CSE322
Halting Problem of TM & PCP

Lecture #39

Non Deterministic Turing Machines

&

Halting Problem of Turing Machine

&

Post Correspondence Problem

Decidability

- When Turing Machine reaches at some final state it “HALTS”.
- There are TMs that never halt on some inputs in any state.
- So distinction between language can be made as:-
  - TM that Halts on all input strings.
  - Tm that never Halts on some input string.

Definition A problem with two answers (Yes/No) is decidable if the corresponding language is recursive. In this case, the language L is also called *decidable*.

Definition A problem/language is undecidable if it is not decidable.

Note: A decidable problem is called a solvable problem and an undecidable problem an unsolvable problem by some authors.

Halting Problem of Turing Machine

In this section we introduce the reduction technique. This technique is used to prove the undecidability of halting problem of Turing machine.

We say that problem A is reducible to problem B if a solution to problem B can be used to solve problem A.

For example, if A is the problem of finding some root of $x^4 - 3x^2 + 2 = 0$ and B is the problem of finding some root of $x^2 - 2 = 0$, then A is reducible to B. As $x^2 - 2$ is a factor of $x^4 - 3x^2 + 2$, a root of $x^2 - 2 = 0$ is also a root of $x^4 - 3x^2 + 2 = 0$.

Note: If A is reducible to B and B is decidable then A is decidable. If A is reducible to B and A is undecidable, then B is undecidable.

Post Correspondence Problem

The Post Correspondence Problem (PCP) was first introduced by Emil Post in 1946. Later, the problem was found to have many applications in the theory of formal languages. The problem over an alphabet Σ belongs to a class of yes/no problems and is stated as follows: Consider the two lists $x = (x_1 \ldots x_n)$, $y = (y_1 \ldots y_n)$ of nonempty strings over an alphabet $\Sigma = \{0, 1\}$. The PCP is to determine whether or not there exist $i_1, \ldots, i_m$ where $1 \leq i_j \leq n$, such that

$$x_{i_1} \ldots x_{i_m} = y_{i_1} \ldots y_{i_m}$$

Note: The indices i_j's need not be distinct and m may be greater than n. Also, if there exists a solution to PCP, there exist infinitely many solutions.