Gauss Jordan Method

Q Using the Gauss-Jordan Method, find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$

$A = AI$

$I = AB$

or

$A = IA$

$I = BA$ colm oper

Sol Consider $A = AI$

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix} = A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to R_2 - R_1, \quad R_3 \to R_3 + 2R_1$

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 2 & -6 \\ 0 & -2 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

operate $R_2 \to \frac{1}{2} R_2, \quad R_3 \to \frac{1}{2} R_3$

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & -3 \\ 0 & -1 & 1 \end{bmatrix} = A \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1/2 & 0 \\ 1 & 0 & 1/2 \end{bmatrix}$$

$R_1 \to R_1 - R_2, \quad R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & -3 \\ 0 & 0 & -2 \end{bmatrix} = A \begin{bmatrix} 3/2 & -1/2 & 0 \\ -1/2 & 1/2 & 0 \\ 1/2 & 1/2 & 1/2 \end{bmatrix}$$

$R_3 \to \frac{1}{-2} R_3$

$$\begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & -3 \end{bmatrix} = A \begin{bmatrix} 3/2 & -1/2 & 0 \\ -1/2 & 1/2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} = A \begin{bmatrix} 3/2 & -1/2 & 0 \\ -1/2 & 1/2 & 0 \\ -1/4 & -1/4 & -1/4 \end{bmatrix}$$

$R_1 \to R_1 - 6R_3$, $R_2 \to R_2 + 3R_3$

$-\frac{1}{2}$ $-\frac{3}{4}$

$\frac{1}{2}$ $-\frac{3}{4}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = A \begin{bmatrix} 3 & 1 & 3/2 \\ -5/4 & -1/4 & -3/4 \\ -1/4 & -1/4 & -1/4 \end{bmatrix}$$

$\boxed{I = AB}$

$\therefore$ Inverse of $A = \begin{bmatrix} 3 & 1 & 3/2 \\ -5/4 & -1/4 & -3/4 \\ -1/4 & -1/4 & -1/4 \end{bmatrix}$ Ans

$\boxed{A^{-1} = \frac{1}{|A|} \operatorname{Adj} A}$, $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$

(i) $|A| = -8$

Cofactors of elements of I^{st} row: $-24, 10, 2$

2^{nd} row: $-8, 2, 2$

3^{rd} row: $-12, 6, 2$

$$\operatorname{Adj} A = \begin{bmatrix} -24 & 10 & 2 \\ -8 & 2 & 2 \\ -12 & 6 & 2 \end{bmatrix}' = \begin{bmatrix} -24 & -8 & -12 \\ 10 & 2 & 6 \\ 2 & 2 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-8} \begin{bmatrix} -24 & -8 & -12 \\ 10 & 2 & 6 \\ 2 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 1 & 3/2 \\ -5/4 & -1/4 & -3/4 \\ -1/4 & -1/4 & -1/4 \end{bmatrix}$$

odd even

The function $\sin nx \cos nx$ is

a) odd function b) even function c) cannot determined d) none of these

odd

The value of $\int_{-2}^{2} \sin nx \, dx$ is

a) 0 b) 1 c) 2 d) 3

If $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$ be defined in the interval $(\alpha, \alpha + 2\pi)$ then the value of b_n

a) $\int_{\alpha}^{\alpha+2\pi} f(x) \sin nx \; dx$ b) $\int_{\alpha}^{\alpha+2\pi} f(x) \cos \; nx \; dx$

c) $\frac{2}{\pi} \int_{o}^{\alpha+2\pi} f(x) \sin nx \; dx$ d) $\frac{1}{\pi} \int_{\alpha}^{\alpha+2\pi} f(x) \sin nx \; dx$

$$b_n = \frac{1}{\pi} \int_{\alpha}^{\alpha+2\pi} f(x) \sin nx \, dx$$

If $f(x) = \sum_{n=1}^{\infty} b_n \sin nx$ be defined in the interval $(0, \pi)$ then the value of b_n is

a) $\int_0^{\pi} f(x) \sin nx \; dx$ b) $\int_0^{\pi} f(x) \cos \; nx \; dx$

c) $\frac{2}{\pi} \int_0^{\pi} f(x) \sin nx \; dx$ d) $\frac{1}{\pi} \int_0^{\pi} f(x) \sin nx \; dx$

$$b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx \, dx$$

In the Fourier series expansion of $f(x) = x\sin x$ in the interval $0 < x < 2\pi$, the value of a_0 is

