CSE322
Finite Automata

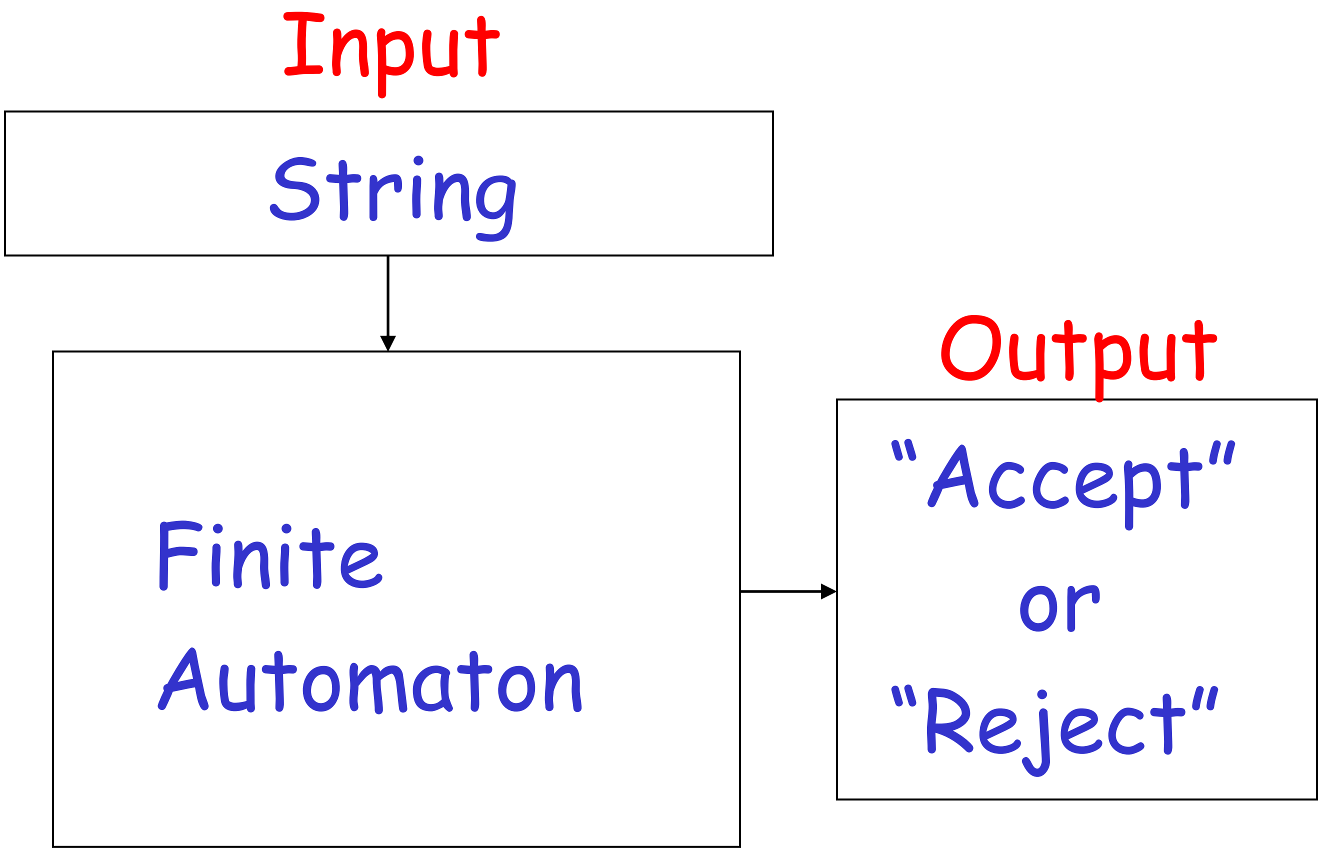
Lecture #2

Topics

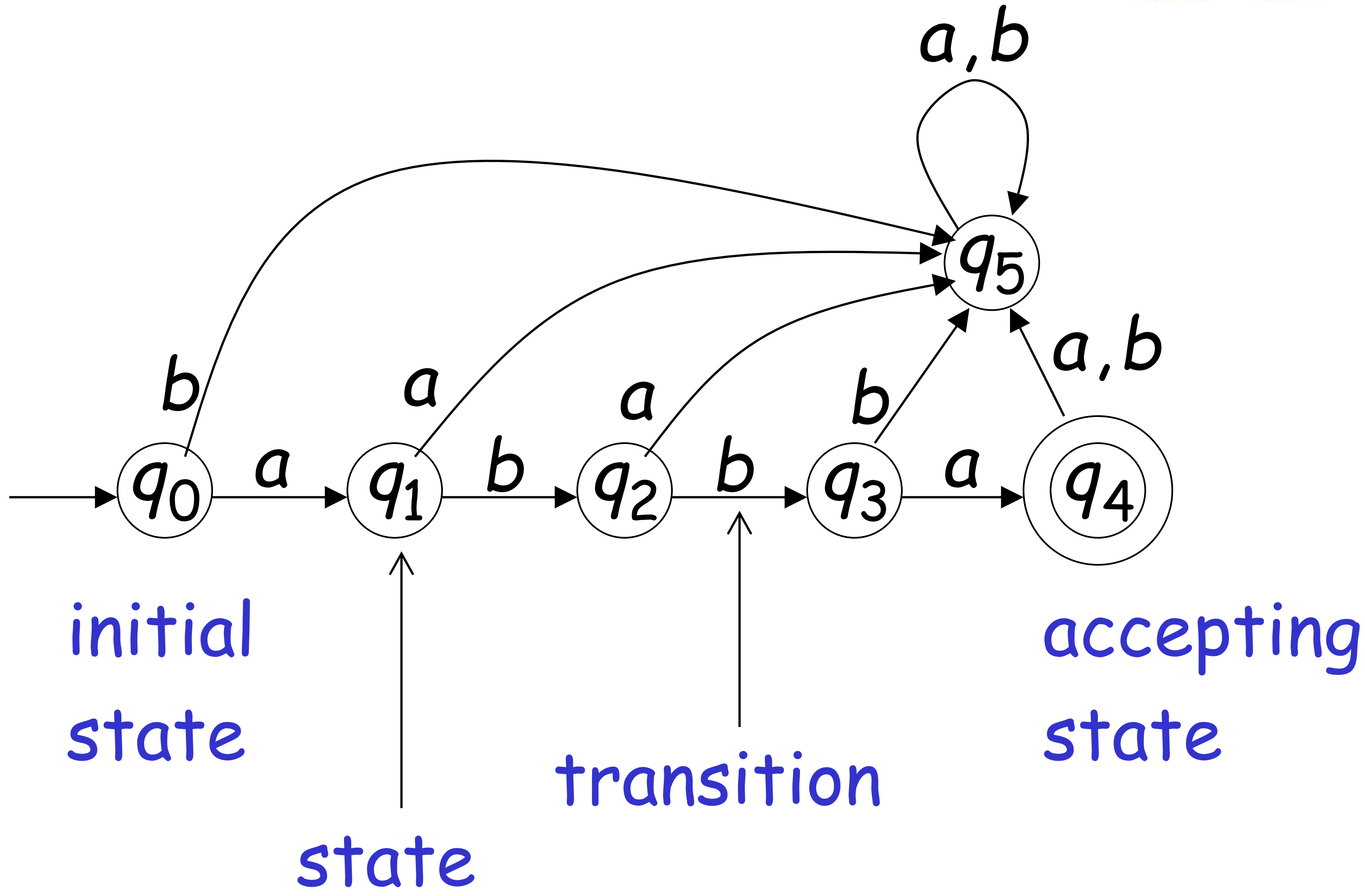
- Acceptability of a String by a Finite Automaton

- Transition Graph and Properties of Transition Functions

Finite Automaton



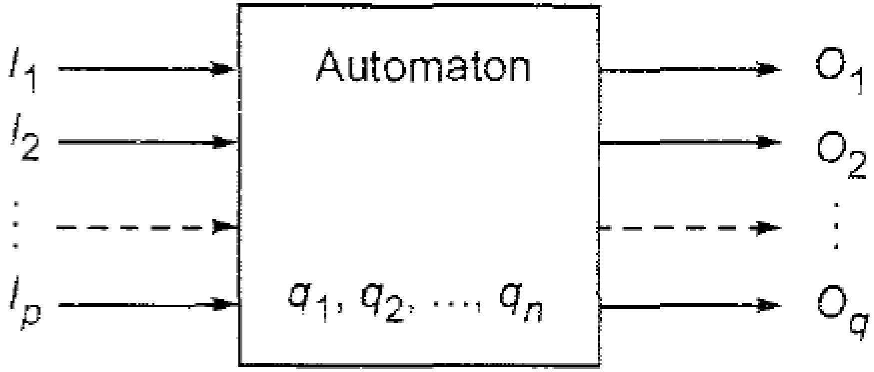
Transition Graph



Automaton:

An automaton is defined as a system where

- energy, materials and information are
- transformed, transmitted and used
- for performing some functions without direct participation of man.



Formal Definition of Finite Automaton

$$\boxed{M = (Q, \Sigma, \delta, q_0, F)}$$ where

Q : Finite non-empty set of states

Σ : Finite non empty set of input alphabets

δ : (direct) transition function that maps **Q x ∑ -> Q**

q_0 : initial state

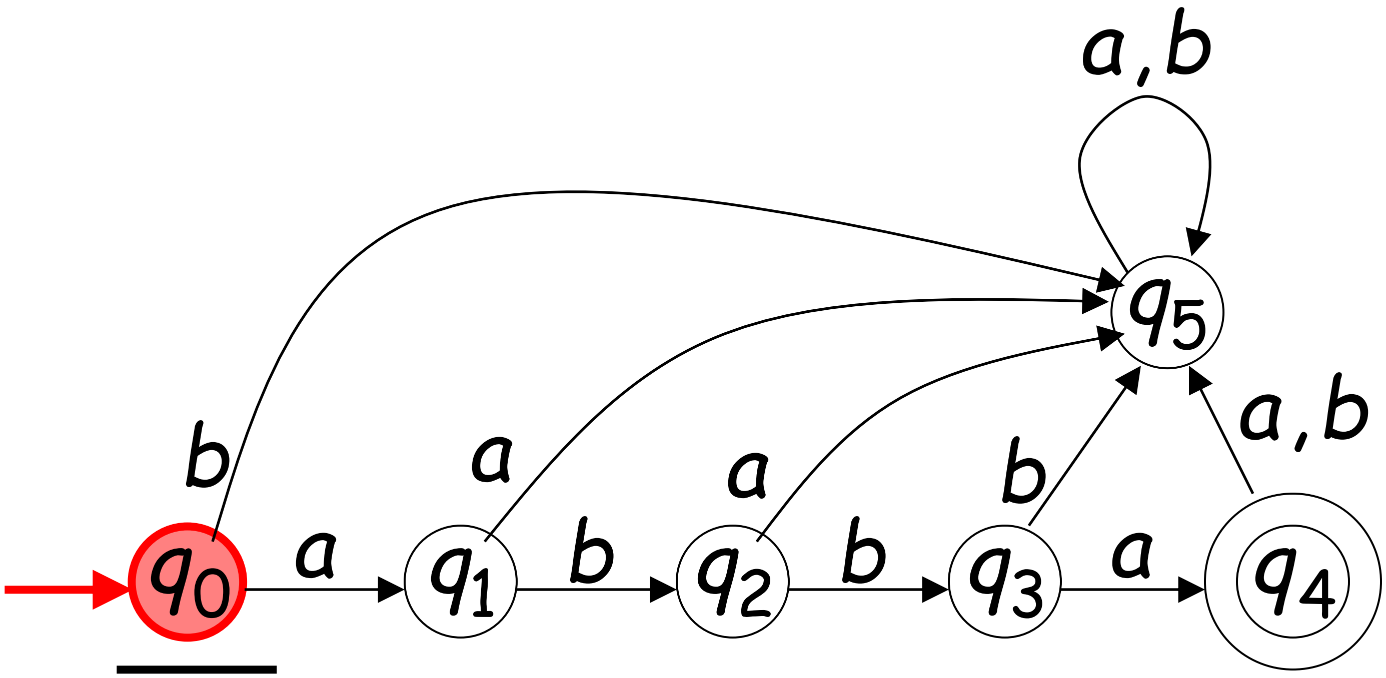
F : set of final states

There are numerous applications of Formal languages and Automata Theory like:

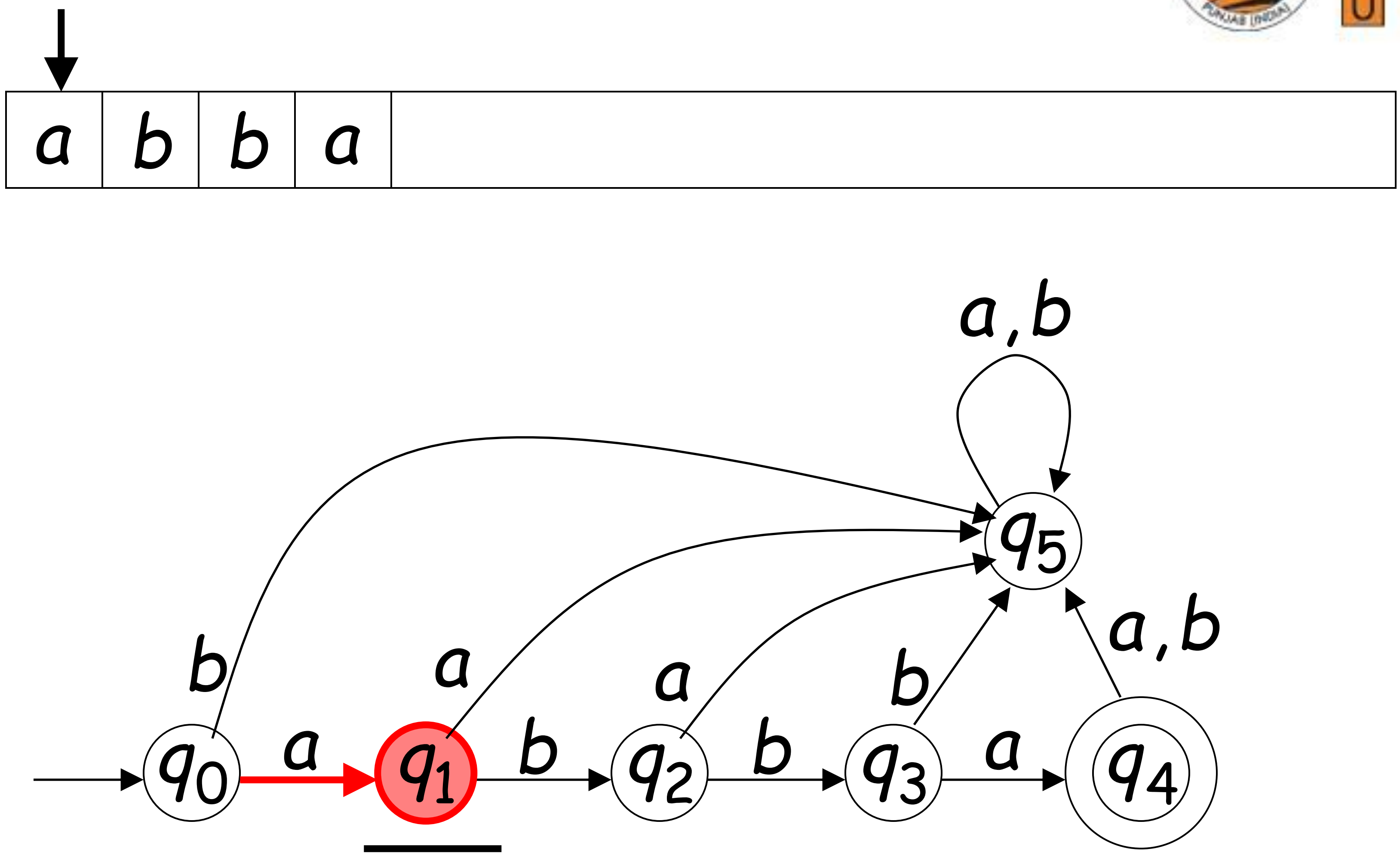
- Text processing, Compilers and Hardware Design
- Motors and Vending machines
- Sensors and Transducers
- Automata Simulators
- And many more ….

