Time and Work

Contents:

1. Introduction:

Time and Work problems are most frequently asked problems in quantitative aptitude. To solve these problems very quickly, you should understand the concept of Time and Work and some shortcut methods.

2. **Basic Concepts:**

In solving the problems based on time and work, we need to calculate the following parameters.

(A) **Time** :- Time taken to complete an assigned job.

(B) **Individual time** :- Time needed by single person to complete a job.

(C) **Work:-** It is the amount of work done actually.

- If a man can do a piece of work in 5 days, then he will finish 1/5th of the work in one day.
- If a man can finish 1/5th of the work in one day then he will take 5 days to complete the work.
- If a man 5/6th of work in one hour then he will take 6/5 hours to complete the full work.
- If A works three times faster than B then A takes 1/3rd the time taken by B.

3. Rules and Tricks:

Rule 1: Universal Rule

This rule can be used in almost every problems.

- If M_1 persons can do W_1 work in D_1 days and M_2 persons can do W_2 works in D_2days then we can say
  $M_1D_1W_2 = M_2D_2W_1$
- If the persons work T_1 and T_2 hours per day respectively then the equation gets modified to
  $M_1D_1T_1W_2 = M_2D_2T_2W_1$
- If the persons has efficiency of E_1 and E_2 respectively then,
  $M_1D_1T_1E_1W_2 = M_2D_2T_2E_2W_1$

Example:

- 5 men can prepare 10 cycles in 6 days working 6 hours a day. Then in how many days can 12 men prepare 16 cycles working 8 hours a day?

Sol:

$M_1D_1T_1W_2 = M_2D_2T_2W_1$

$5 \times 6 \times 6 \times 16 = 12 \times D_2 \times 8 \times 10$

$D_2 = 3$ Days.

So they will complete the work in 3 days.

Rule 2:

If A can do a piece of work in n days, then
The work done by A in one day = 1/n

Example:

If A can repair 50 cycles in 5 days then A can repair 50/5 = 10 cycles in one day.

Rule 3:

If A can do a piece of work in X days and B can do the same work in Y days, then both of them working together will do the same work in

$$\frac{\mathbf{XY}}{\mathbf{X + y}} \text{ days.}$$

Example:

If A can do a piece of work in 10 days and B can do the same work in 15 days then how long will they take if they both work together?

Solution 1:

A can finish the work in D_1 = 10 days.

B can finish the work in D_2 = 15 days.

$$\text{A and B working together} = \frac{D1 * D2}{D1 + D2}$$

$$= 10 * 15/ 10 + 15$$

$$= 150/25 = 6 \text{ days.}$$

Solution 2: (LCM METHOD)

Let the total work be assumed as LCM(10,15)= 30 Units

Now to complete 30 units A takes 10 days

To complete 30 units B takes 15 days

Units done in 1 day by A = 3

Units done in 1 day by B = 2

Units done in 1 day by A & B = 5

To complete 30 units they will take 30/5 = 6days

Rule 4:

If A is twice as good a workman as B, then A will take half of the time taken by B to complete a piece of work.

Example:

A is twice as good a workman as B. Together, they finish the work in 14 days. In how many days can it be done by each separately?

Solution 1:

Let's assume that A alone can finish the work in x days.

It is given that A is twice as good a workman as B so B alone can finish the work in 2x days

$= x * 2x / x + 2x = 2x^2/3x = 14.$

So x = 21 days.

So A can finish the work in 21 days and B can finish the work in 42 days.

Solution 2:

If A and B working together can finish a work in X days and A is K times efficient than B, then the time taken by

(i) A, working alone, to complete the work is

$$\left(\frac{\mathbf{K+1}}{\mathbf{K}}\right)\mathbf{X} \quad \text{days.}$$

Here, X = 14, K = 2

Hence , = (3/2)14

= 21 days.

(ii) B, working alone, to complete the work is

(K+1)X days.

Hence,

= 3 * 14 = 42 days.

Rule 5:

If A can finish a work in X days and B is K times efficient than A, then the time taken by both A and B working together to complete the work is

$$\frac{\mathbf{X}}{\mathbf{1 + K}} \text{ days.}$$

Example:

If A can finish a work in 20 days and B is 3 times efficient than A, then the time taken by both A and B working together to complete the work is

Solution: = 20 / (1+3) = 5 days.

