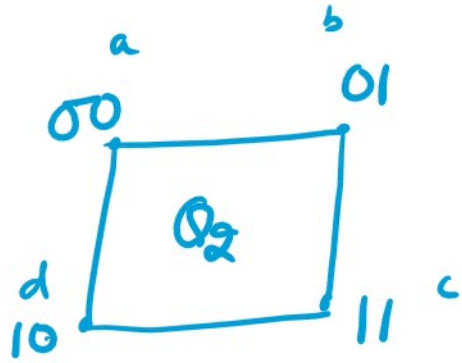
Lecture 27

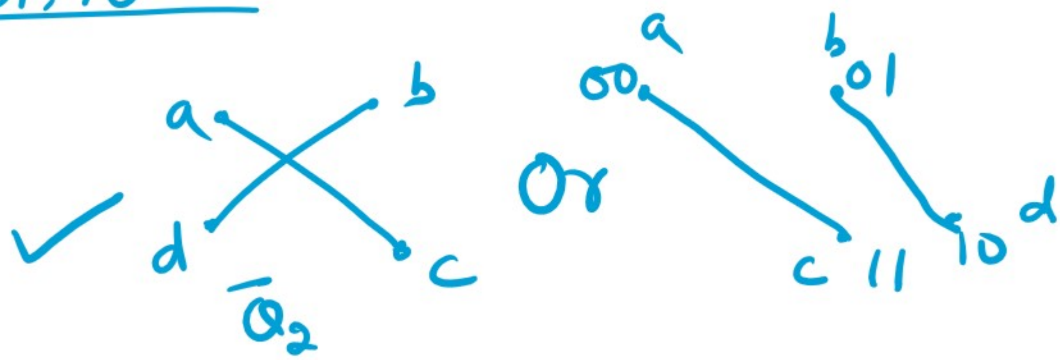
26 October 2021 08:58

Q_2 bit strings of length 2

00, 01, 10, 11

00	01, 10
01	00, 11
10	00, 11
11	01, 10



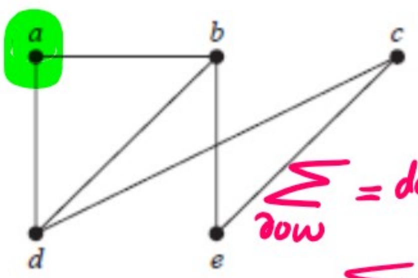


- **Adjacency Matrix**

For Simple Graph

$$a_{ij} = \begin{cases} 1, & \text{if } \{v_i, v_j\} \text{ is an edge of } G \\ 0, & \text{otherwise} \end{cases}$$

For undirected graph, Symmetric matrix



$\sum_{row} = deg(v)$

$\sum_{column} = degree$

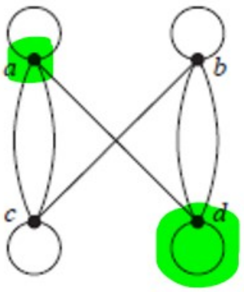
$$\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array} \begin{array}{c} \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

For Pseudograph

$$a_{ij} = \begin{cases} m_{ij}, & \text{if } \{v_i, v_j\} \text{ is an edge with multiplicity } m_{ij} \\ 0, & \text{otherwise} \end{cases}$$

a b c d

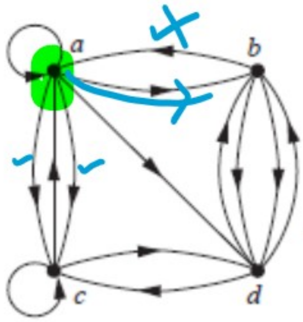
0, otherwise



$$\begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 0 & 2 & 1 \\ b & 0 & 1 & 1 & 2 \\ c & 2 & 1 & 1 & 0 \\ d & 1 & 2 & 0 & 1 \end{array}$$

$\sum_{row}$ sum = degree − no. of loops

$\sum_{colum}$ sum = degree − no. of loops



2.

Terminal

$$\begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 1 & 2 & 1 \\ b & 1 & 0 & 0 & 2 \\ c & 1 & 0 & 1 & 1 \\ d & 0 & 2 & 1 & 0 \end{array}$$

What is the sum of the entries in a row of the adjacency matrix for an undirected graph? For a directed graph?

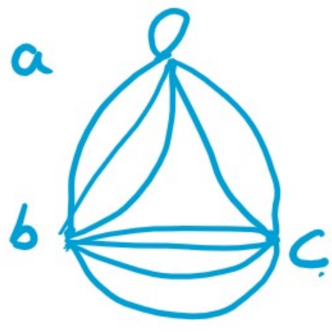
degree − no. of loops $\qquad$ $\deg^+(v)$

What is the sum of the entries in a column of the adjacency matrix for an undirected graph? For a directed graph?

degree − no. of loops $\qquad$ $\deg^-(v)$

Q17. Draw undirected graph using the adjacency matrix.

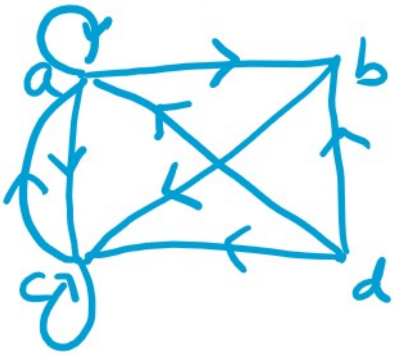
$$\begin{array}{c|ccc} & & b & \\ \hline a & 1 & 3 & 2 \\ & 3 & 0 & 4 \\ & 2 & 4 & 0 \end{array}$$



Q18. Draw directed graph using the adjacency matrix.

