Unit-1 Electromagnetic theory

PHY109 – ENGINEERING PHYSICS

Brief introduction to the course

- **L: 3 T:1 P:0 Credits:4**
- **Unit 1: Electromagnetic theory [7 lectures]**
- **Unit 2: Lasers and applications [6 lectures]**
- **Unit 3: Fiber optics [5 lectures]**
- **Unit 4: Quantum mechanics [7 lectures]**
- **Unit 5: Waves [5 lectures]**
- **Unit 6: Solid state physics [6 lectures]**

Prepared by: Dr. Dipal B. Patel

Unit-1 Electromagnetic theory

Contents:

- Scalar and vectors fields
- Concept of gradient, divergence and curl
- Gauss theorem and Stokes theorem (qualitative),
- Poisson and Laplace equations
- Continuity equation
- Maxwell electromagnetic equations
- Physical significance of Maxwell equations
- Ampere Circuital Law
- Maxwell displacement current and correction in Ampere Circuital Law
- Dielectric constant

Prepared by: Dr. Dipal B. Patel

Electromagnetism

- The phenomenon which deals with the *interaction between an Electric field and a magnetic Field*.
- Separation of charges in a system lead to an electric field and moving charges generate to current and hence a magnetic field
- When these fields are varying with time, they coupled with each other through Maxwell's equations
- Further, Maxwell's eqns help us to investigate propagation of EM waves in different media

Scalar/vector

- **Scalar:** Scalar is a quantity which can be expressed by a single number representing its **magnitude.**

  Example: mass, density and temperature.

- **Vector**: Vector is a quantity which is specified by **both magnitude and direction**.

  Example: Force, Velocity and Displacement.

- **Fields:** If at each point of a region, there is a value for some physical function, the region is called a field.

Prepared by: Dr. Dipal B. Patel

Scalar fields

Scalar Field

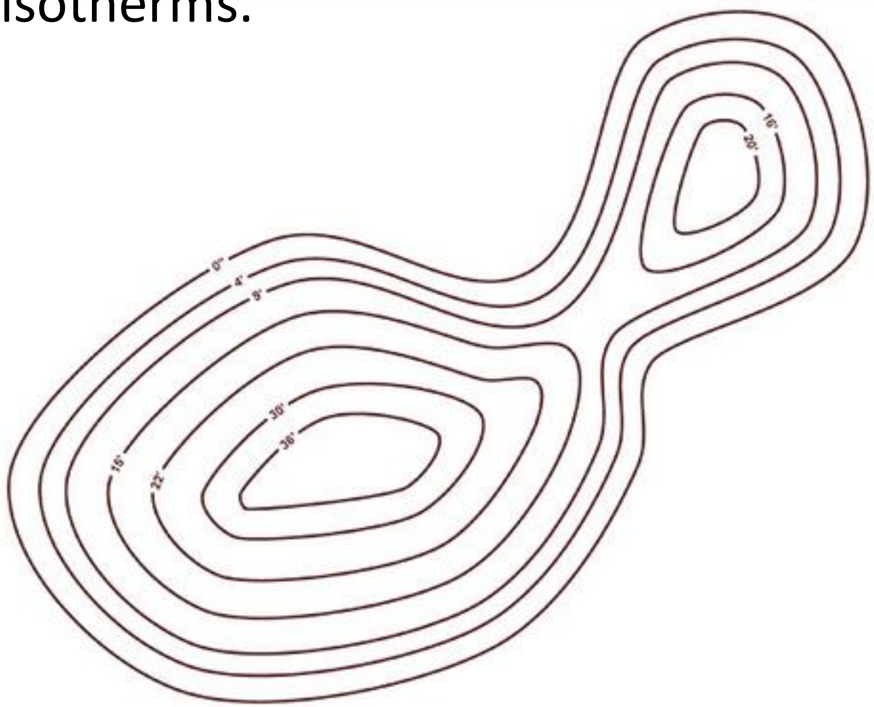
- If at every point in a region, a scalar function has a defined value, the region is called a **scalar field**.

Example: Temperature distribution in a rod, sound intensity in a theatre, height of the surface of the earth above sea-level, electric potential in a region, gravitational potential.

- A scalar field independent of time is called a *stationary or steady-state scalar fields*.

Scalar fields

- Graphically scalar fields can be represented by **contours** which are imaginary surfaces drawn through all points for which field has same value.
- For temperature field the contours are called isothermal surfaces or isotherms.



