RATIO AND PROPORTION

The ratio of two quantities *a* and *b* in the same units, is the fraction $\frac{a}{b}$ and we write it as *a* : *b*.
Ratio of any number is expressed after removing all the common factors in the terms. For example, if there are two quantities having values of 8 and 6, then their ratios will be "4 : 3" because a common factor of 2 was removed from both the terms.

- In the ratio a : b, we call a as the first term or antecedent and b, the second term or consequent.
  Eg: In the ratio 5 : 9, antecedent = 5 and consequent = 9.
- The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.
  Eg. 4 : 5 = 8 : 10 = 12 : 15. Also, 4 : 6 = 2 : 3.

Types of Ratio

1. **Duplicate Ratio**: If a: b is a ratio, then its duplicate ratio is a^2: b^2
   Example: If 2: 3 is a ratio, then its duplicate ratio is 2^2: 3^2 i.e. 4:9
2. **Sub-duplicate Ratio**: If a: b is a ratio, then its sub-duplicate ratio is √ a: √b
   Example: If 16: 25 is a ratio, then its sub-duplicate ratio is √16: √25 = 4: 5
3. **Triplicate Ratio**: If a: b is a ratio, then its triplicate ratio is a^3: b^3
   Example: If 2: 3 is a ratio, then its triplicate ratio is 2^3: 3^3 = 8: 27
4. **Sub-triplicate Ratio**: If a: b is a ratio, then its sub-triplicate ratio is a1/3: b1/3
   Example: If 8: 27 is a ratio, then its sub-triplicate ratio is 81/3: 271/3 = 2: 3
5. **Inverse or Reciprocal Ratio**: The inverse ratio of a: b is 1/a: 1/b
   Example: If 2: 3 is a ratio, then its inverse ratio is (1/2) : (1/3)
6. **Compounded Ratio**: Compound ratio is the ratio of the products, of the corresponding terms of two or more simple ratios.
   Example: The compounded ratio of the ratios: (A : B), (C : D), (E : F) is (ACE : BDF).

Proportion

The equality of two ratios is called proportion.
If a : b = c : d, we write a : b :: c : d and we say that a, b, c, d are in proportion.
Here a and d are called ***extreme terms***, while b and c are called ***mean terms***.
Product of mean terms = Product of extreme terms.
Thus, a : b :: c : d => (b x c) = (a x d).
If a/b=c/d, then:

i. Invertendo - b:a = d:c
ii. Alternendo - a:c = b:d
iii. Componendo - (a+b):b = (c+d):d
iv. Dividendo - (a-b):b = (c-d):d
v. Componendo & Dividendo - (a+b)(a-b) = (c+d)(c-d)

Types of Proportions

1. Fourth Proportional: If a : b = c : d, then d is called the fourth proportional to a, b, c.
2. Third Proportional: If a : b = c : d, then c is called the third proportion to a and b.
Similarly, If the given proportion is a : b :: b : c then c is said to be the third proportion of a and b.
3. Mean Proportional: If the given ratio is a : b :: b : c , then b is said to be the mean proportion.
b= √ac

Example 1: If a : b = 2 : 3 and b : c = 4 : 3, then find a : b : c ?
Solution: a : b = 2 : 3

b : c = 4 : 3 = (4∗(3/4) : 3 ∗ (3/4)) = 3 : (9/4)
a :b : c=2 : 3 : (9/4) = 8 : 12 : 9

Example 2: The sum of two numbers is 72. If the two numbers are in the ratio of 5:3. Find the two numbers.
Solution: As discussed in the theory of this topic, if the two numbers are in the ratio 5:3, let the actual number is 5k and 3k. the sum of two numbers is 72. We have,
5k + 3k = 72
K = (72/8) = 9
Hence 5k = 45 and 3k = 27

