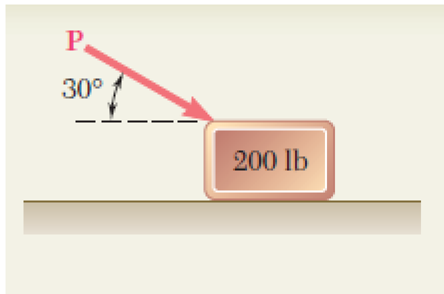
Tutorial sheet 6

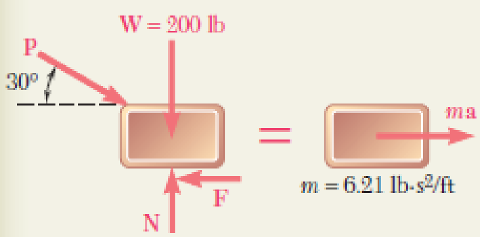
1. A 200 lb block rests on a horizontal plane. Find the magnitude of the force **P** required to give the block an acceleration of 10 ft/s^2 to the right. The coefficient of kinetic friction between the block and the plane is $\mu_K = 0.25$. **[P = 151 lb]**



SOLUTION

The mass of the block is

$$m = \frac{W}{g} = \frac{200 \text{ lb}}{32.2 \text{ ft/s}^2} = 6.21 \text{ lb} \cdot \text{s}^2/\text{ft}$$



We note that $F = \mu_k N = 0.25N$ and that $a = 10$ ft/s^2. Expressing that the forces acting on the block are equivalent to the vector $m\mathbf{a}$, we write

$\xrightarrow{+} \Sigma F_x = ma$: $\quad P \cos 30° - 0.25N = (6.21 \text{ lb} \cdot \text{s}^2/\text{ft})(10 \text{ ft/s}^2)$

$P \cos 30° - 0.25N = 62.1 \text{ lb}$ (1)

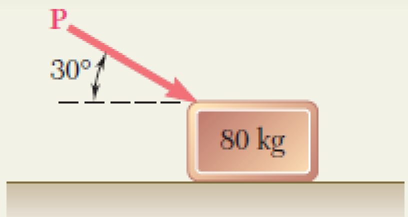
$+\uparrow \Sigma F_y = 0$: $\quad N - P \sin 30° - 200 \text{ lb} = 0$ (2)

Solving (2) for N and substituting the result into (1), we obtain

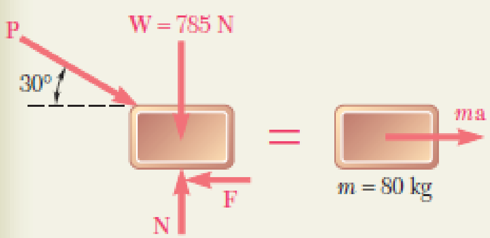
$$N = P \sin 30° + 200 \text{ lb}$$

$$P \cos 30° - 0.25(P \sin 30° + 200 \text{ lb}) = 62.1 \text{ lb} \qquad P = 151 \text{ lb}$$

2. An 80 kg block rests on a horizontal plane. Find the magnitude of the force **P** required to give the block an acceleration of 2.5 m/s^2 to the right. The coefficient of kinetic friction between the block and the plane is μ_K = 0.25.**[P = 535N]**



SOLUTION



The weight of the block is

$$W = mg = (80 \text{ kg})(9.81 \text{ m/s}^2) = 785 \text{ N}$$

We note that $F = \mu_k N = 0.25N$ and that $a = 2.5 \text{ m/s}^2$. Expressing that the forces acting on the block are equivalent to the vector $m\mathbf{a}$, we write

$\xrightarrow{+}\Sigma F_x = ma$: $\quad P \cos 30° - 0.25N = (80 \text{ kg})(2.5 \text{ m/s}^2)$

$$P \cos 30° - 0.25N = 200 \text{ N} \quad (1)$$

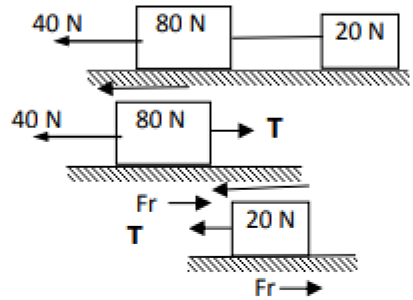
$+\uparrow\Sigma F_y = 0$: $\quad N - P \sin 30° - 785 \text{ N} = 0 \quad (2)$