Rule 6:

If A, B and C, while working alone, can complete a work in X, Y and Z days respectively, then they will together complete the work in

$$\frac{\mathbf{XYZ}}{\mathbf{XY + YZ + ZX}} \text{ days.}$$

Example:

A, B and C can complete a piece of work in 5, 10 and 15 days respectively. Working together, they will complete the same work in:

Solution 1:

By using the formula $= \dfrac{5 * 10 * 15}{(5*10)+(10*15)+(5*15)}$

$= 30/11$ days.

Solution 2: (LCM METHOD)

Let the total work be assumed as LCM(5,10,15)= 30 Units

Now to complete 30 units A takes 5 days

To complete 30 units B takes 10 days

To complete 30 units C takes 15 days

Units done in 1 day by A = 6

Units done in 1 day by B = 3

Units done in 1 day by C = 2

Units done in 1 day by A+B+C = 11

To complete 30 units they will take 30/11 = $2\frac{8}{11}$days

Rule 7:

Two persons A and B, working together, can complete a piece of work in X days. If A, working alone, can complete the work in Y days, then B, working alone, will complete the work in

$$\frac{\mathbf{XY}}{\mathbf{Y - X}} \text{ days.}$$

Example:

Jack and Jill together can do a piece of work in 10 days. Jack alone can do it in 15 days. In how many days can Jill alone do it?

Solution 1:

By using formula,

$$= \frac{10 * 15}{15 - 10}$$

$$= 30 \text{ days.}$$

Solution 2: (UNITARY METHOD)

Let the total work be assumed as LCM(10,15)= 30 Units

Now to complete 30 units Jack and Jill takes 10 days

To complete 30 units Jack takes 15 days

Units done in 1 day by Jack + Jill = 3

Units done in 1 day by Jack = 2

Units done in 1 day by Jill = 1

To complete 30 units they will take 30/1 = 30 days

Rule 8:

If A and B, working together, can finish a piece of work in X days, B and C in Y days, C and A in Z days, then

(a) A, B and C working together, will finish the job in $\mathbf{\dfrac{2XYZ}{XY + YZ + ZX}}$ days.

(b) A alone will finish the job in $\mathbf{\dfrac{2XYZ}{XY + YZ - ZX}}$ days.

(c) B alone will finish the job in

$$\frac{\mathbf{2XYZ}}{\mathbf{YZ + ZX - XY}} \text{ days.}$$

(d) C alone will finish the job in

$$\frac{\mathbf{2XYZ}}{\mathbf{ZX + XY - YZ}} \text{ days.}$$

Note: This type can be solved under unitary method also.

Rule 9:

If A working alone takes X days more than A and B together, then B working alone takes Y days more than A and B together, then the number of days taken by A and B, working together, to finish a job is given by **$\sqrt{XY}$**

Rule 10:

If the number of men are changed in the ratio of m:n, then the time taken to complete the work will change in the ratio n:m.

Rule 11:

If A is K times more efficient than B and is therefore able to finish a work in X days less than B, then

B, working alone, can finish the work in

$$\frac{\mathbf{KX}}{\mathbf{K - 1}} \text{ days.}$$

Example:

A is thrice as good as workman as B and takes 10 days less to do a piece of work than B takes. Find the time in which B alone can complete the work.

Solution 1:

Here, k = 3 and X = 10.

Therefore ,Time taken by B, working alone, to complete the work

= (3 * 10) / (3-1) = 15 days.

Solution 2:

Efficiency ratio = A : B

3 : 1

Time ratio = 1 : 3 (Inverse of efficiency)

Diff = 2 parts

2 parts = 10 days

A = 1 part = 5 days

B = 3 parts = 15 days.

Rule 12:

If A is K times more efficient than B and is therefore able to finish a work in X days less than B, then

A, working alone, can finish the work in

$$\frac{\mathbf{X}}{\mathbf{K - 1}} \text{ days.}$$

Rule 13:

If A is K times more efficient than B and is therefore able to finish a work in X days less than B, then

A and B, working together, can finish the work in $\dfrac{\mathbf{KX}}{\mathbf{K^2 - 1}}$ days.

Rule 14:

If A can complete a/b part of work in X days, then c/d part of the work will be done in

$$\frac{\mathbf{b * c * X}}{\mathbf{a * d}} \text{ days.}$$


Thank you