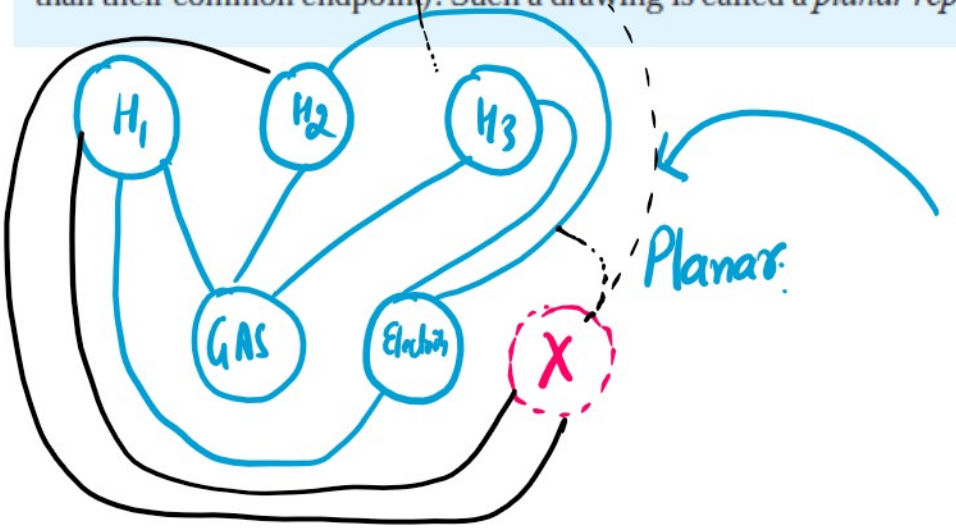
Lecture 31

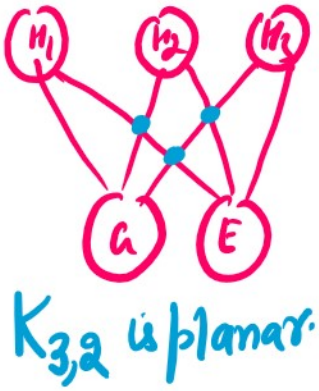
08 November 2021 16:57

# Unit V: Graphs theory II

## Planar Graph:

A graph is called *planar* if it can be drawn in the plane without any edges crossing (where a crossing of edges is the intersection of the lines or arcs representing them at a point other than their common endpoint). Such a drawing is called a *planar representation* of the graph.



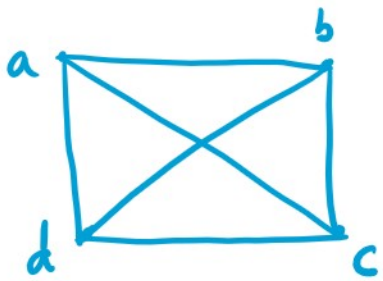



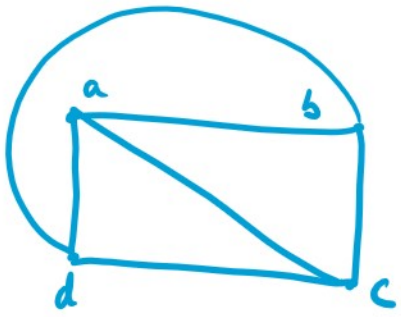

$K_{3,2}$ is planar.

$K_{3,3}$ is not planar

Q1. Determine whether the following graphs are planar?

