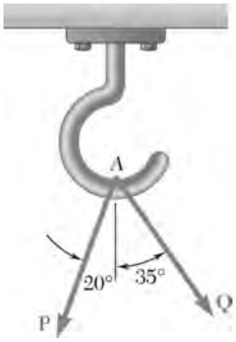
CHAPTER 2

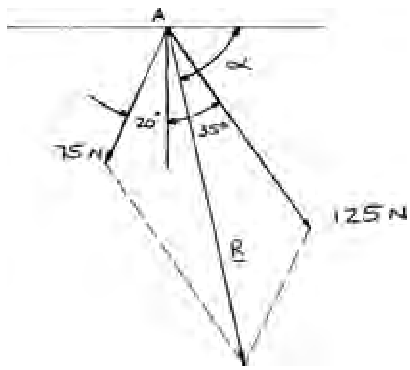


PROBLEM 2.1

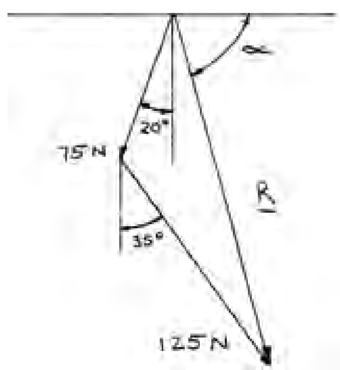
Two forces **P** and **Q** are applied as shown at Point A of a hook support. Knowing that $P = 75$ N and $Q = 125$ N, determine graphically the magnitude and direction of their resultant using (a) the parallelogram law, (b) the triangle rule.

SOLUTION

(a) Parallelogram law:

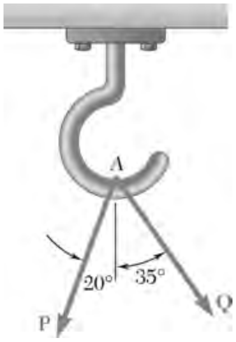


(b) Triangle rule:



We measure: $R = 179$ N, $\alpha = 75.1°$

$\mathbf{R} = 179$ N ↘ 75.1° ◀

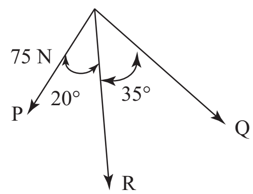


PROBLEM 2.2

For the hook support of Problem 2.1, knowing that the magnitude of **P** is 75-N, determine (*a*) the required magnitude of the force **Q** if the resultant **R** of the two forces applied at *A* is to be vertical, (*b*) the corresponding magnitude of **R**.

SOLUTION

(*a*) Since *R* is vertical, *x* component of *R* is zero.



$$R_x = -P \sin 20^\circ + Q \sin 35^\circ = 0$$

$$\Rightarrow P \sin 20^\circ = Q \sin 35^\circ$$

$$Q = \frac{P \sin 20^\circ}{\sin 35^\circ} = 75\,\text{N}\,\frac{\sin 20^\circ}{\sin 35^\circ}$$

Q = 44.7 N ◀

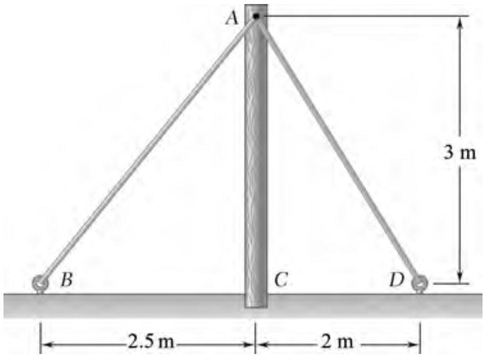
(*b*) Magnitude of **R**:

$$\left|\vec{R}\right| = P \cos 20^\circ + Q \cos 35^\circ$$

$$= 107.093\ \text{N}$$

$$\text{R} = 107.1\ \text{N}$$

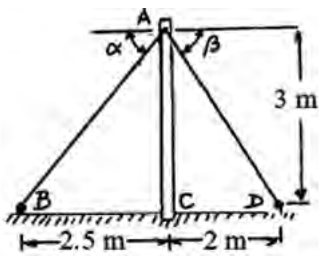
R = 107.1 N ◀



PROBLEM 2.3

The cable stays *AB* and *AD* help support pole *AC*. Knowing that the tension is 600 N in *AB* and 200 N in *AD*, determine graphically the magnitude and direction of the resultant of the forces exerted by the stays at *A* using (*a*) the parallelogram law, (*b*) the triangle rule.

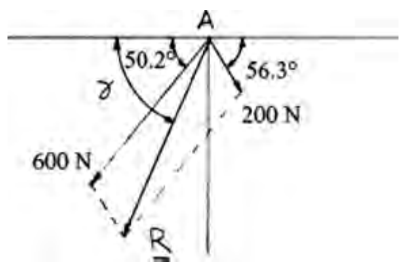
SOLUTION



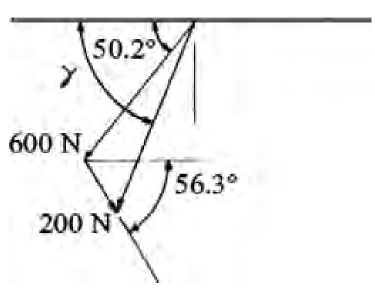
We measure: $\alpha = 50.2°$

$\beta = 56.3°$

(*a*) Parallelogram law:

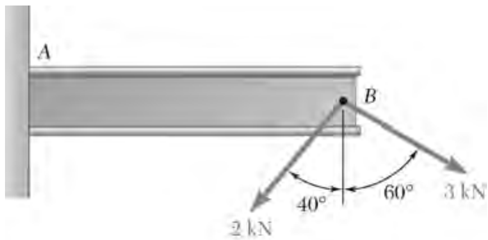


(*b*) Triangle rule:



We measure: $R = 684$ N, $\gamma = 66.5°$

$R = 684$ N 66.5° ◀

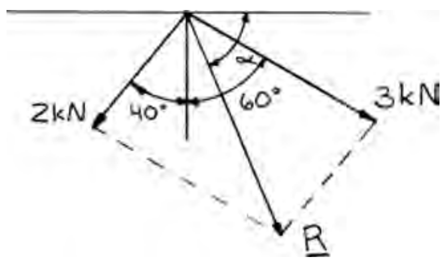


PROBLEM 2.4

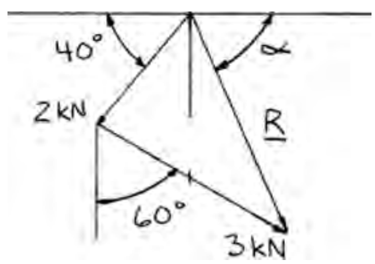
Two forces are applied at Point *B* of beam *AB*. Determine graphically the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.

