Basics of matrix

$$A = \begin{bmatrix} 1 & 5 & 9 \\ 2 & 6 & 10 \\ 3 & 7 & 11 \\ 4 & 8 & 12 \end{bmatrix}$$

list, list, list, list

(4×3) dimension.

It is called List of Lists means there will be multiple lists.

Que If we want to fetch the particular no from the matrix? How we can do this.

Ans: we can fetch that by the help of the variable name and how we can give the variable name to all the elements of Matrix

$$A_{ij} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ & \text{and so on.} & \end{bmatrix}$$

Suppose if I want this 12 No. for this we need of rows no. and column no. that is A_{43} to get this No.

TRANSPOSE OF the MATRIX

this concept we will use in Machine learning.

$$A = [a_1, a_2, a_3, a_4, a_5]$$

$$A^T = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix}$$

means it will be converted from Row Vector to column Vector, and vice versa.

1. Addition
2. Subtraction
3. Scalar product
4. Inner product
5. outer product

How do you apply these operations?
How do you multiply the multiplication of scalar to the vector.
What is inner product and how to do product.

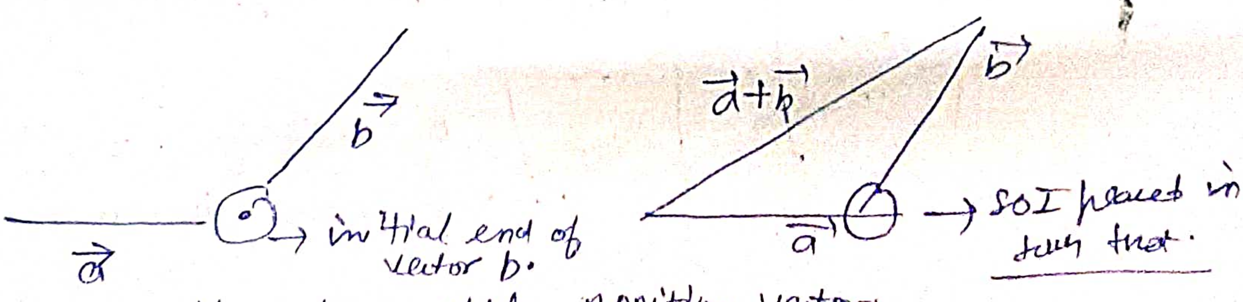
Ans: Addition and subtraction of two vectors Results in a New vector.

let us understand with the help of an Example
$a = (x_1, y_1)$ is a vector in 2 D space.
$b = (x_2, y_2)$

$a+b = (x_1+x_2, b_1+b_2)$
$a-b = (x_1-x_2, y_1-y_2)$

∴ So now to apply to the Addition or Subtraction and any other operation vectors must be 2-Dimensional Space.
Here, in this case both are in of same dimensions. and we can say they are compatible.
and simply add unit to the corresponding elements of these two vectors.
and then y_1 and y_2, simply add them up.

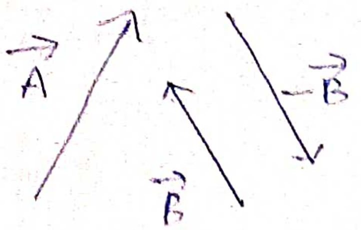
let understand it graphically in 2 Dimension Representation

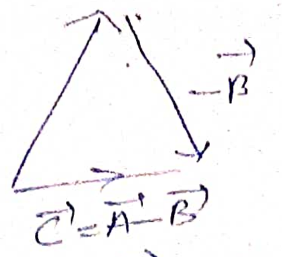


So, initially How do we apply ADDITION vector.
So, I position these vectors such that
what will initial end of b is then we can say like placed at the terminal end of $\vec{a}$. So I place in such that as you can see in Fig (b) the new vector is having the initial position of a and terminal position of $\vec{B}$.

(2) Subtraction

Here I am going to subtraction...





1. If $\vec{B}$ is the given vector, so how we can get $-\vec{B}$, for that it is the with same magnitude, but in a Reverse Direction.
So, Now $\vec{B}$ and $-\vec{B}$ are Represented.

