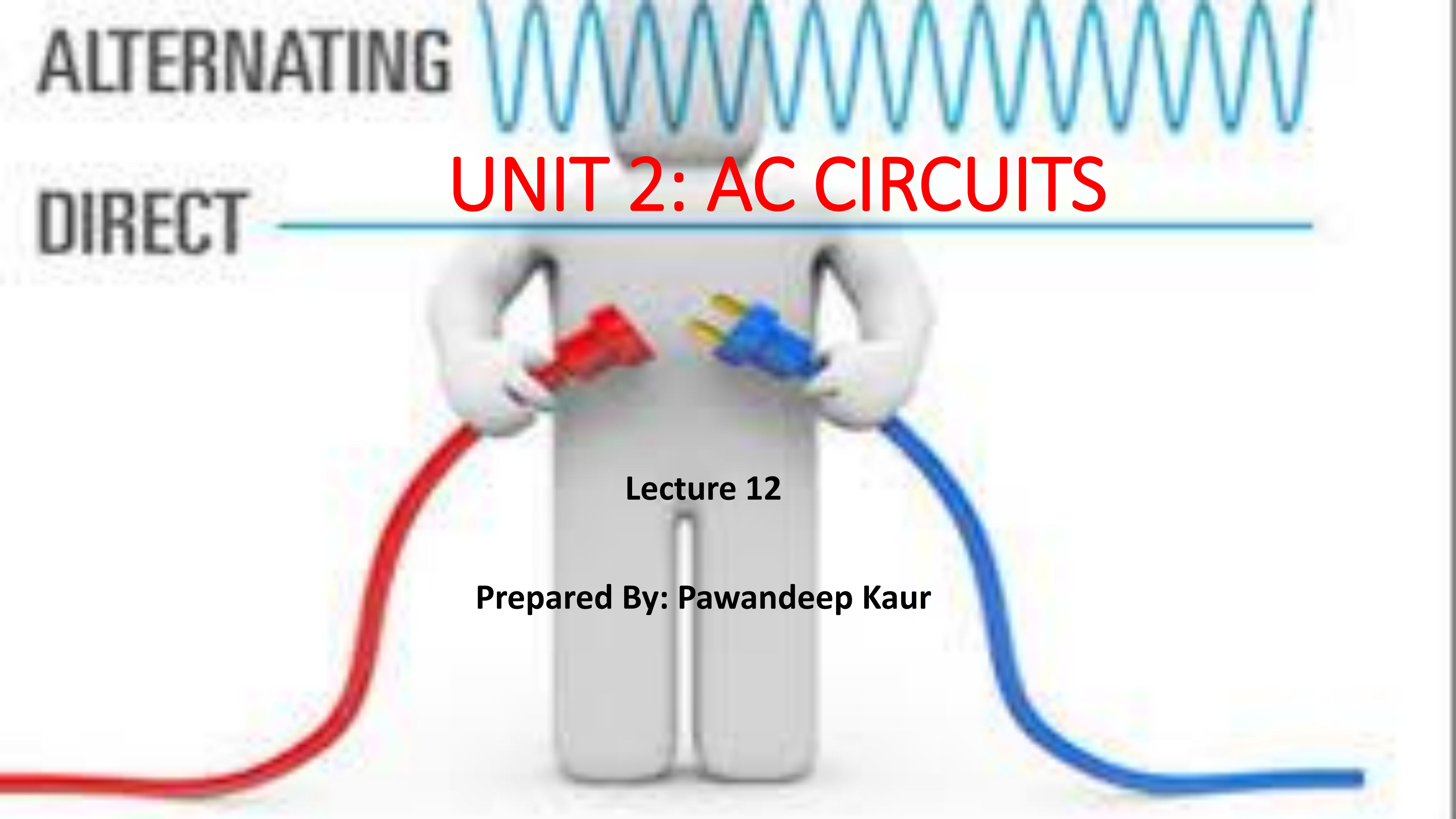


UNIT 2: AC CIRCUITS

Lecture 12

Prepared By: Pawandeep Kaur

Recap Quick Quiz(POLL)

A sine wave has a frequency of 50 Hz. Its angular frequency is ________radian/second.

A. 100 π
B. 50 π
C. 25 π
D. 5 π

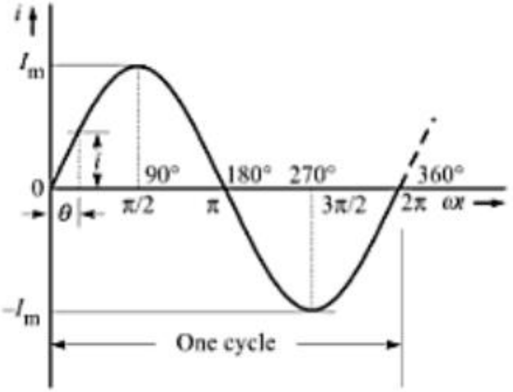
Average Value

Algebraic sum of all the values divided by the total number of values.

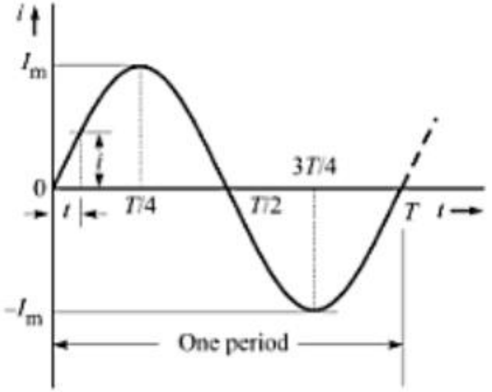
Same concept is applicable for a waveform that cvaries with time.

$$V_{av} = \frac{\text{Area under full cycle}}{\text{Length of one cycle}} = \frac{\int_0^{2\pi} v\,d\theta}{2\pi} = \frac{1}{2\pi}\int_0^{2\pi} v\,d\theta = \frac{1}{2\pi}\int_0^{2\pi} v\,d(\omega t)$$

$$V_{av} = \frac{1}{T}\int_0^{T} v\,dt$$



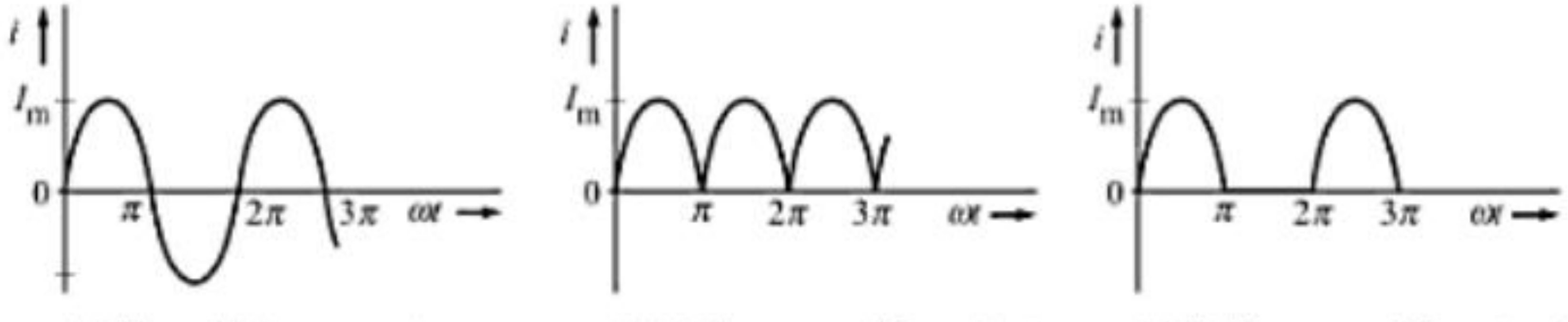
(a) Current i versus angle ωt.



(b) Current i versus time t.

Thus, the average value over full cycle is ZERO

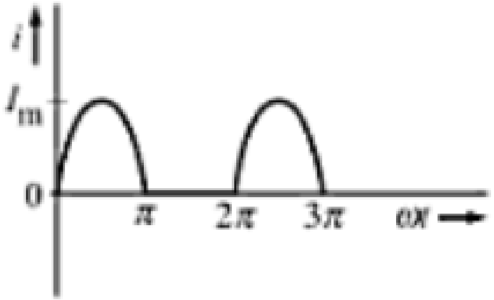
However, an average value can be defined for the half-cycle (positive or negative) for a sinusoidal signal.



