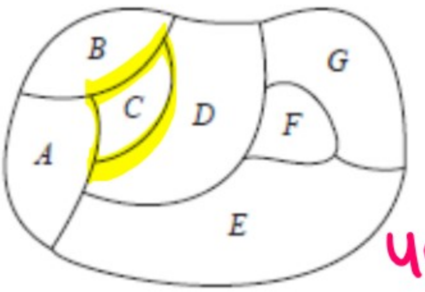
Lecture 33

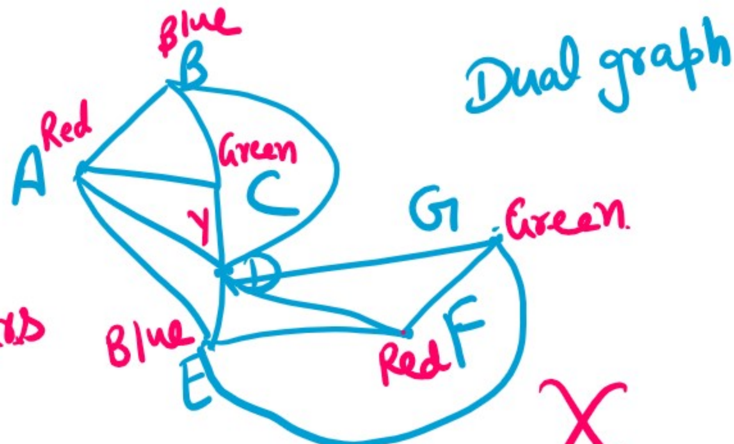
12 November 2021 08:58

Graph Coloring

A *coloring* of a simple graph is the assignment of a color to each vertex of the graph so that no two adjacent vertices are assigned the same color.



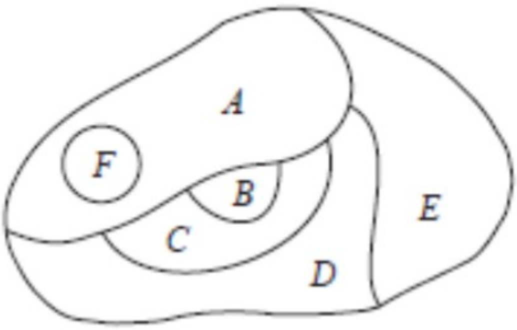
4 colours

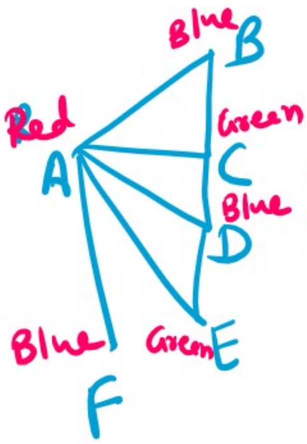


The **chromatic number** of a graph is the least number of colors needed for a coloring of this graph. It is denoted by $\chi(G)$.

Q6. Construct the dual graph for the following map and find the least number of colors needed to color the map so that no two adjacent regions have the same color.

(i)

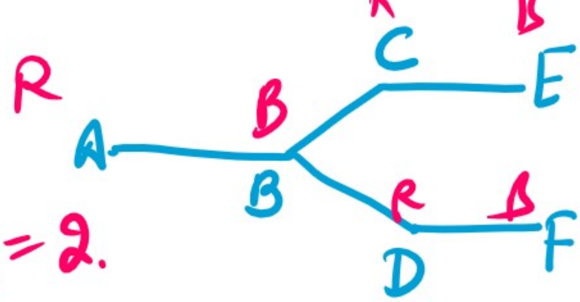




$\chi(G) = 3$

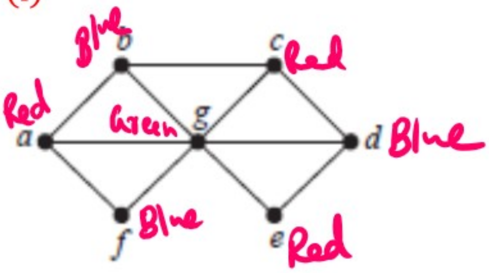
(ii)

$\chi(G) = 2.$



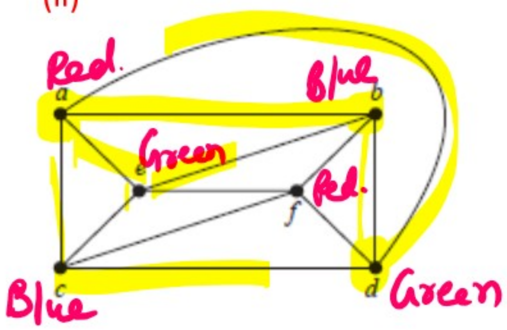
Q7. Find chromatic number of following planar graph.