Prepared by: Dr. Dipal B. Patel___

Vector fields

Vector Field

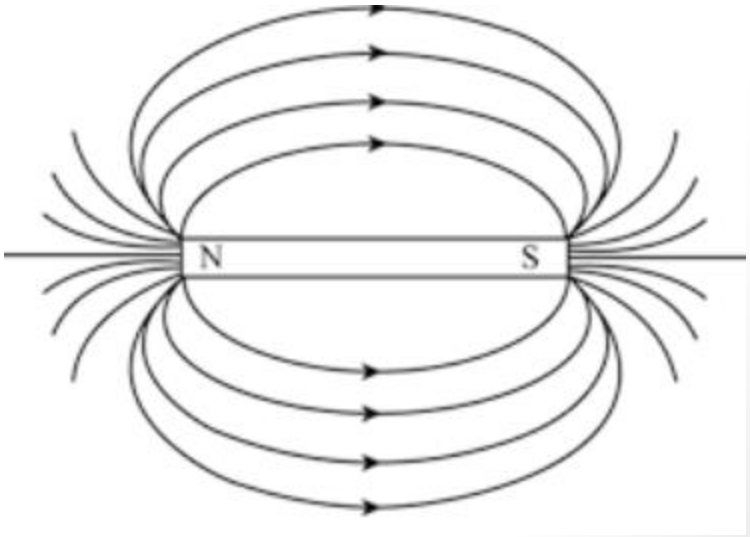
- If at every point in a region, a vector function has a defined value, the region is called a **vector field**.

Example: velocity field of a flowing fluid, intensity of electric, magnetic and gravitational force on a body in a space, wind velocity in an atmosphere, the force on a charged body placed in an electric field**.**

- A time independent vector is called *stationary vector field*.

Vector fields

- Graphically vector field are represented by lines known as field or *flux lines*. These lines are drawn in the field in such a way that tangent at any point of the line gives the direction of vector field at that point.
- To express the *magnitude of vector field* at any point first draw an infinitesimal area perpendicular to the field line. The number of field lines passing through this area element gives the magnitude of vector field.
- The lines presenting vector field cannot cross because if they cross they would give non unique field direction at the point of interaction.



Recall…

- Differentiation
- d/dx…(d/dy or d/dz)
- 1^{st} order…
- 2^{nd} order…
- Significance of differentiation..
  - 1^{st} order: slope
  - 2^{nd} order: maxima, minima
- Types of differentiation
  - Scalar
  - Vector

Vector differentiation

- In order to understand vector differentiation, we introduce an operator known as **del operator** also known as differential vector operator.
- Itself it is nit a vector, but when it operates on a scalar function it provides the resultant as a vector.

<u>Differential Vector Operator (∇), or Del Operator</u>

The differential vector operator (∇), or **del** or **nabla** is defined as,

$$\nabla = \frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z$$

∇ Operations…

$$\nabla = \frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z$$

Operation	Function	Result
Gradient	Scalar	Vector
Divergence	Vector	Scalar
Curl	Vector	Vector

Gradient?

- Derivative of a function of one variable simply tells us how fast the function varies if we move a small distance (d***l***)
- Gradient is the rate of change of a quantity with distance
- However, for a function of three variables the situation is more complicated, as it will depend on what direction we choose to move. For a function F(x, y, z) of three variables, we obtain from a theorem on partial derivative

$$dF = \frac{\partial F}{\partial x} dx + \frac{\partial F}{\partial y} dy + \frac{\partial F}{\partial z} dz$$

- Where ***dF*** is a measure of changes in ***F*** that occurs when we alter all three variables by small amounts *dx, dy* and *dz*

Gradient

- The expression for ***dF*** in terms of a dot product of vectors can be written as,

$$dF = \vec{\nabla} F \cdot d\vec{l}$$

Where, $\vec{\nabla} F = \hat{i}\frac{\partial F}{\partial x} + \hat{j}\frac{\partial F}{\partial y} + \hat{k}\frac{\partial F}{\partial z}$ is nothing but **gradient of F**.

- Clearly the gradient is a vector quantity

$$dF = \vec{\nabla} F \cdot d\vec{l}$$

$$dF = |\vec{\nabla} F||d\vec{l}|\cos\alpha$$

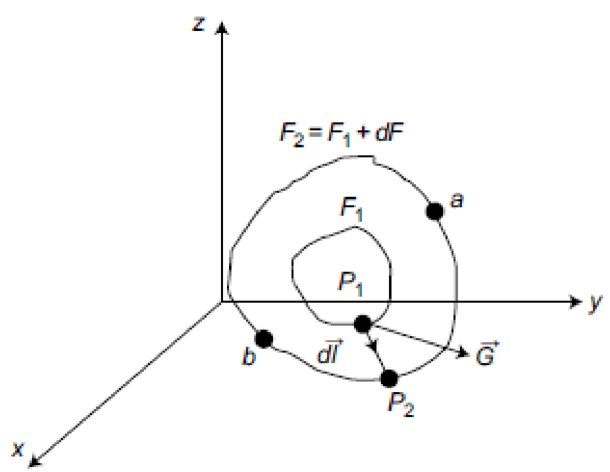
- Hence, upon fixing the small ***dl***, and to get maximum value of dF occurs in the direction of $\alpha = 0$
- **∇F** points in the direction of maximum increase of the function **F**

Prepared by: Dr. Dipal B. Patel

Gradient of a scalar

Definition: The gradient of a scalar function is both the magnitude and the direction of the maximum space rate of change of that function.

