CSE322
PROPERTIES OF REGULAR LANGUAGES

Lecture #12

Properties of
Regular Languages

For regular languages L_1 and L_2 we will prove that:

Union: $L_1 \cup L_2$

Concatenation: $L_1 L_2$

Star: $L_1 *$

Reversal: L_1^R

Complement: $\overline{L_1}$

Intersection: $L_1 \cap L_2$

Are regular Languages

We say: Regular languages are **closed under**

Union: $L_1 \cup L_2$

Concatenation: $L_1 L_2$

Star: $L_1{}^*$

Reversal: $L_1{}^R$

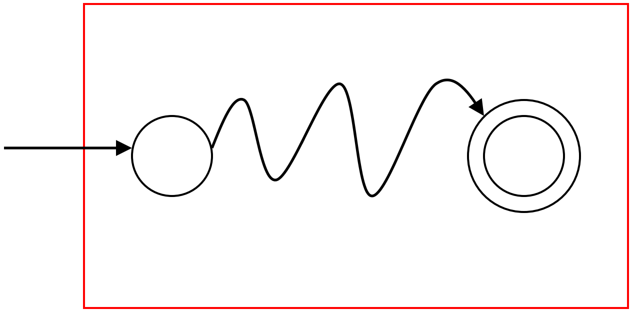
Complement: $\overline{L_1}$

Intersection: $L_1 \cap L_2$

Regular language L_1

$$L(M_1) = L_1$$

NFA M_1

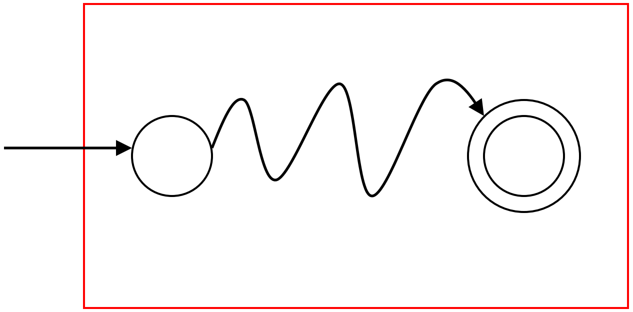


Single accepting state

Regular language L_2

$$L(M_2) = L_2$$

NFA M_2



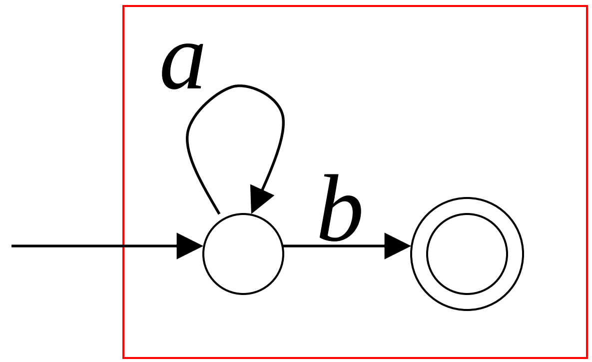