Example 3: A number is divided into parts such that 4 times the first part, 3 times the second part, 6 times the third part and the 8 times the four parts are all equal. In what ratio is the number divided?
Solution: Let the four parts into which the number is divided is a, b, c and d.
4a = 3b = 6c = 8d= e (let)
A = (e/4), b = (e/3), c = (e/6), d = (e/8)
Hence, a : b : c : d = (e/4) : (e/3) : (e/6) : (e/8)
= (1/4) : (1/3) : (1/6) : (1/8)
= (24/4) : (24/3) : (24/6) : (24/8)
= 6 : 8 : 4 : 3

Example 4: a : b = 3: 4; b : c = 6 : 7. Find a : b : c?
Solution:

a	b	c
3	4	
	6	7

a : b : c = 3 × 6 : 6 × 4 : 4 × 7 = 9 : 12 : 14

Example 5: 36% of first number is 28% of the second number. What is the respective ratio of the first number to the second number?
Solution: Let the numbers be x and y.
36% of x = 28% of y
$\frac{x}{y}=\frac{28}{36}=\frac{7}{9}$
∴ x : y = 7 : 9

Example 6: The average of their ages is 30 years. What will be the ratio of their ages after 4 years?
Solution: Average age = 30 years
Total age = 2 × 30 = 60 years.
Let their present ages be 7x and 3x years
∴ 7x + 3x = 60 $\Rightarrow$ x = $\frac{60}{10}$ = 6
∴ Their present ages are 7 × 6 and 3 × 6 = 42 and 18.
∴ Their ages after 4 years
= 42 + 4 and 18 + 4 = 46 and 22 years
∴ ratio = 46 : 22 = 23 : 11

Example 7: In a bowl there is 30 litre mixtures of milk and water. The ratio of milk and water is 7:3. How much water must be added to it so that the ratio of milk to the water be 3:7?

Solution: Milk quantity in the mixture

$= \frac{7}{10} \times 30 = 21$ litres

$\therefore$ Water = 30 - 21 = 9 litres

New ratio = 3 : 7

$\therefore$ 3 parts of milk is 21 litres (There is no difference in the milk quantity of new mixture)

$\therefore$ Water quantity in the mixture

$= \frac{7}{3} \times 21 = 49$ litres

$\therefore$ 49 - 9 = 40 litres water is to be added in the new mixture

Example 8: A bag contains of one rupee, 50 paise and 25 paise coins. if these coins are in the ratio of 5 : 6 : 8, and the total amount of coins is Rs. 210, find the number of 50 paise coins in the bag.

Solution: Let the number of one rupee, 50 paise, 25 paise coins be 5, 6 and 8 respectively

The value of one rupee coins

= Rs. 1×5 = Rs. 5

The value of fifty paise coins

= Rs. 0.50×6 = Rs. 3

The value of twenty five paise coins

= Rs. 0.25×8 = Rs. 2

Total value = 5 + 3 + 2 = Rs. 10

If the total value is Rs. 10

there are 6 coins of fifty paise

if the total value is Rs. 210, then the number of 50 coins = $\frac{210}{10} \times 6 = 126$

VARIATION

Direct Variation

One quantity A is said to vary directly as another quantity B if the two quantities depend upon each other in such a manner that if B is increased in a certain ratio, A also increases in the same ratio.

This is denoted as A α B (A varies directly as B).

If A α B then A = kB, where k is a constant. It is called the constant of proportionality.

Inverse Variation

A quantity A is said to vary inversely as another quantity B if the two quantities depend upon each other in such a manner that if B is increased in a certain ratio, A gets decreased in the same ratio and if B is decreased in a certain ratio, then A gets increased in the same ratio.

It is the same as saying that A varies directly with 1/B. It is denoted as A α 1/B i.e., A = k/B where k is k the constant of proportionality.

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Joint Variation

If there are three quantities A, B and C such that A varies with B when C is constant and varies with C when B is constant, then A is said to vary jointly with B and C when both B and C are varying i.e. A α B when C is constant and A α BC A α BC $\Rightarrow$ A = kBC where k is the constant of proportionality.

