CART Algorithm

Decision Tree Learning

CART Algorithm

- It can handle both classification and regression tasks.
- This algorithm uses a new metric named gini index to create decision points for classification tasks.

Gini Index

- It stores sum of squared probabilities of each class.
- Gini = $1 - \Sigma (Pi)^2$ for i=1 to number of classes.

Dataset

Day	Outlook	Temp.	Humidity	Wind	Decision
1	Sunny	Hot	High	Weak	No
2	Sunny	Hot	High	Strong	No
3	Overcast	Hot	High	Weak	Yes
4	Rain	Mild	High	Weak	Yes
5	Rain	Cool	Normal	Weak	Yes
6	Rain	Cool	Normal	Strong	No
7	Overcast	Cool	Normal	Strong	Yes
8	Sunny	Mild	High	Weak	No
9	Sunny	Cool	Normal	Weak	Yes
10	Rain	Mild	Normal	Weak	Yes
11	Sunny	Mild	Normal	Strong	Yes
12	Overcast	Mild	High	Strong	Yes
13	Overcast	Hot	Normal	Weak	Yes
14	Rain	Mild	High	Strong	No

Outlook for Decision

- Outlook is a nominal feature. It can be sunny, overcast or rain.

Outlook	Yes	No	Number of instances
Sunny	2	3	5
Overcast	4	0	4
Rain	3	2	5

- Gini(Outlook=Sunny) = $1 – (2/5)^2 – (3/5)^2 = 1 – 0.16 – 0.36 = 0.48$
- Gini(Outlook=Overcast) = $1 – (4/4)^2 – (0/4)^2 = 0$
- Gini(Outlook=Rain) = $1 – (3/5)^2 – (2/5)^2 = 1 – 0.36 – 0.16 = 0.48$
- Then, we will calculate weighted sum of gini indexes for outlook feature.
- Gini(Outlook) = (5/14) x 0.48 + (4/14) x 0 + (5/14) x 0.48 = 0.171 + 0 + 0.171 = 0.342

Temperature

- Similarly, temperature is a nominal feature and it could have 3 different values: Cool, Hot and Mild.

Temperature	Yes	No	Number of instances
Hot	2	2	4
Cool	3	1	4
Mild	4	2	6

- Gini(Temp=Hot) = $1 – (2/4)^2 – (2/4)^2 = 0.5$
- Gini(Temp=Cool) = $1 – (3/4)^2 – (1/4)^2 = 1 – 0.5625 – 0.0625 = 0.375$
- Gini(Temp=Mild) = $1 – (4/6)^2 – (2/6)^2 = 1 – 0.444 – 0.111 = 0.445$
- We'll calculate weighted sum of gini index for temperature feature.
- Gini(Temp) = $(4/14) \times 0.5 + (4/14) \times 0.375 + (6/14) \times 0.445 = 0.142 + 0.107 + 0.190 = 0.439$

Humidity

- Humidity is a binary class feature. It can be high or normal.

Humidity	Yes	No	Number of instances
High	3	4	7
Normal	6	1	7

- Gini(Humidity=High) = $1 – (3/7)^2 – (4/7)^2 = 1 – 0.183 – 0.326 = 0.489$
- Gini(Humidity=Normal) = $1 – (6/7)^2 – (1/7)^2 = 1 – 0.734 – 0.02 = 0.244$
- Weighted sum for humidity feature will be calculated next:
- Gini(Humidity) = (7/14) x 0.489 + (7/14) x 0.244 = 0.367

Wind

- Wind is a binary class similar to humidity. It can be weak and strong.

Wind	Yes	No	Number of instances
Weak	6	2	8
Strong	3	3	6

- Gini(Wind=Weak) = $1 – (6/8)^2 – (2/8)^2 = 1 – 0.5625 – 0.062 = 0.375$
- Gini(Wind=Strong) = $1 – (3/6)^2 – (3/6)^2 = 1 – 0.25 – 0.25 = 0.5$
- Gini(Wind) = $(8/14) \times 0.375 + (6/14) \times 0.5 = 0.428$

Time to decide

- We’ve calculated gini index values for each feature.

Feature	Gini index
Outlook	0.342
Temperature	0.439
Humidity	0.367
Wind	0.428

- The winner will be outlook feature because its cost is the lowest.

