Friday, August 21, 2020 10:00 AM

$A = [a_{ij}]_{n \times n} \rightarrow$ complex matrix $\longrightarrow \bar{A} = [\bar{a}_{ij}]$

Hermitian matrix ✓

$$(\bar{A})^T = A$$

or $\overline{(A^T)} = A$

Skew Hermitian

$$(\bar{A})^T = -A$$

$$\overline{(A^T)} = -A$$

Q → let A be a square matrix, then it can be expressed as sum of symmetric & skew symmetric matrices uniquely.

Sol let $A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$

$= P + Q$

let $P = \frac{1}{2}(A + A^T)$ ✓

Taking Transpose on both sides

$$P^T = \left[\frac{1}{2}(A + A^T)\right]^T$$ ✓

$$= \frac{1}{2}(A + A^T)^T$$ ✓

$$= \frac{1}{2}(A^T + (A^T)^T)$$

$$= \frac{1}{2}(A^T + A)$$

$$= P$$

$\boxed{(A^T)^T = A}$ ✓

$\Rightarrow P^T = P$ ✓

$\Rightarrow P$ is symmetric matrix

also $Q = \frac{1}{2}(A - A^T)$

$$Q^T = \frac{1}{2}(A - A^T)^T$$

$$= \frac{1}{2}(A^T - (A^T)^T)$$

$$= \frac{1}{2}(A^T - A)$$

$$= -\frac{1}{2}(A - A^T)$$

$$= -Q$$

$Q^T = -Q \Rightarrow Q$ is skew symmetric

let $A = \underline{R + S}$ — (1), R is symmetric i.e. $R^T = R$ &
S is skew symmetric i.e. $S^T = -S$

$$A^T = (R+S)^T$$
$$= R^T + S^T$$
$$A^T = R - S \quad \text{— (2)}$$

on solving (1) & (2)

$$A + A^T = 2R \quad \text{or} \quad R = \frac{1}{2}(A + A^T) = P$$

also $A - A^T = 2S$ or $S = \frac{1}{2}(A - A^T) = Q$

$\Rightarrow$ $\boxed{\text{Uniqueness}}$

Q The matrix $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}_{2\times 2}$ is

(a) unit matrix (b) a diagonal matrix

(c) a symmetric matrix (d) a skew symmetric matrix

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ & $D = \begin{bmatrix} & \\ & \end{bmatrix}$ $\quad$ $A^T = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = A$

$\Rightarrow$ A is symmetric

$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ & $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Q $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$$A^T = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = -A$$

$\therefore$ $\boxed{A^T = -A}$ $\rightarrow$ skew symmetric

Q Express the matrix $A = \begin{bmatrix} -2 & 3 & 1 \\ 1 & 3 & 2 \\ 5 & -4 & 5 \end{bmatrix}$ as the sum

of symmetric & skew symmetric matrix

Q let A & B be two symmetric matrices of the same order show that AB is symmetric iff $AB = BA$

Sol It is given that A & B are symmetric matrices

$\therefore A^T = A$ & $B^T = B$ — (1)

let AB is symmetric

$\therefore (AB)^T = AB$

or $B^T A^T = AB$

or $\boxed{BA = AB}$

Conversely let $AB = BA$

or $(AB)^T = (BA)^T = A^T B^T = AB$

$\boxed{(AB)^T = AB} \Rightarrow AB$ is symmetric

Q If A & B are symmetric. Show that $AB + BA$ is symmetric & $AB - BA$ is skew symmetric.