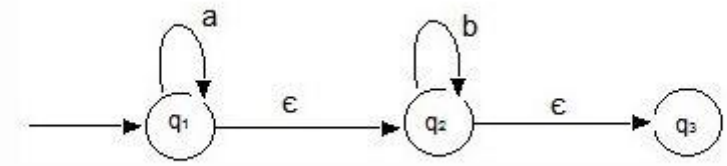
CSE322
Mealy and Moore Machine

Lecture #4

Null Transition

- An NFA with null transition is allowed to make transition not only on input from the alphabet but also with null input, i.e. without any input symbol. This transition without input is called null transition.

Null Transition



Null Transition

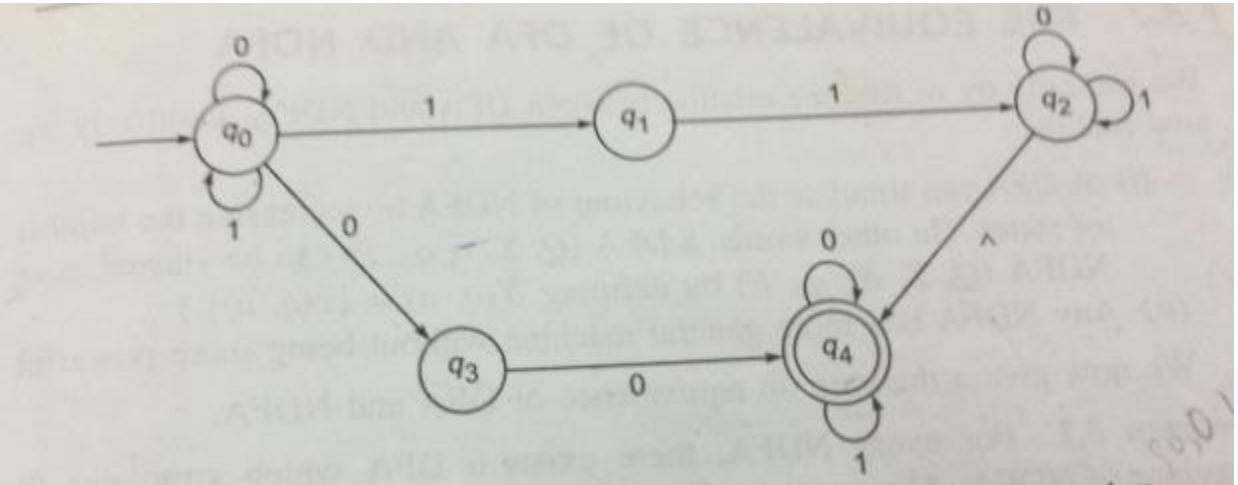
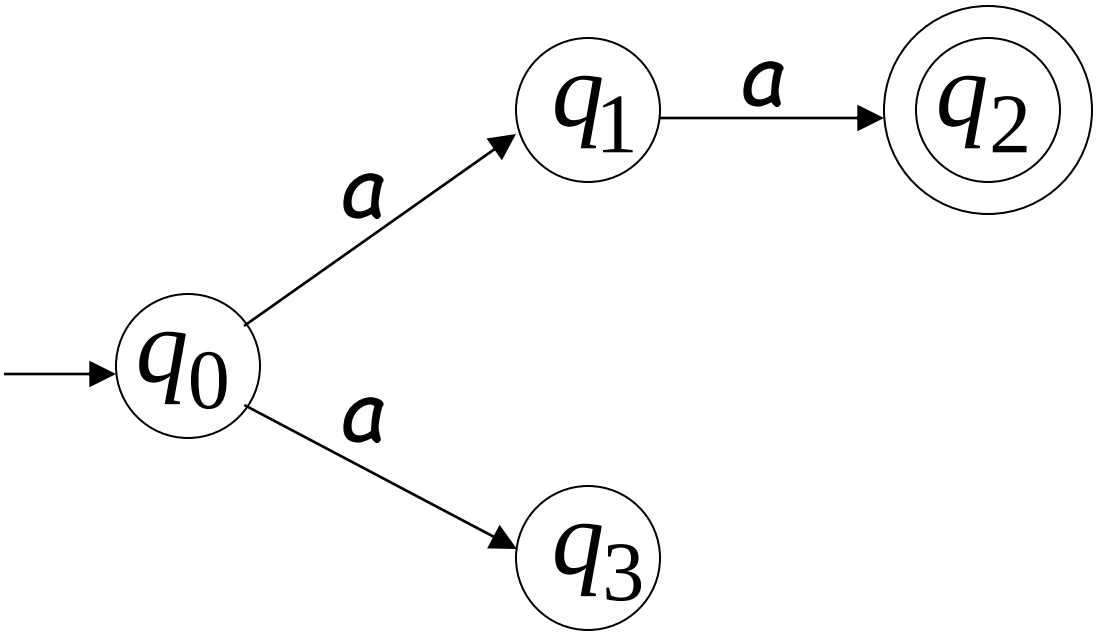
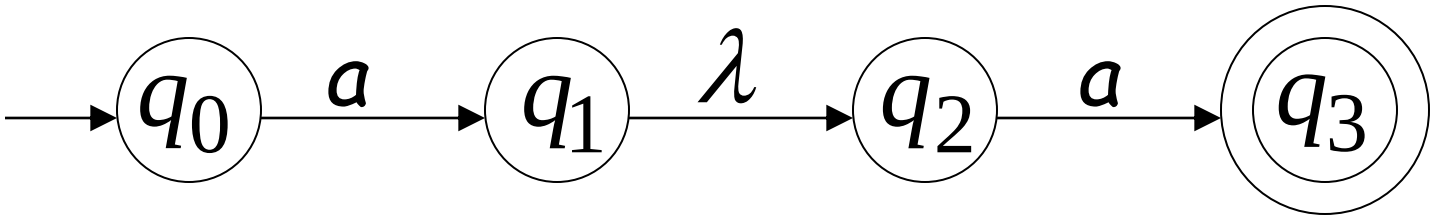


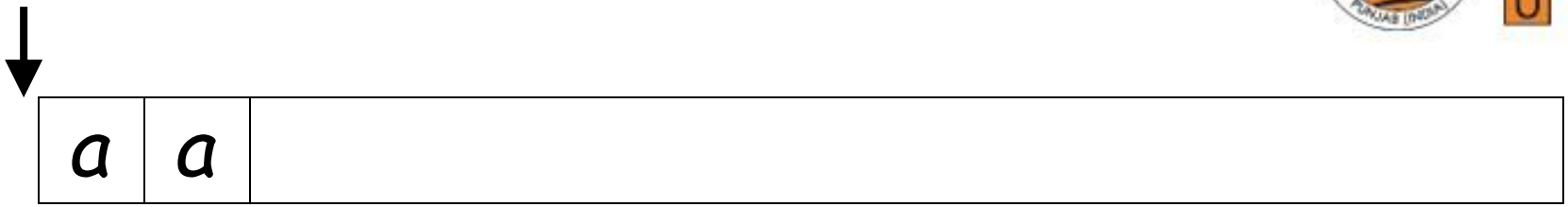
Fig. 3.8 Transition system for a nondeterministic automaton.