Solving (2) for N and substituting the result into (1), we obtain

$$N = P \sin 30° + 785 \text{ N}$$

$$P \cos 30° - 0.25(P \sin 30° + 785 \text{ N}) = 200 \text{ N} \qquad P = 535 \text{ N}$$

3. Two blocks of mass 80 N and 20 N are connected by the string and move along a rough horizontal surface when a force of 40 N is applied towards the right at the block of 80 N Apply D’ Alembert principle to determine the acceleration of the blocks and tension the string. Assume that the coefficient of friction between the sliding surface of the blocks and the plane is 0.3. **[a= 0.98 m/s2, T= 7.99 N]**



Consider 80N weight body

Frictional force $F_r = \mu \times R_N$

$$= \mu \times w \quad = \quad 0.3 \times 80$$

$$= 24N$$

By D' Alembert Principle

$$\sum F = m\,a$$

$$F - T - F_r = m\,a$$

$$40 - T - 24 = \frac{80}{9.81} \times a$$

$$16 - T = 8.15\,a - - - \rightarrow \ (1)$$

Consider 20N weight body

$$F_r = \mu \times R_N = \mu \times w \quad = 0.3 \times 20 = 6N$$

By D' Alembert Principle

$$\sum F = m \times a$$

$$T - F_r = m \times a$$

$$T - 6 = \frac{20}{9.81} \times a$$

$$T - 6 = 2.03\,a - - - \rightarrow \ (2)$$

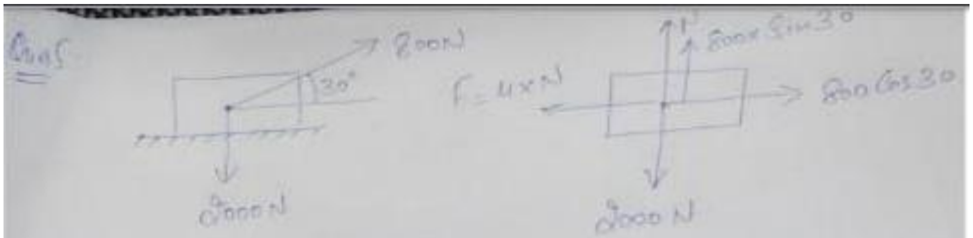
$$(1) + (2) \rightarrow 10 = 10.18\,a$$

$$a = 0.98 m/sec$$

Substitute a value in eq (2) T = $2.03 \times 0.98 + 6 \quad = 7.99N$

4. A block of weight 2000 N rest on a rough horizontal surface (μ = 0.2) and pulled by a force of 800 N at an angle of 300 to the horizontal. Determine the velocity attained by the block after it has moved 20 m starting from rest. Proceed to calculate the further distance moved by the body if the pull is removed. Use work-Energy relation. **[v= 8.55 m/s, s = 18.63 m]**

Ans



$F = \mu \times N$

$= 0.2 \times (2000 - 800 \times \sin 30)$

$\Rightarrow 0.2 \times 1600$

$\Rightarrow 320N$

a) Velocity attained by block after it has moved 20m starting from rest

Net workdone = Change in kinetic energy

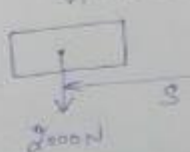
$(800 \times \cos 30 - 320) \times 20 = \left(\frac{1}{2} \times \frac{2000}{9.8} \times V_2^2 - \frac{1}{2} \times \frac{2000}{9.8} \times 0\right)$

$(692.8 - 320) \times 20 = \frac{1}{2} \times \frac{2000}{9.8} \times V_2^2$

$V_2 = \sqrt{73.07}$ $\boxed{V_2 = 8.55 m/s}$

b) Further distance moved by body if pull force is removed

$V_1 = 8.55 m/s$ $V_2 = 0$



Net workdone = $\Delta K.E.$

$(0 - F) \times S = \frac{1}{2} m V_2^2 - \frac{1}{2} m V_1^2$

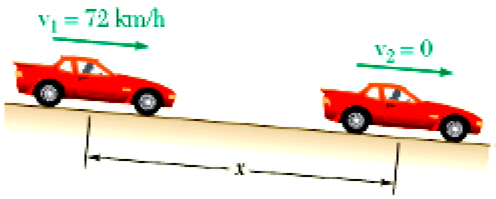
$(0 - \mu \times N) \times S = \frac{1}{2} \times \frac{2000}{9.8} \times 0 - \frac{1}{2} \times \frac{2000}{9.8} \times 8.55^2$