So now how to apply addition of 2 vectors then can we write like $\vec{A} + (-\vec{B})$ to get subtraction of vectors.

(3) Scalar product:

Let $X = \begin{pmatrix} x_1 \\ x_2 \\ \cdots \\ x_n \end{pmatrix}$ and a be the scalar.

— this we have considered the vector in n dimension space.

How do you multiply the scalar to given vector

$$aX = a \begin{pmatrix} x_1 \\ x_2 \\ \cdots \\ x_n \end{pmatrix} = \begin{pmatrix} ax_1 \\ ax_2 \\ \cdots \\ ax_n \end{pmatrix}$$

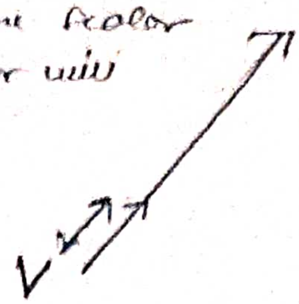
How do you get the result?

By Applying or multiply the scalar with every element of vector.

So, now we will see what happened if we multiply the scalar to the vector.

(INT254) Lect 1.

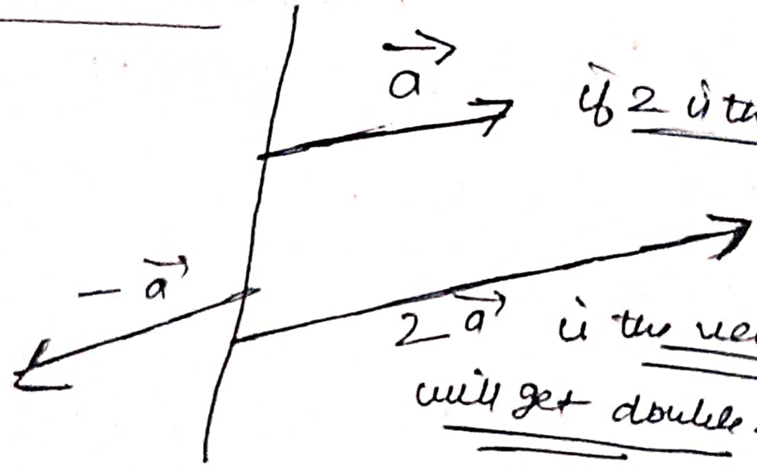
the vector V depending upon the scalar a. your magnitude of the vector will get changed.



av is this vector. But here we have seen that the direction of vector will not get changed.

But what will be happened if the scalar will be -ve. like $-\vec{a}$.

1). if a is positive then we can see the magnitude will not changed.
2.) if a is negative then the resultant will be exactly in the reverse direction.



Que. if scalar is 0.5 then $0.5\vec{a}$, then it will be half of the given ~~magn~~ magnitude.

Matrix—vector product:

$$\begin{bmatrix} 2 & 1 & 2 \\ 3 & 2 & 3 \\ 4 & 3 & 1 \end{bmatrix}_{m \times n} \times \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}_{n \times 1} = \begin{bmatrix} 2*1 + 1*0 + 2*1 \\ \\ \\ \end{bmatrix}$$

So we can say vector with be multiplied with the matrix and vice versa.

I have taken Matrix.

:- Remember when you multiply the scalar with vector then result will be scalar only.

:- vector with the matrix then result will be vector ~~[illegible]~~ only.

1st Row Matrix will be multiplied with the corresponding vector that is X. then they are summed up.

How do you get the 2nd element

3*1 + 2*0 + 3*1

Similarly the 3rd

Lec 1 (INTRO)

Inner (dot) Product of Vectors.

The Inner product captures direction relationship b/w x and y.

So, far we have seen how to multiply vectors with Matrix and scalar with vector.

Now we will see How do we multiply 2 different vectors.

consider, X and y are 2 vectors.

$X = [x_1, x_2 - - - x_n]$ which is n dimensional shape

$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$ it is also a n dimensional space.

