Tuesday, August 18, 2020 9:33 AM

Chapter 1 : Matrices and Eigenvalue Problems

Matrices : An $m \times n$ matirx is an arrangement of mn objects (not necessarily distinct) in m rows and n columns in the form

order of matrix

$A = (a_{ij})$

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$$

Real Matrix : Real

Complex Matrix : At least one of the element is complex

Types of Matrices

Row Vector: $\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \end{bmatrix}$ $1 \times n$ → order

Column Vector: $\begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix}$ $m \times 1$ → order

Rectangular Matrix: $m \neq n$ $m \neq n$ $m > n$ or $n > m$

Square Matrix: $m = n$ $m = n$ b, $l \neq b$ l

Null Matrix: zero $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}_{2\times 2}$ zero matrix

Diagonal Matrix: $m = n$

$a_{ij} = 0, \ i \neq j$ $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ 3×3

$a_{ij} = \alpha, \ i = j$ → scalar matrix

Identity Matrix:

$a_{ij} = 1, \ i = j$ & $a_{ij} = 0, \ i \neq j$

unit matrix

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Equal Matrix:

(1) order $A = (a_{ij})_{m \times n}$ & $B = (b_{ij})_{m \times n}$

(2) corresponding $a_{ij} = b_{ij}$ $A = \begin{bmatrix} a_{11} & a_{12} & \cdots \\ & & \end{bmatrix}_{m \times n}$ & $B = \begin{bmatrix} b_{11} & b_{12} & \cdots \\ & & \end{bmatrix}_{m \times n}$

Sub matrix:

A ~~$\{a_{11} \ a_{12} \ a_{13}\}$~~ $[a \ \ a \ ]$

Sub matrix:

$$A = \begin{bmatrix} \cancel{a_{11}} & \cancel{a_{12}} & \cancel{a_{13}} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad \checkmark \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix}_{2\times 2}$$

Matrix Algebra

The basic operations allowed on matrices are :
1) Multiplication of a Matrix by Scalar,
2) Addition/Subtraction of two matrices,
3) Multiplication of two matrices.

Note : There is no concept of dividing a matrix by a matrix. Therefore the operation A/B, where A and B are matrices is not defined.

1) Multiplication of a Matrix by Scalar :

$$A = (a_{ij})_{m\times n}$$

$$\alpha A = (\alpha a_{ij})_{m\times n}$$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}, \quad 6A = \begin{bmatrix} 6 & 12 \\ 18 & 12 \end{bmatrix}$$

2) Addition/Subtraction of two matrices :

$$A = (a_{ij})_{m\times n} \ \& \ B = (b_{ij})_{m\times n}$$

corresponding element $\quad A - B = \begin{bmatrix} -1 & -1 \\ 2 & 1 \end{bmatrix}_{2\times 2}$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}_{2\times 2} \ \& \ B = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}_{2\times 2} \quad A + B = \begin{bmatrix} 3 & 5 \\ 4 & 3 \end{bmatrix}_{\substack{2\times 2 \\ m\times n}}$$

Properties of Matrix addition and scalar multiplication :

let A, B, C are the matrices of the same order, α, β be any scalar,

1. $A + B = B + A$ — commutative law
2. $(A+B)+C = A+(B+C)$ — Associative law
3. $A + O = A = O + A$ — Existence of additive identity
4. $A + (-A) = O = (-A) + A$ — Existence of additive inverse
5. $\alpha(A+B) = \alpha A + \alpha B$
6. $(\alpha+\beta)A = \alpha A + \beta A$
7. $I \times A = A$
8. $O \times A = O$

3) Multiplication of two matrices :

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}_{2\times 2} \ \& \ B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}_{2\times 1}$$

No

$$AB = \begin{bmatrix} 1\times 1 + 2\times 2 \\ 3\times 1 + 2\times 2 \end{bmatrix}_{2\times 1} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}_{2\times 1}$$

MCQ

$BA \to 1$
$\to 2$

MCQ

A matrix $\mathbf{A} = (\mathbf{a}_{ij})_{m\times n}$ *is said to be rectangular if*
a) $\mathbf{m = n}$ *b)* $\mathbf{m \neq n}$ *c)* $\mathbf{m = p}$ *d)* $\mathbf{m = r}$

Transpose of matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 1 & 5 \end{bmatrix}_{3\times 3}$$

$$A^T \text{ or } A' = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 3 & 3 & 5 \end{bmatrix}_{3\times 3}$$

$$(A')' = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 1 & 5 \end{bmatrix}_{3\times 3} = A$$

① $(AB)' = B'A'$

defined

If A and B are arbitrary square matrices of same order, then
a) $\mathbf{(AB)' = A'B'}$ *b)* $\mathbf{(A')'(B')' = B'A'}$ *c)* $\mathbf{(A+B)' = A' - B'}$ *d)* $\mathbf{(AB)' = B'A'}$

AB $(AB)' = B'A'$

$(AB)' \neq A'B'$

$(A')'(B')' = AB$

$(A+B)' = A' + B'$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times 3}$$

lower triangular

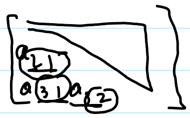
upper triangle

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}_{3\times 3}$$

A matrix $\mathbf{A} = (\mathbf{a}_{ij})_{m\times n}$ *is said to be a square matrix if*
a) $\mathbf{m = n}$ *b)* $\mathbf{m \leq n}$ *c)* $\mathbf{m \geq n}$ *d)* $\mathbf{m < n}$

In an upper triangular matrix $A = (a_{ij})_{n \times n}$, the element $a_{ij} = 0$ for

a) $i < j$ b) $i > j$ c) $i = j$ d) $i \leq j$



$a_{21} = 0$ $2 > 1$ $i > j$

$a_{31} = 0$ $3 > 1$

$a_{32} = 0$ $3 > 2$