Linear Recurrence relations with Constant Coefficients

A linear recurrence relation of degree k with constant coefficients is a recurrence relation of the form

$$c_0 a_n + c_1 a_{n-1} + c_2 a_{n-2} + \dots + c_k a_{n-k} = R(n)$$

Consider the constant $c_0 c_0 \neq 0$

where $c_0, c_1, c_2, \dots, c_k$ are real numbers and $c_0, c_k \neq 0$

Non- home -> R(n) +0

Classify fle following R.R. as Linear/Non-linear,

Homo/Non-homo, Constant/Vanash. If it is lingary find defice.

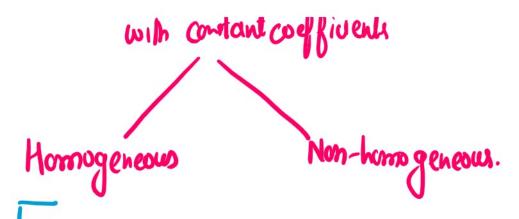
$$0_{n} = a_{n-1} + 2a_{n-3}$$

$$0_{n} - a_{n-1} - 2a_{n-3} = 0$$

Linear Kecuprenu relations

with contant coefficients

Degree = n-(n-3) = 3



Shift Operator:

$$E(a_n) = a_{n+1}$$
, $E^2(a_n) = a_{n+2}$
 $E(a_{n+1}) = a_n$, $E^k(a_{n+1}) = a_n$

Forward Difference Operator:

$$\triangle (a_n) = a_{n+1} - a_n$$

$$\triangle (a_n) = E(a_n) - a_n$$

$$\triangle (a_n) = (E-1)a_n$$

$$\triangle = E-1 \quad \text{or} \quad E = 1+\Delta$$

Linear Homogeneous R.R. with constant coefficient

$$C_0Q_{n+}C_1Q_{n-1}+...+C_kQ_{n-k}=0$$
 $C_0,C_1,...,C_k \rightarrow real no., C_0,C_k \neq 0$
 $Degree=k$

 $a_0 a_{ntk} + a_1 a_{ntk-1} + \dots + a_k a_n = 0$ Rules to solve

1) Make least inder as n.

(2) Represent R.R. in the form of shift operator E.

3) Find auxilliary roots.

Care I: If roots are real and distinct.

 $a_n = \beta_1(x_1)^n + \beta_2(x_2)^n + \dots + \beta_k(x_k)^n$

Cave II. If nots are verland repeated

021, org, org, org, ry, ... Jek-6

 $a_n = \beta_1 x_1^n + (\beta_2 + \beta_3 n) x_2^n + (\beta_4 + \beta_5 n + \beta_6 n^3) x_3^n$

+ . Bx (2x-6)

Cave III If roots are complex. atib=

n = Pn (B connot Bosmno)

Q₁₀ =
$$R^{11}$$
 | R^{11} | R

Try solving the recurrence relation of Tower of Hanoi problem using its initial conditions.

Q13. Solve
$$y_{n+3} - 2y_{n+2} - 5y_{n+1} + 6y_n = 0$$

Q14. Solve
$$y_{n+3} + 3y_{n+2} + 3y_{n+1} + y_n = 0$$
, $y_0 = 1$, $y_1 = -2$, $y_2 = -1$
Q11. Solve $y_n - 6y_{n-1} + 9y_{n-2} = 0$, $y_0 = 1$, $y_1 = 6$

Q12. Solve
$$y_n + 2y_{n-1} + 4y_{n-2} = 0$$