(a) Sinusoidal ac current. (b) Full-wave rectifier output. (c) Half-wave rectifier output.

Average Value

Half Wave Rectifier

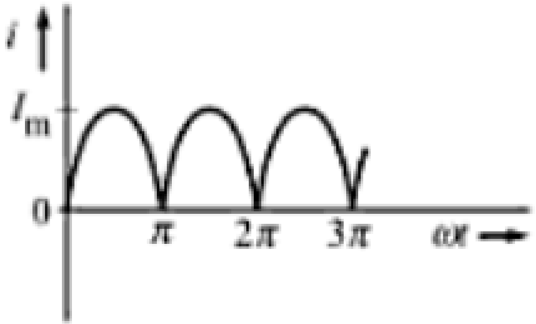


Half-wave rectifier output.

$$I_{av} = \frac{1}{2\pi}\int_0^{2\pi} i\,d(\omega t) = \frac{1}{2\pi}\left[\int_0^{\pi} I_m \sin\omega t\, d(\omega t) + \int_{\pi}^{2\pi} 0\, d(\omega t)\right]$$

$$= \frac{I_m}{2\pi}[-\cos\omega t]_0^{\pi} = \frac{I_m}{\pi} = \mathbf{0.318}\,I_m$$

Full Wave Rectifier



Full-wave rectifier output.

$$I_{av} = \frac{\text{Area under half cycle}}{\text{Length of half cylcle}} = \frac{1}{\pi}\int_0^{\pi} i\,d(\omega t) = \frac{1}{\pi}\int_0^{\pi} I_m \sin\omega t\, d(\omega t)$$

$$= \frac{I_m}{\pi}[-\cos\omega t]_0^{\pi} = \frac{I_m}{\pi}[-\cos\pi + \cos 0°] = \frac{2I_m}{\pi} = \mathbf{0.637}\,I_m$$

Average Value

QUICK QUIZ (POLL)

The average value of current in a sinusoidal signal over full cycle is:

$A.$ $I_m/2$

$B.$ $I_m/\sqrt{2}$

C. 1

D. 0

Recap QUICK QUIZ (POLL)

Based on the previous results, we can say that the average value of full wave rectifier is__________ than half wave rectifier.

A. Double
B. Half
C. Same
D. None of these

RMS (Effective) Value

- The r.m.s. value of an alternating current is given by that steady (d.c.) current which when flowing through a given circuit for a given time produces the same heat as produced by the alternating current when flowing through the same circuit for the same time.

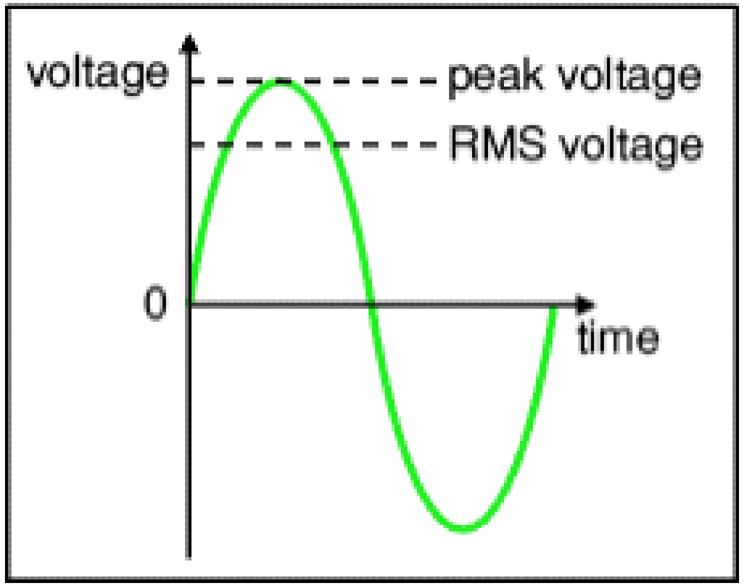
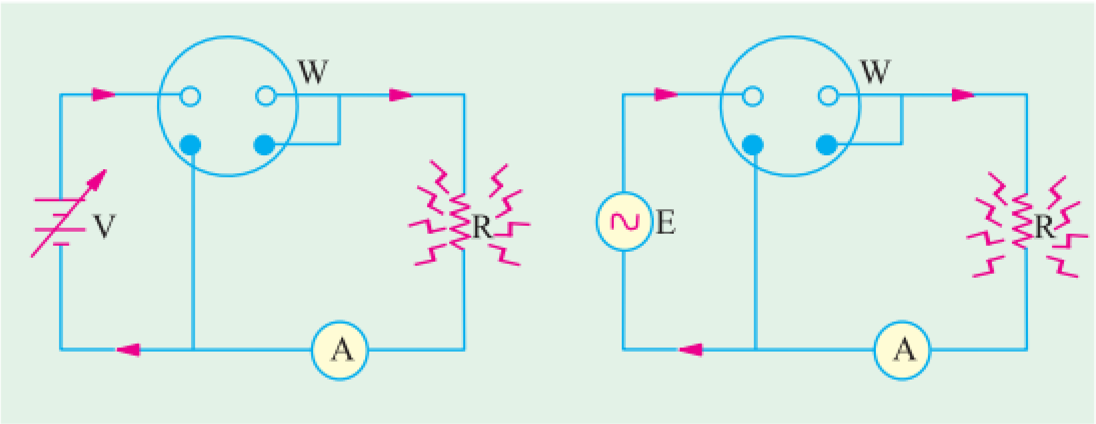
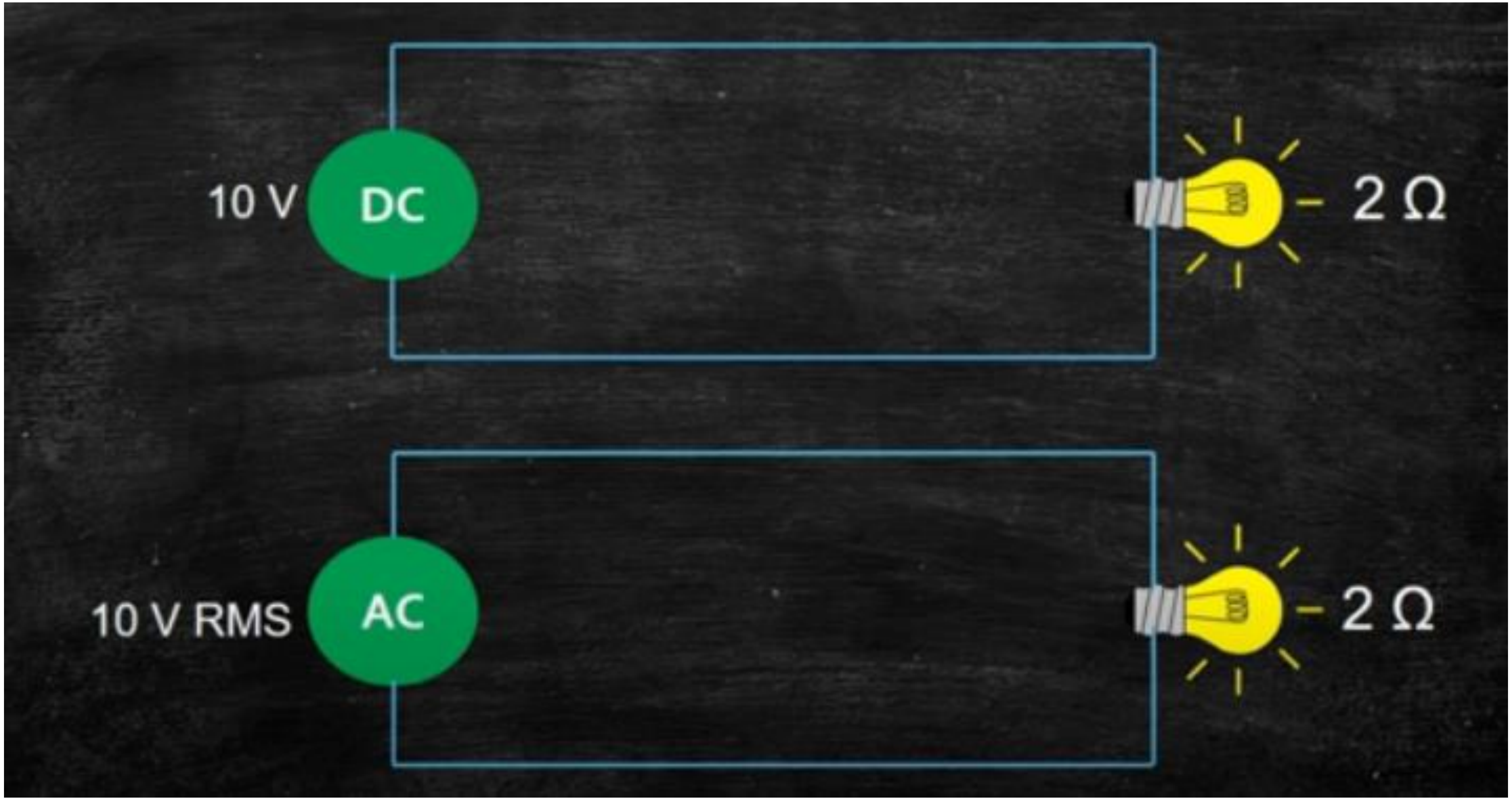
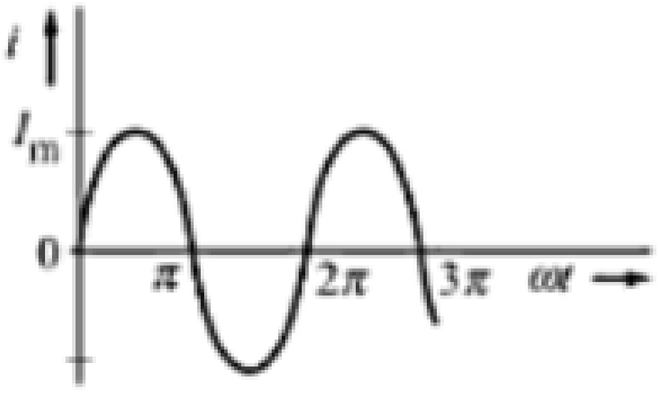