a) 2 b) -2 c) 1 d) -1

$$a_0 = \frac{1}{\pi} \int_0^{2\pi} x \sin x \, dx$$

$$= \frac{1}{\pi} \left[x(-\cos x) - (1)(-\sin x) \right]_0^{2\pi}$$

$$= \frac{1}{\pi} \left[(-2\pi \times 1 - 0) - (0) \right] = -2$$

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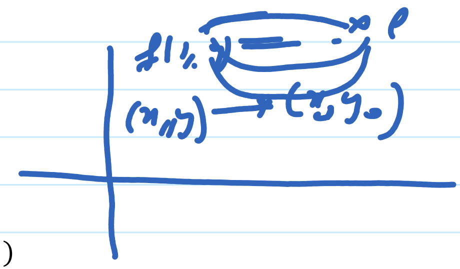


$\lim_{(x,y)\to(x_o,y_o)} f(x,y)$ will exist if

(a) Limit is path dependent ✗
(b) Limit is not finite ✗
✓ (c) Limit is both finite and unique along all possible paths reaching (x_o, y_o)
(d) None of these

If $f(x,y,z) = (xy)^{\sin z}$ then value of $\frac{\partial f}{\partial x}$ at $\left(3, 5, \frac{\pi}{2}\right)$ is			
(a) 3	(b) 5 ✓	(c) 0	(d) 15

$$\frac{\partial f}{\partial x} = (\sin z)(xy)^{\sin z - 1} \times y$$
$$= 1 \times 5 = 5$$

If $f(x,y) = x^y, (x,y) \neq (0,0)$ then the value of f_{xy} is

✓ (a) $x^{y-1}(1 + y\log x)$ (b) $x^y(1 + y\log x)$ (c) $y^{x-1}(1 + y\log x)$ (d) $x^{y-1}(1 + \log x)$

$$\boxed{f_{xy} = f_{yx}}$$

$$f_y = x^y \log x$$
$$f_{xy} = x^y \frac{1}{x} + y x^{y-1} \log x$$
$$= x^{y-1} + y x^{y-1} \log x$$
$$= x^{y-1}[1 + y\log x]$$

$\lim_{h\to 0} \frac{f(x+h,y)-f(x,y)}{h}$ if exist, called partial derivative of $f(x,y)$ with respect to

(a) x at (a,b) (b) y at (a,b) ✓ (c) x at (x,y) (d) y at (x,y)

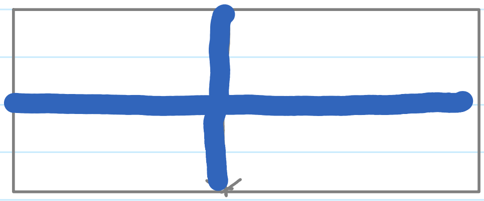
Which of the following limits are suitably representing a rectangular region in XY plane

✗ (a) $y \le x \le y^2, 0 \le y \le 1$ ✗ (b) $0 \le x \le 1, 0 \le y \le x$ ✗ (c) $0 \le y \le x, y \le x \le 1$

Which of the following limits are suitably representing a rectangular region in XY plane

(a) $y \le x \le y^2, 0 \le y \le 1$ (b) $0 \le x \le 1, 0 \le y \le x$ (c) $0 \le y \le x, y \le x \le 1$

(d) $0 \le x \le 1, 0 \le y \le 2$



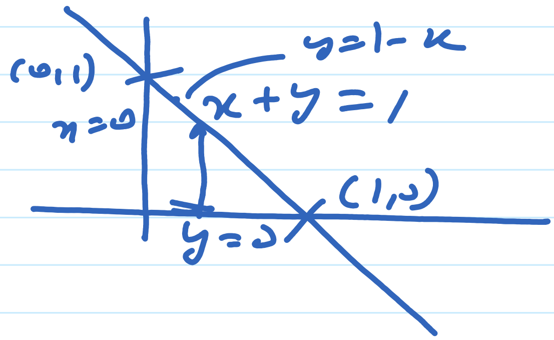
If a region R is bounded by the curves $x = 0, y = 0, x + y = 1$ then which of the following limits correctly justify region R

(a) $0 \le x \le 1, 0 \le y \le 1$ (b) $0 \le x \le 1, 0 \le y \le x$ (c) $0 \le x \le 1, 0 \le y \le x - 1$

(d) $0 \le x \le 1, 0 \le y \le 1 - x$

$0 \le y \le 1-x,$

$0 \le x \le 1$



What is the formula of area of region R in polar coordinates ?

