$\alpha x \qquad \sigma y \qquad n-1$

## OBJECTIVE TYPE QUESTIONS

**Choose the correct alternative:**

1. The value of $\lim_{(x,y)\to(0,0)} (x+y)\sin\frac{1}{(x+y)}, x\neq 0, y\neq 0$ is

   (*i*) limit does not exist (*ii*) 0 (*iii*) 1 (*iv*) $-1$ **Ans.** (*i*)

2. The value of the $\lim_{(x,y)\to(0,0)} \frac{x+\sqrt{y}}{\sqrt{(x^2+y)}}, x\neq 0, y\neq 0$ is

   (*i*) limit does not exist (*ii*) 0 (*iii*) 1 (*iv*) $-1$

   (*AMIETE, Dec.* 2007) **Ans.** (*iii*)

3. The value of $\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^4+y^2}$ is

   (*i*) 0 (*ii*) $\frac{1}{2}$ (*iii*) 1 (*iv*) Does not exist **Ans.** (*iv*)

4. The value of $\lim_{(x,y)\to(0,0)} \frac{x.\sin(x^2+y^2)}{x^2+y^2}$ is

   (*i*) 0 (*ii*) 1 (*iii*) $-1$ (*iv*) Does not exist **Ans.** (*i*)

5. The value of limit $\lim_{\substack{x\to 1\\ y\to 1}} \frac{8x^2y}{x^2+y^2+5}$ is

   (*i*) $\frac{3}{7}$ (*ii*) $\frac{8}{5}$ (*iii*) $\frac{8}{7}$ (*iv*) None of these **Ans.** (*iii*)

6. The value of $\lim_{\substack{x\to 1\\ y\to 2}} \frac{4xy}{6x^2+y^2}$ is

   (*i*) $\frac{4}{5}$ (*ii*) $\frac{2}{3}$ (*iii*) $\frac{3}{10}$ (*iv*) None of these **Ans.** (*i*)

**7.** The value of $\lim_{\substack{y\to 0\\ x\to 1}} \frac{2x^2+y}{4x-y}$ is

(*i*) $\frac{3}{2}$ (*ii*) $\frac{1}{2}$ (*iii*) 1 (*iv*) None of these **Ans.** (*ii*)

**8.** The value of $\lim_{\substack{x\to 0\\ y\to 0}} \frac{2x^2+y}{4x^2-y}$ is

(*i*) $-1$ (*ii*) $\frac{1}{2}$ (*iii*) 1 (*iv*) Limit does not exist **Ans.** (*iv*)

**9.** The value of $\lim_{\substack{y\to 0\\ x\to 0}} \frac{2x^2+y}{4x^2-y}$ is

(*i*) $\frac{3}{4}$ (*ii*) $\frac{2}{1}$ (*iii*) $\frac{1}{2}$ (*iv*) None of these **Ans.** (*iv*)

**10.** If $u = x^2 + y^2$ then the value of $\frac{\partial^2 u}{\partial x \partial y}$ is equal to

(*i*) 0 (*ii*) 2 (*iii*) $2x + 2y$ (*iv*) $y\,x^{y-1}$

*(A.M.I.E.T.E. Dec. 2008)* **Ans.** (*i*)

**11.** If $u = y^x$, then $\frac{\partial u}{\partial x}$ is

(*i*) $xy^{x-1}$ (*ii*) 0 (*iii*) $y^x \log y$ (*iv*) none of these **Ans.** (*iii*)

**12.** If $u = \log\left(\frac{x^2}{y}\right)$, then the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ *is*

(*i*) $2u$ (*ii*) $u$ (*iii*) 0 (*iv*) 1

*(A.M.I.E.T.E. Dec. 2008)* **Ans.** (*iv*)

**13.** If $x = r\cos\theta$, $y = r\sin\theta$, then

(*i*) $\frac{\partial x}{\partial r} = \frac{\partial r}{\partial x}$ (*ii*) $\frac{\partial x}{\partial \theta} = 0$ (*iii*) $\frac{\partial x}{\partial r} = 0$ (*iv*) $\frac{\partial x}{\partial r} = \frac{1}{\partial r/\partial x}$ **Ans.** (*i*)

**14.** If $u = y^x$ then $\frac{\partial u}{\partial y}$ is

(*i*) $xy^{x-1}$ (*ii*) $y^x \log y$ (*iii*) 0 (*iv*) none of these **Ans.** (*i*)