Figure- Difference between peak and RMS voltage



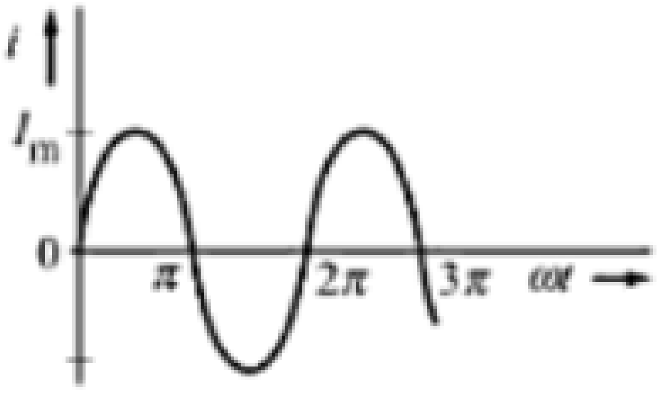

10 V
DC
2 Ω
10 V RMS
AC
2 Ω

RMS (Effective) Value



Sinusoidal ac current.

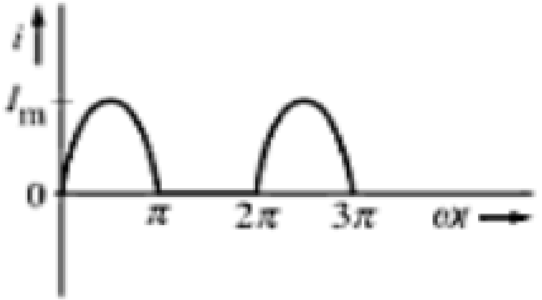
RMS (Effective) Value



Sinusoidal ac current.

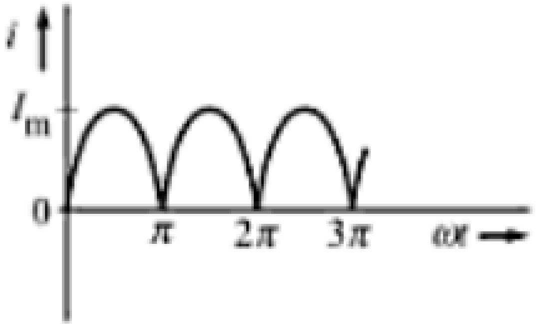
RMS Value

Half Wave Rectifier



Half-wave rectifier output.

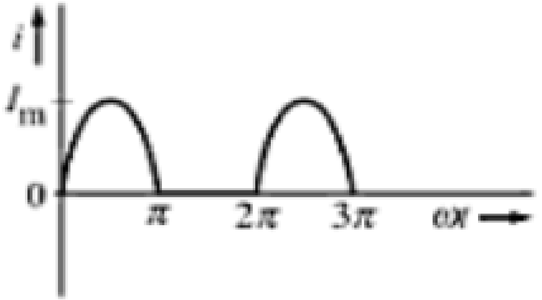
Full Wave Rectifier



Full-wave rectifier output.

RMS Value

Half Wave Rectifier

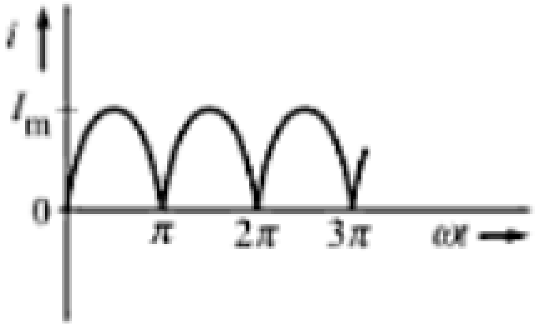


Half-wave rectifier output.

$$I_{eff} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2\, d(\omega t)} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} I_m^2 \sin^2 \omega t\, d(\omega t) + \frac{1}{2\pi}\int_{\pi}^{2\pi} 0\, d(\omega t)}$$

$$= \sqrt{\frac{I_m^2}{4\pi}\int_0^{\pi} (1 - \cos 2\omega t)\, d(\omega t) + 0} = \sqrt{\frac{I_m^2}{4\pi}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_0^{\pi}} = \frac{I_m}{\sqrt{4}} = \frac{I_m}{2}$$

Full Wave Rectifier



Full-wave rectifier output.

$$I_{eff} = \sqrt{\frac{1}{\pi}\int_0^{\pi} i^2\, d(\omega t)} = \sqrt{\frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2 \omega t\, d(\omega t)} = \sqrt{\frac{I_m^2}{2\pi}\int_0^{\pi} (1 - \cos 2\omega t)\, d(\omega t)}$$

$$= \sqrt{\frac{I_m^2}{2\pi}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_0^{\pi}} = \frac{I_m}{\sqrt{2}}$$

RMS Value

QUICK QUIZ (POLL)

What is the effective value of current?

a) RMS current

b) Average current

c) Instantaneous current

d) Total current

Recap QUICK QUIZ (POLL)

The voltage of domestic supply is 220V. This figure represents

A. Mean value

B. R.M.S value

C. Peak value

D. Average value

Form Factor and Peak Factor

- Form Factor, $K_f = \dfrac{V_{rms}}{V_{avg}}$

- Peak Factor or Crest Factor, $K_p = \dfrac{V_m}{V_{rms}}$

Let us calculate these two factors for ***a sinusoidal voltage waveform***,

$$K_f = \frac{V_{rms}}{V_{av}} = \frac{V_m/\sqrt{2}}{2V_m/\pi} = \frac{0.707\,V_m}{0.637\,V_m} = \mathbf{1.11}$$

And

$$K_p = \frac{V_m}{V_{rms}} = \frac{V_m}{V_m/\sqrt{2}} = \sqrt{2} = \mathbf{1.414}$$

QUICK QUIZ (POLL)