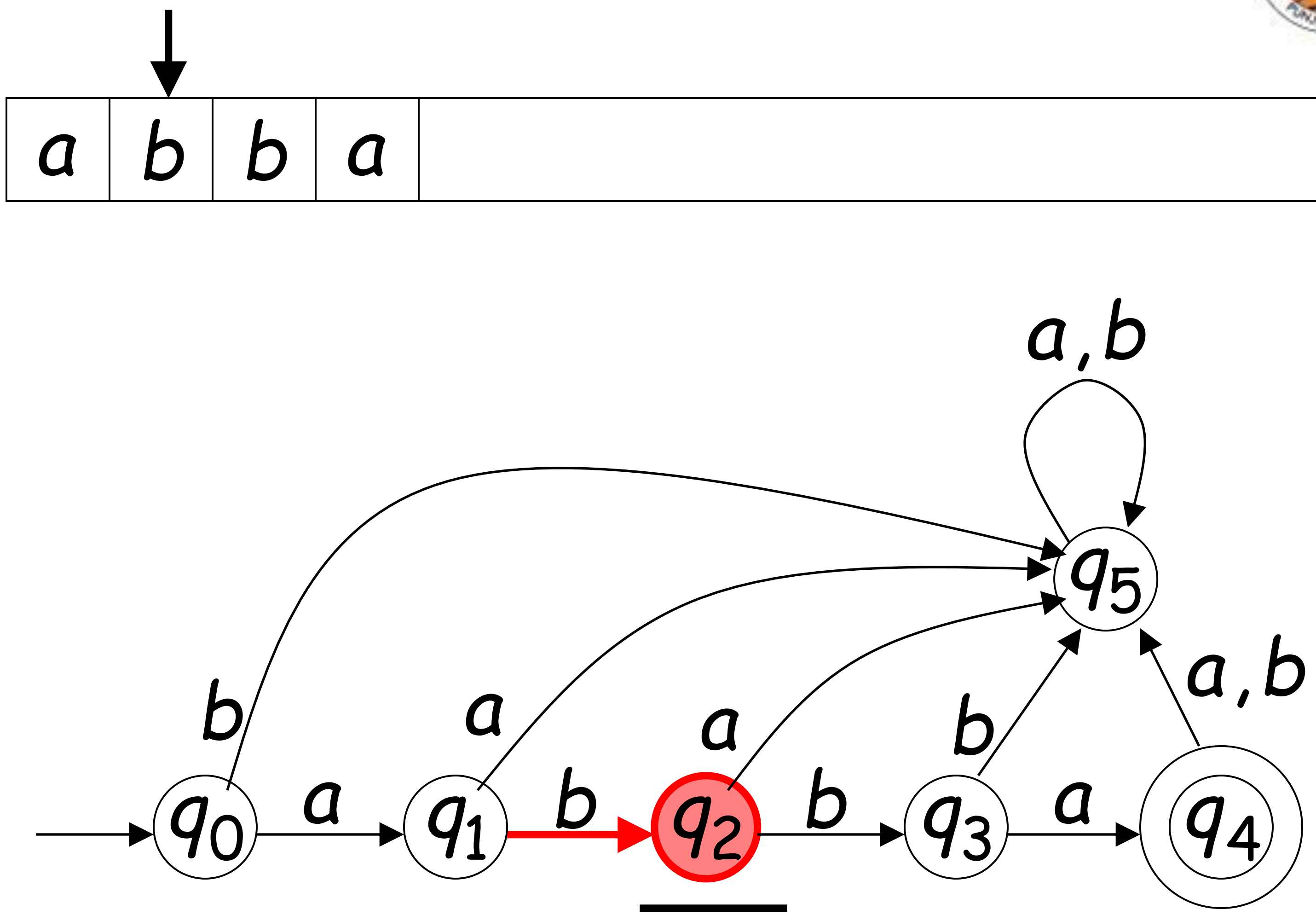
Initial Configuration

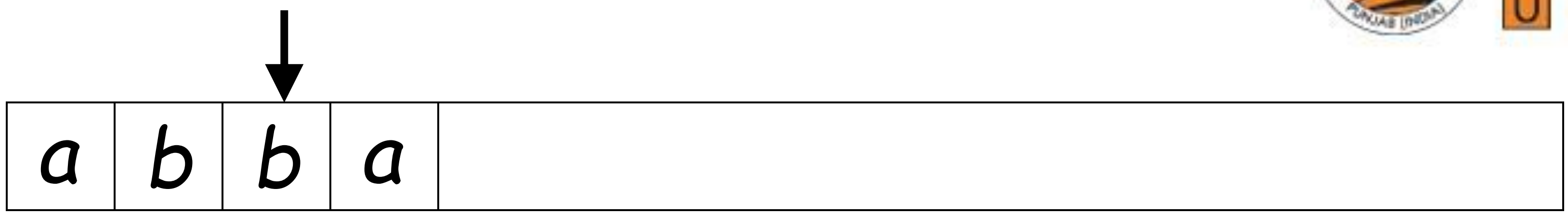
Input String

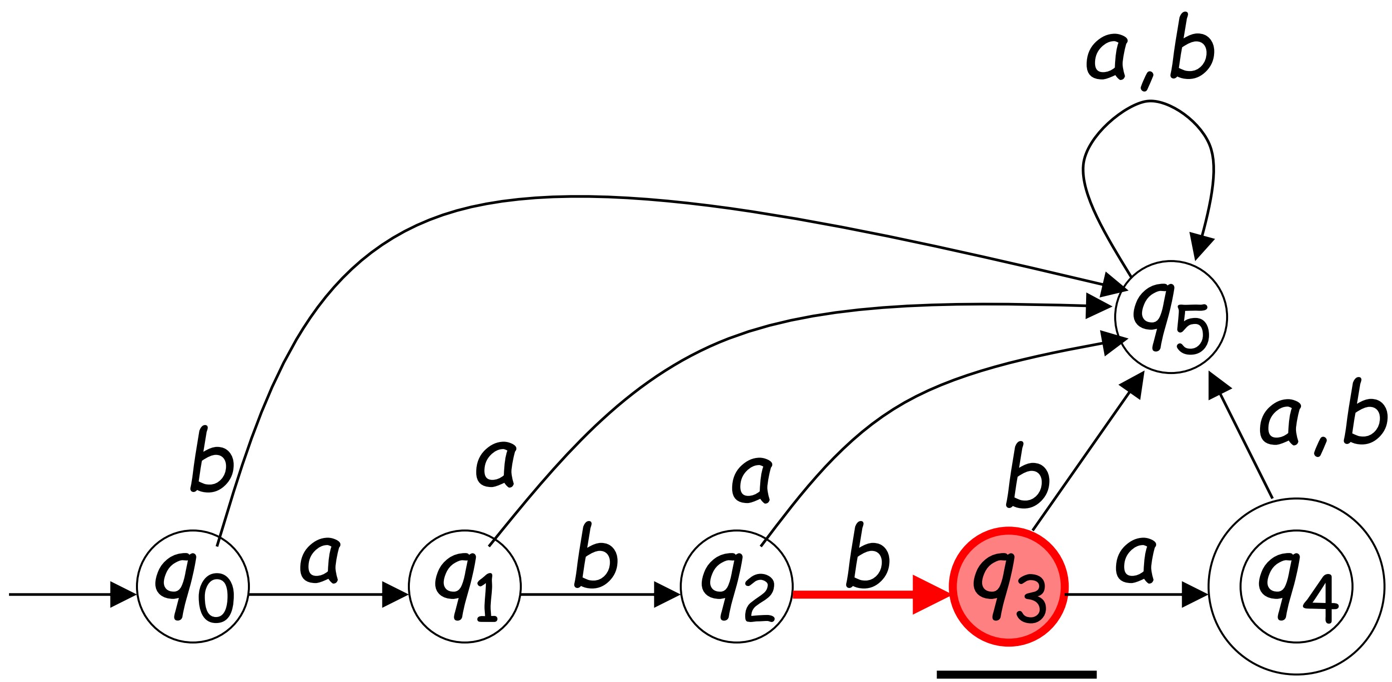


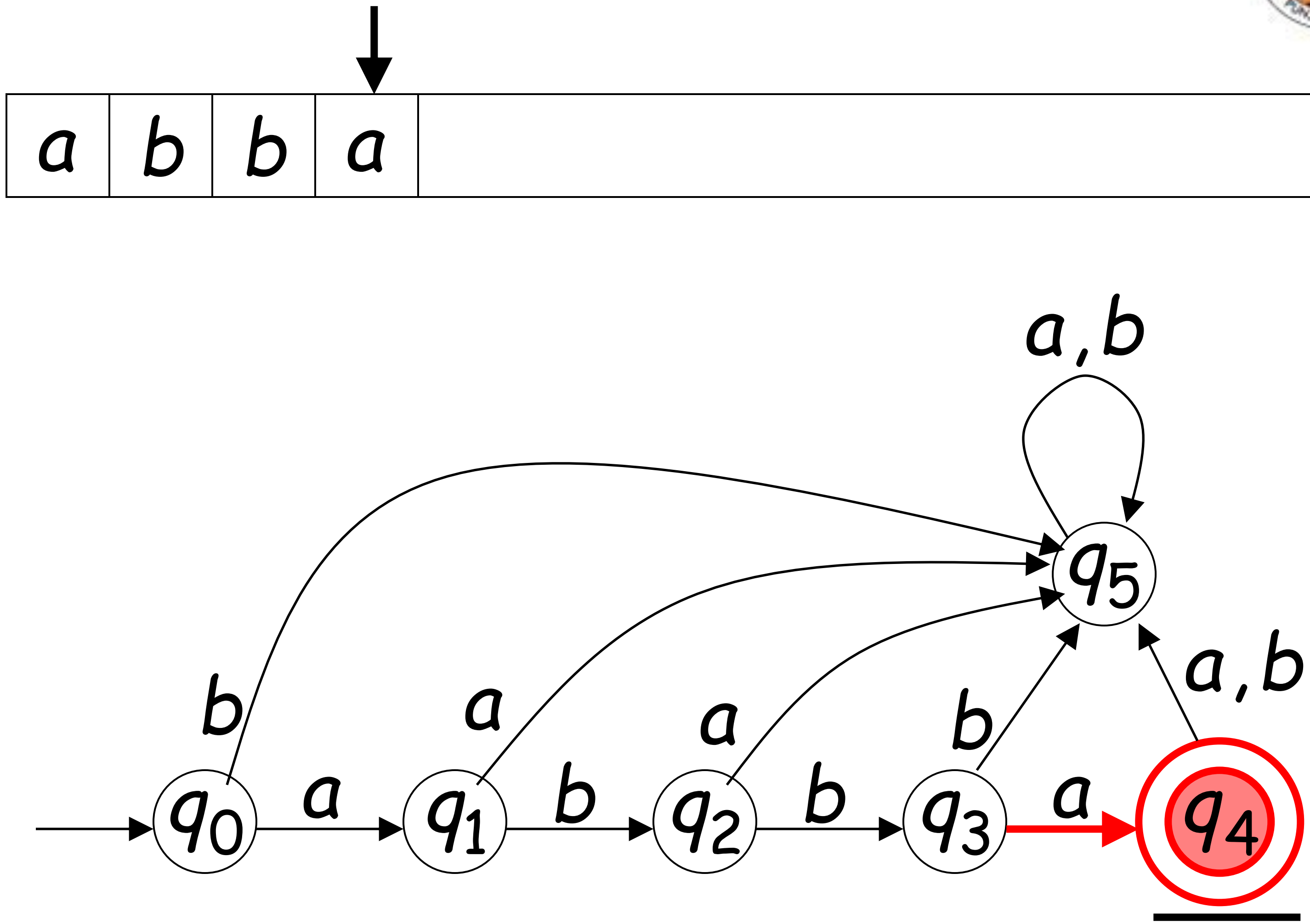
Reading the Input



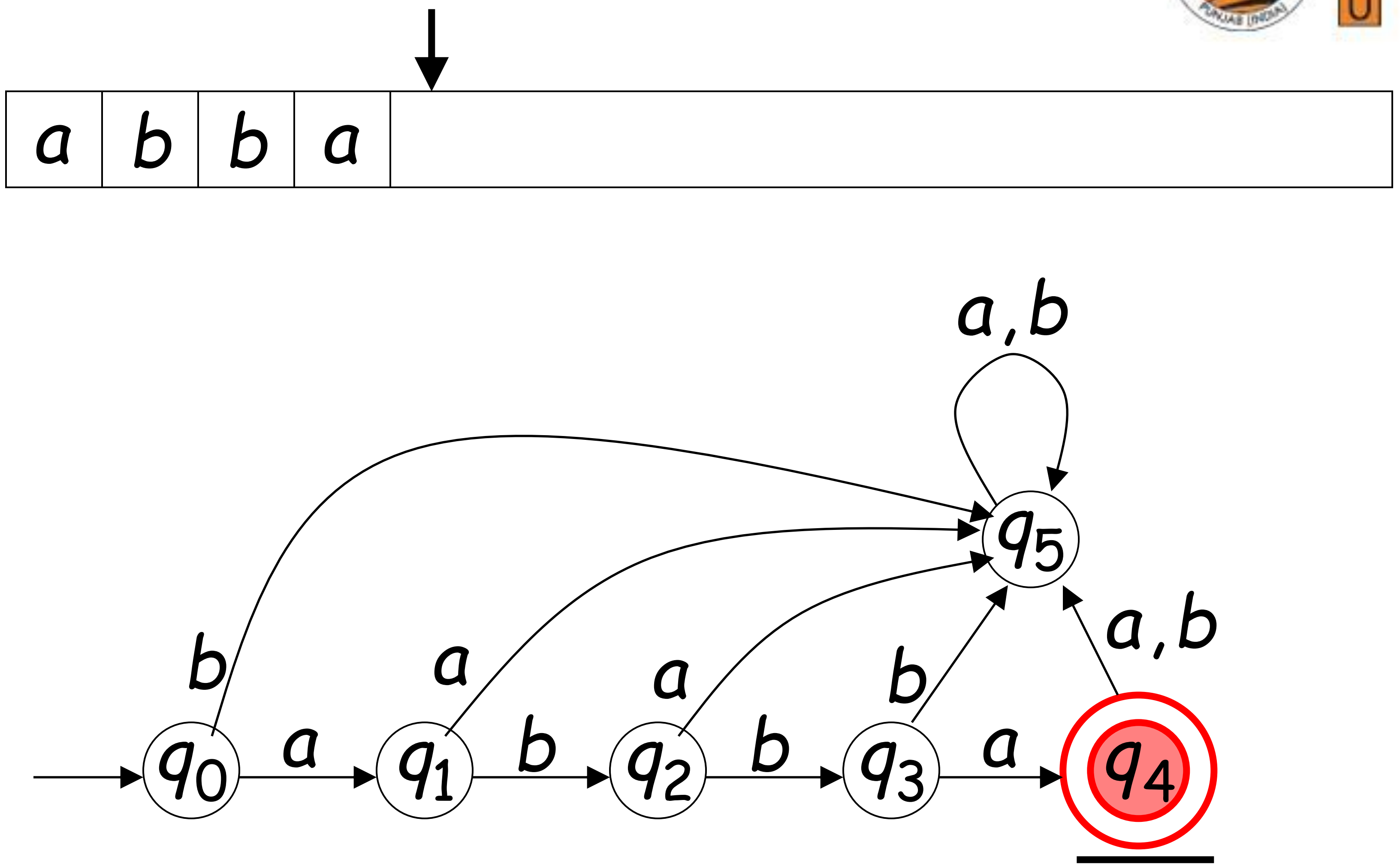

a
b
b
a
a,b
q5
b
a
a
b
a,b
q0
a
q1
b
q2
b
q3
a
q4


a
b
b
a


a,b
q_5
b
a
a
b
a,b
q_0
a
q_1
b
q_2
b
q_3
a
q_4

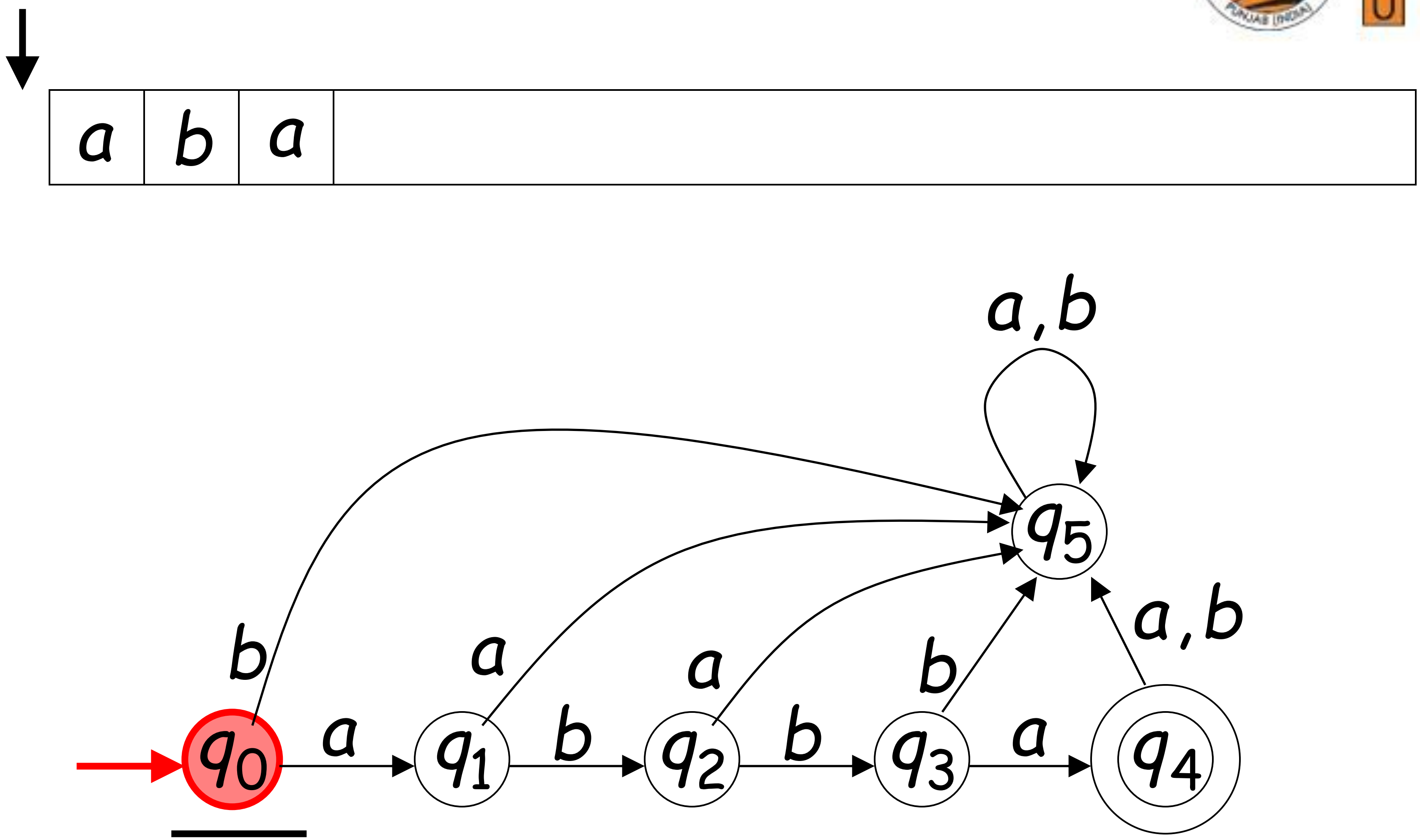

a
b
b
a
a,b
q5
b
a
a
b
a,b
q0
a
q1