**15.** If $u = x^y$ then the value of $\frac{\partial u}{\partial y}$ is equal to

(*i*) 0 (*ii*) $x^y \log(x)$ (*iii*) $xy^{x-1}$ (*iv*) $yx^{y-1}$

*(A.M.I.E.T.E. Dec. 2007)* **Ans.** (*ii*)

**16.** If $x = r\cos\theta$, $y = r\sin\theta$, then $\frac{\partial r}{\partial x}$ is equal to

(*i*) $\sec\theta$ (*ii*) $\sin\theta$ (*iii*) $\cos\theta$ (*iv*) $\operatorname{cosec}\theta$ **Ans.** (*iii*)

**17.** If $u = \tan^{-1}(x + y)$, then $(u_x - u_y)$ equals

(*i*) 0 (*ii*) 1 (*iii*) $-1$ (*iv*) $\sin x \cos y$ **Ans.** (*i*)

**18.** If $P = r\tan\theta$, then $\frac{\partial P}{\partial r}$ is equal to

(*i*) $\tan\theta$ (*ii*) $\sec^2\theta$ (*iii*) $\tan\theta + r\sec^2\theta$ (*iv*) $\frac{1}{2}\tan\theta$ **Ans.** (*i*)

**19.** If $Q = r \cot \theta$, then $\frac{\partial Q}{\partial r}$ is equal to

(*i*) $\cot \theta$ (*ii*) $-\operatorname{cosec}^2 \theta$ (*iii*) $\cot \theta - r \operatorname{cosec}^2 \theta$ (*iv*) $\frac{1}{2}\cot \theta$ Ans. (*i*)

**20.** If $f(x, y, z) = 0$, then the value of $\frac{\partial x}{\partial y}\cdot\frac{\partial y}{\partial z}\cdot\frac{\partial z}{\partial x}$ is:

(*i*) 1 (*ii*) –1 (*iii*) 0 (*iv*) None of these Ans. (*ii*)

**21.** If $f(x, y) = 0$, then $\frac{dy}{dx}$ is equal to

(*i*) $\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}$ (*ii*) $\frac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial x}}$ (*iii*) $-\frac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial x}}$ (*iv*) $-\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}$ Ans. (*iv*)

**22.** If $f(x, y, z) = \frac{x^2}{y^2}+\frac{y^2}{z^2}+\frac{z^2}{x^2}$, then $x f_x + y f_y + z f_z$ is

(*i*) 0 (*ii*) –1 (*iii*) 1 (*iv*) 2 Ans. (*i*)

**23.** If $x = r \cos \theta$, $y = r \sin \theta$ then $\frac{\partial r}{\partial x}$ is equal to

(*i*) $\sec \theta$ (*ii*) $\sin \theta$ (*iii*) $\cos \theta$ (*iv*) $\operatorname{cosec} \theta$ Ans. (*iii*)

**24.** If $u = ax^2 + 2hxy + by^2$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ is equal to

(*i*) $2u$ (*ii*) $u$ (*iii*) 0 (*iv*) None of these Ans. (*i*)

**25.** If $P = s \tan \theta$, $q = s \cot \theta$, then

(*a*) $\frac{\partial P}{\partial S}$ is equal to

(*i*) $\tan \theta$ (*ii*) $\sec^2 \theta$ (*iii*) $\tan \theta + s \sec^2 \theta$ (*iv*) $\frac{1}{2}\tan \theta$ Ans. (*i*)

(*b*) $\frac{\partial q}{\partial s}$ is equal to

(*i*) $\cot \theta$ (*ii*) $-\operatorname{cosec}^2 \theta$ (*iii*) $\cot \theta - s \operatorname{cosec}^2 \theta$ (*iv*) $\frac{1}{2}\cot \theta$ Ans. (*i*)

(*c*) $\frac{\partial s}{\partial p}$ is equal to

(*i*) $\cot \theta$ (*ii*) $\cos^2 \theta$ (*iii*) $\frac{1}{\tan\theta + s\sec^2\theta}$ (*iv*) $\frac{1}{2}\cot \theta$ Ans. (*iv*)

(*d*) $\frac{\partial s}{\partial q}$ is equal to

(*i*) $\tan \theta$ (*ii*) $-\sin^2 \theta$ (*iii*) $\frac{1}{\cot\theta + s\sec^2\theta}$ (*iv*) $\frac{1}{2}\tan \theta$ Ans. (*iv*)

**26.** If $u = f\left(\frac{x}{y}\right)$ then

(*i*) $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = 0$ (*ii*) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$