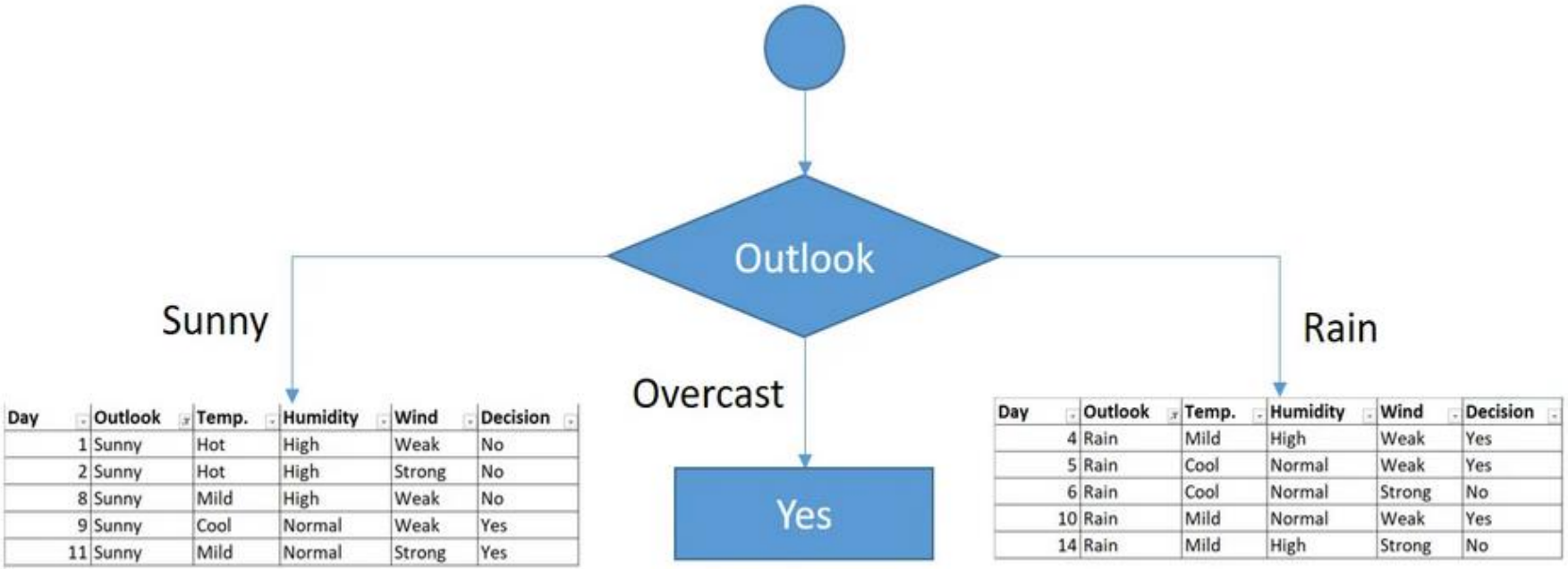
Root Node

Day	Outlook	Temp.	Humidity	Wind	Decision
1	Sunny	Hot	High	Weak	No
2	Sunny	Hot	High	Strong	No
8	Sunny	Mild	High	Weak	No
9	Sunny	Cool	Normal	Weak	Yes
11	Sunny	Mild	Normal	Strong	Yes

Day	Outlook	Temp.	Humidity	Wind	Decision
4	Rain	Mild	High	Weak	Yes
5	Rain	Cool	Normal	Weak	Yes
6	Rain	Cool	Normal	Strong	No
10	Rain	Mild	Normal	Weak	Yes
14	Rain	Mild	High	Strong	No

Day	Outlook	Temp.	Humidity	Wind	Decision
3	Overcast	Hot	High	Weak	Yes
7	Overcast	Cool	Normal	Strong	Yes
12	Overcast	Mild	High	Strong	Yes
13	Overcast	Hot	Normal	Weak	Yes

- You might realize that sub dataset in the overcast leaf has only yes decisions. This means that overcast leaf is over.



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Day	Outlook	Temp.	Humidity	Wind	Decision
4	Rain	Mild	High	Weak	Yes
5	Rain	Cool	Normal	Weak	Yes
6	Rain	Cool	Normal	Strong	No
10	Rain	Mild	Normal	Weak	Yes
14	Rain	Mild	High	Strong	No

Second Level (Outlook=Sunny)

- Focus on the sub dataset for sunny outlook.