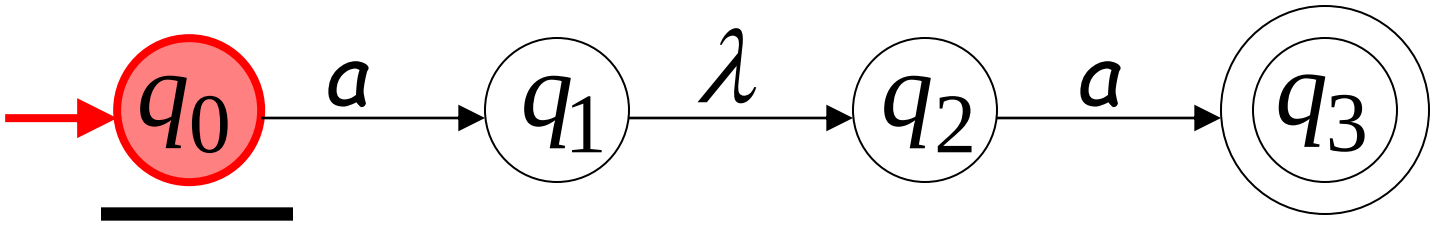
Language accepted: $L = \{aa\}$

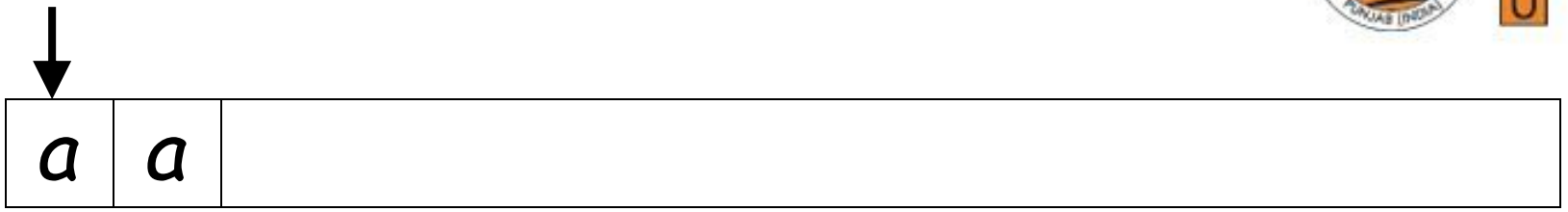


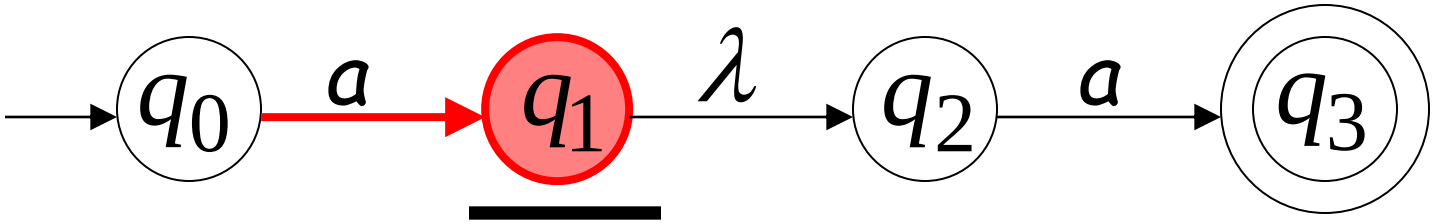
Lambda Transitions



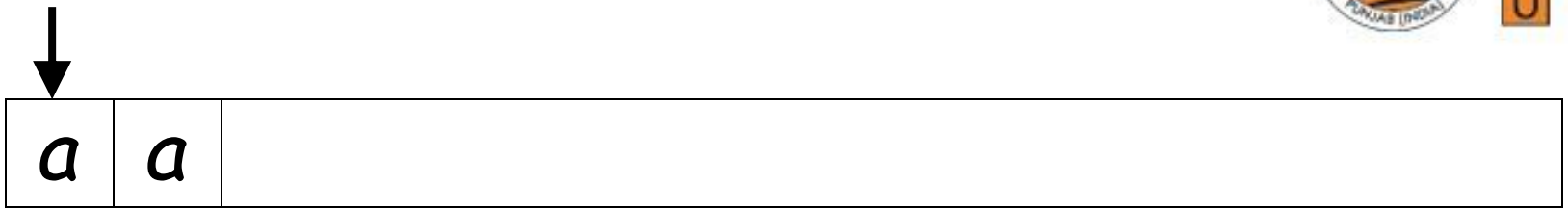

a
a

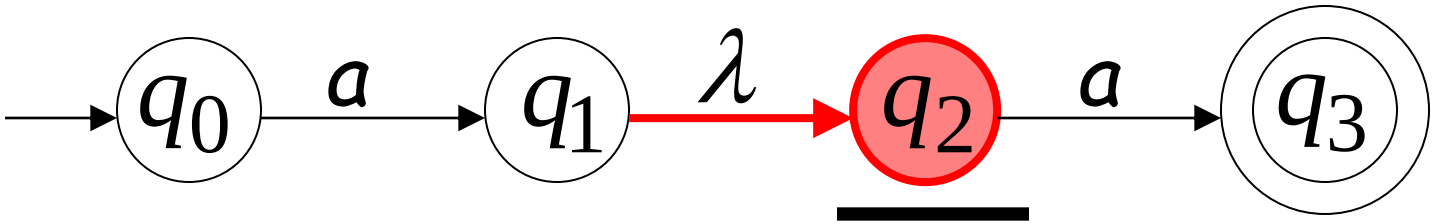

q_0
a
q_1
λ
q_2
a
q_3

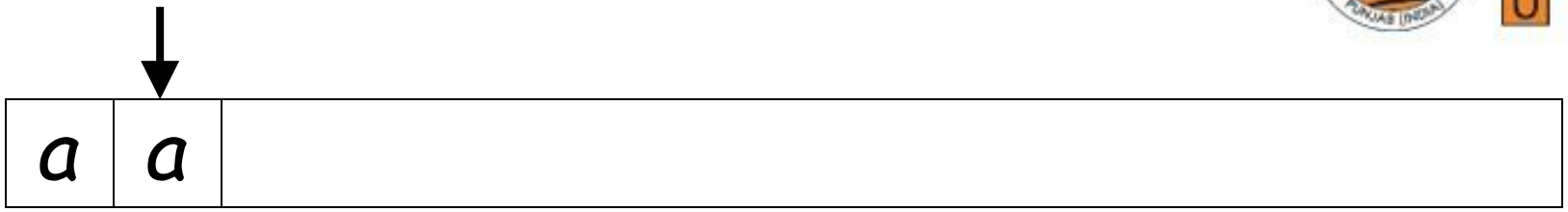

a
a

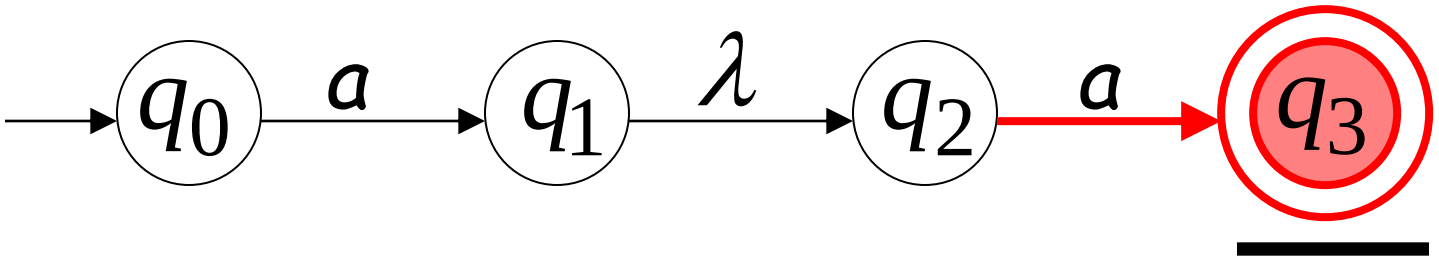

q_0
a
q_1
λ
q_2
a
q_3

(read head does not move)

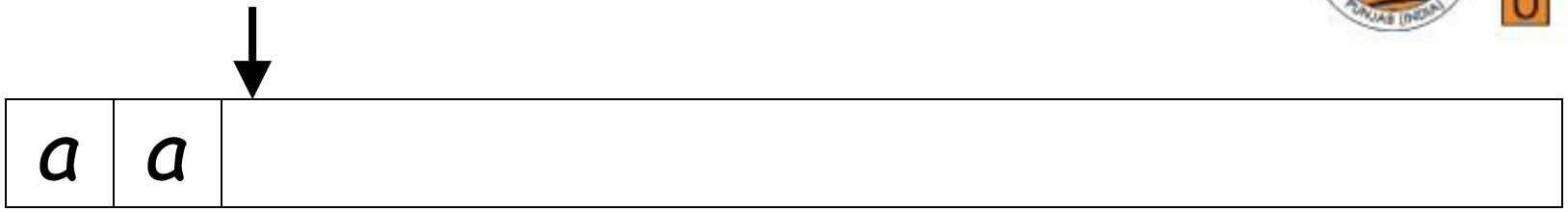




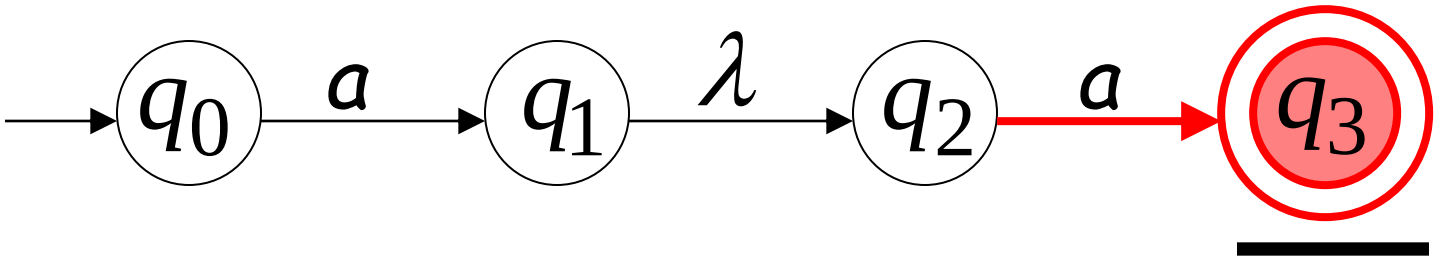

a
a


q_0
a
q_1
λ
q_2
a
q_3

all input is consumed

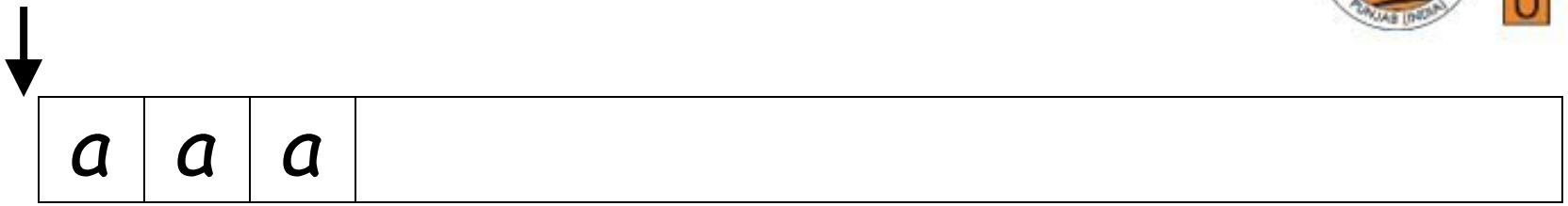


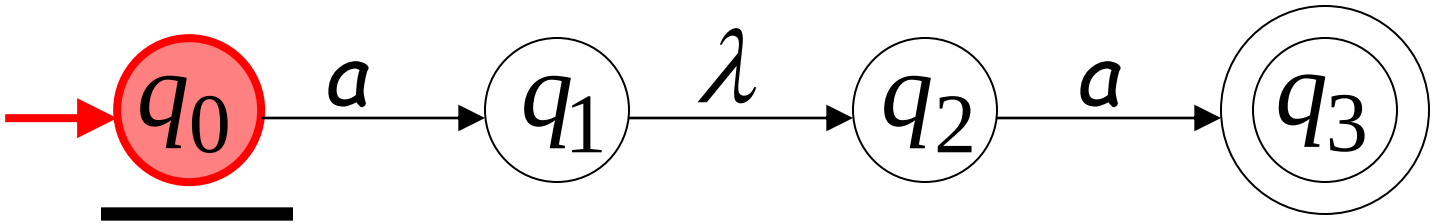
“accept”

