Lecture 19

01 October 2021 22:53

Combining Relation

There are many relations defined from $\underset{m}{A}$ to $\underset{n}{B}$, so two relations from A to B can be combined in following ways.

$R: A \to B$ No. of relations from A to B $= 2^{mn}$

$R_1 \cup R_2$, $R_1 \cap R_2$, $R_1 - R_2 = \text{Only } R_1 = R_1 - (R_1 \cap R_2)$

$R_2 - R_1 = \text{Only } R_2 = R_2 - (R_1 \cap R_2)$

$R_1 \oplus R_2 = (R_1 - R_2) + (R_2 - R_1) = (R_1 \cup R_2) - (R_1 \cap R_2)$

$\overline{R_1} = \{(a,b) \notin R_1 ; a \in A, b \in B\}$

$R_1^{-1} = \{(b,a) ; (a,b) \in R_1\}$ $\quad R_1 : A \to B$

$R_1^{-1} : B \to A$

Q3.

Let $R_1 = \{(1, 2), (2, 3), (3, 4)\}$ and $R_2 = \{(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (3, 4)\}$ be relations from $\{1, 2, 3\}$ to $\{1, 2, 3, 4\}$. Find

$R_1 \cup R_2 = \{(1,2), (2,3), (3,4), (1,1), (2,1), (2,2), (3,1), (3,2), (3,3)\} = R_2$

$R_1 \cap R_2 = \{(1,2), (2,3), (3,4)\} = R_1$

$R_1 - R_2 = \Phi$

$R_2 - R_1 = \{(1,1), (2,1), (2,2), (3,1), (3,2), (3,3)\}$

$R_1 \oplus R_2 = \{(1,1), (2,1), (2,2), (3,1), (3,2), (3,3)\}$

$R_1 \oplus R_2 = \{(1,1), (2,1), \ldots$

$\overline{R_2} = \{(1,3), (1,4), (2,4)\}$ $\quad$ $R_1^{-1} = \{(2,1), (3,2), (4,3)\}$

Q4.

$R_1 = \{(a, b) \in \mathbf{R}^2 \mid a > b\}$, the "greater than" relation,

$R_2 = \{(a, b) \in \mathbf{R}^2 \mid a \geq b\}$, the "greater than or equal to" relation,

$R_3 = \{(a, b) \in \mathbf{R}^2 \mid a < b\}$, the "less than" relation,

$R_4 = \{(a, b) \in \mathbf{R}^2 \mid a \leq b\}$, the "less than or equal to" relation,

$R_5 = \{(a, b) \in \mathbf{R}^2 \mid a = b\}$, the "equal to" relation,

$R_6 = \{(a, b) \in \mathbf{R}^2 \mid a \neq b\}$, the "unequal to" relation.

Find

a) $R_1 \cup R_3$.
b) $R_1 \cup R_5$.
c) $R_2 \cap R_4$. $= R_5$
d) $R_3 \cap R_5$.
e) $R_1 - R_2$. $= \phi$
f) $R_2 - R_1$.
g) $R_1 \oplus R_3$.
h) $R_2 \oplus R_4$.

(a) $R_1 \cup R_3 = R_6$ $\quad \{(a,b) \in \mathbb{R}^2, a \neq b\}$

(b) $R_1 \cup R_5 = R_2$

(d) $R_3 \cap R_5 = \phi$

(f) $R_2 - R_1 = R_5$

(g) $R_1 \oplus R_3 = R_6$

(h) $R_2 \oplus R_4 = R_6$

Composition

Let R be a relation from A to B and S a relation from B to C. The composite of R and S is the relation consisting of ordered pairs (a, c) where $a \in A, c \in C$, and for which there exists an element $b \in B$ such that $(a, b) \in R$ and $(b, c) \in S$. We denote the composite of R and S by $S \circ R$.

$S \circ R : A \to C$

$R : A \to B$
$S : B \to C$

Q5. $\quad A = B = C = \{1,2,3,4\}$

Let R be the relation $\{(1, 2), (1, 3), (2, 3), (2, 4), (3, 1)\}$, and let S be the relation $\{(2, 1), (3, 1), (3, 2), (4, 2)\}$.

Let R be the relation $\{(1, 2), (1, 3), (2, 3), (2, 4), (3, 1)\}$, and let S be the relation $\{(2, 1), (3, 1), (3, 2), (4, 2)\}$. Find $S \circ R$.

$S \circ R = \{(1,1), (1,2), (2,1), (2,2)\}$

R	S	S∘R
(1,2)	(2,1)	(1,1)
(1,3)	(3,1) (3,2)	(1,1) (1,2)
(2,3)	(3,1), (3,2)	(2,1), (2,2)
(2,4)	(4,2)	(2,2)
(3,1)	No	X

R∘S

S	R	R∘S
(2,1)	(1,2), (1,3)	(2,2), (2,3)
(3,1)	(1,2), (1,3)	(3,2), (3,3)
(3,2)	(2,3), (2,4)	(3,3), (3,4)
(4,2)	(2,3), (2,4)	(4,3), (4,4)

Properties of Relation

R is relation from A to B

$R: A \to A \leftarrow$ R is relation on the set A

Reflexive: $(a,a) \in R$ for $a \in A$

Irreflexive $(a,a) \notin R$ for $a \in A$

Irreflexive $(a,a) \notin R$ for $a \in A$

Symmetric: $(a,b) \in R \Rightarrow (b,a) \in R \quad , a,b \in A$

Asymmetric $(a,b) \in R \Rightarrow (b,a) \notin R \quad , a,b \in A$

Antisymmetric $(a,b), (b,a) \in R \Rightarrow a=b$

Transitive:

$$(a,b) \in R, \quad (b,c) \in R$$
$$\Downarrow$$
$$(a,c) \in R$$

Q6.

For each of these relations on the set $\{1, 2, 3, 4\}$, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.

a) $\{(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$
b) $\{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$
c) $\{(2, 4), (4, 2)\}$
d) $\{(1, 2), (2, 3), (3, 4)\}$
e) $\{(1, 1), (2, 2), (3, 3), (4, 4)\}$
f) $\{(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)\}$

(a) Reflexive: X $(1,1) \notin R$

Irreflexive: X $(2,2) \in R$

Symmetric: $(2,4) \in R, \ (4,2) \notin R$ X

Asymmetric: $(2,2) \in R$ X

Asymmetric: $(2,2) \in R$ X

Antisymmetric: X $(2,3), (3,2) \in R$, $2 \neq 3$

Transitive: $\{(2,2), (2,3), (2,4), (3,2), (3,3), (3,4)\}$ Yes Transitive

R∘R

$(2,2)$	$(2,2)$ $(2,3)$ $(2,4)$	$(2,2), (2,3), (2,4) \in R$
$(2,3)$	$(3,2), (3,3), (3,4)$	$(2,2), (2,3), (2,4) \in R$
$(2,4)$	X	
$(3,2)$	$(2,2), (2,3), (2,4)$	$(3,2), (3,3), (3,4) \in R$
$(3,3)$	$(3,2), (3,3), (3,4)$	$(3,2), (3,3), (3,4) \in R$