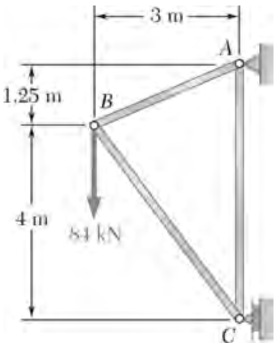
CHAPTER 6



PROBLEM 6.1

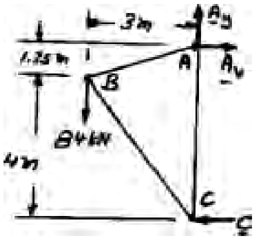
Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

$$AB = \sqrt{3^2 + 1.25^2} = 3.25 \text{ m}$$

$$BC = \sqrt{3^2 + 4^2} = 5 \text{ m}$$

Reactions:



$$+\circlearrowleft \Sigma M_A = 0: \quad (84 \text{ kN})(3 \text{ m}) - C(5.25 \text{ m}) = 0$$

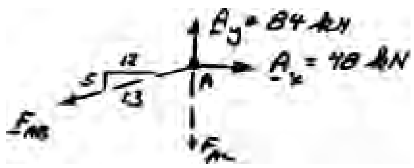
$$\mathbf{C} = 48 \text{ kN} \leftarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - C = 0$$

$$\mathbf{A}_x = 48 \text{ kN} \rightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y = 84 \text{ kN} = 0$$

$$\mathbf{A}_y = 84 \text{ kN} \uparrow$$



Joint A:

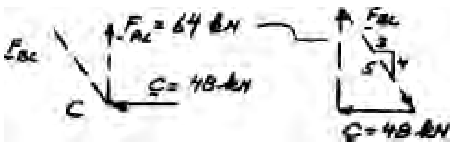
$$\xrightarrow{+} \Sigma F_x = 0: \quad 48 \text{ kN} - \frac{12}{13} F_{AB} = 0$$

$$F_{AB} = +52 \text{ kN} \qquad F_{AB} = 52 \text{ kN} \quad T \blacktriangleleft$$

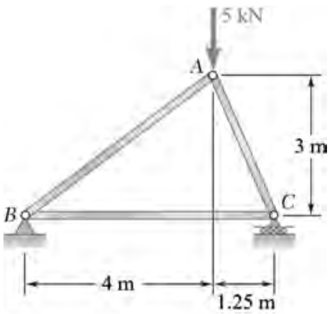
$$+\uparrow \Sigma F_y = 0: \quad 84 \text{ kN} - \frac{5}{13}(52 \text{ kN}) - F_{AC} = 0$$

$$F_{AC} = +64.0 \text{ kN} \qquad F_{AC} = 64.0 \text{ kN} \quad T \blacktriangleleft$$

Joint C:



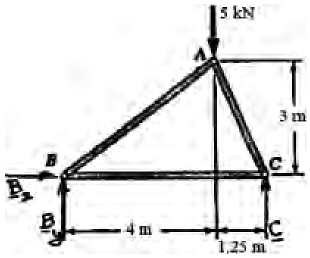
$$\frac{F_{BC}}{5} = \frac{48 \text{ kN}}{3} \qquad F_{BC} = 80.0 \text{ kN} \quad C \blacktriangleleft$$



PROBLEM 6.2

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION



Free body: Entire truss

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x = 0$$

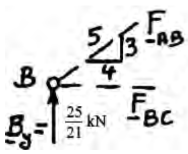
$$\circlearrowleft + \Sigma M_B = 0: \quad C(5.25 \text{ m}) - (5 \text{ kN})(4 \text{ m}) = 0$$

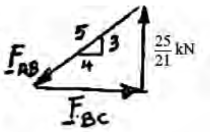
$$\mathbf{C} = \frac{80}{21} \text{ kN} \uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad B_y + \frac{80}{21} \text{ kN} - 5 \text{ kN} = 0$$

$$\mathbf{B}_y = \frac{25}{21} \text{ kN} \uparrow$$

Free body: Joint B:



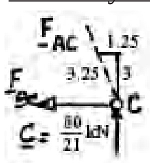


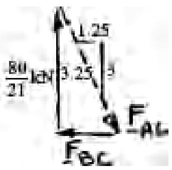
$$\frac{F_{AB}}{5} = \frac{F_{BC}}{4} = \frac{25/21}{3} \text{kN} = \frac{25}{63}$$

$$F_{AB} = \frac{125}{68} \text{ kN} = 1.9841 \text{ kN} \qquad F_{AB} = 1.984 \text{ kN} \quad C \blacktriangleleft$$

$$F_{BC} = \frac{100}{63} \text{ kN} = 1.5873 \text{ kN} \qquad F_{BC} = 1.587 \text{ kN} \quad T \blacktriangleleft$$

Free body: Joint C:

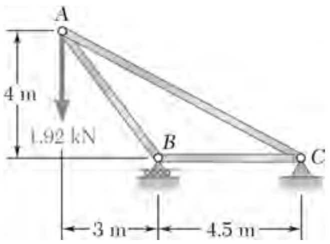




$$\frac{F_{AC}}{3.25} = \frac{F_{BC}}{1.25} = \frac{80/21}{3} \text{kN} = \frac{80}{63} \text{kN}$$

$$F_{AC} = \frac{100}{63} \text{kN} = 1.5873 \text{ kN} \qquad F_{AC} = 1.587 \text{ kN} \quad C \blacktriangleleft$$

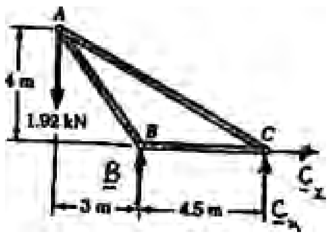
$$F_{BC} = \frac{100}{63} \text{ kN} \quad T \quad \text{(Checks)}$$



PROBLEM 6.3

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION



Free body: Entire truss

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x = 0 \quad \mathbf{C}_x = 0$$

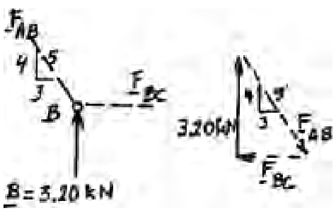
$$\circlearrowleft + \Sigma M_B = 0: \quad (1.92 \text{ kN})(3 \text{ m}) + C_y(4.5 \text{ m}) = 0$$

$$C_y = -1.28 \text{ kN} \qquad \mathbf{C}_y = 1.28 \text{ kN} \downarrow$$

$$+\uparrow \Sigma F_y = 0: \quad B - 1.92 \text{ kN} - 1.28 \text{ kN} = 0$$

$$\mathbf{B} = 3.20 \text{ kN} \uparrow$$

Free body: Joint *B:*

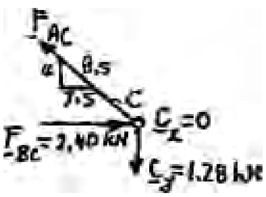


$$\frac{F_{AB}}{5} = \frac{F_{BC}}{3} = \frac{3.20 \text{ kN}}{4}$$

$F_{AB} = 4.00$ kN $\quad C$ ◀

$F_{BC} = 2.40$ kN $\quad C$ ◀

Free body: Joint *C:*

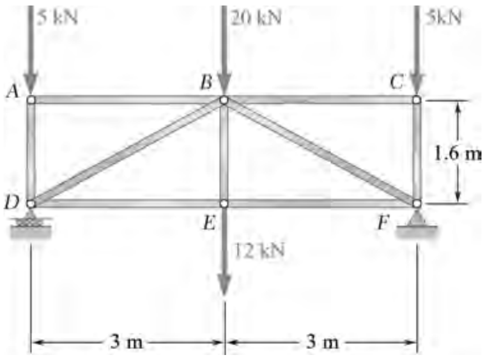


$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{7.5}{8.5} F_{AC} + 2.40 \text{ kN} = 0$$

$$F_{AC} = +2.72 \text{ kN}$$

$F_{AC} = 2.72$ kN $\quad T$ ◀

$$+\uparrow \Sigma F_y = \frac{4}{8.5}(2.72 \text{ kN}) - 1.28 \text{ kN} = 0 \text{ (Checks)}$$

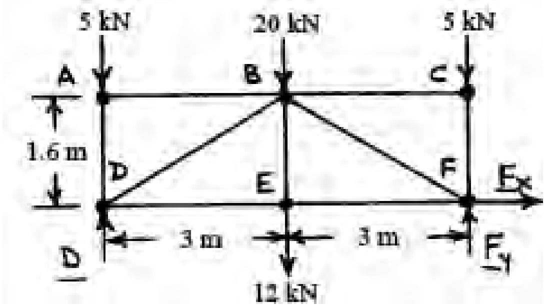


PROBLEM 6.4

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

Reactions:



$$\circlearrowleft + \Sigma M_D = 0: \quad F_y(6) - (20+12)(3) - (5)(6) = 0$$

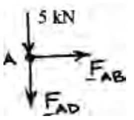
$$\mathbf{F}_y = 21 \text{ kN} \uparrow$$

$$\Sigma F_x = 0: \quad \mathbf{F}_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad D - (5+20+5+12) + (21) = 0$$

$$\mathbf{D} = 21 \text{ kN} \uparrow$$

Joint A:

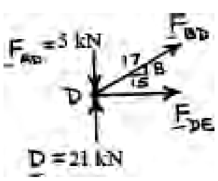


$$\Sigma F_x = 0: \quad F_{AB} = 0 \qquad F_{AB} = 0 \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -5 - F_{AD} = 0$$

$$F_{AD} = -5 \text{ kN} \qquad F_{AD} = 5.00 \text{ kN} \quad C \blacktriangleleft$$

Joint D:



$$+\uparrow \Sigma F_y = 0: \quad -5 + 21 + \frac{8}{17} F_{BD} = 0$$

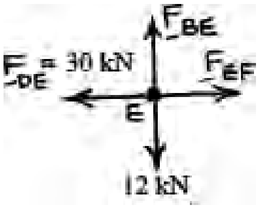
$$F_{BD} = -34 \text{ kN} \qquad F_{BD} = 34.0 \text{ kN} \quad C \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{15}{17}(-34) + F_{DE} = 0$$

$$F_{DE} = +30 \text{ kN} \qquad F_{DE} = 30.0 \text{ kN} \quad T \blacktriangleleft$$

PROBLEM 6.4 (Continued)

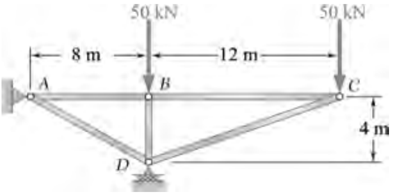
Joint E:



$$+\uparrow \Sigma F_y = 0: \quad F_{BE} - 12 = 0$$

$$F_{BE} = +12 \text{ kN} \qquad\qquad F_{BE} = 12.00 \text{ kN} \quad T \blacktriangleleft$$

Truss and loading symmetrical about ℄

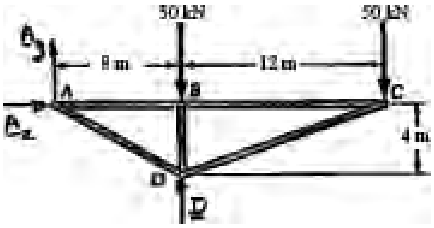


PROBLEM 6.5

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

<u>Free body: Truss</u>



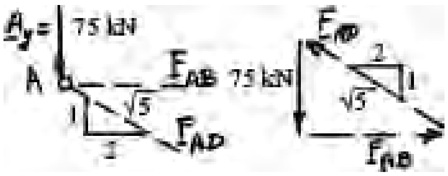
$$\xrightarrow{+} \Sigma F_x = 0: \quad \mathbf{A}_x = 0$$

$$\circlearrowleft + \Sigma M_A = 0: \quad D(8) - (50 \text{ kN})(8) - (50 \text{ kN})(20) = 0$$

$$\mathbf{D} = 175 \text{ kN} \uparrow$$

$$\Sigma F_y = 0: \quad \mathbf{A}_y = 75 \text{ kN} \downarrow$$

<u>Free body: Joint *A*:</u>

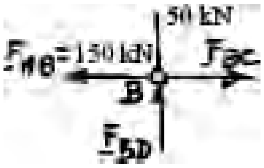


$$\frac{F_{AB}}{2} = \frac{F_{AD}}{\sqrt{5}} = \frac{75 \text{ kN}}{1}$$

$F_{AB} = 150.0 \text{ kN} \quad T$ ◀

$F_{AD} = 167.7 \text{ kN} \quad C$ ◀

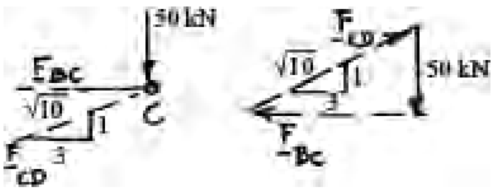
<u>Free body: Joint *B*:</u>



$\Sigma F_x = 0$: $\qquad F_{BC} = 150.0 \text{ kN} \quad T$ ◀

$\Sigma F_y = 0$: $\qquad F_{BD} = 50 \text{ kN} \quad C$ ◀

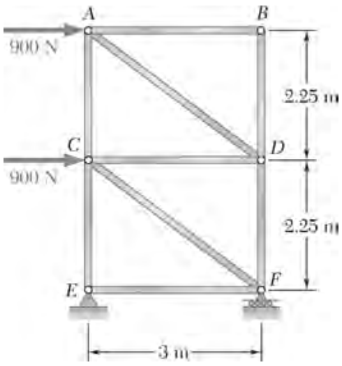
<u>Free body: Joint *C*:</u>



$$\frac{F_{CD}}{\sqrt{10}} = \frac{F_{BC}}{3} = \frac{50 \text{ kN}}{1}$$

$F_{CD} = 158.1 \text{ kN} \quad C$ ◀

$F_{BC} = 150 \text{ kN} \quad T \quad$ (Checks)

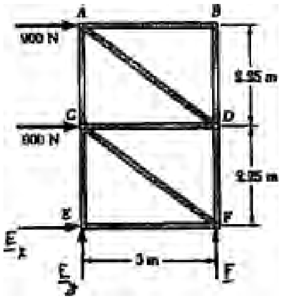


PROBLEM 6.6

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

<u>Free Body: Truss</u>



$$\circlearrowleft + \Sigma M_E = 0: \quad F(3\text{ m}) - (900\text{ N})(2.25\text{ m}) - (900\text{ N})(4.5\text{ m}) = 0$$

$$\mathbf{F} = 2025\text{ N} \uparrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad E_x + 900\text{ N} + 900\text{ N} = 0$$

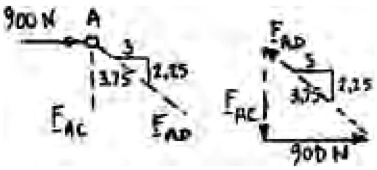
$$E_x = -1800\text{ N} \quad \mathbf{E}_x = 1800\text{ N} \leftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad E_y + 2025\text{ N} = 0$$

$$E_y = -2025\text{ N} \quad \mathbf{E}_y = 2025\text{ N} \downarrow$$

We note that *AB* and *BD* are zero-force members: $F_{AB} = F_{BD} = 0$ ◀

<u>Free body: Joint *A*:</u>

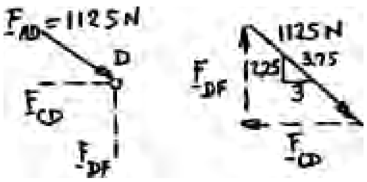


$$\frac{F_{AC}}{2.25} = \frac{F_{AD}}{3.75} = \frac{900\text{ N}}{3}$$

$F_{AC} = 675\text{ N}$ *T* ◀

$F_{AD} = 1125\text{ N}$ *C* ◀

<u>Free body: Joint *D*:</u>

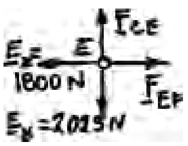


$$\frac{F_{CD}}{3} = \frac{F_{DE}}{2.23} = \frac{1125\text{ N}}{3.75}$$

$F_{CD} = 900\text{ N}$ *T* ◀

$F_{DF} = 675\text{ N}$ *C* ◀

<u>Free body: Joint *E*:</u>



$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{EF} - 1800\text{ N} = 0$$

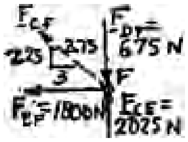
$F_{EF} = 1800\text{ N}$ *T* ◀

$$+\uparrow \Sigma F_y = 0: \quad F_{CE} - 2025\text{ N} = 0$$

$F_{CE} = 2025\text{ N}$ *T* ◀

PROBLEM 6.6 (Continued)

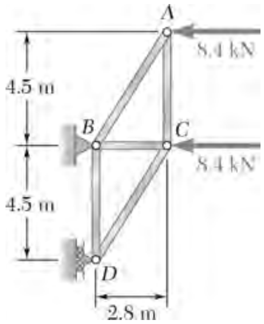
Free body: Joint F:



$$+\uparrow \Sigma F_y = 0: \quad \frac{2.25}{3.75} F_{CF} + 2025 \text{ N} - 675 \text{ N} = 0$$

$$F_{CF} = -2250 \text{ N} \qquad F_{CF} = 2250 \text{ N} \quad C \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = -\frac{3}{3.75}(-2250 \text{ N}) - 1800 \text{ N} = 0 \quad \text{(Checks)}$$

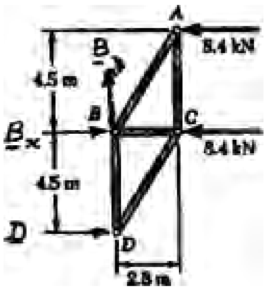


PROBLEM 6.7

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss



$$+\uparrow \Sigma F_y = 0: \quad \mathbf{B}_y = 0$$

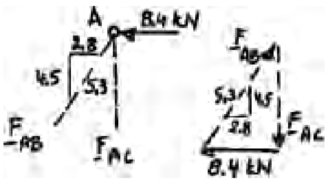
$$+\circlearrowleft \Sigma M_B = 0: \quad D(4.5 \text{ m}) + (8.4 \text{ kN})(4.5 \text{ m}) = 0$$

$$D = -8.4 \text{ kN} \qquad \mathbf{D} = 8.4 \text{ kN} \leftarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x - 8.4 \text{ kN} - 8.4 \text{ kN} - 8.4 \text{ kN} = 0$$

$$B_x = +25.2 \text{ kN} \quad \mathbf{B}_x = 25.2 \text{ kN} \rightarrow$$

Free body: Joint A:

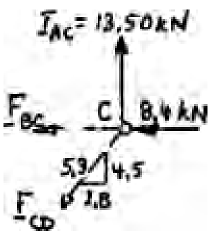


$$\frac{F_{AB}}{5.3} = \frac{F_{AC}}{4.5} = \frac{8.4 \text{ kN}}{2.8}$$

$$F_{AB} = 15.90 \text{ kN} \quad C \blacktriangleleft$$

$$F_{AC} = 13.50 \text{ kN} \quad T \blacktriangleleft$$

Free body: Joint C:



$$+\uparrow \Sigma F_y = 0: \quad 13.50 \text{ kN} - \frac{4.5}{5.3} F_{CD} = 0$$

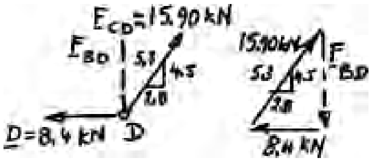
$$F_{CD} = +15.90 \text{ kN} \qquad F_{CD} = 15.90 \text{ kN} \quad T \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -F_{BC} - 8.4 \text{ kN} - \frac{2.8}{5.3}(15.90 \text{ kN}) = 0$$

$$F_{BC} = -16.80 \text{ kN} \qquad F_{BC} = 16.80 \text{ kN} \quad C \blacktriangleleft$$

PROBLEM 6.7 (Continued)

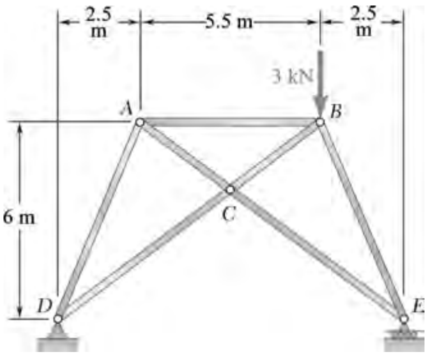
Free body: Joint *D*:



$$\frac{F_{BD}}{4.5} = \frac{8.4 \text{ kN}}{2.8} \qquad F_{BD} = 13.50 \text{ kN} \quad C \blacktriangleleft$$

We can also write the proportion

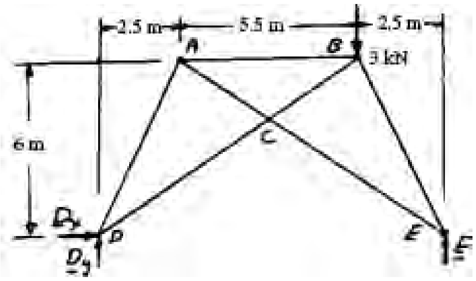
$$\frac{F_{BD}}{4.5} = \frac{15.90 \text{ kN}}{5.3} \qquad F_{BD} = 13.50 \text{ kN} \quad C \quad \text{(Checks)}$$



PROBLEM 6.8

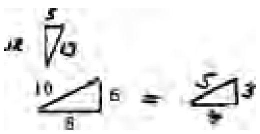
Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION



$$AD = \sqrt{(2.5)^2 + (6)^2} = 6.5 \text{ m}$$

$$BCD = \sqrt{(6)^2 + (8)^2} = 10 \text{ m}$$



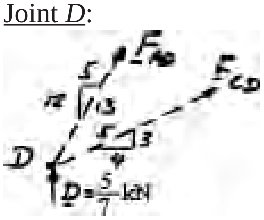
Reactions: $\Sigma F_x = 0\!: \quad D_x = 0$

$$\circlearrowleft^{+} \Sigma M_E = 0\!: \quad D_y(10.5 \text{ m}) - (3 \text{ kN})(2.5 \text{ m}) = 0 \Rightarrow D_y = \frac{5}{7} \text{ kN} \qquad \mathbf{D}_y = 0.714 \text{ kN} \uparrow$$

$$+\uparrow \Sigma F_y = 0\!: \quad \frac{5}{7} \text{ kN} - 3 \text{ kN} + E = 0 \qquad \mathbf{E} = 2.286 \text{ kN} \uparrow$$

$$E = \frac{16}{7} \text{ kN}$$

Joint D:



$$\xleftarrow{+} \Sigma F_x = 0\!: \quad \frac{5}{13} F_{AD} + \frac{4}{5} F_{DC} = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0\!: \quad \frac{12}{13} F_{AD} + \frac{3}{5} F_{DC} + \frac{5}{7} \text{ kN} = 0 \qquad (2)$$

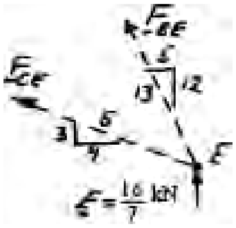
Solving (1) and (2), simultaneously:

$$F_{AD} = -\frac{260}{231} \text{ kN} \qquad F_{AD} = 1.126 \text{ kN} \quad C \blacktriangleleft$$

$$F_{DC} = +\frac{125}{231} \text{ kN} \qquad F_{DC} = 0.541 \text{ kN} \quad T \blacktriangleleft$$

PROBLEM 6.8 (Continued)

Joint E:



$$\overset{+}{\leftarrow} \Sigma F_x = 0: \quad \frac{5}{13}F_{BE} + \frac{4}{5}F_{CE} = 0 \qquad (3)$$

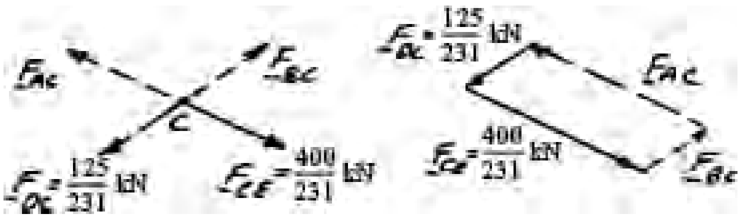
$$+\!\uparrow \Sigma F_y = 0: \quad \frac{12}{13}F_{BE} + \frac{3}{5}F_{CE} + \frac{16}{7}\text{ kN} = 0 \qquad (4)$$

Solving (3) and (4), simultaneously:

$$F_{BE} = -\frac{832}{231}\text{ kN} \qquad F_{BE} = 3.60\text{ kN} \quad C \blacktriangleleft$$

$$F_{CE} = +\frac{400}{231}\text{ kN} \qquad F_{CE} = 1.732\text{ kN} \quad T \blacktriangleleft$$

Joint C:

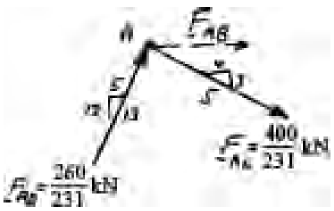


Force polygon is a parallelogram (see Fig. 6.11 p. 209)

$$F_{AC} = 1.732\text{ kN} \quad T \blacktriangleleft$$

$$F_{BC} = 0.541\text{ kN} \quad T \blacktriangleleft$$

Joint A:

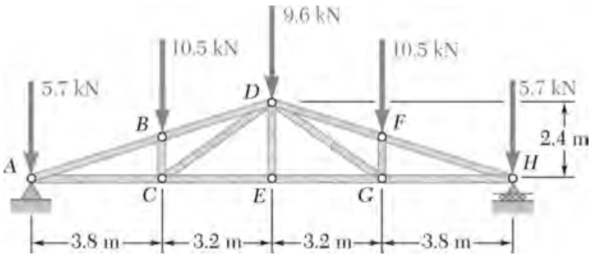


$$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad \frac{5}{13}\left(\frac{260}{231}\text{ kN}\right) + \frac{4}{5}\left(\frac{400}{231}\text{ kN}\right) + F_{AB} = 0$$

$$F_{AB} = -\frac{20}{11}\text{ kN} \qquad F_{AB} = 1.818\text{ kN} \quad C \blacktriangleleft$$

$$+\!\uparrow \Sigma F_y = 0: \quad \frac{12}{13}\left(\frac{260}{231}\text{ kN}\right) - \frac{3}{5}\left(\frac{400}{231}\text{ kN}\right) = 0$$

$$0 = 0 \quad \text{(Checks)}$$

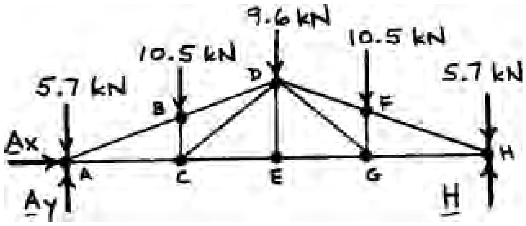


PROBLEM 6.9

Determine the force in each member of the Pratt roof truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss

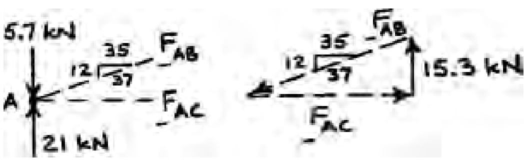


$$\Sigma F_x = 0: \quad A_x = 0$$

Due to symmetry of truss and load

$$A_y = H = \frac{1}{2} \text{ total load} = 21 \text{ kN} \uparrow$$

Free body: Joint A:

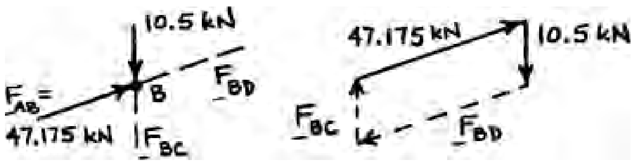


$$\frac{F_{AB}}{37} = \frac{F_{AC}}{35} = \frac{15.3 \text{ kN}}{12}$$

$F_{AB} = 47.175$ kN $\quad F_{AC} = 44.625$ kN $\qquad$ $F_{AB} = 47.2$ kN $\quad C$ ◀

$F_{AC} = 44.6$ kN $\quad T$ ◀

Free body: Joint B:

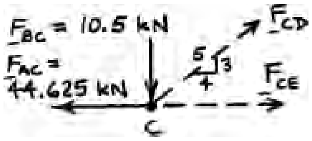


From force polygon: $\quad F_{BD} = 47.175$ kN, $\quad F_{BC} = 10.5$ kN $\qquad$ $F_{BC} = 10.50$ kN $\quad C$ ◀

$F_{BD} = 47.2$ kN $\quad C$ ◀

PROBLEM 6.9 (Continued)

Free body: Joint C:



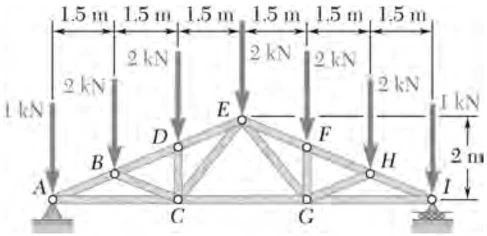
$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5}F_{CD} - 10.5 = 0$ $\qquad F_{CD} = 17.50 \text{ kN} \quad T$ ◀

$\xrightarrow{+} \Sigma F_x = 0: \quad F_{CE} + \frac{4}{5}(17.50) - 44.625 = 0$

$F_{CE} = 30.625 \text{ kN} \qquad F_{CE} = 30.6 \text{ kN} \quad T$ ◀

Free body: Joint E: $\qquad DE$ is a zero-force member $\qquad F_{DE} = 0$ ◀

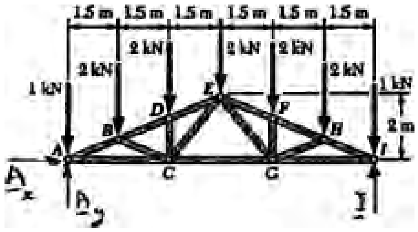
Truss and loading symmetrical about ℄



PROBLEM 6.10

Determine the force in each member of the fan roof truss shown. State whether each member is in tension or compression.

SOLUTION



<u>Free body: Truss</u>

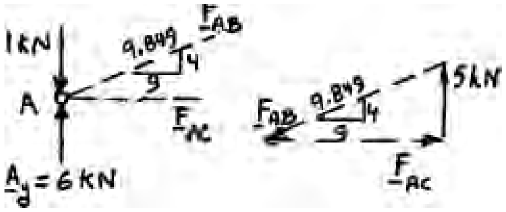
$$\Sigma F_x = 0: \quad \mathbf{A}_x = 0$$

From symmetry of truss and loading:

$$\mathbf{A}_y = \mathbf{I} = \frac{1}{2} \quad \text{Total load}$$

$$\mathbf{A}_y = \mathbf{I} = 6 \text{ kN}\uparrow$$

<u>Free body: Joint *A*</u>:

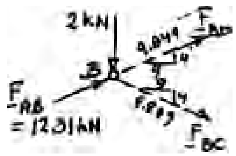


$$\frac{F_{AB}}{9.849} = \frac{F_{AC}}{9} = \frac{5 \text{ kN}}{4}$$

$F_{AB} = 12.31 \text{ kN} \quad C$ ◀

$F_{AC} = 11.25 \text{ kN} \quad T$ ◀

<u>Free body: Joint *B*</u>:



$$\xrightarrow{+} \Sigma F_x = \frac{9}{9.849}(12.31 \text{ kN} + F_{BD} + F_{DC}) = 0$$

or $$F_{BD} + F_{BC} = -12.31 \text{ kN} \tag{1}$$

$$+\uparrow \Sigma F_y = \frac{4}{9.849}(12.31 \text{ kN} + F_{BD} - F_{BC}) - 2 \text{ kN} = 0$$

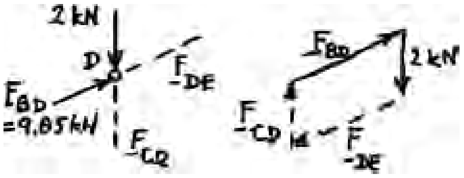
or $$F_{BD} - F_{BC} = -7.386 \text{ kN} \tag{2}$$

Add (1) and (2): $2F_{BD} = -19.70 \text{ kN}$ $\quad F_{BD} = 9.85 \text{ kN} \quad C$ ◀

Subtract (2) from (1): $2F_{BC} = -4.924 \text{ kN}$ $\quad F_{BC} = 2.46 \text{ kN} \quad C$ ◀

PROBLEM 6.10 (Continued)

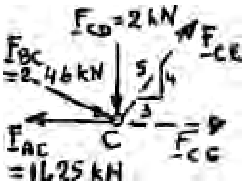
Free body: Joint *D*:



From force polygon: $F_{CD} = 2.00 \text{ kN} \quad C$ ◀

$F_{DE} = 9.85 \text{ kN} \quad C$ ◀

Free body: Joint *C*:



$$+\uparrow \Sigma F_y = \frac{4}{5}F_{CE} - \frac{4}{9.849}(2.46 \text{ kN}) - 2 \text{ kN} = 0 \qquad F_{CE} = 3.75 \text{ kN} \quad T \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{CG} + \frac{3}{5}(3.75 \text{ kN}) + \frac{9}{9.849}(2.46 \text{ kN}) - 11.25 \text{ kN} = 0$$

$$F_{CG} = +6.75 \text{ kN} \qquad F_{CG} = 6.75 \text{ kN} \quad T \blacktriangleleft$$

From the symmetry of the truss and loading:

$$F_{EF} = F_{DE} \qquad F_{EF} = 9.85 \text{ kN} \quad C \blacktriangleleft$$

$$F_{EG} = F_{CE} \qquad F_{EG} = 3.75 \text{ kN} \quad T \blacktriangleleft$$

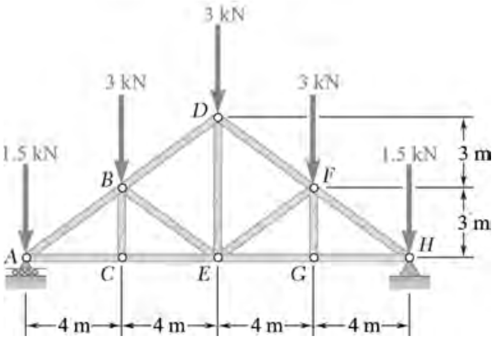
$$F_{FG} = F_{CD} \qquad F_{FG} = 2.00 \text{ kN} \quad C \blacktriangleleft$$

$$F_{FH} = F_{BD} \qquad F_{FH} = 9.85 \text{ kN} \quad C \blacktriangleleft$$

$$F_{GH} = F_{BC} \qquad F_{GH} = 2.46 \text{ kN} \quad C \blacktriangleleft$$

$$F_{GI} = F_{AC} \qquad F_{GI} = 11.25 \text{ kN} \quad T \blacktriangleleft$$

$$F_{HI} = F_{AB} \qquad F_{HI} = 12.31 \text{ kN} \quad C \blacktriangleleft$$

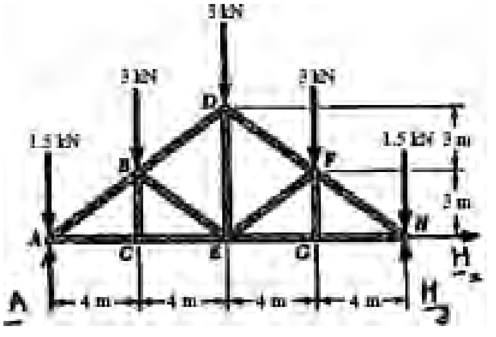


PROBLEM 6.11

Determine the force in each member of the Howe roof truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss



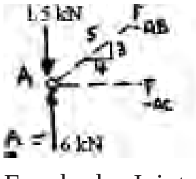
$$\Sigma F_x = 0: \quad \mathbf{H}_x = 0$$

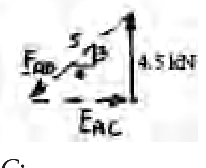
Because of the symmetry of the truss and loading:

$$A = H_y = \frac{1}{2} \quad \text{Total load}$$

$$\mathbf{A} = \mathbf{H}_y = 6 \text{ kN} \uparrow$$

Free body: Joint A:





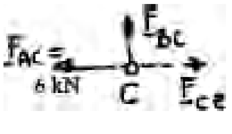
$$\frac{F_{AB}}{5} = \frac{F_{AC}}{4} = \frac{4.5 \text{ kN}}{3}$$

$\mathbf{F}_{AB} = 7.50 \text{ kN} \quad C$ ◀

$\mathbf{F}_{AC} = 6.00 \text{ kN} \quad T$ ◀

Free body: Joint C:

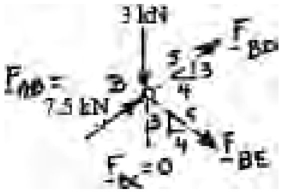
BC is a zero-force member



$\mathbf{F}_{BC} = 0$

$\mathbf{F}_{CE} = 6.00 \text{ kN} \quad T$ ◀

PROBLEM 6.11 (Continued)



Free body: Joint B:

$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{4}{5}F_{BD} + \frac{4}{5}F_{BC} + \frac{4}{5}(7.5 \text{ kN}) = 0$$

or $$F_{BD} + F_{BE} = -7.5 \text{ kN} \qquad (1)$$

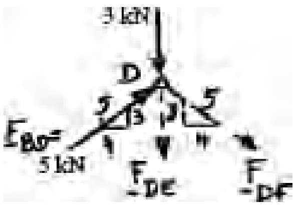
$$+\!\uparrow \Sigma F_y = 0: \quad \frac{3}{5}F_{BD} - \frac{3}{5}F_{BE} + \frac{3}{5}(7.5 \text{ kN}) - 3 \text{ kN} = 0$$

or $$F_{BD} - F_{BE} = -2.5 \text{ kN} \qquad (2)$$

Add Eqs. (1) and (2): $2F_{BD} = -10$ kN $\qquad F_{BD} = 5.00 \text{ kN} \quad C$ ◀

Subtract (2) from (1): $2F_{BE} = -5$ kN $\qquad F_{BE} = 2.50 \text{ kN} \quad C$ ◀

Free Body: Joint D:



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{4}{5}(5 \text{ kN}) + \frac{4}{5}F_{DF} = 0$$

$F_{DF} = -5$ kN $\qquad F_{DF} = 5.00 \text{ kN} \quad C$ ◀

$$+\!\uparrow \Sigma F_y = 0: \quad \frac{3}{5}(5 \text{ kN}) - \frac{3}{5}(-5 \text{ kN}) - 3 \text{ kN} - F_{DE} = 0$$

$F_{DE} = +3$ kN $\qquad F_{DE} = 3.00 \text{ kN} \quad T$ ◀

Because of the symmetry of the truss and loading, we deduce that

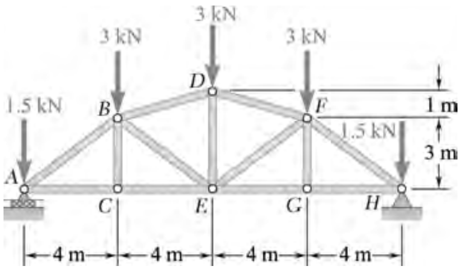
$F_{EF} = F_{BE}$ $\qquad F_{EF} = 2.50 \text{ kN} \quad C$ ◀

$F_{EG} = F_{CE}$ $\qquad F_{EG} = 6.00 \text{ kN} \quad T$ ◀

$F_{FG} = F_{BC}$ $\qquad F_{FG} = 0$ ◀

$F_{FH} = F_{AB}$ $\qquad F_{FH} = 7.50 \text{ kN} \quad C$ ◀

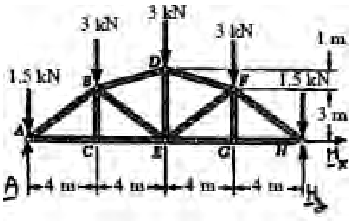
$F_{GH} = F_{AC}$ $\qquad F_{GH} = 6.00 \text{ kN} \quad T$ ◀



PROBLEM 6.12

Determine the force in each member of the Gambrel roof truss shown. State whether each member is in tension or compression.

SOLUTION



Free body: Truss

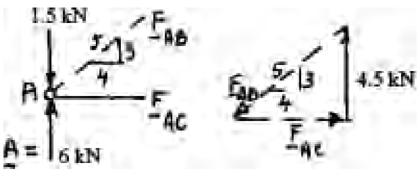
$$\Sigma F_x = 0: \quad \mathbf{H}_x = 0$$

Because of the symmetry of the truss and loading

$$\mathbf{A} = \mathbf{H}_y = \frac{1}{2} \quad \text{Total load}$$

$$\mathbf{A} = \mathbf{H}_y = 6 \text{ kN}\uparrow$$

Free body: Joint *A*:

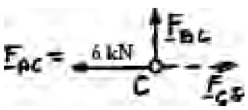


$$\frac{F_{AB}}{5} = \frac{F_{AC}}{4} = \frac{4.5 \text{ kN}}{3}$$

$\mathbf{F}_{AB} = 7.50 \text{ kN} \quad C$ ◀

$\mathbf{F}_{AC} = 6.00 \text{ kN} \quad T$ ◀

Free body: Joint *C*:

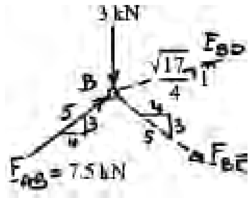


BC is a zero-force member

$$\mathbf{F}_{BC} = 0$$

$F_{CE} = 6.00 \text{ kN} \quad T$ ◀

Free body: Joint *B*:



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{4}{\sqrt{17}} F_{BD} + \frac{4}{5} F_{BE} + \frac{4}{5}(7.5 \text{ kN}) = 0$$

or $$0.9701 F_{BD} + 0.8 F_{BE} = -6 \text{ kN} \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{17}} F_{BD} - \frac{3}{5} F_{BE} + \frac{3}{5}(7.5 \text{ kN}) - 3 \text{ kN} = 0$$

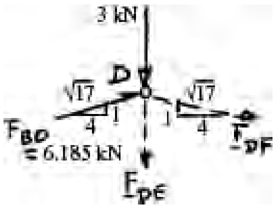
or $$0.2425 F_{BD} - 0.6 F_{BE} = -1.5 \text{ kN} \qquad (2)$$

Solving (1) and (2)

$$F_{BD} = -6.185 \text{ kN} \qquad F_{BD} = 6.19 \text{ kN} \quad C \blacktriangleleft$$

$$F_{BE} = 1.9328 \times 10^{-4} \text{ kN} \qquad F_{BE} \approx 0 \text{ kN} \quad C \blacktriangleleft$$

Free body: Joint *D*:



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{4}{\sqrt{17}}(6.185 \text{ kN}) + \frac{4}{\sqrt{17}} F_{DF} = 0$$

$$F_{DF} = -6.185 \text{ kN} \qquad F_{DF} = 6.19 \text{ kN} \quad C \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{17}}(6.185 \text{ kN}) - \frac{1}{\sqrt{17}}(-6.185 \text{ kN}) - 3 \text{ kN} - F_{DE} = 0$$

$$F_{DE} = 1.6568 \times 10^{-4} \text{ kN} \qquad F_{DE} \approx 0 \text{ kN} \quad T \blacktriangleleft$$

Because of the symmetry of the truss and loading, we deduce that

$$F_{EF} = F_{BE} \qquad F_{EF} = 0 \text{ kN} \quad C \blacktriangleleft$$

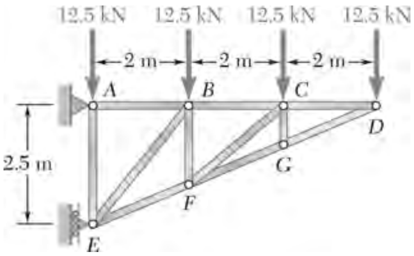
$$F_{EG} = F_{CE} \qquad F_{EG} = 6.00 \text{ kN} \quad T \blacktriangleleft$$

$$F_{FG} = F_{BC} \qquad F_{FG} = 0 \blacktriangleleft$$

$$F_{FH} = F_{AB} \qquad F_{FH} = 7.50 \text{ kN} \quad C \blacktriangleleft$$

$$F_{GH} = F_{AC} \qquad F_{GH} = 6.00 \text{ kN} \quad T \blacktriangleleft$$

Note: Compare results with those of Problem 6.9.



PROBLEM 6.13

Determine the force in each member of the truss shown.

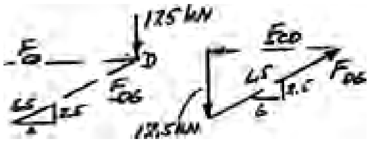
SOLUTION

Joint D:

$$\frac{12.5\text{ kN}}{2.5} = \frac{F_{CD}}{6} = \frac{F_{DG}}{6.5}$$

$F_{CD} = 30\text{ kN}\quad T$ ◀

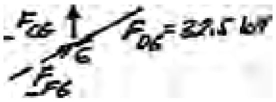
$F_{DG} = 32.5\text{ kN}\quad C$ ◀



Joint G:

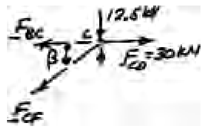
$\nwarrow \Sigma F = 0:\quad F_{CG} = 0$ ◀

$\nearrow \Sigma F = 0:\quad F_{FG} = 32.5\text{ kN}\quad C$ ◀



Joint C:

$$BF = \frac{2}{3}(2.5\text{ m}) = 1.6667\text{ m}\qquad \beta = \angle BCF = \tan^{-1}\frac{BF}{2} = 39.81°$$



$$+\uparrow \Sigma F_y = 0:\quad -12.5\text{ kN} - F_{CF}\sin\beta = 0$$

$$-12.5\text{ kN} - F_{CF}\sin 39.81° = 0$$

$$F_{CF} = -19.526\text{ kN}$$

$F_{CF} = 19.53\text{ kN}\quad C$ ◀

$$\xrightarrow{+} \Sigma F_x = 0:\quad 30\text{ kN} - F_{BC} - F_{CF}\cos\beta = 0$$

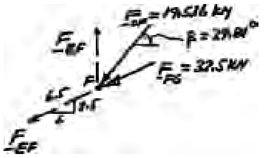
$$30\text{ kN} - F_{BC} - (-19.526\text{ kN})\cos 39.81° = 0$$

$$F_{BC} = +45.0\text{ kN}$$

$F_{BC} = 45.0\text{ kN}\quad T$ ◀

Joint F:

$$\xrightarrow{+} \Sigma F_x = 0:\quad -\frac{6}{6.5}F_{EF} - \frac{6}{6.5}(32.5\text{ kN}) - F_{CF}\cos\beta = 0$$



$$F_{EF} = -32.5\text{ kN} - \left(\frac{6.5}{6}\right)(19.526\text{ kN})\cos 39.81°$$

$$F_{EF} = -48.75\text{ kN}$$

$F_{EF} = 48.8\text{ kN}\quad C$ ◀

$$+\uparrow \Sigma F_y = 0:\quad F_{BF} - \frac{2.5}{6.5}F_{EF} - \frac{2.5}{6.5}(32.5\text{ kN}) - (19.526\text{ kN})\sin 39.81° = 0$$

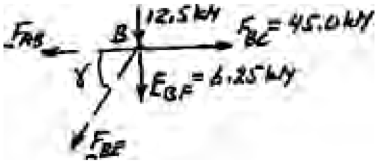
$$F_{BF} - \frac{2.5}{6.5}(-48.75\text{ kN}) - 12.5\text{ kN} - 12.5\text{ kN} = 0$$

$$F_{BF} = +6.25\text{ kN}$$

$F_{BF} = 6.25\text{ kN}\quad T$ ◀

PROBLEM 6.13 (Continued)

Joint B:



$$\tan\alpha = \frac{2.5\text{ m}}{2\text{ m}}; \quad \gamma = 51.34^\circ$$

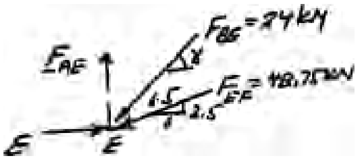
$$+\uparrow \Sigma F_y = 0: \quad -12.5\text{ kN} - 6.25\text{ kN} - F_{BE}\sin 51.34^\circ = 0$$

$$F_{BE} = -24.0\text{ kN} \qquad\qquad F_{BE} = 24.0\text{ kN} \quad C \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad 45.0\text{ kN} - F_{AB} + (24.0\text{ kN})\cos 51.34^\circ = 0$$

$$F_{AB} = +60\text{ kN} \qquad\qquad F_{AB} = 60.0\text{ kN} \quad T \blacktriangleleft$$

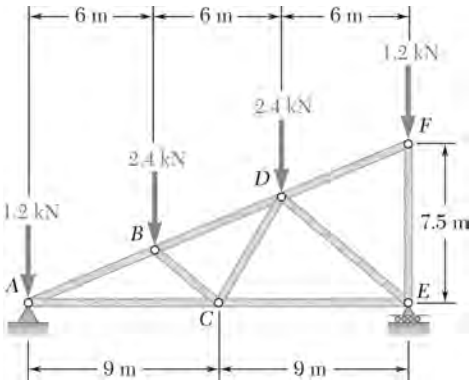
Joint E:



$$\gamma = 51.34^\circ$$

$$\xrightarrow{+} \Sigma F_y = 0: \quad F_{AE} - (24\text{ kN})\sin 51.34^\circ - (48.75\text{ kN})\frac{2.5}{6.5} = 0$$

$$F_{AE} = +37.5\text{ kN} \qquad\qquad F_{AE} = 37.5\text{ kN} \quad T \blacktriangleleft$$

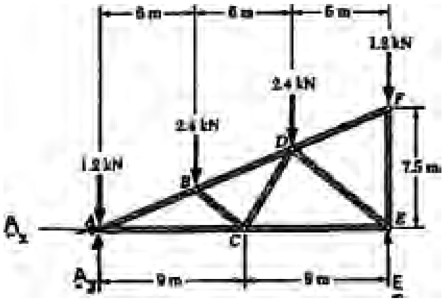


PROBLEM 6.14

Determine the force in each member of the roof truss shown. State whether each member is in tension or compression.

SOLUTION

<u>Free body: Truss</u>



$$\Sigma F_x = 0: \quad \mathbf{A}_x = 0$$

From symmetry of loading:

$$\mathbf{A}_y = \mathbf{E} = \frac{1}{2} \text{ Total load}$$

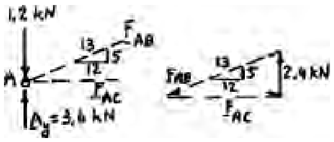
$$\mathbf{A}_y = \mathbf{E} = 3.6 \text{ kN} \uparrow$$

We note that *DF* is a zero-force member and that *EF* is aligned with the load. Thus

$$F_{DF} = 0 \blacktriangleleft$$

$$F_{EF} = 1.2 \text{ kN} \quad C \blacktriangleleft$$

<u>Free body: Joint *A*</u>:

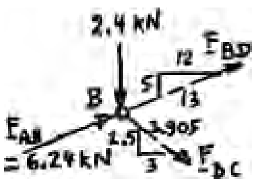


$$\frac{F_{AB}}{13} = \frac{F_{AC}}{12} = \frac{2.4 \text{ kN}}{5}$$

$$F_{AB} = 6.24 \text{ kN} \quad C \blacktriangleleft$$

$$F_{AC} = 2.76 \text{ kN} \quad T \blacktriangleleft$$

<u>Free body: Joint *B*</u>:



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{3}{3.905} F_{BC} + \frac{12}{13} F_{BD} + \frac{12}{13}(6.24 \text{ kN}) = 0 \qquad (1)$$

$$+\!\uparrow \Sigma F_y = 0: \quad -\frac{2.5}{3.905} F_{BC} + \frac{5}{13} F_{BD} + \frac{5}{13}(6.24 \text{ kN}) - 2.4 \text{ kN} = 0 \qquad (2)$$

PROBLEM 6.14 (Continued)

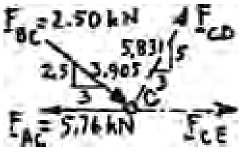
Multiply (1) by 2.5, (2) by 3, and add:

$$\frac{45}{13}F_{BD} + \frac{45}{13}(6.24\text{ kN}) - 7.2\text{ kN} = 0, \quad F_{BD} = -4.16\text{ kN}, \qquad F_{BD} = 4.16\text{ kN} \quad C \blacktriangleleft$$

Multiply (1) by 5, (2) by –12, and add:

$$\frac{45}{3.905}F_{BC} + 28.8\text{ kN} = 0, \quad F_{BC} = -2.50\text{ kN}, \qquad F_{BC} = 2.50\text{ kN} \quad C \blacktriangleleft$$

Free body: Joint C:



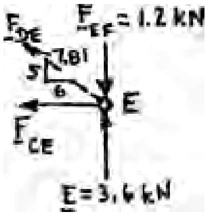
$$+\uparrow \Sigma F_y = 0: \quad \frac{5}{5.831}F_{CD} - \frac{2.5}{3.905}(2.50\text{ kN}) = 0$$

$$F_{CD} = 1.867\text{ kN} \quad T \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{CE} - 5.76\text{ kN} + \frac{3}{3.905}(2.50\text{ kN}) + \frac{3}{5.831}(1.867\text{ kN}) = 0$$

$$F_{CE} = 2.88\text{ kN} \quad T$$

Free body: Joint E:

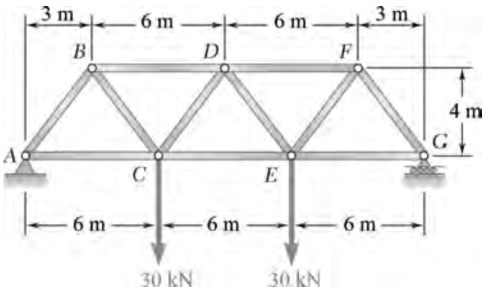


$$+\uparrow \Sigma F_y = 0: \quad \frac{5}{7.81}F_{DE} + 3.6\text{ kN} - 1.2\text{ kN} = 0$$

$$F_{DE} = -3.75\text{ kN} \qquad F_{DE} = 3.75\text{ kN} \quad C \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -F_{CE} - \frac{6}{7.81}(-3.75\text{ kN}) = 0$$

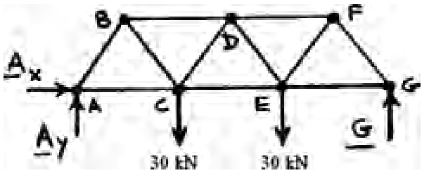
$$F_{CE} = +2.88\text{ kN} \qquad F_{CE} = 2.88\text{ kN} \quad T \qquad \text{(Checks)}$$



PROBLEM 6.15

Determine the force in each member of the Warren bridge truss shown. State whether each member is in tension or compression.

SOLUTION



Free body: Truss $\qquad \Sigma F_x = 0: \quad A_x = 0$

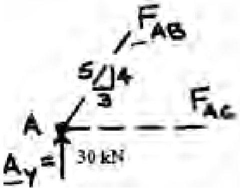
Due to symmetry of truss and loading

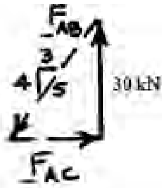
$$A_y = G = \frac{1}{2}\text{Total load} = 30 \text{ kN}\uparrow$$

Free body: Joint *A*: $\qquad \dfrac{F_{AB}}{5} = \dfrac{F_{AC}}{3} = \dfrac{30}{4}\text{ kN}$

$F_{AB} = 37.5 \text{ kN} \quad C$ ◀

$F_{AC} = 22.5 \text{ kN} \quad T$ ◀

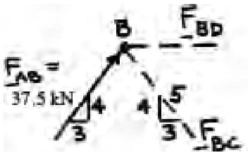


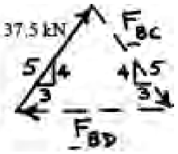


Free body: Joint *B*: $\qquad \dfrac{F_{BC}}{5} = \dfrac{F_{BD}}{6} = \dfrac{37.5}{5}\text{ kN}$

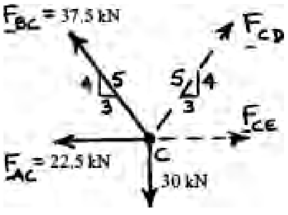
$F_{BC} = 37.5 \text{ kN} \quad T$ ◀

$F_{BD} = 45 \text{ kN} \quad C$ ◀





Free body: Joint *C*:



$$+\uparrow \Sigma F_y = 0: \quad \frac{4}{5}(37.5) + \frac{4}{5}F_{CD} - 30 = 0$$

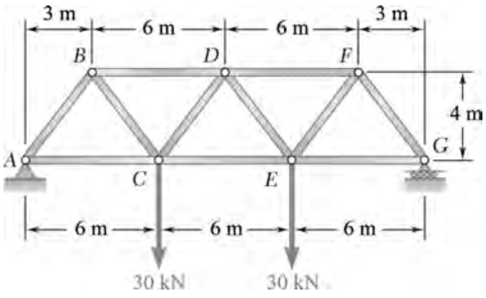
$F_{CD} = 0$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{CE} - 22.5 - \frac{3}{5}(37.5) = 0$$

$$+\uparrow F_{CE} = +45 \text{ kN}$$

$F_{CE} = 45 \text{ kN} \quad T$ ◀

Truss and loading symmetrical about ℄



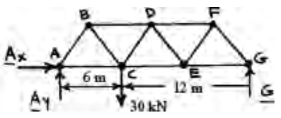
PROBLEM 6.16

Solve Problem 6.15 assuming that the load applied at E has been removed.

PROBLEM 6.15 Determine the force in each member of the Warren bridge truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss

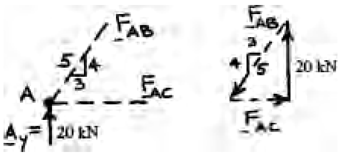


$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\circlearrowleft \Sigma M_G = 0: \quad 30(12) - A_y(18) = 0 \quad \mathbf{A}_y = 20 \text{ kN} \uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad 20 - 30 + G = 0 \quad \mathbf{G} = 10 \text{ kN} \uparrow$$

Free body: Joint A:

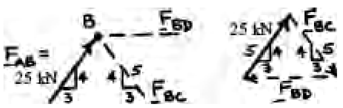


$$\frac{F_{AB}}{5} = \frac{F_{AC}}{3} = \frac{20 \text{ kN}}{4}$$

$F_{AB} = 25.0 \text{ kN} \quad C$ ◀

$F_{AC} = 15.00 \text{ kN} \quad T$ ◀

Free body Joint B:

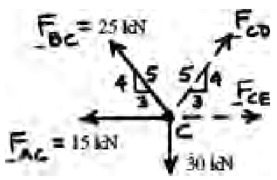


$$\frac{F_{BC}}{5} = \frac{F_{BD}}{6} = \frac{25 \text{ kN}}{5}$$

$F_{BC} = 25.0 \text{ kN} \quad T$ ◀

$F_{BD} = 30.0 \text{ kN} \quad C$ ◀

Free body Joint C:



$$+\uparrow \Sigma F_y = 0: \quad \frac{4}{5}(25) + \frac{4}{5}F_{CD} - 30 = 0$$

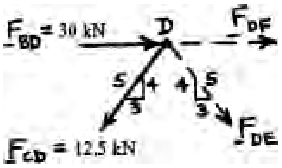
$F_{CD} = 12.50 \text{ kN} \quad T$ ◀

$$+\uparrow \Sigma F_x = 0: \quad F_{CE} + \frac{3}{5}(12.5) - \frac{3}{5}(25) - 15 = 0$$

$F_{CE} = 22.5 \text{ kN} \quad T$ ◀

PROBLEM 6.16 (Continued)

Free body: Joint *D*:



$$+\uparrow \Sigma F_y = 0: \quad -\frac{4}{5}(12.5) - \frac{4}{5}F_{DE} = 0$$

$$F_{DE} = -12.5 \text{ kN} \qquad F_{DE} = 12.50 \text{ kN} \quad C \blacktriangleleft$$

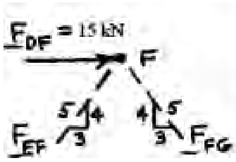
$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{DF} + 30 - \frac{3}{5}(12.5) - \frac{3}{5}(12.5) = 0$$

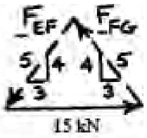
$$F_{DF} = -15 \text{ kN} \qquad F_{DF} = 15.00 \text{ kN} \quad C \blacktriangleleft$$

Free body: Joint *F*:

$$\frac{F_{EF}}{5} = \frac{F_{FG}}{5} = \frac{15 \text{ kN}}{6} \qquad F_{EF} = 12.50 \text{ kN} \quad T \blacktriangleleft$$

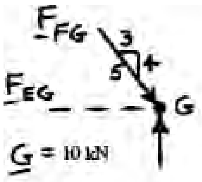
$$F_{FG} = 12.50 \text{ kN} \quad C \blacktriangleleft$$

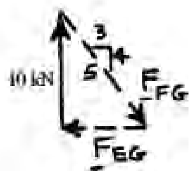




Free body: Joint *G*:

$$\frac{F_{EG}}{3} = \frac{10 \text{ kN}}{4} \qquad F_{EG} = 7.50 \text{ kN} \quad T \blacktriangleleft$$

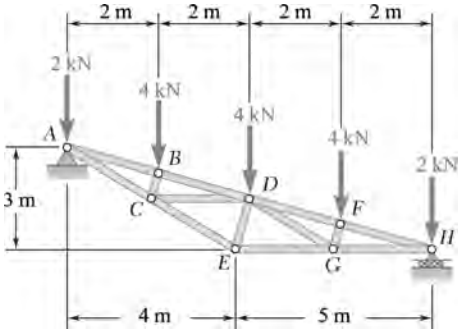




Also:

$$\frac{F_{FG}}{5} = \frac{10 \text{ kN}}{4}$$

$$F_{FG} = 12.50 \text{ kN} \quad C \quad \text{(Checks)}$$

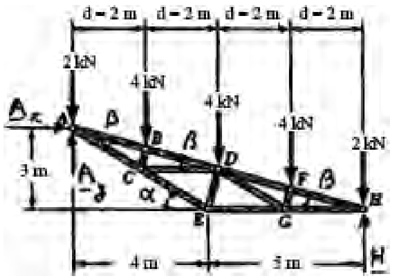


PROBLEM 6.17

Determine the force in member *DE* and in each of the members located to the left of *DE* for the inverted Howe roof truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss:



$$\Sigma F_x = 0: \quad \mathbf{A}_x = 0$$

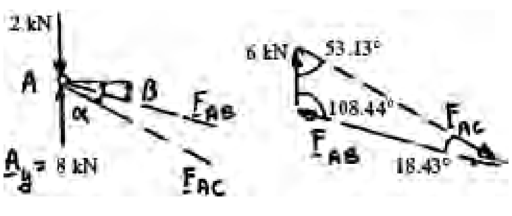
$$\stackrel{+}{\circlearrowleft} \Sigma M_H = 0: \quad (2\text{ kN})(4d) + (4\text{ kN})(3d) + (4\text{ kN})(2d) + (4\text{ kN})d - A_y(4d) = 0$$

$$A_y = 8\text{ kN}\uparrow$$

Angles: $\tan\alpha = \dfrac{3}{4} \quad \alpha = 36.87°$

$\tan\beta = \dfrac{3}{9} \quad \beta = 18.43°$

Free body: Joint *A*:



$$\frac{F_{AB}}{\sin 53.13°} = \frac{F_{AC}}{\sin 108.44°} = \frac{6\text{ kN}}{\sin 18.43°}$$

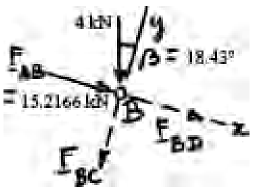
$F_{AB} = 15.2166\text{ kN} \quad C$

$F_{AC} = 18.004\text{ kN} \quad T$

$F_{AB} = 15.22\text{ kN} \quad C \quad F_{AC} = 18.00\text{ kN} \quad T$ ◄

PROBLEM 6.17 (Continued)

Free body: Joint B:



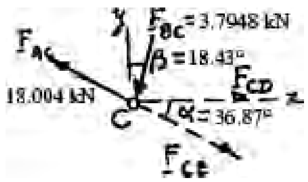
$$+\nearrow \Sigma F_y = 0: \quad -F_{BC} - (4\text{ kN})\cos 18.43° = 0$$

$$F_{BC} = -3.7948\text{ kN} \qquad F_{BC} = 3.79\text{ kN} \quad C \blacktriangleleft$$

$$+\searrow \Sigma F_x = 0: \quad F_{BD} + 15.2166\text{ kN} + (4\text{ kN})\sin 18.43° = 0$$

$$F_{BD} = -16.4812\text{ kN} \qquad F_{BD} = 16.48\text{ kN} \quad C \blacktriangleleft$$

Free body: Joint C:



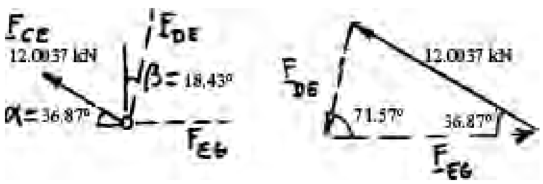
$$+\uparrow \Sigma F_y = 0: \quad -F_{CE}\sin 36.87° + (18.004\text{ kN})\sin 36.87° - (3.7948\text{ kN})\cos 18.43° = 0$$

$$F_{CE} = 12.0037\text{ kN} \qquad F_{CE} = 12.00\text{ kN} \quad T \blacktriangleleft$$

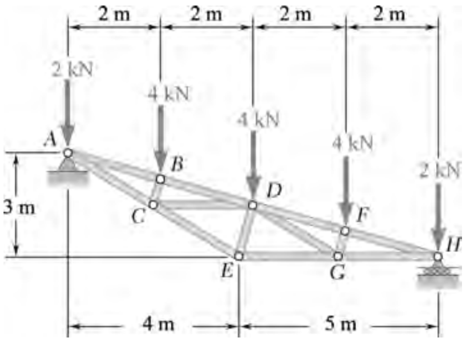
$$\overset{+}{\longrightarrow} \Sigma F_x = 0: \quad F_{CD} - (18.004\text{ kN})\cos 36.87° + (12.0037\text{ kN})\cos 36.87° - (3.7948\text{ kN})\sin 18.43° = 0$$

$$F_{CD} = 5.9999\text{ kN} \qquad F_{CD} = 6.00\text{ kN} \quad T \blacktriangleleft$$

Free body: Joint E:



$$\frac{F_{DE}}{\sin 36.87°} = \frac{12.0037\text{ kN}}{\sin 71.57°} \qquad F_{DE} = 7.59\text{ kN} \quad C \blacktriangleleft$$

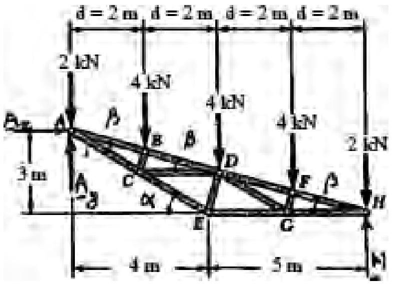


PROBLEM 6.18

Determine the force in each of the members located to the right of *DE* for the inverted Howe roof truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss



$$+\circlearrowleft \Sigma M_A = 0: \quad H(4d) - (4\text{ kN})d - (4\text{ kN})(2d) - (4\text{ kN})(3d) - (2\text{ kN})(4d) = 0$$

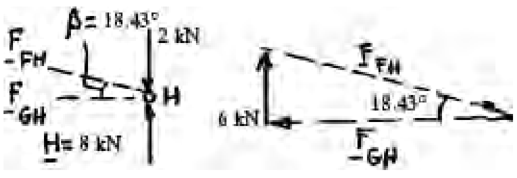
$$\mathbf{H} = 8\text{ kN} \uparrow$$

Angles:

$$\tan\alpha = \frac{3}{4} \quad \alpha = 36.87°$$

$$\tan\beta = \frac{3}{9} \quad \beta = 18.43°$$

Free body: Joint *H*:



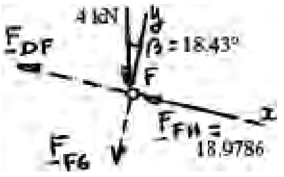
$$F_{GH} = (6\text{ kN})\cot 18.43°$$

$$F_{GH} = 18.0052\text{ kN} \quad T \qquad\qquad F_{GH} = 18.00\text{ kN} \quad T \blacktriangleleft$$

$$F_{FH} = \frac{6\text{ kN}}{\sin 18.43°} = 18.9786\text{ kN} \qquad\qquad F_{FH} = 18.98\text{ kN} \quad C \blacktriangleleft$$

PROBLEM 6.18 (Continued)

Free body Joint *F*:



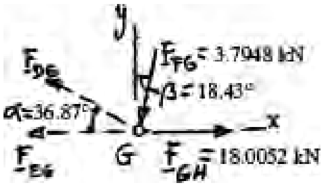
$$+\nearrow \Sigma F_y = 0: \quad -F_{FG} - (4 \text{ kN})\cos 18.43° = 0$$

$$F_{FG} = -3.7948 \text{ kN} \qquad F_{FG} = 3.79 \text{ kN} \quad C \blacktriangleleft$$

$$+\searrow \Sigma F_x = 0: \quad -F_{DF} - 18.9786 \text{ kN} + (4 \text{ kN})\sin 18.43° = 0$$

$$F_{DF} = -17.7140 \text{ kN} \qquad F_{DF} = 17.71 \text{ kN} \quad C \blacktriangleleft$$

Free body: Joint *G*:



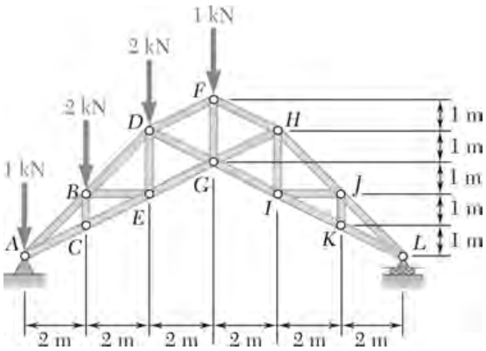
$$+\uparrow \Sigma F_y = 0: \quad F_{DG} \sin 36.87° - (3.7948 \text{ kN})\cos 18.43° = 0$$

$$F_{DG} = +6.000 \text{ kN}$$

$$F_{DG} = 6.00 \text{ kN} \quad T \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -F_{EG} + 18.0052 \text{ kN} - (3.7948 \text{ kN})\sin 18.43° - (6.00 \text{ kN})\cos 36.87° = 0$$

$$F_{EG} = +12.0054 \text{ kN} \qquad F_{EG} = 12.01 \text{ kN} \quad T \blacktriangleleft$$

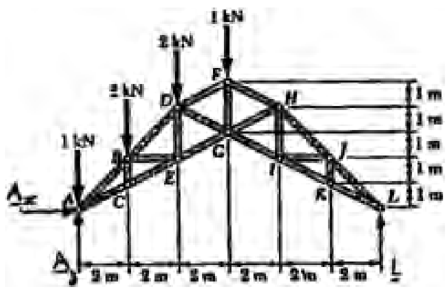


PROBLEM 6.19

Determine the force in each of the members located to the left of FG for the scissors roof truss shown. State whether each member is in tension or compression.

