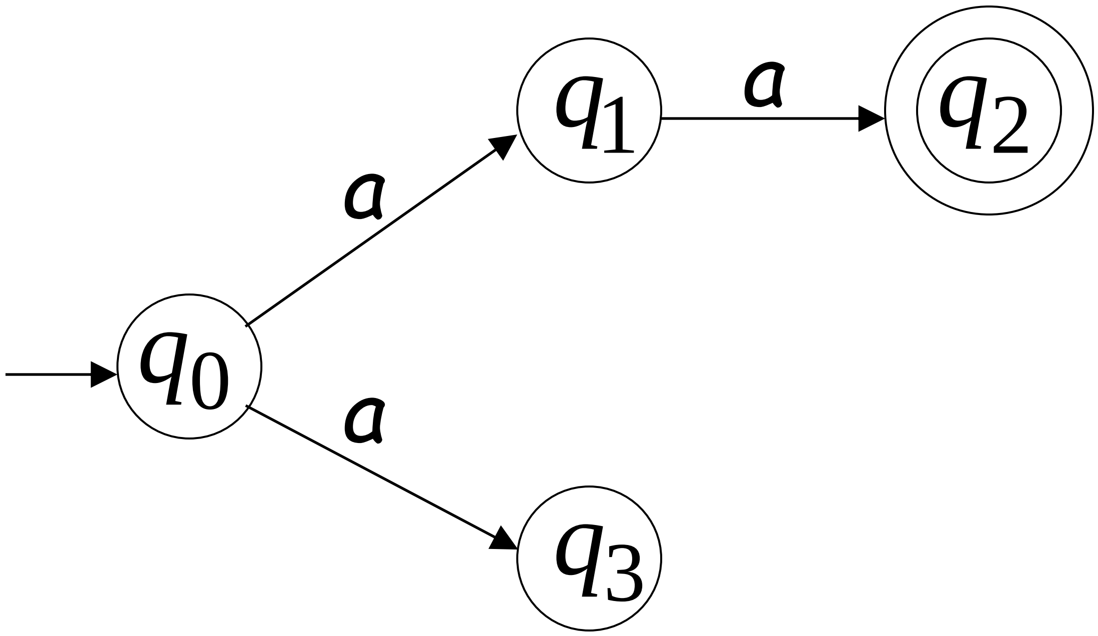
CSE322
DFA and NDFA

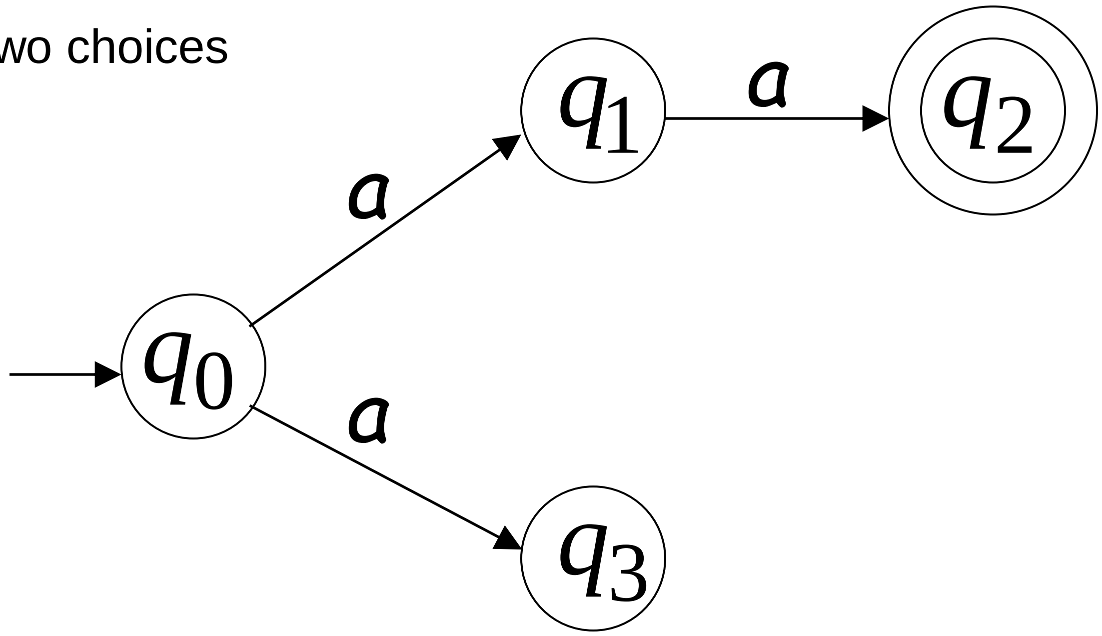
Lecture #3

Alphabet = $\{a\}$



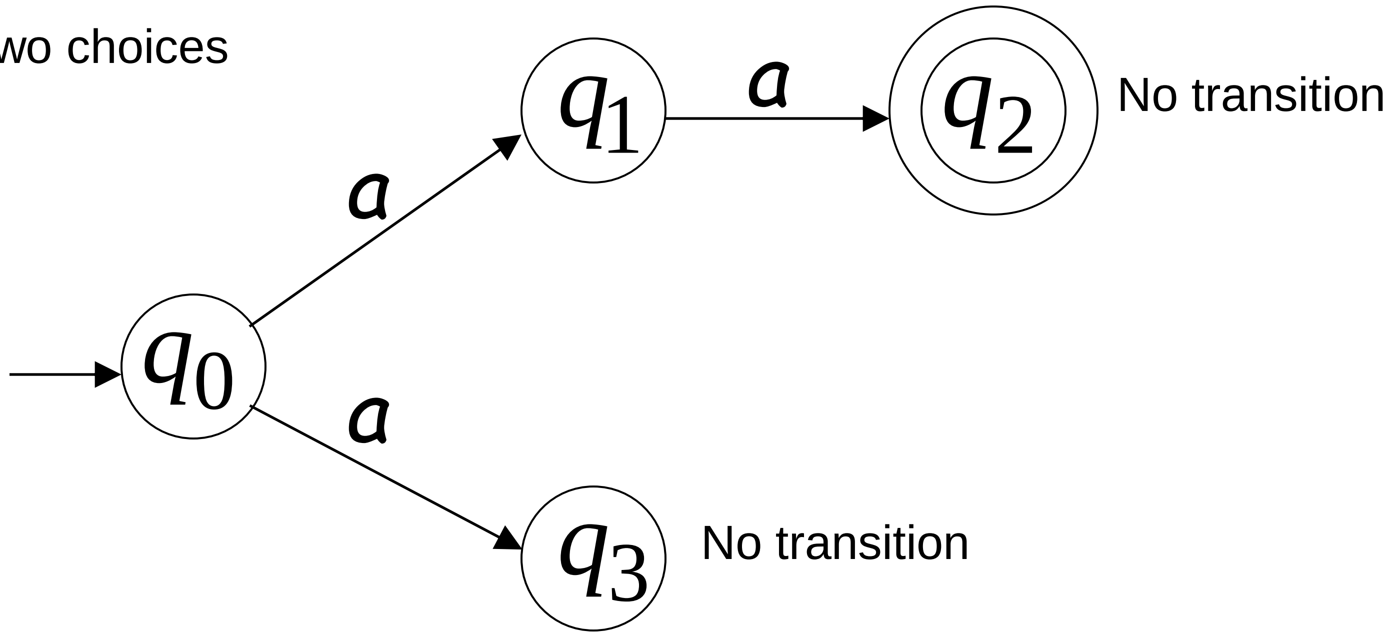
Alphabet = $\{a\}$

Two choices



Alphabet = $\{a\}$

Two choices



NFA and DFA

A Nondeterministic Finite Automata (NFA) is defined by a 5-tuple

$$M = (Q, \Sigma, \delta, q_0, F)$$

where $Q, \Sigma, \delta, q_0, F$ are defined as follows:

Q = Finite set of internal states

Σ = Finite set of symbols called "Input alphabet"

$\delta = Q \times (\Sigma \cup \{\lambda\}) \rightarrow 2^Q$

$q_0 \in Q$ is the Initial states

$F \subseteq Q$ is a set of Final states

Example 1.1.3: Sketch the DFA given

$$M = \left(\{q_1, q_2\}, \{0,1\}, \delta, q_1, \{q_2\} \right)$$

and δ is given by

$$\delta(q_1, 0) = q_1 \text{ and } \delta(q_2, 0) = q_1$$
$$\delta(q_1, 1) = q_2 \quad \delta(q_2, 1) = q_2$$

Determine a Language $L(M)$, that the DFA recognizes.

Example 1.1.5: Obtain the state table diagram and state transition diagram (DFA Schematic) of the finite state Automaton $M = (Q, \Sigma, \delta, q_0, F)$, where $Q = \{q_0, q_1, q_2, q_3\}$, $\Sigma = \{a, b\}$, q_0 is the initial state, F is the final state with the transition defined by

$$\delta(q_0, a) = q_2 \quad \delta(q_3, a) = q_1 \quad \delta(q_2, b) = q_3$$

$$\delta(q_1, a) = q_3 \quad \delta(q_0, b) = q_1 \quad \delta(q_3, b) = q_2$$

$$\delta(q_2, a) = q_0 \quad \delta(q_1, b) = q_0$$

Example 1.2.5: Sketch the NFA state diagram for

$$M = (\{q_0, q_1, q_2, q_3\}, \{0,1\}, \delta, q_0, \{q_3\})$$

with the state table as given below.