(i)



$\chi(G)=3$

(ii)



$\chi(G)=3$

Q8. What is chromatic number of

(i) K_n

$\chi(K_n)=n$ $\chi(K_{501})=501$

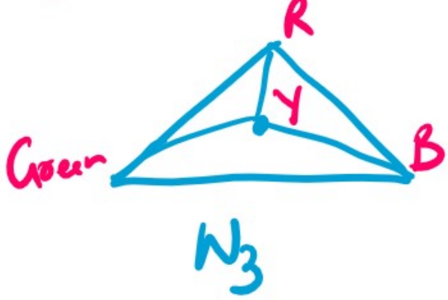
(ii) $K_{m,n}$

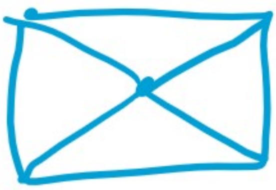
$\chi(K_{m,n})=2$

(iii) C_n

$C_n=\begin{cases}2, & n \text{ is even}\\ 3 & n \text{ is odd}\end{cases}$

(iv) W_n





$\chi(W_n)=\begin{cases}4 & n \text{ odd}\\ 3 & n \text{ even}\end{cases}$

χ(W$_n$ with even no of vertices) = 4

$n+1 = \text{even}$

$n = \text{odd}$

Theorem 2:

THE FOUR COLOR THEOREM The chromatic number of a planar graph is no greater than four.

$\chi(G) > 4 \Rightarrow G$ is non-planar

Trees

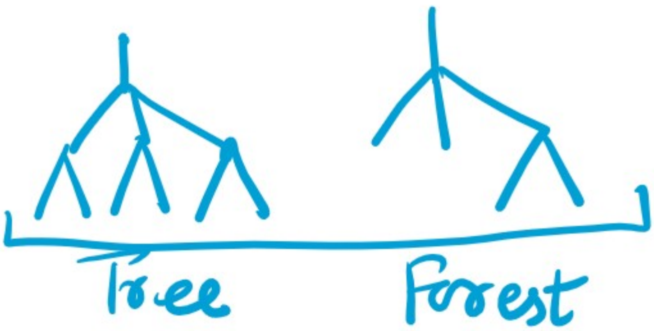
Chapter-11

- A connected undirected graph that contains no simple circuits is called tree.
- Graphs containing no simple circuits, not necessarily connected, are called forests.

Tree → Connected

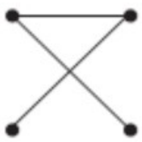
→ Simple

→ No circuits



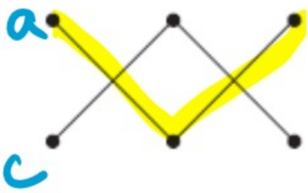
Q9. Which of the following is tree?

(i)



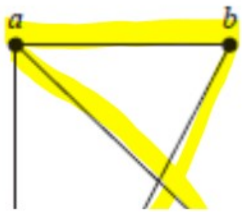
Connected ✓
Simple ✓
No circuit ✓
} Yes tree

(ii)

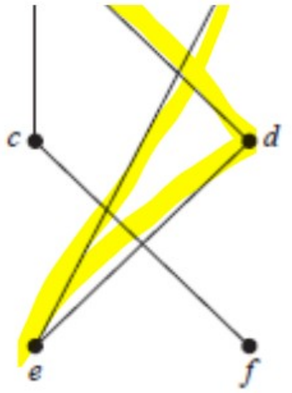


Connected ✗ Not tree

(iii)



Connected ✓
Simple ✓
} Not tree



Simple ✓
No circuit X

Theorem 3:

An undirected graph is a tree if and only if there is a unique simple path between any two of its vertices.

