Center for Professional Enhancement

Mensuration

Mensuration is the branch of mathematics which deals with the study of Geometric shapes, their area, volume and related parameters.

Learning Outcomes

Through this Topic Candidates should be able to

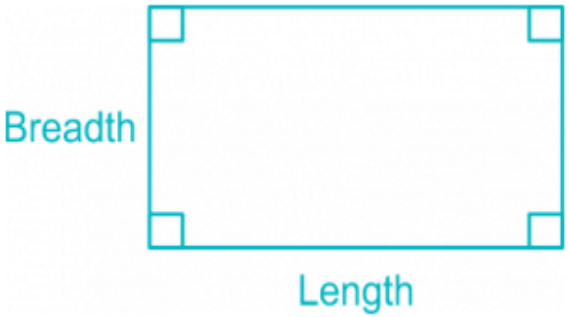
- understand the basic concepts of mensuration
- observe the data given and interpret it from the given problem
- apply the Concepts to solve Company Specific Aptitude tests
- apply the Concepts to solve questions asked in various government exams like SSC CGL, Banking Sector etc.

Contents

- **2D Figures: Area, Perimeter.**
- **3D Figures: Volume, Curved surface Area and Total surface Area.**

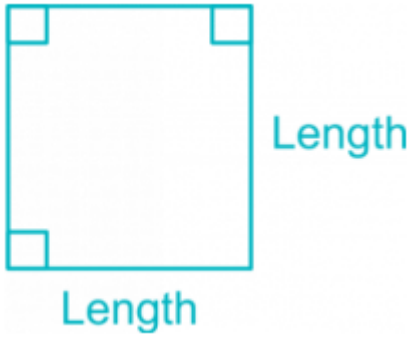
2D Figures	3D Figures
Rectangle	Cuboid
Square	Cube
Triangle	Cylinder
Circle	Cone
Parallelogram	Sphere
Trapezium	Hemisphere

2D Figures: Rectangle



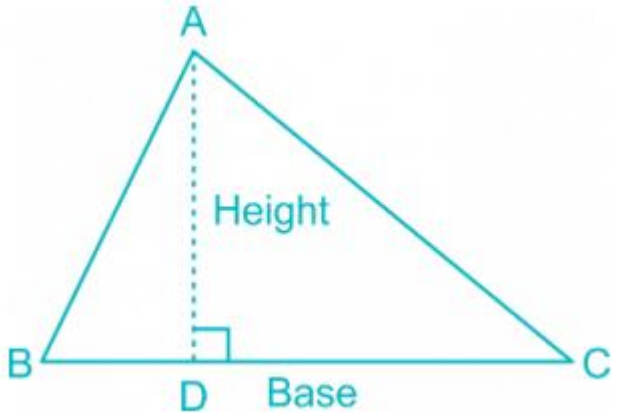
- Area of Rectangle = Length × Breadth.
- Perimeter of a Rectangle = 2 × (Length + Breadth)
- Length of the Diagonal = $\sqrt{(\text{Length}^2 + \text{Breadth}^2)}$

Square



- Area of a Square = Length × Length = $(\text{Length})^2$
- Perimeter of a square = 4 × Length
- Length of the Diagonal = $\sqrt{2}$ × Length

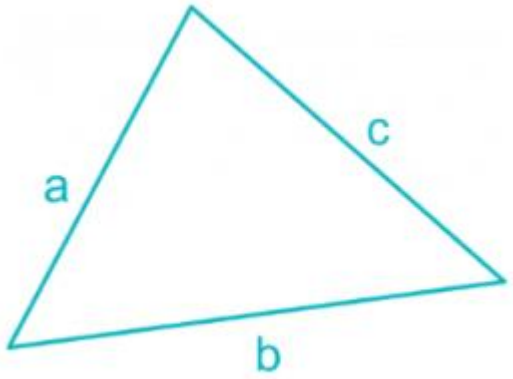
Triangle



- Area of a triangle=(1/2)(Base × Height)=(1/2)(BC×AD)