SOLUTION

<u>Free Body: Truss</u>



$$\Sigma F_x = 0: \quad \mathbf{A}_x = 0$$

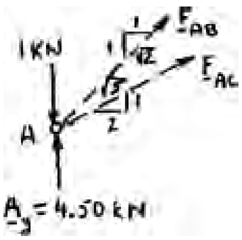
$$+\circlearrowleft \Sigma M_L = 0: \quad (1\text{ kN})(12\text{ m}) + (2\text{ kN})(10\text{ m}) + (2\text{ kN})(8\text{ m}) + (1\text{ kN})(6\text{ m}) - A_y(12\text{ m}) = 0$$

$$\mathbf{A}_y = 4.50\text{ kN} \uparrow$$

We note that BC is a zero-force member: $F_{BC} = 0$ ◀

Also: $F_{CE} = F_{AC}$ (1)

<u>Free body: Joint A</u>:



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{1}{\sqrt{2}} F_{AB} + \frac{2}{\sqrt{5}} F_{AC} = 0 \qquad (2)$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{2}} F_{AB} + \frac{1}{\sqrt{5}} F_{AC} + 3.50\text{ kN} = 0 \qquad (3)$$

Multiply (3) by –2 and add (2):

$$-\frac{1}{\sqrt{2}} F_{AB} - 7\text{ kN} = 0 \qquad F_{AB} = 9.90\text{ kN} \quad C \;◀$$

PROBLEM 6.19 (Continued)

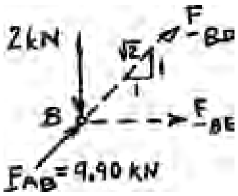
Subtract (3) from (2):

$$\frac{1}{\sqrt{5}}F_{AC} - 3.50\text{ kN} = 0 \quad F_{AC} = 7.826\text{ kN} \qquad F_{AC} = 7.83\text{ kN} \quad T \blacktriangleleft$$

From (1):

$$F_{CE} = F_{AC} = 7.826\text{ kN} \qquad F_{CE} = 7.83\text{ kN} \quad T \blacktriangleleft$$

Free body: Joint *B*:



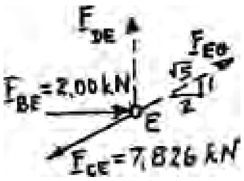
$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{2}}F_{BD} + \frac{1}{\sqrt{2}}(9.90\text{ kN}) - 2\text{ kN} = 0$$

$$F_{BD} = -7.071\text{ kN} \qquad F_{BD} = 7.07\text{ kN} \quad C \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{BE} + \frac{1}{\sqrt{2}}(9.90 - 7.071)\text{kN} = 0$$

$$F_{BE} = -2.000\text{ kN} \qquad F_{BE} = 2.00\text{ kN} \quad C \blacktriangleleft$$

Free body: Joint *E*:



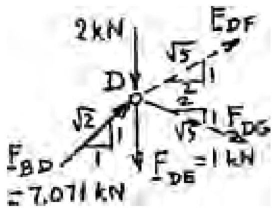
$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{2}{\sqrt{5}}(F_{EG} - 7.826\text{ kN}) + 2.00\text{ kN} = 0$$

$$F_{EG} = 5.590\text{ kN} \qquad F_{EG} = 5.59\text{ kN} \quad T \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad F_{DE} - \frac{1}{\sqrt{5}}(7.826 - 5.590)\text{kN} = 0$$

$$F_{DE} = 1.000\text{ kN} \qquad F_{DE} = 1.000\text{ kN} \quad T \blacktriangleleft$$

Free body: Joint *D*:



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{2}{\sqrt{5}}(F_{DF} + F_{DG}) + \frac{1}{\sqrt{2}}(7.071\text{ kN})$$

or

$$F_{DF} + F_{DG} = -5.590\text{ kN} \qquad (4)$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}}(F_{DF} - F_{DG}) + \frac{1}{\sqrt{2}}(7.071\text{ kN}) = 2\text{ kN} - 1\text{ kN} = 0$$

or

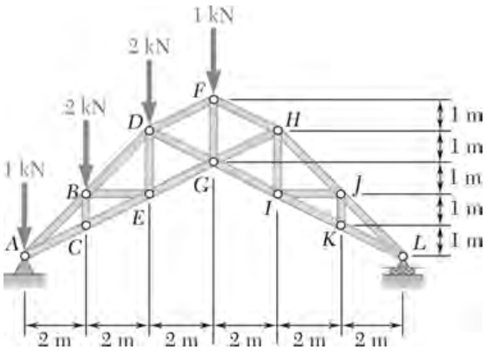
$$F_{DE} - F_{DG} = -4.472 \qquad (5)$$

Add (4) and (5):

$$2F_{DF} = -10.062\text{ kN} \qquad F_{DF} = 5.03\text{ kN} \quad C \blacktriangleleft$$

Subtract (5) from (4):

$$2F_{DG} = -1.1180\text{ kN} \qquad F_{DG} = 0.559\text{ kN} \quad C \blacktriangleleft$$

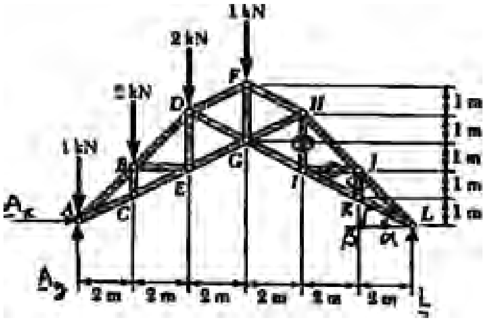


PROBLEM 6.20

Determine the force in member FG and in each of the members located to the right of FG for the scissors roof truss shown. State whether each member is in tension or compression.

SOLUTION

<u>Free body: Truss</u>



$$\circlearrowleft^{+} \Sigma M_A = 0: \quad L(12 \text{ m}) - (2 \text{ kN})(2 \text{ m}) - (2 \text{ kN})(4 \text{ m}) - (1 \text{ kN})(6 \text{ m}) = 0$$

$$\mathbf{L} = 1.500 \text{ kN} \uparrow$$

<u>Angles</u>:

$$\tan\alpha = 1 \qquad \alpha = 45°$$

$$\tan\beta = \frac{1}{2} \qquad \beta = 26.57°$$

<u>Zero-force members</u>:

Examining successively joints K, J, and I, we note that the following members to the right of FG are zero-force members: JK, IJ, and HI.

Thus: $$F_{HI} = F_{IJ} = F_{JK} = 0 \blacktriangleleft$$

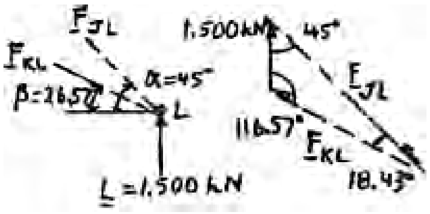
We also note that

$$F_{GI} = F_{IK} = F_{KL} \qquad (1)$$

and $$F_{HJ} = F_{JL} \qquad (2)$$

PROBLEM 6.20 (Continued)

Free body: Joint *L*:



$$\frac{F_{JL}}{\sin 116.57°} = \frac{F_{KL}}{\sin 45°} = \frac{1.500 \text{ kN}}{\sin 18.43°}$$

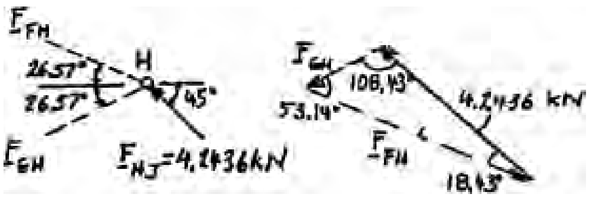
$F_{JL} = 4.2436$ kN $\qquad\qquad$ $F_{JL} = 4.24$ kN C ◀

$F_{KL} = 3.35$ kN T ◀

From Eq. (1): $\quad F_{GI} = F_{IK} = F_{KL} \qquad\qquad F_{GI} = F_{IK} = 3.35$ kN T ◀

From Eq. (2): $\quad F_{HJ} = F_{JL} = 4.2436$ kN $\qquad\qquad F_{HJ} = 4.24$ kN T ◀

Free body: Joint *H*:

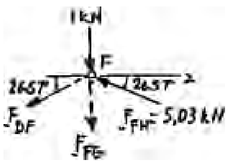


$$\frac{F_{FH}}{\sin 108.43°} = \frac{F_{GH}}{\sin 18.43°} = \frac{4.2436}{\sin 53.14°}$$

$F_{FH} = 5.03$ kN C ◀

$F_{GH} = 1.677$ kN T ◀

Free body: Joint *F*:

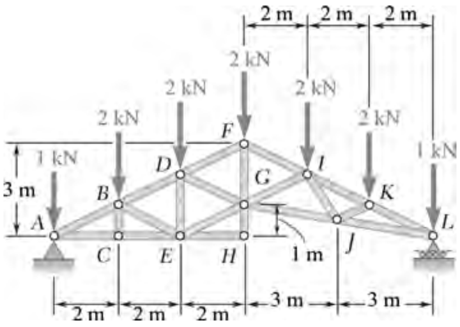


$$\xrightarrow{+} \Sigma F_x = 0: \quad -F_{DF} \cos 26.57° - (5.03 \text{ kN}) \cos 26.57° = 0$$

$$F_{DF} = -5.03 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0: \quad -F_{FG} - 1 \text{ kN} + (5.03 \text{ kN}) \sin 26.57° - (-5.03 \text{ kN}) \sin 26.57° = 0$$

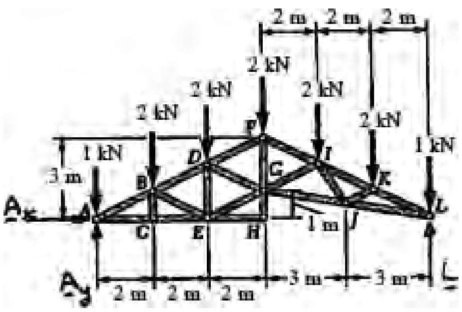
$F_{FG} = 3.500$ kN $\qquad\qquad$ $F_{FG} = 3.50$ kN T ◀



PROBLEM 6.21

Determine the force in each of the members located to the left of line *FGH* for the studio roof truss shown. State whether each member is in tension or compression.

SOLUTION



<u>Free body: Truss</u> $\Sigma F_x = 0: \quad \mathbf{A}_x = 0$

Because of symmetry of loading:

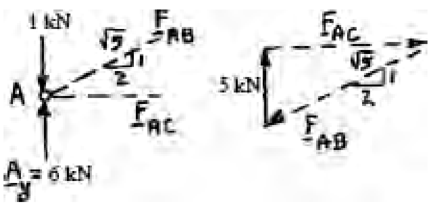
$$A_y = L = \frac{1}{2} \text{ Total load}$$

$$\mathbf{A}_y = \mathbf{L} = 6 \text{ kN} \uparrow$$

<u>Zero-Force Members</u>. Examining joints *C* and *H*, we conclude that *BC*, *EH*, and *GH* are zero-force members. Thus

$$\mathbf{F}_{BC} = \mathbf{F}_{EH} = 0 \quad ◀$$

Also, $\quad F_{CE} = F_{AC} \quad (1)$



<u>Free body: Joint *A*</u>

$$\frac{F_{AB}}{\sqrt{5}} = \frac{F_{AC}}{2} = \frac{5 \text{ kN}}{1}$$

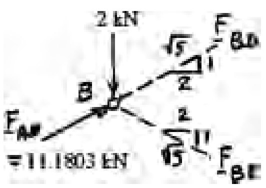
$$F_{AB} = 11.1803 \text{ kN} \quad C$$

$$F_{AB} = 11.18 \text{ kN} \quad C \quad ◀$$

$$F_{AC} = 10.00 \text{ kN} \quad T \quad ◀$$

From Eq. (1): $\quad F_{CE} = 10.00 \text{ kN} \quad T \quad ◀$

<u>Free body: Joint *B*</u>



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{2}{\sqrt{5}} F_{BD} + \frac{2}{\sqrt{5}} F_{BE} + \frac{2}{\sqrt{5}}(11.1803 \text{ kN}) = 0$$

or $\quad F_{BD} + F_{BE} = -11.1803 \text{ kN} \quad (2)$

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}} F_{BD} - \frac{1}{\sqrt{5}} F_{BE} + \frac{1}{\sqrt{5}}(11.1803 \text{ kN}) - 2 \text{ kN} = 0$$

or $\quad F_{BD} - F_{BE} = -6.7082 \text{ kN} \quad (3)$

PROBLEM 6.21 (Continued)

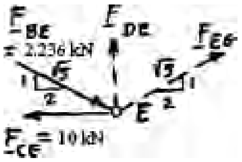
Add (2) and (3): $2F_{BD} = \;\; 17.8885 \text{ kN}$

$$F_{BD} = 8.94425 \text{ kN} \qquad\qquad F_{BD} = 8.94 \text{ kN} \quad C \blacktriangleleft$$

Subtract (3) from (1): $2F_{BE} = -4.4721 \text{ kN}$

$$F_{BE} = -2.236 \text{ kN} \qquad\qquad F_{BE} = 2.24 \text{ kN} \quad C \blacktriangleleft$$

Free body: Joint E



$$\xrightarrow{+} F_x = 0: \quad \frac{2}{\sqrt{5}} F_{EG} + \frac{2}{\sqrt{5}}(2.236 \text{ kN}) \quad 10 \text{ kN} = 0$$

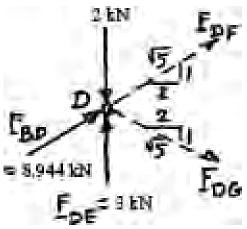
$$F_{EG} = 8.94 \text{ kN} \quad T \blacktriangleleft$$

$$+\!\uparrow F_y = 0: \quad F_{DE} + \frac{1}{\sqrt{5}}(8.944 \text{ kN}) \quad \frac{1}{\sqrt{5}}(2.236 \text{ kN}) = 0$$

$$F_{DE} = -3.00 \text{ kN} \qquad\qquad F_{DE} = 3.00 \text{ kN} \quad C \blacktriangleleft$$

Free body: Joint D

$$\xrightarrow{+} \Sigma F_x = 0: \; \frac{2}{\sqrt{5}} F_{DF} + \frac{2}{\sqrt{5}} F_{DG} + \frac{2}{\sqrt{5}}(8.944 \text{ kN}) = 0$$



or $$F_{DF} + F_{DG} = -8.944 \text{ kN} \qquad (4)$$

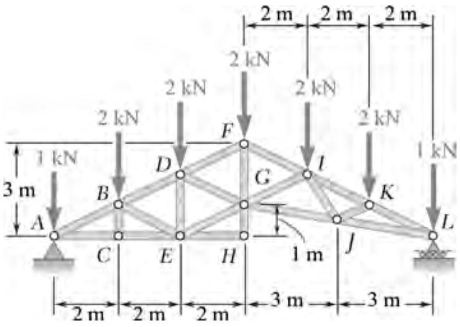
$$+\!\uparrow \Sigma F_y = 0: \; \frac{1}{\sqrt{5}} F_{DF} - \frac{1}{\sqrt{5}} F_{DG} + \frac{1}{\sqrt{5}}(8.944 \text{ kN})$$

$$+ 3 \text{ kN} - 2 \text{ kN} = 0$$

or $$F_{DF} - F_{DG} = -11.1803 \text{ kN} \qquad (5)$$

Add (4) and (5): $2F_{DF} = -20.1243 \text{ kN}$ $\qquad F_{DF} = 10.06 \text{ kN} \quad C \blacktriangleleft$

Subtract (5) from (4): $2F_{DG} = 2.2363 \text{ kN}$ $\qquad F_{DG} = 1.118 \text{ kN} \quad T \blacktriangleleft$



PROBLEM 6.22

Determine the force in member *FG* and in each of the members located to the right of *FG* for the studio roof truss shown. State whether each member is in tension or compression.

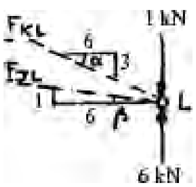
SOLUTION

Reaction at L: Because of the symmetry of the loading,

$$L = \frac{1}{2}\text{Total load, } \mathbf{L} = 6 \text{ kN}$$

(See F.B.D. to the left for more details)

Free body: Joint L



$$\alpha = \tan^{-1}\frac{3}{6} = 26.57°$$

$$\beta = \tan^{-1}\frac{1}{6} = 9.46°$$

$$\frac{F_{JL}}{\sin 63.43°} = \frac{F_{KL}}{\sin 99.46°} = \frac{5 \text{ kN}}{\sin 17.11°}$$

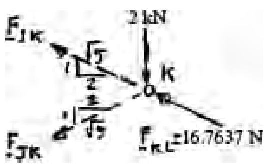
$F_{JL} = 15.20 \text{ kN} \quad C$ $\qquad$ $F_{JL} = 15.20 \text{ kN} \quad T$ ◄

$F_{KL} = 16.7637 \text{ kN} \quad C$ $\qquad$ $F_{KL} = 16.76 \text{ kN} \quad C$ ◄

Free body: Joint K

$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{2}{\sqrt{5}}F_{IK} - \frac{2}{\sqrt{5}}F_{JK} - \frac{2}{\sqrt{5}}(16.7637 \text{ kN}) = 0$$

or: $\qquad F_{IK} + F_{JK} = -16.7637 \text{ kN} \qquad (1)$



$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}}F_{IK} - \frac{1}{\sqrt{5}}F_{JK} + \frac{1}{\sqrt{5}}(16.7637) - 2 = 0$$

or: $\qquad F_{IK} - F_{JK} = -12.2916 \text{ kN} \qquad (2)$

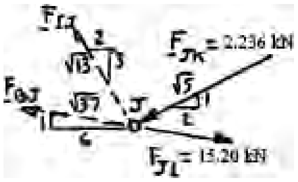
Add (1) and (2): $\qquad 2F_{IK} = -29.0553$

$F_{IK} = -14.5277 \text{ kN}$ $\qquad$ $F_{IK} = 14.53 \text{ kN} \quad C$ ◄

Subtract (2) from (1): $\qquad 2F_{JK} = -4.4721$

$F_{JK} = -2.236 \text{ kN}$ $\qquad$ $F_{JK} = 2.24 \text{ kN} \quad C$ ◄

PROBLEM 6.22 (Continued)



Free body: Joint J

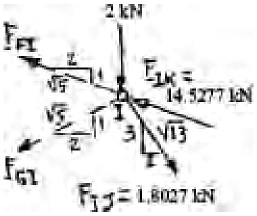
$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{2}{\sqrt{13}}F_{IJ} - \frac{6}{\sqrt{37}}F_{GJ} + \frac{6}{\sqrt{37}}(15.20 \text{ kN}) - \frac{2}{\sqrt{5}}(2.236) = 0 \tag{3}$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{\sqrt{13}}F_{IJ} + \frac{1}{\sqrt{37}}F_{GJ} - \frac{1}{\sqrt{37}}(15.20 \text{ kN}) - \frac{1}{\sqrt{5}}(2.236) = 0 \tag{4}$$

Solving (3) and (4) we obtain:

$F_{IJ} = 1.8027$ kN $\qquad F_{IJ} = 1.803$ kN T ◀

$F_{GJ} = 12.1587$ kN $\qquad F_{GJ} = 12.16$ kN T ◀



Free body: Joint I

$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{2}{\sqrt{5}}F_{FI} - \frac{2}{\sqrt{5}}F_{GI} - \frac{2}{\sqrt{5}}(14.5277) + \frac{2}{\sqrt{13}}(1.8027) = 0$$

or $\qquad F_{FI} + F_{GI} = -13.4097$ kN $\qquad (5)$

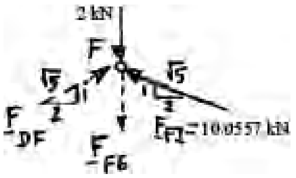
$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}}F_{FI} - \frac{1}{\sqrt{5}}F_{GI} + \frac{1}{\sqrt{5}}(14.5277) - \frac{3}{\sqrt{13}}(1.8027) - 2 = 0$$

or $\qquad F_{FI} - F_{GI} = -6.7016$ kN $\qquad (6)$

Add (5) and (6): $\quad 2F_{FI} = -20.1113$

$F_{FI} = -10.0557$ kN $\qquad F_{FI} = 10.06$ kN C ◀

Subtract (6) from (5): $\quad 2F_{GI} = -6.7081$ kN $\qquad F_{GI} = 3.35$ kN C ◀

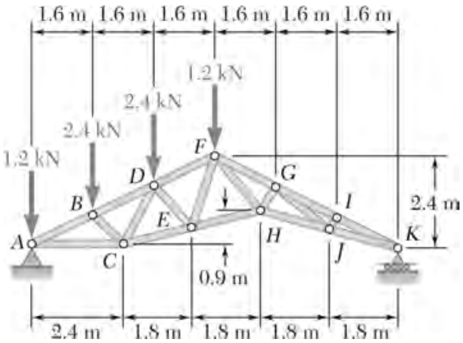


Free body: Joint F

From $\quad \Sigma F_x = 0: \quad F_{DF} = F_{FI} = 10.0557$ kN C

$$+\uparrow \Sigma F_y = 0: \quad F_{FG} - 2 \text{ kN} + 2\left(\frac{1}{\sqrt{5}}\right) = 0$$

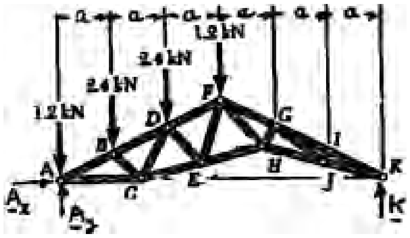
$F_{FG} = +7.00$ kN $\qquad F_{FG} = 7.00$ kN T ◀



PROBLEM 6.23

Determine the force in each of the members connecting joints A through F of the vaulted roof truss shown. State whether each member is in tension or compression.

SOLUTION

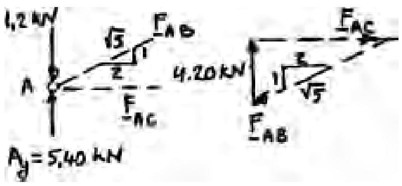


<u>Free body: Truss</u>

$$\Sigma F_x = 0: \quad \mathbf{A}_x = 0$$

$$+\circlearrowleft \Sigma M_K = 0: \quad (1.2 \text{ kN})6a + (2.4 \text{ kN})5a + (2.4 \text{ kN})4a + (1.2 \text{ kN})3a$$

$$-A_y(6a) = 0 \quad \mathbf{A}_y = 5.40 \text{ kN} \uparrow$$

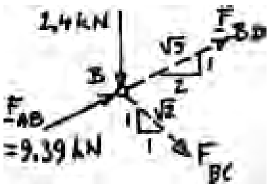


<u>Free body: Joint A</u>

$$\frac{F_{AB}}{\sqrt{5}} = \frac{F_{AC}}{2} = \frac{4.20 \text{ kN}}{1}$$

$$F_{AB} = 9.3915 \text{ kN} \qquad F_{AB} = 9.39 \text{ kN} \quad C \blacktriangleleft$$

$$F_{AC} = 8.40 \text{ kN} \quad T \blacktriangleleft$$



<u>Free body: Joint B</u>

$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{2}{\sqrt{5}} F_{BD} + \frac{1}{\sqrt{2}} F_{BC} + \frac{2}{\sqrt{5}}(9.3915) = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}} F_{BD} - \frac{1}{\sqrt{2}} F_{BC} + \frac{1}{\sqrt{5}}(9.3915) - 2.4 = 0 \qquad (2)$$

Add (1) and (2):

$$\frac{3}{\sqrt{5}} F_{BD} + \frac{3}{\sqrt{5}}(9.3915 \text{ kN}) - 2.4 \text{ kN} = 0$$

$$F_{BD} = -7.6026 \text{ kN} \qquad F_{BD} = 7.60 \text{ kN} \quad C \blacktriangleleft$$

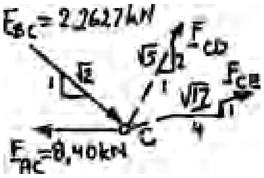
Multiply (2) by –2 and add (1):

$$\frac{3}{\sqrt{2}} F_B + 4.8 \text{ kN} = 0$$

$$F_{BC} = -2.2627 \text{ kN} \qquad F_{BC} = 2.26 \text{ kN} \quad C \blacktriangleleft$$

PROBLEM 6.23 (Continued)

<u>Free body: Joint C</u>



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{1}{\sqrt{5}} F_{CD} + \frac{4}{\sqrt{17}} F_{CE} + \frac{1}{\sqrt{2}}(2.2627) - 8.40 = 0 \qquad (3)$$

$$+\!\uparrow \Sigma F_y = 0: \quad \frac{2}{\sqrt{5}} F_{CD} + \frac{1}{\sqrt{17}} F_{CE} - \frac{1}{\sqrt{2}}(2.2627) = 0 \qquad (4)$$

Multiply (4) by -4 and add (1):

$$-\frac{7}{\sqrt{5}} F_{CD} + \frac{5}{\sqrt{2}}(2.2627) - 8.40 = 0$$

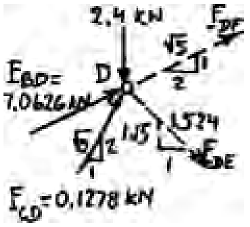
$$F_{CD} = -0.1278 \text{ kN} \qquad F_{CD} = 0.128 \text{ kN} \quad C \blacktriangleleft$$

Multiply (1) by 2 and subtract (2):

$$\frac{7}{\sqrt{17}} F_{CE} + \frac{3}{\sqrt{2}}(2.2627) - 2(8.40) = 0$$

$$F_{CE} = 7.068 \text{ kN} \qquad F_{CE} = 7.07 \text{ kN} \quad T \blacktriangleleft$$

<u>Free body: Joint D</u>



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{2}{\sqrt{5}} F_{DF} + \frac{1}{1.524} F_{DE} + \frac{2}{\sqrt{5}}(7.6026) + \frac{1}{\sqrt{5}}(0.1278) = 0 \qquad (5)$$

$$+\!\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}} F_{DF} - \frac{1.15}{1.524} F_{DE} + \frac{1}{\sqrt{5}}(7.6026) + \frac{2}{\sqrt{5}}(0.1278) - 2.4 = 0 \qquad (6)$$

Multiply (5) by 1.15 and add (6):

$$\frac{3.30}{\sqrt{5}} F_{DF} + \frac{3.30}{\sqrt{5}}(7.6026) + \frac{3.15}{\sqrt{5}}(0.1278) - 2.4 = 0$$

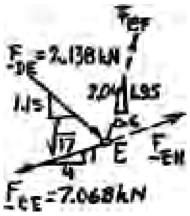
$$F_{DF} = -6.098 \text{ kN} \qquad F_{DF} = 6.10 \text{ kN} \quad C \blacktriangleleft$$

Multiply (6) by –2 and add (5):

$$\frac{3.30}{1.524} F_{DE} - \frac{3}{\sqrt{5}}(0.1278) + 4.8 = 0$$

$$F_{DE} = -2.138 \text{ kN} \qquad F_{DE} = 2.14 \text{ kN} \quad C \blacktriangleleft$$

PROBLEM 6.23 (Continued)



Free body: Joint E

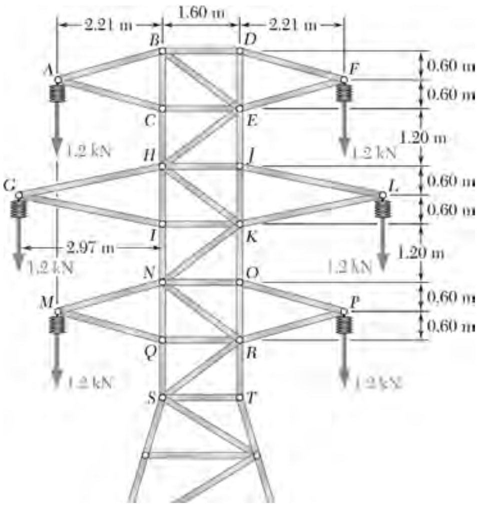
$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{0.6}{2.04}F_{EF} + \frac{4}{\sqrt{17}}(F_{EH} - F_{CE}) + \frac{1}{1.524}(2.138) = 0 \qquad (7)$$

$$+\!\uparrow \Sigma F_y = 0: \quad \frac{1.95}{2.04}F_{EF} + \frac{1}{\sqrt{17}}(F_{EH} - F_{CE}) - \frac{1.15}{1.524}(2.138) = 0 \qquad (8)$$

Multiply (8) by 4 and subtract (7):

$$\frac{7.2}{2.04}F_{EF} - 7.856 \text{ kN} = 0$$

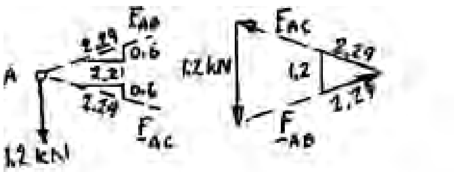
$F_{EF} = 2.23$ kN $\quad T$ ◀



PROBLEM 6.24

The portion of truss shown represents the upper part of a power transmission line tower. For the given loading, determine the force in each of the members located above *HJ*. State whether each member is in tension or compression.

SOLUTION

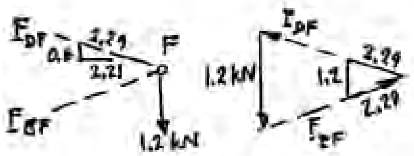


Free body: Joint *A*

$$\frac{F_{AB}}{2.29} = \frac{F_{AC}}{2.29} = \frac{1.2\text{ kN}}{1.2}$$

$F_{AB} = 2.29\text{ kN}\quad T$ ◀

$F_{AC} = 2.29\text{ kN}\quad C$ ◀

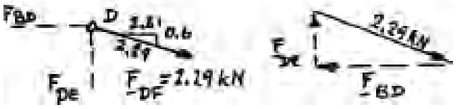


Free body: Joint *F*

$$\frac{F_{DF}}{2.29} = \frac{F_{EF}}{2.29} = \frac{1.2\text{ kN}}{2.1}$$

$F_{DF} = 2.29\text{ kN}\quad T$ ◀

$F_{EF} = 2.29\text{ kN}\quad C$ ◀

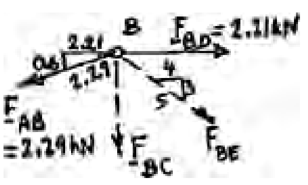


Free body: Joint *D*

$$\frac{F_{BD}}{2.21} = \frac{F_{DE}}{0.6} = \frac{2.29\text{ kN}}{2.29}$$

$F_{BD} = 2.21\text{ kN}\quad T$ ◀

$F_{DE} = 0.600\text{ kN}\quad C$ ◀



Free body: Joint *B*

$$\xrightarrow{+}\Sigma F_x = 0:\quad \frac{4}{5}F_{BE} + 2.21\text{ kN} - \frac{2.21}{2.29}(2.29\text{ kN}) = 0$$

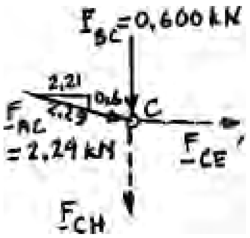
$F_{BE} = 0$ ◀

$$+\!\uparrow\Sigma F_y = 0:\quad -F_{BC} - \frac{3}{5}(0) - \frac{0.6}{2.29}(2.29\text{ kN}) = 0$$

$$F_{BC} = -0.600\text{ kN}$$

$F_{BC} = 0.600\text{ kN}\quad C$ ◀

PROBLEM 6.24 (Continued)



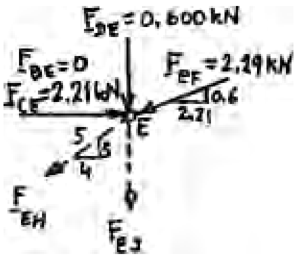
Free body: Joint C

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{CE} + \frac{2.21}{2.29}(2.29 \text{ kN}) = 0$$

$$F_{CE} = -2.21 \text{ kN} \qquad F_{CE} = 2.21 \text{ kN} \quad C \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -F_{CH} - 0.600 \text{ kN} - \frac{0.6}{2.29}(2.29 \text{ kN}) = 0$$

$$F_{CH} = -1.200 \text{ kN} \qquad F_{CH} = 1.200 \text{ kN} \quad C \blacktriangleleft$$



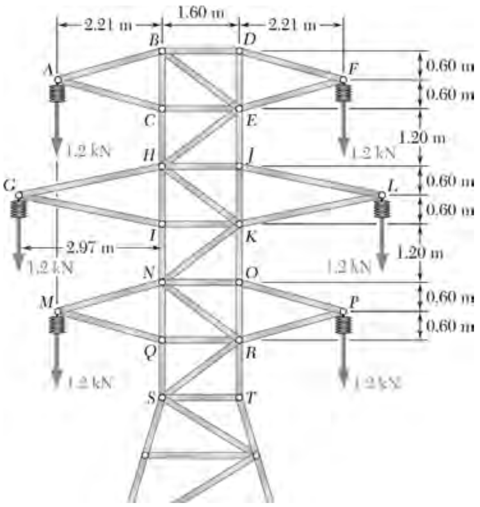
Free body: Joint E

$$\xrightarrow{+} \Sigma F_x = 0: \quad 2.21 \text{ kN} - \frac{2.21}{2.29}(2.29 \text{ kN}) - \frac{4}{5} F_{EH} = 0$$

$$F_{EH} = 0 \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -F_{EJ} - 0.600 \text{ kN} - \frac{0.6}{2.29}(2.29 \text{ kN}) - 0 = 0$$

$$F_{EJ} = -1.200 \text{ kN} \qquad F_{EJ} = 1.200 \text{ kN} \quad C \blacktriangleleft$$

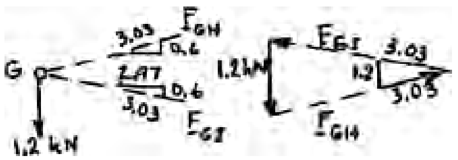


PROBLEM 6.25

For the tower and loading of Problem 6.24 and knowing that $F_{CH} = F_{EJ} = 1.2$ kN *C* and $F_{EH} = 0$, determine the force in member *HJ* and in each of the members located between *HJ* and *NO*. State whether each member is in tension or compression.

PROBLEM 6.24 The portion of truss shown represents the upper part of a power transmission line tower. For the given loading, determine the force in each of the members located above *HJ*. State whether each member is in tension or compression.

SOLUTION

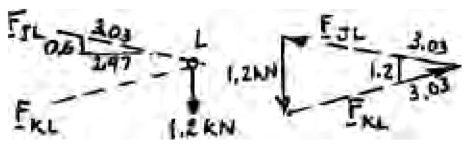


Free body: Joint *G*

$$\frac{F_{GH}}{3.03} = \frac{F_{GI}}{3.03} = \frac{1.2 \text{ kN}}{1.2}$$

$F_{GH} = 3.03$ kN *T* ◀

$F_{GI} = 3.03$ kN *C* ◀

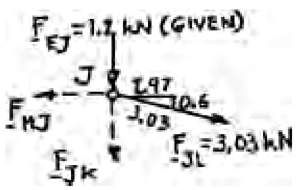


Free body: Joint *L*

$$\frac{F_{JL}}{3.03} = \frac{F_{KL}}{3.03} = \frac{1.2 \text{ kN}}{1.2}$$

$F_{JL} = 3.03$ kN *T* ◀

$F_{KL} = 3.03$ kN *C* ◀



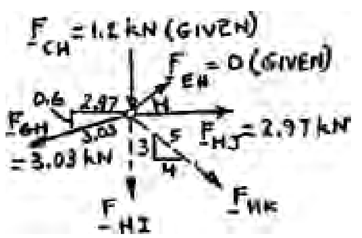
Free body: Joint *J*

$$\xrightarrow{+} \Sigma F_x = 0: \quad -F_{HJ} + \frac{2.97}{3.03}(3.03 \text{ kN}) = 0$$

$F_{HJ} = 2.97$ kN *T* ◀

$$+\!\uparrow F_y = 0: \quad -F_{JK} - 1.2 \text{ kN} - \frac{0.6}{3.03}(3.03 \text{ kN}) = 0$$

$F_{JK} = -1.800$ kN $\qquad F_{JK} = 1.800$ kN *C* ◀



Free body: Joint *H*

$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{4}{5}F_{HK} + 2.97 \text{ kN} - \frac{2.97}{3.03}(3.03 \text{ kN}) = 0$$

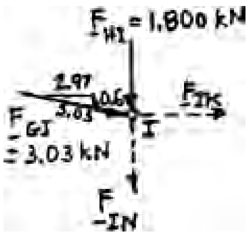
$F_{HK} = 0$ ◀

$$+\!\uparrow \Sigma F_y = 0: \quad -F_{HI} - 1.2 \text{ kN} - \frac{0.6}{3.03}(3.03) \text{ kN} - \frac{3}{5}(0) = 0$$

$F_{HI} = -1.800$ kN $\qquad F_{HI} = 1.800$ kN *C* ◀

PROBLEM 6.25 (Continued)

Free body: Joint I



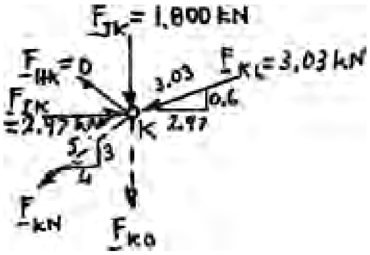
$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{IK} + \frac{2.97}{3.03}(3.03 \text{ kN}) = 0$$

$$F_{IK} = -2.97 \text{ kN} \qquad F_{IK} = 2.97 \text{ kN} \quad C \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -F_{IN} - 1.800 \text{ kN} - \frac{0.6}{3.03}(3.03 \text{ kN}) = 0$$

$$F_{IN} = -2.40 \text{ kN} \qquad F_{IN} = 2.40 \text{ kN} \quad C \blacktriangleleft$$

Free body: Joint K

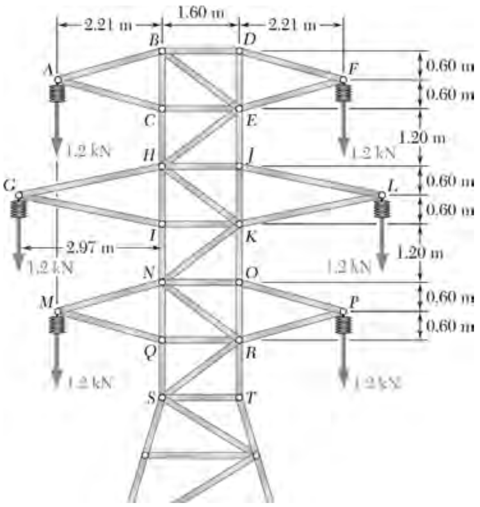


$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{4}{5}F_{KN} + 2.97 \text{ kN} - \frac{2.97}{3.03}(3.03 \text{ kN}) = 0$$

$$F_{KN} = 0 \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -F_{KD} - \frac{0.6}{3.03}(3.03 \text{ kN}) - 1.800 \text{ kN} - \frac{3}{5}(0) = 0$$

$$F_{KD} = -2.40 \text{ kN} \qquad F_{KD} = 2.40 \text{ kN} \quad C \blacktriangleleft$$



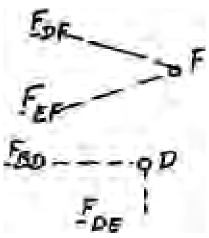
PROBLEM 6.26

Solve Problem 6.24 assuming that the cables hanging from the right side of the tower have fallen to the ground.

PROBLEM 6.24 The portion of truss shown represents the upper part of a power transmission line tower. For the given loading, determine the force in each of the members located above *HJ*. State whether each member is in tension or compression.

SOLUTION

<u>Zero-Force Members.</u>



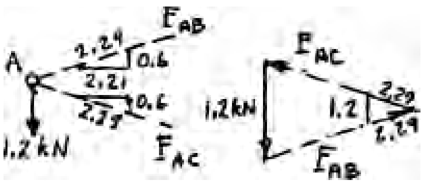
Considering joint *F*, we note that *DF* and *EF* are zero-force members:

$$F_{DF} = F_{EF} = 0 \blacktriangleleft$$

Considering next joint *D*, we note that *BD* and *DE* are zero-force members:

$$F_{BD} = F_{DE} = 0 \blacktriangleleft$$

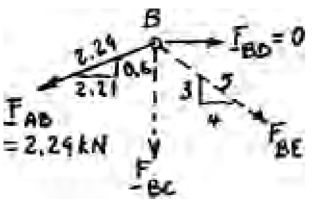
<u>Free body: Joint *A*</u>



$$\frac{F_{AB}}{2.29} = \frac{F_{AC}}{2.29} = \frac{1.2\text{ kN}}{1.2} \qquad F_{AB} = 2.29\text{ kN} \quad T \blacktriangleleft$$

$$F_{AC} = 2.29\text{ kN} \quad C \blacktriangleleft$$

<u>Free body: Joint *B*</u>



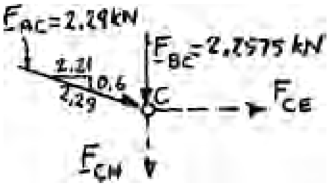
$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{4}{5}F_{BE} - \frac{2.21}{2.29}(2.29\text{ kN}) = 0$$

$$F_{BE} = 2.7625\text{ kN} \qquad F_{BE} = 2.76\text{ kN} \quad T \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -F_{BC} - \frac{0.6}{2.29}(2.29\text{ kN}) - \frac{3}{5}(2.7625\text{ kN}) = 0$$

$$F_{BC} = -2.2575\text{ kN} \qquad F_{BC} = 2.26\text{ kN} \quad C \blacktriangleleft$$

PROBLEM 6.26 (Continued)



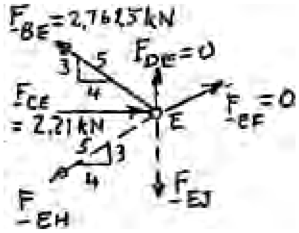
Free body: Joint C

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{CE} + \frac{2.21}{2.29}(2.29 \text{ kN}) = 0$$

$$F_{CE} = 2.21 \text{ kN} \quad C \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -F_{CH} - 2.2575 \text{ kN} - \frac{0.6}{2.29}(2.29 \text{ kN}) = 0$$

$$F_{CH} = -2.8575 \text{ kN} \qquad F_{CH} = 2.86 \text{ kN} \quad C \blacktriangleleft$$



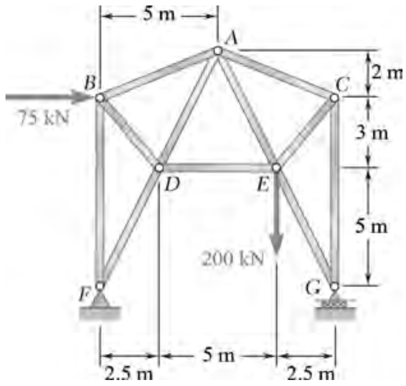
Free body: joint E

$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{4}{5}F_{EH} - \frac{4}{5}(2.7625 \text{ kN}) + 2.21 \text{ kN} = 0$$

$$F_{EH} = 0 \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -F_{EJ} + \frac{3}{5}(2.7625 \text{ kN}) - \frac{3}{5}(0) = 0$$

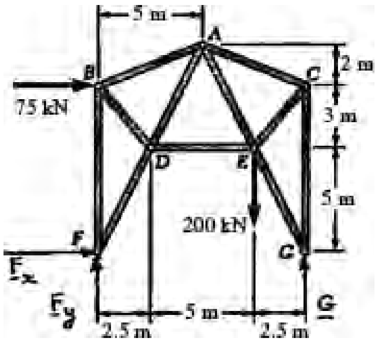
$$F_{EJ} = +1.6575 \text{ kN} \qquad F_{EJ} = 1.658 \text{ kN} \quad T \blacktriangleleft$$



PROBLEM 6.27

Determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION



Free body: Truss

$$+\circlearrowleft \Sigma M_F = 0: \quad G(10\text{ m}) - (75\text{ kN})(8\text{ m}) - (200\text{ kN})(7.5\text{ m}) = 0$$

$$\mathbf{G} = 210\text{ kN}\uparrow$$

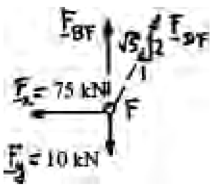
$$\xrightarrow{+} \Sigma F_x = 0: \quad F_x + 75\text{ kN} = 0$$

$$\mathbf{F}_x = 75\text{ kN}\leftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad F_y - 200\text{ kN} + 210\text{ kN} = 0$$

$$\mathbf{F}_y = 10\text{ kN}\downarrow$$

Free body: Joint *F*



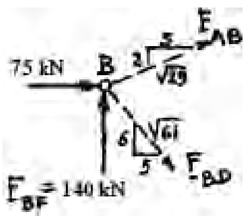
$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{1}{\sqrt{5}}F_{DF} - 75\text{ kN} = 0$$

$$F_{DF} = 167.705\text{ kN} \qquad F_{DF} = 167.7\text{ kN} \quad T \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad F_{BF} - 10\text{ kN} + \frac{2}{\sqrt{5}}(167.705\text{ kN}) = 0$$

$$F_{BF} = -140.00\text{ kN} \qquad F_{BF} = 140.0\text{ kN} \quad C \blacktriangleleft$$

Free body: Joint *B*



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{5}{\sqrt{29}}F_{AB} + \frac{5}{\sqrt{61}}F_{BD} + 75\text{ kN} = 0 \tag{1}$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{2}{\sqrt{29}}F_{AB} - \frac{6}{\sqrt{61}}F_{BD} + 140\text{ kN} = 0 \tag{2}$$

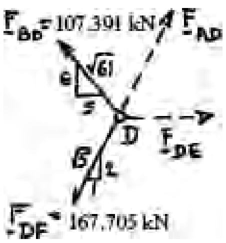
PROBLEM 6.27 (Continued)

Solving (1) and (2)

$$F_{AB} = -154.823 \text{ kN} \qquad F_{AB} = 154.8 \text{ kN} \quad C \blacktriangleleft$$

$$F_{BD} = 107.391 \text{ kN} \qquad F_{BD} = 107.4 \text{ kN} \quad T \blacktriangleleft$$

<u>Free body: joint *D*</u>



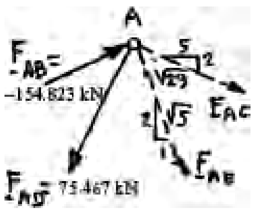
$$+\uparrow \Sigma F_y = 0: \quad \frac{2}{\sqrt{5}} F_{AD} - \frac{2}{\sqrt{5}}(167.705) + \frac{6}{\sqrt{61}}(107.391) = 0$$

$$F_{AD} = 75.467 \text{ kN} \quad T \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{DE} + \frac{1}{\sqrt{5}}(75.467 - 167.705) - \frac{5}{\sqrt{61}}(107.391) = 0$$

$$F_{DE} = 110.00 \text{ kN} \qquad F_{DE} = 110.0 \text{ kN} \quad T \blacktriangleleft$$

<u>Free body: joint *A*</u>



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{5}{\sqrt{29}} F_{AC} + \frac{1}{\sqrt{5}} F_{AE} + \frac{5}{\sqrt{29}}(154.823) - \frac{1}{\sqrt{5}}(75.467) = 0 \qquad (3)$$

$$+\uparrow \Sigma F_y = 0: \quad -\frac{2}{\sqrt{29}} F_{AC} - \frac{2}{\sqrt{5}} F_{AE} + \frac{2}{\sqrt{29}}(154.823) - \frac{2}{\sqrt{5}}(75.467) = 0 \qquad (4)$$

Multiply (3) by 2 and add (4):

$$\frac{8}{\sqrt{29}} F_{AC} + \frac{12}{\sqrt{29}}(154.823) - \frac{4}{\sqrt{5}}(75.467) = 0$$

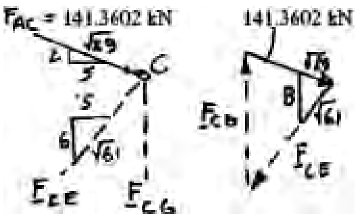
$$F_{AC} = -141.3602 \text{ kN}, \qquad F_{AC} = 141.4 \text{ kN} \quad C \blacktriangleleft$$

Multiply (3) by 2 (4) by 5 and add:

$$-\frac{8}{\sqrt{5}} F_{AE} + \frac{20}{\sqrt{29}}(154.823) - \frac{12}{\sqrt{5}}(75.467) = 0$$

$$F_{AE} = 47.5164 \text{ kN} \qquad F_{AE} = 47.5 \text{ kN} \quad T \blacktriangleleft$$

PROBLEM 6.27 (Continued)

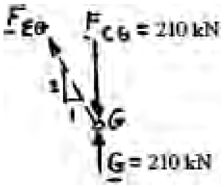


Free body: Joint C

From force triangle

$$\frac{F_{CE}}{\sqrt{61}} = \frac{F_{CG}}{8} = \frac{141.3602 \text{ kN}}{\sqrt{29}} \qquad F_{CE} = 205 \text{ kN} \quad T \blacktriangleleft$$

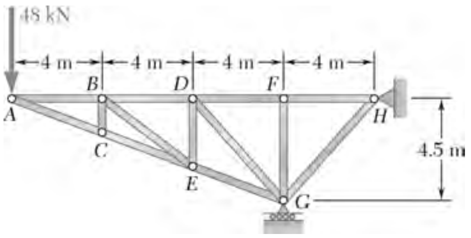
$$F_{CG} = 210.0 \text{ kN} \qquad F_{CG} = 210 \text{ kN} \quad C \blacktriangleleft$$



Free body: Joint G

$$\xrightarrow{+} \Sigma F_x = 0: \qquad F_{EG} = 0 \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 210 \text{ kN} - 210 \text{ kN} = 0 \qquad \text{(Checks)}$$

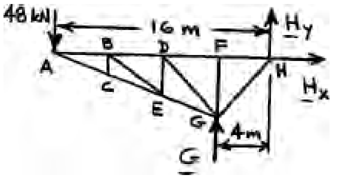


PROBLEM 6.28

Determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

<u>Free body: Truss</u>



$$\Sigma F_x = 0: \quad \mathbf{H}_x = 0$$

$$+\circlearrowleft \Sigma M_H = 0: \quad 48(16) - G(4) = 0 \quad \mathbf{G} = 192 \text{ kN} \uparrow$$

$$+\uparrow \Sigma F_y = 0: \quad 192 - 48 + H_y = 0$$

$$H_y = -144 \text{ kN} \quad \mathbf{H}_y = 144 \text{ kN} \downarrow$$

<u>Zero-Force Members</u>:

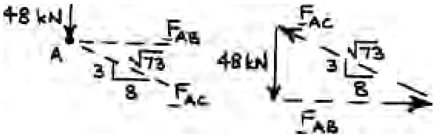
Examining successively joints *C*, *B*, *E*, *D*, and *F*, we note that the following are zero-force members: *BC*, *BE*, *DE*, *DG*, *FG*

Thus, $F_{BC} = F_{BE} = F_{DE} = F_{DG} = F_{FG} = 0$

Also note: $F_{AB} = F_{BD} = F_{DF} = F_{FH}$ (1)

$F_{AC} = F_{CE} = F_{EG}$ (2)

<u>Free body: Joint *A*</u>: $\dfrac{F_{AB}}{8} = \dfrac{F_{AC}}{\sqrt{73}} = \dfrac{48 \text{ kN}}{3}$



$F_{AB} = 128 \text{ kN}$ $\qquad F_{AB} = 128.0 \text{ kN} \quad T$ ◀

$F_{AC} = 136.704 \text{ kN}$ $\qquad F_{AC} = 136.7 \text{ kN} \quad C$ ◀

From Eq. (1): $F_{BD} = F_{DF} = F_{FH} = 128.0 \text{ kN} \quad T$ ◀

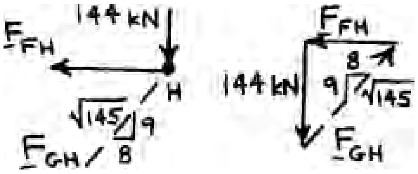
From Eq. (2): $F_{CE} = F_{EG} = 136.7 \text{ kN} \quad C$ ◀

PROBLEM 6.28 (Continued)

Free body: Joint H

$$\frac{F_{GH}}{\sqrt{145}} = \frac{144 \text{ kN}}{9}$$

$F_{GH} = 192.7 \text{ kN} \quad C$ ◀

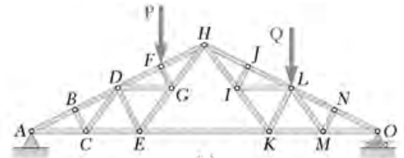


Also

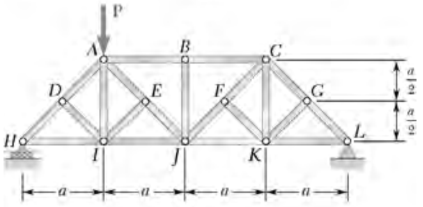
$$\frac{F_{FH}}{8} = \frac{144 \text{ kN}}{9}$$

$$F_{FH} = 128.0 \text{ kN} \quad T \quad \text{(Checks)}$$

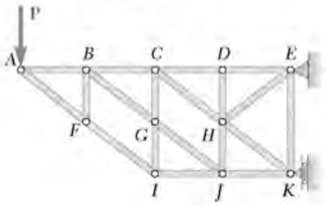
PROBLEM 6.29



(a)



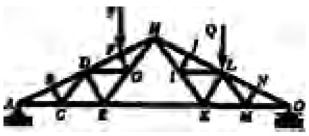
(a)



(a)

Determine whether the trusses of Problems 6.31*a*, 6.32*a*, and 6.33*a* are simple trusses.

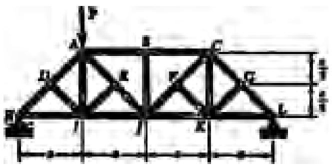
SOLUTION



Truss of Problem 6.31*a*

Starting with triangle *ABC* and adding two members at a time, we obtain joints *D*, *E*, *G*, *F*, and *H*, but cannot go further thus, this truss

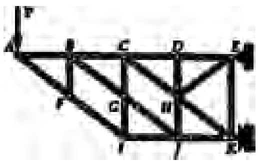
is not a simple truss ◀



Truss of Problem 6.32*a*

Starting with triangle *HDI* and adding two members at a time, we obtain successively joints *A*, *E*, *J*, and *B*, but cannot go further. Thus, this truss

is not a simple truss ◀

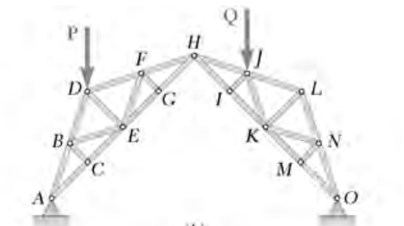


Truss of Problem 6.33*a*

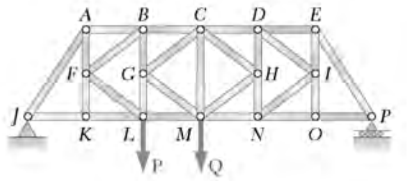
Starting with triangle *EHK* and adding two members at a time, we obtain successively joints *D*, *J*, *C*, *G*, *I*, *B*, *F*, and *A*, thus completing the truss. Therefore, this truss

is a simple truss ◀

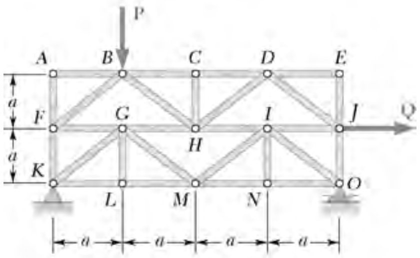
PROBLEM 6.30



(*b*)



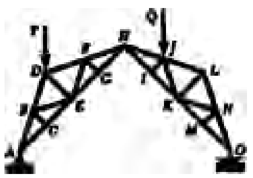
(*b*)



(*b*)

Determine whether the trusses of Problems 6.31*b*, 6.32*b*, and 6.33*b* are simple trusses.

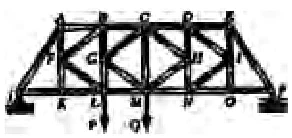
SOLUTION



<u>Truss of Problem 6.31*b*</u>

Starting with triangle *ABC* and adding two members at a time, we obtain successively joints *E*, *D*, *F*, *G*, and *H*, but cannot go further. Thus, this truss

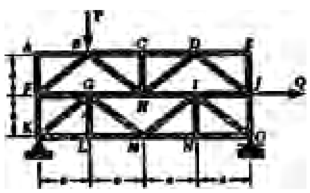
is not a simple truss ◀



<u>Truss of Problem 6.32*b*</u>

Starting with triangle *CGH* and adding two members at a time, we obtain successively joints *B*, *L*, *F*, *A*, *K*, *J*, then *H*, *D*, *N*, *I*, *E*, *O*, and *P*, thus completing the truss.

Therefore, this truss is a simple truss ◀

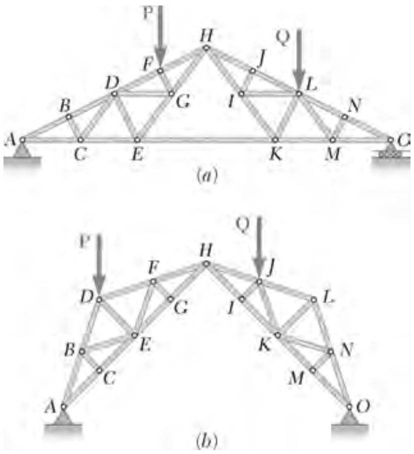


<u>Truss of Problem 6.33*b*</u>

Starting with triangle *GLM* and adding two members at a time, we obtain joints *K* and *I* but cannot continue, starting instead with triangle *BCH*, we obtain joint *D* but cannot continue, thus, this truss

is not a simple truss ◀

PROBLEM 6.31



For the given loading, determine the zero-force members in each of the two trusses shown.

SOLUTION

<u>Truss (*a*)</u>

FB: Joint B: $F_{BC} = 0$

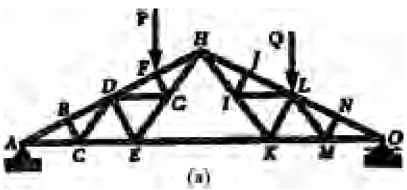
FB: Joint C: $F_{CD} = 0$

FB: Joint J: $F_{IJ} = 0$

FB: Joint I: $F_{IL} = 0$

FB: Joint N: $F_{MN} = 0$

FB: Joint M: $F_{LM} = 0$



The zero-force members, therefore, are BC, CD, IJ, IL, LM, MN ◀

<u>Truss (*b*)</u>

FB: Joint C: $F_{BC} = 0$

FB: Joint B: $F_{BE} = 0$

FB: Joint G: $F_{FG} = 0$

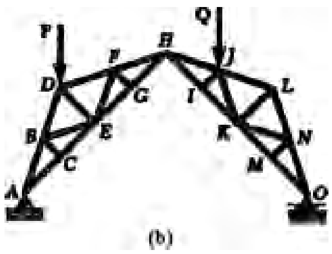
FB: Joint F: $F_{EF} = 0$

FB: Joint E: $F_{DE} = 0$

FB: Joint I: $F_{IJ} = 0$

FB: Joint M: $F_{MN} = 0$

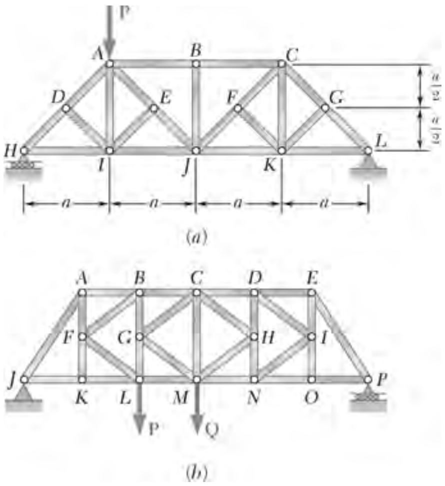
FB: Joint N: $F_{KN} = 0$



The zero-force members, therefore, are $BC, BE, DE, EF, FG, IJ, KN, MN$ ◀

PROBLEM 6.32

For the given loading, determine the zero-force members in each of the two trusses shown.



SOLUTION

<u>Truss (*a*)</u>

FB: Joint B: $F_{BJ} = 0$

FB: Joint D: $F_{DI} = 0$

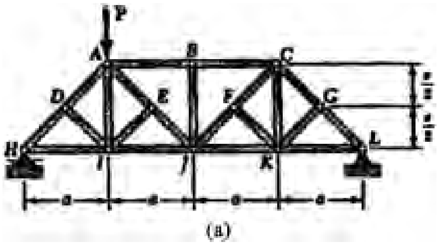
FB: Joint E: $F_{EI} = 0$

FB: Joint I: $F_{AI} = 0$

FB: Joint F: $F_{FK} = 0$

FB: Joint G: $F_{GK} = 0$

FB: Joint K: $F_{CK} = 0$

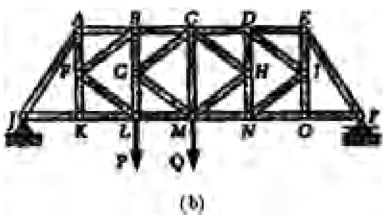


The zero-force members, therefore, are $AI, BJ, CK, DI, EI, FK, GK$ ◀

<u>Truss (*b*)</u>

FB: Joint K: $F_{FK} = 0$

FB: Joint O: $F_{IO} = 0$

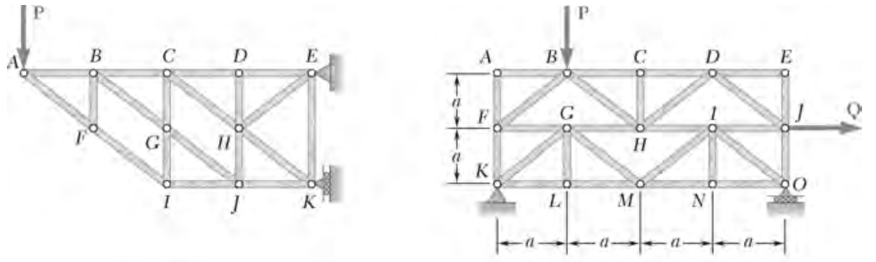


The zero-force members, therefore, are FK and IO ◀

All other members are either in tension or compression.

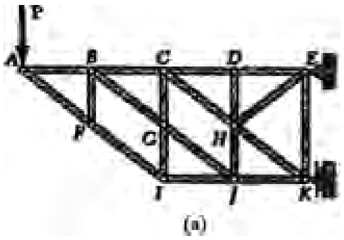
PROBLEM 6.33

For the given loading, determine the zero-force members in each of the two trusses shown.



(a) (b)

SOLUTION



Truss (*a*)

FB: Joint F: $F_{BF} = 0$

FB: Joint B: $F_{BG} = 0$

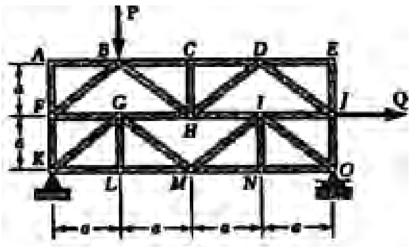
FB: Joint G: $F_{GJ} = 0$

FB: Joint D: $F_{DH} = 0$

FB: Joint J: $F_{HJ} = 0$

FB: Joint H: $F_{EH} = 0$

The zero-force members, therefore, are BF, BG, DH, EH, GJ, HJ ◀



Truss (*b*)

FB: Joint A: $F_{AB} = F_{AF} = 0$

FB: Joint C: $F_{CH} = 0$

FB: Joint E: $F_{DE} = F_{EJ} = 0$

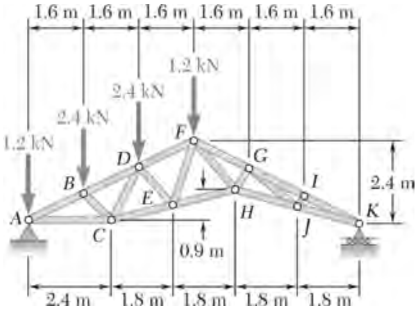
FB: Joint L: $F_{GL} = 0$

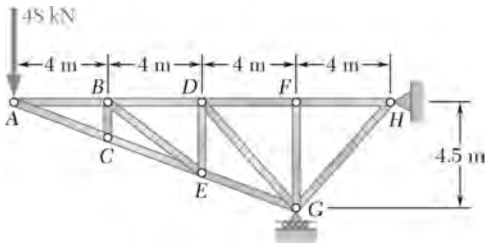
FB: Joint N: $F_{IN} = 0$

The zero-force members, therefore, are $AB, AF, CH, DE, EJ, GL, IN$ ◀

PROBLEM 6.34

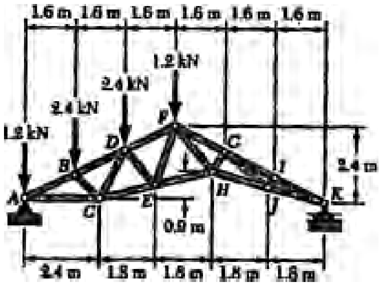
Determine the zero-force members in the truss of (*a*) Problem 6.23, (*b*) Problem 6.28.





SOLUTION

(*a*) <u>Truss of Problem 6.23</u>



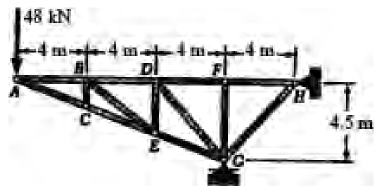
$$FB\text{: Joint } I\text{: } F_{IJ} = 0$$

$$FB\text{: Joint } J\text{: } F_{GJ} = 0$$

$$FB\text{: Joint } G\text{: } F_{GH} = 0$$

The zero-force members, therefore, are GH, GJ, IJ ◀

(*b*) <u>Truss of Problem 6.28</u>



$$FB\text{: Joint } C\text{: } F_{BC} = 0$$

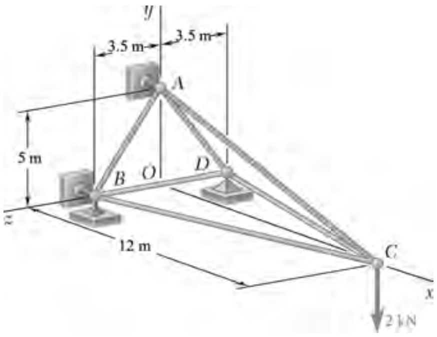
$$FB\text{: Joint } B\text{: } F_{BE} = 0$$

$$FB\text{: Joint } E\text{: } F_{DE} = 0$$

$$FB\text{: Joint } D\text{: } F_{DG} = 0$$

$$FB\text{: Joint } F\text{: } F_{FG} = 0$$

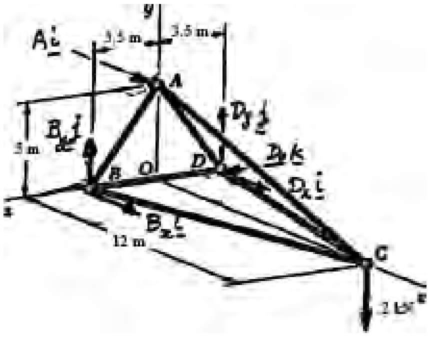
The zero-force members, therefore, are BC, BE, DE, DG, FG ◀



PROBLEM 6.35*

The truss shown consists of six members and is supported by a short link at A, two short links at B, and a ball and socket at D. Determine the force in each of the members for the given loading.

SOLUTION



Free body: Truss

From symmetry:

$$D_x = B_x \quad \text{and} \quad D_y = B_y$$

$$\Sigma M_z = 0: \quad -A(5\text{ m}) - (2\text{ kN})(12\text{ m}) = 0$$

$$A = -4.8\text{ kN}$$

$$\Sigma F_x = 0: \quad B_x + D_x + A = 0$$

$$2B_x - 4.8\text{ kN} = 0, \quad B_x = 2.4\text{ kN}$$

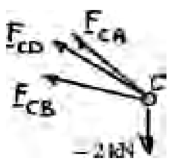
$$\Sigma F_y = 0: \quad B_y + D_y - 2\text{ kN} = 0$$

$$2B_y = 2\text{ kN}$$

$$B_y = +1\text{ kN}$$

Thus $\mathbf{B} = (2.40\text{ kN})\mathbf{i} + (1.000\text{ kN})\mathbf{j}$ ◁

Free body: C



$$F_{CA} = F_{AC}\frac{\overrightarrow{CA}}{CA} = \frac{F_{AC}}{13}(-12\mathbf{i} + 5\mathbf{j})$$

$$F_{CB} = F_{BC}\frac{\overrightarrow{CB}}{CB} = \frac{F_{BC}}{12.5}(-12\mathbf{i} + 3.5\mathbf{k})$$

$$F_{CD} = F_{CD}\frac{\overrightarrow{CD}}{CD} = \frac{F_{CD}}{12.5}(-12\mathbf{i} - 3.5\mathbf{k})$$

$$\Sigma\mathbf{F} = 0: \quad \mathbf{F}_{CA} + \mathbf{F}_{CB} + \mathbf{F}_{CD} - (2\text{ kN})\mathbf{j} = 0$$

PROBLEM 6.35* (Continued)

Substituting for $\mathbf{F}_{CA}, \mathbf{F}_{CB}, \mathbf{F}_{CD}$, and equating to zero the coefficients of $\mathbf{i}, \mathbf{j}, \mathbf{k}$:

$$\mathbf{i}: \quad -\frac{12}{13}F_{AC} - \frac{12}{12.5}(F_{BC} + F_{CD}) = 0 \qquad (1)$$

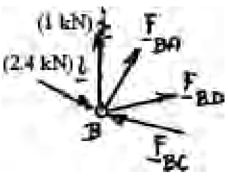
$$\mathbf{j}: \quad \frac{5}{13}F_{AC} - 2\text{ kN} = 0 \qquad F_{AC} = 5.20\text{ kN} \quad T \blacktriangleleft$$

$$\mathbf{k}: \quad \frac{3.5}{12.5}(F_{BC} - F_{CD}) = 0 \quad F_{CD} = F_{BC}$$

Substitute for F_{AC} and F_{CD} in Eq. (1):

$$-\frac{12}{13}(5.20\text{ kN}) - \frac{12}{12.5}(2\,F_{BC}) = 0 \quad F_{BC} = -2.5\text{ kN} \qquad F_{BC} = F_{CD} = 2.50\text{ kN} \quad C \blacktriangleleft$$

<u>Free body: *B*</u>



$$F_{BC} = (2.5\text{ kN})\frac{\overrightarrow{CB}}{CB} = -(2.4\text{ kN})\mathbf{i} + (0.7\text{ kN})\mathbf{k}$$

$$F_{BA} = F_{AB}\frac{\overrightarrow{BA}}{BA} = \frac{F_{AB}}{6.1033}(5\mathbf{j} - 3.5\mathbf{k})$$

$$F_{BD} = -F_{BD}\mathbf{k}$$

$$\Sigma\mathbf{F} = 0: \quad \mathbf{F}_{BA} + \mathbf{F}_{BD} + \mathbf{F}_{BC} + (2.4\text{ kN})\mathbf{i} + (1\text{ kN})\mathbf{j} = 0$$

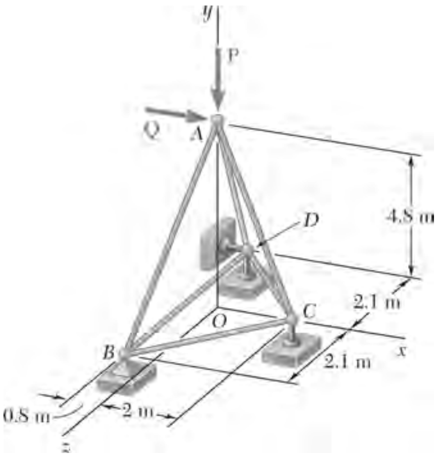
Substituting for $\mathbf{F}_{BA}, \mathbf{F}_{BD}, \mathbf{F}_{BC}$ and equating to zero the coefficients of $\mathbf{j}$ and $\mathbf{k}$:

$$\mathbf{j}: \quad \frac{5}{6.1033}F_{AB} + 1\text{ kN} = 0 \quad F_{AB} = -1.22066\text{ kN} \qquad F_{AB} = 1.221\text{ kN} \quad C \blacktriangleleft$$

$$\mathbf{k}: \quad -\frac{3.5}{6.1033}F_{AB} - F_{BD} + 0.7\text{ kN} = 0$$

$$F_{BD} = -\frac{3.5}{6.1033}(-1.22066\text{ kN}) + 0.7\text{ kN} = +1.400\text{ kN} \qquad F_{BD} = 1.400\text{ kN} \quad T \blacktriangleleft$$

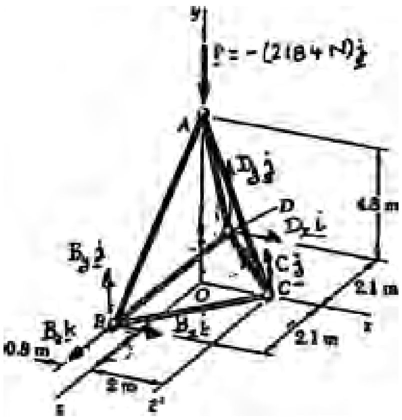
From symmetry: $\quad F_{AD} = F_{AB} \qquad F_{AD} = 1.221\text{ kN} \quad C \blacktriangleleft$



PROBLEM 6.36*

The truss shown consists of six members and is supported by a ball and socket at *B*, a short link at *C*, and two short links at *D*. Determine the force in each of the members for $\mathbf{P} = (-2184\text{ N})\mathbf{j}$ and $\mathbf{Q} = 0$.

