ULTIPLE CHOICE QUESTIONS

For a differentiable function f, the value of Lt $\frac{[f(x+h)]^2 - [f(x)]^2}{2h}$ is

equal to

(a)
$$[f'(x)]^2$$

(c)
$$\frac{1}{2}[f'(x)]^2$$

(d)
$$\frac{1}{2}[f'(x)]^2 - [f(x)]^2$$

The derivative of an even function

If
$$f(x) = \begin{cases} ax^2 + b ; b \neq 0, x \leq 1 \\ bx^2 + ax + c; & x > 1 \end{cases}$$

then f(x) is continuous and differentiable at x = 1 if

$$c = 0, a = 2b$$

(b)
$$a = b$$
, c arbitrary

(c)
$$a = b, c = 0$$

(c)
$$a = 0$$
, $c = 0$
Let $f(x) = \frac{\sin 4\pi [x]}{1 + [x]^2}$ where $[x]$ is the greatest integer $\le x$. Then

- (a) f(x) is not differentiable at some points
- (b) f'(x) exists but is different from 0
- (c) f'(x) = 0 for all x
- (d) f'(x) = 0 but f is not a constant function.

For a real number y, let [y] denote the greatest integer ≤ y. Then

$$f(x) = \frac{\tan(\pi[x-\pi])}{1+[x]^2}$$
 is

- (a) discontinuous at some x
- (b) continuous at all x but the derivative f'(x) does not exist for some x
- (c) f'(x) exists for all x but second derivative f''(x) does not exist
- (d) f'(x) exists for all x

(d)
$$f'(x)$$
 exists for all x
Given $g(0) = 2$, $g'(0) = 1$ and $f(x) = x$ $g(x)$, then $f'(0)$ is

(a) - 2

$$(a) \frac{1}{2}$$

*. A(f(x+y)-f(x)f(y)) for all x and y, f(5)=2 and f'(0)=3, then f'(5) is

(a) 3

(x) 0

(d) None of these

8. Let f be a function satisfying f(x+y)=f(x)+f(y) and $f(x)=x^2g(x)$ for g(x)and y, where g'(x) is a continuous function. Then f''(x) is equal to

(a) g'(x)

(b) g(0)

(e) g(0)+g'(x)

(d) 0

9. If f(a) = 2, f'(a) = 1, g(a) = -1, g'(a) = 2 then the value of

Let
$$\frac{g(x) f(a) - g(a) f(x)}{x - a}$$
 is

(0) -5

(b) $\frac{1}{5}$

(0) 5

(d) none of these

If $f(x) = e^x g(x)$, g(0) = 2, g'(0) = 1, then f'(0) is

(a) 1

(c) 2

(d) 0

Let $f(x) = x(\sqrt{x} - \sqrt{x+1})$. Then

- (a) f is continuous but not differentiable at x = 0
- (b) f is differentiable at x = 0
- (c) f is not differentiable at x = 0
- (d) none of these

If $y = e^{(1 + \log_e x)}$, the $\frac{dy}{dx}$ equals 12.

(a) e

(b) 1

(c) 0

(d) $\log_a x \cdot e^{\log_a x}$

13. If $e^{x^2-y}=x^3$, then $\frac{dy}{dx}$ is equal to

 $(a) = \frac{x}{(1 + \log x)^2}$

 $\frac{1}{(b)} \frac{\log x}{(1 + \log x)^2}$

(d) None of these

The differential coefficient of $f(\log x)$ where $f(x) = \log x$ is

(b) log x

(c) x low x

(d) None of these

If $f(x) = (\log_x 2) (\ln x)$, then f'(x) at x = e is

(a) 0

(6)

