24. $(2y^3xe^y + y^2 + y)\,dx + (y^3x^2e^y - xy - 2x)\,dy = 0.$

25. $y(1 + 3x^3 + 12x^2)\,dx + (x + 4)\,dy = 0.$

26. $y(1 + xy^2)\,dx + 2(x^2y^2 + x + y^4)\,dy = 0.$

27. $(12y + 3y^4 + 4x^3)\,dx + 6x(1 + y^3)\,dy = 0.$

28. $(x^2 + y^2)\,dx - (2xy)\,dy = 0.$

29. $(2x + y)\,dy - (x + 2y)\,dx = 0.$

30. $y^2\,dx + x(x - y)\,dy = 0.$

Solve the following initial value problems.

31. $3x^2y^4dx + 4x^3y^3dy = 0, \quad y(1) = 2.$

32. $(1 + y)\,dy - (1 - x)\,dx = 0, \quad y(1) = 0.$

33. $3y\,dx + 2x\,dy = 0, \quad y(1) = 1.$

34. $2xy\,dx + (x^2 + \pi\cos \pi y)\,dy = 0, \quad y(1) = 1.$

35. $(\cos x + y \sin x)\,dx = (\cos x)\,dy, \quad y(\pi) = 0.$

36. $xe^{x^2+y^2}\,dx + y(1 + e^{x^2+y^2})\,dy = 0, \quad y(0) = 0.$

37. $xy\,dx - (x^2 + y^2)\,dy = 0, \quad y(0) = 1.$

38. $\left(4x^3y^3 + \frac{1}{x}\right)dx + \left(3x^4y^2 - \frac{1}{y}\right)dy = 0, \quad y(1) = 1.$

39. $(x - y\cos x)\,dx - \sin x\,dy = 0, \quad y(\pi/2) = 1.$

40. $(ye^{xy} + 4y^3)\,dx + (xe^{xy} + 12xy^2 - 2y)dy = 0, \quad y(0) = 2.$

41. $(2xy + e^y)\,dx + (x^2 + xe^y)\,dy = 0, \quad y(1) = 1.$

42. $(x^2 + y^2 + x)\,dx + y\,dy = 0, \quad y(1) = 1.$

43. $xy\,dx + (x^2 + 2y^2 + 2)\,dy = 0, \quad y(0) = 1.$

44. Prove that if M and N in $M(x, y)\,dx + N(x, y)\,dy = 0$ satisfy the equation

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} + \frac{k}{x}N$$

then, $F = x^k$ is an integrating factor. Hence, solve $4y\,dx + x\,dy = 0$.

45. Show that $F(x, y)$ is an integrating factor of $M(x, y)\,dx + N(x, y)\,dy = 0$, if and only if

$$\left(M\frac{\partial F}{\partial y} - N\frac{\partial F}{\partial x}\right) + \left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)F = 0.$$