MTH 302 (Probability and Statistics)

Random experiment

A coin is tossed $\{H, T\}$

Dice $\{1, 2, 3, 4, 5, 6\}$

Sample space:

69 students, equally likely

2 students randomly

3 dice   Sample space

$(1, 1, 1), (1, 1, 2)$

$n(S) =$

$6 \times 6 \times 6 = 6^3 = 216$

Example 2: Two balls are to be drawn simultaneously from a set of 3 red and 2 white balls. Find the sample space (Find the $n(s)$)

(a) 6   (b) 10   (c) 5   (d) none of these

$\{R, R, R, W, W\}$

$n(S) = {}^5C_2$

$= \dfrac{\lfloor 5}{\lfloor 2 \lfloor 3}$

$= \dfrac{5 \times 4}{2 \times 1} = 10$

$S = \{1, 2, 3, 4, 5, 6\}$

$E_1 = \{1, 3, 5\}$, $E_3 = \{2, 3, 5\}$

$E_2 = \{2, 4, 6\}$

Event is a subset of sample space

Mutually exclusive events

$E_1 \cap E_2 = \phi$

Mutually exclusive and exhaustive

$$E_1 \cap E_2 = \phi \ , \quad E_1 \cup E_2 = S \qquad \text{erents}$$

Tossing of two coins

$$S = \{ HH, \ HT, \ TH, \ TT \}$$

HH $\longrightarrow$

one H and one tail

Sure event ✓ $\{ 1, 2, 3, 4, 5, 6 \}$ $\quad E \subseteq S$

Impossible event "7" $\longrightarrow$

S

H H H	E: is the event of getting
✓H H T ✓	2 heads and one tail
✓H T H ✓	
H T T	HHT
✓T H H ✓	is a compatible event
T H T	
T T H	G: of getting 3 heads
T T T	HHH elementary event

Random exp, Sample space, Event, MEE, ME and E
Sure event, Impossible

$$\{ 1, 2, 3, 4, 5, 6 \}$$
$$E_1 = \{ 2, 3, 4 \} \qquad E_1 \cap E_2 \neq \phi$$
$$E_2 = \{ 1, 2, 3, 5 \} \qquad E_1 \cup E_2 \neq S$$

$$P(E) = \frac{\text{Number of elements in } E}{\text{Number of elements in } S} = \frac{n(E)}{n(S)}$$

$$P(\phi) = 0 \ , \qquad P(S) = 1$$

$$0 \leq P(E) \leq 1 \quad \ast \qquad\qquad P(G) = 437$$

1) Three coins are tossed simultaneously.
what is the probability that at least
two tails occur?

$$n(S) = 8$$

$$E = \{ TTH, THT, HTT, TTT \}$$

$$n(E) = 4$$

$$P(E) = \frac{4}{8} = \frac{1}{2}$$

Example: Two dice with faces marked 1, 2, 3, 4, 5, 6 are thrown simultaneously and the points on the dice are multiplied together. Find the probability that the product is 12.

(a) $\frac{1}{4}$ (b) $\frac{1}{6}$ (c) $\frac{1}{9}$ ✓   (d) $\frac{1}{8}$

$$2, 4$$
$$\boxed{18}$$

$$n(s) = 36$$

$$E = \{ (2, 6), (3, 4), (4, 3), (6, 2) \}$$

$$P(E) = \frac{n(E)}{n(s)} = \frac{4}{36} = \frac{1}{9}$$

Example: A batch contains 10 articles of which 4 are defective. If 3 articles are chosen at random, what is the probability that none of them is defective?

$$0 \leq P(E) \leq 1$$
$$206$$

$$10 \quad , \quad 4D$$

$$\begin{array}{|ccc|} \hline \checkmark & \checkmark & \checkmark \\ \checkmark & \checkmark & \checkmark \\ \times & \times & \times \\ \hline \end{array}$$

$$n(s) = {}^{10}C_3 = \frac{\underline{10}}{\underline{3} \, \underline{7}}$$

$$= \frac{10 \times 9 \times 8 \times 4}{3 \times 2 \times 1}$$

$$= \frac{10 \times 9 \times 8}{1 \times 2 \times 3}$$

$$= 120$$

$$n(E) = {}^{6}C_3 = \frac{6 \times 5 \times 4}{3 \times 2 \times 1} = 20$$

$$P(E) = \frac{20}{120} = \frac{1}{6} \checkmark$$

12-01-22

Example: What is the probability that all 3 children in a family have different birthdays?
(Assume, 1 year = 365 days)

