Lecture 16

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Q24. Solve $y_n = 6y_{n-1} - 12y_{n-2} + 8y_{n-3} + n^2$

$$(E^3 - 6E^2 + 12E - 8)y_n = (n+3)^2$$

$C.F = (c_1 + c_2 n + c_3 n^2)2^n$

Expected P.I. = $(an^2 + bn + c)$

$$\frac{1}{(E-2)^3}(n+3)^2 \qquad E = 1+\Delta \qquad n^2+6n+9$$

$$\frac{1}{(1+\Delta-2)^3}\left([n]^2+[n]+6[n]+9\right)$$

① ②

$$= \frac{1}{(\Delta-1)^3}\left([n]^2+7[n]+9\right)$$

$(1+x)^n = 1+nx+n(n-1)\frac{x^2}{2!}\ldots$ $\quad -(1-\Delta)^{-3}$

$$-\left(1+3\Delta+(-3)(-3-1)\frac{(-\Delta)^2}{2!}\ldots\right)\left([n]^2+7[n]+9\right)$$

$$= -\left(1+3\Delta+6\Delta^2+\ldots\right)\left([n]^2+7[n]+9\right)$$

$$= -\left([n]^2+7[n]+9 \quad +3(2[n]+7)+6(2)+0\right)$$

$$= -\left([n]^2+13[n]+42\right) = -\left(n(n-1)+13n+42\right)$$

$$= -\left(n^2+12n+42\right)$$

Q25. Solve $(E^2 - 2E + 1)y_n = 2^n n^2 - 1$

$m = 1, 1 \qquad (c_1 + c_2 n)(1)^n$

$2^n(an^2+bn+c)$
$+ d(1)^n \cdot n^2$

P.I.

P.I.

$+\,d(1)^n\cdot n^2$

$$\frac{1}{(E-1)^2}2^n\cdot n^2 - \frac{1}{(E-1)^2}(1)^n \xrightarrow{\text{Put } E=1} [n]\ \frac{1}{2(E-1)}(1)^{n-1}$$

Put E=1

$$2^n\frac{1}{(2E-1)^2}n^2 - [n]^2\frac{1}{2!}(1)^{n-2}$$

$$2^n\frac{1}{\left(2(1+\Delta)-1\right)^2}\left([n]^2+[n]\right) - \frac{n(n-1)}{2}$$

Complete.

Q26. $y_n = 8y_{n-2} - 16y_{n-4} + (-2)^n n^3$

$$y_{n+4} - 8y_{n+2} + 16y_n = (-2)^{n+4}(n+4)^3$$

$$(E^4 - 8E^2 + 16)y_n = 16(-2)^n(n+4)^3$$

Expected

$n^2(-2)^n(an^3+bn^2+cn+d)$

C.F. $m^4 - 8m^2 + 16 = 0 \quad m = 2, 2, -2, -2$

$$(c_1 + c_2 n)(2)^n + (c_3 + c_4 n)(-2)^n$$

$$\frac{1}{(E^4-8E^2+16)}16(-2)^n(n+4)^3$$

$$\frac{1}{(E-2)^2(E+2)^2}16(-2)^n(n+4)^3$$

$$\frac{1}{(E-2)^2(E+2)^2} \quad 16(-2)^n(n+4)^3$$

$$16\cdot(-2)^n \frac{1}{(-2E-2)^2(-2E+2)^2}(n+4)^3 = (-2)^n\frac{1}{(E-1)^2(E+1)^2}(n+4)^3$$

