Lecture 10

07 September 2021 10:02

Q4. Is the sequence $\{a_n\}, a_n = n4^n$ is solution of the recurrence relation $a_n = 8a_{n-1} - 16a_{n-2}$?

$a_n = 8a_{n-1} - 16a_{n-2}$

$a_n = n4^n$

$a_{n-1} = (n-1)4^{n-1}$

$a_{n-2} = (n-2)4^{n-2}$

RHS $8a_{n-1} - 16a_{n-2}$

$= 8(n-1)4^{n-1} - 16(n-2)4^{n-2}$

$= \frac{8(n-1)4^n}{4} - \frac{16(n-2)4^n}{4^2}$

$= 2(n-1)4^n - (n-2)4^n$

$= 4^n(2n-2-n+2) = n4^n = \text{LHS}$

Q5. Find the solution of the following recurrence relation with given initial conditions using iterative approach?

(a) $a_n = na_{n-1}$, $a_0 = 5$

(1) $a_n = n a_{n-1}$

(2) $a_{n-1} = (n-1)a_{n-2}$

(3) $a_n = n(n-1)a_{n-2}$

(4) $a_{n-2} = (n-2)a_{n-3}$

(5) $a_n = (n)(n-1)(n-2)a_{n-3}$

(6) $a_{n-3} = (n-3)a_{n-4}$

(7) $a_n = n(n-1)(n-2)(n-3)a_{n-4}$

$\vdots$

$a_n = \underbrace{n(n-1)(n-2)\cdots 1}\, a_0$

$a_n = (n!)a_0$

$a_n = 5(n!)$

(b) $a_n = 2a_{n-1} - 3, a_0 = -1$

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$$a_n = 2a_{n-1} - 3$$

$$a_{n-1} = 2a_{n-2} - 3 \qquad a_n = 4a_{n-2} - 9 = 3\cdot 3$$

$$a_{n-2} = 2a_{n-3} - 3 \qquad a_n = 8a_{n-3} - 21 = 3\cdot 7$$

$$a_{n-3} = 2a_{n-4} - 3 \qquad a_n = 16a_{n-4} - 45 = 3\cdot 15$$

$$a_{n-4} = 2a_{n-5} - 3 \qquad a_n = 32a_{n-5} - 93 = 3\cdot 31$$

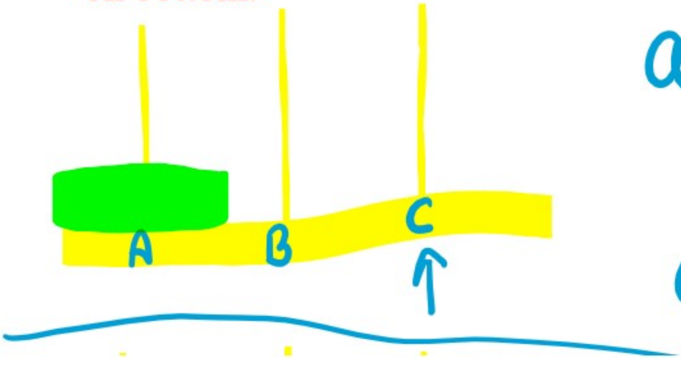
$$a_n = 2^n a_0 - 3\cdot(2^n - 1)$$

$$a_n = 2^n(-1) - 3(2)^n + 3 = 3 - 4\cdot 2^n$$

$$\boxed{a_n = 3 - 2^{n+2}}$$

Modelling with Recurrence relations

Q6. Tower of Hanoi problem
It consists of 3 pegs mounted on a board together with disks of different sizes. Initially these disks are placed on the first peg in the order of size, with the largest on the bottom. The rules allow disks to be moved one at a time from one peg to another as long as disk is never placed on top of smaller disk. The goal of the puzzle is to have all the disks on the second peg in order of size, with largest on bottom.



a_n = no of steps reqd for moving from A to B.

$a_1 = 1$

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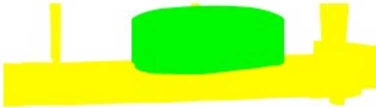
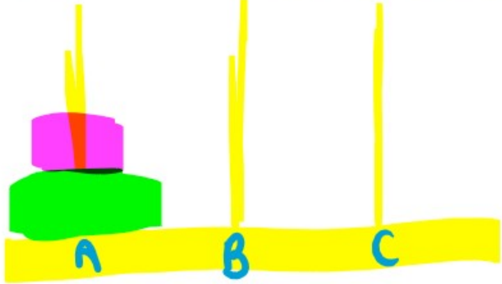
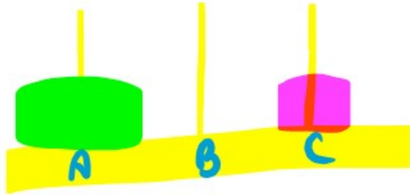
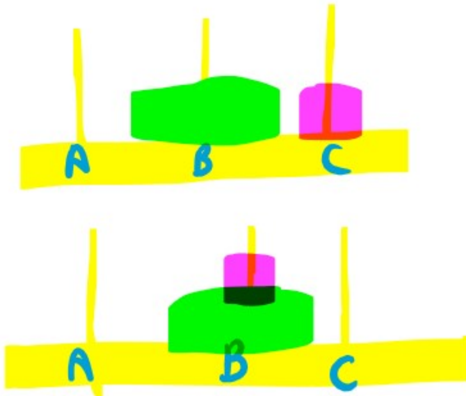




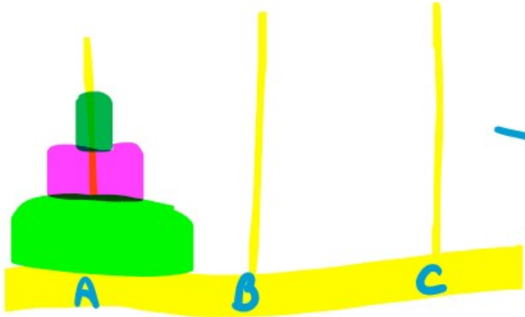
$a_2 = 3$

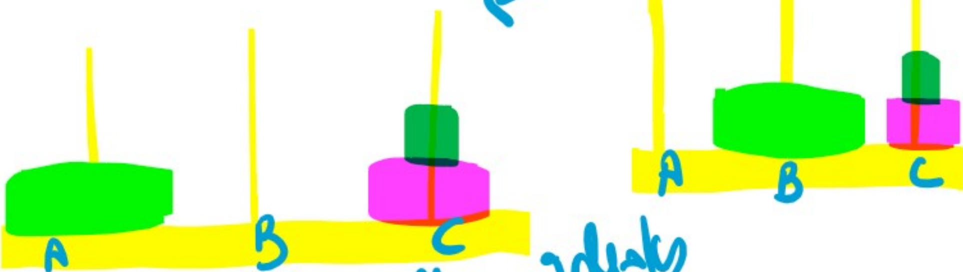




smaller A to B
Purple A to C
smaller B to C

1 step for shuffling largest one from A to B





3 steps for shuffling 2 disks from A to C

3 steps for shuffling 2 disks from C to B

$$a_3 = a_2 + 1 + a_2$$

$$\ldots + 1 + a_{n-1}$$

$$\underset{\text{from A to C}}{a_{n-1}}$$

$$a_n = \underset{\text{from A to C}}{a_{n-1}} + \underset{\substack{n^{th}\text{ disk}\\ \text{from A to B}}}{1} + \underset{\text{from C to B}}{a_{n-1}}$$

$$a_n = 2a_{n-1} + 1, \quad a_1 = 1$$

Find soln. by iterative approach

Try Example on Codeword Enumeration