(c) 1 e

(d) 1/2 e

16. If $y = \sqrt{x + \sqrt{x + \sqrt{x + \dots + \infty}}}$, then $\frac{dy}{dx}$ is equal to

- (a) $\frac{x}{2y-1}$
- (b) $\frac{1}{2y-1}$

(c) $\frac{1}{x\sqrt{y}}$

 $(d) \quad \frac{1}{2y+x}$

17. If $x^m y^n = (x+y)^{m+n}$, then $\frac{dy}{dx}$ is

- (a) $\frac{y}{x}$
- (b) $\frac{x}{y}$

 $(c) = \frac{m}{n}$

(d) n

18. If $x^2 + xy + y^2 = 0$, then $\frac{d^2y}{dx^2}$ is

(a) 0

(5)

(c) $\frac{1}{2}$

(d) $\frac{1}{(2z+y)^2}$

19. If xy = x + y, then $\frac{dy}{dx}$ is

(a) $\frac{xy}{1-x}$

(b) y+1

(c) $\frac{y}{1-xy}$

(4) $\frac{-1}{(a-1)^2}$

M. If $y=x^2$, then $\frac{dy}{dx}$ is

(A) 1 30E 1

(a) xx

XB

(c) $x^{2}(1 + \log x)$

21. If
$$x^m y^n = a$$
, then $\frac{dy}{dx}$ equals

(a)
$$\frac{y}{x}$$

(b)
$$\frac{x}{y}$$

(c)
$$\frac{m x}{n y}$$

$$(d) - \frac{my}{nx}$$

The differential coefficient of $\log \tan x$ 22.

(a)
$$2 \sec 2x$$

(b)
$$2 \csc 2x$$

(c)
$$2 \sec^3 x$$

(d)
$$2 \csc^3 x$$

23. If
$$y = \sin x^{\circ}$$
, then $\frac{dy}{dx}$ is

(a)
$$\cos x^{\circ}$$

(b)
$$\frac{\pi}{180}\cos x^{\circ}$$

(c)
$$\frac{180}{\pi}\cos x^{\circ}$$

none of these (d)

24. If
$$y = \log \cos \sqrt{x}$$
, then $\frac{dy}{dx}$ is

$$(a) - \frac{\tan\sqrt{x}}{2\sqrt{x}}$$

$$(b) \quad \frac{\tan\sqrt{x}}{2\sqrt{x}}$$

(c)
$$\frac{1}{\cos\sqrt{x}}$$

(d)
$$-\tan\sqrt{x}$$

25. If
$$\sin(x+y) = \log(x+y)$$
, then $\frac{dy}{dx}$ is

$$(a)$$
 2

$$(b) - 2$$

$$(b) -2$$

 $(d) -1$

26. If
$$y = \tan^{-1} \left(\frac{\sin x}{1 + \cos x} \right)$$
, then $\frac{dy}{dx}$ is

(a)
$$\frac{1}{4}$$

(b)
$$\frac{1}{2}$$

$$(c) \quad 1 + \cos^2 x$$

(d)
$$-\frac{1}{4}$$

27. If
$$y = \sqrt{\sin x + y}$$
, then $\frac{dy}{dx}$ is

$$(a) \quad \frac{\sin x}{2y-1}$$

$$(b) \quad \frac{\sin x}{1 - 2y}$$

$$(c) \quad \frac{\cos x}{1-2y}$$

$$(d) \quad \frac{\cos x}{2 \ \nu - 1}$$

28. If $y = \tan^{-1}\left(\frac{x+a}{1-xa}\right)$, then $\frac{dy}{dx}$ is

$$(a) \quad \frac{1}{a^2 + x^2}$$

(b)
$$\frac{1}{1+x^2}$$

$$(c) \quad \frac{a}{a^2 + x^2}$$

$$(d) \quad \frac{a^2}{(1-x\,a)^2}$$

29. If $3 \sin(x y) + 4 \cos(x y) = 5$, then $\frac{dy}{dx}$ is equal to

(a)
$$-\frac{y}{x}$$

(b)
$$\frac{3\sin(x\,y) + 4\cos(x\,y)}{3\sin(x\,y) - 4\cos(x\,y)}$$

(c)
$$\frac{3\cos(x y) + 4\sin(x y)}{4\cos(x y) - 3\sin(x y)}$$

(d) none of these.

