Lecture 34

13 November 2021 12:01

Properties of Trees

Theorem 2: A tree with n vertices has $(n-1)$ edges.

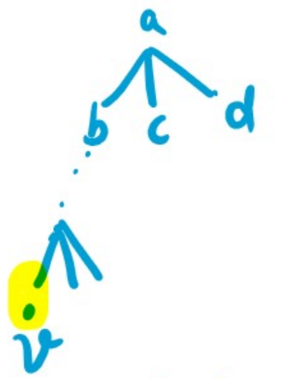
$n=1$ G is a Tree, $e=0$

$e=n-1=1-1=0$

Result is true for $n=1$

Assume that it is true for $n=k$, $e_k=k-1$

To Prove for $n=k+1$



k+1 vertices

let there is a vertex v which is leaf of T

Remove v, New graph T' will be again a Tree with k vertices

Edges in T = Edges in T' + one edge removed

$(k-1)+1=k$

Result is true for $n=k+1$ also

Any Tree containing n vertices will have $(n-1)$ edges.

Definition:

- $m-ary$ **tree**

A rooted tree is called an *m-ary tree* if every internal vertex has no more than m children. The tree is called a *full m-ary tree* if every internal vertex has exactly m children. An m-ary tree with $m=2$ is called a *binary tree*.

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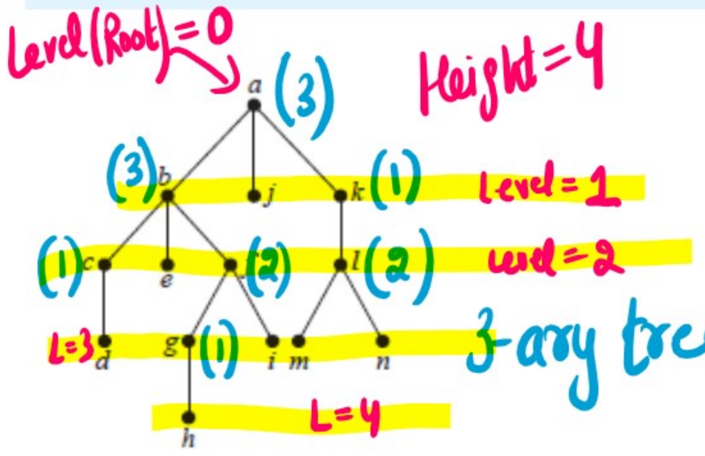
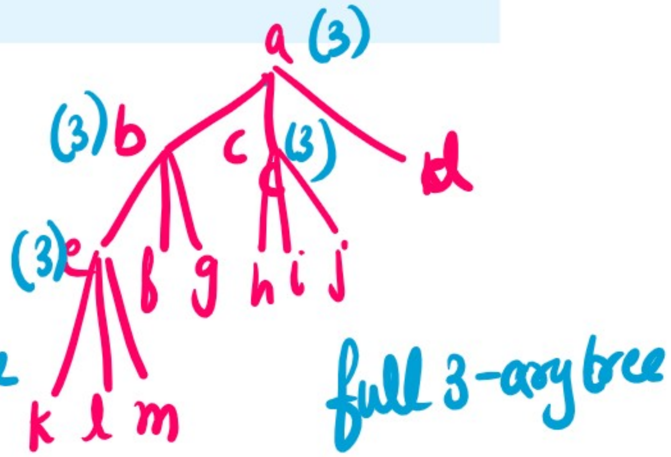
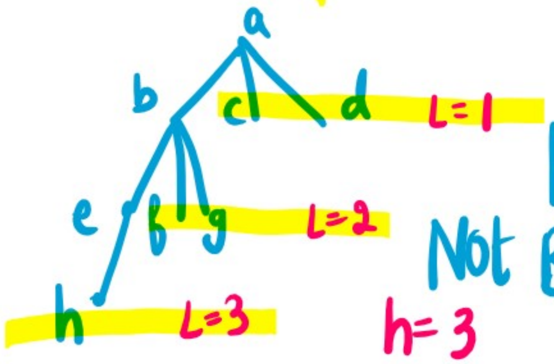




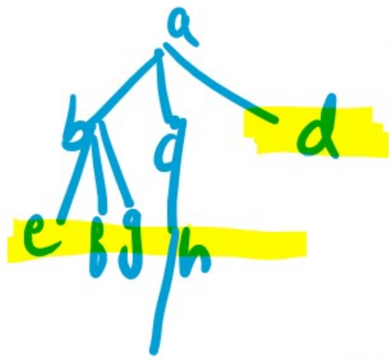
- **Level** of a vertex v in a rooted tree is te length of the unique path from the root to this vertex.

- **Height** of a rooted tree is the maximum of the levels of vertices or we can say it is length of longest path from the root to any vertex.

- A rooted $m - ary$ tree of height h is **Balanced** if all leaves are at levels h or $h - 1$.

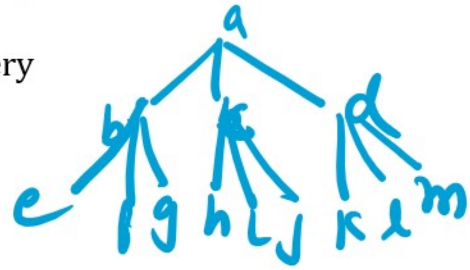


h vertex
Not Balanced
h= 3



h=2

- **Complete $m - ary$** tree is a full $m - ary$ tree in which every leaf is at same level.