Examples 9: The volume of a cylinder varies jointly as its height and the area of its base. When the area of the base is 64 sq. ft. and the height is 10 ft., the volume is 640 cu. ft.. What is the height of the cylinder, whose volume is 360 cu. ft. and area of the base is 72 sq.ft.

Solution: Let V be the volume, a be the area of the base and h be the height.
V = m a h (m is a proportionality constant)
We know a = 64, h = 10 and V = 640
640 = m (64) (10)
m = 1; V = ah
Therefore, 360 = 72×h
=> h = 360/72 = 5 ft.
Hence the height of the cylinder is 5 ft**.**

Examples 10: If x varies directly as y^4+9 and $x=3$ when $y=3$, find x when $y=9$.

Solution: $x \propto (y^4+9)$.

$$c = \frac{x}{y^4+9}.$$

when $x=3, y=3$ (given)

Hence $c = \frac{3}{3^4+9} = \frac{3}{90} = \frac{1}{30}$;

and $x = \frac{1}{30}(y^4+9)$

When $y=9$

$$x = \frac{1}{30}(y^4+9) = \frac{1}{30}(6561+9) = 219.$$

PARTNERSHIP

When two or more than two persons run a business jointly, they are called partners and the deal is known as partnership. The money put in by each of the partners is called his "investment or capital".

Ratio of Division of Gains:

1. If the partners invest *different* amounts each for the *same* period of time, then the profits at the end of the year are shared in the ratio of their investments

Suppose A and B invest Rs x and Rs y respectively for a year in a business, then at the end of the year: (A's share of profit) : (B's share of profit) = x : y

2. If the partners invest the *same* amounts for *different* periods of time, then the profits at the end of the year are shared in the ratio of the time periods for which respectively investments have been in business.

Suppose A and B invest Rs x and Rs x respectively for a time period t1 and t2 in a business, then at the end of the year: (A's share of profit) : (B's share of profit) = t1 : t2

3. If the partners invest *different* amounts and the time period for which their investments are in the business are also *different*, then the profits at the end of the year are shared in the ratio of the product (investment x time period) for each partner.

Suppose A invests Rs x for p months and B invests Rs y for q months, then
(A's share of profit) : (B's share of profit) = xp : yq

4. Working and sleeping partners: A partner who manages the business is known as working partner and the one who simply invests the money is a sleeping partner.

Example 11: P and Q started a business investing Rs 85,000 and Rs 15,000 respectively. In what ratio the profit earned after 2 years be divided between P and Q respectively?
Solution: As time period is same for both, ratio of profit is directly proportional to investment
85,000 : 15,000 = 17 : 3

Example 12: A,B and C started a business by investing Rs 1,20,000, Rs 1,35,000 and Rs 1,50,000.Find the share of each ,out of an annual profit of Rs 56,700?
Solution: Ratio of shares of A, B and C=Ratio of their investments
120,000:135,000:150,000 =8:9:10
A's share=Rs 56,700*(8/27) = Rs 16,800
B's share =Rs 56,700*(9/27) = Rs 18,900
C's share =Rs 56,700*(10/27) = Rs 21,000

Example 13: Ram and Krishna entered into a partnership with Rs 50,000 and Rs 60,000, after 4 months Ram invested Rs 25,000 more while Krishna withdrew Rs 20,000. Find the share of Ram in the annual profit of Rs 289,000.
Solution: Ram : Krishna=50,000*4+75,000*8: 60,000*4+40,000*8 =10:7
Ram's annual profit=289000*(10/17) = Rs 170,000

Example 14: Prerna starts a business with Rs.45,000. Three months later Sanjna joins her with Rs, 30,000. At the end of the year in what ratio should they shared profits?
Solution: Sharing of profits will be in the ratio of investments multiplied by the time period.
Hence the ratio is
(45,000 x 12) : (30,000 x 9) = 2: 1

