text{ lb/ft})x = 0$$

$$M = 1350\text{ lb}\cdot\text{ft} - (900\text{ lb})x + (200\text{ lb/ft})x^2$$

$$M = 450\text{ lb}\cdot\text{ft at } x = 1.5\text{ ft}$$

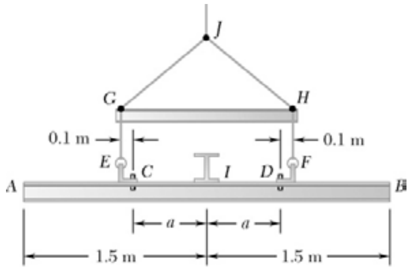
$$M = 337.5\text{ lb}\cdot\text{ft at } x = 2.25\text{ ft}(\text{center})$$

Finish by symmetry

(*b*) From diagrams:

$$|V|_{\max} = 600\text{ lb at } C^- \text{ and } D^+ \blacktriangleleft$$

$$|M|_{\max} = 450\text{ lb}\cdot\text{ft at } C \text{ and } D \blacktriangleleft$$

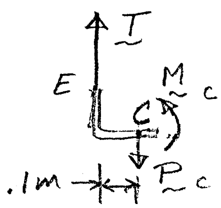


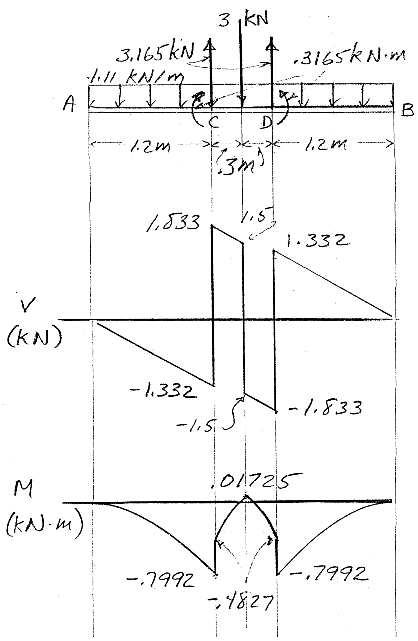
PROBLEM 7.45

Two short angle sections *CE* and *DF* are bolted to the uniform beam *AB* of weight 3.33 kN, and the assembly is temporarily supported by the vertical cables *EG* and *FH* as shown. A second beam resting on beam *AB* at *I* exerts a downward force of 3 kN on *AB*. Knowing that $a = 0.3$ m and neglecting the weight of the ngle sections, (*a*) draw the shear and bending-moment diagrams for beam *AB*, (*b*) determine the maximum absolute values of the shear and bending moment in the beam.

SOLUTION

FBD angle CE:





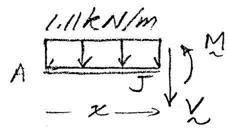
(*a*) By symmetry: $$T = \frac{3.33\text{ kN} + 3\text{ kN}}{2} = 3.165\text{ kN}$$

$$+\uparrow \Sigma F_y = 0: T - P_C = 0 \qquad P_C = T = 3.165\text{ kN}$$

$$\Sigma M_C = 0: M_C - (0.1\text{ m})(3.165\text{ kN}) = 0 \qquad M_C = 0.3165\text{ kN}\cdot\text{m}$$

By symmetry: $$P_D = 3.165\text{ kN}; M_D = 0.3165\text{ kN}\cdot\text{m}$$

Along AC:



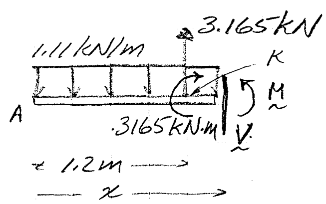
$$+\uparrow \Sigma F_y = 0: -x(1.11\text{ kN/m}) - V = 0$$

$$V = -(1.11\text{ kN/m})x \qquad V = -1.332\text{ kN at } C \quad (x = 1.2\text{ m})$$

$$\Sigma M_J = 0: M + \frac{x}{2}(1.11\text{ kN/m})x = 0$$

$$M = (0.555\text{ kN/m})x^2 \qquad M = -0.7992\text{ kN}\cdot\text{m at } C$$

Along CI:



$$+\uparrow \Sigma F_y = 0: -(1.11\text{ kN/m})x + 3.165\text{ kN} - V = 0$$

$$V = 3.165\text{ kN} - (1.11\text{ kN/m})x \qquad V = 1.5\text{ kN at } I \quad (x = 1.5\text{ m})$$

$$\Sigma M_k = 0:$$

$$M + (1.11\text{ kN/m})x - (x - 1.2\text{ m})(3.165\text{ kN}) - (0.3165\text{ kN}\cdot\text{m}) = 0$$

PROBLEM 7.45 CONTINUED

$$M = 3.4815\ \text{kN}\cdot\text{m} + \left(3.165\ \text{kN}\right)x - \left(0.555\ \text{kN/m}\right)x^2$$

$$M = -0.4827\ \text{kN}\cdot\text{m at } C \qquad M = 0.01725\ \text{kN}\cdot\text{m at } I$$

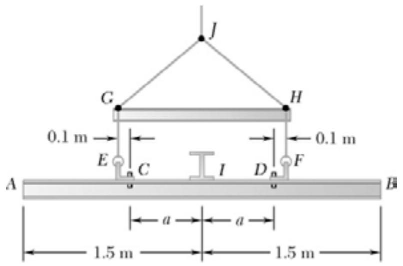
Note: At I, the downward 3 kN force will reduce the shear V by 3 kN, from +1.5 kN to –1.5 kN, with no change in M. From I to B, the diagram can be completed by symmetry.

(b) From diagrams:

$$|V|_{\max} = 1.833\ \text{kN at } C \text{ and } D \blacktriangleleft$$

$$|M|_{\max} = 799\ \text{N}\cdot\text{m at } C \text{ and } D \blacktriangleleft$$

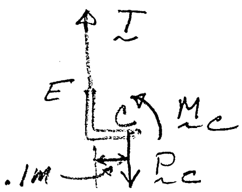
PROBLEM 7.46

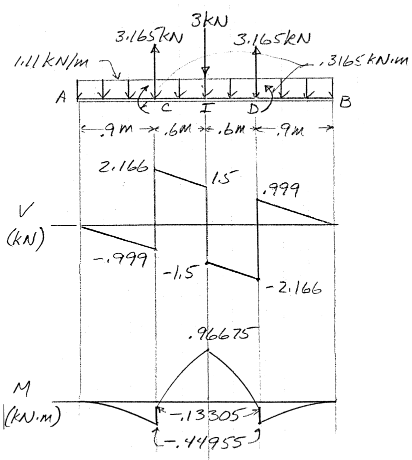


Solve Prob. 7.45 when $a = 0.6$ m.

SOLUTION

FBD angle CE:





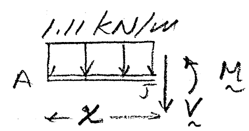
(a) By symmetry: $$T = \frac{3.33 \text{ kN} + 3 \text{ kN}}{2} = 3.165 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0: T - P_C = 0 \qquad P_C = T = 3.165 \text{ kN}$$

$$\circlearrowleft \Sigma M_C = 0: M_C - (0.1 \text{ m})(3.165 \text{ kN}) = 0 \qquad M_C = 0.3165 \text{ kN}\cdot\text{m}$$

By symmetry: $P_D = 3.165$ kN $\qquad M_D = 0.3165$ kN·m

Along AC:



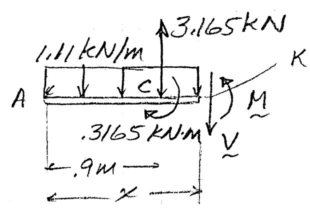
$$+\uparrow \Sigma F_y = 0: -(1.11 \text{ kN/m})x - V = 0$$

$$V = -(1.11 \text{ kN/m})x \qquad V = -0.999 \text{ kN at } C \quad (x = 0.9 \text{ m})$$

$$\circlearrowleft \Sigma M_J = 0: M + \frac{x}{2}(1.11 \text{ kN/m})x = 0$$

$$M = -(0.555 \text{ kN/m})x^2 \qquad M = -0.44955 \text{ kN}\cdot\text{m at } C$$

Along CI:



$$+\uparrow \Sigma F_y = 0: -x(1.11 \text{ kN/m}) + 3.165 \text{ kN} - V = 0$$

$$V = 3.165 \text{ kN} - (1.11 \text{ kN/m})x \qquad V = 2.166 \text{ kN at } C$$

$$V = 1.5 \text{ kN at } I \quad (x = 1.5 \text{ m})$$

$$\circlearrowleft \Sigma M_K = 0:$$

$$M - 0.3165 \text{ kN}\cdot\text{m} + (x - 0.9 \text{ m})(3.165 \text{ kN}) + \frac{x}{2}(1.11 \text{ kN/m})x = 0$$

PROBLEM 7.46 CONTINUED

$$M = -2.532\ \text{kN}\cdot\text{m} + (3.165\ \text{kN})x - (0.555\ \text{kN/m})x^2$$

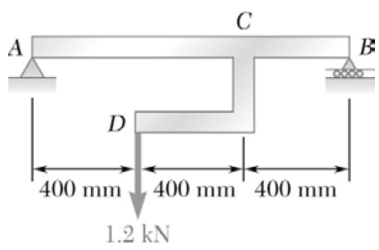
$$M = -\,0.13305\ \text{kN}\cdot\text{m at } C \qquad M = 0.96675\ \text{kN}\cdot\text{m at } I$$

Note: At *I*, the downward 3 kN force will reduce the shear *V* by 3 kN, from +1.5 kN to –1.5 kN, with no change in *M*. From *I* to *B*, the diagram can be completed by symmetry.

(*b*) From diagrams: $|V|_{\max} = 2.17$ kN at *C* and *D* ◀

$|M|_{\max} = 967$ N·m at *I* ◀

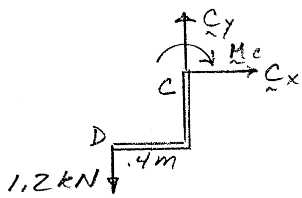
PROBLEM 7.47



Draw the shear and bending-moment diagrams for the beam *AB*, and determine the shear and bending moment (*a*) just to the left of *C*, (*b*) just to the right of *C*.

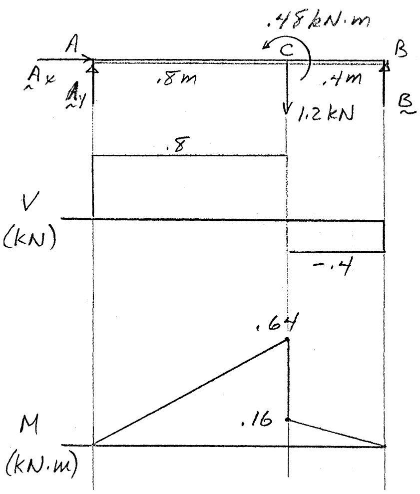
SOLUTION

FBD CD:



$$\uparrow \Sigma F_y = 0: -1.2 \text{ kN} + C_y = 0 \qquad \mathbf{C}_y = 1.2 \text{ kN}\uparrow$$

$$\circlearrowleft \Sigma M_C = 0: (0.4 \text{ m})(1.2 \text{ kN}) - M_C = 0 \qquad M_C = 0.48 \text{ kN}\cdot\text{m}$$



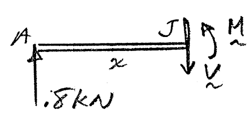
FBD Beam:

$$\circlearrowleft \Sigma M_A = 0: (1.2 \text{ m})B + 0.48 \text{ kN}\cdot\text{m} - (0.8 \text{ m})(1.2 \text{ kN}) = 0$$

$$\mathbf{B} = 0.4 \text{ kN}\uparrow$$

$$\uparrow \Sigma F_y = 0: A_y - 1.2 \text{ kN} + 0.4 \text{ kN} = 0 \qquad \mathbf{A}_y = 0.8 \text{ kN}\uparrow$$

Along AC:

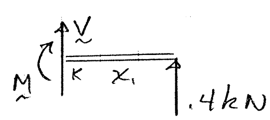


$$\uparrow \Sigma F_y = 0: 0.8 \text{ kN} - V = 0 \qquad V = 0.8 \text{ kN}$$

$$\circlearrowleft \Sigma M_J = 0: M - x(0.8 \text{ kN}) = 0 \qquad M = (0.8 \text{ kN})x$$

$$M = 0.64 \text{ kN}\cdot\text{m at } x = 0.8 \text{ m}$$

Along CB:



$$\uparrow \Sigma F_y = 0: \quad V + 0.4 \text{ kN} = 0 \qquad V = -0.4 \text{ kN}$$

$$\circlearrowleft \Sigma M_K = 0: x_1(0.4 \text{ kN}) - M = 0 \qquad M = (0.4 \text{ kN})x_1$$

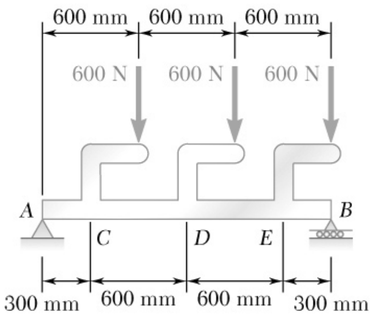
$$M = 0.16 \text{ kN}\cdot\text{m at } x_1 = 0.4 \text{ m}$$

(*a*) Just left of *C*: $V = 800 \text{ N}$ ◀

$M = 640 \text{ N}\cdot\text{m}$ ◀

(*b*) Just right of *C*: $V = -400 \text{ N}$ ◀

$M = 160.0 \text{ N}\cdot\text{m}$ ◀

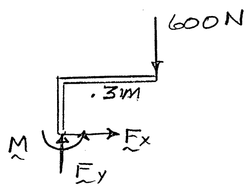


PROBLEM 7.48

Draw the shear and bending-moment diagrams for the beam *AB*, and determine the maximum absolute values of the shear and bending moment.

SOLUTION

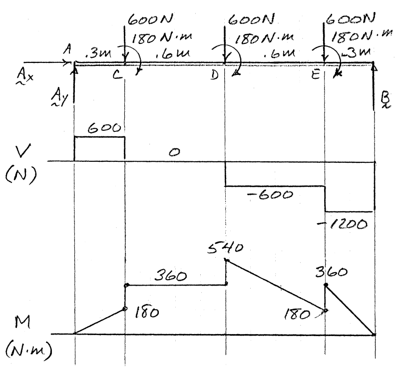
FBD angle:



$$\uparrow \Sigma F_y = 0{:}\ F_y - 600\text{ N} = 0 \qquad F_y = 600\text{ N}$$

$$\circlearrowleft \Sigma M_{\text{Base}} = 0{:}\quad M - (0.3\text{ m})(600\text{ N}) = 0 \qquad M = 180\text{ N}\cdot\text{m}$$

All three angles are the same.



FBD Beam:

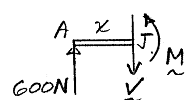
$$\circlearrowleft \Sigma M_A = 0{:}\ (1.8\text{ m})B - 3(180\text{ N}\cdot\text{m})$$

$$-(0.3\text{ m} + 0.9\text{ m} + 1.5\text{ m})(600\text{ N}) = 0$$

$$\mathbf{B} = 1200\text{ N} \uparrow$$

$$\uparrow \Sigma F_y = 0{:}\ A_y - 3(600\text{ N}) + 1200\text{ N} = 0 \qquad \mathbf{A}_y = 600\text{ N}\uparrow$$

Along AC:

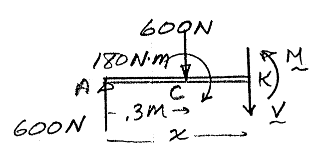


$$\uparrow \Sigma F_y = 0{:}\ 600\text{ N} - V = 0 \qquad V = 600\text{ N}$$

$$\circlearrowleft \Sigma M_J = 0{:}\ M - x(600\text{ N}) = 0$$

$$M = (600\text{ N})x = 180\text{ N}\cdot\text{m at } x = .3\text{ m}$$

Along CD:



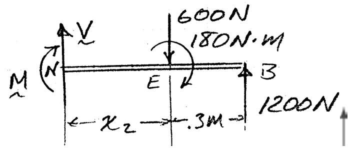
$$\uparrow \Sigma F_y = 0{:}\ 600\text{ N} - 600\text{ N} - V = 0 \qquad V = 0$$

$$\circlearrowleft \Sigma M_K = 0{:}\ M + (x - 0.3\text{ m})(600\text{ N}) - 180\text{ N}\cdot\text{m} - x(600\text{ N}) = 0$$

$$M = 360\text{ N}\cdot\text{m}$$

PROBLEM 7.48 CONTINUED

Along DE:



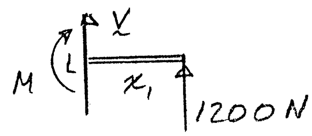
$$\Sigma F_y = 0: V - 600 \text{ N} + 1200 \text{ N} = 0 \qquad V = -600 \text{ N}$$

$$\Sigma M_N = 0: -M - 180 \text{ N}\cdot\text{m} - x_2(600 \text{ N}) + (x_2 + 0.3 \text{ m})(1200 \text{ N}) = 0$$

$$M = 180 \text{ N}\cdot\text{m} + (600 \text{ N})x_2 = 540 \text{ N}\cdot\text{m at D}, x_2 = 0.6 \text{ m}$$

$$M = 180 \text{ N}\cdot\text{m at E } (x_2 = 0)$$

Along EB:



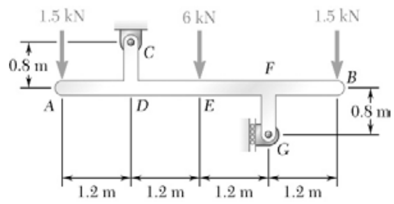
$$\Sigma F_y = 0: V + 1200 \text{ N} = 0 \qquad V = -1200 \text{ N}$$

$$\Sigma M_L = 0: x_1(1200 \text{ N}) - M = 0 \qquad M = (1200 \text{ N})x_1$$

$$M = 360 \text{ N}\cdot\text{m at } x_1 = 0.3 \text{ m}$$

From diagrams: $|V|_{max} = 1200 \text{ N on } EB$ ◀

$|M|_{max} = 540 \text{ N}\cdot\text{m at } D^+$ ◀

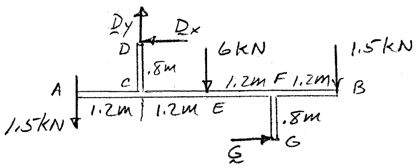


PROBLEM 7.49

Draw the shear and bending-moment diagrams for the beam *AB*, and determine the maximum absolute values of the shear and bending moment.

SOLUTION

FBD Whole:



$$\circlearrowleft \Sigma M_D = 0: (1.2\text{ m})(1.5\text{ kN}) - (1.2\text{ m})(6\text{ kN})$$

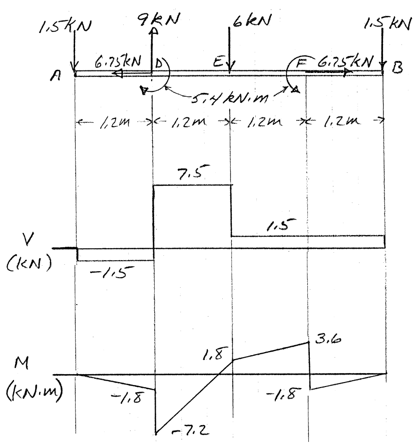
$$- (3.6\text{ m})(1.5\text{ kN}) + (1.6\text{ m})G = 0$$

$$\mathbf{G} = 6.75\text{ kN} \longrightarrow$$

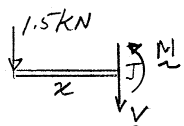
$$\longrightarrow \Sigma F_x = 0: -D_x + G = 0 \qquad \mathbf{D}_x = 6.75\text{ kN} \longleftarrow$$

$$\uparrow \Sigma F_y = 0: D_y - 1.5\text{ kN} - 6\text{ kN} - 1.5\text{ kN} = 0 \qquad \mathbf{D}_y = 9\text{ kN} \uparrow$$

Beam AB, with forces **D** and **G** replaced by equivalent force/couples at *C* and *F*



Along AD:

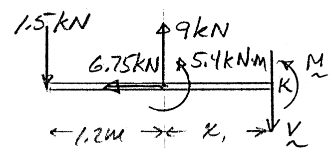


$$\uparrow \Sigma F_y = 0: -1.5\text{ kN} - V = 0 \qquad V = -1.5\text{ kN}$$

$$\circlearrowleft \Sigma M_J = 0: x(1.5\text{ kN}) + M = 0 \qquad M = -(1.5\text{ kN})x$$

$$M = -1.8\text{ kN at } x = 1.2\text{ m}$$

Along DE:



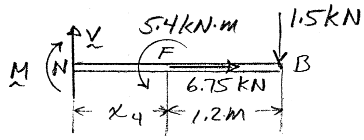
$$\uparrow \Sigma F_y = 0: -1.5\text{ kN} + 9\text{ kN} - V = 0 \qquad V = 7.5\text{ kN}$$

$$\circlearrowleft \Sigma M_K = 0: M + 5.4\text{ kN}\cdot\text{m} - x_1(9\text{ kN}) + (1.2\text{ m} + x_1)(1.5\text{ kN}) = 0$$

$$M = 7.2\text{ kN}\cdot\text{m} + (7.5\text{ kN})x_1 \qquad M = 1.8\text{ kN}\cdot\text{m at } x_1 = 1.2\text{ m}$$

PROBLEM 7.49 CONTINUED

Along EF:



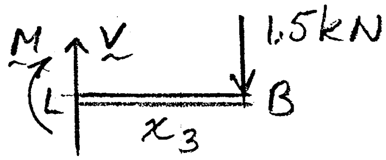
$$+\uparrow \Sigma F_y = 0: V - 1.5 \text{ kN} = 0 \qquad V = 1.5 \text{ kN}$$

$$\circlearrowleft \Sigma M_N = 0: -M + 5.4 \text{ kN}\cdot\text{m} - (x_4 + 1.2 \text{ m})(1.5 \text{ kN})$$

$$M = 3.6 \text{ kN}\cdot\text{ m} - (1.5 \text{ kN})x_4$$

$M = 1.8$ kN·m at $x_4 = 1.2$ m; M = 3.6 kN·m at $x_4 = 0$

Along FB:



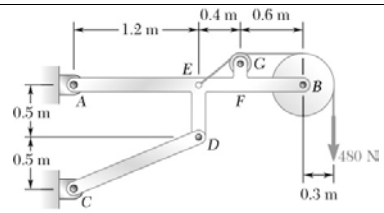
$$+\uparrow \Sigma F_y = 0: V - 1.5 \text{ kN} = 0 \qquad V = 1.5 \text{ kN}$$

$$\circlearrowleft \Sigma M_L = 0: -M - x_3(1.5 \text{ kN}) = 0 \qquad M = (-1.5 \text{ kN})x_3$$

$$M = -1.8 \text{ kN}\cdot\text{m} \text{ at } x_3 = 1.2 \text{ m}$$

From diagrams: $|V|_{\max} = 7.50$ kN on DE ◀

$|M|_{\max} = 7.20$ kN·m at D^{+} ◀

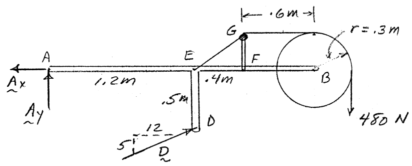


PROBLEM 7.50

Neglecting the size of the pulley at G, (a) draw the shear and bending-moment diagrams for the beam AB, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

FBD Whole:



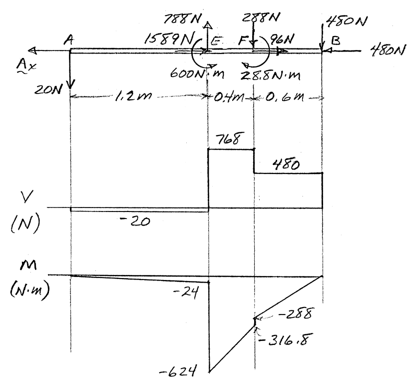
(a) $$\circlearrowleft \Sigma M_A = 0: (0.5\text{ m})\frac{12}{13}D + (1.2\text{ m})\frac{5}{13}D - (2.5\text{ m})(480\text{ N}) = 0$$

$$D = 1300\text{ N}$$

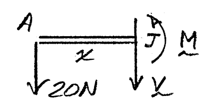
$$\uparrow \Sigma F_y = 0: A_y + \frac{5}{13}(1300\text{ N}) - 480\text{ N} = 0$$

$$A_y = -20\text{ N} \qquad \mathbf{A}_y = 20\text{ N}\downarrow$$

Beam AB with pulley forces and force at D replaced by equivalent force-couples at B, F, E



Along AE:

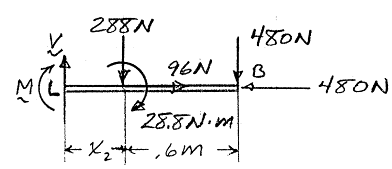


$$\uparrow \Sigma F_y = 0: -20\text{ N} - V = 0 \qquad V = -20\text{ N}$$

$$\circlearrowleft \Sigma M_J = 0: M + x(20\text{ N}) \qquad M = -(20\text{ N})x$$

$$M = -24\text{ N}\cdot\text{m at } x = 1.2\text{ m}$$

Along EF:



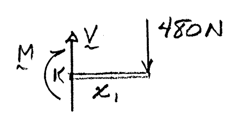
$$\uparrow \Sigma F_y = 0: V - 288\text{ N} - 480\text{ N} = 0 \qquad V = 768\text{ N}$$

$$\circlearrowleft \Sigma M_L = 0: -M - x_2(288\text{ N}) - (28.8\text{ N}\cdot\text{m}) - (x_2 + 0.6\text{ m})(480\text{ N}) = 0$$

$$M = -316.8\text{ N}\cdot\text{m} - (768\text{ N})x_2$$

$$M = -316.8\text{ N}\cdot\text{m at } x_2 = 0; \quad M = -624\text{ N}\cdot\text{m at } x_2 = 0.4\text{ m}$$

Along FB:



$$\uparrow \Sigma F_y = 0: V - 480\text{ N} = 0 \qquad V = 480\text{ N}$$

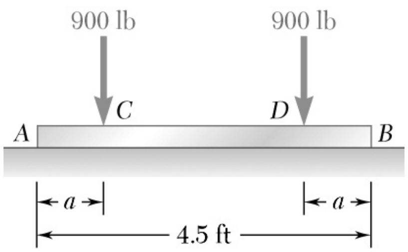
$$\circlearrowleft \Sigma M_K = 0: -M - x_1(480\text{ N}) = 0 \qquad M = -(480\text{ N})x_1$$

$$M = -288\text{ N}\cdot\text{m at } x_1 = 0.6\text{ m}$$

PROBLEM 7.50 CONTINUED

(*b*) From diagrams: $|V|_{\max} = 768\text{ N along } EF$ ◀

$|M|_{\max} = 624\text{ N}\cdot\text{m at } E^{+}$ ◀



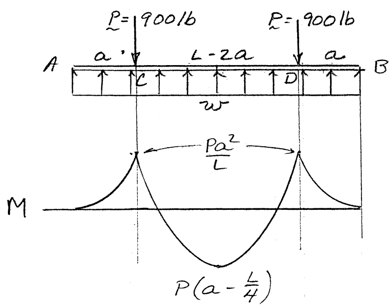
PROBLEM 7.51

For the beam of Prob. 7.43, determine (*a*) the distance *a* for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding value of $|M|_{\max}$.

(*Hint:* Draw the bending-moment diagram and then equate the absolute values of the largest positive and negative bending moments obtained.)

SOLUTION

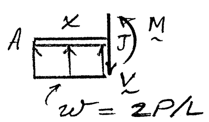
FBD Beam:



$$\uparrow \Sigma F_y = 0: Lw - 2P = 0$$

$$w = 2\frac{P}{L}$$

Along AC:



$$\circlearrowleft \Sigma M_J = 0: M - \frac{x}{2}\left(\frac{2P}{L}x\right) = 0 \qquad M = \frac{P}{L}x^2$$

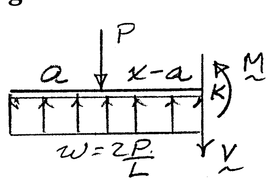
$$M = \frac{P}{L}a^2 \quad \text{at} \quad x = a$$

$$\circlearrowleft \Sigma M_K = 0: M + (x-a)P - \frac{x}{2}\left(\frac{2P}{L}x\right) = 0$$

$$M = P(a-x) + \frac{P}{L}x^2 = \frac{Pa^2}{L} \qquad \text{at} \qquad x = a$$

$$M = P\left(a - \frac{L}{4}\right) \text{ at } x = \frac{L}{2}$$

Along CD:



This is M min by symmetry–see moment diagram completed by symmetry.

For minimum $|M|_{\max}$, set $M_{\max} = -M_{\min}$:

$$P\frac{a^2}{L} = -P\left(a - \frac{L}{4}\right)$$

or

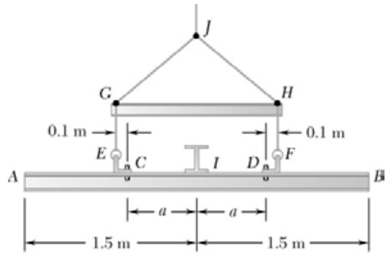
$$a^2 + La - \frac{L^2}{4} = 0$$

Solving:

$$a = \frac{-1 \pm \sqrt{2}}{2}L$$

Positive answer (*a*) $a = 0.20711L = 0.932 \text{ ft}$ ◀

(*b*) $|M|_{\max} = 0.04289PL = 173.7 \text{ lb}\cdot\text{ft}$ ◀



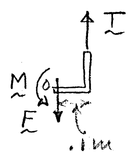
PROBLEM 7.52

For the assembly of Prob. 7.45, determine (*a*) the distance *a* for which the maximum absolute value of the bending moment in beam *AB* is as small as possible, (*b*) the corresponding value of $|M|_{\max}$. (See hint for Prob. 7.51.)

SOLUTION

By symmetry of whole arrangement:

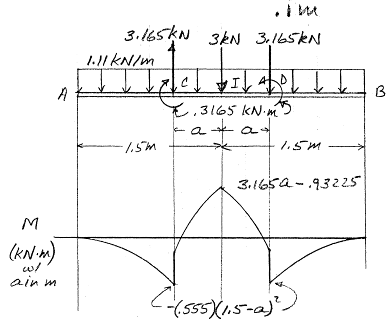
FBD Angle:



$$T = \frac{3.33\text{ kN} + 3\text{ kN}}{2} = 3.165\text{ kN}$$

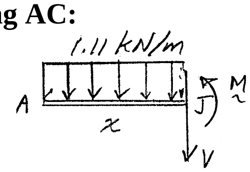
$$\uparrow \Sigma F_y = 0: T - F = 0 \qquad F = 3.165\text{ kN}$$

$$\circlearrowleft \Sigma M_0 = 0: M - (0.1\text{ m})(3.165\text{ kN}) = 0 \qquad M = 0.3165\text{ kN}\cdot\text{m}$$



$$\circlearrowleft \Sigma M_J = 0: M + \frac{x}{2}(1.11\text{ kN/m})x = 0$$

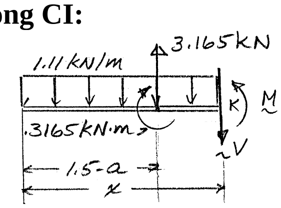
Along AC:



$$M = -(0.555\text{ kN/m})x^2 = -(0.555\text{ kN/m})(1.5\text{ m} - a)^2$$

at C (this is $M_{\min}$)

Along CI:



$$\circlearrowleft \Sigma M_K = 0: M - 0.3165\text{ kN}\cdot\text{m} + \frac{x}{2}(1.11\text{ kN/m})x$$
$$-\left[x - (1.5\text{ m} - a)\right](3.165\text{ kN}) = 0$$

$$M = -4.431\text{ kN}\cdot\text{m} + (3.165\text{ kN})(x + a) - (0.555\text{ kN/m})x^2$$

$$M_{\max}(\text{at } x = 1.5\text{ m}) = -0.93225\text{ kN}\cdot\text{m} + (3.165\text{ kN})a$$

For minimum $|M|_{\max}$, set $M_{\max} = -M_{\min}$:

$$-0.93225\text{ kN}\cdot\text{m} + (3.165\text{ kN})a = (0.555\text{ kN/m})(1.5\text{ m} - a)^2$$

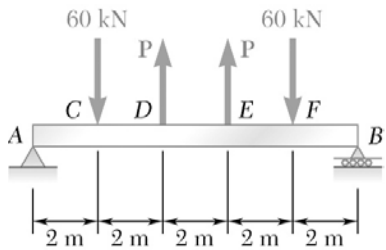
Yielding: $a^2 - (8.7027\text{ m})a + 3.92973\text{ m}^2 = 0$

Solving: $a = 4.3514 \pm \sqrt{13.864} = 0.4778\text{ m},\ 8.075\text{ m}$

Second solution out of range, so (*a*) $a = 0.478\text{ m}$ ◀

$M_{\max} = 0.5801\text{ kN}\cdot\text{m}$ ◀

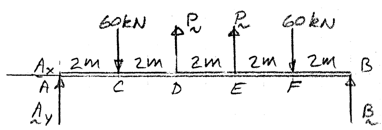
(*b*) $M_{\max} = 580\text{ N}\cdot\text{m}$ ◀

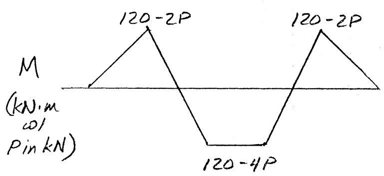


PROBLEM 7.53

For the beam shown, determine (*a*) the magnitude *P* of the two upward forces for which the maximum value of the bending moment is as small as possible, (*b*) the corresponding value of $|M|_{\max}$. (See hint for Prob. 7.51.)

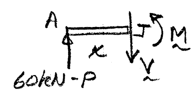
SOLUTION





By symmetry: $A_y = B = 60\text{ kN} - P$

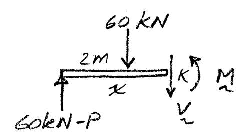
Along AC:



$$\circlearrowleft \Sigma M_J = 0{:}\; M - x(60\text{ kN} - P) = 0 \qquad M = (60\text{ kN} - P)x$$

$$M = 120\text{ kN}\cdot\text{m} - (2\text{ m})P \quad \text{at} \quad x = 2\text{ m}$$

Along CD:

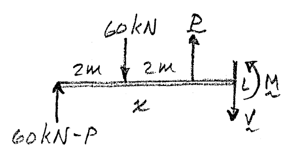


$$\circlearrowleft \Sigma M_K = 0{:}\; M + (x - 2\text{ m})(60\text{ kN}) - x(60\text{ kN} - P) = 0$$

$$M = 120\text{ kN}\cdot\text{m} - Px$$

$$M = 120\text{ kN}\cdot\text{m} - (4\text{ m})P \quad \text{at} \quad x = 4\text{ m}$$

Along DE:



$$\circlearrowleft \Sigma M_L = 0{:}\; M - (x - 4\text{ m})P + (x - 2\text{ m})(60\text{ kN})$$
$$- x(60\text{ kN} - P) = 0$$

$$M = 120\text{ kN}\cdot\text{m} - (4\text{ m})P \qquad \text{(const)}$$

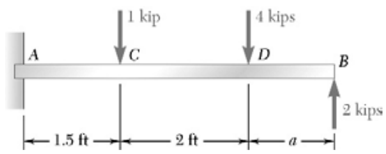
Complete diagram by symmetry

For minimum $|M|_{\max}$, set $M_{\max} = -M_{\min}$

$$120\text{ kN}\cdot\text{m} - (2\text{ m})P = -\left[120\text{ kN}\cdot\text{m} - (4\text{ m})P\right]$$

(*a*) $P = 40.0\text{ kN}$ ◀

$M_{\min} = 120\text{ kN}\cdot\text{m} - (4\text{ m})P$ (*b*) $|M|_{\max} = 40.0\text{ kN}\cdot\text{m}$ ◀

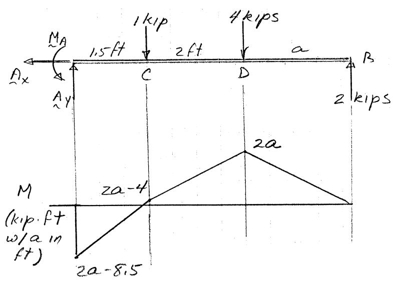


PROBLEM 7.54

For the beam and loading shown, determine (*a*) the distance *a* for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding value of $|M|_{\max}$. (See hint for Prob. 7.51.)

SOLUTION

FBD Beam:



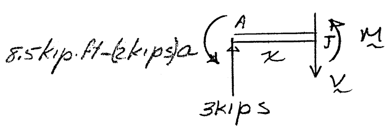
$$\Sigma M_A = 0: M_A - (1.5 \text{ ft})(1 \text{ kip}) - (3.5 \text{ ft})(4 \text{ kips}) + (3.5 \text{ ft} + a)(2 \text{ kips}) = 0$$

$$\mathbf{M}_A = [8.5 \text{ kip}\cdot\text{ft} - (2 \text{ kips})a]$$

$$\Sigma F_y = 0: A_y - 1 \text{ kip} - 4 \text{ kips} + 2 \text{ kips} = 0$$

$$\mathbf{A}_y = 3 \text{ kips}$$

Along AC:



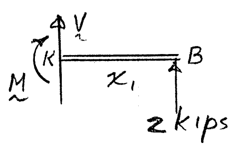
$$\Sigma M_J = 0: M - x(3 \text{ kips}) + 8.5 \text{ kip}\cdot\text{ft} - (2 \text{ kips})a = 0$$

$$M = (3 \text{ kips})x + (2 \text{ kips})a - 8.5 \text{ kip}\cdot\text{ft}$$

$$M = (2 \text{ kips})a - 4 \text{ kip}\cdot\text{ft} \text{ at } C(x = 1.5 \text{ ft})$$

$$M = (2 \text{ kips})a - 8.5 \text{ kip}\cdot\text{ft} \text{ at } A(M_{\min})$$

Along DB:

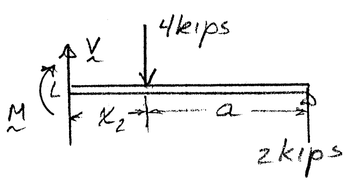


$$\Sigma M_K = 0: -M + x_1(2 \text{ kips}) = 0 \qquad M = (2 \text{ kips})x_1$$

$$M = (2 \text{ kips})a \text{ at } D$$

$$\Sigma M_L = 0: (x_2 + a)(2 \text{ kips}) - x_2(4 \text{ kips}) - M = 0$$

Along CD:



$$M = (2 \text{ kips})a - (2 \text{ kips})x_2$$

$$M = (2 \text{ kips})a - 4 \text{ kip}\cdot\text{ft} \text{ at } C \quad \text{(see above)}$$

For minimum $|M|_{\max}$, set $M_{\max}(\text{at } D) = -M_{\min}(\text{at } A)$

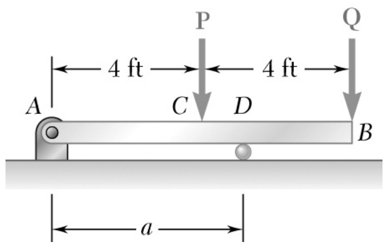
$$(2 \text{ kips})a = -[(2 \text{ kips})a - 8.5 \text{ kip}\cdot\text{ft}]$$

$$4a = 8.5 \text{ ft} \qquad a = 2.125 \text{ ft}$$

(*a*) $a = 2.13 \text{ ft}$ ◀

So $M_{\max} = (2 \text{ kips})a = 4.25 \text{ kip}\cdot\text{ft}$

(*b*) $|M|_{\max} = 4.25 \text{ kip}\cdot\text{ft}$ ◀



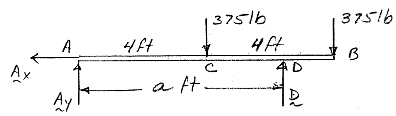
PROBLEM 7.55

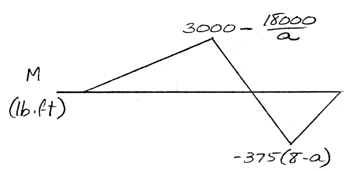
Knowing that $P = Q = 375$ lb, determine (*a*) the distance *a* for which the maximum absolute value of the bending moment in beam *AB* is as small as possible, (*b*) the corresponding value of $|M|_{\max}$. (See hint for Prob. 7.51.)

SOLUTION

$$\circlearrowleft \Sigma M_A = 0: (a \text{ ft})D - (4 \text{ ft})(375 \text{ lb}) - (8 \text{ ft})(375 \text{ lb}) = 0$$

FBD Beam:





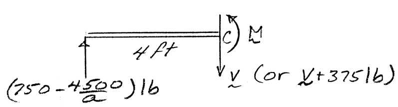
$$\mathbf{D} = \frac{4500}{a} \text{ lb} \uparrow$$

$$\uparrow \Sigma F_y = 0: A_y - 2(375 \text{ lb}) + \frac{4500}{a} \text{ lb} = 0$$

$$\mathbf{A}_y = \left(750 - \frac{4500}{a}\right)\text{lb} \uparrow$$

It is apparent that $M = 0$ at *A* and *B*, and that all segments of the *M* diagram are straight, so the max and min values of *M* must occur at *C* and *D*

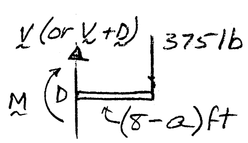
Segment AC:



$$\circlearrowleft \Sigma M_C = 0: M - (4 \text{ ft})\left(750 - \frac{4500}{a}\right)\text{lb} = 0$$

$$M = \left(3000 - \frac{18000}{a}\right)\text{lb} \cdot \text{ft}$$

Segment DB:



$$\circlearrowleft \Sigma M_D = 0: -\left[(8 - a)\text{ft}\right](375 \text{ lb}) - M = 0$$

$$M = -375(8 - a)\text{lb} \cdot \text{ft}$$

For minimum $|M|_{\max}$, set $M_{\max} = -M_{\min}$

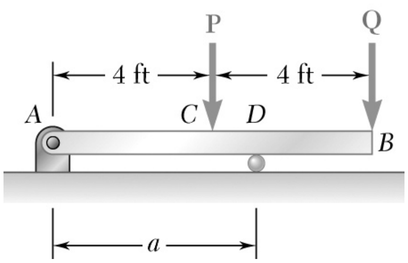
So
$$3000 - \frac{18000}{a} = 375(8 - a)$$

$$a^2 = 48 \qquad a = 6.9282 \text{ ft}$$

(*a*) $a = 6.93$ ft ◀

$$M_{\max} = 375(8 - a) = 401.92 \text{ lb} \cdot \text{ft}$$

(*b*) $|M|_{\max} = 402$ lb·ft ◀

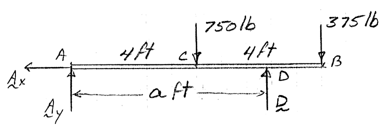


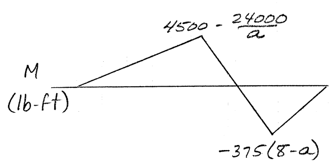
PROBLEM 7.56

Solve Prob. 7.55 assuming that $P = 750$ lb and $Q = 375$ lb.

SOLUTION

FBD Beam:





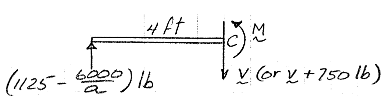
$$\circlearrowleft \Sigma M_D = 0: -(a \text{ ft})A_y + [(a-4)\text{ft}](750 \text{ lb})$$

$$-[(8-a)\text{ft}](375 \text{ lb}) = 0$$

$$\mathbf{A}_y = \left(1125 - \frac{6000}{a}\right)\text{lb} \uparrow$$

It is apparent that $M = 0$ at A and B, and that all segments of the M-diagram are straight, so M_{max} and M_{min} occur at C and D.

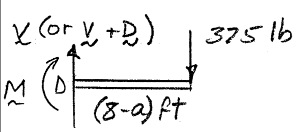
Segment AC:



$$\circlearrowleft \Sigma M_C = 0: M - (4 \text{ ft})\left(1125 - \frac{6000}{a}\right)\text{lb} = 0$$

$$M = \left(4500 - \frac{24000}{a}\right)\text{lb}\cdot\text{ft}$$

Segment DB:



$$\circlearrowleft \Sigma M_D = 0: \ -M - [(8-a)\text{ft}](375 \text{ lb}) = 0$$

$$M = -375(8-a)\text{lb}\cdot\text{ft}$$

For minimum M_{max}, set $M_{\text{max}} = -M_{\text{min}}$

$$4500 - \frac{24000}{a} = 375(8-a)$$

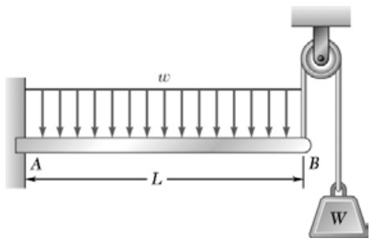
$$a^2 + 4a - 64 = 0 \qquad a = -2 \pm \sqrt{68}\,(\text{need } +)$$

$$a = 6.2462 \text{ ft} \qquad (a) \qquad a = 6.25 \text{ ft} \blacktriangleleft$$

Then $\quad M_{\text{max}} = 375(8-a) = 657.7 \text{ lb}\cdot\text{ft}$

$$(b) \qquad |M|_{\text{max}} = 658 \text{ lb}\cdot\text{ft} \blacktriangleleft$$

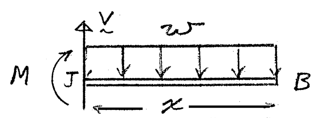
PROBLEM 7.57



In order to reduce the bending moment in the cantilever beam AB, a cable and counterweight are permanently attached at end B. Determine the magnitude of the counterweight for which the maximum absolute value of the bending moment in the beam is as small as possible and the corresponding value of $|M|_{max}$. Consider (a) the case when the distributed load is permanently applied to the beam, (b) the more general case when the distributed load may either be applied or removed.

SOLUTION

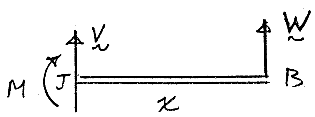
M due to distributed load:



$$\circlearrowleft \Sigma M_J = 0: -M - \frac{x}{2}wx = 0$$

$$M = -\frac{1}{2}wx^2$$

M due to counter weight:

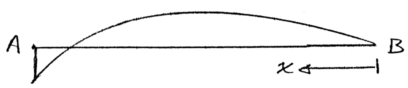


$$\circlearrowleft \Sigma M_J = 0: -M + xw = 0$$

$$M = wx$$

(a) **Both applied:**

$$M = W_x - \frac{w}{2}x^2 \qquad \frac{dM}{dx} = W - wx = 0 \text{ at } x = \frac{W}{w}$$



And here $M = \dfrac{W^2}{2w} > 0$ so M_{max}; M_{min} must be at $x = L$

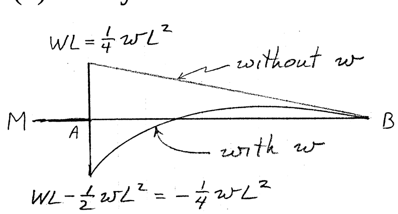
So $M_{min} = WL - \frac{1}{2}wL^2$. For minimum $|M|_{max}$ set $M_{max} = -M_{min}$, so

$$\frac{W^2}{2w} = -WL + \frac{1}{2}wL^2 \text{ or } W^2 + 2wLW - w^2L^2 = 0$$

$$W = -wL \pm \sqrt{2w^2L^2}\,(\text{need} +) \qquad W = \left(\sqrt{2} - 1\right)wL = 0.414wL \blacktriangleleft$$

$$M_{max} = \frac{W^2}{2w} = \frac{\left(\sqrt{2} - 1\right)^2}{2}wL^2 \qquad M_{max} = 0.858wL^2 \blacktriangleleft$$

(b) ***w* may be removed:**



Without w, $\quad M = Wx,\ M_{max} = WL$ at A

With w (see part a) $\quad M = Wx - \dfrac{w}{2}x^2, \quad M_{max} = \dfrac{W^2}{2w}$ at $x = \dfrac{W}{w}$

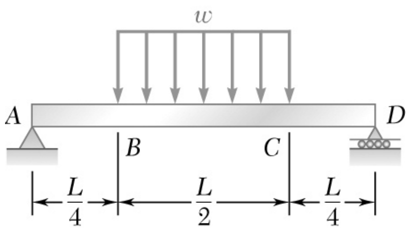
$$M_{min} = WL - \frac{1}{2}wL^2 \text{ at } x = L$$

PROBLEM 7.57 CONTINUED

For minimum M_{max}, set $M_{max}(\text{no } w) = -M_{min}(\text{with } w)$

$$WL = -WL + \frac{1}{2}wL^2 \rightarrow W = \frac{1}{4}wL \rightarrow \qquad M_{max} = \frac{1}{4}wL^2 \blacktriangleleft$$

With $$W = \frac{1}{4}wL \blacktriangleleft$$



PROBLEM 7.58

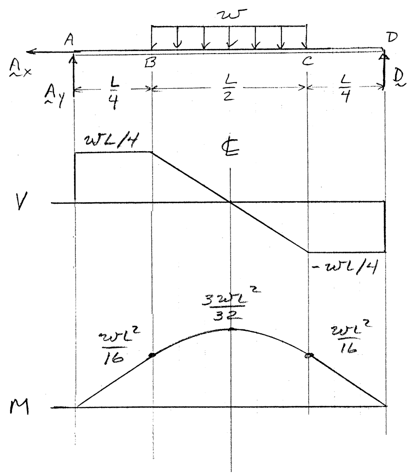
Using the method of Sec. 7.6, solve Prob. 7.29.

SOLUTION

(*a*) and (*b*)

By symmetry: $A_y = D = \frac{1}{2}\left(w\frac{L}{2}\right) = \frac{wL}{4}$ or $\mathbf{A}_y = \mathbf{D} = \frac{wL}{4}\uparrow$

Shear Diag: V jumps to $A_y = \frac{wL}{4}$ at A,



and stays constant (no load) to B. From B to C, V is linear $\left(\frac{dV}{dx} = -w\right)$, and it becomes $\frac{wL}{4} - w\frac{L}{2} = -\frac{wL}{4}$ at C.

(Note: $V = 0$ at center of beam. From C to D, V is again constant.)

Moment Diag: M starts at zero at A

and increases linearly $\left(\frac{dM}{dV} = \frac{wL}{4}\right)$ to B.

$$M_B = 0 + \frac{L}{4}\left(\frac{wL}{4}\right) = \frac{wL^2}{16}.$$

From B to C M is parabolic

$\left(\frac{dM}{dx} = V,\text{ which decreases to zero at center and } -\frac{wL}{4} \text{ at } C\right)$,

M is maximum at center. $M_{\max} = \frac{wL^2}{16} + \frac{1}{2}\left(\frac{L}{4}\right)\left(\frac{wL}{4}\right)$

Then, M is linear with $\frac{dM}{dy} = -\frac{wL}{4}$ to D

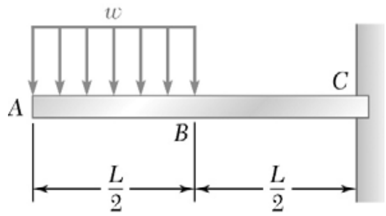
$$M_D = 0$$

$$|V|_{\max} = \frac{wL}{4} \blacktriangleleft$$

$$|M|_{\max} = \frac{3wL^2}{32} \blacktriangleleft$$

Notes: Symmetry could have been invoked to draw second half.
Smooth transitions in M at B and C, as no discontinuities in V.

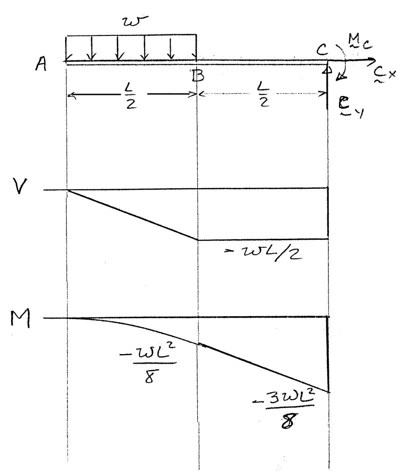
PROBLEM 7.59



Using the method of Sec. 7.6, solve Prob. 7.30.

SOLUTION

(*a*) and (*b*)



Shear Diag: $V = 0$ at A and is linear

$\left(\frac{dV}{dx} = -w\right)$ to $-w\left(\frac{L}{2}\right) = -\frac{wL}{2}$ at B. V is constant $\left(\frac{dV}{dx} = 0\right)$ from B to C.

$$|V|_{\max} = \frac{wL}{2} \blacktriangleleft$$

Moment Diag: $M = 0$ at A and is

parabolic $\left(\frac{dM}{dx}\text{ decreasing with } V\right)$ to B.

$$M_B = \frac{1}{2}\left(\frac{L}{2}\right)\left(-\frac{wL}{2}\right) = -\frac{wL^2}{8}$$

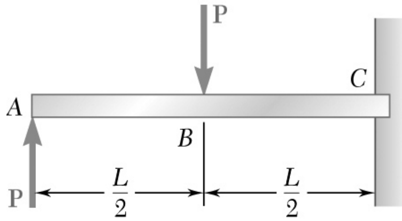
From B to C, M is linear $\left(\frac{dM}{dx} = -\frac{wL}{2}\right)$

$$M_C = -\frac{wL^2}{8} - \left(\frac{L}{2}\right)\left(\frac{wL}{2}\right) = -\frac{3wL^2}{8}$$

$$|M|_{\max} = \frac{3wL^2}{8} \blacktriangleleft$$

Notes: Smooth transition in M at B, as no discontinuity in V.

It was not necessary to predetermine reactions at C.
In fact they are given by $-V_C$ and $-M_C$.

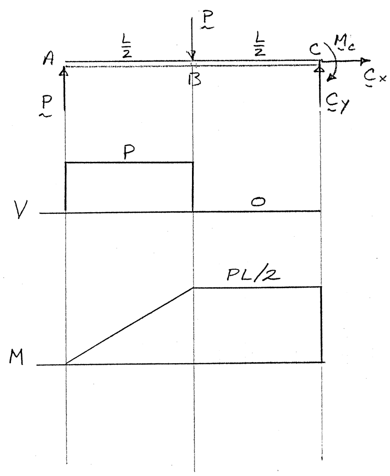


PROBLEM 7.60

Using the method of Sec. 7.6, solve Prob. 7.31.

SOLUTION

(*a*) and (*b*)



Shear Diag:

V jumps to P at A, then is constant $\left(\dfrac{dV}{dx} = 0\right)$ to B. V jumps down P to zero at B, and is constant (zero) to C.

$$|V|_{\max} = P \blacktriangleleft$$

Moment Diag:

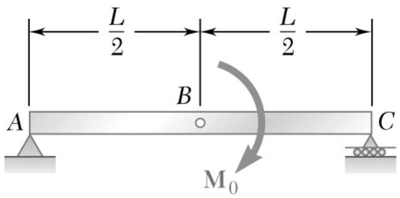
M is linear $\left(\dfrac{dM}{dy} = V = P\right)$ to B.

$$M_B = 0 + \left(\frac{L}{2}\right)(P) = \frac{PL}{2}.$$

M is constant $\left(\dfrac{dM}{dx} = 0\right)$ at $\dfrac{PL}{2}$ to C

$$|M|_{\max} = \frac{PL}{2} \blacktriangleleft$$

Note: It was not necessary to predetermine reactions at C. In fact they are given by $-V_C$ and $-M_C$.



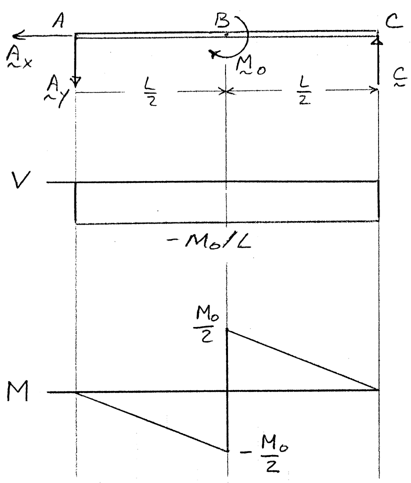
PROBLEM 7.61

Using the method of Sec. 7.6, solve Prob. 7.32.

SOLUTION

(*a*) and (*b*)

$$\circlearrowleft \Sigma M_C = 0: LA_y - M_0 = 0 \qquad \mathbf{A}_y = \frac{M_0}{L} \downarrow$$



Shear Diag:

V jumps to $-\dfrac{M_0}{L}$ at A and is constant $\left(\dfrac{dV}{dx} = 0\right)$ all the way to C

$$|V|_{\max} = \frac{M_0}{L} \blacktriangleleft$$

Moment Diag:

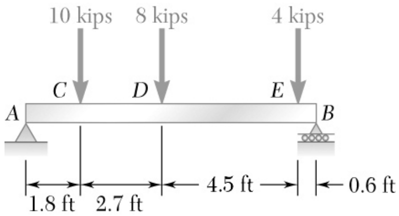
M is zero at A and linear $\left(\dfrac{dM}{dx} = V = -\dfrac{M_0}{L}\right)$ throughout.

$$M_{B^-} = -\frac{L}{2}\left(\frac{M_0}{L}\right) = -\frac{M_0}{2},$$

but M jumps by $+M_0$ to $+\dfrac{M_0}{2}$ at B.

$$M_C = \frac{M_0}{2} - \frac{L}{2}\left(\frac{M_0}{L}\right) = 0$$

$$|M|_{\max} = \frac{M_0}{2} \blacktriangleleft$$

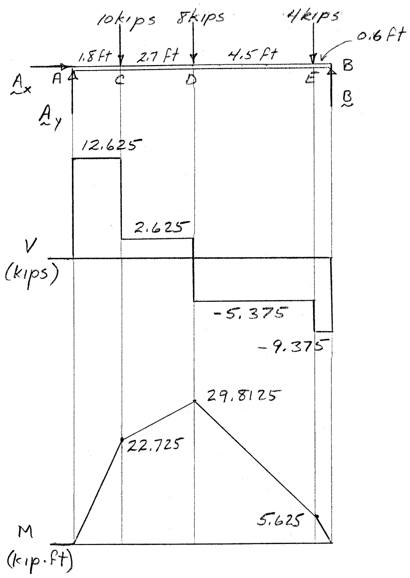


PROBLEM 7.62

Using the method of Sec. 7.6, solve Prob. 7.33.

SOLUTION

(*a*) and (*b*)



$$\circlearrowleft \Sigma M_B = 0: (0.6\text{ ft})(4\text{ kips}) + (5.1\text{ ft})(8\text{ kips}) + (7.8\text{ ft})(10\text{ kips}) - (9.6\text{ ft})A_y = 0$$

$$\mathbf{A}_y = 12.625\text{ kips} \uparrow$$

Shear Diag:

V is piecewise constant, $\left(\dfrac{dV}{dx} = 0\right)$ with discontinuities at each concentrated force. (equal to force)

$$|V|_{\max} = 12.63\text{ kips} \blacktriangleleft$$

Moment Diag:

M is zero at A, and piecewise linear $\left(\dfrac{dM}{dx} = V\right)$ throughout.

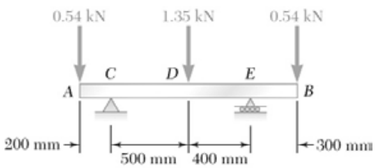
$$M_C = (1.8\text{ ft})(12.625\text{ kips}) = 22.725\text{ kip}\cdot\text{ft}$$

$$M_D = 22.725\text{ kip}\cdot\text{ft} + (2.7\text{ ft})(2.625\text{ kips}) = 29.8125\text{ kip}\cdot\text{ft}$$

$$M_E = 29.8125\text{ kip}\cdot\text{ft} - (4.5\text{ ft})(5.375\text{ kips}) = 5.625\text{ kip}\cdot\text{ft}$$

$$M_B = 5.625\text{ kip}\cdot\text{ft} - (0.6\text{ ft})(9.375\text{ kips}) = 0$$

$$|M|_{\max} = 29.8\text{ kip}\cdot\text{ft} \blacktriangleleft$$

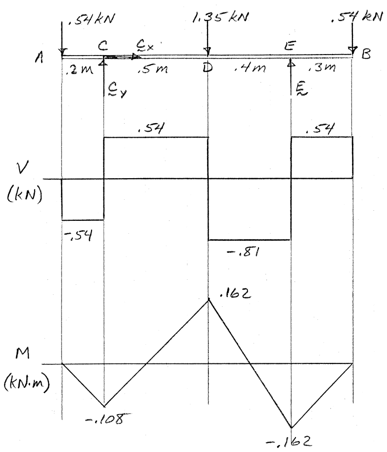


PROBLEM 7.63

Using the method of Sec. 7.6, solve Prob. 7.36.

SOLUTION

(a) and (b)



FBD Beam:

$$\circlearrowleft \Sigma M_E = 0: (1.1\text{ m})(0.54\text{ kN}) - (0.9\text{ m})C_y$$

$$+ (0.4\text{ m})(1.35\text{ kN}) - (0.3\text{ m})(0.54\text{ kN}) = 0$$

$$\mathbf{C}_y = 1.08\text{ kN} \uparrow$$

$$\uparrow \Sigma F_y = 0: -0.54\text{ kN} + 1.08\text{ kN} - 1.35\text{ kN} + E - 0.54\text{ kN} = 0$$

$$\mathbf{E} = 1.35\text{ kN} \uparrow$$

Shear Diag:

V is piecewise constant, $\left(\dfrac{dV}{dx} = 0 \text{ everywhere}\right)$ with discontinuities at each concentrated force. (equal to the force)

$$|V|_{\max} = 810\text{ N} \blacktriangleleft$$

Moment Diag:

M is piecewise linear starting with $M_A = 0$

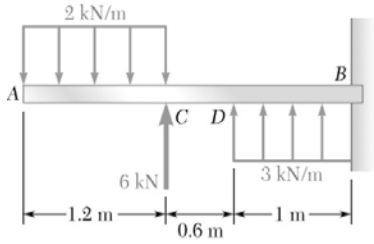
$$M_C = 0 - 0.2\text{ m}(0.54\text{ kN}) = 0.108\text{ kN}\cdot\text{m}$$

$$M_D = 0.108\text{ kN}\cdot\text{m} + (0.5\text{ m})(0.54\text{ kN}) = 0.162\text{ kN}\cdot\text{m}$$

$$M_E = 0.162\text{ kN}\cdot\text{m} - (0.4\text{ m})(0.81\text{ kN}) = -0.162\text{ kN}\cdot\text{m}$$

$$M_B = 0.162\text{ kN}\cdot\text{m} + (0.3\text{ m})(0.54\text{ kN}) = 0$$

$$|M|_{\max} = 0.162\text{ kN}\cdot\text{m} = 162.0\text{ N}\cdot\text{m} \blacktriangleleft$$

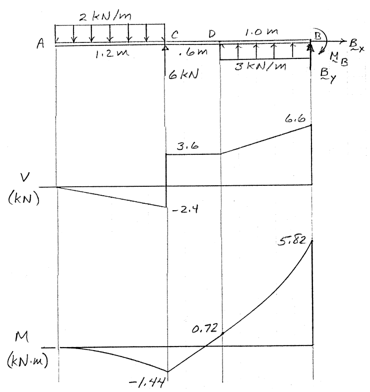


PROBLEM 7.64

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(*a*) and (*b*)



Shear Diag:

$V = 0$ at A and linear $\left(\dfrac{dV}{dx} = -2\text{ kN/m}\right)$ to C

$$V_C = -1.2\text{ m}(2\text{ kN/m}) = -2.4\text{ kN.}$$

At C, V jumps 6 kN to 3.6 kN, and is constant to D. From there, V is linear $\left(\dfrac{dV}{dx} = +3\text{ kN/m}\right)$ to B

$$V_B = 3.6\text{ kN} + (1\text{ m})(3\text{ kN/m}) = 6.6\text{ kN}$$

$$|V|_{\max} = 6.60\text{ kN} \blacktriangleleft$$

Moment Diag: $M_A = 0.$

From A to C, M is parabolic, $\left(\dfrac{dM}{dx}\text{ decreasing with }V\right)$.

$$M_C = -\frac{1}{2}(1.2\text{ m})(2.4\text{ kN}) = -1.44\text{ kN}\cdot\text{m}$$

From C to D, M is linear $\left(\dfrac{dM}{dx} = 3.6\text{ kN}\right)$

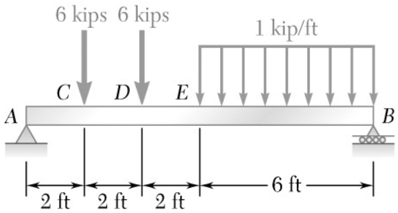
$$M_D = -1.44\text{ kN}\cdot\text{m} + (0.6\text{ m})(3.6\text{ kN})$$
$$= 0.72\text{ kN}\cdot\text{m.}$$

From D to B, M is parabolic $\left(\dfrac{dM}{dx}\text{ increasing with }V\right)$

$$M_B = 0.72\text{ kN}\cdot\text{m} + \frac{1}{2}(1\text{ m})(3.6 + 6.6)\text{ kN}$$
$$= 5.82\text{ kN}\cdot\text{m}$$

$$|M|_{\max} = 5.82\text{ kN}\cdot\text{m} \blacktriangleleft$$

Notes: Smooth transition in M at D. It was unnecessary to predetermine reactions at B, but they are given by $-V_B$ and $-M_B$



PROBLEM 7.65

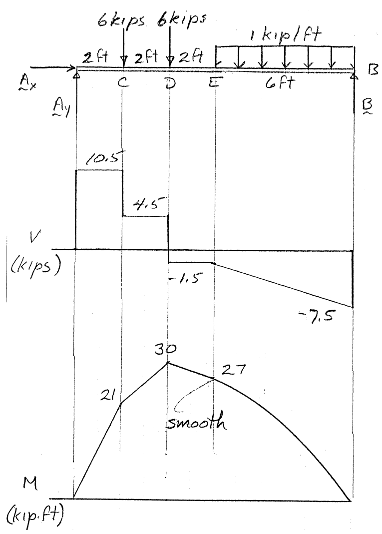
For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(*a*) and (*b*)

$$\circlearrowleft \Sigma M_B = 0: (3\text{ ft})(1\text{ kip/ft})(6\text{ ft}) + (8\text{ ft})(6\text{ kips}) + (10\text{ ft})(6\text{ kips}) - (12\text{ ft})A_y = 0$$

$$\mathbf{A}_y = 10.5\text{ kips}\uparrow$$



Shear Diag:

V is piecewise constant from A to E, with discontinuities at A, C, and E equal to the forces. $V_E = -1.5\text{ kips}$. From E to B, V is linear

$$\left(\frac{dV}{dx} = -1\text{ kip/ft}\right),$$

so

$$V_B = -1.5\text{ kips} - (6\text{ ft})(1\text{ kip/ft}) = -7.5\text{ kips}$$

$$|V|_{\text{max}} = 10.50\text{ kips} \blacktriangleleft$$

Moment Diag: $M_A = 0$, then M is piecewise linear to E

$$M_C = 0 + 2\text{ ft}(10.5\text{ kips}) = 21\text{ kip}\cdot\text{ft}$$

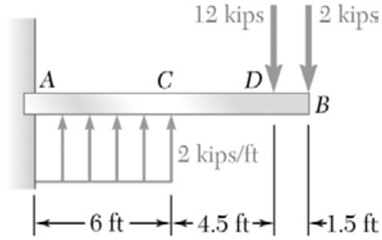
$$M_D = 21\text{ kip}\cdot\text{ft} + (2\text{ ft})(4.5\text{ kips}) = 30\text{ kip}\cdot\text{ft}$$

$$M_E = 30\text{ kip}\cdot\text{ft} - (2\text{ ft})(1.5\text{ kips}) = 27\text{ kip}\cdot\text{ft}$$

From E to B, M is parabolic $\left(\frac{dM}{dx}\text{ decreasing with }V\right)$, and

$$M_B = 27\text{ kip}\cdot\text{ft} - \frac{1}{2}(6\text{ ft})(1.5\text{ kips} + 7.5\text{ kips}) = 0$$

$$|M|_{\text{max}} = 30.0\text{ kip}\cdot\text{ft} \blacktriangleleft$$

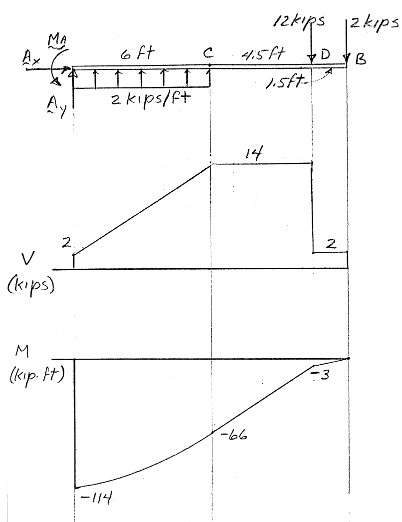


PROBLEM 7.66

Using the method of Sec. 7.6, solve Prob. 7.37.

SOLUTION

(*a*) and (*b*)



FBD Beam:

$$\uparrow \Sigma F_y = 0: \; A_y + (6\text{ ft})(2\text{ kips/ft}) - 12\text{ kips} - 2\text{ kips} = 0$$

$$\mathbf{A}_y = 2\text{ kips} \uparrow$$

$$\circlearrowleft \Sigma M_A = 0: \; M_A + (3\text{ ft})(2\text{ kips/ft})(6\text{ ft})$$
$$-(10.5\text{ ft})(12\text{ kips}) - (12\text{ ft})(2\text{ kips}) = 0$$

$$\mathbf{M}_A = 114\text{ kip}\cdot\text{ft} \circlearrowright$$

Shear Diag:

$V_A = A_y = 2$ kips. Then V is linear $\left(\dfrac{dV}{dx} = 2\text{ kips/ft}\right)$ to C, where

$$V_C = 2\text{ kips} + (6\text{ ft})(2\text{ kips/ft}) = 14\text{ kips}.$$

V is constant at 14 kips to D, then jumps down 12 kips to 2 kips and is constant to B

$$|V|_{\max} = 14.00\text{ kips} \blacktriangleleft$$

Moment Diag: $\qquad M_A = -114\text{ kip}\cdot\text{ft}.$

From A to C, M is parabolic $\left(\dfrac{dM}{dx}\text{ increasing with } V\right)$ and

$$M_C = -114\text{ kip}\cdot\text{ft} + \frac{1}{2}(2\text{ kips} + 14\text{ kips})(6\text{ ft})$$

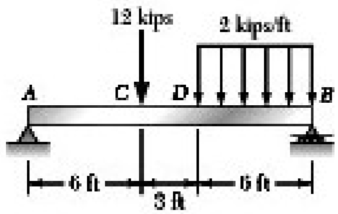
$$M_C = -66\text{ kip}\cdot\text{ft}.$$

Then M is piecewise linear.

$$M_D = -66\text{ kip}\cdot\text{ft} + (14\text{ kips})(4.5\text{ ft}) = -3\text{ kip}\cdot\text{ft}$$

$$M_B = -3\text{ kip}\cdot\text{ft} + (2\text{ kips})(1.5\text{ ft}) = 0$$

$$|M|_{\max} = 114.0\text{ kip}\cdot\text{ft} \blacktriangleleft$$

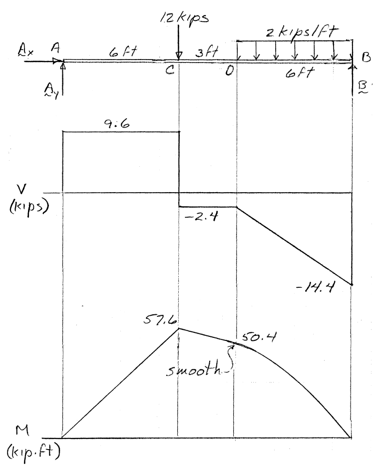


PROBLEM 7.67

Using the method of Sec. 7.6, solve Prob. 7.38.

SOLUTION

(a) and (b)



FBD Beam:

$$\circlearrowleft \Sigma M_B = 0{:}\ (3\text{ ft})\left(2\frac{\text{kips}}{\text{ft}}\right)(6\text{ ft}) + (9\text{ ft})(12\text{ kips}) - (15\text{ ft})A_y = 0$$

$$\mathbf{A}_y = 9.6\text{ kips} \uparrow$$

Shear Diag:

V jumps to $A_y = 9.6$ kips at A, is constant to C, jumps down 12 kips to -2.4 kips at C, is constant to D, and then is linear $\left(\frac{dV}{dx} = -2\text{ kips/ft}\right)$ to B

$$V_B = -2.4\text{ kips} - (2\text{ kips/ft})(6\text{ ft})$$
$$= -14.4\text{ kips}$$

$$|V|_{\max} = 14.40\text{ kips} \blacktriangleleft$$

Moment Diag:

M is linear from A to C $\left(\frac{dM}{dx} = 9.6\text{ kips/ft}\right)$

$$M_C = 9.6\text{ kips}(6\text{ ft}) = 57.6\text{ kip}\cdot\text{ft},$$

M is linear from C to D $\left(\frac{dM}{dx} = -2.4\text{ kips/ft}\right)$

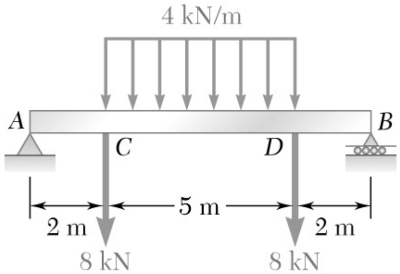
$$M_D = 57.6\text{ kip}\cdot\text{ft} - 2.4\text{ kips}(3\text{ ft})$$
$$M_D = 50.4\text{ kip}\cdot\text{ft}.$$

M is parabolic $\left(\frac{dM}{dx}\text{ decreasing with } V\right)$ to B.

$$M_B = 50.4\text{ kip}\cdot\text{ft} - \frac{1}{2}(2.4\text{ kips} + 14.4\text{ kips})(6\text{ ft}) = 0$$
$$= 0$$

$$|M|_{\max} = 57.6\text{ kip}\cdot\text{ft} \blacktriangleleft$$

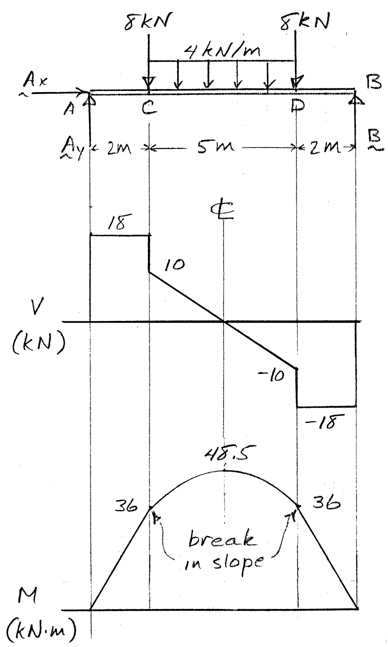
PROBLEM 7.68



Using the method of Sec. 7.6, solve Prob. 7.39.

SOLUTION

(*a*) and (*b*)



FBD Beam:

By symmetry: $A_y = B = \frac{1}{2}(5\text{ m})(4\text{ kN/m}) + 8\text{ kN}$

or $\mathbf{A}_y = \mathbf{B} = 18\text{ kN}\uparrow$

Shear Diag:

V jumps to 18 kN at A, and is constant to C, then drops 8 kN to 10 kN. After C, V is linear $\left(\frac{dV}{dx} = -4\text{ kN/m}\right)$, reaching -10 kN at $D\left[V_D = 10\text{ kN} - (4\text{ kN/m})(5\text{ m})\right]$ passing through zero at the beam center. At D, V drops 8 kN to -18 kN and is then constant to B

$$|V|_{\max} = 18.00\text{ kN} \blacktriangleleft$$

Moment Diag:

$M_A = 0.$ Then M is linear $\left(\frac{dM}{dx} = 18\text{ kN/m}\right)$ to C

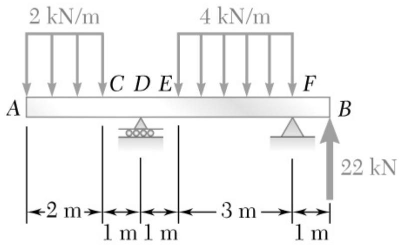
$M_C = (18\text{ kN})(2\text{ m}) = 36\text{ kN}\cdot\text{m},$ M is parabolic to D $\left(\frac{dM}{dx}\text{ decreases with } V \text{ to zero at center}\right)$

$$M_{\text{center}} = 36\text{ kN}\cdot\text{m} + \frac{1}{2}(10\text{ kN})(2.5\text{ m}) = 48.5\text{ kN}\cdot\text{m} = M_{\max}$$

$$|M|_{\max} = 48.5\text{ kN}\cdot\text{m} \blacktriangleleft$$

Complete by invoking symmetry.

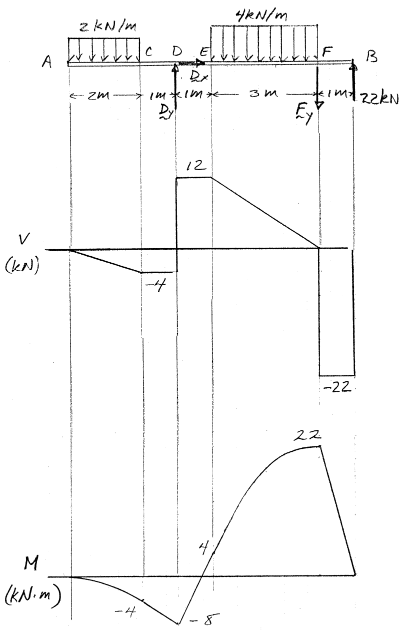
PROBLEM 7.69



Using the method of Sec. 7.6, solve Prob. 7.40.

SOLUTION

(*a*) and (*b*)



FBD Beam:

$$\circlearrowleft \Sigma M_F = 0: (1\text{ m})(22\text{ kN}) + (1.5\text{ m})(4\text{ kN/m})(3\text{ m})$$
$$-(4\text{ m})D_y + (6\text{ m})(2\text{ kN/m})(2\text{ m}) = 0$$
$$\mathbf{D}_y = 16\text{ kN} \uparrow$$

$$\uparrow \Sigma F_y = 0: 16\text{ kN} + 22\text{ kN} - F_y - (2\text{ kN/m})(2\text{ m})$$
$$-(4\text{ kN/m})(3\text{ m}) = 0$$
$$\mathbf{F}_y = 22\text{ kN} \downarrow$$

Shear Diag:

$V_A = 0$, then V is linear $\left(\dfrac{dV}{dx} = -2\text{ kN/m}\right)$ to C;

$$V_C = -2\text{ kN/m}(4\text{ m}) = -4\text{ kN}$$

V is constant to D, jumps 16 kN to 12 kN and is constant to E.

Then V is linear $\left(\dfrac{dV}{dx} = -4\text{ kN/m}\right)$ to F.

$$V_F = 12\text{ kN} - (4\text{ kN/m})(3\text{ m}) = 0.$$

V jumps to -22 kN at F, is constant to B, and returns to zero.

$$|V|_{\max} = 22.0\text{ kN} \blacktriangleleft$$

Moment Diag:

$M_A = 0$, M is parabolic $\left(\dfrac{dM}{dx}\text{ decreases with }V\right)$ to C.

$$M_C = -\frac{1}{2}(4\text{ kN})(2\text{ m}) = -4\text{ kN}\cdot\text{m}.$$

PROBLEM 7.69 CONTINUED

Then M is linear $\left(\frac{dM}{dx} = -4\text{ kN}\right)$ to D.

$$M_D = -4\text{ kN}\cdot\text{m} - (4\text{ kN})(1\text{ m}) = -8\text{ kN}\cdot\text{m}$$

From D to E, M is linear $\left(\frac{dM}{dx} = 12\text{ kN}\right)$, and

$$M_E = -8\text{ kN}\cdot\text{m} + (12\text{ kN})(1\text{m})$$

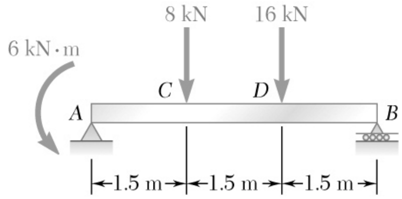
$$M_E = 4\text{ kN}\cdot\text{m}$$

M is parabolic $\left(\frac{dM}{dx}\text{ decreasing with }V\right)$ to F.

$$M_F = 4\text{ kN}\cdot\text{m} + \frac{1}{2}(12\text{ kN})(3\text{ m}) = 22\text{ kN}\cdot\text{m}.$$

Finally, M is linear $\left(\frac{dM}{dx} = -22\text{ kN}\right)$, back to zero at B.

$$|M|_{\max} = 22.0\text{ kN}\cdot\text{m} \blacktriangleleft$$

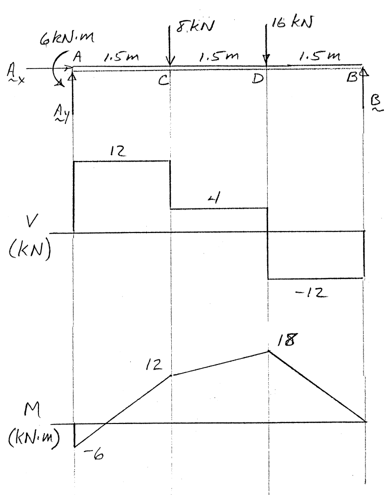


PROBLEM 7.70

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(*a*) and (*b*)



FBD Beam:

$$\circlearrowleft \Sigma M_B = 0: (1.5\text{ m})(16\text{ kN})$$
$$+ (3\text{ m})(8\text{ kN}) + 6\text{ kN}\cdot\text{m} - (4.5\text{ m})A_y = 0$$
$$\mathbf{A}_y = 12\text{ kN} \uparrow$$

Shear Diag:

V is piecewise constant with discontinuities equal to the concentrated forces at A, C, D, B

$$|V|_{\max} = 12.00\text{ kN} \blacktriangleleft$$

Moment Diag:

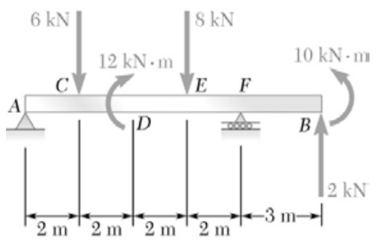
After a jump of $-6\text{ kN}\cdot\text{m}$ at A, M is piecewise linear $\left(\dfrac{dM}{dx} = V\right)$

So $$M_C = -6\text{ kN}\cdot\text{m} + (12\text{ kN})(1.5\text{ m}) = 12\text{ kN}\cdot\text{m}$$

$$M_D = 12\text{ kN}\cdot\text{m} + (4\text{ kN})(1.5\text{ m}) = 18\text{ kN}\cdot\text{m}$$

$$M_B = 18\text{ kN}\cdot\text{m} - (12\text{ kN})(1.5\text{ m}) = 0$$

$$|M|_{\max} = 18.00\text{ kN}\cdot\text{m} \blacktriangleleft$$

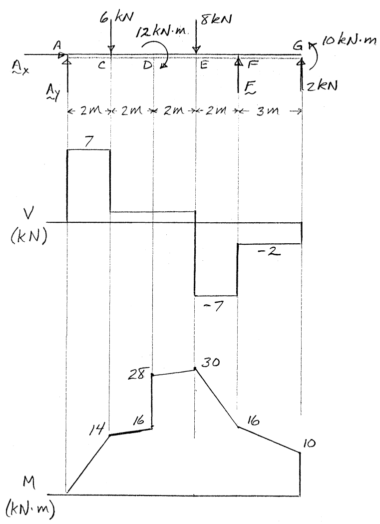


PROBLEM 7.71

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(*a*)



FBD Beam:

$$\circlearrowleft \Sigma M_A = 0: (8\text{ m})F + (11\text{ m})(2\text{ kN}) + 10\text{ kN}\cdot\text{m} - (6\text{ m})(8\text{ kN})$$

$$-12\text{ kN}\cdot\text{m} - (2\text{ m})(6\text{ kN}) = 0 \quad \mathbf{F} = 5\text{ kN}\uparrow$$

$$\uparrow \Sigma F_y = 0: A_y - 6\text{ kN} - 8\text{ kN} + 5\text{ kN} + 2\text{ kN} = 0$$

$$\mathbf{A}_y = 7\text{ kN}\uparrow$$

Shear Diag:

V is piecewise constant with discontinuities equal to the concentrated forces at A, C, E, F, G

Moment Diag:

M is piecewise linear with a discontinuity equal to the couple at D.

$$M_C = (7\text{ kN})(2\text{ m}) = 14\text{ kN}\cdot\text{m}$$

$$M_{D^-} = 14\text{ kN}\cdot\text{m} + (1\text{ kN})(2\text{ m}) = 16\text{ kN}\cdot\text{m}$$

$$M_{D^+} = 16\text{ kN}\cdot\text{m} + 12\text{ kN}\cdot\text{m} = 28\text{ kN}\cdot\text{m}$$

$$M_E = 28\text{ kN}\cdot\text{m} + (1\text{ kN})(2\text{ m}) = 30\text{ kN}\cdot\text{m}$$

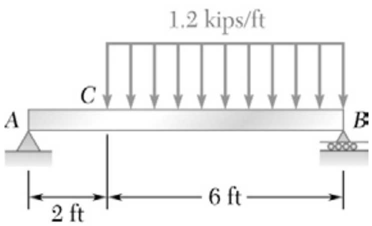
$$M_F = 30\text{ kN}\cdot\text{m} - (7\text{ kN})(2\text{ m}) = 16\text{ kN}\cdot\text{m}$$

$$M_G = 16\text{ kN}\cdot\text{m} - (2\text{ kN})(3\text{ m}) = 10\text{ kN}\cdot\text{m}$$

(*b*) $|V|_{max} = 7.00\text{ kN}$ ◀

$|M|_{max} = 30.0\text{ kN}$ ◀

PROBLEM 7.72



For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum bending moment.

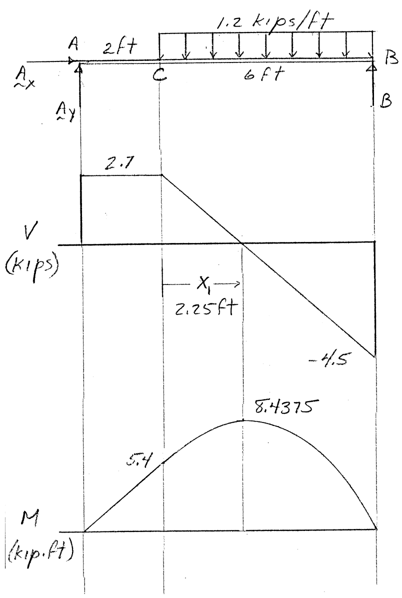
SOLUTION

(*a*)

FBD Beam:

$$\circlearrowleft \Sigma M_B = 0: (3 \text{ ft})(1.2 \text{ kips/ft})(6 \text{ ft}) - (8 \text{ ft})A_y = 0$$

$$\mathbf{A}_y = 2.7 \text{ kips} \uparrow$$



Shear Diag:

$$V = A_y = 2.7 \text{ kips at } A, \text{ is constant to } C, \text{ then linear}$$

$$\left(\frac{dV}{dx} = -1.2 \text{ kips/ft}\right) \text{ to B.} \qquad V_B = 2.7 \text{ kips} - (1.2 \text{ kips/ft})(6 \text{ ft})$$

$$V_B = -4.5 \text{ kips}$$

$$V = 0 = 2.7 \text{ kips} - (1.2 \text{ kips/ft})x_1 \text{ at } \quad x_1 = 2.25 \text{ ft}$$

Moment Diag:

$$M_A = 0, \quad M \text{ is linear } \left(\frac{dM}{dx} = 2.7 \text{ kips}\right) \text{ to } C.$$

$$M_C = (2.7 \text{ kips})(2 \text{ ft}) = 5.4 \text{ kip}\cdot\text{ft}$$

Then M is parabolic $\left(\frac{dM}{dx} \text{ decreasing with } V\right)$

(*b*)

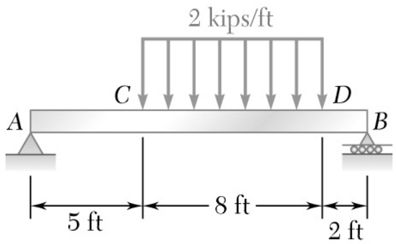
M_{max} occurs where $\frac{dM}{dx} = V = 0$

$$M_{\text{max}} = 5.4 \text{ kip}\cdot\text{ft} + \frac{1}{2}(2.7 \text{ kips})x_1; \qquad x_1 = 2.25 \text{ m}$$

$$M_{\text{max}} = 8.4375 \text{ kip}\cdot\text{ft}$$

$$M_{\text{max}} = 8.44 \text{ kip}\cdot\text{ft, 2.25 m right of } C \blacktriangleleft$$

Check: $$M_B = 8.4375 \text{ kip}\cdot\text{ft} - \frac{1}{2}(4.5 \text{ kips})(3.75 \text{ ft}) = 0$$

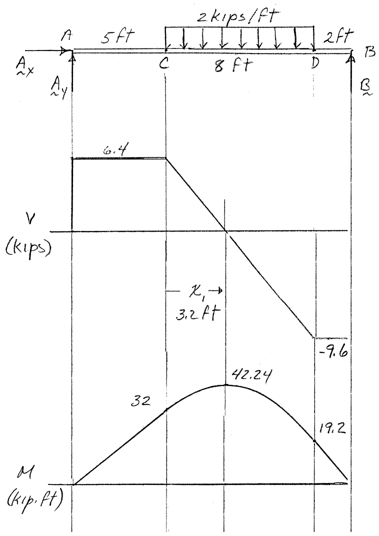


PROBLEM 7.73

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum bending moment.

SOLUTION

(*a*)



FBD Beam:

$$\circlearrowleft \Sigma M_B = 0: (6 \text{ ft})(2 \text{ kips/ft})(8 \text{ ft}) - (15 \text{ ft})A_y = 0$$

$$\mathbf{A}_y = 6.4 \text{ kips} \uparrow$$

Shear Diag:

$V = A_y = 6.4$ kips at A, and is constant to C, then linear $\left(\dfrac{dV}{dx} = -2 \text{ kips/ft}\right)$ to D,

$$V_D = 6.4 \text{ kips} - (2 \text{ kips/ft})(8 \text{ ft}) = -9.6 \text{ kips}$$

$$V = 0 = 6.4 \text{ kips} - (2 \text{ kips/ft})x_1 \text{ at } x_1 = 3.2 \text{ ft}$$

Moment Diag:

$M_A = 0$, then M is linear $\left(\dfrac{dM}{dx} = 6.4 \text{ kips}\right)$ to $M_C = (6.4 \text{ kips})(5 \text{ ft})$.

$M_C = 32$ kip·ft. M is then parabolic $\left(\dfrac{dM}{dx} \text{ decreasing with } V\right)$.

(*b*)

$$M_{\max} \text{ occurs where } \frac{dM}{dx} = V = 0.$$

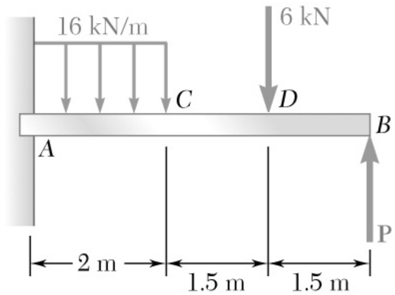
$$M_{\max} = 32 \text{ kip}\cdot\text{ft} + \frac{1}{2}(6.4 \text{ kips})x_1; \qquad x_1 = 3.2 \text{ ft}$$

$$M_{\max} = 42.24 \text{ kip}\cdot\text{ft}$$

$$M_{\max} = 42.2 \text{ kip}\cdot\text{ft, 3.2 ft right of } C \blacktriangleleft$$

$$M_D = 42.24 \text{ kip}\cdot\text{ft} - \frac{1}{2}(9.6 \text{ kips})(4.8 \text{ ft}) = 19.2 \text{ kip}\cdot\text{ft}$$

M is linear from D to zero at B.

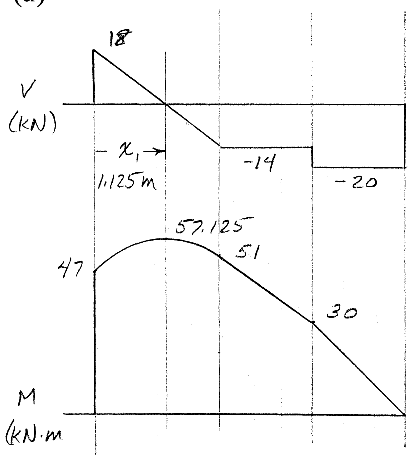


PROBLEM 7.74

For the beam shown, draw the shear and bending-moment diagrams and determine the maximum absolute value of the bending moment knowing that (*a*) $P = 14$ kN, (*b*) $P = 20$ kN.

SOLUTION

(*a*)



FBD Beam:

$$+\uparrow \Sigma F_y = 0: \ A_y - (16 \text{ kN/m})(2 \text{ m}) - 6 \text{ kN} + P = 0$$

$$A_y = 38 \text{ kN} - P$$

(*a*) $\mathbf{A}_y = 24 \text{ kN} \uparrow$

(*b*) $\mathbf{A}_y = 18 \text{ kN} \uparrow$

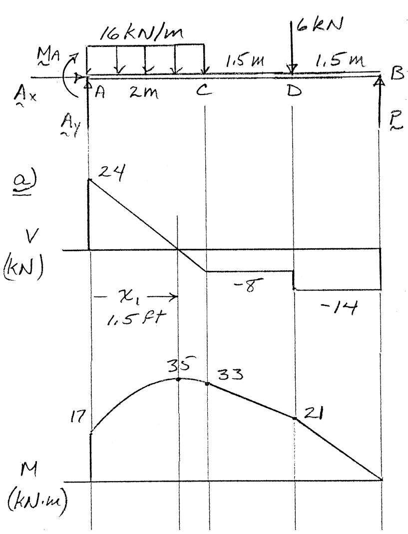
$$\circlearrowleft \Sigma M_A = 0: \ (5 \text{ m})P - (3.5 \text{ m})(6 \text{ kN}) - 1 \text{ m}(16 \text{ kN/m})(2 \text{ m}) - M_A = 0$$

$$M_A = (5 \text{ m})P - 53 \text{ kN}\cdot\text{m}$$

(*a*) $\mathbf{M}_A = 17 \text{ kN}\cdot\text{m} \circlearrowright$

(*b*) $\mathbf{M}_A = 47 \text{ kN}\cdot\text{m} \circlearrowright$

(*b*)



Shear Diags:

$V_A = A_y$. Then V is linear $\left(\dfrac{dV}{dx} = -16 \text{ kN/m}\right)$ to C.

$$V_C = V_A - (16 \text{ kN/m})(2 \text{ m}) = V_A - 32 \text{ kN}$$

(*a*) $V_C = -8$ kN

(*b*) $V_C = -14$ kN

$$V = 0 = V_A - (16 \text{ kN/m})x_1$$

(*a*) $x_1 = 1.5$ m

(*b*) $x_1 = 1.125$ m

V is constant from C to D, decreases by 6 kN at D and is constant to B (at $-P$)

PROBLEM 7.74 CONTINUED

Moment Diags:

$M_A = M_A$ reaction. Then M is parabolic $\left(\frac{dM}{dx} \text{ decreasing with } V\right)$.

The maximum occurs where $V = 0$. $M_{\max} = M_A + \frac{1}{2}V_A x_1$.

(*a*) $$M_{\max} = 17\text{ kN}\cdot\text{m} + \frac{1}{2}(24\text{ kN})(1.5\text{ m}) = 35.0\text{ kN}\cdot\text{m} ◀$$

1.5 ft from A ◀

(*b*) $$M_{\max} = 47\text{ kN}\cdot\text{m} + \frac{1}{2}(18\text{ kN})(1.125\text{ m}) = 57.125\text{ kN}\cdot\text{m}$$

$$M_{\max} = 57.1\text{ kN}\cdot\text{m}\ 1.125\text{ ft from } A ◀$$

$$M_C = M_{\max} + \frac{1}{2}V_C(2\text{ m} - x_1)$$

(*a*) $$M_C = 35\text{ kN}\cdot\text{m} - \frac{1}{2}(8\text{ kN})(0.5\text{ m}) = 33\text{ kN}\cdot\text{m}$$

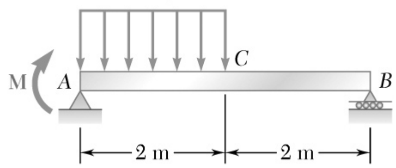
(*b*) $$M_C = 57.125\text{ kN}\cdot\text{m} - \frac{1}{2}(14\text{ kN})(0.875\text{ m}) = 51\text{ kN}\cdot\text{m}$$

M is piecewise linear along C, D, B, with $M_B = 0$ and $M_D = (1.5\text{ m})P$

(*a*) $$M_D = 21\text{ kN}\cdot\text{m}$$

(*b*) $$M_D = 30\text{ kN}\cdot\text{m}$$

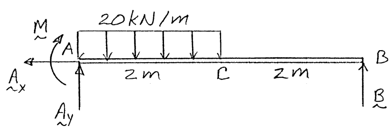
PROBLEM 7.75



For the beam shown, draw the shear and bending-moment diagrams, and determine the magnitude and location of the maximum absolute value of the bending moment knowing that (*a*) $M = 0$, (*b*) $M = 12\ \text{kN}\cdot\text{m}$.

SOLUTION

FBD Beam:



$$\circlearrowleft \Sigma M_A = 0\text{: } (4\text{ m})B - (1\text{ m})(20\text{ kN/m})(2\text{ m}) - M = 0$$

$$B = 10\text{ kN} + \frac{M}{4\text{ m}}$$

(*a*) $\mathbf{B} = 10\text{ kN} \uparrow$

(*b*) $\mathbf{B} = 13\text{ kN} \uparrow$

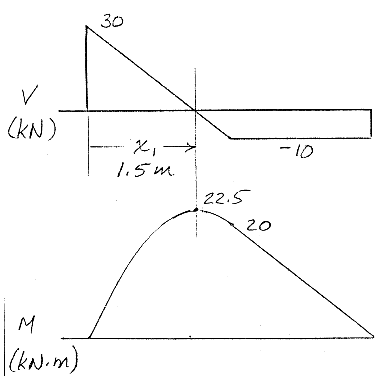
$$\uparrow \Sigma F_y = 0\text{: } A_y - (20\text{ kN/m})(2\text{ m}) + B = 0$$

$$A_y = 40\text{ kN} - B$$

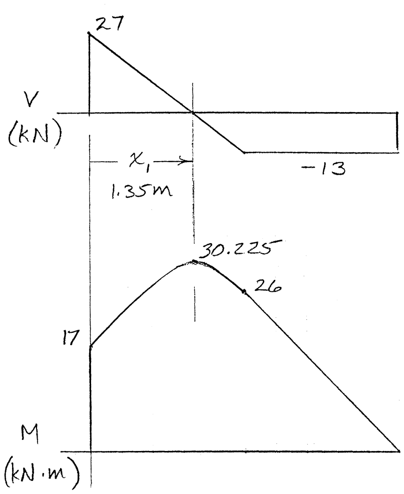
(*a*) $\mathbf{A}_y = 30\text{ kN} \uparrow$

(*b*) $\mathbf{A}_y = 27\text{ kN} \uparrow$

(*a*)



(*b*)



Shear Diags:

$V_A = A_y$, then V is linear $\left(\dfrac{dV}{dx} = -20\text{ kN/m}\right)$ to C.

$$V_C = A_y - (20\text{ kN/m})(2\text{ m}) = A_y - 40\text{ kN}$$

(*a*) $V_C = -10\text{ kN}$

(*b*) $V_C = -13\text{ kN}$

$$V = 0 = A_y - (20\text{ kN/m})x_1 \text{ at } x_1 = \frac{A_y\text{ m}}{20\text{ kN}}$$

(*a*) $x_1 = 1.5\text{ m}$

(*b*) $x_1 = 1.35\text{ m}$

V is constant from C to B.

PROBLEM 7.75 CONTINUED

Moment Diags:

$M_A = $ applied M. Then M is parabolic $\left(\dfrac{dM}{dx} \text{ decreases with } V\right)$

M is max where $V = 0$. $M_{\max} = M + \frac{1}{2}A_y x_1$.

(*a*) $$|M|_{\max} = \frac{1}{2}(30 \text{ kN})(1.5 \text{ m}) = 22.5 \text{ kN}\cdot\text{m} \blacktriangleleft$$

1.500 m from A ◀

(*b*) $$M_{\max} = 12 \text{ kN}\cdot\text{m} + \frac{1}{2}(27 \text{ kN})(1.35 \text{ m}) = 30.225 \text{ kN}\cdot\text{m} \blacktriangleleft$$

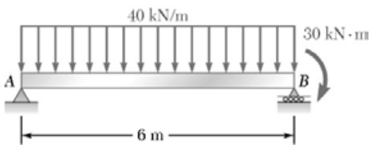
$|M|_{\max} = 30.2$ kN 1.350 m from A ◀

$$M_C = M_{\max} - \frac{1}{2}V_C(2 \text{ m} - x_1)$$

(*a*) $$M_C = 20 \text{ kN}\cdot\text{m}$$

(*b*) $$M_C = 26 \text{ kN}\cdot\text{m}$$

Finally, M is linear $\left(\dfrac{dM}{dx} = V_C\right)$ to zero at B.

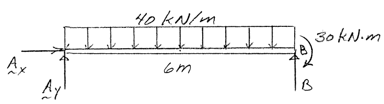


PROBLEM 7.76

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION

(*a*)



FBD Beam:

$$\circlearrowleft \Sigma M_B = 0: (3\text{ m})(40\text{ kN/m})(6\text{ m}) - (30\text{ kN}\cdot\text{m}) - (6\text{ m})A_y = 0$$

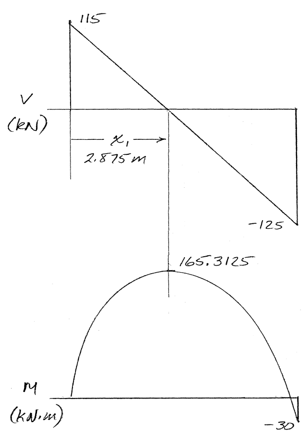
$$\mathbf{A}_y = 115\text{ kN} \uparrow$$

Shear Diag:

$V_A = A_y = 115\text{ kN},$ then V is linear $\left(\dfrac{dM}{dx} = -40\text{ kN/m}\right)$ to B.

$$V_B = 115\text{ kN} - (40\text{ kN/m})(6\text{ m}) = -125\text{ kN.}$$

$$V = 0 = 115\text{ kN} - (40\text{ kN/m})x_1 \text{ at } x_1 = 2.875\text{ m}$$



Moment Diag:

$M_A = 0.$ Then M is parabolic $\left(\dfrac{dM}{dx} \text{ decreasing with } V\right)$. Max M occurs where $V = 0$,

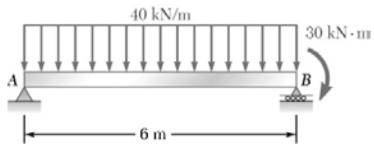
$$M_{\max} = \frac{1}{2}(115\text{ kN/m})(2.875\text{ m}) = 165.3125\text{ kN}\cdot\text{m}$$

$$M_B = M_{\max} - \frac{1}{2}(125\text{ kN})(6\text{ m} - x_1)$$

$$= 165.3125\text{ kN}\cdot\text{m} - \frac{1}{2}(125\text{ kN})(6 - 2.875)\text{m}$$

$$= -30\text{ kN}\cdot\text{m as expected.}$$

(*b*) $\quad |M|_{\max} = 165.3\text{ kN}\cdot\text{m}(2.88\text{ m from } A)$ ◀

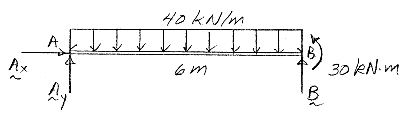


PROBLEM 7.77

Solve Prob. 7.76 assuming that the $30\ \text{kN}\cdot\text{m}$ couple applied at B is counterclockwise

SOLUTION

(*a*)

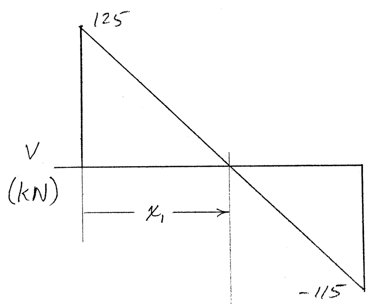


FBD Beam:

$$\circlearrowleft \Sigma M_B = 0: 30\ \text{kN}\cdot\text{m} + (3\ \text{m})(40\ \text{kN/m})(6\ \text{m}) - (6\ \text{m})A_y = 0$$

$$\mathbf{A}_y = 125\ \text{kN} \uparrow$$

Shear Diag:

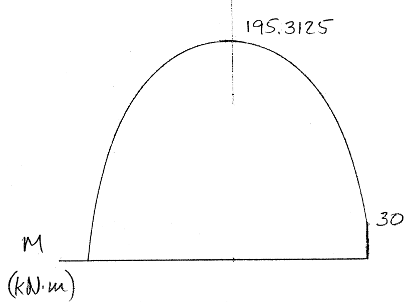


$$V_A = A_y = 125\ \text{kN},\ V \text{ is linear } \left(\frac{dV}{dx} = -40\ \text{kN/m}\right) \text{ to } B.$$

$$V_B = 125\ \text{kN} - (40\ \text{kN/m})(6\ \text{m}) = -115\ \text{kN}$$

$$V = 0 = 115\ \text{kN} - (40\ \text{kN/m})x_1 \text{ at } x_1 = 3.125\ \text{m}$$

Moment Diag:



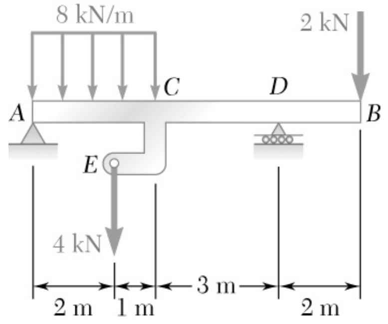
$M_A = 0.$ Then M is parabolic $\left(\dfrac{dM}{dx} \text{ decreases with } V\right)$. Max M occurs where $V = 0$,

$$M_{\max} = \frac{1}{2}(125\ \text{kN})(3.125\ \text{m}) = 195.3125\ \text{kN}\cdot\text{m}$$

(*b*) $\quad |M|_{\max} = 195.3\ \text{kN}\cdot\text{m}(3.125\ \text{m from } A)$ ◀

$$M_B = 195.3125\ \text{kN}\cdot\text{m} - \frac{1}{2}(115\ \text{kN})(6 - 3.125)\text{m}$$

$$M_B = 30\ \text{kN}\cdot\text{m as expected.}$$

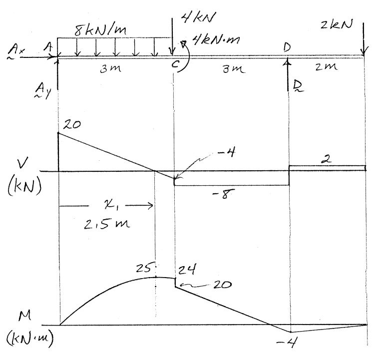


PROBLEM 7.78

For beam AB, (a) draw the shear and bending-moment diagrams, (b) determine the magnitude and location of the maximum absolute value of the bending moment.

SOLUTION

(a)
Replacing the load at E with equivalent force-couple at C:



$$\circlearrowleft \Sigma M_A = 0\colon (6\text{ m})D - (8\text{ m})(2\text{ kN}) - (3\text{ m})(4\text{ kN})$$
$$- (1.5\text{ m})(8\text{ kN/m})(3\text{ m}) - 4\text{ kN}\cdot\text{m} = 0$$

$$\mathbf{D} = 10\text{ kN} \uparrow$$

$$\uparrow \Sigma F_y = 0\colon A_y + 10\text{ kN} - 2\text{ kN} - 4\text{ kN} - (8\text{ kN/m})(3\text{ m}) = 0$$

$$\mathbf{A}_y = 20\text{ kN} \uparrow$$

Shear Diag:

$V_A = A_y = 20\text{ kN}$, then V is linear $\left(\dfrac{dV}{dx} = -8\ \text{kN/m}\right)$ to C.

$$V_C = 20\text{ kN} - (8\text{ kN/m})(3\text{ m}) = -4\text{ kN}$$

$$V = 0 = 20\text{ kN} - (8\text{ kN/m})x_1 \text{ at } x_1 = 2.5\text{ m}$$

At C, V decreases by 4 kN to -8 kN.

At D, V increases by 10 kN to 2 kN.

Moment Diag:

$M_A = 0$, then M is parabolic $\left(\dfrac{dM}{dx} \text{ decreasing with } V\right)$. Max M occurs where $V = 0$.

$$M_{\text{max}} = \frac{1}{2}(20\text{ kN})(2.5\text{ m}) = 25\text{ kN}\cdot\text{m}$$

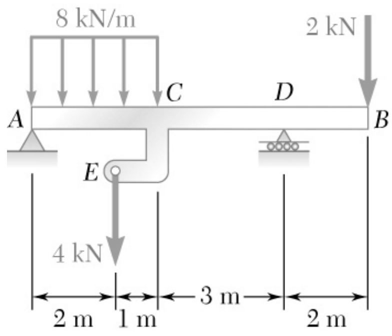
(b) $\quad M_{\text{max}} = 25.0\ \text{kN}\cdot\text{m}$, 2.50 m from A ◀

PROBLEM 7.78 CONTINUED

$$M_C = 25\text{ kN}\cdot\text{m} - \frac{1}{2}(4\text{ kN})(0.5\text{ m}) = 24\text{ kN}\cdot\text{m}.$$

At *C*, *M* decreases by $4\text{ kN}\cdot\text{m}$ to $20\text{ kN}\cdot\text{m}$. From *C* to *B*, *M* is piecewise linear with $\dfrac{dM}{dx} = -8\text{ kN}$ to *D*, then $\dfrac{dM}{dx} = +2\text{ kN}$ to *B*.

$$M_D = 20\text{ kN}\cdot\text{m} - (8\text{ kN})(3\text{ m}) = -4\text{ kN}\cdot\text{m}. \quad M_B = 0$$

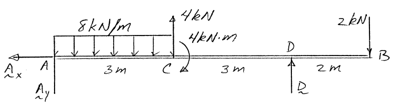


PROBLEM 7.79

Solve Prob. 7.78 assuming that the 4-kN force applied at E is directed upward.

SOLUTION

(*a*)
Replacing the load at E with equivalent force-couple at C.

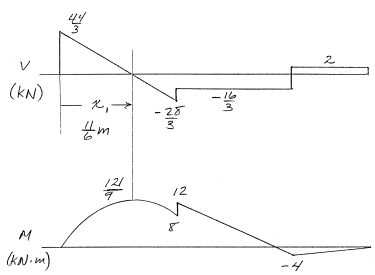


$$\circlearrowleft \Sigma M_A = 0: (6\text{ m})D - (8\text{ m})(2\text{ kN}) + (3\text{ m})(4\text{ kN})$$
$$- 4\text{ kN}\cdot\text{m} - (1.5\text{ m})(8\text{ kN/m})(3\text{ m}) = 0$$

$$\mathbf{D} = \frac{22}{3}\text{ kN} \uparrow$$

$$\uparrow \Sigma F_y = 0: A_y + \frac{22}{3}\text{ kN} - (8\text{ kN/m})(3\text{ m}) + 4\text{ kN} - 2\text{ kN} = 0$$

$$\mathbf{A}_y = \frac{44}{3}\text{ kN} \uparrow$$



Shear Diag:

$$V_A = A_y = \frac{44}{3}\text{ kN, then } V \text{ is linear } \left(\frac{dV}{dx} = -8\text{ kN/m}\right) \text{ to } C.$$

$$V_C = \frac{44}{3}\text{ kN} - (8\text{ kN/m})(3\text{ m}) = -\frac{28}{3}\text{ kN}$$

$$V = 0 = \frac{44}{3}\text{ kN} - (8\text{ kN/m})x_1 \text{ at } x_1 = \frac{11}{6}\text{ m.}$$

At C, V increases 4 kN to $-\dfrac{16}{3}$ kN.

At D, V increases $\dfrac{22}{3}$ kN to 2 kN.

PROBLEM 7.79 CONTINUED

Moment Diag:

$M_A = 0.$ Then M is parabolic $\left(\dfrac{dM}{dx} \text{ decreasing with } V\right)$. Max M occurs where $V = 0.$

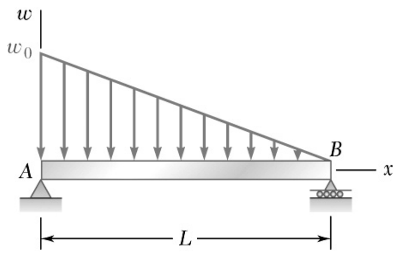
$$M_{\max} = \frac{1}{2}\left(\frac{44}{3}\text{ kN}\right)\left(\frac{11}{6}\text{ m}\right) = \frac{121}{9}\text{ kN}\cdot\text{m}$$

(*b*) $M_{\max} = 13.44\ \text{kN}\cdot\text{m}$ at 1.833 m from A ◀

$$M_C = \frac{121}{9}\text{ kN}\cdot\text{m} - \frac{1}{2}\left(\frac{28}{3}\text{ kN}\right)\left(\frac{7}{6}\text{ m}\right) = 8\text{ kN}\cdot\text{m}.$$

At C, M increases by $4\text{ kN}\cdot\text{m}$ to $12\text{ kN}\cdot\text{m}$. Then M is linear $\left(\dfrac{dM}{dx} = -\dfrac{16}{3}\text{ kN}\right)$ to D.

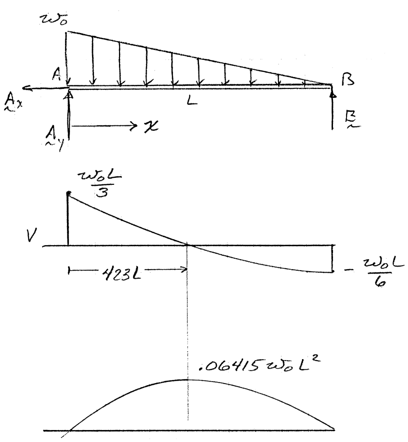
$M_D = 12\text{ kN}\cdot\text{m} - \left(\dfrac{16}{3}\text{ kN}\right)(3\text{ m}) = -4\text{ kN}\cdot\text{m}.$ M is again linear $\left(\dfrac{dM}{dx} = 2\text{ kN}\right)$ to zero at B.



PROBLEM 7.80

For the beam and loading shown, (*a*) derive the equations of the shear and bending-moment curves, (*b*) draw the shear and bending-moment diagrams, (*c*) determine the magnitude and location of the maximum bending moment.

SOLUTION



Distributed load $w = w_0\left(1 - \frac{x}{L}\right) \qquad \left(\text{total} = \frac{1}{2}w_0L\right)$

$$\Sigma M_A = 0: \frac{L}{3}\left(\frac{1}{2}w_0L\right) - LB = 0 \qquad \mathbf{B} = \frac{w_0L}{6}\uparrow$$

$$\uparrow \Sigma F_y = 0: A_y - \frac{1}{2}w_0L + \frac{w_0L}{6} = 0 \qquad \mathbf{A}_y = \frac{w_0L}{3}\uparrow$$

Shear:

$$V_A = A_y = \frac{w_0L}{3},$$

Then $$\frac{dV}{dx} = -w \rightarrow V = V_A - \int_0^x w_0\left(1 - \frac{x}{L}\right)dx$$

$$V = \left(\frac{w_0L}{3}\right) - w_0x + \frac{1}{2}\frac{w_0}{L}x^2 = w_0L\left[\frac{1}{3} - \frac{x}{L} + \frac{1}{2}\left(\frac{x}{L}\right)^2\right]$$

Note: At $x = L,\ V = -\frac{w_0L}{6};$

$V = 0$ at $\left(\frac{x}{L}\right)^2 - 2\left(\frac{x}{L}\right) + \frac{2}{3} = 0 \rightarrow \frac{x}{L} = 1 - \sqrt{\frac{1}{3}}$

Moment:

$$M_A = 0,$$

Then $$\left(\frac{dM}{dx}\right) = V \rightarrow M = \int_0^x V dx = L\int_0^{x/L} V\left(\frac{x}{L}\right)d\left(\frac{x}{L}\right)$$

$$M = w_0L^2\int_0^{x/L}\left[\frac{1}{3} - \frac{x}{L} + \frac{1}{2}\left(\frac{x}{L}\right)^2\right]d\left(\frac{x}{L}\right)$$

$$M = w_0L^2\left[\frac{1}{3}\left(\frac{x}{L}\right) - \frac{1}{2}\left(\frac{x}{L}\right)^2 + \frac{1}{6}\left(\frac{x}{L}\right)^3\right]$$

PROBLEM 7.80 CONTINUED

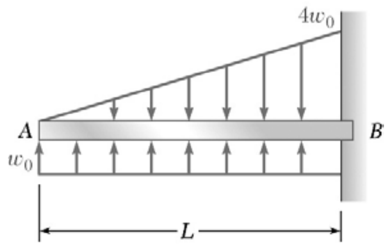
$$M_{\max}\left(\text{at } \frac{x}{L} = 1 - \sqrt{\frac{1}{3}}\right) = 0.06415 w_0 L^2$$

$$(a) \qquad V = w_0 L\left[\frac{1}{3} - \frac{x}{L} + \frac{1}{2}\left(\frac{x}{L}\right)^2\right] \blacktriangleleft$$

$$M = w_0 L^2\left[\frac{1}{3}\left(\frac{x}{L}\right) - \frac{1}{2}\left(\frac{x}{L}\right)^2 + \frac{1}{6}\left(\frac{x}{L}\right)^3\right] \blacktriangleleft$$

$$(c) \qquad M_{\max} = 0.0642\, w_0 L^2 \blacktriangleleft$$

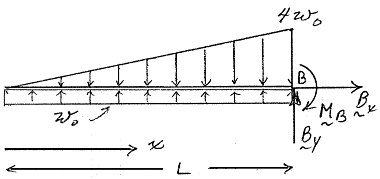
$$\text{at } x = 0.423L \blacktriangleleft$$



PROBLEM 7.81

For the beam and loading shown, (a) derive the equations of the shear and bending-moment curves, (b) draw the shear and bending-moment diagrams, (c) determine the magnitude and location of the maximum bending moment.

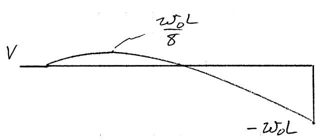
SOLUTION



Distributed load $\qquad w = w_0\left[4\left(\frac{x}{L}\right) - 1\right]$

Shear: $\qquad \frac{dV}{dx} = -w, \text{ and } V(0) = 0, \text{ so}$

$$V = \int_0^x - w dx = - \int_0^{x/L} Lwd\left(\frac{x}{L}\right)$$



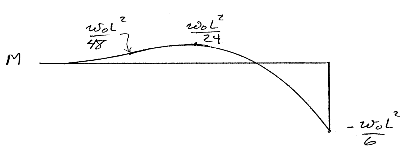
$$V = \int_0^{x/L} w_0L\left[1 - 4\left(\frac{x}{L}\right)\right]d\left(\frac{x}{L}\right) = w_0L\left[\left(\frac{x}{L}\right) - 2\left(\frac{x}{L}\right)^2\right]$$

Notes: At $x = L, V = -w_0L$

And $V = 0$ at $\qquad \left(\frac{x}{L}\right) = 2\left(\frac{x}{L}\right)^2 \quad \text{or} \quad \frac{x}{L} = \frac{1}{2}$

Also V is max where $w = 0 \left(\frac{x}{L} = \frac{1}{4}\right)$

$$V_{max} = \frac{1}{8}w_0L$$



Moment: $\qquad M(0) = 0, \frac{dM}{dx} = V$

$$M = \int_0^x vdx = L\int_0^{x/L} V\left(\frac{x}{L}\right)d\left(\frac{x}{L}\right)$$

$$M = w_0L^2\int_0^{x/L} \left[\left(\frac{x}{L}\right) - 2\left(\frac{x}{L}\right)^2\right]d\left(\frac{x}{L}\right)$$

(a) $\qquad V = w_0L\left[\left(\frac{x}{L}\right) - 2\left(\frac{x}{L}\right)^2\right]$ ◀

$$M = w_0L^2\left[\frac{1}{2}\left(\frac{x}{L}\right)^2 - \frac{2}{3}\left(\frac{x}{L}\right)^3\right]$$ ◀

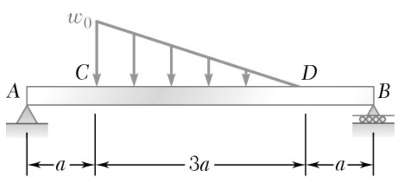
PROBLEM 7.81 CONTINUED

$$M_{\max} = \frac{1}{24} w_0 L^2 \text{ at } \quad x = \frac{L}{2}$$

$$M_{\min} = -\frac{1}{6} w_0 L^2 \text{ at } x = L$$

$$M_{\max} = \frac{w_0 L^2}{24} \text{ at } x = \frac{L}{2}$$

$$(c) \qquad |M|_{\max} = -M_{\min} = \frac{w_0 L^2}{6} \text{ at } B \blacktriangleleft$$

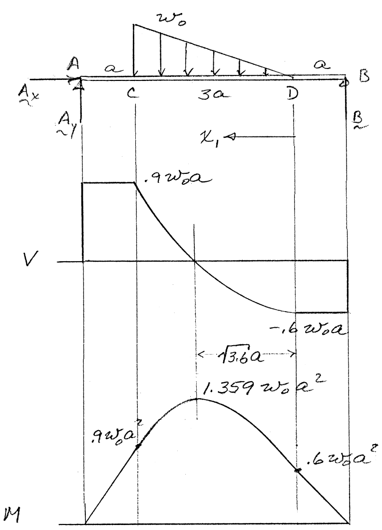


PROBLEM 7.82

For the beam shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the magnitude and location of the maximum bending moment. (*Hint:* Derive the equations of the shear and bending-moment curves for portion *CD* of the beam.)

SOLUTION

(*a*)



FBD Beam:

$$\circlearrowleft \Sigma M_B = 0: \ (3a)\left[\frac{1}{2}w_0(3a)\right] - 5aA_y = 0 \qquad \mathbf{A}_y = 0.9w_0a \uparrow$$

$$\uparrow \Sigma F_y = 0: \ 0.9w_0a - \frac{1}{2}w_0(3a) + B = 0$$

$$\mathbf{B} = 0.6w_0a \uparrow$$

Shear Diag:

$V = A_y = 0.9w_0a$ from A to C and $V = B = -0.6w_0a$ from B to D.

Then from D to C, $w = w_0\frac{x_1}{3a}$. If x_1 is measured right to left,

$\frac{dV}{dx_1} = +w$ and $\frac{dM}{dx_1} = -V$. So, from D, $V = -0.6w_0a + \int_0^{x_1}\frac{w_0}{3a}x_1dx_1$,

$$V = w_0a\left[-0.6 + \frac{1}{6}\left(\frac{x_1}{a}\right)^2\right]$$

$$\text{Note: } V = 0 \text{ at } \left(\frac{x_1}{a}\right)^2 = 3.6, \ x_1 = \sqrt{3.6}a$$

Moment Diag:

$M = 0$ at A, increasing linearly $\left(\frac{dM}{dx_1} = 0.9w_0a\right)$ to $M_C = 0.9w_0a^2$.

Similarly $M = 0$ at B increasing linearly $\left(\frac{dM}{dx} = 0.6w_0a\right)$ to

$M_D = 0.6w_0a^2$. Between C and D

$$M = 0.6w_0a^2 + w_0a\int_0^{x_1}\left[0.6 - \frac{1}{6}\left(\frac{x_1}{a}\right)^2\right]dx_1,$$

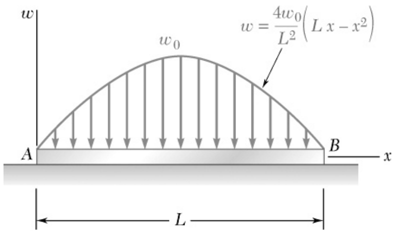
$$M = w_0a^2\left[0.6 + 0.6\left(\frac{x_1}{a}\right) - \frac{1}{18}\left(\frac{x_1}{a}\right)^3\right]$$

PROBLEM 7.82 CONTINUED

(*b*)

At $\dfrac{x_1}{a} = \sqrt{3.6}, \; M = M_{\text{max}} = 1.359 w_0 a^2$ ◀

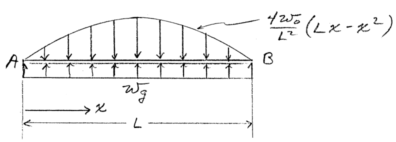
$x_1 = 1.897a$ left of D ◀



PROBLEM 7.83

Beam *AB*, which lies on the ground, supports the parabolic load shown. Assuming the upward reaction of the ground to be uniformly distributed, (*a*) write the equations of the shear and bending-moment curves, (*b*) determine the maximum bending moment.

SOLUTION



(*a*) $$+\uparrow \Sigma F_y = 0: \; w_g L - \int_0^L \frac{4w_0}{L^2}\left(Lx - x^2\right)dx = 0$$

$$w_g L = \frac{4w_0}{L^2}\left(\frac{1}{2}LL^2 - \frac{1}{3}L^3\right) = \frac{2}{3}w_0 L \qquad w_g = \frac{2w_0}{3}$$

Define $\xi = \dfrac{x}{L}$ so $d\xi = \dfrac{dx}{L} \rightarrow$ net load $w = 4w_0\left[\dfrac{x}{L} - \left(\dfrac{x}{L}\right)^2\right] - \dfrac{2}{3}w_0$

or $\quad w = 4w_0\left(-\dfrac{1}{6} + \xi - \xi^2\right)$

$$V = V(0) - \int_0^\xi 4w_0 L\left(-\frac{1}{6} + \xi - \xi^2\right)d\xi =$$

$$0 + 4w_0 L\left(\frac{1}{6}\xi + \frac{1}{2}\xi^2 - \frac{1}{3}\xi^3\right)$$

$$V = \frac{2}{3}w_0 L\left(\xi - 3\xi^2 + 2\xi^3\right) \blacktriangleleft$$

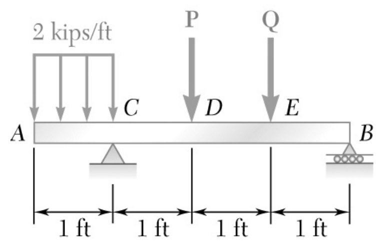
$$M = M_0 + \int_0^x V dx = 0 + \frac{2}{3}w_0 L^2 \int_0^\xi \left(\xi - 3\xi^2 + 2\xi^3\right)d\xi$$

$$= \frac{2}{3}w_0 L^2\left(\frac{1}{2}\xi^2 - \xi^3 + \frac{1}{2}\xi^4\right) = \frac{1}{3}w_0 L^2\left(\xi^2 - 2\xi^3 + \xi^4\right) \blacktriangleleft$$

(*b*) Max *M* occurs where $V = 0 \rightarrow 1 - 3\xi + 2\xi^2 = 0 \rightarrow \xi = \dfrac{1}{2}$

$$M\left(\xi = \frac{1}{2}\right) = \frac{1}{3}w_0 L^2\left(\frac{1}{4} - \frac{2}{8} + \frac{1}{16}\right) = \frac{w_0 L^2}{48}$$

$$M_{\max} = \frac{w_0 L^2}{48} \text{ at center of beam} \blacktriangleleft$$

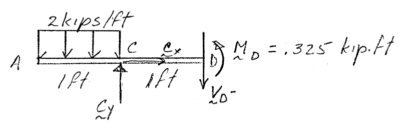


PROBLEM 7.84

The beam *AB* is subjected to the uniformly distributed load shown and to two unknown forces **P** and **Q**. Knowing that it has been experimentally determined that the bending moment is +325 lb ft at *D* and +800 lb ft at *E*, (*a*) determine **P** and **Q**, (*b*) draw the shear and bending-moment diagrams for the beam.

SOLUTION

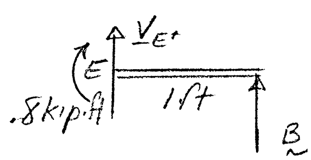
FBD ACD:



$$(a)\ \circlearrowleft\ \Sigma M_{D^-} = 0:\ 0.325\text{ kip}\cdot\text{ft} - (1\text{ ft})C_y + (1.5\text{ ft})(2\text{ kips/ft})(1\text{ ft}) = 0$$

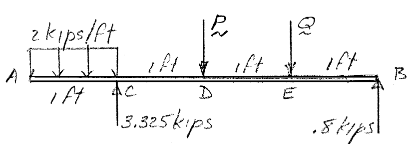
$$\mathbf{C}_y = 3.325\text{ kips}\uparrow$$

FBD EB:



$$\circlearrowleft\ \Sigma M_E = 0:\ (1\text{ ft})B - 0.8\text{ kip}\cdot\text{ft} = 0 \qquad \mathbf{B} = 0.8\text{ kip}\uparrow$$

FBD Beam:



$$\circlearrowleft\ \Sigma M_D = 0:\ (1.5\text{ ft})(2\text{ kips/ft})(1\text{ ft}) - (1\text{ ft})(3.325\text{ kips})$$

$$-(1\text{ ft})Q + 2\text{ ft}(0.8\text{ kips}) = 0$$

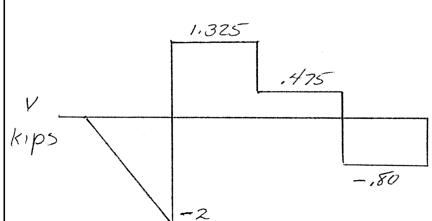
$$Q = 1.275\text{ kips}$$

$$\uparrow\ \Sigma F_y = 0:\ 3.325\text{ kips} + 0.8\text{ kips} - 1.275\text{ kips}$$

$$-(2\text{ kips/ft})(1\text{ft}) - P = 0 \qquad P = 0.85\text{ kip}\downarrow$$

(*a*) $\mathbf{P} = 850\text{ lb}\downarrow$ ◀

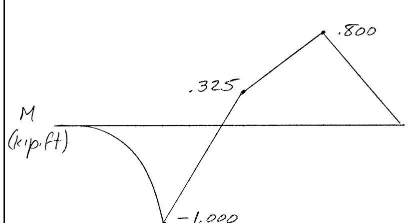
$\mathbf{Q} = 1.275\text{ kips}\downarrow$ ◀



(*b*) **Shear Diag:**

V is linear $\left(\dfrac{dV}{dx} = -2\text{ kips/ft}\right)$ from 0 at *A* to

$-(2\text{ kips/ft})(1\text{ ft}) = -2\text{ kips}$ at *C*. Then *V* is piecewise constant with discontinuities equal to forces at *C*, *D*, *E*, *B*



Moment Diag:

M is parabolic $\left(\dfrac{dM}{dx}\text{ decreasing with } V\right)$ from 0 at *A* to

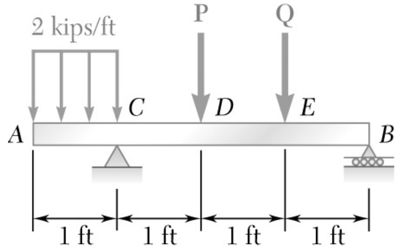
$-\dfrac{1}{2}(2\text{ kips})(1\text{ ft}) = -1\text{ kip}\cdot\text{ft}$ at *C*. Then *M* is piecewise linear with

PROBLEM 7.84 CONTINUED

$$M_D = -1\,\text{kip}\cdot\text{ft} + (1.325\,\text{kips})(1\,\text{ft}) = 0.325\,\text{kip}\cdot\text{ft}$$

$$M_E = 0.325\,\text{kip}\cdot\text{ft} + (0.475\,\text{kips})(1\,\text{ft}) = 0.800\,\text{kip}\cdot\text{ft}$$

$$M_B = 0.8\,\text{kip}\cdot\text{ft} - (0.8\,\text{kip})(1\,\text{ft}) = 0$$

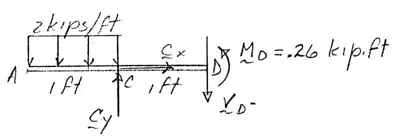


PROBLEM 7.85

Solve Prob. 7.84 assuming that the bending moment was found to be +260 lb ft at *D* and +860 lb ft at *E*.

SOLUTION

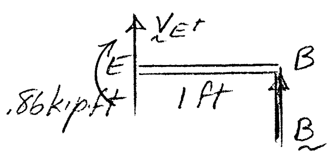
FBD ACD:



(*a*) $\Sigma M_D = 0\!: \; 0.26\,\text{kip}\cdot\text{ft} - (1\,\text{ft})C_y + (1.5\,\text{ft})(2\,\text{kips/ft})(1\,\text{ft}) = 0$

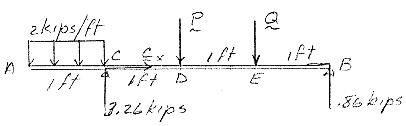
$$\mathbf{C}_y = 3.26\text{ kips} \uparrow$$

FBD DB:



$\Sigma M_E = 0\!: \; (1\,\text{ft})B - 0.86\,\text{kip}\cdot\text{ft} \qquad \mathbf{B} = 0.86\text{ kip} \uparrow$

FBD Beam:



$$\Sigma M_D = 0\!: (1.5\,\text{ft})(2\,\text{kips/ft})(1\,\text{ft})$$

$$- (1\,\text{ft})(3.26\,\text{kips}) + (1\,\text{ft})Q + (2\,\text{ft})(0.86\,\text{kips}) = 0$$

$$Q = 1.460\text{ kips} \qquad \mathbf{Q} = 1.460\text{ kips} \downarrow \blacktriangleleft$$

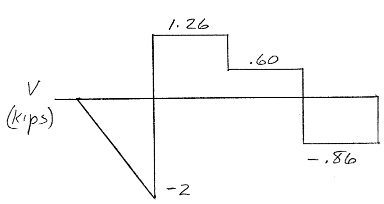
$$\Sigma F_y = 0\!: \; 3.26\,\text{kips} + 0.86\text{ kips} - 1.460\text{ kips}$$

$$-P - (2\,\text{kips/ft})(1\,\text{ft}) = 0$$

$$P = 0.66\text{ kips}$$

$$\mathbf{P} = 660\text{ lb} \downarrow \blacktriangleleft$$

(*b*) **Shear Diag:**

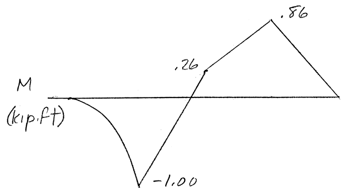


V is linear $\left(\dfrac{dV}{dx} = -2\text{ kips/ft}\right)$ from 0 at *A* to $-(2\,\text{kips/ft})(1\,\text{ft}) = -2\text{ kips}$ at *C*. Then *V* is piecewise constant with discontinuities equal to forces at *C*, *D*, *E*, *B*.

Moment Diag:

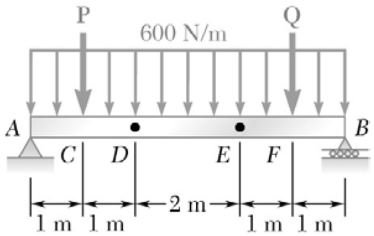
M is parabolic $\left(\dfrac{dM}{dx}\text{ decreasing with } V\right)$ from 0 at *A* to $-\dfrac{1}{2}(2\,\text{kips/ft})(1\,\text{ft}) = -1\,\text{kip}\cdot\text{ft}$ at *C*. Then *M* is piecewise linear with

PROBLEM 7.85 CONTINUED



$$M_0 = 0.26 \text{ kip}\cdot\text{ft}$$

$$M_E = 0.86 \text{ kip}\cdot\text{ft}, \quad M_B = 0$$

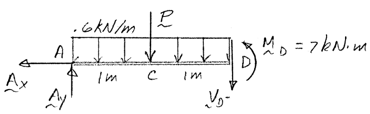


PROBLEM 7.86

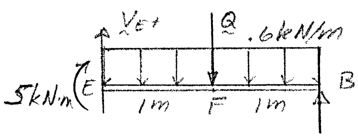
The beam *AB* is subjected to the uniformly distributed load shown and to two unknown forces **P** and **Q**. Knowing that it has been experimentally determined that the bending moment is $+7\text{ kN}\cdot\text{m}$ at *D* and $+5\text{ kN}\cdot\text{m}$ at *E*, (*a*) determine **P** and **Q**, (*b*) draw the shear and bending-moment diagrams for the beam.

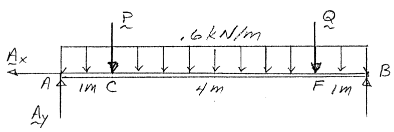
SOLUTION

FBD AD:



FBD EB:





(*a*)

$$\circlearrowleft\ \Sigma M_D = 0:\quad 7\text{ kN}\cdot\text{m} + (1\text{ m})(0.6\text{ kN/m})(2\text{ m}) - (2\text{ m})A_y = 0$$

$$2A_y - P = 8.2\text{ kN} \qquad (1)$$

$$\circlearrowleft\ \Sigma M_E = 0:\ (2\text{ m})B - (1\text{ m})Q - (1\text{ m})(0.6\text{ kN/m})(2\text{ m}) - 5\text{ kN}\cdot\text{m} = 0$$

$$2B - Q = 6.2\text{ kN} \qquad (2)$$

$$\circlearrowleft\ \Sigma M_A = 0:\quad (6\text{ m})B - (1\text{ m})P - (5\text{ m})Q - (3\text{ m})(0.6\text{ kN/m})(6\text{ m}) = 0$$

$$6B - P - 5Q = 10.8\text{ kN} \qquad (3)$$

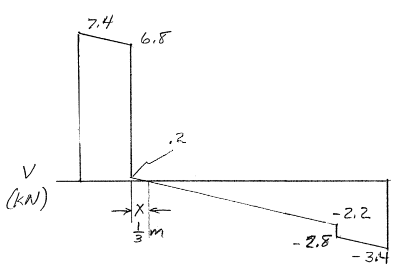
$$\circlearrowleft\ \Sigma M_B = 0:\ (1\text{ m})Q + (5\text{ m})P + (3\text{ m})(0.6\text{ kN/m})(6\text{ m}) - (6\text{ m})A = 0$$

$$6A - Q - 5P = 10.8\text{ kN} \qquad (4)$$

Solving (1)–(4): $\mathbf{P} = 6.60\text{ kN}\downarrow,\ \mathbf{Q} = 600\text{ N}\downarrow$ ◀

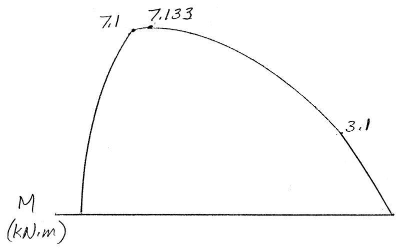
$$\mathbf{A}_y = 7.4\text{ kN}\uparrow, \quad \mathbf{B} = 3.4\text{ kN}\uparrow$$

(*b*) **Shear Diag:**



V is piecewise linear with $\dfrac{dV}{dx} = -0.6\text{ kN/m}$ throughout, and discontinuities equal to forces at *A*, *C*, *F*, *B*.

Note $\quad V = 0 = 0.2\text{ kN} - (0.6\text{ kN/m})x \ \text{ at }\ x = \dfrac{1}{3}\text{ m}$



PROBLEM 7.86 CONTINUED

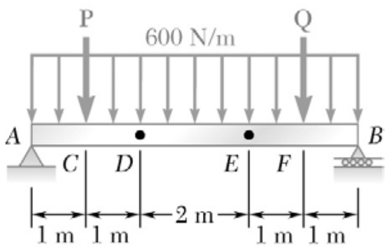
Moment Diag:

M is piecewise parabolic $\left(\dfrac{dM}{dx}\text{ decreasing with } V\right)$ with "breaks" in slope at C and F.

$$M_C = \frac{1}{2}(7.4 + 6.8)\,\text{kN}(1\,\text{m}) = 7.1\,\text{kN}\cdot\text{m}$$

$$M_{\text{max}} = 7.1\,\text{kN}\cdot\text{m} + \frac{1}{2}(0.2\,\text{kN})\left(\frac{1}{3}\,\text{m}\right) = 7.13\dot{3}\,\text{kN}\cdot\text{m}$$

$$M_F = 7.13\dot{3}\,\text{kN}\cdot\text{m} - \frac{1}{2}(2.2\,\text{kN})\left(3\frac{2}{3}\,\text{m}\right) = 3.1\,\text{kN}\cdot\text{m}$$

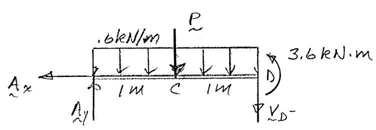


PROBLEM 7.87

Solve Prob. 7.86 assuming that the bending moment was found to be $+3.6\ \text{kN}\cdot\text{m}$ at D and $+4.14\ \text{kN}\cdot\text{m}$ at E.

SOLUTION

FBD AD:

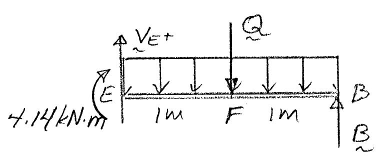


(*a*)

$$\circlearrowleft \Sigma M_D = 0\text{: } 3.6\,\text{kN}\cdot\text{m} + (1\,\text{m})P + (1\,\text{m})(0.6\ \text{kN/m})(2\,\text{m}) - (2\,\text{m})A_y = 0$$

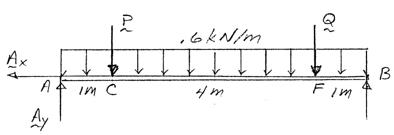
$$2A_y - P = 4.8\,\text{kN} \qquad (1)$$

FBD EB:



$$\circlearrowleft \Sigma M_E = 0\text{: } (2\,\text{m})B - (1\,\text{m})Q - (1\,\text{m})(0.6\ \text{kN/m})(2\,\text{m}) - 4.14\,\text{kN}\cdot\text{m} = 0$$

$$2B - Q = 5.34\,\text{kN} \qquad (2)$$



$$\circlearrowleft \Sigma M_A = 0\text{: } (6\,\text{m})B - (5\,\text{m})Q - (1\,\text{m})P - (3\,\text{m})(0.6\ \text{kN/m})(6\,\text{m}) = 0$$

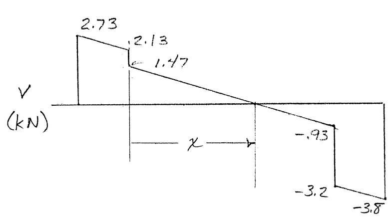
$$6B - P - 5Q = 10.8\,\text{kN} \qquad (3)$$

By symmetry:

$$6A - Q - 5P = 10.8\,\text{kN} \qquad (4)$$

Solving (1)–(4) $\qquad \mathbf{P} = 660\ \text{N}\downarrow,\ \mathbf{Q} = 2.28\ \text{kN}\downarrow$ ◀

$$A_y = 2.73\,\text{kN}\uparrow,\ B = 3.81\,\text{kN}\uparrow$$



(*b*) **Shear Diag:**

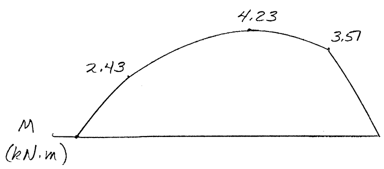
V is piecewise linear with $\left(\dfrac{dV}{dx} = -0.6\ \text{kN/m}\right)$ throughout, and discontinuities equal to forces at A, C, F, B.

Note that $V = 0 = 1.47\ \text{kN} - (0.6\ \text{kN/m})x$ at $x = 2.45\ \text{m}$

Moment Diag:

M is piecewise parabolic $\left(\dfrac{dM}{dx} \text{ decreasing with } V\right)$, with "breaks" in slope at C and F.

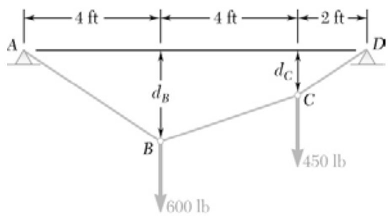
PROBLEM 7.87 CONTINUED



$$M_C = \frac{1}{2}(2.73 + 2.13)\text{kN}(1\text{ m}) = 2.43\text{ kN}\cdot\text{m}$$

$$M_{\max} = 2.43\text{ kN}\cdot\text{m} + \frac{1}{2}(1.47\text{ kN})(2.45\text{ m}) = 4.231\text{ kN}\cdot\text{m}$$

$$M_F = 4.231\text{ kN}\cdot\text{m} - \frac{1}{2}(0.93\text{ kN})(1.55\text{ m}) = 3.51\text{ kN}\cdot\text{m}$$

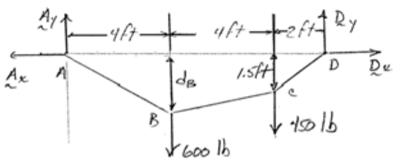


PROBLEM 7.88

Two loads are suspended as shown from cable *ABCD*. Knowing that $d_C = 1.5$ ft, determine (*a*) the distance d_B, (*b*) the components of the reaction at *A*, (*c*) the maximum tension in the cable.

SOLUTION

FBD cable:



$$\circlearrowleft \Sigma M_A = 0: (10\text{ ft})D_y - 8\text{ ft}(450\text{ lb}) - 4\text{ ft}(600\text{ lb}) = 0$$

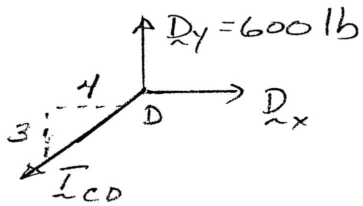
$$D_y = 600\text{ lb} \uparrow$$

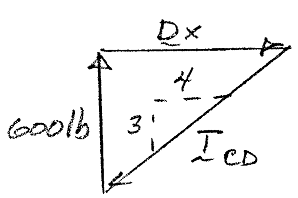
$$\uparrow \Sigma F_y = 0: A_y + 600\text{ lb} - 600\text{ lb} - 450\text{ lb} = 0$$

$$A_y = 450\text{ lb}$$

$$\longleftarrow \Sigma F_x = 0: \quad A_x - D_x = 0 \qquad (1)$$

FBD pt D:



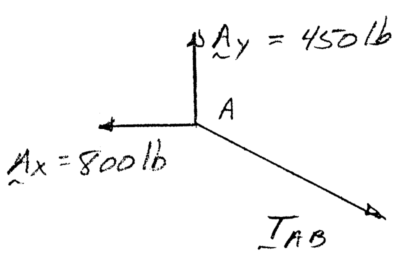


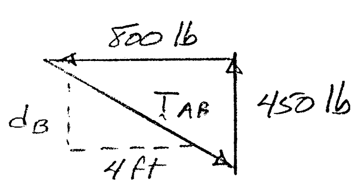
$$\frac{600\text{ lb}}{3} = \frac{D_x}{4} = \frac{T_{CD}}{5}: D_x = 800\text{ lb} \longrightarrow = A_x$$

So

And $$T_{CD} = 1000\text{ lb}$$

FBD pt A:





$$\frac{800\text{ lb}}{4\text{ ft}} = \frac{450\text{ lb}}{d_B}$$

(*a*) $$d_B = 2.25\text{ ft} \blacktriangleleft$$

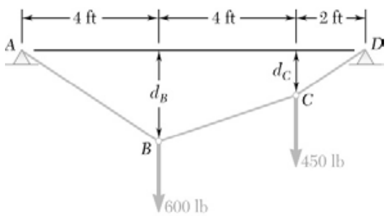
(b) $$\mathbf{A}_x = 800\text{ lb} \longleftarrow \blacktriangleleft$$

$$\mathbf{A}_y = 450\text{ lb} \uparrow \blacktriangleleft$$

$$T_{AB} = \sqrt{(800\text{ lb})^2 + (450\text{ lb})^2} = 918\text{ lb}$$

So (*c*) $$T_{\max} = T_{CD} = 1000\text{ lb} \blacktriangleleft$$

Note: T_{CD} is $T_{\max}$ as cable slope is largest in section *CD*.

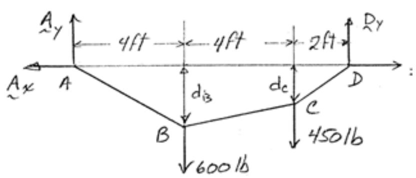


PROBLEM 7.89

Two loads are suspended as shown from cable *ABCD*. Knowing that the maximum tension in the cable is 720 lb, determine (*a*) the distance d_B, (*b*) the distance d_C.

SOLUTION

FBD cable:



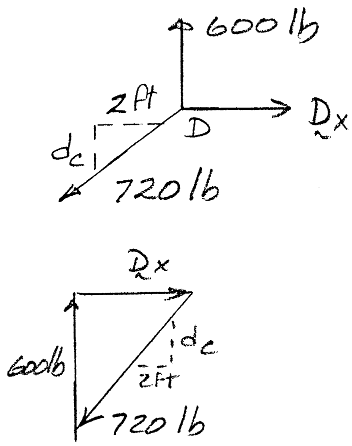
$$\circlearrowleft \Sigma M_A = 0\text{: } (10\text{ ft})D_y - (8\text{ ft})(450\text{ lb}) - (4\text{ ft})(600\text{ lb}) = 0$$

$$\mathbf{D}_y = 600\text{ lb} \uparrow$$

$$\uparrow \Sigma F_y = 0\text{: } A_y + 600\text{ lb} - 600\text{ lb} - 450\text{ lb} = 0$$

$$\mathbf{A}_y = 450\text{ lb} \uparrow$$

FBD pt D:



$$\longleftarrow \Sigma F_x = 0\text{: } A_x - B_x = 0$$

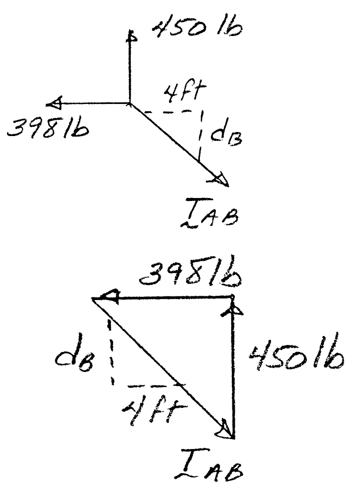
Since $A_x = B_x$; And $D_y > A_y$, Tension $T_{CD} > T_{AB}$

So $T_{CD} = T_{\max} = 720\text{ lb}$

$$D_x = \sqrt{(720\text{ lb})^2 - (600\text{ lb})^2} = 398\text{ lb} = A_x$$

$$\frac{d_C}{600\text{ lb}} = \frac{2\text{ ft}}{398\text{ lb}} \qquad d_C = 3.015\text{ ft}$$

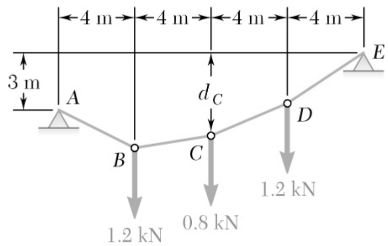
FBD pt. A:



$$\frac{d_B}{450\text{ lb}} = \frac{4\text{ ft}}{398\text{ lb}}$$

(*a*) $d_B = 4.52\text{ ft}$ ◀

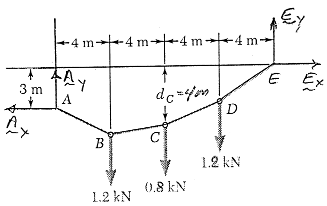
(*b*) $d_C = 3.02\text{ ft}$ ◀

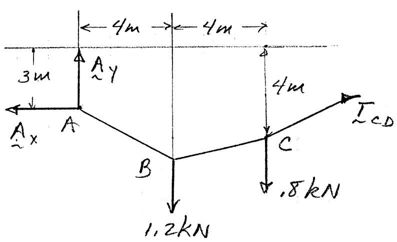


PROBLEM 7.90

Knowing that $d_C = 4$ m, determine (*a*) the reaction at *A*, (*b*) the reaction at *E*.

SOLUTION





(*a*) **FBD cable:**

$$\circlearrowleft \Sigma M_E = 0: \quad (4\text{ m})(1.2\text{ kN}) + (8\text{ m})(0.8\text{ kN}) + (12\text{ m})(1.2\text{ kN})$$

$$-(3\text{ m})A_x - (16\text{ m})A_y = 0$$

$$3A_x + 16A_y = 25.6\text{ kN} \qquad (1)$$

FBD ABC:

$$\circlearrowleft \Sigma M_C = 0: (4\text{ m})(1.2\text{ kN}) + (1\text{ m})A_x - (8\text{ m})A_y = 0$$

$$A_x - 8A_y = -4.8\text{ kN} \qquad (2)$$

Solving (1) and (2) $\qquad A_x = 3.2\text{ kN} \qquad A_y = 1\text{ kN}$

So $\mathbf{A} = 3.35\text{ kN} \measuredangle 17.35°$ ◀

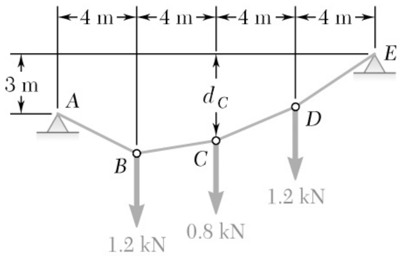
(*b*) cable: $\quad \rightarrow \Sigma F_x = 0: -A_x + E_x = 0$

$$E_x = A_x = 3.2\text{ kN}$$

$$\uparrow \Sigma F_y = 0: A_y - (1.2 + 0.8 + 1.2)\text{ kN} + E_y = 0$$

$$E_y = 3.2\text{ kN} - A_y = (3.2 - 1)\text{ kN} = 2.2\text{ kN}$$

So $\mathbf{E} = 3.88\text{ kN} \measuredangle 34.5°$ ◀

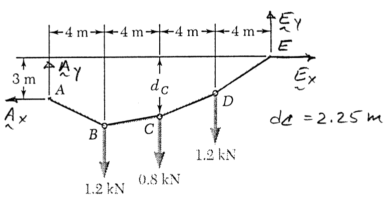


PROBLEM 7.91

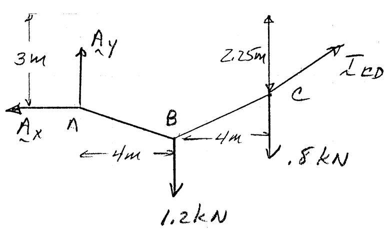
Knowing that $d_C = 2.25$ m, determine (*a*) the reaction at *A*, (*b*) the reaction at *E*.

SOLUTION

FBD Cable:



FBD ABC:



(*a*)

$$\circlearrowleft \Sigma M_E = 0: (4\text{ m})(1.2\text{ kN}) + (8\text{ m})(0.8\text{ kN}) + (12\text{ m})(1.2\text{ kN}) - (3\text{ m})A_x - (16\text{ m})A_y = 0$$

$$3A_x + 16A_y = 25.6\text{ kN} \qquad (1)$$

$$\circlearrowleft \Sigma M_C = 0: (4\text{ m})(1.2\text{ kN}) - (0.75\text{ m})A_x - (8\text{ m})A_y = 0$$

$$0.75A_x + 8A_y = 4.8\text{ kN} \qquad (2)$$

Solving (1) and (2) $\quad A_x = \dfrac{32}{3}\text{ kN}, \quad A_y = -0.4\text{ kN}$

So $\quad \mathbf{A} = 10.67\text{ kN} \measuredangle 2.15°$ ◀

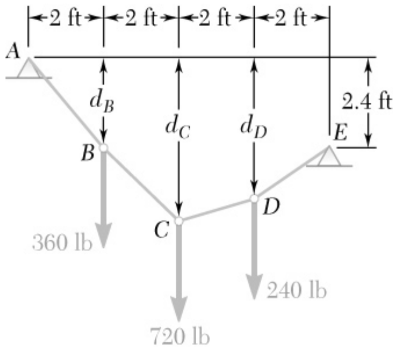
Note: this implies $d_B < 3$ m (in fact $d_B = 2.85$ m)

(*b*) FBD cable: $\quad \rightarrow \Sigma F_x = 0: -\dfrac{32}{3}\text{ kN} + E_x = 0 \qquad E_x = \dfrac{32}{3}\text{ kN}$

$$\uparrow \Sigma F_y = 0: \quad -0.4\text{ kN} - 1.2\text{ kN} - 0.8\text{ kN} - 1.2\text{ kN} + E_y = 0$$

$$E_y = 3.6\text{ kN}$$

$\mathbf{E} = 11.26\text{ kN} \measuredangle 18.65°$ ◀

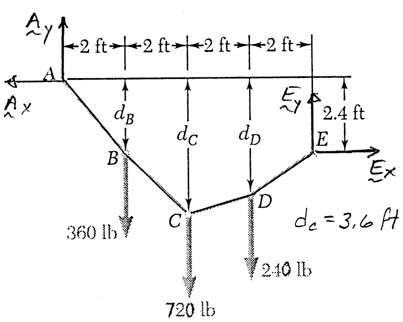


PROBLEM 7.92

Cable *ABCDE* supports three loads as shown. Knowing that $d_C = 3.6$ ft, determine (*a*) the reaction at *E*, (*b*) the distances d_B and d_D.

SOLUTION

FBD Cable:

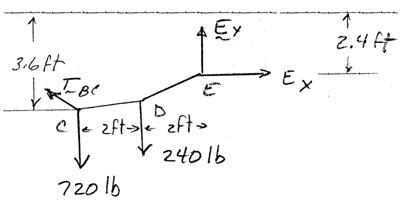


(*a*)

$$\circlearrowleft \Sigma M_A = 0: (2.4\text{ ft})E_x + (8\text{ ft})E_y - (2\text{ ft})(360) - (4\text{ ft})(720\text{ lb}) - (6\text{ ft})(240\text{ lb}) = 0$$

$$0.3E_x + E_y = 630\text{ lb} \qquad (1)$$

FBD CDE:



$$\circlearrowleft \Sigma M_C = 0: -(1.2\text{ ft})E_x + (4\text{ ft})E_y - (2\text{ ft})(240\text{ lb}) = 0$$

$$-0.3E_x + E_y = +120\text{ lb} \qquad (2)$$

Solving (1) and (2) $\qquad E_x = 850\text{ lb} \qquad E_y = 375\text{ lb}$

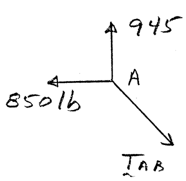
(*a*) $\qquad \mathbf{E} = 929\text{ lb} \measuredangle 23.8° \blacktriangleleft$

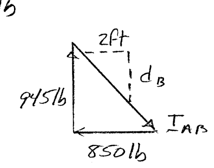
(*b*) cable: $\qquad \xrightarrow{} \Sigma F_x = 0: -A_x + E_x = 0 \qquad A_x = E_x = 850\text{ lb}$

$$\uparrow \Sigma F_y = 0: A_y - 360\text{ lb} - 720\text{ lb} - 240\text{ lb} + 375\text{ lb} = 0$$

$$A_y = 945\text{ lb}$$

Point A:





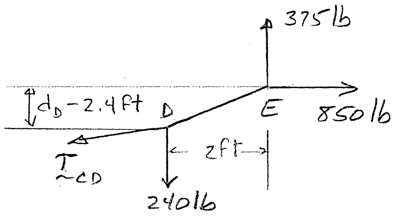
$$\frac{d_B}{2\text{ ft}} = \frac{945\text{ lb}}{850\text{ lb}} \qquad d_B = 2.22\text{ ft} \blacktriangleleft$$

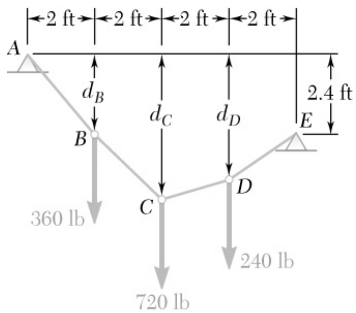
PROBLEM 7.92 CONTINUED

$$\circlearrowleft \Sigma M_D = 0: (2 \text{ ft})(375 \text{ lb}) - (d_D - 2.4 \text{ ft})(850 \text{ lb}) = 0$$

$$d_D = 3.28 \text{ ft} \blacktriangleleft$$

Segment DE:



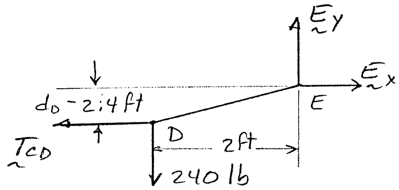


PROBLEM 7.93

Cable *ABCDE* supports three loads as shown. Determine (*a*) the distance d_C for which portion *CD* of the cable is horizontal, (*b*) the corresponding reactions at the supports.

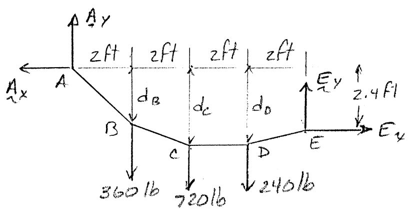
SOLUTION

Segment DE:



$$\uparrow \Sigma F_y = 0: E_y - 240 \text{ lb} = 0 \qquad \mathbf{E}_y = 240 \text{ lb} \uparrow$$

FBD Cable:



$$\circlearrowleft \Sigma M_A = (2.4 \text{ ft})E_x + (8 \text{ ft})(240 \text{ lb}) - (6 \text{ ft})(240 \text{ lb}) - (4 \text{ ft})(720 \text{ lb}) - (2 \text{ ft})(360 \text{ lb}) = 0$$

$$\mathbf{E}_x = 1300 \text{ lb} \longrightarrow$$

So

From Segment DE:

$$\circlearrowleft \Sigma M_D = 0: (2 \text{ ft})E_y - (d_C - 2.4 \text{ ft})E_x = 0$$

$$d_C = 2.4 \text{ ft} + \frac{E_y}{E_x}(2 \text{ ft}) = (2.4 \text{ ft}) + \frac{240 \text{ lb}}{1300 \text{ lb}}(2 \text{ ft}) = 2.7692 \text{ ft}$$

(*a*) $d_C = 2.77$ ft ◀

From FBD Cable:

$$\longrightarrow \Sigma F_x = 0: \quad -A_x + E_x = 0 \qquad \mathbf{A}_x = 1300 \text{ lb} \longleftarrow$$

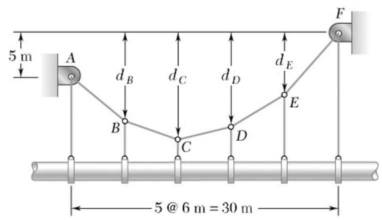
$$\uparrow \Sigma F_y = 0: \; A_y - 360 \text{ lb} - 720 \text{ lb} - 240 \text{ lb} + E_y = 0$$

$$\mathbf{A}_y = 1080 \text{ lb} \uparrow$$

(*b*) $\mathbf{A} = 1.690$ kips ⦪ 39.7° ◀

$\mathbf{E} = 1.322$ kips ∠ 10.46° ◀

PROBLEM 7.94

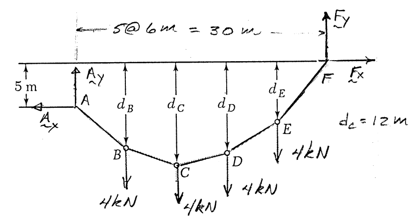


An oil pipeline is supported at 6-m intervals by vertical hangers attached to the cable shown. Due to the combined weight of the pipe and its contents, the tension in each hanger is 4 kN. Knowing that $d_C = 12$ m, determine (*a*) the maximum tension in the cable, (*b*) the distance d_D.

SOLUTION

Note: $\mathbf{A}_y$ and $\mathbf{F}_y$ shown are forces on cable, assuming the 4 kN loads at *A* and *F* act on supports.

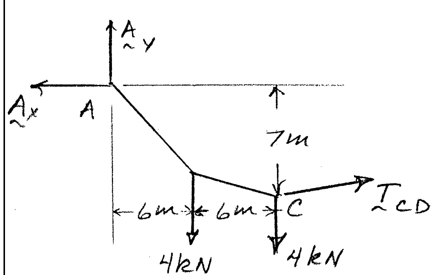
FBD Cable:



$$\circlearrowleft \Sigma M_F = 0: (6 \text{ m})\big[1(4 \text{ kN}) + 2(4 \text{ kN}) + 3(4 \text{ kN}) + 4(4 \text{ kN})\big]$$
$$-(30 \text{ m})A_y - (5 \text{ m})A_x = 0$$

$$A_x + 6A_y = 48 \text{ kN} \qquad (1)$$

FBD ABC:



$$\circlearrowleft \Sigma M_C = 0: (6 \text{ m})(4 \text{ kN}) + (7 \text{ m})A_x - (12 \text{ m})A_y = 0$$

$$7A_x - 12A_y = -24 \text{ kN} \qquad (2)$$

Solving (1) and (2) $\qquad \mathbf{A}_x = 8 \text{ kN} \longrightarrow \qquad \mathbf{A}_y = \dfrac{20}{3} \text{ kN} \uparrow$

From FBD Cable:

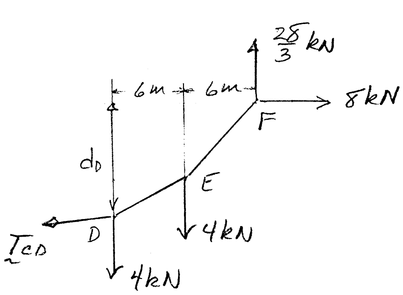
$$\longrightarrow \Sigma F_x = 0: -A_x + F_x = 0 \qquad F_x = A_x = 8 \text{ kN}$$

$$\uparrow \Sigma F_y = 0: A_y - 4(4 \text{ kN}) + F_y = 0$$

$$F_y = 16 \text{ kN} - A_y = \left(16 - \frac{20}{3}\right)\text{kN} = \frac{28}{3} \text{ kN} > A_y$$

So $\qquad T_{EF} > T_{AB} \qquad T_{\max} = T_{EF} = \sqrt{F_x^2 + F_y^2}$

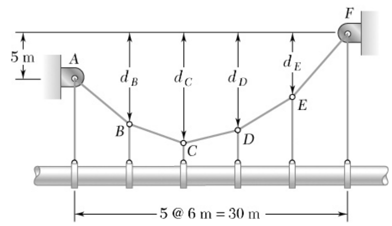
FBD DEF:



(*a*) $\qquad T_{\max} = \sqrt{(18 \text{ kN})^2 + \left(\dfrac{28}{3} \text{ kN}\right)^2} = 12.29 \text{ kN}$ ◀

$$\circlearrowleft \Sigma M_D = 0: (12 \text{ m})\left(\frac{28}{3} \text{ kN}\right) - d_D(8 \text{ kN}) - (6 \text{ m})(4 \text{ kN}) = 0$$

(*b*) $\qquad d_D = 11.00 \text{ m}$ ◀

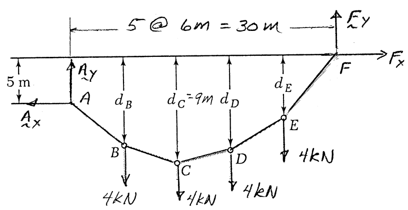


PROBLEM 7.95

Solve Prob. 7.94 assuming that $d_C = 9$ m.

SOLUTION

FBD Cable:



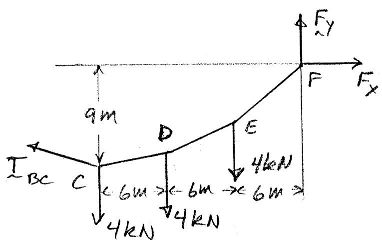
Note: 4 kN loads at A and F act directly on supports, not on cable.

$$\circlearrowleft \Sigma M_A = 0: (30 \text{ m})F_y - (5 \text{ m})F_x$$

$$-(6 \text{ m})\left[1(4 \text{ kN}) + 2(4 \text{ kN}) + 3(4 \text{ kN}) + 4(4 \text{ kN})\right] = 0$$

$$F_x - 6F_y = -48 \text{ kN} \qquad (1)$$

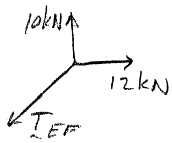
FBD CDEF:



$$\circlearrowleft \Sigma M_C = 0: (18)F_y - (9 \text{ m})F_x - (12 \text{ m})(4 \text{ kN}) - (6 \text{ m})(4 \text{ kN}) = 0$$

$$F_x - 2F_y = -8 \text{ kN} \qquad (2)$$

Solving (1) and (2) $\qquad \mathbf{F}_x = 12 \text{ kN} \longrightarrow \qquad \mathbf{F}_y = 10 \text{ kN} \uparrow$



$$T_{EF} = \sqrt{(10 \text{ kN})^2 + (12 \text{ kN})^2} = 15.62 \text{ kN}$$

Since slope $EF >$ slope AB this is T_{max}

$$(a) \qquad T_{max} = 15.62 \text{ kN} \blacktriangleleft$$

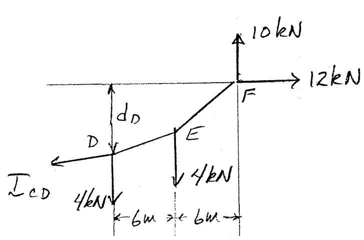
Also could note from FBD cable

$$\uparrow \Sigma F_y = 0: A_y + F_y - 4(4 \text{ kN}) = 0$$

$$A_y = 16 \text{ kN} - 12 \text{ kN} = 4 \text{ kN}$$

Thus $\quad A_y < F_y \quad$ and $\quad T_{AB} < T_{EF}$

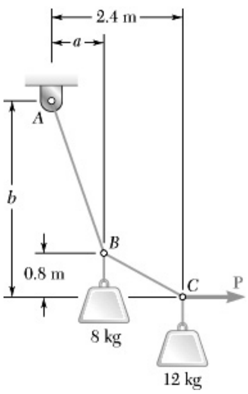
FBD DEF:



(b) $$\circlearrowleft \Sigma M_D = 0: (12 \text{ m})(10 \text{ kN}) - d_D(12 \text{ kN}) - (6 \text{ m})(4 \text{ kN}) = 0$$

$$d_D = 8.00 \text{ m} \blacktriangleleft$$

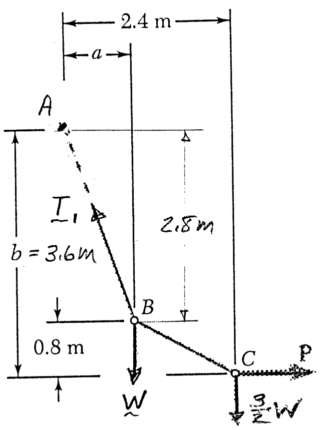
PROBLEM 7.96



Cable *ABC* supports two boxes as shown. Knowing that $b = 3.6$ m, determine (*a*) the required magnitude of the horizontal force **P**, (*b*) the corresponding distance *a*.

SOLUTION

FBD BC:



$$W = (8\text{ kg})(9.81\text{ m/s}^2) = 78.48\text{ N}$$

$$\circlearrowleft \Sigma M_A = 0: (3.6\text{ m})P - (2.4\text{ m})\frac{3W}{2} - aW = 0$$

$$P = W\left(1 + \frac{a}{3.6\text{ m}}\right) \qquad (1)$$

$$\xrightarrow{} \Sigma F_x = 0: -T_{1x} + P = 0 \qquad T_{1x} = P$$

$$\uparrow \Sigma F_y = 0: T_{1y} - W - \frac{3}{2}W = 0 \qquad T_{1y} = \frac{5W}{2}$$

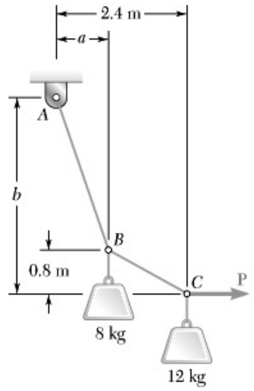
But $\quad \dfrac{T_{1y}}{T_{1x}} = \dfrac{2.8\text{ m}}{a} \quad$ so $\quad \dfrac{5W}{2P} = \dfrac{2.8\text{ m}}{a}$

$$P = \frac{5Wa}{5.6\text{ m}} \qquad (2)$$

Solving (1) and (2): $\quad a = 1.6258$ m, $\quad P = 1.4516W$

So $\quad$ (*a*) $\quad P = 1.4516(78.48) = 113.9\text{ N}$ ◀

(*b*) $\quad a = 1.626$ m ◀

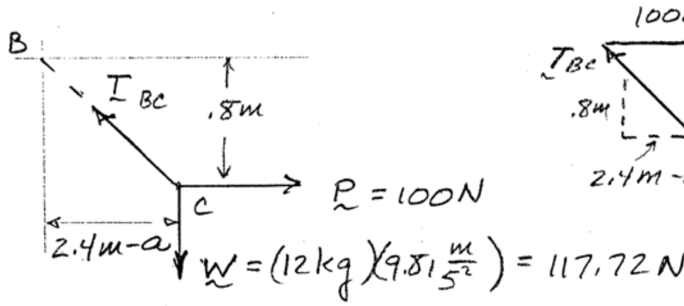


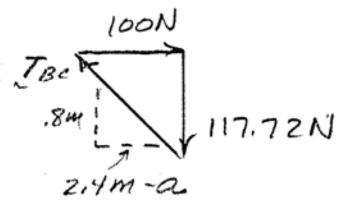
PROBLEM 7.97

Cable *ABC* supports two boxes as shown. Determine the distances *a* and *b* when a horizontal force **P** of magnitude 100 N is applied at *C*.

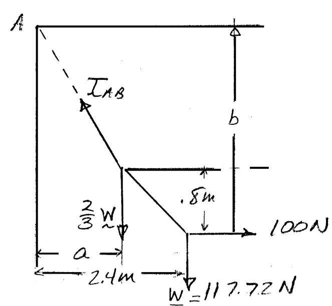
SOLUTION

FBD pt C:





Segment BC:



$$\frac{2.4 \text{ m} - a}{100 \text{ N}} = \frac{0.8 \text{ m}}{117.72 \text{ N}}$$

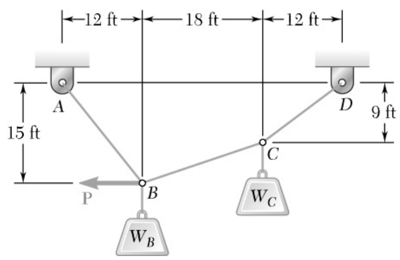
$$a = 1.7204 \text{ m}$$

$$a = 1.720 \text{ m} \blacktriangleleft$$

$$\circlearrowleft \Sigma M_A = 0\text{: } b(100 \text{ N}) - (2.4 \text{ m})(117.72 \text{ N}) - (1.7204 \text{ m})\left(\frac{2}{3}117.72 \text{ N}\right) = 0$$

$$b = 4.1754 \text{ m}$$

$$b = 4.18 \text{ m} \blacktriangleleft$$

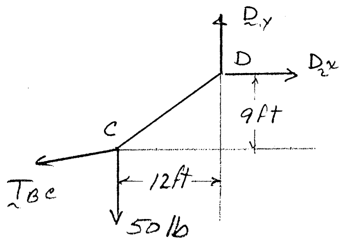


PROBLEM 7.98

Knowing that $W_B = 150$ lb and $W_C = 50$ lb, determine the magnitude of the force **P** required to maintain equilibrium.

SOLUTION

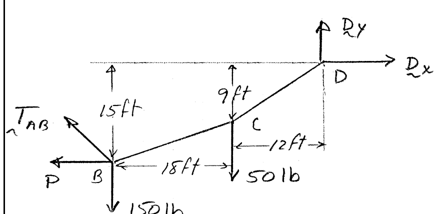
FBD CD:



$$\circlearrowleft \Sigma M_C = 0: (12 \text{ ft})D_y - (9 \text{ ft})D_x = 0$$

$$3D_x = 4D_y \qquad (1)$$

FBD BCD:

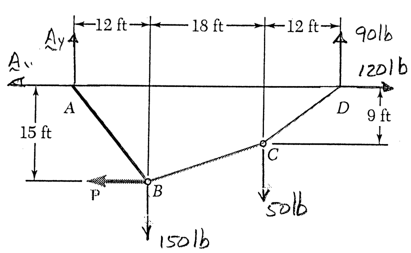


$$\circlearrowleft \Sigma M_B = 0: (30 \text{ ft})D_y - (15 \text{ ft})D_x - (18 \text{ ft})(50 \text{ lb}) = 0$$

$$2D_y - D_x = 60 \text{ lb} \qquad (2)$$

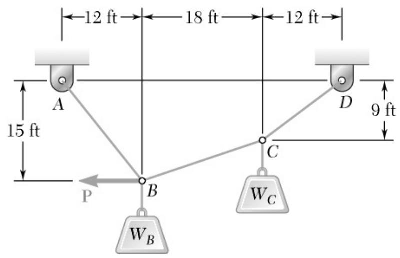
Solving (1) and (2) $\qquad \mathbf{D}_x = 120 \text{ lb} \longrightarrow \qquad \mathbf{D}_y = 90 \text{ lb} \uparrow$

FBD Cable:



$$\circlearrowleft \Sigma M_A = 0: (42 \text{ ft})(90 \text{ lb}) - (30 \text{ ft})(50 \text{ lb})$$
$$- (12 \text{ ft})(150 \text{ lb}) - (15 \text{ ft})P = 0$$

$$P = 32.0 \text{ lb} \blacktriangleleft$$

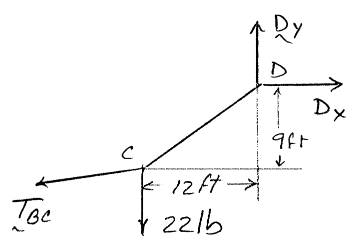


PROBLEM 7.99

Knowing that $W_B = 40$ lb and $W_C = 22$ lb, determine the magnitude of the force **P** required to maintain equilibrium.

SOLUTION

FBD CD:

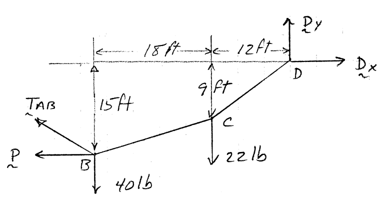


$$\circlearrowleft \Sigma M_C = 0: (12 \text{ ft})D_y - (9 \text{ ft})D_x = 0$$

$$4D_y = 3D_x \qquad (1)$$

$$\circlearrowleft \Sigma M_B = 0: (30 \text{ ft})D_y - (15 \text{ ft})D_x - (18 \text{ ft})(22 \text{ lb}) = 0$$

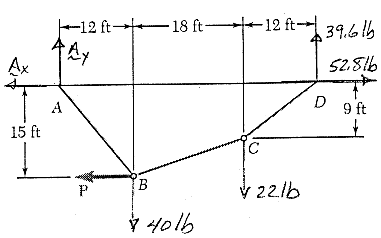
FBD BCD:



$$10D_y - 5D_x = 132 \text{ lb} \qquad (2)$$

Solving (1) and (2) $\quad \mathbf{D}_x = 52.8 \text{ lb} \longrightarrow \quad \mathbf{D}_y = 39.6 \text{ lb} \uparrow$

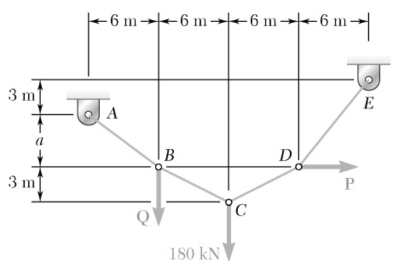
FBD Whole:



$$\circlearrowleft \Sigma M_A = 0: (42 \text{ ft})(39.6 \text{ lb}) - (30 \text{ ft})(22 \text{ lb})$$
$$- (12 \text{ ft})(40 \text{ lb}) - (15 \text{ ft})P = 0$$

$$P = 34.9 \text{ lb} \blacktriangleleft$$

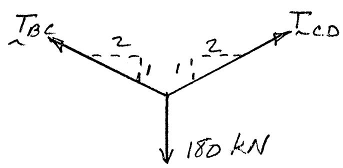
PROBLEM 7.100



If $a = 4.5$ m, determine the magnitudes of **P** and **Q** required to maintain the cable in the shape shown.

SOLUTION

FBD pt C:

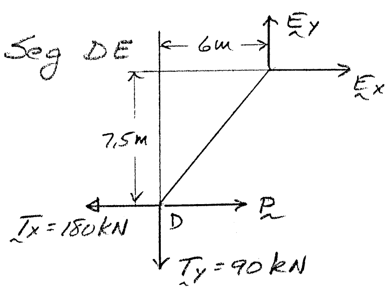


By symmetry: $T_{BC} = T_{CD} = T$

$$\uparrow \Sigma F_y = 0: \; 2\left(\frac{1}{\sqrt{5}}T\right) - 180 \text{ kN} = 0 \qquad T = 90\sqrt{5} \text{ kN}$$

$$T_x = 180 \text{ kN} \qquad T_y = 90 \text{ kN}$$

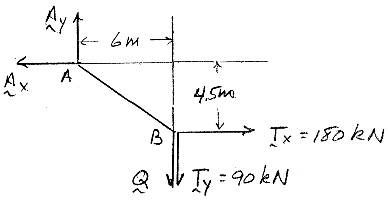
Segment DE:



$$\circlearrowleft \Sigma M_E = 0: \; (7.5 \text{ m})(P - 180 \text{ kN}) + (6 \text{ m})(90 \text{ kN}) = 0$$

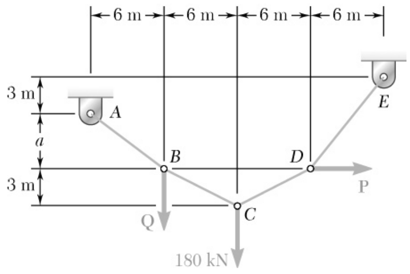
$$P = 108.0 \text{ kN} \blacktriangleleft$$

Segment AB:



$$\circlearrowleft \Sigma M_A = 0: \; (4.5 \text{ m})(180 \text{ kN}) - (6 \text{ m})(Q + 90 \text{ kN}) = 0$$

$$Q = 45.0 \text{ kN} \blacktriangleleft$$

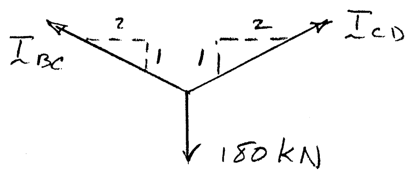


PROBLEM 7.101

If $a = 6$ m, determine the magnitudes of **P** and **Q** required to maintain the cable in the shape shown.

SOLUTION

FBD pt C:

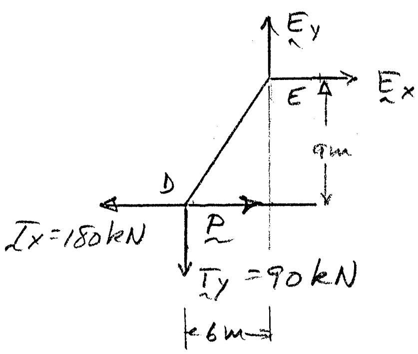


By symmetry: $$T_{BC} = T_{CD} = T$$

$$\uparrow \Sigma F_y = 0{:}\ 2\left(\frac{1}{\sqrt{5}}T\right) - 180\text{ kN} = 0 \qquad T = 90\sqrt{5}\text{ kN}$$

$$T_x = 180\text{ kN} \qquad T_y = 90\text{ kN}$$

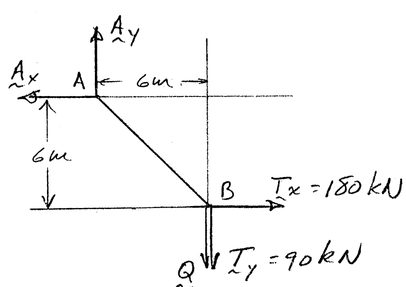
FBD DE:



$$\circlearrowleft \Sigma M_E = 0{:}\ (9\text{ m})(P - 180\text{ kN}) + (6\text{ m})(90\text{ kN}) = 0$$

$$P = 120.0\text{ kN} \blacktriangleleft$$

FBD AB:



$$\circlearrowleft \Sigma M_A = 0{:}\ (6\text{ m})(180\text{ kN}) - (6\text{ m})(Q + 90\text{ kN}) = 0$$

$$Q = 90.0\text{ kN} \blacktriangleleft$$

PROBLEM 7.102

A transmission cable having a mass per unit length of 1 kg/m is strung between two insulators at the same elevation that are 60 m apart. Knowing that the sag of the cable is 1.2 m, determine (*a*) the maximum tension in the cable, (*b*) the length of the cable.

SOLUTION

(*a*) Since $h = 1.2 \text{ m} \ll L = 30 \text{ m}$ we can approximate the load as evenly distributed in the horizontal direction.

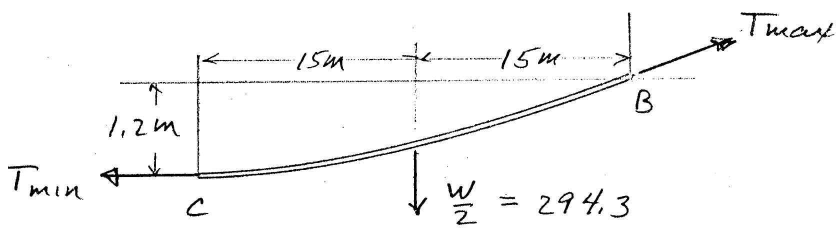
$$w = 1 \text{ kg/m}\left(9.81 \text{ m/s}^2\right) = 9.81 \text{ N/m.}$$

$$w = (60 \text{ m})(9.81 \text{ N/m})$$

$$w = 588.6 \text{ N}$$

Also we can assume that the weight of half the cable acts at the $\frac{1}{4}$ chord point.

FBD half-cable:



$$\Sigma M_B = 0\text{: } (15 \text{ m})(294.3 \text{ N}) - (1.2 \text{ m})T_{\min} = 0$$

$$T_{\min} = 3678.75 \text{ N} = T_{\max\ x}$$

$$\Sigma F_y = 0\text{: } T_{\max\ y} - 294.3 \text{ N} = 0$$

$$T_{\max\ y} = 294.3 \text{ N}$$

$$T_{\max} = 3690.5 \text{ N}$$

$$T_{\max} = 3.69 \text{ kN} \blacktriangleleft$$

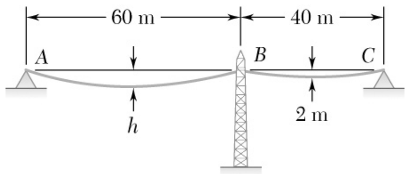
(*b*)

$$s_B = x_B\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2 - \frac{2}{5}\left(\frac{y_B}{x_B}\right)^4 + \cdots\right]$$

$$= (30 \text{ m})\left[1 + \frac{2}{3}\left(\frac{1.2}{30}\right)^2 - \frac{2}{5}\left(\frac{1.2}{30}\right)^4 + \cdots\right] = 30.048 \text{ m} \qquad \text{so} \qquad s = 2s_B = 60.096 \text{ m}$$

$$s = 60.1 \text{ m} \blacktriangleleft$$

Note: The more accurate methods of section 7.10, which assume the load is evenly distributed along the length instead of horizontally, yield $T_{\max} = 3690.5 \text{ N}$ and $s = 60.06$ m. Answers agree to 3 digits at least.

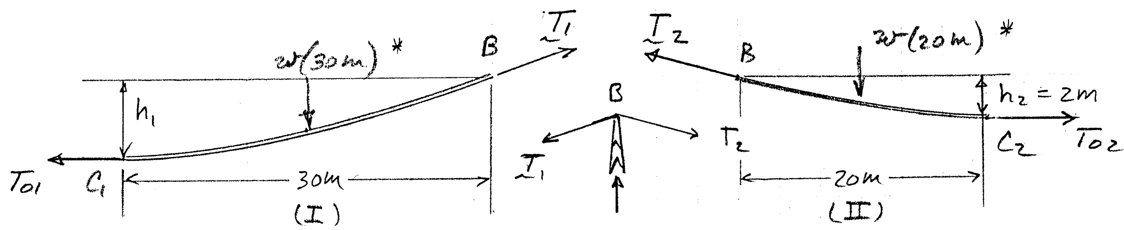


PROBLEM 7.103

Two cables of the same gauge are attached to a transmission tower at *B*. Since the tower is slender, the horizontal component of the resultant of the forces exerted by the cables at *B* is to be zero. Knowing that the mass per unit length of the cables is 0.4 kg/m, determine (*a*) the required sag *h*, (*b*) the maximum tension in each cable.

SOLUTION

Half-cable FBDs:



$T_{1x} = T_{2x}$ to create zero horizontal force on tower $\rightarrow$ thus $T_{01} = T_{02}$

FBD I: $\quad \circlearrowleft \Sigma M_B = 0\colon (15\text{ m})\left[w(30\text{ m})\right] - h_1 T_0 = 0$

$$h_1 = \frac{\left(450\text{ m}^2\right)w}{T_0}$$

FBD II: $\quad \circlearrowleft \Sigma M_B = 0\colon (2\text{ m})T_0 - (10\text{ m})\left[w(20\text{ m})\right] = 0$

$$T_0 = (100\text{ m})w$$

(*a*) $$h_1 = \frac{\left(450\text{ m}^2\right)w}{(100\text{ m})w} = 4.50\text{ m}$$

FBD I: $\quad \xrightarrow{} \Sigma F_x = 0\colon T_{1x} - T_0 = 0$

$$T_{1x} = (100\text{ m})w$$

$$\uparrow \Sigma F_y = 0\colon T_{1y} - (30\text{ m})w = 0$$

$$T_{1y} = (30\text{ m})w$$

$$T_1 = \sqrt{(100\text{ m})^2 + (30\text{ m})^2}\,w$$

$$= (104.4\text{ m})(0.4\text{ kg/m})\left(9.81\text{ m/s}^2\right)$$

$$= 409.7\text{ N}$$

PROBLEM 7.103 CONTINUED

FBD II:

$$\uparrow \Sigma F_y = 0: T_{2y} - (20\text{ m})w = 0$$

$$T_{2y} = (20\text{ m})w$$

$$T_{2x} = T_{1x} = (100\text{ m})w$$

$$T_2 = \sqrt{(100\text{ m})^2 + (20\text{ m})^2}\,w = 400.17\text{ N}$$

(b) $T_1 = 410\text{ N}$ ◀

$T_2 = 400\text{ N}$ ◀

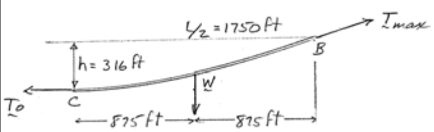
* Since $h \ll L$ it is reasonable to approximate the cable weight as being distributed uniformly along the horizontal. The methods of section 7.10 are more accurate for cables sagging under their own weight.

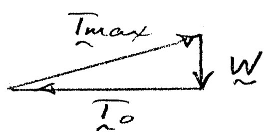
PROBLEM 7.104

The center span of the George Washington Bridge, as originally constructed, consisted of a uniform roadway suspended from four cables. The uniform load supported by each cable was $w = 9.75$ kips/ft along the horizontal. Knowing that the span L is 3500 ft and that the sag h is 316 ft, determine for the original configuration (*a*) the maximum tension in each cable, (*b*) the length of each cable.

SOLUTION

FBD half-span:





$$W = (9.75 \text{ kips/ft})(1750 \text{ ft}) = 17,062.5 \text{ kips}$$

$$\circlearrowleft \Sigma M_B = 0: \ (875 \text{ ft})(17,065 \text{ kips}) - (316 \text{ ft})T_0 = 0$$

$$T_0 = 47,246 \text{ kips}$$

$$T_{\max} = \sqrt{T_0^2 + W^2} = \sqrt{(47,246 \text{ kips})^2 + (17,063 \text{ kips})^2}$$

(*a*) $T_{\max} = 50,200 \text{ kips}$ ◀

$$s = x\left[1 + \frac{2}{3}\left(\frac{y}{x}\right)^2 - \frac{2}{5}\left(\frac{y}{x}\right)^4 + \cdots\right]$$

$$s_B = (1750 \text{ ft})\left[1 + \frac{2}{3}\left(\frac{316 \text{ ft}}{1750 \text{ ft}}\right)^2 - \frac{2}{5}\left(\frac{316 \text{ ft}}{1750 \text{ ft}}\right)^4 + \cdots\right]$$

$$= 1787.3 \text{ ft}$$

(*b*) $l = 2s_B = 3575 \text{ ft}$ ◀

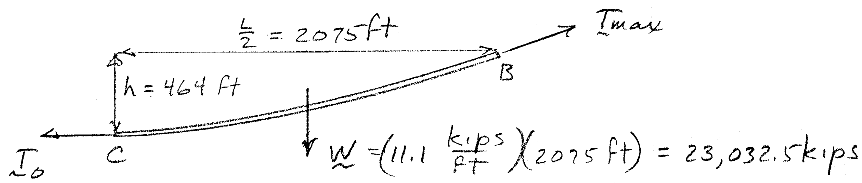
* To get 3-digit accuracy, only two terms are needed.

PROBLEM 7.105

Each cable of the Golden Gate Bridge supports a load $w = 11.1$ kips/ft along the horizontal. Knowing that the span L is 4150 ft and that the sag h is 464 ft, determine (a) the maximum tension in each cable, (b) the length of each cable.

SOLUTION

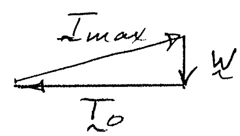
FBD half-span:



(a)

$$\circlearrowleft \Sigma M_B = 0: \left(\frac{2075 \text{ ft}}{2}\right)(23032.5 \text{ kips}) - (464 \text{ ft})T_0 = 0$$

$$T_0 = 47{,}246 \text{ kips}$$



$$T_{\max} = \sqrt{T_0^2 + W^2} = \sqrt{(47{,}246 \text{ kips})^2 + (23{,}033 \text{ kips})^2} = 56{,}400 \text{ kips} \blacktriangleleft$$

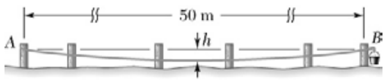
(b)

$$s = x\left[1 + \frac{2}{3}\left(\frac{y}{x}\right)^2 - \frac{2}{5}\left(\frac{y}{x}\right)^4 + \cdots\right]$$

$$s_B = (2075 \text{ ft})\left[1 + \frac{2}{3}\left(\frac{464 \text{ ft}}{2075 \text{ ft}}\right)^2 - \frac{2}{5}\left(\frac{464 \text{ ft}}{2075 \text{ ft}}\right)^4 + \cdots\right]$$

$$s_B = 2142 \text{ ft} \qquad l = 2s_B \qquad l = 4284 \text{ ft} \blacktriangleleft$$

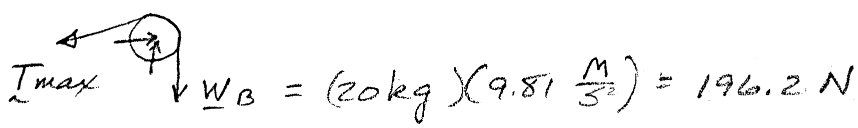
PROBLEM 7.106



To mark the positions of the rails on the posts of a fence, a homeowner ties a cord to the post at *A*, passes the cord over a short piece of pipe attached to the post at *B*, and ties the free end of the cord to a bucket filled with bricks having a total mass of 20 kg. Knowing that the mass per unit length of the rope is 0.02 kg/m and assuming that *A* and *B* are at the same elevation, determine (*a*) the sag *h*, (*b*) the slope of the cable at *B*. Neglect the effect of friction.

SOLUTION

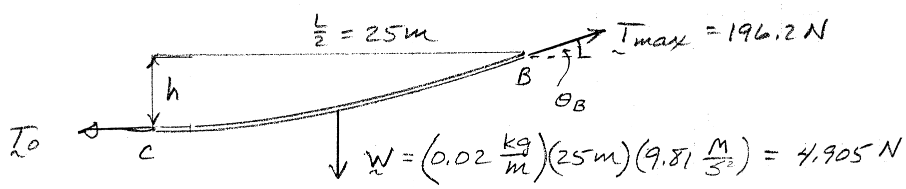
FBD pulley:

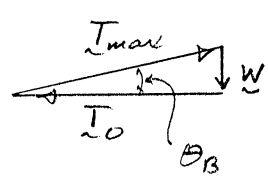


$$\circlearrowleft \Sigma M_P = 0\colon\ (T_{\max} - W_B)r = 0$$

$$T_{\max} = W_B = 196.2 \text{ N}$$

FBD half-span:*





$$T_0 = \sqrt{T_{\max}^2 - W^2} = \sqrt{(196.2 \text{ N})^2 - (4.91 \text{ N})^2} = 196.139 \text{ N}$$

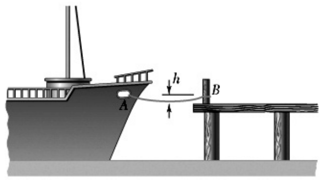
$$\circlearrowleft \Sigma M_B = 0\colon \left(\frac{25 \text{ m}}{2}\right)(4.905 \text{ N}) - h(196.139 \text{ N}) = 0$$

(*a*) $h = 0.3126 \text{ m} = 313 \text{ mm}$ ◀

(*b*) $\theta_B = \sin^{-1}\dfrac{W}{T_{\max}} = \sin^{-1}\left(\dfrac{4.905 \text{ N}}{196.2 \text{ N}}\right) = 1.433°$ ◀

*See note Prob. 7.103

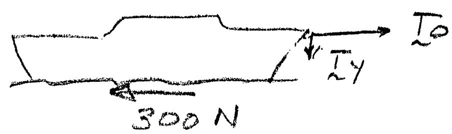
PROBLEM 7.107



A small ship is tied to a pier with a 5-m length of rope as shown. Knowing that the current exerts on the hull of the ship a 300-N force directed from the bow to the stern and that the mass per unit length of the rope is 2.2 kg/m, determine (*a*) the maximum tension in the rope, (*b*) the sag *h*. [*Hint:* Use only the first two terms of Eq. (7.10).]

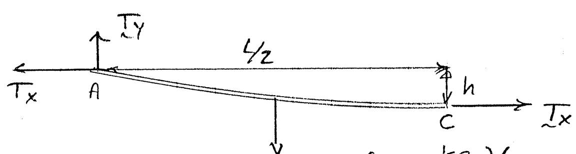
SOLUTION

(*a*) **FBD ship:**



$$\xrightarrow{} \Sigma F_x = 0\text{: } T_0 - 300\text{ N} = 0 \qquad T_0 = 300\text{ N}$$

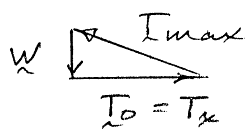
FBD half-span:*



$$W = (2.2\ \tfrac{\text{kg}}{\text{m}})(9.81\ \tfrac{\text{m}}{\text{s}^2})(2.5\text{ m}) = 53.955\text{ N}$$

$$T_{\max} = \sqrt{T_0^2 + W^2} = \sqrt{(300\text{ N})^2 = (54\text{ N})^2} = 305\text{ N} \blacktriangleleft$$

(*b*) $$\circlearrowleft \Sigma M_A = 0\text{: } hT_x - \frac{L}{4}W = 0 \qquad h = \frac{LW}{4T_x}$$



$$s = x\left[1 + \frac{2}{3}\left(\frac{4}{x}\right)^2 + \cdots\right] \qquad \text{but} \qquad y_A = h = \frac{LW}{4T_x} \qquad \text{so} \qquad \frac{y_A}{x_A} = \frac{W}{2T_x}$$

$$(2.5\text{ m}) = \frac{L}{2}\left[1 + \frac{2}{3}\left(\frac{53.955\text{ N}}{600\text{ N}}\right)^2 - \cdots\right] \rightarrow L = 4.9732\text{ m}$$

$$\text{So} \quad h = \frac{LW}{4T_x} = 0.2236\text{ m} \qquad\qquad h = 224\text{ mm} \blacktriangleleft$$

*See note Prob. 7.103

PROBLEM 7.108

The center span of the Verrazano-Narrows Bridge consists of two uniform roadways suspended from four cables. The design of the bridge allowed for the effect of extreme temperature changes which cause the sag of the center span to vary from $h_w = 386$ ft in winter to $h_s = 394$ ft in summer. Knowing that the span is $L = 4260$ ft, determine the change in length of the cables due to extreme temperature changes.

SOLUTION

$$s = x\left[1 + \frac{2}{3}\left(\frac{y}{x}\right)^2 - \frac{2}{5}\left(\frac{y}{x}\right)^4 + \cdots\right]$$

Knowing

$$l = 2s_{\text{TOT}} = L\left[1 + \frac{2}{3}\left(\frac{h}{L/2}\right)^2 - \frac{2}{5}\left(\frac{h}{L/2}\right)^2 + \cdots\right]$$

Winter:

$$l_w = (4260 \text{ ft})\left[1 + \frac{2}{3}\left(\frac{386 \text{ ft}}{2130 \text{ ft}}\right)^2 - \frac{2}{5}\left(\frac{386 \text{ ft}}{2130 \text{ ft}}\right)^4 + \cdots\right] = 4351.43 \text{ ft}$$

Summer:

$$l_s = (4260 \text{ ft})\left[1 + \frac{2}{3}\left(\frac{394 \text{ ft}}{2130 \text{ ft}}\right)^2 - \frac{2}{5}\left(\frac{394 \text{ ft}}{2130 \text{ ft}}\right)^4 + \cdots\right] = 4355.18 \text{ ft}$$

$$\Delta l = l_s - l_w = 3.75 \text{ ft} \blacktriangleleft$$

PROBLEM 7.109

A cable of length $L+\Delta$ is suspended between two points which are at the same elevation and a distance L apart. (*a*) Assuming that Δ is small compared to L and that the cable is parabolic, determine the approximate sag in terms of L and Δ. (*b*) If $L = 30$ m and $\Delta = 1.2$ m, determine the approximate sag. [*Hint:* Use only the first two terms of Eq. (7.10).]

SOLUTION

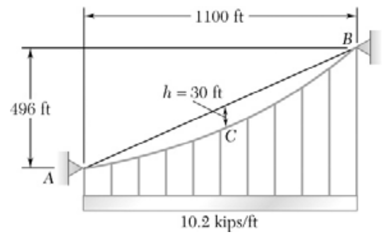
(*a*)

$$s = x\left[1+\frac{2}{3}\left(\frac{y}{x}\right)^2 - \cdots\right]$$

$$L + \Delta = 2s_{\text{TOT}} = L\left[1+\frac{2}{3}\left(\frac{h}{L/2}\right)^2 - \cdots\right]$$

$$\frac{\Delta}{L} = \frac{2}{3}\left(\frac{2h}{L}\right)^2 = \frac{8}{3}\left(\frac{h}{L}\right)^2 \rightarrow h = \sqrt{\frac{3}{8}L\Delta} \blacktriangleleft$$

(*b*) For $L = 30$ m, $\Delta = 1.2$ m $\qquad h = 3.67$ m $\blacktriangleleft$

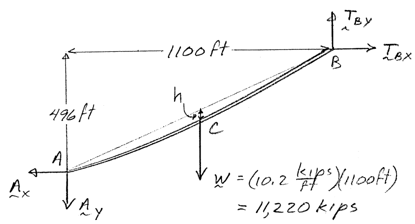


PROBLEM 7.110

Each cable of the side spans of the Golden Gate Bridge supports a load $w = 10.2$ kips/ft along the horizontal. Knowing that for the side spans the maximum vertical distance h from each cable to the chord AB is 30 ft and occurs at midspan, determine (*a*) the maximum tension in each cable, (*b*) the slope at B.

SOLUTION

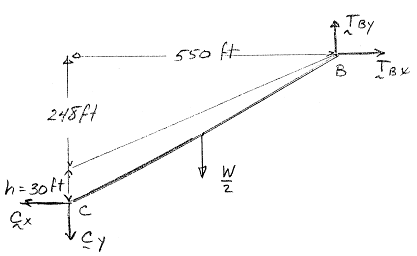
FBD AB:



$$\circlearrowleft \Sigma M_A = 0: (1100 \text{ ft})T_{By} - (496 \text{ ft})T_{Bx} - (550 \text{ ft})W = 0$$

$$11T_{By} - 4.96T_{Bx} = 5.5W \qquad (1)$$

FBD CB:



$$\circlearrowleft \Sigma M_C = 0: (550 \text{ ft})T_{By} - (278 \text{ ft})T_{Bx} - (275 \text{ ft})\frac{W}{2} = 0$$

$$11T_{By} - 5.56T_{Bx} = 2.75W \qquad (2)$$

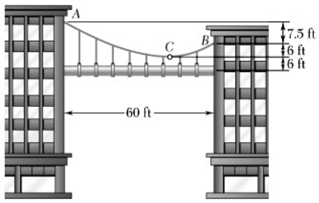
Solving (1) and (2) $\qquad T_{By} = 28,798$ kips

$$T_{Bx} = 51,425 \text{ kips}$$

$$T_{max} = T_B = \sqrt{T_{B_x}^2 + T_{B_y}^2} \qquad \tan\theta_B = \frac{T_{B_y}}{T_{B_x}}$$

So that $\qquad$ (*a*) $\qquad T_{max} = 58,900$ kips ◀

(*b*) $\qquad \theta_B = 29.2°$ ◀

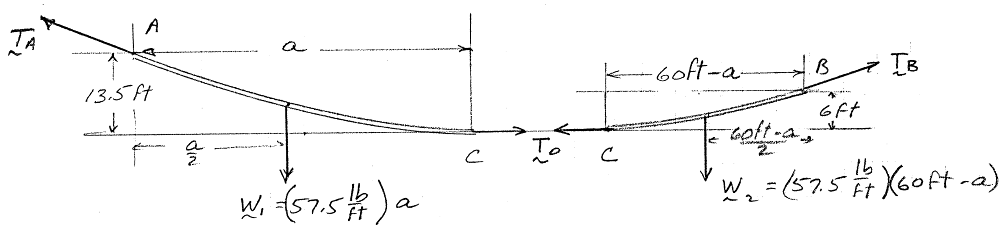


PROBLEM 7.111

A steam pipe weighting 50 lb/ft that passes between two buildings 60 ft apart is supported by a system of cables as shown. Assuming that the weight of the cable is equivalent to a uniformly distributed loading of 7.5 lb/ft, determine (*a*) the location of the lowest point *C* of the cable, (*b*) the maximum tension in the cable.

SOLUTION

FBD AC: **FBD CB:**



$$\Sigma M_A = 0: (13.5 \text{ ft})T_0 - \frac{a}{2}(57.5 \text{ lb/ft})a = 0$$

$$T_0 = (2.12963 \text{ lb/ft}^2)a^2 \qquad (1)$$

$$\Sigma M_B = 0: \frac{60 \text{ ft} - a}{2}(57.5 \text{ lb/ft})(60 \text{ ft} - a) - (6 \text{ ft})T_0 = 0$$

$$6T_0 = (28.75 \text{ lb/ft}^2)\left[3600 \text{ ft}^2 - (120 \text{ ft})a + a^2\right] \qquad (2)$$

Using (1) in (2) $\quad 0.55a^2 - (120 \text{ ft})a + 3600 \text{ ft}^2 = 0$

Solving: $\quad a = (108 \pm 72)$ ft $\qquad a = 36$ ft (180 ft out of range)

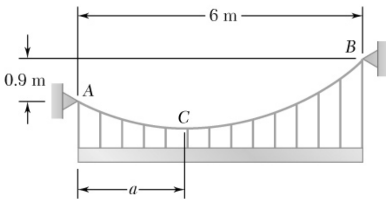
So $\qquad$ *C* is 36 ft from *A*

(*a*) $\quad$ *C* is 6 ft below and 24 ft left of *B* ◀

$$T_0 = 2.1296 \text{ lb/ft}^2(36 \text{ ft})^2 = 2760 \text{ lb}$$

$$W_1 = (57.5 \text{ lb/ft})(36 \text{ ft}) = 2070 \text{ lb}$$

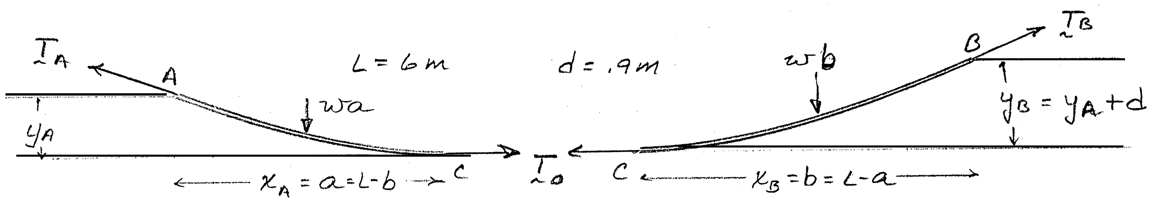
(*b*) $\qquad T_{\max} = T_A = \sqrt{T_0^2 + W_1^2} = \sqrt{(2760 \text{ lb})^2 + (2070 \text{ lb})^2} = 3450 \text{ lb}$ ◀



PROBLEM 7.112

Chain *AB* supports a horizontal, uniform steel beam having a mass per unit length of 85 kg/m. If the maximum tension in the cable is not to exceed 8 kN, determine (*a*) the horizontal distance *a* from *A* to the lowest point *C* of the chain, (*b*) the approximate length of the chain.

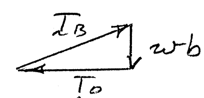
SOLUTION



$$\circlearrowleft \Sigma M_A = 0: \; y_A T_0 - \frac{a}{2} wa = 0 \qquad\qquad \circlearrowleft \Sigma M_B = 0: \; -y_B T_0 + \frac{b}{2} wb = 0$$

$$y_A = \frac{wa^2}{2T_0} \qquad\qquad y_B = \frac{wb^2}{2T_0}$$

$$d = (y_B - y_B) = \frac{w}{2T_0}\left(b^2 - a^2\right)$$



But $$T_0 = \sqrt{T_B^2 - (wb)^2} = \sqrt{T_{\max}^2 - (wb)^2}$$

$$\therefore \quad (2d)^2\left[T_{\max}^2 - (wb)^2\right] = w^2\left(b^2 - a^2\right)^2 = L^2 w^2\left(4b^2 - 4Lb + L^2\right)$$

or $$4\left(L^2 + d^2\right)b^2 - 4L^3 b + \left(L^4 - 4d^2\frac{T_{\max}^2}{w^2}\right) = 0$$

Using $L = 6$ m, $d = 0.9$ m, $T_{\max} = 8$ kN, $w = (85 \text{ kg/m})\left(9.81 \text{ m/s}^2\right) = 833.85$ N/m

yields $b = (2.934 \pm 1.353)$ m $\qquad b = 4.287$ m $\qquad$ (since $b > 3$ m)

(*a*) $\quad a = 6 \text{ m} - b = 1.713 \text{ m}$ ◀

PROBLEM 7.112 CONTINUED

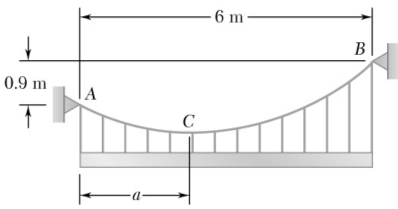
$$T_0 = \sqrt{T_{\max}^2 - (wb)^2} = 7156.9 \text{ N}$$

$$\frac{y_A}{x_A} = \frac{wa}{2T_0} = 0.09979 \qquad \frac{y_B}{x_B} = \frac{wb}{2T_0} = 0.24974$$

$$l = s_A + s_B = a\left[1 + \frac{2}{3}\left(\frac{y_A}{x_A}\right)^2 + \cdots\right] + b\left[1 + \frac{2}{3}\left(\frac{y_B}{x_B}\right)^2 + \cdots\right]$$

$$= (1.713 \text{ m})\left[1 + \frac{2}{3}(0.09979)^2\right] + (4.287 \text{ m})\left[1 + \frac{2}{3}(0.24974)^2\right] = 6.19 \text{ m}$$

(b) $\qquad l = 6.19 \text{ m}$ ◀



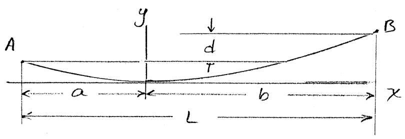
PROBLEM 7.113

Chain AB of length 6.4 m supports a horizontal, uniform steel beam having a mass per unit length of 85 kg/m. Determine (a) the horizontal distance a from A to the lowest point C of the chain, (b) the maximum tension in the chain.