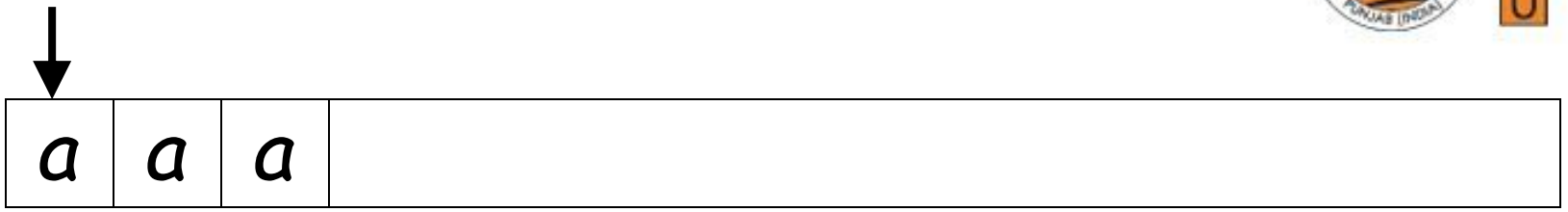


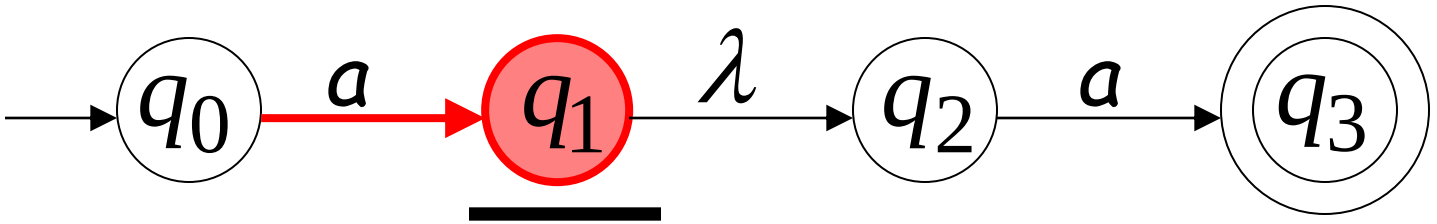
String aa is accepted

Rejection Example

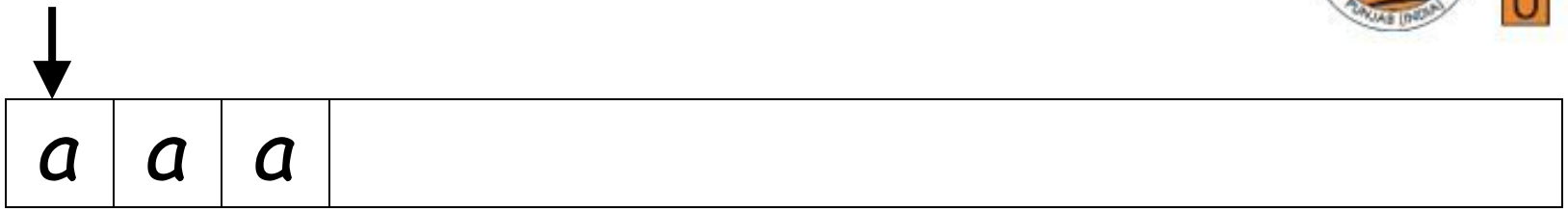


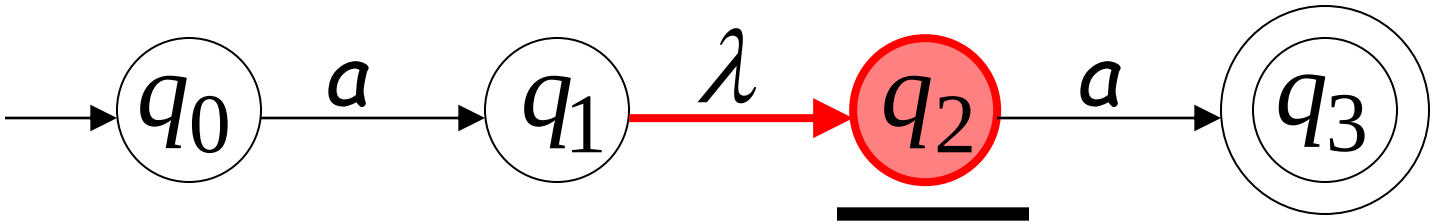


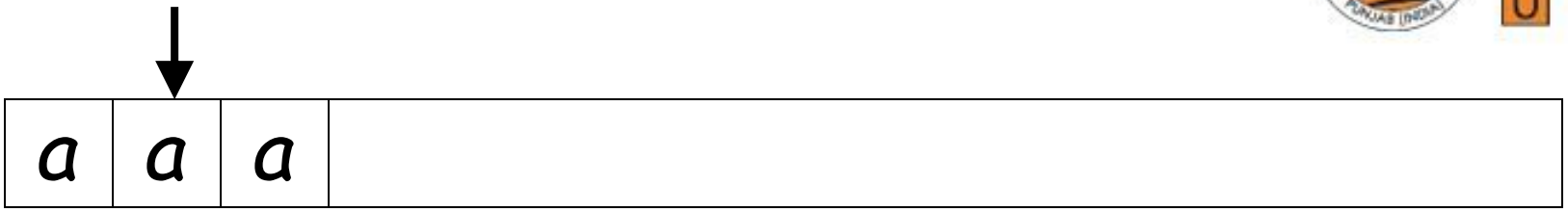

a
a
a

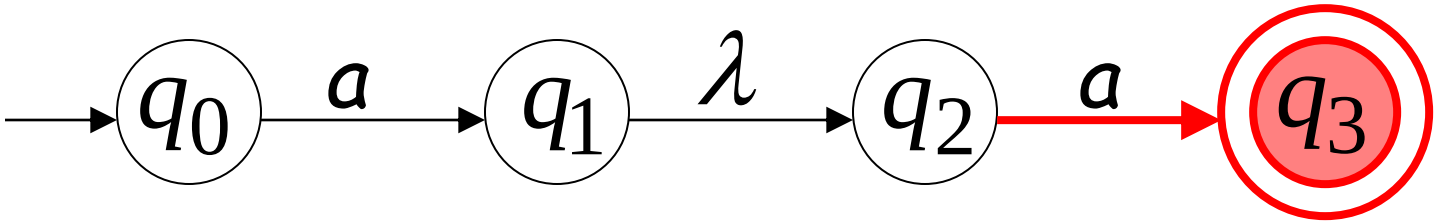

q_0
a
q_1
λ
q_2
a
q_3

(read head doesn't move)





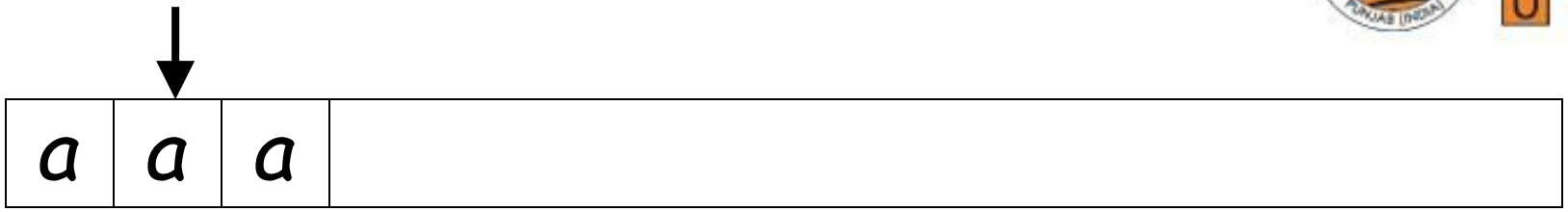




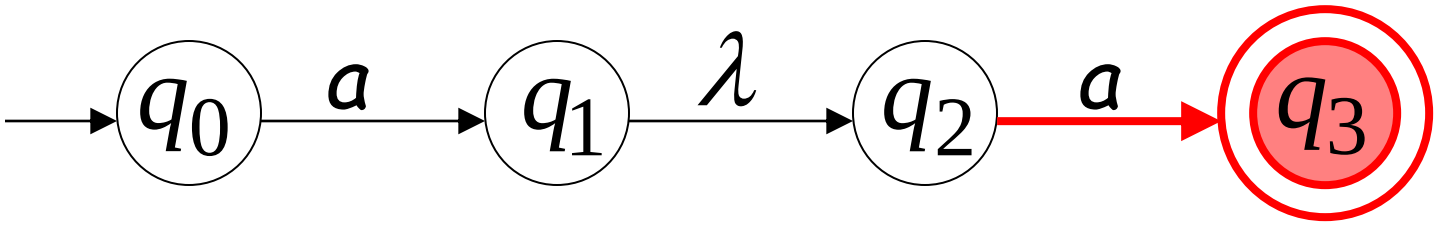
No transition:

the automaton hangs

Input cannot be consumed

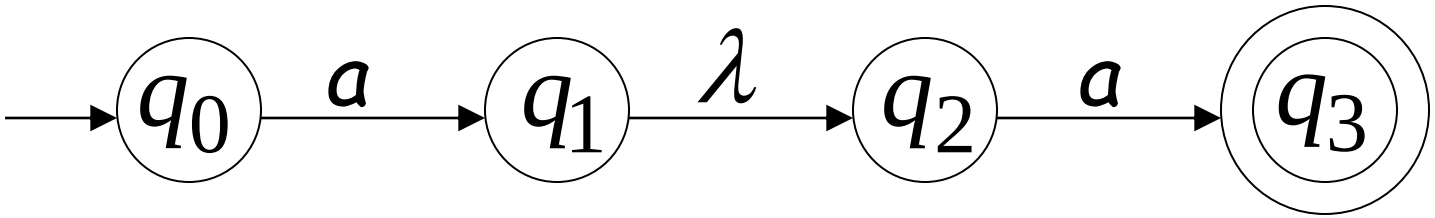


“reject”

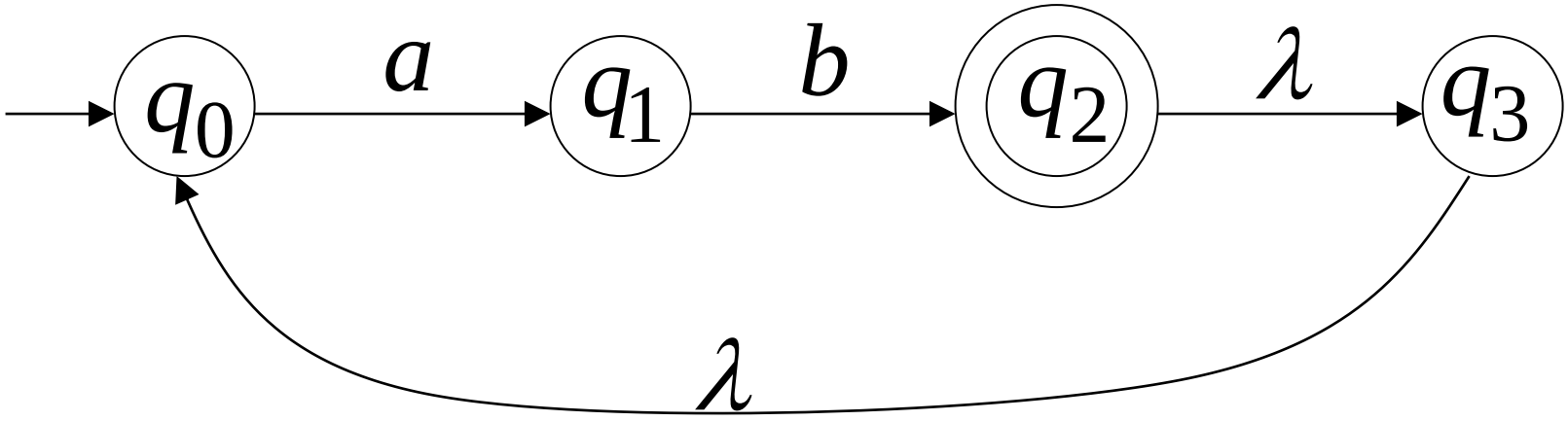


String aaa is rejected

Language accepted: $L = \{aa\}$



Another NFA Example



Consider the finite state machine whose transition function δ is given by Table 3.1 in the form of a transition table. Here, $Q = \{q_0, q_1, q_2, q_3\}$, $\Sigma = \{0, 1\}$, $F = \{q_0\}$. Give the entire sequence of states for the input string 110001.