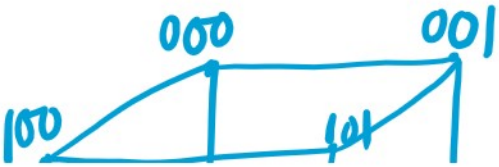
(a) $K_4$

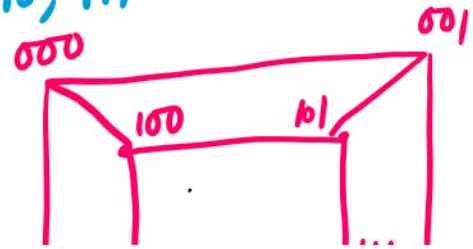


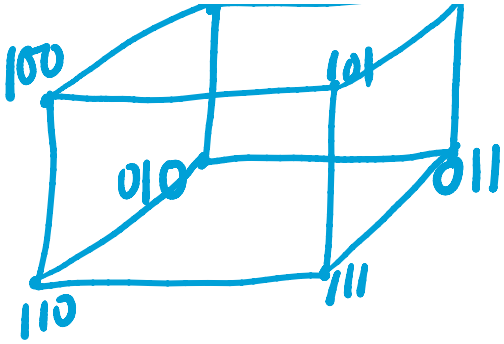



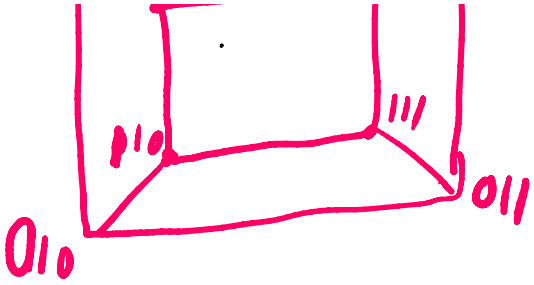

(b) $Q_3$

000, 001, 010, 100, 011, 101, 110, 111

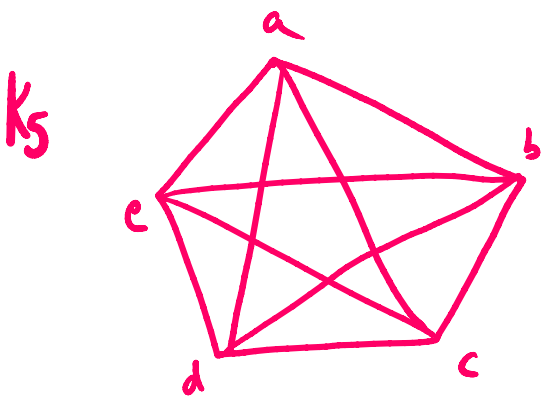


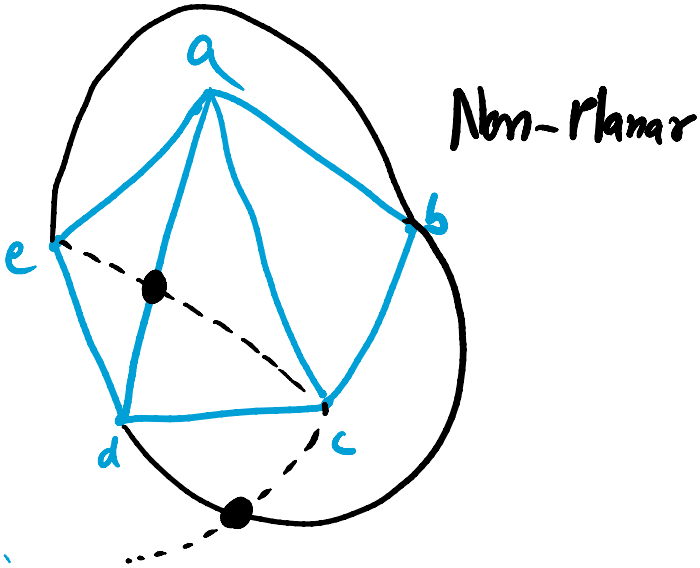