SOLUTION

$$\circlearrowleft \Sigma M_P = 0: \frac{x}{2}wx - yT_0 = 0$$

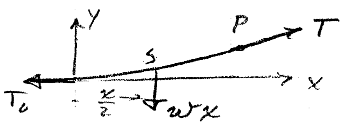
Geometry:



$$y = \frac{wx^2}{2T_0} \qquad \text{so} \qquad \frac{y}{x} = \frac{wx}{2T_0}$$

$$\text{and } d = y_B - y_A = \frac{w}{2T_0}\left(b^2 - a^2\right)$$

FBD Segment:



Also $$l = s_A + s_B = a\left[1 + \frac{2}{3}\left(\frac{y_A}{a}\right)^2\right] + b\left[1 + \frac{2}{3}\left(\frac{y_B}{b}\right)^2\right]$$

$$l - L = \frac{2}{3}\left[\left(\frac{y_A}{a}\right)^2 + \left(\frac{y_B}{b}\right)^2\right] = \frac{w^2}{6T_0^2}\left(a^3 + b^3\right)$$

$$= \frac{1}{6}\frac{4d^2}{\left(b^2 - a^2\right)^2}\left(a^3 + b^3\right) = \frac{2}{3}\frac{d^2\left(a^3 + b^3\right)}{\left(b^2 - a^2\right)^2}$$

Using $l = 6.4$ m, $L = 6$ m, $d = 0.9$ m, $b = 6\text{ m} - a$, and solving for a, knowing that $a < 3$ ft

$a = 2.2196$ m (a) $a = 2.22$ m ◀

Then $$T_0 = \frac{w}{2d}\left(b^2 - a^2\right)$$

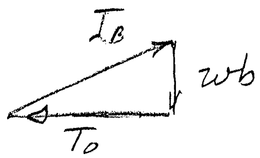
And with $$w = (85\text{ kg/m})\left(9.81\text{ m/s}^2\right) = 833.85\text{ N/m}$$

And $b = 6\text{ m} - a = 3.7804$ m $\quad T_0 = 4338$ N

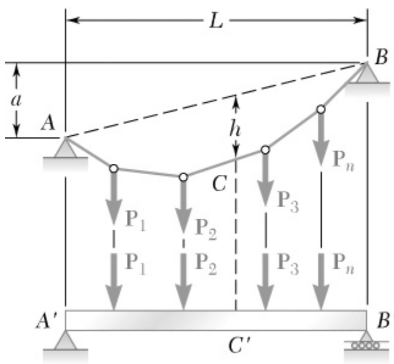
$$T_{\text{max}} = T_B = \sqrt{T_0^2 + (wb)^2}$$

$$= \sqrt{(4338\text{ N})^2 + (833.85\text{ N/m})^2(3.7804\text{ m})^2}$$

$T_{\text{max}} = 5362$ N (b) $T_{\text{max}} = 5.36$ kN ◀



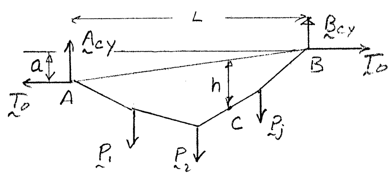
PROBLEM 7.114



A cable AB of span L and a simple beam $A'B'$ of the same span are subjected to identical vertical loadings as shown. Show that the magnitude of the bending moment at a point C' in the beam is equal to the product T_0h where T_0 is the magnitude of the horizontal component of the tension force in the cable and h is the vertical distance between point C and the chord joining the points of support A and B.

SOLUTION

FBD Cable:

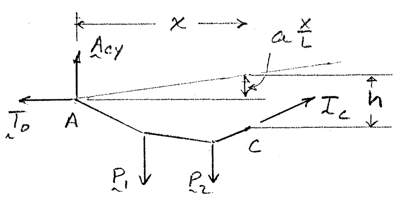


$$\circlearrowleft \Sigma M_B = 0: LA_{Cy} + aT_0 - \Sigma M_{B\text{ loads}} = 0 \qquad (1)$$

(Where $\Sigma M_{B\text{ loads}}$ includes all applied loads)

$$\circlearrowleft \Sigma M_C = 0: xA_{Cy} - \left(h - a\frac{x}{L}\right)T_0 - \Sigma M_{C\text{ left}} = 0 \qquad (2)$$

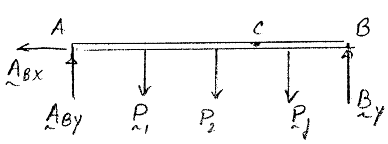
FBD AC:



(Where $\Sigma M_{C\text{ left}}$ includes all loads left of C)

$$\frac{x}{L}(1) - (2): \quad hT_0 - \frac{x}{L}\Sigma M_{B\text{ loads}} + \Sigma M_{C\text{ left}} = 0 \qquad (3)$$

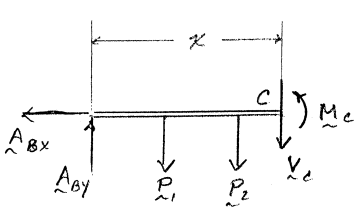
FBD Beam:



$$\circlearrowleft \Sigma M_B = 0: LA_{By} - \Sigma M_{B\text{ loads}} = 0 \qquad (4)$$

$$\circlearrowleft \Sigma M_C = 0: xA_{By} - \Sigma M_{C\text{ left}} - M_C = 0 \qquad (5)$$

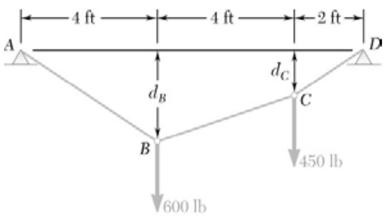
FBD AC:



$$\frac{x}{L}(4) - (5): \quad -\frac{x}{L}\Sigma M_{B\text{ loads}} + \Sigma M_{C\text{ left}} + M_C = 0 \qquad (6)$$

Comparing (3) and (6) $\qquad M_C = hT_0 \qquad$ Q.E.D.

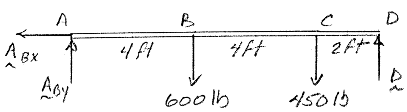
PROBLEM 7.115



Making use of the property established in Prob. 7.114, solve the problem indicated by first solving the corresponding beam problem. Prob. 7.89*a*.

SOLUTION

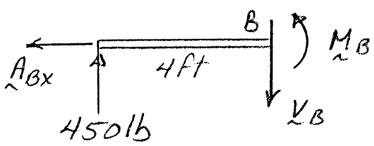
FBD Beam:



$$\circlearrowleft \Sigma M_D = 0\text{: } (2\text{ ft})(450\text{ lb}) + (6\text{ ft})(600\text{ lb}) - (10\text{ ft})\mathbf{A}_{By} = 0$$

$$\mathbf{A}_{By} = 450\text{ lb} \uparrow$$

Section AB:



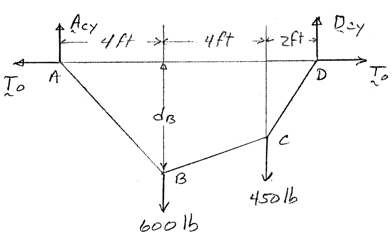
$$\circlearrowleft \Sigma M_B = 0\text{: } M_B - (4\text{ ft})(450\text{ lb}) = 0$$

$$M_B = 1800\text{ lb}\cdot\text{ft}$$

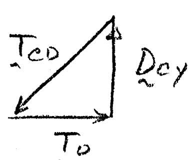
$$\circlearrowleft \Sigma M_A = 0\text{: } (10\text{ ft})D_{Cy} - (8\text{ ft})(450\text{ lb}) - (4\text{ ft})(600\text{ lb}) = 0$$

$$D_{Cy} = 600\text{ lb}$$

Cable:



(Note: $D_y > A_y$ so $T_{max} = T_{CD}$)



$$T_0 = \sqrt{T_{max}^2 - D_{Cy}^2}$$

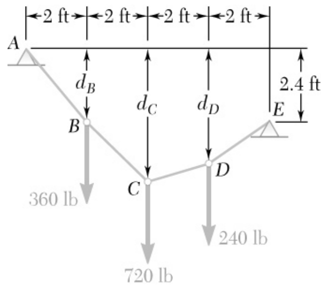
$$T_0 = \sqrt{(720\text{ lb})^2 - (600\text{ lb})^2}$$

$$T_0 = 398\text{ lb}$$

$$d_B = \frac{M_B}{T_0} = \frac{1800\text{ lb}\cdot\text{ft}}{398\text{ lb}} = 4.523\text{ ft}$$

$$d_B = 4.52\text{ ft} \blacktriangleleft$$

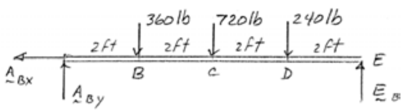
PROBLEM 7.116



Making use of the property established in Prob. 7.114, solve the problem indicated by first solving the corresponding beam problem. Prob. 7.92*b*.

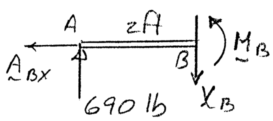
SOLUTION

FBD Beam:



$$\circlearrowleft \Sigma M_E = 0: (2\text{ ft})(240\text{ lb}) + (4\text{ ft})(720\text{ lb}) + (6\text{ ft})(360\text{ lb}) - (8\text{ ft})A_{By} = 0$$

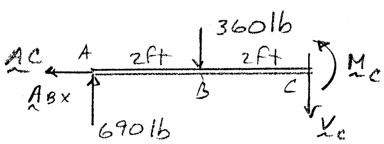
$$\mathbf{A}_{By} = 690\text{ lb}\uparrow$$



$$\circlearrowleft \Sigma M_B = 0: M_B - (2\text{ ft})(690\text{ lb}) = 0$$

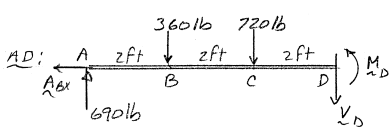
$$M_B = 1380\text{ lb}\cdot\text{ft}$$

$$\circlearrowleft \Sigma M_B = 0: M_C + (2\text{ ft})(360\text{ lb}) - (4\text{ ft})(690\text{ lb}) = 0$$



$$M_C = 2040\text{ lb}\cdot\text{ft}$$

$$\circlearrowleft \Sigma M_D = 0: M_D + (2\text{ ft})(720\text{ lb}) + (4\text{ ft})(360\text{ lb}) - (6\text{ ft})(690\text{ lb}) = 0$$

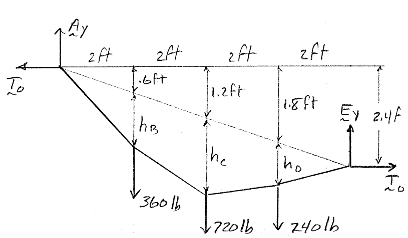


$$M_D = 1260\text{ lb}\cdot\text{ft}$$

$$h_C = d_C - 1.2\text{ ft} = 3.6\text{ ft} - 1.2\text{ ft} = 2.4\text{ ft}$$

$$T_0 = \frac{M_C}{h_C} = \frac{2040\text{ lb}\cdot\text{ft}}{2.4\text{ ft}} = 850\text{ lb}$$

Cable:

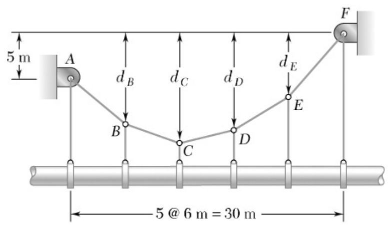


$$h_B = \frac{M_B}{T_0} = \frac{1380\text{ lb}\cdot\text{ft}}{850\text{ lb}} = 1.6235\text{ ft}$$

$$d_B = h_B + 0.6\text{ ft} \qquad d_B = 2.22\text{ ft} \blacktriangleleft$$

$$h_0 = \frac{M_D}{T_0} = \frac{1260\text{ lb}\cdot\text{ft}}{850\text{ lb}} = 1.482\text{ ft}$$

$$d_B = h_0 + 1.8\text{ ft} \qquad d_D = 3.28\text{ ft} \blacktriangleleft$$

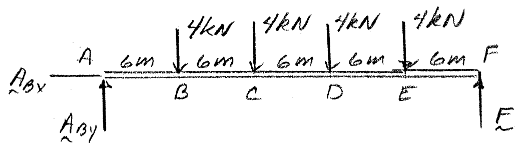


PROBLEM 7.117

Making use of the property established in Prob. 7.114, solve the problem indicated by first solving the corresponding beam problem. Prob. 7.94*b*.

SOLUTION

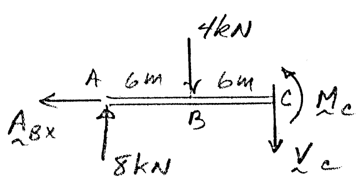
FBD Beam:



By symmetry: $\mathbf{A}_{By} = \mathbf{F} = 8\ kN$

$$M_B = M_E; \qquad M_C = M_D$$

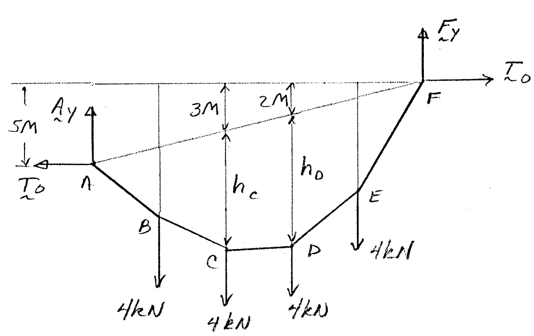
AC:



$$\Sigma M_C = 0\text{: } M_C + (6\text{ m})(4\text{ kN}) - (12\text{ m})(8\text{ kN}) = 0$$

$$M_C = 72\text{ kN}\cdot\text{m} \qquad \text{so} \qquad M_D = 72\text{ kN}\cdot\text{m}$$

Cable:

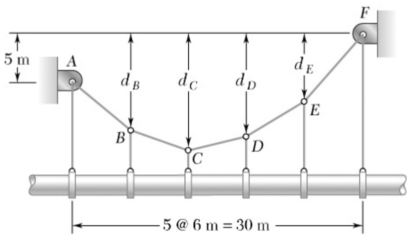


Since $M_D = M_C$

$$h_D = h_C = 12\text{ m} - 3\text{ m} = 9\text{ m}$$

$$d_D = h_D + 2\text{ m} = 11\text{ m}$$

$d_D = 11.00\text{ m}$ ◀

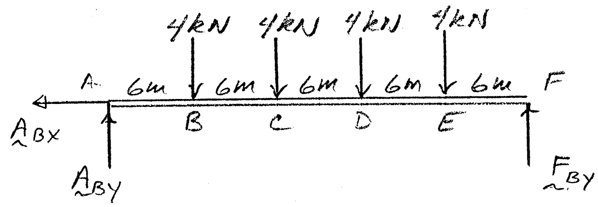


PROBLEM 7.118

Making use of the property established in Prob. 7.114, solve the problem indicated by first solving the corresponding beam problem.
Prob. 7.95*b*.

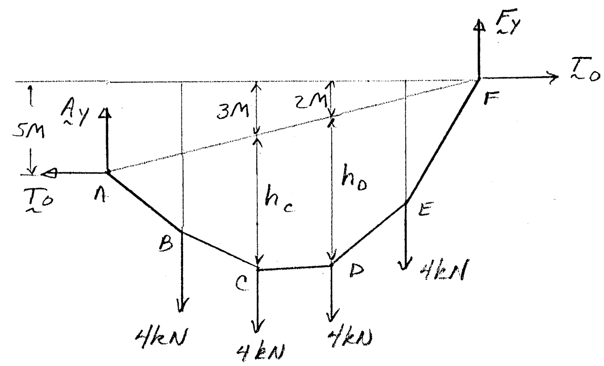
SOLUTION

FBD Beam:



By symmetry: $M_B = M_E$ and $M_C = M_D$

Cable:



Since $M_D = M_C$, $h_D = h_C$

$$h_D = h_C = d_C - 3\text{ m} = 9\text{ m} - 3\text{ m} = 6\text{ m}$$

Then $d_D = h_D + 2\text{ m} = 6\text{ m} + 2\text{ m} = 8\text{ m}$

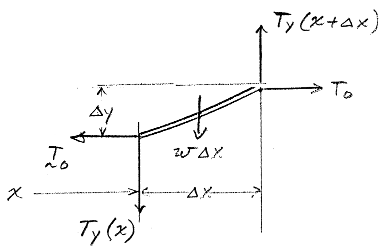
$d_D = 8.00\text{ m}$ ◀

PROBLEM 7.119

Show that the curve assumed by a cable that carries a distributed load $w(x)$ is defined by the differential equation $d^2y/dx^2 = w(x)/T_0$, where T_0 is the tension at the lowest point.

SOLUTION

FBD Elemental segment:



$$+\uparrow \Sigma F_y = 0: T_y(x+\Delta x) - T_y(x) - w(x)\Delta x = 0$$

So
$$\frac{T_y(x+\Delta x)}{T_0} - \frac{T_y(x)}{T_0} = \frac{w(x)}{T_0}\Delta x$$

But
$$\frac{T_y}{T_0} = \frac{dy}{dx}$$

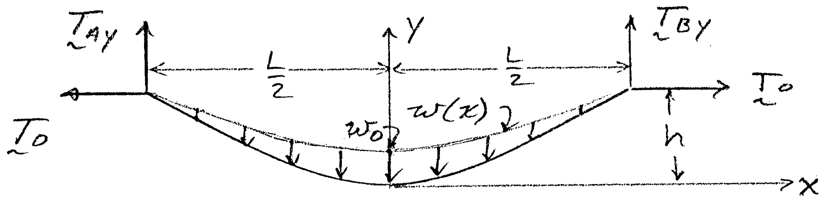
So
$$\frac{\left.\frac{dy}{dx}\right|_{x+\Delta x} - \left.\frac{dy}{dx}\right|_{x}}{\Delta x} = \frac{w(x)}{T_0}$$

In $\lim\limits_{\Delta x \to 0}$:
$$\frac{d^2y}{dx^2} = \frac{w(x)}{T_0}$$
Q.E.D.

PROBLEM 7.120

Using the property indicated in Prob. 7.119, determine the curve assumed by a cable of span L and sag h carrying a distributed load $w = w_0 \cos(\pi x / L)$, where x is measured from mid-span. Also determine the maximum and minimum values of the tension in the cable.

SOLUTION



$$w(x) = w_0 \cos\frac{\pi x}{L}$$

From Problem 7.119

$$\frac{d^2 y}{dx^2} = \frac{w(x)}{T_0} = \frac{w_0}{T_0}\cos\frac{\pi x}{L}$$

So $$\frac{dy}{dx} = \frac{W_0 L}{T_0 \pi}\sin\frac{\pi x}{L} \qquad \left(\text{using } \left.\frac{dy}{dx}\right|_0 = 0\right)$$

$$y = \frac{w_0 L^2}{T_0 \pi^2}\left(1 - \cos\frac{\pi x}{L}\right) \quad \left[\text{using } y(0) = 0\right] \blacktriangleleft$$

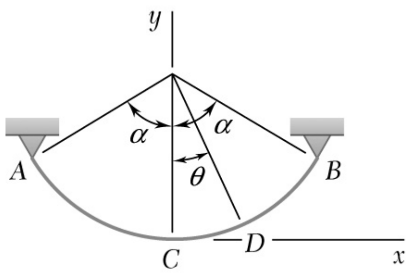
But $$y\left(\frac{L}{2}\right) = h = \frac{w_0 L^2}{T_0 \pi^2}\left(1 - \cos\frac{\pi}{2}\right) \qquad \text{so} \qquad T_0 = \frac{w_0 L^2}{\pi^2 h}$$

And $$T_0 = T_{\min} \qquad \text{so} \qquad T_{\min} = \frac{w_0 L^2}{\pi^2 h} \blacktriangleleft$$

$$T_{\max} = T_A = T_B: \quad \frac{T_{By}}{T_0} = \left.\frac{dy}{dx}\right|_{x=\frac{L}{2}} = \frac{w_0 L}{T_0 \pi}$$

$$T_{By} = \frac{w_0 L}{\pi}$$

$$T_B = \sqrt{T_{By}^2 + T_0^2} = \frac{w_0 L}{\pi}\sqrt{1 + \left(\frac{L}{\pi h}\right)^2} \blacktriangleleft$$

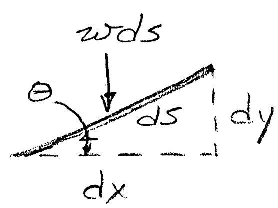


PROBLEM 7.121

If the weight per unit length of the cable *AB* is $w_0 / \cos^2 \theta$, prove that the curve formed by the cable is a circular arc. (*Hint:* Use the property indicated in Prob. 7.119.)

SOLUTION

Elemental Segment:



Load on segment* $\quad w(x)dx = \dfrac{w_0}{\cos^2\theta} ds$

But $\quad dx = \cos\theta ds, \quad$ so $\quad w(x) = \dfrac{w_0}{\cos^3\theta}$

From Problem 7.119 $$\frac{d^2y}{dx^2} = \frac{w(x)}{T_0} = \frac{w_0}{T_0\cos^3\theta}$$

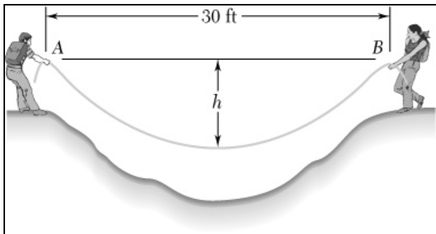
In general $$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(\tan\theta) = \sec^2\theta\frac{d\theta}{dx}$$

So $$\frac{d\theta}{dx} = \frac{w_0}{T_0\cos^3\theta\sec^2\theta} = \frac{w_0}{T_0\cos\theta}$$

or $$\frac{T_0}{w_0}\cos\theta d\theta = dx = rd\theta\cos\theta$$

Giving $r = \dfrac{T_0}{w_0} =$ constant. So curve is circular arc $\quad$ Q.E.D.

*For large sag, it is not appropriate to approximate *ds* by *dx*.

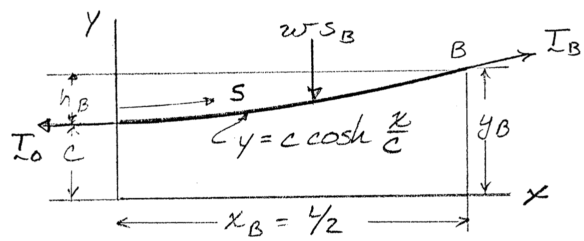


PROBLEM 7.122

Two hikers are standing 30-ft apart and are holding the ends of a 35-ft length of rope as shown. Knowing that the weight per unit length of the rope is 0.05 lb/ft, determine (*a*) the sag *h*, (*b*) the magnitude of the force exerted on the hand of a hiker.

SOLUTION

Half-span:



$$w = 0.05 \text{ lb/ft}, \qquad L = 30 \text{ ft}, \qquad s_B = \frac{35}{2} \text{ ft}$$

$$s_B = c \ \sinh\frac{y_B}{x_B}$$

$$17.5 \text{ ft} = c\sinh\left(\frac{15 \text{ ft}}{c}\right)$$

Solving numerically, $$c = 15.36 \text{ ft}$$

Then $$y_B = c\cosh\frac{x_B}{c} = (15.36 \text{ ft})\cosh\frac{15 \text{ ft}}{15.36 \text{ ft}} = 23.28 \text{ ft}$$

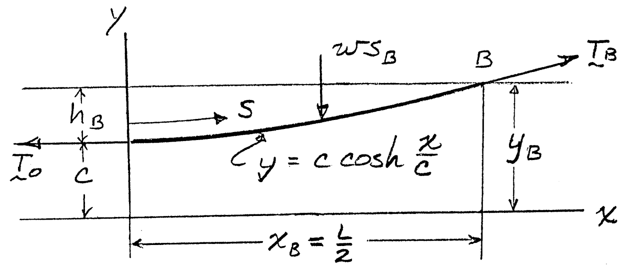
(*a*) $$h_B = y_B - c = 23.28 \text{ ft} - 15.36 \text{ ft} = 7.92 \text{ ft} \blacktriangleleft$$

(*b*) $$T_B = wy_B = (0.05 \text{ lb/ft})(23.28 \text{ ft}) = 1.164 \text{ lb} \blacktriangleleft$$

PROBLEM 7.123

A 60-ft chain weighing 120 lb is suspended between two points at the same elevation. Knowing that the sag is 24 ft, determine (*a*) the distance between the supports, (*b*) the maximum tension in the chain.

SOLUTION



$$s_B = 30 \text{ ft}, \qquad w = \frac{120 \text{ lb}}{60 \text{ ft}} = 2 \text{ lb/ft}$$

$$h_B = 24 \text{ ft}, \qquad x_B = \frac{L}{2}$$

$$y_B^2 = c^2 + s_B^2 = (h_B + c)^2$$
$$= h_B^2 + 2ch_B + c^2$$

$$c = \frac{s_B^2 - h_B^2}{2h_B} = \frac{(30 \text{ ft})^2 - (24 \text{ ft})^2}{2(24 \text{ ft})}$$

$$c = 6.75 \text{ ft}$$

Then
$$s_B = c\sinh\frac{x_B}{c} \rightarrow x_B = c\sinh^{-1}\frac{s_B}{c}$$

$$x_B = (6.75 \text{ ft})\sinh^{-1}\left(\frac{30 \text{ ft}}{6.75 \text{ ft}}\right) = 14.83 \text{ ft}$$

(*a*) $L = 2x_B = 29.7 \text{ ft}$ ◀

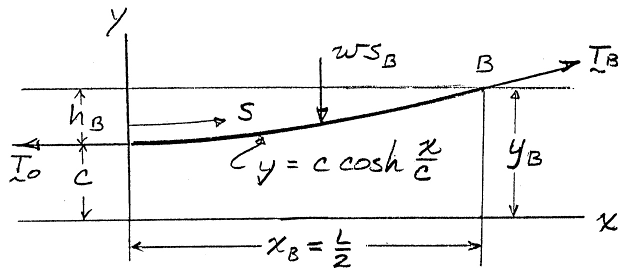
$$T_{\max} = T_B = wy_B = w(c + h_B) = (2 \text{ lb/ft})(6.75 \text{ ft} + 24 \text{ ft}) = 61.5 \text{ lb}$$

(*b*) $T_{\max} = 61.5 \text{ lb}$ ◀

PROBLEM 7.124

A 200-ft steel surveying tape weighs 4 lb. If the tape is stretched between two points at the same elevation and pulled until the tension at each end is 16 lb, determine the horizontal distance between the ends of the tape. Neglect the elongation of the tape due to the tension.

SOLUTION



$$s_B = 100 \text{ ft}, \qquad w = \frac{4 \text{ lb}}{200 \text{ ft}} = 0.02 \text{ lb/ft}$$

$$T_{\max} = 16 \text{ lb}$$

$$T_{\max} = T_B = w y_B$$

$$y_B = \frac{T_B}{w} = \frac{16 \text{ lb}}{0.02 \text{ lb/ft}} = 800 \text{ ft}$$

$$c^2 = y_B^2 - s_B^2$$

$$c = \sqrt{(800 \text{ ft})^2 - (100 \text{ ft})^2} = 793.73 \text{ ft}$$

But

$$y_B = x_B \cosh \frac{x_B}{c} \rightarrow x_B = c \cosh^{-1} \frac{y_B}{c}$$

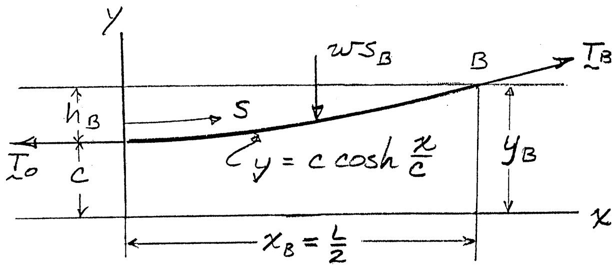
$$= (793.73 \text{ ft}) \ \cosh^{-1}\left(\frac{800 \text{ ft}}{793.73 \text{ ft}}\right) = 99.74 \text{ ft}$$

$$L = 2x_B = 2(99.74 \text{ ft}) = 199.5 \text{ ft} \blacktriangleleft$$

PROBLEM 7.125

An electric transmission cable of length 130 m and mass per unit length of 3.4 kg/m is suspended between two points at the same elevation. Knowing that the sag is 30 m, determine the horizontal distance between the supports and the maximum tension.

SOLUTION



$$s_B = 65 \text{ m}, \qquad h_B = 30 \text{ m}$$

$$w = (3.4 \text{ kg/m})(9.81 \text{ m/s}^2) = 33.35 \text{ N/m}$$

$$y_B^2 = c^2 + s_B^2$$

$$(c + h_B)^2 = c^2 + s_B^2$$

$$c = \frac{s_B^2 - h_B^2}{2h_B} = \frac{(65 \text{ m})^2 - (30 \text{ m})^2}{2(30 \text{ m})}$$

$$= 55.417 \text{ m}$$

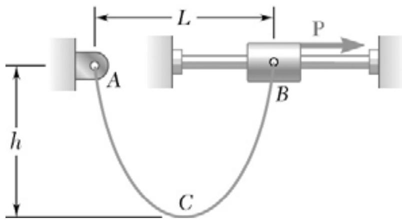
Now
$$s_B = c \sinh \frac{x_B}{c} \rightarrow x_B = c \sinh^{-1} \frac{s_B}{c} = (55.417 \text{ m}) \sinh^{-1}\left(\frac{65 \text{ m}}{55.417 \text{ m}}\right)$$

$$= 55.335 \text{ m}$$

$$L = 2x_B = 2(55.335 \text{ m}) = 110.7 \text{ m} \blacktriangleleft$$

$$T_{\max} = wy_B = w(c + h_B) = (33.35 \text{ N/m})(55.417 \text{ m} + 30 \text{ m}) = 2846 \text{ N}$$

$$T_{\max} = 2.85 \text{ kN} \blacktriangleleft$$

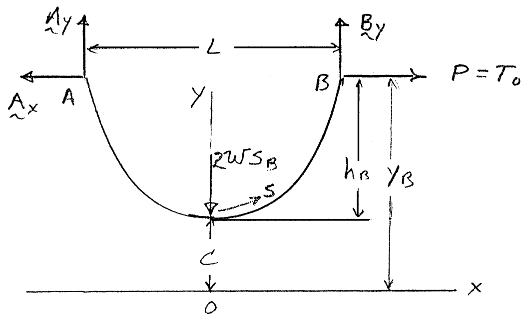


PROBLEM 7.126

A 30-m length of wire having a mass per unit length of 0.3 kg/m is attached to a fixed support at *A* and to a collar at *B*. Neglecting the effect of friction, determine (*a*) the force **P** for which $h = 12$ m, (*b*) the corresponding span *L*.

SOLUTION

FBD Cable:



$$s = 30 \text{ m} \qquad \left(\text{so } s_B = \frac{30 \text{ m}}{2} = 15 \text{ m}\right)$$

$$w = (0.3 \text{ kg/m})(9.81 \text{ m/s}^2) = 2.943 \text{ N/m}$$

$$h_B = 12 \text{ m}$$

$$y_B^2 = (c + h_B)^2 = c^2 + s_B^2$$

So
$$c = \frac{s_B^2 - h_B^2}{2h_B}$$

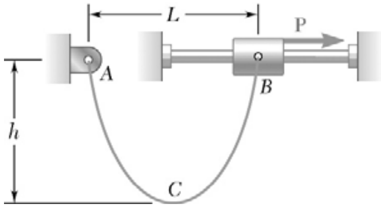
$$c = \frac{(15 \text{ m})^2 - (12 \text{ m})^2}{2(12 \text{ m})} = 3.375 \text{ m}$$

Now
$$s_B = c\sinh\frac{x_B}{c} \to x_B = c\sinh^{-1}\frac{s_B}{c} = (3.375 \text{ m})\sinh^{-1}\left(\frac{15 \text{ m}}{3.375 \text{ m}}\right)$$

$$x_B = 7.4156 \text{ m}$$

$$P = T_0 = wc = (2.943 \text{ N/m})(3.375 \text{ m}) \qquad (a) \qquad \mathbf{P} = 9.93 \text{ N} \longrightarrow \blacktriangleleft$$

$$L = 2x_B = 2(7.4156 \text{ m}) \qquad (b) \qquad L = 14.83 \text{ m} \blacktriangleleft$$

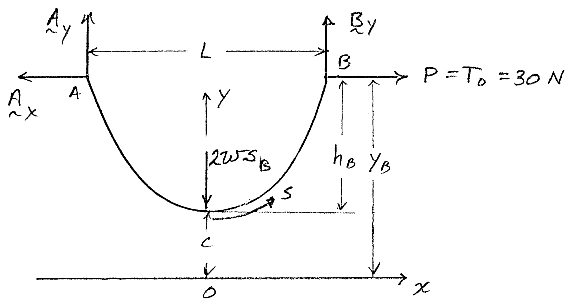


PROBLEM 7.127

A 30-m length of wire having a mass per unit length of 0.3 kg/m is attached to a fixed support at *A* and to a collar at *B*. Knowing that the magnitude of the horizontal force applied to the collar is $P = 30$ N, determine (*a*) the sag *h*, (*b*) the corresponding span *L*.

SOLUTION

FBD Cable:



$$s_T = 30 \text{ m}, \qquad w = (0.3 \text{ kg/m})(9.81 \text{ m/s}^2) = 2.943 \text{ N/m}$$

$$P = T_0 = wc \qquad c = \frac{P}{w}$$

$$c = \frac{30 \text{ N}}{2.943 \text{ N/m}} = 10.1937 \text{ m}$$

$$y_B^2 = (h_B + c)^2 = c^2 + s_B^2$$

$$h^2 + 2ch - s_B^2 = 0 \qquad s_B = \frac{30 \text{ m}}{2} = 15 \text{ m}$$

$$h^2 + 2(10.1937 \text{ m})h - 225 \text{ m}^2 = 0$$

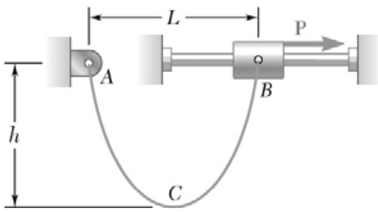
$$h = 7.9422 \text{ m} \qquad (a) \qquad h = 7.94 \text{ m} \blacktriangleleft$$

$$s_B = c \sinh\frac{x_A}{c} \rightarrow x_B = c \sinh^{-1}\frac{s_B}{c} = (10.1937 \text{ m})\sinh^{-1}\left(\frac{15 \text{ m}}{10.1937 \text{ m}}\right)$$

$$= 12.017 \text{ m}$$

$$L = 2x_B = 2(12.017 \text{ m}) \qquad (b) \qquad L = 24.0 \text{ m} \blacktriangleleft$$

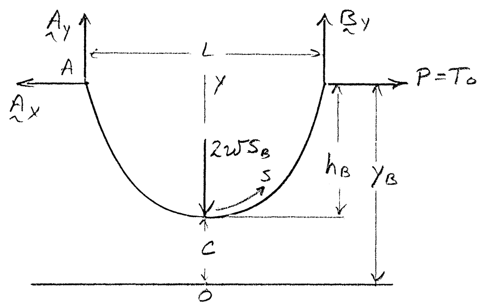
PROBLEM 7.128



A 30-m length of wire having a mass per unit length of 0.3 kg/m is attached to a fixed support at A and to a collar at B. Neglecting the effect of friction, determine (a) the sag h for which $L = 22.5$ m, (b) the corresponding force $\mathbf{P}$.

SOLUTION

FBD Cable:



$$s_T = 30 \text{ m} \rightarrow s_B = \frac{30 \text{ m}}{2} = 15 \text{ m}$$

$$w = (0.3 \text{ kg/m})(9.81 \text{ m/s}^2) = 2.943 \text{ N/m}$$

$$L = 22.5 \text{ m}$$

$$s_B = c \sinh \frac{x_B}{c} = c \sinh \frac{L/2}{c}$$

$$15 \text{ m} = c \sinh \frac{11.25 \text{ m}}{c}$$

Solving numerically: $c = 8.328$ m

$$y_B^2 = c^2 + s_B^2 = (8.328 \text{ m})^2 + (15 \text{ m})^2 = 294.36 \text{ m}^2 \qquad y_B = 17.157 \text{ m}$$

$$h_B = y_B - c = 17.157 \text{ m} - 8.328 \text{ m}$$

(a) $h_B = 8.83 \text{ m}$ ◀

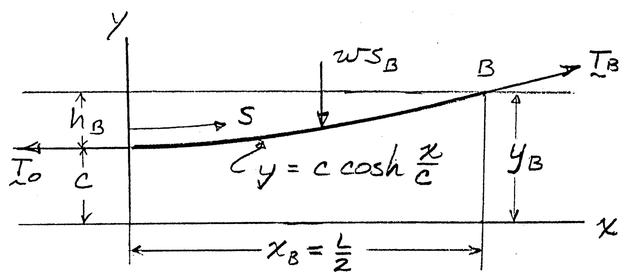
$$P = wc = (2.943 \text{ N/m})(8.328 \text{ m})$$

(b) $\mathbf{P} = 24.5 \text{ N} \longrightarrow$ ◀

PROBLEM 7.129

A 30-ft wire is suspended from two points at the same elevation that are 20 ft apart. Knowing that the maximum tension is 80 lb, determine (*a*) the sag of the wire, (*b*) the total weight of the wire.

SOLUTION



$$L = 20 \text{ ft} \qquad x_B = \frac{20 \text{ ft}}{2} = 10 \text{ ft}$$

$$s_B = \frac{30 \text{ ft}}{2} = 15 \text{ ft}$$

$$s_B = c\sinh\frac{x_B}{c} = c\sinh\frac{10 \text{ ft}}{c}$$

Solving numerically: $\quad c = 6.1647 \text{ ft}$

$$y_B = c\cosh\frac{x_B}{c} = (6.1647 \text{ ft})\cosh\left(\frac{10 \text{ ft}}{1.1647 \text{ ft}}\right)$$

$$y_B = 16.217 \text{ ft}$$

$$h_B = y_B - c = 16.217 \text{ ft} - 6.165 \text{ ft}$$

(*a*) $\qquad h_B = 10.05 \text{ ft}$ ◀

$$T_{\max} = wy_B \qquad \text{and} \qquad W = w(2s_B)$$

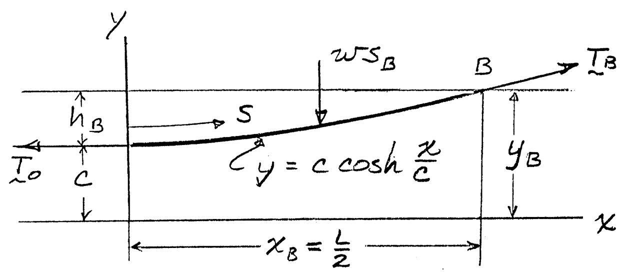
So $\qquad W = \frac{T_{\max}}{y_B}(2s_B) = \frac{80 \text{ lb}}{16.217 \text{ ft}}(30 \text{ ft})$

(*b*) $\qquad \mathbf{W}_m = 148.0 \text{ lb}$ ◀

PROBLEM 7.130

Determine the sag of a 45-ft chain which is attached to two points at the same elevation that are 20 ft apart.

SOLUTION



$$s_B = \frac{45 \text{ ft}}{2} = 22.5 \text{ ft} \qquad L = 20 \text{ ft}$$

$$x_B = \frac{L}{2} = 10 \text{ ft}$$

$$s_B = c \sinh \frac{x_B}{c}$$

$$22.5 \text{ ft} = c \sinh \frac{10 \text{ ft}}{c}$$

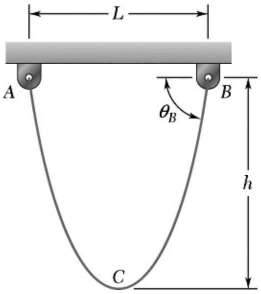
Solving numerically: $c = 4.2023$ ft

$$y_B = c \cosh \frac{x_B}{c}$$

$$= (4.2023 \text{ ft}) \cosh \frac{10 \text{ ft}}{4.2023 \text{ ft}} = 22.889 \text{ ft}$$

$$h_B = y_B - c = 22.889 \text{ ft} - 4.202 \text{ ft}$$

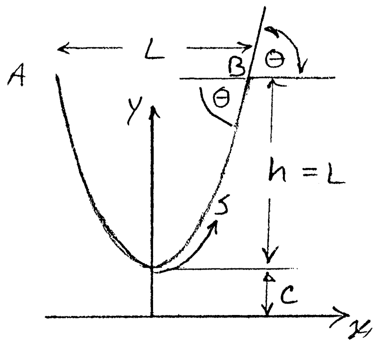
$h_B = 18.69$ ft ◀



PROBLEM 7.131

A 10-m rope is attached to two supports *A* and *B* as shown. Determine (*a*) the span of the rope for which the span is equal to the sag, (*b*) the corresponding angle θ_B.

SOLUTION



We know $\quad y = c\cosh\dfrac{x}{c}$

At *B*, $\quad y_B = c + h = c\cosh\dfrac{h}{2c}$

or $\quad 1 = \cosh\dfrac{h}{2c} - \dfrac{h}{c}$

Solving numerically $\quad \dfrac{h}{c} = 4.933$

$$s_B = c\sinh\frac{x_B}{c} \rightarrow \frac{s_T}{2} = c\sinh\frac{h}{2c}$$

So $\quad c = \dfrac{s_T}{2\sinh\left(\dfrac{h}{2c}\right)} = \dfrac{10\text{ m}}{2\sinh\left(\dfrac{4.933}{2}\right)} = 0.8550\text{ m}$

$h = 4.933c = 4.933(0.8550)\text{ m} = 4.218\text{ m} \qquad h = 4.22\text{ m}$

(*a*) $\quad L = h = 4.22\text{ m}$ ◀

From $\quad y = c\cosh\dfrac{x}{c}, \qquad \dfrac{dy}{dx} = \sinh\dfrac{x}{c}$

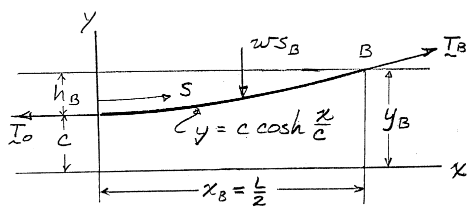
At *B*, $\quad \tan\theta = \left.\dfrac{dy}{dx}\right|_B = \sinh\dfrac{L}{2c} = \sinh\dfrac{4.933}{2} = 5.848$

$\theta = \tan^{-1}5.848 \qquad$ (*b*) $\quad \theta = 80.3°$ ◀

PROBLEM 7.132

A cable having a mass per unit length of 3 kg/m is suspended between two points at the same elevation that are 48 m apart. Determine the smallest allowable sag of the cable if the maximum tension is not to exceed 1800 N.

SOLUTION



$$w = (3\text{ kg/m})(9.81\text{ m/s}^2) = 29.43\text{ N/m}$$

$$L = 48\text{ m}, \qquad T_{\max} \le 1800\text{ N}$$

$$T_{\max} = wy_B \to y_B = \frac{T_{\max}}{w}$$

$$y_B \le \frac{1800\text{ N}}{29.43\text{ N/m}} = 61.162\text{ m}$$

$$y_B = c\cosh\frac{x_B}{c} \qquad 61.162\text{ m} = c\cosh\frac{48\text{ m}/2}{c}\ *$$

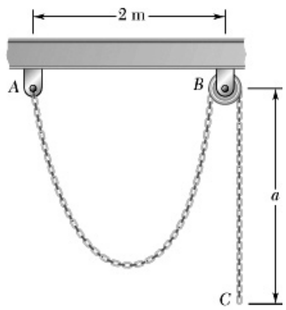
Solving numerically $\qquad c = 55.935\text{ m}$

$$h = y_B - c = 61.162\text{ m} - 55.935\text{ m}$$

$h = 5.23\text{ m}$ ◀

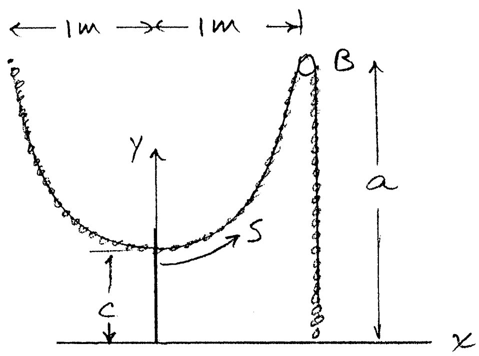
*Note: There is another value of c which will satisfy this equation. It is much smaller, thus corresponding to a much larger h.

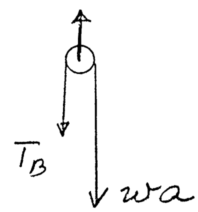
PROBLEM 7.133



An 8-m length of chain having a mass per unit length of 3.72 kg/m is attached to a beam at A and passes over a small pulley at B as shown. Neglecting the effect of friction, determine the values of distance a for which the chain is in equilibrium.

SOLUTION





Neglect pulley size and friction $\qquad T_B = wa$

But $\qquad T_B = wy_B \qquad$ so $\qquad y_B = a$

$$y_B = c\cosh\frac{x_B}{c}$$

$$c\cosh\frac{1\text{ m}}{c} = a$$

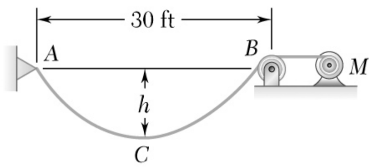
But $\quad s_B = c\sinh\frac{x_B}{c} \qquad \frac{8\text{ m} - a}{2} = c\sinh\frac{1\text{ m}}{c}$

So $$4\text{ m} = c\sinh\frac{1\text{ m}}{c} + \frac{c}{2}\cosh\frac{1\text{ m}}{c}$$

$$16\text{ m} = c\left(3e^{1/c} - e^{-1/c}\right)$$

Solving numerically $\qquad c = 0.3773\text{ m},\ 5.906\text{ m}$

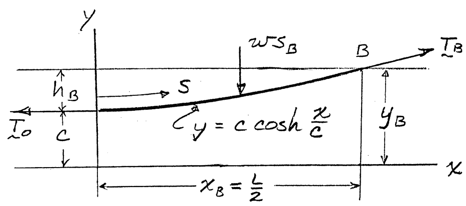
$$a = c\cosh\frac{1\text{ m}}{c} = \begin{cases} (0.3773\text{ m})\cosh\dfrac{1\text{ m}}{0.3773\text{ m}} = 2.68\text{ m} \blacktriangleleft \\ (5.906\text{ m})\cosh\dfrac{1\text{ m}}{5.906\text{ m}} = 5.99\text{ m} \blacktriangleleft \end{cases}$$



PROBLEM 7.134

A motor M is used to slowly reel in the cable shown. Knowing that the weight per unit length of the cable is 0.5 lb/ft, determine the maximum tension in the cable when $h = 15$ ft.

SOLUTION



$$w = 0.5 \text{ lb/ft} \qquad L = 30 \text{ ft} \qquad h_B = 15 \text{ ft}$$

$$y_B = c \cosh \frac{x_B}{c}$$

$$h_B + c = c \cosh \frac{L}{2c}$$

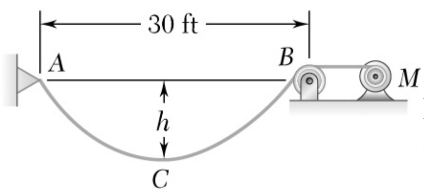
$$15 \text{ ft} = c\left(\cosh \frac{15 \text{ ft}}{c} - 1 \right)$$

Solving numerically $\quad c = 9.281 \text{ ft}$

$$y_B = (9.281 \text{ ft}) \cosh \frac{15 \text{ ft}}{9.281 \text{ ft}} = 24.281 \text{ ft}$$

$$T_{\max} = T_B = w y_B = (0.5 \text{ lb/ft})(24.281 \text{ ft})$$

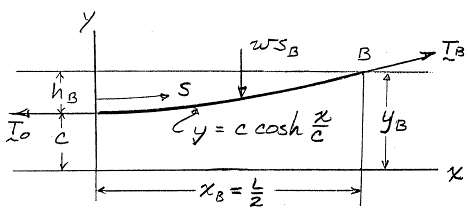
$T_{\max} = 12.14 \text{ lb}$ ◀



PROBLEM 7.135

A motor M is used to slowly reel in the cable shown. Knowing that the weight per unit length of the cable is 0.5 lb/ft, determine the maximum tension in the cable when $h = 9$ ft.

SOLUTION



$$w = 0.5 \text{ lb/ft}, \qquad L = 30 \text{ ft}, \qquad h_B = 9 \text{ ft}$$

$$y_B = h_B + c = c\cosh\frac{x_B}{c} = c\cosh\frac{L}{2c}$$

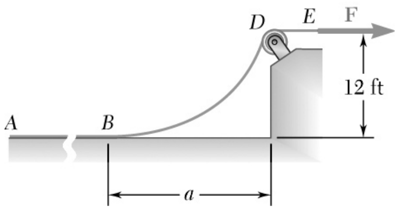
$$9 \text{ ft} = c\left(\cosh\frac{15 \text{ ft}}{c} - 1\right)$$

Solving numerically $c = 13.783$ ft

$$y_B = h_B + c = 9 \text{ ft} + 13.783 \text{ ft} = 21.783 \text{ ft}$$

$$T_{max} = T_B = wy_B = (0.5 \text{ lb/ft})(21.78 \text{ ft})$$

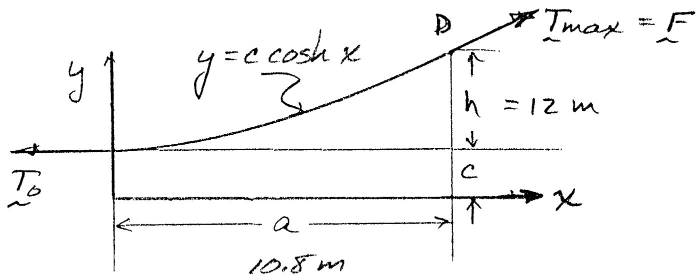
$$T_{max} = 11.39 \text{ lb} \blacktriangleleft$$



PROBLEM 7.136

To the left of point *B* the long cable *ABDE* rests on the rough horizontal surface shown. Knowing that the weight per unit length of the cable is 1.5 lb/ft, determine the force **F** when $a = 10.8$ ft.

SOLUTION



$$y_D = c\cosh\frac{x_D}{c}$$

$$h + c = c\cosh\frac{a}{c}$$

$$12\text{ m} = c\left(\cosh\frac{10.8\text{ m}}{c} - 1\right)$$

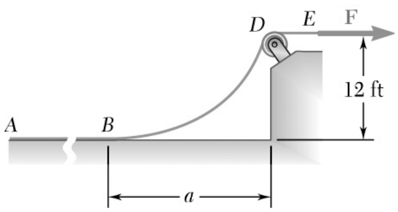
Solving numerically $\qquad c = 6.2136\text{ m}$

Then $\qquad y_B = (6.2136\text{ m})\cosh\frac{10.8\text{ m}}{6.2136\text{ m}} = 18.2136\text{ m}$

$$F = T_{\max} = wy_B = (1.5\text{ lb/ft})(18.2136\text{ m})$$

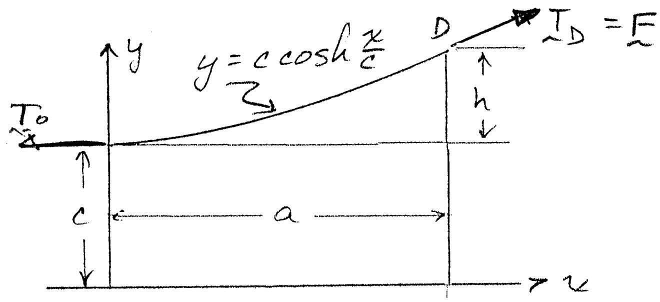
$\mathbf{F} = 27.3\text{ lb} \longrightarrow$ ◀

PROBLEM 7.137



To the left of point *B* the long cable *ABDE* rests on the rough horizontal surface shown. Knowing that the weight per unit length of the cable is 1.5 lb/ft, determine the force **F** when $a = 18$ ft.

SOLUTION



$$y_D = c\cosh\frac{x_D}{c}$$

$$c + h = c\cosh\frac{a}{c}$$

$$h = c\left(\cosh\frac{a}{c} - 1\right)$$

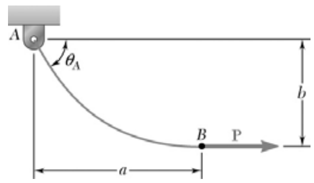
$$12\text{ ft} = c\left(\cosh\frac{18\text{ ft}}{c} - 1\right)$$

Solving numerically $\qquad c = 15.162$ ft

$$y_B = h + c = 12\text{ ft} + 15.162\text{ ft} = 27.162\text{ ft}$$

$$F = T_D = wy_D = (1.5\text{ lb/ft})(27.162\text{ ft}) = 40.74\text{ lb}$$

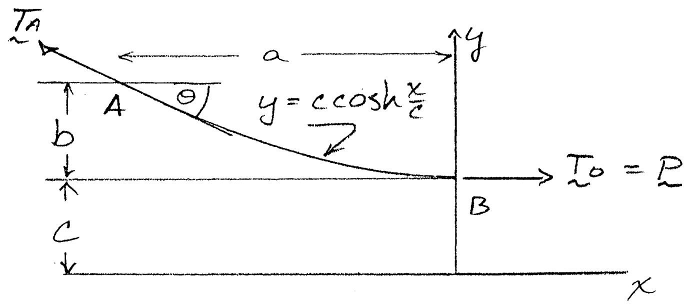
$\mathbf{F} = 40.7$ lb $\longrightarrow$ ◀



PROBLEM 7.138

A uniform cable has a mass per unit length of 4 kg/m and is held in the position shown by a horizontal force **P** applied at *B*. Knowing that $P = 800$ N and $\theta_A = 60°$, determine (*a*) the location of point *B*, (*b*) the length of the cable.

SOLUTION



$$w = 4\text{ kg/m}\left(9.81\text{ m/s}^2\right) = 39.24\text{ N/m}$$

$$P = T_0 = wc \qquad c = \frac{P}{w} = \frac{800\text{ N}}{39.24\text{ N/m}}$$

$$c = 20.387\text{ m}$$

$$y = c\cosh\frac{x}{c}$$

$$\frac{dy}{dx} = \sinh\frac{x}{c}$$

$$\tan\theta = -\left.\frac{dy}{dx}\right|_{-a} = -\sinh\frac{-a}{c} = \sinh\frac{a}{c}$$

$$a = c\sinh^{-1}(\tan\theta) = (20.387\text{ m})\sinh^{-1}(\tan 60°)$$

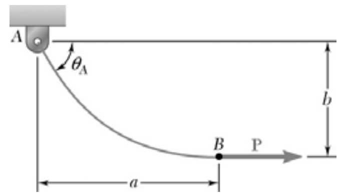
$$a = 26.849\text{ m}$$

$$y_A = c\cosh\frac{a}{c} = (20.387\text{ m})\cosh\frac{26.849\text{ m}}{20.387\text{ m}} = 40.774\text{ m}$$

$$b = y_A - c = 40.774\text{ m} - 20.387\text{ m} = 20.387\text{ m}$$

So (*a*) *B* is 26.8 m right and 20.4 m down from *A* ◀

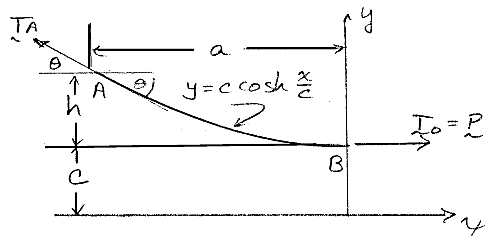
$$s = c\sinh\frac{a}{c} = (20.387\text{ m})\sinh\frac{26.849\text{ m}}{20.387\text{ m}} = 35.31\text{ m}$$ (*b*) $s = 35.3\text{ m}$ ◀



PROBLEM 7.139

A uniform cable having a mass per unit length of 4 kg/m is held in the position shown by a horizontal force **P** applied at *B*. Knowing that $P = 600$ N and $\theta_A = 60°$, determine (*a*) the location of point *B*, (*b*) the length of the cable.

SOLUTION



$$w = (4 \text{ kg/m})(9.81 \text{ m/s}^2) = 39.24 \text{ N/m}$$

$$P = T_0 = wc \qquad c = \frac{P}{w} = \frac{600 \text{ N}}{39.24 \text{ N/m}}$$

$$c = 15.2905 \text{ m}$$

$$y = c\cosh\frac{x}{c} \qquad \frac{dy}{dx} = \sinh\frac{x}{c}$$

At *A*: $\tan\theta = -\left.\frac{dy}{dx}\right|_{-a} = -\sinh\frac{-a}{c} = \sinh\frac{a}{c}$

So $\quad a = c\sinh^{-1}(\tan\theta) = (15.2905 \text{ m})\sinh^{-1}(\tan 60°) = 20.137 \text{ m}$

$$y_B = h + c = c\cosh\frac{a}{c}$$

$$h = c\left(\cosh\frac{a}{c} - 1\right)$$

$$= (15.2905 \text{ m})\left(\cosh\frac{20.137 \text{ m}}{15.2905 \text{ m}} - 1\right)$$

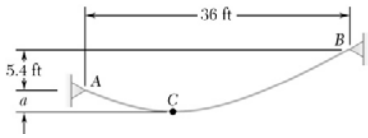
$$= 15.291 \text{ m}$$

So (*a*) *B* is 20.1 m right and 15.29 m down from *A* ◀

$$s = c\sinh\frac{a}{c} = (15.291 \text{ m})\sinh\frac{20.137 \text{ m}}{15.291 \text{ m}} = 26.49 \text{ m}$$

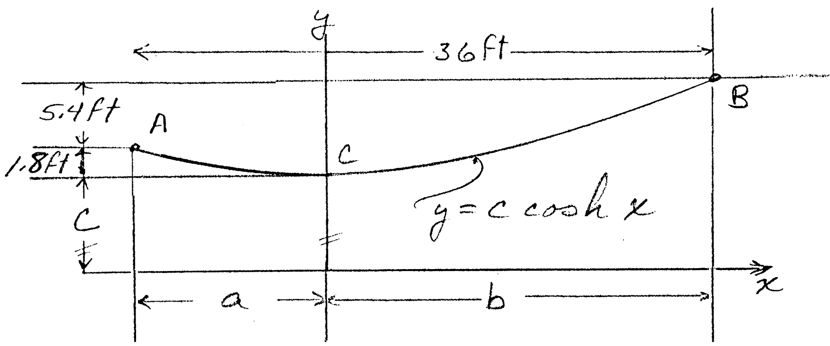
(*b*) $s = 26.5$ m ◀

PROBLEM 7.140



The cable *ACB* weighs 0.3 lb/ft. Knowing that the lowest point of the cable is located at a distance $a = 1.8$ ft below the support *A*, determine (*a*) the location of the lowest point *C*, (*b*) the maximum tension in the cable.

SOLUTION



$$y_A = c\cosh\frac{-a}{c} = c + 1.8\text{ ft}$$

$$a = c\cosh^{-1}\left(1 + \frac{1.8\text{ ft}}{c}\right)$$

$$y_B = c\cosh\frac{b}{c} = c + 7.2\text{ ft}$$

$$b = c\cosh^{-1}\left(1 + \frac{7.2\text{ ft}}{c}\right)$$

But $$a + b = 36\text{ ft} = c\left[\cosh^{-1}\left(1 + \frac{1.8\text{ ft}}{c}\right) + \cosh^{-1}\left(1 + \frac{7.2\text{ ft}}{c}\right)\right]$$

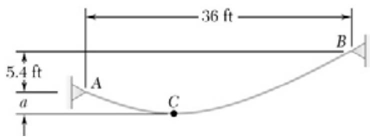
Solving numerically $\quad c = 40.864$ ft

Then $$b = (40.864\text{ ft})\cosh^{-1}\left(1 + \frac{7.2\text{ ft}}{40.864\text{ ft}}\right) = 23.92\text{ ft}$$

(*a*) C is 23.9 ft left of and 7.20 ft below *B* ◀

$$T_{\max} = wy_B = (0.3\text{ lb/ft})(40.864\text{ ft} + 7.2\text{ ft})$$

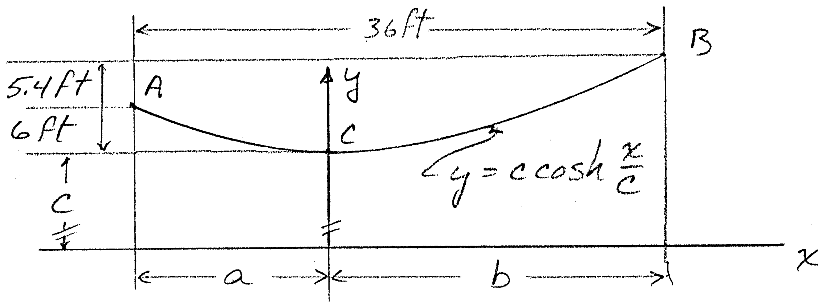
(*b*) $T_{\max} = 14.42$ lb ◀



PROBLEM 7.141

The cable *ACB* weighs 0.3 lb/ft. Knowing that the lowest point of the cable is located at a distance $a = 6$ ft below the support *A*, determine (*a*) the location of the lowest point *C*, (*b*) the maximum tension in the cable.

SOLUTION



$$y_A = c\cosh\frac{-a}{c} = c + 6 \text{ ft}$$

$$a = c\cosh^{-1}\left(1 + \frac{6 \text{ ft}}{c}\right)$$

$$y_B = c\cosh\frac{b}{c} = c + 11.4 \text{ ft}$$

$$b = c\cosh^{-1}\left(1 + \frac{11.4 \text{ ft}}{c}\right)$$

So

$$a + b = c\left[\cosh^{-1}\left(1 + \frac{6 \text{ ft}}{c}\right) + \cosh^{-1}\left(1 + \frac{11.4 \text{ ft}}{\text{c}}\right)\right] = 36 \text{ ft}$$

Solving numerically

$$c = 20.446 \text{ ft}$$

$$b = (20.446 \text{ ft})\cosh^{-1}\left(1 + \frac{11.4 \text{ ft}}{20.446 \text{ ft}}\right) = 20.696 \text{ ft}$$

(a) *C* is 20.7 ft left of and 11.4 ft below *B* ◀

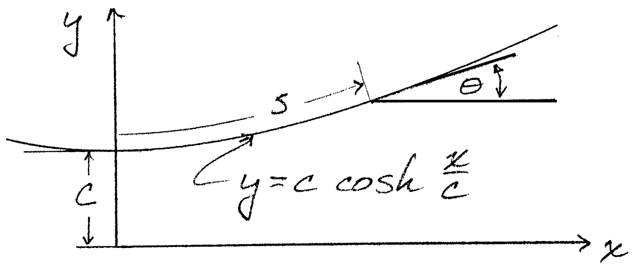
$$T_{\max} = wy_B = (0.3 \text{ lb/ft})(20.446 \text{ ft})\cosh\left(\frac{20.696 \text{ ft}}{20.446 \text{ ft}}\right) = 9.554 \text{ lb}$$

(*b*) $T_{\max} = 9.55$ lb ◀

PROBLEM 7.142

Denoting by θ the angle formed by a uniform cable and the horizontal, show that at any point (*a*) $s = c\tan\theta$, (*b*) $y = c\sec\theta$.

SOLUTION



(*a*) $$\tan\theta = \frac{dy}{dx} = \sinh\frac{x}{c}$$

$$s = c\sinh\frac{x}{c} = c\tan\theta \quad \text{Q.E.D.}$$

(*b*) Also $$y^2 = s^2 + c^2\left(\cosh^2 x = \sinh^2 x + 1\right)$$

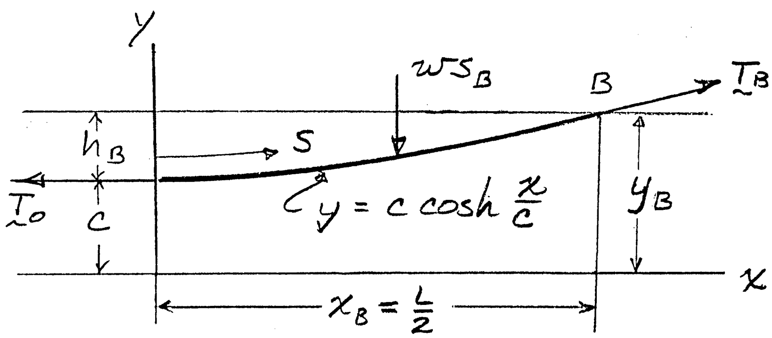
So $$y^2 = c^2\left(\tan^2\theta + 1\right) = c^2\sec^2\theta$$

And $$y = c\sec\theta \quad \text{Q.E.D.}$$

PROBLEM 7.143

(a) Determine the maximum allowable horizontal span for a uniform cable of mass per unit length m' if the tension in the cable is not to exceed a given value T_m. (b) Using the result of part a, determine the maximum span of a steel wire for which $m' = 0.34$ kg/m and $T_m = 32$ kN.

SOLUTION



$$T_B = T_{\max} = wy_B$$

$$= wc\cosh\frac{x_B}{c} = w\frac{L}{2}\left(\frac{2c}{L}\right)\cosh\frac{L}{2c}$$

Let $\quad \xi = \dfrac{L}{2c} \quad$ so $\quad T_{\max} = \dfrac{wL}{2\xi}\cosh\xi$

$$\frac{dT_{\max}}{d\xi} = \frac{wL}{2\xi}\left(\sinh\xi - \frac{1}{\xi}\cosh\xi\right)$$

For $\quad \min T_{\max}, \quad \tanh\xi - \dfrac{1}{\xi} = 0$

Solving numerically $\quad \xi = 1.1997$

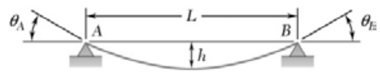
$$(T_{\max})_{\min} = \frac{wL}{2(1.9997)}\cosh(1.1997) = 0.75444wL$$

(a) $\quad L_{\max} = \dfrac{T_{\max}}{0.75444w} = 1.3255\dfrac{T_{\max}}{w}$ ◀

If $\quad T_{\max} = 32$ kN and $w = (0.34 \text{ kg/m})(9.81 \text{ m/s}^2) = 3.3354$ N/m

$$L_{\max} = 1.3255\frac{32.000 \text{ N}}{3.3354 \text{ N/m}} = 12\,717 \text{ m}$$

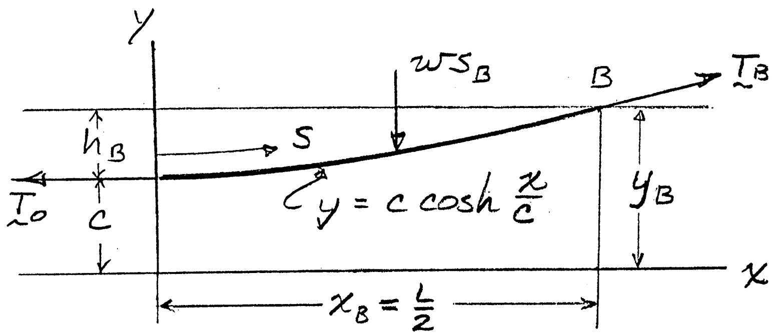
(b) $\quad L_{\max} = 12.72$ km ◀



PROBLEM 7.144

A cable has a weight per unit length of 2 lb/ft and is supported as shown. Knowing that the span L is 18 ft, determine the two values of the sag h for which the maximum tension is 80 lb.

SOLUTION



$$y_{\max} = c\cosh\frac{L}{2c} = h + c$$

$$T_{\max} = wy_{\max} \qquad y_{\max} = \frac{T_{\max}}{w}$$

$$y_{\max} = \frac{80 \text{ lb}}{2 \text{ lb/ft}} = 40 \text{ ft}$$

$$c\cosh\frac{9 \text{ ft}}{c} = 40 \text{ ft}$$

Solving numerically $\quad c_1 = 2.6388$ ft

$$c_2 = 38.958 \text{ ft}$$

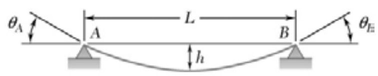
$$h = y_{\max} - c$$

$$h_1 = 40 \text{ ft} - 2.6388 \text{ ft}$$

$h_1 = 37.4$ ft ◀

$$h_2 = 40 \text{ ft} - 38.958 \text{ ft}$$

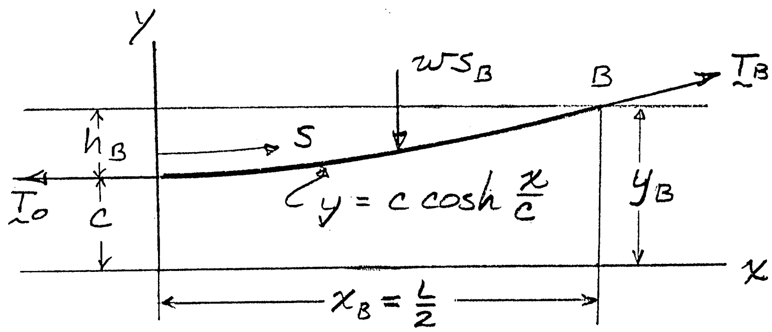
$h_2 = 1.042$ ft ◀



PROBLEM 7.145

Determine the sag-to-span ratio for which the maximum tension in the cable is equal to the total weight of the entire cable *AB*.

SOLUTION



$$T_{\max} = wy_B = 2ws_B$$

$$y_B = 2s_B$$

$$c\cosh\frac{L}{2c} = 2c\sinh\frac{L}{2c}$$

$$\tanh\frac{L}{2c} = \frac{1}{2}$$

$$\frac{L}{2c} = \tanh^{-1}\frac{1}{2} = 0.549306$$

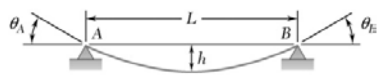
$$\frac{h_B}{c} = \frac{y_B - c}{c} = \cosh\frac{L}{2c} - 1$$

$$= 0.154701$$

$$\frac{h_B}{L} = \frac{h_B/c}{2(L/2c)}$$

$$= \frac{0.5(0.154701)}{0.549306} = 0.14081$$

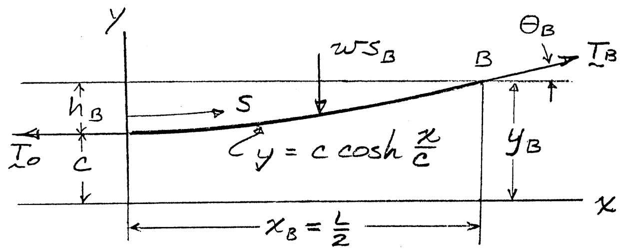
$$\frac{h_B}{L} = 0.1408 \blacktriangleleft$$



PROBLEM 7.146

A cable of weight w per unit length is suspended between two points at the same elevation that are a distance L apart. Determine (a) the sag-to-span ratio for which the maximum tension is as small as possible, (b) the corresponding values of θ_B and T_m.

SOLUTION



(a)

$$T_{\max} = wy_B = wc\cosh\frac{L}{2c}$$

$$\frac{dT_{\max}}{dc} = w\left(\cosh\frac{L}{2c} - \frac{L}{2c}\sinh\frac{L}{2c}\right)$$

For $\quad \min T_{\max}, \quad \dfrac{dT_{\max}}{dc} = 0$

$$\tanh\frac{L}{2c} = \frac{2c}{L} \longrightarrow \frac{L}{2c} = 1.1997$$

$$\frac{y_B}{c} = \cosh\frac{L}{2c} = 1.8102$$

$$\frac{h}{c} = \frac{y_B}{c} - 1 = 0.8102$$

$$\frac{h}{L} = \left[\frac{1}{2}\frac{h}{c}\left(\frac{2c}{L}\right)\right] = \frac{0.8102}{2(1.1997)} = 0.3375$$

$\dfrac{h}{L} = 0.338$ ◀

(b) $\quad T_0 = wc \qquad T_{\max} = wc\cosh\dfrac{L}{2c} \qquad \dfrac{T_{\max}}{T_0} = \cosh\dfrac{L}{2c} = \dfrac{y_B}{c}$

But $\quad T_0 = T_{\max}\cos\theta_B \qquad \dfrac{T_{\max}}{T_0} = \sec\theta_B$

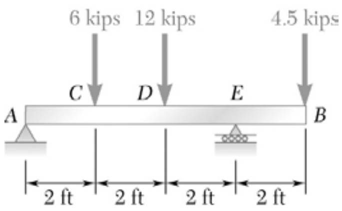
So $\quad \theta_B = \sec^{-1}\left(\dfrac{y_B}{c}\right) = \sec^{-1}(1.8102)$

$$= 56.46°$$

$\theta_B = 56.5°$ ◀

$$T_{\max} = wy_B = w\frac{y_B}{c}\left(\frac{2c}{L}\right)\left(\frac{L}{2}\right) = w(1.8102)\frac{L}{2(1.1997)}$$

$T_{\max} = 0.755wL$ ◀

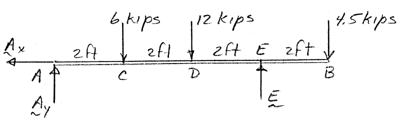


PROBLEM 7.147

For the beam and loading shown, (a) draw the shear and bending moment diagrams, (b) determine the maximum absolute values of the shear and bending moment.

SOLUTION

FBD Beam:



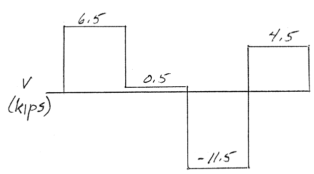
(a)

$$\circlearrowleft \Sigma M_A = 0: (6 \text{ ft})E - (8 \text{ ft})(4.5 \text{ kips}) - (4 \text{ ft})(12 \text{ kips}) - (2 \text{ ft})(6 \text{ kips}) = 0$$

$$\mathbf{E} = 16 \text{ kips} \uparrow$$

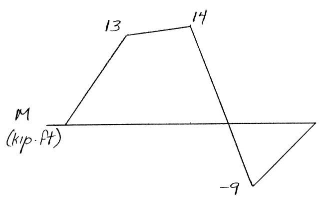
$$\circlearrowleft \Sigma M_E = 0: -(6 \text{ ft})A_y + (4 \text{ ft})(6 \text{ kips}) + (2 \text{ ft})(12 \text{ kips}) - (2 \text{ ft})(4.5 \text{ kips}) = 0$$

$$\mathbf{A}_y = 6.5 \text{ kips} \uparrow$$



Shear Diag: V is piece wise constant with discontinuities equal to the forces at A, C, D, E, B

Moment Diag: M is piecewise linear with slope changes at C, D, E



$$M_A = 0$$

$$M_C = (6.5 \text{ kips})(2 \text{ ft}) = 13 \text{ kip}\cdot\text{ft}$$

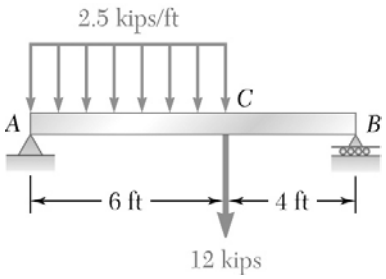
$$M_C = 13 \text{ kip}\cdot\text{ft} + (0.5 \text{ kips})(2 \text{ ft}) = 14 \text{ kip}\cdot\text{ft}$$

$$M_D = 14 \text{ kip}\cdot\text{ft} - (11.5 \text{ kips})(2 \text{ ft}) = -9 \text{ kip}\cdot\text{ft}$$

$$M_B = -9 \text{ kip}\cdot\text{ft} + (4.5 \text{ kips})(2 \text{ ft}) = 0$$

(b) $|V|_{\max} = 11.50$ kips on DE ◀

$|M|_{\max} = 14.00$ kip·ft at D ◀

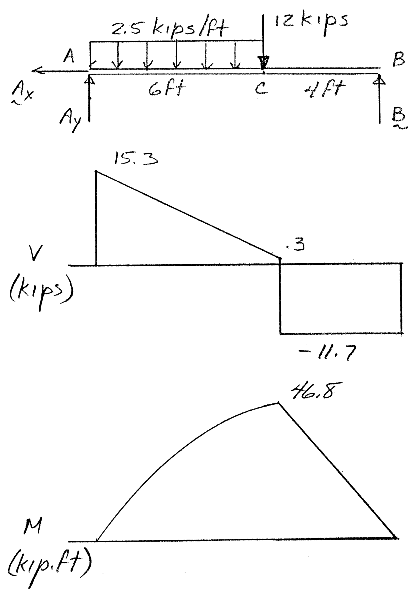


PROBLEM 7.148

For the beam and loading shown, (*a*) draw the shear and bending moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

FBD Beam:



(*a*) $\circlearrowleft \Sigma M_B = 0\colon (4\text{ ft})(12\text{ kips}) + (7\text{ ft})(2.5\text{ kips/ft})(6\text{ ft})$

$- (10\text{ ft})A_y = 0$

$$\mathbf{A}_y = 15.3\text{ kips} \uparrow$$

Shear Diag: $V_A = A_y = 15.3\text{ kips}$, then V is linear

$$\left(\frac{dV}{dx} = -2.5\text{ kips/ft}\right)\text{ to } C.$$

$$V_C = 15.3\text{ kips} - (2.5\text{ kips/ft})(6\text{ ft}) = 0.3\text{ kips}$$

At C, V decreases by 12 kips to -11.7 kips and is constant to B.

Moment Diag: $M_A = 0$ and M is parabolic

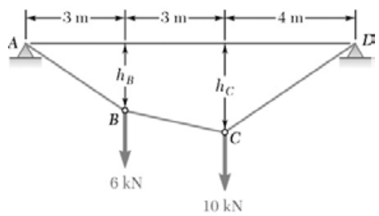
$$\left(\frac{dM}{dx}\text{ decreasing with } V\right)\text{ to } C$$

$$M_C = \frac{1}{2}(15.3\text{ kips} + 0.3\text{ kip})(6\text{ ft}) = 46.8\text{ kip}\cdot\text{ft}$$

$$M_B = 46.8\text{ kip}\cdot\text{ft} - (11.7\text{ kips})(4\text{ ft}) = 0$$

(*b*) $|V|_{\max} = 15.3\text{ kips}$ ◀

$|M|_{\max} = 46.8\text{ kip}\cdot\text{ft}$ ◀

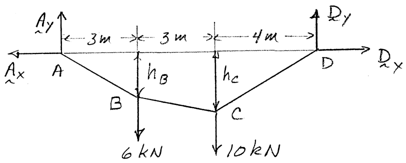


PROBLEM 7.149

Two loads are suspended as shown from the cable *ABCD*. Knowing that $h_B = 1.8$ m, determine (*a*) the distance h_C, (*b*) the components of the reaction at *D*, (*c*) the maximum tension in the cable.

SOLUTION

FBD Cable:



$$\xrightarrow{} \Sigma F_x = 0: -A_x + D_x = 0 \qquad A_x = D_x$$

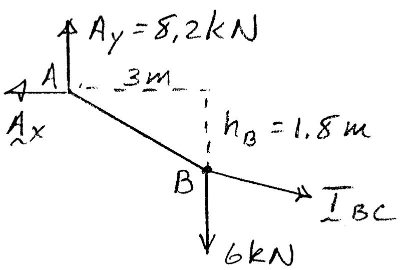
$$\circlearrowleft \Sigma M_A = 0: (10\text{ m})D_y - (6\text{ m})(10\text{ kN}) - (3\text{ m})(6\text{ kN}) = 0$$

$$\mathbf{D}_y = 7.8\text{ kN} \uparrow$$

$$\uparrow \Sigma F_y = 0: A_y - 6\text{ kN} - 10\text{ kN} + 7.8\text{ kN} = 0$$

$$\mathbf{A}_y = 8.2\text{ kN} \uparrow$$

FBD AB:

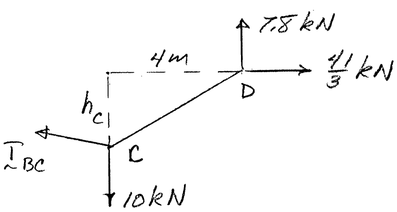


$$\circlearrowleft \Sigma M_B = 0: (1.8\text{ m})A_x - (3\text{ m})(8.2\text{ kN}) = 0$$

$$\mathbf{A}_x = \frac{41}{3}\text{ kN} \longleftarrow$$

From above $\quad D_x = A_x = \dfrac{41}{3}$ kN

FBD CD:



$$\circlearrowleft \Sigma M_C = 0: (4\text{ m})(7.8\text{ kN}) - h_C\left(\frac{41}{3}\text{ kN}\right) = 0$$

$$h_C = 2.283\text{ m}$$

(*a*) $\qquad h_C = 2.28\text{ m}$ ◀

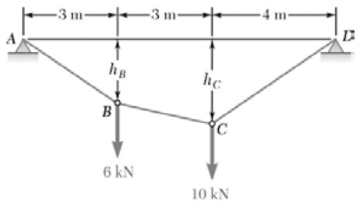
(*b*) $\qquad D_x = 13.67\text{ kN} \longrightarrow$ ◀

$\qquad D_y = 7.80\text{ kN} \uparrow$ ◀

Since $A_x = B_x$ and $A_y > B_y$, max T is T_{AB}

$$T_{AB} = \sqrt{A_x^2 + A_y^2} = \sqrt{\left(\frac{41}{3}\text{ kN}\right)^2 + (8.2\text{ kN})^2}$$

(*c*) $\qquad T_{\max} = 15.94\text{ kN}$ ◀

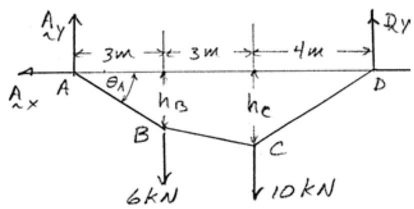


PROBLEM 7.150

Knowing that the maximum tension in cable *ABCD* is 15 kN, determine (*a*) the distance h_B, (*b*) the distance h_C.

SOLUTION

FBD Cable:



$$\xrightarrow{} \Sigma F_x = 0: -A_x + D_x = 0 \qquad A_x = D_x$$

$$\circlearrowleft \Sigma M_A = 0: (10\text{ m})D_y - (6\text{ m})(10\text{ kN}) - (3\text{ m})(6\text{ kN}) = 0$$

$$\mathbf{D}_y = 7.8\text{ kN}\uparrow$$

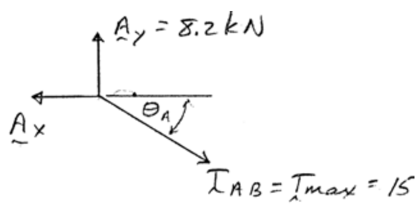
$$\uparrow \Sigma F_y = 0: A_y - 6\text{ kN} - 10\text{ kN} + 7.8\text{ kN} = 0$$

$$\mathbf{A}_y = 8.2\text{ kN}\uparrow$$

Since $\qquad A_x = D_x \qquad$ and $\qquad A_y > D_y, \qquad T_{\max} = T_{AB}$

$$\uparrow \Sigma F_y = 0: 8.2\text{ kN} - (15\text{ kN})\sin\theta_A = 0$$

FBD pt A:

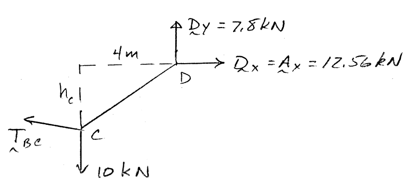


$$\theta_A = \sin^{-1}\frac{8.2\text{ kN}}{15\text{ kN}} = 33.139°$$

$$\xrightarrow{} \Sigma F_x = 0: -A_x + (15\text{ kN})\cos\theta_A = 0$$

$$A_x = (15\text{ kN})\cos(33.139°) = 12.56\text{ kN}$$

FBD CD:

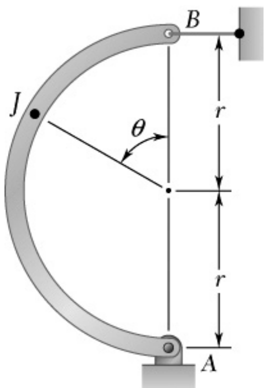


From FBD cable: $\qquad h_B = (3\text{ m})\tan\theta_A = (3\text{ m})\tan(33.139°)$

(*a*) $\qquad h_B = 1.959\text{ m}$ ◀

$$\circlearrowleft \Sigma M_C = 0: (4\text{ m})(7.8\text{ kN}) - h_C(12.56\text{ kN}) = 0$$

(*b*) $\qquad h_C = 2.48\text{ m}$ ◀

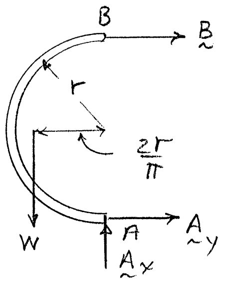


PROBLEM 7.151

A semicircular rod of weight W and uniform cross section is supported as shown. Determine the bending moment at point J when $\theta = 60°$.

SOLUTION

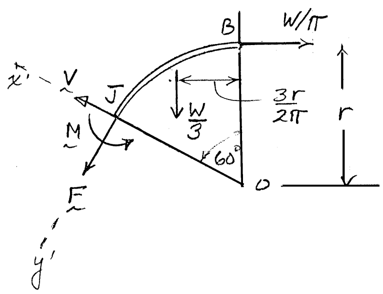
FBD Rod:



$$\circlearrowleft \Sigma M_A = 0: \frac{2r}{\pi}W - 2rB = 0$$

$$\mathbf{B} = \frac{W}{\pi} \longrightarrow$$

FBD BJ:



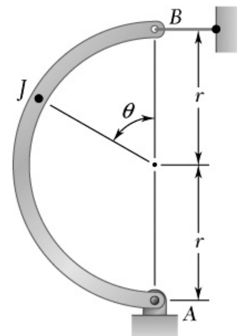
$$\swarrow \Sigma F_{y'} = 0: F + \frac{W}{3}\sin 60° - \frac{W}{\pi}\cos 60° = 0$$

$$F = -0.12952W$$

$$\circlearrowleft \Sigma M_0 = 0: r\left(F - \frac{W}{\pi}\right) + \frac{3r}{2\pi}\left(\frac{W}{3}\right) + M = 0$$

$$M = Wr\left(0.12952 + \frac{1}{\pi} - \frac{1}{2\pi}\right) = 0.28868Wr$$

On BJ $\qquad \mathbf{M}_J = 0.289Wr \circlearrowright$ ◀

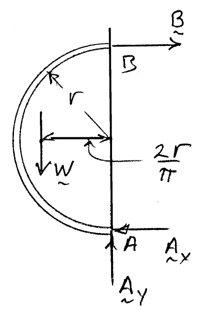


PROBLEM 7.152

A semicircular rod of weight W and uniform cross section is supported as shown. Determine the bending moment at point J when $\theta = 150°$.

SOLUTION

FBD rod:

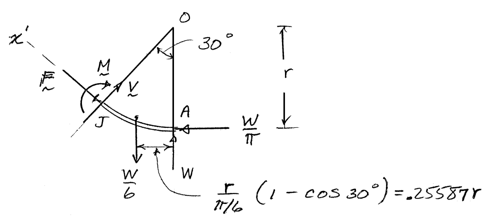


$$\uparrow \Sigma F_y = 0: \; A_y - W = 0 \qquad \mathbf{A}_y = W \uparrow$$

$$\Sigma M_B = 0: \; \frac{2r}{\pi} W - 2rA_x = 0$$

$$\mathbf{A}_x = \frac{W}{\pi} \longleftarrow$$

FBD AJ:



$$\nwarrow \Sigma F_{x'} = 0: \; \frac{W}{\pi}\cos 30° + \frac{5W}{6}\sin 30° - F = 0 \quad F = 0.69233W \searrow$$

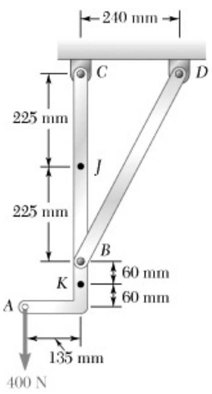
$$\circlearrowleft \Sigma M_0 = 0: 0.25587r\left(\frac{W}{6}\right) + r\left(F - \frac{W}{\pi}\right) - M = 0$$

$$M = Wr\left[\frac{0.25587}{6} + 0.69233 - \frac{1}{\pi}\right]$$

$$M = Wr(0.4166)$$

On AJ $\qquad \mathbf{M} = 0.417Wr \circlearrowright$ ◀

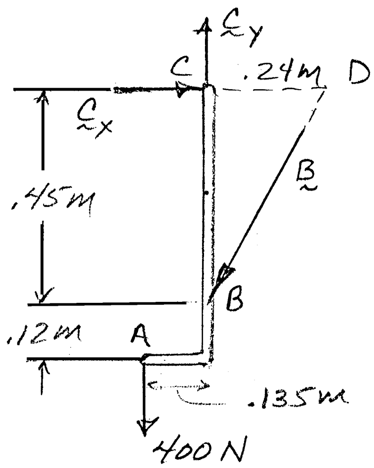
PROBLEM 7.153



Determine the internal forces at point J of the structure shown.

SOLUTION

FBD ABC:



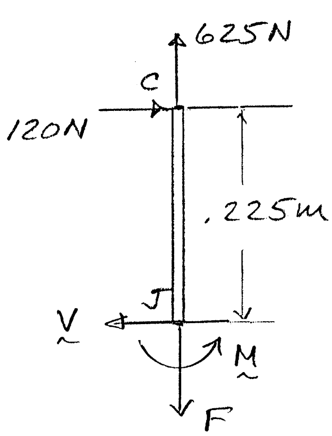
$$\circlearrowleft \Sigma M_D = 0: (0.375 \text{ m})(400 \text{ N}) - (0.24 \text{ m})C_y = 0$$

$$\mathbf{C}_y = 625 \text{ N} \uparrow$$

$$\circlearrowleft \Sigma M_B = 0: -(0.45 \text{ m})C_x + (0.135 \text{ m})(400 \text{ N}) = 0$$

$$\mathbf{C}_x = 120 \text{ N} \longrightarrow$$

FBD CJ:



$$\uparrow \Sigma F_y = 0: 625 \text{ N} - F = 0$$

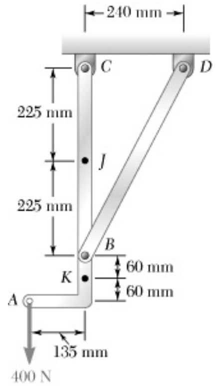
$$\mathbf{F} = 625 \text{ N} \downarrow \blacktriangleleft$$

$$\longrightarrow \Sigma F_x = 0: 120 \text{ N} - V = 0$$

$$\mathbf{V} = 120.0 \text{ N} \longleftarrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_J = 0: M - (0.225 \text{ m})(120 \text{ N}) = 0$$

$$\mathbf{M} = 27.0 \text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

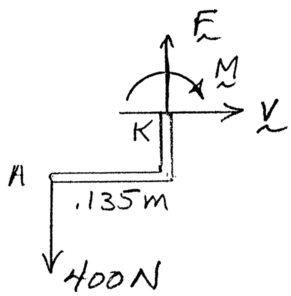


PROBLEM 7.154

Determine the internal forces at point K of the structure shown.

SOLUTION

FBD AK:



$$\longrightarrow \Sigma F_x = 0: V = 0$$

$$\mathbf{V} = 0 \blacktriangleleft$$

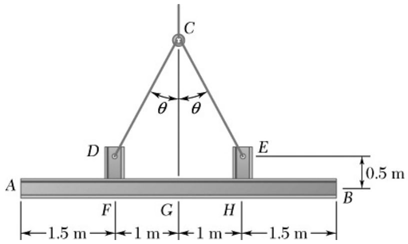
$$\uparrow \Sigma F_y = 0: F - 400 \text{ N} = 0$$

$$\mathbf{F} = 400 \text{ N} \uparrow \blacktriangleleft$$

$$\circlearrowleft \Sigma M_K = 0: (0.135 \text{ m})(400 \text{ N}) - M = 0$$

$$\mathbf{M} = 54.0 \text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

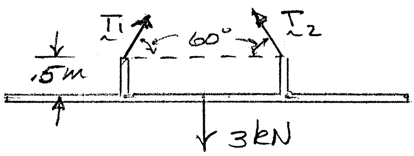
PROBLEM 7.155



Two small channel sections *DF* and *EH* have been welded to the uniform beam *AB* of weight $W = 3$ kN to form the rigid structural member shown. This member is being lifted by two cables attached at *D* and *E*. Knowing the $\theta = 30°$ and neglecting the weight of the channel sections, (*a*) draw the shear and bending-moment diagrams for beam *AB*, (*b*) determine the maximum absolute values of the shear and bending moment in the beam.

SOLUTION

FBD Beam + channels:



(*a*) By symmetry: $T_1 = T_2 = T$

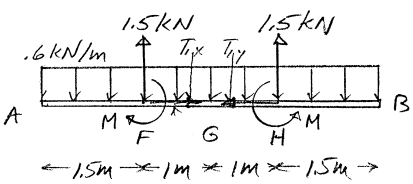
$$\uparrow \Sigma F_y = 0: \; 2T\sin 60° - 3\text{ kN} = 0$$

$$T = \frac{3}{\sqrt{3}}\text{ kN} \qquad T_{1x} = \frac{3}{2\sqrt{3}} \qquad T_{1y} = \frac{3}{2}\text{ kN}$$

$$M = (0.5\text{ m})\frac{3}{2\sqrt{3}}\text{ kN} = 0.433\text{ kN}\cdot\text{m}$$

FBD Beam:

With cable force replaced by equivalent force-couple system at *F* and *G*



Shear Diagram: *V* is piecewise linear

$$\left(\frac{dV}{dx} = -0.6\text{ kN/m}\right) \text{ with 1.5 kN}$$

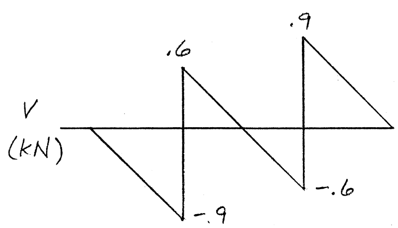
discontinuities at *F* and *H*.

$$V_{F^-} = -(0.6\text{ kN/m})(1.5\text{ m}) = 0.9\text{ kN}$$

V increases by 1.5 kN to $+\;0.6$ kN at F^+

$$V_G = 0.6\text{ kN} - (0.6\text{ kN/m})(1\text{ m}) = 0$$

Finish by invoking symmetry

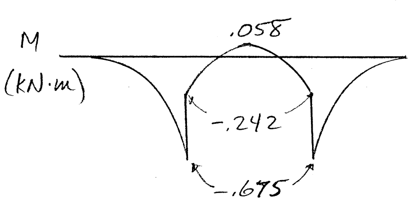


Moment Diagram: *M* is piecewise parabolic

$$\left(\frac{dM}{dx} \text{ decreasing with } V\right) \text{ with discontinuities of .433 kN at } F \text{ and } H.$$

$$M_{F^-} = -\frac{1}{2}(0.9\text{ kN})(1.5\text{ m}) = -0.675\text{ kN}\cdot\text{m}$$

M increases by $0.433\text{ kN}\cdot\text{m}$ to $-0.242\text{ kN}\cdot\text{m}$ at F^+



$$M_G = -0.242\text{ kN}\cdot\text{m} + \frac{1}{2}(0.6\text{ kN})(1\text{ m}) = 0.058\text{ kN}\cdot\text{m}$$

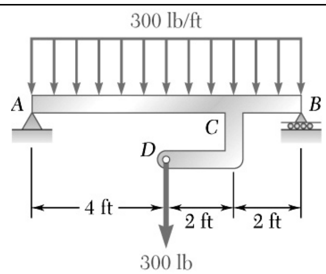
Finish by invoking symmetry

(*b*) $|V|_{max} = 900$ N ◀

at F^- and G^+

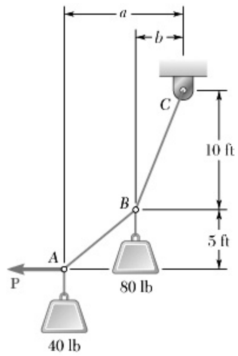
$|M|_{max} = 675$ N·m ◀

at *F* and *G*



PROBLEM 7.156

(*a*) Draw the shear and bending moment diagrams for beam *AB*,
(*b*) determine the magnitude and location of the maximum absolute value of the bending moment.

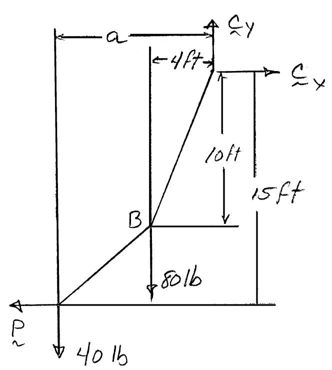


PROBLEM 7.157

Cable *ABC* supports two loads as shown. Knowing that $b = 4$ ft, determine (*a*) the required magnitude of the horizontal force **P**, (*b*) the corresponding distance *a*.

SOLUTION

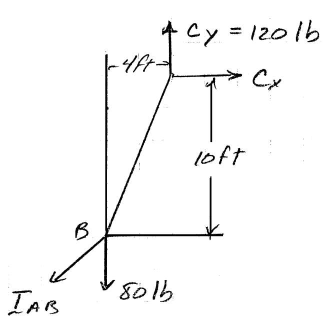
FBD ABC:



$$\uparrow \Sigma F_y = 0: -40 \text{ lb} - 80 \text{ lb} + C_y = 0$$

$$\mathbf{C}_y = 120 \text{ lb} \uparrow$$

FBD BC:



$$\circlearrowleft \Sigma M_B = 0: (4 \text{ ft})(120 \text{ lb}) - (10 \text{ ft})C_x = 0$$

$$\mathbf{C}_x = 48 \text{ lb} \longrightarrow$$

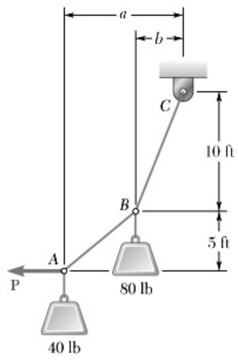
From ABC: $\quad \longrightarrow \Sigma F_x = 0: -P + C_x = 0$

$$P = C_x = 48 \text{ lb}$$

(*a*) $\quad P = 48.0 \text{ lb}$ ◀

$$\circlearrowleft \Sigma M_C = 0: (4 \text{ ft})(80 \text{ lb}) + a(40 \text{ lb}) - (15 \text{ ft})(48 \text{ lb}) = 0$$

(*b*) $\quad a = 10.00 \text{ ft}$ ◀

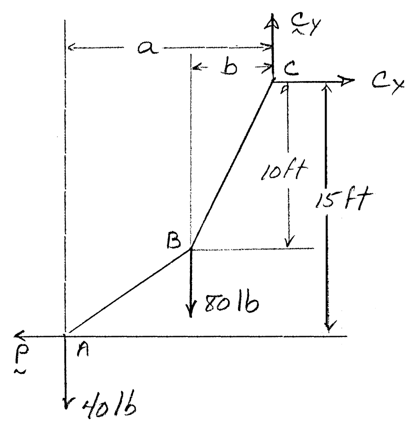


PROBLEM 7.158

Cable *ABC* supports two loads as shown. Determine the distances *a* and *b* when a horizontal force **P** of magnitude 60 lb is applied at *A*.

SOLUTION

FBD ABC:

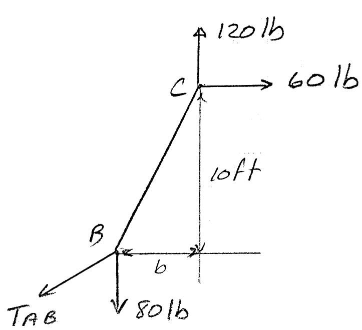


$$\xrightarrow{} \Sigma F_x = 0: C_x - P = 0 \qquad C_x = 60 \text{ lb} \longrightarrow$$

$$\uparrow \Sigma F_y = 0: C_y - 40 \text{ lb} - 80 \text{ lb} = 0$$

$$\mathbf{C}_y = 120 \text{ lb} \uparrow$$

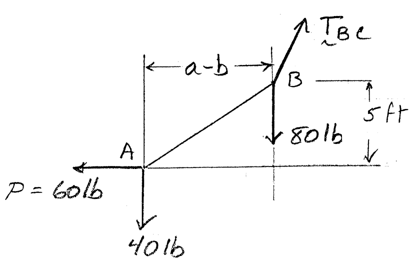
FBD BC:



$$\circlearrowleft \Sigma M_B = 0: b(120 \text{ lb}) - (10 \text{ ft})(60 \text{ lb}) = 0$$

$$b = 5.00 \text{ ft} \blacktriangleleft$$

FBD AB:



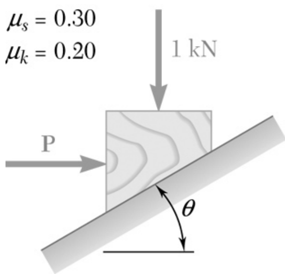
$$\Sigma M_B = 0: (a - b)(40 \text{ lb}) - (5 \text{ ft})60 \text{ lb} = 0$$

$$a - b = 7.5 \text{ ft}$$

$$a = b + 7.5 \text{ ft}$$
$$= 5 \text{ ft} + 7.5 \text{ ft}$$

$$a = 12.50 \text{ ft} \blacktriangleleft$$

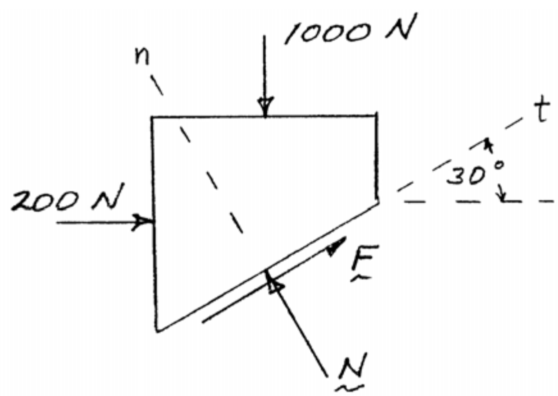
PROBLEM 8.1



Determine whether the block shown is in equilibrium, and find the magnitude and direction of the friction force when $\theta = 30^{\circ}$ and $P = 200\,\text{N}$.

SOLUTION

FBD block:



$$\nwarrow \Sigma F_n = 0: \quad N - (1000\text{ N})\cos 30° - (200\text{ N})\sin 30° = 0$$

$$N = 966.03\text{ N}$$

Assume equilibrium:

$$\nearrow \Sigma F_t = 0: \quad F + (200\text{ N})\cos 30° - (1000\text{ N})\sin 30° = 0$$

$$F = 326.8\text{ N} = F_{\text{eq.}}$$

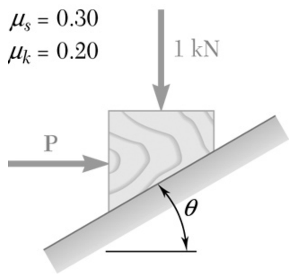
But $$F_{\max} = \mu_s N = (0.3)966\text{ N} = 290\text{ N}$$

$$F_{\text{eq.}} > F_{\max} \qquad \text{impossible} \Rightarrow \text{Block moves} \blacktriangleleft$$

and $$F = \mu_k N$$

$$= (0.2)(966.03\text{ N})$$

Block slides down $$\mathbf{F} = 193.2\text{ N} \nearrow \blacktriangleleft$$

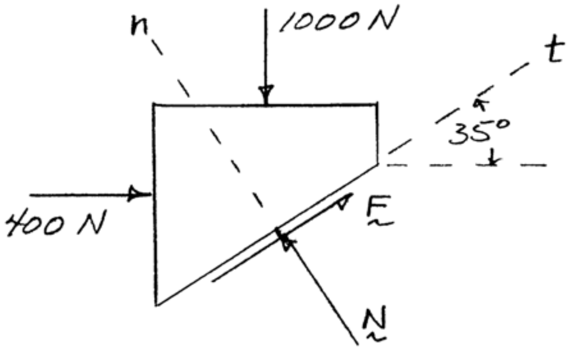


PROBLEM 8.2

Determine whether the block shown is in equilibrium, and find the magnitude and direction of the friction force when $\theta = 35^{\circ}$ and $P = 400$ N.

SOLUTION

FBD block:



$$\nwarrow \Sigma F_n = 0: \quad N - (1000\text{ N})\cos 35° - (400\text{ N})\sin 35° = 0$$

$$N = 1048.6\text{ N}$$

Assume equilibrium:

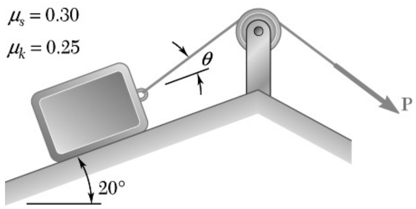
$$\nearrow \Sigma F_t = 0: \quad F - (1000\text{ N})\sin 35° + (400\text{ N})\cos 35° = 0$$

$$F = 246\text{ N} = F_{\text{eq.}}$$

$$F_{\max} = \mu_s N = (0.3)(1048.6\text{ N}) = 314\text{ N}$$

$F_{\text{eq.}} < F_{\max}$ OK equilibrium ◀

$\therefore$ $\mathbf{F} = 246\text{ N} \nearrow$ ◀

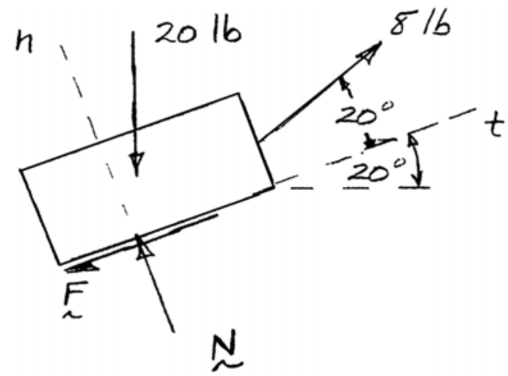


PROBLEM 8.3

Determine whether the 20-lb block shown is in equilibrium, and find the magnitude and direction of the friction force when $P = 8$ lb and $\theta = 20°$.

SOLUTION

FBD block:



$$\nwarrow \Sigma F_n = 0: \quad N - (20\text{ lb})\cos 20° + (8\text{ lb})\sin 20° = 0$$

$$N = 16.0577\text{ lb}$$

$$F_{\max} = \mu_s N = (0.3)(16.0577\text{ lb}) = 4.817\text{ lb}$$

Assume equilibrium:

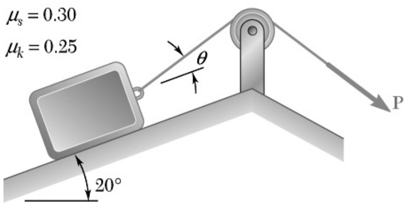
$$\nearrow \Sigma F_t = 0: \quad (8\text{ lb})\cos 20° - (20\text{ lb})\sin 20° - F = 0$$

$$F = 0.6771\text{ lb} = F_{\text{eq.}}$$

$F_{\text{eq.}} < F_{\max}$ OK equilibrium ◀

and $\mathbf{F} = 0.677\text{ lb}$ ↙ ◀

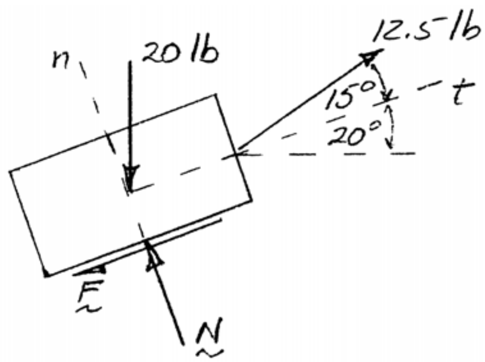
PROBLEM 8.4



Determine whether the 20-lb block shown is in equilibrium, and find the magnitude and direction of the friction force when $P = 12.5$ lb and $\theta = 15°$.

SOLUTION

FBD block:



$$\nwarrow \Sigma F_n = 0: \quad N - (20 \text{ lb})\cos 20° + (12.5 \text{ lb})\sin 15° = 0$$

$$N = 15.559 \text{ lb}$$

$$F_{\max} = \mu_s N = (0.3)(15.559 \text{ lb}) = 4.668 \text{ lb}$$

Assume equilibrium:

$$\nearrow \Sigma F_t = 0: \quad (12.5 \text{ lb})\cos 15° - (20 \text{ lb})\sin 20° - F = 0$$

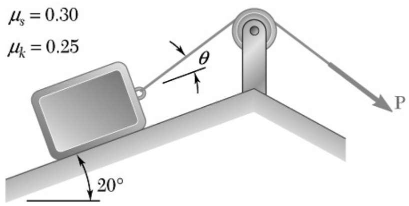
$$F = 5.23 \text{ lb} = F_{\text{eq.}}$$

but $F_{\text{eq.}} > F_{\max}$ impossible, so block slides up ◀

and $F = \mu_k N = (0.25)(15.559 \text{ lb})$

$\mathbf{F} = 3.89$ lb $\swarrow$ ◀

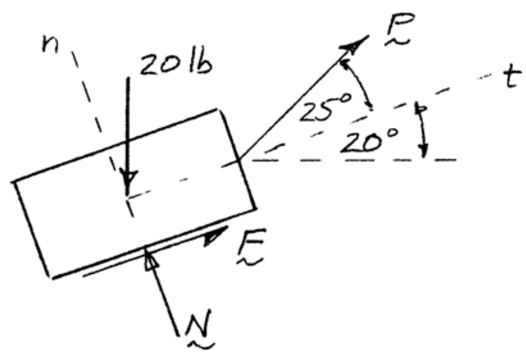
PROBLEM 8.5



Knowing that $\theta = 25°$, determine the range of values of P for which equilibrium is maintained.

SOLUTION

FBD block:



Block is in equilibrium:

$$\nwarrow \Sigma F_n = 0: \quad N - (20\text{ lb})\cos 20° + P\sin 25° = 0$$

$$N = 18.794\text{ lb} - P\sin 25°$$

$$\nearrow \Sigma F_t = 0: \quad F - (20\text{ lb})\sin 20° + P\cos 25° = 0$$

or

$$F = 6.840\text{ lb} - P\cos 25°$$

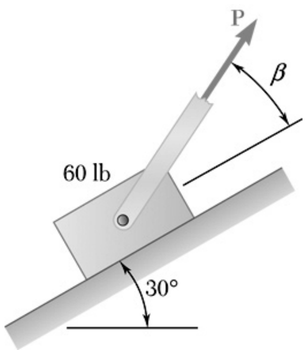
Impending motion up: $F = \mu_s N$; Impending motion down: $F = -\mu_s N$

Therefore,

$$6.840\text{ lb} - P\cos 25° = \pm(0.3)(18.794\text{ lb} - P\sin 25°)$$

$$P_{\text{up}} = 12.08\text{ lb} \qquad P_{\text{down}} = 1.542\text{ lb}$$

$$1.542\text{ lb} \le P_{\text{eq.}} \le 12.08\text{ lb} \blacktriangleleft$$

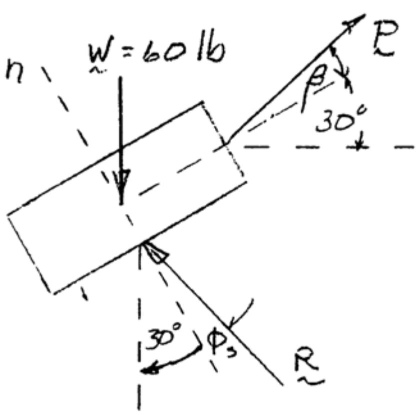


PROBLEM 8.6

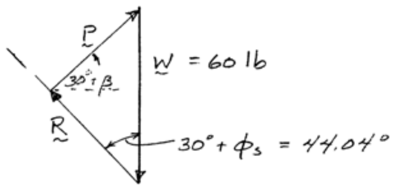
Knowing that the coefficient of friction between the 60-lb block and the incline is $\mu_s = 0.25$, determine (*a*) the smallest value of P for which motion of the block up the incline is impending, (*b*) the corresponding value of β.

SOLUTION

FBD block (impending motion up)



$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.25) = 14.04°$$



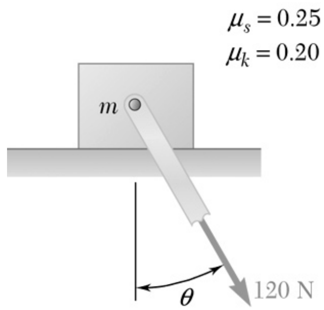
(*a*) Note: For minimum P, $\mathbf{P} \perp \mathbf{R}$ so $\beta = \phi_s$

Then $$P = W\sin(30° + \phi_s)$$

$$= (60 \text{ lb})\sin 44.04° = 41.71 \text{ lb}$$

$P_{\min} = 41.7 \text{ lb}$ ◀

(*b*) Have $\beta = \phi_s$ $\qquad \beta = 14.04°$ ◀

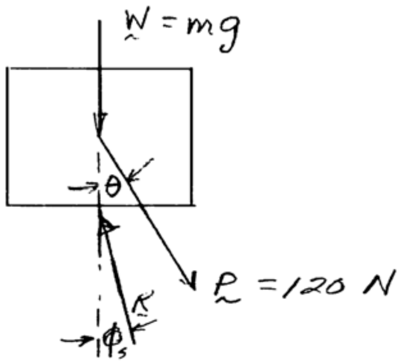


PROBLEM 8.7

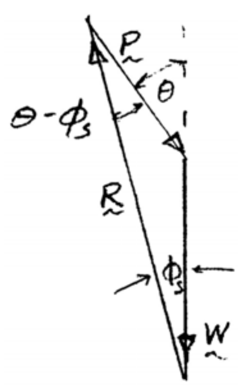
Considering only values of θ less than $90°$, determine the smallest value of θ for which motion of the block to the right is impending when $(a)\, m = 30$ kg, $(b)\, m = 40$ kg.

SOLUTION

FBD block (impending motion to the right)



$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.25) = 14.036°$$



$$\frac{P}{\sin\phi_s} = \frac{W}{\sin(\theta - \phi_s)}$$

$$\sin(\theta - \phi_s) = \frac{W}{P}\sin\phi_s \qquad W = mg$$

(a) $$m = 30\text{ kg}: \quad \theta - \phi_s = \sin^{-1}\left[\frac{(30\text{ kg})(9.81\text{ m/s}^2)}{120\text{ N}}\sin 14.036°\right]$$

$$= 36.499°$$

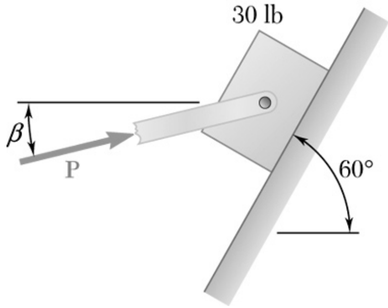
$$\therefore \quad \theta = 36.499° + 14.036° \qquad \text{or} \quad \theta = 50.5° \blacktriangleleft$$

(b) $$m = 40\text{ kg}: \quad \theta - \phi_s = \sin^{-1}\left[\frac{(40\text{ kg})(9.81\text{ m/s}^2)}{120\text{ N}}\sin 14.036°\right]$$

$$= 52.474°$$

$$\therefore \quad \theta = 52.474° + 14.036° \qquad \text{or} \quad \theta = 66.5° \blacktriangleleft$$

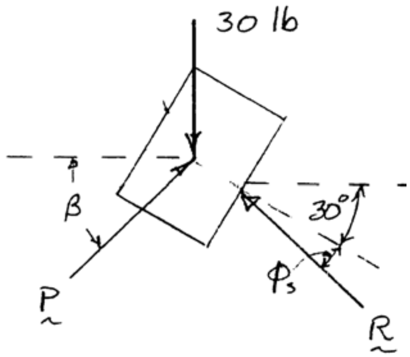
PROBLEM 8.8



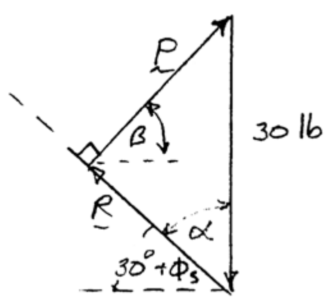
Knowing that the coefficient of friction between the 30-lb block and the incline is $\mu_s = 0.25$, determine (*a*) the smallest value of *P* required to maintain the block in equilibrium, (*b*) the corresponding value of β.

SOLUTION

FBD block (impending motion downward)



$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.25) = 14.036°$$



(*a*) Note: For minimum *P*, $\mathbf{P} \perp \mathbf{R}$

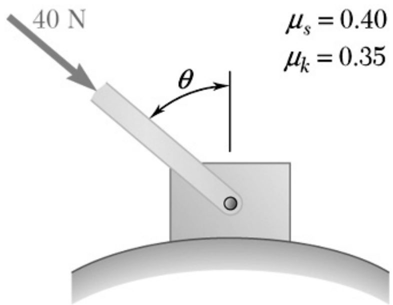
So $\beta = \alpha = 90° - (30° + 14.036°) = 45.964°$

and $P = (30 \text{ lb})\sin\alpha = (30 \text{ lb})\sin(45.964°) = 21.567 \text{ lb}$

$P = 21.6 \text{ lb}$ ◀

(*b*) $\beta = 46.0°$ ◀

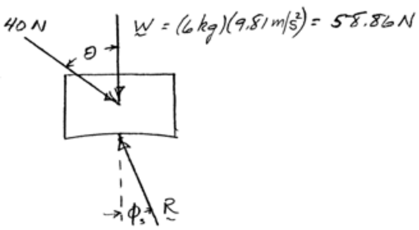
PROBLEM 8.9



A 6-kg block is at rest as shown. Determine the positive range of values of θ for which the block is in equilibrium if (*a*) θ is less than 90°, (*b*) θ is between 90° and 180°.

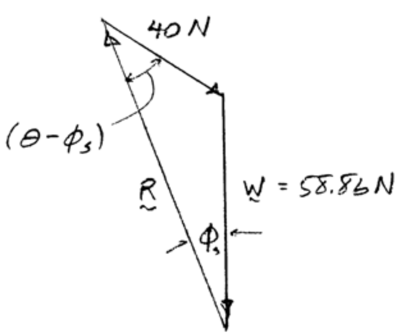
SOLUTION

FBD block (impending motion)



$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.4) = 21.801°$$

(*a*) $0° \leq \theta \leq 90°$:



$$\frac{58.86\text{ N}}{\sin(\theta - \phi_s)} = \frac{40\text{ N}}{\sin\phi_s}$$

$$\theta - \phi_s = \sin^{-1}\frac{58.86\text{ N}}{40\text{ N}}\sin(21.801°)$$

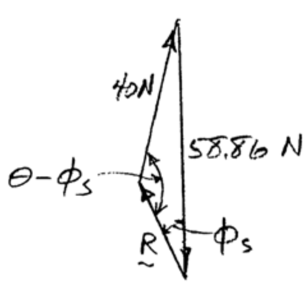
$$= 33.127°,\ 146.873°$$

$$\theta = 54.9° \quad \text{and} \quad \theta = 168.674°$$

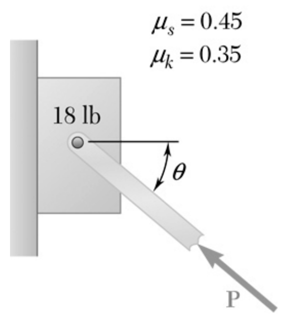
$\therefore$ (*a*) Equilibrium for $0 \leq \theta \leq 54.9°$ ◀

(*b*) $90° \leq \theta \leq 180°$:

(*b*) and for $168.7° \leq \theta \leq 180.0°$ ◀



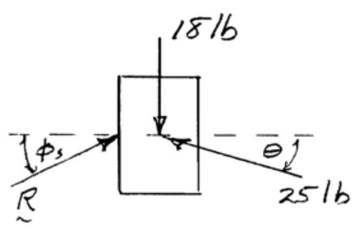
PROBLEM 8.10

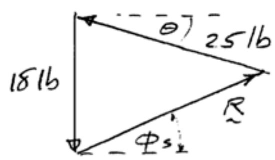


Knowing that $P = 25$ lb, determine the range of values of θ for which equilibrium of the 18-lb block is maintained.

SOLUTION

FBD block (impending motion down)





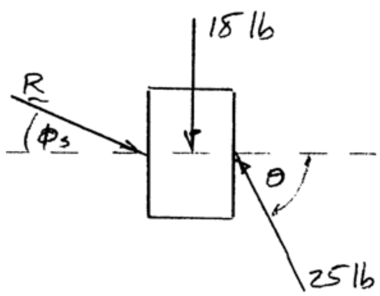
$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.45) = 24.228°$$

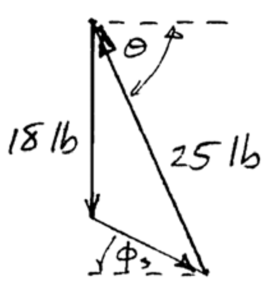
$$\frac{25 \text{ lb}}{\sin(90° - \phi_s)} = \frac{18 \text{ lb}}{\sin(\theta + \phi_s)}$$

$$\theta + \phi_s = \sin^{-1}\left[\frac{18 \text{ lb}}{25 \text{ lb}}\sin(90° - 24.228°)\right] = 41.04°$$

$$\theta = 16.81°$$

Impending motion up:





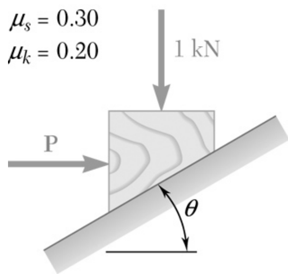
$$\frac{25 \text{ lb}}{\sin(90° + \phi_s)} = \frac{18 \text{ lb}}{\sin(\theta - \phi_s)}$$

$$\theta - \phi_s = \sin^{-1}\left[\frac{18 \text{ lb}}{25 \text{ lb}}\sin(90° + 24.228°)\right] = 41.04°$$

$$\theta = 65.27°$$

Equilibrium for $16.81° \leq \theta \leq 65.3°$ ◀

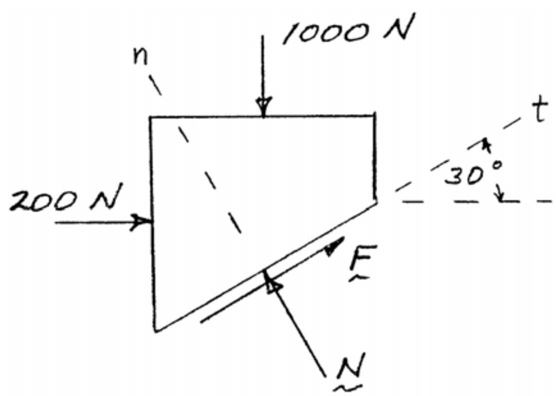
PROBLEM 8.1



Determine whether the block shown is in equilibrium, and find the magnitude and direction of the friction force when $\theta = 30^{\circ}$ and $P = 200\,\text{N}$.

SOLUTION

FBD block:



$$\nwarrow \Sigma F_n = 0: \quad N - (1000\text{ N})\cos 30^{\circ} - (200\text{ N})\sin 30^{\circ} = 0$$

$$N = 966.03\text{ N}$$

Assume equilibrium:

$$\nearrow \Sigma F_t = 0: \quad F + (200\text{ N})\cos 30^{\circ} - (1000\text{ N})\sin 30^{\circ} = 0$$

$$F = 326.8\text{ N} = F_{\text{eq.}}$$

But

$$F_{\max} = \mu_s N = (0.3)966\text{ N} = 290\text{ N}$$

$$F_{\text{eq.}} > F_{\max} \qquad \text{impossible} \Rightarrow \text{Block moves} \blacktriangleleft$$

and

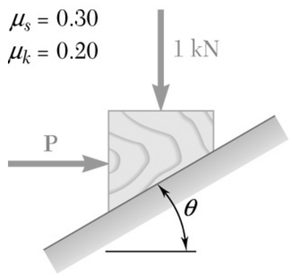
$$F = \mu_k N$$

$$= (0.2)(966.03\text{ N})$$

Block slides down

$$\mathbf{F} = 193.2\text{ N} \nearrow \blacktriangleleft$$

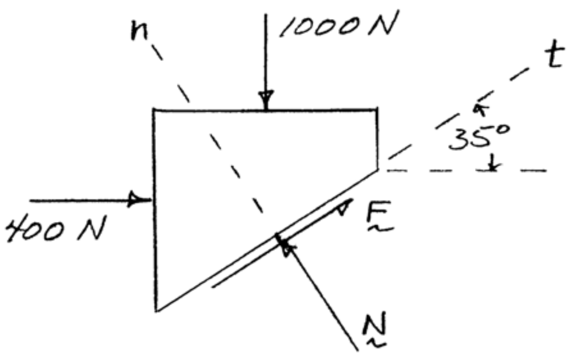
PROBLEM 8.2



Determine whether the block shown is in equilibrium, and find the magnitude and direction of the friction force when $\theta = 35^{\circ}$ and $P = 400$ N.

SOLUTION

FBD block:



$$\searrow \Sigma F_n = 0: \quad N - (1000 \text{ N})\cos 35^\circ - (400 \text{ N})\sin 35^\circ = 0$$

$$N = 1048.6 \text{ N}$$

Assume equilibrium:

$$\nearrow \Sigma F_t = 0: \quad F - (1000 \text{ N})\sin 35^\circ + (400 \text{ N})\cos 35^\circ = 0$$

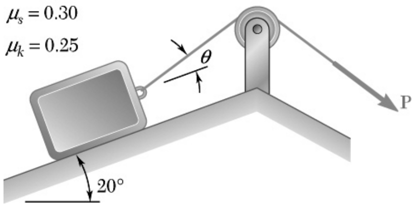
$$F = 246 \text{ N} = F_{\text{eq.}}$$

$$F_{\max} = \mu_s N = (0.3)(1048.6 \text{ N}) = 314 \text{ N}$$

$$F_{\text{eq.}} < F_{\max} \qquad \text{OK} \qquad \text{equilibrium} \blacktriangleleft$$

$$\therefore \; \mathbf{F} = 246 \text{ N} \nearrow \blacktriangleleft$$

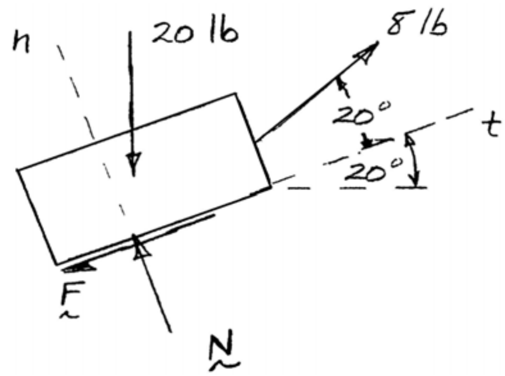
PROBLEM 8.3



Determine whether the 20-lb block shown is in equilibrium, and find the magnitude and direction of the friction force when $P = 8$ lb and $\theta = 20°$.

SOLUTION

FBD block:



$$\nwarrow \Sigma F_n = 0: \quad N - (20 \text{ lb})\cos 20° + (8 \text{ lb})\sin 20° = 0$$

$$N = 16.0577 \text{ lb}$$

$$F_{\max} = \mu_s N = (0.3)(16.0577 \text{ lb}) = 4.817 \text{ lb}$$

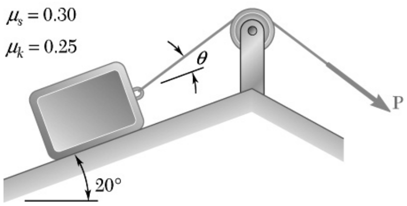
Assume equilibrium:

$$\nearrow \Sigma F_t = 0: \quad (8 \text{ lb})\cos 20° - (20 \text{ lb})\sin 20° - F = 0$$

$$F = 0.6771 \text{ lb} = F_{\text{eq.}}$$

$F_{\text{eq.}} < F_{\max}$ OK equilibrium ◀

and $\mathbf{F} = 0.677$ lb $\swarrow$ ◀

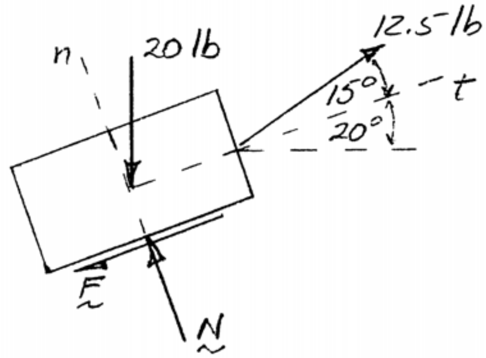


PROBLEM 8.4

Determine whether the 20-lb block shown is in equilibrium, and find the magnitude and direction of the friction force when $P = 12.5$ lb and $\theta = 15°$.

SOLUTION

FBD block:



$$\nwarrow \Sigma F_n = 0: \quad N - (20 \text{ lb})\cos 20° + (12.5 \text{ lb})\sin 15° = 0$$

$$N = 15.559 \text{ lb}$$

$$F_{\max} = \mu_s N = (0.3)(15.559 \text{ lb}) = 4.668 \text{ lb}$$

Assume equilibrium:

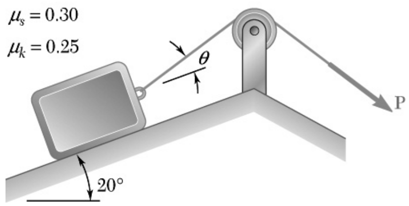
$$\nearrow \Sigma F_t = 0: \quad (12.5 \text{ lb})\cos 15° - (20 \text{ lb})\sin 20° - F = 0$$

$$F = 5.23 \text{ lb} = F_{\text{eq.}}$$

but $F_{\text{eq.}} > F_{\max}$ impossible, so block slides up ◀

and $$F = \mu_k N = (0.25)(15.559 \text{ lb})$$

$$\mathbf{F} = 3.89 \text{ lb} \swarrow$$ ◀

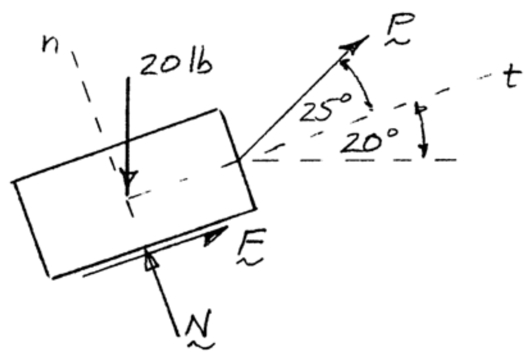


PROBLEM 8.5

Knowing that $\theta = 25°$, determine the range of values of P for which equilibrium is maintained.

SOLUTION

FBD block:



Block is in equilibrium:

$$\nwarrow \Sigma F_n = 0: \quad N - (20\text{ lb})\cos 20° + P\sin 25° = 0$$

$$N = 18.794\text{ lb} - P\sin 25°$$

$$\nearrow \Sigma F_t = 0: \quad F - (20\text{ lb})\sin 20° + P\cos 25° = 0$$

or

$$F = 6.840\text{ lb} - P\cos 25°$$

Impending motion up: $F = \mu_s N$; Impending motion down: $F = -\mu_s N$

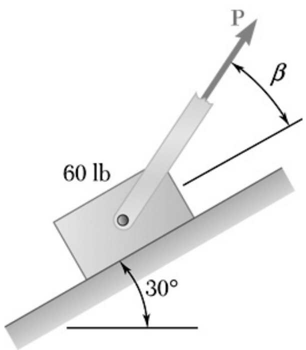
Therefore,

$$6.840\text{ lb} - P\cos 25° = \pm(0.3)(18.794\text{ lb} - P\sin 25°)$$

$$P_{\text{up}} = 12.08\text{ lb} \qquad P_{\text{down}} = 1.542\text{ lb}$$

$$1.542\text{ lb} \le P_{\text{eq.}} \le 12.08\text{ lb} \blacktriangleleft$$

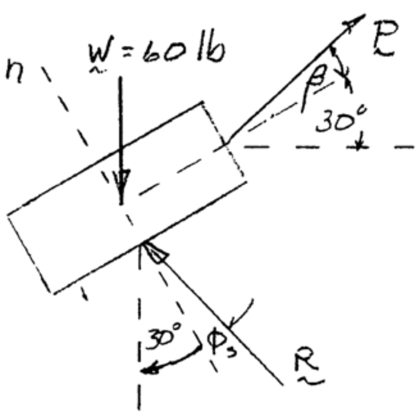
PROBLEM 8.6



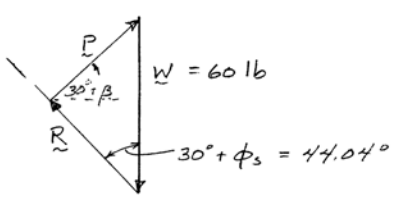
Knowing that the coefficient of friction between the 60-lb block and the incline is $\mu_s = 0.25$, determine (*a*) the smallest value of P for which motion of the block up the incline is impending, (*b*) the corresponding value of β.

SOLUTION

FBD block (impending motion up)



$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.25) = 14.04°$$



(*a*) Note: For minimum P, $\mathbf{P} \perp \mathbf{R}$ so $\beta = \phi_s$

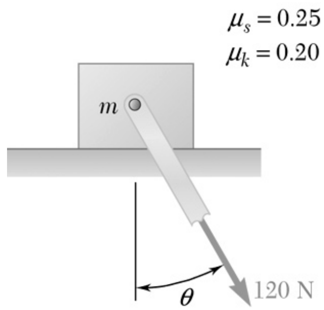
Then $\qquad P = W\sin(30° + \phi_s)$

$$= (60 \text{ lb})\sin 44.04° = 41.71 \text{ lb}$$

$P_{\min} = 41.7 \text{ lb}$ ◀

(*b*) Have $\beta = \phi_s$ $\qquad \beta = 14.04°$ ◀

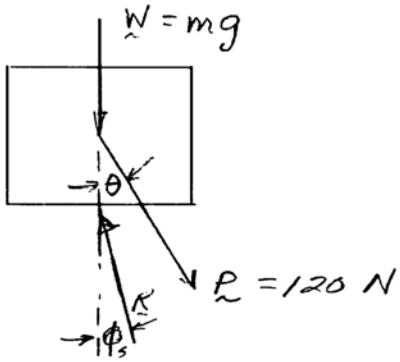
PROBLEM 8.7



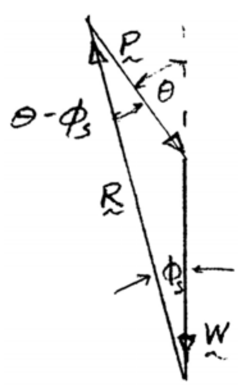
Considering only values of θ less than $90°$, determine the smallest value of θ for which motion of the block to the right is impending when $(a)\, m = 30$ kg, $(b)\, m = 40$ kg.

SOLUTION

FBD block (impending motion to the right)



$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.25) = 14.036°$$



$$\frac{P}{\sin\phi_s} = \frac{W}{\sin(\theta - \phi_s)}$$

$$\sin(\theta - \phi_s) = \frac{W}{P}\sin\phi_s \qquad W = mg$$

(a) $\quad m = 30 \text{ kg}: \quad \theta - \phi_s = \sin^{-1}\left[\frac{(30 \text{ kg})(9.81 \text{ m/s}^2)}{120 \text{ N}}\sin 14.036°\right]$

$$= 36.499°$$

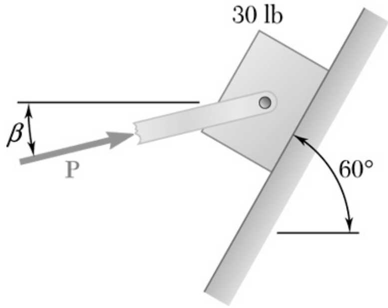
$\therefore \quad \theta = 36.499° + 14.036° \qquad$ or $\quad \theta = 50.5°$ ◀

(b) $\quad m = 40 \text{ kg}: \quad \theta - \phi_s = \sin^{-1}\left[\frac{(40 \text{ kg})(9.81 \text{ m/s}^2)}{120 \text{ N}}\sin 14.036°\right]$

$$= 52.474°$$

$\therefore \quad \theta = 52.474° + 14.036° \qquad$ or $\quad \theta = 66.5°$ ◀

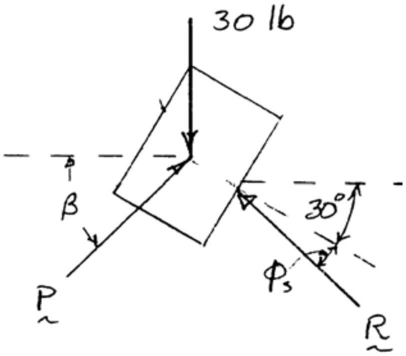
PROBLEM 8.8



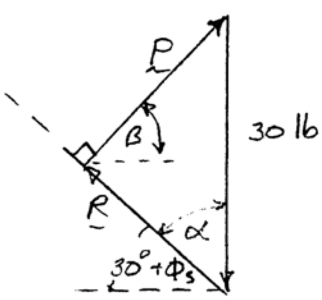
Knowing that the coefficient of friction between the 30-lb block and the incline is $\mu_s = 0.25$, determine (*a*) the smallest value of *P* required to maintain the block in equilibrium, (*b*) the corresponding value of β.

SOLUTION

FBD block (impending motion downward)



$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.25) = 14.036°$$



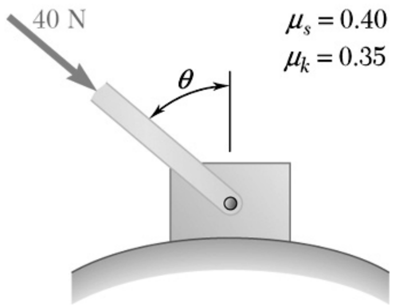
(*a*) Note: For minimum *P*, $\mathbf{P} \perp \mathbf{R}$

So $$\beta = \alpha = 90° - (30° + 14.036°) = 45.964°$$

and $$P = (30 \text{ lb})\sin\alpha = (30 \text{ lb})\sin(45.964°) = 21.567 \text{ lb}$$

$$P = 21.6 \text{ lb} \blacktriangleleft$$

(*b*) $$\beta = 46.0° \blacktriangleleft$$

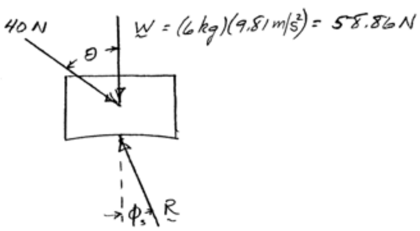


PROBLEM 8.9

A 6-kg block is at rest as shown. Determine the positive range of values of θ for which the block is in equilibrium if (*a*) θ is less than 90°, (*b*) θ is between 90° and 180°.

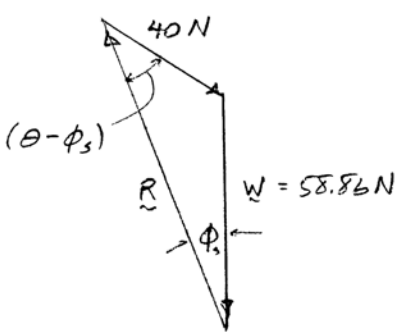
SOLUTION

FBD block (impending motion)



$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.4) = 21.801°$$

(*a*) $0° \leq \theta \leq 90°$:



$$\frac{58.86 \text{ N}}{\sin(\theta - \phi_s)} = \frac{40 \text{ N}}{\sin\phi_s}$$

$$\theta - \phi_s = \sin^{-1}\frac{58.86 \text{ N}}{40 \text{ N}}\sin(21.801°)$$

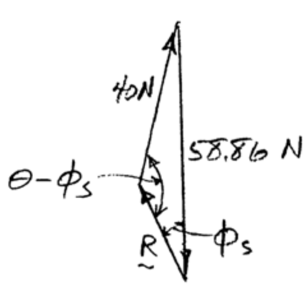
$$= 33.127°,\ 146.873°$$

$$\theta = 54.9° \qquad \text{and} \qquad \theta = 168.674°$$

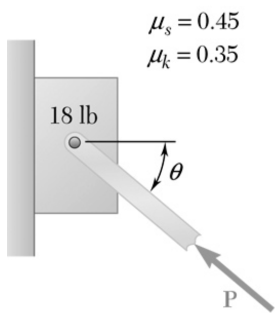
$\therefore$ (*a*) Equilibrium for $0 \leq \theta \leq 54.9°$ ◀

(*b*) $90° \leq \theta \leq 180°$:

(*b*) and for $168.7° \leq \theta \leq 180.0°$ ◀



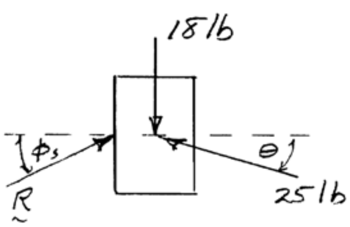
PROBLEM 8.10

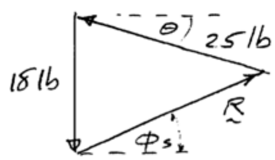


Knowing that $P = 25$ lb, determine the range of values of θ for which equilibrium of the 18-lb block is maintained.

SOLUTION

FBD block (impending motion down)





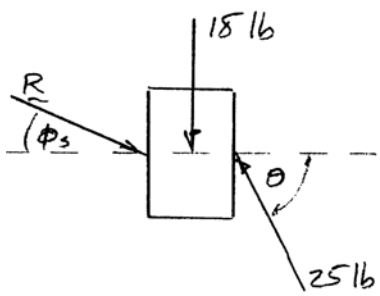
$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.45) = 24.228°$$

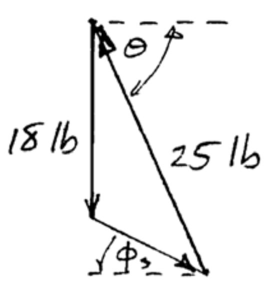
$$\frac{25\text{ lb}}{\sin(90° - \phi_s)} = \frac{18\text{ lb}}{\sin(\theta + \phi_s)}$$

$$\theta + \phi_s = \sin^{-1}\left[\frac{18\text{ lb}}{25\text{ lb}}\sin(90° - 24.228°)\right] = 41.04°$$

$$\theta = 16.81°$$

Impending motion up:





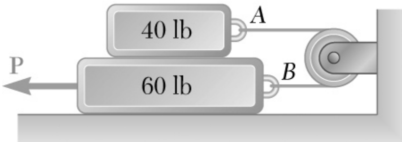
$$\frac{25\text{ lb}}{\sin(90° + \phi_s)} = \frac{18\text{ lb}}{\sin(\theta - \phi_s)}$$

$$\theta - \phi_s = \sin^{-1}\left[\frac{18\text{ lb}}{25\text{ lb}}\sin(90° + 24.228°)\right] = 41.04°$$

$$\theta = 65.27°$$

Equilibrium for $16.81° \leq \theta \leq 65.3°$ ◀

PROBLEM 8.11

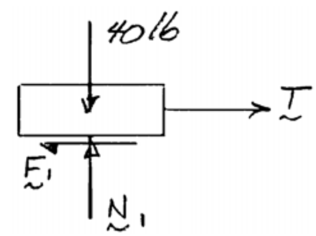


The coefficients of friction are $\mu_s = 0.40$ and $\mu_k = 0.30$ between all surfaces of contact. Determine the force **P** for which motion of the 60-lb block is impending if cable AB (a) is attached as shown, (b) is removed.

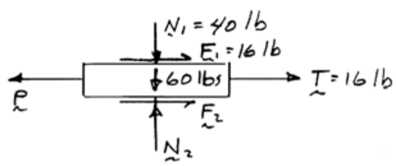
SOLUTION

FBDs

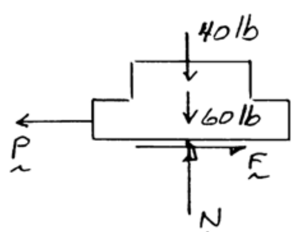
Top block:



Bottom block:



FBD blocks:



(a) Note: With the cable, motion must impend at both contact surfaces.

$$\uparrow \Sigma F_y = 0: \quad N_1 - 40 \text{ lb} = 0 \qquad N_1 = 40 \text{ lb}$$

Impending slip: $\quad F_1 = \mu_s N_1 = 0.4(40 \text{ lb}) = 16 \text{ lb}$

$$\longrightarrow \Sigma F_x = 0: \quad T - F_1 = 0 \qquad T - 16 \text{ lb} = 0 \qquad T = 16 \text{ lb}$$

$$\uparrow \Sigma F_y = 0: \quad N_2 - 40 \text{ lb} - 60 \text{ lb} = 0 \qquad N_2 = 100 \text{ lb}$$

Impending slip: $\quad F_2 = \mu_s N_2 = 0.4(100 \text{ lb}) = 40 \text{ lb}$

$$\longrightarrow \Sigma F_x = 0: \quad -P + 16 \text{ lb} + 16 \text{ lb} + 40 \text{ lb} = 0$$

$$\mathbf{P} = 72.0 \text{ lb} \longleftarrow \blacktriangleleft$$

(b) Without the cable, both blocks will stay together and motion will impend only at the floor.

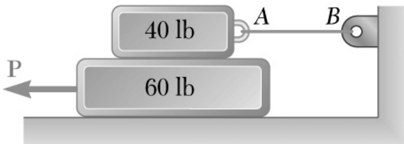
$$\uparrow \Sigma F_y = 0: \quad N - 40 \text{ lb} - 60 \text{ lb} = 0 \qquad N = 100 \text{ lb}$$

Impending slip: $\quad F = \mu_s N = 0.4(100 \text{ lb}) = 40 \text{ lb}$

$$\longrightarrow \Sigma F_x = 0: \quad 40 \text{ lb} - P = 0$$

$$\mathbf{P} = 40.0 \text{ lb} \longleftarrow \blacktriangleleft$$

PROBLEM 8.12

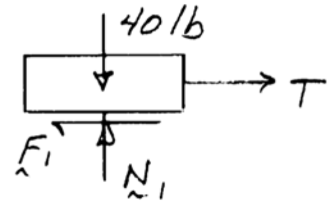


The coefficients of friction are $\mu_s = 0.40$ and $\mu_k = 0.30$ between all surfaces of contact. Determine the force **P** for which motion of the 60-lb block is impending if cable AB (a) is attached as shown, (b) is removed.

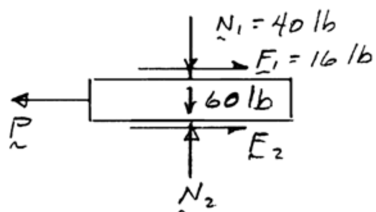
SOLUTION

FBDs

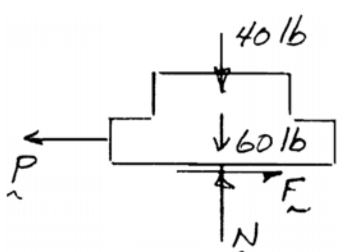
Top block:



Bottom block:



FBD blocks:



(a) With the cable, motion must impend at both surfaces.

$$\uparrow \Sigma F_y = 0: \quad N_1 - 40 \text{ lb} = 0 \qquad N_1 = 40 \text{ lb}$$

Impending slip: $\quad F_1 = \mu_s N_1 = 0.4(40 \text{ lb}) = 16 \text{ lb}$

$$\uparrow \Sigma F_y = 0: \quad N_2 - 40 \text{ lb} - 60 \text{ lb} = 0 \qquad N_2 = 100 \text{ lb}$$

Impending slip: $\quad F_2 = \mu N_2 = 0.4(100 \text{ lb}) = 40 \text{ lb}$

$$\longrightarrow \Sigma F_x = 0: \quad 16 \text{ lb} + 40 \text{ lb} - P = 0 \qquad P = 56 \text{ lb}$$

$\mathbf{P} = 56.0$ lb $\longleftarrow$ ◀

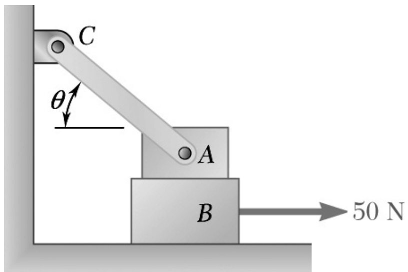
(b) Without the cable, both blocks stay together and motion will impend at the floor surface only.

$$\uparrow \Sigma F_y = 0: \quad N - 40 \text{ lb} - 60 \text{ lb} = 0 \qquad N = 100 \text{ lb}$$

Impending slip: $\quad F = \mu_s N = 0.4(100 \text{ lb}) = 40 \text{ lb}$

$$\longrightarrow \Sigma F_x = 0: \quad -P + 40 \text{ lb} = 0 \qquad P = 40 \text{ lb}$$

$\mathbf{P} = 40.0$ lb $\longleftarrow$ ◀

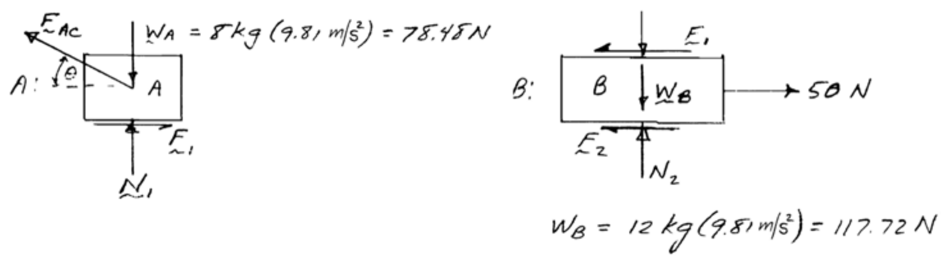


PROBLEM 8.13

The 8-kg block A is attached to link AC and rests on the 12-kg block B. Knowing that the coefficient of static friction is 0.20 between all surfaces of contact and neglecting the mass of the link, determine the value of θ for which motion of block B is impending.

SOLUTION

FBDs:



Motion must impend at both contact surfaces

Block A: $\uparrow \Sigma F_y = 0$: $N_1 - W_A = 0 \qquad N_1 = W_A$

Block B: $\uparrow \Sigma F_y = 0$: $N_2 - N_1 - W_B = 0$

$$N_2 = N_1 + W_B = W_A + W_B$$

Impending motion: $F_1 = \mu_s N_1 = \mu_s W_A$

$$F_2 = \mu_s N_2 = \mu_s(N_1 + W_B)$$

Block B: $\longrightarrow \Sigma F_x = 0$: $50\text{ N} - F_1 - F_2 = 0$

or $\quad 50\text{ N} = \mu_s(N_1 + N_1 + W_B) = 0.2(2N_1 + 117.72\text{ N})$

$$N_1 = 66.14\text{ N} \qquad F_1 = 0.2(66.14\text{ N}) = 13.228\text{ N}$$

Block A: $\longrightarrow \Sigma F_x = 0$: $13.228\text{ N} - F_{AC}\cos\theta = 0$

or $\quad F_{AC}\cos\theta = 13.228\text{ N} \qquad (1)$

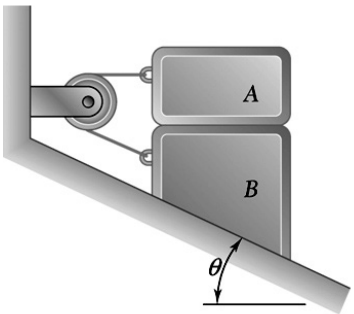
$\uparrow \Sigma F_y = 0$: $66.14\text{ N} - 78.48\text{ N} + F_{AC}\sin\theta = 0$

or $\quad F_{AC}\sin\theta = 78.48\text{ N} - 66.14\text{ N} \qquad (2)$

Then, $\dfrac{\text{Eq. (2)}}{\text{Eq. (1)}}$ $\quad \tan\theta = \dfrac{78.48\text{ N} - 66.14\text{ N}}{13.228\text{ N}}$

$\theta = 43.0°$ ◀

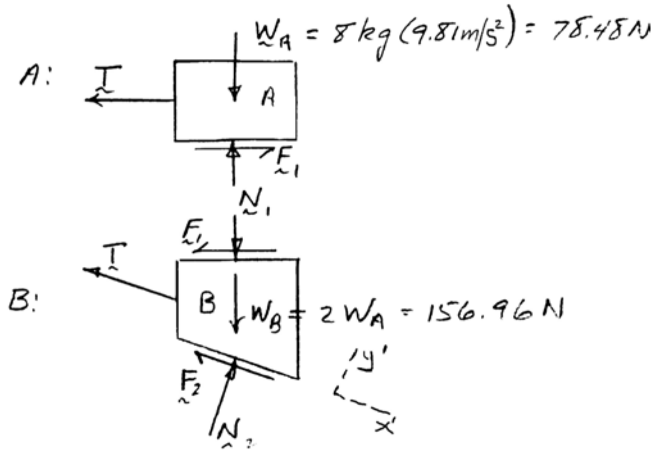
PROBLEM 8.14



The 8-kg block A and the 16-kg block B are at rest on an incline as shown. Knowing that the coefficient of static friction is 0.25 between all surfaces of contact, determine the value of θ for which motion is impending.

SOLUTION

FBDs:



Block A: $\quad \uparrow \Sigma F_y = 0{:}\quad N_1 - W_A = 0 \qquad N_1 = W_A$

Impending motion: $\quad F_1 = \mu_s N_1 = \mu_s W_A$

$$\rightarrow \Sigma F_x = 0{:}\quad F_1 - T = 0 \qquad T = F_1 = \mu_s W_A$$

Block B: $\quad \nearrow \Sigma F_{y'} = 0{:}\quad N_2 - (N_1 + W_B)\cos\theta - F_1\sin\theta = 0$

$$N_2 = 3W_A\cos\theta + \mu_s W_A\sin\theta$$

$$= W_A(3\cos\theta + 0.25\sin\theta)$$

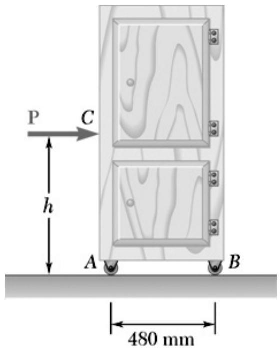
Impending motion: $\quad F_2 = \mu_s N_2 = 0.25W_A(3\cos\theta + 0.25\sin\theta)$

$$\searrow \Sigma F_{x'} = 0{:}\quad -T - F_2 - F_1\cos\theta + (N_1 + W_B)\sin\theta = 0$$

$$\left[-0.25 - 0.25(3\cos\theta + 0.25\sin\theta) - 0.25\cos\theta + 3\sin\theta\right]W_A = 0$$

or $\quad 47\sin\theta - 16\cos\theta - 4 = 0$

Solving numerically $\quad \theta = 23.4° \blacktriangleleft$

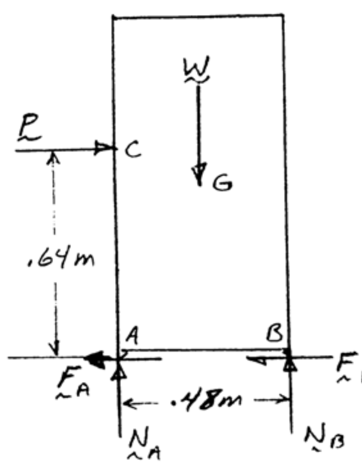


PROBLEM 8.15

A 48-kg cabinet is mounted on casters which can be locked to prevent their rotation. The coefficient of static friction between the floor and each caster is 0.30. Knowing that $h = 640$ mm, determine the magnitude of the force **P** required for impending motion of the cabinet to the right (*a*) if all casters are locked, (*b*) if the casters at *B* are locked and the casters at *A* are free to rotate, (*c*) if the casters at *A* are locked and the casters at *B* are free to rotate.

SOLUTION

FBD cabinet:



$$W = 48\text{ kg}\left(9.81\text{ m/s}^2\right)$$

$$= 470.88\text{ N}$$

$$\mu_s = 0.3$$

Note: For tipping, $N_A = F_A = 0$

$$\circlearrowleft \Sigma M_B = 0: \quad (0.24\text{ m})W - (0.64\text{ m})P_{\text{tip}} = 0 \qquad P_{\text{tip}} = 0.375W$$

(*a*) All casters locked: Impending slip: $F_A = \mu_s N_A,\ F_B = \mu_s N_B$

$$\uparrow \Sigma F_y = 0: \quad N_A + N_B - W = 0 \qquad N_A + N_B = W$$

So $\quad F_A + F_B = \mu_s W$

$$\longrightarrow \Sigma F_x = 0: \quad P - F_A - F_B = 0 \qquad P = F_A + F_B = \mu_s W$$

$$\therefore \quad P = 0.3(470.88\text{ N}) \quad \text{or} \quad P = 141.3\text{ N} \blacktriangleleft$$

$$\left(P = 0.3W < P_{\text{tip}} \qquad \text{OK}\right)$$

(*b*) Casters at *A* free, so $F_A = 0$

Impending slip: $F_B = \mu_s N_B$

$$\longrightarrow \Sigma F_x = 0: \quad P - F_B = 0$$

$$P = F_B = \mu_s N_B \qquad N_B = \frac{P}{\mu_s}$$

$$\circlearrowleft \Sigma M_A = 0: \quad (0.64\text{ m})P + (0.24\text{ m})W - (0.48\text{ m})N_B = 0$$

$$8P + 3W - 6\frac{P}{0.3} = 0 \qquad P = 0.25W$$

$$\left(P = 0.25W < P_{\text{tip}} \qquad \text{OK}\right)$$

$$\therefore \quad P = 0.25(470.88\text{ N}) \qquad P = 117.7\text{ N} \blacktriangleleft$$

PROBLEM 8.15 CONTINUED

(*c*) Casters at *B* free, so $F_B = 0$

Impending slip: $F_A = \mu_s N_A$

$\xrightarrow{}\ \Sigma F_x = 0: \quad P - F_A = 0 \qquad P = F_A = \mu_s N_A$

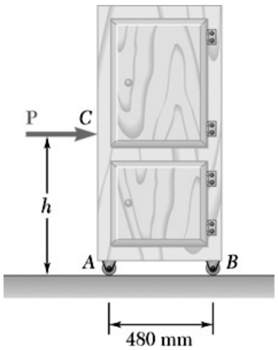
$$N_A = \frac{P}{\mu_s} = \frac{P}{0.3}$$

$$\circlearrowright\ \Sigma M_B = 0: \quad (0.24\text{ m})W - (0.64\text{ m})P - (0.48\text{ m})N_A = 0$$

$$3W - 8P - 6\frac{P}{0.3} = 0 \qquad P = 0.10714W = 50.45\text{ N}$$

$$\left(P < P_{\text{tip}} \qquad \text{OK}\right)$$

$P = 50.5\text{ N}$ ◀

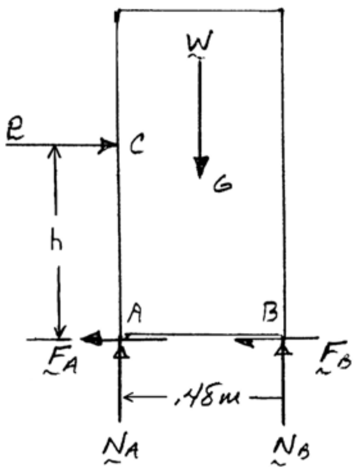


PROBLEM 8.16

A 48-kg cabinet is mounted on casters which can be locked to prevent their rotation. The coefficient of static friction between the floor and each caster is 0.30. Assuming that the casters at *A* and *B* are locked, determine (*a*) the force **P** required for impending motion of the cabinet to the right, (*b*) the largest allowable height *h* if the cabinet in not to tip over.

SOLUTION

FBD cabinet:



$$W = 48\ \text{kg}\left(9.81\ \text{m/s}^2\right)$$

$$= 470.88\ \text{N}$$

(*a*) $\uparrow \Sigma F_y = 0:\quad N_A + N_B - W = 0; \qquad N_A + N_B = W$

Impending slip: $\qquad F_A = \mu_s N_A,\ F_B = \mu_s N_B$

So $\qquad F_A + F_B = \mu_s W$

$\longrightarrow \Sigma F_x = 0:\quad P - F_A - F_B = 0 \qquad P = F_A + F_B = \mu_s W$

$$P = 0.3\left(470.88\ \text{N}\right) = 141.26\ \text{N}$$

$$\mathbf{P} = 141.3\ \text{N} \longrightarrow \blacktriangleleft$$

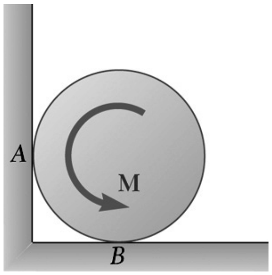
(*b*) For tipping, $\qquad N_A = F_A = 0$

$\circlearrowleft \Sigma M_B = 0:\quad hP - \left(0.24\ \text{m}\right)W = 0$

$$h_{\text{max}} = \left(0.24\ \text{m}\right)\frac{W}{P} = \left(0.24\ \text{m}\right)\frac{1}{\mu_s} = \frac{0.24\ \text{m}}{0.3}$$

$$h_{\text{max}} = 0.800\ \text{m} \blacktriangleleft$$

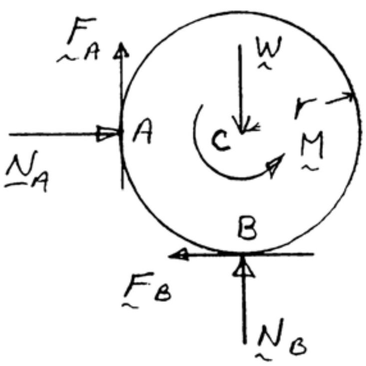
PROBLEM 8.17



The cylinder shown is of weight W and radius r, and the coefficient of static friction μ_s is the same at A and B. Determine the magnitude of the largest couple **M** which can be applied to the cylinder if it is not to rotate.

SOLUTION

FBD cylinder:



For maximum M, motion impends at both A and B

$$F_A = \mu_s N_A,\ F_B = \mu_s N_B$$

$$\longrightarrow \Sigma F_x = 0: \quad N_A - F_B = 0 \qquad \mathrm{N}_A = F_B = \mu_s N_B$$

$$F_A = \mu_s N_A = \mu_s^2 N_B$$

$$\uparrow \Sigma F_y = 0: \quad N_B + F_A - W = 0 \qquad N_B + \mu_s^2 N_B = W$$

or $$N_B = \frac{W}{1+\mu_s^2}$$

and $$F_B = \frac{\mu_s W}{1+\mu_s^2}$$

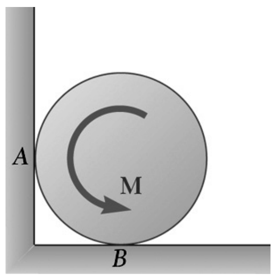
$$F_A = \frac{\mu_s^2 W}{1+\mu^2}$$

$$\Sigma M_C = 0: \quad M - r\left(F_A + F_B\right) = 0$$

$$M = r\left(\mu_s + \mu_s^2\right)\frac{W}{1+\mu_s^2}$$

$$M_{\max} = Wr\mu_s \frac{1+\mu_s}{1+\mu_s^2} \blacktriangleleft$$

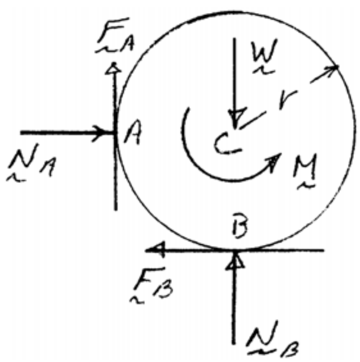
PROBLEM 8.18



The cylinder shown is of weight W and radius r. Express in terms of W and r the magnitude of the largest couple **M** which can be applied to the cylinder if it is not to rotate assuming that the coefficient of static friction is (*a*) zero at A and 0.36 at B, (*b*) 0.30 at A and 0.36 at B.

SOLUTION

FBD cylinder:



For maximum M, motion impends at both A and B

$$F_A = \mu_A N_A; \qquad F_B = \mu_B N_B$$

$$\longrightarrow \Sigma F_x = 0: \quad N_A - F_B = 0 \qquad N_A = F_B = \mu_B N_B$$

$$F_A = \mu_A N_A = \mu_A \mu_B N_B$$

$$\uparrow \Sigma F_y = 0: \quad N_B + F_A - W = 0 \qquad N_B\left(1 + \mu_A \mu_B\right) = W$$

or

$$N_B = \frac{1}{1 + \mu_A \mu_B} W$$

and

$$F_B = \mu_B N_B = \frac{\mu_B}{1 + \mu_A \mu_B} W$$

$$F_A = \mu_A \mu_B N_B = \frac{\mu_A \mu_B}{1 + \mu_A \mu_B} W$$

$$\Sigma M_C = 0: \quad M - r\left(F_A + F_B\right) = 0 \qquad M = W r \mu_B \frac{1 + \mu_A}{1 + \mu_A \mu_B}$$

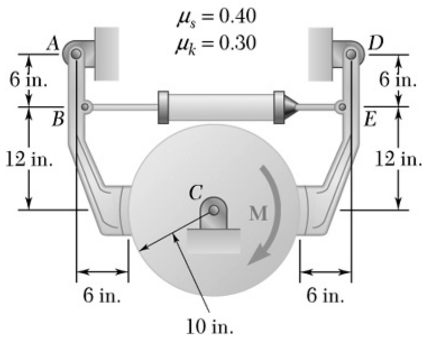
(*a*) For $\mu_A = 0$ and $\mu_B = 0.36$

$M = 0.360Wr$ ◀

(*b*) For $\mu_A = 0.30$ and $\mu_B = 0.36$

$M = 0.422Wr$ ◀

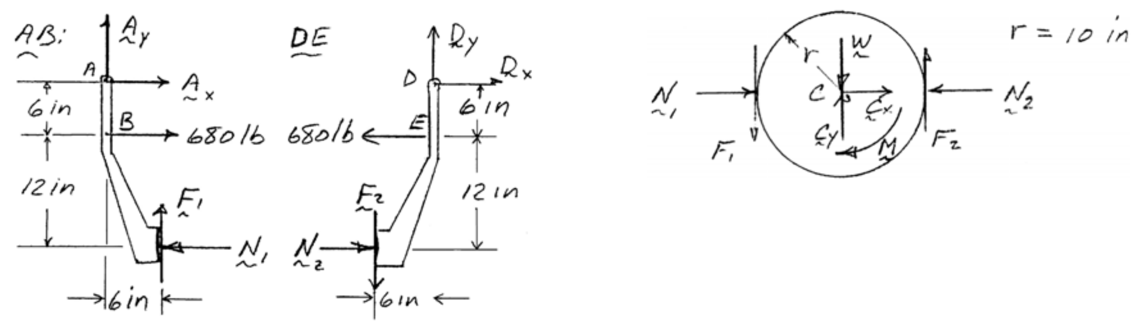
PROBLEM 8.19



The hydraulic cylinder shown exerts a force of 680 lb directed to the right on point *B* and to the left on point *E*. Determine the magnitude of the couple **M** required to rotate the drum clockwise at a constant speed.

SOLUTION

FBDs **Drum:**



Rotating drum ⇒ slip at both sides; constant speed ⇒ equilibrium

$$\therefore \quad F_1 = \mu_k N_1 = 0.3N_1; \qquad F_2 = \mu_k N_2 = 0.3N_2$$

AB: $\circlearrowleft \Sigma M_A = 0{:}\quad (6 \text{ in.})(680 \text{ lb}) + (6 \text{ in.})(F_1) - (18 \text{ in.})N_1 = 0$

$$F_1\left(\frac{18 \text{ in.}}{0.3} - 6 \text{ in.}\right) = (6 \text{ in.})(680 \text{ lb}) \qquad \text{or} \qquad F_1 = 75.555 \text{ lb}$$

DE: $\circlearrowleft \Sigma M_D = 0{:}\quad (6 \text{ in.})F_2 + (18 \text{ in.})N_2 - (6 \text{ in.})(680 \text{ lb}) = 0$

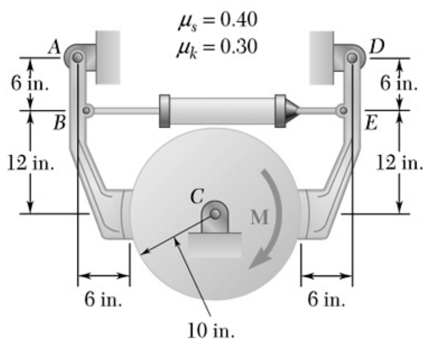
$$F_2\left(6 \text{ in.} + \frac{18 \text{ in.}}{0.3}\right) = (6 \text{ in.})(680 \text{ lb}) \qquad \text{or} \qquad F_2 = 61.818 \text{ lb}$$

Drum: $\circlearrowleft \Sigma M_C = 0{:}\quad r(F_1 + F_2) - M = 0$

$$M = (10 \text{ in.})(75.555 + 61.818)\text{lb}$$

$$M = 1374 \text{ lb}\cdot\text{in.} \blacktriangleleft$$

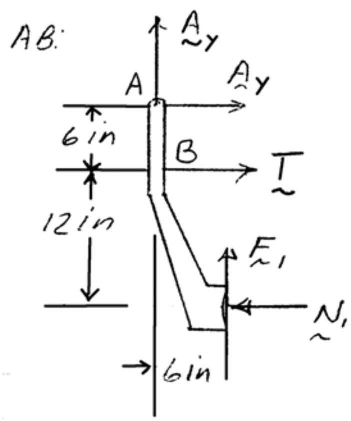
PROBLEM 8.20



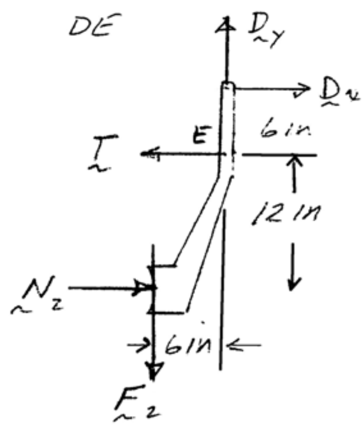
A couple **M** of magnitude $70\text{ lb}\cdot\text{ft}$ is applied to the drum as shown. Determine the smallest force which must be exerted by the hydraulic cylinder on joints B and E if the drum is not to rotate.

SOLUTION

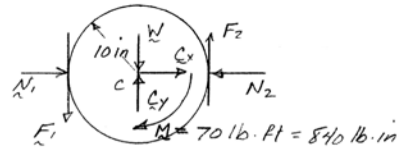
FBDs



DE:



Drum:



For minimum T, slip impends at both sides, so

$$F_1 = \mu_s N_1 = 0.4N_1 \qquad F_2 = \mu_s N_2 = 0.4N_2$$

AB: $\quad \circlearrowleft \Sigma M_A = 0: \quad (6\text{ in.})T + (6\text{ in.})F_1 - (18\text{ in.})N_1 = 0$

$$F_1\left(\frac{18\text{ in.}}{0.4} - 6\text{ in.}\right) = (6\text{ in.})T \qquad \text{or} \qquad F_1 = \frac{T}{6.5}$$

DE: $\quad \circlearrowleft \Sigma M_D = 0: \quad (6\text{ in.})F_2 + (18\text{ in.})N_2 - (6\text{ in.})T = 0$

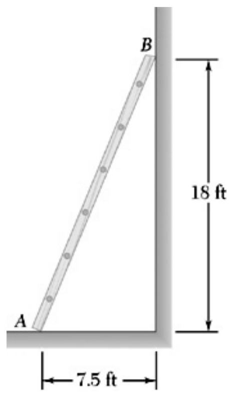
$$F_2\left(6\text{ in.} + \frac{18\text{ in.}}{0.4}\right) = (6\text{ in.})T \qquad \text{or} \qquad F_2 = \frac{T}{8.5}$$

Drum: $\quad \circlearrowleft \Sigma M_C = 0: \quad (10\text{ in.})(F_1 + F_2) - 840\text{ lb}\cdot\text{in.} = 0$

$$T\left(\frac{1}{6.5} + \frac{1}{8.5}\right) = 84\text{ lb}$$

$T = 309\text{ lb}$ ◀

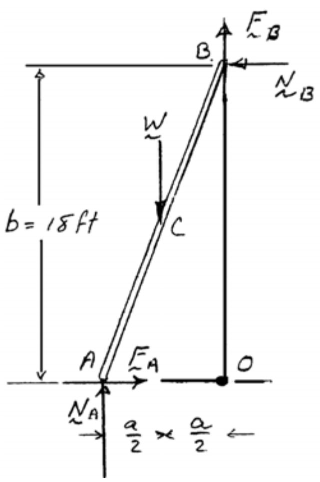
PROBLEM 8.21



A 19.5-ft ladder AB leans against a wall as shown. Assuming that the coefficient of static friction μ_s is the same at A and B, determine the smallest value of μ_s for which equilibrium is maintained.

SOLUTION

FBD ladder:



$a = 7.5$ ft

$b = 18$ ft

Motion impends at both A and B.

$$F_A = \mu_s N_A \qquad F_B = \mu_s N_B$$

$$\rightarrow \Sigma F_x = 0: \quad F_A - N_B = 0 \quad \text{or} \quad N_B = F_A = \mu_s N_A$$

Then $$F_B = \mu_s N_B = \mu_s^2 N_A$$

$$\uparrow \Sigma F_y = 0: \quad N_A - W + F_B = 0 \quad \text{or} \quad N_A\left(1 + \mu_s^2\right) = W$$

$$\Sigma M_O = 0: \quad bN_B + \frac{a}{2}W - aN_A = 0$$

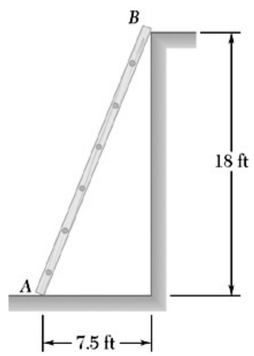
or $$aN_A - b\mu_s N_A = \frac{a}{2}W = \frac{a}{2}N_A\left(1 + \mu_s^2\right)$$

$$\mu_s^2 + \frac{2b}{a}\mu_s - 1 = 0$$

$$\mu_s = -\frac{b}{a} \pm \sqrt{\left(\frac{b}{a}\right)^2 + 1} = -2.4 \pm 2.6$$

The positive root is physically possible. Therefore, $\mu_s = 0.200$ ◀

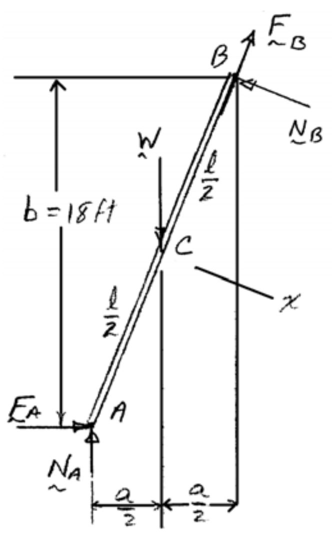
PROBLEM 8.22



A 19.5-ft ladder *AB* leans against a wall as shown. Assuming that the coefficient of static friction μ_s is the same at *A* and *B*, determine the smallest value of μ_s for which equilibrium is maintained.

SOLUTION

FBD ladder:



$$a = 7.5 \text{ ft}$$

$$l = 19.5 \text{ ft}$$

$$\frac{a}{l} = \frac{5}{13}$$

$$\frac{b}{l} = \frac{12}{13}$$

Motion impends at both *A* and *B*, so

$$F_A = \mu_s N_A \qquad \text{and} \qquad F_B = \mu_s N_B$$

$$\circlearrowleft \Sigma M_A = 0: \quad lN_B - \frac{a}{2}W = 0 \qquad \text{or} \qquad N_B = \frac{a}{2l}W = \frac{7.5 \text{ ft}}{39 \text{ ft}}W$$

or
$$N_B = \frac{2.5}{13}W$$

Then
$$F_B = \mu_s N_B = \mu_s \frac{2.5W}{13}$$

$$\longrightarrow \Sigma F_x = 0: \quad F_A + \frac{5}{13}F_B - \frac{12}{13}N_B = 0$$

$$\mu_s N_A + \frac{12.5}{(13)^2}\mu_s W - \frac{30}{(13)^2}W = 0$$

$$N_A - \frac{W}{(13)^2}\frac{(30 - 12.5\mu_s)}{\mu_s}$$

$$\uparrow \Sigma F_y = 0: \quad N_A - W + \frac{12}{13}F_B + \frac{5}{13}N_B = 0$$

$$\left(\frac{30 - 12.5\mu_s}{\mu_s} + 30\mu_s + 12.5\right)\frac{W}{(13)^2} = W$$

or
$$\mu_s^2 - 5.6333\mu_s + 1 = 0$$

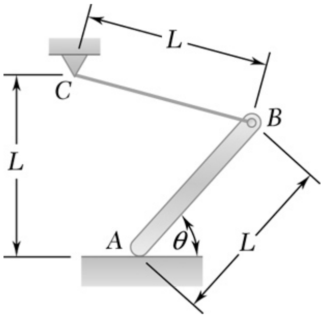
$$\mu_s = 2.8167 \pm 2.6332$$

or
$$\mu_s = 0.1835 \qquad \text{and} \qquad \mu_s = 5.45$$

The larger value is very unlikely unless the surface is treated with some "non-skid" material.

In any event, the smallest value for equilibrium is $\mu_s = 0.1835$ ◀

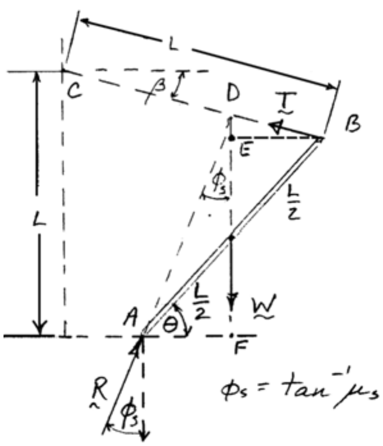
PROBLEM 8.23

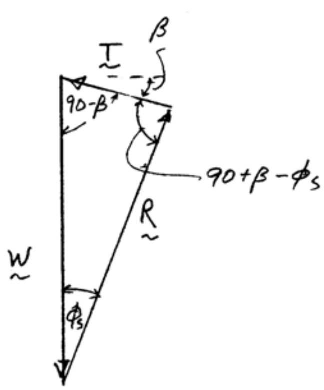


End A of a slender, uniform rod of weight W and length L bears on a horizontal surface as shown, while end B is supported by a cord BC of length L. Knowing that the coefficient of static friction is 0.40, determine (a) the value of θ for which motion is impending, (b) the corresponding value of the tension in the cord.

SOLUTION

FBD rod:





(a) Geometry: $\quad BE = \dfrac{L}{2}\cos\theta \qquad DE = \left(\dfrac{L}{2}\cos\theta\right)\tan\beta$

$$EF = L\sin\theta \qquad DF = \frac{L}{2}\frac{\cos\theta}{\tan\phi_s}$$

So

$$L\left(\frac{1}{2}\cos\theta\tan\beta + \sin\theta\right) = \frac{L}{2}\frac{\cos\theta}{\tan\phi_s}$$

or

$$\tan\beta + 2\tan\theta = \frac{1}{\tan\phi_s} = \frac{1}{\mu_s} = \frac{1}{0.4} = 2.5 \qquad (1)$$

Also,

$$L\sin\theta + L\sin\beta = L$$

or

$$\sin\theta + \sin\beta = 1 \qquad (2)$$

Solving Eqs. (1) and (2) numerically $\quad \theta_1 = 4.62° \qquad \beta_1 = 66.85°$

$$\theta_2 = 48.20° \qquad \beta_2 = 14.75°$$

Therefore, $\quad \theta = 4.62°$ and $\theta = 48.2°$ ◀

(b) Now $\quad \phi_s = \tan^{-1}\mu_s = \tan^{-1}0.4 = 21.801°$

and

$$\frac{T}{\sin\phi_s} = \frac{W}{\sin(90 + \beta - \phi_s)}$$

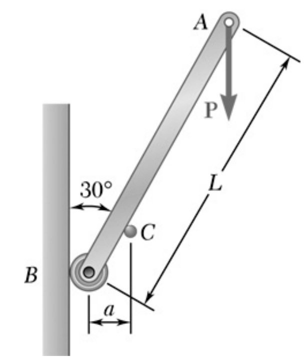
or

$$T = W\frac{\sin\phi_s}{\sin(90 + \beta - \phi_s)}$$

For $\quad \theta = 4.62° \qquad T = 0.526W$ ◀

$\theta = 48.2° \qquad T = 0.374W$ ◀

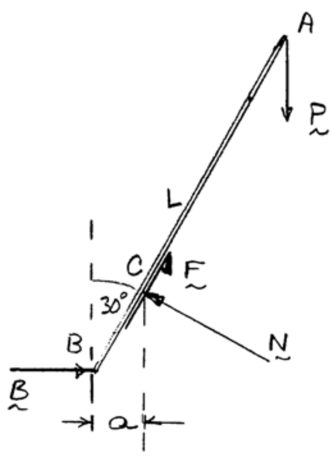
PROBLEM 8.24



A slender rod of length L is lodged between peg C and the vertical wall and supports a load **P** at end A. Knowing that the coefficient of static friction between the peg and the rod is 0.25 and neglecting friction at the roller, determine the range of values of the ratio L/a for which equilibrium is maintained.

SOLUTION

FBD rod:



$$\circlearrowleft \Sigma M_B = 0: \quad \frac{a}{\sin 30°} N - L \sin 30° P = 0$$

$$N = \frac{L}{a} \sin^2 30° P = \frac{L}{a} \frac{P}{4}$$

$$\text{Impending motion at } C: \left.\begin{array}{r} \text{down} \rightarrow F = \mu_s N \\ \text{up} \rightarrow F = -\mu_s N \end{array}\right\} F = \pm \frac{N}{4}$$

$$\uparrow \Sigma F_y = 0: \quad F \cos 30° + N \sin 30° - P = 0$$

$$\pm \frac{L}{a} \frac{P}{16} \frac{\sqrt{3}}{2} + \frac{L}{a} \frac{P}{4} \frac{1}{2} = P$$

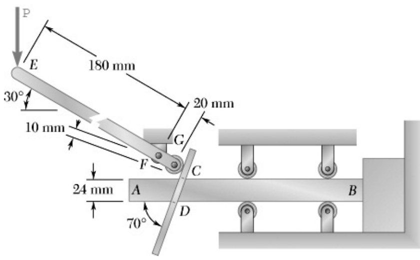
$$\frac{L}{a} \left[\frac{1}{8} \pm \frac{\sqrt{3}}{32} \right] = 1$$

$$\frac{L}{a} = \frac{32}{4 \pm \sqrt{3}}$$

or $\qquad \dfrac{L}{a} = 5.583 \qquad$ and $\qquad \dfrac{L}{a} = 14.110$

For equilibrium: $\qquad 5.58 \le \dfrac{L}{a} \le 14.11$ ◀

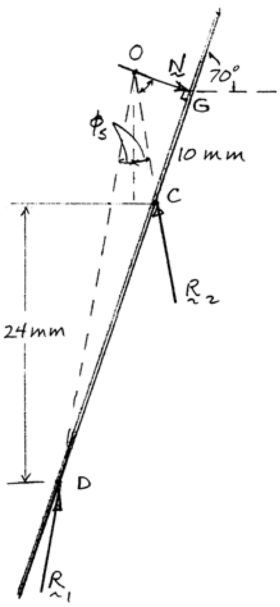
PROBLEM 8.25



The basic components of a clamping device are bar *AB*, locking plate *CD*, and lever *EFG*; the dimensions of the slot in *CD* are slightly larger than those of the cross section of *AB*. To engage the clamp, *AB* is pushed against the workpiece, and then force **P** is applied. Knowing that $P = 160$ N and neglecting the friction force between the lever and the plate, determine the smallest allowable value of the static coefficient of friction between the bar and the plate.

SOLUTION

FBD Plate:



DC is three-force member and motion impends at *C* and *D* (for minimum μ_s).

$$\measuredangle\, OCG = 20° + \phi_s \qquad \measuredangle\, ODG = 20° - \phi_s$$

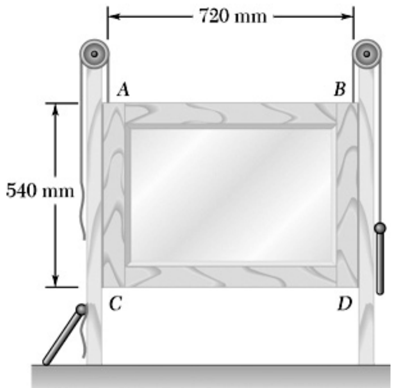
$$\overline{OG} = (10 \text{ mm})\tan(20° + \phi_s) = \left(\frac{24 \text{ mm}}{\sin 70°} + 10 \text{ mm}\right)\tan(20° - \phi_s)$$

or $$\tan(20° + \phi_s) = 3.5540\tan(20° - \phi_s)$$

Solving numerically $$\phi_s = 10.565°$$

Now $$\mu_s = \tan\phi_s$$

so that $$\mu_s = 0.1865 \blacktriangleleft$$



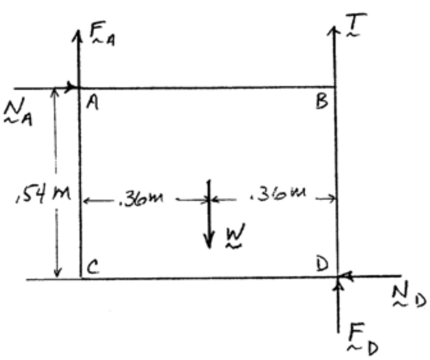
PROBLEM 8.26

A window sash having a mass of 4 kg is normally supported by two 2-kg sash weights. Knowing that the window remains open after one sash cord has broken, determine the smallest possible value of the coefficient of static friction. (Assume that the sash is slightly smaller that the frame and will bind only at points *A* and *D*.)

SOLUTION

FBD window:

$$T = (2\text{ kg})(9.81\text{ m/s}^2) = 19.62\text{ N} = \frac{W}{2}$$



$$W = (4\text{ kg})(9.81\text{ m/s}^2) = 39.24\text{ N}$$

$\rightarrow \Sigma F_x = 0$: $\quad N_A - N_D = 0 \qquad N_A = N_D$

Impending motion: $\quad F_A = \mu_s N_A \qquad F_D = \mu_s N_D$

$\Sigma M_D = 0$: $\quad (0.36\text{ m})W - (0.54\text{ m})N_A - (0.72\text{ m})F_A = 0$

$$W = \frac{3}{2}N_A + 2\mu_s N_A$$

$$N_A = \frac{2W}{3 + 4\mu_s}$$

$\uparrow \Sigma F_y = 0$: $\quad F_A - W + T + F_D = 0$

$$F_A + F_D = W - T$$

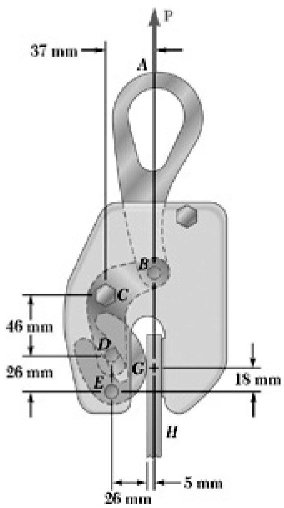
$$= \frac{W}{2}$$

Now $\quad F_A + F_D = \mu_s(N_A + N_D) = 2\mu_s N_A$

Then $\quad \dfrac{W}{2} = 2\mu_s \dfrac{2W}{3 + 4\mu_s}$

or $\quad \mu_s = 0.750$ ◀

PROBLEM 8.27



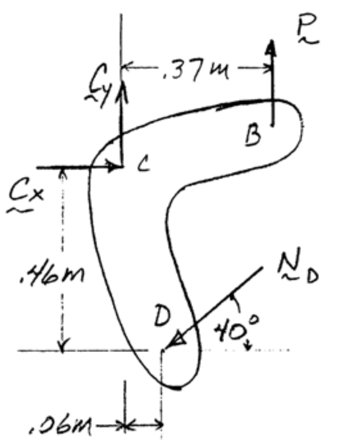
The steel-plate clamp shown is used to lift a steel plate H of mass 250 kg. Knowing that the normal force exerted on steel cam EG by pin D forms an angle of 40° with the horizontal and neglecting the friction force between the cam and the pin, determine the smallest allowable value of the coefficient of static friction.

SOLUTION

(Note: **P** is vertical as AB is two force member; also $P = W$ since clamp + plate is a two force FBD)

FBDs:

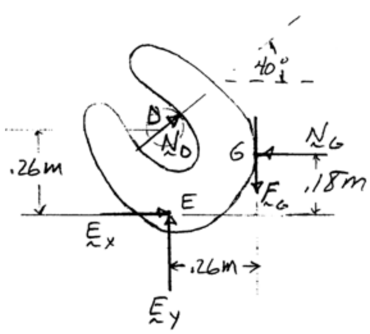
BCD:



$$\circlearrowleft \Sigma M_C = 0: \quad (0.37 \text{ m})P - (0.46 \text{ m})N_D \cos 40° - (0.06 \text{ m})N_D \sin 40° = 0$$

or $$N_D = 0.94642P = 0.94642W$$

EG:



$$\circlearrowleft \Sigma M_E = 0: \quad (0.18 \text{ m})N_G - (0.26 \text{ m})F_G - (0.26 \text{ m})N_D \cos 40° = 0$$

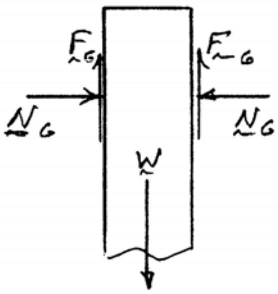
Impending motion: $$F_G = \mu_s N_G$$

Combining $$(18 + 26\mu_s)N_G = 19.9172N_D$$

$$= 18.850W$$

PROBLEM 8.27 CONTINUED

Plate:

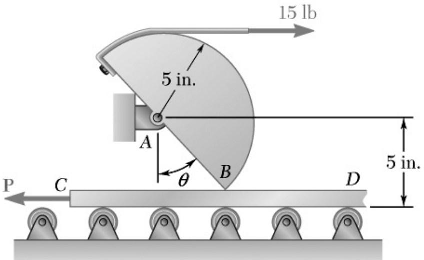


From plate: $F_G = \dfrac{W}{2}$ so that $N_G = \dfrac{W}{2\mu_s}$

Then $\left(18 + 26\mu_s\right)\dfrac{W}{2\mu_s} = 18.85W$

$\mu_s = 0.283$ ◀

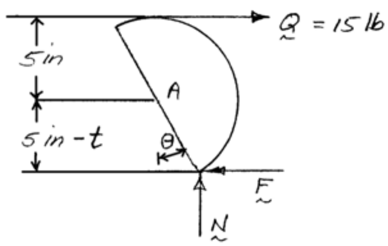
PROBLEM 8.28

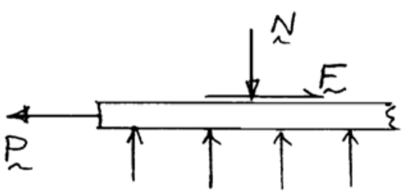


The 5-in.-radius cam shown is used to control the motion of the plate *CD*. Knowing that the coefficient of static friction between the cam and the plate is 0.45 and neglecting friction at the roller supports, determine (*a*) the force **P** for which motion of the plate is impending knowing that the plate is 1 in. thick, (*b*) the largest thickness of the plate for which the mechanism is self-locking, (that is, for which the plate cannot be moved however large the force **P** may be).

SOLUTION

FBDs:





From plate: $\xrightarrow{} \Sigma F_x = 0: \quad F - P = 0 \qquad F = P$

From cam geometry: $\cos\theta = \dfrac{5\text{ in.} - t}{5\text{ in.}}$

$$\circlearrowleft \Sigma M_A = 0: \quad \left[(5\text{ in.})\sin\theta\right]N - \left[(5\text{ in.})\cos\theta\right]F - (5\text{ in.})Q = 0$$

Impending motion: $F = \mu_s N$

So $N\sin\theta - \mu_s N\cos\theta = Q = 15\text{ lb}$

$$N = \frac{Q}{\sin\theta - \mu_s\cos\theta}$$

So $$P = F = \mu_s N = \frac{\mu_s Q}{\sin\theta - \mu_s\cos\theta}$$

(*a*) $$t = 1\text{ in.} \Rightarrow \cos\theta = \frac{4\text{ in.}}{5\text{ in.}} = 0.8; \;\; \sin\theta = 0.6$$

$$P = \frac{(0.45)(15\text{ lb})}{0.6 - (0.45)(0.8)} = 28.125\text{ lb}; \;\; \mathbf{P} = 28.1\text{ lb} \longleftarrow \blacktriangleleft$$

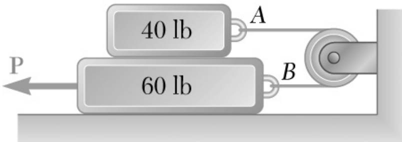
(*b*) $$P \to \infty: \; \sin\theta - \mu_s\cos\theta = \frac{\mu_s Q}{P} \longrightarrow 0$$

Thus $\tan\theta \to \mu_s = 0.45$ so that $\theta = 24.228°$

But $(5\text{ in.})\cos\theta = 5\text{ in.} - t$ or $t = (5\text{ in.})(1 - \cos\theta)$

$$t = 0.440\text{ in.} \blacktriangleleft$$

PROBLEM 8.11

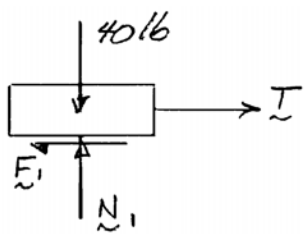


The coefficients of friction are $\mu_s = 0.40$ and $\mu_k = 0.30$ between all surfaces of contact. Determine the force **P** for which motion of the 60-lb block is impending if cable AB (a) is attached as shown, (b) is removed.

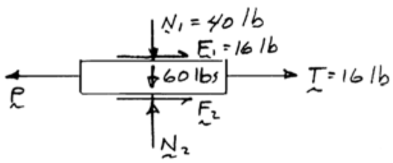
SOLUTION

FBDs

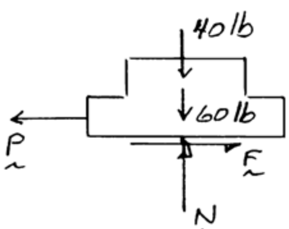
Top block:



Bottom block:



FBD blocks:



(a) Note: With the cable, motion must impend at both contact surfaces.

$$\uparrow \Sigma F_y = 0: \quad N_1 - 40 \text{ lb} = 0 \qquad N_1 = 40 \text{ lb}$$

Impending slip: $\quad F_1 = \mu_s N_1 = 0.4(40 \text{ lb}) = 16 \text{ lb}$

$$\longrightarrow \Sigma F_x = 0: \quad T - F_1 = 0 \qquad T - 16 \text{ lb} = 0 \qquad T = 16 \text{ lb}$$

$$\uparrow \Sigma F_y = 0: \quad N_2 - 40 \text{ lb} - 60 \text{ lb} = 0 \qquad N_2 = 100 \text{ lb}$$

Impending slip: $\quad F_2 = \mu_s N_2 = 0.4(100 \text{ lb}) = 40 \text{ lb}$

$$\longrightarrow \Sigma F_x = 0: \quad -P + 16 \text{ lb} + 16 \text{ lb} + 40 \text{ lb} = 0$$

$$\mathbf{P} = 72.0 \text{ lb} \longleftarrow \blacktriangleleft$$

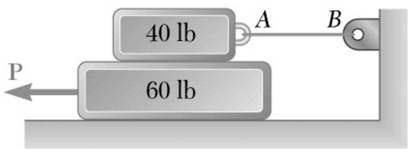
(b) Without the cable, both blocks will stay together and motion will impend only at the floor.

$$\uparrow \Sigma F_y = 0: \quad N - 40 \text{ lb} - 60 \text{ lb} = 0 \qquad N = 100 \text{ lb}$$

Impending slip: $\quad F = \mu_s N = 0.4(100 \text{ lb}) = 40 \text{ lb}$

$$\longrightarrow \Sigma F_x = 0: \quad 40 \text{ lb} - P = 0$$

$$\mathbf{P} = 40.0 \text{ lb} \longleftarrow \blacktriangleleft$$



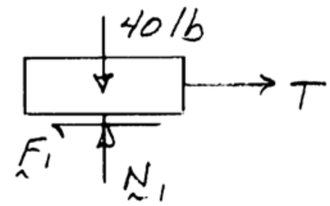
PROBLEM 8.12

The coefficients of friction are $\mu_s = 0.40$ and $\mu_k = 0.30$ between all surfaces of contact. Determine the force **P** for which motion of the 60-lb block is impending if cable AB (a) is attached as shown, (b) is removed.

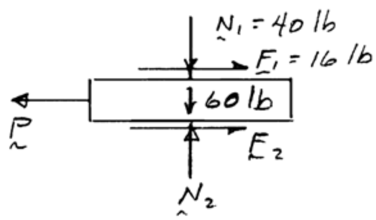
SOLUTION

FBDs

Top block:



Bottom block:



(a) With the cable, motion must impend at both surfaces.

$$\uparrow \Sigma F_y = 0: \quad N_1 - 40 \text{ lb} = 0 \qquad N_1 = 40 \text{ lb}$$

Impending slip: $\quad F_1 = \mu_s N_1 = 0.4(40 \text{ lb}) = 16 \text{ lb}$

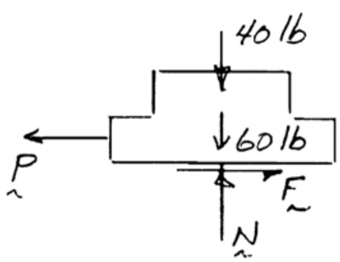
$$\uparrow \Sigma F_y = 0: \quad N_2 - 40 \text{ lb} - 60 \text{ lb} = 0 \qquad N_2 = 100 \text{ lb}$$

Impending slip: $\quad F_2 = \mu N_2 = 0.4(100 \text{ lb}) = 40 \text{ lb}$

$$\longrightarrow \Sigma F_x = 0: \quad 16 \text{ lb} + 40 \text{ lb} - P = 0 \qquad P = 56 \text{ lb}$$

$$\mathbf{P} = 56.0 \text{ lb} \longleftarrow \blacktriangleleft$$

FBD blocks:



(b) Without the cable, both blocks stay together and motion will impend at the floor surface only.

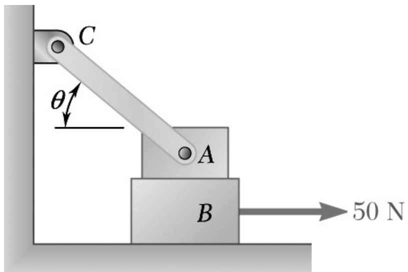
$$\uparrow \Sigma F_y = 0: \quad N - 40 \text{ lb} - 60 \text{ lb} = 0 \qquad N = 100 \text{ lb}$$

Impending slip: $\quad F = \mu_s N = 0.4(100 \text{ lb}) = 40 \text{ lb}$

$$\longrightarrow \Sigma F_x = 0: \quad -P + 40 \text{ lb} = 0 \qquad P = 40 \text{ lb}$$

$$\mathbf{P} = 40.0 \text{ lb} \longleftarrow \blacktriangleleft$$

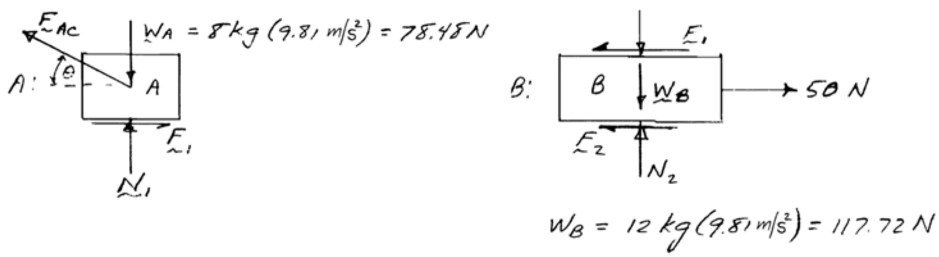
PROBLEM 8.13



The 8-kg block A is attached to link AC and rests on the 12-kg block B. Knowing that the coefficient of static friction is 0.20 between all surfaces of contact and neglecting the mass of the link, determine the value of θ for which motion of block B is impending.

SOLUTION

FBDs:



Motion must impend at both contact surfaces

Block A: $\uparrow \Sigma F_y = 0\text{:} \quad N_1 - W_A = 0 \qquad N_1 = W_A$

Block B: $\uparrow \Sigma F_y = 0\text{:} \quad N_2 - N_1 - W_B = 0$

$$N_2 = N_1 + W_B = W_A + W_B$$

Impending motion:

$$F_1 = \mu_s N_1 = \mu_s W_A$$

$$F_2 = \mu_s N_2 = \mu_s (N_1 + W_B)$$

Block B: $\longrightarrow \Sigma F_x = 0\text{:} \quad 50 \text{ N} - F_1 - F_2 = 0$

or $\quad 50 \text{ N} = \mu_s (N_1 + N_1 + W_B) = 0.2(2N_1 + 117.72 \text{ N})$

$$N_1 = 66.14 \text{ N} \qquad F_1 = 0.2(66.14 \text{ N}) = 13.228 \text{ N}$$

Block A: $\longrightarrow \Sigma F_x = 0\text{:} \quad 13.228 \text{ N} - F_{AC}\cos\theta = 0$

or

$$F_{AC} \cos\theta = 13.228 \text{ N} \tag{1}$$

$$\uparrow \Sigma F_y = 0\text{:} \quad 66.14 \text{ N} - 78.48 \text{ N} + F_{AC}\sin\theta = 0$$

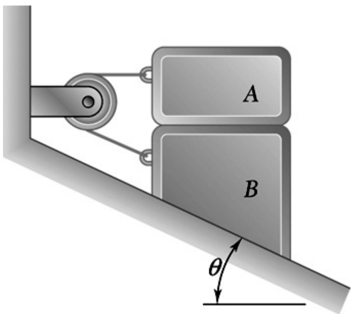
or

$$F_{AC} \sin\theta = 78.48 \text{ N} - 66.14 \text{ N} \tag{2}$$

Then, $\dfrac{\text{Eq. (2)}}{\text{Eq. (1)}}$

$$\tan\theta = \frac{78.48 \text{ N} - 66.14 \text{ N}}{13.228 \text{ N}}$$

$$\theta = 43.0° \blacktriangleleft$$

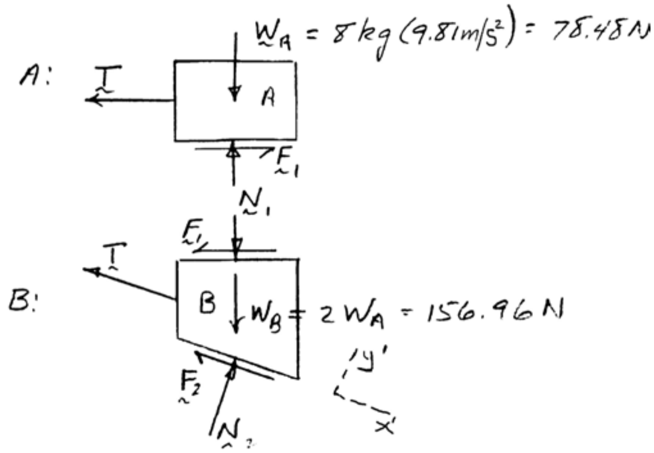


PROBLEM 8.14

The 8-kg block A and the 16-kg block B are at rest on an incline as shown. Knowing that the coefficient of static friction is 0.25 between all surfaces of contact, determine the value of θ for which motion is impending.

SOLUTION

FBDs:



Block A: $\uparrow \Sigma F_y = 0\!: \quad N_1 - W_A = 0 \qquad N_1 = W_A$

Impending motion: $F_1 = \mu_s N_1 = \mu_s W_A$

$\rightarrow \Sigma F_x = 0\!: \quad F_1 - T = 0 \qquad T = F_1 = \mu_s W_A$

Block B: $\nearrow \Sigma F_{y'} = 0\!: \quad N_2 - (N_1 + W_B)\cos\theta - F_1 \sin\theta = 0$

$$N_2 = 3W_A \cos\theta + \mu_s W_A \sin\theta$$

$$= W_A(3\cos\theta + 0.25\sin\theta)$$

Impending motion: $F_2 = \mu_s N_2 = 0.25 W_A (3\cos\theta + 0.25\sin\theta)$

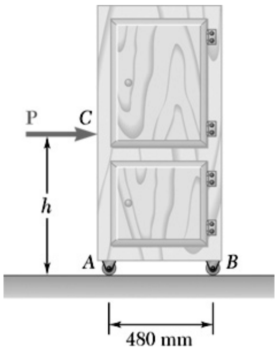
$\searrow \Sigma F_{x'} = 0\!: \quad -T - F_2 - F_1\cos\theta + (N_1 + W_B)\sin\theta = 0$

$$\left[-0.25 - 0.25(3\cos\theta + 0.25\sin\theta) - 0.25\cos\theta + 3\sin\theta\right]W_A = 0$$

or $47\sin\theta - 16\cos\theta - 4 = 0$

Solving numerically $\theta = 23.4° \blacktriangleleft$

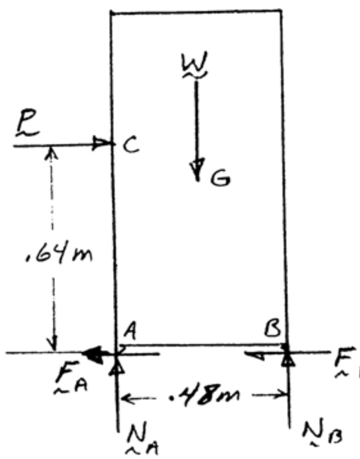
PROBLEM 8.15



A 48-kg cabinet is mounted on casters which can be locked to prevent their rotation. The coefficient of static friction between the floor and each caster is 0.30. Knowing that $h = 640$ mm, determine the magnitude of the force **P** required for impending motion of the cabinet to the right (*a*) if all casters are locked, (*b*) if the casters at *B* are locked and the casters at *A* are free to rotate, (*c*) if the casters at *A* are locked and the casters at *B* are free to rotate.

SOLUTION

FBD cabinet:



$$W = 48 \text{ kg}(9.81 \text{ m/s}^2)$$

$$= 470.88 \text{ N}$$

$$\mu_s = 0.3$$

Note: For tipping, $N_A = F_A = 0$

$$\Sigma M_B = 0: \ (0.24 \text{ m})W - (0.64 \text{ m})P_{\text{tip}} = 0 \qquad P_{\text{tip}} = 0.375W$$

(*a*) All casters locked: Impending slip: $F_A = \mu_s N_A,\ F_B = \mu_s N_B$

$$\Sigma F_y = 0: \ N_A + N_B - W = 0 \qquad N_A + N_B = W$$

So $F_A + F_B = \mu_s W$

$$\Sigma F_x = 0: \ P - F_A - F_B = 0 \qquad P = F_A + F_B = \mu_s W$$

$$\therefore \ P = 0.3(470.88 \text{ N}) \quad \text{or} \quad P = 141.3 \text{ N} \blacktriangleleft$$

$$(P = 0.3W < P_{\text{tip}} \quad \text{OK})$$

(*b*) Casters at *A* free, so $F_A = 0$

Impending slip: $F_B = \mu_s N_B$

$$\Sigma F_x = 0: \ P - F_B = 0$$

$$P = F_B = \mu_s N_B \qquad N_B = \frac{P}{\mu_s}$$

$$\Sigma M_A = 0: \ (0.64 \text{ m})P + (0.24 \text{ m})W - (0.48 \text{ m})N_B = 0$$

$$8P + 3W - 6\frac{P}{0.3} = 0 \qquad P = 0.25W$$

$$(P = 0.25W < P_{\text{tip}} \quad \text{OK})$$

$$\therefore \ P = 0.25(470.88 \text{ N}) \qquad P = 117.7 \text{ N} \blacktriangleleft$$

PROBLEM 8.15 CONTINUED

(c) Casters at B free, so $\qquad F_B = 0$

Impending slip: $\qquad F_A = \mu_s N_A$

$\xrightarrow{} \Sigma F_x = 0\text{:} \quad P - F_A = 0 \qquad P = F_A = \mu_s N_A$

$$N_A = \frac{P}{\mu_s} = \frac{P}{0.3}$$

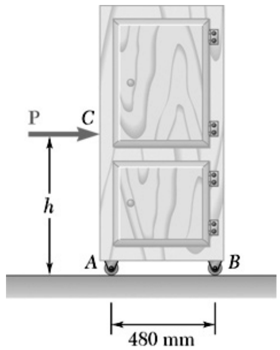
$\circlearrowleft \Sigma M_B = 0\text{:} \quad (0.24\text{ m})W - (0.64\text{ m})P - (0.48\text{ m})N_A = 0$

$$3W - 8P - 6\frac{P}{0.3} = 0 \qquad P = 0.10714W = 50.45\text{ N}$$

$$\left(P < P_{\text{tip}} \qquad \text{OK}\right)$$

$P = 50.5\text{ N}$ ◀

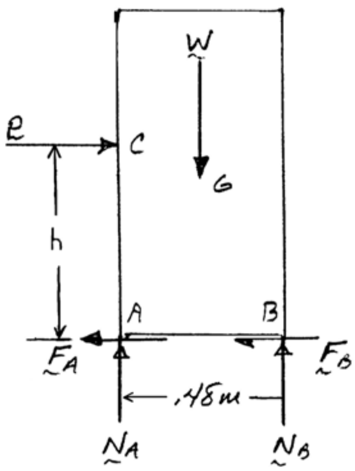
PROBLEM 8.16



A 48-kg cabinet is mounted on casters which can be locked to prevent their rotation. The coefficient of static friction between the floor and each caster is 0.30. Assuming that the casters at A and B are locked, determine (a) the force **P** required for impending motion of the cabinet to the right, (b) the largest allowable height h if the cabinet in not to tip over.

SOLUTION

FBD cabinet:



$$W = 48\text{ kg}\left(9.81\text{ m/s}^2\right)$$

$$= 470.88\text{ N}$$

(a) $\uparrow \Sigma F_y = 0\text{:}\quad N_A + N_B - W = 0;\qquad N_A + N_B = W$

Impending slip: $\quad F_A = \mu_s N_A,\ F_B = \mu_s N_B$

So $\quad F_A + F_B = \mu_s W$

$\longrightarrow \Sigma F_x = 0\text{:}\quad P - F_A - F_B = 0 \qquad P = F_A + F_B = \mu_s W$

$$P = 0.3\left(470.88\text{ N}\right) = 141.26\text{ N}$$

$$\mathbf{P} = 141.3\text{ N} \longrightarrow \blacktriangleleft$$

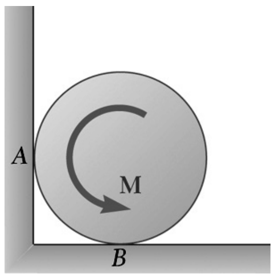
(b) For tipping, $\quad N_A = F_A = 0$

$\circlearrowleft \Sigma M_B = 0\text{:}\quad hP - \left(0.24\text{ m}\right)W = 0$

$$h_{\text{max}} = \left(0.24\text{ m}\right)\frac{W}{P} = \left(0.24\text{ m}\right)\frac{1}{\mu_s} = \frac{0.24\text{ m}}{0.3}$$

$$h_{\text{max}} = 0.800\text{ m} \blacktriangleleft$$

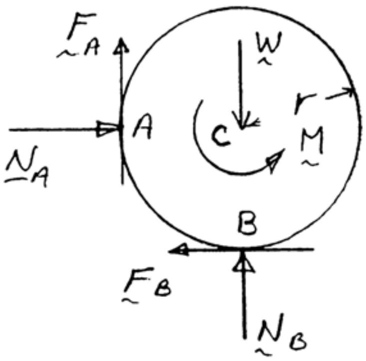
PROBLEM 8.17



The cylinder shown is of weight W and radius r, and the coefficient of static friction μ_s is the same at A and B. Determine the magnitude of the largest couple **M** which can be applied to the cylinder if it is not to rotate.

