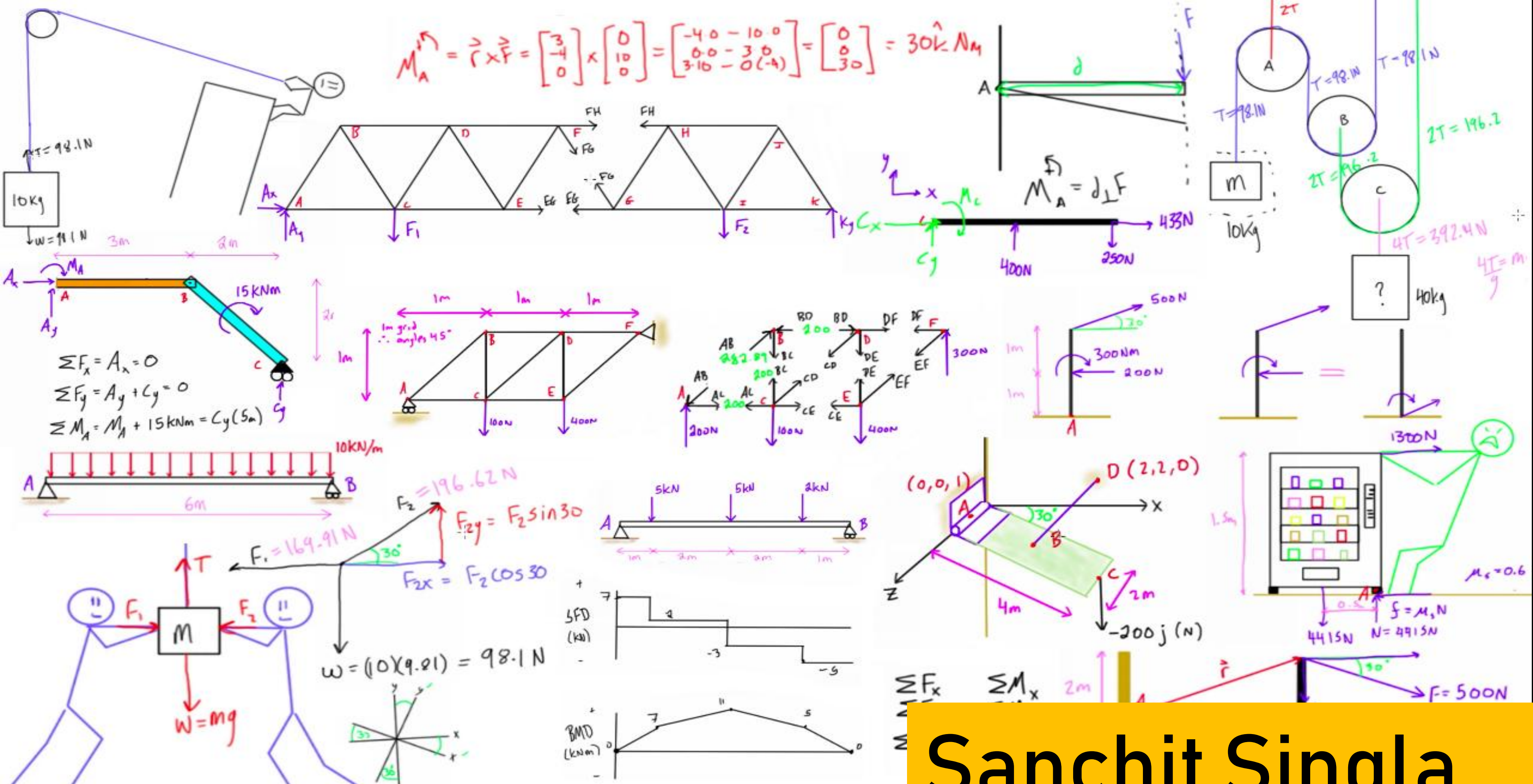

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MEC107

Basic Engineering Mechanics

Learning Outcomes

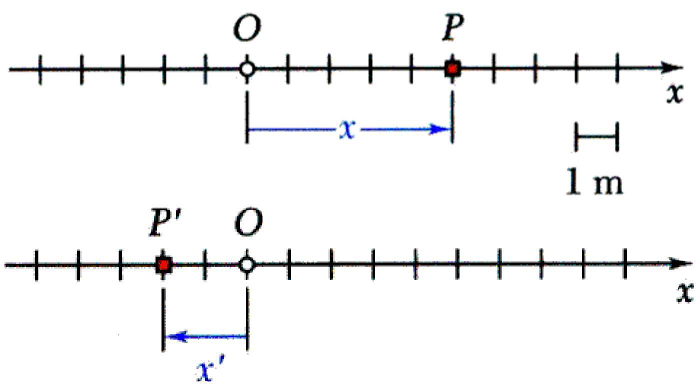
After this lecture, you will be able to

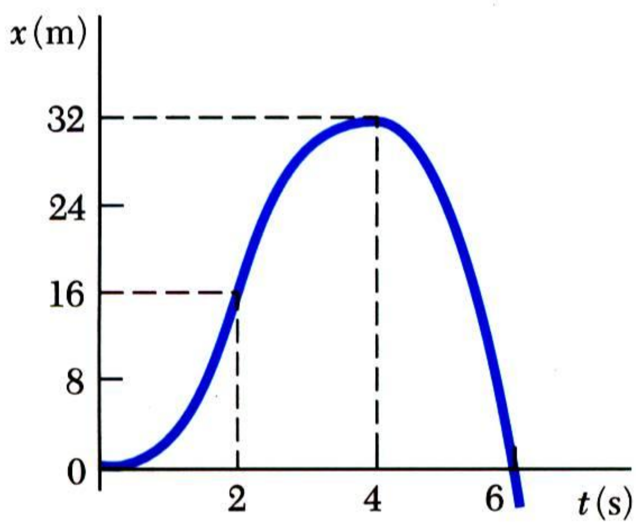
- ✓ learn about Rectilinear Motion.
- ✓ understand Determination of the motion of a Particle.
- ✓ know about Uniform Rectilinear Motion.
- ✓ understand about Uniform Accelerated Rectilinear Motion.

INTRODUCTION

- Dynamics includes:
  - *Kinematics*: study of the geometry of motion. Kinematics is used to relate displacement, velocity, acceleration, and time without reference to the cause of motion.
  - *Kinetics*: study of the relations existing between the forces acting on a body, the mass of the body, and the motion of the body. Kinetics is used to predict the motion caused by given forces or to determine the forces required to produce a given motion.
