Solution of system of Equations:

Consider the System of equations

$$a_1x + b_1y + c_1z = d_1;$$
$$a_2x + b_2y + c_2z = d_2;$$
$$a_3x + b_3y + c_3z = d_3.$$

What do you mean by solution of a polynomial?

$x + 2 = 0$

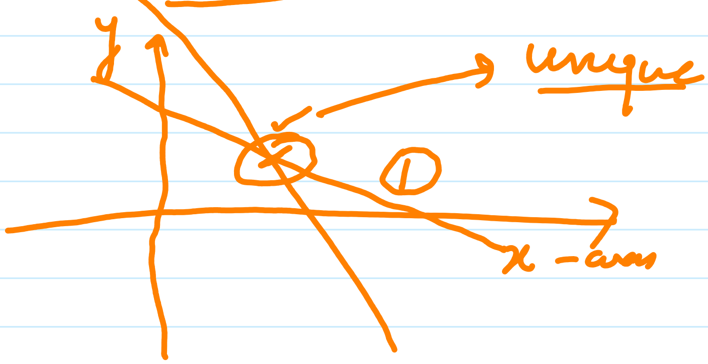
$\boxed{x = -2}$

$-2 + 2 = 0$

$\boxed{0 = 0}$

$$\begin{cases} ax + by = c \\ a_1x + b_1y = c_1 \end{cases}$$

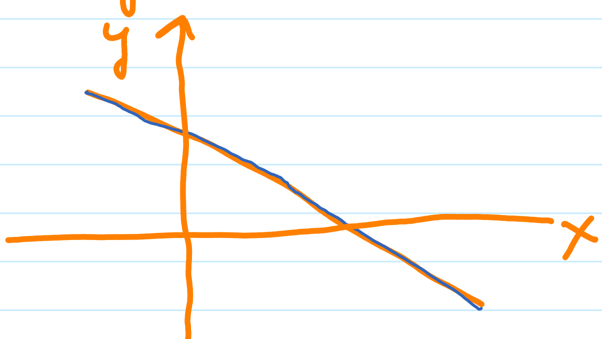
① Consistent ② Inconsistent



Solution of system of equations

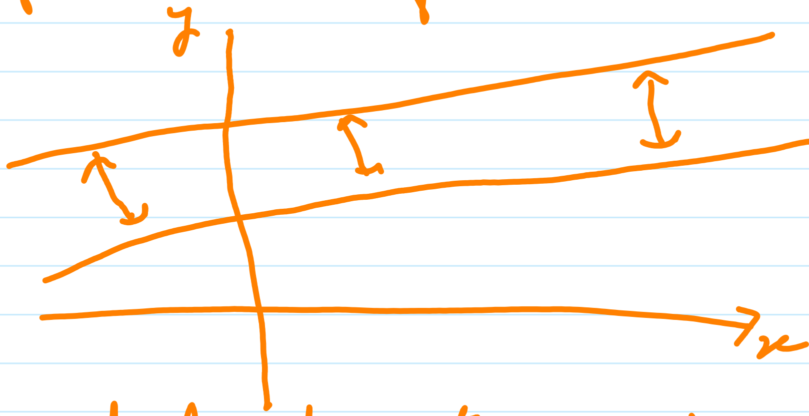
x, y

②



Infinite number of solutions

(3)



No point of intersection, so the system is having no solutions

Consider the System of equations

$a_1x + b_1y + c_1z = d_1;$
$a_2x + b_2y + c_2z = d_2;$
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(1) Consistent (2) Inconsistent

Grammar Rule

$$x = \frac{\Delta_1}{\Delta}, \quad y = \frac{\Delta_2}{\Delta} \;\&\; z = \frac{\Delta_3}{\Delta}$$

Case I: If $\Delta \neq 0$, $\Delta_1, \Delta_2 \,\&\, \Delta_3$ may have any values then the system of equations will have unique solution

Case II: If $\Delta = 0$ and Δ_1 or Δ_2 or $\Delta_3 \neq 0$ then the system will have no solution

Case III: If $\Delta = 0$ and $\Delta_1 = \Delta_2 = \Delta_3 = 0$ then the system will have infinite number of solutions

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$\frac{0}{2} = 0$

$\frac{4}{2} = 2$

$$\Delta_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$$

$$\Delta_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$$

$$\Delta_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

✓ $\frac{2}{0}$

✓ $\frac{0}{0}$

Consider the System of equations

(1) $a_1x + b_1y + c_1z = d_1;$
$a_2x + b_2y + c_2z = d_2;$
$a_3x + b_3y + c_3z = d_3.$

or $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$ or $AX = B$

✓ $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ → Coefficient matrix

✓ $K = [A, B] = \begin{bmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \end{bmatrix}$ → Augmented Matrix

<u>Elementary row Transformation</u>

✓ $K = [A : B] = \begin{bmatrix} a_1 & b_1 & c_1 & d_1 \\ 0 & b_2' & c_2' & d_2' \\ 0 & 0 & c_3' & d_3' \end{bmatrix}$ → <u>Echelon</u>

✓

<u>Rouche's Theorem</u> → The system of equations (1) is consistent iff the coefficient matrix 'A' & the augmented matrix K are of the same rank otherwise is inconsistent

✓ (1) $\rho(A) = \rho(K)$ or $\rho([A; B])$

✓(i) $\rho(A) = \rho(K) =$ Number of unknowns the solution is

① $\rho(A) = \rho(K)$ or $\rho([A;B])$

(1) $\rho(A) = \rho(K)$ = Number of unknowns the solution is unique

② $\rho(A) = \rho(K)$ < Number of unknowns the system will have infinite nr of solutions

② $\rho(A) \neq \rho(K) \rightarrow$ Inconsistent.