(iii) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=u$ (iv) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=1$ **Ans. (ii)**

*(U.P. I Sem. Jan 2011)*

**27.** If $u=x^3\ e^{-\frac{x}{y}}$ then $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}$ is equal to

(i) $3u$ (ii) $6u$ (iii) $9u$ (iv) $-u$ **Ans. (ii)**

**28.** If $f(x, y)=\begin{cases}\frac{x^2+xy}{x+y} & (x,y)\neq(0,0)\\ 0 & (x,y)=(0,0)\end{cases}$ then $f_x(0, 0)$ equals

(i) $-1$ (ii) 0 (iii) 1 (iv) 1/2 **Ans. (iii)**

**29.** If $z=F(x^l y^k)$ satisfies the equation $x\frac{\partial z}{\partial x}-2y\frac{\partial z}{\partial y}=0$, then $\frac{l}{k}$ equals

(i) 1 (ii) 2 (iii) 3 (iv) 4 **Ans. (ii)**

**30.** If $Z=g(x^a y^b)$ satisfies the equation $2x\frac{\partial z}{\partial x}-3y\frac{\partial z}{\partial y}=0$ then $\frac{b}{a}$ satisfies

(i) $3b^2=4a^2$ (ii) $3a^2=4b^2$ (iii) $4b^2=9a^2$ (iv) $9b^2=4a^2$ **Ans. (iv)**

**31.** If $z=f(x+ct)+g(x-ct)$, then

(i) $z_{tt}=z_{xx}$ (ii) $z_t=z_x$ (iii) $z_u=c^2z_{xx}$ (iv) $z_{xx}=c^2z_{tt}$ **Ans. (iii)**

**32.** If $u=x^2-y^2$, $v=xy$ then $\frac{\partial x}{\partial u}$ equals

(i) $\frac{x}{2(x^2+y^2)}$ (ii) $\frac{y}{2(x^2+y^2)}$ (iii) $\frac{y}{x^2+y^2}$ (iv) $\frac{x}{x^2+y^2}$ **Ans. (i)**

**33.** If $z=f(x^j y^k)$ satisfies the equation $x\frac{\partial z}{\partial x}-2y\frac{\partial z}{\partial y}=0$, then $\frac{j}{k}$ equals

(i) 1 (ii) 2 (iii) 3 (iv) 4 **Ans. (ii)**

**34.** If $u=\sin^{-1}\left(\frac{x}{y}\right)$ then $\frac{\partial x}{\partial u}$ equals to

(i) $\frac{1}{\sqrt{y^2-x^2}}$ (ii) $\frac{1}{\sqrt{x^2-y^2}}$ (iii) $\sqrt{1-x^2}$ (iv) None of these **Ans. (i)**

**35.** If $u=\tan^{-1}\left(\frac{y}{x}\right)$ then $\left(\frac{\partial u}{\partial y}\right)$ equals to

(i) $\frac{x^2}{x^2-y^2}$ (ii) $\frac{x}{x^2+y^2}$ (iii) $\frac{y}{x^2+y^2}$ (iv) None of these **Ans. (ii)**

**36.** If $u=\frac{1}{2}\log(x^2+y^2)$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}$ is equal to

(i) $2u$ (ii) $u$ (iii) $x^2+y^2$ (iv) 1 **Ans. (iv)**

**37.** If $x=r\cos\theta$, $y=r\sin\theta$, then

(i) $\left(\frac{\partial r}{\partial x}\right)_y=-\left(\frac{\partial x}{\partial r}\right)_\theta$ (ii) $r\left(\frac{\partial x}{\partial r}\right)_\theta=-\left(\frac{\partial y}{\partial\theta}\right)_r$

(iii) $\left(\frac{\partial x}{\partial \theta}\right)_r = r^2\left(\frac{\partial \theta}{\partial x}\right)_y$ (iv) None of these Ans. (i

**38.** If $v = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$, then $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} + z\frac{\partial v}{\partial z} =$

(i) $-v$ (ii) $v$ (iii) $2v$ (iv) None of these Ans. (i
*(R.G.P.V., Bhopal, Feb. 200*

**39.** $u = \frac{x}{x^2 + y^2}$ then $\frac{\partial u}{\partial y}$ is equal to

(i) $\frac{xy}{(x^2 + y^2)^2}$ (ii) $\frac{2xy}{(x^2 + y^2)^2}$ (iii) $\frac{-2xy}{(x^2 + y^2)^2}$ (iv) $\frac{xy}{(x^2 - y^2)^2}$ Ans. (iii)