Triangle

For a triangle with sides measuring a, b and c, respectively:



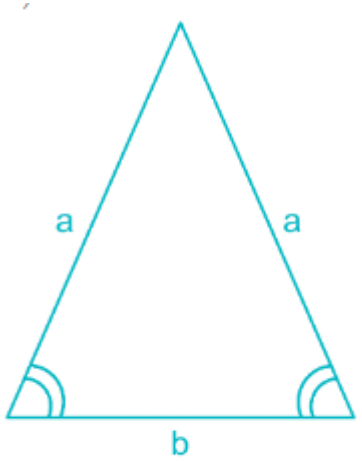
- Perimeter = a + b + c
- s = semi perimeter = perimeter/2 = (a+b+c)/2
- Area of Triangle, A=$\sqrt{s(s-a)(s-b)(s-c)}$

  (This is also known as "Heron's formula")

Isosceles Triangle

- Area of Isosceles Triangle: $\frac{b}{4}\sqrt{4a^2 - b^2}$

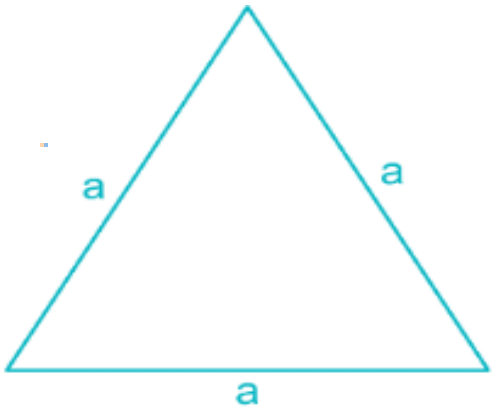
(Where a = length of two equal side, b = length of base of isosceles triangle.)



Isosceles Triangle

Equilateral Triangle

- Area of Equilateral Triangle : $\frac{\sqrt{3}}{4} \times a^2$

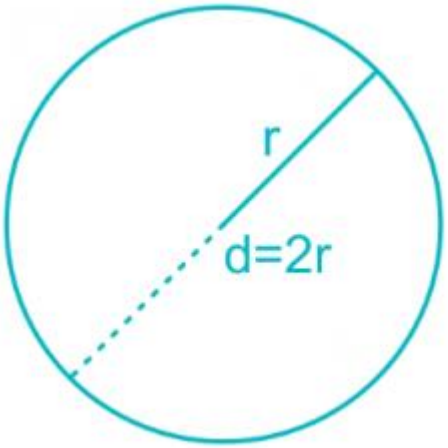


Equilateral Triangle

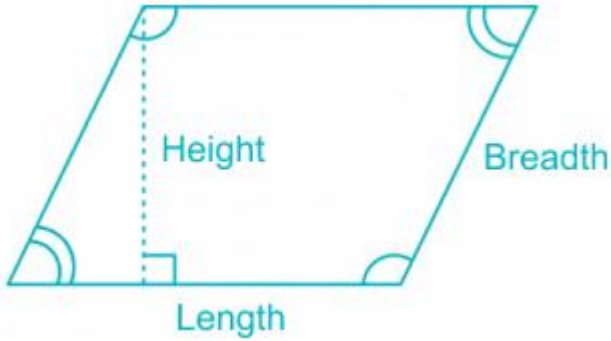
Circle & Semicircle

- Area of a circle = $\pi r^2 = \pi d^2/4$
- Circumference of a circle = $2\pi r = \pi d$
- Circumference of a semicircle = $\pi r + 2r$
- Area of semicircle = $\pi r^2/2$

(In the following formulae, r = radius and d = diameter of the circle)

