## Indeterminate Forms

Consider the ratio  $\frac{f(x)}{g(x)}$  of two functions f(x) and g(x).

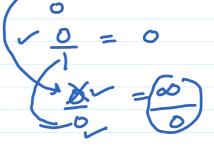
If at any point x = a, f(a) = 0, g(a) = 0.

Then the ratio  $\frac{f(x)}{g(x)}$  takes the form  $\frac{0}{0}$ 

It is called an Indeterminate Form.

If  $f(a) = \pm \infty$ ,  $g(a) = \pm \infty$ , Then the ratio  $\frac{f(x)}{g(x)}$  takes the form  $\frac{\infty}{\infty}$ 

It is another Indeterminate Form

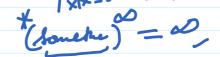




## Other Indeterminate Forms

The other indeterminate forms are:

 $0 \times \infty, 0^0, \infty^0$  and  $\infty - \infty$ 



In each of these cases, we can reduce the ratio function to the from  $\frac{0}{0}$  or  $\frac{\infty}{\infty}$ For the indeterminate forms  $0^0, \infty^0, 1^\infty$ , we take logarithm of the given function and then take the limits.

## <u>Important Note</u>

The functions of the form  $0^{\infty}$ ,  $\infty \times \infty$ ,  $\infty + \infty$ ,  $\infty^{\infty}$ ,  $\infty^{-\infty}$  are not considered as indeterminate forms.

Note that:  $0^{\infty} = 0$ 

$$\infty \times \infty = \infty$$

$$\infty + \infty = \infty$$

$$\infty^{\infty} = \infty$$

$$\infty^{-\infty} = 0$$

## <u>L'Hospital's Rule</u>

L' Hospital rule can be used only when the ratio is of indeterminate from, that is either is of form  $\frac{0}{0}$  or  $\frac{\infty}{\infty}$ 

Suppose that the real valued functions f(x) and g(x) are differentiable in some open interval containing the point x = a, f(a) = 0, g(a) = 0.

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)} = \frac{f'(a)}{g'(a)}, g'(a) \neq 0.$$

Suppose now that f'(a) = 0 = g'(a).

Then we repeat the application of L'Hospital rule on  $\frac{f'(x)}{g'(x)}$  and obtain:

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(a)}{g'(a)} = \lim_{x \to a} \frac{f''(x)}{g''(x)} = \frac{f''(a)}{g''(a)}$$

provided the limit exists.

This application of L'Hospital rule can be continued as long as the indeterminate form is obtained

When both  $f(a) = \pm \infty$ ,  $g(a) = \pm \infty$ , we get another indeterminate form.

In this case also, L'Hospital rule can be applied. We write

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{\left[\frac{1}{g(x)}\right]}{\left[\frac{1}{f(x)}\right]} \quad \text{which is of } \frac{0}{0} \text{ form.}$$

L'Hospital rule can also be applied to find the limits  $x \to \pm \infty$ .

Q Evaluate 
$$LL$$
  $\chi = \frac{1}{2} - \frac{1}{2} - \frac{1}{2} = \frac{1}$ 

Q = Evaluate 
$$U = \frac{\log u}{u + v} = \frac{\log u}{-\log u}$$

$$= \frac{U}{u + v} = \frac{1}{u} \cdot \frac{\log u}{u}$$

$$= -\frac{U}{u + v} \cdot \frac{1}{u} \cdot \frac{\log u}{u}$$

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$$= -\frac{U}{u + v} \cdot \frac{1}{u} \cdot \frac{\log u}{u} - \frac{1}{u} \cdot \frac{\log u}{u}$$

$$= \frac{U}{u + v} \cdot \frac{1}{u} \cdot \frac{1}{u}$$

$$= \frac{U}{u + v} \cdot \frac{1}{u} \cdot \frac{1}{u}$$

$$=-\frac{2}{5}$$
 Ag

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3 Fam reducte to 2 form

I Form 
$$0 \times \infty$$
 It  $f(x) = 0$   $f(x) = \infty$ 

then  $f(x) = 0$   $f(x) = \infty$ 
 $f(x) = 0$ 
 $f(x) = 0$ 
 $f(x) = 0$ 

7. evalueles 
$$4 f(x)g(x) = 4 f(x)$$
 $x \to a f(x)$ 
 $x \to a f(x)g(x)$ 

ol = 
$$\frac{4}{24}$$
  $\frac{g(x)}{1/f(x)}$   $\left[\frac{ab}{ab} f_{min}\right]$ 

I Form 
$$\omega - \omega$$
,  $\mathcal{L}_{x \to a} f(x) = \omega$ ,  $\mathcal{L}_{x \to a} \varphi(x) = \omega$  then
$$\mathcal{L}_{x \to a} \left[ f(x) - \varphi(x) \right] \longrightarrow \omega - \omega$$

$$\frac{4}{x+a}\left[f(x)-f(x)\right] = \frac{4}{x+a}\left[\frac{1}{f(x)}-\frac{1}{f(x)}\right]$$

$$\frac{1}{f(x)}\left[f(x)-f(x)\right]$$

$$= \frac{U}{x-1} \frac{1}{h_{mn}} \frac{1}$$

$$= \frac{4}{2^{-10}} \left[ \frac{x + \frac{1}{3} + \frac{1}{15} x^{4} + -1}{x^{1} + \frac{1}{3} x^{4} + -1} \right] (|x|^{2})$$

$$= \frac{4}{2^{-10}} \left[ 1 + \frac{1}{3} x^{4} + \frac{1}{15} x^{4} + -1 \right] (|x|^{2})$$

$$= \frac{4}{2^{-10}} \left[ 1 + x^{4} + \frac{1}{3} x^{4} + -1 \right] (|x|^{2})$$

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$$= \frac{4}{2^{-10}} \left[ 1 + x^{2} + \frac{1}{3} x^{4} + -1 \right]$$

$$= \frac{4}{2^{-10}} \left[ 1 + x^{4} + \frac{1}{$$