Q18. Draw directed graph using the adjacency matrix.

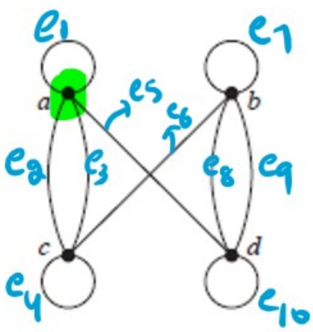
$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$



- **Incidence Matrix**

Let $G = (V, E)$ be an undirected graph. Suppose that $v_1, v_2, \ldots, v_n$ are the vertices and $e_1, e_2, \ldots, e_m$ be the edges of G. Then the incidence matrix w.r.t to this ordering of V and E is the $n \times m$ matrix $M = [m_{ij}]$, where

$$m_{ij} = \begin{cases} 1 & \text{when edge } e_j \text{ is incident with } v_i, \\ 0 & \text{otherwise.} \end{cases}$$



$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 & e_9 & e_{10} \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

What is the sum of the entries in a row of the incidence matrix for an undirected graph? = deg − no. of loops

What is the sum of the entries in a column of the incidence matrix for an undirected graph?

$$= \begin{cases} 1 & \text{if it loop} \\ 2 & \text{not loop} \end{cases}$$

Isomorphism of Graph

The simple graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are *isomorphic* if there exists a one-to-one and onto function f from V_1 to V_2 with the property that a and b are adjacent in G_1 if and only if $f(a)$ and $f(b)$ are adjacent in G_2, for all a and b in V_1. Such a function f is called

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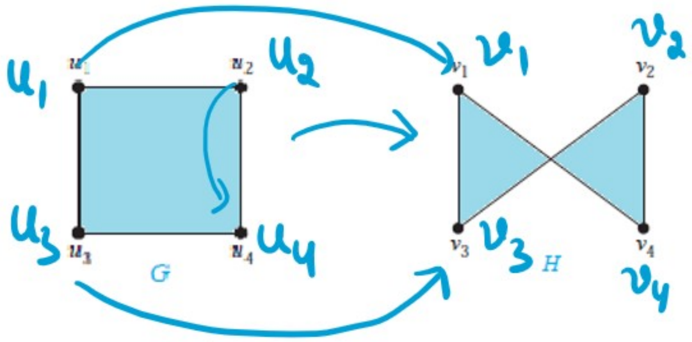


$f(u_1) = v_1$

$f(u_3) = v_3$

$f(u_2) = v_4$

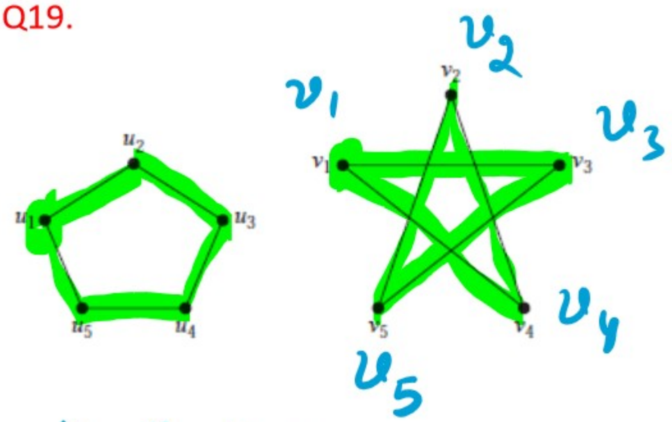
$f(u_4) = v_2$

$$\begin{array}{c|cccc} & u_1 & u_2 & u_3 & u_4 \\ \hline u_1 & 0 & 1 & 1 & 0 \\ u_2 & 1 & 0 & 0 & 1 \\ u_3 & 1 & 0 & 0 & 1 \\ u_4 & 0 & 1 & 1 & 0 \end{array}$$

$$\begin{array}{c|cccc} & v_1 & v_4 & v_3 & v_2 \\ \hline v_1 & 0 & 1 & 1 & 0 \\ v_4 & 1 & 0 & 0 & 1 \\ v_3 & 1 & 0 & 0 & 1 \\ v_2 & 0 & 1 & 1 & 0 \end{array}$$

G and H are isomorphic.

Q19.



$f(u_1) = v_1$

$f(u_2) = v_3$

$f(u_3) = v_5$

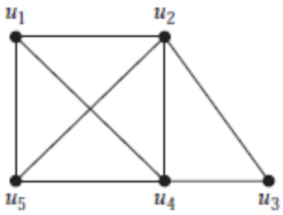
$f(u_4) = v_2$

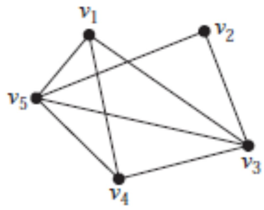
$f(u_5) = v_4$

$$\begin{array}{c|ccccc} & u_1 & u_2 & u_3 & u_4 & u_5 \\ \hline u_1 & & & & & \\ u_2 & & & & & \\ u_3 & & & & & \\ u_4 & & & & & \\ u_5 & & & & & \end{array}$$

$$\begin{array}{c|ccccc} & v_1 & v_2 & v_3 & u_4 & u_5 \\ \hline v_1 & & & & & \\ v_3 & & & & & \\ u_5 & & & & & \\ v_2 & & & & & \\ v_4 & & & & & \end{array}$$

Q20.





Write no. of vertices, no. of edges, degree sequence and adjacency list for both