SOLUTION



<u>Free body: Truss</u>

From symmetry:

$$D_x = B_x \text{ and } D_y = B_y$$

$$\Sigma F_x = 0: \quad 2B_x = 0$$

$$B_x = D_x = 0$$

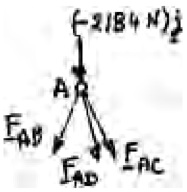
$$\Sigma F_z = 0: \quad B_z = 0$$

$$\Sigma M_{cz} = 0: \quad -2B_y(2.8\text{ m}) + (2184\text{ N})(2\text{ m}) = 0$$

$$B_y = 780\text{ N}$$

Thus $\mathbf{B} = (780\text{ N})\,j$ ◁

<u>Free body: *A*</u>



$$F_{AB} = F_{AB}\frac{\overrightarrow{AB}}{AB} = \frac{F_{AB}}{5.30}(-0.8\mathbf{i} - 4.8\mathbf{j} + 2.1\mathbf{k})$$

$$F_{AC} = F_{AC}\frac{\overrightarrow{AC}}{AC} = \frac{F_{AC}}{5.20}(2\mathbf{i} - 4.8\mathbf{j})$$

$$F_{AD} = F_{AD}\frac{\overrightarrow{AD}}{AD} = \frac{F_{AD}}{5.30}(+0.8\mathbf{i} - 4.8\mathbf{j} - 2.1\mathbf{k})$$

$$\mathbf{\Sigma F} = 0: \quad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} - (2184\text{ N})\mathbf{j} = 0$$

PROBLEM 6.36* (Continued)

Substituting for $\mathbf{F}_{AB}, \mathbf{F}_{AC}, \mathbf{F}_{AD}$, and equating to zero the coefficients of $\mathbf{i}, \mathbf{j}, \mathbf{k}$:

$$\mathbf{i}: \quad -\frac{0.8}{5.30}(F_{AB}+F_{AD})+\frac{2}{5.20}F_{AC}=0 \qquad (1)$$

$$\mathbf{j}: \quad -\frac{4.8}{5.30}(F_{AB}+F_{AD})-\frac{4.8}{5.20}F_{AC}-2184\text{ N}=0 \qquad (2)$$

$$\mathbf{k}: \quad \frac{2.1}{5.30}(F_{AB}-F_{AD})=0 \qquad F_{AD}=F_{AB}$$

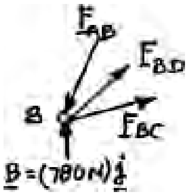
Multiply (1) by –6 and add (2):

$$-\left(\frac{16.8}{5.20}\right)F_{AC}-2184\text{ N}=0, \quad F_{AC}=-676\text{ N} \qquad F_{AC}=676\text{ N} \quad C \blacktriangleleft$$

Substitute for F_{AC} and F_{AD} in (1):

$$-\left(\frac{0.8}{5.30}\right)2F_{AB}+\left(\frac{2}{5.20}\right)(-676\text{ N})=0, \quad F_{AB}=-861.25\text{ N} \qquad F_{AB}=F_{AD}=861\text{ N} \quad C \blacktriangleleft$$

Free body: *B*



$$\mathbf{F}_{AB}=(861.25\text{ N})\frac{\overrightarrow{AB}}{AB}=-(130\text{ N})\mathbf{i}-(780\text{ N})\mathbf{j}+(341.25\text{ N})\mathbf{k}$$

$$\mathbf{F}_{BC}=F_{BC}\left(\frac{2.8i-2.1k}{3.5}\right)=F_{BC}(0.8\mathbf{i}-0.6\mathbf{k})$$

$$\mathbf{F}_{BD}=-F_{BD}\mathbf{k}$$

$$\Sigma\mathbf{F}=0: \quad \mathbf{F}_{AB}+\mathbf{F}_{BC}+\mathbf{F}_{BD}+(780\text{ N})\mathbf{j}=0$$

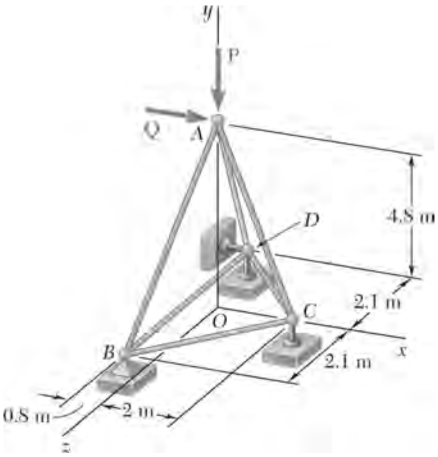
Substituting for $\mathbf{F}_{AB}, \mathbf{F}_{BC}, \mathbf{F}_{BD}$ and equating to zero the coefficients of $\mathbf{i}$ and $\mathbf{k}$:

$$\mathbf{i}: \quad -130\text{ N}+0.8F_{BC}=0 \quad F_{BC}=+162.5\text{ N} \qquad F_{BC}=162.5\text{ N} \quad T \blacktriangleleft$$

$$\mathbf{k}: \quad 341.25\text{ N}-0.6F_{BC}-F_{BD}=0$$

$$F_{BD}=341.25-0.6(162.5)=+243.75\text{ N} \qquad F_{BD}=244\text{ N} \quad T \blacktriangleleft$$

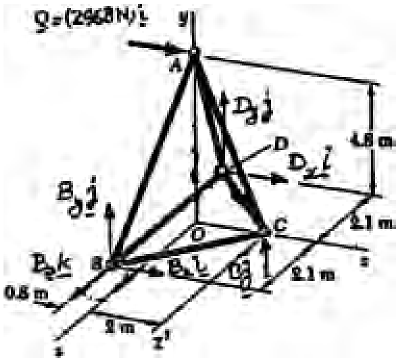
From symmetry: $\quad F_{CD}=F_{BC} \qquad F_{CD}=162.5\text{ N} \quad T \blacktriangleleft$



PROBLEM 6.37*

The truss shown consists of six members and is supported by a ball and socket at B, a short link at C, and two short links at D. Determine the force in each of the members for $\mathbf{P} = 0$ and $\mathbf{Q} = (2968\text{ N})\mathbf{i}$.

SOLUTION



Free body: Truss

From symmetry:

$$D_x = B_x \text{ and } D_y = B_y$$

$$\Sigma F_x = 0\text{:}\quad 2B_x + 2968\text{ N} = 0$$

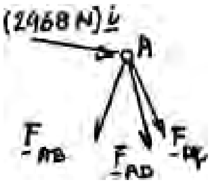
$$B_x = D_x = -1484\text{ N}$$

$$\Sigma M_{cz'} = 0\text{:}\quad -2B_y(2.8\text{ m}) - (2968\text{ N})(4.8\text{ m}) = 0$$

$$B_y = -2544\text{ N}$$

Thus $\qquad \mathbf{B} = -(1484\text{ N})\mathbf{i} - (2544\text{ N})\mathbf{j} \triangleleft$

Free body: A



$$F_{AB} = F_{AB}\frac{\overrightarrow{AB}}{AB}$$

$$= \frac{F_{AB}}{5.30}(-0.8\mathbf{i} - 4.8\mathbf{j} + 2.1\mathbf{k})$$

$$F_{AC} = F_{AC}\frac{\overrightarrow{AC}}{AC} = \frac{F_{AC}}{5.20}(2\mathbf{i} - 4.8\mathbf{j})$$

$$F_{AD} = F_{AD}\frac{\overrightarrow{AD}}{AD}$$

$$= \frac{F_{AD}}{5.30}(-0.8\mathbf{i} - 4.8\mathbf{j} - 2.1\mathbf{k})$$

$$\Sigma\mathbf{F} = 0\text{:}\quad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + (2968\text{ N})\mathbf{i} = 0$$

PROBLEM 6.37* (Continued)

Substituting for $\mathbf{F}_{AB}, \mathbf{F}_{AC}, \mathbf{F}_{AD}$, and equating to zero the coefficients of $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$:

$$\mathbf{i}: \qquad -\frac{0.8}{5.30}(F_{AB}+F_{AD})+\frac{2}{5.20}F_{AC}+2968 \text{ N}=0 \qquad (1)$$

$$\mathbf{j}: \qquad -\frac{4.8}{5.30}(F_{AB}+F_{AD})-\frac{4.8}{5.20}F_{AC}=0 \qquad (2)$$

$$\mathbf{k}: \qquad \frac{2.1}{5.30}(F_{AB}-F_{AD})=0 \qquad\qquad F_{AD}=F_{AB}$$

Multiply (1) by –6 and add (2):

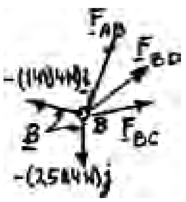
$$-\left(\frac{16.8}{5.20}\right)F_{AC}-6(2968 \text{ N})=0, \quad F_{AC}=-5512 \text{ N} \qquad\qquad F_{AC}=5510 \text{ N} \quad C \blacktriangleleft$$

Substitute for F_{AC} and F_{AD} in (2):

$$-\left(\frac{4.8}{5.30}\right)2F_{AB}-\left(\frac{4.8}{5.20}\right)(-5512 \text{ N})=0, \quad F_{AB}=+2809 \text{ N}$$

$$F_{AB}=F_{AD}=2810 \text{ N} \quad T \blacktriangleleft$$

<u>Free body: *B*</u>



$$\mathbf{F}_{AB}=(2809 \text{ N})\frac{\overrightarrow{BA}}{BA}=(424 \text{ N})\mathbf{i}+(2544 \text{ N})\mathbf{j}-(1113 \text{ N})\mathbf{k}$$

$$\mathbf{F}_{BC}=F_{BC}\left(\frac{2.8\mathbf{i}-2.1\mathbf{k}}{3.5}\right)=F_{BC}(0.8\mathbf{i}-0.6\mathbf{k})$$

$$\mathbf{F}_{BD}=-F_{BD}\mathbf{k}$$

$$\Sigma\mathbf{F}=0: \quad \mathbf{F}_{AB}+\mathbf{F}_{BC}+\mathbf{F}_{BD}-(1484 \text{ N})\mathbf{i}-(2544 \text{ N})\mathbf{j}=0$$

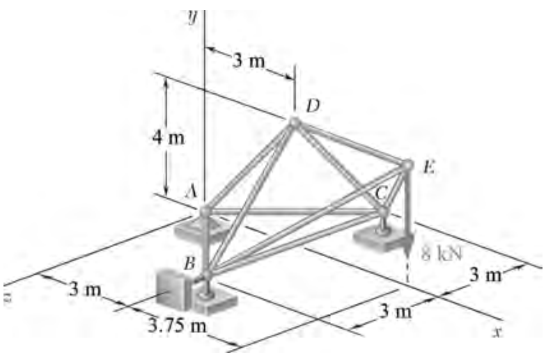
Substituting for $\mathbf{F}_{AB}, \mathbf{F}_{BC}, \mathbf{F}_{BD}$ and equating to zero the coefficients of $\mathbf{i}$ and $\mathbf{k}$:

$$\mathbf{i}: \qquad +24 \text{ N}+0.8F_{BC}-1484 \text{ N}=0, \quad F_{BC}=+1325 \text{ N} \qquad\qquad F_{BC}=1325 \text{ N} \quad T \blacktriangleleft$$

$$\mathbf{k}: \qquad -1113 \text{ N}-0.6F_{BC}-F_{BD}=0$$

$$F_{BD}=-1113 \text{ N}-0.6(1325 \text{ N})=-1908 \text{ N}, \qquad\qquad F_{BD}=1908 \text{ N} \quad C \blacktriangleleft$$

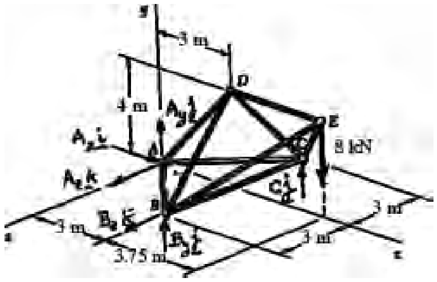
From symmetry: $\qquad F_{CD}=F_{BC} \qquad\qquad F_{CD}=1325 \text{ N} \quad T \blacktriangleleft$



PROBLEM 6.38*

The truss shown consists of nine members and is supported by a ball and socket at A, two short links at B, and a short link at C. Determine the force in each of the members for the given loading.

SOLUTION



Free body: Truss

From symmetry:

$$A_z = B_z = 0$$

$$\Sigma F_x = 0: \quad A_x = 0$$

$$\Sigma M_{BC} = 0: \quad A_y(3 \text{ m}) + (8 \text{ kN})(3.75 \text{ m}) = 0$$

$$A_y = -10 \text{ kN} \qquad \mathbf{A} = -(10.00 \text{ kN})\mathbf{j} \triangleleft$$

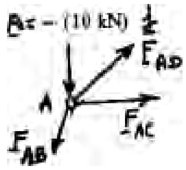
From symmetry:

$$B_y = C$$

$$\Sigma F_y = 0: \quad 2B_y - 10 \text{ kN} - 8 \text{ kN} = 0$$

$$B_y = 9 \text{ kN} \qquad \mathbf{B} = (9.00 \text{ kN})\mathbf{j} \triangleleft$$

Free body: A



$$\Sigma \mathbf{F} = 0: \quad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} - (10 \text{ kN})\mathbf{j} = 0$$

$$F_{AB}\frac{\mathbf{i}+\mathbf{k}}{\sqrt{2}} + F_{AC}\frac{\mathbf{i}-\mathbf{k}}{\sqrt{2}} + F_{AD}(0.6\mathbf{i} + 0.8\mathbf{j}) - (10 \text{ kN})\mathbf{j} = 0$$

Factoring **i**, **j**, **k** and equating their coefficient to zero:

$$\frac{1}{\sqrt{2}}F_{AB} + \frac{1}{\sqrt{2}}F_{AC} + 0.6F_{AD} = 0 \qquad (1)$$

$$0.8F_{AD} - 10 \text{ kN} = 0 \qquad F_{AD} = 12.5 \text{ kN} \quad T \blacktriangleleft$$

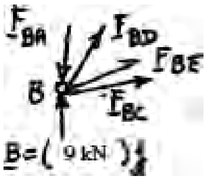
$$\frac{1}{\sqrt{2}}F_{AB} - \frac{1}{\sqrt{2}}F_{AC} = 0 \qquad F_{AC} = F_{AB}$$

PROBLEM 6.38* (Continued)

Substitute for F_{AD} and F_{AC} into (1):

$$\frac{2}{\sqrt{2}}F_{AB} + 0.6(12.5\text{ kN}) = 0, \qquad \mathbf{F}_{AB} = -5.3033\text{ kN}, \qquad F_{AB} = F_{AC} = 5.30\text{ kN} \quad C \blacktriangleleft$$

<u>Free body: *B*</u>



$$\mathbf{F}_{BA} = F_{AB}\frac{\overrightarrow{BA}}{BA} = +(5.3033\text{ kN})\frac{\mathbf{i}+\mathbf{k}}{\sqrt{2}} = (3.75\text{ kN})(\mathbf{i}+\mathbf{k})$$

$$\mathbf{F}_{BC} = -F_{BC}\mathbf{k}$$

$$\mathbf{F}_{BD} = F_{BD}(0.8\mathbf{j} - 0.6\mathbf{k})$$

$$\mathbf{F}_{BE} = F_{BE}\frac{\overrightarrow{BE}}{BE} = \frac{F_{BE}}{6.25}(3.75\mathbf{i} + 4\mathbf{j} - 3\mathbf{k})$$

$$\Sigma\mathbf{F} = 0: \quad \mathbf{F}_{BA} + \mathbf{F}_{BC} + \mathbf{F}_{BD} + \mathbf{F}_{BE} + (9.00\text{ kN})\mathbf{j} = 0$$

Substituting for $\mathbf{F}_{BA}, \mathbf{F}_{BC}, \mathbf{F}_{BD} + \mathbf{F}_{BE}$ and equate to zero the coefficients of $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$:

$\mathbf{i}$: $\quad 3.75\text{ kN} + \left(\frac{3.75}{6.25}\right)F_{BE} = 0, \quad F_{BE} = -6.25\text{ kN} \qquad F_{BE} = 6.25\text{ kN} \quad C \blacktriangleleft$

$\mathbf{j}$: $\quad 0.8\,F_{BD} + \left(\frac{4}{6.25}\right)(-6.25\text{ kN}) + 9.0\text{ kN} = 0 \qquad F_{BD} = 6.25\text{ kN} \quad C \blacktriangleleft$

$\mathbf{k}$: $\quad 3.75\text{ kN} - F_{BC} - 0.6(-6.25\text{ kN}) - \frac{3}{6.25}(-6.25\text{ kN}) = 0$

$$F_{BC} = 10.5\text{ kN} \quad T \blacktriangleleft$$

From symmetry: $\qquad F_{BD} = F_{CD} = 6.25\text{ kN} \quad C \blacktriangleleft$

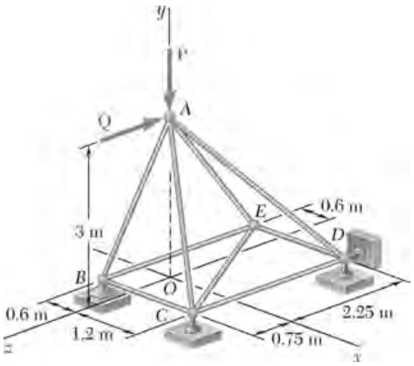
<u>Free body: *D*</u>

$$\Sigma\mathbf{F} = 0: \; \mathbf{F}_{DA} + \mathbf{F}_{DB} + \mathbf{F}_{DC} + \mathbf{F}_{DE}\mathbf{i} = 0$$

We now substitute for $\mathbf{F}_{DA}, \mathbf{F}_{DB}, \mathbf{F}_{DC}$ and equate to zero the coefficient of $\mathbf{i}$. Only $\mathbf{F}_{DA}$ contains $\mathbf{i}$ and its coefficient is

$$-0.6\,F_{AD} = -0.6(12.5\text{ kN}) = -7.5\text{ kN}$$

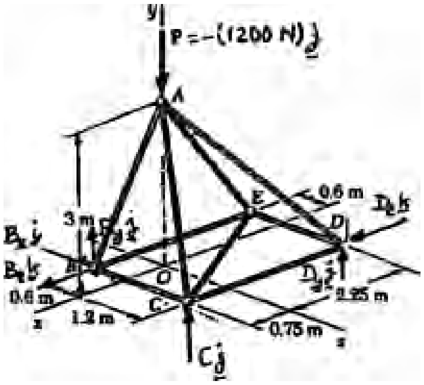
$\mathbf{i}$: $\quad -7.5\text{ kN} + F_{DE} = 0 \qquad F_{DE} = 7.5\text{ kN} \quad T \blacktriangleleft$



PROBLEM 6.39*

The truss shown consists of nine members and is supported by a ball and socket at B, a short link at C, and two short links at D. (a) Check that this truss is a simple truss, that it is completely constrained, and that the reactions at its supports are statically determinate. (b) Determine the force in each member for $\mathbf{P} = (-1200\text{ N})\mathbf{j}$ and $\mathbf{Q} = 0$.

SOLUTION



Free body: Truss

$$\Sigma\mathbf{M}_B = 0: \quad 1.8\mathbf{i}\times C\mathbf{j} + (1.8\mathbf{i} - 3\mathbf{k})\times(D_y\mathbf{j} + D\mathbf{k})$$
$$+(0.6\mathbf{i} - 0.75\mathbf{k})\times(-1200\mathbf{j}) = 0$$
$$-1.8\,C\mathbf{k} + 1.8\,D_y\mathbf{k} - 1.8\,D_z\mathbf{j}$$
$$+3D_y\mathbf{i} - 720\mathbf{k} - 900\mathbf{i} = 0$$

Equate to zero the coefficients of $\mathbf{i}, \mathbf{j}, \mathbf{k}$:

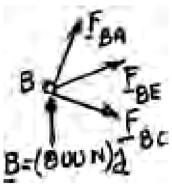
$\mathbf{i}$: $\quad 3D_y - 900 = 0, \quad D_y = 300\text{ N}$

$\mathbf{j}$: $\quad D_z = 0,$ $\quad\quad \mathbf{D} = (300\text{ N})\mathbf{j}$ ◁

$\mathbf{k}$: $\quad 1.8C + 1.8(300) - 720 = 0$ $\quad\quad \mathbf{C} = (100\text{ N})\mathbf{j}$ ◁

$\Sigma\mathbf{F} = 0: \quad \mathbf{B} + 300\mathbf{j} + 100\mathbf{j} - 1200\mathbf{j} = 0$ $\quad\quad \mathbf{B} = (800\text{ N})\mathbf{j}$ ◁

Free body: B



$$\Sigma\mathbf{F} = 0: \quad \mathbf{F}_{BA} + \mathbf{F}_{BC} + \mathbf{F}_{BE} + (800\text{ N})\mathbf{j} = 0, \text{ with}$$

$$\mathbf{F}_{BA} = \mathbf{F}_{AB}\frac{\overrightarrow{BA}}{BA} = \frac{F_{AB}}{3.15}(0.6\mathbf{i} + 3\mathbf{j} - 0.75\mathbf{k})$$

$$\mathbf{F}_{BC} = F_{BC}\mathbf{i} \qquad \mathbf{F}_{BE} = -F_{BE}\mathbf{k}$$

Substitute and equate to zero the coefficient of $\mathbf{j}, \mathbf{i}, \mathbf{k}$:

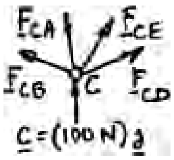
$\mathbf{j}$: $\quad \left(\dfrac{3}{3.315}\right)F_{AB} + 800\text{ N} = 0, \quad F_{AB} = -840\text{ N},$ $\quad\quad F_{AB} = 840\text{ N} \quad C$ ◀

$\mathbf{i}$: $\quad \left(\dfrac{0.6}{3.15}\right)(-840\text{ N}) + F_{BC} = 0$ $\quad\quad F_{BC} = 160.0\text{ N} \quad T$ ◀

$\mathbf{k}$: $\quad \left(-\dfrac{0.75}{3.15}\right)(-840\text{ N}) - F_{BE} = 0$ $\quad\quad F_{BE} = 200\text{ N} \quad T$ ◀

PROBLEM 6.39* (Continued)

Free body C:



$$\Sigma \mathbf{F} = 0: \quad \mathbf{F}_{CA} + \mathbf{F}_{CB} + \mathbf{F}_{CD} + \mathbf{F}_{CE} + (100\text{ N})\mathbf{j} = 0, \text{ with}$$

$$\mathbf{F}_{CA} = F_{AC}\frac{\overrightarrow{CA}}{CA} = \frac{F_{AC}}{3.317}(-1.2\mathbf{i} + 3\mathbf{j} - 0.75\mathbf{k})$$

$$F_{CB} = -(160\text{ N})\mathbf{i}$$

$$\mathbf{F}_{CD} = -F_{CD}\mathbf{k} \quad \mathbf{F}_{CE} = \mathbf{F}_{CE}\frac{\overrightarrow{CE}}{CE} = \frac{F_{CE}}{3,499}(-1.8\mathbf{i} - 3\mathbf{k})$$

Substitute and equate to zero the coefficient of $\mathbf{j}, \mathbf{i}, \mathbf{k}$:

$\mathbf{j}$: $$\left(\frac{3}{3.317}\right)F_{AC} + 100\text{ N} = 0, \quad F_{AC} = -110.57\text{ N} \qquad F_{AC} = 110.6\text{ N} \quad C \blacktriangleleft$$

$\mathbf{i}$: $$-\frac{1.2}{3.317}(-110.57) - 160 - \frac{1.8}{3.499}F_{CE} = 0, \quad F_{CE} = -233.3 \qquad F_{CE} = 233\text{ N} \quad C \blacktriangleleft$$

$\mathbf{k}$: $$-\frac{0.75}{3.317}(-110.57) - F_{CD} - \frac{3}{3.499}(-233.3) = 0 \qquad F_{CD} = 225\text{ N} \quad T \blacktriangleleft$$

Free body: D

$$\Sigma \mathbf{F} = 0: \quad \mathbf{F}_{DA} + \mathbf{F}_{DC} + \mathbf{F}_{DE} + (300\text{ N})\mathbf{j} = 0, \text{ with}$$

$$\mathbf{F}_{DA} = F_{AD}\frac{\overrightarrow{DA}}{DA} = \frac{F_{AD}}{3.937}(-1.2\mathbf{i} + 3\mathbf{j} + 2.25\mathbf{k})$$

$$\mathbf{F}_{DC} = F_{CD}\mathbf{k} = (225\text{ N})\mathbf{k} \qquad F_{DE} = -F_{DE}\mathbf{i}$$

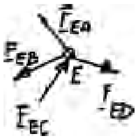
Substitute and equate to zero the coefficient of $\mathbf{j}, \mathbf{i}, \mathbf{k}$:

$\mathbf{j}$: $$\left(\frac{3}{3.937}\right)F_{AD} + 300\text{ N} = 0, \qquad F_{AD} = -393.7\text{ N}, \qquad F_{AD} = 394\text{ N} \quad C \blacktriangleleft$$

$\mathbf{i}$: $$\left(-\frac{1.2}{3.937}\right)(-393.7\text{ N}) - F_{DE} = 0 \qquad F_{DE} = 1200\text{ N} \quad T \blacktriangleleft$$

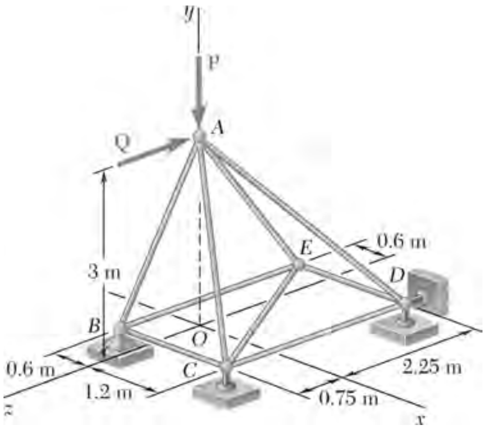
$\mathbf{k}$: $$\left(\frac{2.25}{3.937}\right)(-393.7\text{ N}) + 225\text{ N} = 0 \text{ (Checks)}$$

Free body: E



Member AE is the only member at E which does not lie in the xz plane. Therefore, it is a zero-force member.

$$F_{AE} = 0 \blacktriangleleft$$

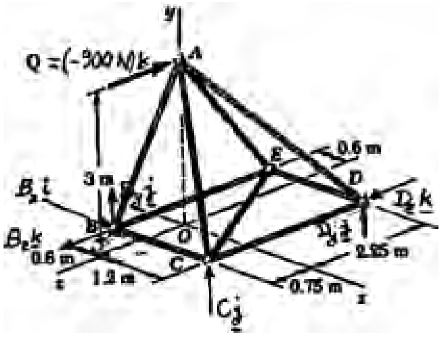


PROBLEM 6.40*

Solve Problem 6.39 for $\mathbf{P} = 0$ and $\mathbf{Q} = (-900\text{ N})\mathbf{k}$.

PROBLEM 6.39* The truss shown consists of nine members and is supported by a ball and socket at *B*, a short link at *C*, and two short links at *D*. (*a*) Check that this truss is a simple truss, that it is completely constrained, and that the reactions at its supports are statically determinate. (*b*) Determine the force in each member for $\mathbf{P} = (-1200\text{ N})\mathbf{j}$ and $\mathbf{Q} = 0$.

SOLUTION



Free body: Truss

$$\Sigma \mathbf{M}_B = 0: \quad 1.8\mathbf{i} \times C\mathbf{j} + (1.8\mathbf{i} - 3\mathbf{k}) \times (D_y\mathbf{j} + D_z\mathbf{k})$$
$$+ (0.6\mathbf{i} + 3\mathbf{j} - 0.75\mathbf{k}) \times (-900\text{N})\mathbf{k} = 0$$
$$1.8C\mathbf{k} + 1.8D_y\mathbf{k} - 1.8D_z\mathbf{j}$$
$$+ 3D_y\mathbf{i} + 540\mathbf{j} - 2700\mathbf{i} = 0$$

Equate to zero the coefficient of **i**, **j**, **k**:

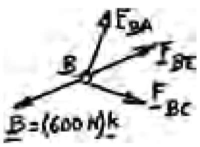
$$3D_y - 2700 = 0 \quad D_y = 900\text{ N}$$
$$-1.8D_z + 540 = 0 \quad D_z = 300\text{ N}$$
$$1.8C + 1.8D_y = 0 \quad C = -D_y = -900\text{ N}$$

Thus: $\mathbf{C} = -(900\text{ N})\mathbf{j} \quad \mathbf{D} = (900\text{ N})\mathbf{j} + (300\text{ N})\mathbf{k}$ ◁

$\Sigma \mathbf{F} = 0: \quad \mathbf{B} - 900\mathbf{j} + 900\mathbf{j} + 300\mathbf{k} - 900\mathbf{k} = 0$

$\mathbf{B} = (600\text{ N})\mathbf{k}$ ◁

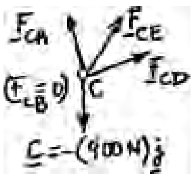
Free body: *B*



Since **B** is aligned with member *BE*:

$F_{AB} = F_{BC} = 0, \quad F_{BE} = 600\text{ N} \quad T$ ◀

Free body: *C*



$\Sigma \mathbf{F} = 0: \; \mathbf{F}_{CA} + \mathbf{F}_{CD} + \mathbf{F}_{CE} - (900\text{ N})\mathbf{j} = 0,$ with

PROBLEM 6.40* (Continued)

$$\mathbf{F}_{CA} = F_{AC}\frac{\overrightarrow{CA}}{CA} = \frac{F_{AC}}{3.317}(-1.2\mathbf{i} + 3\mathbf{j} - 0.75\mathbf{k})$$

$$\mathbf{F}_{CD} = -F_{CD}\mathbf{k} \quad \mathbf{F}_{CE} = F_{CE}\frac{\overrightarrow{CE}}{CE} = \frac{F_{CE}}{3.499}(-1.8\mathbf{i} - 3\mathbf{k})$$

Substitute and equate to zero the coefficient of $\mathbf{j}, \mathbf{i}, \mathbf{k}$:

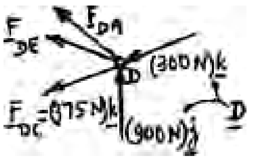
$\mathbf{j}$: $\left(\frac{3}{3.317}\right)F_{AC} - 900\text{ N} = 0, \quad F_{AC} = 995.1\text{ N}$ $\qquad F_{AC} = 995\text{ N} \quad T$ ◀

$\mathbf{i}$: $-\frac{1.2}{3.317}(995.1) - \frac{1.8}{3.499}F_{CE} = 0, \quad F_{CE} = -699.8\text{ N}$ $\qquad F_{CE} = 700\text{ N} \quad C$ ◀

$\mathbf{k}$: $-\frac{0.75}{3.317}(995.1) - F_{CD} - \frac{3}{3.499}(-699.8) = 0$ $\qquad F_{CD} = 375\text{ N} \quad T$ ◀

Free body: D

$$\Sigma\mathbf{F} = 0: \quad \mathbf{F}_{DA} + \mathbf{F}_{DE} + (375\text{ N})\mathbf{k} + (900\text{ N})\mathbf{j} + (300\text{ N})\mathbf{k} = 0$$



with $\quad \mathbf{F}_{DA} = \mathbf{F}_{AD}\frac{\overrightarrow{DA}}{DA} = \frac{F_{AD}}{3.937}(-1.2\mathbf{i} + 3\mathbf{j} + 2.25\mathbf{k})$

and $\quad F_{DE} = -F_{DE}\,\mathbf{i}$

Substitute and equate to zero the coefficient $\mathbf{j}, \mathbf{i}, \mathbf{k}$:

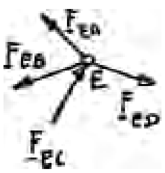
$\mathbf{j}$: $\left(\frac{3}{3.937}\right)F_{AD} + 900\text{ N} = 0, \quad F_{AD} = -1181.1\text{ N}$ $\qquad F_{AD} = 1181\text{N} \quad C$ ◀

$\mathbf{i}$: $-\left(\frac{1.2}{3.937}\right)(-1181.1\text{ N}) - F_{DE} = 0$ $\qquad F_{DE} = 360\text{ N} \quad T$ ◀

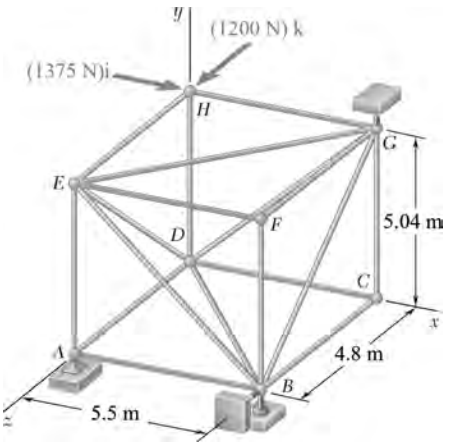
$\mathbf{k}$: $\left(\frac{2.25}{3.937}\right)(-1181.1\text{ N} + 375\text{ N} + 300\text{ N} = 0)$ $\qquad$ (Checks)

Free body: E

Member AE is the only member at E which does not lie in the XZ plane. Therefore, it is a zero-force member.



$F_{AE} = 0$ ◀



PROBLEM 6.41*

The truss shown consists of 18 members and is supported by a ball and socket at A, two short links at B, and one short link at G. (a) Check that this truss is a simple truss, that it is completely constrained, and that the reactions at its supports are statically determinate. (b) For the given loading, determine the force in each of the six members joined at E.

SOLUTION

(a) Check simple truss.

(1) start with tetrahedron $BEFG$

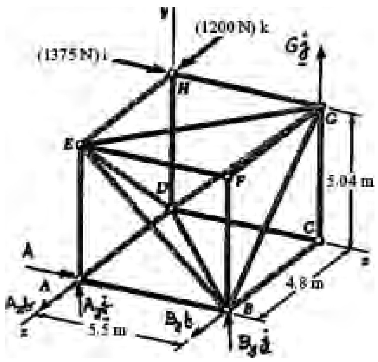
(2) Add members BD, ED, GD joining at D.

(3) Add members BA, DA, EA joining at A.

(4) Add members DH, EH, GH joining at H.

(5) Add members BC, DC, GC joining at C.

Truss has been completed: It is a simple truss



Free body: Truss

Check constraints and reactions:

Six unknown reactions-ok-however supports at A and B constrain truss to rotate about AB and support at G prevents such a rotation. Thus,

Truss is completely constrained and reactions are statically determinate

Determination of Reactions:

$$\Sigma \mathbf{M}_A = 0: \quad 5.5\mathbf{i} \times (B_y\mathbf{j} + B_z\mathbf{k}) + (5.51\mathbf{i} - 4.8\mathbf{k}) \times G\mathbf{j} + (5.04\mathbf{j} - 4.8\mathbf{k}) \times (1375\mathbf{i} + 1200\mathbf{k}) = 0$$

$$5.5B_y\mathbf{k} - 5.5B_y\mathbf{j} + 5.5G\mathbf{k} + 1200G\mathbf{i} - (5.04)(1375)\mathbf{k} + (5.04)(1200)\mathbf{i} - (4.8)(1375)\mathbf{j} = 0$$

Equate to zero the coefficient of $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$:

$\mathbf{i}$: $4.8G + (5.04)(1200) = 0 \quad G = -1260\text{ N}$ $\qquad \mathbf{G} = (-1260\text{ N})\mathbf{j}$ ◁

$\mathbf{j}$: $-5.5B_z - (4.8)(1375) = 0 \quad B_z = -1200\text{ N}$

$\mathbf{k}$: $5.5B_y + 5.5(-1260) - (5.04)(1375) = 0, \quad B_y = 2520\text{ N}$ $\qquad \mathbf{B} = (2520\text{ N})\mathbf{j} - (1200\text{ N})\mathbf{k}$ ◁

$$\Sigma \mathbf{F} = 0: \quad \mathbf{A} + (2520 \text{ N})\mathbf{j} - (1200 \text{ N})\mathbf{k} - (1260 \text{ N})\mathbf{j} + (1375 \text{ N})\mathbf{i} + (1200 \text{ N})\mathbf{k} = 0$$

$$\mathbf{A} = -(1375 \text{ N})\mathbf{i} - (1260 \text{ N})\mathbf{j} \triangleleft$$

Zero-force members

The determination of these members will facilitate our solution.

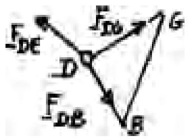
FB: *C*. Writing $\Sigma F_x = 0, \quad \Sigma F_y = 0, \quad \Sigma F_z = 0$ Yields $F_{BC} = F_{CD} = F_{CG} = 0 \triangleleft$

FB: *F*. Writing $\Sigma F_x = 0, \quad \Sigma F_y = 0, \quad \Sigma F_z = 0$ Yields $F_{BF} = F_{EF} = F_{FG} = 0 \triangleleft$

FB: *A*: Since $A_z = 0$, writing $\Sigma F_z = 0$ Yields $F_{AD} = 0 \triangleleft$

FB: *H*: Writing $\Sigma F_y = 0$ Yields $F_{DH} = 0 \triangleleft$

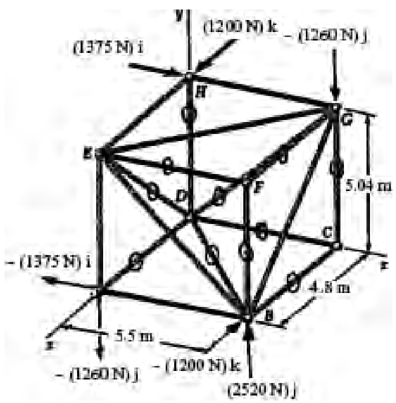
FB: *D*: Since $F_{AD} = F_{CD} = F_{DH} = 0$, we need consider only members *DB*, *DE*, and *DG*.



Since F_{DE} is the only force not contained in plane *BDG*, it must be zero. Simple reasonings show that the other two forces are also zero.

$$F_{BD} = F_{DE} = F_{DG} = 0 \triangleleft$$

The results obtained for the reactions at the supports and for the zero-force members are shown on the figure below. Zero-force members are indicated by a zero ("0").



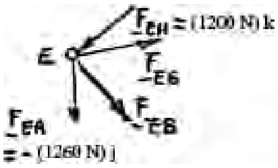
(*b*) Force in each of the members joined at *E*

We already found that $F_{DE} = F_{EF} = 0$ ◀

Free body: *A* $\Sigma F_y = 0$ Yields $F_{AE} = 1260 \text{ N} \quad T$ ◀

Free body: *H* $\Sigma F_z = 0$ Yields $F_{EH} = 1200 \text{ N} \quad C$ ◀

PROBLEM 6.41* (Continued)



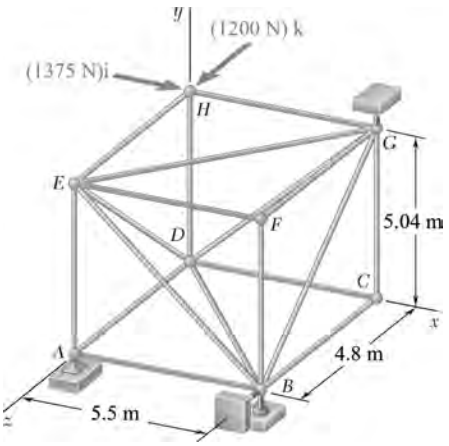
<u>Free body: *E*</u> $\qquad \Sigma \mathbf{F} = 0: \quad \mathbf{F}_{EB} + \mathbf{F}_{EG} + (1200\text{ N})\mathbf{k} - (1260\text{ N})\mathbf{j} = 0$

$$\frac{F_{BE}}{7.46}(5.5\mathbf{i} - 5.04\mathbf{j}) + \frac{F_{EG}}{7.3}(5.5\mathbf{i} - 4.8\mathbf{k}) + 1200\mathbf{k} - 1260\mathbf{j} = 0$$

Equate to zero the coefficient of **y** and **k**:

$$\mathbf{j}: \quad -\left(\frac{5.04}{7.46}\right)F_{BE} - 1260 = 0 \qquad F_{BE} = 1865\text{ N} \quad C \blacktriangleleft$$

$$\mathbf{k}: \quad -\left(\frac{4.8}{7.3}\right)F_{EG} + 1200 = 0 \qquad F_{EG} = 1825\text{ N} \quad T \blacktriangleleft$$



PROBLEM 6.42*

The truss shown consists of 18 members and is supported by a ball and socket at A, two short links at B, and one short link at G. (a) Check that this truss is a simple truss, that it is completely constrained, and that the reactions at its supports are statically determinate. (b) For the given loading, determine the force in each of the six members joined at G.

SOLUTION

See solution to Problem 6.41 for Part (a) and for reactions and zero-force members.

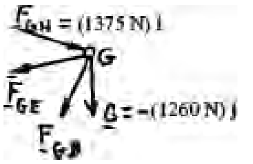
(b) Force in each of the members joined at G.

We already know that

$$F_{CG} = F_{DG} = F_{FG} = 0 \blacktriangleleft$$

Free body: H $\qquad \Sigma F_x = 0 \qquad$ Yields $F_{GH} = 1375 \text{ N} \quad C \blacktriangleleft$

Free body: G $\qquad \Sigma \mathbf{F} = 0: \quad \mathbf{F}_{GB} + \mathbf{F}_{GE} + (1375 \text{ N})\mathbf{i} - (1260 \text{ N})\mathbf{j} = 0$



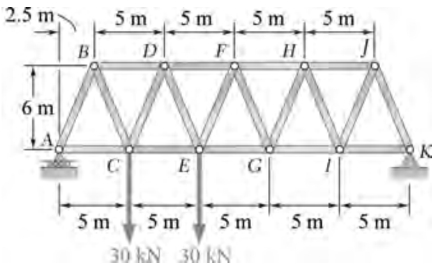
$$\frac{F_{BG}}{6.96}(-5.04\mathbf{j} + 4.8\mathbf{k}) + \frac{F_{EG}}{7.3}(-5.5\mathbf{i} + 4.8\mathbf{k}) + 1375\mathbf{i} - 1260\mathbf{j} = 0$$

Equate to zero the coefficient of $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$:

$$\mathbf{i}: \quad -\left(\frac{5.5}{7.3}\right)F_{EG} + 1375 = 0 \qquad F_{EG} = 1825 \text{ N} \quad T \blacktriangleleft$$

$$\mathbf{j}: \quad -\left(\frac{5.04}{6.96}\right)F_{BG} - 1260 = 0 \qquad F_{BG} = 1740 \text{ N} \quad C \blacktriangleleft$$

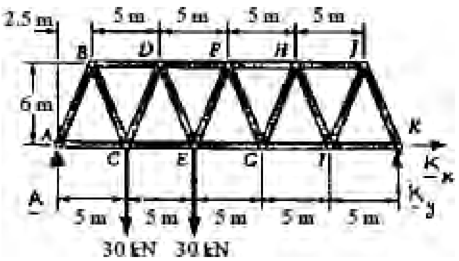
$$\mathbf{k}: \quad \left(\frac{4.8}{6.96}\right)(-1740) + \left(\frac{4.8}{7.3}\right)(1825) = 0 \qquad \text{(Checks)}$$



PROBLEM 6.43

A Warren bridge truss is loaded as shown. Determine the force in members *CE*, *DE*, and *DF*.

SOLUTION



Free body: Truss

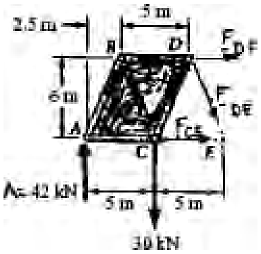
$$\xrightarrow{+} \Sigma F_x = 0: \quad \mathbf{K}_x = 0$$

$$\circlearrowleft + \Sigma M_A = 0: \quad \mathbf{K}_y(25 \text{ m}) - (30 \text{ kN})(5 \text{ m}) - (30 \text{ kN})(10 \text{ m}) = 0$$

$$\mathbf{K} = \mathbf{K}_y = 18.00 \text{ kN} \uparrow \triangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad A + 18 \text{ kN} - 30 \text{ kN} - 30 \text{ kN} = 0$$

$$\mathbf{A} = 42 \text{ kN} \uparrow \triangleleft$$



We pass a section through members *CE*, *DE*, and *DF* and use the free body shown.

$$\circlearrowleft + \Sigma M_D = 0: \quad F_{CE}(6 \text{ m}) - (42 \text{ kN})(7.5 \text{ m}) + (30 \text{ kN})(2.5 \text{ m}) = 0$$

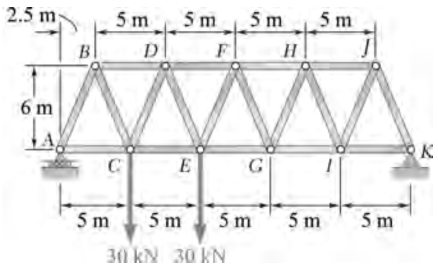
$$F_{CE} = +40 \text{ kN} \qquad F_{CE} = 40.0 \text{ kN} \quad T \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 42 \text{ kN} - 30 \text{ kN} - \frac{6}{6.5} F_{DE} = 0$$

$$F_{DE} = +13 \text{ kN} \qquad F_{DE} = 13.00 \text{ kN} \quad T \blacktriangleleft$$

$$\circlearrowleft + \Sigma M_E = 0: \quad 30 \text{ kN}(5 \text{ m}) - (42 \text{ kN})(10 \text{ m}) - F_{DF}(6 \text{ m}) = 0$$

$$F_{DF} = -45 \text{ kN} \qquad F_{DF} = 45.0 \text{ kN} \quad C \blacktriangleleft$$



PROBLEM 6.44

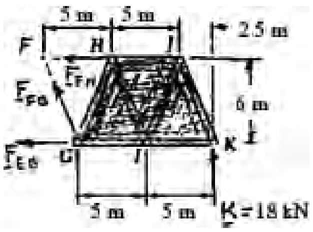
A Warren bridge truss is loaded as shown. Determine the force in members *EG*, *FG*, and *FH*.

SOLUTION

See solution of Problem 6.43 for free-body diagram of truss and determination of reactions:

$$\mathbf{A} = 42 \text{ kN} \qquad \mathbf{K} = \mathbf{K}_y = 18 \text{ kN} \triangleleft$$

We pass a section through members *EG*, *FG*, and *FH*, and use the free body shown.



$$+\circlearrowleft \Sigma M_F = 0: \quad (18 \text{ kN})(12.5 \text{ m}) - F_{EG}(6 \text{ m}) = 0$$

$$F_{EG} = +37.5 \text{ kN} \qquad F_{EG} = 37.5 \text{ kN} \quad T \blacktriangleleft$$

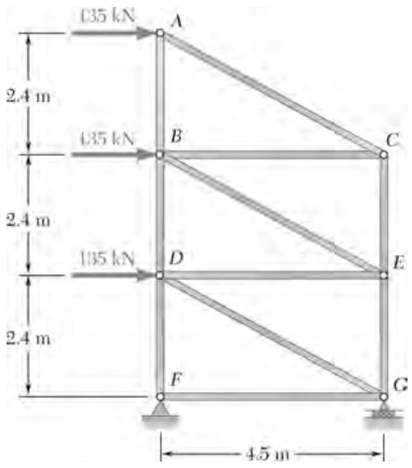
$$+\Sigma F_y = 0: \quad \frac{6}{6.5} F_{FG} + 18 \text{ kN} = 0$$

$$F_{FG} = -19.5 \text{ kN} \qquad F_{FG} = 19.50 \text{ kN} \quad C \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_G = 0: \quad F_{FH}(6 \text{ m}) + (18 \text{ kN})(10 \text{ m}) = 0$$

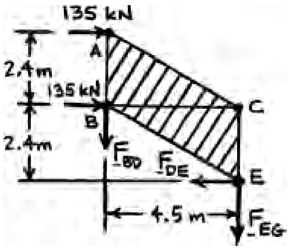
$$F_{FH} = -30 \text{ kN} \qquad F_{FH} = 30.0 \text{ kN} \quad C \blacktriangleleft$$

PROBLEM 6.45



Determine the force in members *BD* and *DE* of the truss shown.

SOLUTION



Member *BD*:

$$+\circlearrowleft \Sigma M_E = 0: \quad F_{BD}(4.5 \text{ m}) - (135 \text{ kN})(4.8 \text{ m}) - (135 \text{ kN})(2.4 \text{ m}) = 0$$

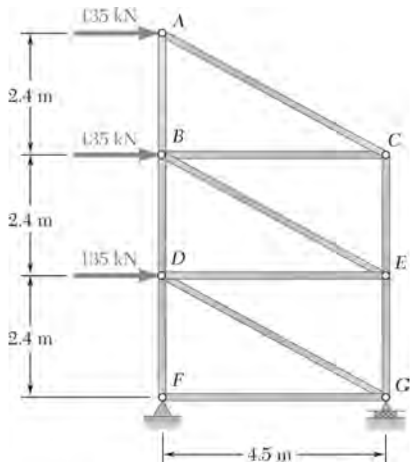
$$F_{BD} = +216 \text{ kN} \qquad F_{BD} = 216 \text{ kN} \quad T \blacktriangleleft$$

Member *DE*:

$$\xrightarrow{+} \Sigma F_x = 0: \quad 135 \text{ kN} + 135 \text{ kN} - F_{DE} = 0$$

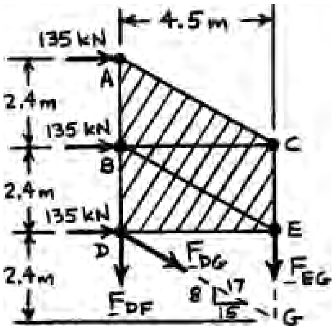
$$F_{DE} = +270 \text{ kN} \qquad F_{DE} = 270 \text{ kN} \quad T \blacktriangleleft$$

PROBLEM 6.46



Determine the force in members *DG* and *EG* of the truss shown.

SOLUTION



Member <u>*DG*</u>:

$$\xrightarrow{+} \Sigma F_x = 0: \quad 135 \text{ kN} + 135 \text{ kN} + 135 \text{ kN} + \frac{15}{17} F_{DG} = 0$$

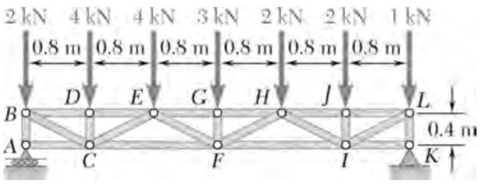
$$F_{DG} = -459 \text{ kN} \qquad F_{DG} = 459 \text{ kN} \quad C \blacktriangleleft$$

Member <u>*EG*</u>:

$$+\circlearrowleft \Sigma M_D = 0: \quad (135 \text{ kN})(4.8 \text{ m}) + (135 \text{ kN})(2.4 \text{ m}) + F_{EG}(4.5 \text{ m}) = 0$$

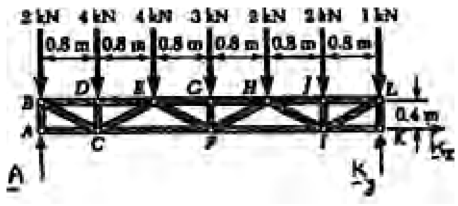
$$F_{EG} = -216 \text{ kN} \qquad F_{EG} = 216 \text{ kN} \quad C \blacktriangleleft$$

PROBLEM 6.47



A floor truss is loaded as shown. Determine the force in members *CF*, *EF*, and *EG*.

SOLUTION



Free body: Truss

$$\xrightarrow{+} \Sigma F_x = 0: \quad k_x = 0$$

$$+\circlearrowleft \Sigma M_A = 0: \quad k_y(4.8 \text{ m}) - (4 \text{ kN})(0.8 \text{ m}) - (4 \text{ kN})(1.6 \text{ m}) - (3 \text{ kN})(2.4 \text{ m}) - (2 \text{ kN})(3.2 \text{ m}) - (2 \text{ kN})(4 \text{ m}) - (1 \text{ kN})(4.8 \text{ m}) = 0$$

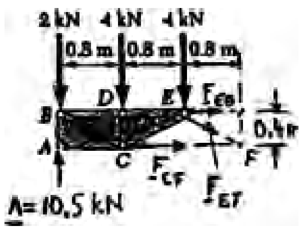
$$k_y = 7.5 \text{ kN}$$

Thus: $\mathbf{k} = 7.5 \text{ kN} \uparrow$ ◁

$$+\uparrow \Sigma F_y = 0: \quad A + 7.5 \text{ kN} - 18 \text{ kN} = 0 \quad A = 10.5 \text{ kN}$$

$\mathbf{A} = 10.5 \text{ kN} \uparrow$ ◁

We pass a section through members *CF*, *EF*, and *EG* and use the free body shown.



$$+\circlearrowleft \Sigma M_E = 0: \quad F_{CF}(0.4 \text{ m}) - (10.5 \text{ kN})(1.6 \text{ m}) + (2 \text{ kN})(1.6 \text{ m}) + (4 \text{ kN})(0.8 \text{ m}) = 0$$

$$F_{CF} = +26.0 \text{ kN}$$

$F_{CF} = 26.0 \text{ kN} \quad T$ ◀

$$+\uparrow \Sigma F_y = 0: \quad 10.5 \text{ kN} - 2 \text{ kN} - 4 \text{ kN} - 4 \text{ kN} - \frac{1}{\sqrt{5}} F_{EF} = 0$$

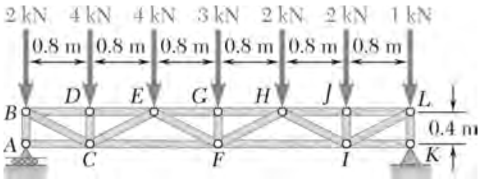
$$F_{EF} = +1.118 \text{ kN}$$

$F_{EF} = 1.118 \text{ kN} \quad T$ ◀

$$+\circlearrowleft \Sigma M_F = 0: \quad (2 \text{ kN})(2.4 \text{ m}) + (4 \text{ kN})(1.6 \text{ m}) + (4 \text{ kN})(0.8 \text{ m}) - (10.5 \text{ kN})(2.4 \text{ m}) - F_{EG}(0.4 \text{ m}) = 0$$

$$F_{EG} = -27.0 \text{ kN}$$

$F_{EG} = 27.0 \text{ kN} \quad C$ ◀



PROBLEM 6.48

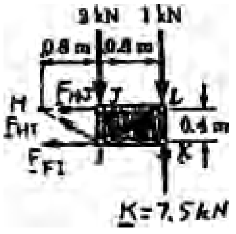
A floor truss is loaded as shown. Determine the force in members *FI*, *HI*, and *HJ*.

SOLUTION

See solution of Problem 6.47 for free-body diagram of truss and determination of reactions:

$$\mathbf{A} = 10.5 \text{ kN}\uparrow, \ \mathbf{k} = 7.5 \text{ kN}\uparrow \ \triangleleft$$

We pass a section through members *FI*, *HI*, and *HJ*, and use the free body shown.



$$+\circlearrowleft \Sigma M_H = 0: \quad (7.5 \text{ kN})(1.6 \text{ m}) - (2 \text{ kN})(0.8 \text{ m}) - (1 \text{ kN})(1.6 \text{ m}) - F_{FI}(0.4 \text{ m}) = 0$$

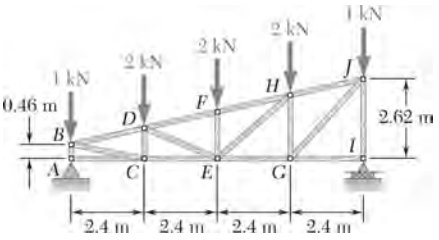
$$F_{FI} = +22.0 \text{ kN} \qquad F_{FI} = 22.0 \text{ kN} \quad T \ \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}} F_{HI} - 2 \text{ kN} - 1 \text{ kN} + 7.5 \text{ kN} = 0$$

$$F_{HI} = -10.06 \text{ kN} \qquad F_{HI} = 10.06 \text{ kN} \quad C \ \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_I = 0: \quad F_{HJ}(0.4 \text{ m}) + (7.5 \text{ kN})(0.8 \text{ m}) - (1 \text{ kN})(0.8 \text{ m}) = 0$$

$$F_{HJ} = -13.00 \text{ kN} \qquad F_{HJ} = 13.00 \text{ kN} \quad C \ \blacktriangleleft$$



PROBLEM 6.49

A pitched flat roof truss is loaded as shown. Determine the force in members *CE*, *DE*, and *DF*.

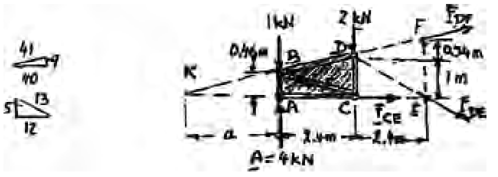
SOLUTION

<u>Reactions at supports</u>: Because of the symmetry of the loading,

$$A_x = 0$$

$$A_y = I = \frac{1}{2}\text{ (Total load)} = \frac{1}{2}(8 \text{ kN}) \qquad \mathbf{A} = \mathbf{I} = 4 \text{ kN}\uparrow \triangleleft$$

We pass a section through members *CD*, *DE*, and *DF*, and use the <u>free body shown</u>.
(We moved F_{DE} to *E* and F_{DF} to *F*)



$$\text{Slope} \quad BJ = \frac{2.16 \text{ m}}{9.6 \text{ m}} = \frac{9}{40}$$

$$\text{Slope} \quad DE = \frac{-1 \text{ m}}{2.4 \text{ m}} = \frac{-5}{12}$$

$$a = \frac{0.46 \text{ m}}{\text{Slope } BJ} = \frac{0.46 \text{ m}}{\frac{9}{40}} = 2.0444 \text{ m}$$

$$+\circlearrowleft \Sigma M_D = 0: \quad F_{CE}(1 \text{ m}) + (1 \text{ kN})(2.4 \text{ m}) - (4 \text{ kN})(2.4 \text{ m}) = 0$$

$$F_{CE} = +7.20 \text{ kN} \qquad F_{CE} = 7.20 \text{ kN} \quad T \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_K = 0: \quad (4 \text{ kN})(2.0444 \text{ m}) - (1 \text{ kN})(2.0444 \text{ m})$$

$$-(2 \text{ kN})(4.4444 \text{ m}) - \left(\frac{5}{13}F_{DE}\right)(6.8444 \text{ m}) = 0$$

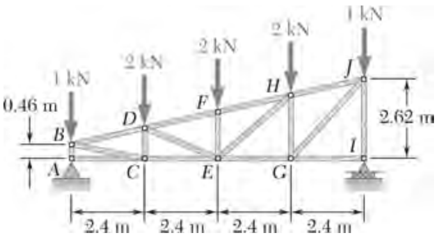
$$F_{DE} = -1.047 \text{ kN} \qquad F_{DE} = 1.047 \text{ kN} \quad C \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_E = 0: \quad (1 \text{ kN})(4.8 \text{ m}) + (2 \text{ kN})(2.4 \text{ m}) - (4 \text{ kN})(4.8 \text{ m})$$

$$-\left(\frac{40}{41}F_{DF}\right)(1.54 \text{ m}) = 0$$

$$F_{DF} = -6.39 \text{ kN} \qquad F_{DF} = 6.39 \text{ kN} \quad C \blacktriangleleft$$

PROBLEM 6.50



A pitched flat roof truss is loaded as shown. Determine the force in members *EG*, *GH*, and *HJ*.

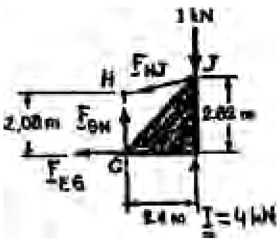
SOLUTION

<u>Reactions at supports</u>: Because of the symmetry of the loading,

$$A_x = 0$$

$$A_y = I = \frac{1}{2}\text{(Total load)} = \frac{1}{2}(8 \text{ kN}) \qquad \mathbf{A} = \mathbf{I} = 4 \text{ kN} \uparrow \triangleleft$$

We pass a section through members *EG*, *GH*, and *HJ*, and use the free body shown.



$$+\circlearrowleft \Sigma M_H = 0: \quad (4 \text{ kN})(2.4 \text{ m}) - (1 \text{ kN})(2.4 \text{ m}) - F_{EG}(2.08 \text{ m}) = 0$$

$$F_{EG} = +3.4615 \text{ kN} \qquad F_{EG} = 3.46 \text{ kN} \quad T \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -F_{GH}(2.4 \text{ m}) - F_{EG}(2.62 \text{ m}) = 0$$

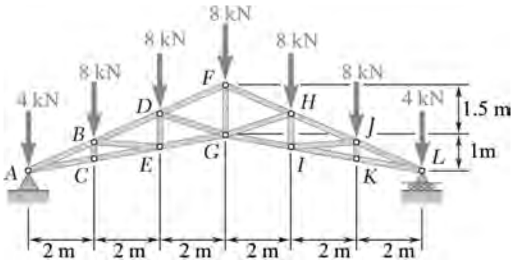
$$F_{GH} = -\frac{2.62}{2.4}(3.4615 \text{ kN})$$

$$F_{GH} = -3.7788 \text{ kN} \qquad F_{GH} = 3.78 \text{ kN} \quad C \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -F_{EG} - \frac{2.4}{2.46}F_{HJ} = 0$$

$$F_{HJ} = -\frac{2.46}{2.4}F_{EG} = -\frac{2.46}{2.4}(3.4615 \text{ kN})$$

$$F_{HJ} = -3.548 \text{ kN} \qquad F_{HJ} = 3.55 \text{ kN} \quad C \blacktriangleleft$$



PROBLEM 6.51

A Howe scissors roof truss is loaded as shown. Determine the force in members *DF*, *DG*, and *EG*.

SOLUTION

Reactions at supports.

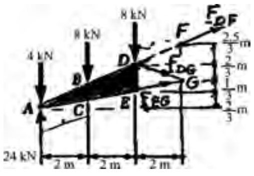
Because of symmetry of loading.

$$A_x = 0, \quad A_y = L = \frac{1}{2}(\text{Total load}) = \frac{1}{2}(48 \text{ kN}) = 24 \text{ kN}$$

$$\mathbf{A} = \mathbf{L} = 24.0 \text{ kN} \uparrow \blacktriangleleft$$

We pass a section through members *DF*, *DG*, and *EG*, and use the free body shown.

We slide $\mathbf{F}_{DF}$ to apply it at *F*:



$$+\circlearrowleft \Sigma M_G = 0: \quad (4 \text{ kN})(6 \text{ m}) + (8 \text{ N})(4 \text{ m}) + (8 \text{ kN})(2 \text{ m})$$

$$-(24 \text{ kN})(6 \text{ m}) - \frac{2F_{DF}}{\sqrt{2^2 + \left(\frac{2.5}{3}\right)^2}}(1.5 \text{ m}) = 0$$

$$F_{DF} = -52.0 \text{ kN}, \quad F_{DF} = 52.0 \text{ kN} \quad C \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -(8 \text{ kN})(2 \text{ m}) - (8 \text{ kN})(4 \text{ m})$$

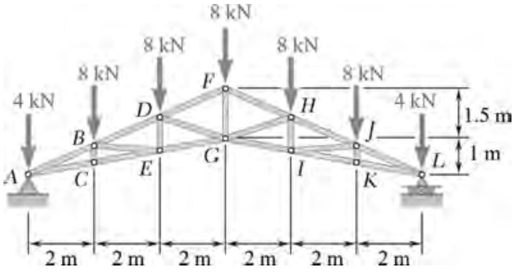
$$-\frac{2/3F_{DG}}{\sqrt{2^2 + (2/3)^2}}(4 \text{ m}) - \frac{2F_{DG}}{\sqrt{2^2 + (2/3)^2}}\left(\frac{5}{3} \text{ m}\right) = 0$$

$$F_{DG} = -16.8654 \text{ kN}, \quad F_{DG} = 16.87 \text{ kN} \quad C \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_D = 0: \quad (4 \text{ kN})(4 \text{ m}) + (8 \text{ kN})(2 \text{ m}) - (24 \text{ kN})(4 \text{ m}) - \frac{2F_{EG}}{\sqrt{2^2 + \left(\frac{1}{3}\right)^2}}(1 \text{ m}) = 0$$

$$F_{EG} = -64.882 \text{ kN}, \quad F_{EG} = 64.9 \text{ kN} \quad C \blacktriangleleft$$

PROBLEM 6.52



A Howe scissors roof truss is loaded as shown. Determine the force in members *GI*, *HI*, and *HJ*.

SOLUTION

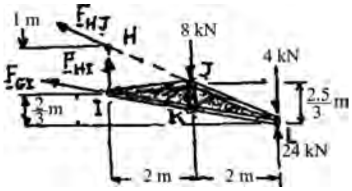
Reactions at supports. Because of symmetry of loading:

$$A_x = 0, \quad A_y = L = \frac{1}{2}(\text{Total load})$$

$$= \frac{1}{2}(48 \text{ kN})$$

$$= 24 \text{ kN} \qquad \mathbf{A} = \mathbf{L} = 24.0 \text{ kN}\uparrow \blacktriangleleft$$

We pass a section through members *GI*, *HI*, and *HJ*, and use the free body shown.



$$+\circlearrowleft \Sigma M_H = 0: \quad \frac{4F_{GI}}{\sqrt{4^2 + \left(\frac{2}{3}\right)^2}}(1 \text{ m}) + (24 \text{ kN})(4 \text{ m}) - (4 \text{ m})(4 \text{ m}) - (8 \text{ kN})(2 \text{ m}) = 0$$

$$F_{GI} = +64.883 \text{ kN} \qquad F_{GI} = 64.9 \text{ kN} \quad T \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_L = 0: \quad (8 \text{ kN})(2 \text{ m}) - F_{HI}(4 \text{ m}) = 0$$

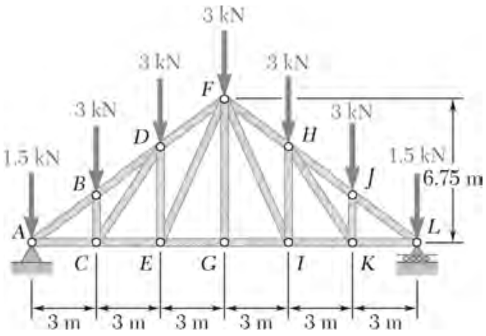
$$F_{HI} = +4.0 \text{ kN}$$

$$F_{HI} = 4.00 \text{ kN} \quad T \blacktriangleleft$$

We slide $\mathbf{F}_{HG}$ to apply it at *H*.

$$+\circlearrowleft \Sigma M_I = 0: \quad \frac{2F_{HJ}}{\sqrt{2^2 + \left(\frac{2.5}{3}\right)^2}}(1 \text{ m}) + (24 \text{ kN})(4 \text{ m}) - (8 \text{ kN})(2 \text{ m}) - (4 \text{ kN})(4 \text{ m}) = 0$$

$$F_{HJ} = -69.333 \text{ kN} \qquad F_{HJ} = 69.3 \text{ kN} \quad C \blacktriangleleft$$

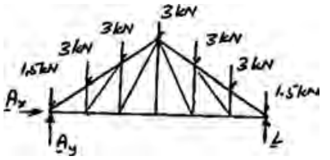


PROBLEM 6.53

A Pratt roof truss is loaded as shown. Determine the force in members *CE*, *DE*, and *DF*.

SOLUTION

<u>Free body: Entire truss</u>



$$\Sigma F_x = 0: \quad A_x = 0$$

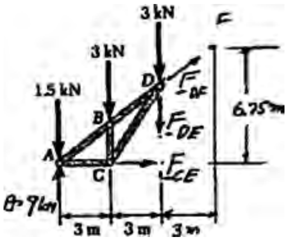
$$\text{Total load} = 5(3 \text{ kN}) + 2(1.5 \text{ kN})$$
$$= 18 \text{ kN}$$

By symmetry:

$$A_y = L = \frac{1}{2}(18 \text{ kN})$$

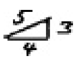
$\mathbf{A} = \mathbf{L} = 9$ kN ◀

<u>Free body: Portion *ACD*</u>



Note: Slope of *ABDF* is

$$\frac{6.75}{9.00} = \frac{3}{4}$$



<u>Force in *CE*:</u>

$$+\circlearrowleft \Sigma M_D = 0: \quad F_{CE}\left(\frac{2}{3} \times 6.75 \text{ m}\right) - (9 \text{ kN})(6 \text{ m}) + (1.5 \text{ kN})(6 \text{ m}) + (3 \text{ kN})(3 \text{ m}) = 0$$

$$F_{CE}(4.5 \text{ m}) - 36 \text{ kN}\cdot\text{m} = 0$$

$$F_{CE} = +8 \text{ kN}$$

$F_{CE} = 8$ kN T ◀

<u>Force in *DE*:</u>

$$+\circlearrowleft \Sigma M_A = 0: \quad F_{DE}(6 \text{ m}) + (3 \text{ kN})(6 \text{ m}) + (3 \text{ kN})(3 \text{ m}) = 0$$

$$F_{DE} = -4.5 \text{ kN}$$

$F_{DE} = 4.5$ kN C ◀

PROBLEM 6.53 (Continued)

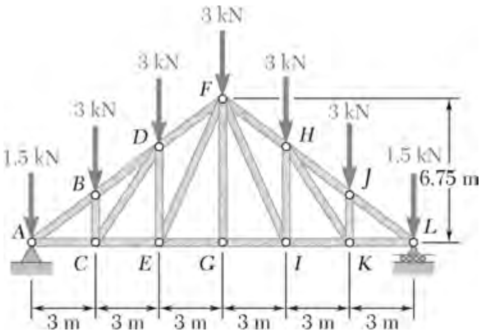
Force in DF:

Sum moments about E where F_{CE} and F_{DE} intersect.

$$+\circlearrowleft \Sigma M_E = 0: \quad (1.5 \text{ kN})(6 \text{ m}) - (9 \text{ kN})(6 \text{ m}) + (3 \text{ kN})(3 \text{ m}) + \frac{4}{5}F_{CE}\left(\frac{2}{3}\times 6.75 \text{ m}\right) = 0$$

$$\frac{4}{5}F_{CE}(4.5 \text{ m}) - 36 \text{ kN}\cdot\text{m}$$

$$F_{CE} = -10.00 \text{ kN} \qquad\qquad F_{CE} = 10.00 \text{ kN} \quad C \blacktriangleleft$$

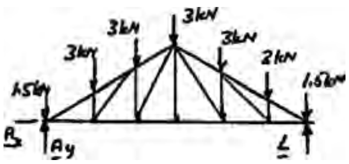


PROBLEM 6.54

A Pratt roof truss is loaded as shown. Determine the force in members *FH*, *FI*, and *GI*.

SOLUTION

<u>Free body: Entire truss</u>



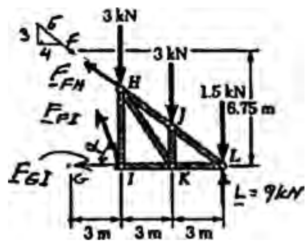
$$\Sigma F_x = 0: \quad A_x = 0$$

$$\text{Total load} = 5(3\text{ kN}) + 2(1.5\text{ kN}) = 18\text{ kN}$$

By symmetry:

$$\mathbf{A}_y = \mathbf{L} = \frac{1}{2}(18) = 9\text{ kN}\uparrow$$

<u>Free body: Portion *HIL*</u>



Slope of *FHJL*

$$\frac{6.75}{9.00} = \frac{3}{4}$$

$$\tan\alpha = \frac{FG}{GI} = \frac{6.75\text{ m}}{3\text{ m}} \qquad \alpha = 66.04°$$

<u>Force in *FH*</u>:

$$\circlearrowleft + \Sigma M_I = 0: \quad \frac{4}{5}F_{FH}\left(\frac{2}{3}\times 6.75\text{ m}\right) + (9\text{ kN})(6\text{ m}) - (1.5\text{ kN})(6\text{ m}) - (3\text{ kN})(3\text{ m}) = 0$$

$$\frac{4}{5}F_{FH}(4.5\text{ m}) + 36\text{ kN}\cdot\text{m}$$

$$F_{FH} = -10.00\text{ kN} \qquad\qquad F_{FH} = 10.00\text{ kN} \quad C \blacktriangleleft$$

<u>Force in *FI*</u>:

$$\circlearrowleft + \Sigma M_L = 0: \quad F_{FI}\sin\alpha(6\text{ m}) - (3\text{ kN})(6\text{ m}) - (3\text{ kN})(3\text{ m}) = 0$$

$$F_{FI}\sin 66.04°(6\text{ m}) = 27\text{ kN}\cdot\text{m}$$

$$F_{FI} = +4.92\text{ kN} \qquad\qquad F_{FI} = 4.92\text{ kN} \quad T \blacktriangleleft$$

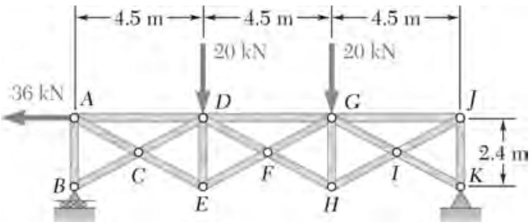
PROBLEM 6.54 (Continued)

<u>Force in *GI*</u>:

$$+\circlearrowleft \Sigma M_H = 0: \quad F_{GI}(6.75 \text{ m}) + (3 \text{ kN})(3 \text{ m}) + (3 \text{ kN})(6 \text{ m}) + (1.5 \text{ kN})(9 \text{ m}) - (9 \text{ kN})(9 \text{ m}) = 0$$

$$F_{GI}(6.75 \text{ m}) = +40.5 \text{ kN}\cdot\text{m}$$

$$F_{GI} = +6.00 \text{ kN} \qquad F_{GI} = 6.00 \text{ kN} \quad T \blacktriangleleft$$

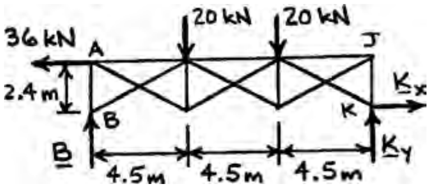


PROBLEM 6.55

Determine the force in members AD, CD, and CE of the truss shown.