TABLE 3.1 Transition Function Table for Example 3.5

State	*Input*	
	0	1
→ q_0	q_2	q_1
q_1	q_3	q_0
q_2	q_0	q_3
q_3	q_1	q_2

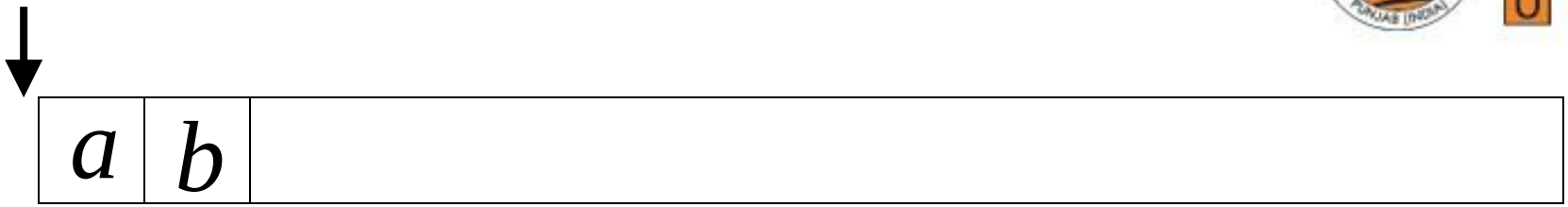
How to accept a string

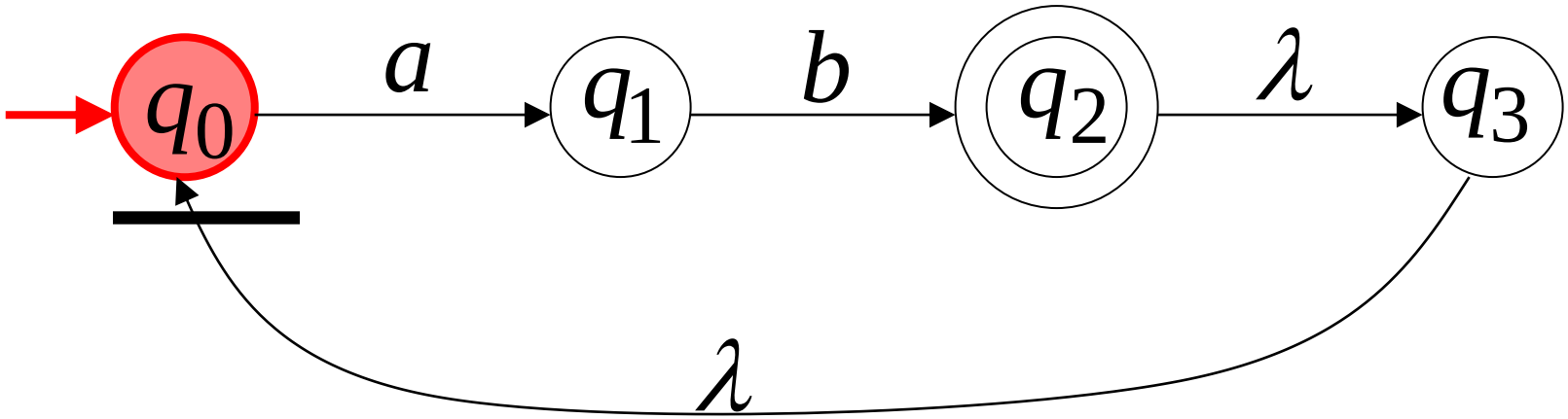
Solution

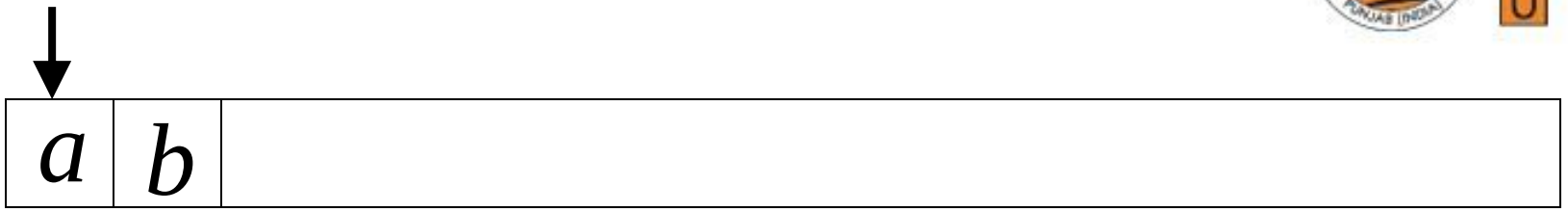
$$\begin{aligned}
\delta(q_0, \overset{\downarrow}{1}10101) &= \delta(q_1, \overset{\downarrow}{1}0101) \\
&= \delta(q_0, \overset{\downarrow}{0}101) \\
&= \delta(q_2, \overset{\downarrow}{1}01) \\
&= \delta(q_3, \overset{\downarrow}{0}1) \\
&= \delta(q_1, \overset{\downarrow}{1}) \\
&= \delta(q_0, \Lambda) \\
&= q_0
\end{aligned}$$

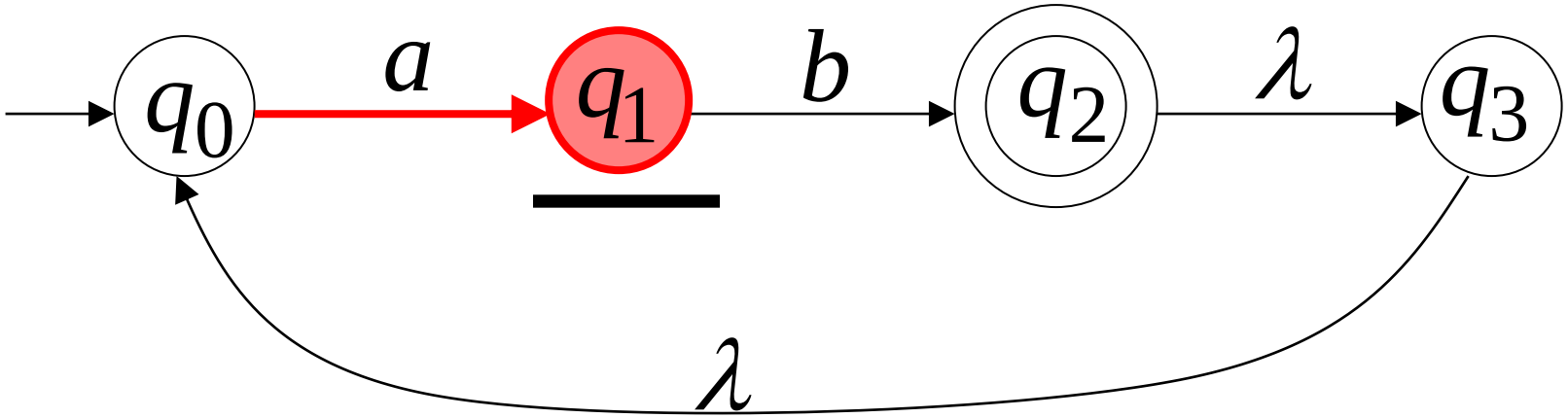
Hence,

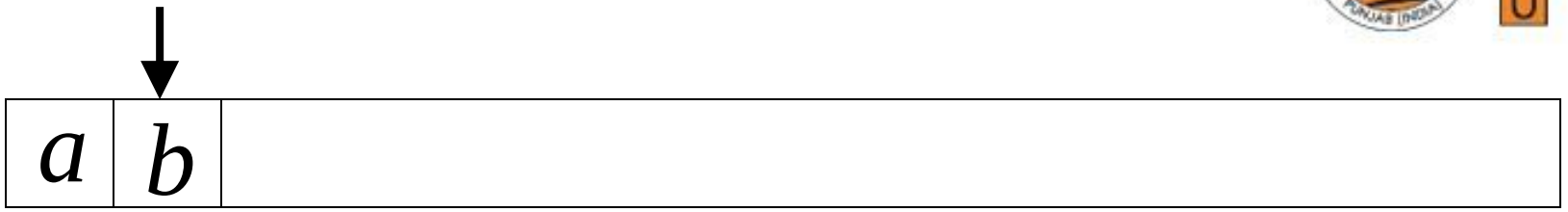
$$q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_0 \xrightarrow{0} q_2 \xrightarrow{1} q_3 \xrightarrow{0} q_1 \xrightarrow{1} q_0$$