Day	Outlook	Temp.	Humidity	Wind	Decision
1	Sunny	Hot	High	Weak	No
2	Sunny	Hot	High	Strong	No
8	Sunny	Mild	High	Weak	No
9	Sunny	Cool	Normal	Weak	Yes
11	Sunny	Mild	Normal	Strong	Yes

- We need to find the gini index scores for temperature, humidity and wind features respectively.

Gini of temperature for sunny outlook

Temperature	Yes	No	Number of instances
Hot	0	2	2
Cool	1	0	1
Mild	1	1	2

- Gini(Outlook=Sunny and Temp.=Hot) = $1 - (0/2)^2 - (2/2)^2 = 0$
- Gini(Outlook=Sunny and Temp.=Cool) = $1 - (1/1)^2 - (0/1)^2 = 0$
- Gini(Outlook=Sunny and Temp.=Mild) = $1 - (1/2)^2 - (1/2)^2 = 1 - 0.25 - 0.25 = 0.5$
- Gini(Outlook=Sunny and Temp.) = $(2/5)x0 + (1/5)x0 + (2/5)x0.5 = 0.2$

Gini of humidity for sunny outlook

Humidity	Yes	No	Number of instances
High	0	3	3
Normal	2	0	2

- Gini(Outlook=Sunny and Humidity=High) = $1 – (0/3)^2 – (3/3)^2 = 0$
- Gini(Outlook=Sunny and Humidity=Normal) = $1 – (2/2)^2 – (0/2)^2 = 0$
- Gini(Outlook=Sunny and Humidity) = $(3/5)x0 + (2/5)x0 = 0$

Gini of wind for sunny outlook

Wind	Yes	No	Number of instances
Weak	1	2	3
Strong	1	1	2

- Gini(Outlook=Sunny and Wind=Weak) = $1 – (1/3)^2 – (2/3)^2 = 0.266$
- Gini(Outlook=Sunny and Wind=Strong) = $1- (1/2)^2 – (1/2)^2 = 0.2$
- Gini(Outlook=Sunny and Wind) = $(3/5)x0.266 + (2/5)x0.2 = 0.466$

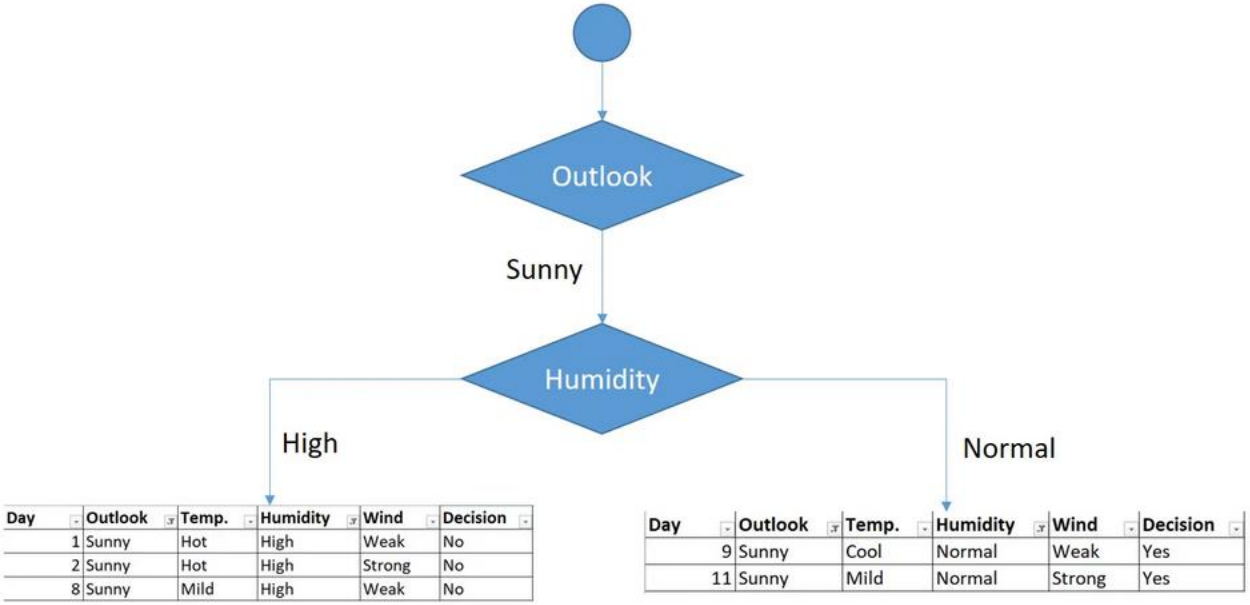
Decision for sunny outlook

- We’ve calculated gini index scores for feature when outlook is sunny.

