CSE408
Brute Force(String Matching, Closest pair, Convex hull,Exhaustive,Voronori diagrams

Lecture # 7&8

Brute Force

A straightforward approach, usually based directly on the problem's statement and definitions of the concepts involved

Examples:

1. Computing a^n ($a > 0$, n a nonnegative integer)

2. Computing $n!$

3. Multiplying two matrices

4. Searching for a key of a given value in a list

Brute-Force Sorting Algorithm

Selection Sort Scan the array to find its smallest element and swap it with the first element. Then, starting with the second element, scan the elements to the right of it to find the smallest among them and swap it with the second elements. Generally, on pass i ($0 \le i \le n$-2), find the smallest element in $A[i..n\text{-}1]$ and swap it with $A[i]$:

$$A[0] \le \ldots \le A[i\text{-}1] \mid A[i], \ldots, A[min], \ldots, A[n\text{-}1]$$

in their final positions

Example: 7 3 2 5

Analysis of Selection Sort

ALGORITHM *SelectionSort*$(A[0..n-1])$

```
//Sorts a given array by selection sort
//Input: An array A[0..n − 1] of orderable elements
//Output: Array A[0..n − 1] sorted in ascending order
for i ← 0 to n − 2 do
    min ← i
    for j ← i + 1 to n − 1 do
        if A[j] < A[min]   min ← j
    swap A[i] and A[min]
```

Time efficiency: **Θ(*n*^2)**

Space efficiency: **Θ(*1*), so in place**

Stability: **yes**

Brute-Force String Matching

- *pattern*: a string of *m* characters to search for
- *text*: a (longer) string of *n* characters to search in
- problem: find a substring in the text that matches the pattern

Brute-force algorithm

Step 1 Align pattern at beginning of text

Step 2 Moving from left to right, compare each character of pattern to the corresponding character in text until

- all characters are found to match (successful search); or
- a mismatch is detected

Step 3 While pattern is not found and the text is not yet exhausted, realign pattern one position to the right and repeat Step 2

Examples of Brute-Force String Matching

1. Pattern: 001011

   Text: 10010101101001100101111010

2. Pattern: happy

   Text: It is never too late to have a happy childhood

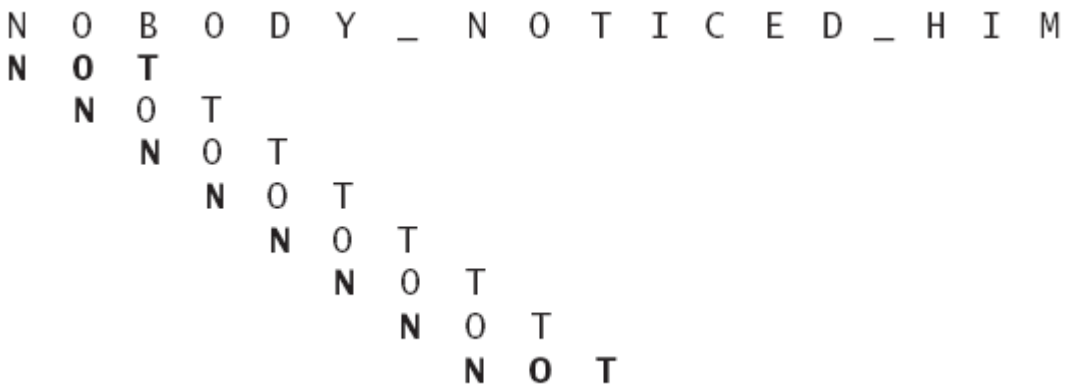
Pseudocode and Efficiency

ALGORITHM *BruteForceStringMatch*$(T[0..n-1], P[0..m-1])$

```
//Implements brute-force string matching
//Input: An array T[0..n − 1] of n characters representing a text and
//       an array P[0..m − 1] of m characters representing a pattern
//Output: The index of the first character in the text that starts a
//        matching substring or −1 if the search is unsuccessful
for i ← 0 to n − m do
    j ← 0
    while j < m and P[j] = T[i + j] do
        j ← j + 1
    if j = m return i
return −1
```

Time efficiency: **Θ(mn) comparisons (in the worst case)**

Why?