b
q2
b
q3
a
q4

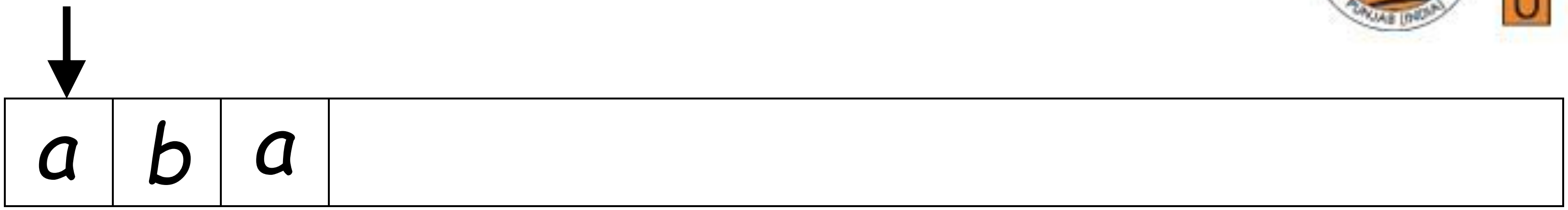
Input finished

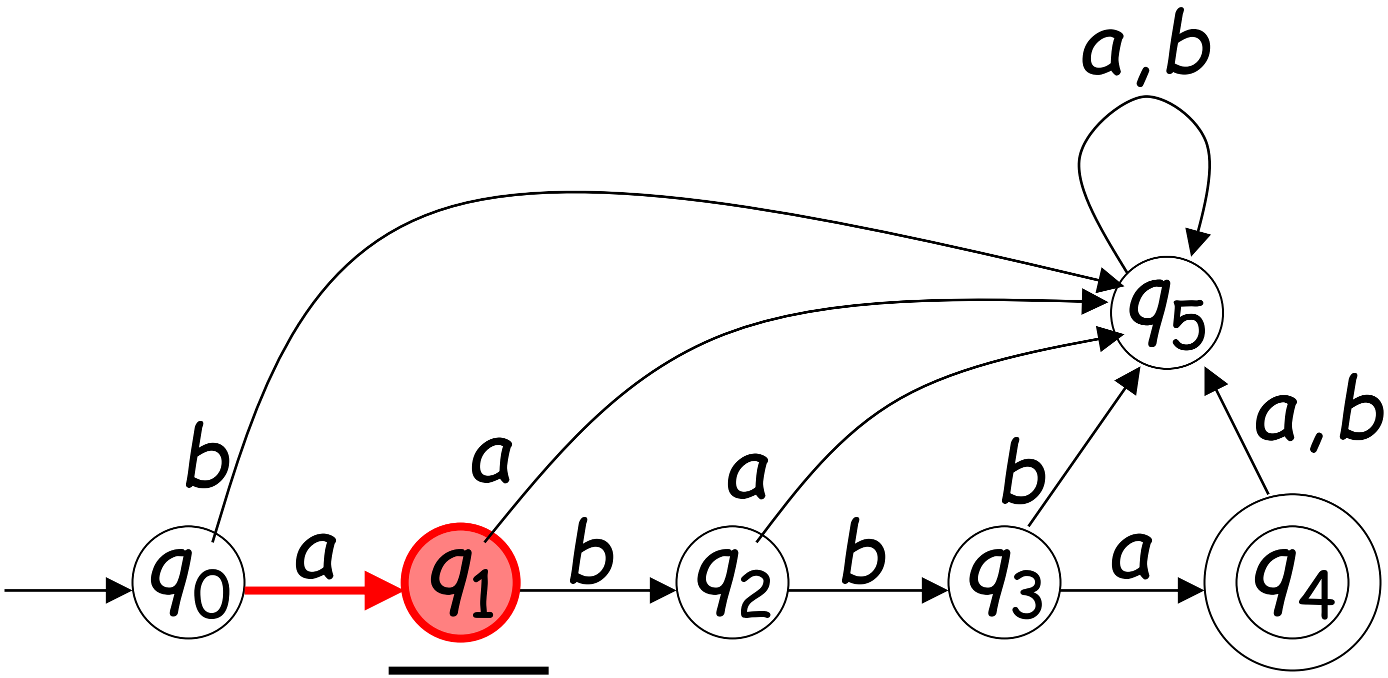


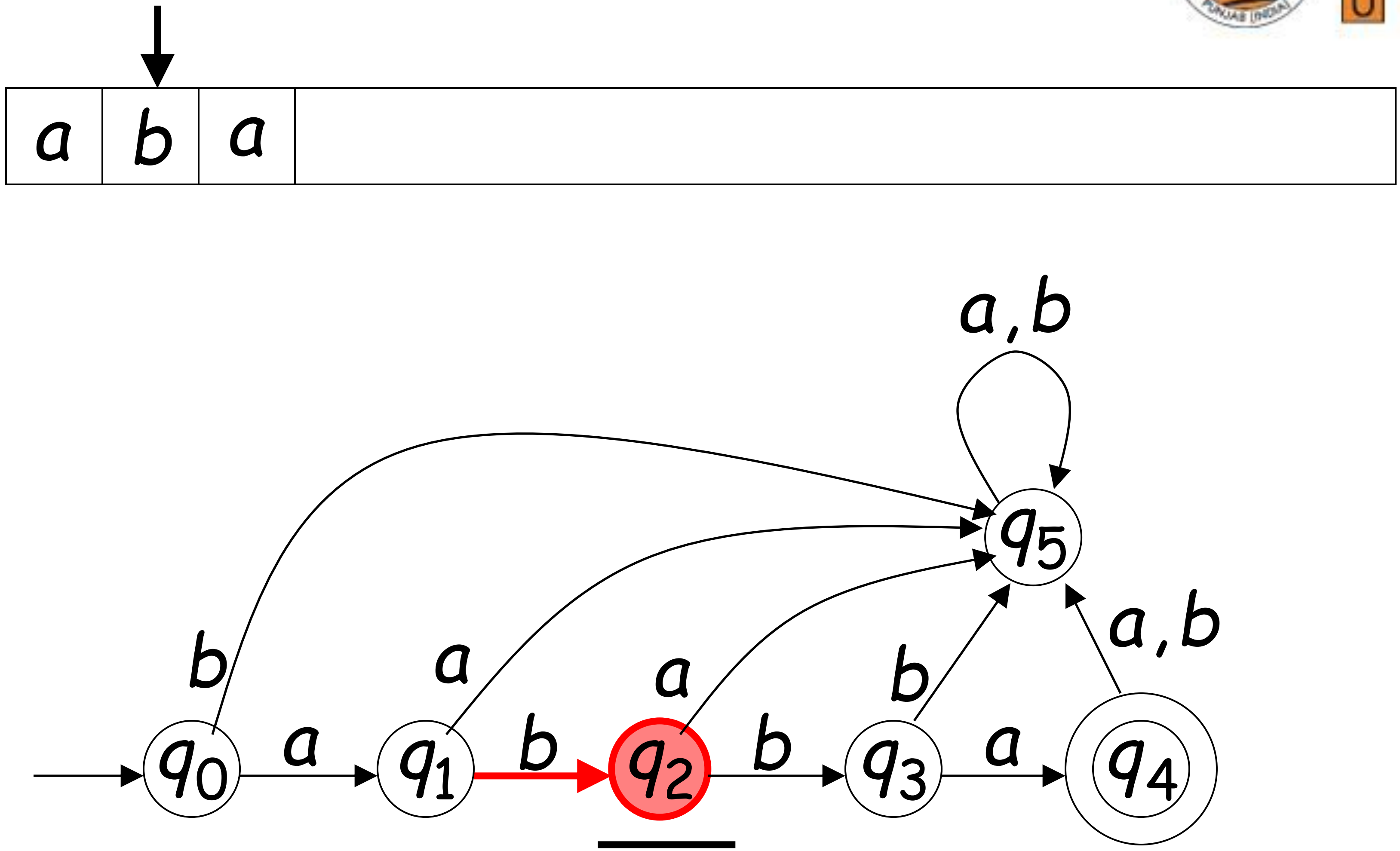
accept

Rejection

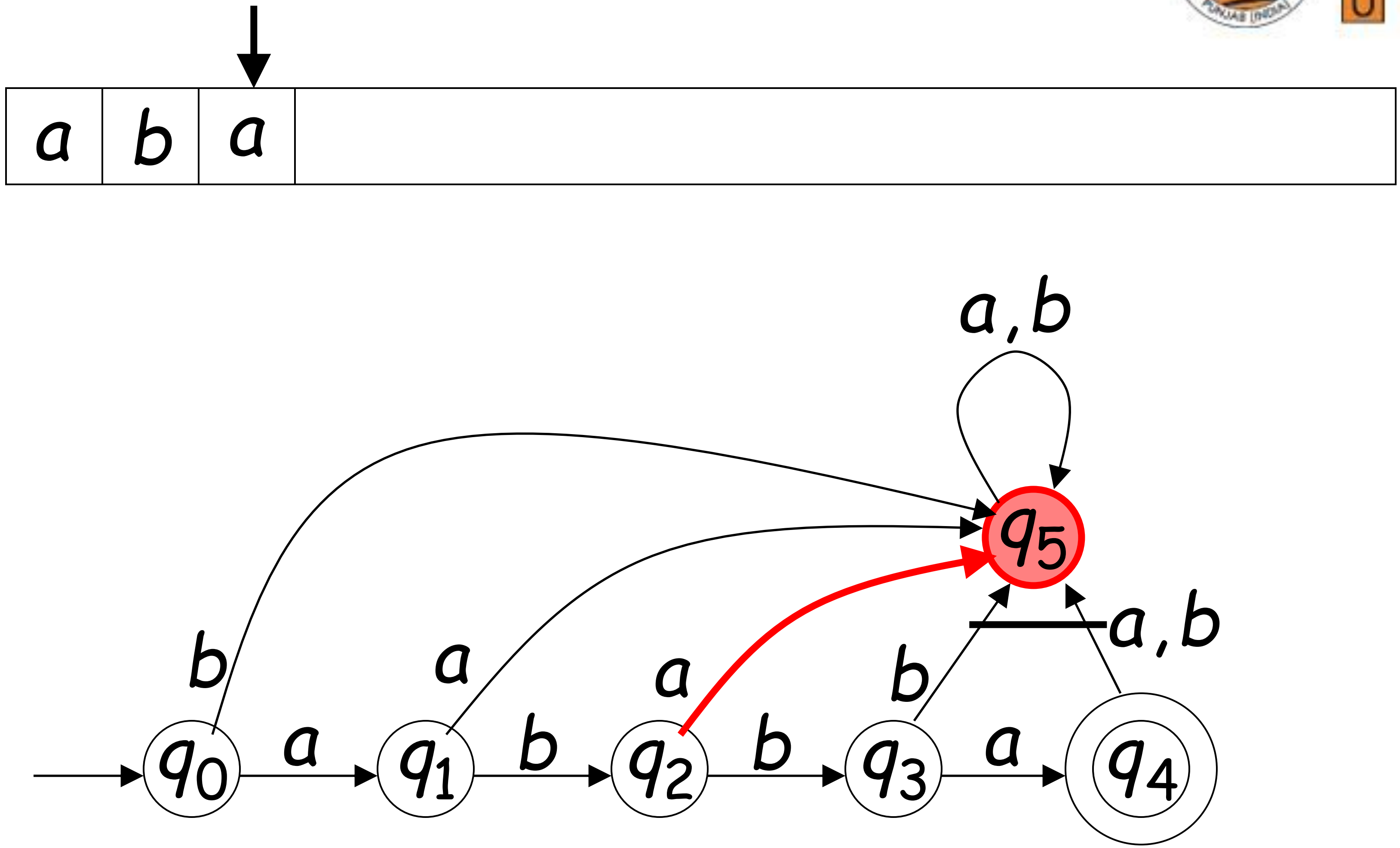



a
b
a


a,b
b
a
a
b
a,b
a
b
b
a
q0
q1
q2
q3
q4
q5

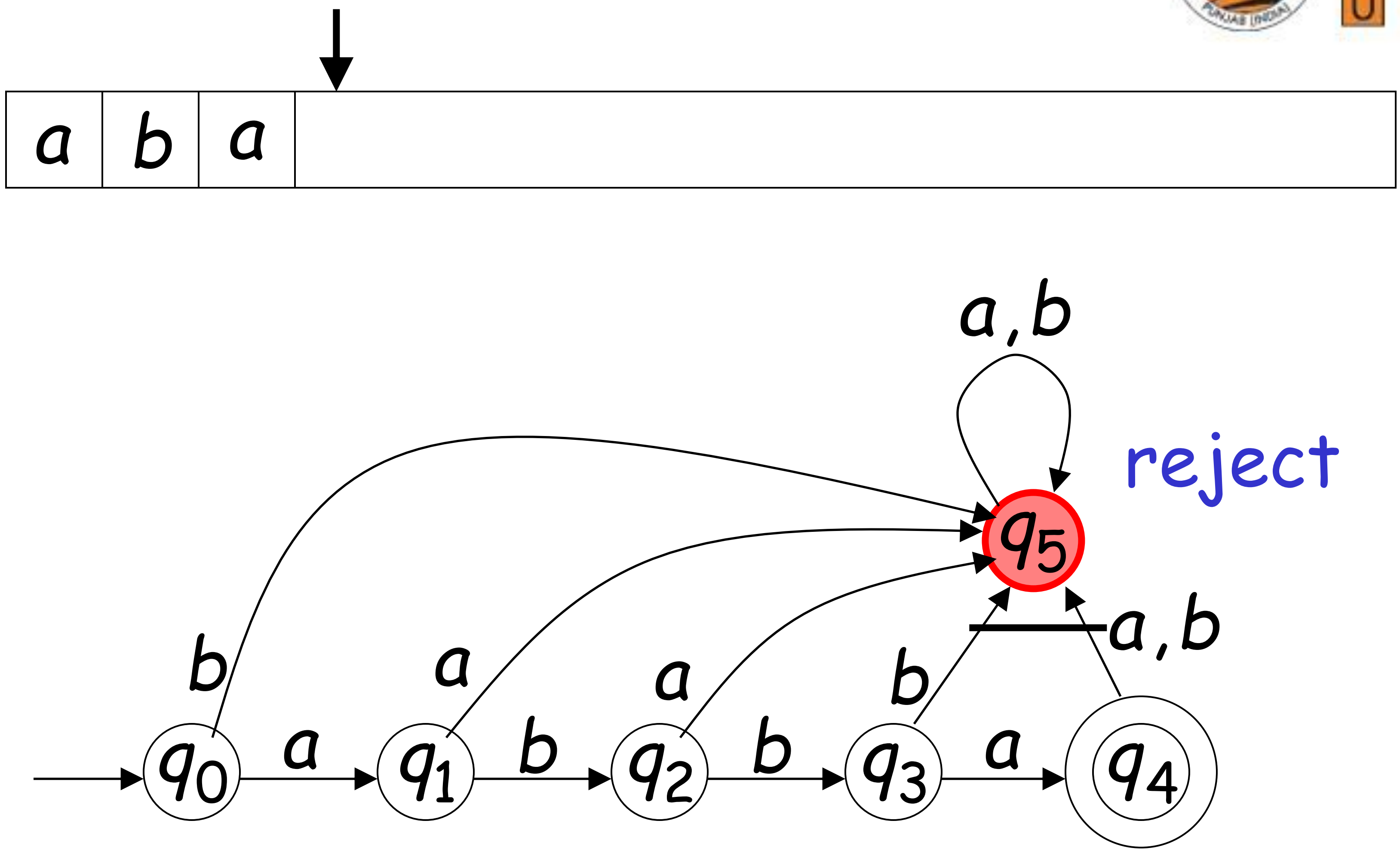


a
b
a
a,b
q5
b
a
a
b
a,b
q0
a
q1
b
q2
b
q3
a
q4



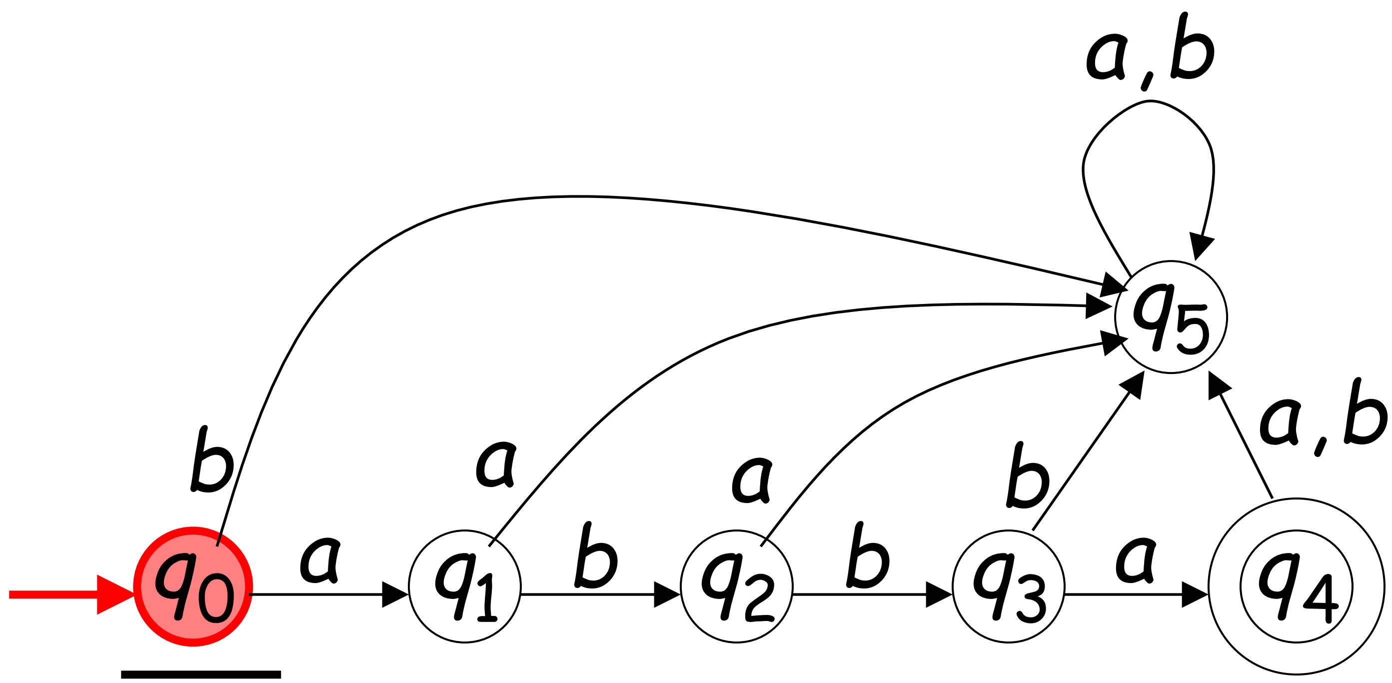
a
b
a
a,b
b
a
a
b
a,b
q0
q1
q2
q3
q4
q5
a
b
b
a

