REMAINDERS
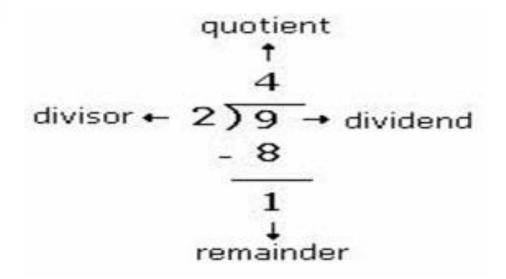
Remainders

- Concept of positive and negative remainders
- Concept of remainders of higher powers
- Fermat's theorem
- Wilson Theorem
- Euler's theorem
- Questions related to positive and negative remainder
- Questions related to all the three theorems
- Miscellaneous Questions

Introduction of Remainder:

REMAINDER

Concept of Negative Remainder:

Example: What is the remainder when 123 × 124 × 125 is divided by 9.

Solution

Remainder obtained when 123 is divided by 9 = -3
Remainder obtained when 124 is divided by 9 = -2
Remainder obtained when 123 is divided by 9 = -1
Final remainder = (-3)(-2)(-1) = -6. The required positive remainder = 9-6 = 3.

Remainder of higher power terms:

We can find out the remainder of higher power term by using Binomial expansion.
Let us suppose we have to find remainder of X^n when divided by 'a'.

For example:
Example 1: What will be remainder if 10^20 is divided by 9.
Solution: using binomial expansion
{(9+1)^20}/9
So remainder will be 1^20 = 1

REMAINDER

Special case:
I. Fermat's Theorem (when divisor is prime):

It states that if X^{Y-1}/Y , where Y is a prime number and (X,Y) are co prime numbers, then remainder will always be 1.

Example 1: Find the remainder when 23^6 is divided by 7.
Solution: 7 is prime number.

So $23^{7-1}/7$

or $23^6/7$ gives remainder 1.

REMAINDER

II. Wilson Theorem (when divisor is prime):

It states that for any prime number 'p', (p-1)! divided by p leaves a remainder of p – 1.

For example,

16! divided by 17, remainder is 16.

12! divided by 13, remainder is 12

10! by 11, remainder is 10

III. Euler's Theorem (when the divisor is either prime or composite):

What is Totient number: The number of co-prime pair less than given number is called totient number of that number.

Example 1:**Find the totient number of 6.**
We will check how many number less than 6 which are co-prime with 6. Since 1,5 are less than 6 and co-prime with 6. So totient number of 6 will be 2.

Example 2.Find the totient no. of 5 .
 1,2,3,4 all are co-prime with 5. So totient number of 5 is 4.

In case of **Prime number** , the totient number of any prime number is (Prime no. – 1)

In case of **Composite number -**
Let the no. is $n=a^p b^q c^r$ (Prime Factorization of n)

Then the totient number of $n = n(1-1/a)(1-1/b)(1-1/c)$

For Example, Let $36 = 2^2 * 3^2$
Totient number of $36 = 36(1-1/2) \, (1-1/3)$
$= 36 * 1/2 * 2/3 = 12$

(it means there are 12 numbers which are less than and co-prime with 36)

Statement: It states that if, for $X^{Y(\emptyset)}/Y$, where X and Y are co-prime numbers and Y($\emptyset$) is the totient number of Y, then the remainder will always be 1.

Example 1: Find the remainder when 23^{16} is divided by 8.
Solution:
Divisor is 8 (composite number) and 23 & 8 are co-prime so we will find the totient number of divisor 8.

    Prime Factorization of 8=2^3

    So totient number of 8 = 8(1-1/2) = 4

    Now Rem[23^4/8] = 1

        $(23^4)^4$/8 = 1^4/8 = 1

REMAINDER

1. Find the remainder when 40*118*160 is divided by 13?

A] 9   B] 4

C] 3   D] 1

REMAINDER

2.Find the remainder when 44*85*148 is divided by 21?

A]7   B]1

C]2   D]4

3. Find the remainder when 44*89*148 is divided by 15?

A]7   B]1

C]13   D]4

4. Find the remainder when 42*87*151 is divided by 22?

A]17   B]16

C]12   D]14

5. Find the remainder when 52*96*123*177*223 is divided by 100?

A]37   B]36

C]62   D]64

6. What is the remainder when 17^2004 is divided by 18?

A]1   B]17

C]5   D]18

7. What is the remainder when 17^2003 is divided by 18?

A]1   B]17

C]5   D]18

8. What is the remainder when 2^2001 is divided by 9?

A]1   B]7

C]5   D]8

REMAINDER

9. What is the remainder when 17^2001 is divided by 290?

A]1   B]17

C]5   D]38

10. Find the remainder when (47^27+47) is divided by 23?

A]1   B]2
C]3   D]6

11. What is the remainder of $2^{18}/7$?

A]1   B]2

C]3   D]6

12. What is the remainder of $2^{101}/11$?

A]1   B]2
C]3   D]6

13. What is the remainder of 5^34/17?

A]17   B]25

C]23   D]8

14. What is the remainder of $15^{94}/47$?

A]37   B]25
C]23   D]38

15. What is the remainder when (1!+2!+3!+4!+…….+12!) is divided by 5?

A] 5   B] 7

C]0   D] 3

REMAINDER

16. What is the remainder when 16! is divided by 17?

A]16   B]0

C]17   D]3

17. What is the remainder when 25! is divided by 529?

A]46   B]480

C]23   D]483

18. What is the remainder when 37! Is divided by 41?

A]7   B]41

C]6   D]47

19. What is remainder obtained if 455^18 is divided by 19?

A]7   B] 1

C]6   D] 8

20. What is the remainder when 3^164 is divided by 162?

A]51   B]81

C]60   D]93

21. Find out the remainder when 7^73 is divided by 30?

A]1   B]0

C]7   D]3

22. What is the remainder of (121) ^(121) divided by 144?

A]121   B]120

C]119   D]113

23. What is the remainder of $(1^7+2^7+3^7+4^7+5^7+6^7+7^7)$ divided by 8?

A]4　　　　　　　　　　　　　B]0

C]2　　　　　　　　　　　　　D]6

REMAINDER

24. Find the remainder if 70170270370470 5…..797798799800 is divided by 7?

A]4   B]0

C]1   D]None

Any Doubts???