SOLUTION

(*a*) Parallelogram law:

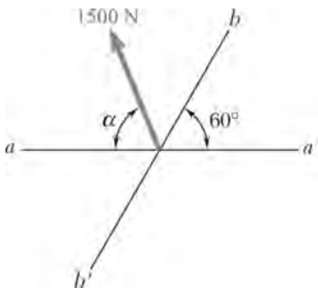


(*b*) Triangle rule:



We measure: $R = 3.30\ \text{kN}, \quad \alpha = 66.6°$

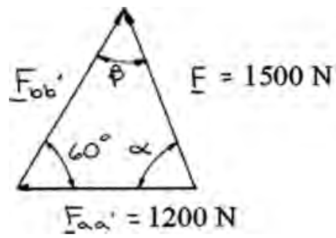
$\mathbf{R} = 3.30$ kN ↘ 66.6° ◀



PROBLEM 2.5

The 1500-N force is to be resolved into components along lines *a*-*a*′ and *b*-*b*′. (*a*) Determine the angle α by trigonometry knowing that the component along line *a*-*a*′ is to be 1200-N. (*b*) What is the corresponding value of the component along *b*-*b*′?

SOLUTION



(*a*) Using the triangle rule and law of sines:

$$\frac{\sin\beta}{1200\text{ N}} = \frac{\sin 60°}{1500\text{ N}}$$

$$\sin\beta = 0.69282$$

$$\beta = 43.854°$$

$$\alpha + \beta + 60° = 180°$$

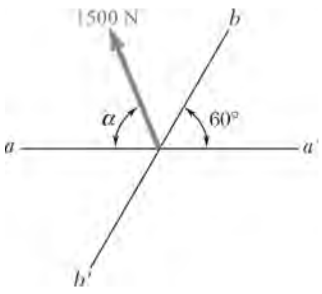
$$\alpha = 180° - 60° - 43.854°$$

$$= 76.146°$$

$\alpha = 76.1°$ ◀

(*b*) Law of sines: $\dfrac{F_{bb'}}{\sin 76.146°} = \dfrac{1500\text{ N}}{\sin 60°}$

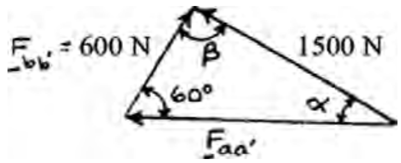
$F_{bb'} = 1682$ N ◀



PROBLEM 2.6

The 1500-N force is to be resolved into components along lines *a*-*a*′ and *b*-*b*′. (*a*) Determine the angle α by trigonometry knowing that the component along line *b*-*b*′ is to be 600 N. (*b*) What is the corresponding value of the component along *a*-*a*′?

SOLUTION



Using the triangle rule and law of sines:

(*a*)

$$\frac{\sin\alpha}{600\text{ N}} = \frac{\sin 60°}{1500\text{ N}}$$

$$\sin\alpha = 0.34641$$

$$\alpha = 20.268°$$

$\alpha = 20.3°$ ◀

(*b*)

$$\alpha + \beta + 60° = 180°$$

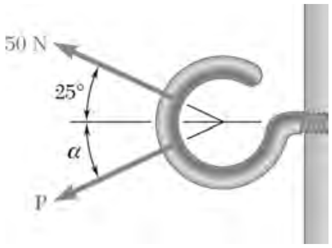
$$\beta = 180° - 60° - 20.268°$$

$$= 99.732°$$

$$\frac{F_{aa'}}{\sin 99.732°} = \frac{1500\text{ N}}{\sin 60°}$$

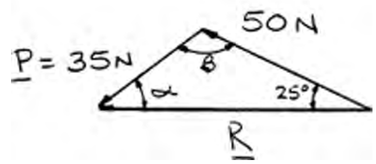
$F_{aa'} = 1707$ N ◀

PROBLEM 2.7



Two forces are applied as shown to a hook support. Knowing that the magnitude of **P** is 35 N, determine by trigonometry (*a*) the required angle α if the resultant **R** of the two forces applied to the support is to be horizontal, (*b*) the corresponding magnitude of **R**.

SOLUTION



Using the triangle rule and law of sines:

(*a*)

$$\frac{\sin \alpha}{50\text{ N}} = \frac{\sin 25^\circ}{35\text{ N}}$$

$$\sin \alpha = 0.60374$$

$$\alpha = 37.138^\circ$$

$\alpha = 37.1^\circ$ ◀

(*b*)

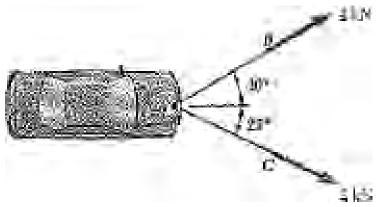
$$\alpha + \beta + 25^\circ = 180^\circ$$

$$\beta = 180^\circ - 25^\circ - 37.138^\circ$$

$$= 117.86^\circ$$

$$\frac{R}{\sin 117.86} = \frac{35\text{ N}}{\sin 25^\circ}$$

$R = 73.2$ N ◀

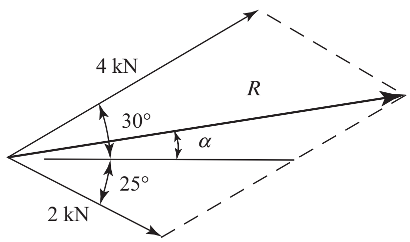


PROBLEM 2.8

A disabled automobile is pulled by means of ropes subjected to the two forces as shown. Determine graphically the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.