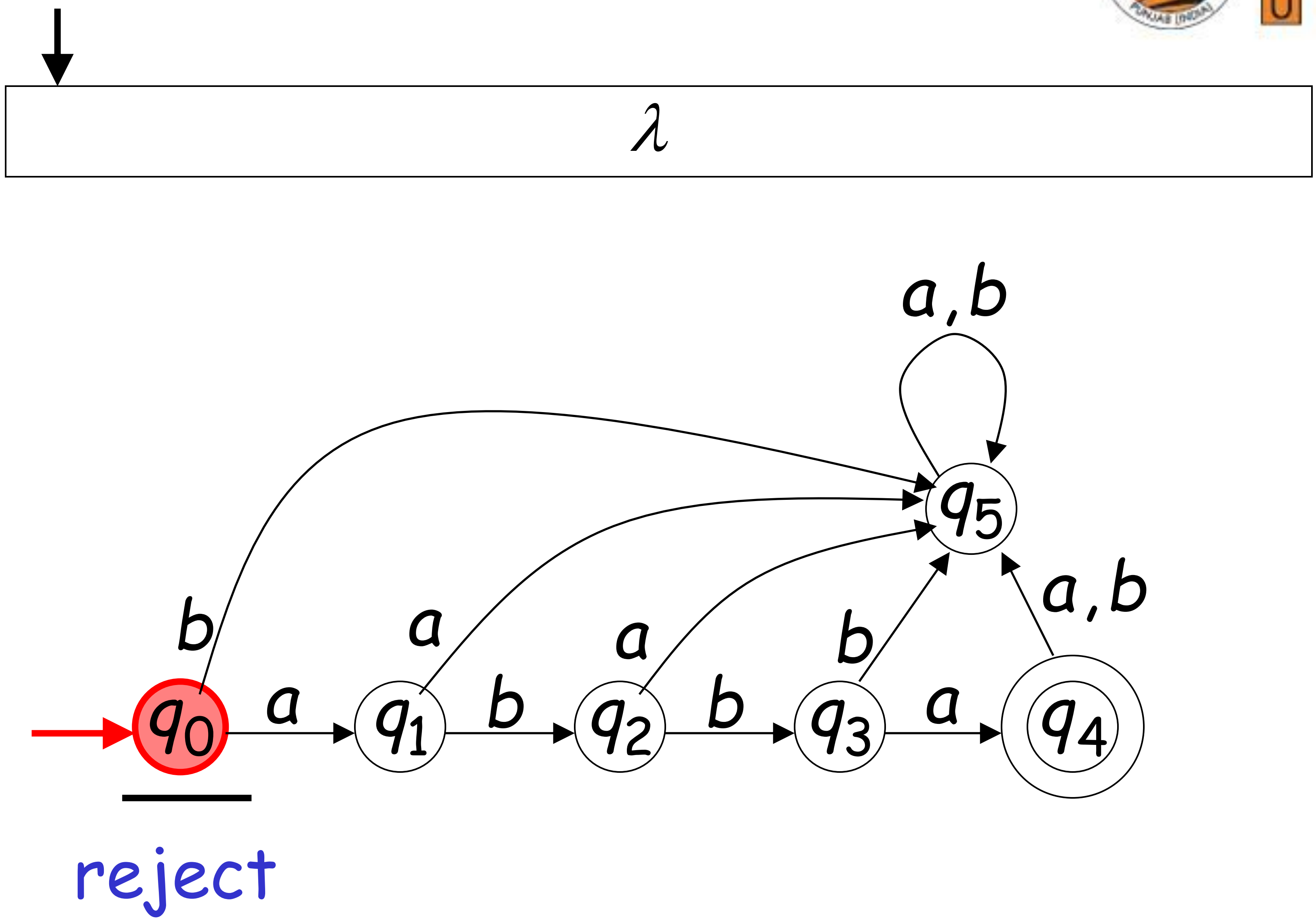
Input finished



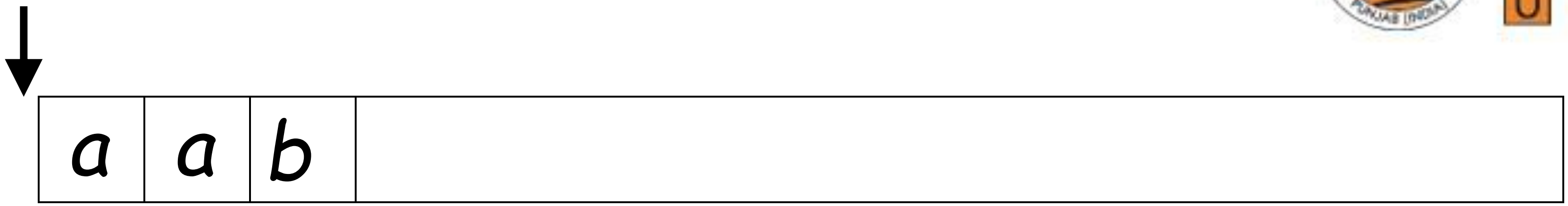
Another Rejection

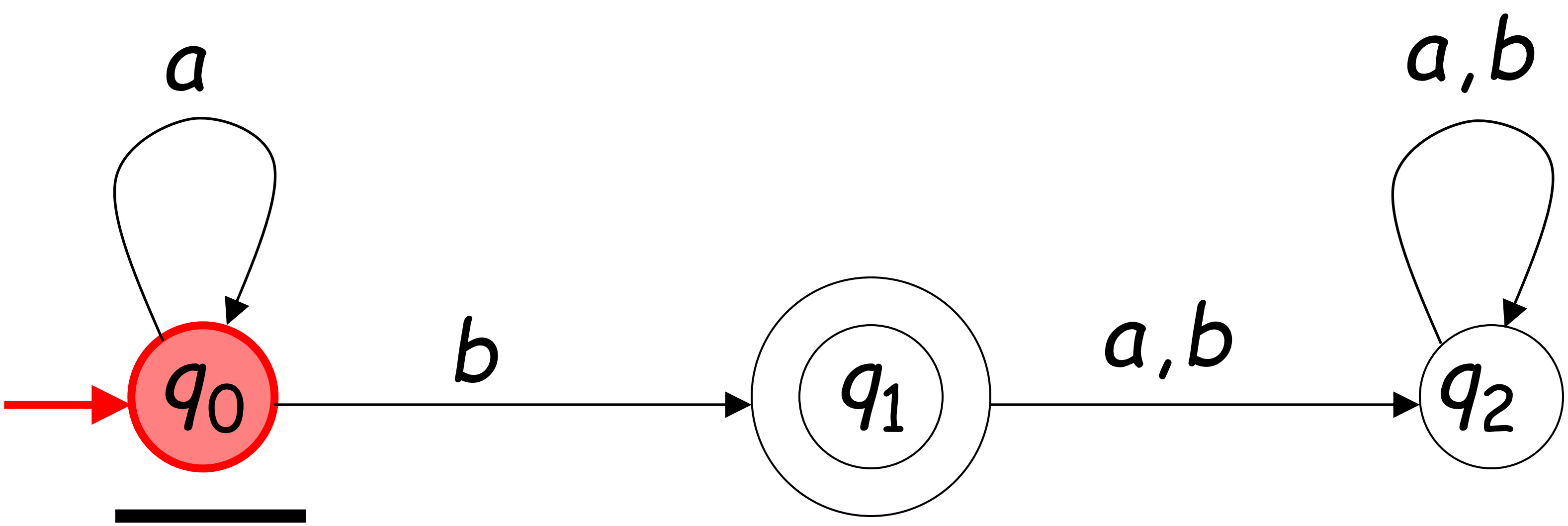
λ

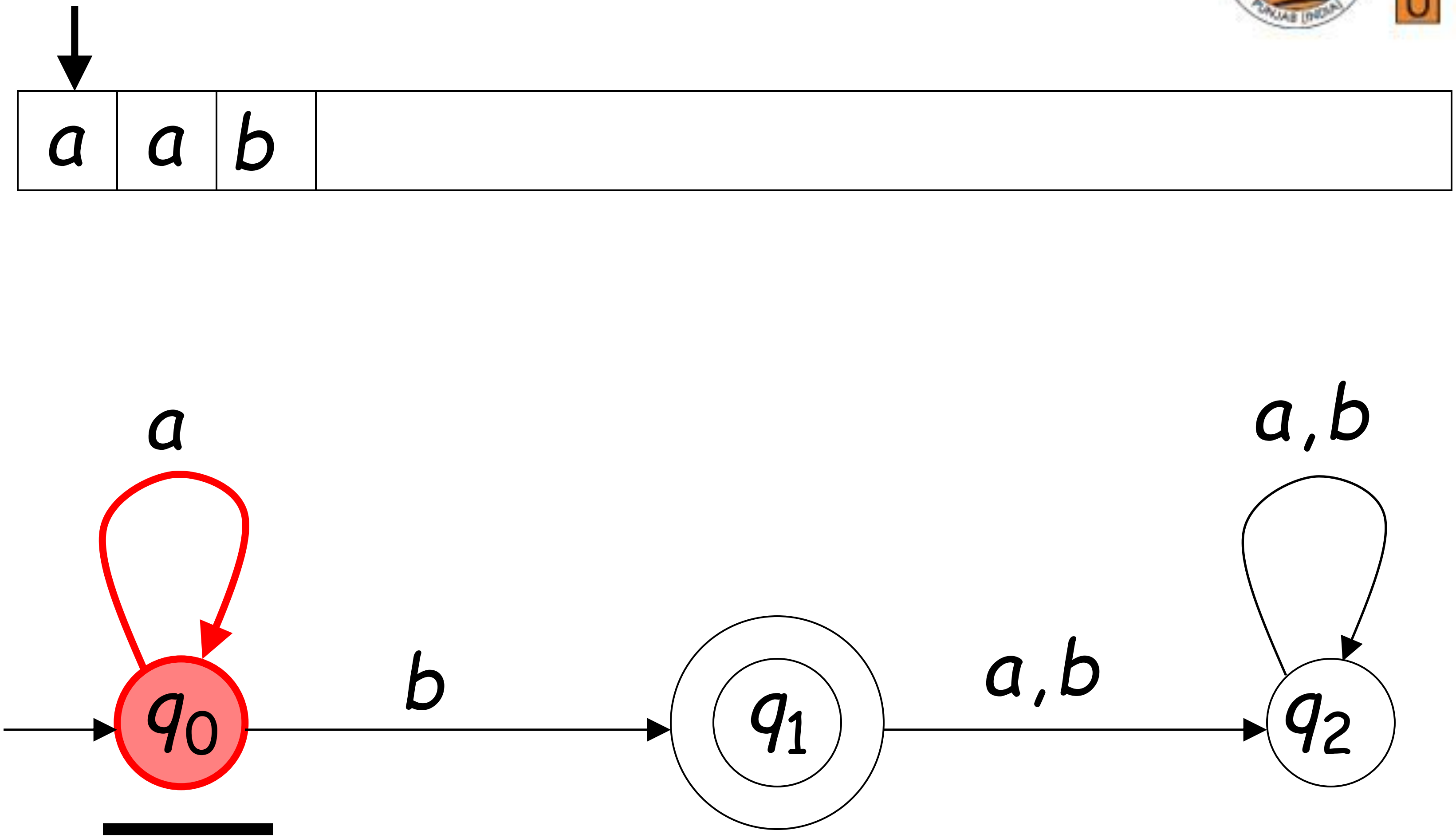


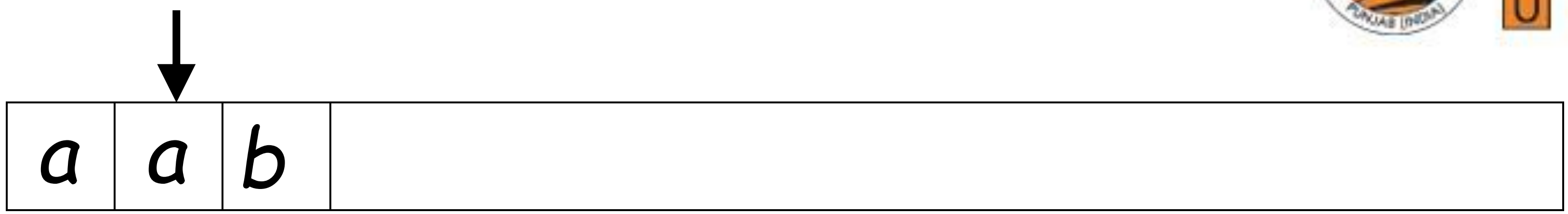

λ
a, b
b
a
a
b
a, b
q0
a
q1
b
q2
b
q3
a
q4
q5
reject

Another Example

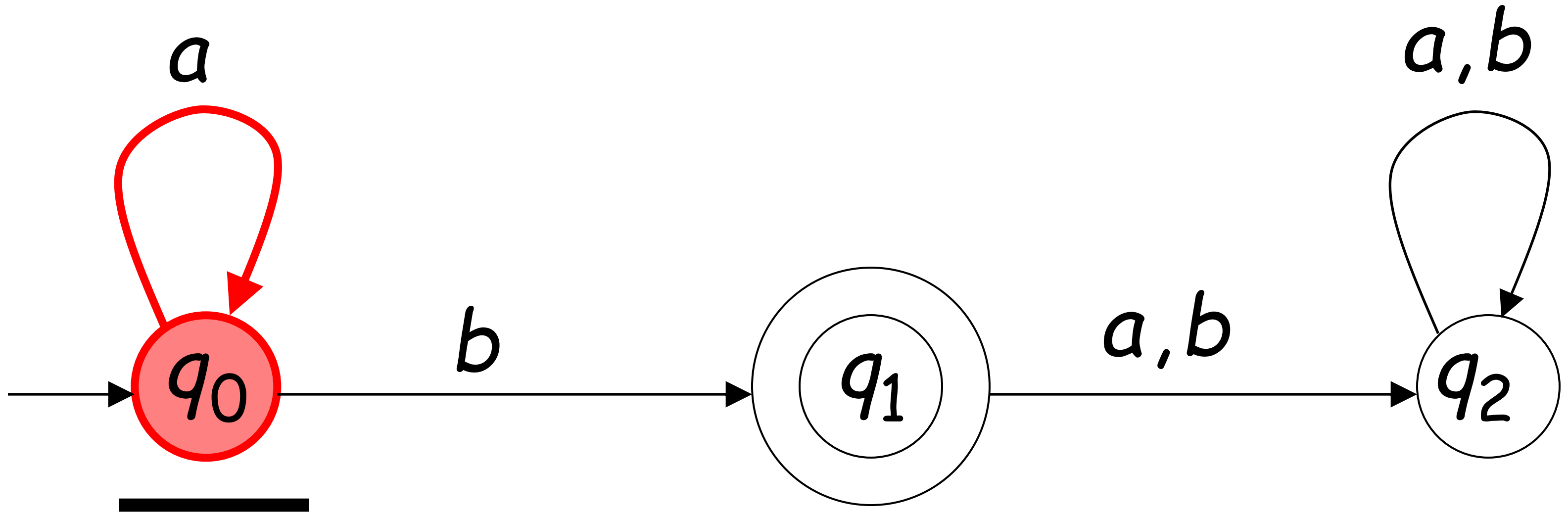




