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F-Test for Equality of two population variances:

Suppose we want to test (I) whether two independent samples x_i, $(i=1,2,\dots n_1)$ and y_j $(j=1,2,\dots n_2)$ have been drawn from the normal populations with the same variance σ^2 (say), or

(II) whether the two independent estimates of the population variances are homogeneous or not.

Under the null hypothesis (H_0) that

(I) $\sigma_x^2 = \sigma_y^2 = \sigma^2$ i.e. the population variances are equal

or, (II) Two independent estimates of the population variance are homogeneous, the statistic F is given by

$$F = \frac{S_x^2}{S_y^2} \quad \text{———} \quad (*)$$

where $S_x^2 = \frac{1}{n_1-1}\sum_{i=1}^{n_1}(x_i-\bar{x})^2$

and $S_y^2 = \frac{1}{n_2-1}\sum_{j=1}^{n_2}(y_j-\bar{y})^2$

are unbiased estimates of the common population variance σ^2 obtained from two independent samples and it follows F-distribution with (ν_1, ν_2) degree of freedom where $\nu_1 = n_1 - 1$ and $\nu_2 = n_2 - 1$

~~Example ①~~ Remark In (*), greater of the two variances S_x^2 and S_y^2 is to be taken in the numerator and n_1 corresponds to the greater variance.

Example (1) In one sample of 8 observations, the sum of squares of deviations of the sample values from the sample mean was 84.4 and in the other sample of 10 observations it was 102.6. Test whether this difference is significant at 5% level, given that the 5 percent point of F for $n_1 = 7$ and $n_2 = 9$ degrees of freedom is 3.29

Solution

Here $n_1 = 8$, $n_2 = 10$, $\Sigma(x-\bar{x})^2 = 84.4$

$\Sigma(y-\bar{y})^2 = 102.6$

$$S_x^2 = \frac{1}{n_1-1}\Sigma(x-\bar{x})^2 = \frac{1}{7}\times 84.4 = 12.057$$

$$S_y^2 = \frac{1}{n_2-1}\Sigma(y-\bar{y})^2 = \frac{1}{9}\times 102.6 = 11.4$$

Under H_0: $\sigma_x^2 = \sigma_y^2 = \sigma^2$ i.e, the estimates of σ^2 given by the samples are homogeneous, the test statistics is

$$F = \frac{S_x^2}{S_y^2} = \frac{12.057}{11.4} = 1.057$$

Tabulated $F_{0.05}$ for (7, 9) d.f is 3.29

Since calculated $F < F_{0.05}$; H_0 may be accepted at 5% level of significance.

Example (2) Two random samples gave the following results

Sample	Size	Sample mean	Sum of squares of deviations from the mean
1	10	15	90
2	12	14	108

Test whether the samples come from the same normal population at 5% level of significance.

[$F_{0.05}(9,11) = 2.90$, $F_{0.05}(11,9) = 3.10$, $t_{0.05}(20) = 2.086$, $t_{0.05}(22) = 2.07$

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<u>Solution</u>

A normal population has two parameters namely mean μ and variance σ^2. To test if two independent samples have been drawn from the same normal population, we have to test (i) the equality of population means, and (ii) the equality of population variances.

<u>Null hypothesis</u>: The two samples have been drawn from the same normal population, i.e.

$H_0: \mu_1 = \mu_2$ and $\sigma_1^2 = \sigma_2^2$

Equality of means will be tested by applying t-test and equality of variances will be tested by applying F-test. <u>Since t-test assumes $\sigma_1^2 = \sigma_2^2$, we shall first apply F-test and then t-test</u>

Given, $n_1 = 10$, $n_2 = 12$, $\bar{x}_1 = 15$
$\bar{x}_2 = 14$, $\Sigma(x_1-\bar{x}_1)^2 = 90$, $\Sigma(x_2-\bar{x}_2)^2 = 108$