SOLUTION

(*a*) Parallelogram law:

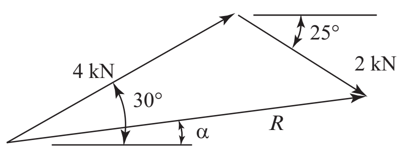


We measure:

$R = 5.4 \text{ kN} \quad \alpha = 12^\circ$

$\mathbf{R} = 5.4 \text{ kN} \measuredangle\ 12^\circ$ ◀

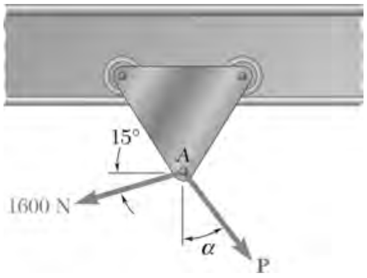
(*b*) Triangle rule:



We measure:

$R = 5.4 \text{ kN} \quad \alpha = 12^\circ$

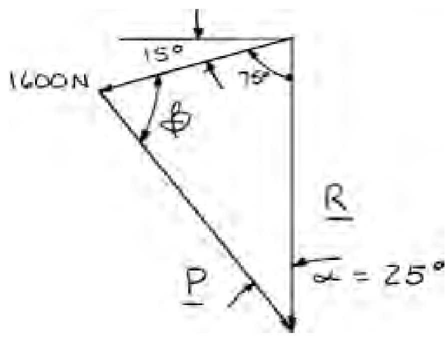
$\mathbf{R} = 5.4 \text{ kN} \measuredangle\ 12^\circ$ ◀



PROBLEM 2.9

A trolley that moves along a horizontal beam is acted upon by two forces as shown. (*a*) Knowing that $\alpha = 25°$, determine by trigonometry the magnitude of the force **P** so that the resultant force exerted on the trolley is vertical. (*b*) What is the corresponding magnitude of the resultant?

SOLUTION



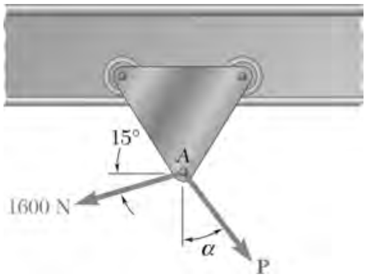
Using the triangle rule and the law of sines:

(*a*) $$\frac{1600 \text{ N}}{\sin 25°} = \frac{P}{\sin 75°}$$ $P = 3660 \text{ N}$ ◀

(*b*) $$25° + \beta + 75° = 180°$$
$$\beta = 180° - 25° - 75°$$
$$= 80°$$

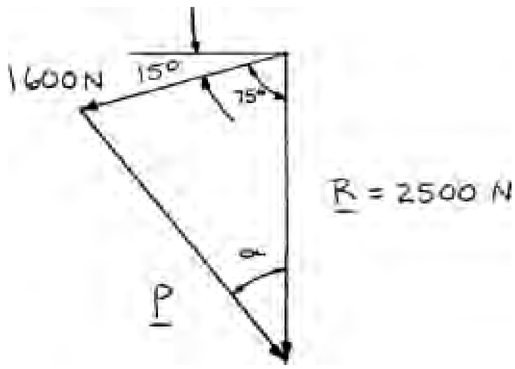
$$\frac{1600 \text{ N}}{\sin 25°} = \frac{R}{\sin 80°}$$ $R = 3730 \text{ N}$ ◀

PROBLEM 2.10



A trolley that moves along a horizontal beam is acted upon by two forces as shown. Determine by trigonometry the magnitude and direction of the force **P** so that the resultant is a vertical force of 2500 N.

SOLUTION



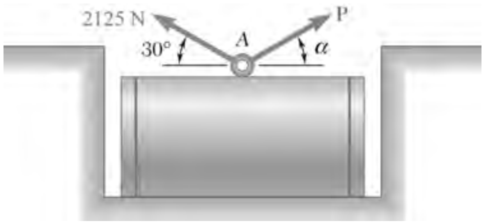
Using the law of cosines: $P^2 = (1600\text{ N})^2 + (2500\text{ N})^2 - 2(1600\text{ N})(2500\text{ N})\cos 75°$

$P = 2596\text{ N}$

Using the law of sines: $\dfrac{\sin\alpha}{1600\text{ N}} = \dfrac{\sin 75°}{2596\text{ N}}$

$\alpha = 36.5°$

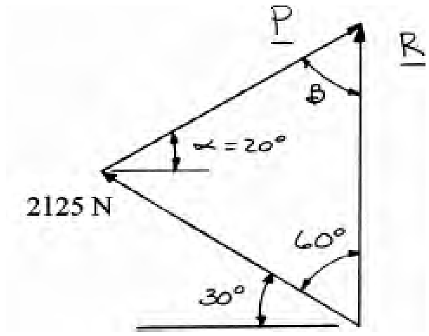
P is directed $90° - 36.5°$ or $53.5°$ below the horizontal. **P** = 2600 N ⦪ 53.5° ◀



PROBLEM 2.11

A steel tank is to be positioned in an excavation. Knowing that $\alpha = 20°$, determine by trigonometry (*a*) the required magnitude of the force **P** if the resultant **R** of the two forces applied at *A* is to be vertical, (*b*) the corresponding magnitude of **R**.