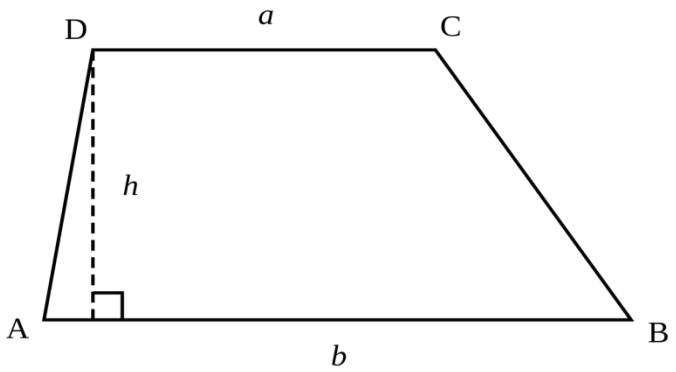


Parallelogram

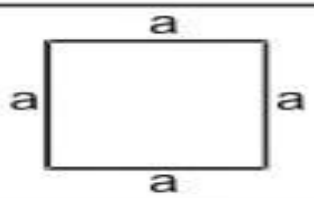
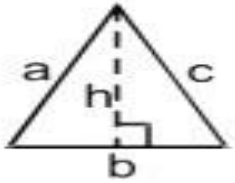
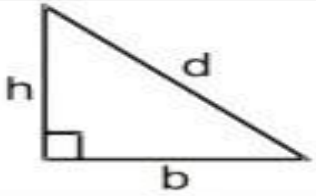
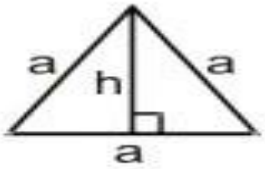
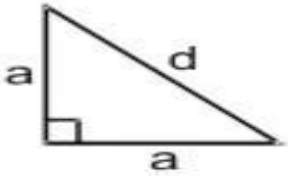


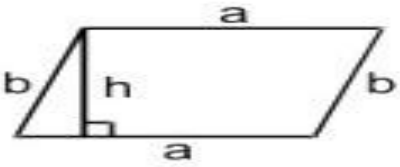
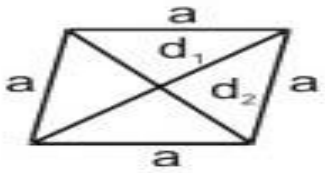
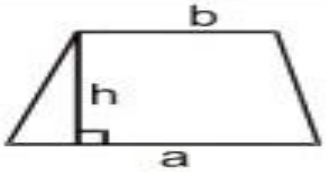
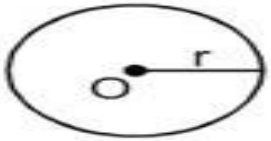
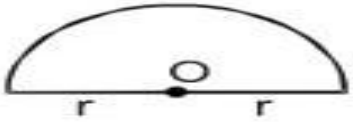
- Area of a Parallelogram = Length × Height
- Perimeter of a Parallelogram = 2 × (Length + Breadth)

Trapezium



- Area of a trapezium =(1/2)× sum of parallel sides × distance between parallel sides

Name	Figure	Perimeter	Area
Rectangle		2 (a + b)	ab
Square		4a	a^2
Triangle		a + b + c = 2s	$1 = \frac{1}{2} \times b \times h$ $2 = \sqrt{s(s-a)(s-b)(s-c)}$
Right triangle		b + h + d	$\frac{1}{2}$ bh
Equilateral triangle		3a	1. $\frac{1}{2}$ ah 2. $\frac{\sqrt{3}}{4} a^2$
Isosceles right triangle		2a + d	$\frac{1}{2} a^2$

Parallelogram		2 (a + b)	ah
Rhombus		4a	$\frac{1}{2} d_1 d_2$
Trapezium		Sum of its four sides	$\frac{1}{2}$ h (a + b)
Circle		$2\pi r$	πr^2
Semicircle		$\pi r + 2r$	$\frac{1}{2} \pi r^2$
Ring (shaded region)		----	$\pi (R^2 - r^2)$
Sector of a circle		l + 2r where $l = (\theta/360) \times 2\pi r$	$\theta/360° \times \pi r^2$

Practice Questions:

1. The area of a square is 4096 sq. cm. Find the ratio of the breadth and length of a rectangle whose length is twice the side of the square and breadth is 24 cm less than the side of the square

A. 5:32

B. 7:16

C. 5:16

D. None of these

Ans. C

2. A wire in the form of a circle of radius 3.5 m is bent in the form of a rectangle whose length and breadth in the ratio of 6:5. What is the area of rectangle.