SOLUTION

FBD cylinder:



For maximum M, motion impends at both A and B

$$F_A = \mu_s N_A,\ F_B = \mu_s N_B$$

$$\rightarrow \Sigma F_x = 0: \quad N_A - F_B = 0 \qquad N_A = F_B = \mu_s N_B$$

$$F_A = \mu_s N_A = \mu_s^2 N_B$$

$$\uparrow \Sigma F_y = 0: \quad N_B + F_A - W = 0 \qquad N_B + \mu_s^2 N_B = W$$

or

$$N_B = \frac{W}{1 + \mu_s^2}$$

and

$$F_B = \frac{\mu_s W}{1 + \mu_s^2}$$

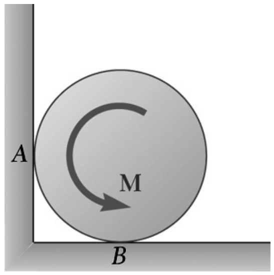
$$F_A = \frac{\mu_s^2 W}{1 + \mu^2}$$

$$\Sigma M_C = 0: \quad M - r\left(F_A + F_B\right) = 0$$

$$M = r\left(\mu_s + \mu_s^2\right)\frac{W}{1 + \mu_s^2}$$

$$M_{\max} = Wr\mu_s \frac{1 + \mu_s}{1 + \mu_s^2} \blacktriangleleft$$

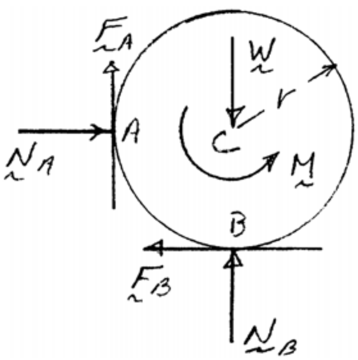
PROBLEM 8.18



The cylinder shown is of weight W and radius r. Express in terms of W and r the magnitude of the largest couple **M** which can be applied to the cylinder if it is not to rotate assuming that the coefficient of static friction is (*a*) zero at A and 0.36 at B, (*b*) 0.30 at A and 0.36 at B.

SOLUTION

FBD cylinder:



For maximum M, motion impends at both A and B

$$F_A = \mu_A N_A; \qquad F_B = \mu_B N_B$$

$$\longrightarrow \Sigma F_x = 0: \quad N_A - F_B = 0 \qquad N_A = F_B = \mu_B N_B$$

$$F_A = \mu_A N_A = \mu_A \mu_B N_B$$

$$\uparrow \Sigma F_y = 0: \quad N_B + F_A - W = 0 \qquad N_B\left(1 + \mu_A \mu_B\right) = W$$

or

$$N_B = \frac{1}{1 + \mu_A \mu_B} W$$

and

$$F_B = \mu_B N_B = \frac{\mu_B}{1 + \mu_A \mu_B} W$$

$$F_A = \mu_A \mu_B N_B = \frac{\mu_A \mu_B}{1 + \mu_A \mu_B} W$$

$$\Sigma M_C = 0: \quad M - r\left(F_A + F_B\right) = 0 \qquad M = Wr\mu_B \frac{1 + \mu_A}{1 + \mu_A \mu_B}$$

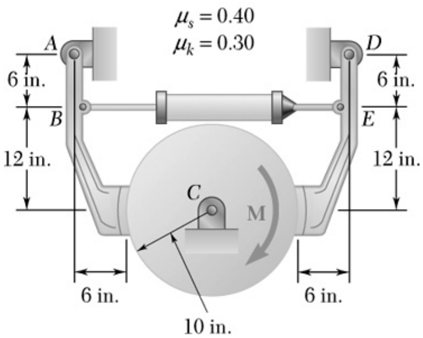
(*a*) For $\mu_A = 0$ and $\mu_B = 0.36$

$$M = 0.360Wr \blacktriangleleft$$

(*b*) For $\mu_A = 0.30$ and $\mu_B = 0.36$

$$M = 0.422Wr \blacktriangleleft$$

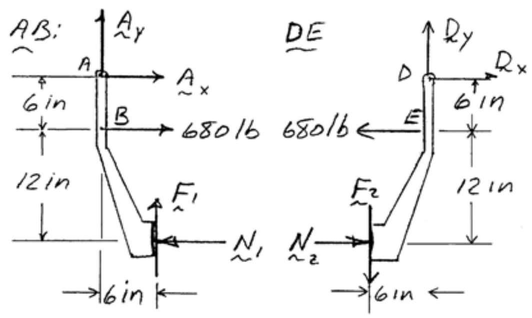
PROBLEM 8.19

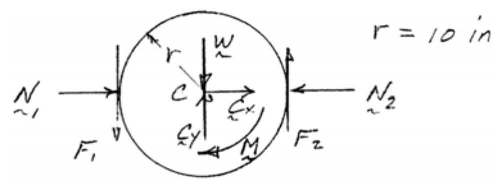


The hydraulic cylinder shown exerts a force of 680 lb directed to the right on point *B* and to the left on point *E*. Determine the magnitude of the couple **M** required to rotate the drum clockwise at a constant speed.

SOLUTION

FBDs **Drum:**





Rotating drum $\Rightarrow$ slip at both sides; constant speed $\Rightarrow$ equilibrium

$$\therefore \quad F_1 = \mu_k N_1 = 0.3N_1; \qquad F_2 = \mu_k N_2 = 0.3N_2$$

AB: $\quad \circlearrowleft \Sigma M_A = 0: \quad (6\text{ in.})(680\text{ lb}) + (6\text{ in.})(F_1) - (18\text{ in.})N_1 = 0$

$$F_1\left(\frac{18\text{ in.}}{0.3} - 6\text{ in.}\right) = (6\text{ in.})(680\text{ lb}) \qquad \text{or} \qquad F_1 = 75.555\text{ lb}$$

DE: $\quad \circlearrowleft \Sigma M_D = 0: \quad (6\text{ in.})F_2 + (18\text{ in.})N_2 - (6\text{ in.})(680\text{ lb}) = 0$

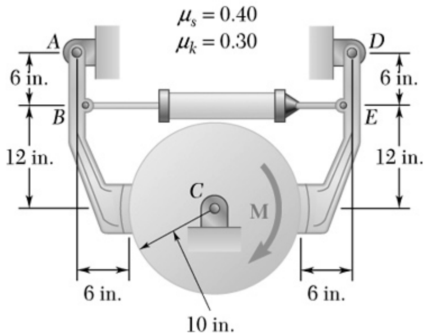
$$F_2\left(6\text{ in.} + \frac{18\text{ in.}}{0.3}\right) = (6\text{ in.})(680\text{ lb}) \qquad \text{or} \qquad F_2 = 61.818\text{ lb}$$

Drum: $\quad \circlearrowleft \Sigma M_C = 0: \quad r(F_1 + F_2) - M = 0$

$$M = (10\text{ in.})(75.555 + 61.818)\text{lb}$$

$M = 1374$ lb·in. ◀

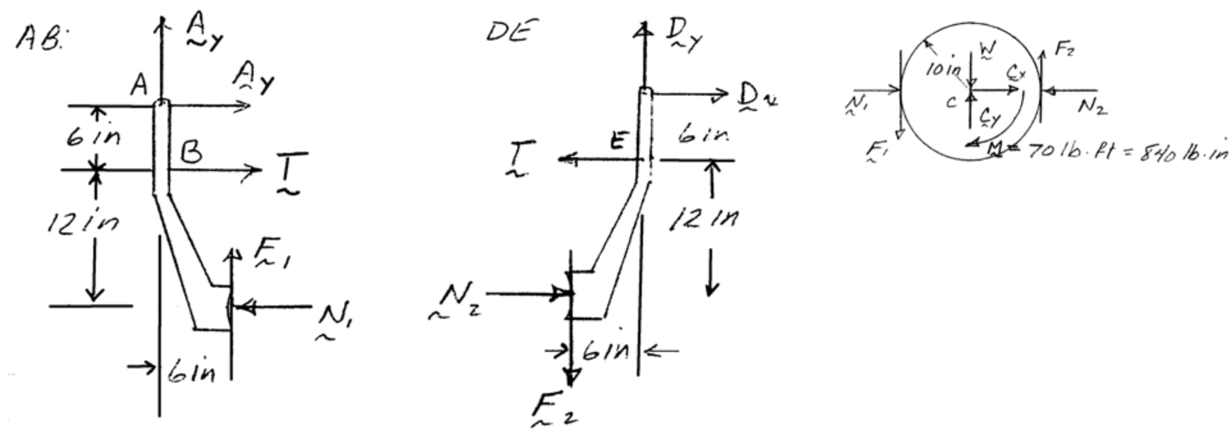
PROBLEM 8.20



A couple **M** of magnitude 70 lb·ft is applied to the drum as shown. Determine the smallest force which must be exerted by the hydraulic cylinder on joints *B* and *E* if the drum is not to rotate.

SOLUTION

FBDs **DE:** **Drum:**



For minimum *T*, slip impends at both sides, so

$$F_1 = \mu_s N_1 = 0.4 N_1 \qquad F_2 = \mu_s N_2 = 0.4 N_2$$

AB: $\circlearrowleft \Sigma M_A = 0$: $(6 \text{ in.})T + (6 \text{ in.})F_1 - (18 \text{ in.})N_1 = 0$

$$F_1\left(\frac{18 \text{ in.}}{0.4} - 6 \text{ in.}\right) = (6 \text{ in.})T \quad \text{or} \quad F_1 = \frac{T}{6.5}$$

DE: $\circlearrowleft \Sigma M_D = 0$: $(6 \text{ in.})F_2 + (18 \text{ in.})N_2 - (6 \text{ in.})T = 0$

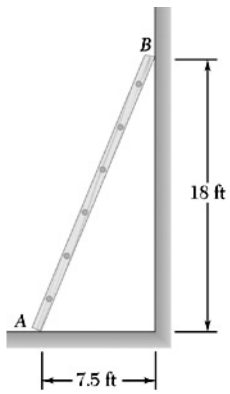
$$F_2\left(6 \text{ in.} + \frac{18 \text{ in.}}{0.4}\right) = (6 \text{ in.})T \quad \text{or} \quad F_2 = \frac{T}{8.5}$$

Drum: $\circlearrowleft \Sigma M_C = 0$: $(10 \text{ in.})(F_1 + F_2) - 840 \text{ lb}\cdot\text{in.} = 0$

$$T\left(\frac{1}{6.5} + \frac{1}{8.5}\right) = 84 \text{ lb}$$

$T = 309$ lb ◀

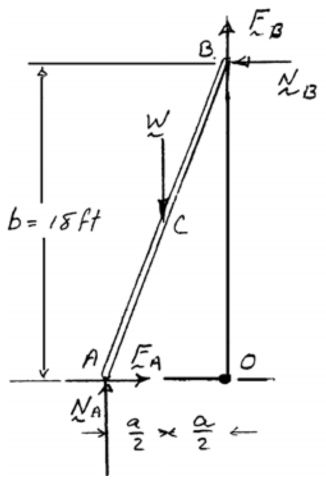
PROBLEM 8.21



A 19.5-ft ladder AB leans against a wall as shown. Assuming that the coefficient of static friction μ_s is the same at A and B, determine the smallest value of μ_s for which equilibrium is maintained.

SOLUTION

FBD ladder:



$a = 7.5$ ft

$b = 18$ ft

Motion impends at both A and B.

$$F_A = \mu_s N_A \qquad F_B = \mu_s N_B$$

$$\rightarrow \Sigma F_x = 0: \quad F_A - N_B = 0 \quad \text{or} \quad N_B = F_A = \mu_s N_A$$

Then $$F_B = \mu_s N_B = \mu_s^2 N_A$$

$$\uparrow \Sigma F_y = 0: \quad N_A - W + F_B = 0 \quad \text{or} \quad N_A\left(1 + \mu_s^2\right) = W$$

$$\Sigma M_O = 0: \quad bN_B + \frac{a}{2}W - aN_A = 0$$

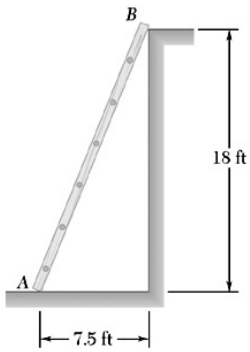
or $$aN_A - b\mu_s N_A = \frac{a}{2}W = \frac{a}{2}N_A\left(1 + \mu_s^2\right)$$

$$\mu_s^2 + \frac{2b}{a}\mu_s - 1 = 0$$

$$\mu_s = -\frac{b}{a} \pm \sqrt{\left(\frac{b}{a}\right)^2 + 1} = -2.4 \pm 2.6$$

The positive root is physically possible. Therefore, $\mu_s = 0.200$ ◀

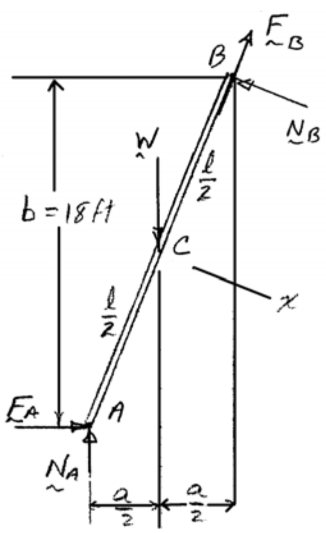
PROBLEM 8.22



A 19.5-ft ladder AB leans against a wall as shown. Assuming that the coefficient of static friction μ_s is the same at A and B, determine the smallest value of μ_s for which equilibrium is maintained.

SOLUTION

FBD ladder:



$$a = 7.5 \text{ ft}$$

$$l = 19.5 \text{ ft}$$

$$\frac{a}{l} = \frac{5}{13}$$

$$\frac{b}{l} = \frac{12}{13}$$

Motion impends at both A and B, so

$$F_A = \mu_s N_A \qquad \text{and} \qquad F_B = \mu_s N_B$$

$$\circlearrowleft \Sigma M_A = 0: \quad lN_B - \frac{a}{2}W = 0 \qquad \text{or} \qquad N_B = \frac{a}{2l}W = \frac{7.5 \text{ ft}}{39 \text{ ft}}W$$

or

$$N_B = \frac{2.5}{13}W$$

Then

$$F_B = \mu_s N_B = \mu_s \frac{2.5W}{13}$$

$$\longrightarrow \Sigma F_x = 0: \quad F_A + \frac{5}{13}F_B - \frac{12}{13}N_B = 0$$

$$\mu_s N_A + \frac{12.5}{(13)^2}\mu_s W - \frac{30}{(13)^2}W = 0$$

$$N_A - \frac{W}{(13)^2}\frac{(30 - 12.5\mu_s)}{\mu_s}$$

$$\uparrow \Sigma F_y = 0: \quad N_A - W + \frac{12}{13}F_B + \frac{5}{13}N_B = 0$$

$$\left(\frac{30 - 12.5\mu_s}{\mu_s} + 30\mu_s + 12.5\right)\frac{W}{(13)^2} = W$$

or

$$\mu_s^2 - 5.6333\mu_s + 1 = 0$$

$$\mu_s = 2.8167 \pm 2.6332$$

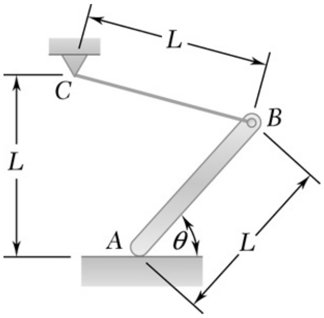
or

$$\mu_s = 0.1835 \qquad \text{and} \qquad \mu_s = 5.45$$

The larger value is very unlikely unless the surface is treated with some “non-skid” material.

In any event, the smallest value for equilibrium is $\mu_s = 0.1835$ ◀

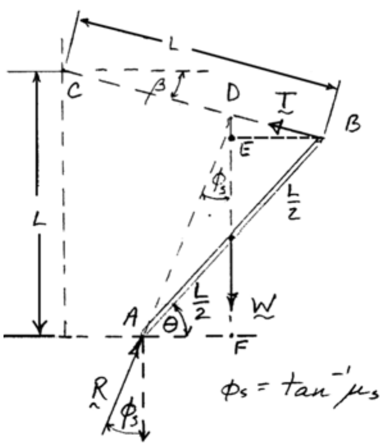
PROBLEM 8.23

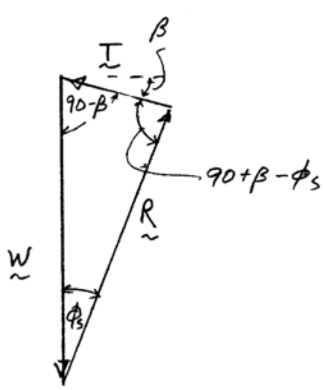


End A of a slender, uniform rod of weight W and length L bears on a horizontal surface as shown, while end B is supported by a cord BC of length L. Knowing that the coefficient of static friction is 0.40, determine (a) the value of θ for which motion is impending, (b) the corresponding value of the tension in the cord.

SOLUTION

FBD rod:





(a) Geometry: $BE = \dfrac{L}{2}\cos\theta \qquad DE = \left(\dfrac{L}{2}\cos\theta\right)\tan\beta$

$$EF = L\sin\theta \qquad DF = \frac{L}{2}\frac{\cos\theta}{\tan\phi_s}$$

So
$$L\left(\frac{1}{2}\cos\theta\tan\beta + \sin\theta\right) = \frac{L}{2}\frac{\cos\theta}{\tan\phi_s}$$

or
$$\tan\beta + 2\tan\theta = \frac{1}{\tan\phi_s} = \frac{1}{\mu_s} = \frac{1}{0.4} = 2.5 \qquad (1)$$

Also,
$$L\sin\theta + L\sin\beta = L$$

or
$$\sin\theta + \sin\beta = 1 \qquad (2)$$

Solving Eqs. (1) and (2) numerically $\theta_1 = 4.62° \qquad \beta_1 = 66.85°$

$$\theta_2 = 48.20° \qquad \beta_2 = 14.75°$$

Therefore, $\theta = 4.62°$ and $\theta = 48.2°$ ◀

(b) Now
$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.4 = 21.801°$$

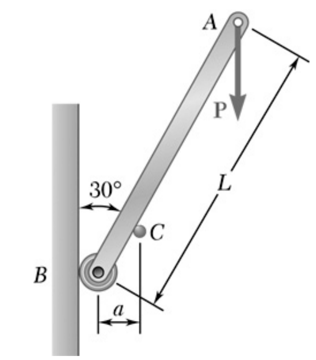
and
$$\frac{T}{\sin\phi_s} = \frac{W}{\sin(90 + \beta - \phi_s)}$$

or
$$T = W\frac{\sin\phi_s}{\sin(90 + \beta - \phi_s)}$$

For $\theta = 4.62° \qquad T = 0.526W$ ◀

$\theta = 48.2° \qquad T = 0.374W$ ◀

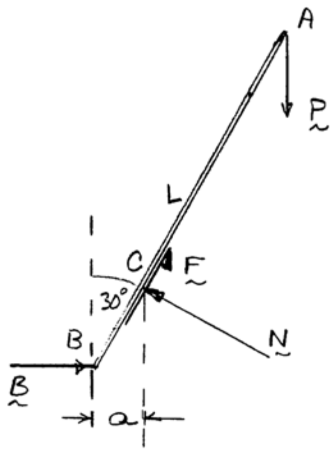
PROBLEM 8.24



A slender rod of length L is lodged between peg C and the vertical wall and supports a load **P** at end A. Knowing that the coefficient of static friction between the peg and the rod is 0.25 and neglecting friction at the roller, determine the range of values of the ratio L/a for which equilibrium is maintained.

SOLUTION

FBD rod:



$$\circlearrowleft \Sigma M_B = 0: \quad \frac{a}{\sin 30°} N - L \sin 30° P = 0$$

$$N = \frac{L}{a} \sin^2 30° P = \frac{L}{a} \frac{P}{4}$$

$$\text{Impending motion at } C\text{: } \left.\begin{array}{l} \text{down} \rightarrow F = \mu_s N \\ \text{up} \rightarrow F = -\mu_s N \end{array}\right\} F = \pm \frac{N}{4}$$

$$\uparrow \Sigma F_y = 0: \quad F \cos 30° + N \sin 30° - P = 0$$

$$\pm \frac{L}{a} \frac{P}{16} \frac{\sqrt{3}}{2} + \frac{L}{a} \frac{P}{4} \frac{1}{2} = P$$

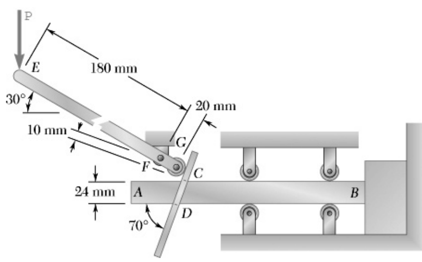
$$\frac{L}{a} \left[\frac{1}{8} \pm \frac{\sqrt{3}}{32} \right] = 1$$

$$\frac{L}{a} = \frac{32}{4 \pm \sqrt{3}}$$

or $\quad \dfrac{L}{a} = 5.583 \quad$ and $\quad \dfrac{L}{a} = 14.110$

For equilibrium: $\quad 5.58 \leq \dfrac{L}{a} \leq 14.11$ ◀

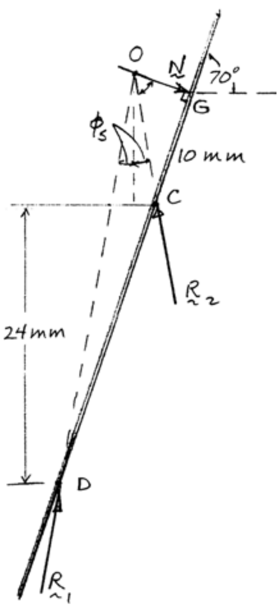
PROBLEM 8.25



The basic components of a clamping device are bar *AB*, locking plate *CD*, and lever *EFG*; the dimensions of the slot in *CD* are slightly larger than those of the cross section of *AB*. To engage the clamp, *AB* is pushed against the workpiece, and then force **P** is applied. Knowing that $P = 160$ N and neglecting the friction force between the lever and the plate, determine the smallest allowable value of the static coefficient of friction between the bar and the plate.

SOLUTION

FBD Plate:



DC is three-force member and motion impends at *C* and *D* (for minimum μ_s).

$$\measuredangle\, OCG = 20° + \phi_s \qquad \measuredangle\, ODG = 20° - \phi_s$$

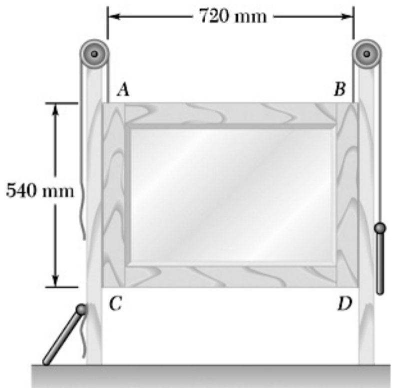
$$\overline{OG} = (10\text{ mm})\tan(20° + \phi_s) = \left(\frac{24\text{ mm}}{\sin 70°} + 10\text{ mm}\right)\tan(20° - \phi_s)$$

or $\qquad \tan(20° + \phi_s) = 3.5540\tan(20° - \phi_s)$

Solving numerically $\qquad \phi_s = 10.565°$

Now $\qquad \mu_s = \tan\phi_s$

so that $\qquad \mu_s = 0.1865$ ◀



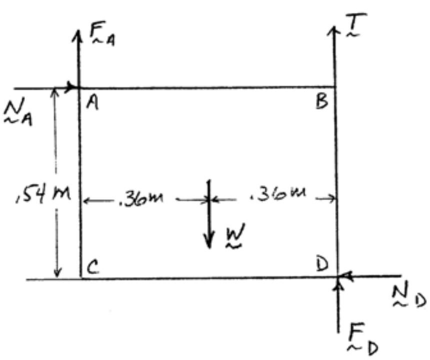
PROBLEM 8.26

A window sash having a mass of 4 kg is normally supported by two 2-kg sash weights. Knowing that the window remains open after one sash cord has broken, determine the smallest possible value of the coefficient of static friction. (Assume that the sash is slightly smaller that the frame and will bind only at points A and D.)

SOLUTION

FBD window:

$$T = (2\text{ kg})(9.81\text{ m/s}^2) = 19.62\text{ N} = \frac{W}{2}$$



$$W = (4\text{ kg})(9.81\text{ m/s}^2) = 39.24\text{ N}$$

$\longrightarrow \Sigma F_x = 0$: $\quad N_A - N_D = 0 \qquad N_A = N_D$

Impending motion: $\quad F_A = \mu_s N_A \qquad F_D = \mu_s N_D$

$\Sigma M_D = 0$: $\quad (0.36\text{ m})W - (0.54\text{ m})N_A - (0.72\text{ m})F_A = 0$

$$W = \frac{3}{2}N_A + 2\mu_s N_A$$

$$N_A = \frac{2W}{3 + 4\mu_s}$$

$\uparrow \Sigma F_y = 0$: $\quad F_A - W + T + F_D = 0$

$$F_A + F_D = W - T$$

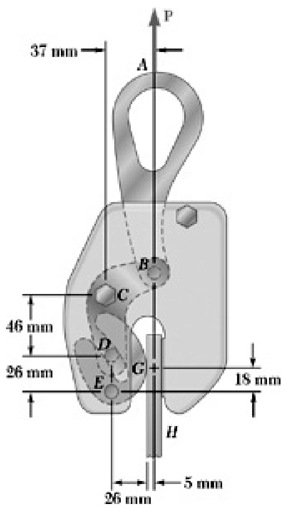
$$= \frac{W}{2}$$

Now $\quad F_A + F_D = \mu_s(N_A + N_D) = 2\mu_s N_A$

Then $\quad \dfrac{W}{2} = 2\mu_s \dfrac{2W}{3 + 4\mu_s}$

or $\quad \mu_s = 0.750$ ◀

PROBLEM 8.27



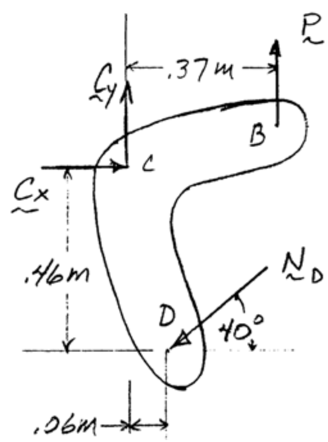
The steel-plate clamp shown is used to lift a steel plate H of mass 250 kg. Knowing that the normal force exerted on steel cam EG by pin D forms an angle of 40° with the horizontal and neglecting the friction force between the cam and the pin, determine the smallest allowable value of the coefficient of static friction.

SOLUTION

(Note: $\mathbf{P}$ is vertical as AB is two force member; also $P = W$ since clamp + plate is a two force FBD)

FBDs:

BCD:

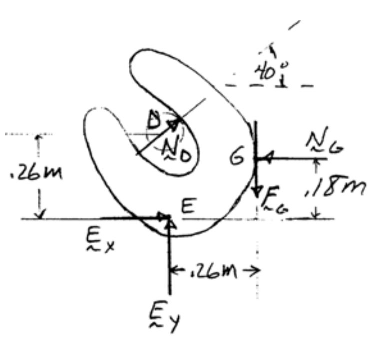


$$\circlearrowleft \Sigma M_C = 0: \quad (0.37\text{ m})P - (0.46\text{ m})N_D \cos 40^\circ$$

$$- (0.06\text{ m})N_D \sin 40^\circ = 0$$

or $$N_D = 0.94642P = 0.94642W$$

EG:



$$\circlearrowleft \Sigma M_E = 0: \quad (0.18\text{ m})N_G - (0.26\text{ m})F_G - (0.26\text{ m})N_D \cos 40^\circ = 0$$

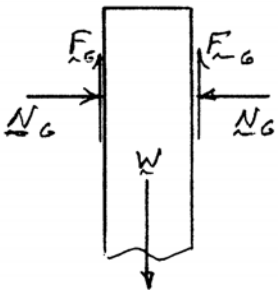
Impending motion: $$F_G = \mu_s N_G$$

Combining $$(18 + 26\mu_s)N_G = 19.9172N_D$$

$$= 18.850W$$

PROBLEM 8.27 CONTINUED

Plate:

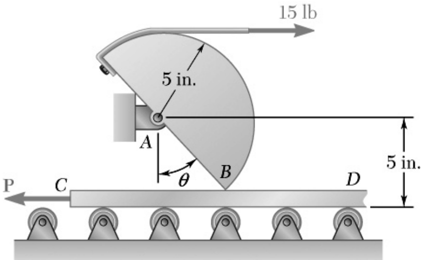


From plate: $F_G = \dfrac{W}{2}$ so that $N_G = \dfrac{W}{2\mu_s}$

Then $\left(18 + 26\mu_s\right)\dfrac{W}{2\mu_s} = 18.85W$

$\mu_s = 0.283$ ◀

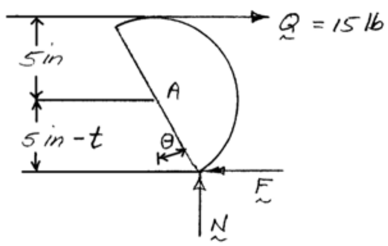
PROBLEM 8.28

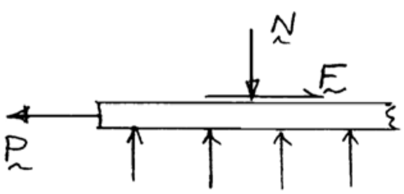


The 5-in.-radius cam shown is used to control the motion of the plate *CD*. Knowing that the coefficient of static friction between the cam and the plate is 0.45 and neglecting friction at the roller supports, determine (*a*) the force **P** for which motion of the plate is impending knowing that the plate is 1 in. thick, (*b*) the largest thickness of the plate for which the mechanism is self-locking, (that is, for which the plate cannot be moved however large the force **P** may be).

SOLUTION

FBDs:





From plate: $\xrightarrow{} \Sigma F_x = 0: \quad F - P = 0 \qquad F = P$

From cam geometry: $\cos\theta = \dfrac{5\text{ in.} - t}{5\text{ in.}}$

$$\circlearrowleft \Sigma M_A = 0: \quad \left[(5\text{ in.})\sin\theta\right]N - \left[(5\text{ in.})\cos\theta\right]F - (5\text{ in.})Q = 0$$

Impending motion: $F = \mu_s N$

So $N\sin\theta - \mu_s N\cos\theta = Q = 15\text{ lb}$

$$N = \frac{Q}{\sin\theta - \mu_s\cos\theta}$$

So $P = F = \mu_s N = \dfrac{\mu_s Q}{\sin\theta - \mu_s\cos\theta}$

(*a*) $t = 1\text{ in.} \Rightarrow \cos\theta = \dfrac{4\text{ in.}}{5\text{ in.}} = 0.8; \ \sin\theta = 0.6$

$$P = \frac{(0.45)(15\text{ lb})}{0.6 - (0.45)(0.8)} = 28.125\text{ lb}; \ \mathbf{P} = 28.1\text{ lb} \longleftarrow \blacktriangleleft$$

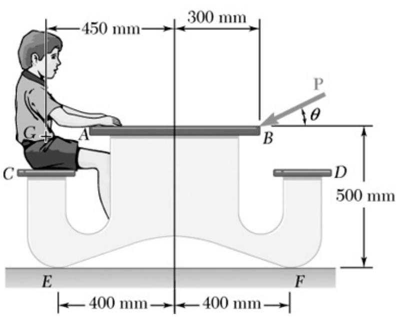
(*b*) $P \to \infty: \ \sin\theta - \mu_s\cos\theta = \dfrac{\mu_s Q}{P} \longrightarrow 0$

Thus $\tan\theta \to \mu_s = 0.45$ so that $\theta = 24.228°$

But $(5\text{ in.})\cos\theta = 5\text{ in.} - t$ or $t = (5\text{ in.})(1 - \cos\theta)$

$$t = 0.440\text{ in.} \blacktriangleleft$$

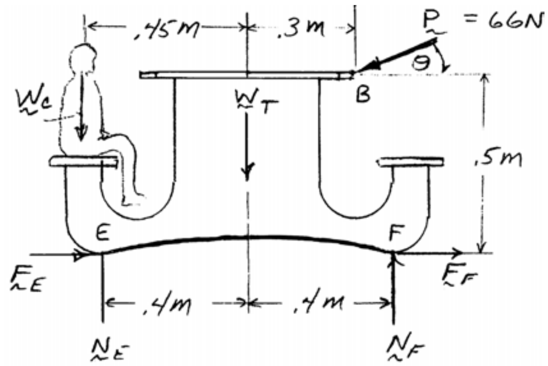
PROBLEM 8.29



A child having a mass of 18 kg is seated halfway between the ends of a small, 16-kg table as shown. The coefficient of static friction is 0.20 between the ends of the table and the floor. If a second child pushes on edge B of the table top at a point directly opposite to the first child with a force $\mathbf{P}$ lying in a vertical plane parallel to the ends of the table and having a magnitude of 66 N, determine the range of values of θ for which the table will (*a*) tip, (*b*) slide.

SOLUTION

FBD table + child:



$$W_C = 18\text{ kg}\left(9.81\text{ m/s}^2\right) = 176.58\text{ N}$$

$$W_T = 16\text{ kg}\left(9.81\text{ m/s}^2\right) = 156.96\text{ N}$$

(*a*) Impending tipping about E, $N_F = F_F = 0$, and

$$\circlearrowleft \Sigma M_E = 0: \quad (0.05\text{ m})(176.58\text{ N}) - (0.4\text{ m})(156.96\text{ N}) + (0.5\text{ m})P\cos\theta - (0.7\text{ m})P\sin\theta = 0$$

$$33\cos\theta - 46.2\sin\theta = 53.955$$

Solving numerically $\qquad \theta = -36.3° \quad \text{and} \quad \theta = -72.6°$

Therefore $\qquad -72.6° \le \theta \le -36.3°$ ◀

Impending tipping about F is not possible

(*b*) For impending slip: $\qquad F_E = \mu_s N_E = 0.2N_E \qquad F_F = \mu_s N_F = 0.2N_F$

$$\xrightarrow{} \Sigma F_x = 0: \quad F_E + F_F - P\cos\theta = 0 \quad \text{or} \quad 0.2\left(N_E + N_F\right) = (66\text{ N})\cos\theta$$

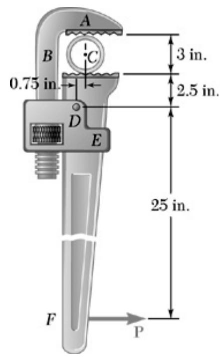
$$\uparrow \Sigma F_y = 0: \quad N_E + N_F - 176.58\text{ N} - 156.96\text{ N} - P\sin\theta = 0$$

$$N_E + N_F = \left(66\sin\theta + 333.54\right)\text{ N}$$

So $\qquad 330\cos\theta = 66\sin\theta + 333.54$

Solving numerically, $\qquad \theta = -3.66° \quad \text{and} \quad \theta = -18.96°$

Therefore, $\qquad -18.96° \le \theta \le -3.66°$ ◀

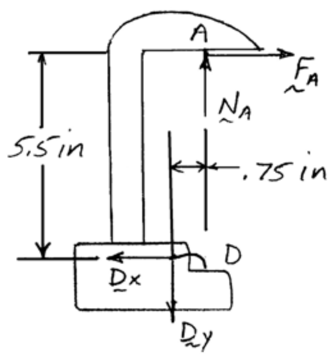


PROBLEM 8.30

A pipe of diameter 3 in. is gripped by the stillson wrench shown. Portions *AB* and *DE* of the wrench are rigidly attached to each other, and portion *CF* is connected by a pin at *D*. If the wrench is to grip the pipe and be self-locking, determine the required minimum coefficients of friction at *A* and *C*.

SOLUTION

FBD ABD:



$$\circlearrowleft \Sigma M_D = 0: \quad (0.75\text{ in.})N_A - (5.5\text{ in.})F_A = 0$$

Impending motion: $F_A = \mu_A N_A$

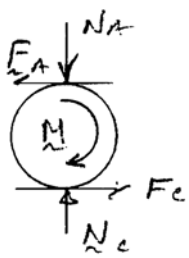
Then $0.75 - 5.5\mu_A = 0$

or $\mu_A = 0.13636$

$$\mu_A = 0.1364 \blacktriangleleft$$

$$\rightarrow \Sigma F_x = 0: \quad F_A - D_x = 0 \qquad D_x = F_A$$

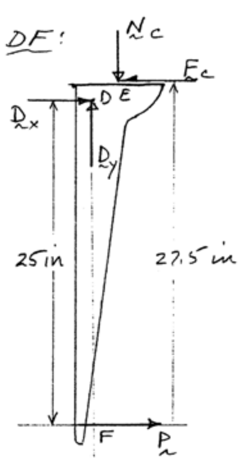
Pipe:



$$\uparrow \Sigma F_y = 0: \quad N_C - N_A = 0$$

$$N_C = N_A$$

FBD DF:



$$\circlearrowleft \Sigma M_F = 0: \quad (27.5\text{ in.})F_C - (0.75\text{ in.})N_C - (25\text{ in.})D_x = 0$$

Impending motion: $F_C = \mu_C N_C$

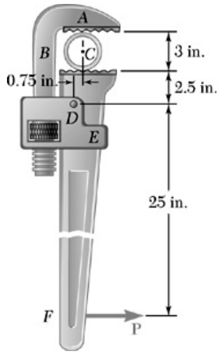
Then $27.5\mu_C - 0.75 = 25\dfrac{F_A}{N_C}$

But $N_C = N_A$ and $\dfrac{F_A}{N_A} = \mu_A = 0.13636$

So $27.5\mu_C = 0.75 + 25(0.13636)$

$$\mu_C = 0.1512 \blacktriangleleft$$

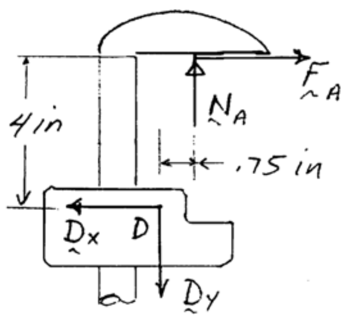
PROBLEM 8.31



Solve Problem 8.30 assuming that the diameter of the pipe is 1.5 in.

SOLUTION

FBD ABD:



$$\circlearrowleft \Sigma M_D = 0: \quad (0.75 \text{ in.})N_A - (4 \text{ in.})F_A = 0$$

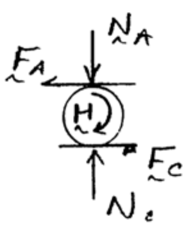
Impending motion: $F_A = \mu_A N_A$

Then $0.75 \text{ in.} - (4 \text{ in.})\mu_A = 0$ $\mu_A = 0.1875$ ◀

$$\xrightarrow{} \Sigma F_x = 0: \quad F_A - D_x = 0$$

so that $D_x = F_A = 0.1875 N_A$

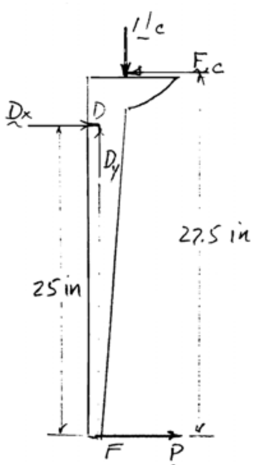
FBD Pipe:



$$\uparrow \Sigma F_y = 0: \quad N_C - N_A = 0$$

$$N_C = N_A$$

FBD DF:



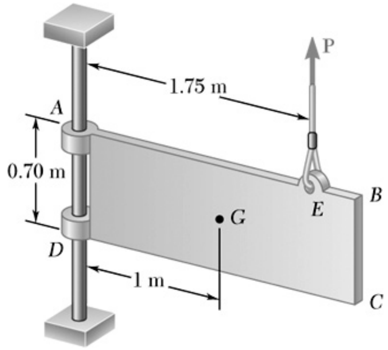
$$\circlearrowleft \Sigma M_F = 0: \quad (27.5 \text{ in.})F_C - (0.75 \text{ in.})N_C - (25 \text{ in.})D_x = 0$$

Impending motion: $F_C = \mu_C N_C$

$$27.5\mu_C - 0.75 = 25(0.1875)\frac{N_A}{N_C}$$

But $N_A = N_C$ (from pipe *FBD*) so $\frac{N_A}{N_C} = 1$

and $\mu_C = 0.1977$ ◀

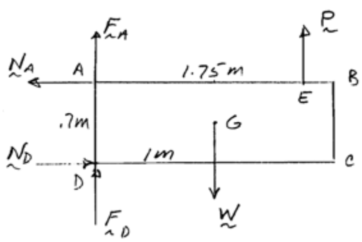


PROBLEM 8.32

The 25-kg plate *ABCD* is attached at *A* and *D* to collars which can slide on the vertical rod. Knowing that the coefficient of static friction is 0.40 between both collars and the rod, determine whether the plate is in equilibrium in the position shown when the magnitude of the vertical force applied at *E* is (*a*) $P = 0$, (*b*) $P = 80$ N.

SOLUTION

FBD plate:



$$W = 25\text{ kg}(9.81\text{ N/kg})$$

$$= 245.25\text{ N}$$

(*a*) $P = 0$; assume equilibrium:

$$\circlearrowleft \Sigma M_A = 0: \quad (0.7\text{ m})N_D - (1\text{ m})W = 0 \qquad N_D = \frac{10W}{7}$$

$$\longrightarrow \Sigma F_x = 0: \quad N_D - N_A = 0 \qquad N_A = N_D = \frac{10W}{7}$$

$$(F_A)_{\max} = \mu_s N_A \qquad (F_D)_{\max} = \mu_s N_D$$

So $$(F_A + F_D)_{\max} = \mu_s(N_A + N_D) = \frac{20\mu_s W}{7} = 1.143W$$

$$\uparrow \Sigma F_y = 0: \quad F_A + F_D - W = 0$$

$$\therefore \quad F_A + F_D = W < (F_A + F_D)_{\max} \qquad \text{OK.}$$

Plate is in equilibrium ◀

(*b*) $P = 80$ N; assume equilibrium:

$$\circlearrowleft \Sigma M_A = 0: \quad (1.75\text{ m})P + (0.7\text{ m})N_D - (1\text{ m})W = 0$$

or $$N_D = \frac{W - 1.75P}{0.7}$$

$$\longrightarrow \Sigma F_x = 0: \quad N_D - N_A = 0 \qquad N_D = N_A = \frac{W - 1.75P}{0.7}$$

$$(F_A)_{\max} = \mu_s N_A \qquad (F_B)_{\max} = \mu_s N_B$$

So $$(F_A + F_B)_{\max} = 0.4\frac{2W - 3.5P}{0.7} = 120.29\text{ N}$$

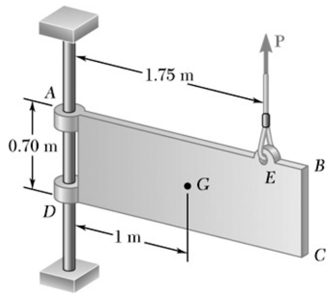
$$\uparrow \Sigma F_y = 0: \quad F_A + F_D - W + P = 0$$

$$F_A + F_D = W - P = 165.25\text{ N}$$

$$(F_A + F_D)_{\text{equil}} > (F_A + F_D)_{\max}$$

Impossible, so plate slides downward ◀

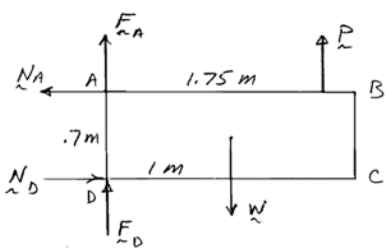
PROBLEM 8.33



In Problem 8.32, determine the range of values of the magnitude P of the vertical force applied at E for which the plate will move downward.

SOLUTION

FBD plate:



$$W = (25 \text{ kg})(9.81 \text{ m/s}^2)$$

$$= 245.25 \text{ N}$$

$$\circlearrowleft \Sigma M_A = 0: \quad (0.7 \text{ m})N_D - (1 \text{ m})W + (1.75 \text{ m})P = 0$$

$$N_D = \frac{W - 1.75P}{0.7}$$

$$\longrightarrow \Sigma F_x = 0: \quad N_D - N_A = 0 \quad \text{so that} \quad N_A = N_D = \frac{W - 1.75P}{0.7}$$

Note: N_A and N_D will be > 0 if $P < \frac{4}{7}W$ and < 0 if $P > \frac{4}{7}W$.

Impending motion downward: F_A and F_B are both > 0, so

$$F_A = \mu_s |N_A| = \frac{0.4}{0.7}|W - 1.75P| = \left|\frac{4}{7}W - P\right|$$

$$F_D = \mu_S |N_D| = \left|\frac{4}{7}W - P\right|$$

$$\uparrow \Sigma F_y = 0: \quad F_A + F_D - W + P = 0$$

$$2\left|\frac{4}{7}W - P\right| - W + P = 0$$

For $P < \frac{4}{7}W$; $\qquad P = \frac{W}{7} = 35.04 \text{ N}$

For $P > \frac{4}{7}W$; $\qquad P = \frac{5W}{7} = 175.2 \text{ N}$

Downward motion for $35.0 \text{ N} < P < 175.2 \text{ N}$ ◀

Alternative Solution

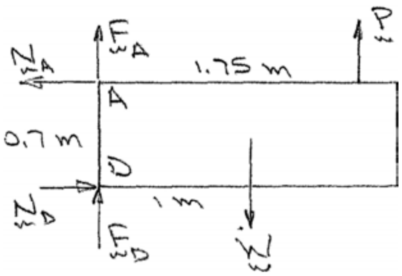
We first observe that for smaller values of the magnitude of **P** that (Case 1) the inner left-hand and right-hand surfaces of collars A and D, respectively, will contact the rod, whereas for larger values of the magnitude of **P** that (Case 2) the inner right-hand and left-hand surfaces of collars A and D, respectively, will contact the rod.

First note: $\qquad W = (25 \text{ kg})(9.81 \text{ m/s}^2)$

$$= 245.25 \text{ N}$$

PROBLEM 8.33 CONTINUED

Case 1



$$\circlearrowleft \Sigma M_D = 0: \quad (0.7 \text{ m})N_A - (1 \text{ m})(245.25 \text{ N}) + (1.75 \text{ m})P = 0$$

or $$N_A = \frac{10}{7}\left(245.25 - \frac{7}{4}P\right) \text{ N}$$

$$\rightarrow \Sigma F_x = 0: \quad -N_A + N_D = 0$$

or $$N_D = N_A$$

$$\uparrow \Sigma F_y = 0: \quad F_A + F_D + P - 245.25 \text{ N} = 0$$

or $$F_A + F_D = (245.25 - P) \text{ N}$$

Now $$(F_A)_{\max} = \mu_s N_A \qquad (F_D)_{\max} = \mu_s N_D$$

so that $$(F_A)_{\max} + (F_D)_{\max} = \mu_s (N_A + N_D)$$

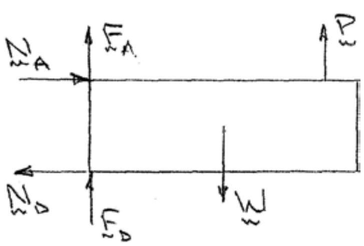
$$= 2(0.4)\left[\frac{10}{7}\left(245.25 - \frac{7}{4}P\right)\right]$$

For motion: $$F_A + F_D > (F_A)_{\max} + (F_D)_{\max}$$

Substituting $$245.25 - P > \frac{8}{7}\left(245.25 - \frac{7}{4}P\right)$$

or $$P > 35.0 \text{ N}$$

Case 2



From Case 1: $$N_D = N_A$$

$$F_A + F_D = (245.25 - P) \text{ N}$$

$$(F_A)_{\max} + (F_D)_{\max} = 2\mu_s N_A$$

$$\circlearrowleft \Sigma M_D = 0: \quad -(0.7 \text{ m})N_A - (1 \text{ m})(245.25 \text{ N}) + (1.75 \text{ m})P = 0$$

or $$N_A = \frac{10}{7}\left(\frac{7}{4}P - 245.25\right) \text{ N}$$

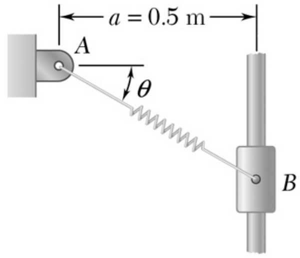
For motion: $$F_A + F_D > (F_A)_{\max} + (F_D)_{\max}$$

Substituting: $$245.25 - P > 2(0.4)\left[\frac{10}{7}\left(\frac{7}{4}P - 245.25\right)\right]$$

or $$P < 175.2 \text{ N}$$

Therefore, have downward motion for $35.0 \text{ N} < P < 175.2 \text{ N}$ ◀

PROBLEM 8.34

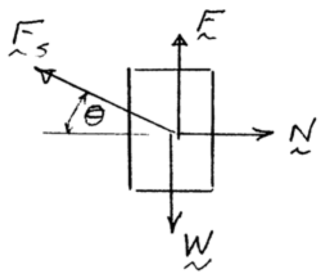


A collar B of weight W is attached to the spring AB and can move along the rod shown. The constant of the spring is 1.5 kN/m and the spring is unstretched when $\theta = 0$. Knowing that the coefficient of static friction between the collar and the rod is 0.40, determine the range of values of W for which equilibrium is maintained when (*a*) $\theta = 20^\circ$, (*b*) $\theta = 30^\circ$.

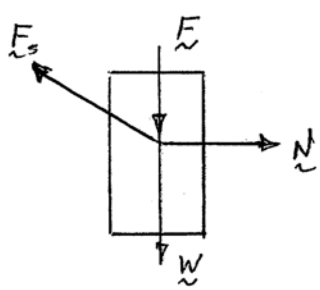
SOLUTION

FBD collar:

Impending motion down:



Impending motion up:



Stretch of spring $x = \overline{AB} - a = \dfrac{a}{\cos\theta} - a$

$$F_s = kx = k\left(\frac{a}{\cos\theta} - a\right) = (1.5 \text{ kN/m})(0.5 \text{ m})\left(\frac{1}{\cos\theta} - 1\right)$$

$$= (0.75 \text{ kN})\left(\frac{1}{\cos\theta} - 1\right)$$

$\longrightarrow \Sigma F_x = 0$: $\quad N - F_s\cos\theta = 0$

$$N = F_s\cos\theta = (0.75 \text{ kN})(1 - \cos\theta)$$

Impending slip: $\quad F = \mu_s N = (0.4)(0.75 \text{ kN})(1 - \cos\theta)$

$$= (0.3 \text{ kN})(1 - \cos\theta)$$

$+$ down, $-$ up

$\uparrow \Sigma F_y = 0$: $\quad F_s\sin\theta \pm F - W = 0$

$$(0.75 \text{ kN})(\tan\theta - \sin\theta) \pm (0.3 \text{ kN})(1 - \cos\theta) - W = 0$$

or $\quad W = (0.3 \text{ kN})[2.5(\tan\theta - \sin\theta) \pm (1 - \cos\theta)]$

(*a*) $\theta = 20°$: $\quad W_{\text{up}} = -0.00163 \text{ kN} \quad$ (impossible)

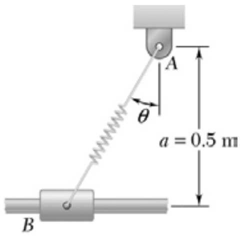
$$W_{\text{down}} = 0.03455 \text{ kN} \quad (\text{OK})$$

Equilibrium if $0 \le W \le 34.6$ N ◀

(*b*) $\theta = 30°$: $\quad W_{\text{up}} = 0.01782 \text{ kN} \quad$ (OK)

$$W_{\text{down}} = 0.0982 \text{ kN} \quad (\text{OK})$$

Equilibrium if $17.82 \text{ N} \le W \le 98.2$ N ◀

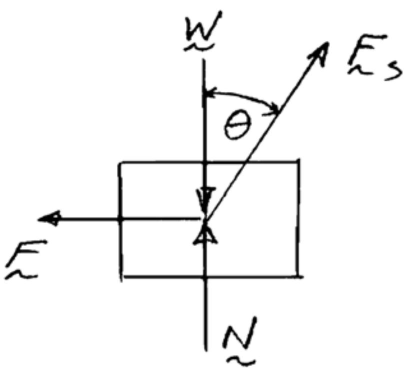


PROBLEM 8.35

A collar B of weight W is attached to the spring AB and can move along the rod shown. The constant of the spring is 1.5 kN/m and the spring is unstretched when $\theta = 0$. Knowing that the coefficient of static friction between the collar and the rod is 0.40, determine the range of values of W for which equilibrium is maintained when (*a*) $\theta = 20^\circ$, (*b*) $\theta = 30^\circ$.

SOLUTION

FBD collar:



Stretch of spring $x = \overline{AB} - a = \dfrac{a}{\cos\theta} - a$

$$F_s = k\left(\frac{a}{\cos\theta} - a\right) = (1.5\text{ kN/m})(0.5\text{ m})\left(\frac{1}{\cos\theta} - 1\right)$$

$$= (0.75\text{ kN})\left(\frac{1}{\cos\theta} - 1\right) = (750\text{ N})(\sec\theta - 1)$$

$$\uparrow \Sigma F_y = 0: \quad F_s\cos\theta - W + N = 0$$

or $$W = N + (750\text{ N})(1 - \cos\theta)$$

Impending slip:

$$F = \mu_s|N| \quad (F \text{ must be } +, \text{ but } N \text{ may be positive or negative})$$

$$\longrightarrow \Sigma F_x = 0: \quad F_s\sin\theta - F = 0$$

or $$F = F_s\sin\theta = (750\text{ N})(\tan\theta - \sin\theta)$$

(*a*) $\theta = 20^\circ$: $\quad F = (750\text{ N})(\tan 20^\circ - \sin 20^\circ) = 16.4626\text{ N}$

Impending motion: $\quad |N| = \dfrac{F}{\mu_s} = \dfrac{16.4626\text{ N}}{0.4} = 41.156\text{ N}$

(Note: for $|N| < 41.156$ N, motion will occur, equilibrium for $|N| > 41.156$)

But $\quad W = N + (750\text{ N})(1 - \cos 20^\circ) = N + 45.231\text{ N}$

So equilibrium for $W \le 4.07$ N and $W \ge 86.4$ N ◀

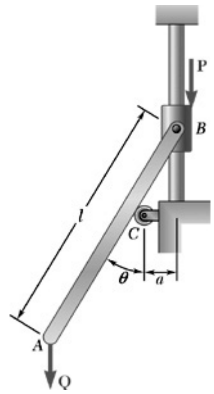
(*b*) $\theta = 30^\circ$: $\quad F = (750\text{ N})(\tan 30^\circ - \sin 30^\circ) = 58.013\text{ N}$

Impending motion: $\quad |N| = \dfrac{F}{\mu_s} = \dfrac{58.013}{0.4} = 145.032\text{ N}$

$$W = N + (750\text{ N})(1 - \cos 30^\circ) = N \pm 145.03\text{ N}$$

$$= -44.55\text{ N}(\text{impossible}),\ 245.51\text{ N}$$

Equilibrium for $W \ge 246$ N ◀

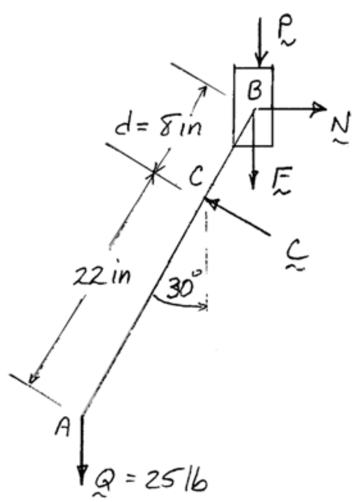


PROBLEM 8.36

The slender rod AB of length $l = 30$ in. is attached to a collar at B and rests on a small wheel located at a horizontal distance $a = 4$ in. from the vertical rod on which the collar slides. Knowing that the coefficient of static friction between the collar and the vertical rod is 0.25 and neglecting the radius of the wheel, determine the range of values of P for which equilibrium is maintained when $Q = 25$ lb and $\theta = 30^\circ$.

SOLUTION

FBD rod + collar:



Note: $d = \dfrac{a}{\sin\theta} = \dfrac{4\text{ in.}}{\sin 30^\circ} = 8\text{ in.},$ so $AC = 22$ in.

Neglect weights of rod and collar.

$$\circlearrowleft \Sigma M_B = 0: \quad (30\text{ in.})(\sin 30^\circ)(25\text{ lb}) - (8\text{ in.})C = 0$$

$$C = 46.875\text{ lb}$$

$$\longrightarrow \Sigma F_x = 0: \quad N - C\cos 30^\circ = 0$$

$$N = (46.875\text{ lb})\cos 30^\circ = 40.595\text{ lb}$$

Impending motion up: $\quad F = \mu_s N = 0.25(40.595\text{ lb})$

$$= 10.149\text{ lb}$$

$$\uparrow \Sigma F_y = 0: \quad -25\text{ lb} + (46.875\text{ lb})\sin 30^\circ - P - 10.149\text{ lb} = 0$$

or $\quad P = -1.563\text{ lb} - 10.149\text{ lb} = -11.71\text{ lb}$

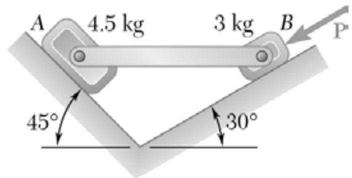
Impending motion down: Direction of **F** is now upward, but still have

$$|F| = \mu_s N = 10.149\text{ lb}$$

$$\uparrow \Sigma F_y = 0: \quad -25\text{ lb} + (46.875\text{ lb})\sin 30^\circ - P + 10.149\text{ lb} = 0$$

or $\quad P = -1.563\text{ lb} + 10.149\text{ lb} = 8.59\text{ lb}$

$\therefore$ Equilibrium for $-11.71\text{ lb} \le P \le 8.59\text{ lb}$ ◀



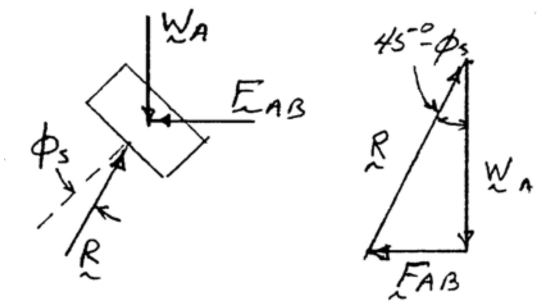
PROBLEM 8.37

The 4.5-kg block A and the 3-kg block B are connected by a slender rod of negligible mass. The coefficient of static friction is 0.40 between all surfaces of contact. Knowing that for the position shown the rod is horizontal, determine the range of values of P for which equilibrium is maintained.

SOLUTION

FBDs: Note: $\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.4 = 21.801°$

(a) Block A impending slip ↘

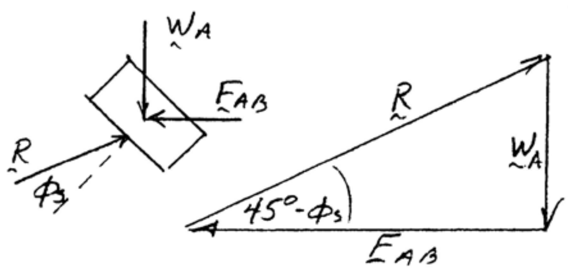


$$F_{AB} = W_A \tan(45° - \phi_s)$$

$$= (4.5\text{ kg})(9.81\text{ m/s}^2)\tan(23.199°)$$

$$= 18.9193\text{ N}$$

(b) Block A impending slip ↖

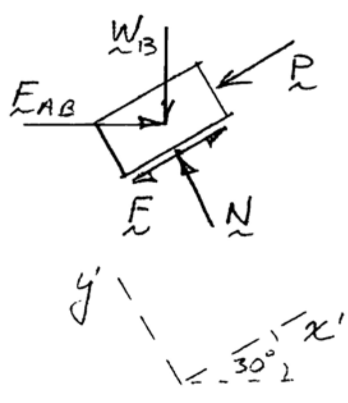


$$F_{AB} = W_A \operatorname{ctn}(45° - \phi_s)$$

$$= (4.5\text{ kg})(9.81\text{ m/s}^2)\operatorname{ctn}(23.199°)$$

$$= 103.005\text{ N}$$

Block B:



$$W_B = (3\text{ kg})(9.81\text{ m/s}^2)$$

$$= 29.43\text{ N}$$

From Block B: ↖ $\Sigma F_{y'} = 0$: $N - W_B \cos 30° - F_{AB} \sin 30° = 0$

PROBLEM 8.37 CONTINUED

Case (a) $$N = (29.43\text{ N})\cos 30° + (18.9193\text{ N})\sin 30° = 34.947\text{ N}$$

Impending motion: $$F = \mu_s N = 0.4(34.947\text{ N}) = 13.979\text{ N}$$

$$\nearrow \Sigma F_{x'} = 0: \quad F_{AB}\cos 30° - W_B \sin 30° - 13.979\text{ N} - P = 0$$

$$P = (18.9193\text{ N})\cos 30° - (29.43\text{ N})\sin 30° - 13.979\text{ N}$$

$$= -12.31\text{ N}$$

Case (b) $$N = (29.43\text{ N})\cos 30° + (103.005\text{ N})\sin 30° = 76.9896\text{ N}$$

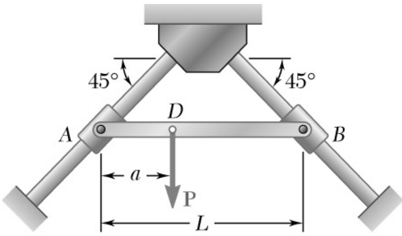
Impending motion: $$F = 0.4(76.9896\text{ N}) = 30.7958\text{ N}$$

$$\nearrow \Sigma F_{x'} = 0: \quad (103.005\text{ N})\cos 30° - (29.43\text{ N})\sin 30° + 30.7958\text{ N} - P = 0$$

$$P = 105.3\text{ N}$$

For equilibrium $-12.31\text{ N} \le P \le 105.3\text{ N}$ ◀

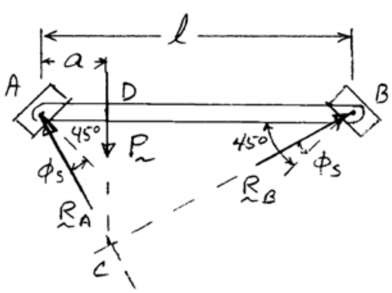
PROBLEM 8.38



Bar AB is attached to collars which can slide on the inclined rods shown. A force **P** is applied at point *D* located at a distance *a* from end *A*. Knowing that the coefficient of static friction μ_s between each collar and the rod upon which it slides is 0.30 and neglecting the weights of the bar and of the collars, determine the smallest value of the ratio *a/L* for which equilibrium is maintained.

SOLUTION

FBD bar + collars:



Impending motion

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.3 = 16.6992°$$

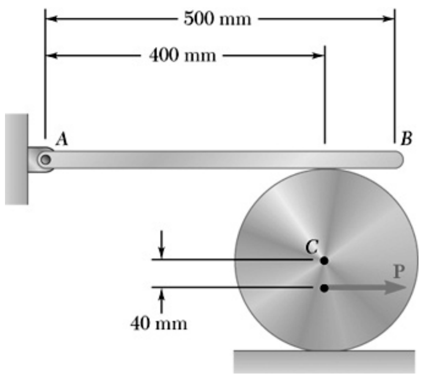
Neglect weights: 3-force *FBD* and $\measuredangle ACB = 90°$

So $$AC = \frac{a}{\cos(45° + \phi_s)} = l\sin(45° - \phi_s)$$

$$\frac{a}{l} = \sin(45° - 16.6992°)\cos(45° + 16.6992°)$$

$$\frac{a}{l} = 0.225 \blacktriangleleft$$

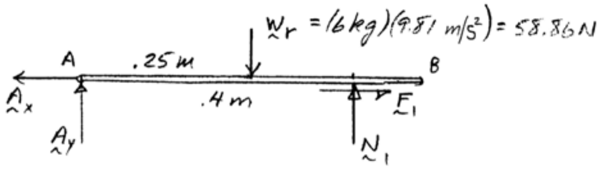
PROBLEM 8.39



The 6-kg slender rod AB is pinned at A and rests on the 18-kg cylinder C. Knowing that the diameter of the cylinder is 250 mm and that the coefficient of static friction is 0.35 between all surfaces of contact, determine the largest magnitude of the force **P** for which equilibrium is maintained.

SOLUTION

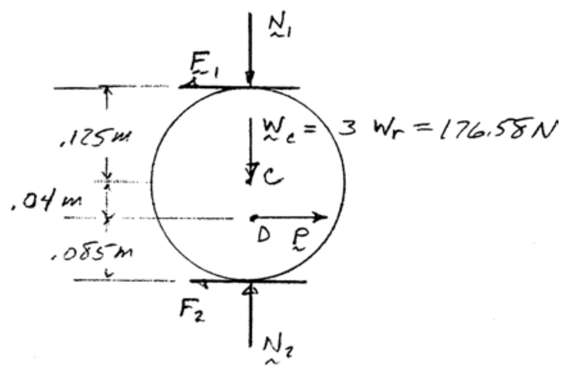
FBD rod:



$$\circlearrowleft \Sigma M_A = 0: \quad (0.4 \text{ m})N_1 - (0.25 \text{ m})W_r = 0$$

$$N_1 = 0.625W_r = 36.7875 \text{ N}$$

FBD cylinder:



Cylinder:

$$\uparrow \Sigma F_y = 0: \quad N_2 - N_1 - W_C = 0 \qquad \text{or} \qquad N_2 = 0.625W_r + 3W_r = 3.625W_r = 5.8N_1$$

$$\circlearrowleft \Sigma M_D = 0: \quad (0.165 \text{ m})F_1 - (0.085 \text{ m})F_2 = 0 \qquad \text{or} \qquad F_2 = 1.941F_1$$

Since $\mu_{s1} = \mu_{s2}$, motion will impend first at top of the cylinder

So $$F_1 = \mu_s N_1 = 0.35(36.7875 \text{ N}) = 12.8756 \text{ N}$$

and $$F_2 = 1.941(12.8756 \text{ N}) = 24.992 \text{ N}$$

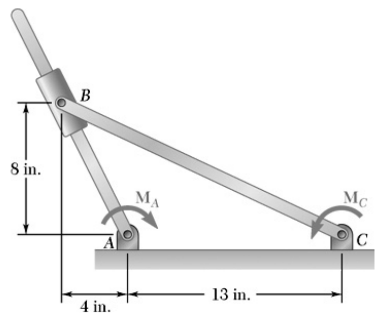
$$\left[\text{Check } F_2 = 25 \text{ N} < \mu_S N_2 = 74.7 \text{ N} \qquad \text{OK}\right]$$

$$\longrightarrow \Sigma F_x = 0: \quad P - F_1 - F_2 = 0$$

or $$P = 12.8756 \text{ N} + 24.992 \text{ N}$$

or $P = 37.9 \text{ N}$ ◀

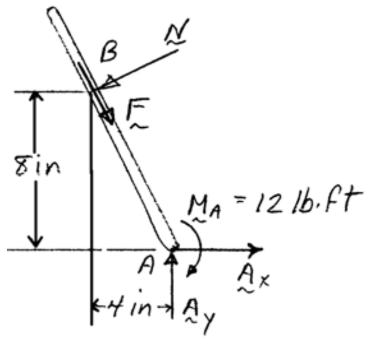
PROBLEM 8.40



Two rods are connected by a collar at B. A couple $\mathbf{M}_A$ of magnitude $12\text{ lb}\cdot\text{ft}$ is applied to rod AB. Knowing that $\mu_s = 0.30$ between the collar and rod AB, determine the largest couple $\mathbf{M}_C$ for which equilibrium will be maintained.

SOLUTION

FBD AB:



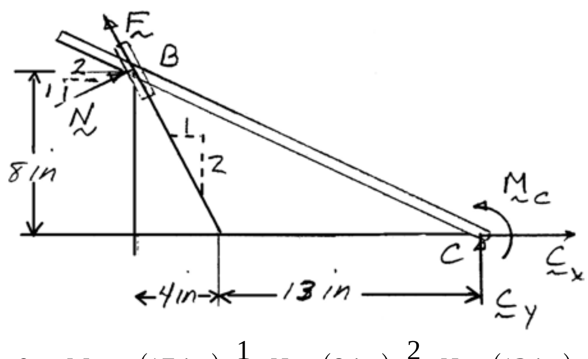
$$\circlearrowleft \Sigma M_A = 0: \quad \sqrt{8\text{ in}^2 + 4\text{ in}^2}\,(N) - M_A = 0$$

$$N = \frac{(12\text{ lb}\cdot\text{ft})(12\text{ in./ft})}{8.9443\text{ in.}} = 16.100\text{ lb}$$

Impending motion: $\qquad F = \mu_s N = 0.3(16.100\text{ lb}) = 4.83\text{ lb}$

(Note: For max, M_C, need F in direction shown; see FBD BC.)

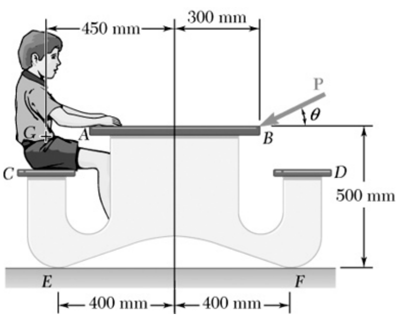
FBD BC + collar:



$$\circlearrowleft \Sigma M_C = 0: \quad M_C - (17\text{ in.})\frac{1}{\sqrt{5}}N - (8\text{ in.})\frac{2}{\sqrt{5}}N - (13\text{ in.})\frac{2}{\sqrt{5}}F = 0$$

or $$M_C = \frac{17\text{ in.}}{\sqrt{5}}(16.100\text{ lb}) + \frac{16\text{ in.}}{\sqrt{5}}(16.100\text{ lb}) + \frac{26\text{ in.}}{\sqrt{5}}(4.830\text{ lb}) = 293.77\text{ lb}\cdot\text{in.}$$

$$(\mathbf{M}_C)_{\max} = 24.5\text{ lb}\cdot\text{ft} \circlearrowright \blacktriangleleft$$

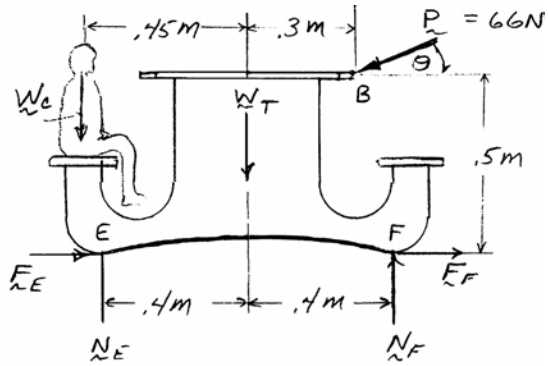


PROBLEM 8.29

A child having a mass of 18 kg is seated halfway between the ends of a small, 16-kg table as shown. The coefficient of static friction is 0.20 between the ends of the table and the floor. If a second child pushes on edge B of the table top at a point directly opposite to the first child with a force $\mathbf{P}$ lying in a vertical plane parallel to the ends of the table and having a magnitude of 66 N, determine the range of values of θ for which the table will (*a*) tip, (*b*) slide.

SOLUTION

FBD table + child:



$$W_C = 18\text{ kg}\left(9.81\text{ m/s}^2\right) = 176.58\text{ N}$$

$$W_T = 16\text{ kg}\left(9.81\text{ m/s}^2\right) = 156.96\text{ N}$$

(*a*) Impending tipping about E, $N_F = F_F = 0$, and

$$\circlearrowleft \Sigma M_E = 0: \quad (0.05\text{ m})(176.58\text{ N}) - (0.4\text{ m})(156.96\text{ N}) + (0.5\text{ m})P\cos\theta - (0.7\text{ m})P\sin\theta = 0$$

$$33\cos\theta - 46.2\sin\theta = 53.955$$

Solving numerically $\qquad \theta = -36.3° \quad \text{and} \quad \theta = -72.6°$

Therefore $\qquad -72.6° \le \theta \le -36.3°$ ◀

Impending tipping about F is not possible

(*b*) For impending slip: $\qquad F_E = \mu_s N_E = 0.2N_E \qquad F_F = \mu_s N_F = 0.2N_F$

$$\longrightarrow \Sigma F_x = 0: \quad F_E + F_F - P\cos\theta = 0 \quad \text{or} \quad 0.2\left(N_E + N_F\right) = (66\text{ N})\cos\theta$$

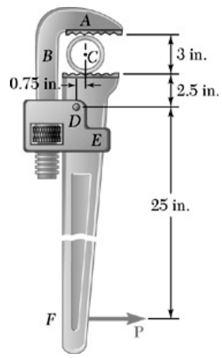
$$\uparrow \Sigma F_y = 0: \quad N_E + N_F - 176.58\text{ N} - 156.96\text{ N} - P\sin\theta = 0$$

$$N_E + N_F = \left(66\sin\theta + 333.54\right)\text{ N}$$

So $\qquad 330\cos\theta = 66\sin\theta + 333.54$

Solving numerically, $\qquad \theta = -3.66° \quad \text{and} \quad \theta = -18.96°$

Therefore, $\qquad -18.96° \le \theta \le -3.66°$ ◀

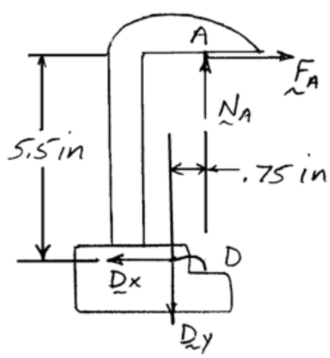


PROBLEM 8.30

A pipe of diameter 3 in. is gripped by the stillson wrench shown. Portions *AB* and *DE* of the wrench are rigidly attached to each other, and portion *CF* is connected by a pin at *D*. If the wrench is to grip the pipe and be self-locking, determine the required minimum coefficients of friction at *A* and *C*.

SOLUTION

FBD ABD:



$$\circlearrowleft \Sigma M_D = 0: \quad (0.75\text{ in.})N_A - (5.5\text{ in.})F_A = 0$$

Impending motion: $\quad F_A = \mu_A N_A$

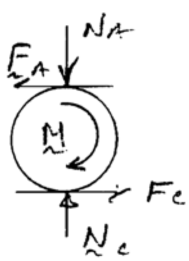
Then $\quad 0.75 - 5.5\mu_A = 0$

or $\quad \mu_A = 0.13636$

$$\mu_A = 0.1364 \blacktriangleleft$$

$$\rightarrow \Sigma F_x = 0: \quad F_A - D_x = 0 \qquad D_x = F_A$$

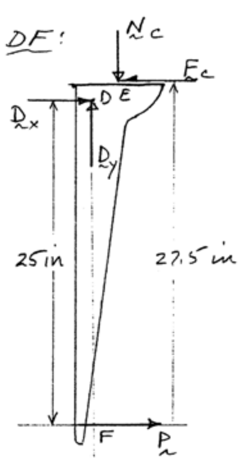
Pipe:



$$\uparrow \Sigma F_y = 0: \quad N_C - N_A = 0$$

$$N_C = N_A$$

FBD DF:



$$\circlearrowleft \Sigma M_F = 0: \quad (27.5\text{ in.})F_C - (0.75\text{ in.})N_C - (25\text{ in.})D_x = 0$$

Impending motion: $\quad F_C = \mu_C N_C$

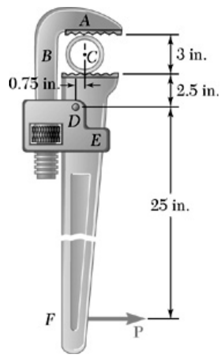
Then $\quad 27.5\mu_C - 0.75 = 25\dfrac{F_A}{N_C}$

But $\quad N_C = N_A \quad$ and $\quad \dfrac{F_A}{N_A} = \mu_A = 0.13636$

So $\quad 27.5\mu_C = 0.75 + 25(0.13636)$

$$\mu_C = 0.1512 \blacktriangleleft$$

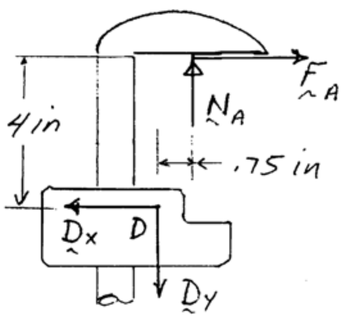
PROBLEM 8.31



Solve Problem 8.30 assuming that the diameter of the pipe is 1.5 in.

SOLUTION

FBD ABD:



$$\circlearrowleft \Sigma M_D = 0: \quad (0.75 \text{ in.})N_A - (4 \text{ in.})F_A = 0$$

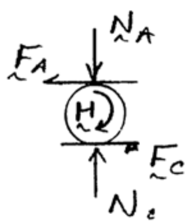
Impending motion: $\qquad F_A = \mu_A N_A$

Then $\qquad 0.75 \text{ in.} - (4 \text{ in.})\mu_A = 0 \qquad \mu_A = 0.1875$ ◀

$$\xrightarrow{} \Sigma F_x = 0: \quad F_A - D_x = 0$$

so that $\qquad D_x = F_A = 0.1875 N_A$

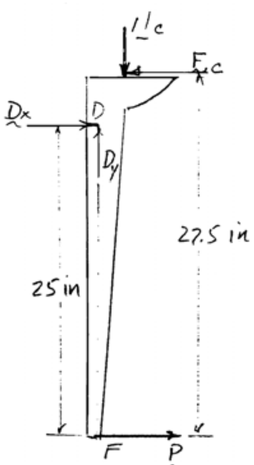
FBD Pipe:



$$\uparrow \Sigma F_y = 0: \quad N_C - N_A = 0$$

$$N_C = N_A$$

FBD DF:



$$\circlearrowleft \Sigma M_F = 0: \quad (27.5 \text{ in.})F_C - (0.75 \text{ in.})N_C - (25 \text{ in.})D_x = 0$$

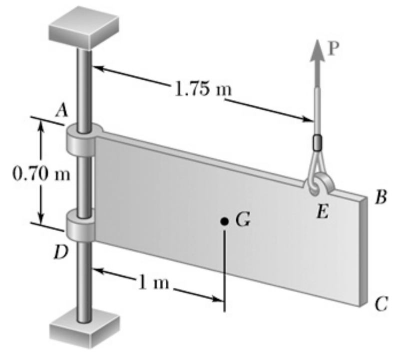
Impending motion: $\qquad F_C = \mu_C N_C$

$$27.5\mu_C - 0.75 = 25(0.1875)\frac{N_A}{N_C}$$

But $N_A = N_C$ (from pipe *FBD*) so $\qquad \dfrac{N_A}{N_C} = 1$

and $\mu_C = 0.1977$ ◀

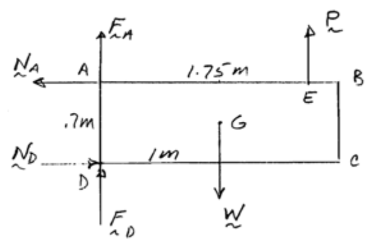
PROBLEM 8.32



The 25-kg plate *ABCD* is attached at *A* and *D* to collars which can slide on the vertical rod. Knowing that the coefficient of static friction is 0.40 between both collars and the rod, determine whether the plate is in equilibrium in the position shown when the magnitude of the vertical force applied at *E* is (*a*) $P = 0$, (*b*) $P = 80$ N.

SOLUTION

FBD plate:



$$W = 25\text{ kg}(9.81\text{ N/kg})$$

$$= 245.25\text{ N}$$

(*a*) $P = 0$; assume equilibrium:

$$\circlearrowleft \Sigma M_A = 0: \quad (0.7\text{ m})N_D - (1\text{ m})W = 0 \qquad N_D = \frac{10W}{7}$$

$$\longrightarrow \Sigma F_x = 0: \quad N_D - N_A = 0 \qquad N_A = N_D = \frac{10W}{7}$$

$$(F_A)_{\max} = \mu_s N_A \qquad (F_D)_{\max} = \mu_s N_D$$

So $$(F_A + F_D)_{\max} = \mu_s(N_A + N_D) = \frac{20\mu_s W}{7} = 1.143W$$

$$\uparrow \Sigma F_y = 0: \quad F_A + F_D - W = 0$$

$$\therefore \quad F_A + F_D = W < (F_A + F_D)_{\max} \qquad \text{OK.}$$

Plate is in equilibrium ◀

(*b*) $P = 80$ N; assume equilibrium:

$$\circlearrowleft \Sigma M_A = 0: \quad (1.75\text{ m})P + (0.7\text{ m})N_D - (1\text{ m})W = 0$$

or $$N_D = \frac{W - 1.75P}{0.7}$$

$$\longrightarrow \Sigma F_x = 0: \quad N_D - N_A = 0 \qquad N_D = N_A = \frac{W - 1.75P}{0.7}$$

$$(F_A)_{\max} = \mu_s N_A \qquad (F_B)_{\max} = \mu_s N_B$$

So $$(F_A + F_B)_{\max} = 0.4\frac{2W - 3.5P}{0.7} = 120.29\text{ N}$$

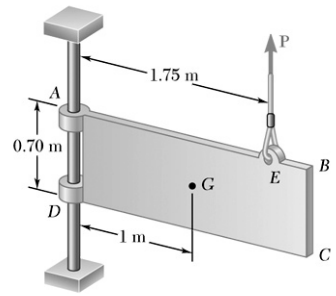
$$\uparrow \Sigma F_y = 0: \quad F_A + F_D - W + P = 0$$

$$F_A + F_D = W - P = 165.25\text{ N}$$

$$(F_A + F_D)_{\text{equil}} > (F_A + F_D)_{\max}$$

Impossible, so plate slides downward ◀

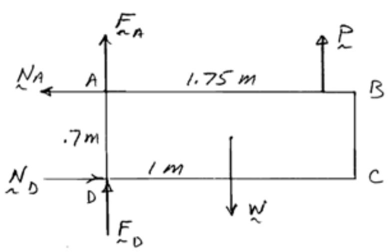
PROBLEM 8.33



In Problem 8.32, determine the range of values of the magnitude P of the vertical force applied at E for which the plate will move downward.

SOLUTION

FBD plate:



$$W = (25\text{ kg})(9.81\text{ m/s}^2)$$

$$= 245.25\text{ N}$$

$$\circlearrowleft \Sigma M_A = 0: \quad (0.7\text{ m})N_D - (1\text{ m})W + (1.75\text{ m})P = 0$$

$$N_D = \frac{W - 1.75P}{0.7}$$

$$\longrightarrow \Sigma F_x = 0: \quad N_D - N_A = 0 \quad \text{so that} \quad N_A = N_D = \frac{W - 1.75P}{0.7}$$

Note: N_A and N_D will be > 0 if $P < \frac{4}{7}W$ and < 0 if $P > \frac{4}{7}W$.

Impending motion downward: F_A and F_B are both > 0, so

$$F_A = \mu_s |N_A| = \frac{0.4}{0.7}|W - 1.75P| = \left|\frac{4}{7}W - P\right|$$

$$F_D = \mu_S |N_D| = \left|\frac{4}{7}W - P\right|$$

$$\uparrow \Sigma F_y = 0: \quad F_A + F_D - W + P = 0$$

$$2\left|\frac{4}{7}W - P\right| - W + P = 0$$

For $P < \frac{4}{7}W$; $\qquad P = \frac{W}{7} = 35.04\text{ N}$

For $P > \frac{4}{7}W$; $\qquad P = \frac{5W}{7} = 175.2\text{ N}$

Downward motion for $35.0\text{ N} < P < 175.2\text{ N}$ ◀

Alternative Solution

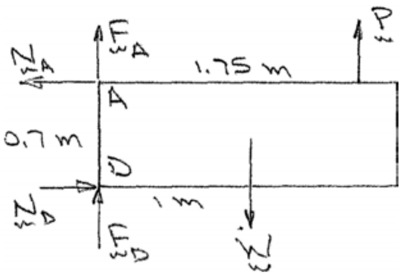
We first observe that for smaller values of the magnitude of **P** that (Case 1) the inner left-hand and right-hand surfaces of collars A and D, respectively, will contact the rod, whereas for larger values of the magnitude of **P** that (Case 2) the inner right-hand and left-hand surfaces of collars A and D, respectively, will contact the rod.

First note: $\qquad W = (25\text{ kg})(9.81\text{ m/s}^2)$

$$= 245.25\text{ N}$$

PROBLEM 8.33 CONTINUED

Case 1



$\circlearrowleft \Sigma M_D = 0$: $(0.7 \text{ m})N_A - (1 \text{ m})(245.25 \text{ N}) + (1.75 \text{ m})P = 0$

or $\quad N_A = \frac{10}{7}\left(245.25 - \frac{7}{4}P\right) \text{ N}$

$\rightarrow \Sigma F_x = 0$: $-N_A + N_D = 0$

or $\quad N_D = N_A$

$\uparrow \Sigma F_y = 0$: $F_A + F_D + P - 245.25 \text{ N} = 0$

or $\quad F_A + F_D = (245.25 - P) \text{ N}$

Now $\quad (F_A)_{\max} = \mu_s N_A \qquad (F_D)_{\max} = \mu_s N_D$

so that $\quad (F_A)_{\max} + (F_D)_{\max} = \mu_s (N_A + N_D)$

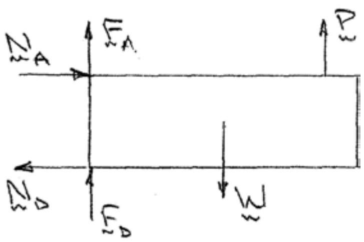
$$= 2(0.4)\left[\frac{10}{7}\left(245.25 - \frac{7}{4}P\right)\right]$$

For motion: $\quad F_A + F_D > (F_A)_{\max} + (F_D)_{\max}$

Substituting $\quad 245.25 - P > \frac{8}{7}\left(245.25 - \frac{7}{4}P\right)$

or $\quad P > 35.0 \text{ N}$

Case 2



From Case 1: $\quad N_D = N_A$

$F_A + F_D = (245.25 - P) \text{ N}$

$(F_A)_{\max} + (F_D)_{\max} = 2\mu_s N_A$

$\circlearrowleft \Sigma M_D = 0$: $-(0.7 \text{ m})N_A - (1 \text{ m})(245.25 \text{ N}) + (1.75 \text{ m})P = 0$

or $\quad N_A = \frac{10}{7}\left(\frac{7}{4}P - 245.25\right) \text{ N}$

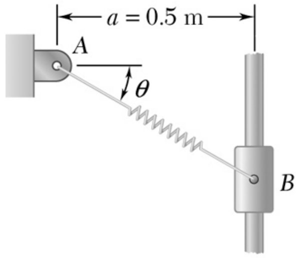
For motion: $\quad F_A + F_D > (F_A)_{\max} + (F_D)_{\max}$

Substituting: $\quad 245.25 - P > 2(0.4)\left[\frac{10}{7}\left(\frac{7}{4}P - 245.25\right)\right]$

or $\quad P < 175.2 \text{ N}$

Therefore, have downward motion for $\quad 35.0 \text{ N} < P < 175.2 \text{ N}$ ◀

PROBLEM 8.34

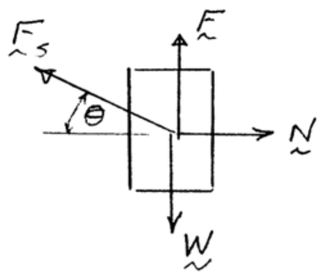


A collar B of weight W is attached to the spring AB and can move along the rod shown. The constant of the spring is 1.5 kN/m and the spring is unstretched when $\theta = 0$. Knowing that the coefficient of static friction between the collar and the rod is 0.40, determine the range of values of W for which equilibrium is maintained when (*a*) $\theta = 20^\circ$, (*b*) $\theta = 30^\circ$.

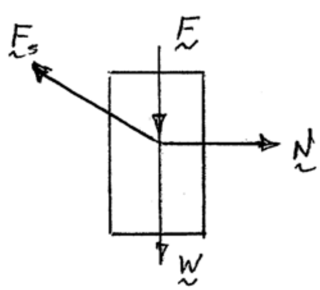
SOLUTION

FBD collar:

Impending motion down:



Impending motion up:



Stretch of spring $x = \overline{AB} - a = \dfrac{a}{\cos\theta} - a$

$$F_s = kx = k\left(\frac{a}{\cos\theta} - a\right) = (1.5 \text{ kN/m})(0.5 \text{ m})\left(\frac{1}{\cos\theta} - 1\right)$$

$$= (0.75 \text{ kN})\left(\frac{1}{\cos\theta} - 1\right)$$

$$\longrightarrow \Sigma F_x = 0: \quad N - F_s\cos\theta = 0$$

$$N = F_s\cos\theta = (0.75 \text{ kN})(1 - \cos\theta)$$

Impending slip: $\quad F = \mu_s N = (0.4)(0.75 \text{ kN})(1 - \cos\theta)$

$$= (0.3 \text{ kN})(1 - \cos\theta)$$

$+$ down, $-$ up

$$\uparrow \Sigma F_y = 0: \quad F_s\sin\theta \pm F - W = 0$$

$$(0.75 \text{ kN})(\tan\theta - \sin\theta) \pm (0.3 \text{ kN})(1 - \cos\theta) - W = 0$$

or $\quad W = (0.3 \text{ kN})[2.5(\tan\theta - \sin\theta) \pm (1 - \cos\theta)]$

(*a*) $\theta = 20^\circ$: $\quad W_{\text{up}} = -0.00163 \text{ kN} \quad$ (impossible)

$W_{\text{down}} = 0.03455 \text{ kN} \quad$ (OK)

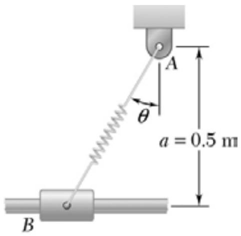
Equilibrium if $0 \le W \le 34.6$ N ◀

(*b*) $\theta = 30^\circ$: $\quad W_{\text{up}} = 0.01782 \text{ kN} \quad$ (OK)

$W_{\text{down}} = 0.0982 \text{ kN} \quad$ (OK)

Equilibrium if $17.82 \text{ N} \le W \le 98.2$ N ◀

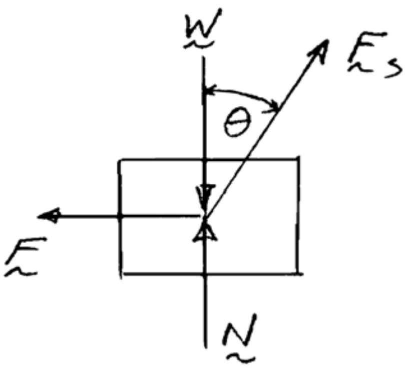
PROBLEM 8.35



A collar B of weight W is attached to the spring AB and can move along the rod shown. The constant of the spring is 1.5 kN/m and the spring is unstretched when $\theta = 0$. Knowing that the coefficient of static friction between the collar and the rod is 0.40, determine the range of values of W for which equilibrium is maintained when (*a*) $\theta = 20^{\circ}$, (*b*) $\theta = 30^{\circ}$.

SOLUTION

FBD collar:



Stretch of spring $x = \overline{AB} - a = \dfrac{a}{\cos\theta} - a$

$$F_s = k\left(\frac{a}{\cos\theta} - a\right) = (1.5\text{ kN/m})(0.5\text{ m})\left(\frac{1}{\cos\theta} - 1\right)$$

$$= (0.75\text{ kN})\left(\frac{1}{\cos\theta} - 1\right) = (750\text{ N})(\sec\theta - 1)$$

$$\uparrow \Sigma F_y = 0: \quad F_s\cos\theta - W + N = 0$$

or
$$W = N + (750\text{ N})(1 - \cos\theta)$$

Impending slip:

$$F = \mu_s|N| \quad (F \text{ must be } +, \text{ but } N \text{ may be positive or negative})$$

$$\longrightarrow \Sigma F_x = 0: \quad F_s\sin\theta - F = 0$$

or
$$F = F_s\sin\theta = (750\text{ N})(\tan\theta - \sin\theta)$$

(*a*) $\theta = 20^{\circ}$: $\quad F = (750\text{ N})(\tan 20^{\circ} - \sin 20^{\circ}) = 16.4626\text{ N}$

Impending motion: $\quad |N| = \dfrac{F}{\mu_s} = \dfrac{16.4626\text{ N}}{0.4} = 41.156\text{ N}$

(Note: for $|N| < 41.156$ N, motion will occur, equilibrium for $|N| > 41.156$)

But $\quad W = N + (750\text{ N})(1 - \cos 20^{\circ}) = N + 45.231\text{ N}$

So equilibrium for $W \le 4.07$ N and $W \ge 86.4$ N ◀

(*b*) $\theta = 30^{\circ}$: $\quad F = (750\text{ N})(\tan 30^{\circ} - \sin 30^{\circ}) = 58.013\text{ N}$

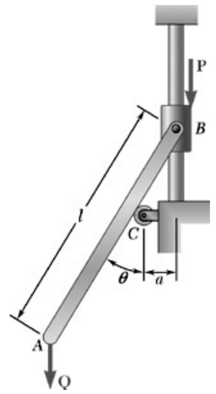
Impending motion: $\quad |N| = \dfrac{F}{\mu_s} = \dfrac{58.013}{0.4} = 145.032\text{ N}$

$$W = N + (750\text{ N})(1 - \cos 30^{\circ}) = N \pm 145.03\text{ N}$$

$$= -44.55\text{ N}(\text{impossible}), \; 245.51\text{ N}$$

Equilibrium for $W \ge 246$ N ◀

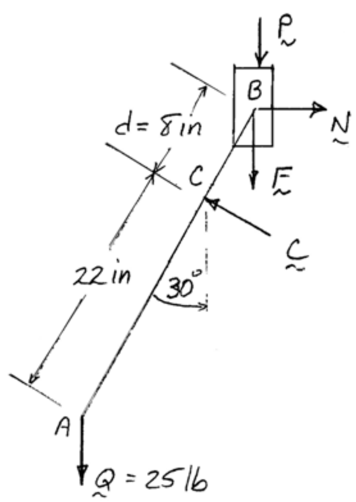
PROBLEM 8.36



The slender rod AB of length $l = 30$ in. is attached to a collar at B and rests on a small wheel located at a horizontal distance $a = 4$ in. from the vertical rod on which the collar slides. Knowing that the coefficient of static friction between the collar and the vertical rod is 0.25 and neglecting the radius of the wheel, determine the range of values of P for which equilibrium is maintained when $Q = 25$ lb and $\theta = 30^{\circ}$.

SOLUTION

FBD rod + collar:



Note: $d = \dfrac{a}{\sin\theta} = \dfrac{4 \text{ in.}}{\sin 30^{\circ}} = 8$ in., so $AC = 22$ in.

Neglect weights of rod and collar.

$$\circlearrowleft \Sigma M_B = 0: \quad (30 \text{ in.})(\sin 30^{\circ})(25 \text{ lb}) - (8 \text{ in.})C = 0$$

$$C = 46.875 \text{ lb}$$

$$\longrightarrow \Sigma F_x = 0: \quad N - C\cos 30^{\circ} = 0$$

$$N = (46.875 \text{ lb})\cos 30^{\circ} = 40.595 \text{ lb}$$

Impending motion up: $\quad F = \mu_s N = 0.25(40.595 \text{ lb})$

$$= 10.149 \text{ lb}$$

$$\uparrow \Sigma F_y = 0: \quad -25 \text{ lb} + (46.875 \text{ lb})\sin 30^{\circ} - P - 10.149 \text{ lb} = 0$$

or $\quad P = -1.563 \text{ lb} - 10.149 \text{ lb} = -11.71 \text{ lb}$

Impending motion down: Direction of **F** is now upward, but still have

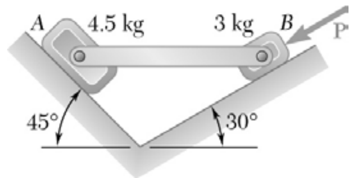
$$|F| = \mu_s N = 10.149 \text{ lb}$$

$$\uparrow \Sigma F_y = 0: \quad -25 \text{ lb} + (46.875 \text{ lb})\sin 30^{\circ} - P + 10.149 \text{ lb} = 0$$

or $\quad P = -1.563 \text{ lb} + 10.149 \text{ lb} = 8.59 \text{ lb}$

$\therefore$ Equilibrium for $-11.71 \text{ lb} \le P \le 8.59 \text{ lb}$ ◀

PROBLEM 8.37

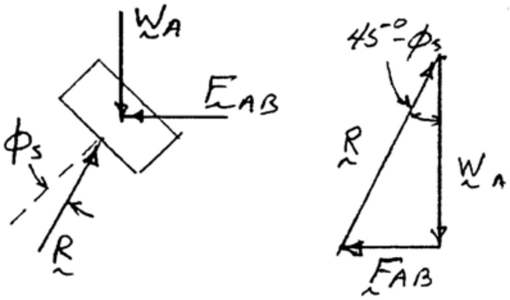


The 4.5-kg block A and the 3-kg block B are connected by a slender rod of negligible mass. The coefficient of static friction is 0.40 between all surfaces of contact. Knowing that for the position shown the rod is horizontal, determine the range of values of P for which equilibrium is maintained.

SOLUTION

FBDs: Note: $\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.4 = 21.801°$

(a) Block A impending slip ↘

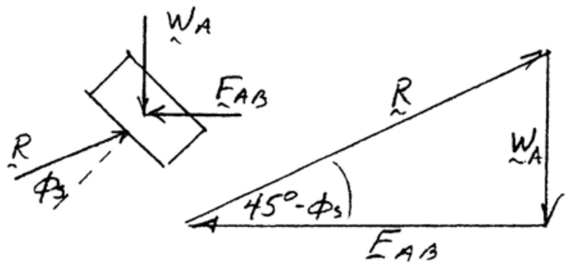


$$F_{AB} = W_A \tan(45° - \phi_s)$$

$$= (4.5\text{ kg})(9.81\text{ m/s}^2)\tan(23.199°)$$

$$= 18.9193\text{ N}$$

(b) Block A impending slip ↖

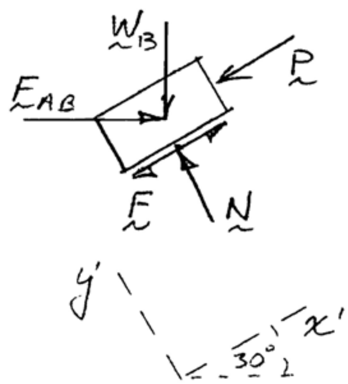


$$F_{AB} = W_A \operatorname{ctn}(45° - \phi_s)$$

$$= (4.5\text{ kg})(9.81\text{ m/s}^2)\operatorname{ctn}(23.199°)$$

$$= 103.005\text{ N}$$

Block B:



$$W_B = (3\text{ kg})(9.81\text{ m/s}^2)$$

$$= 29.43\text{ N}$$

From Block B: ↖ $\Sigma F_{y'} = 0$: $N - W_B\cos30° - F_{AB}\sin30° = 0$

PROBLEM 8.37 CONTINUED

Case (a) $$N = (29.43\text{ N})\cos 30° + (18.9193\text{ N})\sin 30° = 34.947\text{ N}$$

Impending motion: $$F = \mu_s N = 0.4(34.947\text{ N}) = 13.979\text{ N}$$

$\nearrow \Sigma F_{x'} = 0$: $F_{AB}\cos 30° - W_B \sin 30° - 13.979\text{ N} - P = 0$

$$P = (18.9193\text{ N})\cos 30° - (29.43\text{ N})\sin 30° - 13.979\text{ N}$$

$$= -12.31\text{ N}$$

Case (b) $$N = (29.43\text{ N})\cos 30° + (103.005\text{ N})\sin 30° = 76.9896\text{ N}$$

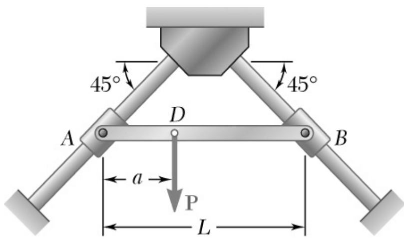
Impending motion: $$F = 0.4(76.9896\text{ N}) = 30.7958\text{ N}$$

$\nearrow \Sigma F_{x'} = 0$: $(103.005\text{ N})\cos 30° - (29.43\text{ N})\sin 30° + 30.7958\text{ N} - P = 0$

$$P = 105.3\text{ N}$$

For equilibrium $-12.31\text{ N} \le P \le 105.3\text{ N}$ ◀

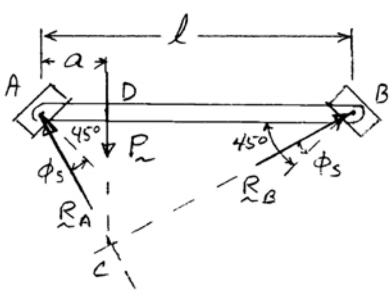
PROBLEM 8.38



Bar AB is attached to collars which can slide on the inclined rods shown. A force **P** is applied at point *D* located at a distance *a* from end *A*. Knowing that the coefficient of static friction μ_s between each collar and the rod upon which it slides is 0.30 and neglecting the weights of the bar and of the collars, determine the smallest value of the ratio *a/L* for which equilibrium is maintained.

SOLUTION

FBD bar + collars:



Impending motion

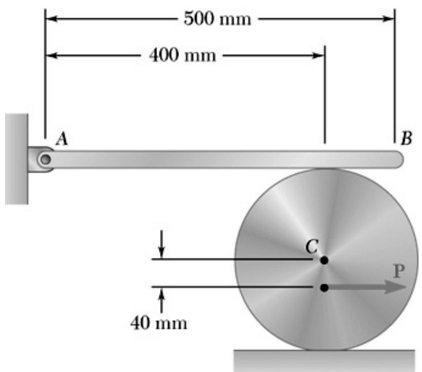
$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.3 = 16.6992°$$

Neglect weights: 3-force *FBD* and $\measuredangle ACB = 90°$

So $$AC = \frac{a}{\cos(45° + \phi_s)} = l\sin(45° - \phi_s)$$

$$\frac{a}{l} = \sin(45° - 16.6992°)\cos(45° + 16.6992°)$$

$$\frac{a}{l} = 0.225 \blacktriangleleft$$

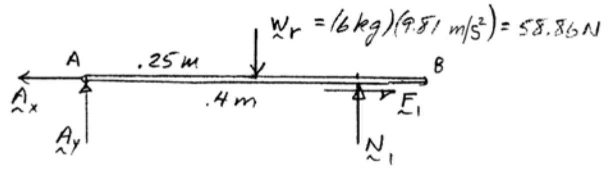


PROBLEM 8.39

The 6-kg slender rod *AB* is pinned at *A* and rests on the 18-kg cylinder *C*. Knowing that the diameter of the cylinder is 250 mm and that the coefficient of static friction is 0.35 between all surfaces of contact, determine the largest magnitude of the force **P** for which equilibrium is maintained.

SOLUTION

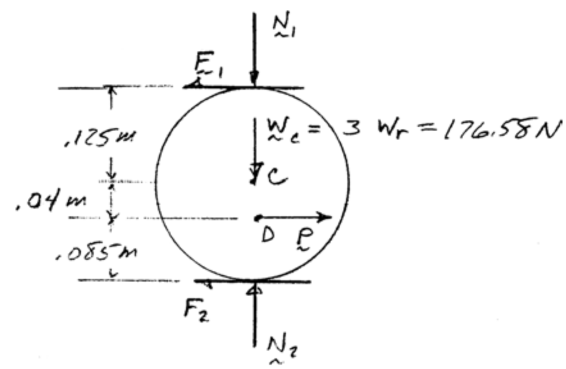
FBD rod:



$$\circlearrowleft \Sigma M_A = 0: \quad (0.4\text{ m})N_1 - (0.25\text{ m})W_r = 0$$

$$N_1 = 0.625W_r = 36.7875\text{ N}$$

FBD cylinder:



Cylinder:

$$\uparrow \Sigma F_y = 0: \quad N_2 - N_1 - W_C = 0 \quad \text{or} \quad N_2 = 0.625W_r + 3W_r = 3.625W_r = 5.8N_1$$

$$\circlearrowleft \Sigma M_D = 0: \quad (0.165\text{ m})F_1 - (0.085\text{ m})F_2 = 0 \quad \text{or} \quad F_2 = 1.941F_1$$

Since $\mu_{s1} = \mu_{s2}$, motion will impend first at top of the cylinder

So $$F_1 = \mu_s N_1 = 0.35(36.7875\text{ N}) = 12.8756\text{ N}$$

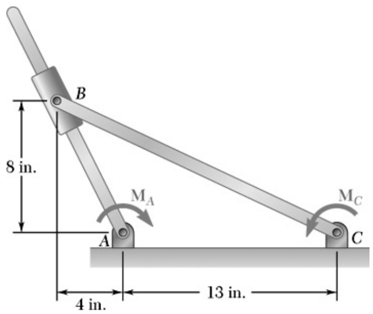
and $$F_2 = 1.941(12.8756\text{ N}) = 24.992\text{ N}$$

$$\left[\text{Check } F_2 = 25\text{ N} < \mu_S N_2 = 74.7\text{ N} \qquad \text{OK}\right]$$

$$\longrightarrow \Sigma F_x = 0: \quad P - F_1 - F_2 = 0$$

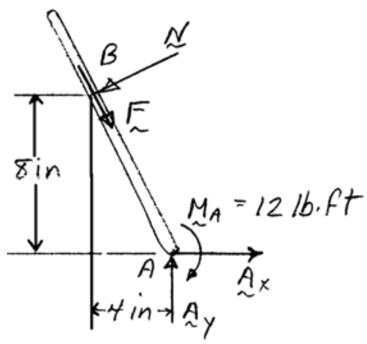
or $$P = 12.8756\text{ N} + 24.992\text{ N}$$

or $P = 37.9\text{ N}$ ◀

	**PROBLEM 8.40** Two rods are connected by a collar at B. A couple $\mathbf{M}_A$ of magnitude $12\text{ lb}\cdot\text{ft}$ is applied to rod AB. Knowing that $\mu_s = 0.30$ between the collar and rod AB, determine the largest couple $\mathbf{M}_C$ for which equilibrium will be maintained.

SOLUTION

FBD AB:



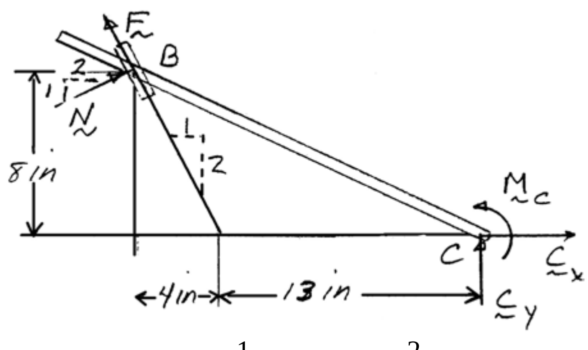
$$\circlearrowleft \Sigma M_A = 0: \quad \sqrt{8\text{ in}^2 + 4\text{ in}^2}\,(N) - M_A = 0$$

$$N = \frac{(12\text{ lb}\cdot\text{ft})(12\text{ in./ft})}{8.9443\text{ in.}} = 16.100\text{ lb}$$

Impending motion: $\quad F = \mu_s N = 0.3(16.100\text{ lb}) = 4.83\text{ lb}$

(Note: For max, M_C, need F in direction shown; see FBD BC.)

FBD BC + collar:

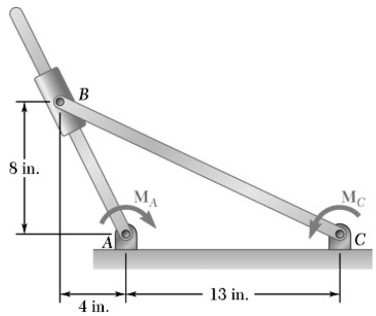


$$\circlearrowleft \Sigma M_C = 0: \quad M_C - (17\text{ in.})\frac{1}{\sqrt{5}}N - (8\text{ in.})\frac{2}{\sqrt{5}}N - (13\text{ in.})\frac{2}{\sqrt{5}}F = 0$$

or $$M_C = \frac{17\text{ in.}}{\sqrt{5}}(16.100\text{ lb}) + \frac{16\text{ in.}}{\sqrt{5}}(16.100\text{ lb}) + \frac{26\text{ in.}}{\sqrt{5}}(4.830\text{ lb}) = 293.77\text{ lb}\cdot\text{in.}$$

$$(\mathbf{M}_C)_{\max} = 24.5\text{ lb}\cdot\text{ft} \circlearrowright \blacktriangleleft$$

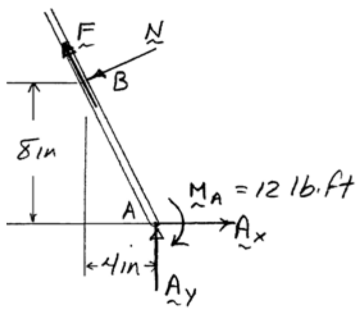
PROBLEM 8.41



In Problem 8.40, determine the smallest couple $\mathbf{M}_C$ for which equilibrium will be maintained.

SOLUTION

FBD AB:



$$\circlearrowleft \Sigma M_A = 0: \quad N\left(\sqrt{8\text{ in}^2 + 4\text{ in}^2}\right) - M_A = 0$$

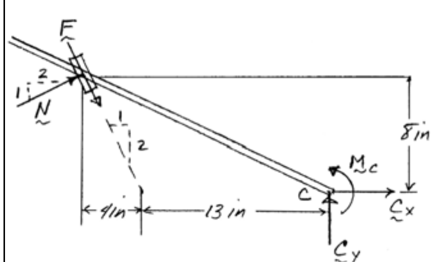
$$N = \frac{(12\text{ lb}\cdot\text{ft})(12\text{ in./ft})}{8.9443\text{ in.}} = 16.100\text{ lb}$$

Impending motion: $\quad F = \mu_s N = 0.3(16.100\text{ lb})$

$$= 4.830\text{ lb}$$

(Note: For min. M_C, need F in direction shown; see FBD BC.)

FBD BC + collar:

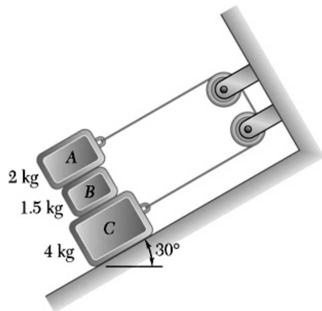


$$\circlearrowleft \Sigma M_C = 0: \quad M_C - (17\text{ in.})\frac{1}{\sqrt{5}}N - (8\text{ in.})\frac{2}{\sqrt{5}}N + (13\text{ in.})\frac{2}{\sqrt{5}}F = 0$$

$$M_C = \frac{1}{\sqrt{5}}\left[(17\text{ in.} + 16\text{ in.})(16.100\text{ lb}) - (26\text{ in.})(4.830\text{ lb})\right]$$

$$= 181.44\text{ lb}\cdot\text{in.}$$

$$(\mathbf{M}_C)_{\min} = 15.12\text{ lb}\cdot\text{ft} \circlearrowleft \blacktriangleleft$$



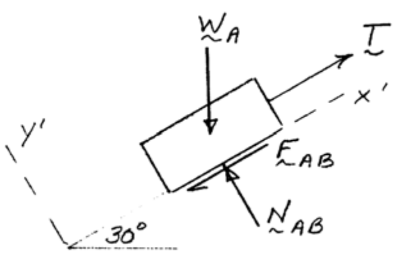
PROBLEM 8.42

Blocks A, B, and C having the masses shown are at rest on an incline. Denoting by μ_s the coefficient of static friction between all surfaces of contact, determine the smallest value of μ_s for which equilibrium is maintained.

SOLUTION

For impending motion, C will start down and A will start up. Since, the normal force between B and C is larger than that between A and B, the corresponding friction force can be larger as well. Thus we assume that motion impends between A and B.

FBD A:



$$\nwarrow \Sigma F_{y'} = 0: \quad N_{AB} - W_A \cos 30° = 0; \;\; N_{AB} = \frac{\sqrt{3}}{2} W_A$$

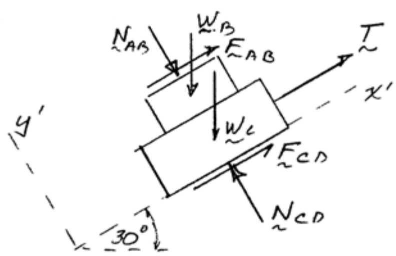
Impending motion: $$F_{AB} = \mu_s N_{AB} = \frac{\sqrt{3}}{2} W_A \mu_s$$

$$\nearrow \Sigma F_{x'} = 0: \quad T - F_{AB} - W_A \sin 30° = 0$$

or $$T = \left(\sqrt{3}\mu_s + 1\right)\frac{W_A}{2}$$

$$\nwarrow \Sigma F_{y'} = 0: \quad N_{CD} - N_{AB} - \left(W_B + W_C\right)\cos 30° = 0$$

FBD B + C:



or $$N_{CD} = \frac{\sqrt{3}}{2}\left(W_A + W_B + W_C\right)$$

Impending motion: $$F_{CD} = \mu_s N_{CD} = \frac{\sqrt{3}}{2}\left(W_A + W_B + W_C\right)\mu_s$$

$$\nearrow \Sigma F_{x'} = 0: \quad T + F_{AB} + F_{CD} - \left(W_B + W_C\right)\sin 30° = 0$$

$$T = \frac{W_B + W_C}{2} - \frac{\sqrt{3}}{2}\mu_s\left(2W_A + W_B + W_C\right)$$

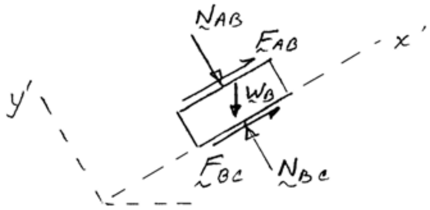
Equating T's: $$\sqrt{3}\mu_s\left(3W_A + W_B + W_C\right) = W_B + W_C - W_A$$

$$\mu_s = \frac{m_B + m_C - m_A}{\left(3m_A + m_B + m_C\right)\sqrt{3}} = \frac{1.5 \text{ kg} + 4 \text{ kg} - 2 \text{ kg}}{\left(6 \text{ kg} + 1.5 \text{ kg} + 4 \text{ kg}\right)\sqrt{3}}$$

$$\mu_s = 0.1757 \blacktriangleleft$$

PROBLEM 8.42 CONTINUED

FBD B:



$$\nwarrow \Sigma F_{y'} = 0: \quad N_{BC} - N_{AB} - W_B \cos 30° = 0$$

or $$N_{BC} = \frac{\sqrt{3}}{2}(W_A + W_B)$$

$$(F_{BC})_{\max} = \mu_s N_{BC} = 0.1757\frac{\sqrt{3}}{2}(W_A + W_B)$$

$$= 0.1522(m_A + m_B)g = 0.1522(3.5 \text{ kg})(9.81 \text{ m/s}^2)$$

$$= 5.224 \text{ N}$$

$$\nearrow \Sigma F_{x'} = 0: \quad F_{AB} + F_{BC} - W_B \sin 30° = 0$$

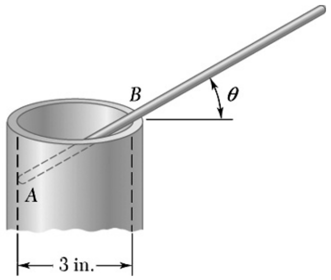
or $$F_{BC} = -F_{AB} + \frac{1}{2}W_B = -\frac{\sqrt{3}}{2}W_A(0.1757) + \frac{W_B}{2}$$

$$= (-0.1522m_A + 0.5m_B)g$$

$$= [-0.1522(2 \text{ kg}) + 0.5(1.5 \text{ kg})](9.81 \text{ m/s}^2)$$

$$= 4.37 \text{ N}$$

$$F_{BC} < F_{BC\max} \qquad \text{OK}$$

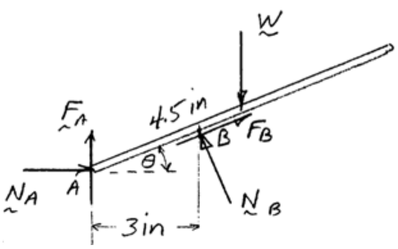


PROBLEM 8.43

A slender steel rod of length 9 in. is placed inside a pipe as shown. Knowing that the coefficient of static friction between the rod and the pipe is 0.20, determine the largest value of θ for which the rod will not fall into the pipe.

SOLUTION

FBD rod:



$$\circlearrowleft \Sigma M_A = 0: \quad \frac{3 \text{ in.}}{\cos\theta} N_B - \left[(4.5 \text{ in.})\cos\theta\right] W = 0$$

or

$$N_B = \left(1.5\cos^2\theta\right)W$$

Impending motion:

$$F_B = \mu_s N_B = \left(1.5\mu_s \cos^2\theta\right)W$$

$$= \left(0.3\cos^2\theta\right)W$$

$$\xrightarrow{} \Sigma F_x = 0: \quad N_A - N_B \sin\theta + F_B \cos\theta = 0$$

or

$$N_A = \left(1.5\cos^2\theta\right)W\left(\sin\theta - 0.2\cos\theta\right)$$

Impending motion:

$$F_A = \mu_s N_A$$

$$= \left(0.3\cos^2\theta\right)W\left(\sin\theta - 0.2\cos\theta\right)$$

$$\uparrow \Sigma F_y = 0: \quad F_A + N_B\cos\theta + F_B\sin\theta - W = 0$$

or

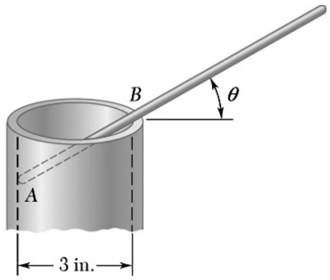
$$F_A = W\left(1 - 1.5\cos^3\theta - 0.3\cos^2\theta\sin\theta\right)$$

Equating F_A's

$$0.3\cos^2\theta\left(\sin\theta - 0.2\cos\theta\right) = 1 - 1.5\cos^3\theta - 0.3\cos^2\theta\sin\theta$$

$$0.6\cos^2\theta\sin\theta + 1.44\cos^3\theta = 1$$

Solving numerically $\qquad \theta = 35.8° \blacktriangleleft$

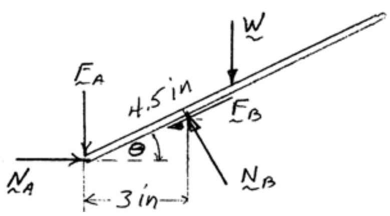


PROBLEM 8.44

In Problem 8.43, determine the smallest value of θ for which the rod will not fall out of the pipe.

SOLUTION

FBD rod:



$$\circlearrowleft \Sigma M_A = 0: \quad \frac{3\text{ in.}}{\cos\theta} N_B - \left[(4.5\text{ in.})\cos\theta\right]W = 0$$

or $$N_B = 1.5W\cos^2\theta$$

Impending motion: $$F_B = \mu_s N_B = 0.2\left(1.5W\cos^2\theta\right)$$

$$= 0.3W\cos^2\theta$$

$$\longrightarrow \Sigma F_x = 0: \quad N_A - N_B\sin\theta - F_B\cos\theta = 0$$

or $$N_A = W\cos^2\theta(1.5\sin\theta + 0.3\cos\theta)$$

Impending motion: $$F_A = \mu_s N_A$$

$$= W\cos^2\theta(0.3\sin\theta + 0.06\cos\theta)$$

$$\uparrow \Sigma F_y = 0: \quad N_B\cos\theta - F_B\sin\theta - W - F_A = 0$$

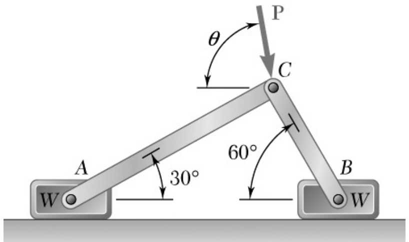
or $$F_A = W\left[\cos^2\theta(1.5\cos\theta - 0.3\sin\theta) - 1\right]$$

Equating F_A's:

$$\cos^2\theta(1.44\cos\theta - 0.6\sin\theta) = 1$$

Solving numerically $\theta = 20.5°$ ◀

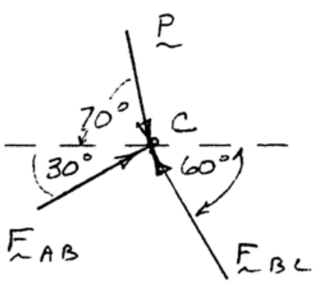
PROBLEM 8.45

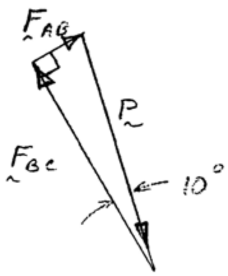


Two slender rods of negligible weight are pin-connected at C and attached to blocks A and B, each of weight W. Knowing that $\theta = 70^{\circ}$ and that the coefficient of static friction between the blocks and the horizontal surface is 0.30, determine the largest value of P for which equilibrium is maintained.

SOLUTION

FBD pin C:





$$F_{AB} = P\sin 10^{\circ} = 0.173648P$$

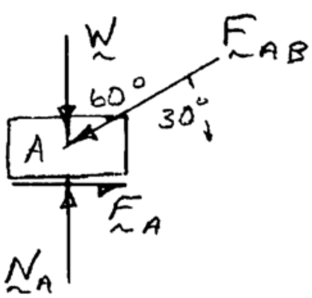
$$F_{BC} = P\cos 10^{\circ} = 0.98481P$$

$$\uparrow \Sigma F_y = 0: \quad N_A - W - F_{AB}\sin 30^{\circ} = 0$$

or

$$N_A = W + 0.173648P\sin 30^{\circ} = W + 0.086824P$$

FBD block A:



$$\rightarrow \Sigma F_x = 0: \quad F_A - F_{AB}\cos 30^{\circ} = 0$$

or

$$F_A = 0.173648P\cos 30^{\circ} = 0.150384P$$

For impending motion at A: $\quad F_A = \mu_s N_A$

Then $\quad N_A = \dfrac{F_A}{\mu_s}: \quad W + 0.086824P = \dfrac{0.150384}{0.3}P$

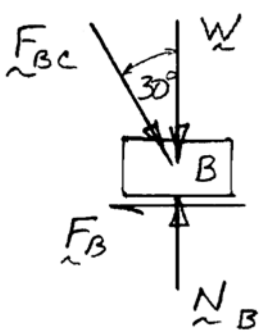
or

$$P = 2.413W$$

$$\uparrow \Sigma F_y = 0: \quad N_B - W - F_{BC}\cos 30^{\circ} = 0$$

$$N_B = W + 0.98481P\cos 30^{\circ} = W + 0.85287P$$

FBD block B:



$$\rightarrow \Sigma F_x = 0: \quad F_{BC}\sin 30^{\circ} - F_B = 0$$

$$F_B = 0.98481P\sin 30^{\circ} = 0.4924P$$

For impending motion at B: $\quad F_B = \mu_s N_B$

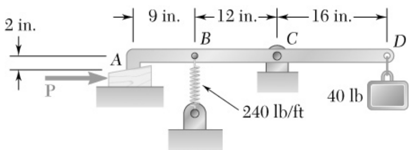
Then $\quad N_B = \dfrac{F_B}{\mu_s}: \quad W + 0.85287P = \dfrac{0.4924P}{0.3}$

or

$$P = 1.268W$$

Thus, maximum P for equilibrium $\quad P_{max} = 1.268W$ ◀

PROBLEM 8.46

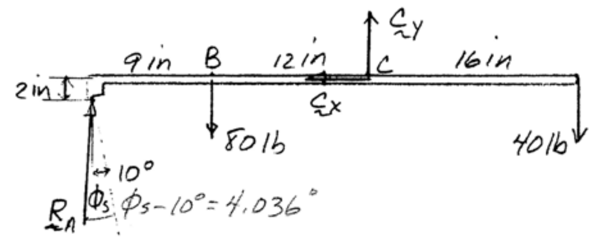


A 40-lb weight is hung from a lever which rests against a 10° wedge at A and is supported by a frictionless hinge at C. Knowing that the coefficient of static friction is 0.25 at both surfaces of the wedge and that for the position shown the spring is stretched 4 in., determine (a) the magnitude of the force **P** for which motion of the wedge is impending, (b) the components of the corresponding reaction at C.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.25 = 14.036° \qquad F_s = kx = (240 \text{ lb/ft})\left(\frac{4 \text{ in.}}{12 \text{ in./ft}}\right) = 80 \text{ lb}$$

FBD lever:



$$\circlearrowleft \Sigma M_C = 0: \quad (12 \text{ in.})(80 \text{ lb}) - (16 \text{ in.})(40 \text{ lb}) - (21 \text{ in.})R_A\cos(\phi_s - 10°)$$

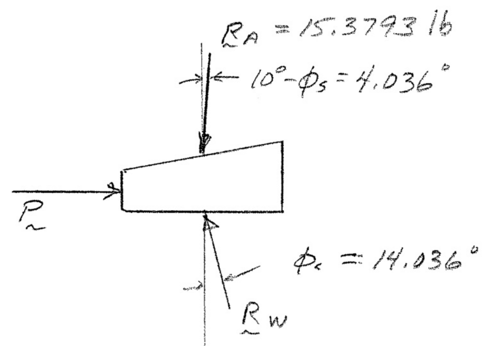
$$+ (2 \text{ in.})R_A\sin(\phi_s - 10°) = 0$$

or $\quad R_A = 15.3793 \text{ lb}$

(b) $\quad \rightarrow \Sigma F_x = 0: \quad (15.379 \text{ lb})\sin(4.036°) - C_x = 0 \qquad \mathbf{C}_x = 1.082 \text{ lb} \leftarrow$ ◀

$$\uparrow \Sigma F_y = 0: \quad (15.379 \text{ lb})\cos(4.036°) - 80 \text{ lb} - 40 \text{ lb} + C_y = 0 \qquad \mathbf{C}_y = 104.7 \text{ lb} \uparrow$$ ◀

FBD wedge:



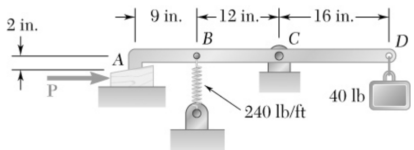
$$\uparrow \Sigma F_y = 0: \quad R_W\cos 14.036° - (15.3793 \text{ lb})\cos 4.036° = 0$$

or $\quad R_W = 15.8133 \text{ lb}$

(a) $\quad \rightarrow \Sigma F_x = 0: \quad P - (15.3793 \text{ lb})\sin 4.036° - (15.8133 \text{ lb})\sin 14.036° = 0$

$P = 4.92 \text{ lb}$ ◀

PROBLEM 8.47

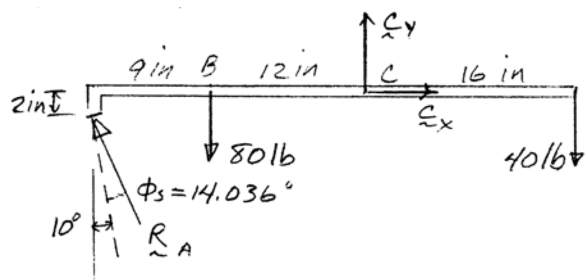


Solve Problem 8.46 assuming that force **P** is directed to the left.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.25 = 14.036° \qquad F_s = kx = (240\text{ lb/ft})\left(\frac{4\text{ in.}}{12\text{ in./ft}}\right) = 80\text{ lb}$$

FBD lever:



$$\circlearrowleft \Sigma M_C = 0: \quad (12\text{ in.})(80\text{ lb}) - (16\text{ in.})(40\text{ lb}) - (21\text{ in.})R_A\cos 24.036°$$

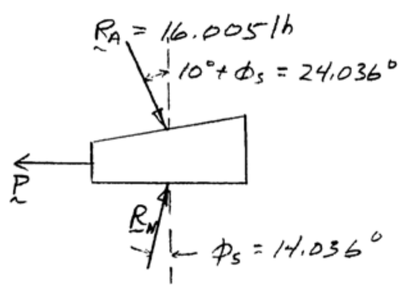
$$- (2\text{ in.})R_A\sin 24.036° = 0$$

or $\qquad R_A = 16.005\text{ lb}$

(*b*) $\qquad \rightarrow \Sigma F_x = 0: \quad C_x - (16.005\text{ lb})\sin 24.036° = 0 \qquad\qquad \mathbf{C}_x = 6.52\text{ lb} \longrightarrow$ ◀

$\uparrow \Sigma F_y = 0: \quad C_y - 80\text{ lb} - 40\text{ lb} + (16.005\text{ lb})\cos(24.036°) = 0 \qquad\qquad \mathbf{C}_y = 105.4\text{ lb} \uparrow$ ◀

FBD wedge:



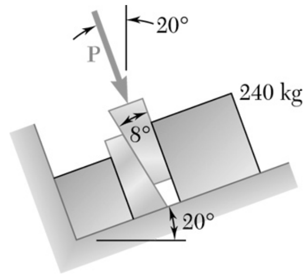
$$\uparrow \Sigma F_y = 0: \quad R_W\cos 14.036° - (16.005\text{ lb})\cos 24.036° = 0$$

or $\qquad R_W = 15.067\text{ lb}$

(*a*) $\qquad \rightarrow \Sigma F_x = 0: \quad (16.005\text{ lb})\sin 24.036° + (15.067\text{ lb})\sin 14.036° - P = 0$

$P = 10.17\text{ lb}$ ◀

PROBLEM 8.48

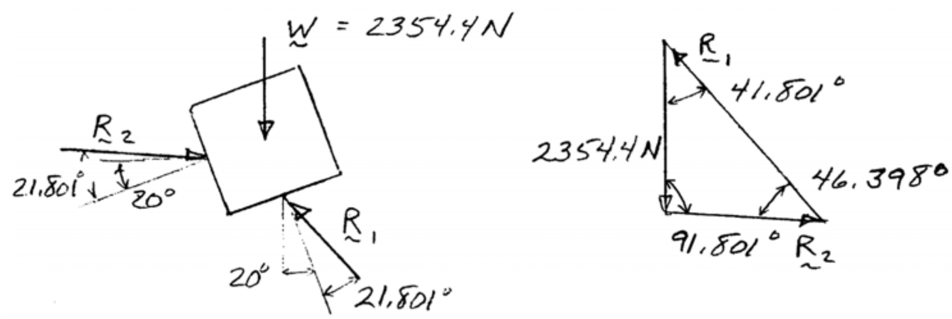


Two 8° wedges of negligible mass are used to move and position a 240-kg block. Knowing that the coefficient of static friction is 0.40 at all surfaces of contact, determine the magnitude of the force **P** for which motion of the block is impending.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.4 = 21.801^\circ \qquad W = 240\text{ kg}\left(9.81\text{ m/s}^2\right) = 2354.4\text{ N}$$

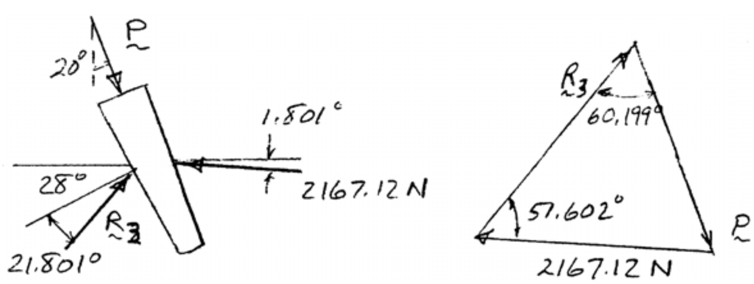
FBD block:



$$\frac{R_2}{\sin 41.801^\circ} = \frac{2354.4\text{ N}}{\sin 46.398^\circ}$$

$$R_2 = 2167.12\text{ N}$$

FBD wedge:

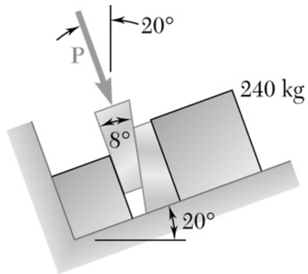


$$\frac{P}{\sin 51.602^\circ} = \frac{2167.12\text{ N}}{\sin 60.199^\circ}$$

$$P = 1957\text{ N}$$

$P = 1.957$ kN ◀

PROBLEM 8.49

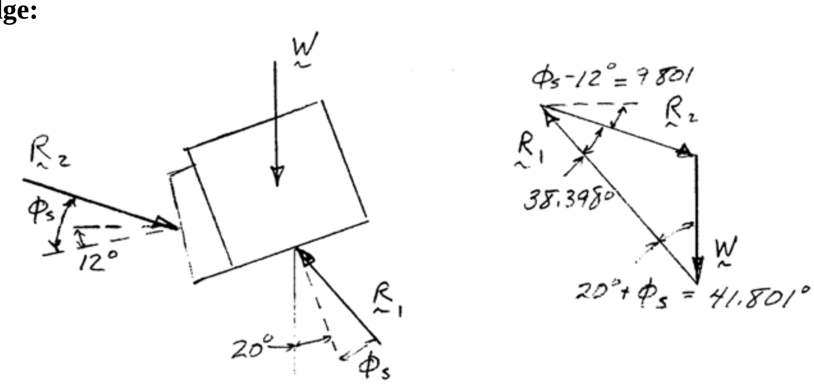


Two 8° wedges of negligible mass are used to move and position a 240-kg block. Knowing that the coefficient of static friction is 0.40 at all surfaces of contact, determine the magnitude of the force **P** for which motion of the block is impending.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.4 = 21.801° \qquad W = 240\text{ kg}\left(9.81\text{ m/s}^2\right) = 2354.4\text{ N}$$

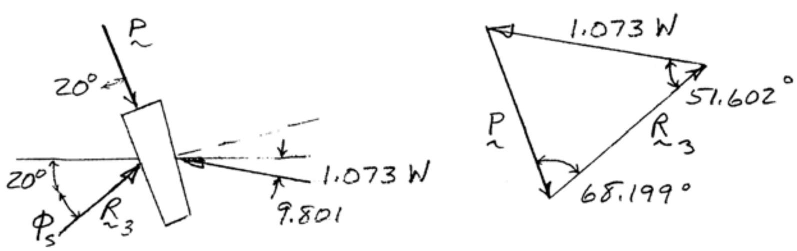
FBD block + wedge:



$$\frac{R_2}{\sin 41.801°} = \frac{2354.4\text{ N}}{\sin 38.398°}$$

$$R_2 = 2526.6\text{ N}$$

FBD wedge:

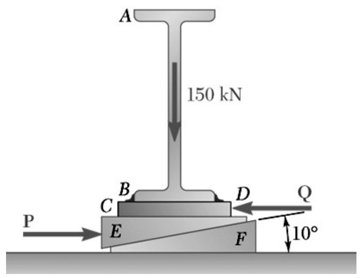


$$\frac{P}{\sin 51.602°} = \frac{2526.6\text{ N}}{\sin 68.199°}$$

$$P = 2132.7\text{ N}$$

$$P = 2.13\text{ kN} \blacktriangleleft$$

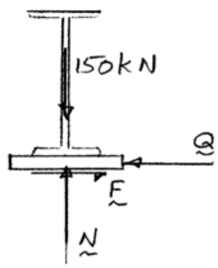
PROBLEM 8.50



The elevation of the end of the steel beam supported by a concrete floor is adjusted by means of the steel wedges *E* and *F*. The base plate *CD* has been welded to the lower flange of the beam, and the end reaction of the beam is known to be 150 kN. The coefficient of static friction is 0.30 between the two steel surfaces and 0.60 between the steel and the concrete. If the horizontal motion of the beam is prevented by the force **Q**, determine (*a*) the force **P** required to raise the beam, (*b*) the corresponding force **Q**.

SOLUTION

FBD AB + CD:



$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.3 = 16.70° \text{ for steel on steel}$$

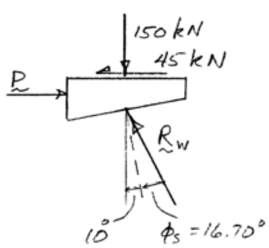
$$\uparrow \Sigma F_y = 0: \quad N - 150 \text{ kN} = 0 \qquad N = 150 \text{ kN}$$

Impending motion: $F = \mu_s N = 0.3(150 \text{ kN}) = 45 \text{ kN}$

$$\longrightarrow \Sigma F_x = 0: \quad F - Q = 0$$

(*b*) $\mathbf{Q} = 45.0 \text{ kN} \longleftarrow$ ◀

FBD top wedge:



Assume bottom wedge doesn't move:

$$\uparrow \Sigma F_y = 0: \quad R_W \cos(10° + 16.70°) - 150 \text{ kN} = 0$$

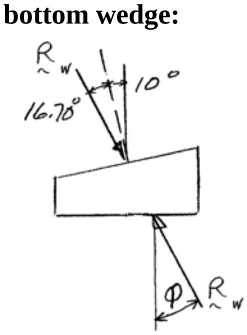
$$R_W = 167.9 \text{ kN}$$

$$\longrightarrow \Sigma F_x = 0: \quad P - 45 \text{ kN} - (167.9 \text{ kN})\sin 26.70° = 0$$

$$P = 120.44 \text{ kN}$$

(*a*) $\mathbf{P} = 120.4 \text{ kN} \longrightarrow$ ◀

FBD bottom wedge:

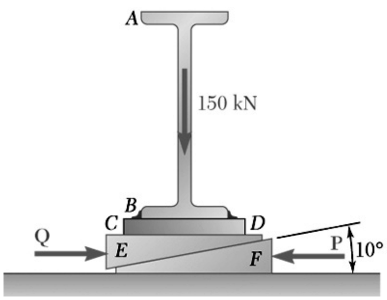


Bottom wedge is two-force member, so $\phi = 26.70°$ for equilibrium, but

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.6 = 31.0° \ (\text{steel on concrete})$$

So $\phi < \phi_s$ OK.

PROBLEM 8.51

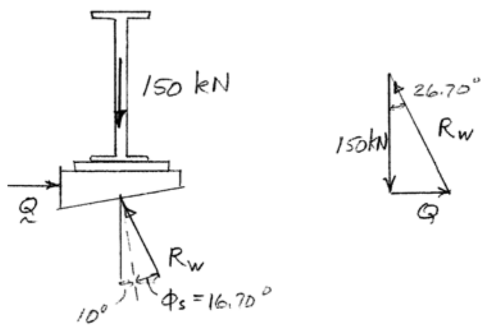


The elevation of the end of the steel beam supported by a concrete floor is adjusted by means of the steel wedges *E* and *F*. The base plate *CD* has been welded to the lower flange of the beam, and the end reaction of the beam is known to be 150 kN. The coefficient of static friction is 0.30 between the two steel surfaces and 0.60 between the steel and the concrete. If the horizontal motion of the beam is prevented by the force **Q**, determine (*a*) the force **P** required to raise the beam, (*b*) the corresponding force **Q**.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.3 = 16.70° \text{ for steel on steel}$$

FBD AB + CD + top wedge: Assume top wedge doesn't move

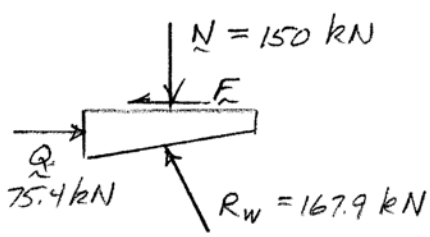


$$R_w = \frac{150 \text{ kN}}{\cos 26.70°} = 167.90 \text{ kN}$$

$$Q = (150 \text{ kN})\tan 26.70° = 75.44 \text{ kN}$$

(*b*) $\mathbf{Q} = 75.4 \text{ kN} \longrightarrow$ ◀

FBD top wedge:

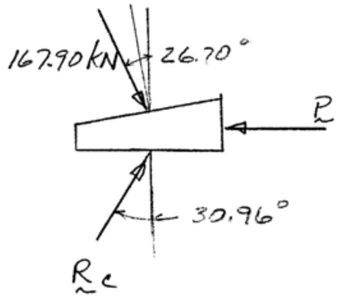


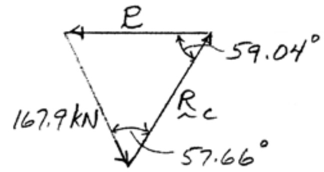
$$\xrightarrow{} \Sigma F_x = 0: \quad 75.44 \text{ kN} - 167.9 \text{ kN} \sin 26.70° - F = 0$$

$$F = 0 \text{ as expected.}$$

PROBLEM 8.51 CONTINUED

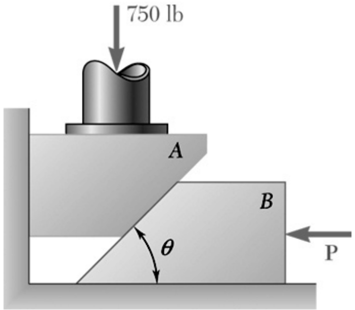
FBD bottom wedge: $\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.6 = 30.96°$ steel on concrete





$$\frac{P}{\sin 57.66°} = \frac{167.90 \text{ kN}}{\sin 59.04°}$$

(*a*) $\mathbf{P} = 165.4$ kN ← ◀



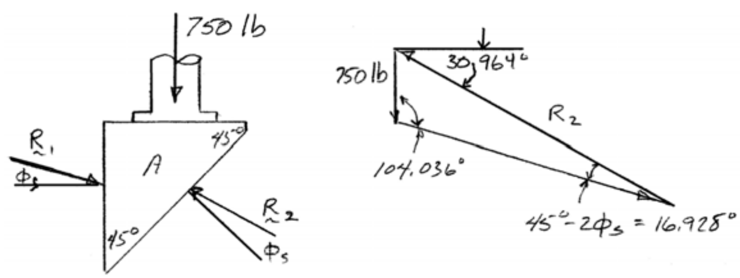
PROBLEM 8.52

Block *A* supports a pipe column and rests as shown on wedge *B*. Knowing that the coefficient of static friction at all surfaces of contact is 0.25 and that $\theta = 45°$, determine the smallest force **P** required to raise block *A*.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.25 = 14.036°$$

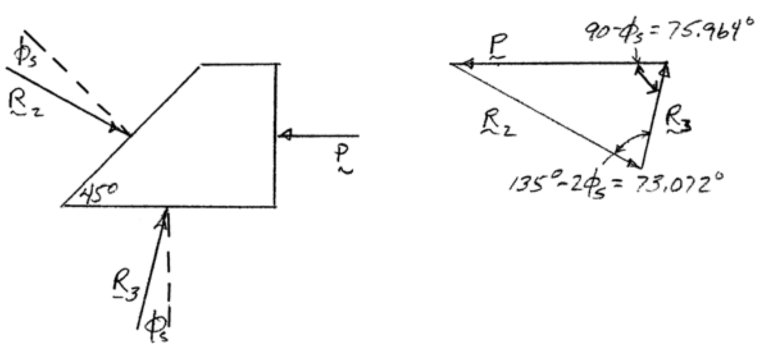
FBD block A:



$$\frac{R_2}{\sin 104.036°} = \frac{750 \text{ lb}}{\sin 16.928°}$$

$$R_2 = 2499.0 \text{ lb}$$

FBD wedge B:

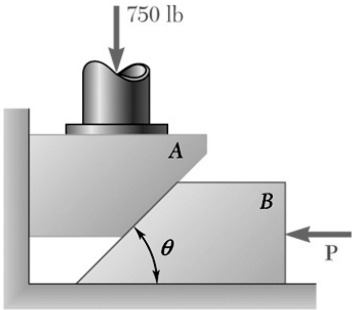


$$\frac{P}{\sin 73.072°} = \frac{2499.0}{\sin 75.964°}$$

$$P = 2464 \text{ lb}$$

$\mathbf{P} = 2.46$ kips ⟵ ◀

PROBLEM 8.53

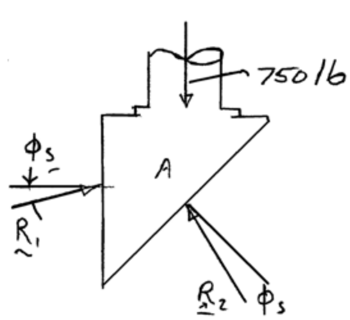


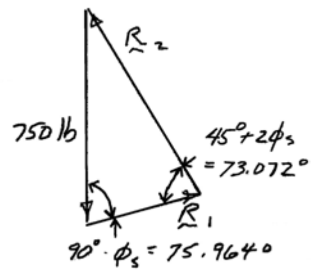
Block *A* supports a pipe column and rests as shown on wedge *B*. Knowing that the coefficient of static friction at all surfaces of contact is 0.25 and that $\theta = 45°$, determine the smallest force **P** for which equilibrium is maintained.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.25 = 14.036°$$

FBD block A:

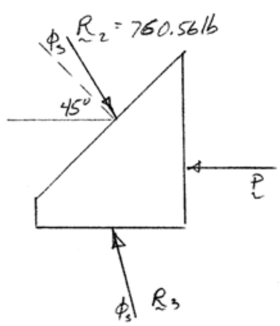


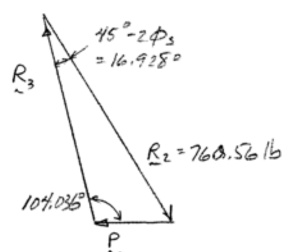


$$\frac{R_2}{\sin(75.964°)} = \frac{750 \text{ lb}}{\sin(73.072°)}$$

$$R_2 = 760.56 \text{ lb}$$

FBD wedge B:



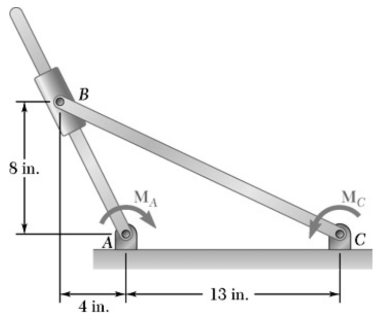


$$\frac{P}{\sin 16.928°} = \frac{760.56}{\sin 104.036°}$$

$$P = 228.3 \text{ lb}$$

$$\mathbf{P} = 228 \text{ lb} \longleftarrow$$

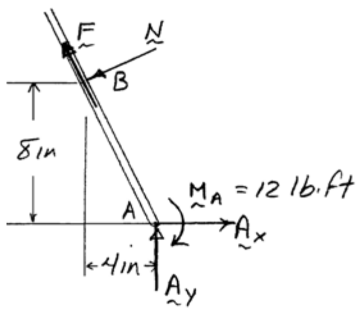
PROBLEM 8.41



In Problem 8.40, determine the smallest couple $\mathbf{M}_C$ for which equilibrium will be maintained.

SOLUTION

FBD AB:



$$\circlearrowleft \Sigma M_A = 0: \quad N\left(\sqrt{8 \text{ in}^2 + 4 \text{ in}^2}\right) - M_A = 0$$

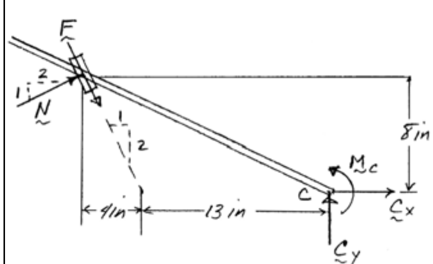
$$N = \frac{(12 \text{ lb}\cdot\text{ft})(12 \text{ in./ft})}{8.9443 \text{ in.}} = 16.100 \text{ lb}$$

Impending motion: $\quad F = \mu_s N = 0.3(16.100 \text{ lb})$

$$= 4.830 \text{ lb}$$

(Note: For min. M_C, need F in direction shown; see FBD BC.)

FBD BC + collar:



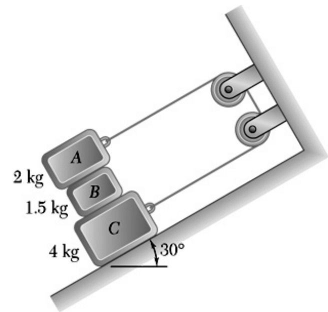
$$\circlearrowleft \Sigma M_C = 0: \quad M_C - (17 \text{ in.})\frac{1}{\sqrt{5}}N - (8 \text{ in.})\frac{2}{\sqrt{5}}N + (13 \text{ in.})\frac{2}{\sqrt{5}}F = 0$$

$$M_C = \frac{1}{\sqrt{5}}\left[(17 \text{ in.} + 16 \text{ in.})(16.100 \text{ lb}) - (26 \text{ in.})(4.830 \text{ lb})\right]$$

$$= 181.44 \text{ lb}\cdot\text{in.}$$

$$(\mathbf{M}_C)_{\min} = 15.12 \text{ lb}\cdot\text{ft} \circlearrowleft \blacktriangleleft$$

PROBLEM 8.42

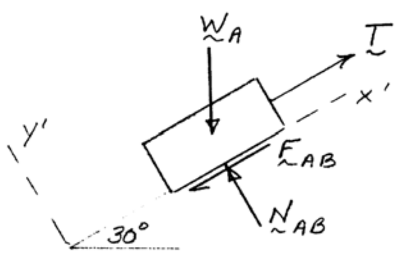


Blocks *A*, *B*, and *C* having the masses shown are at rest on an incline. Denoting by μ_s the coefficient of static friction between all surfaces of contact, determine the smallest value of μ_s for which equilibrium is maintained.

SOLUTION

For impending motion, *C* will start down and *A* will start up. Since, the normal force between *B* and *C* is larger than that between *A* and *B*, the corresponding friction force can be larger as well. Thus we assume that motion impends between *A* and *B*.

FBD A:



$$\nwarrow \Sigma F_{y'} = 0: \quad N_{AB} - W_A \cos 30° = 0; \; N_{AB} = \frac{\sqrt{3}}{2} W_A$$

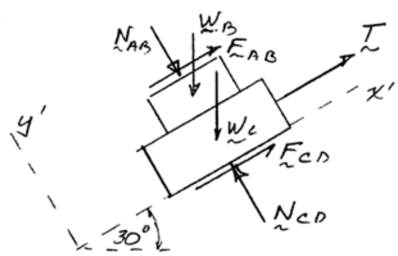
Impending motion: $$F_{AB} = \mu_s N_{AB} = \frac{\sqrt{3}}{2} W_A \mu_s$$

$$\nearrow \Sigma F_{x'} = 0: \quad T - F_{AB} - W_A \sin 30° = 0$$

or $$T = \left(\sqrt{3}\mu_s + 1\right)\frac{W_A}{2}$$

$$\nwarrow \Sigma F_{y'} = 0: \quad N_{CD} - N_{AB} - \left(W_B + W_C\right)\cos 30° = 0$$

FBD B + C:



or $$N_{CD} = \frac{\sqrt{3}}{2}\left(W_A + W_B + W_C\right)$$

Impending motion: $$F_{CD} = \mu_s N_{CD} = \frac{\sqrt{3}}{2}\left(W_A + W_B + W_C\right)\mu_s$$

$$\nearrow \Sigma F_{x'} = 0: \quad T + F_{AB} + F_{CD} - \left(W_B + W_C\right)\sin 30° = 0$$

$$T = \frac{W_B + W_C}{2} - \frac{\sqrt{3}}{2}\mu_s\left(2W_A + W_B + W_C\right)$$

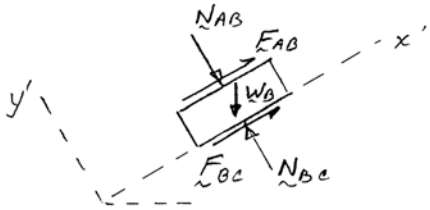
Equating *T*'s: $$\sqrt{3}\mu_s\left(3W_A + W_B + W_C\right) = W_B + W_C - W_A$$

$$\mu_s = \frac{m_B + m_C - m_A}{\left(3m_A + m_B + m_C\right)\sqrt{3}} = \frac{1.5 \text{ kg} + 4 \text{ kg} - 2 \text{ kg}}{\left(6 \text{ kg} + 1.5 \text{ kg} + 4 \text{ kg}\right)\sqrt{3}}$$

$$\mu_s = 0.1757 \blacktriangleleft$$

PROBLEM 8.42 CONTINUED

FBD B:



$$\nwarrow \Sigma F_{y'} = 0: \quad N_{BC} - N_{AB} - W_B \cos 30° = 0$$

or $$N_{BC} = \frac{\sqrt{3}}{2}(W_A + W_B)$$

$$(F_{BC})_{\max} = \mu_s N_{BC} = 0.1757 \frac{\sqrt{3}}{2}(W_A + W_B)$$

$$= 0.1522(m_A + m_B)g = 0.1522(3.5 \text{ kg})(9.81 \text{ m/s}^2)$$

$$= 5.224 \text{ N}$$

$$\nearrow \Sigma F_{x'} = 0: \quad F_{AB} + F_{BC} - W_B \sin 30° = 0$$

or $$F_{BC} = -F_{AB} + \frac{1}{2}W_B = -\frac{\sqrt{3}}{2}W_A(0.1757) + \frac{W_B}{2}$$

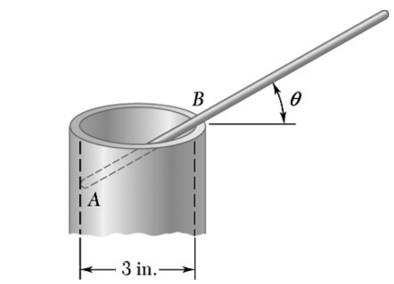
$$= (-0.1522 m_A + 0.5 m_B)g$$

$$= [-0.1522(2 \text{ kg}) + 0.5(1.5 \text{ kg})](9.81 \text{ m/s}^2)$$

$$= 4.37 \text{ N}$$

$$F_{BC} < F_{BC\max} \qquad \text{OK}$$

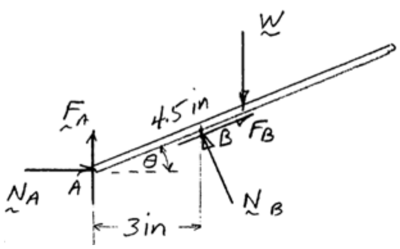
PROBLEM 8.43



A slender steel rod of length 9 in. is placed inside a pipe as shown. Knowing that the coefficient of static friction between the rod and the pipe is 0.20, determine the largest value of θ for which the rod will not fall into the pipe.

SOLUTION

FBD rod:



$$\circlearrowleft \Sigma M_A = 0: \quad \frac{3\text{ in.}}{\cos\theta} N_B - \left[(4.5\text{ in.})\cos\theta\right] W = 0$$

or $$N_B = \left(1.5\cos^2\theta\right)W$$

Impending motion: $$F_B = \mu_s N_B = \left(1.5\mu_s\cos^2\theta\right)W$$

$$= \left(0.3\cos^2\theta\right)W$$

$$\longrightarrow \Sigma F_x = 0: \quad N_A - N_B\sin\theta + F_B\cos\theta = 0$$

or $$N_A = \left(1.5\cos^2\theta\right)W\left(\sin\theta - 0.2\cos\theta\right)$$

Impending motion: $$F_A = \mu_s N_A$$

$$= \left(0.3\cos^2\theta\right)W\left(\sin\theta - 0.2\cos\theta\right)$$

$$\uparrow \Sigma F_y = 0: \quad F_A + N_B\cos\theta + F_B\sin\theta - W = 0$$

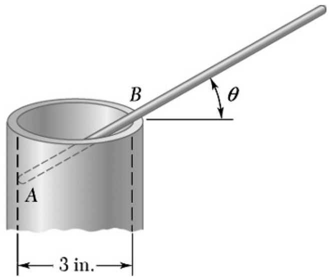
or $$F_A = W\left(1 - 1.5\cos^3\theta - 0.3\cos^2\theta\sin\theta\right)$$

Equating F_A's

$$0.3\cos^2\theta\left(\sin\theta - 0.2\cos\theta\right) = 1 - 1.5\cos^3\theta - 0.3\cos^2\theta\sin\theta$$

$$0.6\cos^2\theta\sin\theta + 1.44\cos^3\theta = 1$$

Solving numerically $\theta = 35.8°$ ◀

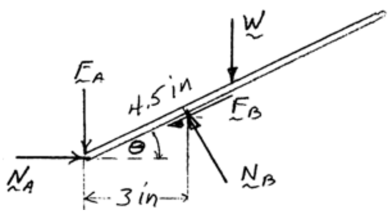


PROBLEM 8.44

In Problem 8.43, determine the smallest value of θ for which the rod will not fall out of the pipe.

SOLUTION

FBD rod:



$$\circlearrowleft \Sigma M_A = 0: \quad \frac{3\text{ in.}}{\cos\theta} N_B - \left[(4.5\text{ in.})\cos\theta\right] W = 0$$

or $$N_B = 1.5W\cos^2\theta$$

Impending motion: $$F_B = \mu_s N_B = 0.2\left(1.5W\cos^2\theta\right)$$

$$= 0.3W\cos^2\theta$$

$$\longrightarrow \Sigma F_x = 0: \quad N_A - N_B\sin\theta - F_B\cos\theta = 0$$

or $$N_A = W\cos^2\theta(1.5\sin\theta + 0.3\cos\theta)$$

Impending motion: $$F_A = \mu_s N_A$$

$$= W\cos^2\theta(0.3\sin\theta + 0.06\cos\theta)$$

$$\uparrow \Sigma F_y = 0: \quad N_B\cos\theta - F_B\sin\theta - W - F_A = 0$$

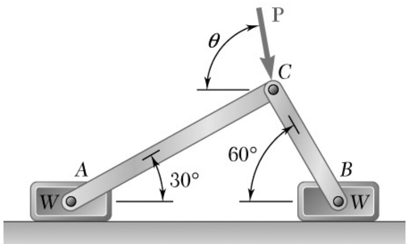
or $$F_A = W\left[\cos^2\theta(1.5\cos\theta - 0.3\sin\theta) - 1\right]$$

Equating F_A's:

$$\cos^2\theta(1.44\cos\theta - 0.6\sin\theta) = 1$$

Solving numerically $\theta = 20.5° \blacktriangleleft$

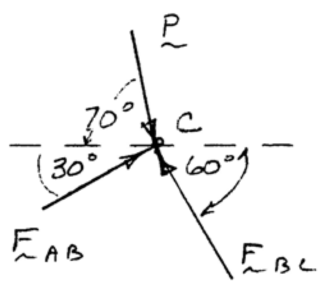
PROBLEM 8.45

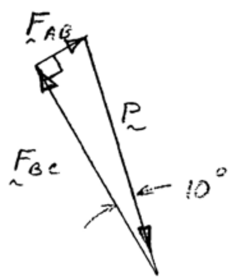


Two slender rods of negligible weight are pin-connected at C and attached to blocks A and B, each of weight W. Knowing that $\theta = 70^\circ$ and that the coefficient of static friction between the blocks and the horizontal surface is 0.30, determine the largest value of P for which equilibrium is maintained.

SOLUTION

FBD pin C:





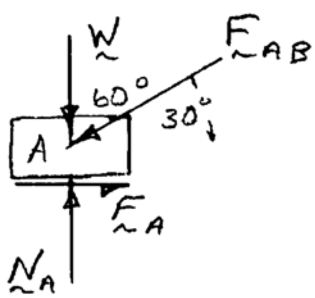
$$F_{AB} = P\sin 10^\circ = 0.173648P$$

$$F_{BC} = P\cos 10^\circ = 0.98481P$$

$$\uparrow \Sigma F_y = 0: \quad N_A - W - F_{AB}\sin 30^\circ = 0$$

or
$$N_A = W + 0.173648P\sin 30^\circ = W + 0.086824P$$

FBD block A:



$$\longrightarrow \Sigma F_x = 0: \quad F_A - F_{AB}\cos 30^\circ = 0$$

or
$$F_A = 0.173648P\cos 30^\circ = 0.150384P$$

For impending motion at A: $\quad F_A = \mu_s N_A$

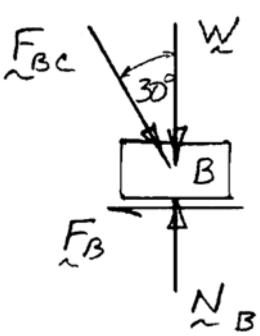
Then
$$N_A = \frac{F_A}{\mu_s}: \quad W + 0.086824P = \frac{0.150384}{0.3}P$$

or
$$P = 2.413W$$

$$\uparrow \Sigma F_y = 0: \quad N_B - W - F_{BC}\cos 30^\circ = 0$$

$$N_B = W + 0.98481P\cos 30^\circ = W + 0.85287P$$

FBD block B:



$$\longrightarrow \Sigma F_x = 0: \quad F_{BC}\sin 30^\circ - F_B = 0$$

$$F_B = 0.98481P\sin 30^\circ = 0.4924P$$

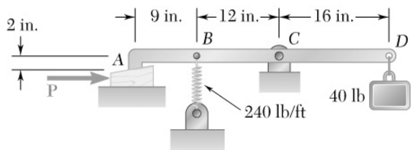
For impending motion at B: $\quad F_B = \mu_s N_B$

Then
$$N_B = \frac{F_B}{\mu_s}: \quad W + 0.85287P = \frac{0.4924P}{0.3}$$

or
$$P = 1.268W$$

Thus, maximum P for equilibrium $\quad P_{max} = 1.268W$ ◀

PROBLEM 8.46

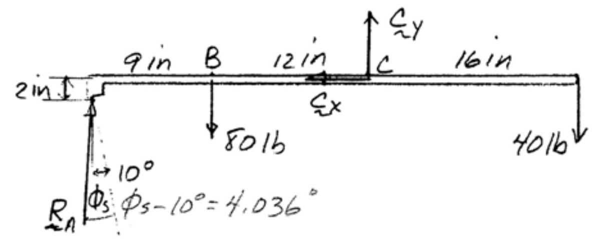


A 40-lb weight is hung from a lever which rests against a 10° wedge at A and is supported by a frictionless hinge at C. Knowing that the coefficient of static friction is 0.25 at both surfaces of the wedge and that for the position shown the spring is stretched 4 in., determine (a) the magnitude of the force **P** for which motion of the wedge is impending, (b) the components of the corresponding reaction at C.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.25 = 14.036° \qquad F_s = kx = (240 \text{ lb/ft})\left(\frac{4 \text{ in.}}{12 \text{ in./ft}}\right) = 80 \text{ lb}$$

FBD lever:



$$\circlearrowleft \Sigma M_C = 0\text{:} \quad (12 \text{ in.})(80 \text{ lb}) - (16 \text{ in.})(40 \text{ lb}) - (21 \text{ in.})R_A\cos(\phi_s - 10°)$$

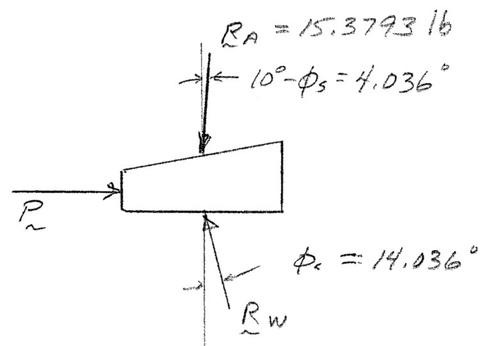
$$+ (2 \text{ in.})R_A\sin(\phi_s - 10°) = 0$$

or $\quad R_A = 15.3793 \text{ lb}$

(b) $\quad \rightarrow \Sigma F_x = 0\text{:} \quad (15.379 \text{ lb})\sin(4.036°) - C_x = 0 \qquad$ $\mathbf{C}_x = 1.082 \text{ lb} \longleftarrow$ ◀

$\uparrow \Sigma F_y = 0\text{:} \quad (15.379 \text{ lb})\cos(4.036°) - 80 \text{ lb} - 40 \text{ lb} + C_y = 0 \qquad$ $\mathbf{C}_y = 104.7 \text{ lb} \uparrow$ ◀

FBD wedge:

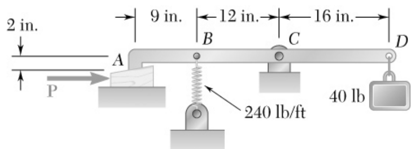


$$\uparrow \Sigma F_y = 0\text{:} \quad R_W\cos 14.036° - (15.3793 \text{ lb})\cos 4.036° = 0$$

or $\quad R_W = 15.8133 \text{ lb}$

(a) $\quad \rightarrow \Sigma F_x = 0\text{:} \quad P - (15.3793 \text{ lb})\sin 4.036° - (15.8133 \text{ lb})\sin 14.036° = 0$

$P = 4.92 \text{ lb}$ ◀



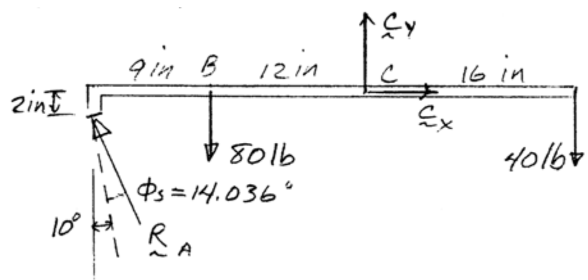
PROBLEM 8.47

Solve Problem 8.46 assuming that force **P** is directed to the left.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.25 = 14.036° \qquad F_s = kx = (240 \text{ lb/ft})\left(\frac{4 \text{ in.}}{12 \text{ in./ft}}\right) = 80 \text{ lb}$$

FBD lever:



$$\circlearrowleft \Sigma M_C = 0: \quad (12 \text{ in.})(80 \text{ lb}) - (16 \text{ in.})(40 \text{ lb}) - (21 \text{ in.})R_A\cos 24.036°$$

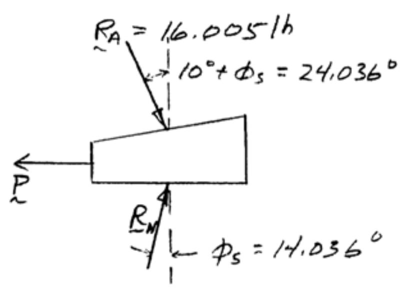
$$- (2 \text{ in.})R_A\sin 24.036° = 0$$

or $\qquad R_A = 16.005$ lb

(*b*) $\qquad \xrightarrow{} \Sigma F_x = 0: \quad C_x - (16.005 \text{ lb})\sin 24.036° = 0 \qquad\qquad$ $\mathbf{C}_x = 6.52 \text{ lb} \longrightarrow$ ◀

$\uparrow \Sigma F_y = 0: \quad C_y - 80 \text{ lb} - 40 \text{ lb} + (16.005 \text{ lb})\cos(24.036°) = 0 \qquad\qquad$ $\mathbf{C}_y = 105.4 \text{ lb} \uparrow$ ◀

FBD wedge:



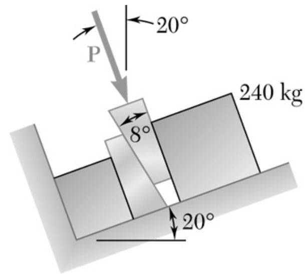
$$\uparrow \Sigma F_y = 0: \quad R_W\cos 14.036° - (16.005 \text{ lb})\cos 24.036° = 0$$

or $\qquad R_W = 15.067$ lb

(*a*) $\qquad \xrightarrow{} \Sigma F_x = 0: \quad (16.005 \text{ lb})\sin 24.036° + (15.067 \text{ lb})\sin 14.036° - P = 0$

$P = 10.17$ lb ◀

PROBLEM 8.48

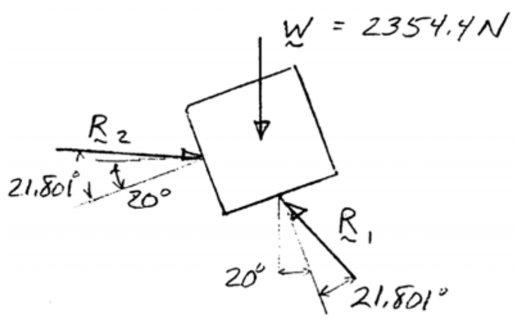


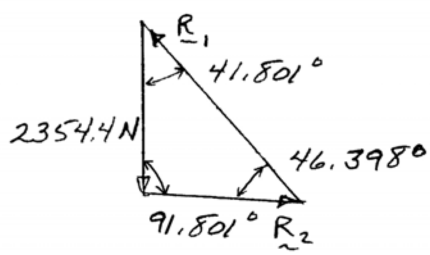
Two 8° wedges of negligible mass are used to move and position a 240-kg block. Knowing that the coefficient of static friction is 0.40 at all surfaces of contact, determine the magnitude of the force **P** for which motion of the block is impending.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.4 = 21.801^\circ \qquad W = 240\text{ kg}\left(9.81\text{ m/s}^2\right) = 2354.4\text{ N}$$

FBD block:

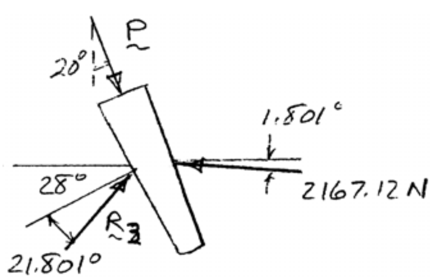


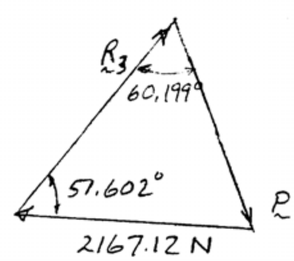


$$\frac{R_2}{\sin 41.801^\circ} = \frac{2354.4\text{ N}}{\sin 46.398^\circ}$$

$$R_2 = 2167.12\text{ N}$$

FBD wedge:

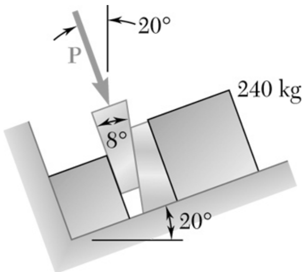




$$\frac{P}{\sin 51.602^\circ} = \frac{2167.12\text{ N}}{\sin 60.199^\circ}$$

$P = 1957$ N

$P = 1.957$ kN ◀



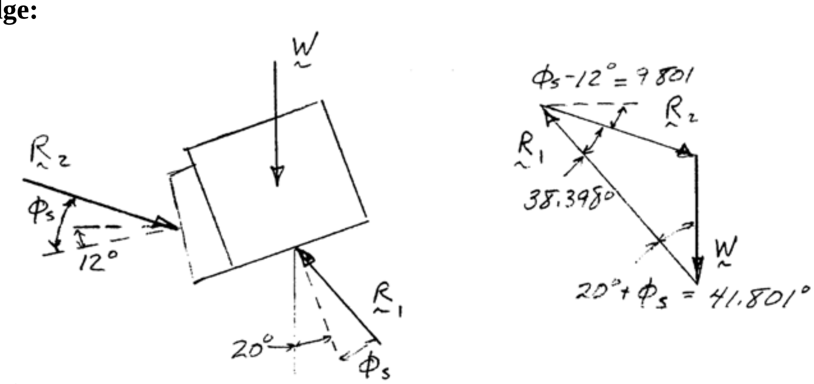
PROBLEM 8.49

Two 8° wedges of negligible mass are used to move and position a 240-kg block. Knowing that the coefficient of static friction is 0.40 at all surfaces of contact, determine the magnitude of the force **P** for which motion of the block is impending.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.4 = 21.801° \qquad W = 240\text{ kg}\left(9.81\text{ m/s}^2\right) = 2354.4\text{ N}$$

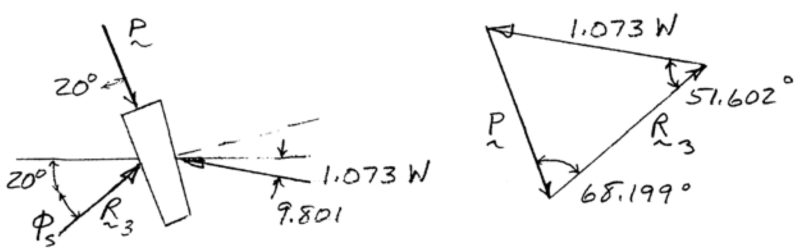
FBD block + wedge:



$$\frac{R_2}{\sin 41.801°} = \frac{2354.4\text{ N}}{\sin 38.398°}$$

$$R_2 = 2526.6\text{ N}$$

FBD wedge:

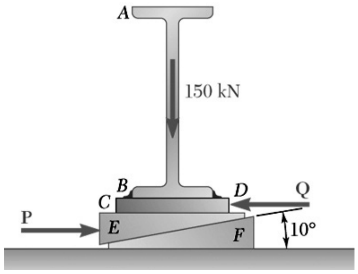


$$\frac{P}{\sin 51.602°} = \frac{2526.6\text{ N}}{\sin 68.199°}$$

$$P = 2132.7\text{ N}$$

$P = 2.13$ kN ◀

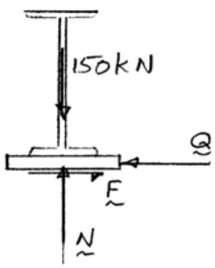
PROBLEM 8.50



The elevation of the end of the steel beam supported by a concrete floor is adjusted by means of the steel wedges *E* and *F*. The base plate *CD* has been welded to the lower flange of the beam, and the end reaction of the beam is known to be 150 kN. The coefficient of static friction is 0.30 between the two steel surfaces and 0.60 between the steel and the concrete. If the horizontal motion of the beam is prevented by the force **Q**, determine (*a*) the force **P** required to raise the beam, (*b*) the corresponding force **Q**.

SOLUTION

FBD AB + CD:



$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.3 = 16.70° \text{ for steel on steel}$$

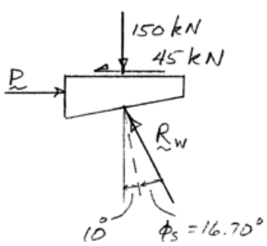
$$\uparrow \Sigma F_y = 0: \quad N - 150\text{ kN} = 0 \qquad N = 150\text{ kN}$$

Impending motion: $\quad F = \mu_s N = 0.3(150\text{ kN}) = 45\text{ kN}$

$$\longrightarrow \Sigma F_x = 0: \quad F - Q = 0$$

$$(b)\ \mathbf{Q} = 45.0\text{ kN} \longleftarrow \blacktriangleleft$$

FBD top wedge:



Assume bottom wedge doesn't move:

$$\uparrow \Sigma F_y = 0: \quad R_W \cos(10° + 16.70°) - 150\text{ kN} = 0$$

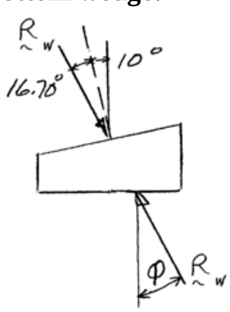
$$R_W = 167.9\text{ kN}$$

$$\longrightarrow \Sigma F_x = 0: \quad P - 45\text{ kN} - (167.9\text{ kN})\sin 26.70° = 0$$

$$P = 120.44\text{ kN}$$

$$(a)\ \mathbf{P} = 120.4\text{ kN} \longrightarrow \blacktriangleleft$$

FBD bottom wedge:

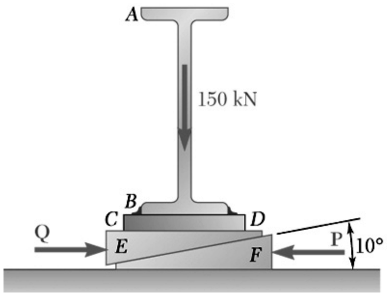


Bottom wedge is two-force member, so $\phi = 26.70°$ for equilibrium, but

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.6 = 31.0° \ (\text{steel on concrete})$$

So $\qquad \phi < \phi_s \qquad$ OK.

PROBLEM 8.51

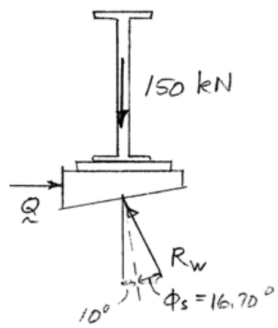


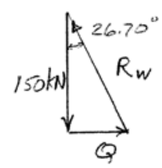
The elevation of the end of the steel beam supported by a concrete floor is adjusted by means of the steel wedges *E* and *F*. The base plate *CD* has been welded to the lower flange of the beam, and the end reaction of the beam is known to be 150 kN. The coefficient of static friction is 0.30 between the two steel surfaces and 0.60 between the steel and the concrete. If the horizontal motion of the beam is prevented by the force **Q**, determine (*a*) the force **P** required to raise the beam, (*b*) the corresponding force **Q**.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.3 = 16.70° \text{ for steel on steel}$$

FBD AB + CD + top wedge: Assume top wedge doesn't move



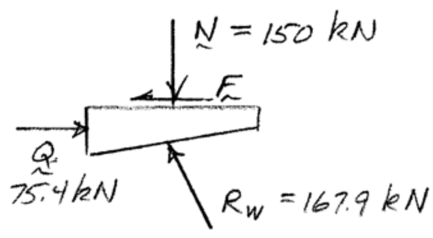


$$R_w = \frac{150 \text{ kN}}{\cos 26.70°} = 167.90 \text{ kN}$$

$$Q = (150 \text{ kN})\tan 26.70° = 75.44 \text{ kN}$$

(*b*) $\mathbf{Q} = 75.4 \text{ kN} \longrightarrow$ ◀

FBD top wedge:

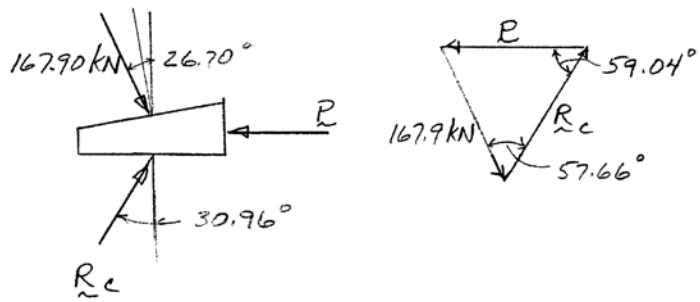


$$\longrightarrow \Sigma F_x = 0: \quad 75.44 \text{ kN} - 167.9 \text{ kN} \sin 26.70° - F = 0$$

$F = 0$ as expected.

PROBLEM 8.51 CONTINUED

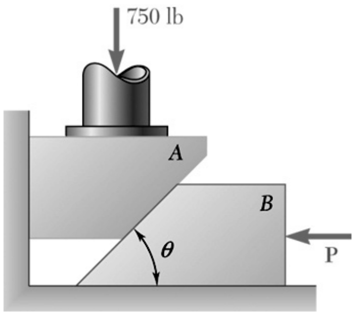
FBD bottom wedge: $\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.6 = 30.96°$ steel on concrete



$$\frac{P}{\sin 57.66°} = \frac{167.90\text{ kN}}{\sin 59.04°}$$

(*a*) **P** = 165.4 kN ⟵ ◀

PROBLEM 8.52

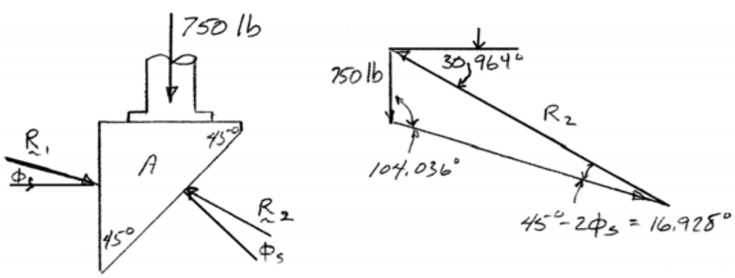


Block *A* supports a pipe column and rests as shown on wedge *B*. Knowing that the coefficient of static friction at all surfaces of contact is 0.25 and that $\theta = 45°$, determine the smallest force **P** required to raise block *A*.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.25 = 14.036°$$

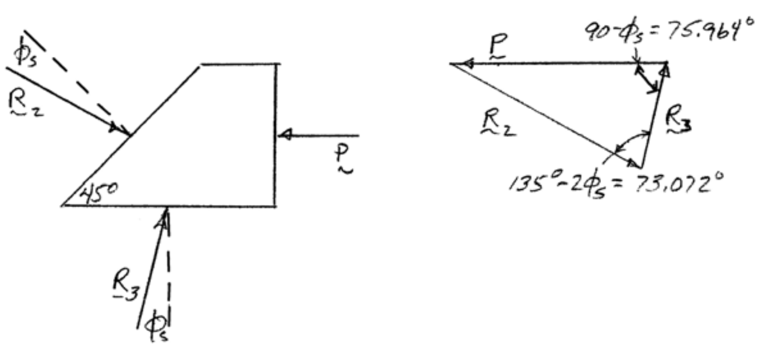
FBD block A:



$$\frac{R_2}{\sin 104.036°} = \frac{750 \text{ lb}}{\sin 16.928°}$$

$$R_2 = 2499.0 \text{ lb}$$

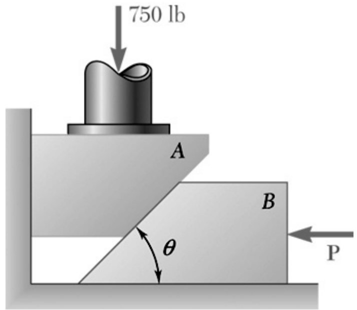
FBD wedge B:



$$\frac{P}{\sin 73.072°} = \frac{2499.0}{\sin 75.964°}$$

$$P = 2464 \text{ lb}$$

$\mathbf{P} = 2.46$ kips ⟵ ◀



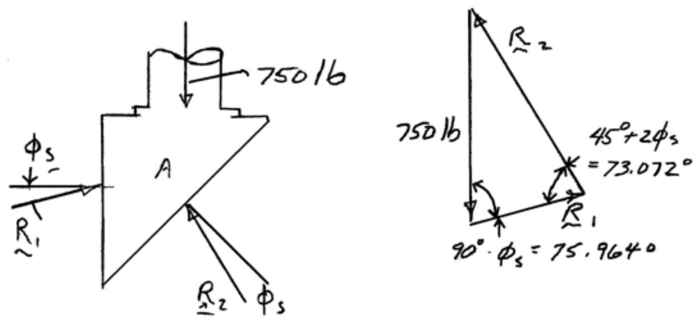
PROBLEM 8.53

Block A supports a pipe column and rests as shown on wedge B. Knowing that the coefficient of static friction at all surfaces of contact is 0.25 and that $\theta = 45°$, determine the smallest force **P** for which equilibrium is maintained.

SOLUTION

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.25 = 14.036°$$

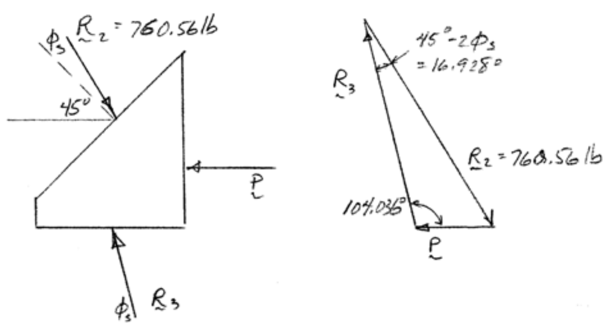
FBD block A:



$$\frac{R_2}{\sin(75.964°)} = \frac{750\text{ lb}}{\sin(73.072°)}$$

$$R_2 = 760.56\text{ lb}$$

FBD wedge B:

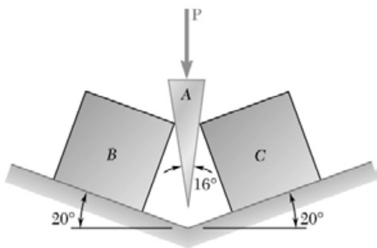


$$\frac{P}{\sin 16.928°} = \frac{760.56}{\sin 104.036°}$$

$$P = 228.3\text{ lb}$$

$\mathbf{P} = 228\text{ lb} \longleftarrow$ ◀

PROBLEM 8.54



A 16° wedge *A* of negligible mass is placed between two 80-kg blocks *B* and *C* which are at rest on inclined surfaces as shown. The coefficient of static friction is 0.40 between both the wedge and the blocks and block *C* and the incline. Determine the magnitude of the force **P** for which motion of the wedge is impending when the coefficient of static friction between block *B* and the incline is (*a*) 0.40, (*b*) 0.60.

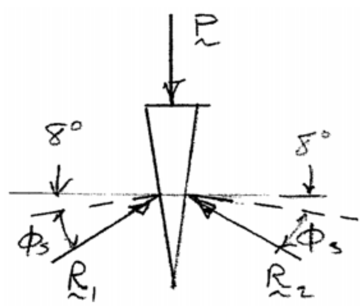
SOLUTION

(*a*)

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.4 = 21.8014°;$$

$$W = 80\text{ kg}\left(9.81\text{ m/s}^2\right) = 784.8\text{ N}$$

FBD wedge:



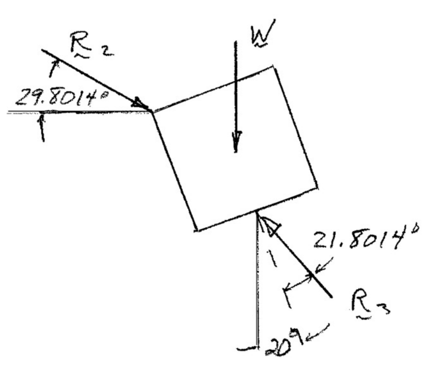
By symmetry:

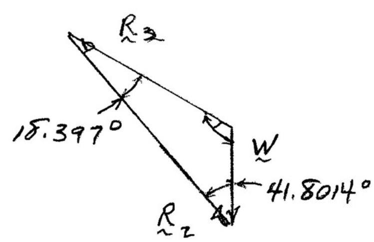
$$\mathbf{R}_1 = \mathbf{R}_2$$

$$\uparrow \Sigma F_y = 0: \quad 2R_2\sin\left(8° + 21.8014°\right) - P = 0$$

$$P = 0.99400R_2$$

FBD block C:





$$\frac{R_2}{\sin 41.8014°} = \frac{W}{\sin 18.397°}$$

$$R_2 = 2.112\,W$$

PROBLEM 8.54 CONTINUED

$$P = 0.994R_2 = (0.994)(2.112W)$$

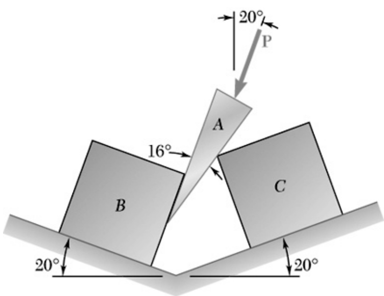
$$P = 2.099(784.8\text{ N}) = 1647.5\text{ N}$$

(*a*) $P = 1.648\text{ kN}$ ◀

(*b*) Note that increasing the friction between block B and the incline has no effect on the above calculations. The physical effect is that slip of B will not impend.

(*b*) $P = 1.648\text{ kN}$ ◀

PROBLEM 8.55

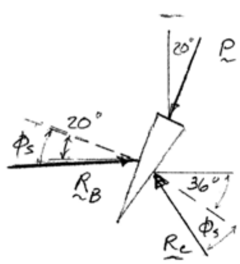


A 16° wedge A of negligible mass is placed between two 80-kg blocks B and C which are at rest on inclined surfaces as shown. The coefficient of static friction is 0.40 between both the wedge and the blocks and block C and the incline. Determine the magnitude of the force **P** for which motion of the wedge is impending when the coefficient of static friction between block B and the incline is (a) 0.40, (b) 0.60.

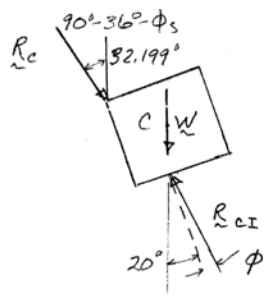
SOLUTION

(a) $\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.4 = 21.801°$ $\qquad W = 80\text{ kg}\left(9.81\text{ m/s}^2\right) = 784.8\text{ N}$

FBD wedge:



FBD block C:

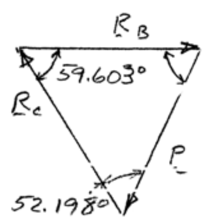


Note that, since $\left(R_{CI}\right)_y > \left(R_C\right)_y$, while the horizontal components are equal,

$$20° + \phi < 32.199°$$

$$\phi < 12.199° < \phi_s$$

Therefore, motion of C is *not* impending; thus, motion of B up the incline is impending.

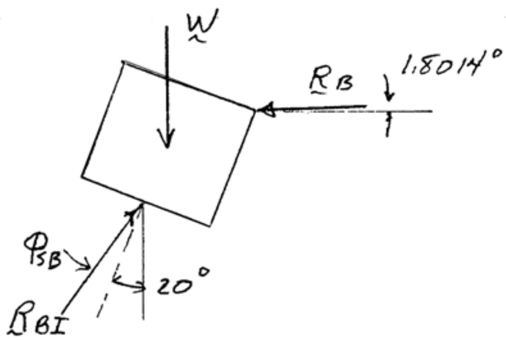


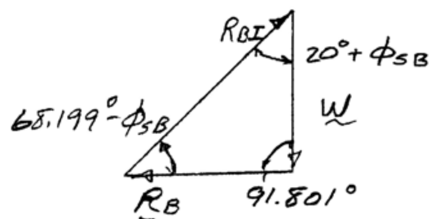
$$\frac{R_B}{\sin 52.198°} = \frac{P}{\sin 59.603°}$$

$$P = 1.0916R_B$$

PROBLEM 8.55 CONTINUED

FBD block B:





$$\frac{R_B}{\sin(20° + \phi_{sB})} = \frac{W}{\sin(68.199° - \phi_{sB})}$$

or
$$R_B = \frac{W\sin(20° + \phi_{sB})}{\sin(68.199° - \phi_{sB})}$$

(a) Have $\phi_{sB} = \phi_s = 21.801°$

Then
$$R_B = \frac{(784.8 \text{ N})\sin(20° + 21.801°)}{\sin(68.199° - 21.801°)} = 722.37 \text{ N}$$

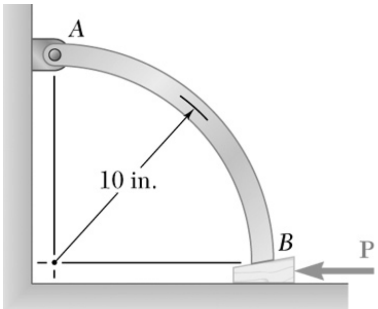
and
$$P = 1.0916(722.37 \text{ N})$$
or $P = 789$ N ◀

(b) Have $\phi_{sB} = \tan^{-1}\mu_{sB} = \tan^{-1}0.6 = 30.964°$

Then
$$R_B = \frac{(784.8 \text{ N})\sin(20° + 30.964°)}{\sin(68.199° - 30.964°)} = 1007.45 \text{ N}$$

and
$$P = 1.0916(1007.45 \text{ N})$$
or $P = 1100$ N ◀

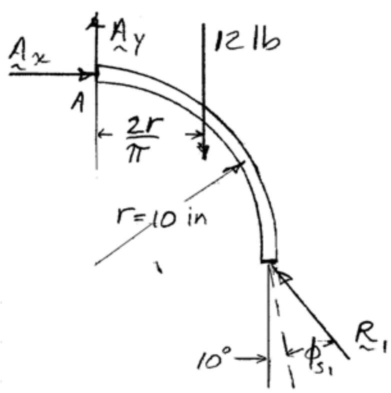
PROBLEM 8.56



A 10° wedge is to be forced under end *B* of the 12-lb rod *AB*. Knowing that the coefficient of static friction is 0.45 between the wedge and the rod and 0.25 between the wedge and the floor, determine the smallest force **P** required to raise end *B* of the rod.

SOLUTION

FBD AB:

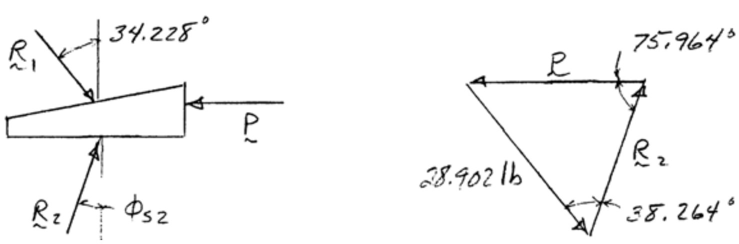


$$\phi_{s1} = \tan^{-1}(\mu_s)_1 = \tan^{-1} 0.45 = 24.228°$$

$$\circlearrowleft \Sigma M_A = 0: \quad rR_1 \cos(10° + 24.228°) - rR_1 \sin(10° + 24.228°) - \frac{2r}{\pi}(12 \text{ lb}) = 0$$

$$R_1 = 28.902 \text{ lb}$$

FBD wedge:

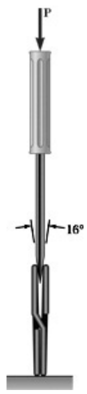


$$\phi_{s2} = \tan^{-1}(\mu_s)_2 = \tan^{-1} 0.25 = 14.036°$$

$$\frac{P}{\sin(38.264°)} = \frac{28.902 \text{ lb}}{\sin 75.964°};$$

$\mathbf{P} = 22.2 \text{ lb} \longleftarrow$ ◀

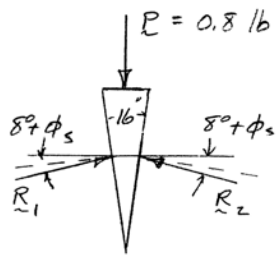
PROBLEM 8.57



A small screwdriver is used to pry apart the two coils of a circular key ring. The wedge angle of the screwdriver blade is 16° and the coefficient of static friction is 0.12 between the coils and the blade. Knowing that a force **P** of magnitude 0.8 lb was required to insert the screwdriver to the equilibrium position shown, determine the magnitude of the forces exerted on the ring by the screwdriver immediately after force **P** is removed.

SOLUTION

FBD wedge:



By symmetry: $R_1 = R_2$

$$\uparrow \Sigma F_y = 0\text{:} \quad 2R_1 \sin(8° + \phi_s) - P = 0$$

Have $\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.12 = 6.843°$ $\quad P = 0.8\text{ lb}$

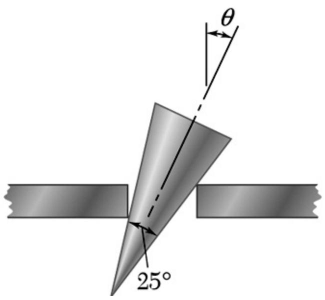
So $R_1 = R_2 = 1.5615\text{ lb}$

When **P** is removed, the vertical components of R_1 and R_2 vanish, leaving the horizontal components, $R_1\cos(14.843°)$, only

Therefore, side forces are 1.509 lb ◀

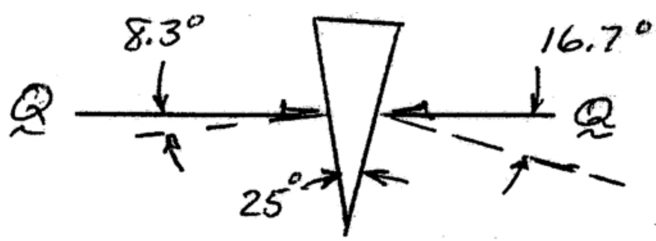
But these will occur only instantaneously as the angle between the force and the wedge normal is $8° > \phi_s = 6.84°$, so the screwdriver will slip out.

PROBLEM 8.58



A conical wedge is placed between two horizontal plates that are then slowly moved toward each other. Indicate what will happen to the wedge (a) if $\mu_s = 0.20$, (b) if $\mu_s = 0.30$.

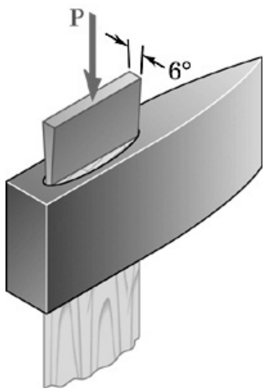
SOLUTION



As the plates are moved, the angle θ will decrease.

(a) $\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.2 = 11.31°$. As θ decreases, the minimum angle at the contact approaches $12.5° > \phi_s = 11.31°$, so the wedge will slide up and out from the slot. ◀

(b) $\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.3 = 16.70°$. As θ decreases, the angle at one contact reaches $16.7°$. (At this time the angle at the other contact is $25° - 16.7° = 8.3° < \phi_s$) The wedge binds in the slot. ◀

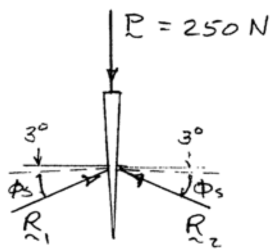


PROBLEM 8.59

A 6° steel wedge is driven into the end of an ax handle to lock the handle to the ax head. The coefficient of static friction between the wedge and the handle is 0.35. Knowing that a force **P** of magnitude 250 N was required to insert the wedge to the equilibrium position shown, determine the magnitude of the forces exerted on the handle by the wedge after force **P** is removed.

SOLUTION

FBD wedge:



By symmetry $R_1 = R_2$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.35 = 19.29°$$

$$\uparrow \Sigma F_y = 0: \quad 2R\sin(19.29° + 3°) - P = 0$$

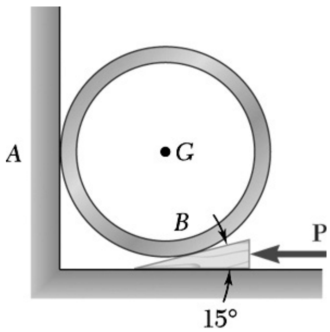
$$R_1 = R_2 = 329.56 \text{ N}$$

When force **P** is removed, the vertical components of R_1 and R_2 vanish, leaving only the horizontal components $H_1 = H_2 = R\cos(22.29°)$

$$H_1 = H_2 = 305 \text{ N} \blacktriangleleft$$

Since the wedge angle $3° < \phi_s = 19.3°$, the wedge is "self-locking" and will remain seated.

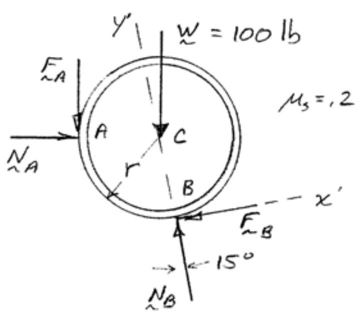
PROBLEM 8.60



A 15° wedge is forced under a 100-lb pipe as shown. The coefficient of static friction at all surfaces is 0.20. Determine (*a*) at which surface slipping of the pipe will first occur, (*b*) the force **P** for which motion of the wedge is impending.

SOLUTION

FBD pipe:



(*a*) $\Sigma M_C = 0{:}\quad rF_A - rF_B = 0$

or $$F_A = F_B$$

But it is apparent that $N_B > N_A$, so since $(\mu_s)_A = (\mu_s)_B$,

motion must first impend at A ◀

and $$F_B = F_A = \mu_s N_A = 0.2N_A$$

(*b*) $\Sigma M_B = 0{:}\quad (r\sin 15°)W + r(1+\sin 15°)F_A - (r\cos 15°)N_A = 0$

$$0.2588(100\text{ lb}) + 1.2588(0.2N_A) - 0.9659N_A = 0$$

or $$N_A = 36.24\text{ lb}$$

and $$F_A = 7.25\text{ lb}$$

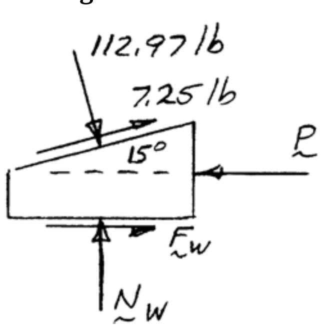
$\Sigma F_{y'} = 0{:}\quad N_B - N_A\sin 15° - F_A\cos 15° - W\cos 15° = 0$

$$N_B = (36.24\text{ lb})\sin 15° + (7.25\text{ lb} + 100\text{ lb})\cos 15°$$

$$= 112.97\text{ lb}$$

(note $N_B > N_A$ as stated, and $F_B < \mu_s N_B$)

FBD wedge:



$\Sigma F_y = 0{:}\quad N_W + (7.25\text{ lb})\sin 15° - (112.97\text{ lb})\cos 15° = 0$

$$N_W = 107.24\text{ lb}$$

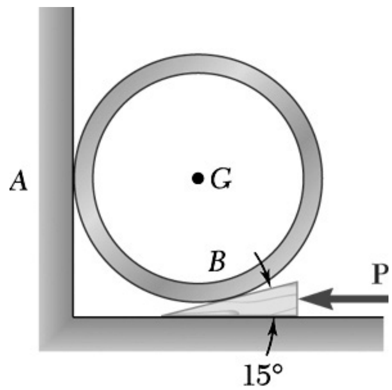
Impending slip: $F_W = \mu_s N_W = 0.2(107.24) = 21.45\text{ lb}$

$\Sigma F_x = 0{:}\quad 21.45\text{ lb} + (7.25\text{ lb})\cos 15° + (112.97\text{ lb})\sin 15° - P = 0$

$\mathbf{P} = 57.7\text{ lb} \longleftarrow$ ◀

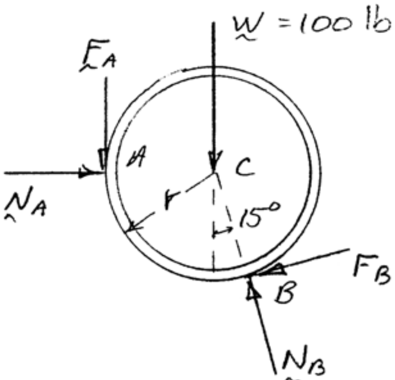
PROBLEM 8.61

A 15° wedge is forced under a 100-lb pipe as shown. Knowing that the coefficient of static friction at both surfaces of the wedge is 0.20, determine the largest coefficient of static friction between the pipe and the vertical wall for which slipping is impending at *A*.



SOLUTION

FBD pipe:



$$\circlearrowleft \Sigma M_C = 0: \quad rF_A - rF_B = 0$$

or $$F_A = F_B$$

It is apparent that $N_B > N_A$, so if $(\mu_s)_A = (\mu_s)_B$, motion must impend first at *A*. As $(\mu_s)_A$ is increased to some $(\mu_s^*)_A$, motion will impend simultaneously at *A* and *B*.

Then $$F_A = F_B = \mu_{sB} N_B = 0.2N_B$$

$$\uparrow \Sigma F_y = 0: \quad N_B \cos 15° - F_B \sin 15° - F_A - 100 \text{ lb} = 0$$

$$N_B \cos 15° - 0.2N_B \sin 15° - 0.2N_B = 100 \text{ lb}$$

or $$N_B = 140.024 \text{ lb}$$

So $$F_A = F_B = 0.2N_B = 28.005 \text{ lb}$$

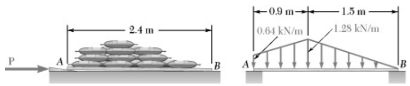
$$\longrightarrow \Sigma F_x = 0: \quad N_A - N_B \sin 15° - F_B \cos 15° = 0$$

$$N_A = 140.024 \sin 15° + 28.005 \cos 15° = 63.29 \text{ lb}$$

Then $$(\mu_s^*)_A = \frac{F_A}{N_A} = \frac{28.005 \text{ lb}}{63.29 \text{ lb}}$$

or $$(\mu_s^*)_A = 0.442 \blacktriangleleft$$

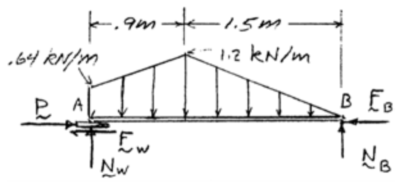
PROBLEM 8.62



Bags of grass seed are stored on a wooden plank as shown. To move the plank, a 9° wedge is driven under end *A*. Knowing that the weight of the grass seed can be represented by the distributed load shown and that the coefficient of static friction is 0.45 between all surfaces of contact, (*a*) determine the force **P** for which motion of the wedge is impending, (*b*) indicate whether the plank will slide on the floor.

SOLUTION

FBD plank + wedge:



(*a*) $\Sigma M_A = 0$: $(2.4 \text{ m})N_B - (0.45 \text{ m})(0.64 \text{ kN/m})(0.9 \text{ m})$

$$-(0.6 \text{ m})\frac{1}{2}(0.64 \text{ kN/m})(0.9 \text{ m})$$

$$-(1.4 \text{ m})\frac{1}{2}(1.28 \text{ kN/m})(1.5 \text{ m}) = 0$$

or $$N_B = 0.740 \text{ kN} = 740 \text{ N}$$

$\Sigma F_y = 0$: $N_W - (0.64 \text{ kN/m})(0.9 \text{ m}) - \frac{1}{2}(0.64 \text{ kN/m})(0.9 \text{ m})$

$$-\frac{1}{2}(1.28 \text{ kN/m})(1.5 \text{ m}) = 0$$

or $$N_W = 1.084 \text{ kN} = 1084 \text{ N}$$

Assume impending motion of the wedge on the floor and the plank on the floor at *B*.

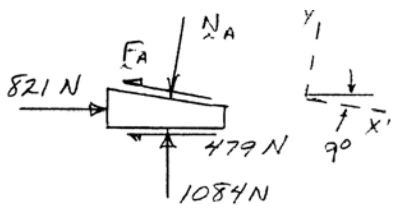
So $$F_W = \mu_s N_W = 0.45(1084 \text{ N}) = 478.8 \text{ N}$$

and $$F_B = \mu_s N_B = 0.45(740 \text{ N}) = 333 \text{ N}$$

$\rightarrow \Sigma F_x = 0$: $P - F_W - F_B = 0$

or $$P = 478.8 \text{ N} + 333 \text{ N} \qquad P = 821 \text{ N} \blacktriangleleft$$

Check wedge:



(*b*) $\Sigma F_y = 0$: $(1084 \text{ N})\cos 9° + (821 \text{ N} - 479 \text{ N})\sin 9° - N_A = 0$

or $$N_A = 1124 \text{ N}$$

$\Sigma F_x = 0$: $(821 \text{ N} - 479 \text{ N})\cos 9° - (1084 \text{ N})\sin 9° - F_A = 0$

or $$F_A = 168 \text{ N}$$

$$F_A < \mu_s N_A = 0.45(1124 \text{ N}) = 506 \text{ N}$$

So, no impending motion at wedge/plank

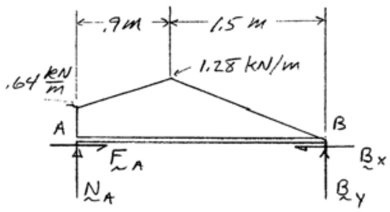
∴ Impending motion of plank on floor at *B* ◀

PROBLEM 8.63

Solve Problem 8.62 assuming that the wedge is driven under the plank at *B* instead of at *A*.

SOLUTION

FBD plank:



$\longrightarrow \Sigma F_x = 0$: $F_A - B_x = 0$

$$F_A = B_x$$

(*a*) $\circlearrowleft \Sigma M_A = 0$: $(2.4 \text{ m})B_y - (0.45 \text{ m})(0.64 \text{ kN/m})(0.9 \text{ m})$

$$- (0.6 \text{ m})\frac{1}{2}(0.64 \text{ kN/m})(0.9 \text{ m})$$

$$- (1.4 \text{ m})\frac{1}{2}(1.28 \text{ kN/m})(1.5 \text{ m}) = 0$$

or $\quad B_y = 0.740 \text{ kN} = 740 \text{ N}$

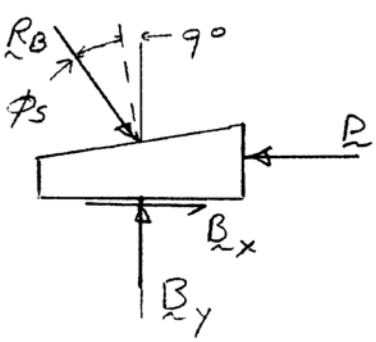
$\uparrow \Sigma F_y = 0$: $N_A - (0.64 \text{ kN/m})(0.9 \text{ m}) - \frac{1}{2}(0.64 \text{ kN/m})(0.9 \text{ m})$

$$-\frac{1}{2}(1.28 \text{ kN/m})(1.5 \text{ m}) = 0$$

or $\quad N_A = 1.084 \text{ kN} = 1084 \text{ N}$

Since $B_y < N_A$, assume impending motion of the wedge under the plank at *B*.

FBD wedge:



$(R_B)_y = B_y = 740 \text{ N}$ and $B_x = \mu_s B_y = 0.45(740 \text{ N}) = 333 \text{ N}$

$$(R_B)_x = (R_B)_y \tan(9° + \phi_s)$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1} 0.45 = 24.228°$$

So $\quad (R_B)_x = (740 \text{ N})\tan(9° + 24.228°) = 485 \text{ N}$

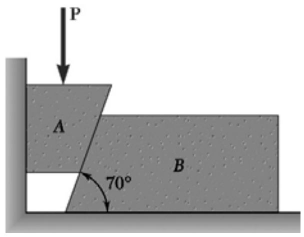
$\longrightarrow \Sigma F_x = 0$: $485 \text{ N} - 333 \text{ N} - P = 0$

$$\mathbf{P} = 818 \text{ N} \longleftarrow \blacktriangleleft$$

(*b*) Check:

$F_A = B_x = 333 \text{ N}$ and $\dfrac{F_A}{N_A} = \dfrac{333}{1084} = 0.307 < \mu_s \quad$ OK

No impending slip of plank at *A* ◀

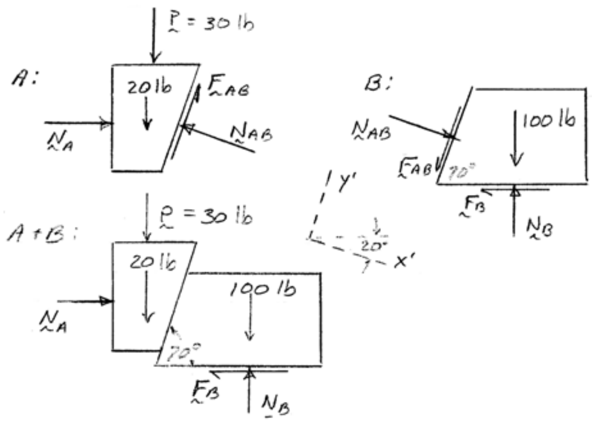


PROBLEM 8.64

The 20-lb block A is at rest against the 100-lb block B as shown. The coefficient of static friction μ_s is the same between blocks A and B and between block B and the floor, while friction between block A and the wall can be neglected. Knowing that $P = 30$ lb, determine the value of μ_s for which motion is impending.

SOLUTION

FBD's:



Impending motion at all surfaces $\qquad F_{AB} = \mu_s N_{AB}$

$$F_B = \mu_s N_B$$

$A + B$: $\quad \uparrow \Sigma F_y = 0: \quad N_B - 30 \text{ lb} - 20 \text{ lb} - 100 \text{ lb} = 0$

or $\qquad N_B = 150 \text{ lb}$

and $\qquad F_B = \mu_s N_B = (150 \text{ lb})\mu_s$

$\longrightarrow \Sigma F_x = 0: \quad N_A - F_B = 0 \qquad$ so that $\qquad N_A = (150 \text{ lb})\mu_s$

A: $\quad \searrow \Sigma F_{x'} = 0: \quad N_A \cos 20° + (30 \text{ lb} + 20 \text{ lb})\sin 20° - N_{AB} = 0$

or $\qquad N_{AB} = 17.1010 \text{ lb} + \mu_s(140.954 \text{ lb})$

$\nearrow \Sigma F_{y'} = 0: \quad F_{AB} + N_A \sin 20° - (30 \text{ lb} + 20 \text{ lb})\cos 20° = 0$

or $\qquad F_{AB} = 46.985 \text{ lb} - \mu_s(51.303 \text{ lb})$

But $\qquad F_{AB} = \mu_s N_{AB}: \quad 46.985 - 51.303\mu_s = 17.101\mu_s + 140.954\mu_s^2$

$$\mu_s^2 + 0.4853\mu_s - 0.3333 = 0$$

$$\mu_s = -0.2427 \pm 0.6263$$

$\mu_s > 0 \qquad$ so $\qquad \mu_s = 0.384$ ◀

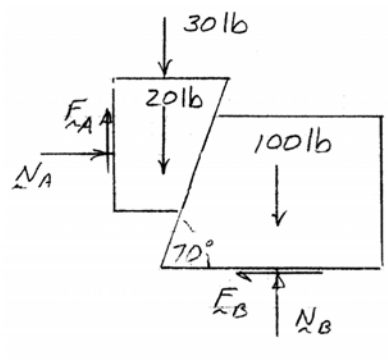
PROBLEM 8.65

Solve Problem 8.64 assuming that μ_s is the coefficient of static friction between all surfaces of contact.