30. If $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, then $\frac{dy}{dx}$ is

(a)
$$-\frac{2}{1+x^2}$$

$$(b) \quad \frac{2}{1+x^2}$$

$$(c) \quad \frac{1}{2-x^2}$$

$$(d) \quad \frac{2}{2-x^2}$$

If $x = a \cos^3 \theta$, $y = a \sin^3 \theta$, then $\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ is equal to

(a) $\tan^2 \theta$

 $(d) \mid \sec \theta \mid$

If $x = a (\cos \theta + \theta \sin \theta)$, $y = a (\sin \theta - \theta \cos \theta)$, then $\frac{dy}{dx}$ is equal to

(a) $\cot \theta$

(c) $\tan \frac{\theta}{2}$

(d) $\cot \frac{\theta}{2}$

33. If $y = \log\left(\frac{1-x^2}{1+x^2}\right)$, $\frac{dy}{dx}$ is

(a)
$$\frac{4}{1-x^2}$$

$$(b) \quad -\frac{4x}{1-x^4}$$

(c)
$$\frac{4x^3}{1-x^4}$$

$$(d) = \frac{4x^3}{1 - x^4}$$

34. For the curve $\sqrt{x} + \sqrt{y} = 1$, $\frac{dy}{dx}$ at $\left(\frac{1}{4}, \frac{1}{4}\right)$ is

(a)
$$\frac{1}{2}$$

$$(c) -1$$

35. If $x = a(\cos \theta + \theta \sin' \theta)$, $y = a(\sin \theta - \theta \cos \theta)$ then $\frac{dy}{dx}$ is

(a)
$$\frac{y}{x}$$

(b)
$$\sin \theta$$

(c)
$$\cos \theta$$

(d)
$$\tan \theta$$

36. The differential coefficient of x^6 w.r.t. x^3 is

(a)
$$6x^6$$

(b)
$$3x^2$$

(c)
$$2x^3$$

$$(d)$$
 x^2

37. The derivative of $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ w.r.t. $\sqrt{1+3x}$ at $x=-\frac{1}{3}$ is

(a)
$$\frac{1}{2}$$

(b)
$$\frac{1}{3}$$

$$(c)$$
 0

38. If $y = \sec^{-1}\left(\frac{x+1}{x-1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$, then $\frac{dy}{dx}$ is

$$(c) \quad \frac{x-1}{x+1}$$

(d)
$$\frac{x+1}{x-1}$$

39. $\frac{d}{dx} \left[\tan^{-1} \left(\sec x + \tan x \right) \right]$ is

(b)
$$\frac{1}{2}$$

(d)
$$\sec x - \tan x$$

40. Let $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, 0 < x < 1 and $0 < y < \frac{\pi}{2}$, then $\frac{dy}{dx}$ is equal to

(a)
$$\frac{2}{1+x^2}$$

(b)
$$\frac{2x}{1+x^2}$$

(c)
$$-\frac{2}{1+x^2}$$

41. If $y = \tan^{-1} \left(\frac{\sqrt{1 + z^2 - 1}}{z} \right)$, then y'(0) equals

(5) 0

(d) none of these

42. The differential coeff. of tam^{-1} $\left(\frac{\cos x + \sin x}{\cos x - \sin x}\right)$ is

(d) none of these

43. The differential coefficient of $\sqrt{\sec \sqrt{x}}$ w.i.t. x is

- (a) $\frac{1}{4\sqrt{x}} \left(\sec \sqrt{x} \right)^{\frac{3}{2}} \sin \sqrt{x}$
- (b) $\frac{1}{4\sqrt{x}} \sec \sqrt{x} \sin \sqrt{x}$
- (c) $\frac{1}{2}\sqrt{x}\left(\sec\sqrt{x}\right)^{\frac{3}{2}}\sin\sqrt{x}$ (d) $\frac{1}{2}\sec\sqrt{x}\sin\sqrt{x}$

