# PERMUTATION AND COMBINATION

These two words permutation and combination, at the initial level are very confusing and are generally used interchangeably. So let's take them one by one and understand them.

**Combination**

Combination means from the given certain objects (may be alike or different) selecting one or more objects. Combination can also be replaced by the words – selection, collection or committee.

For Example: Combination of top 5 cricket players from the team of 11 players is the selection of 5 players (in any order).

The sequence in which they have to be selected is not important here. Also we can say that the order of selection is not the concern in the case of combination

**Permutation**

Permutation means arrangement of the alike or different objects taken some or all at a time. So we can observe the word 'arrangement' used in the definition of permutation. Here the arrangement means selection as well as ordering. That means the order in which the objects are selected have also been taken care of in this case.

For Example: The number of 5 digit numbers which can be formed using the digits 0, 1, 2, 3, 4 and 5. In this example, we just not have to select the 5 digits out of given 6 digits but also have to see the number of possible cases for the different arrangement. So the numbers 34251, 21034, 42351 are all different cases.

**Factorial**

In Mathematics, the factorial is represented by the symbol '!' i.e. if we have to write 5 factorial, so it will be written as 5! So in general factorial of any positive number n will be represented by n!

Mathematically,

$$n! = \begin{cases} 1 & if\ n = 0 \\ (n-1)!\ n & if\ n > 0 \end{cases}$$ where n is any positive integer.

Similarly we can say for any positive integer 'n'

n! = n x (n – 1) x (n – 2) x …….. x 3 x 2 x 1.

i.e. the product of all the positive integers less than or equal to n.

Just see below for the factorial of few frequently used numbers.

0! = 1

1! = 1

2! = 2 x 1 = 2

3! = 3 x 2 x 1 = 6

4! = 4 x 3 x 2 x 1 = 24

5! = 5 x 4 x 3 x 2 x 1 = 120 and so on.

**Difference between Permutation and Combination**

1. The very basic difference in permutation and combination is the order of the objects considered. In combination, the order is not considered at all while for permutation it is must. So the permutation is the ordered arrangement while the combination is the unordered selection.

From the three alphabets A, B and C, the permutation of these 3 letters will be ABC, ACB, BAC, BCA, CBA and CAB. While the combination of 3 letters will be just (A, B, C).

2. Permutation gives the answer to the number of arrangements while the combination explains the possible number of selections.

3. Permutation of a single combination can be multiple but the combination of a single permutation is unique (considering all at a time).

**Fundamental Principles of Counting**

**Multiplication Theorem**

If an operation can be performed in m different ways and following which a second operation can be performed in n different ways, then the two operations in succession can be performed in m*n different ways.

**Example 1:** In a class of 5 girls and 4 boys, the teacher has to select 1 girl AND 1 boy. In how many ways can she make her selection?

**Solution:** Here the teacher has to choose the pair of a girl AND a boy
For selecting a boy she has 8 options/ways AND that for a girl 10 options/ways
For 1st boy ------- any one of the 5 girls ----------- 5 ways
For 2nd boy ------- any one of the 5 girls ----------- 5 ways….
For 4th boy ------- any one of the 5 girls ----------- 5 ways
Total number of ways 5 + 5 + 5 + 5 = 20 ways OR 5*4 = 20 ways.

**Addition Theorem**

If an operation can be performed in m different ways and a second independent operation can be performed in n different ways, either of the two operations can be performed in (m + n) ways.

**Example 2:** In a class of 5 girls and 4 boys, the teacher has to select either a girl OR a boy. In how many ways can he make his selection?

**Solution:** Here the teacher has to choose either a girl OR a boy (Only 1 student)
For selecting a boy he has 4 options/ways OR that for a girl 5 options/ways.
The first of these can be performed in 4 ways and the second in 5 ways.
Therefore, by fundamental principle of addition either of the two jobs can be performed in (4 + 5) ways.
Hence, the teacher can make the selection of a student in 9 ways.