Single accepting state

Example

M_1

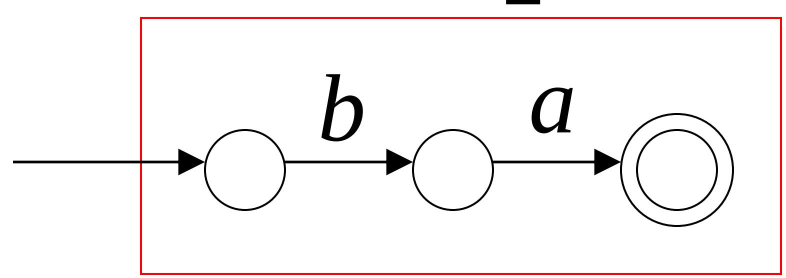
$n \geq 0$

$L_1 = \{a^n b\}$



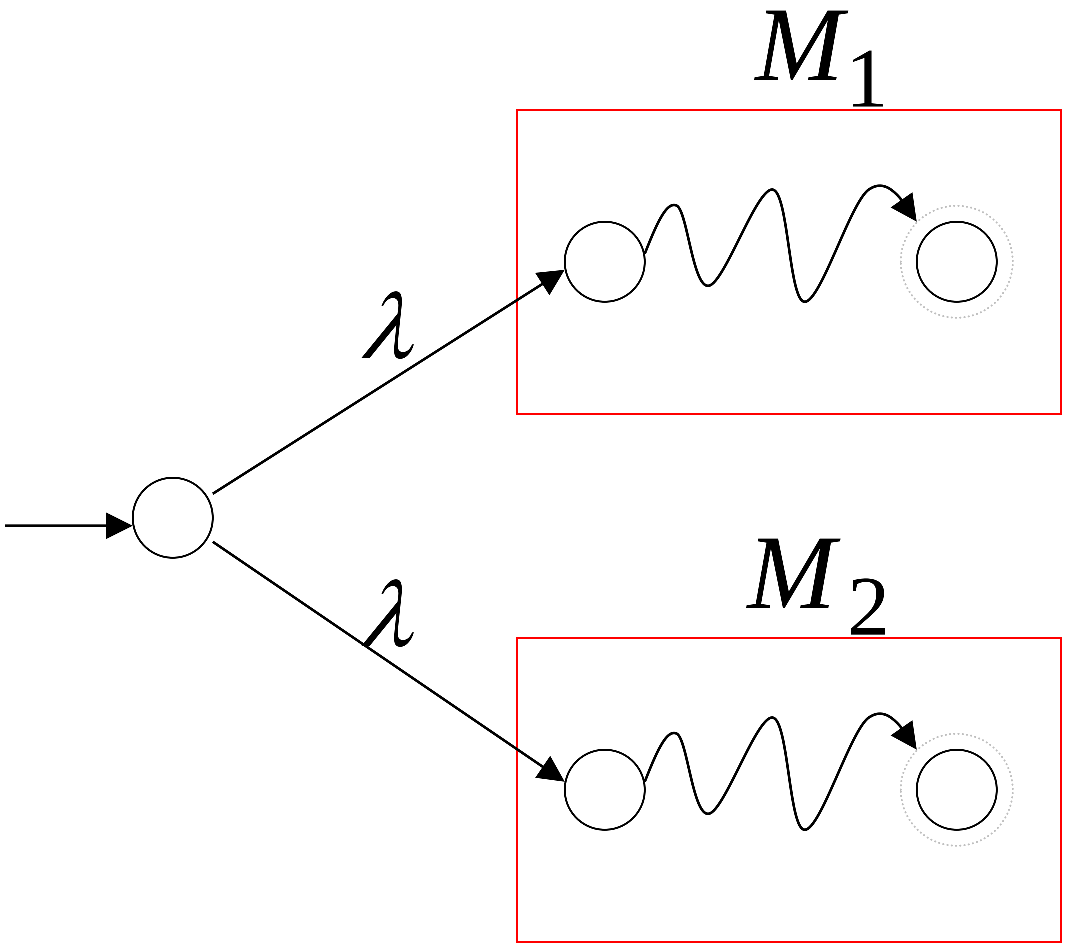
M_2

$L_2 = \{ba\}$



Union

NFA for $L_1 \cup L_2$

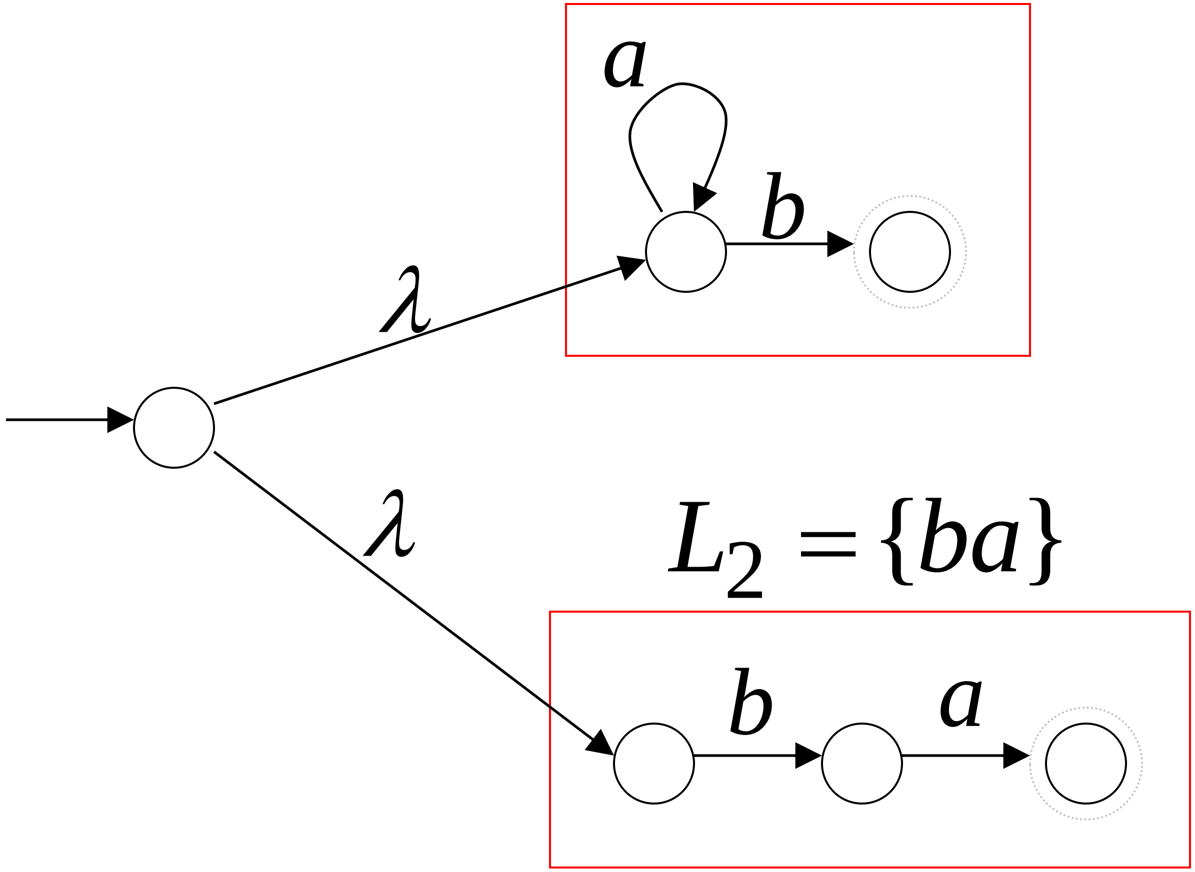


Example

NFA for $L_1 \cup L_2 = \{a^n b\} \cup \{ba\}$

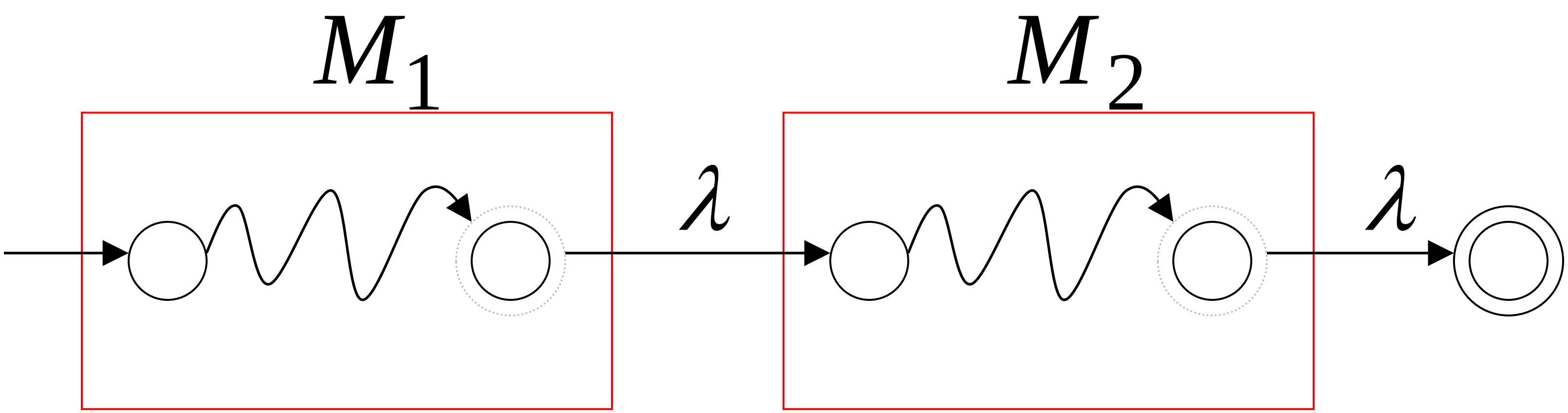
$L_1 = \{a^n b\}$

