Lecture 6

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- **Uniqueness Quantifier**

There exists unique /exactly one/ one and only one element for which P(x) is true.

$\exists_1 x\, P(x)$ or $\exists! x\, P(x)$

(1) $\exists_1 x\ (x=-x)$ Domain of Real No.

(2) $\exists_1 x\ (x^2=x)$ Domain of +ve Real no.

- **Quantifiers with Restricted domains**

A condition is put on domain.

We write the condition just next to Quantifier and after that P(x) is written in brackets.

Eg: $\forall x\ (x^2 \geq x)$ false Domain of Real No.

$\forall x\ |x| \geq 1\ \ (x^2 \geq x)$ True or

$\forall x\ (|x| \geq 1 \longrightarrow x^2 \geq x)$

$\exists x\ x>0\ \ (x^2=x)$ or

$\exists x\ (x>0 \wedge x^2=x)$

Q12. Let $P(x)$ be the statement "x spends more than five hours every weekday in class."

Q12. Let $P(x)$ be the statement "x spends more than five hours every weekday in class." where the domain consisits of all students. Express each of these quantifications in English.

(A) $\exists x\, P(x)$ There exists a student who spends ——

(B) $\forall x\ P(x)$ All students spends ——.

(C) $\exists x\ \neg P(x)$ There exist a student who doesn't spend —

(D) $\forall x\ \neg P(x)$ None of student spends more ——.

Negating Quantifiers

There exist x such that $\neg P(x)$

$\exists x\ \neg P(x)$

For no x $P(x)$
or
For all x, $\neg P(x)$

$\forall x\ \neg P(x)$

(E) $\neg\exists x\, P(x)$

$\exists x P(x)$ → There exists a student who spends more than 5 hours ——

$\neg \exists x P(x)$ → None of student spends more than ——

(F) $\neg\forall x\ P(x)$

$\forall x P(x)$ → Every student spends more than ——

$\neg \forall x P(x)$ → Some students don't spend more ——

$\neg \forall x P(x) \rightarrow$ Some students don't spend more

English sentences -------> Logical Expressions

$\neg \exists x P(x)$ $\equiv \forall x \neg P(x)$

$P(x)$: x is funny , $\neg P(x)$: x is not funny

$\exists x \neg P(x) \rightarrow$ There is a x who is not funny

$\neg(\exists x P(x)) \Rightarrow$ None of x is funny.

Q13. Let $P(x)$ be the statement "x can speak Russian" and let $Q(x)$ be the statement "x knows the computer language C++". Write the followig using proposition, logical connectives, quantifiers. The domain consists of all students at your school.

(A) There is a student at your school who can speak Russian and who knows C++.

$\exists x \quad P(x) \wedge Q(x)$

(B) There is a student at your school who can speak Russian but does not know C++.

$\exists x \; P(x) \wedge \neg Q(x)$

(C) Every student at your school either can speak Russian or knows C++.

$\forall x \quad P(x) \vee Q(x)$

(D) No student at your school can speak Russian or knows C++.

10. Let $C(x)$ be the statement "x has a cat," let $D(x)$ be the statement "x has a dog," and let $F(x)$ be the statement "x has a ferret." Express each of these statements in terms of $C(x)$, $D(x)$, $F(x)$, quantifiers, and logical connectives. Let the domain consist of all students in your class.

a) A student in your class has a cat, a dog, and a ferret.
b) All students in your class have a cat, a dog, or a ferret.
c) Some student in your class has a cat and a ferret, but not a dog.
d) No student in your class has a cat, a dog, and a ferret.
e) For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has one of these animals as a pet.