SOLUTION



Using the triangle rule and the law of sines:

(*a*)

$$\beta + 50° + 60° = 180°$$

$$\beta = 180° - 50° - 60°$$

$$= 70°$$

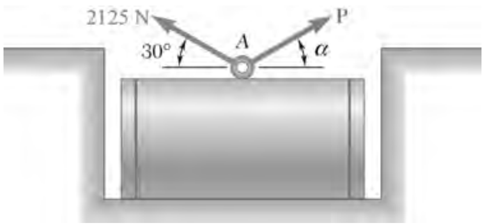
$$\frac{2125 \text{ N}}{\sin 70°} = \frac{P}{\sin 60°}$$

$P = 1958$ N ◀

(*b*)

$$\frac{2125 \text{ N}}{\sin 70°} = \frac{R}{\sin 50°}$$

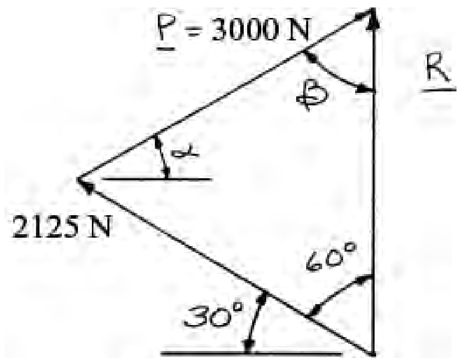
$R = 1732$ N ◀



PROBLEM 2.12

A steel tank is to be positioned in an excavation. Knowing that the magnitude of **P** is 3000 N, determine by trigonometry (*a*) the required angle α if the resultant **R** of the two forces applied at *A* is to be vertical, (*b*) the corresponding magnitude of **R**.

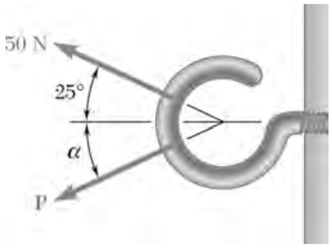
SOLUTION



Using the triangle rule and the law of sines:

(*a*)
$$(\alpha + 30°) + 60° + \beta = 180°$$
$$\beta = 180° - (\alpha + 30°) - 60°$$
$$\beta = 90° - \alpha$$
$$\frac{\sin(90° - \alpha)}{2125 \text{ N}} = \frac{\sin 60°}{3000 \text{ N}}$$
$$90° - \alpha = 37.8° \qquad \alpha = 52.2° \blacktriangleleft$$

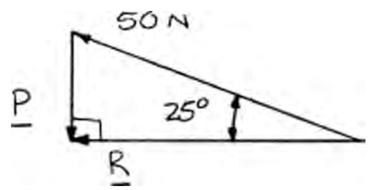
(*b*)
$$\frac{R}{\sin(52.2° + 30°)} = \frac{3000 \text{ N}}{\sin 60°} \qquad R = 3430 \text{ N} \blacktriangleleft$$



PROBLEM 2.13

For the hook support of Problem 2.7, determine by trigonometry (*a*) the magnitude and direction of the smallest force **P** for which the resultant **R** of the two forces applied to the support is horizontal, (*b*) the corresponding magnitude of **R**.

SOLUTION

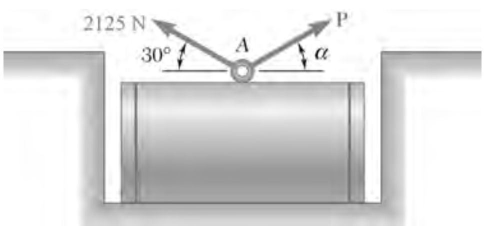


The smallest force P will be perpendicular to R.

(*a*) $P = (50\text{ N})\sin 25°$ $\quad$ $\mathbf{P} = 21.1\text{ N} \downarrow$ ◀

(*b*) $R = (50\text{ N})\cos 25°$ $\quad$ $R = 45.3\text{ N}$ ◀

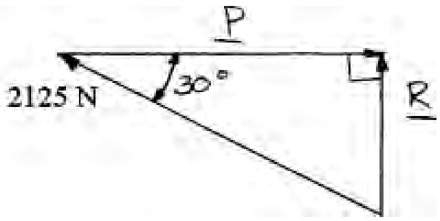
PROBLEM 2.14



For the steel tank of Problem 2.11, determine by trigonometry (*a*) the magnitude and direction of the smallest force **P** for which the resultant **R** of the two forces applied at *A* is vertical, (*b*) the corresponding magnitude of **R**.

PROBLEM 2.11 A steel tank is to be positioned in an excavation. Knowing that $\alpha = 20°$, determine by trigonometry (*a*) the required magnitude of the force **P** if the resultant **R** of the two forces applied at *A* is to be vertical, (*b*) the corresponding magnitude of **R**.

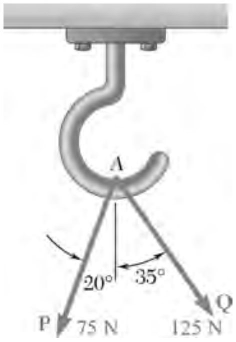
SOLUTION



The smallest force *P* will be perpendicular to *R*.

(*a*) $P = (2125 \text{ N})\cos 30°$ **P** = 1840 N → ◀

(*b*) $R = (2125 \text{ N})\sin 30°$ *R* = 1062.5 N ◀



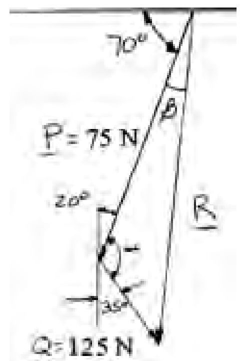
PROBLEM 2.15

Solve Problem 2.1 by trigonometry.

PROBLEM 2.1 Two forces **P** and **Q** are applied as shown at Point *A* of a hook support. Knowing that $P = 75$ N and $Q = 125$ N, determine graphically the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.