A. 30

B. 60

C. 120

D. None of these

Ans. A

3. The circumference of two circles are 264 m and 352 m. Find the difference between area of the larger and smaller circles.

A. 4123

B. 8642

C. 4312

D. 2612

Ans. C

4. What would be the cost of building 7 m wide garden around a circular field with diameter equal to 280m, if the cost per sq. m for building the garden is Rs. 21

A. Rs. 156242

B. Rs. 248521

C. Rs. 132594

D. None of these

Ans. C

5. A cow is tied on the corner of a rectangular field of size 30m*20m by a 14 m long rope. The area of the region that she can graze is

A. 350 sq. m

B. 196 sq. m

C. 154 sq. m

D. 22 sq. m

Ans. C

6. At each corner of a triangular field of side 26m, 28m and 30m a cow is tethered by a rope of length 7m. The area un-grazed by the cow is

A. 336 sq. m

B. 259 sq. m

C. 154 sq. m

D. 77 sq. m

Ans. B

7. The radius of a circular field is equal to the side of a square field. If the difference between the perimeter of the circular field and that of the square field is 32m, what is the perimeter of the square field?

A. 84m

B. 95m

C. 56m

D. 28m

Ans. C

8. Two equal maximum sized circular plates are cut-off from a circular paper-sheet of circumference 352 cm. The circumference of each circular plate is.

A. 176 cm

B. 180 cm

C. 165 cm

D. 150 cm

Ans. A

9. A wire, when bent in the form of a square, encloses a region having area 121 cm2. If the same wire is bent into the form of a circle, then the area of the circle is?

A. 144 sq. cm

B. 180 sq. cm

C. 154 sq. cm

D. 176 sq. cm

Ans. C

10. Four circles having equal radii are drawn with centres at the four corners of a square. Each circle touches the other two adjacent circle. If remaining area of the square is 168 cm square, what is the size of the radius of the circle?