N O B O D Y _ N O T I C E D _ H I M
N O T
N O T
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N O T
N O T

Brute-Force Polynomial Evaluation

Problem: Find the value of polynomial

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x^1 + a_0$$

at a point $x = x_0$

Brute-force algorithm

```
p ← 0.0
for i ← n downto 0 do
    power ← 1
        for j ← 1 to i do    //compute x^i
            power ← power * x
        p ← p + a[i] * power
return p
```

Efficiency:

$\Sigma_{0 \le i \le n}\, i = \Theta(n^2)$ multiplications

Polynomial Evaluation: Improvement

We can do better by evaluating from right to left:

Better brute-force algorithm

$p \leftarrow a[0]$
$power \leftarrow 1$
for $i \leftarrow 1$ **to** n **do**
    $power \leftarrow power * x$
    $p \leftarrow p + a[i] * power$
return p

Efficiency: **$\Theta(n)$ multiplications**

Horner's Rule is another linear time method.

Closest-Pair Problem

Find the two closest points in a set of *n* points (in the two-dimensional Cartesian plane).

Brute-force algorithm

Compute the distance between every pair of distinct points

and return the indexes of the points for which the distance is the smallest.

For simplicity, we consider the two-dimensional case of the closest-pair problem. We assume that the points in question are specified in a standard fashion by their (x, y) Cartesian coordinates and that the distance between two points $p_i(x_i, y_i)$ and $p_j(x_j, y_j)$ is the standard Euclidean distance

$$d(p_i, p_j) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}.$$

The brute-force approach to solving this problem leads to the following obvious algorithm: compute the distance between each pair of distinct points and find a pair with the smallest distance. Of course, we do not want to compute the distance between the same pair of points twice. To avoid doing so, we consider only the pairs of points (p_i, p_j) for which $i < j$.

Closest-Pair Brute-Force Algorithm (cont.)

ALGORITHM *BruteForceClosestPoints*(P)

//Input: A list P of n $(n \geq 2)$ points $P_1 = (x_1, y_1), \ldots, P_n = (x_n, y_n)$
//Output: Indices *index*1 and *index*2 of the closest pair of points
$dmin \leftarrow \infty$
for $i \leftarrow 1$ **to** $n - 1$ **do**
    for $j \leftarrow i + 1$ **to** n **do**
        $d \leftarrow sqrt((x_i - x_j)^2 + (y_i - y_j)^2)$ //*sqrt* is the square root function
        if $d < dmin$
            $dmin \leftarrow d$; $index1 \leftarrow i$; $index2 \leftarrow j$
return *index*1, *index*2

Efficiency: **Θ(n^2) multiplications (or sqrt)**

How to make it faster? Using divide-and-conquer!

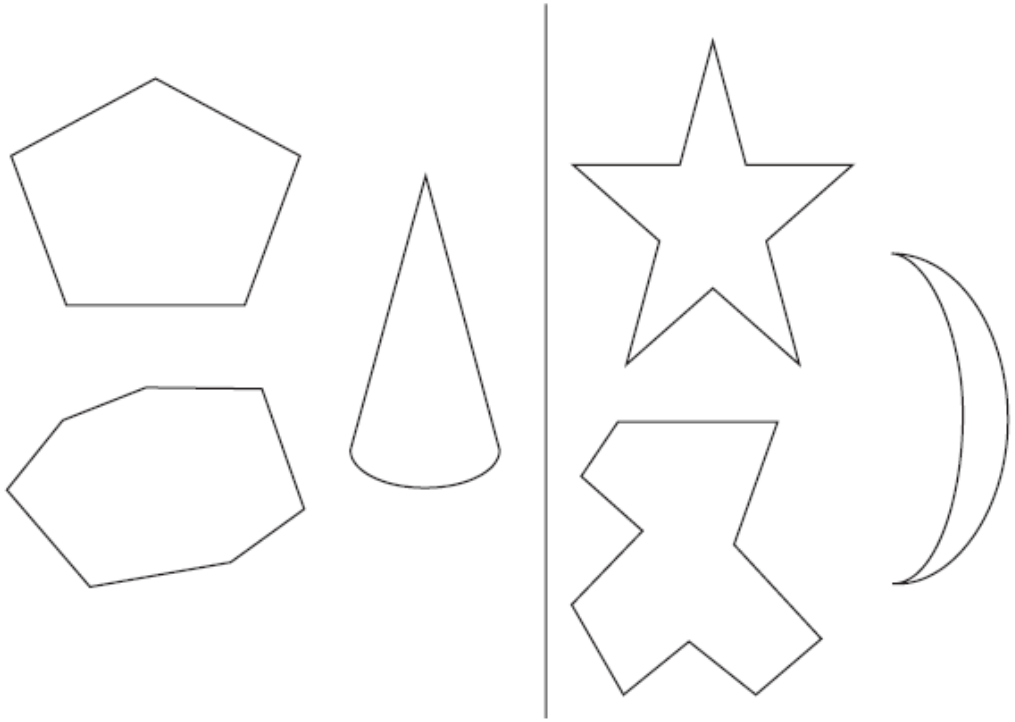
Then the basic operation of the algorithm will be squaring a number. The number of times it will be executed can be computed as follows:

$$C(n) = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} 2 = 2 \sum_{i=1}^{n-1} (n-i)$$

$$= 2[(n-1) + (n-2) + \cdots + 1] = (n-1)n \in \Theta(n^2).$$

Convex Hull Problem

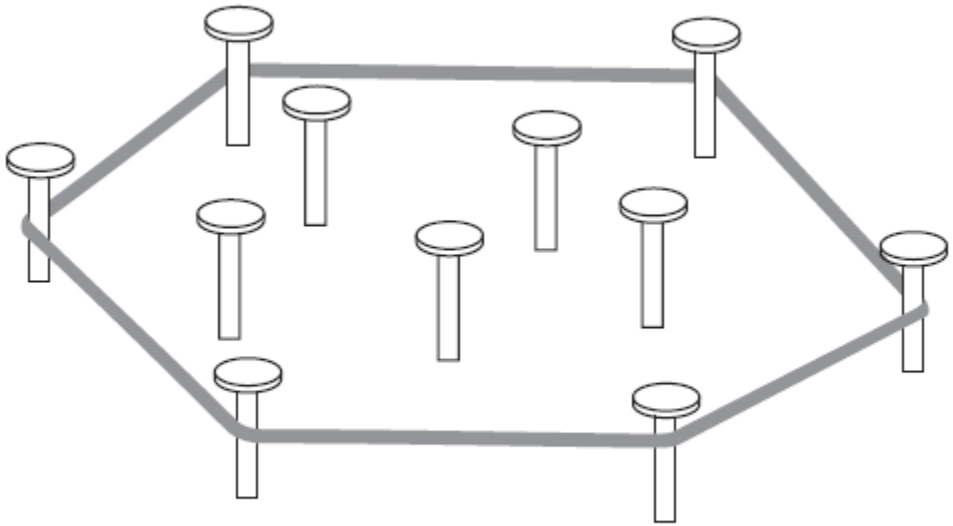
DEFINITION A set of points (finite or infinite) in the plane is called ***convex*** if for any two points p and q in the set, the entire line segment with the endpoints at p and q belongs to the set.



(a) (b)

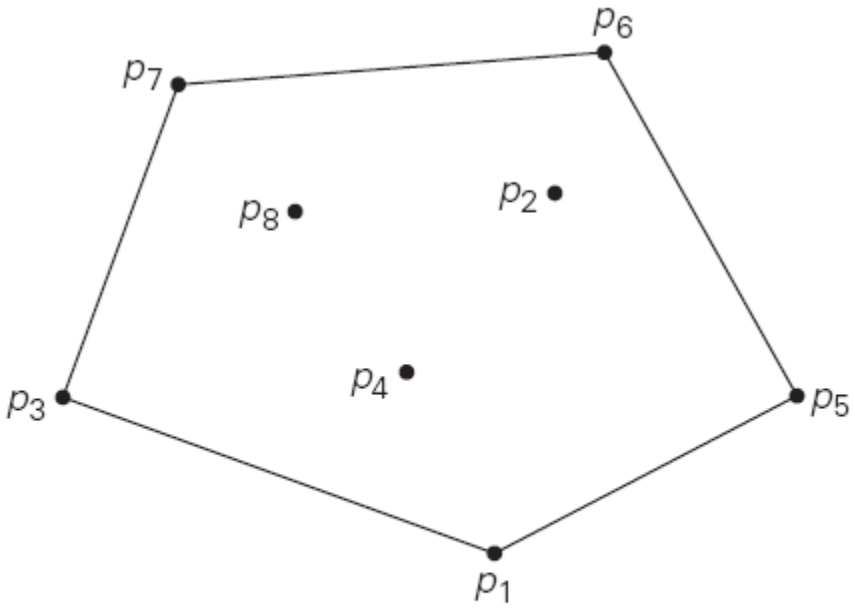
(a) Convex sets. (b) Sets that are not convex.