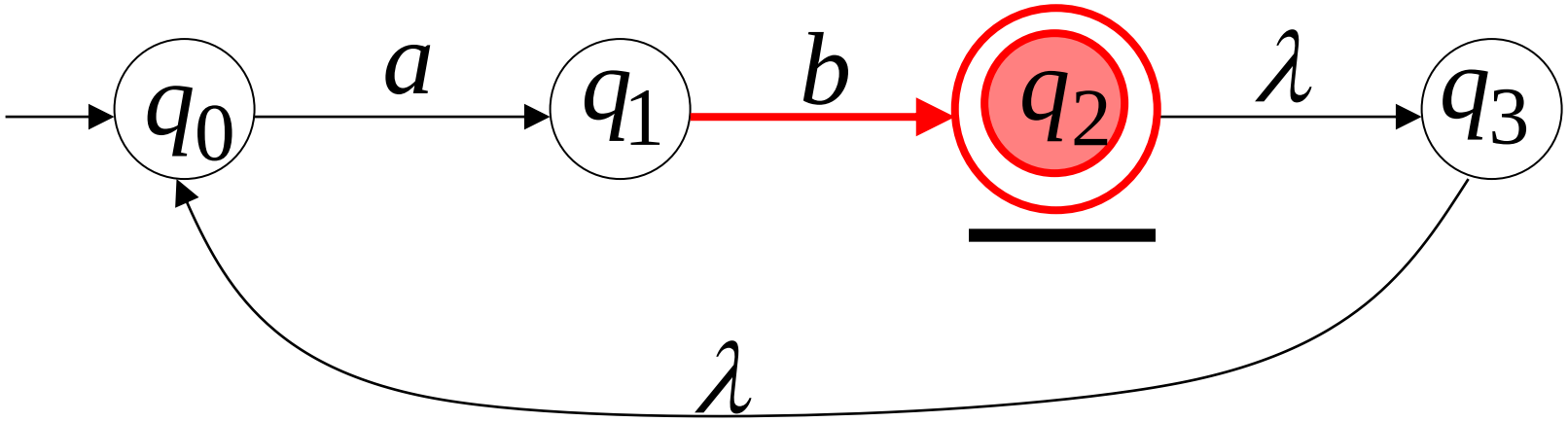

a
b

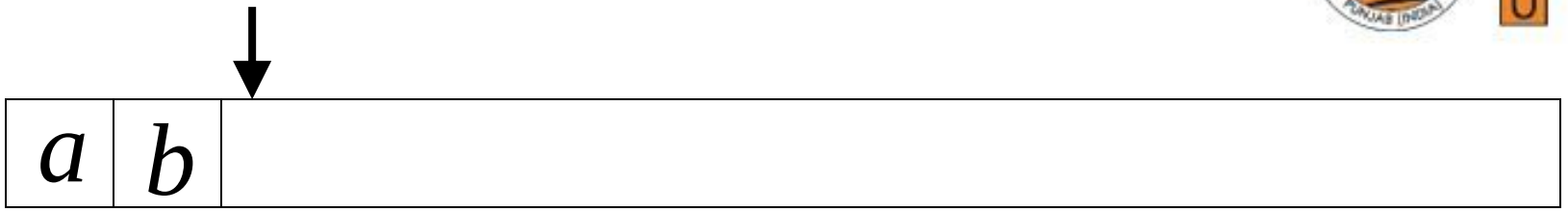

q_0
a
q_1
b
q_2
λ
q_3
λ

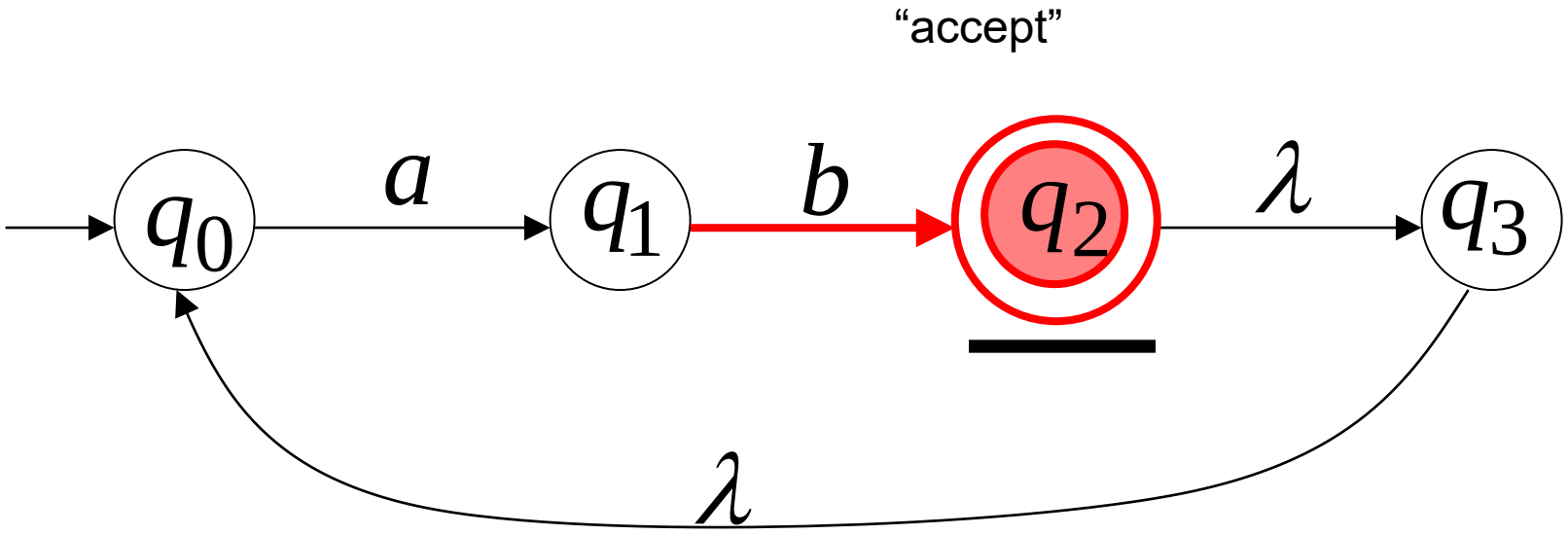

a
b

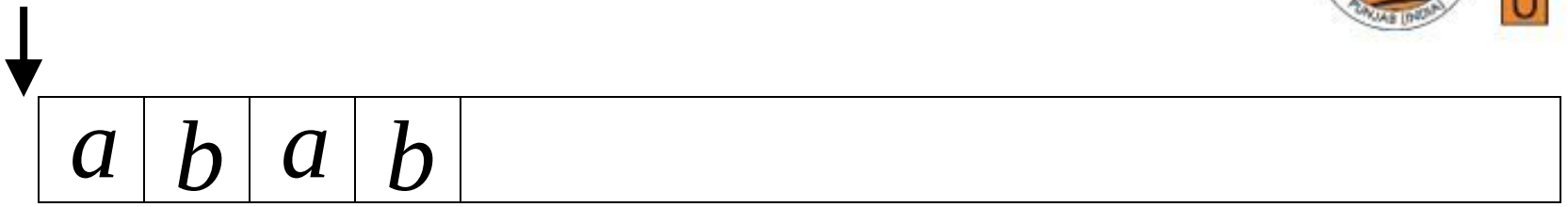

q_0
a
q_1
b
q_2
λ
q_3
λ

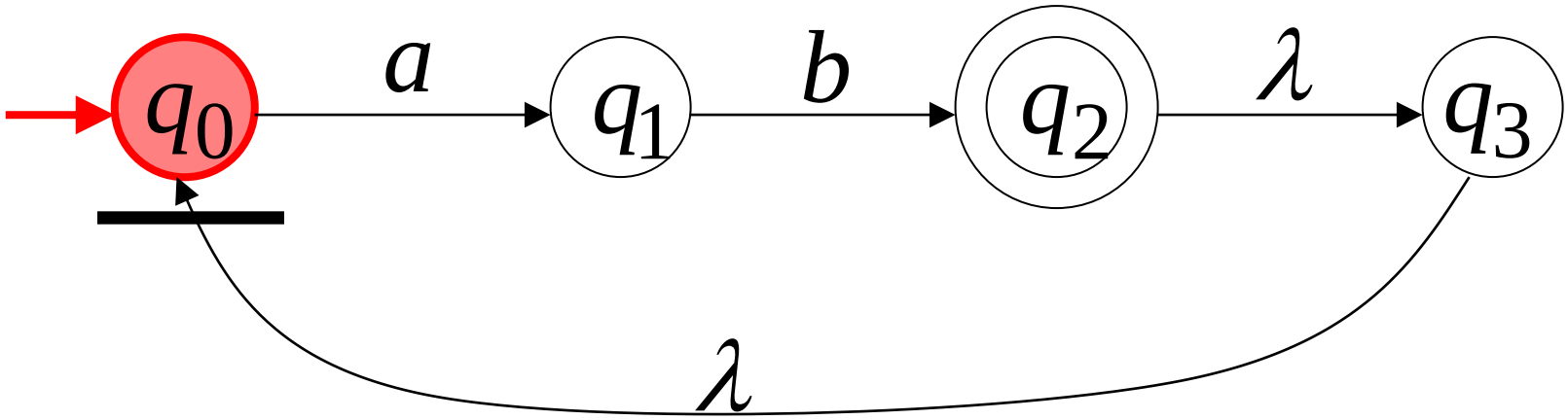

a
b

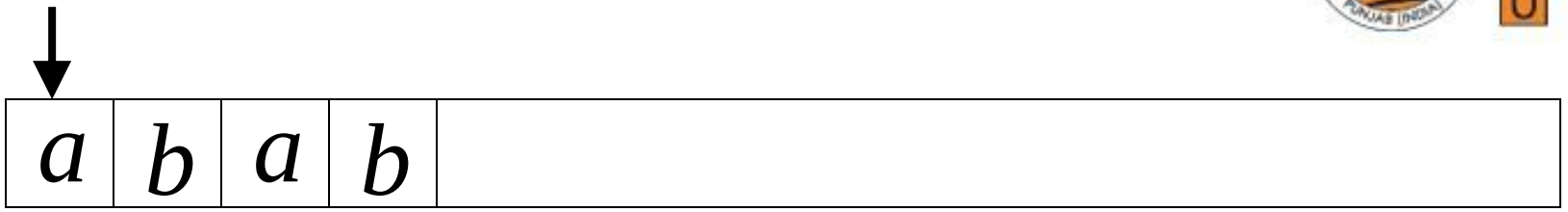

q_0
a
q_1
b
q_2
λ
q_3
λ

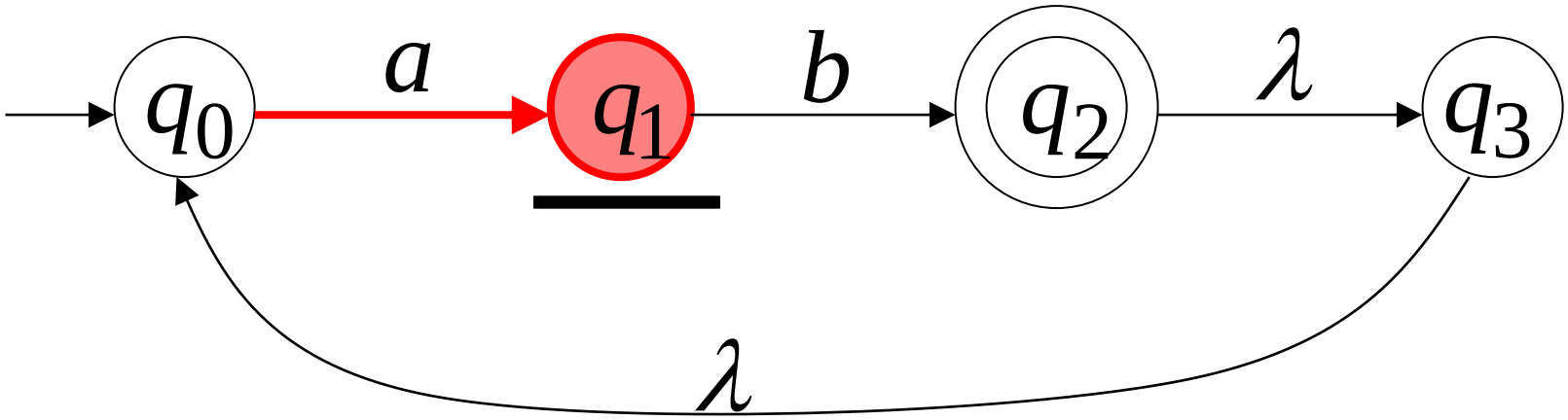

a
b

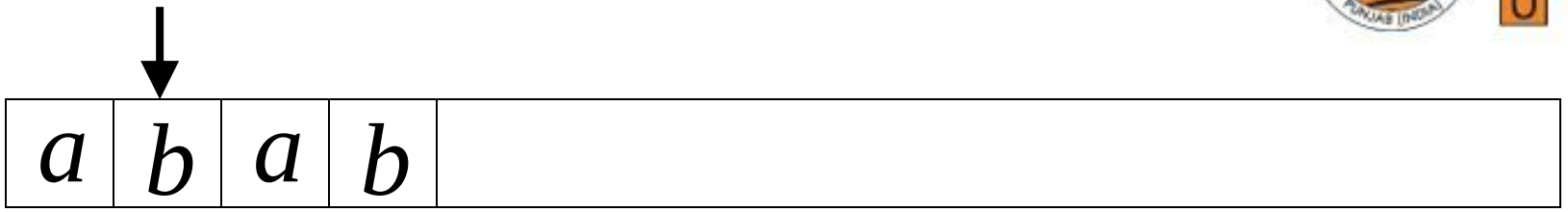

"accept"
q_0
a
q_1
b
q_2
λ
q_3
λ

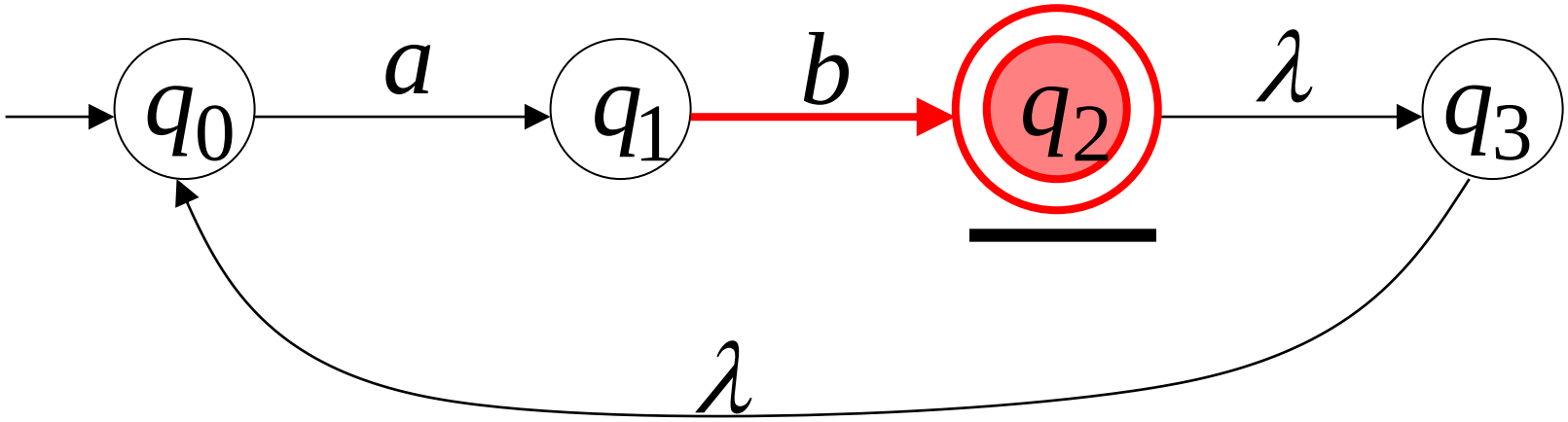

a
b
a
b

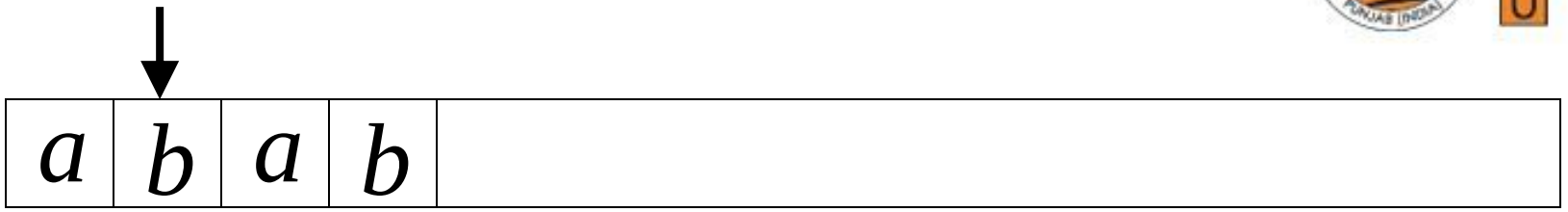

q_0
a
q_1
b
q_2
λ
q_3
λ

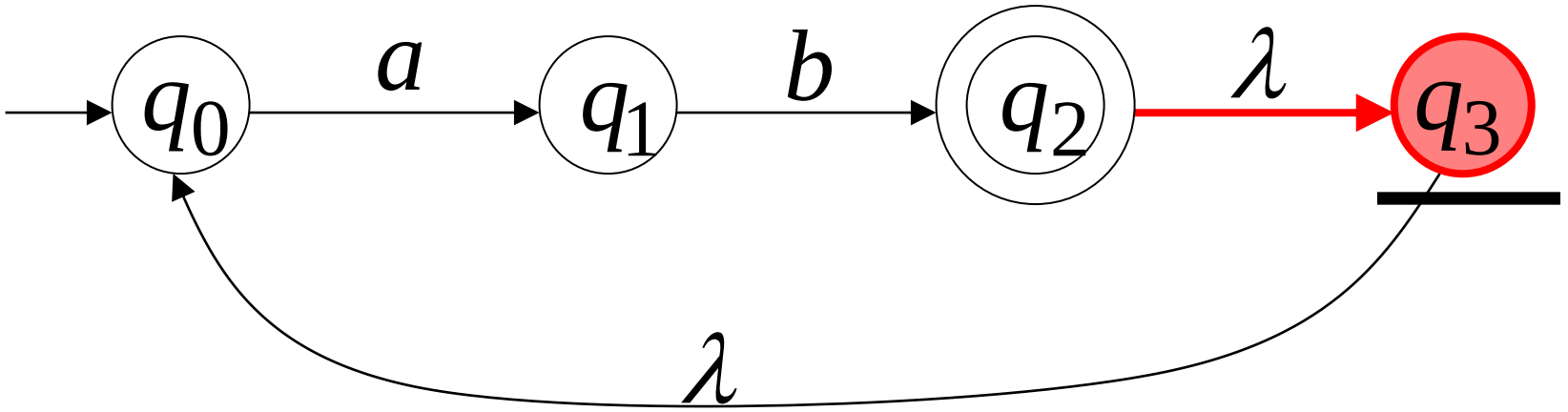

a
b
a
b

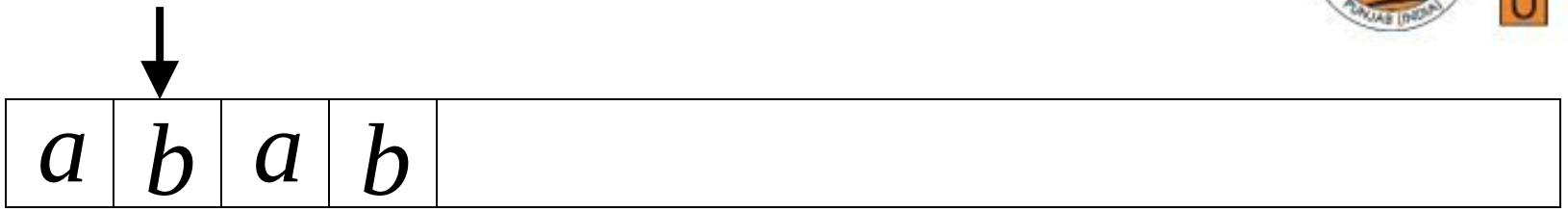

q_0
a
q_1
b
q_2
λ
q_3
λ

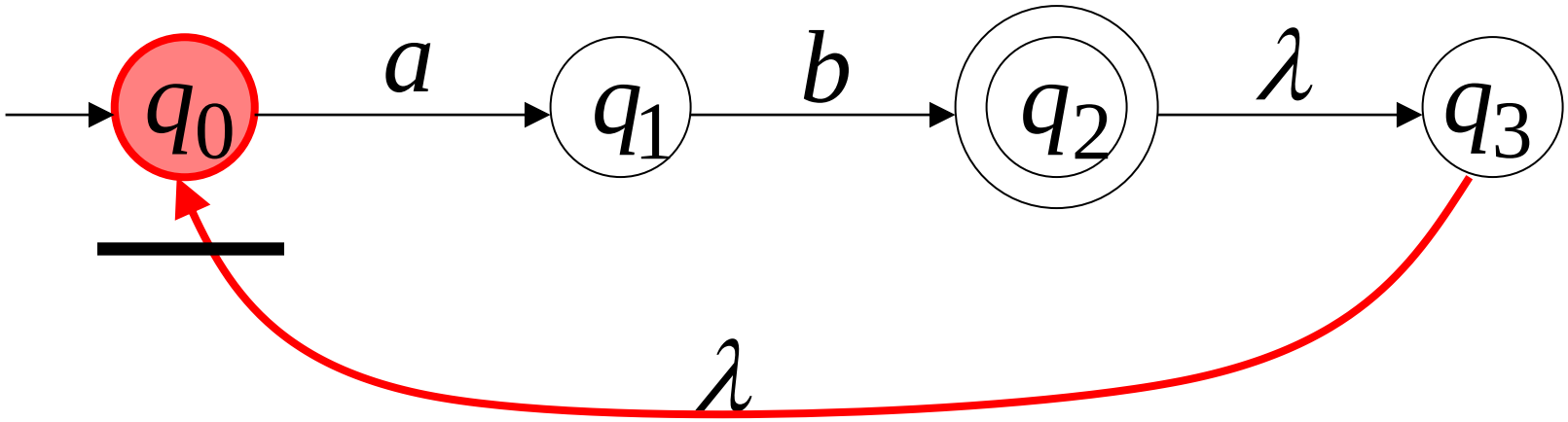

a
b
a
b

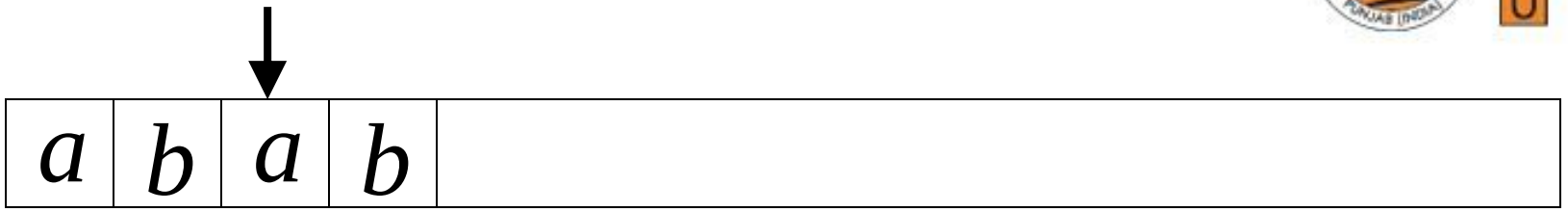

q_0
a
q_1
b
q_2
λ
q_3
λ

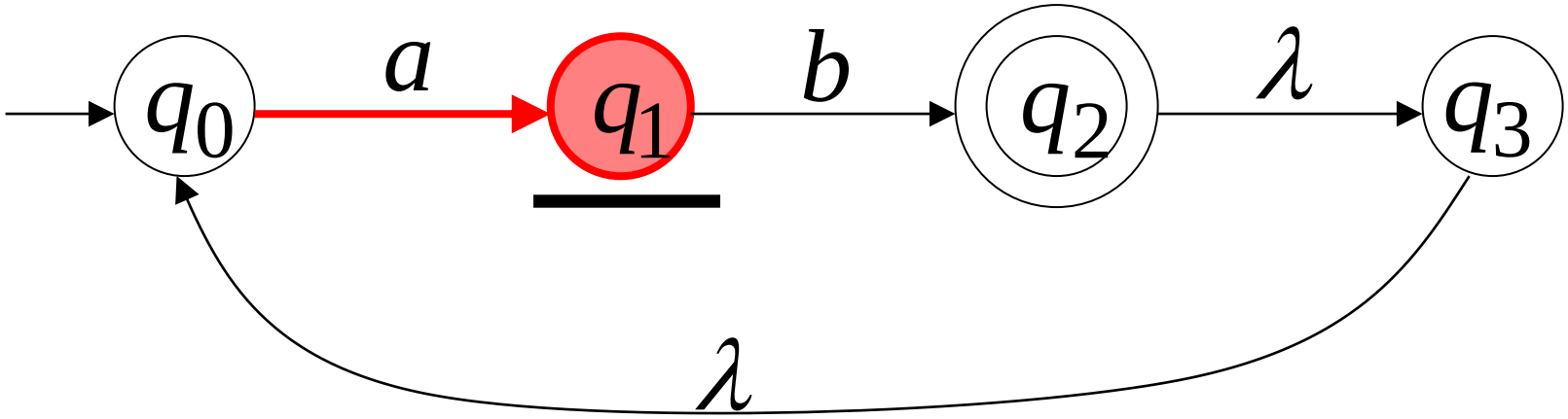

a
b
a
b

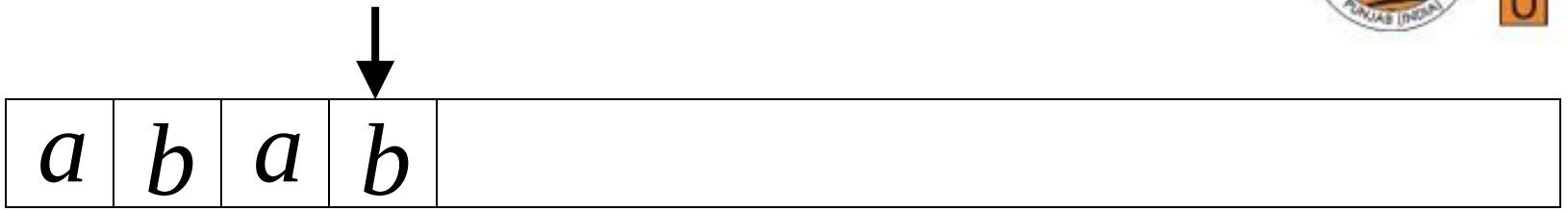

q_0
a
q_1
b
q_2
λ
q_3
λ

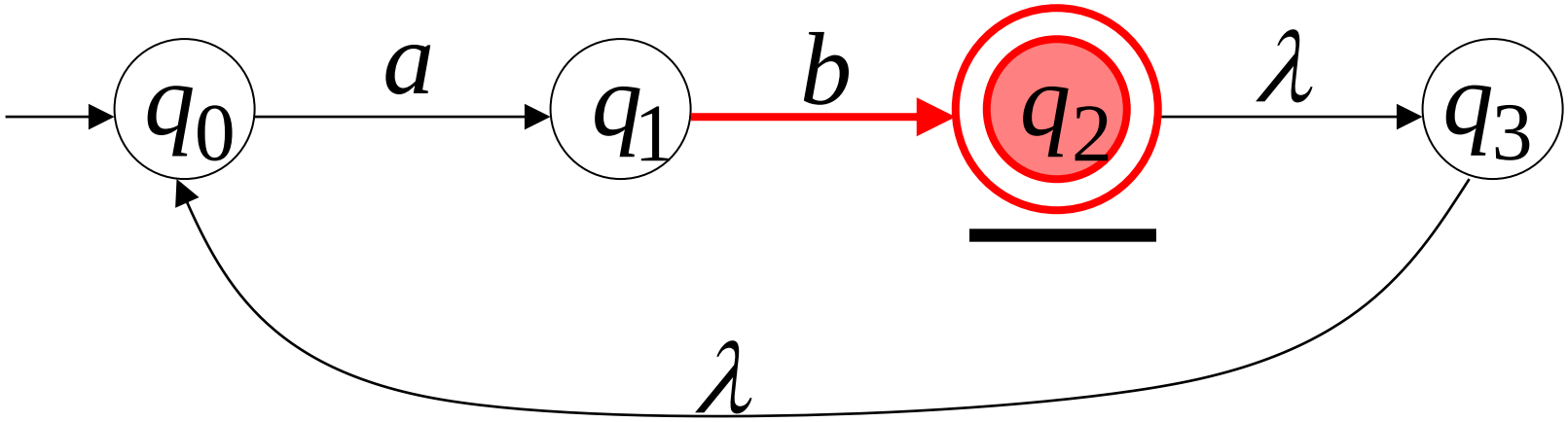

a
b
a
b

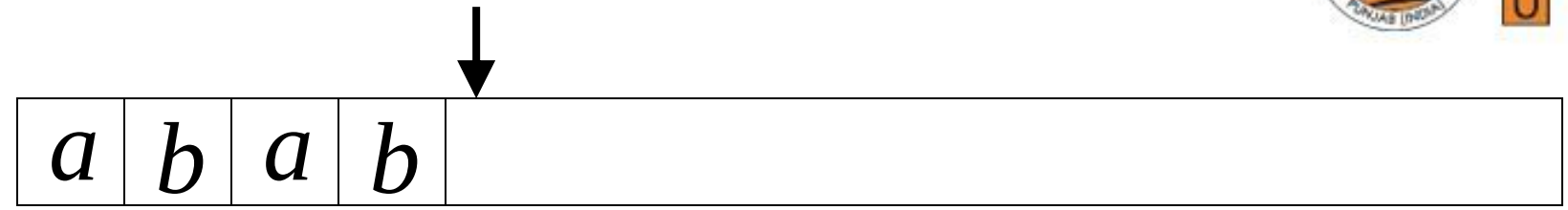

