## 15.8 Answers and Hints

### Exercise 15.1

1. Parallel planes.
2. Concentric Spheres.
3. Paraboloids of revolution.
4. Ellipsoids.
5. $\mathbf{r}(t) = (1 + t)\mathbf{i} + 2(1 + t)\mathbf{j} + (3 + 2t)\mathbf{k}$.
6. $\mathbf{r}(t) = (1 + t)(\mathbf{i} - \mathbf{j}) + \mathbf{k}$.
7. $\mathbf{r}(t) = t(\mathbf{i} + \mathbf{j} + \mathbf{k})$.
8. $\mathbf{r}(t) = (3 - 2t)\mathbf{i} + t\mathbf{j} + t\mathbf{k}$.
9. $(7 + t)\mathbf{i} + 2(1 + t)\mathbf{j} + t\mathbf{k}$.
10. $(1 - t/2)\mathbf{i} - t\mathbf{j} + t\mathbf{k}$.
11. $2t^2\mathbf{i} + t\mathbf{j} + t\mathbf{k}$.
12. $9\cos^2 t\,\mathbf{i} + 3\sin t\,\mathbf{j} \pm 3\cos t\,\mathbf{k}$.
13. $2\sin t\,\mathbf{i} + 4\mathbf{j} \pm 2\cos t\,\mathbf{k}$
14. $a(1 + \sqrt{2}\sin t)\mathbf{j} + a(1 \pm \sqrt{2}\cos t)\mathbf{k}$.
15. $(5t^2\cos t + 10t\sin t)\mathbf{i} + (t\cos t + \sin t)\mathbf{j} + (t^3\cos t + 3t^2\sin t)\,\mathbf{k}$.
16. $4\cos 4t + 3t^2$.
17. $-4t(1 + 3t^2)\mathbf{i} - 27t^2\mathbf{j} + 2t(1 + 12t^2)\mathbf{k}$.
18. $(3t^2 - 2e^{2t})\mathbf{i} - [(1 - 2t) - (2 + t)e^t]\mathbf{j} - [te^t + t(2 + 3t)]\mathbf{k}$.
19. $2t\,\mathbf{u}(t^2) + 2t^3\mathbf{u}'(t^2)$.
20. $a\mathbf{u}'(at) - (a/t^2)\mathbf{v}'(a/t)$.
21. $\mathbf{u}(t) \times \mathbf{u}'''(t) + \mathbf{u}'(t) \times \mathbf{u}''(t)$.
22. $\mathbf{u}(t) \cdot \mathbf{u}'(t) \times \mathbf{u}'''(t)$.
23. $x(t) = 2 + t$, $y(t) = 8(1 + t)$, $z(t) = 12(2 + 3t)$.
24. $x(t) = (1 + t)/\sqrt{2}$, $y(t) = (1 - t)/\sqrt{2}$, $z(t) = (\pi/4) + t$.
25. $x(t) = 3 + 4t$, $y(t) = 3 + t$, $z(t) = (6 + t)/9$.
26. $x(t) = 1 + t$, $y(t) = (1 + t)e$, $z(t) = 1$.
27. $3a/2$.
28. $(p\pi/2) + 2\log(\pi + p) - 2\log 2$, $p = \sqrt{4 + \pi^2}$.
29. $2\pi a$, $\mathbf{r}(s) = a\cos(s/a)\,\mathbf{i} + a\sin(s/a)\,\mathbf{j}$, $0 \le s \le 2\pi a$.
30. $4\pi\sqrt{10}$, $\mathbf{r}(s) = \cos(s/\sqrt{10})\,\mathbf{i} + \sin(s/\sqrt{10})\,\mathbf{j} + (3s/\sqrt{10})\mathbf{k}$, $-2\pi\sqrt{10} \le s \le 2\pi\sqrt{10}$.
31. $(5\sqrt{5} - 1)/3$, $\mathbf{r}(s) = [(s^*)^2/2]\,\mathbf{i} + [(s^*)^3/3]\,\mathbf{k}$, $s^* = [(3s + 1)^{2/3} - 1]^{1/2}$, $0 \le s \le (5\sqrt{5} - 1)/3$.
32. $[\sqrt{2} + \log(1 + \sqrt{2})]/2$.
33. Unit tangent vector $= (2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k})/\sqrt{22}$. The given curve is a straight line. $\mathbf{T}$ is independent of $t$ and $d\mathbf{T}/dt = 0$.
34. $\mathbf{v}(t) = (\cos t - \sin t)\,\mathbf{i} + (\cos t + \sin t)\,\mathbf{j} + \mathbf{k}$, speed $= \sqrt{3}$, $\mathbf{a}(t) = -[(\sin t + \cos t)\,\mathbf{i} + (\sin t - \cos t)\mathbf{j}]$
35. $|\mathbf{v}(t)|^2 = c^2$ or $\mathbf{v}(t) \cdot \mathbf{v}(t) = c^2$. Then, $d[\mathbf{v}(t) \cdot \mathbf{v}(t)]/dt = 0$ or $\mathbf{a}(t) \cdot \mathbf{v}(t) = 0$ for all $t$.

