Q.4. Write Logical Statements.

p: You drive over 65 miles per hour.

q: You get a speeding ticket.

(A) You will get a speeding ticket, if you drive over 65 miles per hour.

(B) You get a speeding ticket, but you don't drive over 65 miles per hour.

(C) You get a speeding ticket unless you are not driving over 65 miles per hour.

(A) $p \rightarrow q$ (B) $\neg p \wedge q$ (C) $p \rightarrow q$

Propositional Equivalences

A compound proposition that is always true, no matter what the truth values of the propositions that occur in it, it is called a **Tautology**. A compound proposition that is always false is called **Contradiction**. A compound proposition that is neither a tautology or contradiction is called a **Contingency.**

Logical Equivalences

The compound propositions p and q are called logically equivalent if $p \leftrightarrow q$ is a tautogy.

They have same truth table

$$p \equiv q$$

Q5. Show that $p \vee (p \wedge q) \equiv p$

p	q	$p \wedge q$	$p \vee (p \wedge q)$	$p \leftrightarrow p \vee (p \wedge q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	F	T
F	F	F	F	T

$$F \mid F \mid F \mid , F \mid T$$

Imp Laws:

- **Identity Law** $p \wedge T \equiv p \quad , \quad p \vee F \equiv p$

- **Domination Law** $p \vee T \equiv T \quad , \quad p \wedge F \equiv F$

- **Idempotent Law** $p \vee p \equiv p \quad , \quad p \wedge p \equiv p$

- **Double Negation Law** $\neg(\neg p) \equiv p$

- **Commutative Law** $p \vee q \equiv q \vee p, \quad p \wedge q \equiv q \wedge p$

- **Associative Law** $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$
  $p \vee (q \vee r) \equiv (p \vee q) \vee r$

- **Distributive Law** $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
  $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

- **De Morgan's Law**
  $\neg(p \wedge q) = \neg p \vee \neg q$
  $\neg(p \vee q) = \neg p \wedge \neg q$

- **Absorption Law**
  $p \wedge (q \vee p) \equiv p$
  $p \vee (q \wedge p) \equiv p$

- **Negation Law**

p	$\neg p$
T	F
F	T

  $p \vee \neg p \equiv T \quad , \quad p \wedge \neg p \equiv F$

Q6. Use De Morgan's laws to find the negation of each of the following sentences.
(A) Kwame will take a job in industry or go to graduate school.

Kwame will not take a job in industry and will not go to graduate school.

(B) James is young and strong.

James is either not young or not strong.

James is either not young or not strong.

Q7. Show that each of these conditional statements is tautology by using truth tables

(A) $\neg(p \rightarrow q) \rightarrow p$

(B) $[\neg p \land (p \lor q)] \rightarrow q$

Complete it

(C) $[(p \lor q) \land (p \rightarrow r) \land (q \rightarrow r)] \rightarrow r$

Q8. Show that the conditional statement mentioned in above question is a tautology without using truth tables.

(A) $\neg(p \rightarrow q) \rightarrow p$

$p \rightarrow q \equiv \neg p \lor q$

$\equiv \neg(\neg p \lor q) \rightarrow p$

$\equiv (p \land \neg q) \rightarrow p$ Using De-Morgan's Law

$\equiv \neg(p \land \neg q) \lor p$ Use def^n of conditional

$\equiv (\neg p \lor q) \lor p$ Using De Morgan's Law

$\equiv q \lor (\neg p \lor p)$ By Associative Law

$\equiv q \lor T$ Negation Law

$\equiv T$ Domination Law