q_0
a
q_1
b
q_2
λ
q_3
λ

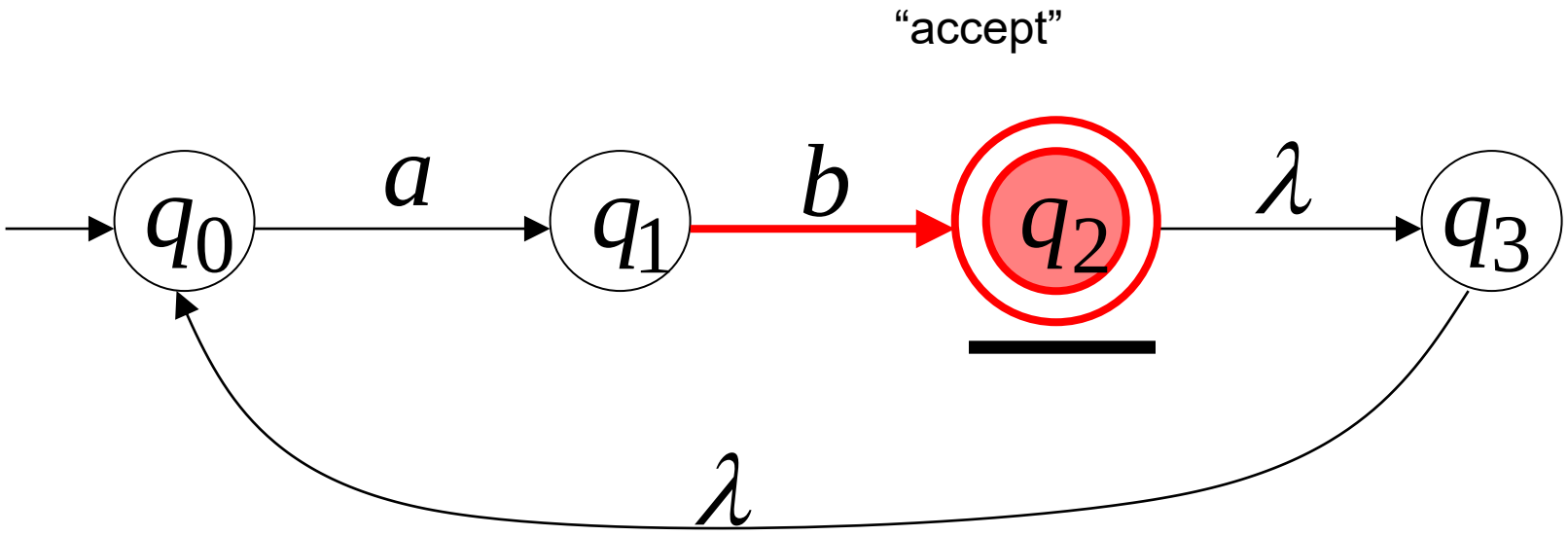

a
b
a
b


q_0
a
q_1
b
q_2
λ
q_3
λ


a
b
a
b

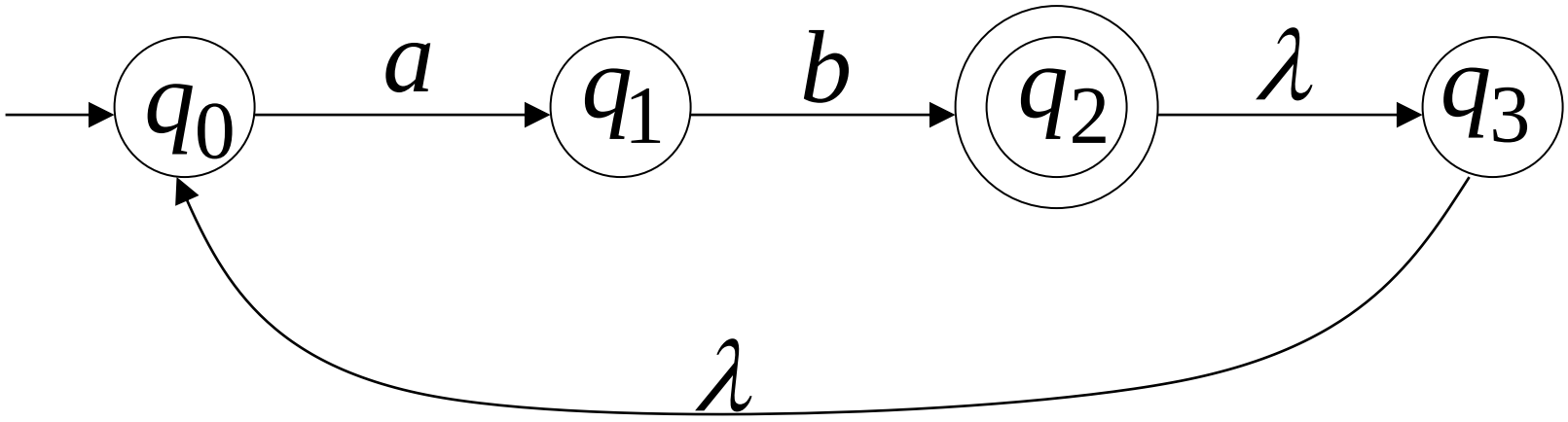

q_0
a
q_1
b
q_2
λ
q_3
λ


a
b
a
b

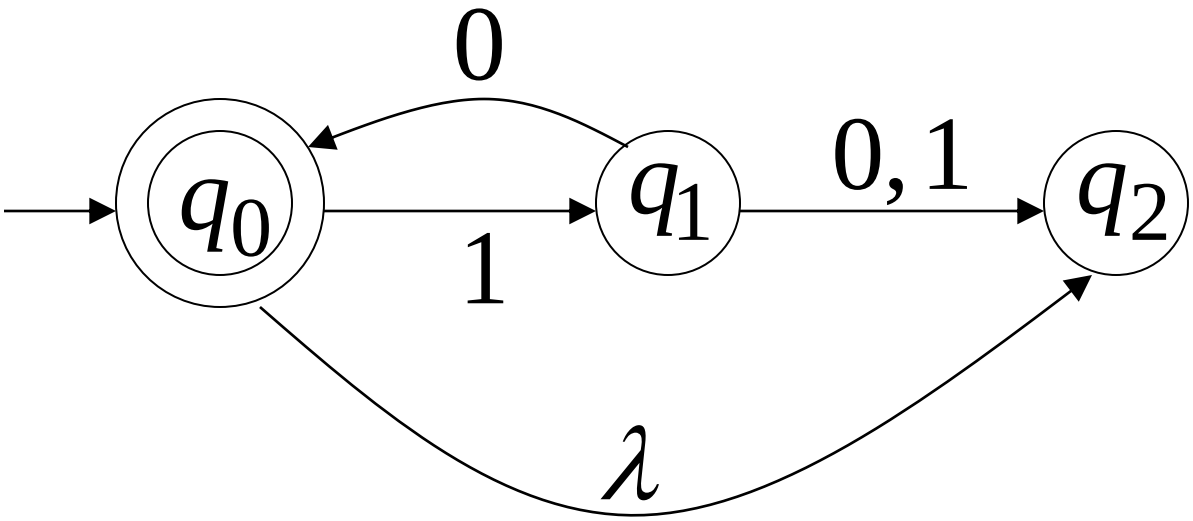

"accept"
q_0
q_1
q_2
q_3
a
b
λ
λ

$$L = \{ab,\ abab,\ ababab,\ ...\}$$

$$= \{ab\}^{+}$$

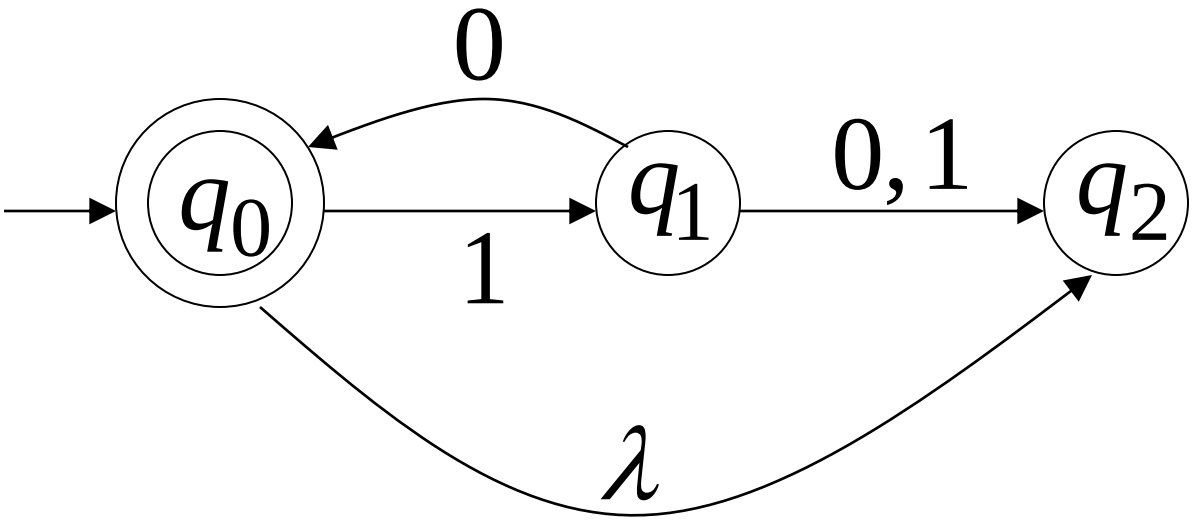


Another NFA Example



$$L(M) = \{\lambda,\ 10,\ 1010,\ 101010,\ \ldots\}$$

$$= \{10\}*$$



(redundant state)

Mealy and Moore Model

- In finite Automata acceptability was decided on the basis of reach ability of the final state by initial state.
- This restriction are removed and new model is given in which output can be chosen from some other alphabet.

- The value of the output function $Z(t)$ is a function of present state $q(t)$ and the present input $x(t)$
- $Z(t) = \lambda(q(t), x(t))$ *Mealy Machine*

- The value of the output function $Z(t)$ is a function of present state q(t) only and is independent of the current input
- $Z(t) = \lambda(q(t))$ *Moore Machine*

Moore Machine

Moore Machine is six-tuple ($Q,\sum,\Delta,\delta,\lambda,q_0$):

(i) Q is a finite set of states

(ii) $\sum$ is the input alphabet

(iii) Δ is the output alphabet

(iv) δ is the transition function from $\sum$ X Q into Q

(v) λ is the output function mapping Q into Δ and

(vi) q_0 is the initial state

Mealy Machine

Mealy Machine is six-tuple ($Q,\sum,\Delta,\delta,\lambda,q_0$):

(i) Q is a finite set of states

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(iv) δ is the transition function from $\sum$ X Q into Q

(v) λ is the output function mapping $\sum$ X Q into Δ and

(vi) q_0 is the initial state

Example of Moore Machine

Present state	Next state δ		Output
	$a = 0$	$a = 1$	λ
$\rightarrow q_0$	q_3	q_1	0
q_1	q_1	q_2	1
q_2	q_2	q_3	0
q_3	q_3	q_0	0

For the input string 0111, the transition of states is given by $q_0 \rightarrow q_3 \rightarrow q_0 \rightarrow q_1 \rightarrow q_2$. The output string is 00010. For the input string Λ, the output is $\lambda(q_0) = 0$.

