HEIGHT AND DISTANCE

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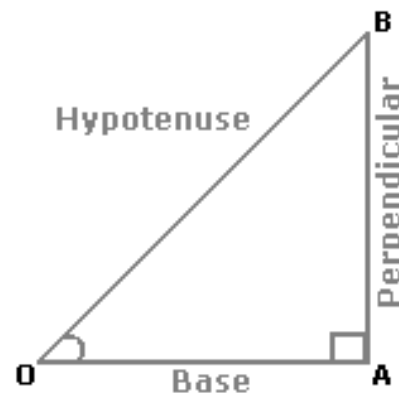vi. **Calculating time and distance**

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1) Introduction

This chapter deals with finding the heights, distances and angles using trigonometric values

1.i) Trigonometry:

Trigonometry:

In a right angled △ OAB, where ∠BOA = θ,

i. $\sin\theta = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}} = \dfrac{AB}{OB};$

ii. $\cos\theta = \dfrac{\text{Base}}{\text{Hypotenuse}} = \dfrac{OA}{OB};$

iii. $\tan\theta = \dfrac{\text{Perpendicular}}{\text{Base}} = \dfrac{AB}{OA};$

iv. $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta} = \dfrac{OB}{AB};$

v. $\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{OB}{OA};$

vi. $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{OA}{AB};$

1.ii) Trigonometry Identities:

$$\sin^2 \theta + \cos^2 \theta = 1.$$
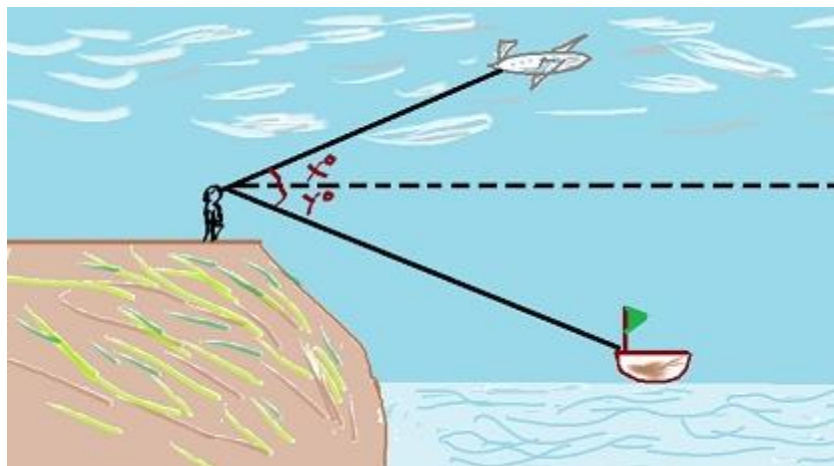$$1 \quad + \tan^2 \theta = \sec^2 \theta.$$
$$1 \quad + \cot^2 \theta = \operatorname{cosec}^2 \theta$$

1.iii) Values of T-ratios:

	0°	30°	45°	60°	90°
sin θ	0	1/2	1/√2	√3/2	1
cos θ	1	√3/2	1/√2	1/2	0
tan θ	0	1/√3	1	√3	Not defined

1.iv) Angle of elevation and depression

x – angle of elevation
y – angle of depression

Note: *The base line for angle of elevation and angle of depression will always be the horizontal line.*

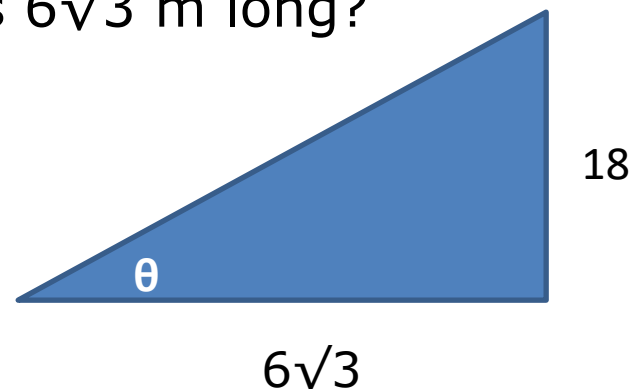# 2) Problems

2.i) Two of the sides given

Example: Find the angle of elevation of the sun when the shadow of a pole of 18 m height is 6√3 m long?

Given:
Perpendicular = 18 m
Base   = 6√3 m
Angle   = ?