Mathematical Expression of Gradient: Considering a scalar function *F*, a mathematical expression for the gradient can be obtained by finding the difference in the field dF between the points P_1 and P_2.



Gradient of a scalar quantity

Cont…

<u>Properties of Gradient:</u>

I. The magnitude of the gradient of a scalar function is the maximum rate of change of the function per unit distance.

II. The direction of the gradient of a scalar function is in the direction in which the function changes the most.

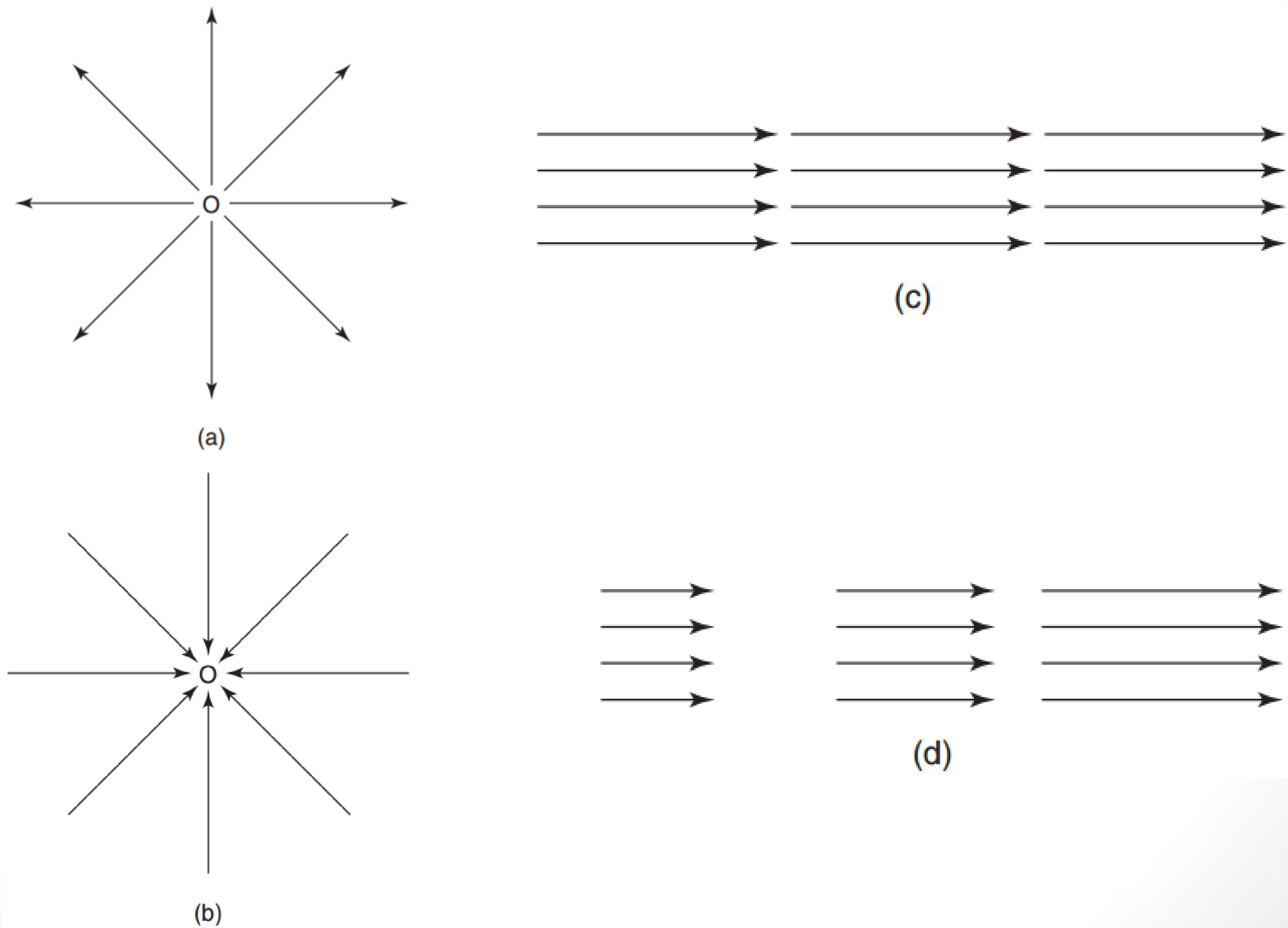
Divergence?

- Considering the net flux $\oint \vec{A} \cdot d\vec{S}$ of a vector field **A** from a closed surface S

- The divergence of A is defined as the ***net outward flux per unit volume over a closed surface***

- The divergence of A at a given point is a measure of how much the vector A spreads out, i.e., diverges, from that point

Divergence?