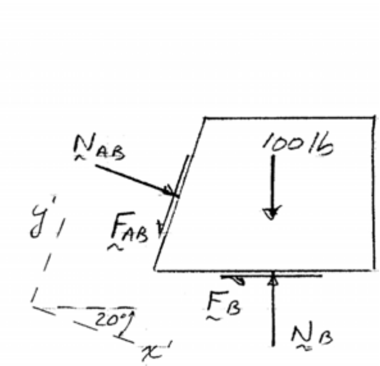
SOLUTION

FBD's:

$A + B$:



B:



Impending motion at all surfaces, so

$$F_A = \mu_s N_A$$

$$F_B = \mu_s N_B$$

$$F_{AB} = \mu_s N_{AB}$$

$A + B$: $\longrightarrow \Sigma F_x = 0$: $N_A - F_B = 0$ or $N_A = F_B = \mu_s N_B$

$\uparrow \Sigma F_y = 0$: $F_A - 30\text{ lb} - 20\text{ lb} - 100\text{ lb} + N_B = 0$ or $\mu_s N_A + N_B = 150\text{ lb}$

So

$$N_B = \frac{150\text{ lb}}{1 + \mu_s^2} \quad \text{and} \quad F_B = \frac{\mu_s}{1 + \mu_s^2}(150\text{ lb})$$

B: $\searrow \Sigma F_{x'} = 0$: $N_{AB} + (100\text{ lb} - N_B)\sin 20° - F_B \cos 20° = 0$

or

$$N_{AB} = N_B \sin 20° + F_B \cos 20° - (100\text{ lb})\sin 20°$$

$\nearrow \Sigma F_{y'} = 0$: $-F_{AB} + (N_B - 100\text{ lb})\cos 20° - F_B \sin 20° = 0$

or

$$F_{AB} = N_B \cos 20° - F_B \sin 20° - (100\text{ lb})\cos 20°$$

PROBLEM 8.65 CONTINUED

Now $\quad F_{AB} = \mu_s N_{AB}: \quad \frac{150\text{ lb}}{1+\mu_s^2}\cos 20° - \frac{\mu_s}{1+\mu_s^2}(150\text{ lb})\sin 20° - (100\text{ lb})\cos 20°$

$$= \frac{\mu_s}{1+\mu_s^2}(150\text{ lb})\sin 20° + \frac{\mu_s^2}{1+\mu_s^2}(150\text{ lb})\cos 20° - \mu_s(100\text{ lb})\sin 20°$$

$$2\mu_s^3 - 5\mu_s^2\text{ctn}\,20° - 4\mu_s + \text{ctn}\,20° = 0$$

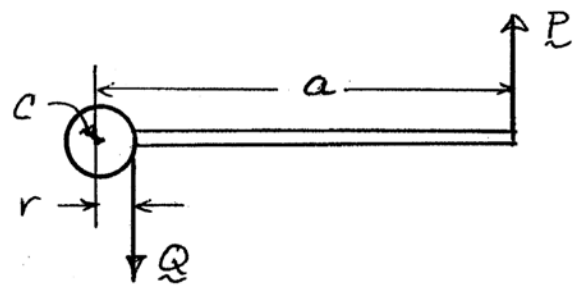
Solving numerically: $\mu_s = 0.330$ ◀

PROBLEM 8.66

Derive the following formulas relating the load **W** and the force **P** exerted on the handle of the jack discussed in Section 8.6. (*a*) $P = (Wr/a)\tan(\theta + \phi_s)$, to raise the load; (*b*) $P = (Wr/a)\tan(\phi_s - \theta)$, to lower the load if the screw is self-locking; (*c*) $P = (Wr/a)\tan(\theta - \phi_s)$, to hold the load if the screw is not self-locking.

SOLUTION

FBD jack handle:

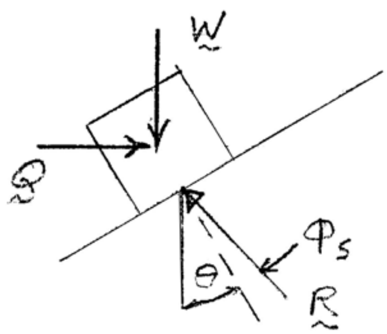


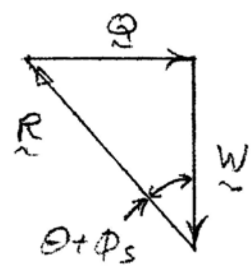
See Section 8.6

$$\circlearrowleft \Sigma M_C = 0: \quad aP - rQ = 0 \text{ or } P = \frac{r}{a}Q$$

FBD block on incline:

(*a*) Raising load



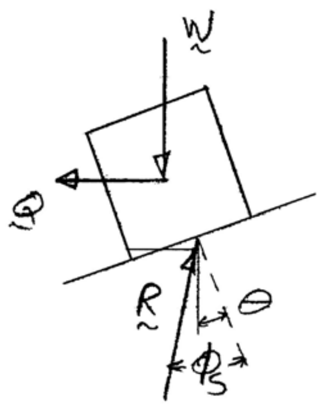


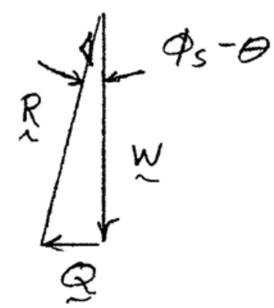
$$Q = W\tan(\theta + \phi_s)$$

$$P = \frac{r}{a}W\tan(\theta + \phi_s) \blacktriangleleft$$

PROBLEM 8.66 CONTINUED

(b) Lowering load if screw is self-locking (i.e.: if $\phi_s > \theta$)

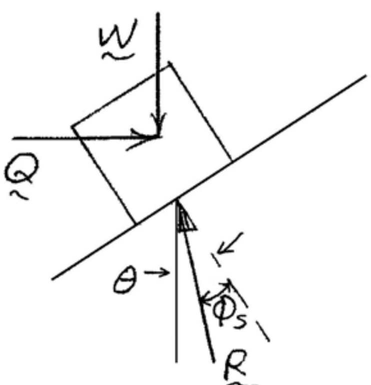


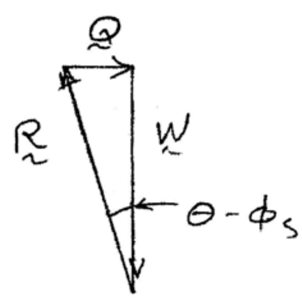


$$Q = W\tan(\phi_s - \theta)$$

$$P = \frac{r}{a}W\tan(\phi_s - \theta) \blacktriangleleft$$

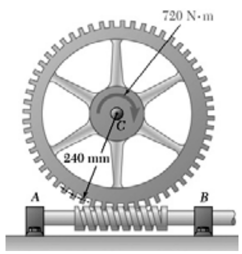
(c) Holding load is screw is not self-locking (i.e. if $\phi_s < \theta$)





$$Q = W\tan(\theta - \phi_s)$$

$$P = \frac{r}{a}W\tan(\theta - \phi_s) \blacktriangleleft$$

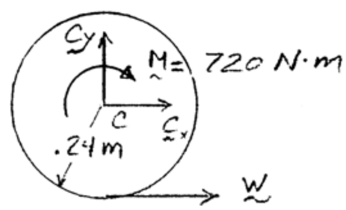


PROBLEM 8.67

The square-threaded worm gear shown has a mean radius of 30 mm and a lead of 7.5 mm. The larger gear is subjected to a constant clockwise couple of 720 N·m. Knowing that the coefficient of static friction between the two gears is 0.12, determine the couple that must be applied to shaft *AB* in order to rotate the large gear counterclockwise. Neglect friction in the bearings at *A*, *B*, and *C*.

SOLUTION

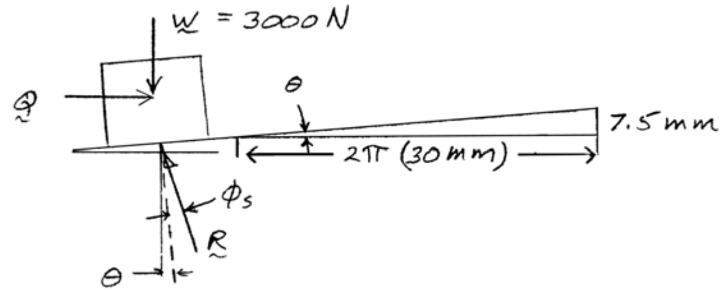
FBD large gear:



$$\Sigma M_C = 0: \quad (0.24 \text{ m})W - 720 \text{ N}\cdot\text{m} = 0$$

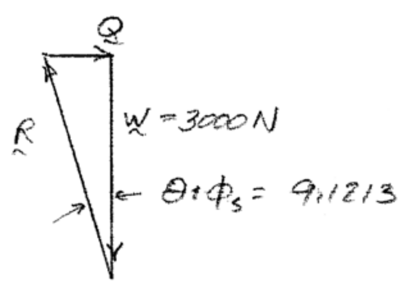
$$W = 3000 \text{ N}$$

Block on incline:



$$\theta = \tan^{-1}\frac{7.5 \text{ mm}}{2\pi(30 \text{ mm})} = 2.2785°$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.12 = 6.8428°$$

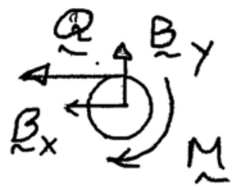


$$Q = (3000 \text{ N})\tan 9.1213°$$

$$= 481.7 \text{ N}$$

PROBLEM 8.67 CONTINUED

Worm gear:



$$r = 30 \text{ mm}$$

$$= 0.030 \text{ m}$$

$$\circlearrowright \Sigma M_B = 0: \quad rQ - M = 0$$

$$M = rQ = (0.030 \text{ m})(481.7 \text{ N})$$

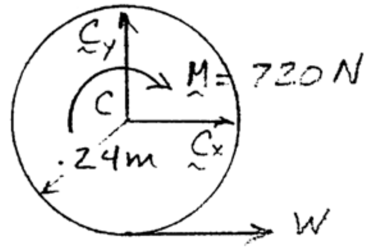
$M = 14.45 \text{ N}\cdot\text{m}$ ◀

PROBLEM 8.68

In Problem 8.67, determine the couple that must be applied to shaft *AB* in order to rotate the gear clockwise.

SOLUTION

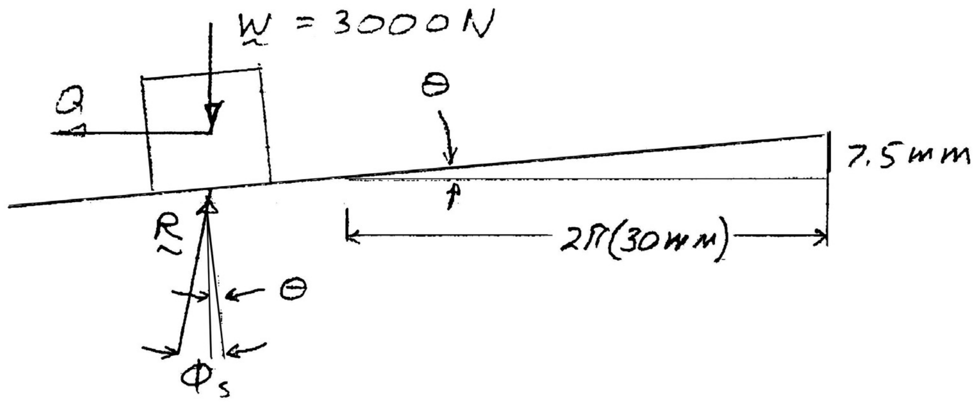
FBD large gear:



$$\circlearrowleft \Sigma M_C = 0: \quad (0.24 \text{ m})W - 720 \text{ N}\cdot\text{m} = 0$$

$$W = 3000 \text{ N}$$

Block on incline:



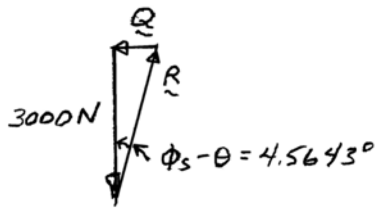
$$\theta = \tan^{-1}\frac{7.5 \text{ mm}}{2\pi(30 \text{ mm})} = 2.2785°$$

$$\phi_s = \tan^{-1}\mu = \tan^{-1} 0.12$$

$$\phi_s = 6.8428°$$

$$\phi_s - \theta = 4.5643°$$

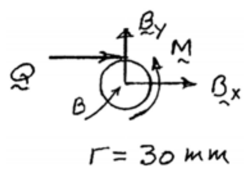
PROBLEM 8.68 CONTINUED



$$Q = (3000 \text{ N}) \tan 4.5643°$$

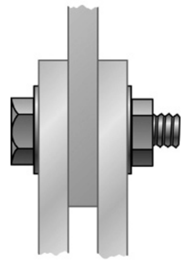
$$= 239.5 \text{ N}$$

Worm gear:



$$\circlearrowleft \Sigma M_B = 0: \quad M - rQ = 0$$

$$M = rQ = (0.030 \text{ m})(239.5 \text{ N}) = 7.18 \text{ N} \cdot \text{m} \blacktriangleleft$$

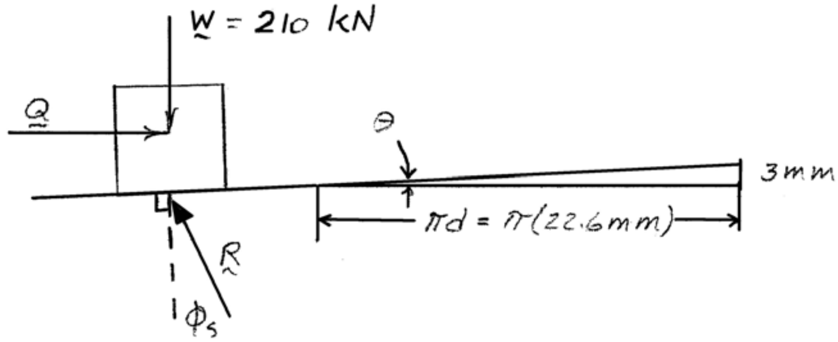


PROBLEM 8.69

High-strength bolts are used in the construction of many steel structures. For a 24-mm-nominal-diameter bolt the required minimum bolt tension is 210 kN. Assuming the coefficient of friction to be 0.40, determine the required couple that should be applied to the bolt and nut. The mean diameter of the thread is 22.6 mm, and the lead is 3 mm. Neglect friction between the nut and washer, and assume the bolt to be square-threaded.

SOLUTION

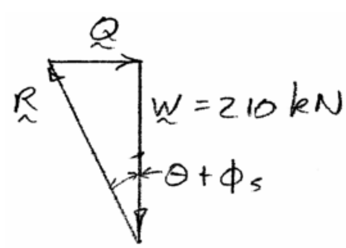
FBD block on incline:



$$\theta = \tan^{-1}\frac{3\text{ mm}}{(22.6\text{ mm})\pi} = 2.4195^\circ$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.40$$

$$\phi_s = 21.8014^\circ$$



$$Q = (210\text{ kN})\tan(21.8014^\circ + 2.4195^\circ)$$

$$Q = 94.47\text{ kN}$$

$$\text{Torque} = \frac{d}{2}Q = \frac{22.6\text{ mm}}{2}(94.47\text{ kN})$$

$$= 1067.5\text{ N}\cdot\text{m}$$

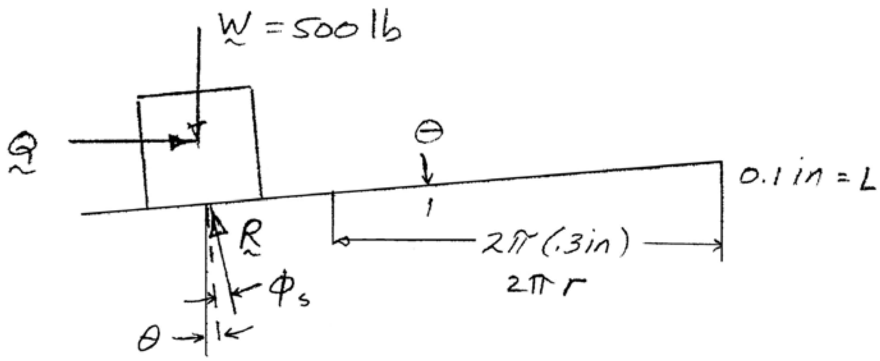
$$\text{Torque} = 1.068\text{ kN}\cdot\text{m} \blacktriangleleft$$

PROBLEM 8.70

The ends of two fixed rods A and B are each made in the form of a single-threaded screw of mean radius 0.3 in. and pitch 0.1 in. Rod A has a right-handed thread and rod B a left-handed thread. The coefficient of static friction between the rods and the threaded sleeve is 0.12. Determine the magnitude of the couple that must be applied to the sleeve in order to draw the rods closer together.

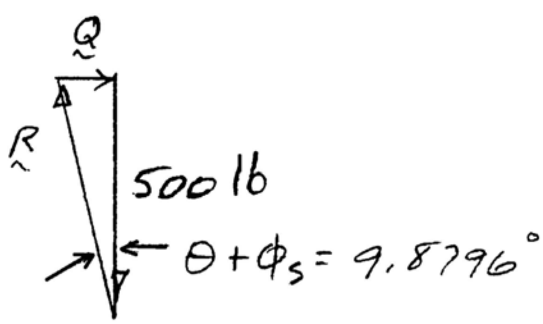
SOLUTION

Block on incline:



$$\theta = \tan^{-1}\frac{0.1\text{ in.}}{2\pi(0.3\text{ in.})} = 3.0368°$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.12 = 6.8428°$$



$$Q = (500\text{ lb})\tan 9.8796° = 87.08\text{ lb}$$

Couple on each side

$$M = rQ = (0.3\text{ in.})(87.08\text{ lb}) = 26.12\text{ lb}\cdot\text{in.}$$

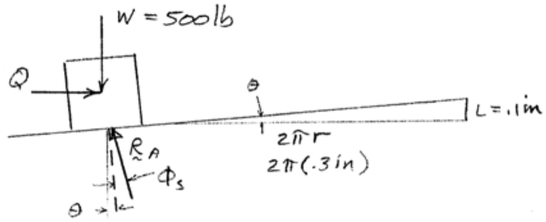
Couple to turn $= 2M = 52.2$ lb·in. ◀

PROBLEM 8.71

Assuming that in Problem 8.70 a right-handed thread is used on *both* rods *A* and *B*, determine the magnitude of the couple that must be applied to the sleeve in order to rotate it.

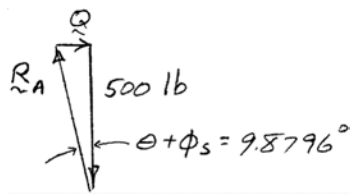
SOLUTION

Block on incline *A*:



$$\theta = \tan^{-1}\frac{0.1\text{ in.}}{2\pi(0.3\text{ in.})} = 3.0368°$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.12 = 6.8428°$$



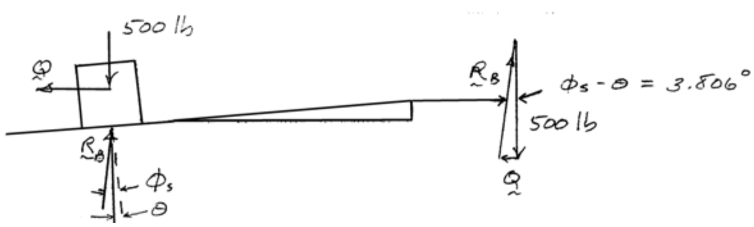
$$Q = (500\text{ lb})\tan 9.8796°$$

$$= 87.08\text{ lb}$$

$$\text{Couple at } A = (0.3\text{ in.})(87.08\text{ lb})$$

$$= 26.124\text{ lb}\cdot\text{in.}$$

Block on incline *B*:



$$Q = (500\text{ lb})\tan 3.806°$$

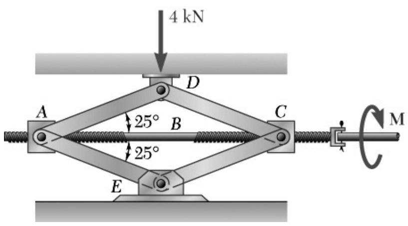
$$= 33.26\text{ lb}$$

$$\text{Couple at } B = (0.3\text{ in.})(33.26\text{ lb})$$

$$= 9.979\text{ lb}\cdot\text{in.}$$

$$\text{Total couple} = 26.124\text{ lb}\cdot\text{in.} + 9.979\text{ lb}\cdot\text{in.}$$

Couple to turn $= 36.1\text{ lb}\cdot\text{in.}$ ◀

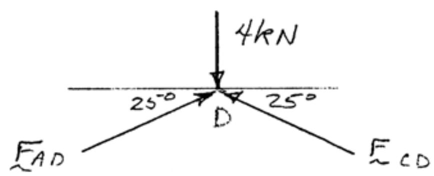


PROBLEM 8.72

The position of the automobile jack shown is controlled by a screw *ABC* that is single-threaded at each end (right-handed thread at *A*, left-handed thread at *C*). Each thread has a pitch of 2 mm and a mean diameter of 7.5 mm. If the coefficient of static friction is 0.15, determine the magnitude of the couple **M** that must be applied to raise the automobile.

SOLUTION

FBD joint *D*:

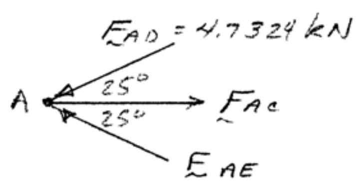


By symmetry:
$$F_{AD} = F_{CD}$$

$$\uparrow \Sigma F_y = 0: \quad 2F_{AD}\sin 25° - 4\text{ kN} = 0$$

$$F_{AD} = F_{CD} = 4.7324\text{ kN}$$

FBD joint *A*:

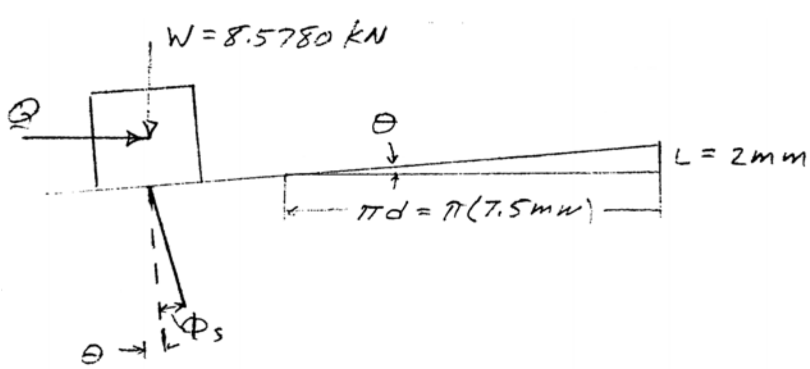


By symmetry:
$$F_{AE} = F_{AD}$$

$$\longrightarrow \Sigma F_x = 0: \quad F_{AC} - 2(4.7324\text{ kN})\cos 25° = 0$$

$$F_{AC} = 8.5780\text{ kN}$$

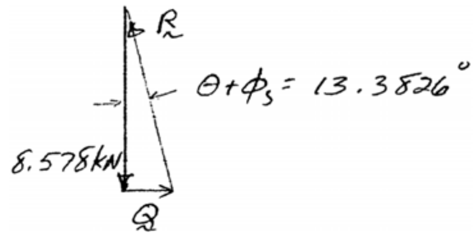
Block and incline *A*:



$$\theta = \tan^{-1}\frac{2\text{ mm}}{\pi(7.5\text{ mm})} = 4.8518°$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.15 = 8.5308°$$

PROBLEM 8.72 CONTINUED



$$Q = (8.578 \text{ kN})\tan(13.3826°)$$

$$= 2.0408 \text{ kN}$$

Couple at *A*: $$M_A = rQ$$

$$= \left(\frac{7.5}{2} \text{ mm}\right)(2.0408 \text{ kN})$$

$$= 7.653 \text{ N}\cdot\text{m}$$

By symmetry: Couple at *C*: $$M_C = 7.653 \text{ N}\cdot\text{m}$$

Total couple $M = 2(7.653 \text{ N}\cdot\text{m})$

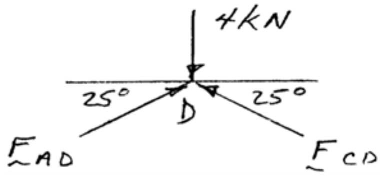
$M = 15.31 \text{ N}\cdot\text{m}$ ◀

PROBLEM 8.73

For the jack of Problem 8.72, determine the magnitude of the couple **M** that must be applied to lower the automobile.

SOLUTION

FBD joint *D*:



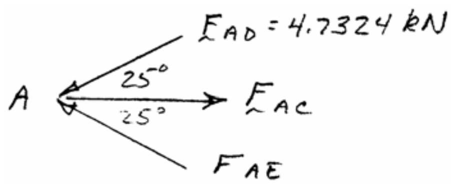
By symmetry:

$$F_{AD} = F_{CD}$$

$$\uparrow \Sigma F_y = 0: \quad 2F_{AD} \sin 25° - 4 \text{ kN} = 0$$

$$F_{AD} = F_{CD} = 4.7324 \text{ kN}$$

FBD joint *A*:



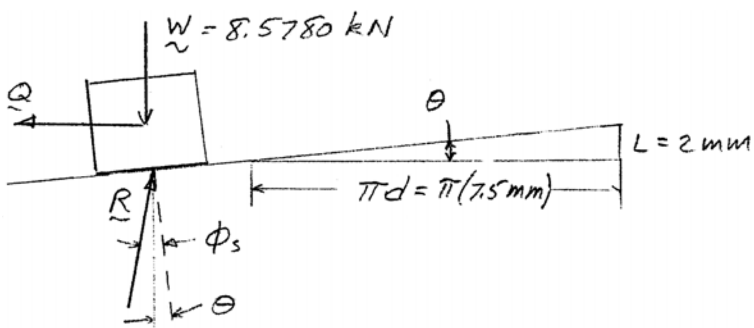
By symmetry:

$$F_{AE} = F_{AD}$$

$$\longrightarrow \Sigma F_x = 0: \quad F_{AC} - 2(4.7324 \text{ kN})\cos 25° = 0$$

$$F_{AC} = 8.5780 \text{ kN}$$

Block and incline at *A*:

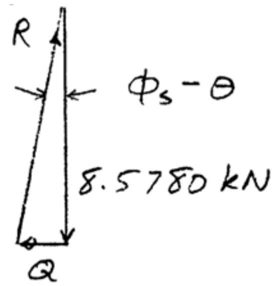


$$\theta = \tan^{-1} \frac{2 \text{ mm}}{\pi(7.5 \text{ mm})} = 4.8518°$$

$$\phi_s = \tan^{-1} \mu_s = \tan^{-1} 0.15$$

$$\phi_s = 8.5308°$$

PROBLEM 8.73 CONTINUED



$$\phi_s - \theta = 3.679°$$

$$Q = (8.5780\text{ kN})\tan 3.679°$$

$$Q = 0.55156\text{ kN}$$

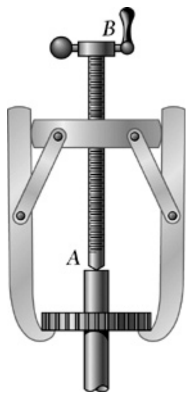
Couple at A: $M_A = Qr$

$$= (0.55156\text{ kN})\left(\frac{7.5\text{ mm}}{2}\right)$$

$$= 2.0683\text{ N}\cdot\text{m}$$

By symmetry: Couple at C: $M_C = 2.0683\text{ N}\cdot\text{m}$

Total couple $M = 2(2.0683\text{ N}\cdot\text{m})$ $M = 4.14\text{ N}\cdot\text{m}$ ◀

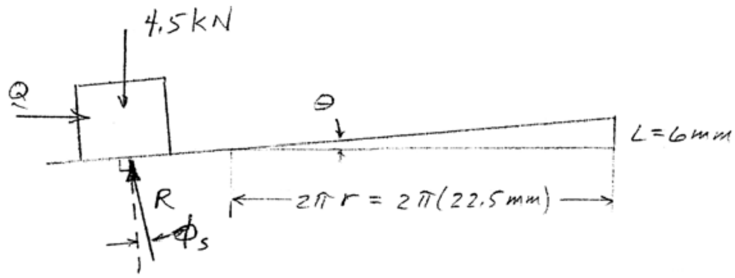


PROBLEM 8.74

In the gear-pulling assembly shown, the square-threaded screw *AB* has a mean radius of 22.5 mm and a lead of 6 mm. Knowing that the coefficient of static friction is 0.10, determine the couple which must be applied to the screw in order to produce a force of 4.5 kN on the gear. Neglect friction at end *A* of the screw.

SOLUTION

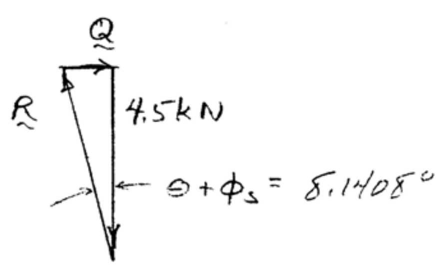
Block on incline:



$$\theta = \tan^{-1}\frac{6\text{ mm}}{2\pi(22.5\text{ mm})} = 2.4302°$$

$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.1$$

$$\phi_s = 5.7106°$$



$$Q = (4.5\text{ kN})\tan 8.1408°$$

$$= 0.6437\text{ kN}$$

$$\text{Couple } M = rQ$$

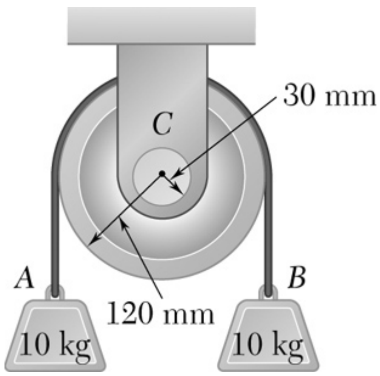
$$= (22.5\text{ mm})(0.6437\text{ kN})$$

$$= 14.483\text{ N}\cdot\text{m}$$

$$M = 14.48\text{ N}\cdot\text{m} ◀$$

NOTE FOR PROBLEMS 8.75–8.89

Note to instructors: In this manual, the singular $\sin(\tan^{-1}\mu) \approx \mu$ is NOT used in the solution of journal bearing and axle friction problems. While this approximation may be valid for very small values of μ, there is little if any reason to use it, and the error may be significant. For example, in Problems 8.76–8.79, $\mu_s = 0.40$, and the error made by using the approximation is about 7.7%.

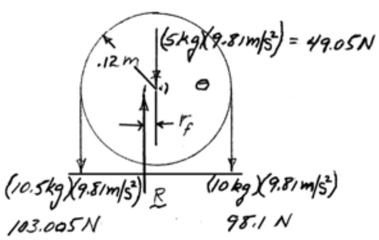


PROBLEM 8.75

A 120-mm-radius pulley of mass 5 kg is attached to a 30-mm-radius shaft which fits loosely in a fixed bearing. It is observed that the pulley will just start rotating if a 0.5-kg mass is added to block *A*. Determine the coefficient of static friction between the shaft and the bearing.

SOLUTION

FBD pulley:



$$+\uparrow \Sigma F_y = 0: \quad R - 103.005\text{ N} - 49.05\text{ N} - 98.1\text{ N} = 0$$

$$R = 250.155\text{ N}$$

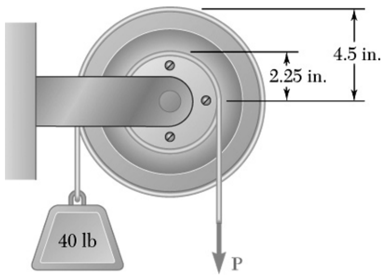
$$\circlearrowleft \Sigma M_O = 0: \quad (0.12\text{ m})(103.005\text{ N} - 98.1\text{ N}) - r_f(250.155\text{ N}) = 0$$

$$r_f = 0.0023529\text{ m} = 2.3529\text{ mm}$$

$$\phi_s = \sin^{-1}\frac{r_f}{r_s}$$

$$\mu_s = \tan\phi_s = \tan\left(\sin^{-1}\frac{r_f}{r_s}\right) = \tan\left(\sin^{-1}\frac{2.3529\text{ mm}}{30\text{ mm}}\right)$$

$$\mu_s = 0.0787 \blacktriangleleft$$

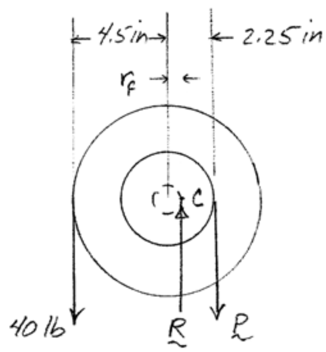


PROBLEM 8.76

The double pulley shown is attached to a 0.5-in.-radius shaft which fits loosely in a fixed bearing. Knowing that the coefficient of static friction between the shaft and the poorly lubricated bearing is 0.40, determine the magnitude of the force **P** required to start raising the load.

SOLUTION

FDB pulley:



$$r_f = r_s \sin\phi_s = r_s \sin\left(\tan^{-1}\mu_s\right)^*$$

$$r_f = (0.5 \text{ in.})\sin\left(\tan^{-1} 0.40\right) = 0.185695 \text{ in.}$$

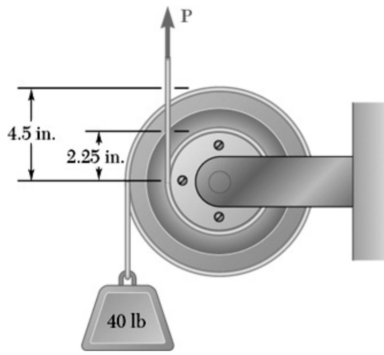
$$\circlearrowleft \Sigma M_C = 0: \quad (4.5 \text{ in.} + 0.185695 \text{ in.})(40 \text{ lb})$$

$$- (2.25 \text{ in.} - 0.185695 \text{ in.})P = 0$$

$$P = 90.8 \text{ lb} \blacktriangleleft$$

* See note before Problem 8.75.

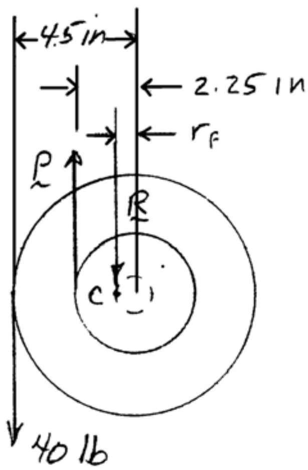
PROBLEM 8.77



The double pulley shown is attached to a 0.5-in.-radius shaft which fits loosely in a fixed bearing. Knowing that the coefficient of static friction between the shaft and the poorly lubricated bearing is 0.40, determine the magnitude of the force **P** required to start raising the load.

SOLUTION

FBD pulley:



$$r_f = r_s \sin\phi_s = r_s \sin\left(\tan^{-1}\mu_s\right) = (0.5 \text{ in.})\sin\left(\tan^{-1} 0.4\right)^{*}$$

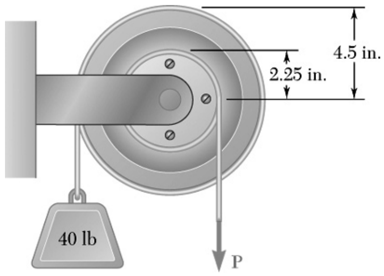
$$r_f = 0.185695 \text{ in.}$$

$$\circlearrowleft \Sigma M_C = 0: \quad (4.5 \text{ in.} - 0.185695 \text{ in.})(40 \text{ lb})$$

$$- (2.25 \text{ in.} - 0.185695 \text{ in.})P = 0$$

$$P = 83.6 \text{ lb} \blacktriangleleft$$

* See note before Problem 8.75.

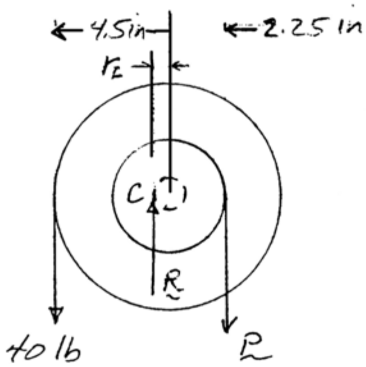


PROBLEM 8.78

The double pulley shown is attached to a 0.5-in.-radius shaft which fits loosely in a fixed bearing. Knowing that the coefficient of static friction between the shaft and the poorly lubricated bearing is 0.40, determine the magnitude of the force **P** required to maintain equilibrium.

SOLUTION

FBD pulley:



$$r_f = r_s \sin\phi_s = r_s \sin\left(\tan^{-1}\mu_s\right)^*$$

$$r_f = (0.5 \text{ in.})\sin\left(\tan^{-1} 0.40\right) = 0.185695 \text{ in.}$$

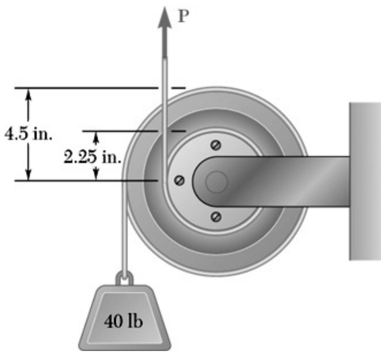
$$\circlearrowleft \Sigma M_C = 0: \quad (4.5 \text{ in.} - 0.185695 \text{ in.})(40 \text{ lb})$$

$$-(2.25 \text{ in.} + 0.185695 \text{ in.})(P) = 0$$

$$P = 70.9 \text{ lb} \blacktriangleleft$$

* See note before Problem 8.75.

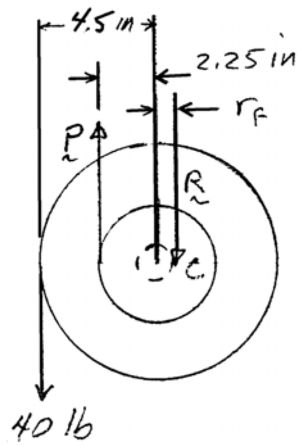
PROBLEM 8.79



The double pulley shown is attached to a 0.5-in.-radius shaft which fits loosely in a fixed bearing. Knowing that the coefficient of static friction between the shaft and the poorly lubricated bearing is 0.40, determine the magnitude of the force **P** required to maintain equilibrium.

SOLUTION

FBD pulley:



$$r_f = r_s \sin\phi_s = r_s \sin\left(\tan^{-1}\mu_s\right)^*$$

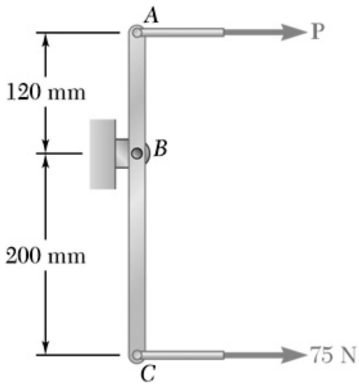
$$r_f = (0.5\text{ in.})\sin\left(\tan^{-1}0.4\right) = 0.185695\text{ in.}$$

$$\circlearrowleft \Sigma M_C = 0:\quad (4.5\text{ in.} + 0.185695\text{ in.})(40\text{ lb})$$

$$-(2.25\text{ in.} + 0.185695\text{ in.})P = 0$$

$$P = 77.0\text{ lb} \blacktriangleleft$$

* See note before Problem 8.75.

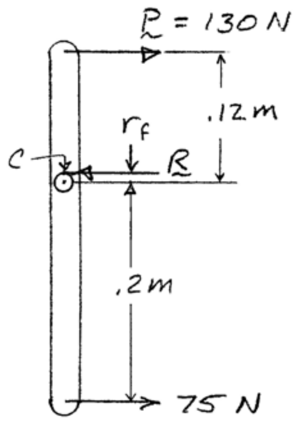


PROBLEM 8.80

Control lever ABC fits loosely on a 18-mm-diameter shaft at support B. Knowing that $P = 130$ N for impending clockwise rotation of the lever, determine (a) the coefficient of static friction between the pin and the lever, (b) the magnitude of the force **P** for which counterclockwise rotation of the lever is impending.

SOLUTION

(a) **FBD lever** (Impending CW rotation):



$$\circlearrowleft \Sigma M_C = 0: \quad (0.2 \text{ m} + r_f)(75 \text{ N}) - (0.12 \text{ m} - r_f)(130 \text{ N}) = 0$$

$$r_f = 0.0029268 \text{ m} = 2.9268 \text{ mm}$$

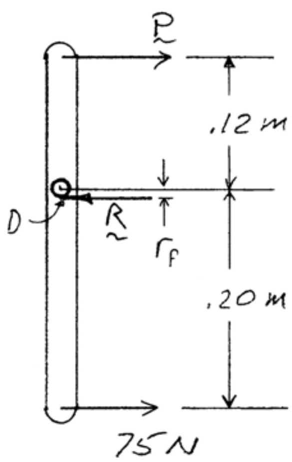
$$\sin\phi_s = \frac{r_f}{r_s}$$

$$\mu_s = \tan\phi_s = \tan\left(\sin^{-1}\frac{r_f}{r_s}\right) = \tan\left(\sin^{-1}\frac{2.9268 \text{ mm}}{18 \text{ mm}}\right)^*$$

$$= 0.34389$$

$$\mu_s = 0.344 \blacktriangleleft$$

(b) **FBD lever** (Impending CCW rotation):



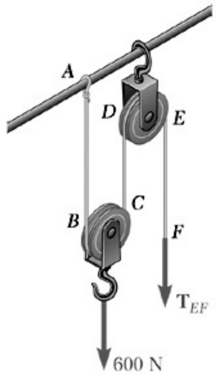
$$\circlearrowleft \Sigma M_D = 0: \quad (0.20 \text{ m} - 0.0029268 \text{ m})(75 \text{ N})$$

$$- (0.12 \text{ m} + 0.0029268 \text{ m})P = 0$$

$$P = 120.2 \text{ N} \blacktriangleleft$$

* See note before Problem 8.75.

PROBLEM 8.81

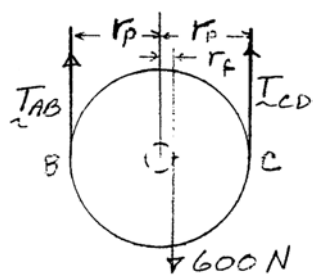


The block and tackle shown are used to raise a 600-N load. Each of the 60-mm-diameter pulleys rotates on a 10-mm-diameter axle. Knowing that the coefficient of kinetic friction is 0.20, determine the tension in each portion of the rope as the load is slowly raised.

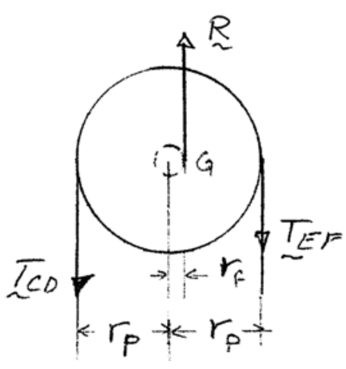
SOLUTION

Pulley FBD's:

Left:



Right:



$$r_p = 30 \text{ mm}$$

$$r_f = r_{\text{axle}} \sin\phi_k = r_{\text{axle}} \sin\left(\tan^{-1}\mu_k\right)^*$$

$$= (5 \text{ mm})\sin\left(\tan^{-1} 0.2\right)$$

$$= 0.98058 \text{ mm}$$

Left:

$$\circlearrowleft \Sigma M_C = 0: \quad \left(r_p - r_f\right)(600 \text{ lb}) - 2r_p T_{AB} = 0$$

or $$T_{AB} = \frac{30 \text{ mm} - 0.98058 \text{ mm}}{2(30 \text{ mm})}(600 \text{ N}) = 290.19 \text{ N}$$

$$T_{AB} = 290 \text{ N} \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \quad 290.19 \text{ N} - 600 \text{ N} + T_{CD} = 0$$

or $$T_{CD} = 309.81 \text{ N} \qquad T_{CD} = 310 \text{ N} \blacktriangleleft$$

Right:

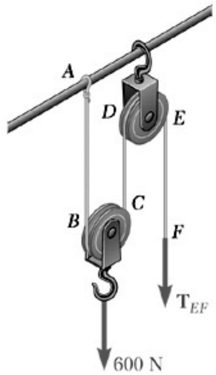
$$\circlearrowleft \Sigma M_G = 0: \quad \left(r_p + r_f\right)T_{CD} - \left(r_p - r_f\right)T_{EF} = 0$$

or $$T_{EF} = \frac{30 \text{ mm} + 0.98058 \text{ mm}}{30 \text{ mm} - 0.98058 \text{ mm}}(309.81 \text{ N}) = 330.75 \text{ N}$$

$$T_{EF} = 331 \text{ N} \blacktriangleleft$$

* See note before Problem 8.75.

PROBLEM 8.82

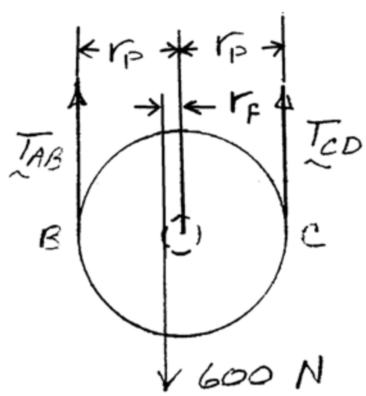


The block and tackle shown are used to lower a 600-N load. Each of the 60-mm-diameter pulleys rotates on a 10-mm-diameter axle. Knowing that the coefficient of kinetic friction is 0.20, determine the tension in each portion of the rope as the load is slowly lowered.

SOLUTION

Pulley FBDs:

Left:



$$r_p = 30 \text{ mm}$$

$$r_f = r_{\text{axle}} \sin\phi_k = r_{\text{axle}} \sin\left(\tan^{-1}\mu_k\right)^*$$

$$= (5 \text{ mm})\sin\left(\tan^{-1} 0.2\right)$$

$$= 0.98058 \text{ mm}$$

$$\Sigma M_C = 0: \quad (r_p + r_f)(600 \text{ N}) - 2r_p T_{AB} = 0$$

or $$T_{AB} = \frac{30 \text{ mm} + 0.98058 \text{ mm}}{2(30 \text{ mm})}(600 \text{ N}) = 309.81 \text{ N}$$

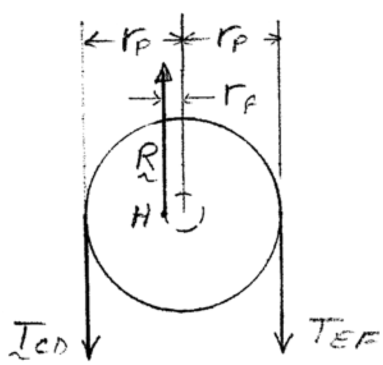
$$T_{AB} = 310 \text{ N} \blacktriangleleft$$

$$\Sigma F_y = 0: \quad T_{AB} - 600 \text{ N} + T_{CD} = 0$$

Right:

or $$T_{CD} = 600 \text{ N} - 309.81 \text{ N} = 290.19 \text{ N}$$

$$T_{CD} = 290 \text{ N} \blacktriangleleft$$



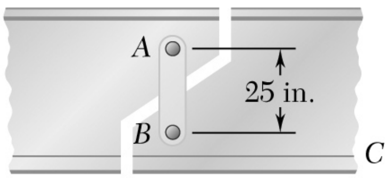
$$\Sigma M_H = 0: \quad (r_p - r_f)T_{CD} - (r_p + r_f)T_{EF} = 0$$

or $$T_{EF} = \frac{30 \text{ mm} - 0.98058 \text{ mm}}{30 \text{ mm} + 0.98058 \text{ mm}}(290.19 \text{ N})$$

$$T_{EF} = 272 \text{ N} \blacktriangleleft$$

* See note before Problem 8.75.

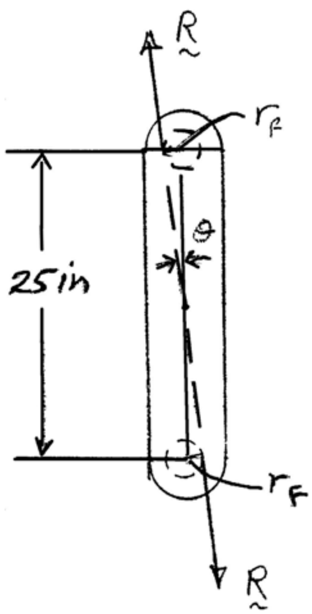
PROBLEM 8.83



The link arrangement shown is frequently used in highway bridge construction to allow for expansion due to changes in temperature. At each of the 3-in.-diameter pins A and B the coefficient of static friction is 0.20. Knowing that the vertical component of the force exerted by BC on the link is 50 kips, determine (a) the horizontal force which should be exerted on beam BC to just move the link, (b) the angle that the resulting force exerted by beam BC on the link will form with the vertical.

SOLUTION

FBD link AB:



Note that AB is a two force member. For impending motion, the pin forces are tangent to the friction circles.

$$\theta = \sin^{-1}\frac{r_f}{25\text{ in.}}$$

where $$r_f = r_p \sin\phi_s = r_p \sin\left(\tan^{-1}\mu_s\right)^{*}$$

$$= (1.5\text{ in.})\sin\left(\tan^{-1}0.2\right) = 0.29417\text{ in.}$$

Then $$\theta = \sin^{-1}\frac{0.29417\text{ in.}}{12.5\text{ in.}} = 1.3485°$$

(b) $\theta = 1.349°$ ◀

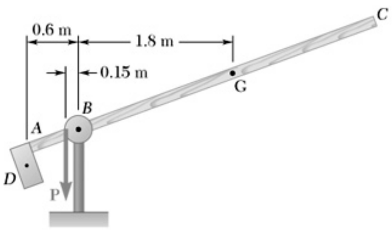
$$R_{\text{vert}} = R\cos\theta \qquad R_{\text{horiz}} = R\sin\theta$$

$$R_{\text{horiz}} = R_{\text{vert}}\tan\theta = (50\text{ kips})\tan 1.3485° = 1.177\text{ kips}$$

(a) $R_{\text{horiz}} = 1.177\text{ kips}$ ◀

* See note before Problem 8.75.

PROBLEM 8.84

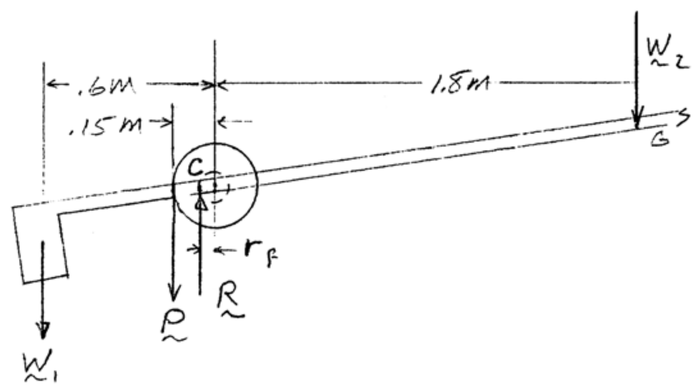


BEER • JOHNSTON Fig. P8-84 and P8-86
Vector Mechanics for Engineers: Statics & Dynamics, 7e
100% of size Fine Line Illustrations (516) 501-0400

A gate assembly consisting of a 24-kg gate *ABC* and a 66-kg counterweight *D* is attached to a 24-mm-diameter shaft *B* which fits loosely in a fixed bearing. Knowing that the coefficient of static friction is 0.20 between the shaft and the bearing, determine the magnitude of the force **P** for which counterclockwise rotation of the gate is impending.

SOLUTION

FBD gate:



$$W_1 = 66\text{ kg}\left(9.81\text{ m/s}^2\right) = 647.46\text{ N}$$

$$W_2 = 24\text{ kg}\left(9.81\text{ m/s}^2\right) = 235.44\text{ N}$$

$$r_f = r_s \sin\phi_s = r_s \sin\left(\tan^{-1}\mu_s\right)$$

$$= \left(0.012\text{ m}\right)\sin\left(\tan^{-1}0.2\right) = 0.0023534\text{ m}$$

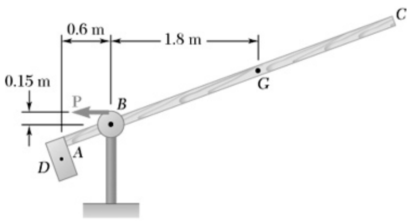
$$\circlearrowleft \Sigma M_C = 0: \quad \left(0.6\text{ m} - r_f\right)W_1 + \left(0.15\text{ m} - r_f\right)P - \left(1.8\text{ m} + r_f\right)W_2 = 0$$

$$P = \frac{\left(1.80235\text{ m}\right)\left(235.44\text{ N}\right) - \left(0.59765\text{ m}\right)\left(647.46\text{ N}\right)}{\left(0.14765\text{ m}\right)}$$

$$= 253.2\text{ N}$$

$$P = 253\text{ N} \blacktriangleleft$$

PROBLEM 8.85



BEER • JOHNSTON Fig. P8-85 and P8-87
Vector Mechanics for Engineers: Statics & Dynamics, 7e
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A gate assembly consisting of a 24-kg gate *ABC* and a 66-kg counterweight *D* is attached to a 24-mm-diameter shaft *B* which fits loosely in a fixed bearing. Knowing that the coefficient of static friction is 0.20 between the shaft and the bearing, determine the magnitude of the force **P** for which counterclockwise rotation of the gate is impending.

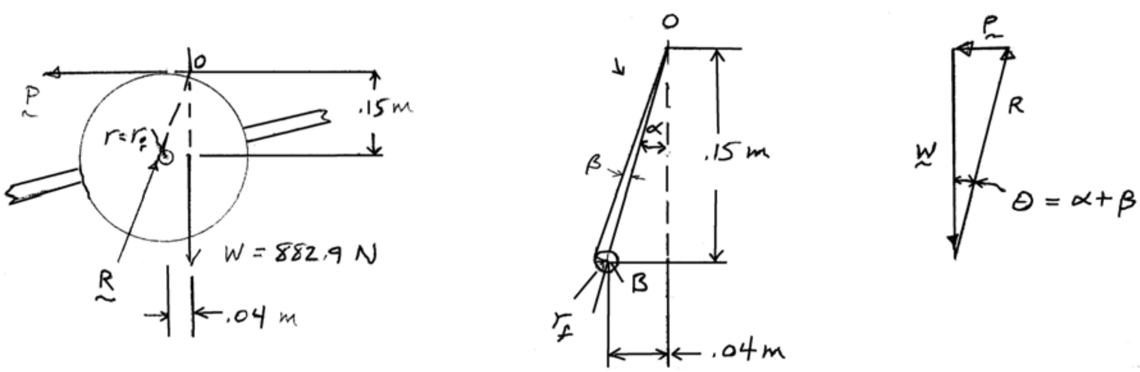
SOLUTION

It is convenient to replace the $(66\text{ kg})g$ and $(24\text{ kg})g$ weights with a single combined weight of $(90\text{ kg})(9.81\text{ m/s}^2) = 882.9\text{ N}$, located at a distance $x = \dfrac{(1.8\text{ m})(24\text{ kg}) - (0.6\text{ m})(24\text{ kg})}{90\text{ kg}} = 0.04\text{ m}$ to the right of *B*.

$$r_f = r_s \sin\phi_s = r_s \sin\left(\tan^{-1}\mu_s\right)^* = (0.012\text{ m})\sin\left(\tan^{-1}0.2\right)$$

$$= 0.0023534\text{ m}$$

FBD pulley + gate:



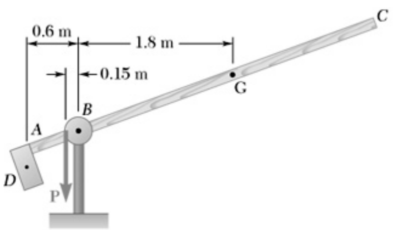
$$\alpha = \tan^{-1}\frac{0.04\text{ m}}{0.15\text{ m}} = 14.931° \qquad OB = \frac{0.15}{\cos\alpha} = 0.15524\text{ m}$$

$$\beta = \sin^{-1}\frac{r_f}{OB} = \sin^{-1}\frac{0.0023534\text{ m}}{0.15524\text{ m}} = 0.8686° \qquad \text{then} \qquad \theta = \alpha + \beta = 15.800°$$

$$P = W\tan\theta = 248.9\text{ N}$$

$P = 250\text{ N}$ ◀

* See note before Problem 8.75.



BEER • JOHNSTON Fig. P8-84 and P8-86
Vector Mechanics for Engineers: Statics & Dynamics, 7e
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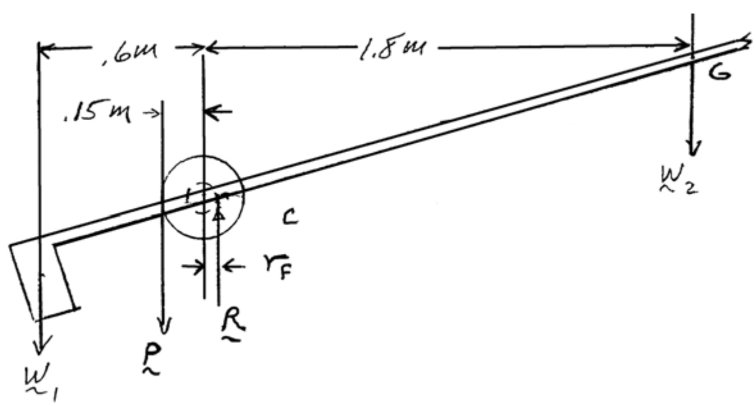
PROBLEM 8.86

A gate assembly consisting of a 24-kg gate *ABC* and a 66-kg counterweight *D* is attached to a 24-mm-diameter shaft *B* which fits loosely in a fixed bearing. Knowing that the coefficient of static friction is 0.20 between the shaft and the bearing, determine the magnitude of the force **P** for which clockwise rotation of the gate is impending.

SOLUTION

FBD gate:



$$W_1 = 66\text{ kg}\left(9.81\text{ m/s}^2\right) = 647.46\text{ N}$$

$$W_2 = 24\text{ kg}\left(9.81\text{ m/s}^2\right) = 235.44\text{ N}$$

$$r_f = r_s \sin\phi_s = r_s \sin\left(\tan^{-1}\mu_s\right)^*$$

$$= \left(0.012\text{ m}\right)\sin\left(\tan^{-1}0.2\right) = 0.0023534\text{ m}$$

$$\circlearrowleft \Sigma M_C = 0: \quad \left(0.6\text{ m} + r_f\right)W_1 + \left(0.15\text{ m} + r_f\right)P - \left(1.8\text{ m} - r_f\right)W_2 = 0$$

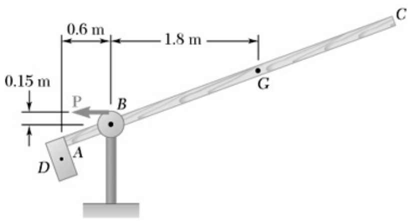
$$P = \frac{(1.79765\text{ m})(235.44\text{ N}) - (0.60235\text{ m})(647.46\text{ N})}{0.15235\text{ m}}$$

$$= 218.19\text{ N}$$

$P = 218\text{ N}$ ◀

* See note before Problem 8.75.

PROBLEM 8.87



BEER • JOHNSTON Fig. P8-85 and P8-87
Vector Mechanics for Engineers: Statics & Dynamics, 7e
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A gate assembly consisting of a 24-kg gate *ABC* and a 66-kg counterweight *D* is attached to a 24-mm-diameter shaft *B* which fits loosely in a fixed bearing. Knowing that the coefficient of static friction is 0.20 between the shaft and the bearing, determine the magnitude of the force **P** for which clockwise rotation of the gate is impending.

SOLUTION

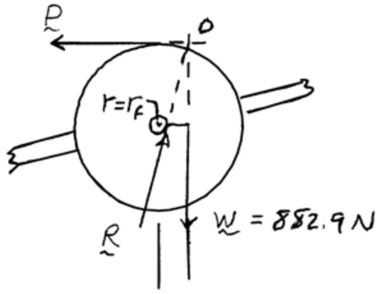
It is convenient to replace the $(66\text{ kg})g$ and $(24\text{ kg})g$ weights with a single weight of

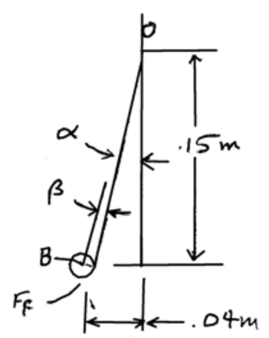
$(90\text{ kg})(9.81\text{ N/kg}) = 882.9\text{ N}$, located at a distance $x = \dfrac{(1.8\text{ m})(24\text{ kg}) - (0.15\text{ m})(66\text{ kg})}{90\text{ kg}} = 0.04\text{ m}$ to the right of *B*.

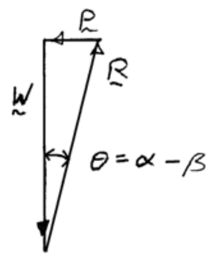
FBD pulley + gate:

$$r_f = r_s \sin\phi_s = r_s \sin\left(\tan^{-1}\mu_s\right)^* = (0.012\text{ m})\sin\left(\tan^{-1}0.2\right)$$

$$r_f = 0.0023534\text{ m}$$







$$\alpha = \tan^{-1}\frac{0.04\text{ m}}{0.15\text{ m}} = 14.931° \qquad OB = \frac{0.15\text{ m}}{\cos\alpha} = 0.15524\text{ m}$$

$$\beta = \sin^{-1}\frac{r_f}{OB} = \sin^{-1}\frac{0.0023534\text{ m}}{0.15524\text{ m}} = 0.8686° \qquad \text{then} \qquad \theta = \alpha - \beta = 14.062°$$

$$P = W\tan\theta = 221.1\text{ N}$$

$$P = 221\text{ N} \blacktriangleleft$$

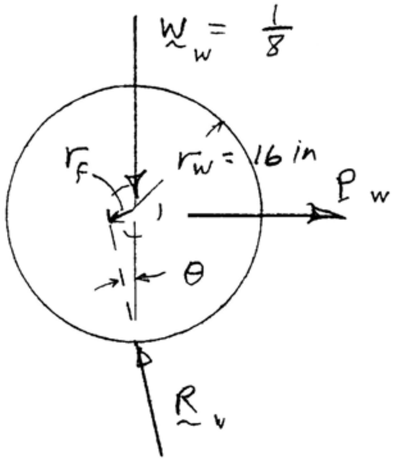
* See note before Problem 8.75.

PROBLEM 8.88

A loaded railroad car has a weight of 35 tons and is supported by eight 32-in.-diameter wheels with 5-in.-diameter axles. Knowing that the coefficients of friction are $\mu_s = 0.020$ and $\mu_k = 0.015$, determine the horizontal force required (*a*) for impending motion of the car, (*b*) to keep the car moving at a constant speed. Neglect rolling resistance between the wheels and the track.

SOLUTION

FBD wheel:



$$W_w = \frac{1}{8}W_c = \frac{1}{8}(35\text{ ton}) = \frac{1}{8}(70{,}000)\text{lb}$$

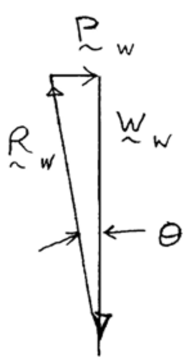
$$r_f = r_a \sin\phi = r_a \sin\left(\tan^{-1}\mu\right)^*$$

$$\theta = \sin^{-1}\frac{r_f}{r_w} = \sin^{-1}\left[\frac{(2.5\text{ in.})\sin\left(\tan^{-1}\mu\right)}{16\text{ in.}}\right]$$

$$= \sin^{-1}\left[0.15625\sin\left(\tan^{-1}\mu\right)\right]$$

(*a*) For impending motion use $\mu_s = 0.02$: then $\theta_s = 0.179014°$

(*b*) For steady motion use $\mu_k = 0.15$: then $\theta_k = 0.134272°$



$$P_w = W_w \tan\theta \qquad P_c = W_c \tan\theta = 8W_w \tan\theta$$

(*a*) $$P_c = (70{,}000\text{ lb})\tan(0.179014°)$$

$$P_c = 219\text{ lb} ◀$$

(*b*) $$P_c = (70{,}000\text{ lb})\tan(0.134272°)$$

$$P_c = 164.0\text{ lb} ◀$$

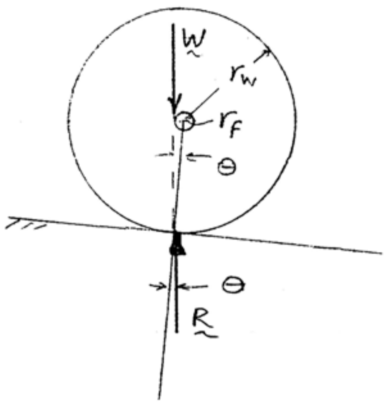
* See note before Problem 8.75.

PROBLEM 8.89

A scooter is designed to roll down a 2 percent slope at a constant speed. Assuming that the coefficient of kinetic friction between the 1-in.-diameter axles and the bearing is 0.10, determine the required diameter of the wheels. Neglect the rolling resistance between the wheels and the ground.

SOLUTION

FBD wheel:



Note: The wheel is a two-force member in equilibrium, so **R** and **W** must be collinear and tangent to friction circle.

$$2\% \text{ slope} \Rightarrow \tan\theta = 0.02$$

Also $$\sin\theta = \frac{r_f}{r_w}\sin\left(\tan^{-1}0.02\right) = 0.019996$$

But $$r_f = r_a \sin\phi_k = r_a \sin\left(\tan^{-1}\mu_k\right)^*$$

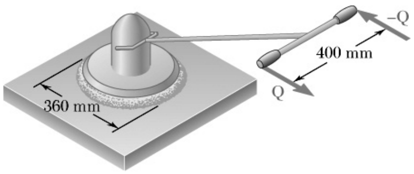
$$= (1\text{ in.})\sin\left(\tan^{-1}0.1\right) = 0.099504\text{ in.}$$

Then $$r_w = \frac{r_f}{\sin\theta} = \frac{0.099504}{0.019996} = 4.976\text{ in.}$$

and $$d_w = 2r_w \qquad d_w = 9.95\text{ in.} \blacktriangleleft$$

* See note before Problem 8.75.

PROBLEM 8.90



A 25-kg electric floor polisher is operated on a surface for which the coefficient of kinetic friction is 0.25. Assuming that the normal force per unit area between the disk and the floor is uniformly distributed, determine the magnitude Q of the horizontal forces required to prevent motion of the machine.

SOLUTION

Couple exerted on handle $$M_H = dQ = (0.4\text{ m})Q$$

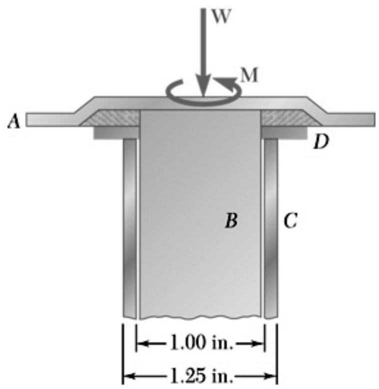
Couple exerted on floor $$M_F = \frac{2}{3}\mu_k PR \qquad \text{(Equation 8.9)}$$

where $$\mu_k = 0.25, \quad P = (25\text{ kg})\left(9.81\text{ m/s}^2\right) = 245.25\text{ N}, \quad R = 0.18\text{ m}$$

For equilibrium $$M_H = M_F,$$

so $$Q = \frac{\frac{2}{3}(0.25)(245.25\text{ N})(0.18\text{ m})}{0.4\text{ m}}$$

$$Q = 18.39\text{ N} \blacktriangleleft$$



PROBLEM 8.91

The pivot for the seat of a desk chair consists of the steel plate *A*, which supports the seat, the solid steel shaft *B* which is welded to *A* and which turns freely in the tubular member *C*, and the nylon bearing *D*. If a person of weight $W = 180$ lb is seated directly above the pivot, determine the magnitude of the couple **M** for which rotation of the seat is impending knowing that the coefficient of static friction is 0.15 between the tubular member and the bearing.

SOLUTION

For an annular bearing area $$M = \frac{2}{3}\mu_s P \frac{R_2^3 - R_1^3}{R_2^2 - R_1^2} \quad \text{(Equation 8.8)}$$

Since $R = \dfrac{D}{2}$ $$M = \frac{1}{3}\mu_s P \frac{D_2^3 - D_1^3}{D_2^2 - D_1^2}$$

Now $$\mu_s = 0.15,\ P = W = 180 \text{ lb},\ D_1 = 1.00 \text{ in.},\ D_2 = 1.25 \text{ in.}$$

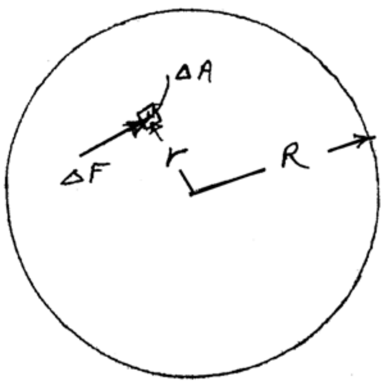
$$M = \frac{0.15}{3}(180 \text{ lb})\frac{(1.25 \text{ in.})^3 - (4 \text{ in.})^3}{(1.25 \text{ in.})^2 - (1 \text{ in.})^2}$$

$$M = 15.25 \text{ lb}\cdot\text{in.} \blacktriangleleft$$

PROBLEM 8.92

As the surfaces of a shaft and a bearing wear out, the frictional resistance of a thrust bearing decreases. It is generally assumed that the wear is directly proportional to the distance traveled by any given point of the shaft and thus to the distance r from the point to the axis of the shaft. Assuming, then, that the normal force per unit area is inversely proportional to r, show that the magnitude M of the couple required to overcome the frictional resistance of a worn-out end bearing (with contact over the full circular area) is equal to 75 percent of the value given by formula (8.9) for a new bearing.

SOLUTION



Let the normal force on ΔA be ΔN, and $\dfrac{\Delta N}{\Delta A} = \dfrac{k}{r}$

As in the text $\Delta F = \mu \Delta N, \ \Delta M = r\Delta F$

The total normal force

$$P = \lim_{\Delta A \to 0} \Sigma \Delta N = \int_0^{2\pi}\left(\int_0^R \frac{k}{r} r dr\right) d\theta$$

$$P = 2\pi\left(\int_0^R k dr\right) = 2\pi k R \qquad \text{or} \qquad k = \frac{P}{2\pi R}$$

The total couple $$M_{\text{worn}} = \lim_{\Delta A \to 0} \Sigma \Delta M = \int_0^{2\pi}\left(\int_0^R r\mu \frac{k}{r} r dr\right) d\theta$$

$$M_{\text{worn}} = 2\pi\mu k \int_0^R r dr = 2\pi\mu k \frac{R^2}{2} = 2\pi\mu \frac{P}{2\pi R}\frac{R^2}{2}$$

or $$M_{\text{worn}} = \frac{1}{2}\mu P R$$

Now $$M_{\text{new}} = \frac{2}{3}\mu P R \qquad \text{[Eq. (8.9)]}$$

Thus $$\frac{M_{\text{worn}}}{M_{\text{new}}} = \frac{\frac{1}{2}\mu P R}{\frac{2}{3}\mu P R} = \frac{3}{4} = 75\% \blacktriangleleft$$

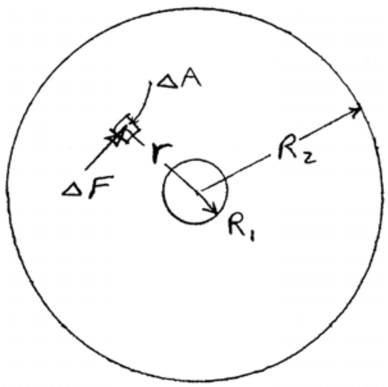
PROBLEM 8.93

Assuming that bearings wear out as indicated in Problem 8.92, show that the magnitude M of the couple required to overcome the frictional resistance of a worn-out collar bearing is $M = \frac{1}{2}\mu_k P(R_1 + R_2)$

where $P =$ magnitude of the total axial force

$R_1,\ R_2 =$ inner and outer radii of collar

SOLUTION



Let normal force on ΔA be ΔN, and $\dfrac{\Delta N}{\Delta A} = \dfrac{k}{r}$

As in the text $\qquad \Delta F = \mu \Delta N,\ \Delta M = r\Delta F$

The total normal force P is

$$P = \lim_{\Delta A \to 0} \Sigma \Delta N = \int_0^{2\pi}\left(\int_{R_1}^{R_2} \frac{k}{r} r dr\right) d\theta$$

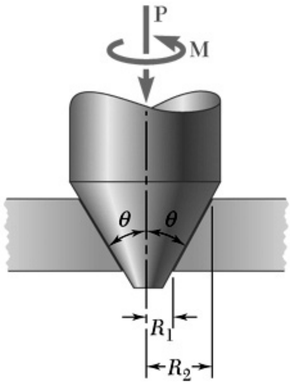
$$P = 2\pi \int_{R_1}^{R_2} k dr = 2\pi k (R_2 - R_1) \qquad \text{or} \qquad k = \frac{P}{2\pi(R_2 - R_1)}$$

The total couple is $\qquad M_{\text{worn}} = \lim_{\Delta A \to 0} \Sigma \Delta M = \int_0^{2\pi}\left(\int_{R_1}^{R_2} r\mu \frac{k}{r} r dr\right) d\theta$

$$M_{\text{worn}} = 2\pi\mu k \int_{R_1}^{R_2} (r dr) = \pi\mu k\left(R_2^2 - R_1^2\right) = \frac{\pi\mu P\left(R_2^2 - R_1^2\right)}{2\pi(R_2 - R_1)}$$

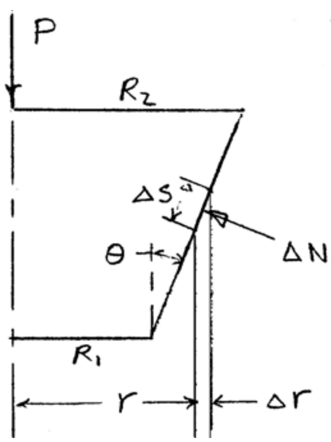
$$M_{\text{worn}} = \frac{1}{2}\mu P(R_2 + R_1) \blacktriangleleft$$

PROBLEM 8.94



Assuming that the pressure between the surfaces of contact is uniform, show that the magnitude M of the couple required to overcome frictional resistance for the conical bearing shown is $M = \dfrac{2}{3}\dfrac{\mu_k P}{\sin\theta}\dfrac{R_2^3 - R_1^3}{R_2^2 - R_1^2}$

SOLUTION



Let normal force on ΔA be ΔN, and $\dfrac{\Delta N}{\Delta A} = k$,

so $\qquad \Delta N = k\Delta A \qquad \Delta A = r\Delta s\Delta\phi \qquad \Delta s = \dfrac{\Delta r}{\sin\theta}$

where ϕ is the azimuthal angle around the symmetry axis of rotation

$$\Delta F_y = \Delta N\sin\theta = kr\Delta r\Delta\phi$$

Total vertical force $\qquad P = \lim_{\Delta A\to 0}\Sigma\Delta F_y$

$$P = \int_0^{2\pi}\left(\int_{R_1}^{R_2} krdr\right)d\phi = 2\pi k\int_{R_1}^{R_2} rdr$$

$$P = \pi k\left(R_2^2 - R_1^2\right) \qquad \text{or} \qquad k = \frac{P}{\pi\left(R_2^2 - R_1^2\right)}$$

Friction force $\qquad \Delta F = \mu\Delta N = \mu k\Delta A$

Moment $\qquad \Delta M = r\Delta F = r\mu kr\dfrac{\Delta r}{\sin\theta}\Delta\phi$

Total couple $\qquad M = \lim_{\Delta A\to 0}\Sigma\Delta M = \int_0^{2\pi}\left(\int_{R_1}^{R_2}\dfrac{\mu k}{\sin\theta}r^2dr\right)d\phi$

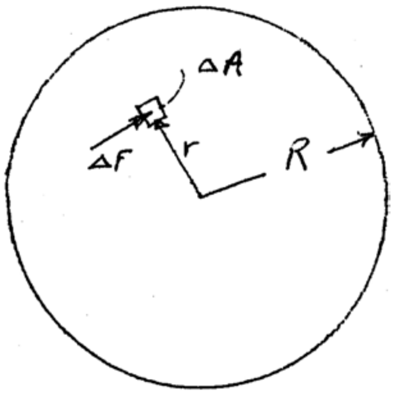
$$M = 2\pi\frac{\mu k}{\sin\theta}\int_{R_1}^{R_2} r^2dr = \frac{2}{3}\frac{\pi\mu}{\sin\theta}\frac{P}{\pi\left(R_2^2 - R_3^2\right)}\left(R_2^3 - R_3^3\right)$$

$$M = \frac{2}{3}\frac{\mu P}{\sin\theta}\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2} \blacktriangleleft$$

PROBLEM 8.95

Solve Problem 8.90 assuming that the normal force per unit area between the disk and the floor varies linearly from a maximum at the center to zero at the circumference of the disk.

SOLUTION



Let normal force on ΔA be ΔN, and $\dfrac{\Delta N}{\Delta A} = k\left(1 - \dfrac{r}{R}\right)$

$$\Delta F = \mu \Delta N = \mu k\left(1 - \frac{r}{R}\right)\Delta A = \mu k\left(1 - \frac{r}{R}\right) r \Delta r \Delta \theta$$

$$P = \lim_{\Delta A \to 0} \Sigma \Delta N = \int_0^{2\pi}\left[\int_0^R k\left(1 - \frac{r}{R}\right) r dr\right] d\theta$$

$$P = 2\pi k \int_0^R \left(1 - \frac{r}{R}\right) r dr = 2\pi k\left(\frac{R^2}{2} - \frac{R^3}{3R}\right)$$

$$P = \frac{1}{3}\pi k R^2 \qquad \text{or} \qquad k = \frac{3P}{\pi R^2}$$

$$M = \lim_{\Delta A \to 0} \Sigma r \Delta F = \int_0^{2\pi}\left[\int_0^R r \mu k\left(1 - \frac{r}{R}\right) r dr\right] d\theta$$

$$= 2\pi \mu k \int_0^R \left(r^2 - \frac{r^3}{R}\right) dr = 2\pi \mu k\left(\frac{R^3}{3} - \frac{R^4}{4R}\right) = \frac{1}{6}\pi \mu k R^3$$

$$= \frac{\pi \mu}{6}\frac{3P}{\pi R^2} R^3 = \frac{1}{2}\mu P R$$

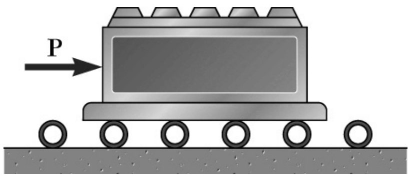
where $\qquad \mu = \mu_k = 0.25 \qquad R = 0.18 \text{ m}$

$$P = W = (25 \text{ kg})(9.81 \text{ m/s}^2) = 245.25 \text{ N}$$

Then $\qquad M = \dfrac{1}{2}(0.25)(245.25 \text{ N})(0.18 \text{ m}) = 5.5181 \text{ N}\cdot\text{m}$

Finally, $\qquad Q = \dfrac{M}{d} = \dfrac{5.5181 \text{ N}\cdot\text{m}}{0.4 \text{ m}} \qquad Q = 13.80 \text{ N}$ ◀

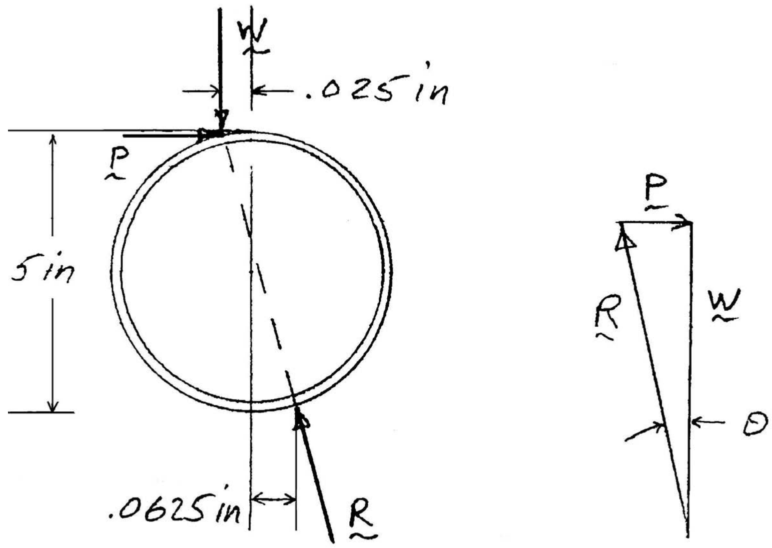
PROBLEM 8.96



A 1-ton machine base is rolled along a concrete floor using a series of steel pipes with outside diameters of 5 in. Knowing that the coefficient of rolling resistance is 0.025 in. between the pipes and the base and 0.0625 in. between the pipes and the concrete floor, determine the magnitude of the force **P** required to slowly move the base along the floor.

SOLUTION

FBD pipe:



$$\theta = \sin^{-1}\frac{0.025\text{ in.} + 0.0625\text{ in.}}{5\text{ in.}} = 1.00257^\circ$$

$P = W\tan\theta$ for each pipe, so also for total

$$P = (2000\text{ lb})\tan(1.00257^\circ)$$

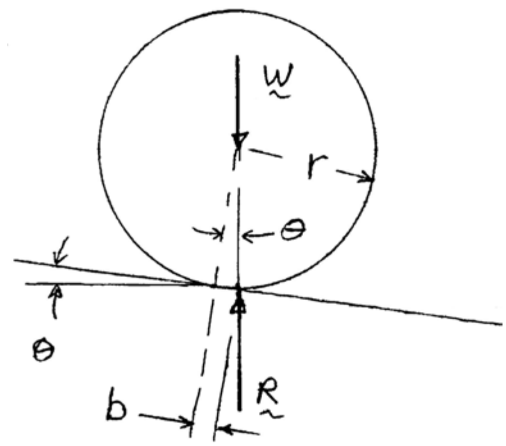
$P = 35.0\text{ lb}$ ◀

PROBLEM 8.97

Knowing that a 120-mm-diameter disk rolls at a constant velocity down a 2 percent incline, determine the coefficient of rolling resistance between the disk and the incline.

SOLUTION

FBD disk:



$$\tan\theta = \text{slope} = 0.02$$

$$b = r\tan\theta = (60\text{ mm})(0.02)$$

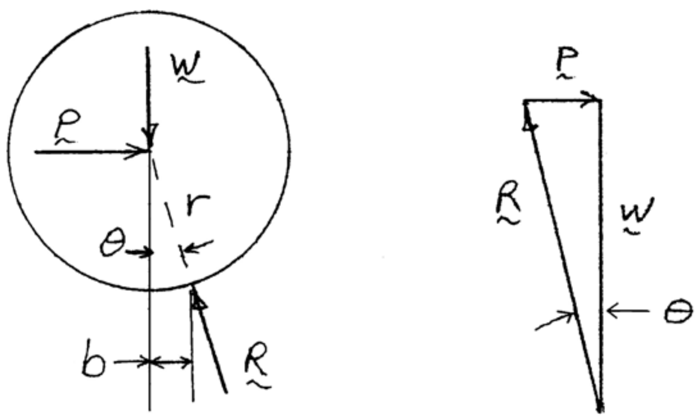
$$b = 1.200\text{ mm} \blacktriangleleft$$

PROBLEM 8.98

Determine the horizontal force required to move a 1-Mg automobile with 460-mm-diameter tires along a horizontal road at a constant speed. Neglect all forms of friction except rolling resistance, and assume the coefficient of rolling resistance to be 1 mm.

SOLUTION

FBD wheel:



$$r = 230 \text{ mm}$$

$$b = 1 \text{ mm}$$

$$\theta = \sin^{-1}\frac{b}{r}$$

$P = W\tan\theta = W\tan\left(\sin^{-1}\frac{b}{r}\right)$ for each wheel, so for total

$$P = (1000 \text{ kg})(9.81 \text{ m/s}^2)\tan\left(\sin^{-1}\frac{1}{230}\right)$$

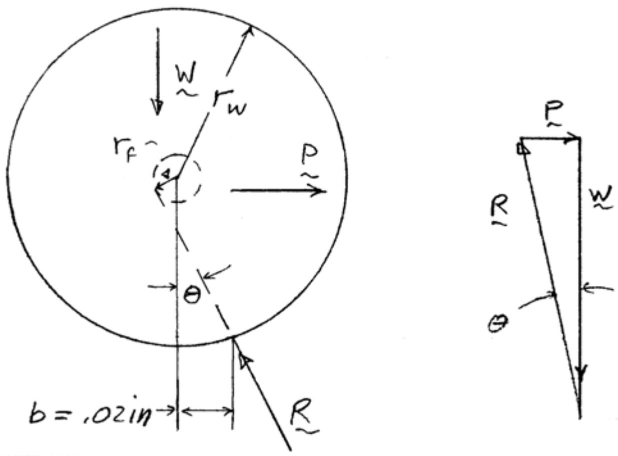
$P = 42.7 \text{ N}$ ◀

PROBLEM 8.99

Solve Problem 8.88 including the effect of a coefficient of rolling resistance of 0.02 in.

SOLUTION

FBD wheel:



$$r_f = r_a \sin\phi = r_a \sin\left(\tan^{-1}\mu\right)$$

$$= (2.5 \text{ in.}) \sin\left(\tan^{-1}\mu\right)$$

$P = W\tan\theta$ for each wheel, so also for total $P = W\tan\theta$

$$\tan\theta \approx \frac{b + r_f}{r_w} \text{ for small } \theta$$

So
$$P = (70{,}000 \text{ lb})\frac{(0.02 \text{ in.}) + r_f}{16 \text{ in.}}$$

(*a*) For impending motion, use $\mu_s = 0.02$:

Then $r_f = 0.04999$ in. and $P = 306$ lb ◀

(*b*) For steady motion, use $\mu_k = 0.015$:

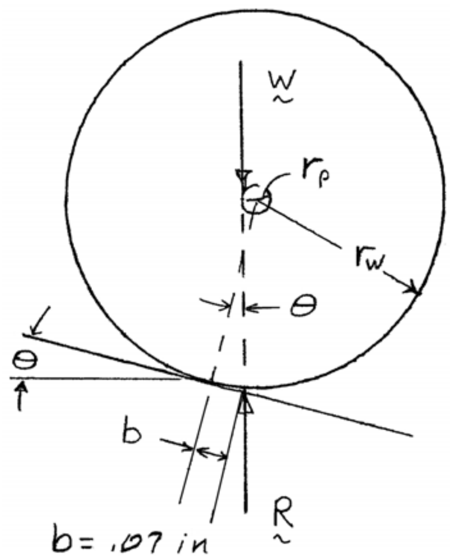
Then $r_f = 0.037496$ in. and $P = 252$ lb ◀

PROBLEM 8.100

Solve Problem 8.89 including the effect of a coefficient of rolling resistance of 0.07 in.

SOLUTION

FBD wheel:



The wheel is a two-force body, so **R** and **W** are colinear and tangent to the friction circle.

$$\tan\theta = \text{slope} = 0.02$$

$$\tan\theta \approx \frac{b + r_f}{r_w} \qquad \text{or} \qquad r_w \approx \frac{b + r_f}{\tan\theta}$$

Now

$$r_f = r_a \sin\phi_k = r_a \sin\left(\tan^{-1}\mu_k\right)$$

$$= (0.5\text{ in.})\sin\left(\tan^{-1}0.1\right)$$

$$= 0.049752$$

$$\therefore \quad r_w \approx \frac{0.07\text{ in.} + 0.049752\text{ in.}}{0.02} = 5.9876\text{ in.}$$

$$d = 2r_w$$

$d = 11.98$ in. ◀

PROBLEM 8.101

A hawser is wrapped two full turns around a bollard. By exerting a 320-N force on the free end of the hawser, a dockworker can resist a force of 20 kN on the other end of the hawser. Determine (*a*) the coefficient of static friction between the hawser and the bollard, (*b*) the number of times the hawser should be wrapped around the bollard if a 80-kN force is to be resisted by the same 320-N force.

SOLUTION

Two full turns of rope $\rightarrow$ $\qquad \beta = 4\pi$ rad

(*a*)

$$\mu_s \beta = \ln \frac{T_2}{T_1} \qquad \text{or} \qquad \mu_s = \frac{1}{\beta} \ln \frac{T_2}{T_1}$$

$$\mu_s = \frac{1}{4\pi} \ln \frac{20\,000 \text{ N}}{320 \text{ N}} = 0.329066$$

$$\mu_s = 0.329 \blacktriangleleft$$

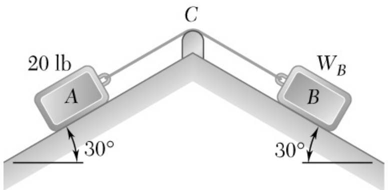
(*b*)

$$\beta = \frac{1}{\mu_s} \ln \frac{T_2}{T_1}$$

$$= \frac{1}{0.329066} \ln \frac{80\,000 \text{ N}}{320 \text{ N}}$$

$$= 16.799 \text{ rad}$$

$$\beta = 2.67 \text{ turns} \blacktriangleleft$$

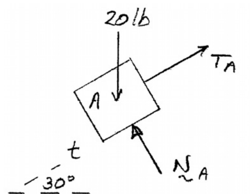


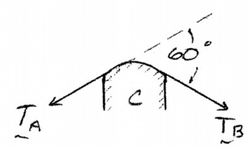
PROBLEM 8.102

Blocks A and B are connected by a cable that passes over support C. Friction between the blocks and the inclined surfaces can be neglected. Knowing that motion of block B up the incline is impending when $W_B = 16$ lb, determine (a) the coefficient of static friction between the rope and the support, (b) the largest value of W_B for which equilibrium is maintained. (*Hint:* See Problem 8.128.)

SOLUTION

FBD A:

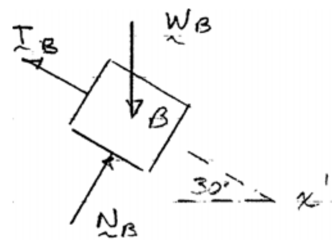




$$\nearrow \Sigma F_t = 0: \quad T_A - 20 \text{ lb} \sin 30° = 0$$

$$T_A = 10 \text{ lb}$$

FBD B:



$$\searrow \Sigma F_{x'} = 0: \quad W_B \sin 30° - T_B = 0$$

$$T_B = \frac{W_B}{2}$$

From hint, $\beta = 60° = \frac{\pi}{3}\text{rad}$ regardless of shape of support C

(a) For motion of B up incline when $W_B = 16$ lb, $\quad T_B = \frac{W_B}{2} = 8 \text{ lb}$

and $\quad \mu_s \beta = \ln \frac{T_A}{T_B} \quad$ or $\quad \mu_s = \frac{1}{\beta} \ln \frac{T_A}{T_B} = \frac{3}{\pi} \ln \frac{10 \text{ lb}}{8 \text{ lb}} = 0.213086$

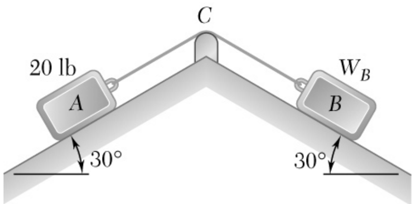
$$\mu_s = 0.213 \blacktriangleleft$$

(b) For maximum W_B, motion of B impends down and $T_B > T_A$

So $\quad T_B = T_A e^{\mu_s \beta} = (10 \text{ lb}) e^{0.213086\pi/3} = 12.500 \text{ lb}$

Now $\quad W_B = 2T_B$

So that $\quad W_B = 25.0 \text{ lb} \blacktriangleleft$

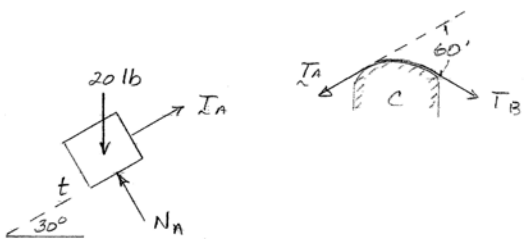


PROBLEM 8.103

Blocks A and B are connected by a cable that passes over support C. Friction between the blocks and the inclined surfaces can be neglected. Knowing that the coefficient of static friction between the rope and the support is 0.50, determine the range of values of W_B for which equilibrium is maintained. (*Hint:* See Problem 8.128.)

SOLUTION

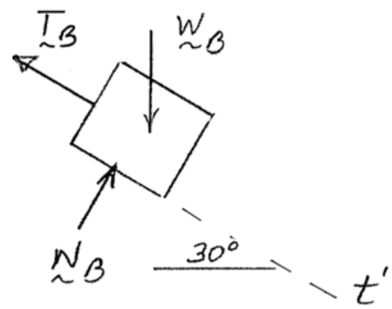
FBD A:



$$\Sigma F_t = 0: \quad T_A - 20 \text{ lb} \sin 30° = 0$$

$$T_A = 10 \text{ lb}$$

FBD B:



$$\Sigma F_{t'} = 0: \quad W_B \sin 30° - T_B = 0$$

$$T_B = \frac{W_B}{2}$$

From hint, $\beta = 60° = \dfrac{\pi}{3}$ rad, regardless of shape of support C.

For impending motion of B up, $T_A > T_B$, so

$$T_A = T_B e^{\mu_s \beta} \qquad \text{or} \qquad T_B = T_A e^{-\mu_s \beta} = (10 \text{ lb}) e^{-0.5\pi/3} = 5.924 \text{ lb}$$

$$W_B = 2T_B = 11.85 \text{ lb}$$

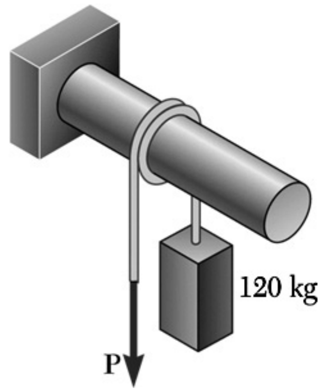
For impending motion of B down, $T_B > T_B$, so

$$T_B = T_A e^{\mu_s \beta} = (10 \text{ lb}) e^{0.5\pi/3} = 16.881 \text{ lb}$$

$$W_B = 2T_B = 33.76 \text{ lb}$$

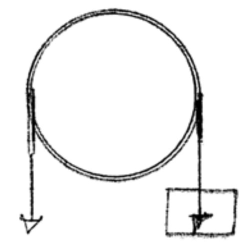
For equilibrium $\qquad 11.85 \text{ lb} \le W_B \le 33.8 \text{ lb}$ ◀

PROBLEM 8.104



A 120-kg block is supported by a rope which is wrapped $1\frac{1}{2}$ times around a horizontal rod. Knowing that the coefficient of static friction between the rope and the rod is 0.15, determine the range of values of P for which equilibrium is maintained.

SOLUTION



$\underset{\sim}{P}$ $\quad \underset{\sim}{W} = (120\text{ kg})(9.81\text{ m/s}^2)$

$= 1177.2\text{ N}$

$$\beta = 1.5\text{ turns} = 3\pi\text{ rad}$$

For impending motion of W up

$$P = We^{\mu_s \beta} = (1177.2\text{ N})e^{(0.15)3\pi}$$

$$= 4839.7\text{ N}$$

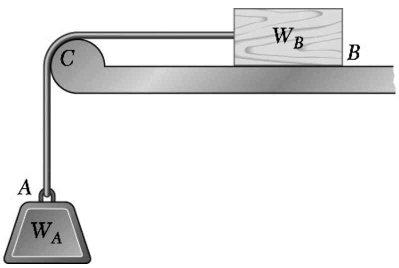
For impending motion of W down

$$P = We^{-\mu_s \beta} = (1177.2\text{ N})e^{-(0.15)3\pi}$$

$$= 286.3\text{ N}$$

For equilibrium

$286\text{ N} \le P \le 4.84\text{ kN}$ ◀

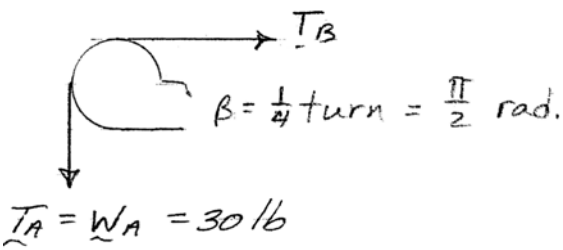


PROBLEM 8.105

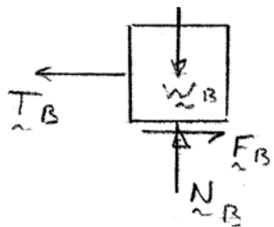
The coefficient of static friction between block B and the horizontal surface and between the rope and support C is 0.40. Knowing that $W_A = 30\text{ lb}$, determine the smallest weight of block B for which equilibrium is maintained.

SOLUTION

Support at C:



FBD block B:



$$\uparrow \Sigma F_y = 0: \quad N_B - W_B = 0 \qquad \text{or} \qquad N_B = W_B$$

Impending motion $$F_B = \mu_s N_B = 0.4 N_B = 0.4 W_B$$

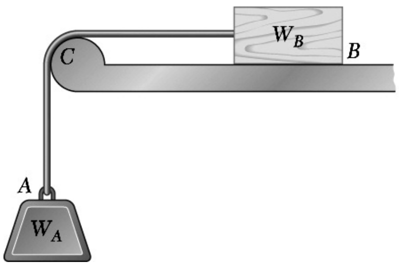
$$\longrightarrow \Sigma F_x = 0: \quad F_B - T_B = 0 \qquad \text{or} \qquad T_B = F_B = 0.4 W_B$$

At support, for impending motion of W_A down, $$W_A = T_B e^{\mu_s \beta}$$

so $$T_B = W_A e^{-\mu_s \beta} = (30\text{ lb}) e^{-(0.4)\pi/2} = 16.005\text{ lb}$$

Now $$W_B = \frac{T_B}{0.4}$$

so that $$W_B = 40.0\text{ lb} \blacktriangleleft$$

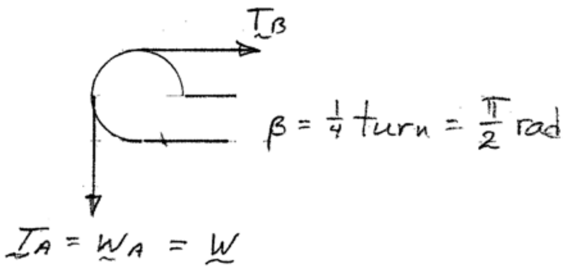


PROBLEM 8.106

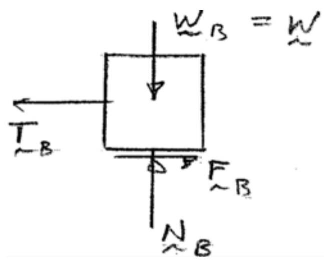
The coefficient of static friction μ_s is the same between block B and the horizontal surface and between the rope and support C. Knowing that $W_A = W_B$, determine the smallest value of μ_s for which equilibrium is maintained.

SOLUTION

Support at C



FBD B:



$$\uparrow \Sigma F_y = 0: \quad N_B - W = 0 \qquad \text{or} \qquad N_B = W$$

Impending motion: $\quad F_B = \mu_s N_B = \mu_s W$

$$\longrightarrow \Sigma F_x = 0: \quad F_B - T_B = 0 \qquad \text{or} \qquad T_B = F_B = \mu_s W$$

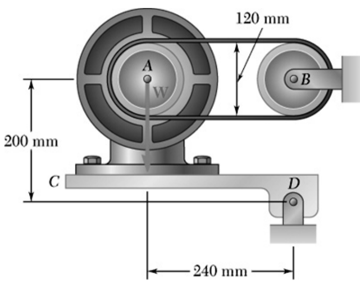
Impending motion of rope on support: $\quad W = T_B e^{\mu_s \beta} = \mu_s W e^{\mu_s \beta}$

or $\quad 1 = \mu_s e^{\mu_s \beta}$

or $\quad \mu_s e^{\frac{\pi}{2}\mu_s} = 1$

Solving numerically: $\quad \mu_s = 0.475$ ◀

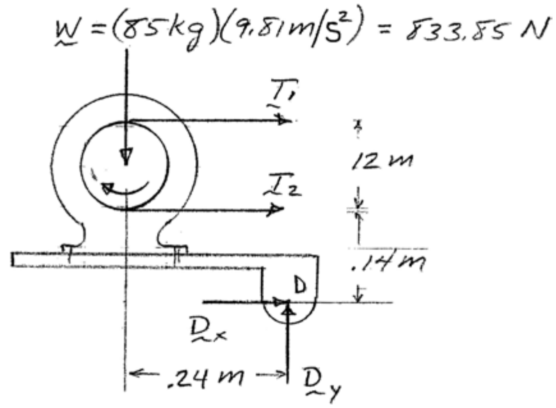
PROBLEM 8.107



In the pivoted motor mount shown, the weight **W** of the 85-kg motor is used to maintain tension in the drive belt. Knowing that the coefficient of static friction between the flat belt and drums *A* and *B* is 0.40, and neglecting the weight of platform *CD*, determine the largest torque which can be transmitted to drum *B* when the drive drum *A* is rotating clockwise.

SOLUTION

FBD motor + mount:



For impending slipping of belt,
$$T_2 = T_1 e^{\mu_s \beta} = T_1 e^{0.4\pi} = 3.5136 T_1$$

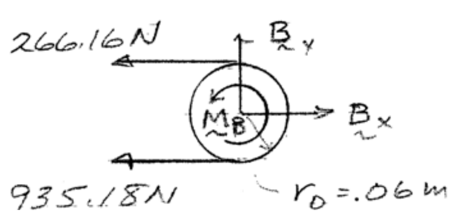
$$\circlearrowleft \Sigma M_D = 0: \quad (0.24 \text{ m})(833.85 \text{ N}) - (0.14 \text{ m})T_2 - (0.26 \text{ m})T_1 = 0$$

$$\left[(0.14 \text{ m})(3.5136) + 0.26 \text{ m}\right]T_1 = 200.124 \text{ N}\cdot\text{m}$$

or
$$T_1 = 266.16 \text{ N}$$

and
$$T_2 = 3.5136 T_1 = 935.18 \text{ N}$$

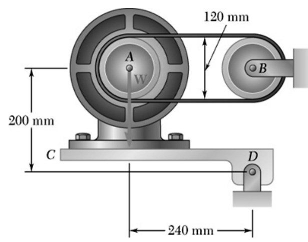
FBD drum:



$$\circlearrowleft \Sigma M_B = 0: \quad M_B - (0.06\text{m})(266.16 \text{ N} - 935.18 \text{ N}) = 0$$

$$M_B = 40.1 \text{ N}\cdot\text{m} \blacktriangleleft$$

(Compare to $M_B = 81.7 \text{ N}\cdot\text{m}$ using V-belt, Problem 8.130.)

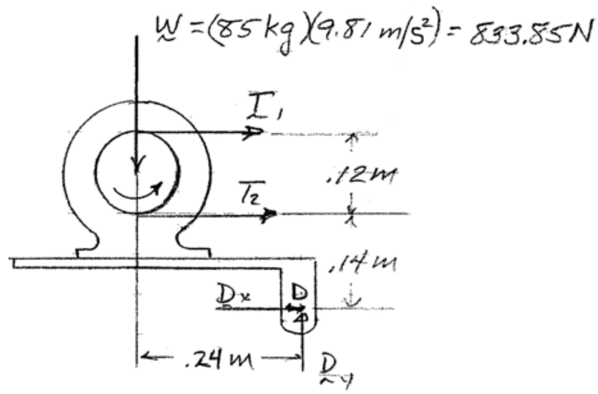


PROBLEM 8.108

Solve Problem 8.107 assuming that the drive drum *A* is rotating counterclockwise.

SOLUTION

FBD motor + mount:



Impending slipping of belt: $$T_1 = T_2 e^{\mu_s \beta} = T_1 e^{0.4\pi} = 3.5136 T_2$$

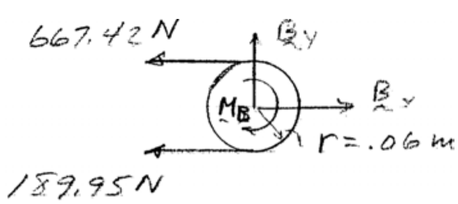
$$\circlearrowleft \Sigma M_D = 0: \quad (0.24\text{ m})W - (0.26\text{ m})T_1 - (0.14\text{ m})T_2 = 0$$

$$\left[(0.26\text{ m})(3.5136) + 0.14\text{ m}\right]T_2 = (0.24\text{ m})(833.85\text{ N})$$

or $$T_2 = 189.95\text{ N}$$

and $$T_1 = 667.42\text{ N}$$

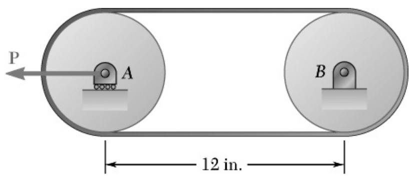
FBD drum:



$$\circlearrowleft \Sigma M_B = 0: \quad (0.06\text{ m})(667.42\text{ N} - 189.95\text{ N}) - M_B = 0$$

$$M_B = 28.6\text{ N}\cdot\text{m} \blacktriangleleft$$

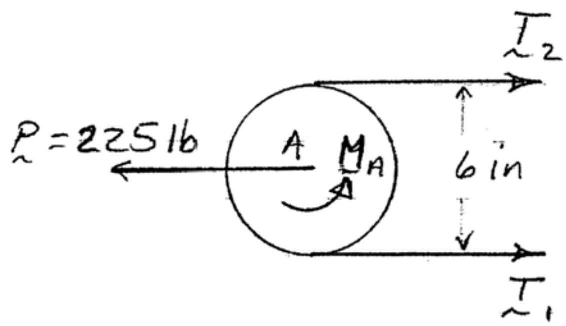
PROBLEM 8.109



A flat belt is used to transmit a torque from pulley A to pulley B. The radius of each pulley is 3 in., and a force of magnitude $P = 225\,\text{lb}$ is applied as shown to the axle of pulley A. Knowing that the coefficient of static friction is 0.35, determine (a) the largest torque which can be transmitted, (b) the corresponding maximum value of the tension in the belt.

SOLUTION

FBD pulley A:



Impending slipping of belt: $$T_2 = T_1 e^{\mu_s \beta}$$

$$T_2 = T_1 e^{0.35\pi} = 3.0028 T_1$$

$$\longrightarrow \Sigma F_x = 0: \quad T_1 + T_2 - 225\text{ lb} = 0$$

$$T_1(1 + 3.0028) = 225\text{ lb} \qquad \text{or} \qquad T_1 = 56.211\text{ lb}$$

$$T_2 = 3.0028 T_1 \qquad \text{or} \qquad T_2 = 168.79\text{ lb}$$

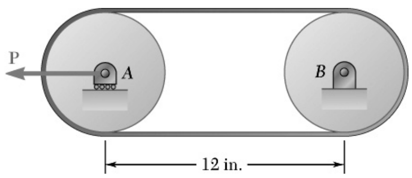
(a) $$\Sigma M_A = 0: \quad M_A + (6\text{ in.})(T_1 - T_2) = 0 \qquad \text{or} \qquad M_A = (3\text{ in.})(168.79\text{ lb} - 56.21\text{ lb})$$

$$\therefore \quad \text{max. torque: } M_A = 338\text{ lb}\cdot\text{in.} \blacktriangleleft$$

(b) $$\text{max. tension: } T_2 = 168.8\text{ lb} \blacktriangleleft$$

(Compare with $M_A = 638\text{ lb}\cdot\text{in.}$ with V-belt, Problem 8.131.)

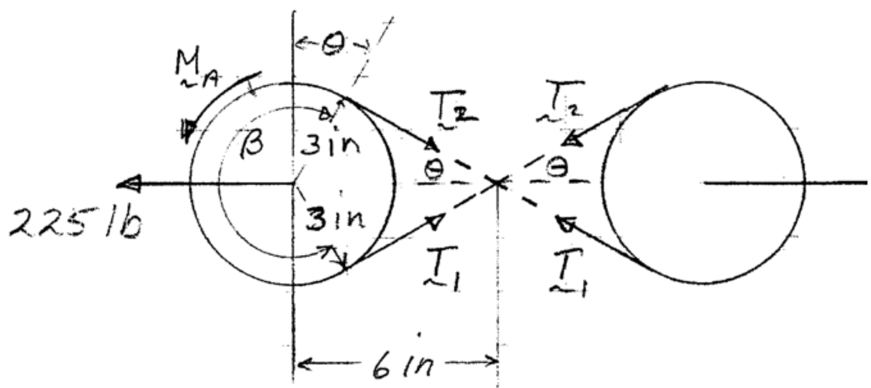
PROBLEM 8.110



Solve Problem 8.109 assuming that the belt is looped around the pulleys in a figure eight.

SOLUTION

FBDs pulleys:



$$\theta = \sin^{-1}\frac{3\text{ in.}}{6\text{ in.}} = 30° = \frac{\pi}{6}\text{rad.}$$

$$\beta = \pi + 2\frac{\pi}{6} = \frac{4\pi}{3}$$

Impending belt slipping: $T_2 = T_1 e^{\mu_s \beta}$

$$T_2 = T_1 e^{(0.35)4\pi/3} = 4.3322T_1$$

$$\longrightarrow \Sigma F_x = 0: \quad T_1 \cos 30° + T_2 \cos 30° - 225 \text{ lb} = 0$$

$$(T_1 + 4.3322T_1)\cos 30° = 225 \text{ lb} \qquad \text{or} \qquad T_1 = 48.7243 \text{ lb}$$

$$T_2 = 4.3322T_1 \qquad \text{so that} \qquad T_2 = 211.083 \text{ lb}$$

(*a*) $\circlearrowleft \Sigma M_A = 0: \quad M_A + (3\text{ in.})(T_1 - T_2) = 0 \qquad \text{or} \qquad M_A = (3\text{ in.})(211.083\text{ lb} - 48.224\text{ lb})$

$$M_{max} = M_A = 487 \text{ lb}\cdot\text{in.} \blacktriangleleft$$

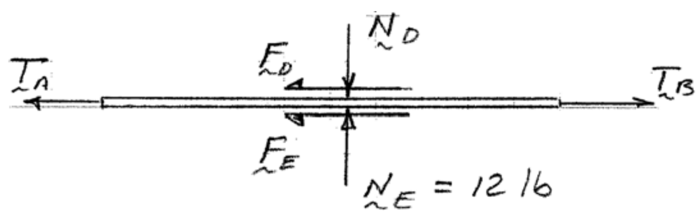
(*b*) $T_{max} = T_2 = 211 \text{ lb} \blacktriangleleft$

PROBLEM 8.111

A couple $\mathbf{M}_B$ of magnitude $2\text{ lb}\cdot\text{ft}$ is applied to the drive drum B of a portable belt sander to maintain the sanding belt C at a constant speed. The total downward force exerted on the wooden workpiece E is 12 lb, and $\mu_k = 0.10$ between the belt and the sanding platen D. Knowing that $\mu_s = 0.35$ between the belt and the drive drum and that the radii of drums A and B are 1.00 in., determine (*a*) the minimum tension in the lower portion of the belt if no slipping is to occur between the belt and the drive drum, (*b*) the value of the coefficient of kinetic friction between the belt and the workpiece.

SOLUTION

FBD lower portion of belt:



$$\uparrow \Sigma F_y = 0: \quad N_E - N_D = 0$$

or

$$N_D = N_E = 12\text{ lb}$$

Slipping:

$$F_D = (\mu_k)_{\text{belt/platen}} N_D$$

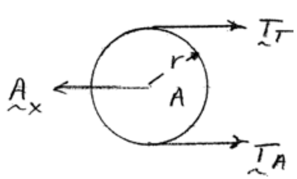
$$F_D = 0.1(12\text{ lb}) = 1.2\text{ lb}$$

and

$$F_E = (\mu_k)_{\text{belt/wood}} N_E$$

$$F = (12\text{ lb})(\mu_k)_{\text{belt/wood}} \qquad (1)$$

FBD drum A: (assumed free to rotate)

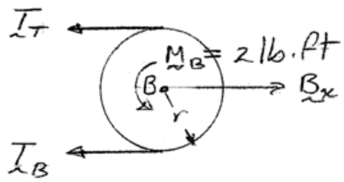


$$\longrightarrow \Sigma F_x = 0: \quad T_B - T_A - F_D - F_E = 0 \qquad (2)$$

$$\circlearrowleft \Sigma M_A = 0: \quad r_A(T_A - T_T) = 0 \qquad \text{or} \qquad T_T = T_A$$

PROBLEM 8.111 CONTINUED

FBD drum B:



$$\circlearrowleft \Sigma M_B = 0: \quad M_B + r\left(T_T - T_B\right) = 0$$

or $$T_B - T_T = \frac{M_B}{r} = \left(\frac{2 \text{ lb}\cdot\text{ft}}{1 \text{ in.}}\right)\left(\frac{12 \text{ in.}}{\text{ft}}\right) = 24 \text{ lb}$$

Impending slipping: $$T_B = T_T e^{\mu_s \beta} = T_T e^{0.35\pi}$$

So $$\left(e^{0.35\pi} - 1\right)T_T = 24 \text{ lb} \qquad \text{or} \qquad T_T = 11.983 \text{ lb}$$

Now $$T_A = T_T = 11.983 \text{ lb} \text{ then } T_B = \left(11.983 \text{ lb}\right)e^{0.35\pi} = 35.983 \text{ lb}$$

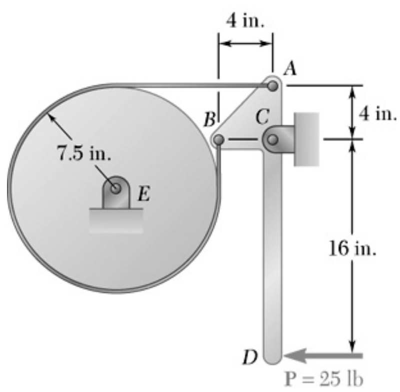
From Equation (2): $$35.983 \text{ lb} - 11.983 \text{ lb} - 1.2 \text{ lb} = F_E = 22.8 \text{ lb}$$

From Equation (1): $$\left(\mu_k\right)_{\text{belt/wood}} = \frac{F_E}{12 \text{ lb}} = \frac{22.8 \text{ lb}}{12 \text{ lb}} = 1.900$$

Therefore

(a) $T_{\min} = T_A = 11.98 \text{ lb}$ ◀

(b) $\left(\mu_k\right)_{\text{belt/wood}} = 1.900$ ◀

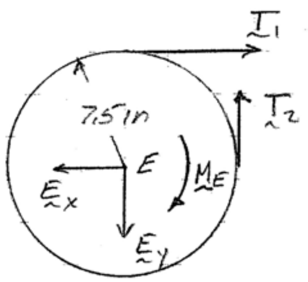


PROBLEM 8.112

A band belt is used to control the speed of a flywheel as shown. Determine the magnitude of the couple being applied to the flywheel knowing that the coefficient of kinetic friction between the belt and the flywheel is 0.25 and that the flywheel is rotating clockwise at a constant speed. Show that the same result is obtained if the flywheel rotates counterclockwise.

SOLUTION

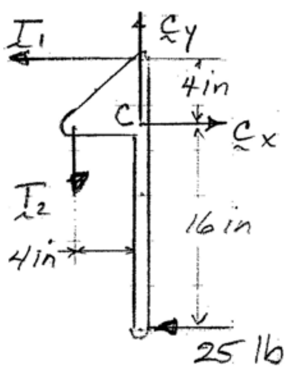
FBD wheel:



$$\Sigma M_E = 0: \quad -M_E + (7.5\text{ in.})(T_2 - T_1) = 0$$

or $$M_E = (7.5\text{ in.})(T_2 - T_1)$$

FBD lever:



$$\Sigma M_C = 0: \quad (4\text{ in.})(T_1 + T_2) - (16\text{ in.})(25\text{ lb}) = 0$$

or $$T_1 + T_2 = 100\text{ lb}$$

Impending slipping: $$T_2 = T_1 e^{\mu_s \beta}$$

or $$T_2 = T_1 e^{0.25\left(\frac{3\pi}{2}\right)} = 3.2482 T_1$$

So $$T_1(1 + 3.2482) = 100\text{ lb}$$

$$T_1 = 23.539\text{ lb}$$

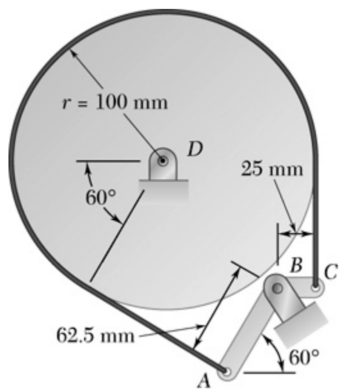
and $$M_E = (7.5\text{ in.})(3.2482 - 1)(23.539\text{ lb}) = 396.9\text{ lb}\cdot\text{in.}$$

$$M_E = 397\text{ lb}\cdot\text{in.} \blacktriangleleft$$

Changing the direction of rotation will change the direction of M_E and will switch the magnitudes of T_1 and T_2.

The magnitude of the couple applied will not change. ◀

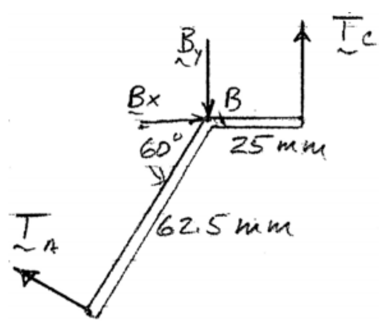
PROBLEM 8.113



The drum brake shown permits clockwise rotation of the drum but prevents rotation in the counterclockwise direction. Knowing that the maximum allowed tension in the belt is 7.2 kN, determine (*a*) the magnitude of the largest counterclockwise couple that can be applied to the drum, (*b*) the smallest value of the coefficient of static friction between the belt and the drum for which the drum will not rotate counterclockwise.

SOLUTION

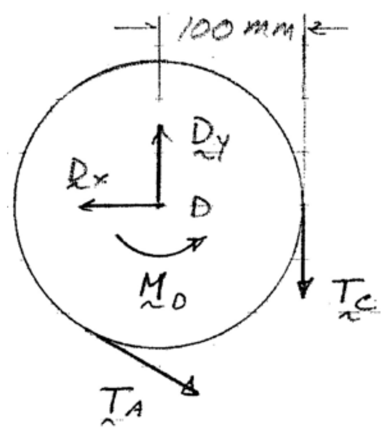
FBD lever:



$$\circlearrowleft \Sigma M_B = 0: \quad (25\text{ mm})T_C - (62.5\text{ mm})T_A = 0$$

$$T_C = 2.5T_A$$

FBD lever:



Impending ccw rotation:

(*a*) $$T_C = T_{\max} = 7.2\text{ kN}$$

But $$T_C = 2.5T_A$$

So $$T_A = \frac{7.2\text{ kN}}{2.5} = 2.88\text{ kN}$$

$$\circlearrowleft \Sigma M_D = 0: \quad M_D + (100\text{ mm})(T_A - T_C) = 0$$

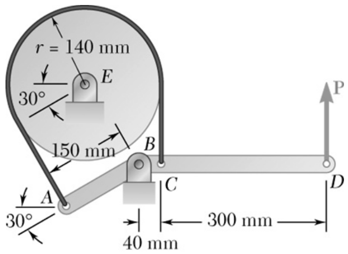
$$M_D = (100\text{ mm})(7.2 - 2.88)\text{ kN}$$

$$M_D = 432\text{ N}\cdot\text{m} \blacktriangleleft$$

(*b*) Also, impending slipping: $$\mu_s\beta = \ln\frac{T_C}{T_A}$$

$$\mu_s = \frac{1}{\beta}\ln\frac{T_C}{T_A} = \frac{1}{\frac{4\pi}{3}}\ln 2.5 = 0.2187$$

Therefore, $$(\mu_s)_{\min} = 0.219 \blacktriangleleft$$

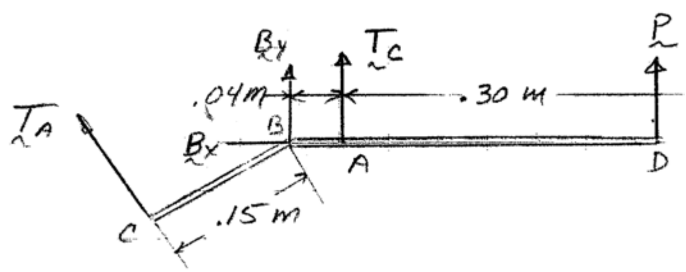


PROBLEM 8.114

A differential band brake is used to control the speed of a drum which rotates at a constant speed. Knowing that the coefficient of kinetic friction between the belt and the drum is 0.30 and that a couple of magnitude is $150\ \text{N}\cdot\text{m}$ applied to the drum, determine the corresponding magnitude of the force **P** that is exerted on end *D* of the lever when the drum is rotating (*a*) clockwise, (*b*) counterclockwise.

SOLUTION

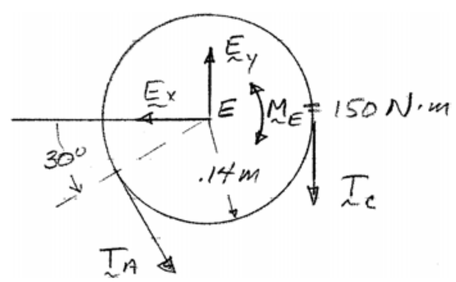
FBD lever:



$$\circlearrowleft \Sigma M_B = 0: \quad (0.34\ \text{m})P + (0.04\ \text{m})T_C - (0.15\ \text{m})T_A = 0$$

$$P = \frac{15T_A - 4T_C}{34} \qquad (1)$$

FBD drum:



(*a*) For cw rotation, $M_E \circlearrowright$

$$\circlearrowleft \Sigma M_E = 0: \quad (0.14\ \text{m})(T_A - T_C) - M_E = 0$$

$$T_A - T_C = \frac{150\ \text{N}\cdot\text{m}}{0.14\ \text{m}} = 1071.43\ \text{N}$$

Impending slipping: $\quad T_A = T_C e^{\mu_k \beta} = T_C e^{(0.3)\frac{7\pi}{6}}$

$$T_A = 3.00284 T_C$$

So $\quad (3.00284 - 1)T_C = 1071.43\ \text{N} \quad$ or $\quad T_C = 534.96\ \text{N}$

and $\quad T_A = 1606.39\ \text{N}$

PROBLEM 8.114 CONTINUED

From Equation (1):

$$P = \frac{15(1606.39\text{ N}) - 4(534.96\text{ N})}{34}$$

$$P = 646\text{ N} \blacktriangleleft$$

(*b*) For ccw rotation, $M_E \circlearrowleft$ and $\Sigma M_E = 0 \Rightarrow T_C - T_A = 1071.43\text{ N}$

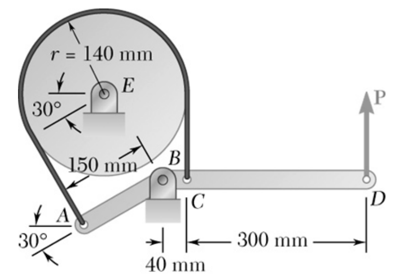
Also, impending slip $\Rightarrow$ $T_C = 3.00284T_A$, so $T_A = 534.96\text{ N}$

and $T_C = 1606.39\text{ N}$

And Equation (1) $\Rightarrow$

$$P = \frac{15(534.96\text{ N}) - 4(1606.39\text{ N})}{34}$$

$$P = 47.0\text{ N} \blacktriangleleft$$

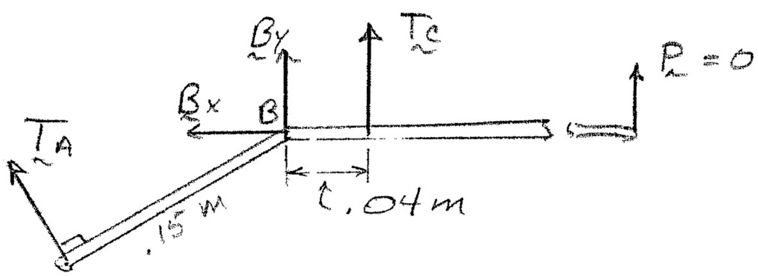


PROBLEM 8.115

A differential band brake is used to control the speed of a drum. Determine the minimum value of the coefficient of static friction for which the brake is self-locking when the drum rotates counterclockwise.

SOLUTION

FBD lever:

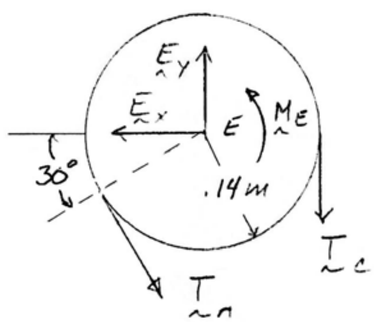


For self-locking $\mathbf{P} = 0$

$$\circlearrowleft \Sigma M_B = 0: \quad (0.04\text{ m})T_C - (0.15\text{ m})T_A = 0$$

$$T_C = 3.75T_A$$

FBD drum:



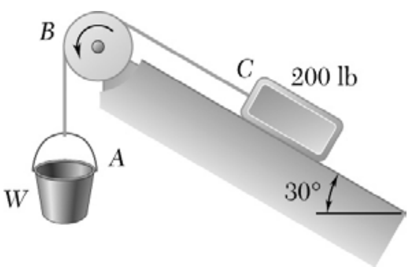
For impending slipping of belt $$T_C = T_A e^{\mu_s \beta}$$

or $$\mu_s \beta = \ln\frac{T_C}{T_A}$$

Then $$\mu_s = \frac{1}{\beta}\ln\frac{T_C}{T_A} = \frac{1}{\frac{7\pi}{6}}\ln 3.75 = 0.3606$$

$$(\mu_s)_{\text{req}} = 0.361 \blacktriangleleft$$

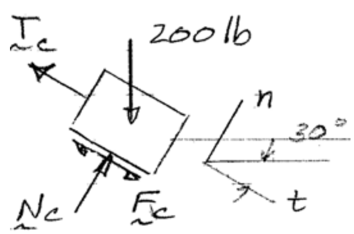
PROBLEM 8.116



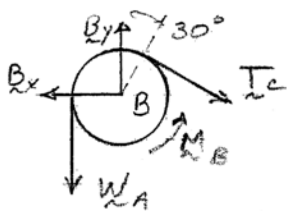
Bucket A and block C are connected by a cable that passes over drum B. Knowing that drum B rotates slowly counterclockwise and that the coefficients of friction at all surfaces are $\mu_s = 0.35$ and $\mu_k = 0.25$, determine the smallest combined weight W of the bucket and its contents for which block C will (*a*) remain at rest, (*b*) be about to move up the incline, (*c*) continue moving up the incline at a constant speed.

SOLUTION

FBD block:



FBD drum:



$$\nearrow \Sigma F_n = 0: \quad N_C - (200\text{ lb})\cos 30° = 0; \; N = 100\sqrt{3}\text{ lb}$$

$$\nwarrow \Sigma F_t = 0: \quad T_C - (200\text{ lb})\sin 30° \mp F_C = 0$$

$$T_C = 100\text{ lb} \pm F_C \qquad (1)$$

where the upper signs apply when F_C acts $\searrow$

(*a*) For impending motion of block $\searrow$, F_C $\nwarrow$, and

$$F_C = \mu_s N_C = 0.35\left(100\sqrt{3}\text{ lb}\right) = 35\sqrt{3}\text{ lb}$$

So, from Equation (1): $\quad T_C = \left(100 - 35\sqrt{3}\right)\text{ lb}$

But belt slips on drum, so $\quad T_C = W_A e^{\mu_k \beta}$

$$W_A = \left[\left(100 - 35\sqrt{3}\right)\text{lb}\right] e^{-0.25\left(\frac{2\pi}{3}\right)}$$

$W_A = 23.3\text{ lb}$ ◀

(*b*) For impending motion of block $\nwarrow$, F_C $\searrow$ and $F_C = \mu_s N_C = 35\sqrt{3}\text{ lb}$

From Equation (1): $\quad T_C = \left(100 + 35\sqrt{3}\right)\text{ lb}$

Belt still slips, so $\quad W_A = T_C e^{-\mu_k \beta} = \left[\left(100 + 35\sqrt{3}\right)\text{ lb}\right] e^{-0.25\left(\frac{2\pi}{3}\right)}$

$W_A = 95.1\text{ lb}$ ◀

PROBLEM 8.116 CONTINUED

(*c*) For steady motion of block $\nwarrow$, F_C $\searrow$, and $F_C = \mu_k N_C = 25\sqrt{3}$ lb

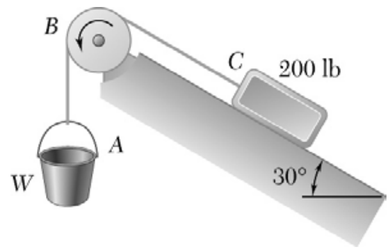
Then, from Equation (1): $T = \left(100 + 25\sqrt{3}\right)$ lb.

Also, belt is not slipping on drum, so

$$W_A = T_C e^{-\mu_s \beta} = \left[\left(100 + 25\sqrt{3}\right)\text{ lb}\right] e^{-0.35\left(\frac{2\pi}{3}\right)}$$

$W_A = 68.8$ lb ◀

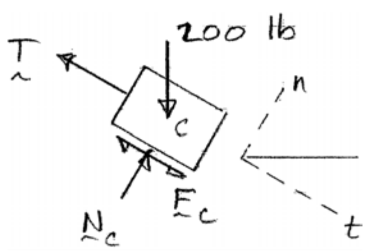
PROBLEM 8.117



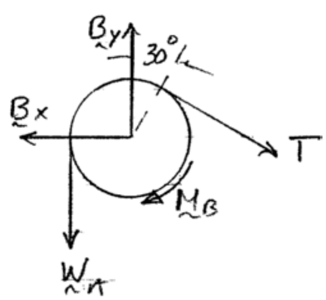
Solve Problem 8.116 assuming that drum B is frozen and cannot rotate.

SOLUTION

FBD block:



FBD drum:



$\nearrow \Sigma F_n = 0: \quad N_C - (200 \text{ lb})\cos 30° = 0; \;\; N_C = 100\sqrt{3} \text{ lb}$

$\searrow \Sigma F_t = 0: \quad \pm F_C + (200 \text{ lb})\sin 30° - T = 0$

$$T = 100 \text{ lb} \pm F_C \qquad (1)$$

where the upper signs apply when F_C acts $\searrow$

(a) For impending motion of block $\searrow$, F_C $\nwarrow$ and $F_C = \mu_s N_C$

So $\quad F_C = 0.35\left(100\sqrt{3} \text{ lb}\right) = 35\sqrt{3} \text{ lb}$

and $\quad T = 100 \text{ lb} - 35\sqrt{3} \text{ lb} = 39.375 \text{ lb}$

Also belt slipping is impending so $\quad T = W_A e^{\mu_s \beta}$

or $\quad W_A = Te^{-\mu_s \beta} = (39.378 \text{ lb})e^{-0.35\left(\frac{2\pi}{3}\right)}$

$$W_A = 18.92 \text{ lb} \blacktriangleleft$$

(b) For impending motion of block $\nwarrow$, F_C $\searrow$, and

$F_C = \mu_s N_C = 35\sqrt{3} \text{ lb}$

But $\quad T = \left(100 + 35\sqrt{3}\right) \text{ lb} = 160.622 \text{ lb}.$

Also belt slipping is impending

So $\quad W_A = Te^{+\mu_s \beta} = (160.622 \text{ lb})e^{0.35\left(\frac{2\pi}{3}\right)};$

$$W_A = 334 \text{ lb} \blacktriangleleft$$

(c) For steady motion of block $\nwarrow$, F_C $\searrow$, and $F_C = \mu_k N_C = 25\sqrt{3} \text{ lb}$

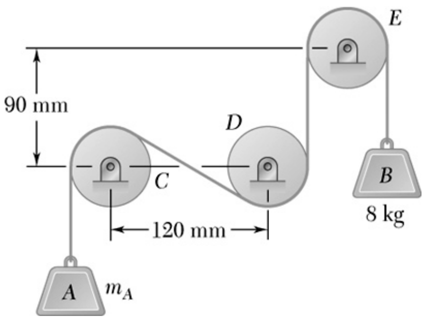
Then $\quad T = \left(100 \text{ lb} + 25\sqrt{3} \text{ lb}\right) = 143.301 \text{ lb}.$

Now belt is slipping

So $\quad W_A = Te^{\mu_k \beta} = (143.301 \text{ lb})e^{0.25\left(\frac{2\pi}{3}\right)}$

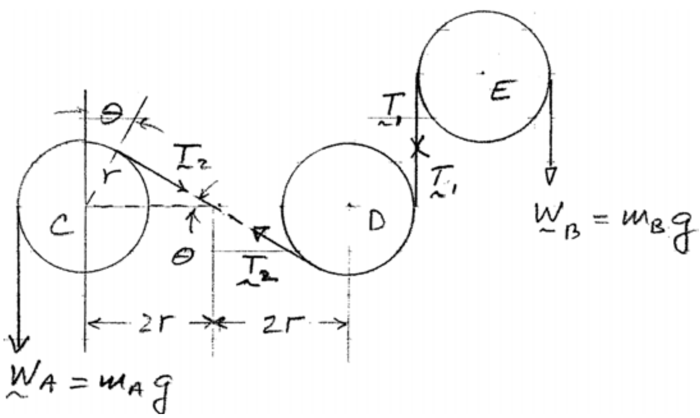
$$W_A = 242 \text{ lb} \blacktriangleleft$$

PROBLEM 8.118



A cable passes around three 30-mm-radius pulleys and supports two blocks as shown. Pulleys C and E are locked to prevent rotation, and the coefficients of friction between the cable and the pulleys are $\mu_s = 0.20$ and $\mu_k = 0.15$. Determine the range of values of the mass of block A for which equilibrium is maintained (a) if pulley D is locked, (b) if pulley D is free to rotate.

SOLUTION



Note: $$\theta = \sin^{-1}\frac{r}{2r} = 30° = \frac{\pi}{6}\text{ rad}$$

So $$\beta_C = \beta_D = \frac{2\pi}{3} \qquad \text{and} \qquad \beta_E = \pi$$

(a) All pulleys locked $\Rightarrow$ slipping impends at all surface simultaneously.

If A impends $\uparrow$, $$T_2 = W_A e^{\mu_s \beta_C};\ T_1 = T_2 e^{\mu_s \beta_D};\ W_B = T_1 e^{\mu_s \beta_E}$$

So $$W_B = W_A e^{\mu_s(\beta_C + \beta_D + \beta_E)} \qquad \text{or} \qquad W_A = W_B e^{-\mu_s(\beta_C + \beta_D + \beta_E)}$$

Then $$m_A = m_B e^{-\mu_s(\beta_C + \beta_D + \beta_E)} = (8\text{ kg})e^{-0.2\left(\frac{2\pi}{3} + \frac{2\pi}{3} + \pi\right)} = 1.847\text{ kg}$$

If A impends $\downarrow$, $$W_A = T_2 e^{\mu_s \beta_C} = T_1 e^{\mu_s \beta_D} e^{\mu_s \beta_C} = W_B e^{\mu_s(\beta_E + \beta_D + \beta_C)}$$

So $$m_A = m_B e^{\mu_s(\beta_E + \beta_D + \beta_C)} = (8\text{ kg})e^{0.2\left(\pi + \frac{2\pi}{3} + \frac{2\pi}{3}\right)} = 34.7\text{ kg}$$

Equilibrium for $1.847\text{ kg} \le m_A \le 34.7\text{ kg}$ ◀

PROBLEM 8.118 CONTINUED

(*b*) Pulleys *C* & *E* locked, pulley *D* free $\Rightarrow T_1 = T_2$, other relations remain the same.

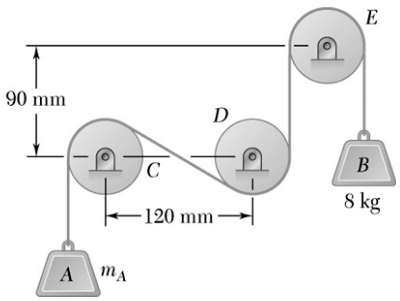
If *A* impends $\uparrow$, $\quad T_2 = W_A e^{\mu_s \beta_C} = T_1 \qquad W_B = T_1 e^{\mu_s \beta_E} = W_A e^{\mu_s(\beta_C + \beta_E)}$

So $\quad m_A = m_B e^{-\mu_s(\beta_C + \beta_E)} = (8\text{ kg})e^{-0.2\left(\frac{2\pi}{3} + \pi\right)} = 2.807\text{ kg}$

If *A* impends $\downarrow$ slipping is reversed, $\quad W_A = W_B e^{+\mu_s(\beta_C + \beta_E)}$

Then $\quad m_A = m_B e^{\mu_s(\beta_C + \beta_E)} = (8\text{ kg})e^{0.2\left(\frac{5\pi}{3}\right)} = 22.8\text{ kg}$

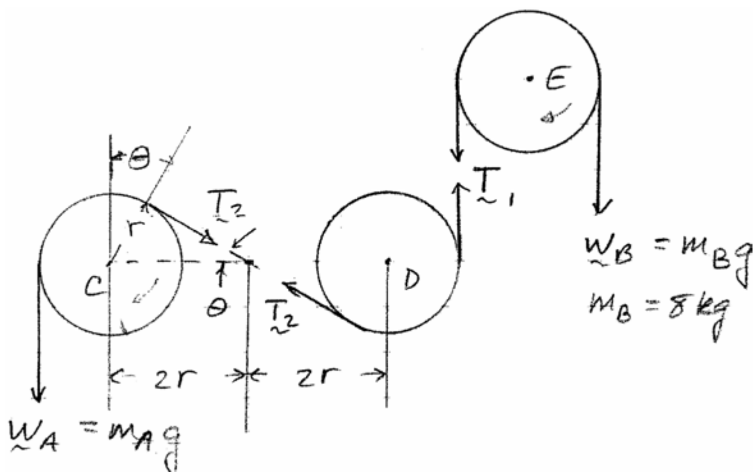
Equilibrium for $2.81\text{ kg} \le m_A \le 22.8\text{ kg}$ ◀



PROBLEM 8.119

A cable passes around three 30-mm-radius pulleys and supports two blocks as shown. Two of the pulleys are locked to prevent rotation, while the third pulley is rotated slowly at a constant speed. Knowing that the coefficients of friction between the cable and the pulleys are $\mu_s = 0.20$ and $\mu_k = 0.15$, determine the largest mass m_A which can be raised (*a*) if pulley *C* is rotated, (*b*) if pulley *E* is rotated.

SOLUTION



Note: $$\theta = \sin^{-1}\frac{r}{2r} = 30° = \frac{\pi}{6}\text{ rad}$$

$$\beta_C = \beta_D = \frac{2\pi}{3} \qquad \text{and} \qquad \beta_E = \pi$$

Mass *A* moves up

(*a*) *C* rotates ↻, for maximum W_A have no belt slipping on *C*, so

$$W_A = T_2 e^{\mu_s \beta_C}$$

D and *E* are fixed, so $$T_1 = T_2 e^{\mu_k \beta_D}$$

and $$W_B = T_1 e^{\mu_k \beta_E} = T_2 e^{\mu_k(\beta_D + \beta_E)} \Rightarrow T_2 = W_B e^{-\mu_k(\beta_D + \beta_E)}$$

Thus $$m_A g = m_B g e^{\mu_s \beta_C - \mu_k(\beta_D + \beta_E)} \qquad \text{or} \qquad m_A = (8\text{ kg})e^{\left(\frac{0.4\pi}{3} - 0.1\pi - 0.15\pi\right)}$$

$$m_A = 5.55\text{ kg} ◀$$

PROBLEM 8.119 CONTINUED

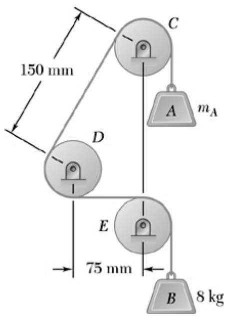
(*b*) E rotates ↻, no belt slip on E, so $$T_1 = W_B e^{\mu_s \beta_E}$$

C and D fixed, so $$T_1 = T_2 e^{\mu_k \beta_D} = W_A e^{\mu_k (\beta_C + \beta_D)}$$

or $$m_A g = T_1 e^{-\mu_k (\beta_C + \beta_D)} = m_B g e^{\mu_s \beta_E - \mu_k (\beta_C + \beta_D)}$$

Then $$m_A = (8 \text{ kg}) e^{(0.2\pi - 0.1\pi - 0.1\pi)} = 8.00 \text{ kg}$$

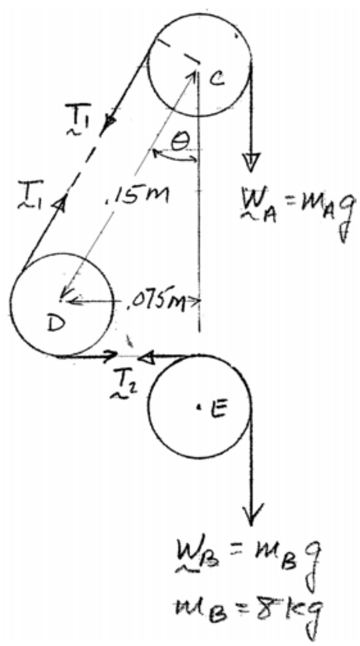
$$m_A = 8.00 \text{ kg} \blacktriangleleft$$



PROBLEM 8.120

A cable passes around three 30-mm-radius pulleys and supports two blocks as shown. Pulleys C and E are locked to prevent rotation, and the coefficients of friction between the cable and the pulleys are $\mu_s = 0.20$ and $\mu_k = 0.15$. Determine the range of values of the mass of block A for which equilibrium is maintained (*a*) if pulley D is locked, (*b*) if pulley D is free to rotate.

SOLUTION



Note: $$\theta = \sin^{-1}\frac{0.075\text{ m}}{0.15\text{ m}} = 30° = \frac{\pi}{6}\text{ rad}$$

So $$\beta_C = \frac{5}{6}\pi,\ \beta_D = \frac{2}{3}\pi,\ \beta_E = \frac{1}{2}\pi$$

(*a*) All pulleys locked, slipping at all surfaces.

For m_A impending $\uparrow$, $$T_1 = W_A e^{\mu_s \beta_C},$$

$$T_2 = T_1 e^{\mu_s \beta_D}, \qquad \text{and} \qquad W_B = T_2 e^{\mu_k \beta_E},$$

So $$m_B g = m_A g e^{\mu_s(\beta_C + \beta_D + \beta_E)}$$

$$8\text{ kg} = m_A e^{0.2\left(\frac{5}{6}+\frac{2}{3}+\frac{1}{2}\right)\pi} \qquad \text{or} \qquad m_A = 2.28\text{ kg}$$

For m_A impending down, all tension ratios are inverted, so

$$m_A = (8\text{ kg})e^{0.2\left(\frac{5}{6}+\frac{2}{3}+\frac{1}{2}\right)\pi} = 28.1\text{ kg}$$

Equilibrium for $2.28\text{ kg} \le m_A \le 28.1\text{ kg}$ ◀

(*b*) Pulleys C and E locked, D free $\Rightarrow T_1 = T_2$, other ratios as in (*a*)

m_A impending $\uparrow$, $$T_1 = W_A e^{\mu_s \beta_C} = T_2$$

and $$W_B = T_2 e^{\mu_s \beta_E} = W_A e^{\mu_s(\beta_C + \beta_E)}$$

So $$m_B g = m_A g\, e^{\mu(\beta_C + \beta_E)} \qquad \text{or} \qquad 8\text{ kg} = m_A e^{0.2\left(\frac{5}{6}+\frac{1}{2}\right)\pi}$$

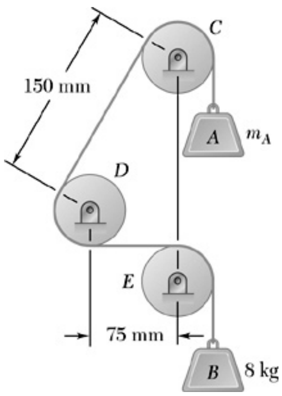
$$m_A = 3.46\text{ kg}$$

m_A impending $\downarrow$, all tension ratios are inverted, so

$$m_A = 8\text{ kg}\, e^{0.2\left(\frac{5}{6}+\frac{1}{2}\right)\pi}$$
$$= 18.49\text{ kg}$$

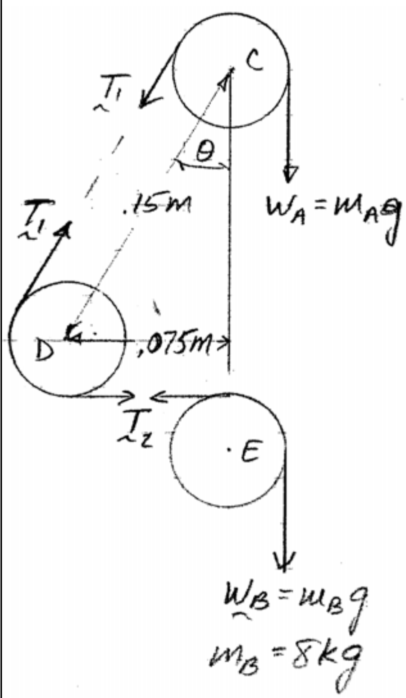
Equilibrium for $3.46\text{ kg} \le m_A \le 18.49\text{ kg}$ ◀

PROBLEM 8.121



A cable passes around three 30-mm-radius pulleys and supports two blocks as shown. Two of the pulleys are locked to prevent rotation, while the third pulley is rotated slowly at a constant speed. Knowing that the coefficients of friction between the cable and the pulleys are $\mu_s = 0.20$ and $\mu_k = 0.15$, determine the largest mass m_A which can be raised (*a*) if pulley *C* is rotated, (*b*) if pulley *E* is rotated.

SOLUTION



Note: $$\theta = \sin^{-1}\frac{0.075 \text{ m}}{0.15 \text{ m}} = 30° = \frac{\pi}{6} \text{ rad}$$

So $$\beta_C = \frac{5}{6}\pi, \;\; \beta_D = \frac{2}{3}\pi, \;\; \beta_E = \frac{1}{2}\pi$$

(*a*) To raise maximum m_A, with C rotating $W_A = T_1 e^{\mu_s \beta_C}$. If D and E are fixed, cable must slip there, so $T_2 = T_1 e^{\mu_k \beta_D}$

and $$W_B = T_2 e^{\mu_k \beta_E} = T_1 e^{\mu_k(\beta_D + \beta_E)}$$

$$= W_A e^{-\mu_s \beta_C} e^{\mu_k(\beta_D + \beta_E)}$$

$$(8 \text{ kg})g = m_A g e^{-0.2\left(\frac{5}{6}\pi\right)} e^{0.15\left(\frac{2}{3}+\frac{1}{2}\right)\pi}$$

$$m_A = 7.79 \text{ kg} \blacktriangleleft$$

(*b*) With E rotating, $T_2 = W_B e^{\mu_s \beta_E}$. With C and D fixed.

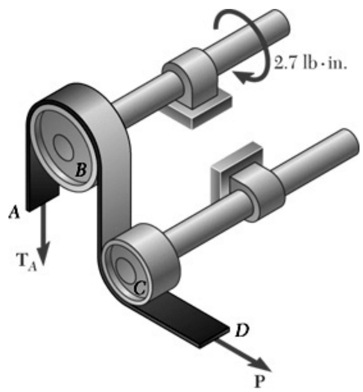
$$T_1 = W_A e^{\mu_k \beta_C} \quad \text{and} \quad T_2 = T_1 e^{\mu_k \beta_D} = W_A e^{\mu_k(\beta_C + \beta_D)}$$

so $$W_B = W_A e^{\mu_k(\beta_C + \beta_D)} e^{-\mu_s \beta_E}$$

$$(8 \text{ kg})g = m_A g e^{0.15\left(\frac{5}{6}+\frac{2}{3}\right)\pi} e^{-0.2\left(\frac{1}{2}\pi\right)}$$

$$m_A = 5.40 \text{ kg} \blacktriangleleft$$

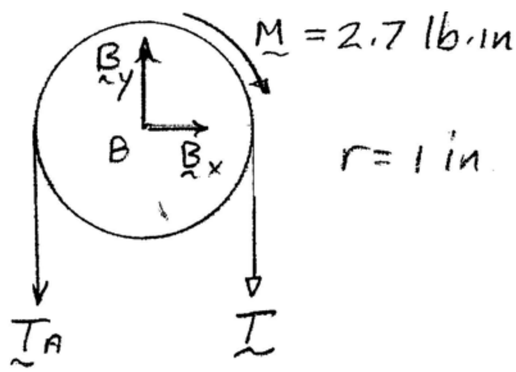
PROBLEM 8.122



A recording tape passes over the 1-in.-radius drive drum B and under the idler drum C. Knowing that the coefficients of friction between the tape and the drums are $\mu_s = 0.40$ and $\mu_k = 0.30$ and that drum C is free to rotate, determine the smallest allowable value of P if slipping of the tape on drum B is not to occur.

SOLUTION

FBD drive drum:



$$\circlearrowleft \Sigma M_B = 0: \quad r(T_A - T) - M = 0$$

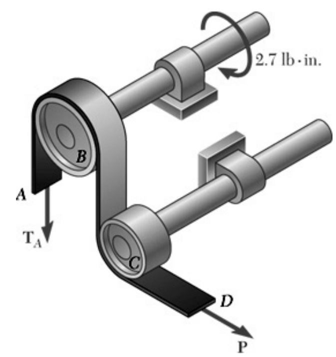
$$T_A - T = \frac{M}{r} = \frac{2.7 \text{ lb} \cdot \text{in.}}{1 \text{ in.}} = 2.7 \text{ lb}$$

Impending slipping: $$T_A = Te^{\mu_S \beta} = Te^{0.4\pi}$$

So $$T\left(e^{0.4\pi} - 1\right) = 2.7 \text{ lb}$$

or $$T = 1.0742 \text{ lb}$$

If C is free to rotate, $P = T$ $$P = 1.074 \text{ lb} \blacktriangleleft$$

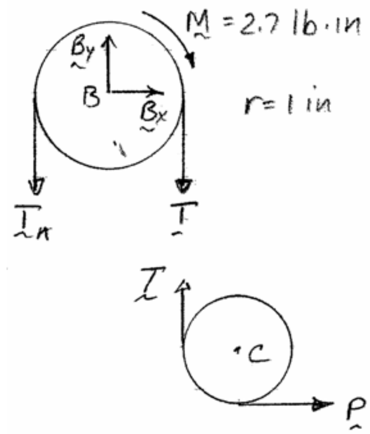


PROBLEM 8.123

Solve Problem 8.122 assuming that the idler drum C is frozen and cannot rotate.

SOLUTION

FBD drive drum:



$$\circlearrowleft \Sigma M_B = 0: \quad r(T_A - T) - M = 0$$

$$T_A - T = \frac{M}{r} = \frac{2.7 \text{ lb} \cdot \text{in.}}{1 \text{ in.}} = 2.7 \text{ lb}$$

Impending slipping: $$T_A = Te^{\mu_s \beta} = Te^{0.4\pi}$$

So $$\left(e^{0.4\pi} - 1\right)T = 2.7 \text{ lb}$$

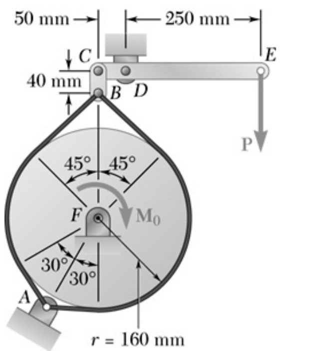
or $$T = 1.07416 \text{ lb}$$

If C is fixed, the tape must slip $\circlearrowleft$

So $$P = Te^{\mu_k \beta_C} = 1.07416 \text{ lb } e^{0.3\frac{\pi}{2}} = 1.7208 \text{ lb}$$

$$P = 1.721 \text{ lb} \blacktriangleleft$$

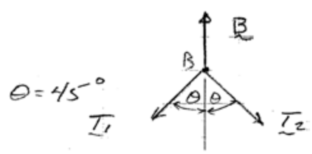
PROBLEM 8.124

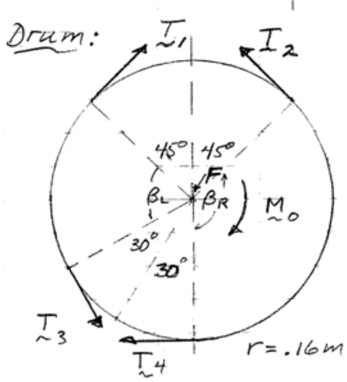


For the band brake shown, the maximum allowed tension in either belt is 5.6 kN. Knowing that the coefficient of static friction between the belt and the 160-mm-radius drum is 0.25, determine (*a*) the largest clockwise moment $\mathbf{M}_0$ that can be applied to the drum if slipping is not to occur, (*b*) the corresponding force **P** exerted on end *E* of the lever.

SOLUTION

FBD pin B:





(*a*) By symmetry: $T_1 = T_2$

$$\uparrow \Sigma F_y = 0: \quad B - 2\left(\frac{\sqrt{2}}{2}T_1\right) = 0 \qquad \text{or} \qquad B = \sqrt{2}T_1 = \sqrt{2}T_2 \qquad (1)$$

For impending rotation ↻:

$$T_3 > T_1 = T_2 > T_4, \text{ so } T_3 = T_{\text{max}} = 5.6 \text{ kN}$$

Then $$T_1 = T_3 e^{-\mu_s \beta_L} = (5.6 \text{ kN})e^{-0.25\left(\frac{\pi}{4}+\frac{\pi}{6}\right)}$$

or $$T_1 = 4.03706 \text{ kN} = T_2$$

and $$T_4 = T_2 e^{-\mu_s \beta_R} = (4.03706 \text{ kN})e^{-0.25\left(\frac{3\pi}{4}\right)}$$

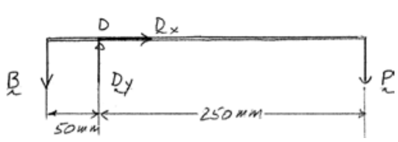
or $$T_4 = 2.23998 \text{ kN}$$

$$\circlearrowleft \Sigma M_F = 0: \quad M_0 + r(T_4 - T_3 + T_2 - T_1) = 0$$

or $$M_0 = (0.16 \text{ m})(5.6 \text{ kN} - 2.23998 \text{ kN}) = 0.5376 \text{ kN}\cdot\text{m}$$

$$\mathbf{M}_0 = 538 \text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

Lever:



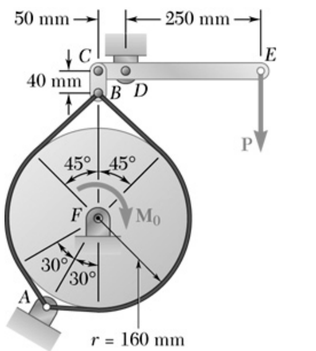
(*b*) Using Equation (1)

$$B = \sqrt{2}T_1 = \sqrt{2}(4.03706 \text{ kN})$$
$$= 5.70927 \text{ kN}$$

$$\circlearrowleft \Sigma M_D = 0: \quad (0.05 \text{ m})(5.70927 \text{ kN}) - (0.25 \text{ m})P = 0$$

$$\mathbf{P} = 1.142 \text{ kN} \downarrow \blacktriangleleft$$

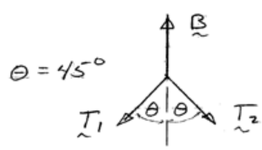
PROBLEM 8.125

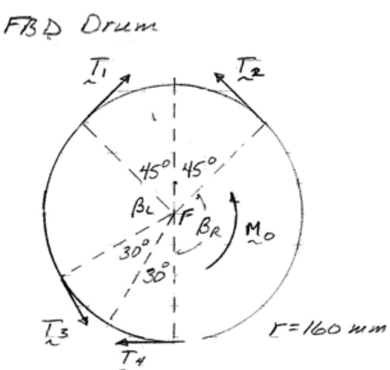


Solve Problem 8.124 assuming that a counterclockwise moment is applied to the drum.

SOLUTION

FBD pin B:





(*a*) By symmetry: $T_1 = T_2$

$$\uparrow \Sigma F_y = 0: \quad B - 2\left(\frac{\sqrt{2}}{2}T_1\right) = 0 \qquad \text{or} \qquad B = \sqrt{2}T_1 \qquad (1)$$

For impending rotation ⟲:

$$T_4 > T_2 = T_1 > T_3, \text{ so } T_4 = T_{\max} = 5.6 \text{ kN}$$

Then $\quad T_2 = T_4 e^{-\mu_s \beta_R} = (5.6 \text{ kN})e^{-0.25\left(\frac{3\pi}{4}\right)}$

or $\quad T_2 = 3.10719 \text{ kN} = T_1$

and $\quad T_3 = T_1 e^{-\mu_s \beta_L} = (3.10719 \text{ kN})e^{-0.25\left(\frac{\pi}{4}+\frac{\pi}{6}\right)}$

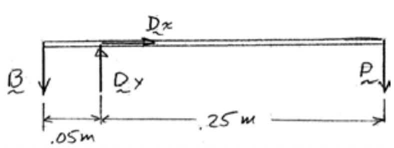
or $\quad T_3 = 2.23999 \text{ kN}$

$$\circlearrowleft \Sigma M_F = 0: \quad M_0 + r(T_2 - T_1 + T_3 - T_4) = 0$$

$$M_0 = (160 \text{ mm})(5.6 \text{ kN} - 2.23999 \text{ kN}) = 537.6 \text{ N}\cdot\text{m}$$

$$\mathbf{M}_0 = 538 \text{ N}\cdot\text{m} \circlearrowleft \blacktriangleleft$$

FBD Lever:



(*b*) Using Equation (1)

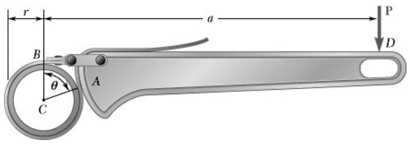
$$B = \sqrt{2}T_1 = \sqrt{2}(3.10719 \text{ kN})$$

$$B = 4.3942 \text{ kN}$$

$$\circlearrowleft \Sigma M_D = 0: \quad (0.05 \text{ m})(4.3942 \text{ kN}) - (0.25 \text{ m})P = 0$$

$$\mathbf{P} = 879 \text{ N} \downarrow \blacktriangleleft$$

PROBLEM 8.126

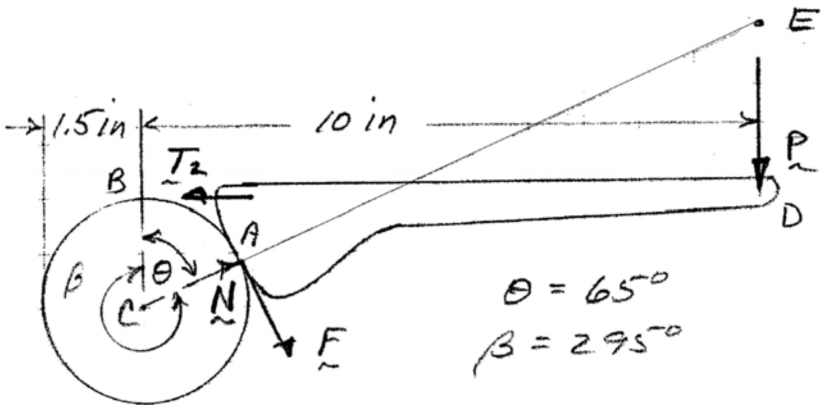


The strap wrench shown is used to grip the pipe firmly without marring the surface of the pipe. Knowing that the coefficient of static friction is the same for all surfaces of contact, determine the smallest value of μ_s for which the wrench will be self-locking when $a = 10$ in., $r = 1.5$ in., and $\theta = 65^{\circ}$.

SOLUTION

For the wrench to be self-locking, friction must be sufficient to maintain equilibrium as P is increased from zero to $P_{\max}$, as well as to prevent slipping of the belt on the pipe.

FBD wrench:



$$\circlearrowleft \Sigma M_E = 0: \quad \left(\frac{10 \text{ in.}}{\sin 65^{\circ}} - 1.5 \text{ in.}\right)F - \left(\frac{10 \text{ in.}}{\tan 65^{\circ}} - 1.5 \text{ in.}\right)T_2 = 0$$

$$9.5338F = 3.1631 T_2 \qquad \text{or} \qquad 3.01406 = \frac{T_2}{F} \tag{1}$$

$$\longrightarrow \Sigma F_x = 0: \quad -T_2 + N \sin 65^{\circ} + F \cos 65^{\circ} = 0$$

Impending slipping: $\quad N = F/\mu_s$

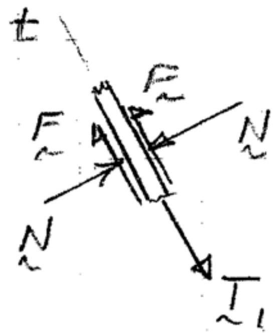
So $$F\left(\frac{\sin 65^{\circ}}{\mu_s} + \cos 65^{\circ}\right) = T_2$$

or $$\frac{0.90631}{\mu_s} + 0.42262 = \frac{T_2}{F} \tag{2}$$

Solving Equations (1) and (2) yields $\mu_s = 0.3497$; must still check belt on pipe.

PROBLEM 8.126 CONTINUED

Small portion of belt at A:



$$\nwarrow \Sigma F_t = 0: \quad 2F - T_1 = 0$$

or $$T_1 = 2F$$

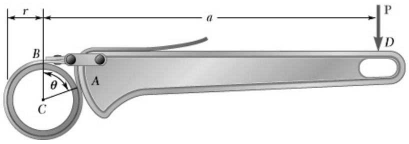
Belt impending slipping: $$\ln\frac{T_2}{T_1} = \mu_s \beta$$

So $$\mu_s = \frac{1}{\beta}\ln\frac{T_2}{T_1} = \frac{1}{\beta}\ln\frac{T_2}{2F}$$

Using Equation (1) $$\mu_s = \frac{180}{295\pi}\ln 1.50703$$

$$= 0.0797$$

$\therefore$ for self-locking, need $\mu_s = 0.350$ ◀



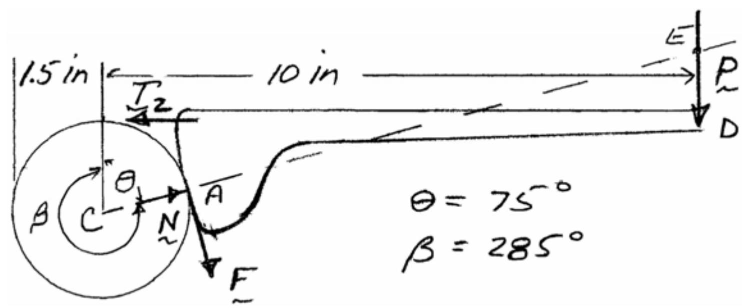
PROBLEM 8.127

Solve Problem 8.126 assuming that $\theta = 75^{\circ}$.

SOLUTION

For the wrench to be self-locking, friction must be sufficient to maintain equilibrium as P is increased from zero to $P_{\max}$, as well as to prevent slipping of the belt on the pipe.

FBD wrench:



$$\circlearrowleft \Sigma M_E = 0: \quad \left(\frac{10\text{ in.}}{\sin 75^{\circ}} - 1.5\text{ in.}\right)F - \left(\frac{10\text{ in.}}{\tan 75^{\circ}} - 1.5\text{ in.}\right)T_2 = 0$$

or $$\frac{T_2}{F} = 7.5056 \qquad (1)$$

$$\xrightarrow{} \Sigma F_x = 0: \quad -T_2 + N\sin 75^{\circ} + F\cos 75^{\circ} = 0$$

Impending slipping: $$N = F/\mu_s$$

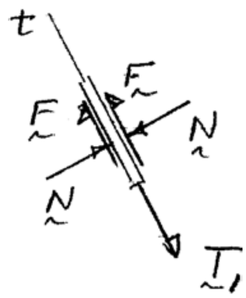
So $$F\left(\frac{\sin 75^{\circ}}{\mu_s} + \cos 75^{\circ}\right) = T_2$$

or $$\frac{T_2}{F} = \frac{0.96593}{\mu_s} + 0.25882 \qquad (2)$$

Solving Equations (1) and (2): $\mu_s = 0.13329$; must still check belt on pipe.

PROBLEM 8.127 CONTINUED

Small portion of belt at A:



$$\nwarrow \Sigma F_t = 0: \quad 2F - T_1 = 0$$

or $$T_1 = 2F$$

Impending belt slipping: $$\ln \frac{T_2}{T_1} = \mu_s \beta$$

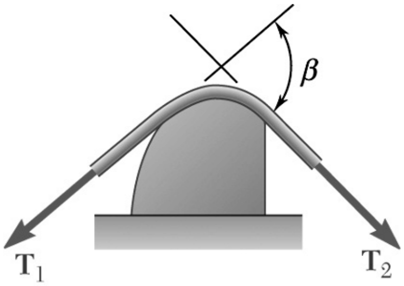
So $$\mu_s = \frac{1}{\beta} \ln \frac{T_2}{T_1} = \frac{1}{\beta} \ln \frac{T_2}{2F}$$

Using Equation (1): $$\mu_s = \frac{180}{285\pi} \ln \frac{7.5056}{2}$$

$$= 0.2659$$

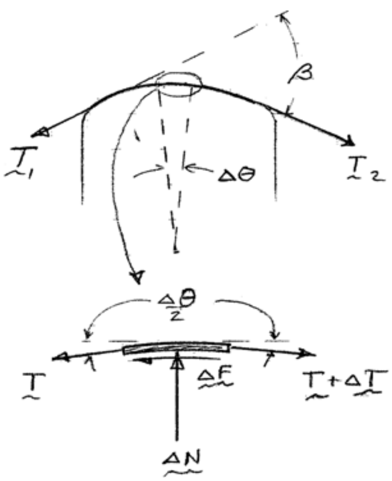
$\therefore$ for self-locking, $\mu_s = 0.266$ ◀

PROBLEM 8.128



Prove that Equatins (8.13) and (8.14) are valid for any shape of surface provided that the coefficient of friction is the same at all points of contact.

SOLUTION



$$\uparrow \Sigma F_n = 0: \quad \Delta N - \left[T + \left(T + \Delta T\right)\right]\sin\frac{\Delta\theta}{2} = 0$$

or $$\Delta N = \left(2T + \Delta T\right)\sin\frac{\Delta\theta}{2}$$

$$\longrightarrow \Sigma F_t = 0: \quad \left[\left(T + \Delta T\right) - T\right]\cos\frac{\Delta\theta}{2} - \Delta F = 0$$

or $$\Delta F = \Delta T\cos\frac{\Delta\theta}{2}$$

Impending slipping: $$\Delta F = \mu_s \Delta N$$

So $$\Delta T\cos\frac{\Delta\theta}{2} = \mu_s 2T\sin\frac{\Delta\theta}{2} + \mu_s \Delta T\frac{\sin\Delta\theta}{2}$$

In limit as $\Delta\theta \to 0$: $dT = \mu_s T d\theta$, or $\dfrac{dT}{T} = \mu_s d\theta$

So $$\int_{T_1}^{T_2}\frac{dT}{T} = \int_0^{\beta}\mu_s d\theta;$$

and $$\ln\frac{T_2}{T_1} = \mu_s\beta$$

or $T_2 = T_1 e^{\mu_s\beta}$ ◀

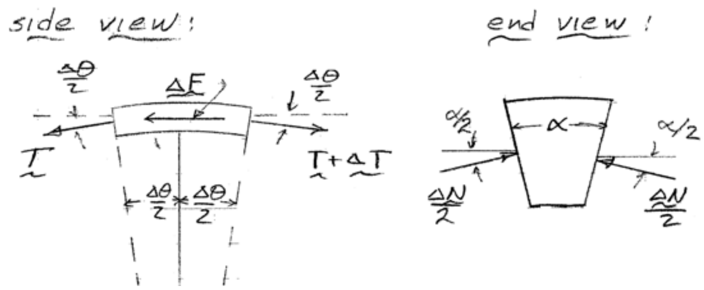
Note: Nothing above depends on the shape of the surface, except it is assumed smooth.

PROBLEM 8.129

Complete the derivation of Equation (8.15), which relates the tension in both parts of a V belt.

SOLUTION

Small belt section:



$$\uparrow \Sigma F_y = 0: \quad 2\frac{\Delta N}{2}\sin\frac{\alpha}{2} - \left[T + \left(T + \Delta T\right)\right]\sin\frac{\Delta\theta}{2} = 0$$

$$\longrightarrow \Sigma F_x = 0: \quad \left[\left(T + \Delta T\right) - T\right]\cos\frac{\Delta\theta}{2} - \Delta F = 0$$

Impending slipping: $$\Delta F = \mu_s \Delta N \Rightarrow \Delta T\cos\frac{\Delta\theta}{2} = \mu_s \frac{2T + \Delta T}{\sin\frac{\alpha}{2}}\sin\frac{\Delta\theta}{2}$$

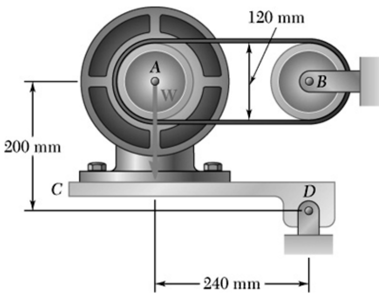
In limit as $\Delta\theta \to 0$: $$dT = \frac{\mu_s T d\theta}{\sin\frac{\alpha}{2}} \qquad \text{or} \qquad \frac{dT}{T} = \frac{\mu_s}{\sin\frac{\alpha}{2}}d\theta$$

So $$\int_{T_1}^{T_2}\frac{dT}{T} = \frac{\mu_s}{\sin\frac{\alpha}{2}}\int_0^{\beta} d\theta$$

or $$\ln\frac{T_2}{T_1} = \frac{\mu_s \beta}{\sin\frac{\alpha}{2}}$$

or $$T_2 = T_1 e^{\mu_s\beta/\sin\frac{\alpha}{2}} \blacktriangleleft$$

PROBLEM 8.130

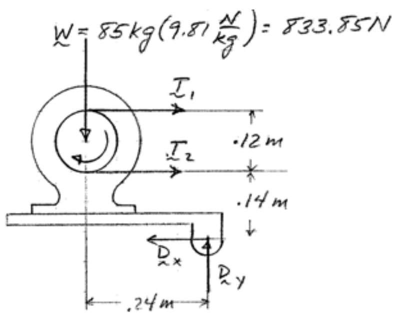


Solve Problem 8.107 assuming that the flat belt and drums are replaced by a V belt and V pulleys with $\alpha = 36^\circ$. (The angle α is as shown in Figure 8.15*a*.)

SOLUTION

FBD motor + mount:

$$\circlearrowleft \Sigma M_D = 0: \quad (0.24\text{ m})W - (0.26\text{ m})T_1 - (0.14\text{ m})T_2 = 0$$



Impending slipping: $$T_2 = T_1 e^{\mu_S \beta/\sin\frac{\alpha}{2}}$$

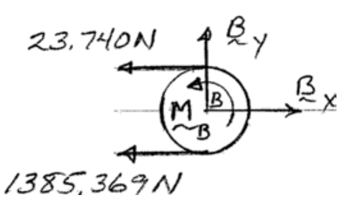
$$T_2 = T_1 e^{\frac{0.4\pi}{\sin 18^\circ}} = 58.356 T_1$$

Thus $$(0.24\text{ m})(833.85\text{ N}) - \left[0.26\text{ m} + (0.14\text{ m})(58.356)\right]T_1 = 0$$

$$T_1 = 23.740\text{ N}$$

$$T_2 = 1385.369\text{ N}$$

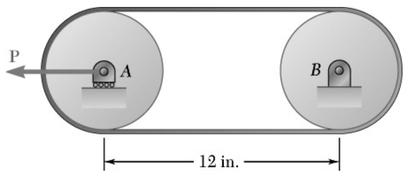
FBD Drum:



$$\circlearrowleft \Sigma M_B = 0: \quad M_B + (0.06\text{ m})(23.740\text{ N} - 1385.369\text{ N}) = 0$$

$$M_B = 81.7\text{ N}\cdot\text{m} \blacktriangleleft$$

(Compare to $M_B = 40.1\text{ N}\cdot\text{m}$ using flat belt, Problem 8.107.)

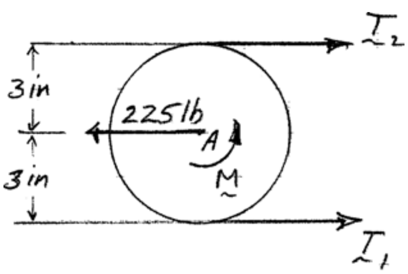


PROBLEM 8.131

Solve Problem 8.109 assuming that the flat belt and drums are replaced by a V belt and V pulleys with $\alpha = 36^{\circ}$. (The angle α is as shown in Figure 8.15*a*.)

SOLUTION

FBD pulley A:



Impending slipping: $$T_2 = T_1 e^{\mu_s \beta / \sin\frac{\alpha}{2}}$$

$$T_2 = T_1 e^{0.35\pi/\sin 18^{\circ}} = 35.1015 T_1$$

$$\xrightarrow{\quad} \Sigma F_x = 0: \quad T_1 + T_2 - 225 \text{ lb} = 0$$

$$T_1(1 + 35.1015) = 225 \text{ lb}$$

So $$T_1 = 6.2324 \text{ lb}$$

$$T_2 = 218.768 \text{ lb} = T_{max}$$

$$\Sigma M_A = 0: \quad M + (3 \text{ in.})(T_1 - T_2) = 0$$

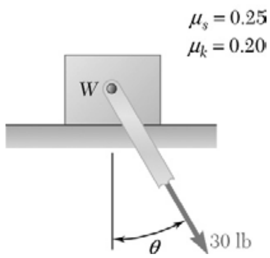
$$M = (3 \text{ in.})(218.768 \text{ lb} - 6.232 \text{ lb})$$

(*a*) $$M = 638 \text{ lb}\cdot\text{in.} ◀$$

(Compare to 338 lb·in. with flat belt, Problem 8.109.)

(*b*) $$T_{max} = 219 \text{ lb} ◀$$

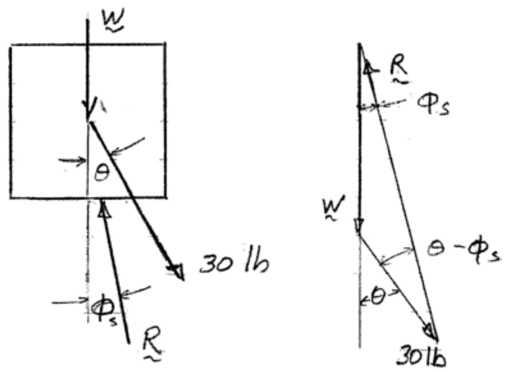
PROBLEM 8.132



Considering only values of θ less than $90°$, determine the smallest value of θ required to start the block moving to the right when (*a*) $W = 75$ lb, (*b*) $W = 100$ lb.

SOLUTION

FBD block: (motion impending)



$$\phi_s = \tan^{-1}\mu_s = 14.036°$$

$$\frac{30 \text{ lb}}{\sin\phi_s} = \frac{W}{\sin(\theta - \phi_s)}$$

$$\sin(\theta - \phi_s) = \frac{W \sin 14.036°}{30 \text{ lb}}$$

or
$$\sin(\theta - 14.036°) = \frac{W}{123.695 \text{ lb}}$$

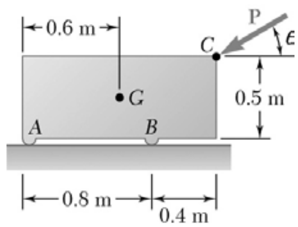
(*a*)
$$W = 75 \text{ lb:} \quad \theta = 14.036° + \sin^{-1}\frac{75 \text{ lb}}{123.695 \text{ lb}}$$

$\theta = 51.4°$ ◀

(*b*)
$$W = 100 \text{ lb:} \quad \theta = 14.036° + \sin^{-1}\frac{100 \text{ lb}}{123.695 \text{ lb}}$$

$\theta = 68.0°$ ◀

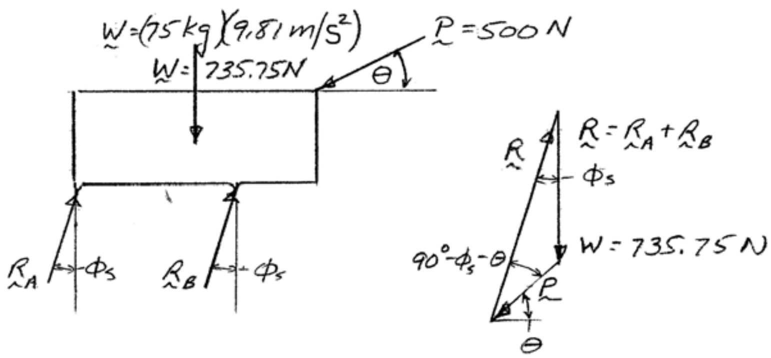
PROBLEM 8.133



The machine base shown has a mass of 75 kg and is fitted with skids at *A* and *B*. The coefficient of static friction between the skids and the floor is 0.30. If a force **P** of magnitude 500 N is applied at corner *C*, determine the range of values of θ for which the base will not move.

SOLUTION

FBD machine base (slip impending):



$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.3 = 16.699°$$

$$\frac{W}{\sin(90° - \phi_s - \theta)} = \frac{P}{\sin\phi_s}$$

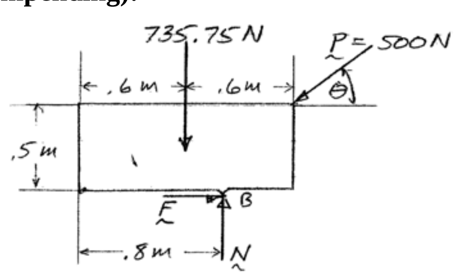
$$\sin(90° - \phi_s - \theta) = \frac{W\sin 16.699°}{P}$$

$$90° - 16.699° - \theta = \sin^{-1}\left[\frac{735.75\text{ lb}}{500\text{ lb}}(0.28734)\right]$$

$$\theta = 73.301° - 25.013°$$

$$\theta = 48.3°$$

FBD machine base (tip about B impending):



PROBLEM 8.133 CONTINUED

$$\Sigma M_B = 0: \quad (0.2\text{ m})(735.75\text{ N}) + (0.5\text{ m})\left[(500\text{ N})\cos\theta\right]$$

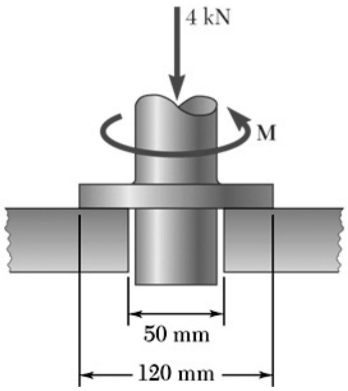
$$-(0.4\text{ m})\left[(500\text{ N})\sin\theta\right] = 0$$

$$0.8\sin\theta - \cos\theta = 0.5886$$

Solving numerically $\qquad \theta = 78.7°$

So, for equilibrium $\qquad 48.3° \le \theta \le 78.7°$ ◀

PROBLEM 8.134



Knowing that a couple of magnitude $30\ \mathrm{N\cdot m}$ is required to start the vertical shaft rotating, determine the coefficient of static friction between the annular surfaces of contact.

SOLUTION

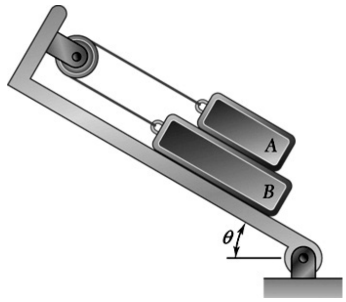
For annular contact regions, use Equation 8.8 with impending slipping:

$$M = \frac{2}{3}\mu_s N \frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}$$

So,

$$30\ \mathrm{N\cdot m} = \frac{2}{3}\mu_s (4000\ \mathrm{N}) \frac{(0.06\ \mathrm{m})^3 - (0.025\ \mathrm{m})^3}{(0.06\ \mathrm{m})^2 - (0.025\ \mathrm{m})^2}$$

$$\mu_s = 0.1670 \blacktriangleleft$$



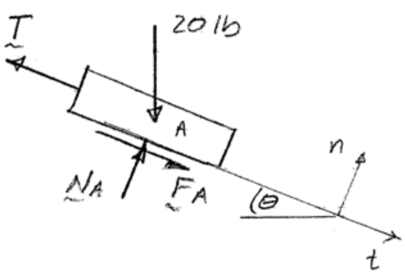
PROBLEM 8.135

The 20-lb block A and the 30-lb block B are supported by an incline which is held in the position shown. Knowing that the coefficient of static friction is 0.15 between the two blocks and zero between block B and the incline, determine the value of θ for which motion is impending.

SOLUTION

FBD's

Block A:



A: $\nearrow \Sigma F_n = 0\text{:} \quad N_A - (20 \text{ lb})\cos\theta = 0 \qquad \text{or} \qquad N_A = (20 \text{ lb})\cos\theta$

B: $\nearrow \Sigma F_n = 0\text{:} \quad N_B - N_A - (30 \text{ lb})\cos\theta = 0$

or $\quad N_B = N_A + (30 \text{ lb})\cos\theta = (50 \text{ lb})\cos\theta$

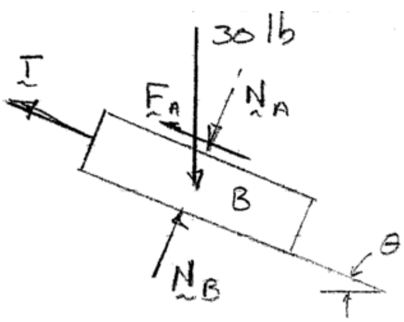
Impending motion at all surfaces:

$$F_A = \mu_s N_A$$

$$= 0.15(20 \text{ lb})\cos\theta$$

$$= (3 \text{ lb})\cos\theta$$

Block B:



A: $\searrow \Sigma F_t = 0\text{:} \quad F_A + (20 \text{ lb})\sin\theta - T = 0$

B: $\searrow \Sigma F_t = 0\text{:} \quad -F_A + (30 \text{ lb})\sin\theta - T = 0$

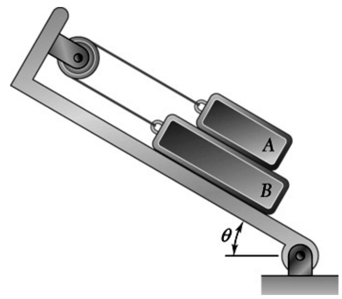
So $\quad (10 \text{ lb})\sin\theta - 2F_A = 0$

$$(10 \text{ lb})\sin\theta = 2(3 \text{ lb})\cos\theta$$

$$\theta = \tan^{-1}\frac{6 \text{ lb}}{10 \text{ lb}} = 30.96°$$

$\theta = 31.0°$ ◀

PROBLEM 8.136

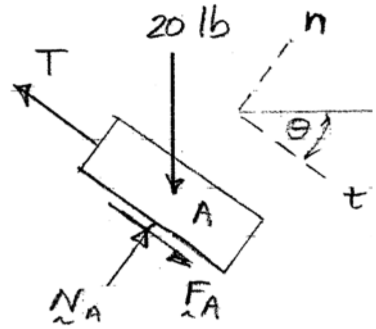


The 20-lb block A and the 30-lb block B are supported by an incline which is held in the position shown. Knowing that the coefficient of static friction is 0.15 between all surfaces of contact, determine the value of θ for which motion is impending.

SOLUTION

FBD's

Block A:



A: $\nearrow \Sigma F_n = 0$: $N_A - (20\text{ lb})\cos\theta = 0$ or $N_A = (20\text{ lb})\cos\theta$

B: $\nearrow \Sigma F_n = 0$: $N_B - N_A - (30\text{ lb})\cos\theta = 0$

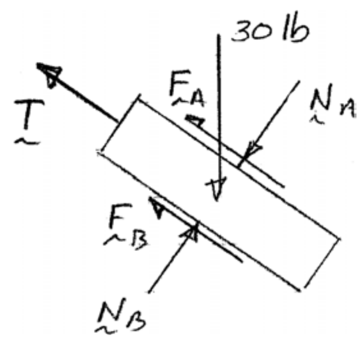
or $$N_B = N_A + (30\text{ lb})\cos\theta = (50\text{ lb})\cos\theta$$

Impending motion at all surfaces; B impends $\searrow$:

$$F_A = \mu_s N_A = (0.15)(20\text{ lb})\cos\theta = (3\text{ lb})\cos\theta$$

$$F_B = \mu_s N_B = (0.15)(50\text{ lb})\cos\theta = (7.5\text{ lb})\cos\theta$$

Block B:



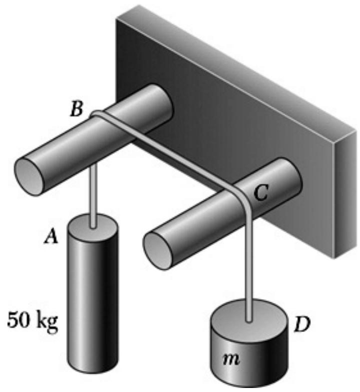
A: $\searrow \Sigma F_t = 0$: $(20\text{ lb})\sin\theta + F_A - T = 0$

B: $\searrow \Sigma F_t = 0$: $(30\text{ lb})\sin\theta - F_A - F_B - T = 0$

So $$(10\text{ lb})\sin\theta - 2F_A - F_B = 0$$

$$(10\text{ lb})\sin\theta = 2(3\text{ lb})\cos\theta + (7.5\text{ lb})\cos\theta$$

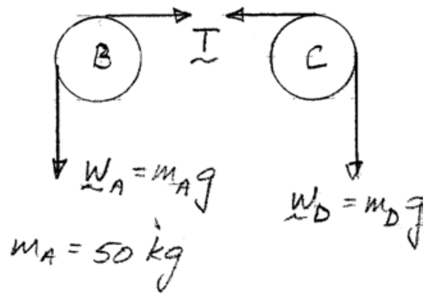
$$\tan\theta = \frac{13.5\text{ lb}}{10\text{ lb}} = 1.35; \qquad \theta = 53.5° \blacktriangleleft$$



PROBLEM 8.137

Two cylinders are connected by a rope that passes over two fixed rods as shown. Knowing that the coefficient of static friction between the rope and the rods is 0.40, determine the range of values of the mass m of cylinder D for which equilibrium is maintained.

SOLUTION



For impending motion of A up: $$T = W_A e^{\mu_s \beta_B}$$

and $$W_D = T e^{\mu_s \beta_C} = W_A e^{\mu_s(\beta_B + \beta_C)}$$

or $$m_D g = (50 \text{ kg}) g e^{0.4\left(\frac{\pi}{2}+\frac{\pi}{2}\right)}$$

$$m_D = 175.7 \text{ kg}$$

For impending motion of A down, the tension ratios are inverted, so

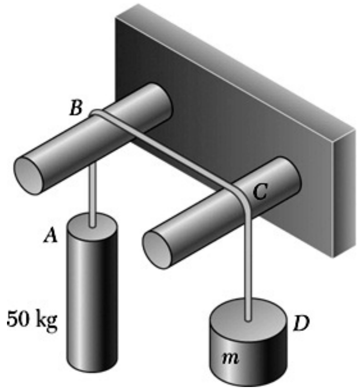
$$W_A = W_D e^{\mu_s(\beta_C + \beta_B)}$$

$$(50 \text{ kg}) g = m_D g e^{0.4\left(\frac{\pi}{2}+\frac{\pi}{2}\right)}$$

$$m_D = 14.23 \text{ kg}$$

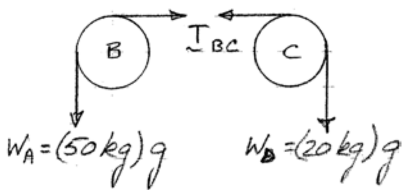
For equilibrium: $$14.23 \text{ kg} \le m_D \le 175.7 \text{ kg} \blacktriangleleft$$

PROBLEM 8.138



Two cylinders are connected by a rope that passes over two fixed rods as shown. Knowing that for cylinder D upward motion impends when $m = 20$ kg, determine (a) the coefficient of static friction between the rope and the rods, (b) the corresponding tension in portion BC of the rope.

SOLUTION



(a) Motion of D impends upward, so

$$T_{BC} = W_D e^{\mu_s \beta_C} \tag{1}$$

$$W_A = T_{BC} e^{\mu_s \beta_B} = W_D e^{\mu_s(\beta_C + \beta_B)}$$

So $$\mu_s\left(\frac{\pi}{2} + \frac{\pi}{2}\right) = \ln\frac{W_A}{W_D} = \ln\left(\frac{50\text{ kg}}{20\text{ kg}}\right)$$

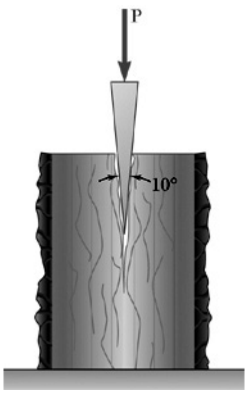
$$\mu_s = 0.29166$$

$$\mu_s = 0.292 \blacktriangleleft$$

(b) From Equation (1): $$T_{BC} = (20\text{ kg})(9.81\text{ m/s}^2)e^{0.29166\pi/2}$$

$$T_{BC} = 310\text{ N} \blacktriangleleft$$

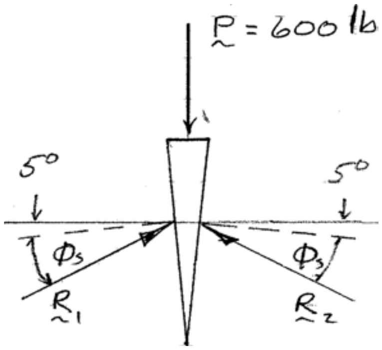
PROBLEM 8.139



A $10°$ wedge is used to split a section of a log. The coefficient of static friction between the wedge and the log is 0.35. Knowing that a force **P** of magnitude 600 lb was required to insert the wedge, determine the magnitude of the forces exerted on the wood by the wedge after insertion.

SOLUTION

FBD wedge (impending motion $\downarrow$):



$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.35 = 19.29°$$

By symmetry: $$R_1 = R_2$$

$$\uparrow \Sigma F_y = 0: \quad 2R_1 \sin(5° + \phi_s) - 600 \text{ lb} = 0$$

or $$R_1 = R_2 = \frac{300 \text{ lb}}{\sin(5° + 19.29°)} = 729.30 \text{ lb}$$

When P is removed, the vertical components of R_1 and R_2 vanish, leaving the horizontal components

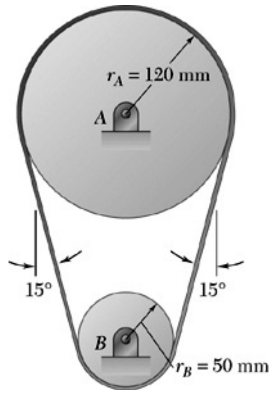
$$R_{1x} = R_{2x} = R_1 \cos(5° + \phi_s)$$

$$= (729.30 \text{ lb})\cos(5° + 19.29°)$$

$$R_{1x} = R_{2x} = 665 \text{ lb} \blacktriangleleft$$

(Note that $\phi_s > 5°$, so wedge is self-locking.)

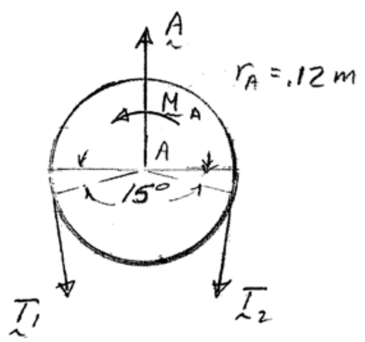
PROBLEM 8.140

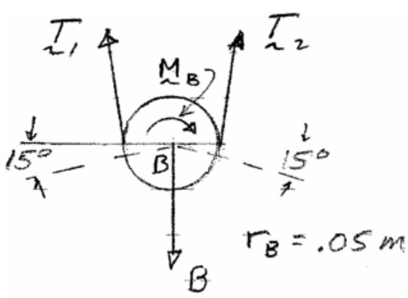


A flat belt is used to transmit a torque from drum B to drum A. Knowing that the coefficient of static friction is 0.40 and that the allowable belt tension is 450 N, determine the largest torque that can be exerted on drum A.

SOLUTION

FBD's drums:





$$\beta_A = 180° + 30° = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$$

$$\beta_B = 180° - 30° = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

Since $\beta_B < \beta_A$, slipping will impend first on B (friction coefficients being equal)

So $$T_2 = T_{\max} = T_1 e^{\mu_s \beta_B}$$

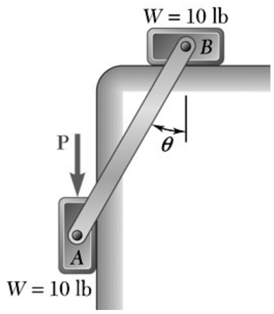
$$450 \text{ N} = T_1 e^{(0.4)5\pi/6} \qquad \text{or} \qquad T_1 = 157.914 \text{ N}$$

$$\circlearrowleft \Sigma M_A = 0: \quad M_A + (0.12 \text{ m})(T_1 - T_2) = 0$$

$$M_A = (0.12 \text{ m})(450 \text{ N} - 157.914 \text{ N}) = 35.05 \text{ N}\cdot\text{m}$$

$$M_A = 35.1 \text{ N}\cdot\text{m} \blacktriangleleft$$

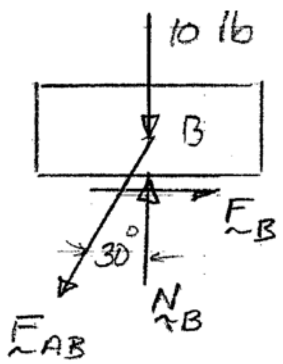
PROBLEM 8.141



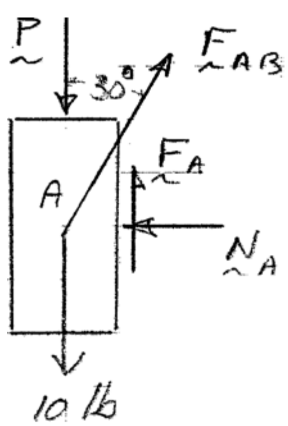
Two 10-lb blocks A and B are connected by a slender rod of negligible weight. The coefficient of static friction is 0.30 between all surfaces of contact, and the rod forms an angle $\theta = 30°$ with the vertical. (a) Show that the system is in equilibrium when $P = 0$. (b) Determine the largest value of P for which equilibrium is maintained.

SOLUTION

FBD block B:



FBD block A:



(b) For $P_{\max}$, motion impends at both surfaces

B: $\quad \uparrow \Sigma F_y = 0: \quad N_B - 10 \text{ lb} - F_{AB}\cos 30° = 0$

$$N_B = 10 \text{ lb} + \frac{\sqrt{3}}{2}F_{AB} \qquad (1)$$

Impending motion: $\quad F_B = \mu_s N_B = 0.3N_B$

$$\xrightarrow{\quad} \Sigma F_x = 0: \quad F_B - F_{AB}\sin 30° = 0$$

$$F_{AB} = 2F_B = 0.6N_B \qquad (2)$$

Solving (1) and (2) $\quad N_B = 10 \text{ lb} + \frac{\sqrt{3}}{2}(0.6N_B)$

$$= 20.8166 \text{ lb}$$

Then $\quad F_{AB} = 0.6N_B = 12.4900 \text{ lb}$

A: $\quad \xrightarrow{\quad} \Sigma F_x = 0: \quad F_{AB}\sin 30° - N_A = 0$

$$N_A = \frac{1}{2}F_{AB} = \frac{1}{2}(12.4900 \text{ lb}) = 6.2450 \text{ lb}$$

Impending motion: $\quad F_A = \mu_s N_A = 0.3(6.2450 \text{ lb}) = 1.8735 \text{ lb}$

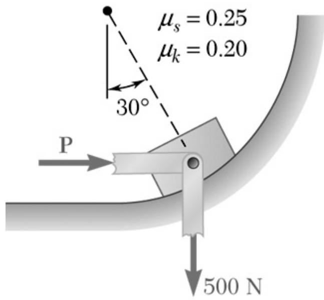
$$\uparrow \Sigma F_y = 0: \quad F_A + F_{AB}\cos 30° - P - 10 \text{ lb} = 0$$

$$P = F_A + \frac{\sqrt{3}}{2}F_{AB} - 10 \text{ lb}$$

$$= 1.8735 \text{ lb} + \frac{\sqrt{3}}{2}(12.4900 \text{ lb}) - 10 \text{ lb} = 2.69 \text{ lb}$$

$$P = 2.69 \text{ lb} \blacktriangleleft$$

(a) Since $P = 2.69$ lb to initiate motion, equilibrium exists with $P = 0$ ◀

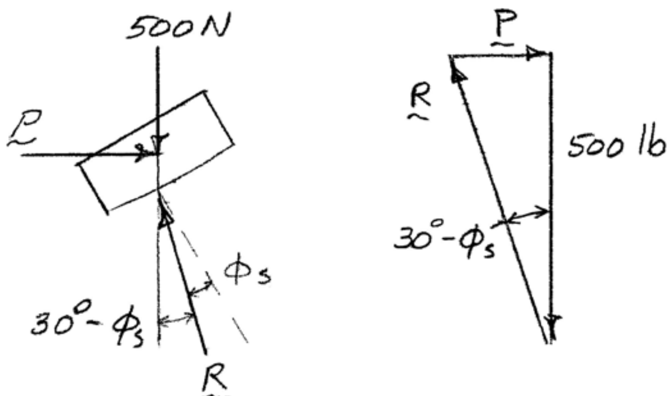


PROBLEM 8.142

Determine the range of values of P for which equilibrium of the block shown is maintained.

SOLUTION

FBD block (Impending motion down):

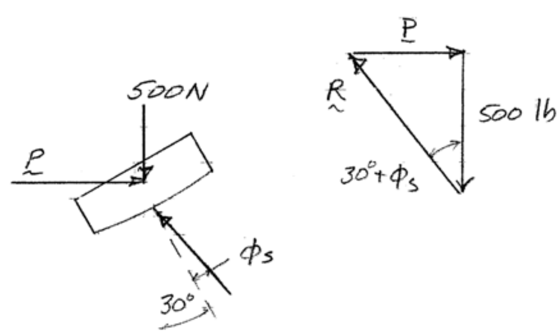


$$\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.25$$

$$P = (500 \text{ lb})\tan\left(30° - \tan^{-1}0.25\right)$$

$$= 143.03 \text{ lb}$$

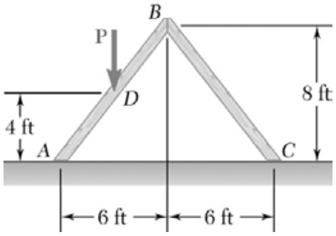
(Impending motion up):



$$P = (500 \text{ lb})\tan\left(30° + \tan^{-1}0.25\right)$$

$$= 483.46 \text{ lb}$$

Equilibrium for $143.0 \text{ lb} \le P \le 483 \text{ lb}$ ◀

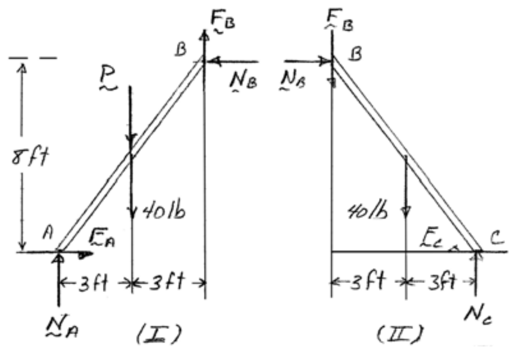


PROBLEM 8.143

Two identical uniform boards, each of weight 40 lb, are temporarily leaned against each other as shown. Knowing that the coefficient of static friction between all surfaces is 0.40, determine (*a*) the largest magnitude of the force **P** for which equilibrium will be maintained, (*b*) the surface at which motion will impend.

SOLUTION

Board FBDs:



Assume impending motion at *C*, so

$$F_C = \mu_s N_C$$
$$= 0.4 N_C$$

FBD II:

$$\circlearrowleft \Sigma M_B = 0: \quad (6\text{ ft})N_C - (8\text{ ft})F_C - (3\text{ ft})(40\text{ lb}) = 0$$

$$\left[6\text{ ft} - 0.4(8\text{ ft})\right]N_C = (3\text{ ft})(40\text{ lb})$$

or

$$N_C = 42.857\text{ lb}$$

and

$$F_C = 0.4N_C = 17.143\text{ lb}$$

$$\longrightarrow \Sigma F_x = 0: \quad N_B - F_C = 0$$

$$N_B = F_C = 17.143\text{ lb}$$

$$\uparrow \Sigma F_y = 0: \quad -F_B - 40\text{ lb} + N_C = 0$$

$$F_B = N_C - 40\text{ lb} = 2.857\text{ lb}$$

Check for motion at *B*:

$$\frac{F_B}{N_B} = \frac{2.857\text{ lb}}{17.143\text{ lb}} = 0.167 < \mu_s, \quad \text{OK, no motion.}$$

PROBLEM 8.143 CONTINUED

FBD I: $\circlearrowleft \Sigma M_A = 0$: $(8 \text{ ft})N_B + (6 \text{ ft})F_B - (3 \text{ ft})(P + 40 \text{ lb}) = 0$

$$P = \frac{(8 \text{ ft})(17.143 \text{ lb}) + (6 \text{ ft})(2.857 \text{ lb})}{3 \text{ ft}} - 40 \text{ lb} = 11.429 \text{ lb}$$

Check for slip at A (unlikely because of P)

$\longrightarrow \Sigma F_x = 0$: $F_A - N_B = 0$ or $F_A = N_B = 17.143 \text{ lb}$

$\uparrow \Sigma F_y = 0$: $N_A - P - 40 \text{ lb} + F_B = 0$ or $N_A = 11.429 \text{ lb} + 40 \text{ lb} - 2.857 \text{ lb}$

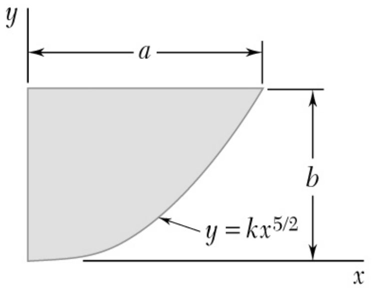
$= 48.572 \text{ lb}$

Then $\dfrac{F_A}{N_A} = \dfrac{17.143 \text{ lb}}{48.572 \text{ lb}} = 0.353 < \mu_s$, OK, no slip $\Rightarrow$ assumption is correct

Therefore, (*a*) $P_{\max} = 11.43 \text{ lb}$ ◀

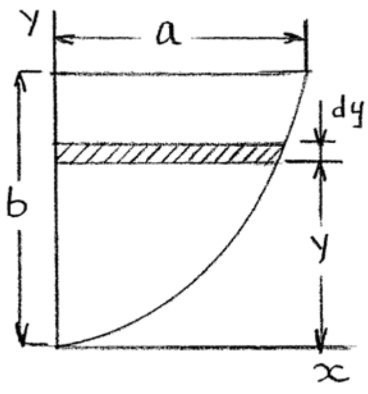
(*b*) Motion impends at C ◀

PROBLEM 9.1



Determine by direct integration the moment of inertia of the shaded area with respect to the y axis.

SOLUTION



At $\qquad x = a,\ \ y = b: \quad b = ka^{\frac{5}{2}} \qquad \text{or} \qquad k = \dfrac{b}{a^{\frac{5}{2}}}$

$$\therefore\ y = \frac{b}{a^{\frac{5}{2}}}x^{\frac{5}{2}} \qquad \text{or} \qquad x = \frac{a}{b^{\frac{2}{5}}}y^{\frac{2}{5}}$$

$$dI_y = \frac{1}{3}x^3dy$$

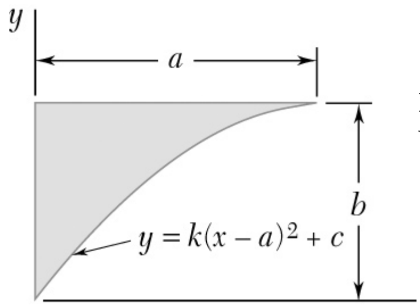
$$= \frac{1}{3}\frac{a^3}{b^{\frac{6}{5}}}y^{\frac{6}{5}}dy$$

Then $\qquad I_y = \dfrac{1}{3}\dfrac{a^3}{b^{\frac{6}{5}}}\displaystyle\int_0^b y^{\frac{6}{5}}dy$

$$= \frac{1}{3}\frac{5}{11}\frac{a^3}{b^{\frac{6}{5}}}y^{\frac{11}{5}}\Big|_0^b$$

$$= \frac{5}{33}\frac{a^3}{b^{\frac{6}{5}}}b^{\frac{11}{5}}$$

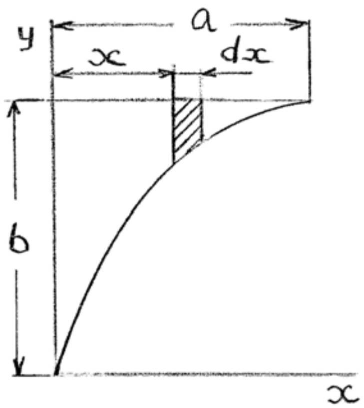
or $\quad I_y = \dfrac{5}{33}a^3b$ ◀



PROBLEM 9.2

Determine by direct integration the moment of inertia of the shaded area with respect to the *y* axis.

SOLUTION



At $$x = 0,\ y = 0:\quad 0 = ka^2 + c$$

$$k = -\frac{c}{a^2}$$

$$x = a,\quad y = b:\qquad b = c$$

$$\therefore k = -\frac{b}{a^2}$$

Then $$y = -\frac{b}{a^2}(x-a)^2 + b$$

$$= -\frac{b}{a^2}\left(x^2 - 2ax + a^2\right) + b$$

Now $$dI_y = x^2 dA = x^2(ydx) = \left(-\frac{b}{a^2}x^4 + \frac{2b}{a}x^3 - bx^2 + bx^2\right)dx$$

$$= \left(-\frac{b}{a^2}x^4 + \frac{2b}{a}x^3\right)dx$$

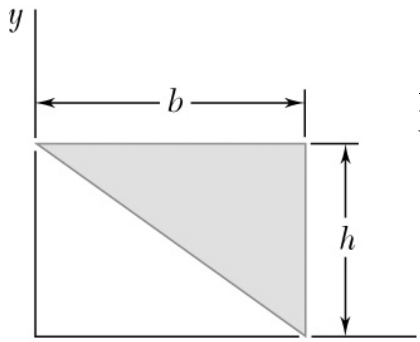
Then $$I_y = \int dI_y = \int_0^a \left(-\frac{b}{a^2}x^4 + \frac{2b}{a}x^3\right)dx$$

$$= b\left[-\frac{1}{a^2}\frac{x^5}{5} + \frac{2}{a}\frac{x^4}{4}\right]_0^a$$

$$= b\left(\frac{a^3}{5} + \frac{2a^3}{4}\right) = ba^3\left(\frac{1}{2} - \frac{1}{5}\right)$$

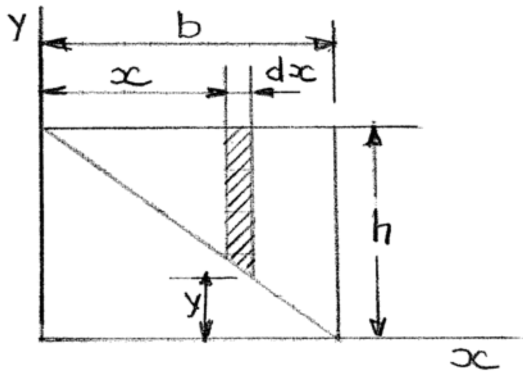
$$I_y = \frac{3a^3b}{10} \blacktriangleleft$$

PROBLEM 9.3



Determine by direct integration the moment of inertia of the shaded area with respect to the y axis.

SOLUTION



By observation
$$y = h - \frac{h}{b}x$$

$$= h\left(1 - \frac{x}{b}\right)$$

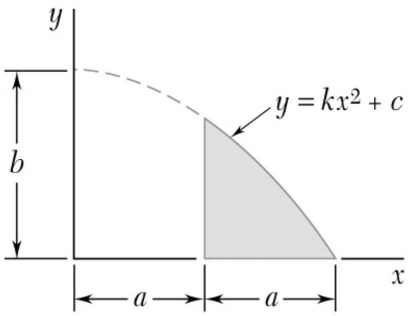
Now
$$dI_y = x^2 dA = x^2\left[\left(h - y\right)dx\right]$$

$$= x^2\left[h - h\left(1 - \frac{x}{b}\right)\right]dx$$

$$= \frac{hx^3}{b}dx$$

Then
$$I_y = \int dI_y = \int_0^b \frac{hx^3}{b}dx = \frac{hx^4}{4b}\Bigg|_0^b = \frac{hb^4}{4b}$$

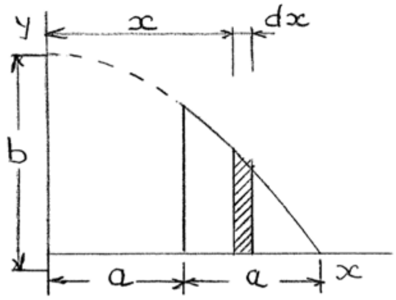
$$I_y = \frac{b^3h}{4} \blacktriangleleft$$



PROBLEM 9.4

Determine by direct integration the moment of inertia of the shaded area with respect to the y axis.

SOLUTION



Have $$y = kx^2 + c$$

At $$x = 0, y = b: \quad b = k(0) + c$$

or $$c = b$$

At $$x = 2a, \; y = 0: \quad 0 = k(2a)^2 + b$$

or $$k = -\frac{b}{4a^2}$$

Then $$y = -\frac{b}{4a^2}x^2 + b$$

$$= \frac{b}{4a^2}\left(4a^2 - x^2\right)$$

Then $$I_y = \int x^2 dA, \qquad dA = ydx = \frac{b}{4a^2}\left(4a^2 - x^2\right)dx$$

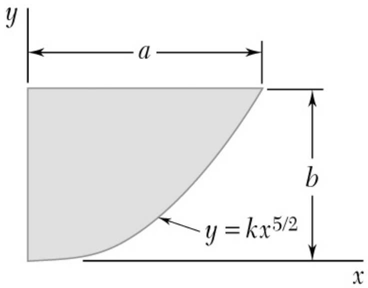
$$I_y = \int_a^{2a} x^2 dA = \frac{b}{4a^2}\int_a^{2a} x^2\left(4a^2 - x^2\right)dx$$

$$= \frac{b}{4a^2}\left[4a^2\frac{x^3}{3} - \frac{x^5}{5}\right]_a^{2a}$$

$$= \frac{b}{3}\left(8a^3 - a^3\right) - \frac{b}{20a^2}\left(32a^5 - a^5\right)$$

$$= \frac{7a^3b}{3} - \frac{31a^3b}{20}$$

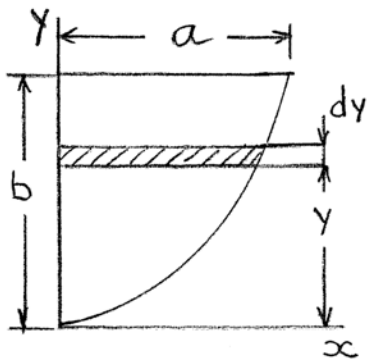
$$I_y = \frac{47}{60}a^3b \blacktriangleleft$$



PROBLEM 9.5

Determine by direct integration the moment of inertia of the shaded area with respect to the *x* axis.