For a pure sinusoidal waveform the Form Factor will always be equal to:

A. $\frac{1}{\sqrt{2}}$

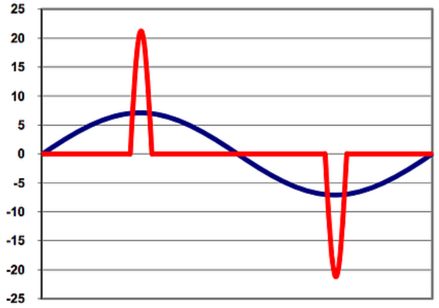
B. 0.637

C. 1.11

D. 1.414

Importance of Form Factor and Peak Factor

- Actually some of our meters are designed to measure the RMS values but that is of pure sinusoidal waveforms, if there comes any distortion in the waveform, the meter **won't give the correct** RMS value. For meter the waveform is still a sinusoidal but it doesn't detect the distortion that's why we use form factor to get accurate value of RMS by just multiplying form factor with the average value of that distorted waveform. It is helpful in finding the RMS values of waveforms other than pure sinusoidal.
- Similarly, Some loads, such as switching power supplies or lamp ballasts, have current waveforms that are not sinusoidal. They draw a high current for a short period of time, and their crest factors, therefore, can be quite a bit higher than 1.414.

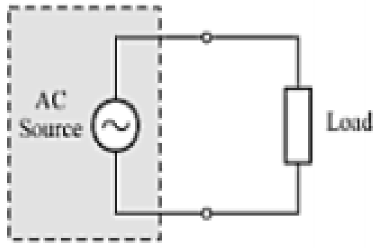


Power and Power Factor

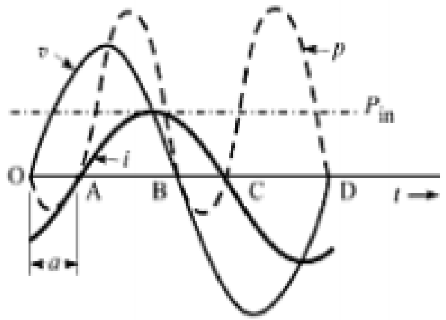
- In a general ac circuit, we assume:
- $v = v_m \sin \omega t \; and \; i = i_m \sin(\omega t - \theta)$

INSTANTANEOUS POWER:

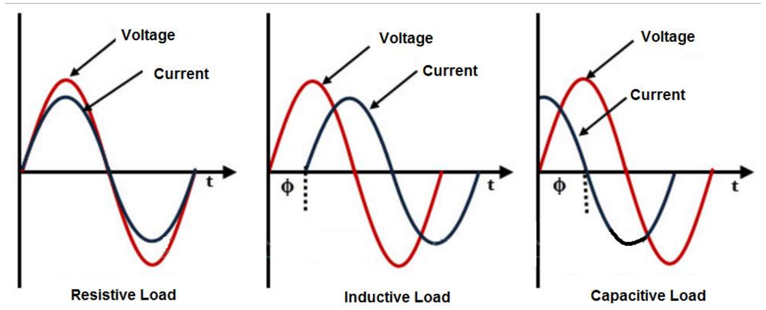
$p = vi = V_{rms} I_{rms} [cos\theta - \cos(2\omega t - \theta)]]$

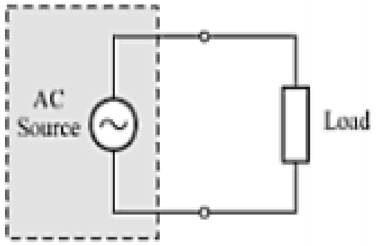


(a) AC source delivering power to a load.

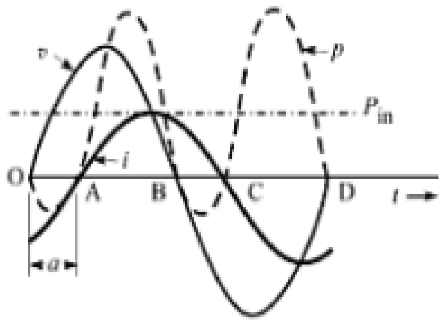


(b) Waveforms of voltage, current and power.



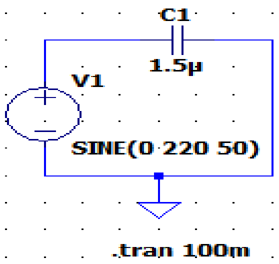


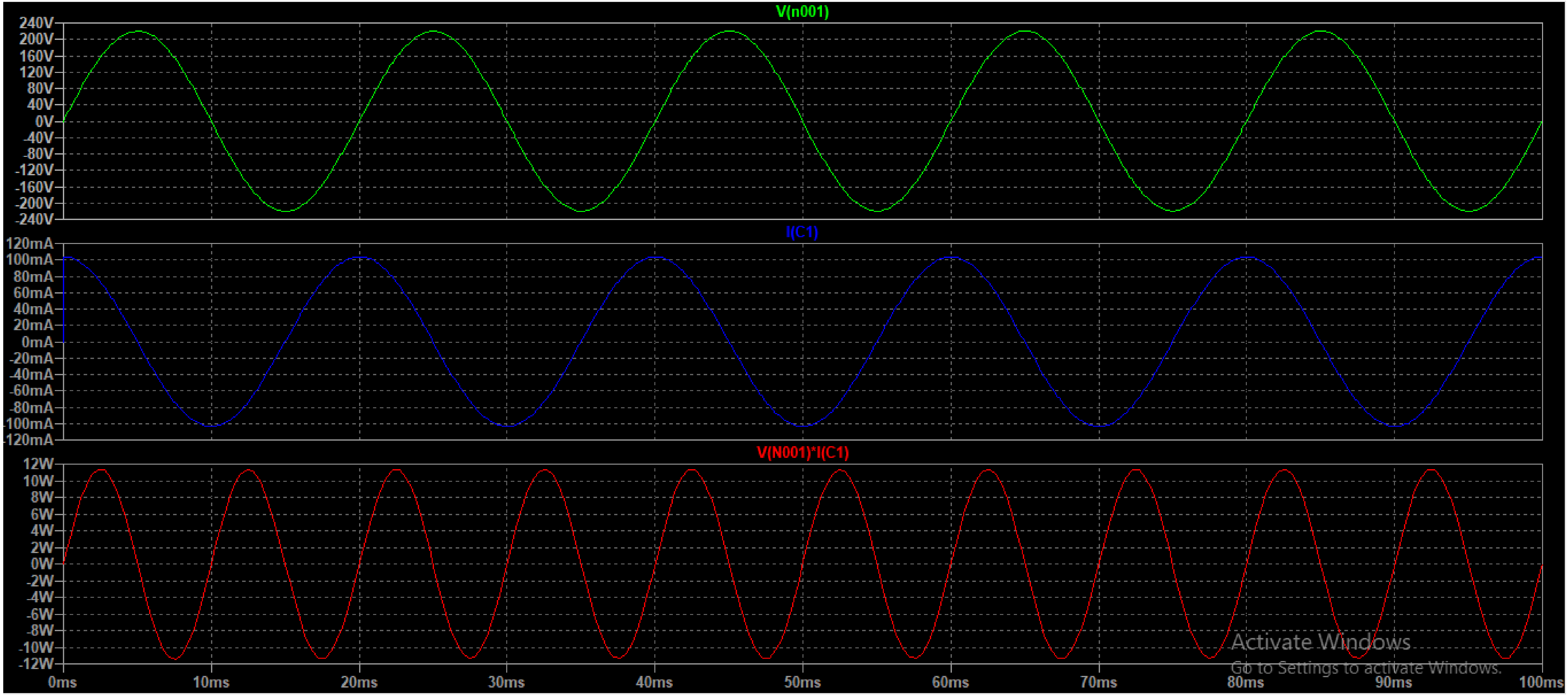
(a) AC source delivering power to a load.