$(-0.2 \times 2000) \times S = -\frac{1}{2} \times \frac{2000}{9.8} \times 8.55^2$

$S = \frac{146205}{7840} \Rightarrow \boxed{18.64 m}$

5. A body of 5 kg mass is initially at rest on a rough horizontal surface (μ = 0.2) and is acted upon by a 20 N pull applied horizontally. Calculate: (a) the work done by the net force on the body in 5 seconds, (b) change in kinetic energy of the body in 5 seconds. **[work done= 259.59 Nm, Change in K.E.= 259.59]**

6. An automobile weighing 1000 Kg is driven down a 5° incline at a speed of 72 km/hr when the brakes are applied, causing a constant total braking force (applied by the road on the tires) of 5000 N. Determine the distance traveled by the automobile as it comes to a stop.



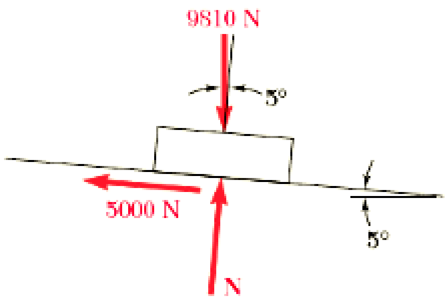
SOLUTION:

- Evaluate the change in kinetic energy.

$$v_1 = \left(72\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{km}}\right)\left(\frac{1\text{h}}{3600\text{ s}}\right) = 20\,\text{m/s}$$

$$T_1 = \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}(1000\,\text{kg})(20\text{m/s})^2 = 20{,}000\text{ j}$$

$$v_2 = 0 \qquad T_2 = 0$$

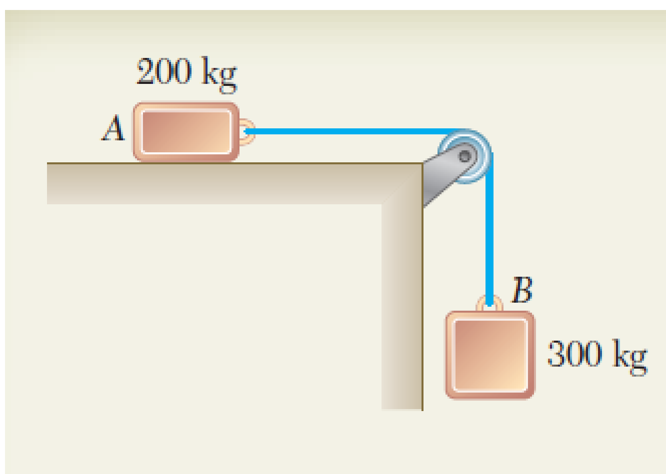


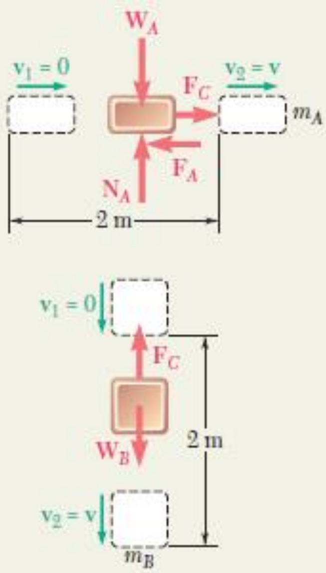
- Determine the distance required for the work to equal the kinetic energy change.

$$U_{1\to2} = -5000\,x + (1000\,\text{kg})(9.81\text{m/s}^2)(\sin 5°)x$$
$$= -4145\,x$$

$$T_1 + U_{1\to2} = T_2$$
$$200{,}000 - 4145x = 0$$
$$\boxed{x = 48.25\,\text{m}}$$

7. Two blocks are joined by an inextensible cable as shown in figure. If the system is released from rest, determine the velocity of block A after it has moved 2 m. Assume that the coefficient of kinetic friction between block A and the plane is 0.25 and that the pulley is weightless and frictionless. **[v=4.43 m/sec]**