(a) $\iint dxdy$ (b) $\iint dydx$ (c) $\iint drd\theta$ (d) $\iint rdrd\theta$

If region R is defined as $0 \le y \le \sqrt{1 - x^2}, 0 \le x \le 1$, then limits of R in polar coordinates are

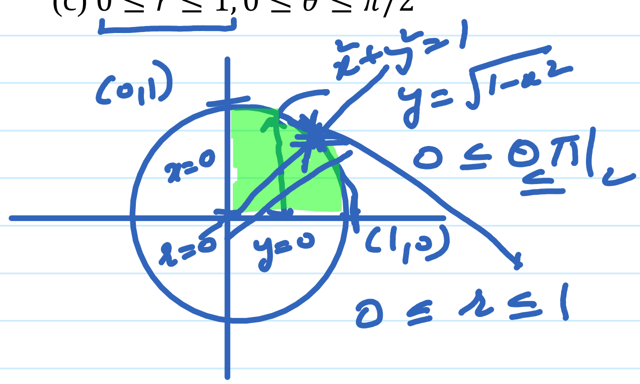
(a) $0 \le r \le a, 0 \le \theta \le \pi$ (b) $0 \le r \le 1, 0 \le \theta \le \pi$ (c) $0 \le r \le 1, 0 \le \theta \le \pi/2$

(d) $0 \le r \le 1, 0 \le \theta \le 2\pi$

$y = \sqrt{1-x^2}$

$y^2 = 1-x^2$

$x^2+y^2 = 1$



$\int_{x=0}^{1} \int_{y=0}^{\sqrt{1-x^2}} f(x, y) dydx$, if we change the order of integration then which of the following

$\int_{x=0}^{1}\int_{y=0}^{\sqrt{1-x^2}} f(x,y)\,dy\,dx$, if we change the order of integration then which of the following limits will be correct

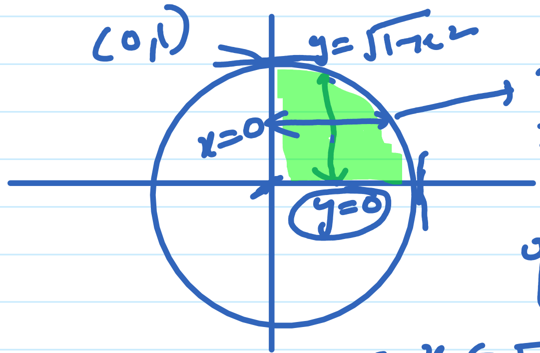
(a) $0 \le y \le \sqrt{1-x^2}, 0 \le x \le 1$ ✗ (b) $0 \le x \le 1, 0 \le y \le 1$ ✗

(c) $0 \le y \le 1, 0 \le x \le \sqrt{1-y^2}$ ✓ (d) $0 \le x \le \sqrt{1-y^2}, 0 \le y \le \sqrt{1-x^2}$

$y = \sqrt{1-x^2}$

SBS $y^2 = 1 - x^2$

or $\boxed{x^2 + y^2 = 1}$



$x^2 + y^2 = 1$

$x^2 = 1 - y^2$

$x = \pm\sqrt{1-y^2}$

or $\boxed{x = \sqrt{1-y^2}}$

$0 \le x \le \sqrt{1-y^2}$

$0 \le y \le 1$

Area of the region bounded by $0 \le x \le 1, 0 \le y \le x$

(a) 1 (b) 1/2 ✓ (c) 1/4 (d) None of these

$\text{Area} = \int_0^1\int_0^x dy\,dx$

$= \int_0^1 |x|_0^x\,dx$

$= \int_0^1 x\,dx$

$\frac{x^2}{2}\Big|_0^1 = 1/2$

Which of the following limits are suitable for defining a cube

(a) $0 \le x \le 1, 0 \le y \le x, 0 \le z \le y$ ✗ (b) $0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1$ ✓

(c) $0 \le x \le 1, 0 \le y \le 1, 0 \le z \le y$ ✗ (d) $0 \le x \le 1, 0 \le y \le x, 0 \le z \le 1$ ✗