18

θ

6√3

Solution:
tan θ = perpendicular/base
        = 18/ 6√3
        = 3/√3
        = √3
θ   = 60º
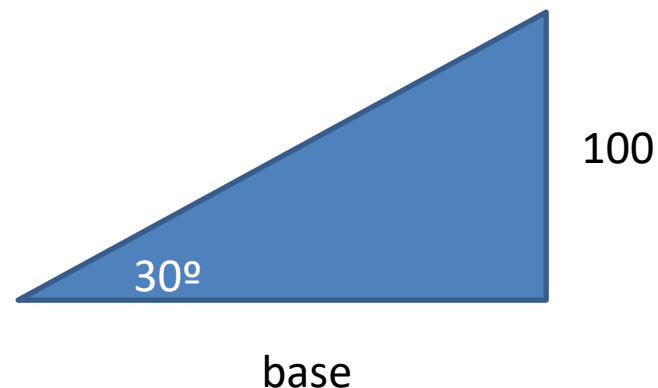
2.ii) One angle and one of the sides given

Example: From a point P on a level ground, the angle of elevation of the top tower is 30º. If the tower is 100 m high, then what is the distance of point P from the foot of the tower.

Given:
Perpendicular = 100 m
Angle   = 30º
Base   = ?

Solution:

tan θ   = perpendicular/base
tan 30º = 100/base
1/ √3   = 100/base
Base   = 100 √3
            = 173 m
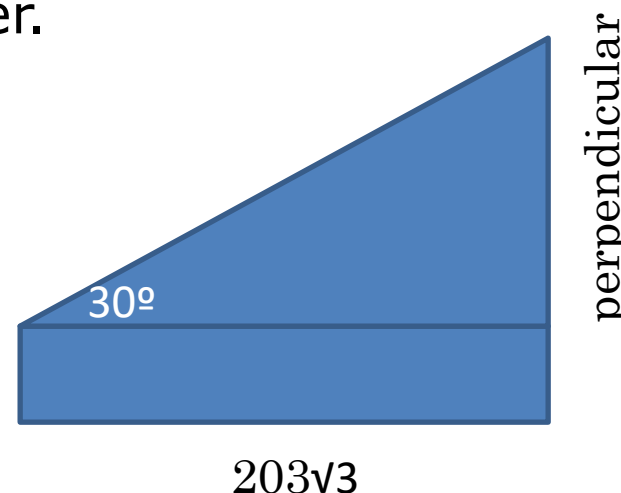
100

30º

base

2.iii) Two heights and one angle

Example: An observer 1.6 m tall is 203√3 m away from a tower. The angle of elevation from his eye to the top of the tower is 30º. Find the height of the tower.

Given:
Base = 203√3 m
Angle = 30º

Height = perpendicular + 1.6 = ?

Solution:

tan θ	**= perpendicular / base**
tan 30º	= perpendicular / 203√3
1/ √3	= perpendicular / 203√3
Perpendicular	= 203√3 / √3

Perpendicular = 203 m

Height of the tower = perpendicular + 1.6
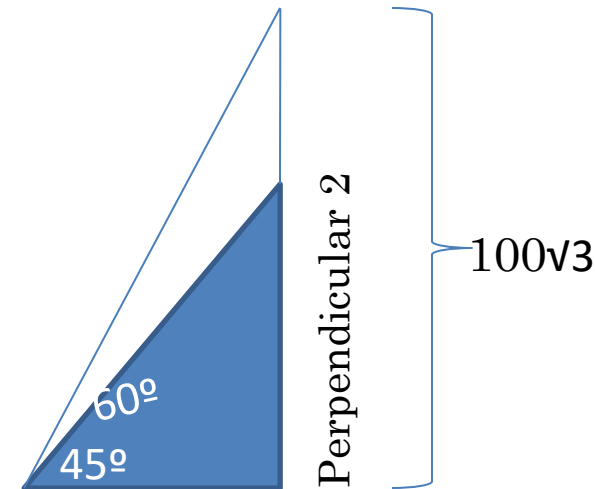$$= 203 + 1.6$$
= 204.6m

2.iv) Two angles and one height

Example: An aeroplane when 100√3 m high passes vertically above another aeroplane at an instant when their angles of elevation at same observing point are 60° and 45° respectively. Approximately, how many meters higher is the one than the other?