**40.** If $u = x^2 \tan^{-1}\left(\frac{y}{x}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ at $x = y = 1$ is

(i) $\frac{\pi}{4}$ (ii) $\frac{\pi}{2}$ (iii) $\pi$ (iv) $-\frac{\pi}{4}$ Ans. (ii)

**41.** If $u = \frac{x^2 + y^2 + xy}{x + y}$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ equals

(i) 0 (ii) 1 (iii) $u$ (iv) $2u$ Ans. (iii)

**42.** If $z = \log [(x^3 + y^3)/(x + y)]$, then $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$ is equal to

(i) 2 (ii) $2z$ (iii) $2e^z$ (iv) 0 Ans. (i)

**43.** If $z = \frac{x^3 + y^3}{xy}$, then the value of $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$ equals

(i) 1 (ii) $2z$ (iii) $\frac{x^3 + y^3}{xy}$ (iv) None of these Ans. (iii)

**44.** Let $u(x, y) = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right), x \neq 0, y \neq 0$ then $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$ equals

(i) 0 (ii) $2u$ (iii) $u$ (iv) $3u$ Ans. (ii)

**45.** If $u = \frac{x^2y^2}{x^2 + y^2}\log\frac{y}{x}$ and $v = \cos^{-1}\left(\frac{xy}{x^2 - y^2}\right)$ and $z = u + v$ then $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$ equals

(i) $4v$ (ii) $4u$ (iii) $2u$ (iv) $4u + v$ Ans. (iii)

**46.** If $u = \frac{x^3 + y^3}{x + y}$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to

(i) 0 (ii) $u$ (iii) $2u$ (iv) $3u$ Ans. (iii)

**47.** Euler's Theorem on Homogeneous function if $z$ is a Homogeneous of $x$, $y$ of order $n$, then:

(i) $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nz$ (ii) $x^2\frac{\partial z}{\partial y} + y^2\frac{\partial z}{\partial x} = nz$

(iii) $x\frac{\partial z}{\partial y} + y\frac{\partial z}{\partial x} = nz$ (iv) $y^2\frac{\partial z}{\partial x} + x^2\frac{\partial z}{\partial y} = nz$ Ans. (i)

*(R.G.P.V., Bhopal, Feb. 2006)*

48. If $u = \dfrac{x^{1/4} + y^{1/4}}{x^{1/5} + y^{1/5}}$, then the value of $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$ is

(*i*) $4u$ (*ii*) $5u$ (*iii*) $20u$ (*iv*) $\dfrac{1}{20}u$ **Ans. (*iv*)**

*(R.G.P.V., Bhopal, Ist Semester, June 2007)*

**Fill in the blanks:**

49. If $u = xf\left(\dfrac{y}{x}\right) + g\left(\dfrac{y}{x}\right)$ then $x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x \partial y} + y^2\dfrac{\partial^2 u}{\partial y^2}$ = ..... **Ans. 0**

50. If $x = e^r \cos\theta$, $y = e^r \sin\theta$, then $\dfrac{\partial\theta}{\partial x}$ is equal to ..... and $\dfrac{\partial\theta}{\partial y}$ is equal to .....

**Ans.** $\dfrac{\partial\theta}{\partial x} = -\dfrac{\sin^2\theta}{y}$, $\dfrac{\partial\theta}{\partial y} = \dfrac{\cos^2\theta}{x}$

51. If $z = x^3 \cos(y/x)$ then $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y}$ is equal to ..... **Ans. $3z$**

52. If $x = u + v$, $y = u - v$, then $\dfrac{\partial u}{\partial x}$ is equal to ..... and $\dfrac{\partial v}{\partial y}$ is equal to .....

**Ans.** $\dfrac{\partial u}{\partial x} = \dfrac{1}{2}$, $\dfrac{\partial v}{\partial y} = -\dfrac{1}{2}$

53. If $u = \log\left(\dfrac{x}{y}\right) + \tan\left(\dfrac{x}{y}\right)$ then $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$ = ..... **Ans. 0**

54. If $u = x^3 \cos\left(\dfrac{y}{x}\right)$, then $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$ is equal to ..... **Ans. $3u$**

55. State, whether the statement is

(*i*) If $u = \dfrac{x^2 + y^2}{x^2 - y^2} + 4$ then $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 4$

(*i*) True (*ii*) False (*iii*) Could be either (*iv*) Do not know. **Ans. (*ii*)**