SOLUTION

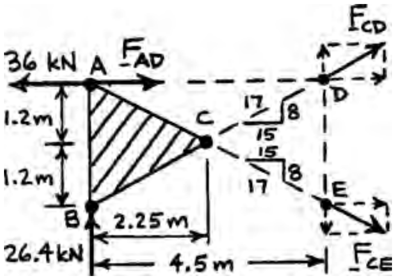
<u>Reactions</u>:



$$\circlearrowleft + \Sigma M_k = 0: \quad 36(2.4) - B(13.5) + 20(9) + 20(4.5) = 0 \qquad \mathbf{B} = 26.4 \text{ kN} \uparrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -36 + K_x = 0 \qquad \mathbf{K}_x = 36 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0: \quad 26.4 - 20 - 20 + K_y = 0 \quad \mathbf{K}_y = 13.6 \text{ kN} \uparrow$$



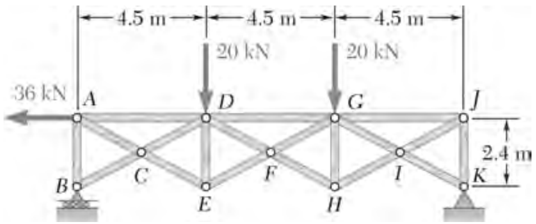
$$\circlearrowleft + \Sigma M_C = 0: \quad 36(1.2) - 26.4(2.25) - F_{AD}(1.2) = 0$$

$$F_{AD} = -13.5 \text{ kN} \qquad F_{AD} = 13.5 \text{ kN} \quad C \blacktriangleleft$$

$$\circlearrowleft + \Sigma M_A = 0: \quad \left(\frac{8}{17} F_{CD}\right)(4.5) = 0 \qquad F_{CD} = 0 \blacktriangleleft$$

$$\circlearrowleft + \Sigma M_D = 0: \quad \left(\frac{15}{17} F_{CE}\right)(2.4) - 26.4(4.5) = 0$$

$$F_{CE} = +56.1 \text{ kN} \qquad F_{CE} = 56.1 \text{ kN} \quad T \blacktriangleleft$$



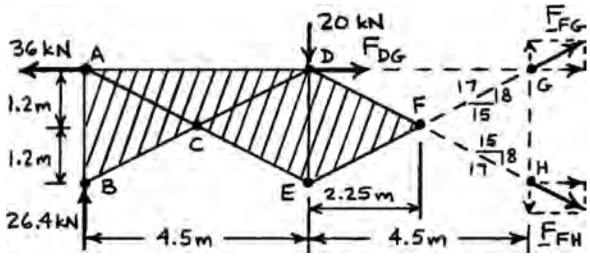
PROBLEM 6.56

Determine the force in members *DG*, *FG*, and *FH* of the truss shown.

SOLUTION

See the solution to Problem 6.55 for free-body diagram and analysis to determine the reactions at the supports *B* and *K*.

$$\mathbf{B} = 26.4 \text{ kN}\uparrow; \quad \mathbf{K}_x = 36.0 \text{ kN} \quad ; \quad \mathbf{K}_y = 13.60 \text{ kN}\uparrow$$



$$+\circlearrowleft \Sigma M_F = 0: \quad 36(1.2) - 26.4(6.75) + 20(2.25) - F_{DG}(1.2) = 0$$

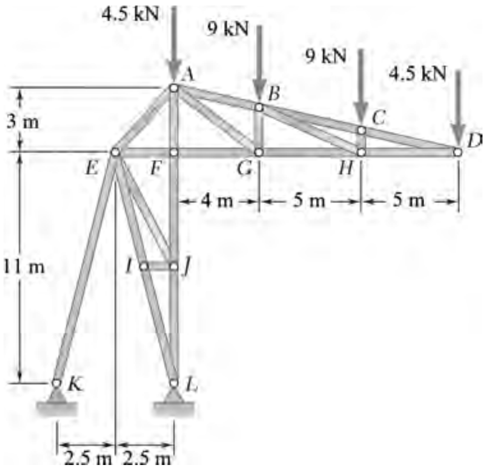
$$F_{DG} = -75 \text{ kN} \qquad F_{DG} = 75.0 \text{ kN} \quad C \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_D = 0: \quad -26.4(4.5) + \left(\frac{8}{17} F_{FG}\right)(4.5) = 0$$

$$F_{FG} = +56.1 \text{ kN} \qquad F_{FG} = 56.1 \text{ kN} \quad T \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_G = 0: \quad 20(4.5) - 26.4(9) + \left(\frac{15}{17} F_{FH}\right)(2.4) = 0$$

$$F_{FH} = +69.7 \text{ kN} \qquad F_{FH} = 69.7 \text{ kN} \quad T \blacktriangleleft$$

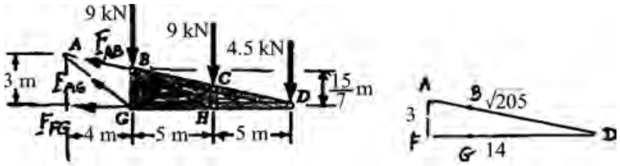


PROBLEM 6.57

A stadium roof truss is loaded as shown. Determine the force in members *AB*, *AG*, and *FG*.

SOLUTION

We pass a section through members *AB*, *AG*, and *FG*, and use the free body shown.



$$\circlearrowleft^{+}\ \Sigma M_G = 0: \quad \left(\frac{14}{\sqrt{205}} F_{AB}\right)\left(\frac{15}{7}\ \text{m}\right) - (9\ \text{kN})(5\ \text{m}) - (4.5\ \text{kN})(10\ \text{m}) = 0$$

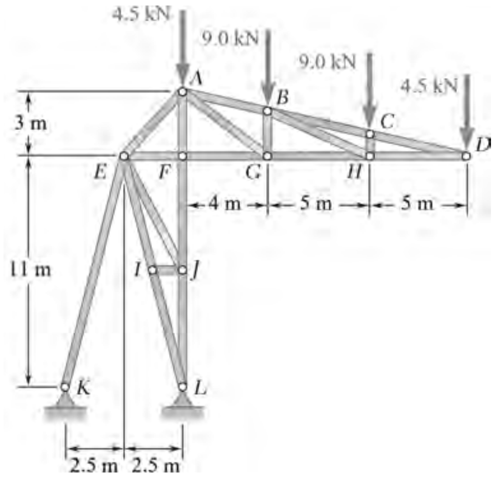
$F_{AB} = +42.953$ kN $\qquad F_{AB} = 43.0$ kN T ◀

$$\circlearrowleft^{+}\ \Sigma M_D = 0: \quad -\left(\frac{3}{5} F_{AG}\right)(10\ \text{m}) + (9\ \text{kN})(10\ \text{m}) + (9\ \text{kN})(5\ \text{m}) = 0$$

$F_{AG} = +22.5$ kN $\qquad F_{AG} = 22.5$ kN T ◀

$$\circlearrowleft^{+}\ M_A = 0: \quad -F_{FG}(3\ \text{m}) - (9\ \text{kN})(4\ \text{m}) - (9\ \text{kN})(9\ \text{m}) - (4.5\ \text{kN})(14\ \text{m}) = 0$$

$F_{FG} = -60$ kN $\qquad F_{FG} = 60.0$ kN C ◀

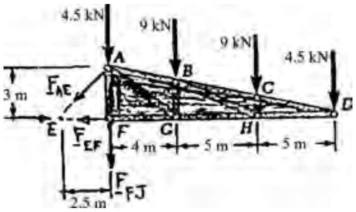


PROBLEM 6.58

A stadium roof truss is loaded as shown. Determine the force in members *AE*, *EF*, and *FJ*.

SOLUTION

We pass a section through members *AE*, *EF*, and *FJ*, and use the free body shown.



$$\circlearrowleft + \Sigma M_F = 0: \quad \left(\frac{25}{\sqrt{2.5^2+3^2}} F_{AE}\right)(3\text{ m}) - (9\text{ kN})(4\text{ m}) - (9\text{ kN})(9\text{ m}) - (4.5\text{ kN})(14\text{ m}) = 0$$

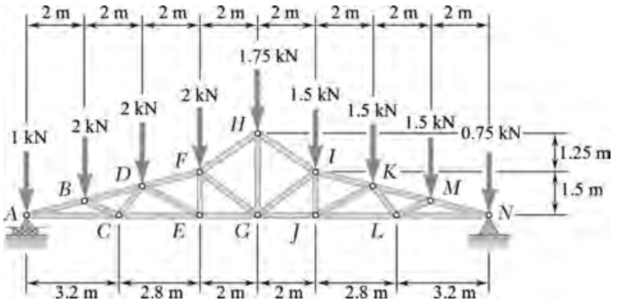
$$F_{AE} = +71.4898\text{ kN} \qquad F_{AE} = 71.5\text{ kN} \quad T \blacktriangleleft$$

$$\circlearrowleft + \Sigma M_A = 0: \quad -F_{EF}(3\text{ m}) - (9\text{ kN})(4\text{ m}) - (9\text{ kN})(9\text{ m}) - (4.5\text{ kN})(14\text{ m}) = 0$$

$$F_{EF} = -60.0\text{ kN} \qquad F_{EF} = 60.0\text{ kN} \quad C \blacktriangleleft$$

$$\circlearrowleft + \Sigma M_E = 0: \quad -F_{FJ}(2.5\text{ m}) - (4.5\text{ kN})(2.5\text{ m}) - (9\text{ kN})(6.5\text{ m}) - (9\text{ kN})(11.5\text{ m}) - (4.5\text{ kN})(16.5\text{ m}) = 0$$

$$F_{FJ} = -99.0\text{ kN} \qquad F_{FJ} = 99.0\text{ kN} \quad C \blacktriangleleft$$

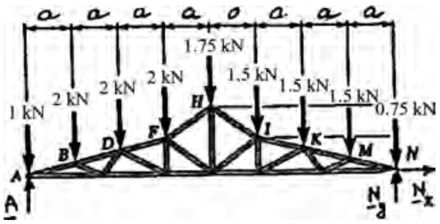


PROBLEM 6.59

A Polynesian, or duopitch, roof truss is loaded as shown. Determine the force in members *DF*, *EF*, and *EG*.

SOLUTION

Free body: Truss



$$\Sigma F_x = 0: \quad N_x = 0$$

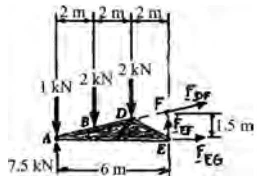
$$+\circlearrowleft \Sigma M_N = 0: \quad (1 \text{ kN})(8a) + (2 \text{ kN})(7a + 6a + 5a) + (1.75 \text{ kN})(4a) + (1.5 \text{ kN})(3a + 2a + a) - A(8a) = 0$$

$$\mathbf{A} = 7.50 \text{ kN} \uparrow \triangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 7.5 \text{ kN} - 1 \text{ kN} - 3(2 \text{ kN}) - 1.75 \text{ kN} - 3(1.5 \text{ kN}) - 0.75 \text{ kN} + N_y = 0$$

$$N_y = 6.50 \text{ kN} \qquad \mathbf{N} = 6.50 \text{ kN} \uparrow \triangleleft$$

We pass a section through *DF*, *EF*, and *EG*, and use the free body shown.



(We apply $\mathbf{F}_{DF}$ at F)

$$+\circlearrowleft \Sigma M_E = 0: \quad (1 \text{ kN})(6 \text{ m}) + (2 \text{ kN})(4 \text{ m}) + (2 \text{ kN})(2 \text{ m}) - (7.5 \text{ kN})(6 \text{ m})$$

$$-\left(\frac{6}{\sqrt{6^2 + 1.5^2}} F_{DF}\right)(1.5 \text{ m}) = 0$$

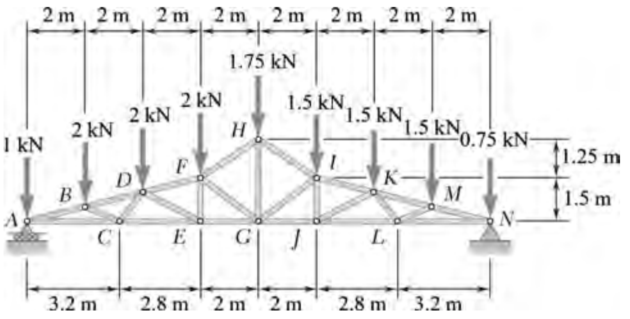
$$F_{DF} = -18.554 \text{ kN} \qquad F_{DF} = 18.6 \text{ kN} \quad C \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_A = 0: \quad F_{EF}(6 \text{ m}) - (2 \text{ kN})(2 \text{ m}) - (2 \text{ kN})(4 \text{ m}) = 0$$

$$F_{EF} = +2.00 \text{ kN} \qquad F_{EF} = 2.00 \text{ kN} \quad T \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_F = 0: \quad F_{EG}(1.5 \text{ m}) - (7.5 \text{ kN})(6 \text{ m}) + (1 \text{ kN})(6 \text{ m}) + (2 \text{ kN})(4 \text{ m}) + (2 \text{ kN})(2 \text{ m}) = 0$$

$$F_{EG} = +18.0 \text{ kN} \qquad F_{EG} = 18.0 \text{ kN} \quad T \blacktriangleleft$$



PROBLEM 6.60

A Polynesian, or duopitch, roof truss is loaded as shown. Determine the force in members *HI*, *GI*, and *GJ*.

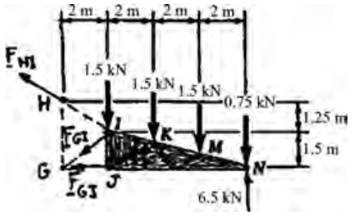
SOLUTION

See solution of Problem 6.59 for reactions:

$$\mathbf{A} = 7.5\text{ kN}\uparrow, \quad \mathbf{N} = 6.5\text{ kN}\uparrow \triangleleft$$

We pass a section through *HI*, *GI*, and *GJ*, and use the free body shown.

(We apply F_{HI} at *H*.)



$$+\circlearrowleft \Sigma M_G = 0: \quad \left(\frac{2}{\sqrt{2^2+(1.25)^2}}F_{HI}\right)(275\text{ m}) + (6.5\text{ kN})(8\text{ m}) - (1.5\text{ kN})(2\text{ m})$$

$$-(1.5\text{ kN})(4\text{ m}) - (1.5\text{ kN})(6\text{ m}) - (0.75\text{ kN})(8\text{ m}) = 0$$

$$F_{HI} = -12.007\text{ kN} \quad F_{HI} = 12.01\text{ kN} \quad C \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_I = 0: \quad (6.5\text{ kN})(6\text{ m}) - (1.5\text{ kN})(2\text{ m}) - (1.5\text{ kN})(4\text{ m})$$

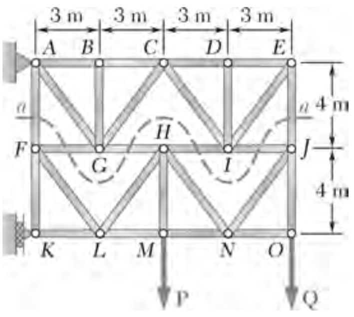
$$-(0.75\text{ kN})(6\text{ m}) - F_{GJ}(1.5\text{ m}) = 0$$

$$F_{GJ} = +17.00\text{ kN} \quad F_{GJ} = 17.00\text{ kN} \quad T \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{4}{5}F_{GI} - \frac{2}{\sqrt{2^2+(1.25)^2}}(-12.007\text{ kN}) - 17.00\text{ kN} = 0$$

$$F_{GI} = -8.5226\text{ kN} \quad F_{GI} = 8.52\text{ kN} \quad C \blacktriangleleft$$

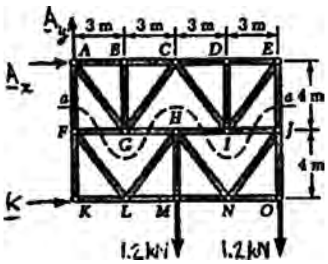
PROBLEM 6.61



Determine the force in members *AF* and *EJ* of the truss shown when $P = Q = 1.2$ kN. (*Hint:* Use section *aa*.)

SOLUTION

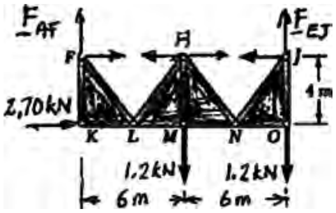
Free body: Entire truss



$$\circlearrowleft^{+} \Sigma M_A = 0: \quad K(8 \text{ m}) - (1.2 \text{ kN})(6 \text{ m}) - (1.2 \text{ kN})(12 \text{ m}) = 0$$

$K = +2.70$ kN $\qquad \mathbf{K} = 2.70 \text{ kN} \rightarrow \triangleleft$

Free body: Lower portion

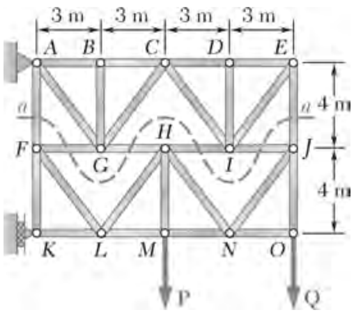


$$\circlearrowleft^{+} \Sigma M_F = 0: \quad F_{EJ}(12 \text{ m}) + (2.70 \text{ kN})(4 \text{ m}) - (1.2 \text{ kN})(6 \text{ m}) - (1.2 \text{ kN})(12 \text{ m}) = 0$$

$F_{EJ} = +0.900$ kN $\qquad F_{EJ} = 0.900 \text{ kN} \quad T \blacktriangleleft$

$$+\uparrow \Sigma F_y = 0: \quad F_{AF} + 0.9 \text{ kN} - 1.2 \text{ kN} - 1.2 \text{ kN} = 0$$

$F_{AF} = +1.500$ kN $\qquad F_{AF} = 1.500 \text{ kN} \quad T \blacktriangleleft$

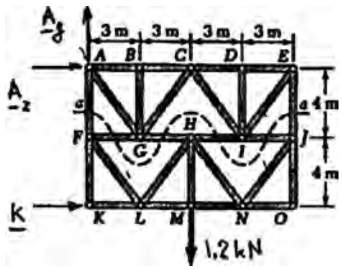


PROBLEM 6.62

Determine the force in members AF and EJ of the truss shown when $P = 1.2$ kN and $Q = 0$. (*Hint:* Use section aa.)

SOLUTION

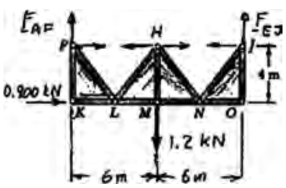
<u>Free body: Entire truss</u>



$$+\circlearrowleft \Sigma M_A = 0: \quad K(8 \text{ m}) - (1.2 \text{ kN})(6 \text{ m}) = 0$$

$$K = +0.900 \text{ kN} \qquad \mathbf{K} = 0.900 \text{ kN} \quad \triangleleft$$

<u>Free body: Lower portion</u>



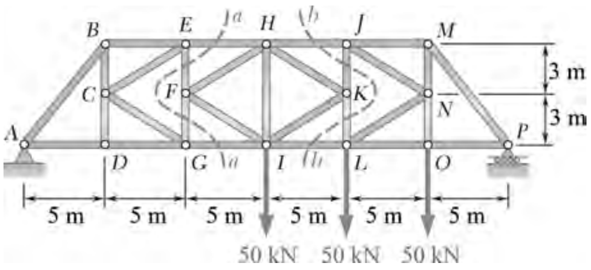
$$+\circlearrowleft \Sigma M_F = 0: \quad F_{EJ}(12 \text{ m}) + (0.900 \text{ kN})(4 \text{ m}) - (1.2 \text{ kN})(6 \text{ m}) = 0$$

$$F_{EJ} = +0.300 \text{ kN} \qquad F_{EJ} = 0.300 \text{ kN} \quad T \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad F_{AF} + 0.300 \text{ kN} - 1.2 \text{ kN} = 0$$

$$F_{AF} = +0.900 \text{ kN} \qquad F_{AF} = 1.900 \text{ kN} \quad T \blacktriangleleft$$

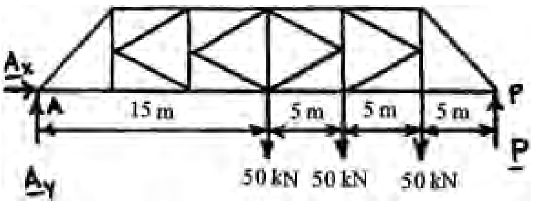
PROBLEM 6.63



Determine the force in members *EH* and *GI* of the truss shown. (*Hint:* Use section *aa*.)

SOLUTION

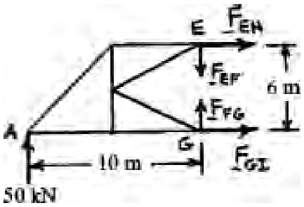
Reactions:



$$\Sigma F_x = 0: \quad A_x = 0$$

$$+\circlearrowleft \Sigma M_P = 0: \quad 50(15) + 50(10) + 50(5) - A_y(30) = 0$$

$$\mathbf{A}_y = 50 \text{ kN} \uparrow$$



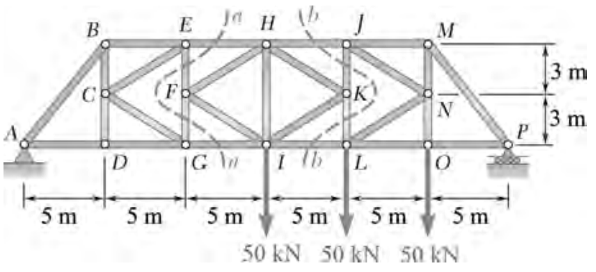
$$+\uparrow \Sigma F_y = 0: \quad 50 - 50 - 50 - 50 + P = 0 \qquad \mathbf{P} = 100 \text{ kN} \uparrow$$

$$+\circlearrowleft \Sigma M_G = 0: \quad -(50 \text{ kN})(10 \text{ m}) - F_{EH}(6 \text{ m}) = 0$$

$$F_{EH} = -83.33 \text{ kN} \qquad F_{EH} = 83.3 \text{ kN} \quad C \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{GI} - 83.33 \text{ kN} = 0 \qquad F_{GI} = 83.3 \text{ kN} \quad T \blacktriangleleft$$

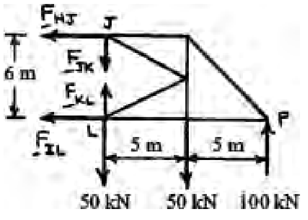
PROBLEM 6.64



Determine the force in members *HJ* and *IL* of the truss shown. (*Hint:* Use section *bb*.)

SOLUTION

See the solution to Problem 6.63 for free body diagram and analysis to determine the reactions at supports *A* and *P*.



$$\mathbf{A}_x = 0; \quad \mathbf{A}_y = 50 \text{ kN}\uparrow; \quad \mathbf{P} = 100 \text{ kN}\uparrow$$

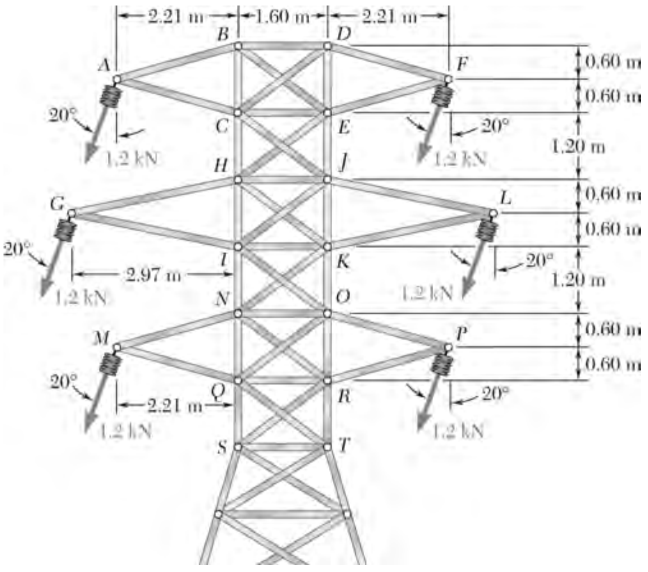
$$\circlearrowleft + \Sigma M_L = 0: \quad F_{HJ}(6 \text{ m}) - (50 \text{ kN})(5 \text{ m}) + (100 \text{ kN})(10 \text{ m}) = 0$$

$$F_{HJ} = -125.0 \text{ kN} \qquad F_{HJ} = 125.0 \text{ kN} \quad C \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -125 \text{ kN} - F_{IL} = 0$$

$$F_{IL} = +125 \text{ kN} \qquad F_{IL} = 125.0 \text{ kN} \quad T \blacktriangleleft$$

PROBLEM 6.65



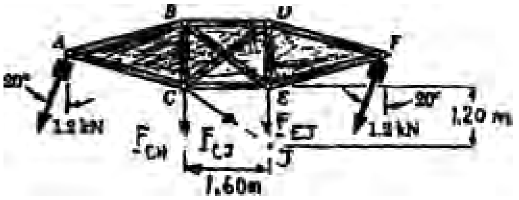
The diagonal members in the center panels of the power transmission line tower shown are very slender and can act only in tension; such members are known as *counters*. For the given loading, determine (*a*) which of the two counters listed below is acting, (*b*) the force in that counter.

Counters *CJ* and *HE*.

SOLUTION

Free body: Portion *ABDFEC* of tower

We assume that counter *CJ* is acting and show the forces exerted by that counter and by members *CH* and *EJ*.



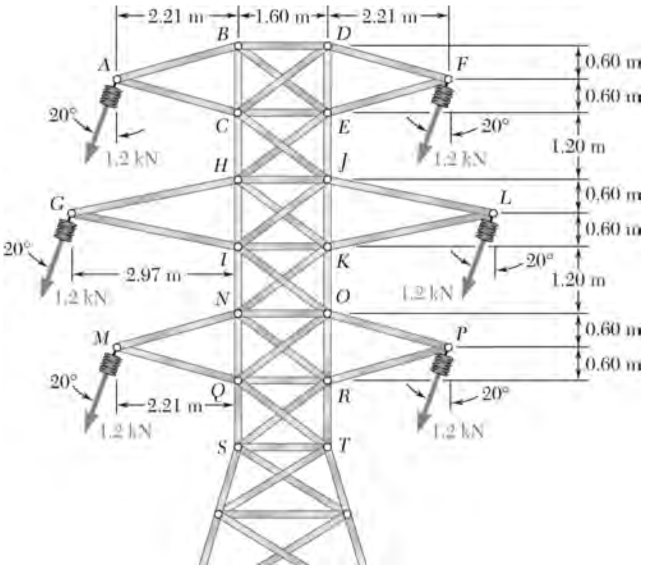
$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{4}{5}F_{CJ} - 2(1.2 \text{ kN})\sin 20° = 0 \qquad F_{CJ} = +1.026 \text{ kN}$$

Since *CJ* is found to be in tension, our assumption was correct. Thus, the answers are

(*a*) *CJ* ◀

(*b*) 1.026 kN *T* ◀

PROBLEM 6.66

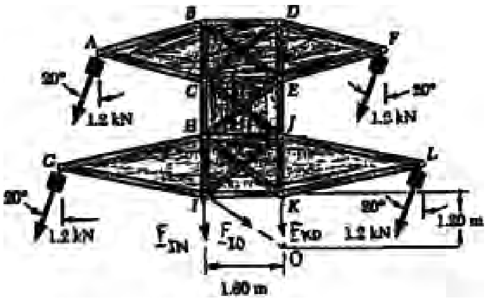


The diagonal members in the center panels of the power transmission line tower shown are very slender and can act only in tension; such members are known as *counters*. For the given loading, determine (*a*) which of the two counters listed below is acting, (*b*) the force in that counter.

Counters *IO* and *KN*.

SOLUTION

Free body: Portion of tower shown



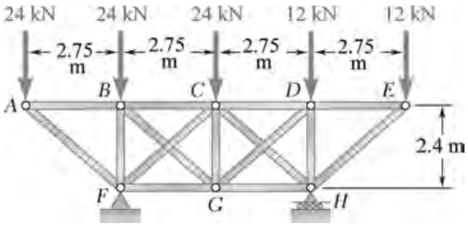
We assume that counter *IO* is acting and show the forces exerted by that counter and by members *IN* and *KO*.

$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{4}{5}F_{IO} - 4(1.2 \text{ kN})\sin 20° = 0 \qquad F_{IO} = +2.05 \text{ kN}$$

Since *IO* is found to be in tension, our assumption was correct. Thus, the answers are

(*a*) *IO* ◀

(*b*) 2.05 kN *T* ◀

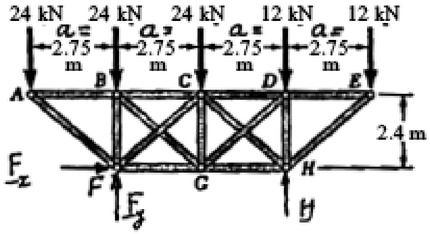


PROBLEM 6.67

The diagonal members in the center panels of the truss shown are very slender and can act only in tension; such members are known as *counters*. Determine the forces in the counters that are acting under the given loading.

SOLUTION

Free body: Truss



$$\Sigma F_x = 0: \quad F_x = 0$$

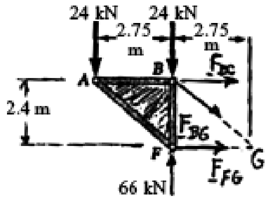
$$+\circlearrowleft \Sigma M_H = 0: \quad 24(3a) + 24(2a) + 24a - 12a - F_y(2a) = 0$$

$$F_y = +66 \text{ kN} \qquad \mathbf{F} = 66.0 \text{ kN}\uparrow \triangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad H + 66 \text{ kN} - 3(24 \text{ kN}) - 2(12 \text{ kN}) = 0$$

$$H = +30.0 \text{ kN} \qquad \mathbf{H} = 30.0 \text{ kN}\uparrow \triangleleft$$

Free body: *ABF*



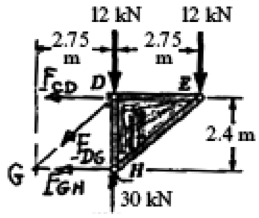
We assume that counter *BG* is acting.

$$+\uparrow \Sigma F_y = 0: \quad -\frac{2.4}{3.65} F_{BG} + 66 - 2(24) = 0$$

$$F_{BG} = +27.375 \text{ kN} \qquad F_{BG} = 27.4 \text{ kN} \quad T \blacktriangleleft$$

Since *BG* is in tension, our assumption was correct

Free body: *DEH*

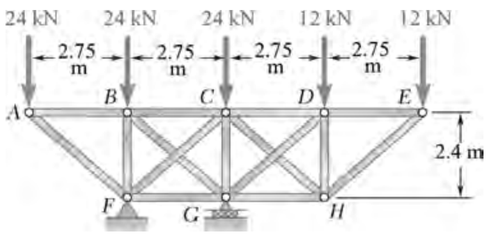


We assume that counter *DG* is acting.

$$+\Sigma F_y = 0: \quad -\frac{2.4}{3.65} F_{DG} + 30 - 2(12) = 0$$

$$F_{DG} = +9.125 \text{ kN} \qquad F_{DG} = 9.13 \text{ kN} \quad T \blacktriangleleft$$

Since *DG* is in tension, O.K.

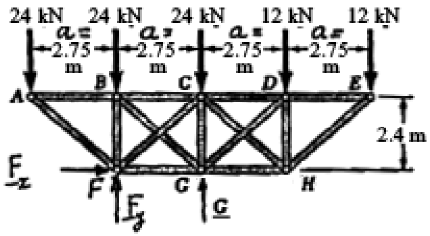


PROBLEM 6.68

The diagonal members in the center panels of the truss shown are very slender and can act only in tension; such members are known as *counters*. Determine the forces in the counters that are acting under the given loading.

SOLUTION

<u>Free body: Truss</u>



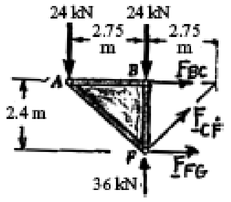
$$\Sigma F_x = 0: \quad F_x = 0$$

$$+\circlearrowleft \Sigma M_G = 0: \quad -F_y a + 24(2a) + 24a - 12a - 12(2a) = 0$$

$$F_y = 36.0 \text{ kN}$$

$F = 36.0 \text{ kN}\uparrow$ ◁

<u>Free body: *ABF*</u>



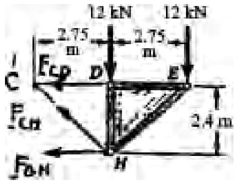
We assume that counter *CF* is acting.

$$+\uparrow \Sigma F_y = 0: \frac{2.4}{3.65} F_{CF} + 36 - 2(24) = 0$$

$$F_{CF} = +18.25 \text{ kN}$$

$F_{CF} = 18.25 \text{ kN} \quad T$ ◀

Since *CF* is in tension, O.K.



<u>Free body: *DEH*</u>

We assume that counter *CH* is acting.

$$+\Sigma F_y = 0: \quad \frac{2.4}{3.65} F_{CH} - 2(12 \text{ kN}) = 0$$

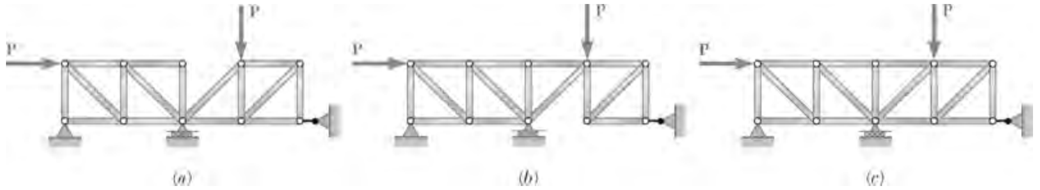
$$F_{CH} = +36.5 \text{ kN}$$

$F_{CH} = 36.5 \text{ kN} \quad T$ ◀

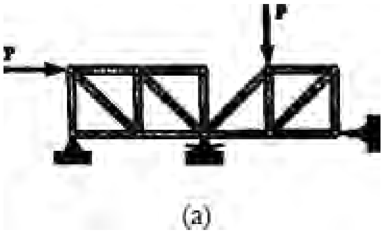
Since *CH* is in tension, O.K.

PROBLEM 6.69

Classify each of the structures shown as completely, partially, or improperly constrained; if completely constrained, further classify as determinate or indeterminate. (All members can act both in tension and in compression.)



SOLUTION



Structure (a)

Number of members:

$$m = 16$$

Number of joints:

$$n = 10$$

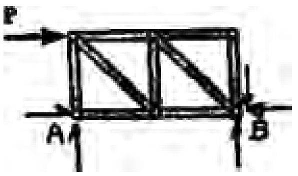
Reaction components:

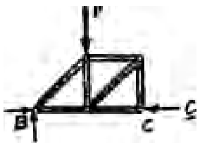
$$r = 4$$

$$m + r = 20, \quad 2n = 20$$

Thus: $m + r = 2n$ ◁

To determine whether the structure is actually completely constrained and determinate, we must try to find the reactions at the supports. We divide the structure into two simple trusses and draw the free-body diagram of each truss.



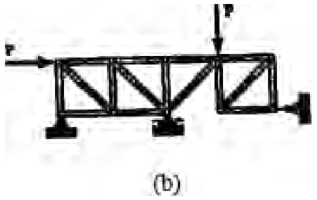


This is a properly supported simple truss – O.K.

This is an improperly supported simple truss. Reaction at C passes through B. Thus, Eq. $\Sigma M_B = 0$ cannot be satisfied.)

Structure is improperly constrained ◄

Structure (b)



$$m = 16$$

$$n = 10$$

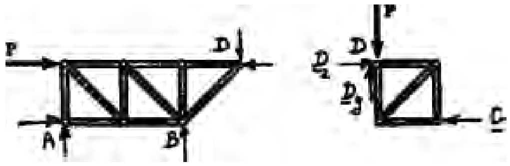
$$r = 4$$

$$m + r = 20, \quad 2n = 20$$

Thus: $m + r = 2n$ ◁

PROBLEM 6.69 (Continued)

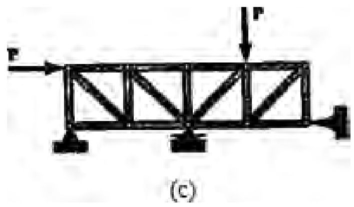
We must again try to find the reactions at the supports dividing the structure as shown.



Both portions are simply supported simple trusses.

Structure is completely constrained and determinate ◀

<u>Structure (c)</u>



$$m = 17$$
$$n = 10$$
$$r = 4$$
$$m + r = 21, \quad 2n = 20$$

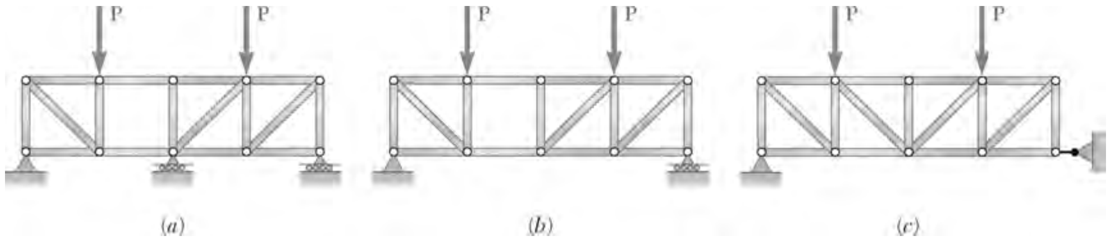
Thus: $m + r > 2n$ ◁

This is a simple truss with an extra support which causes reactions (and forces in members) to be indeterminate.

Structure is completely constrained and indeterminate ◀

PROBLEM 6.70

Classify each of the structures shown as completely, partially, or improperly constrained; if completely constrained, further classify as determinate or indeterminate. (All members can act both in tension and in compression.)



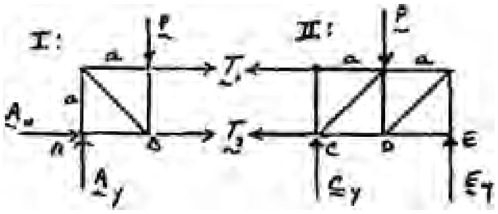
SOLUTION

Structure (a):

Non-simple truss with $r = 4,\ m = 16,\ n = 10$

so $m + r = 20 = 2n,$ but must examine further.

FBD Sections:



FBD	I:	$\Sigma M_A = 0$	$\Rightarrow T_1$
	II:	$\Sigma F_x = 0$	$\Rightarrow T_2$
	I:	$\Sigma F_x = 0$	$\Rightarrow A_x$
	I:	$\Sigma F_y = 0$	$\Rightarrow A_y$
	II:	$\Sigma M_E = 0$	$\Rightarrow C_y$
	II:	$\Sigma F_y = 0$	$\Rightarrow E_y$

Since each section is a simple truss with reactions determined,

structure is completely constrained and determinate. ◀

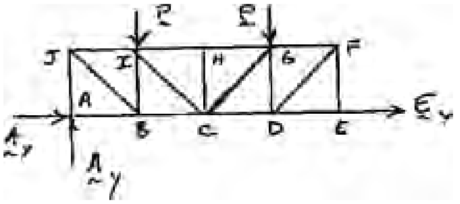
Non-simple truss with $r = 3,\ m = 16,\ n = 10$

Structure (b):

so $m + r = 19 < 2n = 20$ structure is partially constrained ◀

PROBLEM 6.70 (Continued)

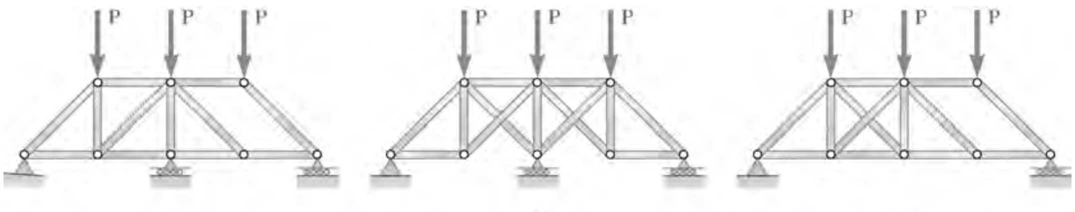
Structure (c):



Simple truss with $r = 3$, $m = 17$, $n = 10$ $m + r = 20 = 2n$, but the horizontal reaction forces A_x and E_x are collinear and no equilibrium equation will resolve them, so the structure is improperly constrained and indeterminate ◀

PROBLEM 6.71

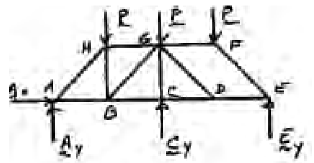
Classify each of the structures shown as completely, partially, or improperly constrained; if completely constrained, further classify as determinate or indeterminate. (All members can act both in tension and in compression.)

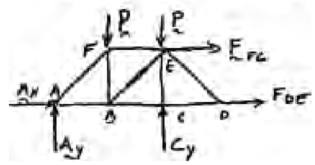


(a) (b) (c)

SOLUTION

Structure (a):





Non-simple truss with $r = 4, m = 12, n = 8$ so $r + m = 16 = 2n$,

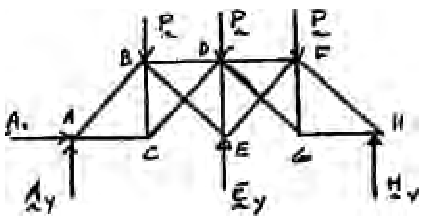
check for determinacy:

One can solve joint F for forces in EF, FG and then solve joint E for $\mathbf{E}_y$ and force in DE.

This leaves a simple truss $ABCDGH$ with

$r = 3, m = 9, n = 6$ so $r + m = 12 = 2n$ Structure is completely constrained and determinate ◀

Structure (b):

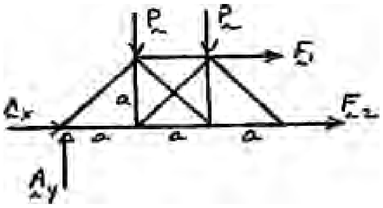


Simple truss (start with ABC and add joints alphabetically to complete truss) with $r = 4, m = 13, n = 8$

so $r + m = 17 > 2n = 16$ Constrained but indeterminate ◀

PROBLEM 6.71 (Continued)

Structure (c):



Non-simple truss with $r = 3, m = 13, n = 8$ so $r + m = 16 = 2n$. To further examine, follow procedure in Part (a) above to get truss at left.

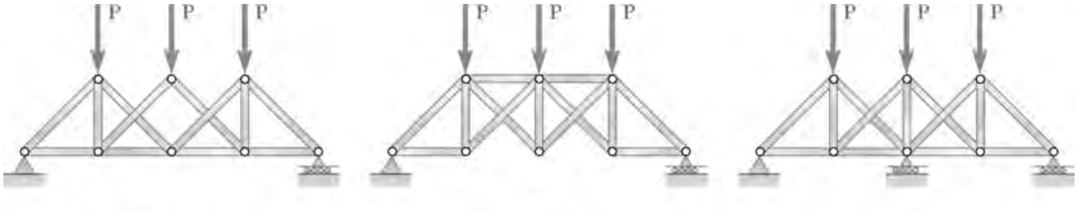
Since $\mathbf{F}_1 \neq 0$ (from solution of joint F),

$\Sigma M_A = aF_1 \circlearrowright \neq 0$ and there is no equilibrium.

Structure is improperly constrained ◄

PROBLEM 6.72

Classify each of the structures shown as completely, partially, or improperly constrained; if completely constrained, further classify as determinate or indeterminate. (All members can act both in tension and in compression.)



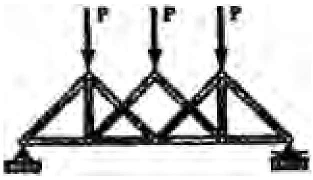
(a) (b) (c)

SOLUTION

Structure (a)

Number of members:

$$m = 12$$



Number of joints:

$$n = 8$$

Reaction components:

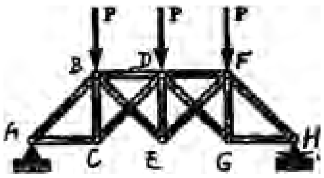
$$r = 3$$

$$m + r = 15, \quad 2n = 16$$

Thus: $m + r < 2n$ ◁

Structure is partially constrained ◀

Structure (b)



$$m = 13, \; n = 8$$

$$r = 3$$

$$m + r = 16, \quad 2n = 16$$

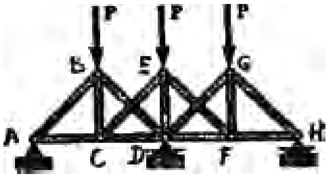
Thus: $m + r = 2n$ ◁

To verify that the structure is actually completely constrained and determinate, we observe that it is a simple truss (follow lettering to check this) and that it is simply supported by a pin-and-bracket and a roller. Thus:

Structure is completely constrained and determinate ◀

PROBLEM 6.72 (Continued)

Structure (c)



$$m = 13, \quad n = 8$$

$$r = 4$$

$$m + r = 17, \quad 2n = 16$$

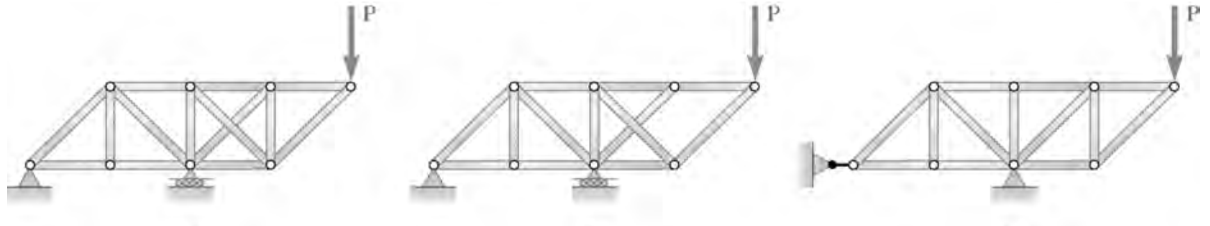
Thus: $m + r > 2n$ ◁

Structure is completely constrained and indeterminate ◀

This result can be verified by observing that the structure is a simple truss (follow lettering to check thus), therefore rigid, and that its supports involve 4 unknowns.

PROBLEM 6.73

Classify each of the structures shown as completely, partially, or improperly constrained; if completely constrained, further classify as determinate or indeterminate. (All members can act both in tension and in compression.)



(a) (b) (c)

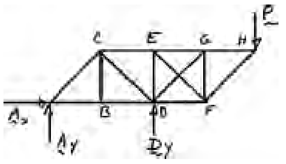
SOLUTION

Structure (a): Rigid truss with $r = 3,\ m = 14,\ n = 8$

so $r + m = 17 > 2n = 16$

so completely constrained but indeterminate ◄

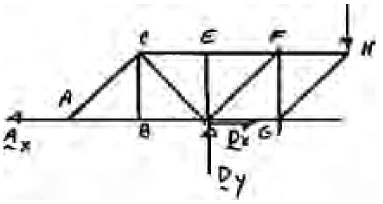
Structure (b): Simple truss (start with *ABC* and add joints alphabetically), with



$r = 3,\ m = 13,\ n = 8$ so $r + m = 16 = 2n$

so completely constrained and determinate ◄

Structure (c):

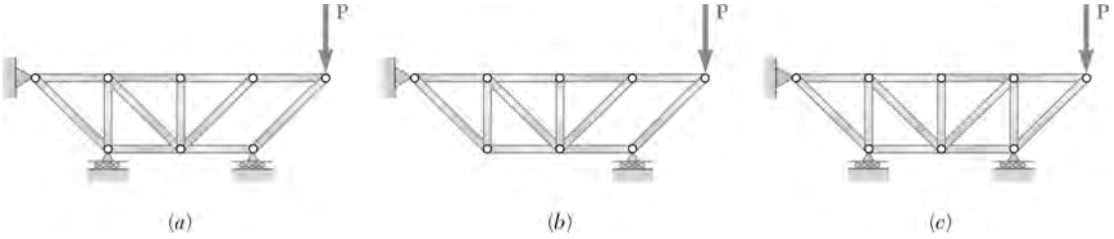


Simple truss with $r = 3,\ m = 13,\ n = 8$ so $r + m = 16 = 2n$, but horizontal reactions (A_x and D_x) are collinear so cannot be resolved by any equilibrium equation.

structure is improperly constrained ◄

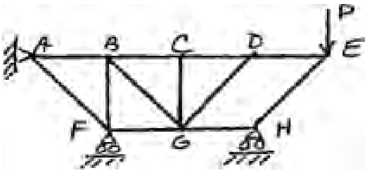
PROBLEM 6.74

Classify each of the structures shown as completely, partially, or improperly constrained; if completely constrained, further classify as determinate or indeterminate. (All members can act both in tension and in compression.)



SOLUTION

Structure (a):

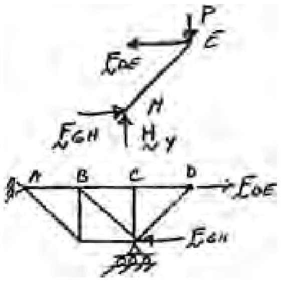


No. of members $m = 12$

No. of joints $n = 8 \quad m + r = 16 = 2n$

No. of react. comps. $r = 4 \quad$ unks $=$ eqns

FBD of EH:

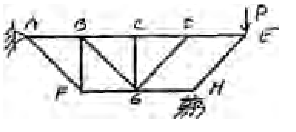


$$\Sigma M_H = 0 \rightarrow F_{DE}; \Sigma F_x = 0 \rightarrow F_{GH}; \Sigma F_y = 0 \rightarrow H_y$$

Then *ABCDGF* is a simple truss and all forces can be determined.

This example is completely constrained and determinate. ◀

Structure (b):



No. of members $m = 12$

No. of joints $n = 8 \quad m + r = 15 < 2n = 16$

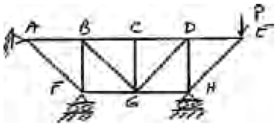
No. of react. comps. $r = 3 \quad$ unks $<$ eqns

partially constrained ◀

Note: Quadrilateral *DEHG* can collapse with joint *D* moving downward: in (a) the roller at *F* prevents this action.

PROBLEM 6.74 (Continued)

Structure (c):

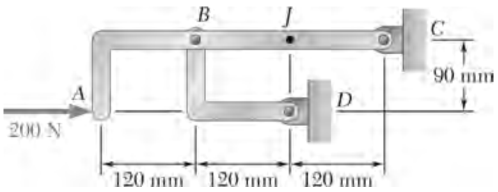


No. of members $m = 13$

No. of joints $n = 8 \quad m + r = 17 > 2n = 16$

No. of react. comps. $r = 4 \quad \text{unks} > \text{eqns}$

completely constrained but indeterminate ◄

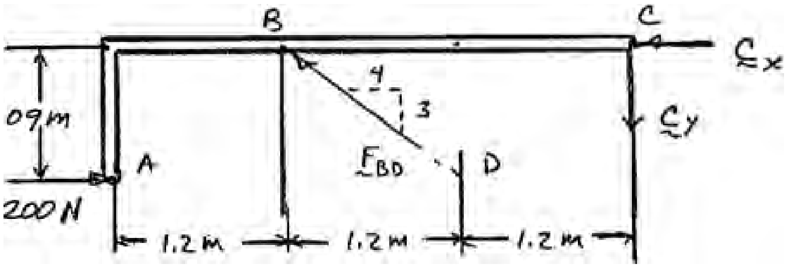


PROBLEM 6.75

For the frame and loading shown, determine the force acting on member *ABC* (*a*) at *B*, (*b*) at *C*.

SOLUTION

FBD ABC:



Note: *BD* is two-force member

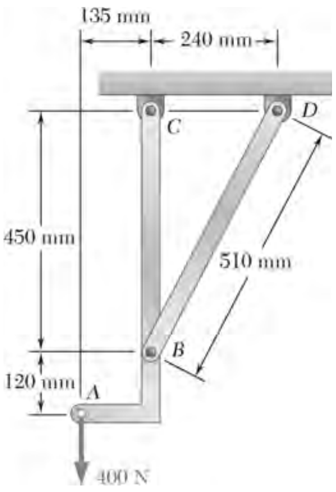
(*a*) $$\circlearrowright \Sigma M_C = 0: \quad (0.09\text{ m})(200\text{ N}) - (2.4\text{ m})\left(\frac{3}{5}F_{BD}\right) = 0$$

$$\mathbf{F}_{BD} = 125.0\text{ N} \; \nwarrow \; 36.9° \blacktriangleleft$$

(*b*) $$\rightarrow \Sigma F_x = 0: \quad 200\text{ N} - \frac{4}{5}(125\text{ N}) - C_x = 0 \qquad \mathbf{C}_x = 100\text{ N} \leftarrow$$

$$\uparrow \Sigma F_y = 0: \quad \frac{3}{5}F_{BD} - C_y = 0 \qquad \mathbf{C}_y = \frac{3}{5}(125\text{ N}) = 75\text{ N} \downarrow$$

$$\mathbf{C} = 125.0\text{ N} \; \swarrow \; 36.9° \blacktriangleleft$$



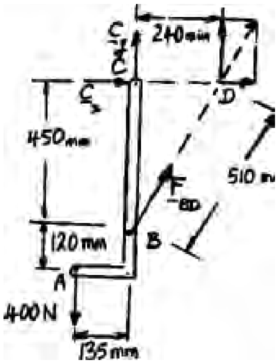
PROBLEM 6.76

Determine the force in member *BD* and the components of the reaction at *C*.

SOLUTION

We note that *BD* is a two-force member. The force it exerts on *ABC*, therefore, is directed along the *BD*.

Free body: *ABC*



Attaching $\mathbf{F}_{BD}$ at *D* and resolving it into components, we write

$$+\circlearrowleft \Sigma M_C = 0: \quad (400 \text{ N})(135 \text{ mm}) + \left(\frac{450}{510} F_{BD}\right)(240 \text{ mm}) = 0$$

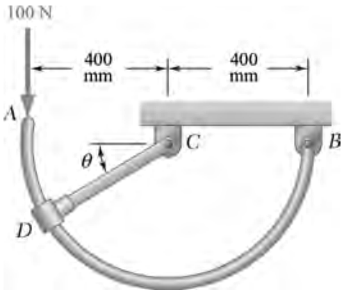
$$F_{BD} = -255 \text{ N} \qquad F_{BD} = 255 \text{ N} \quad C \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + \frac{240}{510}(-255 \text{ N}) = 0$$

$$C_x = +120.0 \text{ N} \qquad \mathbf{C}_x = 120.0 \text{ N} \rightarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - 400 \text{ N} + \frac{450}{510}(-255 \text{ N}) = 0$$

$$C_y = +625 \text{ N} \qquad \mathbf{C}_y = 625 \text{ N} \uparrow \blacktriangleleft$$

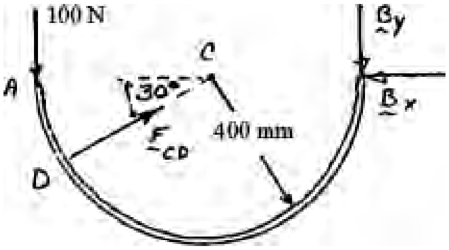


PROBLEM 6.77

Rod *CD* is fitted with a collar at *D* that can be moved along rod *AB*, which is bent in the shape of an arc of circle. For the position when $\theta = 30°$, determine (*a*) the force in rod *CD*, (*b*) the reaction at *B*.

SOLUTION

FBD:



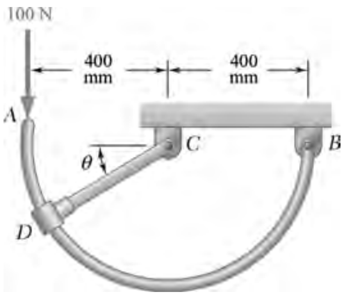
(*a*) $\circlearrowright \Sigma M_C = 0: \quad (400 \text{ mm})(100 \text{ N} - B_y) = 0$ $\qquad \mathbf{B}_y = 100 \text{ N} \downarrow$

$\uparrow \Sigma F_y = 0: \; -100 \text{ N} + F_{CD} \sin 30° - 100 \text{ N} = 0$ $\qquad F_{CD} = 400 \text{ N} \quad T$ ◀

(*b*) $\rightarrow \Sigma F_x = 0: \quad (400 \text{ N})\cos 30° - B_x = 0$

$$\mathbf{B}_x = 346.411 \text{ N} \leftarrow$$

$B = \sqrt{Bx^2 + By^2} = 360.27 \text{ N}$ $\qquad$ so $\mathbf{B} = 360 \text{ N} \measuredangle\, 16.10°$ ◀



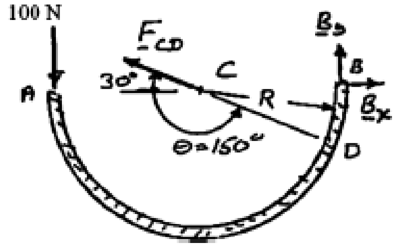
PROBLEM 6.78

Solve Problem 6.77 when $\theta = 150°$.

PROBLEM 6.77 Rod *CD* is fitted with a collar at *D* that can be moved along rod *AB*, which is bent in the shape of an arc of circle. For the position when $\theta = 30°$, determine (*a*) the force in rod *CD*, (*b*) the reaction at *B*.

SOLUTION

Note that *CD* is a two-force member, $\mathbf{F}_{CD}$ must be directed along *DC*.



(*a*) $\circlearrowleft^{+} \Sigma M_B = 0: \quad (100 \text{ N})(2R) - (F_{CD} \sin 30°)R = 0$

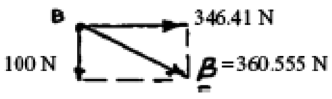
$$F_{CD} = 400 \text{ N} \qquad\qquad F_{CD} = 400 \text{ N} \quad T \blacktriangleleft$$

(*b*) $\circlearrowleft^{+} \Sigma M_C = 0: \quad (100 \text{ N})R + (B_y)R = 0$

$$B_y = 100 \text{ N} \qquad\qquad \mathbf{B}_y = 100 \text{ N} \downarrow$$

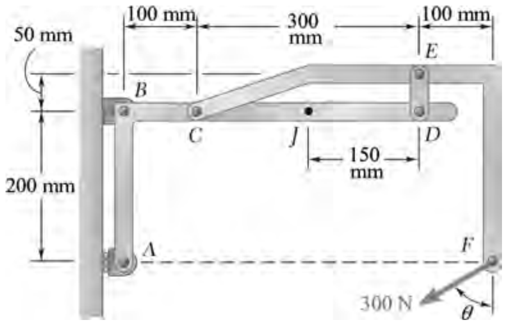
$\xrightarrow{+} \Sigma F_x = 0: \quad -F_{CD} \cos 30° + B_x = 0$

$$-(400 \text{ N}) \cos 30° + B_x = 0$$



$$B_x = 346.41 \text{ N} \qquad \mathbf{B}_x = 346.41 \text{ N} \rightarrow$$

$$\mathbf{B} = 361 \text{ N} \searrow 16.10° \blacktriangleleft$$

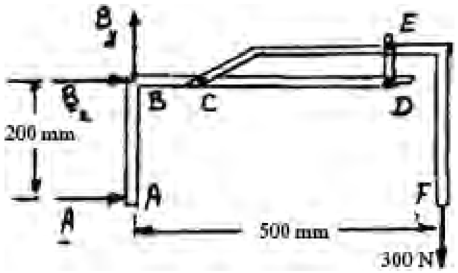


PROBLEM 6.79

Determine the components of all forces acting on member *ABCD* when $\theta = 0$.

SOLUTION

Free body: Entire assembly



$$+\circlearrowleft \Sigma M_B = 0: \quad A(200 \text{ mm}) - (300 \text{ N})(500 \text{ mm}) = 0$$

$A = 750$ N $\qquad$ $\mathbf{A} = 750 \text{ N} \rightarrow$ ◀

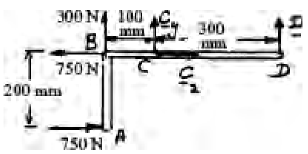
$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x + 750 \text{ N} = 0 \quad B_x = -750 \text{ N}$$

$\mathbf{B}_x = 750 \text{ N} \leftarrow$ ◀

$$+\uparrow \Sigma F_y = 0: \quad B_y - 300 \text{ N} = 0 \quad B_y = +300 \text{ N}$$

$\mathbf{B}_y = 300 \text{ N} \uparrow$ ◀

Free body: Member *ABCD* $\qquad$ We note that **D** is directed along *DE*, since *DE* is a two-force member.



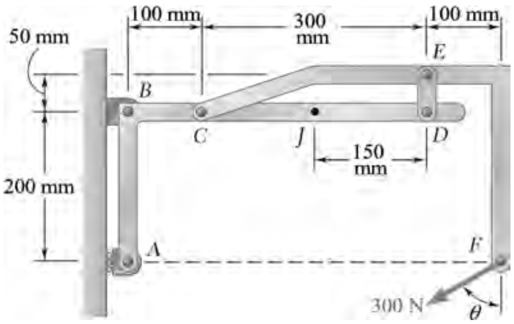
$$+\circlearrowleft \Sigma M_C = 0: \quad D(300) - (300 \text{ N})(100) + (750 \text{ N})(200) = 0$$

$D = -400$ N $\quad$ $\mathbf{D} = 400 \text{ N} \downarrow$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + 750 - 750 = 0 \qquad C_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad C_y + 300 - 400 = 0 \qquad C_y = +100.0 \text{ N}$$

$\mathbf{C} = 100.0 \text{ N} \uparrow$ ◀

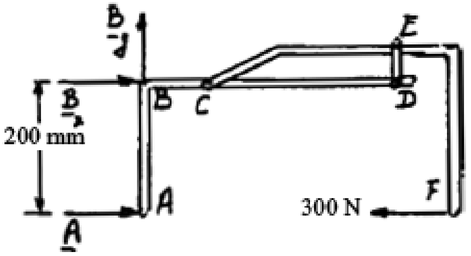


PROBLEM 6.80

Determine the components of all forces acting on member $ABCD$ when $\theta = 90°$.

SOLUTION

Free body: Entire assembly



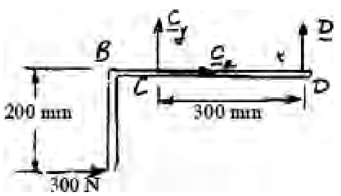
$$+\circlearrowleft \Sigma M_B = 0: \quad A(200 \text{ mm}) - (300 \text{ N})(200 \text{ mm}) = 0$$

$$A = +300 \text{ N} \qquad \mathbf{A} = 300 \text{ N} \rightarrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x + 300 \text{ N} - 300 \text{ N} = 0 \quad B_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad B_y = 0 \qquad \mathbf{B} = 0 \blacktriangleleft$$

Free body: Member *ABCD* We note that **D** is directed along *DE*, since *DE* is a two-force member.



$$+\circlearrowleft \Sigma M_C = 0: \quad D(300 \text{ mm}) + (300 \text{ N})(200 \text{ mm}) = 0$$

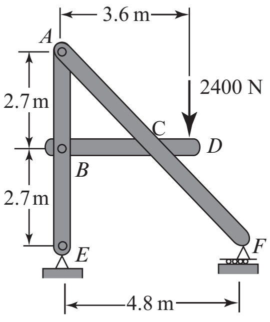
$$D = -200 \text{ N} \quad \mathbf{D} = 200 \text{ N} \downarrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + 300 \text{ N} = 0$$

$$C_x = -300 \text{ N} \qquad \mathbf{C}_x = 300 \text{ N} \leftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - 200 \text{ N} = 0$$

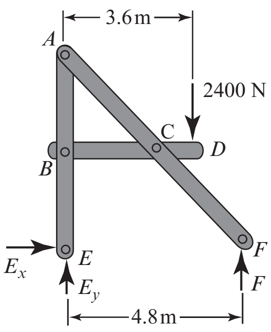
$$C_y = +200 \text{ N} \qquad \mathbf{C}_y = 200 \text{ N} \uparrow \blacktriangleleft$$



PROBLEM 6.81

Determine the components of the forces acting on each member of the frame shown.

SOLUTION



Free body: Entire frame.

$$+\circlearrowleft \Sigma M_E = 0: \quad -(2400 \text{ N})(3.6 \text{ m}) + F(4.8 \text{ m}) = 0$$

$$F = +1800 \text{ N} \qquad \mathbf{F} = 1800 \text{ N} \uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -2400 \text{ N} + 1800 \text{ N} + E_y = 0$$

$$E_y = +600 \text{ N} \qquad \mathbf{E}_y = 600 \text{ N} \uparrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad E_x = 0 \qquad \mathbf{E}_x = 0 \blacktriangleleft$$

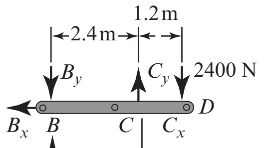
Members

Free body: Member BCD

$$+\circlearrowleft \Sigma M_B = 0: \quad -(2400 \text{ N})(3.6 \text{ m}) + C_y(2.4 \text{ m}) = 0 \qquad C_y = +3600 \text{ N} \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_C = 0: \quad -(2400 \text{ N})(1.2 \text{ m}) + B_y(2.4 \text{ m}) = 0 \qquad B_y = +1200 \text{ N} \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -B_x + C_x = 0$$



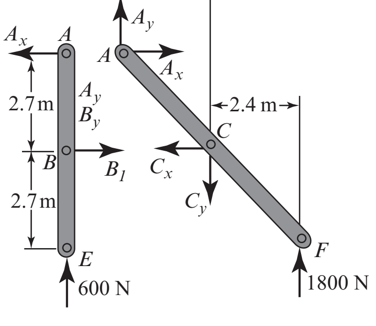
Free body: Member ABE

$$+\circlearrowleft \Sigma M_A = 0: \quad B_x(2.7 \text{ m}) = 0 \qquad B_x = 0 \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad +B_x - A_x = 0 \qquad A_x = 0 \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -A_y + B_y + 600 \text{ N} = 0$$

$$-A_y + 1200 \text{ N} + 600 \text{ N} = 0 \qquad A_y = +1800 \text{ N} \blacktriangleleft$$



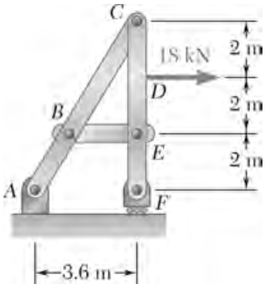
Free body: Member BCD. Returning now to member *BCD*, we write

$$\xrightarrow{+} \Sigma F_x = 0: \quad -B_x + C_x = 0 \qquad 0 + C_x = 0 \qquad C_x = 0 \blacktriangleleft$$

***Free body: Member ACF* (Check).** All unknown components have now been found, to check the results, we verify that members *ACF* is in equilibrium.

$$+\circlearrowleft \Sigma M_C = (1800 \text{ N})(2.4 \text{ m}) + A_y(2.4 \text{ m}) - A_x(2.7 \text{ m})$$

$$= (1800 \text{ N})(2.4 \text{ m}) - (1800 \text{ N})(2.4 \text{ m}) - 0 = 0 \qquad \text{(checks)}$$



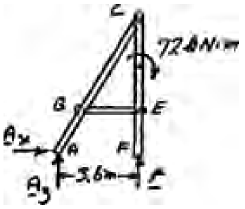
PROBLEM 6.82

Solve Sample Problem 6.5 assuming that the 18-kN load is replaced by a clockwise couple of magnitude 72 kN · m applied to member *CDEF* at Point *D*.

SAMPLE PROBLEM 6.5 For the frame and loading shown, determine the components of all forces acting on member *ABC*.

SOLUTION

Free body: Entire frame



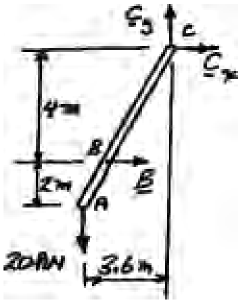
$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$

$+\circlearrowleft \Sigma M_F = 0: \quad -72 \text{ kN} \cdot \text{m} - A_y(3.6 \text{ m}) = 0$

$A_y = -20 \text{ kN} \quad \mathbf{A}_y = 20 \text{ kN}\downarrow$ $\qquad \mathbf{A} = 20.0 \text{ kN}\downarrow$ ◀

Free body: Member *ABC*

Note: BE is a two-force member. Thus **B** is directed along line *BE*.



$+\circlearrowleft \Sigma M_C = 0: \quad B(4 \text{ m}) + (20 \text{ kN})(3.6 \text{ m}) = 0$

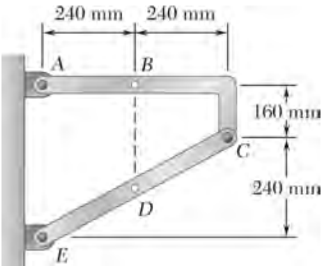
$B = -18 \text{ kN}$ $\qquad \mathbf{B} = 18.00 \text{ kN} \leftarrow$ ◀

$\xrightarrow{+} F_x = 0: \quad -18 \text{ kN} + C_x = 0$

$C_x = 18 \text{ kN}$ $\qquad \mathbf{C}_x = 18.00 \text{ kN} \rightarrow$ ◀

$+\uparrow \Sigma F_y = 0: \quad C_y - 20 \text{ kN} = 0$

$C_y = 20 \text{ kN}$ $\qquad \mathbf{C}_y = 20.0 \text{ kN} \uparrow$ ◀



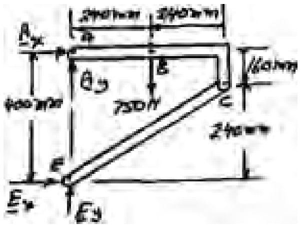
PROBLEM 6.83

Determine the components of the reactions at A and E if a 750-N force directed vertically downward is applied (a) at B, (b) at D.

SOLUTION

Free-body: Entire Frame

The following analysis is valid for both (a) and (b) since position of load on its line of action is immaterial.



$$+\circlearrowleft \Sigma M_E = 0: \quad -(750 \text{ N})(240 \text{ mm}) - A_x(400 \text{ mm}) = 0$$

$$A_x = -450 \text{ N} \quad \mathbf{A}_x = 450 \text{ N} \leftarrow$$

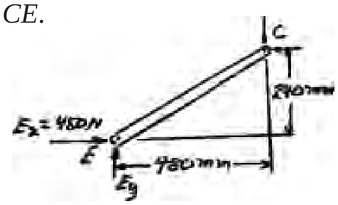
$$\xrightarrow{+} \Sigma F_x = 0: \quad E_x - 450 \text{ N} = 0 \quad E_x = 450 \text{ N} \quad \mathbf{E}_x = 450 \text{ N} \rightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + E_y - 750 \text{ N} = 0 \qquad (1)$$

(a) Load applied at B.

Free body: Member CE

CE is a two-force member. Thus, the reaction at E must be directed along CE.



$$\frac{E_y}{450 \text{ N}} = \frac{240 \text{ mm}}{480 \text{ mm}}; \quad \mathbf{E}_y = 225 \text{ N}\uparrow$$

From Eq. (1): $A_y + 225 - 750 = 0; \quad \mathbf{A}_y = 525 \text{ lb}\uparrow$

Thus, reactions are:

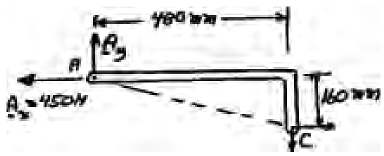
$$\mathbf{A}_x = 450 \text{ N} \leftarrow, \quad \mathbf{A}_y = 525 \text{ lb}\uparrow \blacktriangleleft$$

$$\mathbf{E}_x = 450\text{N} \rightarrow, \quad \mathbf{E}_y = 225 \text{ lb}\uparrow \blacktriangleleft$$

(b) Load applied at D.

Free body: Member AC

AC is a two-force member. Thus, the reaction at A must be directed along AC.



$$\frac{A_y}{450 \text{ N}} = \frac{160 \text{ mm}}{480 \text{ mm}} \quad \mathbf{A}_y = 150.0 \text{ N}\uparrow$$

From Eq. (1):

$$A_y + E_y - 750 \text{ N} = 0$$

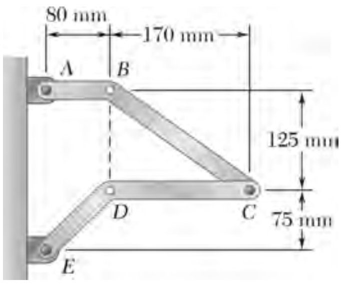
$$150 \text{ N} + E_y - 750 \text{ N} = 0$$

$$E_y = 600 \text{ N} \quad \mathbf{E}_y = 600 \text{ N}\uparrow$$

Thus, reactions are:

$$\mathbf{A}_x = 450 \text{ N} \leftarrow, \quad \mathbf{A}_y = 150.0 \text{ N}\uparrow \blacktriangleleft$$

$$\mathbf{E}_x = 450 \text{ N} \rightarrow, \quad \mathbf{E}_y = 600 \text{ N}\uparrow \blacktriangleleft$$



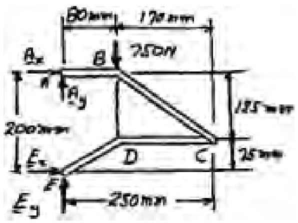
PROBLEM 6.84

Determine the components of the reactions at A and E if a 750-N force directed vertically downward is applied (a) at B, (b) at D.