Feature	Gini index
Temperature	0.2
Humidity	0
Wind	0.466

- The winner is humidity because it has the lowest value.

Second Level (Outlook = sunny)

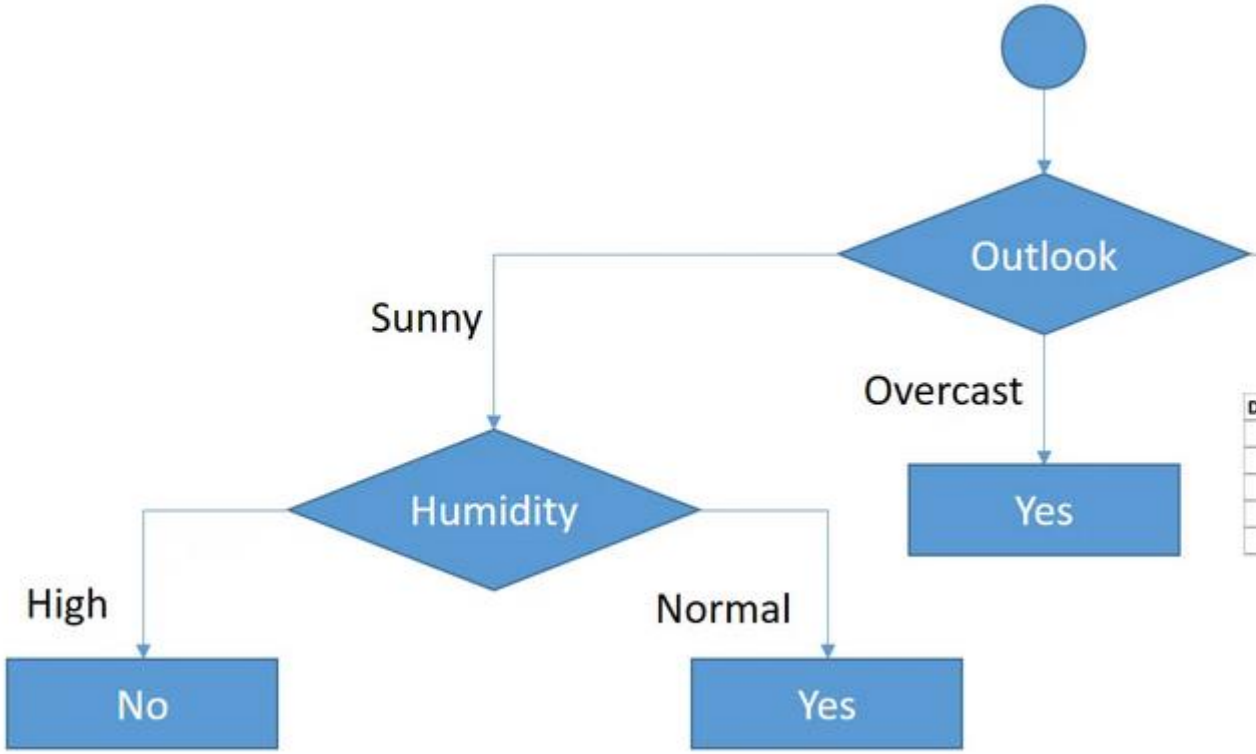


Day	Outlook	Temp.	Humidity	Wind	Decision
1	Sunny	Hot	High	Weak	No
2	Sunny	Hot	High	Strong	No
8	Sunny	Mild	High	Weak	No

Day	Outlook	Temp.	Humidity	Wind	Decision
9	Sunny	Cool	Normal	Weak	Yes
11	Sunny	Mild	Normal	Strong	Yes

- As seen, decision is always no for high humidity and sunny outlook.
- On the other hand, decision will always be yes for normal humidity and sunny outlook.

Second Level (Outlook = sunny)



Day	Outlook	Temp.	Humidity	Wind	Decision
4	Rain	Mild	High	Weak	Yes
5	Rain	Cool	Normal	Weak	Yes
6	Rain	Cool	Normal	Strong	No
10	Rain	Mild	Normal	Weak	Yes
14	Rain	Mild	High	Strong	No

Second Level (Outlook = rain)

- Now, we need to focus on rain outlook.