SOLUTION

Work and Energy for Block A. We denote the friction force by $\mathbf{F}_A$ and the force exerted by the cable by $\mathbf{F}_C$, and write

$$m_A = 200 \text{ kg} \qquad W_A = (200 \text{ kg})(9.81 \text{ m/s}^2) = 1962 \text{ N}$$
$$F_A = \mu_k N_A = \mu_k W_A = 0.25(1962 \text{ N}) = 490 \text{ N}$$
$$T_1 + U_{1\to 2} = T_2: \qquad 0 + F_C(2 \text{ m}) - F_A(2 \text{ m}) = \tfrac{1}{2} m_A v^2$$
$$F_C(2 \text{ m}) - (490 \text{ N})(2 \text{ m}) = \tfrac{1}{2}(200 \text{ kg})v^2 \qquad (1)$$

Work and Energy for Block B. We write

$$m_B = 300 \text{ kg} \qquad W_B = (300 \text{ kg})(9.81 \text{ m/s}^2) = 2940 \text{ N}$$
$$T_1 + U_{1\to 2} = T_2: \qquad 0 + W_B(2 \text{ m}) - F_C(2 \text{ m}) = \tfrac{1}{2} m_B v^2$$
$$(2940 \text{ N})(2 \text{ m}) - F_C(2 \text{ m}) = \tfrac{1}{2}(300 \text{ kg})v^2 \qquad (2)$$

Adding the left-hand and right-hand members of (1) and (2), we observe that the work of the forces exerted by the cable on A and B cancels out:

$$(2940 \text{ N})(2 \text{ m}) - (490 \text{ N})(2 \text{ m}) = \tfrac{1}{2}(200 \text{ kg} + 300 \text{ kg})v^2$$
$$4900 \text{ J} = \tfrac{1}{2}(500 \text{ kg})v^2 \qquad v = 4.43 \text{ m/s} \blacktriangleleft$$

8. A 1300-kg small hybrid car is traveling at 108 km/h. Determine (a) the kinetic energy of the vehicle, (b) the speed required for a 9000-kg truck to have the same kinetic energy as the car. **[K.E.= 585 KJ and v=11.4 m/sec]**

SOLUTION

$$v = 108 \text{ km/h} = 30 \text{ m/s}$$

(a) $$T_{car} = \frac{1}{2} m_c v^2 = \frac{1}{2}(1300)\,(30)^2 = 585 \times 10^3 \text{ J} \qquad T_{car} = 585 \text{ kJ} \blacktriangleleft$$

(b) $$T_{truck} = \frac{1}{2} m_{truck} v_{truck}^2$$

$$v_{truck}^2 = \frac{2T_{truck}}{m_{truck}} = \frac{(2)(585 \times 10^3)}{9000} = 130 \text{ m}^2/\text{s}^2$$

$$v_{truck} = 11.40 \text{ m/s} \qquad v_{truck} = 41.0 \text{ km/h} \blacktriangleleft$$

9. A 450 kg satellite is placed in a circular orbit 6400 km above the surface of the earth. At this elevation, the acceleration of gravity is 2.4 m/s^2. Determine the kinetic energy of the satellite, knowing that its orbital speed is 20,200 km/h. **[K.E= 7.08 × 10^9 J]**

SOLUTION

$$v = 20,200 \text{ km/h}$$
$$v = 5611.11 \text{ m/s}$$
$$\text{Mass of satellite } m = 450 \text{ kg}$$
$$T = \frac{1}{2}mv^2$$
$$= \frac{1}{2}(450)\,(5611.11)^2$$
$$T = 7.08402 \times 10^9 \text{ J} \qquad T = 7.08 \times 10^9 \text{ J} \blacktriangleleft$$

Note: Acceleration of gravity has no effect on the mass of the satellite.

10. A 4-kg stone is dropped from a height h and strikes the ground with a velocity of 25 m/s. (a) Find the kinetic energy of the stone as it strikes the ground and the height h from which it was dropped. (b) From what height h_1 must a 1 kg stone be dropped so it has the same kinetic energy? **[(a) K.E.=1250 J, h=31.9 m (b) h=127.4 m]**

SOLUTION

(*a*) <u>On the earth</u>.

$$T = \frac{1}{2}mv^2 = \frac{1}{2}(4 \text{ kg})\,(25 \text{ m/s})^2 = 1250 \text{ N}\cdot\text{m} \qquad T = 1250 \text{ J} \blacktriangleleft$$

$$W = mg = (4 \text{ kg})\,(9.81 \text{ m/s}^2) = 39.240 \text{ N}$$
$$T_1 + U_{1-2} = T_2 \quad T_1 = 0 \quad U_{1-2} = Wh \quad T_2 = 39.240 \text{ N}$$
$$h = \frac{T_2}{W} = \frac{(1250 \text{ N}\cdot\text{m})}{(39.240 \text{ N})} = 31.855 \text{ m} \qquad h = 31.9 \text{ m} \blacktriangleleft$$