Example of Mealy Machine

Present state	Next state			
	a = 0		a = 1	
	state	output	state	output
$\rightarrow q_1$	q_3	0	q_2	0
q_2	q_4	1	q_4	0
q_3	q_2	1	q_1	1
q_4	q_4	1	q_3	0

Note: For the input string 0011. the transition of states is given by $q_1 \rightarrow q_3 \rightarrow q_2 \rightarrow q_4 \rightarrow q_3$, and the output string is 0100. In the case of a Mealy machine, we get an output only on the application of an input symbol. So for the input string Λ, the output is only Λ. It may be observed that in the case of a Moore machine, we get $\lambda(q_0)$ for the input string Λ.

Transforming Mealy to Moore Machine

Consider the Mealy machine described by the transition table given by Table 3.10. Construct a Moore machine which is equivalent to the Mealy machine.

TABLE 3.10 Mealy Machine of Example 3.9

Present state	*Next state*			
	Input a = 0		*Input a = 1*	
	state	*output*	*state*	*output*
$\rightarrow q_1$	q_3	0	q_2	0
q_2	q_1	1	q_4	0
q_3	q_2	1	q_1	1
q_4	q_4	1	q_3	0

Solution

Present state	Next state			
	Input a = 0		Input a = 1	
	state	output	state	output
$\rightarrow q_1$	q_3	0	q_{20}	0
q_{20}	q_1	1	q_{40}	0
q_{21}	q_1	1	q_{40}	0
q_3	q_{21}	1	q_1	1
q_{40}	q_{41}	1	q_3	0
q_{41}	q_{41}	1	q_3	0

Present state	Next state		Output
	a = 0	a = 1	
$\rightarrow q_1$	q_3	q_{20}	1
q_{20}	q_1	q_{40}	0
q_{21}	q_1	q_{40}	1
q_3	q_{21}	q_1	0
q_{40}	q_{41}	q_3	0
q_{41}	q_{41}	q_3	1

Transforming Moore to Mealy Machine

Construct a Mealy Machine which is equivalent to the Moore machine given by Table 3.14.

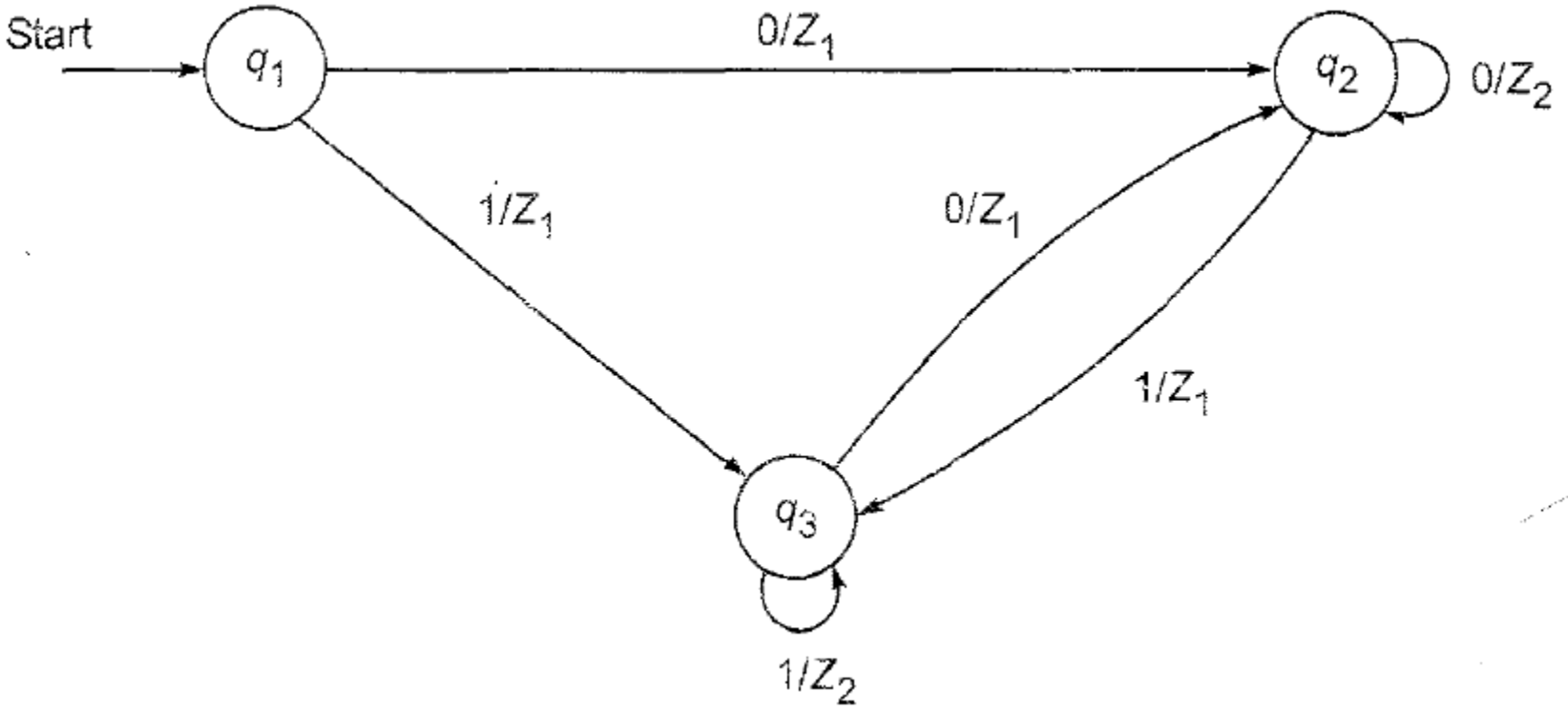
TABLE 3.14 Moore Machine of Example 3.10

Present state	*Next state*		*Output*
	$a = 0$	$a = 1$	
$\rightarrow q_0$	q_3	q_1	0
q_1	q_1	q_2	1
q_2	q_2	q_3	0
q_3	q_3	q_0	0

Solution

Present state	Next state			
	a = 0		a = 1	
	state	output	state	output
$\rightarrow q_0$	q_3	0	q_1	1
q_1	q_3	1	q_2	0
q_2	q_2	0	q_3	0
q_3	q_3	0	q_0	0

Consider a Mealy machine represented by Fig. 3.10. Construct a Moore machine equivalent to this Mealy machine.



Solution

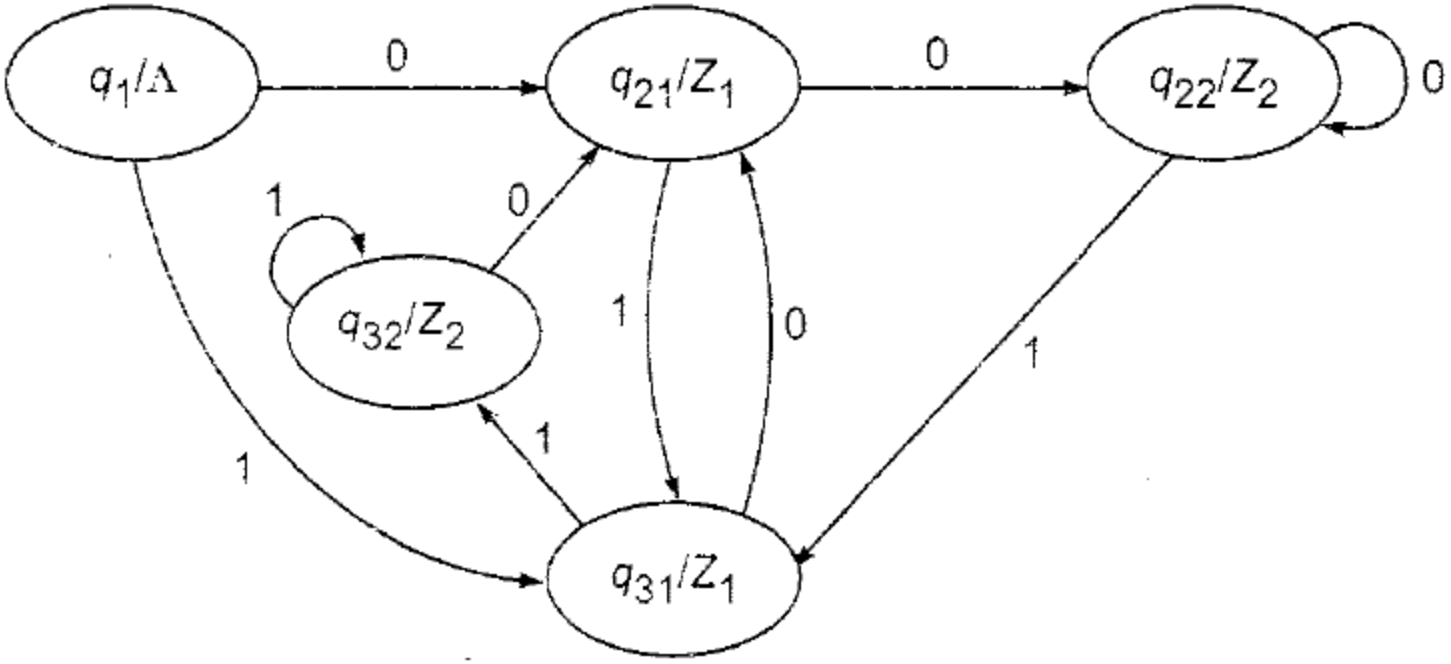
Present state	Next state			
	$a = 0$		$a = 1$	
	state	output	state	output
$\rightarrow q_1$	q_2	Z_1	q_3	Z_1
q_2	q_2	Z_2	q_3	Z_1
q_3	q_2	Z_1	q_3	Z_2

Present state	Next state		Output
	$a = 0$	$a = 1$	
$\rightarrow q_1$	q_{21}	q_{31}	
q_{21}	q_{22}	q_{31}	Z_1
q_{22}	q_{22}	q_{31}	Z_2
q_{31}	q_{21}	q_{32}	Z_1
q_{32}	q_{21}	q_{32}	Z_2

Answer Justification

- Here there is no output associated with q1 so, It will be ^.
- q2 is associated with 2 outputs z1 and z2 so 2 states will be considered q21,q22.
- q3 is associated with 2 outputs z1 and z2 so 2 states will be considered q31,q32

Answer



Consider the Moore machine described by the transition table given

Present state	Next state		Output
	a = 0	a = 1	
$\rightarrow q_1$	q_1	q_2	0
q_2	q_1	q_3	0
q_3	q_1	q_3	1

Solution

Present state	Next state			
	a = 0		a = 1	
	state	output	state	output
$\rightarrow q_1$	q_1	0	q_2	0
q_2	q_1	0	q_3	1
q_3	q_1	0	q_3	1

Convert it to Moore Machine

Present state	*Next state*			
	a = 0		*a* = 1	
	state	*output*	*state*	*output*
$\rightarrow q_1$	q_1	0	q_2	0
q_2	q_1	0	q_2	1

Convert it to mealy machine