CSE322
LEFT & RIGHT LINEAR REGULAR GRAMMAR

Lecture #18

Grammars

Grammars

Grammars express languages

Example: the English language

$$\langle sentence \rangle \rightarrow \langle noun\_phrase \rangle \ \langle predicate \rangle$$

$$\langle noun\_phrase \rangle \rightarrow \langle article \rangle \ \langle noun \rangle$$

$$\langle predicate \rangle \rightarrow \langle verb \rangle$$

$$\langle article \rangle \rightarrow a$$

$$\langle article \rangle \rightarrow the$$

$$\langle noun \rangle \rightarrow boy$$

$$\langle noun \rangle \rightarrow dog$$

$$\langle verb \rangle \rightarrow runs$$

$$\langle verb \rangle \rightarrow walks$$

A derivation of "the boy walks":

$$\langle sentence \rangle \Rightarrow \langle noun\_phrase \rangle \ \langle predicate \rangle$$
$$\Rightarrow \langle noun\_phrase \rangle \ \langle verb \rangle$$
$$\Rightarrow \langle article \rangle \ \langle noun \rangle \ \langle verb \rangle$$
$$\Rightarrow the \ \langle noun \rangle \ \langle verb \rangle$$
$$\Rightarrow the \ boy \ \langle verb \rangle$$
$$\Rightarrow the \ boy \ walks$$

A derivation of "a dog runs":

$$
\begin{aligned}
\langle sentence \rangle &\Rightarrow \langle noun\_phrase \rangle \ \langle predicate \rangle \\
&\Rightarrow \langle noun\_phrase \rangle \ \langle verb \rangle \\
&\Rightarrow \langle article \rangle \ \langle noun \rangle \ \langle verb \rangle \\
&\Rightarrow a \ \langle noun \rangle \ \langle verb \rangle \\
&\Rightarrow a \ dog \ \langle verb \rangle \\
&\Rightarrow a \ dog \ runs
\end{aligned}
$$

Language of the grammar:

L = { "a boy runs",
"a boy walks",
"the boy runs",
"the boy walks",
"a dog runs",
"a dog walks",
"the dog runs",
"the dog walks" }

Notation

$$\langle noun \rangle \rightarrow boy$$

$$\langle noun \rangle \rightarrow dog$$

Variable
or
Non-terminal

Production
rule

Terminal

Another Example

Grammar: $S \to aSb$

$S \to \lambda$

Derivation of sentence ab:

$$S \Rightarrow aSb \Rightarrow ab$$

$S \to aSb$ $\qquad\qquad$ $S \to \lambda$

Grammar: $S \to aSb$

$S \to \lambda$

Derivation of sentence $aabb$:

$$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aabb$$

$S \to aSb$ $\qquad\qquad$ $S \to \lambda$

Other derivations:

$$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aaaSbbb \Rightarrow aaabbb$$

$$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aaaSbbb$$
$$\Rightarrow aaaaSbbbb \Rightarrow aaaabbbb$$

Language of the grammar

$$S \to aSb$$

$$S \to \lambda$$

$$L = \{a^n b^n : n \geq 0\}$$

More Notation

Grammar $G = (V, T, S, P)$

$V:$ Set of variables

$T:$ Set of terminal symbols

$S:$ Start variable

$P:$ Set of Production rules

Example

Grammar G :

$$S \to aSb$$

$$S \to \lambda$$

$$G = (V, T, S, P)$$

$$V = \{S\} \qquad T = \{a, b\}$$

$$P = \{S \to aSb,\ S \to \lambda\}$$

More Notation

Sentential Form:

A sentence that contains variables and terminals

Example:

$$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aaaSbbb \Rightarrow aaabbb$$

Sentential Forms   sentence

We write: $S \overset{*}{\Rightarrow} aaabbb$

Instead of:

$$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aaaSbbb \Rightarrow aaabbb$$

