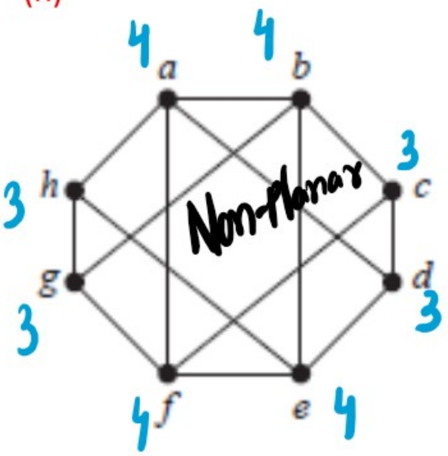
Lecture 32

10 November 2021 09:02

(h)



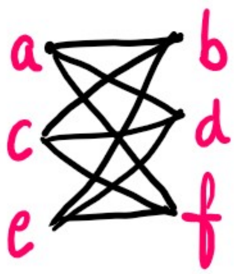
8 vertices — 4 are of degree 4

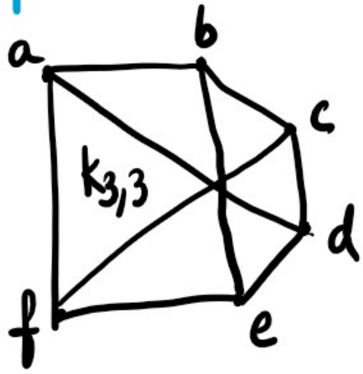
4 are of degree 3

$K_{3,3}$

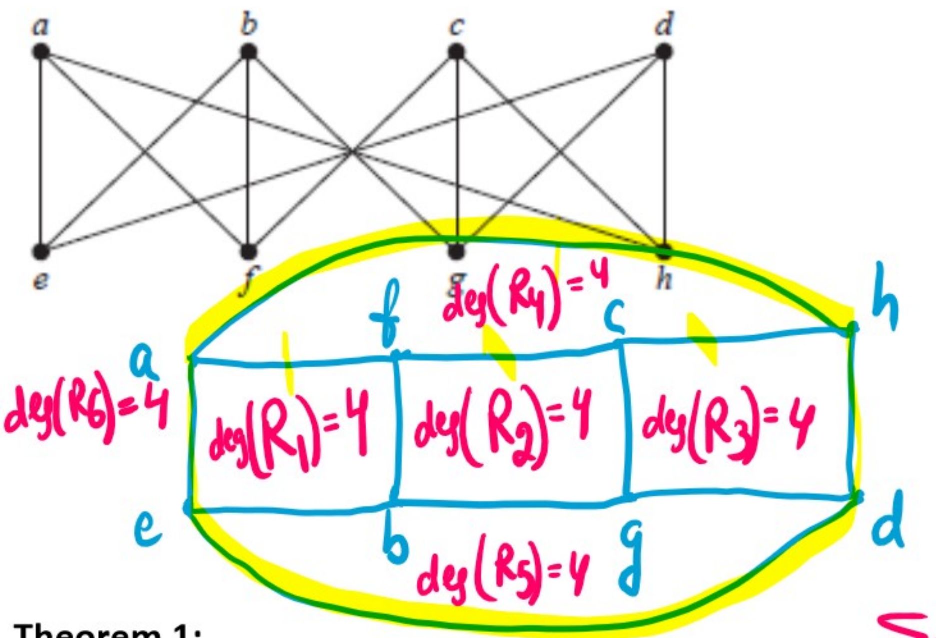
a	bdfh
b	aceg
c	bdf
d	ace

e	bdfh
f	aceg
g	bfh
h	aeg





(i)



Planar

$e = 12$

$v = 8$

$r = 6$

$12 - 8 + 2 = r$

$\sum \deg(\text{region}) = 2e$

Theorem 1:

EULER'S FORMULA Let G be a connected planar simple graph with e edges and v vertices. Let r be the number of regions in a planar representation of G. Then $r = e - v + 2$.

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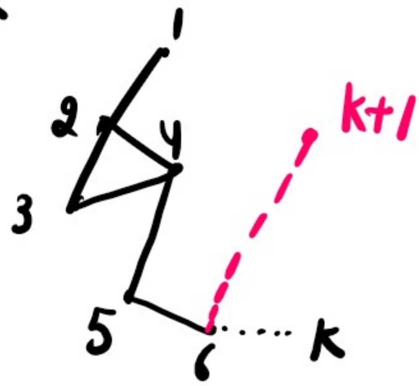
By Method of mathematical induction.