DEFINITION The *convex hull* of a set S of points is the smallest convex set containing S. (The "smallest" requirement means that the convex hull of S must be a subset of any convex set containing S.)



Rubber-band interpretation of the convex hull.

THEOREM The convex hull of any set S of $n > 2$ points not all on the same line is a convex polygon with the vertices at some of the points of S. (If all the points do lie on the same line, the polygon degenerates to a line segment but still with the endpoints at two points of S.)



The convex hull for this set of eight points is the convex polygon with vertices at p_1, p_5, p_6, p_7, and p_3.

- The *convex-hull problem is the problem of constructing the convex hull for* a given set *S of n points*
- To solve it, we need to find the points that will serve as the vertices of the polygon in question.
- Mathematicians call the vertices of such a polygon “extreme points.”
- By definition, an *extreme point of a convex set is a* point of this set that is not a middle point of any line segment with endpoints in the set.

- how can we solve the convex-hull problem in a brute-force manner?
- Nevertheless, there is a simple but inefficient algorithm that is based on the following observation about line segments making up the boundary of a convex hull
- a line segment connecting two points *pi and pj of a set of n points is a part of the convex hull's boundary if and* only if all the other points of the set lie on the same side of the straight line through these two points

A few elementary facts from analytical geometry are needed to implement this algorithm. First, the straight line through two points (x_1, y_1), (x_2, y_2) in the coordinate plane can be defined by the equation

$$ax + by = c,$$

where $a = y_2 - y_1, b = x_1 - x_2, c = x_1y_2 - y_1x_2$.

Second, such a line divides the plane into two half-planes: for all the points in one of them, $ax + by > c$, while for all the points in the other, $ax + by < c$. (For the points on the line itself, of course, $ax + by = c$.) Thus, to check whether certain points lie on the same side of the line, we can simply check whether the expression $ax + by - c$ has the same sign for each of these points. We leave the implementation details as an exercise.

Brute-Force Strengths and Weaknesses

- Strengths
  - wide applicability
  - simplicity
  - yields reasonable algorithms for some important problems (e.g., matrix multiplication, sorting, searching, string matching)

- Weaknesses
  - rarely yields efficient algorithms
  - some brute-force algorithms are unacceptably slow
  - not as constructive as some other design techniques

Exhaustive Search

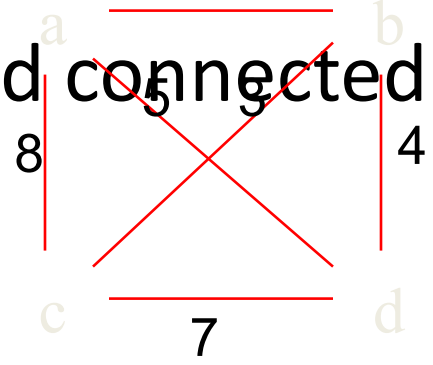
A brute force solution to a problem involving search for an element with a special property, usually among combinatorial objects such as permutations, combinations, or subsets of a set.

Method:

- generate a list of all potential solutions to the problem in a systematic manner (see algorithms in Sec. 5.4)
- evaluate potential solutions one by one, disqualifying infeasible ones and, for an optimization problem, keeping track of the best one found so far
- when search ends, announce the solution(s) found

Example 1: Traveling Salesman Problem

- Given *n* cities with known distances between each pair, find the shortest tour that passes through all the cities exactly once before returning to the starting city
- Alternatively: Find shortest *Hamiltonian circuit* in a weighted connected graph
- Example:



How do we represent a solution (Hamiltonian circuit)?

TSP by Exhaustive Search

Tour	Cost_____
a→b→c→d→a	2+3+7+5 = 17
a→b→d→c→a	2+4+7+8 = 21
a→c→b→d→a	8+3+4+5 = 20
a→c→d→b→a	8+7+4+2 = 21
a→d→b→c→a	5+4+3+8 = 20
a→d→c→b→a	5+7+3+2 = 17

$\Theta((n-1)!)$

Efficiency:

Chapter 5 discusses how to generate permutations fast.