SOLUTION

Using the triangle rule and the law of cosines:



$$20° + 35° + \alpha = 180°$$

$$\alpha = 125°$$

$$R^2 = P^2 + Q^2 - 2PQ\cos\alpha$$

$$R^2 = (75\text{ N})^2 + (125\text{ N})^2 - 2(75\text{ N})(125\text{ N})\cos 125°$$

$$R^2 = 32004.56$$

$$R = 178.898\text{ N}$$

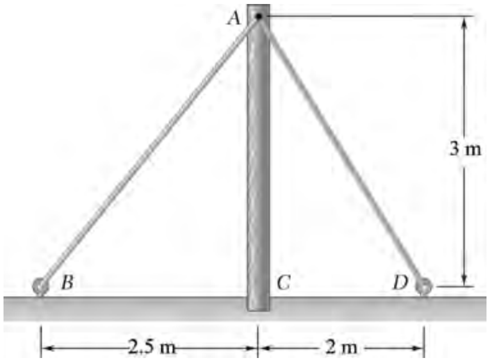
Using the law of sines:

$$\frac{\sin\beta}{125\text{ N}} = \frac{\sin 125°}{178.898\text{ N}}$$

$$\beta = 34.915°$$

$$70° + \beta = 104.915°$$

R = 178.9 N ⦨ 104.9º ◀

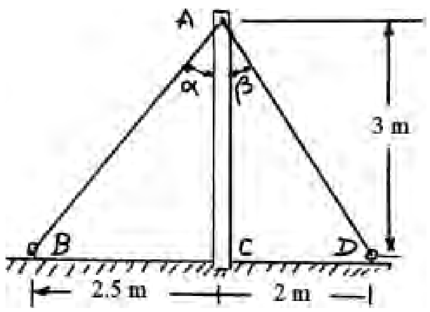


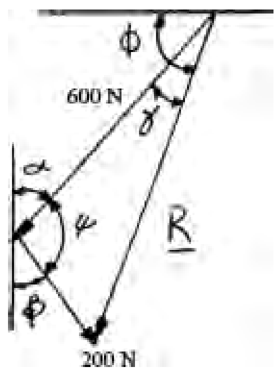
PROBLEM 2.16

Solve Problem 2.3 by trigonometry.

PROBLEM 2.3 The cable stays *AB* and *AD* help support pole *AC*. Knowing that the tension is 600 N in *AB* and 200 N in *AD*, determine graphically the magnitude and direction of the resultant of the forces exerted by the stays at *A* using (*a*) the parallelogram law, (*b*) the triangle rule.

SOLUTION





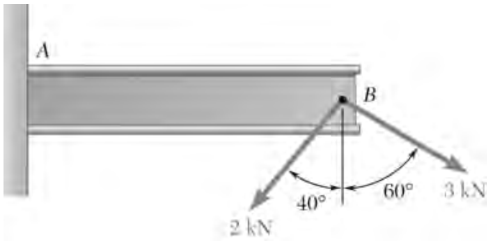
$$\tan\alpha = \frac{2.5}{3}$$
$$\alpha = 39.81°$$
$$\tan\beta = \frac{2}{3}$$
$$\beta = 33.69°$$

Using the triangle rule:
$$\alpha + \beta + \psi = 180°$$
$$39.81° + 33.69° + \psi = 180°$$
$$\psi = 106.5°$$

Using the law of cosines:
$$R^2 = (600\text{ N})^2 + (200\text{ N})^2 - 2(600\text{ N})(200\text{ N})\cos 106.5°$$
$$R = 684.22\text{ N}$$

Using the law of sines:
$$\frac{\sin\gamma}{200\text{ N}} = \frac{\sin 106.5°}{684.22\text{ N}}$$
$$\gamma = 16.28°$$
$$\phi = (90° - \alpha) + \gamma$$
$$\phi = (90° - 39.81°) + 16.28°$$
$$\phi = 66.47°$$

R = 684 N ↙ 66.5° ◀

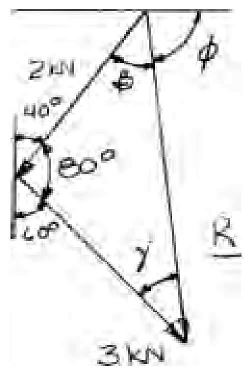


PROBLEM 2.17

Solve Problem 2.4 by trigonometry.

PROBLEM 2.4 Two forces are applied at Point *B* of beam *AB*. Determine graphically the magnitude and direction of their resultant using (*a*) the parallelogram law, (*b*) the triangle rule.

SOLUTION



Using the law of cosines:

$$R^2 = (2 \text{ kN})^2 + (3 \text{ kN})^2 - 2(2 \text{ kN})(3 \text{ kN})\cos 80°$$

$$R = 3.304 \text{ kN}$$

Using the law of sines:

$$\frac{\sin\gamma}{2 \text{ kN}} = \frac{\sin 80°}{3.304 \text{ kN}}$$

$$\gamma = 36.59°$$

$$\beta + \gamma + 80° = 180°$$

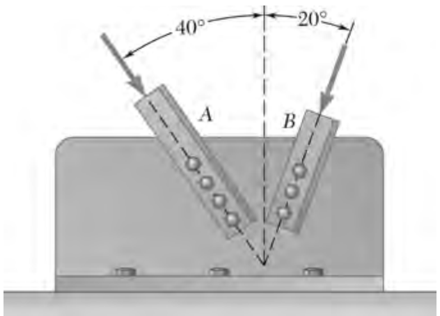
$$\gamma = 180° - 80° - 36.59°$$

$$\gamma = 63.41°$$

$$\phi = 180° - \beta + 50°$$

$$\phi = 66.59°$$

R = 3.30 kN ⦨ 66.6° ◀

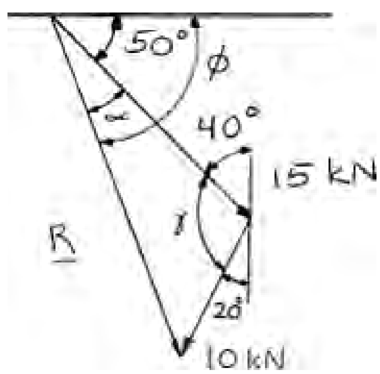


PROBLEM 2.18

Two structural members *A* and *B* are bolted to a bracket as shown. Knowing that both members are in compression and that the force is 15 kN in member *A* and 10 kN in member *B*, determine by trigonometry the magnitude and direction of the resultant of the forces applied to the bracket by members *A* and *B*.