**Permutations (Arrangement)**

The different arrangements which can be made by taking some or all of the given things or objects at a time is called Permutation. In permutations the order of arrangement is taken into account; When the order is changed, a different permutation is obtained.

Eg. A number of permutations of three elements a, b, c by taking three at a time will be abc, acb, bac, bca, cab, cba.

The formula of calculating number of permutations is ${}^{n}P_{r}$ i.e. number of all permutations all n distinct things taken r at a time $(0 \le r \le n) = {}^{n}P_{r} = n! / (n-r)!$

A permutation of n taken r at a time is defined as an ordered selection of r out of the n items. The total number of all the possible permutations is denoted as:

nPr = n! (n−1)(n-2) ….. (n-r+1)! Where n ≥ r

Note: This is valid only when repetition is not allowed.

**Example 3:** In how many different ways would you arrange 5 persons on 3 chairs?

**Solution:** Here n= 5 and r= 3

Arranging 5 persons on 3 chairs is same as filling 3 places when we have 5 different things at our disposal. The first place can be filled in 5 ways (by Fundamental Principal of Multiplication).

After filling it, there are 4 things left and anyone of these 4 things can be used to fill second place. So the second place can be filled in 4 ways.

Hence by fundamental principal of multiplication, the first two places can be filled in 5 * 4 ways.

Now, there are 3 things left, so that the third place be filled in 3 ways
So the total number of arrangements will be 5 * 4 * 3.

As per the formula of permutation it will be 5P3 = 5! / (5−3)! = 5! / 2! = 5 X 4 X 3

**Rule 1: Continuing in this manner we can say that number of permutations (or arrangements) of n things taken all at a time will be n!**
e.g. In how many ways can 6 persons stand in a queue? Here n= r=6 so total number of permutations will be 6! = 720

**Example 4:** How many four digit numbers are there with distinct digits n = 10 i.e. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 and r = 4
**Solution:** Total Number of arrangements = 10! / (10−4)!
But these arrangements also include those numbers which have zero (0) at thousand's place.
Such numbers are not four digit numbers and hence need to be excluded.
When 0 is fixed at thousand's place, we have to arrange remaining 9 digits by taking 3 at a time in a way 9! / (9−3)!
Hence total number of four digit numbers = 10! / (10−4)! – 9! / (9−3)! = 5040 – 504 = 4536

**Example 5:** There are 6 periods in each working day of a school. In how many ways can one arrange 5 subjects such that each subject is allowed at least one period?
**Solution:** 5 periods can be arranged in 6 periods in ${}^{6}P_{5}$ ways.
Now one period is left and it can be allotted to any one of the 5 subjects. So number of ways in which remaining one period can be arranged is 5.
Total Number of arrangements = ${}^{6}P_{5}$ X 5 = 3600

**Permutations under Certain Conditions**
1. Permutations of n different objects taken r at a time, when a particular object is to be always included in each arrangement is r. ${}^{(n-1)}P_{(r-1)}$
2. Permutations of n different objects taken r at a time, when a particular object is never taken in each arrangement is ${}^{(n-1)}P_{r}$

**Example 6:** Make all arrangement of letters of the word PENCIL so that
i) N is always next to E
ii) N and E are always together.
**Solution:** i) Let's keep EN together and consider it one letter.
Now we have 5 letters which can be arranged in a row in ${}^{5}P_{5}$ = 5! = 120 ways.

ii) Solve as above. Just keep in mind that now E and N can interchange their places in 2! ways. So total arrangements = 5! X 2! = 240

**Rule 2: Permutation of n different things taken r at a time when repetition is allowed**
**n x n x n x…. r times = $n^r$ ways**

**Rule 3: The number of mutually distinguishable permutations of n things, taken all at a time, of which p are alike of one kind, q are alike of second**
**such that p + q = n, is n! / (p! x q!)**

**Rule 4: The total number of permutations of n things, of which p are alike of one kind, q are alike of second kind and remaining all are distinct, is n! / (p! x q!)**

