Grammar In TOC

- **Gramma** - Standard way of representing a language eg

eg: from English

My name is Dr. Tarak. ✓

Is my name Dr. Tarak? ✓

My is Dr. name Tarak. X

$G = \{V, \underset{\Sigma}{T}, P, S\}$

V = Veriables (capital letter), for use again agian.

Σ/T = Terminal (small letter), for terminate a ~~state~~ string

P = Production rule.

S = Starting symbol.

eg:
$V = \{\langle S\rangle, \langle N\rangle, \langle V\rangle, \langle A\rangle\}$
$\Sigma = \{$Ram, Sam, ate, sang, well$\}$
$S = \{S\}$
$P \Rightarrow \langle S\rangle \Rightarrow \langle N\rangle\langle V\rangle$
$\langle S\rangle \Rightarrow \langle N\rangle\langle V\rangle A$

$\langle N\rangle \rightarrow$ Ram
$\langle N\rangle \rightarrow$ Sam
$\langle V\rangle \rightarrow$ Ate $\quad \langle V\rangle \rightarrow$ Sang
$\langle A\rangle \rightarrow$ well.

eg: 4.2 $G = \{\{S\}, \{a, b\}, \{S \rightarrow aSb, S \rightarrow \lambda\}, S)$ find $L(G)$.

$S \rightarrow aSb \mid \lambda$

$\{\lambda,\ a\underline{S}b,\ aa\underline{S}bb,\ aaaSbbb, \dots a^n b^n\}$

$ab \qquad aabb \qquad a^3b^3$

a^2b^2

$$\boxed{L(G) = \{a^n b^n \mid n \geq 0\}}$$

⇒ a followed by equal no. of b.

⇒ Can we generate $\underline{abab}$ from $L(G)$. (No) eg: 4.4.

$G = \{\{S, C\}, \{a, b\}, P, S\}$. where P consists of $S \to aCa$, $C \to aCa \mid b$. find $L(G)$.

$S \Rightarrow aCa \Rightarrow aba$. so, $aba \in L(G)$.

$S \Rightarrow aCa$

$S \overset{*}{\Rightarrow} a^nCa^n$ ($C \to aCa$ $(n-1)$ times).

$\Rightarrow a^nba^n$

so. $a^nba^n \in L(G)$ $n \geq 1$

$$\boxed{L(G) = \{a^nba^n \mid n \geq 1\}}$$

eg: 4.5.

If G is $S \to aS \mid bS \mid a \mid b$ find $L(G)$.

$S \to a$

$S \to aS$

~~$S \to b$~~ $\{a, aS, aaS, a^2aS, aS \dots a^n\}$

aa $\quad a^3$ $\quad a^3a$

a^4

~~$S \to$~~

$S \to b$

$S \to bS$

$\{b, bS, bbS, bbbS \dots b^n\}$.

bb $\quad b^3$ $\quad b^4$

$L(G) \subseteq \{a,b\}^* - \lambda = \{a,b\}^+$

$$\boxed{L(G) = \{a,b\}^+}$$

Language to grammar :-

eg: $L = \{aa, ab, ba, bb\}$ Finite language.

→ $S \rightarrow \{aa \mid ab \mid ba \mid bb\}$

→ RE → $\underbrace{(a+b)}_{A}\ \underbrace{(a+b)}_{B}$

$$\boxed{S \rightarrow AB \quad A \rightarrow a|b \quad B \rightarrow a|b}$$

$$\boxed{G_1 = \{\{S, A, B\}, \{a, b\}, P, \{S\}\} \quad P = S \rightarrow AB,\ A \rightarrow a|b,\ B \rightarrow a|b}$$

→ Same RE → $\underset{A}{(a+b)}\ \underset{A}{(a+b)}$.

$$\boxed{S \rightarrow AA \quad A \rightarrow a|b}$$

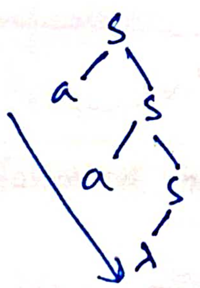
$$\boxed{G_2 = \{\{S, A, B\}, \{a, b\}, S \rightarrow AA, A \rightarrow a|b, \{S\}\}}$$

eg: $L = a^n \mid n \geq 0$

$L = \{\lambda, a, aa, aaa \ldots\}$

$$\boxed{S \rightarrow aS|\epsilon}$$

Check 'aa' is belongs to G.