- *Rectilinear* motion: position, velocity, and acceleration of a particle as it moves along a straight line.
- *Curvilinear* motion: position, velocity, and acceleration of a particle as it moves along a curved line in two or three dimensions.

Rectilinear Motion: Position, Velocity & Acceleration



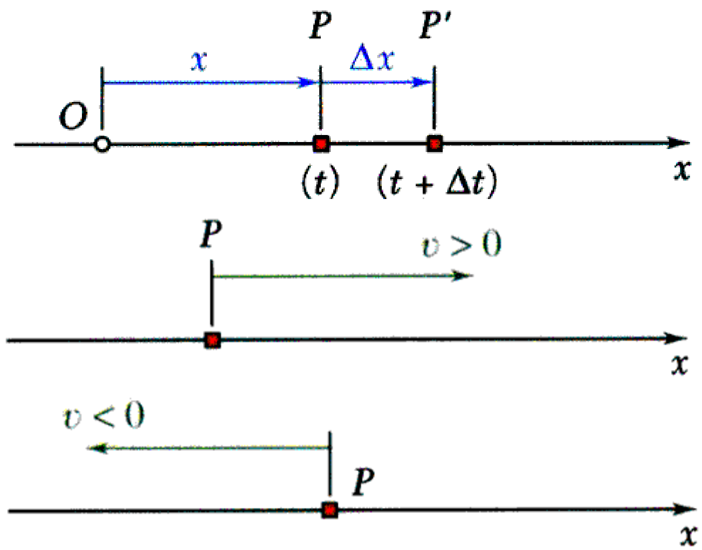


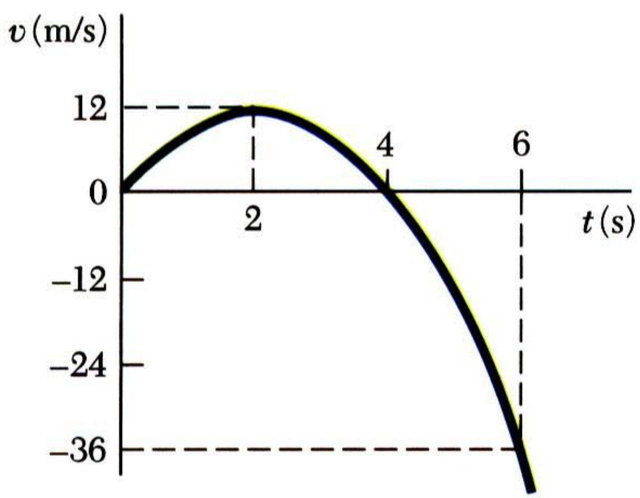
- Particle moving along a straight line is said to be in *rectilinear motion*.
- *Position coordinate* of a particle is defined by positive or negative distance of particle from a fixed origin on the line.
- The *motion* of a particle is known if the position coordinate for particle is known for every value of time *t*. Motion of the particle may be expressed in the form of a function, e.g.,