<u>F-test</u>

$$S_1^2 = \frac{1}{n_1-1}\Sigma(x_1-\bar{x}_1)^2 = \frac{1}{9}\times 90 = 10$$

$$S_2^2 = \frac{1}{n_2-1}\Sigma(x_2-\bar{x}_2)^2 = \frac{108}{11} = 9.82$$

Since $S_1^2 > S_2^2$, under $H_0: \sigma_1^2 = \sigma_2^2$, the test statistics is

$$F = \frac{S_1^2}{S_2^2} = \frac{10}{9.82} = 1.018$$

Tabulated $F_{0.05}(9, 11) = 2.90$ (Given)

~~Since tabulated F is less t~~

Since calculated F is less than tabulated F, F is not significant. Hence null hypothesis of equality of population variances may be accepted.

Since $\sigma_1^2 = \sigma_2^2$, we can now apply t-test for testing $H_0 : \mu_1 = \mu_2$

t-test

: Under $H_0' : \mu_1 = \mu_2$ against the alternative hypothesis $H_1' : \mu_1 \neq \mu_2$, the test statistic is

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{S^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where

$$S^2 = \frac{1}{n_1 + n_2 - 2}\left[\Sigma(x_1 - \bar{x}_1)^2 + \Sigma(x_2 - \bar{x}_2)^2\right]$$

$$= \frac{1}{20}(90 + 108) = 9.9$$

$$\therefore t = \frac{15 - 14}{\sqrt{9.9\left(\frac{1}{10} + \frac{1}{12}\right)}} = \frac{1}{\sqrt{9.9 \times \frac{11}{60}}} = 0.742$$

Tabulated $t_{0.05}$ for 20 d.f $= 2.086$

Since $|t| < t_{0.05}$, it is not significant. Hence the hypothesis $H_0' : \mu_1 = \mu_2$ may be accepted.

So we conclude that the given samples have been drawn from the same normal population.

(3) Two independent samples of eight and seven items respectively had the following values of the variables

Sample 1 : 9 11 13 11 15 9 12 14

Sample 2 : 10 12 10 14 9 8 10

Do the two estimates of population variance differ significantly at 5% level of significance?

[Given $F_{0.05}(7, 6) = 4.21$]

~~Sol~~

Solution

x_1	x_1^2	x_2	x_2^2
9	81	10	100
11	121	12	144
13	169	10	100
11	121	14	196
15	225	9	81
9	81	8	64
12	144	10	100
14	196	$\Sigma x_2 = 73$	$\Sigma x_2^2 = 785$
$\Sigma x_1 = 94$	$\Sigma x_1^2 = 1138$		

$$s_1^2 = \frac{1}{n_1}\Sigma x_1^2 - \left(\frac{1}{n_1}\Sigma x_1\right)^2 = \frac{1}{8}\times 1138 - \left(\frac{1}{8}\times 94\right)^2$$
$$= 142.25 - 138.06 = 4.19$$

$$s_2^2 = \frac{1}{n_2}\Sigma x_2^2 - \left(\frac{1}{n_2}\Sigma x_2\right)^2 = \frac{1}{7}\times 785 - \left(\frac{1}{7}\times 73\right)^2$$
$$= 112.142 - 108.755$$
$$= 3.39$$

Now, $n_1 s_1^2 = (n_1 - 1) S_1^2$

$$\Rightarrow S_1^2 = \frac{n_1 s_1^2}{(n_1-1)} = \frac{1}{7}\times 8 \times (4.19) = 4.79$$

and $n_2 s_2^2 = (n_2 - 1) S_2^2$

$$\Rightarrow S_2^2 = \frac{n_2 s_2^2}{(n_2-1)} = \frac{1}{6}\times 7 \times 3.39 = 3.96$$

Null hypothesis $H_0 : \sigma_1^2 = \sigma_2^2 = \sigma^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2$

$$F = \frac{S_1^2}{S_2} = \frac{4.79}{3.96} = 1.21$$

Calculate F < Tabulated F $(1.21 < 4.21)$

So Null hypothesis is accepted

So the two estimates ab population variances do not debber sqnebicantly.