Both are compatible. but they should be same in size

then we can apply the Inner product,

we can represent that $X^T Y \in R$

$\begin{bmatrix} x_1 \\ x_2 \\ x_n \end{bmatrix}$ X transpose this $\rightarrow [x_1, x_2\, x_n -]$

the result of this will be scalar. if I apply the Inner product on 2 vectors..

$x_1 * y_1 + x_2 * y_2 + x_n * y_n$. that give me

the scalar value. $X = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ and $y = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ then

$$= \begin{pmatrix} 1 & 3 \end{pmatrix} \begin{pmatrix} 2 \\ 5 \end{pmatrix}$$

$$= -1 * 2 + 3 * 5 = 1$$

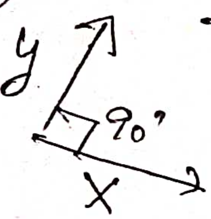
:- Cosine

Angle θ between vectors x and y that can be computed with that $\cos\theta = \dfrac{x^T y}{|x|\,|y|}$ — which is equal to Inner product — this is Norm of vector is always represented with pipeline symbols.

| cosine similarity will give the |

:- Norm of vector will be denominator,
: Inner product is Numerator.

① $\dfrac{x^T y}{|x|\,|y|} = 0$



$\cos\theta = 0$

$x^T y = 0$

x orthogonal to y
or
⇒ we say vector are orthogonal to each other

$x \perp y$

$\theta = 90°$ then only we will get $\boxed{\cos\theta = 0}$

②

$\cos\theta = 1$

$x^T y = |x|\,|y| > 0$

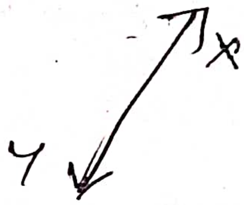
if $\cos\theta = 1$ then we replace.

$\cos\theta = \dfrac{x^T y}{|x|\,|y|}$

$|x|\,|y| = x^T y$

means they are moving in a same direction.

③



$\cos\theta = -1$

$x^T y = -|x|\,|y| < 0$

they are moving in opposite direction

angle b/w them vectors are 180°.

Lect 7. / INT254)

Outer product -

Result of 2 vectors is Matrix.

$$\underline{xy^T} = \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_n \end{bmatrix} [y_1, y_2 - - - y_n] = \begin{bmatrix} x_1y_1 & x_1y_2 & x_1y_n \\ x_2y_1 & x_2y_2 & x_2y_n \\ x_3y_1 & x_3y_2 & x_3y_n \end{bmatrix}$$

$x = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ and $y = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

then

$$xy^T = \begin{pmatrix} -1 \\ 3 \end{pmatrix} (2 \quad 5)$$

$$= \begin{pmatrix} -1 \times 2 & -1 \times 5 \\ 3 \times 2 & 3 \times 5 \end{pmatrix} = \begin{bmatrix} -2 & -5 \\ 6 & 15 \end{bmatrix}$$

How do you find a Norm of a vector (NORM)

① I am given a no. 5, 7 and which one is ~~bigger~~ bigger
So we can say $5 < 7$

② 20 and 5 and how do we can compare 20 and 5.
$\underline{20 > 5.}$

How do we compare the vector for that we need of
Magnitude. you should compute the magnitude,
our How do we compare the magnitude of vectors by
the help of Norm.

A Norm is a function. In this your vector will be represented as $|x|$ and $||x||$. this function is converting n dimension ~~the~~ features to scalar. That satisfies these given 2 ~~[illegible]~~ properties.

1) $||x|| \geq 0$ and $||x|| = 0$ only if $x = 0$.

Norm of the vector should no ~~[illegible]~~ greater than 0,

2) $||x+y|| \leq ||x|| + ||y||$

If you consider 2 vector x and y. If we have add then you want to compute the norm of 2 vectors. the Resultant should be addition of vectors then it satisfy the individual norm addition should be equal to 0 and

If Any Norm will satisfy these 3 properties then only it will ~~[illegible]~~ be called as Norm function.