(*b*) K.E. of 1 kg stone = 1250 J = T_2

$$T_1 + U_{1-2} = T_2,\ U_{1-2} = (1)\,(9.81)\,h$$
$$h = \frac{T_2}{W} = \frac{1250}{(1)\,(9.81)} = 127.421 \text{m} \qquad h = 127.4 \text{ m} \blacktriangleleft$$

11. A 1 kg stone is dropped from a height h and strikes the ground with a velocity of 25 m/s. (a) Find the kinetic energy of the stone as it strikes the ground and the height h from which it was dropped, (b) Solve Part a assuming that the same stone is dropped on the moon. (Acceleration of gravity on the moon = 1.62 m/s^2.) **[(a) K.E= 313 J, h=31.9 m, (b) K.E.= 313 J, h=192.9 m]**

SOLUTION

For the stone, $m = 1$ kg

(*a*) $T_2 = \frac{1}{2}mv^2 = \frac{1}{2}(1)(25)^2 = 312.5$ J $\qquad T_2 = 313$ J ◀

On the earth. $g = 9.81$ m/s²

$$T_1 + U_{1\to 2} = T_2 \quad \text{or} \quad 0 + Wh = T_2$$

$$h = \frac{T_2}{W} = \frac{312.5}{(1)(9.81)} = 31.855 \text{ m} \qquad h = 31.9 \text{ m} ◀$$

On the moon. $g = 1.62$ m/s²

$$W = mg = 1.62 \text{ N}$$

(*b*) $T_2 = \frac{1}{2}mv^2 = \frac{1}{2}(1)(25)^2$ (T remains same) $\qquad T_2 = 313$ J ◀

$$h = \frac{T_2}{W} = \frac{312.5}{1.62} = 192.901 \text{ m} \qquad h = 192.9 \text{ m} ◀$$

12. Determine the maximum theoretical speed that may be achieved over a distance of 100 m by a car starting from rest, assuming there is no slipping. The coefficient of static friction between the tires and pavement is 0.75, and 60 percent of the weight of the car is distributed over its front wheels and 40 percent over its rear wheels. Assume (a) front-wheel drive, (b) rear-wheel drive.**[(a)v=29.714 m/sec, (b) v=24.261 m/sec]**

Let W be the weight and m the mass. $W = mg$

(*a*) Front wheel drive. $N = 0.60\ W$

$\mu_s = 0.75$

Maximum friction force without slipping:

$$F = \mu_s N = 0.45\,W$$

$$U_{1\to 2} = Fd$$

$$= (0.45W)\ (100)$$

$$= 45\ W\,\mathrm{J}$$

$$T_1 = 0, \qquad T_2 = \frac{1}{2} m v_2^2$$

Principle of work and energy:

$$T_1 + U_{1\to 2} = T_2$$

$$0 + 45\ \mathbf{W} = \frac{1}{2}\frac{W}{g} v_2^2$$

$$v_2^2 = (2)(9.81)(45)$$

$$= 882.9\ \mathrm{m}^2/\mathrm{s}^2$$

$$v_2 = 29.714\ \mathrm{m/s}$$

$v_2 = 107.0\ \mathrm{km/h}$ ◀

(*b*) Rear wheel drive. $N = 0.40\,W$

$\mu_s = 0.75$

Maximum friction force without slipping:

$$F = \mu_s N = 0.30\ W$$

$$U_{1\to 2} = Fd$$

$$= (0.30\,W)(100)$$

$$= 30\ \mathrm{W\,J}$$

$$T_1 = 0$$

$$T_2 = \frac{1}{2} m v_2^2$$

Principle of work and energy:

$$T_1 + U_{1\to2} = T_2$$

$$0 + 30\,\mathbf{W} = \frac{1}{2}\frac{W}{g}v_2^2$$

$$v_2^2 = (2)(9.81)(30)$$

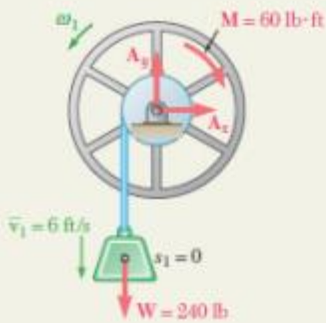
$$= 588.6\ \mathrm{m^2/s^2}$$

$$v_2 = 24.261\ \mathrm{m/s} \qquad v_2 = 87.3\ \mathrm{km/h} \blacktriangleleft$$