44. Derivative of tan^{-1} $\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w.r.t. $tan^{-1}x$ is

(a) $\frac{1}{1+x^2}$

(b) $\sqrt{1+x^2-1}$

45. The derivative of $\cos^{-1}(2x^2-1)$ w.r.t. $\cos^{-1}x$ is

(a) 2

(b) ²/₋

(c) $\frac{1}{2\sqrt{1-x^2}}$

The derivative of $\sin^{-1} x$ w.r.t. $\cos^{-1} \sqrt{1-x^2}$ is

(a) $\frac{1}{\sqrt{1-x^2}}$

- (d) $\tan^{-1} \frac{x}{\sqrt{1-x^2}}$

47. Let $f(x) = \frac{x^2}{1 - x^2}$, $x \neq 0$, ± 1 , then derivative of f(x) w.r.t. x^2 is

(a)
$$\frac{2x}{(1-x^2)^2}$$

(b)
$$\frac{1}{(1-x^2)^2}$$

(c)
$$\frac{1}{(2+x^2)^2}$$

(d)
$$\frac{1}{(2-x^2)^2}$$

48. The derivative of $\sec^{-1}\left(\frac{1}{2 x^2 - 1}\right)$ w.r.t. $\sqrt{1 - x^2}$ at $x = \frac{1}{2}$ is

49. If f(x) = |x - 2| and g'(x) = f[f(x)], then for x > 20, g'(x) is

50. If $y^2 = p(x)$, a polynomial of degree 3, then $2 \frac{d}{dx} \left(y^3 \frac{d^2 y}{dx^2} \right)$ is

(a)
$$p''(x) + p'(x)$$

(b)
$$p''(x)p'''(x)$$

(c)
$$p(x)p'''(x)$$

51. If $u = \sin^{-1} \frac{2x}{1+x^2}$ and $v = \tan^{-1} \frac{2x}{1-x^2}$, then $\frac{dy}{dx}$ is

$$(a) \quad \frac{1-x^2}{1+x^2}$$

(c)
$$\frac{1}{2}$$

$$(d)$$
 x

52. If $x = a \cos n t - b \sin n t$, then $\frac{d^2 x}{dt^2}$ is

$$(a) - nx$$

$$(b)$$
 nx

(c)
$$n^2 x$$

$$(d)-n^2x$$

53. If $y = a x^{n+1} + b x^{-n}$, then $x^2 \frac{d^2 y}{dx^2}$ is

(a)
$$n(n-1)y$$

(b)
$$n(n+1)y$$

(d)
$$n^2 v$$

If $x = at^2$, y = 2at, then $\frac{d^2y}{dx^2}$ is

(a)
$$-\frac{1}{t^2}$$

(b)
$$\frac{1}{2t^3}$$

(c)
$$-\frac{1}{t^3}$$

(d)
$$-\frac{1}{2at^3}$$

If $y = a^x$, then $\frac{d^2y}{dx^2}$ equals

$$(a) \quad a^x \left(\log a\right)^2$$

(b)
$$a^{2x}(\log a)$$

(c)
$$a^x \log a$$

(d) none of these

If $x = t^2$, $y = t^3$, then $\frac{d^2y}{dx^2}$ is equal to

(a)
$$\frac{3}{2}$$

$$(b) \quad \frac{3}{4t}$$

(c)
$$\frac{3}{21}$$

$$(d) \quad \frac{3t}{2}$$

If $x = 2 \cos t - \cos 2t$, $y = 2 \sin t - \sin 2t$, then the value of $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{2}$ is

(a)
$$\frac{3}{2}$$

(b)
$$-\frac{5}{2}$$

(c)
$$\frac{5}{2}$$

$$(d) -\frac{3}{2}$$

58. $\frac{d^{20}}{d^{20}}$ (2 cos x cos 3 x) is

(a)
$$2^{20} (\cos 2x - 2^{20} \cos 4x)$$

(b)
$$2^{20} (\cos 2x + 2^{20} \cos 4x)$$

(c)
$$2^{20} (\sin 2 x + 2^{20} \sin 4 x)$$

(d)
$$2^{20} (\sin 2x - 2^{20} \sin 4x)$$

Let [x] denotes the greatest integer $\leq x$, and $f(x) = [\tan^2 x]$. Then 59.