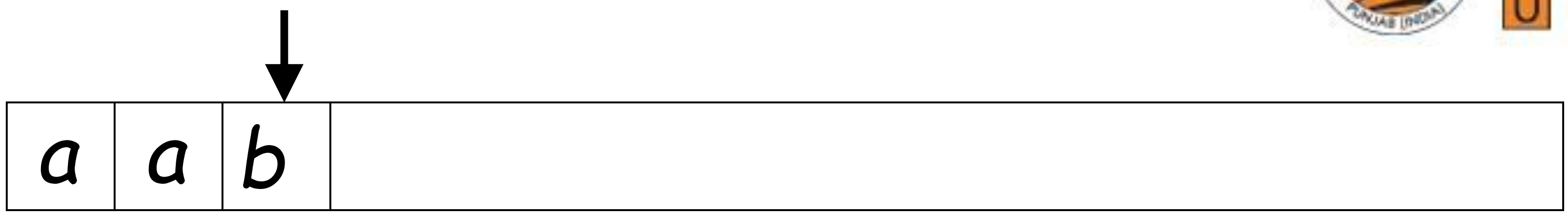

a a b
a
b
a,b
a,b
q0
q1
q2

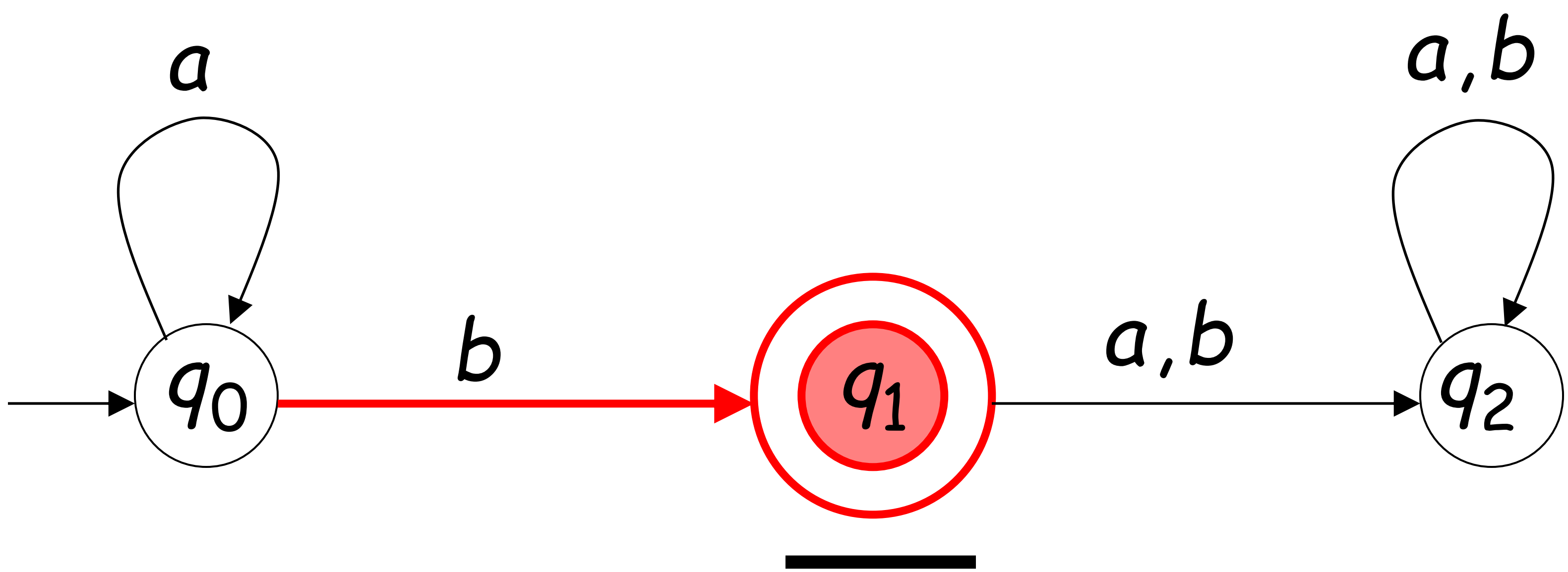


a
a
b

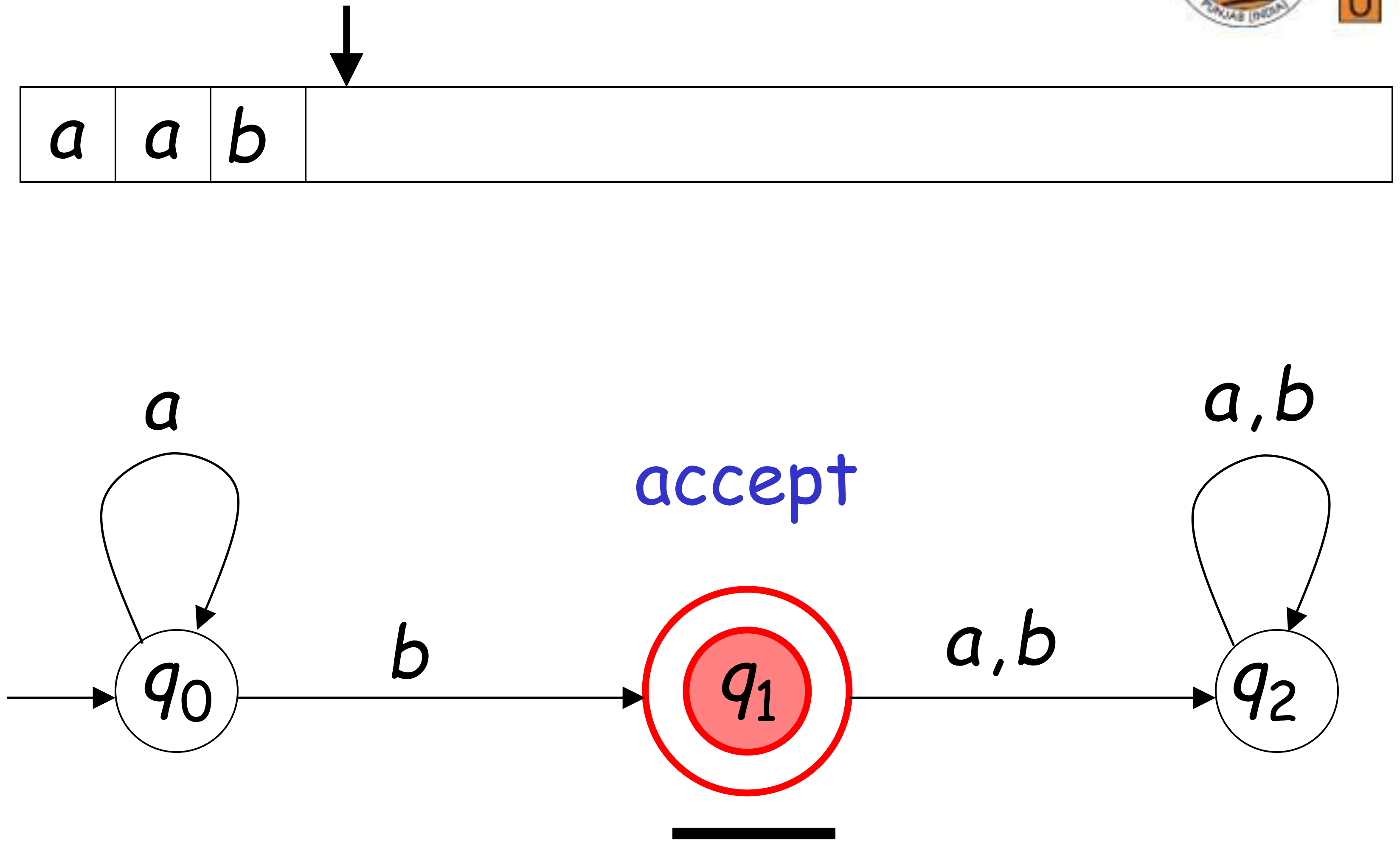


a
a,b
b
a,b
q_0
q_1
q_2

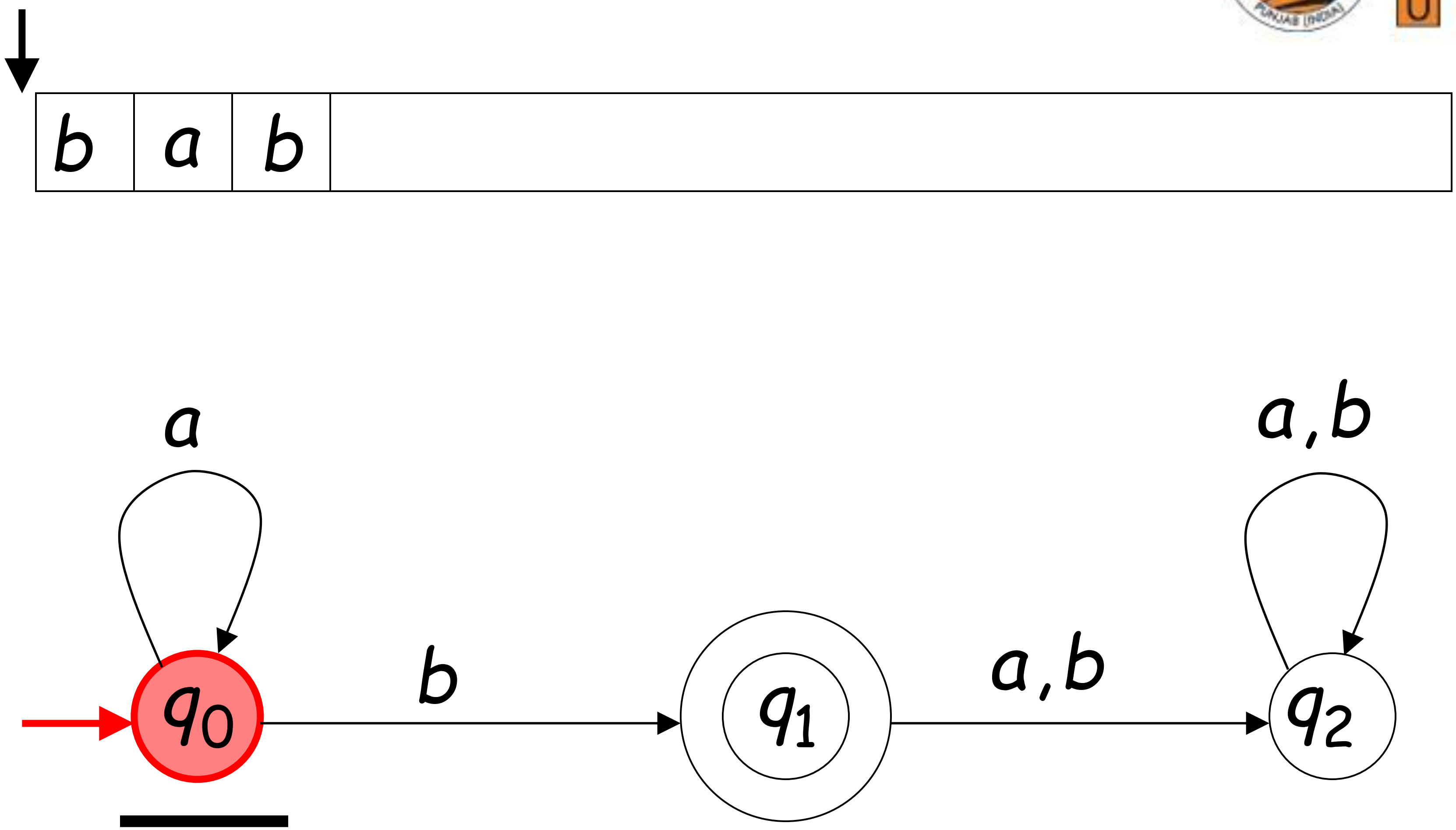

a
a
b

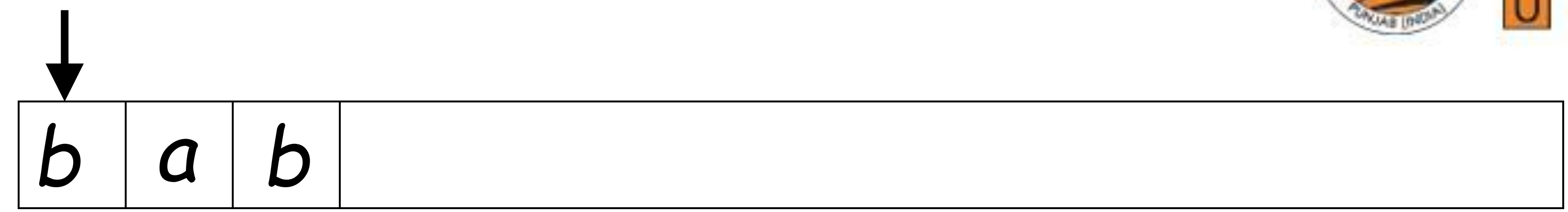

a
a,b
b
a,b
q0
q1
q2

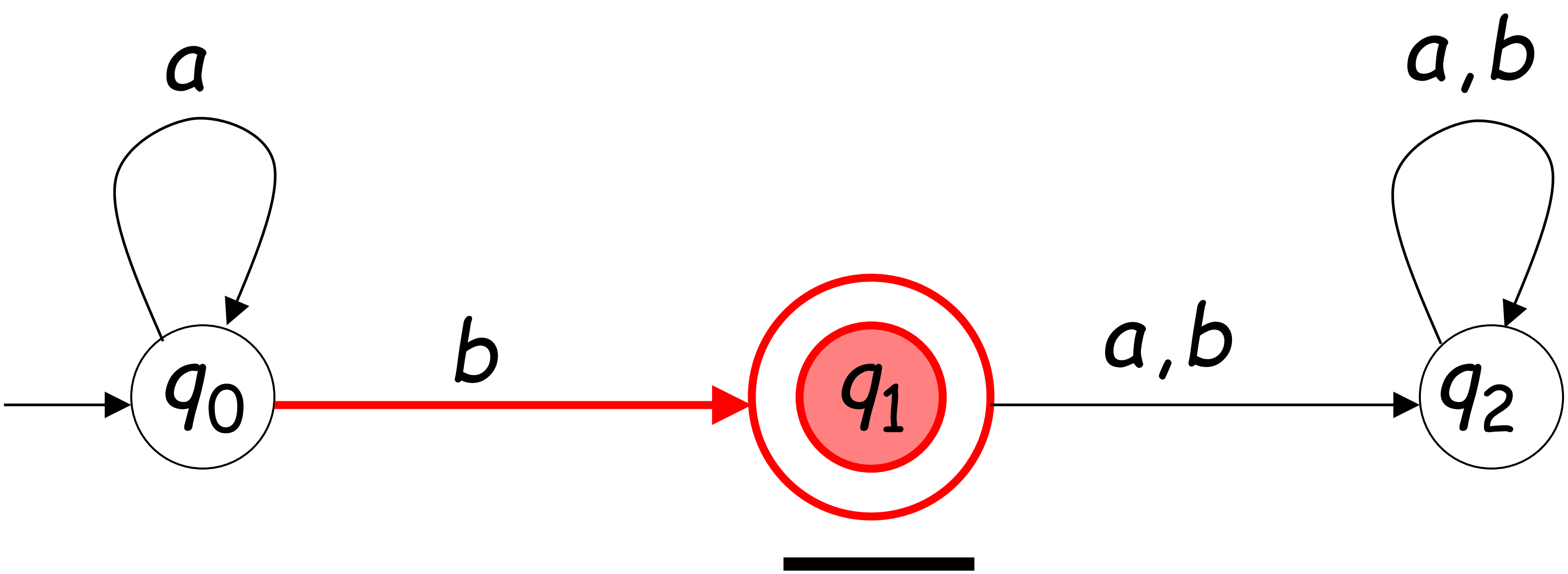