$v = 1$, $e = 0$, $r = 1$, $e - v + 2 = 0 - 1 + 2 = 1 = r$

Result is true for $v = 1$

Assume that it is true for $v = k$

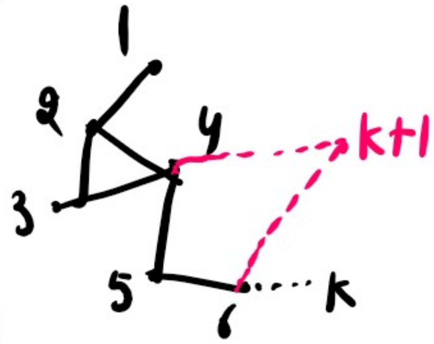
$r_k = e_k - k + 2$



let $v = k+1$, $e_{k+1} = e_k + 1$

$r_{k+1} = r_k$

$$(e_k + 1) - (k+1) + 2 = e_k - k + 2 = r_k = r_{k+1}$$



$e_{k+1} = e_k + 2$

$r_{k+1} = r_k + 1$

$$(e_k + 2) - (k+1) + 2 = e_k - k + 3$$

$$= (e_k - k + 2) + 1 = r_k + 1 = r_{k+1}$$

Result holds for $v = k+1$ also. So By mathematical induction, it will be true for any no. of vertices.

Corollary 1:

Corollary 1:

If G is a connected planar simple graph with e edges and v vertices, where $v \geq 3$, then $e \leq 3v - 6$.

minimum degree of region $= 3$, $\deg(r_1) \geqslant 3$

$$2e = \sum \deg(\text{region}) \geqslant 3r$$

$$2e \geqslant 3r, \quad r \leq \frac{2}{3}e$$

$$r = e - v + 2 \leq \frac{2}{3}e$$

$$3e - 3v + 6 \leq 2e$$

$$e \leq 3v - 6$$

K_5: $v = 5$, $e = \frac{5(5-1)}{2} = 10$

$10 \leq 3(5) - 6$, $10 \leq 9$ False

Non-Planar.

Corollary 2:

If G is a connected planar simple graph, then G has a vertex of degree not exceeding five.

G has all vertices of degree exceeding 5.

$$\deg(v) \geq 6$$

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$2e = \sum \deg(v) \geq 6v$

$\underbrace{e \geq 3v \text{ , Acc. in Cor 1 } e \leq 3v-6}_{\text{Not Possible}}$

Q2.

Suppose that a connected planar graph has eight vertices, each of degree three. Into how many regions is the plane divided by a planar representation of this graph?

$v = 8, \deg(v) = 3, r = ??, e =$

$2e = 8 \times 3 = 24, \quad e = 12$

$r = e - v + 2 = 12 - 8 + 2 = 6$

Corollary 3:

If a connected planar simple graph has e edges and v vertices with $v \geq 3$ and no circuits of length three, then $e \leq 2v - 4$.

$\deg(\text{region}) \geq 4$

$e \leq 3v-6$

$2e = \sum \deg(\text{region}) \geq 4r$

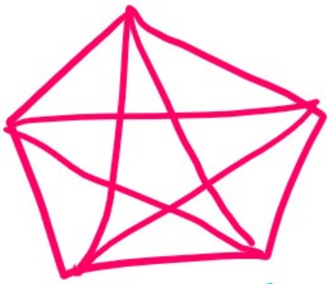
$e \leq 2v-4$

$r \leq \frac{e}{2}, \quad r = e - v + 2 \leq \frac{e}{2}$

$2e - 2v + 4 \leq e$

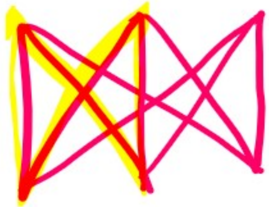
$e \leq 2v - 4$

K_5



$e \leq 3v-6$

$K_{3,3}$



$e \leq 2v-4$

$v=6,\ e=9,\ 9 \leq 2(6)-4$

$9 \leq 8$

Q3.

Suppose that a connected bipartite planar simple graph has e edges and v vertices. Show that $e \leq 2v-4$ if $v \geq 3$.

Q4.

Suppose that a connected planar simple graph with e edges and v vertices contains no simple circuits of length 4 or less. Show that $e \leq (5/3)v - (10/3)$ if $v \geq 4$.

$\deg(r) \geq 5$

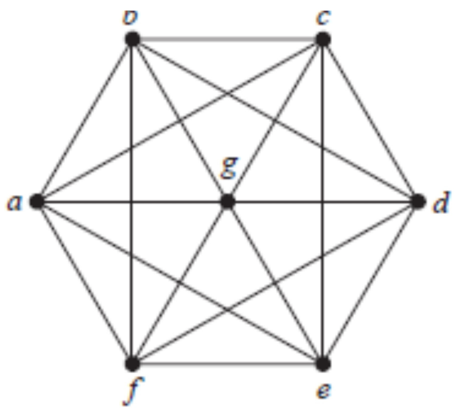
$2e = \sum \deg(r) \geq 5r$

$r \leq \frac{2}{5}e$

$e - v + 2 \leq \frac{2}{5}e$

Q5. Use Euler's formula to show the given graph is non-planar.

$e \leq 3v-6 \rightarrow$ circuit of length 3



$e \leq 3v - 6 \to$ len 3

$e \leq 2v - 4 \to$ circuit of min len 4

$e \leq \frac{5}{3}v - \frac{10}{3} \to$ circuit of min le 5.