Prepared by: Dr. Dipal B. Patel___

Divergence of a vector

<u>Definition: </u>Mathematically, divergence of a vector at any point is defined as the limit of its surface integral per unit volume as the volume enclosed by the surface around the point shrinks to zero.

$$\operatorname{div} \vec{F} = \nabla . \vec{F} = \lim_{v \to 0} \left(\frac{\oint_S \vec{F} . d\vec{S}}{v} \right) = \lim_{v \to 0} \left(\frac{\oint_S \vec{F} . \hat{a}_n dS}{v} \right)$$

Where v is the volume, S is the surface of that volume, and the integral is a surface integral with $\mathbf{a_n}$ being the outward normal to the surface.

Hence, the total net outward flow per unit volume is given as,

$$\frac{\oint_S \vec{F}.d\vec{S}}{\Delta v} = \left(\frac{\partial V_1}{\partial x} + \frac{\partial V_2}{\partial y} + \frac{\partial V_3}{\partial z}\right)$$

By definition, this is the divergence of the vector.

$$\boxed{\text{div } \vec{F} = \nabla.\vec{F} = \left(\frac{\partial V_1}{\partial x} + \frac{\partial V_2}{\partial y} + \frac{\partial V_3}{\partial z}\right)}$$

Physical Interpretations: The divergence of a vector field is the outward normal flux of vector field from a closed surface.

- **If divergence of any vector function is zero,** then the flux of vector function entering into a region must be equal to that leaving it. Such vector function **is called solenoidal !**

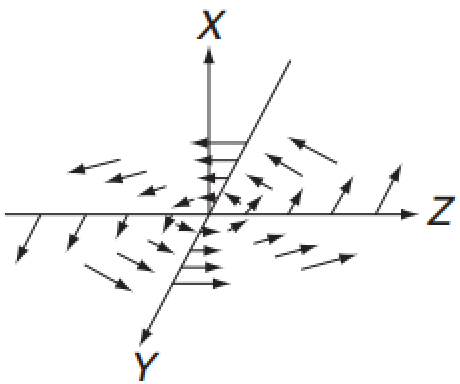
Cont…

<u>Properties of Divergence:</u>

I. The result of the divergence of a vector field is a scalar.
II. Divergence of a scalar field has no meaning.
III. Divergence may be positive, negative or zero.
IV. A vector field with constant *zero divergence* is called *solenoidal*; in this case, no net flow can occur across any closed surface.

Curl?

- Circulation of a vector field **A** around a closed path $\oint \vec{A} \cdot d\vec{l}$
- The curl of **A** is a rotational vector
- Its magnitude would be the maximum circulation of **A** per unit area
- Its direction is the normal direction of the area when the area is oriented so as to make the circulation maximum
- Actually curl of **A** at some point **O** is a measure of how much the vector <u>*A curls around the point O*</u>



Prepared by: Dr. Dipal B. Patel
Disclaimer: This material is not sufficient for the sake of taking exam, supplement it with the text book reading.

Curl of a vector

Definition: The curl of a vector field, denoted as curl **F** or ∇×**F**, is defined as the vector field having magnitude equal to the maximum circulation at each point and oriented perpendicularly to this plane of circulation for each point.