Input finished

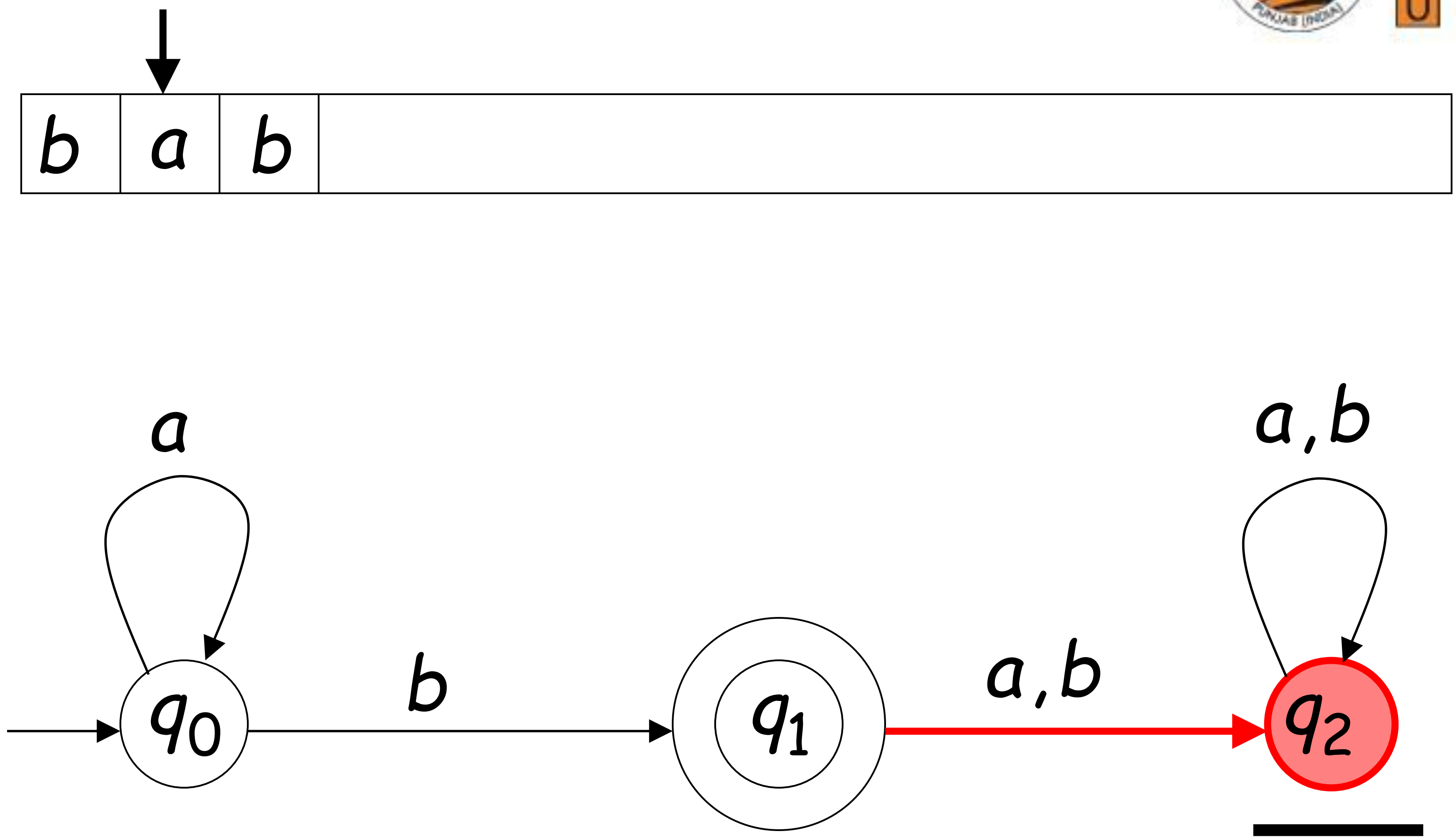


Rejection Example

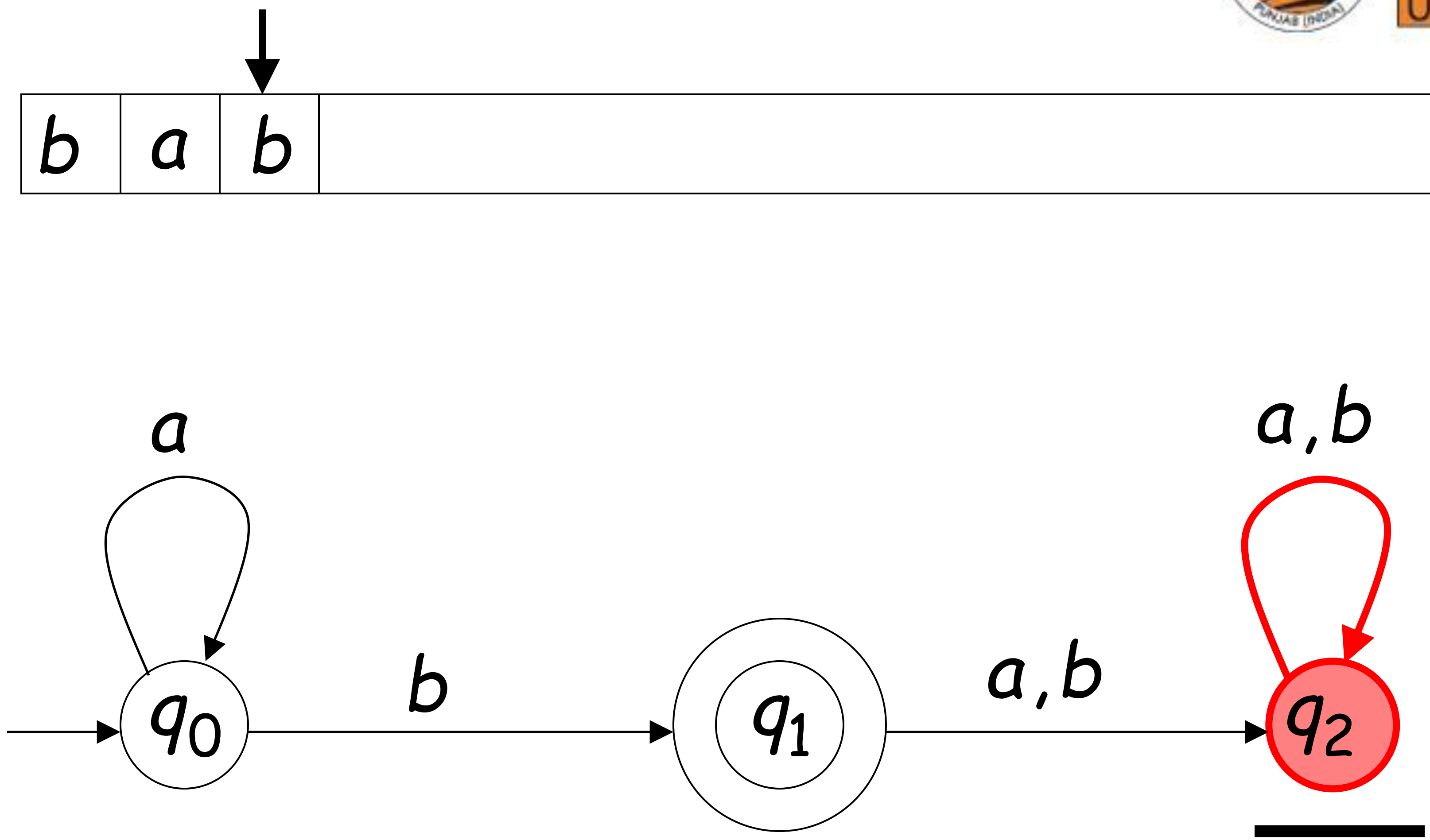



b
a
b

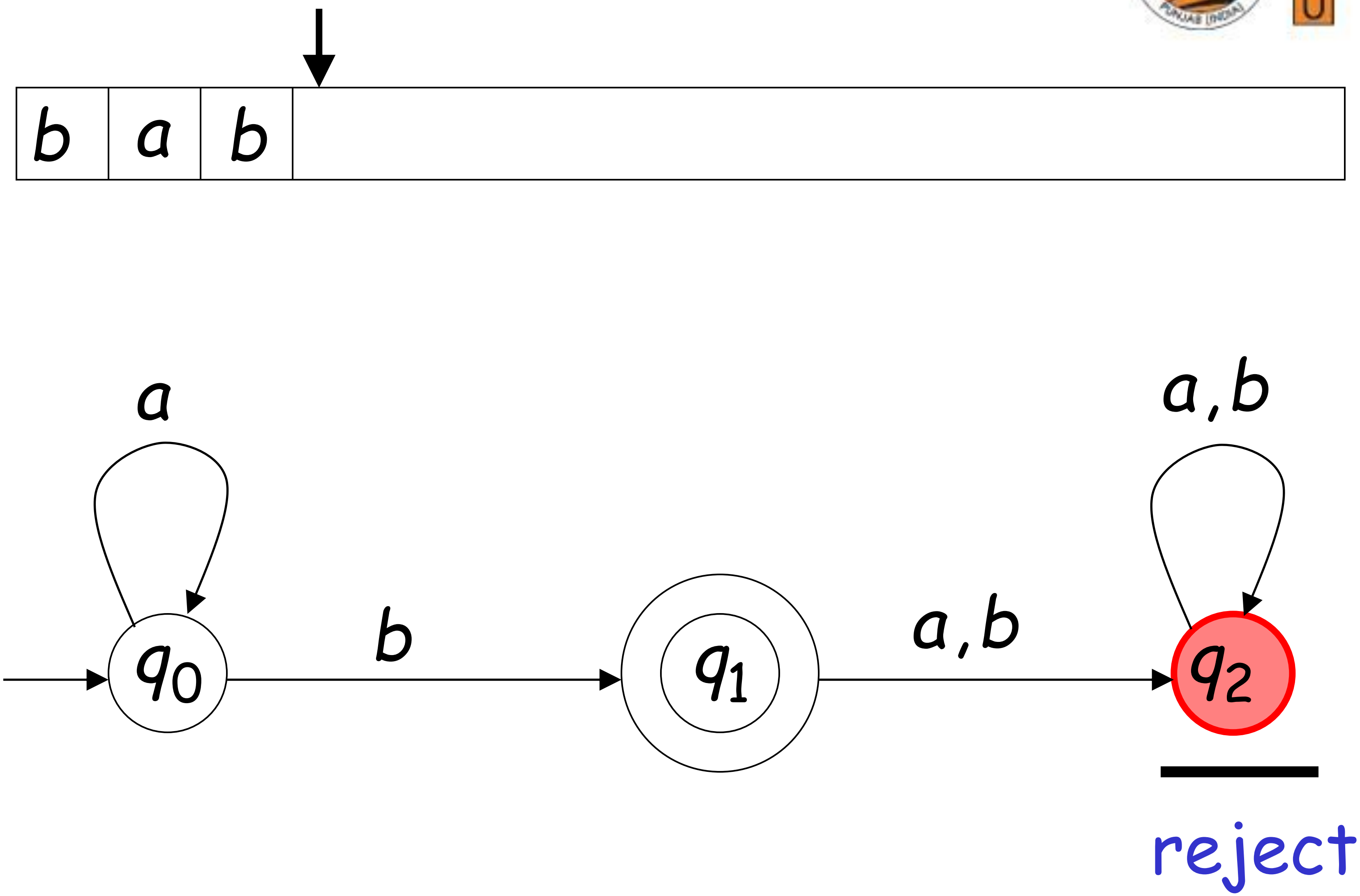

a
a,b
b
a,b
q_0
q_1
q_2



b
a
b
a
a,b
b
a,b
q0
q1
q2


b
a
b
a
a,b
b
a,b
q_0
q_1
q_2

Input finished



Languages Accepted by FAs

FA M

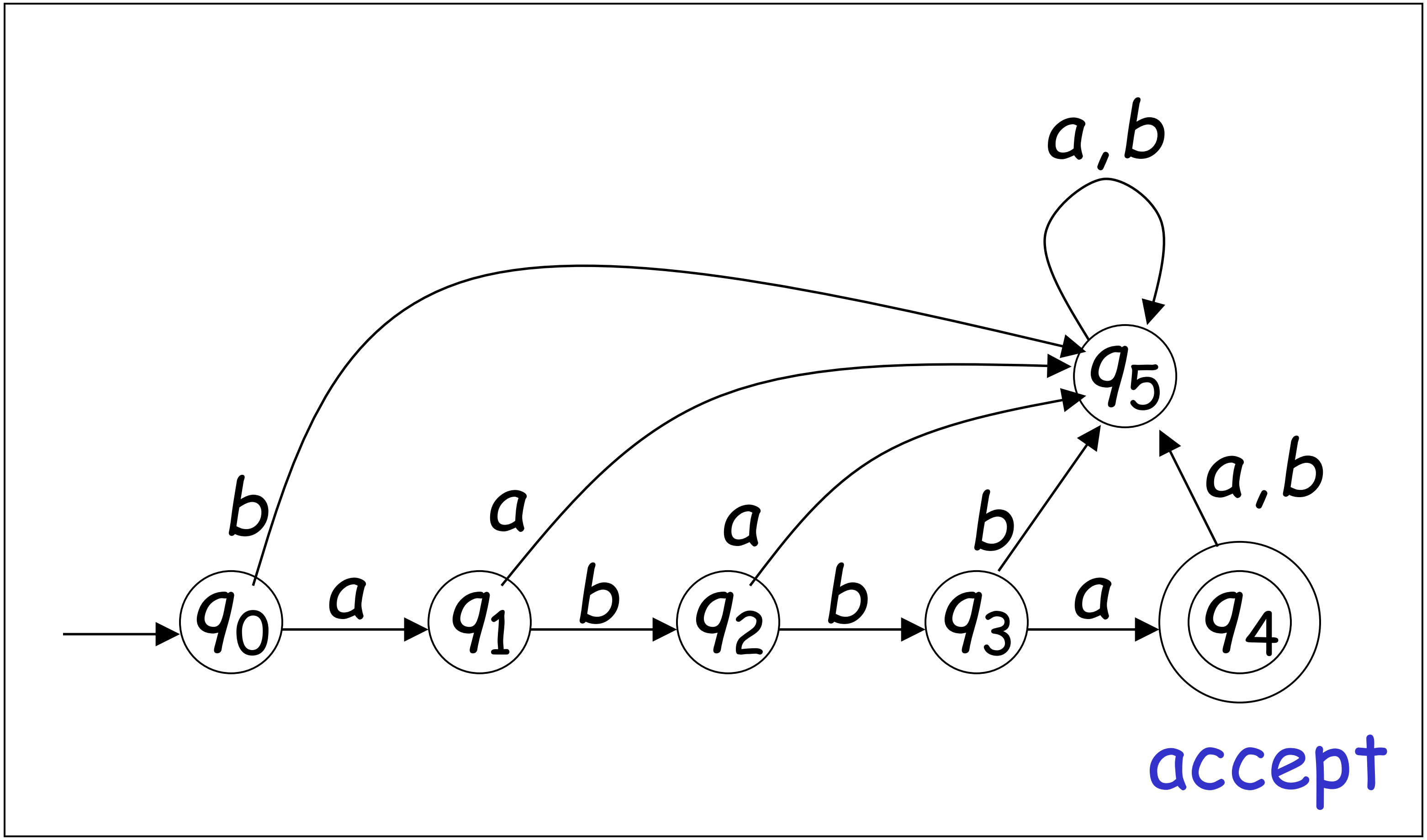
Definition:

The language $L(M)$ contains all input strings accepted by M

$L(M)$ = { strings that bring M to an accepting state}

$$L(M) = \{abba\}$$

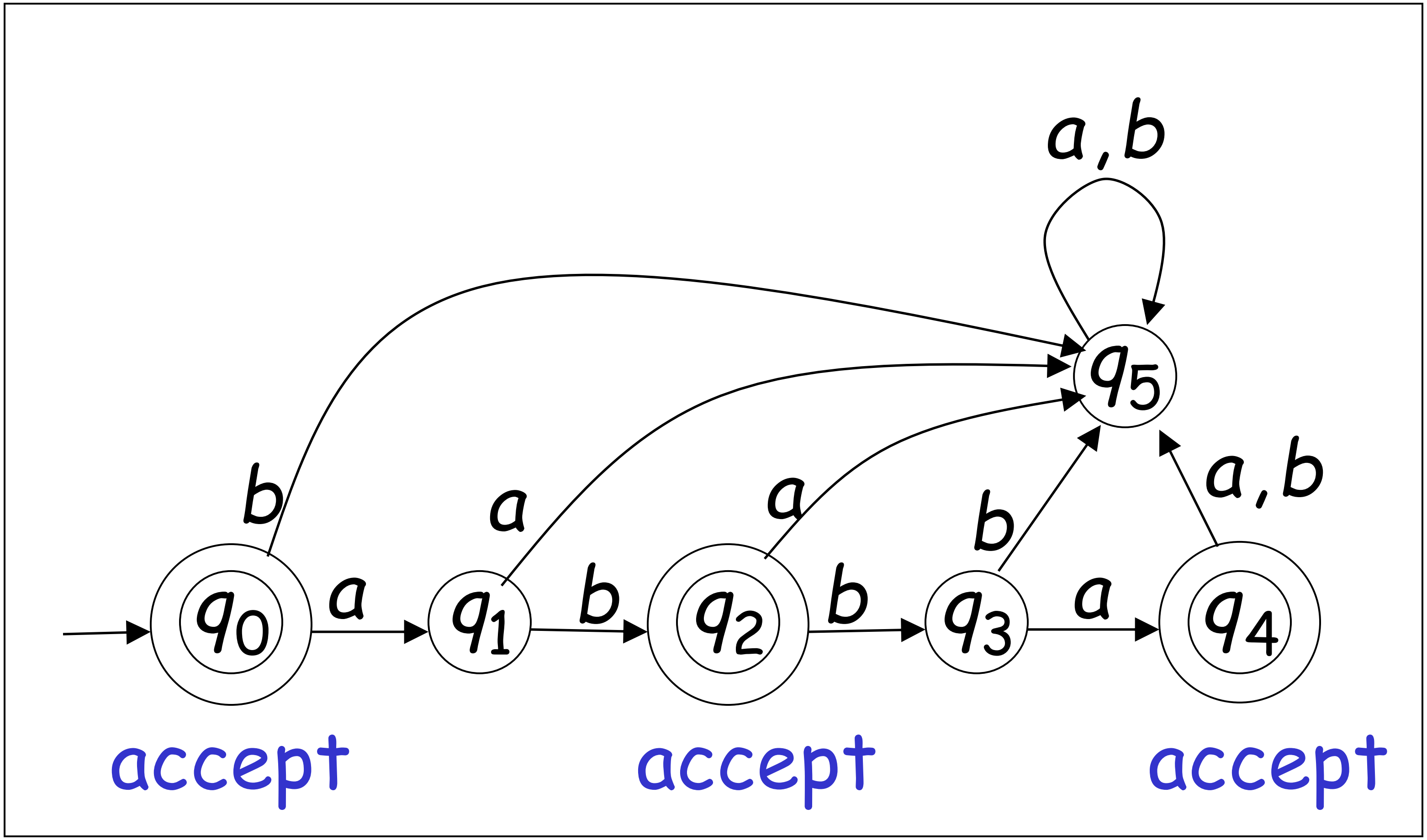
M



Example

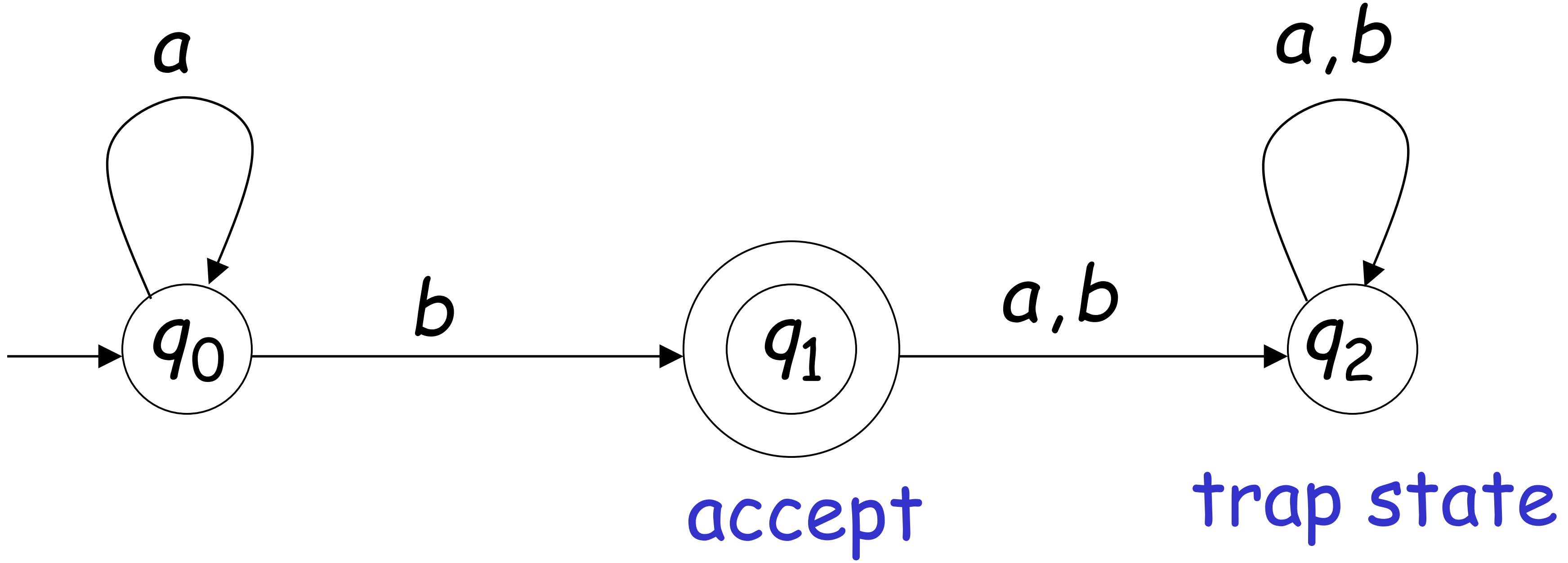
$$L(M) = \{\lambda, ab, abba\}$$

M



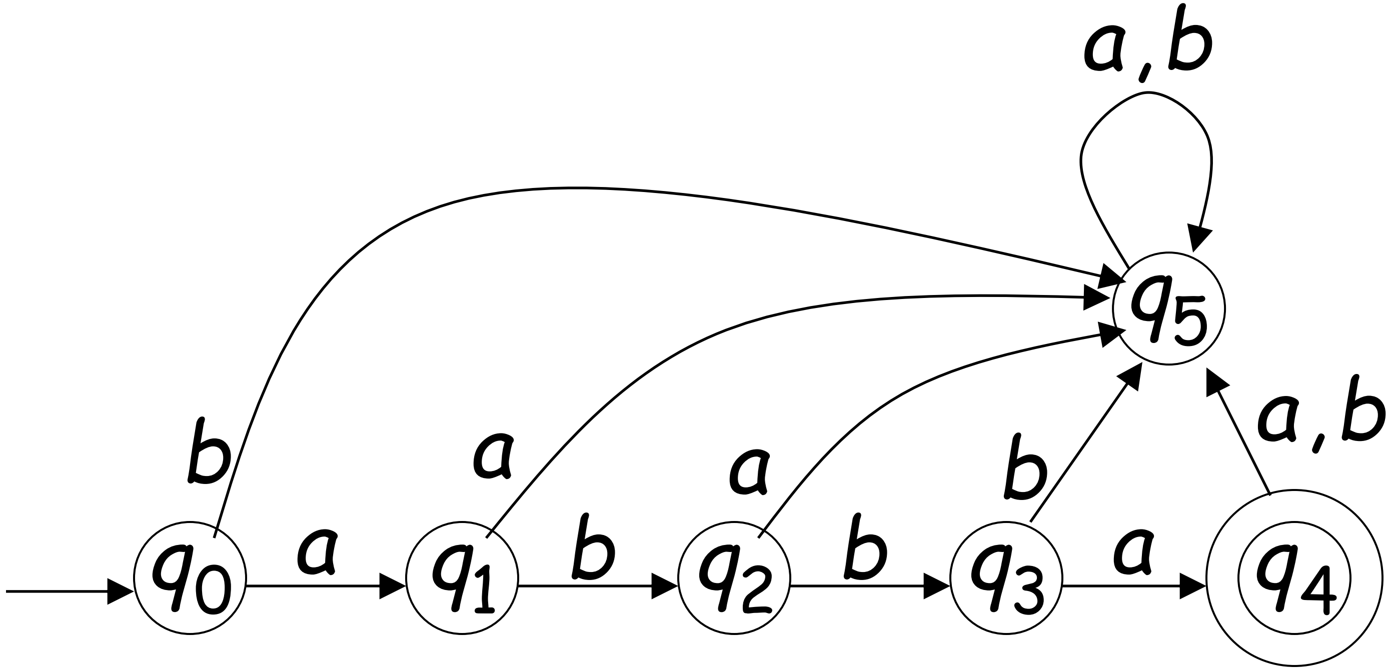
Example

$$L(M) = \{a^n b : n \geq 0\}$$

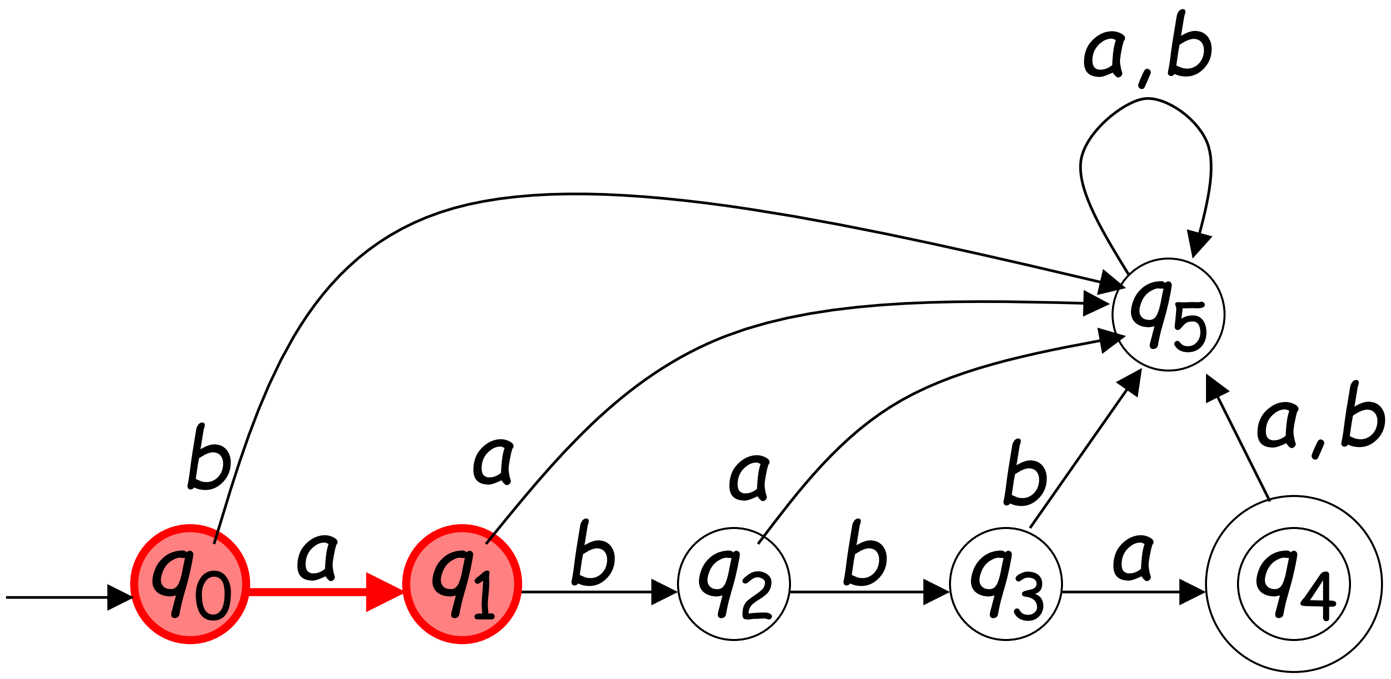


Transition Function δ

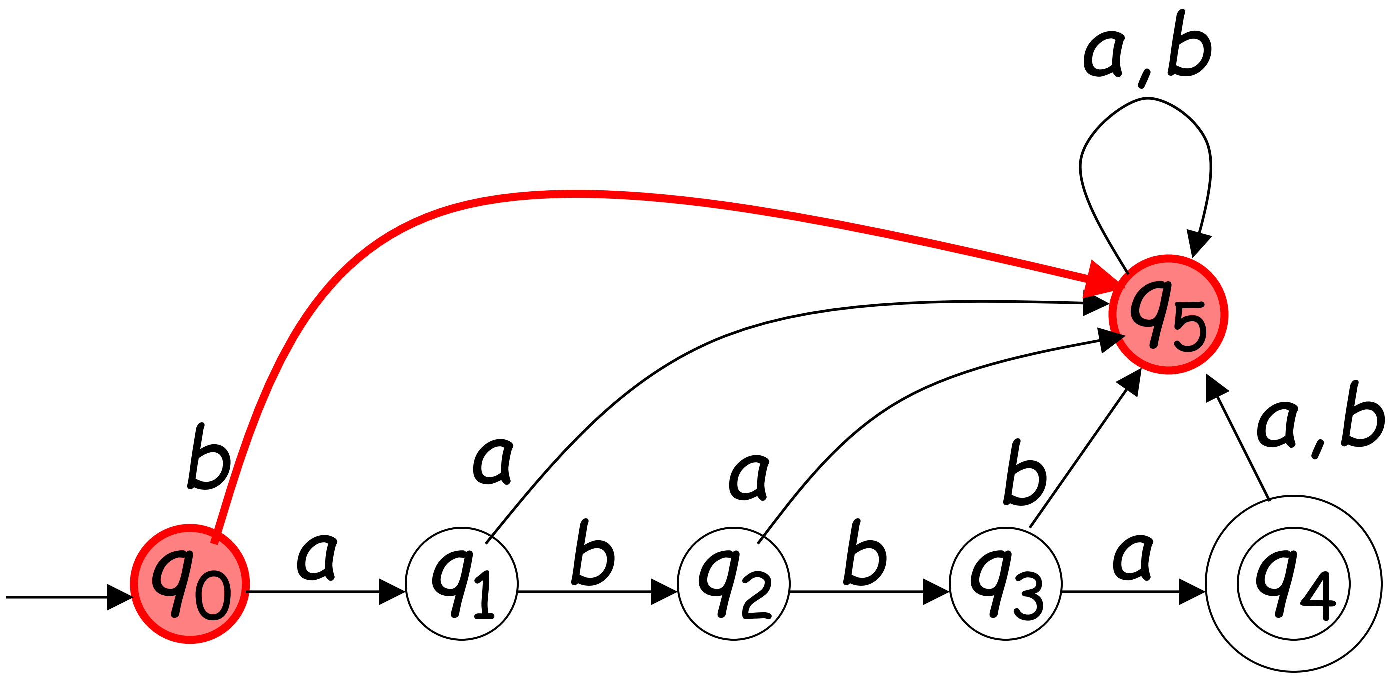
$$\delta : Q \times \Sigma \rightarrow Q$$



