



CSE322

NDFA WITH NULL MOVES AND

REGULAR EXPRESSION

Lecture #7

Theorem 5.2 (Kleene's theorem) If \mathbf{R} is a regular expression over Σ representing $L \subseteq \Sigma^*$, then there exists an NDFA M with Λ -moves such that $L = T(M)$.

Basis. Let the number of characters in \mathbf{R} be 1. Then $\mathbf{R} = \Lambda$, or $\mathbf{R} = \emptyset$, or $\mathbf{R} = a_i$, $a_i \in \Sigma$. The transition systems given in Fig. 5.5 will recognize these regular expressions.

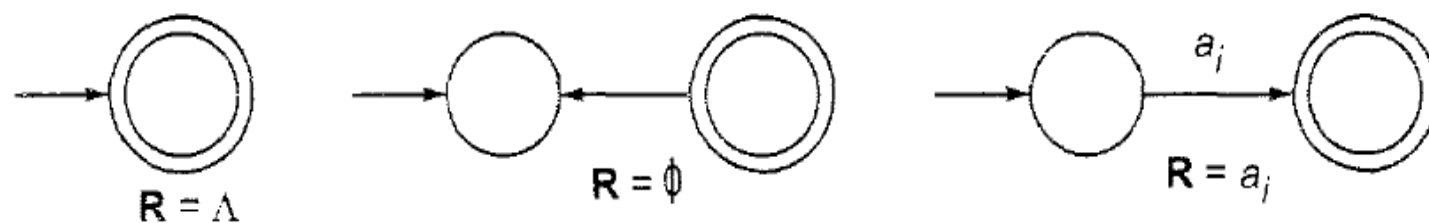


Fig. 5.5 Transition systems for recognizing elementary regular sets.

NDFA WITH NULL MOVES & RE



Induction step. Assume that the theorem is true for regular expressions having n characters. Let \mathbf{R} be a regular expression having $n + 1$ characters. Then,

$$\mathbf{R} = \mathbf{P} + \mathbf{Q} \quad \text{or} \quad \mathbf{R} = \mathbf{PQ} \quad \text{or} \quad \mathbf{R} = \mathbf{P}^*$$

according as the last operator in \mathbf{R} is $+$, product or closure. Also \mathbf{P} and \mathbf{Q} are regular expressions having n characters or less. By induction hypothesis, $L(\mathbf{P})$ and $L(\mathbf{Q})$ are recognized by M_1 and M_2 where M_1 and M_2 are NDFAs with Λ -moves, such that $L(\mathbf{P}) = T(M_1)$ and $L(\mathbf{Q}) = T(M_2)$. M_1 and M_2 are represented in Fig. 5.6.

