04 September 2020 09:47

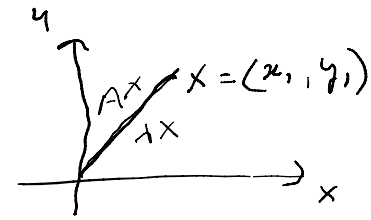
Eigen Values and Eigen Vector

let $A = (a_{ij})$ be a square matrix of order n. The matrix A may singular or non singular. Consider the homogeneous system of equations

$$AX = \lambda X$$

or $AX - \lambda X = 0$

or $\boxed{(A - \lambda I)X = 0}$



Solution depends on $(A - \lambda I)$

If $A - \lambda I \neq 0$ then we will get zero solution

For the non zero solution $|A - \lambda I| = 0$

characteristic equation

$AX = B \to$ Non homog

$AX = \to$ homogen

$\boxed{AX = 0} \to x, y, z$

$x = \frac{\Delta_1}{\Delta}, \; y = \frac{\Delta_2}{\Delta},$

$z = \frac{\Delta_3}{\Delta}, \; \Delta \neq 0$

$x = 0, y = 0, z = 0$

Ex $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Consider $A - \lambda I = \begin{bmatrix} a & b \\ c & d \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} a - \lambda & b \\ c & d - \lambda \end{bmatrix}$

$\therefore$ The characteristic equation is

$|A - \lambda I| = 0$

$\therefore \begin{vmatrix} a - \lambda & b \\ c & d - \lambda \end{vmatrix} = 0$

or $(a - \lambda)(d - \lambda) - bc = 0$

or $ad - a\lambda - d\lambda + \lambda^2 - bc = 0$

or $\lambda^2 - (a + d)\lambda + (ad - bc) = 0$

$\lambda = \alpha \,\&\, \beta$

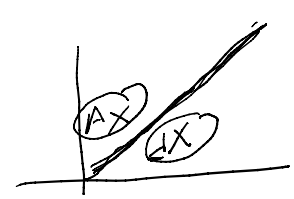
The λ' are called eigen values.

Eigen values are the roots of characteristic equation.

Eigen vector

$\boxed{AX = \lambda X}$

for eigen value



Eigen vector

$$\boxed{AX = \lambda X}$$

Eigen vector eigen value

(AX) (λX)

Q Find the eigen values and eigen vectors of $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$

Soln Consider $A - \lambda I = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{bmatrix}$$

The characteristic equation is

$$|A - \lambda I| = 0$$

ie $\begin{vmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{vmatrix} = 0$

$$\lambda^2 - 7\lambda + 6 = 0$$

$\therefore$ the roots are 1 & 6

$\therefore$ The eigen values are $\lambda = 1$ & 6

let $X = \begin{bmatrix} x \\ y \end{bmatrix}$ be the eigen vector corresponding to the eigen value $\underline{\lambda = 1}$

Consider $(A - \lambda I) X = 0$

Here $\lambda = 1$

$$(A - I) X = 0$$

$$\begin{bmatrix} 4 & 4 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 - \frac{1}{4} R_1$

$$\begin{bmatrix} 4 & 4 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{—(1)}$$

$AX = 0$

$\rho(A) = \underline{Nr}$

$\rho(A) < \underline{Nr}$

Rank = 1 & order of A = 2

$\#$ [Number of eigen vectors = order − Rank
= 2 − 1
= 1]

from eq (1) $4x + 4y = 0$

or $\boxed{x = -y}$

$$\therefore X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -y \\ y \end{bmatrix} = \underbrace{\begin{bmatrix} -1 \\ 1 \end{bmatrix}}_{} y$$

Here $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ is the eigen vector corresponding to the eigen value $\lambda = 1$

or If $y = -x$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ -x \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} x$$

... to the

or

If $y = -x$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ -x \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} x$$

$\therefore \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is the eigen vector corresponding to the eigen value $\lambda = 1$

Note If X is the eigen vector corresponding to the eigen value λ then αX is also the eigen vector " " " " " λ.

$A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ $\boxed{\lambda = 1}$ is

(a) $\begin{bmatrix} 2 \\ -2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$ (c) $\begin{bmatrix} 3 \\ 9 \end{bmatrix}$ (d) None of these

$\begin{bmatrix} 1 \\ -1 \end{bmatrix} \to 2\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$

Q let $X = \begin{bmatrix} x \\ y \end{bmatrix}$ be the eigen vector corresponding to the eigen value $\lambda = 6$

$\therefore$ Consider $(A - 6I)X = 0$

$$\begin{bmatrix} -1 & 4 \\ 1 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad R_2 \to R_2 + R_1$$

or $$\begin{bmatrix} -1 & 4 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

or $-x + 4y = 0$

or $\boxed{x = 4y}$

$$\therefore X = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4y \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix} y$$

$\therefore \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ is the eigen vector corresponding to eigen value $\lambda = 6$

Q $\lambda = 1, 6 \longrightarrow A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} \longrightarrow A^5 \to \lambda = 1^5 \,\&\, 6^5$

$\frac{1}{1} \,\&\, \frac{1}{6}$ $\quad A^{-1}$

Q $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

Q

$3 - 1 = 2$

$x \to 7$