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$$n^3+12n^2+48n+64$$

$$(-2)^n \frac{1}{(1+\Delta-1)^2(1+\Delta+1)^2}\left([n]^3+3[n]^2+[n] + 12([n]^2+[n]) + 48[n]+64\right)$$

$$(-2)^n \frac{1}{\Delta^2}\,\frac{1}{(\Delta+2)^2}\left(\quad '' \quad\right)$$

$$(-2)^n \frac{1}{\Delta^2}\,\frac{1}{4}\left(1+\frac{\Delta}{2}\right)^{-2}\left(\quad '' \quad\right)$$

integrate twice

Three degree polynomial

Suppose that $\{a_n\}$ satisfies the linear nonhomogeneous recurrence relation

$$a_n = c_1a_{n-1} + c_2a_{n-2} + \cdots + c_ka_{n-k} + F(n),$$

where $c_1, c_2, \ldots, c_k$ are real numbers, and

$$F(n) = (b_tn^t + b_{t-1}n^{t-1} + \cdots + b_1n + b_0)s^n,$$

where $b_0, b_1, \ldots, b_t$ and s are real numbers. When s is not a root of the characteristic equation of the associated linear homogeneous recurrence relation, there is a particular solution of the form

$$(p_tn^t + p_{t-1}n^{t-1} + \cdots + p_1n + p_0)s^n.$$

When s is a root of this characteristic equation and its multiplicity is m, there is a particular

$$(p_t n^t + p_{t-1} n^{t-1} + \cdots + p_1 n + p_0) s^n.$$

When s is a root of this characteristic equation and its multiplicity is m, there is a particular solution of the form

$$n^m (p_t n^t + p_{t-1} n^{t-1} + \cdots + p_1 n + p_0) s^n.$$

Generating Function

The *generating function for the sequence* $a_0, a_1, \ldots, a_k, \ldots$ of real numbers is the infinite series

$$G(x) = a_0 + a_1 x + \cdots + a_k x^k + \cdots = \sum_{k=0}^{\infty} a_k x^k.$$

$a_n = \{1\}$

$1, 1, 1, 1, \ldots$

$$G(x) = 1 \cdot x^0 + 1 \cdot x^1 + 1 \cdot x^2 + 1 \cdot x^3 + \cdots$$

$$= 1 + x + x^2 + x^3 + \cdots$$

$$G(x) = \left(\frac{1}{1-x}\right) = (1-x)^{-1}$$

$1, -1, 1, -1, 1, -1, 1, \ldots$

$$G(x) = 1 - x + x^2 - x^3 + x^4 \ldots$$

$$G(x) = (1+x)^{-1}$$

TABLE 1 Useful Generating Functions.	
$G(x)$	a_k
$(1+x)^n = \sum_{k=0}^{n} C(n,k) x^k = 1 + C(n,1)x + C(n,2)x^2 + \cdots + x^n$	$C(n,k)$
$(1+ax)^n = \sum_{k=0}^{n} C(n,k) a^k x^k = 1 + C(n,1)ax + C(n,2)a^2x^2 + \cdots + a^n x^n$	$C(n,k)a^k$
$(1+x^r)^n = \sum_{k=0}^{n} C(n,k) x^{rk}$	$C(n,k/r)$ if $r \mid k$; 0 otherwise

$(1+x^r)^n = \sum_{k=0}^{n} C(n,k)x^{rk} = 1 + C(n,1)x^r + C(n,2)x^{2r} + \cdots + x^{rn}$	$C(n, k/r)$ if $r \mid k$; 0 otherwise
$\frac{1-x^{n+1}}{1-x} = \sum_{k=0}^{n} x^k = 1 + x + x^2 + \cdots + x^n$	1 if $k \leq n$; 0 otherwise
$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k = 1 + x + x^2 + \cdots$	1
$\frac{1}{1-ax} = \sum_{k=0}^{\infty} a^k x^k = 1 + ax + a^2x^2 + \cdots$	a^k
$\frac{1}{1-x^r} = \sum_{k=0}^{\infty} x^{rk} = 1 + x^r + x^{2r} + \cdots$	1 if $r \mid k$; 0 otherwise
$\frac{1}{(1-x)^2} = \sum_{k=0}^{\infty} (k+1)x^k = 1 + 2x + 3x^2 + \cdots$	$k+1$
$\frac{1}{(1-x)^n} = \sum_{k=0}^{\infty} C(n+k-1,k)x^k = 1 + C(n,1)x + C(n+1,2)x^2 + \cdots$	$C(n+k-1,k) = C(n+k-1,n-1)$