$L_2 = \{ba\}$



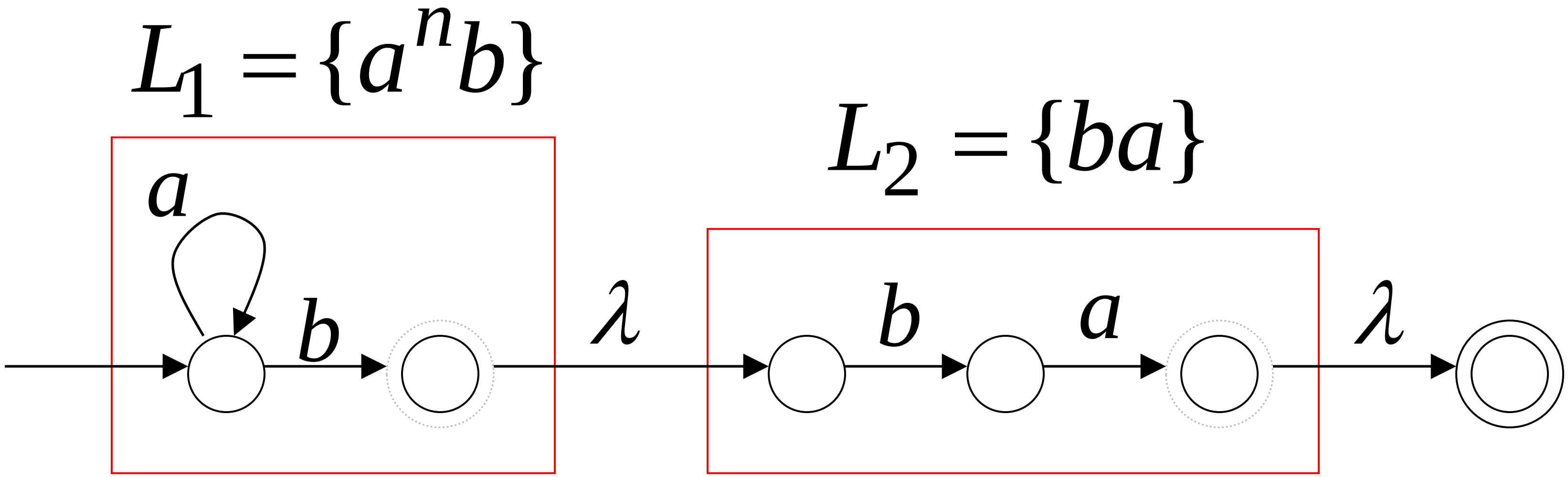
Concatenation

NFA for $L_1 L_2$



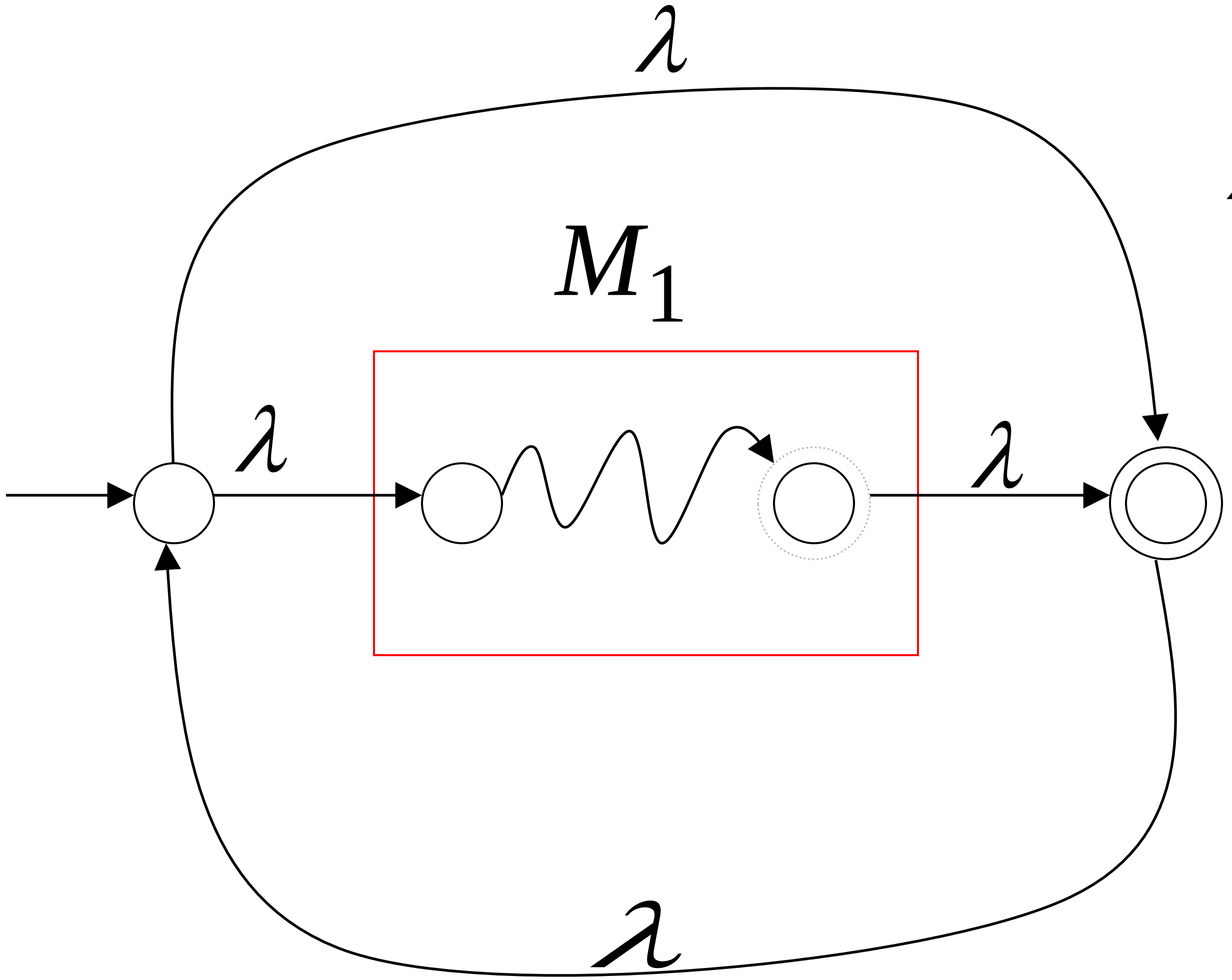
Example

NFA for $L_1 L_2 = \{a^n b\}\{ba\} = \{a^n bba\}$



Star Operation

NFA for $L_1{}^*$



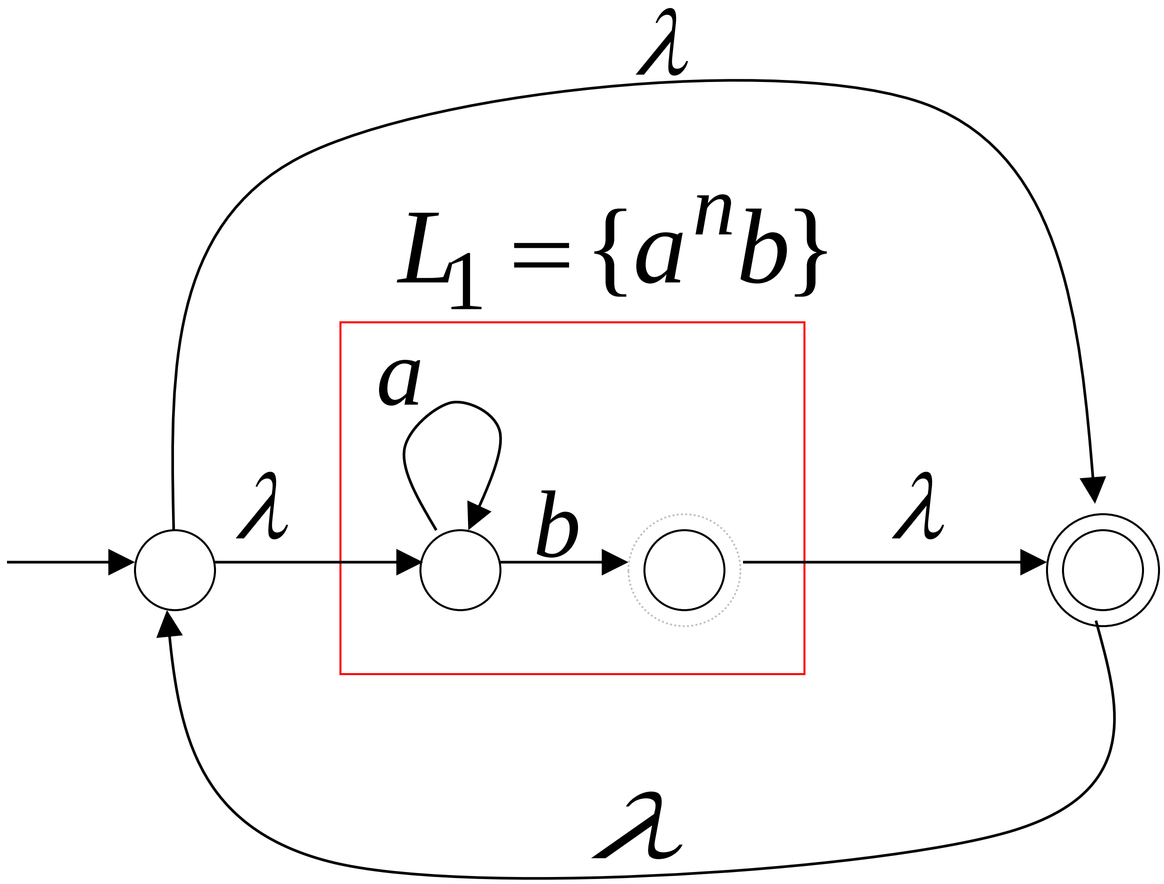
$\lambda \in L_1{}^*$

Example

NFA for $L_1^* = \{a^n b\}^*$

$w = w_1 w_2 \cdots w_k$

