Lecture 18

28 September 2021 11:06

Q30. Use Generating function to solve recurrence relation $a_k = a_{k-1} + 4^{k-1},\ a_0 = 2.$

$$a_{k+1} - a_k = 4^k \qquad G(x) = \sum_{k=0}^{\infty} a_k x^k$$

$$\sum_{k=0}^{\infty} a_{k+1} x^k - \sum_{k=0}^{\infty} a_k x^k = \sum_{k=0}^{\infty} 4^k x^k$$

$$a_1 + a_2 x + a_3 x^2 + \dots \quad - G(x) = (1-4x)^{-1}$$

$$\frac{a_1 x + a_2 x^2 + a_3 x^3 + \dots + a_0 - a_0}{x} - G(x) = \frac{1}{1-4x}$$

$$\frac{G(x) - 2}{x} - G(x) = \frac{1}{1-4x}$$

$$G(x) - 2 - xG(x) = \frac{x}{1-4x}$$

$$(1-x)G(x) = \frac{x}{1-4x} + 2 = \frac{x + 2 - 8x}{1-4x}$$

$$G(x) = \frac{2-7x}{(1-4x)(1-x)} = \frac{A}{1-4x} + \frac{B}{1-x}$$

$$A = \frac{2-\frac{7}{4}}{1-\frac{1}{4}} = \frac{1}{3}$$

$$B = \frac{2-7}{1-4} = \frac{5}{3}$$

$$G(x) = \frac{\frac{1}{3}}{\dots} \quad \frac{5}{\dots} \quad \frac{1}{\dots}(1-4x)^{-1} + 5(1-x)^{-1}$$

$$G(x) = \frac{\frac{1}{3}}{1-4x} + \frac{\frac{5}{3}}{1-x} = \frac{1}{3}(1-4x)^{-1} + \frac{5}{3}(1-x)^{-1}$$

$$a_k = \frac{1}{3}(4)^k + \frac{5}{3}(1)^k \Rightarrow a_k = \frac{4^k+5}{3}$$

Q31. Use Generating function to solve recurrence relation $a_k = 5a_{k-1} - 6a_{k-2}, a_0 = 6, a_1 = 30$.

$$a_{k+2} - 5a_{k+1} + 6a_k = 0$$

$$\sum_{k=0}^{\infty} a_{k+2}x^k - \sum_{k=0}^{\infty} 5a_{k+1}x^k + \sum_{k=0}^{\infty} 6a_kx^k = 0$$

$a_2 + a_3x + a_4x^2 + \dots$ $\quad$ $a_1 + a_2x + a_3x^2 + \dots$

$$\frac{G(x) - a_0 - a_1x}{x^2} - 5\frac{G(x) - a_0}{x} + 6G(x) = 0$$

$$\frac{G(x) - 6 - 30x}{x^2} - 5\left(\frac{G(x) - 6}{x}\right) + 6G(x) = 0$$

$$\frac{G(x) - 6 - 30x - 5x(G(x) - 6) + 6x^2G(x)}{x^2} = 0$$

$$(1 - 5x + 6x^2)G(x) - 6 - 30x + 30x = 0$$

$$G(x) = \frac{6}{(1-5x+6x^2)} = \frac{6}{(1-2x)(1-3x)}$$

$$G(x) = \frac{\frac{6}{1-\frac{3}{2}}}{1-2x} + \frac{\frac{6}{1-\frac{2}{3}}}{1-3x} = \frac{-12}{1-2x} + \frac{18}{1-3x}$$

$$\boxed{(1-2x)^{-1} \quad (1-3x)^{-1}}$$

$$a_k = -12(2)^k + 18(3)^k$$

$$a_k = -12(2)^k + 18(3)^k$$

Unit III: COUNTING TECHNIQUES AND RELATIONS

Ch-6 Ch-9

Relations

Let A and B be sets. A binary relation from A to B is a subset of $A \times B$.

For $A = \{0, 1, 2, 3, 4\}, B = \{0, 1, 2, 3\}$. Write $A \times B$

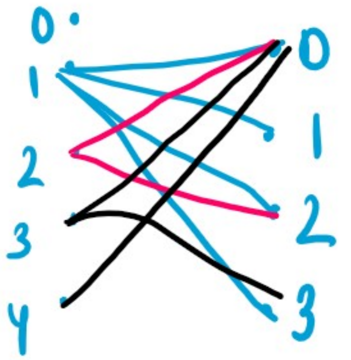
$$A \times B = \{(0,0), (0,1), (0,2), (0,3), (1,0), (1,1), (1,2), (1,3), (2,0), (2,1), (2,2), (2,3), (3,0), (3,1), (3,2), (3,3), (4,0), (4,1), (4,2), (4,3)\}$$

$$R_1 = \{(0,1), (0,2), (0,3), (1,2), (1,3), (2,3)\}$$

Q1. List the ordered pairs in the relation R from A to B and also find graphical and tabular representation of relation, where $(a, b) \in R$ iff

(i) $a | b$ a divides b

$$\{(1,0), (1,1), (1,2), (1,3), (2,0), (2,2), (3,0), (3,3), (4,0)\}$$



$\xrightarrow{\quad} B$

A ↓	0	1	2	3
0				
1	X	X	X	X
2	X		X	
3	X			X
4				

(ii) $\gcd(a, b) = 1$ $A = \{0, 1, 2, 3, 4\}$ $B = \{0, 1, 2, 3\}$

$\gcd(a,b)=1$

$\{(0,1),(1,0),(1,2),(1,3),(2,1),(2,3),$
$(3,1),(3,2),(4,1),(4,3)\}$

- If A has m elements and B has n elements, then no. of elements in $A \times B =$ ......mn......
- If A has m elements and B has n elements, then no. of relations from A to B are........2^{mn}
- A relation on set A is the relation from A to A. No. of elts $A \times A = m^2$
- No. of relations on A=........2^{m^2}

Q2. If A has 5 elements and B has 3 elements, then

(i) $n(A \times B) =$ 15 $A \times B \neq B \times A$

(ii) $n(B \times A) =$ 15

(iii) $n(A \times A) =$ 25

(iv) no. of relations from A to B= 2^{15}

(v) no. of relations on A= 2^{25}