Lecture 15

24 September 2021 08:59

Q22. $y_n - 7y_{n-1} + 10y_{n-2} = 7(3)^n + e^{2n} + 5^n$

$y_{n+2} - 7y_{n+1} + 10y_n = 63\cdot(3)^n + e^4\cdot e^{2n} + 25\cdot(5)^n$

$(E^2 - 7E + 10)y_n = 63\cdot(3)^n + e^4\cdot e^{2n} + 25\cdot(5)^n$

$m = 2, 5$ $\quad$ C.F $= C_1(2)^n + C_2(5)^n$

P.I.

$$\frac{1}{E^2-7E+10} 63(3)^n \xrightarrow{\text{Put } E=3} -\frac{63}{2}(3)^n$$

$$+ \frac{1}{E^2-7E+10} e^4\cdot e^{2n} \xrightarrow{\text{Put } E=e^2} \frac{e^4\, e^{2n}}{e^4-7e^2+10}$$

$$+ \frac{1}{E^2-7E+10} 25\cdot(5)^n \xrightarrow[\text{Case fails}]{\text{Put } E=5} [n]\frac{1}{2E-7} 25\cdot(5)^{n-1}$$

Put $E=5$

$$\frac{5n(5)^n}{3} = [n]\frac{1}{3} 25\cdot(5)^{n-1}$$

$$y_n = C_1(2)^n + C_2(5)^n - \frac{63}{2}(3)^n + \frac{e^4}{e^4-7e^2+10} + \frac{5n(5)^n}{3}$$

Q23. Solve $y_n = 8y_{n-2} - 16y_{n-4} + (-2)^n$

Q23. Solve $y_n = 8y_{n-2} - 16y_{n-4} + (-2)^n$

$n \to n+4$

$y_{n+4} = 8y_{n+2} - 16y_n + (-2)^{n+4}$

$y_{n+4} - 8y_{n+2} + 16y_n = (-2)^{n+4}$

Expected P.S. $= n^2 A(-2)^n$

$(E^4 - 8E^2 + 16)y_n = 16\cdot(-2)^n$

$(m^2)^2 - 8(m^2) + 16 = 0$

$m^2 = 4, 4 \Rightarrow m = \pm 2, \pm 2$

$C.F = (c_1 + c_2 n)(2)^n + (c_3 + c_4 n)(-2)^n$

P.I. $\dfrac{1}{E^4 - 8E^2 + 16} 16(-2)^n \xrightarrow[\text{Case fails}]{\text{Put } E=-2} [n] \dfrac{1}{4E^3 - 16E} 16(-2)^{n-1}$

Case fails $\downarrow$ Put $E=-2$

$[n]^2 \dfrac{1}{32} 16(-2)^{n-2} \xleftarrow{\text{Put } E=-2} [n]^2 \dfrac{1}{12E^2 - 16} 16(-2)^{n-2}$

$\dfrac{n(n-1)(-2)^n}{8}$

$y_n = (c_1 + c_2 n)(2)^n + (c_3 + c_4 n)(-2)^n + \dfrac{n(n-1)(-2)^n}{8}$

Q24. Solve $y_n = 6y_{n-1} - 12y_{n-2} + 8y_{n-3} + n^2$

$$y_{n+3} - 6y_{n+2} + 12y_{n+1} - 8y_n = (n+3)^2$$

$$(E^3 - 6E^2 + 12E - 8)y_n = (n+3)^2$$

Expected P.I.

$an^2 + bn + c$

$$m = 2, 2, 2$$

$$C.F = (c_1 + c_2 n + c_3 n^2)(2)^n$$

Case III: If RHS $= n^m$

$$\frac{1}{f(E)} n^m = \boxed{\frac{1}{f(1+\Delta)}}$$

(change into factorial notation)

↓

Expand using binomial expansion upto Δ^m

$$(1+x)^n = 1 + nx + n(n-1)\frac{x^2}{2!} + n(n-1)(n-2)\frac{x^3}{3!} + \dots$$

$$\Delta([n]^2) = 2[n]$$

$$\Delta^2([n]^4) = 12[n]^2$$

$$n = [n], \quad [n]^2 = n(n-1) = n^2 - n$$

$$[n]^2 + n = n^2 \Rightarrow n^2 = [n]^2 + [n]$$

$$[n]^2+n = n^2 \Rightarrow n^2 = [n]^2+[n]$$

$$[n]^3 = n(n-1)(n-2) = n^3-3n^2+2n$$

$$n^3 = [n]^3+3n^2-2n$$

$$= [n]^3+3\left([n]^2+[n]\right)-2[n]$$

$$= [n]^3+3[n]^2+[n]$$

Case 4: If $R(n) = a^n V(n)$, where $V(n)$ =polynomial in n.

P.I.=$\frac{1}{f(E)}\, a^n V(n) = a^n \frac{1}{f(aE)} V(n)$