Number Systems

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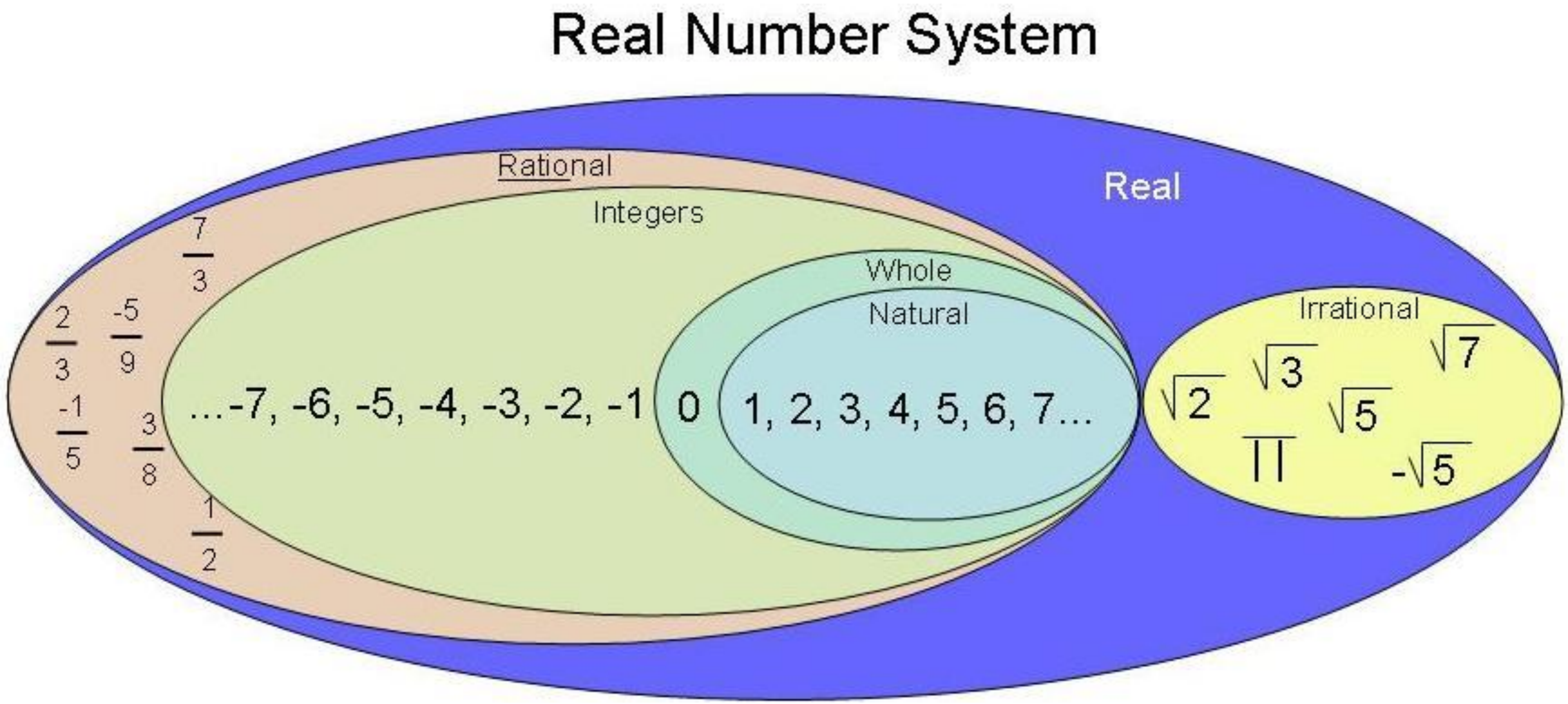
1. Types of numbers



What is a rational number?

A **rational number** is a number which can be expressed in the form of **p/q** where **p & q are integers** and **q≠0.**

Numbers other than rational numbers are called **irrational numbers** which is **non-terminating and non-repeating.**

What are prime numbers?

Prime number is a number which has **exactly two factors** which is **1 and itself**.

Numbers other than prime is called **composite numbers** which has **more than two factors**.

Divisibility Rules

A number is divisible by

- **2** If the last digit is even.
- **3** If the sum of the digits is divisible by 3.
- **4** If the last two digits of the number divisible by 4.
- **5** If the last digit is a 5 or a 0.
- **6** If the number is divisible by both 3 and 2.
- **7** If the number formed by subtracting twice the last digit with the number formed by;
  rest of the digits is divisible by 7. Example: 343. 34-(3x2) = 28 is divisible by 7.
- **8** If the last three digits form a number divisible by 8.
- **9** If the sum of the digits is divisible by 9.
- **10** If the last digit of number is 0.
- **11** If the difference between sum of digits in even places and the sum of the digits in odd places is 0 or divisible by 11.
  Example: 365167484
- (3+5+6+4+4) – (6+1+7+8) = 0
- ∴ 365167484 is divisible by 11.
- **12** If the number is divisible by both 3 and 4.