$0 \le x \le a$

$0 \le y \le b$

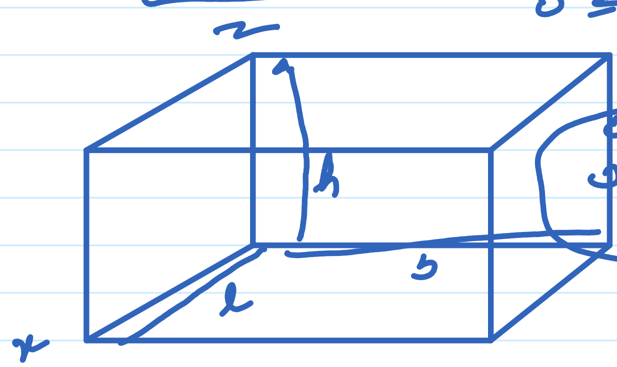
$0 \le z \le c$

$a = b = c$

$0 \le x \le a$

$0 \le y \le a$

$0 \le z \le a$



A solid is bounded $x = 0, y = 0, z = 0, x + y + z = 1$ then which of the following limits are correct for the given solid

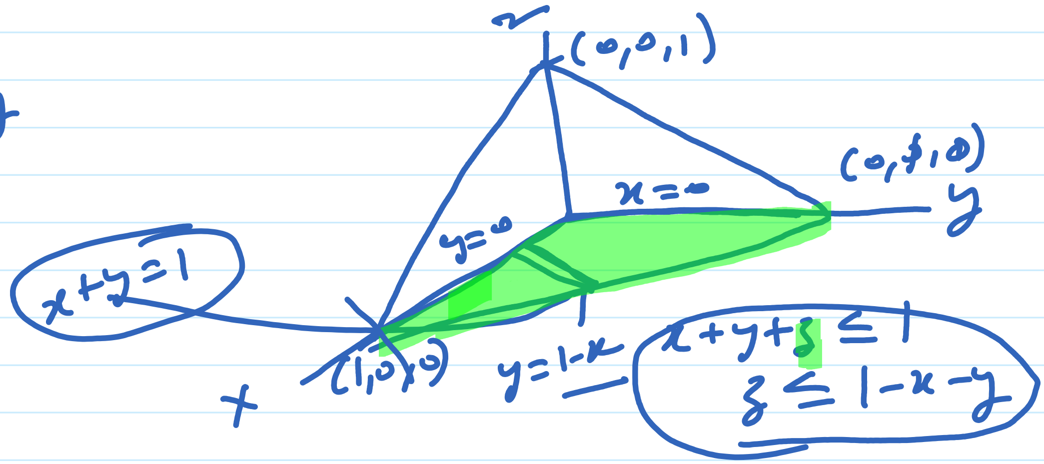
(a) $0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1$

(b) $0 \le x \le 1, 0 \le y \le 1 - x, 0 \le z \le 1 - y - x$

(c) $0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1 - x - y$

(d) $0 \le x \le 1, 0 \le y \le 1 - x, 0 \le z \le 1 - y$

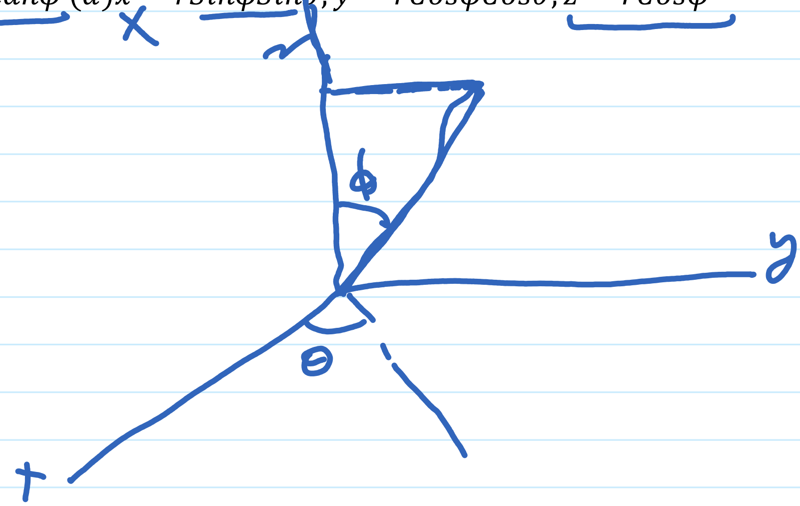
$0 \le x \le 1$
$0 \le y \le 1 - x$
$0 \le z \le 1 - x - y$



