CSE322
Recursive and Recursively enumerable sets

Lecture #15

Definitions

- A *procedure for solving a problem is a finite sequence of instructions* which can be mechanically carried out given any input.
- An *algorithm is a procedure that terminates after a finite number of steps* for any input.

Definitions

- A set X is recursive if we have an algorithm to determine whether a given element belongs to X or not.
- A recursively enumerable set is a set X for which we have a procedure to determine whether a given element belongs to X or not.
- A context sensitive language is recursive

NUMERICAL

Consider the grammar G given by $S \rightarrow 0SA_1 2$, $S \rightarrow 012$, $2A_1 \rightarrow A_1 2$, $1A_1 \rightarrow 11$. Test whether (a) $00112 \in L(G)$ and (b) $001122 \in L(G)$.

Solution

(a) To test whether $w = 00112 \in L(G)$, we construct the sets W_0, W_1, W_2 etc. $|w| = 5$.

$$W_0 = \{S\}$$

$$W_1 = \{012,\ S,\ 0SA_1 2\}$$

$$W_2 = \{012,\ S,\ 0SA_1 2\}$$

As $W_2 = W_1$, we terminate. (Although $0SA_1 2 \Rightarrow 0012A_1 2$, we cannot include $0012A_1 2$ in W_1 as its length is > 5.) Then $00112 \notin W_1$. Hence, $00112 \notin L(G)$.

SOULTION

(b) To test whether $w = 001122 \in L(G)$. Here, $|w| = 6$. We construct W_0, W_1, W_2, etc.

$W_0 = \{S\}$

$W_1 = \{012.\ S,\ 0SA_12\}$

$W_2 = \{012,\ S,\ 0SA_12,\ 0012A_12\}$

$W_3 = \{012,\ S,\ 0SA_12,\ 0012A_12,\ 001A_122\}$

$W_4 = \{012,\ S,\ 0SA_12,\ 0012A_12,\ 001A_122,\ 001122\}$

$W_5 = \{012,\ S.\ 0SA_12.\ 0012A_12,\ 001A_122,\ 001122\}$

As $W_5 = W_4$, we terminate. Then $001122 \in W_4$. Thus, $001122 \in L(G)$.