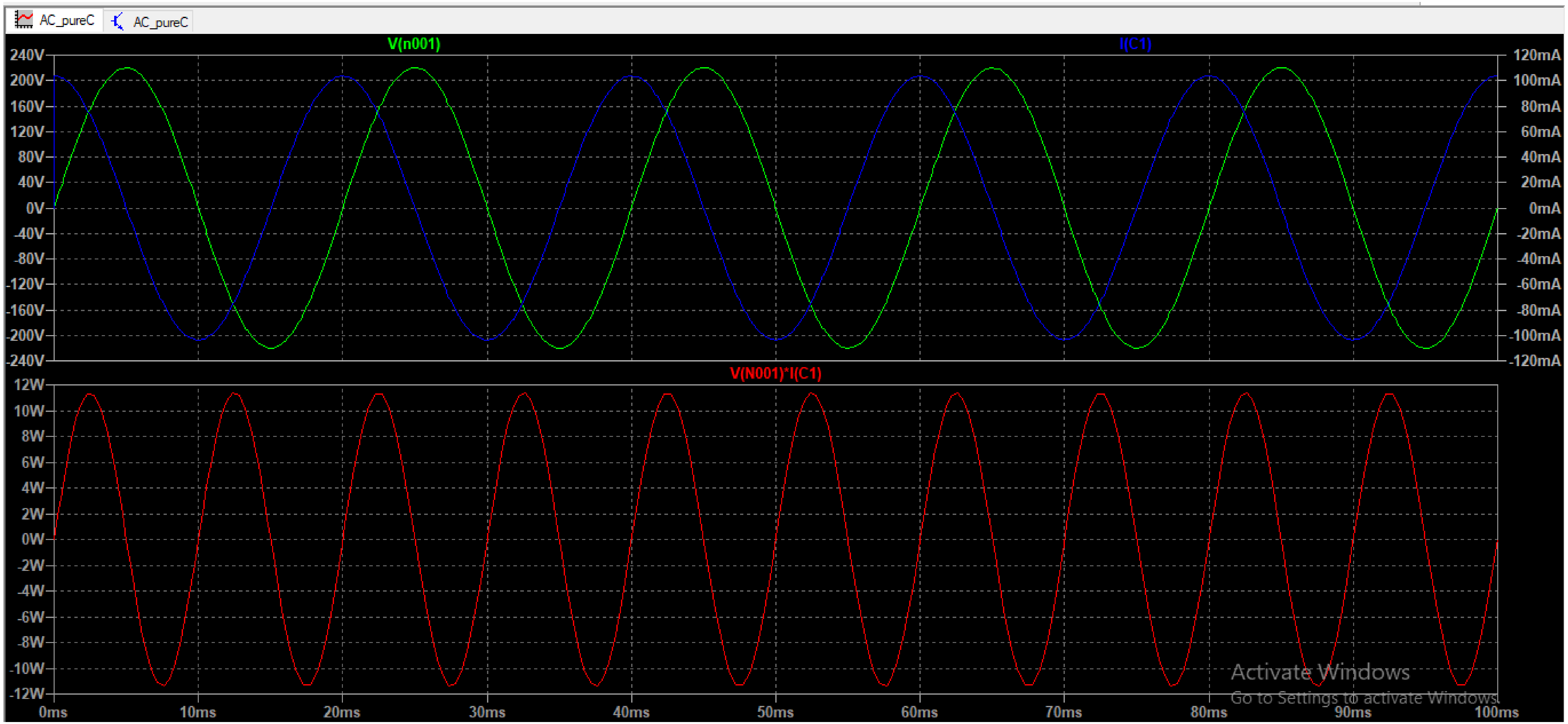


(*b*) Waveforms of voltage, current and power.

LT Spice Simulation







AC_pureC
AC_pureC
V(n001)
I(C1)
240V
200V
160V
120V
80V
40V
0V
-40V
-80V
-120V
-160V
-200V
-240V
120mA
100mA
80mA
60mA
40mA
20mA
0mA
-20mA
-40mA
-60mA
-80mA
-100mA
-120mA
V(N001)*I(C1)
12W
10W
8W
6W
4W
2W
0W
-2W
-4W
-6W
-8W
-10W
-12W
0ms
10ms
20ms
30ms
40ms
50ms
60ms
70ms
80ms
90ms
100ms
Activate Windows
Go to Settings to activate Windows.

Average Power

- Also called as True Power or Real Power.
- It represents the actual amount of power being used, or dissipated, in a circuit.
- Recall that : $p = vi = V_{rms}I_{rms}[cos\theta - 2\cos(\omega t - \theta)]$
- Average value of $\cos(\omega t - \theta)$ is ZERO for a full cycle.
- Therefore, average power:

$$P_{avg} = V_{rms}I_{rms}cos\theta$$

- Units: Watts

Apparent Power

- Recall that : $p = vi = V_{rms}I_{rms}[cos\theta - 2\cos(\omega t - \theta)]$
- Average value of $\cos(\omega t - \theta)$ is ZERO for a full cycle.
- Therefore, Apparent power:

$$P_{apr} = V_{rms}I_{rms}$$

- Units: Volt-Amperes (VA)

Power Factor

- Recall that : $p = vi = V_{rms} I_{rms}[cos\theta - 2\cos(\omega t - \theta)]$
- Average Power: $P_{avg} = V_{rms} I_{rms} cos\theta$
- And, Apparent power:$P_{apr} = V_{rms} I_{rms}$

Therefore,

Average Power = Apparent Power × $cos\theta$

Implies,

$cos\theta$=$\frac{Average\ Power}{Apparent\ Power} = Power\ Factor$

QUICK QUIZ (POLL)

The S.I unit for Apparent power is:

A. Watts
B. VAr
C. VA
D. Watt-hour

QUICK QUIZ (POLL)

The power factor angle of a purely inductive circuit is:

A. 0 degree
B. 45 degree
C. 90 degree
D. Can’t be determined

Numerical Practice

In an ac circuit, the instantaneous voltage and current are given as

$$v = 55 \sin \omega t \text{ V} \qquad \text{and} \qquad i = 6.1 \sin (\omega t - \pi/5) \text{ A}$$

Determine the average power, the apparent power, the instantaneous power when ωt (in radians) equals 0.3, and the power factor in percentage.

Numerical Practice

In an ac circuit, the instantaneous voltage and current are given as

$$v = 55\sin \omega t \text{ V} \qquad \text{and} \qquad i = 6.1 \sin (\omega t - \pi/5) \text{ A}$$

Determine the average power, the apparent power, the instantaneous power when ωt (in radians) equals 0.3, and the power factor in percentage.

Reactive Power

- This “**phantom power**” is called reactive power, and it is measured in a unit called Volt-Amps-Reactive (VAR), rather than watts.
- As a rule, true power is a function of a circuit’s dissipative elements, usually resistances (R). Reactive power is a function of a circuit’s reactance (X).
- Therefore, we can say that an Apparent power is a function of a circuit’s total impedance (Z).

Reactive Power

- We know that reactive loads such as inductors and capacitors dissipate zero power, yet the fact that they drop voltage and draw current gives the deceptive impression that they actually do dissipate power.
- This “**phantom power**” is called reactive power, and it is measured in a unit called Volt-Amps-Reactive (VAR), rather than watts.
- As a rule, true power is a function of a circuit’s dissipative elements, usually resistances (R). Reactive power is a function of a circuit’s reactance (X).
- Therefore, we can say that an Apparent power is a function of a circuit’s total impedance (Z).