Which of the following relations correctly define relation between Cartesian coordinates (x, y, z) and spherical polar coordinates (r, θ, ϕ)

(a) $x = rSin\phi Cos\theta, y = rCos\phi Cos\theta, z = rCos\phi$ (b) $x = rSin\phi Cos\theta, y = rSin\phi Sin\theta, z = rCos\phi$

(c) $x = rSin\phi Cos\theta, y = rCos\phi Cos\theta, z = rtan\phi$ (d) $x = rSin\phi Sin\theta, y = rCos\phi Cos\theta, z = rCos\phi$



$x^2+y^2+z^2=r^2$

3^2

Volume of a sphere $\boxed{x^2+y^2+z^2=9}$ is

(a) 27π cubic units (b) 18π cubic units (c) 108π cubic units (d) 36π cubic units

$8\iint$

$x = r\cos\theta\sin\phi$
$y = r\sin\theta\sin\phi$
$z = r\cos\phi$

$0 \le \phi \le \pi/2$
$0 \le \theta \le \pi/2$
$0 \le r \le 1$

$\frac{4}{3}\pi r^3$

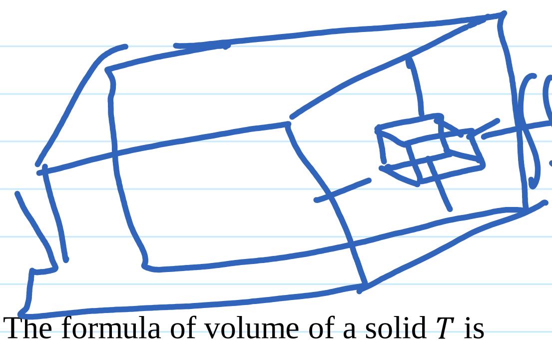
$\frac{4}{3}\pi \times 3^3$

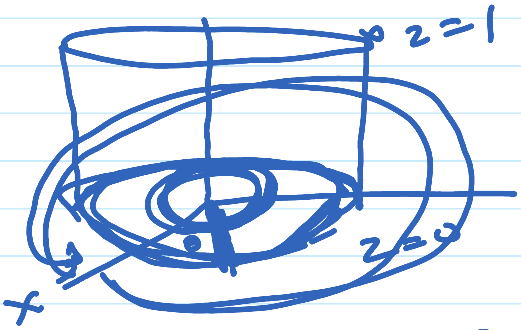
36π

A solid is bounded by $x^2+y^2=1, 0 \le z \le 1$. which of the following limits are correct for the given solid

(a) $0 \le z \le 1, 0 \le r \le 1, 0 \le \theta \le \pi$ (b) $0 \le z \le 1, 0 \le r \le 1, 0 \le \theta \le \pi/2$

(c) $0 \le z \le 1, 0 \le r \le 1, 0 \le \theta \le 2\pi$ (d) $0 \le z \le 1, 0 \le r \le 1, 0 \le \theta \le \pi/4$





$\iiint dxdydz$

$0 \le r \le 1 \quad 0 \le \theta \le 2\pi$

$\iint z\,dxdy$

The formula of volume of a solid T is

(a) $\iint dxdy$ (b) $\iiint dxdydz$ (c) $\iiint z\, dxdydz$ (d) $\iiint y\, dxdydz$

The value of $\int_1^e\int_1^e\int_1^e \frac{1}{xyz}dxdydz$ is

The value of $\int_1^e \int_1^e \int_1^e \frac{1}{xyz} dxdydz$ is

a) 0 b) $\frac{1}{3}$ c) 1 d) None of these

$|\log x|_1^e \; |\log y|_1^e \; |\log z|_1^e$

$1 \times 1 \times 1 = 1$

l $\cancel{x}=1$ $\quad$ 2 b $\quad$ 1 h

$h = l \neq b$

If a solid is defined as $0 \leq x \leq 1, 2 \leq y \leq 4, 0 \leq z \leq 1$, then it represents

(a) A cylinder (b) A sphere (c) A cuboid (d) A cube

Which of the following limits correctly justify the triangular region with vertices (0,0), (0,1) and (1,1)

(a) $0 \leq y \leq 1, y \leq x \leq 1$ (b) $0 \leq y \leq 1, 0 \leq x \leq 1$

(c) $0 \leq y \leq 1, 0 \leq x \leq y$ (d) $0 \leq y \leq 1, 0 \leq x \leq 1 - y$