**Example 7:** How many words would you form with the letters of the word MISSISSIPPI ?

**Solution:** Total letters =11, S = 4 times, I = 4 times, P = 2 times

So, total number of permutations = 11! / (4! X 4! X 2!) = 34650

**Rule 5: Sum of all the numbers which can be formed by using the n digits without repetition is: (n-1)! * (sum of the digits) * (111…..n times).**

**Rule 6: Sum of all the numbers which can be formed by using the n digits (repetition being allowed) is: $n^{n-1}$*(sum of the digits)*(111…..n times).**

**Example 8:** Find the sum of all the numbers that can be formed with the digits 2, 3, 4, and 5 taken all at a time.

(i) If repetition is not allowed

(ii) If repetition is allowed

**Solution:** i) (sum of digits) (n-1)! (11111….. n times)

= (2 + 3 + 4 + 5) (4-1)! (1111) = 93324

ii) $n^{n-1}$*(sum of the digits)*(111…..n times)

= 64 * (2 + 3 + 4 + 5) * 1111 = 896 * 1111 = 995456

**Circular permutations**

If the objects are arranged in a circular manner this distinguished ordering no longer exists, that is, there is no "first element" in the arrangement, any element can be considered as the start of the arrangement. The arrangements of objects in a circular manner are called circular permutations.

The number of circular permutations of a set S with n elements is (n - 1)!

The basic difference between linear permutations and circulations is that every linear arrangement has a beginning and an end, but there is nothing like beginning or end in a circular permutation. Thus, in circular permutations, we consider one object as fixed and the remaining objects are arranged as in case of linear arrangement.

**Rule 7:** There are certain arrangements in which clockwise and anticlockwise arrangements are not distinct. e.g. arrangements of beads in a necklace, arrangement of flowers in a garland etc.

**In such cases number of circular permutations of n distinct objects is 1/2 * [(n-1)!]**

**Example 9:** i) Arrange 6 persons around a circular table.

ii) In how many of these arrangements will two particular persons be next to each other?

**Solution:** i. (6-1)! = 5! = 120

ii. Consider two particular persons as one person. We have 5 persons in all.

These 5 persons can be seated around a circular table in (5-1)! = 4! Ways.

But two Particular persons can be arranged between themselves in 2! Ways

So Total number or arrangements = 4! X 2! = 48

**Combinations (Selection)**

Each of the different selections or groups which are made by taking some or all of a number of things or objects at a time is called combination. ${}^nc_r = n! / \{r!\,(n-r)!\}$

Suppose there are three objects namely x, y, and z. Now we are asked to calculate the combinations (selections) of these objects taking 2 at a time.

x y, y z, x z = Total 3 Combinations.

Note the important difference here. In later case we did not differ between x y and y x, as we did in the first case. This is the only difference between Permutations and Combinations.

In Combinations we find different ways of choosing r objects from n given objects while in Permutations we find different ways of choosing r objects from n given objects and ways of arranging these r objects.

The formula of permutations (${}^nP_r$) itself says first selection (${}^nc_r$) and then arrangement (r!) i.e.

$$ {}^nP_r = {}^nc_r * r! $$

**Other Important Rules:**

i. If ${}^nc_x = {}^nc_y$, then either x = y or x + y = n

ii. ${}^nc_r = {}^nc_{(n-r)}$

iii. ${}^nc_n = 1$

iv. ${}^nc_0 = 1$

v. Selection of any number of things out of n distinct things. = $2^n$

vi. Selection of any number of things out of n identical things = n+1

Zero ball selected = 1 way

One ball selected = 1 way

Two balls selected =1 way

Total ways = (2 +1) = 3 ways.

vii. Number of diagonals of n sided polygon = ${}^nc_2 - n = n(n-3) / 2$

viii. Number of straight lines formed by n points of which r are collinear = ${}^nc_2 - {}^rc_2 + 1$

ix. Number of triangles formed by n points of which r are collinear = ${}^nc_3 - {}^rc_3$

x. With m parallel lines intersected by n parallel lines number of parallelograms can be formed =