Any other numbers can be written in terms of the numbers whose divisibility is already known.

Example: 15 = 3 x 5
18 = 2 x 9
33 = 3 x 11

Note: The numbers expressed should be co-prime (i.e., the HCF of the two numbers should be 1)

Example: 40 = 4 x 10 is wrong because HCF(4,10) is 2.
∴ 40 = 5 x 8 because HCF(5,8) is 1.

Q. What should come in place of x if 563x5 is divisible by 9?

Q. What should come in place of x if 4857x is divisible by 88?

Unit Digit Concept

	Power			
Base	1	2	3	4
2	2	4	8	6
3	3	9	7	1
7	7	9	3	1
8	8	4	2	6
4	4	6		
9	9	1		

Number	Cyclicity
1	1
2	4
3	4
4	2
5	1
6	1
7	4
8	4
9	2
10	1

Choose the nth value in the cycle if the remainder is n except for the last value whose remainder should be 0.

Note: *The last digit of an expression will always depend on the unit digit of the values.*

Example: *The unit digit of 123 x 456 x 789 = 3 x 6 x 9*

= 18 x 9

= 8 x 9

= 2

Example 2: *What is the unit digit of (123)^42?*

The unit digit pattern of 3 repeats four times. So find the remainder when the power value is divided by 4.

*42/4= R(**2**)*

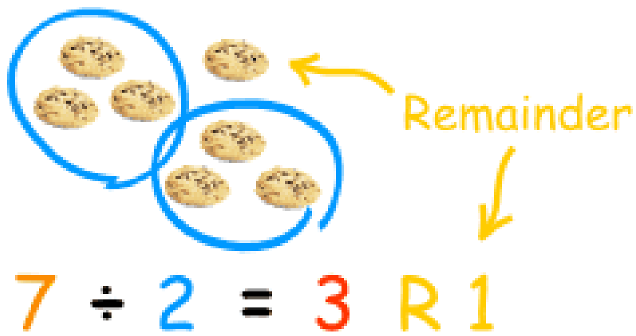
2^{nd} value in 3 cycle is 9.

∴ Unit digit of (123)^42 is 9

. Remainder theorem

Type 1: *Numerator in terms of powers*

The remainder pattern should be found starting from the power of 1. The same procedure should be followed as done in the unit digit concept.



Example: What is the remainder when 2^202 is divided by 7?

2^1/7 = R(2)
2^2/7= R(4)
2^3/7 = R(1)

The next three remainder values will be the same. i.e., The remainder pattern is 2,4,1, 2,4,1, 2,4,1.....

The size of the pattern is 3.

Now divide the power by number of repeating values (3) to choose the remainder.

Choose the nth value in the cycle if the remainder is n except for the last value whose remainder should be 0.

202/3 = R(**1**).

The 1st value in the cycle is 2.

Note: *While finding the remainder pattern if the remainder becomes 1, then the process can be stopped as the it will always repeat after 1.*

∴ 2^202/7 = R(2)

Note: *While finding the remainder pattern if the remainder becomes 1, then the process can be stopped as the it will always repeat after 1.*

Type 2: *Different numerator values*

Replace each of the values of the numerator by its remainder when divided by the denominator and simplify.

Example: What is the remainder when 13 x 14 x 16 is divided by 6.

13/6 = R(1) ∴ replace 13 by 1

Similarly replace 14 and 16 by 2 and 4 respectively.

