Q Determine the rank of

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}_{3\times 3}$$

Here $|A| = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{vmatrix} = 0$

$\therefore$ rank of A or $\rho(A) < 3$

Consider minors of order 2

Minor of 1 $= \begin{vmatrix} 4 & 2 \\ 6 & 5 \end{vmatrix} = 8 \neq 0$

Here we got a minor of order 2, which is non zero

$\therefore$ $\boxed{\rho(A) = 2}$

Q If rank of matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & \mu \end{bmatrix}$ is '2', find the value of μ

As rank is '2'

$\therefore$ $|A| = 0$

$$\begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & \mu \end{vmatrix} = 0$$

$\Rightarrow 2\mu = 0$

$\Rightarrow \boxed{\mu = 0}$

Check $\begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{vmatrix} = 0$

Minor of '0' i.e. $a_{31} = \begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix} = -4 \neq 0$

Rank using elementary row operations

Echelon form

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times 3}$$

Rank → The number of non-zero rows in the echelon form of matrix.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - 2R_1$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & 2 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$$

operate $R_3 \to R_3 - R_2$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

which is the required echelon form.

By definition $\boxed{\rho(A) = 2}$

Q $$A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 1 \end{bmatrix}_{3\times 4}$$

Note → $\rho(A) \le \min\{3, 4\}$

Possible sub matrices of order 3 are

$$\begin{vmatrix} 1 & 3 & 4 \\ 3 & 9 & 12 \\ 1 & 3 & 4 \end{vmatrix} = 0, \quad \begin{vmatrix} 1 & 3 & 3 \\ 3 & 9 & 3 \\ 1 & 3 & 1 \end{vmatrix} = 0, \quad \begin{vmatrix} 1 & 4 & 3 \\ 3 & 12 & 3 \\ 1 & 4 & 1 \end{vmatrix} = 0, \quad \begin{vmatrix} 3 & 4 & 3 \\ 9 & 12 & 3 \\ 3 & 4 & 1 \end{vmatrix} = 0$$

So $\rho(A) < 3$

Consider $\begin{vmatrix} 1 & 3 \\ 3 & 3 \end{vmatrix} = 3 - 9 = -6 \neq 0$

$\therefore \boxed{\rho(A) = 2}$

$$A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 1 \end{bmatrix}$$

operation $R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - R_1$

$$\sim \begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

$R_3 \to R_3 - \frac{1}{3} R_2$

$$\sim \begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 0 & 0 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\text{Rank}(A) = 2$

Q $A = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}_{3 \times 1}$