SOLUTION



At $x = a,\ \ y = b$: $\quad b = ka^{\frac{3}{2}}$

or $\ k = \dfrac{b}{a^{\frac{3}{2}}}$

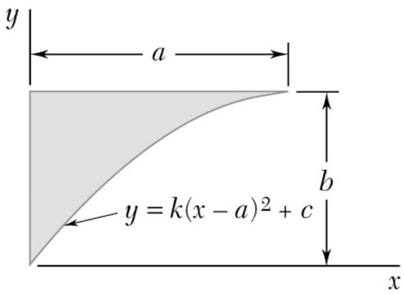
$\therefore\ y = \dfrac{b}{a^{\frac{3}{2}}}\ x^{\frac{3}{2}}$

$I_x = \int y^2 dA \qquad dA = xdy$

$$= \int_0^b y^2 \left[\frac{a}{b^{\frac{2}{5}}} y^{\frac{2}{5}} dy \right]$$

$$= \frac{a}{b^{\frac{2}{5}}} \times \frac{5}{17} y^{\frac{17}{5}} \Big|_0^b = \frac{5a}{17} \frac{b^{\frac{17}{5}}}{b^{\frac{2}{5}}}$$

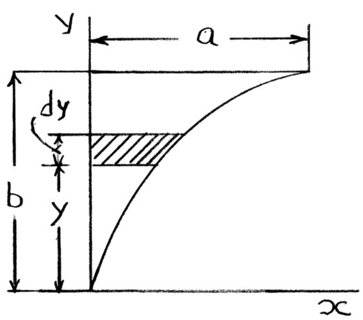
or $\ I_x = \dfrac{5}{17} ab^3$ ◀



PROBLEM 9.6

Determine by direct integration the moment of inertia of the shaded area with respect to the x axis.

SOLUTION



At $$x = 0, \quad y = 0: \quad 0 = ka^2 + c$$

$$k = -\frac{c}{a^2}$$

$$x = a, \quad y = b \quad b = c$$

$$k = -\frac{b}{a^2}$$

Then $$y = b - \frac{b}{a^2}(x - a)^2$$

Now $$dI_x = y^2 dA = y^2(x dy)$$

From above $$(x - a)^2 = \frac{a^2}{b}(b - y)$$

Then $$x - a = a^2\sqrt{1 - \frac{y}{b}}$$

and $$x = a^2\sqrt{1 - \frac{y}{b}} + a$$

Then $$dI_x = ay^2\left(1 + \sqrt{1 - \frac{y}{b}}\right)dy$$

and $$I_x = \int dI_x = a\int_0^b y^2\left(1 + \sqrt{1 - \frac{y}{b}}\right)dy$$

$$= a\frac{y^3}{3}\bigg|_0^b + a\int_0^b y^2\left(\sqrt{1 - \frac{y}{6}}\right)dy$$

PROBLEM 9.6 CONTINUED

For the second integral use substitution

$$u = 1 - \frac{y}{b} \Rightarrow du = -\frac{1}{b}dy,\ y = b(1-u)$$

$$y = 0 \quad u = 1$$
$$y = b \quad u = 0$$

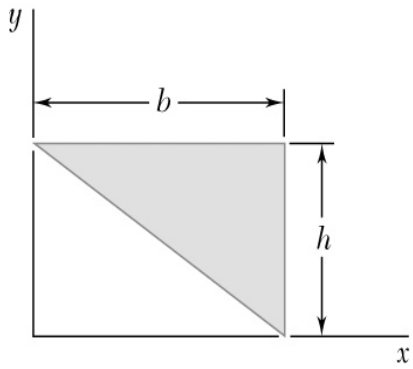
Now
$$\int_0^b y^2\left(\sqrt{1-\frac{y}{b}}\right)dy = -\int_0^b b^2(1-u)^2u^{\frac{1}{2}}du$$

$$= -b^3\int_1^0 \left(u^{\frac{1}{2}} - 2u^{\frac{3}{2}} + u^{\frac{5}{2}}\right)du = -b^3\left(\frac{2}{3}u^{\frac{3}{2}} - \frac{4}{5}u^{\frac{5}{2}} + \frac{2}{7}u^{\frac{7}{2}}\right)\Bigg|_1^0$$

$$= +b^3\left(\frac{2}{3} - \frac{4}{5} + \frac{2}{7}\right) = b^3\left(\frac{70-84+30}{105}\right) = \frac{16b^3}{105}$$

Then
$$I_x = a\frac{b^3}{3} + \frac{16ab^3}{105} = \frac{51}{105}ab^3$$

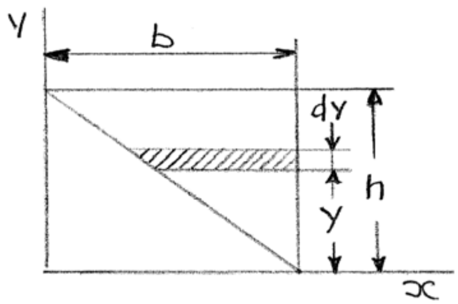
or $I_x = \dfrac{17}{35}ab^3$ ◀



PROBLEM 9.7

Determine by direct integration the moment of inertia of the shaded area with respect to the *x* axis.

SOLUTION



By observation

$$y = h - \frac{h}{b}x$$

$$= h\left(1 - \frac{x}{b}\right)$$

or

$$x = b\left(1 - \frac{y}{h}\right)$$

Now

$$dI_x = y^2 dA = y^2(b - x)dy$$

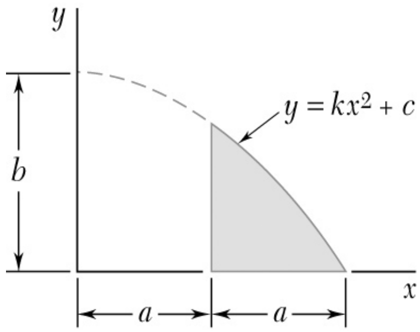
$$= y^2\left(b - b + \frac{by}{h}dy\right)$$

$$= \frac{by^3}{h}dy$$

Then

$$I_x = \int_0^h \frac{by^3}{h}dy = \frac{by^4}{4h}\bigg|_0^h = \frac{bh^4}{4h}$$

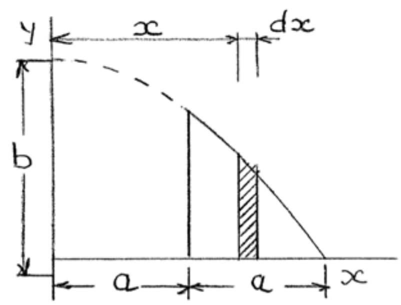
or $I_x = \dfrac{bh^3}{4}$ ◀



PROBLEM 9.8

Determine by direct integration the moment of inertia of the shaded area with respect to the x axis.

SOLUTION



Have $$y = kx^2 + c$$

At $$x = 0, \quad y = b: \quad b = k(0) + c$$

or $$c = b$$

At $$x = 2a, \quad y = 0: \quad 0 = k(2a)^2 + b$$

or $$k = -\frac{b}{4a^2}$$

Then $$y = \frac{b}{4a^2}\left(4a^2 - x^2\right)$$

Now $$dI_x = \frac{1}{3}y^3 dx$$

$$= \frac{1}{3}\frac{b^3}{64a^6}\left(4a^2 - x^2\right)^3 dx$$

PROBLEM 9.8 CONTINUED

Then $$I_x = \int dI_x$$

$$= \frac{1}{3}\frac{b^3}{64a^6}\int_a^{2a}\left(4a^2 - x^2\right)^3 dx$$

$$= \frac{b^3}{192a^6}\int_a^{2a}\left(64a^6 - 48a^4x^2 + 12a^2x^4 - x^6\right)dx$$

$$= \frac{b^3}{192a^6}\left[64a^6x - 16a^4x^3 + \frac{12}{5}a^2x^5 - \frac{x^7}{7}\right]_a^{2a}$$

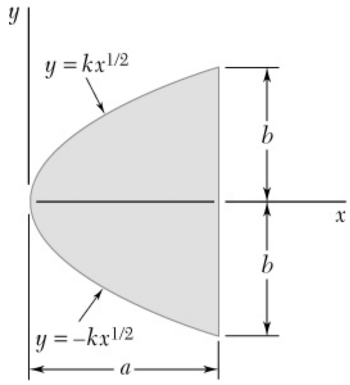
$$= \frac{b^3}{192a^6}\left[64a^7(2-1) - 16a^7(8-1)\right.$$

$$\left. + \frac{12}{5}a^7(32-1) - \frac{1}{7}(128-1)\right]$$

$$= \frac{ab^3}{192}\left(64 - 112 + \frac{372}{5} - \frac{127}{7}\right) = 0.043006ab^3$$

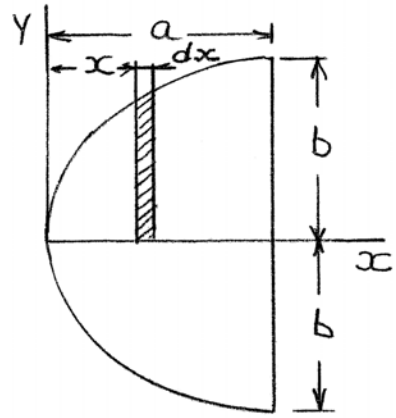
$$I_x = 0.0430ab^3 \blacktriangleleft$$

PROBLEM 9.9



Determine by direct integration the moment of inertia of the shaded area with respect to the x axis.

SOLUTION



At $\quad x = a, \; y = b: \quad b = ka^{\frac{1}{2}}$

or $\quad k = \dfrac{b}{\sqrt{a}}$

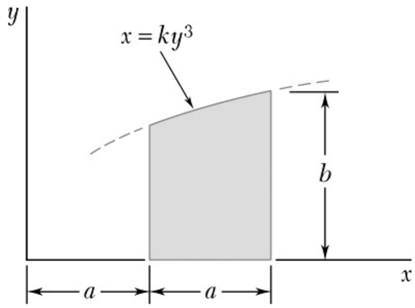
Then $\quad y = \dfrac{b}{\sqrt{a}} x^{\frac{1}{2}}$

Now $\quad dI_x = \dfrac{1}{3} y^3 dx = \dfrac{1}{3}\left(\dfrac{b}{\sqrt{a}}\right)^3 x^{\frac{3}{2}} dx$

Then $\quad I_x = 2\int_0^a dI_x = 2\int_0^a \dfrac{1}{3}\left(\dfrac{b}{\sqrt{a}}\right)^3 x^{\frac{3}{2}} dx$

$$= \frac{2}{3}\left(\frac{b}{\sqrt{a}}\right)^3 \frac{2}{5} x^{\frac{5}{2}} \Bigg|_0^a = \frac{4}{15} \frac{b^3}{a^{\frac{3}{2}}} a^{\frac{5}{2}}$$

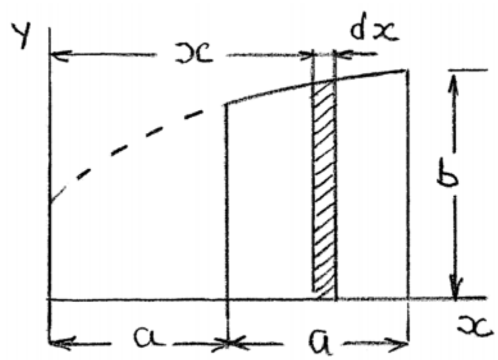
$$I_x = \frac{4}{15} ab^3 \blacktriangleleft$$



PROBLEM 9.10

Determine by direct integration the moment of inertia of the shaded area with respect to the x axis.

SOLUTION



At $$x = 2a,\ y = b: \qquad 2a = kb^3$$

or $$k = \frac{2a}{b^3}$$

Then $$x = \frac{2a}{b^3}y^3$$

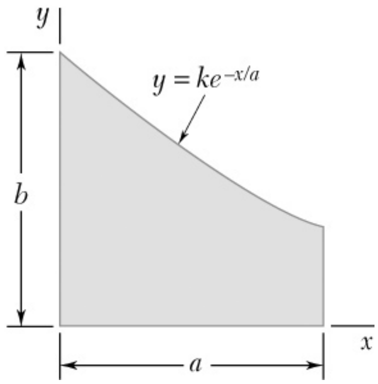
or $$y = \frac{b}{(2a)^{\frac{1}{3}}}x^{\frac{1}{3}}$$

Now $$dI_x = \frac{1}{3}y^3dx = \frac{1}{3}\frac{b^3}{2a}xdx$$

Then $$I_x = \int dI_x = \frac{1}{3}\frac{b^3}{2a}\int_a^{2a} xdx = \frac{1}{6}\frac{b^3}{a}\frac{1}{2}x^2\Big|_a^{2a}$$

$$= \frac{b^3}{12a}\left(4a^2 - a^2\right)$$

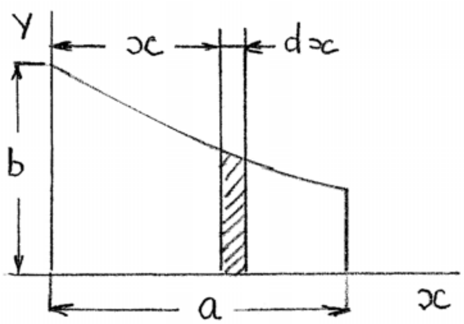
$$I_x = \frac{1}{4}ab^3 \blacktriangleleft$$



PROBLEM 9.11

Determine by direct integration the moment of inertia of the shaded area with respect to the x axis.

SOLUTION



At $$x = 0, y = b: \quad b = ke^0 = k$$

Then $$y = be^{-\frac{x}{a}}$$

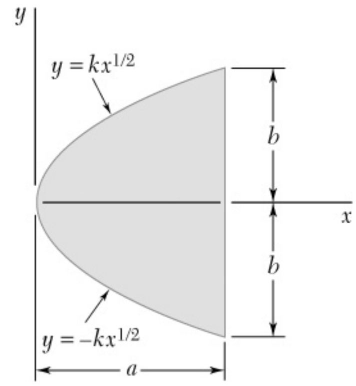
Now $$dI_x = \frac{1}{3}y^3 dx = \frac{b^3}{3}\left(e^{-\frac{x}{a}}\right)^3 dx$$

Then $$I_x = \int dI_x = \int_0^a \frac{b^3}{3}\left(e^{-\frac{x}{a}}\right)^3 dx$$

$$= \frac{b^3}{3}\int e^{-\frac{3x}{a}} dx = \frac{b^3}{3}\left(\frac{-a}{3}\right)e^{-\frac{3x}{a}}\Big|_0^a = -\frac{b^3 a}{9}\left(e^{-3} - e^0\right)$$

$$= \frac{ab^3}{9}(0.95021) = 0.10558ab^3$$

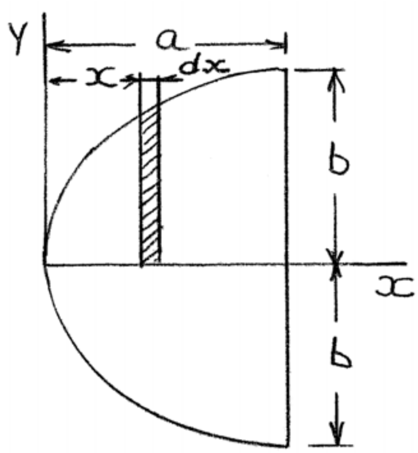
or $I_x = 0.1056ab^3$ ◀



PROBLEM 9.12

Determine by direct integration the moment of inertia of the shaded area with respect to the *y* axis.

SOLUTION



At $\quad x = a, \quad y = b: \quad b = ka^{\frac{1}{2}} \quad$ or $\quad k = \dfrac{b}{\sqrt{a}}$

Then $$y = \frac{b}{\sqrt{a}}x^{\frac{1}{2}}$$

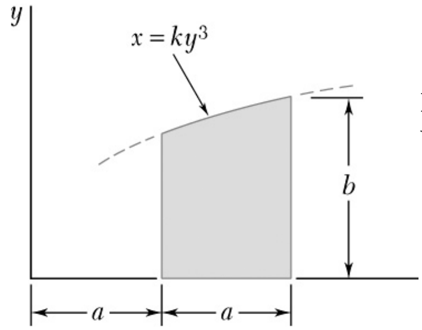
Now $$dI_y = x^2 dA, \qquad dA = ydx$$

$$dI_y = x^2 ydx = \frac{b}{\sqrt{a}}x^{\frac{5}{2}}dx$$

Then $$I_y = \int dI_y = 2\int_0^a \frac{b}{\sqrt{a}}x^{\frac{5}{2}}dx = \frac{4}{7}\frac{b}{\sqrt{a}}x^{\frac{7}{2}}\Big|_0^a$$

$$= \frac{4}{7}\frac{b}{a^{\frac{1}{2}}}a^{\frac{7}{2}}$$

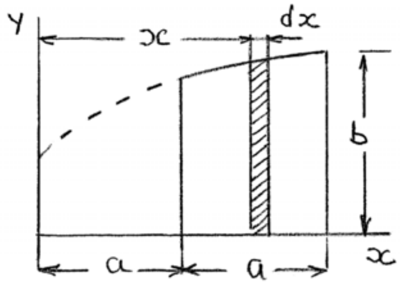
or $I_y = \dfrac{4}{7}a^3 b$ ◀



PROBLEM 9.13

Determine by direct integration the moment of inertia of the shaded area with respect to the y axis.

SOLUTION



At $x = 2a,\quad y = b:\quad 2a = kb^3$ or s

Then
$$x = \frac{2a}{b^3}y^3$$

or
$$y = \frac{b}{(2a)^{\frac{1}{3}}}x^{\frac{1}{3}}$$

Now
$$I_y = \int x^2 dA \qquad dA = ydx$$

Then
$$I_y = \int_a^{2a} x^2 \frac{b}{(2a)^{\frac{1}{3}}}x^{\frac{1}{3}}dx$$

$$= \frac{b}{(2a)^{\frac{1}{3}}}\int_a^{2a} x^{\frac{5}{3}}dx$$

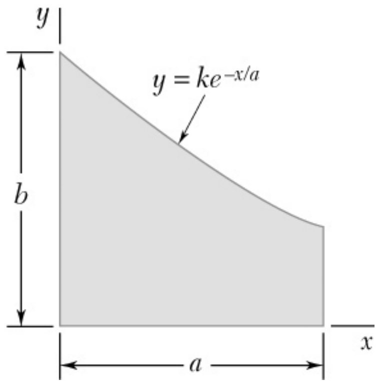
$$= \frac{b}{(2a)^{\frac{1}{3}}}\frac{3}{10}x^{\frac{10}{3}}\Bigg|_a^{2a}$$

$$= \frac{3b}{10(2a)^{\frac{1}{3}}}\left[(2a)^{\frac{10}{3}} - a^{\frac{10}{3}}\right]$$

$$= \frac{3ba^3}{10(2)^{\frac{1}{3}}}\left(2^{\frac{10}{3}} - 1^{\frac{10}{3}}\right)$$

$$= 2.1619a^3b$$

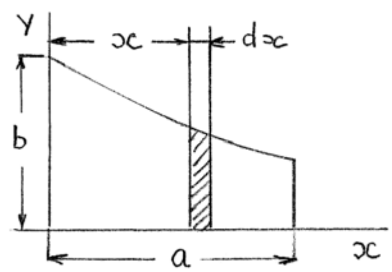
or $I_y = 2.16a^3b$ ◀



PROBLEM 9.14

Determine by direct integration the moment of inertia of the shaded area with respect to the y axis.

SOLUTION



At $$x = 0, \quad y = b: \quad b = ke^0 = k$$

Then $$y = be^{-\frac{x}{a}}$$

Now $$dI_y = x^2 dA = x^2 y dx$$

$$= x^2 be^{-\frac{x}{a}} dx$$

Then $$I_y = \int dI_y = \int_0^a bx^2 e^{-\frac{x}{a}} dx = b\int_0^a x^2 e^{-\frac{x}{a}} dx$$

Use integration by parts

$$u = x^2 \qquad dv = e^{-\frac{x}{a}} dx$$

$$du = 2x dx \qquad v = -ae^{-\frac{x}{a}}$$

Then $$I_y = \int_0^a x^2 e^{-\frac{x}{a}} dx = b\left[-ax^2 e^{-\frac{x}{a}} \Big|_0^a - \int_0^a \left(-ae^{-\frac{x}{a}} \right) 2x dx \right]$$

$$= b\left(-a^3 e^{-1} + 2a\int_0^a xe^{-\frac{x}{a}} dx \right)$$

Again use integration by parts:

$$u = x \qquad dv = e^{-\frac{x}{a}} dx$$

$$du = dx \qquad v = -ae^{-\frac{x}{a}}$$

PROBLEM 9.14 CONTINUED

Then $\quad \int_0^a xe^{-\frac{x}{a}}dx = -axe^{-\frac{x}{a}}\Big|_0^a - \int_0^a\left(-ae^{-\frac{x}{a}}\right)dx$

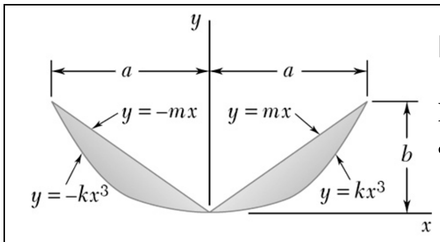
$$= -a^2e^{-1} - a^2e^{-\frac{x}{a}}\Big|_0^a = -a^2e^{-1} - a^2e^{-1} + a^2e^0$$

$$= -2a^2e^{-1} + a^2$$

Finally, $\quad I_y = b\left[-a^3e^{-1} + 2a\left(-2a^2e^{-1} + a^2\right)\right] = ba^3\left(-5e^{-1} + 2\right)$

$$= 0.1606ba^3$$

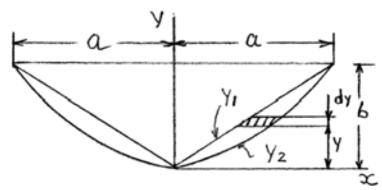
or $\; I_y = 0.1606ba^3$ ◀



PROBLEM 9.15

Determine the moment of inertia and the radius of gyration of the shaded area shown with respect to the x axis.

SOLUTION



At $\quad x = a, \quad y_1 = y_2 = b$

$$y_1: \quad b = ma \quad \text{or} \quad m = \frac{b}{a}$$

$$y_2: \quad b = ka^3 \quad \text{or} \quad k = \frac{b}{a^3}$$

Then
$$y_1 = \frac{b}{a}x \quad \text{or} \quad x_1 = \frac{a}{b}y$$

$$y_2 = \frac{b}{a^3}x^3 \quad \text{or} \quad x_2 = \left(\frac{a}{b^{\frac{1}{3}}}\right)y^{\frac{1}{3}}$$

Now
$$dA = (x_2 - x_1)dy = \left(\frac{a}{b^{\frac{1}{3}}}y^{\frac{1}{3}} - \frac{a}{b}y\right)dy$$

$$A = 2\int dA = 2\int_0^b\left(\frac{a}{b^{\frac{1}{3}}}y^{\frac{1}{3}} - \frac{a}{b}y\right)dy = 2\left[\frac{a}{b^{\frac{1}{3}}}\frac{3}{4}y^{\frac{4}{3}} - \frac{a}{b}\frac{1}{2}y^2\right]_0^b$$

$$= \frac{3ab}{2} - ab = \frac{1}{2}ab$$

Then
$$dI_x = y^2 dA = y^2\left(\frac{a}{b^{\frac{1}{3}}}y^{\frac{1}{3}} - \frac{a}{b}y\right)dy$$

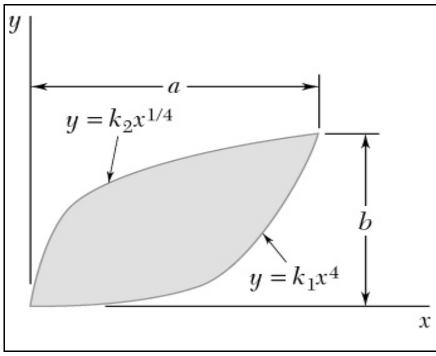
Now
$$I_x = 2\int dI_x = 2a\int_0^b\left(\frac{y^{\frac{7}{3}}}{b^{\frac{1}{3}}} - \frac{y^3}{b}\right)dy = 2a\left[\frac{3}{10}\frac{y^{\frac{10}{3}}}{b^{\frac{1}{3}}} - \frac{y^4}{4b}\right]_0^b$$

$$= 2a\left(\frac{3}{10}b^3 - \frac{b^3}{4}\right) = 2ab^3\left(\frac{3}{10} - \frac{1}{4}\right)$$

$$\text{or} \quad I_x = \frac{1}{10}ab^3 \blacktriangleleft$$

And
$$k_x^2 = \frac{I_x}{A} = \frac{\frac{1}{10}ab^3}{\frac{1}{2}ab} = \frac{1}{5}b^2$$

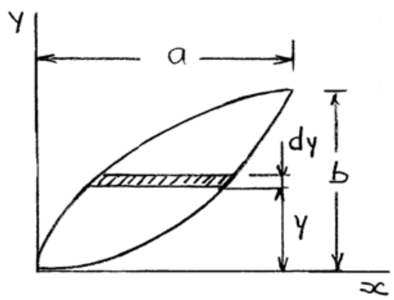
$$k_x = \frac{b}{\sqrt{5}} \blacktriangleleft$$



PROBLEM 9.16

Determine the moment of inertia and the radius of gyration of the shaded area shown with respect to the *x* axis.

SOLUTION



At $\qquad x = a,\ y = b: \qquad b = k_1a^4 \qquad b = k_2a^{\frac{1}{4}}$

or
$$k_1 = \frac{b}{a^4} \qquad k_2 = \frac{b}{a^{\frac{1}{4}}}$$

Then
$$y_1 = \frac{b}{a^4}x^4 \qquad y_2 = \frac{b}{a^{\frac{1}{4}}}x^{\frac{1}{4}}$$

and
$$x_1 = \frac{a}{b^{\frac{1}{4}}}y^{\frac{1}{4}} \qquad x_2 = \frac{a}{b^4}y^4$$

Now
$$A = \int(y_2 - y_1)\,dx = b\int_0^a\left(\frac{x^{\frac{1}{4}}}{a^{\frac{1}{4}}} - \frac{x^4}{a^4}\right)dx$$

$$= b\left[\frac{4}{5}\frac{x^{\frac{5}{4}}}{a^{\frac{1}{4}}} - \frac{1}{5}\frac{x^5}{a^4}\right]_0^a = \frac{3}{5}ab$$

Then
$$I_x = \int y^2 dA \qquad dA = (x_1 - x_2)\,dy$$

$$I_x = \int_0^b y^2\left(\frac{a}{b^{\frac{1}{4}}}y^{\frac{1}{4}} - \frac{a}{b^4}y^4\right)dy$$

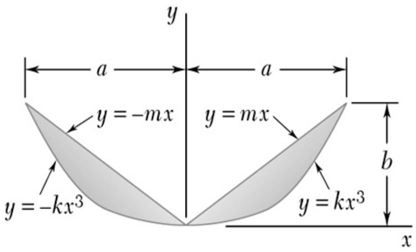
$$= a\left[\frac{4}{13}\frac{y^{\frac{13}{4}}}{b^{\frac{1}{4}}} - \frac{1}{7}\frac{y^7}{b^4}\right]_0^b$$

$$= ab^3\left(\frac{4}{13} - \frac{1}{7}\right)$$

or $\quad I_x = \dfrac{15}{91}ab^3$ ◀

Now
$$k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{\frac{15}{91}ab^3}{\frac{3}{5}ab}} = \sqrt{\frac{25}{91}b^2} = 0.52414b$$

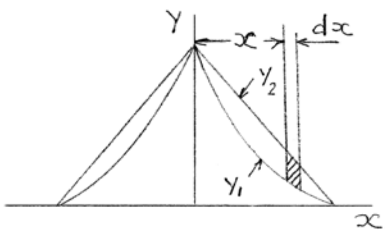
or $\quad k_x = 0.524b$ ◀



PROBLEM 9.17

Determine the moment of inertia and the radius of gyration of the shaded area shown with respect to the *y* axis.

SOLUTION



At $$x = a, \qquad y_1 = y_2 = b$$

$$y_1\!: \quad b = ka^3 \qquad \text{or} \qquad k = \frac{b}{a^3}$$

$$y_2\!: \quad b = ma \qquad \text{or} \qquad m = \frac{b}{a}$$

Then $$y_1 = \frac{b}{a^3}x^3$$

$$y_2 = \frac{b}{a}x$$

Now $$dA = \left(y_2 - y_1\right)dx = \left(\frac{b}{a}x - \frac{b}{a^3}x^3\right)dx$$

$$A = \int dA = 2\frac{b}{a}\int_0^a\left(x - \frac{x^3}{a^2}\right)dx = 2\frac{b}{a}\left[\frac{1}{2}x^2 - \frac{1}{4a^2}x^4\right]_0^a$$

$$= 2\frac{b}{a}\left[\frac{a^2}{2} - \frac{1}{4a^2}a^4\right] = \frac{1}{2}ab$$

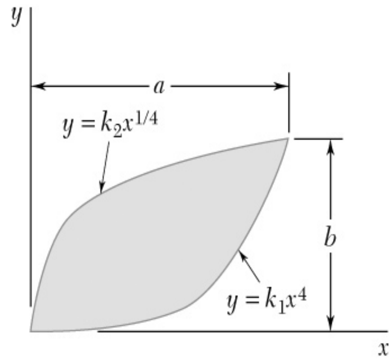
Now $$dI_y = x^2 dA = \frac{b}{a}\left[\left(x^3 - \frac{x^5}{a^2}\right)dx\right]$$

Then $$I_y = \int_0^a dI_y = 2\frac{b}{a}\int_0^a\left(x^3 - \frac{x^5}{a^2}\right)dx$$

$$= 2\frac{b}{a}\left[\frac{1}{4}x^4 - \frac{1}{6a^2}x^6\right]_0^a = 2\frac{b}{a}\left(\frac{a^4}{4} - \frac{1}{6}\frac{a^6}{a^2}\right)$$

$$= \frac{1}{6}a^3b \qquad\qquad \text{or} \quad I_y = \frac{1}{6}a^3b \ ◀$$

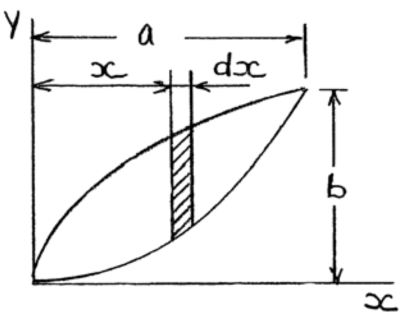
And $$k_y^2 = \frac{I_y}{A} = \frac{\frac{1}{6}a^3b}{\frac{1}{2}ab} = \frac{1}{3}a^2 \qquad\qquad \text{or} \quad k_y = \frac{a}{\sqrt{3}} \ ◀$$



PROBLEM 9.18

Determine the moment of inertia and the radius of gyration of the shaded area shown with respect to the y axis.

SOLUTION



At $\quad x = a,\; y = b: \quad b = k_1 a^4 \qquad b = k_2 a^{\frac{1}{4}}$

or $$k_1 = \frac{b}{a^4} \qquad k_2 = \frac{b}{a^{\frac{1}{4}}}$$

Then $$y_1 = \frac{b}{a^4}x^4 \qquad \text{and} \qquad y_2 = \frac{b}{a^{\frac{1}{4}}}x^{\frac{1}{4}}$$

Now $$A = \int (y_2 - y_1)\,dx = b\int_0^a \left(\frac{x^{\frac{1}{4}}}{a^{\frac{1}{4}}} - \frac{x^4}{a^4}\right)dx$$

$$= b\left[\frac{4}{5}\frac{x^{\frac{5}{4}}}{a^{\frac{1}{4}}} - \frac{1}{5}\frac{x^5}{a^4}\right]_0^a = \frac{3}{5}ab$$

Now $$I_y = \int x^2 dA \qquad dA = (y_2 - y_1)\,dx$$

Then $$I_y = \int_0^a x^2\left(\frac{b}{a^{\frac{1}{4}}}x^{\frac{1}{4}} - \frac{b}{a^4}x^4\right)dx$$

$$= b\int_0^a \left(\frac{x^{\frac{9}{4}}}{a^{\frac{1}{4}}} - \frac{x^6}{a^4}\right)dx$$

$$= b\left[\frac{4}{13}\frac{x^{\frac{13}{4}}}{a^{\frac{1}{4}}} - \frac{1}{7}\frac{x^7}{a^4}\right]_0^a$$

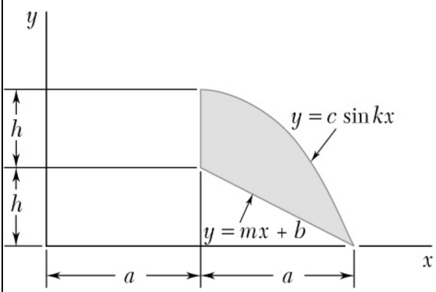
$$= b\left(\frac{4}{13}a^3 - \frac{1}{7}a^3\right)$$

or $I_y = \dfrac{15}{91}a^3 b$ ◀

Now $$k_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{\frac{15}{91}a^3 b}{\frac{3}{5}ab}} = \sqrt{\frac{25}{91}}a = 0.52414a$$

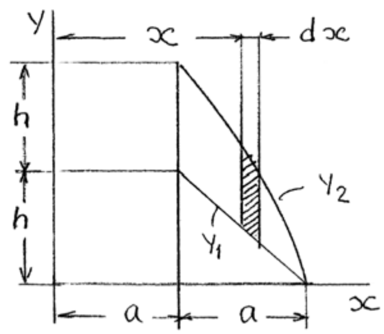
or $k_y = 0.524a$ ◀

PROBLEMS 9.19 AND 9.20



P 9.19 Determine the moment of inertia and the radius of gyration of the shaded area shown with respect to the *x* axis.
P 9.20 Determine the moment of inertia and the radius of gyration of the shaded area shown with respect to the *y* axis.

SOLUTION



First determine constants *m*, *b*, and *c*

$$y_1: \quad \text{at} \quad x = 2a, \qquad y = 0$$

$$0 = m(2a) + b$$

At $\qquad x = a, \qquad y = h$

$$h = m(a) + b$$

Solving yields $\qquad m = -\frac{h}{a} \qquad b = 2h$

Then $\qquad y_1 = -\frac{h}{a}x + 2h$

$$y_2: \quad \text{at} \quad x = 2a, \qquad y = 0$$

$$0 = c\sin k(2a)$$

At $\qquad x = a, \qquad y = 2h$

$$2h = c\sin ka$$

Solving $\qquad c\sin k(2a) = 0 \qquad c \neq 0$

$$\sin k(2a) = 0, \;\; k(2a) = \pi, \;\; k = \frac{\pi}{2a}$$

Substitute *k*, $\quad 2h = c\sin ka \quad$ yields $\quad 2h = c\sin\frac{\pi}{2} \quad$ or $\quad c = 2h$

Then $\qquad y_2 = 2h\sin\frac{\pi}{2a}x$

To calculate the area of shaded surface, a differential strip parallel to the *y* axis is chosen to be *dA*.

$$dA = (y_2 - y_1)dx = \left[2h\sin\frac{\pi}{2a}x - \left(-\frac{h}{a}x + 2h\right)\right]dx$$

PROBLEMS 9.19 AND 9.20 CONTINUED

$$A = \int dA = \int_0^{2a}\left(2h\sin\frac{\pi}{2a}x - 2hx + \frac{h}{a}x\right)dx$$

$$= h\left[-\frac{4a}{\pi}\cos\frac{\pi}{2a}x - 2x + \frac{x^2}{2a}\right]_a^{2a}$$

$$= h\left[-\frac{4a}{\pi}\cos\frac{\pi}{2a}2a - 2(2a) + \frac{(2a)^2}{2a}\right]$$

$$-h\left[-\frac{4a}{\pi}\cos\frac{\pi}{2a}a - 2(a) + \frac{a^2}{2a}\right]$$

$$= h\left(\frac{4a}{\pi} - 4a + 2a\right) - h\left(-2a + \frac{a}{2}\right) = ah\left(\frac{4}{\pi} - \frac{1}{2}\right)$$

$$A = 0.77324ah$$

PROBLEM 9.19

Moment of inertia $\quad I_x = \int_a^{2a} dI_x$

where $\quad dI_x = \frac{1}{3}\left(y_2^3 - y_1^3\right)dx$

Now $\quad dI_x = \frac{1}{3}\left[\left(2h\sin\frac{\pi}{2a}x\right)^3 - \left(2h - \frac{h}{a}x\right)^3\right]dx$

$$= \frac{1}{3}\left[8h^3\sin^3\frac{\pi}{2a}x - h^3\left(2 - \frac{x}{a}\right)^3\right]dx$$

Then $\quad I_x = \frac{8h^3}{3}\int_{-a}^{2a}\sin^3\frac{\pi}{2a}xdx - \frac{h^3}{3}\int_a^{2a}\left(2 - \frac{x}{a}\right)^3 dx$

Now $\quad \int\sin^3\frac{\pi}{2a}xdx = \int\sin\frac{\pi}{2a}x\left(1 - \cos^2\frac{\pi}{2a}x\right)dx$

$$= \int\left(\sin\frac{\pi}{2a}x\right)dx - \int\left(\sin\frac{\pi}{2a}x\cos^2\frac{\pi}{2a}x\right)dx$$

$$= -\frac{2a}{\pi}\cos\frac{\pi}{2a}x + \frac{2a}{3\pi}\cos^3\frac{\pi}{2a}x$$

Then $\quad \int_a^{2a}\left(\sin^3\frac{\pi}{2}x\right)dx = -\frac{2a}{\pi}\left[\cos\frac{\pi}{2a}x - \frac{1}{3}\cos^3\frac{\pi}{2a}x\right]_a^{2a}$

$$= -\frac{2a}{\pi}\left(-1 + \frac{1}{3}\right) = \frac{4a}{3\pi}$$

PROBLEMS 9.19 AND 9.20 CONTINUED

And $$\int_a^{2a}\left(2-\frac{x}{a}\right)^3 dx = -\frac{a}{4}\left(2-\frac{x}{a}\right)^4\Bigg|_a^{2a}$$

$$= -\frac{a}{4}\left(2-\frac{2a}{a}\right)^4 + \frac{a}{4}\left(2-\frac{a}{a}\right)^4 = \frac{a}{4}$$

Then $$I_x = \frac{8h^3}{3}\left(\frac{4a}{3\pi}\right) - \frac{h^3}{3}\left(\frac{a}{4}\right) = \frac{h^3 a}{3}\left(\frac{32}{3\pi}-\frac{1}{4}\right)$$

$$I_x = 1.0484ah^3 \blacktriangleleft$$

and $$k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{1.0484ah^3}{0.77324ah}} = 1.1644h$$

$$k_x = 1.164h \blacktriangleleft$$

<u>PROBLEM 9.20</u>

$$I_y = \int dI_y$$

$$dI_y = x^2 dA \qquad dA = (y_2 - y_1)dx$$

From Problem 9.19 $$y_1 = 2h - \frac{h}{a}x \qquad y_2 = 2h\sin\frac{\pi}{2a}x$$

Now $$dI_y = \left[2hx^2\sin\frac{\pi}{2a}x - h\left(2x^2 - \frac{x^3}{a}\right)\right]dx$$

Then $$I_y = \int_a^{2a} dI_y = h\int_a^{2a}\left(2x^2\sin\frac{\pi x}{2a} - 2x^2 + \frac{x^3}{a}\right)dx$$

Now using integration by parts

$$u = x^2 \qquad dv = \sin\frac{\pi}{2a}x\,dx$$

$$du = 2x\,dx \qquad v = -\frac{2a}{\pi}\cos\frac{\pi}{2a}x$$

Then $$\int x^2\sin\frac{\pi}{2a}x\,dx = x^2\left(-\frac{2a}{\pi}\cos\frac{\pi}{2a}x\right) - \int\left(-\frac{2a}{\pi}\cos\frac{\pi}{2a}x\right)2x\,dx$$

PROBLEM 9.20 CONTINUED

Now let

$$u = x \qquad dv = \cos\frac{\pi}{2a}x\,dx$$

$$du = dx \qquad v = \frac{2a}{\pi}\sin\frac{\pi}{2a}x$$

Then

$$\int x^2\sin\frac{\pi}{2a}x\,dx = -\frac{2a}{\pi}x^2\cos\frac{\pi}{2a}x + \frac{4a}{\pi}\left[x\left(\frac{2a}{\pi}\sin\frac{\pi}{2a}x\right) - \int\left(\frac{2a}{\pi}\sin\frac{\pi}{2a}x\right)dx\right]$$

Finally,

$$I_y = 2h\left[\left(-\frac{2a}{\pi}x^2\cos\frac{\pi}{2a}x + \frac{8a^2}{\pi^2}x\sin\frac{\pi}{2a}x + \frac{4a^2}{\pi^2}\frac{2a}{\pi}\cos\frac{\pi}{2a}x\right) - \frac{1}{3}x^3 + \frac{1}{8a}x^4\right]_a^{2a}$$

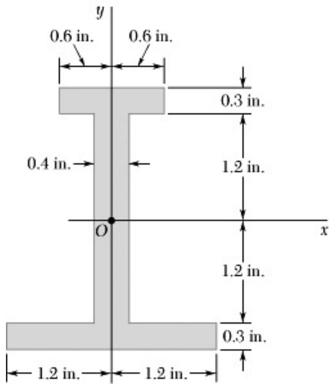
$$= 2h\left[\frac{2a}{\pi}(2a)^2 - \frac{16a^3}{\pi^3} - \frac{(2a)^3}{3} + \frac{1}{8a}(2a)^4 - \frac{8a^2}{\pi^2}a + \frac{a^3}{3} - \frac{a^4}{8a}\right] = 1.5231a^3h$$

$$I_y = 1.523a^3h \blacktriangleleft$$

and

$$k_y^2 = \frac{I_y}{A} = \frac{1.5231a^3h}{0.77324} = 1.4035a^2$$

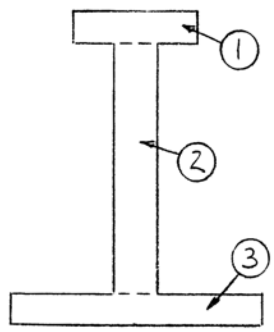
$$k_y = 1.404a \blacktriangleleft$$



PROBLEM 9.31

Determine the moment of inertia and the radius of gyration of the shaded area with respect to the *x* axis.

SOLUTION



First note that

$$A = A_1 + A_2 + A_3$$
$$= (1.2\text{ in.})(0.3\text{ in.}) + (2.4\text{ in.})(0.4\text{ in.}) + (2.4\text{ in.})(0.3\text{ in.})$$
$$= (0.36 + 0.96 + 0.72)\text{in}^2$$
$$= 2.04\text{ in}^2$$

Now

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3$$

where

$$(I_x)_1 = \frac{1}{12}(1.2\text{ in.})(0.3\text{ in.})^3 + (0.36\text{ in}^2)(1.36\text{ in.})^2 = 0.6588\text{ in}^4$$

$$(I_x)_2 = \frac{1}{12}(0.4\text{ in.})(2.4\text{ in.})^3 = 0.4608\text{ in}^4$$

$$(I_x)_3 = \frac{1}{12}(2.4\text{ in.})(0.3\text{ in.})^3 + (0.72\text{ in}^2)(1.35\text{ in.})^2 = 1.3176\text{ in}^4$$

Then

$$I_x = 0.6588\text{ in}^4 + 0.4608\text{ in}^4 + 1.3176\text{ in}^4 = 2.4372\text{ in}^4$$

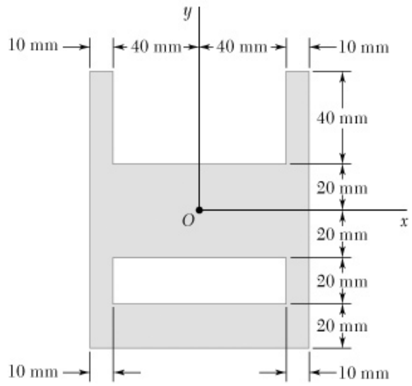
or $I_x = 2.44\text{ in}^4$ ◀

and

$$k_x^2 = \frac{I_x}{A} = \frac{2.4372\text{ in}^4}{2.04\text{ in}^2} = 1.1947\text{ in}^2$$

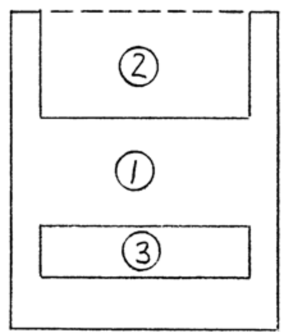
or $k_x = 1.093\text{ in.}$ ◀

PROBLEM 9.32



Determine the moment of inertia and the radius of gyration of the shaded area with respect to the x axis.

SOLUTION



First note that $A = A_1 - A_2 - A_3$

$$= (100 \text{ mm})(120 \text{ mm}) - (80 \text{ mm})(40 \text{ mm}) - (80 \text{ mm})(20 \text{ mm}) = 7200 \text{ mm}^2$$

$$= (12\ 000 - 3200 - 1600)\text{mm}^2 = 7200 \text{ mm}$$

Now $$I_x = (I_x)_1 - (I_x)_2 - (I_x)_3$$

where $$(I_x)_1 = \frac{1}{12}(100 \text{ mm})(120 \text{ mm})^3 = 14.4 \times 10^6 \text{ mm}^4$$

$$(I_x)_2 = \frac{1}{12}(80 \text{ mm})(40 \text{ mm})^3 + (3200 \text{ mm}^2)(40 \text{ mm})^2 = 5.5467 \times 10^6 \text{ mm}^4$$

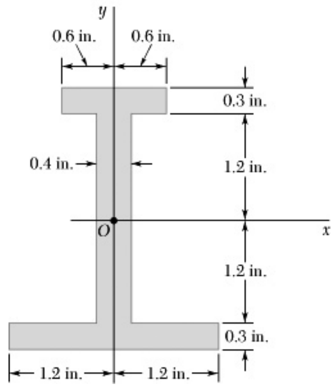
$$(I_x)_3 = \frac{1}{12}(80 \text{ mm})(20 \text{ mm})^2 + (1600 \text{ mm}^2)(30 \text{ mm})^2 = 1.4933 \times 10^6 \text{ mm}^4$$

Then $$I_x = (14.4 - 5.5467 - 1.4933) \times 10^6 \text{ mm}^4 = 7.36 \times 10^6 \text{ mm}^4$$

or $I_x = 7.36 \times 10^6 \text{ mm}^4$ ◀

and $$k_x^2 = \frac{I_x}{A} = \frac{7.36 \times 10^6}{7200} = 1022.2 \text{ mm}^2$$

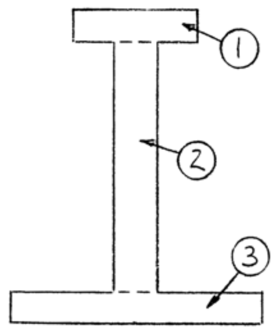
or $k_x = 32.0 \text{ mm}$ ◀



PROBLEM 9.33

Determine the moment of inertia and the radius of gyration of the shaded area with respect to the *y* axis.

SOLUTION



First note that

$$A = A_1 + A_2 + A_3$$

$$= (1.2\text{ in.})(0.3\text{ in.}) + (2.4\text{ in.})(0.4\text{ in.}) + (2.4\text{ in.})(0.3\text{ in.})$$

$$= (0.36 + 0.96 + 0.72)\text{ in}^2 = 2.04\text{ in}^2$$

Now

$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3$$

Where:

$$(I_y)_1 = \frac{1}{12}(0.3\text{ in.})(1.2\text{ in.})^3 = 0.0432\text{ in}^4$$

$$(I_y)_2 = \frac{1}{12}(2.4\text{ in.})(0.4\text{ in.})^3 = 0.0128\text{ in}^4$$

$$(I_y)_3 = \frac{1}{12}(0.3\text{ in.})(2.4\text{ in.})^3 = 0.3456\text{ in}^4$$

Then

$$I_y = (0.0432 + 0.0128 + 0.3456)\text{ in}^4 = 0.4016\text{ in}^4$$

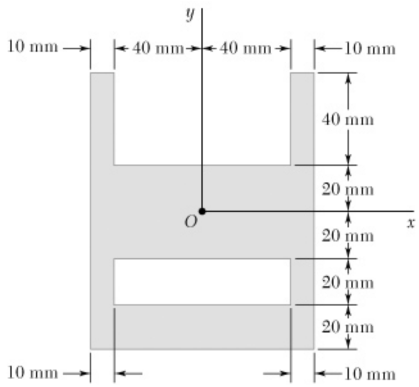
or $I_y = 0.402\text{ in}^4$ ◀

And

$$k_y^2 = \frac{I_y}{A} = \frac{0.4016}{2.04\text{ in}^2} = 0.19686\text{ in}^2$$

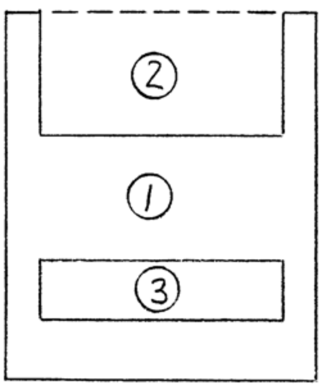
or $k_y = 0.444\text{ in.}$ ◀

PROBLEM 9.34



Determine the moment of inertia and the radius of gyration of the shaded area with respect to the y axis.

SOLUTION



First note that $A = A_1 - A_2 - A_3$

$$= (100\text{ mm})(120\text{ mm}) - (80\text{ mm})(40\text{ mm})$$

$$-(80\text{ mm})(20\text{ mm}) = 7200\text{ mm}^2$$

$$= (12\ 000 - 3200 - 1600)\text{mm}^2 = 7200\text{ mm}^2$$

Now $I_y = (I_y)_1 - (I_y)_2 - (I_y)_3$

where $(I_y)_1 = \frac{1}{12}(120\text{ mm})(100\text{ mm})^3 = 10 \times 10^6\text{ mm}^4$

$$(I_y)_2 = \frac{1}{12}(40\text{ mm})(80\text{ mm})^3 = 1.7067 \times 10^6\text{ mm}^4$$

$$(I_y)_3 = \frac{1}{12}(20\text{ mm})(80\text{ mm})^3 = 0.8533 \times 10^6\text{ mm}^4$$

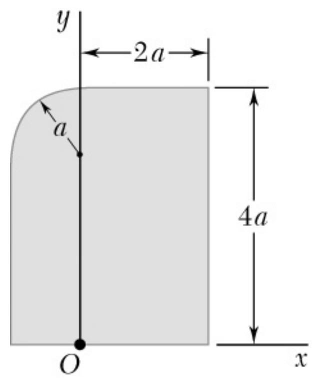
Then $I_y = (10 - 1.7067 - 0.8533) \times 10^6\text{ mm}^4 = 7.44 \times 10^6\text{ mm}^4$

or $I_y = 7.44 \times 10^6\text{ mm}^4$ ◀

And $k_y^2 = \frac{I_y}{A} = \frac{7.44 \times 10^6\text{mm}^4}{7200\text{ mm}^2} = 1033.33\text{ mm}^2$

$$k = 32.14550\text{ mm}$$

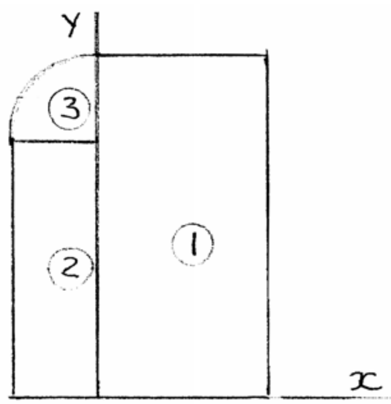
or $k_y = 32.1\text{ mm}$ ◀



PROBLEM 9.35

Determine the moments of inertia of the shaded area shown with respect to the x and y axes.

SOLUTION



Have $$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3$$

$$= \left[\frac{1}{3}(2a)(4a)^3\right] + \left[\frac{1}{3}(a)(3a)^3\right]$$

$$+ \left\{\left[\frac{\pi}{16}a^4 - \frac{\pi}{4}a^2\left(\frac{4a}{3\pi}\right)^2\right] + \frac{\pi}{4}a^2\left(3a + \frac{4a}{3\pi}\right)^2\right\}$$

$$= \left(\frac{128}{3}a^4\right) + \left(\frac{27}{3}a^4\right) + \left(\frac{\pi}{16} - \frac{4}{9\pi} + \frac{9\pi}{4} + 2 + \frac{4}{9\pi}\right)a^4$$

$$= \left(\frac{161}{3} + \frac{37}{\pi}\right)a^4 = 60.9316a^4$$

or $I_x = 60.9a^4$ ◀

Also $$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3$$

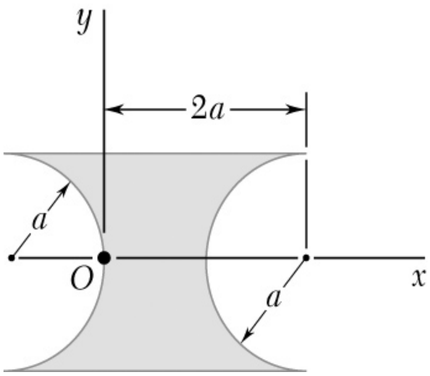
$$= \left[\frac{1}{3}(4a)(2a)^3\right] + \left[\frac{1}{3}(3a)(a)^3\right] + \left[\frac{\pi}{16}a^4\right]$$

$$= \left(\frac{32}{3} + 1 + \frac{\pi}{16}\right)a^4 = 11.8630a^4$$

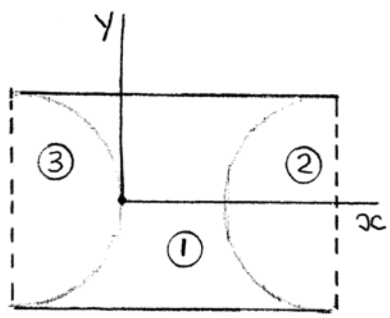
or $I_y = 11.86a^4$ ◀

PROBLEM 9.36

Determine the moments of inertia of the shaded area shown with respect to the x and y axes.



SOLUTION

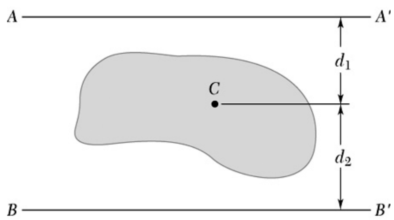


Have
$$I_x = (I_x)_1 - (I_x)_2 - (I_x)_3$$
$$= \left[\frac{1}{12}(3a)(2a)^3\right] - \left[\frac{\pi}{8}a^4\right] - \left[\frac{\pi}{8}a^4\right]$$
$$= \left(2 - \frac{\pi}{8} - \frac{\pi}{8}\right)a^4 = \left(2 - \frac{\pi}{4}\right)a^4$$

or $I_x = 1.215a^4$ ◀

Also
$$I_y = (I_y)_1 - (I_y)_2 - (I_y)_3$$
$$= \left[\frac{1}{12}(2a)(3a)^3 + (3a)(2a)\left(\frac{a}{2}\right)^2\right]$$
$$-\left\{\left[\frac{\pi}{8}a^4 - \frac{\pi}{2}a^2\left(\frac{4a}{3\pi}\right)^2\right] + \frac{\pi}{2}a^2\left(2a - \frac{4a}{3\pi}\right)^2\right\}$$
$$-\left\{\left[\frac{\pi}{8}a^4 - \frac{\pi}{2}a^2\left(\frac{4a}{3\pi}\right)^2\right] + \frac{\pi}{2}a^2\left(a - \frac{4a}{3\pi}\right)^2\right\}$$
$$= \left(\frac{9}{2} + \frac{3}{2}\right)a^4 - \left(\frac{\pi}{8} - \frac{8}{9\pi} + 2\pi - \frac{8}{3} + \frac{8}{9\pi}\right)a^4$$
$$-\left(\frac{\pi}{8} - \frac{8}{9\pi} + \frac{\pi}{2} - \frac{4}{3} + \frac{8}{9\pi}\right)a^4 = \left(10 - \frac{11\pi}{4}\right)a^4$$
$$= 1.3606a^4$$

or $I_y = 1.361a^4$ ◀



PROBLEM 9.37

For the 6-in^2 shaded area shown, determine the distance d_2 and the moment of inertia with respect to the centroidal axis parallel to AA' knowing that the moments of inertia with respect to AA' and BB' are 30 in^4 and 58 in^4, respectively, and that d_1 = 1.25 in.

SOLUTION

Have
$$I_{AA'} = \bar{I} + Ad_1^2$$

and
$$I_{BB'} = \bar{I} + Ad_2^2$$

subtracting
$$I_{AA'} - I_{BB'} = A\left(d_1^2 - d_2^2\right)$$

or
$$(30 - 58)\,\text{in}^4 = \left(6\ \text{in}^2\right)\left[(1.25\ \text{in.})^2 - d_2^2\right]$$

Solve for d_2

$$d_2^2 = \left(1.25^2 + 4.6667\right)\text{in}^2 = 6.2292\ \text{in}^2$$

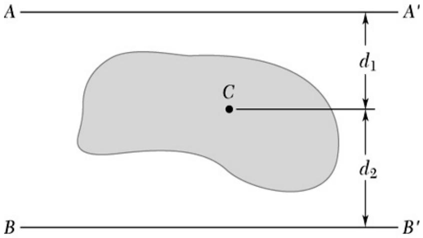
Then
$$d_2 = 2.4958\ \text{in.}$$

or $d_2 = 2.50$ in. ◀

and
$$\bar{I} = I_{AA'} - Ad_1^2 = 30\ \text{in}^4 - \left(6\ \text{in}^2\right)(1.25\ \text{in.})^2 = 20.625\ \text{in}^4$$

or $\bar{I} = 20.6$ in^4 ◀

PROBLEM 9.38



Determine for the shaded region the area and the moment of inertia with respect to the centroidal axis parallel to BB' knowing that $d_1 = 1.25$ in. and $d_2 = 0.75$ in. and that the moments of inertia with respect to AA' and BB' are 20 in^4 and 15 in^4, respectively.

SOLUTION

Have $$I_{AA'} = \bar{I} + Ad_1^2$$

and $$I_{BB'} = \bar{I} + Ad_2^2$$

subtracting $$I_{AA'} - I_{BB'} = A\left(d_1^2 - d_2^2\right)$$

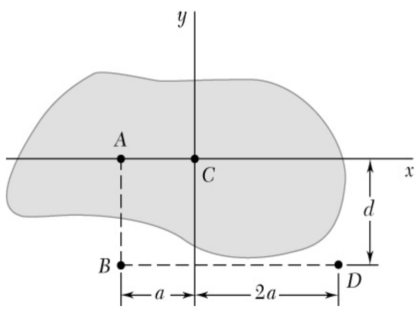
$$20\text{ in}^4 - 15\text{ in}^4 = A\left[(1.25)^2 - (0.75)^2\right]\text{in}^2$$

$$5\text{ in}^4 = A[1.5625 - 0.5625]\text{ in}^2$$

or $A = 5\text{ in}^2$ ◀

and $$\bar{I} = I_{AA'} - Ad^2 = 20\text{ in}^4 - \left(5\text{ in}^2\right)(1.25\text{ in.})^2 = 12.1875\text{ in}^4$$

or $\bar{I} = 12.19\text{ in}^4$ ◀



PROBLEM 9.39

The centroidal polar moment of inertia $\bar{J}_C$ of the 15.5×10^3 mm^2 shaded region is 250×10^6 mm^4. Determine the polar moments of inertia J_B and J_D of the shaded region knowing that $J_D = 2J_B$ and $d = 100$ mm.

SOLUTION

Have $$J_B = \bar{J}_C + Ad_{CB}^2$$

and $$J_D = \bar{J}_C + Ad_{CD}^2$$

Now $$J_D = 2J_B$$

Then $$\bar{J}_C + Ad_{CD}^2 = 2\left(\bar{J}_C + Ad_{CB}^2\right)$$

Now $$d_{CB}^2 = a^2 + d^2 \qquad \text{and} \qquad d_{CD}^2 = (2a)^2 + d^2$$

Substituting $$A\left(4a^2 + d^2\right) = \bar{J}_C + 2A\left(a^2 + d^2\right)$$

or $$a^2 = \frac{1}{2}\left(\frac{\bar{J}_C}{A} + d^2\right)$$

$$= \frac{1}{2}\left[\frac{250 \times 10^6 \text{ mm}^4}{15.5 \times 10^3 \text{ mm}^2} + (100 \text{ mm})^2\right] = 13064.5 \text{ mm}^2$$

or $$a = 114.300 \text{ mm}$$

Then $$J_B = 250 \times 10^6 \text{ mm}^4 + \left(15.5 \times 10^3 \text{ mm}^2\right)\left[(114.300 \text{ mm})^2 + (100 \text{ mm})^2\right]$$

$$= \left(250 \times 10^6 + 357.5 \times 10^6\right)\text{mm}^4 = 607.5 \times 10^6 \text{ mm}^4$$

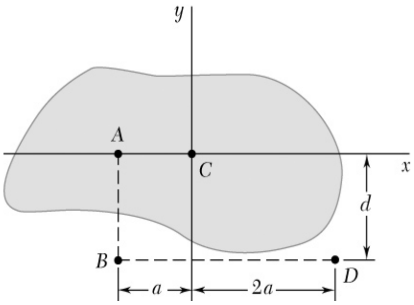
or $J_B = 608 \times 10^6$ mm^4 ◀

And $$J_D = 250 \times 10^6 \text{ mm}^4 + \left(15.5 \times 10^3 \text{ mm}^2\right)\left[(228.60 \text{ mm})^2 + (100 \text{ mm})^2\right]$$

$$= \left(250 \times 10^6 + 964.99 \times 10\right)\text{mm}^4 = 1214.99 \times 10^6 \text{ mm}^4$$

or $J_D = 1215 \times 10^6$ mm^4 ◀

PROBLEM 9.40



Determine the centroidal polar moment of inertia $\bar{J}_C$ of the 10×10^3 mm^2 shaded area knowing that the polar moments of inertia of the area with respect to points *A*, *B*, and *D* are $J_A = 45 \times 10^6$ mm^4, $J_B = 130 \times 10^6$ mm^4, and $J_D = 252 \times 10^6$ mm^4, respectively.

SOLUTION

Have $\quad J_A = \bar{J}_C + Ad_{CA}^2 \quad$ where $\quad d_{CA}^2 = a^2$

Then
$$J_A = \bar{J}_C + Aa^2 \qquad (1)$$

Have $\quad J_B = \bar{J}_C + Ad_{CB}^2 \quad$ where $\quad d_{CB}^2 = a^2 + d^2$

Then
$$J_B = \bar{J}_C + A\left(a^2 + d^2\right) \qquad (2)$$

Have $\quad J_D = \bar{J}_C + Ad_{CD}^2 \quad$ where $\quad d_{CD}^2 = 4a^2 + d^2$

Then
$$J_D = \bar{J}_C + A\left(4a^2 + d^2\right) \qquad (3)$$

Then Equation (3) – Equation(2):
$$J_D - J_B = 3Aa^2 \qquad (4)$$

and Equation(4) – 3[Equation(1)]:
$$\left(J_D - J_B\right) - 3J_A = -3\bar{J}_C$$

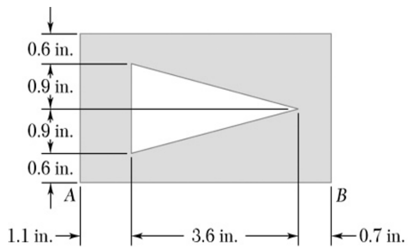
or
$$\bar{J}_C = J_A - \frac{1}{3}\left(J_D - J_B\right)$$

$$= 45 \times 10^6 \text{ mm}^4 - \frac{1}{3}\left(252 \times 10^6 - 130 \times 10^6\right) \text{mm}^4 = 4.3333 \times 10^6 \text{ mm}^4$$

or $\bar{J}_C = 4.33 \times 10^6$ mm^4 ◀

Note
$$a = 63.77 \text{ mm}$$

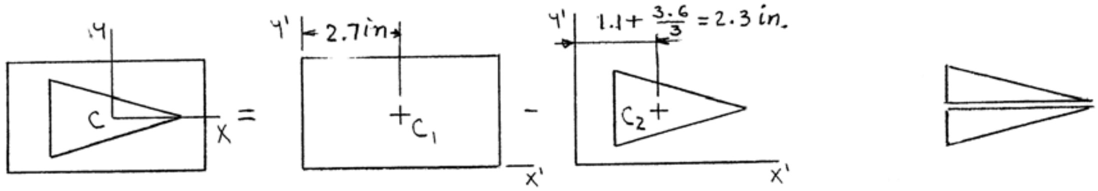
and
$$d = 92.195 \text{ mm}$$



PROBLEM 9.41

Determine the moments of inertia $\bar{I}_x$ and $\bar{I}_y$ of the area shown with respect to centroidal axes respectively parallel and perpendicular to side *AB*.

SOLUTION



First calculate the centroid *C* of the area

From symmetry $\bar{Y} = 0.6\text{ in.} + 0.9\text{ in.} = 1.5\text{ in.}$

To compute $\bar{X}$ use the equation $\bar{X}A = \Sigma Ax$

or

$$\bar{X} = \frac{\left[(3\times 5.4)\text{ in}^2\right]\times(2.7\text{ in.}) - \left[\frac{1}{2}(1.8\times 3.6)\text{ in}^2\right]\times(2.3\text{ in.})}{(3\times 5.4)\text{ in}^2 - \frac{1}{2}(1.8\times 3.6)\text{ in}^2}$$

$$= 2.8\text{ in.}$$

The moment of inertia of the composite area is obtained by subtracting the moment of inertia of the triangle from the moment of inertia of the rectangle

$$\bar{I}_x = (I_x)_1 - (I_x)_2$$

where

$$(I_x)_1 = \frac{1}{12}(5.4\text{ in.})(3\text{ in.})^3 = 12.15\text{ in}^4$$

and

$$(I_x)_2 = 2\left[\frac{1}{12}(3.6\text{ in.})(0.9\text{ in.})^3\right] = 0.4374\text{ in}^4$$

Then

$$\bar{I}_x = (12.15 - 0.4374)\text{ in}^4 = 11.7126\text{ in}^4$$

or $\bar{I}_x = 11.71\text{ in}^4$ ◀

Similarly,

$$\bar{I}_y = (I_y)_1 - (I_y)_2$$

where

$$(I_y)_1 = \frac{1}{12}(3\text{ in.})(5.4\text{ in.})^3 + \left[(3\times 5.4)\text{ in}^2\right](2.8\text{ in.} - 2.7\text{ in.})^2$$

$$= 39.582\text{ in}^4$$

PROBLEM 9.41 CONTINUED

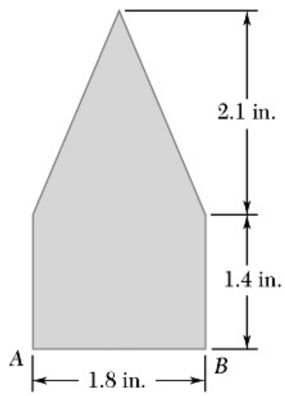
and
$$\left(I_y\right)_2 = \frac{1}{36}(1.8\text{ in.})(3.6\text{ in.})^3 + \left[\frac{1}{2}(1.8)(3.6)\text{ in}^2\right](2.8\text{ in.} - 2.3\text{ in.})^2$$

$$= 3.1428\text{ in}^4$$

Then
$$\bar{I}_y = (39.582 - 3.1428)\text{ in}^4 = 36.4392\text{ in}^4$$

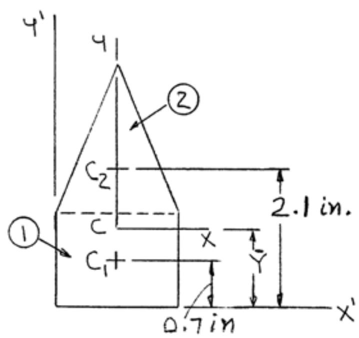
$$\text{or }\ \bar{I}_y = 36.4\text{ in}^4 \blacktriangleleft$$

PROBLEM 9.42



Determine the moments of inertia $\bar{I}_x$ and $\bar{I}_y$ of the area shown with respect to centroidal axes respectively parallel and perpendicular to side *AB*.

SOLUTION



By symmetry $\bar{X} = 0.9 \text{ in.}$

Have $A\bar{Y} = \Sigma \bar{y} A$

Where $A = (1.8 \text{ in.})(1.4 \text{ in.}) + \frac{1}{2}(1.8 \text{ in.})(2.1 \text{ in.})$

$$= (2.52 + 1.89)\text{in}^2 = 4.41 \text{ in}^2$$

Then $(4.41 \text{ in}^2)\bar{Y} = (0.7 \text{ in.})(2.52 \text{ in}^2) + (2.1 \text{ in.})(1.89 \text{ in}^2)$

$$= 5.733 \text{ in}^3$$

or $\bar{Y} = 1.3 \text{ in.}$

Now $\bar{I}_x = (I_x)_1 + (I_x)_2$

where $(I_x)_1 = \frac{1}{12}(1.8 \text{ in.})(1.4 \text{ in.})^3$

$$+ (2.52 \text{ in}^2)(1.3 \text{ in.} - 0.7 \text{ in.})^2 = 1.3188 \text{ in}^4$$

And $(I_x)_2 = \frac{1}{36}(1.8 \text{ in.})(2.1 \text{ in.})^3$

$$+ (1.89 \text{ in}^2)(2.1 \text{ in.} - 1.3 \text{ in.})^2 = 1.67265 \text{ in}^4$$

Then $\bar{I}_x = (1.3188 + 1.67265)\text{ in}^4 = 2.99145 \text{ in}^4$

or $\bar{I}_x = 2.99 \text{ in}^4$ ◀

Also $\bar{I}_y = (I_y)_1 + (I_y)_2$

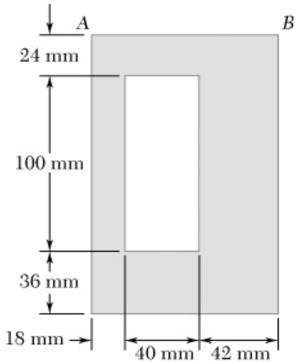
where $(I_y)_1 = \frac{1}{12}(1.4 \text{ in.})(1.8 \text{ in.})^3 = 0.6804 \text{ in}^4$

PROBLEM 9.42 CONTINUED

and
$$\left(I_y\right)_2 = 2\left[\frac{1}{36}(2.1\text{ in.})(0.9\text{ in.})^3 + \left(\frac{1}{2}\times 1.89\text{ in}^2\right)(0.3\text{ in.})^2\right] = 0.25515\text{ in}^4$$

Then
$$\bar{I}_y = (0.6804 + 0.25515)\text{ in}^4$$

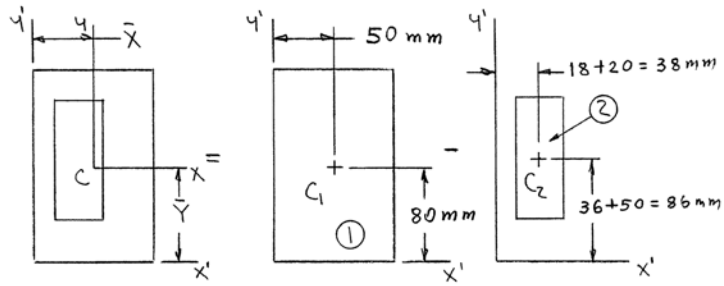
or $\bar{I}_y = 0.936\text{ in}^4$ ◀



PROBLEM 9.43

Determine the moments of inertia $\bar{I}_x$ and $\bar{I}_y$ of the area shown with respect to centroidal axes respectively parallel and perpendicular to side *AB*.

SOLUTION



$$A = A_1 - A_2$$

$$= (100\text{ mm})(160\text{ mm}) - (40\text{ mm})(100\text{ mm})$$

$$= (16\ 000 - 4000)\text{mm}^2$$

$$= 12\ 000\text{ mm}^2$$

First locate the centroid:

Have $$A\bar{X} = \Sigma A\bar{x}$$

or $$\left(12\ 000\text{ mm}^2\right)\bar{X} = \left[(16\ 000)(50) - (4000)(38)\right]\text{mm}^3 = 648\ 000\text{ mm}^3$$

or $$\bar{X} = \frac{648\ 000\text{ mm}^3}{12\ 000\text{ mm}^2} = 54\text{ mm}$$

And $$A\bar{Y} = \Sigma A\bar{y}$$

or $$\left(12\ 000\text{ mm}^2\right)\bar{Y} = \left[(16\ 000)(86) - (4000)(86)\right]\text{mm}^3 = 936\ 000\text{ mm}^3$$

or $$\bar{Y} = \frac{936\ 000\text{ mm}^3}{12\ 000\text{ mm}^2} = 78\text{ mm}$$

PROBLEM 9.43 CONTINUED

Now $$\bar{I}_x = (I_x)_1 - (I_x)_2$$

where $$(I_x)_1 = \frac{1}{12}(40 \text{ mm})(100 \text{ mm})^3 + (16\ 000 \text{ mm}^2)(80 \text{ mm} - 78 \text{ mm})^2$$

$$= 34.197 \times 10^6 \text{ mm}^4$$

$$(I_x)_2 = \frac{1}{12}(40 \text{ mm})(100 \text{ mm})^3 + (4000 \text{ mm}^2)(80 \text{ mm} - 78 \text{ mm})^2$$

$$= 3.5893 \times 10^6 \text{ mm}^4$$

Then $$\bar{I}_x = (34.197 - 3.5893) \times 10^6 \text{ mm}^4$$

or $\bar{I}_x = 30.6 \times 10^6 \text{ mm}^4$ ◀

Also $$\bar{I}_y = (I_y)_1 - (I_y)_2$$

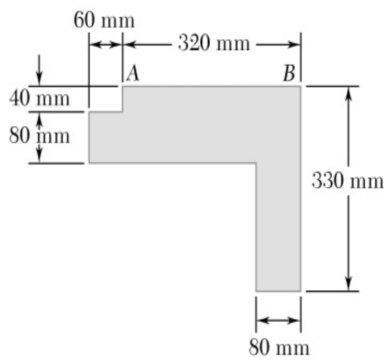
where $$(I_y)_1 = \frac{1}{12}(160 \text{ mm})(100 \text{ mm})^3 + (16\ 000 \text{ mm}^2)(54 \text{ mm} - 50 \text{ mm})^2 = 13.589 \times 10^6 \text{ mm}^4$$

$$(I_y)_2 = \frac{1}{12}(100 \text{ mm})(40 \text{ mm})^3 + (4000 \text{ mm}^2)(54 \text{ mm} - 38 \text{ mm})^2 = 1.5573 \times 10^6 \text{ mm}^4$$

Then $$\bar{I}_y = (13.589 - 1.5573) \times 10^6 \text{ mm}^4$$

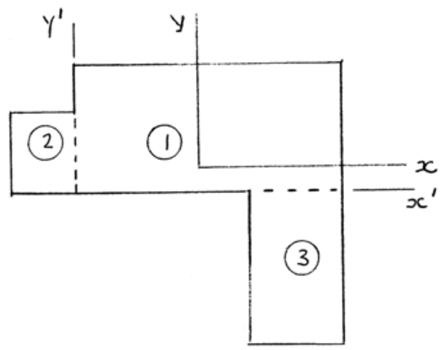
or $\bar{I}_y = 12.03 \times 10^6 \text{ mm}^4$ ◀

PROBLEM 9.44



Determine the moments of inertia $\bar{I}_x$ and $\bar{I}_y$ of the area shown with respect to centroidal axes respectively parallel and perpendicular to side *AB*.

SOLUTION



First locate centroid

$$\bar{x}_1 = 160 \text{ mm} \qquad \bar{y}_1 = 60 \text{ mm}$$

$$A_1 = 320 \text{ mm} \times 120 \text{ mm} = 38\ 400 \text{ mm}^2$$

$$\bar{x}_2 = -30 \text{ mm} \qquad \bar{y}_2 = 40 \text{ mm}$$

$$A_2 = 60 \text{ mm} \times 80 \text{ mm} = 4800 \text{ mm}^2$$

$$\bar{x}_3 = 280 \text{ mm} \qquad \bar{y}_3 = -105 \text{ mm}$$

$$A_3 = 80 \text{ mm} \times 210 \text{ mm} = 16\ 800 \text{ mm}^2$$

Then

$$\bar{X} = \frac{\Sigma \bar{x} A}{\Sigma A}$$

$$= \frac{\left[160(38\ 400) - 30(4800) + 280(16\ 800)\right] \text{mm}^3}{(38\ 400 + 4800 + 16\ 800) \text{ mm}^2}$$

$$= 178.4 \text{ mm}$$

And

$$\bar{Y} = \frac{\Sigma \bar{y} A}{\Sigma A}$$

$$= \frac{\left[60(38\ 400) + 40(4800) - 105(16\ 800)\right] \text{mm}^3}{(38\ 400 + 4800 + 16\ 800) \text{ mm}^2}$$

$$= 12.20 \text{ mm}$$

PROBLEM 9.44 CONTINUED

Then $\bar{I}_x = (I_x)_1 + (I_x)_2 + (I_x)_3$

$$= \left[\frac{1}{12}(320 \text{ mm})(120 \text{ mm})^3 + (38\,400 \text{ mm}^2)(60 \text{ mm} - 12.2 \text{ mm})^2\right]$$

$$+ \left[\frac{1}{12}(60 \text{ mm})(80 \text{ mm})^3 + (4800 \text{ mm}^2)(40 \text{ mm} - 12.2 \text{ mm})^2\right]$$

$$+ \left[\frac{1}{12}(80 \text{ mm})(210 \text{ mm})^3 + (16\,800 \text{ mm}^2)(105 \text{ mm} + 12.2 \text{ mm})^2\right]$$

$$= \left[(46.080 + 87.7379) + (2.5600 + 3.7096) + (61.7400 + 230.7621)\right] \times 10^6 \text{ mm}^4$$

$$= 432.5896 \times 10^6 \text{ mm}^4$$

or $\bar{I}_x = 433 \times 10^6 \text{ mm}^4$ ◀

And $\bar{I}_y = (I_y)_1 + (I_y)_2 + (I_y)_3$

$$= \left[\frac{1}{12}(120 \text{ mm})(320 \text{ mm})^3 + (38\,400 \text{ mm}^2)(178.4 \text{ mm} - 160 \text{ mm})^2\right]$$

$$+ \left[\frac{1}{12}(80 \text{ mm})(60 \text{ mm})^3 + (4800 \text{ mm}^2)(30 \text{ mm} + 178.4 \text{ mm})^2\right]$$

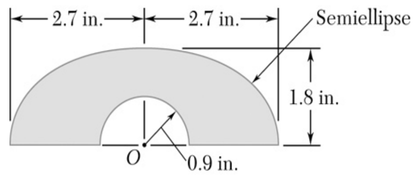
$$+ \left[\frac{1}{12}(210 \text{ mm})(80 \text{ mm})^3 + (16\,800 \text{ mm}^2)(280 \text{ mm} - 178.4 \text{ mm})^2\right]$$

$$= \left[(327.6800 + 13.0007) + (1.4400 + 208.4667) + (8.9600 + 173.4190)\right] \times 10^6 \text{ mm}^4$$

$$= 732.9664 \times 10^6 \text{ mm}^4$$

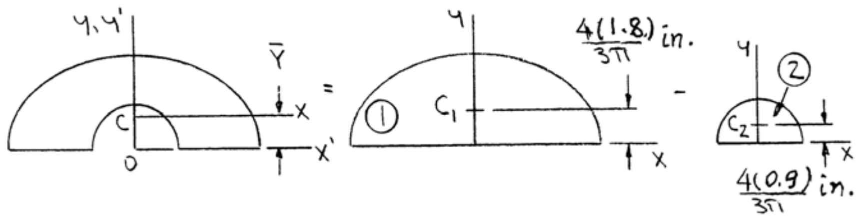
or $\bar{I}_y = 733 \times 10^6 \text{ mm}^4$ ◀

PROBLEM 9.45



Determine the polar moment of inertia of the area shown with respect to (*a*) point *O*, (*b*) the centroid of the area.

SOLUTION



First locate centroid *C* of the area

	A, in^2	$\bar{y}$, in.	$\bar{y}A$, in^3
1	$\frac{\pi}{2}(2.7)(1.8) = 7.6341$	0.76394	5.8319
2	$-\frac{\pi}{2}(0.9)^2 = -1.2723$	0.38197	0.4860
Σ	6.3618		5.3460

Then $\quad \bar{Y}\Sigma A = \Sigma \bar{y}A: \qquad \bar{Y} = \dfrac{5.3460 \text{ in}^2}{6.3618 \text{ in}^2} = 0.84033 \text{ in.}$

(*a*) $$J_O = (J_O)_1 - (J_O)_2 = \frac{\pi}{8}(2.7 \text{ in.})(1.8 \text{ in.})\left[(2.7)^2 + (1.8)^2\right]\text{in}^2 - \frac{\pi}{4}(0.9 \text{ in.})^4$$

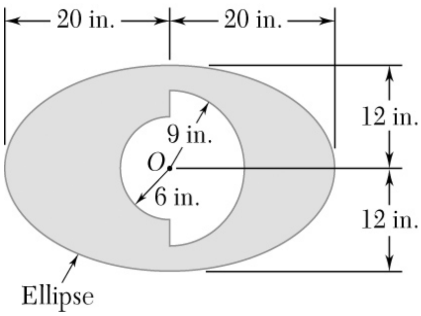
$$= 19.5814 \text{ in}^4$$

or $J_O = 19.58 \text{ in}^4$ ◀

(*b*) $$J_O = \bar{J}_C + A(\bar{y})^2$$

or $$\bar{J}_C = 19.5814 \text{ in}^4 - (6.3618 \text{ in}^2)(0.84033 \text{ in.})^2 = 15.0890 \text{ in}^4$$

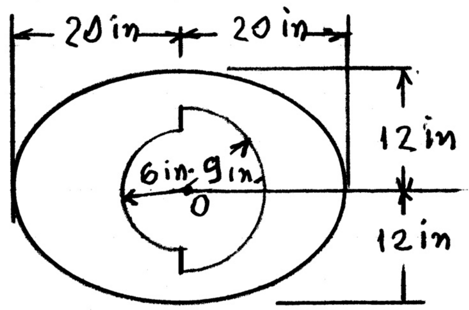
$\bar{J}_C = 15.09 \text{ in}^4$ ◀



PROBLEM 9.46

Determine the polar moment of inertia of the area shown with respect to (*a*) point *O*, (*b*) the centroid of the area.

SOLUTION



First locate centroid

Symmetry implies $$\bar{Y} = 0$$

$$\bar{x}_1 = 0 \quad A_1 = \pi(20\text{ in.})(12\text{ in.})$$

$$= (240\pi)\text{ in}^2$$

$$\bar{x}_2 = \frac{4(9\text{ in.})}{3\pi} = (12\pi)\text{ in.}$$

$$A_2 = -\frac{\pi}{2}(9\text{ in.})^2 = -(40.5\pi)\text{ in}^2$$

$$\bar{x}_3 = -\frac{4(6\text{ in.})}{3\pi} = -\frac{8}{\pi}\text{ in.}$$

$$A_3 = -\frac{\pi}{2}(6\text{ in.})^2 = -(18\pi)\text{ in}^2$$

Then $$\bar{X} = \frac{\Sigma\bar{x}A}{\Sigma A} = \frac{(0)(240\pi\text{ in}^2) + (12\pi\text{ in.})(-40.5\pi\text{ in}^2) - \frac{8\text{ in.}}{\pi}(-18\pi\text{ in}^2)}{240\pi\text{ in}^2 - 40.5\pi\text{ in}^2 - 18\pi\text{ in}^2}$$

$$= \frac{-486\text{ in}^3 + 144\text{ in}^3}{181.5\pi\text{ in}^2} = \frac{-342\text{ in}^3}{181.5\pi\text{ in}^2} = -0.59979\text{ in.}$$

PROBLEM 9.46 CONTINUED

(a) Have

$$J_O = \left(J_O\right)_1 - \left(J_O\right)_2 - \left(J_O\right)_3$$

$$= \frac{\pi}{4}(20\text{ in.})(12\text{ in.})\left[(20\text{ in.})^2 + (12\text{ in.})^2\right] - \left[\frac{\pi}{4}(9\text{ in.})^4\right] - \left[\frac{\pi}{4}(6\text{ in.})^4\right]$$

$$= \pi(32640 - 1640.25 - 324.00)\text{ in}^4 = 96,371\text{ in}^4$$

or $J_O = 96.4 \times 10^3\text{ in}^4$ ◀

(b) Have

$$J_O = \bar{J}_C + A\bar{x}^2$$

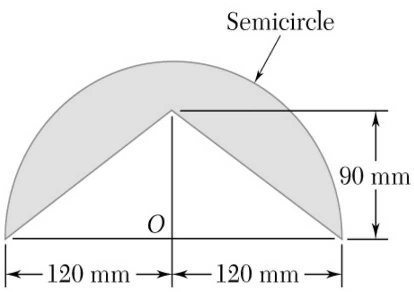
Then

$$\bar{J}_C = 96,371\text{ in}^4 - \left(181.5\pi\text{ in}^2\right)(-0.59979\text{ in.})^2$$

$$= 96,371\text{ in}^4 - 204.5629\text{ in}^4 = 96,166.4379\text{ in}^4$$

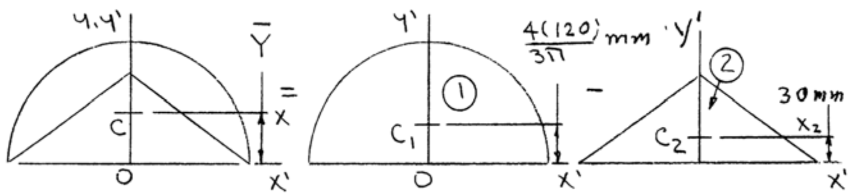
or $\bar{J}_C = 96.2 \times 10^3\text{ in}^4$ ◀

PROBLEM 9.47



Determine the polar moment of inertia of the area shown with respect to (*a*) point *O*, (*b*) the centroid of the area.

SOLUTION



	A, mm^2	$\bar{y}$, mm	$\bar{y}A$, mm^3
1	$\frac{\pi}{2}(120)^2 = 22\ 619.5$	50.9296	1.1520×10^6
2	$-\frac{1}{2}(240)(90) = -10\ 800$	30	-0.324×10^6
Σ	11 819.5		0.828×10^6

Now
$$\bar{Y} = \frac{\Sigma A\bar{Y}}{\Sigma A} = \frac{0.828 \times 10^6 \text{ mm}^3}{11819.5 \text{ mm}^2} = 70.054 \text{ mm}$$

(*a*)
$$J_O = (J_O)_1 - (J_O)_2$$

where
$$(J_O)_1 = \frac{\pi}{4}\left(120 \text{ mm}^4\right) = 162.86 \times 10^6 \text{ mm}^4$$

and
$$(J_O)_2 = (I_{x'})_2 + (I_{y'})_2 = \frac{1}{12}(240 \text{ mm})(90 \text{ mm})^3 + 2\left[\frac{1}{12}(90 \text{ mm})(120 \text{ mm})^3\right]$$

$$= 40.5 \times 10^6 \text{ mm}^4$$

Then
$$J_O = (162.86 - 40.5) \times 10^6 \text{ mm}^4 = 122.36 \times 10^6 \text{ mm}^4$$

or $J_O = 122.4 \times 10^6 \text{ mm}^4$ ◀

PROBLEM 9.47 CONTINUED

(*b*)

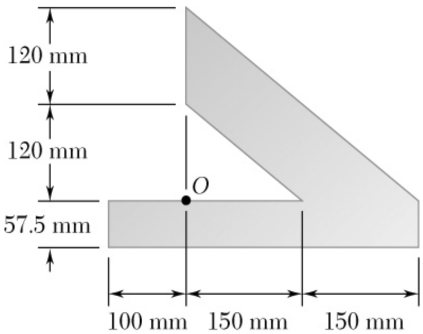
$$J_O = \bar{J}_C + A\bar{y}^2$$

or

$$\bar{J}_C = 122.36 \times 10^6 \text{ mm}^4 - \left(11\,819.5 \text{ mm}^2\right)\left(70.054 \text{ mm}\right)^2$$

$$= \left(122.36 - 58.005\right)10^6 \text{ mm}^4$$

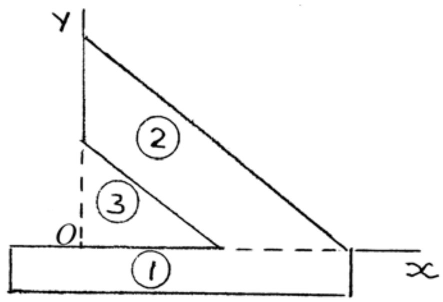
or $\bar{J}_C = 64.4 \times 10^6 \text{ mm}^4$ ◀



PROBLEM 9.48

Determine the polar moment of inertia of the area shown with respect to (*a*) point *O*, (*b*) the centroid of the area.

SOLUTION



First locate centroid

$$\bar{x}_1 = 100 \text{ mm} \qquad \bar{y}_1 = -28.75 \text{ mm}$$

$$A_1 = (400 \text{ mm})(57.5 \text{ mm}) = 23\ 000 \text{ mm}^2$$

$$\bar{x}_2 = 100 \text{ mm} \qquad \bar{y}_2 = 80 \text{ mm}$$

$$A_2 = \frac{1}{2}(300 \text{ mm})(240 \text{ mm}) = 36\ 000 \text{ mm}^2$$

$$\bar{x}_3 = 50 \text{ mm} \qquad \bar{y}_3 = 40 \text{ mm}$$

$$A_3 = -\frac{1}{2}(150 \text{ mm})(120 \text{ mm}) = -9000 \text{ mm}^2$$

Then $$\bar{X} = \frac{\Sigma \bar{x}A}{\Sigma A} = \frac{(100 \text{ mm})(23\ 000 \text{ mm}^2) + (100 \text{ mm})(36\ 000 \text{ mm}^2) + (50 \text{ mm})(-9000 \text{ mm}^2)}{23\ 000 \text{ mm}^2 + 36\ 000 \text{ mm}^2 - 9000 \text{ mm}^2}$$

$$= \frac{(2.3 + 3.6 - 0.45) \times 10^6 \text{ mm}^3}{50 \times 10^3 \text{ mm}^2} = 109.0 \text{ mm}$$

And $$\bar{Y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{(-28.75 \text{ mm})(23\ 000 \text{ mm}^2) + (80 \text{ mm})(36\ 000 \text{ mm}^2) + (40 \text{ mm})(-9000 \text{ mm}^2)}{50 \times 10^3 \text{ mm}^2}$$

$$= \frac{(-661.25 + 2880 - 360) \times 10^3 \text{ mm}^3}{50 \times 10^3 \text{ mm}^2} = 37.175 \text{ mm}$$

PROBLEM 9.48 CONTINUED

(*a*) Now $$J_O = I_x + I_y$$

where $$I_x = (I_x)_1 + (I_x)_2 - (I_x)_3$$

$$(I_x)_1 = \frac{1}{3}(400\text{ mm})(57.5\text{ mm})^3 = 25.3479 \times 10^6\text{ mm}^4$$

$$(I_x)_2 = \frac{1}{12}(300\text{ mm})(240\text{ mm})^3 = 345.6000 \times 10^6\text{ mm}^4$$

$$(I_x)_3 = \frac{1}{12}(150\text{ mm})(120\text{ mm})^3 = 21.6000 \times 10^6\text{ mm}^4$$

Then $$I_x = (25.3479 + 345.6000 - 21.6000) \times 10^6\text{ mm}^4$$

$$= 349.348 \times 10^6\text{ mm}^4$$

Also $$I_y = (I_y)_1 + (I_y)_2 - (I_y)_3$$

where $$(I_y)_1 = \frac{1}{12}(57.5\text{ mm})(400\text{ mm})^3 + (23\ 000\text{ mm}^2)(100\text{ mm})^2$$

$$= (306.6667 + 230.0000) \times 10^6\text{ mm}^4 = 536.6667 \times 10^6\text{ mm}^4$$

$$(I_y)_2 = \frac{1}{12}(240\text{ mm})(300\text{ mm})^3 = 540.0000 \times 10^6\text{ mm}^4$$

$$(I_y)_3 = \frac{1}{12}(150\text{ mm})(120\text{ mm})^3 = 33.7500 \times 10^6\text{ mm}^4$$

Then $$I_y = (536.6667 + 540 - 33.75) \times 10^6\text{ mm}^4 = 1042.917 \times 10^6\text{ mm}^4$$

Finally, $$J_O = (349.348 + 1042.917) \times 10^6\text{ mm}^4 = 1392.265 \times 10^6\text{ mm}^4$$

or $J_O = 1392 \times 10^6\text{ mm}^4$ ◀

(*b*) Have $$J_O = \bar{J}_C + Ad^2 \qquad \text{where} \qquad d^2 = \bar{X}^2 + \bar{Y}^2$$

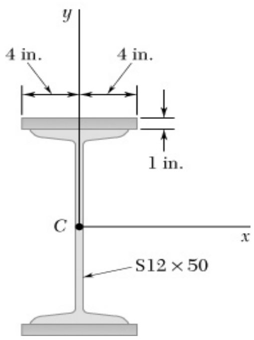
Then $$\bar{J}_C = 1392.265 \times 10^6\text{ mm}^4 - (50 \times 10^3\text{ mm}^2)\left[(109.0\text{ mm})^2 + (37.175\text{ mm})^2\right]$$

$$= (1392.265 - 594.050 - 69.099) \times 10^6\text{ mm}^4$$

$$= 729.1660 \times 10^6\text{ mm}^4$$

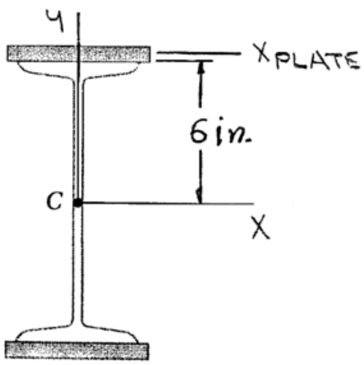
or $\bar{J}_C = 729 \times 10^6\text{ mm}^4$ ◀

PROBLEM 9.49



Two 1-in. steel plates are welded to a rolled S section as shown. Determine the moments of inertia and the radii of gyration of the section with respect to the centroidal *x* and *y* axes.

SOLUTION



S-section

$$A = 14.7\text{ in}^2$$

$$\bar{I}_x = 305\text{ in}^4$$

$$\bar{I}_y = 15.7\text{ in}^4$$

$$A = A_S + 2A_{\text{plate}}$$

$$= 14.7\text{ in}^2 + 2(8\text{ in.})(1\text{ in.}) = 30.7\text{ in}^2$$

$$\bar{I}_x = \left(\bar{I}_x\right)_S + 2\left(\bar{I}_x\right)_{\text{plate}}$$

$$= 305\text{ in}^4 + 2\left[\frac{(8\text{ in.})(1\text{ in.})^3}{12} + (8\text{ in.})(1\text{ in.})(6.5\text{ in.})^2\right]$$

$$= (305 + 677.33)\text{ in}^4 = 982.33\text{ in}^4$$

or $\bar{I}_x = 9.82\text{ in}^4$ ◀

and $\bar{k}_x^2 = \dfrac{\bar{I}_x}{A} = \dfrac{982.33\text{ in}^4}{30.7\text{ in}^4} = 31.998\text{ in}^2$

or $\bar{k}_x = 5.66\text{ in.}$ ◀

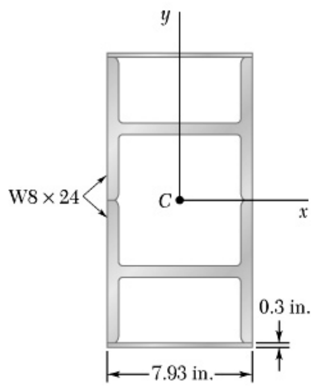
Also $\bar{I}_y = \left(\bar{I}_y\right)_S + 2\left(\bar{I}_y\right)_{\text{plate}} = 15.7\text{ in}^4 + 2\left[\dfrac{(1\text{ in.})(8\text{ in.})^3}{12}\right]$

$$= (15.7 + 85.333)\text{ in}^4 = 101.03\text{ in}^4$$

or $\bar{I}_y = 101.0\text{ in}^4$ ◀

and $\bar{k}_y^2 = \dfrac{\bar{I}_y}{A} = \dfrac{101.03\text{ in}^4}{30.7\text{ in}^2} = 3.29098\text{ in}^2$

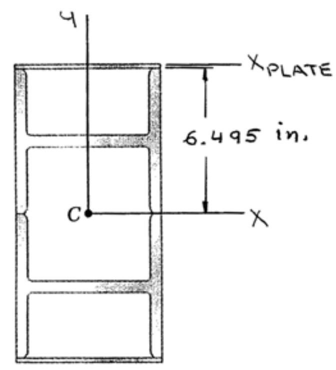
or $\bar{k}_y = 1.814\text{ in.}$ ◀



PROBLEM 9.50

To form a reinforced box section, two rolled W sections and two plates are welded together. Determine the moments of inertia and the radii of gyration of the combined section with respect to the centroidal axes shown.

SOLUTION



W-section

$$A = 7.08\text{ in}^2$$

$$\bar{I}_x = 18.3\text{ in}^4$$

$$\bar{I}_y = 82.8\text{ in}^4$$

$$A = 2A_W + 2A_{\text{plate}}$$

$$= 2\left[7.08\text{ in}^2 + (7.93\text{ in.})(0.3\text{ in.})\right]$$

$$= 18.918\text{ in}^2$$

Now

$$\bar{I}_x = 2\left(\bar{I}_x\right)_W + 2\left(\bar{I}_x\right)_{\text{plate}}$$

$$= 2\left[18.3\text{ in}^4 + \left(7.08\text{ in}^2\right)\left(\frac{6.495\text{ in.}}{2}\right)^2\right]$$

$$+ 2\left\{\frac{(7.93\text{ in.})(0.3\text{ in.})^3}{12} + \left[(7.93\text{ in.})(0.3\text{ in.})\right](6.495\text{ in.} + 0.15\text{ in.})^2\right\}$$

$$= 2\left[92.967\text{ in}^4\right] + 2\left[105.07\text{ in}^4\right] = 396.07\text{ in}^4$$

or $\bar{I}_x = 396\text{ in}^4$ ◀

and $$\bar{k}_x^2 = \frac{\bar{I}_x}{A} = \frac{396.07\text{ in}^4}{18.918\text{ in}^2} = 20.936\text{ in}^2$$

or $\bar{k}_x = 4.58\text{ in.}$ ◀

PROBLEM 9.50 CONTINUED

Also

$$\bar{I}_y = 2\left(\bar{I}_y\right)_W + 2\left(\bar{I}_y\right)_{\text{plate}}$$

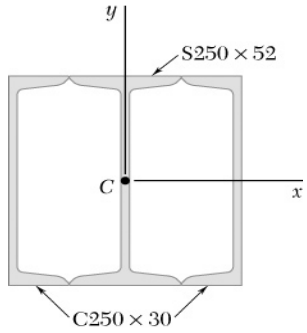
$$= 2\left(82.8\text{ in}^4\right) + 2\left[\frac{(0.3\text{ in.})(7.93\text{ in.})^3}{12}\right] = (165.60 + 24.9339)\text{ in}^4$$

$$= 190.53\text{ in}^4$$

or $\bar{I}_y = 190.5\text{ in}^4$ ◀

and $$\bar{k}_y^2 = \frac{\bar{I}_y}{A} = \frac{190.53\text{ in}^4}{18.918\text{ in}^2} = 10.072$$

or $\bar{k}_x = 3.17$ in. ◀



PROBLEM 9.51

Two C250 × 30 channels are welded to a 250 × 52 rolled S section as shown. Determine the moments of inertia and the radii of gyration of the combined section with respect to its centroidal *x* and *y* axes.

SOLUTION

Use Figure 9.13B (textbook) properties of rolled-steel shapes (SI units) to get the values for C250 and S250 shapes

S250 × 52 section:

$$A = 6670 \text{ mm}^2$$
$$I_x = 61.2 \times 10^6 \text{ mm}^4$$
$$I_y = 3.59 \times 10^6 \text{ mm}^4$$

C250 × 30 section:

$$A = 3780 \text{ mm}^2$$
$$I_x = 32.6 \times 10^6 \text{ mm}^4$$
$$I_y = 1.14 \times 10^6 \text{ mm}^4$$

How, for the combined section:

$$A = A_S + 2A_C$$
$$= \left[6670 + 2(3780)\right]\text{mm}^2$$
$$= 14\ 230 \text{ mm}^2$$

$$\bar{I}_x = (I_x)_S + 2(I_x)_C$$
$$= \left[61.2 \times 10^6 + 2\left(32.6 \times 10^6\right)\right]\text{mm}^4$$

or $\bar{I}_x = 126.4 \times 10^6 \text{ mm}^4$ ◀

$$\bar{I}_y = (I_y)_S + 2\left[(I_y)_C + A_C d^2\right]$$

where *d* is the distance from the centroid of the C section to the centroid *C* of the combined section

Now
$$\bar{I}_y = 3.59 \times 10^6 \text{ mm}^4 + 2\left[\left(1.14 \times 10^6 \text{ mm}^4\right) + \left(3780 \text{ mm}^2\right)\left(\frac{126}{2} + 69 - 15.3\right)^2 \text{ mm}^2\right]$$

$$= (3.59 + 2.28 + 102.9588) \times 10^6 \text{ mm}^4$$

or $\bar{I}_y = 108.8 \times 10^6 \text{ mm}^4$ ◀

Also
$$\bar{k}_x = \sqrt{\frac{\bar{I}_x}{A}}$$

$$= \sqrt{\frac{126.4 \times 10^6 \text{ mm}^4}{14\ 230 \text{ mm}^2}}$$

or $\bar{k}_x = 94.2 \text{ mm}$ ◀

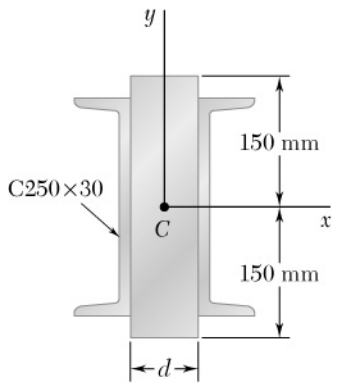
PROBLEM 9.51 CONTINUED

And $$\bar{k}_y = \sqrt{\frac{\bar{I}_y}{A}}$$

$$= \sqrt{\frac{108.8 \times 10^6 \text{ mm}^4}{14\ 230 \text{ mm}^2}}$$

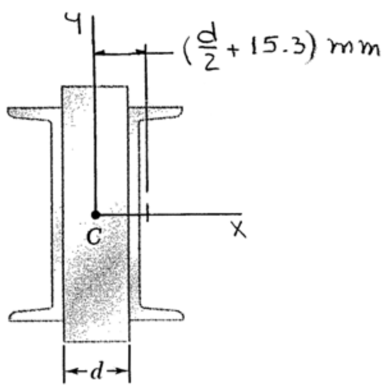
or $\bar{k}_y = 87.5 \text{ mm}$ ◀

PROBLEM 9.52



Two channels are welded to a $d \times$ 300-mm steel plate as shown. Determine the width d for which the ratio $\bar{I}_x / \bar{I}_y$ of the centroidal moments of inertia of the section is 16.

SOLUTION



Channel: $A = 3780 \text{ mm}^2$

$\bar{I}_x = 32.6 \times 10^6 \text{ mm}^4$

$\bar{I}_y = 1.14 \times 10^6 \text{ mm}^4$

Now $\bar{I}_x = 2(\bar{I}_x)_C + (\bar{I}_x)_{\text{plate}}$

$$= 2(32.6 \times 10^6 \text{ mm}^4) + \frac{d}{12}(300 \text{ mm})^3$$

$$= (65.2 \times 10^6 + 2.25d \times 10^6) \text{ mm}^4$$

And

$$\bar{I}_y = 2(I_y)_{\text{channel}} + (\bar{I}_y)_{\text{plate}}$$

$$= 2\left[1.14 \times 10^6 \text{ mm}^4 + (3780 \text{ mm}^2)\left(\frac{d}{2} + 15.3 \text{ mm}\right)^2\right] + \frac{(300 \text{ mm})d^3}{12}$$

$$= \left[(2.28 \times 10^6 + 1890d + 115.668 \times 10^3 d + 1.7697 \times 10^6) + 25d^3\right] \text{mm}^4$$

$$= (25d^3 + 1890d^2 + 115.67 \times 10^3 d + 4.0497 \times 10^6) \text{ mm}^4$$

Given $\bar{I}_x = 16\bar{I}_y$

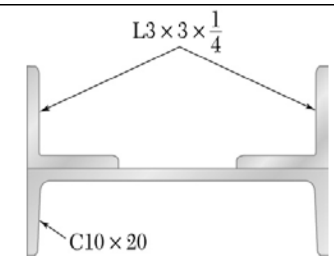
Then $65.2 \times 10^6 + 2.25d \times 10^6$

$$= 16(25d^3 + 1890d^2 + 115.67 \times 10^3 d + 4.0497 \times 10^6)$$

or $25d^3 + 1890d^2 - 24.955d - 25300 = 0$

Solving numerically $d = 12.2935 \text{ mm}$

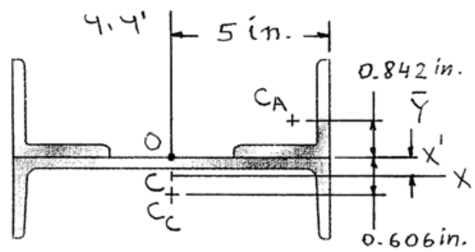
or $d = 12.29 \text{ mm}$ ◀



PROBLEM 9.53

Two L3 × 3 × $\frac{1}{4}$ -in. angles are welded to a C10 × 20 channel. Determine the moments of inertia of the combined section with respect to centroidal axes respectively parallel and perpendicular to the web of the channel.

SOLUTION



Angle: $A = 1.44\text{ in}^2$

$\bar{I}_x = \bar{I}_y = 1.24\text{ in}^4$

Channel: $A = 5.88\text{ in}^2$

$\bar{I}_x = 2.81\text{ in}^4 \qquad \bar{I}_y = 78.9\text{ in}^4$

Locate the centroid

$$\bar{X} = 0$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{2\left[\left(1.44\text{ in}^2\right)\left(0.842\text{ in.}\right)\right] + \left(5.88\text{ in}^2\right)\left(-0.606\text{ in.}\right)}{2\left(1.44\text{ in}^2\right) + 5.88\text{ in}^2}$$

$$= \frac{\left(2.42496 - 3.5638\right)\text{in}^3}{8.765\text{ in}^4} = -0.12995\text{ in.}$$

Now

$$\left(\bar{I}_x\right) = 2\left(I_x\right)_{\text{L}} + \left(I_x\right)_{\text{C}} = 2\left[1.24\text{ in}^4 + \left(1.44\text{ in}^2\right)\left(0.842\text{ in.} + 0.12995\text{ in.}\right)^2\right]$$

$$+\left[2.81\text{ in}^4 + \left(5.88\text{ in}^2\right)\left(0.606\text{ in.} - 0.12995\text{ in.}\right)^2\right]$$

$$= 2\left(2.6003\right)\text{in}^4 + 4.1425\text{ in}^4 = 9.3431\text{ in}^4$$

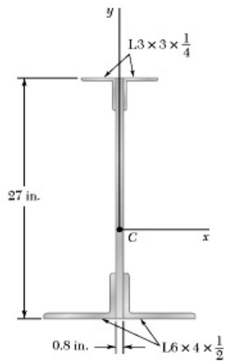
or $\bar{I}_x = 9.34\text{ in}^4$ ◀

Also

$$\left(\bar{I}_y\right) = 2\left(I_y\right)_{\text{L}} + \left(\bar{I}_y\right)_{\text{C}} = 2\left[2.14\text{ in}^4 + 1.44\text{ in}^2\left(5\text{ in.} - 0.842\text{ in.}\right)^2\right] + 7.89\text{ in}^4$$

$$= 2\left(26.136\right)\text{in}^4 + 78.9\text{ in}^4 = 131.17\text{ in}^4$$

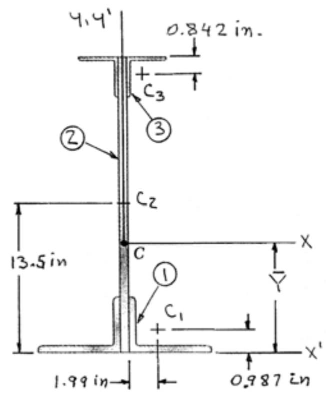
or $\bar{I}_y = 131.2\text{ in}^4$ ◀



PROBLEM 9.54

To form an unsymmetrical girder, two L3 × 3 × $\frac{1}{4}$ -in. angles and two L6 × 4 × $\frac{1}{2}$ -in. angles are welded to a 0.8-in. steel plate as shown. Determine the moments of inertia of the combined section with respect to its centroidal *x* and *y* axes.

SOLUTION



Angle: $$\text{L}3 \times 3 \times \frac{1}{4}:$$

$$A = 1.44\text{ in}^2 \qquad \bar{I}_x = \bar{I}_y = 1.24\text{ in}^4$$

$$\text{L}6 \times 4 \times \frac{1}{2}:$$

$$A = 4.75\text{ in}^2 \qquad \bar{I}_x = 6.27\text{ in}^4$$

$$\bar{I}_y = 17.4\text{ in}^4$$

Plate: $$A = (27\text{ in.})(0.8\text{ in.}) = 21.6\text{ in}^2$$

$$\bar{I}_x = \frac{1}{12}(0.8\text{ in.})(27\text{ in.})^3 = 1312.2\text{ in}^4$$

$$\bar{I}_y = \frac{1}{12}(27\text{ in.})(0.8\text{ in.})^3 = 1.152\text{ in}^4$$

PROBLEM 9.54 CONTINUED

Centroid: $\bar{X} = 0$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A}$$

or
$$\bar{Y} = \frac{2\left[\left(1.44\text{ in}^2\right)\left(27\text{ in.} - 0.84\text{ in.}\right)\right] + 2\left[\left(4.75\text{ in}^2\right)\left(0.987\text{ in.}\right)\right] + \left(21.6\text{ in}^2\right)\left(13.5\text{ in.}\right)^2}{2\left(1.44\text{ in}^2 + 4.75\text{ in}^2\right) + 21.6\text{ in}^2}$$

$$= \frac{376.31\text{ in}^3}{33.98\text{ in}^2} = 11.0745\text{ in.}$$

Now
$$\bar{I}_x = 2\left(I_x\right)_1 + 2\left(I_x\right)_3 + \left(I_x\right)_2$$

$$= 2\left[6.25 + 4.75\left(11.075 - 0.987\right)^2\right]\text{in}^4 + 2\left[1.24 + 1.44\left(27 - 0.842 - 11.075\right)^2\right]\text{in}^4$$

$$+\left[1312.2 + 21.6\left(13.5 - 11.075\right)^2\right]\text{in}^4$$

$$= 2\left(489.67\right)\text{in}^4 + 2\left(328.84\right)\text{in}^4 + 1439.22\text{ in}^4 = 3076.24\text{ in}^4$$

or $\bar{I}_x = 3076\text{ in}^4$ ◀

Also
$$\left(\bar{I}_y\right) = 2\left(I_y\right)_1 + 2\left(I_y\right)_3 + \left(I_y\right)_2$$

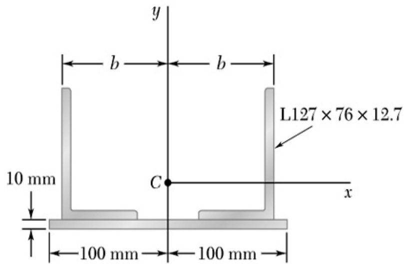
$$= 2\left[17.4 + 4.75\left(0.4 + 1.99\right)^2\right]\text{in}^4 + 2\left[1.24 + 1.44\left(0.4 + 0.842\right)^2\right]\text{in}^4 + 1.152\text{ in}^4$$

$$= 2\left(44.532\right)\text{in}^4 + 2\left(3.4613\right)\text{in}^4 + 1.152\text{ in}^4$$

$$= 97.139\text{ in}^4$$

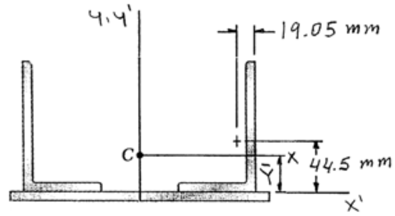
or $\bar{I}_y = 97.1\text{ in}^4$ ◀

PROBLEM 9.55



Two L127 × 76 × 12.7-mm angles are welded to a 10-mm steel plate. Determine the distance *b* and the centroidal moments of inertia $\bar{I}_x$ and $\bar{I}_y$ of the combined section knowing that $\bar{I}_y = 3\bar{I}_x$.

SOLUTION



Angle:

$$A = 2420 \text{ mm}^2$$

$$\bar{I}_x = 3.93 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = 1.074 \times 10^6 \text{ mm}^4$$

Plate:

$$A = (200 \text{ mm})(10 \text{ mm}) = 2000 \text{ mm}^2$$

$$\bar{I}_x = \frac{1}{12}(200 \text{ mm})(10 \text{ mm})^3 = 0.01667 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = \frac{1}{12}(10 \text{ mm})(200 \text{ mm})^3 = 6.6667 \times 10^6 \text{ mm}^4$$

Centroid

$$\bar{X} = 0$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A}$$

or

$$\bar{Y} = \frac{2\left(2420 \text{ mm}^2\right)(44.5 \text{ mm}) + 2000 \text{ mm}^2(-5 \text{ mm})}{\left[2(2420) + 2000\right] \text{mm}^2} = \frac{205.380 \text{ mm}^3}{6840 \text{ mm}^2}$$

$$= 30.026 \text{ mm}$$

Now

$$\bar{I}_x = 2(I_x)_{\text{angle}} + (I_x)_{\text{plate}}$$

$$= 2\left[3.93 \times 10^6 + (2420)(44.5 - 30.026)^2\right] \text{mm}^4$$

$$+\left[0.01667 \times 10^6 + (2000)(30.026 + 5)^2\right] \text{mm}^4$$

$$= 2\left(4.43698 \times 10^6\right) \text{mm}^4 + 2.4703 \times 10^6 \text{ mm}^4$$

$$= 11.344 \times 10^6 \text{ mm}^4$$

or $\bar{I}_x = 11.34 \times 10^6 \text{ mm}^4$ ◀

PROBLEM 9.55 CONTINUED

Also $$\bar{I}_y = 2\left(I_y\right)_{\text{angle}} + \left(\bar{I}_y\right)_{\text{plate}}$$

Where $$\left(\bar{I}_y\right)_{\text{angle}} = 1.074 \times 10^6 \text{ mm}^4 + \left(2420 \text{ mm}^2\right)\left(b - 19.05 \text{ mm}\right)^2$$

$$= \left[1.074 \times 10^6 + \left(2420\right)\left(b^2 - 38.1b + 362.9\right)\right] \text{mm}^4$$

$$= \left[2420b^2 - 92202b + 1.9522 \times 10^6\right] \text{mm}^4$$

and $$\left(\bar{I}_y\right)_{\text{plate}} = 6.6667 \times 10^6 \text{ mm}^4$$

Now $$\bar{I}_y = 3\left(\bar{I}_x\right)$$

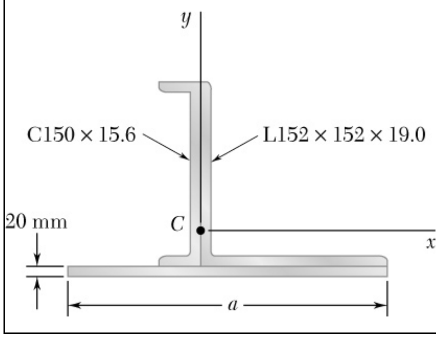
Then $$2\left[2420b^2 - 92202b + 1.9522 \times 10^6\right] \text{mm}^4 + 6.6667 \times 10^6 \text{ mm}^4 = 3\left[\left(11.34 \times 10^6\right) \text{mm}^4\right]$$

or $$2420b^2 - 9.2202b + 1.9522 \times 10^6 - 13.6767 \times 10^6 = 0$$

$$b^2 - 38.1b - 4844.8 = 0$$

$$b = 91.2144 \text{ mm}$$

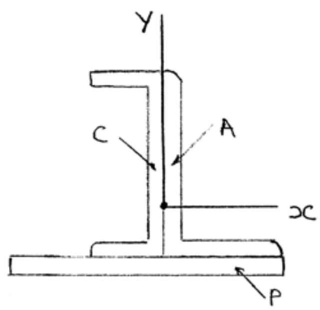
or $b = 91.2$ mm ◀



PROBLEM 9.56

A channel and an angle are welded to an $a \times 20$-mm steel plate. Knowing that the centroidal y axis is located as shown, determine (a) the width a, (b) the moments of inertia with respect to the centroidal x and y axes.

SOLUTION



(a) Using Figure 9.13B

From the geometry of $L152 \times 152 \times 19$, $C150 \times 15.6$, plate $a \times 20$ mm and how they are welded

$$x_A = 44.9 \text{ mm} \quad A_A = 5420 \text{ mm}^2$$

$$x_C = -12.5 \text{ mm} \quad A_C = 1980 \text{ mm}^2$$

$$x_P = -\left(\frac{a}{2} - 152\right) \text{mm} \quad A_P = (20a) \text{ mm}^2$$

From the condition $\bar{X} = \dfrac{\Sigma \bar{x} A}{\Sigma A} = 0$

$$(44.9 \text{ mm})(5420 \text{ mm}^2) - (12.5 \text{ mm})(1980 \text{ mm}^2) - \left[\left(\frac{a}{2} - 152\right)\text{mm}\right](20a \text{ mm}^2) = 0$$

or $a^2 - 304a - 21860.8 = 0 \qquad a = 364.05 \text{ mm}$

or $a = 364 \text{ mm}$ ◀

And $A_P = (20 \text{ mm})(364 \text{ mm})$

$$= 7280 \text{ mm}^2$$

PROBLEM 9.56 CONTINUED

(*b*) Locate the centroid

$$Y = \frac{\Sigma A\bar{y}}{\Sigma A}$$

$$= \frac{\left(5420\text{ mm}^2\right)\left(44.9\text{ mm}\right) + \left(1980\text{ mm}^2\right)\left(\frac{152}{2}\text{ mm}\right) + \left(7280\text{ mm}^2\right)\left(-10\text{ mm}\right)}{\left(5420 + 1980 + 7280\right)\text{ mm}^2}$$

$$= 21.867\text{ mm}$$

Now

$$\bar{I}_x = \left(I_x\right)_A + \left(I_x\right)_C + \left(I_x\right)_P$$

$$= \left[11.6 \times 10^6\text{ mm}^4 + \left(5420\text{ mm}^2\right)\left(44.9\text{ mm} - 21.867\text{ mm}\right)^2\right]$$

$$+\left[6.21 \times 10^6\text{ mm}^4 + \left(1980\text{ mm}^2\right)\left(76\text{ mm} - 21.867\text{ mm}\right)^2\right]$$

$$+\left[\frac{1}{12}\left(364.05\text{ mm}\right)\left(20\text{ mm}\right)^3 + \left(7281\text{ mm}^2\right)\left(10\text{ mm} + 21.867\text{ mm}\right)^2\right]$$

$$= \left[\left(11.6 + 2.8754\right) + \left(6.21 + 5.8022\right) + \left(0.2427 + 7.3939\right)\right] \times 10^6\text{ mm}^4$$

$$= \left(14.4754 + 12.0122 + 7.6366\right) \times 10^6\text{ mm}^4$$

$$= 34.1242 \times 10^6\text{ mm}^4$$

or $\bar{I}_x = 34.1 \times 10^6\text{ mm}^4$ ◀

And

$$\bar{I}_y = \left(I_y\right)_A + \left(I_y\right)_C + \left(I_y\right)_P$$

$$= \left[11.6 \times 10^6\text{ mm}^4 + \left(5420\text{ mm}^2\right)\left(44.9\text{ mm}\right)^2\right]$$

$$+\left[0.347 \times 10^6\text{ mm}^4 + \left(1980\text{ mm}^2\right)\left(12.5\text{ mm}\right)^2\right]$$

$$+\left[\frac{1}{12}\left(20\text{ mm}\right)\left(364.05\text{ mm}\right)^3 + \left(7821\text{ mm}^2\right)\left(\frac{364.05\text{ mm}}{2} - 152\text{ mm}\right)^2\right]$$

$$= \left(11.6 + 10.9268\right) \times 10^6\text{ mm}^4 + \left(0.347 + 0.3094\right) \times 10^6\text{ mm}^4$$

$$+\left(80.4140 + 6.5638\right) \times 10^6\text{ mm}^4$$

$$= \left(22.5268 + 0.6564 + 86.9778\right) \times 10^6\text{ mm}^4$$

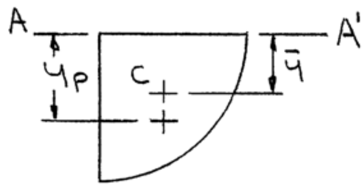
$$= 110.161 \times 10^{-6}\text{ mm}^4$$

or $\bar{I}_y = 110.2 \times 10^6\text{ mm}^4$ ◀

PROBLEM 9.57

The panel shown forms the end of a trough which is filled with water to the line AA'. Referring to Sec. 9.2, determine the depth of the point of application of the resultant of the hydrostatic forces acting on the panel (the center of pressure).

SOLUTION



Using the equation developed on page 491 of the text have

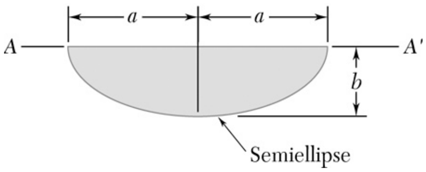
$$y_P = \frac{I_{AA'}}{\bar{y}A}$$

For a quarter circle $$I_{AA'} = \frac{\pi}{16}r^4$$

and $$\bar{y} = \frac{4r}{3\pi}, \; A = \frac{\pi}{4}r^2$$

Then $$y_P = \frac{\frac{\pi}{16}r^4}{\left(\frac{4r}{3\pi}\right)\left(\frac{\pi}{4}r^2\right)}$$

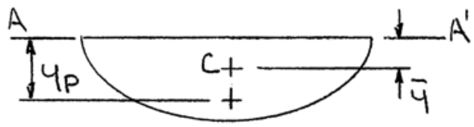
or $y_P = \dfrac{3\pi}{16}r$ ◀



PROBLEM 9.58

The panel shown forms the end of a trough which is filled with water to the line AA'. Referring to Sec. 9.2, determine the depth of the point of application of the resultant of the hydrostatic forces acting on the panel (the center of pressure).

SOLUTION



Using the equation developed on page 491 of the text have

$$y_P = \frac{I_{AA'}}{\bar{y}A}$$

For a semiellipse

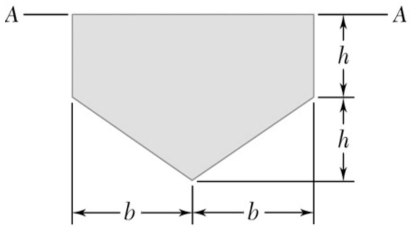
$$I_{AA'} = \frac{\pi}{8}ab^3$$

$$\bar{y} = \frac{4b}{3\pi}, \quad A = \frac{\pi}{2}ab$$

Then

$$y_P = \frac{\frac{\pi}{8}ab^3}{\left(\frac{4b}{3\pi}\right)\left(\frac{\pi}{2}ab\right)}$$

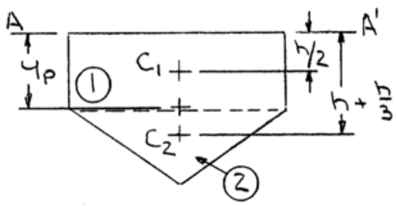
or $\quad y_P = \dfrac{3\pi}{16}b$ ◀



PROBLEM 9.59

The panel shown forms the end of a trough which is filled with water to the line AA'. Referring to Sec. 9.2, determine the depth of the point of application of the resultant of the hydrostatic forces acting on the panel (the center of pressure).

SOLUTION



Using the equation developed on page 491 of the text

Have $$y_P = \frac{I_{AA'}}{\bar{y}A}$$

Now $$\bar{Y}A = \Sigma\bar{y}A$$

$$= \frac{h}{2}(2b \times h) + \frac{4}{3}h\left(\frac{1}{2} \times 2b \times h\right)$$

$$= \frac{7}{3}bh^2$$

And $$I_{AA'} = (I_{AA'})_1 + (I_{AA'})_2$$

where

$$(I_{AA'})_1 = \frac{1}{3}(2b)(h)^3 = \frac{2}{3}bh^3$$

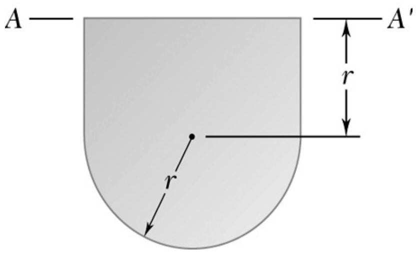
$$(I_{AA'})_2 = \bar{I}_x + Ad^2 = \frac{1}{36}(2b)(h)^3 + \left(\frac{1}{2} \times 2b \times h\right)\left(\frac{4}{3}h\right)^2$$

$$= \frac{11}{6}bh^3$$

Then $$I_{AA'} = \frac{2}{3}bh^3 + \frac{11}{6}bh^3 = \frac{5}{2}bh^3$$

Finally, $$y_P = \frac{\frac{5}{2}bh^3}{\frac{7}{3}bh^2}$$

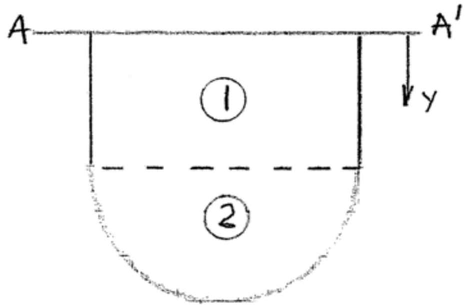
or $y_P = \frac{15}{14}h$ ◀



PROBLEM 9.60

The panel shown forms the end of a trough which is filled with water to the line AA'. Referring to Sec. 9.2, determine the depth of the point of application of the resultant of the hydrostatic forces acting on the panel (the center of pressure).

SOLUTION



Using the equation developed on page 491 of the text

Have $$y_P = \frac{I_{AA'}}{\bar{y}A}$$

where $$I_{AA'} = (I_{AA'})_1 + (I_{AA'})_2$$

$$= \left[\frac{1}{3}(2r)(r)^3\right] + \left\{\left[\frac{\pi}{8}r^4 - \frac{\pi}{2}r^2\left(\frac{4r}{3\pi}\right)^2\right] + \frac{\pi}{2}r^2\left(r + \frac{4r}{3\pi}\right)^2\right\}$$

$$= \frac{2}{3}r^4 + \left(\frac{\pi}{8} - \frac{8}{9\pi} + \frac{\pi}{2} + \frac{4}{3} + \frac{9}{8\pi}\right)r^4 = \left(2 + \frac{5\pi}{8}\right)r^4$$

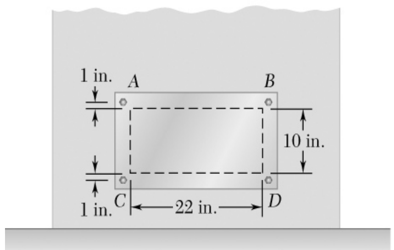
And $$\bar{Y}A = \Sigma\bar{y}A = \left[\frac{r}{2}(2r \times r)\right] + \left[\left(r + \frac{4r}{3\pi}\right)\left(\frac{\pi}{2}r^2\right)\right]$$

$$= \left(1 + \frac{\pi}{2} + \frac{2}{3}\right)r^3 = \left(\frac{5}{3} + \frac{\pi}{2}\right)r^3$$

Then $$y_P = \frac{\left(2 + \frac{5\pi}{8}\right)r^4}{\left(\frac{5}{3} + \frac{\pi}{2}\right)r^3} = 1.2242r$$

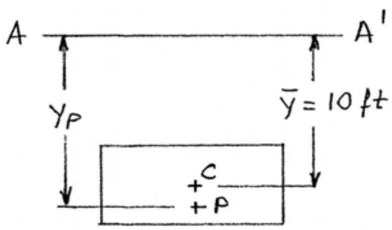
or $y_P = 1.224r$ ◀

PROBLEM 9.61



The cover for a 10 × 22-in. access hole in an oil storage tank is attached to the outside of the tank with four bolts as shown. Knowing that the specific weight of the oil is 57.4 lb/ft^3 and that the center of the cover is located 10 ft below the surface of the oil, determine the additional force on each bolt because of the pressure of the oil.

SOLUTION



Using the equation developed on page 491 of the text have

$$y_P = \frac{I_{AA'}}{\bar{y}A} \qquad R = \gamma \bar{y} A$$

Then $$R = 57.4 \text{ lb/ft}^3 \times 10 \text{ ft} \times (22 \times 10) \text{ in}^2 \times \frac{1 \text{ ft}^2}{144 \text{ in}^2}$$

$$= 876.94 \text{ lb}$$

and $$I_{AA'} = \frac{1}{12}(22 \text{ in.})(10 \text{ in.})^3 + [(22 \text{ in.})(10 \text{ in.})](120 \text{ in.})^2$$

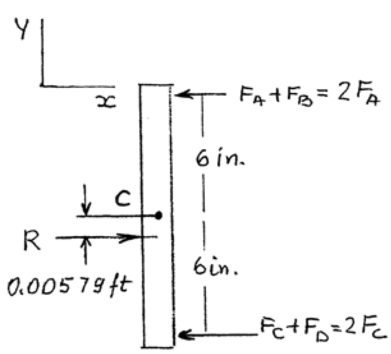
$$= 3.169833 \times 10^6 \text{ in}^4 = 152.8662 \text{ ft}^4$$

and $$\bar{y}A = 10 \text{ ft} \times (22 \times 10) \text{ in}^2 \times \frac{1 \text{ ft}^2}{144 \text{ in}^2} = 15.27778 \text{ ft}^3$$

Then $$y_p = \frac{152.8662 \text{ ft}^4}{15.27778 \text{ ft}^2} = 10.00579 \text{ ft}$$

Now symmetry implies $F_A = F_B$ and $F_C = F_D$

Equilibrium



$$\Sigma M_{CD} = 0: \quad (1 \text{ ft})(2FAa) - (0.5 - 0.00579) \text{ ft} \times 876.94 \text{ lb} = 0$$

$$F_A = 216.70 \text{ lb}$$

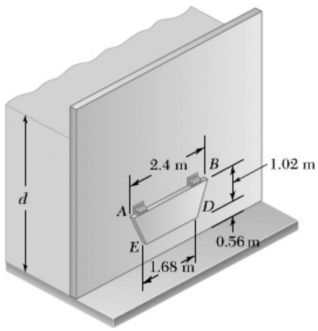
or $F_A = F_B = 217$ lb ◀

Also $$\Sigma F_x = 0:$$

or $$-2(216.70) + 876.94 - 2F_C = 0$$

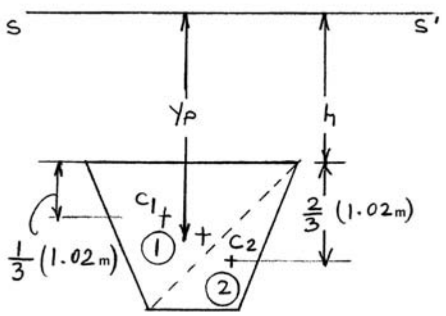
or $F_C = F_D = 222$ lb ◀

PROBLEM 9.62



A vertical trapezoidal gate that is used as an automatic valve is held shut by two springs attached to hinges located along edge *AB*. Knowing that each spring exerts a couple of magnitude $8.50\ \text{kN}\cdot\text{m}$, determine the depth *d* of water for which the gate will open.

SOLUTION



$$d = (h + 1.58)\text{ m}$$

From page 491 $\quad y_p = \dfrac{I_{ss'}}{\bar{y}A} \qquad \gamma = \rho g \qquad R = \gamma\bar{y}A$

Now $\quad \bar{y}A = \Sigma\bar{y}A$

$$= \left[(h + 0.34\text{ m})\right]\left(\frac{1}{2}\times 2.4\text{ m}\times 1.02\text{ m}\right) + \left[(h + 0.68\text{ m})\right]\left(\frac{1}{2}\times 1.68\text{ m}\times 1.02\text{ m}\right)$$

$$= (2.0808h + 0.99878)\text{m}^3$$

Also, $$R = \left(10^3\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)(2.0808h + 0.99878)\text{m}^3$$

$$= 20\ 413(h + 0.480)\text{ N}$$

And $$I_{ss'} = \left(I_{ss'}\right)_1 + \left(I_{ss'}\right)_2 = \left\{\frac{1}{36}(2.4\text{ m})(1.02\text{ m})^3 + \left[\frac{1}{2}(2.4\text{ m})(1.02\text{ m})\right](h + 0.34)^2\text{ m}^2\right\}$$

$$+\left\{\frac{1}{36}(1.68\text{ m})(1.08\text{ m})^3 + \left[\frac{1}{2}(1.68\text{ m})(1.02\text{ m})\right](h + 0.68)^2\text{ m}^2\right\}$$

$$= \left[0.07075 + 1.224(h + 0.34)^2 + 0.04952 + 0.8568(h + 0.68)^2\right]\text{m}^4$$

$$= \left[0.12027 + 1.224\left(h^2 + 0.68h + 0.1156\right) + 0.8568\left(h^2 + 1.36h + 0.4624\right)\right]\text{m}^4$$

$$= \left(2.0808h^2 + 1.9976h + 0.65795\right)\text{m}^4$$

PROBLEM 9.62 CONTINUED

Then $$y_p = \frac{\left(2.0808h^2 + 1.9976h + 0.65795\right)\text{ m}^4}{\left(2.0808h + 0.99878\right)\text{ m}^3}$$

$$= \frac{h^2 + 0.960h + 0.3162}{h + 0.480}\text{ m}$$

For gate to open $$\Sigma M_{AB} = 0\text{:}\quad M_{\text{open}} - \left(y_p - h\right)R = 0$$

$$2\left(8500\text{ N}\cdot\text{m}\right) - \left[\left(\frac{h^2 + 0.960h + 0.3162}{h + 0.480} - h\right)\text{m}\right]\left[\left(20\ 413\right)\left(h + 0.48\right)\right]\text{N} = 0$$

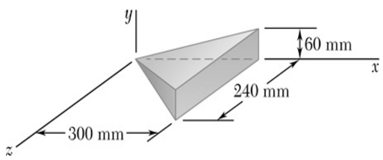
or $$17\ 000 = 20\ 413\left(h^2 + 0.96h + 0.3162 - h^2 - 0.480h\right)$$

or $$0.48h - 0.5166 = 0$$

$$h = 1.0763\text{ m}$$

Now $$d = h + 1.58\text{ m} = \left(1.0763 + 1.58\right)\text{m} = 2.6563\text{ m}$$

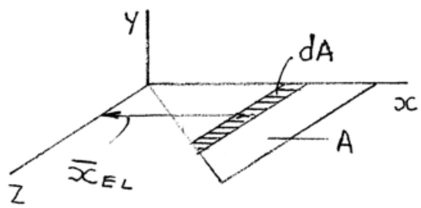
or $d = 2.66\text{ m}$ ◀



PROBLEM 9.63

Determine the *x* coordinate of the centroid of the volume shown. (*Hint:* The height *y* of the volume is proportional to the *x* coordinate; consider an analogy between this height and the water pressure on a submerged surface.)

SOLUTION



Have $$\bar{x} = \frac{\int \bar{x}_{EL} dV}{\int dV}$$

where $$dV = y dA \qquad \text{and} \qquad \bar{x}_{EL} = x$$

Now $$y = \frac{60}{300}x = \frac{1}{5}x$$

Then $$\bar{x} = \frac{\int x\left(\frac{1}{5}x\right)dA}{\int\left(\frac{1}{5}x\right)dA}$$

$$= \frac{\int x^2 dA}{\int x dA} = \frac{(I_z)_A}{(\bar{x}A)_A}$$

where $(I_z)_A$ is the moment of inertia of the area with respect to the *z* axis, and $\bar{x}$ is analogous to y_p

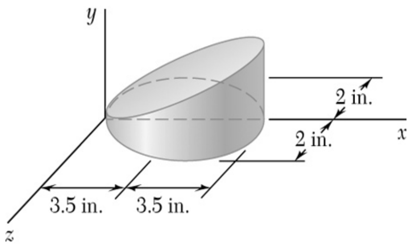
Now $$(I_z)_A = \frac{1}{36}(240\text{ mm})(300\text{ mm})^3 + \frac{1}{2}\left[(240\text{ mm})(300\text{ mm})\right](200\text{ mm})^2$$

$$= 1.620 \times 10^9 \text{ mm}^4$$

and $$\bar{x}A = (200\text{ mm})\left[\frac{1}{2}(240\text{ mm})(300\text{ mm})\right] = 7.20 \times 10^6 \text{ mm}^3$$

Then $$\bar{x} = \frac{1.620 \times 10^9 \text{ mm}^4}{7.20 \times 10^6 \text{ mm}^3}$$

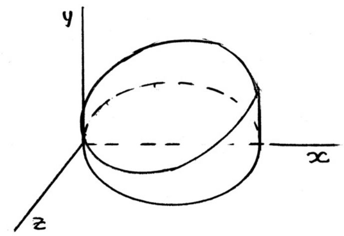
or $\bar{x} = 225$ mm ◀



PROBLEM 9.64

Determine the x coordinate of the centroid of the volume shown; this volume was obtained by intersecting an elliptic cylinder with an oblique plane. (*Hint:* The height y of the volume is proportional to the x coordinate; consider an analogy between this height and the water pressure on a submerged surface.)

SOLUTION



Have $$y = \frac{h}{a}x$$

$$\bar{x}\int dV = \int \bar{x}_{EL}\,dV, \qquad \text{where} \qquad \bar{x}_{EL} = x$$

And $$dV = y\,dA = \left(\frac{h}{a}x\right)dA$$

Now $$\bar{x} = \frac{\int x\left(\frac{h}{a}x\right)dA}{\int\left(\frac{h}{a}x\right)dA} = \frac{\int x^2 dA}{\int x\,dA} = \frac{(I_z)_A}{(\bar{x}A)_A}$$

For the given volume

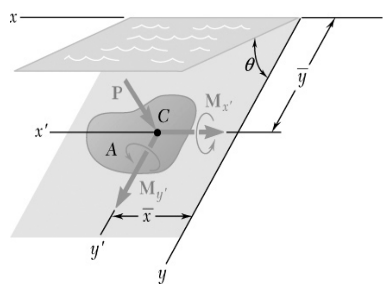
$$(I_z)_A = \frac{\pi}{4}(2\text{ in.})(3.5\text{ in.})^3 + \left[\pi(3.5\text{ in.})(2\text{ in.})\right](3.5\text{ in.})^2$$

$$= (21.4375\pi + 85.7500\pi)\text{ in}^4 = 336.74\text{ in}^4$$

and $$(\bar{x}A)_A = 3.5\text{ in.}\left[\pi(3.5\text{ in.})(2\text{ in.})\right] = 76.969\text{ in}^3$$

Then $$\bar{x} = \frac{336.74\text{ in}^4}{76.969\text{ in}^3} = 4.375\text{ in.}$$

or $\bar{x} = 4.38\text{ in.}$ ◀



PROBLEM 9.65

Show that the system of hydrostatic forces acting on a submerged plane area A can be reduced to a force **P** at the centroid C of the area and two couples. The force **P** is perpendicular to the area and is of magnitude $P = \gamma A\bar{y}\sin\theta$, where γ is the specific weight of the liquid, and the couples are $\mathbf{M}_{x'} = \left(\gamma \bar{I}_{x'} \sin\theta\right)\mathbf{i}$ and $\mathbf{M}_{y'} = \left(\gamma \bar{I}_{x'y'} \sin\theta\right)\mathbf{j}$, where $\bar{I}_{x'y'} = \int x'y'\,dA$ (see Sec. 9.8). Note that the couples are independent of the depth at which the area is submerged.

SOLUTION

The pressure p at an arbitrary depth $\left(y\sin\theta\right)$ is

$$p = \gamma\left(y\sin\theta\right)$$

so that the hydrostatic force dF exerted on an infinitesimal area dA is

$$dF = \left(\gamma y\sin\theta\right)dA$$

Equivalence of the force **P** and the system of infinitesimal forces dF requires

$$\Sigma F: \quad P = \int dF = \int \gamma y \sin\theta dA = \gamma\sin\theta \int y dA$$

$$\text{or} \quad P = \gamma A\bar{y}\sin\theta \blacktriangleleft$$

Equivalence of the force and couple $\left(\mathbf{P},\ \mathbf{M}_{x'} + \mathbf{M}_{y'}\right)$ and the system of infinitesimal hydrostatic forces requires

$$\Sigma M_x: \quad -\bar{y}P - M_{x'} = \int\left(-ydF\right)$$

Now
$$-\int ydF = -\int y\left(\gamma y\sin\theta\right)dA = -\gamma\sin\theta\int y^2 dA$$

$$= -\left(\gamma\sin\theta\right)I_x$$

Then
$$-\bar{y}P - M_{x'} = -\left(\gamma\sin\theta\right)I_x$$

or
$$M_{x'} = \left(\gamma\sin\theta\right)I_x - \bar{y}\left(\gamma A\bar{y}\sin\theta\right)$$

$$= \gamma\sin\theta\left(I_x - A\bar{y}^2\right)$$

$$\text{or} \quad M_{x'} = \gamma \bar{I}_{x'}\sin\theta \blacktriangleleft$$

$$\Sigma M_y: \quad \bar{x}P + M_{y'} = \int xdF$$

Now
$$\int xdF = \int x\left(\gamma y\sin\theta\right)dA = \gamma\sin\theta\int xydA$$

$$= \left(\gamma\sin\theta\right)I_{xy} \qquad \text{(Equation 9.12)}$$

PROBLEM 9.65 CONTINUED

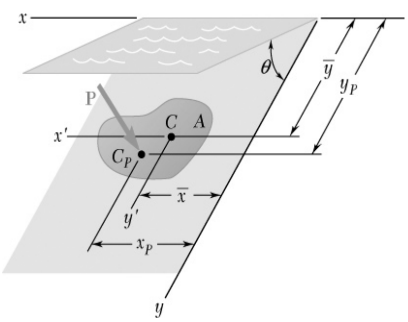
Then
$$\bar{x}P + M_{y'} = (\gamma \sin\theta) I_{xy}$$

or
$$M_{y'} = (\gamma \sin\theta) I_{xy} - \bar{x}(\gamma A\bar{y} \sin\theta)$$

$$= \gamma \sin\theta \left(I_{xy} - A\bar{x}\,\bar{y}\right)$$

or, using Equation 9.13,

or $M_{y'} = \gamma \bar{I}_{x'y'} \sin\theta$ ◀



PROBLEM 9.66

Show that the resultant of the hydrostatic forces acting on a submerged plane area A is a force **P** perpendicular to the area and of magnitude $P = \gamma A\bar{y}\sin\theta = \bar{p}A$, where γ is the specific weight of the liquid and $\bar{p}$ is the pressure at the centroid C of the area. Show that **P** is applied at a point C_p, called the center of pressure, whose coordinates are $x_p = I_{xy}/A\bar{y}$ and $y_p = I_x/A\bar{y}$, where $\bar{I}_{xy} = \int xy\,dA$ (see Sec. 9.8). Show also that the difference of ordinates $y_p - \bar{y}$ is equal to $\bar{k}_{x'}^2/\bar{y}$ and thus depends upon the depth at which the area is submerged.

SOLUTION

The pressure p at an arbitrary depth $(y\sin\theta)$ is

$$p = \gamma(y\sin\theta)$$

so that the hydrostatic force dP exerted on an infinitesimal area dA is

$$dP = (\gamma y\sin\theta)dA$$

The magnitude **P** of the resultant force acting on the plane area is then

$$P = \int dP = \int \gamma y\sin\theta dA = \gamma\sin\theta\int ydA$$

$$= \gamma\sin\theta(\bar{y}A)$$

Now $$\bar{p} = \gamma\bar{y}\sin\theta \qquad \therefore\ P = \bar{p}A \blacktriangleleft$$

Next observe that the resultant **P** is equivalent to the system of infinitesimal forces d**P**. Equivalence then requires

$$\Sigma M_x:\quad -y_P P = -\int ydP$$

Now $$\int ydP = \int y(\gamma y\sin\theta)dA = \gamma\sin\theta\int y^2 dA$$

$$= (\gamma\sin\theta)I_x$$

Then $$y_P P = (\gamma\sin\theta)I_x$$

or $$y_P = \frac{(\gamma\sin\theta)I_x}{\gamma\sin\theta(\bar{y}A)}$$

$$\text{or}\quad y_P = \frac{I_x}{A\bar{y}} \blacktriangleleft$$

$$\Sigma M_y:\quad x_P P = \int xdP$$

Now $$\int xdP = \int x(\gamma y\sin\theta)dA = \gamma\sin\theta\int xydA$$

$$= (\gamma\sin\theta)I_{xy} \qquad \text{(Equation 9.12)}$$

PROBLEM 9.66 CONTINUED

Then $$x_P P = (\gamma \sin\theta) I_{xy}$$

or $$x_P = \frac{(\gamma \sin\theta) I_{xy}}{\gamma \sin\theta (\bar{y} A)}$$

$$\text{or} \quad x_P = \frac{I_{xy}}{A\bar{y}} \blacktriangleleft$$

Now $$I_x = \bar{I}_{x'} + A\bar{y}^2$$

From above $$I_x = (A\bar{y}) y_P$$

By definition $$\bar{I}_{x'} = \bar{k}_{x'}^2 A$$

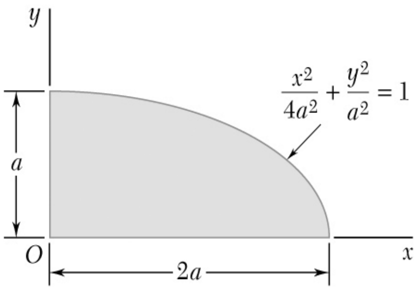
Substituting $$(A\bar{y}) y_P = \bar{k}_{x'}^2 A + A\bar{y}^2$$

Rearranging yields $$y_P - \bar{y} = \frac{\bar{k}_{x'}^2}{\bar{y}} \blacktriangleleft$$

Although $\bar{k}_{x'}$ is not a function of the depth of the area (it depends only on the shape of A), $\bar{y}$ is dependent on the depth.

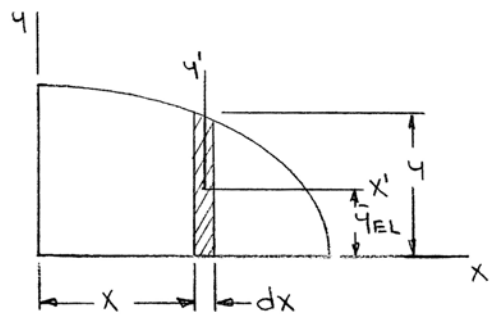
$$\therefore (y_P - \bar{y}) = f(\text{depth})$$

PROBLEM 9.67



Determine by direct integration the product of inertia of the given area with respect to the x and y axes.

SOLUTION



First note

$$y = a\sqrt{1 - \frac{x^2}{4a^2}}$$

$$= \frac{1}{2}\sqrt{4a^2 - x^2}$$

Have

$$dI_{xy} = d\bar{I}_{x'y'} + \bar{x}_{EL}\bar{y}_{EL}dA$$

where

$$d\bar{I}_{x'y'} = 0 \qquad (\text{symmetry}) \qquad \bar{x}_{EL} = x$$

$$\bar{y}_{EL} = \frac{1}{2}y = \frac{1}{4}\sqrt{4a^2 - x^2}$$

$$dA = ydx = \frac{1}{2}\sqrt{4a^2 - x^2 dx}$$

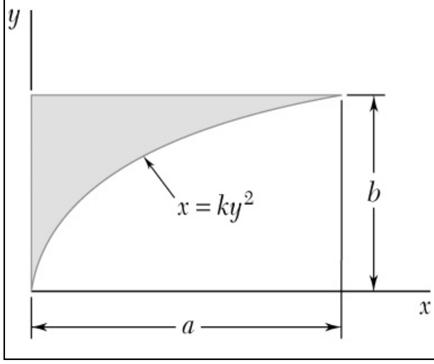
Then

$$I_{xy} = \int dI_{xy} = \int_0^{2a} x\left(\frac{1}{4}\sqrt{4a^2 - x^2}\right)\left(\frac{1}{2}\sqrt{4a^2 - x^2}\right)dx$$

$$= \frac{1}{8}\int_0^{2a}\left(4a^2x - x^3\right)dx = \frac{1}{8}\left[2a^2x^2 - \frac{1}{4}x^4\right]_0^{2a}$$

$$= \frac{a^4}{8}\left[2(2)^2 - \frac{1}{4}(2)^4\right]$$

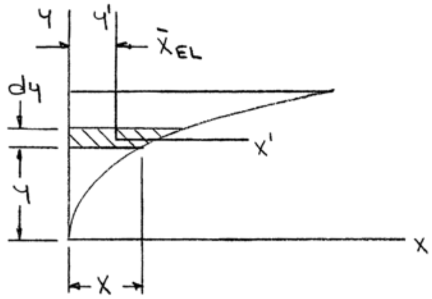
or $I_{xy} = \frac{1}{2}a^4$ ◀



PROBLEM 9.68

Determine by direct integration the product of inertia of the given area with respect to the x and y axes.

SOLUTION



At $\quad x = a,\ y = b: \quad a = kb^2$

or $\quad k = \dfrac{a}{b^2}$

Then $\quad x = \dfrac{a}{b^2}y^2$

Have $\quad dI_{xy} = d\bar{I}_{x'y'} + \bar{x}_{EL}\bar{y}_{EL}dA$

where $\quad d\bar{I}_{x'y'} = 0 \quad (\text{symmetry}) \qquad \bar{x}_{EL} = \dfrac{1}{2}x = \dfrac{a}{2b^2}y^2$

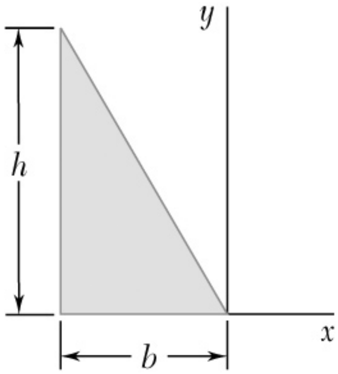
$$\bar{y}_{EL} = y \qquad dA = xdy = \frac{a}{b^2}y^2dy$$

Then $\quad I_{xy} = \int dI_{xy} = \int_0^b \left(\dfrac{a}{2b^2}y^2\right)(y)\left(\dfrac{a}{b^2}y^2dy\right)$

$$= \frac{a^2}{2b^4}\int_0^b y^5dy = \frac{a^2}{2b^4}\left[\frac{1}{6}y^6\right]_0^b$$

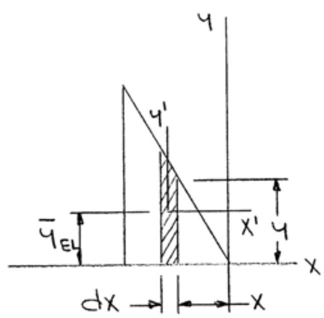
or $\quad I_{xy} = \dfrac{1}{12}a^2b^2$ ◀

PROBLEM 9.69



Determine by direct integration the product of inertia of the given area with respect to the *x* and *y* axes.

SOLUTION



First note that

$$y = -\frac{h}{b}x$$

Now $$dI_{xy} = d\bar{I}_{x'y'} + \bar{x}_{EL}\bar{y}_{EL}dA$$

where $$d\bar{I}_{x'y'} = 0 \ \text{(symmetry)}$$

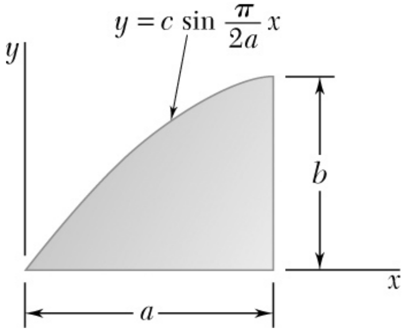
$$\bar{x}_{EL} = x \qquad \bar{y}_{EL} = \frac{1}{2}y = -\frac{1}{2}\frac{h}{b}x$$

$$dA = ydx = -\frac{h}{b}xdx$$

Then $$I_{xy} = \int dI_{xy} = \int_{-b}^{0} x\left(-\frac{1}{2}\frac{h}{b}x\right)\left(-\frac{h}{b}xdx\right)$$

$$= \frac{1}{2}\frac{h^2}{b^2}\int_{-b}^{0} x^3dx = \frac{1}{2}\frac{h^2}{b^2}\left[\frac{1}{4}x^4\right]_{-b}^{0}$$

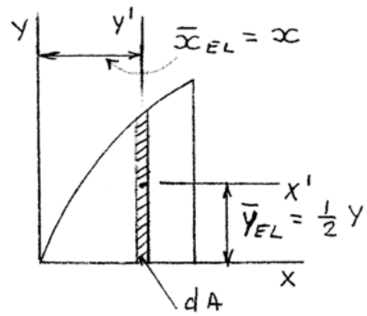
or $I_{xy} = -\frac{1}{8}b^2h^2$ ◀



PROBLEM 9.70

Determine by direct integration the product of inertia of the given area with respect to the *x* and *y* axes.

SOLUTION



At $\qquad x = a,\ y = b: \quad b = c\sin\left(\frac{\pi}{2a}a\right) \qquad \text{or} \qquad c = b$

Now $\qquad y = b\sin\frac{\pi}{2a}x$

Have $\qquad d\bar{I}_{x'y'} = 0 \qquad \text{(symmetry)}$

$$dA = ydx$$

Now $\qquad dI_{xy} = \cancelto{0}{d\bar{I}_{x'y'}} + \bar{x}_{\text{EL}}\bar{y}_{\text{EL}}dA$

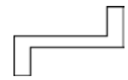
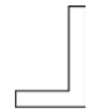
or $\qquad I_{xy} = \int_0^a x\left(\frac{1}{2}y\right)(ydx) = \frac{1}{2}\int_0^a xb^2\sin^2\frac{\pi}{2a}xdx$

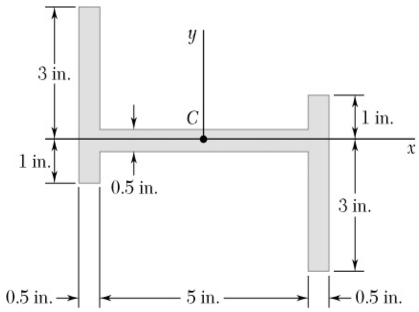
$$= \frac{b^2}{2}\left[\frac{x^2}{4} - \frac{x\sin\frac{\pi}{a}x}{4\left(\frac{\pi}{2a}\right)} - \frac{\cos\frac{\pi}{a}x}{8\left(\frac{\pi}{2a}\right)^2}\right]_0^a$$

$$= \frac{b^2}{2}\left(\frac{a^2}{4} + \frac{4a^2}{8\pi^2} + \frac{4a^2}{8\pi^2}\right)$$

or $\quad I_{xy} = \dfrac{a^2b^2}{8\pi^2}\left(4 + \pi^2\right)$ ◀

The following table is provided for the convenience of the instructor, as many problems in this and the next lesson are related.

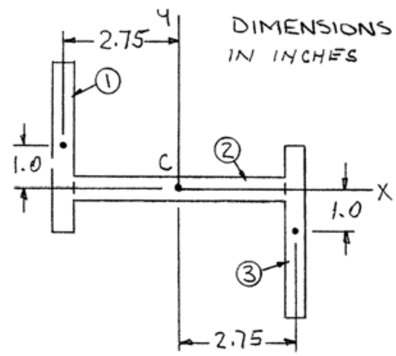
Type of Problem						
Compute I_x and I_y	Fig. 9.12			Fig. 9.13B		Fig. 9.13A
Compute I_{xy}	9.67	9.72	9.73	9.74	9.75	9.78
$I_{x'}$, $I_{y'}$, $I_{x'y'}$ by equations	*9.79*	9.80	*9.81*	9.83	9.82	9.84
Principal axes by equations	*9.85*	9.86	*9.87*	9.89	9.88	9.90
$I_{x'}$, $I_{y'}$, $I_{x'y'}$ by Mohr's circle	*9.91*	9.92	*9.93*	9.95	9.94	9.96
Principal axes by Mohr’s circle	9.97	9.98	9.100	9.101	9.103	9.106



PROBLEM 9.71

Using the parallel-axis theorem, determine the product of inertia of the area shown with respect to the centroidal *x* and *y* axes.

SOLUTION



Have $$\bar{I}_{xy} = (I_{xy})_1 + (\bar{I}_{xy})_2 + (I_{xy})_3$$

Symmetry implies $$(I_{xy})_2 = 0$$

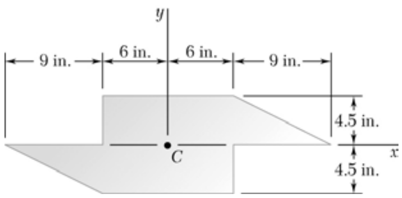
For the other rectangles $$I_{xy} = \bar{I}_{x'y'} + \bar{x}\,\bar{y}A$$

Where symmetry implies $$I_{x'y'} = 0$$

	A in^2	$\bar{x}$, in.	$\bar{y}$, in.	$A\bar{x}\,\bar{y}$ in^4
1	$4(0.5) = 2$	–2.75	1.0	–5.5
3	$4(0.5) = 2$	2.75	–1.0	–5.5
Σ				–11.00

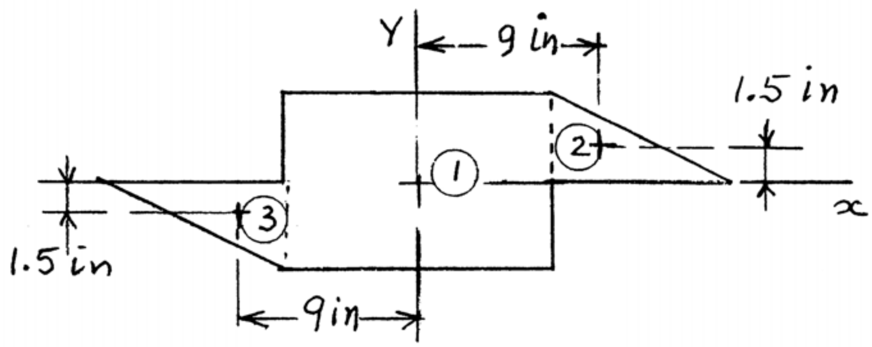
or $\bar{I}_{xy} = -11.00\text{ in}^4$ ◀

PROBLEM 9.72



Using the parallel-axis theorem, determine the product of inertia of the area shown with respect to the centroidal x and y axes.

SOLUTION



Note: Orientation of A_3 corresponding to a 180° rotation of the axes. Equation 9.20 then yields

$$I_{x'y'} = I_{xy}$$

Symmetry implies $\quad (I_{xy})_1 = 0$

Using Sample Problem 9.6 $\quad (\bar{I}_{x'y'})_2 = -\frac{1}{72}(9\text{ in.})^2(4.5\text{ in.})^2 = -22.78125\text{ in}^4$

and $\quad \bar{X}_2 = 9\text{ in.} \qquad \bar{Y}_2 = 1.5\text{ in.} \qquad A_2 = \frac{1}{2}(9\text{ in.})(4.5\text{ in.}) = 20.25\text{ in}^2$

Similarly, $\quad (\bar{I}_{x'y'})_3 = -\frac{1}{72}(9\text{ in.})^2(4.5\text{ in.})^2 = -22.78125\text{ in}^4$

and $\quad \bar{X}_3 = -9\text{ in.} \qquad \bar{Y}_2 = -1.5\text{ in.} \qquad A_3 = \frac{1}{2}(9\text{ in.})(4.5\text{ in.}) = 20.25\text{ in}^2$

Then $\quad \bar{I}_{xy} = \cancelto{0}{(I_{xy})_1} + (I_{xy})_2 + (I_{xy})_3 \qquad \text{with} \qquad (I_{xy})_2 = (I_{xy})_3$

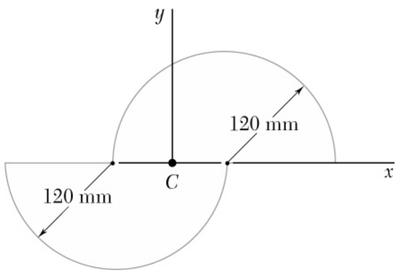
and $\quad I_{xy} = \bar{I}_{x'y'} + \bar{x}\,\bar{y}\,A$

Therefore, $\quad \bar{I}_{xy} = 2\left[-22.78125 + (9)(1.5)(20.25)\right]\text{in}^4$

$$= 501.1875\text{ in}^4$$

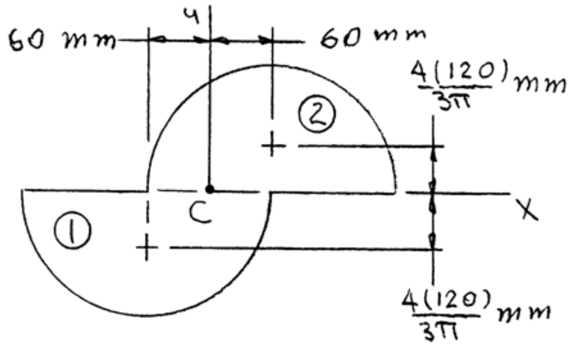
or $\bar{I}_{xy} = 501\text{ in}^4$ ◀

PROBLEM 9.73



Using the parallel-axis theorem, determine the product of inertia of the area shown with respect to the centroidal x and y axes.

SOLUTION



Have $\quad \bar{I}_{xy} = \left(I_{xy}\right)_1 + \left(I_{xy}\right)_2$

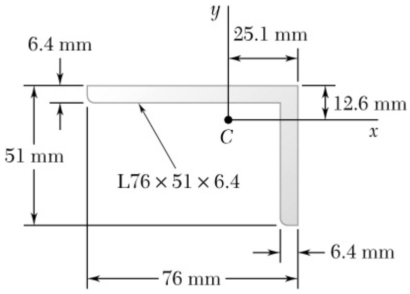
For each semicircle $\quad I_{xy} = \bar{I}_{x'y'} + \bar{x}\,\bar{y}\,A \quad$ and $\quad \bar{I}_{x'y'} = 0$ (symmetry)

Thus $\quad \bar{I}_{xy} = \Sigma \bar{x}\,\bar{y}\,A$

	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$A\bar{x}\,\bar{y}$, mm^4
1	$\frac{\pi}{2}(120)^2 = 7200\pi$	-60	$-\frac{160}{\pi}$	69.12×10^6
2	$\frac{\pi}{2}(120)^2 = 7200\pi$	60	$\frac{160}{\pi}$	69.12×10^6
Σ				138.24×10^6

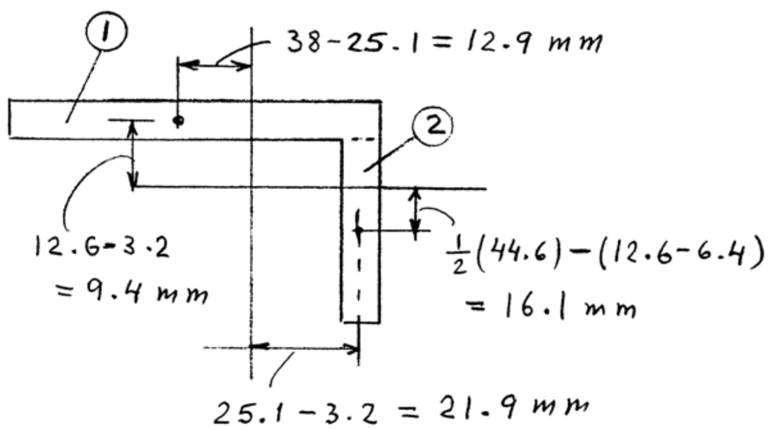
or $\bar{I}_{xy} = 138.2 \times 10^6 \text{ mm}^4$ ◀

PROBLEM 9.74



Using the parallel-axis theorem, determine the product of inertia of the area shown with respect to the centroidal x and y axes.

SOLUTION



Have $$I_{xy} = (I_{xy})_1 + (I_{xy})_2$$

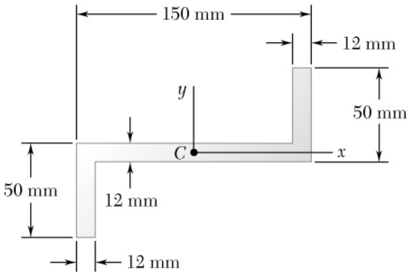
For each rectangle $$I_{xy} = \bar{I}_{x'y'} + A\bar{x}\,\bar{y} \qquad \text{and} \qquad \bar{I}_{x'y'} = 0 \text{ (symmetry)}$$

Thus $$\bar{I}_{xy} = \Sigma \bar{x}\,\bar{y}\,A$$

	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$A\bar{x}\,\bar{y}$, mm^4
1	$76(6.4) = 486.4$	–12.9	9.4	–58 980.86
2	$44.6(6.4) = 285.44$	21.9	–16.1	–100 643.29
Σ				–159 624.15

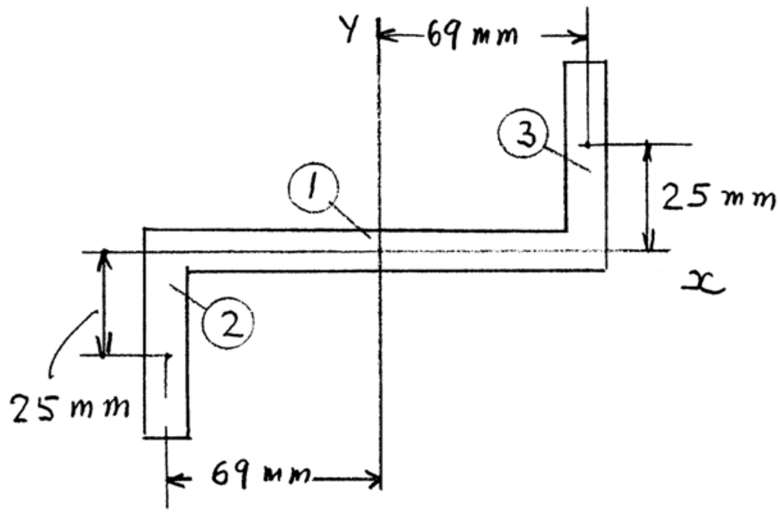
or $\bar{I}_{xy} = -0.1596 \times 10^6 \text{ mm}^4$ ◀

PROBLEM 9.75



Using the parallel-axis theorem, determine the product of inertia of the area shown with respect to the centroidal x and y axes.

SOLUTION



Have $\qquad \bar{I}_{xy} = \left(\bar{I}_{xy}\right)_1 + \left(I_{xy}\right)_2 + \left(I_{xy}\right)_3$

Now symmetry implies $\qquad \left(\bar{I}_{xy}\right)_1 = 0$

and for the other rectangles $\qquad I_{xy} = \bar{I}_{x'y'} + \bar{x}\,\bar{y}A \qquad$ where $\qquad \bar{I}_{x'y'} = 0$ (symmetry)

Thus

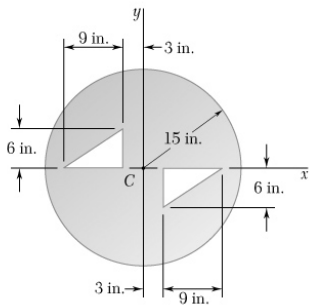
$$\bar{I}_{xy} = \left(\bar{x}\,\bar{y}A\right)_2 + \left(\bar{x}\,\bar{y}\right)A_3$$

$$= (-69 \text{ mm})(-25 \text{ mm})\left[(12 \text{ mm})(38 \text{ mm})\right]$$

$$+ (69 \text{ mm})(25 \text{ mm})\left[(12 \text{ mm})(38 \text{ mm})\right]$$

$$= (786\ 600 + 786\ 600) \text{ mm}^4 = 1\ 573\ 200 \text{ mm}^4$$

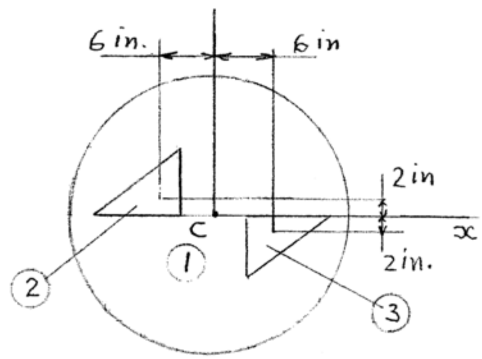
or $\bar{I}_{xy} = 1.573 \times 10^6 \text{ mm}^4$ ◀



PROBLEM 9.76

Using the parallel-axis theorem, determine the product of inertia of the area shown with respect to the centroidal *x* and *y* axes.

SOLUTION



Symmetry implies $\qquad (I_{xy})_1 = 0$

Using Sample Problem 9.6 and Equation 9.20, note that the orientation of A_2 corresponds to a 90° rotation of the axes; thus $\qquad (\bar{I}_{x'y'})_2 = \frac{1}{72}b^2h^2$

Also, the orientation of A_3 corresponds to a 270° rotation of the axes; thus $\qquad (\bar{I}_{x'y'})_3 = \frac{1}{72}b^2h^2$

Then $\qquad (\bar{I}_{x'y'})_2 = \frac{1}{72}(9\text{ in.})^2(6\text{ in.})^2 = 40.5\text{ in}^4$

and $\qquad \bar{x}_2 = 6\text{ in.}, \quad \bar{y}_2 = -2\text{ in.}, \quad A_2 = \frac{1}{2}(9\text{ in.})(6\text{ in.}) = 27\text{ in}^2$

Also $\qquad (\bar{I}_{x'y'})_3 = (\bar{I}_{x'y'})_2 = 40.5\text{ in}^4$

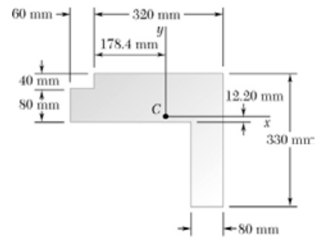
and $\qquad \bar{x}_3 = -6\text{ in.}, \quad \bar{y}_3 = 2\text{ in.}, \quad A_3 = A_2 = 27\text{ in}^2$

Now $\qquad \bar{I}_{xy} = \cancel{(I_{xy})_1}^{\,0} - (I_{xy})_2 - (I_{xy})_3 \quad$ and $\quad I_{xy} = \bar{I}_{x'y'} + \bar{x}\,\bar{y}A \quad (I_{xy})_2 = (I_{xy})_3$

Then $\qquad \bar{I}_{xy} = -2\left[40.5\text{ in}^4 + (6\text{ in.})(-2\text{ in.})(27\text{ in}^2)\right]$

$$= -2(40.5 - 324)\text{ in}^4$$

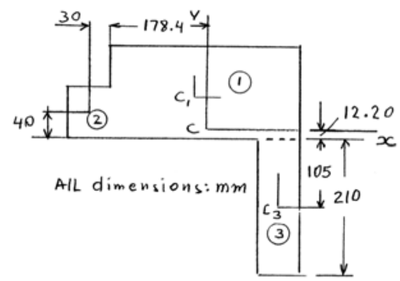
or $\bar{I}_{xy} = 567\text{ in}^4$ ◀



PROBLEM 9.77

Using the parallel-axis theorem, determine the product of inertia of the area shown with respect to the centroidal *x* and *y* axes.

SOLUTION



Have $$I_{xy} = \bar{I}_{x'y'} + \bar{x}\,\bar{y}A$$

Where $\bar{I}_{x'y'} = 0$ for each rectangle

Then $$\bar{I}_{xy} = (I_{xy})_1 + (I_{xy})_2 + (I_{xy})_3$$
$$= \Sigma\bar{x}\,\bar{y}A$$

Now $$\bar{x}_1 = -(178.4 \text{ mm} - 160 \text{ mm}) = -18.4 \text{ mm}$$

$$\bar{y}_1 = 60 \text{ mm} - 12.2 \text{ mm} = 47.8 \text{ mm}$$

$$A_1 = 320 \text{ mm} \times 120 \text{ mm} = 38400 \text{ mm}^2$$

and $$\bar{x}_2 = -(178.4 \text{ mm} + 30 \text{ mm}) = -208.4 \text{ mm}$$

$$\bar{y}_2 = 40 \text{ mm} - 12.2 \text{ mm} = 27.8 \text{ mm}$$

$$A_2 = 60 \text{ mm} \times 80 \text{ mm} = 4800 \text{ mm}^2$$

and $$\bar{x}_3 = (320 \text{ mm} - 178.4 \text{ mm}) - 40 \text{ mm} = 101.6 \text{ mm}$$

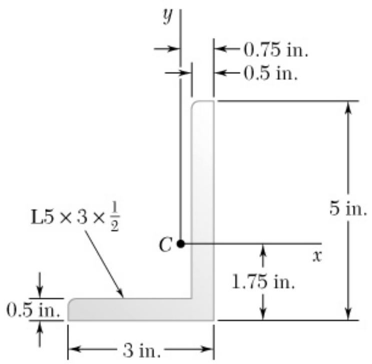
$$\bar{y}_3 = -(12.2 \text{ mm} + 105 \text{ mm}) = -117.2 \text{ mm}$$

$$A_3 = (80 \text{ mm} \times 210 \text{ mm}) = 16800 \text{ mm}^2$$

Then $$\bar{I}_{xy} = \left[(-18.4 \text{ mm})(47.8 \text{ mm})(38400 \text{ mm}^2)\right] + \left[(-208.4 \text{ mm})(27.8 \text{ mm})(4800 \text{ mm}^2)\right]$$
$$+ \left[(101.6 \text{ mm})(-117.2 \text{ mm})(16800 \text{ mm}^2)\right]$$
$$= -(33.7736 + 27.8089 + 200.0463) \times 10^6 \text{ mm}^4$$
$$= -261.6288 \times 10^6 \text{ mm}^4$$

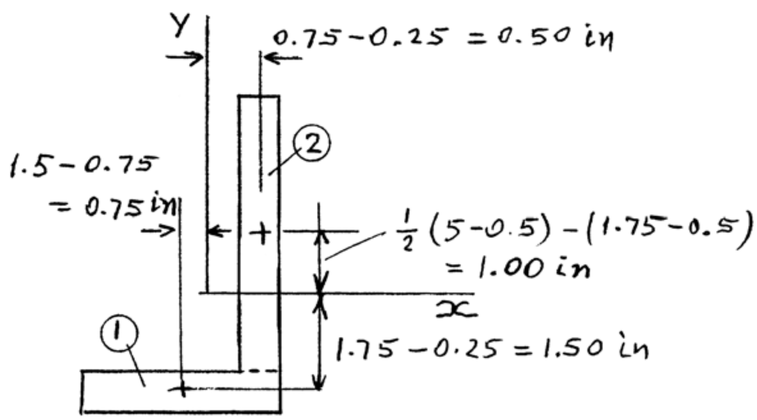
or $\bar{I}_{xy} = -262 \times 10^6 \text{ mm}^4$ ◀

PROBLEM 9.78



Using the parallel-axis theorem, determine the product of inertia of the area shown with respect to the centroidal *x* and *y* axes.

SOLUTION



Have
$$\bar{I}_{xy} = \left(I_{xy}\right)_1 + \left(I_{xy}\right)_2$$

For each rectangle

$$I_{xy} = \bar{I}_{x'y'} + \bar{x}\,\bar{y}A \qquad \text{and} \qquad \bar{I}_{x'y'} = 0 \text{ (symmetry)}$$

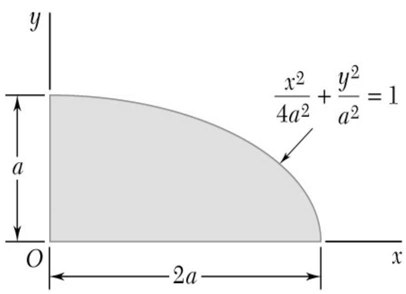
Then
$$\bar{I}_{xy} = \Sigma\bar{x}\,\bar{y}A = (-0.75\text{ in.})(-1.5\text{ in.})\left[(3\text{ in.})(0.5\text{ in.})\right]$$

$$+ (0.5\text{ in.})(1.00\text{ in.})\left[(4.5\text{ in.})(0.5\text{ in.})\right]$$

$$= (1.6875 + 1.125)\text{in}^4 = 2.8125\text{ in}^4$$

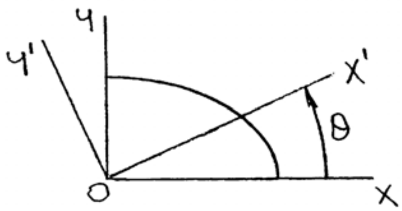
or $\bar{I}_{xy} = 2.81\text{ in}^4$ ◀

PROBLEM 9.79



Determine for the quarter ellipse of Problem 9.67 the moments of inertia and the product of inertia with respect to new axes obtained by rotating the x and y axes about O (a) through 45° counterclockwise, (b) through 30° clockwise.

SOLUTION



From Figure 9.12:
$$I_x = \frac{\pi}{16}(2a)(a)^3$$
$$= \frac{\pi}{8}a^4$$
$$I_y = \frac{\pi}{16}(2a)^3(a)$$
$$= \frac{\pi}{2}a^4$$

From Problem 9.67:
$$I_{xy} = \frac{1}{2}a^4$$

First note
$$\frac{1}{2}(I_x + I_y) = \frac{1}{2}\left(\frac{\pi}{8}a^4 + \frac{\pi}{2}a^4\right) = \frac{5}{16}\pi a^4$$
$$\frac{1}{2}(I_x - I_y) = \frac{1}{2}\left(\frac{\pi}{8}a^4 - \frac{\pi}{2}a^4\right) = -\frac{3}{16}\pi a^4$$

Now use Equations (9.18), (9.19), and (9.20).

Equation (9.18):
$$I_{x'} = \frac{1}{2}(I_x + I_y) + \frac{1}{2}(I_x - I_y)\cos 2\theta - I_{xy}\sin 2\theta$$
$$= \frac{5}{16}\pi a^4 - \frac{3}{16}\pi a^4\cos 2\theta - \frac{1}{2}a^4\sin 2\theta$$

Equation (9.19):
$$I_{y'} = \frac{1}{2}(I_x + I_y) - \frac{1}{2}(I_x - I_y)\cos 2\theta + I_{xy}\sin 2\theta$$
$$= \frac{5}{16}\pi a^4 + \frac{3}{16}\pi a^4\cos 2\theta + \frac{1}{2}a^4\sin 2\theta$$

Equation (9.20):
$$I_{x'y'} = \frac{1}{2}(I_x - I_y)\sin 2\theta + I_{xy}\cos 2\theta$$
$$= -\frac{3}{16}\pi a^4\sin 2\theta + \frac{1}{2}a^4\cos 2\theta$$

PROBLEM 9.79 CONTINUED

(*a*) $\theta = +45°$: $$I_{x'} = \frac{5}{16}\pi a^4 - \frac{3}{16}\pi a^4 \cos 90° - \frac{1}{2}a^4 \sin 90°$$

or $I_{x'} = 0.482a^4$ ◀

$$I_{y'} = \frac{5}{16}\pi + \frac{3}{16}\pi a^4 \cos 90° + \frac{1}{2}a^4$$

or $I_{y'} = 1.482a^4$ ◀

$$I_{x'y'} = -\frac{3}{16}\pi a^4 \sin 90° + \frac{1}{2}a^4 \cos 90°$$

or $I_{x'y'} = -0.589a^4$ ◀

(*b*) $\theta = -30°$:

$$I_{x'} = \frac{5}{16}\pi a^4 - \frac{3}{16}\pi a^4 \cos(-60°) - \frac{1}{2}a^4 \sin(-60°)$$

or $I_{x'} = 1.120a^4$ ◀

$$I_{y'} = \frac{5}{16}\pi a^4 + \frac{3}{16}\pi a^4 \cos(-60°) + \frac{1}{2}a^4 \sin(-60°)$$

or $I_{y'} = 0.843a^4$ ◀

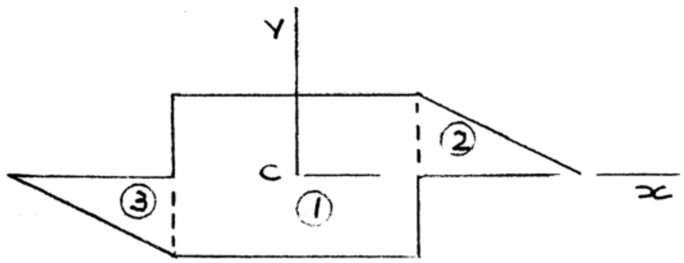
$$I_{x'y'} = -\frac{3}{16}\pi a^4 \sin(-60°) + \frac{1}{2}a^4 \cos(-60°)$$

or $I_{x'y'} = 0.760a^4$ ◀

PROBLEM 9.80

Determine the moments of inertia and the product of inertia of the area of Problem 9.72 with respect to new centroidal axes obtained by rotating the x and y axes 45° clockwise.

SOLUTION



From the solution to Problem 9.72

$$\bar{I}_{xy} = 501.1875 \text{ in}^4$$

$$A_2 = A_3 = 20.25 \text{ in}^2$$

First compute the moment of inertia

$$\bar{I}_x = (I_x)_1 + (I_x)_2 + (I_x)_3 \qquad \text{with} \qquad (I_x)_2 = (I_x)_3$$

$$= \left[\frac{1}{12}(12 \text{ in.})(9 \text{ in.})^3\right] + 2\left[\frac{1}{12}(9 \text{ in.})(4.5\text{in.})^3\right]$$

$$= (729 + 136.6875) \text{ in}^4 = 865.6875 \text{ in}^4$$

and

$$\bar{I}_y = (I_y)_1 + (I_y)_2 + (I_y)_3 \qquad \text{with} \qquad (I_y)_2 = (I_y)_3$$

$$= \left[\frac{1}{12}(9 \text{ in.})(12 \text{ in.})^3\right] + 2\left[\frac{1}{36}(4.5 \text{ in.})(9 \text{ in.})^3 + (20.25 \text{ in}^2)(9 \text{ in.})^2\right]$$

$$= (1296 + 182.25 + 3280.5)\text{in}^4 = 4758.75 \text{ in}^4$$

From Equation 9.18

$$\bar{I}_{x'} = \frac{\bar{I}_x + \bar{I}_y}{2} + \frac{\bar{I}_x - \bar{I}_y}{2}\cos 2\theta - \bar{I}_{xy}\sin 2\theta$$

$$= \frac{865.6875 \text{ in}^4 + 4758.75 \text{ in}^4}{2} + \frac{865.6875 \text{ in}^4 - 4758.75 \text{ in}^4}{2}\cos[2(-45°)]$$

$$-501.1875 \text{ in}^4 \sin[2(-45°)]$$

$$= (2812.21875 + 501.1875)\text{in}^4 = 3313.4063 \text{ in}^4$$

or $\bar{I}_{x'} = 3.31 \times 10^3 \text{ in}^4$ ◀

PROBLEM 9.80 CONTINUED

Similarly

$$\bar{I}_{y'} = \frac{\bar{I}_x + \bar{I}_y}{2} - \frac{\bar{I}_x - \bar{I}_y}{2}\cos 2\theta + \bar{I}_{xy}\sin 2\theta$$

$$= (2812.21875 - 501.1875)\text{in}^4 = 2311.0313 \text{ in}^4$$

$$\text{or } \bar{I}_{y'} = 2.31 \times 10^3 \text{ in}^4 \blacktriangleleft$$

and

$$\bar{I}_{x'y'} = \frac{\bar{I}_x - \bar{I}_y}{2}\sin 2\theta + \bar{I}_{xy}\cos 2\theta$$

$$= \frac{865.6875 \text{ in}^4 - 4758.75 \text{ in}^4}{2}\sin\left[2(-45°)\right]$$

$$+ 501.1875\cos\left[2(-45°)\right]$$

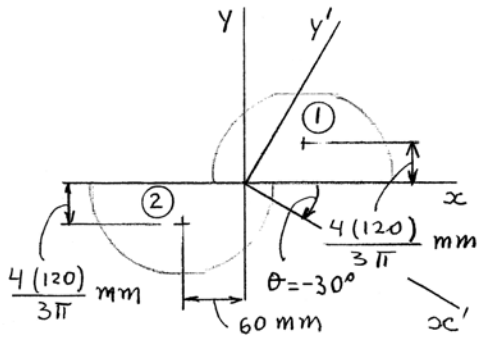
$$= (-1946.53125)(-1)\text{in}^4 = 1946.53125 \text{ in}^4$$

$$\text{or } \bar{I}_{x'y'} = 1.947 \times 10^3 \text{ in}^4 \blacktriangleleft$$

PROBLEM 9.81

Determine the moments of inertia and the product of inertia of the area of Problem 9.73 with respect to new centroidal axes obtained by rotating the x and y axes through 30° clockwise.

SOLUTION



From Problem 9.73, $\bar{I}_{xy} = 138.24 \times 10^6 \text{ mm}^4$

$$\bar{I}_x = (I_x)_1 + (I_x)_2 \qquad (I_x)_1 = (I_x)_2$$

$$= 2\left[\frac{\pi}{8}(120 \text{ mm})^4\right]$$

$$= 51.84\pi \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = (I_y)_1 + (I_y)_2 \qquad (I_y)_1 = (I_y)_2$$

$$= 2\left[\frac{\pi}{8}(120 \text{ mm})^4 + \frac{\pi}{2}(120 \text{ mm})^2 (60 \text{ mm})^2\right]$$

$$= 103.68\pi \times 10^6 \text{ mm}^4$$

Have $\bar{I}_x = 2\left(25.92\pi \times 10^6\right) = 51.84\pi \times 10^6 \text{ mm}^4$

and $\bar{I}_y = 2\left(51.84\pi \times 10^6\right) = 103.68\pi \times 10^6 \text{ mm}^4$

Then $\frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = 77.76\pi \times 10^6 \text{ mm}^4$

and $\frac{1}{2}\left(\bar{I}_x - \bar{I}_y\right) = -25.92\pi \times 10^6 \text{ mm}^4$

PROBLEM 9.81 CONTINUED

Now, from Equations 9.18, 9.19, and 9.20

Equation 9.18: $$\bar{I}_{x'} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) + \frac{1}{2}\left(\bar{I}_x - \bar{I}_y\right)\cos 2\theta - \bar{I}_{xy}\sin 2\theta$$

$$= \left[77.76\pi \times 10^6 - 25.92\pi \times 10^6 \cos\left(-60°\right) - 138.24 \times 10^6 \sin\left(-60°\right)\right]\text{mm}^4$$

$$= 323.29 \times 10^6 \text{ mm}^4$$

or $\bar{I}_x = 323 \times 10^6 \text{ mm}^4$ ◀

Equation 9.19: $$\bar{I}_{y'} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) - \frac{1}{2}\left(\bar{I}_x - \bar{I}_y\right)\cos 2\theta + \bar{I}_{xy}\sin 2\theta$$

$$= \left[77.76\pi \times 10^6 + 25.92\pi \times 10^6 \cos\left(-60°\right) + 138.24 \times 10^6 \sin\left(-60°\right)\right]\text{mm}^4$$

$$= 165.29 \times 10^6 \text{ mm}^4$$

or $\bar{I}_{y'} = 165.29 \times 10^6 \text{ mm}^4$ ◀

Equation 9.20: $$\bar{I}_{x'y'} = \frac{1}{2}\left(\bar{I}_x - \bar{I}_y\right)\sin 2\theta + \bar{I}_{xy}\cos 2\theta$$

$$= \left[-25.92\pi \times 10^6 \sin\left(-60°\right) + 138.24 \times 10^6 \cos\left(-60°\right)\right]\text{mm}^4$$

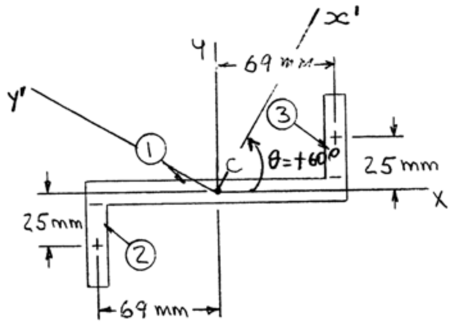
$$= 139.64 \times 10^6 \text{ mm}^4$$

or $\bar{I}_{x'y'} = 139.6 \times 10^4 \text{ mm}^4$ ◀

PROBLEM 9.82

Determine the moments of inertia and the product of inertia of the area of Problem 9.75 with respect to new centroidal axes obtained by rotating the x and y axes through 60° counterclockwise.

SOLUTION



From Problem 9.75 $\quad \bar{I}_{xy} = 1.5732 \times 10^6 \text{ mm}^4$

Now $\quad \bar{I}_x = (I_x)_1 + (I_x)_2 + (I_x)_3$

where $\quad (I_x)_1 = \frac{1}{12}(150 \text{ mm})(12 \text{ mm})^3 = 21\,600 \text{ mm}^4$

and $\quad (I_x)_2 = (I_x)_3 = \frac{1}{12}(12 \text{ mm})(38 \text{ mm})^3 + \left[(12 \text{ mm})(38 \text{ mm})\right](25 \text{ mm})^2$

$$= 339\,872 \text{ mm}^4$$

Then $\quad \bar{I}_x = \left[21\,600 + 2(339\,872)\right]\text{mm}^4 = 701\,344 \text{ mm}^4 = 0.70134 \times 10^6 \text{ mm}^4$

Also $\quad \bar{I}_y = (I_y)_1 + (I_y)_2 + (I_y)_3$

where $\quad (I_y)_1 = \frac{1}{12}(12 \text{ mm})(150 \text{ mm})^3 = 3.375 \times 10^6 \text{ mm}^4$

and $\quad (I_y)_2 = (I_y)_3 = \frac{1}{12}(38 \text{ mm})(12 \text{ mm})^3 + \left[(12 \text{ mm})(38 \text{ mm})\right](69 \text{ mm})^2$

$$= 2.1765 \times 10^6 \text{ mm}^4$$

Then $\quad \bar{I}_y = \left[(3.375 + 2(2.1765)\right] \times 10^6 \text{ mm}^4 = 7.728 \times 10^6 \text{ mm}^4$

Now $\quad \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = 4.2146 \times 10^6 \text{ mm}^4$

and $\quad \frac{1}{2}\left(\bar{I}_x - \bar{I}_y\right) = -3.5133 \times 10^6 \text{ mm}^4$

PROBLEM 9.82 CONTINUED

Using Equations 9.18, 9.19, and 9.20

From Equation 9.18:

$$\bar{I}_{x'} = \frac{\bar{I}_x + \bar{I}_y}{2} + \frac{\bar{I}_x - \bar{I}_y}{2}\cos 2\theta - \bar{I}_{xy}\sin 2\theta$$

$$= \left[4.2147\times 10^6 + \left(-3.5133\times 10^6\right)\cos\left(120°\right) - 1.5732\times 10^6\sin\left(120°\right)\right]\text{mm}^4$$

$$= 4.6089\times 10^6 \text{ mm}^4$$

or $\bar{I}_{x'} = 4.61\times 10^6 \text{ mm}^4$ ◀

From Equation 9.19:

$$\bar{I}_{y'} = \frac{\bar{I}_x + \bar{I}_y}{2} - \frac{\bar{I}_x - \bar{I}_y}{2}\cos 2\theta + \bar{I}_{xy}\sin 2\theta$$

$$= \left[4.2147\times 10^6 - \left(-3.5133\times 10^6\right)\cos\left(120°\right) + 1.5732\times 10^6\sin\left(120°\right)\right]\text{mm}^4$$

$$= 3.8205\times 10^6 \text{ mm}^4$$

or $\bar{I}_{y'} = 3.82\times 10^6 \text{ mm}^4$ ◀

From Equation 9.20:

$$\bar{I}_{x'y'} = \frac{\bar{I}_x - \bar{I}_y}{2}\sin 2\theta + \bar{I}_{xy}\cos 2\theta$$

$$= \left[-3.5133\times 10^6\sin\left(120°\right) + 1.5732\times 10^6\cos\left(120°\right)\right]\text{mm}^4$$

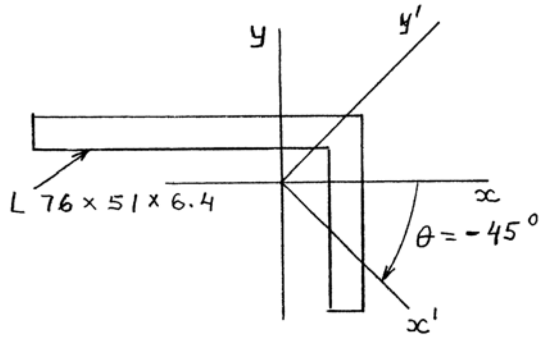
$$= -3.8292\times 10^6 \text{ mm}^4$$

or $\bar{I}_{x'y'} = -3.83\times 10^6 \text{ mm}^4$ ◀

PROBLEM 9.83

Determine the moments of inertia and the product of inertia of the L76 × 51 × 6.4-mm angle cross section of Problem 9.74 with respect to new centroidal axes obtained by rotating the x and y axes through 45^{o} clockwise.

SOLUTION



From Problem 9.74

$$\bar{I}_{xy} = -0.1596 \times 10^6 \text{ mm}^4$$

From Figure 9.13

$$\bar{I}_x = 0.166 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = 0.453 \times 10^6 \text{ mm}^4$$

Now
$$\frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = 0.3095 \times 10^6 \text{ mm}^4$$

$$\frac{1}{2}\left(\bar{I}_x - \bar{I}_y\right) = -0.1453 \times 10^6 \text{ mm}^4$$

Using Equations (9.18), (9.19), and (9.20)

Equation (9.18):

$$\bar{I}_{x'} = \frac{\bar{I}_x + \bar{I}_y}{2} + \frac{\bar{I}_x - \bar{I}_y}{2}\cos 2\theta - \bar{I}_{xy}\sin 2\theta$$

$$= \left[0.3095 \times 10^6 + \left(-0.1435 \times 10^6\right)\cos\left(-90°\right) - \left(-0.1596 \times 10^6\right)\sin\left(-90°\right)\right]\text{mm}^4$$

$$= 0.1499 \times 10^6 \text{ mm}^4$$

or $\bar{I}_{x'} = 0.1499 \times 10^6 \text{ mm}^4$ ◀

PROBLEM 9.83 CONTINUED

Equation (9.19):

$$\bar{I}_{y'} = \frac{\bar{I}_x + \bar{I}_y}{2} - \frac{\bar{I}_x - \bar{I}_y}{2}\cos 2\theta + \bar{I}_{xy}\sin 2\theta$$

$$= \left[0.3095 \times 10^6 - \left(-0.1435 \times 10^6\right)\cos\left(-90°\right) + \left(-0.1596 \times 10^6\right)\sin\left(-90°\right)\right]\text{mm}^4$$

$$= 0.4691 \times 10^6 \text{ mm}^4$$

or $\bar{I}_{y'} = 0.469 \times 10^6 \text{ mm}^4$ ◀

Equation (9.20):

$$\bar{I}_{x'y'} = \frac{\bar{I}_x - \bar{I}_y}{2}\sin 2\theta + \bar{I}_{xy}\cos 2\theta$$

$$= \left[-0.1435 \times 10^6 \sin\left(-90°\right) + 0.1596 \times 10^6 \cos\left(-90°\right)\right]\text{mm}^4$$

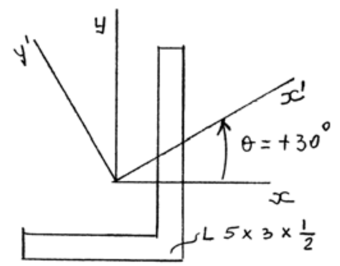
$$= 0.1435 \times 10^6 \text{ mm}^4$$

or $\bar{I}_{x'y'} = 0.1435 \times 10^6 \text{ mm}^4$ ◀

PROBLEM 9.84

Determine the moments of inertia and the product of inertia of the L5 × 3 × $\frac{1}{2}$ -in. angle cross section of Problem 9.78 with respect to new centroidal axes obtained by rotating the x and y axes through 30° counterclockwise.

SOLUTION



From Problem 9.78

$$\bar{I}_{xy} = 2.8125 \text{ in}^4$$

From Figure 9.13

$$\bar{I}_x = 9.45 \text{ in}^4, \qquad \bar{I}_y = 2.58 \text{ in}^4$$

Now

$$\frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = 6.015 \text{ in}^4$$

$$\frac{1}{2}\left(\bar{I}_x - \bar{I}_y\right) = 3.435 \text{ in}^4$$

Using Equations (9.18), (9.19), and (9.20)

Equation (9.18):

$$\bar{I}_{x'} = \frac{\bar{I}_x + \bar{I}_y}{2} + \frac{\bar{I}_x - \bar{I}_y}{2}\cos 2\theta - \bar{I}_{xy}\sin 2\theta$$

$$= \left[6.015 + 3.435\cos(60^\circ) - 2.8125\sin(60^\circ)\right]\text{in}^4 = 5.2968 \text{ in}^4$$

or $\bar{I}_{x'} = 5.30 \text{ in}^4$ ◀

Equation (9.19):

$$\bar{I}_{y'} = \frac{\bar{I}_x + \bar{I}_y}{2} - \frac{\bar{I}_x - \bar{I}_y}{2}\cos 2\theta + \bar{I}_{xy}\sin 2\theta$$

$$= \left[6.015 - 3.435\cos(60^\circ) + 2.8125\sin(60^\circ)\right]\text{in}^4 = 6.7332 \text{ in}^4$$

or $\bar{I}_{y'} = 6.73 \text{ in}^4$ ◀

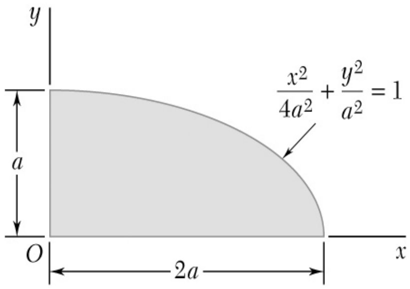
Equation (9.20):

$$\bar{I}_{x'y'} = \frac{\bar{I}_x - \bar{I}_y}{2}\sin 2\theta + \bar{I}_{xy}\cos 2\theta$$

$$= \left[3.435\sin(60^\circ) + 2.8125\cos(60^\circ)\right]\text{in}^4 = 4.3810 \text{ in}^4$$

or $\bar{I}_{x'y'} = 4.38 \text{ in}^4$ ◀

PROBLEM 9.85



For the quarter ellipse of Problem 9.67, determine the orientation of the principal axes at the origin and the corresponding values of the moments of inertia.

SOLUTION

From Problem 9.79: $I_x = \frac{\pi}{8}a^4 \qquad I_y = \frac{\pi}{2}a^4$

Problem 9.67: $I_{xy} = \frac{1}{2}a^4$

Now, Equation (9.25): $$\tan 2\theta_m = -\frac{2I_{xy}}{I_x - I_y} = -\frac{2\left(\frac{1}{2}a^4\right)}{\frac{\pi}{8}a^4 - \frac{\pi}{2}a^4}$$

$$= \frac{8}{3\pi} = 0.84883$$

Then $2\theta_m = 40.326°$ and $220.326°$

or $\theta_m = 20.2°$ and $110.2°$ ◀

Also, Equation (9.27): $$I_{\max,\min} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$$

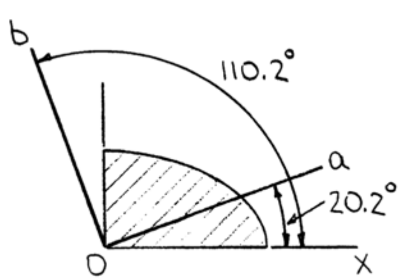
$$= \frac{1}{2}\left(\frac{\pi}{8}a^4 + \frac{\pi}{2}a^4\right)$$

$$\pm \sqrt{\left[\frac{1}{2}\left(\frac{\pi}{8}a^4 - \frac{\pi}{2}a^4\right)\right]^2 + \left(\frac{1}{2}a^4\right)^2}$$

$$= (0.981748 \pm 0.772644)a^4$$

or $I_{\max} = 1.754a^4$ ◀

and $I_{\min} = 0.209a^4$ ◀



By inspection, the a axis corresponds to $I_{\min}$ and the b axis corresponds to $I_{\max}$.

PROBLEM 9.86

For the area indicated, determine the orientation of the principal axes at the origin and the corresponding values of the moments of inertia.

Area of Problem 9.72

SOLUTION

From the solutions to Problem 9.72 and 9.80

$$\bar{I}_{xy} = 501.1875 \text{ in}^4 \qquad \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = 2812.21875 \text{ in}^4$$

$$\frac{1}{2}\left(\bar{I}_x - \bar{I}_y\right) = -1946.53125 \text{ in}^4$$

Then Equation (9.25):
$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} = -\frac{501.1875}{-1946.53125} = 0.257477$$

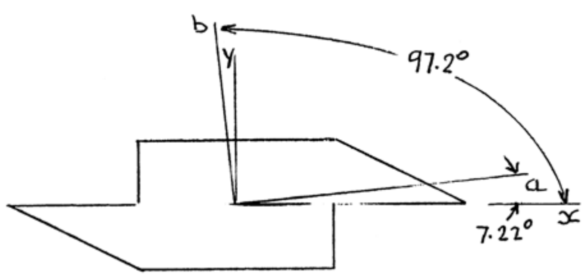
or
$$2\theta_m = 14.4387° \quad \text{and} \quad 194.4387°$$

or $\theta_m = 7.22°$ and $97.2°$ ◀

Equation (9.27):
$$\bar{I}_{\max,\min} = \frac{\bar{I}_x + \bar{I}_y}{2} \pm \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2}$$

$$= 2812.21875 \pm \sqrt{(-1946.53125)^2 + (501.1875)^2}$$

$$= (2812.21875 \pm 2010.0181)\text{in}^4$$



or $\bar{I}_{\max} = 4.82 \times 10^3 \text{ in}^4$ ◀

and $\bar{I}_{\min} = 802 \text{ in}^4$ ◀

By inspection, the *a* axis corresponds to $\bar{I}_{\min}$ and the *b* axis corresponds to $\bar{I}_{\max}$.

PROBLEM 9.87

For the area indicated, determine the orientation of the principal axes at the origin and the corresponding values of the moments of inertia.

Area of Problem 9.73

SOLUTION

From Problems 9.73 and 9.81

$$\bar{I}_x = 51.84\pi \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = 103.68\pi \times 10^6 \text{ mm}^4$$

$$\bar{I}_{xy} = 138.24 \times 10^6 \text{ mm}^4$$

Equation (9.25):

$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} = -\frac{2(138.24 \times 10^6)}{51.84\pi \times 10^6 - 103.68\pi \times 10^6}$$

$$= 1.69765$$

$$2\theta_m = 59.500° \quad \text{and} \quad 239.500°$$

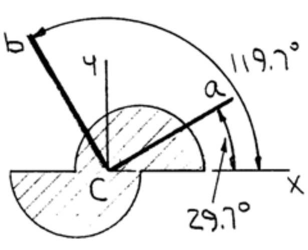
or $\theta_m = 29.7°$ and $119.7°$ ◀

Then

$$\bar{I}_{\max,\min} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) \pm \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2}$$

$$= \frac{(51.84 + 103.68)\pi \times 10^6}{2} \pm \sqrt{\left[\frac{(51.84 - 103.68)\pi \times 10^6}{2}\right]^2 + (138.24 \times 10^6)^2}$$

$$= (244.29 \pm 160.44) \times 10^6 \text{ mm}^4$$



or $\bar{I}_{\max} = 405 \times 10^6 \text{ mm}^4$ ◀

and $\bar{I}_{\min} = 83.9 \times 10^6 \text{ mm}^4$ ◀

Note: By inspection the *a* axis corresponds to $\bar{I}_{\min}$ and the *b* axis corresponds to $I_{\max}$.

PROBLEM 9.88

For the area indicated, determine the orientation of the principal axes at the origin and the corresponding values of the moments of inertia.

Area of Problem 9.75

SOLUTION

From Problems 9.75 and 9.82

$$\bar{I}_x = 0.70134 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = 7.728 \times 10^6 \text{ mm}^4$$

$$\bar{I}_{xy} = 1.5732 \times 10^6 \text{ mm}^4$$

Then
$$\frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = 4.2147 \times 10^6 \text{ mm}^4$$

$$\frac{1}{2}\left(\bar{I}_x - \bar{I}_y\right) = -3.5133 \times 10^6 \text{ mm}^4$$

Equation (9.25):
$$\tan 2\theta = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} = -\frac{2\left(1.5732 \times 10^6\right)}{0.70134 \times 10^6 - 7.728 \times 10^6}$$

$$= 0.44778$$

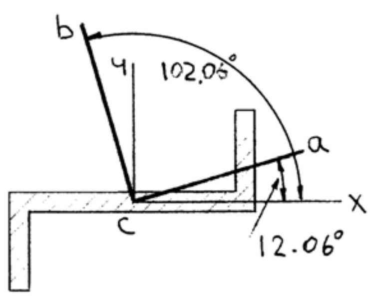
Then
$$2\theta_m = 24.12° \quad \text{and} \quad 204.12°$$

or $\theta_m = 12.06°$ and $102.1°$ ◀

Also, Equation (9.27):
$$\bar{I}_{\max,\min} = \frac{\bar{I}_x + \bar{I}_y}{2} \pm \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 \bar{I}_{xy}^2}$$

$$= 4.2147 \times 10^6 \pm \sqrt{\left(-3.5133 \times 10^6\right)^2 + \left(1.5732 \times 10^6\right)^2}$$

$$= \left(4.2147 \pm 3.8494\right) \times 10^6 \text{ mm}^4$$



or $\bar{I}_{\max} = 8.06 \times 10^6 \text{ mm}^4$ ◀

and $\bar{I}_{\min} = 0.365 \times 10^6 \text{ mm}^4$ ◀

By inspection, the *a* axis corresponds to $\bar{I}_{\min}$ and the *b* axis corresponds to $\bar{I}_{\max}$.

PROBLEM 9.89

For the angle cross section indicated, determine the orientation of the principal axes at the origin and the corresponding values of the moments of inertia.

The L76 × 51 × 6.4-mm angle cross section of Problem 9.74

SOLUTION

From Problems 9.74 and 9.83

$$\bar{I}_x = 0.166 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = 0.453 \times 10^6 \text{ mm}^4$$

$$\bar{I}_{xy} = -0.1596 \times 10^6 \text{ mm}^4$$

Then

$$\frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = 0.3095 \times 10^6 \text{ mm}^4$$

$$\frac{1}{2}\left(\bar{I}_x - \bar{I}_y\right) = -0.1435 \times 10^6 \text{ mm}^4$$

Equation (9.25):

$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{I_x - I_y} = -\frac{2\left(-0.1596 \times 10^6\right)}{\left(0.166 - 0.453\right) \times 10^6} = -1.1122$$

Then

$$2\theta_m = -48.041° \quad \text{and} \quad 131.96°$$

or

$$\theta_m = -24.0° \text{ and } 66.0° \blacktriangleleft$$

Also, Equation (9.27):

$$\bar{I}_{\max,\min} = \frac{\left(\bar{I}_x + \bar{I}_y\right)}{2} \pm \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2}$$

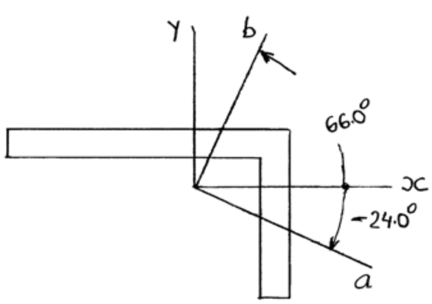
$$= 0.3095 \times 10^6 \pm \sqrt{\left(-0.1435 \times 10^6\right)^2 + \left(-0.1596 \times 10^6\right)^2}$$

$$= \left(0.3095 \pm 0.21463\right) \times 10^6 \text{ mm}^4$$

or

$$\bar{I}_{\max} = 0.524 \times 10^6 \text{ mm}^4 \blacktriangleleft$$

$$\bar{I}_{\min} = 0.0949 \times 10^6 \text{ mm}^4 \blacktriangleleft$$



By inspection, the a axis corresponds to $\bar{I}_{\min}$ and the b axis corresponds to $\bar{I}_{\max}$.

PROBLEM 9.90

For the angle cross section indicated, determine the orientation of the principal axes at the origin and the corresponding values of the moments of inertia.

The $L5 \times 3 \times \frac{1}{2}$-in. angle cross section of Problem 9.78

SOLUTION

From Problems 9.78 and 9.84

$$\bar{I}_{xy} = 2.81 \text{ in}^4$$

$$\bar{I}_x = 9.45 \text{ in}^4$$

$$\bar{I}_y = 2.58 \text{ in}^4$$

Then
$$\frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = 6.015 \text{ in}^4$$

$$\frac{1}{2}\left(\bar{I}_x - \bar{I}_y\right) = 3.435 \text{ in}^4$$

Equation (9.25):
$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} = -\frac{2(2.81)}{9.45 - 2.58} = -0.8180$$

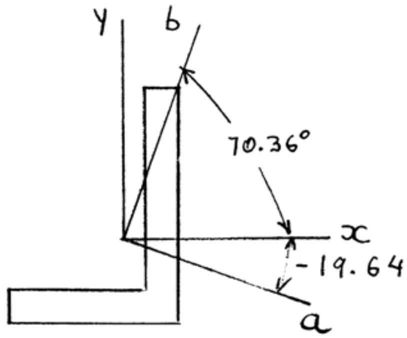
Then
$$2\theta_m = -39.2849 \quad \text{and} \quad 140.7151$$

or $\theta_m = -19.64$ and 70.36 ◀

Also, Equation (9.27):
$$\bar{I}_{\max,\min} = \frac{\left(\bar{I}_x + \bar{I}_y\right)}{2} \pm \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + I_{xy}^2}$$

$$= 6.015 \pm \sqrt{3.435^2 - 2.81^2}$$

$$= (6.015 \pm 4.438)\text{in}^4$$



or $\bar{I}_{\max} = 10.45 \text{ in}^4$ ◀

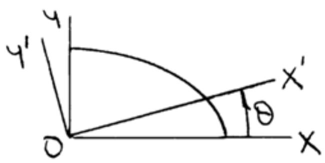
and $\bar{I}_{\min} = 1.577 \text{ in}^4$ ◀

Note: By inspection, the *a* axis corresponds to $I_{\max}$ and the *b* axis corresponds to $I_{\min}$.

PROBLEM 9.91

Using Mohr's circle, determine for the quarter ellipse of Problem 9.67 the moments of inertia and the product of inertia with respect to new axes obtained by rotating the x and y axes about O (a) through 45° counterclockwise, (b) through 30° clockwise.

