Lecture 39

30 November 2021 10:01

- **A positive integer is perfect if it equals the sum of its positive divisors other than itself.**
  Eg. 6, 28

6   1, 2, 3, 6 $1+2+3=6$

28   1, 2, 4, 7, 14 $1+2+4+7+14=28$

The Euclidean Algorithm

Lemma 1:

divide, divisor   divisor, remainder

Let $a = bq + r$, where a, b, q, and r are integers. Then $\gcd(a, b) = \gcd(b, r)$.

Q16. Find gcd using Euclidean Algorithm

(i) gcd(111, 201)

$201 = 111(1) + 90$

$111 = 90(1) + 21$

$90 = 21(4) + 6$

$21 = 6(3) + 3$

$6 = 3(2) + 0$

$\gcd(201, 111) = 3$

(ii) gcd(1000, 5040)

$5040 = 1000(5) + 40$

$1000 = 40(25) + 0$

$\gcd(1000, 5040) = 40$

(iii) gcd(1529, 14039)

$14039 = 1529(9) + 278$

$1529 = 278(5) + 139$

$278 = 139(2) + 0$

$\gcd(14039, 1529) = \gcd(1529, 278)$

$\gcd = 139$

Q17. How many divisions are required to find gcd(34, 55) using the Euclidean algorithm?

$55 = 34(1) + 21$

$34 = 21(1) + 13$

$8 = 5(1) + 3$

$5 = 3(1) + 2$

$34 = 21(1) + 13$ $\quad$ $5 = 3(1) + 2$

$21 = 13(1) + 8$ $\quad$ $3 = 2(1) + 1$

$13 = 8(1) + 5$ $\quad$ $2 = 1(2) + 0$

Theorem 10:

BÉZOUT'S THEOREM If a and b are positive integers, then there exist integers s and t such that $\gcd(a, b) = sa + tb$.

If a and b are positive integers, then integers s and t such that $\gcd(a, b) = sa + tb$ are called *Bézout coefficients* of a and b (after Étienne Bézout, a French mathematician of the eighteenth century). Also, the equation $\gcd(a, b) = sa + tb$ is called *Bézout's identity.*

Q18. Find gcd and Bezout coefficients for

(i) (123, 277)

$277 = 123(2) + 31$, $31 = 277 - 123(2)$ $\quad$ $1 = (277 - 123(2))(4) - 123(1)$

$123 = 31(3) + 30$, $30 = 123 - 31(3)$ $\quad$ $1 = 31(4) - 123(1)$

$31 = 30(1) + 1$, $1 = 31 - 30(1)$ $\quad$ $1 = 31 - (123 - 31(3))(1)$

$30 = 1(30) + 0$ $\quad$ $1 = 31 - 123(1) + 31(3)$

Do Back substitu

$1 = 277(4) - 123(9)$

$1 = 277(4) + 123(-9)$

4, −9 are Bezout's coeff.

(ii) (144, 89)

$144 = 89(1) + 55$ $\quad$ $55 = 144 - 89(1)$ $\quad$ $1 = 144(34) - 89(55)$

$89 = 55(1) + 34$ $\quad$ $34 = 89 - 55(1)$ $\quad$ $1 = 55(34) - 89(21)$

$55 = 34(1) + 21$ $\quad$ $21 = 55 - 34(1)$ $\quad$ $1 = 55(13) - 34(21)$

$13 = 34 - 21(1)$ $\quad$ $1 = 21(13) - 34(8)$

$55 = 34(1) + 21$

$34 = 21(1) + 13$

$21 = 13(1) + 8$

$13 = 8(1) + 5$

$8 = 5(1) + 3$

$5 = 3(1) + 2$

$3 = 2(1) + 1$

$13 = 34 - 21(1)$

$8 = 21 - 13(1)$

$5 = 13 - 8(1)$

$3 = 8 - 5(1)$

$2 = 5 - 3(1)$

$1 = 3 - 2(1)$

$3 - (5 - 3(1))(1)$

$3 - 5(1) + 3(2)$

$1 = 21(13) - 34(8)$

$1 = 21(5) - 13(8)$

$1 = 8(5) - 13(3)$

$1 = 8(2) - 5(3)$

$1 = 3(2) - 5(1)$

$1 = 144(34) + 89(-55)$

(iii) (3454, 4666)