Day	Outlook	Temp.	Humidity	Wind	Decision
4	Rain	Mild	High	Weak	Yes
5	Rain	Cool	Normal	Weak	Yes
6	Rain	Cool	Normal	Strong	No
10	Rain	Mild	Normal	Weak	Yes
14	Rain	Mild	High	Strong	No

- We'll calculate gini index scores for temperature, humidity and wind features when outlook is rain.

Gini of temprature for rain outlook

Temperature	Yes	No	Number of instances
Cool	1	1	2
Mild	2	1	3

- Gini(Outlook=Rain and Temp.=Cool) = $1 – (1/2)^2 – (1/2)^2 = 0.5$
- Gini(Outlook=Rain and Temp.=Mild) = $1 – (2/3)^2 – (1/3)^2 = 0.444$
- Gini(Outlook=Rain and Temp.) = $(2/5)x0.5 + (3/5)x0.444 = 0.466$

Gini of humidity for rain outlook

Humidity	Yes	No	Number of instances
High	1	1	2
Normal	2	1	3

- Gini(Outlook=Rain and Humidity=High) = $1 – (1/2)^2 – (1/2)^2 = 0.5$
- Gini(Outlook=Rain and Humidity=Normal) = $1 – (2/3)^2 – (1/3)^2 = 0.444$
- Gini(Outlook=Rain and Humidity) = $(2/5)x0.5 + (3/5)x0.444 = 0.466$

Gini of wind for rain outlook

Wind	Yes	No	Number of instances
Weak	3	0	3
Strong	0	2	2

- Gini(Outlook=Rain and Wind=Weak) = $1 – (3/3)^2 – (0/3)^2 = 0$
- Gini(Outlook=Rain and Wind=Strong) = $1 – (0/2)^2 – (2/2)^2 = 0$
- Gini(Outlook=Rain and Wind) = $(3/5)x0 + (2/5)x0 = 0$

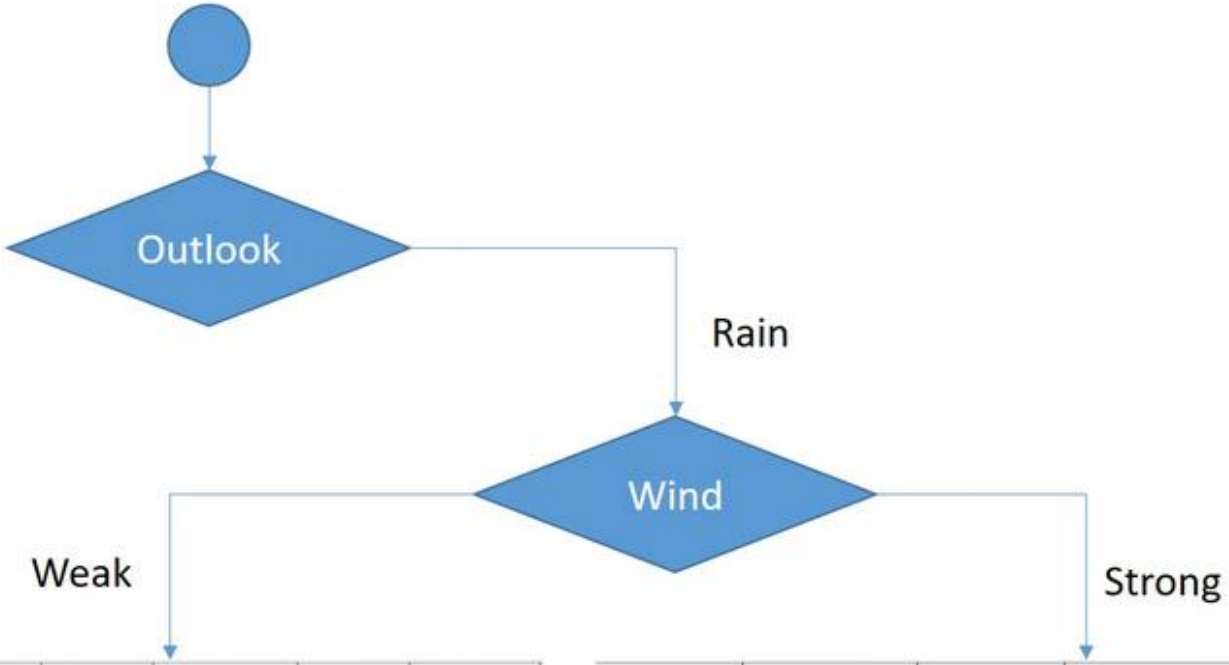
Decision for rain outlook

Feature	Gini index
Temperature	0.466
Humidity	0.466
Wind	0

- The winner is wind feature for rain outlook because it has the minimum gini index score in features.