δ	0	1
q_0	q_0, q_1	q_0, q_2
q_1	q_3	$\varnothing$
q_2	$\varnothing$	q_3
q_3	q_3	q_3

Consider the finite state machine whose transition function δ is given by Table 3.1 in the form of a transition table. Here, $Q = \{q_0, q_1, q_2, q_3\}$, $\Sigma = \{0, 1\}$, $F = \{q_0\}$. Give the entire sequence of states for the input string 110001.

TABLE 3.1 Transition Function Table for Example 3.5

State	*Input*	
	0	1
$\rightarrow q_0$	q_2	q_1
q_1	q_3	q_0
q_2	q_0	q_3
q_3	q_1	q_2

Solution

$$\delta(\overset{\downarrow}{q_0}, 110101) = \delta(\overset{\downarrow}{q_1}, 10101)$$

$$\begin{aligned}
&= \delta(q_0, 0101) \\
&= \delta(q_2, 101) \\
&= \delta(q_3, 01) \\
&= \delta(q_1, 1) \\
&= \delta(q_0, \Lambda) \\
&= q_0
\end{aligned}$$

Hence,

$$q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_0 \xrightarrow{0} q_2 \xrightarrow{1} q_3 \xrightarrow{0} q_1 \xrightarrow{1} q_0$$

Real Life examples

- example can be any person's career. a person can learn hours and study various subjects but will still land on a career less predictable like a mathematics student may become a soldier, a botanist may become cook etc.
- Downloading a movie

CSE322
Equivalence of DFA and NDFA

Lecture #3

Acceptablity

- NFAs and DFAs are equivalent in that if a language is recognized by an NFA, it is also recognized by a DFA and vice versa

DFA and NDFA

- Every NDFA is DFA but vice versa is not true

Equivalence of DFA and NDFA

We naturally try to find the relation between DFA and NDFA. Intuitively we now feel that:

(i) A DFA can simulate the behaviour of NDFA by increasing the number of states. (In other words, a DFA $(Q, \Sigma, \delta, q_0, F)$ can be viewed as an NDFA $(Q, \Sigma, \delta', q_0, F)$ by defining $\delta'(q, a) = \{\delta(q, a)\}$.)

(ii) Any NDFA is a more general machine without being more powerful.

Equivalence of NFA and DFA

Theorem 3.1 For every NDFA, there exists a DFA which simulates the behaviour of NDFA. Alternatively, if L is the set accepted by NDFA, then there exists a DFA which also accepts L.

Proof Let $M = (Q, \Sigma, \delta, q_0, F)$ be an NDFA accepting L. We construct a DFA M' as:

$$M' = (Q', \Sigma, \delta, q'_0, F')$$

where

(i) $Q' = 2^Q$ (any state in Q' is denoted by $[q_1, q_2, \ldots, q_j]$, where $q_1, q_2, \ldots, q_j \in Q$);

(ii) $q'_0 = [q_0]$; and

(iii) F' is the set of all subsets of Q containing an element of F.

Problem

Construct a deterministic automaton equivalent to

$$M = (\{q_0, \quad q_1\}, \{0, 1\}, \delta, q_0, \{q_0\})$$

where δ is defined by its state table (see Table 3.2).

TABLE 3.2 State Table for Example 3.6

State/Σ	0	1
$\rightarrow q_0$	q_0	q_1
q_1	q_1	q_0, q_1

Solution

For the deterministic automaton M_1,

(i) the states are subsets of $\{q_0, q_1\}$, i.e. $\emptyset$, $[q_0]$, $[q_0, q_1]$, $[q_1]$;
(ii) $[q_0]$ is the initial state;
(iii) $[q_0]$ and $[q_0, q_1]$ are the final states as these are the only states containing q_0; and
(iv) δ is defined by the state table given by Table 3.3.

TABLE 3.3 State Table of M_1 for Example 3.6

State/Σ	0	1
$\emptyset$	$\emptyset$	$\emptyset$
$[q_0]$	$[q_0]$	$[q_1]$
$[q_1]$	$[q_1]$	$[q_0, q_1]$
$[q_0, q_1]$	$[q_0, q_1]$	$[q_0, q_1]$

Problem

Find a deterministic acceptor equivalent to

$$M = (\{q_0, q_1, q_2\}, \{a, b\}, \delta, q_0, \{q_2\})$$

where δ is as given by Table 3.4.