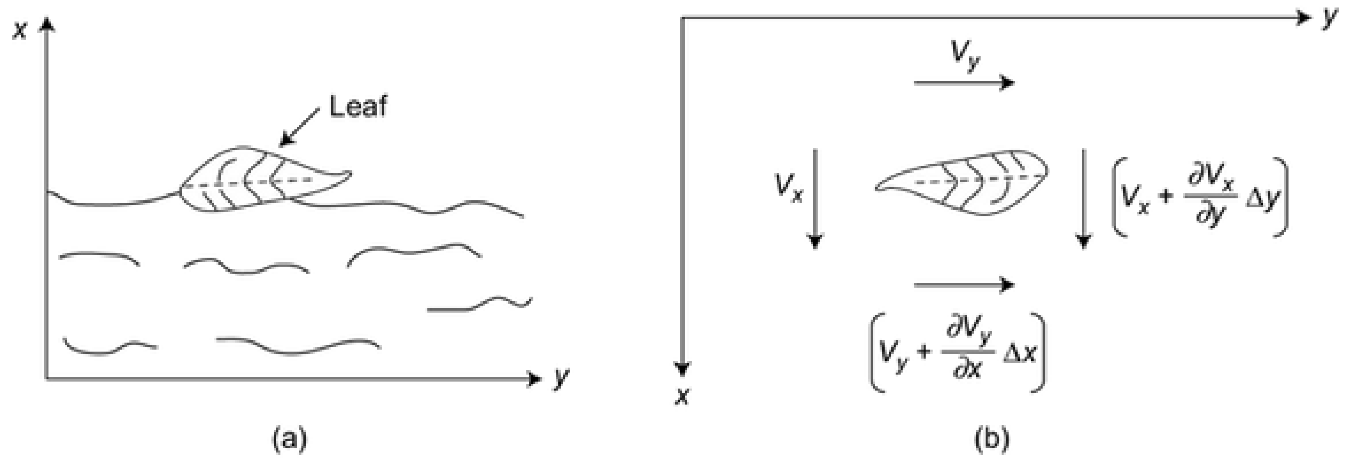
Mathematically,

$$\text{curl } \vec{F} = \lim_{v \to 0} \left(\frac{\oint_S \vec{F} \times \hat{a}_n dS}{v} \right) = \lim_{v \to 0} \left(\frac{\oint_S \vec{F} \times d\vec{S}}{v} \right)$$

Curl of a vector

<u>Physical Interpretation:</u> It provides a measure of the amount of rotation or angular momentum of the vector around the point.

We consider a stream on the surface of which floats a leaf, in the xy-plane.



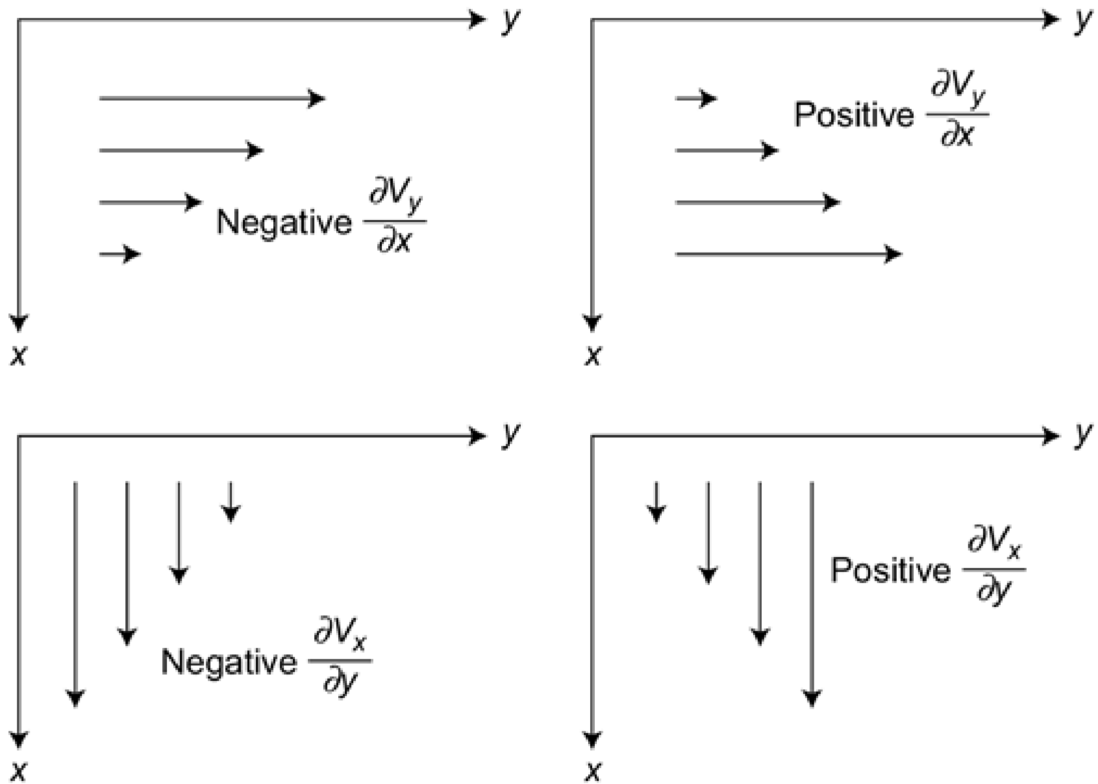
Rotation of a floating leaf and interpretation of curl

Prepared by: Dr. Dipal B. Patel___

If the velocity at the surface if only in the y-direction and is uniform over the surface, there will be no circulation of the leaf.

But, if there are eddies in the stream, there will be rotational movement of the leaf.

The rate of rotation or angular velocity at any point is a measure of the curl of the velocity of the stream at that point.



Interpretation of positive and negative velocity gradients

Cont…

<u>Properties of Curl</u>

I. The result of the curl of a vector field is another vector field.
II. Curl of a scalar field has no meaning.
III. If the value of the curl of a vector field is zero then the vector field is said to be an **irrotational** or **conservative** field.

Numerical (Gradient, divergence, curl)

(1) If $\phi = x^{3/2} + y^{3/2} + z^{3/2}$, find grad of the given function.

Numerical (Gradient, divergence, curl)

(2) If $\phi = 3x^2y - yz^2$, find grad of the function at (1, 2, -1).

