Lecture 14

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Q19. $y_{n+4} + 16y_n = 0$

$(E^4 + 16)y_n = 0$, $m^4 + 16 = 0$, $m^4 = -16$

$m^4 = 16e^{i\pi}$ $e^{i\theta} = \cos\theta + i\sin\theta$ $\tan^{-1}\left(\frac{0}{-16}\right)$

$\pi-\alpha$	α
$\alpha-\pi$	$-\alpha$

$m^4 = 16(\cos\pi + i\sin\pi)$

$m = 2(\cos\pi + i\sin\pi)^{1/4}$

$m = 2[\cos(\pi + 2n\pi) + i\sin(\pi + 2n\pi)]^{1/4}$

$m = 2\left[\cos\left(\frac{\pi+2n\pi}{4}\right) + i\sin\left(\frac{\pi+2n\pi}{4}\right)\right]$, $n = 0, 1, 2, 3$

$m = 2e^{\frac{i\pi}{4}}, 2e^{\frac{i3\pi}{4}}, 2e^{\frac{i5\pi}{4}} = -\frac{i3\pi}{4}, 2e^{\frac{i7\pi}{4}} = -\frac{i\pi}{4}$

$m = 2\left[\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right], 2\left[-\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right], 2\left(-\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right), 2\left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right)$

Linear Non-Homogeneous Recurrence Relations with Constant Coefficients

$C_0 a_{n+k} + C_1 a_{n+k-1} + \dots + C_k a_n = R(n)$

$R(n) \neq 0$

$C_0, C_1, C_2, \ldots C_k \to$ real constants

$R(n) \neq 0$

$C_0, C_k \neq 0$

degree $= k$

$$(C_0E^k + C_1E^{k-1} + \ldots + C_k)\, a_n = R(n)$$

$$f(E)\, a_n = R(n)$$

P.I. $\quad a_n = \frac{1}{f(E)} R(n)$

Case I: If $R(n) = a^n$

$$\frac{1}{f(E)} a^n \xrightarrow{\text{Put } E = a} \frac{1}{f(a)} \cdot a^n, \text{ provided } f(a) \neq 0$$

Case II: If $R(n) = \cos \alpha n \,/\, \sin \alpha n$

$$\cos \alpha n = \frac{e^{i\alpha n} + e^{-i\alpha n}}{2}, \quad \sin \alpha n = \frac{e^{i\alpha n} - e^{-i\alpha n}}{2i}$$

$$e^{i\alpha n} = (e^{i\alpha})^n, \quad e^{-i\alpha n} = (e^{-i\alpha})^n$$

$-i\alpha$

Put $E = e^{i\alpha}$ Put $E = e^{-i\alpha}$

For Case of failure:

$$\frac{1}{E-a}\, a^n = [n]\,\frac{1}{1!}\, a^{n-1}$$

$$\frac{1}{(E-a)^2}\, a^n = [n]^2\,\frac{1}{2!}\, a^{n-2}$$

$$\frac{1}{(E-a)^3}\, a^n = [n]^3\,\frac{1}{3!}\, a^{n-3}$$

$$[n] = n,\quad [n]^2 = n(n-1),\quad [n]^3 = n(n-1)(n-2)$$

Q20. Solve Tower of Hanoi Problem

$$a_n = 2a_{n-1} + 1, \quad a_1 = 1$$

$$n \to n+1$$

$$a_{n+1} = 2a_n + 1$$

$$a_{n+1} - 2a_n = 1, \qquad (E-2)a_n = 1 + 0$$

C.F. $\quad CF = C(2)^n$

C.F. $\quad m-2=0, \quad m=2 \quad C.F. = C_1(2)^n$

P.I. $\quad \frac{1}{E-2} 1 = \frac{1}{E-2}(1)^n \xrightarrow{\text{Put } E=1} (-1)$

G.S. $\quad y_n = C.F. + P.I. = C_1(2)^n - 1$

$$y_1 = 1 \Rightarrow C_1 = 1$$

$$y_n = 2^n - 1$$

Q21. Solve $y_n - 4y_{n-1} + 3y_{n-2} = 5^n$

$$n-2 \to n, \quad n \to n+2$$

$$y_{n+2} - 4y_{n+1} + 3y_n = 5^{n+2} = 25\cdot(5)^n$$

$$(E^2 - 4E + 3)y_n = 25\cdot(5)^n$$

C.F.: $\quad m^2 - 4m + 3 = 0, \quad m = 3, 1$

$$C.F = C_1(3)^n + C_2(1)^n$$

P.I. $\quad \frac{1}{E^2-4E+3} 25\cdot(5)^n \xrightarrow{\text{Put } E=5} \frac{1}{25-20+3} 25(5)^n$

$$= \frac{25}{8}(5)^n$$

$$y_n = C_1(3)^n + C_2 + \frac{25}{8}(5)^n$$

Q.2 $\quad y_n - 7y_{n-1} + 10y_{n-2} = 7(3)^n + e^{2n} + 5^n$