Lecture 8

03 September 2021

13:03

Methods of Proving Theorems

Pose pat If b then q.

Direct Proof

Assume b is true

!

We will have q is true

Q15. Give a direct proof of the theorem "If n is an odd integer, then n^2 is odd."

p:n is an odd integer,
$$q: n^3$$
 is odd
Nasume p is true
 $n = 0dd$
 $n = 2k+1$
 $n^3 = (2k+1)^3 = 4k^2+4k+1$
 $n^3 = 2(2k^2+2k)+1$
 $n^3 = 0dd$
 q is true. $p \rightarrow q$ is true.

Q16. Use a direct proof to show that every odd integer is difference of two squares.

p=nis odd integer, q=nisdelf of two squares Assume p is true

Q17. Prove that if n is an integer and 3n + 2 is even, then n is even.

Proof by Contraposition

Assume q is false, then we will show by is false.

$$n = ak+1$$
 $3n+a = 3(ak+1)+a$
 $= 6k+5$
 $= a(3k+a)+1$
 $3n+a = 0dd$

p is false.

Hence $b \rightarrow q$ is true.

Q18. Prove that if $x + y \geq 2$, where x, y are real numbers, then $x \geq 1$ or $y \geq 1$.

P: x + y > 3Q: $x \geq || or y > ||$ Nssume q is false. $x \leq 1$ and $y \leq 1$ $x + y \leq 3$ P is false.

P $\Rightarrow q$ is true.

Proof by Contradiction

pg is false when pis true and q is false

Passure p is true and q is false

Contradiction.

(919(a): Prove that Broduct of two rational nos is rational.

b: a, b are two rational nos.

9: ab is rational no.

$$a = \frac{m}{n}$$
, $b = \frac{3r}{s}$ $m_1 n_2 r_3 a^{re}$ integer

$$ab = \frac{mr}{ns}$$
,

ab = mr, mr, ms are integers

ab is rational no.

q is bone.

b→q is true.

Q19. Prove or disprove that product of a nonzero rational number and an irrational number is irrational.

p: a is non-zero rational no., b is irrational no.

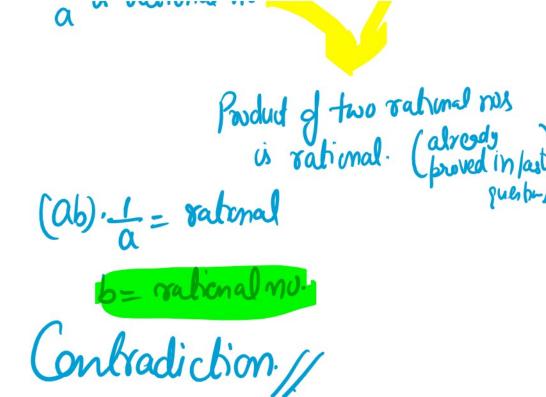
q: ab is irrational no.

Nssume p is true, q is false

$$a = \frac{m}{n}$$
, m, n are integers $ab = rational$

b = Irrational no

$$\frac{1}{\alpha} = \frac{\eta}{m} \quad ; \quad \eta, m \text{ are 1 ld grow } \\ m \neq 0, m \neq 0$$



Q20. Show that at least 10 of any 64 days chosen must fall on the same day of the week.

p: Total days chosen are 64
q: atkast po fall on same day of week.

Assume q is false, atmost q fall on same day of week.

Max. no. of days = 9x7 = 63

b is false.

þ-)q is brue.

Vacuous Proof

The false then bog is ?? true

Trivial Proof

Tygistrue, Then bog is ?? true

Jed man Conno bile g: Prove The proposition P(0), where P(n) is The proposition "If n is positive integer greater than I, then n'sn" Which kind of proof you will use? P(0): "If o is positive integer greater man, then 0 > 0" False -> Vacuous Boof. Mistakes in Proof (3). Brove that 1= 2 multiply b ab=b Sulfrad a ab-a2 = b2-a2 a(b-a) = (b+a)(b-a) (ancelly common) a= b+a wing a=b a= 2a Q2. To find som of Jax2-1 22-1 = 2

January the eph.

January 1 =
$$x^2$$

S.B.S.

Not Reversible

 $x^2 - 1 = x^2$
 $x^2 - 1 = 0$

(x-1) (x+1) = 0

X=1 or x=-1

But x=-1 doesn't valisty the eph.