$$x = 6t^2 - t^3$$

or in the form of a graph *x* vs. *t*.

Rectilinear Motion: Position, Velocity & Acceleration





- Consider particle which occupies position *P* at time *t* and *P'* at *t*+*Δt*,

$$\text{Average velocity} = \frac{\Delta x}{\Delta t}$$

$$\text{Instantaneous velocity} = v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

- Instantaneous velocity may be positive or negative. Magnitude of velocity is referred to as *particle speed*.

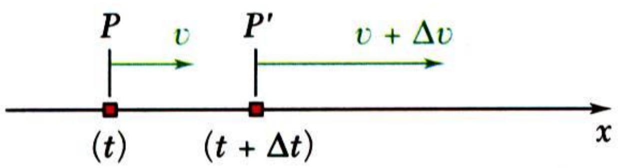
- From the definition of a derivative,

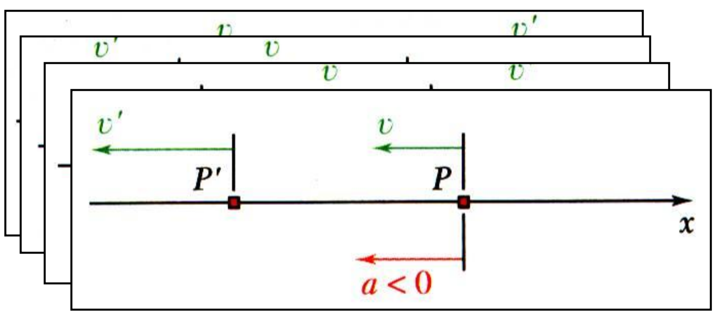
$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

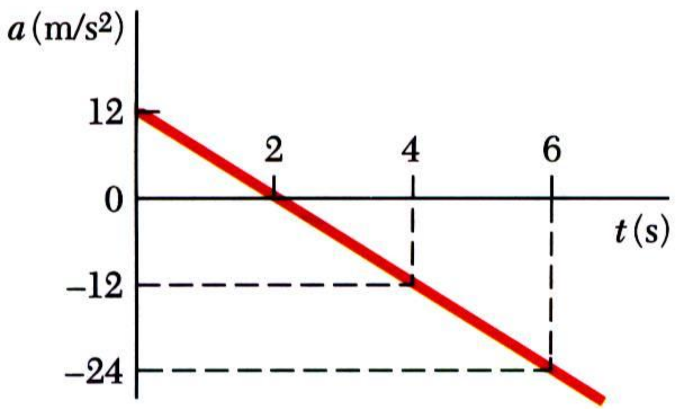
e.g., $x = 6t^2 - t^3$

$$v = \frac{dx}{dt} = 12t - 3t^2$$

Rectilinear Motion: Position, Velocity & Acceleration







- Consider particle with velocity v at time t and v' at $t+\Delta t$,

$$\textit{Instantaneous acceleration} = a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