(*ii*) If $u = \dfrac{x^3 - x^2y + xy^2 + y^3}{x^2 - xy - y^2}$ then $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = u$

(*i*) True (*ii*) False (*iii*) Could be either (*iv*) Do not know. **Ans. (*i*)**

(*c*) If $u = \log_e \dfrac{x^4 - y^4}{x^3 + y^3}$, then $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \dfrac{4}{3}$

(*i*) True (*ii*) False (*iii*) Could be either (*iv*) Do not know. **Ans. (*ii*)**

(*d*) If $f(x, y) = \dfrac{1}{x^3} + \dfrac{1}{x^2 y} + \dfrac{1}{x^3 + 5y^3}$, then $x\dfrac{\partial f}{\partial x} + y\dfrac{\partial f}{\partial y} + 3f = 0$

(*i*) True (*ii*) False (*iii*) Could be either (*iv*) Do not know. **Ans. (*i*)**

5. Show that the plane $ax + by + cz + d = 0$ touches the surface $px^2 + qy^2 + 2z = 0$, if $\frac{a^2}{p} + \frac{b^2}{q} + 2cd = 0$.

## OBJECTIVE TYPE QUESTIONS

**Choose the correct answer :**

1. If $f = x^2 + y^2$, $x = r + 3s$, $y = 2r - s$, then $\frac{\partial f}{\partial r}$ is

(*i*) $4x + 2y$ (*ii*) $2x + y$ (*iii*) $2x + 4y$ (*iv*) $x + 4y$ **Ans. (*iii*)**

2. If $f = x + 4y$, $x = 2s + t$, $y = s + 2t$, then $\frac{\partial f}{\partial t}$ is

(*i*) 9 (*ii*) 8 (*iii*) 7 (*iv*) $-7$ **Ans. (*i*)**

3. If $z = xy$, $x = e^r \cos\theta$, $y = e^\theta \sin r$, then $\frac{\partial z}{\partial r}$ is

(*i*) $xy - x\,e^\theta \cos r$ (*ii*) $xy + x\,e^\theta \cos r$
(*iii*) $xy + x\,e^\theta \sin r$ (*iv*) $xy + y\,e^\theta \cos r$ **Ans. (*ii*)**

4. If $z = x^2 + y^2$ and $x = r + t$, $y = r^2 + t^2$, then $\frac{\partial z}{\partial t}$ is

(*i*) $x + 6yt$ (*ii*) $2x + 2yt$ (*iii*) $x + 4yt$ (*iv*) $2x + 4yt$ **Ans. (*iv*)**

5. If $z = x + y$, $x = e^{r\cos\theta}$, $y = e^{r\sin\theta}$, then $\frac{\partial z}{\partial \theta}$ is

(*i*) $r\,(\cos\theta\, e^{r\cos\theta} - \sin\theta\, e^{r\sin\theta})$ (*ii*) $r\,(\cos\theta\, e^{r\sin\theta} - \sin\theta\, e^{r\cos\theta})$
(*iii*) $r\,e^r(\cos\theta - \sin\theta)$ (*iv*) $r\,(\cos\theta\, e^{r\sin\theta} + \sin\theta\, e^{r\cos\theta})$ **Ans. (*ii*)**

6. If $z = x^2 y^2$, and $x = s \log r$, $y = r \log s$ then $\frac{\partial z}{\partial r}$ is

(*i*) $2xy\left(\frac{xs}{r} + y\log s\right)$ (*ii*) $2xy\,(ys + x\log s)$
(*iii*) $2xy\left(\frac{ys}{r} + x\log s\right)$ (*iv*) $2xy\left(\frac{ys}{r} - x\log s\right)$ **Ans. (*iii*)**

7. If $z = f(x, y)$, $x = r\cos\theta$, $y = r\sin\theta$, then $\frac{\partial z}{\partial r}$ is

(*i*) $\frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta$ (*ii*) $\frac{\partial f}{\partial x}\sin\theta + \frac{\partial f}{\partial y}\cos\theta$
(*iii*) $\frac{\partial f}{\partial x}\cos\theta - \frac{\partial f}{\partial y}\sin\theta$ (*iv*) $\frac{\partial f}{\partial x}\sin\theta - \frac{\partial f}{\partial y}\cos\theta$ **Ans. (*i*)**

