Friday, August 21, 2020 10:00 AM

Chapter 1 : Matrices and Eigenvalue Problems

Some Important results of Matrices :

① It is possible that product of two matrices AB is defined but BA may not be defined

$A = 2 \times 3$ & $B \rightarrow 3 \times 4 \rightarrow \underline{AB}$

$B \rightarrow 3 \times 4$ & $A = 2 \times 3 \longrightarrow BA$

If AB & BA both are defined then AB & BA are square matrices.

for ex $A \rightarrow 3 \times 4$ & $B \rightarrow 4 \times 3$

$AB \longrightarrow 3 \times 3$ & $BA \rightarrow$

$B \rightarrow 4 \times 3$ & $A \rightarrow 3 \times 4$

$BA \rightarrow 4 \times 4$ $\underline{\text{Square}}$

$$A = \begin{bmatrix} 1 & 2 & 0 & 2 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 2 \end{bmatrix}_{3 \times 4} \quad \& \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 1 & 2 & 0 \\ 1 & 1 & 1 \end{bmatrix}_{4 \times 3}$$

$$AB = \begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix}_{3 \times 3}$$

If $AB \neq BA$, in case $AB = BA \rightarrow \underline{A \,\&\, B}$ Commute each other.

② $AB = 0$ then A&B may not be zero | $xy = 0 \Rightarrow x = 0$ or $y = 0$

$$A = \begin{bmatrix} x & 0 \\ y & 0 \end{bmatrix}_{2 \times 2} \quad \& \quad B = \begin{bmatrix} 0 & 0 \\ a & b \end{bmatrix}_{2 \times 2}$$

$$AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

③ $A^K = \underbrace{A \times A \times A \times \cdots \times A}_{K \text{ times}}$

Transpose of Matrix:

$A = [a_{11} \quad a_{12} \quad a_{13}]$

Transpose of Matrix:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times 3}$$

$$A^T =$$

In case $\underbrace{A \cdot A \cdots A}_{k \text{ times}} = A^k = 0 \rightarrow$ Nilpotent matrix

The smallest value of K is said to be index of nilpotency of matrix

Symmetric and Skew Symmetric matrices :

$$A = (a_{ij})$$

$$A^T = A$$

$$(a_{ij}) = (a_{ji})$$

Symmetric & if $a_{ij} = -a_{ji}$ or $A^T = -A$ skew symmetric

(4) $A^2 = A \rightarrow$ idempotent matrix

(5) $A^2 = I \rightarrow$ Involutary matrix

(6) $AB = AC \not\Rightarrow B = C$

Conjugate Matrix:

$$A = (a_{ij})$$

$$\bar{A} = (\bar{a}_{ij})$$

$$\bar{3} = 3$$

$$\overline{x + iy} = x - iy$$

(1) $a_{ii} = -a_{ii} \Rightarrow 2a_{ii} = 0$

$a_{ii} = 0 \quad \forall i$

Diagonal zero

Hermitian amd Skew-Hermitian Matrices:

(2) If $A \rightarrow$ sym & skew sym

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \cdots \end{bmatrix} \rightarrow \text{Null Matrix}$$

Ex

Every square matrix can be expressed as sum of symmetric & skew symm

$$A = \underbrace{\frac{1}{2}(A + A^T)}_{\text{sym}} + \underbrace{\frac{1}{2}(A - A^T)}_{\text{skw}}$$

Identify the following matrices

$$1)\begin{bmatrix} \mathbf{a} & \mathbf{b} & \mathbf{e} \\ \mathbf{b} & \mathbf{d} & \mathbf{e} \\ \mathbf{c} & \mathbf{e} & \mathbf{f} \end{bmatrix} \qquad 2)\begin{bmatrix} \mathbf{0} & \mathbf{b} & \mathbf{e} \\ -\mathbf{b} & \mathbf{0} & \mathbf{e} \\ -\mathbf{c} & -\mathbf{e} & \mathbf{0} \end{bmatrix}$$

3) $\begin{bmatrix} 1 & 2+4i & 1-i \\ 2-4i & -5 & 3-5i \\ 1+i & 3+5i & 6 \end{bmatrix}$ 4) $\begin{bmatrix} 0 & 2+4i & 1-i \\ -2+4i & 0 & 3-5i \\ -1-i & -3-5i & 0 \end{bmatrix}$

If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$ then A^2 is equal to

a) unit matrix **b) null matrix** **c) A** **d) −A**

Hermitian and Skew Hermitian

$(A)^T = A \rightarrow$ Symmetric

① $(\bar{A})^T = A \rightarrow$ ~~Hermitian matrix~~

② $(\bar{A})^T = -A \rightarrow$ skew Hermitian matrix

① $A \rightarrow$ real $\quad \bar{A} = A$ skew Sym matrix

① Sym $\rightarrow$ Herm $\rightarrow A$

Sk Sy $\rightarrow$ Skew Her $\rightarrow A$

② $\bar{a}_{ii} = a_{ii}$ $\quad a_{ii} = x_j + iy_j$

$x_j - iy_j = x_j + iy_j$

$\Rightarrow y_j = 0$

$a_{ii} = x_j \rightarrow$ real no

In case of Hermitian matrix the elements on the main diagonal are real nrs

(3) $\bar{a}_{ii} = -a_{ii}$

$x_j - iy_j = -x_j + iy_j$. $\Rightarrow x_j = 0$

$a_{ii} = iy_j$ → purely imaginary

$y_j = 0$

$a_{ii} = 0$

If