SOLUTION

<u>Free body: Entire frame</u>

The following analysis is valid for both (a) and (b) since position of load on its line of action is immaterial.



$$+\circlearrowleft \Sigma M_E = 0: \quad -(750\text{ N})(80\text{ mm}) - A_x(200\text{ mm}) = 0$$

$$A_x = -300\text{ N} \quad \mathbf{A}_x = 300\text{ N} \leftarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad E_x - 300\text{ N} = 0 \quad E_x = 300\text{ N} \quad \mathbf{E}_x = 300\text{ N} \rightarrow$$

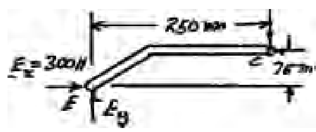
$$+\uparrow \Sigma F_y = 0: \quad A_y + E_y - 750\text{ N} = 0 \qquad (1)$$

(a) <u>Load applied at B.</u>

<u>Free body: Member CE</u>

CE is a two-force member. Thus, the reaction at E must be directed along CE.

$$\frac{E_y}{300\text{ N}} = \frac{75\text{ mm}}{250\text{ mm}} \quad E_y = 90\text{ N}\uparrow$$



From Eq. (1): $\quad A_y + 90\text{ N} - 750\text{ N} = 0 \qquad A_y = 660\text{ N}\uparrow$

Thus reactions are:

$$\mathbf{A}_x = 300\text{ N}\leftarrow, \qquad \mathbf{A}_y = 660\text{ N}\uparrow \blacktriangleleft$$

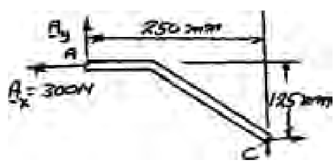
$$\mathbf{E}_x = 300\text{ N}\rightarrow, \qquad \mathbf{E}_y = 90.0\text{ N}\uparrow \blacktriangleleft$$

(b) <u>Load applied at D.</u>

<u>Free body: Member AC</u>

AC is a two-force member. Thus, the reaction at A must be directed along AC.

$$\frac{A_y}{300\text{ N}} = \frac{125\text{ mm}}{250\text{ mm}} \quad A_y = 150\text{ N}\uparrow$$



PROBLEM 6.84 (Continued)

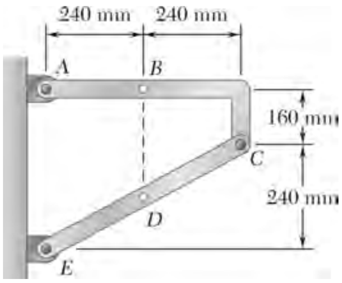
From Eq. (1): $A_y + E_y - 750 \text{ N} = 0$

$$150 \text{ N} + E_y - 750 \text{ N} = 0$$

$$E_y = 600 \text{ N} \quad \mathbf{E}_y = 600 \text{ N} \uparrow$$

Thus, reactions are: $\mathbf{A}_x = 300 \text{ N} \leftarrow, \quad \mathbf{A}_y = 150.0 \text{ N} \uparrow$ ◀

$\mathbf{E}_x = 300 \text{ N} \rightarrow, \quad \mathbf{E}_y = 600 \text{ N} \uparrow$ ◀



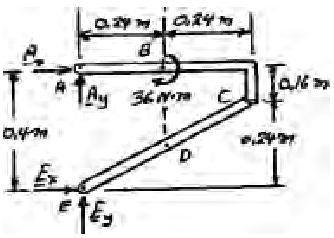
PROBLEM 6.85

Determine the components of the reactions at A and E if the frame is loaded by a clockwise couple of magnitude 36 N · m applied (a) at B, (b) at D.

SOLUTION

Free body: Entire frame

The following analysis is valid for both (a) and (b) since the point of application of the couple is immaterial.



$$\circlearrowleft + \Sigma M_E = 0: \quad -36\text{ N}\cdot\text{m} - A_x(0.4\text{ m}) = 0$$

$$A_x = -90\text{ N} \quad \mathbf{A}_x = 90.0\text{ N} \leftarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -90 + E_x = 0$$

$$E_x = 90\text{ N} \quad \mathbf{E}_x = 90.0\text{ N} \rightarrow$$

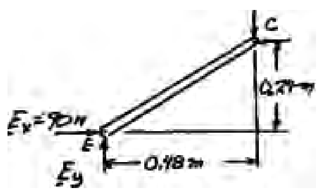
$$+\!\uparrow \Sigma F_y = 0: \quad A_y + E_y = 0 \qquad (1)$$

(a) Couple applied at B.

Free body: Member CE

AC is a two-force member. Thus, the reaction at E must be directed along EC.

$$\frac{E_y}{90\text{ N}} = \frac{0.24\text{ m}}{0.48\text{ m}}; \quad \mathbf{E}_y = 45.0\text{ N} \uparrow$$



From Eq. (1): $A_y + 45\text{ N} = 0$

$$A_y = -45\text{ N} \quad \mathbf{A}_y = 45.0\text{ N} \downarrow$$

Thus, reactions are

$$\mathbf{A}_x = 90.0\text{ N} \leftarrow, \quad \mathbf{A}_y = 45.0\text{ N} \downarrow \blacktriangleleft$$

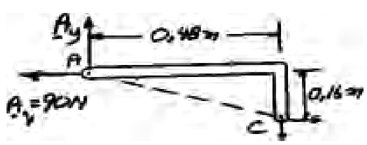
$$\mathbf{E}_x = 90.0\text{ N} \rightarrow, \quad \mathbf{E}_y = 45.0\text{ N} \uparrow \blacktriangleleft$$

(b) Couple applied at D.

Free body: Member AC

AC is a two-force member. Thus, the reaction at A must be directed along AC.

$$\frac{A_y}{90\text{ N}} = \frac{0.16\text{ m}}{0.48\text{ m}}; \quad \mathbf{A}_y = 30\text{ N} \uparrow$$



PROBLEM 6.85 (Continued)

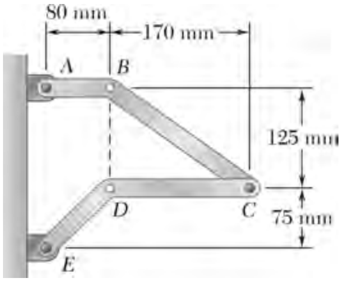
From Eq. (1): $A_y + E_y = 0$

$$30\text{ N} + E_y = 0$$

$$E_y = -30\text{ N} \quad \mathbf{E}_y = 30\text{ N}\downarrow$$

Thus, reactions are: $\mathbf{A}_x = 90.0\text{ N}\leftarrow, \quad \mathbf{A}_y = 30.0\text{ N}\uparrow$ ◀

$\mathbf{E}_x = -90.0\text{ N}\rightarrow, \quad \mathbf{E}_y = 30.0\text{ N}\downarrow$ ◀



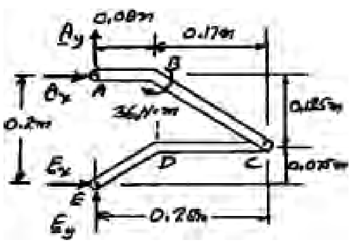
PROBLEM 6.86

Determine the components of the reactions at A and E if the frame is loaded by a clockwise couple of magnitude 36 N·m applied (a) at B, (b) at D.

SOLUTION

<u>Free body: Entire frame</u>

The following analysis is valid for both (a) and (b) since the point of application of the couple is immaterial.



$$+\circlearrowleft \Sigma M_E = 0: \quad -36\ \text{N}\cdot\text{m} - A_x(0.2\ \text{m}) = 0$$

$$A_x = -180\ \text{N} \qquad \mathbf{A}_x = 180\ \text{N} \leftarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -180\ \text{N} + E_x = 0$$

$$E_x = 180\ \text{N} \qquad \mathbf{E}_x = 180\ \text{N} \rightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + E_y = 0 \tag{1}$$

(a) <u>Couple applied at B.</u>

<u>Free body: Member CE</u>

AC is a two-force member. Thus, the reaction at E must be directed along EC.

$$\frac{E_y}{180\ \text{N}} = \frac{0.075\ \text{m}}{0.25\ \text{m}} \qquad \mathbf{E}_y = 54\ \text{N}\uparrow$$

From Eq. (1): $\qquad A_y + 54\ \text{N} = 0$

$$A_y = -54\ \text{N} \qquad \mathbf{A}_y = 54.0\ \text{N}\downarrow$$

Thus reactions are

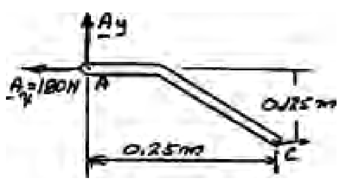
$$\mathbf{A}_x = 180.0\ \text{N}\leftarrow, \qquad \mathbf{A}_y = 54.0\ \text{N}\downarrow \blacktriangleleft$$

$$\mathbf{E}_x = 180.0\ \text{N}\rightarrow, \qquad \mathbf{E}_y = 54.0\ \text{N}\uparrow \blacktriangleleft$$

(b) <u>Couple applied at D.</u>

<u>Free body: Member AC</u>

AC is a two-force member. Thus, the reaction at A must be directed along EC.



$$\frac{A_y}{180\ \text{N}} = \frac{0.125\ \text{m}}{0.25\ \text{m}} \qquad A_y = 90\ \text{N}\uparrow$$

PROBLEM 6.86 (Continued)

From Eq. (1): $A_y + E_y = 0$

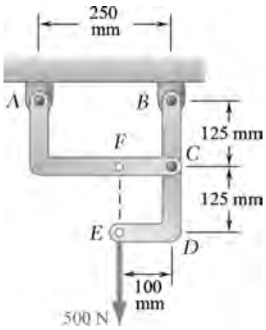
$$90\text{ N} + E_y = 0$$

$$E_y = -90\text{ N} \quad \mathbf{E}_y = 90\text{ N}\downarrow$$

Thus, reactions are

$$\mathbf{A}_x = 180.0\text{ N}\leftarrow, \qquad \mathbf{A}_y = 90.0\text{ N}\uparrow \blacktriangleleft$$

$$\mathbf{E}_x = -180.0\text{ N}\rightarrow, \qquad \mathbf{E}_y = 90.0\text{ N}\downarrow \blacktriangleleft$$

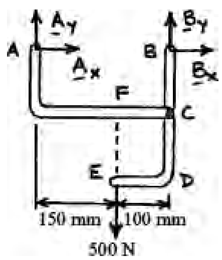


PROBLEM 6.87

Determine the components of the reactions at A and B, (a) if the 500-N load is applied as shown, (b) if the 500-N load is moved along its line of action and is applied at Point F.

SOLUTION

<u>Free body: Entire frame</u>



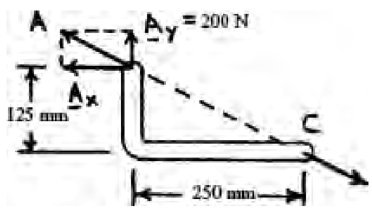
Analysis is valid for either (a) or (b), since position of 500 N load on its line of action is immaterial.

$$+\circlearrowleft \Sigma M_A = 0: \quad B_y(250) - (500 \text{ N})(150) = 0 \quad B_y = +300 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + 300 - 500 = 0 \qquad A_y = +200 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + B_x = 0 \tag{1}$$

(a) <u>Load applied at E.</u>
<u>Free body: Member AC</u>



Since AC is a two-force member, the reaction at A must be directed along CA. We have

$$\frac{A_x}{250 \text{ mm}} = \frac{200 \text{ N}}{125 \text{ mm}} \qquad \mathbf{A}_x = 400 \text{ N} \leftarrow, \qquad \mathbf{A}_y = 200 \text{ N} \uparrow ◀$$

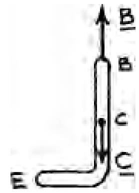
From Eq. (1): $\quad -400 + B_x = 0 \quad B_x = +400 \text{ N}$

Thus, $\qquad \mathbf{B}_x = 400 \text{ N} \rightarrow, \qquad \mathbf{B}_y = 300 \text{ N} \uparrow ◀$

PROBLEM 6.87 (Continued)

(*b*) Load applied at *F*.

Free body: Member *BCD*



Since *BCD* is a two-force member (with forces applied at *B* and *C* only), the reaction at *B* must be directed along *CB*. We have therefore

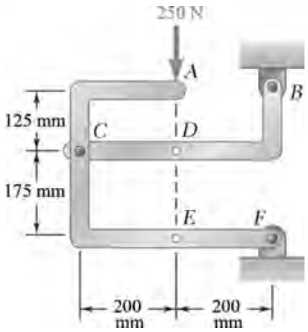
$$B_x = 0$$

The reaction at *B* is $\mathbf{B}_x = 0$ $\mathbf{B}_y = 300\ \text{N}\uparrow$ ◀

From Eq. (1): $A_x + 0 = 0 \quad A_x = 0$

The reaction at *A* is $\mathbf{A}_x = 0$ $\mathbf{A}_y = 200\ \text{N}\uparrow$ ◀

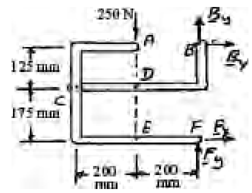
PROBLEM 6.88



The 250 N load can be moved along the line of action shown and applied at *A*, *D*, or *E*. Determine the components of the reactions at *B* and *F* if the 250 N load is applied (*a*) at *A*, (*b*) at *D*, (*c*) at *E*.

SOLUTION

Free body: Entire frame



The following analysis is valid for (*a*), (*b*) and (*c*) since the position of the load along its line of action is immaterial.

$$+\circlearrowleft \Sigma M_F = 0: \quad (250\text{ N})(200\text{ mm}) - B_x(300\text{ mm}) = 0$$

$$B_x = \frac{500}{3}\text{ N} = 166.67\text{ N} \qquad \mathbf{B}_x = 166.7\text{ N} \rightarrow$$

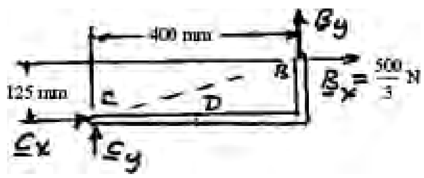
$$\xrightarrow{+} \Sigma F_x = 0: \quad 166.67\text{ N} + F_x = 0$$

$$F_x = -166.67\text{N} \qquad \mathbf{F}_x = 166.67\text{N} \leftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad B_y + F_y - 250\text{ N} = 0 \tag{1}$$

(*a*) Load applied at *A*.

Free body: Member *CDB*



CDB is a two-force member. Thus, the reaction at *B* must be directed along *BC*.

$$\frac{B_y}{500/3\text{ N}} = \frac{125\text{ mm}}{400\text{ mm}} \qquad \mathbf{B}_y = \frac{625}{12}\text{ N} = 52.083\text{ N}$$

$$\mathbf{B}_y = 52.1\text{ N} \uparrow$$

From Eq. (1): $\dfrac{625}{12}\text{ N} + F_y - 250\text{ N} = 0$

$$F_y = 197.92\text{ N} \qquad \mathbf{F}_y = 197.9\text{ N} \uparrow$$

Thus reactions are:

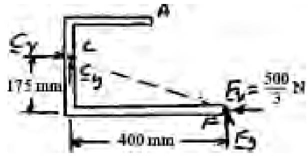
$$\mathbf{B}_x = 166.7\text{ N} \rightarrow, \qquad \mathbf{B}_y = 52.1\text{ N} \uparrow \blacktriangleleft$$

$$\mathbf{F}_x = 166.7\text{ N} \leftarrow, \qquad \mathbf{F}_y = 197.9\text{ N} \uparrow \blacktriangleleft$$

PROBLEM 6.88 (Continued)

(*b*) Load applied at *D*.

Free body: Member *ACF*.



ACF is a two-force member. Thus, the reaction at *F* must be directed along *CF*.

$$\frac{F_y}{500/3\text{ N}} = \frac{175\text{ mm}}{400\text{ mm}} \qquad \mathbf{F}_y = \frac{875}{12}\text{ N} = 72.92\text{ N}\uparrow$$

From Eq. (1): $\qquad B_y + \frac{875}{12}\text{ N} - 250\text{ N} = 0$

$$B_y = 177.083\text{ N} \quad B_y = 177.1\text{ N}\uparrow$$

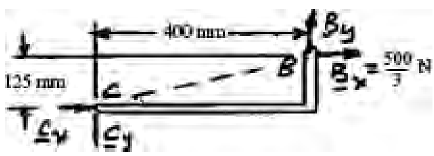
Thus, reactions are:

$$\mathbf{B}_x = 166.7\text{ N}\leftarrow, \qquad \mathbf{B}_y = 177.1\text{ N}\uparrow \blacktriangleleft$$

$$\mathbf{F}_x = 166.7\text{ N}\rightarrow, \qquad \mathbf{F}_y = 72.9\text{ N}\uparrow \blacktriangleleft$$

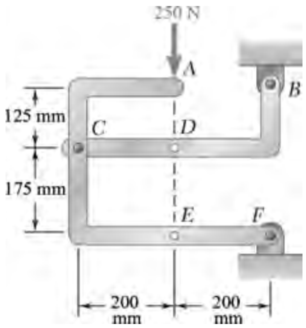
(*c*) Load applied at *E*.

Free body: Member *CDB*



This is the same free body as in Part (*a*).

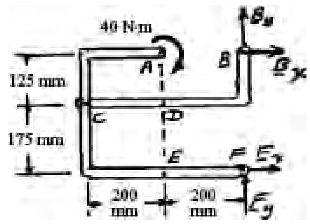
Reactions are same as (*a*) ◀



PROBLEM 6.89

The 250-N load is removed and a 40 N·m. clockwise couple is applied successively at A, D, and E. Determine the components of the reactions at B and F if the couple is applied (a) at A, (b) at D, (c) at E.

SOLUTION



Free body: Entire frame

The following analysis is valid for (a), (b), and (c), since the point of application of the couple is immaterial.

$$+\circlearrowleft \Sigma M_F = 0: \quad -40 \text{ N}\cdot\text{m} - B_x(0.3 \text{ m}) = 0$$

$$B_x = -\frac{400}{3} \text{ N} \qquad \mathbf{B}_x = 133.33 \text{ N} \leftarrow$$

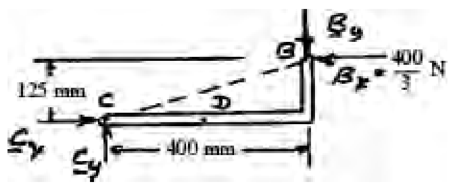
$$\xrightarrow{+} \Sigma F_x = 0: \quad -400/3 \text{ N} + F_x = 0$$

$$F_x = \frac{400}{3} \text{ N} \qquad \mathbf{F}_x = 133.33 \text{ N} \rightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad B_y + F_y = 0 \tag{1}$$

(a) Couple applied at A.

Free body: Member CDB



CDB is a two-force member. Thus, reaction at B must be directed along BC.

$$\frac{B_y}{400/3 \text{ N}} = \frac{125 \text{ mm}}{400 \text{ mm}} \qquad \mathbf{B}_y = \frac{125}{3} \text{ N} = 41.667 \text{ N} \downarrow$$

From Eq. (1): $\quad -\frac{125}{3} \text{ N} + F_y = 0$

$$F_y = 125/3 \text{ N} \qquad \mathbf{F}_y = 41.667 \text{ N} \uparrow$$

Thus, reactions are:

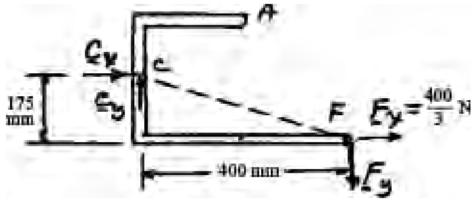
$$\mathbf{B}_x = 133.3 \text{ N} \leftarrow, \qquad \mathbf{B}_y = 41.7 \text{ N} \downarrow \blacktriangleleft$$

$$\mathbf{F}_x = 133.3 \text{ N} \rightarrow, \qquad \mathbf{F}_y = 41.7 \text{ N} \uparrow \blacktriangleleft$$

PROBLEM 6.89 (Continued)

(*b*) Couple applied at *D*.

Free body: Member *ACF*.



ACF is a two-force member. Thus, the reaction at *F* must be directed along *CF*.

$$\frac{F_y}{\frac{400}{3}\text{ N}} = \frac{175\text{ mm}}{400\text{ mm}} \qquad \mathbf{F}_y = \frac{175}{3}\text{ N} = 58.333\text{ N}\downarrow$$

From Eq. (1): $\quad B_y - \frac{175}{3}\text{ N} = 0$

$$B_y = +\frac{175}{3}\text{ N} \qquad \mathbf{B}_y = 58.333\text{ N}\uparrow$$

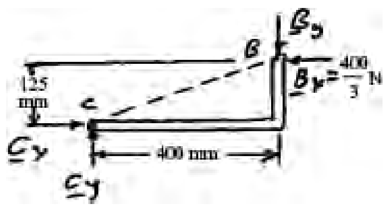
Thus, reactions are:

$$\mathbf{B}_x = 133.3\text{ N}\leftarrow, \qquad \mathbf{B}_y = 58.30\text{ N}\uparrow \blacktriangleleft$$

$$\mathbf{F}_x = 133.3\text{ N}\rightarrow, \qquad \mathbf{F}_y = 58.3\text{ N}\downarrow \blacktriangleleft$$

(*c*) Couple applied at *E*.

Free body: Member *CDB*

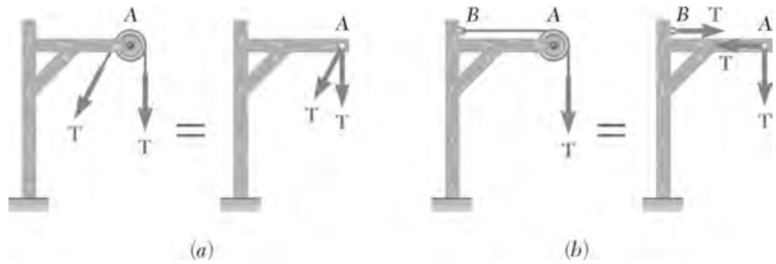


This is the same free body as in Part (*a*).

Reactions are same as in (*a*) ◀

PROBLEM 6.90

(*a*) Show that when a frame supports a pulley at *A*, an equivalent loading of the frame and of each of its component parts can be obtained by removing the pulley and applying at *A* two forces equal and parallel to the forces that the cable exerted on the pulley. (*b*) Show that if one end of the cable is attached to the frame at a point *B*, a force of magnitude equal to the tension in the cable should also be applied at *B*.

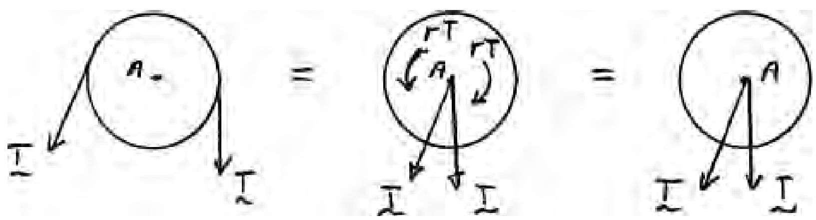


SOLUTION

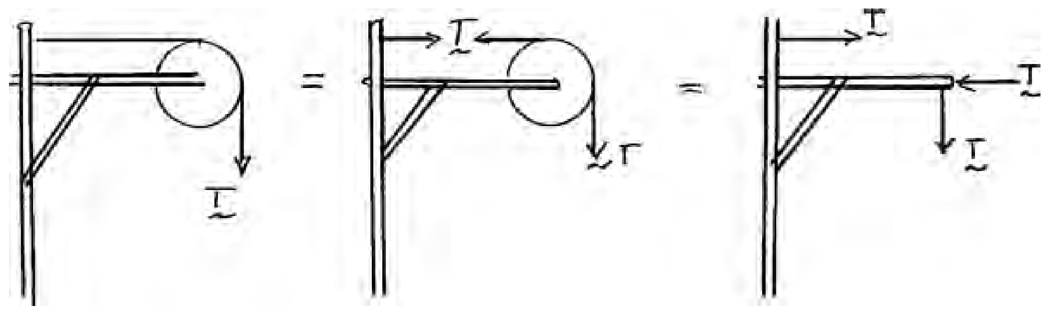
First note that, when a cable or cord passes over a *frictionless*, *motionless* pulley, the tension is unchanged.

$$\circlearrowleft \Sigma M_C = 0: \quad rT_1 - rT_2 = 0 \quad T_1 = T_2$$

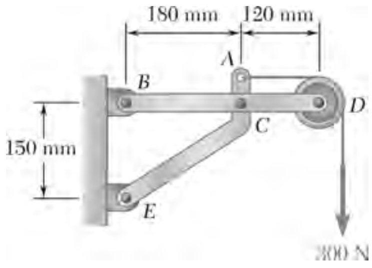
(*a*) Replace each force with an equivalent force-couple.



(*b*) Cut cable and replace forces on pulley with equivalent pair of forces at *A* as above.



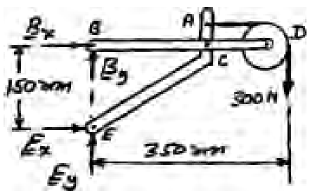
PROBLEM 6.91



Knowing that the pulley has a radius of 50 mm, determine the components of the reactions at *B* and *E*.

SOLUTION

Free body: Entire assembly



$$+\circlearrowleft \Sigma M_E = 0: \quad -(300 \text{ N})(350 \text{ mm}) - B_x(150 \text{ mm}) = 0$$

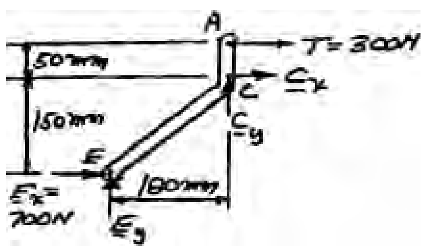
$$B_x = -700 \text{ N} \qquad \mathbf{B}_x = 700 \text{ N} \leftarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -700 \text{N} + E_x = 0$$

$$E_x = 700 \text{ N} \qquad \mathbf{E}_x = 700 \text{ N} \rightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad B_y + E_y - 300 \text{ N} = 0 \qquad (1)$$

Free body: Member *ACE*



$$+\circlearrowleft \Sigma M_C = 0: \quad (700 \text{ N})(150 \text{ mm}) - (300 \text{ N})(50 \text{ mm}) - E_y(180 \text{ mm}) = 0$$

$$E_y = 500 \text{ N} \qquad \mathbf{E}_y = 500 \text{ N} \uparrow$$

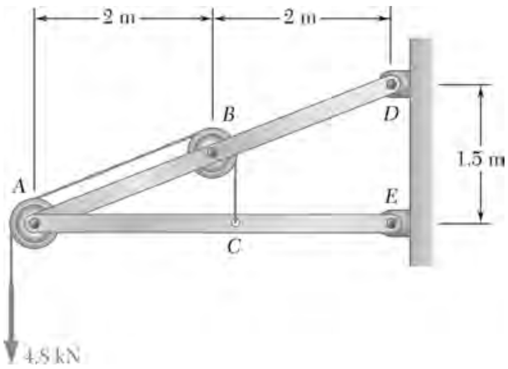
From Eq. (1): $B_y + 500 \text{ N} - 300 \text{ N} = 0$

$$B_y = -200 \text{ N} \qquad \mathbf{B}_y = 200 \text{ N} \downarrow$$

Thus, reactions are:

$$\mathbf{B}_x = 700 \text{ N} \leftarrow, \qquad \mathbf{B}_y = 200 \text{ N} \downarrow \blacktriangleleft$$

$$\mathbf{E}_x = 700 \text{ N} \rightarrow, \qquad \mathbf{E}_y = 500 \text{ N} \uparrow \blacktriangleleft$$

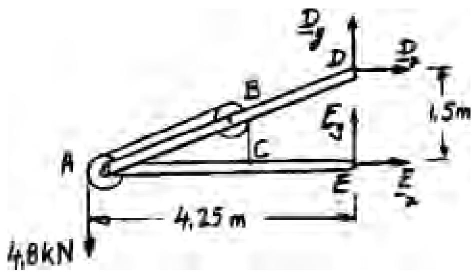


PROBLEM 6.92

Knowing that each pulley has a radius of 250 mm, determine the components of the reactions at D and E.

SOLUTION

<u>Free body: Entire assembly</u>



$$+\circlearrowleft \Sigma M_E = 0: \quad (4.8 \text{ kN})(4.25 \text{ m}) - D_x(1.5 \text{ m}) = 0$$

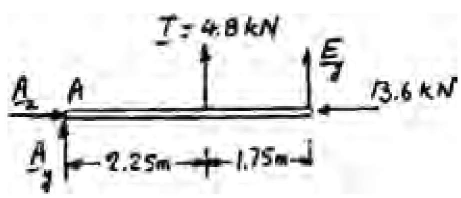
$$D_x = +13.60 \text{ kN} \qquad \mathbf{D}_x = 13.60 \text{ kN} \rightarrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad E_x + 13.60 \text{ kN} = 0$$

$$E_x = -13.60 \text{ kN} \qquad \mathbf{E}_x = 13.60 \text{ kN} \leftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad D_y + E_y - 4.8 \text{ kN} = 0 \qquad (1)$$

<u>Free body: Member *ACE*</u>



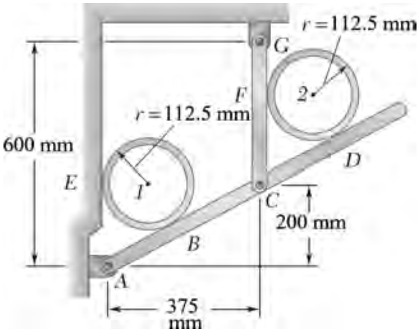
$$+\circlearrowleft \Sigma M_A = 0: \quad (4.8 \text{ kN})(2.25 \text{ m}) + E_y(4 \text{ m}) = 0$$

$$E_y = -2.70 \text{ kN} \qquad \mathbf{E}_y = 2.70 \text{ kN} \downarrow \blacktriangleleft$$

From Eq. (1):

$$D_y - 2.70 \text{ kN} - 4.80 \text{ kN} = 0$$

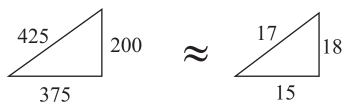
$$D_y = +7.50 \text{ kN} \qquad \mathbf{D}_y = 7.50 \text{ kN} \uparrow \blacktriangleleft$$



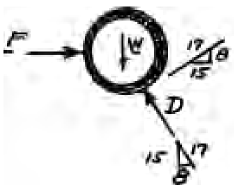
PROBLEM 6.93

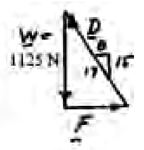
Two 225-mm diameter pipes (pipe *1* and pipe *2*) are supported every 2.5 m by a small frame like that shown. Knowing that the combined weight of each pipe and its contents is 450 N/m and assuming frictionless surfaces, determine the components of the reactions at *A* and *G*.

SOLUTION



Free-body: Pipe 2





$$W = (450\text{ N/m})(2.5\text{ m}) = 1125\text{ N}$$

$$\frac{F}{8} = \frac{D}{17} = \frac{1125\text{ N}}{15}$$

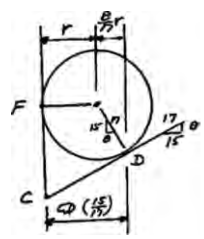
$$\mathbf{F} = 600\text{ N} \rightarrow$$

$$\mathbf{D} = 1275\text{ N}$$

Geometry of pipe 2 $\qquad r = 112.5\text{ mm}$

By symmetry: $\qquad CF = CD \qquad (1)$

Equate horizontal distance:



$$r + \frac{8}{17}r = CD\left(\frac{15}{17}\right)$$

$$\frac{25}{17}r = CD\left(\frac{15}{17}\right)$$

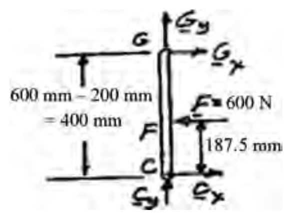
$$CD = \frac{25}{15}r = \frac{5}{3}r$$

From Eq. (1):

$$CF = \frac{5}{3}r = \frac{5}{3}(112.5\text{ mm})$$

$$CF = 187.5\text{ mm}$$

Free-body: Member *CFG*

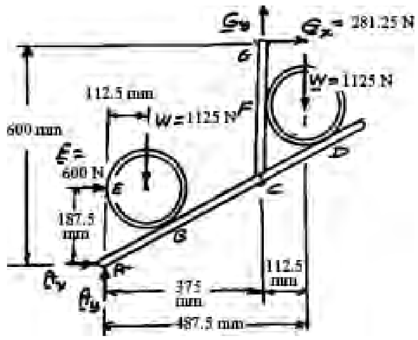


$$+\circlearrowleft \Sigma M_C = 0: \quad (600\text{ N})(187.5\text{ mm}) - G_x(400\text{ mm}) = 0$$

$$G_x = 281.25\text{ N} \qquad \mathbf{G}_x = 281\text{ N} \rightarrow \blacktriangleleft$$

PROBLEM 6.93 (Continued)

Free body: Frame and pipes



Note: Pipe 2 is similar to pipe 1.

$$AE = CF = 187.5 \text{ mm}$$
$$\mathbf{E} = \mathbf{F} = 600 \text{ N}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad G_y(375 \text{ mm}) - (281.25 \text{ N})(600 \text{ mm}) - (1125 \text{ N})(112.5 \text{ mm})$$
$$-(1125 \text{ N})(487.5 \text{ mm}) - (600 \text{ N})(187.5 \text{ mm}) = 0$$

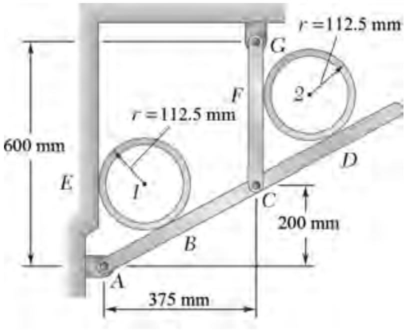
$$G_y = 2550 \text{ N} \qquad \mathbf{G}_y = 2550 \text{ N}\uparrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + 600 \text{ N} + 281.25 \text{ N} = 0$$

$$A_x = -881.25 \text{ N} \qquad \mathbf{A}_x = 881 \text{ N} \leftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad A_y + 2550 \text{ N} - 1125 \text{ N} - 1125 \text{ N} = 0$$

$$A_y = -300 \text{ N} \qquad \mathbf{A}_y = 300 \text{ N}\downarrow \blacktriangleleft$$



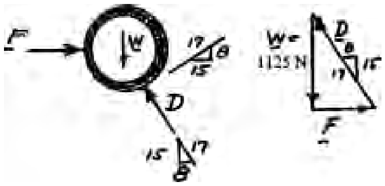
PROBLEM 6.94

Solve Problem 6.93 assuming that pipe *1* is removed and that only pipe *2* is supported by the frames.

PROBLEM 6.93 Two 225-mm diameter pipes (pipe *1* and pipe *2*) are supported every 2.5 m by a small frame like that shown. Knowing that the combined weight of each pipe and its contents is 450 N/m and assuming frictionless surfaces, determine the components of the reactions at *A* and *G*.

SOLUTION

Free-body: Pipe 2



$$W = (450 \text{ N/m})(2.5 \text{ m}) = 1125 \text{ N}$$

$$\frac{F}{8} = \frac{D}{17} = \frac{1125 \text{ N}}{15}$$

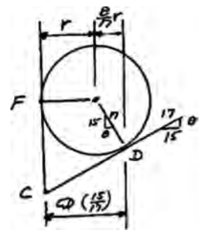
$$\mathbf{F} = 600 \text{ N} \rightarrow$$

$$\mathbf{D} = 1275 \text{ N}$$

Geometry of pipe 2 $\qquad r = 112.5$ mm

By symmetry: $\qquad CF = CD \qquad (1)$

Equate horizontal distance:



$$r + \frac{8}{17}r = CD\left(\frac{15}{17}\right)$$

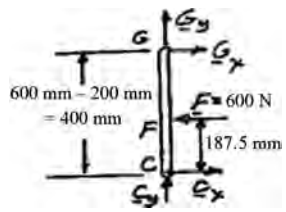
$$\frac{25}{17}r = CD\left(\frac{15}{17}\right)$$

$$CD = \frac{25}{15}r = \frac{5}{3}r$$

From Eq. (1):

$$CF = \frac{5}{3}r = \frac{5}{3}(112.5 \text{ mm})$$

$$CF = 187.5 \text{ mm}$$



Free-body: Member *CFG*

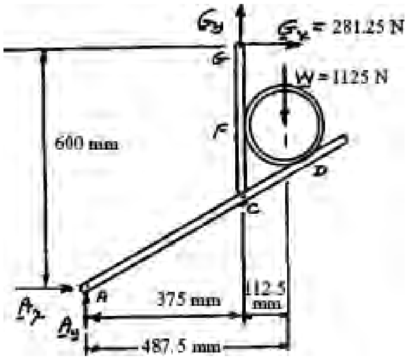
$$+\circlearrowleft \Sigma M_C = 0: \quad (600 \text{ N})(187.5 \text{ mm}) - G_x(400 \text{ mm}) = 0$$

$$G_x = 281.25 \text{ N} \qquad \mathbf{G}_x = 281 \text{ N} \rightarrow \blacktriangleleft$$

PROBLEM 6.94 (Continued)

Free body: Frame and pipe 2

$$\mathbf{G}_x = 281 \text{ N} \rightarrow \blacktriangleleft$$



$+\circlearrowleft \Sigma M_A = 0$: $\quad G_y(375 \text{ mm}) - (281.25 \text{ N})(600 \text{ mm}) - (1125 \text{ N})(487.5 \text{ mm}) = 0$

$$G_y = 1912.5 \text{ N} \qquad \mathbf{G}_y = 1913 \text{ N} \uparrow \blacktriangleleft$$

$\xrightarrow{+} \Sigma F_x = 0$: $\quad A_x + 281.25 \text{ N} = 0$

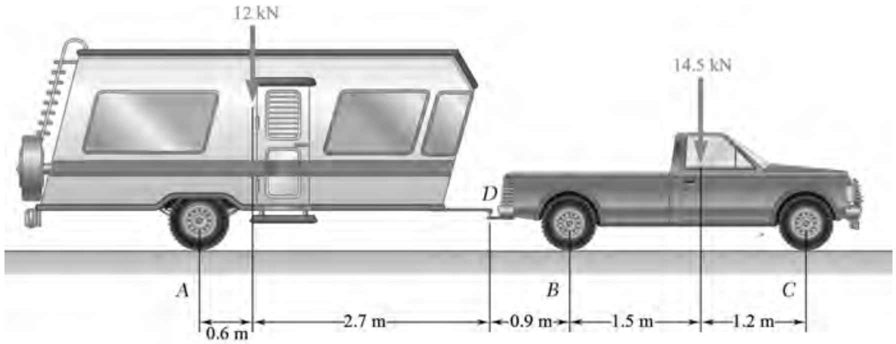
$$A_x = -281.25 \text{ N} \qquad \mathbf{A}_x = 281 \text{ N} \leftarrow \blacktriangleleft$$

$+\uparrow \Sigma F_y = 0$: $\quad A_y + 1912.5 \text{ N} - 1125 \text{ N} = 0$

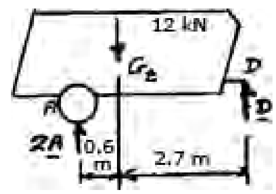
$$A_y = -787.5 \text{ N} \qquad \mathbf{A}_y = 788 \text{ N} \downarrow \blacktriangleleft$$

PROBLEM 6.95

A trailer weighing 12 kN is attached to a 14.5 kN pickup truck by a ball-and-socket truck hitch at *D*. Determine (*a*) the reactions at each of the six wheels when the truck and trailer are at rest, (*b*) the additional load on each of the truck wheels due to the trailer.



SOLUTION



(*a*) Free body: Trailer

(We shall denote by *A*, *B*, *C* the reaction at one wheel)

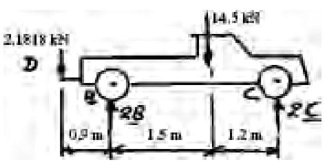
$$\circlearrowleft + \Sigma M_A = 0: \quad -(12 \text{ kN})(0.6 \text{ m}) + D(3.3 \text{ m}) = 0$$

$$D = 2.1818 \text{ kN}$$

$$+\uparrow \Sigma F_y = 0: \quad 2A - 12 \text{ kN} + 2.1818 \text{ kN} = 0$$

$$A = 4.9091 \text{ kN} \qquad \mathbf{A} = 4.91 \text{ kN} \uparrow ◀$$

Free body: Truck



$$\circlearrowleft + \Sigma M_B = 0: \quad (2.1818 \text{ kN})(0.9 \text{ m}) - (14.5 \text{ kN})(1.5 \text{ m}) + 2C(2.7 \text{ m}) = 0$$

$$C = 3.6641 \text{ kN} \qquad \mathbf{C} = 3.66 \text{ kN} \uparrow ◀$$

$$+\uparrow \Sigma F_y = 0: \quad 2B - 2.1818 \text{ kN} - 14.5 \text{ kN} + 2(3.6641 \text{ kN}) = 0$$

$$B = 4.6768 \text{ kN} \qquad \mathbf{B} = 4.68 \text{ kN} \uparrow ◀$$

(*b*) Additional load on truck wheels

Use free body diagram of truck without 14.5 kN

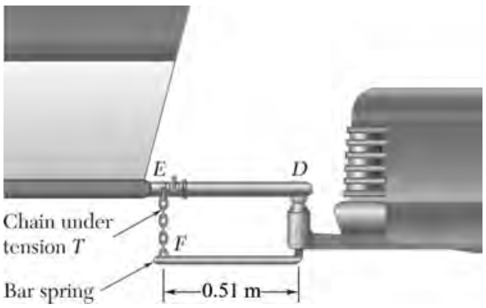
$$\circlearrowleft + \Sigma M_B = 0: \quad (2.1818 \text{ kN})(0.9 \text{ m}) + 2C(2.7 \text{ m}) = 0$$

$$C = -0.3636 \text{ kN} \qquad \Delta C = -0.364 \text{ kN} ◀$$

$$+\uparrow \Sigma F_y = 0: \quad 2B - 2.1818 \text{ N} - 2(0.3636 \text{ kN}) = 0$$

$$B = 1.4545 \text{ kN} \qquad \Delta B = +1.455 \text{ kN} ◀$$

PROBLEM 6.96

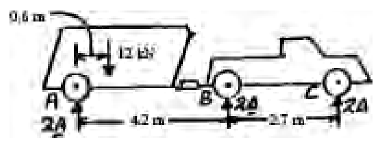


In order to obtain a better weight distribution over the four wheels of the pickup truck of Problem 6.95, a compensating hitch of the type shown is used to attach the trailer to the truck. The hitch consists of two bar springs (only one is shown in the figure) that fit into bearings inside a support rigidly attached to the truck. The springs are also connected by chains to the trailer frame, and specially designed hooks make it possible to place both chains in tension. (*a*) Determine the tension T required in each of the two chains if the additional load due to the trailer is to be evenly distributed over the four wheels of the truck. (*b*) What are the resulting reactions at each of the six wheels of the trailer-truck combination?

PROBLEM 6.95 A trailer weighing 12 kN is attached to a 14.5 kN pickup truck by a ball-and-socket truck hitch at D. Determine (*a*) the reactions at each of the six wheels when the truck and trailer are at rest, (*b*) the additional load on each of the truck wheels due to the trailer.

SOLUTION

(*a*) We shall first find the additional reaction Δ at each wheel due the trailer.



Free body diagram (Same Δ at each truck wheel)

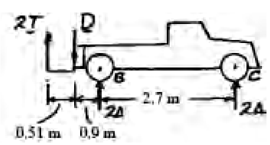
$$\circlearrowleft^{+} \Sigma M_A = 0: \quad -(12\text{ kN})(0.6\text{ m}) + 2\Delta(4.2\text{ m}) + 2\Delta(6.9\text{ m}) = 0$$

$$\Delta = 0.32432\text{ kN}$$

$$+\uparrow \Sigma F_Y = 0: \quad 2A - 12\text{ kN} + 4(0.32432\text{ kN}) = 0;$$

$$A = 5.3514\text{ kN}; \qquad \mathbf{A} = 5.35\text{ kN} \uparrow$$

Free body: Truck
(Trailer loading only)



$$\circlearrowleft^{+} \Sigma M_D = 0: \quad 2\Delta(3.6\text{ m}) + 2\Delta(0.9\text{ m}) - 2T(0.51\text{ m}) = 0$$

$$T = 8.824\Delta$$

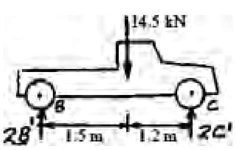
$$= 8.824(0.32432\text{ kN})$$

$$T = 2.8616\text{ kN}$$

$$T = 2.86\text{ kN} \blacktriangleleft$$

Free body: Truck

(Truck weight only)



$$\circlearrowleft^{+} \Sigma M_B = 0: \quad -(14.5\text{ kN})(1.5\text{ m}) + 2C'(2.7\text{ m}) = 0$$

$$C' = 4.02778\text{ kN} \qquad \mathbf{C}' = 4.03\text{ kN} \uparrow$$

PROBLEM 6.96 (Continued)

$$+\uparrow \Sigma F_y = 0: \quad 2B' - 14.5 \text{ kN} + 2(4.02778 \text{ kN}) = 0$$

$$B' = 3.2222 \text{ kN} \qquad \mathbf{B'} = 3.22 \text{ kN}\uparrow$$

<u>Actual reactions</u>

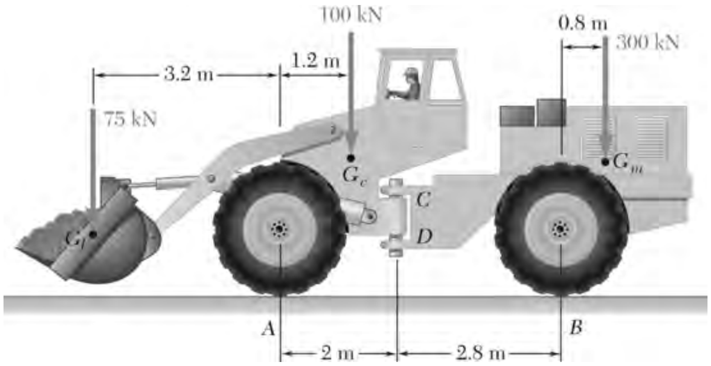
$$B = B' + \Delta = 3.2222 \text{ kN} + 0.32432 \text{ kN} = 3.5465 \text{ kN} \qquad \mathbf{B} = 3.55 \text{ kN}\uparrow \blacktriangleleft$$

$$C = C' + \Delta = 4.02778 \text{ kN} + 0.32432 \text{ kN} = 4.3521 \text{ kN} \qquad \mathbf{C} = 4.35 \text{ kN}\uparrow \blacktriangleleft$$

(From Part *a*): $\mathbf{A} = 5.35 \text{ kN}\uparrow \blacktriangleleft$

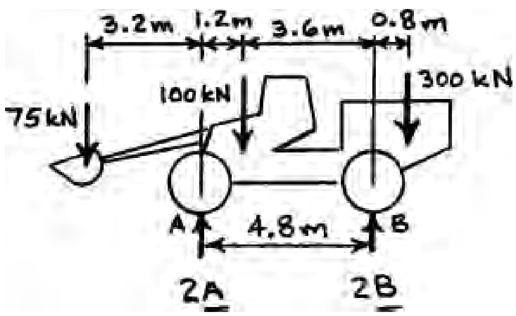
PROBLEM 6.97

The cab and motor units of the front-end loader shown are connected by a vertical pin located 2 m behind the cab wheels. The distance from C to D is 1 m. The center of gravity of the 300 kN motor unit is located at G_m, while the centers of gravity of the 100 kN cab and 75 kN load are located, respectively, at G_c and G_l. Knowing that the machine is at rest with its brakes released, determine (*a*) the reactions at each of the four wheels, (*b*) the forces exerted on the motor unit at C and D.



SOLUTION

(*a*) Free body: Entire machine



A = Reaction at each front wheel

B = Reaction at each rear wheel

$$+\circlearrowleft \Sigma M_A = 0: \quad 75(3.2 \text{ m}) - 100(1.2 \text{ m}) + 2B(4.8 \text{ m}) - 300(5.6 \text{ m}) = 0$$

$$2B = 325 \text{ kN} \qquad \mathbf{B} = 162.5 \text{ kN}\uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 2A + 325 - 75 - 100 - 300 = 0$$

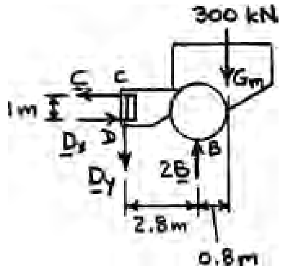
$$2A = 150 \text{ kN} \qquad \mathbf{A} = 75.0 \text{ kN}\uparrow \blacktriangleleft$$

PROBLEM 6.97 (Continued)

(*b*) Free body: Motor unit

$$\stackrel{+}{\circlearrowleft} \Sigma M_D = 0: \quad C(1\text{ m}) + 2B(2.8\text{ m}) - 300(3.6\text{ m}) = 0$$

$$C = 1080 - 5.6B \qquad (1)$$



Recalling $B = 162.5\text{ kN}, \quad C = 1080 - 5.6(162.5) = 170\text{ kN}$

$$\mathbf{C} = 170.0\text{ kN} \leftarrow \blacktriangleleft$$

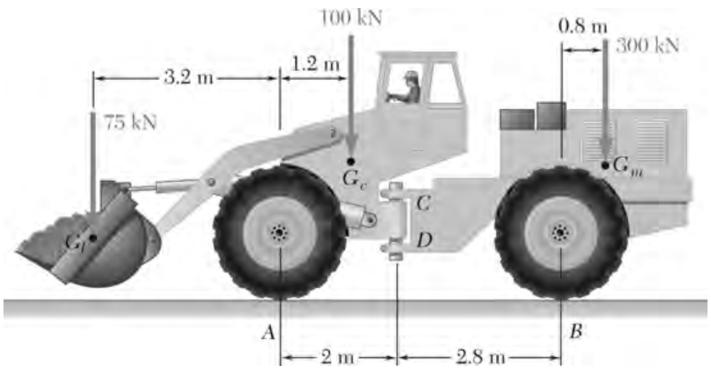
$$\xrightarrow{+} \Sigma F_x = 0: \quad D_x - 170 = 0 \qquad \mathbf{D}_x = 170.0\text{ kN} \rightarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 2(162.5) - D_y - 300 = 0 \qquad \mathbf{D}_y = 25.0\text{ kN} \downarrow \blacktriangleleft$$

PROBLEM 6.98

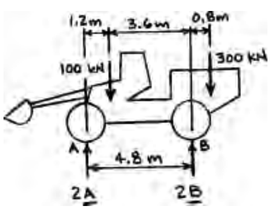
Solve Problem 6.97 assuming that the 75 kN load has been removed.

PROBLEM 6.97 The cab and motor units of the front-end loader shown are connected by a vertical pin located 2 m behind the cab wheels. The distance from C to D is 1 m. The center of gravity of the 300-kN motor unit is located at G_m, while the centers of gravity of the 100-kN cab and 75-kN load are located, respectively, at G_c and G_l. Knowing that the machine is at rest with its brakes released, determine (*a*) the reactions at each of the four wheels, (*b*) the forces exerted on the motor unit at C and D.



SOLUTION

(*a*) Free body: Entire machine



A = Reaction at each front wheel

B = Reaction at each rear wheel

$$+\circlearrowleft \Sigma M_A = 0: \quad 2B(4.8 \text{ m}) - 100(1.2 \text{ m}) - 300(5.6 \text{ m}) = 0$$

$$2B = 375 \text{ kN} \qquad \mathbf{B} = 187.5 \text{ kN} \uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 2A + 375 - 100 - 300 = 0$$

$$2A = 25 \text{ kN} \qquad \mathbf{A} = 12.50 \text{ kN} \uparrow \blacktriangleleft$$

(*b*) Free body: Motor unit

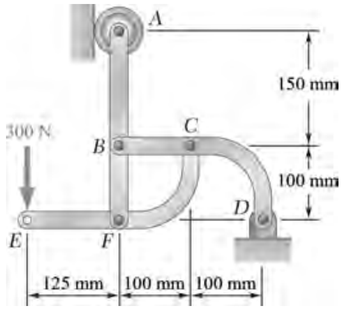
See solution of Problem 6.97 for free body diagram and derivation of Eq. (1). With $B = 187.5$ kN, we have

$$C = 1080 - 5.6(187.5) = 30 \text{ kN}$$

$$\mathbf{C} = 30.0 \text{ kN} \leftarrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad D_x - 30 = 0 \qquad \mathbf{D}_x = 30.0 \text{ kN} \rightarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad 2(187.5) - D_y - 300 = 0 \qquad \mathbf{D}_y = 75.0 \text{ kN} \downarrow \blacktriangleleft$$

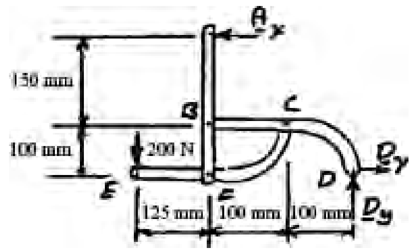


PROBLEM 6.99

For the frame and loading shown, determine the components of the forces acting on member *CFE* at *C* and *F*.

SOLUTION

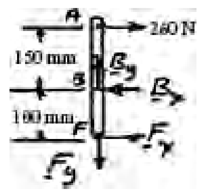
Free body: Entire frame



$$\circlearrowleft \Sigma M_D = 0: \quad (200 \text{ N})(325 \text{ mm}) + A_x(250 \text{ mm}) = 0$$

$$A_x = -260 \text{ N}, \qquad \mathbf{A}_x = 260 \text{ N} \rightarrow$$

Free body: Member *ABF*

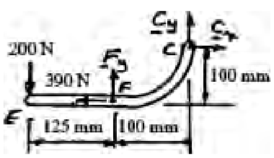


$$\circlearrowleft \Sigma M_B = 0: \quad -(260 \text{ N})(150 \text{ mm}) + F_x(100 \text{ mm}) = 0$$

$$F_x = +390 \text{ N}$$

Free body: Member *CFE*

From above: $\mathbf{F}_x = 390 \text{ N} \leftarrow$ ◀



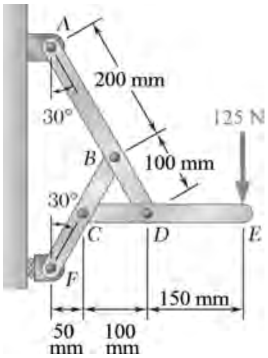
$$\circlearrowleft \Sigma M_C = 0: \quad (200 \text{ N})(225 \text{ mm}) - (390 \text{ N})(100 \text{ mm}) - F_y(100 \text{ mm}) = 0$$

$$F_y = +60 \text{ N} \qquad \mathbf{F}_y = 60.0 \text{ N} \uparrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - 390 \text{ N} = 0$$

$$C_x = +390 \text{ N} \qquad \mathbf{C}_x = 390 \text{ N} \rightarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -200 \text{ N} + 60 \text{ N} + C_y = 0; \quad C_y = +140 \text{ N} \qquad \mathbf{C}_y = 140.0 \text{ N} \uparrow \blacktriangleleft$$

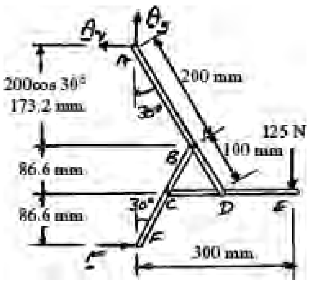


PROBLEM 6.100

For the frame and loading shown, determine the components of the forces acting on member *CDE* at *C* and *D*.

SOLUTION

<u>Free body: Entire frame</u>



$$+\uparrow \Sigma M_y = 0: \quad A_y - 125 \text{ N} = 0$$

$$A_y = 125 \text{ N} \qquad \mathbf{A}_y = 125.0 \text{ N} \uparrow$$

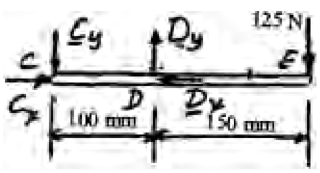
$$+\circlearrowleft \Sigma M_F = 0: \quad A_x(173.2 + 2\times 86.6) - (125 \text{ N})(300 \text{ mm}) = 0$$

$$A_x = 108.256 \text{ N} \qquad \mathbf{A}_x = 108.3 \text{ N} \leftarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad F - 108.256 \text{ N} = 0$$

$$F = 108.256 \text{ N} \qquad \mathbf{F} = 108.3 \text{ N} \rightarrow$$

<u>Free body: Member *CDE*</u>



$$+\circlearrowleft \Sigma M_C = 0: \quad D_y(100 \text{ mm}) - (125 \text{ N})(250 \text{ mm}) = 0$$

$$D_y = +312.5 \text{ N} \qquad \mathbf{D}_y = 313 \text{ N} \uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -C_y + 312.5 \text{ N} - 125 \text{ N} = 0$$

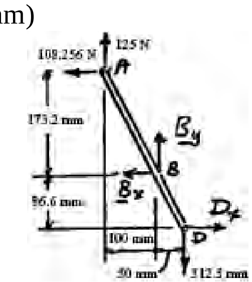
$$C_y = +187.5 \text{ N} \qquad \mathbf{C}_y = 187.5 \text{ N} \downarrow \blacktriangleleft$$

<u>Free body: Member *ABD*</u>

$$+\circlearrowleft \Sigma M_B = 0: \quad D_x(86.66 \text{ mm}) + (108.256 \text{ N})(173.2 \text{ mm})$$

$$-(125 \text{ N})(100 \text{ mm}) - (312.5 \text{ N})(50 \text{ mm})$$

$$D_x = +108.182 \text{ N}$$



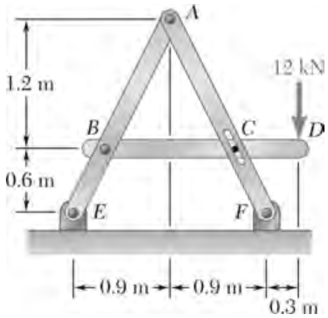
<u>Return to free body: Member *CDE*</u>

From above

$$D_x = +108.182 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - 108.182 \text{ N} \qquad \mathbf{D}_x = 108.2 \text{ N} \blacktriangleleft$$

$$C_x = +108.182 \text{ N} \qquad \mathbf{C}_x = 108.2 \text{ N} \rightarrow \blacktriangleleft$$

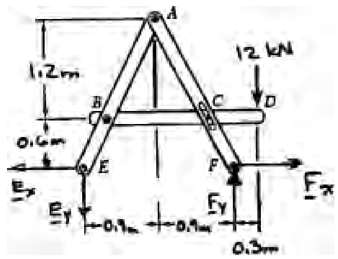


PROBLEM 6.101

For the frame and loading shown, determine the components of all forces acting on member *ABE*.

SOLUTION

FBD Frame:



$$\circlearrowleft \Sigma M_E = 0: \quad (1.8\text{ m})F_y - (2.1\text{ m})(12\text{ kN}) = 0$$

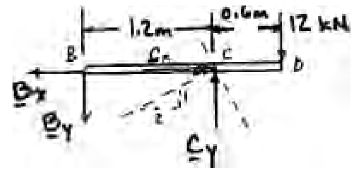
$$\mathbf{F}_y = 14.00\text{ kN}\uparrow$$

$$\uparrow \Sigma F_y = 0: \quad -E_y + 14.00\text{ kN} - 12\text{ kN} = 0$$

$$E_y = 2\text{ kN}\downarrow$$

$$\mathbf{E}_y = 2.00\text{ kN}\downarrow \blacktriangleleft$$

FBD member *BCD*:



$$\circlearrowleft \Sigma M_B = 0: \quad (1.2\text{ m})C_y - (12\text{ kN})(1.8\text{ m}) = 0 \quad C_y = 18.00\text{ kN}\uparrow$$

But **C** is $\perp ACF$, so $C_x = 2C_y$; $\quad C_x = 36.0\text{ kN}\rightarrow$

$$\rightarrow \Sigma F_x = 0: \quad -B_x + C_x = 0 \quad B_x = C_x = 36.0\text{ kN}$$

$$B_x = 36.0\text{ kN}\leftarrow \text{ on } BCD$$

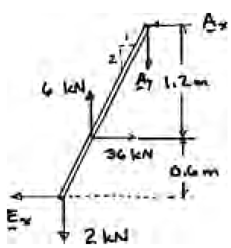
$$\uparrow \Sigma F_y = 0: \quad -B_y + 18.00\text{ kN} - 12\text{ kN} = 0 \quad B_y = 6.00\text{ kN}\downarrow \text{ on } BCD$$

on *ABE*:

$$\mathbf{B}_x = 36.0\text{ kN}\rightarrow \blacktriangleleft$$

$$\mathbf{B}_y = 6.00\text{ kN}\uparrow \blacktriangleleft$$

FBD member *ABE*:

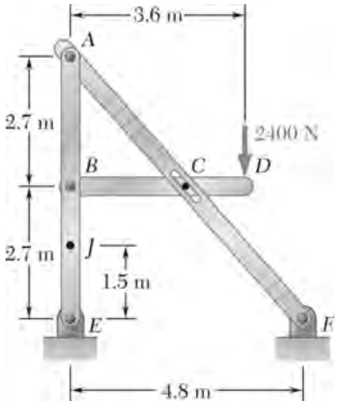


$$\circlearrowleft \Sigma M_A = 0: \quad (1.2\text{ m})(36.0\text{ kN}) - (0.6\text{ m})(6.00\text{ kN}) + (0.9\text{ m})(2.00\text{ kN}) - (1.8\text{ m})(E_x) = 0$$

$$\mathbf{E}_x = 23.0\text{ kN}\leftarrow \blacktriangleleft$$

$$\rightarrow \Sigma F_x = 0: \quad -23.0\text{ kN} + 36.0\text{ kN} - A_x = 0 \qquad \mathbf{A}_x = 13.00\text{ kN}\leftarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \quad -2.00\text{ kN} + 6.00\text{ kN} - A_y = 0 \qquad \mathbf{A}_y = 4.00\text{ kN}\downarrow \blacktriangleleft$$



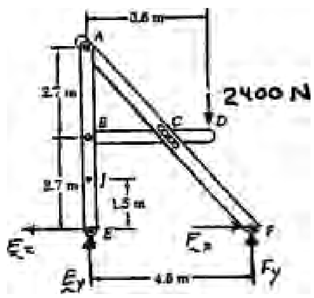
PROBLEM 6.102

For the frame and loading shown, determine the components of all forces acting on member *ABE*.

PROBLEM 6.101 For the frame and loading shown, determine the components of all forces acting on member *ABE*.

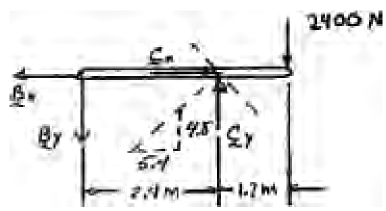
SOLUTION

FBD Frame:



$\circlearrowleft M_F = 0{:}\quad (1.2\text{ m})(2400\text{ N}) \quad (4.8\text{ m})E_y = 0$ $\qquad \mathbf{E}_y = 600\text{ N}\uparrow$ ◀

FBD member *BC*:



$$C_y = \frac{4.8}{5.4}C_x = \frac{8}{9}C_x$$

$\circlearrowleft \Sigma M_C = 0{:}\quad (2.4\text{ m})B_y - (1.2\text{ m})(2400\text{ N}) = 0 \quad B_y = 1200\text{ N}\downarrow$

on *ABE*: $\qquad \mathbf{B}_y = 1200\text{ N}\uparrow$ ◀

$\uparrow \Sigma F_y = 0{:}\quad -1200\text{ N} + C_y - 2400\text{ N} = 0 \quad C_y = 3600\text{ N}\uparrow$

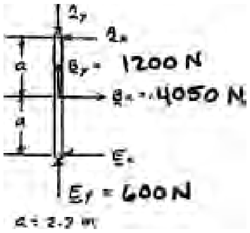
so $\qquad C_x = \dfrac{9}{8}C_y \quad C_x = 4050\text{ N}\rightarrow$

$\rightarrow \Sigma F_x = 0{:}\quad -B_x + C_x = 0 \quad B_x = 4050\text{ N}\leftarrow$ on *BC*

on *ABE*: $\qquad \mathbf{B}_x = 4050\text{ N}\rightarrow$ ◀

PROBLEM 6.102 (Continued)

FBD member *AB0E*:

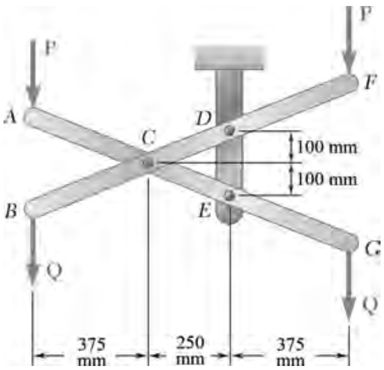


$$\circlearrowleft \Sigma M_A = 0: \quad a(4050 \text{ N}) - 2aE_x = 0$$

$$E_x = 2025 \text{ N} \qquad \mathbf{E}_x = 2025 \text{ N} \leftarrow \blacktriangleleft$$

$$\rightarrow \Sigma F_x = 0: \quad -A_x + (4050 - 2025) \text{ N} = 0 \qquad \mathbf{A}_x = 2025 \text{ N} \leftarrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \quad 600 \text{ N} + 1200 \text{ N} - A_y = 0 \qquad \mathbf{A}_y = 1800 \text{ N} \downarrow \blacktriangleleft$$



PROBLEM 6.103

Knowing that $P = 75$ N and $Q = 325$ N, determine the components of the forces exerted (*a*) on member *BCDF* at *C* and *D*, (*b*) on member *ACEG* at *E*.

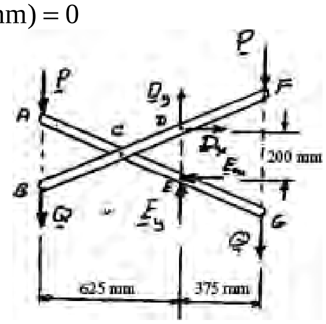
SOLUTION

<u>Free body: Entire frame</u>

$$\circlearrowleft^{+}\ \Sigma M_E = 0: \quad (P+Q)(625\text{ mm}) - (P+Q)(375\text{ mm}) - D_x(200\text{ mm}) = 0$$

$$D_x = (P+Q)\frac{5}{4}$$

$$\mathbf{D}_x = (P+Q)\frac{5}{4} \rightarrow$$



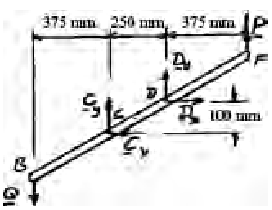
$$\xrightarrow{+} \Sigma F_x = 0: \quad -E_x + (P+Q)\frac{5}{4} = 0$$

$$E_x = (P+Q)\frac{5}{4}; \quad \mathbf{E}_x = (P+Q)\frac{5}{4} \leftarrow$$

<u>Free body: Member *BCDF*</u>

From above:

$$\mathbf{D}_x = (P+Q)\frac{5}{4} \rightarrow \triangleleft$$



$$\xrightarrow{+} \Sigma F_x = 0: \quad -C_x + D_x = 0 \qquad \mathbf{C}_x = (P+Q)\frac{5}{4} \leftarrow \triangleleft$$

$$\circlearrowleft^{+}\ \Sigma M_D = 0: \quad -(P+Q)\frac{5}{4}(100\text{ mm}) - P(375\text{ mm}) + Q(625\text{ mm}) - C_y(250\text{ mm})$$

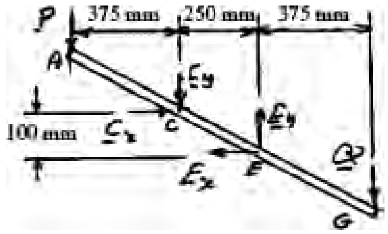
$$C_y = 2Q - 2P \qquad \mathbf{C}_y = 2Q - 2P \uparrow \triangleleft$$

$$+\!\uparrow \Sigma F_y = 0: \quad D_y + (2Q - 2P) = P + Q = 0$$

$$D_y = -Q + 3P \qquad \mathbf{D}_y = -Q + 3P \uparrow \triangleleft$$

PROBLEM 6.103 (Continued)

Free body: Member *ACEG*



From above

$$\mathbf{E}_x = (P+Q)\frac{5}{4} \leftarrow \triangleleft$$

and $$C_y = 2Q - 2P$$

$$+\uparrow \Sigma F_y = 0: \quad E_y - C_y - P - Q = 0$$

$$E_y - (2Q - 2P) - P - Q = 0$$

$$E_y = 3Q - P \qquad \mathbf{E}_y = 3Q - P \uparrow \triangleleft$$

$$P = 75 \text{ N} \quad \text{and} \quad Q = 325 \text{ N}$$

$$C_x = (P+Q)\frac{5}{4} = (400)\frac{5}{4} = +500 \text{ N} \qquad \mathbf{C}_x = 500 \text{ N} \leftarrow \blacktriangleleft$$

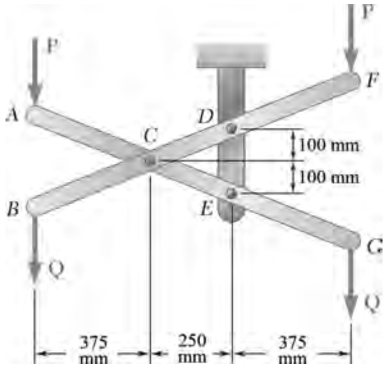
$$C_y = 2Q - 2P = 2(325) - 2(75) = +500 \text{ N} \qquad \mathbf{C}_y = 500 \text{ N} \uparrow \blacktriangleleft$$

$$D_x = (P+Q)\frac{5}{4} = (400)\frac{5}{4} = +500 \text{ N} \qquad \mathbf{D}_x = 500 \text{ N} \rightarrow \blacktriangleleft$$

$$D_y = -Q + 3P = -325 + 3(75) = -100 \text{ N} \qquad \mathbf{D}_y = 100.0 \text{ N} \downarrow \blacktriangleleft$$

$$E_x = (P+Q)\frac{5}{4} = (400)\frac{5}{4} = +500 \text{ N} \qquad \mathbf{E}_x = 500 \text{ N} \leftarrow \blacktriangleleft$$

$$E_y = 3Q - P = 3(325) - 75 = +900 \text{ N} \qquad \mathbf{E}_y = 900 \text{ N} \uparrow \blacktriangleleft$$



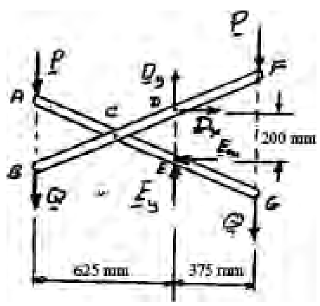
PROBLEM 6.104

Knowing that $P = 125$ N and $Q = 275$ N, determine the components of the forces exerted (*a*) on member *BCDF* at *C* and *D*, (*b*) on member *ACEG* at *E*.