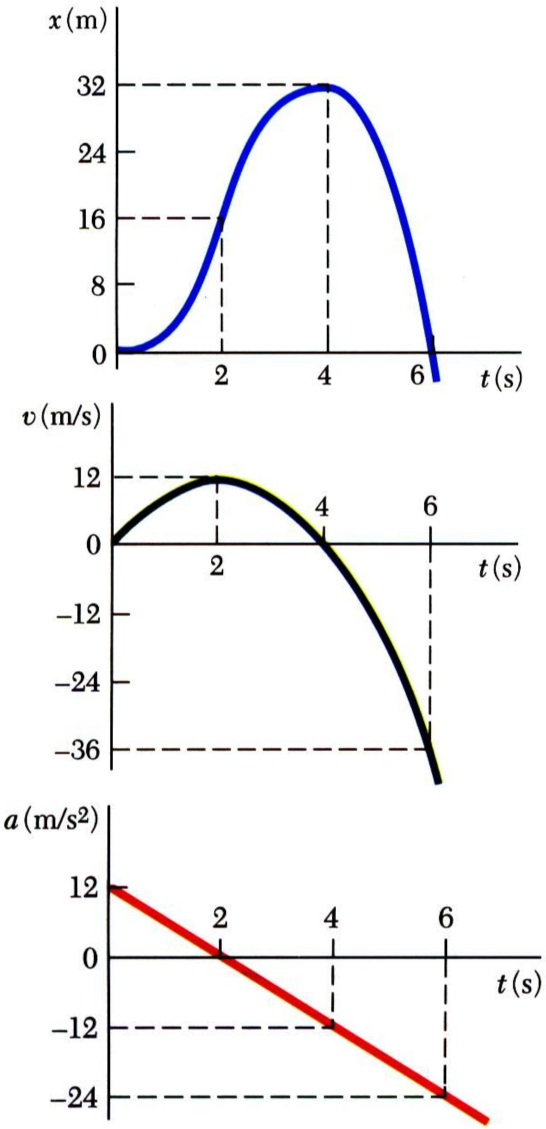
- Instantaneous acceleration may be:
  - positive: increasing positive velocity
    or decreasing negative velocity
  - negative: decreasing positive velocity
    or increasing negative velocity.
- From the definition of a derivative,

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2 x}{dt^2}$$

e.g. $v = 12t - 3t^2$

$$a = \frac{dv}{dt} = 12 - 6t$$

Rectilinear Motion: Position, Velocity & Acceleration



- Consider particle with motion given by

$$x = 6t^2 - t^3$$

$$v = \frac{dx}{dt} = 12t - 3t^2$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = 12 - 6t$$

- at $t = 0$, $x = 0$, $v = 0$, $a = 12$ m/s^2
- at $t = 2$ s, $x = 16$ m, $v = v_{max} = 12$ m/s, $a = 0$
- at $t = 4$ s, $x = x_{max} = 32$ m, $v = 0$, $a = -12$ m/s^2
- at $t = 6$ s, $x = 0$, $v = -36$ m/s, $a = 24$ m/s^2

Determination Of The Motion Of A Particle

- Recall, *motion* of a particle is known if position is known for all time t.
- Typically, conditions of motion are specified by the type of acceleration experienced by the particle. Determination of velocity and position requires two successive integrations.
- Three classes of motion may be defined for:
  - acceleration given as a function of *time*, $a = f(t)$
  - acceleration given as a function of *position*, $a = \mathrm{f}(x)$
  - acceleration given as a function of *velocity*, $a = \mathrm{f}(v)$

Determination Of The Motion Of A Particle

- Acceleration given as a function of *time*, $a = f(t)$:

$$\frac{dv}{dt} = a = f(t) \qquad dv = f(t)dt \qquad \int_{v_0}^{v(t)} dv = \int_0^t f(t)dt \qquad v(t) - v_0 = \int_0^t f(t)dt$$

$$\frac{dx}{dt} = v(t) \qquad dx = v(t)dt \qquad \int_{x_0}^{x(t)} dx = \int_0^t v(t)dt \qquad x(t) - x_0 = \int_0^t v(t)dt$$

- Acceleration given as a function of *position*, $a = f(x)$:

$$v = \frac{dx}{dt} \text{ or } dt = \frac{dx}{v} \qquad a = \frac{dv}{dt} \text{ or } a = v\frac{dv}{dx} = f(x)$$

$$v\,dv = f(x)dx \qquad \int_{v_0}^{v(x)} v\,dv = \int_{x_0}^{x} f(x)dx \qquad \tfrac{1}{2}v(x)^2 - \tfrac{1}{2}v_0^2 = \int_{x_0}^{x} f(x)dx$$

Determination Of The Motion Of A Particle

- Acceleration given as a function of velocity, $a = f(v)$:

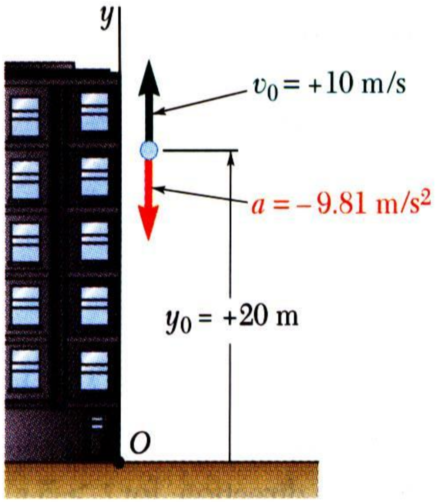
$$\frac{dv}{dt} = a = f(v) \qquad \frac{dv}{f(v)} = dt \qquad \int_{v_0}^{v(t)} \frac{dv}{f(v)} = \int_0^t dt$$

$$\int_{v_0}^{v(t)} \frac{dv}{f(v)} = t$$

$$v\frac{dv}{dx} = a = f(v) \qquad dx = \frac{v\,dv}{f(v)} \qquad \int_{x_0}^{x(t)} dx = \int_{v_0}^{v(t)} \frac{v\,dv}{f(v)}$$

$$x(t) - x_0 = \int_{v_0}^{v(t)} \frac{v\,dv}{f(v)}$$

Numerical



Ball tossed with 10 m/s vertical velocity from window 20 m above ground.

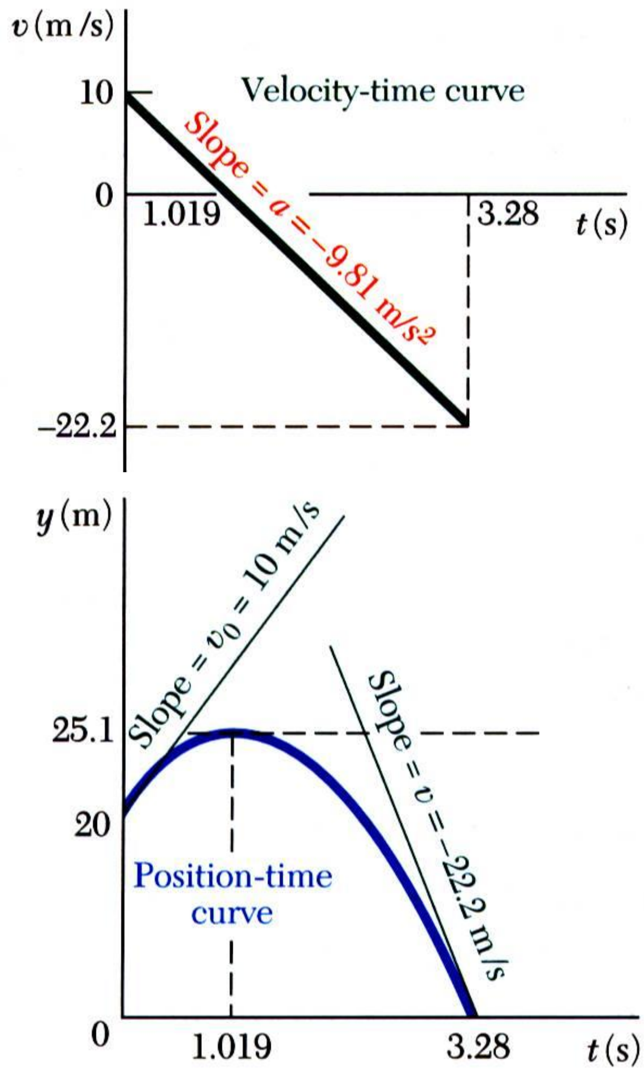
Determine:

- velocity and elevation above ground at time *t*,
- highest elevation reached by ball and corresponding time, and
- time when ball will hit the ground and corresponding velocity.

SOLUTION:

- Integrate twice to find $v(t)$ and $y(t)$.
- Solve for *t* at which velocity equals zero (time for maximum elevation) and evaluate corresponding altitude.
- Solve for *t* at which altitude equals zero (time for ground impact) and evaluate corresponding velocity.

Numerical



SOLUTION:

- Integrate twice to find $v(t)$ and $y(t)$.

$$\frac{dv}{dt} = a = -9.81\,\mathrm{m/s^2}$$

$$\int_{v_0}^{v(t)} dv = -\int_0^t 9.81\,dt \qquad v(t) - v_0 = -9.81t$$

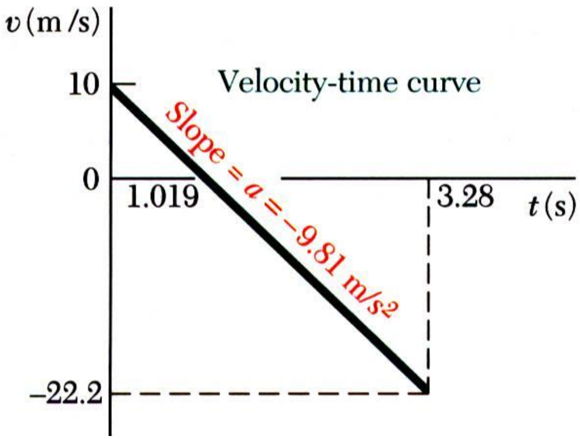
$$\boxed{v(t) = 10\,\frac{\mathrm{m}}{\mathrm{s}} - \left(9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)t}$$