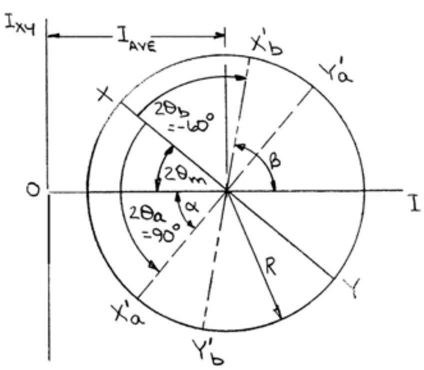
SOLUTION



From Problem 9.79: $$I_x = \frac{\pi}{8}a^4$$

$$I_y = \frac{\pi}{2}a^4$$

Problem 9.67: $$I_{xy} = \frac{1}{2}a^4$$



The Mohr's circle is defined by the diameter XY, where

$$X\left(\frac{\pi}{8}a^4,\ \frac{1}{2}a^4\right) \quad \text{and} \quad Y\left(\frac{\pi}{2}a^4,\ -\frac{1}{2}a^4\right)$$

Now $$I_{\text{ave}} = \frac{1}{2}\left(I_x + I_y\right) = \frac{1}{2}\left(\frac{\pi}{8}a^4 + \frac{\pi}{2}a^4\right) = \frac{5}{16}\pi a^4 = 0.98175a^4$$

and $$R = \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2} = \sqrt{\left[\frac{1}{2}\left(\frac{\pi}{8}a^4 - \frac{\pi}{2}a^4\right)\right]^2 + \left(\frac{1}{2}a^4\right)^2}$$

$$= 0.77264a^4$$

The Mohr's circle is then drawn as shown.

$$\tan 2\theta_m = -\frac{2I_{xy}}{I_x - I_y}$$

$$= -\frac{2\left(\frac{1}{2}a^4\right)}{\frac{\pi}{8}a^4 - \frac{\pi}{2}a^4}$$

$$= 0.84883$$

or $$2\theta_m = 40.326^\circ$$

PROBLEM 9.91 CONTINUED

Then $\alpha = 90° - 40.326°$

$= 49.674°$

$\beta = 180° - (40.326° + 60°)$

$= 79.674°$

(*a*) $I_{x'} = I_{\text{ave}} - R\cos\alpha = 0.98175a^4 - 0.77264a^4 \cos 49.674°$

or $I_{x'} = 0.482a^4$ ◀

$I_{y'} = I_{\text{ave}} + R\cos\alpha = 0.98175a^4 + 0.77264a^4 \cos 49.674°$

or $I_{y'} = 1.482a^4$ ◀

$I_{x'y'} = -R\sin\alpha = -0.77264a^4 \sin 49.674°$

or $I_{x'y'} = -0.589a^4$ ◀

(*b*) $I_{x'} = I_{\text{ave}} + R\cos\beta = 0.98175a^4 + 0.77264a^4 \cos 79.674°$

or $I_{x'} = 1.120a^4$ ◀

$I_{y'} = I_{\text{ave}} - R\cos\beta = 0.98175a^4 - 0.77264a^4 \cos 79.674°$

or $I_{y'} = 0.843a^4$ ◀

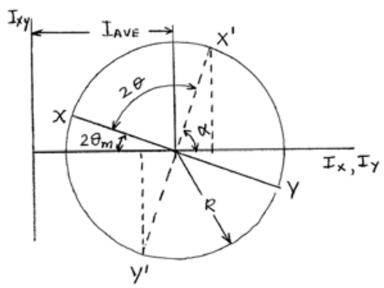
$I_{x'y'} = R\sin\beta = 0.77264a^4 \sin 79.674°$

or $I_{x'y'} = 0.760a^4$ ◀

PROBLEM 9.92

Using Mohr's circle, determine the moments of inertia and the product of inertia of the area of Problem 9.72 with respect to new centroidal axes obtained by rotating the x and y axes 45° clockwise.

SOLUTION



From the solution to

Problem 9.72: $\bar{I}_{xy} = 501.1875 \text{ in}^4$

Problem 9.80: $\bar{I}_x = 865.6875 \text{ in}^4$

$\bar{I}_y = 4758.75 \text{ in}^4$

Now $$\frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = 2812.21875 \text{ in}^4$$

$$\frac{1}{2}\left(\bar{I}_x - \bar{I}_y\right) = -1946.53125 \text{ in}^4$$

The Mohr's circle is defined by the points X and Y where

$$X: \ \left(\bar{I}_x, \bar{I}_{xy}\right) \qquad Y: \ \left(\bar{I}_y, -\bar{I}_{xy}\right)$$

Now $$I_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = 2812.2 \text{ in}^4$$

and $$R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2} = \sqrt{(-1946.53125)^2 + 501.1875^2} \text{ in}^4$$

$$= 2010.0 \text{ in}^4$$

Also, $$\tan 2\theta_m = \frac{\bar{I}_{xy}}{\left|\frac{\bar{I}_x - \bar{I}_y}{2}\right|} = \frac{501.1875}{1946.53125} = 0.2575$$

or $$2\theta_m = 14.4387°$$

Then $$\alpha = 180° - (14.4387° + 90°) = 75.561°$$

PROBLEM 9.92 CONTINUED

Then $\qquad \bar{I}_{x'}, \bar{I}_{y'} = I_{\text{ave}} \pm R\cos\alpha = 2812.2 \pm 2010.0\cos 75.561°$

or $\bar{I}_{x'} = 3.31 \times 10^3 \text{ in}^4$ ◀

and $\bar{I}_{y'} = 2.31 \times 10^3 \text{ in}^4$ ◀

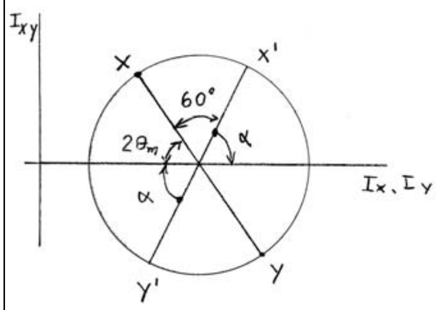
and $\qquad \bar{I}_{x'y'} = R\sin\alpha = 2010.0\sin 75.561°$

or $\bar{I}_{x'y'} = 1.947 \times 10^3 \text{ in}^4$ ◀

PROBLEM 9.93

Using Mohr's circle, determine the moments of inertia and the product of inertia of the area of Problem 9.73 with respect to new centroidal axes obtained by rotating the x and y axes through 30° clockwise.

SOLUTION



From Problems 9.73 and 9.81

$$\bar{I}_{xy} = 138.24 \times 10^6 \text{ mm}^4$$

$$\bar{I}_x = 51.84\pi \times 10^6 \text{ mm}^4$$

$$= 162.86 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = 103.68\pi \times 10^6 \text{ mm}^4$$

$$= 325.72 \times 10^6 \text{ mm}^4$$

Now $$\bar{I}_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right)$$

$$= 244.29 \times 10^6 \text{ mm}^4$$

$$R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2}$$

$$= 160.4405 \times 10^6 \text{ mm}^4$$

From Problem 9.87 $$2\theta_m = 59.5^{\circ}$$

Then $$\alpha = 180 - 60^{\circ} - 2\theta_m = 60.5^{\circ}$$

Then $$\bar{I}_{x'} = \bar{I}_{\text{ave}} + R\cos\alpha = 244.29 + 160.4405\cos 60.5^{\circ}$$

$$= 323.29 \times 10^6 \text{ mm}^4$$

or $\bar{I}_{x'} = 323 \times 10^6 \text{ mm}^4$ ◀

$$\bar{I}_{y'} = \bar{I}_{\text{ave}} - R\cos\alpha = 244.24 - 160.4405\cos 60.5^{\circ}$$

$$= 165.29 \times 10^6 \text{ mm}^4$$

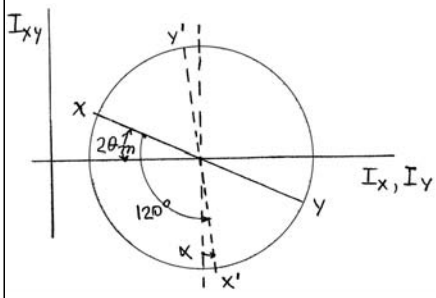
or $\bar{I}_{y'} = 165.3 \times 10^6 \text{ mm}^4$ ◀

$\bar{I}_{x'y'} = R\sin\alpha = 160.44\sin 60.5^{\circ} = 139.6 \times 10^6 \text{ mm}^4$ ◀

PROBLEM 9.94

Using Mohr's circle, determine the moments of inertia and the product of inertia of the area of Problem 9.75 with respect to new centroidal axes obtained by rotating the x and y axes through 60° counterclockwise.

SOLUTION



From Problems 9.75 and 9.82

$$\bar{I}_x = 0.70134 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = 7.728 \times 10^6 \text{ mm}^4$$

$$\bar{I}_{xy} = 1.5732 \times 10^6 \text{ mm}^4$$

Now $$\bar{I}_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = 4.2147 \times 10^6 \text{ mm}^4$$

and $$R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2} = 3.8494 \times 10^6 \text{ mm}^4$$

Then $$2\theta_m = \tan^{-1}\left[\frac{-2(1.5732)}{0.70134 - 7.728}\right] = 24.12^{\circ}$$

and $$\alpha = 120^{\circ} - 24.12^{\circ} - 90 = 5.88^{\circ}$$

Then $$\bar{I}_{x'} = \bar{I}_{\text{ave}} + R\sin\alpha = (4.2147 + 3.8494\sin 5.88^{\circ}) \times 10^6 \text{ mm}^4$$

$$= 4.6091 \times 10^6 \text{ mm}^4$$

or $\bar{I}_{x'} = 4.61 \times 10^6 \text{ mm}^4$ ◀

$$\bar{I}_{y'} = \bar{I}_{\text{ave}} - R\sin\alpha = (4.2147 - 3.8494\sin 5.88^{\circ}) \times 10^6 \text{ mm}^4$$

$$= 3.8203 \times 10^6 \text{ mm}^4$$

or $\bar{I}_{y'} = 3.82 \times 10^6 \text{ mm}^4$ ◀

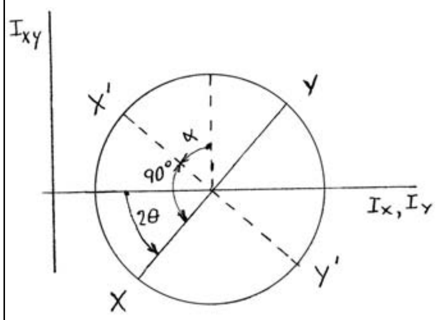
$$\bar{I}_{x'y'} = -R\cos\alpha = -3.8494\cos 5.88^{\circ} = -3.8291 \times 10^6 \text{ mm}^4$$

or $\bar{I}_{x'y'} = -3.83 \times 10^6 \text{ mm}^4$ ◀

PROBLEM 9.95

Using Mohr's circle, determine the moments of inertia and the product of inertia of the L76 × 51 × 6.4-mm angle cross section of Problem 9.74 with respect to new centroidal axes obtained by rotating the x and y axes through 45° clockwise.

SOLUTION



From Problems 9.74 and 9.83

$$\bar{I}_x = 0.166 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = 0.453 \times 10^6 \text{ mm}^4$$

$$\bar{I}_{xy} = -0.1596 \times 10^6 \text{ mm}^4$$

Now $$I_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = 0.3095 \times 10^6 \text{ mm}^4$$

and $$R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2}$$

$$= 0.21463 \times 10^6 \text{ mm}^4$$

Then $$2\theta_m = \tan^{-1}\left[\frac{-2(-0.1596)}{0.166 - 0.453}\right] = -48.04^{\circ}$$

and $$\alpha + 90^{\circ} - 2\theta = 90^{\circ}; \ \alpha = 2\theta_m$$

Then

$$\bar{I}_{x'} = \bar{I}_{\text{ave}} - R\sin\alpha = \left(0.3095 - 0.21463\sin 48.04^{\circ}\right) \times 10^6 \text{ mm}^4$$

$$= 0.14989 \times 10^6 \text{ mm}^4$$

or $\bar{I}_{x'} = 0.1499 \times 10^6 \text{ mm}^4$ ◀

and

$$\bar{I}_{y'} = \bar{I}_{\text{ave}} + R\sin\alpha = \left(0.3095 + 0.21463\sin 48.04^{\circ}\right) \times 10^6 \text{ mm}^4$$

$$= 0.46910 \times 10^6 \text{ mm}^4$$

or $\bar{I}_{y'} = 0.4690 \times 10^6 \text{ mm}^4$ ◀

and

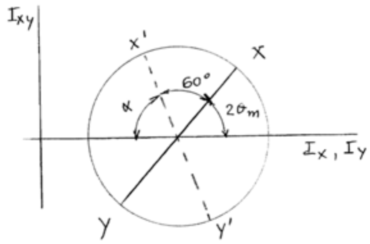
$$\bar{I}_{x'y'} = R\cos\alpha = 0.21463\cos 48.04^{\circ} = 0.1435 \times 10^6 \text{ mm}^4$$

or $\bar{I}_{x'y'} = 0.1435 \times 10^6 \text{ mm}^4$ ◀

PROBLEM 9.96

Using Mohr's circle, determine the moments of inertia and the product of inertia of the L5 × 3 × $\frac{1}{2}$-in. angle cross section of Problem 9.78 with respect to new centroidal axes obtained by rotating the x and y axes through 30° counterclockwise.

SOLUTION



Have $\bar{I}_x = 9.45 \text{ in}^4$

$\bar{I}_y = 2.58 \text{ in}^4$

From Problem 9.78 $\bar{I}_{xy} = 2.8125 \text{ in}^4$

Now $$\bar{I}_{\text{ave}} = \frac{\bar{I}_x + \bar{I}_y}{2} = 6.015 \text{ in}^4$$

and $$R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \left(\bar{I}_{xy}\right)^2}$$

$$= 4.43952 \text{ in}^4$$

Then $$2\theta_m = \tan^{-1}\left[\frac{-2(2.8125)}{9.45 - 2.58}\right] = -39.31^{\circ}$$

$$2\theta_m + 60 + \alpha = 180^{\circ}, \qquad \alpha = 80.69^{\circ}$$

Then $$\bar{I}_{x'} = \bar{I}_{\text{ave}} - R\cos\alpha = 6.015 \text{ in}^4 - \left(4.43952 \text{ in}^4\right)\cos 80.69^{\circ}$$

$$= 5.29679 \text{ in}^4$$

or $\bar{I}_{x'} = 5.30 \text{ in}^4$ ◀

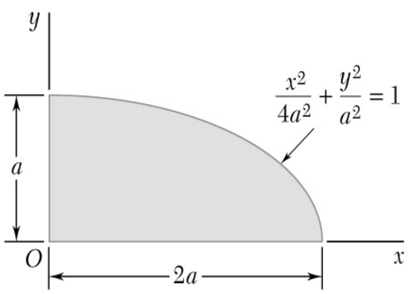
$$\bar{I}_{y'} = \bar{I}_{\text{ave}} + R\cos\alpha = 6.015 \text{ in}^4 + \left(4.43952 \text{ in}^4\right)\cos 80.69^{\circ}$$

$$= 6.73321 \text{ in}^4$$

or $\bar{I}_{y'} = 6.73 \text{ in}^4$ ◀

$$\bar{I}_{x'y'} = R\sin\alpha = \left(4.43952 \text{ in}^4\right)\sin 80.69^{\circ} = 4.38104 \text{ in}^4$$

or $\bar{I}_{x'y'} = 4.38 \text{ in}^4$ ◀



PROBLEM 9.97

For the quarter ellipse of Problem 9.67, use Mohr's circle to determine the orientation of the principal axes at the origin and the corresponding values of the moments of inertia.

SOLUTION

From Problem 9.79: $\quad I_x = \frac{\pi}{8}a^4 \qquad I_y = \frac{\pi}{2}a^4$

Problem 9.67: $\quad I_{xy} = \frac{1}{2}a^4$

The Mohr's circle is defined by the diameter *XY*, where

$$X\left(\frac{\pi}{8}a^4,\ \frac{1}{2}a^4\right) \quad \text{and} \quad Y\left(\frac{\pi}{2}a^4,\ -\frac{1}{2}a^4\right)$$

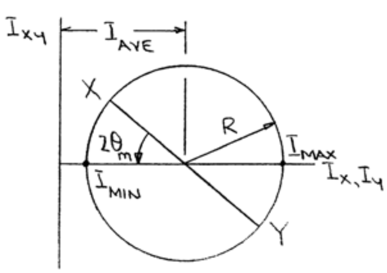
Now $\quad I_{\text{ave}} = \frac{1}{2}\left(I_x + I_y\right) = \frac{1}{2}\left(\frac{\pi}{8}a^4 + \frac{\pi}{2}a^4\right) = 0.98175a^4$

and

$$R = \sqrt{\left[\frac{1}{2}\left(I_x - I_y\right)\right]^2 + I_{xy}^2} = \sqrt{\left[\frac{1}{2}\left(\frac{\pi}{8}a^4 - \frac{\pi}{2}a^4\right)\right]^2 + \left(\frac{1}{2}a^4\right)^2}$$

$$= 0.77264a^4$$

The Mohr's circle is then drawn as shown.



$$\tan 2\theta_m = -\frac{2I_{xy}}{I_x - I_y}$$

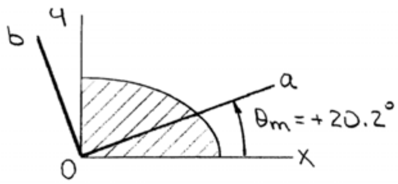
$$= -\frac{2\left(\frac{1}{2}a^4\right)}{\frac{\pi}{8}a^4 - \frac{\pi}{2}a^4}$$

$$= 0.84883$$

or $\quad 2\theta_m = 40.326°$

and $\quad \theta_m = 20.2°$

PROBLEM 9.97 CONTINUED



$\therefore$ The principal axes are obtained by rotating the xy axes through

$20.2°$ counterclockwise ◀

About O.

Now $$I_{\max,\min} = I_{\text{ave}} \pm R = 0.98175a^4 \pm 0.77264a^4$$

or $I_{\max} = 1.754a^4$ ◀

and $I_{\min} = 0.209a^4$ ◀

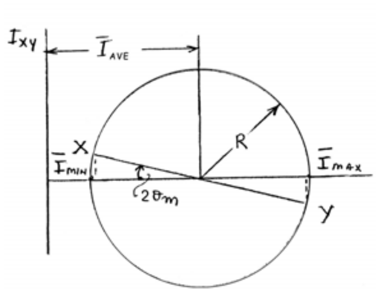
From the Mohr's circle it is seen that the a axis corresponds to $I_{\min}$ and the b axis corresponds to $I_{\max}$.

PROBLEM 9.98

Using Mohr's circle, determine for the area indicated the orientation of the principal centroidal axes and the corresponding values of the moments of inertia.

Area of Problem 9.72

SOLUTION



From the solution to Problem 9.72:

$$\bar{I}_{xy} = 501.1875 \text{ in}^4$$

From the solution to Problem 9.80:

$$\bar{I}_x = 865.6875 \text{ in}^4$$

$$\bar{I}_y = 4758.75 \text{ in}^4$$

$$\frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = 2812.21875 \text{ in}^2$$

$$\frac{1}{2}\left(\bar{I}_x - \bar{I}_y\right) = -1946.53125 \text{ in}^4$$

The Mohr's circle is defined by the point $\quad X: \ \left(\bar{I}_x, \bar{I}_{xy}\right), \qquad Y: \ \left(\bar{I}_y, -\bar{I}_{xy}\right)$

Now $\quad I_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = 2812.2 \text{ in}^4$

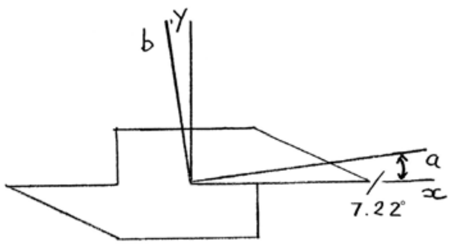
and $\quad R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}} = \sqrt{\left(-1946.53125\right)^2 + 501.1875^2} = 2010.0 \text{ in}^4$

PROBLEM 9.98 CONTINUED

$$\tan 2\theta_m = -\frac{\bar{I}_{xy}}{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)} = -\frac{501.1875}{-1946.53125} = 0.2575, \qquad 2\theta_m = 14.4387°$$

or $\theta_m = 7.22°$ counterclockwise ◀

Then $\bar{I}_{\max,\min} = \bar{I}_{\text{ave}} \pm R = (2812.2 \pm 2010.0)\text{in}^4$



or $\bar{I}_{\max} = 4.82 \times 10^3 \text{ in}^4$ ◀

and $\bar{I}_{\min} = 802 \text{ in}^4$ ◀

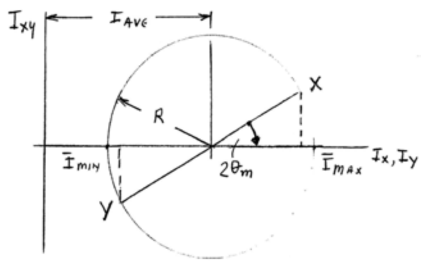
Note: From the Mohr's circle it is seen that the *a* axis corresponds to $\bar{I}_{\min}$ and the *b* axis corresponds to $\bar{I}_{\max}$.

PROBLEM 9.99

Using Mohr's circle, determine for the area indicated the orientation of the principal centroidal axes and the corresponding values of the moments of inertia.

Area of Problem 9.76

SOLUTION



From the solution to Problem 9.76 $\qquad \bar{I}_{xy} = 576\text{ in}^4$

Now $\qquad \bar{I}_x = (I_x)_1 - (I_x)_2 - (I_x)_3, \quad \text{where} \quad (I_x)_2 = (I_x)_3$

$$= \frac{\pi}{4}(15\text{ in.})^4 - 2\left[\frac{1}{12}(9\text{ in.})(6\text{ in.})^3\right] = (39761 - 324)\text{in}^4$$

$$= 39,437\text{ in}^4$$

and $\qquad \bar{I}_y = (I_y)_1 - (I_y)_2 - (I_y)_3, \quad \text{where} \quad (I_y)_2 = (I_y)_3$

$$= \frac{\pi}{4}(15\text{ in.})^4 - 2\left[\frac{1}{36}(6\text{ in.})(9\text{ in.})^3 + \frac{1}{2}(9\text{ in.})(6\text{ in.})(6\text{ in.})^2\right]$$

$$= (39,761 - 243 - 1944)\text{in}^4 = 37,574\text{ in}^4$$

The Mohr's circle is defined by the point (X, Y) where

$$X: \ (\bar{I}_x, \bar{I}_{xy}) \qquad Y: \ (\bar{I}_y, -\bar{I}_{xy})$$

Now $\qquad \bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(39,437 + 37,574)\text{in}^4 = 38,506\text{ in}^4$

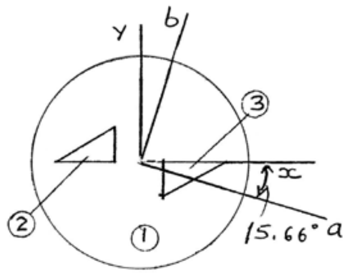
and $\qquad R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2} = \sqrt{\left[\frac{1}{2}(39,437 - 37,574)\right]^2 + 567^2} = 1090.5\text{ in}^4$

PROBLEM 9.99 CONTINUED

$$\tan 2\theta_m = \frac{-\bar{I}_{xy}}{\frac{\bar{I}_x - \bar{I}_y}{2}} = \frac{-567}{\frac{1}{2}(39,437 - 37,574)} = -0.6087$$

or $\theta_m = -15.66°$ clockwise ◀

Then $$\bar{I}_{\max,\min} = \bar{I}_{\text{ave}} \pm R = (38,506 \pm 1090.50)\text{in}^4$$



or $\bar{I}_{\max} = 39.6 \times 10^3 \text{ in}^4$ ◀

and $\bar{I}_{\min} = 37.4 \times 10^3 \text{ in}^4$ ◀

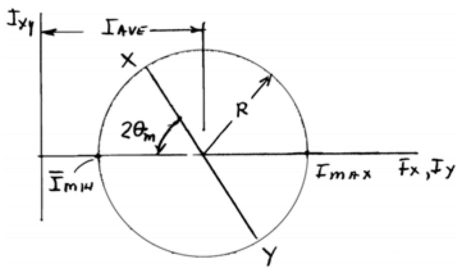
Note: From the Mohr's circle it is seen that the *a* axis corresponds to the $\bar{I}_{\max}$ and the *b* axis corresponds to $\bar{I}_{\min}$.

PROBLEM 9.100

Using Mohr's circle, determine for the area indicated the orientation of the principal centroidal axes and the corresponding values of the moments of inertia.

Area of Problem 9.73

SOLUTION



From Problems 9.73 and 9.81

$$\bar{I}_x = 162.86 \times 10^6 \text{ mm}^4$$

$$\bar{I}_y = 325.72 \times 10^6 \text{ mm}^4$$

$$\bar{I}_{xy} = 138.24 \times 10^6 \text{ mm}^4$$

Define points $X(162.86, 138.24) \times 10^6 \text{ mm}^4 \qquad Y(325.72, -138.24) \times 10^6 \text{ mm}^4$

Now $I_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(162.86 + 325.72) \times 10^6 \text{ mm}^4$

$$= 244.29 \times 10^6 \text{ mm}^4$$

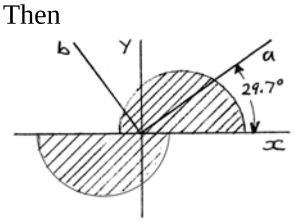
and $R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + I_{xy}^2} = \sqrt{\left[\frac{(162.86 - 325.72)}{2} \times 10^6\right]^2 + (138.24 \times 10^6)^2}$

$$= 160.44 \times 10^6 \text{ mm}^4$$

and $2\theta_m = \tan^{-1}\left[\frac{-2(138.24) \times 10^6}{(162.86 - 325.72) \times 10^6}\right] = 59.4999°$

or $\theta_m = 29.7°$ counterclockwise ◀

Then $\bar{I}_{\text{max, min}} = \bar{I}_{\text{ave}} \pm R = (244.29 \times 10^6 \pm 160.44 \times 10^6) \text{ mm}^4$



or $\bar{I}_{\text{max}} = 405 \times 10^6 \text{ mm}^4$ ◀

and $\bar{I}_{\text{min}} = 83.9 \times 10^6 \text{ mm}^4$ ◀

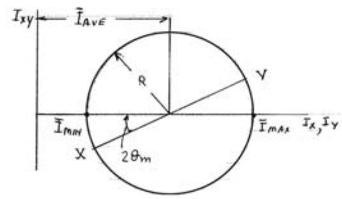
Note: From the Mohr's circle it is seen that the *a* axis corresponds to $\bar{I}_{\text{min}}$ and the *b* axis corresponds to $\bar{I}_{\text{max}}$.

PROBLEM 9.101

Using Mohr's circle, determine for the area indicated the orientation of the principal centroidal axes and the corresponding values of the moments of inertia.

Area of Problem 9.74

SOLUTION



From Problems 9.74 and 9.83

$$\bar{I}_x = 0.166 \times 10^6 \text{ mm}^4, \qquad \bar{I}_y = 0.453 \times 10^6 \text{ mm}^4, \qquad \bar{I}_{xy} = -0.1596 \times 10^6 \text{ mm}^4$$

Define points $\quad X\left(0.166, -0.1596\right) \times 10^6 \text{ mm}^4 \quad$ and $\quad Y\left(0.453, -0.1596\right) \times 10^6 \text{ mm}^4$

Now $$\bar{I}_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = \frac{1}{2}\left(0.166 + 0.453\right) \times 10^6 \text{ mm}^4$$

$$= 0.3095 \times 10^6 \text{ mm}^4$$

and $$R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2} = \sqrt{\left[\frac{\left(0.166 - 0.453\right)10^6}{2}\right]^2 + \left(-0.1596 \times 10^6\right)^2}$$

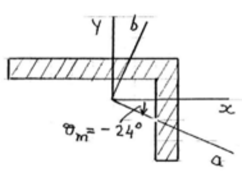
$$= 0.21463 \times 10^6 \text{ mm}^4$$

Also $$2\theta_m = \tan^{-1}\left(\frac{-2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y}\right) = \tan^{-1}\left[\frac{-2\left(-0.1596\right)}{0.166 - 0.453}\right] = -48.04°$$

$$\theta_m = -24.02°$$

or $\theta = -24.0°$ clockwise ◀

Then $$\bar{I}_{\text{max, min}} = \bar{I}_{\text{ave}} \pm R = \left(0.3095 \pm 0.21463\right) \times 10^6 \text{ mm}^4$$



or $\bar{I}_{\text{max}} = 0.524 \times 10^6 \text{ mm}^4$ ◀

and $\bar{I}_{\text{min}} = 0.0949 \times 10^6 \text{ mm}^4$ ◀

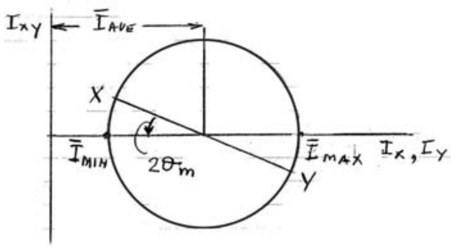
Note: From the Mohr's circle it is seen that the *a* axis corresponds to $\bar{I}_{\text{min}}$ and the *b* axis corresponds to $\bar{I}_{\text{max}}$.

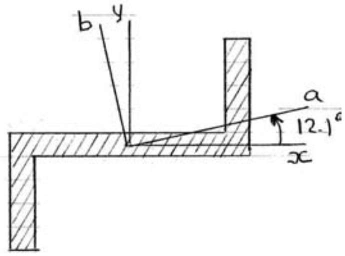
PROBLEM 9.102

Using Mohr's circle, determine for the area indicated the orientation of the principal centroidal axes and the corresponding values of the moments of inertia.

Area of Problem 9.75

SOLUTION





From Problems 9.75 and 9.82

$$\bar{I}_x = 0.70134 \times 10^6 \text{ mm}^4, \qquad \bar{I}_y = 7.728 \times 10^6 \text{ mm}^4, \qquad \bar{I}_{xy} = 1.5732 \times 10^6 \text{ mm}^4$$

Now $$\bar{I}_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = \frac{1}{2}(0.70134 + 7.728) \times 10^6 \text{ mm}^4 = 4.2147 \times 10^6 \text{ mm}^4$$

and $$R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2} = \sqrt{\left[\frac{(0.70134 - 7.728) \times 10^6}{2}\right]^2 + \left(1.5732 \times 10^6\right)^2}$$

$$= 3.8495 \times 10^6 \text{ mm}^4$$

Define points $$X\left(0.70134,\ 15732\right) \times 10^6 \text{ mm}$$

$$Y\left(7.728,\ \ -1.5732\right) \times 10^6 \text{ mm}$$

Also $$2\theta_m = \tan^{-1}\left[\frac{-2(1.5732)}{0.70134 - 7.728}\right] = 24.122°,\ \theta_m = 12.06°$$

or $\theta_m = 12.06°$ counterclockwise ◀

Then $$\bar{I}_{\text{max, min}} = \bar{I}_{\text{ave}} \pm R = \left(4.2147 \pm 3.8495\right) \times 10^6 \text{ mm}^4$$

or $\bar{I}_{\text{max}} = 8.06 \times 10^6 \text{ mm}^4$ ◀

and $\bar{I}_{\text{min}} = 0.365 \times 10^6 \text{ mm}^4$ ◀

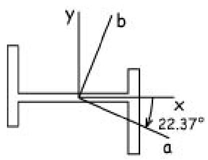
Note: From the Mohr's circle it is seen that the *a* axis corresponds to $\bar{I}_{\text{min}}$ and the *b* axis corresponds to $\bar{I}_{\text{max}}$.

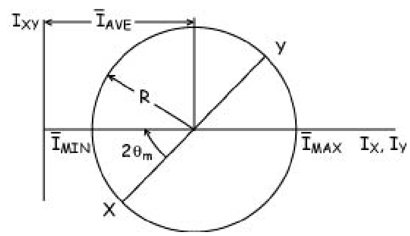
PROBLEM 9.103

Using Mohr's circle, determine for the area indicated the orientation of the principal centroidal axes and the corresponding values of the moments of inertia.

Area of Problem 9.71

SOLUTION





From Problem 9.71 $\quad \bar{I}_{xy} = -11.0 \text{ in}^4$

Compute $\bar{I}_x$ and $\bar{I}_y$ for area of Problem 9.71

$$\bar{I}_x = \frac{5 \text{ in.} \times (0.5 \text{ in.})^3}{12} + 2\left[\frac{(0.5 \text{ in.})(4 \text{ in.})^3}{12} + (4 \text{ in.} \times 0.5 \text{ in.})(1.0 \text{ in.})^2\right]$$

$$= 9.38542 \text{ in}^4$$

$$\bar{I}_y = 2\left[\frac{(0.5 \text{ in.})^3(4 \text{ in.})}{12} + (4 \text{ in.} \times 0.5 \text{ in.})(2.75 \text{ in.})^2\right] + \frac{0.5 \text{ in.} \times (5 \text{ in.})^3}{12}$$

$$= 35.54167 \text{ in}^4$$

Define points $\quad X(9.38542,\ -11), \quad$ and $\quad Y(35.54167,\ 11)$

Now
$$I_{\text{ave}} = \frac{\bar{I}_x + \bar{I}_y}{2} = \frac{9.38542 \text{ in}^4 + 35.54167 \text{ in}^4}{2} = 22.46354 \text{ in}^4$$

and
$$R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \left(\bar{I}_{xy}\right)^2} = \sqrt{\left(\frac{9.38542 - 35.54167}{2}\right)^2 + (11.0)^2}$$

$$= 17.08910 \text{ in}^4$$

Also
$$2\theta_m = \tan^{-1}\left[\frac{-2(-11.0)}{9.38542 - 35.54167}\right] = -40.067 \qquad \text{or } \theta_m = -20.033° \text{ clockwise} \blacktriangleleft$$

Then
$$\bar{I}_{\max,\min} = \bar{I}_{\text{ave}} \pm R = 22.46354 \pm 17.08910$$

$$= 39.55264,\ 5.37444 \qquad \text{or } \bar{I}_{\max} = 39.55 \text{ in}^4 \blacktriangleleft$$

$$\bar{I}_{\min} = 5.37 \text{ in}^4 \blacktriangleleft$$

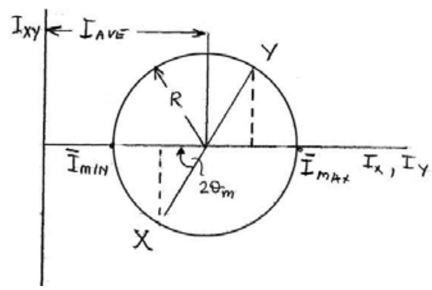
Note: The *a* axis corresponds to $\bar{I}_{\min}$ and *b* axis corresponds to $\bar{I}_{\max}$.

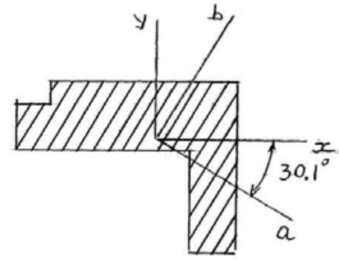
PROBLEM 9.104

Using Mohr's circle, determine for the area indicated the orientation of the principal centroidal axes and the corresponding values of the moments of inertia.

Area of Problem 9.77

SOLUTION





From Problems 9.44 and 9.77

$$\bar{I}_x = 432.59 \times 10^6 \text{ mm}^4, \qquad \bar{I}_y = 732.97 \times 10^6 \text{ mm}^4, \qquad \bar{I}_{xy} = -261.63 \times 10^6 \text{ mm}^4$$

Define points $X(432.59, \ -261.63) \times 10^6 \text{ mm}^4$

$$Y(732.97, \ 261.63) \times 10^6 \text{ mm}^4$$

Now $$\bar{I}_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = \frac{1}{2}(432.59 + 732.97) \times 10^6 = 582.78 \times 10^6 \text{ mm}^4$$

and $$R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2} = \frac{1}{2}\sqrt{\left(\frac{432.59 - 732.97}{2} \times 10^6\right)^2 + \left(-261.63 \times 10^6\right)^2}$$

$$= 301.67 \times 10^6 \text{ mm}^4$$

Also $$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} = \frac{-2(-261.63) \times 10^6}{(432.59 - 732.97) \times 10^6} = -60.14°$$

or $\theta_m = -30.1°$ clockwise ◀

Then $$\bar{I}_{\text{max, min}} = \bar{I}_{\text{ave}} \pm R = (582.78 \pm 301.67) \times 10^6 \text{ mm}^4$$

or $\bar{I}_{\text{max}} = 884 \times 10^6 \text{ mm}^4$ ◀

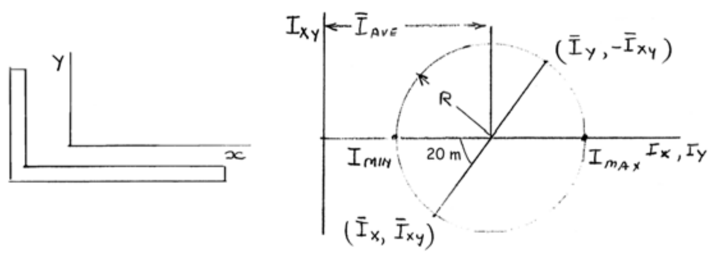
and $\bar{I}_{\text{min}} = 281 \times 10^6 \text{ mm}^4$ ◀

Note: From the Mohr's circle it is seen that the *a* axis corresponds to $\bar{I}_{\text{min}}$ and the *b* axis corresponds to $\bar{I}_{\text{max}}$.

PROBLEM 9.105

The moments and product of inertia for an L102 × 76 × 6.4-mm angle cross section with respect to two rectangular axes x and y through C are, respectively, $\bar{I}_x = 0.166 \times 10^6 \text{ mm}^4$, $\bar{I}_y = 0.453 \times 10^6 \text{ mm}^4$, and $\bar{I}_{xy} < 0$, with the minimum value of the moment of inertia of the area with respect to any axis through C being $\bar{I}_{\min} = 0.051 \times 10^6 \text{ mm}^4$. Using Mohr's circle, determine (a) the product of inertia $\bar{I}_{xy}$ of the area, (b) the orientation of the principal axes, (c) the value of $\bar{I}_{\max}$.

SOLUTION



Given: $\bar{I}_x = 0.166 \times 10^6 \text{ mm}^4$, $\bar{I}_y = 0.453 \times 10^6 \text{ mm}^4$ and $\bar{I}_{xy} < 0$

Note: A review of a table of rolled-steel shapes reveals that the given values of $\bar{I}_x$ and $\bar{I}_y$ are obtained when the 102 mm leg of the angle is parallel to the x axis. For $\bar{I}_{xy} < 0$ the angle must be oriented as shown.

(a) Now

$$\bar{I}_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = \frac{1}{2}(0.166 + 0.453) \times 10^6 \text{ mm}^4$$

$$= 0.3095 \times 10^6 \text{ mm}^4$$

Now

$$\bar{I}_{\min} = \bar{I}_{\text{ave}} - R \qquad \text{or} \qquad R = \bar{I}_{\text{ave}} - \bar{I}_{\min}$$

Then

$$R = (0.3095 - 0.051) \times 10^6 \text{ mm}^4$$

$$= 0.2585 \times 10^6 \text{ mm}^4$$

From

$$R^2 = \left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \left(\bar{I}_{xy}\right)^2$$

$$\bar{I}_{xy} = \sqrt{\left[(0.2585)^2 - \left(\frac{0.166 - 0.453}{2}\right)^2\right]} \times 10^6 \text{ mm}^4$$

$$\bar{I}_{xy} = \pm 0.21501 \times 10^6 \text{ mm}^4$$

Since $\bar{I}_{xy} < 0$, $\bar{I}_{xy} = -0.21501 \times 10^6 \text{ mm}^4$ or $\bar{I}_{xy} = -0.215 \times 10^6 \text{ mm}^4$ ◀

PROBLEM 9.105 CONTINUED

(*b*)

$$2\theta_m = \tan^{-1}\left[\frac{-2(-0.21501)}{0.166 - 0.453}\right] = -56.28°$$

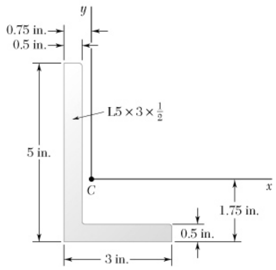
or $\theta_m = -28.1$ clockwise ◀

(*c*)

$$\bar{I}_{\max} = \bar{I}_{\text{ave}} + R = (0.3095 + 0.2585) \times 10^6 \text{ mm}^4$$

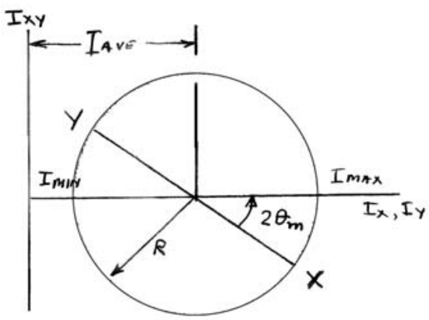
or $\bar{I}_{\max} = 0.568 \times 10^6 \text{ mm}^4$ ◀

PROBLEM 9.106



Using Mohr's circle, determine for the cross section of the rolled-steel angle shown the orientation of the principal centroidal axes and the corresponding values of the moments of inertia. (Properties of the cross sections are given in Fig. 9.13.)

SOLUTION



From Figure 9.13 $\quad \bar{I}_x = 9.45 \text{ in}^4$

$\bar{I}_y = 2.58 \text{ in}^4$

From Problem 9.78 $\quad \bar{I}_{xy} = -2.81 \text{ in}^4$

The Mohr's circle is defined by the diameter XY where

$$X(9.45,\ -2.81) \text{ in}^4$$

$$Y(2.58,\ 2.81) \text{ in}^4$$

Now $\quad \bar{I}_{\text{ave}} = \frac{1}{2}(\bar{I}_x + \bar{I}_y) = \frac{1}{2}(9.45 \text{ in}^4 + 2.58 \text{ in}^4)$

$$= 6.015 \text{ in}^4$$

and $\quad R = \sqrt{\left[\frac{1}{2}(\bar{I}_x - \bar{I}_y)\right]^2 + \bar{I}_{xy}^2}$

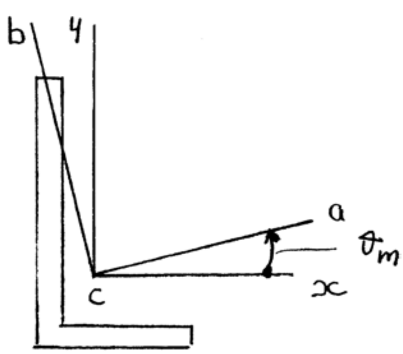
$$= \sqrt{\frac{1}{2}(9.45 \text{ in}^4 - 2.58 \text{ in}^4)^2 + (2.81 \text{ in}^4)^2}$$

$$= 5.612 \text{ in}^4$$

$$\tan 2\theta_m = \frac{-2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} = \frac{-2(-2.81 \text{ in}^4)}{9.45 \text{ in}^4 - 2.58 \text{ in}^4} = 0.81805$$

or $\quad 2\theta_m = 32.285°$

or $\theta_m = 19.643°$ counterclockwise ◀



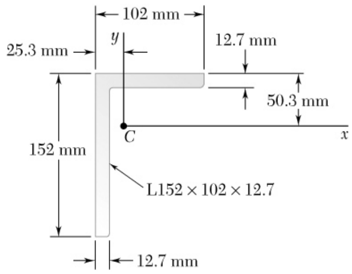
About C.

Now $\quad \bar{I}_{\text{max, min}} = \bar{I}_{\text{ave}} \pm R = (6.015 \pm 5.612)\text{in}^4$

or $\bar{I}_{\text{max}} = 11.63 \text{ in}^4$ ◀

and $\bar{I}_{\text{min}} = 0.403 \text{ in}^4$ ◀

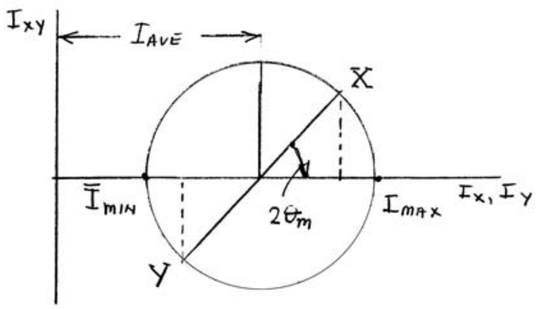
From the Mohr's circle it is seen that the a axis corresponds to $\bar{I}_{\text{max}}$ and the b axis corresponds to $\bar{I}_{\text{min}}$.



PROBLEM 9.107

Using Mohr's circle, determine for the cross section of the rolled-steel angle shown the orientation of the principal centroidal axes and the corresponding values of the moments of inertia. (Properties of the cross sections are given in Fig. 9.13.)

SOLUTION



From Figure 9.13B: $\bar{I}_x = 7.20 \times 10^6 \text{ mm}^4, \quad \bar{I}_y = 2.64 \times 10^6 \text{ mm}^4$

Have $\bar{I}_{xy} = (I_{xy})_1 + (I_{xy})_2,$ where $I_{xy} = \bar{I}_{x'y'} + \bar{x}\,\bar{y}A$ and $\bar{I}_{x'y'} = 0$

Now $\bar{x}_1 = \dfrac{102}{2} - 25.3 = 25.7 \text{ mm}, \quad \bar{y}_1 = 50.3 - \dfrac{12.7}{2} = 43.95 \text{ mm}$

$$A_1 = 102 \times 12.7 = 1295.4 \text{ mm}^2$$

$$\bar{x}_2 = -25.3 - \frac{12.7}{2} = -18.95 \text{ mm} \qquad \bar{y}_2 = -\left[\frac{1}{2}(152 - 12.7) - (50.3 - 12.7)\right] = 32.05 \text{ mm}$$

$$A_2 = (12.7)(152 - 12.7) = 1769.11 \text{ mm}^2$$

Then $\bar{I}_{xy} = \left\{\left[(25.7 \text{ mm})(43.95 \text{ mm})(1295.4 \text{ mm}^2)\right] + \left[(-18.95 \text{ mm})(-32.05 \text{ mm})(1769.11 \text{ mm}^2)\right]\right\} \times 10^6$

$$= (1.46317 + 1.07446) \times 10^6 \text{ mm}^4 = 2.5376 \times 10^6 \text{ mm}^4$$

The Mohr's circle is defined by points X and Y, where

$$X\left(\bar{I}_x,\ \bar{I}_{xy}\right),\ Y\left(\bar{I}_y,\ -\bar{I}_{xy}\right)$$

Now $\bar{I}_{\text{ave}} = \dfrac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = \dfrac{1}{2}(7.20 + 2.64) \times 10^6 \text{ mm}^4 = 4.92 \times 10^6 \text{ mm}^4$

PROBLEM 9.107 CONTINUED

and $$R = \sqrt{\left(\frac{\overline{I}_x - \overline{I}_y}{2}\right) + \overline{I}_{xy}^2} = \left[\sqrt{\frac{1}{2}(7.20 - 2.64)^2 + 2.5376^2}\right] \times 10^6 \text{ mm}^4$$

$$= 3.4114 \times 10^6 \text{ mm}^4$$

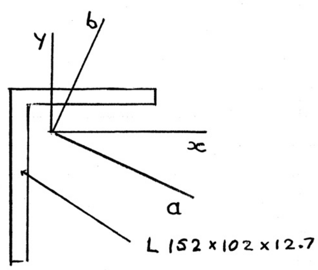
$$\tan\theta_m = -\frac{2\overline{I}_{xy}}{\overline{I}_x - \overline{I}_y} = -\frac{2(2.5376)}{(7.20 - 2.64)} = -1.11298, \qquad 2\theta = -48.0607°$$

or $$\theta = -24.0° \text{ clockwise} ◀$$

Now $$\overline{I}_{\text{max, min}} = \overline{I}_{\text{ave}} \pm R = (4.92 \pm 3.4114) \times 10^6 \text{ mm}^4$$

or $$\overline{I}_{\text{max}} = 8.33 \times 10^6 \text{ mm}^4 ◀$$

and $$\overline{I}_{\text{min}} = 1.509 \times 10^6 \text{ mm}^4 ◀$$



Note: From the Mohr's circle it is seen that the *a* axis corresponds to $\overline{I}_{\text{max}}$ and the *b* axis corresponds to $\overline{I}_{\text{min}}$.

PROBLEM 9.108

For a given area the moments of inertia with respect to two rectangular centroidal x and y axes are $\bar{I}_x = 640\ \text{in}^4$ and $\bar{I}_y = 280\ \text{in}^4$, respectively. Knowing that after rotating the x and y axes about the centroid 60° clockwise the product of inertia relative to the rotated axes is $-180\ \text{in}^4$, use Mohr's circle to determine (*a*) the orientation of the principal axes, (*b*) the centroidal principal moments of inertia.

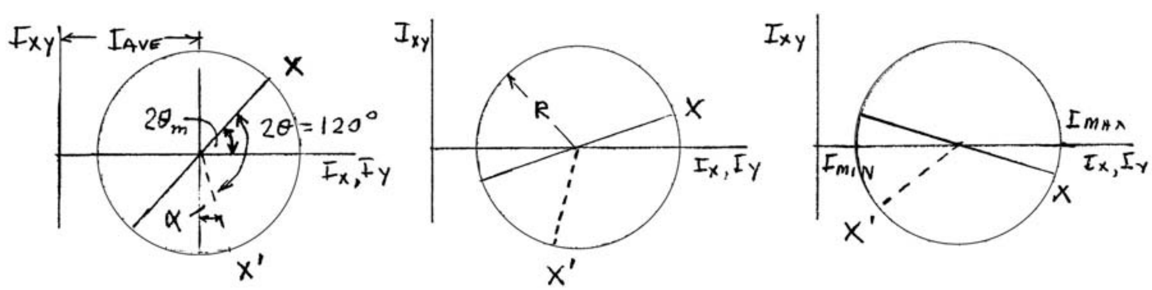
SOLUTION

Have $$I_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = \frac{1}{2}\left(640\ \text{in}^4 + 280\ \text{in}^4\right) = 460\ \text{in}^4$$

$$\frac{1}{2}\left(\bar{I}_x - \bar{I}_y\right) = \frac{1}{2}\left(640\ \text{in}^4 - 280\ \text{in}^4\right) = 180\ \text{in}^4$$

Also have $$\bar{I}_{x'y'} = -180\ \text{in}^4, \qquad 2\theta = -120°, \qquad I_x > I_y$$

Letting the points $\left(\bar{I}_x,\ \bar{I}_{xy}\right)$ and $\left(\bar{I}_{x'},\ \bar{I}_{x'y'}\right)$ be denoted by X an X', respectively, three possible Mohr's circles can be constructed



Assume the first case applies

Then $$\frac{\bar{I}_x - \bar{I}_y}{2} = R\cos 2\theta_m \qquad \text{or} \qquad R\cos 2\theta_m = 180\ \text{in}^4$$

Also $$\left|\bar{I}_{x'y'}\right| = R\cos\alpha \qquad \text{or} \qquad R\cos\alpha = 180\ \text{in}^4$$

$$\therefore \quad \alpha = \pm 2\theta_m$$

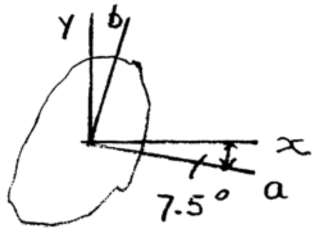
Also have $$120° = 2\theta_m + \left(90° - \alpha\right) \qquad \text{or} \qquad 2\theta_m - \alpha = 30°$$

$$\therefore \quad \alpha = -2\theta_m \qquad \text{and} \qquad 2\left(2\theta_m\right) = 30° \qquad \text{or} \qquad 2\theta_m = |\alpha| = 15°$$

Note $$\left.\begin{matrix} 2\theta_m > 0 \\ \alpha < 0 \end{matrix}\right\} \text{ implies case 2 applies}$$

PROBLEM 9.108 CONTINUED

(a) Therefore, $\theta_m = 7.5°$ clockwise ◀



(b) Have $R\cos 15° = 180$ or $R = 186.35\text{ in}^4$

Then $\bar{I}_{\text{max, min}} = \bar{I}_{\text{ave}} \pm R = 460 \pm 186.350$

or $\bar{I}_{\text{max}} = 646\text{ in}^4$ ◀

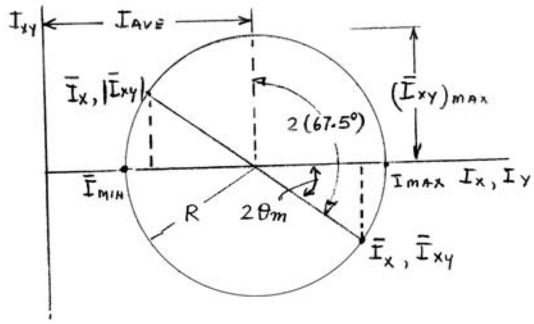
and $\bar{I}_{\text{min}} = 274\text{ in}^4$ ◀

Note: From the Mohr's circle it is seen that the a axis corresponds to $\bar{I}_{\text{max}}$ and the b axis corresponds to $\bar{I}_{\text{min}}$.

PROBLEM 9.109

It is known that for a given area $\bar{I}_y = 300 \text{ in}^4$ and $\bar{I}_{xy} = -125 \text{ in}^4$, where the x and y axes are rectangular centroidal axes. If the axis corresponding to the maximum product of inertia is obtained by rotating the x axis 67.5° counterclockwise about C, use Mohr's circle to determine (*a*) the moment of inertia $\bar{I}_x$ of the area, (*b*) the principal centroidal moments of inertia.

SOLUTION



First assume $\bar{I}_x > \bar{I}_y$

(Note: Assuming $\bar{I}_x < \bar{I}_y$ is not consistent with the requirement that the axis corresponding to the $\left(\bar{I}_{xy}\right)_{\max}$ is obtained by rotating the x axis through $67.5°$ counterclockwise)

From Mohr's circle have $$2\theta_m = 2(67.5°) - 90° = 45°$$

(*a*) From $$\tan 2\theta_m = \frac{2\left|\bar{I}_{xy}\right|}{\bar{I}_x - \bar{I}_y}$$

Have $$\bar{I}_x = \bar{I}_y + 2\frac{\left|\bar{I}_{xy}\right|}{\tan 2\theta_m} = 300 \text{ in}^4 + 2\frac{125 \text{ in}^4}{\tan 45°} = 550 \text{ in}^4$$

or $\bar{I}_x = 550 \text{ in}^4$ ◀

(*b*) Now $$I_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right) = \frac{550 + 300}{2} \text{ in}^4 = 425 \text{ in}^4$$

and $$R = \frac{\left|\bar{I}_{xy}\right|}{\sin 2\theta_m} = \frac{125 \text{ in}^4}{\sin 45°} = 176.78 \text{ in}^4$$

Then $$\bar{I}_{\max,\ \min} = \bar{I}_{\text{ave}} \pm R = (425 \pm 176.76)\text{in}^4$$
$$= (601.78,\ 248.22)\text{in}^4$$

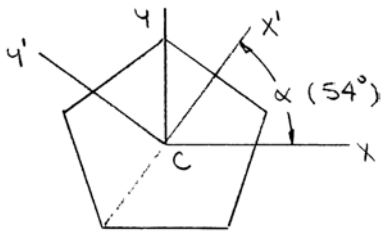
or $\bar{I}_{\max} = 602 \text{ in}^4$ ◀

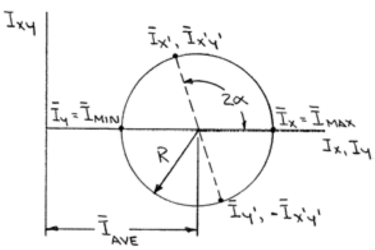
and $\bar{I}_{\min} = 248 \text{ in}^4$ ◀

PROBLEM 9.110

Using Mohr's circle, show that for any regular polygon (such as a pentagon) (*a*) the moment of inertia with respect to every axis through the centroid is the same, (*b*) the product of inertia with respect to every pair of rectangular axes through the centroid is zero.

SOLUTION





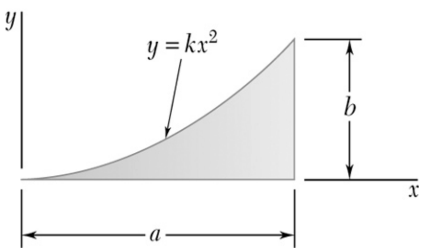
Consider the regular pentagon shown, with centroidal axes *x* and *y*.

Because the *y* axis is an axis of symmetry, it follows that $\bar{I}_{xy} = 0.$ Since $\bar{I}_{xy} = 0,$ the *x* and *y* axes must be principal axes. Assuming $\bar{I}_x = \bar{I}_{\max}$ and $\bar{I}_y = \bar{I}_{\min},$ the Mohr's circle is then drawn as shown.

Now rotate the coordinate axes through an angle α as shown; the resulting moments of inertia, $\bar{I}_{x'}$ and $\bar{I}_{y'},$ and product of inertia, $\bar{I}_{x'y'},$ are indicated on the Mohr's circle. However, the x' axis is an axis of symmetry, which implies $\bar{I}_{x'y'} = 0.$ For this to be possible on the Mohr's circle, the radius *R* must be equal to zero (thus, the circle degenerates into a point). With $R = 0,$ it immediately follows that

(*a*) $\bar{I}_x = \bar{I}_y = \bar{I}_{x'} = \bar{I}_{y'} = \bar{I}_{\text{ave}}$ (for all moments of inertia with respect to an axis through *C*) ◀

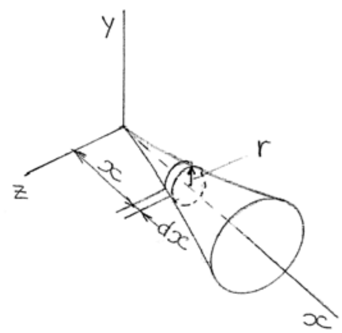
(*b*) $\bar{I}_{xy} = \bar{I}_{x'y'} = 0$ (for all products of inertia with respect to all pairs of rectangular axes with origin at *C*)◀



PROBLEM 9.121

The parabolic spandrel shown is revolved about the x axis to form a homogeneous solid of revolution of mass m. Using direct integration, express the moment of inertia of the solid with respect to the x axis in terms of m and b.

SOLUTION



At $\qquad x = a,\ y = b: \qquad b = ka^2 \qquad \text{or} \qquad k = \dfrac{b}{a^2}$

Then $$y = \frac{b}{a^2}x^2$$

Now $$dm = \rho\left(\pi r^2\right)dx$$

$$= \pi\rho\left(\frac{b}{a^2}x^2\right)^2 dx$$

Then $$m = \pi\rho\frac{b^2}{a^4}\int_0^a x^4 dx$$

$$= \frac{1}{5}\pi\rho\frac{b^2}{a^4}x^5\Big|_0^a$$

$$= \frac{1}{5}\pi\rho ab^2 \qquad \text{or} \qquad \pi\rho = \frac{5m}{ab^2}$$

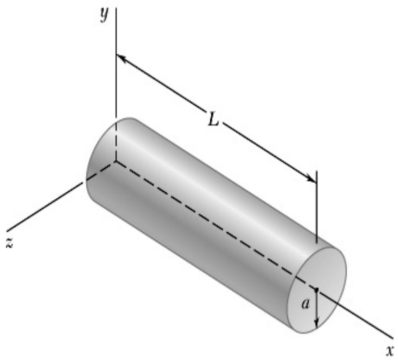
Now $$d\,\overline{I}_x = \left(\frac{1}{2}r^2\right)dm = \frac{1}{2}\left(\frac{b}{a^2}x^2\right)^2\left[\pi\rho\left(\frac{b}{a^2}x^2\right)^2 dx\right]$$

$$= \frac{5m}{ab^2}\times\frac{1}{2}\frac{b^2}{a^4}x^4\times\frac{b^2}{a^4}x^4dx = \frac{5}{2}m\frac{b^2}{a^9}x^8dx$$

Then.. $$\overline{I}_x = \frac{5}{2}m\frac{b^2}{a^9}\int_0^a x^8dx = \frac{5}{2}m\frac{b^2}{a^9}\times\frac{1}{9}x^9\Big|_0^a$$

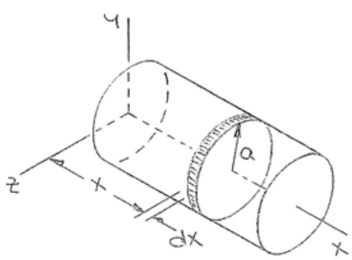
$$\text{or } \overline{I}_x = \frac{5}{18}mb^2 \blacktriangleleft$$

PROBLEM 9.122



Determine by direct integration the moment of inertia with respect to the z axis of the right circular cylinder shown assuming that it has a uniform density and a mass m.

SOLUTION



For the cylinder $\quad m = \rho V = \rho \pi a^2 L$

For the element shown $\quad dm = \rho \pi a^2 dx$

$$= \frac{m}{L} dx$$

and $\quad dI_z = d\bar{I}_z + x^2 dm$

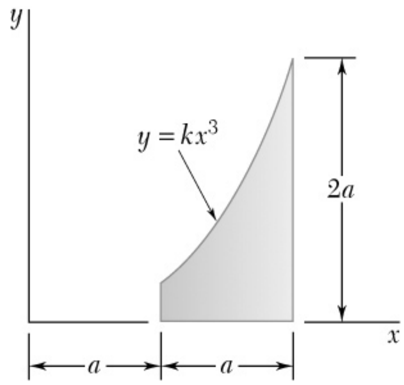
$$= \frac{1}{4} a^2 dm + x^2 dm$$

Then $\quad I_z = \int dI_z = \int_0^L \left(\frac{1}{4}a^2 + x^2\right)\left(\frac{m}{L}dx\right) = \frac{m}{L}\left[\frac{1}{4}a^2 x + \frac{1}{3}x^3\right]_0^L$

$$= \frac{m}{L}\left(\frac{1}{4}a^2 L + \frac{1}{3}L^3\right)$$

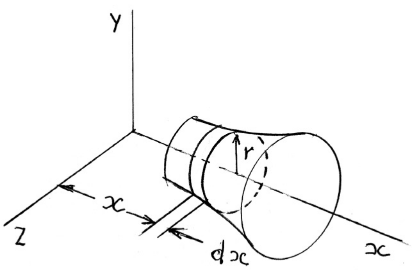
or $\quad I_z = \frac{1}{12} m\left(3a^2 + 4L^2\right)$ ◀

PROBLEM 9.123



The area shown is revolved about the *x* axis to form a homogeneous solid of revolution of mass *m*. Determine by direct integration the moment of inertia of the solid with respect to (*a*) the *x* axis, (*b*) the *y* axis. Express your answers in terms of *m* and *a*.

SOLUTION



At $\quad x = 2a \qquad 2a = k(2a)^3 \qquad$ or $\qquad k = \dfrac{1}{4a^2}$

Then $$y = \frac{1}{4a^2}x^3$$

Now $$dm = \rho\left(\pi r^2 dx\right)$$

$$= \pi\rho\left(\frac{1}{4a^2}x^3\right)^2 dx = \frac{\pi\rho}{16a^4}x^6 dx$$

Then $$m = \frac{\pi\rho}{16a^4}\int_a^{2a} x^6 dx$$

$$= \frac{\pi\rho}{16a^4}\frac{1}{7}x^7\Big|_a^{2a} = \frac{\pi\rho}{112a^4}\left[(2a)^7 - (a)^7\right] = \frac{127}{112}\pi\rho a^3$$

or $$\pi\rho = \frac{112m}{127a^3}$$

(*a*) Now $$d\bar{I}_x = \left(\frac{1}{2}r^2\right)dm = \frac{1}{2}\left(\frac{1}{4a^2}x^3\right)^2\left(\frac{\pi\rho}{16a^4}x^6 dx\right)$$

$$= \frac{1}{32a^4}x^6 \times \frac{112m}{127a^3} \times \frac{x^6}{16a^4}dx = \frac{7m}{4064a^{11}}x^{12}dx$$

Then

$$\bar{I}_x = \frac{7m}{4064a^{11}}\int_a^{2a} x^{12}dx = \frac{7m}{4064a^{11}}\frac{1}{13}x^{13}\Big|_a^{2a}$$

$$= \frac{7m}{52832a^{11}}\left[(2a)^{13} - (a)^{13}\right] = \frac{57337}{52832}ma^2 = 1.0853ma^2$$

or $\bar{I}_x = 1.085ma^2$ ◀

PROBLEM 9.123 CONTINUED

Have $$d\bar{I}_y = \left(\frac{1}{4}r^2 + x^2\right)dm = \left[\frac{1}{4}\left(\frac{1}{4a^2}x^3\right)^2 + x^2\right]\frac{\pi\rho}{16a^4}x^6dx$$

$$= \frac{1}{16a^4} \times \frac{112m}{127a^3}\left(\frac{1}{64a^4}x^{12} + x^8\right)dx$$

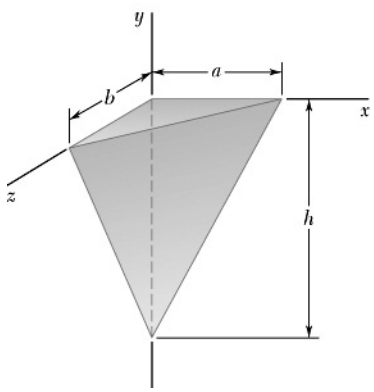
Then $$\bar{I}_y = \frac{7m}{127a^7}\int_a^{2a}\left(\frac{1}{64a^4}x^{12} + x^8\right)dx = \frac{7m}{127a^7}\left(\frac{1}{832a^4}x^{13} + \frac{1}{9}x^8\right)\Bigg|_a^{2a}$$

$$= \frac{7m}{127a^7}\left[\frac{1}{832a^4}(2a)^{13} + \frac{1}{9}(2a)^9 - \frac{1}{832a^4}(a)^{13} - \frac{1}{9}(a)^9\right]$$

$$= \frac{7m}{127a^7}\left(\frac{8191}{832}a^9 + \frac{511}{9}a^9\right) = 3.67211ma^2$$

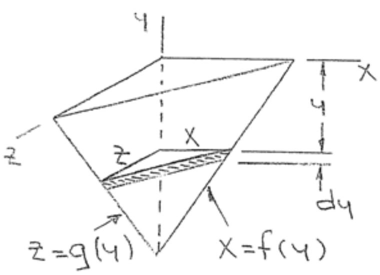
or $\bar{I}_y = 3.67ma^2$ ◀

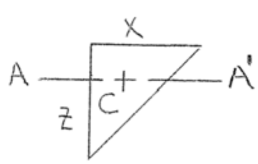
PROBLEM 9.124



Determine by direct integration the moment of inertia with respect to the x axis of the tetrahedron shown assuming that it has a uniform density and a mass m.

SOLUTION





Have $$x = \frac{a}{h}y + a = a\left(1 + \frac{y}{h}\right)$$

and $$z = \frac{b}{h}y + b = b\left(1 + \frac{y}{h}\right)$$

For the element shown $$dm = \rho\left(\frac{1}{2}xzdy\right) = \frac{1}{2}\rho ab\left(1 + \frac{y}{h}\right)^2 dy$$

Then $$m = \int dm = \int_{-h}^{0} \frac{1}{2}\rho ab\left(1 + \frac{y}{h}\right)^2 dy$$

$$= \frac{1}{2}\rho ab \times \frac{h}{3}\left[\left(1 + \frac{y}{h}\right)^3\right]_{-h}^{0}$$

$$= \frac{1}{6}\rho abh\left[(1)^3 - (1-1)^3\right]$$

$$= \frac{1}{6}\rho abh$$

Now, for the element $$I_{AA',\text{area}} = \frac{1}{36}xz^3 = \frac{1}{36}ab^3\left(1 + \frac{y}{h}\right)^4$$

Then $$dI_{AA',\text{ mass}} = \rho t I_{AA',\text{ area}} = \rho(dy)\left[\frac{1}{3}ab^3\left(1 + \frac{y}{h}\right)^4\right]$$

PROBLEM 9.124 CONTINUED

Now

$$dI_x = dI_{AA',\text{mass}} + \left[y^2 + \left(\frac{1}{3}z\right)^2\right]dm$$

$$= \frac{1}{36}\rho ab^3\left(1 + \frac{y}{h}\right)^4 dy$$

$$+ \left\{y^2 + \left[\frac{1}{3}b\left(1 + \frac{y}{h}\right)\right]^2\right\}\left[\frac{1}{2}\rho ab\left(1 + \frac{y}{h}\right)^2 dy\right]$$

$$= \frac{1}{12}\rho ab^3\left(1 + \frac{y}{h}\right)^4 dy + \frac{1}{2}\rho ab\left(y^2 + 2\frac{y^3}{h} + \frac{y^4}{h^2}\right)dy$$

Now $$m = \frac{1}{6}\rho abh$$

Then $$dI_x = \left[\frac{1}{2}m\frac{b^2}{h}\left(1 + \frac{y}{h}\right)^4 + \frac{3m}{h}\left(y^2 + 2\frac{y^3}{h} + \frac{y^4}{h^2}\right)\right]dy$$

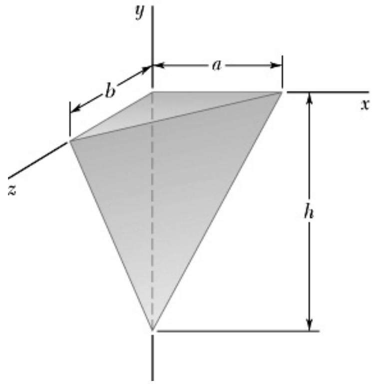
and

$$I_x = \int dI_x = \int_{-h}^{0}\frac{m}{2h}\left[b^2\left(1 + \frac{y}{h}\right)^4 + 6\left(y^2 + 2\frac{y^3}{h} + \frac{y^4}{h^2}\right)\right]dy$$

$$= \frac{m}{2h}\left[b^2 \times \frac{h}{5}\left(1 + \frac{y}{h}\right)^5 + 6\left(\frac{1}{3}y^3 + \frac{1}{2}\frac{y^4}{h} + \frac{y^5}{5h^2}\right)\right]_{-h}^{0}$$

$$= \frac{m}{2h}\left\{\frac{1}{5}b^2h(1)^5 - 6\left[\frac{1}{3}(-h)^3 + \frac{1}{2h}(-h)^4 + \frac{1}{5h^2}(-h)^5\right]\right\}$$

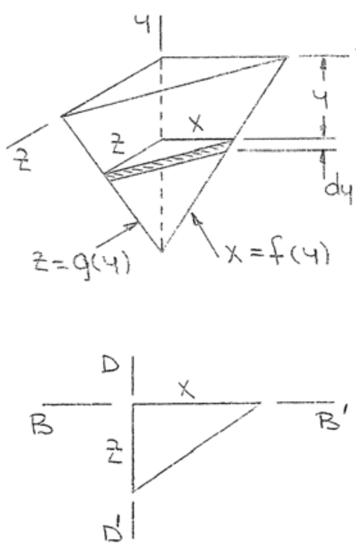
$$\text{or } I_x = \frac{1}{10}m\left(b^2 + h^2\right) \blacktriangleleft$$



PROBLEM 9.125

Determine by direct integration the moment of inertia with respect to the y axis of the tetrahedron shown assuming that it has a uniform density and a mass m.

SOLUTION



Have $$x = \frac{a}{h}y + a = a\left(1 + \frac{y}{h}\right)$$

and $$z = \frac{b}{h}y + b = b\left(1 + \frac{y}{h}\right)$$

For the element shown $$dm = \rho\left(\frac{1}{2}xzdy\right) = \frac{1}{2}\rho ab\left(1 + \frac{y}{h}\right)^2 dy$$

Then $$m = \int dm = \int_{-h}^{0} \frac{1}{2}\rho ab\left(1 + \frac{y}{h}\right)^2 dy$$

$$= \frac{1}{2}\rho ab \times \frac{h}{3}\left[\left(1 + \frac{y}{h}\right)^3\right]_{-h}^{0}$$

$$= \frac{1}{6}\rho abh\left[(1)^3 - (1-1)^3\right]$$

$$= \frac{1}{6}\rho abh$$

Also $$I_{BB',\text{area}} = \frac{1}{12}xz^3 \qquad I_{DD',\text{area}} = \frac{1}{12}zx^3$$

Then, using $I_{\text{mass}} = \rho t I_{\text{area}}$ have

$$dI_{BB',\text{mass}} = \rho(dy)\left(\frac{1}{12}xz^3\right) \qquad dI_{DD',\text{mass}} = \rho(dy)\left(\frac{1}{12}zx^3\right)$$

PROBLEM 9.125 CONTINUED

Now

$$dI_y = dI_{BB',\text{mass}} + dI_{DD',\text{mass}}$$

$$= \frac{1}{12}\rho xz\left(x^2 + z^2\right)dy$$

$$= \frac{1}{12}\rho ab\left(1 + \frac{y}{h}\right)^2\left[\left(a^2 + b^2\right)\left(1 + \frac{y}{h}\right)^2\right]dy$$

Have $$m = \frac{1}{6}\rho abh \Rightarrow dI_y = \frac{m}{2h}\left(a^2 + b^2\right)\left(1 + \frac{y}{h}\right)^4 dy$$

Then

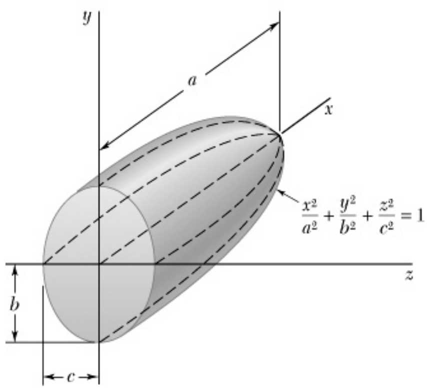
$$I_y = \int dI_y = \int_{-h}^{0}\frac{m}{2h}\left(a^2 + b^2\right)\left(1 + \frac{y}{h}\right)^4 dy$$

$$= \frac{m}{2h}\left(a^2 + b^2\right)\times\frac{h}{5}\left[\left(1 + \frac{y}{h}\right)^5\right]_{-h}^{0}$$

$$= \frac{m}{10}\left(a^2 + b^2\right)\left[(1)^5 - (1-1)^5\right]$$

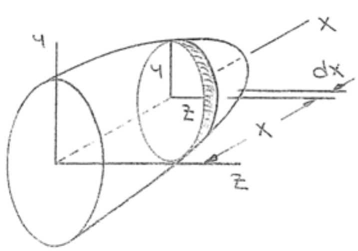
or $I_y = \dfrac{1}{10}m\left(a^2 + b^2\right)$ ◀

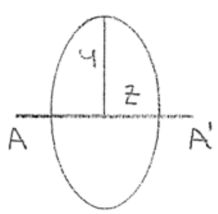
PROBLEM 9.126



Determine by direct integration the moment of inertia with respect to the z axis of the semiellipsoid shown assuming that it has a uniform density and a mass m.

SOLUTION





First note that when

$$z = 0: \qquad y = b\left(1 - \frac{x^2}{a^2}\right)^{\frac{1}{2}}$$

$$y = 0: \qquad z = c\left(1 - \frac{x^2}{a^2}\right)^{\frac{1}{2}}$$

For the element shown $\qquad dm = \rho(\pi yzdx) = \pi\rho bc\left(1 - \frac{x^2}{a^2}\right)dx$

Then $\qquad m = \int dm = \int_0^a \pi\rho bc\left(1 - \frac{x^2}{a^2}\right)dx$

$$= \pi\rho bc\left[x - \frac{1}{3a^2}x^3\right]_0^a = \frac{2}{3}\pi\rho abc$$

For the element $\qquad I_{AA',\text{area}} = \frac{\pi}{4}zy^3$

Then $\qquad dI_{AA',\text{mass}} = \rho t I_{AA',\text{area}} = \rho(dx)\left(\frac{\pi}{4}zy^3\right)$

Now $\qquad dI_z = dI_{AA',\text{mass}} + x^2 dm$

$$= \frac{\pi}{4}\rho b^3 c\left(1 - \frac{x^2}{a^2}\right)^2 dx + x^2\left[\pi\rho bc\left(1 - \frac{x^2}{a^2}\right)dx\right]$$

$$= \frac{3m}{2a}\left[\frac{b^2}{4}\left(1 - 2\frac{x^2}{a^2} + \frac{x^4}{a^4}\right) + \left(x^2 - \frac{x^4}{a^2}\right)\right]dx$$

PROBLEM 9.126 CONTINUED

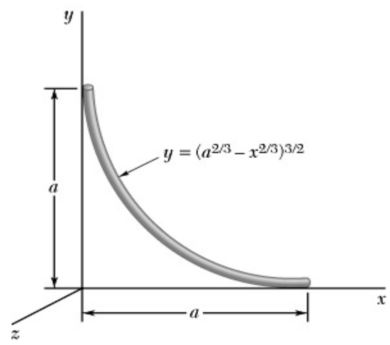
Finally $\qquad I_z = \int dI_z$

$$= \frac{3m}{2a}\int_0^a \left[\frac{b^2}{4}\left(1 - 2\frac{x^2}{a^2} + \frac{x^4}{a^4}\right) + \left(x^2 - \frac{x^4}{a^2}\right)\right]dx$$

$$= \frac{3m}{2a}\left[\frac{b^2}{4}\left(x - \frac{2}{3}\frac{x^3}{a^2} + \frac{1}{5}\frac{x^5}{a^4}\right) + \left(\frac{1}{3}x^3 - \frac{1}{5}\frac{x^5}{a^2}\right)\right]_0^a$$

$$= \frac{3}{2}m\left[\frac{b^2}{4}\left(1 - \frac{2}{3} + \frac{1}{5}\right) + a^2\left(\frac{1}{3} - \frac{1}{5}\right)\right]$$

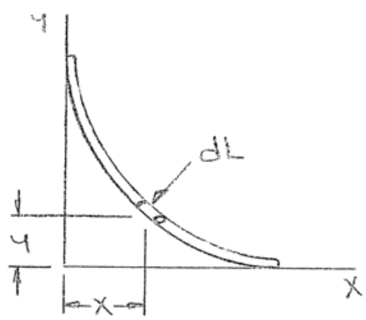
or $I_z = \frac{1}{5}m\left(a^2 + b^2\right)$ ◀



PROBLEM 9.127

A thin steel wire is bent into the shape shown. Denoting the mass per unit length of the wire by m', determine by direct integration the moment of inertia of the wire with respect to each of the coordinate axes.

SOLUTION



First note $$\frac{dy}{dx} = -x^{-\frac{1}{3}}\left(a^{\frac{2}{3}} - x^{\frac{2}{3}}\right)^{\frac{1}{2}}$$

Then $$1 + \left(\frac{dy}{dx}\right)^2 = 1 + x^{-\frac{2}{3}}\left(a^{\frac{2}{3}} - x^{\frac{2}{3}}\right)$$

$$= \left(\frac{a}{x}\right)^{\frac{2}{3}}$$

For the element shown $$dm = m'dL = m'\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

$$= m'\left(\frac{a}{x}\right)^{\frac{1}{3}} dx$$

Then $$m = \int dm = \int_0^a m'\frac{a^{\frac{1}{3}}}{x^{\frac{1}{3}}}dx = \frac{3}{2}m'a^{\frac{1}{3}}\left[x^{\frac{2}{3}}\right]_0^a = \frac{3}{2}m'a$$

Now $$I_x = \int y^2 dm = \int_0^a \left(a^{\frac{2}{3}} - x^{\frac{2}{3}}\right)^3 \left(m'\frac{a^{\frac{1}{3}}}{x^{\frac{1}{3}}}dx\right)$$

$$= m'a^{\frac{1}{3}}\int_0^a \left(\frac{a^2}{x^{\frac{1}{3}}} - 3a^{\frac{4}{3}}x^{\frac{1}{3}} + 3a^{\frac{2}{3}}x - x^{\frac{5}{3}}\right)dx$$

$$= m'a^{\frac{1}{3}}\left[\frac{3}{2}a^2x^{\frac{2}{3}} - \frac{9}{4}a^{\frac{4}{3}}x^{\frac{4}{3}} + \frac{3}{2}a^{\frac{2}{3}}x^2 - \frac{3}{8}x^{\frac{8}{3}}\right]_0^a$$

$$= m'a^3\left(\frac{3}{2} - \frac{9}{4} + \frac{3}{2} - \frac{3}{8}\right) = \frac{3}{8}m'a^3$$

or $I_x = \frac{1}{4}ma^2$ ◀

Symmetry implies $I_y = \frac{1}{4}ma^2$ ◀

PROBLEM 9.127 CONTINUED

Alternative Solution

$$I_y = \int x^2 dm = \int_0^a x^2 \left(m' \frac{a^{\frac{1}{3}}}{x^{\frac{1}{3}}} dx \right) = m'a^{\frac{1}{3}} \int_0^a x^{\frac{5}{3}} dx$$

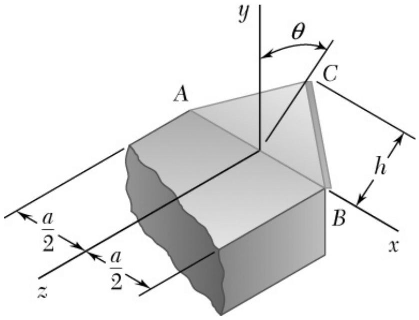
$$= m'a^{\frac{1}{3}} \times \frac{3}{8} \left[x^{\frac{8}{3}} \right]_0^a = \frac{3}{8} m'a^3$$

$$= \frac{1}{4} ma^2$$

Also $$I_z = \int \left(x^2 + y^2 \right) dm = I_y + I_x$$

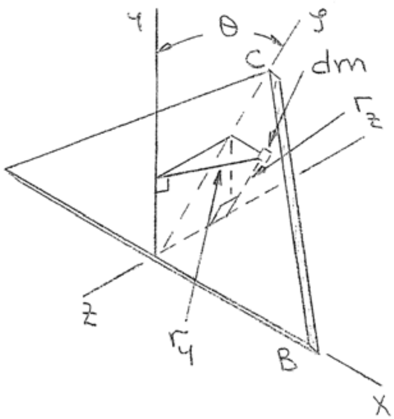
or $I_z = \dfrac{1}{2} ma^2$ ◀

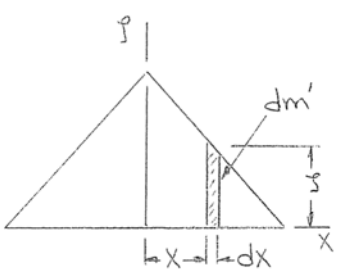
PROBLEM 9.128



A thin triangular plate of mass m is welded along its base AB to a block as shown. Knowing that the plate forms an angle θ with the y axis, determine by direct integration the mass moment of inertia of the plate with respect to (a) the x axis, (b) the y axis, (c) the z axis.

SOLUTION





For line BC

$$\zeta = -\frac{h}{\frac{a}{2}}x + h$$

$$= \frac{h}{a}(a - 2x)$$

Also

$$m = \rho V = \rho t\left(\frac{1}{2}ah\right)$$

$$= \frac{1}{2}\rho tah$$

(a) Have

$$dI_x = \frac{1}{12}\zeta^2 dm' + \left(\frac{\zeta}{2}\right)^2 dm'$$

$$= \frac{1}{3}\zeta^2 dm'$$

where

$$dm' = \rho t\zeta\, dx$$

Then

$$I_x = \int dI_x = 2\int_0^{\frac{a}{2}} \frac{1}{3}\zeta^2(\rho t\zeta\, dx)$$

$$= \frac{2}{3}\rho t\int_0^{\frac{a}{2}}\left[\frac{h}{a}(a-2x)\right]^3 dx$$

$$= \frac{2}{3}\rho t\frac{h^3}{a^3}\times\frac{1}{4}\left(-\frac{1}{2}\right)\left[(a-2x)^4\right]_0^{\frac{a}{2}}$$

$$= -\frac{1}{12}\rho t\frac{h^3}{a^3}\left[(a-a)^4 - (a)^4\right]$$

$$= \frac{1}{12}\rho tah^3$$

or $I_x = \dfrac{1}{6}mh^2$ ◀

PROBLEM 9.128 CONTINUED

Now $$I_{\zeta} = \int x^2 dm$$

and $$I_{\zeta} = \int x^2 dm' = 2\int_0^{\frac{a}{2}} x^2 \left(\rho t \zeta\, dx\right)$$

$$= 2\rho t \int_0^{\frac{a}{2}} x^2 \left[\frac{h}{a}\left(a - 2x\right)\right] dx$$

$$= 2\rho t \frac{h}{a}\left[\frac{a}{3}x^3 - \frac{1}{4}x^4\right]_0^{\frac{a}{2}}$$

$$= 2\rho t \frac{h}{a}\left[\frac{a}{3}\left(\frac{a}{2}\right)^3 - \frac{1}{4}\left(\frac{a}{2}\right)^4\right]$$

$$= \frac{1}{48}\rho t a^3 h = \frac{1}{24} m a^2$$

(*b*) Have $$I_y = \int r_y^2 dm = \int\left[x^2 + \left(\zeta \sin\theta\right)^2\right] dm$$

$$= \int x^2 dm + \sin^2\theta \int \zeta^2 dm$$

Now $$I_x = \int \zeta^2 dm \Rightarrow I_y = I_{\zeta} + I_x \sin^2\theta$$

$$= \frac{1}{24} m a^2 + \frac{1}{6} m h^2 \sin^2\theta$$

$$\text{or } I_y = \frac{m}{24}\left(a^2 + 4h^2 \sin^2\theta\right) \blacktriangleleft$$

(*c*) Have $$I_z = \int r_z^2 dm = \int\left(x^2 + y^2\right) dm$$

$$= \int\left[x^2 + \left(\zeta \cos\theta\right)^2\right] dm$$

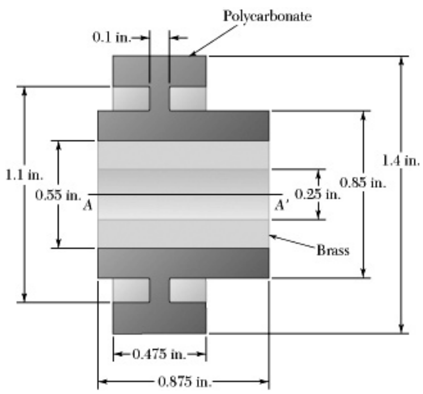
$$= \int x^2 dm + \cos^2\theta \int \zeta^2 dm$$

$$= I_{\zeta} + I_x \cos^2\theta$$

$$= \frac{1}{24} m a^2 + \frac{1}{6} m h^2 \cos^2\theta$$

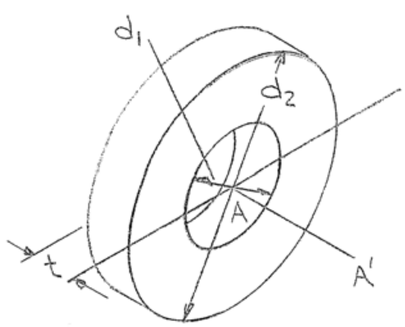
$$\text{or } I_z = \frac{m}{24}\left(a^2 + 4h^2 \cos^2\theta\right) \blacktriangleleft$$

PROBLEM 9.129



Shown is the cross section of a molded flat-belt pulley. Determine its mass moment of inertia and its radius of gyration with respect to the axis AA'. (The specific weight of brass is 0.306 lb/in^3 and the specific weight of the fiber-reinforced polycarbonate used is 0.0433 lb/in^3.)

SOLUTION



First note for the cylindrical ring shown that

$$m = \rho V = \rho t \times \frac{\pi}{4}\left(d_2^2 - d_1^2\right)$$

and, using Figure 9.28, that

$$I_{AA'} = \frac{1}{2}m_2\left(\frac{d_2}{2}\right)^2 - \frac{1}{2}m_1\left(\frac{d_1}{2}\right)^2$$

$$= \frac{1}{8}\left[\left(\rho t \times \frac{\pi}{4}d_2^2\right)d_2^2 - \left(\rho t \times \frac{\pi}{4}d_1^2\right)d_1^2\right]$$

$$= \frac{1}{8}\left(\frac{\pi}{4}\rho t\right)\left(d_2^4 - d_1^4\right)$$

$$= \frac{1}{8}\left(\frac{\pi}{4}\rho t\right)\left(d_2^2 - d_1^2\right)\left(d_2^2 + d_1^2\right)$$

$$= \frac{1}{8}m\left(d_1^2 + d_2^2\right)$$

Now treat the pulley as four concentric rings and, working from the brass outward, have

$$m = \frac{\pi}{4}\left\{\frac{0.306\,\text{lb/in}^3}{32.2\text{ ft/s}^2}(0.875\text{ in.})\left[(0.55\text{ in.})^2 - (0.25\text{ in.})^2\right]\right\}$$

$$+ \frac{\pi}{4}\frac{1.0433\text{ lb/in}^3}{32.2\text{ ft/s}^2}\left\{(0.875\text{ in.})\left[(0.85\text{ in.})^2 - (0.55\text{ in.})^2\right]\right.$$

$$+ (0.10\text{ in.})\left[(1.1\text{ in.})^2 - (0.85\text{ in.})^2\right]$$

$$\left. + (0.475\text{ in.})\left[(1.4\text{ in.})^2 - (1.1\text{ in.})^2\right]\right\}$$

$$= \frac{\pi}{128.8}(0.06426 + 0.01593 + 0.00211 + 0.015426)\text{lb}\cdot\text{s}^2\text{/ft}$$

PROBLEM 9.129 CONTINUED

Now $$m = \left(1567.38\cdot 10^{-6} + 388.553\cdot 10^{-6} + 51.465\cdot 10^{-6} + 376.259\cdot 10^{-6}\right)\text{ lb}\cdot\text{s}^2/\text{ft} = 2383.657\cdot 10^{-6}\text{ lb}\cdot\text{s}^2/\text{ft}$$

Then $$I_{AA'} = \frac{1}{8}\left\{1567.38\cdot 10^{-6}\text{ lb}\cdot\text{s}^2/\text{ft}\left[\left(\frac{0.25}{12}\text{ ft}\right)^2 + \left(\frac{0.55}{12}\text{ ft}\right)^2\right]\right.$$

$$+ 388.553\cdot 10^{-6}\text{ lb}\cdot\text{s}^2/\text{ft}\left[\left(\frac{0.55}{12}\text{ ft}\right)^2 + \left(\frac{0.85}{12}\text{ ft}\right)^2\right]$$

$$+ 51.465\cdot 10^{-6}\text{ lb}\cdot\text{s}^2/\text{ft}\left[\left(\frac{0.85}{12}\text{ ft}\right)^2 + \left(\frac{1.1}{12}\text{ ft}\right)^2\right]$$

$$\left.+ 376.259\cdot 10^{-6}\text{ lb}\cdot\text{s}^2/\text{ft}\left[\left(\frac{1.1}{12}\text{ ft}\right)^2 + \left(\frac{1.4}{12}\text{ ft}\right)^2\right]\right\}$$

$$= \frac{1}{8}\left(3.9728 + 2.7657 + 0.69067 + 8.2829\right)\cdot 10^{-6}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 1.96401\times 10^{-6}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_{AA'} = 1.964\cdot 10^{-6}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

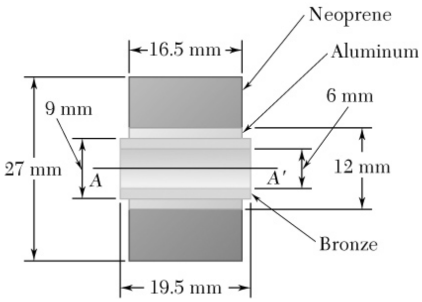
and $$k_{AA'}^2 = \frac{I_{AA'}}{m} = \frac{1.96401\times 10^{-6}\text{ lb}\cdot\text{ft}\cdot\text{s}^2}{2383.657\times 10^{-6}\text{ lb}\cdot\text{s}^2/\text{ft}}$$

$$= 8.23947\times 10^{-4}\text{ ft}^2$$

$$k_{AA'} = 2.87044\cdot 10^{-2}\text{ ft} = 0.34445\text{ in.}$$

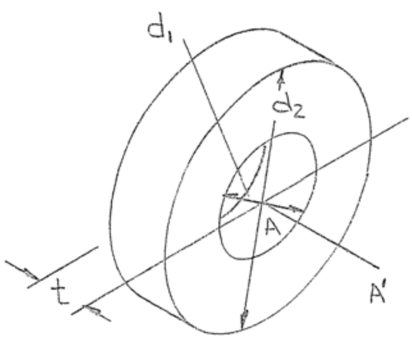
or $k_{AA'} = 0.344$ in. ◀

PROBLEM 9.130



Shown is the cross section of an idler roller. Determine its moment of inertia and its radius of gyration with respect to the axis AA'. (The density of bronze is 8580 kg/m^3; of aluminum, 2770 kg/m^3; and of neoprene, 1250 kg/m^3.)

SOLUTION



First note for the cylindrical ring shown that

$$m = \rho V = \rho t \times \frac{\pi}{4}\left(d_2^2 - d_1^2\right) = \frac{\pi}{4}\rho t\left(d_2^2 - d_1^2\right)$$

and, using Figure 9.28, that

$$I_{AA'} = \frac{1}{2}m_2\left(\frac{d_2}{2}\right)^2 - \frac{1}{2}m_1\left(\frac{d_1}{2}\right)^2$$

$$= \frac{1}{8}\left[\left(\rho t \times \frac{\pi}{4}d_2^2\right)d_2^2 - \left(\rho t \times \frac{\pi}{4}d_1^2\right)d_1^2\right]$$

$$= \frac{1}{8}\left(\frac{\pi}{4}\rho t\right)\left(d_2^4 - d_1^4\right)$$

$$= \frac{1}{8}\left(\frac{\pi}{4}\rho t\right)\left(d_2^2 - d_1^2\right)\left(d_2^2 + d_1^2\right)$$

$$= \frac{1}{8}m\left(d_1^2 + d_2^2\right)$$

Now treat the roller as three concentric rings and, working from the bronze outward, have

Have $$m = \frac{\pi}{4}\Big\{\left(8580\text{ kg/m}^3\right)(0.0195\text{ m})\left[(0.009\text{ m})^2 - (0.006\text{ m})^2\right]$$

$$+ \left(2770\text{ kg/m}^3\right)(0.0165\text{ m})\left[(0.012\text{ m})^2 - (0.009\text{ m})^2\right]$$

$$+ \left(1250\text{ kg/m}^3\right)(0.0165\text{ m})\left[(0.027\text{ m})^2 - (0.012\text{ m})^2\right]\Big\}$$

$$= \frac{\pi}{4}[7.52895 + 2.87942 + 12.06563] \times 10^{-3}\text{ kg}$$

$$= 5.9132 \times 10^{-3}\text{ kg} + 2.26149 \times 10^{-3}\text{ kg}$$

$$+ 9.47632 \times 10^{-3}\text{ kg}$$

$$= 17.6510 \times 10^{-3}\text{ kg}$$

PROBLEM 9.130 CONTINUED

And $$I_{AA'} = \frac{1}{8}\left\{\left(5.9132\times10^{-3}\text{ kg}\right)\left[(0.006)^2 + (0.009)^2\right]\text{m}^2\right.$$

$$+\left(2.26149\times10^{-3}\text{ kg}\right)\left[(0.009)^2 + (0.012)^2\right]\text{m}^2$$

$$\left.+\left(9.47632\times10^{-3}\text{ kg}\right)\left[(0.012)^2 + (0.027)^2\right]\text{m}^2\right\}$$

$$= \frac{1}{8}\left(691.844 + 508.835 + 8272.827\right)10^{-9}\text{ kg}\cdot\text{m}^2$$

$$= 1.18419\times10^{-6}\text{ kg}\cdot\text{m}^2$$

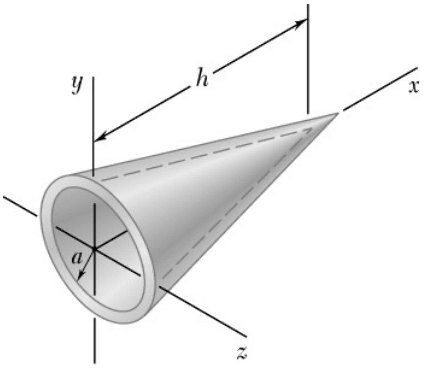
or $I_{AA'} = 1.184\times10^{-6}\text{ kg}\cdot\text{m}^2$ ◀

Now $$k_{AA'}^2 = \frac{I_{AA'}}{m} = \frac{1.18419\times10^{-6}\text{ kg m}^2}{17.6510\times10^{-3}\text{ kg}}$$

$$= 67.08902\times10^{-6}\text{ m}^2$$

$$k_{AA'} = 8.19079\times10^{-3}\text{ m}$$

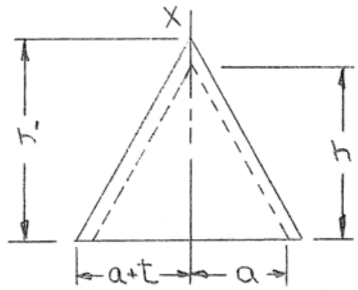
or $k_{AA'} = 8.19\text{ mm}$ ◀



PROBLEM 9.131

Given the dimensions and the mass m of the thin conical shell shown, determine the moment of inertia and the radius of gyration of the shell with respect to the x axis. (*Hint:* Assume that the shell was formed by removing a cone with a circular base of radius a from a cone with a circular base of radius $a+t$. In the resulting expressions, neglect terms containing t^2, t^3, etc. Do not forget to account for the difference in the heights of the two cones.)

SOLUTION



First note
$$\frac{h'}{a+t} = \frac{h}{a}$$

or
$$h' = \frac{h}{a}(a+t)$$

For a cone of height H whose base has a radius r, have

where
$$I_x = \frac{3}{10}mr^2$$

$$m = \rho V$$

$$= \rho \times \frac{\pi}{3}r^2 H$$

Then
$$I_x = \frac{3}{10}\left(\frac{\pi}{3}\rho r^2 H\right)r^2$$

$$= \frac{\pi}{10}\rho r^4 H$$

Now, following the hint have

$$m_{\text{shell}} = m_{\text{outer}} - m_{\text{inner}} = \frac{\pi}{3}\rho\left[(a+t)^2 h' - a^2 h\right]$$

$$= \frac{\pi}{3}\rho\left[(a+t)^2 \times \frac{h}{a}(a+t) - a^2 h\right]$$

$$= \frac{\pi}{3}\rho a^2 h\left[\left(1+\frac{t}{a}\right)^3 - 1\right] = \frac{\pi}{3}\rho a^2 h\left(1 + 3\frac{t}{a} + \ldots - 1\right)$$

Neglecting the t^2 and t^3 terms obtain

$$m_{\text{shell}} \approx \pi\rho a h t$$

PROBLEM 9.131 CONTINUED

Also $$\left(I_x\right)_{\text{shell}} = \left(I_x\right)_{\text{outer}} - \left(I_x\right)_{\text{inner}}$$

$$= \frac{\pi}{10}\rho\left[\left(a + t\right)^4 h' - a^4 h\right]$$

$$= \frac{\pi}{10}\rho\left[\left(a + t\right)^4 \times \frac{h}{a}\left(a + t\right) - a^4 h\right]$$

$$= \frac{\pi}{10}\rho a^4 h\left[\left(1 + \frac{t}{a}\right)^5 - 1\right]$$

$$= \frac{\pi}{10}\rho a^4 h\left(1 + 5\frac{t}{a} + \ldots - 1\right)$$

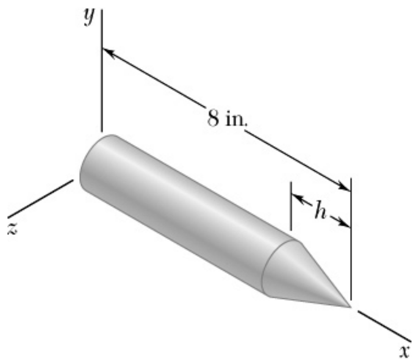
Neglecting t^2 and higher order terms, obtain

$$\left(I_x\right)_{\text{shell}} \approx \frac{\pi}{2}\rho a^3 ht$$

or $I_x = \frac{1}{2}ma^2$ ◀

Now $$k_x^2 = \frac{I_x}{m} = \frac{\frac{1}{2}ma^2}{m}$$

or $k_x = \frac{a}{\sqrt{2}}$ ◀



PROBLEM 9.132

A portion of an 8-in.-long steel rod of diameter 1.50 in. is turned to form the conical section shown. Knowing that the turning process reduces the moment of inertia of the rod with respect to the x axis by 20 percent, determine the height h of the cone.

SOLUTION

8-in. rod $\quad (\bar{I}_x)_0 = \frac{1}{2}m_0 a^2 \quad$ where $\quad m_0 = \rho V_0 = \rho(\pi a^2 L)$

and $\quad a = 0.75$ in. $\quad$ and $\quad L = 8$ in.

Therefore,
$$(\bar{I}_x)_0 = \frac{1}{2}\rho\pi a^4 L$$

Rod and cone
$$\bar{I}_x = (\bar{I}_x)_{\text{cyl}} + (\bar{I}_x)_{\text{cone}} = \frac{1}{2}m_{\text{cyl}}a^2 + \frac{3}{10}m_{\text{cone}}a^2$$

where
$$m_{\text{cyl}} = \rho V_{\text{cyl}} = \rho\left[\pi a^2(L-h)\right]$$

and
$$m_{\text{cone}} = \rho V_{\text{cone}} = \rho\left[\frac{1}{3}\pi a^2 h\right]$$

Then
$$\bar{I}_x = \frac{1}{2}\rho\pi a^4(L-h) + \frac{1}{10}\rho\pi a^4 h$$

Given
$$\bar{I}_x = 0.8(\bar{I}_x)_0$$

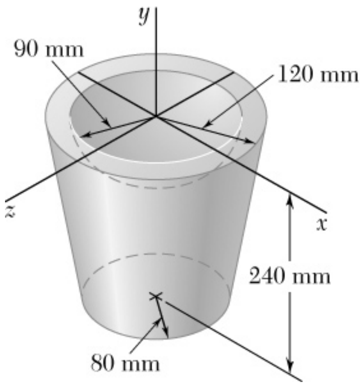
Then
$$\frac{1}{2}\rho\pi a^4(L-h) + \frac{1}{10}\rho\pi a^4 h = 0.8\left(\frac{1}{2}\rho\pi a^4 L\right)$$

or
$$\frac{5}{10}L - \frac{5}{10}h + \frac{1}{10}h = \frac{4}{10}L$$

or
$$h = \frac{1}{4}L = \frac{1}{4}(8.00 \text{ in.}) = 2.00 \text{ in.}$$

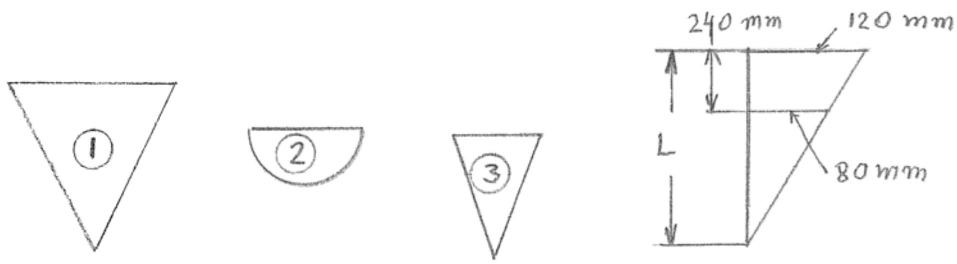
or $h = 2.00$ in. ◀

PROBLEM 9.133



The steel machine component shown is formed by machining a hemisphere into the base of a truncated cone. Knowing that the density of steel is 7850 kg/m^3, determine the mass moment of inertia of the component with respect to the y axis.

SOLUTION



First note $$\frac{L}{120} = \frac{L-240}{80} \qquad \text{or} \qquad L = 720 \text{ mm}$$

and $$m = \rho_{st}V$$

Now $$m_1 = \rho_{st}\left(\frac{1}{3}\pi a_1^2 h_1\right) = \frac{\pi}{3} \times 7850 \text{ kg/m}^3 \times (0.120 \text{ m})^2 (0.720 \text{ m})$$

$$= 85.230 \text{ kg}$$

$$m_2 = \rho_{st}\left(\frac{2}{3}\pi a_2^2\right) = \frac{2}{3}\pi \times 7850 \text{ kg/m}^3 \times (0.090 \text{ m})^2 = 11.9855 \text{ kg}$$

$$m_3 = \rho_{st}\left(\frac{1}{3}\pi a_3^2 h_3\right) = \frac{\pi}{3} \times 7850 \text{ kg/m}^3 \times (0.080 \text{ m})^2 (0.720 - 0.240)\text{m}$$

$$= 25.253 \text{ kg}$$

Now $$\bar{I}_y = (\bar{I}_y)_1 - (\bar{I}_y)_2 - (\bar{I}_y)_3$$

PROBLEM 9.133 CONTINUED

where (using Figure 9.28)

$$\left(\bar{I}_y\right)_1 = \frac{3}{10}m_1a_1^2 = \frac{3}{10}\left(85.230 \text{ kg}\right)\left(0.120 \text{ m}\right)^2 = 0.36819 \text{ kg}\cdot\text{m}^2$$

$$\left(\bar{I}_y\right)_2 = \frac{1}{2}\left(\bar{I}_y\right)_{\text{sphere}} = \frac{1}{2}\left(\frac{2}{5}m_{\text{sphere}}a_2^2\right) \qquad \text{where} \qquad m_{\text{sphere}} = 2m_{\text{hemisphere}}$$

$$= \frac{1}{2}\left(\frac{2}{5}\ 2\times 11.9855 \text{ kg}\right)\left(0.090 \text{ m}\right)^2$$

$$= 0.038833 \text{ kg}\cdot\text{m}^2$$

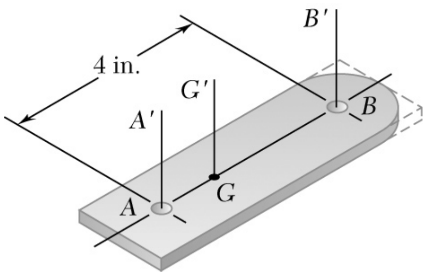
$$\left(\bar{I}_y\right)_3 = \frac{3}{10}m_3a_3^2 = \frac{3}{10}\left(25.253 \text{ kg}\right)\left(0.080 \text{ m}\right)^2 = 0.048486 \text{ kg}\cdot\text{m}^2$$

Then

$$\bar{I}_y = \left(0.36819 - 0.038833 - 0.048486\right)\text{kg}\cdot\text{m}^2 = 0.28087 \text{ kg}\cdot\text{m}^2$$

$$= 0.281 \text{ kg}\cdot\text{m}^2$$

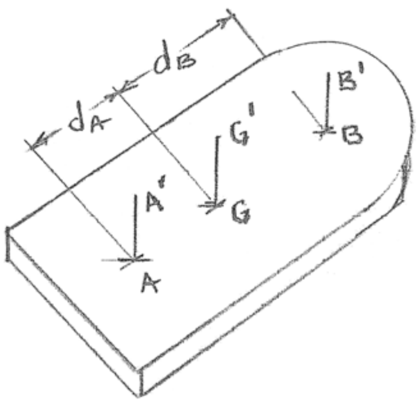
or $\bar{I}_y = 281\times 10^{-3} \text{ kg}\cdot\text{m}^2$ ◀



PROBLEM 9.134

After a period of use, one of the blades of a shredder has been worn to the shape shown and is of weight 0.4 lb. Knowing that the moments of inertia of the blade with respect to the AA' and BB' axes are 0.6×10^{-3} lb · ft · s^2 and 1.26×10^{-3} lb · ft · s^2, respectively, determine (*a*) the location of the centroidal axis GG', (*b*) the radius of gyration with respect to axis GG'.

SOLUTION



Have

(*a*)
$$d_B = \frac{4}{12} - d_A = (0.33333 - d_A)\text{ ft}$$

and
$$I_{AA'} = \bar{I}_{GG'} + md_A^2$$

$$I_{BB'} = \bar{I}_{GG'} + md_B^2$$

Then
$$I_{BB'} - I_{AA'} = m\left(d_B^2 - d_A^2\right)$$

$$= m\left[\left(0.33333 - d_A\right)^2 - d_A^2\right]$$

$$= m\left(0.11111 - 0.66666d_A\right)$$

Then
$$(1.26 - 0.6) \times 10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= \frac{0.40\text{ lb}}{32.2\text{ ft/s}^2}\left(0.11111 - 0.66666d_A\right)\text{ ft}^2$$

or
$$d_A = 0.08697\text{ ft}$$

or
$$d_A = 1.044\text{ in.} \blacktriangleleft$$

(*b*)
$$I_{AA'} = \bar{I}_{GG'} + md_A^2$$

or
$$\bar{I}_{GG'} = 0.6 \times 10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2 - \frac{0.4\text{ lb}}{32.2\text{ ft/s}^2}\left(0.08697\text{ ft}\right)^2$$

$$= 0.50604 \times 10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

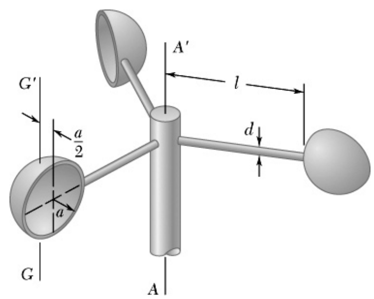
Then
$$\bar{k}_{GG'}^2 = \frac{\bar{I}_{GG'}}{m} = \frac{0.50604 \times 10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2}{\dfrac{0.4\text{ lb}}{32.2\text{ ft/s}^2}}$$

$$= 0.04074\text{ ft}^2$$

$$\bar{k}_{GG'} = 0.20183\text{ ft} = 2.4219\text{ in.}$$

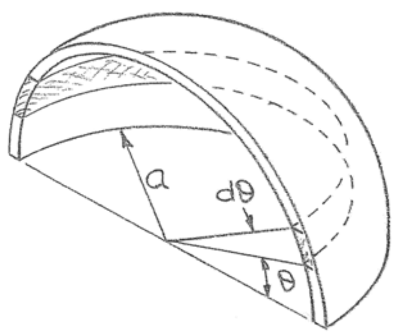
or $\bar{k}_{GG'} = 2.42$ in. $\blacktriangleleft$

PROBLEM 9.135



The cups and the arms of an anemometer are fabricated from a material of density ρ. Knowing that the moment of inertia of a thin, hemispherical shell of mass m and thickness t with respect to its centroidal axis GG', is $5ma^2/12$, determine (a) the moment of inertia of the anemometer with respect to the axis AA', (b) the ratio of a to l for which the centroidal moment of inertia of the cups is equal to 1 percent of the moment of inertia of the cups with respect to the axis AA'.

SOLUTION



(a) First note $\quad m_{\text{arm}} = \rho V_{\text{arm}} = \rho \times \frac{\pi}{4}d^2 l$

and $\quad dm_{\text{cup}} = \rho dV_{\text{cup}}$

$$= \rho\left[(2\pi a\cos\theta)(t)(a d\theta)\right]$$

Then $\quad m_{\text{cup}} = \int dm_{\text{cup}} = \int_0^{\frac{\pi}{2}} 2\pi\rho a^2 t\cos\theta d\theta$

$$= 2\pi\rho a^2 t\left[\sin\theta\right]_0^{\frac{\pi}{2}}$$

$$= 2\pi\rho a^2 t$$

Now $\quad (I_{AA'})_{\text{anem.}} = (I_{AA'})_{\text{cups}} + (I_{AA'})_{\text{arms}}$

Using the parallel-axis theorem and assuming the arms are slender rods, have

$$(I_{AA'})_{\text{anem.}} = 3\left[(I_{GG'})_{\text{cup}} + m_{\text{cup}}d_{AG}^2\right]$$

$$+ 3\left[\bar{I}_{\text{arm}} + m_{\text{arm}}d_{AG_{\text{arm}}}\right]$$

$$= 3\left\{\frac{5}{12}m_{\text{cup}}a^2 + m_{\text{cup}}\left[(l+a)^2 + \left(\frac{a}{2}\right)^2\right]\right\}$$

$$+3\left[\frac{1}{2}m_{\text{arm}}l^2 + m_{\text{arm}}\left(\frac{l}{2}\right)^2\right]$$

$$= 3m_{\text{cup}}\left(\frac{5}{3}a^2 + 2la + l^2\right) + m_{\text{arm}}l^2$$

$$= 3(2\pi\rho a^2 t)\left(\frac{5}{3}a^2 + 2la + l^2\right) + \left(\frac{\pi}{4}\rho d^2 l\right)(l^2)$$

or $\quad (I_{AA'})_{\text{anem}} = \pi\rho l^2\left[6a^2 t\left(\frac{5}{3}\frac{a^2}{l^2} + 2\frac{a}{l} + 1\right) + \frac{d^2 l}{4}\right]$ ◀

PROBLEM 9.135 CONTINUED

(*b*) Have $$\frac{\left(I_{GG'}\right)_{\text{cup}}}{\left(I_{AA'}\right)_{\text{cup}}} = 0.01$$

or $$\frac{5}{12}m_{\text{cup}}a^2 = 0.01m_{\text{cup}}\left(\frac{5}{3}a^2 + 2la + l^2\right) \quad (\text{From Part a})$$

Now let $\zeta = \dfrac{a}{l}$

Then $$5\zeta^2 = 0.12\left(\frac{5}{3}\zeta^2 + 2\zeta + 1\right)$$

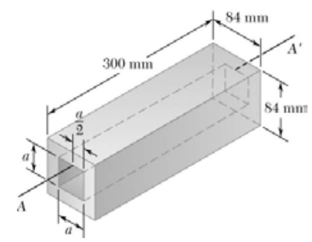
or $$40\zeta^2 - 2\zeta - 1 = 0$$

Then $$\zeta = \frac{2 \pm \sqrt{(-2)^2 - 4(40)(-1)}}{2(40)}$$

or $$\zeta = 0.1851 \qquad \text{and} \qquad \zeta = -0.1351$$

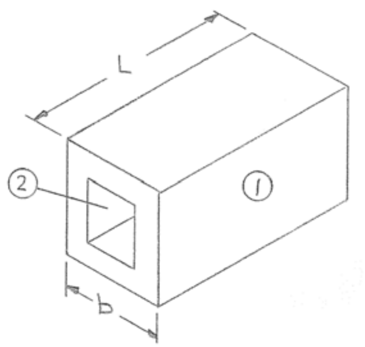
$$\therefore \quad \frac{a}{l} = 0.1851 \blacktriangleleft$$

PROBLEM 9.136



A square hole is centered in and extends through the aluminum machine component shown. Determine (*a*) the value of *a* for which the mass moment of inertia of the component with respect to the axis AA', which bisects the top surface of the hole, is maximum, (*b*) the corresponding values of the mass moment of inertia and the radius of gyration with respect to the axis AA'. (The density of aluminum is 2800 kg/m^3.)

SOLUTION



First note $\qquad m_1 = \rho V_1 = \rho b^2 L$

And $\qquad m_2 = \rho V_2 = \rho a^2 L$

(*a*) Using Figure 9.28 and the parallel-axis theorem, have

$$I_{AA'} = (I_{AA'})_1 - (I_{AA'})_2$$

$$= \left[\frac{1}{12}m_1(b^2+b^2) + m_1\left(\frac{a}{2}\right)^2\right]$$

$$-\left[\frac{1}{12}m_2(a^2+a^2) + m_2\left(\frac{a}{2}\right)^2\right]$$

$$= (\rho b^2 L)\left(\frac{1}{6}b^2 + \frac{1}{4}a^2\right) - (\rho a^2 L)\left(\frac{5}{12}a^2\right)$$

$$= \frac{\rho L}{12}(2b^4 + 3b^2a^2 - 5a^4)$$

Then $$\frac{dI_{AA'}}{da} = \frac{\rho L}{12}(6b^2a - 20a^3) = 0$$

or $\qquad a = 0 \qquad$ and $\qquad a = b\sqrt{\frac{3}{10}}$

Also.. $$\frac{d^2I_{AA'}}{da^2} = \frac{\rho L}{12}(6b^2 - 60a^2) = \frac{1}{2}\rho L(b^2 - 10a^2)$$

Now, for

$a = 0, \quad \dfrac{d^2I_{AA'}}{da^2} > 0 \qquad$ and for $\qquad a = b\sqrt{\dfrac{3}{10}}, \quad \dfrac{d^2I_{AA'}}{da^2} < 0$

$\therefore \quad (I_{AA'})_{max} \qquad$ occurs when $\qquad a = b\sqrt{\dfrac{3}{10}}$

$$a = 84\sqrt{\frac{3}{10}} = 46.009 \text{ mm}$$

or $\qquad a = 46.0 \text{ mm}$ ◀

PROBLEM 9.136 CONTINUED

(*b*) From part (*a*)

$$\left(I_{AA'}\right)_{\text{mass}} = \frac{\rho L}{12}\left[2b^4 + 3b^2\left(b\sqrt{\frac{3}{10}}\right)^2 - 5\left(b\sqrt{\frac{3}{10}}\right)^4\right]$$

$$= \frac{49}{240}\rho L b^4 = \frac{49}{240}\left(2800\ \text{kg/m}^3\right)\left(0.3\ \text{m}\right)\left(0.4\ \text{m}\right)^4$$

$$= 8.5385 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

or $\left(I_{AA'}\right)_{\text{mass}} = 8.54 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

and $$k_{AA'}^2 = \frac{\left(I_{AA'}\right)_{\text{mass}}}{m}$$

where

$$m = m_1 - m_2 = \rho L\left(b^2 - a^2\right) = \rho L\left[b^2 - \left(b\sqrt{\frac{3}{10}}\right)^2\right] = \frac{7}{10}\rho L b^2$$

Then

$$k_{AA'}^2 = \frac{\frac{49}{240}\rho L b^4}{\frac{7}{10}\rho L b^2} = \frac{7}{24}b^2 = \frac{7}{24}\left(84\ \text{mm}\right)^2 = 2058\ \text{mm}^2$$

$$k_{AA'} = 45.3652\ \text{mm}$$

or $k_{AA'} = 45.4\ \text{mm}$ ◀

To the instructor:

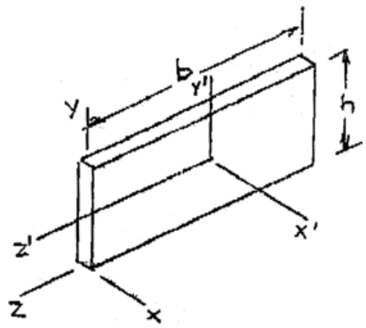
The following formulas for the mass moment of inertia of thin plates and a half cylindrical shell are derived at this time for use in the solutions of Problems 9.137–9.142

Thin rectangular plate

$$\left(I_x\right)_m = \left(\bar{I}_{x'}\right)_m + md^2$$

$$= \frac{1}{12}m\left(b^2 + h^2\right) + m\left[\left(\frac{b}{2}\right)^2 + \left(\frac{h}{2}\right)^2\right]$$

$$= \frac{1}{3}m\left(b^2 + h^2\right)$$



$$\left(I_y\right)_m = \left(\bar{I}_{y'}\right)_m + md^2$$

$$= \frac{1}{12}mb^2 + m\left(\frac{b}{2}\right)^2$$

$$= \frac{1}{3}mb^2$$

$$I_z = \left(\bar{I}_{z'}\right)_m + md^2$$

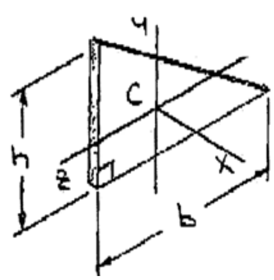
$$= \frac{1}{12}mh^2 + m\left(\frac{h}{2}\right)^2$$

$$= \frac{1}{3}mh^2$$

Thin triangular plate

Have $\quad m = \rho V = \rho\left(\frac{1}{2}bht\right)$

and $\quad \bar{I}_{z,\text{ area}} = \frac{1}{36}bh^3$



Then $\quad \bar{I}_{z,\text{ mass}} = \rho t I_{z,\text{ area}}$

$$= \rho t \times \frac{1}{36}bh^3$$

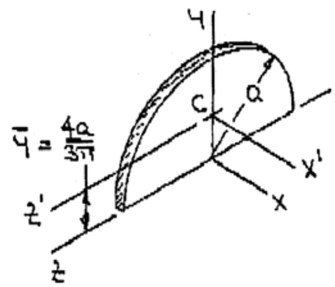
$$= \frac{1}{18}mh^2$$

Similarly, $$\bar{I}_{y,\text{ mass}} = \frac{1}{18}mb^2$$

Now $$\bar{I}_{x,\text{ mass}} = \bar{I}_{y,\text{ mass}} + \bar{I}_{z,\text{ mass}} = \frac{1}{18}m\left(b^2 + h^2\right)$$

Thin semicircular plate

Have $$m = \rho V = \rho\left(\frac{\pi}{2}a^2 t\right)$$



And $$\bar{I}_{y,\text{ area}} = I_{z,\text{ area}} = \frac{\pi}{8}a^4$$

Then $$\bar{I}_{y,\text{ mass}} = I_{z,\text{ mass}} = \rho t\bar{I}_{y,\text{ area}}$$

$$= \rho t \times \frac{\pi}{8}a^4$$

$$= \frac{1}{4}ma^2$$

Now $$I_{x,\text{ mass}} = \bar{I}_{y,\text{ mass}} + I_{z,\text{ mass}} = \frac{1}{2}ma^2$$

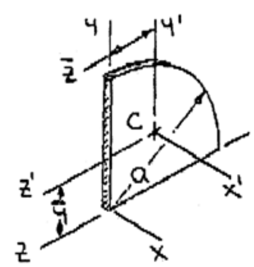
Also $$I_{x,\text{ mass}} = \bar{I}_{x',\text{ mass}} + m\bar{y}^2 \quad \text{or} \quad \bar{I}_{x',\text{ mass}} = m\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)a^2$$

And $$I_{z,\text{ mass}} = \bar{I}_{z',\text{ mass}} + m\bar{y}^2 \quad \text{or} \quad \bar{I}_{z',\text{ mass}} = m\left(\frac{1}{4} - \frac{16}{9\pi^2}\right)a^2$$

Thin Quarter-Circular Plate

$$\bar{y} = \bar{z} = \frac{4a}{3\pi}$$

Have $$m = \rho V = \rho\left(\frac{\pi}{4}a^2 t\right)$$



and $$I_{y,\text{ area}} = I_{z,\text{ area}} = \frac{\pi}{16}a^4$$

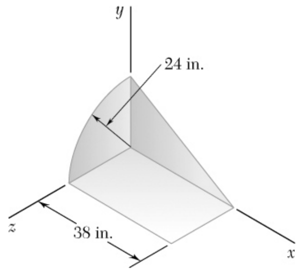
Then $$I_{y,\text{ mass}} = I_{z,\text{ mass}} = \rho t I_{y,\text{ area}}$$

$$= \rho t \times \frac{\pi}{16}a^4$$

$$= \frac{1}{4}ma^2$$

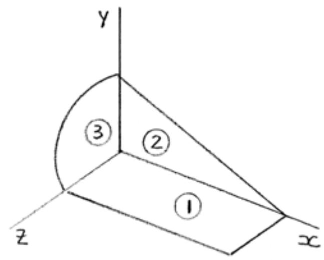
Now	$I_{x,\text{ mass}} = I_{y,\text{ mass}} + I_{z,\text{ mass}} = \frac{1}{2}ma^2$
Also	$I_{x,\text{ mass}} = I_{x',\text{ mass}} + m\left(\overline{y}^2 + \overline{z}^2\right)$
or	$\overline{I}_{x',\text{ mass}} = m\left(\frac{1}{2} - \frac{32}{9\pi^2}\right)a^2$
and	$I_{y,\text{ mass}} = I_{y',\text{ mass}} + m\overline{z}^2$
or	$\overline{I}_{y',\text{ mass}} = m\left(\frac{1}{4} - \frac{16}{9\pi^2}\right)a^2$

PROBLEM 9.137



A 0.1-in-thick piece of sheet metal is cut and bent into the machine component shown. Knowing that the specific weight of steel is $0.284\ \text{lb/in}^3$, determine the moment of inertia of the component with respect to each of the coordinate axes.

SOLUTION



$$m_1 = \rho V = \frac{\gamma}{g} tA$$

$$= \frac{0.284\ \text{lb/in}^3}{32.2\ \text{ft/s}^2}(0.10\ \text{in.})(38\ \text{in.})(24\ \text{in.})$$

$$= 0.80438\ \text{lb}\cdot\text{s}^2/\text{ft}$$

$$m_2 = \frac{0.284}{32.2}(0.1\ \text{in.})\left[\left(\frac{1}{2}\right)(38\ \text{in.})(24\ \text{in.})\right]$$

$$= 0.402186\ \text{lb}\cdot\text{s}^2/\text{ft}$$

$$m_3 = \frac{0.284}{32.2}(0.1\ \text{in.})\left[\left(\frac{\pi}{4}\right)(24\ \text{in.})^2\right] = 0.39900\ \text{lb}\cdot\text{s}^2/\text{ft}$$

Using Fig. 9.28 for component 1 and the equations derived for components 2 and 3, have

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3$$

$$= \left(0.80438\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left[\frac{1}{12}\left(\frac{24}{12}\right)^2 + \left(\frac{24}{2\times 12}\right)^2\right]\text{ft}^2 + \left(0.402186\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left[\frac{1}{18}\left(\frac{24}{12}\right)^2 + \left(\frac{24}{3\times 12}\right)^2\right]\text{ft}^2$$

$$+ \left(0.39900\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left[\frac{1}{2}\left(\frac{24}{12}\right)^2\right]\text{ft}^2$$

$$= (1.0725 + 0.26812 + 0.7980)\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 2.13862\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_x = 2.14\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

PROBLEM 9.137 CONTINUED

Also

$$I_y = \left(I_y\right)_1 + \left(I_y\right)_2 + \left(I_y\right)_3$$

$$= \left(0.80438 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[\left(\frac{38}{12}\right)^2 + \left(\frac{24}{12}\right)^2\right]\text{ft}^2 + \left[\left(\frac{38}{2\times 12}\right)^2 + \left(\frac{24}{2\times 12}\right)^2\right]\text{ft}^2\right\}$$

$$+ \left(0.402186 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left[\frac{1}{18}\left(\frac{38}{12}\right)^2 + \left(\frac{38}{3\times 12}\right)^2\right]\text{ft}^2$$

$$+ \left(0.39900 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left[\frac{1}{4}\left(\frac{24}{12}\right)^2\right]\text{ft}^2$$

$$= \left(3.76122 + 0.672172 + 0.3990\right)\text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 4.83234 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_y = 4.83 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

$$I_z = \left(I_z\right)_1 + \left(I_z\right)_2 + \left(I_z\right)_3$$

$$= \left(0.80438 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left[\frac{1}{12}\left(\frac{38}{12}\right)^2 + \left(\frac{38}{2\times 12}\right)^2\right]\text{ft}^2$$

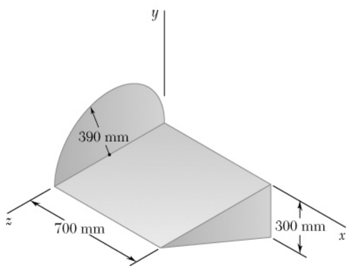
$$+ \left(0.401286 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{18}\left[\left(\frac{38}{12}\right)^2 + \left(\frac{24}{12}\right)^2\right] + \left[\left(\frac{38}{3\times 12}\right)^2 + \left(\frac{24}{3\times 12}\right)^2\right]\right\}\text{ft}^2$$

$$+ \left(0.3990 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left[\frac{1}{4}\left(\frac{24}{12}\right)^2\right]\text{ft}^2$$

$$= \left(2.68871 + 0.940296 + 0.3990\right)\text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 4.0280 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

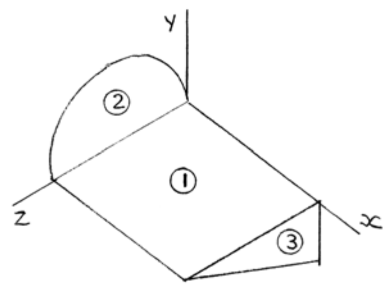
or $I_z = 4.03 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀



PROBLEM 9.138

A 3-mm-thick piece of sheet metal is cut and bent into the machine component shown. Knowing that the density of steel is $7850 \text{ kg}/\text{m}^3$, determine the moment of inertia of the component with respect to each of the coordinate axes.

SOLUTION



First compute the mass of each component.

Have $m = \rho_{st}V = \rho_{st}tA$

$$m_1 = \left(7850 \text{ kg/m}^3\right)(0.003 \text{ m})(0.70 \text{ m})(0.780 \text{ m}) = 12.858 \text{ kg}$$

$$m_2 = \left(7850 \text{ kg/m}^3\right)(0.003 \text{ m})\left[\left(\frac{\pi}{2}\right)(0.39 \text{ m})^2\right] = 5.6265 \text{ kg}$$

$$m_3 = \left(7850 \text{ kg/m}^3\right)(0.003 \text{ m})\left[\left(\frac{1}{2}\right)(0.780 \text{ m})(0.3 \text{ m})\right] = 2.7554 \text{ kg}$$

Using Fig. 9-28 for component 1 and the equations derived above for components 2 and 3 have

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3$$

$$= (12.858 \text{ kg})\left[\frac{1}{12}(0.78)^2 + \left(\frac{0.78}{2}\right)^2\right] \text{m}^2$$

$$+ (5.6265 \text{ kg})\left\{\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)(0.39)^2 + \left[\left(\frac{4 \times 0.39}{3\pi}\right)^2 + (0.39)^2\right]\right\} \text{m}^2$$

$$+ (2.7554 \text{ kg})\left\{\frac{1}{18}\left[(0.78)^2 + (0.30)^2\right] + \left[\left(\frac{0.78}{3}\right)^2 + \left(\frac{0.30}{3}\right)^2\right]\right\} \text{m}^2$$

$$= (2.6076 + 1.2836 + 0.3207) \text{ kg}\cdot\text{m}^2$$

$$= 4.2119 \text{ kg}\cdot\text{m}^2$$

or $I_x = 4.21 \text{ kg}\cdot\text{m}^2$ ◀

PROBLEM 9.138 CONTINUED

And

$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3$$

$$= (12.858\text{ kg})\left\{\frac{1}{12}\left[(0.7)^2 + (0.78)^2\right] + \frac{1}{4}\left[(0.7)^2 + (0.78)^2\right]\right\}\text{m}^2$$

$$+ (5.6265\text{ kg})\left[\frac{1}{4}(0.39)^2 + (0.39)^2\right]\text{m}^2$$

$$+ (2.7554\text{ kg})\left\{\frac{1}{18}(0.78)^2 + \left[(0.7)^2 + \left(\frac{0.78}{3}\right)^2\right]\right\}\text{m}^4$$

$$= (4.7077 + 1.0697 + 1.6295)\text{ kg}\cdot\text{m}^2$$

$$= 7.4069\text{ kg}\cdot\text{m}^2$$

or $I_y = 7.41\text{ kg}\cdot\text{m}^2$ ◀

And

$$I_z = (I_z)_1 + (I_z)_2 + (I_z)_3$$

$$= (12.858\text{ kg})\left[(0.7)^2\left(\frac{1}{12} + \frac{1}{4}\right)\right]\text{m}^2$$

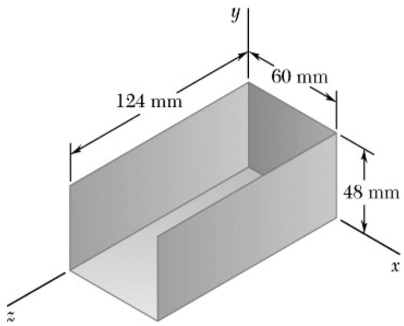
$$+ (5.6265\text{ kg})\left[\frac{1}{4}(0.39)^2\right]\text{m}^2$$

$$+ (2.7554\text{ kg})\left\{\frac{1}{18}(0.3)^2 + \left[(0.70)^2 + \left(\frac{0.30}{3}\right)^2\right]\right\}\text{m}^2$$

$$= (2.1001 + 0.21395 + 1.39145)\text{ kg}\cdot\text{m}^2 = 3.7055\text{ kg}\cdot\text{m}^2$$

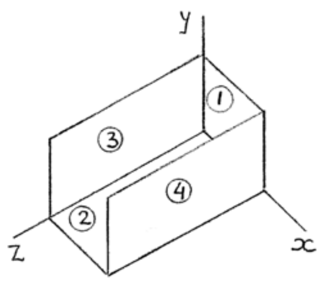
or $I_z = 3.71\text{ kg}\cdot\text{m}^2$ ◀

PROBLEM 9.139



The cover of an electronic device is formed from sheet aluminum that is 2 mm thick. Determine the mass moment of inertia of the cover with respect to each of the coordinate axes. (The density of aluminum is 2770 kg/m^3.)

SOLUTION



Have $$m = \rho V$$

Now $$m_1 = \left(2770 \text{ kg/m}^3\right)(0.002 \text{ m})(0.048 \text{ m})(0.06 \text{ m})$$
$$= 0.015955 \text{ kg}$$
$$m_2 = \left(2770 \text{ kg/m}^3\right)(0.002 \text{ m})(0.06 \text{ m})(0.124 \text{ m})$$
$$= 0.041218 \text{ kg}$$
$$m_3 = \left(2770 \text{ kg/m}^3\right)(0.002 \text{ m})(0.048 \text{ m})(0.124 \text{ m})$$
$$= 0.032974 \text{ kg}$$

Using Fig. 9.28 and the parallel axis theorem, have

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3 + (I_x)_4 \qquad \text{and} \qquad (I_x)_3 = (I_x)_4$$

$$= (0.015955 \text{ kg})\left[(0.048)^2\left(\frac{1}{12} + \frac{1}{4}\right)\right]\text{m}^2 + (0.041218 \text{ kg})\left[(0.124)^2\left(\frac{1}{12} + \frac{1}{4}\right)\right]\text{m}^2$$

$$+ 2(0.032974 \text{ kg})\left\{\left[(0.048)^2 + (0.124)^2\right]\left(\frac{1}{12} + \frac{1}{4}\right)\right\}\text{m}^2$$

$$= \left(0.012253 \times 10^{-3} + 0.211256 \times 10^{-3} + 0.388654 \times 10^{-3}\right)\text{kg}\cdot\text{m}^2 = 0.61216 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

or $I_x = 0.612 \times 10^{-3} \text{ kg}\cdot\text{m}^2$ ◀

PROBLEM 9.139 CONTINUED

$$I_y = \left(I_y\right)_1 + \left(I_y\right)_2 + \left(I_y\right)_3 + \left(I_y\right)_4$$

$$= (0.015955\text{ kg})\left[\frac{1}{3}(0.06)^2\right]\text{m}^2 + (0.041218\text{ kg})\left\{\left[(0.06)^2 + (0.124)^2\right]\left(\frac{1}{3}\right)\right\}\text{m}^2$$

$$+ (0.032974\text{ kg})\left[(0.124)^2\left(\frac{1}{3}\right)\right]\text{m}^2 + (0.032974\text{ kg})\left[\frac{1}{12}(0.124)^2 + (0.06)^2 + \frac{1}{4}(0.124)^2\right]\text{m}^2$$

$$= \left[(0.019146 + 0.260718 + 0.16900 + 0.287709)\times 10^{-3}\right]\text{kg}\cdot\text{m}^2 = 0.73657\times 10^{-3}\text{kg}\cdot\text{m}^2$$

or $I_y = 0.737\times 10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

$$I_z = \left(I_z\right)_1 + \left(I_z\right)_2 + \left(I_z\right)_3 + \left(I_z\right)_4$$

$$= (0.015955\text{ kg})\left\{\left[(0.06)^2 + (0.048)^2\right]\left(\frac{1}{3}\right)\right\}\text{m}^2 + (0.041218\text{ kg})\left[(0.06)^2\left(\frac{1}{3}\right)\right]\text{m}^2$$

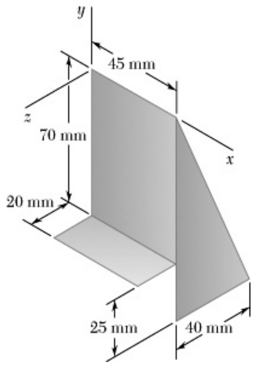
$$+ (0.032974\text{ kg})\left[(0.048)^2\left(\frac{1}{3}\right)\right]\text{m}^2 + (0.032974\text{ kg})\left[\left(\frac{1}{3}\right)(0.048)^2 + (0.06)^2\right]\text{m}^2$$

$$= \left[(0.031399 + 0.049462 + 0.025324 + 0.14403)\times 10^{-3}\right]\text{kg}\cdot\text{m}^2$$

$$= 0.250215\times 10^{3}\ \text{kg}\cdot\text{m}^2$$

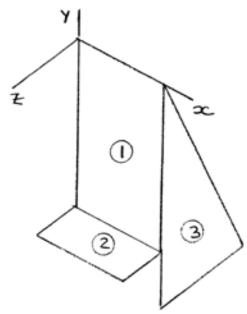
or $I_z = 0.250\times 10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

PROBLEM 9.140



A framing anchor is formed of 2-mm-thick galvanized steel. Determine the mass moment of inertia of the anchor with respect to each of the coordinate exes. (The density of galvanized steel is 7530 kg/m^3.)

SOLUTION



First compute the mass of each component

Have $$m = \rho V = \rho A t$$

Now $$m_1 = \left(7530 \text{ kg/m}^3\right)(0.002 \text{ m})(0.045 \text{ m})(0.070 \text{ m}) = 0.047439 \text{ kg}$$

$$m_2 = \left(7530 \text{ kg/m}^3\right)(0.002 \text{ m})(0.045 \text{ m})(0.020 \text{ m}) = 0.013554 \text{ kg}$$

$$m_3 = \left(7530 \text{ kg/m}^3\right)(0.002 \text{ m}) \times \frac{1}{2}(0.04 \text{ m})(0.095 \text{ m}) = 0.028614 \text{ kg}$$

Using Fig. 9.28 for components 1 and 2 and the equations derived above for components 3, have

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3$$

$$= (0.047439 \text{ kg})\left[\frac{1}{3}(0.07)^2\right]\text{m}^2 + (0.013554 \text{ kg})\left\{\frac{1}{12}(0.020)^2 + \left[(0.07)^2 + (0.01)^2\right]\right\}\text{m}^2$$

$$+ (0.028614 \text{ kg})\left\{\frac{1}{18}\left[(0.095)^2 + (0.04)^2\right] + \frac{1}{9}\left[(2 \times 0.095)^2 + (0.040)^2\right]\right\}\text{m}^2$$

$$= \left[(0.077484 + 0.068222 + 0.136751) \times 10^{-3}\right]\text{kg}\cdot\text{m}^2 = 0.282457 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

or $I_x = 0.2825 \times 10^{-3} \text{ kg}\cdot\text{m}^2$ ◀

PROBLEM 9.140 CONTINUED

$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3$$

$$= (0.047439\text{ kg})\left[\frac{1}{3}(0.045)^2\right]\text{m}^2 + (0.013554\text{ kg})\left\{\left[(0.045)^2 + (0.02)^2\right]\left(\frac{1}{3}\right)\right\}\text{m}^2$$

$$+ (0.028614\text{ kg})\left[\frac{1}{18}(0.04)^2(0.045)^2 + \frac{1}{9}(0.04)^2\right]\text{m}^2$$

$$= \left[(0.03202 + 0.010956 + 0.065574)\times 10^{-3}\right]\text{kg}\cdot\text{m}^2 = 0.10855\times 10^{3}\text{ kg}\cdot\text{m}^2$$

or $I_y = 0.1086\times 10^{3}\text{ kg}\cdot\text{m}^2$ ◀

$$I_z = (I_z)_1 + (I_z)_2 + (I_z)_3$$

$$= (0.047439\text{ kg})\left\{\left[(0.045)^2 + (0.070)^2\right]\left(\frac{1}{3}\right)\right\}\text{m}^2 + (0.013554\text{ kg})\left[(0.045)^2\left(\frac{1}{3}\right) + (0.070)^2\right]\text{m}^2$$

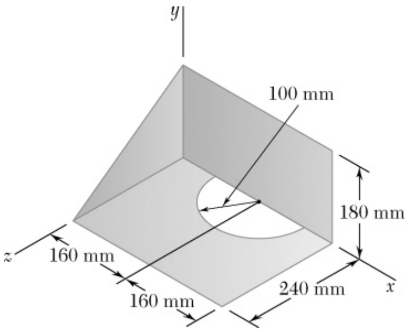
$$+ (0.028614\text{ kg})\left[\frac{1}{18}(0.095)^2 + 0.045^2 + \left(\frac{2}{3}0.095\right)^2\right]\text{m}^2$$

$$= \left[(0.0109505 + 0.075564 + 0.187064)\times 10^{-3}\right]\text{kg}\cdot\text{m}^3$$

$$= 0.37213\times 10^{-3}\text{ kg}\cdot\text{m}^2$$

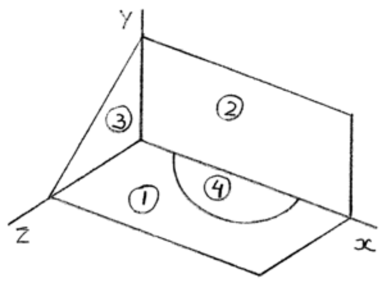
or $I_z = 0.372\times 10^{-3}\text{ kg}\cdot\text{m}^2$ ◀

PROBLEM 9.141



A 2-mm-thick piece of sheet steel is cut and bent into the machine component shown. Knowing that the density of steel is 7850 kg/m^3, determine the moment of inertia of the component with respect to each of the coordinate axes.

SOLUTION



Have $$m = \rho_{st}V = \rho_{st}tA$$

Then $$m_1 = 7850\text{ kg/m}^3 \times 0.002\text{ m} \times (0.320 \times 0.240)\text{m}^2$$

$$= 1.20576\text{ kg}$$

$$m_2 = 7850\text{ kg/m}^3 \times 0.002\text{ m} \times (0.320 \times 0.180)\text{m}^2$$

$$= 0.90432\text{ kg}$$

$$m_3 = 7850\text{ kg/m}^3 \times 0.002\text{ m} \times \left(\frac{1}{2} \times 0.180 \times 0.240\right)\text{m}^2$$

$$= 0.33912\text{ kg}$$

$$m_4 = 7850\text{ kg/m}^3 \times 0.002\text{ m} \times \left[\frac{\pi}{2}(0.100\text{ m})^2\right]$$

$$= 0.24662\text{ kg}$$

Using Fig. 9-2B for components 1 and 2 and the equations derived above for components 3 and 4, have

Now $$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3 - (I_x)_4$$

where $$(I_x)_1 = \frac{1}{3}(1.20576\text{ kg})(0.240\text{ m})^2$$

$$= 23.151 \times 10^{-3}\text{ kg}\cdot\text{m}^2$$

$$(I_x)_2 = \frac{1}{3}(0.90432\text{ kg})(0.180\text{ m})^2$$

$$= 9.7667 \times 10^{-3}\text{ kg}\cdot\text{m}^2$$

$$(I_x)_3 = \frac{1}{6}(0.33912\text{ kg})\left[(0.180)^2 + (0.240)^2\right]\text{m}^2$$

$$= 5.0868 \times 10^{-3}\text{ kg}\cdot\text{m}^2$$

PROBLEM 9.141 CONTINUED

$$(I_x)_4 = \frac{1}{4}(0.24662 \text{ kg})(0.100 \text{ m})^2$$

$$= 0.61655 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

Then $$I_x = \left[(23.151 + 9.7667 + 5.0868 - 0.61655) \times 10^{-3}\right] \text{kg} \cdot \text{m}^2$$

$$\text{or } I_x = 37.4 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \blacktriangleleft$$

And $$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3 - (I_y)_4$$

where $$(I_y)_1 = \frac{1}{3}(1.20576 \text{ kg})\left[(0.320)^2 + (0.240)^2\right]\text{m}^2$$

$$= 64.307 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$(I_y)_2 = \frac{1}{3}(0.90432 \text{ kg})(0.320 \text{ m})^2$$

$$= 30.867 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$(I_y)_3 = \frac{1}{6}(0.33912 \text{ kg})(0.240 \text{ m})^2$$

$$= 3.2556 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$(I_y)_4 = (0.24662 \text{ kg})\left\{\left[\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)(0.100 \text{ m})^2\right] + \left[(0.160)^2 + \left(\frac{4 \times 0.100}{3\pi}\right)^2\right]\text{m}^2\right\}$$

$$= 7.5466 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

Then $$I_y = \left[(64.307 + 30.067 + 3.2556 - 7.5466)10^{-3}\right]\text{kg} \cdot \text{m}^2$$

$$\text{or } I_y = 90.9 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \blacktriangleleft$$

And $$I_z = (I_z)_1 + (I_z)_2 + (I_z)_3 - (I_z)_4$$

where $$(I_z)_1 = \frac{1}{3}(1.20576 \text{ kg})(0.320 \text{ m})^2 = 41.157 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$(I_z)_2 = \frac{1}{3}(0.90432 \text{ kg})\left[(0.320)^2 + (0.180)^2\right]\text{m}^2$$

$$= 40.634 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

$$(I_z)_3 = \frac{1}{6}(0.33912 \text{ kg})(0.180 \text{ m})^2 = 1.83125 \times 10^{-3} \text{ kg} \cdot \text{m}^2$$

PROBLEM 9.141 CONTINUED

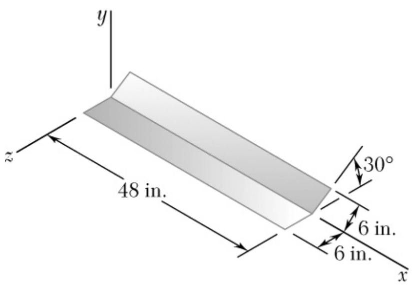
$$\left(I_z\right)_4 = \left(0.24662 \text{ kg}\right)\left\{\left[\frac{1}{4}\left(0.100 \text{ m}\right)^2\right] + \left[\left(0.160 \text{ m}\right)^2\right]\right\}$$

$$= 6.9300 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

Then $$\left(I_z\right)_z = \left[\left(41.157 + 40.634 + 1.83125 - 6.9300\right) \times 10^{-3}\right]\text{kg}\cdot\text{m}^2$$

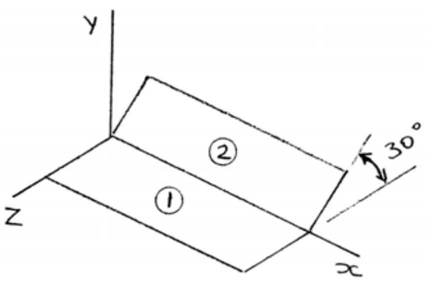
or $I_z = 76.7 \times 10^{-3} \text{ kg}\cdot\text{m}^2$ ◀

PROBLEM 9.142



The piece of roof flashing shown is formed from sheet copper that is 0.032 in. thick. Knowing that the specific weight of copper is 558 lb/ft^3, determine the mass moment of inertia of the flashing with respect to each of the coordinate axes.

SOLUTION



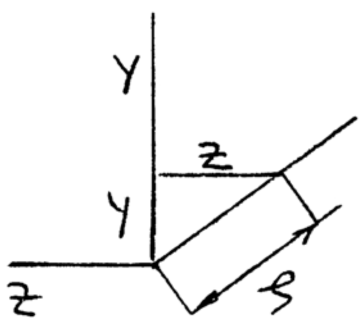
Have $$m = \rho_{\text{copper}} V$$

$$= \frac{\gamma_{\text{copper}}}{g} tA$$

Then $$m_1 = m_2$$

$$= \frac{558 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2} \times 0.032 \text{ in.} \times (48 \times 6) \text{ in}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3$$

$$= 0.092422 \text{ lb}\cdot\text{s}^2/\text{ft}$$



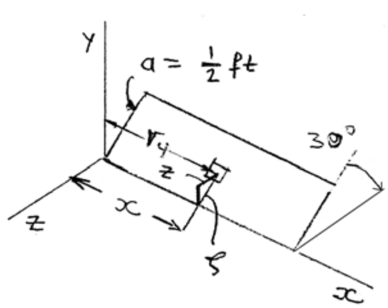
Using Fig. 9-2B for components 1 and 2, have

Now $$I_x = (I_x)_1 + (I_x)_2 \qquad \text{and} \qquad (I_x)_1 = (I_x)_2$$

Then $$I_x = 2\left[\frac{1}{3}(0.092422 \text{ lb}\cdot\text{s}^2/\text{ft})\left(\frac{6}{12} \text{ ft}\right)^2\right] = 1.54037 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_x = 1.54 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

$$I_y = (I_y)_1 + (I_y)_2$$



where $$(I_y)_1 = \frac{1}{3}(0.092422 \text{ lb}\cdot\text{s}^2/\text{ft})\left[\left(\frac{48}{12}\right)^2 + \left(\frac{6}{12}\right)^2\right] \text{ ft}^2$$

$$= 500.62 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

and $$(I_y)_2 = \int r_y^2 dm$$

where $$r_y^2 = x^2 + z^2$$

$$= x^2 + (\zeta \cos 30)^2$$

PROBLEM 9.142 CONTINUED

and $$dm = \rho dV = \frac{\gamma_{\text{copper}}}{g} t d\zeta dx$$

Then $$(I_y)_2 = \frac{\gamma_{\text{copper}}}{g} t \int_0^L \int_0^a \left(x^2 + \zeta^2 \cos^2 30°\right) d\zeta dx$$

$$= \frac{\gamma_{\text{copper}}}{g} t \int_0^L \left(ax^2 + \frac{1}{3}a^3 \cos^2 30°\right) dx$$

$$= \frac{1}{3}\frac{\gamma_{\text{copper}}}{g} t \left(aL^3 + a^3 L \cos^2 30°\right) \qquad A = aL$$

$$= \frac{1}{3} m_2 \left(L^2 + a^2 \cos^2 30°\right)$$

$$= \frac{1}{3}\left(0.092422 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left[(4)^2 + \left(\frac{1}{2}\cos 30°\right)^2\right] \text{ft}^2$$

$$= 498.69 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

Finally, $$I_y = (500.62 + 498.69) \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_y = 999 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

$$I_z = (I_z)_1 + (I_z)_2$$

where $$(I_z)_1 = \frac{1}{3}\left(0.092422 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left(\frac{48}{12} \text{ ft}\right)^2$$

$$= 492.92 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

and $$(I_z)_2 = \int r_z^2 dm$$

where $$r_z^2 = x^2 + y^2$$

and $$y = \zeta \sin 30°$$

Then $$(I_z)_2 = \int \left[x^2 + (\zeta \sin 30°)^2\right] dm$$

PROBLEM 9.142 CONTINUED

Similarly, as $\left(I_y\right)_2$

$$\left(I_z\right)_2 = \frac{1}{3}m_2\left(L^2 + a^2\sin^2 30°\right)$$

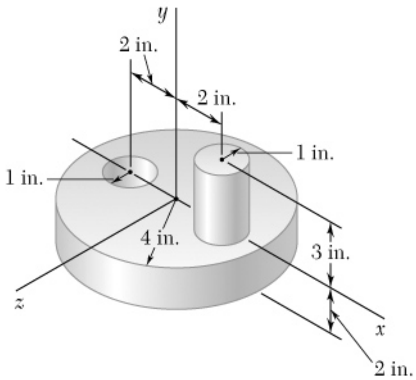
$$= \frac{1}{3}\left(0.092422 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left[\left(4\right)^2 + \left(\frac{1}{2}\right)^2\sin^2 30°\right]\text{ft}^2$$

$$= 494.84\times10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

Then $$I_x = \left(492.92 + 494.84\right)\times10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

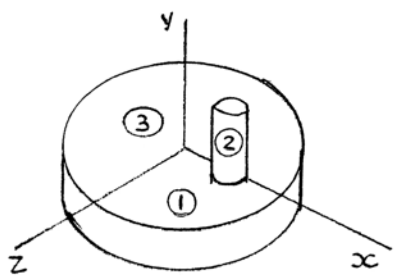
or $I_z = 988\times10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

PROBLEM 9.143



The machine element shown is fabricated from steel. Determine the mass moment of inertia of the assembly with respect to (*a*) the *x* axis, (*b*) the *y* axis, (*c*) the *z* axis. (The specific weight of steel is 0.284 lb/in^3.)

SOLUTION



Have
$$m = \rho V = \frac{\gamma}{g} V = \frac{0.284 \text{ lb/in}^3}{32.2 \text{ ft/s}^2} \times V$$

$$= \left(0.0088199 \text{ lb}\cdot\text{s}^2/\text{ft}\cdot\text{in}^3\right) V$$

Then
$$m_1 = \left(0.0088199 \text{ lb}\cdot\text{s}^2/\text{ft}\cdot\text{in}^3\right)\left[\pi(4)^2(2)\right]\text{in}^3 = 0.88667 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m_2 = \left(0.0088199 \text{ lb}\cdot\text{s}^2/\text{ft}\cdot\text{in}^3\right)\left[\pi(1)^2(3)\right]\text{in}^3 = 0.083126 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m_3 = \left(0.0088199 \text{ lb}\cdot\text{s}^2/\text{ft}\cdot\text{in}^3\right)\left[\pi(1)^2(2)\right]\text{in}^3 = 0.055417 \text{ lb}\cdot\text{s}^2/\text{ft}$$

Using Fig. 9-28 and the parallel theorem, have

(*a*)

$$I_x = (I_x)_1 + (I_x)_2 - (I_x)_3$$

$$= \left(0.88667 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[3(4)^2 + (2)^2\right] + (1)^2\right\}\text{in}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$+ \left(0.083126 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[3(1)^2 + (3)^2\right] + (1.5)^2\right\}\text{in}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$- \left(0.055417 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[3(1)^2 + (2)^2\right] + (1)^2\right\}\text{in.} \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$= 0.034106 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_x = 0.0341 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

PROBLEM 9.143 CONTINUED

(*b*)

$$I_y = \left(I_y\right)_1 + \left(I_y\right)_2 - \left(I_y\right)_3$$

$$= \left(0.88667 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left[\frac{1}{2}(4)^2\right]\text{in}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$+ \left(0.083126 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left[\frac{1}{2}(1)^2 + (2)^2\right]\text{in}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$- \left(\ 0.055417 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left[\frac{1}{2}(1)^2 + (2)^2\right]\text{in}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$= 5.0125 \times 10^{-2} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_y = 0.0501 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

(*c*)

$$I_z = \left(I_z\right)_1 + \left(I_z\right)_2 - \left(I_z\right)_3$$

$$= \left(0.88667 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[3(4)^2 + (2)^2\right] + (1)^2\right\}\text{in}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$+ \left(0.083126 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[3(1)^2 + (3)^2\right] + \left[(2)^2 + (1.5)^2\right]\right\}\text{in}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

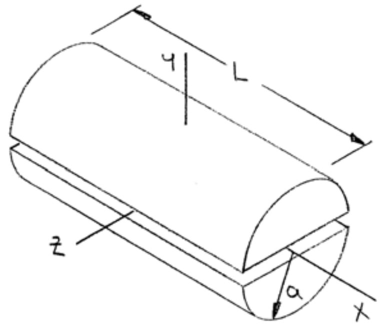
$$- \left(0.055417 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[3(1)^2 + (2)^2\right] + \left[(2)^2 + (1)^2\right]\right\}\text{in}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$= 0.034876 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_z = 0.0349 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

To the Instructor:

The following formulas for the mass of inertia of a semicylinder are derived at this time for use in the solutions of Problems 9.144–9.147.



From Figure 9.28

Cylinder:
$$(I_x)_{\text{cyl}} = \frac{1}{2}m_{\text{cyl}}a^2$$

$$(I_y)_{\text{cyl}} = (I_z)_{\text{cyl}} = \frac{1}{12}m_{\text{cyl}}\left(3a^2 + L^2\right)$$

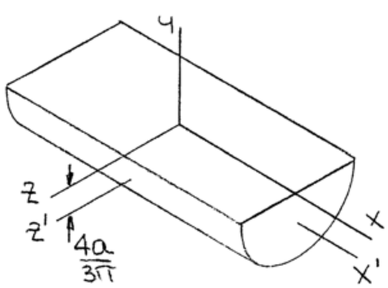
Symmetry and the definition of the mass moment of inertia $\left(I = \int r^2 dm\right)$ imply

$$(I)_{\text{semicylinder}} = \frac{1}{2}(I)_{\text{cylinder}}$$

$$\therefore \quad (I_x)_{\text{sc}} = \frac{1}{2}\left(\frac{1}{2}m_{\text{cyl}}a^2\right)$$

and
$$(I_y)_{\text{sc}} = (I_z)_{\text{sc}} = \frac{1}{2}\left[\frac{1}{12}m_{\text{cyl}}\left(3a^2 + L^2\right)\right]$$

However,
$$m_{\text{sc}} = \frac{1}{2}m_{\text{cyl}}$$



Thus,
$$(I_x)_{\text{sc}} = \frac{1}{2}m_{\text{sc}}a^2$$

and
$$(I_y)_{\text{sc}} = (I_z)_{\text{sc}} = \frac{1}{12}m_{\text{sc}}\left(3a^2 + L^2\right)$$

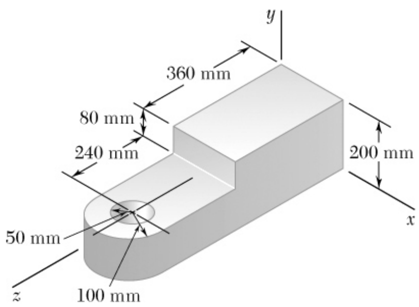
Also, using the parallel axis theorem find

$$\bar{I}_{x'} = m_{\text{sc}}\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)a^2$$

$$\bar{I}_{z'} = m_{\text{sc}}\left[\left(\frac{1}{4} - \frac{16}{9\pi^2}\right)a^2 + \frac{1}{12}L^2\right]$$

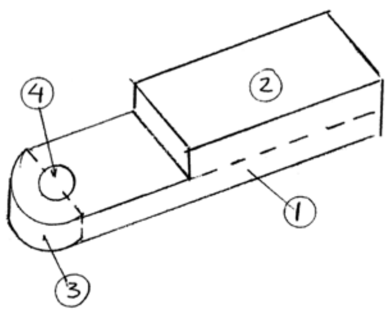
where x' and z' are centroidal axes.

PROBLEM 9.144



Determine the mass moment of inertia of the steel machine element shown with respect to the y axis. (The density of steel is 7850 kg/m^3.)

SOLUTION



Have $$m = \rho_{\text{steel}} V$$

Then $$m_1 = 7850\text{ kg/m}^3(0.200 \times 0.120 \times 0.600)\text{m}^3$$

$$= 113.040\text{ kg}$$

$$m_2 = 7850\text{ kg/m}^3 \times (0.200 \times 0.080 \times 0.360)\text{m}^3$$

$$= 45.216\text{ kg}$$

$$m_3 = 7850\text{ kg/m}^3 \times \left[\frac{\pi}{2}(0.100)^2(0.120)\right]\text{m}^3$$

$$= 14.7969\text{ kg}$$

$$m_4 = 7850\text{ kg/m}^3 \times \left[\pi(0.050)^2(0.120)\right]\text{m}^3$$

$$= 7.3985\text{ kg}$$

Using Figure 9.28 for components 1 and 2 and the equations derived above for components 3 and 4, have

Now $$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3$$

where $$(I_y)_1 = (113.040\text{ kg})\left\{\frac{1}{12}\left[(0.600)^2 + (0.200)^2\right] + \left[\left(\frac{0.600}{2}\right)^2 + \left(\frac{0.200}{2}\right)^2\right]\right\}\text{m}^2$$

$$= 15.0720\text{ kg}\cdot\text{m}^2$$

PROBLEM 9.144 CONTINUED

$$\left(I_y\right)_2 = \left(45.216 \text{ kg}\right)\left\{\frac{1}{12}\left[\left(0.360\right)^2 + \left(0.200\right)^2\right] + \left[\left(\frac{0.360}{2}\right)^2 + \left(\frac{0.200}{2}\right)^2\right]\right\}\text{m}^2$$

$$= 2.5562 \text{ kg}\cdot\text{m}^2$$

$$\left(I_y\right)_3 = \left(14.7969 \text{ kg}\right)\left\{\left[\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)\left(0.100\right)^2\right] + \left[\left(0.100\right)^2 + \left(0.600 + \frac{4\times 0.100}{3\pi}\right)^2\right]\right\}\text{m}^2$$

$$= 6.3024 \text{ kg}\cdot\text{m}^2$$

$$\left(I_y\right)_4 = \left(7.3985 \text{ kg}\right)\left\{\left[\frac{1}{2}\left(0.050\right)^2\right] + \left[\left(0.100\right)^2 + \left(0.600\right)^2\right]\right\}\text{m}^2$$

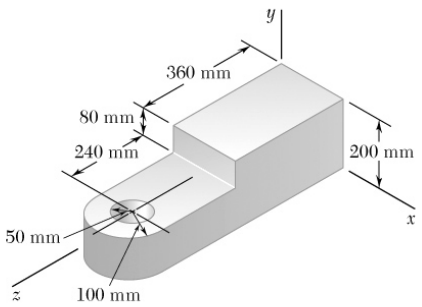
$$= 2.7467 \text{ kg}\cdot\text{m}^2$$

Then
$$I_y = \left(15.0720 + 2.5562 + 6.3024 - 2.7467\right)\text{kg}\cdot\text{m}^2$$

$$= 21.1839 \text{ kg}\cdot\text{m}^2$$

or $I_y = 21.2 \text{ kg}\cdot\text{m}^2$ ◀

PROBLEM 9.145



Determine the mass moment of inertia of the steel machine element shown with respect to the z axis. (The density of steel is 7850 kg/m^3.)

SOLUTION

See machine elements shown in Problem 9.145

Also note
$$m_1 = 113.040 \text{ kg}$$
$$m_2 = 45.216 \text{ kg}$$
$$m_3 = 14.7969 \text{ kg}$$
$$m_4 = 7.3985 \text{ kg}$$

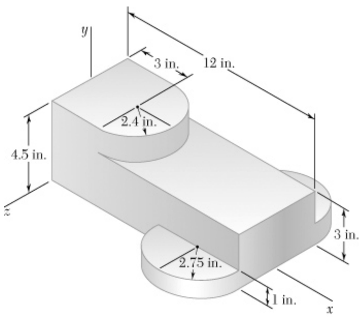
Using Fig. 9.28 for components 1 and 2 and the equations derived above for components 3 and 4, have

Now
$$I_z = (I_z)_1 + (I_z)_2 + (I_z)_3 - (I_z)_4$$

where
$$(I_z)_1 = (113.040 \text{ kg})\left\{\frac{1}{12}\left[(0.200)^2 + (0.120)^2\right] + \left[\left(\frac{0.200}{2}\right)^2 + \left(\frac{0.120}{2}\right)^2\right]\right\}\text{m}^2$$
$$= 2.0498 \text{ kg}\cdot\text{m}^2$$
$$(I_z)_2 = (45.216 \text{ kg})\left\{\frac{1}{12}\left[(0.200)^2 + (0.080)^2\right] + \left[(0.100)^2 + (0.160)^2\right]\right\}\text{m}^2$$
$$= 1.78453 \text{ kg}\cdot\text{m}^2$$
$$(I_z)_3 = (14.7969 \text{ kg})\left\{\frac{1}{12}\left[3(0.100)^2 + (0.120)^2\right] + \left[(0.100)^2 + (0.060)^2\right]\right\}\text{m}^2$$
$$= 0.25599 \text{ kg}\cdot\text{m}^2$$
$$(I_z)_4 = (7.3985 \text{ kg})\left\{\frac{1}{12}\left[3(0.050)^2 + (0.120)^2\right] + \left[(0.100)^2 + (0.060)^2\right]\right\}\text{m}^2$$
$$= 0.114122 \text{ kg}\cdot\text{m}^2$$

Then
$$I_z = (2.0498 + 1.78453 + 0.25599 - 0.114122) \text{ kg}\cdot\text{m}^2$$
$$= 3.97629 \text{ kg}\cdot\text{m}^2$$

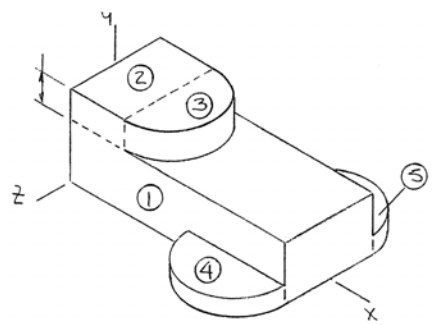
or $I_z = 3.98 \text{ kg}\cdot\text{m}^2$ ◀



PROBLEM 9.146

An aluminum casting has the shape shown. Knowing that the specific weight of aluminum is $0.100\ \text{lb/in}^3$, determine the moment of inertia of the casting with respect to the z axis.

SOLUTION



Have
$$m = \rho V = \frac{\gamma}{g}V = \frac{0.10\ \text{lb/in}^3}{32.2\ \text{ft/s}^2}V$$

$$= \left(0.0031056\ \text{lb}\cdot\text{s}^2/\text{ft}\cdot\text{in}^3\right)V$$

Then
$$m_1 = \left(0.0031056\ \text{lb}\,\text{s}^2/\text{ft}\cdot\text{in}^3\right)(12\ \text{in.})(3\ \text{in.})(4.8\ \text{in.})$$

$$= 0.53665\ \text{lb}\cdot\text{s}^2/\text{ft}$$

$$m_2 = \left(0.0031056\ \text{lb}\cdot\text{s}^2/\text{ft}\cdot\text{in}^3\right)(1.5\ \text{in.})(4.8\ \text{in.})(3\ \text{in.}) = 0.06708\ \text{lb}\cdot\text{s}^2/\text{ft}$$

$$m_3 = \left(0.0031056\ \text{lb}\cdot\text{s}^2/\text{ft}\cdot\text{in}^3\right)\left[\frac{\pi}{2}(2.4\ \text{in.})^2 \times (1.5\ \text{in.})\right]$$

$$= 0.042148\ \text{lb}\cdot\text{s}^2/\text{ft}$$

$$m_4 = m_5 = \left(0.0031056\ \text{lb}\cdot\text{s}^2/\text{ft}\cdot\text{in}^3\right)\left[\frac{\pi}{2}(2.75\ \text{in.})^2 \times (1.0\ \text{in.})\right]$$

$$= 0.036892\ \text{lb}\cdot\text{s}^2/\text{ft}$$

PROBLEM 9.146 CONTINUED

Using Fig. 9.28 for components 1 and 2 and the equations derived above for components 3, 4, and 5, have

$$I_z = (I_z)_1 + (I_z)_2 + (I_z)_3 + (I_z)_4 + (I_z)_5 \qquad \text{where} \qquad (I_z)_4 = (I_z)_5$$

$$I_z = \left(0.53665\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[(12\ \text{in.})^2 + (3\ \text{in.})^2\right] + \left(\frac{12\ \text{in.}}{2}\right)^2 + \left(\frac{3\ \text{in.}}{2}\right)^2\right\} \times \left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$+\left(0.06708\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[(3\ \text{in.})^2 + (1.5\ \text{in.})^2\right] + (1.5\ \text{in.})^2 + (4.5\ \text{in.} - 0.75\ \text{in.})^2\right\} \times \left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

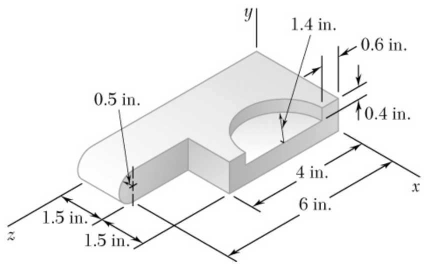
$$+\left(0.042148\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left\{\left(\frac{1}{4} - \frac{16}{4\pi}\right)(2.4\ \text{in.})^2 + \frac{1}{12}(1.5\ \text{in.})^2\right.$$

$$\left. +\left(3\ \text{in.} + \frac{4 \times 2.4\ \text{in.}}{3\pi}\right)^2 + (4.5\ \text{in.} - 0.75\ \text{in.})^2\right\} \times \left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$+ 2\left(0.036892\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[3(2.75\ \text{in.})^2 + (1.0\ \text{in.})^2\right] + \left[(12\ \text{in} - 2.75\ \text{in})^2 + (0.5\ \text{in})^2\right]\right\} \times \left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^2$$

$$= 0.252096\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

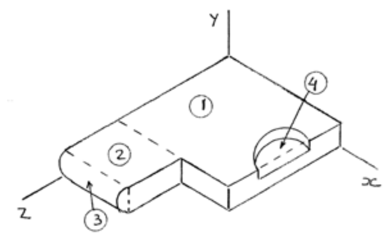
or $I_z = 0.252\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀



PROBLEM 9.147

Determine the moment of inertia of the steel machine element shown with respect to (*a*) the *x* axis, (*b*) the *y* axis, (*c*) the *z* axis. (The specific weight of steel is 490 lb/ft^3.)

SOLUTION



Have $$m = \rho_{ST} V = \frac{\delta_{ST}}{g} V$$

Then $$m_1 = \frac{490\text{ lb/ft}^3}{32.2\text{ ft/s}^2} \times (3 \times 1 \times 4)\text{ in}^3 \times \left(\frac{1\text{ ft}}{12\text{ in.}}\right)^3$$

$$= 105.676 \times 10^{-3}\text{ lb}\cdot\text{s}^2\text{/ft}$$

$$m_2 = \frac{490\text{ lb/ft}^3}{32.2\text{ ft/s}^2} \times (1.5 \times 1 \times 2)\text{ in}^3 \times \left(\frac{1\text{ ft}}{12\text{ in.}}\right)^3 = 26.419 \times 10^{-3}\text{ lb}\cdot\text{s}^2\text{/ft}$$

$$m_3 = \frac{490\text{ lb/ft}^3}{32.2\text{ ft/s}^2} \times \left[\frac{\pi}{2}(0.5)^2 \times 1.5\right]\text{in}^3 \times \left(\frac{1\text{ ft}}{12\text{ in.}}\right)^3 = 5.1874 \times 10^3\text{ lb}\cdot\text{s}^2\text{/ft}$$

$$m_4 = \frac{490\text{ lb/ft}^3}{32.2\text{ ft/s}^2} \times \left[\frac{\pi}{2}(1.4)^2 \times 0.4\right]\text{in}^3 \times \left(\frac{1\text{ ft}}{12\text{ in.}}\right)^3$$

$$= 10.8491 \times 10^{-3}\text{ lb}\cdot\text{s}^2\text{/ft}$$

(*a*) Using Fig. 9.28 for components 1 and 2 and the equations derived above for components 3 and 4, have

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3 - (I_x)_4$$

where $$(I_x)_1 = (105.676 \times 10^{-3}\text{ lb}\cdot\text{s}^2\text{/ft})\left\{\frac{1}{12}\left[(1)^2 + (4)^2\right] + \left[\left(\frac{1}{2}\right)^2 + \left(\frac{4}{2}\right)^2\right]\text{in}^2 \times \left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2\right\}$$

$$= 4.1585 \times 10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$(I_x)_2 = (26.419 \times 10^{-3}\text{ lb}\cdot\text{s}^2\text{/ft})\left\{\frac{1}{12}\left[(1)^2 + (2)^2\right] + \left[(0.5)^2 + (5)^2\right]\right\}\text{in}^2 \times \left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$= 4.7089 \times 10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

PROBLEM 9.147 CONTINUED

$$\left(I_x\right)_3 = \left(5.1874\times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left\{\left[\left(\frac{1}{2}-\frac{16}{9\pi^2}\right)(0.5)^2\right]+\left[(0.5)^2+\left(6+\frac{4\times 0.5}{3\pi}\right)^2\right]\right\}\text{in}^2\times\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$= 1.40209\times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$\left(I_x\right)_4 = \left(10.8451\times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[3(1.4)^2+(0.4)^2\right]+\left(6\times\frac{4\times 0.5}{3\pi}\right)^2\right\}\text{in}^2\times\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$= 0.38736\times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

Then $$I_x = \left[(4.1585+4.7089+1.40209-0.38736)\times 10^{-3}\right]\text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 9.8821\times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_x = 9.88\times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

(*b*) Have $$I_y = \left(I_y\right)_1+\left(I_y\right)_2+\left(I_y\right)_3-\left(I_y\right)_4$$

where

$$\left(I_y\right)_1 = \left(105.676\times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[(3)^2+(4)^2\right]+\left[\left(\frac{3}{2}\right)^2+\left(\frac{4}{2}\right)^2\right]\right\}\text{in}^2\times\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$= 6.1155\times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$\left(I_y\right)_2 = \left(26.419\times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[(1.5)^2+(2)^2\right]+\left[(0.75)^2+(5)^2\right]\right\}\text{in}^2\times\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$= 4.7854\times 10^{-5}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$\left(I_y\right)_3 = \left(5.1874\times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left\{\left[\left(\frac{1}{4}-\frac{16}{9\pi^2}\right)(0.5)^2+\frac{1}{12}(1.5)^2\right]+\left[(0.75)^2+\left(6+\frac{4.05}{3\pi}\right)^2\right]\right\}\text{in}^2\times\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$= 1.4178\,\text{s}\times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$\left(I_y\right)_4 = \left(10.8451\times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left\{\left[\left(\frac{1}{2}-\frac{16}{9\pi^2}\right)(1.4)^2\right]+\left[\left(3-\frac{4\times 1.4}{3\pi}\right)^2+(2)^2\right]\right\}\text{in}^2\times\left(\frac{1\text{ ft}}{12\text{ in.}}\right)^2$$

$$= 0.78438\times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

PROBLEM 9.147 CONTINUED

Then $$I_y = \left[\left(16.1155 + 4.7854 + 1.41785 - 0.78438\right) \times 10^{-3}\right] \text{lb} \cdot \text{ft} \cdot \text{s}^2$$

$$= 11.5344 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

or $I_y = 11.53 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$ ◀

(*c*) Have $$I_z = (I_z)_1 + (I_z)_2 + (I_z)_3 - (I_z)_4$$

where

$$(I_z)_1 = \left(105.676 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[(3)^2 + (1)^2\right] + \left[\left(\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2\right]\right\}\text{in}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$= 2.4462 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

$$(I_z)_2 = \left(26.419 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[(1.5)^2 + (1)^2\right] + \left[\left(\frac{1.5}{2}\right)^2 + \left(\frac{1}{2}\right)^2\right]\right\}\text{in}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$= 0.198754 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

$$(I_z)_3 = \left(5.1874 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}\right)\left\{\frac{1}{12}\left[3(0.5)^2 + (1.5)^2\right] + \left[(0.75)^2 + (0.5)^2\right]\right\}\text{in}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$= 0.038275 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

$$(I_z)_4 = \left(10.8451 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}\right)\left\{\left[\left(\frac{1}{4} - \frac{16}{9\pi^2}\right)(1.4)^2 + \frac{1}{12}(0.4)^2\right] + \left[\left(3 - \frac{4 \times 1.4}{3\pi}\right)^2 + (0.8)^2\right]\right\}\text{in}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$= 0.49543 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

Then $$I_z = \left[\left(2.4462 + 0.198754 + 0.038275 - 0.49543\right) \times 10^{-3}\right] \text{lb} \cdot \text{ft} \cdot \text{s}^2$$

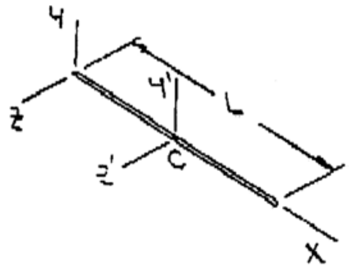
$$= 2.1878 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$$

or $I_z = 2.19 \times 10^{-3} \text{ lb} \cdot \text{ft} \cdot \text{s}^2$ ◀

To the instructor:

The following formulas for the mass moment of inertia of wires are derived or summarized at this time for use in the solutions of problems 9.148–9.150

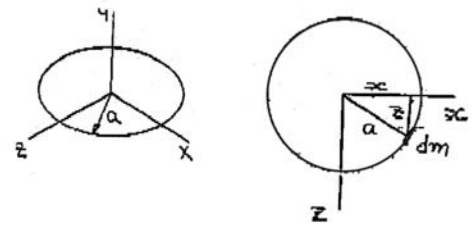
Slender Rod



$$I_x = 0 \qquad \bar{I}_{y'} = \bar{I}_{z'} = \frac{1}{12}mL^2 \text{ (Fig. 9.28)}$$

$$I_y = I_z = \frac{1}{3}mL^2 \text{ (Sample Problem 9.9)}$$

Circle



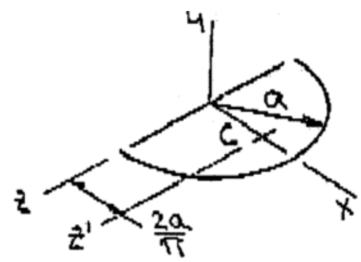
Have $\qquad \bar{I}_y = \int r^2 dm = ma^2$

Now $\qquad \bar{I}_y = \bar{I}_x + \bar{I}_z$

And symmetry implies $\qquad \bar{I}_x = \bar{I}_z$

$$\therefore \quad \bar{I}_x = \bar{I}_z = \frac{1}{2}ma^2$$

Semicircle



Following the above arguments for a circle, have

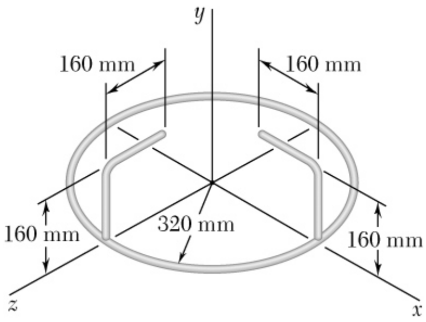
$$\bar{I}_x = I_z = \frac{1}{2}ma^2 \qquad I_y = ma^2$$

Using the parallel-axis theorem

$$I_z = \bar{I}_{z'} + m\bar{x}^2 \qquad x = \frac{2a}{\pi}$$

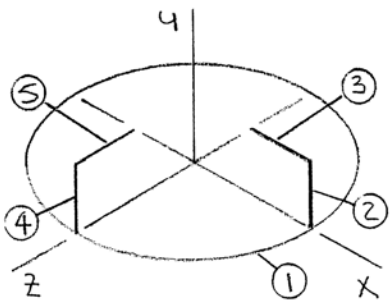
or $\qquad I_{z'} = m\left(\frac{1}{2} - \frac{4}{\pi^2}\right)a^2$

PROBLEM 9.148



Aluminum wire with a mass per unit length of 0.049 kg/m is used to form the circle and the straight members of the figure shown. Determine the mass moment of inertia of the assembly with respect to each of the coordinate axes.

SOLUTION



First compute the mass of each component.

Have $$m = \rho L$$

Then $$m_1 = (0.049 \text{ kg/m})\left[2\pi(0.32 \text{ m})\right]$$

$$= 0.09852 \text{ kg}$$

$$m_2 = m_3 = m_4 = m_5$$

$$= (0.049\,\text{kg/m})(0.160\,\text{m}) = 0.00784\,\text{kg}$$

Using the equation given above and the parallel axis theorem, have

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3 + (I_x)_5$$

$$= (0.09852\,\text{kg})\left[\left(\frac{1}{2}\right)(0.32 \text{ m})^2\right] + (0.00784\,\text{kg})\left[\left(\frac{1}{3}\right)(0.160\,\text{m})^2\right]$$

$$+ (0.00784\,\text{kg})\left[0 + (0.160\,\text{m})^2\right]$$

$$+ (0.00784\,\text{kg})\left[\left(\frac{1}{12}\right)(0.16\,\text{m})^2 + (0.08 \text{ m})^2 + (0.32 \text{ m})^2\right]$$

$$+ (0.00784 \text{ kg})\left[\frac{1}{12}(0.16\,\text{m})^2 + (0.16\,\text{m})^2 + (0.32\,\text{m} - 0.08\,\text{m})^2\right]$$

$$= \left[(5.0442 + 0.06690 + 0.2007 + 0.86972 + 0.66901) \times 10^{-3}\right]\text{kg}\cdot\text{m}^2$$

$$= 6.8505 \times 10^{-3}\text{ kg}\cdot\text{m}^2 \qquad \text{or} \quad I_x = 6.85 \times 10^{-3}\,\text{kg}\cdot\text{m}^2 \blacktriangleleft$$

PROBLEM 9.148 CONTINUED

Have $$I_y = \left(I_y\right)_1 + \left(I_y\right)_2 + \left(I_y\right)_3 + \left(I_y\right)_4 + \left(I_y\right)_5$$

where $$\left(I_y\right)_2 = \left(I_y\right)_4 \qquad \text{and} \qquad \left(I_y\right)_3 = \left(I_y\right)_5$$

Then $$I_y = \left(0.09852\,\text{kg}\right)\left[\left(0.32\text{ m}\right)^2\right] + 2\left(0.00784\,\text{kg}\right)\left[0 + \left(0.32\,\text{m}\right)^2\right]$$

$$+ 2\left(0.00784\,\text{kg}\right)\left[\frac{1}{12}\left(0.16\,\text{m}\right)^2 + \left(0.24\,\text{m}\right)^2\right]$$

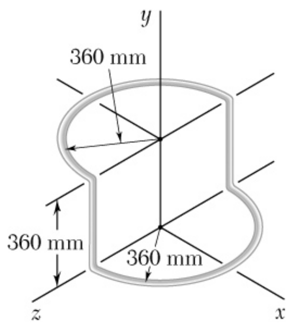
$$= \left[10.088 + 2\left(0.80282\right) + 2\left(0.46831\right)\right] \times 10^{-3}\,\text{kg}\cdot\text{m}^2$$

$$= 12.6303 \times 10^{-3}\,\text{kg}\cdot\text{m}^2$$

or $I_y = 12.63 \times 10^{-3}\text{ kg}\cdot\text{m}^2$ ◀

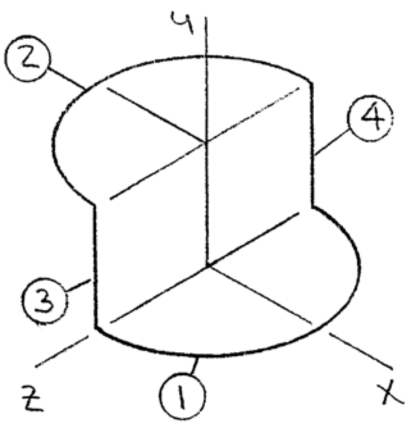
By symmetry $I_z = I_x$ or $I_z = 6.85 \times 10^{-3}\,\text{kg}\cdot\text{m}^2$ ◀

PROBLEM 9.149



The figure shown is formed of 3-mm-diameter steel wire. Knowing that the density of the steel is $7850\ \text{kg/m}^3$, determine the mass moment of inertia of the wire with respect to each of the coordinate axes.

SOLUTION



Have $$m = \rho V = \rho AL$$

Then

$$m_1 = m_2 = \left(7850\ \text{kg/m}^3\right)\left[\pi\left(0.0015\ \text{m}\right)^2\right] \times \left(\pi \times 0.36\ \text{m}\right)$$

$$m_2 = m_1 = 0.062756\ \text{kg}$$

$$m_3 = m_4 = \left(7850\ \text{kg/m}^3\right)\left[\pi\left(0.0015\ \text{m}\right)^2\right] \times \left(0.36\ \text{m}\right)$$

$$= 0.019976\ \text{kg}$$

Using the equations given above and the parallel axis theorem, have

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3 + (I_x)_4$$

where $$(I_x)_3 = (I_x)_4$$

Then

$$I_x = \left(0.062756\ \text{kg}\right)\left[\frac{1}{2}\left(0.36\ \text{m}\right)^2\right] + \left(0.062756\right)\left[\frac{1}{2}\left(0.36\ \text{m}\right)^2 + \left(0.36\ \text{m}\right)^2\right]$$

$$+ 2\left(0.019976\ \text{kg}\right)\left[\frac{1}{12}\left(0.36\ \text{m}\right)^2 + \left(0.18\ \text{m}\right)^2 + \left(0.36\ \text{m}\right)^2\right]$$

$$= \left[4.06659 + 12.19977 + 2\left(3.45185\right)\right] \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

$$= 23.1701 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

or $I_x = 23.2 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

Have $$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3 + (I_y)_4$$

where $(I_y)_1 = (I_y)_2$ and $(I_y)_3 = (I_y)_4$

PROBLEM 9.149 CONTINUED

Then

$$I_y = 2(0.062756 \text{ kg})\left[(0.36 \text{ m})^2\right] + 2(0.019976 \text{ kg})\left[0 + (0.36 \text{ m})^2\right]$$

$$= 2\left(8.13318 \times 10^{-3} \text{ kg}\cdot\text{m}^2\right) + 2\left(2.58889 \times 10^{-3} \text{ kg}\cdot\text{m}^2\right)$$

$$= 21.44414 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

or $I_y = 21.4 \times 10^{-3} \text{ kg}\cdot\text{m}^2$ ◀

Have

$$I_z = (I_z)_1 + (I_z)_2 + (I_z)_3 + (I_z)_4$$

where $(I_z)_3 = (I_z)_4$

Then

$$I_z = (0.062756 \text{ kg})\left[\frac{1}{2}(0.36 \text{ m})^2\right]$$

$$+ (0.062756 \text{ kg})\left[\left(\frac{1}{2} - \frac{4}{\pi^2}\right)(0.36 \text{ m})^2 + \left(\frac{2 \times 0.36 \text{ m}^2}{\pi}\right) + (0.36 \text{ m})^2\right]$$

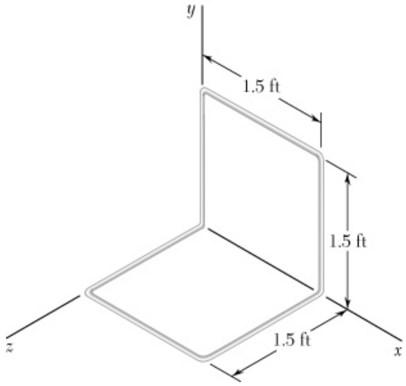
$$+ 2(0.019976 \text{ kg})\left[\frac{1}{3}(0.36 \text{ m})^2\right]$$

$$= \left[4.06659 + 12.1998 + 2(0.86296)\right]10^{-3} \text{ kg}\cdot\text{m}^2$$

$$= 17.9923 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

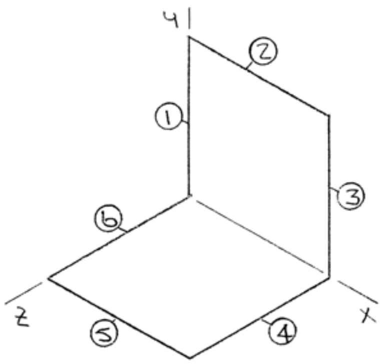
or $I_z = 17.99 \times 10^{-3} \text{ kg}\cdot\text{m}^2$ ◀

PROBLEM 9.150



A homogeneous wire with a weight per unit length of 0.041 lb/ft is used to form the figure shown. Determine the moment of inertia of the wire with respect to each of the coordinate axes.

SOLUTION



First compute the mass of each component. Mass of each component is

indentical

$$m = \frac{(m/L)}{g} L$$

Have
$$= \frac{(0.041\ \text{lb/ft})(1.5\ \text{ft})}{32.2\ \text{ft/s}^2}$$

$$= 0.00190994\ \text{lb}\cdot\text{s}^2/\text{ft}$$

Using the equations given above and the parallel axis theorem, have

$$(I_x)_1 = (I_x)_3 + (I_x)_4 = (I_x)_6 \qquad \text{and} \qquad (I_x)_2 = (I_x)_5$$

Then
$$I_x = 4(I_x)_1 + 2(I_x)_2$$

$$I_x = 4(0.00190994\ \text{lb}\cdot\text{s}^2/\text{ft})\left[\frac{1}{3}(5\ \text{ft})^2\right]$$

$$+ 2(0.00190994\ \text{lb}\cdot\text{s}^2/\text{ft})\left[0 + (1.5\ \text{ft})^2\right]$$

$$= 0.0143246\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_x = 14.32 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

Now
$$(I_y)_1 = 0 \qquad (I_y)_2 = (I_y)_6 \qquad (I_y)_4 = (I_y)_5$$

PROBLEM 9.150 CONTINUED

Then

$$I_y = 2(I_y)_2 + (I_y)_3 + 2(I_y)_4$$

$$= \left(0.0019094 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left\{\left[2\left(\frac{1}{3}\right)(1.5 \text{ ft})^2\right] + \left[0 + (1.5 \text{ ft})^2\right]\right.$$

$$\left. + 2\left[\frac{1}{12}(1.5 \text{ ft})^2 + (1.5 \text{ ft})^2 + (0.75 \text{ ft})^2\right]\right\}$$

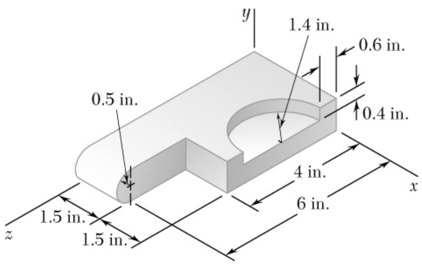
$$= 0.0019094(1.5 + 2.25 + 6) \text{ lb}\cdot\text{ft}\cdot\text{s}^2 = 0.0186219 \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_y = 18.62 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2 \blacktriangleleft$$

By symmetry $\qquad I_z = I_y$

$$I_z = 18.62 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2 \blacktriangleleft$$

PROBLEM 9.151



Determine the products of inertia I_{xy}, I_{yz}, and I_{zx} of the steel machine element shown. (The specific weight of steel is 490 lb/ft^3.)

SOLUTION

From the solution to Problem 9.147

$$m_1 = 105.676 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft} \qquad m_3 = 5.1874 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}$$

$$m_2 = 26.419 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft} \qquad m_4 = 10.8451 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}$$

First note that symmetry implies $\bar{I}_{x'y'} = \bar{I}_{y'z'} = \bar{I}_{z'x'} = 0$ for each component

Now $$I_{uv} = \overset{0}{\cancel{\bar{I}_{u'v'}}} + m\bar{u}\,\bar{v} = m\bar{u}\,\bar{v}$$

so that $$(I_{uv})_{\text{body}} = \Sigma m\bar{u}\,\bar{v}$$

Then $$I_{xy} = \Sigma m\bar{x}\,\bar{y} = (105.676 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})\left(\frac{1.5}{12}\text{ft}\right)\left(\frac{0.5}{12}\text{ft}\right) + (26.419 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})\left(\frac{0.75}{12}\text{ft}\right)\left(\frac{0.5}{12}\text{ft}\right)$$

$$+(5.1874 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})\left(\frac{0.75}{12}\text{ft}\right)\left(\frac{0.5}{12}\text{ft}\right)$$

$$-(10.8451 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft})\left[\left(3\ \text{in.} - \frac{4 \times 1.4\ \text{in.}}{3\pi}\right)\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)\right]\left(\frac{0.8}{12}\text{ft}\right)$$

$$= (550.40 + 68.799 + 13.5089 - 144.952) \times 10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 487.76 \times 10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_{xy} = 0.488 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

PROBLEM 9.151 CONTINUED

$$I_{yz} = \Sigma m\overline{y}\,\overline{z} = \left(105.676\times10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft}\right)\left(\frac{0.5}{12}\text{ ft}\right)\left(\frac{2}{12}\text{ ft}\right) + \left(26.419\times10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft}\right)\left(\frac{0.5}{12}\text{ ft}\right)\left(\frac{5}{12}\text{ ft}\right)$$

$$+\left(5.1874\times10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft}\right)\left(\frac{0.5}{12}\text{ ft}\right)\left[\left(6\text{ in.} + \frac{4\times0.5\text{ in.}}{3\pi}\right)\left(\frac{1\text{ ft}}{12\text{ in.}}\right)\right]$$

$$-\left(10.8451\times10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft}\right)\left(\frac{0.8}{12}\text{ ft}\right)\left(\frac{2}{12}\text{ ft}\right)$$

$$= \left(733.86 + 458.66 + 111.893 - 120.501\right)\times10^{-6}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 1183.91\times10^{-6}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_{yz} = 1.184\times10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

$$I_{zx} = \Sigma m\overline{z}\,\overline{x} = \left(105.676\times10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft}\right)\left(\frac{2}{12}\text{ ft}\right)\left(\frac{1.5}{12}\text{ ft}\right) + \left(26.419\times10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft}\right)\left(\frac{5}{12}\text{ ft}\right)\left(\frac{0.75}{12}\text{ ft}\right)$$

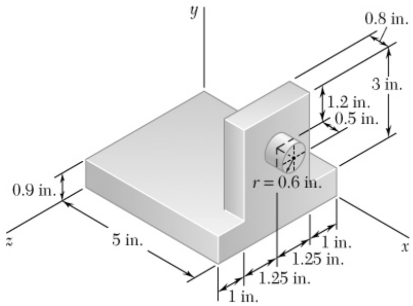
$$+\left(5.1874\times10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft}\right)\left[\left(6 + \frac{4\times0.5}{3\pi}\right)\text{in.}\times\left(\frac{1\text{ ft}}{12\text{ in.}}\right)\right]\left(\frac{0.75}{12}\text{ ft}\right)$$

$$-\left(10.8451\times10^{-3}\text{ lb}\cdot\text{s}^2/\text{ft}\right)\left(\frac{2}{12}\text{ ft}\right)\left[\left(3 - \frac{4\times1.4}{3\pi}\right)\text{in.}\times\left(\frac{1\text{ ft}}{12\text{ in.}}\right)\right]$$

$$= \left(2201.6 + 687.99 + 167.840 - 362.38\right)\times10^{-6}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 2695.1\times10^{-6}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

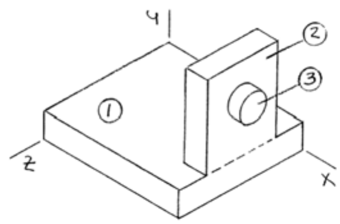
or $I_{zx} = 2.70\times10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀



PROBLEM 9.152

Determine the products of inertia I_{xy}, I_{yz}, and I_{zx} of the steel machine element shown. (The specific weight of steel is $0.284\ \text{lb}/\text{in}^3$.)

SOLUTION



First compute the mass of each component $\qquad m = \dfrac{\gamma}{g}V$

Then

$$m_1 = \frac{0.284\ \text{lb/in}^3}{32.2\ \text{ft/s}^2}\left(5\ \text{in.} \times 4.5\ \text{in.} \times 0.9\ \text{in.}\right) = 0.1786\ \text{lb}\cdot\text{s}^2/\text{ft}$$

$$m_2 = \frac{0.284\ \text{lb/in}^3}{32.2\ \text{ft/s}^2}\left(3\ \text{in.} \times 2.5\ \text{in.} \times 0.8\ \text{in.}\right) = 0.05292\ \text{lb}\cdot\text{s}^2/\text{ft}$$

$$m_3 = \frac{0.284\ \text{lb/in}^3}{32.2\ \text{ft/s}^2}\left[\pi\left(0.6\ \text{in.}\right)^2 \times 0.5\ \text{in.}\right] = 0.0049875\ \text{lb}\cdot\text{s}^2/\text{ft}$$

Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}$, $\bar{I}_{y'z'}$, and $\bar{I}_{z'x'}$, of each component are zero because of symmetry. Now $I_{uv} = \cancel{\bar{I}_{u'v'}}^{\,0} + m\bar{u}\bar{v}$ so that $\left(I_{uv}\right)_{\text{body}} = \Sigma m\bar{u}\,\bar{v}$.

	m, $\text{lb}\cdot\text{s}^2/\text{ft}$	$\bar{x}$, ft	$\bar{y}$, ft	$\bar{z}$, ft	$m\bar{x}\,\bar{y}$ $\text{lb}\cdot\text{ft}\cdot\text{s}^2$	$m\bar{y}\,\bar{z}$ $\text{lb}\cdot\text{ft}\cdot\text{s}^2$	$m\bar{z}\,\bar{x}$ $\text{lb}\cdot\text{ft}\cdot\text{s}^2$
1	0.1786	2.5 0.20833	0.45 0.0375	2.25 0.1875	0.20093 $1.39531\cdot10^{-3}$	0.18083 $1.25578\cdot10^{-3}$	1.0046 $6.97656\cdot10^{-3}$
2	0.05292	4.6 0.38333	2.40 0.20	2.25 0.1875	0.58424 $4.0572\cdot10^{-3}$	0.28577 $1.98451\cdot10^{-3}$	0.54772 $3.80362\cdot10^{-3}$
3	0.0049875	5.25 0.4375	2.70 0.225	2.25 0.1875	0.07069 $0.49095\cdot10^{-3}$	0.03030 $0.21041\cdot10^{-3}$	0.05891 $0.40913\cdot10^{-3}$
Σ					0.85586 $5.94347\cdot10^{-3}$	0.4969 $3.45069\cdot10^{-3}$	1.61123 $11.18909\cdot10^{-3}$

PROBLEM 9.152 CONTINUED

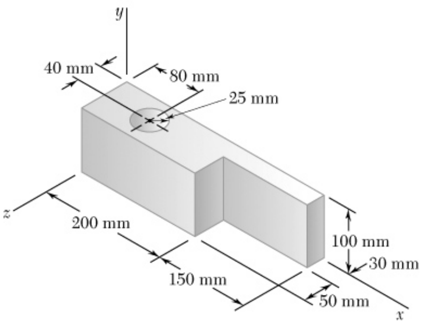
Then

or $I_{xy} = 5.94 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

or $I_{yz} = 3.45 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

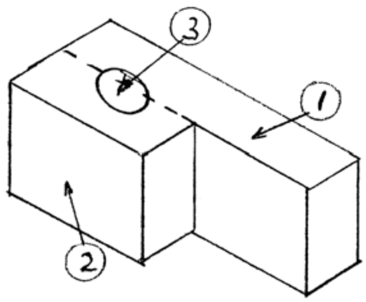
or $I_{zx} = 11.19 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

PROBLEM 9.153



Determine the mass products of inertia I_{xy}, I_{yz}, and I_{zx} of the cast aluminum machine component shown. (The density of aluminum is 2700 kg/m^3.)

SOLUTION



Have $$m = \rho_{\text{al}} V$$

Then $$m_1 = \left(2700\,\frac{\text{kg}}{\text{m}^3}\right)(0.350 \times 0.100 \times 0.030)\,\text{m}^3$$

$$= 2.8350\,\text{kg}$$

$$m_2 = \left(2700\,\frac{\text{kg}}{\text{m}^3}\right)(0.200 \times 0.100 \times 0.050)\,\text{m}^3$$

$$= 2.7000\,\text{kg}$$

$$m_3 = \left(2700\,\frac{\text{kg}}{\text{m}^3}\right)\left[\pi(0.025)^2 \times 0.100\right]\text{m}^3$$

$$= 0.53014\,\text{kg}$$

First note that symmetry implies $\bar{I}_{x'y'} = \bar{I}_{y'z'} = I_{z'x'} = 0$ for each component

Now $$I_{uv} = I_{u'v'} + m\bar{u}\,\bar{v}$$

where $$\bar{I}_{u'v'} = 0$$

$$I_{xy} = \Sigma m\bar{x}\,\bar{y} = (2.8350\,\text{kg})(0.175\,\text{m})(0.050\,\text{m})$$

$$+(2.7000\,\text{kg})(0.100\,\text{m})(0.050\,\text{m}) - (0.53014\,\text{kg})(0.080\,\text{m})(0.050\,\text{m})$$

$$= (24.806 + 13.500 - 2.1206) \times 10^{-3}\,\text{kg}\cdot\text{m}^2 = 36.1854 \times 10^3\,\text{kg}\,\text{m}^2$$

or $I_{xy} = 36.2 \times 10^{-3}\,\text{kg}\cdot\text{m}^2$ ◀

$$I_{yz} = \Sigma m\bar{y}\,\bar{z} = (2.8350\,\text{kg})(0.050\,\text{m})(0.015\,\text{m})$$

$$+(2.7000\,\text{kg})(0.050\,\text{m})(0.055\,\text{m}) - (0.53014\,\text{kg})(0.050\,\text{m})(0.040\,\text{m})$$

$$= (2.1263 + 7.4250 - 1.06028) \times 10^{-3}\,\text{kg}\cdot\text{m}^2 = 8.49102 \times 10^{-3}\,\text{kg}\cdot\text{m}^2$$

or $I_{yz} = 8.49 \times 10^{-3}\,\text{kg}\cdot\text{m}^2$ ◀

PROBLEM 9.153 CONTINUED

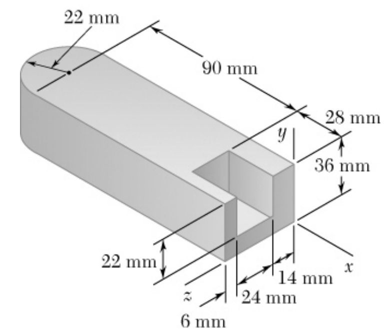
$$I_{zx} = \Sigma m\bar{z}\,\bar{x} = (2.8350\,\text{kg})(0.015\,\text{m})(0.175\,\text{m})$$

$$+(\,2.7000\,\text{kg})(0.055\,\text{m})(0.100\,\text{m}) - (0.53014\,\text{kg})(0.040\,\text{m})(0.080\,\text{m})$$

$$= (7.4419 + 14.850 - 1.69645)10^{-3}\,\text{kg}\cdot\text{m}^2 = 20.59545\times 10^{-3}\,\text{kg}\cdot\text{m}^2$$

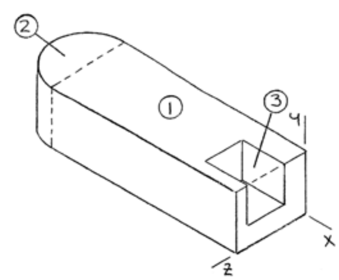
or $I_{zx} = 20.6\times 10^{-3}\,\text{kg}\cdot\text{m}^2$ ◀

PROBLEM 9.154



Determine the mass products of inertia I_{xy}, I_{yz}, and I_{zx} of the cast aluminum machine component shown. (The density of aluminum is 2700 kg/m^3.)

SOLUTION



Have $m = \rho V$

Then

$$m_1 = \left(2700 \text{ kg/m}^3\right)\left(0.118 \times 0.036 \times 0.044\right) \text{m}^3 = 0.50466 \text{ kg}$$

$$m_2 = \left(2700 \text{ kg/m}^3\right)\left[\frac{\pi}{2}(0.022)^2 \times 0.036\right] \text{m}^3 = 0.07389 \text{ kg}$$

$$m_3 = \left(2700 \text{ kg/m}^3\right)\left(0.028 \times 0.022 \times 0.024\right) \text{m}^3 = 0.03992 \text{ kg}$$

Now observe that $\bar{I}_{x'y'}, \bar{I}_{y'z'}$ and $\bar{I}_{z'x'}$ are zero because of symmetry

Now

$$\bar{x}_2 = -\left(0.118 + \frac{4 \times 0.023}{3\pi}\right) \text{m} = -0.12734 \text{ m}$$

$$\bar{y}_3 = \left(0.036 - \frac{0.062}{2}\right) \text{m} = 0.025 \text{ m}$$

	m, kg	$\bar{x}$, m	$\bar{y}$, m	$\bar{z}$, m	$m\bar{x}\,\bar{y}$ kg·m^2	$m\bar{y}\,\bar{z}$ kg·m^2	$m\bar{z}\,\bar{x}$ kg·m^2
1	0.50466	–0.059	0.018	0.022	-0.53595×10^{-3}	0.19985×10^{-3}	-0.65505×10^{-3}
2	0.07389	–0.12734	0.018	0.022	-0.16932×10^{-3}	0.02926×10^{-3}	-0.20695×10^{-3}
3	0.03992	–0.041	0.025	0.026	-0.01397×10^{-3}	0.02594×10^{-3}	-0.01453×10^{-3}

PROBLEM 9.154 CONTINUED

And

$$I_{xy} = \Sigma\left(\cancel{I_{x'y'}}^{0} + m\bar{x}\,\bar{y}\right)$$

$$I_{yz} = \Sigma\left(\cancel{I_{y'z'}}^{0} + m\bar{y}\,\bar{z}\right)$$

$$I_{zx} = \Sigma\left(\cancel{I_{z'x'}}^{0} + m\bar{x}\,\bar{z}\right)$$

Finally $$I_{xy} = \left(I_{xy}\right)_1 + \left(I_{xy}\right)_2 - \left(I_{xy}\right)_3 = -0.6913 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

or $I_{xy} = -0.691 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

$$I_{yz} = \left(I_{yz}\right)_1 + \left(I_{yz}\right)_2 - \left(I_{yz}\right)_3 = 0.20317 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

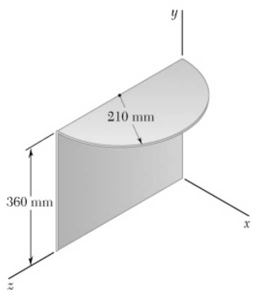
or $I_{yz} = 0.203 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

$$I_{zx} = \left(I_{zx}\right)_1 + \left(I_{zx}\right)_2 - \left(I_{zx}\right)_3 = -0.84747 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

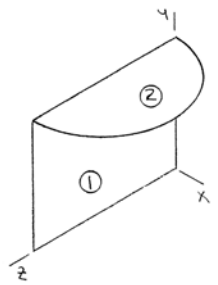
or $I_{zx} = -0.848 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

PROBLEM 9.155

A section of sheet steel 3 mm thick is cut and bent into the machine component shown. Knowing that the density of the steel is $7860\ \text{kg/m}^3$, determine the mass products of inertia I_{xy}, I_{yz}, and I_{zx} of the component.



SOLUTION



Have $$m = \rho v = \rho_{st} t A$$

Then $$m_1 = \left(7860\,\text{kg/m}^3\right)\left(0.003 \times 0.420 \times 0.360\right)\text{m}^3 = 3.5653\,\text{kg}$$

$$m_2 = \left(7860\,\text{kg/m}^3\right)\left(0.003\ \text{m}\right)\left[\frac{\pi}{2}\left(0.210\,\text{m}\right)^2\right] = 1.6334\,\text{kg}$$

$$\bar{x}_2 = \frac{4\left(0.210\,\text{m}\right)}{3\pi} = 0.089127\,\text{m}$$

Now observe that $$\bar{I}_{x'y'} = \bar{I}_{y'z'} = \bar{I}_{z'x'} = 0$$

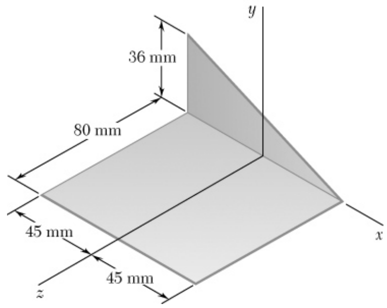
	m, kg	$\bar{x}$, m	$\bar{y}$, m	$\bar{z}$, m	$m\bar{x}\,\bar{y}$, $\text{kg}\cdot\text{m}^2$	$m\bar{y}\,\bar{z}$, $\text{kg}\cdot\text{m}^2$	$m\bar{z}\,\bar{x}$, $\text{kg}\cdot\text{m}^2$
1	3.5653	0	0.8	0.21	0	134.768×10^{-3}	0
2	1.6334	0.089127	0.36	0.21	52.409×10^{-3}	123.485×10^{-3}	30.572×10^{-3}
Σ					52.409×10^{-3}	258.253×10^{-3}	30.572×10^{-3}

Then $$I_{xy} = \Sigma\left(\cancel{\bar{I}_{x'y'}}^{\,0} + m\bar{x}\,\bar{y}\right)$$ or $I_{xy} = 52.4 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

$$I_{yz} = \Sigma\left(\cancel{\bar{I}_{y'z'}}^{\,0} + m\bar{y}\,\bar{z}\right)$$ or $I_{yz} = 258 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

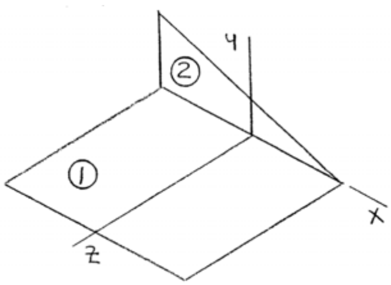
$$I_{zx} = \Sigma\left(\cancel{\bar{I}_{z'x'}}^{\,0} + m\bar{z}\,\bar{x}\right)$$ or $I_{zx} = 30.6 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀

PROBLEM 9.156



A section of sheet steel 3 mm thick is cut and bent into the machine component shown. Knowing that the density of the steel is $7860\ \text{kg/m}^3$, determine the mass products of inertia I_{xy}, I_{yz}, and I_{zx} of the component.

SOLUTION



First compute the mass of each component

Have $$m = \rho_{sT} V = \rho_{sT} tA$$

Then $$m_1 = \left(7860\ \text{kg/m}^3\right)\left[(0.003)(0.08)(0.09)\right]\text{m}^3$$

$$= 0.169776\ \text{kg}$$

$$m_2 = 7860\ \text{kg/m}^3\left[(0.003)\left(\frac{1}{2}\times 0.09\times 0.036\right)\right]\text{m}^3$$

$$= 0.03820\ \text{kg}$$

Now observe that

$$\left(\bar{I}_{x'y'}\right)_1 = \left(\bar{I}_{y'z'}\right)_1 = \left(\bar{I}_{z'x'}\right)_1 = 0$$

$$\left(\bar{I}_{y'z'}\right)_2 = \left(\bar{I}_{z'x'}\right)_2 = 0$$

From Sample Problem 9.6 $$\left(\bar{I}_{x'y'}\right)_{2,\text{area}} = -\frac{1}{72}b_2^2h_2^2$$

Then $$\left(\bar{I}_{x'y'}\right)_2 = \rho_{sT} t\left(I_{x'y'}\right)_{2,\text{area}} = \rho_{sT} t\left(-\frac{1}{72}b_2^2h_2^2\right) = -\frac{1}{36}m_2b_2h_2$$

Also $$\bar{x}_1 = \bar{y}_1 = \bar{z}_2 = 0 \qquad \bar{x}_2 = \left(-0.045 + \frac{0.09}{3}\right)\text{m} = -0.015\ \text{m}$$

PROBLEM 9.156 CONTINUED

Finally..

$$I_{xy} = \Sigma\left(\bar{I}_{xy} + m\bar{x}\,\bar{y}\right) = (0+0) + \left[-\frac{1}{36}(0.03820\,\text{kg})(0.09\,\text{m})(0.036\,\text{m})\right.$$

$$\left. + (0.03820\,\text{kg})(-0.015\,\text{m})\left(\frac{0.036\,\text{m}}{3}\right)\right]$$

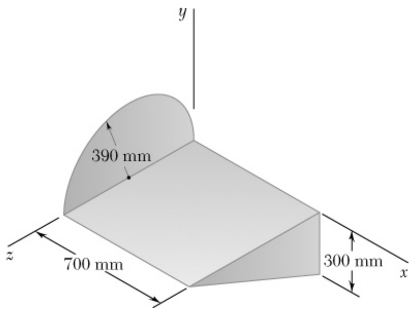
$$= \left(-3.4379\times10^{-6} - 6.876\times10^{-6}\right)\text{kg}\cdot\text{m}^2 = -10.3139\times10^{-6}\ \text{kg}\cdot\text{m}^2$$

or $I_{xy} = -10.31\times10^{-6}\ \text{kg}\cdot\text{m}^2$ ◀

And $I_{yz} = \Sigma\left(\bar{I}_{y'z'} + m\,\bar{y}\,\bar{z}\right) = (0+0) + (0+0) = 0$ or $I_{yz} = 0$ ◀

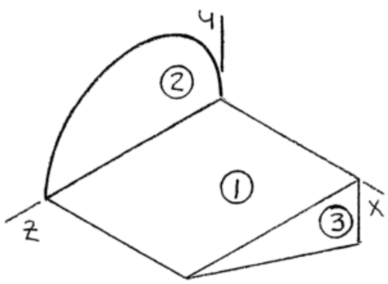
$I_{zx} = \Sigma\left(\bar{I}_{zx'} + m\,\bar{z}\,\bar{x}\right) = (0+0) + (0+0) = 0$ or $I_{zx} = 0$ ◀

PROBLEM 9.157



A section of sheet steel 3 mm thick is cut and bent into the machine component shown. Knowing that the density of the steel is $7860\ \text{kg/m}^3$, determine the mass products of inertia I_{xy}, I_{yz}, and I_{zx} of the component.