Prepared by: Dr. Dipal B. Patel

Numerical (Gradient, divergence, curl)

(3) Find divergence of $V(x,y,z) = x^2y\ i + 3y\ j$

Prepared by: Dr. Dipal B. Patel

Prepared by: Dr. Dipal B. Patel

Disclaimer: This material is not sufficient for the sake of taking exam, supplement it with the text book reading.

Qualitatively (without derivation)

GAUSS’ THEOREM

Background

- Formulated by *Carl Friedrich Gauss* in 1813
- **It is** also known as **Gauss's flux theorem**, which relates the distribution of electric charge to the resulting electric field.
- It can be applied on closed surface enclosing a volume such as a spherical surface.
- It is one of Maxwell's four equations (the basis of classical electrodynamics).
- Gauss's law can be used to derive Coulomb's law, and vice versa.

Gauss' law

- Statement:

'The net electric flux through any hypothetical closed surface is equal to 1/ε times the net electric charge within that closed surface'.

- The law can be expressed mathematically using vector calculus in integral form and differential form; both are equivalent since they are related by the divergence theorem, also called Gauss's theorem

Can be expressed in two ways:

(1) In terms of a relation between the electric field **E** and the total electric charge **q**, or

(2) In terms of the electric displacement field **D** and the *free* electric charge

Integral form of Gauss' law

- Gauss's law can be expressed as:

$$\Phi_E = Q/\varepsilon_0$$

where Φ_E is the electric flux through a closed surface *S* enclosing any volume *V*, Q is the total charge enclosed within *V*, and ε_0 is the electric constant.

- The electric flux Φ_E is defined as a surface integral of the electric field:

$$\Phi_E = \oint\!\!\!\oint_S \mathbf{E} \cdot d\mathbf{A}$$

where **E** is the electric field, d**A** is a vector representing an infinitesimal element of area of the surface, and · represents the dot product of two vectors.

- Since the flux is defined as an *integral* of the electric field, this expression of Gauss's law is called the *integral form*

Differential form of Gauss' law

- By the divergence theorem, Gauss's law can alternatively be written in the differential form:

$$\nabla \cdot E = \rho/\varepsilon_0$$

Where,

$\nabla \cdot E$ is the divergence of the electric field

ε_0 is the electric constant, and

ρ is the total electric charge density (charge per unit volume).

Gauss divergence theorem

- Statement: The flux of a vector field F, over any closed surface S, is equal to the volume integral of the divergence of that vector field over the volume V enclosed by the surface S.

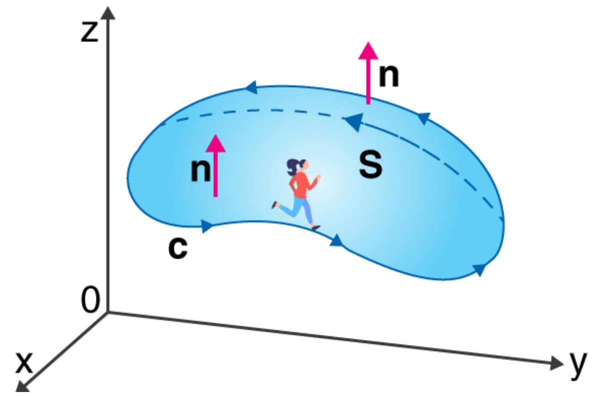
$$\int_S \boldsymbol{F}.ds = \int_V div\,\boldsymbol{F}\,dV$$

(Relation between surface integration and volume integration)

Stokes' theorem

- Statement: The surface integral of the curl of a vector field **A**, taken over any surface S, is equal to the line integral of **A** around the closed curve forming the periphery of the surface.

$$\iint_S (\nabla \times \boldsymbol{A}).\, dS = \oint \boldsymbol{A}.\, dl$$



✓ Right hand rule…

✓ Doesn't depend on shape of the surface

✓ Depends only on the line integral

Poisson's and Laplace's equations: Why??

- Need of Poisson's and Laplace's equations?
  - To find electric field (E) and potential (V)
- Limitations of Gauss's law?
  - Uniform distribution of charges
- Means, for nonuniform distribution of charges, one cannot use Gauss's law to find electric field (E) in any real world problem.

Poisson's & Laplace's equation

- Gauss's law:

$$\boxed{\nabla \cdot E = \rho/\varepsilon_0}$$

Where,

$\nabla \cdot E$ is the divergence of the electric field

ε_0 is the electric constant, and

ρ is the total electric charge density (charge per unit volume).

$$E = -\nabla V$$

Where,

V = electric potential (Scalar)

- $\nabla^2 V = -\rho/\varepsilon_0$: Poisson's eqn. for a homogeneous region.
- $\nabla^2 V = 0$: Laplace's eqn. for a charge free region.

Laplace's eqn…

- Applicable to those electrostatic problems, where,

1. The entire charge resides on the surface of the conductor

2. Entire charges concentrated in the form of point charges, line charges, or surface charges at a <u>single position</u>.