$$\frac{dy}{dt} = v = 10 - 9.81t$$

$$\int_{y_0}^{y(t)} dy = \int_0^t (10 - 9.81t)\,dt \qquad y(t) - y_0 = 10t - \tfrac{1}{2}9.81t^2$$

$$\boxed{y(t) = 20\,\mathrm{m} + \left(10\,\frac{\mathrm{m}}{\mathrm{s}}\right)t - \left(4.905\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)t^2}$$

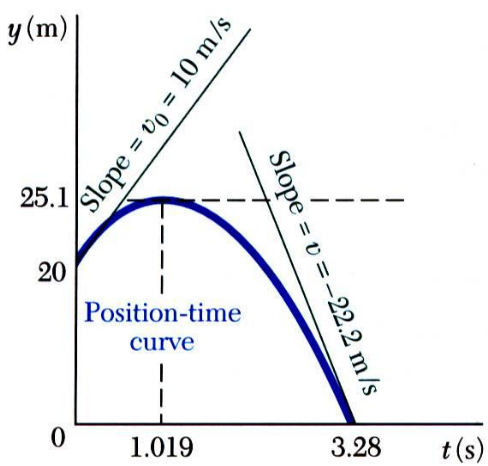
Numerical



- Solve for t at which velocity equals zero and evaluate corresponding altitude.

$$v(t) = 10\,\frac{\mathrm{m}}{\mathrm{s}} - \left(9.81\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)t = 0$$

$$\boxed{t = 1.019\,\mathrm{s}}$$



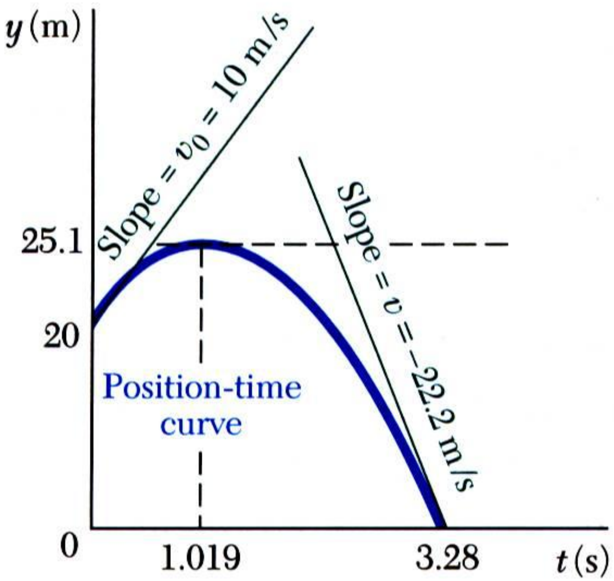
- Solve for t at which altitude equals zero and evaluate corresponding velocity.

$$y(t) = 20\,\mathrm{m} + \left(10\,\frac{\mathrm{m}}{\mathrm{s}}\right)t - \left(4.905\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)t^2$$

$$y = 20\,\mathrm{m} + \left(10\,\frac{\mathrm{m}}{\mathrm{s}}\right)(1.019\,\mathrm{s}) - \left(4.905\,\frac{\mathrm{m}}{\mathrm{s}^2}\right)(1.019\,\mathrm{s})^2$$

$$\boxed{y = 25.1\,\mathrm{m}}$$

SAMPLE PROBLEM 11.2



- Solve for t at which altitude equals zero and evaluate corresponding velocity.