13. A 240-lb block is suspended from an inextensible cable which is wrapped around a drum of 1.25-ft radius rigidly attached to a flywheel in figure. The drum and flywheel have a combined centroidal moment of inertia is 10.5 lb.ft.s^2. At the instant shown, the velocity of the block is 6 ft/s directed downward. Knowing that the bearing at A is poorly lubricated and that the bearing friction is equivalent to a couple M of magnitude 60 lb.ft, determine the velocity of the block after it has moved 4 ft downward. **[V=12.01 ft/s]**

SOLUTION

We consider the system formed by the flywheel and the block. Since the cable is inextensible, the work done by the internal forces exerted by the cable cancels. The initial and final positions of the system and the external forces acting on the system are as shown.



Kinetic Energy. ***Position 1.***

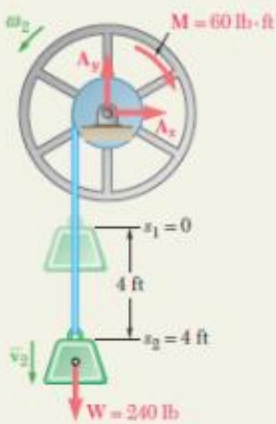
Block: $\bar{v}_1 = 6$ ft/s

Flywheel: $\omega_1 = \dfrac{\bar{v}_1}{r} = \dfrac{6 \text{ ft/s}}{1.25 \text{ ft}} = 4.80$ rad/s

$$T_1 = \tfrac{1}{2}m\bar{v}_1^2 + \tfrac{1}{2}\bar{I}\omega_1^2$$
$$= \frac{1}{2}\frac{240 \text{ lb}}{32.2 \text{ ft/s}^2}(6 \text{ ft/s})^2 + \tfrac{1}{2}(10.5 \text{ lb}\cdot\text{ft}\cdot\text{s}^2)(4.80 \text{ rad/s})^2$$
$$= 255 \text{ ft}\cdot\text{lb}$$

Position 2. Noting that $\omega_2 = \bar{v}_2/1.25$, we write

$$T_2 = \tfrac{1}{2}m\bar{v}_2^2 + \tfrac{1}{2}\bar{I}\omega_2^2$$
$$= \frac{1}{2}\frac{240}{32.2}(\bar{v}_2)^2 + (\tfrac{1}{2})(10.5)\left(\frac{\bar{v}_2}{1.25}\right)^2 = 7.09\bar{v}_2^2$$



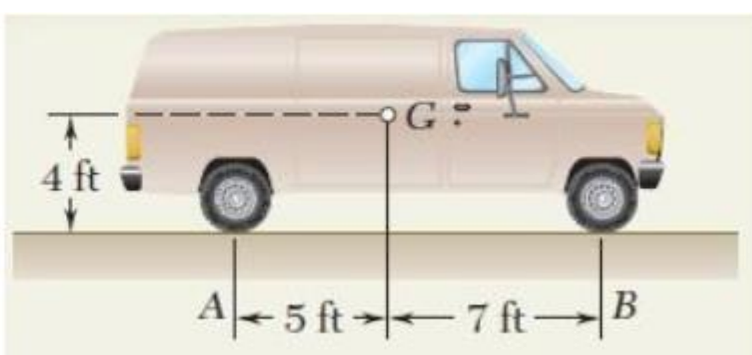
Work. During the motion, only the weight **W** of the block and the friction couple **M** do work. Noting that **W** does positive work and that the friction couple **M** does negative work, we write

$$s_1 = 0 \qquad s_2 = 4 \text{ ft}$$
$$\theta_1 = 0 \qquad \theta_2 = \frac{s_2}{r} = \frac{4 \text{ ft}}{1.25 \text{ ft}} = 3.20 \text{ rad}$$
$$U_{1\to2} = W(s_2 - s_1) - M(\theta_2 - \theta_1)$$
$$= (240 \text{ lb})(4 \text{ ft}) - (60 \text{ lb}\cdot\text{ft})(3.20 \text{ rad})$$
$$= 768 \text{ ft}\cdot\text{lb}$$

