Lecture 30

02 November 2021 10:01

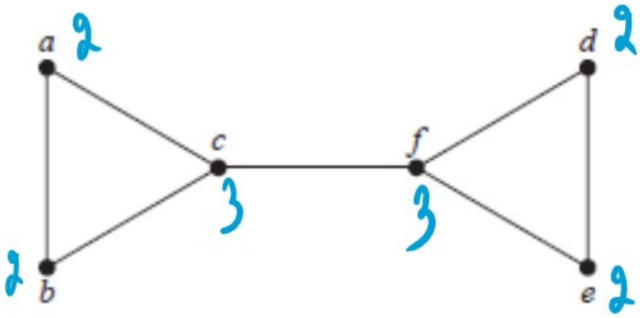
Hamilton Paths and Circuits

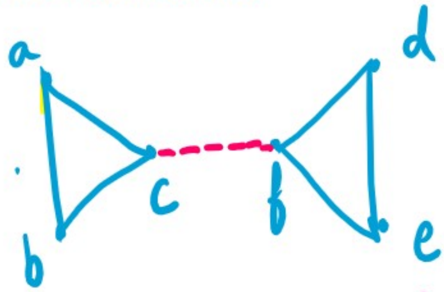
A simple path in a graph G that passes through every vertex exactly once is called a *Hamilton path*, and a simple circuit in a graph G that passes through every vertex exactly once is called a *Hamilton circuit*. That is, the simple path $x_0, x_1, \ldots, x_{n-1}, x_n$ in the graph $G = (V, E)$ is a Hamilton path if $V = \{x_0, x_1, \ldots, x_{n-1}, x_n\}$ and $x_i \neq x_j$ for $0 \leq i < j \leq n$, and the simple circuit $x_0, x_1, \ldots, x_{n-1}, x_n, x_0$ (with $n > 0$) is a Hamilton circuit if $x_0, x_1, \ldots, x_{n-1}, x_n$ is a Hamilton path.

Important points to remember

- If a graph has pendant vertex, then it doesn't have Hamilton Circuit.
- All edges corresponding to degree 2 vertices will always be considered for constructing Hamilton Circuit.
- No more than two edges will be used corresponding to any vertex in G for constructing Hamilton Circuit.
- Hamilton circuit cannot contain smaller circuit in it.

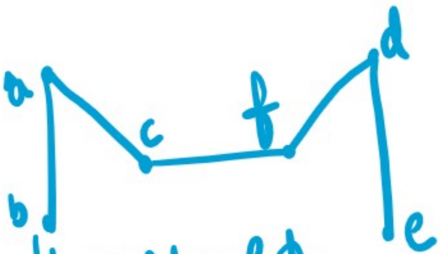
Q33. Determine whether the graphs has Hamilton Circuit, construct such if exists. If no, determine whether graph has Hamilton path, construct such if exists.



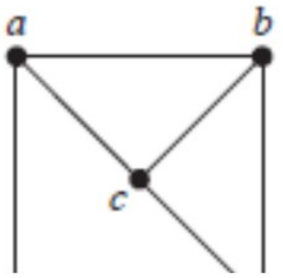


No Hamilton Circuit

a, b, d, e are vertices of degree 2 which leads to disconnected graph.
If we consider {c, f} edge, it leads to more than two edges for c, f.

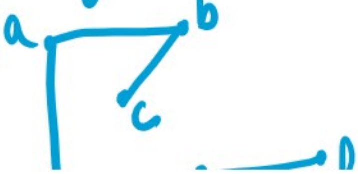


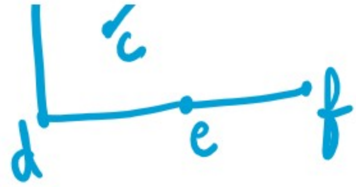
Hamilton Path.