A. 1.4 cm

B. 14 cm

C. 35 cm

D. 21 cm

Ans. B

11. The area of a triangle is 216 cm^2 and its sides are in the ratio 3 : 4 : 5. The perimeter of the triangle is

A. 6 cm

B. 12 cm

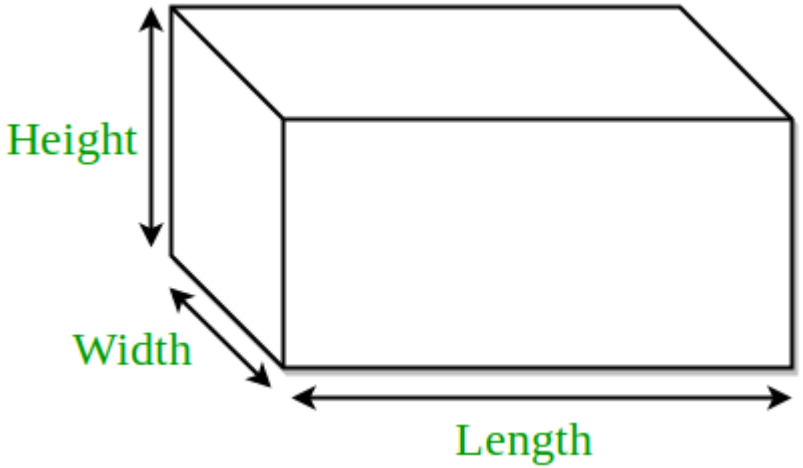
C. 36 cm

D. 72 cm

Ans. D

3D Figures: Cuboid

Let
l= length
b= breadth
h= height. Then,

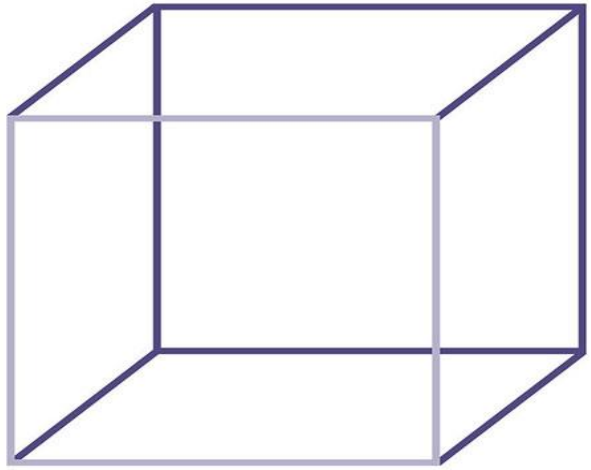


- **Volume** = (l x b x h) cubic units.
- **Lateral Surface area** = 2(b + l) h sq. units.
- **Surface area** = 2(lb + bh + lh) sq. units.
- **Diagonal** = $\sqrt{(l^2 + b^2 + h^2)}$ units.

Cube

Let each edge of a cube be of length a. Then,

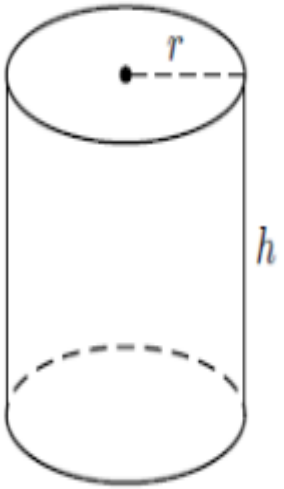
- **Volume** = a^3 cubic units.
- **Lateral Surface area** = $4a^2$ sq. units
- **Surface area** = $6a^2$ sq. units.
- **Diagonal** = $\sqrt{3}a$ units.



Cylinder

Let radius of base = r and Height (or length) = h. Then,

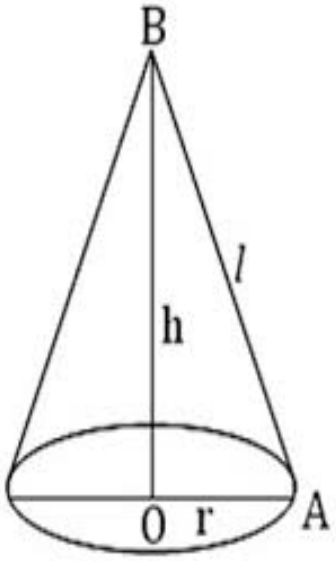
- **Volume** = ($\pi r^2 h$) cubic units.
- **Curved surface area =** ($2\pi r\, h$) sq. units.
- **Total surface area** = $2\ \pi\ r(h + r)$ sq. units.



Cone

Let radius of base = r and Height = h. Then,

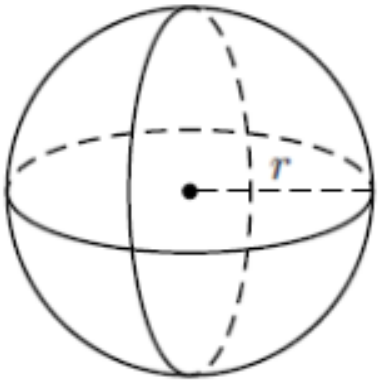
- **Slant height,** $l = \sqrt{h^2 + r^2}$ units.
- **Volume** = (1/3) π r^2h cubic units.
- **Curved surface area** = (π rl) sq. units.
- **Total surface area** = (π rl + π r^2) sq. units.