8. If $z = f(x, y)$, $x = r\cos\theta$, $y = r\sin\theta$, then $\frac{1}{r}\frac{\partial z}{\partial \theta} =$

(*i*) $\frac{\partial f}{\partial x}\sin\theta + \frac{\partial f}{\partial y}\cos\theta$ (*ii*) $-\frac{\partial f}{\partial x}\sin\theta + \frac{\partial f}{\partial y}\cos\theta$
(*iii*) $\frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta$ (*iv*) $\frac{\partial f}{\partial x}\cos\theta - \frac{\partial f}{\partial y}\sin\theta$ **Ans. (*ii*)**

**9.** If $z = f(x, y)$, $x = s + t$, $y = s - t$ then $\frac{\partial z}{\partial s}$ is equal to

(*i*) $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}$ (*ii*) $\frac{\partial f}{\partial x} - \frac{\partial f}{\partial y}$

(*iii*) $\frac{\partial f}{\partial y} - \frac{\partial f}{\partial x}$ (*iv*) None of these

Ans. (*i*)

**10.** If $z = x + y$, $x = 2t$, $y = t^2$ then $\frac{\partial z}{\partial t}$ is equal to

(*i*) $2 + 2t$ (*ii*) $2 - 2t$ (*iii*) $2t - 2$ (*iv*) $2t + 5$ Ans. (*i*)

**11.** If $z = x^2 + y^2$, $x = t^2$ and $y = t^3$ then $\frac{\partial z}{\partial t}$ is equal to

(*i*) $4t^3 - 6t^5$ (*ii*) $4t^3 + 6t^5$ (*iii*) $6t^3 - 4t^5$ (*iv*) $6t - 5$ Ans. (*ii*)

**12.** If $z = 2x + 3y$, $x = \sin\theta$ and $y = \cos\theta$ then $\frac{\partial z}{\partial \theta}$ is equal to

(*i*) $2\cos\theta + 3\sin\theta$ (*ii*) $3\sin\theta - 2\cos\theta$

(*iii*) $2\cos\theta - 3\sin\theta$ (*iv*) $3\cos\theta + 2\sin\theta$ Ans. (*iii*)

**13.** If $z = x^2 y^2$, $x = t$ and $y = 2t$ then $\frac{\partial z}{\partial t}$ is equal to

(*i*) $2xy(2x - y)$ (*ii*) $xy(2x + y)$

(*iii*) $2xy(x + 2y)$ (*iv*) $2xy(2x + y)$ Ans. (*iv*)

**14.** If $z = x^3y^3$ then $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}$ is equal to

(*i*) $6xy(x^2 + y^2)$ (*ii*) $6xy(x + y)$ (*iii*) $6xy(x - y)$ (*iv*) $xy(x^2 + y^2)$ Ans. (*i*)

**15.** If $z = \sqrt{xy}$ then $\frac{\partial^2 z}{\partial x \partial y}$ is equal to

(*i*) $4z$ (*ii*) $\frac{1}{4z}$ (*iii*) $\frac{z}{4}$ (*iv*) $\frac{4}{z}$ Ans. (*ii*)

**16.** If $u = x^2 + y^2 + z^2$, $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$ and $z = r\cos\theta$ then $\frac{\partial u}{\partial r}$ is equal to

(*i*) $r$ (*ii*) $2r$ (*iii*) $r^2$ (*iv*) $2r^2$ Ans. (*ii*)

**17.** If $y = e^x + \sin x$, then $\frac{d^2y}{dx^2}$ is equal to .

(*i*) $e^x + \sin x$ (*ii*) $e^x - \sin x$ (*iii*) $e^x - \cos x$ (*iv*) None of these

Ans. (*ii*)

**18.** If $y = \tan x + \sec x$ then $\frac{d^2y}{dx^2}$ is equal to

(*i*) $\sec x(\tan^2 x + \sec^2 x)$ (*ii*) $\sec x(\sec x \tan x + \tan^2 x \sec^2 x)$

(*iii*) $\sec x(2\sec x \tan x + \tan^2 x + \sec^2 x)$ (*iv*) $2\sec x \tan x + \tan^2 x + \sec^2 x$

Ans. (*iii*)

**19.** If $f(x, y, z) = 0$, then $\frac{\partial x}{\partial y}\cdot\frac{\partial y}{\partial z}\cdot\frac{\partial z}{\partial x}$ is equal to

(*i*) 1 (*ii*) 2 (*iii*) $-1$ (*iv*) 0 Ans. (*iii*)

**20.** If $z = f(x, y)$ where $x = \phi(t)$, $y = \psi(t)$, then $\frac{dz}{dt}$ is equal to

(*i*) $\frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\cdot\frac{dy}{dt}$ (*ii*) $\frac{\partial z}{\partial x}\frac{\partial x}{\partial t} - \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t}$