(a) Lt
$$f(x)$$
 does not exist

(b)
$$f(x)$$
 is continuous at $x = 0$

(c)
$$f(x)$$
 is not differentiable at $x = 0$

(d)
$$f'(0) = 1$$

Let f(x) be a quadratic expression which is positive for all real x.

If g(x) = f(x) + f'(x) + f''(x), then for any real x,

(a)
$$g(x) < 0$$

(b)
$$g(x) > 0$$

$$(c) \quad g(x) = 0$$

(d)
$$g(x) \ge 0$$

61. If
$$f: (-\infty, -1) \cup [1, \infty) \rightarrow [0, \pi]$$
 is $f(x) = \sec^{-1} x$, then $f'(x)$ is

$$(a) \frac{1}{x\sqrt{x^2-1}}$$

$$(b) \quad -\frac{1}{x\sqrt{x^2-1}}$$

$$(c) \quad \frac{1}{|x|\sqrt{x^2-1}}$$

$$(d) \quad \frac{1}{|x|\sqrt{x^2-1}}$$

If the function is defined by $f(x) = \frac{x}{1+|x|}$, then at what points is 62. differentiable?

(a) everywhere

(b) except at $x = \pm 1$

(c) except at x = 0

- (d) except at x = 0 or ± 1 .
- The value of the derivative of |x-1| + |x-3| at x = 2 is 63.

(c) 2 (d) not defined

64.
$$\frac{d}{dx} \left\{ \operatorname{cosec}^{-1} \left(\frac{1+x^2}{2x} \right) \right\}$$
 is equal to

(a)
$$-\frac{2}{1+x^2}, x \neq 0$$

$$(b) \quad \frac{2}{1+x^2}, x \neq 0$$

(c)
$$\frac{2(1-x^2)}{(1+x^2)(1-x^2)}$$
, $x \neq 0, \pm 1$ (d) none of these

- Let f(x) = x |x|. The set of point where f(x) is twice differentiable is

(a) $(-\infty, \infty)$

(b) $(-\infty, 0)$

(c) $(0, \infty)$

(d) $R - \{0\}$

66. Let
$$f(x) = \begin{cases} (x-1)\sin\frac{1}{x-1}, & \text{if } x \neq 1 \\ 0, & \text{if } x = 1 \end{cases}$$

Then which one of the following is true?

- (a) f is differentiable at x = 1 but not at x = 0
- (b) f is neither differentiable at x = 0 nor at x = 1
- (c) f is differentiable at x = 0 and at x = 1
- (d) f is differentiable at x = 0 but not at x = 1
- A value of c for which the conclusion of Mean Value Theorem holds for cfunction $f(x) = \log_{\epsilon} x$ on the interval [1, 3] is

(a)
$$2\log_3 e$$

$$(b) \quad \frac{1}{2}\log_e 3$$

the set of points, where fifth a fitting in differentiable is

 $(1/2)^n y^n = (x+y)^{n+n}$, then $\frac{dy}{dx}$ is

Let f(x) be differentiable for all x. If f(1) = -2 and $f'(x) \ge 2$ for $x \in [1, 6]$. there.