SOLUTION



First compute the mass of each component

Have..

$$m = \rho_{sT} V = \rho_{sT} tA$$

Then

$$m_1 = (7860\ \text{kg/m}^3)\left[(0.003)(0.7)(0.78)\right]\text{m}^3 = 12.785\ \text{kg}$$

$$m_2 = (7860\ \text{kg/m}^3)\left[(0.003)\left(\frac{\pi}{2}\times 0.39^2\right)\right]\text{m}^3 = 5.6337\ \text{kg}$$

$$m_3 = (7860\ \text{kg/m}^3)\left[(0.003)\left(\frac{1}{2}\times 0.78\times 0.3\right)\right]\text{m}^3 = 2.7589\ \text{kg}$$

Now observe that because of symmetry the centroidal products of inertia of components 1 and 2 are zero and $\left(\bar{I}_{x'y'}\right)_3 = \left(\bar{I}_{z'x'}\right)_3 = 0$

Also

$$\left(\bar{I}_{y'z'}\right)_{3,\text{mass}} = \rho_{sT} t \left(\bar{I}_{y'z'}\right)_{3,\text{area}}$$

Using the results of Sample Problem 9.6 and noting that the orientation of the axes corresponds to a $90°$ rotation, have

$$\left(\bar{I}_{y'z'}\right)_{3,\text{area}} = \frac{1}{72} b_3^2 h_3^2$$

Then

$$\left(\bar{I}_{y'z'}\right)_3 = \rho_{sT} t\left(\frac{1}{72} b_3^2 h_3^2\right) = \frac{1}{36} m_3 b_3 h_3$$

Also $\quad \bar{y}_1 = \bar{x}_2 = 0 \qquad \bar{y}_2 = \dfrac{4\times 0.39\ \text{m}}{3\pi} = 0.16552\ \text{m}$

Finally

$$I_{xy} = \Sigma\left(\bar{I}_{x'y'} + m\bar{x}\,\bar{y}\right) = (0+0) + (0+0)$$

$$+\left[0 + (2.7589\ \text{kg})(0.7\ \text{m})\left(\frac{0.3\,\text{m}}{2}\right)\right] = -0.19312\ \text{kg}\cdot\text{m}^2$$

or $\quad I_{xy} = -0.1931\ \text{kg}\cdot\text{m}^2$ ◀

PROBLEM 9.157 CONTINUED

$$I_{yz} = \Sigma\left(\bar{I}_{y'z'} + m\bar{y}\,\bar{z}\right)$$

$$= (0+0) + \left[0 + (5.6337\text{ kg})(0.16552\text{ m})(0.39\text{ m})\right]$$

$$+ (2.7589\text{ kg})\left[\frac{1}{36}(0.78\text{ m})(0.3\text{ m}) + \left(\frac{-0.3\text{ m}}{3}\right)\left(\frac{0.78\text{ m}}{3}\right)\right]$$

$$= (0.36367 + 0.017933 - 0.07173)\text{ kg}\cdot\text{m}^2 = 0.30987\text{ kg}\cdot\text{m}^2$$

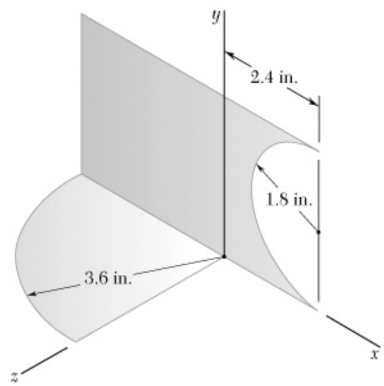
or $I_{yz} = 0.310\text{ kg}\cdot\text{m}^2$ ◀

$$I_{zx} = \Sigma\left(\bar{I}_{z'x'} + m\bar{z}\,\bar{x}\right) = \left[0 + (12.875\text{ kg})(0.35\text{ m})(0.39\text{ m})\right]$$

$$+(0+0) + \left[0 + (2.7589\text{ kg})\left(\frac{0.78\text{ m}}{2}\right)(0.7\text{ m})\right]$$

$$= (1.75744 + 0.50212)\text{ kg}\cdot\text{m}^2 = 2.25956\text{ kg}\cdot\text{m}^2$$

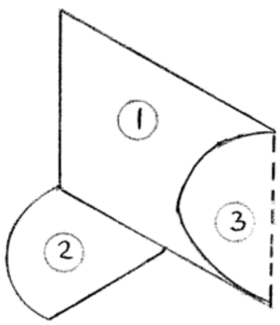
or $I_{zx} = 2.26\text{ kg}\cdot\text{m}^2$ ◀



PROBLEM 9.158

A section of sheet steel 0.08 in. thick is cut and bent into the machine component shown. Knowing that the specific weight of steel is 490 lb/ft^3, determine the mass products of inertia I_{xy}, I_{yz}, and I_{zx} of the component.

SOLUTION



First note $$m = \rho_{sT} V = \frac{\gamma_{sT}}{g} tA$$

Then $$m_1 = \left(\frac{490\ \text{lb/ft}^3}{32.2\ \text{ft/s}^2}\right)(0.08\ \text{in.})\left[(6 \times 3.6)\text{in}^2\right]\left(\frac{1\,\text{ft}}{12\,\text{in.}}\right)^3$$

$$= 15.2174 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}$$

$$m_2 = \left(\frac{490\ \text{lb/ft}^3}{32.2\ \text{ft/s}^2}\right)(0.08\ \text{in.})\left[\frac{\pi}{2}(1.8\ \text{in.})^2\right]\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^3$$

$$= 3.5855 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}$$

$$m_3 = \left(\frac{490\ \text{lb/ft}^3}{32.2\ \text{ft/s}^2}\right)(0.08\ \text{in.})\left[\frac{\pi}{4}(3.6\ \text{in.})^2\right]\left(\frac{1\ \text{ft}}{12\ \text{in.}}\right)^3$$

$$= 7.1710 \times 10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}$$

Note that symmetry implies $$\left(\bar{I}_{x'y'}\right)_{1,2} = \left(\bar{I}_{y'z'}\right)_{1,2} = \left(\bar{I}_{z'x'}\right)_{1,2} = 0$$

$$\left(\bar{I}_{x'y'}\right)_3 = \left(\bar{I}_{y'z'}\right)_3 = 0$$

Now $$I_{uv} = \bar{I}_{u'v'} + m\bar{u}\,\bar{v}$$

PROBLEM 9.158 CONTINUED

Thus $$I_{xy} = \Sigma m\bar{x}\,\bar{y}$$

$$= m_1\bar{x}_1\bar{y}_1 - m_2\bar{x}_2\bar{y}_2 + \cancelto{0}{m_3x_3\bar{y}_3}$$

$$= \left(15.2174\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left(-\frac{0.6}{12}\ \text{ft}\right)\left(\frac{1.8}{12}\ \text{ft}\right)$$

$$-\left(3.5855\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left[\left(2.4-\frac{4\times1.8}{3\pi}\right)\text{in.}\times\frac{1\ \text{ft}}{12\ \text{in.}}\right]\left(\frac{1.8}{12}\ \text{ft}\right)$$

$$= \left(-114.131 - 73.326\right)\times10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$ or $I_{xy} = -187.5\times10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

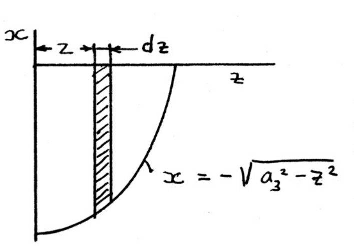
Now $$I_{yz} = \Sigma m\bar{y}\,\bar{z} = \cancelto{0}{m_1\bar{y}_1\bar{z}_1} - \cancelto{0}{m_2\bar{y}_2\bar{z}_2} + \cancelto{0}{m_3\bar{y}_3\bar{z}_3}$$ or $I_{yz} = 0$ ◀

Also $$I_{zx} = \left(I_{zx}\right)_1 - \left(I_{zx}\right)_2 + \left(I_{zx}\right)_3$$

$$= \cancelto{0}{m_1\bar{z}_1\bar{x}_1} - \cancelto{0}{m_2\bar{z}_2\bar{x}_2} + \left(I_{zx}\right)_3$$

Now determine $\left(I_{zx}\right)_3$

Have $$\left(dI_{zx}\right)_3 = \left(\cancelto{0}{d\bar{I}_{z'x'}}\right)_3 + \bar{z}\,\bar{x}\,dm$$



$$= (z)\left(-\frac{x}{2}\right)\left(\frac{\gamma_{sT}}{g}t\left|x\right|dz\right)$$

$$= -\frac{1}{2}\frac{\gamma_{sT}}{g}tz\left(a_3^2 - z^2\right)dz$$

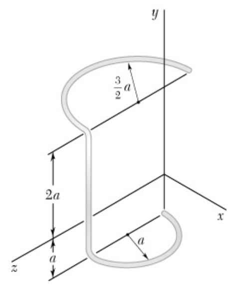
Now $$m_3 = \frac{\gamma_{sT}}{g}t\left(\frac{\pi}{4}a_3^2\right)$$

or $$\frac{\gamma_{sT}}{g}t = \frac{4m_3}{\pi a_3^2}$$

Therefore, $$\left(I_{zx}\right)_3 = \frac{2m_3}{\pi a_3^2}\int_0^a\left(a_3^2z - z^3\right)dz = -\frac{2m_3}{\pi a_3^2}\left(\frac{1}{2}a_3^2z^2 - \frac{1}{4}z^4\right)\Bigg|_0^a$$

$$= -\frac{1}{2\pi}m_3a_3^2$$

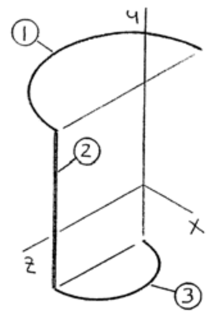
Finally $$I_{zx} = -\frac{1}{2\pi}\left(7.1710\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}\right)\left(\frac{3.6}{12}\text{ft}\right)^2$$ or $I_{zx} = -102.7\times10^{-6}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀



PROBLEM 9.159

Brass wire with a weight per unit length w is used to form the figure shown. Determine the products of inertia I_{xy}, I_{yz}, and I_{zx} of the wire figure.

SOLUTION



First compute the mass of each component. Have

$$m = \frac{W}{g} = \frac{1}{g} wL$$

Then..

$$m_1 = \frac{w}{g}\left(\pi \times \frac{3}{2}a\right) = \frac{3}{2}\pi\frac{w}{g}a$$

$$m_2 = \frac{w}{g}(3a) = 3\frac{w}{g}a$$

$$m_3 = \frac{w}{g}(\pi \times a) = \pi\frac{w}{g}a$$

Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}, \bar{I}_{y'z'}$, and $\bar{I}_{z'x'}$, of each component are zero because of symmetry.

	m	$\bar{x}$	$\bar{y}$	$\bar{z}$	$m\bar{x}\,\bar{y}$	$m\bar{y}\,\bar{z}$	$m\bar{z}\,\bar{x}$
1	$\frac{3}{2}\pi\frac{w}{g}a$	$-\frac{2}{\pi}\left(\frac{3}{2}a\right)$	$2a$	$\frac{1}{2}a$	$-9\frac{w}{g}a^3$	$\frac{3}{2}\pi\frac{w}{g}a^3$	$-\frac{9}{4}\frac{w}{g}a^3$
2	$3\frac{w}{g}a$	0	$\frac{1}{2}a$	$2a$	0	$3\frac{w}{g}a^3$	0
3	$\pi\frac{w}{g}a$	$\frac{2}{\pi}(a)$	$-a$	a	$-2\frac{w}{g}a^3$	$-\pi\frac{w}{g}a^3$	$2\frac{w}{g}a^3$
Σ					$-11\frac{w}{g}a^3$	$\frac{w}{g}\left(\frac{\pi}{2}+3\right)a^3$	$-\frac{1}{4}\frac{w}{g}a^3$

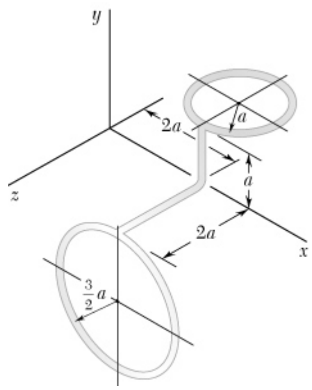
PROBLEM 9.159 CONTINUED

Then

$$I_{xy} = \Sigma\left(\cancelto{0}{\bar{I}_{x'y'}} + m\bar{x}\,\bar{y}\right) \qquad \text{or } I_{xy} = -11\frac{w}{g}a^3 \blacktriangleleft$$

$$I_{yz} = \Sigma\left(\cancelto{0}{\bar{I}_{y'z'}} + m\bar{y}\,\bar{z}\right) \qquad \text{or } I_{yz} = \frac{1}{2}\frac{w}{g}a^3(\pi + b) \blacktriangleleft$$

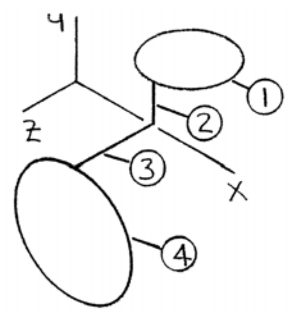
$$I_{zx} = \Sigma\left(\cancelto{0}{\bar{I}_{z'x'}} + m\bar{z}\,\bar{x}\right) \qquad \text{or } I_{zx} = -\frac{1}{4}\frac{w}{g}a^3 \blacktriangleleft$$



PROBLEM 9.160

Brass wire with a weight per unit length w is used to form the figure shown. Determine the products of inertia I_{xy}, I_{yz}, and I_{zx} of the wire figure.

SOLUTION



First compute the mass of each component. Have

$$m = \frac{W}{g} = \frac{1}{g} wL$$

Then

$$m_1 = \frac{w}{g}(2\pi \times a) = 2\pi \frac{w}{g} a$$

$$m_2 = \frac{w}{g}(a) = \frac{w}{g} a$$

$$m_3 = \frac{w}{g}(2a) = 2\frac{w}{g} a$$

$$m_4 = \frac{w}{g}\left(2\pi \times \frac{3}{2} a\right) = 3\pi \frac{w}{g} a$$

Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}, \bar{I}_{y'z'},$ and $\bar{I}_{z'x'},$ of each component are zero because of symmetry.

PROBLEM 9.160 CONTINUED

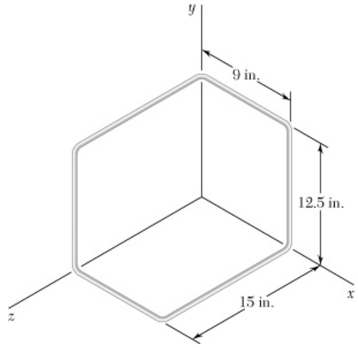
	m	$\bar{x}$	$\bar{y}$	$\bar{z}$	$m\bar{x}\,\bar{y}$	$m\bar{y}\,\bar{z}$	$m\bar{z}\,\bar{x}$
1	$2\pi\frac{w}{g}a$	$2a$	a	$-a$	$4\pi\frac{w}{g}a^3$	$-2\pi\frac{w}{g}a^3$	$-4\pi\frac{w}{g}a^3$
2	$\frac{w}{g}a$	$2a$	$\frac{1}{2}a$	0	$\frac{w}{g}a^3$	0	0
3	$2\frac{w}{g}a$	$2a$	0	a	0	0	$4\frac{w}{g}a^3$
4	$3\pi\frac{w}{g}a$	$2a$	$-\frac{3}{2}a$	$2a$	$-9\pi\frac{w}{g}a^3$	$-9\pi\frac{w}{g}a^3$	$12\pi\frac{w}{g}a^3$
Σ					$\frac{w}{g}(1-5\pi)a^3$	$-11\pi\frac{w}{g}a^3$	$4\frac{w}{g}(1+2\pi)a^3$

Then $I_{xy} = \Sigma(\cancelto{0}{\bar{I}_{x'y'}} + m\bar{x}\,\bar{y})$ or $I_{xy} = \frac{w}{g}a^3(1-5\pi)$ ◀

$I_{yz} = \Sigma(\cancelto{0}{\bar{I}_{y'z'}} + m\bar{y}\,\bar{z})$ or $I_{yz} = -11\pi\frac{w}{g}a^3$ ◀

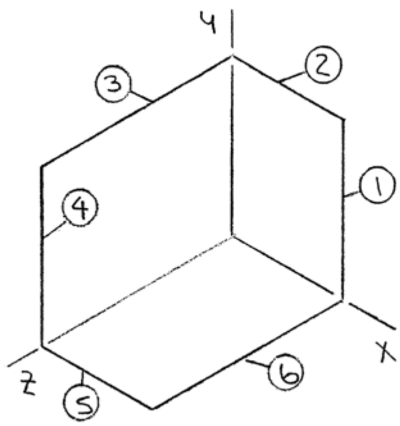
$I_{zx} = \Sigma(\cancelto{0}{\bar{I}_{z'x'}} + m\bar{z}\,\bar{x})$ or $I_{zx} = 4\frac{w}{g}a^3(1+2\pi)$ ◀

PROBLEM 9.161



The figure shown is formed of 0.075-in.-diameter aluminum wire. Knowing that the specific weight of aluminum is $0.10\ \text{lb/in}^3$, determine the products of inertia I_{xy}, I_{yz}, and I_{zx} of the wire figure.

SOLUTION



First note $$m = \rho V = \frac{\gamma}{g}V = \frac{\gamma}{g}AL$$

Specific weight of aluminium $= 0.10\ \text{lb/in}^3 = 172.8\ \text{lb/ft}^3$

Then $$m = \frac{172.8\ \text{lb/ft}^3}{32.2\ \text{ft/s}^2}\left[\frac{\pi}{4}\frac{(0.075\ \text{ft})^2}{12}\right]L$$

$$= \left(0.16464\times10^{-3}\right)L\ \text{lb}\cdot\text{s}^2/\text{ft}^2$$

Now $$L_1 = L_4 = 12.5\ \text{in.} = 1.04167\ \text{ft}$$

$$m_1 = m_4 = 0.17150\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}$$

$$L_2 = L_5 = 9\ \text{in.} = 0.75\ \text{ft}$$

$$m_2 = m_5 = 0.12348\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}$$

$$L_3 = L_6 = 15\ \text{in.} = 1.25\ \text{ft}$$

$$m_3 = m_6 = 0.20580\times10^{-3}\ \text{lb}\cdot\text{s}^2/\text{ft}$$

and $$\bar{I}_{x'y'} = \bar{I}_{y'z'} = \bar{I}_{z'x'} = 0$$

PROBLEM 9.161 CONTINUED

	m, lb·s²/ft	$\bar{x}$, ft	$\bar{y}$, ft	$\bar{z}$, ft	$m\bar{x}\,\bar{y}$, lb·ft·s²	$m\bar{y}\,\bar{z}$, lb·ft·s²	$m\bar{z}\,\bar{x}$, lb·ft·s²
1	0.17150×10^{-3}	0.75	0.5208	0	0.06699×10^{-3}	0	
2	0.12348×10^{-3}	0.375	1.04167	0	0.04823×10^{-3}	0	
3	0.20580×10^{-3}	0	1.04167	0.625	0	0.13398×10^{-3}	
4	0.17150×10^{-3}	0	0.5208	1.25	0	0.111646×10^{-3}	
5	0.12348×10^{-3}	0.375	0	1.25	0	0	0.05788×10^{-3}
6	0.20580×10^{-3}	0.75	0	0.625	0	0	0.09647×10^{-3}
Σ					0.11522×10^{-3}	0.24563×10^{-3}	0.15435×10^{-3}

$$I_{xy} = \Sigma\left(\cancelto{0}{I_{x'y'}} + m\bar{x}\,\bar{y}\right) = 0.115222 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

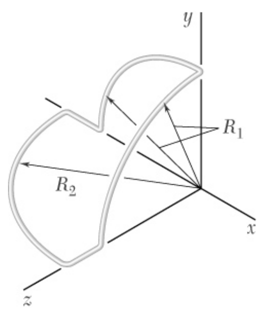
or $I_{xy} = 0.1152 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

$$I_{yz} = \Sigma\left(\cancelto{0}{I_{y'z'}} + m\bar{y}\,\bar{z}\right) = 0.24563 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_{yz} = 0.246 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

$$I_{zx} = \Sigma\left(\cancelto{0}{I_{z'x'}} + m\bar{z}\,\bar{x}\right) = 0.15435 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

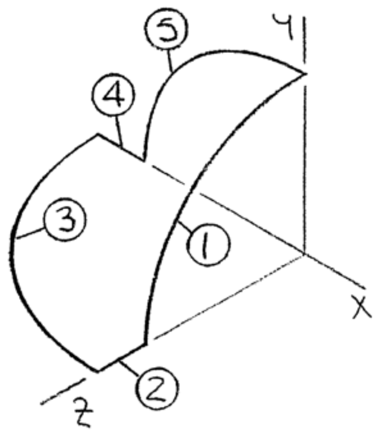
or $I_{zx} = 0.1543 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀



PROBLEM 9.162

Thin aluminum wire of uniform diameter is used to form the figure shown. Denoting by m' the mass per unit length of wire, determine the products of inertia I_{xy}, I_{yz}, and I_{zx} of the wire figure.

SOLUTION



First compute the mass of each component. Have

$$m = \left(\frac{m}{L}\right)L = m'L$$

Then $$m_1 = m_5 = m'\left(\frac{\pi}{2}R_1\right) = \frac{\pi}{2}m'R_1$$

$$m_2 = m_4 = m'(R_2 - R_1)$$

$$m_3 = m'\left(\frac{\pi}{2}R_2\right) = \frac{\pi}{2}m'R_2$$

Now observe that because of symmetry the centroidal products of inertia, $\bar{I}_{x'y'}$, $\bar{I}_{y'z'}$, and $\bar{I}_{z'x'}$, of components 2 and 4 are zero and

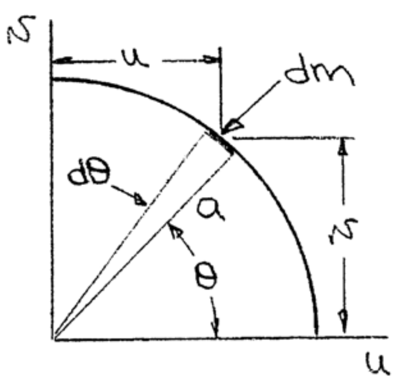
$$\left(\bar{I}_{x'y'}\right)_1 = \left(\bar{I}_{z'x'}\right)_1 = 0 \qquad \left(\bar{I}_{x'y'}\right)_3 = \left(\bar{I}_{y'z'}\right)_3 = 0$$

$$\left(\bar{I}_{y'z'}\right)_5 = \left(\bar{I}_{z'x'}\right)_5 = 0$$

Also $\quad \bar{x}_1 = \bar{x}_2 = 0 \qquad \bar{y}_2 = \bar{y}_3 = \bar{y}_4 = 0 \qquad \bar{z}_4 = \bar{z}_5 = 0$

Using the parallel-axis theorem [Equations (9.47)], it follows that

$I_{xy} = I_{yz} = I_{zx}$ for components 2 and 4.



To determine I_{uv} for one quarter of a circular arc have $\qquad dI_{uv} = uvdm$

where $$u = a\cos\theta \qquad v = a\sin\theta$$

and $$dm = \rho dV = \rho\left[A(ad\theta)\right]$$

where A is the cross-sectional area of the wire. Now

$$m = m'\left(\frac{\pi}{2}a\right) = \rho A\left(\frac{\pi}{2}a\right)$$

so that $$dm = m'ad\theta$$

and $$dI_{uv} = (a\cos\theta)(a\sin\theta)(m'ad\theta)$$

$$= m'a^3\sin\theta\cos\theta d\theta$$

PROBLEM 9.162 CONTINUED

Then $$I_{uv} = \int dI_{uv} = \int_0^{\frac{\pi}{2}} m'a^3 \sin\theta \cos\theta d\theta$$

$$= m'a^3 \left[\frac{1}{2}\sin^2\theta\right]_0^{\frac{\pi}{2}} = \frac{1}{2}m'a^3$$

Thus, $$\left(I_{yz}\right)_1 = \frac{1}{2}m'R_1^3$$

and $$\left(I_{zx}\right)_3 = -\frac{1}{2}m'R_2^3 \qquad \left(I_{xy}\right)_5 = -\frac{1}{2}m'R_1^3$$

Because of $90°$ rotations of the coordinate axes. Finally,

$$I_{xy} = \Sigma\left(I_{xy}\right) = \left[\cancelto{0}{\left(\bar{I}_{x'y'}\right)_1} + m_1\bar{x}_1\bar{y}_1\right] + \left[\cancelto{0}{\left(\bar{I}_{x'y'}\right)_3} + m_3\bar{x}_3\bar{y}_3\right] + \left(I_{xy}\right)_5$$

or $I_{xy} = -\frac{1}{2}m'R_1^3$ ◀

$$I_{yz} = \Sigma\left(I_{yz}\right) = \left(I_{yz}\right)_1 + \left[\cancelto{0}{\left(\bar{I}_{y'z'}\right)_3} + m_3\bar{y}_3\bar{z}_3\right] + \left[\cancelto{0}{\left(\bar{I}_{y'z'}\right)_5} + m_5\bar{y}_5\bar{z}_5\right]$$

or $I_{yz} = \frac{1}{2}m'R_1^3$ ◀

$$I_{zx} = \Sigma\left(I_{zx}\right) = \left[\cancelto{0}{\left(\bar{I}_{z'x'}\right)_1} + m_1\bar{z}_1\bar{x}_1\right] + \left(I_{zx}\right)_3 + \left[\cancelto{0}{\left(\bar{I}_{z'x'}\right)_5} + m_5\bar{z}_5\bar{x}_5\right]$$

or $I_{zx} = -\frac{1}{2}m'R_2^3$ ◀

PROBLEM 9.163

Complete the derivation of Eqs. (9.47), which express the parallel-axis theorem for mass products of inertia.

SOLUTION

Have $$I_{xy} = \int xydm \qquad I_{yz} = \int yzdm \qquad I_{zx} = \int zxdm \tag{9.45}$$

and $$x = x' + \overline{x} \qquad y = y' + \overline{y} \qquad z = z' + \overline{z} \tag{9.31}$$

Consider $$I_{xy} = \int xydm$$

Substituting for x and for y

$$I_{xy} = \int (x' + \overline{x})(y' + \overline{y})dm$$

$$= \int x'y'dm + \overline{y}\int x'dm + \overline{x}\int y'dm + \overline{x}\,\overline{y}\int dm$$

By definition $$\overline{I}_{x'y'} = \int x'y'dm$$

and $$\int x'dm = m\overline{x}\,'$$

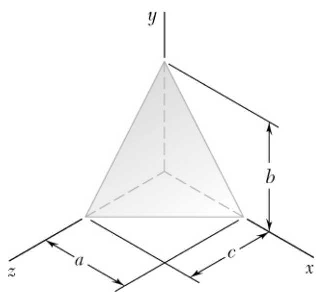
$$\int y'dm = m\overline{y}\,'$$

However, the origin of the primed coordinate system coincides with the mass center G, so that

$$\overline{x}\,' = \overline{y}\,' = 0$$

$$\therefore\ I_{xy} = \overline{I}_{x'y'} + m\overline{x}\,\overline{y} \quad \text{Q.E.D.} \blacktriangleleft$$

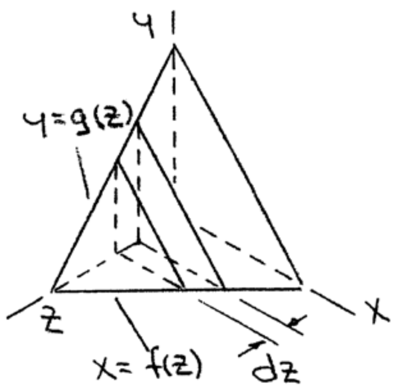
The expressions for I_{yz} and I_{zx} are obtained in a similar manner.



PROBLEM 9.164

For the homogeneous tetrahedron of mass m shown, (*a*) determine by direct integration the product of inertia I_{zx}, (*b*) deduce I_{yz} and I_{xy} from the results obtained in part *a*.

SOLUTION



(*a*) First divide the tetrahedron into a series of thin vertical slices of thickness dz as shown.

Now $$x = -\frac{a}{c}z + a = a\left(1 - \frac{z}{c}\right)$$

and $$y = -\frac{b}{c}z + b = b\left(1 - \frac{z}{c}\right)$$

The mass dm of the slab is

$$dm = \rho dV = \rho\left(\frac{1}{2}xydz\right) = \frac{1}{2}\rho ab\left(1 - \frac{z}{c}\right)^2 dz$$

Then $$m = \int dm = \int_0^c \frac{1}{2}\rho ab\left(1 - \frac{z}{c}\right)^2 dz = \frac{1}{2}\rho ab\left[\left(-\frac{c}{3}\right)\left(1 - \frac{z}{c}\right)^3\right]_0^c$$

$$= \frac{1}{6}\rho abc$$

Now $$dI_{zx} = d\bar{I}_{z'x'} + \bar{z}_{EL}\bar{x}_{EL}dm$$

where $$d\bar{I}_{z'x'} = 0 \qquad (\text{symmetry})$$

and $$\bar{z}_{EL} = z \qquad \bar{x}_{EL} = \frac{1}{3}x = \frac{1}{3}a\left(1 - \frac{z}{c}\right)$$

Then $$I_{zx} = \int dI_{zx} = \int_0^c z\left[\frac{1}{3}a\left(1 - \frac{z}{c}\right)\right]\left[\frac{1}{2}\rho ab\left(1 - \frac{z}{c}\right)^2 dz\right]$$

$$= \frac{1}{6}\rho a^2 b\int_0^c\left(z - 3\frac{z^2}{c} + 3\frac{z^3}{c^2} - \frac{z^4}{c^3}\right)dz$$

$$= \frac{m}{c}a\left[\frac{1}{2}z^2 - \frac{z^3}{c} + \frac{3}{4}\frac{z^4}{c^2} - \frac{1}{5}\frac{z^5}{c^3}\right]_0^c$$

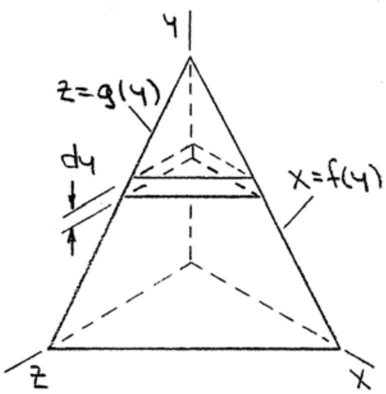
or $I_{zx} = \dfrac{1}{20}mac$ ◀

PROBLEM 9.164 CONTINUED

(*b*) Because of the symmetry of the body, I_{xy} and I_{yz} can be deduced by considering the circular permutation of (x, y, z) and (a, b, c). Thus

$$I_{xy} = \frac{1}{20}mab \blacktriangleleft$$

$$I_{yz} = \frac{1}{20}mbc \blacktriangleleft$$



Alternative solution for part *a*
First divide the tetrahedron into a series of thin horizontal slices of thickness dy as shown.

Now $\qquad x = -\frac{a}{b}y + a = a\left(1 - \frac{y}{b}\right)$

and $\qquad z = -\frac{c}{b}y + c = c\left(1 - \frac{y}{b}\right)$

The mass dm of the slab is

$$dm = \rho dV = \rho\left(\frac{1}{2}xzdy\right) = \frac{1}{2}\rho ac\left(1 - \frac{y}{b}\right)^2 dy$$

Now $\qquad dI_{zx} = \rho t dI_{zx,\text{ Area}}$

where $\qquad t = dy$

and $dI_{zx,\text{ Area}} = \frac{1}{24}x^2z^2$ from the results of Sample Problem 9.6

Then $\qquad dIzx = \rho(dy)\left\{\frac{1}{24}\left[a\left(1 - \frac{y}{b}\right)^2\right]\left[c\left(1 - \frac{y}{b}\right)\right]^2\right\}$

$$= \frac{1}{24}\rho a^2c^2\left(1 - \frac{y}{b}\right)^4 dy = \frac{1}{4}\frac{m}{b}ac\left(1 - \frac{y}{b}\right)^4 dy$$

Finally $\qquad I_{zx} = \int dI_{zx} = \int_0^b \frac{1}{4}\frac{m}{b}ac\left(1 - \frac{y}{b}\right)^4 dy$

$$= \frac{1}{4}\frac{m}{b}ac\left[\left(-\frac{b}{5}\right)\left(1 - \frac{y}{b}\right)^5\right]_0^b$$

or $\quad I_{zx} = \frac{1}{20}mac \blacktriangleleft$

PROBLEM 9.164 CONTINUED

Alternative solution for part a

The equation of the included face of the tetrahedron is

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

so that
$$y = b\left(1 - \frac{x}{a} - \frac{z}{c}\right)$$

For an infinitesimal element of sides $dx, dy,$ and dz

$$dm = \rho dV = \rho dydxdz$$

From part a
$$x = a\left(1 - \frac{z}{c}\right)$$

Now

$$I_{zx} = \int zxdm = \int_0^c \int_0^{a\left(1-\frac{z}{c}\right)} \int_0^{b\left(1-\frac{x}{a}-\frac{z}{c}\right)} zx\left(\rho dydxdz\right)$$

$$= \rho\int_0^c \int_0^{a\left(1-\frac{z}{c}\right)} zx\left[b\left(1 - \tfrac{x}{a} - \tfrac{z}{c}\right)\right]dxdz$$

$$= \rho b\int_0^c z\left[\frac{1}{2}x^2 - \frac{1}{3}\frac{x^3}{a} - \frac{1}{2}\frac{z}{c}x^2\right]_0^{a\left(1-\frac{z}{c}\right)} dz$$

$$= \rho b\int_0^c z\left[\frac{1}{2}a^2\left(1 - \frac{z}{c}\right)^2 - \frac{1}{3a}a^3\left(1 - \frac{z}{c}\right)^3 - \frac{1}{2}\frac{z}{c}a^2\left(1 - \frac{z}{c}\right)^2\right]dz$$

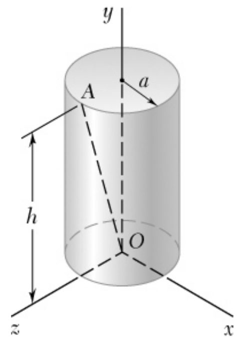
$$= \rho b\int_0^c \frac{1}{6}a^2 z\left(1 - \frac{z}{c}\right)^3 dz$$

$$= \frac{1}{6}\rho a^2 b\int_0^c\left(z - 3\frac{z^2}{c} + 3\frac{z^3}{c^2} - \frac{z^4}{c^3}\right)dz$$

$$= \frac{m}{c}a\left[\frac{1}{2}z^2 - \frac{z^3}{c} + \frac{3}{4}\frac{z^4}{c^2} - \frac{1}{5}\frac{z^5}{c^3}\right]_0^c$$

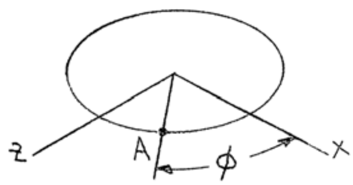
or $I_{zx} = \dfrac{1}{20}mac$ ◀

PROBLEM 9.165



The homogeneous circular cylinder shown has a mass *m*. Determine the moment of inertia of the cylinder with respect to the line joining the origin *O* and point *A* which is located on the perimeter of the top surface of the cylinder.

SOLUTION



From Figure 9.28 $\quad I_y = \frac{1}{2}ma^2$

and using the parallel-axis theorem

$$I_x = I_z = \frac{1}{12}m\left(3a^2 + h^2\right) + m\left(\frac{h}{2}\right)^2 = \frac{1}{12}m\left(3a^2 + 4h^2\right)$$

Symmetry implies $\quad I_{xy} = I_{yz} = I_{zx} = 0$

For convenience, let point *A* lie in the *yz* plane. Then

$$\lambda_{OA} = \frac{1}{\sqrt{h^2 + a^2}}\left(h\mathbf{j} + a\mathbf{k}\right)$$

With the mass products of inertia equal to zero, Equation (9.46) reduces to

$$I_{OA} = I_x\cancelto{0}{\lambda_x^2} + I_y\lambda_y^2 + I_z\lambda_z^2$$

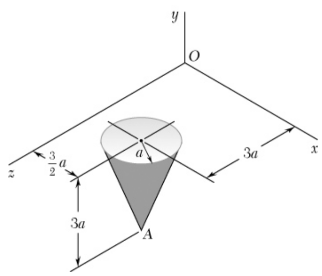
$$= \frac{1}{2}ma^2\left(\frac{h}{\sqrt{h^2 + a^2}}\right)^2 + \frac{1}{12}m\left(3a^2 + 4h^2\right)\left(\frac{a}{\sqrt{h^2 + a^2}}\right)^2$$

$$\text{or} \quad I_{OA} = \frac{1}{12}ma^2\,\frac{10h^2 + 3a^2}{h^2 + a^2} \blacktriangleleft$$

Note: For point *A* located at an arbitrary point on the perimeter of the top surface, λ_{OA} is given by

$$\lambda_{OA} = \frac{1}{\sqrt{h^2 + a^2}}\left(a\cos\phi\,\mathbf{i} + h\mathbf{j} + a\sin\phi\,\mathbf{k}\right)$$

which results in the same expression for I_{OA}



PROBLEM 9.166

The homogeneous circular cone shown has a mass m. Determine the moment of inertia of the cone with respect to the line joining the origin O and point A.

SOLUTION

First note that
$$d_{OA} = \sqrt{\left(\frac{3}{2}a\right)^2 + (-3a)^2 + (3a)^2} = \frac{9}{2}a$$

Then
$$\lambda_{OA} = \frac{1}{\frac{9}{2}a}\left(\frac{3}{2}a\mathbf{i} - 3a\mathbf{j} + 3a\mathbf{k}\right) = \frac{1}{3}(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$$

For a rectangular coordinate system with origin at point A and axes aligned with the given x, y, z axes, have (using Figure 9.28)

$$I_x = I_z = \frac{3}{5}m\left[\frac{1}{4}a^2 + (3a)^2\right] \qquad I_y = \frac{3}{10}ma^2$$
$$= \frac{111}{20}ma^2$$

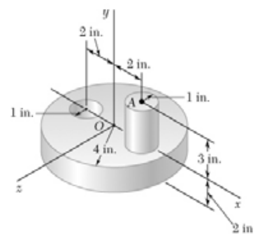
Also, symmetry implies
$$I_{xy} = I_{yz} = I_{zx} = 0$$

With the mass products of inertia equal to zero, Equation (9.46) reduces to

$$I_{OA} = I_x\lambda_x^2 + I_y\lambda_y^2 + I_z\lambda_z^2$$
$$= \frac{111}{20}ma^2\left(\frac{1}{3}\right)^2 + \frac{3}{10}ma^2\left(-\frac{2}{3}\right)^2 + \frac{111}{20}ma^2\left(\frac{2}{3}\right)^2$$
$$= \frac{193}{60}ma^2$$

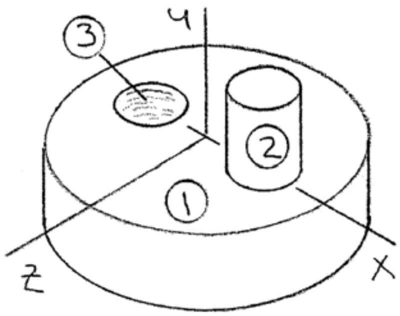
or $I_{OA} = 3.22ma^2$ ◀

PROBLEM 9.167



Shown is the machine element of Problem 9.143. Determine its moment of inertia with respect to the line joining the origin O and point A.

SOLUTION



First compute the mass of each component

Have $$m = \rho_{sT} V = \frac{0.284 \text{ lb/in}^3}{32.2 \text{ ft/s}^2} V = \left(0.008819 \text{ lb}\cdot\text{s}^2/\text{ft}\cdot\text{in}^3\right)V$$

Then

$$m_1 = 0.008819 \text{ lb}\cdot\text{s}^2/\text{ft}\cdot\text{in}^3 \left[\pi(4 \text{ in.})^2(2 \text{ in.})\right] = 0.88667 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m_2 = 0.008819 \text{ lb}\cdot\text{s}^2/\text{ft}\cdot\text{in}^3 \left[\pi(1 \text{ in.})^2(3 \text{ in.})\right] = 0.083125 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m_3 = 0.008819 \text{ lb}\cdot\text{s}^2/\text{ft}\cdot\text{in}^3 \left[\pi(1 \text{ in.})^2(2 \text{ in.})\right] = 0.055417 \text{ lb}\cdot\text{s}^2/\text{ft}$$

Symmetry implies $$I_{yz} = I_{zx} = 0 \quad \left(I_{xy}\right)_1 = 0$$

and $$\left(\bar{I}_{x'y'}\right)_2 = \left(\bar{I}_{x'y'}\right)_3 = 0$$

Now

$$I_{xy} = \Sigma\left(\bar{I}_{x'y'} + m\bar{x}\,\bar{y}\right) = m_2\bar{x}_2\bar{y}_2 - m_3\bar{x}_3\bar{y}_3$$

$$= \left(0.083125 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left[(2 \text{ in.})(1.5 \text{ in.})\right]\times\left(\frac{1 \text{ ft}^2}{144 \text{ in}^2}\right)$$

$$-\left(0.055417 \text{ lb}\cdot\text{s}^2/\text{ft}\right)\left[(-2 \text{ in.})(-1 \text{ in.})\right]\times\left(\frac{1 \text{ ft}^2}{144 \text{ in}^2}\right)$$

$$= 0.96209 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

From the solution to Problem 9.143:

$$I_x = 34.106 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_y = 50.125 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_z = 34.876 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

PROBLEM 9.167 CONTINUED

By observation $\boldsymbol{\lambda}_{OA} = \dfrac{1}{\sqrt{13}}(2\mathbf{i} + 3\mathbf{j})$

Then

$$I_{OA} = I_x\lambda_x^2 + I_y\lambda_y^2 + \cancelto{0}{I_z\lambda_z^2} - 2I_{xy}\lambda_x\lambda_y - \cancelto{0}{2I_{yz}\lambda_y\lambda_z} - \cancelto{0}{2I_{zx}\lambda_z\lambda_x}$$

$$= \left(34.106\times10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2\right)\left(\frac{2}{\sqrt{13}}\right)^2$$

$$+\left(50.125\times10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2\right)\left(\frac{3}{\sqrt{13}}\right)^2$$

$$-2\left(0.96209\times10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2\right)\left(\frac{2}{\sqrt{13}}\right)\left(\frac{3}{\sqrt{13}}\right)$$

$$= (10.4942 + 34.7019 - 0.8881)\times10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 44.308\times10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

or $I_{OA} = 44.3\times10^{-3}\text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

PROBLEM 9.168

Determine the moment of inertia of the steel machine element of Problems 9.147 and 9.151 with respect to the axis through the origin which forms equal angles with the *x*, *y*, and *z* axes.

SOLUTION

From Problem 9.147: $I_x = 9.8821 \times 10^{-3}\ \text{lb} \cdot \text{ft} \cdot \text{s}^2$

$I_y = 11.5344 \times 10^{-3}\ \text{lb} \cdot \text{ft} \cdot \text{s}^2$

$I_z = 2.1878 \times 10^{-3}\ \text{lb} \cdot \text{ft} \cdot \text{s}^2$

Problem 9.151: $I_{xy} = 0.48776 \times 10^{-3}\ \text{lb} \cdot \text{ft} \cdot \text{s}^2$

$I_{yz} = 1.18391 \times 10^{-3}\ \text{lb} \cdot \text{ft} \cdot \text{s}^2$

$I_{zx} = 2.6951 \times 10^{-3}\ \text{lb} \cdot \text{ft} \cdot \text{s}^2$

Now $\lambda_x = \lambda_y = \lambda_z$

and $\lambda_x^2 + \lambda_y^2 + \lambda_z^2 = 1$

Therefore, $3\lambda_x^2 = 1$

or $\lambda_x = \lambda_y = \lambda_z = \dfrac{1}{\sqrt{3}}$

Equation 9.46

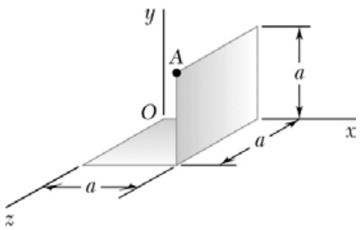
$$I_{OL} = I_x\lambda_x^2 + I_y\lambda_y^2 + I_z\lambda_z^2 - 2I_{xy}\lambda_x\lambda_y - 2I_{yz}\lambda_y\lambda_z - 2I_{zx}\lambda_z\lambda_x$$

$$= \left[9.8821\left(\frac{1}{\sqrt{3}}\right)^2 + 11.5344\left(\frac{1}{\sqrt{3}}\right)^2 + 2.1878\left(\frac{1}{\sqrt{3}}\right)^2 - 2(0.48776)\left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right)\right.$$

$$\left. - 2(1.18391)\left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right) - 2(2.6951)\left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right)\right] \times 10^{-3}\ \text{lb} \cdot \text{ft} \cdot \text{s}^2$$

$$= 4.95692 \times 10^{-3}\ \text{lb} \cdot \text{ft} \cdot \text{s}^2$$

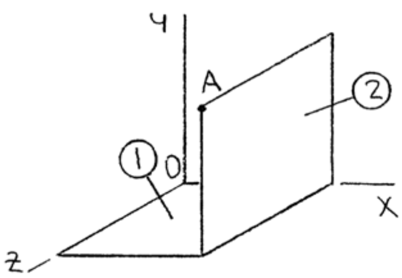
or $I_{OL} = 4.96 \times 10^{-3}\ \text{lb} \cdot \text{ft} \cdot \text{s}^2$ ◀



PROBLEM 9.169

The thin bent plate shown is of uniform density and weight W. Determine its mass moment of inertia with respect to the line joining the origin O and point A.

SOLUTION



First note that $$m_1 = m_2 = \frac{1}{2}\frac{W}{g}$$

And that $$\lambda_{OA} = \frac{1}{\sqrt{3}}(\mathbf{i} + \mathbf{j} + \mathbf{k})$$

Using Figure 9.28 and the parallel-axis theorem have

$$I_x = (I_x)_1 + (I_x)_2$$

$$= \left[\frac{1}{12}\left(\frac{1}{2}\frac{W}{g}\right)a^2 + \frac{1}{2}\frac{W}{g}\left(\frac{a}{2}\right)^2\right]$$

$$+\left\{\frac{1}{12}\left(\frac{1}{2}\frac{W}{g}\right)(a^2 + a^2) + \frac{1}{2}\frac{W}{g}\left[\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2\right]\right\}$$

$$= \frac{1}{2}\frac{W}{g}\left[\left(\frac{1}{12} + \frac{1}{4}\right)a^2 + \left(\frac{1}{6} + \frac{1}{2}\right)a^2\right] = \frac{1}{2}\frac{W}{g}a^2$$

$$I_y = (I_y)_1 + (I_y)_2$$

$$= \left\{\frac{1}{12}\left(\frac{1}{2}\frac{W}{g}\right)(a^2 + a^2) + \frac{1}{2}\frac{W}{g}\left[\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2\right]\right\}$$

$$+\left\{\frac{1}{12}\left(\frac{1}{2}\frac{W}{g}\right)a^2 + \frac{1}{2}\frac{W}{g}\left[(a)^2 + \left(\frac{a}{2}\right)^2\right]\right\}$$

$$= \frac{1}{2}\frac{W}{g}\left[\left(\frac{1}{6} + \frac{1}{2}\right)a^2 + \left(\frac{1}{12} + \frac{5}{4}\right)a^2\right] = \frac{W}{g}a^2$$

PROBLEM 9.169 CONTINUED

$$I_z = (I_z)_1 + (I_z)_2$$

$$= \left[\frac{1}{12}\left(\frac{1}{2}\frac{W}{g}\right)a^2 + \frac{1}{2}\frac{W}{g}\left(\frac{a}{2}\right)^2\right]$$

$$+\left\{\frac{1}{12}\left(\frac{1}{2}\frac{W}{g}\right)a^2 + \frac{1}{2}\frac{W}{g}\left[(a)^2 + \left(\frac{a}{2}\right)^2\right]\right\}$$

$$= \frac{1}{2}\frac{W}{g}\left[\left(\frac{1}{12} + \frac{1}{4}\right)a^2 + \left(\frac{1}{12} + \frac{5}{4}\right)a^2\right] = \frac{5}{6}\frac{W}{g}a^2$$

Now observe that the centroidal products of inertia, $\bar{I}_{x'y'}, \bar{I}_{y'z'},$ and $\bar{I}_{z'x'}$, of both components are zero because of symmetry. Also, $\bar{y}_1 = 0$

Then $$I_{xy} = \Sigma\left(\cancelto{0}{\bar{I}_{x'y'}} + m\bar{x}\,\bar{y}\right) = m_2\bar{x}_2\bar{y}_2 = \frac{1}{2}\frac{W}{g}(a)\left(\frac{a}{2}\right) = \frac{1}{4}\frac{W}{g}a^2$$

$$I_{yz} = \Sigma\left(\cancelto{0}{\bar{I}_{y'z'}} + m\bar{y}\,\bar{z}\right) = m_2\bar{y}_2\bar{z}_2 = \frac{1}{2}\frac{W}{g}\left(\frac{a}{2}\right)\left(\frac{a}{2}\right) = \frac{1}{8}\frac{W}{g}a^2$$

$$I_{zx} = \Sigma\left(\cancelto{0}{\bar{I}_{z'x'}} + m\bar{z}\,\bar{x}\right) = m_1\bar{z}_1\bar{x}_1 + m_2\bar{z}_2\bar{x}_2$$

$$= \frac{1}{2}\frac{W}{g}\left(\frac{a}{2}\right)\left(\frac{a}{2}\right) + \frac{1}{2}\frac{W}{g}\left(\frac{a}{2}\right)(a) = \frac{3}{8}\frac{W}{g}a^2$$

Substituting into Equation (9.46)

$$I_{OA} = I_x\lambda_x^2 + I_y\lambda_y^2 + I_z\lambda_z^2 - 2I_{xy}\lambda_x\lambda_y - 2I_{yz}\lambda_y\lambda_z - 2I_{zx}\lambda_z\lambda_x$$

Noting that

$$\lambda_x^2 = \lambda_y^2 = \lambda_z^2 = \lambda_x\lambda_y = \lambda_y\lambda_z = \lambda_z\lambda_x = \frac{1}{3}$$

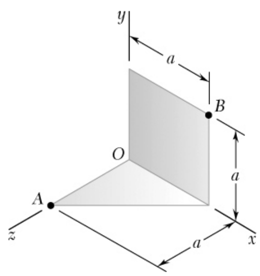
Have

$$I_{OA} = \frac{1}{3}\left[\frac{1}{2}\frac{W}{g}a^2 + \frac{W}{g}a^2 + \frac{5}{6}\frac{W}{g}a^2\right.$$

$$\left. - 2\left(\frac{1}{4}\frac{W}{g}a^2 + \frac{1}{8}\frac{W}{g}a^2 + \frac{3}{8}\frac{W}{g}a^2\right)\right]$$

$$= \frac{1}{3}\left[\frac{14}{6} - 2\left(\frac{3}{4}\right)\right]\frac{W}{g}a^2$$

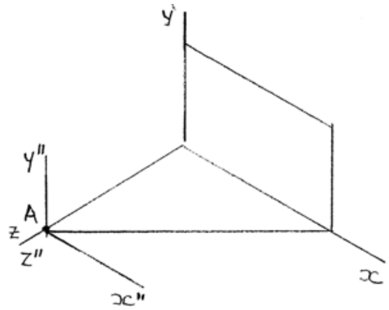
or $I_{OA} = \dfrac{5}{18}\dfrac{W}{g}a^2$ ◀



PROBLEM 9.170

A piece of sheet metal of thickness t and density ρ is cut and bent into the shape shown. Determine its mass moment of inertia with respect to a line joining points A and B.

SOLUTION



Have $$m = \rho V = \rho t A$$

Then $$m_1 = \rho t a^2 \qquad m_2 = \frac{1}{2}\rho t a^2$$

Compute moments and moments of inertia with respect to point A

Now

$$I_{x''} = \left(I_{x''}\right)_1 + \left(I_{x''}\right)_2$$

$$= \rho t a^2 \left\{\left(\frac{1}{2}a^2\right) + \left[\left(\frac{a}{2}\right)^2 + (a)^2\right]\right\} + \frac{1}{2}\rho t a^2 \left\{\left[\frac{1}{18}a^2\right] + \left[\left(\frac{2}{3}a\right)^2\right]\right\}$$

$$= \frac{19}{12}\rho t a^4$$

$$I_{y''} = \left(I_{y''}\right)_1 + \left(I_{y''}\right)_2$$

$$= \rho t a^2 \left\{\left[\frac{1}{12}a^2\right] + \left[\left(\frac{a}{2}\right)^2 + (a)^2\right]\right\}$$

$$+\frac{1}{2}\rho t a^2 \left\{\frac{1}{18}\left[a^2 + a^2\right] + \left[\left(\frac{b}{3}\right)^2 + \left(\frac{2^2}{3}\right)^2\right]\right\}$$

$$= \frac{5}{3}\rho t a^4$$

$$I_{z''} = \left(I_{z''}\right)_1 + \left(I_{z''}\right)_2$$

$$= \rho t a^2 \left[\frac{1}{3}\left(a^2 + a^2\right)\right] + \frac{1}{2}\rho t a^2 \left(\frac{1}{6}a^2\right)$$

$$= \frac{3}{4}\rho t a^4$$

Now note symmetry implies $$\left(\bar{I}_{x'y'}\right)_1 = \left(\bar{I}_{y'z'}\right)_1 = \left(\bar{I}_{z'x'}\right)_1 = 0$$

$$\left(\bar{I}_{x'y}\right)_2 = \left(\bar{I}_{y'z'}\right)_2 = 0$$

PROBLEM 9.170 CONTINUED

Now $$I_{uv} = I_{u'v'} + m\bar{u}\,\bar{v}$$

Therefore $$I_{x''y''} = m_1\bar{x}_1''\bar{y}_1'' + \cancelto{0}{m_2\bar{x}_2''\bar{y}_2''} = \rho t a^2\left(\frac{a}{2}\right)\left(\frac{a}{2}\right) = \frac{1}{4}\rho t a^4$$

$$I_{y''z''} = m_1\bar{y}_1''\,\bar{z}_1'' + \cancelto{0}{m_2\bar{y}_2''}\bar{z}_2'' = \rho t a^2\left(\frac{a}{2}\right)(-a) = -\frac{1}{2}\rho t a^4$$

$$I_{z''x''} = m_1\bar{z}_1''\,\bar{x}_1'' + \left[\left(\bar{I}_{z'x'}\right)_2 + m_2\bar{z}_2''\,\bar{x}_2''\right]$$

From Sample Problem 9.6 $$\left[\left(I_{z'x'}\right)_2\right]_{\text{area}} = -\frac{1}{72}a^4$$

Then $$\left(\bar{I}_{z''x''}\right)_2 = \rho t\left[\left(\bar{I}_{z'x'}\right)_2\right]_{\text{area}} = -\frac{1}{72}\rho t a^4$$

Then $$I_{z'x'} = \rho\frac{1}{2}a^2(-a)\left(\frac{a}{2}\right)$$

$$+\left[-\frac{1}{72}\rho\frac{1}{2}ta^4 + \frac{1}{2}\rho t a^2\left(-\frac{2}{3}a\right)\left(\frac{1}{3}a\right)\right]$$

$$= -\frac{5}{8}\rho t a^4$$

By observation $$\boldsymbol{\lambda}_{AB} = \frac{1}{\sqrt{3}}(\mathbf{i} + \mathbf{j} - \mathbf{k})$$

Now, Equation 9.46

$$I_{AB} = I_{x''}\lambda_x^2 + I_{y''}\lambda_y^2 + I_{z''}\lambda_z^2 - 2I_{x''y''}\lambda_{x''y''} - 2I_{y''z''}\lambda_{y''}\lambda_{z''} - 2I_{z''x''}\lambda_{z''}\lambda_{x''}$$

$$= \rho t a^4\left[\frac{19}{12}\left(\frac{1}{\sqrt{3}}\right)^2 + \frac{5}{3}\left(\frac{1}{\sqrt{3}}\right)^2 + \frac{3}{4}\left(-\frac{1}{\sqrt{3}}\right)^2\right.$$

$$-2\left(\frac{1}{4}\right)\left(\frac{1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right) - 2\left(-\frac{1}{2}\right)\left(\frac{1}{\sqrt{3}}\right)\left(-\frac{1}{\sqrt{3}}\right)$$

$$\left.-2\left(-\frac{5}{8}\right)\left(-\frac{1}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{3}}\right)\right]$$

or $$I_{AB} = \frac{5}{12}\rho t a^4 \blacktriangleleft$$

PROBLEM 9.171

Determine the mass moment of inertia of the machine component of Problems 9.138 and 9.157 with respect to the axis through the origin characterized by the unit vector $\boldsymbol{\lambda} = (-4\mathbf{i} + 8\mathbf{j} + \mathbf{k})/9$.

SOLUTION

From Problem 9.138:

$$I_x = 4.212 \text{ kg}\cdot\text{m}^2$$

$$I_y = 7.407 \text{ kg}\cdot\text{m}^2$$

$$I_z = 3.7055 \text{ kg}\cdot\text{m}^2$$

From Problem 9.157:

$$I_{xy} = -0.19312 \text{ kg}\cdot\text{m}^2$$

$$I_{yz} = 0.30987 \text{ kg}\cdot\text{m}^2$$

$$I_{zx} = 2.25956 \text{ kg}\cdot\text{m}^2$$

Now

$$\boldsymbol{\lambda}_{OL} = \frac{1}{9}(-4\mathbf{i} + 8\mathbf{j} + \mathbf{k})$$

Eq. (9.46):

$$I_{OL} = I_x\lambda_x^2 + I_y\lambda_y^2 + I_z\lambda_z^2 - 2I_{xy}\lambda_x\lambda_y - 2I_{yz}\lambda_y\lambda_z - 2I_{zx}\lambda_z\lambda_x$$

$$= \left[4.212\left(-\frac{4}{9}\right)^2 + 7.407\left(\frac{8}{9}\right)^2 + 3.7055\left(\frac{1}{9}\right)^2\right.$$

$$- 2(-0.19312)\left(-\frac{4}{9}\right)\left(\frac{8}{9}\right) - 2(0.3098)\left(\frac{8}{9}\right)\left(\frac{1}{9}\right)$$

$$\left.- 2(2.25956)\left(\frac{1}{9}\right)\left(-\frac{4}{9}\right)\right] \text{kg}\cdot\text{m}^2$$

$$= (0.832 + 5.85244 + 0.04575 - 0.15259 - 0.061195 + 0.22317) \text{ kg}\cdot\text{m}^2$$

$$= 6.73957 \text{ kg}\cdot\text{m}^2$$

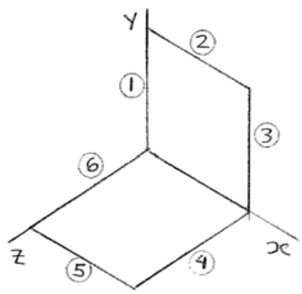
$$I_{OL} = 6.74 \text{ kg}\cdot\text{m}^2 \blacktriangleleft$$

PROBLEM 9.172

For the wire figure of the problem indicated, determine the mass moment of inertia of the figure with respect to the axis through the origin characterized by the unit vector $\boldsymbol{\lambda} = \left(-3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}\right)/7$.

Problem 9.150

SOLUTION



Mass of each leg is identical:

$$m = \left(\frac{W/L}{g}\right)L$$

$$= \frac{0.041\ \text{lb/ft}\left(1.5\ \text{ft}\right)}{32.2\ \text{ft/s}^2} = 0.00190994\ \text{lb}\cdot\text{s}^2\text{/ft}$$

Also, $\bar{I}_{x'y'} = \bar{I}_{y'z'} = \bar{I}_{z'x'} = 0$ for each leg,

and $\quad \bar{x}_1 = \bar{x}_6 = 0 \qquad \bar{y}_4 = \bar{y}_5 = \bar{y}_6 = 0 \qquad \bar{z}_1 = \bar{z}_2 = \bar{z}_3 = 0$

Now

$$I_{xy} = \Sigma\left(\cancelto{0}{\bar{I}_{x'y'}} + m\bar{x}\,\bar{y}\right) = m_2\bar{x}_2\bar{y}_2 + m_3\bar{x}_3\bar{y}_3$$

$$= \left(0.00190994\ \text{lb}\cdot\text{s}^2\text{/ft}\right)\left[\left(0.75\right)\left(1.5\right) + \left(1.5\right)\left(0.75\right)\right]\text{ft}^2$$

$$= 0.0042974\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 4.2974\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_{yz} = 0$$

$$I_{zx} = \Sigma\left(\bar{I}_{z'x'} + m\bar{z}\,\bar{x}\right) = m_4\bar{z}_4\,\bar{x}_4 + m_5\bar{z}_5\bar{x}_5$$

$$= \left(0.00190994\ \text{lb}\cdot\text{s}^2\text{/ft}\right)\left[\left(0.75\right)\left(1.5\right) + \left(1.5\right)\left(0.75\right)\right]\text{ft}^2$$

$$= 0.0042974\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 4.2974\times10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

PROBLEM 9.172 CONTINUED

From Problem 9.150 $\quad I_x = 14.32 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$

$$I_y = I_z = 18.62 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

Now $\quad \boldsymbol{\lambda}_{OL} = \dfrac{1}{7}\left(-3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}\right)$ and then

$$I_{OL} = I_x\lambda_x^2 + I_y\lambda_y^2 + I_z\lambda_z^2 - 2I_{xy}\lambda_x\lambda_y - 2\cancelto{0}{I_{yz}\lambda_y\lambda_z} - 2I_{zx}\lambda_z\lambda_x \qquad \left[\text{Eq. } (9.46)\right]$$

$$= \left\{\left[\left(14.32 \times 10^{-3}\right)\left(-\frac{3}{7}\right)^2 + \left(18.62 \times 10^{-3}\right)\left[\left(\frac{-6}{7}\right)^2 + \left(\frac{2}{7}\right)^2\right] - 2\left(4.2974 \times 10^{-3}\right)\left[\left(-\frac{3}{7}\right)\left(\frac{-6}{7}\right)\right]\right.\right.$$

$$\left.\left. - 2\left(4.2974 \times 10^{-3}\right)\left[\left(\frac{2}{7}\right)\left(\frac{-3}{7}\right)\right]\right]\right\}\text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= \left(2.6302 \times 10^{-3} + 15.20 \times 10^{-3} - 3.1573 \times 10^{-3} + 1.05242 \times 10^{-3}\right)\text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$= 15.725 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

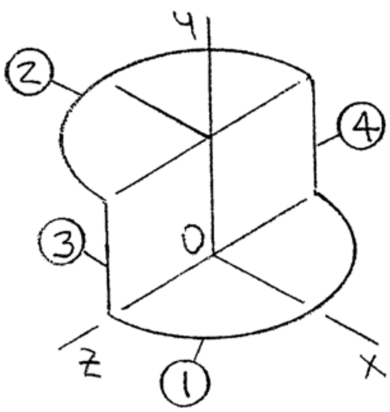
or $\quad I_{OL} = 15.73\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

PROBLEM 9.173

For the wire figure of the problem indicated, determine the mass moment of inertia of the figure with respect to the axis through the origin characterized by the unit vector $\boldsymbol{\lambda} = (-3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k})/7$.

Problem 9.149

SOLUTION



First compute the mass of each component

Have $$m = \rho_{st}V = mAL$$

$$= \left(7850\ \text{kg/m}^3\right)\left[\pi\left(0.0015\ \text{m}\right)^2\right]L$$

$$= \left(0.055488L\right)\ \text{kg/m}$$

Then $$m_1 = m_2 = 0.055488\ \text{kg/m}\left(\pi \times 0.36\ \text{m}\right)$$

$$= 0.062756\ \text{kg}$$

$$m_3 = m_4 = 0.055488\ \text{kg/m}\left(0.36\ \text{m}\right)$$

$$= 0.019976\ \text{kg}$$

Now observe that the centroidal products of inertia $\bar{I}_{x'y'} = \bar{I}_{y'z'}$ $= \bar{I}_{z'x'} = 0$ for each component.

Also $$\bar{x}_3 = \bar{x}_4 = 0, \qquad \bar{y}_1 = 0, \qquad \bar{z}_1 = \bar{z}_2 = 0$$

Then

$$I_{xy} = \Sigma\left(\cancelto{0}{I_{x'y'}} + m\bar{x}\,\bar{y}\right) = m_2\bar{x}_2\bar{y}_2$$

$$= \left(0.062756\ \text{kg}\right)\left(-\frac{2 \times 0.36\ \text{m}}{\pi}\right)\left(0.36\ \text{m}\right) = -5.1777 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

$$I_{yz} = \Sigma\left(\cancelto{0}{I_{y'z'}} + m\bar{y}\,\bar{z}\right) = m_3\bar{y}_3\bar{z}_3 + m_4\bar{y}_4\bar{z}_4$$

where $m_3 = m_4$, $\bar{y}_3 = \bar{y}_4$, $\bar{z}_4 = -\bar{z}_3$, so that $I_{yz} = 0$

$$I_{zx} = \Sigma\left(I_{z'x'} + m\bar{z}\,\bar{x}\right) = m_1\bar{z}_1\bar{x}_1 + m_2\bar{z}_2\bar{x}_2 = 0$$

PROBLEM 9.173 CONTINUED

From the solution to Problem 9.149

$$I_x = 23.170 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

$$I_y = 21.444 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

$$I_z = 17.992 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

Now $$\boldsymbol{\lambda}_{OL} = \frac{1}{7}(-3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k})$$

Have

$$I_{OL} = I_x\lambda_x^2 + I_y\lambda_y^2 + I_z\lambda_z^2 - 2I_{xy}\lambda_x\lambda_y - 2\cancelto{0}{I_{yz}}\lambda_y\lambda_z - 2\cancelto{0}{I_{zx}}\lambda_z\lambda_x \quad \left[\text{Eq. } (9.46)\right]$$

$$= \left[23.170\left(-\frac{3}{7}\right)^2 + 21.444\left(-\frac{6}{7}\right)^2 + 17.992\left(\frac{2}{7}\right)^2\right.$$

$$\left. - 2(-5.1777)\left(-\frac{3}{2}\right)\left(-\frac{6}{7}\right)\right] \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

$$= (4.2557 + 15.755 + 1.4687 + 3.8040) \times 10^{-3} \text{ kg}\cdot\text{m}$$

$$= 25.283 \times 10^{-3} \text{ kg}\cdot\text{m}^2$$

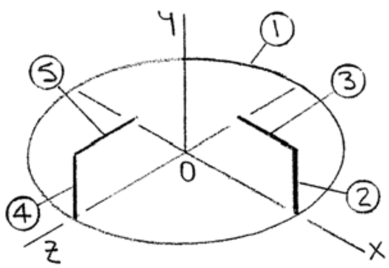
or $I_{OL} = 25.3 \times 10^{-3} \text{ kg}\cdot\text{m}^2$ ◀

PROBLEM 9.174

For the wire figure of the problem indicated, determine the mass moment of inertia of the figure with respect to the axis through the origin characterized by the unit vector $\boldsymbol{\lambda} = (-3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k})/7$.

Problem 9.148

SOLUTION



First compute the mass of each component.

Have $$m = (m/L)L$$

$$= (0.049\text{ kg/m})L$$

Then $$m_1 = (0.049\text{ kg/m})(2\pi \times 0.32\text{ m})$$

$$= 0.09852\text{ kg}$$

$$m_2 = m_3 = m_4 = m_5 = (0.049\text{ kg})(0.160\text{ m})$$

$$= 0.00784\text{ kg}$$

Now observe that $\bar{I}_{x'y'} = \bar{I}_{y'z'} = \bar{I}_{z'x'} = 0$ for each component.

Also, $\bar{x}_1 = \bar{x}_4 = \bar{x}_5 = 0, \quad \bar{y}_1 = 0, \quad \bar{z}_1 = \bar{z}_2 = \bar{z}_3 = 0$

Then

$$I_{xy} = \Sigma\left(\cancelto{0}{\bar{I}_{x'y'}} + m\bar{x}\,\bar{y}\right) = m_2\bar{x}_2\bar{y}_2 + m_3\bar{x}_3\bar{y}_3$$

$$= (0.00784\text{ kg})\left[(0.32\text{ m})(0.08\text{ m}) + (0.24\text{ m})(0.16\text{ m})\right]$$

$$= 0.50176 \times 10^{-3}\text{ kg}\cdot\text{m}^2$$

By symmetry $$I_{yz} = I_{xy} = 0.50176 \times 10^{-3}\text{ kg}\cdot\text{m}^2$$

Now $$I_{zx} = \Sigma\left(I_{z'x'} + m\bar{z}\,\bar{x}\right) = 0$$

From the solution to Problem 9.148

$$I_x = I_z = 6.8505 \times 10^{-3}\text{ kg}\cdot\text{m}^2$$

$$I_y = 12.630 \times 10^{-3}\text{ kg}\cdot\text{m}^2$$

PROBLEM 9.174 CONTINUED

Now $$\boldsymbol{\lambda}_{OL} = \frac{1}{7}\left(-3\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}\right)$$

Have

$$I_{OL} = I_x\lambda_x^2 + I_y\lambda_y^2 + I_z\lambda_z^2 - 2I_{xy}\lambda_x\lambda_y - 2I_{yz}\lambda_y\lambda_z - 2\cancelto{0}{I_{zx}}\lambda_z\lambda_x \quad \left[\text{Eq.}(9.46)\right]$$

$$= \left\{(6.8505)\left[\left(-\frac{3}{7}\right)^2 + \left(-\frac{6}{7}\right)^2\right]\right\} \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

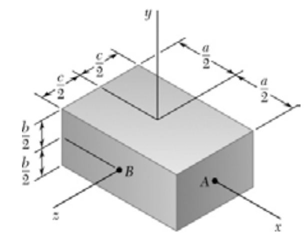
$$- \left\{(12.63)\left(\frac{2}{7}\right)^2\right] \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

$$-2\left(0.50176\right)\left[\left(-\frac{3}{7}\right)\left(-\frac{6}{7}\right) + \left(-\frac{6}{3}\right)\left(\frac{2}{7}\right)\right]\right\} \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

$$= \left(6.29128 + 1.03102 - 0.12288\right) \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

$$= 0.719942\ \text{kg}\cdot\text{m}^2$$

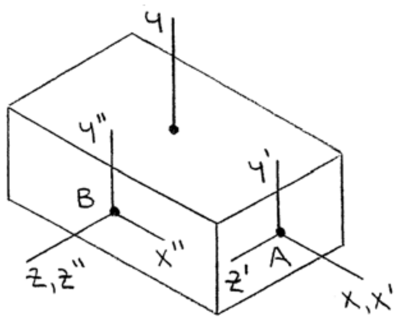
or $I_{OL} = 7.2 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$ ◀



PROBLEM 9.175

For the rectangular prism shown, determine the values of the ratios b/a and c/a so that the ellipsoid of inertia of the prism is a sphere when computed (*a*) at point *A*, (*b*) at point *B*.

SOLUTION



(*a*) Using Figure 9.28 and the parallel-axis theorem have at point *A*..

$$I_{x'} = \frac{1}{12}m\left(b^2 + c^2\right)$$

$$I_{y'} = \frac{1}{12}m\left(a^2 + c^2\right) + m\left(\frac{a}{2}\right)^2 = \frac{1}{12}m\left(4a^2 + c^2\right)$$

$$I_{z'} = \frac{1}{12}m\left(a^2 + b^2\right) + m\left(\frac{a}{2}\right)^2 = \frac{1}{12}m\left(4a^2 + b^2\right)$$

Now observe that symmetry implies

$$I_{x'y'} = I_{y'z'} = I_{z'x'} = 0$$

Using Equation (9.48), the equation of the ellipsoid of inertia is then

$$I_{x'}x^2 + I_{y'}y^2 + I_{z'}z^2 = 1$$

or $\frac{1}{12}m\left(b^2 + c^2\right)x^2 + \frac{1}{12}m\left(4a^2 + c^2\right)y^2 + \frac{1}{12}m\left(4a^2 + b^2\right)z^2 = 1$

For the ellipsoid to be a sphere, the coefficients must be equal. Therefore,

$$\frac{1}{12}m\left(b^2 + c^2\right) = \frac{1}{12}m\left(4a^2 + c^2\right) = \frac{1}{12}m\left(4a^2 + b^2\right)$$

Then $\qquad b^2 + c^2 = 4a^2 + c^2 \qquad$ or $\frac{b}{a} = 2$ ◀

and $\qquad b^2 + c^2 = 4a^2 + b^2 \qquad$ or $\frac{c}{a} = 2$ ◀

(*b*) Using Figure 9.28 and the parallel-axis theorem, we have at point *B*

$$I_{x''} = \frac{1}{12}m\left(b^2 + c^2\right) + m\left(\frac{c}{2}\right)^2 = \frac{1}{12}m\left(b^2 + 4c^2\right)$$

$$I_{y''} = \frac{1}{12}m\left(a^2 + c^2\right) + m\left(\frac{c}{2}\right)^2 = \frac{1}{12}m\left(a^2 + 4c^2\right)$$

$$I_{z''} = \frac{1}{12}m\left(a^2 + b^2\right)$$

PROBLEM 9.175 CONTINUED

Now observe that symmetry implies

$$I_{x''y''} = I_{y''z''} = I_{z''x''} = 0$$

From part *a* it then immediately follows that

$$\frac{1}{12}m\left(b^2 + 4c^2\right) = \frac{1}{12}m\left(a^2 + 4c^2\right) = \frac{1}{12}m\left(a^2 + b^2\right)$$

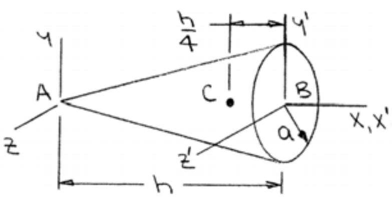
Then $\qquad b^2 + 4c^2 = a^2 + 4c^2 \qquad$ or $\quad \dfrac{b}{a} = 1 \blacktriangleleft$

and $\qquad b^2 + 4c^2 = a^2 + b^2 \qquad$ or $\quad \dfrac{c}{a} = \dfrac{1}{2} \blacktriangleleft$

PROBLEM 9.176

For the right circular cone of Sample Prob. 9.11, determine the value of the ratio a/h for which the ellipsoid of inertia of the cone is a sphere when computed (a) at the apex of the cone, (b) at the center of the base of the cone.

SOLUTION



(a) From sample Problem 9.11, we have at the apex A

$$I_x = \frac{3}{10} ma^2$$

$$I_y = I_z = \frac{3}{5} m\left(\frac{1}{4}a^2 + h^2\right)$$

Now observe that symmetry implies $I_{xy} = I_{yz} = I_{zx} = 0$

Using Equation (9.48), the equation of the ellipsoid of inertia is then

$$I_x x^2 + I_y y^2 + I_z z^2 = 1$$

or $$\frac{3}{10} ma^2 x^2 + \frac{3}{5} m\left(\frac{1}{4}a^2 + h^2\right) y^2 + \frac{3}{5} m\left(\frac{1}{4}a^2 + h^2\right) z^2 = 1$$

For the ellipsoid to be a sphere, the coefficients must be equal. Therefore,

$$\frac{3}{10} ma^2 = \frac{3}{5} m\left(\frac{1}{4}a^2 + h^2\right) \qquad \text{or} \quad \frac{a}{h} = 2 \blacktriangleleft$$

(b) From Sample Problem 9.11, we have

$$I_{x'} = \frac{3}{10} ma^2$$

and at the centroid C $$\bar{I}_{y''} = \frac{3}{20} m\left(a^2 + \frac{1}{4}h^2\right)$$

Then $$I_{y'} = I_{z'} = \frac{3}{20} m\left(a^2 + \frac{1}{4}h^2\right) + m\left(\frac{h}{4}\right)^2$$

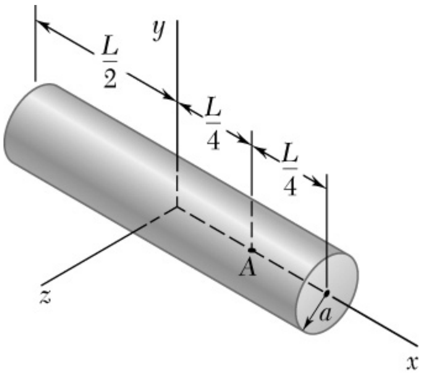
$$= \frac{1}{20} m\left(3a^2 + 2h^2\right)$$

Now observe that symmetry implies

$$I_{x'y'} = I_{y'z'} = I_{z'x'} = 0$$

From part a it then immediately follows that

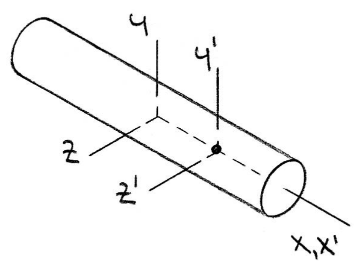
$$\frac{3}{10} ma^2 = \frac{1}{20} m\left(3a^2 + 2h^2\right) \qquad \text{or} \quad \frac{a}{h} = \sqrt{\frac{2}{3}} \blacktriangleleft$$



PROBLEM 9.177

For the homogeneous circular cylinder shown, of radius *a* and length *L*, determine the value of the ration *a*/*L* for which the ellipsoid of inertia of the cylinder is a sphere when computed (*a*) at the centroid of the cylinder, (*b*) at point *A*.

SOLUTION



(*a*) From Figure 9.28

$$\bar{I}_x = \frac{1}{2}ma^2 \qquad \bar{I}_y = \bar{I}_z = \frac{1}{12}m\left(3a^2 + L^2\right)$$

Now observe that symmetry implies

$$I_{xy} = I_{yz} = I_{zx} = 0$$

Using Equation (9.48), the equation of the ellipsoid of inertia is then

$$I_x x^2 + I_y y^2 + I_z z^2 = 1: \ \frac{1}{2}ma^2x^2 + \frac{1}{12}m\left(3a^2 + L^2\right)y^2 + \frac{1}{12}m\left(3a^2 + L^2\right) = 1$$

For the ellipsoid to be a sphere, the coefficients must be equal. Therefore,

$$\frac{1}{2}ma^2 = \frac{1}{12}m\left(3a^2 + L^2\right) \qquad \text{or } \frac{a}{L} = \frac{1}{\sqrt{3}} \blacktriangleleft$$

(*b*) Using Fig. 9.28 and the parallel-axis theorem

Have $I_{x'} = \frac{1}{2}ma^2$

$$I_{y'} = I_{z'} = \frac{1}{12}m\left(3a^2 + L^2\right) + m\left(\frac{L}{4}\right)^2 = m\left(\frac{1}{4}a^2 + \frac{7}{48}L^2\right)$$

Now observe that symmetry implies

$$I_{x'y'} = I_{y'z'} = I_{z'x'} = 0$$

From Part a it then immediately follows that

$$\frac{1}{2}ma^2 = m\left(\frac{1}{4}a^2 + \frac{7}{48}L^2\right) \qquad \text{or } \frac{a}{L} = \sqrt{\frac{7}{12}} \blacktriangleleft$$

PROBLEM 9.178

Given an arbitrary body and three rectangular axes *x*, *y*, and *z*, prove that the moment of inertia of the body with respect to any one of the three axes cannot be larger than the sum of the moments of inertia of the body with respect to the other two axes. That is, prove that the inequality $I_x \leq I_y + I_z$ and the two similar inequalities are satisfied. Further, prove that $I_y \geq \frac{1}{2} I_x$ if the body is a homogeneous solid of revolution, where *x* is the axis of revolution and *y* is a transverse axis.

SOLUTION

(i) To prove $$I_y + I_z \geq I_x$$

By definition $$I_y = \int \left(z^2 + x^2\right) dm \qquad I_z = \int \left(x^2 + y^2\right) dm$$

Then $$I_y + I_z = \int \left(z^2 + x^2\right) dm + \int \left(x^2 + y^2\right) dm$$

$$= \int \left(y^2 + z^2\right) dm + 2\int x^2 dm$$

Now.. $$\int \left(y^2 + z^2\right) dm = I_x \qquad \text{and} \qquad \int x^2 dm \geq 0$$

$$\therefore \quad I_y + I_z \geq I_x \qquad \text{Q.E.D.}$$

The proofs of the other two inequalities follow similar steps.

(ii) If the *x* axis is the axis of revolution, then

$$I_y = I_z$$

and from part (i) $$I_y + I_z \geq I_x$$

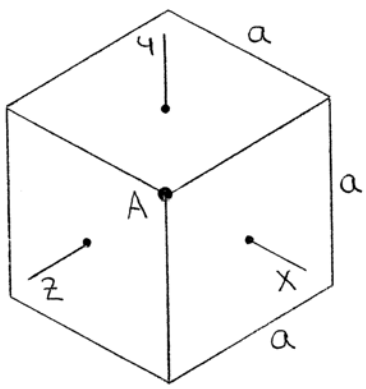
or $$2I_y \geq I_x$$

or $$I_y \geq \frac{1}{2} I_x \qquad \text{Q.E.D.}$$

PROBLEM 9.179

Consider a cube of mass m and side a. (a) Show that the ellipsoid of inertia at the center of the cube is a sphere, and use this property to determine the moment of inertia of the cube with respect to one of its diagonals. (b) Show that the ellipsoid of inertia at one of the corners of the cube is an ellipsoid of revolution, and determine the principal moments of inertia of the cube at that point.

SOLUTION



(a) At the center of the cube have (using Figure 9.28)

$$I_x = I_y = I_z = \frac{1}{12}m\left(a^2 + a^2\right) = \frac{1}{6}ma^2$$

Now observe that symmetry implies $\qquad I_{xy} = I_{yz} = I_{zx} = 0$

Using Equation (9.48), the equation of the ellipsoid of inertia is

$$\left(\frac{1}{6}ma^2\right)x^2 + \left(\frac{1}{6}ma^2\right)y^2 + \left(\frac{1}{6}ma^2\right)z^2 = 1$$

or
$$x^2 + y^2 + z^2 = \frac{6}{ma^2} \left(= R^2\right) \blacktriangleleft$$

which is the equation of a sphere.

Since the ellipsoid of inertia is a sphere, the moment of inertia with respect to any axis OL through the center O of the cube must always be the same $\left(R = \dfrac{1}{\sqrt{I_{OL}}}\right)$.

$$\therefore \quad I_{OL} = \frac{1}{6}ma^2 \blacktriangleleft$$

(b) The above sketch of the cube is the view seen if the line of sight is along the diagonal that passes through corner A. For a rectangular coordinate system at A and with one of the coordinate axes aligned with the diagonal, an ellipsoid of inertia at A could be constructed. If the cube is then rotated 120° about the diagonal, the mass distribution will remain unchanged. Thus, the ellipsoid will also remain unchanged after it is rotated. As noted at the end of section 9.17, this is possible only if the ellipsoid is an ellipsoid of revolution, where the diagonal is both the axis of revolution and a principal axis.

It then follows that
$$I_{x'} = I_{OL} = \frac{1}{6}ma^2 \blacktriangleleft$$

PROBLEM 9.179 CONTINUED

In addition, for an ellipsoid of revolution, the two transverse principal moments of inertia are equal and any axis perpendicular to the axis of revolution is a principal axis. Then, applying the parallel-axis theorem between the center of the cube and corner A for any perpendicular axis

$$I_{y'} = I_{z'} = \frac{1}{6}ma^2 + m\left(\frac{\sqrt{3}}{2}a\right)^2$$

$$\text{or } I_{y'} = I_{z'} = \frac{11}{12}ma^2 \blacktriangleleft$$

Note: Part b can also be solved using the method of Section 9.18. First note that at corner A

$$I_x = I_y = I_z = \frac{2}{3}ma^2 \qquad I_{xy} = I_{yz} = I_{zx} = \frac{1}{4}ma^2$$

Substituting into Equation (9.56) yields

$$k^3 - 2ma^2k^2 + \frac{55}{48}m^2a^6k - \frac{121}{864}m^3a^9 = 0$$

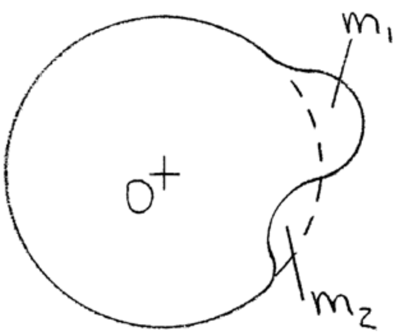
For which the roots are

$$k_1 = \frac{1}{6}ma^2 \qquad k_2 = k_3 = \frac{11}{12}ma^2$$

PROBLEM 9.180

Given a homogeneous body of mass m and of arbitrary shape and three rectangular axes x, y, and z with origin at O, prove that the sum $I_x + I_y + I_z$ of the moments of inertia of the body cannot be smaller than the similar sum computed for a sphere of the same mass and the same material centered at O. Further, using the results of Prob. 9.178, prove that if the body is a solid of revolution, where x is the axis of revolution, its moment of inertia I_y about a transverse axis y cannot be smaller than $\frac{3ma^2}{10}$, where a is the radius of the sphere of the same mass and the same material.

SOLUTION



(i) Using Equation (9.30), we have

$$I_x + I_y + I_z = \int\left(y^2 + z^2\right)dm + \int\left(z^2 + x^2\right)dm + \int\left(x^2 + y^2\right)dm$$

$$= 2\int\left(x^2 + y^2 + z^2\right)dm$$

$$= 2\int r^2 dm$$

where r is the distance from the origin O to the element of mass dm. Now assume that the given body can be formed by adding and subtracting appropriate volumes V_1 and V_2 from a sphere of mass m and radius a which is centered at O; it then follows that $m_1 = m_2 \left(m_{\text{body}} = m_{\text{sphere}} = m\right)$.

Then

$$\left(I_x + I_y + I_z\right)_{\text{body}} = \left(I_x + I_y + I_z\right)_{\text{sphere}} + \left(I_x + I_y + I_z\right)_{V_1}$$

$$- \left(I_x + I_y + I_z\right)_{V_2}$$

or

$$\left(I_x + I_y + I_z\right)_{\text{body}} = \left(I_x + I_y + I_z\right)_{\text{sphere}} + 2\int_{m_1} r^2 dm - 2\int_{m_2} r^2 dm$$

Now, $m_1 = m_2$ and $r_1 \geq r_2$ for all elements of mass dm in volumes 1 and 2.

$$\therefore \quad \int_{m_1} r^2 dm - \int_{m_2} r^2 dm \geq 0$$

so that $\quad \left(I_x + I_y + I_z\right)_{\text{body}} \geq \left(I_x + I_y + I_z\right)_{\text{sphere}}$ Q.E.D.

PROBLEM 9.180 CONTINUED

(ii) First note from Figure 9.28 that for a sphere

$$I_x = I_y = I_z = \frac{2}{5}ma^2$$

Thus,
$$\left(I_x + I_y + I_z\right)_{\text{sphere}} = \frac{6}{5}ma^2$$

For a solid of revolution, where the *x* axis is the axis of revolution, have

$$I_y = I_z$$

Then, using the results of part i

$$\left(I_x + 2I_y\right)_{\text{body}} \geq \frac{6}{5}ma^2$$

From Problem 9.178 have
$$I_y \geq \frac{1}{2}I_x$$

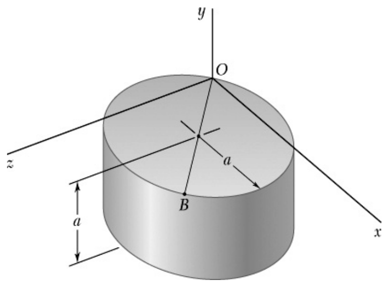
or
$$\left(2I_y - I_x\right)_{\text{body}} \geq 0$$

Adding the last two inequalities yields

$$\left(4I_y\right)_{\text{body}} \geq \frac{6}{5}ma^2$$

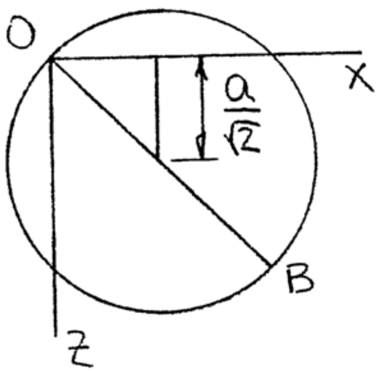
or
$$\left(I_y\right)_{\text{body}} \geq \frac{3}{10}ma^2 \quad \text{Q.E.D.}$$

PROBLEM 9.181



The homogeneous circular cylinder shown has a mass m, and the diameter OB of its top surface forms 45° angles with the x and z axes. (a) Determine the principal moments of inertia of the cylinder at the origin O. (b) Compute the angles that the principal axes of inertia at O form with the coordinate axes. (c) Sketch the cylinder, and show the orientation of the principal axes of inertia relative to the x, y, and z axes.

SOLUTION



(a) First compute the moments of inertia using Figure 9.28 and the parallel-axis theorem.

$$I_x = I_z = \frac{1}{12}m\left(3a^2 + a^2\right) + m\left[\left(\frac{a}{\sqrt{2}}\right)^2 + \left(\frac{a}{2}\right)^2\right] = \frac{13}{12}ma^2$$

$$I_y = \frac{1}{2}ma^2 + m(a)^2 = \frac{3}{2}ma^2$$

Next observe that the centroidal products of inertia are zero because of symmetry. Then

$$I_{xy} = \cancelto{0}{\bar{I}_{x'y'}} + m\bar{x}\,\bar{y} = m\left(\frac{a}{\sqrt{2}}\right)\left(-\frac{a}{2}\right) = -\frac{1}{2\sqrt{2}}ma^2$$

$$I_{yz} = \cancelto{0}{\bar{I}_{y'z'}} + m\bar{y}\,\bar{z} = m\left(-\frac{a}{2}\right)\left(\frac{a}{\sqrt{2}}\right) = -\frac{1}{2\sqrt{2}}ma^2$$

$$I_{zx} = \cancelto{0}{\bar{I}_{z'x'}} + m\bar{z}\,\bar{x} = m\left(\frac{a}{\sqrt{2}}\right)\left(\frac{a}{\sqrt{2}}\right) = \frac{1}{2}ma^2$$

Substituting into Equation (9.56)

$$K^3 - \left(\frac{13}{12} + \frac{3}{2} + \frac{13}{12}\right)ma^2K^2$$

$$+\left[\left(\frac{13}{12}\times\frac{3}{2}\right) + \left(\frac{3}{2}\times\frac{13}{12}\right) + \left(\frac{13}{12}\times\frac{13}{12}\right) - \left(-\frac{1}{2\sqrt{2}}\right)^2 - \left(-\frac{1}{2\sqrt{2}}\right)^2 - \left(\frac{1}{2}\right)^2\right]\left(ma^2\right)^2 K$$

$$-\left[\left(\frac{13}{12}\times\frac{3}{2}\times\frac{13}{12}\right) - \left(\frac{13}{12}\right)\left(-\frac{1}{2\sqrt{2}}\right)^2 - \left(\frac{3}{2}\right)\left(\frac{1}{2}\right)^2 - \left(\frac{13}{12}\right)\left(-\frac{1}{2\sqrt{2}}\right)^2 - 2\left(-\frac{1}{2\sqrt{2}}\right)\left(-\frac{1}{2\sqrt{2}}\right)\left(\frac{1}{2}\right)\right]\left(ma^2\right)^3 = 0$$

Simplifying and letting $K = ma^2\zeta$ yields

$$\zeta^3 - \frac{11}{3}\zeta^2 + \frac{565}{144}\zeta - \frac{95}{96} = 0$$

PROBLEM 9.181 CONTINUED

Solving yields

$$\zeta_1 = 0.363383 \qquad \zeta_2 = \frac{19}{12} \qquad \zeta_3 = 1.71995$$

The principal moments of inertia are then

$$K_1 = 0.363ma^2 \blacktriangleleft$$

$$K_2 = 1.583ma^2 \blacktriangleleft$$

$$K_3 = 1.720ma^2 \blacktriangleleft$$

(*b*) To determine the direction cosines $\lambda_x, \lambda_y, \lambda_z$ of each principal axis, we use two of the equations of Equations (9.54) and Equation (9.57). Thus

$$\left(I_x - K\right)\lambda_x - I_{xy}\lambda_y - I_{zx}\lambda_z = 0 \qquad (9.54a)$$

$$-I_{zx}\lambda_x - I_{yz}\lambda_y + \left(I_z - K\right)\lambda_z = 0 \qquad (9.54c)$$

$$\lambda_x^2 + \lambda_y^2 + \lambda_z^2 = 1 \qquad (9.57)$$

Note: Since $I_{xy} = I_{yz}$, Equations (9.54a) and (9.54c) were chosen to simplify the “elimination” of λ_y during the solution process.

Substituting for the moments and products of inertia in Equations (9.54a) and (9.54c)

$$\left(\frac{13}{12}ma^2 - K\right)\lambda_x - \left(-\frac{1}{2\sqrt{2}}ma^2\right)\lambda_y - \left(\frac{1}{2}ma^2\right)\lambda_z = 0$$

$$-\left(\frac{1}{2}ma^2\right)\lambda_x - \left(-\frac{1}{2\sqrt{2}}ma^2\right)\lambda_y + \left(\frac{13}{12}ma^2 - K\right)\lambda_z = 0$$

or

$$\left(\frac{13}{12} - \zeta\right)\lambda_x + \frac{1}{2\sqrt{2}}\lambda_y - \frac{1}{2}\lambda_z = 0 \qquad (i)$$

and

$$-\frac{1}{2}\lambda_x + \frac{1}{2\sqrt{2}}\lambda_y + \left(\frac{13}{12} - \zeta\right)\lambda_z = 0 \qquad (ii)$$

Observe that these equations will be identical, so that one will need to be replaced, if

$$\frac{13}{12} - \zeta = -\frac{1}{2} \qquad \text{or} \qquad \zeta = \frac{19}{12}$$

Thus, a third independent equation will be needed when the direction cosines associated with K_2 are determined. Then for K_1 and K_3

PROBLEM 9.181 CONTINUED

Eq.(i) – Eq.(ii) $\left[\frac{13}{12} - \zeta - \left(-\frac{1}{2}\right)\right]\lambda_x + \left[-\frac{1}{2} - \left(\frac{13}{12} - \zeta\right)\right]\lambda_z = 0$

or $\lambda_z = \lambda_x$

Substituting into Eq.(i) $\left(\frac{13}{12} - \zeta\right)\lambda_x + \frac{1}{2\sqrt{2}}\lambda_y - \frac{1}{2}\lambda_x = 0$

or $\lambda_y = 2\sqrt{2}\left(\zeta - \frac{7}{12}\right)\lambda_x$

Substituting into Equation (9.57)

$$\lambda_x^2 + \left[2\sqrt{2}\left(\zeta - \frac{7}{12}\right)\lambda_x\right]^2 + (\lambda_x)^2 = 1$$

or
$$\left[2 + 8\left(\zeta - \frac{7}{12}\right)^2\right]\lambda_x^2 = 1 \qquad \text{(iii)}$$

$\mathbf{K}_1$: Substituting the value of ζ_1 into Eq.(iii)

$$\left[2 + 8\left(0.363383 - \frac{7}{12}\right)^2\right](\lambda_x)_1^2 = 1$$

or $(\lambda_x)_1 = (\lambda_z)_1 = 0.647249$

and then $(\lambda_y)_1 = 2\sqrt{2}\left(0.363383 - \frac{7}{12}\right)(0.647249)$

$= -0.402662$

$\therefore \quad (\theta_x)_1 = (\theta_z)_1 = 49.7° \quad (\theta_y)_1 = 113.7°$ ◀

$\mathbf{K}_3$: Substituting the value of ζ_3 into Eq.(iii)

$$\left[2 + 8\left(1.71995 - \frac{7}{12}\right)^2\right](\lambda_x)_3^2 = 1$$

or $(\lambda_x)_3 = (\lambda_z)_3 = 0.284726$

and then $(\lambda_y)_3 = 2\sqrt{2}\left(1.71995 - \frac{7}{12}\right)(0.284726)$

$= 0.915348$

$\therefore \quad (\theta_x)_3 = (\theta_z)_3 = 73.5° \quad (\theta_y)_3 = 23.7°$ ◀

PROBLEM 9.181 CONTINUED

K$_2$: For this case, the set of equations to be solved consists of Equations (9.54a), (9.54b), and (9.57). Now

$$-I_{xy}\lambda_x + \left(I_y - K\right)\lambda_y - I_{yz}\lambda_z = 0 \qquad (9.54b)$$

Substituting for the moments and products of inertia.

$$-\left(-\frac{1}{2\sqrt{2}}ma^2\right)\lambda_x + \left(\frac{3}{2}ma^2 - K\right)\lambda_y - \left(-\frac{1}{2\sqrt{2}}ma^2\right)\lambda_z = 0$$

or
$$\frac{1}{2\sqrt{2}}\lambda_x + \left(\frac{3}{2} - \zeta\right)\lambda_y + \frac{1}{2\sqrt{2}}\lambda_z = 0 \qquad (iv)$$

Substituting the value of ζ_2 into Eqs.(i) and (iv)

$$\left(\frac{13}{12} - \frac{19}{12}\right)\left(\lambda_x\right)_2 + \frac{1}{2\sqrt{2}}\left(\lambda_y\right)_2 - \frac{1}{2}\left(\lambda_z\right)_2 = 0$$

$$\frac{1}{2\sqrt{2}}\left(\lambda_x\right)_2 + \left(\frac{3}{2} - \frac{19}{12}\right)\left(\lambda_y\right)_2 + \frac{1}{2\sqrt{2}}\left(\lambda_z\right)_2 = 0$$

or
$$-\left(\lambda_x\right)_2 + \frac{1}{\sqrt{2}}\left(\lambda_y\right)_2 - \left(\lambda_z\right)_2 = 0$$

and
$$\left(\lambda_x\right)_2 - \frac{\sqrt{2}}{6}\left(\lambda_y\right)_2 + \left(\lambda_z\right)_2 = 0$$

Adding yields
$$\left(\lambda_y\right)_2 = 0$$

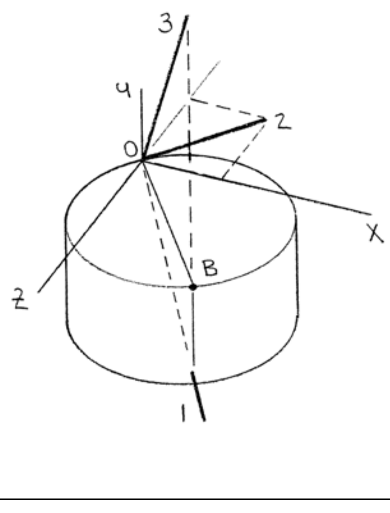
and then
$$\left(\lambda_z\right)_2 = -\left(\lambda_x\right)_2$$

Substituting into Equation (9.57)

$$\left(\lambda_x\right)_2^2 + \cancelto{0}{\left(\lambda_y\right)_2^2} + \left(-\lambda_x\right)_2^2 = 1$$

or
$$\left(\lambda_x\right)_2 = \frac{1}{\sqrt{2}} \qquad \text{and} \qquad \left(\lambda_z\right)_2 = -\frac{1}{\sqrt{2}}$$

$$\therefore \quad \left(\theta_x\right)_2 = 45.0° \quad \left(\theta_y\right)_2 = 90.0° \quad \left(\theta_z\right)_2 = 135.0° \blacktriangleleft$$



(*c*) Principal axes 1 and 3 lie in the vertical plane of symmetry passing through points *O* and *B*. Principal axis 2 lies in the *xz* plane.

PROBLEM 9.182

Prob. 9.167

For the component described in the problem indicated, determine (*a*) the principal moments of inertia at the origin, (*b*) the principal axes of inertia at the origin. Sketch the body and show the orientation of the principal axes of inertia relative to the *x*, *y*, and *z* axes.

SOLUTION

From the solution to Problem 9.143 and 9.167

$$I_x = 34.106 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_y = 50.125 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_z = 34.876 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_{xy} = 0.96211 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_{yz} = I_{zx} = 0$$

(*a*) From Equation 9.55

$$\begin{vmatrix} I_x - K & I_{xy} & 0 \\ I_{xy} & I_y - K & 0 \\ 0 & 0 & I_z - K \end{vmatrix} = 0$$

or $$(I_x - K)(I_y - K)(I_z - K) - (I_z - K)I_{xy}^2 = 0$$

or $$(I_z - K)\left[(I_x - K)(I_y - K) - I_{xy}^2\right] = 0$$

Then $$I_z - K = 0 \text{ and } I_xI_y - (I_x + I_y)K + K^2 - I_{xy}^2 = 0$$

Now $$K_1 = I_z = 34.876 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

or $K_1 = 34.9 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

and

$$(34.876 \times 10^{-3})(50.125 \times 10^{-3}) - (34.106 \times 10^{-3} + 50.125 \times 10^{-3})K$$
$$+ K^2 - (0.96211 \times 10^{-3})^2 = 0$$

or $$1.70864 \times 10^{-3} - 84.231 \times 10^{-3}K + K^2 = 0$$

Solving yields $$K_2 = 34.0486 \times 10^{-3} \quad K_3 = 50.1824 \times 10^{3}$$

or $K_2 = 34.0 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

and $K_3 = 50.2 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

PROBLEM 9.182 CONTINUED

(*b*) To determine the directions cosines λ_x, λ_y and λ_z of each principal axis use two of the Equations 9.54 and Equation 9.57

$\mathbf{K}_1$: Using Equation 9.54(a) and Equation 9.54(b) with $I_{yz} = I_{zx} = 0$, we have

$$\left(I_x - K_1\right)\left(\lambda_x\right)_1 - I_{xy}\left(\lambda_y\right)_1 = 0$$

$$-I_{xy}\left(\lambda_x\right)_1 + \left(I_y - K_1\right)\left(\lambda_y\right)_1 = 0$$

Substituting

$$\left(34.106 \times 10^{-3} - 34.876 \times 10^{-3}\right)\left(\lambda_x\right)_1 - 0.96211 \times 10^{-3}\left(\lambda_y\right)_1 = 0$$

$$-0.96211 \times 10^{-3}\left(\lambda_x\right)_1 + \left(50.125 \times 10^{-3} - 34.876 \times 10^{-3}\right)\left(\lambda_y\right)_1 = 0$$

or

$$-0.770 \times 10^{-3}\left(\lambda_x\right)_1 - 0.96211 \times 10^{-3}\left(\lambda_y\right)_1 = 0$$

$$-0.96211 \times 10^{-3}\left(\lambda_x\right)_1 + 15.249 \times 10^{-3}\left(\lambda_y\right)_1 = 0$$

Solving yields $\left(\lambda_x\right)_1 = \left(\lambda_y\right)_1 = 0$

From Equation 9.57 $\cancelto{0}{\left(\lambda_x\right)_1^2} + \cancelto{0}{\left(\lambda_y\right)_1^2} + \left(\lambda_z\right)_1^2 = 1$ or $\left(\lambda_z\right)_1 = 1$

and $\left(\theta_x\right)_1 = 90.0°$, $\left(\theta_y\right)_1 = 90.0°$, $\left(\theta_z\right)_1 = 0°$ ◀

$\mathbf{K}_2$: Using Equation 9.54(b) and Equation 9.54(c) with $I_{yz} = I_{zx} = 0$

$$-I_{xz}\left(\lambda_x\right)_2 + \left(I_y - k_2\right)\left(\lambda_y\right)_2 = 0$$

$$\left(I_z - K_2\right)\left(\lambda_z\right)_2 = 0$$

Now $I_z \neq K_2 \Rightarrow \left(\lambda_z\right)_2 = 0$

Substituting

$$-0.96211 \times 10^{-3}\left(\lambda_x\right)_2 + \left(50.125 \times 10^{-3} - 34.0486 \times 10^{-3}\right)\left(\lambda_y\right)_2 = 0$$

or $\left(\lambda_y\right)_2 = 0.05985\left(\lambda_x\right)_2$

PROBLEM 9.182 CONTINUED

Then $$(\lambda_x)_2^2 + \left[0.05985(\lambda_x)_2\right]^2 + \cancel{(\lambda_z)_2}^{\,0} = 1$$

$$(\lambda_x)_2 = 0.99821$$

$$(\lambda_y)_2 = 0.05974$$

and $$(\theta_x)_2 = 3.43°,\quad (\theta_y)_2 = 86.6°,\quad (\theta_z)_2 = 90.0° \blacktriangleleft$$

$\mathbf{K}_3$: $$-I_{xy}(\lambda_x)_3 + (I_y - K_3)(\lambda_y)_3 = 0$$

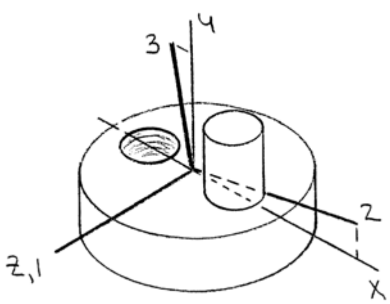
$$(I_z - K_3)(\lambda_z)_3 = 0$$

Now $$I_z \neq K_3 \Rightarrow (\lambda_z)_3 = 0$$

Substituting

$$-0.96211\times10^{-3}(\lambda_x)_3 + \left(50.125\times10^{-3} - 50.1824\times10^{-3}\right)(\lambda_y)_3 = 0$$

$$-0.96211\times10^{-3}(\lambda_x)_3 - 0.0574\times10^{-3}(\lambda_y)_3 = 0$$



or $$(\lambda_y)_3 = -16.7615(\lambda_x)_3$$

Have $$(\lambda_x)_3^2 + \left[-16.7615(\lambda_x)_3\right]^2 + \cancel{(\lambda_z)_3^2}^{\,0} = 1$$

yields $$(\lambda_x)_3 = -0.059555 \quad \text{and} \quad (\lambda_y)_3 = 0.998231$$

and $$(\theta_x)_3 = 93.4°,\quad (\theta_y)_3 = 3.41°,\quad \theta_z = 90.0° \blacktriangleleft$$

(*c*) Principal axis 1 coincides with the *z* axis, while the principal axes 2 and 3 lie in the *xy* plane

PROBLEM 9.183

Prob. 9.147 and 9.151

For the component described in the problem indicated, determine (*a*) the principal moments of inertia at the origin, (*b*) the principal axes of inertia at the origin. Sketch the body and show the orientation of the principal axes of inertia relative to the *x*, *y*, and *z* axes.

SOLUTION

From Problem 9.147: $I_x = 9.8821 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$

$$I_y = 11.5344 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_z = 2.1878 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

From Problem 9.151: $I_{xy} = 0.48776 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$

$$I_{yz} = 1.18391 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_{zx} = 2.6951 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

(*a*) From Equation 9.56

$$K^3 - \left(I_x + I_y + I_z\right)K^2 + \left(I_xI_y + I_yI_z + I_zI_x - I_{xy}^2 - I_{yz}^2 - I_{zx}^2\right)K$$

$$-\left(I_xI_yI_z - I_xI_{yz}^2 - I_yI_{zx}^2 - I_zI_{xy}^2 - 2I_{xy}I_{yz}I_{zx}\right) = 0$$

Substituting

$$K^3 - \left[(9.8821 + 11.5344 + 2.1878) \times 10^{-3}\right]K^2 + \left\{\left[(9.8821)(11.5344) + (11.5344)(2.1878)\right.\right.$$

$$\left.\left. + (2.1878)(9.8821) - (0.48776)^2 - (1.18391)^2 - (2.6951)^2\right] \times 10^{-6}\right\}K$$

$$-\left[(9.8821)(11.5344)(2.1878) - (9.8821)(1.18391)^2 - (11.5344)(2.6951)^2\right.$$

$$\left. - (2.1878)(0.48776)^2 - 2(0.48776)(1.18391)(2.6951)\right] \times 10^{-9} = 0$$

or

$$K^3 - \left(23.6043 \times 10^{-3}\right)K^2 + \left(151.9360 \times 10^{-6}\right)K - 148.1092 \times 10^{-9} = 0$$

Solving numerically

$K_1 = 1.180481 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ or $K_1 = 1.180 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

$K_2 = 10.72017 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ or $K_2 = 10.72 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

$K_3 = 11.70365 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ or $K_3 = 11.70 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

PROBLEM 9.183 CONTINUED

Solving numerically

$$K_1 = 1.180481 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2 \qquad \text{or} \qquad K_1 = 1.180 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2 \blacktriangleleft$$

$$K_2 = 10.72017 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2 \qquad \text{or} \qquad K_2 = 10.72 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2 \blacktriangleleft$$

$$K_3 = 11.70365 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2 \qquad \text{or} \qquad K_3 = 11.70 \times 10^{-3} \text{ lb}\cdot\text{ft}\cdot\text{s}^2 \blacktriangleleft$$

(*b*) From Equations 9.54(a) and 9.54(b)

$$(I_x - K)(\lambda_x) - I_{xy}(\lambda_y) - I_{zx}(\lambda_z) = 0$$

$$-I_{xy}(\lambda_x) + (I_y - K_1)(\lambda_y) - I_{yz}(\lambda_z) = 0$$

$\mathbf{K}_1$: Substitute K_1 and solve for λ to get (λ_x), (λ_{xy}) and $(\lambda_y)_3$.

$$\left[(9.8821 - 1.180481)(\lambda_x)_1 - 0.48776(\lambda_y)_1 - 2.6951(\lambda_z)_1\right] \times 10^{-3} = 0$$

$$\left[-0.48776(\lambda_x)_1 + (11.5344 - 1.180481)(\lambda_y)_1 - 1.18391(\lambda_z)_1\right] \times 10^{-3} = 0$$

or
$$17.83996(\lambda_x)_1 - (\lambda_y)_1 - 5.52546(\lambda_z)_1 = 0$$

$$-0.0471(\lambda_x)_1 + (\lambda_{y1})_1 - 0.11434(\lambda_z)_1 = 0$$

Then
$$(\lambda_z)_1 = 3.1549(\lambda_x)_1$$

and
$$(\lambda_y)_1 = 0.40769(\lambda_x)_1$$

Equation 9.57:
$$(\lambda_x)_1^2 + (\lambda_y)_1^2 + (\lambda_z)_1^2 = 1$$

Substituting
$$(\lambda_x)_1^2 + \left[0.40769(\lambda_x)_1\right]^2 + \left[3.1549(\lambda_x)_1\right]^2 = 1$$

$$\text{or} \quad (\lambda_x)_1 = 0.29989 \qquad \text{then} \qquad (\theta_x)_1 = 72.5° \blacktriangleleft$$

$$\text{and} \quad (\lambda_y)_1 = 0.122262 \qquad \text{then} \qquad (\theta_y)_1 = 83.0° \blacktriangleleft$$

$$(\lambda_z)_1 = 0.94612 \qquad \text{then} \qquad (\theta_z)_1 = 18.89° \blacktriangleleft$$

$\mathbf{K}_2$: Substitute K_2 and solve for λ.

$$\left[(9.8821 - 10.72017)(\lambda_x)_2 - 0.48776(\lambda_y)_2 - 2.6951(\lambda_z)_2\right] \times 10^{-3} = 0$$

$$\left[-0.48776(\lambda_x)_2 + (11.5344 - 10.72017)(\lambda_y)_2 - 1.18391(\lambda_z)_2\right] \times 10^{-3} = 0$$

PROBLEM 9.183 CONTINUED

or $$-1.718202(\lambda_x)_2 - (\lambda_y)_2 - 5.52546(\lambda_z)_2 = 0$$

$$-0.599045(\lambda_x)_2 + (\lambda_y)_2 - 1.45402(\lambda_z)_2 = 0$$

Then $$(\lambda_z)_2 = -0.33201(\lambda_x)_2$$

and $$(\lambda_y)_2 = 0.116306(\lambda_x)_2$$

Then $$(\lambda_x)_2^2 + \left[0.116306(\lambda_x)_2\right]^2 + \left[-0.33201(\lambda_x)_2\right]^2 = 1$$

or $(\lambda_x)_2 = 0.94333$ then $(\theta_x)_2 = 19.38°$ ◀

And $(\lambda_y)_2 = 0.109715$ then $(\theta_y)_2 = 83.7°$ ◀

$(\lambda_z)_2 = -0.31320$ then $(\theta_z)_2 = 108.3°$ ◀

K$_3$: Substitute K_3 and solve for λ.

$$\left[(9.8821 - 11.70365)(\lambda_x)_3 - 0.48776(\lambda_y)_3 - 2.6951(\lambda_z)_3\right] \times 10^{-3} = 0$$

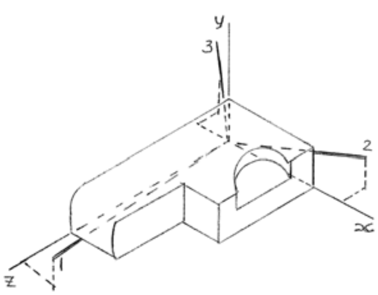
$$\left[-0.48776(\lambda_x)_3 + (11.5344 - 11.70365)(\lambda_y)_3 - 1.18391(\lambda_z)_3\right] \times 10^{-3} = 0$$

or $$-3.73452(\lambda_x)_3 - (\lambda_y)_3 - 5.52546(\lambda_z)_3 = 0$$

$$2.88189(\lambda_x)_3 + (\lambda_y)_3 + 6.99504(\lambda_z)_3 = 0$$

Then $$(\lambda_z)_3 = 0.58019(\lambda_x)_3$$

and $$(\lambda_y)_3 = -6.9403(\lambda_y)_3$$



$$(\lambda_x)_3^2 + \left[-0.69403(\lambda_x)_3\right]^2 + \left[0.58019(\lambda_x)_3\right]^2 = 1$$

or $(\lambda_x)_3 = -0.142128^*$ then $(\theta_x)_3 = 98.2°$ ◀

$(\lambda_y)_3 = 0.98641$ then $(\theta_y)_3 = 9.46°$ ◀

$(\lambda_z)_3 = -0.082461$ then $(\theta_z)_3 = 94.7°$ ◀

*Note: the negative root of $(\lambda_x)_3$ is taken so that axes 1, 2, 3 form a right-handed set.

PROBLEM 9.184

Prob. 9.169

For the component described in the problem indicated, determine (*a*) the principal moments of inertia at the origin, (*b*) the principal axes of inertia at the origin. Sketch the body and show the orientation of the principal axes of inertia relative to the *x*, *y*, and *z* axes.

SOLUTION

(*a*) From the solution of Problem 9.169 have

$$I_x = \frac{1}{2}\frac{W}{g}a^2 \qquad I_{xy} = \frac{1}{4}\frac{W}{g}a^2$$

$$I_y = \frac{W}{g}a^2 \qquad I_{yz} = \frac{1}{8}\frac{W}{g}a^2$$

$$I_z = \frac{5}{6}\frac{W}{g}a^2 \qquad I_{zx} = \frac{3}{8}\frac{W}{g}a^2$$

Substituting into Equation (9.56)

$$K^3 - \left[\left(\frac{1}{2} + 1 + \frac{5}{6}\right)\left(\frac{W}{g}a^2\right)\right]K^2 + \left[\left(\frac{1}{2}\right)(1) + (1)\left(\frac{5}{6}\right) + \left(\frac{5}{6}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{4}\right)^2 - \left(\frac{1}{8}\right)^2 - \left(\frac{3}{8}\right)^2\right]\left(\frac{W}{g}a^2\right)^2 K$$

$$-\left[\left(\frac{1}{2}\right)(1)\left(\frac{5}{6}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{8}\right)^2 - (1)\left(\frac{3}{8}\right)^2 - \left(\frac{5}{6}\right)\left(\frac{1}{4}\right)^2 - 2\left(\frac{1}{4}\right)\left(\frac{1}{8}\right)\left(\frac{3}{8}\right)\right]\left(\frac{W}{g}a^2\right)^3 = 0$$

Simplifying and letting $K = \frac{W}{g}a^2\zeta$ yields

$$\zeta^3 - 2.33333\zeta^2 + 1.53125\zeta - 0.192708 = 0$$

Solving yields

$$\zeta_1 = 0.163917 \qquad \zeta_2 = 1.05402 \qquad \zeta_3 = 1.11539$$

The principal moments of inertia are then

$$K_1 = 0.1639\frac{W}{g}a^2 \blacktriangleleft$$

$$K_2 = 1.054\frac{W}{g}a^2 \blacktriangleleft$$

$$K_3 = 1.115\frac{W}{g}a^2 \blacktriangleleft$$

PROBLEM 9.184 CONTINUED

(*b*) To determine the direction cosines $\lambda_x,\ \lambda_y,\ \lambda_z$ of each principal axis, use two of the equations of Equations (9.54) and (9.57). Then

$\mathbf{K}_1$: Begin with Equations (9.54a) and (9.54b).

$$(I_x - K_1)(\lambda_x)_1 - I_{xy}(\lambda_y)_1 - I_{zx}(\lambda_z)_1 = 0$$

$$-I_{xy}(\lambda_x)_1 + (I_y - K_2)(\lambda_y)_1 - I_{yz}(\lambda_z)_1 = 0$$

Substituting

$$\left[\left(\frac{1}{2} - 0.163917\right)\left(\frac{W}{g}a^2\right)\right](\lambda_x)_1 - \left(\frac{1}{4}\frac{W}{g}a^2\right)(\lambda_y)_1 - \left(\frac{3}{8}\frac{W}{g}a^2\right)(\lambda_z)_1 = 0$$

$$-\left(\frac{1}{4}\frac{W}{g}a^2\right)(\lambda_x)_1 + \left[(1 - 0.163917)\left(\frac{W}{g}a^2\right)\right](\lambda_y)_1 - \left(\frac{1}{8}\frac{W}{g}a^2\right)(\lambda_z)_1 = 0$$

Simplifying yields

$$1.34433(\lambda_x)_1 - (\lambda_y)_1 - 1.5(\lambda_z)_1 = 0$$

$$-0.299013(\lambda_x)_1 + (\lambda_y)_1 - 0.149507(\lambda_z)_1 = 0$$

Adding and solving for $(\lambda_z)_1$

$$(\lambda_z)_1 = 0.633715(\lambda_x)_1$$

and then
$$(\lambda_y)_1 = [1.34433 - 1.5(0.633715)](\lambda_x)_1$$

$$= 0.393758(\lambda_x)_1$$

Now substitute into Equation (9.57)

$$(\lambda_x)_1^2 + [0.393758(\lambda_x)_1]^2 + [0.633715(\lambda_x)_1]^2 = 1$$

or
$$(\lambda_x)_1 = 0.801504$$

and
$$(\lambda_y)_1 = 0.315599 \qquad (\lambda_z)_1 = 0.507925$$

$$\therefore\quad (\theta_x)_1 = 36.7° \qquad (\theta_y)_1 = 71.6° \qquad (\theta_z)_1 = 59.5° ◀$$

$\mathbf{K}_2$: Begin with Equations (9.54a) and (9.54b).

$$(I_x - K_2)(\lambda_x)_2 - I_{xy}(\lambda_y)_2 - I_{zx}(\lambda_z)_2 = 0$$

$$-I_{xy}(\lambda_x)_2 + (I_y - K_2)(\lambda_y)_2 - I_{yz}(\lambda_z)_2 = 0$$

PROBLEM 9.184 CONTINUED

Substituting

$$\left[\left(\frac{1}{2} - 1.05402\right)\left(\frac{W}{g}a^2\right)\right](\lambda_x)_2 - \left(\frac{1}{4}\frac{W}{g}a^2\right)(\lambda_y)_2 - \left(\frac{3}{8}\frac{W}{g}a^2\right)(\lambda_z)_2 = 0$$

$$-\left(\frac{1}{4}\frac{W}{g}a^2\right)(\lambda_x)_2 + \left[(1 - 1.05402)\left(\frac{W}{g}a^2\right)\right](\lambda_y)_2 - \left(\frac{1}{8}\frac{W}{g}a^2\right)(\lambda_z)_2 = 0$$

Simplifying yields

$$-2.21608(\lambda_x)_2 - (\lambda_y)_2 - 1.5(\lambda_z)_2 = 0$$

$$4.62792(\lambda_x)_2 + (\lambda_y)_2 + 2.31396(\lambda_z)_2 = 0$$

Adding and solving for $(\lambda_z)_2$

$$(\lambda_z)_2 = -2.96309(\lambda_x)_2$$

and then

$$(\lambda_y)_2 = [-2.21608 - 1.5(-2.96309)](\lambda_x)_2$$

$$= 2.22856(\lambda_x)_2$$

Now substitute into Equation (9.57)

$$(\lambda_x)_2^2 + \left[2.22856(\lambda_x)_2\right]^2 + \left[-2.96309(\lambda_x)_2\right]^2 = 1$$

or

$$(\lambda_x)_2 = 0.260410$$

and

$$(\lambda_y)_2 = 0.580339 \qquad (\lambda_z)_2 = -0.771618$$

$$\therefore \quad (\theta_x)_2 = 74.9° \qquad (\theta_y)_2 = 54.5° \qquad (\theta_z)_2 = 140.5° \blacktriangleleft$$

$\mathbf{K}_3$: Begin with Equations (9.54a) and (9.54b).

$$(I_x - K_3)(\lambda_x)_3 - I_{xy}(\lambda_y)_3 - I_{zx}(\lambda_z)_3 = 0$$

$$-I_{xy}(\lambda_x)_3 + (I_y - K_3)(\lambda_y)_3 - I_{yz}(\lambda_z)_3 = 0$$

Substituting

$$\left[\left(\frac{1}{2} - 1.11539\right)\left(\frac{W}{g}a^2\right)\right](\lambda_x)_3 - \left(\frac{1}{4}\frac{W}{g}a^2\right)(\lambda_y)_3 - \left(\frac{3}{8}\frac{W}{g}a^2\right)(\lambda_z)_3 = 0$$

$$-\left(\frac{1}{4}\frac{W}{g}a^2\right)(\lambda_x)_3 + \left[(1 - 1.11539)\left(\frac{W}{g}a^2\right)\right](\lambda_y)_3 - \left(\frac{1}{8}\frac{W}{g}a^2\right)(\lambda_z)_3 = 0$$

PROBLEM 9.184 CONTINUED

Simplifying yields

$$-2.46156(\lambda_x)_3 - (\lambda_y)_3 - 1.5(\lambda_z)_3 = 0$$

$$2.16657(\lambda_x)_3 + (\lambda_y)_3 + 1.08328(\lambda_z)_3 = 0$$

Adding and solving for $(\lambda_z)_3$

$$(\lambda_z)_3 = -0.707885(\lambda_x)_3$$

and then

$$(\lambda_y)_3 = [-2.46156 - 1.5(-0.707885)](\lambda_x)_3$$

$$= -1.39973(\lambda_x)_3$$

Now substitute into Equation (9.57)

$$(\lambda_x)_3^2 + [-1.39973(\lambda_x)_3]^2 + [-0.707885(\lambda_x)_3]^2 = 1 \qquad \text{(i)}$$

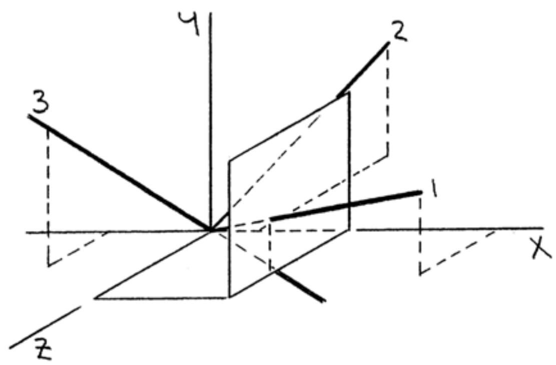
or

$$(\lambda_x)_3 = 0.537577$$

and

$$(\lambda_y)_3 = -0.752463 \qquad (\lambda_z)_3 = -0.380543$$

$$\therefore \quad (\theta_x)_3 = 57.5° \qquad (\theta_y)_3 = 138.8° \qquad (\theta_z)_3 = 112.4° \blacktriangleleft$$



(*c*) Note: Principal axis 3 has been labeled so that the principal axes form a right-handed set. To obtain the direction cosines corresponding to the labeled axis, the negative root of Equation (i) must be chosen; that is, $(\lambda_x)_3 = -0.537577$

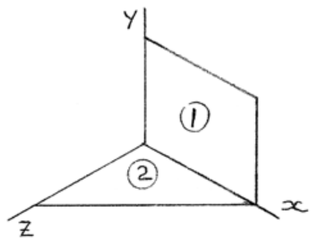
Then

$$(\theta_x)_3 = 122.5° \qquad (\theta_y)_3 = 41.2° \qquad (\theta_z)_3 = 67.6°$$

PROBLEM 9.185

Prob. 9.170
For the component described in the problem indicated, determine (*a*) the principal moments of inertia at the origin, (*b*) the principal axes of inertia at the origin. Sketch the body and show the orientation of the principal axes of inertia relative to the *x*, *y*, and *z* axes.

SOLUTION



From Problem 9.170 $\quad m_1 = \rho t a^2 \qquad m_2 = \frac{1}{2}\rho t a^2$

Now
$$I_x = (I_x)_1 + (I_x)_2 = \frac{1}{3}(\rho t a^2)a^2 + \frac{1}{6}\left(\frac{1}{2}\rho t a^2\right)a^2 = \frac{5}{12}\rho t a^4$$

$$I_y = (I_y)_1 + (I_y)_2 = \frac{1}{3}(\rho t a^2)a^2 + \frac{1}{6}\left(\frac{1}{2}\rho t a^2\right)(a^2 + a^2) = \frac{1}{2}\rho t a^4$$

$$I_z = (I_z)_1 + (I_z)_2 = \frac{1}{3}(\rho t a^2)(a^2 + a^2) + \frac{1}{6}\left(\frac{1}{2}\rho t a^2\right)a^2 = \frac{3}{4}\rho t a^4$$

Now note that symmetry implies
$$(\bar{I}_{x'y'})_1 = (\bar{I}_{y'z'})_1 = (\bar{I}_{z'x'})_1 = 0$$

$$(\bar{I}_{x'y'})_2 = (\bar{I}_{y'z'})_2 = 0$$

Have
$$I_{uv} = \bar{I}_{u'v'} + m\bar{u}\,\bar{v}$$

PROBLEM 9.185 CONTINUED

Then
$$I_{xy} = m_1\bar{x}_1\bar{y}_1 + \cancelto{0}{m_2\bar{x}_2\bar{y}_2} = \rho t a^2\left(\frac{a}{2}\right)\left(\frac{a}{2}\right) = \frac{1}{4}\rho t a^4$$

$$I_{yx} = \cancelto{0}{m_1\bar{y}_1\bar{z}_1} + \cancelto{0}{m_2\bar{y}_2 z_2}$$

$$I_{zx} = \cancelto{0}{m_1\bar{z}_1\bar{x}_1} + \left[\left(\bar{I}_{z'x'}\right)_2 + m_2\bar{z}_2\bar{x}_2\right]$$

From Problem 9.170
$$\left(\bar{I}_{z'x'}\right)_2 = -\frac{1}{72}\rho t a^4$$

Then
$$I_{zx} = -\frac{1}{72}\rho t a^4 + \left(\frac{1}{2}\rho t a^2\right)\left(\frac{1}{3}a\right)\left(\frac{1}{3}a\right) = \frac{1}{24}\rho t a^4$$

(*a*) Equation 9.56

$$K^3 - \left(I_x + I_y + I_z\right)K^2 + \left(I_x I_y + I_y I_z + I_z I_x\right) - I_{xy}^2 - I_{yz}^2 - I_{zx}^2\Big)K$$

$$-\left(I_x I_y I_z - I_x I_{yz}^2 - I_y I_{zx}^2 - I_z I_{xy}^2 - 2I_{xy}I_{yz}I_{zx} = 0\right.$$

Substituting

$$K^3 - \left[\left(\frac{5}{12} + \frac{1}{2} + \frac{3}{4}\right)\rho t a^4\right]K^2 + \left\{\left[\left(\frac{5}{12}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{3}{4}\right) + \left(\frac{3}{4}\right)\left(\frac{5}{12}\right) - \left(\frac{1}{4}\right)^2 - 0 - \left(\frac{1}{24}\right)^2\right]\left(\rho t a^4\right)^2\right\}K$$

$$-\left\{\left[\left(\frac{5}{12}\right)\left(\frac{1}{2}\right)\left(\frac{3}{4}\right) - 0 - \left(\frac{1}{2}\right)\left(\frac{1}{24}\right)^2 - \left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^2 - 0\right]\left(\rho t a^4\right)^3\right\} = 0$$

Simplifying and letting $\quad K = \rho t a^4 \zeta$

yields
$$\zeta^3 = \frac{5}{3}\zeta^2 + \frac{479}{576}\zeta - \frac{125}{1152} = 0$$

Solving numerically...

$\zeta_1 = 0.203032$ or $K_1 = 0.203\rho t a^4$ ◀

$\zeta_2 = 0.698281$ or $K_2 = 0.698\rho t a^4$ ◀

$\zeta_3 = 0.765354$ or $K_3 = 0.765\rho t a^4$ ◀

(*b*) Equations 9.54a and 9.54b

$$\left(I_x - K\right)\left(\lambda_x\right) - I_{xy}\left(\lambda_y\right) - I_{zx}\left(\lambda_z\right) = 0$$

$$-I_{xy}\left(\lambda_x\right) + \left(I_y - K\right)\left(\lambda_y\right) - I_{yz}\left(\lambda_z\right) = 0$$

PROBLEM 9.185 CONTINUED

Substituting K_1

$$\left[\left(\frac{5}{12} - 0.203032\right)(\lambda_x)_1 - \frac{1}{4}(\lambda_y)_1 - \frac{1}{24}(\lambda_z)_1\right]\rho t a^4 = 0$$

$$\left[-\frac{1}{4}(\lambda_x)_1 + \left(\frac{1}{2} - 0.203032\right)(\lambda_y)_1 - 0\right]\rho t a^4 = 0$$

or $$(\lambda_y)_1 = 0.841842(\lambda_x)_1$$

and $$(\lambda_z)_1 = 0.0761800(\lambda_x)_1$$

Equation 9.57 $$(\lambda_x)_1^2 + (\lambda_y)_1^2 + (\lambda_z)_1^2 = 1$$

Substituting $$(\lambda_x)_1^2 + \left[0.841842(\lambda_x)_1\right]^2 + \left[0.0761800(\lambda_x)_1\right]^2 = 1$$

or $(\lambda_x)_1 = 0.763715$ then $(\theta_x)_1 = 40.2°$ ◀

$(\lambda_y)_1 = 0.642927$ then $(\theta_y)_1 = 50.0°$ ◀

$(\lambda_z)_1 = 0.0581798$ then $(\theta_z)_1 = 86.7°$ ◀

Substituting K_2

$$\left[\left(\frac{5}{12} - 0.698281\right)(\lambda_x)_2 - \frac{1}{4}(\lambda_y)_2 - \frac{1}{24}(\lambda_z)_2\right]\rho t a^4 = 0$$

$$\left[-\frac{1}{4}(\lambda_x)_2 + \left(\frac{1}{2} - 0.698281\right)(\lambda_y)_2 - 0\right]\rho t a^4 = 0$$

or $$(\lambda_y)_2 = -1.260837(\lambda_x)_2$$

and $$(\lambda_z)_2 = 0.806278(\lambda_x)_2$$

Then $$(\lambda_x)_2^2 + \left[-1.260837(\lambda_x)_2\right]^2 + \left[0.806278(\lambda_x)_2\right]^2 = 1$$

or $(\lambda_x)_2 = 0.555573$ then $(\theta_x)_2 = 56.2°$ ◀

$(\lambda_y)_2 = -0.700487$ then $(\theta_y)_2 = 134.5°$ ◀

$(\lambda_z)_2 = 0.447946$ then $(\theta_z)_2 = 63.4°$ ◀

PROBLEM 9.185 CONTINUED

Substituting K_3

$$\left[\left(\frac{5}{12} - 0.765354\right)(\lambda_x)_3 - \frac{1}{4}(\lambda_y)_3 - \frac{1}{24}(\lambda_z)_3\right]\rho t a^4 = 0$$

$$\left[-\frac{1}{4}(\lambda_x)_3 + \left(\frac{1}{2} - 0.765354\right)(\lambda_y)_3 - 0\right]\rho t a^4 = 0$$

or $$(\lambda_y)_3 = -0.942138(\lambda_x)_3$$

And $$(\lambda_z)_3 = -2.71567(\lambda_x)_3$$

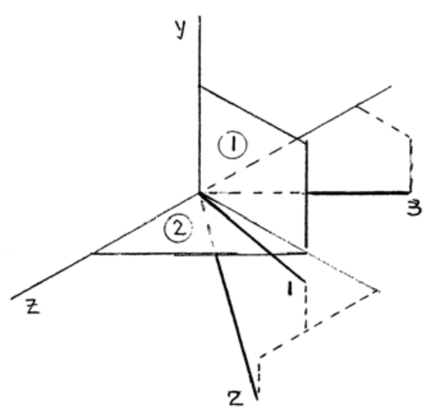
Then $$(\lambda_x)_3^2 + \left[-0.942138(\lambda_x)_3\right]^2 + \left[-2.71567(\lambda_x)_3\right]^2 = 1$$

or $(\lambda_x)_3 = 0.328576$ then $(\theta_x)_3 = 70.8°$ ◀

$(\lambda_y)_3 = -0.309564$ then $(\theta_y)_3 = 108.0°$ ◀

$(\lambda_z)_3 = -0.892304$ then $(\theta_z)_3 = 153.2°$ ◀

(*c*)



PROBLEM 9.186

For the component described in the problem indicated, determine (*a*) the principal moments of inertia at the origin, (*b*) the principal axes of inertia at the origin. Sketch the body and show the orientation of the principal axes of inertia relative to the *x*, *y*, and *z* axes.

Problem 9.150 and 9.172

SOLUTION

(*a*) From the solutions to Problem 9.150

$$I_x = 14.32 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

$$I_y = I_z = 18.62 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$$

From Problem 9.172 $\quad I_{xy} = I_{zx} = 4.297 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2,\ I_{xz} = 0$

Substituting into Eq. (9.56) and using $\quad I_y = I_z, \quad I_{xy} = I_{zx}, \quad I_{yz} = 0$

$$K^3 - \left(I_x + 2I_y\right)K^2 + \left[I_x\left(2I_y\right) + I_y^2 - 2\left(I_{xy}\right)^2\right]K - \left(I_x I_y^2 - 2I_y I_{xy}^2\right) = 0$$

$$K^3 - \left[14.32 \times 10^{-3} + 2\left(18.62 \times 10^{-2}\right)\right]K^2 + \left[\left(14.32 \times 10^{-3}\right)(2)\left(18.62 \times 10^{-3}\right) + \left(18.62 \times 10^{-3}\right)^2\right.$$

$$\left. - 2\left(4.297 \times 10^{-3}\right)^2\right]K - \left[\left(14.32 \times 10^{-3}\right)\left(18.62 \times 10^{-3}\right)^2 - 2\left(18.62 \times 10^{-3}\right)\left(4.297 \times 10^{-3}\right)^2\right] = 0$$

or $\quad K^3 - 51.56 \times 10^{-3}K^2 + 0.84305 \times 10^{-3}K - 0.004277 \times 10^{-3} = 0$

Solving: $\quad K_1 = 0.010022\ \text{lb}\cdot\text{ft}\cdot\text{s}^2 \quad$ or $\quad K_1 = 10.02 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

$K_2 = 0.018624\ \text{lb}\cdot\text{ft}\cdot\text{s}^2 \quad$ or $\quad K_2 = 18.62 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

$K_3 = 0.022914\ \text{lb}\cdot\text{ft}\cdot\text{s}^2 \quad$ or $\quad K_3 = 22.9 \times 10^{-3}\ \text{lb}\cdot\text{ft}\cdot\text{s}^2$ ◀

(*b*) To determine the direction cosines $\lambda_x,\ \lambda_y,\ \lambda_z$ of each principal axis, use two of the equations of Equations (9.54) and Equation (9.57). Then

$\mathbf{K}_1$: Begin with Equations (9.54b) and (9.54c):

$$-I_{xy}(\lambda_x)_1 + (I_y - k_1)(\lambda_y)_1 - \cancelto{0}{I_{yz}}(\lambda_z) = 0$$

$$-I_{zx}(\lambda_x)_1 - I_{yz}(\lambda_y)_1 + (I_z - K_1)(\lambda_z) = 0$$

or $\quad -4.297 \times 10^{-3}(\lambda_x)_1 + \left(18.62 \times 10^{-3} - 10.02 \times 10^{-3}\right)(\lambda_y)_1 = 0$

$$-4.297 \times 10^{-3}(\lambda_x)_1 + \left(18.62 \times 10^{-3} - 10.02 \times 10^{-3}\right)(\lambda_z)_1 = 0$$

PROBLEM 9.186 CONTINUED

$$(\lambda_y)_1 = (\lambda_z)_1 = 0.49965(\lambda_x)_1$$

$$(\lambda_x)_1^2 + 2\left[0.49965(\lambda_x)_1\right]^2 = 1$$

$$(\lambda_x)_1 = 0.81669$$

$$(\lambda_y)_1 = (\lambda_z)_1 = 0.40806$$

$$(\theta_x)_1 = 35.2°; \quad (\theta_y)_1 = (\theta_z)_1 = 65.9° \blacktriangleleft$$

K$_2$: Begin with Equations (9.54a) and (9.54b):

$$(I_x - K_2)(\lambda_x)_2 - I_{xy}(\lambda_y)_2 - I_{zx}(\lambda_z)_2 = 0$$

$$-I_{xy}(\lambda_x)_2 + (I_y - K_2)(\lambda_y)_2 - \cancelto{0}{I_{yz}}(\lambda_z)_2 = 0$$

Substituting:

$$\left(14.32 \times 10^{-3} - 18.62 \times 10^{-3}\right)(\lambda_x)_2 - 4.297 \times 10^{-3}\left[(\lambda_y)_2 - (\lambda_z)_2\right] = 0 \qquad \text{(i)}$$

$$-4.297 \times 10^{-3}(\lambda_x)_2 + \left(18.62 \times 10^{-3} - 18.62 \times 10^{-3}\right)(\lambda_y)_2 = 0 \qquad \text{(ii)}$$

From (ii) $\quad (\lambda_x)_2 = 0$

From (i) $\quad (\lambda_y)_2 = -(\lambda_z)_2$

Substituting: $\quad (\lambda_x)_2^2 + (\lambda_y)_2^2 + \left[-(\lambda_z)_2\right]^2 = 1$

$$(\lambda_y)_2 = \frac{1}{\sqrt{2}}$$

$$(\theta_x)_2 = 90.0°, \ (\theta_y)_2 = 45.0°, \ (\theta_z)_2 = 135.0° \blacktriangleleft$$

K$_3$: Begin with Equations (9.54b) and (9.54c)

$$-I_{xy}(\lambda_x)_3 + (I_y - K_3)(\lambda_y)_3 + I_{yz}(\lambda_z)_3 = 0$$

$$-I_{zx}(\lambda_x)_3 - \cancelto{0}{I_{yz}}(\lambda_y) + (I_z - K_3)(\lambda_z)_3 = 0$$

Substituting:

$$-4.297 \times 10^{-3}(\lambda_x)_3 + \left(18.62 \times 10^{-3} - 22.9 \times 10^{-3}\right)(\lambda_z)_3 = 0$$

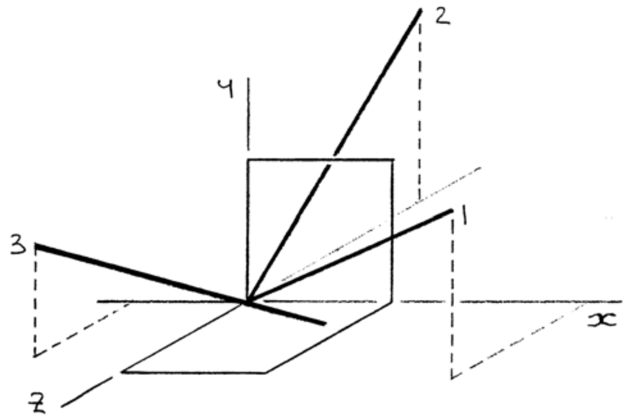
$$-4.297 \times 10^{-3}(\lambda_x)_3 + \left(18.62 \times 10^{-3} - 22.9 \times 10^{-3}\right)(\lambda_z)_3 = 0$$

Simplifying: $\quad (\lambda_y)_3 = (\lambda_z)_3 = -(\lambda_x)_3$

$$(\lambda_x)_3^2 + 2\left[-(\lambda_x)_3\right]^2 = 1 \Rightarrow (\lambda_x)_3 = \frac{1}{\sqrt{3}} \quad \text{and} \quad (\lambda_y)_3 = (\lambda_z)_3 = -\frac{1}{\sqrt{3}}$$

$$(\theta_x)_3 = 54.7°, \ (\theta_y)_3 = (\theta_z)_3 = 125.3° \blacktriangleleft$$

PROBLEM 9.186 CONTINUED

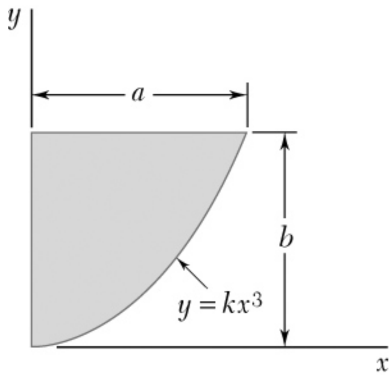


(c) Note: Principal axis 3 has been labeled so that the principal axes form a right-handed set to obtain the direction cosines corresponding to the labeled axis, the negative root of Equation (i) must be chosen; that is:

$$(\lambda_x)_3 = -\frac{1}{\sqrt{3}}$$

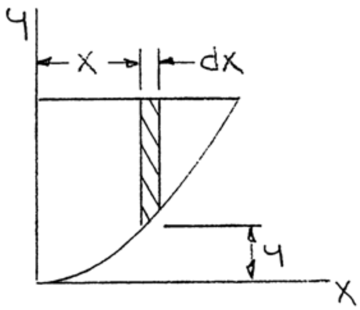
Then: $(\theta_x)_3 = 125.3°\quad (\theta_y)_3 = (\theta_z)_3 = 54.7°$ ◀

PROBLEM 9.187



Determine by direct integration the moment of inertia of the shaded area with respect to the y axis.

SOLUTION



At $\qquad x = a,\ y = b: \quad b = ka^3$

or $\qquad k = \dfrac{b}{a^3}$

Then $\qquad y = \dfrac{b}{a^3}x^3$

Now $\qquad dI_y = x^2 dA = x^2\left[(b - y)dx\right]$

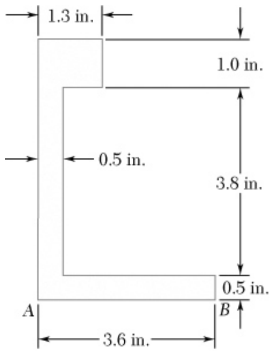
$$= bx^2\left(1 - \frac{x^3}{a^3}\right)dx$$

Then $\qquad I_y = \int dI_y = \int_0^a bx^2\left(1 - \dfrac{x^3}{a^3}\right)dx$

$$= b\left[\frac{1}{3}x^3 - \frac{x^6}{6a^3}\right]_0^a$$

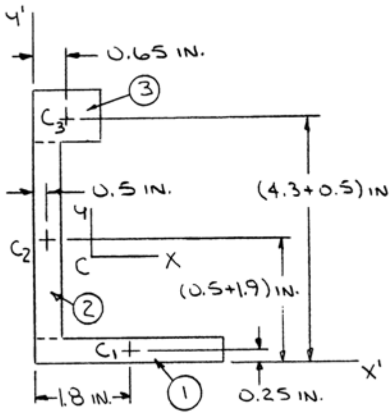
or $\qquad I_y = \dfrac{1}{6}a^3 b$ ◀

PROBLEM 9.188



Determine the moments of inertia $\bar{I}_x$ and $\bar{I}_y$ of the area shown with respect to centroidal axes respectively parallel and perpendicular to side AB.

SOLUTION



First locate centroid C of the area.

	A, in^2	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}A$, in^3	$\bar{y}A$, in^3
1	$3.6 \times 0.5 = 1.8$	1.8	0.25	3.24	0.45
2	$0.5 \times 3.8 = 1.9$	0.25	2.4	0.475	4.56
3	$1.3 \times 1 = 1.3$	0.65	4.8	0.845	6.24
Σ	5.0			4.560	11.25

Then $\quad \bar{X}\Sigma A = \Sigma \bar{x}A: \quad \bar{X}(5\ \text{in}^2) = 4.560\ \text{in}^3$

or $\quad \bar{X} = 0.912\ \text{in.}$

And $\quad \bar{Y}\Sigma A = \Sigma \bar{y}A: \quad \bar{Y}(5\ \text{in}^2) = 11.25\ \text{in}^3$

or $\quad \bar{Y} = 2.25\ \text{in.}$

Now $\quad \bar{I}_x = (I_x)_1 + (I_x)_2 + (I_x)_3$

where
$$(I_x)_1 = \frac{1}{12}(3.6\ \text{in.})(0.5\ \text{in.})^3 + (1.8\ \text{in}^2)\left[(2.25 - 0.25)\text{in.}\right]^2$$
$$= (0.0375 + 7.20)\ \text{in}^4 = 7.2375\ \text{in}^4$$
$$(I_x)_2 = \frac{1}{12}(0.5\ \text{in.})(3.8\ \text{in.})^3 + (1.9\ \text{in}^2)\left[(2.4 - 2.25)\ \text{in.}\right]^2$$
$$= (2.2863 + 0.0428)\ \text{in}^4 = 2.3291\ \text{in}^4$$
$$(I_x)_3 = \frac{1}{12}(1.3\ \text{in.})(1\ \text{in.})^3 + (1.3\ \text{in}^2)\left[(4.8 - 2.25\ \text{in.})\right]^2$$
$$= (0.1083 + 8.4533)\ \text{in}^4 = 8.5616\ \text{in}^4$$

Then
$$\bar{I}_x = (7.2375 + 2.3291 + 8.5616)\ \text{in}^4 = 18.1282\ \text{in}^4$$

or $\bar{I}_x = 18.13\ \text{in}^4$ ◀

PROBLEM 9.188 CONTINUED

Also $$\bar{I}_y = \left(I_y\right)_1 + \left(I_y\right)_2 + \left(I_y\right)_3$$

where $$\left(I_y\right)_1 = \frac{1}{12}\left(0.5\text{ in.}\right)\left(3.6\text{ in.}\right)^3 + \left(1.8\text{ in}^2\right)\left[\left(1.8 - 0.912\right)\text{in.}\right]^2$$

$$= \left(1.9440 + 1.4194\right)\text{in}^4 = 3.3634\text{ in}^4$$

$$\left(I_y\right)_2 = \frac{1}{12}\left(3.8\text{ in.}\right)\left(0.5\text{ in.}\right)^3 + \left(1.9\text{ in}^2\right)\left[\left(0.912 - 0.25\right)\text{in.}\right]^2$$

$$= \left(0.0396 + 0.8327\right)\text{in}^4 = 0.8723\text{ in}^4$$

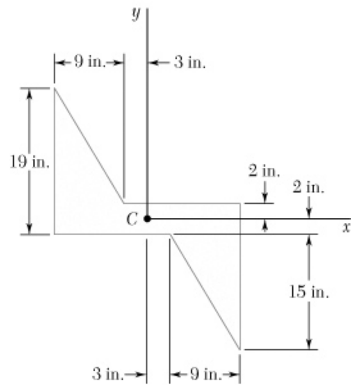
$$\left(I_y\right)_3 = \frac{1}{12}\left(1\text{ in.}\right)\left(1.3\text{ in.}\right)^3 + \left(1.3\text{ in}^2\right)\left[\left(0.912 - 0.65\right)\text{in.}\right]^2$$

$$= \left(0.1831 + 0.0892\right)\text{in}^4 = 0.2723\text{ in}^4$$

Then $$\bar{I}_y = \left(3.3634 + 0.8723 + 0.2723\right)\text{in}^4 = 4.5080\text{ in}^4$$

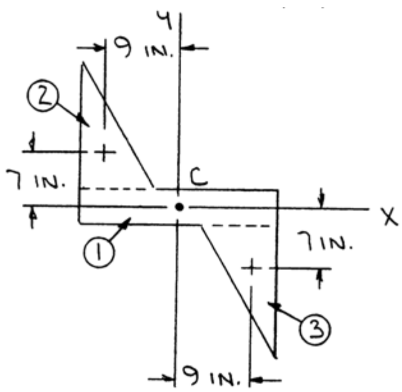
or $\bar{I}_y = 4.51\text{ in}^4$ ◀

PROBLEM 9.189



Using the parallel-axis theorem, determine the product of inertia of the area shown with respect to the centroidal *x* and *y* axes.

SOLUTION



Have $$\bar{I}_{xy} = \left(\bar{I}_{xy}\right)_1 + \left(I_{xy}\right)_2 + \left(I_{xy}\right)_3$$

Now, symmetry implies

$$\left(\bar{I}_{xy}\right)_1 = 0$$

and for each triangle $$I_{xy} = \bar{I}_{x'y'} + \bar{x}\,\bar{y}A$$

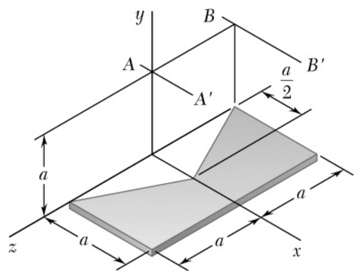
where, using the results of Sample Problem 9.6, $\bar{I}_{x'y'} = -\frac{1}{72}b^2h^2$ for both triangles. Note that the sign of $\bar{I}_{x'y'}$ is unchanged because the angles of rotation are $0°$ and $180°$ for triangles 2 and 3, respectively.
Now

	A, in^2	$\bar{x}$, in.	$\bar{y}$, in.	$\bar{x}\,\bar{y}A$, in^4
2	$\frac{1}{2}(9)(15) = 67.5$	–9	7	–4252.5
3	$\frac{1}{2}(9)(15) = 67.5$	9	–7	–4252.5
Σ				–8505

Then $$\bar{I}_{xy} = 2\left[-\frac{1}{72}(9\text{ in.})^2(15\text{ in.})^2\right] - 8505\text{ in}^4$$

$$= -9011.25\text{ in}^4$$

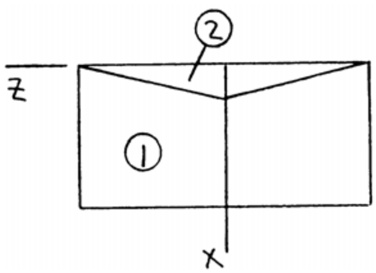
or $\bar{I}_{xy} = -9010\text{ in}^4$ ◀



PROBLEM 9.190

A piece of thin, uniform sheet metal is cut to form the machine component shown. Denoting the mass of the component by m, determine its moment of inertia with respect to (a) the x axis, (b) the y axis.

SOLUTION



First note

$$\text{mass} = m = \rho V = \rho t A$$

$$= \rho t\left[(2a)(a) - \frac{1}{2}(2a)\left(\frac{a}{2}\right)\right]$$

$$= \frac{3}{2}\rho t a^2$$

Also

$$I_{\text{mass}} = \rho t I_{\text{area}}$$

$$= \frac{2m}{3a^2} I_{\text{area}}$$

(a) Now

$$\bar{I}_{x,\text{area}} = (I_x)_{1,\text{area}} - 2(I_x)_{2,\text{area}}$$

$$= \frac{1}{12}(a)(2a)^3 - 2\left[\frac{1}{12}\left(\frac{a}{2}\right)(a)^3\right]$$

$$= \frac{7}{12}a^4$$

Then

$$\bar{I}_{x,\text{mass}} = \frac{2m}{3a^2} \times \frac{7}{12}a^4$$

or $\bar{I}_x = \frac{7}{18}ma^2$ ◀

(b) Have

$$\bar{I}_{z,\text{ area}} = (I_z)_{1,\text{ area}} - 2(I_z)_{2,\text{ area}}$$

$$= \frac{1}{3}(2a)(a)^3 - 2\left[\frac{1}{12}(a)\left(\frac{a}{2}\right)^3\right]$$

$$= \frac{31}{48}a^4$$

Then

$$I_{z,\text{mass}} = \frac{2m}{3a^2} \times \frac{31}{48}a^4$$

$$= \frac{31}{72}ma^2$$

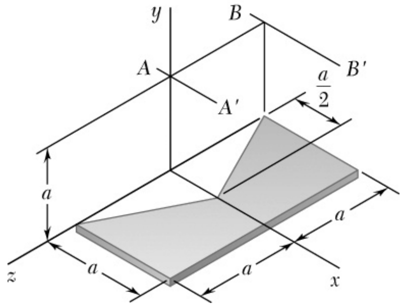
PROBLEM 9.190 CONTINUED

Finally, $$I_{y,\,\text{mass}} = \bar{I}_{x,\,\text{mass}} + I_{z,\,\text{mass}}$$

$$= \frac{7}{18}ma^2 + \frac{31}{72}ma^2$$

$$= \frac{59}{72}ma^2$$

or $I_y = 0.819\ \text{ma}^2$ ◀



PROBLEM 9.191

A piece of thin, uniform sheet metal is cut to form the machine component shown. Denoting the mass of the component by m, determine its moment of inertia with respect to (a) the axis AA' (b) the axis BB' where the AA' and BB' axes are parallel to the x axis and lie in a plane parallel to and at a distance a above the zx plane.

SOLUTION

First note that the x axis is a centroidal axis so that

$$I = \bar{I}_{x,\,\text{mass}} + md^2$$

and that from the solution to Problem 9.115,

$$\bar{I}_{x,\,\text{mass}} = \frac{7}{18}ma^2$$

(a) Have

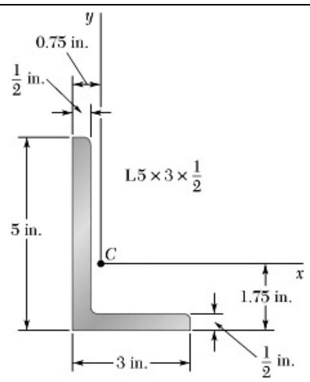
$$I_{AA',\,\text{mass}} = \frac{7}{18}ma^2 + m(a)^2$$

or $I_{AA'} = 1.389ma^2$ ◀

(b) Have

$$I_{BB',\,\text{mass}} = \frac{7}{18}ma^2 + m\left(a\sqrt{2}\right)^2$$

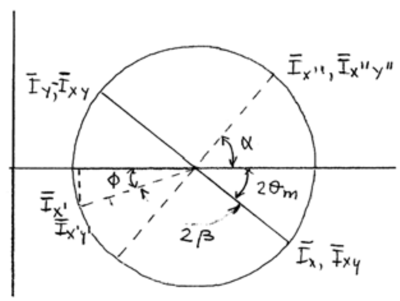
or $I_{BB'} = 2.39ma^2$ ◀



PROBLEM 9.192

For the $5 \times 3 \times \frac{1}{2}$-in. angle cross section shown, use Mohr's circle to determine (*a*) the moments of inertia and the product of inertia with respect to new centroidal axes obtained by rotating the *x* and *y* axes 45° counterclockwise, (*b*) the orientation of new centroidal axes for which $\bar{I}_x = 2\text{ in}^4$ and $\bar{I}_{x'y'} < 0$.

SOLUTION



From Figure 9.13A $\qquad \bar{I}_x = 9.45\text{ in}^4$

$$\bar{I}_y = 2.58\text{ in}^4$$

From Problem 9.106 $\qquad \bar{I}_{xy} = -2.8125\text{ in}^4$

Now

$$\bar{I}_{\text{ave}} = \frac{1}{2}\left(\bar{I}_x + \bar{I}_y\right)$$

$$= \frac{1}{2}(9.45 + 2.58)\text{ in}^4 = 6.015\text{ in}^4$$

$$R = \sqrt{\left(\frac{\bar{I}_x - \bar{I}_y}{2}\right)^2 + \bar{I}_{xy}^2}$$

$$= \left(\frac{9.45 - 2.58}{2}\right)^2 + (-2.8125)^2$$

$$= 4.4395\text{ in}^4$$

$$\tan 2\theta_m = -\frac{2\bar{I}_{xy}}{\bar{I}_x - \bar{I}_y} = -\frac{2(-2.8125)\text{ in}^4}{(9.45 - 2.58)\text{ in}^4} = 0.81877$$

$$2\theta_m = \tan^{-1}(0.81877) = 39.310°$$

PROBLEM 9.192 CONTINUED

(*a*) First note $\alpha = 90° - 39.310° = 50.690°$

Then $\bar{I}_{x''}, \bar{I}_{y''} = \bar{I}_{\text{ave}} \pm R\cos\alpha = (6.015 \pm 4.4395\cos\alpha)\text{ in}^4$

or $\bar{I}_{x''} = 8.83\text{ in}^4$ ◀

and $\bar{I}_{y''} = 3.20\text{ in}^4$ ◀

Also $\bar{I}_{x''y''} = R\sin\alpha = 4.4395\sin\alpha$

or $\bar{I}_{x''y''} = 3.43\text{ in}^4$ ◀

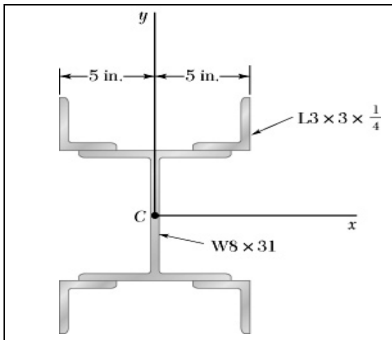
(*b*) Have $\bar{I}_{x'} = \bar{I}_{\text{ave}} - R\cos\phi$

or $2 = 6.015 - 4.4395\cos\phi$ or $\phi = 25.260°$

Then $2\beta = 180° - (39.310° + 25.260°) = 115.430°$

or $\beta = 57.715°$

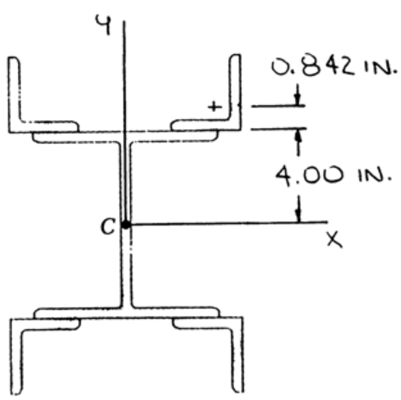
Rotate the centroidal axis $57.7°$ clockwise. ◀



PROBLEM 9.193

Four $3 \times 3 \times \frac{1}{4}$-in. angles are welded to a rolled W section as shown. Determine the moments of inertia and the radii of gyration of the combined section with respect to its centroidal *x* and *y* axes.

SOLUTION



W section: $A = 9.13\,\text{in}^2$

$$\bar{I}_x = 110\,\text{in}^4$$

$$\bar{I}_y = 37.1\,\text{in}^4$$

Angle: $A = 1.44\,\text{in}^2$

$$\bar{I}_x = \bar{I}_y = 1.24\,\text{in}^4$$

Note: $A_{\text{total}} = A_{\text{W}} + 4A_{\text{angle}}$

$$= \left[9.13 + 4(1.44)\right]\text{in}^2$$

$$= 14.89\,\text{in}^2$$

Now $\bar{I}_x = \left(\bar{I}_x\right)_{\text{W}} + 4\left(I_x\right)_{\text{angle}}$

where $\left(I_x\right)_{\text{angle}} = \bar{I}_x + Ad^2$

$$= 1.24\,\text{in}^4 + \left(1.44\,\text{in}^2\right)\left[(4.00 + 0.842)\,\text{in.}\right]^2$$

$$= 35.0007\,\text{in}^4$$

Then $\bar{I}_x = \left[110 + 4(35.0007)\right]\text{in}^4 = 250.0028\,\text{in}^4$

or $\bar{I}_x = 250\,\text{in}^4$ ◀

and $\bar{k}_x^2 = \dfrac{\bar{I}_x}{A_{\text{total}}} = \dfrac{250.0028\,\text{in}^4}{14.89\,\text{in}^2}$

or $\bar{k}_x = 4.10\,\text{in.}$ ◀

also $\bar{I}_y = \left(\bar{I}_y\right)_{\text{W}} + 4\left(I_y\right)_{\text{angle}}$

PROBLEM 9.193 CONTINUED

where $\left(I_y\right)_{\text{angle}} = \bar{I}_y + Ad^2$

$$= 1.24 \text{ in}^4 + \left(1.44 \text{ in}^2\right)\left[\left(5 - 0.842\right)\text{in.}\right]^2$$

$$= 26.1361 \text{ in}^4$$

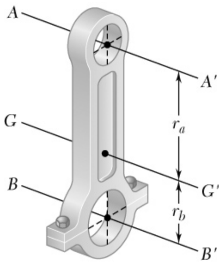
Then $\bar{I}_y = \left[37.1 + 4\left(26.1361\right)\right]\text{in}^4 = 141.6444 \text{ in}^4$

or $\bar{I}_y = 141.6 \text{ in}^4$ ◀

and $\bar{k}_y^2 = \dfrac{\bar{I}_y}{A_{\text{total}}} = \dfrac{141.6444 \text{ in}^4}{14.89 \text{ in}^2}$

or $\bar{k}_y = 3.08 \text{ in.}$ ◀

PROBLEM 9.194



For the 2-kg connecting rod shown, it has been experimentally determined that the mass moments of inertia of the rod with respect to the center-line axes of the bearings AA' and BB' are, respectively, $I_{AA'} = 78\text{ gm}^2$ and $I_{BB'} = 41\text{ gm}^2$. Knowing that $r_a + r_b = 290\text{ mm}$, determine (*a*) the location of the centroidal axis GG' (*b*) the radius of gyration with respect to axis GG'

SOLUTION

(a) Have $\quad I_{AA'} = \bar{I}_{GG'} + mr_a^2 \qquad r_a + r_b = 290\text{ mm}$

$$= 0.29\text{ m}$$

and $\quad I_{BB'} = \bar{I}_{GG'} + mr_b^2$

Subtracting $\quad I_{BB'} - I_{AA'} = m\left(r_b^2 - r_a^2\right)$

$$(41 - 78)\text{g}\cdot\text{m}^2 = (2000\text{ g})(r_b + r_a)(r_b - r_a)$$

or $\quad -37 = (2000)(0.29)(r_b - r_a)$

or $\quad r_a - r_b = 63.793 \times 10^{-3}\text{ m}$

now $\quad r_a + r_b = 0.29\text{ m}$

so that $\quad 2r_a = 0.35379\text{ m}$

$$r_a = 0.17689\text{ m}$$

or $r_a = 176.9\text{ mm}$ ◀

(b) Have $\quad I_{AA'} = \bar{I}_{GG'} + mr_a^2$

Then $\quad \bar{I}_{GG'} = 78\text{ g}\cdot\text{m}^2 - (2000\text{ g})(0.17689\text{ m})^2$

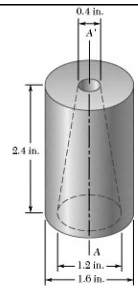
$$= 15.420\text{ g}\cdot\text{m}^2$$

Finally, $\quad \bar{k}_{GG'}^2 = \dfrac{\bar{I}_{GG'}}{m} = \dfrac{15.420\text{ g}\cdot\text{m}^2}{2000\text{ g}} = 0.007710\text{ m}^2$

$$\bar{k}_{GG'} = 0.08781\text{ m}$$

$\bar{k}_{GG'} = 87.8\text{ mm}$ ◀

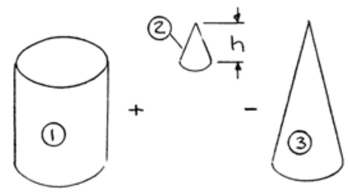
PROBLEM 9.195



Determine the mass moment of inertia of the 0.9-lb machine component shown with respect to the axis AA'.

SOLUTION

First note that the given shape can be formed adding a small cone to a cylinder and then removing a larger cone as indicated.



Now $$\frac{h}{0.4} = \frac{h + 2.4}{1.2} \quad \text{or} \quad h = 1.2 \text{ in.}$$

The weight of the body is given by

$$W = mg = g\left(m_1 + m_2 - m_3\right) = \rho g\left(V_1 + V_2 - V_3\right)$$

or $$0.9 \text{ lb} = \rho \times 32.2 \text{ ft/s}^2\left[\pi(0.8)^2(2.4) + \frac{\pi}{3}(0.2)^2(1.2) - \frac{\pi}{3}(0.6)^2(3.6)\right]\text{in}^3 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^3$$

$$= \rho \times 32.2 \text{ ft/s}^2\left(2.79253 + 0.02909 - 0.78540\right) \times 10^{-3} \text{ ft}^3$$

or $$\rho = 13.7266 \text{ lb}\cdot\text{s}^2/\text{ft}^4$$

Then $$m_1 = (13.7266)\left(2.79253 \times 10^{-3}\right) = 0.038332 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m_2 = (13.7266)\left(0.02909 \times 10^{-3}\right) = 0.000399 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$m_3 = (13.7266)\left(0.78540 \times 10^{-3}\right) = 0.010781 \text{ lb}\cdot\text{s}^2/\text{ft}$$

Finally, using Figure 9.28 have,

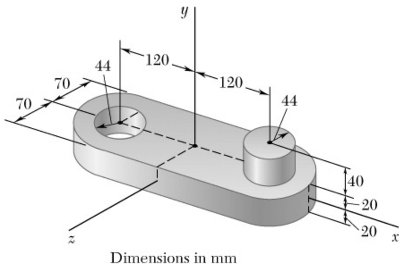
$$I_{AA'} = \left(I_{AA'}\right)_1 + \left(I_{AA'}\right)_2 - \left(I_{AA'}\right)_3$$

$$= \frac{1}{2}m_1a_1^2 + \frac{3}{10}m_2a_2^2 - \frac{3}{10}m_3a_3^2$$

$$= \left[\frac{1}{2}(0.038332)(0.8)^2 + \frac{3}{10}(0.000399)(0.2)^2 - \frac{3}{10}(0.010781)(0.6)^2\right](\text{lb}\cdot\text{s}^2/\text{ft}) \times \text{in}^2 \times \left(\frac{1 \text{ ft}}{12 \text{ in.}}\right)^2$$

$$= \left(85.1822 + 0.0333 - 8.0858\right) \times 10^{-6} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$$

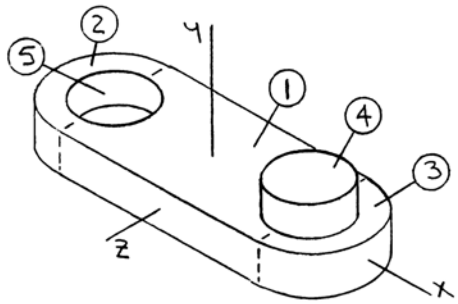
or $I_{AA'} = 77.1 \times 10^{-6} \text{ lb}\cdot\text{ft}\cdot\text{s}^2$ ◀



PROBLEM 9.196

Determine the moments of inertia and the radii of gyration of the steel machine element shown with respect to the *x* and *y* axes. (The density of steel is 7850 kg/m^3.)

SOLUTION



First compute the mass of each component. Have

$$m = \rho_{st} V$$

Then
$$m_1 = \left(7850 \text{ kg/m}^3\right)\left(0.24 \times 0.04 \times 0.14\right)\text{m}^3$$

$$= 10.5504 \text{ kg}$$

$$m_2 = m_3 = \left(7850 \text{ kg/m}^3\right)\left[\frac{\pi}{2}(0.07)^2 \times 0.04\right]\text{m}^3 = 2.41683 \text{ kg}$$

$$m_4 = m_5 = \left(7850 \text{ kg/m}^3\right)\left[\pi(0.044)^2 \times (0.04)\right]\text{m}^3 = 1.90979 \text{ kg}$$

Using Figure 9.28 for components 1, 4, and 5 and the equations derived above (before the solution to Problem 9.144) for a semicylinder, have

$$I_x = (I_x)_1 + (I_x)_2 + (I_x)_3 + (I_x)_4 - (I_x)_5 \qquad \text{where} \qquad (I_x)_2 = (I_x)_3$$

$$= \left[\frac{1}{12}(10.5504 \text{ kg})\left(0.04^2 + 0.14^2\right)\text{m}^2\right] + 2\left\{\frac{1}{12}(2.41683 \text{ kg})\left[3(0.07 \text{ m})^2 + (0.04 \text{ m})^2\right]\right\}$$

$$+ \left\{\frac{1}{12}(1.90979 \text{ kg})\left[3(0.044 \text{ m})^2 + (0.04 \text{ m})^2\right] + (1.90979 \text{ kg})(0.04 \text{ m})^2\right\}$$

$$- \left\{\frac{1}{12}(1.90979 \text{ kg})\left[3(0.044 \text{ m})^2 + (0.04 \text{ m})^2\right]\right\}$$

$$= \left[(0.0186390) + 2(0.0032829) + (0.0011790 + 0.0030557) - (0.0011790)\right]\text{kg}\cdot\text{m}^2$$

$$= 0.0282605 \text{ kg}\cdot\text{m}^2$$

or $I_x = 28.3 \times 10^{-3} \text{ kg}\cdot\text{m}^2$ ◀

PROBLEM 9.196 CONTINUED

$$I_y = (I_y)_1 + (I_y)_2 + (I_y)_3 + (I_y)_4 - (I_y)_5$$

where $$(I_y)_2 = (I_y)_3 \qquad (I_y)_4 = \left|(I_y)_5\right|$$

Then $$I_y = \left[\frac{1}{12}(10.5504\text{ kg})\left(0.24^2 + 0.14^2\right)\text{m}^2\right]$$

$$+ 2\left[(2.41683\text{ kg})\left(\frac{1}{2} - \frac{16}{9\pi^2}\right)\left(0.07\text{ m}^2\right) + (2.41683\text{ kg})\left(0.12 + \frac{4 \times 0.07}{3\pi}\right)^2\text{m}^2\right]$$

$$= \left[(0.0678742) + 2(0.0037881 + 0.0541678)\right]\text{kg}\cdot\text{m}^2$$

$$= 0.1837860\text{ kg}\cdot\text{m}^2$$

or $I_y = 183.8 \times 10^{-3}\text{ kg}\cdot\text{m}^2$ ◀

Also $$m = m_1 + m_2 + m_3 + m_4 - m_5 \text{ where } m_2 = m_3,\ m_4 = |m_5|$$

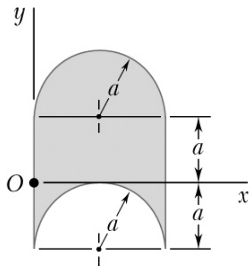
$$= (10.5504 + 2 \times 2.41683)\text{kg} = 15.38406\text{ kg}$$

Then $$k_x^2 = \frac{I_x}{m} = \frac{0.0282605\text{ kg}\cdot\text{m}^2}{15.38406\text{ kg}}$$

or $k_x = 42.9\text{ mm}$ ◀

and $$k_y^2 = \frac{I_y}{m} = \frac{0.1837860\text{ kg}\cdot\text{m}^2}{15.38406\text{ kg}}$$

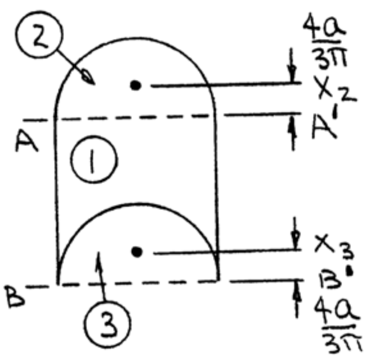
or $k_y = 109.3\text{ mm}$ ◀



PROBLEM 9.197

Determine the moments of inertia of the shaded area shown with respect to the x and y axes.

SOLUTION



Have $$I_x = (I_x)_1 + (I_x)_2 - (I_x)_3$$

where $$(I_x)_1 = \frac{1}{12}(2a)(2a)^3 = \frac{4}{3}a^4$$

Now $$(I_{AA})_2 = (I_{BB})_3 = \frac{\pi}{8}a^4$$

and $$(I_{AA})_2 = (\bar{I}_{xz})_2 + Ad^2$$

or $$(\bar{I}_{x_2})_2 = (\bar{I}_{x_3})_3 = \frac{\pi}{8}a^4 - \left(\frac{\pi}{2}a^2\right)\left(\frac{4a}{3\pi}\right)^2$$

$$= \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)a^4$$

Then $$(I_x)_2 = (\bar{I}_{x_2})_2 + Ad_2^2 = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)a^4 + \left(\frac{\pi}{2}a^2\right)\left(a + \frac{4a}{3\pi}\right)^2$$

$$= \left(\frac{4}{3} + \frac{5\pi}{8}\right)a^4$$

$$(I_x)_3 = (\bar{I}_{x_3})_3 + Ad_3^2 = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)a^4 + \left(\frac{\pi}{2}a^2\right)\left(a - \frac{4a}{3\pi}\right)^2$$

$$= \left(-\frac{4}{3} + \frac{5\pi}{8}\right)a^4$$

Finally $$I_x = \frac{4}{3}a^4 + \left[\left(\frac{4}{3} + \frac{5\pi}{8}\right)a^4\right] - \left[\left(-\frac{4}{3} + \frac{5\pi}{8}\right)a^4\right]$$

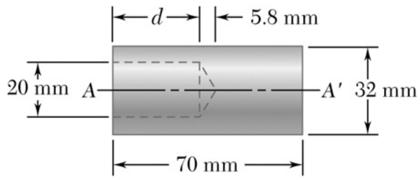
or $I_x = 4a^4$ ◀

Also $$I_y = (I_y)_1 + (I_y)_2 - (I_y)_3$$

where $$(I_y)_1 = \frac{1}{12}(2a)(2a)^3 + (2a)^2(a)^2 = \frac{16}{3}a^4$$

$$(I_y)_2 = (I_y)_3 \quad \left[= \frac{\pi}{8}a^4 + \left(\frac{\pi}{2}a^2\right)(a)^2\right] \quad \therefore\ I_y = \frac{16}{3}a^4$$ ◀

PROBLEM 9.198



A 20-mm-diameter hole is bored in a 32-mm-diameter rod as shown. Determine the depth d of the hole so that the ratio of the moments of inertia of the rod with and without the hole with respect to the axis AA' is 0.96.

SOLUTION

First note

Cylinder:
$$I_{AA'} = \frac{1}{2}ma^2 \qquad m = \rho V = \rho \times \left(\pi a^2 L\right)$$
$$= \frac{1}{2}\pi\rho a^4 L$$

Cone:
$$I_{AA'} = \frac{3}{10}ma^2 \qquad m = \rho V = \rho \times \left(\frac{\pi}{3}a^2 h\right)$$
$$= \frac{1}{10}\pi\rho a^4 h$$

Now
$$\frac{\left(I_{AA'}\right)_{\text{bored}}}{\left(I_{AA'}\right)_{\text{solid}}} = 0.96$$

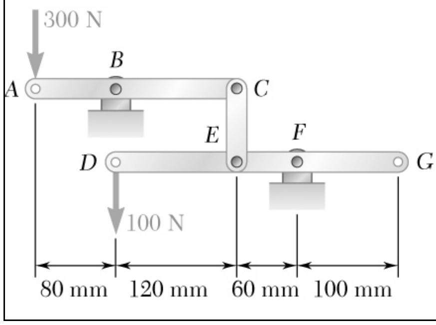
or
$$0.96\left(I_{AA'}\right)_{\text{solid}} = \left(I_{AA'}\right)_{\text{solid}} - \left(I_{AA'}\right)_{\text{hole}}$$

or
$$0.04\left(I_{AA'}\right)_{\text{solid}} = \left[\left(I_{AA'}\right)_{\text{cylinder}} + \left(I_{AA'}\right)_{\text{cone}}\right]_{\text{hole}}$$

Then
$$0.04\left(\frac{1}{2}\pi\rho a_{\text{rod}}^4 L_{\text{rod}}\right) = \frac{1}{2}\pi\rho a_{\text{hole}}^4 d + \frac{1}{10}\pi\rho a_{\text{hole}}^4 h_{\text{cone}}$$

or
$$d = 0.04\left(\frac{a_{\text{rod}}}{a_{\text{hole}}}\right)^4 L_{\text{rod}} - \frac{1}{5}h_{\text{cone}}$$
$$= 0.04\left(\frac{16\text{ mm}}{10\text{ mm}}\right)^4 (70\text{ mm}) - \frac{1}{5}(5.8\text{ mm})$$

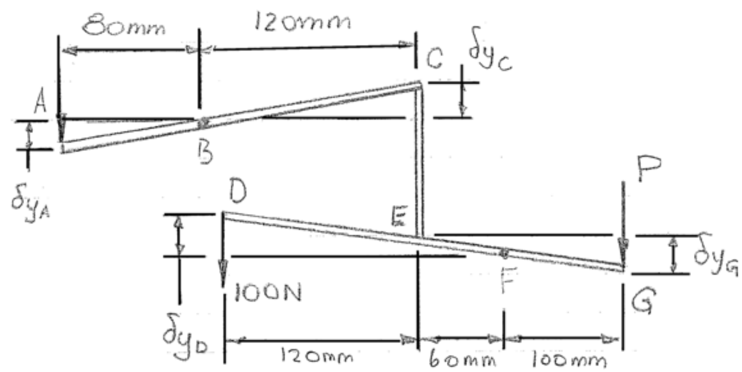
or $d = 17.19$ mm ◀



PROBLEM 10.1

Determine the vertical force **P** which must be applied at G to maintain the equilibrium of the linkage.

SOLUTION



Assuming

$$\delta y_A \downarrow$$

it follows

$$\delta y_C = \frac{120}{80}\delta y_A = 1.5\delta y_A \uparrow$$

$$\delta y_E = \delta y_C = 1.5\delta y_A \uparrow$$

$$\delta y_D = \frac{180}{60}\delta y_A = 3(1.5\delta y_A) = 4.5\delta y_A \uparrow$$

$$\delta y_G = \frac{100}{60}\delta y_A = \frac{100}{60}(1.5\delta y_A) = 2.5\delta y_A \downarrow$$

Then, by Virtual Work

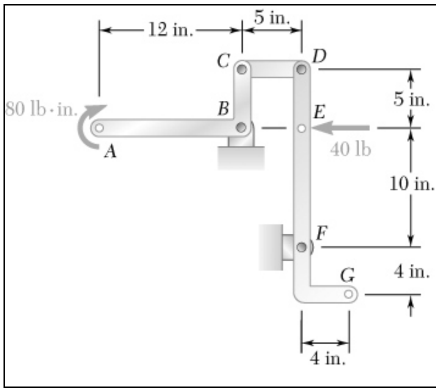
$$\delta U = 0: \quad (300\text{ N})\delta y_A - (100\text{ N})\delta y_D + P\delta y_G = 0$$

$$300\delta y_A - 100(4.5\delta y_A) + P(2.5\delta y_A) = 0$$

$$300 - 450 + 2.5P = 0$$

$$P = +60\text{ N}$$

$\mathbf{P} = 60\text{ N} \downarrow$ ◀

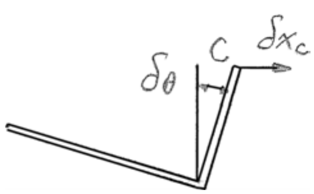


PROBLEM 10.2

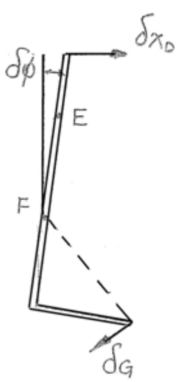
Determine the vertical force **P** which must be applied at G to maintain the equilibrium of the linkage.

SOLUTION

Link ABC



Link DEFG



Assume

$$\delta\theta \text{ clockwise}$$

Then for point C

$$\delta x_C = (5\delta\theta) \text{ in.} \longrightarrow$$

and for point D

$$\delta x_D = \delta x_C = (5\delta\theta) \text{ in.} \longrightarrow$$

And for link $DEFG$

$$\delta x_D = 15\delta\phi$$

$$\therefore \quad 5\delta\theta = 15\delta\phi$$

or $$\delta\phi = \frac{1}{3}\delta\theta$$

Then $$\delta_G = 4\sqrt{2}\delta\phi = \left(\frac{4}{3}\sqrt{2}\delta\theta\right) \text{ in.}$$

Now $$\delta y_G = \delta_G \cos 45°$$

$$= \left(\frac{4}{3}\sqrt{2}\delta\theta\right)\cos 45°$$

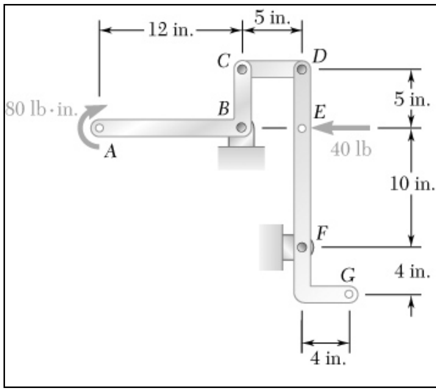
$$= \left(\frac{4}{3}\delta\theta\right) \text{ in.}$$

Then, by Virtual Work.

$$\delta U = 0\text{:} \quad (80 \text{ lb}\cdot\text{in.})\delta\theta - (40 \text{ lb})\delta x_E (\text{in.}) + P\delta y_G (\text{in.}) = 0$$

$$80\delta\theta - 40\left(\frac{10}{3}\delta\theta\right) + P\left(\frac{4}{3}\delta\theta\right) = 0$$

or $\mathbf{P} = 40 \text{ lb} \downarrow$ ◀

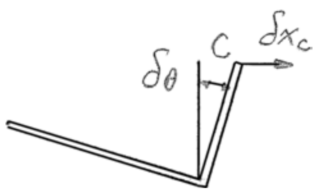


PROBLEM 10.3

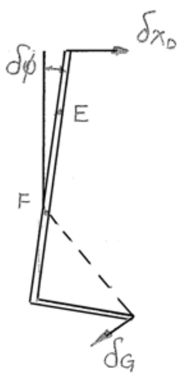
Determine the couple **M** which must be applied to member *DEFG* to maintain the equilibrium of the linkage.

SOLUTION

Link ABC



Link DEFG



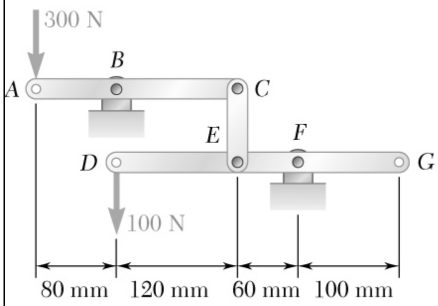
Following the kinematic analysis of Problem 10.2, we have $U = 0$:

$$\delta U = 0\!: \quad (80\text{ lb}\cdot\text{in.})\delta\theta - (40\text{ lb})\delta x_E(\text{in.}) + M\delta\phi = 0$$

$$80\delta\theta - 40\left(\frac{10}{3}\delta\theta\right) + M\left(\frac{1}{3}\delta\theta\right) = 0$$

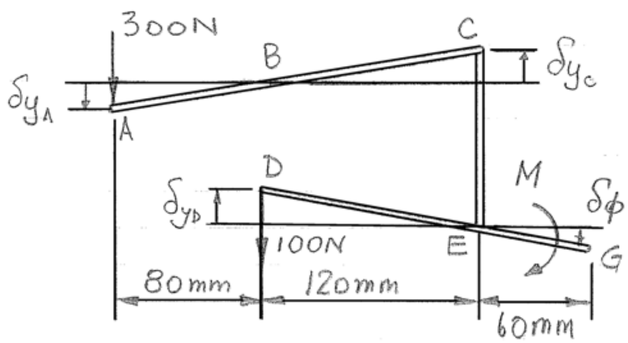
or $\mathbf{M} = 160\text{ lb}\cdot\text{in.}$ ◀

PROBLEM 10.4



Determine the couple **M** which must be applied to member *DEFG* to maintain the equilibrium of the linkage.

SOLUTION



Assuming

$$\delta y_A \downarrow$$

it follows

$$\delta y_C = \frac{120}{80}\delta y_A = 1.5\delta y_A \uparrow$$

$$\delta y_E = \delta y_C = 1.5\delta y_A \uparrow$$

$$\delta y_D = \frac{180}{60}\delta y_A = 3(1.5\delta y_A) = 4.5\delta y_A \uparrow$$

$$\delta\phi = \frac{\delta y_E}{60} = \frac{1.5\delta y_A}{60} = \frac{1}{40}\delta y_A$$

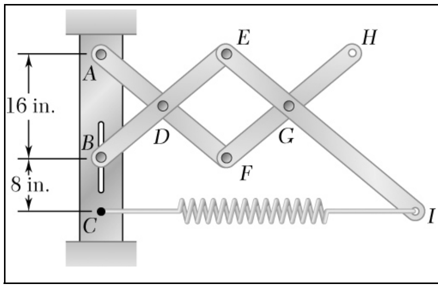
Then, by Virtual Work:

$$\delta U = 0: \quad (300\text{ N})\delta y_A - (100\text{ N})\delta y_D + M\delta\phi = 0$$

$$300\delta y_A - 100(4.5\delta y_A) + M\left(\frac{1}{40}\delta y_A\right) = 0$$

$$300 - 450 + \frac{1}{40}M = 0$$

$$M = +6000\text{ N}\cdot\text{mm} \circlearrowright \qquad \mathbf{M} = 6.00\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$



PROBLEM 10.5

An unstretched spring of constant 4 lb/in. is attached to pins at points C and I as shown. The pin at B is attached to member BDE and can slide freely along the slot in the fixed plate. Determine the force in the spring and the horizontal displacement of point H when a 20-lb horizontal force directed to the right is applied (*a*) at point G, (*b*) at points G and H.

SOLUTION

First note:

$$x_G = 3x_D \Rightarrow \delta x_G = 3\delta x_D$$

$$x_H = 4x_D \Rightarrow \delta x_H = 4\delta x_D$$

$$x_I = 5x_D \Rightarrow \delta x_I = 5\delta x_D$$

(*a*) Virtual Work $\delta U = 0$: $F_G \delta x_G - F_{SP} \delta x_I = 0$

or $$(20 \text{ lb})(3\delta x_D) - F_{SP}(5\delta x_D) = 0$$

thus, $F_{SP} = 12.00 \text{ lb } T$ ◀

Now $$F_{SP} = k\Delta x_I$$

or $$12.00 \text{ lb} = (4 \text{ lb/in.})\Delta x_I$$

Thus, $$\Delta x_I = 3 \text{ in.}$$

and $$\delta x_D = \frac{1}{4}\delta x_H = \frac{1}{5}\delta x_I$$

$$\therefore \quad \Delta x_H = \frac{4}{5}\Delta x_I$$

$$= \frac{4}{5}(3 \text{ in.})$$

or $\Delta x_H = 2.40 \text{ in.} \longrightarrow$ ◀

(*b*) Virtual Work: $\delta U = 0$: $F_G \delta x_G + F_H \delta x_H - F_{SP} \delta x_I = 0$

or $$(20 \text{ lb})(3\delta x_D) + (20 \text{ lb})(4\delta x_D) - F_{SP}(5\delta x_D) = 0$$

thus, $F_{SP} = 28.0 \text{ lb } T$ ◀

Now $$F_{SP} = k\Delta x_I$$

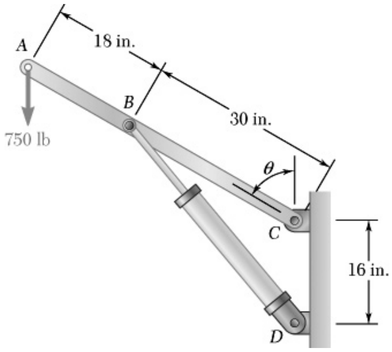
or $$28.0 \text{ lb} = (4 \text{ lb/in.})\Delta x_I$$

Thus, $$\Delta x_I = 7 \text{ in.}$$

From part (*a*) $$\Delta x_H = \frac{4}{5}\Delta x_I$$

$$= \frac{4}{5}(7 \text{ in.})$$

or $\Delta x_H = 5.60 \text{ in.} \longrightarrow$ ◀



PROBLEM 10.6

An unstretched spring of constant 4 lb/in. is attached to pins at points *C* and *I* as shown. The pin at *B* is attached to member *BDE* and can slide freely along the slot in the fixed plate. Determine the force in the spring and the horizontal displacement of point *H* when a 20-lb horizontal force directed to the right is applied (*a*) at point *E*, (*b*) at points *D* and *E*.

SOLUTION

First note:

$$x_G = 3x_D \Rightarrow \delta x_G = 3\delta x_D$$

$$x_H = 4x_D \Rightarrow \delta x_H = 4\delta x_D$$

$$x_I = 5x_D \Rightarrow \delta x_I = 5\delta x_D$$

(*a*) Virtual Work: $\delta U = 0$: $F_E \delta x_E - F_{SP} \delta x_I = 0$

or $$(20 \text{ lb})(2\delta x_D) - F_{SP}(5\delta x_D) = 0$$

thus, $F_{SP} = 8.00 \text{ lb } T$ ◀

Now $$F_{SP} = k\Delta x_I$$

or $$8.00 \text{ lb} = (4 \text{ lb/in.})\Delta x_I$$

Thus, $$\Delta x_I = 2 \text{ in.}$$

And $$\delta x_D = \frac{1}{4}\delta x_H = \frac{1}{5}\delta x_I$$

$$\therefore \quad \Delta x_H = \frac{4}{5}\Delta x_I$$

$$= \frac{4}{5}(2 \text{ in.})$$

or $\Delta x_H = 1.600 \text{ in.} \longrightarrow$ ◀

(*b*) Virtual Work: $\delta U = 0$: $F_D \delta x_D + F_E \delta x_E - F_{SP} \delta x_I = 0$

or $$(20 \text{ lb})\delta x_D + (20 \text{ lb})(2\delta x_D) - F_{SP}(5\delta x_D) = 0$$

thus, $F_{SP} = 12.00 \text{ lb } T$ ◀

PROBLEM 10.6 CONTINUED

Now $$F_{SP} = k\Delta x_I$$

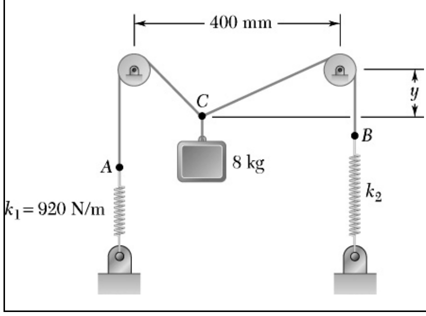
or $$12.00 \text{ lb} = (4 \text{ lb/in.})\Delta x_I$$

Thus, $$\Delta x_I = 3 \text{ in.}$$

From part (*a*) $$\Delta x_H = \frac{4}{5}\Delta x_I$$

$$= \frac{4}{5}(3 \text{ in.})$$

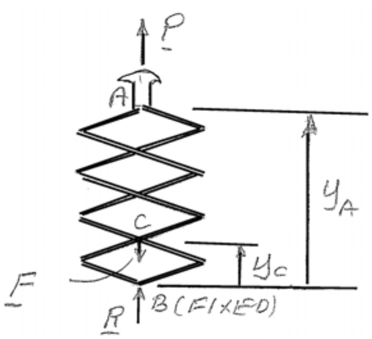
or $\Delta x_H = 2.40 \text{ in.} \longrightarrow$ ◀



PROBLEM 10.7

Knowing that the maximum friction force exerted by the bottle on the cork is 300 N, determine (*a*) the force **P** which must be applied to the corkscrew to open the bottle, (*b*) the maximum force exerted by the base of the corkscrew on the top of the bottle.

SOLUTION



From sketch

$$y_A = 4y_C$$

Thus, $$\delta y_A = 4\delta y_C$$

(*a*) Virtual Work:

$$\delta U = 0: \quad P\delta y_A - F\delta y_C = 0$$

$$P = \frac{1}{4}F$$

$$F = 300 \text{ N}: \quad P = \frac{1}{4}(300 \text{ N}) = 75 \text{ N}$$

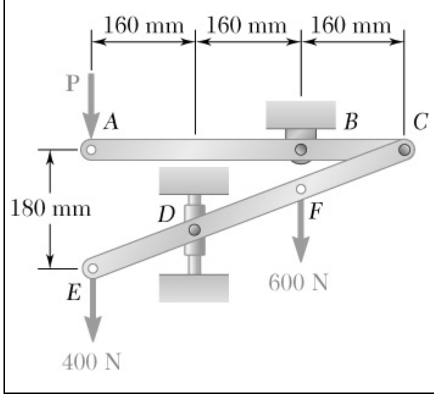
$$\mathbf{P} = 75.0 \text{ N} \uparrow \blacktriangleleft$$

(*b*) Free body: Corkscrew

$$+\uparrow \Sigma F_y = 0: \quad R + P - F = 0$$

$$R + 75 \text{ N} - 300 \text{ N} = 0$$

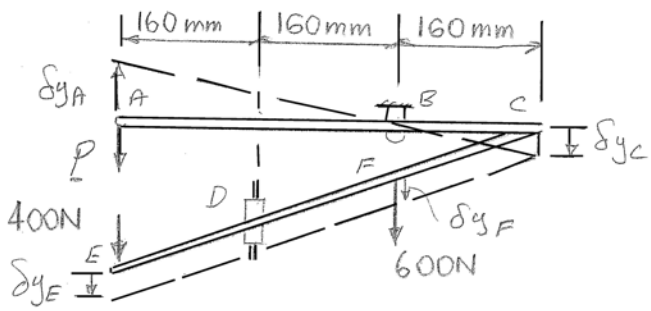
$$\mathbf{R} = 225 \text{ N} \downarrow \blacktriangleleft$$



PROBLEM 10.8

The two-bar linkage shown is supported by a pin and bracket at B and a collar at D that slides freely on a vertical rod. Determine the force **P** required to maintain the equilibrium of the linkage.

SOLUTION



Assume $\delta y_A \uparrow$

Have $\delta y_C = \dfrac{160}{320}\delta y_A$ or $\delta y_C = \dfrac{1}{2}\delta y_A \downarrow$

Since bar CD moves in translation

$$\delta y_E = \delta y_F = \delta y_C$$

or

$$\delta y_E = \delta y_F = \frac{1}{2}\delta y_A \downarrow$$

Virtual Work:

$$\delta U = 0: \quad -P\delta y_A + (400\text{ N})\delta y_E + (600\text{ N})\delta y_F = 0$$

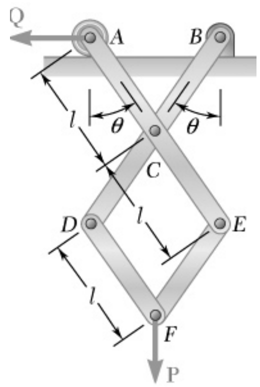
$$-P\delta y_A + (400\text{ N})\left(\frac{1}{2}\delta y_A\right) + (600\text{ N})\left(\frac{1}{2}\delta y_A\right) = 0$$

or

$$P = 500\text{ N}$$

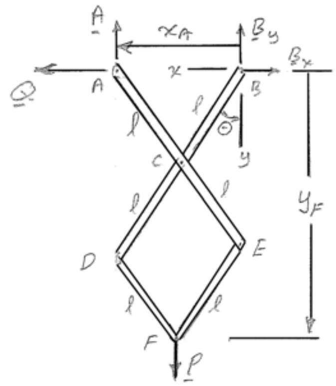
$\mathbf{P} = 500\text{ N} \downarrow$ ◀

PROBLEM 10.9



The mechanism shown is acted upon by the force **P**; derive an expression for the magnitude of the force **Q** required for equilibrium.

SOLUTION



Virtual Work:

Have $$x_A = 2l\sin\theta$$

$$\delta x_A = 2l\cos\theta\,\delta\theta$$

and $$y_F = 3l\cos\theta$$

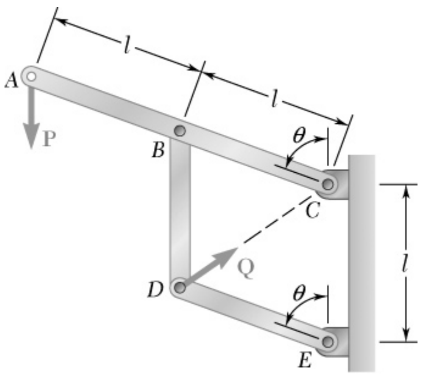
$$\delta y_F = -3l\sin\theta\,\delta\theta$$

Virtual Work: $$\delta U = 0:\quad Q\delta x_A + P\delta y_F = 0$$

$$Q\left(2l\cos\theta\,\delta\theta\right) + P\left(-3l\sin\theta\,\delta\theta\right) = 0$$

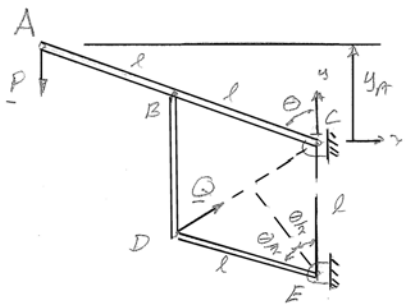
$$Q = \frac{3}{2}P\tan\theta \blacktriangleleft$$

PROBLEM 10.10



Knowing that the line of action of the force **Q** passes through point C, derive an expression for the magnitude of **Q** required to maintain equilibrium

SOLUTION



Have $\qquad y_A = 2l\cos\theta; \qquad \delta y_A = -2l\sin\theta\,\delta\theta$

$$CD = 2l\sin\frac{\theta}{2}; \qquad \delta(CD) = l\cos\frac{\theta}{2}\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad -P\delta y_A - Q\delta(CD) = 0$$

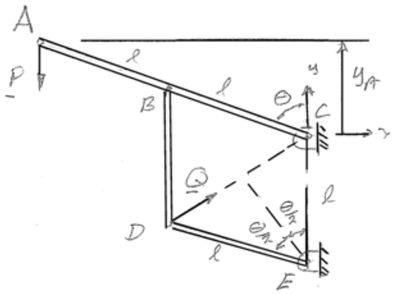
$$-P(-2l\sin\theta\,\delta\theta) - Q\left(l\cos\frac{\theta}{2}\delta\theta\right) = 0$$

$$Q = 2P\frac{\sin\theta}{\cos(\theta/2)} \blacktriangleleft$$

PROBLEM 10.11

Solve Problem 10.10 assuming that the force **P** applied at point *A* acts horizontally to the left.

SOLUTION



Have $\qquad x_A = 2l\sin\theta; \qquad \delta x_A = 2l\cos\theta\delta\theta$

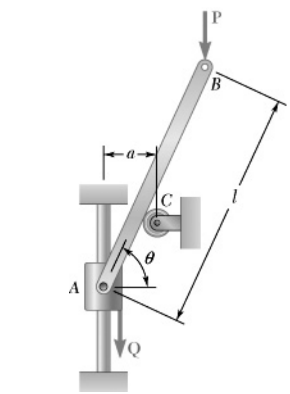
$$CD = 2l\sin\frac{\theta}{2}; \qquad \delta(CD) = l\cos\frac{\theta}{2}\delta\theta$$

Virtual Work: $\qquad \delta U = 0{:} \quad P\delta x_A - Q\delta(CD) = 0$

$$P(2l\cos\theta\delta\theta) - Q\left(l\cos\frac{\theta}{2}\delta\theta\right) = 0$$

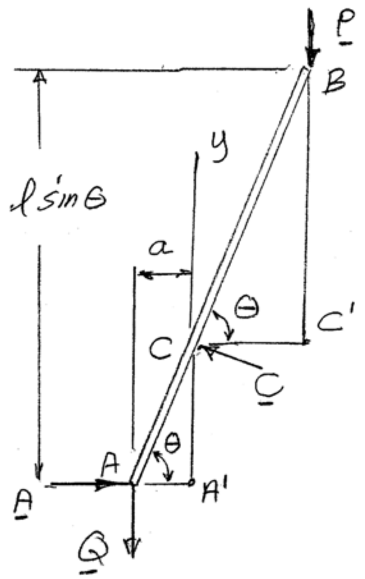
$$Q = 2P\frac{\cos\theta}{\cos(\theta/2)} \blacktriangleleft$$

PROBLEM 10.12



The slender rod AB is attached to a collar A and rests on a small wheel at C. Neglecting the radius of the wheel and the effect of friction, derive an expression for the magnitude of the force **Q** required to maintain the equilibrium of the rod.

SOLUTION



For $\Delta AA'C$:

$$A'C = a\tan\theta$$

$$y_A = -(A'C) = -a\tan\theta$$

$$\delta y_A = -\frac{a}{\cos^2\theta}\delta\theta$$

For $\Delta CC'B$:

$$BC' = l\sin\theta - A'C$$

$$= l\sin\theta - a\tan\theta$$

$$y_B = BC' = l\sin\theta - a\tan\theta$$

$$\delta y_B = l\cos\theta\delta\theta - \frac{a}{\cos^2\theta}\delta\theta$$

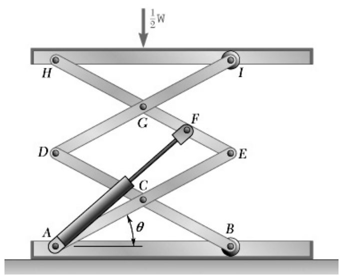
Virtual Work:

$$\delta U = 0: \quad Q\delta y_A - P\delta y_B = 0$$

$$-Q\left(-\frac{a}{\cos^2\theta}\right)\delta\theta - P\left(l\cos\theta - \frac{a}{\cos^2\theta}\right)\delta\theta = 0$$

$$Q\left(\frac{a}{\cos^2\theta}\right) = P\left(l\cos\theta - \frac{a}{\cos^2\theta}\right)$$

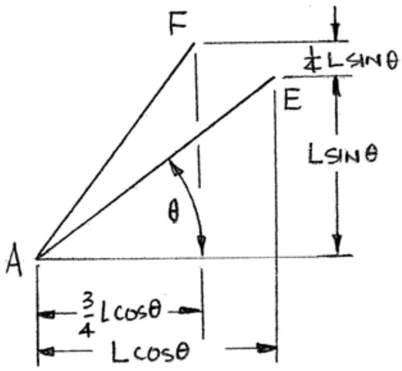
$$Q = P\left(\frac{l}{a}\cos^3\theta - 1\right) \blacktriangleleft$$



PROBLEM 10.13

A double scissor lift table is used to raise a 1000-lb machine component. The table consists of a platform and two identical linkages on which hydraulic cylinders exert equal forces. (Only one linkage and one cylinder are shown.) Each member of the linkage is of length 24 in., and pins C and G are at the midpoints of their respective members. The hydraulic cylinder is pinned at A to the base of the table and at F which is 6 in. from E. If the component is placed on the table so that half of its weight is supported by the system shown, determine the force exerted by each cylinder when $\theta = 30°$.

SOLUTION



First note $\quad y_H = 2L\sin\theta \qquad L = 24$ in. (length of link)

Then
$$\delta y_H = 2L\cos\theta\delta\theta$$

Now
$$d_{AF} = \sqrt{\left(\frac{3}{4}L\cos\theta\right)^2 + \left(\frac{5}{4}L\sin\theta\right)^2}$$

$$= \frac{1}{4}L\sqrt{9 + 16\sin^2\theta}$$

Then
$$\delta d_{AF} = \frac{L}{4}\frac{2(16\sin\theta\cos\theta)}{2\sqrt{9 + 16\sin^2\theta}}\delta\theta$$

$$= 4L\frac{\sin\theta\cos\theta}{\sqrt{9 + 16\sin^2\theta}}\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad F_{\text{cyl}}\delta d_{AF} - \left(\frac{1}{2}W\right)\delta y_H = 0$$

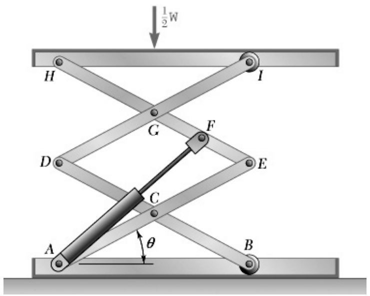
or
$$F_{\text{cyl}}\left(4L\frac{\sin\theta\cos\theta}{\sqrt{9 + 16\sin^2\theta}}\right)\delta\theta - (500 \text{ lb})(2L\cos\theta\delta\theta) = 0$$

and
$$F_{\text{cyl}}\frac{\sin\theta}{\sqrt{9 + 16\sin^2\theta}} = 250 \text{ lb}$$

Finally,
$$F_{\text{cyl}}\frac{\sin 30°}{\sqrt{9 + 16\sin^2 30°}} = 250 \text{ lb}$$

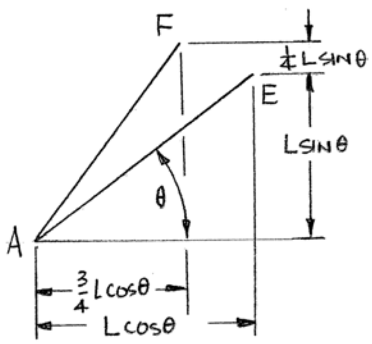
or $F_{\text{cyl}} = 1803$ lb ◀

PROBLEM 10.14



A double scissor lift table is used to raise a 1000-lb machine component. The table consists of a platform and two identical linkages on which hydraulic cylinders exert equal forces. (Only one linkage and one cylinder are shown.) Each member of the linkage is of length 24 in., and pins C and G are at the midpoints of their respective members. The hydraulic cylinder is pinned at A to the base of the table and at F which is 6 in. from E. If the component is placed on the table so that half of its weight is supported by the system shown, determine the smallest allowable value of θ knowing that the maximum force each cylinder can exert is 8 kips.

SOLUTION



From the results of the Problem 10.13

$$F_{\text{cyl}} \frac{\sin\theta}{\sqrt{9 + 16\sin^2\theta}} = 250 \text{ lb}$$

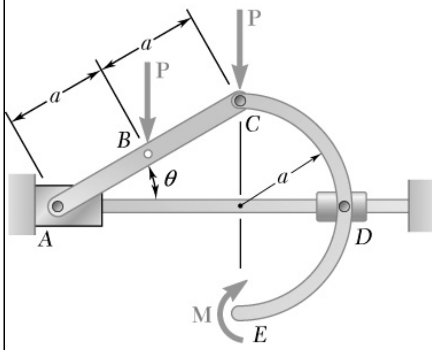
Then
$$(8000 \text{ lb}) \frac{\sin\theta}{\sqrt{9 + 16\sin^2\theta}} = 250 \text{ lb}$$

or
$$(32\sin\theta)^2 = 9 + 16\sin^2\theta$$

Thus,
$$\sin^2\theta = \frac{9}{1008}$$

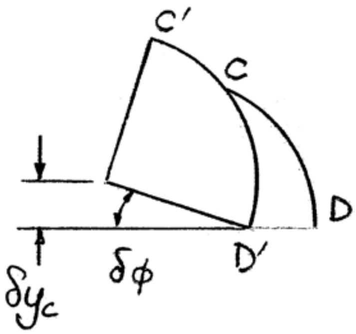
$$\theta = 5.42° \blacktriangleleft$$

PROBLEM 10.15



Derive an expression for the magnitude of the couple **M** required to maintain the equilibrium of the linkage shown.

SOLUTION



ABC: $y_B = a\sin\theta \Rightarrow \delta y_B = a\cos\theta\delta\theta$

$$y_C = 2a\sin\theta \Rightarrow \delta y_C = 2a\cos\theta\delta\theta$$

CDE: Note that as *ABC* rotates counterclockwise, *CDE* rotates clockwise while it moves to the left.

Then $\delta y_C = a\delta\phi$

or $2a\cos\theta\delta\theta = a\delta\phi$

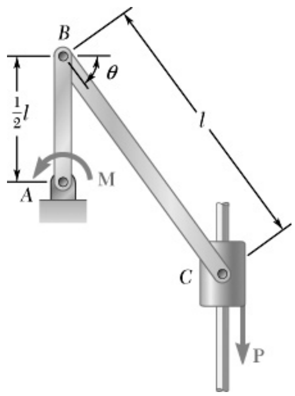
or $\delta\phi = 2\cos\theta\delta\theta$

Virtual Work:

$$\delta U = 0: \quad -P\delta y_B - P\delta y_C + M\delta\phi = 0$$

$$-P(a\cos\theta\delta\theta) - P(2a\cos\theta\delta\theta) + M(2\cos\theta\delta\theta) = 0$$

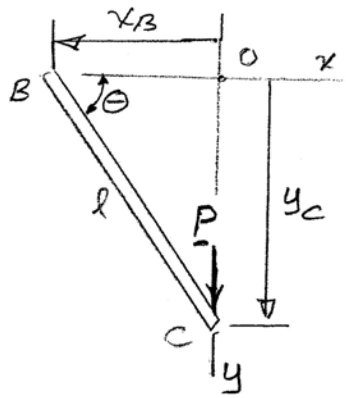
or $M = \dfrac{3}{2}Pa$ ◀

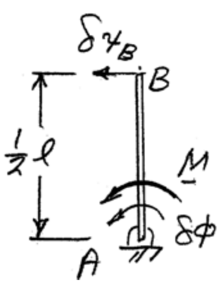


PROBLEM 10.16

Derive an expression for the magnitude of the couple **M** required to maintain the equilibrium of the linkage shown.

SOLUTION





Have

$$x_B = l\cos\theta$$

$$\delta x_B = -l\sin\theta\delta\theta \qquad (1)$$

$$y_C = l\sin\theta$$

$$\delta y_C = l\cos\theta\delta\theta$$

Now $$\delta x_B = \frac{1}{2}l\delta\theta$$

Substituting from Equation (1)

$$-l\sin\theta\delta\theta = \frac{1}{2}l\delta\phi$$

or $$\delta\phi = -2\sin\theta\delta\theta$$

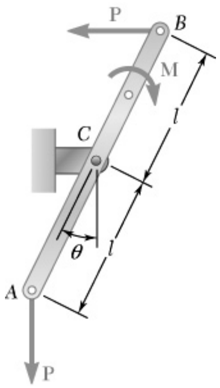
Virtual Work:

$$\delta U = 0: \quad M\delta\varphi + P\delta y_C = 0$$

$$M\left(-2\sin\theta\delta\theta\right) + P\left(l\cos\theta\delta\theta\right) = 0$$

or $$M = \frac{1}{2}Pl\frac{\cos\theta}{\sin\theta}$$

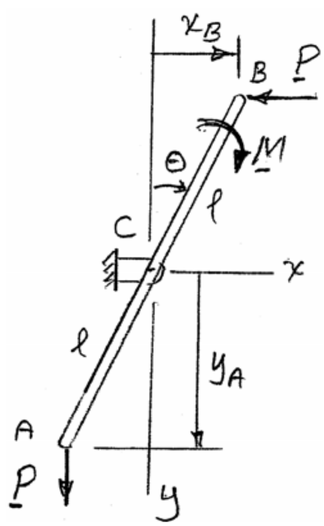
$$M = \frac{Pl}{2\tan\theta} \blacktriangleleft$$



PROBLEM 10.17

Derive an expression for the magnitude of the couple **M** required to maintain the equilibrium of the linkage shown.