Second Level (Outlook = rain)

- Put the wind feature for rain outlook branch and monitor the new sub data sets.

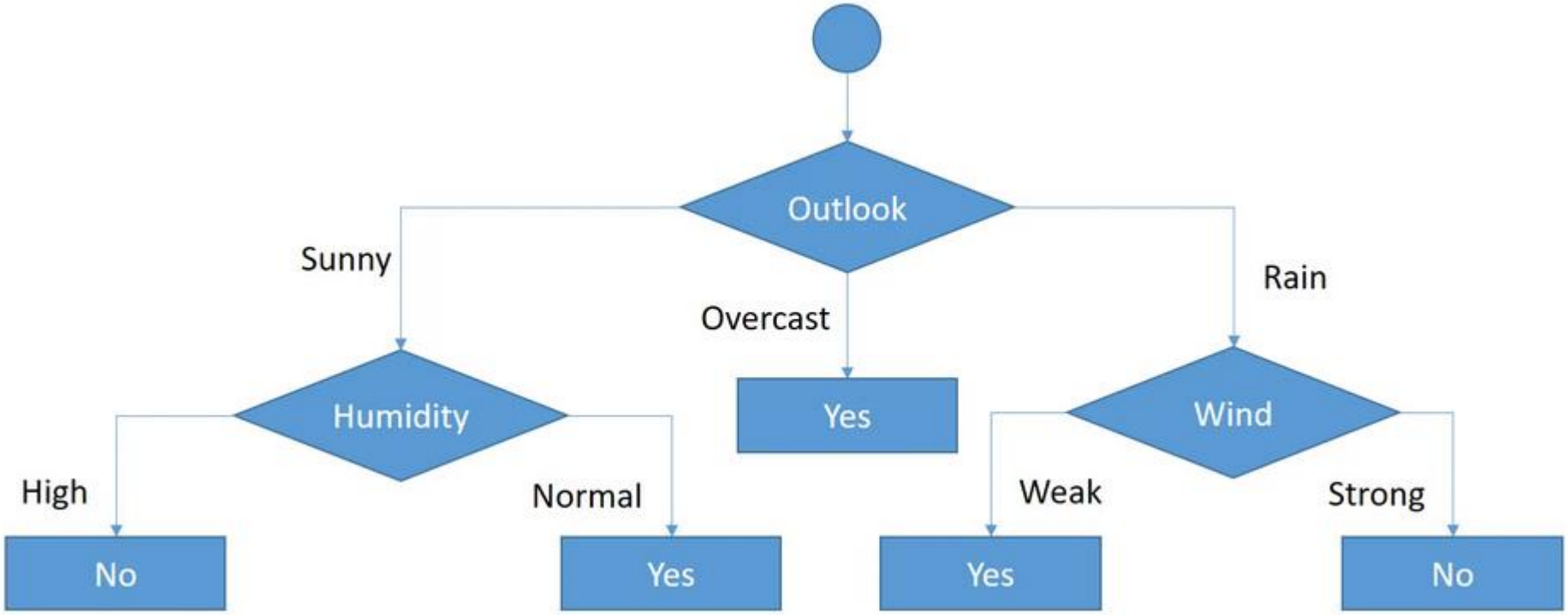


Day	Outlook	Temp.	Humidity	Wind	Decision
4	Rain	Mild	High	Weak	Yes
5	Rain	Cool	Normal	Weak	Yes
10	Rain	Mild	Normal	Weak	Yes

Day	Outlook	Temp.	Humidity	Wind	Decision
6	Rain	Cool	Normal	Strong	No
14	Rain	Mild	High	Strong	No

Second Level (Outlook = rain)

- As seen, decision is always yes when wind is weak.
- On the other hand, decision is always no if wind is strong.



Thank You !!!