### Exercise 15.2

1. $-21\,\mathbf{i}$.
2. $\mathbf{i}(\pi + \sqrt{2})/2$.
3. $\pi(\mathbf{i} - \mathbf{j}) + \mathbf{k}$.
4. $(3\mathbf{i} - 4\mathbf{j} + 5\mathbf{k})/25$.
5. $(\mathbf{i} + \mathbf{j} + \mathbf{k})/\sqrt{3}$.
6. $e^2(5\mathbf{i} + 9\mathbf{j} + \mathbf{k})$.
7. $(\mathbf{i} + \mathbf{j} + \mathbf{k})/2$.
8. $3\mathbf{i} - (\pi^2/54)(9\sqrt{3} + 4\pi)\mathbf{j} + 2\mathbf{k}$.

9. $16(\mathbf{i}-\mathbf{j}), (\mathbf{i}-\mathbf{j})/\sqrt{2}$.

10. $2(3\mathbf{i}+4\mathbf{j}), (3\mathbf{i}+4\mathbf{j})/5$.

11. $4(2\mathbf{i}-\mathbf{j}), (2\mathbf{i}-\mathbf{j})/\sqrt{5}$.

12. $2(\mathbf{i}+2\mathbf{j}+\mathbf{k}), (\mathbf{i}+2\mathbf{j}+\mathbf{k})/\sqrt{6}$.

13. $-(2\mathbf{i}+\mathbf{j}+\mathbf{k}), -(2\mathbf{i}+\mathbf{j}+\mathbf{k})/\sqrt{6}$.

14. $2(2\mathbf{i}-\mathbf{j}-\sqrt{3}\mathbf{k}), (2\mathbf{i}-\mathbf{j}-\sqrt{3}\mathbf{k})/\sqrt{8}$.

21. $-11/\sqrt{5}$.

22. $-1$.

23. $-3/\sqrt{6}$.

24. $-3$.

25. $2\sqrt{18}$.

26. $58/5\sqrt{2}$.

27. $(-\mathbf{i}+2\mathbf{j})/\sqrt{2}, \sqrt{5/2}$.

28. $2(\mathbf{j}+4\mathbf{k}), 2\sqrt{17}$.

29. $6(2\mathbf{i}-\mathbf{j}-2\mathbf{k})$, 18.

30. $13\mathbf{i}+5\mathbf{j}-10\mathbf{k}, \sqrt{294}$.

31. $-(11\mathbf{i}+7\mathbf{j}), -\sqrt{170}$.

32. $-8(\sqrt{\pi/3})(\mathbf{i}+\mathbf{j}), -8\sqrt{2\pi/3}$.

33. $-2(\mathbf{i}-2\mathbf{j}+\mathbf{k}), -2\sqrt{6}$.

34. $-(e/2)(\mathbf{i}+\mathbf{j}+8\mathbf{k}), -\sqrt{66}(e/2)$.

35. All points on the line $(2-\sqrt{3})x+(2\sqrt{3}-1)y=\sqrt{3}$.

36. $(3\mathbf{i}+4\mathbf{j})/5, -(3\mathbf{i}+4\mathbf{j})/5$.

37. $3\mathbf{i}-\mathbf{j}$.

38. $a=-2, b=2, c=1$.

39. In the direction of maximum rate of decrease, $-(8\mathbf{i}+8\mathbf{j}-\mathbf{k})$.

40. $3x-3y-2z=2$.

41. $2x+6y+z=26$.

42. $x+3y+2=0$.

43. $x-2\sqrt{3}\,y+z=(6-\pi\sqrt{3})/6$.

44. $(xx_0/a^2)+(yy_0/b^2)+(zz_0/c^2)=1$.

45. $(xx_0/a^2)-(yy_0/b^2)+(zz_0/c^2)=1$.

46. $x^2y^2z+c$.

47. $\sqrt{x^2+y^2+z^2}+c$.

48. $6x^2-5y^3+z+c$.

49. $e^{xyz}+c$.

50. $f=k$, constant, No.

51. $\cos^{-1}(\sqrt{21/101})$.

52. $\cos^{-1}(\sqrt{2/3})$.

53. $\cos^{-1}(8/(3\sqrt{21}))$.

54. $x(t)=2(1+6t), y(t)=1-4t, z(t)=10-t$.

55. $x(t)=2(1+2t), y(t)=1+4t, z(t)=-(1+8t)$.

## Exercise 15.3

1. 4, 0.

2. $x+y+z, -(\mathbf{i}y+\mathbf{j}z+\mathbf{k}x)$.

3. $6(x^2+y^2+z^2)^{3/2}$, 0.

4. $2(x+y+z)$, 0.

5. $(1-2z)e^{-y}, [2\mathbf{i}(xy-e^{-y})-\mathbf{j}y^2+\mathbf{k}xe^{-y}]$.