∴ (13 x 14 x 16)/6 = (1 x 2 x 4)/6

= 8/6

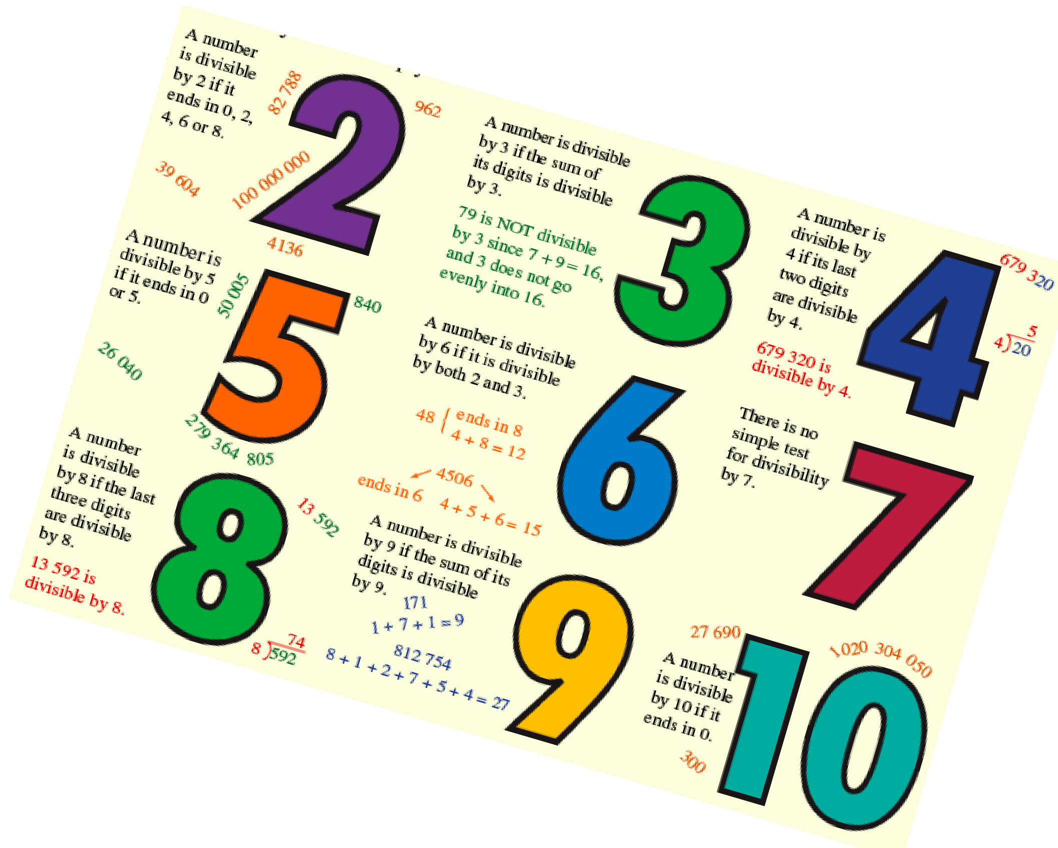
= R(2)

Note: *Do not cancel any numerator value with the denominator value as the remainder will differ.*

R(6/4) ≠ R(3/2)
6/4 = R(2)
But 3/2 = R(1)

Q) What is the remainder when 3 to the power 7 is divided by 8?

A)3
B)4
C)5
D)7
E)none



Q) Remainder when 17^23 is divided by 16?

A)1

B)2

C)3

D)4

Factors

Factors of a number are the values that divides the number completely.

Example: Factors of 10 are 1, 2, 5 and 10.

Multiple of a number is the product of that number and any other whole number.

Example: multiples of 10 are 10, 20, 30,…..

- ***i) Number of factors:***

- **Example:** 3600

- **Step 1:** Prime factorize the given number
- $3600 = 36 \times 100$
- $= 6^2 \times 10^2$
- $= 2^2 \times 3^2 \times 2^2 \times 5^2$
- $= 2^4 \times 3^2 \times 5^2$

- **Step 2:** Add 1 to the powers and multiply.
- $(4+1) \times (2+1) \times (2+1)$
- $= 5 \times 3 \times 3$
- $= 45$
- ∴ Number of factors of 3600 is 45.

ii) Sum of factors:

Example: 45

Step 1: Prime factorize the given number

$45 = 3^2 \times 5^1$

Step 2: Split each prime factor as sum of every distinct factors.

$(3^0 + 3^1 + 3^2) \times (5^0 + 5^1)$

The following result will be the sum of the factors

$= 78$

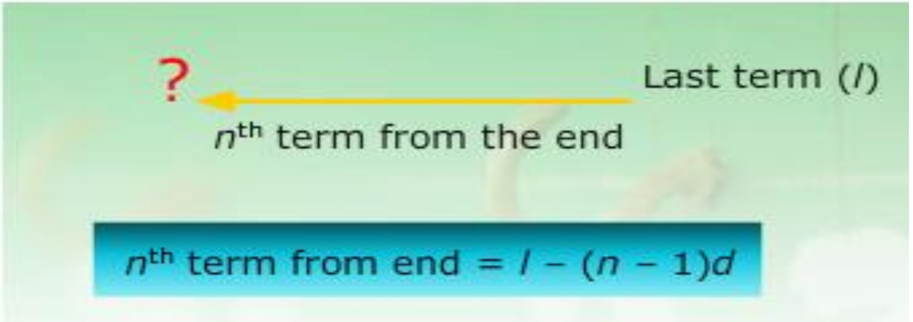
Q.How many zeros are there in 100!?