${}^mc_2 * {}^nc_2 = m\,n(m-1)(n-1) / 4$

**Example 10:** Find the number of ways in which Asha can eat sweets out of 5 distinct sweets at a party?

**Solution:** Number of ways Asha can or cannot eat sweets at a party out of 5 distinct sweets available at party = $2^5 = 32$

**Corollary:** When zero selections are not allowed i.e. If Asha has asked to select at least one sweet = $2^n - 1 = 2^5 - 1 = 31$

**Example 11:** 3 men and 3 women are candidates for 2 vacancies. A voter has to vote for 2 candidates. In how many ways can one cast her/his vote?

**Solution:** In all there are 6 candidates and a voter has to vote for any 2 of them.

So he can select 2 candidates from 6 candidates in ${}^6C_2$ ways

= 6! / 4! X 2! = (6X5) / 2 = 5 X 3 = 15

**Example 12:** In how many ways can a cricket eleven be chosen out of a batch of 15 players if

**Solution:** i) There is no restriction on the selection = ${}^{15}C_{11}$

ii) A Particular Player is always chosen = ${}^{14}C_{10}$

iii) A Particular Player is never chosen = ${}^{14}C_{11}$

## Division and Distribution

### A. Distinct Objects

**Case 1:** Number of ways in which n distinct things can be divided into r unequal groups containing $a_1$, $a_2$, $a_3$, ......, $a_r$ things (different number of things in each group and the groups are unmarked, i.e., not distinct)

$$= {}^{n}C_{a1} \times {}^{(n-a_1)}C_{a2} \times \ldots \times {}^{(n-a_1-a_2-\ldots-a_{r-1})}C_{ar}$$

$$= n!/(a_1!\ a_2!\ a_3!\ \ldots\ a_r!)$$

(Here $a_1 + a_2 + \ldots + a_r = n$)

**Case 2:** Number of ways in which n distinct things can be distributed among r persons such that some person get $a_1$ things, another person get $a_2$ things . . . . and similarly someone gets $a_r$ things (each person gets different number of things)

= Number of ways in which n distinct things can be divided into r unequal groups containing $a_1$, $a_2$, $a_3$, ......, $a_r$ things (different number of objects in each group and the groups are numbered, i.e., distinct)

$$=n!\ r!/(\ a_1!\ a_2!\ a_3!\ \ldots\ a_r!)$$

(Here $a_1 + a_2 + a_3 + \ldots + a_r = n$)

**Case 3: Number of ways of grouping dissimilar things**

In how many ways can you divide 4 different things (say a, b, c and d) into two groups having two things each? Your answer would be to select two things out of the four and two would be left behind for selecting next 2, i.e. ${}^{4}C_2 \times {}^{2}C_2= 6$. But are there really 6 ways?

Actually, there are only 3 ways of dividing the four things into two groups of two. When you selected two things out of the four, the things selected were ab, ac, ad, bc, bd, and cd. But the last three groups are already formed when you select the first three groups, i.e. when you select ab, you automatically get cd. When you select ac, you automatically get bd, and so on.

If 'n x a' different things are divided into n groups of 'a' things each, the number of ways of grouping = ${}^{na}C_a \times {}^{(na-a)}C_a \times {}^{(na-2a)}C_a \times \ldots\ldots\ldots \times {}^{2a}C_a= na!/\{n!\ (a!)^n\}$

**Case 4:** Number of ways in which m × n distinct things can be distributed equally among n persons (each person gets a number of things)

= Number of ways in which m × n distinct things can be divided equally into n groups (each group will have m things and the groups are numbered, i.e., distinct) = $(n \times a)!\ /\ (a!)^n$

### B. Identical Objects

**Case 1:** Number of ways of Non negative integral distribution of n identical items among r persons i.e. each one of whom can receive 0,1,2 or more items ($\leq n$) = ${}^{(n+r-1)}C_{(r-1)}$