Principle of Work and Energy

$$T_1 + U_{1\to2} = T_2$$
$$255 \text{ ft}\cdot\text{lb} + 768 \text{ ft}\cdot\text{lb} = 7.09\bar{v}_2^2$$
$$\bar{v}_2 = 12.01 \text{ ft/s} \qquad \bar{\mathbf{v}}_2 = 12.01 \text{ ft/s}\downarrow$$

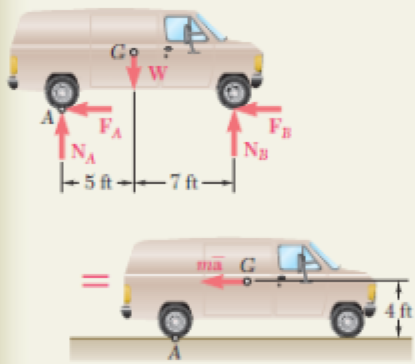
14. When the forward speed of the truck shown was 30 ft/s, the brakes were suddenly applied, causing all four wheels to stop rotating as shown in figure. It was observed that the truck skidded to rest in 20 ft. Determine the magnitude of the normal reaction and of the friction force at each wheel as the truck skidded to rest. **[N_A=0.175W, N_B= 0.325W, F_A= 0.122W, F_B=0.227W]**



Kinematics of Motion. Choosing the positive sense to the right and using the equations of uniformly accelerated motion, we write

$$\overline{v}_0 = +30 \text{ ft/s} \qquad \overline{v}^2 = \overline{v}_0^2 + 2\overline{a}\overline{x} \qquad 0 = (30)^2 + 2\overline{a}(20)$$
$$\overline{a} = -22.5 \text{ ft/s}^2 \qquad \overline{\mathbf{a}} = 22.5 \text{ ft/s}^2 \leftarrow$$

Equations of Motion. The external forces consist of the weight $\mathbf{W}$ of the truck and of the normal reactions and friction forces at the wheels. (The vectors $\mathbf{N}_A$ and $\mathbf{F}_A$ represent the sum of the reactions at the rear wheels, while $\mathbf{N}_B$ and $\mathbf{F}_B$ represent the sum of the reactions at the front wheels.) Since the truck is in translation, the effective forces reduce to the vector $m\overline{\mathbf{a}}$ attached at G. Three equations of motion are obtained by expressing that the system of the external forces is equivalent to the system of the effective forces.



$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \qquad N_A + N_B - W = 0$$

Since $F_A = \mu_k N_A$ and $F_B = \mu_k N_B$, where μ_k is the coefficient of kinetic friction, we find that

$$F_A + F_B = \mu_k(N_A + N_B) = \mu_k W$$
$$\xrightarrow{+} \Sigma F_x = \Sigma (F_x)_{\text{eff}}: \qquad -(F_A + F_B) = -m\overline{a}$$
$$-\mu_k W = -\frac{W}{32.2 \text{ ft/s}^2}(22.5 \text{ ft/s}^2)$$
$$\mu_k = 0.699$$
$$+\circlearrowleft \Sigma M_A = \Sigma (M_A)_{\text{eff}}: \qquad -W(5 \text{ ft}) + N_B(12 \text{ ft}) = m\overline{a}(4 \text{ ft})$$
$$-W(5 \text{ ft}) + N_B(12 \text{ ft}) = \frac{W}{32.2 \text{ ft/s}^2}(22.5 \text{ ft/s}^2)(4 \text{ ft})$$
$$N_B = 0.650W$$
$$F_B = \mu_k N_B = (0.699)(0.650W) \qquad F_B = 0.454W$$
$$+\uparrow \Sigma F_y = \Sigma (F_y)_{\text{eff}}: \qquad N_A + N_B - W = 0$$
$$N_A + 0.650W - W = 0$$
$$N_A = 0.350W$$
$$F_A = \mu_k N_A = (0.699)(0.350W) \qquad F_A = 0.245W$$

Reactions at Each Wheel. Recalling that the values computed above represent the sum of the reactions at the two front wheels or the two rear wheels, we obtain the magnitude of the reactions at each wheel by writing

$$N_{\text{front}} = \tfrac{1}{2}N_B = 0.325W \qquad N_{\text{rear}} = \tfrac{1}{2}N_A = 0.175W$$
$$F_{\text{front}} = \tfrac{1}{2}F_B = 0.227W \qquad F_{\text{rear}} = \tfrac{1}{2}F_A = 0.122W$$