3) $||ax|| = |a| \, ||x||$

if you multiply vector with scalar then it should be equal to the modulus of scalar that is Absolute value and with the multiplied with Norm of vector.

The Most of used Vector Norms belong to the family of P-norms or Lp-norms.

$$||x||_p = \sqrt[p]{\sum_{i=1}^{n} |x_i|^p}$$

Projection:

Let's Assume that I have 2 vectors a and b.
Let's Assume the angle b/w both of them is θ.

We know that

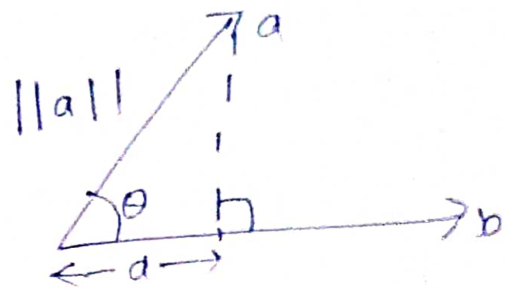
Projection of a on b $= d = ||a|| \cos\theta$

We also know $\vec{a}$ and $\vec{b}$ is Nothing

You take this vector and make it ~~prependic~~ perpendicular to b.

and this d distance is Nothing

but projection of a on b.



Means the distance you get d is and known projection of a on b.

As we see projection of a on b $= d = ||a|| \cos\theta$

We also know $a \cdot b = \text{sum of } (a_i b_i) \quad i = 1, 2 \ldots n$

$$= ||a||\,||b|| \cos\theta$$

So $$(a \cdot b) / ||b|| = \frac{||a||\,\cancel{||b||} \cos\theta}{\cancel{||b||}}$$

$$(a \cdot b) / ||b|| = d$$

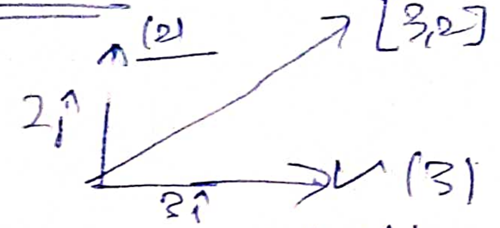
so we can compute the projection of a on b by using this formula, in case you don't know θ.

Change of basis.

① If I have vector in 2D space.

② we have standard way to describe it with coordinates. In this case, the vector has coordinates [3,2].

. the more Linear Algebra oriented way we have to describe the coordinate. As per Linear Algebra, is to think each of these numbers as a scalar.



You think of that first coordinate as scaling. the vector with length 1, pointing to the right while second coordinate scale j hat, the vector with length 1, pointing straight up.

∴ to sum of these two scaled vectors is to tell what the coordinate are meant to Describe.

First coordinate that indicate the right motion
Second " " " the upward motion.

So, to translate b/w vectors and set of numbers is called coordinate systems

And the two special vectors, i hat and j hat are called basis vectors of our standard coordinate system

Q what if we used different basis vector?
is the idea of using a different set of basis vector

Like we are using different set of basis vector which I will call b_1 and b_2

first basis vector b_1 points up into the right a little bit.
second " " b_2 " left and up.

Describe the ~~coordinate~~ vector with coordinate that is
[5/3, 1/3]

this is the way to get that vector.

To scale b_1 by 5/3. Then Add them both together
" " b_2 " 1/3.

I will show you how you could have figured out these numbers 5/3 and 1/3.

If we think coordinate b_1 is -1 $\begin{bmatrix}-1\\2\end{bmatrix}$
" " b_2 is 2

$2-1=1$

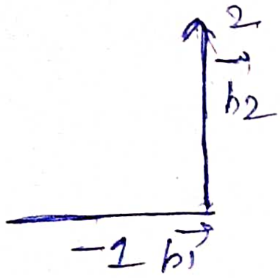


Fig (a)