**Case 2:** Number of ways in which n identical things can be distributed among r persons, each one of whom can receive at least 1 item ($\leq n$) i.e. positive integral distribution = ${}^{(n-1)}C_{(r-1)}$

## LEVEL – I

1. In how many ways can the letters of the word SPECIAL be arranged using all the letters?
A. 5010 B. 5020 C. 5040 D. 5080

2. In how many ways can the letters of the word SPECIAL be arranged using only 4 letters at a time?
A. 810 B. 850 C. 830 D. 840

3. How many distinguishable permutations of the letters in the word BANANA are there?
A. 720 B. 120 C. 60 D. 360

4. How many ways a 6 member team can be formed having 3 men and 3 ladies from a group of 6 men and 7 ladies?
A. 700 B. 720 C. 120 D. 500

5. The value of ${}^{75}C_2$ is:
A. 2775 B. 2315 C. 1215 D. 1675

6. What is the number of possible words that can be made using the word “EASYQUIZ” such that the vowels always come together?
A. 120 B. 720 C. 2880 D. 4320

7. What is the number of possible words that can be made using the word “QUIZ” such that the vowels never come together?
A. 8 B. 12 C. 16 D. 24

8. How many words can be made from the word “APPLE” using all the alphabets with repetition and without repetition respectively?
A. 1024, 60 B. 60, 1024 C. 1024, 1024 D. 240, 1024

9. In how many different ways can the alphabets of the word ‘SCORING’ be arranged so that the vowels always come together?
A. 120 B. 720 C. 240 D. 1440

10. In how many ways can the alphabets of the word ‘DERAIL’ be arranged so that the vowels come at the odd positions only?
A. 12 B. 18 C. 24 D. 36

11. In how many ways can an interview panel of 3 members be formed from 3 engineers, 2 psychologists and 3 managers if at least 1 engineer must be included?
A. 30 B. 15 C. 46 D. 45

12. Find the number of squares on a chessboard?
A. 204 B. 100 C. 1296 D. 64

13. A box contains 2 red coins, 3 green coins and 4 blue coins. In how many ways can 3 coins be chosen such that at least one coin is green?
A. 16 B. 32 C. 64 D. 128

14. Out of 6 engineers and 4 doctors, how many groups of 4 professionals can be formed such that at least 1 engineer is always there?
A. 129 B. 109 C. 229 D. 209

15. From a group of 9 different books, 4 books are to be selected and arranged on a shelf. How many arrangements are possible?
A. 3023 B. 3024 C. 3025 D. 3026

16. How many different combinations are possible if 10 numbers are grouped five at a time?
A. 252 B. 242 C. 232 D. 282

17. There are 3 questions in a question paper. If the questions have 4, 3 and 2 solutions respectively, find the total number of solutions.
A. 22 B. 23 C. 24 D. 28

18. In how many ways can a party of 4 men and 4 women be seated at a circular table so that no two women are adjacent?
A. 576 B. 144 C. 36 D. 16

19. There are 5 boys and 5 girls. In how any ways they can be seated in a row so that all the girls do not sit together?
A. 5! X 5! B. 10! - 5! X 5! C. 6! X 5! D. 10! - 6! X 5!

20. In a party, every guest shakes hand with every other guest. If there were total of 66 handshakes in the party, find the number of persons present in the party?
A. 12 B. 33 C. 4376 D. 66

## LEVEL – II

1. How many 4-digit numbers can be formed from the digits 1, 2, 3, 4, 5, 6 and 7 which are divisible by 5 when none of the digits are repeated?
A. 120 B. 35 C. 24 D. 720

2. In how many ways can 20 boys and 18 girls make a queue such that no two girls are together?
A. $20!*\ ^{20}C_{18}$ B. $20!*\ ^{20}P_{18}$ C. $20!*\ ^{21}C_{18}$ D. $20!*\ ^{21}P_{18}$

3. In how many ways can 3 prizes be distributed among 4 boys,

i) When no boy gets more than one prize?
A. 12 B. 64 C. 24 D. 56

ii) When a boy may get any number of prizes?
A. 12 B. 64 C. 24 D. 56

iii) When no boy gets all the prizes?