Reactive Power

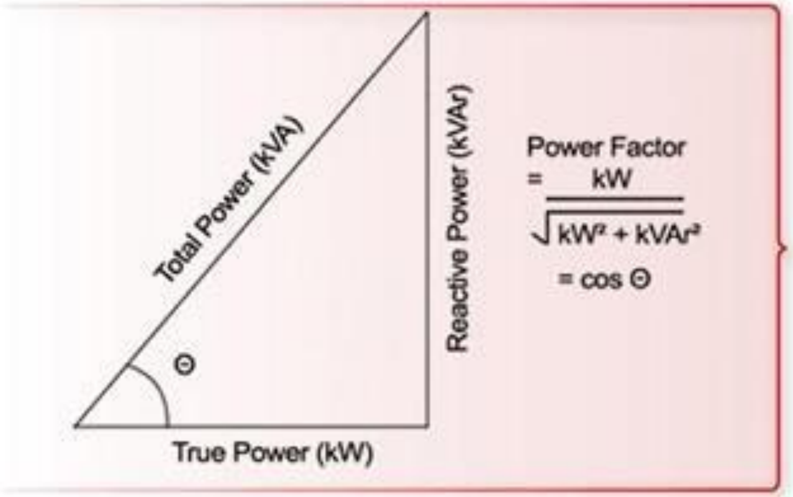
- It has been seen that power is consumed only in resistance. A pure inductor and a pure capacitor do not consume any power since in a half cycle whatever power is received from the source by these components, the same power is returned to the source. This power which returns and flows in both the direction in the circuit, is called Reactive power. This reactive power does not perform any useful work in the circuit.
- In a purely resistive circuit, the current is in phase with the applied voltage, whereas in a purely inductive and capacitive circuit the current is 90 degrees out of phase, i.e., if the inductive load is connected in the circuit the current lags voltage by 90 degrees and if the capacitive load is connected the current leads the voltage by 90 degrees.
- Hence, from all the above discussion, it is concluded that the current in phase with the voltage produces true or active power, whereas, the current 90 degrees out of phase with the voltage contributes to reactive power.

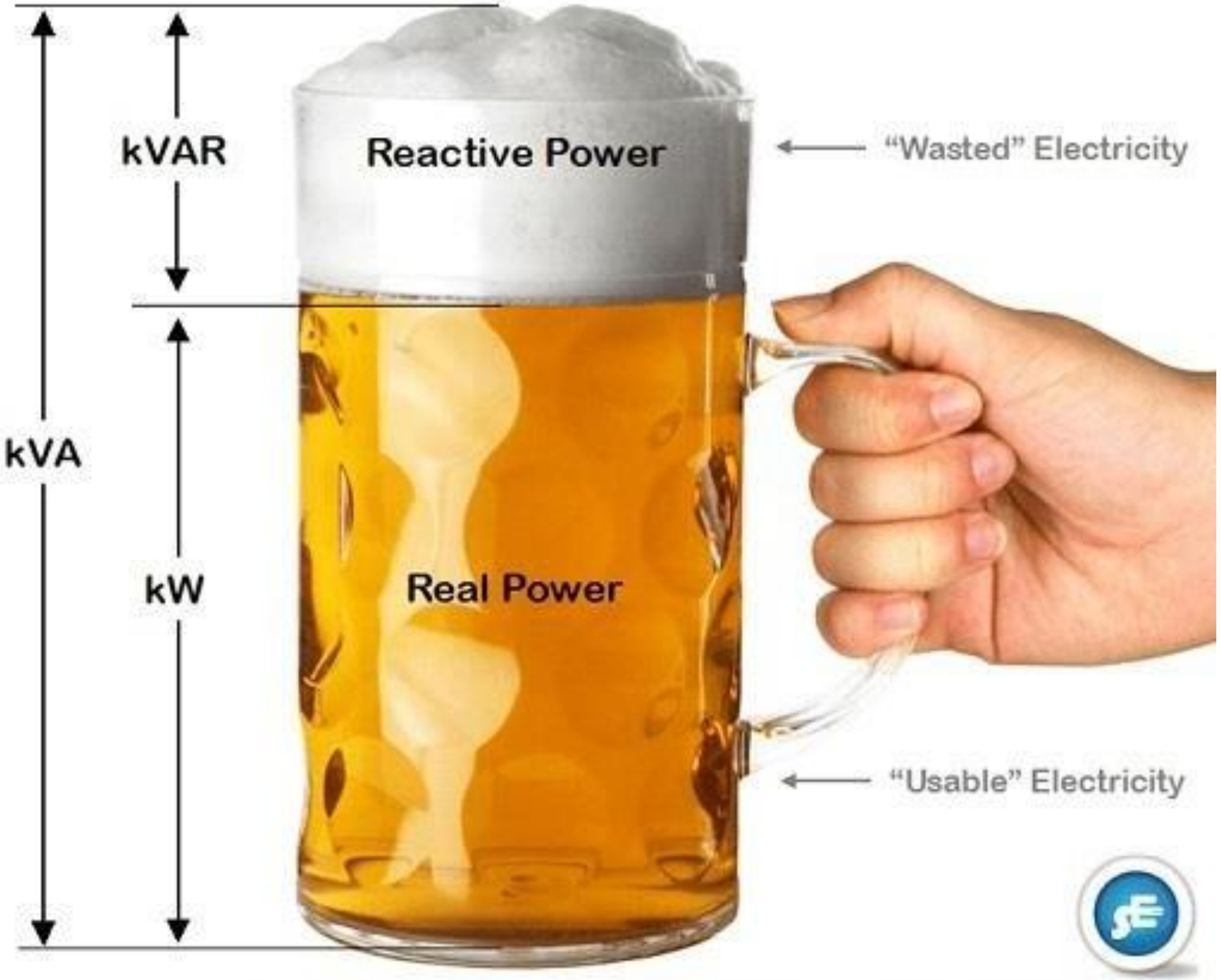
Therefore,

- ✔ True power = voltage x current in phase with the voltage
- ✔ Reactive power = voltage x current out of phase with the voltage

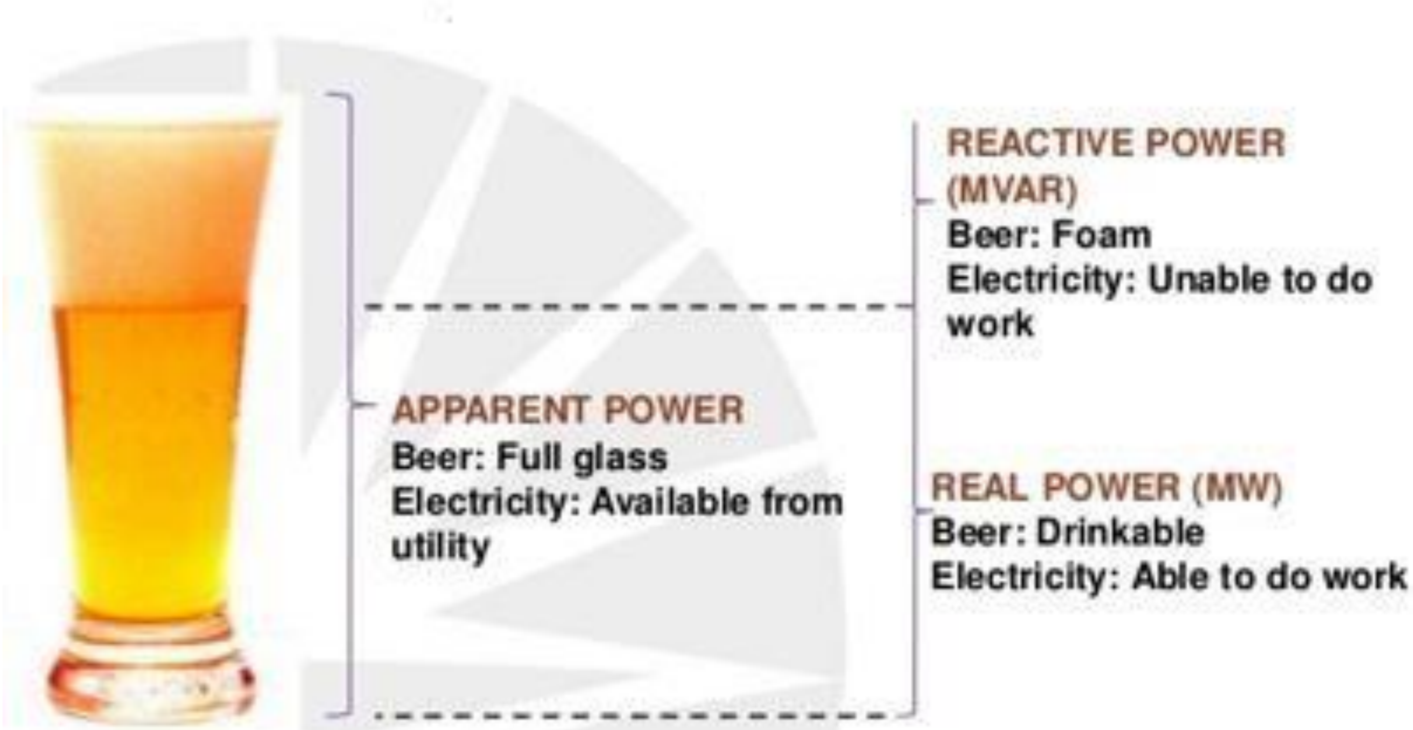
What is Power Factor?