(*iii*) $\frac{\partial z}{\partial x} + \frac{\partial y}{\partial t}\frac{\partial z}{\partial y}$ (*iv*) $\frac{dx}{dt} + \frac{\partial z}{\partial t}\frac{dx}{dt}$ **Ans.** (*i*)

**21.** If $f(x, y) = 0$, then $\frac{dy}{dx}$ is equal to

(*i*) $\frac{\frac{\partial y}{\partial f}}{\frac{\partial x}{\partial f}}$ (*ii*) $-\frac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial x}}$ (*iii*) $-\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}$ (*iv*) $\frac{\partial y}{\partial x}\cdot\frac{\partial f}{\partial y}$ **Ans.** (*iii*)

**22.** If $f(x, y) = 0$ and $\phi(y, z) = 0$, then $\frac{\partial f}{\partial y}\cdot\frac{\partial \phi}{\partial z}\cdot\frac{\partial z}{\partial x}$ is equal to

(*i*) $\frac{\partial x}{\partial y}\cdot\frac{\partial \phi}{\partial y}$ (*ii*) $\frac{\partial f}{\partial x}\cdot\frac{\partial y}{\partial \phi}$ (*iii*) $\frac{\partial f}{\partial y}\cdot\frac{\partial \phi}{\partial x}$ (*iv*) $\frac{\partial f}{\partial x}\cdot\frac{\partial \phi}{\partial y}$ **Ans.** (*iv*)

**23.** If $f(x, y) = 0$, then $\frac{d^2y}{dx^2}$ is equal to

(*i*) $-\frac{q^2r - 2pqs + p^2t}{q^3}$ (*ii*) $\frac{q^2r - 2pqs + p^2t}{q}$

(*iii*) $\frac{q^2r - 2pqs - p^2t}{q^3}$ (*iv*) $\frac{q^2r - 2pqs + p^2t}{q^3}$ **Ans.** (*i*)

**24.** The equation of the tangent plane to the surface $x^2 + y^2 + z^2 = 14$ at (1, 2, 3) is

(*i*) $2x + 4y + 6z = 14$ (*ii*) $x + 2y + 3z = 0$

(*iii*) $x + 2y + 3z = 1$ (*iv*) $x + 2y + 3z = 14$ **Ans.** (*iv*)

**25.** The equation of the normal to the tangent plane $x^2 + y^2 + z^2 = 6$ at (–1, –2, – 1) is

(*i*) $\frac{x+1}{2} = \frac{y+2}{-4} = \frac{z+1}{-2}$ (*ii*) $\frac{x+1}{1} = \frac{y+2}{2} = \frac{z+1}{1}$

(*iii*) $\frac{x+1}{1} = \frac{y+2}{2} = \frac{z+1}{-1}$ (*iv*) $\frac{x+1}{1} = \frac{y+2}{-2} = \frac{z+1}{1}$ **Ans.** (*ii*)

3. If $\frac{\partial(u, v)}{\partial(x, y)} \times \frac{\partial(x, y)}{\partial(u, v)}$ is equal to

(i) 1 (ii) $-1$ (iii) zero (iv) none of these **Ans. (i)**

4. If $x = r\cos\theta, y = r\sin\theta$ then $J\left(\frac{x, y}{r, \theta}\right) J\left(\frac{r, \theta}{x, y}\right)$ is equal to

(i) 1 (ii) $-1$ (iii) 0 (iv) none of these **Ans. (i)**

5. If $x = r\cos\theta, y = r\sin\theta$, then $\frac{\partial(r, \theta)}{\partial(x, y)}$ is equal to

(i) 1 (ii) $r$ (iii) $\frac{1}{r}$ (iv) 0 **Ans. (iii)**

6. If $x = r\cos\theta, y = r\sin\theta, z = z$ then $\frac{\partial(x, y, z)}{\partial(r, \theta, z)}$ is equal to

(i) $r$ (ii) $\frac{1}{r}$ (iii) $r^2\sin\theta$ (iv) none of these **Ans. (i)**

7. Jacobian $\frac{(u, v)}{(r, s)}$ Jacobian $\frac{(r, s)}{(x, y)} =$

(i) $J\frac{(x, y)}{(u, v)}$ (ii) $J\frac{(ur, vs)}{(rx, xy)}$ (iii) $J\frac{(u, v)}{(x, y)}$ (iv) $J\frac{(u, y)}{(x, v)}$ **Ans. (iii)**