Example 15: The working partner of a business gets as his commission 10% of the profits left after his commission is paid. If the working partner's commission is Rs. 30,000 then, find the total profit.
Solution: Let total profit be P.
The profit left after the working partner's commission of Rs. 30,000 is (P - 30,000).
10% of this is the working partner's commission. So we have (0.1) (P – 30,000) = 30,000
$\Rightarrow$ (0.1)P = 33,000
$\therefore$ P = Rs. 3,30,000

LEVEL – I

1. If 15% of x is the same as 20% of y, then x : y is?
   A] 3 : 4 B] 17 : 16 C] 4 : 3 D] 16 : 17

2. In a college, the ratio of the number of boys to girls is 8 : 5. If there are 160 girls, the total number of students in the college is:
   A] 100 B] 260 C] 250 D] 416

3. The ratio of income of A to that of B is 5 : 4 and the expenditure of A to that of B is 3: 2. If at the end of the year, each saves Rs, 800, the income of A is:
   A] Rs. 1600 B] Rs. 2000 C] Rs. 1800 D] Rs. 2200

4. If p : q = 3 : 4 and q : r= 8 : 9, then p : r is:
   A] 1: 3 B] 2 : 3 C] 3 : 2 D] 1: 2

5. If a+b : b+c : c+a = 6 : 7 : 8 and a + b + c = 14, then the value of c is : .
   A] 6 B] 8 C] 7 D] 2

6. Two numbers are respectively 20% and 50% more than a third number. The ratio of the two numbers is?
   A] 5:4 B] 3:2 C] 4:5 D] 2:3

7. If three numbers in the ratio 3 : 2: 5 be such that the sum of their squares is 1862, the middle number will be?
   A] 7 B] 21 C] 14 D] 35

8. A certain amount was divided between Salim and Rahim in the ratio of 4 : 3. If Rahim's share was Rs. 2400, the total amount was.
   A] Rs. 5600 B] Rs. 9600 C] Rs. 3200 D] Rs. 16800

9. A profit of Rs. 30000 is to be distributed among A, B, C in the proportion 3 : 5 : 7. What will be the difference between B's and C's shares?
   A] Rs. 2000 B] Rs. 10000 C] Rs. 4000 D] Rs. 14000

10. An amount of money is to be distributed among P, Q and R in the ratio 3 : 5 : 7. If Qs share is Rs. 1500, what is the difference between Ps and Rs shares?
    A] Rs. 1200 B] Rs. 1600 C] Rs, 1500 D] Rs. 1900

11. Rs. 120 are divided among A, B, C such that A's share is Rs. 20 more than B's and Rs. 20 less than C's. What is B's share?
    A] Rs. 10 B] Rs. 20 C] Rs. 15 D] Rs. 25

12. The compounded ratio of (2 : 3), (6: 11) and (11 :2) is.
    A] 1: 2 B] 11: 24 C] 2: 1 D] 36: 121

13. What number should be added to each of the numbers 8, 21, 13 and 31 so that the resulting numbers, in this order form a proportion?
    A] 2 B] 5 C] 3 D] 7

14. An alloy is to contain copper and zinc in the ratio 9 :4. The zinc required (in kg) to be melted with 24 kg of copper, is
    A] 10.66 B] 9.66 C] 10.33 D] 9

15. What number should be subtracted from both the terms of the ratio 15 : 19 so as to make it as 3 : 4 ?
A] 3 B] 6 C] 5 D] 9

16. Rs.432 is divided amongst three workers A, B and C such that 8 times A's share is equal to 12 times B's share which is equal to 6 times C's share. How much did A get?
A] Rs.192 B] Rs.133 C] Rs.144 D] Rs.128

17. The price of Computer and CD player are in the ratio 6:5. If the computer costs Rs 5000 more than CD player. What is the price of the computer?
A] 25,000 B] 15,000 C] 50,000 D] 30,000

18. If Rs. 582 be divided into three parts, proportional to 1/2:2/3:3/4, then the first part is?
A] 161 B] 151.8 C] 142 D] 153