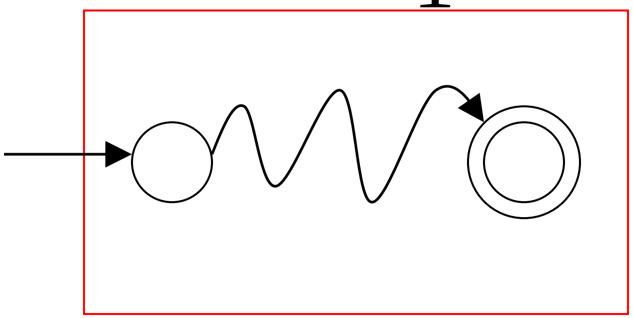
$w_i \in L_1$



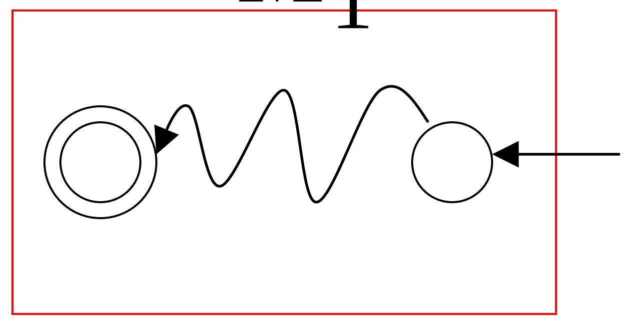
Reverse

NFA for L_1^R

L_1 M_1



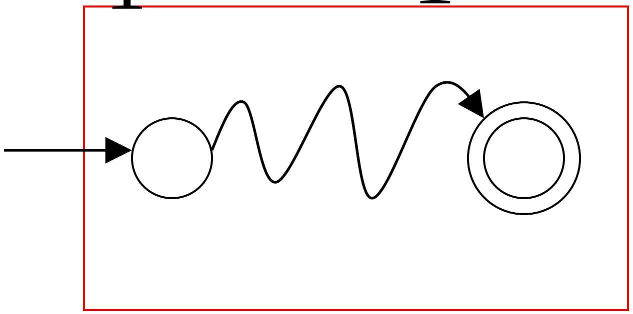
M_1'



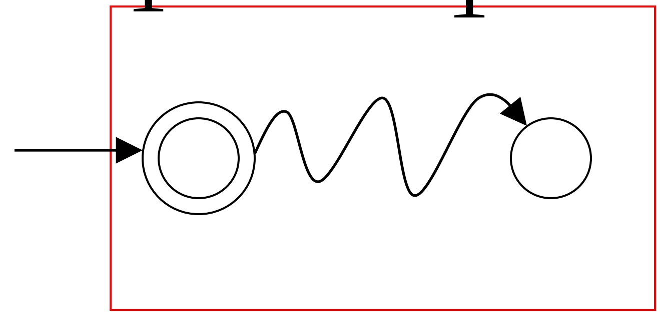
Homework 2

Complement

L_1 M_1



$\overline{L_1}$ M_1'



Homework 2

Intersection

L_1 regular

L_2 regular

We show

$L_1 \cap L_2$

regular

DeMorgan's Law:

$$\overline{L_1 \cap L_2} = \overline{\overline{L_1} \cup \overline{L_2}}$$

$L_1\,,\;L_2$ regular

$\Rightarrow$ $\overline{L_1}\,,\;\overline{L_2}$ regular

$\Rightarrow$ $\overline{L_1} \cup \overline{L_2}$ regular

$\Rightarrow$ $\overline{\overline{L_1} \cup \overline{L_2}}$ regular

$\Rightarrow$ $L_1 \cap L_2$ regular

Example

$L_1 = \{a^n b\}$ regular

$L_2 = \{ab, ba\}$ regular

$$\Rightarrow \quad L_1 \cap L_2 = \{ab\}$$

regular

Another Proof for Intersection Closure

Machine M_1

FA for L_1

Machine M_2

FA for L_2

Construct a new FA M that accepts $L_1 \cap L_2$

M simulates in parallel M_1 and M_2

States in M

q_i, p_j

State in M_1

State in M_2

FA M_1

$$q_1 \xrightarrow{a} q_2$$

transition

FA M_2

$$p_1 \xrightarrow{a} p_2$$

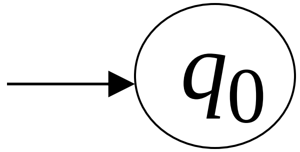
transition

FA M

$$q_1, p_1 \xrightarrow{a} q_2, p_2$$

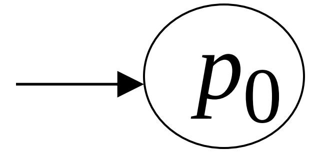
transition

FA M_1



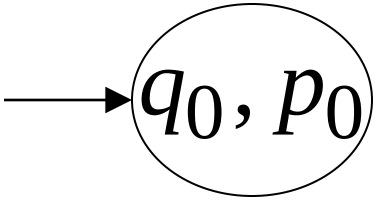
initial state

FA M_2

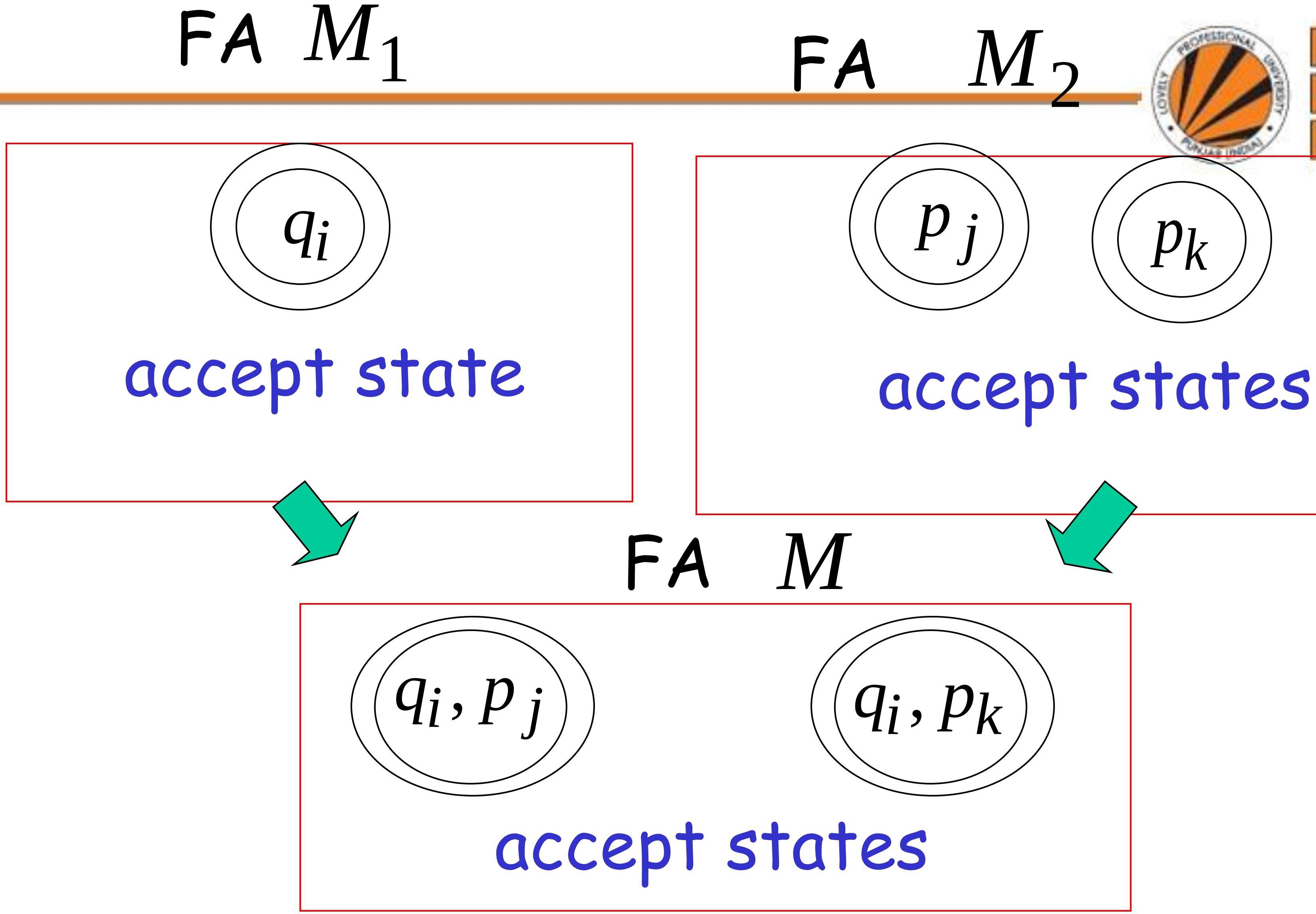


initial state

FA M



Initial state



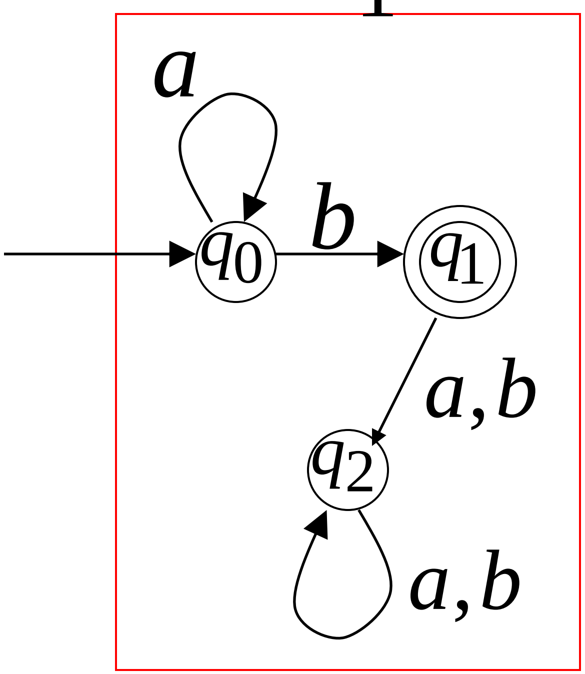
Both constituents must be accepting states

Example:

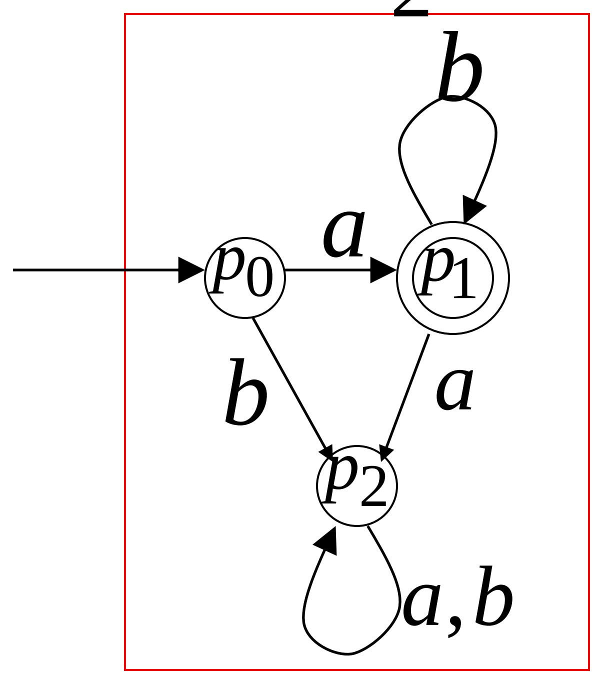
$$L_1 = \{a^n b\} \quad n \geq 0$$

$$L_2 = \{ab^m\} \quad m \geq 0$$

M_1

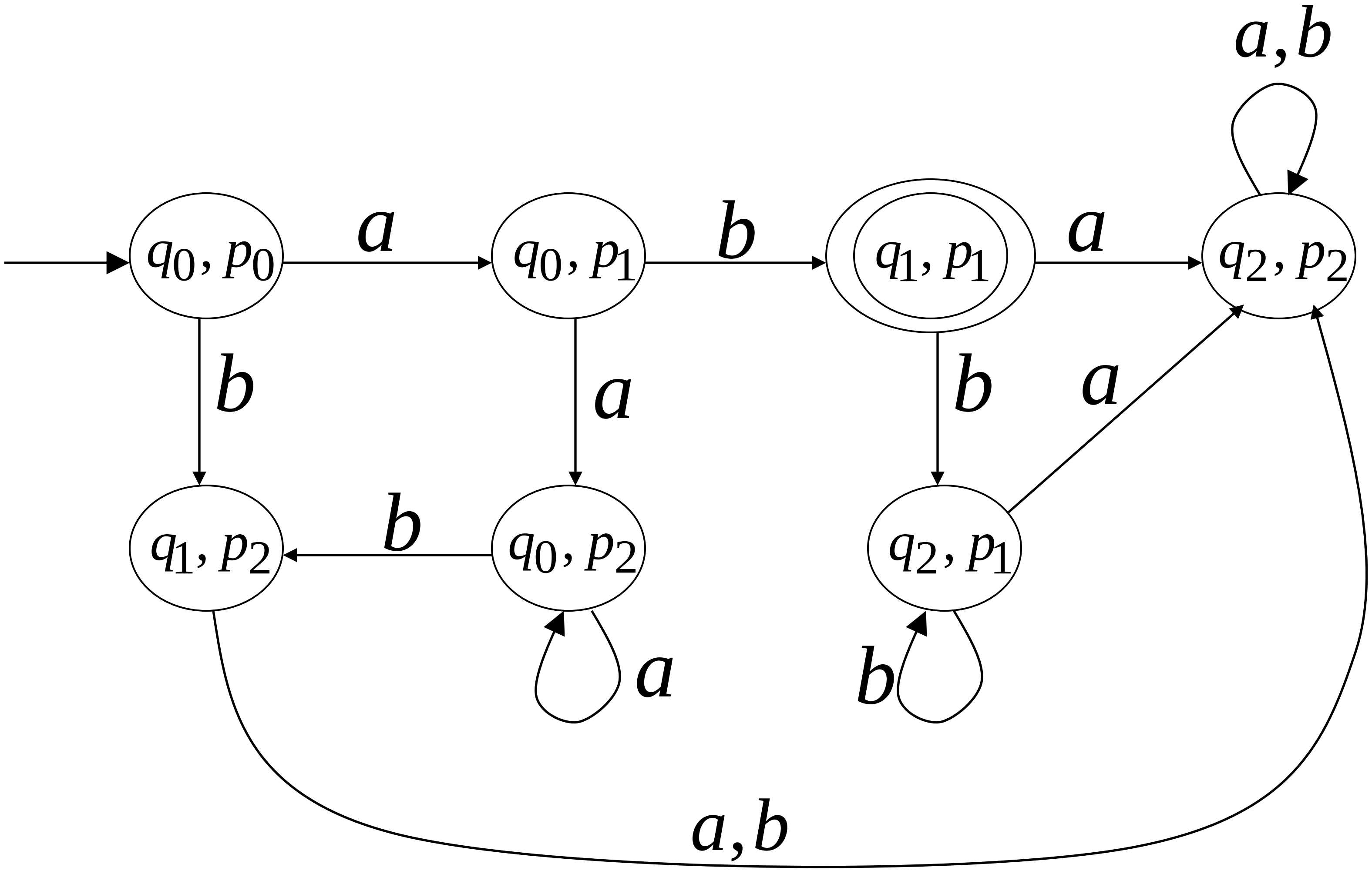


M_2



Automaton for intersection

$$L = \{a^n b\} \cap \{ab^n\} = \{ab\}$$



M simulates in parallel M_1 and M_2

M accepts string w if and only if

M_1 accepts string w and

M_2 accepts string w

$$L(M) = L(M_1) \cap L(M_2)$$