8. Cylindrical coordinates are

(i) $x = r\sin\theta, y = r\cos\theta, z = z$

(ii) $x = r\cos\theta, y = r\sin\theta, z = z$

(iii) $x = r\cos\theta, y = r\sin\theta, z = r\sin\theta\cos\theta$

(iv) $x = r\cos\theta, y = r\sin\theta, z = \cos\theta$ **Ans. (ii)**

9. Spherical coordinates are

(i) $x = r\sin\theta, \cos\phi, y = r\cos\theta\sin\phi, z = r\sin\theta$

(ii) $x = r\cos\theta, y = r\sin\theta, z = r\cos\phi$

(iii) $x = r\cos\theta, \sin\phi. y = r\cos\theta, \cos\phi\, z = r\sin\theta$

(iv) $x = r\sin\theta, \cos\phi, y = r\sin\theta, \sin\theta\, z = r\cos\theta$ **Ans. (ii)**

10. If $x = r\sin\theta\cos\phi, y = r\sin\theta\sin\phi, z = r\cos\theta$, then the value of the Jacobian $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)} =$

(i) $r^2$ (ii) $r^2\sin\theta$ (iii) $r^2\cos\theta$ (iv) $r^2\cos\phi$ **Ans. (ii)**

**Fill in the blanks:**

11. If $x = r\cos\theta, y = r\sin\theta$, then the value of the Jacobian $\frac{\partial(x, y)}{\partial(r, \theta)}$ is .... **Ans. $r$**

12. If $u = x(1 - y), v = xy$, then the value of the Jacobian $\frac{\partial(u, v)}{\partial(x, y)}$ is .... **Ans. $x$**

13. $\frac{\partial(u,v)}{\partial(r,s)} \times \frac{\partial(r,s)}{\partial(x,y)} =$ ..... Ans. $\frac{\partial(u,v)}{\partial(x,y)}$

14. $\frac{\partial(x,y)}{\partial(r,\theta)} \times \frac{\partial(r,\theta)}{\partial(x,y)} =$ ..... Ans. 1

**Indicate True or False for the following:**

15. If $u = 2axy$, $v = a(x^2 - y^2)$, where $x = r\cos\theta$, $y = r\sin\theta$ then Jacobian $\frac{\partial(x,y)}{\partial(r,\theta)}$ is $-4a^2r^3$. Ans. True

16. If $x = \sqrt{vw}$, $y = \sqrt{wu}$, $z = \sqrt{uv}$ and $u = r\sin\theta\cos\phi$, $v = r\sin\theta\sin\phi$, $w = r\cos\theta$ then the value of the Jacobian $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$ is $-\frac{1}{4}$. Ans. False

17. If $u = x + y$, $y = uv$ then Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$ is $(x+y)^{-1}$ Ans. True

18. If $u = \frac{x+y}{1-xy}$, $v = \tan^{-1}x + \tan^{-1}y$ then Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$ is 0. Ans. True

19. If $x = u(1-v)$, $y = uv$ then the value of Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$ is $\frac{1}{u}$ Ans. False

**Match the following:**

20.

| | |
|---|---|
| (i) $\frac{\partial(r,\theta,z)}{\partial(x,y,z)}$ | (a) $\frac{\partial(u,v)}{\partial(r,s)} \times \frac{\partial(r,s)}{\partial(x,y)}$ |
| (ii) $\frac{\partial(r,\theta,\phi)}{\partial(x,y,z)}$ | (b) 1 |
| (iii) $\frac{\partial(u,v)}{\partial(x,y)}$ | (c) $\frac{1}{r^2\sin\theta}$ |
| (iv) $\frac{\partial(u,v)}{\partial(x,y)} \times \frac{\partial(x,y)}{\partial(u,v)}$ | (d) $\frac{1}{r}$ |

Ans. (i) → (d)
(ii) → (c)
(iii) → (a)
(iv) → (b)

21.

| | |
|---|---|
| (i) $u = x^2$, $v = y^2$ | (a) $J(u,v) = x - y$ |
| (ii) $u = x + y$, $v = xy$ | (b) $J(u,v) = -29$ |
| (iii) $u = x + y$, $v = \frac{y}{x+y}$ | (c) $J(u,v)\frac{1}{x+y}$ |
| (iv) $u = 3x + 5y$, $v = 4x - 3y$ | (d) $J(u,v) = 4xy$ |

Ans. (i) → (d)
(ii) → (a)
(iii) → (c)
(iv) → (b)