Lecture 8

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Methods of Proving Theorems

Prove that If p then q.

Direct Proof

Assume p is true

We will prove q is true

Q15. Give a direct proof of the theorem "If n is an odd integer, then n^2 is odd."

p: n is an odd integer, q: n^2 is odd

Assume p is true

$n = \text{Odd}$

$n = 2k+1$

$n^2 = (2k+1)^2 = 4k^2 + 4k + 1$

$n^2 = 2(2k^2 + 2k) + 1$

$n^2 = \text{Odd}$

q is true. $p \to q$ is true.

Q16. Use a direct proof to show that every odd integer is difference of two squares.

p = n is odd integer, q = n is diff of two squares

Assume p is true

Assume p is true

$n = \text{Odd}$

$n = 2k+1$

$n = 2k+1+k^2-k^2$

$n = (k+1)^2 - k^2$

$n = \text{diff of two squares}$

q is true.

$p \to q$ is true.

Q17. Prove that if n is an integer and $3n + 2$ is even, then n is even.

p: $3n+2$ is even

q: n is even

Proof by Contraposition

$$p \to q \equiv \neg q \to \neg p$$

Assume q is false, then we will show p is false.

assume n is odd

$n = 2k+1$

$3n+2 = 3(2k+1)+2$

$= 6k+5$

$= 2(3k+2)+1$

$3n+2 = \text{Odd}$

p is false.

p is false.

Hence $p \to q$ is true.

Q18. Prove that if $x + y \geq 2$, where x, y are real numbers, then $x \geq 1$ or $y \geq 1$.

$p: x+y \geq 2$ $\quad$ $q: x \geq 1$ or $y \geq 1$

Assume q is false.

$x<1$ and $y<1$

$x+y<2$

p is false.

$p \to q$ is true.

Proof by Contradiction

$p \to q$ is False when p is true and q is false

$$p \wedge \neg q \longrightarrow F$$

Assume p is true and q is false

$\Downarrow$

Contradiction.

Q19(a): Prove that Product of two rational no.s is rational.

p: a, b are two rational no.s.

q: ab is rational no.

q: ab is rational no.

Assume p is true.

$a=\frac{m}{n}$, $b=\frac{r}{s}$ m, n, r, s are integers

n, s ≠ 0

$ab=\frac{mr}{ns}$, mr, ns are integers
ns ≠ 0

ab is rational no.

q is true.

$p \to q$ is true.

Q19. Prove or disprove that product of a nonzero rational number and an irrational number is irrational.

p: a is non-zero rational no., b is irrational no.

q: ab is irrational no.

Assume p is true, q is false

$a=\frac{m}{n}$, m, n are integers
m, n ≠ 0

ab = rational no.

b = irrational no.

$\frac{1}{a}=\frac{n}{m}$; n, m are integers
m ≠ 0, n ≠ 0

$\frac{1}{a}$ is rational no.

a

Product of two rational nos is rational. (already proved in last question)

$(ab)\cdot\frac{1}{a}$ = rational

b = rational no.

Contradiction //

Q20. Show that at least 10 of any 64 days chosen must fall on the same day of the week.

p: Total days chosen are 64

q: atleast 10 fall on same day of week.

Assume q is false, atmost 9 fall on same day of week.

Max. no. of days = 9x7 = 63

p is false.

$p \to q$ is true.

Vacuous Proof

If p is false, then $p \to q$ is ?? true

Trivial Proof

If q is true, then $p \to q$ is ?? true

If q is true, then p → q ...

Q: Prove the proposition P(0), where P(n) is the proposition "If n is positive integer greater than 1, then $n^2 > n$"

Which kind of proof you will use?

P(0): "If 0 is positive integer greater than 1, then $0^2 > 0$"

↓

False ⟶ Vacuous Proof.

Mistakes in Proof

Q1. Prove that $1 = 2$

$$a = b$$

$$ab = b^2 \quad \text{multiply } b$$

$$ab - a^2 = b^2 - a^2 \quad \text{Subtract } a^2$$

$$a(\cancel{b-a}) = (b+a)(\cancel{b-a}) \quad \text{Cancelling common factor as long as it is non-zero}$$

$$a = b + a \quad \text{using } a = b$$

$$a = 2a$$

$$1 = 2$$

Q2. To find soln of $\sqrt{2x^2 - 1} = x$

$$\sqrt{2x^2 - 1} = x$$

→ Not Reversible

$\sqrt{2x-1} = x$

S.B.S. → Not Reversible

$2x^2-1 = x^2$

$x^2-1=0$

$(x-1)(x+1)=0$

$x=1$ or $x=-1$

But $x=-1$ doesn't satisfy the eqn.