SOLUTION



Have

$$x_B = l\sin\theta$$

$$\delta x_B = l\cos\theta\delta\theta$$

$$y_A = l\cos\theta$$

$$\delta y_A = -l\sin\theta\delta\theta$$

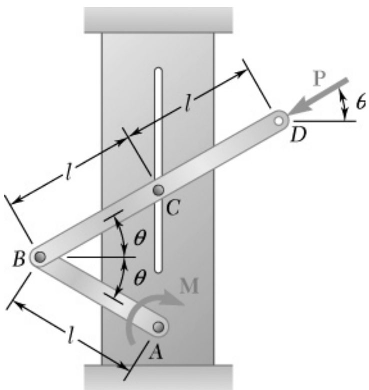
Virtual Work:

$$\delta U = 0: \quad M\delta\theta - P\delta x_B + P\delta y_A = 0$$

$$M\delta\theta - P(l\cos\theta\delta\theta) + P(-l\sin\theta\delta\theta) = 0$$

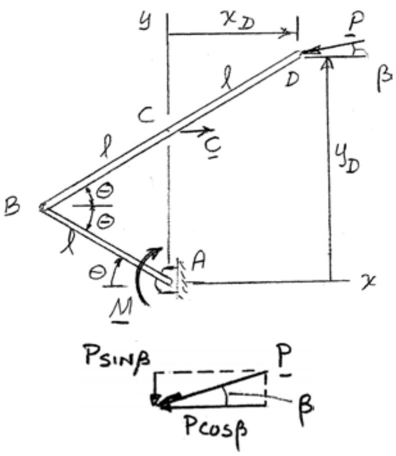
$$M = Pl(\sin\theta + \cos\theta) \blacktriangleleft$$

PROBLEM 10.18



The pin at C is attached to member BCD and can slide along a slot cut in the fixed plate shown. Neglecting the effect of friction, derive an expression for the magnitude of the couple **M** required to maintain equilibrium when the force **P** which acts at D is directed (a) as shown, (b) vertically downward, (c) horizontally to the right.

SOLUTION



Have
$$x_D = l\cos\theta$$

$$\delta x_D = -l\sin\theta\delta\theta$$

$$y_D = 3l\sin\theta$$

$$\delta y_D = 3l\cos\theta\delta\theta$$

Virtual Work: $\delta U = 0$: $M\delta\theta - (P\cos\beta)\delta x_D - (P\sin\beta)\delta y_D = 0$

$$M\delta\theta - (P\cos\beta)(-l\sin\theta\delta\theta) - (P\sin\beta)(3l\cos\theta\delta\theta) = 0$$

$$M = Pl(3\sin\beta\cos\theta - \cos\beta\sin\theta) \qquad (1)$$

(a) For **P** directed along BCD, $\beta = \theta$

Equation (1): $M = Pl(3\sin\theta\cos\theta - \cos\theta\sin\theta)$

$$M = Pl(2\sin\theta\cos\theta) \qquad M = Pl\sin 2\theta \blacktriangleleft$$

(b) For **P** directed $\downarrow$, $\beta = 90°$

Equation (1): $M = Pl(3\sin 90°\cos\theta - \cos 90°\sin\theta)$

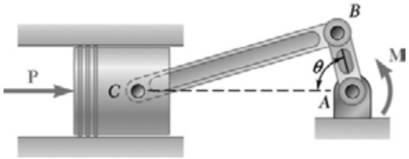
$$M = 3Pl\cos\theta \blacktriangleleft$$

(c) For **P** directed $\longrightarrow$, $\beta = 180°$

Equation (1): $M = Pl(3\sin 180°\cos\theta - \cos 180°\sin\theta)$

$$M = Pl\sin\theta \blacktriangleleft$$

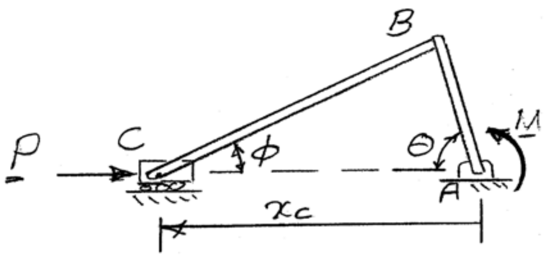
PROBLEM 10.19



A 1-kip force **P** is applied as shown to the piston of the engine system. Knowing that $AB = 2.5$ in. and $BC = 10$ in., determine the couple **M** required to maintain the equilibrium of the system when (*a*) $\theta = 30°$, (*b*) $\theta = 150°$.

SOLUTION

Analysis of the geometry:



Law of Sines

$$\frac{\sin\phi}{AB} = \frac{\sin\theta}{BC}$$

$$\sin\phi = \frac{AB}{BC}\sin\theta \tag{1}$$

Now

$$x_C = AB\cos\theta + BC\cos\phi$$

$$\delta x_C = -AB\sin\theta\delta\theta - BC\sin\phi\delta\phi \tag{2}$$

Now, from Equation (1)

$$\cos\phi\delta\phi = \frac{AB}{BC}\cos\theta\delta\theta$$

or

$$\delta\phi = \frac{AB}{BC}\frac{\cos\theta}{\cos\phi}\delta\theta \tag{3}$$

From Equation (2)

$$\delta x_C = -AB\sin\theta\delta\theta - BC\sin\phi\left(\frac{AB}{BC}\frac{\cos\theta}{\cos\phi}\delta\theta\right)$$

or

$$\delta x_C = -\frac{AB}{\cos\phi}(\sin\theta\cos\phi + \sin\phi\cos\theta)\delta\theta$$

Then

$$\delta x_C = -\frac{AB\sin(\theta + \phi)}{\cos\phi}\delta\theta$$

PROBLEM 10.19 CONTINUED

Virtual Work: $\delta U = 0$: $\quad -P\delta x_C - M\delta\theta = 0$

$$-P\left[-\frac{AB\sin(\theta + \phi)}{\cos\phi}\delta\theta\right] - M\delta\theta = 0$$

Thus, $$M = AB\frac{\sin(\theta + \phi)}{\cos\phi}P \qquad (4)$$

For the given conditions: $P = 1.0$ kip $= 1000$ lb, $AB = 2.5$ in., and $BC = 10$ in.:

(*a*) When $\theta = 30°$: $\quad \sin\phi = \frac{2.5}{10}\sin 30°$, $\qquad \phi = 7.181°$

$$M = (2.5 \text{ in.})\frac{\sin(30° + 7.181°)}{\cos 7.181°}(1.0 \text{ kip}) = 1.5228 \text{ kip}\cdot\text{in.}$$

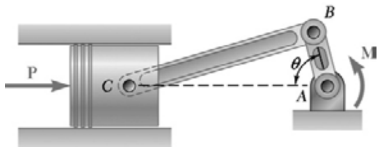
$$= 0.1269 \text{ kip}\cdot\text{ft}$$

or $\mathbf{M} = 126.9$ lb·ft ◀

(*b*) When $\theta = 150°$: $\quad \sin\phi = \frac{2.5}{10}\sin 150°$, $\qquad \phi = 7.181°$

$$M = (2.5 \text{ in.})\frac{\sin(150° + 7.181°)}{\cos 7.181°}(1.0 \text{ kip}) = 0.97722 \text{ kip}\cdot\text{in.}$$

or $\mathbf{M} = 81.4$ lb·ft ◀



PROBLEM 10.20

A couple **M** of magnitude $75\text{ lb}\cdot\text{ft}$ is applied as shown to the crank of the engine system. Knowing that $AB = 2.5$ in. and $BC = 10$ in., determine the force **P** required to maintain the equilibrium of the system when (*a*) $\theta = 60°$, (*b*) $\theta = 120°$.

SOLUTION

From the analysis of Problem 10.19,

$$M = AB\frac{\sin(\theta + \phi)}{\cos\phi}P$$

Now, with $M = 75\text{ lb}\cdot\text{ft} = 900\text{ lb}\cdot\text{in.}$

(*a*) For $\theta = 60°$

$$\sin\phi = \frac{2.5}{10}\sin 60°, \qquad \phi = 12.504°$$

$$(900\text{ lb}\cdot\text{in.}) = (2.5\text{ in.})\frac{\sin(60° + 12.504°)}{\cos 12.504°}(P)$$

or $$P = 368.5\text{ lb}$$

$\mathbf{P} = 369\text{ lb} \longrightarrow$ ◀

(*b*) For $\theta = 120°$

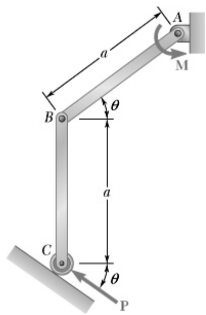
$$\sin\phi = \frac{2.5}{10}\sin 120°, \qquad \phi = 12.504°$$

$$(900\text{ lb}\cdot\text{in.}) = (2.5\text{ in.})\frac{\sin(120° + 12.504°)}{\cos 12.504°}(P)$$

or $$P = 476.7\text{ lb}$$

$\mathbf{P} = 477\text{ lb} \longrightarrow$ ◀

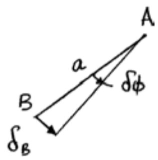
PROBLEM 10.21



For the linkage shown, determine the force **P** required for equilibrium when $a = 18\text{ in.}$, $M = 240\text{ lb}\cdot\text{in.}$, and $\theta = 30°$.

SOLUTION

Consider a virtual counterclockwise rotation $\delta\phi$ of link AB.

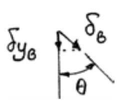


Then $$\delta_B = a\delta\phi$$

Note that

$$\delta y_B = \delta_B \cos\theta$$

$$= a\cos\theta\,\delta\phi$$



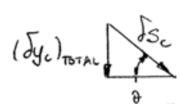
If the incline were removed, point C would move down δy_C as a result of the virtual rotation, where

$$\delta y_C = \delta y_B = a\cos\theta\,\delta\phi$$

For the roller to remain on the incline, the vertical link BC would then have to rotate counterclockwise. Thus, to first order:

$$(\delta y_C)_{\text{total}} \approx \delta y_C$$

Then $$\delta S_C = \frac{(\delta y_C)_{\text{total}}}{\sin\theta}$$



$$= \frac{a\cos\theta\,\delta\phi}{\sin\theta}$$

$$= \frac{a}{\tan\theta}\delta\phi$$

Now, by Virtual Work: $$\delta U = 0: \quad M\delta\phi - P\delta S_C = 0$$

or $$M\delta\phi - P\left(\frac{a}{\tan\theta}\delta\phi\right) = 0$$

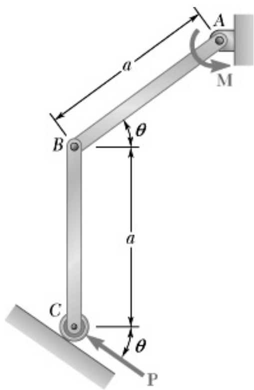
or $$M\tan\theta = Pa$$

With $$M = 240\text{ lb}\cdot\text{in.},\ a = 18\text{ in.},\ \text{and } \theta = 30°$$

$$(240\text{ lb}\cdot\text{in.})\tan 30° = P(18\text{ in.})$$

or $\mathbf{P} = 7.70\text{ lb}$ ⦣ $30.0°$ ◀

PROBLEM 10.22



For the linkage shown, determine the couple **M** required for equilibrium when $a = 2$ ft, $P = 30$ lb, and $\theta = 40°$.

SOLUTION

From the analysis of Problem 10.21, $M \tan\theta = Pa$

Now, with $P = 30$ lb, $a = 2$ ft, and $\theta = 40°$, we have

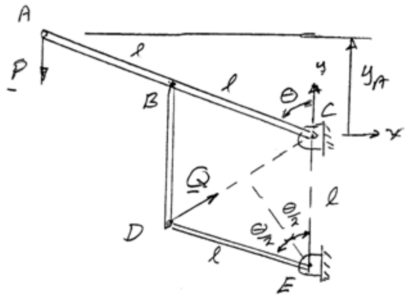
$$M \tan 40° = (30 \text{ lb})(2 \text{ ft})$$

or $\mathbf{M} = 71.5 \text{ lb}\cdot\text{ft}$ ↻ ◀

PROBLEM 10.23

Determine the value of θ corresponding to the equilibrium position of the mechanism of Problem 10.10 when $P = 60$ lb and $Q = 75$ lb.

SOLUTION



From geometry

$$y_A = 2l\cos\theta, \qquad \delta y_A = -2l\sin\theta\,\delta\theta$$

$$CD = 2l\sin\frac{\theta}{2}, \qquad \delta(CD) = l\cos\frac{\theta}{2}\delta\theta$$

Virtual Work:

$$\delta U = 0{:}\quad -P\delta y_A - Q\delta(CD) = 0$$

$$-P(-2l\sin\theta\,\delta\theta) - Q\left(l\cos\frac{\theta}{2}\delta\theta\right) = 0$$

or
$$Q = 2P\frac{\sin\theta}{\cos(\theta/2)}$$

With
$$P = 60 \text{ lb}, \qquad Q = 75 \text{ lb}$$

$$(75 \text{ lb}) = 2(60 \text{ lb})\frac{\sin\theta}{\cos(\theta/2)}$$

$$\frac{\sin\theta}{\cos(\theta/2)} = 0.625$$

or
$$\frac{2\sin(\theta/2)\cos(\theta/2)}{\cos(\theta/2)} = 0.625$$

$$\theta = 36.42°$$

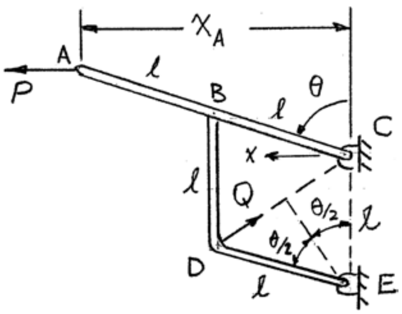
$$\theta = 36.4° ◀$$

(Additional solutions discarded as not applicable are $\theta = \pm 180°$)

PROBLEM 10.24

Determine the value of θ corresponding to the equilibrium position of the mechanism of Problem 10.11 when $P = 20$ lb and $Q = 25$ lb.

SOLUTION



$$x_A = 2l\sin\theta$$

$$\delta x_A = 2l\cos\theta\,\delta\theta$$

$$CD = 2l\sin\frac{\theta}{2}$$

$$\delta(CD) = l\cos\frac{\theta}{2}\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad P\delta x_A - Q\delta(CD) = 0$$

$$P(2l\cos\theta\,\delta\theta) - Q\left(l\cos\frac{\theta}{2}\delta\theta\right) = 0$$

or $$Q = 2P\frac{\cos\theta}{\cos(\theta/2)}$$

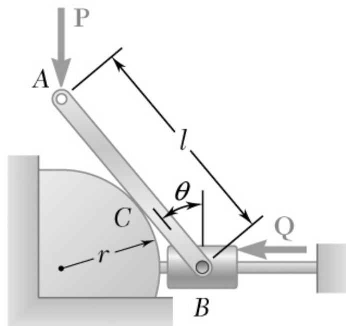
With $P = 20$ lb and $Q = 25$ lb

$$(25\text{ lb}) = 2(20\text{ lb})\frac{\cos\theta}{\cos(\theta/2)}$$

or $$\frac{\cos\theta}{\cos(\theta/2)} = 0.625$$

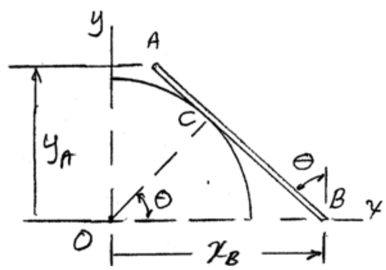
Solving numerically, $\theta = 56.615°$ $\qquad \theta = 56.6° \blacktriangleleft$

PROBLEM 10.25



A slender rod of length l is attached to a collar at B and rests on a portion of a circular cylinder of radius r. Neglecting the effect of friction, determine the value of θ corresponding to the equilibrium position of the mechanism when $l = 300$ mm, $r = 90$ mm, $P = 60$ N, and $Q = 120$ N.

SOLUTION



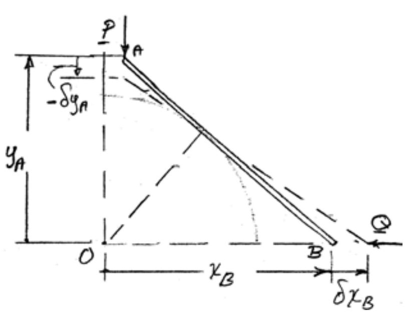
Geometry $$OC = r$$

$$\cos\theta = \frac{OC}{OB} = \frac{r}{x_B}$$

$$x_B = \frac{r}{\cos\theta}$$

$$\delta x_B = \frac{r\sin\theta}{\cos^2\theta}\delta\theta$$

$$y_A = l\cos\theta; \qquad \delta y_A = -l\sin\theta\delta\theta$$



Virtual Work:

$$\delta U = 0: \quad P(-\delta y_A) - Q\delta x_B = 0$$

$$Pl\sin\theta\,\delta\theta - Q\frac{r\sin\theta}{\cos^2\theta}\delta\theta = 0$$

$$\cos^2\theta = \frac{Qr}{Pl} \tag{1}$$

Then, with $l = 300$ mm, $r = 90$ mm, $P = 60$ N, and $Q = 120$ N

$$\cos^2\theta = \frac{(120\text{ N})(90\text{ mm})}{(60\text{ N})(300\text{ mm})} = 0.6$$

or $\theta = 39.231°$ $\qquad$ $\theta = 39.2°$ ◀

PROBLEM 10.26

A slender rod of length l is attached to a collar at B and rests on a portion of a circular cylinder of radius r. Neglecting the effect of friction, determine the value of θ corresponding to the equilibrium position of the mechanism when $l = 280$ mm, $r = 100$ mm, $P = 300$ N, and $Q = 600$ N.

SOLUTION

From the analysis of Problem 10.25

$$\cos^2\theta = \frac{Qr}{Pl}$$

Then with $l = 280$ mm, $r = 100$ mm, $P = 300$ N, and $Q = 600$ N

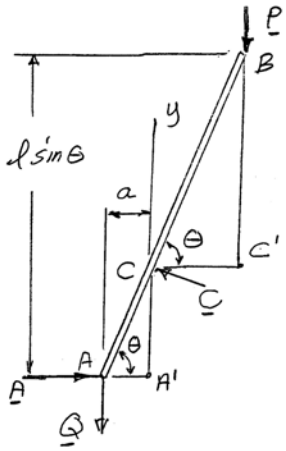
$$\cos^2\theta = \frac{(600\text{ N})(100\text{ mm})}{(300\text{ N})(280\text{ mm})} = 0.71429$$

or $$\theta = 32.311°$$ $\theta = 32.3°$ ◀

PROBLEM 10.27

Determine the value of θ corresponding to the equilibrium position of the mechanism of Problem 10.12 when $l = 600$ mm, $a = 100$ mm, $P = 100$ N, and $Q = 160$ N.

SOLUTION



For $\Delta AA'C$:

$$A'C = a\tan\theta$$

$$y_A = -(A'C) = -a\tan\theta$$

$$\delta y_A = -\frac{a}{\cos^2\theta}\delta\theta$$

For $\Delta CC'B$:

$$BC' = l\sin\theta - A'C$$

$$= l\sin\theta - a\tan\theta$$

$$y_B = BC' = l\sin\theta - a\tan\theta$$

$$\delta y_B = l\cos\theta\,\delta\theta - \frac{a}{\cos^2\theta}\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad -Q\delta y_A - P\delta y_B = 0$$

$$-Q\left(-\frac{a}{\cos^2\theta}\right)\delta\theta - P\left(l\cos\theta - \frac{a}{\cos^2\theta}\right)\delta\theta = 0$$

$$Q\left(\frac{a}{\cos^2\theta}\right) = P\left(l\cos\theta - \frac{a}{\cos^2\theta}\right)$$

or

$$Q = P\left(\frac{l}{a}\cos^3\theta - 1\right)$$

With $l = 600$ mm, $a = 100$ mm, $P = 100$ N, and $Q = 160$ N

$$(160\text{ N}) = (100\text{ N})\left(\frac{600\text{ mm}}{100\text{ mm}}\cos^3\theta - 1\right)$$

or

$$\cos^3\theta = 0.4333$$

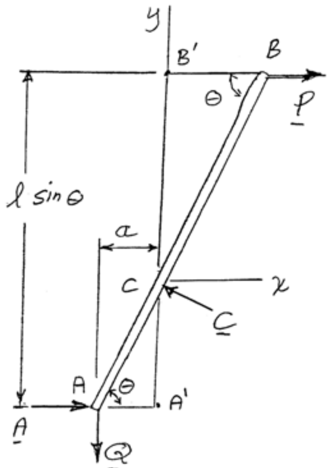
$$\theta = 40.82°$$

$\theta = 40.8°$ ◀

PROBLEM 10.28

Determine the value of θ corresponding to the equilibrium position of the mechanism of Problem 10.13 when $l = 900$ mm, $a = 150$ mm $P = 75$ N, and $Q = 135$ N.

SOLUTION



For $\Delta AA'C$: $$A'C = a\tan\theta$$

$$y_A = -(A'C) = -a\tan\theta$$

$$\delta y_A = -\frac{a}{\cos^2\theta}\delta\theta$$

For $\Delta BB'C$:

$$B'C = l\sin\theta - A'C$$

$$= l\sin\theta - a\tan\theta$$

$$BB' = \frac{B'C}{\tan\theta} = \frac{l\sin\theta - a\tan\theta}{\tan\theta}$$

$$x_B = BB' = l\cos\theta - a$$

$$\delta x_B = -l\sin\theta\,\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad P\delta x_B - Q\delta y_A = 0$$

$$P(-l\sin\theta\,\delta\theta) - Q\left(-\frac{a}{\cos^2\theta}\delta\theta\right) = 0$$

$$Pl\sin\theta\cos^2\theta = Qa$$

or $$Q = P\frac{l}{a}\sin\theta\cos^2\theta$$

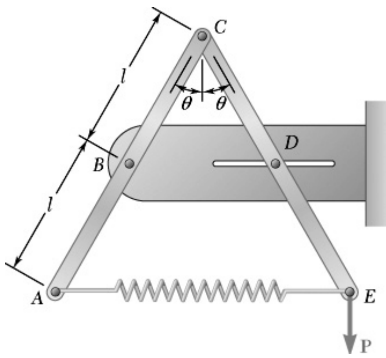
With $l = 900$ mm, $a = 150$ mm, $P = 75$ N, and $Q = 135$ N

$$135\text{ N} = (75\text{ N})\frac{900\text{ mm}}{150\text{ mm}}\sin\theta\cos^2\theta$$

or $$\sin\theta\cos^2\theta = 0.300$$

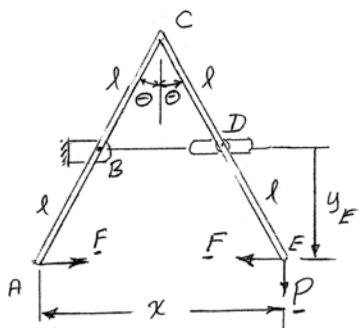
Solving numerically, $\theta = 19.81°$ and $51.9°$ ◀

PROBLEM 10.29



Two rods *AC* and *CE* are connected by a pin at *C* and by a spring *AE*. The constant of the spring is *k*, and the spring is unstretched when $\theta = 30°$. For the loading shown, derive an equation in $P, \theta,\ l$, and *k* that must be satisfied when the system is in equilibrium.

SOLUTION



$$y_E = l\cos\theta$$

$$\delta y_E = -l\sin\theta\,\delta\theta$$

Spring:

$$\text{Unstretched length} = 2l$$

$$x = 2(2l\sin\theta) = 4l\sin\theta$$

$$\delta x = 4l\cos\theta\,\delta\theta$$

$$F = k(x - 2l)$$

$$F = k(4l\sin\theta - 2l)$$

Virtual Work:

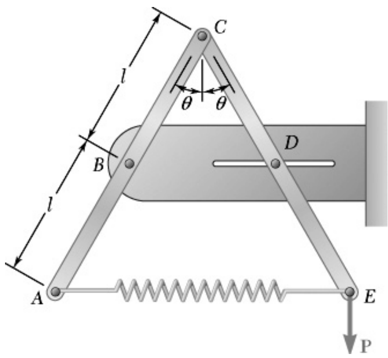
$$\delta U = 0: \quad P\delta y_E - F\delta x = 0$$

$$P(-l\sin\theta\,\delta\theta) - k(4l\sin\theta - 2l)(4l\cos\theta\,\delta\theta) = 0$$

$$-P\sin\theta - 8kl(2\sin\theta - 1)\cos\theta = 0$$

or $$\frac{P}{8kl} = (1 - 2\sin\theta)\frac{\cos\theta}{\sin\theta} \qquad \frac{P}{8kl} = \frac{1 - 2\sin\theta}{\tan\theta} \blacktriangleleft$$

PROBLEM 10.30



Two rods AC and CE are connected by a pin at C and by a spring AE. The constant of the spring is 300 N/m, and the spring is unstretched when $\theta = 30°$. Knowing that $l = 200$ mm and neglecting the mass of the rods, determine the value of θ corresponding to equilibrium when $P = 160$ N.

SOLUTION

From the analysis of Problem 10.29,

$$\frac{P}{8kl} = \frac{1 - 2\sin\theta}{\tan\theta}$$

Then with $P = 160$ N, $l = 0.2$ m, and $k = 300$ N/m

$$\frac{160 \text{ N}}{8(300 \text{ N/m})(0.2 \text{ m})} = \frac{1 - 2\sin\theta}{\tan\theta}$$

or

$$\frac{1 - 2\sin\theta}{\tan\theta} = \frac{1}{3} = 0.3333$$

Solving numerically,

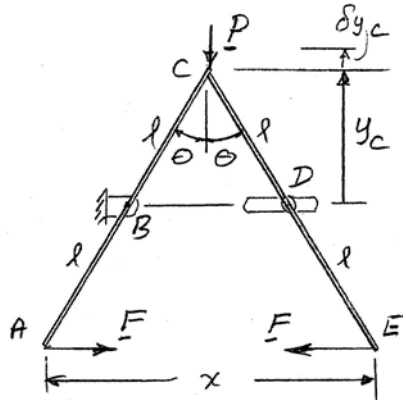
$$\theta = 24.98°$$

$\theta = 25.0°$ ◀

PROBLEM 10.31

Solve Problem 10.30 assuming that force **P** is moved to *C* and acts vertically downward.

SOLUTION



$$y_C = l\cos\theta, \qquad \delta y_C = -l\sin\theta\delta\theta$$

Spring:

$$\text{Unstretched length} = 2l$$

$$x = 2(2l\sin\theta) = 4l\sin\theta$$

$$\delta x = 4l\cos\theta\delta\theta$$

$$F = k(x - 2l)$$

$$F = k(4l\sin\theta - 2l)$$

Virtual Work:

$$\delta U = 0: \quad -P\delta y_C - F\delta x$$

$$-P(-l\sin\theta\delta\theta) - k(4l\sin\theta - 2l)(4l\cos\theta\delta\theta) = 0$$

$$P\sin\theta - 8kl(2\sin\theta - 1)\cos\theta = 0$$

or

$$\frac{P}{8kl} = (2\sin\theta - 1)\frac{\cos\theta}{\sin\theta}$$

With $l = 200$ mm, $k = 300$ N/m, and $P = 160$ N

$$\frac{(160 \text{ N})}{8(300 \text{ N/m})(0.2)} = (2\sin\theta - 1)\frac{\cos\theta}{\sin\theta}$$

or

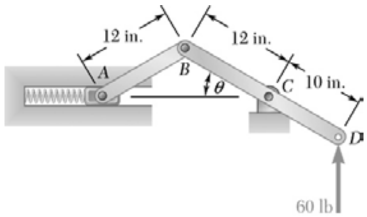
$$(2\sin\theta - 1)\frac{\cos\theta}{\sin\theta} = \frac{1}{3}$$

Solving numerically, $\theta = 39.65°$

and $\theta = 68.96°$

$\theta = 39.7°$ ◀

and $\theta = 69.0°$ ◀



PROBLEM 10.32

For the mechanism shown, block A can move freely in its guide and rests against a spring of constant 15 lb/in. that is undeformed when $\theta = 45°$. For the loading shown, determine the value of θ corresponding to equilibrium.

SOLUTION

First note $$y_D = 10\sin\theta\,(\text{in.})$$

Then $$\delta y_D = 10\cos\theta\delta\theta\ (\text{in.})$$

Also $$x_A = 2(12\cos\theta)\ \text{in.}$$

Then $$(x_A)_0 = (24\text{ in.})\cos 45°$$

and $$\delta x_A = -24\sin\theta\delta\theta\ (\text{in.})$$

With $\delta\theta < 0$: Virtual Work: $$\delta U = 0:\quad (60\text{ lb})\delta y_D - F_{SP}\left|\delta x_A\right| = 0$$

where $$F_{SP} = k\left[x_A - (x_A)_0\right]$$

$$= (15\text{ lb/in.})(24\cos\theta - 24\cos 45°)(\text{in.})$$

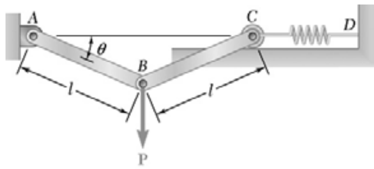
$$= (360\text{ lb})(\cos\theta - \cos 45°)$$

Then $$(60)(10\cos\theta\delta\theta) - \left[360(\cos\theta - \cos 45°)\right](24\sin\theta\delta\theta) = 0$$

or $$5 - 72\tan\theta(\cos\theta - \cos 45°) = 0$$

Solving numerically, $\theta = 15.03°$ and $\theta = 36.9°$ ◀

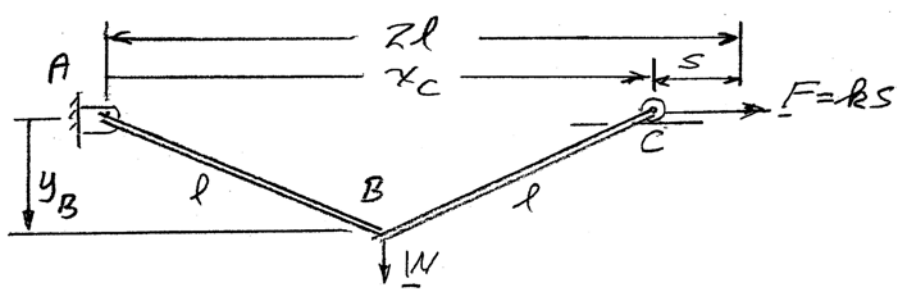
PROBLEM 10.33 AND 10.34



10.33: A force **P** of magnitude 150 lb is applied to the linkage at *B*. The constant of the spring is 12.5 lb/in., and the spring is unstretched when *AB* and *BC* are horizontal. Neglecting the weight of the linkage and knowing that $l = 15$ in., determine the value of θ corresponding to equilibrium.

10.34: A vertical force **P** is applied to the linkage at *B*. The constant of the spring is *k*, and the spring is unstretched when *AB* and *BC* are horizontal. Neglecting the weight of the linkage, derive an equation in θ, *P*, *l*, and *k* that must be satisfied when the linkage is in equilibrium.

SOLUTION



$$x_C = 2l\cos\theta \qquad \delta x_C = -2l\sin\theta\delta\theta$$

$$y_B = l\sin\theta \qquad \delta y_B = l\cos\theta\delta\theta$$

$$F = ks = k(2l - x_C) = 2kl(1 - \cos\theta)$$

Virtual Work: $\delta U = 0$: $F\delta x_C + W\delta y_B = 0$

$$2kl(1 - \cos\theta)(-2l\sin\theta\delta\theta) + W(l\cos\theta\delta\theta) = 0$$

$$4kl^2(1 - \cos\theta)\sin\theta = Wl\cos\theta$$

or $$(1 - \cos\theta)\tan\theta = \frac{W}{4kl}$$

Problem 10.33: Given: $l = 0.3$ m, $W = 600$ N, $k = 2500$ N/m

Then $$(1 - \cos\theta)\tan\theta = \frac{600 \text{ N}}{4(2500 \text{ N/m})(0.3 \text{ m})}$$

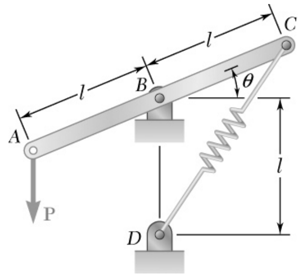
or $$(1 - \cos\theta)\tan\theta = 0.2$$

Solving numerically, $$\theta = 40.22°$$

$$\theta = 40.2° \blacktriangleleft$$

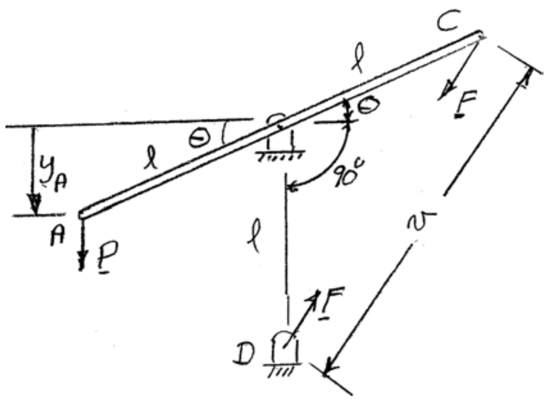
Problem 10.34: From above $$(1 - \cos\theta)\tan\theta = \frac{W}{4kl} \blacktriangleleft$$

PROBLEM 10.35



Knowing that the constant of spring CD is k and that the spring is unstretched when rod ABC is horizontal, determine the value of θ corresponding to equilibrium for the data indicated. $P = 150$ lb, $l = 30$ in., $k = 40$ lb/in.

SOLUTION



$$y_A = l\sin\theta$$

$$\delta y_A = l\cos\theta\delta\theta$$

Spring: $v = CD$

Unstretched when $\theta = 0$

so that $v_0 = \sqrt{2}l$

For θ:

$$v = 2l\sin\left(\frac{90° + \theta}{2}\right)$$

$$\delta v = l\cos\left(45° + \frac{\theta}{2}\right)\delta\theta$$

Stretched length:

$$s = v - v_0 = 2l\sin\left(45° + \frac{\theta}{2}\right) - \sqrt{2}l$$

Then

$$F = ks = kl\left[2\sin\left(45° + \frac{\theta}{2}\right) - \sqrt{2}\right]$$

Virtual Work:

$$\delta U = 0: \quad P\delta y_A - F\delta v = 0$$

$$Pl\cos\theta\delta\theta - kl\left[2\sin\left(45° + \frac{\theta}{2}\right) - \sqrt{2}\right]l\cos\left(45° + \frac{\theta}{2}\right)\delta\theta = 0$$

or

$$\frac{P}{kl} = \frac{1}{\cos\theta}\left[2\sin\left(45° + \frac{\theta}{2}\right)\cos\left(45° + \frac{\theta}{2}\right) - \sqrt{2}\cos\left(45° + \frac{\theta}{2}\right)\right]$$

$$= \frac{1}{\cos\theta}\left[2\sin\left(45° + \frac{\theta}{2}\right)\cos\left(45° + \frac{\theta}{2}\right)\cos\theta - \sqrt{2}\cos\left(45° + \frac{\theta}{2}\right)\right]$$

$$= 1 - \sqrt{2}\frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta}$$

PROBLEM 10.35 CONTINUED

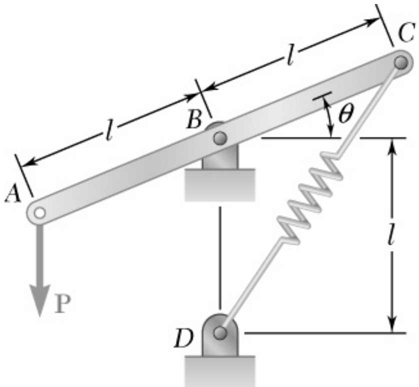
Now, with $P = 150$ lb, $l = 30$ in., and $k = 40$ lb/in.

$$\frac{(150 \text{ lb})}{(40 \text{ lb/in.})(30 \text{ in.})} = 1 - \sqrt{2}\frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta}$$

or $$\frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta} = 0.61872$$

Solving numerically, $\theta = 17.825°$ $\theta = 17.83°$ ◀

PROBLEM 10.36



Knowing that the constant of spring CD is k and that the spring is unstretched when rod ABC is horizontal, determine the value of θ corresponding to equilibrium for the data indicated. $P = 600$ N, $l = 800$ mm, $k = 4$ kN/m.

SOLUTION

From the analysis of Problem 10.35, we have

$$\frac{P}{kl} = 1 - \sqrt{2}\,\frac{\cos\left(45° + \dfrac{\theta}{2}\right)}{\cos\theta}$$

With $P = 600$ N, $l = 800$ mm, and $k = 4$ kN/m

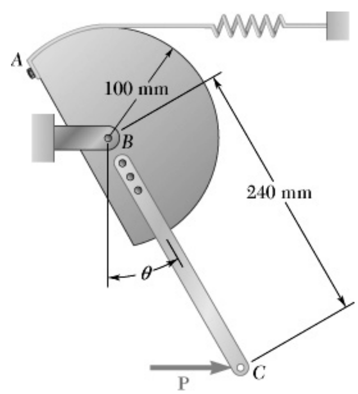
$$\frac{(600\text{ N})}{(4000\text{ N/m})(0.8\text{ m})} = 1 - \sqrt{2}\,\frac{\cos\left(45° + \dfrac{\theta}{2}\right)}{\cos\theta}$$

or

$$\frac{\cos\left(45° + \dfrac{\theta}{2}\right)}{\cos\theta} = 0.57452$$

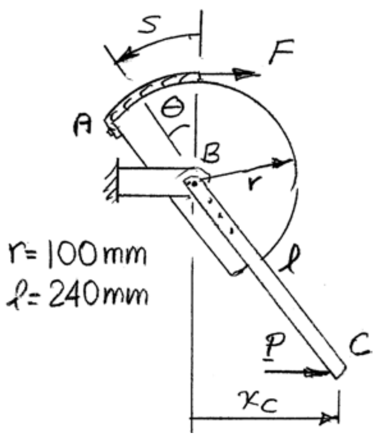
Solving numerically, $\theta = 30.98°$ $\qquad\theta = 31.0°$ ◀

PROBLEM 10.37



A horizontal force **P** of magnitude 160 N is applied to the mechanism at C. The constant of the spring is $k = 1.8\,\text{kN/m}$, and the spring is unstretched when $\theta = 0$. Neglecting the mass of the mechanism, determine the value of θ corresponding to equilibrium.

SOLUTION



Have $\quad s = r\theta \qquad \delta s = r\delta\theta$

$$F = ks = kr\theta$$

and $\quad x_C = l\sin\theta$

$$\delta x_C = l\cos\theta\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad P\delta x_C - F\delta s = 0$$

$$Pl\cos\theta\delta\theta - kr\theta(r\delta\theta) = 0$$

or

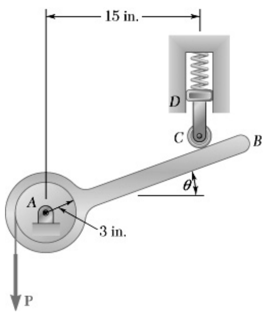
$$\frac{Pl}{kr^2} = \frac{\theta}{\cos\theta}$$

$$\frac{(160\text{ N})(0.24\text{ m})}{(1800\text{ N/m})(0.1\text{ m})^2} = \frac{\theta}{\cos\theta}$$

$$2.1333 = \frac{\theta}{\cos\theta}$$

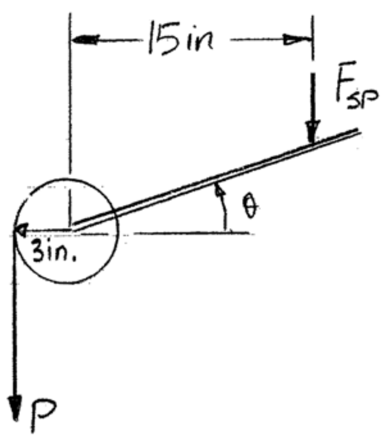
Solving numerically, $\quad \theta = 1.054\text{ rad} = 60.39° \qquad \theta = 60.4°$ ◀

PROBLEM 10.38



A cord is wrapped around drum *A* which is attached to member *AB*. Block *D* can move freely in its guide and is fastened to link *CD*. Neglecting the weight of *AB* and knowing that the spring is of constant 4 lb/in. and is undeformed when $\theta = 0$, determine the value of θ corresponding to equilibrium when a downward force **P** of magnitude 96 lb is applied to the end of the cord.

SOLUTION



Have $$y_C = 15\tan\theta \ \text{(in.)}$$

Then $$\delta y_C = 15\sec^2\theta\delta\theta \ \text{(in.)}$$

Virtual Work: $$\delta U = 0: \quad P\delta s_P - F_{SP}\delta y_C = 0$$

where $$\delta s_P = (3 \text{ in.})\delta\theta$$

and $$F_{SP} = ky_C$$

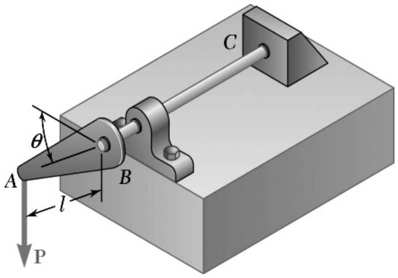
$$= (4 \text{ lb/in.})(15 \text{ in.})\tan\theta$$

$$= 60\tan\theta \ \text{(lb)}$$

Then $$(96 \text{ lb})(3 \text{ in.})\delta\theta - \left[(60\tan\theta) \text{ lb}\right]\left[(15\sec^2\theta\delta\theta) \text{ in.}\right] = 0$$

or $$3.125\tan\theta\sec^2\theta = 1$$

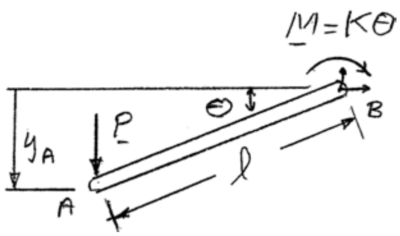
Solving numerically, $$\theta = 16.41° \blacktriangleleft$$



PROBLEM 10.39

The lever AB is attached to the horizontal shaft BC which passes through a bearing and is welded to a fixed support at C. The torsional spring constant of the shaft BC is K; that is, a couple of magnitude K is required to rotate end B through 1 rad. Knowing that the shaft is untwisted when AB is horizontal, determine the value of θ corresponding to the position of equilibrium when $P = 400$ lb, $l = 10$ in., and $K = 150$ lb · ft/rad.

SOLUTION



Have

$$y_A = l\sin\theta$$

$$\delta y_A = l\cos\theta\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad P\delta y_A - M\delta\theta = 0$$

$$Pl\cos\theta\delta\theta - K\theta\delta\theta = 0$$

or

$$\frac{\theta}{\cos\theta} = \frac{Pl}{K} \tag{1}$$

With $P = 400$ lb, $l = 10$ in., and $K = 150$ lb·ft/rad

$$\frac{\theta}{\cos\theta} = \frac{(400\text{ lb})\left(\dfrac{10\text{ in.}}{12\text{ in./ft}}\right)}{150\text{ lb}\cdot\text{ft/rad}}$$

or

$$\frac{\theta}{\cos\theta} = 2.2222$$

Solving numerically, $\theta = 61.25°$

$\theta = 61.2°$ ◀

PROBLEM 10.40

Solve Problem 10.39 assuming that $P = 1.26$ kips, $l = 10$ in., and $K = 150$ lb · ft/rad. Obtain answers in each of the following quadrants: $0 < \theta < 90°$, $270° < \theta < 360°$, and $360° < \theta < 450°$.

SOLUTION

Using Equation (1) of Problem 10.39 and

$$P = 1.26 \text{ kip}, \; l = 10 \text{ in., and } K = 150 \text{ lb}\cdot\text{ft/rad}$$

we have

$$\frac{\theta}{\cos\theta} = \frac{(1260 \text{ lb})\left(\dfrac{10 \text{ in.}}{12 \text{ in./ft}}\right)}{150 \text{ lb}\cdot\text{ft/rad}}$$

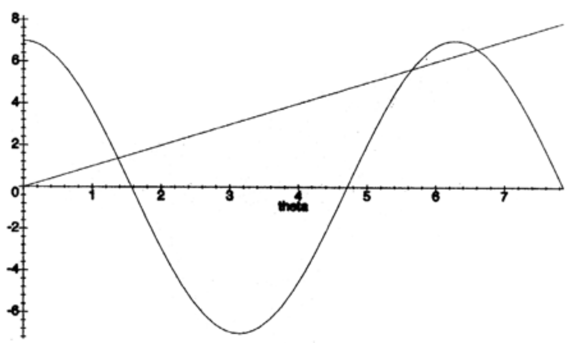
or

$$\frac{\theta}{\cos\theta} = 7 \qquad \text{or} \qquad \theta = 7\cos\theta \tag{1}$$

The solutions to this equation can be shown graphically using any appropriate graphing tool, such as Maple, with the command: plot({theta, 7 * cos(theta)}, t = 0..5 * Pi/2);

Thus, we plot $y = \theta$ and $y = 7\cos\theta$ in the range

$$0 \le \theta \le \frac{5\pi}{2}$$



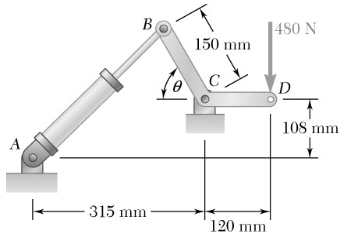
We observe that there are three points of intersection, which implies that Equation (1) has three roots in the specified range of θ.

$0 \le \theta \le 90°\left(\dfrac{\pi}{2}\right)$; $\quad \theta = 1.37333$ rad, $\quad \theta = 78.69°$ $\qquad \theta = 78.7°$ ◀

$270 \le \theta \le 360°\left(\dfrac{3\pi}{2} \le \theta \le 2\pi\right)$; $\quad \theta = 5.65222$ rad, $\quad \theta = 323.85°$ $\qquad \theta = 324°$ ◀

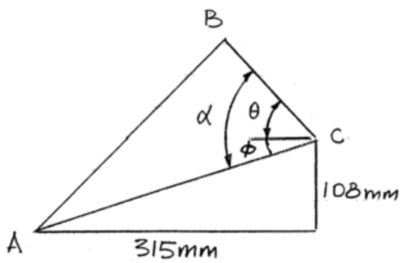
$360 \le \theta \le 450°\left(2\pi \le \theta \le \dfrac{5\pi}{2}\right)$; $\quad \theta = 6.61597$ rad, $\quad \theta = 379.07$ $\qquad \theta = 379°$ ◀

PROBLEM 10.41



The position of crank *BCD* is controlled by the hydraulic cylinder *AB*. For the loading shown, determine the force exerted by the hydraulic cylinder on pin *B* knowing that $\theta = 60°$.

SOLUTION



Have

$$d_{AC} = \sqrt{315^2 + 108^2} = 333 \text{ mm}$$

$$\tan\phi = \frac{108}{315}$$

or $$\phi = 18.9246°$$

Now, let $$\alpha = \theta + \phi$$

Then, by the Law of Cosines

$$d_{AB} = 333^2 + 150^2 - 2(333)(150)\cos\alpha$$

or $$d_{AB} = \sqrt{(13.3389 - 9.990\cos\alpha)} \times 10^2 \ (\text{mm})$$

and

$$\delta d_{AB} = \frac{499.5\sin\alpha}{\sqrt{(13.3389 - 9.990\cos\alpha)}}\delta\alpha \ (\text{mm})$$

With $$\delta\alpha > 0$$

Virtual Work: $$\delta U = 0: \ P\delta y_D - F_{\text{cyl}}\delta d_{AB} = 0$$

where $P = 480$ N, and

$$\delta y_D = d_{CD}\delta\alpha$$

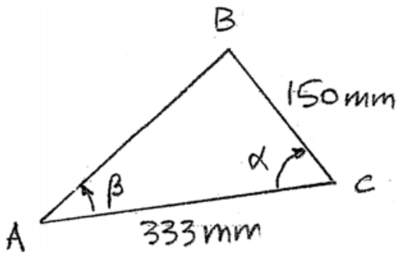
Then

$$(480 \text{ N})(120 \text{ mm})\delta\alpha - F_{\text{cyl}}\left\{\left[\frac{499.5\sin\alpha}{\sqrt{(13.3389 - 9.990\cos\alpha)}}\right]\text{mm}\right\}\delta\alpha = 0$$

or $$(499.5\sin\alpha)F_{\text{cyl}} = (57.6 \times 10^3)\sqrt{13.3389 - 9.990\cos\alpha}$$

With $$\theta = 60°: \quad \alpha = 60° + 18.9246°$$

PROBLEM 10.41 CONTINUED



have

$$\left[499.5\sin\left(60° + 18.9246°\right)\right]F_{\text{cyl}}$$

$$= \left(57.6 \times 10^3\right)\sqrt{13.3389 - 9.990\cos\left(60° + 18.9246°\right)}$$

or

$$F_{\text{cyl}} = 397.08 \text{ N}$$

and

$$d_{AB} = 100\sqrt{13.3389 - 9.990\cos 78.9246°} = 337.93 \text{ mm}$$

Then, by the Law of Sines

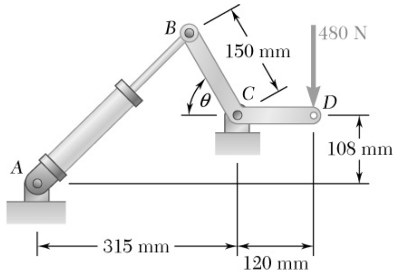
$$\frac{150}{\sin\beta} = \frac{337.93}{\sin 78.9246°}$$

or

$$\beta = 25.824°$$

$\mathbf{F}_{\text{cyl}} = 397 \text{ N} \measuredangle 44.7°$ ◀

PROBLEM 10.42



The position of crank *BCD* is controlled by the hydraulic cylinder *AB*. Determine the angle θ knowing that the hydraulic cylinder exerts a 420-N force on pin *B* when the crank is in the position shown.

SOLUTION

From Problem 10.41, we have

$$(499.5\sin\alpha)F_{\text{cyl}} = (57.6\times10^3)\sqrt{13.3389 - 9.990\cos\alpha}$$

Then, with $$F_{\text{cyl}} = 420\text{ N}$$

We have

$$499.5\sin\alpha(420) = (57.6\times10^3)\sqrt{13.3389 - 9.990\cos\alpha}$$

or

$$(3.64219\sin\alpha)^2 = 13.3389 - 9.990\cos\alpha$$

or

$$13.2655(1-\cos^2\alpha) = 13.3389 - 9.990\cos\alpha$$

or

$$13.2655\cos^2\alpha - 9.990\cos\alpha + 0.0734 = 0$$

Then

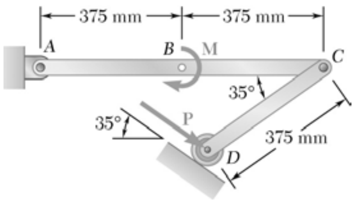
$$\cos\alpha = \frac{9.990 \pm \sqrt{(-9.990)^2 - 4(13.2655)(0.0734)}}{2(13.2655)}$$

or

$$\alpha = 41.7841° \quad \text{and} \quad \alpha = 89.5748°$$

Now $$\theta = \alpha - \phi \quad \text{and} \quad \phi = 18.9246°$$

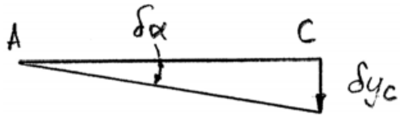
so that $$\theta = 22.9° \text{ and } \theta = 70.7° \blacktriangleleft$$



PROBLEM 10.43

For the linkage shown, determine the force **P** required for equilibrium when $M = 40\ \text{N}\cdot\text{m}$.

SOLUTION

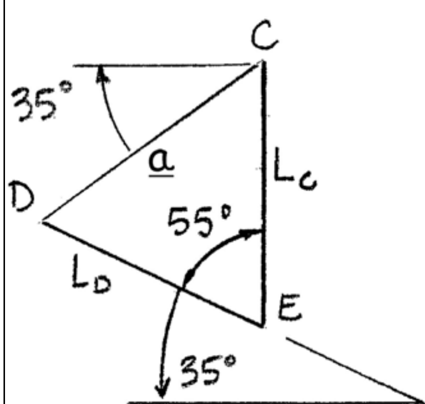


For bar *ABC*, we have

$$\delta\alpha = \frac{\delta y_c}{2a} \qquad \text{where} \qquad a = 375\ \text{mm}$$

and for bar *CD*, using the Law of Cosines

$$a^2 = L_C^2 + L_D^2 - 2L_C L_D \cos 55°$$



Then, noting that $a = \text{constant},$ we have

$$0 = 2L_C\delta L_C + 2L_D\delta L_D - 2(\delta L_C)L_D\cos 55° - 2L_C(\delta L_D)\cos 55°$$

Then, because $\delta L_C = -\delta y_C$:

$$(L_C - L_D\cos 55°)\delta y_C = (L_D - L_C\cos 55°)\delta L_D$$

For the given position of member *CD*, ΔCDE is isosceles.

$$\therefore\ L_D = a \qquad \text{and} \qquad L_C = 2a\cos 55°$$

Then

$$(2a\cos 55° - a\cos 55°)\delta y_C = \left(a - 2a\cos^2 55°\right)\delta L_D$$

or

$$\delta L_D = \frac{\cos 55°}{1 - 2\cos^2 55°}\delta y_C$$

Now, Virtual Work: $\qquad \delta U = 0\text{:}\ M\delta a - P\delta L_D = 0$

or

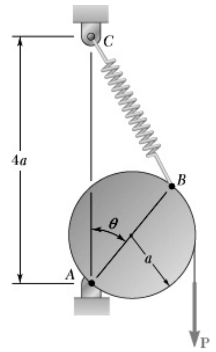
$$M\left(\frac{\delta y_C}{2a}\right) - P\left(\frac{\cos 55°}{1 - 2\cos^2 55°}\right)\delta y_C = 0$$

which gives
$$P = \frac{M}{2a}\frac{1 - 2\cos^2 55°}{\cos 55°}$$

Then
$$P = \frac{40\ \text{N}\cdot\text{m}}{2(0.375\ \text{m})}\frac{1 - 2\cos^2 55°}{\cos 55°}$$

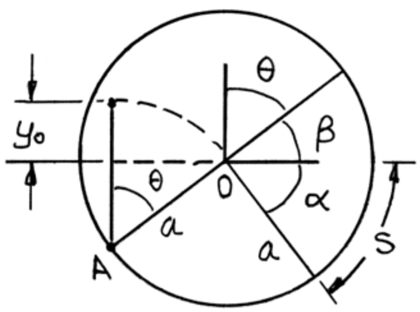
or $\mathbf{P} = 31.8\ \text{N}$ ◸35.0° ◀

PROBLEM 10.44



A cord is wrapped around a drum of radius a that is pinned at A. The constant of the spring is $3\,\text{kN/m}$, and the spring is unstretched when $\theta = 0$. Knowing that $a = 150\,\text{mm}$ and neglecting the mass of the drum, determine the value of θ corresponding to equilibrium when a downward force **P** of magnitude 48 N is applied to the end of the cord.

SOLUTION



First note

$$\theta + \beta = 90°$$

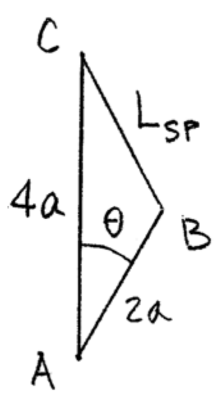
$$\alpha + \beta = 90° \qquad \Rightarrow \qquad \alpha = \theta$$

$$\therefore \quad s = a\theta \qquad \text{Length of cord unwound for rotation } \theta$$

Now $y_0 = a(1 - \cos\theta)$, the distance O moves down for rotation θ

$$y_P = y_O + s$$

$\therefore \quad y_P = a\theta + a(1 - \cos\theta)$ is the distance P moves down for rotation θ



Then $$\delta y_P = (a + a\sin\theta)\delta\theta$$

Now, by the Law of Cosines

$$L_{SP}^2 = (4a)^2 + (2a)^2 - 2(4a)(2a)\cos\theta$$

or

$$L_{SP} = 2a\sqrt{5 - 4\cos\theta}$$

Then

$$\delta L_{SP} = 2a\frac{4\sin\theta}{2\sqrt{5 - 4\cos\theta}}\delta\theta$$

$$= \frac{4a\sin\theta}{\sqrt{5 - 4\cos\theta}}\delta\theta$$

Finally

$$F_{SP} = k\left[L_{SP} - (L_{SP})_0\right]$$

$$= k\left(2a\sqrt{5 - 4\cos\theta} - 2a\right)$$

$$= 2ka\left(\sqrt{5 - 4\cos\theta} - 1\right)$$

Thus, by Virtual Work: $$\delta U = 0: \; P\delta y_P - F_{SP}\delta L_{SP} = 0$$

PROBLEM 10.44 CONTINUED

or

$$Pa(1+\sin\theta)\delta\theta - 2ka\left(\sqrt{5-4\cos\theta}-1\right)\left(\frac{4a\sin\theta}{\sqrt{5-4\cos\theta}}\delta\theta\right) = 0$$

or

$$\left[\frac{P}{8ka}(1+\sin\theta)-\sin\theta\right]\sqrt{5-4\cos\theta}+\sin\theta = 0$$

Substituting given values:

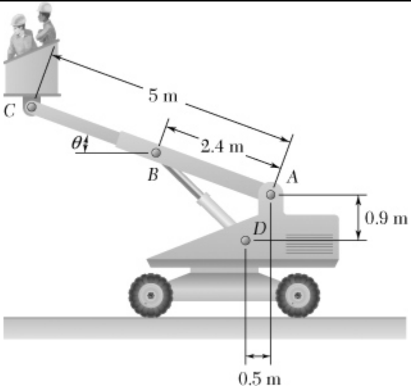
$$\left[\frac{48\text{ N}}{8(3000\text{ N/m})(0.15\text{ m})}(1+\sin\theta)-\sin\theta\right]\sqrt{5-4\cos\theta}+\sin\theta = 0$$

or

$$\left[\frac{1}{75}(1+\sin\theta)-\sin\theta\right]\sqrt{5-4\cos\theta}+\sin\theta = 0$$

Solving numerically,

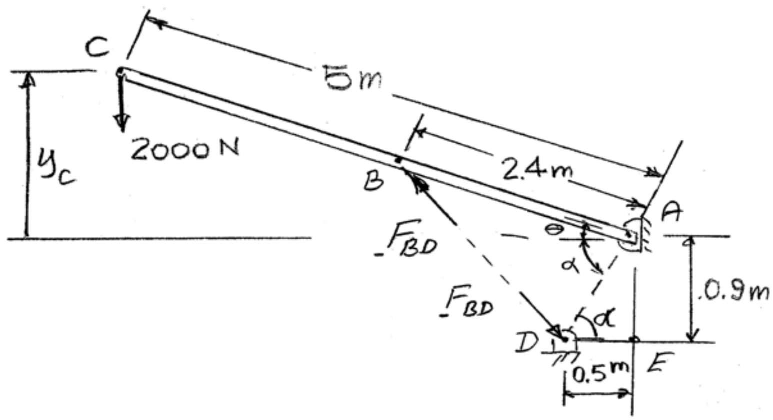
$\theta = 15.27°$ ◀



PROBLEM 10.45

The telescoping arm *ABC* is used to provide an elevated platform for construction workers. The workers and the platform together have a mass of 204 kg, and their combined center of gravity is located directly above *C*. For the position when $\theta = 20^\circ$, determine the force exerted on pin *B* by the single hydraulic cylinder *BD*.

SOLUTION



In ΔADE:

$$\tan\alpha = \frac{AE}{DE} = \frac{0.9 \text{ m}}{0.5 \text{ m}}$$

$$\alpha = 60.945^\circ$$

$$AD = \frac{0.9 \text{ m}}{\sin 60.945^\circ} = 1.0296 \text{ m}$$

From the geometry:

$$y_C = (5 \text{ m})\sin\theta, \qquad \delta y_C = (5 \text{ m})\cos\theta\delta\theta$$

Then, in triangle *BAD*: Angle $BAD = \alpha + \theta$

Law of Cosines:

$$BD^2 = AB^2 + AD^2 - 2(AB)(AD)\cos(\alpha + \theta)$$

or $$BD^2 = (2.4 \text{ m})^2 + (1.0296 \text{ m})^2 - 2(2.4 \text{ m})(1.0296 \text{ m})\cos(\alpha + \theta)$$

$$BD^2 = 6.82 \text{ m}^2 - \left(4.942\cos(\alpha + \theta)\right) \text{ m}^2 \tag{1}$$

PROBLEM 10.45 CONTINUED

And then

$$2(BD)(\delta BD) = (4.942\sin(\alpha + \theta))\delta\theta$$

$$\delta BD = \frac{4.942\sin(\alpha + \theta)}{2(BD)}\delta\theta$$

Virtual work: $\delta U = 0$: $-P\delta y_C + F_{BD}\delta BD = 0$ Substituting $-(2000\text{ N})(5\text{ m})\cos\theta\delta\theta + F_{B}$

or $$F_{BD} = \left[4047\frac{\cos\theta}{\sin(\alpha + \theta)}BD\right]\text{N/m} \qquad (2)$$

Now, with $\theta = 20°$ and $\alpha = 60.945°$

Equation (1):

$$BD^2 = 6.82 - 4.942\cos(60.945° + 20°)$$

$$BD^2 = 6.042$$

$$BD = 2.46\text{ m}$$

Equation (2)

$$F_{BD} = \left[4047\frac{\cos 20°}{\sin(60.945° + 20°)}(2.46\text{ m})\right]\text{N/m}$$

or $F_{BD} = 9473\text{ N}$ $\qquad \mathbf{F}_{BD} = 9.47\text{ kN}$ ◀

PROBLEM 10.46

Solve Problem 10.45 assuming that the workers are lowered to a point near the ground so that $\theta = -20^{o}$.

SOLUTION

Using the figure and analysis of Problem 10.45, including Equations (1) and (2), and with $\theta = -20°$, we have

Equation (1):

$$BD^2 = 6.82 - 4.942\cos(60.945° - 20°)$$

$$BD^2 = 3.087$$

$$BD = 1.757 \text{ m}$$

Equation (2):

$$F_{BD} = 4047\frac{\cos(-20°)}{\sin(60.945° - 20°)}(1.757)$$

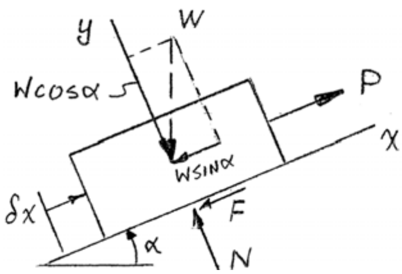
$$F_{BD} = 10196 \text{ N}$$

or $\mathbf{F}_{BD} = 10.20 \text{ kN}$ ◀

PROBLEM 10.47

A block of weight W is pulled up a plane forming an angle α with the horizontal by a force **P** directed along the plane. If μ is the coefficient of friction between the block and the plane, derive an expression for the mechanical efficiency of the system. Show that the mechanical efficiency cannot exceed $\frac{1}{2}$ if the block is to remain in place when the force **P** is removed.

SOLUTION



$$\text{Input work} = P\delta x$$

$$\text{Output work} = (W\sin\alpha)\delta x$$

Efficiency:

$$\eta = \frac{W\sin\alpha\delta x}{P\delta x} \qquad \text{or} \qquad \eta = \frac{W\sin\alpha}{P} \tag{1}$$

$$+\nearrow \Sigma F_x = 0: \quad P - F - W\sin\alpha = 0 \quad \text{or} \quad P = W\sin\alpha + F \tag{2}$$

$$+\nwarrow \Sigma F_y = 0: \quad N - W\cos\alpha = 0 \quad \text{or} \quad N = W\cos\alpha$$

$$F = \mu N = \mu W\cos\alpha$$

Equation (2): $\quad P = W\sin\alpha + \mu W\cos\alpha = W(\sin\alpha + \mu\cos\alpha)$

Equation (1): $\quad \eta = \dfrac{W\sin\alpha}{W(\sin\alpha + \mu\cos\alpha)} \quad \text{or} \quad \eta = \dfrac{1}{1+\mu\cot\alpha}$ ◀

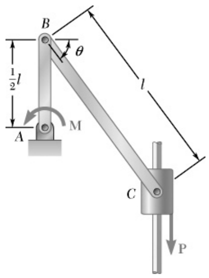
If block is to remain in place when $P = 0$, we know (see page 416) that $\phi_s \geq \alpha$ or, since

$$\mu = \tan\phi_s, \qquad \mu \geq \tan\alpha$$

Multiply by $\cot\alpha$: $\quad \mu\cot\alpha \geq \tan\alpha\cot\alpha = 1$

Add 1 to each side: $\quad 1 + \mu\cot\alpha \geq 2$

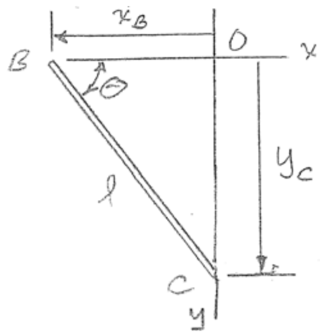
Recalling the expression for η, we find $\quad \eta \leq \dfrac{1}{2}$ ◀

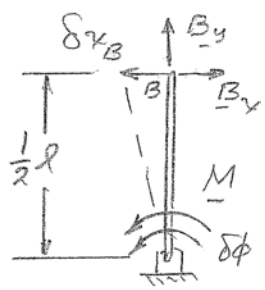


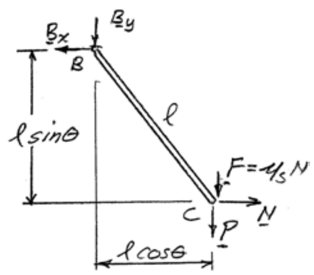
PROBLEM 10.48

Denoting by μ_s the coefficient of static friction between collar *C* and the vertical rod, derive an expression for the magnitude of the largest couple **M** for which equilibrium is maintained in the position shown. Explain what happens if $\mu_s \geq \tan\theta$.

SOLUTION







Member *BC*: Have $x_B = l\cos\theta$

$$\delta x_B = -l\sin\theta\delta\theta \tag{1}$$

and $y_C = l\sin\theta$

$$\delta y_C = l\cos\theta\delta\theta \tag{2}$$

Member *AB*: Have $\delta x_B = \frac{1}{2}l\delta\phi$

Substituting from Equation (1),

$$-l\sin\theta\delta\theta = \frac{1}{2}l\delta\phi$$

or

$$\delta\phi = -2\sin\theta\delta\theta \tag{3}$$

Free body of rod *BC*

For $M_{\max}$, motion of collar *C* impends upward

$$+\circlearrowleft \Sigma M_B = 0: \quad Nl\sin\theta - (P + \mu_s N)(l\cos\theta) = 0$$

$$N\tan\theta - \mu_s N = P$$

$$N = \frac{P}{\tan\theta - \mu_s}$$

Virtual Work

$$\delta U = 0: \quad M\delta\phi + (P + \mu_s N)\delta y_C = 0$$

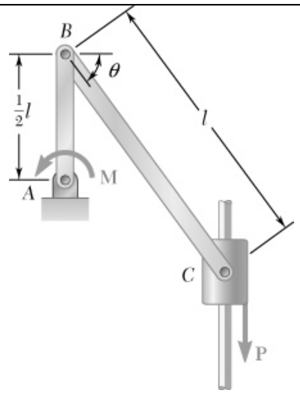
$$M(-2\sin\theta\delta\theta) + (P + \mu_s N)l\cos\theta\delta\theta = 0$$

$$M_{\max} = \frac{(P + \mu_s N)}{2\tan\theta}l = \frac{P + \mu_s \dfrac{P}{\tan\theta - \mu_s}}{2\tan\theta}l$$

or

$$M_{\max} = \frac{Pl}{2(\tan\theta - \mu_s)} \blacktriangleleft$$

If $\mu_s = \tan\theta$, $M = \infty$, system becomes *self-locking*



PROBLEM 10.49

Knowing that the coefficient of static friction between collar C and the vertical rod is 0.40, determine the magnitude of the largest and smallest couple **M** for which equilibrium is maintained in the position shown when $\theta = 35^{\text{o}}$, $l = 30$ in., and $P = 1.2$ kips.

SOLUTION

From the analysis of Problem 10.48, we have $$M_{\text{max}} = \frac{Pl}{2(\tan\theta + \mu_s)}$$

With $$\theta = 35°, \quad l = 30 \text{ in.}, \quad P = 1.25 \text{ kips}$$

$$M_{\text{max}} = \frac{(1200 \text{ lb})(30 \text{ in.})}{2(\tan 35° - 0.4)} = 59{,}958.5 \text{ lb}\cdot\text{in.}$$

$$= 4996.5 \text{ lb}\cdot\text{ft}$$

$$= 4.9965 \text{ kip}\cdot\text{ft}$$

$M_{\text{max}} = 5.00 \text{ kip}\cdot\text{ft}$ ◀

For M_{min}, motion of C impends downward and F acts upward. The equations of Problem 10.48 can still be used if we replace μ_s by $-\mu_s$. Then

$$M_{\text{min}} = \frac{Pl}{2(\tan\theta + \mu_s)}$$

Substituting, $$M_{\text{min}} = \frac{(1200 \text{ lb})(30 \text{ in.})}{2(\tan 35° + 0.4)} = 16{,}360.5 \text{ lb}\cdot\text{in.}$$

$$= 1363.4 \text{ lb}\cdot\text{ft}$$

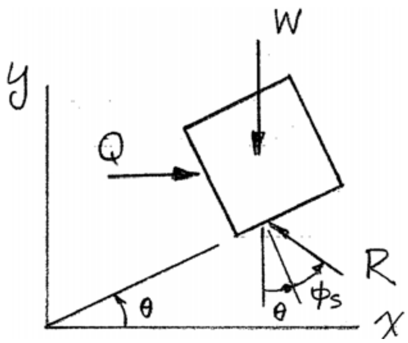
$$= 1.3634 \text{ kip}\cdot\text{ft}$$

$M_{\text{min}} = 1.363 \text{ kip}\cdot\text{ft}$ ◀

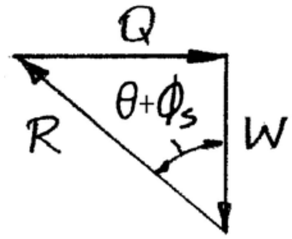
PROBLEM 10.50

Derive an expression for the mechanical efficiency of the jack discussed in Section 8.6. Show that if the jack is to be self-locking, the mechanical efficiency cannot exceed $\frac{1}{2}$.

SOLUTION



Recall Figure 8.9a. Draw force triangle



$$Q = W\tan(\theta + \phi_s)$$

$$y = x\tan\theta \text{ so that } \delta y = \delta x\tan\theta$$

$$\text{Input work } = Q\delta x = W\tan(\theta + \phi_s)\delta x$$

$$\text{Output work } = W\delta y = W(\delta x)\tan\theta$$

Efficiency: $$\eta = \frac{W\tan\theta\delta x}{W\tan(\theta + \phi_s)\delta x}; \qquad \eta = \frac{\tan\theta}{\tan(\theta + \phi_s)} \blacktriangleleft$$

From page 432, we know the jack is self-locking if

$$\phi_s \geq \theta$$

Then $$\theta + \phi_s \geq 2\theta$$

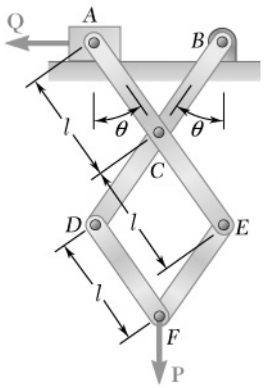
so that $$\tan(\theta + \phi_s) \geq \tan 2\theta$$

From above $$\eta = \frac{\tan\theta}{\tan(\theta + \phi_s)}$$

It then follows that $$\eta \leq \frac{\tan\theta}{\tan 2\theta}$$

But $$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

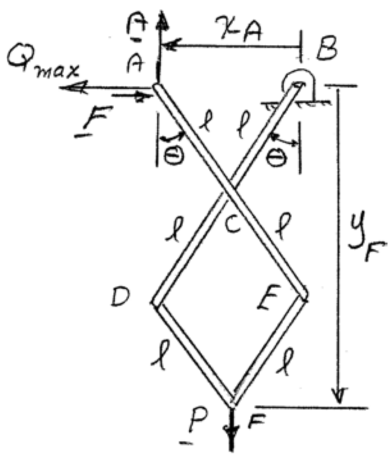
Then $$\eta \leq \frac{\tan\theta(1 - \tan^2\theta)}{2\tan\theta} = \frac{1 - \tan^2\theta}{2} \qquad \therefore \quad \eta \leq \frac{1}{2} \blacktriangleleft$$



PROBLEM 10.51

Denoting by μ_s the coefficient of static friction between the block attached to rod ACE and the horizontal surface, derive expressions in terms of P, μ_s, and θ for the largest and smallest magnitudes of the force **Q** for which equilibrium is maintained.

SOLUTION



For the linkage:

$$+\circlearrowleft \Sigma M_B = 0: \quad -x_A + \frac{x_A}{2}P = 0 \qquad \text{or} \qquad \mathbf{A} = \frac{P}{2} \uparrow$$

Then: $$F = \mu_s A = \mu_s \frac{P}{2} = \frac{1}{2}\mu_s P$$

Now $$x_A = 2l\sin\theta$$

$$\delta x_A = 2l\cos\theta\,\delta\theta$$

and $$y_F = 3l\cos\theta$$

$$\delta y_F = -3l\sin\theta\,\delta\theta$$

Virtual Work:

$$\delta U = 0: \quad (Q_{\max} - F)\delta x_A + P\delta y_F = 0$$

$$\left(Q_{\max} - \frac{1}{2}\mu_s P\right)(2l\cos\theta\,\delta\theta) + P(-3l\sin\theta\,\delta\theta) = 0$$

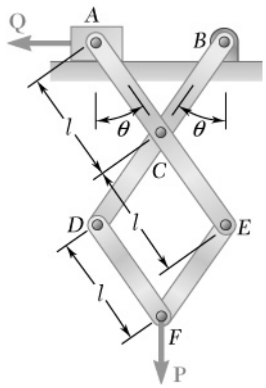
or $$Q_{\max} = \frac{3}{2}P\tan\theta + \frac{1}{2}\mu_s P$$

$$Q_{\max} = \frac{P}{2}(3\tan\theta + \mu_s) \blacktriangleleft$$

For $Q_{\min}$, motion of A impends to the right and **F** acts to the left. We change μ_s to $-\mu_s$ and find

$$Q_{\min} = \frac{P}{2}(3\tan\theta - \mu_s) \blacktriangleleft$$

PROBLEM 10.52



Knowing that the coefficient of static friction between the block attached to rod *ACE* and the horizontal surface is 0.15, determine the magnitudes of the largest and smallest force **Q** for which equilibrium is maintained when $\theta = 30^\circ$, $l = 8$ in., and $P = 160$ lb.

SOLUTION

Using the results of Problem 10.52 with

$$\theta = 30°,\ l = 8\text{ in.},\ P = 160\text{ lb, and } \mu_s = 0.15$$

We have

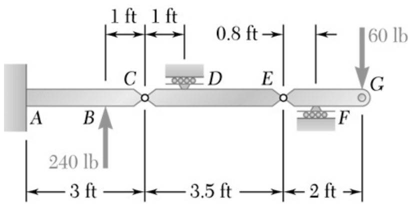
$$Q_{\max} = \frac{(160\text{ lb})}{2}(3\tan 30° + 0.15) = 150.56\text{ lb}$$

$Q_{\max} = 150.6$ lb ◀

and

$$Q_{\min} = \frac{(160\text{ lb})}{2}(3\tan 30° - 0.15) = 126.56\text{ lb}$$

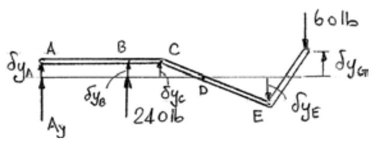
$Q_{\min} = 126.6$ lb ◀



PROBLEM 10.53

Using the method of virtual work, determine separately the force and the couple representing the reaction at A.

SOLUTION



A_y: Consider an upward displacement δy_A of ABC

ABC: $$\delta y_A = \delta y_B = \delta y_C$$

CDE: $$\frac{\delta y_C}{1\text{ ft}} = \frac{\delta y_E}{2.5\text{ ft}}$$

or $$\delta y_E = 2.5\delta y_A$$

EFG: $$\frac{\delta y_E}{0.8\text{ ft}} = \frac{\delta y_G}{1.2\text{ ft}}$$

or $$\delta y_G = \frac{1.2\text{ ft}}{0.8\text{ ft}}(2.5\delta y_A)$$

$$= 3.75\delta y_A$$

Virtual Work:

$$\delta U = 0: \quad A_y\delta y_A + (240\text{ lb})\delta y_B - (60\text{ lb})\delta y_G = 0$$

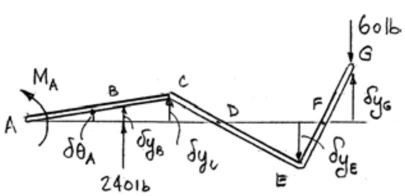
or $$A_y\delta y_A + (240\text{ lb})\delta y_A - (60\text{ lb})3.75\delta y_A = 0$$

or $$\mathbf{A}_y = 15\text{ lb}\downarrow$$

A_x: Consider a horizontal displacement δx_A:

Virtual Work: $$\delta U = 0: \quad A_x\delta x_A = 0$$

or $A_x = 0$ $\quad\therefore\quad \mathbf{A} = 15.00\text{ lb}\downarrow$ ◀



M_A: Consider a counterclockwise rotation about A:

ABC: $$\delta y_B = 2\delta\theta_A, \qquad \delta y_C = 3\delta\theta_A$$

CDE: $$\frac{\delta y_C}{1\text{ ft}} = \frac{\delta y_E}{2.5\text{ ft}}$$

or $$\delta y_E = 2.5(3\delta\theta_A)$$

$$= 7.5\delta\theta_A$$

EFG: $$\frac{\delta y_E}{0.8\text{ ft}} = \frac{\delta y_G}{1.2\text{ ft}}$$

PROBLEM 10.53 CONTINUED

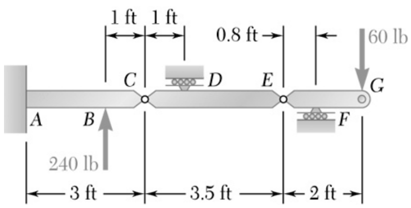
or $$\delta y_G = \frac{(1.2 \text{ ft})}{(0.8 \text{ ft})}(7.5\delta\theta_A)$$

$$= 11.25\delta\theta_A$$

Virtual Work: $\delta U = 0$: $M_A\delta\theta_A + (240 \text{ lb})\delta y_B - (60 \text{ lb})\delta y_G = 0$

or $$M_A\delta\theta_A + (240 \text{ lb})(2\delta\theta_A) - (60 \text{ lb})(11.25\delta\theta_A) = 0$$

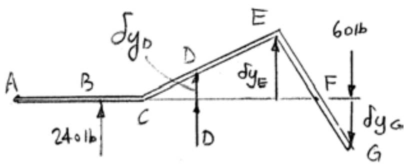
or $\mathbf{M}_A = 195.0 \text{ lb}\cdot\text{ft}$ ↻ ◀



PROBLEM 10.54

Using the method of virtual work, determine the reaction at D.

SOLUTION



Consider an upward displacement δy_E of pin E.

CDE: $$\frac{\delta y_D}{1\text{ ft}} = \frac{\delta y_E}{3.5\text{ ft}}$$

or $$\delta y_D = \frac{1}{3.5}\delta E$$

EFG: $$\frac{\delta y_E}{0.8\text{ ft}} = \frac{\delta y_G}{1.2\text{ ft}}$$

or $$\delta y_G = 1.5\delta y_E$$

Virtual Work:

$$\delta U = 0: \quad D\delta y_D + 60\delta y_G = 0$$

or $$D\left(\frac{1}{3.5}\delta y_E\right) + (60\text{ lb})(1.5\delta y_E) = 0$$

or $$\mathbf{D} = 315\text{ lb}\downarrow \blacktriangleleft$$

PROBLEM 10.55

Referring to Problem 10.41 and using the value found for the force exerted by the hydraulic cylinder *AB*, determine the change in the length of *AB* required to raise the 480-N load 18 mm.

SOLUTION

From the solution to Problem 10.41

$$F_{\text{cyl}} = 397.08 \text{ N}$$

And, Virtual Work: $\delta U = 0: \quad F_{\text{cyl}} \delta S_{AB} - P \delta y_D = 0$

where $\delta S_{AB} < 0$ for $\delta y_D > 0$

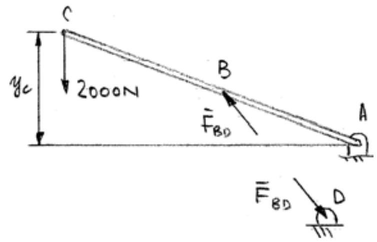
Then $(397.08 \text{ N}) \delta S_{AB} - (480 \text{ N})(18 \text{ mm}) = 0$

or $\delta S_{AB} = 21.8 \text{ mm (shortened)}$ ◀

PROBLEM 10.56

Referring to Problem 10.45 and using the value found for the force exerted by the hydraulic cylinder *BD*, determine the change in the length of *BD* required to raise the platform attached at *C* by 50 mm.

SOLUTION



Virtual Work: Assume that both δy_C and δ_{BD} increase

$$\delta U = 0: -(2000 \text{ N})\delta y_C + F_{BD}\delta_{BD} = 0$$

$$\delta y_C = 0.05 \text{ m} \qquad \text{and} \qquad F_{BD} = 9473 \text{ N } (\text{from Problem 10.45})$$

$$-2000(0.05 \text{ m}) + 9473\delta_{BD} = 0$$

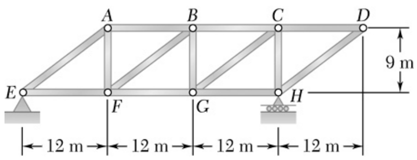
$$\delta_{BD} = 0.010556 \text{ m}$$

$$= 10.556 \text{ mm}$$

The positive sign indicates that *BD* gets longer.

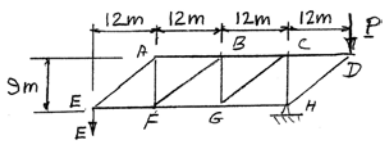
$$\delta_{BD} = 10.56 \text{ mm} \blacktriangleleft$$

PROBLEM 10.57



Determine the vertical movement of joint *D* if the length of member *BF* is increased by 75 mm. (*Hint:* Apply a vertical load at joint *D*, and, using the methods of Chapter 6, compute the force exerted by member *BF* on joints *B* and *F*. Then apply the method of virtual work for a virtual displacement resulting in the specified increase in length of member *BF*. This method should be used only for small changes in the lengths of members.)

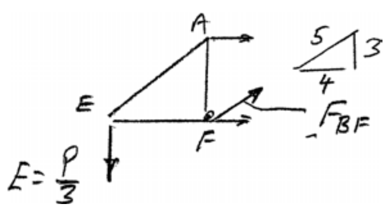
SOLUTION



Apply vertical load *P* at *D*.

$$+\circlearrowleft \Sigma M_H = 0: \quad -P(12 \text{ m}) + E(36 \text{ m}) = 0$$

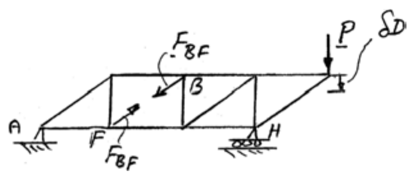
$$\mathbf{E} = \frac{P}{3} \downarrow$$



$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}F_{BF} - \frac{P}{3} = 0$$

$$F_{BF} = \frac{5}{9}P$$

Virtual Work:

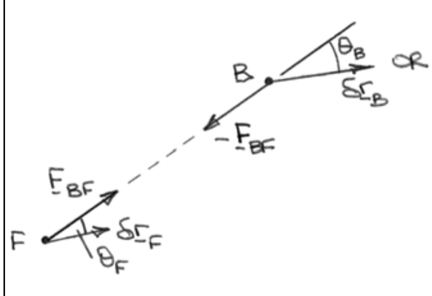


We remove member *BF* and replace it with forces $\mathbf{F}_{BF}$ and $-\mathbf{F}_{BF}$ at pins *F* and *B*, respectively. Denoting the virtual displacements of points *B* and *F* as $\delta\mathbf{r}_B$ and $\delta\mathbf{r}_F$, respectively, and noting that **P** and $\overrightarrow{\delta D}$ have the same direction, we have

Virtual Work: $\quad \delta U = 0: \quad P\delta D + \mathbf{F}_{BF} \cdot \delta\mathbf{r}_F + (-\mathbf{F}_{BF}) \cdot \delta\mathbf{r}_B = 0$

$$P\delta D + F_{BF}\delta r_F \cos\theta_F - F_{BF}\delta r_B \cos\theta_B = 0$$

$$P\delta D - F_{BF}\left(\delta r_B \cos\theta_B - \delta r_F \cos\theta_F\right) = 0$$



where $\left(\delta r_B \cos\theta_B - \delta r_F \cos\theta_F\right) = \delta_{BF}$, which is the change in length of member *BF*. Thus,

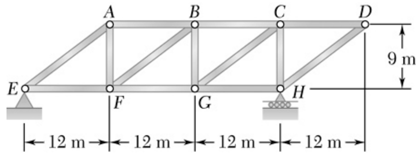
$$P\delta D - F_{BF}\delta_{BF} = 0$$

$$P\delta D - \left(\frac{5}{9}P\right)(75 \text{ mm}) = 0$$

$$\delta D = +41.67 \text{ mm}$$

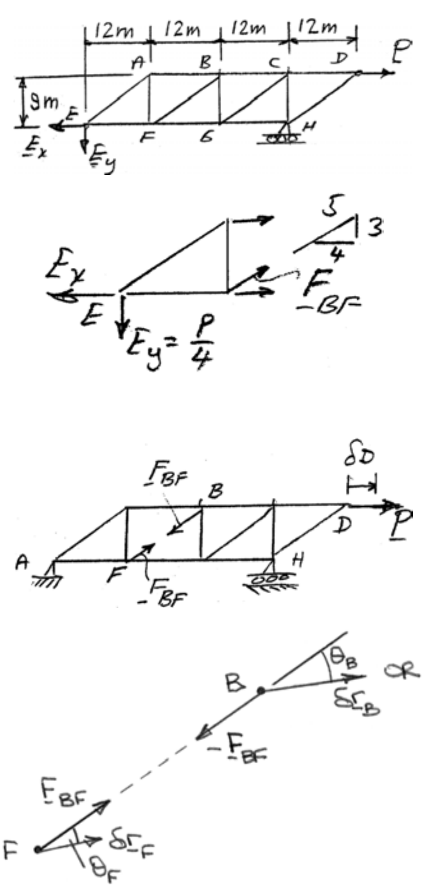
$$\delta D = 41.7 \text{ mm} \downarrow \blacktriangleleft$$

PROBLEM 10.58



Determine the horizontal movement of joint D if the length of member BF is increased by 75 mm. (See the hint for Problem 10.57.)

SOLUTION



Apply horizontal load P at D.

$$+\circlearrowleft \Sigma M_H = 0: \quad P(9 \text{ m}) - E_y(36 \text{ m}) = 0$$

$$E_y = \frac{P}{4} \downarrow$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}F_{BF} - \frac{P}{4} = 0$$

$$F_{BF} = \frac{5}{12}P$$

We remove member BF and replace it with forces $\mathbf{F}_{BF}$ and $-\mathbf{F}_{BF}$ at pins F and B, respectively. Denoting the virtual displacements of points B and F as $\delta\mathbf{r}_B$ and $\delta\mathbf{r}_F$, respectively, and noting that $\mathbf{P}$ and $\overrightarrow{\delta D}$ have the same direction, we have

Virtual Work: $\quad \delta U = 0: \quad P\delta D + \mathbf{F}_{BF}\cdot\delta\mathbf{r}_F + (-\mathbf{F}_{BF})\cdot\delta\mathbf{r}_B = 0$

$$P\delta D + F_{BF}\delta r_F \cos\theta_F - F_{BF}\delta r_B \cos\theta_B = 0$$

$$P\delta D - F_{BF}\left(\delta r_B \cos\theta_B - \delta r_F \cos\theta_F\right) = 0$$

where $(\delta r_B \cos\theta_B - \delta r_F \cos\theta_F) = \delta_{BF}$, which is the change in length of member BF. Thus,

$$P\delta D - F_{BF}\delta_{BF} = 0$$

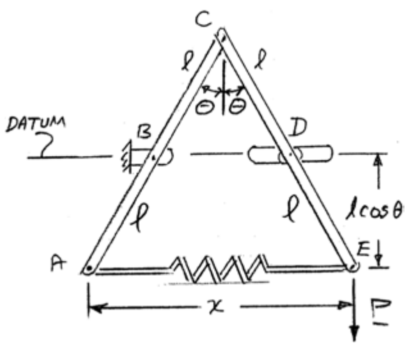
$$P\delta D - \left(\frac{5}{9}P\right)(75 \text{ mm}) = 0$$

$$\delta D = 31.25 \text{ mm} \qquad \delta D = 31.3 \text{ mm} \longrightarrow \blacktriangleleft$$

PROBLEM 10.59

Using the method of Section 10.8, solve Problem 10.29.

SOLUTION



Spring:

$$AE = x = 2(2l\sin\theta) = 4l\sin\theta$$

Unstretched length:

$$x_0 = 4l\sin 30° = 2l$$

Deflection of spring

$$s = x - x_0$$

$$s = 2l(2\sin\theta - 1)$$

$$V = \frac{1}{2}ks^2 + Py_E$$

$$V = \frac{1}{2}k\left[2l(2\sin\theta - 1)\right]^2 + P(-l\cos\theta)$$

$$\frac{dV}{d\theta} = 4kl^2(2\sin\theta - 1)2\cos\theta + Pl\sin\theta = 0$$

$$(2\sin\theta - 1)\frac{\cos\theta}{\sin\theta} + \frac{P}{8kl} = 0$$

$$\frac{P}{8kl} = \frac{1 - 2\sin\theta}{\tan\theta} \blacktriangleleft$$

PROBLEM 10.60

Using the method of Section 10.8, solve Problem 10.30.

SOLUTION

Using the result of Problem 10.59, with

$$P = 160 \text{ N},\ l = 200 \text{ mm, and } k = 300 \text{ N/m}$$

$$\frac{P}{8kl} = \frac{1 - 2\sin\theta}{\tan\theta}$$

or

$$\frac{1 - 2\sin\theta}{\tan\theta} = \frac{160 \text{ N}}{8(300 \text{ N/m})(0.2 \text{ m})}$$

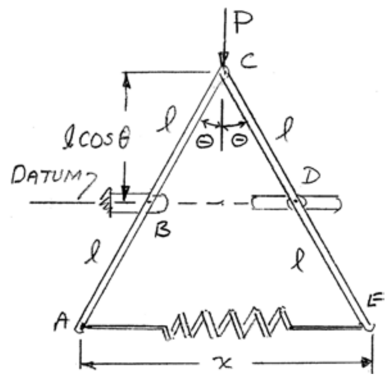
$$= \frac{1}{3}$$

Solving numerically, $\theta = 25.0° \blacktriangleleft$

PROBLEM 10.61

Using the method of Section 10.8, solve Problem 10.31.

SOLUTION



Spring: $AE = x = 2(2l\sin\theta) = 4l\sin\theta$

Unstretched length: $x_0 = 4l\sin 30° = 2l$

Deflection of spring $s = x - x_0$

$$s = 2l(2\sin\theta - 1)$$

$$V = \frac{1}{2}ks^2 + Py_C$$

$$= \frac{1}{2}k\left[2l(2\sin\theta - 1)\right]^2 + P(l\cos\theta)$$

$$V = 2kl^2(2\sin\theta - 1)^2 + Pl\cos\theta$$

$$\frac{dV}{d\theta} = 4kl^2(2\sin\theta - 1)2\cos\theta - Pl\sin\theta = 0$$

$$(1 - 2\sin\theta)\frac{\cos\theta}{\sin\theta} + \frac{P}{8kl} = 0$$

$$\frac{P}{8kl} = \frac{2\sin\theta - 1}{\tan\theta}$$

PROBLEM 10.61 CONTINUED

With $P = 160\text{ N}, l = 200\text{ mm}, \quad \text{and} \quad k = 300\text{ N/m}$

Have
$$\frac{(160\text{ N})}{8(300\text{ N/m})(0.2\text{ m})} = \frac{2\sin\theta - 1}{\tan\theta}$$

or
$$\frac{2\sin\theta - 1}{\tan\theta} = \frac{1}{3}$$

Solving numerically, $\theta = 39.65° \quad \text{and} \quad 68.96°$

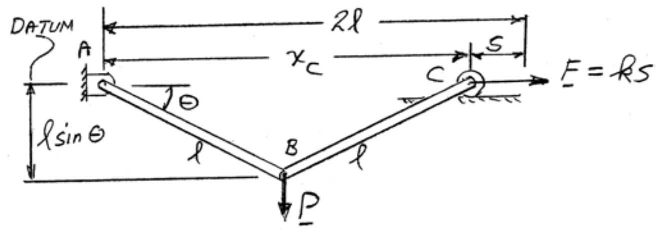
$\theta = 39.7°$ ◀
$\theta = 69.0°$

PROBLEMS 10.62 AND 10.63

10.62: Using the method of Section 10.8, solve Problem 10.33.

10.63: Using the method of Section 10.8, solve Problem 10.34.

SOLUTION



Problem 10.62

Have $$P = 150 \text{ lb}, \ l = 15 \text{ in.}, \text{ and } k = 12.5 \text{ lb/in.}$$

Then $$(1 - \cos\theta)\tan\theta = \frac{150 \text{ lb}}{4(12.5 \text{ lb/in.})(15 \text{ in.})}$$

$$= 0.2$$

Solving numerically, $$\theta = 40.2° \blacktriangleleft$$

Problem 10.63 $$V = \frac{1}{2}ks^2 + Py_B$$

$$V = \frac{1}{2}k(2l - x_C)^2 + Py_B$$

$$x_C = 2l\cos\theta \qquad \text{and} \qquad y_B = -l\sin\theta$$

Thus, $$V = \frac{1}{2}k(2l - 2l\cos\theta)^2 - Pl\sin\theta$$

$$= 2kl^2(1 - \cos\theta)^2 - Pl\sin\theta$$

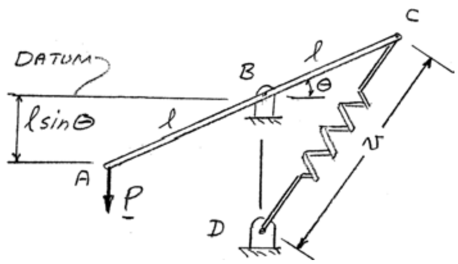
$$\frac{dV}{d\theta} = 2kl^2 2(1 - \cos\theta)\sin\theta - Pl\cos\theta = 0$$

or $$(1 - \cos\theta)\tan\theta = \frac{P}{4kl} \blacktriangleleft$$

PROBLEM 10.64

Using the method of Section 10.8, solve Problem 10.35.

SOLUTION



Spring
$$v = 2l\sin\left(\frac{90° + \theta}{2}\right)$$

$$v = 2l\sin\left(45° + \frac{\theta}{2}\right)$$

Unstretched $(\theta = 0)$

$$v_0 = 2l\sin 45° = \sqrt{2}l$$

Deflection of spring
$$s = v - v_0 = 2l\sin\left(45° + \frac{\theta}{2}\right) - \sqrt{2}l$$

$$V = \frac{1}{2}ks^2 + Py_A = \frac{1}{2}kl^2\left[2\sin\left(45° + \frac{\theta}{2}\right) - \sqrt{2}\right]^2 + P(-l\sin\theta)$$

$$\frac{dV}{d\theta} = kl^2\left[2\sin\left(45° + \frac{\theta}{2}\right) - \sqrt{2}\right]\cos\left(45° + \frac{\theta}{2}\right) - Pl\cos\theta = 0$$

$$\left[2\sin\left(45° + \frac{\theta}{2}\right)\cos\left(45° + \frac{\theta}{2}\right) - \sqrt{2}\cos\left(45° + \frac{\theta}{2}\right)\right] = \frac{P}{kl}\cos\theta$$

$$\cos\theta - \sqrt{2}\cos\left(45° + \frac{\theta}{2}\right) = \frac{P}{kl}\cos\theta$$

Divide each member by $\cos\theta$

$$1 - \sqrt{2}\frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta} = \frac{P}{kl}$$

PROBLEM 10.64 CONTINUED

Then with $P = 150$ lb, $l = 30$ in. and $k = 40$ lb/in.

$$1 - \sqrt{2}\frac{\cos\left(45^\circ + \dfrac{\theta}{2}\right)}{\cos\theta} = \frac{150 \text{ lb}}{(40 \text{ lb/in.})(30 \text{ in.})}$$

$$= 0.125$$

or
$$\frac{\cos\left(45^\circ + \dfrac{\theta}{2}\right)}{\cos\theta} = 0.618718$$

Solving numerically, $\theta = 17.83^\circ$ ◀

PROBLEM 10.65

Using the method of Section 10.8, solve Problem 10.36.

SOLUTION

Using the results of Problem 10.64 with $P = 600$ N, $l = 800$ mm, and $k = 4$ kN/m , have

$$1 - \sqrt{2}\frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta} = \frac{P}{kl}$$

$$= \frac{600 \text{ N}}{(4000 \text{ N/m})(0.8 \text{ m})}$$

$$= 0.1875$$

or

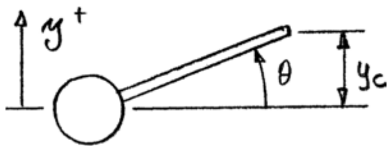
$$\frac{\cos\left(45° + \frac{\theta}{2}\right)}{\cos\theta} = 0.57452$$

Solving numerically, $\theta = 30.985°$ $\qquad \theta = 31.0°$ ◀

PROBLEM 10.66

Using the method of Section 10.8, solve Problem 10.38.

SOLUTION



Spring $$V_{SP} = \frac{1}{2}ky_C^2$$

where $$y_C = d_{AC}\tan\theta \qquad d_{AC} = 15 \text{ in.}$$

$$\therefore \quad V_{SP} = \frac{1}{2}kd_{AC}^2\tan^2\theta$$

Force P: $$V_P = -Py_P$$

where $$y_P = r\theta \qquad r = 3 \text{ in.}$$

$$\therefore \quad V_P = -Pr\theta$$

Then $$V = V_{SP} + V_P$$

$$= \frac{1}{2}kd_{AC}^2\tan^2\theta - Pr\theta$$

Equilibrium $$\frac{dV}{d\theta} = 0: \ kd_{AC}^2\tan\theta\sec^2\theta - Pr = 0$$

or $$(4 \text{ lb/in.})(15 \text{ in.})^2\tan\theta\sec^2\theta - (96 \text{ lb})(3 \text{ in.}) = 0$$

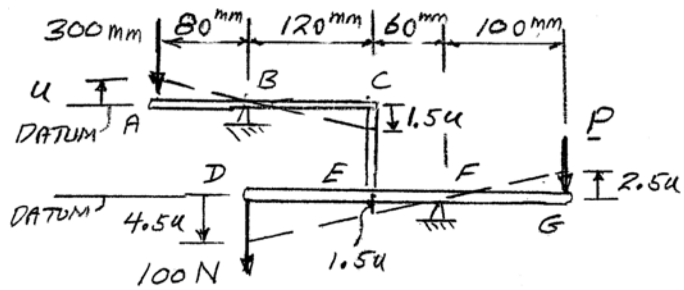
or $$3.125\tan\theta\sec^2\theta - 1 = 0$$

Solving numerically, $\theta = 16.4079°$ $\qquad \theta = 16.41° \blacktriangleleft$

PROBLEM 10.67

Show that the equilibrium is neutral in Problem 10.1.

SOLUTION



We have $$y_A = u$$

$$y_D = -4.5u$$

$$y_G = 2.5u$$

Have $$V = (300\text{ N})y_A + (100\text{ N})y_D + P(y_E) = 0$$

$$V = 300u + 100(-4.5u) + P(2.5u) = 0$$

$$V = (-150 + 2.5P)u$$

$$\frac{dV}{du} = -150 + 2.5P = 0 \text{ so that } \qquad P = 60\text{ N}$$

Substitute $P = 60$ N in expression for V: $$V = [-150 + 2.5(60)]u$$

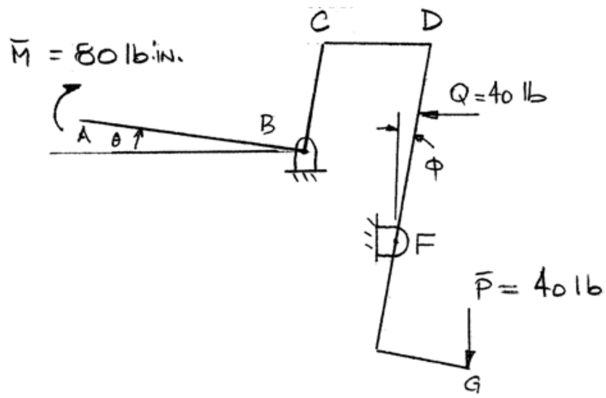
$$= 0$$

$\therefore$ V is constant and equilibrium is neutral ◀

PROBLEM 10.68

Show that the equilibrium is neutral in Problem 10.2.

SOLUTION



Consider a small disturbance of the system so that $\theta \ll 1$

Have $$x_C = x_D, \qquad 5\theta \simeq 15\phi$$

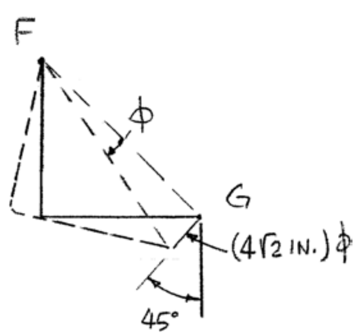
or $$\phi = \frac{\theta}{3}$$

Potential energy $$V = M\theta - Qx_E + Py_G$$

where $$x_E = (10 \text{ in.})\phi$$

$$= \left(\frac{10}{3}\theta\right) \text{ in.}$$

and $$y_G = \left[\left(4\sqrt{2} \text{ in.}\right)\phi\right]\cos 45°$$



PROBLEM 10.68 CONTINUED

Then $$V = M\theta - \frac{10}{3}Q\theta + \frac{4}{3}P\theta$$

$$= \left(M + \frac{10}{3}Q + \frac{4}{3}P\right)\theta$$

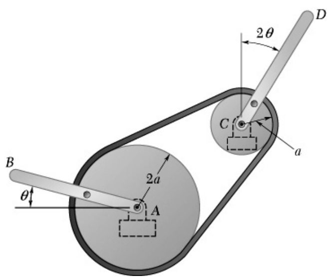
and $$\frac{dV}{d\theta} = M - \frac{10}{3}Q + \frac{4}{3}P$$

For equilibrium $$\frac{dV}{d\theta} = 0: \quad M - \frac{10}{3}Q + \frac{4}{3}P = 0$$

$\therefore$ At equilibrium, $V = 0$, a constant, for all values of θ.

Hence, equilibrium is neutral

Q.E.D.◀



PROBLEM 10.69

Two identical uniform rods, each of weight W and length L, are attached to pulleys that are connected by a belt as shown. Assuming that no slipping occurs between the belt and the pulleys, determine the positions of equilibrium of the system and state in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION

Let each rod be of length L and weight W. Then the potential energy V is

$$V = W\left(\frac{L}{2}\sin\theta\right) + W\left(\frac{L}{2}\cos 2\theta\right)$$

Then

$$\frac{dV}{d\theta} = \frac{W}{2}L\cos\theta - WL\sin 2\theta$$

For equilibrium

$$\frac{dV}{d\theta} = 0: \quad \frac{W}{2}L\cos\theta - WL\sin 2\theta = 0$$

or

$$\cos\theta - 2\sin 2\theta = 0$$

Solving numerically or using a computer algebra system, such as Maple, gives four solutions:

$$\theta = 1.570796327 \text{ rad} = 90.0°$$

$$\theta = -1.570796327 \text{ rad} = 270°$$

$$\theta = 0.2526802551 \text{ rad} = 14.4775°$$

$$\theta = 2.888912399 \text{ rad} = 165.522°$$

Now

$$\frac{d^2V}{d\theta^2} = -\frac{1}{2}WL\sin\theta - 2WL\cos 2\theta$$

$$= -WL\left(\frac{1}{2}\sin\theta + 2\cos 2\theta\right)$$

PROBLEM 10.69 CONTINUED

At $\theta = 14.4775°$

$$\frac{d^2V}{d\theta^2} = -WL\left\{\frac{1}{2}\sin 14.4775° + 2\cos\left[2\left(14.4775°\right)\right]\right\}$$

$$= -1.875WL \ \left(< 0\right) \qquad \therefore \quad \theta = 14.48°, \ \text{Unstable} ◀$$

At $\theta = 90°$

$$\frac{d^2V}{d\theta^2} = -WL\left\{\frac{1}{2}\sin 90° + 2\cos 180°\right\}$$

$$= 1.5WL \ \left(> 0\right) \qquad \therefore \quad \theta = 90°, \ \text{Stable} ◀$$

At $\theta = 165.522°$

$$\frac{d^2V}{d\theta^2} = -WL\left\{\frac{1}{2}\sin 165.522° + 2\cos\left(2 \times 165.522°\right)\right\}$$

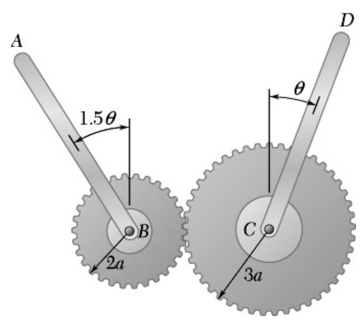
$$= -1.875WL \ \left(< 0\right) \qquad \therefore \quad \theta = 165.5°, \ \text{Unstable} ◀$$

At $\theta = 270°$

$$\frac{d^2V}{d\theta^2} = -WL\left(\frac{1}{2}\sin 270° + 2\cos 540°\right)$$

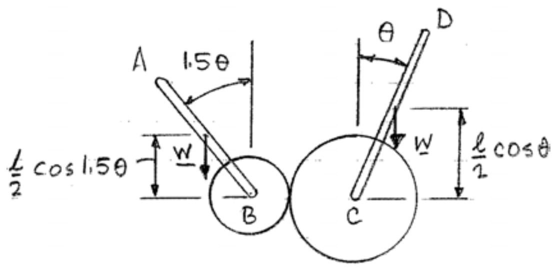
$$= 2.5WL \ \left(> 0\right) \qquad \therefore \quad \theta = 270°, \ \text{Stable} ◀$$

PROBLEM 10.70



Two uniform rods, each of mass m and length l, are attached to gears as shown. For the range $0 \le \theta \le 180^\circ$, determine the positions of equilibrium of the system and state in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION



Potential energy $$V = W\left(\frac{l}{2}\cos 1.5\theta\right) + W\left(\frac{l}{2}\cos\theta\right) \qquad W = mg$$

$$\frac{dV}{d\theta} = \frac{Wl}{2}(-1.5\sin 1.5\theta) + \frac{Wl}{2}(-\sin\theta)$$

$$= -\frac{Wl}{2}(1.5\sin 1.5\theta + \sin\theta)$$

$$\frac{d^2V}{d\theta^2} = -\frac{Wl}{2}(2.25\cos 1.5\theta + \cos\theta)$$

For equilibrium $$\frac{dV}{d\theta} = 0\text{: } 1.5\sin 1.5\theta + \sin\theta = 0$$

Solutions: One solution, by inspection, is $\theta = 0$, and a second angle less than 180° can be found numerically:

$$\theta = 2.4042 \text{ rad} = 137.8^\circ$$

Now $$\frac{d^2V}{d\theta^2} = -\frac{Wl}{2}(2.25\cos 1.5\theta + \cos\theta)$$

PROBLEM 10.70 CONTINUED

At $\theta = 0$:

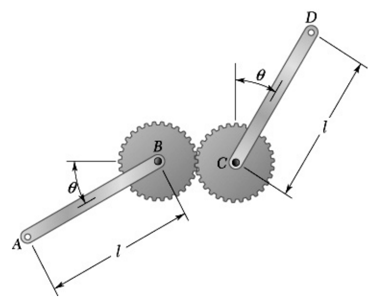
$$\frac{d^2V}{d\theta^2} = -\frac{Wl}{2}\left(2.25\cos 0° + \cos 0°\right)$$

$$= -\frac{Wl}{2}\left(3.25\right) \ \left(< 0\right) \qquad \therefore\ \theta = 0,\ \text{Unstable} \blacktriangleleft$$

At $\theta = 137.8°$:

$$\frac{d^2V}{d\theta^2} = -\frac{Wl}{2}\left[2.25\cos\left(1.5 \times 137.8°\right) + \cos 137.8°\right]$$

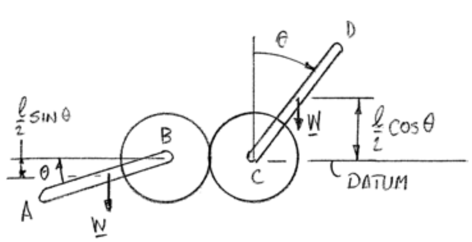
$$= \frac{Wl}{2}\left(2.75\right) \ \left(> 0\right) \qquad \therefore\ \theta = 137.8°,\ \text{Stable} \blacktriangleleft$$



PROBLEM 10.71

Two uniform rods, each of mass m, are attached to gears of equal radii as shown. Determine the positions of equilibrium of the system and state in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION



Potential Energy

$$V = W\left(-\frac{l}{2}\sin\theta\right) + W\left(\frac{l}{2}\cos\theta\right) \qquad W = mg$$

$$= W\frac{l}{2}(\cos\theta - \sin\theta)$$

$$\frac{dV}{d\theta} = \frac{Wl}{2}(-\sin\theta - \cos\theta)$$

$$\frac{d^2V}{d\theta^2} = \frac{Wl}{2}(\sin\theta - \cos\theta)$$

For Equilibrium: $\frac{dV}{d\theta} = 0: \quad \sin\theta = -\cos\theta$

or $\tan\theta = -1$

Thus $\theta = -45.0°$ and $\theta = 135.0°$

Stability:

At $\theta = -45.0°$:

$$\frac{d^2V}{d\theta^2} = \frac{Wl}{2}\left[\sin(-45°) - \cos 45°\right]$$

$$= \frac{Wl}{2}\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right) < 0$$

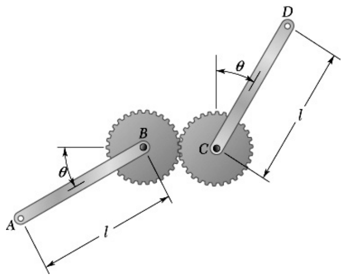
$\therefore \quad \theta = -45.0°$, Unstable ◀

At $\theta = 135.0°$:

$$\frac{d^2V}{d\theta^2} = \frac{Wl}{2}(\sin 135° - \cos 135°)$$

$$= \frac{Wl}{2}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) > 0$$

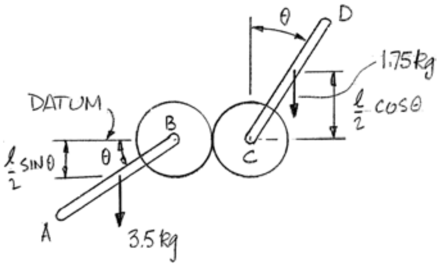
$\therefore \quad \theta = 135.0°$, Stable ◀



PROBLEM 10.72

Two uniform rods, *AB* and *CD*, are attached to gears of equal radii as shown. Knowing that $m_{AB} = 3.5\text{ kg}$ and $m_{CD} = 1.75\text{ kg}$, determine the positions of equilibrium of the system and state in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION



Potential Energy

$$V = \left(3.5\text{ kg} \times 9.81\text{ m/s}^2\right)\left(-\frac{l}{2}\sin\theta\right) + \left(1.75\text{ kg} \times 9.81\text{ m/s}^2\right)\left(\frac{l}{2}\cos\theta\right)$$

$$= \left(8.5838\text{ N}\right)l\left(-2\sin\theta + \cos\theta\right)$$

$$\frac{dV}{d\theta} = \left(8.5838\text{ N}\right)l\left(-2\cos\theta - \sin\theta\right)$$

$$\frac{d^2V}{d\theta^2} = \left(8.5838\text{ N}\right)l\left(2\sin\theta - \cos\theta\right)$$

Equilibrium: $\frac{dV}{d\theta} = 0$: $\quad -2\cos\theta - \sin\theta = 0$

or $\tan\theta = -2$

Thus $\theta = -63.4°$ and $116.6°$

Stability

At $\theta = -63.4°$: $\frac{d^2V}{d\theta^2} = \left(8.5838\text{ N}\right)l\left[2\sin\left(-63.4°\right) - \cos\left(-63.4°\right)\right]$

$$= \left(8.5838\text{ N}\right)l\left(-1.788 - 0.448\right) < 0$$

$\therefore \quad \theta = -63.4°$, Unstable ◀

At $\theta = 116.6°$: $\frac{d^2V}{d\theta^2} = \left(8.5838\text{ N}\right)l\left[2\sin\left(116.6°\right) - \cos\left(116.6°\right)\right]$

$$= \left(8.5838\text{ N}\right)l\left(1.788 + 0.447\right) > 0$$

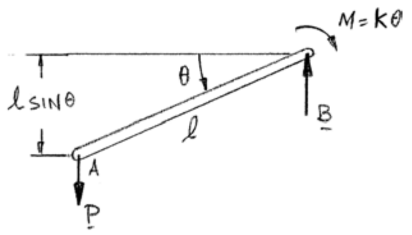
$\therefore \quad \theta = 116.6°$, Stable ◀

PROBLEM 10.73

Using the method of Section 10.8, solve Problem 10.39. Determine whether the equilibrium is stable, unstable or neutral. (*Hint:* The potential energy corresponding to the couple exerted by a torsional spring is $\frac{1}{2}K\theta^2$, where K is the torsional spring constant and θ is the angle of twist.)

SOLUTION

Potential Energy



$$V = \frac{1}{2}K\theta^2 - Pl\sin\theta$$

$$\frac{dV}{d\theta} = K\theta - Pl\cos\theta$$

$$\frac{d^2V}{d\theta^2} = K + Pl\sin\theta$$

Equilibrium: $\frac{dV}{d\theta} = 0$: $\cos\theta = \frac{K}{Pl}\theta$

For $P = 400$ lb, $l = 10$ in., $K = 150$ lb·ft/rad

$$\cos\theta = \frac{150\text{ lb}\cdot\text{ft/rad}}{(400\text{ lb})\left(\frac{10}{12}\text{ ft}\right)}\theta$$

$$= 0.450\theta$$

Solving numerically, we obtain

$$\theta = 1.06896\text{ rad} = 61.247°$$

$\theta = 61.2°$ ◀

Stability

$$\frac{d^2V}{d\theta^2} = (150\text{ lb}\cdot\text{ft/rad}) + (400\text{ lb})\left(\frac{10}{12}\text{ ft}\right)\sin 61.2° > 0$$

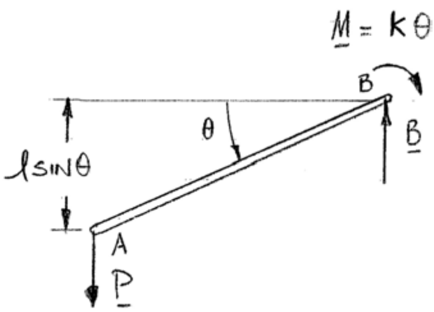
∴ Stable ◀

PROBLEM 10.74

In Problem 10.40, determine whether each of the positions of equilibrium is stable, unstable, of neutral. (See the hint for Problem 10.73.)

SOLUTION

Potential Energy



$$V = \frac{1}{2}K\theta^2 - Pl\sin\theta$$

$$\frac{dV}{d\theta} = K\theta - Pl\cos\theta$$

$$\frac{d^2V}{d\theta^2} = K + Pl\sin\theta$$

Equilibrium $\qquad \frac{dV}{d\theta} = 0: \quad \cos\theta = \frac{K}{Pl}\theta$

For $\qquad P = 1260 \text{ lb}, \ l = 10 \text{ in., and } K = 150 \text{ lb}\cdot\text{ft/rad}$

$$\cos\theta = \frac{150 \text{ lb}\cdot\text{ft/rad}}{(1260 \text{ lb})\left(\frac{10}{12}\text{ ft}\right)}\theta$$

or $\qquad \cos\theta = \frac{\theta}{7}$

Solving numerically, $\qquad \theta = 1.37333 \text{ rad, } 5.652 \text{ rad, and } 6.616 \text{ rad}$

or $\qquad \theta = 78.7°, \ 323.8°, \ 379.1°$

Stability At $\theta = 78.7°$: $\qquad \frac{d^2V}{d^2\theta^2}$ hmm

$$\frac{d^2V}{d\theta^2} = (150 \text{ lb}\cdot\text{ft/rad}) + (1260 \text{ lb})\left(\frac{10}{12}\text{ ft}\right)\sin 78.7°$$

$$= 1179.6 \text{ ft}\cdot\text{lb} > 0 \qquad \therefore \quad \theta = 78.7°, \ \text{Stable} \blacktriangleleft$$

At $\theta = 323.8°$:

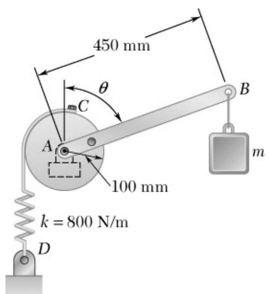
$$\frac{d^2V}{d\theta^2} = (150 \text{ lb}\cdot\text{ft/rad}) + (1260 \text{ lb})\left(\frac{10}{12}\text{ ft}\right)\sin 323.8°$$

$$= -470 \text{ ft}\cdot\text{lb} < 0 \qquad \therefore \quad \theta = 324°, \ \text{Unstable} \blacktriangleleft$$

At $\theta = 379.1°$:

$$\frac{d^2V}{d\theta^2} = (150 \text{ lb}\cdot\text{ft/rad}) + (1260 \text{ lb})\left(\frac{10}{12}\text{ ft}\right)\sin 379.1°$$

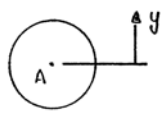
$$= 493.5 \text{ ft}\cdot\text{lb} > 0 \qquad \therefore \quad \theta = 379°, \ \text{Stable} \blacktriangleleft$$



PROBLEM 10.75

Angle θ is equal to 45° after a block of mass m is hung from member AB as shown. Neglecting the mass of AB and knowing that the spring is unstretched when $\theta = 20°$, determine the value of m and state whether the equilibrium is stable, unstable, or neutral.

SOLUTION



Potential Energy

Have $$V = \frac{1}{2}kx_{SP}^2 + mgy_B$$

where $x_{SP} = r(\theta - \theta_0)$, $r = 100$ mm, $\theta_0 = 20° = \frac{\pi}{9}$ rad

$$y_B = L_{AB}\cos\theta, \qquad L_{AB} = 450 \text{ mm}$$

Then $$V = \frac{1}{2}kr^2(\theta - \theta_0)^2 + mgL_{AB}\cos\theta$$

and $$\frac{dV}{d\theta} = kr^2(\theta - \theta_0) - mgL_{AB}\sin\theta$$

$$\frac{d^2V}{d\theta^2} = kr^2 - mgL_{AB}\cos\theta$$

With $k = 800$ N/m, $\theta = 45°$

Equilibrium: $$\frac{dV}{d\theta} = 0: \quad (800 \text{ N/m})(0.1 \text{ m})^2\left(\frac{\pi}{4} - \frac{\pi}{9}\right) - m(9.81 \text{ m/s}^2)(0.45 \text{ m})\sin\frac{\pi}{4} = 0$$

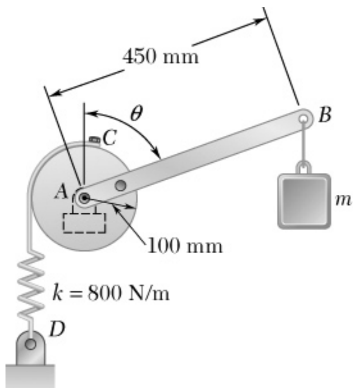
Then $m = 1.11825$ kg $\qquad m = 1.118 \text{ kg}$ ◀

Stability

Now $$\frac{d^2V}{d\theta^2} = (800 \text{ N/m})(0.1 \text{ m})^2 - (1.118 \text{ kg})(9.81 \text{ m/s}^2)(0.45 \text{ m})\cos\frac{\pi}{4}$$

$$= 4.51 \text{ J} > 0$$

∴ Stable ◀



PROBLEM 10.76

A block of mass m is hung from member AB as shown. Neglecting the mass of AB and knowing that the spring is unstretched when $\theta = 20°$, determine the value of θ corresponding to equilibrium when $m = 3\text{ kg}$. State whether the equilibrium is stable, unstable, or neutral.

SOLUTION

Using the general results of Problem 10.76 and noting that now

$$m = 3\text{ kg}, \qquad \text{and} \qquad \theta_0 = 20°$$

we have

Equilibrium
$$\frac{dV}{d\theta} = 0: \quad kr^2(\theta - \theta_0) - mgL_{AB}\sin\theta = 0$$

$$(800\text{ N/m})(0.1\text{ m})^2\left(\theta - \frac{\pi}{9}\right) - (3\text{ kg})(9.81\text{ m/s}^2)(0.45\text{ m})\sin\theta = 0$$

or
$$\left(\theta - \frac{\pi}{9}\right) - 1.65544\sin\theta = 0$$

Solving numerically,
$$\theta = 1.91011\text{ rad}$$
$$= 109.441°$$

or $\theta = 109.4°$ ◀

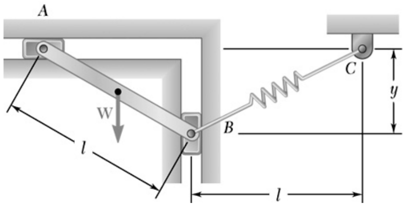
Stability

$$\frac{d^2v}{d\theta^2} = kr^2 - mgL_{AB}\cos\theta$$

$$= (800\text{ N/m})(0.1\text{ m})^2 - (3\text{ kg})(9.81\text{ m/s})(0.45\text{ m})\cos(109.4°)$$

$$= 12.41\text{ J} > 0$$

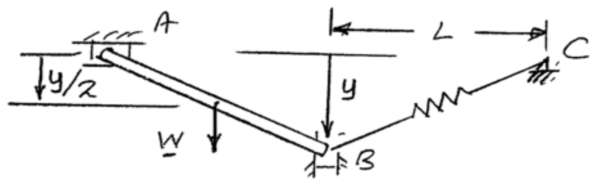
$\therefore$ Stable ◀



PROBLEM 10.77

A slender rod AB, of mass m, is attached to two blocks A and B which can move freely in the guides shown. Knowing that the spring is unstretched when $y = 0$, determine the value of y corresponding to equilibrium when $m = 12$ kg, $l = 750$ mm, and $k = 900$ N/m.

SOLUTION



Deflection of spring = s, where

$$s = \sqrt{l^2 + y^2} - l$$

$$\frac{ds}{dy} = \frac{y}{\sqrt{l^2 - y^2}}$$

Potential Energy:

$$V = \frac{1}{2}ks^2 - W\frac{y}{2}$$

$$\frac{dV}{dy} = ks\frac{ds}{dy} - \frac{1}{2}W$$

$$\frac{dV}{dy} = k\left(\sqrt{l^2 + y^2} - l\right)\frac{y}{\sqrt{l^2 + y^2}} - \frac{1}{2}W$$

$$= k\left(1 - \frac{l}{\sqrt{l^2 + y^2}}\right)y - \frac{1}{2}W$$

Equilibrium

$$\frac{dV}{dy} = 0: \quad \left(1 - \frac{l}{\sqrt{l^2 + y^2}}\right)y = \frac{1}{2}\frac{W}{k}$$

Now

$$W = mg = (12 \text{ kg})\left(9.81 \text{ m/s}^2\right) = 117.72 \text{ N}, \ l = 0.75 \text{ m, and } k = 900 \text{ N/m}$$

Then

$$\left(1 - \frac{0.75 \text{ m}}{\sqrt{(0.75 \text{ m})^2 + y^2}}\right)y = \frac{1}{2}\frac{(117.72 \text{ N})}{(900 \text{ N/m})}$$

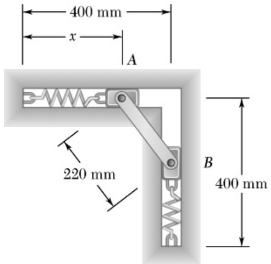
or

$$\left(1 - \frac{0.75}{\sqrt{0.5625 + y^2}}\right)y = 0.6540$$

Solving numerically,

$$y = 0.45342 \text{ m}$$

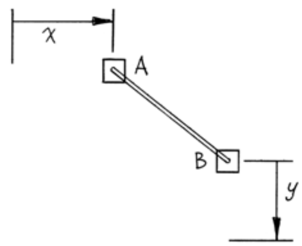
$$y = 453 \text{ mm} \blacktriangleleft$$



PROBLEM 10.78

The slender rod *AB* of negligible mass is attached to two 4-kg blocks *A* and *B* that can move freely in the guides shown. Knowing that the constant of the springs is 160 N/m and that the unstretched length of each spring is 150 mm, determine the value of *x* corresponding to equilibrium.

SOLUTION



First note

$$y = \left[0.4 - \sqrt{(0.22)^2 - (0.4 - x)^2}\right]\text{m}$$

$$= \left(0.4 - \sqrt{-x^2 + 0.8x - 0.1116}\right)\text{m}$$

Now, the Potential Energy is

$$V = \frac{1}{2}k(x - 0.15)^2 + \frac{1}{2}k(y - 0.15)^2 + 0.4m_A g + m_B gy$$

$$= \frac{1}{2}k(x - 0.15)^2 + \frac{1}{2}k\left(0.25 - \sqrt{-x^2 + 0.8x - 0.1116}\right)^2$$

$$+0.4m_A g + m_B g\left(0.4 - \sqrt{-x^2 + 0.8x - 0.1116}\right)$$

For Equilibrium

$$\frac{dV}{d\theta} = 0: \quad k(x - 0.15) + k\left(0.25 - \sqrt{-x^2 + 0.8x - 0.1116}\right)\left(-\frac{0.8 - 2x}{2\sqrt{-x^2 + 0.8x - 0.1116}}\right)$$

$$- m_B g\frac{0.8 - 2x}{2\sqrt{-x^2 + 0.8x - 0.1116}} = 0$$

Simplifying,

$$k(x - 0.4) + \sqrt{-x^2 + 0.8x - 0.1116} + 4m_B g(x - 0.4) = 0$$

Substituting the masses, $m_A = m_B = 0.4$ kg, and the spring constant $k = 160$ N/m:

$$(160\text{ N/m})\left(x - 0.4 + \sqrt{-x^2 + 0.8x - 0.1116}\right)\text{m}^2 + 4(4\text{ kg})(9.81\text{ m/s}^2)(x - 0.4)\text{m} = 0$$

PROBLEM 10.78 CONTINUED

or
$$\left(x - 0.4 + \sqrt{-x^2 + 0.8x - 0.1116}\right) + 0.981(x - 0.4) = 0$$

Simplifying,
$$\left(0.8x - x^2 - 0.1116\right)^2 = (0.7924 - 1.981x)^2$$

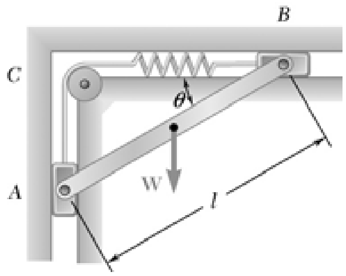
or
$$4.92436^2 - 3.93949x + 0.739498 = 0$$

Then
$$x = \frac{3.93949 \pm \sqrt{(-3.93949)^2 - 4(4.92436)(0.739498)}}{2(4.92436)}$$

or
$$x = 0.49914 \text{ m} \quad \text{and} \quad x = 0.30086 \text{ m}$$

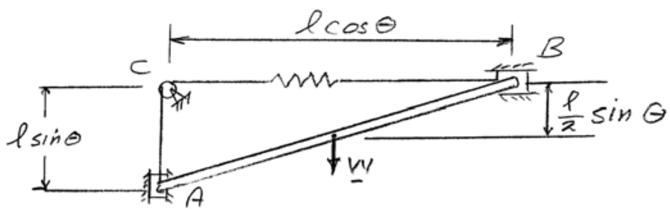
Now
$$x \leq 0.4 \text{ m} \Rightarrow x = 301 \text{ mm} \blacktriangleleft$$

PROBLEM 10.79



A slender rod *AB*, of mass *m*, is attached to two blocks *A* and *B* that can move freely in the guides shown. The constant of the spring is *k*, and the spring is unstretched when *AB* is horizontal. Neglecting the weight of the blocks, derive an equation in θ, *m*, *l*, and *k* that must be satisfied when the rod is in equilibrium.

SOLUTION



Elongation of Spring:

$$s = l\sin\theta + l\cos\theta - l$$

$$s = l(\sin\theta + \cos\theta - 1)$$

Potential Energy: $V = \frac{1}{2}ks^2 - W\frac{l}{2}\sin\theta \qquad W = mg$

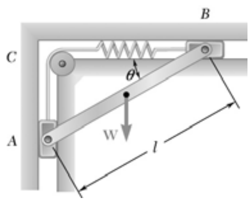
$$= \frac{1}{2}kl^2(\sin\theta + \cos\theta - 1)^2 - mg\frac{l}{2}\sin\theta$$

$$\frac{dV}{d\theta} = kl^2(\sin\theta + \cos\theta - 1)(\cos\theta - \sin\theta) - \frac{1}{2}mgl\cos\theta \tag{1}$$

Equilibrium:

$$\frac{dV}{d\theta} = 0: (\sin\theta + \cos\theta - 1)(\cos\theta - \sin\theta) - \frac{mg}{2kl}\cos\theta = 0$$

or $$\cos\theta\left[(\sin\theta + \cos\theta - 1)(1 - \tan\theta) - \frac{mg}{2kl}\right] = 0 \blacktriangleleft$$



PROBLEM 10.80

A slender rod *AB*, of mass *m*, is attached to two blocks *A* and *B* that can move freely in the guides shown. Knowing that the spring is unstretched when *AB* is horizontal, determine three values of θ corresponding to equilibrium when $m = 125$ kg, $l = 320$ mm, and $k = 15$ kN/mm. State in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION

Using the results of Problem 10.79, particularly the condition of equilibrium

$$\cos\theta\left[(\sin\theta + \cos\theta - 1)(1 - \tan\theta) - \frac{mg}{2kl}\right] = 0$$

Now, with $W = mg = (125 \text{ kg})(9.81 \text{ m/s}^2) = 1226.25 \text{ N}$, $l = 320 \text{ mm}$, and $k = 15 \text{ kN/m}$,

Now
$$\frac{W}{2kl} = \frac{1226.25 \text{ N}}{2(15000 \text{ N/m})(0.32 \text{ m})} = 1.2773$$

so that
$$\cos\theta\left[(\sin\theta + \cos\theta - 1)(1 - \tan\theta) - 1.2773\right] = 0$$

By inspection, one solution is $\cos\theta = 0$ or $\theta = 90.0°$

Solving numerically: $\theta = 0.38338 \text{ rad} = 9.6883°$ and $\theta = 0.59053 \text{ rad} = 33.8351°$

Stability

$$\frac{d^2V}{d\theta^2} = kl^2\left[(\cos\theta - \sin\theta)(\cos\theta - \sin\theta) + (\sin\theta + \cos\theta - 1)(-\sin\theta - \cos\theta)\right] + \frac{1}{2}mgl\sin\theta$$

$$= kl^2\left[\cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta - \sin^2\theta - \cos^2\theta - 2\sin\theta\cos\theta + \sin\theta + \cos\theta + \frac{mg}{2kl}\sin\theta\right]$$

$$= kl^2\left[\left(1 + \frac{mg}{2kl}\right)\sin\theta + \cos\theta - 2\sin 2\theta\right]$$

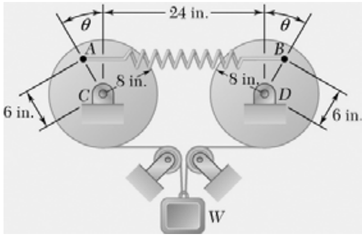
$$= (15 \text{ N/m})(0.32 \text{ m})^2\left[(1 - 127.73)\sin\theta + \cos\theta - 2\sin 2\theta\right]$$

Thus, at

At $\theta = 90°$: $\frac{d^2V}{d\theta^2} = 89.7 > 0$ $\therefore\ \theta = 90.0°$, Stable ◀

At $\theta = 9.6883°$: $\frac{d^2V}{d\theta^2} = 0.512 > 0$ $\therefore\ \theta = 9.69°$, Stable ◀

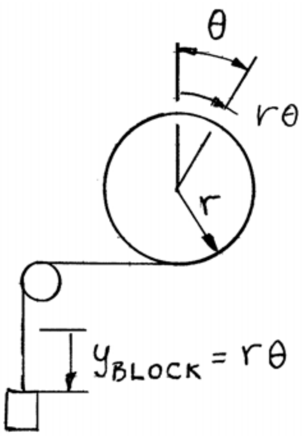
At $\theta = 33.8351°$: $\frac{d^2V}{d\theta^2} = -0.391 < 0$ $\therefore\ \theta = 33.8°$, Unstable ◀



PROBLEM 10.81

Spring AB of constant 10 lb/in. is attached to two identical drums as shown. Knowing that the spring is unstretched when $\theta = 0$, determine (a) the range of values of the weight W of the block for which a position of equilibrium exists, (b) the range of values of θ for which the equilibrium is stable.

SOLUTION



Have $$V = \frac{1}{2}kx_{SP}^2 - W_{y_{\text{block}}}$$

where $$x_{SP} = 2r_A \sin\theta, \qquad r_A = 6 \text{ in.}$$

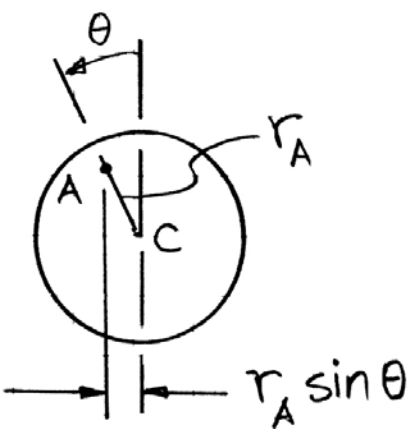
and $$y_{\text{block}} = r\theta, \qquad r = 8 \text{ in.}$$

Then $$V = \frac{1}{2}k\left(2r_A \sin\theta\right)^2 - Wr\theta$$

$$= 2kr_A^2 \sin^2\theta - Wr\theta$$

and $$\frac{dV}{d\theta} = 2kr_A^2\left(2\sin\theta\cos\theta\right) - Wr$$

$$= 2kr_A^2 \sin 2\theta - Wr$$



$$\frac{d^2V}{d\theta^2} = 4kr_A^2 \cos 2\theta \tag{1}$$

For equilibrium $$\frac{dV}{d\theta} = 0: \quad 2kr_A^2 \sin 2\theta - Wr = 0$$

Substituting, $$2(10 \text{ lb/in.})(6 \text{ in.})^2 \sin 2\theta - W(8 \text{ in.}) = 0$$

or $$W = 90 \sin 2\theta \text{ (lb)}$$

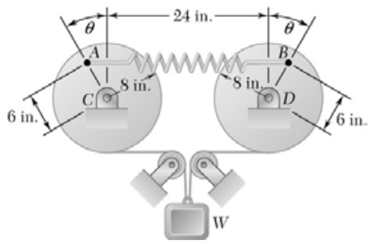
(a) From Equation (2), with $W \geq 0$: $0 \leq W \leq 90$ lb ◀

(b) From Stable equilibrium $$\frac{d^2V}{d\theta^2} > 0$$

Then from Equation (1), $$\cos 2\theta > 0$$

or $0 \leq \theta \leq 45°$ ◀

PROBLEM 10.82



Spring AB of constant 10 lb/in. is attached to two identical drums as shown. Knowing that the spring is unstretched when $\theta = 0$ and that $W = 40\,\text{lb}$, determine the values of θ less than 180° corresponding to equilibrium. State in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION

See sketch, Problem 10.81.

Using Equation (2) of Problem 10.81, with $W = 40$ lb

$$40 = 90\sin 2\theta \quad \text{(for equilibrium)}$$

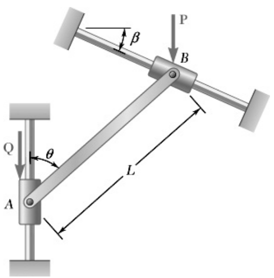
Solving $\theta = 13.1939°$ and $\theta = 76.806°$

Using Equation (1) of Problem 10.81, we have

At $\theta = 13.1939°$: $$\frac{d^2V}{d\theta^2} = 4kr_A^2\cos\left(2 \times 13.1939°\right) > 0$$ $\therefore \quad \theta = 13.19°,$ Stable ◀

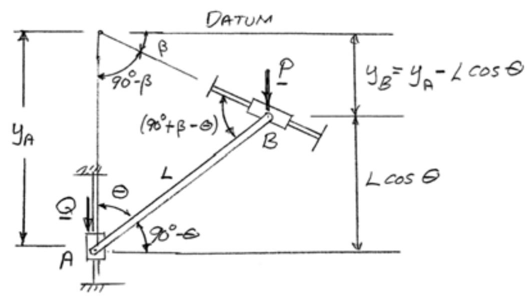
At $\theta = 76.806°$: $$\frac{d^2V}{d\theta^2} = 4kr_A^2\cos\left(2 \times 76.806°\right) < 0$$ $\therefore \quad \theta = 76.8°,$ Unstable ◀

PROBLEM 10.83



A slender rod *AB* of negligible weight is attached to two collars *A* and *B* that can move freely along the guide rods shown. Knowing that $\beta = 30^{\circ}$ and $P = Q = 100$ lb, determine the value of the angle θ corresponding to equilibrium.

SOLUTION



Law of Sines
$$\frac{y_A}{\sin(90^{\circ} + \beta - \theta)} = \frac{L}{\sin(90 - \beta)}$$

$$\frac{y_A}{\cos(\theta - \beta)} = \frac{L}{\cos\beta}$$

or
$$y_A = L\frac{\cos(\theta - \beta)}{\cos\beta}$$

From the figure:
$$y_B = L\frac{\cos(\theta - \beta)}{\cos\beta} - L\cos\theta$$

Potential Energy:
$$V = -Py_B - Qy_A = -P\left[L\frac{\cos(\theta - \beta)}{\cos\beta} - L\cos\theta\right] - QL\frac{\cos(\theta - \beta)}{\cos\beta}$$

$$\frac{dV}{d\theta} = -PL\left[-\frac{\sin(\theta - \beta)}{\cos\beta} + \sin\theta\right] + QL\frac{\sin(\theta - \beta)}{\cos\beta}$$

$$= L(P + Q)\frac{\sin(\theta - \beta)}{\cos\beta} - PL\sin\theta$$

Equilibrium
$$\frac{dV}{d\theta} = 0: \quad L(P + Q)\frac{\sin(\theta - \beta)}{\cos\beta} - PL\sin\theta = 0$$

or
$$(P + Q)\sin(\theta - \beta) = P\sin\theta\cos\beta$$

$$(P + Q)(\sin\theta\cos\beta - \cos\theta\sin\beta) = P\sin\theta\cos\beta$$

PROBLEM 10.83 CONTINUED

or
$$-(P + Q)\cos\theta\sin\beta + Q\sin\theta\cos\beta = 0$$

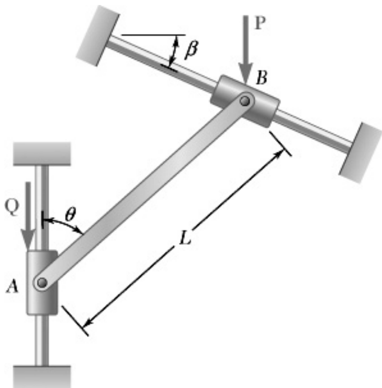
$$-\frac{P + Q}{Q}\frac{\sin\beta}{\cos\beta} + \frac{\sin\theta}{\cos\theta} = 0$$

$$\tan\theta = \frac{P + Q}{Q}\tan\beta \qquad (2)$$

With
$$P = Q = 100 \text{ lb}, \qquad \beta = 30°$$

$$\tan\theta = \frac{200 \text{ lb}}{100 \text{ lb}}\tan 30° = 1.1547$$

$$\theta = 49.1° \blacktriangleleft$$



PROBLEM 10.84

A slender rod AB of negligible weight is attached to two collars A and B that can move freely along the guide rods shown. Knowing that $\beta = 30^{\circ}$, $P = 40$ lb, and $Q = 10$ lb, determine the value of the angle θ corresponding to equilibrium.

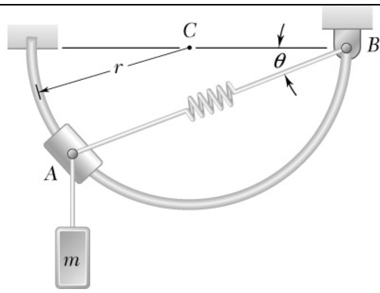
SOLUTION

Using Equation (2) of Problem 10.83, with $P = 40$ lb, $Q = 10$ lb, and $\beta = 30°$, we have

$$\tan\theta = \frac{(40 \text{ lb})(10 \text{ lb})}{(10 \text{ lb})}\tan 30° = 2.88675$$

$$\theta = 70.89°$$

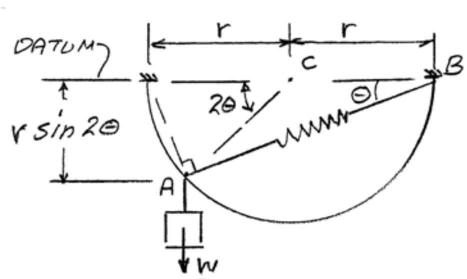
$\theta = 70.9°$ ◀



PROBLEM 10.85

Collar A can slide freely on the semicircular rod shown. Knowing that the constant of the spring is k and that the unstretched length of the spring is equal to the radius r, determine the value of θ corresponding to equilibrium when $m = 20$ kg, $r = 180$ mm, and $k = 3$ N/mm.

SOLUTION



Stretch of Spring

$$s = AB - r$$

$$s = 2(r\cos\theta) - r$$

$$s = r(2\cos\theta - 1)$$

Potential Energy:

$$V = \frac{1}{2}ks^2 - Wr\sin 2\theta \qquad W = mg$$

$$V = \frac{1}{2}kr^2(2\cos\theta - 1)^2 - Wr\sin 2\theta$$

$$\frac{dV}{d\theta} = -kr^2(2\cos\theta - 1)2\sin\theta - 2Wr\cos 2\theta$$

Equilibrium

$$\frac{dV}{d\theta} = 0: -kr^2(2\cos\theta - 1)\sin\theta - Wr\cos 2\theta = 0$$

$$\frac{(2\cos\theta - 1)\sin\theta}{\cos 2\theta} = -\frac{W}{kr}$$

Now

$$\frac{W}{kr} = \frac{(20\text{ kg})(9.81\text{ m/s}^2)}{(3000\text{ N/m})(0.180\text{ m})} = 0.36333$$

Then

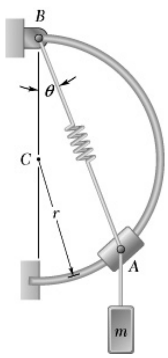
$$\frac{(2\cos\theta - 1)\sin\theta}{\cos 2\theta} = -0.36333$$

Solving numerically,

$$\theta = 0.9580\text{ rad} = 54.9°$$

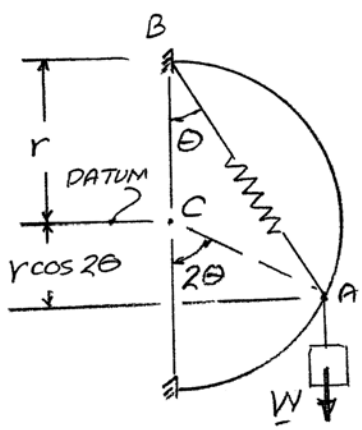
$\theta = 54.9°$ ◀

PROBLEM 10.86



Collar A can slide freely on the semicircular rod shown. Knowing that the constant of the spring is k and that the unstretched length of the spring is equal to the radius r, determine the value of θ corresponding to equilibrium when $m = 20$ kg, $r = 180$ mm, and $k = 3$ N/mm.

SOLUTION



Stretch of spring

$$s = AB - r = 2(r\cos\theta) - r$$

$$s = r(2\cos\theta - 1)$$

$$V = \frac{1}{2}ks^2 - Wr\cos 2\theta$$

$$= \frac{1}{2}kr^2(2\cos\theta - 1)^2 - Wr\cos 2\theta$$

$$\frac{dV}{d\theta} = -kr^2(2\cos\theta - 1)2\sin\theta + 2Wr\sin 2\theta$$

Equilibrium

$$\frac{dV}{d\theta} = 0: \quad -kr^2(2\cos\theta - 1)\sin\theta + Wr\sin 2\theta = 0$$

$$-kr^2(2\cos\theta - 1)\sin\theta + Wr(2\sin\theta\cos\theta) = 0$$

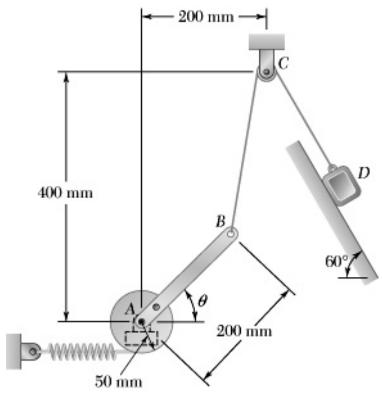
or $$\frac{(2\cos\theta - 1)\sin\theta}{2\cos\theta} = \frac{W}{kr}$$

Now $$\frac{W}{kr} = \frac{(20\text{ kg})(9.81\text{ m/s}^2)}{(3000\text{ N/m})(0.180\text{ m})} = 0.36333$$

Then $$\frac{2\cos\theta - 1}{2\cos\theta} = 0.36333$$

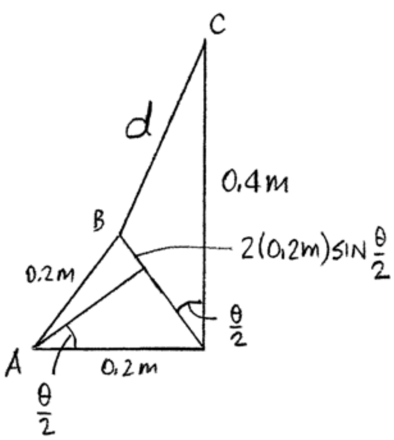
Solving $\theta = 38.2482°$ $\qquad \theta = 38.2° \blacktriangleleft$

PROBLEM 10.87



The 12-kg block D can slide freely on the inclined surface. Knowing that the constant of the spring is 480 N/m and that the spring is unstretched when $\theta = 0$, determine the value of θ corresponding to equilibrium.

SOLUTION



First note, by Law of Cosines

$$d^2 = (0.4)^2 + \left(0.4\sin\frac{\theta}{2}\right)^2 - 2(0.4)\left(0.4\sin\frac{\theta}{2}\right)\cos\frac{\theta}{2}$$

or
$$d = 0.4\sqrt{1 + \sin^2\frac{\theta}{2} - \sin\theta} \text{ m}$$

Now

$$V = \frac{1}{2}kx_{SP}^2 - m_D g y_D$$

$$= \frac{1}{2}k(r_A\theta)^2 - m_D g\left[(y_D)_0 + (0.4 - d)\sin 60°\right]$$

$$= \frac{1}{2}kr_A^2\theta^2 - m_D g\left[(y_D)_0 + \left(0.4 - 0.4\sqrt{1 + \sin^2\frac{\theta}{2} - \sin\theta}\right)\sin 60°\right]$$

For equilibrium $\quad \dfrac{dV}{d\theta} = 0$:

$$kr_A^2\theta + 0.4m_D g\sin 60°\frac{2\left(\frac{1}{2}\right)\left(\sin\frac{\theta}{2}\cos\frac{\theta}{2} - \cos\theta\right)}{2\sqrt{1 + \sin^2\frac{\theta}{2} - \sin\theta}} = 0$$

or
$$kr_A^2\theta + 0.1m_D g\sin 60°\frac{\sin\theta - 2\cos\theta}{\sqrt{1 + \sin^2\frac{\theta}{2} - \sin\theta}} = 0$$

PROBLEM 10.87 CONTINUED

Substituting,

$$(480\text{ N/m})(0.050\text{ m})^2\theta\sqrt{1+\sin^2\frac{\theta}{2}}-\sin\theta$$

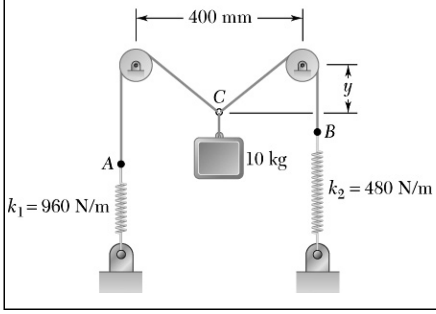
$$+(0.1\text{ m})(12\text{ kg})(9.81\text{ m/s}^2)\frac{\sqrt{3}}{2}(\sin\theta-2\cos\theta)=0$$

or $$\theta\sqrt{1+\sin^2\frac{\theta}{2}}-\sin\theta+8.4957(\sin\theta-2\cos\theta)=0$$

Solving numerically, $\theta=1.07223\text{ rad}$

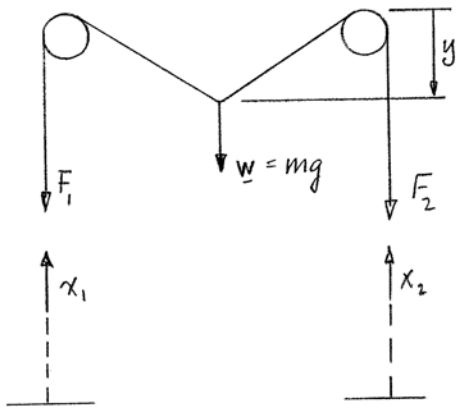
or $\theta=61.4°$ ◀

PROBLEM 10.88



Cable *AB* is attached to two springs and passes through a ring at *C*. Knowing that the springs are unstretched when $y = 0$, determine the distance *y* corresponding to equilibrium.

SOLUTION



First note that the tension in the cable is the same throughout.

$$\therefore\ F_1 = F_2$$

or

$$k_1x_1 = k_2x_2$$

or

$$x_2 = \frac{k_1}{k_2}x_1$$

$$= \frac{960 \text{ N/m}}{480 \text{ N/m}}x_1$$

$$= 2x_1$$

Now, point *C* is midway between the pulleys.

$$\therefore\ \ y^2 = \left[0.2 + \frac{1}{2}(x_1 + x_2)^2\right] - (0.2)^2$$

$$= 0.2(x_1 + x_2) + \frac{1}{4}(x_1 + x_2)^2$$

$$= 0.2(x_1 + 2x_1) + \frac{1}{4}(x_1 + 2x_1)^2$$

$$= 0.6x_1 + \frac{9}{4}x_1^2\ (\text{m}^2)$$

PROBLEM 10.88 CONTINUED

Now

$$V = \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2x_2^2 - mgy$$

$$= \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2\left(2x_1\right)^2 - mg\left(\frac{1}{4}\sqrt{2.4x_1 + 9x_1^2}\right)$$

$$= \frac{1}{2}\left(k_1 + 4k_2\right)x_1^2 - mg\left(\frac{1}{4}\sqrt{2.4x_1 + 9x_1^2}\right)$$

For equilibrium

$$\frac{dV}{dx_1} = 0: \quad \left(k_1 + 4k_2\right)x_1 - mg\left(\frac{2.4 + 18x_1}{2\sqrt{2.4x_1 + 9x_1^2}}\right) = 0$$

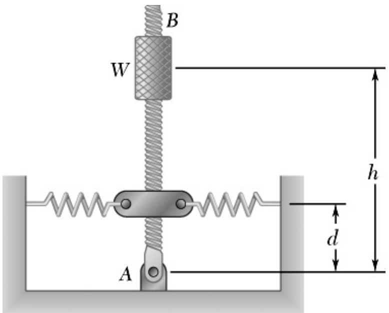
or $$\left(980 + 4 \times 490\right)\text{N/m} \times \left(x_1\right)\left(\text{m}\right)\left(\sqrt{2.4x_1 + 9x_1^2}\right)\left(\text{m}\right) - \frac{1}{2}\left(10\text{ kg}\right)\left(9.81\text{ m/s}^2\right)\left(1.2 + 9x_1\right)\left(\text{m}\right) = 0$$

or $$288x_1\sqrt{2.4x_1 + 9x_1^2} - 5.886\left(1 + 7.5x_1\right) = 0$$

Solving, $$x_1 = 0.068151\text{ m}$$

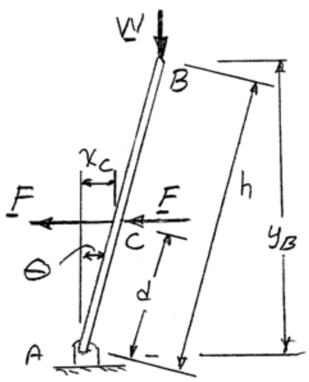
Then $$y^2 = 0.6\left(0.068151\right) + \frac{9}{4}\left(0.068151\right)^2$$ or $y = 227\text{ mm}$ ◀

PROBLEM 10.89



Rod AB is attached to a hinge at A and to two springs, each of constant k. If $h = 50$ in., $d = 24$ in., and $W = 160$ lb, determine the range of values of k for which the equilibrium of the rod is stable in the position shown. Each spring can act in either tension or compression.

SOLUTION



Have $\quad x_C = d\sin\theta \qquad y_B = h\cos\theta$

Potential Energy: $\quad V = 2\left(\frac{1}{2}kx_C^2 + Wy_B\right)$

$$= kd^2\sin^2\theta + Wh\cos\theta$$

Then
$$\frac{dV}{d\theta} = 2kd^2\sin\theta\cos\theta - Wh\sin\theta$$

$$= kd^2\sin 2\theta - Wh\sin\theta$$

and
$$\frac{d^2V}{d\theta^2} = 2kd^2\cos 2\theta - Wh\cos\theta \qquad (1)$$

For equilibrium position $\theta = 0$ to be stable, we must have

$$\frac{d^2V}{d\theta^2} = 2kd^2 - Wh > 0$$

or
$$kd^2 > \frac{1}{2}Wh \qquad (2)$$

Note: For $kd^2 = \frac{1}{2}Wh$, we have $\frac{d^2V}{d\theta^2} = 0$, so that we must determine which is the first derivative that is not equal to zero. Differentiating Equation (1), we write

$$\frac{d^3V}{d\theta^3} = -4kd^2\sin 2\theta + Wh\sin\theta = 0 \qquad \text{for } \theta = 0$$

$$\frac{d^4V}{d\theta^2} = -8kd^2\cos 2\theta + Wh\cos\theta$$

PROBLEM 10.89 CONTINUED

For $\theta = 0$: $$\frac{d^4V}{d\theta^4} = -8kd^2 + Wh$$

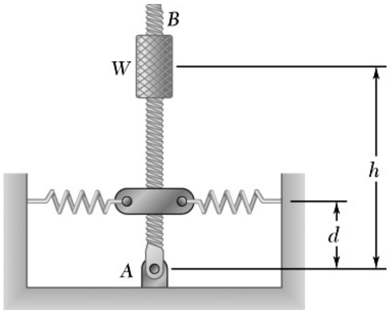
Since $kd^2 = \frac{1}{2}Wh$, $\frac{d^4V}{d\theta^4} = -4Wh + Wh < 0$, we conclude that the equilibrium is unstable for $kd^2 = \frac{1}{2}Wh$ and the $>$ sign in Equation (2) is correct.

With $$W = 160 \text{ lb}, h = 50 \text{ in., and } d = 24 \text{ in.}$$

Equation (2) gives $$k(24 \text{ in.})^2 > \frac{1}{2}(160 \text{ lb})(50 \text{ in.})$$

or $$k > 6.944 \text{ lb/in.}$$

$k > 6.94$ lb/in. ◀



PROBLEM 10.90

Rod AB is attached to a hinge at A and to two springs, each of constant k. If $h = 30$ in., $k = 4$ lb/in., and $W = 40$ lb, determine the smallest distance d for which the equilibrium of the rod is stable in the position shown. Each spring can act in either tension or compression.

SOLUTION

Using Equation (2) of Problem 10.89 with

$$h = 30 \text{ in.}, k = 4 \text{ lb/in., and } W = 40 \text{ lb}$$

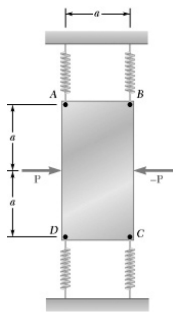
$$(4 \text{ lb/in.})d^2 > \frac{1}{2}(40 \text{ lb})(30 \text{ in.})$$

or

$$d^2 > 150 \text{ in}^2$$

$$d > 12.247 \text{ in.}$$

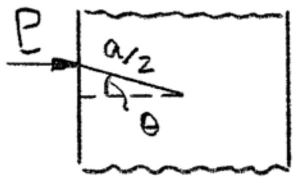
smallest $d = 12.25$ in. ◀



PROBLEM 10.91

The uniform plate *ABCD* of negligible mass is attached to four springs of constant k and is in equilibrium in the position shown. Knowing that the springs can act in either tension or compression and are undeformed in the given position, determine the range of values of the magnitude P of two equal and opposite horizontal forces **P** and **–P** for which the equilibrium position is stable.

SOLUTION

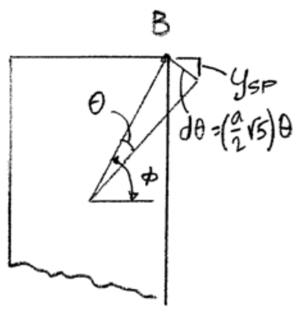


Consider a small clockwise rotation θ of the plate about its center.

Then $$V = 2V_P + 4V_{SP}$$

where $$V_P = P\left(\frac{a}{2}\cos\theta\right)$$

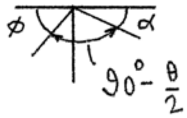
$$= \frac{1}{2}(Pa\cos\theta)$$



and $$V_{SP} = \frac{1}{2}ky_{SP}^2$$

Now $$d = \sqrt{\left(\frac{a}{2}\right)^2 + a^2}$$

$$= \frac{a}{2}\sqrt{5}$$



and $$\alpha = 180^\circ - \left[\phi + \left(90^\circ - \frac{\theta}{2}\right)\right]$$

$$= 90^\circ - \left(\phi - \frac{\theta}{2}\right)$$

Then $$y_{SP} = \left[\left(\frac{a}{2}\sqrt{5}\right)\theta\right]\sin\alpha$$

$$= \frac{a}{2}\theta\sqrt{5}\sin\left[90^\circ - \left(\phi - \frac{\theta}{2}\right)\right]$$

$$= \frac{a}{2}\theta\sqrt{5}\cos\left(\phi - \frac{\theta}{2}\right)$$

PROBLEM 10.91 CONTINUED

and
$$V_{SP} = \frac{1}{2}k\left[\frac{a}{2}\theta\sqrt{5}\cos\left(\phi - \frac{\theta}{2}\right)\right]^2$$
$$= \frac{5}{8}ka^2\theta^2\cos^2\left(\phi - \frac{\theta}{2}\right)$$
$$\therefore\ V = Pa\cos\theta + \frac{5}{2}ka^2\theta^2\cos^2\left(\phi - \frac{\theta}{2}\right)$$

Then
$$\frac{dV}{d\theta} = -Pa\sin\theta + \frac{5}{8}ka^2\left[2\theta\cos^2\left(\phi - \frac{\theta}{2}\right)\right.$$
$$\left. + \theta^2\left(-\frac{1}{2}\right)\cos\left(\phi - \frac{\theta}{2}\right)\sin\left(\phi - \frac{\theta}{2}\right)\right]$$
$$= -Pa\sin\theta + \frac{5}{2}ka^2\left[2\theta\cos^2\left(\phi - \frac{\theta}{2}\right) + \frac{1}{2}\theta^2\sin(2\phi - \theta)\right]$$
$$\frac{d^2V}{d\theta^2} = -Pa\cos\theta + \frac{5}{2}ka^2\left[2\cos^2\left(\phi - \frac{\theta}{2}\right)\right.$$
$$-2\theta\left(-\frac{1}{2}\right)\cos\left(\phi - \frac{\theta}{2}\right)\sin\left(\phi - \frac{\theta}{2}\right) + \theta\sin(2\phi - \theta)$$
$$\left.-\frac{1}{2}\theta^2\cos(2\phi - \theta)\right]$$
$$= -Pa\cos\theta + \frac{5}{2}ka^2\left[2\cos^2\left(\phi - \frac{\theta}{2}\right) + \frac{3}{2}\theta\sin(2\phi - \theta)\right.$$
$$\left.-\frac{1}{2}\theta^2\cos(2\phi - \theta)\right]$$
$$\frac{d^2V}{d\theta^3} = Pa\sin\theta + \frac{5}{2}ka^2\left[4\left(-\frac{1}{2}\right)\cos\left(\phi - \frac{\theta}{2}\right)\sin\left(\phi - \frac{\theta}{2}\right) + \frac{3}{2}\sin(2\phi - \theta)\right.$$
$$\left.-\frac{3}{2}\theta\cos(2\phi - \theta) - \theta\cos(2\phi - \theta) + \frac{1}{2}\theta^2\sin(2\phi - \theta)\right]$$
$$= Pa\sin\theta + \frac{5}{2}ka^2\left[\frac{1}{2}\sin(2\phi - \theta) - \frac{5}{2}\theta\cos(2\phi - \theta)\right.$$
$$\left.+\frac{1}{2}\theta^2\sin(2\phi - \theta)\right]$$

PROBLEM 10.91 CONTINUED

When $\theta = 0$, $\dfrac{dV}{d\theta} = 0$ for all values of P.

For stable equilibrium when $\theta = 0$, require

$$\frac{d^2V}{d\theta^2} > 0: \quad -Pa + \frac{5}{2}ka^2\left(2\cos^2\phi\right) > 0$$

Now, when $\theta = 0$,
$$\cos\phi = \frac{\frac{a}{2}}{\frac{a}{2}\sqrt{5}} = \frac{1}{\sqrt{5}}$$

$$\therefore \quad -Pa + 5ka^2\left(\frac{1}{5}\right) > 0$$

or
$$P < ka$$

When $P = ka$ $\left(\text{for } \theta = 0\right)$:

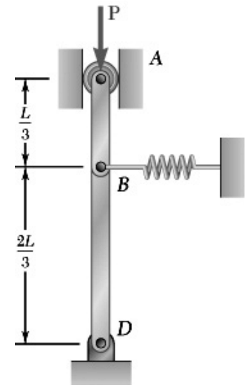
$$\frac{dV}{d\theta} = 0$$

$$\frac{d^2V}{d\theta^2} = 0$$

$$\frac{d^3V}{d\theta^3} = \frac{5}{4}ka^2\sin 2\phi > 0 \Rightarrow \text{unstable}$$

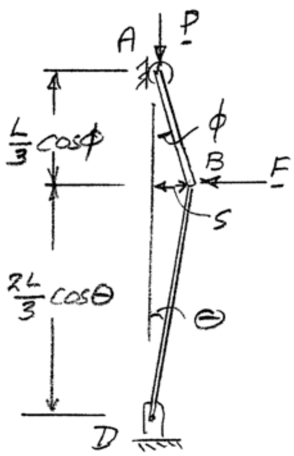
$\therefore$ Stable equilibrium for $0 \le P < ka$ ◀

PROBLEM 10.92



Two bars are attached to a single spring of constant k that is unstretched when the bars are vertical. Determine the range of values of P for which the equilibrium of the system is stable in the position shown.

SOLUTION



Spring: $$s = \frac{L}{3}\sin\phi = \frac{2L}{3}\sin\theta$$

For small values of ϕ and θ: $$\phi = 2\theta$$

$$V = P\left(\frac{L}{3}\cos\phi + \frac{2L}{3}\cos\theta\right) + \frac{1}{2}ks^2$$

$$= \frac{PL}{3}(\cos 2\theta + 2\cos\theta) + \frac{1}{2}k\left(\frac{2L}{3}\sin\theta\right)^2$$

$$\frac{dV}{d\theta} = \frac{PL}{3}(-2\sin 2\theta - 2\sin\theta) + \frac{2}{9}kL^2\sin\theta\cos\theta$$

$$= -\frac{PL}{3}(2\sin 2\theta + 2\sin\theta) + \frac{2}{9}kL^2\sin 2\theta$$

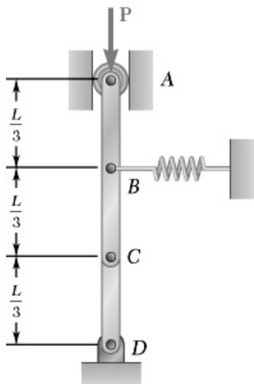
$$\frac{d^2V}{d\theta^2} = -\frac{PL}{3}(4\cos 2\theta + 2\cos\theta) + \frac{4}{9}kL^2\cos 2\theta$$

When $$\theta = 0: \quad \frac{d^2V}{d\theta^2} = -\frac{6PL}{3} + \frac{4}{9}kL^2$$

For stability: $$\frac{d^2V}{d\theta^2} > 0: \quad -2PL + \frac{4}{9}kL^2 > 0$$

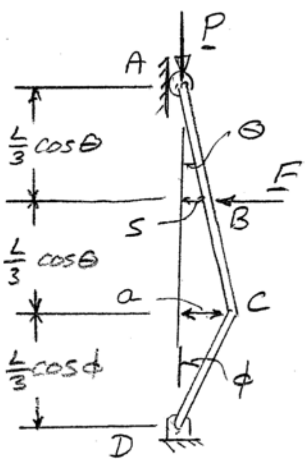
$$0 \le P < \frac{2}{9}kL \blacktriangleleft$$

PROBLEM 10.93



Two bars are attached to a single spring of constant k that is unstretched when the bars are vertical. Determine the range of values of P for which the equilibrium of the system is stable in the position shown.

SOLUTION



First note $$a = \frac{2L}{3}\sin\theta = \frac{L}{3}\sin\phi$$

and $$s = \frac{L}{3}\sin\theta$$

For small values of ϕ and θ: $$\phi = 2\theta$$

$$V = P\left(\frac{2L}{3}\cos\theta + \frac{L}{3}\cos\phi\right) + \frac{1}{2}ks^2$$

$$= \frac{PL}{3}\left(2\cos\theta + \cos 2\theta\right) + \frac{1}{2}k\left(\frac{L}{3}\sin\theta\right)^2$$

$$\frac{dV}{d\theta} = \frac{PL}{3}\left(-2\sin\theta - 2\sin 2\theta\right) + \frac{kL^2}{9}\sin\theta\cos\theta$$

$$= -\frac{2PL}{3}\left(\sin\theta + \sin 2\theta\right) + \frac{kL^2}{18}\sin 2\theta$$

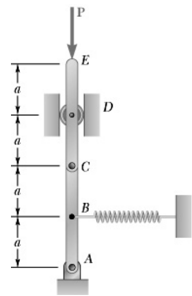
$$\frac{d^2V}{d\theta^2} = -\frac{2PL}{3}\left(\cos\theta + 2\cos 2\theta\right) + \frac{kL^2}{9}\cos 2\theta$$

When $$\theta = 0: \quad \frac{d^2V}{d\theta^2} = -2PL + \frac{kL^2}{9}$$

For stability: $$\frac{d^2V}{d\theta^2} > 0: \quad -2PL + \frac{kL^2}{9} > 0$$

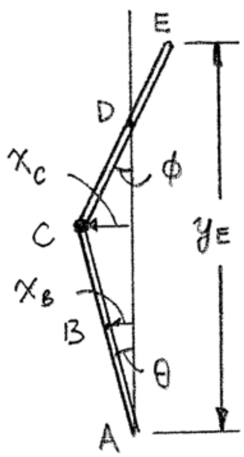
$$0 \le P < \frac{1}{18}kL \blacktriangleleft$$

PROBLEM 10.94



Bar AC is attached to a hinge at A and to a spring of constant k that is undeformed when the bar is vertical. Knowing that the spring can act in either tension or compression, determine the range of values of P for which the equilibrium of the system is stable in the position shown.

SOLUTION



Consider a small disturbance of the system defined by the angle θ.

Have

$$x_C = 2a\sin\theta = a\sin\phi$$

For small θ: $$2\theta = \phi$$

Now, the Potential Energy is

$$V = \frac{1}{2}kx_B^2 + Py_E$$

where $$x_B = a\sin\theta$$

and $$y_E = y_C + y_{E/C}$$
$$= 2a\cos\theta + 2a\cos\phi$$
$$= 2a(\cos\theta + \cos 2\theta)$$

Then $$V = \frac{1}{2}ka^2\sin^2\theta + 2Pa(\cos\theta + \cos 2\theta)$$

and $$\frac{dV}{d\theta} = \frac{1}{2}ka^2(2\sin\theta\cos\theta) - 2Pa(\sin\theta + 2\sin 2\theta)$$
$$= \frac{1}{2}ka^2\sin 2\theta - 2Pa(\sin\theta + 2\sin 2\theta)$$
$$\frac{d^2V}{d\theta^2} = ka^2\cos 2\theta - 2Pa(\cos\theta + 4\cos 2\theta)$$

For $\theta = 0$ and for stable equilibrium:

$$\frac{d^2V}{d\theta^2} > 0$$

or $$ka^2 - 2Pa(1+4) > 0$$

PROBLEM 10.94 CONTINUED

or $$P < \frac{1}{10} ka$$

$$\therefore \ 0 \le P < \frac{1}{10} ka \blacktriangleleft$$

Check stability for $$P = \frac{ka}{10}$$

$$\frac{d^3V}{d\theta^3} = -2ka^2 \sin 2\theta + 2Pa\left(\sin\theta + 8\sin 2\theta\right)$$

$$\frac{d^4V}{d\theta^4} = -4ka^2 \cos 2\theta + 2Pa\left(\cos\theta + 16\cos 2\theta\right)$$

Then, with $\theta = 0$ and $P = \dfrac{ka}{10}$

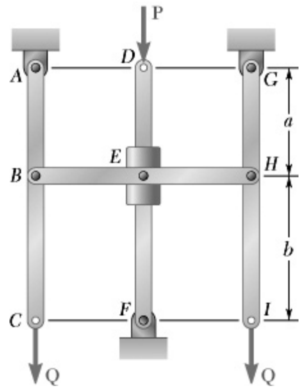
$$\frac{dV}{d\theta} = 0$$

$$\frac{d^2V}{d\theta^2} = 0$$

$$\frac{d^3V}{d\theta^3} = 0$$

$$\frac{d^4V}{d\theta^4} = -4ka^2 + 2\left(\frac{1}{10} ka\right)(a)(1 + 16)$$

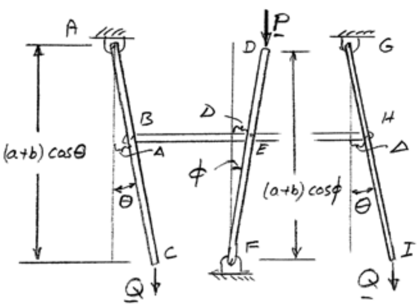
$$= -0.6ka^2 < 0 \Rightarrow \text{Unstable}$$



PROBLEM 10.95

The horizontal bar *BEH* is connected to three vertical bars. The collar at *E* can slide freely on bar *DF*. Determine the range of values of *P* for which the equilibrium of the system is stable in the position shown when $a = 300$ mm, $b = 400$ mm, and $Q = 90$ N.

SOLUTION



First note $\quad A = a\sin\theta = b\sin\phi$

For small values of θ and ϕ: $\quad a\theta = b\phi$

or $$\phi = \frac{a}{b}\theta$$

$$V = P(a+b)\cos\phi - 2Q(a+b)\cos\theta$$

$$= (a+b)\left[P\cos\left(\frac{a}{b}\theta\right) - 2Q\cos\theta\right]$$

$$\frac{dV}{d\theta} = (a+b)\left[-\frac{a}{b}P\sin\left(\frac{a}{b}\theta\right) + 2Q\sin\theta\right]$$

$$\frac{d^2V}{d\theta^2} = (a+b)\left[-\frac{a^2}{b^2}P\cos\left(\frac{a}{b}\theta\right) + 2Q\cos\theta\right]$$

When $\theta = 0$: $$\frac{d^2V}{d\theta^2} = (a+b)\left(-\frac{a^2}{b^2}P + 2Q\right)$$

Stability: $$\frac{d^2V}{d\theta^2} > 0: \quad -\frac{a^2}{b^2}P + 2Q > 0$$

$$P < 2\frac{b^2}{a^2}Q \qquad (1)$$

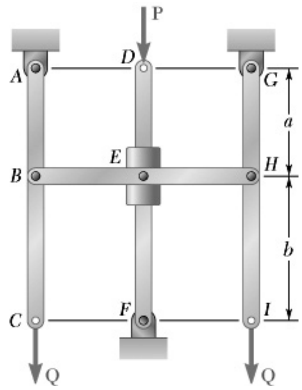
or $$Q > \frac{a^2}{2b^2}P \qquad (2)$$

With $\quad Q = 90$ N, $a = 300$ mm, and $b = 400$ mm

Equation (1): $$P < 2\frac{(400\text{ mm})^2}{(300\text{ mm})^2}(90\text{ N}) = 320\text{ N}$$

For stability $\quad 0 \le P < 320\text{ N}$ ◀

PROBLEM 10.96



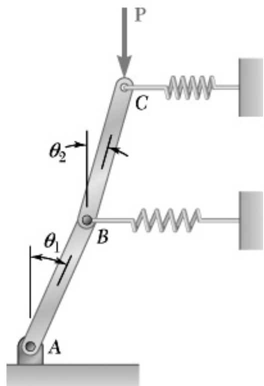
The horizontal bar *BEH* is connected to three vertical bars. The collar at *E* can slide freely on bar *DF*. Determine the range of values of Q for which the equilibrium of the system is stable in the position shown when $a = 480$ mm, $b = 400$ mm, and $P = 600$ N.

SOLUTION

Using Equation (2) of Problem 10.95 with $P = 600$ N, $a = 480$ mm, and $b = 400$ mm

Equation (2) $$Q > \frac{1}{2}\frac{(480 \text{ mm})^2}{(400 \text{ mm})^2}(600 \text{ N}) = 432 \text{ N}$$

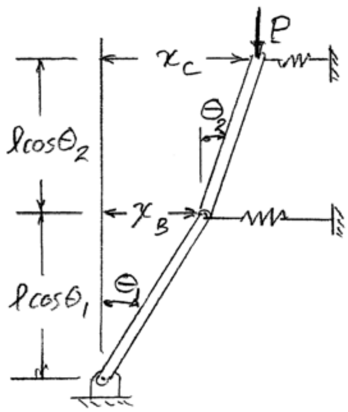
For stability $Q > 432$ N ◀



PROBLEM 10.97

Bars AB and BC, each of length l and of negligible weight, are attached to two springs, each of constant k. The springs are undeformed and the system is in equilibrium when $\theta_1 = \theta_2 = 0$. Determine the range of values of P for which the equilibrium position is stable.

SOLUTION



Have $$x_B = l\sin\theta$$

$$x_C = l\sin\theta_1 + l\sin\theta_2$$

$$y_C = l\cos\theta_1 + l\cos\theta_2$$

$$V = Py_C + \frac{1}{2}kx_B^2 + \frac{1}{2}kx_C^2$$

or $$V = Pl\left(\cos\theta_1 + \cos\theta_2\right) + \frac{1}{2}kl^2\left[\sin^2\theta_1 + \left(\sin\theta_1 + \sin\theta_2\right)^2\right]$$

For small values of θ_1 and θ_2:

$$\sin\theta_1 \approx \theta_1, \qquad \sin\theta_2 \approx \theta_2, \qquad \cos\theta_1 \approx 1 - \frac{1}{2}\theta_1^2, \qquad \cos\theta_2 \approx 1 - \frac{1}{2}\theta_2^2$$

Then $$V = Pl\left(1 - \frac{\theta_1^2}{2} + 1 - \frac{\theta_2^2}{2}\right) + \frac{1}{2}kl^2\left[\theta_1^2 + \left(\theta_1 + \theta_2\right)^2\right]$$

and $$\frac{\partial V}{\partial\theta_1} = -Pl\theta_1 + kl^2\left[\theta_1 + \left(\theta_1 + \theta_2\right)\right]$$

$$\frac{\partial V}{\partial\theta_2} = -Pl\theta_2 + kl^2\left(\theta_1 + \theta_2\right)$$

$$\frac{\partial^2 V}{\partial\theta_1^2} = -Pl + 2kl^2 \qquad \frac{\partial^2 V}{\partial\theta_2^2} = -Pl + kl^2$$

$$\frac{\partial^2 V}{\partial\theta_1\partial\theta_2} = kl^2$$

PROBLEM 10.97 CONTINUED

Stability Conditions for stability (see page 583).

For
$$\theta_1 = \theta_2 = 0: \quad \frac{\partial V}{\partial \theta_1} = \frac{\partial V}{\partial \theta_2} = 0 \quad \text{(condition satisfied)}$$

$$\left(\frac{\partial^2 V}{\partial \theta_1 \partial \theta_2}\right)^2 - \frac{\partial^2 V}{\partial \theta_1^2}\frac{\partial^2 V}{\partial \theta_2^2} < 0$$

Substituting,
$$\left(kl^2\right)^2 - \left(-Pl + 2kl^2\right)\left(-Pl + kl\right) < 0$$

$$k^2 l^4 - P^2 l^2 + 3Pkl^3 - 2k^2 l^4 < 0$$

$$P^2 - 3klP + k^2 l^2 > 0$$

Solving,
$$P < \frac{3 - \sqrt{5}}{2}kl \qquad \text{or} \qquad P > \frac{3 + \sqrt{5}}{2}kl$$

or
$$P < 0.382kl \qquad \text{or} \qquad P > 2.62kl$$

$$\frac{\partial^2 V}{\partial \theta_1^2} > 0: \quad -Pl + 2kl^2 > 0$$

or
$$P < \frac{1}{2}kl$$

$$\frac{\partial^2 V}{\partial \theta_2^2} > 0: \quad -Pl + kl^2 > 0$$

or
$$P < kl$$

Therefore, all conditions for stable equilibrium are satisfied when

$$0 \le P < 0.382kl \blacktriangleleft$$

PROBLEM 10.98

Solve Problem 10.97 knowing that $l = 400 \text{ mm}$ and $k = 1.25 \text{ kN/m}$.

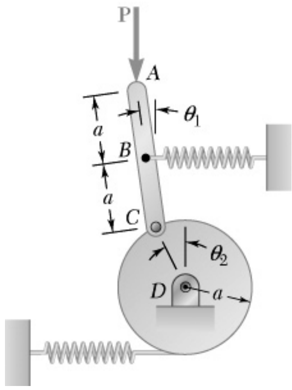
SOLUTION

From the analysis of Problem 10.98 with

$$l = 400 \text{ mm} \qquad \text{and} \qquad k = 1.25 \text{ kN/m}$$

$$P < 0.382kl = 0.382(1250 \text{ N/m})(0.4 \text{ m}) = 191 \text{ N}$$

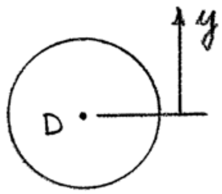
$$0 \leq P < 191.0 \text{ N} \blacktriangleleft$$



PROBLEM 10.99

Bar ABC of length $2a$ and negligible weight is hinged at C to a drum of radius a as shown. Knowing that the constant of each spring is k and that the springs are undeformed when $\theta_1 = \theta_2 = 0$, determine the range of values of P for which the equilibrium position $\theta_1 = \theta_2 = 0$ is stable.

SOLUTION



Have $$V = \frac{1}{2}k\left(a\theta_2\right)^2 + \frac{1}{2}k\left(a\sin\theta_1 + a\sin\theta_2\right)^2 + P\left(2a\cos\theta_1 + a\cos\theta_2\right)$$

Then $$\frac{\partial V}{\partial\theta_1} = ka^2\left(\sin\theta_1 + \sin\theta_2\right)\cos\theta_1 - 2Pa\sin\theta_1$$

$$= ka^2\left(\frac{1}{2}\sin 2\theta_1 + \cos\theta_1\sin\theta_2\right) - 2Pa\sin\theta_1$$

and $$\frac{\partial^2 V}{\partial\theta_1^2} = ka^2\left(\cos 2\theta_1 - \sin\theta_1\sin\theta_2\right) - 2Pa\cos\theta_1$$

$$\frac{\partial^2 V}{\partial\theta_1\partial\theta_2} = ka^2\cos\theta_1\cos\theta_2$$

Also $$\frac{\partial V}{\partial\theta_2} = ka^2\theta_2 + ka^2\left(\sin\theta_1 + \sin\theta_2\right)\cos\theta_2 - Pa\sin\theta_2$$

$$= ka^2\theta_2 + ka^2\left(\sin\theta_1\cos\theta_2 + \frac{1}{2}\sin 2\theta_2\right) - Pa\sin\theta_2$$

and $$\frac{\partial^2 V}{\partial\theta_2^2} = ka^2 + ka^2\left(-\sin\theta_1\sin\theta_2 + \cos 2\theta_2\right) - Pa\cos\theta_2$$

When $$\theta_1 = \theta_2 = 0$$

$$\frac{\partial V}{\partial\theta_1} = 0 \qquad \frac{\partial^2 V}{\partial\theta_1\partial\theta_2} = ka^2 \qquad \frac{\partial V}{\partial\theta_2} = 0$$

$$\frac{\partial^2 V}{\partial\theta_1^2} = ka^2 - 2Pa \qquad \frac{\partial^2 V}{\partial\theta_2^2} = ka^2 + ka^2 - Pa = 2ka^2 - Pa$$

PROBLEM 10.99 CONTINUED

Apply Equations 10.24 $\quad \dfrac{\partial V}{\partial \theta_1} = 0$: condition satisfied

$\dfrac{\partial V}{\partial \theta_2} = 0$: condition satisfied

$$\left(\frac{\partial^2 V}{\partial \theta_1 \partial \theta_2}\right)^2 - \frac{\partial^2 V}{\partial \theta_1^2}\frac{\partial^2 V}{\partial \theta_2^2} < 0: \quad \left(ka^2\right)^2 - \left(ka^2 - 2Pa\right)\left(2ka^2 - Pa\right) < 0$$

or
$$k^2a^2 - \left(ka - 2P\right)\left(2ka - P\right) < 0$$

Expanding
$$k^2a^2 - 2ka^2 + 5kaP - 2P^2 < 0$$

or
$$2P^2 - 5kaP + k^2a^2 > 0$$

or
$$P < \frac{5 - \sqrt{17}}{4}ka \quad \text{and} \quad P > \frac{5 + \sqrt{17}}{4}ka$$

or
$$P < 0.21922ka \quad \text{and} \quad P > 2.2808ka$$

Also
$$\frac{\delta^2 V}{\delta \theta_1^2} > 0: ka^2 - 2Pa > 0 \quad \text{or} \quad \frac{\delta^2 V}{\delta \theta_2^2} > 0: 2ka^2 - Pa > 0$$

or
$$P < \frac{1}{2}ka \quad \text{or} \quad P < 2ka$$

$\therefore$ For stable equilibrium when $\theta_1 = \theta_2 = 0$:

$$0 \le P < 0.219ka \blacktriangleleft$$

PROBLEM 10.100

Solve Problem 10.99 knowing that $k = 10$ lb/in. and $a = 14$ in.

SOLUTION

From the solution to Problem 10.99, with $k = 10$ lb/in. and $a = 14$ in.

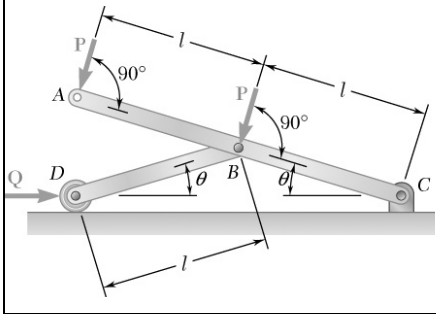
$$0 \leq P < 0.21922ka$$

or

$$0 \leq P < 0.21922(10 \text{ lb/in.})(14 \text{ in.})$$

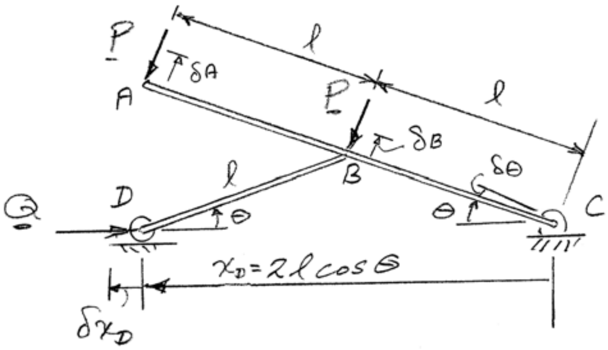
or $0 \leq P < 30.7$ lb ◀

PROBLEM 10.101



Derive an expression for the magnitude of the force **Q** required to maintain the equilibrium of the mechanism shown.

SOLUTION



Have $$x_D = 2l\cos\theta \quad \text{so that} \quad \delta x_D = -2l\sin\theta\delta\theta$$

$$\delta A = 2l\delta\theta$$

$$\delta B = l\delta\theta$$

Virtual Work:

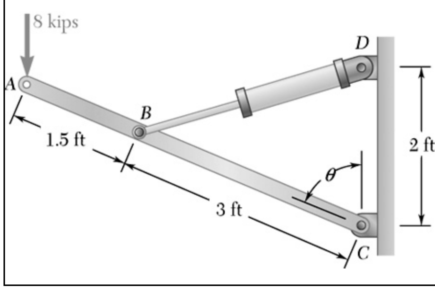
$$\delta U = 0: \quad -Q\delta x_D - P\delta A - P\delta B = 0$$

$$-Q(-2l\sin\theta\delta\theta) - P(2l\delta\theta) - P(l\delta\theta) = 0$$

$$2Ql\sin\theta - 3Pl = 0$$

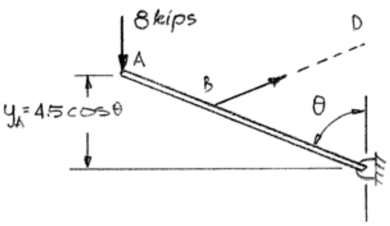
$$Q = \frac{3}{2}\frac{P}{\sin\theta} \blacktriangleleft$$

PROBLEM 10.102



The position of boom *ABC* is controlled by the hydraulic cylinder *BD*. For the loading shown, determine the force exerted by the hydraulic cylinder on pin *B* when $\theta = 70°$.

SOLUTION



First note, by the Law of Cosines

$$DB^2 = (3\text{ ft})^2 + (2\text{ ft})^2 - 2(3\text{ ft})(2\text{ ft})\cos\theta$$

$$= [13 - 12\cos\theta](\text{ft}^2)$$

$$DB = \sqrt{13 - 12\cos\theta}$$

Then $$\delta_B = \delta DB = \frac{1}{2}\frac{(-12)(\sin\theta)}{\sqrt{13-12\cos\theta}}\delta\theta$$

or $$\delta_B = \frac{6\sin\theta}{\sqrt{13-12\cos\theta}}\delta\theta$$

Also $$y_A = 4.5\cos\theta$$

Then $$\delta y_A = -4.5\sin\theta\delta\theta$$

Virtual Work

$$\delta U = 0: \quad -(8\text{ kips})\delta y_A - F_{DB}\delta_B = 0$$

Then $$-8(-4.5\sin\theta)\delta\theta - F_{DB}\left(\frac{6\sin\theta}{\sqrt{13-12\cos\theta}}\right)\delta\theta = 0$$

or $$F_{DB} = \frac{(8)(4.5\sin\theta)}{6\sin\theta}\sqrt{13-12\cos\theta}$$

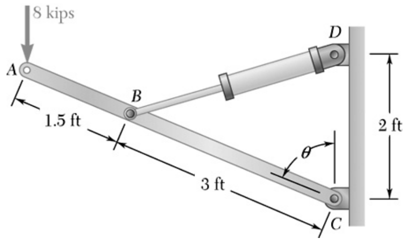
or $$F_{DB} = 6\sqrt{13-12\cos\theta}$$

For $$\theta = 70°$$

$$F_{DB} = 17.895\text{ kips}$$

$$\mathbf{F}_{DB} = 17.90\text{ kips} \nearrow ◀$$

PROBLEM 10.103



The position of boom *ABC* is controlled by the hydraulic cylinder *BD*. For the loading shown, determine the largest allowable value of the angle θ if the maximum force that the cylinder can exert on pin *B* is 25 kips.

SOLUTION

From the analysis of Problem 10.102, we have

$$F_{AB} = 6\sqrt{13 - 12\cos\theta}$$

For $$F_{AB} = 25 \text{ kips}$$

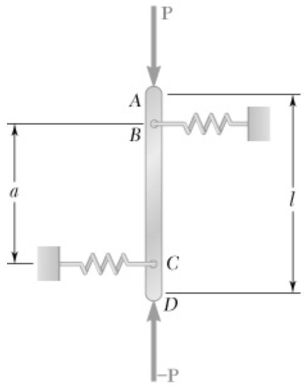
$$6\sqrt{13 - 12\cos\theta} = 25$$

or $$\cos\theta = \frac{-17.36 + 13}{12} = -0.3633$$

$$\theta = 111.31°$$

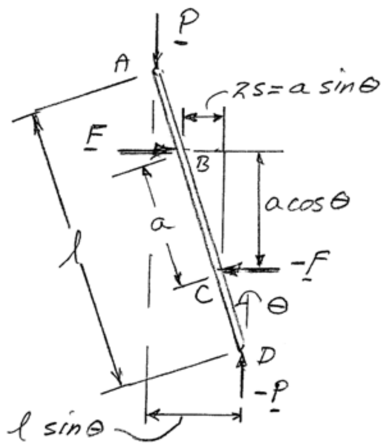
$$\theta = 111.3° \blacktriangleleft$$

PROBLEM 10.104



A vertical bar *AD* is attached to two springs of constant *k* and is in equilibrium in the position shown. Determine the range of values of the magnitude *P* of two equal and opposite vertical forces **P** and **–P** for which the equilibrium position is stable if (*a*) $AB = CD$, (*b*) $AB = 2CD$.

SOLUTION



For both (*a*) and (*b*): Since **P** and **–P** are vertical, they form a couple of moment

$$M_P = +Pl\sin\theta$$

The forces **F** and **–F** exerted by springs must, therefore, also form a couple, with moment

$$M_F = -Fa\cos\theta$$

We have

$$dU = M_P d\theta + M_F d\theta$$

$$= \left(Pl\sin\theta - Fa\cos\theta\right)d\theta$$

but $$F = ks = k\left(\frac{1}{2}a\sin\theta\right)$$

Thus, $$dU = \left(Pl\sin\theta - \frac{1}{2}ka^2\sin\theta\cos\theta\right)d\theta$$

From Equation (10.19), page 580, we have

$$dV = -dU = -Pl\sin\theta d\theta + \frac{1}{4}ka^2\sin 2\theta d\theta$$

or $$\frac{dV}{d\theta} = -Pl\sin\theta + \frac{1}{4}ka^2\sin 2\theta$$

and $$\frac{d^2V}{d\theta^2} = -Pl\cos\theta + \frac{1}{2}ka^2\cos 2\theta \qquad (1)$$

PROBLEM 10.104 CONTINUED

For $\theta = 0$: $$\frac{d^2V}{d\theta^2} = -Pl + \frac{1}{2}ka^2$$

For Stability: $$\frac{d^2V}{d\theta^2} > 0, \qquad -Pl + \frac{1}{2}ka^2 > 0$$

or (for parts *a* and *b*) $$P < \frac{ka^2}{2l} \blacktriangleleft$$

Note: To check that equilibrium is unstable for $P = \dfrac{ka^2}{2l}$, we differentiate (1) twice:

$$\frac{d^3V}{d\theta^3} = +Pl\sin\theta - ka^2\sin 2\theta = 0, \qquad \text{for} \qquad \theta = 0,$$

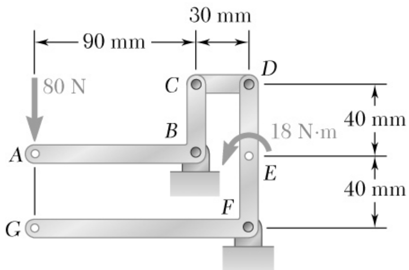
$$\frac{d^4V}{d\theta^4} = Pl\cos\theta - 2ka^2\cos 2\theta$$

For $\theta = 0$ $$\frac{d^4V}{d\theta^4} = Pl - 2ka^2 = \frac{ka^2}{2} - 2ka^2 < 0$$

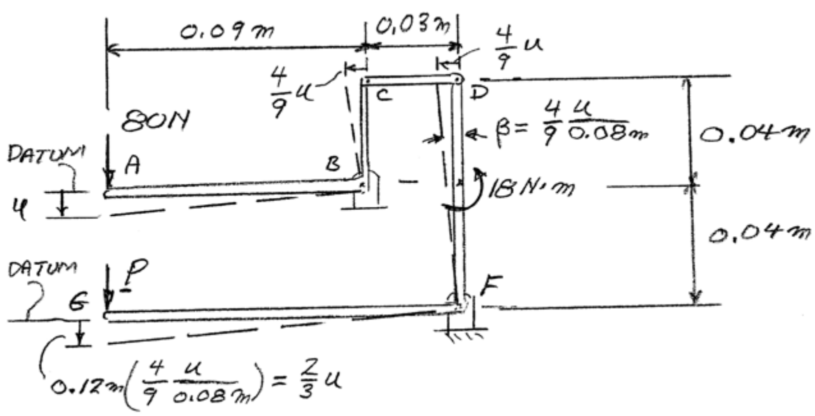
Thus, equilibrium is unstable when $$P = \frac{ka^2}{2l}$$

PROBLEM 10.105

Determine the vertical force **P** which must be applied at G to maintain the equilibrium of the linkage.



SOLUTION



$$y_A = -u, \qquad y_G = -\frac{2}{3}u, \qquad \beta = \frac{4u}{0.72}$$

$$V = (80\text{ N})y_A + P(y_G) - (18\text{ N}\cdot\text{m})\beta$$

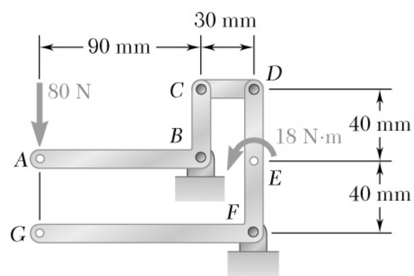
$$= 80(-u) + P\left(-\frac{2}{3}u\right) - (18)\frac{4u}{0.72}$$

$$\frac{dV}{du} = -80 - \frac{2}{3}P - 100 = 0$$

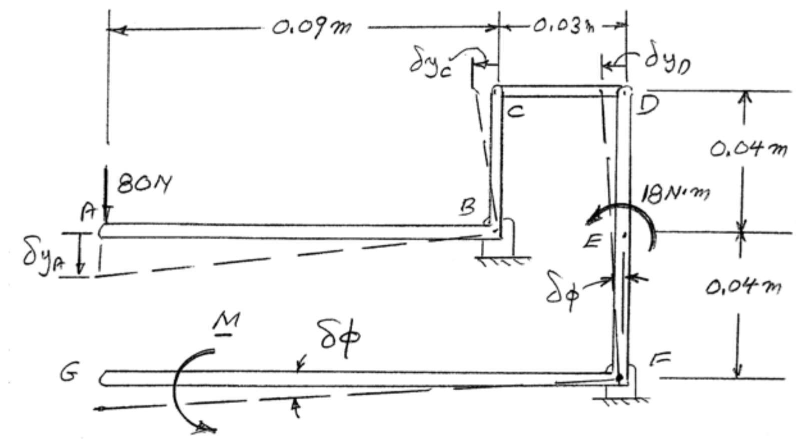
$$P = -270\text{ N}$$

$\mathbf{P} = 270\text{ N} \uparrow$ ◀

Substituting $P = -270\text{ N}$ into the expression for V, we have $V = 0$. Thus V is constant and *equilibrium is neutral.*

	**PROBLEM 10.106** Determine the couple **M** which must be applied to member *DEFG* to maintain the equilibrium of the linkage.

SOLUTION



Assume $\quad \delta y_A \downarrow: \delta y_C = \dfrac{0.04}{0.09}\delta y_A = \dfrac{4}{9}\delta y_A \longleftarrow, \qquad \delta y_D = \delta y_C = \dfrac{4}{9}\delta y_A \longleftarrow$

$$\delta\phi = \frac{\delta y_C}{0.08} = \frac{4}{9}\frac{\delta y_A}{0.08} = \frac{4}{0.72}\delta y_A = \frac{50}{9}\delta y_A \circlearrowright$$

Virtual Work:

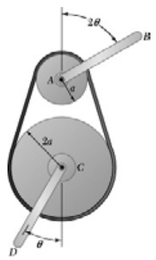
$$\delta U = 0: \quad (80\text{ N})\delta y_A + (18\text{ N}\cdot\text{m})\delta\phi + M\delta\phi = 0$$

$$80\delta y_A + 18\left(\frac{50}{9}\delta y_A\right) + M\left(\frac{50}{9}\delta y_A\right) = 0$$

$$80 + 100 + \frac{50}{9}M = 0$$

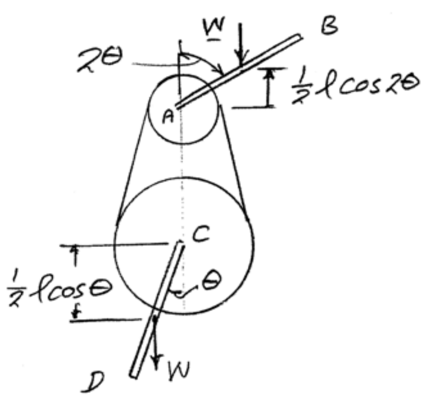
$M = -32.4\text{ N}\cdot\text{m}$ $\qquad\qquad$ $\mathbf{M} = 32.4\text{ N}\cdot\text{m} \circlearrowright$ ◀

PROBLEM 10.107



Two uniform rods, each of mass m and length l, are attached to drums that are connected by a belt as shown. Assuming that no slipping occurs between the belt and the drums, determine the positions of equilibrium of the system and state in each case whether the equilibrium is stable, unstable, or neutral.

SOLUTION



$$W = mg$$

$$V = W\left(\frac{l}{2}\cos 2\theta\right) - W\left(\frac{l}{2}\cos\theta\right)$$

$$\frac{dV}{d\theta} = W\frac{l}{2}(-2\sin 2 + \sin\theta)$$

$$\frac{d^2V}{d\theta^2} = W\frac{l}{2}(-4\cos 2\theta - \cos\theta)$$

Equilibrium:

$$\frac{dV}{d\theta} = 0: \frac{Wl}{2}(-2\sin 2\theta + \sin\theta) = 0$$

or $$\sin\theta(-4\cos\theta + 1) = 0$$

Solving, $$\theta = 0, 75.5°,\ 180°,\ \text{ and } \ 284.5°$$

Stability:

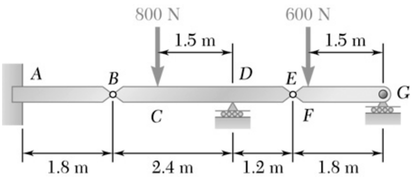
$$\frac{d^2V}{d\theta^2} = W\frac{l}{2}(-4\cos 2\theta - \cos\theta)$$

At $\theta = 0$: $\quad \frac{d^2V}{d\theta^2} = W\frac{l}{2}(-4 - 1) < 0$ $\quad \therefore\ \theta = 0$, Unstable ◀

At $\theta = 75.5°$: $\quad \frac{d^2V}{d\theta^2} = W\frac{l}{2}(-4(-.874) - .25) > 0$ $\quad \therefore\ \theta = 75.5°$, Stable ◀

At $\theta = 180°$: $\quad \frac{d^2V}{d\theta^2} = W\frac{l}{2}(-4 + 1) < 0$ $\quad \therefore\ \theta = 180.0°$, Unstable ◀

At $\theta = 284.5°$: $\quad \frac{d^2V}{d\theta^2} = W\frac{l}{2}(-4(-.874) - .25) > 0$ $\quad \therefore\ \theta = 285°$, Stable ◀

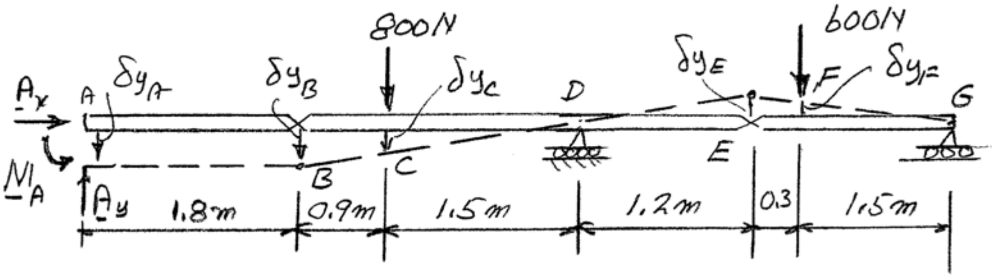


PROBLEM 10.108

Using the method of virtual work, determine separately the force and the couple representing the reaction at A.

SOLUTION

Vertical component at A. Move point A downward *without* rotation.



Since AB remains horizontal, $\delta y_A = \delta y_B$

$$\delta y_C = \frac{5}{8}\delta y_B; \qquad \delta y_E = \frac{1}{2}\delta y_B; \qquad \delta y_E = \frac{5}{6}\delta y_E = \frac{5}{6}\left(\frac{1}{2}\delta y_B\right) = \frac{5}{12}\delta y_B$$

Virtual Work: $\delta U = 0: \ -A\delta y_A + (800\text{ N})\delta y_C - (600)\delta y_F = 0$

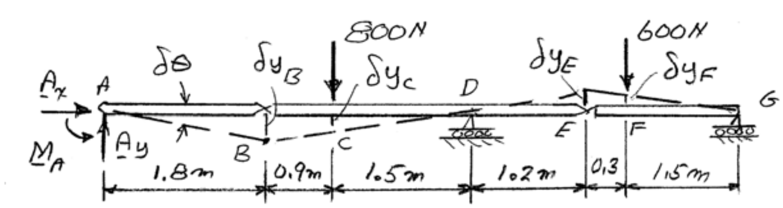
$$-A_y\delta y_B + 800\left(\frac{5}{8}\delta y_B\right) - 600\left(\frac{5}{12}\delta y_B\right) = 0$$

$$A_y = +250\text{ N} \qquad \mathbf{A}_y = 250\text{ N}\uparrow$$

For horizontal motion $\delta x_A,\ \delta U = 0 = A_x\delta x_A; \qquad A_x = 0$

$$\therefore \ \mathbf{A} = 250\text{ N}\uparrow \blacktriangleleft$$

For couple $\mathbf{M}_A$, we rotate AB about A through $\delta\theta$



PROBLEM 10.108 CONTINUED

$$\delta y_B = 1.8\delta\theta; \qquad \delta y_E = \frac{1}{2}\delta y_B = \frac{1}{2}(1.8\delta\theta) = 0.9\delta\theta$$

$$\delta y_C = \frac{5}{8}\delta y_B = \frac{5}{8}(1.8\delta\theta) = 1.25\delta\theta$$

$$\delta y_F = \frac{5}{6}\delta y_E = \frac{5}{6}(0.98\theta) = 0.75\delta\theta$$

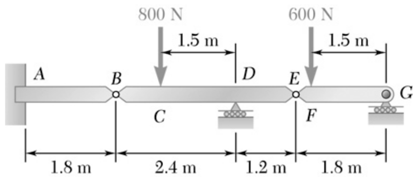
Virtual Work:

$$\delta U = 0: \quad -M_A\delta\theta + (800 \text{ N})\delta y_C - (600 \text{ N})\delta y_F = 0$$

$$-M_A\delta\theta + 800(1.125\delta\theta) - 600(0.75\delta\theta) = 0$$

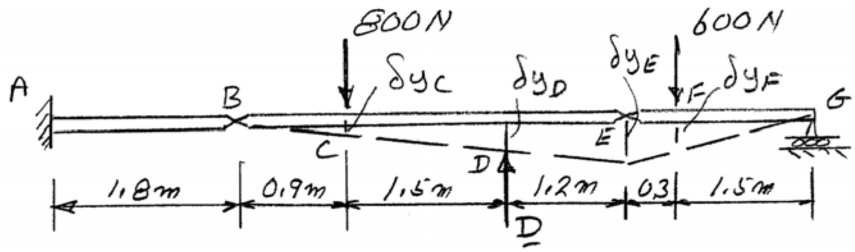
$$M_A = +450 \text{ N}\cdot\text{m} \qquad \mathbf{M}_A = 450 \text{ N}\cdot\text{m} \blacktriangleleft$$

PROBLEM 10.109



Using the method of virtual work, determine the reaction at *D*.

SOLUTION



We move point *D* downward a distance δy_D

$$\delta y_C = \frac{3}{8}\delta y_D \qquad \delta y_E = \frac{3}{2}\delta y_D$$

$$\delta y_F = \frac{5}{6}\delta y_E = \frac{5}{6}\left(\frac{3}{2}\delta y_D\right) = \frac{5}{4}\delta y_D$$

Virtual Work:

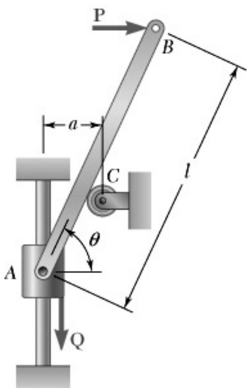
$$\delta U = 0: \quad -D\delta y_D + (800\text{ N})\delta y_C + (600\text{ N})\delta y_F = 0$$

$$-D\delta y_D + 800\left(\frac{3}{8}\delta y_D\right) + 600\left(\frac{5}{4}\delta y_D\right) = 0$$

$$D = +1050\text{ N}$$

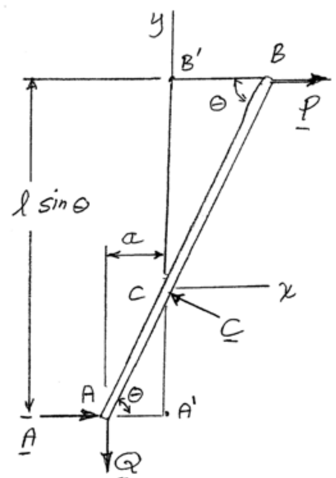
$\mathbf{D} = 1050\text{ N}\uparrow$ ◀

PROBLEM 10.110



The slender rod AB is attached to a collar A and rests on a small wheel at C. Neglecting the radius of the wheel and the effect of friction, derive an expression for the magnitude of the force **Q** required to maintain the equilibrium of the rod.

SOLUTION



For $\Delta AA'C$: $\quad A'C = a\tan\theta$

$$y_A = -(A'C) = -a\tan\theta$$

$$\delta y_A = -\frac{a}{\cos^2\theta}\delta\theta$$

For $\Delta BB'C$:

$$B'C = l\sin\theta - A'C$$

$$= l\sin\theta - a\tan\theta$$

$$BB' = \frac{B'C}{\tan\theta} = \frac{l\sin\theta - a\tan\theta}{\tan\theta}$$

$$x_B = BB' = l\cos\theta - a$$

$$\delta x_B = -l\sin\theta\,\delta\theta$$

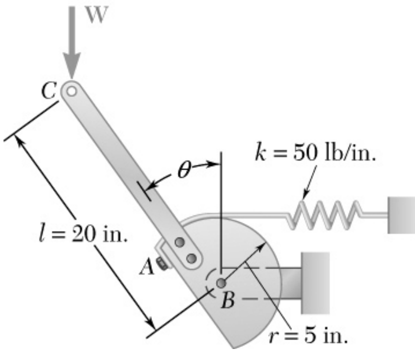
Virtual Work:

$$\delta U = 0: \quad P\delta x_B - Q\delta y_A = 0$$

$$P(-l\sin\theta\,\delta\theta) - Q\left(-\frac{a}{\cos^2\theta}\delta\theta\right) = 0$$

or $\quad Pl\sin\theta\cos^2\theta = Qa$

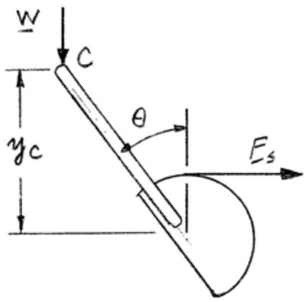
$$Q = P\frac{l}{a}\sin\theta\cos^2\theta \blacktriangleleft$$



PROBLEM 10.111

A load **W** of magnitude 100 lb is applied to the mechanism at C. Knowing that the spring is unstretched when $\theta = 15°$, determine the value of θ corresponding to equilibrium and check that the equilibrium is stable.

SOLUTION



Have $$y_C = l\cos\theta$$

$$V = \frac{1}{2}k\left[r(\theta - \theta_0)\right]^2 + Wy_C \qquad \theta_0 = 15° = \frac{\pi}{12}\text{ rad}$$

$$= \frac{1}{2}kr^2(\theta - \theta_0)^2 + Wl\cos\theta$$

$$\frac{dV}{d\theta} = kr^2(\theta - \theta_0) - Wl\sin\theta$$

Equilibrium $$\frac{dV}{d\theta} = 0:\quad kr^2(\theta - \theta_0) - wl\sin\theta = 0 \tag{1}$$

With $W = 100$ lb, $R = 50$ lb/in., $l = 20$ in., and $r = 5$ in.

$$(50\text{ lb/in.})(25\text{ in}^2)\left(\theta - \frac{\pi}{12}\right) - (100\text{ lb})(20\text{ in.})\sin\theta = 0$$

or $$0.625\theta - \sin\theta = 0.16362$$

Solving numerically, $$\theta = 1.8145\text{ rad} = 103.97°$$

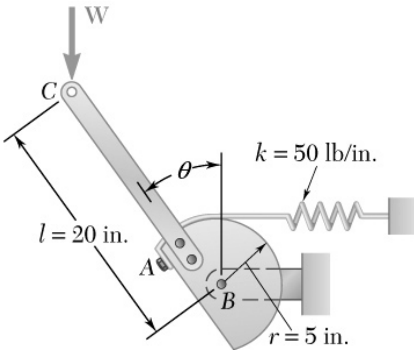
$$\theta = 104.0° \blacktriangleleft$$

Stability $$\frac{d^2V}{d\theta^2} = kr^2 - Wl\cos\theta \tag{2}$$

or $$= 1250 - 2000\cos\theta$$

For $\theta = 104.0°$: $$= 1734\text{ in.}\cdot\text{lb} > 0 \qquad \therefore\ \text{Stable} \blacktriangleleft$$

PROBLEM 10.112



A load **W** of magnitude 100 lb is applied to the mechanism at C. Knowing that the spring is unstretched when $\theta = 30°$, determine the value of θ corresponding to equilibrium and check that the equilibrium is stable.

SOLUTION

Using the solution of Problem 10.111, particularly Equations (1), with 15° replace by $30°\left(\frac{\pi}{6}\text{ rad}\right)$:

For equilibrium

$$kr^2\left(\theta - \frac{\pi}{6}\right) - Wl\sin\theta = 0$$

With

$$k = 50 \text{ lb/in.}, W = 100 \text{ lb}, r = 5 \text{ in., and } l = 20 \text{ in.}$$

$$(50 \text{ lb/in.})(25 \text{ in.}^2)\left(\theta - \frac{\pi}{6}\right) - (100 \text{ lb})(20 \text{ in.})\sin\theta = 0$$

or

$$1250\theta - 654.5 - 2000\sin\theta = 0$$

Solving numerically,

$$\theta = 1.9870 \text{ rad} = 113.8°$$

$\theta = 113.8°$ ◀

Stability: Equation (2), Problem 111:

$$\frac{d^2V}{d\theta^2} = kr^2 - Wl\cos\theta$$

or

$$= 1250 - 2000\cos\theta$$

For $\theta = 113.8°$:

$$= 2057 \text{ in.}\cdot\text{lb} > 0$$

$\therefore$ Stable ◀