SOLUTION

<u>Free body: Entire frame</u>

$$+\circlearrowleft \Sigma M_E = 0: \quad (P+Q)(625\text{ mm}) - (P+Q)(375\text{ mm}) - D_x(200\text{ mm}) = 0$$



$$D_x = (P+Q)\frac{5}{4}$$

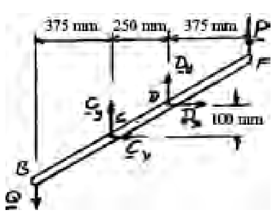
$$\mathbf{D}_x = (P+Q)\frac{5}{4} \rightarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -E_x + (P+Q)\frac{5}{4} = 0$$

$$E_x = (P+Q)\frac{5}{4}; \quad \mathbf{E}_x = (P+Q)\frac{5}{4} \leftarrow$$

<u>Free body: Member *BCDF*</u>

From above:



$$\mathbf{D}_x = (P+Q)\frac{5}{4} \rightarrow \triangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -C_x + D_x = 0 \qquad \mathbf{C}_x = (P+Q)\frac{5}{4} \leftarrow \triangleleft$$

$$+\circlearrowleft \Sigma M_D = 0: \quad -(P+Q)\frac{5}{4}(100\text{ mm}) - P(375\text{ mm}) + Q(625\text{ mm}) - C_y(250\text{ mm})$$

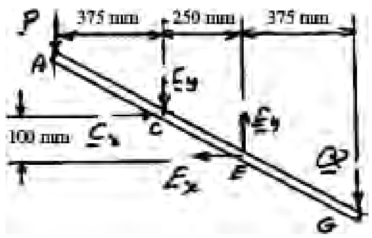
$$C_y = 2Q - 2P \qquad \mathbf{C}_y = 2Q - 2P \uparrow \triangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad D_y + (2Q - 2P) = P + Q = 0$$

$$D_y = -Q + 3P \qquad \mathbf{D}_y = -Q + 3P \uparrow \triangleleft$$

PROBLEM 6.104 (Continued)

Free body: Member *ACEG*



From above

$$\mathbf{E}_x = (P+Q)\frac{5}{4} \leftarrow \triangleleft$$

and $$C_y = 2Q - 2P$$

$$+\uparrow \Sigma F_y = 0: \quad E_y - C_y - P - Q = 0$$

$$E_y - (2Q - 2P) - P - Q = 0$$

$$E_y = 3Q - P \qquad \mathbf{E}_y = 3Q - P \uparrow \triangleleft$$

$$P = 125 \text{ N} \quad \text{and} \quad Q = 275 \text{ N}$$

$$C_x = (P+Q)\frac{5}{4} = (400)\frac{5}{4} = +500 \text{ N} \qquad \mathbf{C}_x = 500 \text{ N} \leftarrow \blacktriangleleft$$

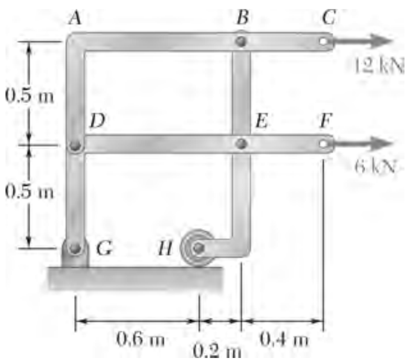
$$C_y = 2Q - 2P = 2(275) - 2(125) = +300 \text{ N} \qquad \mathbf{C}_y = 300 \text{ N} \uparrow \blacktriangleleft$$

$$D_x = (P+Q)\frac{5}{4} = (400)\frac{5}{4} = +500 \text{ N} \qquad \mathbf{D}_x = 500 \text{ N} \rightarrow \blacktriangleleft$$

$$D_y = -Q + 3P = -275 + 3(125) = +100 \text{ N} \qquad \mathbf{D}_y = 100.0 \text{ N} \uparrow \blacktriangleleft$$

$$E_x = (P+Q)\frac{5}{4} = (400)\frac{5}{4} = +500 \text{ N} \qquad \mathbf{E}_x = 500 \text{ N} \leftarrow \blacktriangleleft$$

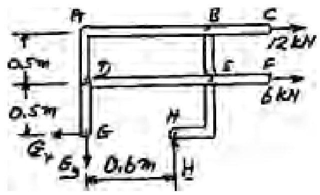
$$E_y = 3Q - P = 3(275) - 125 = +700 \text{ N} \qquad \mathbf{E}_y = 700 \text{ N} \uparrow \blacktriangleleft$$



PROBLEM 6.105

For the frame and loading shown, determine the components of the forces acting on member *DABC* at *B* and *D*.

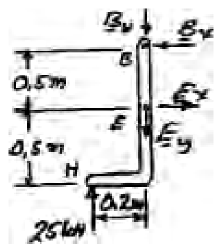
SOLUTION



<u>Free body: Entire frame</u>

$$\circlearrowleft^{+} \Sigma M_G = 0: \quad H(0.6 \text{ m}) - (12 \text{ kN})(1 \text{ m}) - (6 \text{ kN})(0.5 \text{ m}) = 0$$

$$H = 25 \text{ kN} \qquad \mathbf{H} = 25 \text{ kN} \uparrow$$



<u>Free body: Member *BEH*</u>

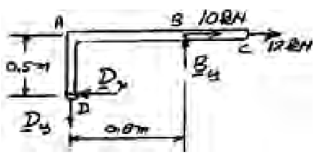
$$\circlearrowleft^{+} \Sigma M_F = 0: \quad B_x(0.5 \text{ m}) - (25 \text{ kN})(0.2 \text{ m}) = 0$$

$$B_x = +10 \text{ kN}$$

<u>Free body: Member *DABC*</u>

From above: $\mathbf{B}_x = 10.00 \text{ kN} \rightarrow$ ◀

$$\circlearrowleft^{+} \Sigma M_D = 0: \quad -B_y(0.8 \text{ m}) + (10 \text{ kN} + 12 \text{ kN})(0.5 \text{ m}) = 0$$



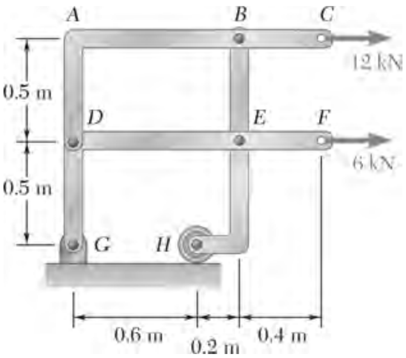
$$B_y = +13.75 \text{ kN} \qquad \mathbf{B}_y = 13.75 \text{ kN} \uparrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -D_x + 10 \text{ kN} + 12 \text{ kN} = 0$$

$$D_x = +22 \text{ kN} \qquad \mathbf{D}_x = 22.0 \text{ kN} \leftarrow \blacktriangleleft$$

$$+\!\uparrow \Sigma F_y = 0: \quad -D_y + 13.75 \text{ kN} = 0$$

$$D_y = +13.75 \text{ kN} \qquad \mathbf{D}_y = 13.75 \text{ kN} \downarrow \blacktriangleleft$$

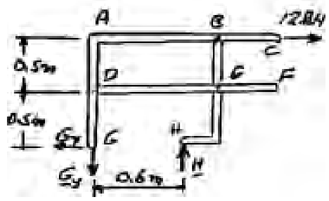


PROBLEM 6.106

Solve Problem 6.105 assuming that the 6-kN load has been removed.

PROBLEM 6.105 For the frame and loading shown, determine the components of the forces acting on member *DABC* at *B* and *D*.

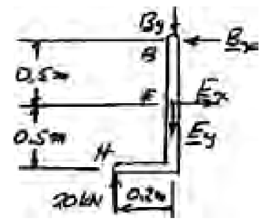
SOLUTION



Free body: Entire frame

$$\overset{+}{\circlearrowleft}\Sigma M_G = 0: \quad H(0.6\text{ m}) - (12\text{ kN})(1\text{ m}) = 0$$

$$H = 20\text{ kN} \quad \mathbf{H} = 20\text{ kN}\uparrow$$



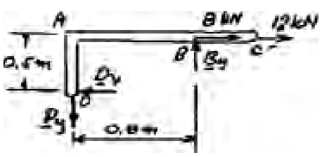
Free body: Member *BEH*

$$\overset{+}{\circlearrowleft}\Sigma M_E = 0: \quad B_x(0.5\text{ m}) - (20\text{ kN})(0.2\text{ m}) = 0$$

$$B_x = +8\text{ kN}$$

Free body: Member *DABC*

From above: $\mathbf{B}_x = -8.00\text{ kN}\rightarrow$ ◀



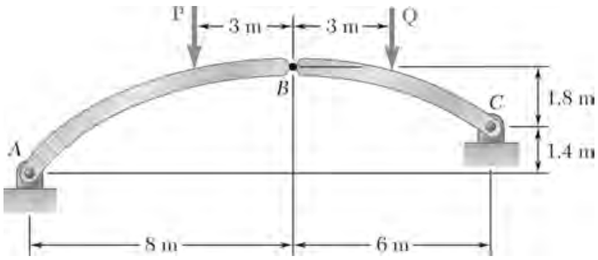
$$\overset{+}{\circlearrowleft}\Sigma M_D = 0: \quad -B_y(0.8\text{ m}) + (8\text{ kN} + 12\text{ kN})(0.5\text{ m}) = 0$$

$$B_y = +12.5\text{ kN} \qquad \mathbf{B}_y = 12.50\text{ kN}\uparrow \blacktriangleleft$$

$$\xrightarrow{+}\Sigma F_x = 0: \quad -D_x + 8\text{ kN} + 12\text{ kN} = 0$$

$$D_x = +20\text{ kN} \qquad \mathbf{D}_x = 20.0\text{ kN}\leftarrow \blacktriangleleft$$

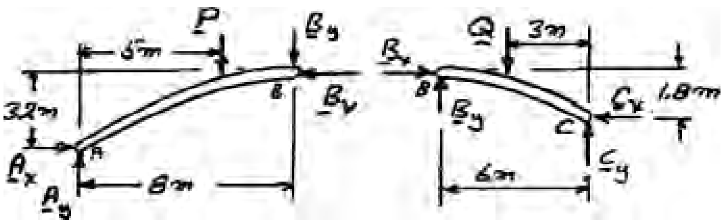
$$+\uparrow\Sigma F_y = 0: \quad -D_y + 12.5\text{ kN} = 0; \quad D_y = +12.5\text{ kN} \qquad \mathbf{D}_y = 12.50\text{ kN}\downarrow \blacktriangleleft$$



PROBLEM 6.107

The axis of the three-hinge arch *ABC* is a parabola with vertex at *B*. Knowing that $P = 112$ kN and $Q = 140$ kN, determine (*a*) the components of the reaction at *A*, (*b*) the components of the force exerted at *B* on segment *AB*.

SOLUTION



Free body: Segment *AB*:

$$+\circlearrowleft \Sigma M_A = 0: \quad B_x(3.2 \text{ m}) - B_y(8 \text{ m}) - P(5 \text{ m}) = 0 \qquad (1)$$

0.75 (Eq. 1) $\quad B_x(2.4 \text{ m}) - B_y(6 \text{ m}) - P(3.75 \text{ m}) = 0 \qquad (2)$

Free body: Segment *BC*:

$$+\circlearrowright \Sigma M_C = 0: \quad B_x(1.8 \text{ m}) + B_y(6 \text{ m}) - Q(3 \text{ m}) = 0 \qquad (3)$$

Add (2) and (3): $\quad 4.2B_x - 3.75P - 3Q = 0$

$$B_x = (3.75P + 3Q)/4.2 \qquad (4)$$

Eq. (1): $\quad (3.75P + 3Q)\dfrac{3.2}{4.2} - 8B_y - 5P = 0$

$$B_y = (-9P + 9.6Q)/33.6 \qquad (5)$$

Given that $P = 112$ kN and $Q = 140$ kN

(*a*) Reaction at *A*:

Considering again *AB* as a free body

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - B_x = 0; \quad A_x = B_x = 200 \text{ kN} \qquad \mathbf{A}_x = 200 \text{ kN} \rightarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - P - B_y = 0$$

$$A_y - 112 \text{ kN} - 10 \text{ kN} = 0$$

$$A_y = +122 \text{ kN} \qquad \mathbf{A}_y = 122.0 \text{ kN} \uparrow \blacktriangleleft$$

PROBLEM 6.107 (Continued)

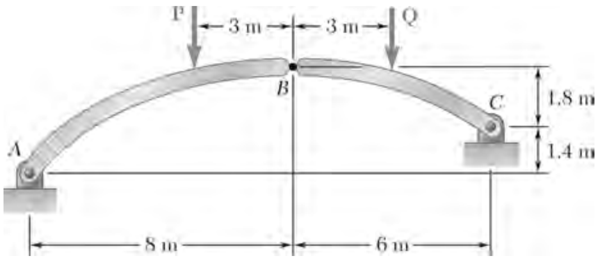
(*b*) Force exerted at *B* on *AB*

Eq. (4): $B_x = (3.75 \times 112 + 3 \times 140)/4.2 = 200 \text{ kN}$

$\mathbf{B}_x = 200 \text{ kN} \leftarrow$ ◀

Eq. (5): $B_y = (-9 \times 112 + 9.6 \times 140)/33.6 = +10 \text{ kN}$

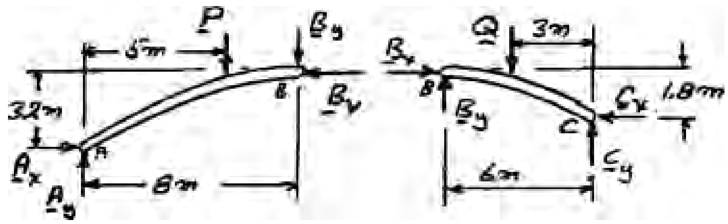
$\mathbf{B}_y = 10.00 \text{ kN} \downarrow$ ◀



PROBLEM 6.108

The axis of the three-hinge arch *ABC* is a parabola with vertex at *B*. Knowing that P = 140 kN and Q = 112 kN, determine (*a*) the components of the reaction at *A*, (*b*) the components of the force exerted at *B* on segment *AB*.

SOLUTION



Free body: Segment *AB*:

$$\circlearrowleft^{+} \Sigma M_A = 0: \quad B_x(3.2 \text{ m}) - B_y(8 \text{ m}) - P(5 \text{ m}) = 0 \qquad (1)$$

0.75 (Eq. 1) $$B_x(2.4 \text{ m}) - B_y(6 \text{ m}) - P(3.75 \text{ m}) = 0 \qquad (2)$$

Free body: Segment *BC*:

$$\circlearrowright^{+} \Sigma M_C = 0: \quad B_x(1.8 \text{ m}) + B_y(6 \text{ m}) - Q(3 \text{ m}) = 0 \qquad (3)$$

Add (2) and (3): $$4.2B_x - 3.75P - 3Q = 0$$

$$B_x = (3.75P + 3Q)/4.2 \qquad (4)$$

Eq. (1): $$(3.75P + 3Q)\frac{3.2}{4.2} - 8B_y - 5P = 0$$

$$B_y = (-9P + 9.6Q)/33.6 \qquad (5)$$

Given that $P = 140$ kN and $Q = 112$ kN

(*a*) Reaction at *A*:

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - B_x = 0; \quad A_x = B_x = 205 \text{ kN}$$

$$\mathbf{A}_x = 205 \text{ kN} \rightarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - P - B_y = 0$$

$$A_y - 140 \text{ kN} - (-5.5 \text{ kN}) = 0$$

$$A_y = 134.5 \text{ kN}$$

$$\mathbf{A}_y = 134.5 \text{ kN} \uparrow \blacktriangleleft$$

PROBLEM 6.108 (Continued)

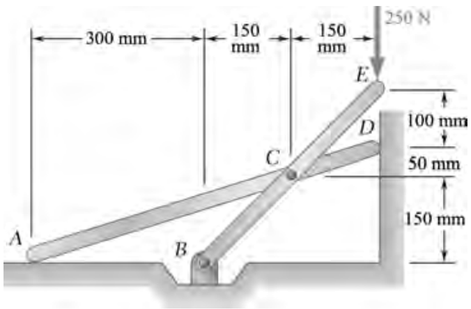
(*b*) Force exerted at B on AB

Eq. (4): $B_x = (3.75\times 140 + 3\times 112)/4.2 = 205$ kN

$$\mathbf{B}_x = 205 \text{ kN} \leftarrow ◀$$

Eq. (5): $B_y = (-9\times 140 + 9.6\times 112)/33.6 = -5.5$ kN

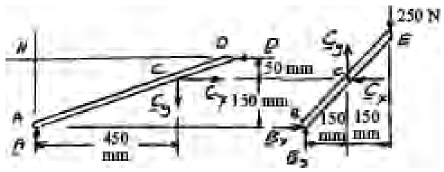
$$\mathbf{B}_y = 5.5 \text{ kN} \uparrow ◀$$



PROBLEM 6.109

Knowing that the surfaces at A and D are frictionless, determine the forces exerted at B and C on member BCE.

SOLUTION



Free body of Member ACD

$$\circlearrowleft + \Sigma M_H = 0: \quad C_x(50 \text{ mm}) - C_y(450 \text{ mm}) = 0 \quad C_x = 9C_y \qquad (1)$$

Free body of Member BCE

$$\circlearrowleft + \Sigma M_B = 0: \quad C_x(150 \text{ mm}) + C_y(150 \text{ mm}) - (250 \text{ N})(300 \text{ mm}) = 0$$

Substitute from (1):

$$9C_y(150) + C_y(150) - 75000 = 0$$

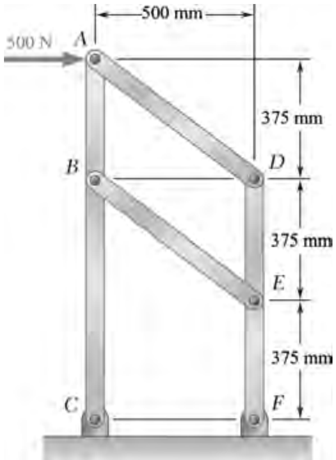
$$C_y = +50 \text{ N}; \quad C_x = 9C_y = 9(50 \text{ N}) = +450 \text{ N}$$

$\mathbf{C} = 453 \text{ N} \measuredangle\ 6.34° ◀$

$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x - 450 \text{ N} = 0 \qquad B_x = 450 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad B_y + 50 \text{ N} - 250 \text{ N} = 0 \qquad B_y = 200 \text{ N}$$

$\mathbf{B} = 492 \text{ N} \measuredangle\ 24.0° ◀$

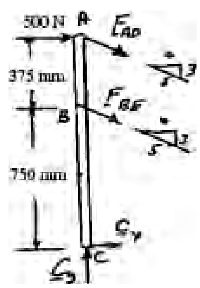


PROBLEM 6.110

For the frame and loading shown, determine (*a*) the reaction at *C*, (*b*) the force in member *AD*.

SOLUTION

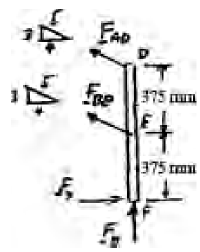
Free body: Member *ABC*



$$+\circlearrowleft \Sigma M_C = 0: \quad +(500 \text{ N})(1125 \text{ mm}) + \frac{4}{5}F_{AD}(1125 \text{ mm}) + \frac{4}{5}F_{BE}(750 \text{ mm}) = 0$$

$$562500 + 900\, F_{AD} + 600\, F_{BF} = 0 \qquad (1)$$

$$\Rightarrow 3F_{AD} + 2F_{BE} = -1875 \text{ N}$$



Free Body: Member *DEF*

$$+\circlearrowleft \Sigma M_F = 0: \quad \frac{4}{5}F_{AD}(750 \text{ mm}) + \frac{4}{5}F_{BE}(375 \text{ mm}) = 0$$

$$F_{BE} = -2F_{AD} \qquad (2)$$

(*a*) Substitute from (2) into (1)

$$3F_{AD} + 2(-2F_{AD}) = -1875 \text{ N}$$

$$F_{AD} = +1875 \text{ N} \qquad F_{AD} = 1875 \text{ N} \blacktriangleleft$$

(2)

$$F_{BE} = -2F_{AD} = -2(1875 \text{ N})$$

$$F_{BE} = -3750 \text{ N} \quad F_{BE} = 3750 \text{ N comp.}$$

PROBLEM 6.110 (Continued)

(*b*) Return to free body of member *ABC*

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + 500 \text{ N} + \frac{4}{5}F_{AD} + \frac{4}{5}F_{BE} = 0$$

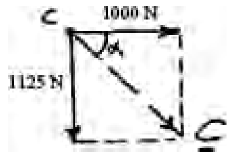
$$C_x + 500 + \frac{4}{5}(1875) + \frac{4}{5}(-3750) = 0$$

$$C_x = +1000 \text{ N} \quad \mathbf{C}_x = 1000 \text{ N} \rightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - \frac{3}{5}F_{AD} - \frac{3}{5}F_{BF} = 0$$

$$C_y - \frac{3}{5}(1875) - \frac{3}{5}(-3750) = 0$$

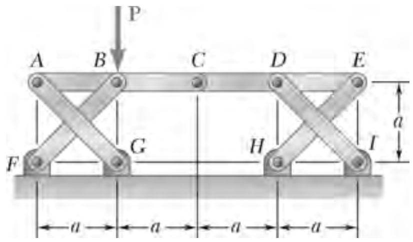
$$C_y = -1125 \text{ N} \quad \mathbf{C}_y = 1125 \text{ N}\downarrow$$



$$\alpha = 48.37°$$

$$C = 1505.2 \text{ N}$$

$\mathbf{C} = 1505 \text{ N} \searrow 48.4°$ ◀

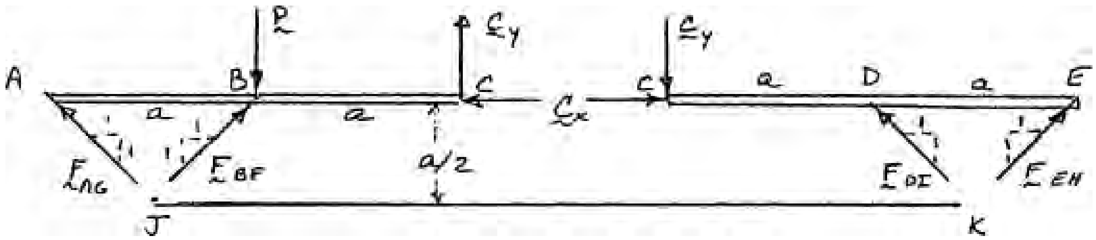


PROBLEM 6.111

Members *ABC* and *CDE* are pin-connected at *C* and supported by four links. For the loading shown, determine the force in each link.

SOLUTION

Member FBDs:



I II

From FBD I: $\circlearrowleft\Sigma M_J = 0$: $\dfrac{a}{2}C_x + \dfrac{3a}{2}C_y - \dfrac{a}{2}P = 0 \quad C_x + 3C_y = P$

FBD II: $\circlearrowleft\Sigma M_K = 0$: $\dfrac{a}{2}C_x - \dfrac{3a}{2}C_y = 0 \quad C_x - 3C_y = 0$

Solving: $C_x = \dfrac{P}{2}$; $C_y = \dfrac{P}{6}$ as drawn

FBD I: $\circlearrowleft\Sigma M_B = 0$: $aC_y - a\dfrac{1}{\sqrt{2}}F_{AG} = 0 \quad F_{AG} = \sqrt{2}C_y = \dfrac{\sqrt{2}}{6}P$ $\qquad F_{AG} = \dfrac{\sqrt{2}}{6}P \quad C$ ◀

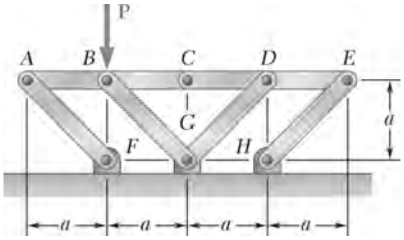
$\rightarrow\Sigma F_x = 0$: $-\dfrac{1}{\sqrt{2}}F_{AG} + \dfrac{1}{\sqrt{2}}F_{BF} - C_x = 0 \quad F_{BF} = F_{AG} + C_x\sqrt{2} = \dfrac{\sqrt{2}}{6}P + \dfrac{\sqrt{2}}{2}P$

$F_{BF} = \dfrac{2\sqrt{2}}{3}P \quad C$ ◀

FBD II: $\circlearrowleft\Sigma M_D = 0$: $a\dfrac{1}{\sqrt{2}}F_{EH} + aC_y = 0 \quad F_{EH} = -\sqrt{2}C_y = -\dfrac{\sqrt{2}}{6}P$ $\qquad F_{EH} = \dfrac{\sqrt{2}}{6}P \quad T$ ◀

$\rightarrow\Sigma F_x = 0$: $C_x - \dfrac{1}{\sqrt{2}}F_{DI} + \dfrac{1}{\sqrt{2}}F_{EH} = 0 \quad F_{DI} = F_{EH} + C_x\sqrt{2} = -\dfrac{\sqrt{2}}{6}P + \dfrac{\sqrt{2}}{2}P$

$F_{DI} = \dfrac{\sqrt{2}}{3}P \quad C$ ◀

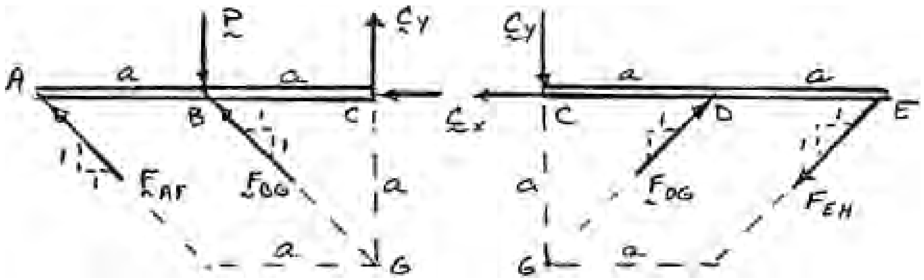


PROBLEM 6.112

Members *ABC* and *CDE* are pin-connected at *C* and supported by four links. For the loading shown, determine the force in each link.

SOLUTION

Member FBDs:



I **II**

FBD I: $\circlearrowleft \Sigma M_B = 0\colon \quad aC_y - a\frac{1}{\sqrt{2}}F_{AF} = 0 \quad F_{AF} = \sqrt{2}C_y$

FBD II: $\circlearrowleft M_D = 0\colon \quad aC_y - a\frac{1}{\sqrt{2}}F_{EH} = 0 \quad F_{EH} = \sqrt{2}C_y$

FBDs combined: $\circlearrowleft \Sigma M_G = 0\colon \quad aP - a\frac{1}{\sqrt{2}}F_{AF} - a\frac{1}{\sqrt{2}}F_{EH} = 0 \quad P = \frac{1}{\sqrt{2}}\sqrt{2}C_y + \frac{1}{\sqrt{2}}\sqrt{2}C_y$

$$C_y = \frac{P}{2} \qquad \text{so } F_{AF} = \frac{\sqrt{2}}{2}P \quad C \blacktriangleleft$$

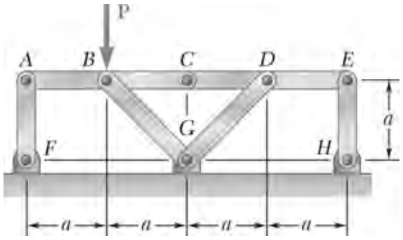
$$F_{EH} = \frac{\sqrt{2}}{2}P \quad T \blacktriangleleft$$

FBD I: $\uparrow \Sigma F_y = 0\colon \quad \frac{1}{\sqrt{2}}F_{AF} + \frac{1}{\sqrt{2}}F_{BG} - P + C_y = 0 \quad \frac{P}{2} + \frac{1}{\sqrt{2}}F_{BG} - P + \frac{P}{2} = 0$

$$F_{BG} = 0 \blacktriangleleft$$

FBD II: $\uparrow \Sigma F_y = 0\colon \quad -C_y + \frac{1}{\sqrt{2}}F_{DG} - \frac{1}{\sqrt{2}}F_{EH} = 0 \quad -\frac{P}{2} + \frac{1}{\sqrt{2}}F_{DG} - \frac{P}{2} = 0$

$$F_{DG} = \sqrt{2}P \quad C \blacktriangleleft$$

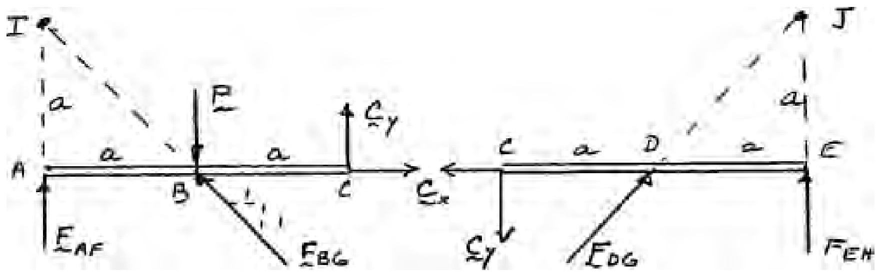


PROBLEM 6.113

Members *ABC* and *CDE* are pin-connected at *C* and supported by four links. For the loading shown, determine the force in each link.

SOLUTION

Member FBDs:



I **II**

FBD I: $\circlearrowleft \Sigma M_I = 0\text{:}\quad 2aC_y + aC_x - aP = 0 \quad 2C_y + C_x = P$

FBD II: $\circlearrowleft \Sigma M_J = 0\text{:}\quad 2aC_y - aC_x = 0 \quad 2C_y - C_x = 0$

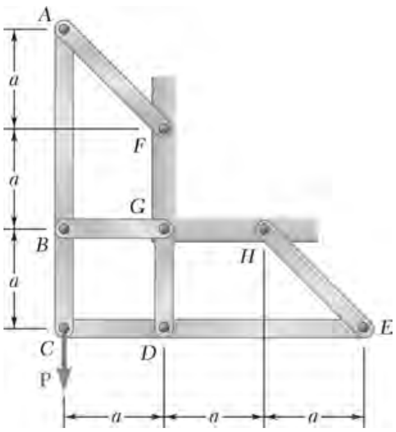
Solving: $C_x = \dfrac{P}{2}; \quad C_y = \dfrac{P}{4}$ as shown

FBD I: $\rightarrow \Sigma F_x = 0\text{:}\quad -\dfrac{1}{\sqrt{2}} F_{BG} + C_x = 0 \quad F_{BG} = C_x\sqrt{2}$ $\qquad F_{BG} = \dfrac{P}{\sqrt{2}} \quad C$ ◀

$\uparrow \Sigma F_y = 0\text{:}\quad F_{AF} - P + \dfrac{1}{\sqrt{2}}\left(\dfrac{\sqrt{2}}{2}P\right) + \dfrac{P}{4} = 0$ $\qquad F_{AF} = \dfrac{P}{4} \quad C$ ◀

FBD II: $\rightarrow \Sigma F_x = 0\text{:}\quad -C_x + \dfrac{1}{\sqrt{2}} F_{DG} = 0 \quad F_{DG} = C_x\sqrt{2}$ $\qquad F_{DG} = \dfrac{P}{\sqrt{2}} \quad C$ ◀

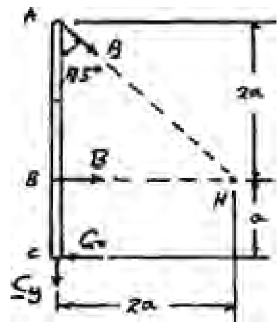
$\uparrow \Sigma F_y = 0\text{:}\quad -C_y + \dfrac{1}{\sqrt{2}}\left(\dfrac{\sqrt{2}}{2}P\right) + F_{EH} = 0 \quad F_{EH} = \dfrac{P}{4} - \dfrac{P}{2} = -\dfrac{P}{4}$ $\qquad F_{EH} = \dfrac{P}{4} \quad T$ ◀



PROBLEM 6.114

Members *ABC* and *CDE* are pin-connected at *C* and supported by the four links *AF*, *BG*, *DG*, and *EH*. For the loading shown, determine the force in each link.

SOLUTION



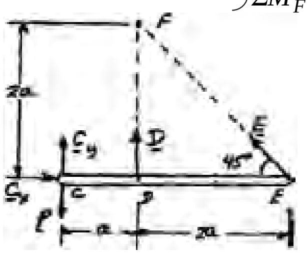
Free body: Member *ABC*

$$\overset{+}{\circlearrowleft}\Sigma M_H = 0: \quad C_x(a) - C_y(2a) = 0$$

$$C_x = 2C_y$$

Note: This checks that for 3-force member the forces are concurrent.

Free body: Member *CDE*



$$\overset{+}{\circlearrowleft}\Sigma M_F = 0: \quad C_x(2a) - C_y(a) + P(a) = 0$$

$$2C_x - C_y + P = 0$$

$$2(2C_y) - C_y + P = 0 \qquad C_y = -\frac{1}{3}P \ \triangleleft$$

$$C_x = 2C_y: \qquad C_x = -\frac{2}{3}P \ \triangleleft$$

$$\xrightarrow{+}\Sigma F = 0: \quad C_x - \frac{E}{\sqrt{2}} = 0; \quad -\frac{2}{3}P - \frac{E}{\sqrt{2}} = 0$$

$$E = -\frac{2\sqrt{2}}{3}P \qquad F_{EH} = \frac{2\sqrt{2}}{3}P \quad \text{comp.} \blacktriangleleft$$

$$\overset{+}{\circlearrowleft}\Sigma M_E = 0: \quad D(2a) + C_y(3a) - P(3a) = 0$$

$$D(2a) - \frac{P}{3}(3a) - P(3a) = 0$$

$$D = +2P \qquad F_{DG} = 2P \quad T \ \blacktriangleleft$$

PROBLEM 6.114 (Continued)

Return to free body of ABC

$$+\downarrow \Sigma F_y = 0: \quad \frac{A}{\sqrt{2}} + C_y = 0$$

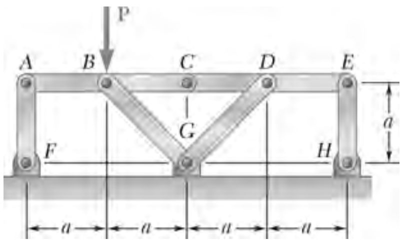
$$\frac{A}{\sqrt{2}} - \frac{P}{3} = 0$$

$$A = +\frac{\sqrt{2}}{3}P \qquad F_{AF} = \frac{\sqrt{2}}{3}P \quad T \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_A = 0: \quad B(2a) - C_x(3a) = 0$$

$$B(2a) + \frac{2}{3}P(3a) = 0$$

$$B = -P \qquad F_{BG} = P \quad C \blacktriangleleft$$



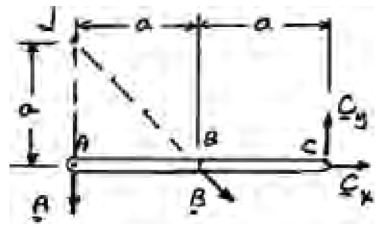
PROBLEM 6.115

Solve Problem 6.113 assuming that the force **P** is replaced by a clockwise couple of moment $\mathbf{M}_0$ applied to member *CDE* at *D*.

PROBLEM 6.113 Members *ABC* and *CDE* are pin-connected at *C* and supported by four links. For the loading shown, determine the force in each link.

SOLUTION

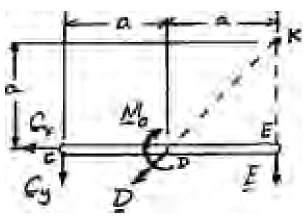
Free body: Member *ABC*



$$\circlearrowleft^{+} \Sigma M_J = 0: \quad C_y(2a) + C_x(a) = 0$$

$$C_x = -2C_y$$

Free body: Member *CDE*



$$\circlearrowleft^{+} \Sigma M_K = 0: \quad C_y(2a) - C_x(a) - M_0 = 0$$

$$C_y(2a) - (-2C_y)(a) - M_0 = 0 \qquad C_y = \frac{M_0}{4a} \triangleleft$$

$$C_x = -2C_y: \qquad C_x = -\frac{M_0}{2a} \triangleleft$$

$$\overset{+}{\longleftarrow} \Sigma F_x = 0: \quad \frac{D}{\sqrt{2}} + C_x = 0; \quad \frac{D}{\sqrt{2}} - \frac{M_0}{2a} = 0$$

$$D = \frac{M_0}{\sqrt{2}a} \qquad F_{DG} = \frac{M_0}{\sqrt{2}a} \quad T \blacktriangleleft$$

$$\circlearrowleft^{+} \Sigma M_D = 0: \quad E(a) - C_y(a) + M_0 = 0$$

$$E(a) - \left(\frac{M_0}{4a}\right)(a) + M_0 = 0$$

$$E = -\frac{3}{4}\frac{M_0}{a} \qquad F_{EH} = \frac{3}{4}\frac{M_0}{a} \quad C \blacktriangleleft$$

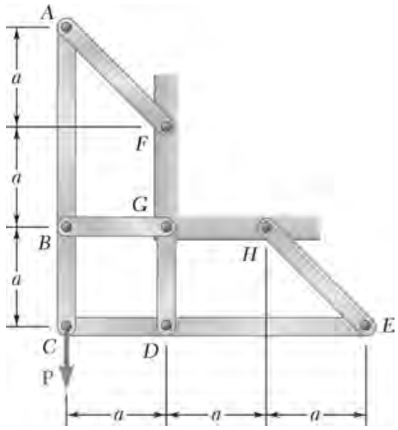
Return to Free Body of *ABC*

$$\overset{+}{\longrightarrow} \Sigma F_x = 0: \quad \frac{B}{\sqrt{2}} + C_x = 0; \quad \frac{B}{\sqrt{2}} - \frac{M_0}{2a} = 0$$

$$B = \frac{M_0}{\sqrt{2}a} \qquad F_{BG} = \frac{M_0}{\sqrt{2}a} \quad T \blacktriangleleft$$

$$\circlearrowleft^{+} \Sigma M_B = 0: \quad A(a) + C_y(a); \quad A(a) + \frac{M_0}{4a}(a) = 0$$

$$A = -\frac{M_0}{4a} \qquad F_{AF} = \frac{M_0}{4a} \quad C \blacktriangleleft$$

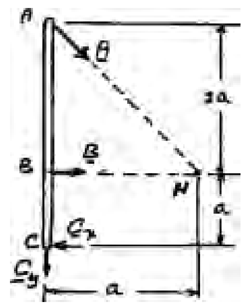


PROBLEM 6.116

Solve Problem 6.114 assuming that the force **P** is replaced by a clockwise couple of moment $\mathbf{M}_0$ applied to member *CDE* at *D*.

PROBLEM 6.114 Members *ABC* and *CDE* are pin-connected at *C* and supported by the four links *AF*, *BG*, *DG*, and *EH*. For the loading shown, determine the force in each link.

SOLUTION

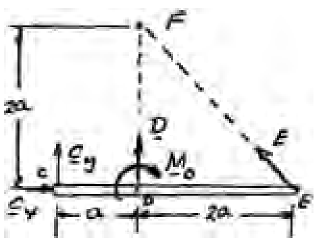


Free body: Member *ABC*

$$+\circlearrowleft \Sigma M_H = 0: \quad C_x(a) - C_y(2a) = 0$$

$$C_x = 2C_y$$

Free body: Member *CDE*



$$+\circlearrowleft \Sigma M_F = 0: \quad C_x(2a) - C_y(a) - M_0 = 0$$

$$(2C_y)(2a) - C_y(a) - M_0 = 0 \qquad C_y = \frac{M_0}{3a} \triangleleft$$

$$C_x = 2C_y: \qquad C_x = \frac{2M_0}{3a} \triangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - \frac{E}{\sqrt{2}} = 0; \quad \frac{2M_0}{3a} - \frac{E}{\sqrt{2}} = 0$$

$$E = \frac{2\sqrt{2}}{3}\frac{M_0}{a} \qquad F_{EH} = \frac{2\sqrt{2}}{3}\frac{M_0}{a} \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad D + \frac{E}{\sqrt{2}} + C_y = 0$$

$$D + \frac{2\sqrt{2}}{3}\frac{M_0}{a}\frac{1}{\sqrt{2}} + \frac{M_0}{3a} = 0$$

$$D = -\frac{M_0}{a} \qquad F_{DG} = \frac{M_0}{a} \quad C \blacktriangleleft$$

PROBLEM 6.116 (Continued)

Return to free body of ABC

$$+\downarrow \Sigma F_y = 0: \quad \frac{A}{\sqrt{2}} + C_y = 0; \quad \frac{A}{\sqrt{2}} + \frac{M_0}{3a} = 0$$

$$A = -\frac{\sqrt{2}}{3}\frac{M_0}{a} \qquad\qquad F_{AF} = \frac{\sqrt{2}}{3}\frac{M_0}{a} \quad C \blacktriangleleft$$

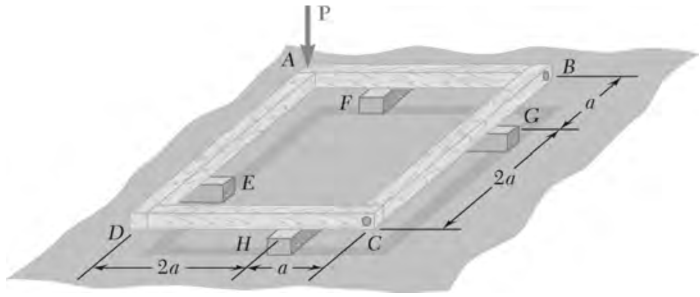
$$+\circlearrowleft \Sigma M_A = 0: \quad B(2a) - C_x(3a) = 0$$

$$B(2a) - \left(\frac{2}{3}\frac{M_0}{a}\right)(3a) = 0$$

$$B = +\frac{M_0}{a} \qquad\qquad F_{BG} = \frac{M_0}{a} \quad T \blacktriangleleft$$

PROBLEM 6.117

Four beams, each of length $3a$, are held together by single nails at A, B, C, and D. Each beam is attached to a support located at a distance a from an end of the beam as shown. Assuming that only vertical forces are exerted at the connections, determine the vertical reactions at E, F, G, and H.



SOLUTION

We shall draw the free body of each member. Force **P** will be applied to member AFB. Starting with member AED, we shall express all forces in terms of reaction E.

Member *AFB*:

$$+\circlearrowleft \Sigma M_D = 0: \quad A(3a) + E(a) = 0$$

$$A = -\frac{E}{3}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -D(3a) - E(2a) = 0$$

$$D = -\frac{2E}{3}$$

Member *DHC*:

$$+\circlearrowleft \Sigma M_C = 0: \quad \left(-\frac{2E}{3}\right)(3a) - H(a) = 0$$

$$H = -2E \qquad (1)$$

$$+\circlearrowleft \Sigma M_H = 0: \quad \left(-\frac{2E}{3}\right)(2a) + C(a) = 0$$

$$C = +\frac{4E}{3}$$

Member *CGB*:

$$+\circlearrowleft \Sigma M_B = 0: \quad +\left(\frac{4E}{3}\right)(3a) - G(a) = 0$$

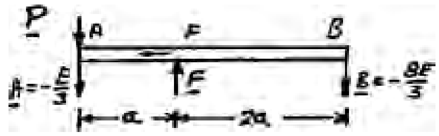
$$G = +4E \qquad (2)$$

PROBLEM 6.117 (Continued)

$$+\circlearrowleft \Sigma M_G = 0: \quad +\left(\frac{4E}{3}\right)(2a) + B(a) = 0$$

$$B = -\frac{8E}{3}$$

<u>Member *AFB*</u>:



$$+\uparrow \Sigma F_y = 0: \quad F - A - B - P = 0$$

$$F - \left(-\frac{E}{3}\right) - \left(-\frac{8E}{3}\right) - P = 0$$

$$F = P - 3E \qquad (3)$$

$$+\circlearrowleft \Sigma M_A = 0: \quad F(a) - B(3a) = 0$$

$$(P - 3E)(a) - \left(-\frac{8E}{3}\right)(3a) = 0$$

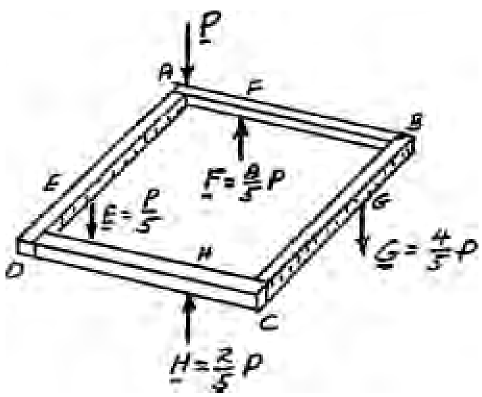
$$P - 3E + 8E = 0; \quad E = -\frac{P}{5} \qquad \mathbf{E} = \frac{P}{5}\downarrow \blacktriangleleft$$

Substitute $E = -\frac{P}{5}$ into Eqs. (1), (2), and (3).

$$H = -2E = -2\left(-\frac{P}{5}\right) \qquad H = +\frac{2P}{5} \qquad \mathbf{H} = \frac{2P}{5}\uparrow \blacktriangleleft$$

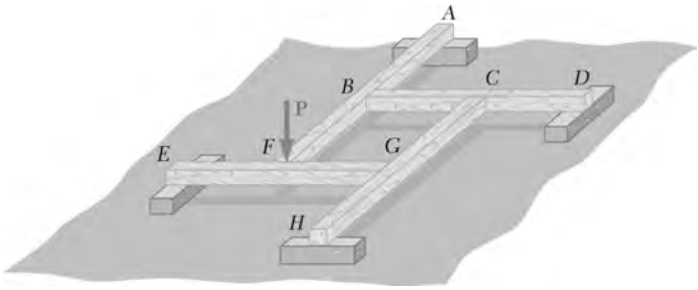
$$G = +4E = 4\left(-\frac{P}{5}\right) \qquad G = -\frac{4P}{5} \qquad \mathbf{G} = \frac{4P}{5}\downarrow \blacktriangleleft$$

$$F = P - 3E = P - 3\left(-\frac{P}{5}\right) \qquad F = +\frac{8P}{5} \qquad \mathbf{F} = \frac{8P}{5}\downarrow \blacktriangleleft$$



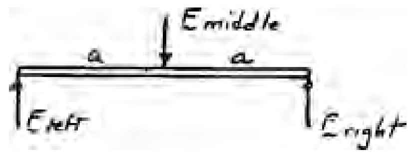
PROBLEM 6.118

Four beams, each of length $2a$, are nailed together at their midpoints to form the support system shown. Assuming that only vertical forces are exerted at the connections, determine the vertical reactions at A, D, E, and H.



SOLUTION

Note that, if we assume P is applied to EG, each individual member FBD looks like



so $$2F_{\text{left}} = 2F_{\text{right}} = F_{\text{middle}}$$

Labeling each interaction force with the letter corresponding to the joint of its application, we see that

$$B = 2A = 2F$$

$$C = 2B = 2D$$

$$G = 2C = 2H$$

$$P + F = 2G (= 4C = 8B = 16F) = 2E$$

From $$P + F = 16F, \quad F = \frac{P}{15}$$ so $\mathbf{A} = \dfrac{P}{15}\uparrow$ ◀

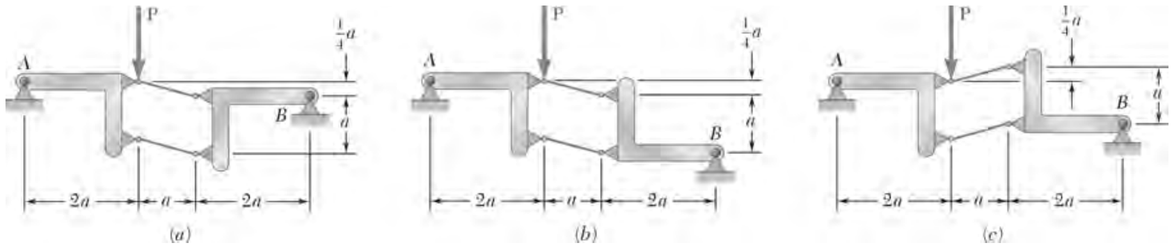
$\mathbf{D} = \dfrac{2P}{15}\uparrow$ ◀

$\mathbf{H} = \dfrac{4P}{15}\uparrow$ ◀

$\mathbf{E} = \dfrac{8P}{15}\uparrow$ ◀

PROBLEM 6.119

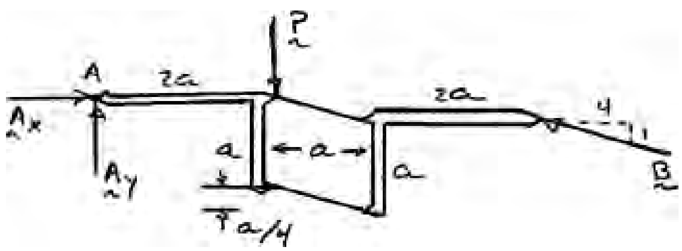
Each of the frames shown consists of two L-shaped members connected by two rigid links. For each frame, determine the reactions at the supports and indicate whether the frame is rigid.



SOLUTION

Note: In all three cases, the right member has only three forces acting, two of which are parallel. Thus the third force, at B, must be parallel to the link forces.

(a) **FBD whole:**



$$\circlearrowleft \Sigma M_A = 0: \quad -2aP - \frac{a}{4}\frac{4}{\sqrt{17}}B + 5a\frac{1}{\sqrt{17}}B = 0 \quad B = 2.06P$$

$\mathbf{B} = 2.06P \;\; 14.04° ◀$

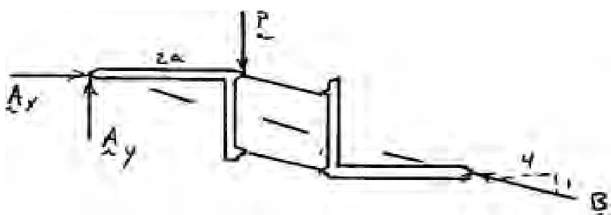
$$\rightarrow \Sigma F_x = 0: \quad A_x - \frac{4}{\sqrt{17}}B = 0 \qquad \mathbf{A}_x = 2P \leftarrow$$

$$\uparrow \Sigma F_y = 0: \quad A_y - P + \frac{1}{\sqrt{17}}B = 0 \quad \mathbf{A}_y = \frac{P}{2}\uparrow$$

$\mathbf{A} = 2.06P \;\; 14.04° ◀$

rigid ◀

(b) **FBD whole:**



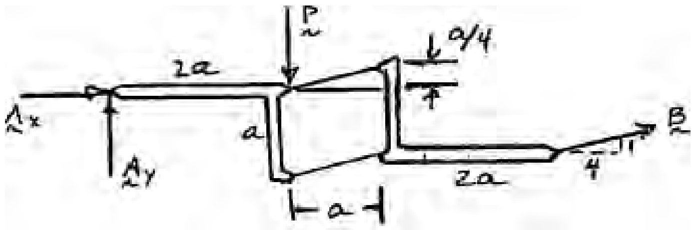
Since **B** passes through A, $\circlearrowright \Sigma M_A = 2aP = 0$ only if $P = 0$

no equilibrium if $P \neq 0$

not rigid ◀

PROBLEM 6.119 (Continued)

(*c*) **FBD whole:**



$$\circlearrowleft\Sigma M_A = 0: \quad 5a\frac{1}{\sqrt{17}}B + \frac{3a}{4}\frac{4}{\sqrt{17}}B - 2aP = 0 \quad B = \frac{\sqrt{17}}{4}P$$

$\mathbf{B} = 1.031P \angle 14.04°$ ◀

$$\rightarrow\Sigma F_x = 0: \quad A_x + \frac{4}{\sqrt{17}}B = 0 \quad A_x = -P$$

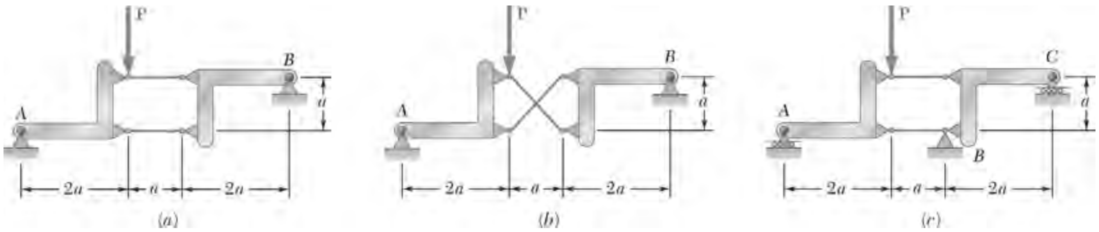
$$\uparrow\Sigma F_y = 0: \quad A_y - P + \frac{1}{\sqrt{17}}B = 0 \quad A_y = P - \frac{P}{4} = \frac{3P}{4}$$

$\mathbf{A} = 1.250P \searrow 36.9°$ ◀

System is rigid ◀

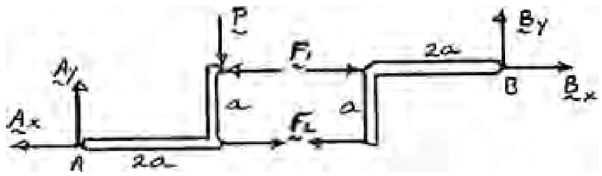
PROBLEM 6.120

Each of the frames shown consists of two L-shaped members connected by two rigid links. For each frame, determine the reactions at the supports and indicate whether the frame is rigid.



SOLUTION

(*a*) **Member FBDs:**



I **II**

FBD I: $\circlearrowleft \Sigma M_A = 0: \quad aF_1 - 2aP = 0 \quad F_1 = 2P; \uparrow \Sigma F_y = 0: \quad A_y - P = 0 \quad \mathbf{A}_y = P \uparrow$

FBD II: $\circlearrowleft \Sigma M_B = 0: \quad -aF_2 = 0 \quad F_2 = 0$

$\rightarrow \Sigma F_x = 0: \quad B_x + F_1 = 0, \quad B_x = -F_1 = -2P \quad \mathbf{B}_x = 2P \rightarrow$

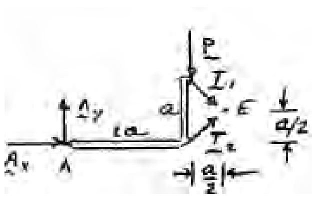
$\uparrow \Sigma F_y = 0: \quad B_y = 0$ so $\mathbf{B} = 2P \rightarrow$ ◀

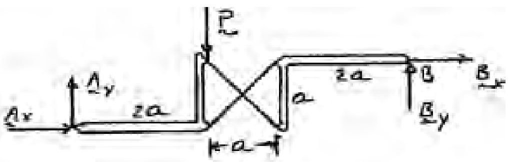
FBD I: $\rightarrow \Sigma F_x = 0: \quad -A_x - F_1 + F_2 = 0 \quad A_x = F_2 - F_1 = 0 - 2P \quad \mathbf{A}_x = 2P \rightarrow$

so $\mathbf{A} = 2.24P \nearrow 26.6°$ ◀

frame is rigid ◀

(*b*) **FBD left:** **FBD whole:**





I **II**

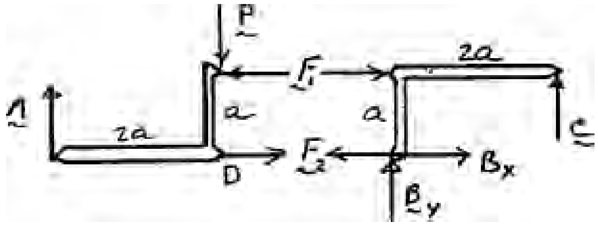
FBD I: $\circlearrowleft \Sigma M_E = 0: \quad \frac{a}{2}P + \frac{a}{2}A_x - \frac{5a}{2}A_y = 0 \quad A_x - 5A_y = -P$

FBD II: $\circlearrowleft \Sigma M_B = 0: \quad 3aP + aA_x - 5aA_y = 0 \quad A_x - 5A_y = -3P$

This is impossible unless $P = 0$ not rigid ◀

PROBLEM 6.120 (Continued)

(*c*) **Member FBDs:**



I **II**

FBD I: $\Sigma F_y = 0$: $A - P = 0$ **A** $= P\uparrow$ ◀

$\circlearrowleft \Sigma M_D = 0$: $aF_1 - 2aA = 0$ $F_1 = 2P$

$\rightarrow \Sigma F_x = 0$: $F_2 - F_1 = 0$ $F_2 = 2P$

FBD II: $\circlearrowleft \Sigma M_B = 0$: $2aC - aF_1 = 0$ $C = \dfrac{F_1}{2} = P$ **C** $= P\uparrow$ ◀

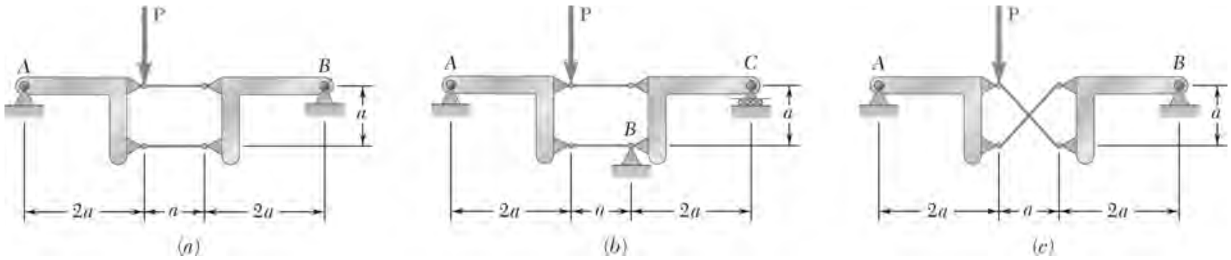
$\rightarrow \Sigma F_x = 0$: $F_1 - F_2 + B_x = 0$ $B_x = P - P = 0$

$\uparrow \Sigma F_x = 0$: $B_y + C = 0$ $B_y = -C = -P$ **B** $= P$ ◀

Frame is rigid ◀

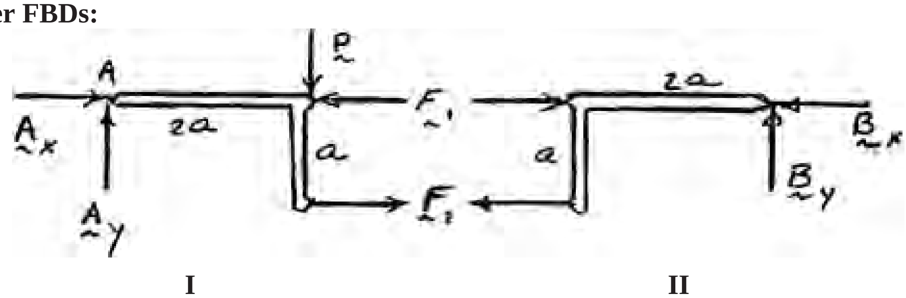
PROBLEM 6.121

Each of the frames shown consists of two L-shaped members connected by two rigid links. For each frame, determine the reactions at the supports and indicate whether the frame is rigid.



SOLUTION

(*a*) **Member FBDs:**

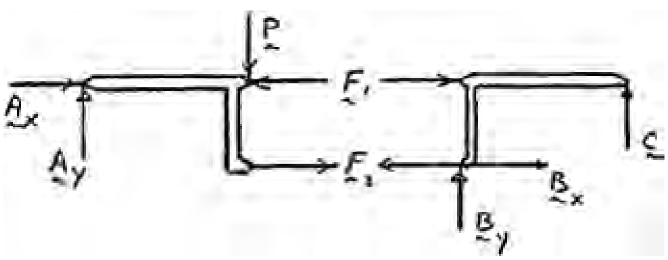


FBD II: $\uparrow \Sigma F_y = 0$: $B_y = 0$ $\circlearrowright \Sigma M_B = 0$: $aF_2 = 0$ $F_2 = 0$

FBD I: $\circlearrowleft \Sigma M_A = 0$: $aF_2 - 2aP = 0$ but $F_2 = 0$

so $P = 0$ not rigid for $P \neq 0$ ◀

(*b*) **Member FBDs:**



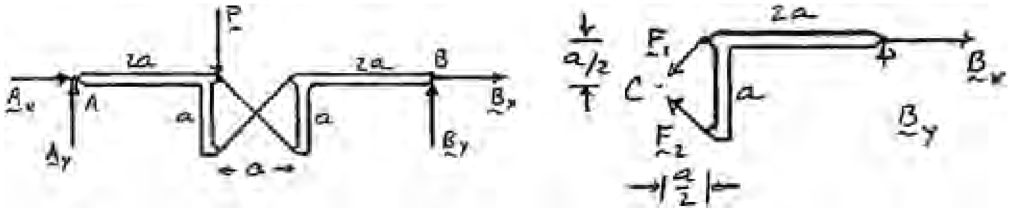
Note: 7 Unknowns $(A_x, A_y, B_x, B_y, F_1, F_2, C)$ but only 6 independent equations.

System is statically indeterminate ◀

System is, however, rigid ◀

PROBLEM 6.121 (Continued)

(c) **FBD whole:** **FBD right:**



I **II**

FBD I: $\circlearrowleft \Sigma M_A = 0: \quad 5aB_y - 2aP = 0 \qquad \mathbf{B}_y = \frac{2}{5}P \uparrow$

$\uparrow \Sigma F_y = 0: \quad A_y - P + \frac{2}{5}P = 0 \qquad \mathbf{A}_y = \frac{3}{5}P \uparrow$

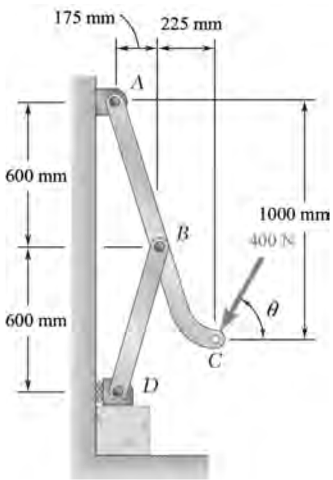
FBD II: $\circlearrowleft \Sigma M_c = 0: \quad \frac{a}{2}B_x - \frac{5a}{2}B_y = 0 \quad B_x = 5B_y \qquad \mathbf{B}_x = 2P \rightarrow$

FBD I: $\rightarrow \Sigma F_x = 0: \quad A_x + B_x = 0 \quad A_x = -B_x \qquad \mathbf{A}_x = 2P \leftarrow$

$\mathbf{A} = 2.09P \;\; \nwarrow 16.70°$ ◀

$\mathbf{B} = 2.04P \;\; \nearrow 11.31°$ ◀

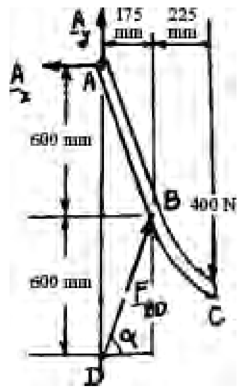
System is rigid ◀



PROBLEM 6.122

A 400-N force is applied to the toggle vise at C. Knowing that $\theta = 90°$, determine (*a*) the vertical force exerted on the block at D, (*b*) the force exerted on member ABC at B.

SOLUTION



We note that BD is a two-force member.

Free body: Member *ABC*

We have $$BD = \sqrt{(175)^2 + (600)^2} = 625 \text{ mm}$$

$$(F_{BD})_x = \frac{7}{25}F_{BD}, \quad (F_{BD})_y = \frac{24}{25}F_{BD}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad (F_{BD})_x(600) + (F_{BD})_y(175) - 400(400) = 0$$

$$\left(\frac{7}{25}F_{BD}\right)(600) + \left(\frac{24}{25}F_{BD}\right)(175) = 400(400)$$

$$336F_{BD} = 160{,}000$$

$$F_{BD} = 476.19 \text{ N}$$

$$\tan\alpha = \frac{600}{175} \quad \alpha = 73.7°$$

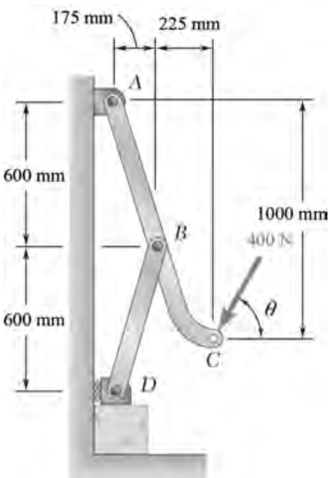
(*b*) Force exerted at *B*: $\mathbf{F}_{BD} = 476 \text{ N} \measuredangle 73.7°$ ◀

(*a*) Vertical force exerted on block

$$(F_{BD})_y = \frac{24}{25}F_{BD} = \frac{24}{25}(476.19 \text{ N}) = 457.14 \text{ N}$$

$(\mathbf{F}_{BD})_y = 457 \text{ N}\downarrow$ ◀

PROBLEM 6.123

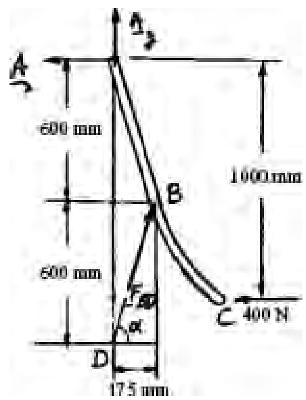


Solve Problem 6.122 when $\theta = 0$.

PROBLEM 6.122 A 400-N force is applied to the toggle vise at *C*. Knowing that $\theta = 90°$, determine (*a*) the vertical force exerted on the block at *D*, (*b*) the force exerted on member *ABC* at *B*.

SOLUTION

We note that *BD* is a two-force member.



Free body: Member *ABC*

We have $$BD = \sqrt{(175)^2 + (600)^2} = 625 \text{ mm}$$

$$(F_{BD})_x = \frac{7}{25}F_{BD}, \quad (F_{BD})_y = \frac{24}{25}F_{BD}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad (F_{BD})_x(600) + (F_{BD})_y(175) - 400(1000) = 0$$

$$\left(\frac{7}{25}F_{BD}\right)(600) + \left(\frac{24}{25}F_{BD}\right)(175) = 400(1000)$$

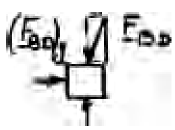
$$336\, F_{BD} = 400{,}000$$

$$F_{BD} = 1190.48 \text{ N}$$

$$\tan\alpha = \frac{24}{7} \quad \alpha = 73.7°$$

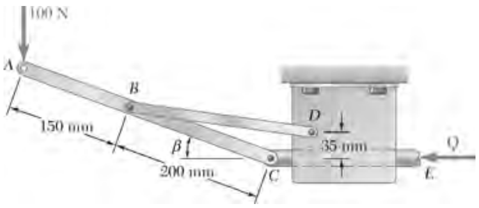
(*b*) Force exerted at *B*: $\mathbf{F}_{BD} = 1190 \text{ N} \measuredangle 73.7°$ ◀

(*a*) Vertical force exerted on block



$$(F_{BD})_y = \frac{24}{25}F_{BD} = \frac{24}{25}(1190.48 \text{ N}) = 1142.86 \text{ N}$$

$$(\mathbf{F}_{BD})_y = 1143 \text{ N} \downarrow$$ ◀



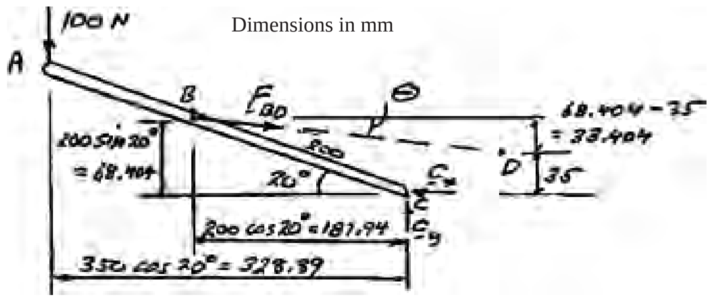
PROBLEM 6.124

The control rod CE passes through a horizontal hole in the body of the toggle system shown. Knowing that link BD is 250 mm long, determine the force $\mathbf{Q}$ required to hold the system in equilibrium when $\beta = 20°$.

SOLUTION

We note that BD is a two-force member.

Free body: Member ABC



Since $\quad BD = 250, \quad \theta = \sin^{-1}\dfrac{33.404}{250}; \quad \theta = 7.679°$

$$\circlearrowleft + \Sigma M_C = 0: \quad (F_{BD}\sin\theta)187.94 - (F_{BD}\cos\theta)68.404 + (100\text{ N})328.89 = 0$$

$$F_{BD}[187.94\sin 7.679° - 68.404\cos 7.679°] = 32889$$

$$F_{BD} = 770.6\text{ N}$$

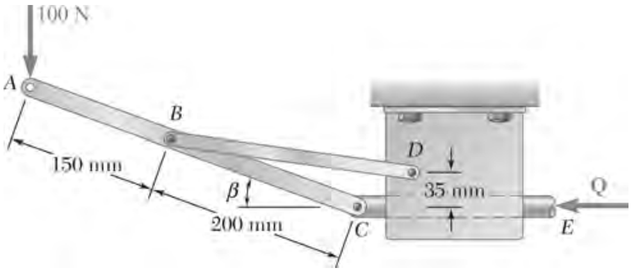
$$\xrightarrow{+} \Sigma F_x = 0: \quad (770.6\text{ N})\cos 7.679° = C_x = 0$$

$$C_x = +763.7\text{ N}$$

Member CQ:

$$\Sigma F_x = 0: \quad Q = C_x = 763.7\text{ N}$$

$\mathbf{Q} = 764\text{ N} \leftarrow$ ◀



PROBLEM 6.125

Solve Problem 6.124 when (*a*) $\beta = 0$, (*b*) $\beta = 6°$.

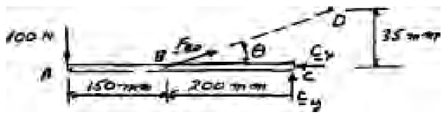
PROBLEM 6.124 The control rod *CE* passes through a horizontal hole in the body of the toggle system shown. Knowing that link *BD* is 250 mm long, determine the force **Q** required to hold the system in equilibrium when $\beta = 20°$.

SOLUTION

We note that *BD* is a two-force member.

(*a*) $\underline{\beta = 0}$:

Free body: Member *ABC*



Since $\quad BD = 250 \text{ mm}, \quad \sin\theta = \dfrac{35 \text{ mm}}{250 \text{ mm}}; \quad \theta = 8.048°$

$$+\circlearrowleft \Sigma M_C = 0: \quad (100 \text{ N})(350 \text{ mm}) - F_{BD}\sin\theta(200 \text{ mm}) = 0$$

$$F_{BD} = 1250 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{BD}\cos\theta - C_x = 0$$

$$(1250 \text{ N})(\cos 8.048°) - C_x = 0 \qquad \mathbf{C}_x = 1237.7 \text{ N}$$

Member *CE*:

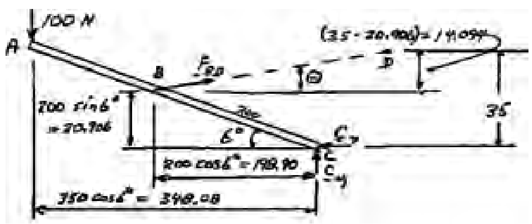
$$\xrightarrow{+} \Sigma F_x = 0: \quad (1237.7 \text{ N}) - Q = 0$$

$$Q = 1237.7 \text{ N}$$

$\mathbf{Q} = 1238 \text{ N} \leftarrow$ ◀

(*b*) $\underline{\beta = 6°}$

Free body: Member *ABC*



Dimensions in mm

Since $\quad BD = 250 \text{ mm}, \quad \theta = \sin^{-1}\dfrac{14.094 \text{ mm}}{250 \text{ mm}}$

$$\theta = 3.232°$$

$$+\circlearrowleft \Sigma M_C = 0: \quad (F_{BD}\sin\theta)198.90 + (F_{BD}\cos\theta)20.906 - (100 \text{ N})348.08 = 0$$

$$F_{BD}[198.90\sin 3.232° + 20.906\cos 3.232°] = 34808$$

$$F_{BD} = 1084.8 \text{ N}$$

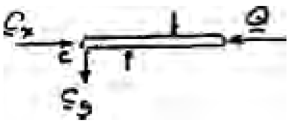
PROBLEM 6.125 (Continued)

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{BD}\cos\theta - C_x = 0$$

$$(1084.8\text{ N})\cos 3.232° - C_x = 0$$

$$C_x = +1083.1\text{ N}$$

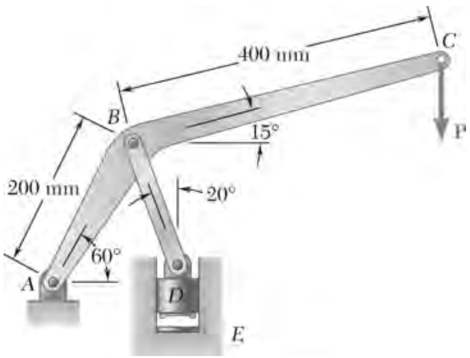
Member *DE*:



$$\Sigma F_x = 0: \quad Q = C_x$$

$$Q = 1083.1\text{ N}$$

$\mathbf{Q} = 1083\text{ N} \leftarrow$ ◀

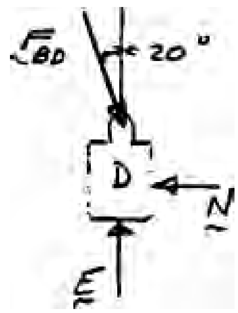


PROBLEM 6.126

The press shown is used to emboss a small seal at E. Knowing that $P = 250$ N, determine (*a*) the vertical component of the force exerted on the seal, (*b*) the reaction at A.

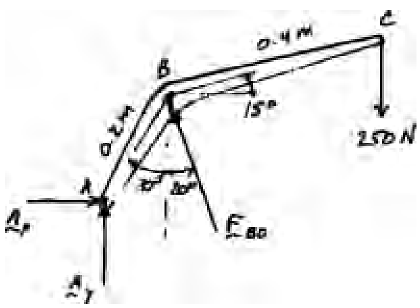
SOLUTION

FBD Stamp *D*:



$$\uparrow \Sigma F_y = 0: \quad E - F_{BD}\cos 20° = 0, \quad E = F_{BD}\cos 20°$$

FBD *ABC*:



$$\circlearrowleft \Sigma M_A = 0: \quad (0.2\text{ m})(\sin 30°)(F_{BD}\cos 20°) + (0.2\text{ m})(\cos 30°)(F_{BD}\sin 20°)$$

$$-[(0.2\text{ m})\sin 30° + (0.4\text{ m})\cos 15°](250\text{ N}) = 0$$

$$F_{BD} = 793.64\text{ N} \quad C$$

and, from above,

$$E = (793.64\text{ N})\cos 20°$$

(*a*) $\mathbf{E} = 746\text{ N}\downarrow$ ◀

$$\rightarrow \Sigma F_x = 0: \quad A_x - (793.64\text{ N})\sin 20° = 0$$

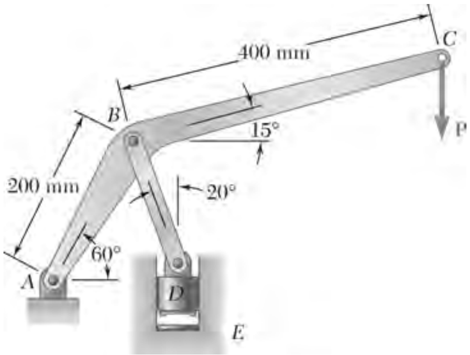
$$\mathbf{A}_x = 271.44\text{ N} \rightarrow$$

$$\uparrow \Sigma F_y = 0: \quad A_y + (793.64\text{ N})\cos 20° - 250\text{ N} = 0$$

$$\mathbf{A}_y = 495.78\text{ N}\downarrow$$

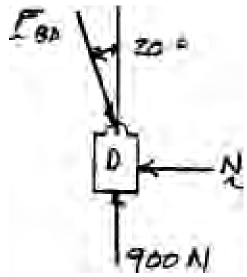
so (*b*) $\mathbf{A} = 565\text{ N} \searrow 61.3°$ ◀

PROBLEM 6.127



The press shown is used to emboss a small seal at E. Knowing that the vertical component of the force exerted on the seal must be 900 N, determine (*a*) the required vertical force **P**, (*b*) the corresponding reaction at A.