SOLUTION

Using the force triangle and the laws of cosines and sines:



We have $$\gamma = 180° - (40° + 20°) = 120°$$

Then $$R^2 = (15\text{ kN})^2 + (10\text{ kN})^2 - 2(15\text{ kN})(10\text{ kN})\cos 120° = 475\text{ kN}^2$$

$$R = 21.794\text{ kN}$$

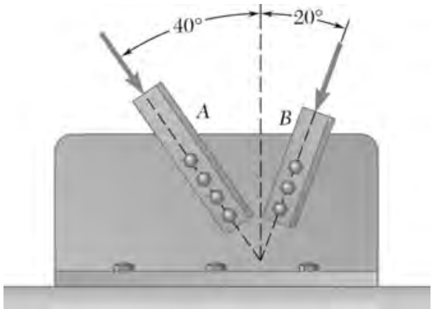
and $$\frac{10\text{ kN}}{\sin\alpha} = \frac{21.794\text{ kN}}{\sin 120°}$$

$$\sin\alpha = \left(\frac{10\text{ kN}}{21.794\text{ kN}}\right)\sin 120° = 0.39737$$

$$\alpha = 23.414$$

Hence: $$\phi = \alpha + 50° = 73.414$$

R = 21.8 kN ◥ 73.4° ◀

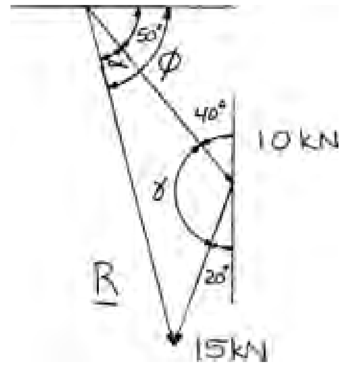


PROBLEM 2.19

Two structural members *A* and *B* are bolted to a bracket as shown. Knowing that both members are in compression and that the force is 10 kN in member *A* and 15 kN in member *B*, determine by trigonometry the magnitude and direction of the resultant of the forces applied to the bracket by members *A* and *B*.

SOLUTION

Using the force triangle and the laws of cosines and sines



We have $\gamma = 180° - (40° + 20°)$

$= 120°$

Then $R^2 = (10\text{ kN})^2 + (15\text{ kN})^2$

$- 2(10\text{ kN})(15\text{ kN})\cos 120°$

$= 475\text{ kN}^2$

$R = 21.794\text{ kN}$

and $$\frac{15\text{ kN}}{\sin\alpha} = \frac{21.794\text{ kN}}{\sin 120°}$$

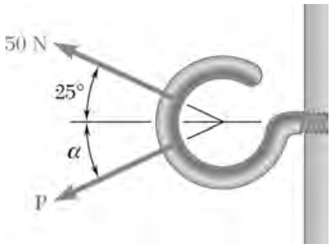
$$\sin\alpha = \left(\frac{15\text{ kN}}{21.794\text{ kN}}\right)\sin 120°$$

$= 0.59605$

$\alpha = 36.588°$

Hence: $\phi = \alpha + 50° = 86.588°$ **R** = 21.8 kN ◥ 86.6° ◀

PROBLEM 2.20

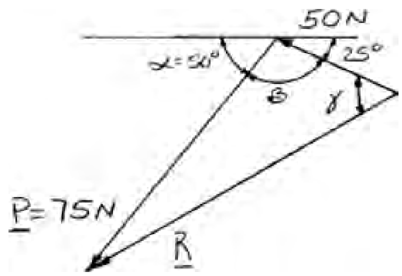


For the hook support of Problem 2.7, knowing that $P = 75\text{ N}$ and $\alpha = 50°$, determine by trigonometry the magnitude and direction of the resultant of the two forces applied to the support.

PROBLEM 2.7 Two forces are applied as shown to a hook support. Knowing that the magnitude of **P** is 35 N, determine by trigonometry (*a*) the required angle α if the resultant **R** of the two forces applied to the support is to be horizontal, (*b*) the corresponding magnitude of **R**.

SOLUTION

Using the force triangle and the laws of cosines and sines:



We have

$$\beta = 180° - (50° + 25°)$$
$$= 105°$$

Then

$$R^2 = (75\text{ N})^2 + (50\text{ N})^2 - 2(75\text{ N})(50\text{ N})\cos 105°$$
$$R^2 = 10066.1\text{ N}^2$$
$$R = 100.330\text{ N}$$

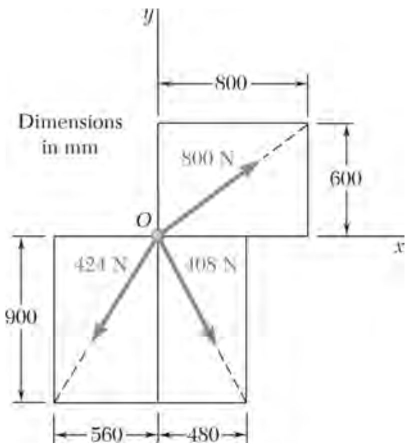
and

$$\frac{\sin\gamma}{75\text{ N}} = \frac{\sin 105°}{100.330\text{ N}}$$
$$\sin\gamma = 0.72206$$
$$\gamma = 46.225°$$

Hence:

$$\gamma - 25° = 46.225° - 25° = 21.225°$$

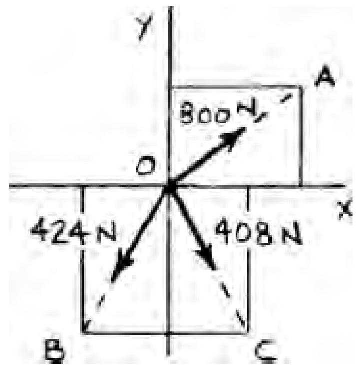
$\mathbf{R} = 100.3\text{ N} \measuredangle\ 21.2°$ ◀



PROBLEM 2.21

Determine the x and y components of each of the forces shown.