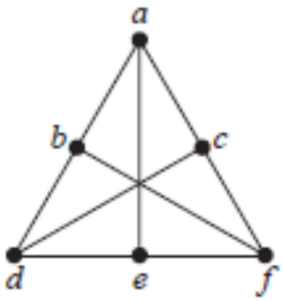



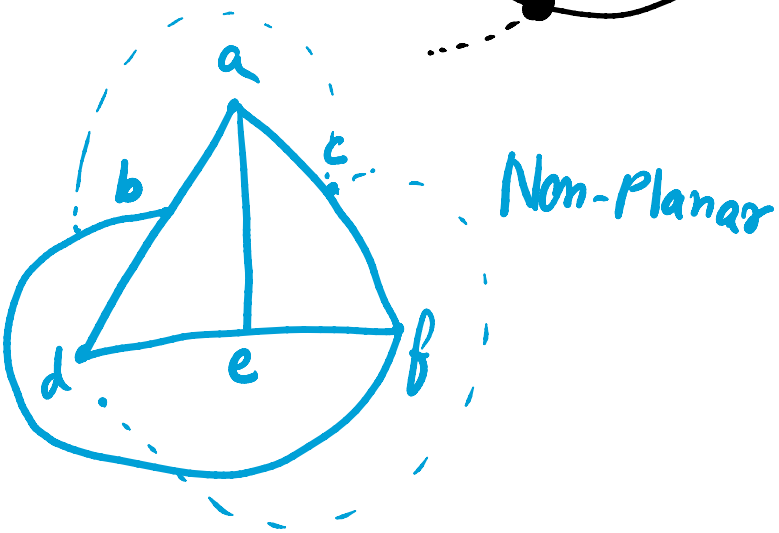

(c) $K_{3,3}$ → Non-Planar



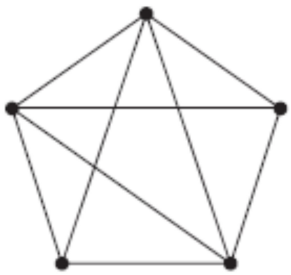



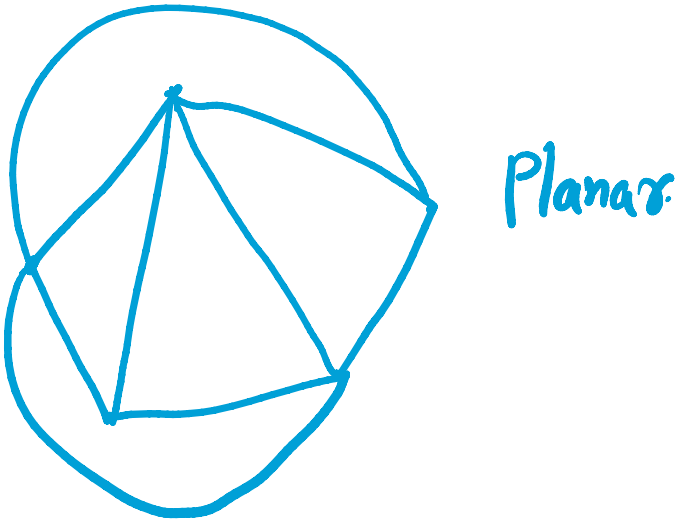

(d)



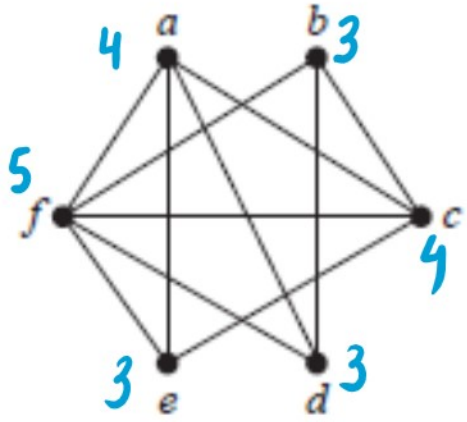

(e)

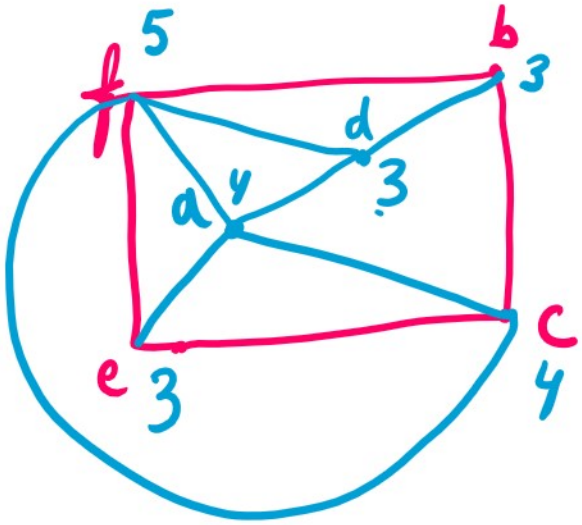





(f)

(f)