6. $yz+2x^2+2xz-y^2, -[2yz\mathbf{i}+(z^2-xy)\mathbf{j}+(xz-4xy)\mathbf{k}]$.

7. $6x, -[\mathbf{i}x+\mathbf{j}(2x-y)-6y\mathbf{k}]$.

8. $2(x+y+z)$, 0.

9. $\mathbf{i}+\mathbf{j}-4z\mathbf{k}$.

10. $(\mathbf{i}+\mathbf{j}+\mathbf{k})\,x\cos(x+y+z)+\mathbf{i}\sin(x+y+z)$.

11. $(\mathbf{i} + \mathbf{j} + \mathbf{k})\, e^{x+y+z}$.

12. $16(y^3z^2\mathbf{i} + 3xy^2z^2\mathbf{j} + 2xy^3z\,\mathbf{k})$.

16. $f\,(\text{div}\,\mathbf{r}) + \nabla f \cdot \mathbf{r} = 5f$.

23. $$\Sigma\,\mathbf{i}\left[-\frac{3y}{r^5}(a_1y - a_2x) - \frac{3z}{r^5}(a_1z - a_3x) + \frac{2a_1}{r^3}\right]$$
$$= \Sigma\,\mathbf{i}\left[-\frac{3}{r^5}\{a_1(x^2 + y^2 + z^2) - x(a_1x + a_2y + a_3z)\} + \frac{2a_1}{r^3}\right]$$
$$= \Sigma\,\mathbf{i}\left[-\frac{3}{r^3}a_1 + \frac{3x}{r^5}(a_1x + a_2y + a_3z) + \frac{2a_1}{r^3}\right] = \frac{3}{r^5}(\mathbf{a}\cdot\mathbf{r})\mathbf{r} - \frac{1}{r^3}\mathbf{a}$$

24. $$\Sigma\,\mathbf{i}\left[a_1\frac{\partial v_2}{\partial y} - a_2\frac{\partial v_1}{\partial y} - a_3\frac{\partial v_1}{\partial z} + a_1\frac{\partial v_3}{\partial z}\right]$$
$$= \Sigma\,\mathbf{i}\left[a_1\left(\frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial y} + \frac{\partial v_3}{\partial z}\right) - \left(a_1\frac{\partial v_1}{\partial x} + a_2\frac{\partial v_1}{\partial y} + a_3\frac{\partial v_1}{\partial z}\right)\right] = (\nabla\cdot\mathbf{v})\,\mathbf{a} - (\mathbf{a}\cdot\nabla)\,\mathbf{v}$$

26. $x(y^2 + y) - (x^3/3) + c$.

27. $x^3y^2z^4 + c$.

28. $e^{xy} + 2e^z + c$.

29. $(1/2)\sin(x^2 + y^2 + z^2) + c$.

30. $a = 2,\ b = c,\ c$ arbitrary.

38. $$\nabla\times(\nabla\times\mathbf{E}) = \nabla\times\left(-\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t}\right) = -\frac{1}{c}\frac{\partial}{\partial t}(\nabla\times\mathbf{H}) = -\frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2}$$

$\nabla\times(\nabla\times\mathbf{E}) = \nabla(\nabla\cdot\mathbf{E}) - \nabla^2\mathbf{E} = -\nabla^2\mathbf{E}$. Therefore, $\dfrac{\partial^2\mathbf{E}}{\partial t^2} = c^2\nabla^2\mathbf{E}$. Similarly for $\mathbf{H}$.

## Exercise 15.4

1. $36\sqrt{14}$.
2. $2291\sqrt{11}/60$.
3. 0.
4. $\frac{3}{4}\left[8\sqrt{65} + \log(8 + \sqrt{65})\right] + \frac{1}{6}\left[(65)^{3/2} - 1\right]$.
5. $10(3\pi + 5)$.
6. $[3\sqrt{37} + (1/2)\log(6 + \sqrt{37})] + [(37)^{3/2} - 1]/12$.
7. $164\sqrt{5}/3$.
8. 316.
9. 36.
10. 0.
11. $(7/2) - \cos 1$.
12. $-20169/35$.
13. $(2e^8 + 3e^2 + 19)/3$.
14. $3\pi$.
15. 0.
16. $-9\sqrt{2}/4,\ 9(4 - \sqrt{2})/4$.
17. $1576/15,\ 2096/3$.
18. $-(8 + 3\pi)/4,\ (9 + 3\pi)/2$.
19. $4(4 - 3\pi)/9,\ 16/9,\ \pi/2$.
20. $16/3,\ 16/3,\ 101/6$.
21. $-5/3,\ -13/10,\ -13/5$.
22. 16.
23. $10/3,\ 17/5$.
24. $70/3$.
25. $105/2$.
26. $-9\pi$.
27. $-2\pi$.
28. $40/3$.
29. $47/2$.
30. $-[\pi - 3\log\sqrt{3}]/3$.