SOLUTION



Compute the following distances:

$$OA = \sqrt{(600)^2 + (800)^2} = 1000 \text{ mm}$$

$$OB = \sqrt{(560)^2 + (900)^2} = 1060 \text{ mm}$$

$$OC = \sqrt{(480)^2 + (900)^2} = 1020 \text{ mm}$$

800-N Force: $F_x = +(800 \text{ N})\dfrac{800}{1000}$ $\quad F_x = +640 \text{ N}$ ◀

$F_y = +(800 \text{ N})\dfrac{600}{1000}$ $\quad F_y = +480 \text{ N}$ ◀

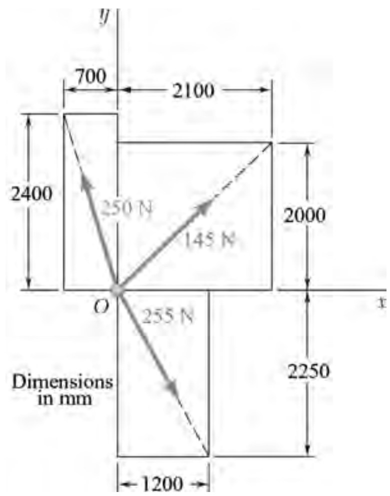
424-N Force: $F_x = -(424 \text{ N})\dfrac{560}{1060}$ $\quad F_x = -224 \text{ N}$ ◀

$F_y = -(424 \text{ N})\dfrac{900}{1060}$ $\quad F_y = -360 \text{ N}$ ◀

408-N Force: $F_x = +(408 \text{ N})\dfrac{480}{1020}$ $\quad F_x = +192.0 \text{ N}$ ◀

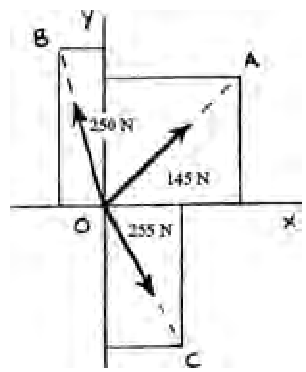
$F_y = -(408 \text{ N})\dfrac{900}{1020}$ $\quad F_y = -360 \text{ N}$ ◀

PROBLEM 2.22



Determine the x and y components of each of the forces shown.

SOLUTION



Compute the following distances:

$$OA = \sqrt{(2100)^2 + (2000)^2}$$
$$= 2900 \text{ mm}$$
$$OB = \sqrt{(700)^2 + (2400)^2}$$
$$= 2500 \text{ mm}$$
$$OC = \sqrt{(2250)^2 + (1200)^2}$$
$$= 2550 \text{ mm}$$

145-N Force: $F_x = +(145 \text{ N})\dfrac{2100}{2900}$ $\quad F_x = +105 \text{ N}$ ◀

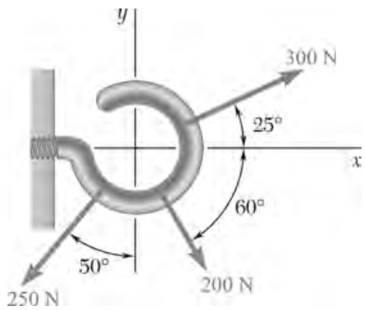
$F_y = +(145 \text{ N})\dfrac{2000}{2900}$ $\quad F_y = 100 \text{ N}$ ◀

250-N Force: $F_x = -(250 \text{ N})\dfrac{700}{2500}$ $\quad F_x = -70 \text{ N}$ ◀

$F_y = +(250 \text{ N})\dfrac{2400}{2500}$ $\quad F_y = 240 \text{ N}$ ◀

255-N Force: $F_x = +(255 \text{ N})\dfrac{1200}{2550}$ $\quad F_x = +120.0 \text{ N}$ ◀

$F_y = -(255 \text{ N})\dfrac{2250}{2550}$ $\quad F_y = -225 \text{ N}$ ◀



PROBLEM 2.23

Determine the x and y components of each of the forces shown.

SOLUTION

200-N Force: $F_x = +(200\text{ N})\cos 60°$ $\quad F_x = 100.0\text{ N}$ ◀

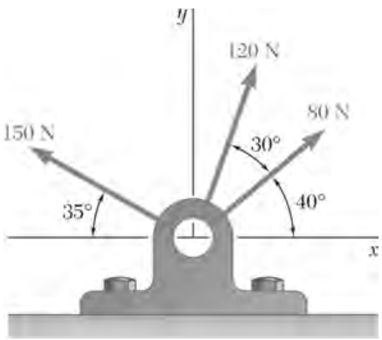
$F_y = -(200\text{ N})\sin 60°$ $\quad F_y = -173.2\text{ N}$ ◀

250-N Force: $F_x = -(250\text{ N})\sin 50°$ $\quad F_x = -191.5\text{ N}$ ◀

$F_y = -(250\text{ N})\cos 50°$ $\quad F_y = -160.7\text{ N}$ ◀

300-N Force: $F_x = +(300\text{ N})\cos 25°$ $\quad F_x = 272\text{ N}$ ◀

$F_y = +(300\text{ N})\sin 25°$ $\quad F_y = 126.8\text{ N}$ ◀



PROBLEM 2.24

Determine the x and y components of each of the forces shown.

SOLUTION

80-N Force: $F_x = +(80\text{ N})\cos 40°$ $\qquad F_x = 61.3\text{ N}$ ◀

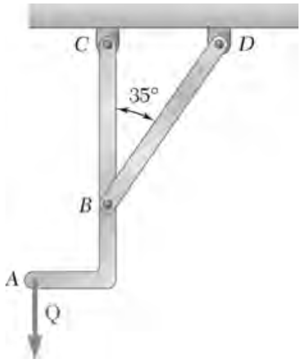
$F_y = +(80\text{ N})\sin 40°$ $\qquad F_y = 51.4\text{ N}$ ◀

120-N Force: $F_x = +(120\text{ N})\cos 70°$ $\qquad F_x = 41.0\text{ N}$ ◀

$F_y = +(120\text{ N})\sin 70°$ $\qquad F_y = 112.8\text{ N}$ ◀

150-N Force: $F_x = -(150\text{ N})\cos 35°$ $\qquad F_x = -122.\ 9\text{ N}$ ◀

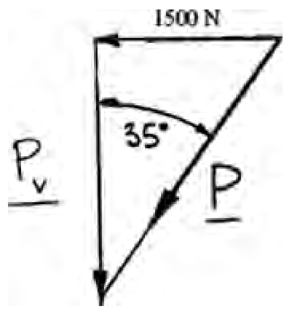
$F_y = +(150\text{ N})\sin 35°$ $\qquad F_y = 86.0\text{ N}$ ◀



PROBLEM 2.25

Member *BD* exerts on member *ABC* a force **P** directed along line *BD*. Knowing that **P** must have a 1500 N horizontal component, determine (*a*) the magnitude of the force **P**, (*b*) its vertical component.