Example 2: Knapsack Problem

Given *n* items:

- weights: w_1 w_2 … w_n
- values: v_1 v_2 … v_n
- a knapsack of capacity *W*

Find most valuable subset of the items that fit into the knapsack

Example: Knapsack capacity W=16

item	weight	value
1	2	$20
2	5	$30
3	10	$50
4	5	$10

Knapsack Problem by Exhaustive Search

Subset	Total weight	Total value
{1}	2	$20
{2}	5	$30
{3}	10	$50
{4}	5	$10
{1,2}	7	$50
{1,3}	12	$70
{1,4}	7	$30
{2,3}	15	$80
{2,4}	10	$40
{3,4}	15	$60
{1,2,3}	17	not feasible
{1,2,4}	12	$60
{1,3,4}	17	not feasible
{2,3,4}	20	not feasible
{1,2,3,4}	22	not feasible

Efficiency: Θ(2^n)

Each subset can be represented by a binary string (bit vector, Ch 5).

Example 3: The Assignment Problem

There are *n* people who need to be assigned to *n* jobs, one person per job. The cost of assigning person *i* to job *j* is C[*i*,*j*]. Find an assignment that minimizes the total cost.

	Job 0	Job 1	Job 2	Job 3
Person 0	9	2	7	8
Person 1	6	4	3	7
Person 2	5	8	1	8
Person 3	7	6	9	4

Algorithmic Plan: Generate all legitimate assignments, compute their costs, and select the cheapest one.

How many assignments are there?

Pose the problem as one about a cost matrix:

Assignment Problem by Exhaustive Search

$$C = \begin{matrix} 9 & 2 & 7 & 8 \\ 6 & 4 & 3 & 7 \\ 5 & 8 & 1 & 8 \\ 7 & 6 & 9 & 4 \end{matrix}$$

Assignment (col.#s)	Total Cost
1, 2, 3, 4	9+4+1+4=18
1, 2, 4, 3	9+4+8+9=30
1, 3, 2, 4	9+3+8+4=24
1, 3, 4, 2	9+3+8+6=26
1, 4, 2, 3	9+7+8+9=33
1, 4, 3, 2	9+7+1+6=23
etc.	

(For this particular instance, the optimal assignment can be found by exploiting the specific features of the number given. It is:

2,1,3,4

Final Comments on Exhaustive Search

- Exhaustive-search algorithms run in a realistic amount of time only on very small instances

- In some cases, there are much better alternatives!
  - Euler circuits
  - shortest paths
  - minimum spanning tree
  - assignment problem

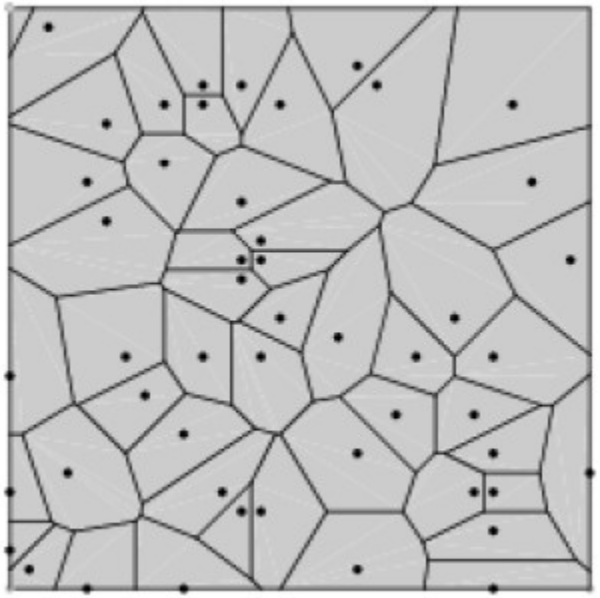
The Hungarian method runs in O(n^3) time.

- In many cases, exhaustive search or its variation is the only known way to get exact solution

Vornoi diagram

The partitioning of a plane with points into convex polygons such that each polygon contains exactly one generating point and every point in a given polygon is closer to its generating point than to any other.

A Voronoi diagram is sometimes also known as a Dirichlet tessellation. The cells are called Dirichlet regions, Thiessen polytopes, or Voronoi polygons.



- Voronoi diagrams were considered as early at 1644 by René Descartes and were used by Dirichlet (1850) in the investigation of positive quadratic forms.

- They were also studied by Voronoi (1907), who extended the investigation of Voronoi diagrams to higher dimensions.

- They find widespread applications in areas such as computer graphics, epidemiology, geophysics, and meteorology