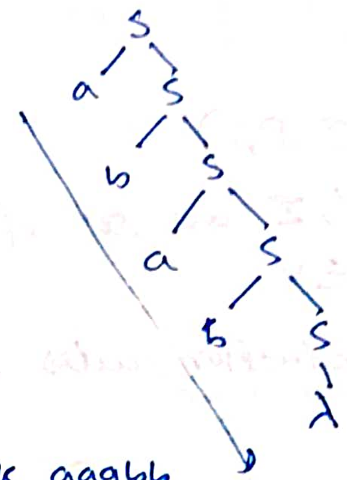


= aa ✓ True.

eg:- $L = \{(a+b)^*$

$\boxed{S \to aS \mid bS \mid \lambda}$

W= abab



abab ✓

eg: $L = a^m b^n \mid m, n \geq 0$

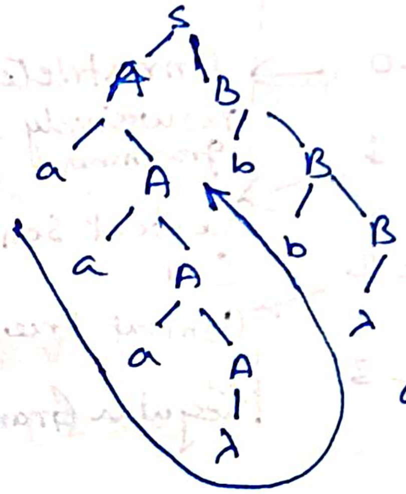
$RE \to a^* b^*$

$S \to AB$

$A \to aA \mid \lambda \to [a^*]$

$B \to bB \mid \lambda \to [b^*]$

check aaabb



aaabb ✓

4.2 Chomsky Classification of Languages

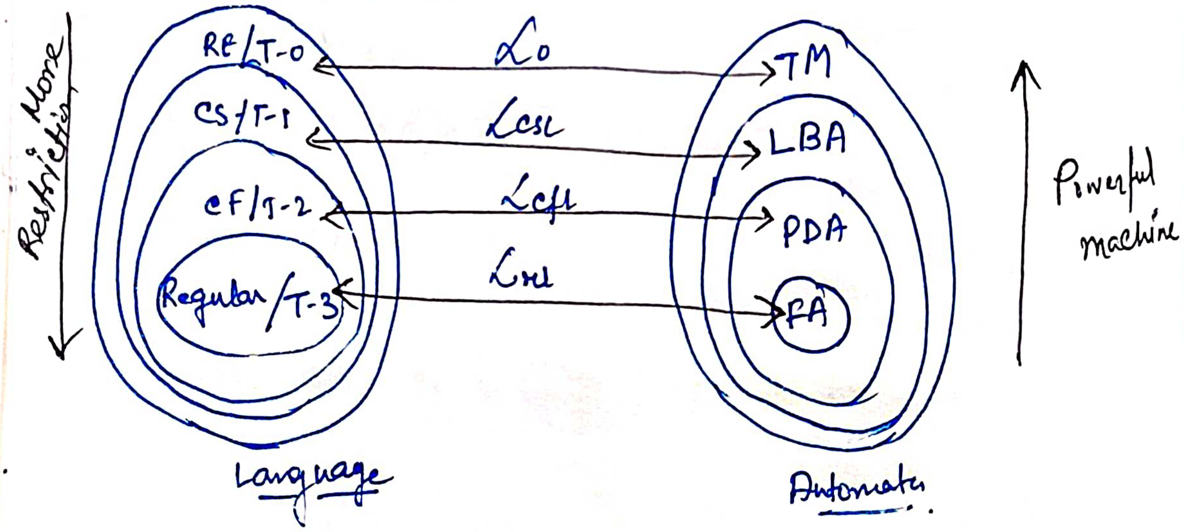
$G = (V, \Sigma, P, S)$

where V, Σ are set of symbols.
& $S \in V$.

$\therefore$ P = production rules are the classifier for grammar.

Type	Grammar	Lang	Automata
Type-0	Unrestricted grammar / recursively enumerable grammar (REG)	REL	Turing Machine (TM)
Type 1	Context Sensitive Grammar (CSG)	CSL	Linear Bounded Automata (LBA)
Type 2	Context free Grammar (CFG)	CFL	Push Down Automata (PDA)
Type 3	Regular Grammar (RG)	RL	Finite Automata (FA)

* Languages & automata



* More restricted language = easy machine to decode.

* Languages and their relations

$$L_{rl} \subseteq L_{cfl} \subseteq L_{csl} \subseteq L_0$$

* Type-0 / Recursively Enumerable Grammar (REG)

$G = (V, \Sigma, \underline{P}, S)$

Production rule

$$\boxed{\underline{\alpha \rightarrow \beta} \qquad \alpha \in (\Sigma \cup V)^* V (\Sigma \cup V)^* \qquad \beta \in (\Sigma \cup V)^*}$$

eg: $S \rightarrow A$ $S \rightarrow a$ $A \rightarrow BB$.   Anything.

$S \rightarrow aAb$ $A \rightarrow Aa$

* Type-1 / CSG :-

$\alpha \rightarrow \beta$

$\alpha \in (\Sigma \cup V)^* V (\Sigma \cup V)^*$

$\beta \in (\Sigma \cup V)^+$

$|\alpha| \leq |\beta|$

eg1 $A \rightarrow \epsilon$

$\underset{1}{|A|} \rightarrow \underset{0}{|\epsilon|}$ Not possible

$\therefore \beta \in (\Sigma \cup V)^+$

eg2 $AaB \rightarrow aa$

$|AaB| \rightarrow |aa|$

$3 \not\leq 2$ Not type1.