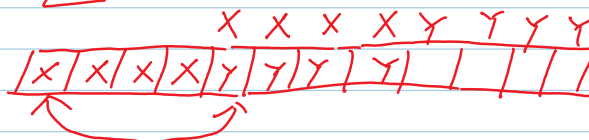Sol.: $n(S) = 365 \times 365 \times 365$ 1 child

$n(E) = 365 \times 364 \times 363$

$P(E) = \dfrac{365 \times 364 \times 363}{365 \times 365 \times 365} = \underline{\quad}$

X and Y stand in a line at random with 10 other people. What is the probability that there are 3 people between X and Y?   X Y , 10

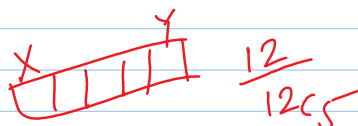Sol.:- $n(S) = \lfloor 12$

X X X X X Y Y Y Y

| X | X | X | X | Y | Y | Y | Y | | | |

$12c_5$ $\lfloor 12$

$n(E) = 2 \times 8 \times \lfloor 10$

| X | | | | | | $\dfrac{\lfloor 12}{12c_5}$

$P(E) = \dfrac{2 \times 8 \times \lfloor 10}{\lfloor 13} = \dfrac{2 \times 8 \times \lfloor 10^{4}}{6\lfloor 12 \times 11 \times \lfloor 10}_{3} = \dfrac{4}{33}$

Example: If 10 persons are arranged at random $\boxed{\lfloor 12}_5$
(I) in a line   (II) in a ring.
Find the probability that 2 particular persons will be next to each other.

A B
B A

a) $\dfrac{2}{5}$ b) $\dfrac{1}{5}$ c) $\dfrac{3}{5}$ d) don't know

I) $n(S) = \lfloor 10$, $n(E) = \lfloor 2 \times \lfloor 2$ $\boxed{8} + \boxed{\lfloor 2}^{1}$

$\bigcirc$ $\lfloor n-1$ $P(E) = \dfrac{\lfloor 2 \times \lfloor 2}{\lfloor 10} = \dfrac{2}{10}$ $\lfloor 9^{9}$

II) $n(S) = \lfloor 10-1 = \lfloor 9$ $\boxed{\lfloor AB}$

$n(E) = \lfloor 8 \times \lfloor 2$ 8H=S

$P = \lfloor 8 \times \lfloor 2 / \lfloor 9 = \dfrac{2}{9}$ ✓

1. What is the Chance that a leap year selected at random will contain 53 Sundays?

Sol:-

$S = \{(mon, Tue), (Tue, wed), (Wed, Thurs)$
$(Thurs, Fri), (Fri, Sat), (Sat, Sun)$
$(Sun, mon)\}$

$\dfrac{2}{7}$ ✓

361
↳ 52 weeks
$52 \times 7 = 364$

2 days

$\{1, 2, 3, 4\}$
$P = \dfrac{2}{4}$

$P(E) = \dfrac{n(E)}{n(S)}$

2. A bag contains 3 red, 6 white and 7 blue balls. What is the probability that two balls drawn are white and blue?

$3 + 6 + 7 = 16$

$n(S) = 16c_2$

$n(E) = 6c_1 \times 7c_1$

$P = \dfrac{6c_1 \times 7c_1}{16c_2} =$ ✓

$\boxed{15}$ $\boxed{5}$

$\boxed{\lfloor 11 \times \lfloor 5}$

3. What is the probability of getting 9 cards of the same suit in one hand at a game of bridge?

mlq If the letters of the word ATTEMPT are written down at random, the chance that all TS are consecutive is...
(a) 1/42   (b) 6/7   ✓(c) 1/7   (d) 1/8

$n(S) = \dfrac{\lfloor 7}{\lfloor 3}$ ✓

$n(E) = \lfloor 5$

(T)

A
(TTT)
E M P T
A EMP TTT
$4 + 1 = 5$ $\lfloor 5 \times \lfloor 3$ / $\lfloor 3$

TTT
TTT

$P = \dfrac{\dfrac{\lfloor 5}{\lfloor 2}}{\dfrac{\lfloor 7}{\lfloor 3}} = \dfrac{\lfloor 5 \times \lfloor 3}{\lfloor 7}$

$= \dfrac{\lfloor 5 \times 3 \times 2 \times 1}{7 \times 6 \times \lfloor 5}$