Sphere

Let the radius of the sphere be r. Then,

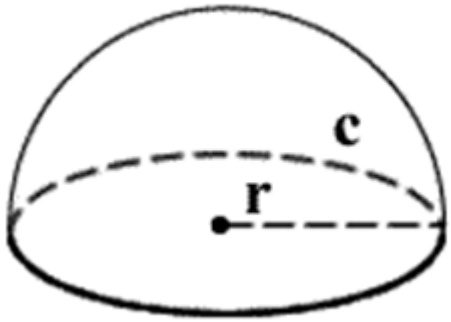
- **Volume** = (4/3) π r^3 cubic units.
- **Surface area** = (4 π r^2) sq. units.



Hemisphere

Let the radius of a hemisphere be *r*. Then,

- **Volume** = (2/3) π r^3 cubic units.
- **Curved surface area** = (2 π r^2) sq. units.
- **Total surface area** = (3 π r^2) sq. units.



Practice Questions:

1. If the areas of the three adjacent faces of a cubical box are 120 cm square, 72 cm square and 60 cm square respectively, then the volume of the box is:

A. 800 cm cube

B. 680 cm cube

C. 700 cm cube

D. 720 cm cube

Ans. D

2. A wooden box measures 20 cm × 12 cm × 10 cm. the thickness of the wood is 1 cm. The volume of the wood required to make the box is:

A. 960 cm cube

B. 900 cm cube

C. 1000 cm cube

D. 1100 cm cube

Ans. A

3. The size of a wooden block is (15 cm × 12 cm × 20 cm). How many such blocks will be required to construct a solid wooden cube of minimum size?

A. 50

B. 40

C. 60

D. 55

Ans. C

Solution

- Side of smallest cube we can form = L.C.M of 15, 12, 20 = 60cm
- Volume of the cube = (60 x 60 x 60)cu.cm = 216000 cu.cm
- Volume of the block= (15 x 12 x 20)cu.cm = 3600 cu.cm
- Number of blocks = (216000/3600) = 60

4. Two solid cylinders of radii 4 cm and 5 cm and lengths 6 cm and 4 cm, respectively are recast into cylindrical disc of thickness 1 cm. The radius of the disc is

A. 7 cm

B. 14 cm

C. 21 cm

D. 28 cm

Ans. B

5. The radius of cross-section of a solid cylindrical rod of iron is 50 cm. The cylinder is melted down and formed into 6 solid spherical balls of the same radius as that of the cylinder. The length of the rod (in m) is

A. 0.8

B. 2

C. 3

D. 4

Ans. D

6. A cone, a hemisphere and a cylinder stand on equal bases of radius R and have equal heights H. Their whole surfaces are in the ratio:

A. $(\sqrt{3}+1) : 3 : 4$

B. $(\sqrt{2}+1) : 7 : 8$

C. $(\sqrt{2}+1) : 3 : 4$

D. None of these

Ans. C

7. A large solid sphere of diameter 15 m is melted and recast into several small spheres of diameter 3 m. What is the percentage increase in the surface area of the smaller spheres over that of the large sphere?

A. 200%

B. 400%

C. 500%

D. can’t be determined

Ans. B

8. A hemispherical basin 150 cm in diameter holds water one hundred and twenty times as much a cylindrical tube. If the height of the tube is 15 cm, then the diameter of the tube (in cm) is:

A. 23

B. 24

C. 25

D. 26

Ans. C

9. A cylinder is circumscribed about a hemisphere and a cone is inscribed in the cylinder so as to have its vertex at the Centre of one end and the other end as its base. The volumes of the cylinder, hemisphere and the cone are respectively in the ratio of:

A. $3 : \sqrt{3} : 2$

B. $3 : 2 : 1$

C. $1 : 2 : 3$

D. $2 : 3 : 1$

Ans. B

10. The height of a cone is 40 cm. The cone is cut parallel to its base such that the volume of the small cone is 1/64 of the cone. Find at which height from base the cone is cut?

A. 20 cm
B. 30 cm
C. 25 cm
D. 22.5 cm

Ans. B


Thank You