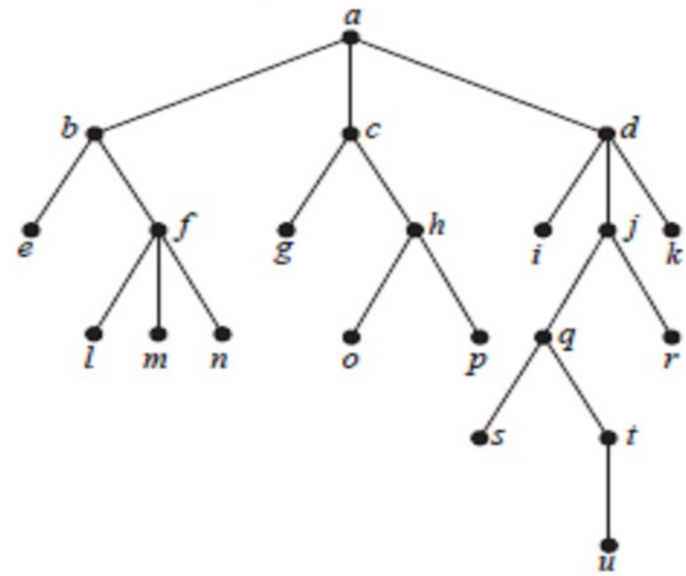
Some Definitions:

- **Rooted tree** is a tree in which one vertex has been designated as the root and every edge is directed away from the root. h i j
- **Parent** of v is the unique vertex u such that there is a directed edge from u to v.
- **Child:** u is parent of v and v is child of u.
- **Siblings:** Vertices with same parent.
- **Ancestors** of a vertex other than the root are the vertices in the path from the root to this vertex. $f \to c, a$
- **Descendants** of a vertex v are those vertices who has v as their ancestor. $c \to f, g, h, i, j$
- **Leaf** has no children. d, e, h, i, j, g
- **Internal vertices** are the vertices that have children.

a, b, c, f

Q10.

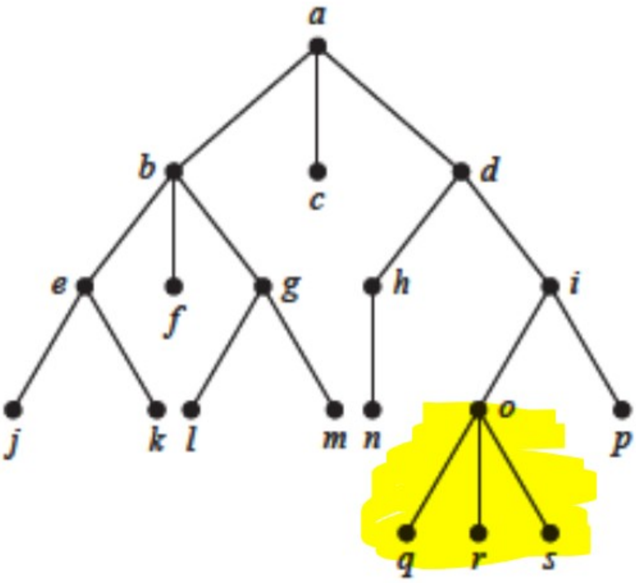
. Answer these questions about the rooted tree illustrated.



a) Which vertex is the root? = a
b) Which vertices are internal? = a, b, c, d, f, h, j, q, t
c) Which vertices are leaves? → e, g, i, k, l, m, n, o, p, r, s, u
d) Which vertices are children of j? q, r
e) Which vertex is the parent of h? c
f) Which vertices are siblings of o? p
g) Which vertices are ancestors of m? f, b, a
h) Which vertices are descendants of b? e, f, l, m, n

Q11.

Left sub-tree of i



a) Which vertex is the root? – a
b) Which vertices are internal?
c) Which vertices are leaves? – c, f, j, k, l, m, n, q, r, s, p
d) Which vertices are children of j? None

b) Which vertices are internal?

c) Which vertices are leaves? – c, f, j, k, l, m, n, q, r, s, p

d) Which vertices are children of j? None

e) Which vertex is the parent of h? d

f) Which vertices are siblings of o? p

g) Which vertices are ancestors of m? g, b, a

h) Which vertices are descendants of b? e, f, g, j, k, l, m