TABLE 3.4 State Table for Example 3.7

State/Σ	a	b
$\rightarrow q_0$	q_0, q_1	q_2
q_1	q_0	q_1
q_2		q_0, q_1

Solution

Solution

The deterministic automaton M_1 equivalent to M is defined as follows:

$$M_1 = (2^Q, \{a, b\}, \delta, [q_0], F')$$

where

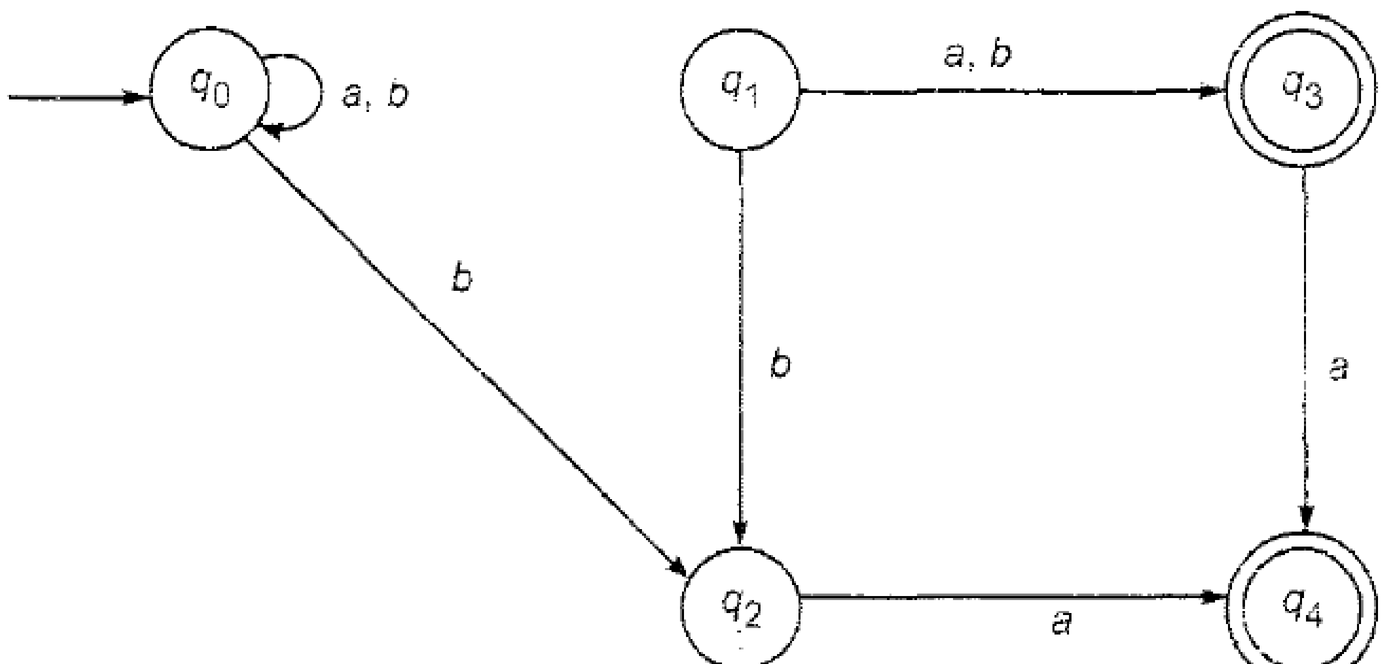
$$F = \{[q_2], [q_0, q_2], [q_1, q_2], [q_0, q_1, q_2]\}$$

We start the construction by considering $[q_0]$ first. We get $[q_2]$ and $[q_0, q_1]$. Then we construct δ for $[q_2]$ and $[q_0, q_1]$. $[q_1, q_2]$ is a new state appearing under the input columns. After constructing δ for $[q_1, q_2]$, we do not get any new states and so we terminate the construction of δ. The state table is given by Table 3.5.

TABLE 3.5 State Table of M_1 for Example 3.7

State/Σ	a	b
$[q_0]$	$[q_0, q_1]$	$[q_2]$
$[q_2]$	$\emptyset$	$[q_0, q_1]$
$[q_0, q_1]$	$[q_0, q_1]$	$[q_1, q_2]$
$[q_1, q_2]$	$[q_0]$	$[q_0, q_1]$

Construct a DFA equivalent to the NDFA M whose transition diagram is given



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