(a)
$$f(6) = 5$$

If f is a real-valued differentiable function satisfying

 $|f(x)-f(y)| \le (x-y)^2$, $x,y \in \mathbb{R}$ and f(0)=0, then f(1) equals

Handred and and the

(a)
$$\frac{x}{1+x}$$

$$(x) = \frac{1-x}{x}$$

Let f(x) be a polynomial function of second degree. If f(1) = f(-1) and a, b, care in A.P., then f (a), f (b) and f (c) are in

(a) AP

(b) GP

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(d) Anthonise-Geometry Progression

If $f(x) = x^{*}$, then the value of

100 - 100 - 100 - 100 - 100 - 100 - 100 a

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75. If
$$f(x) = \begin{cases} x e^{-\left(\frac{1}{|x|} + \frac{1}{x}\right)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

then f(x) is

- continuous as well as differentiable for all x
- continuous for all x but not differentiable at x = 0
- (c) neither differentiable nor continuous at x = 0
- (d)discontinuous everywhere

76. If
$$y = (x + \sqrt{1 + x^2})^n$$
, then $(1 + x^2) \frac{d^2 y}{dx^2} + x \frac{dy}{dx}$ is

77. If
$$\sin y = x \sin (a + y)$$
, then $\frac{dy}{dx}$ is

(a)
$$\frac{\sin a}{\sin^2(a+y)}$$

$$(b) \quad \frac{\sin^2(a+y)}{\sin a}$$

(c)
$$\sin a \sin^2(a+y)$$

(d)
$$\frac{\sin^2(a-y)}{\sin a}$$

78. If
$$x^y = e^{x-y}$$
, then $\frac{dy}{dx}$ is

$$(a) \quad \frac{1+x}{1+\log x}$$

$$(b) \quad \frac{1 - \log x}{1 + \log x}$$

$$(d) \quad \frac{\log x}{(1+\log x)^2}$$

79. The two curves
$$x^3 - 3xy^2 + 2 = 0$$
 and $3x^2y - y^3 - 2 = 0$

(a) cut at right angle

cut at right angle

(b) touch each other

(c) cut at an angle $\frac{\pi}{3}$

(d) cut at an angle $\frac{\pi}{4}$

80. Let
$$f(2) = 4$$
 and $f'(2) = 4$. Then, Lt $x \to 2$ $\frac{x f(2) - 2 f(x)}{x - 2}$ is given by

(b) -2

(d)

Let y be an implicit function of x defined by $x^{2x} - 2x^x \cot y - 1 = 0$. Then \$1. y'(1) equals

(a) -1

(b) 1

(c) log 2

 $(d) - \log 2$

If $f: (-1, 1) \rightarrow \mathbb{R}$ be a differentiable function with f(0) = -1 and f'(0) = 1. Let $g(x) = [f(2 f(x) + 2)]^2$. Then g'(0) is equal to 22.

(a)

(b) -4

0 (c)

(d) - 2

Lt $\frac{1}{h \to 0} \frac{1}{h} f(1+h) = 5$, then f'(1)Suppose f(x) is differentiable at x = 1 and 83.

- equals
- (a)6

5 (b)

4 (c)

67.

73,

79.

(a)

(a)

(a)

68. (b)

74. (c)

80. (c)

3 (d)

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(c) 6. 5. (d) 4. (c) (a) 3. 2. (a) (b) 1. (a)12. 11. (b) 10. (b) (c) 9, 8. (d) (b) 7. (a)18. 17. (a) 16. (b) (a) 15. 14. (c) (c) 13. 24. (a) 23. (b) 22. (b) (d) 21. 20. (c) (d) 19. 29. (a) **30.** (a) 28. (b) (d) 27. 26. (b) 25. (d) 36. (c) 35. (d) 34. (c) (b) 33. 32. (b) 31. (d) 42. (c) 41. (a) 40. (a) (b) 39. 37. 38. (a) (c) (d) 48. 47. (b) 46. (b) (a)45. 43. 44. (d) (b) 54. (d) 53. (b) 52. (d) (b) 51. 49. 50. (c) (b) 60. (b) 59. (b) 58. (b) (d) 55, 57. (a) 56. (b) 66. (d)(d)65. 64. (c) (b) 61. 63. (c) 62. (a) 72. (c) 71. (c)

(d)

(b)

(a)

69.

75.

81.

70. (d)

76. (a)

82. (b)

77. (b)

83. (b)

78. (d)