ARITHMETIC PROGRESSION

An Arithmetic Progression (A.P.) is a sequence in which the difference between any two consecutive terms is constant.

Let a = first term, d = common difference

- Then nth term $a_n = a + (n - 1)d$



Sum of an A.P

The sum of n terms of an A.P. whose first term is a and common difference is d, is given by

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

The sum of n terms of an A.P. whose first term is a and last term is l is given by the formula:

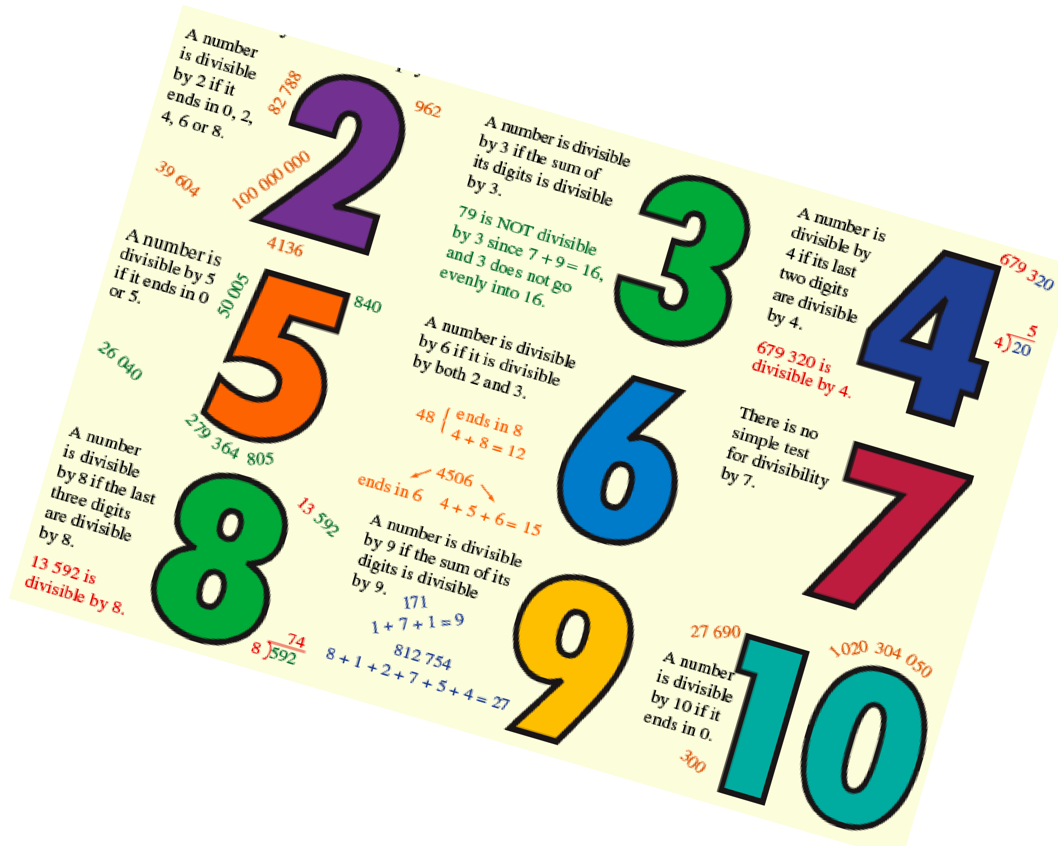
$$S_n = \frac{n}{2}\left[a + l\right]$$

AM (Arithmetic mean)

If a, b, c are in AP then the arithmetic mean is given by

$b = (a+c)/2$

Q) Find the sum of the series 5,8,11,…….. 221



GEOMETRIC PROGRESSION

A geometric sequence are powers r^k of a fixed number r, such as 2^k and 3^k. The general form of a geometric sequence is

The n-th term of a geometric sequence with initial value a and common ratio r is given by

$$a_n = a\,r^{n-1}.$$

Such a geometric sequence also follows the recursive relation

$a_n = r\,a_{n-1}$ for every integer $n \geq 1.$

Sum= $\dfrac{a(1-r^m)}{1-r}$

GM (Geometric mean)

If a, b, c are in GP Then the GM is given by

$b = \sqrt{ab}$

Q) Find the sum of the series 2, 4, 8, 16…. 256.

Next Class Average