CSE322
Concept of Computability

Lecture #40

Computability

- The Problem of finding out whether a given problem is “Solvable” by automata reduces to the evaluation of function on the set of natural numbers or a given alphabet by mechanical means.

Primitive Recursion Function

- Consider the function $exp(x, y) = x^y$
- $x^0 = 1$
- $x^1 = x$
- $x^2 = x . x$
- … $x^y = x. x. x\ldots\ldots$(y occurrences of x)
- $x^{y+1} = x . x^y$

- The two Rewriting Rules:-
  - $X^0 = 1$
  - $X^{y+1} = x.\ x^y$

Reducing exponential to reduce multiplication

$x.0 = 0$

$X(y+1) = x + x\ .\ y$

Primitive Recursion is spirit of

Converting exponential into multiplication or addition.

Or in general we can say complex functions into easy functions.

- The primitive recursive functions are among the number-theoretic functions, which are functions from the natural numbers (nonnegative integers) {0, 1, 2, ...} to the natural numbers. These functions take *n* arguments for some natural number *n* and are called *n*-ary.
- The basic primitive recursive functions are given by these axioms:
- **Constant function or Zero function**: The 0-ary constant function 0 is primitive recursive.
- **Successor function**: The 1-ary successor function *S*, which returns the successor of its argument is primitive recursive. That is, $S(k) = k + 1$.
- **Projection function**: For every $n \geq 1$ and each *i* with $1 \leq i \leq n$, the *n*-ary projection function P_i^n, which returns its *i*-th argument, is primitive recursive.

Initial Functions

The initial functions over N are given in Table 11.1. In particular,

$$S(4) = 5. \quad Z(7) = 0$$

$$U_2^3(2, 4, 7) = 4, \quad U_1^3(2, 4, 7) = 2, \quad U_3^3(2, 4, 7) = 7$$

TABLE Initial Functions Over N

Zero function Z defined by $Z(x) = 0$.
Successor function S defined by $S(x) = x + 1$.
Projection function U_i^n defined by $U_i^n(x_1, \ldots, x_n) = x_i$.

Definition If $f_1, f_2, \ldots, f_k$ are partial functions of n variables and g is a partial function of k variables, then the composition of g with $f_1, f_2, \ldots, f_k$ is a partial function of n variables defined by

$$g(f_1(x_1, x_1, \ldots, x_n), f_2(x_1, x_2, \ldots, x_n), \ldots, f_k(x_1, x_2, \ldots, x_n))$$

If, for example, f_1, f_2 and f_3 are partial functions of two variables and g is a partial function of three variables, then the composition of g with f_1, f_2, f_3 is given by $g(f_1(x_1, x_2), f_2(x_1, x_2), f_3(x_1, x_2))$.

Complexity

- Time Complexity:- Amount of time taken by a Machine to run input string.
- Space Complexity:- Amount of space or memory required by machine to implement input string.

The classes of P and NP problem

Definition A Turing machine M is said to be of time complexity $T(n)$ if the following holds: Given an input w of length n, M halts after making at most $T(n)$ moves.

Note: In this case, M eventually halts. Recall that the standard TM is called a deterministic TM.

Definition A language L is in class **P** if there exists some polynomial $T(n)$ such that $L = T(M)$ for some deterministic TM M of time complexity $T(n)$.

Definition A language L is in class **NP** if there is a nondeterministic TM M and a polynomial time complexity $T(n)$ such that $L = T(M)$ and M executes at most $T(n)$ moves for every input w of length n.

Power of Quantum Computation

In classical complexity theory, the classes **P** and **NP** play a major role, but there are other classes of interest. Some of them are given below:

L — The class of all decision problems which may be decided by a TM running in logarithmic space.

PSPACE — The class of decision problems which may be decided on a Turing machine using a polynomial number of working bits, with no limitation on the amount of time that may be used by the machine.

EXP — The class of all decision problems which may be decided by a TM in exponential time, that is, $O(2^{n^k})$, k being a constant.

The hierarchy of these classes is given by

$$\mathbf{L} \subseteq \mathbf{P} \subseteq \mathbf{NP} \subseteq \mathbf{PSPACE} \subseteq \mathbf{EXP}$$

The inclusions are strongly believed to be strict but none of them has been proved so far in classical complexity theory.