In general we write: $w_1 \overset{*}{\Rightarrow} w_n$

If: $w_1 \Rightarrow w_2 \Rightarrow w_3 \Rightarrow \cdots \Rightarrow w_n$

By default: $w \overset{*}{\Rightarrow} w$

Example

Grammar

$$S \to aSb$$

$$S \to \lambda$$

Derivations

$$S \stackrel{*}{\Rightarrow} \lambda$$

$$S \stackrel{*}{\Rightarrow} ab$$

$$S \stackrel{*}{\Rightarrow} aabb$$

$$S \stackrel{*}{\Rightarrow} aaabbb$$

Example

Grammar

$$S \rightarrow aSb$$

$$S \rightarrow \lambda$$

Derivations

$$S \overset{*}{\Rightarrow} aaSbb$$

$$aaSbb \overset{*}{\Rightarrow} aaaaaSbbbbb$$

Another Grammar Example

Grammar $G:$

$$S \to Ab$$

$$A \to aAb$$

$$A \to \lambda$$

Derivations:

$$S \Rightarrow Ab \Rightarrow b$$

$$S \Rightarrow Ab \Rightarrow aAbb \Rightarrow abb$$

$$S \Rightarrow aAbb \Rightarrow aaAbbb \Rightarrow aabbb$$

More Derivations

$$S \Rightarrow Ab \Rightarrow aAbb \Rightarrow aaAbbb \Rightarrow aaaAbbbb$$

$$\Rightarrow aaaaAbbbbb \Rightarrow aaaabbbbb$$

$$S \overset{*}{\Rightarrow} aaaabbbbb$$

$$S \overset{*}{\Rightarrow} aaaaaabbbbbbb$$

$$S \overset{*}{\Rightarrow} a^n b^n b$$

Language of a Grammar

For a grammar G
with start variable S :

$$L(G) = \{w : \quad S \overset{*}{\Rightarrow} w\}$$

String of terminals

Example

For grammar G : $S \to Ab$

$$A \to aAb$$

$$A \to \lambda$$

$$L(G) = \{a^n b^n b : \quad n \geq 0\}$$

Since: $S \overset{*}{\Rightarrow} a^n b^n b$

A Convenient Notation

$$A \to aAb$$

$$A \to \lambda$$

$$\Longrightarrow \quad A \to aAb \mid \lambda$$

$$\langle article \rangle \to a$$

$$\langle article \rangle \to the$$

$$\Longrightarrow \quad \langle article \rangle \to a \mid the$$

Linear Grammars

Linear Grammars

Grammars with
at most one variable at the right side
of a production

Examples:

$$S \to aSb$$
$$S \to \lambda$$

$$S \to Ab$$
$$A \to aAb$$
$$A \to \lambda$$

A Non-Linear Grammar

Grammar G:

$$S \to SS$$

$$S \to \lambda$$

$$S \to aSb$$

$$S \to bSa$$

$$L(G) = \{w: \quad n_a(w) = n_b(w)\}$$

Another Linear Grammar

Grammar G :

$$S \to A$$

$$A \to aB \mid \lambda$$

$$B \to Ab$$

$$L(G) = \{a^n b^n : n \geq 0\}$$

Right-Linear Grammars

All productions have form: $A \to xB$

or

$A \to x$

Example: $S \to abS$

$S \to a$

Left-Linear Grammars

All productions have form:

$$A \rightarrow Bx$$

or

$$A \rightarrow x$$

Example:

$$S \rightarrow Aab$$

$$A \rightarrow Aab \mid B$$

$$B \rightarrow a$$

Regular Grammars

Regular Grammars

A regular grammar is any right-linear or left-linear grammar

Examples:

$$G_1$$

$$S \to abS$$

$$S \to a$$

$$G_2$$

$$S \to Aab$$

$$A \to Aab \mid B$$

$$B \to a$$

Observation

Regular grammars generate regular languages

Examples:

G_1

$$S \to abS$$

$$S \to a$$

$$L(G_1) = (ab)^* a$$

G_2

$$S \to Aab$$

$$A \to Aab \mid B$$

$$B \to a$$

$$L(G_2) = aab(ab)^*$$

Regular Grammars
Generate
Regular Languages

Theorem

$$\left\{ \begin{array}{l} \text{Languages} \\ \text{Generated by} \\ \text{Regular Grammars} \end{array} \right\} = \left\{ \begin{array}{l} \text{Regular} \\ \text{Languages} \end{array} \right\}$$

Theorem - Part 1

$$\left\{\begin{array}{l}\text{Languages} \\ \text{Generated by} \\ \text{Regular Grammars}\end{array}\right\} \subseteq \left\{\begin{array}{l}\text{Regular} \\ \text{Languages}\end{array}\right\}$$

Any regular grammar generates a regular language

Theorem - Part 2

$$\left\{\begin{array}{l}\text{Languages} \\ \text{Generated by} \\ \text{Regular Grammars}\end{array}\right\} \supseteq \left\{\begin{array}{l}\text{Regular} \\ \text{Languages}\end{array}\right\}$$

Any regular language is generated by a regular grammar