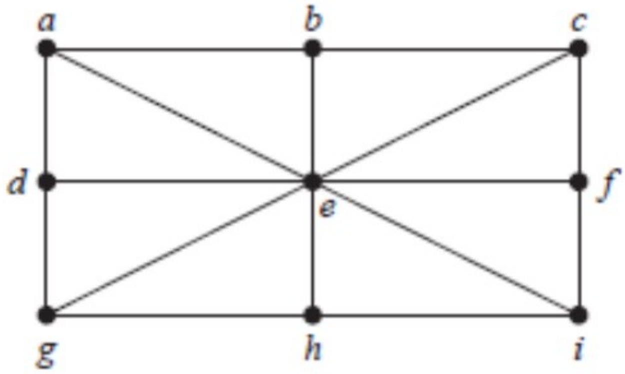


No Hamilton Circuit, f → Pendant vertex

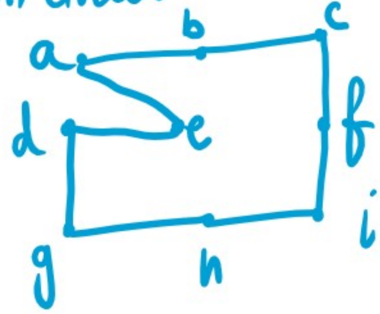
Hamilton Path



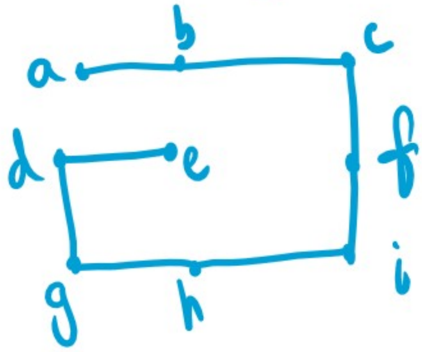




Hamilton Circuit

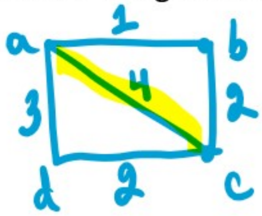


H.P ✓



Shortest path Problems

Graphs that have a number assigned to each edge are called **Weighted Graphs**.



$$\text{a to c}\begin{cases} abc = 1+2=3 \\ ac = 4 \\ adc = 3+2=5 \end{cases}$$

Dijkstra's Algorithm

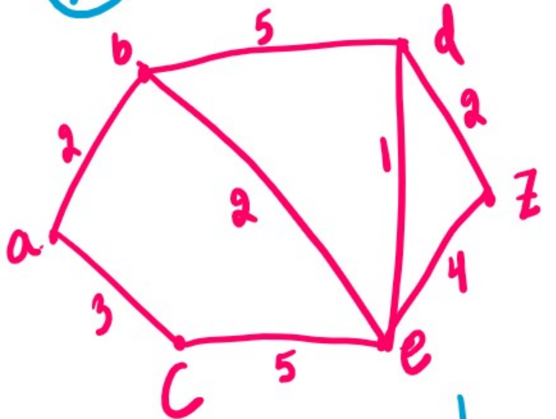
This is used to find the length of shortest path between any pair of vertices in a weighted connected simple graph G.

ALGORITHM 1 Dijkstra's Algorithm.

procedure *Dijkstra*(G: weighted connected simple graph, with
  all weights positive)
{G has vertices $a = v_0, v_1, \ldots, v_n = z$ and lengths $w(v_i, v_j)$
  where $w(v_i, v_j) = \infty$ if $\{v_i, v_j\}$ is not an edge in G}
for $i := 1$ **to** n
  $L(v_i) := \infty$
$L(a) := 0$
$S := \emptyset$
{the labels are now initialized so that the label of a is 0 and all
  other labels are ∞, and S is the empty set}
while $z \notin S$
  $u :=$ a vertex not in S with $L(u)$ minimal
  $S := S \cup \{u\}$
  for all vertices v not in S
    if $L(u) + w(u, v) < L(v)$ **then** $L(v) := L(u) + w(u, v)$
    {this adds a vertex to S with minimal label and updates the
    labels of vertices not in S}
return $L(z)$ {$L(z) =$ length of a shortest path from a to z}

(•) → vertex is closed from back side and open from front.

(X) → Path is cancelled.



To find shortest path b/w a and z

a — b 2 ✓
a — c 3 ✓

b — d 2+5=7 (X)
b — e 2+2=4 ✓

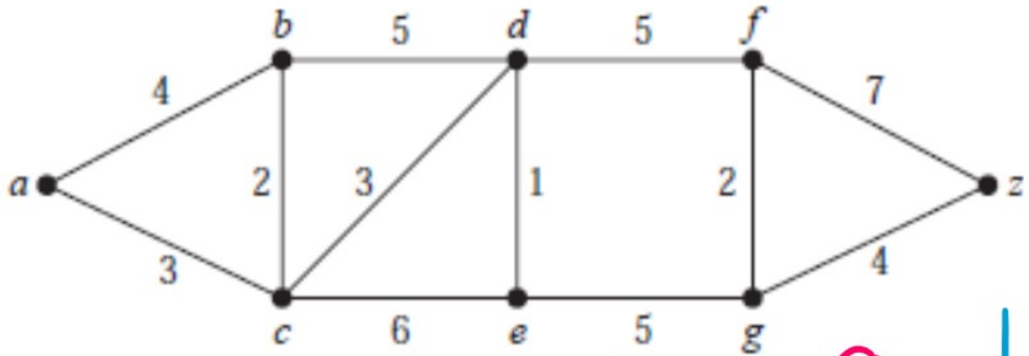
c — e 3+5=8 (X)

e — d 4+1=5 ✓
e — z 4+4=8 (X)

d — z 5+2=7 ✓

a — b — c — d — z = 7

Q34. Find the length of the shortest path between *a* and *z* in the given weighted graph.



a — b 4 ✓
a — c 3 ✓

c — b 3+2=5 ✗
c — d 3+3=6 ✓
c — e 3+6=9 ✗

b — d 4+5=9 ✗

d — e 6+1=7 ✓
d — f 6+5=11 ✓

e — g 7+5=12 ✓

f — g 2+11=13 ✗
f — z 7+11=18 ✗

g — z 12+4=16 ✓

a — c — d — e — g — z = 16

a — c — d — f = 11