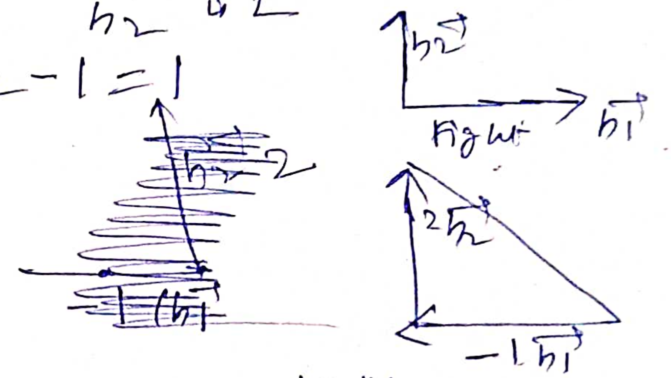
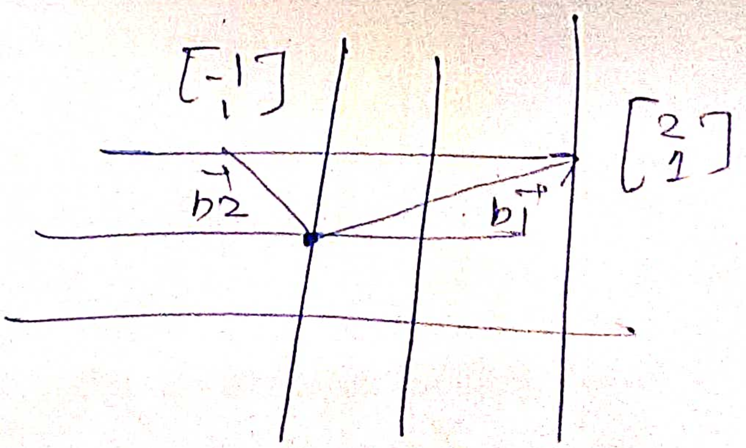


Fig (b)

Example 2



DA

The grid square is just to contrast And a way to visualize our coordinate system

8- Basis Vector: to learn About how to change basis | to 2 and another which can be useful in certain situations

~~Understanding change of Basis~~ X

Finding a Basis

A basis is a set of linearly independent vectors that can be used as building blocks to make any other vector in the vector space.

$$\vec{v_1}, \vec{v_2}, \ldots \vec{v_n} \in V$$

these vectors form a basis for V if:

1/ They are linearly independent.

2/ And the vectors span the space V.

· Here we will focus on part vector space.

the Requirement says that the vector V_1, V_2 to V_n can be combined in some linear combination to express any other vector in the space V.

Let's take simple Example using the space of vectors of length 3, R3.

$$R^3 = \begin{bmatrix}1\\0\\0\end{bmatrix}\begin{bmatrix}0\\1\\0\end{bmatrix}\begin{bmatrix}0\\0\\1\end{bmatrix} \rightarrow \begin{bmatrix}a\\b\\c\end{bmatrix}$$

We must check that a linear combination of this vectors can form any ~~the~~ vector in space, and can Equal any given vector of R3, which we will Express as a vector composed of Scalar a, b, c

The scalar we multiply our vectors by which are c_1, c_2 and c_3 are the variables while the a, b and c that make up of our given vector should be treated as a regular number

$$c_1\begin{bmatrix}1\\0\\0\end{bmatrix}+c_2\begin{bmatrix}0\\1\\0\end{bmatrix}+c_3\begin{bmatrix}0\\0\\1\end{bmatrix}=\begin{bmatrix}a\\b\\c\end{bmatrix}$$

$$\begin{bmatrix}c_1\\0\\0\end{bmatrix}+\begin{bmatrix}0\\c_2\\0\end{bmatrix}+\begin{bmatrix}0\\0\\c_3\end{bmatrix}=\begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix}=\begin{bmatrix}a\\b\\c\end{bmatrix}$$

$c_1 = a$

$c_2 = b$

$c_3 = c$

we can easily check that these three vectors are linearly independent by setting the right side $= 0$.

$$\begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}\Rightarrow \begin{array}{l}c_1 = 0\\ c_2 = 0\\ c_3 = 0\end{array}$$