Power Factor is the percentage of apparent power that does real work. Understand Power Factor using Beer Mug Analogy.

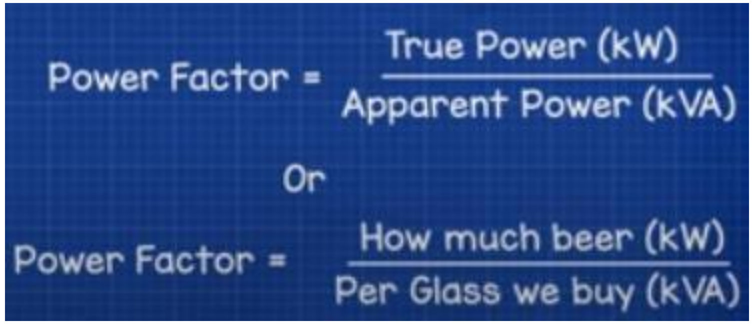




Power Factor Comparison



It is the Active Power that contributes to the energy consumed, or transmitted. Reactive Power does not contribute to the energy. It is an inherent part of the "total power" which is often referred as "Useless Power".



Power Triangle

P = true power $\quad P = I^2R \quad P = \dfrac{E^2}{R}$

*Measured in units of **Watts***

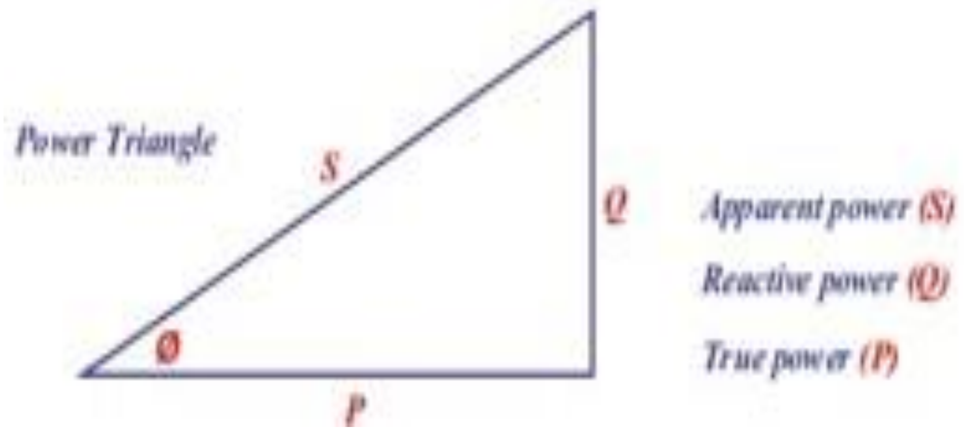
Q = reactive power $\quad Q = I^2X \quad Q = \dfrac{E^2}{X}$

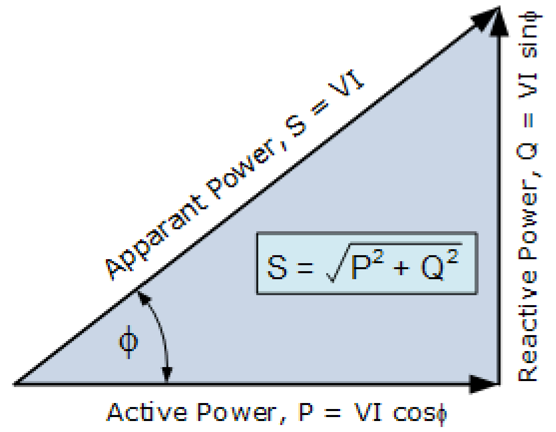
*Measured in units of **Volt-Amps-Reactive (VAR)***

S = apparent power $\quad S = I^2Z \quad Q = \dfrac{E^2}{Z} \quad S = IE$

*Measured in units of **Volt-Amps (VA)***

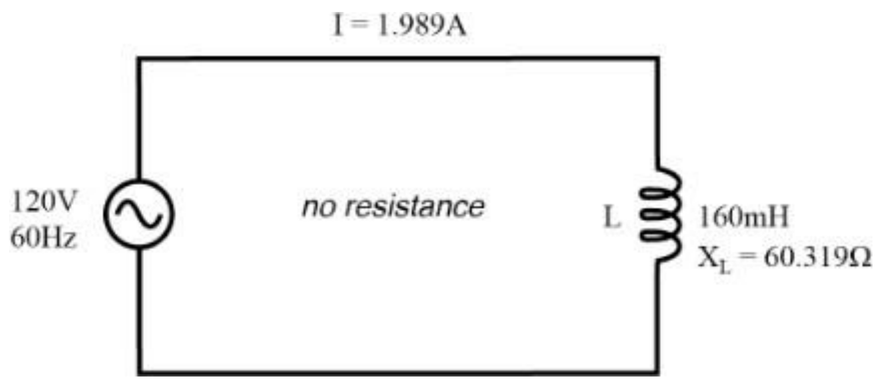
- Active power P = V x I $\cos\phi$ = V I $\cos\phi$
- Reactive power P_r or Q = V x I $\sin\phi$ = V I $\sin\phi$
- Apparent power P_a or S = V x I = VI





Practice Problem

Find the reactive power in the given circuit?



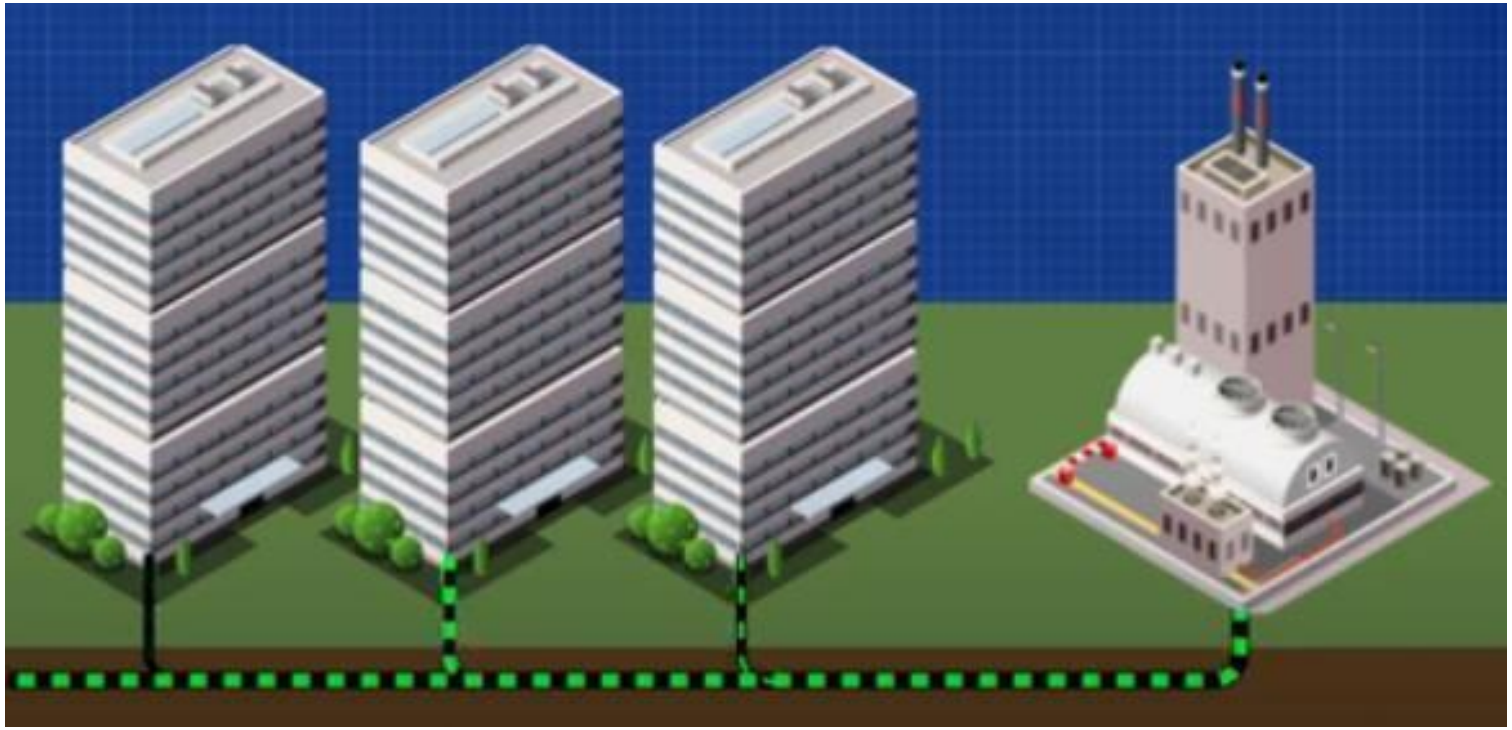
Power Factor Correction

Why is it required?

