Lecture 20

04 October 2021 16:58

For each of these relations on the set {1, 2, 3, 4}, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.

a) {(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)}
b) {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)}
c) {(2, 4), (4, 2)}
d) {(1, 2), (2, 3), (3, 4)}
e) {(1, 1), (2, 2), (3, 3), (4, 4)}
f) {(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)}

	Reflexive	Irreflexive	Sym.	Asy.	Antisy	Transitive
(b)	✓	✗ $(1,1) \in R$	✓	✗	$(1,2),(2,1) \in R$, $1 \neq 2$	✓
(c)	✗	✓	✓	✗	✗	✗ $(2,4),(4,2) \in R$, $(2,2) \notin R$
(d)	✗	✓	✗	✓	✓	$(1,3) \notin R$
(e)	✓	✗	✓	✗	✓	✓
(f)	✗	✓	✗ $(4,1) \notin R$	✗ $(1,3),(3,1) \in R$	✗ $1 \neq 3$	✗ $(1,3),(3,1) \in R$, $(1,1) \notin R$

Representing Relation with Matrices

The relation R can be represented by the matrix $M_R = \{m_{ij}\}$, where

$$m_{ij} = \begin{cases} 1 \text{ if } (a_i, b_j) \in R, \\ 0 \text{ if } (a_i, b_j) \notin R. \end{cases}$$

M_R = matrix of 0, 1 entries

Q7.

Represent each of these relations on {1, 2, 3, 4} with a matrix (with the elements of this set listed in increasing order).

a) {(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)}
b) {(1, 1), (1, 4), (2, 2), (3, 3), (4, 1)}

(a) $\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$

(b) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

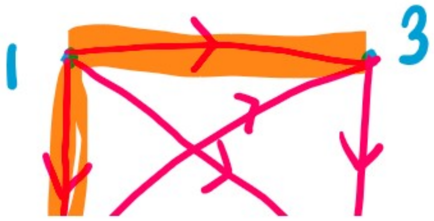
Representing relation as a directed graph

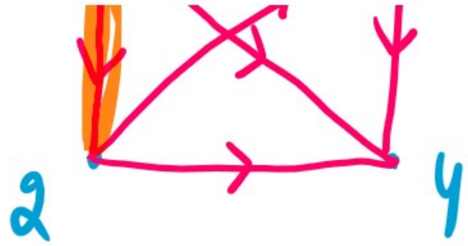
A *directed graph*, or *digraph*, consists of a set V of *vertices* (or *nodes*) together with a set E of ordered pairs of elements of V called *edges* (or *arcs*). The vertex a is called the *initial vertex* of the edge (a, b), and the vertex b is called the *terminal vertex* of this edge.

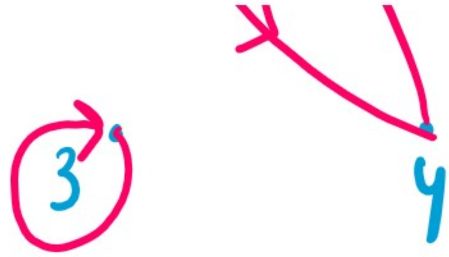
(a, b) $\quad a \longrightarrow b$

Represent each of these relations on {1, 2, 3, 4} with a matrix (with the elements of this set listed in increasing order).

a) {(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)}
b) {(1, 1), (1, 4), (2, 2), (3, 3), (4, 1)}







Q8.

Let R_1 and R_2 be relations on a set A represented by the matrices

$$\mathbf{M}_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{M}_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$

Find the matrices that represent

a) $R_1 \cup R_2$. **b)** $R_1 \cap R_2$. **c)** $R_2 \circ R_1$.
d) $R_1 \circ R_1$. **e)** $R_1 \oplus R_2$.

$$M_{R_1 \cup R_2} = M_{R_1} \vee M_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$M_{R_1 \cap R_2} = M_{R_1} \wedge M_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$M_{R_1 \oplus R_2} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

(c) $$M_{R_2 \circ R_1} = M_{R_1} \odot M_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 1 \end{bmatrix} = [$$

$$= \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \Big[\; 0+0+0 \quad 1+0+0 \quad 0+0+0$$

Counting of Relations

R on set A, A has n elements.

No. of relations $= 2^{n^2}$

No. of reflexive $= \begin{bmatrix} 1 & & \checkmark \\ & 1 \dots & \\ \checkmark & & 1 \end{bmatrix}$ off diagonal $\Rightarrow n^2-n$ $\quad 2^{n^2-n}$

No. of irreflexive $= \begin{bmatrix} 0 & & \\ & 0 \dots & \\ & & 0 \end{bmatrix} = 2^{n^2-n}$

No of symmetric $= \begin{bmatrix} & 1 & 0 \\ 1 & & \\ 0 & & \end{bmatrix} = 2^{\frac{n^2-n}{2}+n}$

No of asymmetric =

No of antisymmetric =