

FACTORS AND FACTORIALS

Factors

- Concepts of Factors and Multiples
- Number of factors
- Sum of factors
- Product of factors
- Number of odd and even factors

Factorials

- Concept of Number of Zeros
- Highest power of a number in a factorial
- Number of zeroes in a factorial

FACTORS

Factors and Multiples : All the numbers that divide a number completely, i.e., without leaving any remainder, are called factors of that number.

For example, 24 is completely divisible by 1, 2, 3, 4, 6, 8, 12, 24. Each of these numbers is called a factor of 24 and 24 is called a multiple of each of these numbers.

FACTORS

These are certain basic formulas pertaining to factors of a number N, such that,

$$N = p^a q^b r^c$$

Where, p, q and r are prime factors of the number n.

a, b and c are non-negative powers/ exponents

- **Number of factors of N** = $(a+1)(b+1)(c+1)$
- **Sum of factors:** $(p^{a+1}-1)(q^{b+1}-1)(r^{c+1}-1) / (p-1)(q-1)(r-1)$
- **Product of factors of N , if N is not a perfect square** = $N^{\text{No. of factors}/2}$
- **Product of factors of N , if N is a perfect square** = $N^{(\text{No. of factors}-1)/2} * \sqrt{N}$

FACTORS

Number of even Factors and odd factors:

Let N as a number.

N in prime factorization = $a^p \times b^q \times c^r$

No. of factors of N = $(p+1)(q+1)(r+1)$

Now suppose that b and c are odd prime numbers in prime factorization of N

Now to find even no of factors you have to find odd no of factors first.

Odd no of factors = $(q+1)(r+1)$

Even no of factors = $(\text{total no of factors}) - (\text{odd no of factors})$.

FACTORS

1. What is the number of factors of 1125?

- A. 8 B. 12 C. 22 D. 24

FACTORS

2. What is the number of factors of $2^6 \times 3^3 \times 7^3$

A. 28

B. 36

C. 64

D. 112

FACTORS

3. What is the number of factors of $4^2 \times 9^3 \times 10^3$?

A. 112

B. 890

C. 224

D. 160

FACTORS

4. What is the sum of the factors of 72?

A. 220

B. 145

C. 195

D. 260

FACTORS

5. What is the sum of the factors of 600?

A. 1560

B. 1650

C. 1770

D. 1860

FACTORS

6. What is the product of the factors of 361?

A. 19^5

B. 19^3

C. 19^{10}

D. None

FACTORS

7. What is the product of the factors of 1024?

A. 2^{58}

B. 2^{36}

C. 2^{55}

D. None

FACTORS

8. What is the product of the factors of 360?

A. 360^{12}

B. 360^8

C. 360^{16}

D. None

FACTORS

9. What is the product of the factors of 524?

A. 524^2

B. 524^3

C. 524^6

D. None

FACTORS

10. Find the smallest number that has exactly 18 factors?

A. 156

B. 180

C. 360

D. 760

FACTORS

11. Find the odd factors of 252?

A. 8

B. 5

C. 7

D. 6

FACTORS

12. How many factors of $2^4 * 5^3 * 7^4$ are odd numbers?

A. 20

B. 25

C. 27

D. 29

FACTORS

13. Number $N = 2^6 * 5^5 * 7^6 * 10^7$; how many factors of N are even numbers?

A. 1183

B. 1173

C. 1673

D. 190

FACTORIALS

Number of zeroes:

It is very easy to find the number of zero at the end , all you have to do is count how many times did 2 and 5 occurred in the question as factor. Number of zeros is equal to the one (2 or 5) which occurred less times.

i.e. $2*5 = 10$

$$2*2*5*5 = 100$$

So the number of zeros depends upon the number of pairs of 2 and 5.

Example 1. How many numbers of zeros will be there at the trail (end) of the $1*2*3*4*5*6*7*8*9*10$?

Solution:

In given expression number of 2's = 8

Number of 5's = 2

So total number of pairs = 2

Two zeroes will be there at the end of the calculation.

FACTORIALS

Number of zeroes in a factorial:

5	25
5	5
5	1
5	0

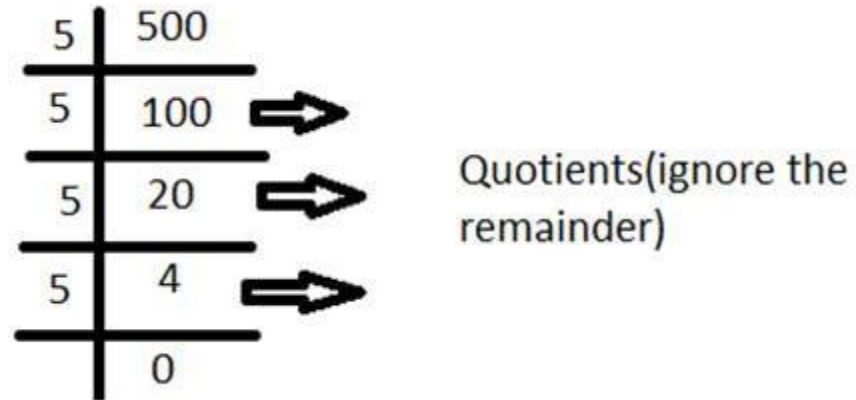
Quotient (Ignore the remainder)

There will be $5+1+0 = 6$ zeroes at the end of $25!$

FACTORIALS

Example . Find the number of zeros at the end of 500!

Solution:



$$\text{Total number of 5's} = 100 + 20 + 4 = 124$$

FACTORIALS

14. The highest power of 3 that completely divides $43!$ is:

- A] 19 B] 26 C] 16 D] 15

FACTORIALS

15. What is the highest power of 7 in 100 factorials?

A] 16

B] 12

C] 21

D] 23

FACTORIALS

16. What is the highest power of 7 in 100 factorials?

A] 16

B] 12

C] 21

D] 23

FACTORIALS

17. What is number of trailing zeroes in 12135000?

A] 0

B] 2

C] 3

D] 5

FACTORIALS

18. What is number of trailing zeroes in 121350001?

- A] 0 B] 2 C] 3 D] 5

FACTORIALS

19. Find the number of zeros in $182!$?

A] 44

B] 42

C] 51

D] 48

FACTORIALS

20. Find the number of zeros in $532!$?

A] 144

B] 142

C] 131

D] 148

FACTORIALS

21. What is the number of trailing zeroes in $1173!$

A] 214

B] 233

C] 265

D] 290

FACTORIALS

22. Find the No. of zeroes at the end of $2^7 \times 3^5 \times 5^8 \times 7^5 \times 8^3 \times 10^5$.

A] 13

B] 17

C] 15

D] 20

FACTORIALS

23. Which of the following cannot be the number of zeroes at the end of any factorial?

A] 25

B] 26

C] 30

D] 36

FACTORIALS

24. If the number of zeros are 117 for the number $x!$, then find the least value of x ?

A] 289

B] 326

C] 430

D] None

Any Doubts???