Lecture 13

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Q11. Solve $y_n - 6y_{n-1} + 9y_{n-2} = 0, \quad y_0 = 1, y_1 = 6$

$n-2 \to n \qquad n \to n+2$

$y_{n+2} - 6y_{n+1} + 9y_n = 0$

$(E^2 - 6E + 9)y_n = 0$

$m^2 - 6m + 9 = 0$

$m = 3, 3$

$y_n = (c_1 + c_2 n)3^n$

$y_0 = 1$, Put $n=0$ $\quad c_1 = 1, \quad c_1 = 1$

$y_1 = 6$, Put $n=1$ $\quad (c_1 + c_2)3 = 6, \; c_2 = 1$

Particular Soln.

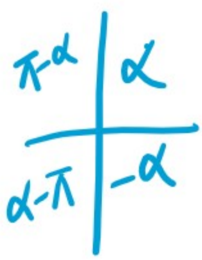
$y_n = (1+n)3^n$

Q12. Solve $y_n + 2y_{n-1} + 4y_{n-2} = 0$

$n-2 \to n \qquad n \to n+2$

$y_{n+2} + 2y_{n+1} + 4y_n = 0 \Rightarrow (E^2 + 2E + 4)y_n = 0$

$m = -1 \pm \sqrt{3}\, i$



$R = \sqrt{1+3} = 2$

$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{-1}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$y_n = 2^n\left[c_1 \cos\left(\frac{2n\pi}{3}\right) + c_2 \sin\left(\frac{2n\pi}{3}\right)\right]$

Q13. Solve $y_{n+3} - 2y_{n+2} - 5y_{n+1} + 6y_n = 0$

Q13. Solve $y_{n+3} - 2y_{n+2} - 5y_{n+1} + 6y_n = 0$

$$(E^3 - 2E^2 - 5E + 6)y_n = 0$$

$$m^3 - 2m^2 - 5m + 6 = 0$$

$$m = 1, 3, -2$$

1	1	-2	-5	6
	0	1	-1	6
	1	-1	-6	0

$$m^2 - m - 6 = 0$$

$$y_n = C_1 + C_2(3)^n + C_3(-2)^n$$

Q14. Solve $y_{n+3} + 3y_{n+2} + 3y_{n+1} + y_n = 0, \quad y_0 = 1, y_1 = -2, y_2 = -1$

$$(E^3 + 3E^2 + 3E + 1)y_n = 0$$

$$m = -1, -1, -1$$

$$m^3 + 3m^2 + 3m + 1 = 0$$

$$m(m^2 + 3m + 3) + 1 = 0$$

$$y_n = (C_1 + C_2 n + C_3 n^2)(-1)^n$$

Find C_1, C_2, C_3

Q15.

A model for the number of lobsters caught per year is based on the assumption that the number of lobsters caught in a year is the average of the number caught in the two previous years.

$$L_n = \frac{L_{n-1} + L_{n-2}}{2}$$

$$2m^2 - m - 1 = 0$$

$$m = -\frac{1}{2}, 1$$

a) Find a recurrence relation for $\{L_n\}$, where L_n is the number of lobsters caught in year n, under the assumption for this model.

b) Find L_n if 100,000 lobsters were caught in year 1 and 300,000 were caught in year 2.

$L_1 = 1{,}00{,}000$

$L_2 = 3{,}00{,}000$

$$C_1\left(\frac{-1}{2}\right)^n + C_2$$

b) Find L_n if 100,000 lobsters were caught in year 1 and 300,000 were caught in year 2.

$$L_n = \frac{L_{n-1} + L_{n-2}}{2} \qquad L_{n+2} = \frac{L_n + L_{n+1}}{2}$$

$$2L_{n+2} - L_{n+1} - L_n = 0$$

Complete it

$$L_1 = 1,00,000$$

$$L_2 = 3,00,000$$

Q16.

What is the general form of the solutions of a linear homogeneous recurrence relation if its characteristic equation has roots 1, 1, 1, 1, −2, −2, −2, 3, 3, −4?

$$m = 1, 1, 1, 1, -2, -2, -2, 3, 3, -4$$

$$y_n = (c_1 + c_2 n + c_3 n^2 + c_4 n^3) + (c_5 + c_6 n + c_7 n^2)(-2)^n + (c_8 + c_9 n)(3^n) + c_{10}(-4)^n$$

Q17. Find L.H.R.R whose characteristics roots are 1, −1, 2, 2.

$$(m-1)(m+1)(m-2)^2 = 0$$

$$(m^2-1)(m^2-4m+4) = 0$$

$$m^4 - 4m^3 + 3m^2 + 4m - 4 = 0$$

$$(E^4 - 4E^3 + 3E^2 + 4E - 4)\, y_n = 0$$

$$+ 4y_{n+1} - 4y_n = 0$$

$y_{n+4} - 4y_{n+3} + 3y_{n+2} + 4y_{n+1} - 4y_n = 0$

Q18. What is lowest degree R.R if its particular solution is

$$5 + 3(-1)^n + n\cos\left(\frac{n\pi}{2}\right) + 2\sin\left(\frac{n\pi}{2}\right)\cdot n^2$$

$$5(1)^n + 3(-1)^n + 1^n\left[\left(0 + 1\cdot n + 0\cdot n^2\right)\cos\frac{n\pi}{2} + \left(0 + 0\cdot n + 1\cdot n^2\right)\sin\frac{n\pi}{2}\right]$$

$$1, -1, 1\cdot e^{\pm i\frac{\pi}{2}}, 1\cdot e^{\pm i\frac{\pi}{2}}$$

$1, -1, \pm i, \pm i$ $\qquad A + iB = Re^{i\theta}$

$(m-1)(m+1)\left((m-i)(m+i)\right)^2$ $\qquad e^{i\theta} = \cos\theta + i\sin\theta$

$(m^2-1)(m^2+0m+1)^2 = 0$ $\qquad (m^2-1)(m^2+1)^2 = 0$

$(m^4-1)(m^2+1) = 0$

$m^6 + m^4 - m^2 - 1 = 0$

$y_{n+6} + y_{n+4} - y_{n+2} - y_n = 0$

or

$y_n + y_{n-2} - y_{n-4} - y_{n-6} = 0$