Given:
Perpendicular 1 = 100√3 m
Angle 1   = 60°
Angle 2   = 45°

Difference b/w the heights =Perpendicular 1 - Perpendicular 2 = ?

Solution:

Angle 1 = Perpendicular 1/ Base
tan 60° = 100√3 / base
√3   = 100√3 /base
Base   = 100√3 / √3
Base   = 100

Angle 2   = Perpendicular 2/ Base
tan 45°   = Perpendicular 2 / 100
1   = Perpendicular 2 / 100
Perpendicular 2 = 100

Difference b/w the heights =Perpendicular 1 - Perpendicular 2
                            = 100√3 – 100
                            = 173 – 100
                            = 73 m
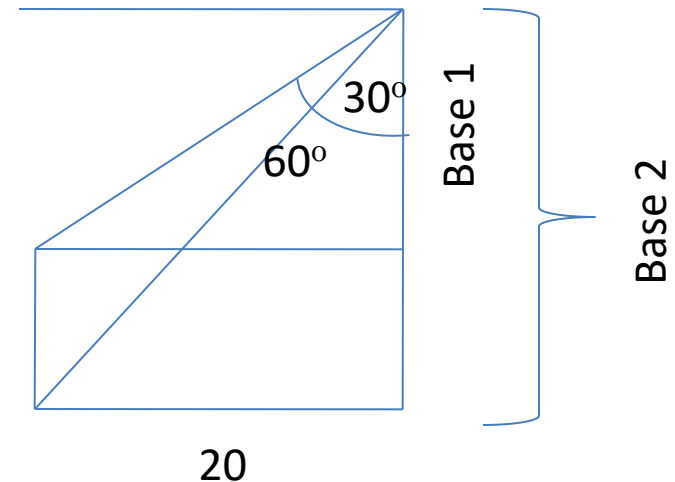
2.v) Two angles and two heights

Example: Two towers face each other separated by a distance d = 20 m. As seen from the top of the first tower, the angle of depression of the second tower's base is 60º and that of the top is 30º. What is the height of the second tower?

Given:

Angle 1 = 90º - 30º = 60º

Angle 2 = 90º - 60º = 30º

Perpendicular = 20m

Height of the second tower = Base 2 – Base 1

Solution:

Angle 2 = perpendicular/base 2
tan 30° = 20 / base 2
1/$\sqrt{3}$ = 20 / base 2
Base 2   = 20$\sqrt{3}$

Angle 1 = perpendicular / base 1
tan 60° = 20 / base 1
$\sqrt{3}$ = 20 / base 1
Base 1 = 20 / $\sqrt{3}$

Height of the second tower = Base 2 – Base 1
$\qquad\qquad\qquad\qquad$ = 20$\sqrt{3}$ – 20 / $\sqrt{3}$
$\qquad\qquad\qquad\qquad$ = 20$\sqrt{3}$ - 20$\sqrt{3}$ / 3
$\qquad\qquad\qquad\qquad$ = 20$\sqrt{3}$ (1 – 1/3)
$\qquad\qquad\qquad\qquad$ = 20$\sqrt{3}$ (2/3)
$\qquad\qquad\qquad\qquad$ **= 40$\sqrt{3}$ / 3**

2.vi) Calculating time and speed

Example: You are stationed at a radar base and you observe an unidentified plane at an altitude h = 2000 m flying towards your radar base at an angle of elevation = 30°. After exactly one minute, your radar sweep reveals that the plane is now at an angle of elevation = 60° maintaining the same altitude. What is the speed (in m/s) of the plane?
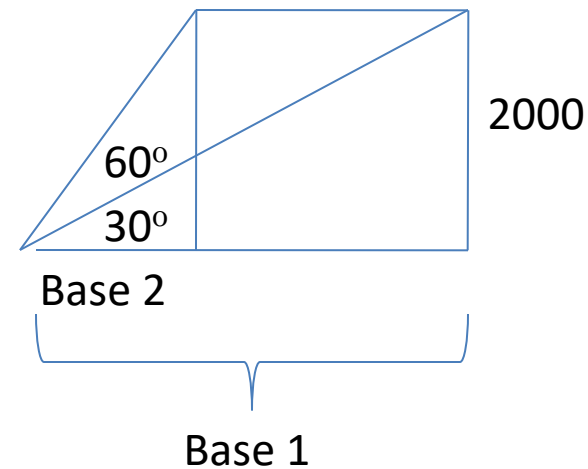
Given:

Angle 1 = 30°
Angle 2 = 60°
Perpendicular = 2000 m
Time = 60 sec
Speed = distance / time = (base 1 – base 2) / time

Solution:

Angle 1 = perpendicular / base 1
tan 30° = 2000 / base 1
1/$\sqrt{3}$ = 2000 / base 1
Base 1   = 2000$\sqrt{3}$

Angle 2 = perpendicular / base 2
Angle 60° = 2000 / base 2
$\sqrt{3}$ = 2000 / base 2
Base 2   = 2000 / $\sqrt{3}$

Speed = distance / time
        = (base 1 – base 2) / time
        = (2000$\sqrt{3}$ - 2000 / $\sqrt{3}$) / 60
        = 200$\sqrt{3}$ / 9 m/s

3) Practice problems

1. Find the angle of elevation of the sun when the shadow of a pole of 18 m height is 6√3 m long?

A. 30°

B. 60°

C. 45°

D. None of these

Ans. B

2. The angle of elevation of the sun, when the length of the shadow of a tree is √3 times the height of tree, is :

A. 30 degree

B. 45 degree

C. 60 degree

D. 9 degree

Ans. A

3. The angle of elevation of a ladder leaning against a wall is 60° and the foot of the ladder is 4.6 m away from the wall. The length of the ladder is:

A. 2.3 m

B. 4.6 m

C. 7.8 m

D. 9.2 m

Ans. D

4. An observer 1.6 m tall is 20√3 away from a tower. The angle of elevation from his eye to the top of the tower is 30°. The heights of the tower is:

A. 21.6 m

B. 23.2 m

C. 24.72 m

D. None of these

Ans. A

5. From a tower of 80 m high, the angle of depression of a bus is 30°. How far is the bus from the tower?

A. 80 m
B. 80√3 m
C. 80/√3m
D. 240 m

Ans. B

6. From a point P on a level ground, the angle of elevation of the top of a tower is 30^0 If the tower is 100 m high, the distance of point P from the foot of the tower is:

A. 149 m

B. 156 m

C. 173 m

D. 200 m

Ans. C

7. The thread of a kite is 120 m long and it is making 30° angular elevation with the ground .What is the height of the kite?

A. 60 m

B. 20 m

C. 40 m

D. 10 m

Ans. A

8. When the sun's altitude changes from 30° to 60°, the length of the shadow of a tower decreases by 70m. What is the height of the tower?

A. 55.6 m

B. 60.6 m

C. 65.6 m

D. 70.6 m

   Ans. B

9. The length of the shadow of a vertical tower on level ground increases by 10 metres when the altitude of the sun changes from 45° to 30°. Then the height of the tower is:

A. 5√3 m

B. 10(√3 + 1) m

C. 5(√3 + 1) m

D. 10√3 m

Ans. C

10. Two ships are sailing in the sea on the two sides of a lighthouse. The angle of elevation of the top of the lighthouse is observed from the ships are 30° and 45° respectively. If the lighthouse is 100 m high, the distance between the two ships is:

A. 173 m

B. 200 m

C. 273 m

D. 300 m

Ans. C

11. A vertical post 15 ft. high is broken at a certain height and its upper part, not completely separated meets the ground angle of 30^0. Find the height at which the post is broken.

A. 10 ft.

B. 5 ft.

C. $15\sqrt{3}$ (2-$\sqrt{3}$) ft.

D. $5\sqrt{3}$ ft.

Ans. B

12. The angle of elevation of the top of a tower from a point on the ground is 30° and moving 70 meters towards the tower it becomes 60°. The height of the tower is:

A. 10 meter

B. 10/√3 meter

C. 10√3 meter

D. 35√3 meter

Ans. D

13. The top of a 15 m high tower makes an angle of elevation of 60 degree with the bottom of an electric pole and an angle of 30 degree with the top of the pole. What is the height of the pole?

A. 12 m

B. 10 m

C. 11 m

D. 5 m

Ans. B

14. Two pillars of equal height are on either side of a road, which is 120m wide. The angles of elevation of the top of the pillars are 60^0 and 30^0 at a point on the road between the pillars. Find the height of the pillars.

A. 10√3 m

B. 30√3 m

C. 20√3 m

D. None of these

Ans. B

THANKS
FOR
LISTENING