Left content   Right content

$\alpha \underline{A} \beta \rightarrow \alpha \underline{\delta} \beta$

$\alpha, \beta \in (\Sigma \cup V)^*$

$A \in V$

$\delta \in (\Sigma \cup V)^+$

eg: $xAy \rightarrow a$

$aAb \rightarrow c$

* As type 2 & type 3 accepts ϵ, type1 must accept ϵ.

$S \rightarrow \epsilon$ [exception]

1. Only start symbol can produce ϵ.
2. 'S' can not come right side.

* Type :

* **Type-2 / CFG :-**

$\alpha \rightarrow \beta$

$\alpha \in V$ [restriction on α)

$|\alpha| = 1$

$\beta \in (\Sigma \cup V)^*$

eg:- $S \rightarrow aSb$
$S \rightarrow ab$

* **Type-3 / RG:-**

Left linear grammar

$A \rightarrow a \mid Ba$

$A, B \in V$

$|A| = |B| = 1$

$a \in \Sigma^*$

Right linear grammar

$A \rightarrow a \mid aB$

$A, B \in V$

$|A| = |B| = 1$

$a \in \Sigma^*$

eg: $A \rightarrow BC$ two variable (BC)
Not type 3. only one required.

$A \rightarrow Bc$ [Yes $B \in V$ and in extreme left]

$A \rightarrow a\underline{B}a$ [No, B should be either left or right]

P-167 5·6 Regular set & Regular grammar

5·6·1 RE to RG

Let $M = (\{q_0, \cdots q_n\}, \Sigma, \delta, q_0, F)$.

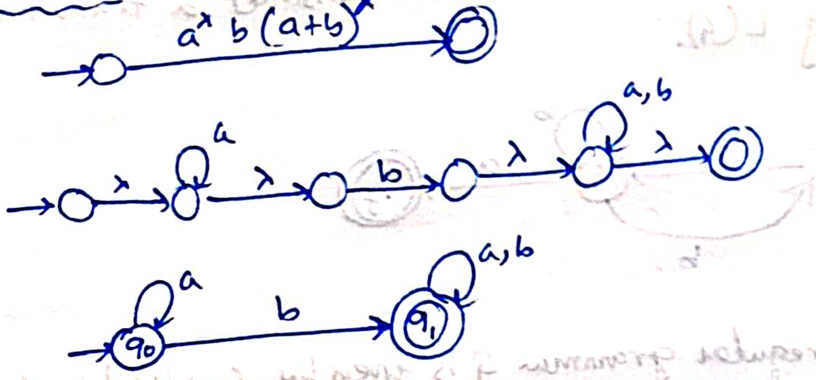
$G = (\{A_0, A_1 \cdots A_n\}, \Sigma, P, /$

Rules

1. $A_i \to a A_j$ is included in P if $\delta(q_i, a) = q_j \notin F$.
2. $A_i \to a A_j$ and $A_i \to a$ are included in P if $\delta(q_i, a) = q_j \in F$.

eg.-5·24 Construct a regular grammar G generating the regular set represented by $P = a^* b(a+b)^*$.

Construct DFA.



$M = (\{q_0, q_1\}, \{a, b\}, q_0, \delta, \{q_1\})$

$G = (\{A_0, A_1\}, \{a, b\}, P, A_0)$ where ~~P is~~

P is $A_0 \to aA_0$ $A_0 \to bA_1$ $A_0 \to b$

$A_1 \to aA_1$ $A_1 \to bA_1$ $A_1 \to a$ $A_1 \to b$.

5.6.2 RG to FA (RE).

Let $G = (\{A_0 A_1 \cdots A_n\}, \Sigma, P, A_0)$

$M = (\{q_0 \cdots q_n, q_f\}, \Sigma, \delta, q_0, \{q_f\})$

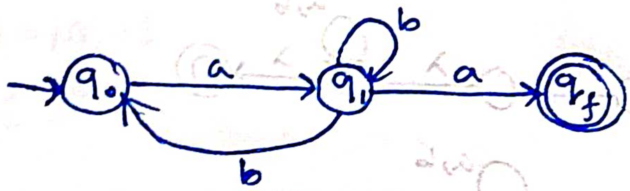
δ defined as followes :-

i) Each production $A_i \to aA_j$ indicates a transaction from q_i to q_j with label 'a'.

ii) Each production ~~$A_k \to a$~~ $A_k \to a$ indicates a transaction from q_k to q_f with Label 'a'.

eg:- 5.25

Let $G = (\{A_0, A_1\}, \{a, b\}, P, A_0)$, where P consists of $A_0 \to aA_1$, $A_1 \to bA_1$, $A_1 \to a$, $A_1 \to bA_0$. Construct a transition system M accepting L(G).



eg: 5.26

If a regular grammar G is given by $S \to aS \mid a$, find M accepting L(G).

$S \to aS$

$S \to a$