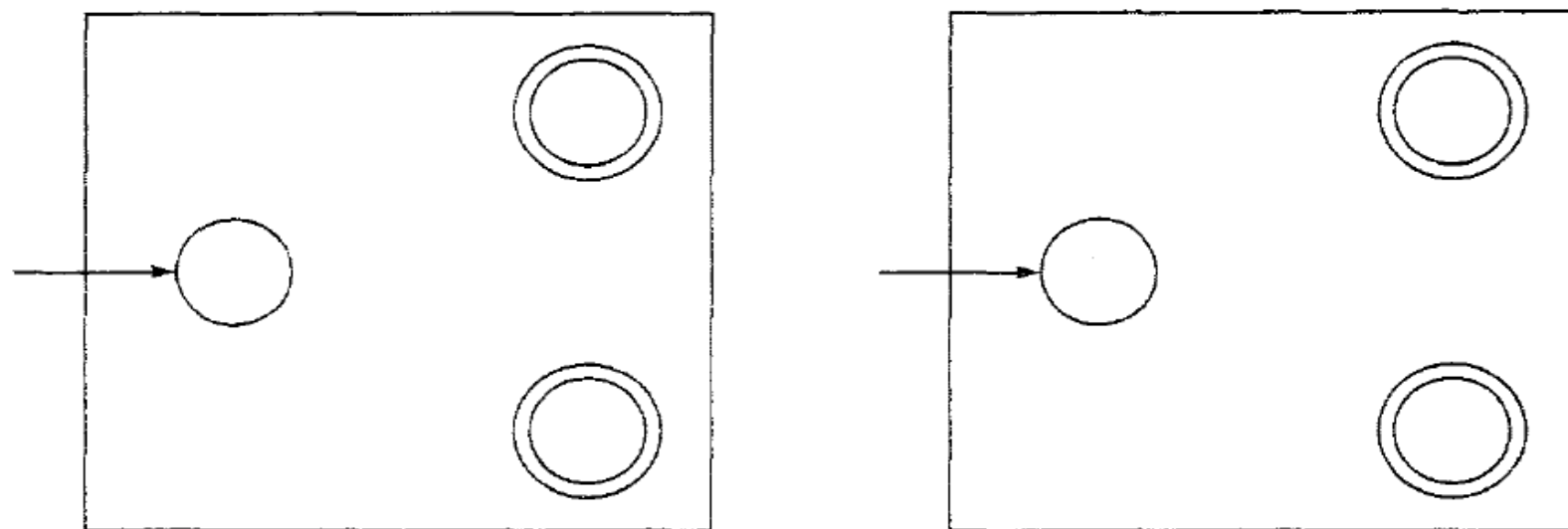
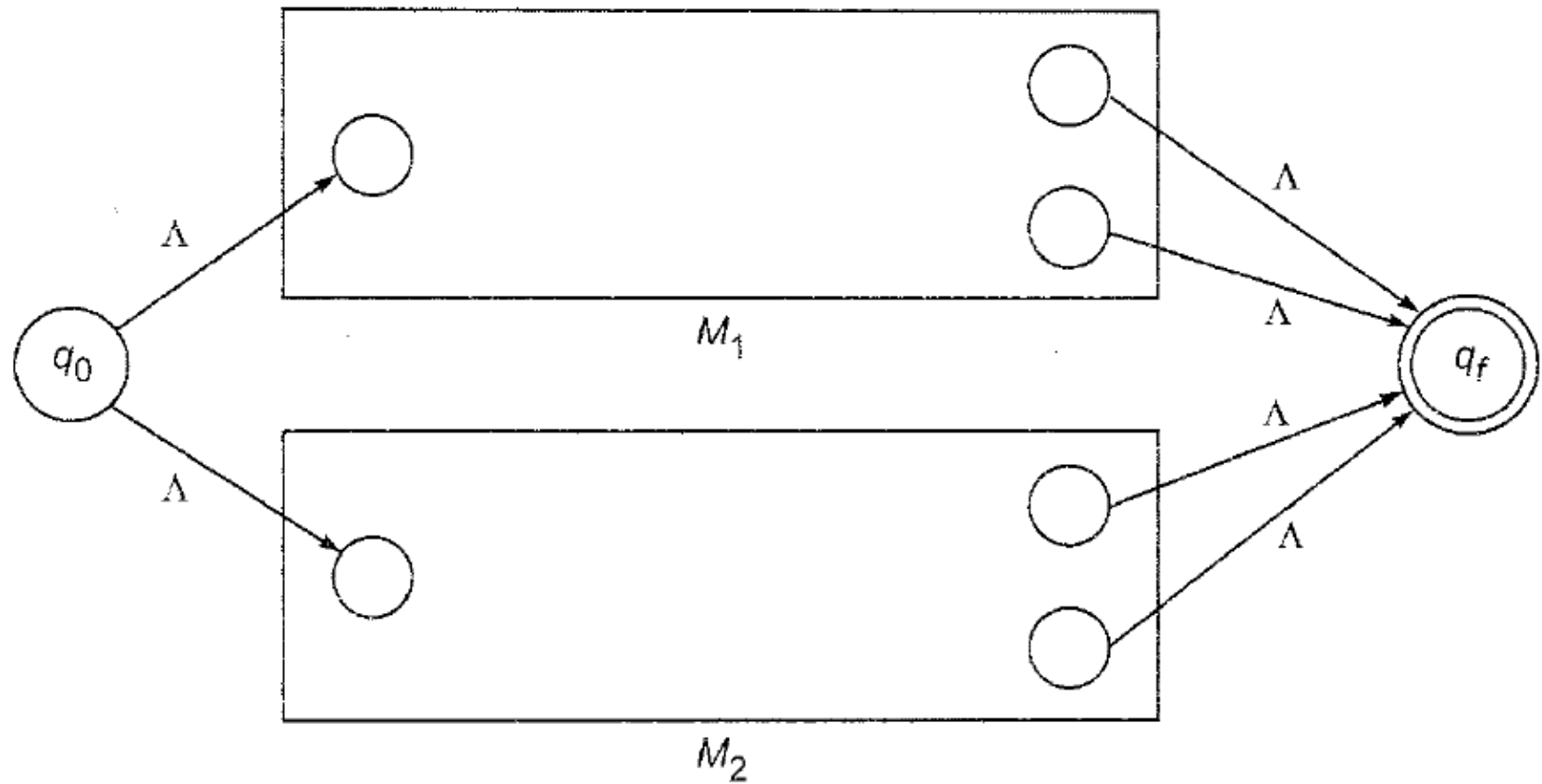


Fig. 5.6 Nondeterministic finite automata M_1 and M_2 .