Theorem 3: A full $m - ary$ tree with i internal vertices contains $n = mi + 1$ vertices.

full m-ary tree, each internal vertex has m children
i Internal —— mi children
Except root, every vertex is child of some.

$n = mi+1$ $\quad l = n - i = (mi+1) - i$

$n = mi + 1$; $l = n - i = (mi+1) - i$
$= (m-1)i + 1$

Theorem 4:

A full m-ary tree with

(*i*) n vertices has $i = (n-1)/m$ internal vertices and $l = [(m-1)n+1]/m$ leaves,
(*ii*) i internal vertices has $n = mi + 1$ vertices and $l = (m-1)i + 1$ leaves,
(*iii*) l leaves has $n = (ml-1)/(m-1)$ vertices and $i = (l-1)/(m-1)$ internal vertices.

$\underline{n, i, l}$ $n = mi + 1$

(i) $n =$ known , $i = \frac{n-1}{m}$ $l = n - i$
$l = n - \left(\frac{n-1}{m}\right)$

Q12.

How many edges does a tree with 10,000 vertices have?

$e = 10{,}000 - 1 = 9999$

Q13.

How many vertices does a full 5-ary tree with 100 internal vertices have?

$m = 5$, $i = 100$, $n = ?$

$n = mi + 1 = 501$

$l = 501 - 100 = 401$

$\log_a a$

$2^9, 2^{10}$

Q14.

How many edges does a full binary tree with 1000 internal vertices have?

$m = 2$, $i = 1000$, $e = ??$

$h = ??$
$h = \lceil \log_2 1001 \rceil$

$m=2, \quad i=1000, \quad e=\,!!$

$n=2001$

$e=2000, \quad l=1001$

$h=\left\lceil \frac{\log 1001}{\log 2} \right\rceil$

$h=\lceil 9.10 \rceil$

$h=10$

Q15.

A chain letter starts when a person sends a letter to five others. Each person who receives the letter either sends it to five other people who have never received it or does not send it to anyone. Suppose that 10,000 people send out the letter before the chain ends and that no one receives more than one letter. How many people receive the letter, and how many do not send it out?

$m=5 \qquad i=10{,}000,$ Receive the letter $= n-1 = 50{,}000$

Don't send out $= l$ $\quad , n = 50{,}001$

$\downarrow$

$l = 40{,}001$

Theorem 5:

There are at most m^h leaves in an m-ary tree of height h.

$h=1$

m-ary

$l = m \leq m^1$ True

$h=k \qquad l \leq m^k$

$h=k+1, \quad l_{k+1} \leq m^k \cdot m \qquad l \leq m^{k+1}$

Corollary:

Corollary:

If an ~~m-ary~~ tree of height h has l leaves, ~~then~~ $h \geq \lceil \log_m l \rceil$. If the m-ary tree is full and balanced, then $h = \lceil \log_m l \rceil$. (We are using the ceiling function here. Recall that $\lceil x \rceil$ is the smallest integer greater than or equal to x.)

$l \leq m^h$, $h \geq \lceil \log_m l \rceil \rightarrow$ ceiling f^n

GINT $\lfloor 2.5 \rfloor = 2$ $\lceil 2.5 \rceil = 3$

-2.5 $\lfloor -2.5 \rfloor = -3$ $\lceil -2.5 \rceil = -2$

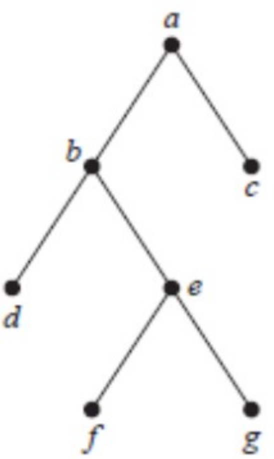
Floor f^n ↑ Top

Tree Traversal

- **preorder** Root Left Right
- **inorder** Left Root Right
- **Postorder** Left Right Root

Q16. Determine the order of vertices in which preorder, inorder, postorder traversal visits the vertices.

(i)

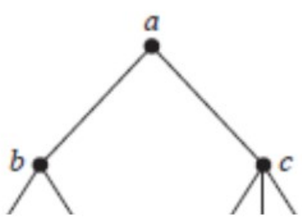


Pre-Order → abdefgc

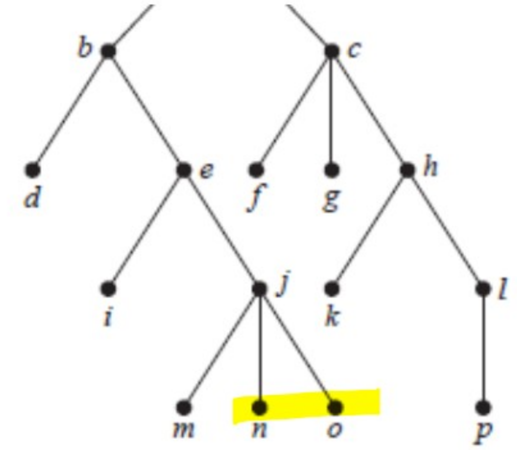
In-Order → dbfegac

Post-order → dfgebca

(ii)



Pre-order – abdeijmnocfghklp



In order- dbiemjnoafcgkhpl

Post order- dimnojebfgkplhca

Infix, Prefix, Postfix Notations

Q17.

\+ * + /

a) Represent the expressions $(x + xy) + (x/y)$ and $x + ((xy + x)/y)$ using binary trees.

Write these expressions in

b) prefix notation.

c) postfix notation.

d) infix notation.