A. 64 B. 63 C. 12 D. 60

4. i. How many different arrangements can be made by using all the letters in the word MATHEMATICS?

A. $11! / (2!)^3$ B. $11! / (2!)^2$ C. $11!$ D. $11! / (2!)^3 \times 5!$

ii. How many of the words begin with I from the word MATHEMATICS?

A. $11! / (2!)^2$ B. $10! / (2!)^3$ C. $10!$ D. $10! / (2!)^3 \times 4!$

5. A telegraph has 5 arms and each arm has four distinct positions including the position of rest. What is the total number of signals that can be made?

A. 511 B. 625 C. 19 D. 1023

6. How many 3 digit numbers can be formed whose unit digit is a prime number? (Repetition is allowed).

A. 500 B. 450 C. 400 D. 360

7. If there are 4 roads from A to B, 5 roads from B to C and 3 roads from C to D, then how many combinations of roads are there from A to D via B and C?

A. 48 B. 60 C. 72 D. 24

8. . If ${}^{n}C_4 = 70$, find n.

A. 5 B. 8 C. 4 D. 7

9. In how many different ways a person can choose his dress among 3 shirts, 4 jeans and 5 trousers? A dress is formed by either a shirt and a jean or a shirt and a trouser.

A. 17 B. 60 C. 27 D. 12

10. Find the number of ways in which a six-lettered code can be formed using the English alphabets and the digits from 0 to 9 such that the first three places and the last three places are to be filled with the numerals and alphabets respectively.

A. $9(10)^2(26)^3$ B. $(234)^3$ C. $(260)^3$ D. $(36)^6$

11. The letter of the word LABOUR are permuted in all possible ways and the words thus formed are arranged as in a dictionary. What is the rank of the word LABOUR?

A. 275 B. 251 C. 240 D. 242

12. A standard deck of cards consists of 4 suits (spades, hearts, diamonds, and clubs) of 13 cards each. In how many ways can 5-card hands be dealt that include 3 diamonds and 2 cards from other suits?

A. 211906 B. 211916 C. 211926 D. 211936

13. The Indian Cricket team consists of 16 players. It includes 2 wicket keepers and 5 bowlers. In how many ways can a cricket eleven be selected if we have to select 1 wicket keeper and at least 4 bowlers?

A. 1011 B. 1092 C. 2092 D. 3092

14. A class photograph has to be taken. The front row consists of 6 girls who are sitting. 20 boys are standing behind. The two corner positions are reserved for the 2 tallest boys. In how many ways can the students be arranged?

A. 9!x1440 B. 10!x1440 C. 11!x1440 D. 18!x1440

15. How many words can be formed from the letters of the word "ENGINEERING", so that vowels always come together?

A. 4200 B. 420 C. 7! X 5! D. 7! X 5! / (2! X 3!)

16. Ram goes to a fruit-seller who is left with 4 apples, 5 bananas and 6 guavas only. In how many ways can Ram make a purchase from the shop?

A. 120 B. 209 C. 119 D. 210

17. How many three-digit numbers divisible by 3 may be formed out of the digits 2, 3, 4 and 6 if the digits are not to be repeated?

A. 24 B. 6 C. 12 D. 36

18. In how many ways 6 girls out of 12 girls in a class may be selected for a team so that 2 particular girls (captain and vice-captain) are always there?

A. 360 B. 210 C. 24 D. 120

19. How many 5 digit numbers can be formed by using the first 9 natural numbers, such that all the 5 digits of the number formed are in ascending order and at the hundredth place the digit is ‘5’?

A. 34 B. 31 C. 37 D. 36

20. In how many different ways can five players A, B, C, D and E be arranged in a line such that A is always to the left of B?

A. 60 B. 120 C. 48 D. 24