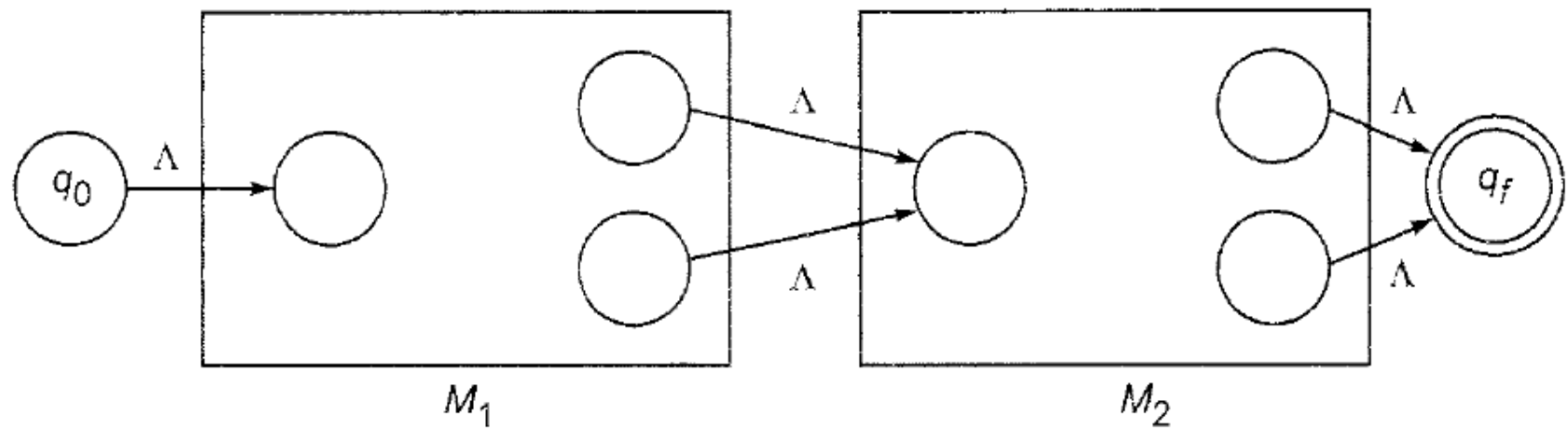
CASE1 : $R=P+Q$



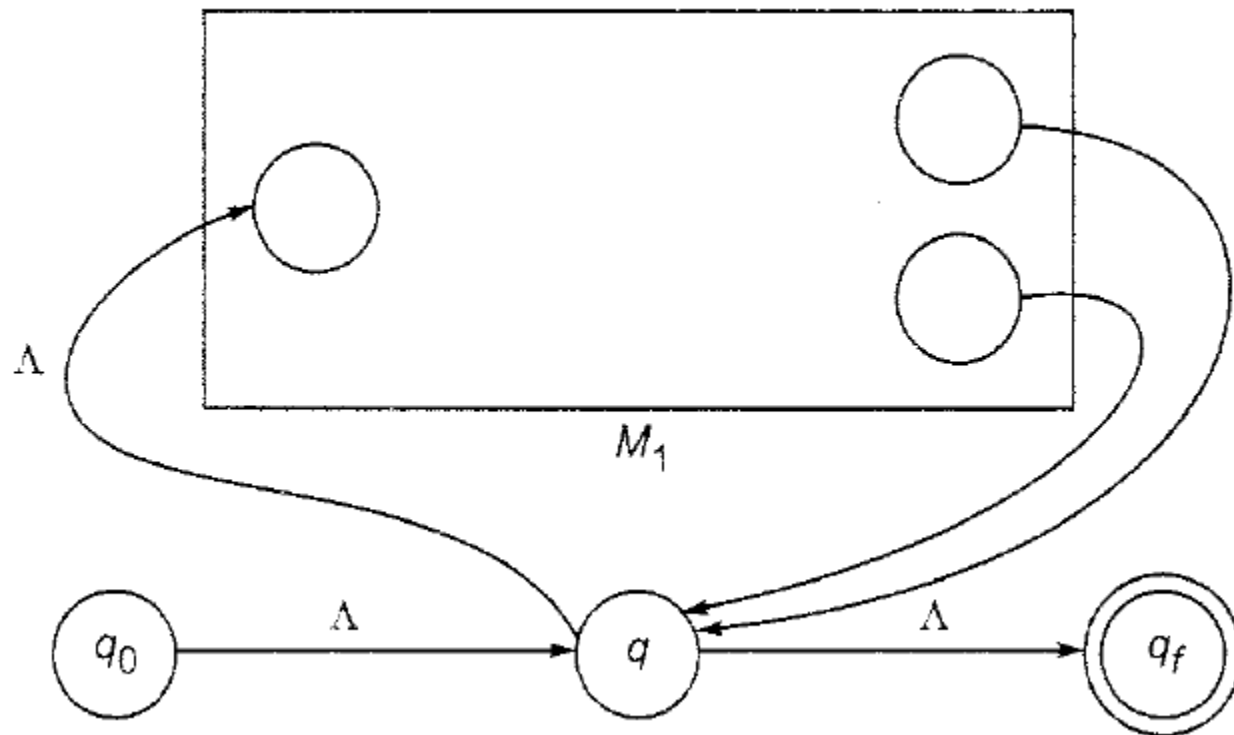
L
P
U



CASE2:R=PQ



CASE 3: $R=(P)^*$



- Step 1: Convert the given transition system into state transition table where each state corresponds to a row and each input symbol corresponds to a column.
- Step 2: Construct the successor table which lists the subsets of states reachable from the set of initial states. Denote this collection of subsets by Q' .
- Step 3: The transition graph given by the successor table is the required deterministic system. The final states contain some final state of NDFA. If possible, reduce the number of states