SOLUTION

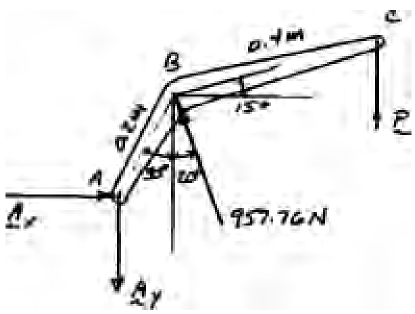


FBD Stamp *D*:

$$\uparrow \Sigma F_y = 0: \quad 900\text{ N} - F_{BD}\cos 20° = 0, \quad F_{BD} = 957.76\text{ N} \quad C$$

(*a*)

FBD *ABC*:



$$\circlearrowleft \Sigma M_A = 0: \quad [(0.2\text{ m})(\sin 30°)](957.76\text{ N})\cos 20° + [(0.2\text{ m})(\cos 30°)](957.76\text{ N})\sin 20°$$

$$-[(0.2\text{ m})\sin 30° + (0.4\text{ m})\cos 15°]P = 0$$

$$P = 301.70\text{ N}, \qquad \mathbf{P} = 302\text{ N}\downarrow \blacktriangleleft$$

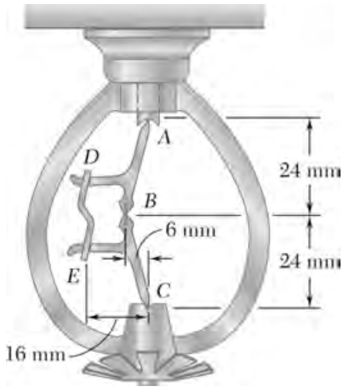
(*b*) $\rightarrow \Sigma F_x = 0: \quad A_x - (957.76\text{ N})\sin 20° = 0$

$$\mathbf{A}_x = 327.57\text{ N}\rightarrow$$

$$\uparrow \Sigma F_y = 0: \quad -A_y + (957.76\text{ N})\cos 20° - 301.70\text{ N} = 0$$

$$\mathbf{A}_y = 598.30\text{ N}\downarrow$$

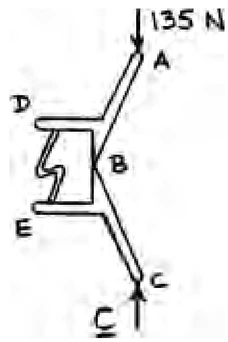
so $\qquad \mathbf{A} = 682\text{ N} \searrow 61.3° \blacktriangleleft$



PROBLEM 6.128

Water pressure in the supply system exerts a downward force of 135 N on the vertical plug at A. Determine the tension in the fusible link DE and the force exerted on member BCE at B.

SOLUTION

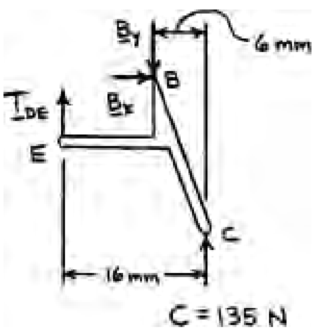


Free body: Entire linkage

$$+\Sigma F_y = 0: \quad C - 135 = 0$$

$$C = +135 \text{ N}$$

Free body: Member *BCE*



$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x = 0$$

$$\circlearrowleft + \Sigma M_B = 0: \quad (135 \text{ N})(6 \text{ mm}) - T_{DE}(10 \text{ mm}) = 0$$

$$T_{DE} = 81.0 \text{ N} \blacktriangleleft$$

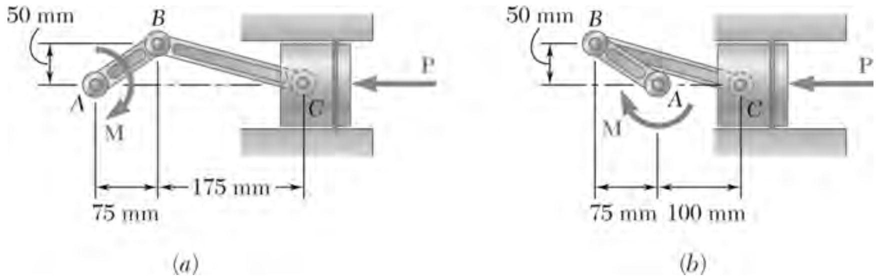
$$+\!\uparrow \Sigma F_y = 0: \quad 135 + 81 - B_y = 0$$

$$B_y = +216 \text{ N}$$

$$\mathbf{B} = 216 \text{ N} \downarrow \blacktriangleleft$$

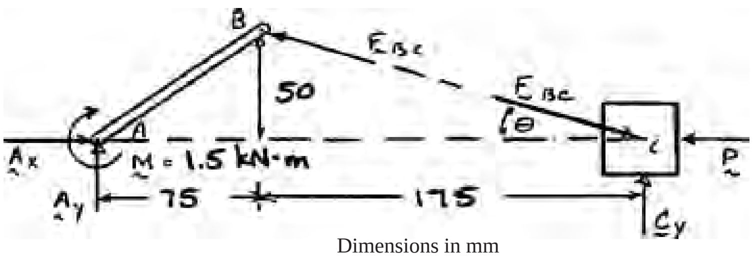
PROBLEM 6.129

A couple **M** of magnitude 1.5 kN·m is applied to the crank of the engine system shown. For each of the two positions shown, determine the force **P** required to hold the system in equilibrium.



SOLUTION

(*a*) **FBDs:**



Dimensions in mm

Note: $\tan\theta = \dfrac{50 \text{ mm}}{175 \text{ mm}}$

$= \dfrac{2}{7}$

FBD whole: $\circlearrowleft \Sigma M_A = 0$: $(0.250 \text{ m})C_y - 1.5 \text{ kN}\cdot\text{m} = 0 \quad C_y = 6.00 \text{ kN}$

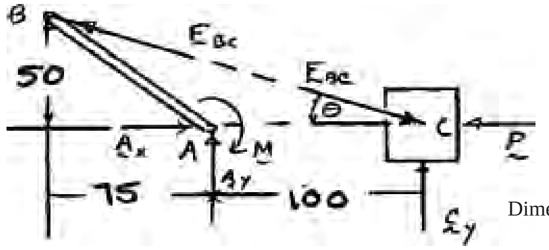
FBD piston: $\uparrow \Sigma F_y = 0$: $C_y - F_{BC}\sin\theta = 0 \quad F_{BC} = \dfrac{C_y}{\sin\theta} = \dfrac{6.00 \text{ kN}}{\sin\theta}$

$\rightarrow \Sigma F_x = 0$: $F_{BC}\cos\theta - P = 0$

$$P = F_{BC}\cos\theta = \frac{6.00 \text{ kN}}{\tan\theta} = 7 \text{ kips}$$

$\mathbf{P} = 21.0 \text{ kN} \leftarrow$ ◀

(*b*) **FBDs:**



Note: $\tan\theta = \dfrac{2}{7}$ as above

FBD whole: $\circlearrowleft \Sigma M_A = 0$: $(0.100 \text{ m})C_y - 1.5 \text{ kN}\cdot\text{m} = 0 \quad C_y = 15 \text{ kN}$

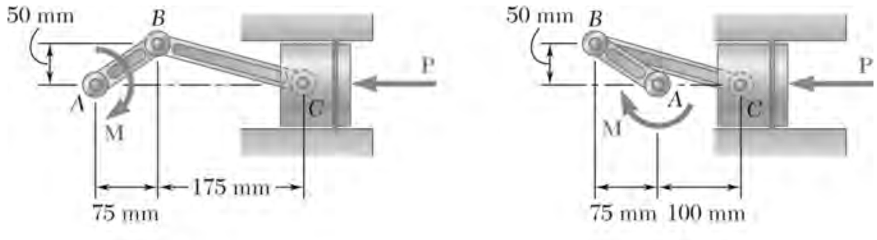
$\Sigma F_y = 0$: $C_y - F_{BC}\sin\theta = 0 \quad F_{BC} = \dfrac{C_y}{\sin\theta}$

$\rightarrow \Sigma F_x = 0$: $F_{BC}\cos\theta - P = 0$

$$P = F_{BC}\cos\theta = \frac{C_y}{\tan\theta} = \frac{15 \text{ kN}}{2/7} \qquad \mathbf{P} = 52.5 \text{ kN} \leftarrow \blacktriangleleft$$

PROBLEM 6.130

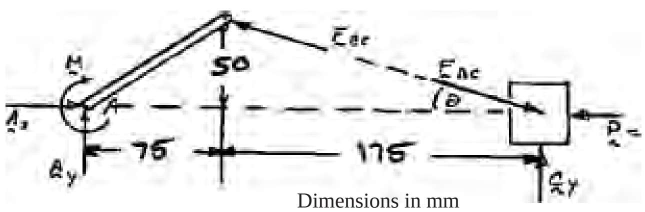
A force **P** of magnitude 16 kN is applied to the piston of the engine system shown. For each of the two positions shown, determine the couple **M** required to hold the system in equilibrium.



(*a*) (*b*)

SOLUTION

(*a*) **FBDs:**



Dimensions in mm

Note: $\tan\theta = \dfrac{50 \text{ mm}}{175 \text{ mm}}$

$= \dfrac{2}{7}$

FBD piston: $\rightarrow \Sigma F_x = 0: \quad F_{BC}\cos\theta - P = 0 \quad F_{BC} = \dfrac{P}{\cos\theta}$

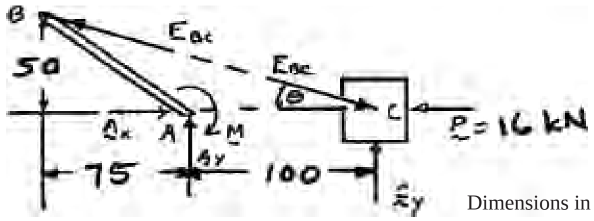
$\uparrow \Sigma F_y = 0: \quad C_y - F_{BC}\sin\theta = 0 \quad C_y = F_{BC}\sin\theta = P\tan\theta = \dfrac{2}{7}P$

FBD whole: $\circlearrowleft \Sigma M_A = 0: \quad (0.250 \text{ m})C_y - M = 0$

$$M = (0.250 \text{ m})\left(\frac{2}{7}\right)(16 \text{ kN})$$

$= 1.14286 \text{ kN}\cdot\text{m}$ $\quad$ $\mathbf{M} = 1143 \text{ N}\cdot\text{m} \circlearrowright$ ◄

(*b*) **FBDs:**



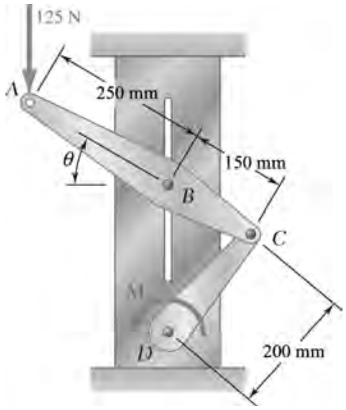
Dimensions in mm

Note: $\tan\theta = \frac{2}{7}$ as above

FBD piston: as above $C_y = P\tan\theta = \frac{2}{7}P$

FBD whole: $\circlearrowleft \Sigma M_A = 0$: $(0.100 \text{ m})C_y - M = 0$ $M = (0.100 \text{ m})\frac{2}{7}(16 \text{ kN})$

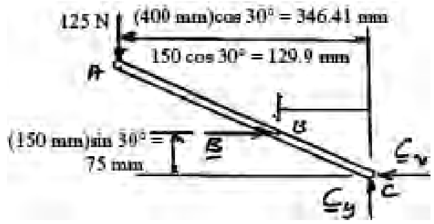
$M = 0.45714 \text{ kN}\cdot\text{m}$ $\mathbf{M} = 457 \text{ N}\cdot\text{m} \circlearrowright$ ◀



PROBLEM 6.131

The pin at B is attached to member ABC and can slide freely along the slot cut in the fixed plate. Neglecting the effect of friction, determine the couple **M** required to hold the system in equilibrium when $\theta = 30°$.

SOLUTION



<u>Free body: Member *ABC*</u>

$$+\circlearrowleft \Sigma M_C = 0: \quad (125\text{ N})(346.41\text{ mm}) - B(75\text{ mm}) = 0$$

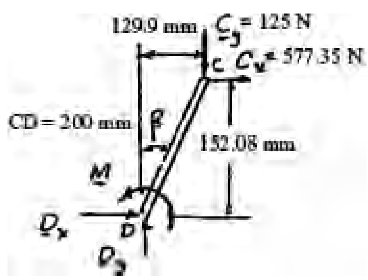
$$B = +577.35\text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad -125\text{ N} + C_y = 0$$

$$C_y = +125\text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad 577.35\text{ N} - C_x = 0$$

$$C_x = +577.35\text{ N}$$



<u>Free body: Member *CD*</u>

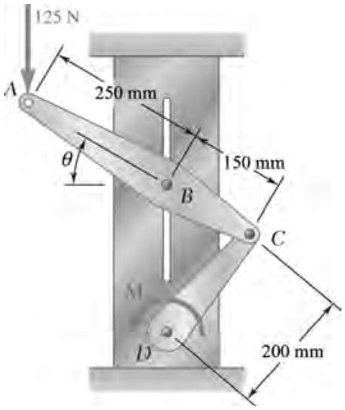
$$\beta = \sin^{-1}\frac{129.9}{200}; \quad \beta = 40.5°$$

$$CD\cos\beta = (200)\cos 40.50° = 152.08\text{ mm}$$

$$+\circlearrowleft \Sigma M_D = 0: \quad M - (125\text{ N})(129.9\text{ mm}) - (577.35\text{ N})(152.08\text{ mm}) = 0$$

$$M = +104040.89\text{ N}\cdot\text{mm}$$
$$= 104.04\text{ N.m}$$

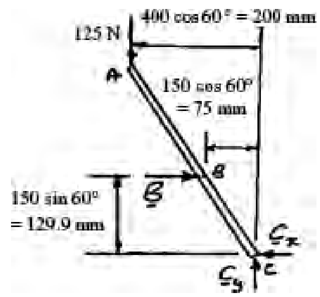
$$\mathbf{M} = 104.0\text{ N}\cdot\text{m} \blacktriangleleft$$



PROBLEM 6.132

The pin at B is attached to member ABC and can slide freely along the slot cut in the fixed plate. Neglecting the effect of friction, determine the couple **M** required to hold the system in equilibrium when $\theta = 60°$.

SOLUTION



Free body: Member ABC

$$+\circlearrowleft \Sigma M_C = 0: \quad (125 \text{ N})(200 \text{ mm}) - B(129.9 \text{ mm}) = 0$$

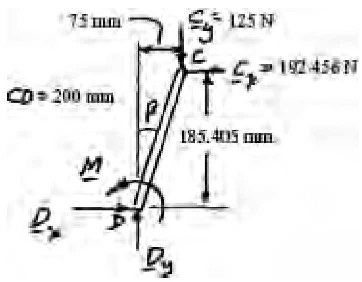
$$B = +192.456 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad 192.456 \text{ N} - C_x = 0$$

$$C_x = +192.456 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad -125 \text{ N} + C_y = 0$$

$$C_y = +125 \text{ N}$$



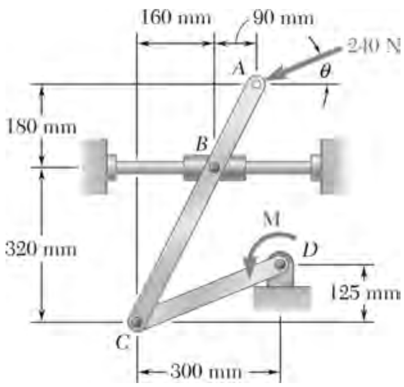
Free body: Member CD

$$\beta = \sin^{-1}\frac{75}{200}; \quad \beta = 22.024°$$

$$CD\cos\beta = (200 \text{ mm})\cos 22.024° = 185.405 \text{ mm}$$

$$+\circlearrowleft \Sigma M_D = 0: \quad M - (125 \text{ N})(75 \text{ mm}) - (192.456 \text{ N})(185.405 \text{ mm}) = 0$$

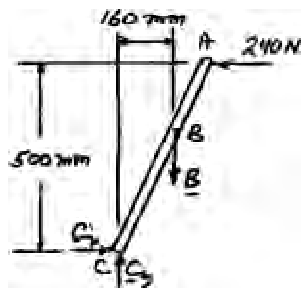
$$M = +45057.3 \text{ N}\cdot\text{mm} \qquad \mathbf{M} = 45.1 \text{ N}\cdot\text{m} \circlearrowleft \blacktriangleleft$$



PROBLEM 6.133

Arm *ABC* is connected by pins to a collar at *B* and to crank *CD* at *C*. Neglecting the effect of friction, determine the couple **M** required to hold the system in equilibrium when $\theta = 0$.

SOLUTION



<u>Free body: Member *ABC*</u>

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - 240\text{ N} = 0$$

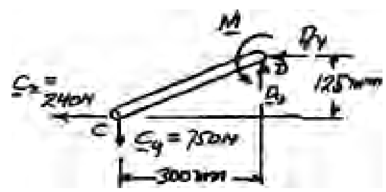
$$C_x = +240\text{ N}$$

$$\circlearrowleft + \Sigma M_C = 0: \quad (240\text{ N})(500\text{ mm}) - B(160\text{ mm}) = 0$$

$$B = +750\text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - 750\text{ N} = 0$$

$$C_y = +750\text{ N}$$

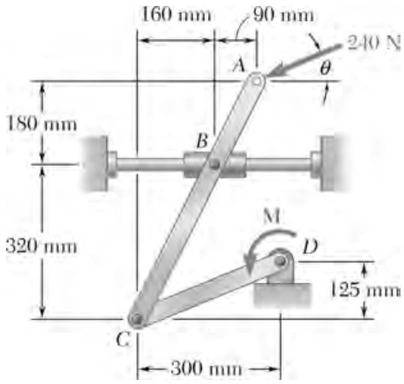


<u>Free body: Member *CD*</u>

$$\circlearrowleft + \Sigma M_D = 0: \quad M + (750\text{ N})(300\text{ mm}) - (240\text{ N})(125\text{ mm}) = 0$$

$$M = -195 \times 10^3\text{ N} \cdot \text{mm}$$

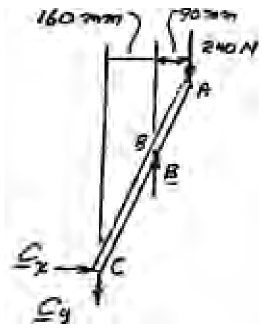
$$\mathbf{M} = 195.0\text{ kN} \cdot \text{m} \circlearrowright \blacktriangleleft$$



PROBLEM 6.134

Arm ABC is connected by pins to a collar at B and to crank CD at C. Neglecting the effect of friction, determine the couple **M** required to hold the system in equilibrium when $\theta = 90°$.

SOLUTION

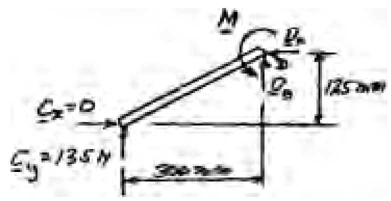


Free body: Member ABC

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x = 0$$

$$\circlearrowleft + \Sigma M_B = 0: \quad C_y(160 \text{ mm}) - (240 \text{ N})(90 \text{ mm}) = 0$$

$$C_y = +135 \text{ N}$$

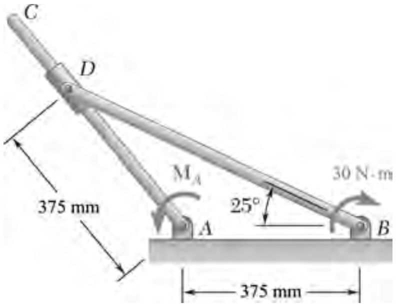


Free body: Member CD

$$\circlearrowleft + \Sigma M_D = 0: \quad M - (135 \text{ N})(300 \text{ mm}) = 0$$

$$M = +40.5 \times 10^3 \text{ N} \cdot \text{mm}$$

$$\mathbf{M} = 40.5 \text{ kN} \cdot \text{m} \circlearrowleft \blacktriangleleft$$



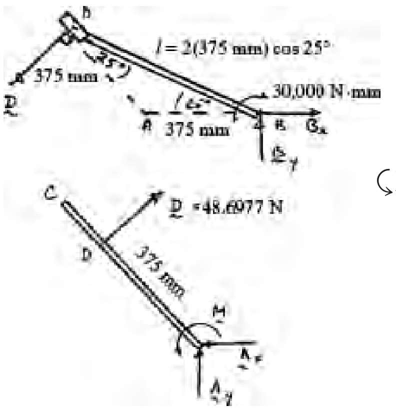
PROBLEM 6.135

Two rods are connected by a slider block as shown. Neglecting the effect of friction, determine the couple $\mathbf{M}_A$ required to hold the system in equilibrium.

SOLUTION

Note: $\mathbf{D} \perp AC$

Member FBDs:



$$\circlearrowleft \Sigma M_B = 0: \quad lD\sin 115° - 30{,}000 \text{ N}\cdot\text{mm} = 0$$

$$D = \frac{30000 \text{ N}\cdot\text{mm}}{2(375 \text{ mm})\cos 25° \sin 115°}$$

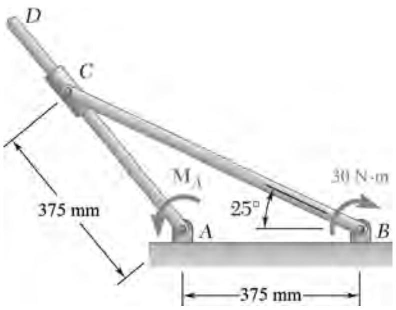
$$= 48.6977 \text{ N}$$

$$\circlearrowleft \Sigma M_A = 0: \quad M_A - (375 \text{ mm})(48.6977 \text{ N}) = 0$$

$$M_A = 18261.64 \text{ N}\cdot\text{mm}$$

$$= 18.26 \text{ N.m}$$

$$\mathbf{M}_A = 18.26 \text{ N}\cdot\text{m} \circlearrowleft \blacktriangleleft$$



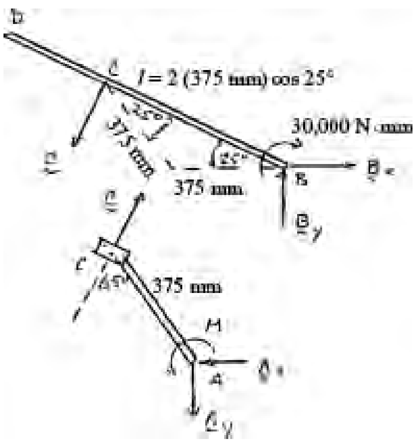
PROBLEM 6.136

Two rods are connected by a slider block as shown. Neglecting the effect of friction, determine the couple $\mathbf{M}_A$ required to hold the system in equilibrium.

SOLUTION

Note: $\mathbf{C} \perp BD$

Member FBD's:



$$\circlearrowleft \Sigma M_B = 0: \quad lC - 30{,}000\ \text{N}\cdot\text{mm} = 0$$

$$C = \frac{30{,}000\ \text{N}\cdot\text{mm}}{2(375\ \text{mm})\cos 25°}$$

$$C = 44.135\ \text{N}$$

$$\circlearrowleft \Sigma M_A = 0: \quad M_A - (375\ \text{mm})C\sin 65° = 0$$

$$M_A = (375\ \text{mm})(44.135\ \text{N})\sin 65°$$

$$M_A = 14999.96\ \text{N}\cdot\text{mm}$$

$$= 15.00\ \text{N.m}$$

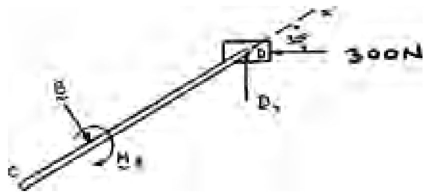
$$\mathbf{M}_A = 15.00\ \text{N}\cdot\text{m} \circlearrowleft \blacktriangleleft$$

PROBLEM 6.137

Rod *CD* is attached to the collar *D* and passes through a collar welded to end *B* of lever *AB*. Neglecting the effect of friction, determine the couple **M** required to hold the system in equilibrium when $\theta = 30°$.

SOLUTION

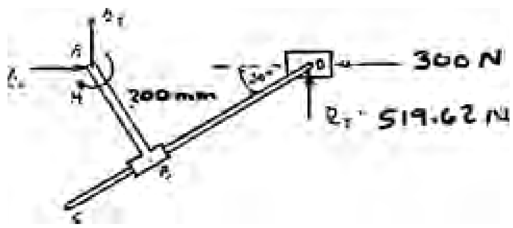
Note: $\mathbf{B} \perp CD$



FBD *DC*: $\nearrow \Sigma F_{x'} = 0: \quad D_y \sin 30° - (300\text{ N})\cos 30° = 0$

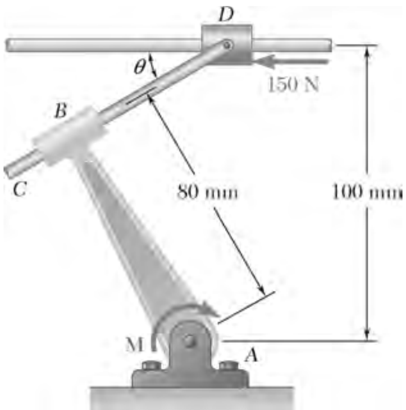
$$D_y = \frac{300\text{ N}}{\tan 30°} = 519.62\text{ N}$$

FBD machine:



$$\circlearrowleft \Sigma M_A = 0: \quad \frac{0.200\text{ m}}{\sin 30°} 519.62\text{ N} - M = 0$$

$$M = 207.85\text{ N}\cdot\text{m} \qquad \mathbf{M} = 208\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

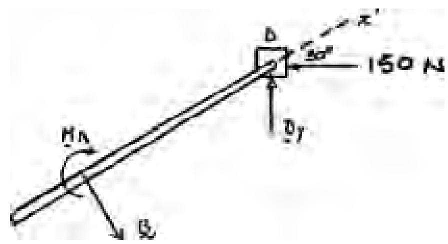


PROBLEM 6.138

Rod CD is attached to the collar D and passes through a collar welded to end B of lever AB. Neglecting the effect of friction, determine the couple $\mathbf{M}$ required to hold the system in equilibrium when $\theta = 30°$.

SOLUTION

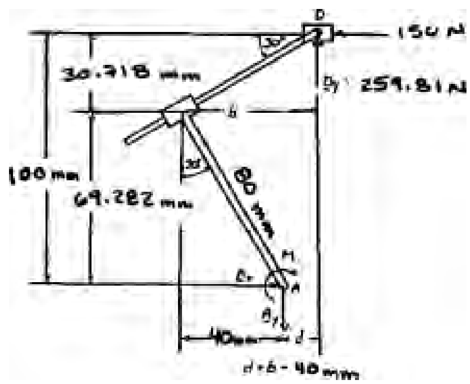
FBD *DC*:



$$\nearrow \Sigma F_{x'} = 0: \quad D_y \sin 30° - (150 \text{ N})\cos 30° = 0$$

$$D_y = (150 \text{ N}) \text{ ctn } 30° = 259.81 \text{ N}$$

FBD machine:



$$\circlearrowleft \Sigma M_A = 0: \quad (0.100 \text{ m})(150 \text{ N}) + d(259.81 \text{ N}) - M = 0$$

$$d = b - 0.040 \text{ m}$$

$$b = \frac{0.030718 \text{ m}}{\tan 30}$$

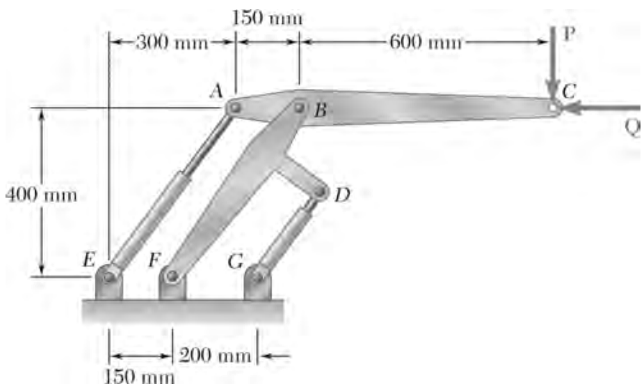
so

$$b = 0.053210 \text{ m}$$

$$d = 0.0132100 \text{ m}$$

$$M = 18.4321 \text{ N} \cdot \text{m}$$

$$\mathbf{M} = 18.43 \text{ N} \cdot \text{m} \circlearrowright \blacktriangleleft$$

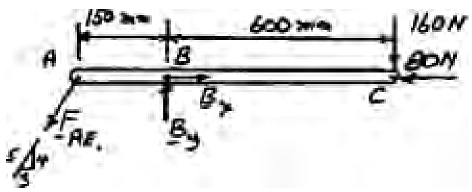


PROBLEM 6.139

Two hydraulic cylinders control the position of the robotic arm *ABC*. Knowing that in the position shown the cylinders are parallel, determine the force exerted by each cylinder when $P = 160$ N and $Q = 80$ N.

SOLUTION

Free body: Member *ABC*



$$\circlearrowleft^{+} \Sigma M_B = 0: \quad \frac{4}{5}F_{AE}(150 \text{ mm}) - (160 \text{ N})(600 \text{ mm}) = 0$$

$F_{AE} = +800$ N $\qquad F_{AE} = 800 \text{ N} \quad T$ ◀

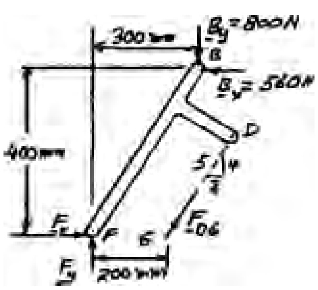
$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{3}{5}(800 \text{ N}) + B_x - 80 \text{ N} = 0$$

$$B_x = +560 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad -\frac{4}{5}(800 \text{ N}) + B_y - 160 \text{ N} = 0$$

$$B_y = +800 \text{ N}$$

Free body: Member *BDF*

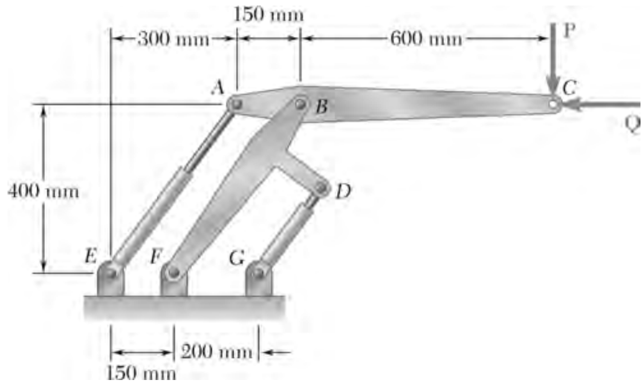


$$\circlearrowleft^{+} \Sigma M_F = 0: \quad (560 \text{ N})(400 \text{ mm}) - (800 \text{ N})(300 \text{ mm}) - \frac{4}{5}F_{DG}(200 \text{ mm}) = 0$$

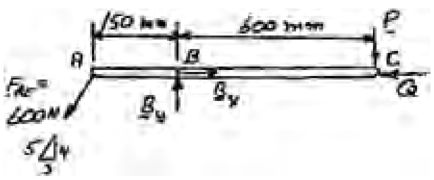
$F_{DG} = -100$ N $\qquad F_{DG} = 100.0 \text{ N} \quad C$ ◀

PROBLEM 6.140

Two hydraulic cylinders control the position of the robotic arm *ABC*. In the position shown, the cylinders are parallel and both are in tension. Knowing the $F_{AE} = 600$ N and $F_{DG} = 50$ N, determine the forces **P** and **Q** applied at *C* to arm *ABC*.



SOLUTION



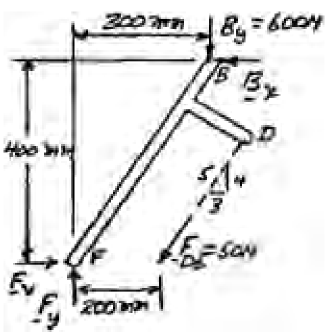
Free body: Member *ABC*

$$+\circlearrowleft \Sigma M_B = 0: \quad \frac{4}{5}(600\text{ N})(150\text{ mm}) - P(600\text{ mm}) = 0$$

$$P = +120\text{ N} \qquad \mathbf{P} = 120.0\text{ N}\downarrow ◀$$

$$+\circlearrowleft \Sigma M_C = 0: \quad \frac{4}{5}(600)(750\text{ mm}) - B_y(600\text{ mm}) = 0$$

$$B_y = +600\text{ N}$$



Free body: Member *BDF*

$$+\circlearrowleft \Sigma M_F = 0: \quad B_x(400\text{ mm}) - (600\text{ N})(300\text{ mm})$$

$$-\frac{4}{5}(50\text{ N})(200\text{ mm}) = 0$$

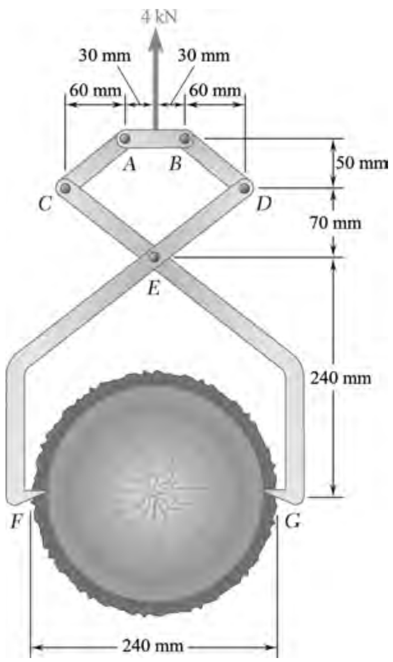
$$B_x = +470\text{ N}$$

Return to free body: Member *ABC*

$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{3}{5}(600\text{ N}) + 470\text{ N} - Q = 0$$

$$Q = +110\text{ N} \qquad \mathbf{Q} = 110.0\text{ N}\leftarrow ◀$$

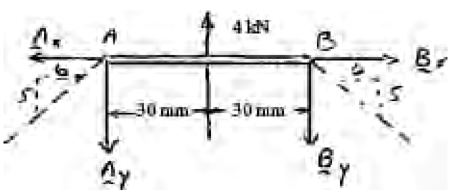
PROBLEM 6.141



A log weighing 4 kN is lifted by a pair of tongs as shown. Determine the forces exerted at *E* and *F* on tong *DEF*.

SOLUTION

FBD *AB*:



By symmetry: $A_y = B_y = 2 \text{ kN}$

and $A_x = B_x$

$$= \frac{6}{5}(2 \text{ kN})$$

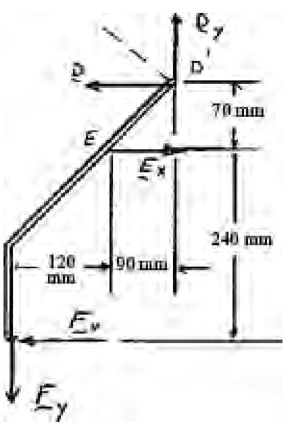
$$= 2.4 \text{ kN}$$

Note: $\mathbf{D} = -\mathbf{B}$

so $D_x = 2.4 \text{ kN}$

$D_y = 2 \text{ kN}$

FBD *DEF*:



$$\circlearrowleft \Sigma M_F = (210 \text{ mm})(2 \text{ kN}) + (310 \text{ mm})(2.4 \text{ kN}) - (240 \text{ mm})E_x = 0$$

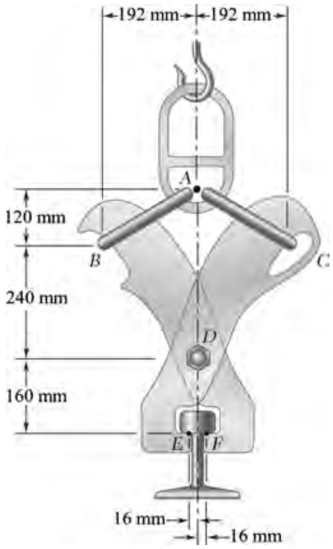
$E_x = 4.85 \text{ kN}$ $\qquad \mathbf{E} = 4.85 \text{ kN} \rightarrow$ ◀

$$\rightarrow \Sigma F_x = 0: \quad -2.4 \text{ kN} + 4.85 \text{ kN} - F_x = 0$$

$$F_x = 2.45 \text{ kN}$$

$$\uparrow \Sigma F_y = 0: \quad 2 \text{ kN} - F_y = 0$$

$F_y = 2 \text{ kN}$ $\qquad \mathbf{F} = 3.16 \text{ kN} \swarrow 39.2°$ ◀

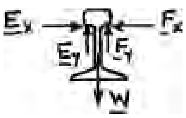


PROBLEM 6.142

A 12 m length of railroad rail of weight 750 N/m is lifted by the tongs shown. Determine the forces exerted at D and F on tong BDF.

SOLUTION

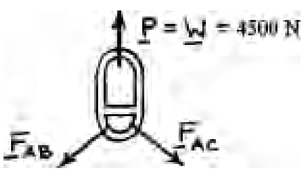
Free body: Rail



$$W = (12\text{ m})(750\text{ N/m}) = 9000\text{ N}$$

By symmetry $\quad E_y = F_y = \frac{1}{2}W = 4500\text{ N}$

Free body: Upper link



By symmetry, $\quad (F_{AB})_y = (F_{AC})_y = \frac{1}{2}W = 4500\text{ N}$

Since AB is a two-force member,

$$\frac{(F_{AB})_x}{192} = \frac{(F_{AB})_y}{120} \qquad (F_{AB})_x = \frac{192}{120}(4500) = 7200\text{ N}$$

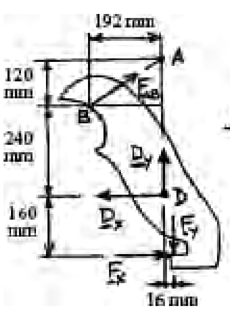
$+\circlearrowleft \Sigma M_D = 0$: (Attach F_{AB} at A)

$$F_x(160) - (F_{AB})_x(360) - F_y(16) = 0$$

$$F_x(160) - (7200\text{ N})(360) - (4500\text{ N})(16) = 0$$

$$F_x = +16650\text{ N} \qquad \mathbf{F} = 17250\text{ N} \searrow 15.12° \blacktriangleleft$$

Free Body: Tong BDF

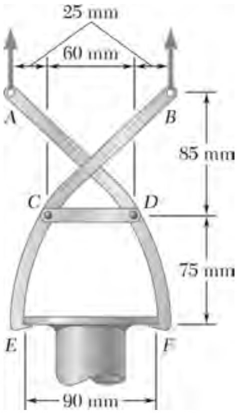


$\xrightarrow{+} \Sigma F_x = 0$: $\quad -D_x + (F_{AB})_x + F_x = 0$

$$D_x = (F_{AB})_x + F_x = 7200 + 16650 = 23850\text{ N}$$

$+\uparrow \Sigma F_y = 0$: $\quad D_y + (F_{AB})_y - F_y = 0$

$$D_y = 0 \qquad \mathbf{D} = 23900\text{ N} \leftarrow \blacktriangleleft$$

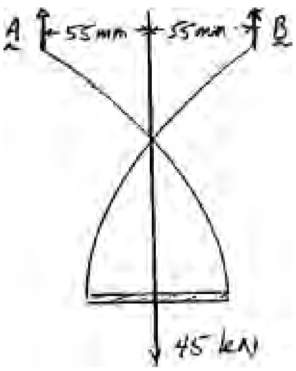


PROBLEM 6.143

The tongs shown are used to apply a total upward force of 45 kN on a pipe cap. Determine the forces exerted at D and F on tong ADF.

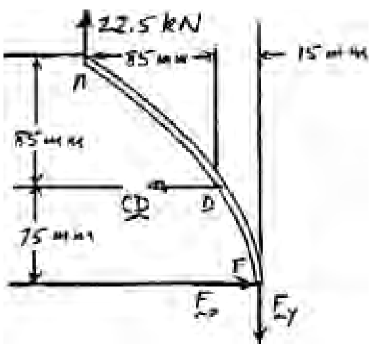
SOLUTION

FBD whole:



By symmetry $\quad \mathbf{A} = \mathbf{B} = 22.5 \text{ kN} \uparrow$

FBD *ADF*:



$$\circlearrowleft \Sigma M_F = 0: \quad (75 \text{ mm})CD - (100 \text{ mm})(22.5 \text{ kN}) = 0$$

$$\mathbf{CD} = 30.0 \text{ kN} \leftarrow \blacktriangleleft$$

$$\rightarrow \Sigma F_x = 0: \quad F_x - CD = 0$$

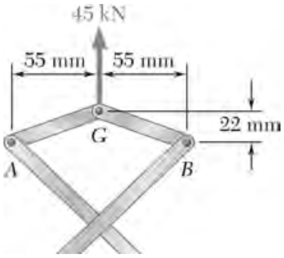
$$F_x = CD = 30 \text{ kN}$$

$$\uparrow \Sigma F_y = 0: \quad 22.5 \text{ kN} - F_y = 0$$

$$F_y = 22.5 \text{ kN}$$

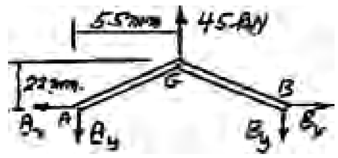
so $\quad \mathbf{F} = 37.5 \text{ kN} \measuredangle 36.9° \blacktriangleleft$

PROBLEM 6.144



If the toggle shown is added to the tongs of Problem 6.143 and a single vertical force is applied at G, determine the forces exerted at D and F on tong ADF.

SOLUTION



Free body: Toggle

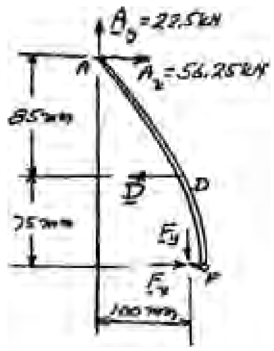
By symmetry $\quad A_y = \frac{1}{2}(45 \text{ kN}) = 22.5 \text{ kN}$

AG is a two-force member

$$\frac{22.5 \text{ kN}}{22 \text{ mm}} = \frac{A_x}{55 \text{ mm}}$$

$$A_x = 56.25 \text{ kN}$$

Free body: Tong *ADF*



$$+\uparrow \Sigma F_y = 0: \quad 22.5 \text{ kN} - F_y = 0$$

$$F_y = +22.5 \text{ kN}$$

$$+\circlearrowleft \Sigma M_F = 0: \quad D(75 \text{ mm}) - (22.5 \text{ kN})(100 \text{ mm}) - (56.25 \text{ kN})(160 \text{ mm}) = 0$$

$$D = +150 \text{ kN}$$

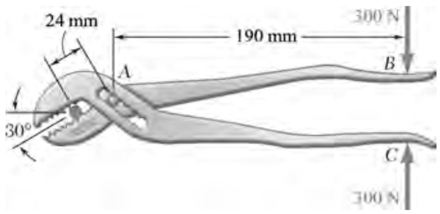
$\mathbf{D} = 150.0 \text{ kN} \leftarrow$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad 56.25 \text{ kN} - 150 \text{ kN} + F_x = 0$$

$$F_x = 93.75 \text{ kN}$$

$\mathbf{F} = 96.4 \text{ kN} \searrow 13.50°$ ◀

PROBLEM 6.145



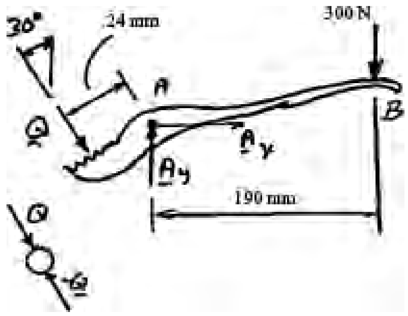
The pliers shown are used to grip a 6 mm-diameter rod. Knowing that two 300 N forces are applied to the handles, determine (*a*) the magnitude of the forces exerted on the rod, (*b*) the force exerted by the pin at *A* on portion *AB* of the pliers.

SOLUTION

<u>Free body: Portion *AB*</u>

(*a*) $\circlearrowleft^{+} \Sigma M_A = 0: \quad Q(24 \text{ mm}) - (300 \text{ N})(190 \text{ mm}) = 0 \Rightarrow Q = 2375 \text{ N}$

$Q = 2380 \text{ N}$ ◀



(*b*) $\xrightarrow{+} \Sigma F_x = 0: \quad Q(\sin 30°) + A_x = 0$

$(2375 \text{ N})(\sin 30°) + A_x = 0$

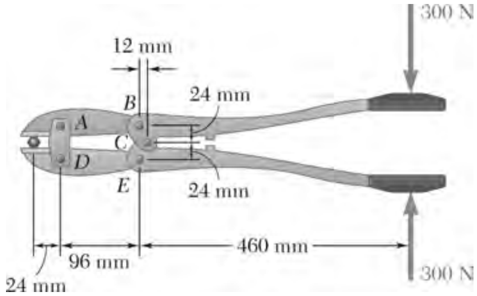
$A_x = -1187.5 \text{ N} \qquad \mathbf{A}_y = 1188 \text{ N} \leftarrow$

$+\uparrow \Sigma F_y = 0: \quad -Q(\cos 30°) + A_y - 300 \text{ N} = 0$

$-(2375 \text{ N})(\cos 30°) + A_y - 300 \text{ N} = 0$

$A_y = +2356.81 \text{ N} \qquad \mathbf{A}_y = 2360 \text{ N} \uparrow$

$\mathbf{A} = 2640 \text{ N} \measuredangle\, 63.3°$ ◀

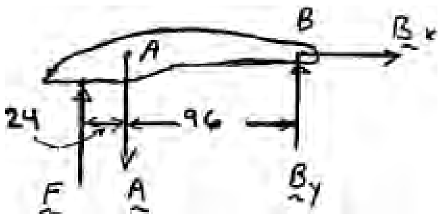


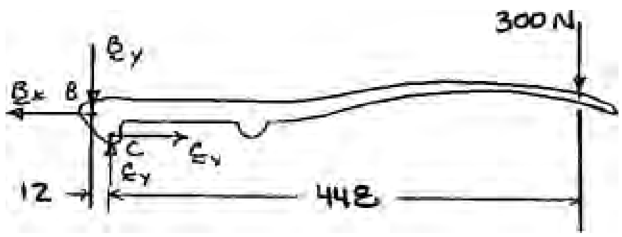
PROBLEM 6.146

In using the bolt cutter shown, a worker applies two 300-N forces to the handles. Determine the magnitude of the forces exerted by the cutter on the bolt.

SOLUTION

FBD cutter *AB*: **FBD handle *BC*:**





I Dimensions in mm **II**

FBD I: $\rightarrow \Sigma F_x = 0: \quad B_x = 0$

FBD II: $\circlearrowleft \Sigma M_C = 0: \quad (12 \text{ mm})B_y - (448 \text{ mm})300 \text{ N} = 0$

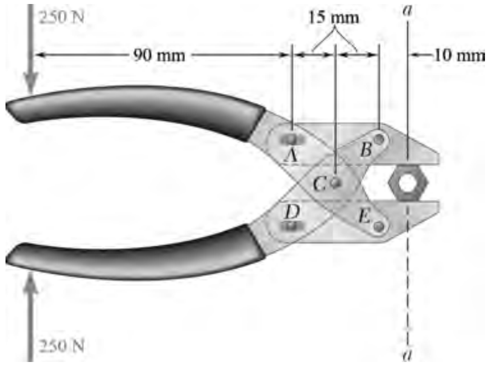
$$B_y = 11{,}200.0 \text{ N}$$

Then FBD I: $\circlearrowleft \Sigma M_A = 0: \quad (96 \text{ mm})B_y - (24 \text{ mm})F = 0$

$$F = 4B_y$$

$$F = 44{,}800 \text{ N} = 44.8 \text{ kN} \blacktriangleleft$$

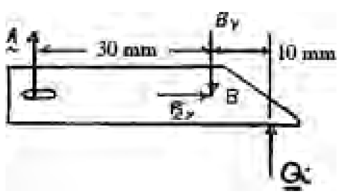
PROBLEM 6.147



Determine the magnitude of the gripping forces exerted along line aa on the nut when two 250 N forces are applied to the handles as shown. Assume that pins A and D slide freely in slots cut in the jaws.

SOLUTION

FBD jaw *AB*:



$\rightarrow \Sigma F_x = 0: \quad B_x = 0$

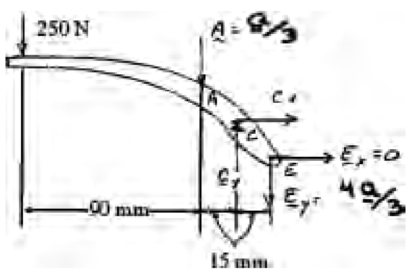
$\circlearrowleft \Sigma M_B = 0: \quad (10 \text{ mm})Q - (30 \text{ mm})A = 0$

$$A = \frac{Q}{3}$$

$\uparrow \Sigma F_y = 0: \quad A + Q - B_y = 0$

$$B_y = A + Q = \frac{4Q}{3}$$

FBD handle *ACE*: By symmetry and FBD jaw DE:



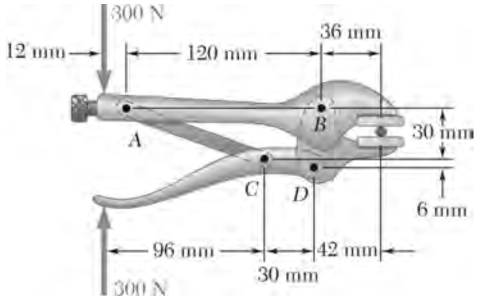
$$D = A = \frac{Q}{3}$$

$$E_x = B_x = 0$$

$$E_y = B_y = \frac{4Q}{3}$$

$\circlearrowleft \Sigma M_C = 0: \quad (105 \text{ mm})(250 \text{ N}) + (15 \text{ mm})\frac{Q}{3} - (15 \text{ mm})\frac{4Q}{3} = 0$

$Q = 1750 \text{ N}$ ◀



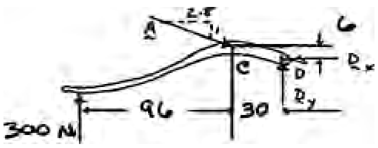
PROBLEM 6.148

Determine the magnitude of the gripping forces produced when two 300 N forces are applied as shown.

SOLUTION

We note that *AC* is a two-force member

FBD handle *CD*:

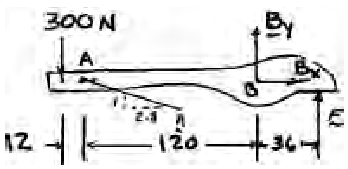


Dimensions in mm

$$\circlearrowleft \Sigma M_D = 0: \quad -(126 \text{ mm})(300 \text{ N}) - (6 \text{ mm})\frac{2.8}{\sqrt{8.84}}A + (30 \text{ mm})\left(\frac{1}{\sqrt{8.84}}A\right) = 0$$

$$A = 2863.6\sqrt{8.84} \text{ N}$$

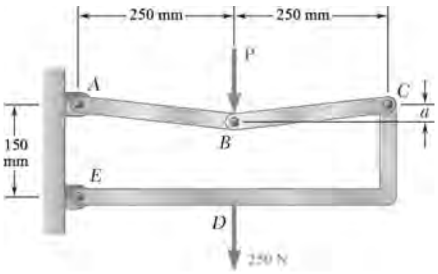
FBD handle *AB*:



Dimensions in mm

$$\circlearrowleft \Sigma M_B = 0: \quad (132 \text{ mm})(300 \text{ N}) - (120 \text{ mm})\frac{1}{\sqrt{8.84}}(2863.6\sqrt{8.84} \text{ N}) + (36 \text{ mm})F = 0$$

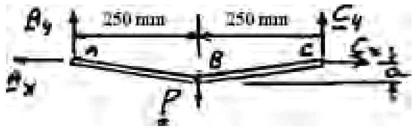
$$F = 8.45 \text{ kN} \blacktriangleleft$$



PROBLEM 6.149

Knowing that the frame shown has a sag at B of $a = 25$ mm, determine the force **P** required to maintain equilibrium in the position shown.

SOLUTION



We note that AB and BC are two-force members

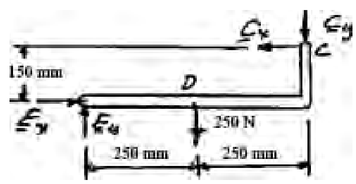
Free body: Toggle

By symmetry:

$$C_y = \frac{P}{2}$$

$$\frac{C_x}{250 \text{ mm}} = \frac{C_y}{a}$$

$$C_x = \frac{250}{a} C_y = \frac{250}{a} \cdot \frac{P}{2} = \frac{125P}{a}$$



Free body: Member *CDE*

$$+\circlearrowleft \Sigma M_E = 0: \quad C_x(150 \text{ mm}) - C_y(500 \text{ mm}) - (250 \text{ N})(250 \text{ mm}) = 0$$

$$\frac{125P}{a}(150) - \frac{P}{2}(500) = 62500$$

$$P\left(\frac{75}{a} - 1\right) = 250 \qquad (1)$$

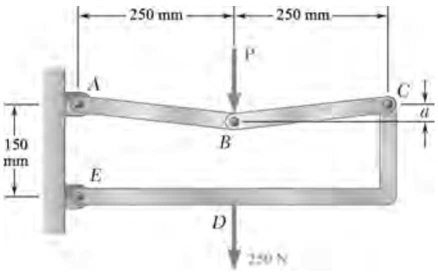
For

$$a = 25 \text{ mm}$$

$$2P = 250$$

$$P = 125 \text{ N}$$

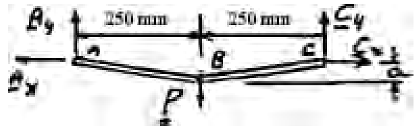
$$\mathbf{P} = 125 \text{ N} \downarrow \blacktriangleleft$$



PROBLEM 6.150

Knowing that the frame shown has a sag at B of $a = 12.5$ mm, determine the force **P** required to maintain equilibrium in the position shown.

SOLUTION



We note that AB and BC are two-force members

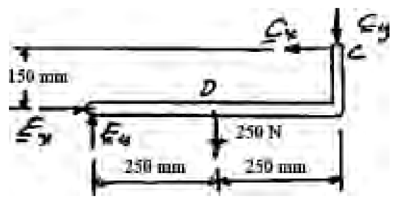
Free body: Toggle

By symmetry:

$$C_y = \frac{P}{2}$$

$$\frac{C_x}{250 \text{ mm}} = \frac{C_y}{a}$$

$$C_x = \frac{250}{a}C_y = \frac{250}{a} \cdot \frac{P}{2} = \frac{125P}{a}$$



Free body: Member CDE

$$+\circlearrowleft \Sigma M_E = 0: \quad C_x(150 \text{ mm}) - C_y(500 \text{ mm}) - (250 \text{ N})(250 \text{ mm}) = 0$$

$$\frac{125P}{a}(150) - \frac{P}{2}(500) = 62500$$

$$P\left(\frac{75}{a} - 1\right) = 250 \qquad (1)$$

For $\qquad a = 12.5$ mm

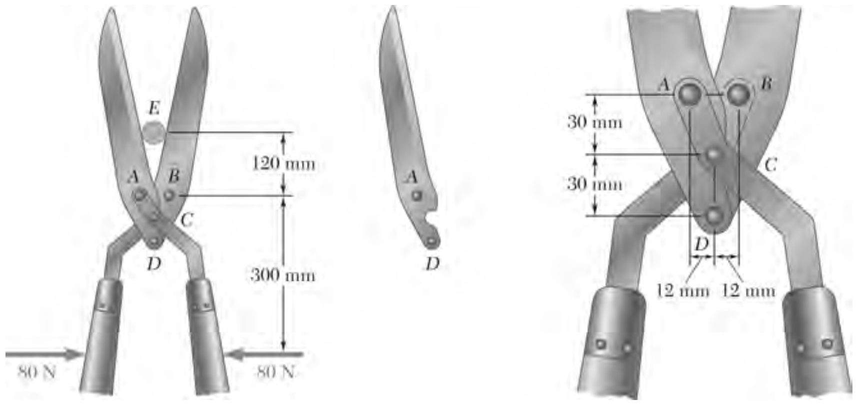
$$5P = 250 \text{ N}$$

$$P = 50 \text{ N}$$

$\mathbf{P} = 50.0 \text{ N}\downarrow$ ◀

PROBLEM 6.151

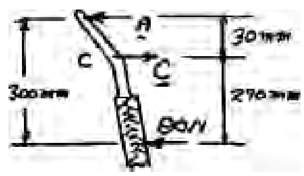
The garden shears shown consist of two blades and two handles. The two handles are connected by pin C and the two blades are connected by pin D. The left blade and the right handle are connected by pin A; the right blade and the left handle are connected by pin B. Determine the magnitude of the forces exerted on the small branch at E when two 80-N forces are applied to the handles as shown.



SOLUTION

By symmetry vertical components C_y, D_y, E_y are 0. Then by considering $\Sigma F_y = 0$ on the blades or handles, we find that A_y and B_y are 0.

Thus forces at A, B, C, D, and E are horizontal.



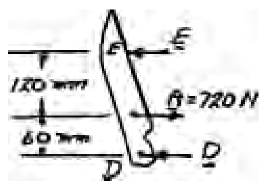
Free body: Right handle

$$\circlearrowleft + \Sigma M_C = 0: \quad A(30 \text{ mm}) - (80 \text{ N})(270 \text{ mm}) = 0$$

$$A = +720 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C - 720 \text{ N} - 80 \text{ N} = 0$$

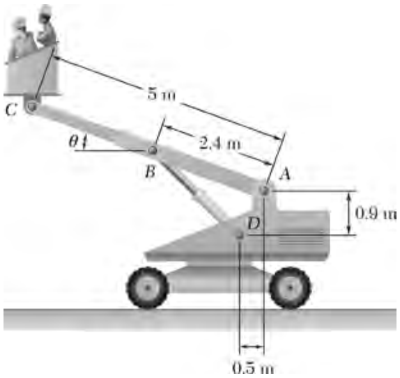
$$C = +800 \text{ N}$$



Free body: Left blade

$$\circlearrowleft + \Sigma M_D = 0: \quad E(180 \text{ mm}) - (720 \text{ N})(60 \text{ mm}) = 0$$

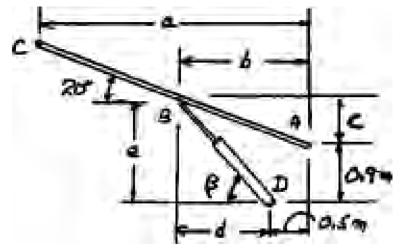
$$E = 240 \text{ N} \blacktriangleleft$$



PROBLEM 6.152

The telescoping arm *ABC* is used to provide an elevated platform for construction workers. The workers and the platform together have a mass of 200 kg and have a combined center of gravity located directly above *C*. For the position when $\theta = 20°$, determine (*a*) the force exerted at *B* by the single hydraulic cylinder *BD*, (*b*) the force exerted on the supporting carriage at *A*.

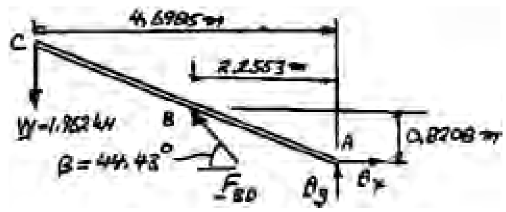
SOLUTION



<u>Geometry</u>:

$$a = (5\text{ m})\cos 20° = 4.6985\text{ m}$$
$$b = (2.4\text{ m})\cos 20° = 2.2553\text{ m}$$
$$c = (2.4\text{ m})\sin 20° = 0.8208\text{ m}$$
$$d = b - 0.5 = 1.7553\text{ m}$$
$$e = c + 0.9 = 1.7208\text{ m}$$

$$\tan\beta = \frac{e}{d} = \frac{1.7208}{1.7553}; \quad \beta = 44.43°$$



<u>Free body: Arm *ABC*</u>

We note that *BD* is a two-force member

$$W = (200\text{ kg})(9.81\text{ m/s}^2) = 1.962\text{ kN}$$

(*a*) $\circlearrowleft\!+\ \Sigma M_A = 0: \quad (1.962\text{ kN})(4.6985\text{ m}) - F_{BD}\sin 44.43°(2.2553\text{ m}) + F_{BD}\cos 44.43(0.8208\text{ m}) = 0$

$$9.2185 - F_{BD}(0.9927) = 0: \quad F_{BD} = 9.2867\text{ kN}$$

$\mathbf{F}_{BD} = 9.29\text{ kN} \nwarrow 44.4°$ ◀

(*b*)

$$\xrightarrow{+}\Sigma F_x = 0: \quad A_x - F_{BD}\cos\beta = 0$$

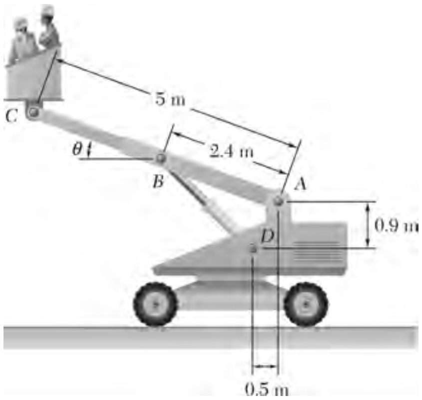
$$A_x = (9.2867\text{ kN})\cos 44.43° = 6.632\text{ kN} \qquad \mathbf{A}_x = 6.632\text{ kN} \rightarrow$$

$$+\!\uparrow\Sigma F_y = 0: \quad A_y - 1.962\text{ kN} + F_{BD}\sin\beta = 0$$

$$A_y = 1.962\text{ kN} - (9.2867\text{ kN})\sin 44.43° = -4.539\text{ kN}$$

$\mathbf{A}_y = 4.539\text{ kN}\downarrow$

$\mathbf{A} = 8.04\text{ kN} \searrow 34.4°$ ◀



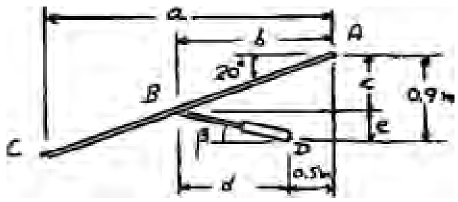
PROBLEM 6.153

The telescoping arm *ABC* can be lowered until end *C* is close to the ground, so that workers can easily board the platform. For the position when $\theta = 20°$, determine (*a*) the force exerted at *B* by the single hydraulic cylinder *BD*, (*b*) the force exerted on the supporting carriage at *A*.

SOLUTION

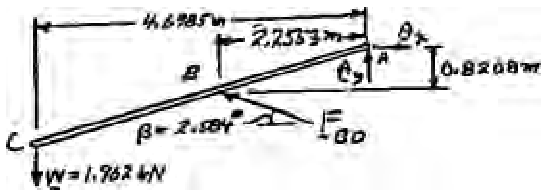
<u>Geometry</u>:

$$a = (5\text{ m})\cos 20° = 4.6985\text{ m}$$
$$b = (2.4\text{ m})\cos 20° = 2.2552\text{ m}$$
$$c = (2.4\text{ m})\sin 20° = 0.8208\text{ m}$$
$$d = b - 0.5 = 1.7553\text{ m}$$
$$e = 0.9 - c = 0.0792\text{ m}$$



$$\tan\beta = \frac{e}{d} = \frac{0.0792}{1.7552}; \quad \beta = 2.584°$$

<u>Free body: Arm *ABC*</u>



We note that *BD* is a two-force member

$$W = (200\text{ kg})(9.81\text{ m/s}^2)$$
$$W = 1962\text{ N} = 1.962\text{ kN}$$

(*a*) $\circlearrowleft\!+\ \Sigma M_A = 0: \quad (1.962\text{ kN})(4.6985\text{ m}) - F_{BD}\sin 2.584°(2.2553\text{ m}) - F_{BD}\cos 2.584°(0.8208\text{ m}) = 0$

$$9.2185 - F_{BD}(0.9216) = 0 \quad F_{BD} = 10.003\text{ kN}$$

(*b*) $\mathbf{F}_{BD} = 10.00\text{ kN} \measuredangle\ 2.58°$ ◀

$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - F_{BD}\cos\beta = 0$

$$A_x = (10.003\text{ kN})\cos 2.583° = 9.993\text{ kN}$$

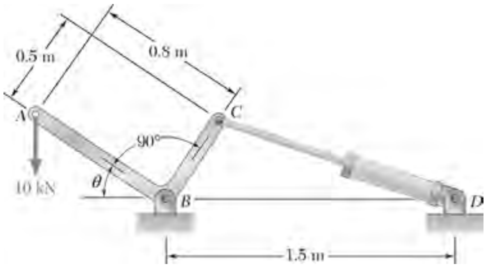
$\mathbf{A}_x = 9.993\text{ kN} \rightarrow$

$+\!\uparrow \Sigma F_y = 0: \quad A_y - 1.962\text{ kN} + F_{BD}\sin\beta = 0$

$$A_y = 1.962\text{ kN} - (10.003\text{ kN})\sin 2.583° = -1.5112\text{ kN}$$

$\mathbf{A}_y = 1.5112\text{ kN} \downarrow$

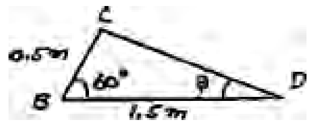
$\mathbf{A} = 10.11\text{ kN} \measuredangle\ 8.60°$ ◀



PROBLEM 6.154

The position of member *ABC* is controlled by the hydraulic cylinder *CD*. Knowing that $\theta = 30°$, determine for the loading shown (*a*) the force exerted by the hydraulic cylinder on pin *C*, (*b*) the reaction at *B*.

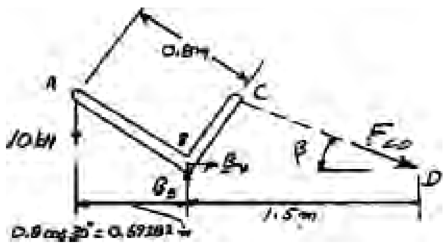
SOLUTION



Geometry: In ΔBCD

Law of cosines
$$(CD)^2 = (0.5)^2 + (1.5)^2 - 2(0.5)(1.5)\cos 60°$$
$$CD = 1.3229 \text{ m}$$

Law of sines
$$\frac{\sin\beta}{0.5 \text{ m}} = \frac{\sin 60°}{1.3229 \text{ m}}$$
$$\sin\beta = 0.3273 \qquad \beta = 19.107°$$



Free body: Entire system

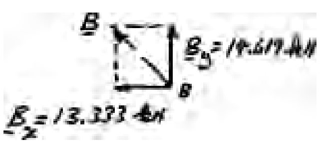
Move force F_{CD} along its line of action so it acts at *D*.

(*a*)
$$+\circlearrowleft \Sigma M_B = 0: \quad (10 \text{ kN})(0.69282 \text{ m}) - F_{CD}\sin\beta(1.5 \text{ m}) = 0$$
$$6.9282 \text{ kN}\cdot\text{m} - F_{CD}\sin 19.107°(1.5 \text{ m}) = 0$$
$$F_{CD} = 14.111 \text{ kN}$$

$$\mathbf{F}_{CD} = 14.11 \text{ kN} \searrow 19.11° \blacktriangleleft$$

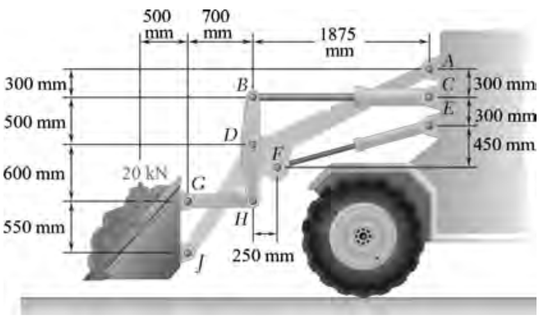
(*b*)
$$\xrightarrow{+} \Sigma F_x = 0: \quad B_x + F_{CD}\cos\beta = 0$$
$$B_x + (14.111 \text{ kN})\cos 19.107° = 0$$
$$B_x = -13.333 \text{ kN} \qquad \mathbf{B}_x = 13.333 \text{ kN} \leftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad B_y - 10 \text{ kN} - F_{CD}\sin 19.107° = 0$$
$$B_y - 10 \text{ kN} - (14.111 \text{ kN})\sin 19.107° = 0$$
$$B_y = +14.619 \text{ kN} \qquad \mathbf{B}_y = 14.619 \text{ kN}\uparrow$$



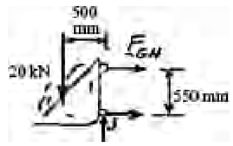
$$\mathbf{B} = 19.79 \text{ kN} \nwarrow 47.6° \blacktriangleleft$$

PROBLEM 6.155



The motion of the bucket of the front-end loader shown is controlled by two arms and a linkage that are pin-connected at *D*. The arms are located symmetrically with respect to the central, vertical, and longitudinal plane of the loader; one arm *AFJ* and its control cylinder *EF* are shown. The single linkage *GHDB* and its control cylinder *BC* are located in the plane of symmetry. For the position and loading shown, determine the force exerted (*a*) by cylinder *BC*, (*b*) by cylinder *EF*.

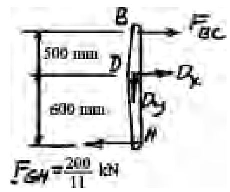
SOLUTION



Free body: Bucket

$$+\circlearrowleft \Sigma M_J = 0: \quad (20\text{ kN})(500\text{ mm}) - F_{GH}(550\text{ mm}) = 0$$

$$F_{GH} = \frac{200}{11}\text{ kN} = 18.1818\text{ kN}$$



Free body: Arm *BDH*

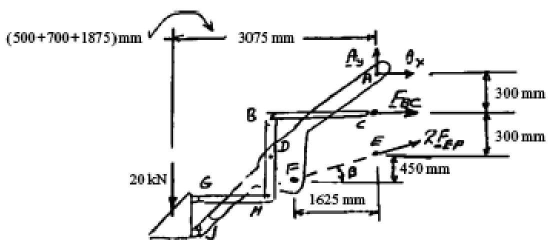
$$+\circlearrowleft \Sigma M_D = 0: \quad -\left(\frac{200}{11}\text{ kN}\right)(600\text{ mm}) - F_{BC}(500\text{ mm}) = 0$$

$$F_{BC} = \frac{-240}{11} = 21.8181\text{ kN}$$

$F_{BC} = 21.8\text{ kN} \quad C$ ◀

Free body: Entire mechanism

(Two arms and cylinders *AFJE*)



Note: Two arms thus $2F_{EF}$

$$\tan\beta = \frac{450\text{ mm}}{1625\text{ mm}}$$

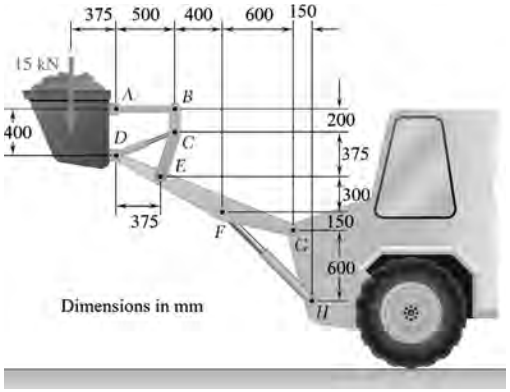
$$\beta = 15.48°$$

$$+\circlearrowleft \Sigma M_A = 0: \quad (20\text{ kN})(3075\text{ mm}) + F_{BC}(300\text{ mm}) + 2F_{EF}\cos\beta(600\text{ mm}) = 0$$

$$(20\text{ kN})(3075\text{ mm}) - \left(\frac{240}{11}\right)(300\text{ mm}) + 2F_{EF}\cos(15.48°)(600\text{ mm}) = 0$$

$$F_{EF} = -47.5193\text{ kN}$$

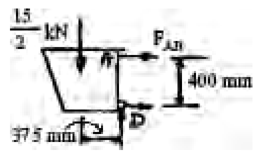
$F_{EF} = 47.5\text{ kN} \quad C$ ◀



PROBLEM 6.156

The bucket of the front-end loader shown carries a 15 kN load. The motion of the bucket is controlled by two identical mechanisms, only one of which is shown. Knowing that the mechanism shown supports one-half of the 15 kN load, determine the force exerted (*a*) by cylinder *CD*, (*b*) by cylinder *FH*.