3. Region between two conductors is filled with one or more homogeneous dielectrics.

Conservation of energy

- Weak version: Energy can neither be created nor destroyed—i.e., the total amount of energy in the universe is fixed.
- This statement does not rule out the possibility that a quantity of energy could disappear from one point while simultaneously appearing at another point.

- Strong version: Energy is *locally* conserved: energy can neither be created nor destroyed, *nor* can it "teleport" from one place to another—it can only move by a ***continuous flow***.

- A *continuity equation* is the mathematical way to express this kind of statement.

Continuity equation

- Describes the transport of some quantity
- Applicable to conserved quantities
- Since mass, energy, momentum, electric charge and other natural quantities are conserved under their respective appropriate conditions, a variety of physical phenomena may be described using continuity equations
- <u>Statement</u>: The continuity equation for electric charge states that the amount of electric charge in any volume of space can only change by the amount of electric current flowing into or out of that volume through its boundaries

Derivation of continuity equation

$$I = \oint_s \vec{J} \cdot d\vec{S}$$

$$I = \frac{dq}{dt} = \frac{d}{dt} \int_v \rho dV$$

$$\oint_s \vec{J} \cdot d\vec{S} = -\frac{dq}{dt} = \frac{-d}{dt} \int_v \rho dV$$

$$\oint_s \vec{J} \cdot d\vec{S} = -\int_v \frac{\partial \rho}{\partial t} dV$$

$$\oint_s \vec{J} \cdot d\vec{S} = \int_v (\operatorname{div} \vec{J}) dV$$

$$\int_v (\operatorname{div} \vec{J})\, dV = -\int_v \frac{\partial \rho}{\partial t} dV$$

$$\operatorname{div} \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

Continuity equation: significance

- In case of stationary currents, i.e., when the charge density at any point within the region remains constant, but the charges are moving

$$\frac{\partial \rho}{\partial t} = 0,$$

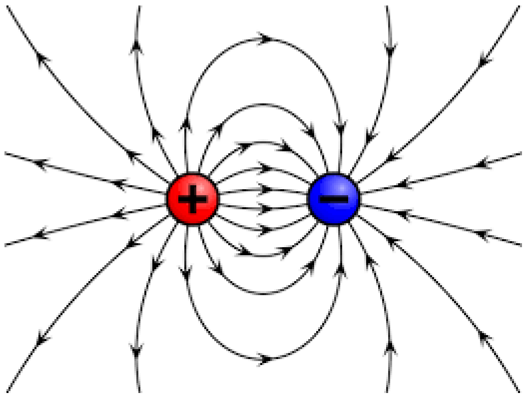
$$\vec{\nabla} \cdot \vec{J} = 0$$

which expresses the fact that there is no net outward flux of current density J

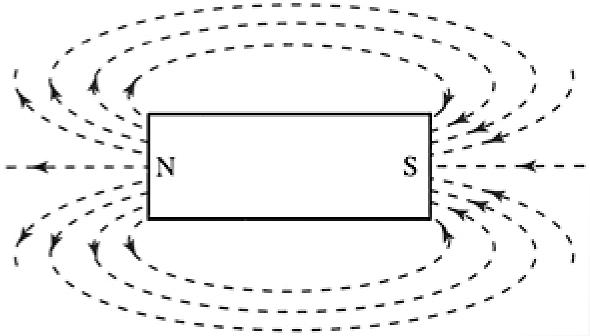
✓ zero divergence….

Ampere's circuital law: Significance

- Electrostatic → Gauss' law
  - To find electric field (E) and potential (V)



- Magnetostatic → Ampere's circuital law
  - To find magnetic field (B)

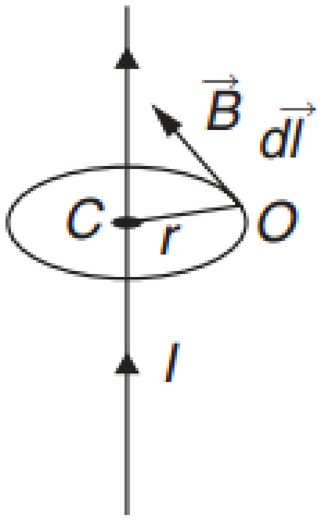


Ampere's circuital law

- Statement: The line integral of magnetic field B around any closed path in vacuum/air is equal to μ_0 times the total current I enclosed by that path.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$B = \frac{\mu_0}{4\pi} \frac{2I}{r}$$



✓ Permeability

$$\oint \vec{B} \cdot d\vec{l} = \oint B dl \cos\theta$$

$$= B \oint dl \qquad [\because \theta = 0°]$$

$$= \frac{\mu_0}{4\pi} \frac{2I}{r} 2\pi r = \mu_0 I$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

Displacement current

- According to Maxwell, it is not only the current in a conductor that produces a magnetic field. A changing electric field in vacuum or in a dielectric also produces a magnetic field.
- This implies that a changing electric field is equivalent to current, which flows till the electric field is changing.
- This equivalent current produces the same magnetic effects as a conventional current in a conductor.
- This equivalent current is known as *displacement current*.