- In the earlier method for automata, we started with $[q_0]$. Here we start with the set of all initial states. The other steps are similar.

PROBLEM



Obtain the deterministic graph (system) equivalent to the transition system given in Fig. 5.11.

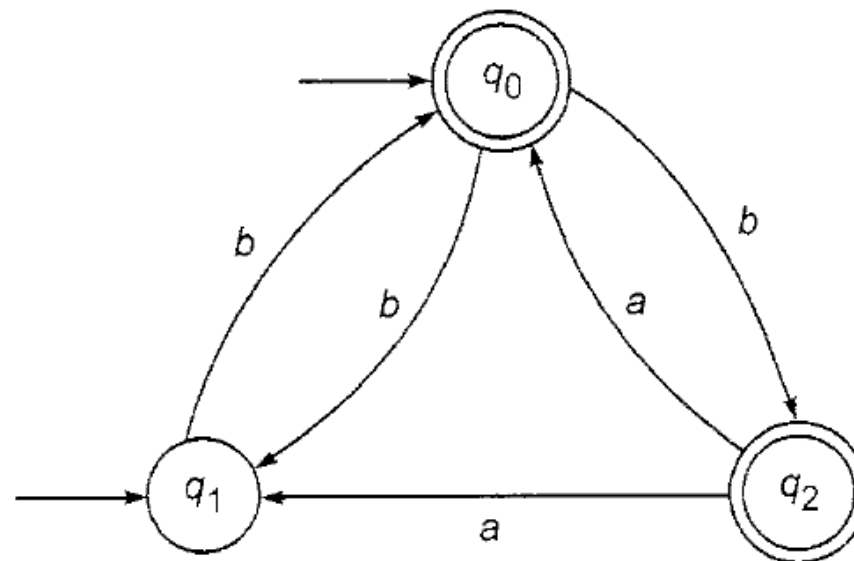


Fig. 5.11 Nondeterministic transition system of Example 5.7.

SOLUTION



L
P
U

State/ Σ	a	b
$\rightarrow q_0$		q_1, q_2
q_1		q_0
q_2	q_0, q_1	

Q	a	b
$[q_0, q_1]$	\emptyset	$[q_0, q_1, q_2]$
$[q_0, q_1, q_2]$	$[q_0, q_1]$	$[q_0, q_1, q_2]$
\emptyset	\emptyset	\emptyset