$$y(t) = 20\,\mathrm{m} + \left(10\frac{\mathrm{m}}{\mathrm{s}}\right)t - \left(4.905\frac{\mathrm{m}}{\mathrm{s}^2}\right)t^2 = 0$$

$$\boxed{\begin{aligned} t &= -1.243\,\mathrm{s} \;\; (\text{meaningless}) \\ t &= 3.28\,\mathrm{s} \end{aligned}}$$

$$v(t) = 10\frac{\mathrm{m}}{\mathrm{s}} - \left(9.81\frac{\mathrm{m}}{\mathrm{s}^2}\right)t$$

$$v(3.28\,\mathrm{s}) = 10\frac{\mathrm{m}}{\mathrm{s}} - \left(9.81\frac{\mathrm{m}}{\mathrm{s}^2}\right)(3.28\,\mathrm{s})$$

$$\boxed{v = -22.2\frac{\mathrm{m}}{\mathrm{s}}}$$

Uniform Rectilinear Motion

For particle in uniform rectilinear motion, the acceleration is zero and the velocity is constant.

$$\frac{dx}{dt} = v = \text{constant}$$

$$\int_{x_0}^{x} dx = v\int_{0}^{t} dt$$

$$x - x_0 = vt$$

$$x = x_0 + vt$$

Uniformly Accelerated Rectilinear Motion

For particle in uniformly accelerated rectilinear motion, the acceleration of the particle is constant.

$$\frac{dv}{dt} = a = \text{constant} \qquad \int_{v_0}^{v} dv = a\int_0^t dt \qquad v - v_0 = at$$

$$v = v_0 + at$$

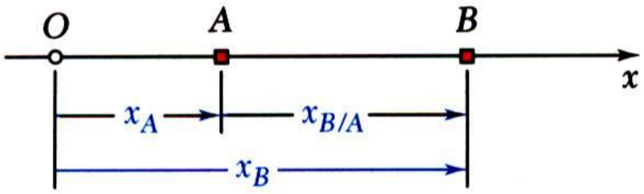
$$\frac{dx}{dt} = v_0 + at \qquad \int_{x_0}^{x} dx = \int_0^t (v_0 + at)dt \qquad x - x_0 = v_0 t + \tfrac{1}{2}at^2$$

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

$$v\frac{dv}{dx} = a = \text{constant} \qquad \int_{v_0}^{v} v\,dv = a\int_{x_0}^{x} dx \qquad \tfrac{1}{2}\left(v^2 - v_0^2\right) = a\left(x - x_0\right)$$

$$v^2 = v_0^2 + 2a\left(x - x_0\right)$$

Motion Of Several Particles: Relative Motion



- For particles moving along the same line, time should be recorded from the same starting instant and displacements should be measured from the same origin in the same direction.

$$x_{B/A} = x_B - x_A = \text{relative position of } B \text{ with respect to } A$$

$$x_B = x_A + x_{B/A}$$

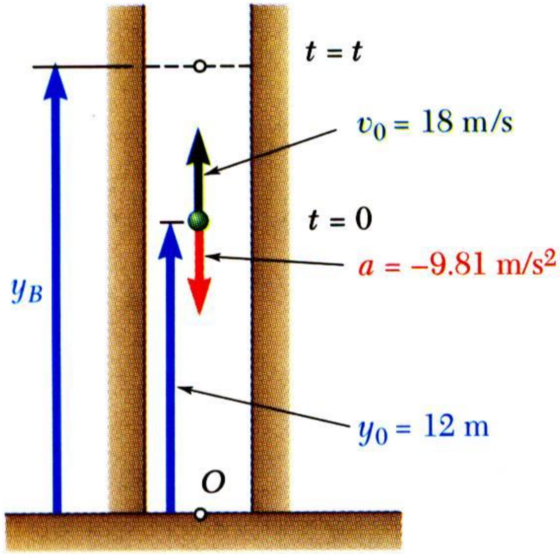
$$v_{B/A} = v_B - v_A = \text{relative velocity of } B \text{ with respect to } A$$

$$v_B = v_A + v_{B/A}$$

$$a_{B/A} = a_B - a_A = \text{relative acceleration of } B \text{ with respect to } A$$

$$a_B = a_A + a_{B/A}$$

Numerical



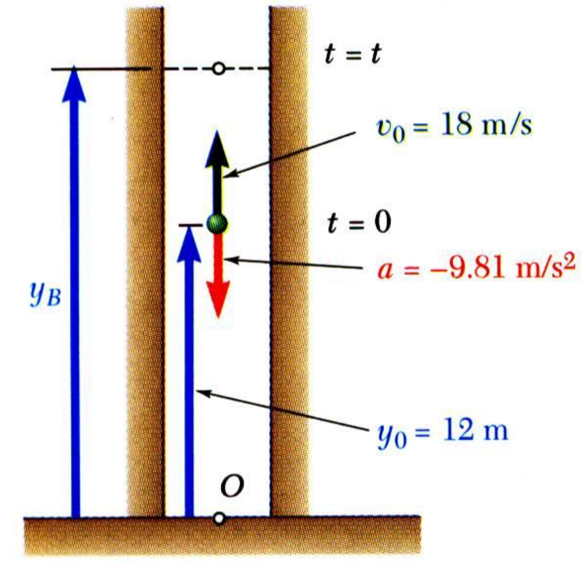
Ball thrown vertically from 12 m level in elevator shaft with initial velocity of 18 m/s. At same instant, open-platform elevator passes 5 m level moving upward at 2 m/s.