19. The price of mixer, grinder and washing machine are in the ratio 2:3:6. After one year the price of the items are increased 20%, 15%, 25% respectively. Then what will be ratio after one year?
A] 16:23:50 B] 17:22:23 C] 16:10:22 D] 18:22:24

20. A person has 25p, 10p and 5p in the ration 1:2:3 in his pocket. If the person has Rs. 30 in all, how many 5 paisa coins are there?
A] 110 B] 150 C] 130 D] 180

21. The partners A, B, C invests Rs. 26000, Rs. 34000 & Rs. 10000 respectively in a business. Out of a profit of Rs. 3500, B's share is
A] Rs. 1300 B] Rs. 1700 C] Rs. 500 D] Rs. 1500

22. Pooja invests Rs. 30000 for one year in a shop. How much her partner Neha should invest in order that the profit after one year may be 2:3
A] Rs. 20000 B] Rs. 40000 C] Rs. 45000 D] Rs. 18000

23. Rs. 700 is divided among A,B, and C so that A receives half as much as B and B half as much as C. Then C's share is
A] Rs. 200 B] Rs. 300 C] Rs. 400 D] Rs. 600

24. P and Q started a business investing Rs 85000 and Rs 15000 resp. In what ratio the profit earned after 2 years be divided between P and Q respectively.
A] 17:5 B] 17:3 C] 17:6 D] 17:7

25. In business, A and C invested amounts in the ratio 2:1, whereas the ratio between amounts invested by A and B was 3:2, If Rs 157300 was their profit, how much amount did B receive.
A] Rs 48000 B] Rs 47000 C] Rs 47400 D] Rs 48400

26. If 4 (A's capital) = 6 (B's capital) = 10 (C's capital), then out of a profit of Rs. 4650, C will receive ____
A] Rs.700 B] Rs.800 C] Rs.900 D] Rs.1000

27. A and B invest in a business in the ratio 3 : 2. If 5% of the total profit goes to charity and A's share is Rs. 855, the total profit is:
A] Rs. 1425 B] Rs. 1500 C] Rs. 1537.50 D] Rs. 1576

28. A starts business with Rs. 3500 and after 5 months, B joins with A as his partner. After a year, the profit is divided in the ratio 2 : 3. What is B's contribution in the capital?
A] Rs. 7500 B] Rs. 8000 C] Rs. 8500 D] Rs. 9000

29. In a partnership, A invests (1/6) of the capital for (1/6) of the time, B invests (1/3) of the capital for (1/3) of the time and C, the rest of the capital for the whole time. Out of a profit of Rs. 4600, B's share is

A] Rs. 800 B] Rs. 1000 C] Rs. 650 D] Rs. 960

30. A, B and C enter into a partnership in the ratio $\frac{7}{2} : \frac{4}{3} : \frac{6}{5}$. After 4 months, A increases his share 50%. If the total profit at the end of one year be Rs. 21,600, then B's share in the profit is:

A] Rs. 2100 B] Rs. 2400 C] Rs. 3600 D] Rs. 4000

LEVEL - II

1. A shopkeeper contains apples, oranges and bananas in the ratio 5:7:8. There is a demand to increase their quantity by 50%, 60% and 70% respectively. What will be ratio of the increased quantity?

A] 25:75:100 B] 26:72:112 C] 75:112:136 D] 76:100:201

2. A packet of sweets is distributed among A, B, C, D in the proportion of 6:8:5:4. If B gets 10 sweets more than D then what is A's share?

A] 16 B] 17 C] 15 D] 18

3. If A's 60% of salary is equal to two-third of B's salary. Now find the ratio of A's salary to B's salary.

A] 9:10 B] 10:9 C] 11:12 D] 13:11

4. In a mixture of 45 litres, the ratio of milk and water is 3 : 2. How much water must be added to make the ratio 9 : 11?

A] 10 liters B] 15 liters C] 17 liters D] 20 liters

5. Seats of physics, Chemistry and Mathematics in a school are in the ratio 4: 5: 6. There is a proposal to increase these seats by 75 in each department. What were the total number of seats in the school finally?