SOLUTION

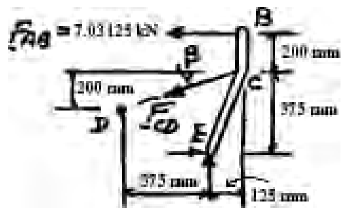


Free body: Bucket (One mechanism)

$$\circlearrowleft + \Sigma M_D = 0: \quad (7.5 \text{ kN})(375 \text{ mm}) - F_{AB}(400 \text{ mm}) = 0$$

$$F_{AB} = 7.03125 \text{ kN}$$

Note: There are 2 identical support mechanisms.



Free body: One arm *BCE*

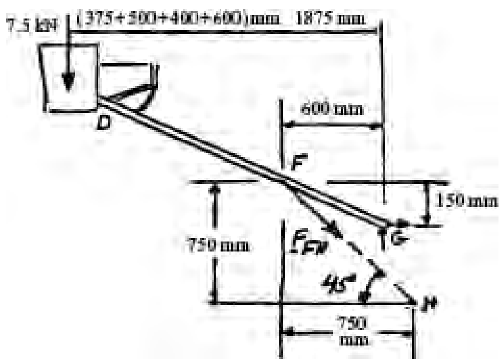
$$\tan \beta = \frac{200}{500}$$

$$\beta = 21.8°$$

$$\circlearrowleft + \Sigma M_E = 0: \quad (7.03125 \text{ kN})(575 \text{ mm}) + F_{CD} \cos 21.8°(375 \text{ mm}) - F_{CD} \sin 21.8°(125 \text{ mm}) = 0$$

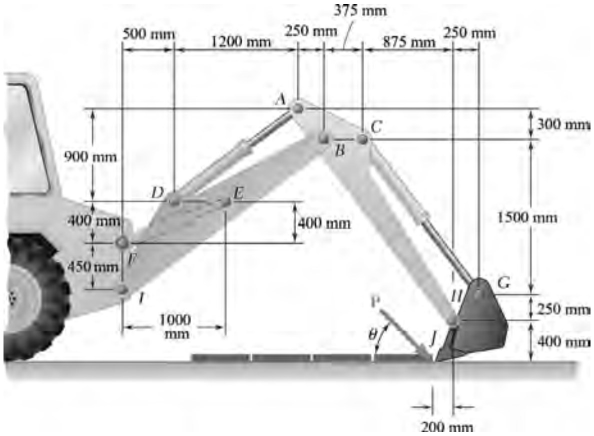
$$F_{CD} = -13.3979 \text{ kN} \qquad\qquad F_{CD} = 13.40 \text{ kN} \quad C \blacktriangleleft$$

Free body: Arm *DFG*



$$\circlearrowleft + \Sigma M_G = 0: \quad (7.5 \text{ kN})(1875 \text{ mm}) + F_{FH} \sin 45°(600 \text{ mm}) - F_{FH} \cos 45°(150 \text{ mm}) = 0$$

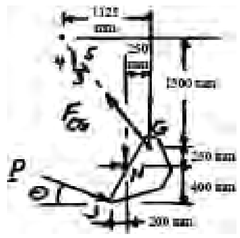
$$F_{FH} = -44.1941 \text{ kN} \qquad\qquad F_{FH} = 44.19 \text{ kN} \quad C \blacktriangleleft$$



PROBLEM 6.157

The motion of the backhoe bucket shown is controlled by the hydraulic cylinders *AD*, *CG*, and *EF*. As a result of an attempt to dislodge a portion of a slab, a 10 kN force **P** is exerted on the bucket teeth at *J*. Knowing that $\theta = 45°$, determine the force exerted by each cylinder.

SOLUTION



Free body: Bucket

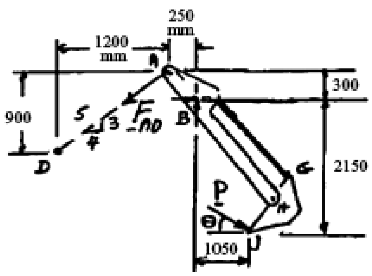
$$\overset{+}{\circlearrowleft} \Sigma M_H = 0 \quad \text{(Dimensions in mm)}$$

$$\frac{4}{5}F_{CG}(250) + \frac{3}{5}F_{CG}(250) + P\cos\theta(400) + P\sin\theta(200) = 0$$

$$F_{CG} = \frac{-4P}{7}(2\cos\theta + \sin\theta) \qquad (1)$$

Free body: Arm *ABH* and bucket

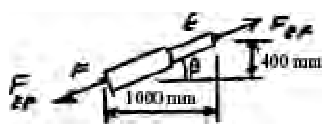
(Dimensions in mm)



$$\overset{+}{\circlearrowleft} \Sigma M_B = 0: \quad \frac{4}{5}F_{AD}(300) + \frac{3}{5}F_{AD}(250) + P\cos\theta(2150) - P\sin\theta(1050) = 0$$

$$F_{AD} = -\frac{5P}{39}(43\cos\theta - 21\sin\theta) \qquad (2)$$

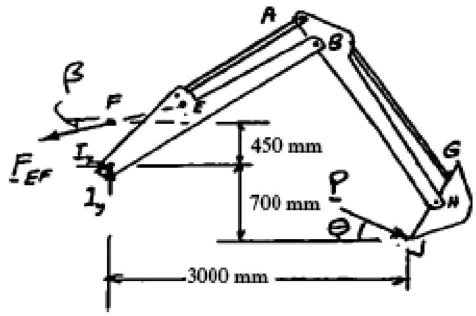
Free body: Bucket and arms *IEB* + *ABH*



Geometry of cylinder *EF* $\qquad \tan\beta = \dfrac{400 \text{ mm}}{1000 \text{ mm}}$

$$\beta = 21.801°$$

PROBLEM 6.157 (Continued)



$$+\circlearrowleft \Sigma M_I = 0: \quad F_{EF} \cos\beta(450 \text{ mm}) + P\cos\theta(700 \text{ mm}) - P\sin\theta(3000 \text{ mm}) = 0$$

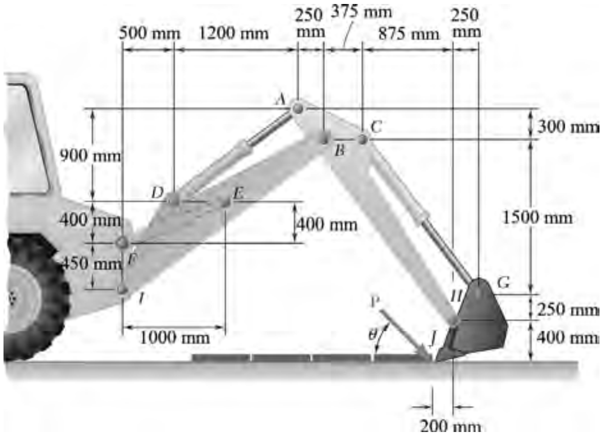
$$F_{EF} = 0.23934P(30\sin\theta - 7\cos\theta) \tag{3}$$

For $P = 10$ kN, $\theta = 45°$

Eq. (1): $F_{CG} = -\frac{4}{7} \times 10(2\cos 45° + \sin 45°) = -12.122$ kN $\qquad F_{CG} = 12.12$ kN C ◀

Eq. (2): $F_{AD} = -\frac{5}{39} \times 10(43\cos 45° - 21\sin 45°) = -19.944$ kN $\qquad F_{AD} = 19.94$ kN C ◀

Eq. (3): $F_{EF} = 0.23934 \times 10(30\sin 45° - 7\cos 45°) = 38.9249$ kN $\qquad F_{EF} = 38.9$ kN T ◀

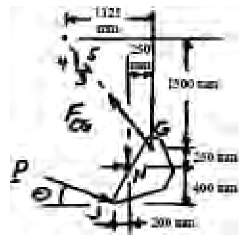


PROBLEM 6.158

Solve Problem 6.157 assuming that the 10 kN force **P** acts horizontally to the right ($\theta = 0$).

PROBLEM 6.157 The motion of the backhoe bucket shown is controlled by the hydraulic cylinders *AD*, *CG*, and *EF*. As a result of an attempt to dislodge a portion of a slab, a 10-kN force **P** is exerted on the bucket teeth at *J*. Knowing that $\theta = 45°$, determine the force exerted by each cylinder.

SOLUTION



Free body: Bucket

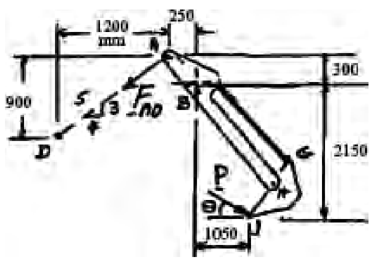
$+\circlearrowleft \Sigma M_H = 0$ (Dimensions in mm)

$$\frac{4}{5}F_{CG}(250) + \frac{3}{5}F_{CG}(250) + P\cos\theta(400) + P\sin\theta(200) = 0$$

$$F_{CG} = \frac{-4P}{7}(2\cos\theta + \sin\theta) \qquad (1)$$

Free body: Arm *ABH* and bucket

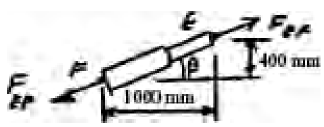
(Dimensions in mm)



$$+\circlearrowleft \Sigma M_B = 0: \quad \frac{4}{5}F_{AD}(300) + \frac{3}{5}F_{AD}(250) + P\cos\theta(2150) - P\sin\theta(1050) = 0$$

$$F_{AD} = \frac{-5P}{39}(43\cos\theta - 21\sin\theta) \qquad (2)$$

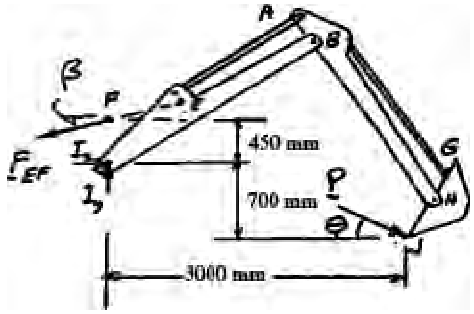
Free body: Bucket and arms *IEB* + *ABH*



Geometry of cylinder *EF* $\qquad \tan\beta = \dfrac{400 \text{ mm}}{1000 \text{ mm}}$

$$\beta = 21.801°$$

PROBLEM 6.158 (Continued)



$$\circlearrowleft^{+} \Sigma M_I = 0: \quad F_{EF} \cos\beta(450 \text{ mm}) + P\cos\theta(700 \text{ mm}) - P\sin\theta(3000 \text{ mm}) = 0$$

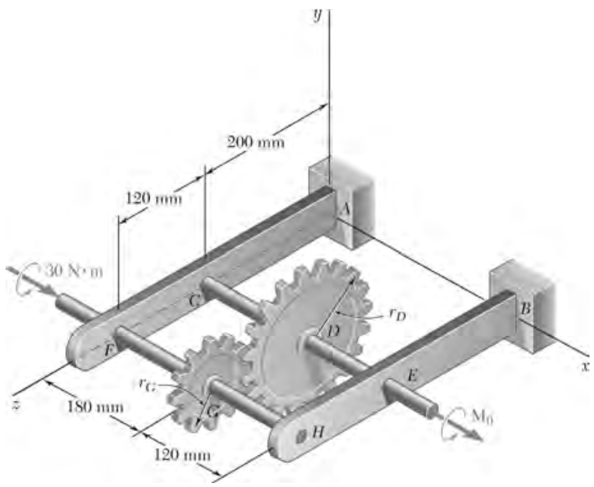
$$F_{EF} = 0.23934P(30\sin\theta - 7\cos\theta) \qquad (3)$$

For $P = 10$ kN, $\theta = 0$

Eq. (1): $F_{CG} = \dfrac{-4\times 10}{7}(2\cos 0° + \sin 0°) = -11.4286$ kN $\qquad F_{CG} = 11.43$ kN C ◀

Eq. (2): $F_{AD} = \dfrac{-5\times 10}{39}(43\cos 0° - 21\sin 0°) = -55.128$ kN $\qquad F_{AD} = 55.1$ kN C ◀

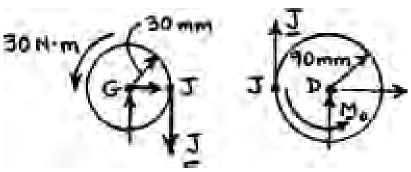
Eq. (3): $F_{EF} = 0.2394\times 10(30\sin 0° - 7\cos 0°) = -16.758$ kN $\qquad F_{EF} = 16.76$ kN C ◀



PROBLEM 6.159

The gears D and G are rigidly attached to shafts that are held by frictionless bearings. If $r_D = 90$ mm and $r_G = 30$ mm, determine (*a*) the couple $\mathbf{M}_0$ that must be applied for equilibrium, (*b*) the reactions at A and B.

SOLUTION



(*a*) Projections on *yz* plane

Free body: Gear G

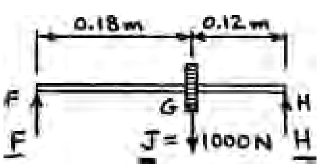
$$+\circlearrowleft \Sigma M_G = 0: \quad 30 \text{ N}\cdot\text{m} - J(0.03 \text{ m}) = 0; \quad J = 1000 \text{ N}$$

Free body: Gear D

$$+\circlearrowleft \Sigma M_D = 0: \quad M_0 - (1000 \text{ N})(0.09 \text{ m}) = 0$$

$$M_0 = 90 \text{ N}\cdot\text{m} \qquad \mathbf{M}_0 = (90.0 \text{ N}\cdot\text{m})\mathbf{i} \blacktriangleleft$$

(*b*) Gear G and axle FH



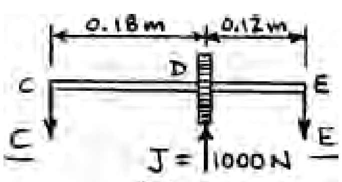
$$+\circlearrowleft \Sigma M_F = 0: \quad H(0.3 \text{ m}) - (1000 \text{ N})(0.18 \text{ m}) = 0$$

$$H = 600 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad F + 600 - 1000 = 0$$

$$F = 400 \text{ N}$$

Gear D and axle CE



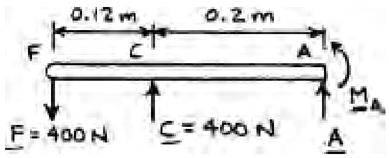
$$+\circlearrowleft \Sigma M_C = 0: \quad (1000 \text{ N})(0.18 \text{ m}) - E(0.3 \text{ m}) = 0$$

$$E = 600 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad 1000 - C - 600 = 0$$

$$C = 400 \text{ N}$$

PROBLEM 6.159 (Continued)

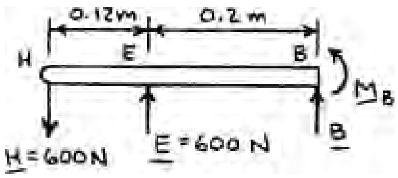


Free body: Bracket *AE*

$+\uparrow \Sigma F_y = 0: \quad A - 400 + 400 = 0$ $\quad \mathbf{A} = 0$ ◀

$\circlearrowleft + \Sigma M_A = 0: \quad M_A + (400 \text{ N})(0.32 \text{ m}) - (400 \text{ N})(0.2 \text{ m}) = 0$

$M_A = -48 \text{ N} \cdot \text{m} \qquad \mathbf{M}_A = -(48.0 \text{ N} \cdot \text{m})\mathbf{i}$ ◀

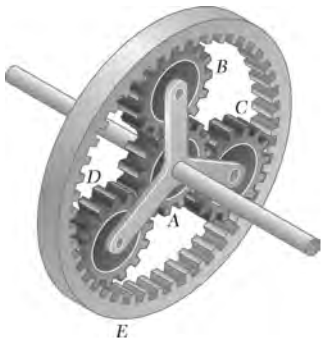


Free body: Bracket *BH*

$+\uparrow \Sigma F_y = 0: \quad B - 600 + 600 = 0$ $\quad \mathbf{B} = 0$ ◀

$\circlearrowleft + \Sigma M_B = 0: \quad M_B + (600 \text{ N})(0.32 \text{ m}) - (600 \text{ N})(0.2 \text{ m}) = 0$

$M_B = -72 \text{ N} \cdot \text{m} \qquad \mathbf{M}_B = -(72.0 \text{ N} \cdot \text{m})\mathbf{i}$ ◀

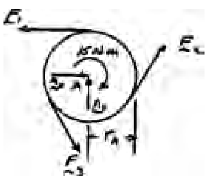


PROBLEM 6.160

In the planetary gear system shown, the radius of the central gear A is $a = 18$ mm, the radius of each planetary gear is b, and the radius of the outer gear E is $(a + 2b)$. A clockwise couple of magnitude $M_A = 10$ N·m is applied to the central gear A and a counterclockwise couple of magnitude $M_S = 50$ N·m is applied to the spider BCD. If the system is to be in equilibrium, determine (a) the required radius b of the planetary gears, (b) the magnitude M_E of the couple that must be applied to the outer gear E.

SOLUTION

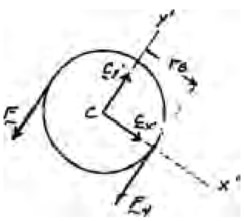
FBD Central Gear:



By symmetry: $F_1 = F_2 = F_3 = F$

$$\circlearrowright \Sigma M_A = 0: \quad 3(r_A F) - 10\ \text{N}\cdot\text{m} = 0, \qquad F = \frac{10}{3r_A}\ \text{N}\cdot\text{m}$$

FBD Gear C:



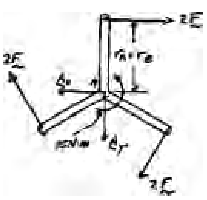
$$\circlearrowright \Sigma M_C = 0: \quad r_B(F - F_4) = 0, \quad F_4 = F$$

$$\searrow \Sigma F_{x'} = 0: \quad C_{x'} = 0$$

$$\nearrow \Sigma F_{y'} = 0: \quad C_{y'} - 2F = 0, \qquad C_{y'} = 2F$$

Gears B and D are analogous, each having a central force of $2F$

FBD Spider:



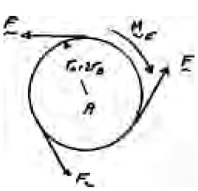
$$\circlearrowleft \Sigma M_A = 0: \quad 50\ \text{N}\cdot\text{m} - 3(r_A + r_B)2F = 0$$

$$50\ \text{N}\cdot\text{m} - 3(r_A + r_B)\frac{20}{r_A}\ \text{N}\cdot\text{m} = 0$$

$$\frac{r_A + r_B}{r_A} = 2.5 = 1 + \frac{r_B}{r_A}, \qquad r_B = 1.5 r_A$$

Since $r_A = 18$ mm,

FBD Outer Gear:



(a) $$r_B = 27.0\ \text{mm} \blacktriangleleft$$

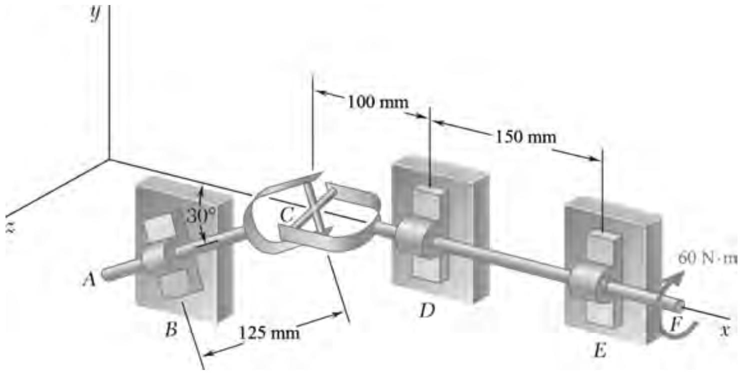
$$\circlearrowleft \Sigma M_A = 0: \quad 3(r_A + 2r_B)F - M_E = 0$$

$$3(18\ \text{mm} + 54\ \text{mm})\frac{10\ \text{N}\cdot\text{m}}{54\ \text{mm}} - M_E = 0$$

(b) $$\mathbf{M}_E = 40.0\ \text{N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

PROBLEM 6.161*

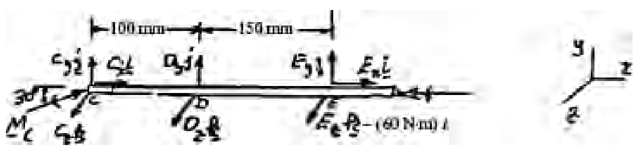
Two shafts *AC* and *CF*, which lie in the vertical *xy* plane, are connected by a universal joint at *C*. The bearings at *B* and *D* do not exert any axial force. A couple of magnitude 60 N · m (clockwise when viewed from the positive *x* axis) is applied to shaft *CF* at *F*. At a time when the arm of the crosspiece attached to shaft *CF* is horizontal, determine (*a*) the magnitude of the couple that must be applied to shaft *AC* at *A* to maintain equilibrium, (*b*) the reactions at *B*, *D*, and *E*. (*Hint:* The sum of the couples exerted on the crosspiece must be zero.)



SOLUTION

We recall from Figure 4.10, that a universal joint exerts on members it connects a force of unknown direction and a couple about an axis perpendicular to the crosspiece.

<u>Free body: Shaft *DF*</u>

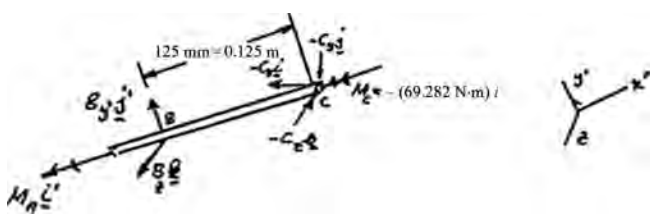


$$\Sigma M_x = 0: \quad M_C \cos 30° - 60 \text{ N} \cdot \text{m} = 0$$

$$M_C = 69.282 \text{ N} \cdot \text{m}$$

<u>Free body: Shaft *BC*</u>

We use here x', y', z with x' along BC



$$\Sigma M_C = 0: \quad -M_A i' - (69.282 \text{ N} \cdot \text{m}) i' + (-0.125 \text{ m}) i' \times (B_y j' + B_z k) = 0$$

PROBLEM 6.161* (Continued)

Equate coefficients of unit vectors to zero:

$\mathbf{i}$: $\quad M_A - 69.282\ \text{N}\cdot\text{m} = 0 \qquad M_A = 69.282\ \text{N}\cdot\text{m}$

$M_A = 69.3\ \text{N}\cdot\text{m}$ ◀

$\mathbf{j}$: $\quad B_z = 0$

$\mathbf{k}$: $\quad B_y = 0$

$\left.\begin{array}{l} B_z = 0 \\ B_y = 0 \end{array}\right\} \rightarrow B = 0 \qquad \mathbf{B} = 0$

$\Sigma\mathbf{F} = 0\text{:}\quad B + C = 0,\quad \text{since } B = 0, \qquad \mathbf{C} = 0$

<u>Return to free body of shaft *DF*</u>

$\Sigma\mathbf{M}_D = 0 \qquad$ (Note that $C = 0$ and $M_C = 69.282\ \text{N}\cdot\text{m}$)

$$(69.282\ \text{N}\cdot\text{m})(\cos 30°\mathbf{i} + \sin 30°\mathbf{j}) - (60\ \text{N}\cdot\text{m})\mathbf{i}$$

$$+(0.15\ \text{m})\mathbf{i} \times (E_x\mathbf{i} + E_y\mathbf{j} + E_z\mathbf{k}) = 0$$

$$(60\ \text{N}\cdot\text{m})\mathbf{i} + (34.641\ \text{N}\cdot\text{m})\mathbf{j} - (60\ \text{N}\cdot\text{m})\mathbf{i}$$

$$+(0.15\ \text{m})E_y\mathbf{k} - (0.15\ \text{m})E_z\mathbf{j} = 0$$

Equate coefficients of unit vectors to zero:

$\mathbf{j}$: $\quad 34.641\ \text{N}\cdot\text{m} - (0.15\ \text{m})E_z = 0 \qquad E_z = 230.94\ \text{N}$

$\mathbf{k}$: $\quad E_y = 0$

$$\Sigma\mathbf{F} = 0\text{:}\quad \mathbf{C} + \mathbf{D} + \mathbf{E} = 0$$

$$0 + D_y\mathbf{j} + D_z\mathbf{k} + E_x\mathbf{i} + (230.94\ \text{N})\mathbf{k} = 0$$

$\mathbf{i}$: $\quad E_x = 0$

$\mathbf{j}$: $\quad D_y = 0$

$\mathbf{k}$: $\quad D_z + 230.94\ \text{N} = 0 \qquad D_z = -230.94\ \text{N}$

<u>Reactions are</u>:

$\mathbf{B} = 0$ ◀

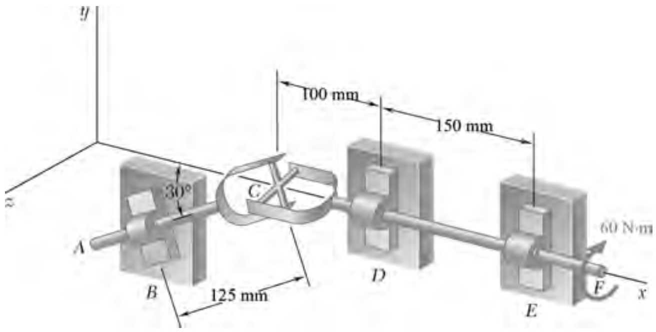
$\mathbf{D} = -(231\ \text{N})\mathbf{k}$ ◀

$\mathbf{E} = (231\ \text{N})\mathbf{k}$ ◀

PROBLEM 6.162*

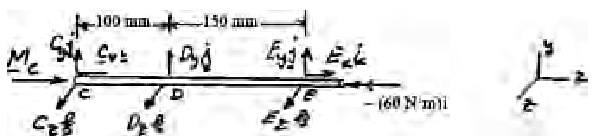
Solve Problem 6.161 assuming that the arm of the crosspiece attached to shaft *CF* is vertical.

PROBLEM 6.161 Two shafts *AC* and *CF*, which lie in the vertical *xy* plane, are connected by a universal joint at *C*. The bearings at *B* and *D* do not exert any axial force. A couple of magnitude 60 N · m (clockwise when viewed from the positive *x* axis) is applied to shaft *CF* at *F*. At a time when the arm of the crosspiece attached to shaft *CF* is horizontal, determine (*a*) the magnitude of the couple that must be applied to shaft *AC* at *A* to maintain equilibrium, (*b*) the reactions at *B*, *D*, and *E*. (*Hint:* The sum of the couples exerted on the crosspiece must be zero.)



SOLUTION

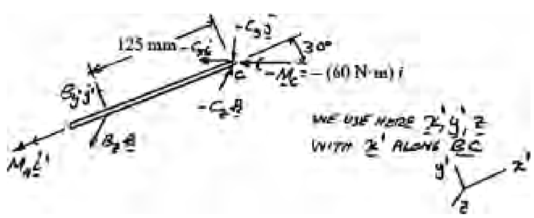
Free body: Shaft *DF*



$$\Sigma M_x = 0: \quad M_C - 60 \text{ N}\cdot\text{m} = 0$$

$$M_C = 60 \text{ N}\cdot\text{m}$$

Free body: Shaft *BC*



We resolve $-(60 \text{ N}\cdot\text{m})\mathbf{i}$ into components along x' and y' axes:

$$-\mathbf{M}_C = -(60 \text{ N}\cdot\text{m})(\cos 30°\mathbf{i}' + \sin 30°\mathbf{j}')$$

$$\Sigma\mathbf{M}_C = 0: \quad M_A\mathbf{i}' - (60 \text{ N}\cdot\text{m})(\cos 30°\mathbf{i}' + \sin 30°\mathbf{j}') + (0.125 \text{ m})\mathbf{i}' \times (B_{y'}\mathbf{j}' + B_z\mathbf{k}) = 0$$

$$M_A\mathbf{i}' - (51.9615 \text{ N}\cdot\text{m})\mathbf{i}' - (30 \text{ N}\cdot\text{m})\mathbf{j} + (0.125 \text{ m})B_{y'}\mathbf{k} - (0.125 \text{ m})B_z\mathbf{j}' = 0$$

PROBLEM 6.162* (Continued)

Equate to zero coefficients of unit vectors:

$$\mathbf{i}':\quad M_A - 51.9615\ \text{N}\cdot\text{m} = 0 \qquad\qquad M_A = 52.0\ \text{N}\cdot\text{m} \blacktriangleleft$$

$$\mathbf{j}':\quad -30\ \text{N}\cdot\text{m} - (0.125\ \text{m})B_z = 0 \qquad\qquad B_z = -240\ \text{N}$$

$$\mathbf{k}:\quad \mathbf{B}_{y'} = 0$$

Reactions at B: $\mathbf{B} = -240\ \text{N}$

$$\Sigma\mathbf{F} = 0:\quad \mathbf{B} - \mathbf{C} = 0$$

$$-(240\ \text{N})\mathbf{k} - C = 0 \qquad\qquad \mathbf{C} = -(240\ \text{N})\text{k}$$

Return to free body of shaft DF:

$$\Sigma\mathbf{M}_D = 0:\quad (0.15\ \text{m})\mathbf{i}\times(E_x\mathbf{i} + E_y\mathbf{j} + E_z\mathbf{k}) - (0.1\ \text{m})\mathbf{i}\times(-240\ \text{N})\mathbf{k}$$

$$-(60\ \text{N}\cdot\text{m})\mathbf{i} + (60\ \text{N}\cdot\text{m})\mathbf{i} = 0$$

$$(0.15\ \text{m})E_y\mathbf{k} - (0.15\ \text{m})E_z\mathbf{j} - (24\ \text{N}\cdot\text{m})\mathbf{j} = 0$$

$$\mathbf{k}:\quad E_y = 0$$

$$\mathbf{j}:\quad -(0.15\ \text{m})E_z - 24\ \text{N}\cdot\text{m} = 0 \qquad\qquad E_z = -160.0\ \text{N}$$

$$\Sigma F = 0:\quad \mathbf{C} + \mathbf{D} + \mathbf{E} = 0$$

$$-(240\ \text{N})\mathbf{k} + D_y\mathbf{j} + D_z\mathbf{k} + E_x\mathbf{i} - (160\ \text{N})\mathbf{k} = 0$$

$$\mathbf{i}:\quad E_x = 0$$

$$\mathbf{k}:\quad -240\ \text{N} - 160\ \text{N} + D_z = 0$$

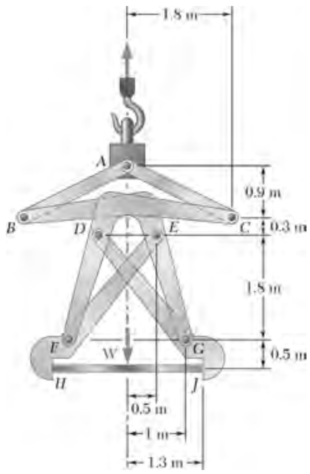
$$D_z = 400\ \text{N}$$

Reactions are:

$$\mathbf{B} = -(240\ \text{N})\mathbf{k} \blacktriangleleft$$

$$\mathbf{D} = (400\ \text{N})\mathbf{k} \blacktriangleleft$$

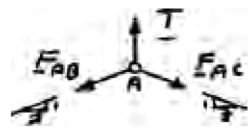
$$\mathbf{E} = -(160.0\ \text{N})\mathbf{k} \blacktriangleleft$$



PROBLEM 6.163*

The large mechanical tongs shown are used to grab and lift a thick 7500 kg steel slab *HJ*. Knowing that slipping does not occur between the tong grips and the slab at *H* and *J*, determine the components of all forces acting on member *EFH*. (*Hint:* Consider the symmetry of the tongs to establish relationships between the components of the force acting at *E* on *EFH* and the components of the force acting at *D* on *CDF*.)

SOLUTION

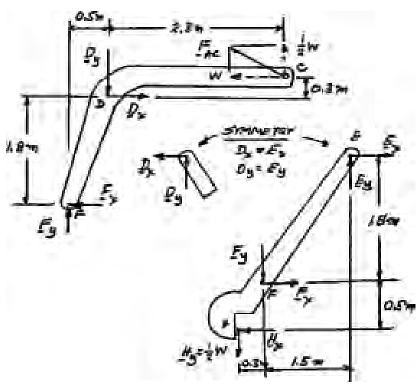


Free body: Pin *A* $\quad T = W = mg = (7500 \text{ kg})(9.81 \text{ m/s}^2) = 73.575 \text{ kN}$

$$\Sigma F_x = 0: \quad (F_{AB})_x = (F_{AC})_x$$

$$\Sigma F_y = 0: \quad (F_{AB})_y = (F_{AC})_y = \frac{1}{2}W$$

Also: $\quad (F_{AC})_x = 2(F_{AC})_y = W$



Free body: Member *CDF*

$$\circlearrowleft \Sigma M_D = 0: \quad W(0.3) + \frac{1}{2}W(2.3) - F_x(1.8) - F_y(0.5 \text{ m}) = 0$$

or $$1.8F_x + 0.5F_y = 1.45W \qquad (1)$$

PROBLEM 6.163* (Continued)

$$\xrightarrow{+} \Sigma F_x = 0: \quad D_x - F_x - W = 0$$

or $$E_x - F_x = W \qquad (2)$$

$$+\uparrow \Sigma F_y = 0: \quad F_y - D_y + \frac{1}{2}W = 0$$

or $$E_y - F_y = \frac{1}{2}W \qquad (3)$$

<u>Free body: Member *EFH*</u>

$$\circlearrowleft + \Sigma M_E = 0: \quad F_x(1.8) + F_y(1.5) - H_x(2.3) + \frac{1}{2}W(1.8 \text{ m}) = 0$$

or: $$1.8F_x + 1.5F_y = 2.3H_x - 0.9W \qquad (4)$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad E_x + F_x - H_x = 0$$

or $$E_x + F_x = H_x \qquad (5)$$

Subtract (2) from (5): $$2F_x = H_x - W \qquad (6)$$

Subtract (4) from $3 \times (1)$: $$3.6F_x = 5.25W - 2.3H_x \qquad (7)$$

Add (7) to $2.3 \times (6)$: $$8.2F_x = 2.95W$$

$$F_x = 0.35976W \qquad (8)$$

Substitute from (8) into (1):

$$(1.8)(0.35976W) + 0.5F_y = 1.45W$$

$$0.5F_y = 1.45W - 0.64756W = 0.80244W$$

$$F_y = 1.6049W \qquad (9)$$

Substitute from (8) into (2): $E_x - 0.35976W = W; \quad E_x = 1.35976W$

Substitute from (9) into (3): $E_y - 1.6049W = \frac{1}{2}W \quad E_y = 2.1049W$

From (5): $H_x = E_x + F_x = 1.35976W + 0.35976W = 1.71952W$

Recall that: $H_y = \frac{1}{2}W$

PROBLEM 6.163* (Continued)

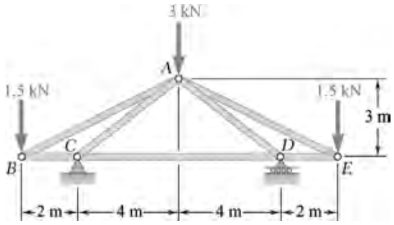
Since all expressions obtained are positive, all forces are directed as shown on the free-body diagrams.

Substitute $W = 73.575 \text{ kN}$

$\mathbf{E}_x = 100.0 \text{ kN} \rightarrow$ $\mathbf{E}_y = 154.9 \text{ kN} \uparrow$ ◀

$\mathbf{F}_x = 26.5 \text{ kN} \rightarrow$ $\mathbf{F}_y = 118.1 \text{ kN} \downarrow$ ◀

$\mathbf{H}_x = 126.5 \text{ kN} \leftarrow$ $\mathbf{H}_y = 36.8 \text{ kN} \downarrow$ ◀



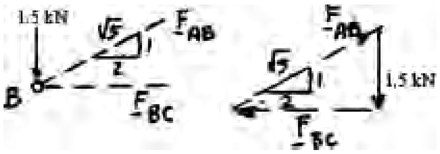
PROBLEM 6.164

Using the method of joints, determine the force in each member of the truss shown. State whether each member is in tension or compression.

SOLUTION

Free body: Truss.

From the symmetry of the truss and loading, we find $\mathbf{C} = \mathbf{D} = 3 \text{ kN} \uparrow$

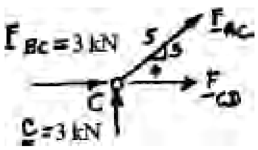


Free body: Joint B

$$\frac{F_{AB}}{\sqrt{5}} = \frac{F_{BC}}{2} = \frac{1.5 \text{ kN}}{1}$$

$$F_{AB} = 3.3541 \text{ N} \quad T \qquad F_{BC} = 3.00 \text{ kN} \quad C \blacktriangleleft$$

Free body: Joint C



$$+\uparrow \Sigma F_y = 0: \quad \frac{3}{5} F_{AC} + 3 \text{ kN} = 0$$

$$F_{AC} = -5 \text{ kN} \qquad F_{AC} = 5.00 \text{ kN} \quad C \blacktriangleleft$$

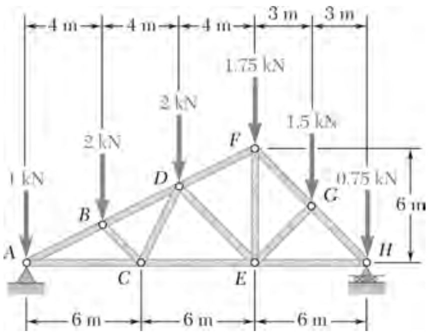
$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{4}{5}(-5 \text{ kN}) + 3 \text{ kN} + F_{CD} = 0 \qquad F_{CD} = 1.000 \text{ kN} \quad T \blacktriangleleft$$

From symmetry:

$$F_{AD} = F_{AC} = 5.00 \text{ kN} \quad C, \quad F_{AE} = F_{AB} = 3.35 \text{ kN} \quad T, \quad F_{DE} = F_{BC} = 3.00 \text{ kN} \quad C \blacktriangleleft$$

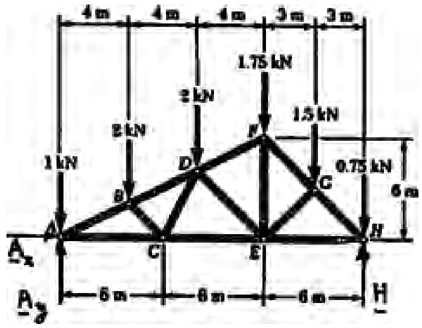
PROBLEM 6.165

Using the method of joints, determine the force in each member of the double-pitch roof truss shown. State whether each member is in tension or compression.



SOLUTION

Free body: Truss



$$+\circlearrowleft \Sigma M_A = 0: \quad H(18 \text{ m}) - (2 \text{ kN})(4 \text{ m}) - (2 \text{ kN})(8 \text{ m}) - (1.75 \text{ kN})(12 \text{ m}) - (1.5 \text{ kN})(15 \text{ m}) - (0.75 \text{ kN})(18 \text{ m}) = 0$$

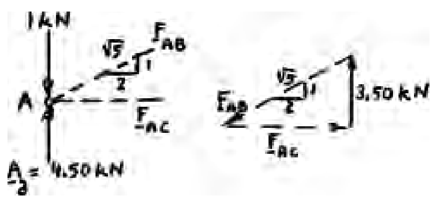
$$\mathbf{H} = 4.50 \text{ kN}\uparrow$$

$$\Sigma F_x = 0: \quad A_x = 0$$

$$\Sigma F_y = 0: \quad A_y + H - 9 = 0$$

$$A_y = 9 - 4.50, \qquad A_y = 4.50 \text{ kN}\uparrow$$

Free body: Joint A

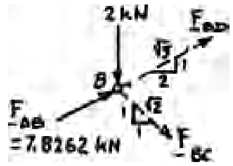


$$\frac{F_{AB}}{\sqrt{5}} = \frac{F_{AC}}{2} = \frac{3.50 \text{ kN}}{1}$$

$$F_{AB} = 7.8262 \text{ kN} \quad C \qquad F_{AB} = 7.83 \text{ kN} \quad C \blacktriangleleft$$

$$F_{AC} = 7.00 \text{ kN} \quad T \blacktriangleleft$$

PROBLEM 6.165 (Continued)



Free body: Joint B

$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{2}{\sqrt{5}} F_{BD} + \frac{2}{\sqrt{5}}(7.8262 \text{ kN}) + \frac{1}{\sqrt{2}} F_{BC} = 0$$

or $\quad F_{BD} + 0.79057 F_{BC} = -7.8262 \text{ kN} \quad (1)$

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}} F_{BD} + \frac{1}{\sqrt{5}}(7.8262 \text{ kN}) - \frac{1}{\sqrt{2}} F_{BC} - 2 \text{ kN} = 0$$

or $\quad F_{BD} - 1.58114 F_{BC} = -3.3541 \quad (2)$

Multiply (1) by 2 and add (2):

$$3F_{BD} = -19.0065$$

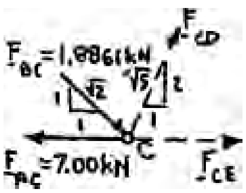
$$F_{BD} = -6.3355 \text{ kN} \qquad F_{BD} = 6.34 \text{ kN} \quad C \blacktriangleleft$$

Subtract (2) from (1):

$$2.37111 F_{BC} = -4.4721$$

$$F_{BC} = -1.8861 \text{ kN} \qquad F_{BC} = 1.886 \text{ kN} \quad C \blacktriangleleft$$

Free body: Joint C

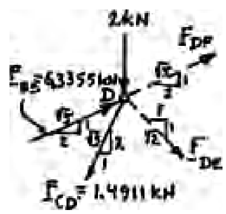


$$+\uparrow \Sigma F_y = 0: \quad \frac{2}{\sqrt{5}} F_{CD} - \frac{1}{\sqrt{2}}(1.8861 \text{ kN}) = 0$$

$$F_{CD} = +1.4911 \text{ kN} \qquad F_{CD} = 1.491 \text{ kN} \quad T \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{CE} - 7.00 \text{ kN} + \frac{1}{\sqrt{2}}(1.8861 \text{ kN}) + \frac{1}{\sqrt{5}}(1.4911 \text{ kN}) = 0$$

$$F_{CE} = 5.000 \text{ kN} \qquad F_{CE} = 5.00 \text{ kN} \quad T \blacktriangleleft$$



Free body: Joint D

$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{2}{\sqrt{5}} F_{DF} + \frac{1}{\sqrt{2}} F_{DE} + \frac{2}{\sqrt{5}}(6.3355 \text{ kN}) - \frac{1}{\sqrt{5}}(1.4911 \text{ kN}) = 0$$

or $\quad F_{DF} + 0.79057 F_{DE} = -5.5900 \text{ kN} \quad (1)$

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{\sqrt{5}} F_{DF} - \frac{1}{\sqrt{2}} F_{DE} + \frac{1}{\sqrt{5}}(6.3355 \text{ kN}) - \frac{2}{\sqrt{5}}(1.4911 \text{ kN}) - 2 \text{ kN} = 0$$

or $\quad F_{DF} - 0.79057 F_{DE} = -1.1188 \text{ kN} \quad (2)$

Add (1) and (2): $\quad 2F_{DF} = -6.7088 \text{ kN}$

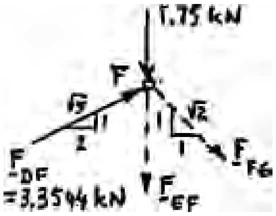
$$F_{DF} = -3.3544 \text{ kN} \qquad F_{DF} = 3.35 \text{ kN} \quad C \blacktriangleleft$$

Subtract (2) from (1): $\quad 1.58114 F_{DE} = -4.4712 \text{ kN}$

$$F_{DE} = -2.8278 \text{ kN} \qquad F_{DE} = 2.83 \text{ kN} \quad C \blacktriangleleft$$

PROBLEM 6.165 (Continued)

Free body: Joint F



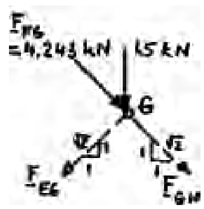
$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{1}{\sqrt{2}} F_{FG} + \frac{2}{\sqrt{5}}(3.3544 \text{ kN}) = 0$$

$$F_{FG} = -4.243 \text{ kN} \qquad F_{FG} = 4.24 \text{ kN} \quad C \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad -F_{EF} - 1.75 \text{ kN} + \frac{1}{\sqrt{5}}(3.3544 \text{ kN}) - \frac{1}{\sqrt{2}}(-4.243 \text{ kN}) = 0$$

$$F_{EF} = 2.750 \text{ kN} \qquad F_{EF} = 2.75 \text{ kN} \quad T \blacktriangleleft$$

Free body: Joint G



$$\xrightarrow{+} \Sigma F_x = 0: \quad \frac{1}{\sqrt{2}} F_{GH} - \frac{1}{\sqrt{2}} F_{EG} + \frac{1}{\sqrt{2}}(4.243 \text{ kN}) = 0$$

or:

$$F_{GH} - F_{EG} = -4.243 \text{ kN} \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad -\frac{1}{\sqrt{2}} F_{GH} - \frac{1}{\sqrt{2}} F_{EG} - \frac{1}{\sqrt{2}}(4.243 \text{ kN}) - 1.5 \text{ kN} = 0$$

or:

$$F_{GH} + F_{EG} = -6.364 \text{ kN} \qquad (2)$$

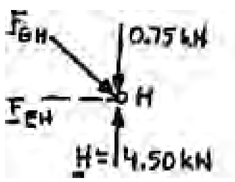
Add (1) and (2): $\quad 2F_{GH} = -10.607$

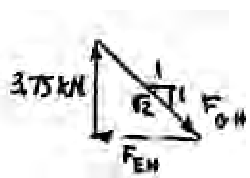
$$F_{GH} = -5.303 \qquad F_{GH} = 5.30 \text{ kN} \quad C \blacktriangleleft$$

Subtract (1) from (2): $\quad 2F_{EG} = -2.121 \text{ kN}$

$$F_{EG} = -1.0605 \text{ kN} \qquad F_{EG} = 1.061 \text{ kN} \quad C \blacktriangleleft$$

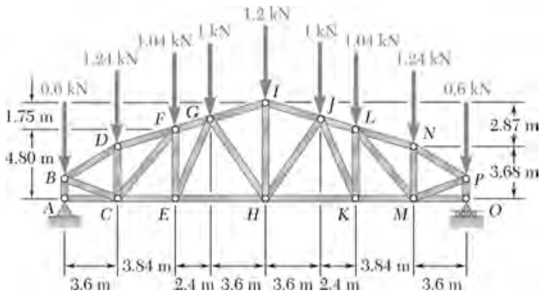
Free body: Joint H





$$\frac{F_{EH}}{1} = \frac{3.75 \text{ kN}}{1} \qquad F_{EH} = 3.75 \text{ kN} \quad T \blacktriangleleft$$

We can also write: $\quad \dfrac{F_{GH}}{\sqrt{2}} = \dfrac{3.75 \text{ kN}}{1} \qquad F_{GH} = 5.30 \text{ kN} \quad C \quad$ (Checks)



PROBLEM 6.166

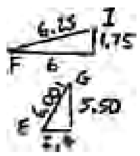
The truss shown was designed to support the roof of a food market. For the given loading, determine the force in members *FG*, *EG*, and *EH*.

SOLUTION

<u>Reactions at supports</u>. Because of the symmetry of the loading

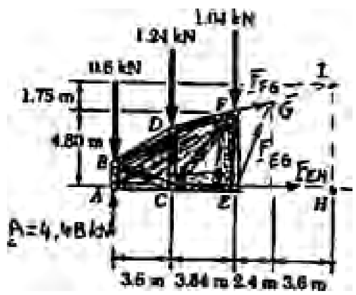
$$A_x = 0, \quad A_y = O = \frac{1}{2}(\text{Total load}) \qquad \mathbf{A} = \mathbf{O} = 4.48 \text{ kN}\uparrow \blacktriangleleft$$

We pass a section through members *FG*, *EG*, and *EH*, and use the <u>Free body shown</u>.



$$\text{Slope } FG = \text{Slope } FI = \frac{1.75 \text{ m}}{6 \text{ m}}$$

$$\text{Slope } EG = \frac{5.50 \text{ m}}{2.4 \text{ m}}$$



$$+\circlearrowleft \Sigma M_E = 0: \quad (0.6 \text{ kN})(7.44 \text{ m}) + (1.24 \text{ kN})(3.84 \text{ m})$$

$$-(4.48 \text{ kN})(7.44 \text{ m})$$

$$-\left(\frac{6}{6.25}F_{FG}\right)(4.80 \text{ m}) = 0$$

$$F_{FG} = -5.231 \text{ kN} \qquad F_{FG} = 5.23 \text{ kN} \quad C \blacktriangleleft$$

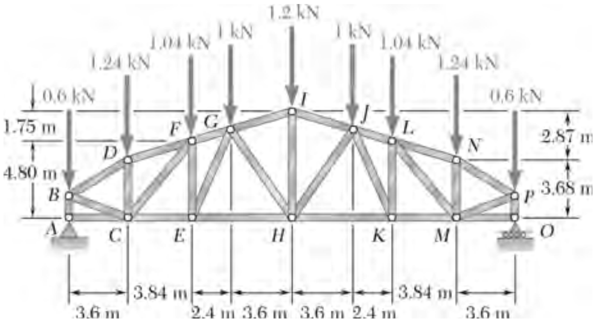
$$+\circlearrowleft \Sigma M_G = 0: \quad F_{EH}(5.50 \text{ m}) + (0.6 \text{ kN})(9.84 \text{ m})$$

$$+(1.24 \text{ kN})(6.24 \text{ m}) + (1.04 \text{ kN})(2.4 \text{ m})$$

$$-(4.48 \text{ kN})(9.84 \text{ m}) = 0 \qquad F_{EH} = 5.08 \text{ kN} \quad T \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad \frac{5.50}{6.001}F_{EG} + \frac{1.75}{6.25}(-5.231 \text{ kN}) + 4.48 \text{ kN} - 0.6 \text{ kN} - 1.24 \text{ kN} - 1.04 \text{ kN} = 0$$

$$F_{EG} = -0.1476 \text{ kN} \qquad F_{EG} = 0.1476 \text{ kN} \quad C \blacktriangleleft$$



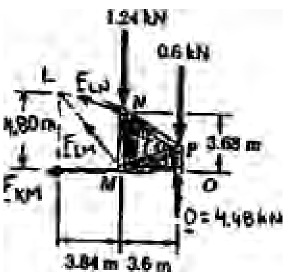
PROBLEM 6.167

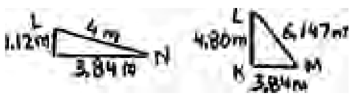
The truss shown was designed to support the roof of a food market. For the given loading, determine the force in members *KM*, *LM*, and *LN*.

SOLUTION

Because of symmetry of loading, $\mathbf{O} = \frac{1}{2}(\text{Load})$ $\qquad\qquad$ $\mathbf{O} = 4.48\text{ kN}\uparrow$ ◁

We pass a section through *KM*, *LM*, *LN*, and use free body shown





$$+\circlearrowleft \Sigma M_M = 0: \quad \left(\frac{3.84}{4}F_{LN}\right)(3.68\text{ m})$$
$$+(4.48\text{ kN} - 0.6\text{ kN})(3.6\text{ m}) = 0$$

$$F_{LN} = -3.954\text{ kN} \qquad\qquad F_{LN} = 3.95\text{ kN} \quad C \blacktriangleleft$$

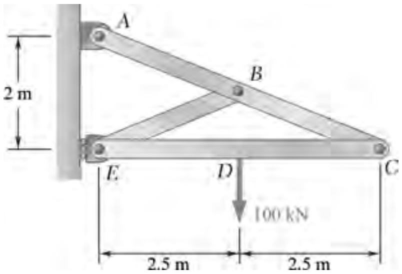
$$+\circlearrowleft \Sigma M_L = 0: \quad -F_{KM}(4.80\text{ m}) - (1.24\text{ kN})(3.84\text{ m})$$
$$+(4.48\text{ kN} - 0.6\text{ kN})(7.44\text{ m}) = 0$$

$$F_{KM} = +5.022\text{ kN} \qquad\qquad F_{KM} = 5.02\text{ kN} \quad T \blacktriangleleft$$

$$+\Sigma F_y = 0: \quad \frac{4.80}{6.147}F_{LM} + \frac{1.12}{4}(-3.954\text{ kN}) - 1.24\text{ kN} - 0.6\text{ kN} + 4.48\text{ kN} = 0$$

$$F_{LM} = -1.963\text{ kN} \qquad\qquad F_{LM} = 1.963\text{ kN} \quad C \blacktriangleleft$$

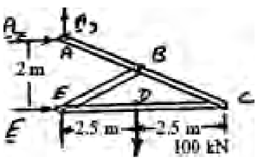
PROBLEM 6.168



For the frame and loading shown, determine the components of all forces acting on member *ABC*.

SOLUTION

Free body: Entire frame



$$+\circlearrowleft \Sigma M_E = 0: \quad -A_x(2) - (100 \text{ kN})(2.5) = 0$$

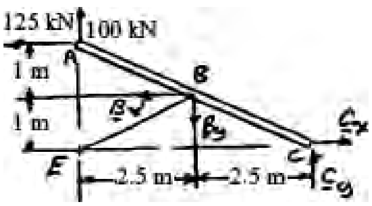
$$A_x = -125 \text{ kN}, \qquad \mathbf{A}_x = 125.0 \text{ kN} \leftarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 100 \text{ kN} = 0$$

$$A_y = 100 \text{ kN} \qquad \mathbf{A}_y = 100.0 \text{ kN} \uparrow \blacktriangleleft$$

Free body: Member *ABC*

Note: BE is a two-force member, thus **B** is directed along line *BE* and $B_y = \dfrac{2}{5} B_x$



$$+\circlearrowleft \Sigma M_C = 0: \quad (125 \text{ kN})(2 \text{ m}) - (100 \text{ kN})(5 \text{ m}) + B_x(1 \text{ m}) + B_y(2.5 \text{ m}) = 0$$

$$-250 \text{ kN}\cdot\text{m} + B_x(1 \text{ m}) + \frac{2}{5} B_x(2.5 \text{ m}) = 0$$

$$B_x = 125 \text{ kN} \qquad \mathbf{B}_x = 125.0 \text{ kN} \leftarrow \blacktriangleleft$$

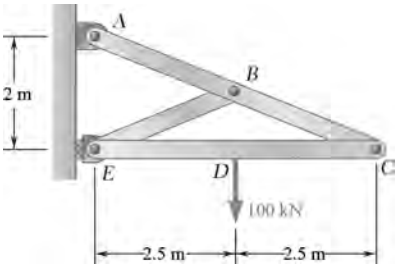
$$B_y = \frac{2}{5}(B_x) = \frac{2}{5}(125) = 50 \text{ kN} \qquad \mathbf{B}_y = 50.0 \text{ kN} \downarrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - 125 \text{ kN} - 125 \text{ kN} = 0$$

$$C_x = 250 \text{ kN} \qquad \mathbf{C}_x = 250 \text{ kN} \rightarrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad C_y + 100 \text{ kN} - 50 \text{ kN} = 0$$

$$C_y = -50 \text{ kN} \qquad \mathbf{C}_y = 50.0 \text{ kN} \downarrow \blacktriangleleft$$



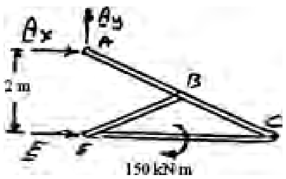
PROBLEM 6.169

Solve Problem 6.168 assuming that the 100 kN load is replaced by a clockwise couple of magnitude 150 kN·m applied to member *EDC* at Point *D*.

PROBLEM 6.168 For the frame and loading shown, determine the components of all forces acting on member *ABC*.

SOLUTION

Free body: Entire frame



$$+\uparrow \Sigma F_y = 0: \quad A_y = 0$$

$$+\circlearrowleft \Sigma M_E = 0: \quad -A_x(2\text{ m}) - 150\text{ kN}\cdot\text{m} = 0$$

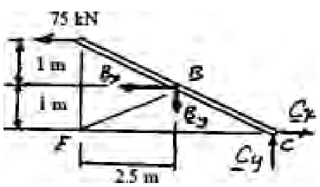
$$A_x = -75\text{ kN} \qquad \mathbf{A}_x = 75\text{ kN} \leftarrow$$

$$\mathbf{A} = 75.0\text{ kN} \leftarrow \blacktriangleleft$$

Free Body: Member *ABC*

Note: BE is a two-force member, thus **B** is directed along line *BE* and $B_y = \frac{2}{5}B_x$

$$+\circlearrowleft \Sigma M_C = 0: \quad (75\text{ kN})(2\text{ m}) + B_x(1\text{ m}) + B_y(2.5\text{ m}) = 0$$



$$150\text{ kN} + B_x(1\text{ m}) + \frac{2}{5}B_x(2.5\text{ m}) = 0$$

$$B_x = -75\text{ kN} \qquad \mathbf{B}_x = 75.0\text{ kN} \rightarrow \blacktriangleleft$$

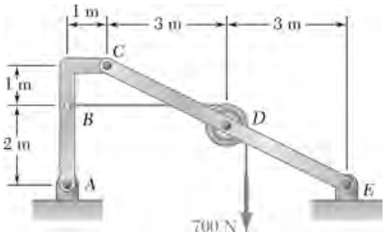
$$B_y = \frac{2}{5}B_x = \frac{2}{5}(-75) = -30\text{ kN} \qquad \mathbf{B}_y = 30.0\text{ kN} \uparrow \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -75\text{ kN} + 75\text{ kN} + C_x = 0 \qquad C_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad +30\text{ kN} + C_y = 0$$

$$C_y = -30\text{ kN} \qquad C_y = 30\text{ kN} \downarrow$$

$$\mathbf{C} = 30.0\text{ kN} \downarrow \blacktriangleleft$$

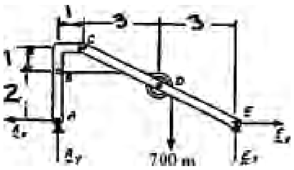


PROBLEM 6.170

Knowing that the pulley has a radius of 0.5 m, determine the components of the reactions at A and E.

SOLUTION

FBD Frame:



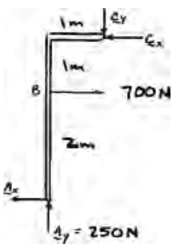
Dimensions in *m*

$$+\circlearrowleft \Sigma M_A = 0: \quad (7\text{ m})E_y - (4.5\text{ m})(700\text{ N}) = 0 \qquad \mathbf{E}_y = 450\text{ N}\uparrow \blacktriangleleft$$

$$\uparrow \Sigma F_y = 0: \quad A_y - 700\text{ N} + 450\text{ N} = 0 \qquad \mathbf{A}_y = 250\text{ N}\uparrow \blacktriangleleft$$

$$\rightarrow \Sigma F_x = 0: \quad A_x - E_x = 0 \qquad A_x = E_x$$

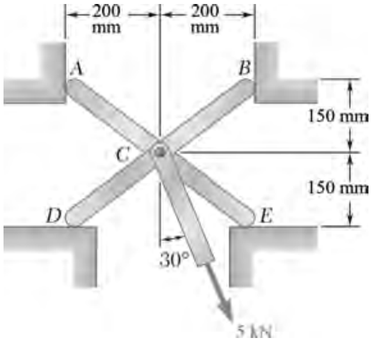
FBD Member ABC:



$$\circlearrowleft \Sigma M_C = 0: \quad (1\text{ m})(700\text{ N}) - (1\text{ m})(250\text{ N}) - (3\text{ m})\,A_x = 0$$

$$\mathbf{A}_x = 150.0\text{ N} \leftarrow \blacktriangleleft$$

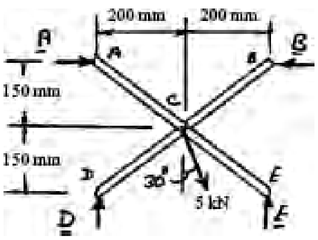
$$\text{so} \quad \mathbf{E}_x = 150.0\text{ N} \rightarrow \blacktriangleleft$$



PROBLEM 6.171

For the frame and loading shown, determine the reactions at A, B, D, and E. Assume that the surface at each support is frictionless.

SOLUTION



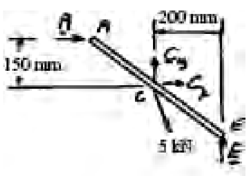
Free body: Entire frame

$$\xrightarrow{+} \Sigma F_x = 0: \quad A - B + (5 \text{ kN})\sin 30° = 0$$

$$A - B + 2.5 \text{ kN} = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad D + E - (5 \text{ kN})\cos 30° = 0$$

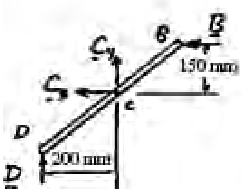
$$D + E - 4.3301 \text{ kN} = 0 \qquad (2)$$



Free body: Member ACE

$$+\circlearrowleft \Sigma M_C = 0: \quad -A(150 \text{ mm}) + E(200 \text{ mm}) = 0$$

$$E = \frac{3}{4}A \qquad (3)$$



Free body: Member BCD

$$+\circlearrowleft \Sigma M_C = 0: \quad -D(200 \text{ mm}) + B(150 \text{ mm}) = 0$$

$$D = \frac{3}{4}B \qquad (4)$$

Substitute E and D from (3) and (4) into (2)

$$\frac{3}{4}A + \frac{3}{4}B - 4.3301 = 0$$

$$A + B - 5.7735 = 0 \qquad (5)$$

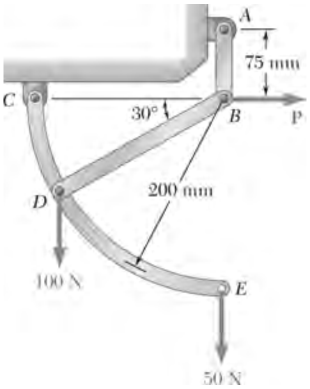
(1) $\quad A - B + 2.5 = 0 \qquad (6)$

(5) + (6) $\quad 2A - 3.2735 = 0 \quad A = 1.6368 \text{ kN} \quad \mathbf{A} = 1.637 \text{ kN} \rightarrow$ ◀

(5) − (6) $\quad 2B - 8.2735 = 0 \quad B = 4.1368 \text{ kN} \quad \mathbf{B} = 4.14 \text{ kN} \leftarrow$ ◀

(4) $\quad D = \frac{3}{4}(4.1368) \quad D = 3.1026 \text{ N} \quad \mathbf{D} = 3.10 \text{ kN}\uparrow$ ◀

(3) $\quad E = \frac{3}{4}(1.6368) \quad E = 1.2276 \text{ kN} \quad \mathbf{E} = 1.228 \text{ kN}\uparrow$ ◀



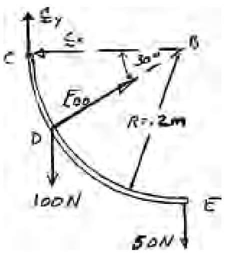
PROBLEM 6.172

For the system and loading shown, determine (*a*) the force **P** required for equilibrium, (*b*) the corresponding force in member *BD*, (*c*) the corresponding reaction at *C*.

SOLUTION

Member FBDs: **FBD I:**

I:



$$\circlearrowleft \Sigma M_C = 0: \quad R(F_{BD}\sin 30°) - [R(1-\cos 30°)](100\text{ N}) - R(50\text{ N}) = 0$$

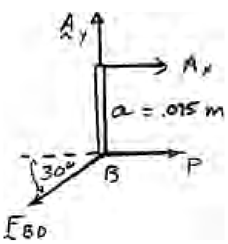
$F_{BD} = 126.795\text{ N}$ (*b*) $F_{BD} = 126.8\text{ N}\quad T$ ◀

$\rightarrow \Sigma F_x = 0: \quad -C_x + (126.795\text{ N})\cos 30° = 0$ $\mathbf{C}_x = 109.808\text{ N} \leftarrow$

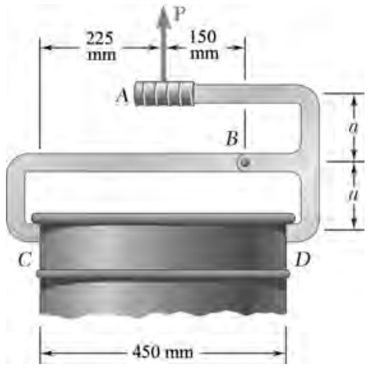
$\uparrow \Sigma F_y = 0: \quad C_y + (126.795\text{ N})\sin 30° - 100\text{ N} - 50\text{ N} = 0$

$\mathbf{C}_y = 86.603\text{ N}\uparrow$ (*c*) so $\mathbf{C} = 139.8\text{ N} \measuredangle\, 38.3°$ ◀

II: **FBD II:**



$\circlearrowleft \Sigma M_A = 0: \quad aP - a[(126.795\text{ N})\cos 30°] = 0$ (*a*) $\mathbf{P} = 109.8\text{ N} \rightarrow$ ◀

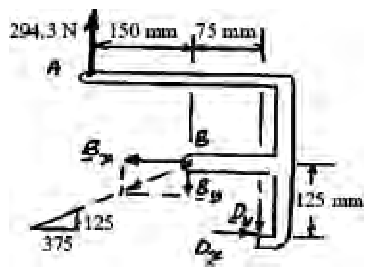


PROBLEM 6.173

A small barrel weighing 30 kg is lifted by a pair of tongs as shown. Knowing that $a = 125$ mm, determine the forces exerted at B and D on tong ABD.

SOLUTION

We note that BC is a two-force member.



$$P = \text{Weight of barrel} = 30 \times 9.81 \text{ N}$$
$$= 294.3 \text{ N}$$

Free body: Tong *ABD*

$$\frac{B_x}{375} = \frac{B_y}{125} \quad B_x = 3B_y$$

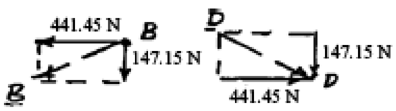
$$+\circlearrowleft \Sigma M_D = 0: \quad B_y(75 \text{ mm}) + 3B_y(125 \text{ mm}) - (294.3 \text{ N})(225 \text{ mm}) = 0$$

$$B_y = 147.15 \text{ N} \downarrow$$

$$B_x = 3B_y: \quad B_x = 441.45 \text{ N} \leftarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -441.45 \text{ N} + D_x = 0 \qquad \mathbf{D}_x = 441 \text{ N} \rightarrow$$

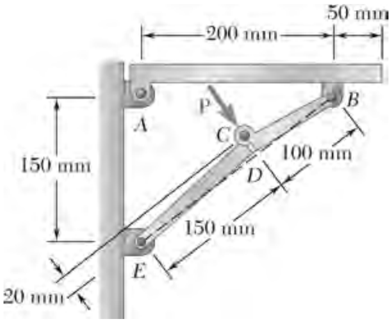
$$+\uparrow \Sigma F_y = 0: \quad 294.3 \text{ N} - 147.15 \text{ N} - D_y = 0 \qquad \mathbf{D}_y = 147.2 \text{ N} \downarrow$$



$\mathbf{B} = 465$ N ↙ $18.43°$ ◀

$\mathbf{D} = 465$ N ↘ $18.43°$ ◀

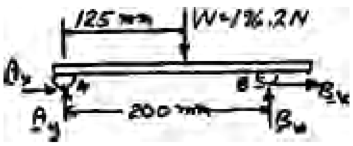
PROBLEM 6.174



A 20-kg shelf is held horizontally by a self-locking brace that consists of two parts *EDC* and *CDB* hinged at *C* and bearing against each other at *D*. Determine the force **P** required to release the brace.

SOLUTION

Free body: Shelf

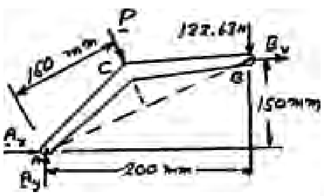


$$W = (20 \text{ kg})(9.81 \text{ m/s}^2) = 196.2 \text{ N}$$

$$+\circlearrowleft \Sigma M_A = 0: \quad B_y(200 \text{ mm}) - (196.2 \text{ N})(125 \text{ mm}) = 0$$

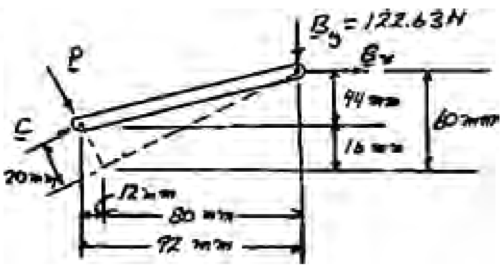
$$B_y = 122.63 \text{ N}$$

Free body: Portion *ACB*



$$+\circlearrowleft \Sigma M_A = 0: \quad -B_x(150 \text{ mm}) - P(150 \text{ mm}) - (122.63 \text{ N})(200 \text{ mm}) = 0$$

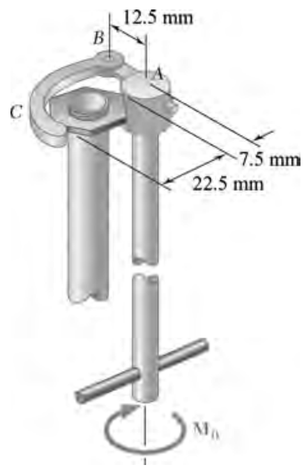
$$B_x = -163.5 - P \qquad (1)$$



$$+\circlearrowright \Sigma M_C = 0: \quad +(122.63 \text{ N})(92 \text{ mm}) + B_x(94 \text{ mm}) = 0$$

$$+(122.63 \text{ N})(92 \text{ mm}) + (-163.5 - P)(44 \text{ mm}) = 0$$

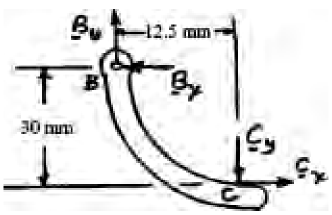
$$P = 92.9 \text{ N} \qquad \mathbf{P} = 92.9 \text{ N} \searrow \blacktriangleleft$$



PROBLEM 6.175

The specialized plumbing wrench shown is used in confined areas (e.g., under a basin or sink). It consists essentially of a jaw BC pinned at B to a long rod. Knowing that the forces exerted on the nut are equivalent to a clockwise (when viewed from above) couple of magnitude 16 N · m, determine (*a*) the magnitude of the force exerted by pin B on jaw BC, (*b*) the couple $\mathbf{M}_0$ that is applied to the wrench.

SOLUTION



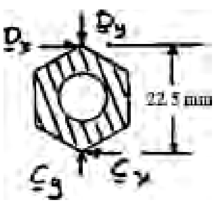
Free body: Jaw BC

This is a two-force member

$$\frac{C_y}{30\text{ mm}} = \frac{C_x}{12.5\text{ mm}} \qquad C_y = 2.4C_x$$

$$\Sigma F_x = 0: \quad B_x = C_x \qquad (1)$$

$$\Sigma F_y = 0: \quad B_y = C_y = 2.4\,C_x \qquad (2)$$



Free body: Nut $\quad \Sigma F_x = 0: \quad C_x = D_x$

$$\circlearrowright \Sigma M = 16\text{ N}\cdot\text{m} = 16000\text{ N}\cdot\text{mm}$$

$$C_x(22.5\text{ mm}) = 16000\text{ N}\cdot\text{mm}$$

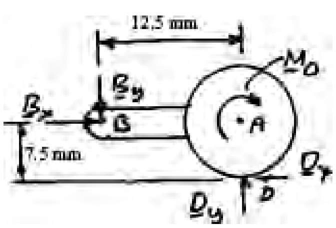
$$C_x = 711.111\text{ N}$$

(*a*) Eq. (1): $\quad B_x = C_x = 711.111\text{ N}$

Eq. (2): $\quad B_y = C_y = 2.4(711.111\text{ N}) = 1706.667\text{ N}$

$$B = \left(B_x^2 + B_y^2\right)^{1/2} = \sqrt{711.111^2 + 1706.667^2}$$

$$= 1848.89\text{ N} \qquad B = 1850\text{ N} \blacktriangleleft$$



(*b*) Free body: Rod

$$+\circlearrowleft \Sigma M_D = 0: \quad -M_0 + B_y(12.5\text{ mm}) - B_x(7.5\text{ mm}) = 0$$

$$-M_0 + (1706.667)(12.5) - (711.111)(7.5) = 0$$

$$M_0 = -16000\text{ N}\cdot\text{mm}$$

$$= -16\text{ N}\cdot\text{m} \qquad \mathbf{M}_0 = 16.00\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$