Determine *(a)* when and where ball hits elevator and *(b)* relative velocity of ball and elevator at contact.

SOLUTION:

- Substitute initial position and velocity and constant acceleration of ball into general equations for uniformly accelerated rectilinear motion.
- Substitute initial position and constant velocity of elevator into equation for uniform rectilinear motion.
- Write equation for relative position of ball with respect to elevator and solve for zero relative position, i.e., impact.
- Substitute impact time into equation for position of elevator and relative velocity of ball with respect to elevator.

Numerical

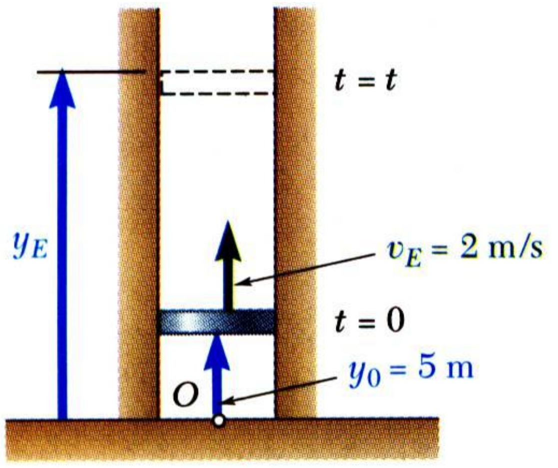


SOLUTION:

- Substitute initial position and velocity and constant acceleration of ball into general equations for uniformly accelerated rectilinear motion.

$$v_B = v_0 + at = 18\frac{\text{m}}{\text{s}} - \left(9.81\frac{\text{m}}{\text{s}^2}\right)t$$

$$y_B = y_0 + v_0 t + \tfrac{1}{2}at^2 = 12\,\text{m} + \left(18\frac{\text{m}}{\text{s}}\right)t - \left(4.905\frac{\text{m}}{\text{s}^2}\right)t^2$$

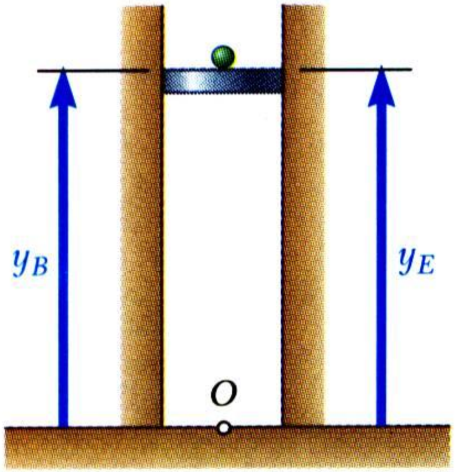


- Substitute initial position and constant velocity of elevator into equation for uniform rectilinear motion.

$$v_E = 2\frac{\text{m}}{\text{s}}$$

$$y_E = y_0 + v_E t = 5\,\text{m} + \left(2\frac{\text{m}}{\text{s}}\right)t$$

Numerical



- Write equation for relative position of ball with respect to elevator and solve for zero relative position, i.e., impact.

$$y_{B/E} = \left(12 + 18t - 4.905t^2\right) - \left(5 + 2t\right) = 0$$

$$\boxed{\begin{aligned} t &= -0.39\,\mathrm{s} \quad (\text{meaningless}) \\ t &= 3.65\,\mathrm{s} \end{aligned}}$$

- Substitute impact time into equations for position of elevator and relative velocity of ball with respect to elevator.

$$y_E = 5 + 2(3.65)$$

$$\boxed{y_E = 12.3\,\mathrm{m}}$$

$$v_{B/E} = (18 - 9.81t) - 2$$

$$= 16 - 9.81(3.65)$$

$$\boxed{v_{B/E} = -19.81\,\frac{\mathrm{m}}{\mathrm{s}}}$$