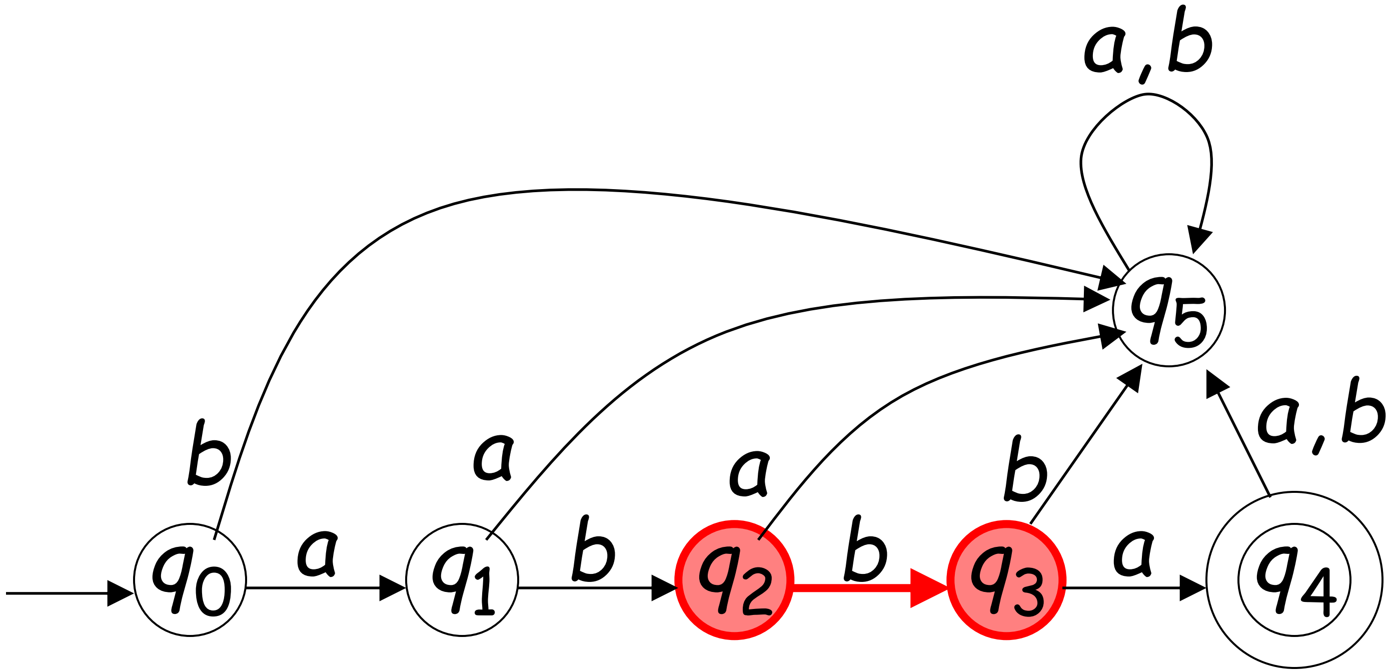
$$\delta(q_0, a) = q_1$$



$$\delta(q_0, b) = q_5$$

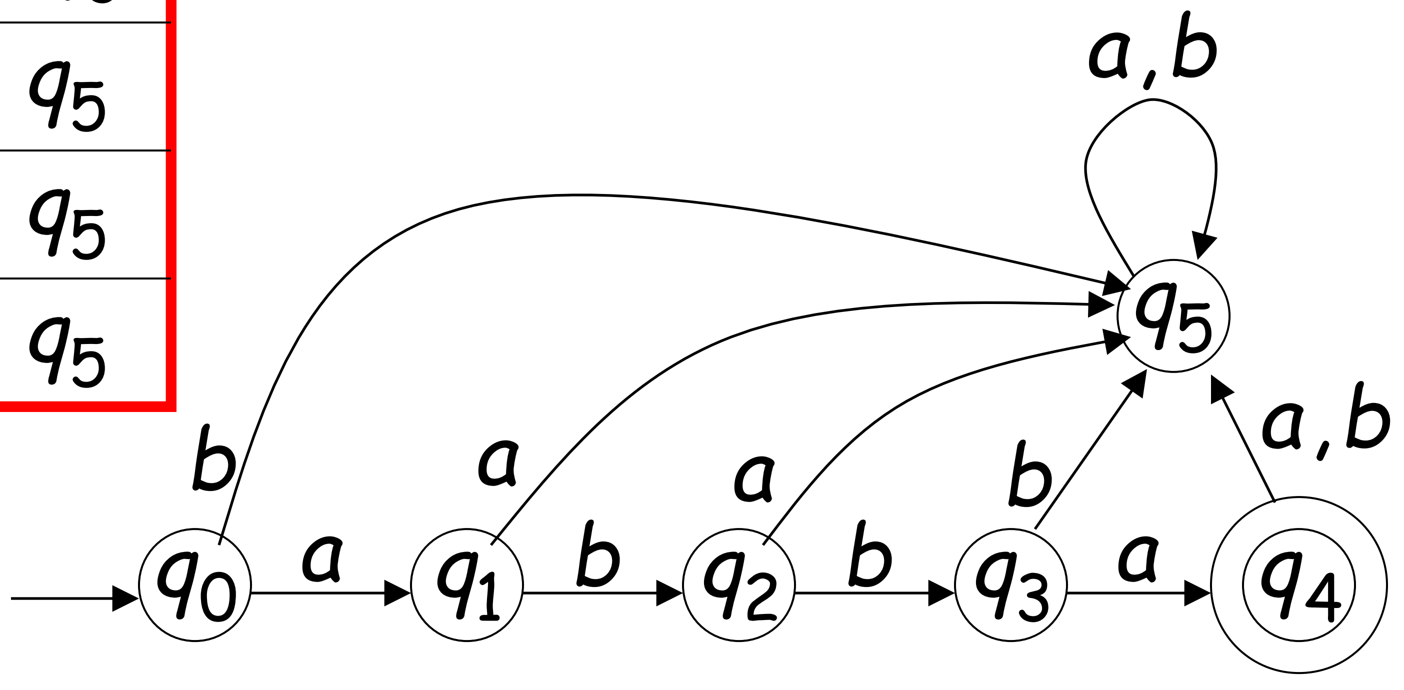


$$\delta(q_2, b) = q_3$$



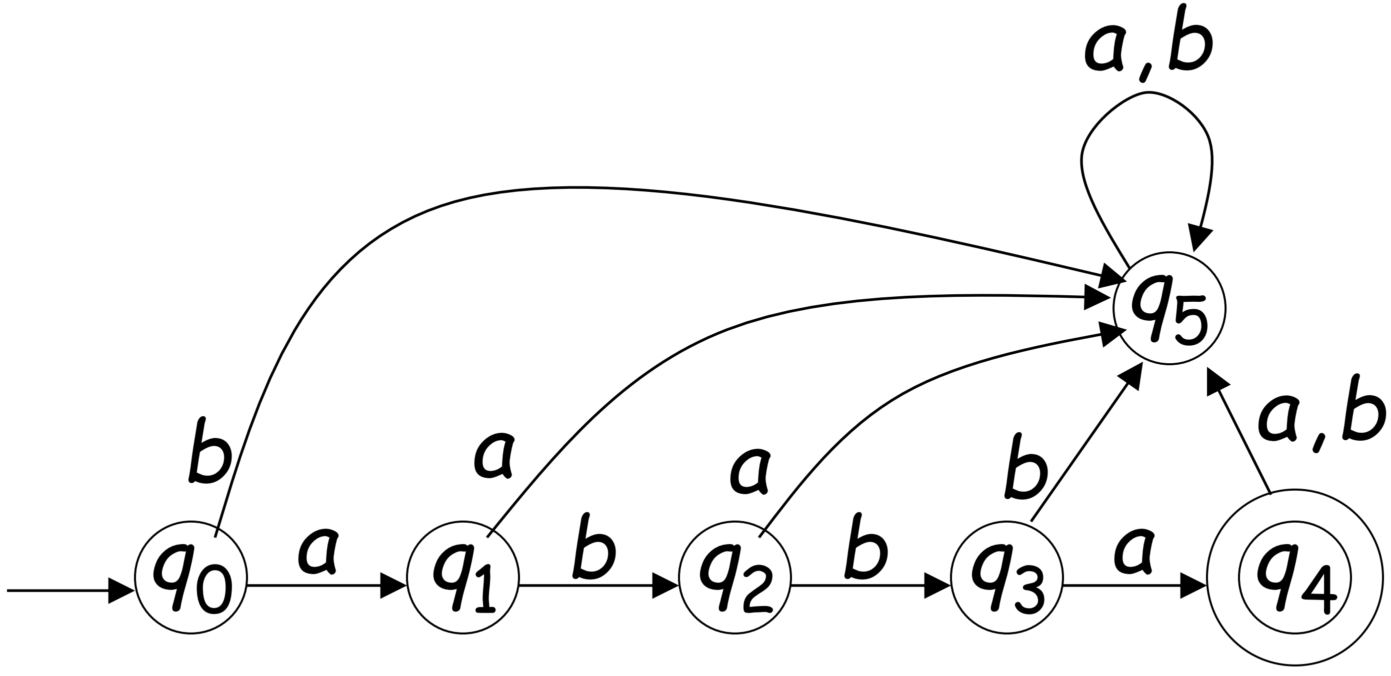
Transition Function δ

δ	a	b
q_0	q_1	q_5
q_1	q_5	q_2
q_2	q_5	q_3
q_3	q_4	q_5
q_4	q_5	q_5
q_5	q_5	q_5

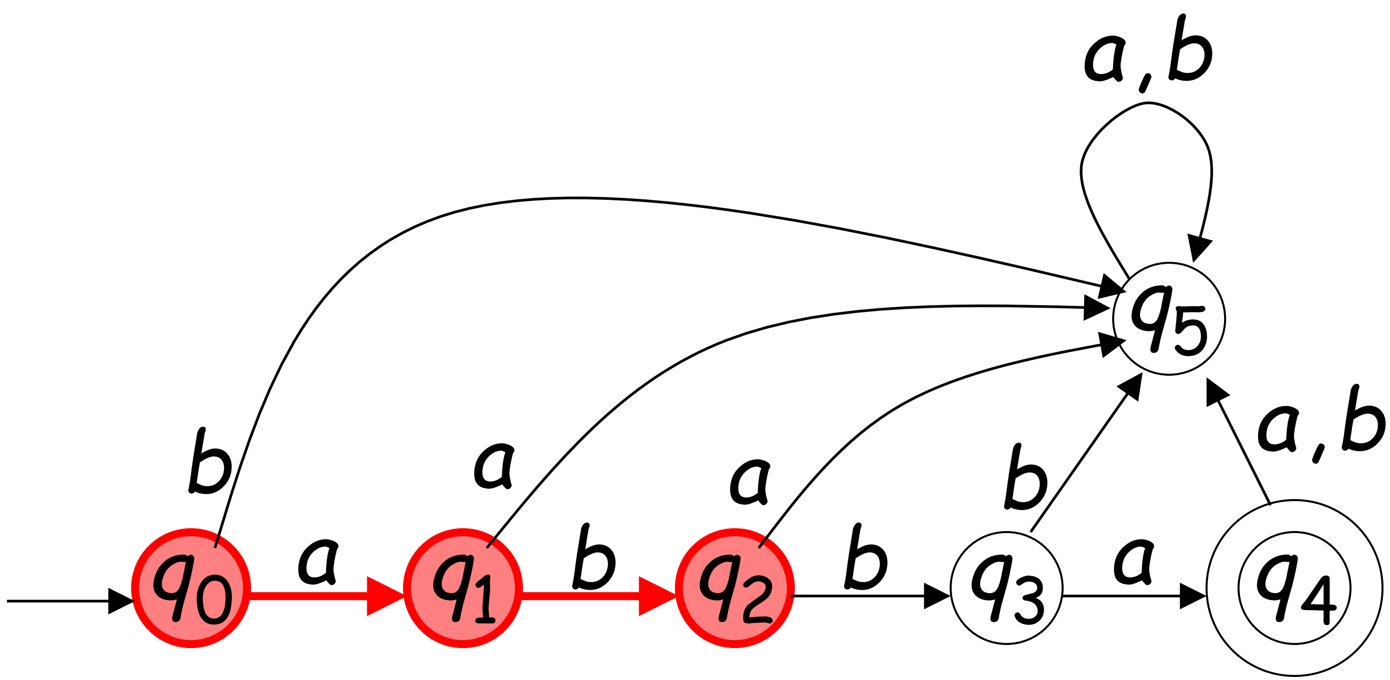


Extended Transition Function δ^*

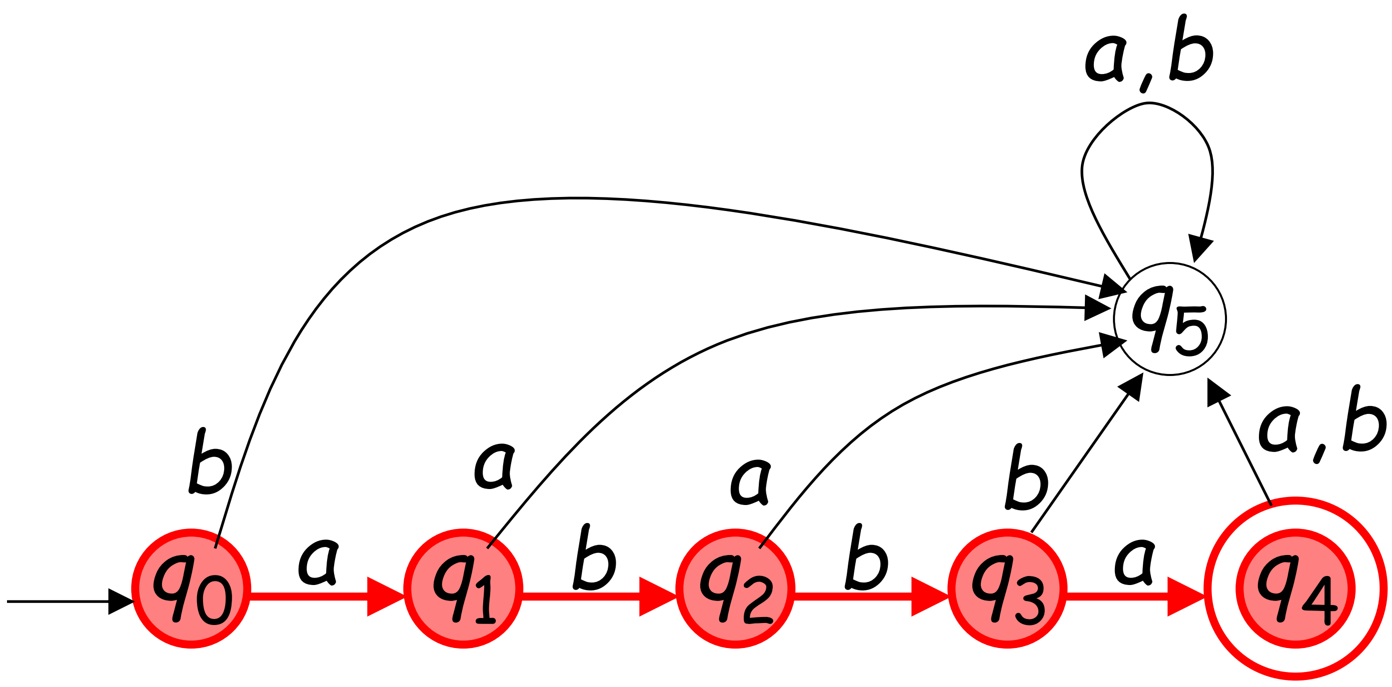
$$\delta^*: Q \times \Sigma^* \to Q$$



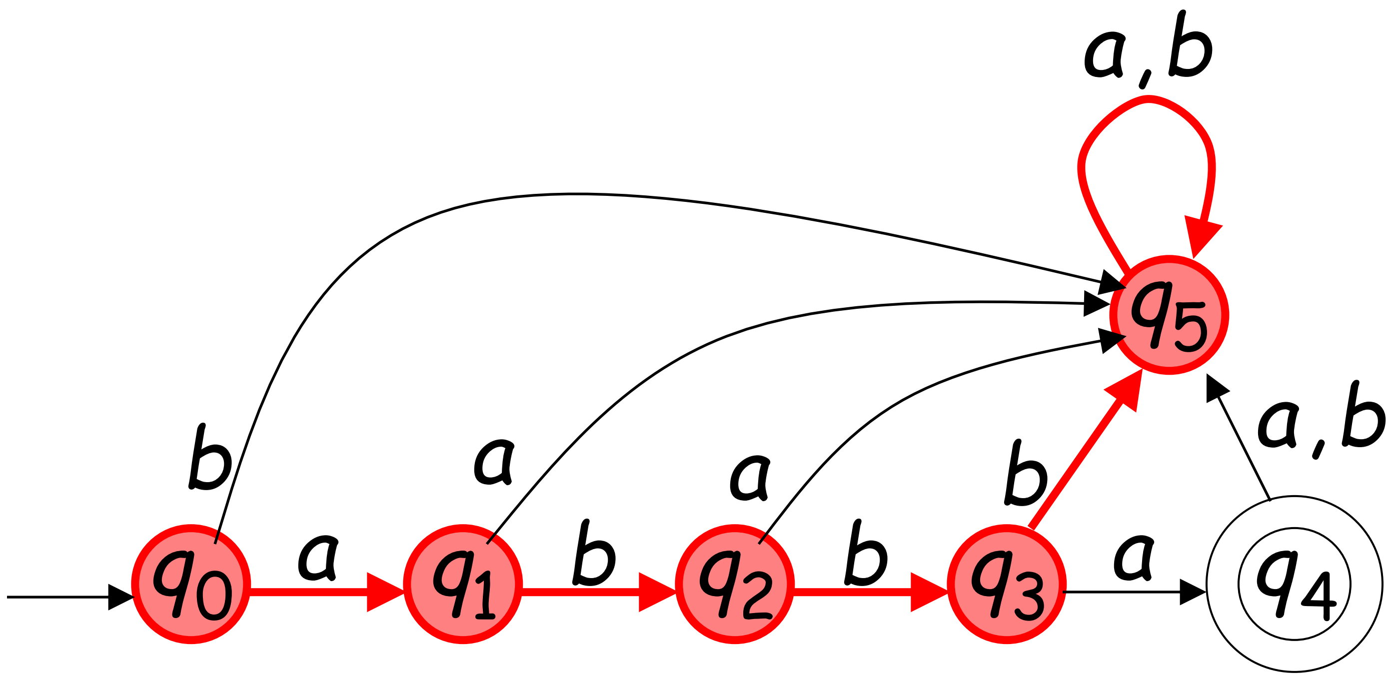
$$\delta^*(q_0, ab) = q_2$$



$$\delta^*(q_0, abba) = q_4$$



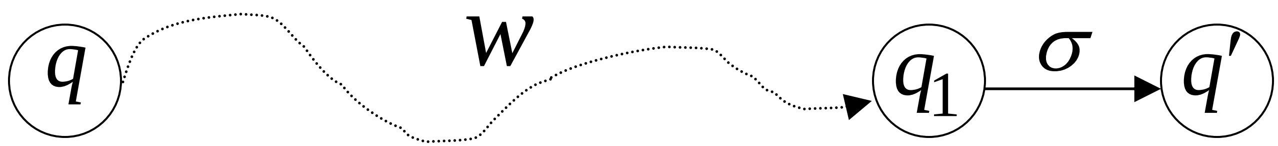
$$\delta^*(q_0, abbbaa) = q_5$$



Recursive Definition

$$\delta^*(q,\lambda)=q$$

$$\delta^*(q,w\sigma)=\delta(\delta^*(q,w),\sigma)$$



$$\left.\begin{array}{l}\delta^*(q,w\sigma)=q' \\ \\ \delta(q_1,\sigma)=q'\end{array}\right\} \Rightarrow \delta^*(q,w\sigma)=\delta(q_1,\sigma)$$

$$\left.\begin{array}{l}\delta^*(q,w\sigma)=\delta(q_1,\sigma) \\ \\ \delta^*(q,w)=q_1\end{array}\right\} \Rightarrow \delta^*(q,w\sigma)=\delta(\delta^*(q,w),\sigma)$$

Consider the finite state machine whose transition function δ is given by Table 3.1 in the form of a transition table. Here, $Q = \{q_0, q_1, q_2, q_3\}$, $\Sigma = \{0, 1\}$, $F = \{q_0\}$. Give the entire sequence of states for the input string 110001.

TABLE 3.1 Transition Function Table for Example 3.5

State	*Input*	
	0	1
$\rightarrow q_0$	q_2	q_1
q_1	q_3	q_0
q_2	q_0	q_3
q_3	q_1	q_2

Solution

$$\delta(q_0, \overset{\downarrow}{1}10101) = \delta(q_1, \overset{\downarrow}{1}0101)$$

$$= \delta(q_0, \overset{\downarrow}{0}101)$$

$$= \delta(q_2, \overset{\downarrow}{1}01)$$

$$= \delta(q_3, \overset{\downarrow}{0}1)$$

$$= \delta(q_1, \overset{\downarrow}{1})$$

$$= \delta(q_0, \Lambda)$$

$$= q_0$$

Hence,

$$q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_0 \xrightarrow{0} q_2 \xrightarrow{1} q_3 \xrightarrow{0} q_1 \xrightarrow{1} q_0$$

The symbol ↓ indicates that the current input symbol is being processed by the machine.