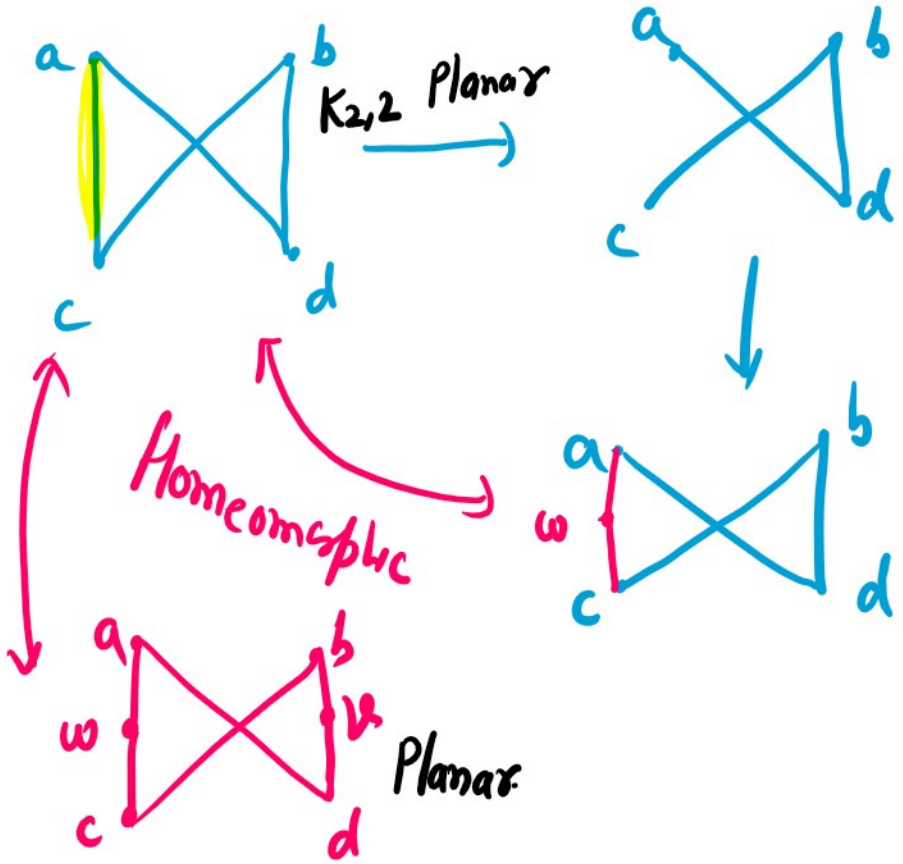



**Kuratowski's Theorem**

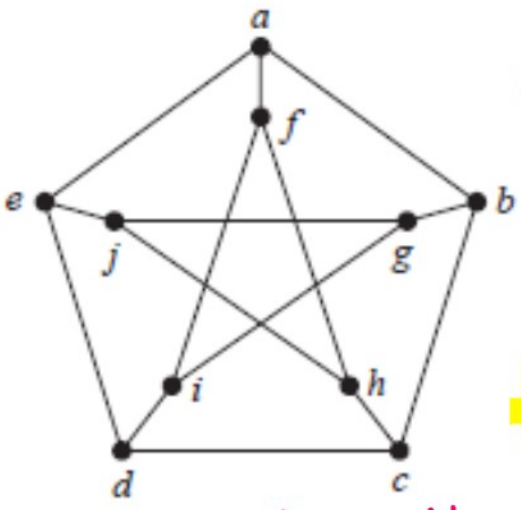
A graph is non-planar if it contains either of these two graphs $K_{3,3}$ or $K_5$.

If a graph is planar, so will be any graph obtained by removing an edge $\{u, v\}$ and adding a new vertex $w$ together with edges $\{u, w\}$ and $\{w, v\}$. Such an operation is called an **elementary subdivision**. The graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are called **homeomorphic** if they can be obtained from the same graph by a sequence of elementary subdivisions.



Q1. Determine whether the following graphs are planar?

(g)



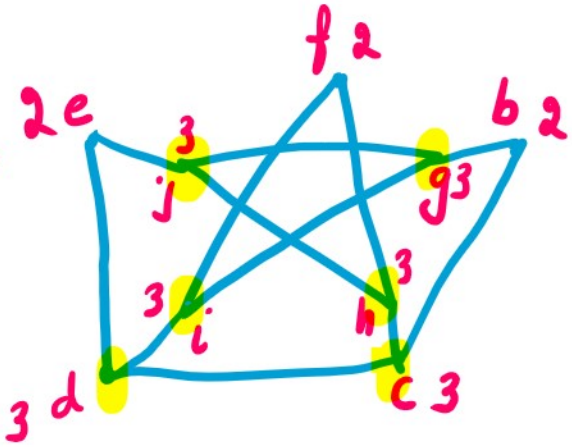

Petersen's Graph.
10 vertices of degree 3

$K_5$ → 5 vertices of degree 4.

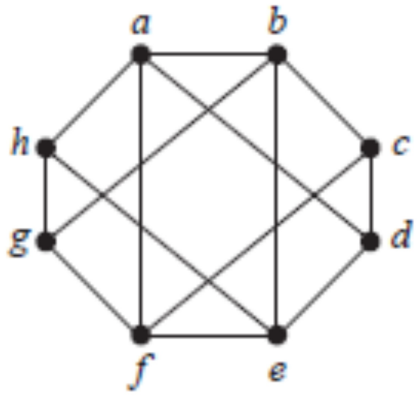
$K_{3,3}$ → 6 vertices of degree 3

3-regular graph

↓ Removing vertex a




(h)


a
b
h
c
g
d
f
e