Power Factor Correction

Why is it required?

Reactive Power Charges



Power Factor Correction

Why is it required?

❖ Poor Power factor means that more power is drawn.

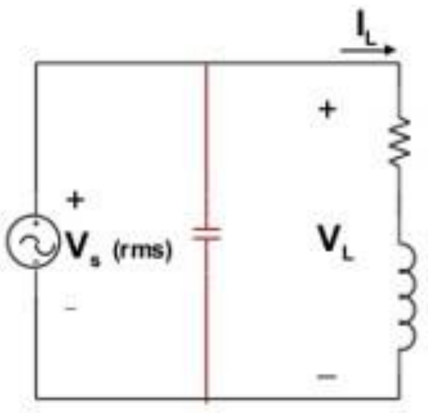
❖ Requires larger cables

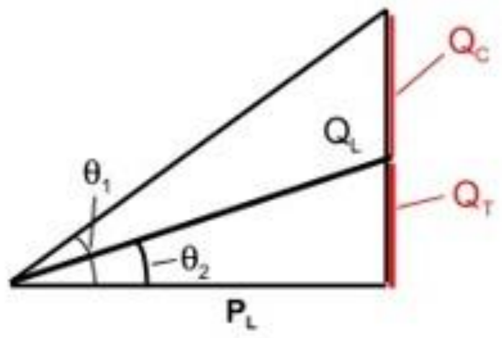
❖ Reactive Power penalty fee

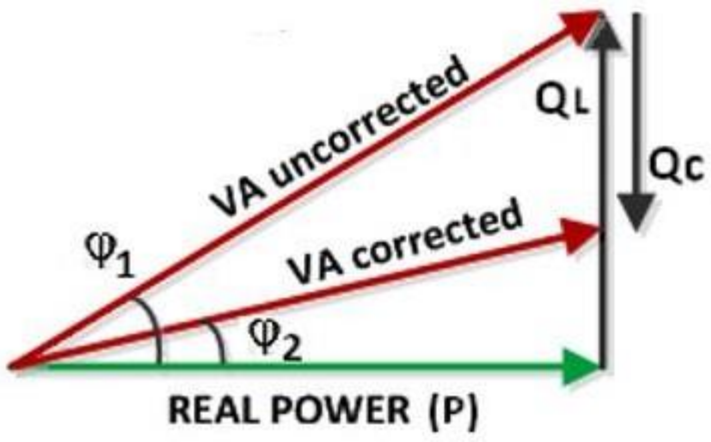
❖ Losses in a transformer

❖ Voltage drops

Example: Power Factor Correction







Before C added, $S = P_L + jQ_L$ p.f. $= \cos\theta_1$

After C added, $S = P_L + j(Q_L - Q_C)$ p.f. $= \cos\theta_2$ i.e. increased

(voltage and current to original load retained)

Cancelling some or all reactive components of power by adding reactance of opposite type to the circuit • This is power factor correction

- In many ways, reactive power can be thought of like the foam head on a pint or glass of beer. You pay the barman for a full glass of beer but you only drink the actual liquid beer itself which, on many occasions, is always less than a full glass.
- This is because the head (or froth) of the beer takes up additional wasted space in the glass leaving less room for the real liquid beer that you consume, and the same idea is true in many ways for reactive power.
- But for many industrial power applications, reactive power is often useful for an electrical circuit to have. While the real or active power is the energy supplied to run a motor, heat a home, or illuminate an electric light bulb, reactive power provides the important function of regulating the voltage thereby helping to move power effectively through the utility grid and transmission lines to where it is required by the load.
- While reducing reactive power to help improve the power factor and system efficiency is a good thing, one of the disadvantages of reactive power is that a sufficient quantity of it is required to control the voltage and overcome the losses in a transmission network. This is because if the electrical network voltage is not high enough, active power cannot be supplied. But having too much reactive power flowing around in the network can cause excess heating (I^2R losses) and undesirable voltage drops and loss of power along the transmission lines.

Importance of Power Factor

Power factor can be an important aspect to consider in an AC circuit because of any power factor less than 1 means that the circuit's wiring has to carry more current than what would be necessary with zero reactance in the circuit to deliver the same amount of (true) power to the resistive load.

If our last example circuit had been purely resistive, we would have been able to deliver a full 169.256 watts to the load with the same 1.410 amps of current, rather than the mere 119.365 watts that it is presently dissipating with that same current quantity.

The poor power factor makes for an inefficient power delivery system.

Poor Power Factor

Poor power factor can be corrected, paradoxically, by adding another load to the circuit drawing an equal and opposite amount of reactive power, to cancel out the effects of the load's inductive reactance.

Inductive reactance can only be canceled by capacitive reactance, so we have to add a *capacitor* in parallel to our example circuit as the additional load.

The effect of these two opposing reactances in parallel is to bring the circuit's total impedance equal to its total resistance (to make the impedance phase angle equal, or at least closer, to zero).

Since we know that the (uncorrected) reactive power is 119.998 VAR (inductive), we need to calculate the correct capacitor size to produce the same quantity of (capacitive) reactive power.

Since this capacitor will be directly in parallel with the source (of known voltage), we'll use the power formula which starts from voltage and reactance:

Concept of Phasors

$z = x + jy$ Rectangular form

$z = r\angle\phi$ Polar form

$z = re^{j\phi}$ Exponential form

$$r = \sqrt{x^2 + y^2}, \qquad \phi = \tan^{-1}\frac{y}{x}$$

On the other hand, if we know r and ϕ, we can obtain x and y as

$$x = r\cos\phi, \qquad y = r\sin\phi$$

Thus, z may be written as

$$\boxed{z = x + jy = r\angle\phi = r(\cos\phi + j\sin\phi)}$$

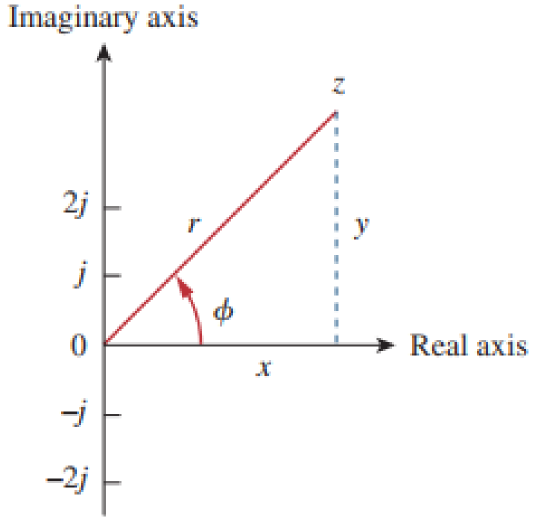


Figure
Representation of a complex number $z = x + jy = r\angle\phi$.

$$v(t) = V_m \cos(\omega t + \phi) \quad \Leftrightarrow \quad \mathbf{V} = V_m \angle \phi$$

(Time-domain representation) (Phasor-domain representation)

Practice Problem

Evaluate the following complex number?

(a) $(40\angle 50^\circ + 20\angle -30^\circ)^{1/2}$