A] 600 B] 750 C] 900 D] Data Inadequate

6. 60 kg of an alloy A is mixed with 100 kg of alloy B. If alloy A has lead and tin in the ratio 3 : 2 and Alloy B has tin and copper in the ration 1 : 4, then the amount of tin in the new alloy is

A] 36 kg B] 44 kg C] 53 kg D] 80 kg

7. A diamond falls and breaks into three pieces whose weights are in the ration 1 : 3 : 6. The value of the diamond is proportional to the square of its weight. If the original value is Rs. 30,000. What is the loss in the value due to the breakage?

A] Rs. 13,000 B] Rs. 16,200 C] Rs. 18,600 D] Rs.19,400

8. W varies inversely as the square of t. If W = 12 when t = 2. Find t when W = 27.

A] 27t = 24 B] $27t^2 = 48$ C] $108 = 12t^2$ D] $12t = 54$

9. Find the mean proportion to 36 and 16?

A] 24 B] 36 C] 18 D] 26

10. Find a if, 4 : a :: a : 9

A] ± 7 B] ± 8 C] ± 6 D] ± 9

11. A disinfecting solution is mixed at a ratio of 2 parts of alcohol to 5 parts of distilled water. If the solution has 0.5 liters of water, how many millilitres of alcohol does it contain?

A] 0.2 ml B] 200 ml C] 500 ml D] 100 ml

12. A bag containing 24 mirrors fell down. Which of the following cannot be the ratio of the broken mirrors to unbroken mirrors?

A] 2:1 B] 1:3 C] 4:3 D] 1:1

13. The ratio of marks obtained by vinod and Basu is 6:5. If the combined average of their percentage is 68.75 and their sum of the marks is 275, find the total marks for which exam was conducted.

A] 150 B] 200 C]400 D] None of these.

14. The marks scored by a student in three subjects are in the ratio of 4 : 5 : 6. If the candidate scored an overall aggregate of 60% of the sum of the maximum marks and the maximum marks in all three subjects is the same, in how many subjects did he score more than 60%?

A] 1 B] 2 C] 3 D] None of the subjects

15. The ratio of the cost prices of two articles A and B is 4:5.The articles are sold at a profit with their selling prices being in the ratio 5:6.If the profit on article A is half of its cost price, find the ratio of the profits on the articles A and B?

A] 7:10 B] 9:11 C] 5:9 D] 10:11

16. Manoj got Rs. 6000 as his share out of a total profit of Rs. 9000 which he and Ramesh earned at the end of one year. If Manoj invested Rs. 20000 for 6 months, whereas Ramesh invested his amount for the whole year, what was the amount invested by Ramesh?

A] Rs. 3000 B] Rs. 3000 C] Rs. 1000 D] Rs. 5000

17. A and B enter into partnership. A invests Rs. 16000 for 8 months and B remains in the business for 4 months. Out of a total profit, B claims 2/7 of the profit, B contributed.

A] Rs. 11900 B] Rs. 10500 C] Rs. 13600 D] Rs. 12800

18. Kamal started a business investing Rs 9000. After five months, Sameer joined with a capital of Rs 8000. If at the end of the year, they earn a profit of Rs. 6970, then what will be the share of Sameer in the profit?

A] Rs 2380 B] Rs 2300 C] Rs 2280 D] Rs 2260

19. A, B, C rent a pasture. A puts 10 oxen for 7 months, B puts 12 oxen for 5 months and C puts 15 oxen for 3 months for grazing. If the rent of the pasture is Rs. 175, how much must C pay as his share of rent?

A] Rs. 45 B] Rs. 50 C] Rs. 55 D] Rs. 60

20. A and B enter into partnership with investments of Rs. 54000 and Rs. 81000 respectively. A stayed for the entire year. If at the end of the year the profit was distributed equally, then for how many months less was B's investment there in the business?

A] 2 B] 4 C] 5 D] 6