Lecture 9

06 September 2021 09:55

Unit II: Recurrence Relations / Advanced Counting Techniques.
Text Book: Ch 8

Many counting problems cannot be solved easily using simple counting techniques. Here we will study a variety of counting problems that can be solved by finding relationship between terms of sequence or by modelling of recurrence relations.

Eg: No. of corona patients doubles every day.
What will be no. of corona patients on 16th day.

$a_2 = 2a_1, \quad a_3 = 2a_2, \ldots, \quad a_n = 2a_{n-1}$

$a_1 = 1$

Solution $a_n = 2^{n-1}$

Recurrence relation

A recurrence relation for the sequence $\{a_n\}$ is an equation that expresses a_n in terms of one or more of the previous terms of the sequence, namely, $a_0, a_1, \ldots, a_{n-1}$ for all integers n with $n \geq n_0$, where n_0 is a nonnegative integer. A sequence is called a solution of a recurrence relation.

Q1. Find the first four terms of the sequence defined by each of these recurrence relations and initial conditions $a_n = 3{a_{n-1}}^2$, $a_0 = 1$

$a_1 = 3a_0^2 = 3 \cdot 1 = 3$

$a_2 = 3a_1^2 = 27$

$a_3 = 3a_2^2 = 3(27)^2 = 2187$

$a_3 = 3a_2^2 = 3(27)^2 = 2187$

$a_4 = 3a_3^2 = 14348907$

Q2. Find 5th term of $a_n = na_{n-1} + a_{n-2}{}^2$, $a_0 = -1$, $a_1 = 0$

$a_2 = 2a_1 + a_0^2 = 1$

$a_3 = 3a_2 + a_1^2 = 3$

$a_4 = 4a_3 + a_2^2 = 13$

$a_5 = 5a_4 + a_3^2 = 74$

Q3. Show that the sequence $\{a_n\}$ is solution of the recurrence solution $a_n = -3a_{n-1} + 4a_{n-2}$, where $a_n = (-4)^n$.

$a_n = -3a_{n-1} + 4a_{n-2}$ LHS $a_n = (-4)^n$

RHS $a_{n-1} = (-4)^{n-1}$, $a_{n-2} = (-4)^{n-2}$

$-3\left((-4)^{n-1}\right) + 4\left((-4)^{n-2}\right)$

$= -3\left((-4)^{n-1}\right) - (-4)(-4)^{n-2}$

$= -3(-4)^{n-1} - (-4)^{n-1}$

$= (-4)^{n-1}(-3-1) = (-4)^n = \text{LHS}$

Q4. Is the sequence $\{a_n\}$, $a_n = n4^n$ is solution of the recurrence relation $a_n =$

$8a_{n-1} - 16a_{n-2}$?

Q5. Find the solution of the following recurrence relation with given initial conditions using iterative approach?

(a) $a_n = 2a_{n-1} - 3, a_0 = -1$

(b) $a_n = na_{n-1}, a_0 = 5$