Modified Ampere's law

- On the basis of the fact that, the magnetic field around a conductor is produced by the current flowing in it.
- Maxwell hypothesized that changing electric field should also induce a magnetic field.
- A changing electric field is equivalent to a current called displacement current (I_d) which flows as long as electric field is changing.
- The displacement current produces the magnetic field the same way as the conductor current (I).
- Thus the total magnetic field (***B***) will be the sum of two terms (1) due to conductor current I($\boldsymbol{B_1}$) and

  (2) due to displacement current I_d($\boldsymbol{B_2}$)

Modified Ampere's law

$$\vec{B} = \vec{B}_1 + \vec{B}_2$$

$$\text{curl } \vec{B} = \text{curl } \vec{B}_1 + \text{curl } \vec{B}_2$$

$$\text{curl } \vec{B} = \mu_0 \vec{J} + \mu_0 \vec{J}_d$$

$$\text{curl } \vec{B} = \mu_0 (\vec{J} + \vec{J}_d)$$

$$\oint_c \vec{B} \cdot d\vec{l} = \mu_0 (I + I_d)$$

$$\text{curl } \vec{B} = \mu_0 \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right)$$

$$\text{curl } \vec{H} = \vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Significance of Maxwell's eqn

- Maxwell's 1st eqn: (Gauss's law of electrostatics)
  - Which says that the electric flux (**E**) out of any closed surface is *proportional* to the total charge (q) enclosed within the surface
  - The integral form of this eqn can be used to find electric field around charged objects
  - By finding out the area integral of the electric field one can measure net charge enclosed

$$\vec{\nabla} \cdot \vec{D} = \rho$$

$$\int_s \vec{D} \cdot \vec{dS} = \int_V \rho dV \quad \text{or} \quad \oint_s \vec{E} \cdot \vec{dS} = q$$

Significance of Maxwell's eqn

- Maxwell's 2nd eqn: (Gauss's law of magnetostatics)
  - Which says that the net magnetic flux out of any closed surface is zero.
  - Magnetic monopoles does not exists.

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\oint_s \vec{B} \cdot \vec{dS} = 0$$

Significance of Maxwell's eqn

- Maxwell's 3rd eqn: (Faraday's law)
  - Faraday's law of electromagnetic induction
  - Changing magnetic field induces electric field
  - Which is equivalent to generated voltage or emf in the loop
  - Integral form states that the line integral of the electric field (**E**) around a closed loop is equal to negative of the rate of change of the magnetic flux (**B**) through the area enclosed by the loop.

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\oint \vec{E} \cdot \vec{dl} = -\frac{\partial}{\partial t} \int \vec{B} \cdot \vec{dS}$$

Significance of Maxwell's eqn

- Maxwell's 4th eqn: (Ampere's law)
  - For static electric field **E**, the 2nd term of RHS of integral form of the eqn vanishes and then this eqn says that the line integral of the magnetic field (**H**) around a closed loop is *proportional* to the electric current flowing through the loop
  - Useful for calculating the magnetic field for simple geometries
  - Says that the changing electric field induces magnetic field (complementing Maxwell's 3rd eqn)

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\oint \vec{H} \cdot d\vec{l} = \int_s \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \cdot \vec{dS}$$

Differential form ←→ Integral form

(i) $\vec{\nabla} \cdot \vec{D} = \rho$

(i) $\int_s \vec{D} \cdot \vec{dS} = \int_V \rho dV$ or $\oint_s \vec{E} \cdot \vec{dS} = q$

(ii) $\vec{\nabla} \cdot \vec{B} = 0$

(ii) $\oint_s \vec{B} \cdot \vec{dS} = 0$

(iii) $\vec{\nabla} \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$

(iii) $\oint \vec{E} \cdot \vec{dl} = -\dfrac{\partial}{\partial t} \int \vec{B} \cdot \vec{dS}$

(iv) $\vec{\nabla} \times \vec{H} = \vec{J} + \dfrac{\partial \vec{D}}{\partial t}$

(iv) $\oint \vec{H} \cdot d\vec{l} = \int_s \left(\vec{J} + \dfrac{\partial \vec{D}}{\partial t} \right) \cdot \vec{dS}$

$$\int_S \boldsymbol{F}.ds = \int_V div\, \boldsymbol{F}\, dV$$

$$\iint_S (\nabla \times \boldsymbol{A}).dS = \oint \boldsymbol{A}.dl$$

Disclaimer: This material is not sufficient for the sake of taking exam, supplement it with the text book reading

END OF UNIT-1