SOLUTION



(*a*) $P \sin 35° = 1500 \text{ N}$

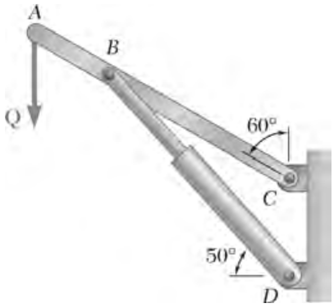
$$P = \frac{1500 \text{ N}}{\sin 35°}$$

$P = 2620 \text{ N}$ ◀

(*b*) Vertical component $P_v = P \cos 35°$

$$= (2615.17) \cos 35°$$

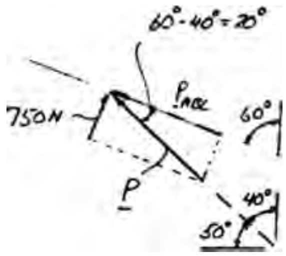
$P_v = 2140 \text{ N}$ ◀



PROBLEM 2.26

The hydraulic cylinder *BD* exerts on member *ABC* a force **P** directed along line *BD*. Knowing that **P** must have a 750-N component perpendicular to member *ABC*, determine (*a*) the magnitude of the force **P**, (*b*) its component parallel to *ABC*.

SOLUTION

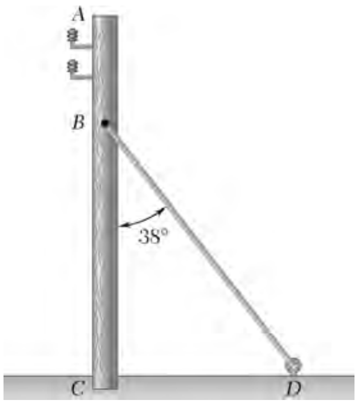


(*a*) $750\text{ N} = P\sin 20°$

$P = 2193\text{ N}$ $\qquad P = 2190\text{ N}$ ◀

(*b*) $P_{ABC} = P\cos 20°$

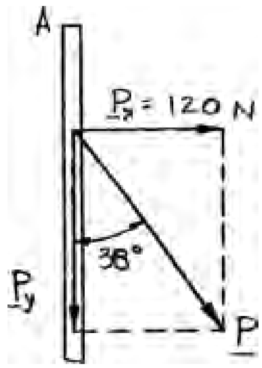
$= (2193\text{ N})\cos 20°$ $\qquad P_{ABC} = 2060\text{ N}$ ◀



PROBLEM 2.27

The guy wire *BD* exerts on the telephone pole *AC* a force **P** directed along *BD*. Knowing that **P** must have a 120-N component perpendicular to the pole *AC*, determine (*a*) the magnitude of the force **P**, (*b*) its component along line *AC*.

SOLUTION



(*a*)

$$P = \frac{P_x}{\sin 38°}$$

$$= \frac{120\ \text{N}}{\sin 38°}$$

$$= 194.91\ \text{N}$$

or $P = 194.9\ \text{N}$ ◀

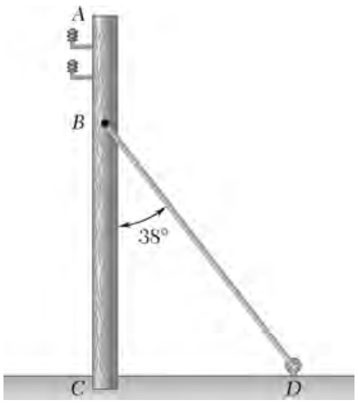
(*b*)

$$P_y = \frac{P_x}{\tan 38°}$$

$$= \frac{120\ \text{N}}{\tan 38°}$$

$$= 153.59\ \text{N}$$

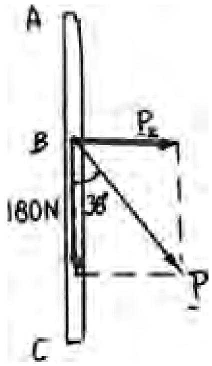
or $P_y = 153.6\ \text{N}$ ◀



PROBLEM 2.28

The guy wire *BD* exerts on the telephone pole *AC* a force **P** directed along *BD*. Knowing that **P** has a 180-N component along line *AC*, determine (*a*) the magnitude of the force **P**, (*b*) its component in a direction perpendicular to *AC*.

SOLUTION



(*a*)

$$P = \frac{P_y}{\cos 38°}$$

$$= \frac{180 \text{ N}}{\cos 38°}$$

$$= 228.4 \text{ N}$$

$P = 228 \text{ N}$ ◀

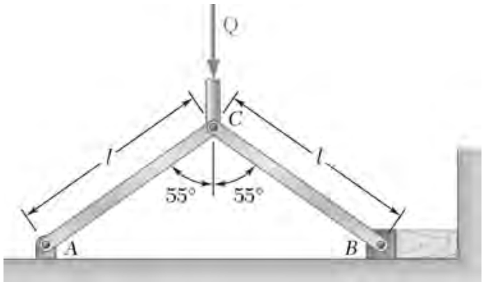
(*b*)

$$P_x = P_y \tan 38°$$

$$= (180 \text{ N}) \tan 38°$$

$$= 140.63 \text{ N}$$

$P_x = 140.6 \text{ N}$ ◀



PROBLEM 2.29

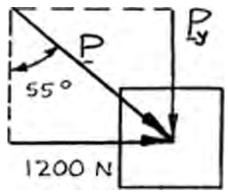
Member *CB* of the vise shown exerts on block *B* a force **P** directed along line *CB*. Knowing that **P** must have a 1200-N horizontal component, determine (*a*) the magnitude of the force **P**, (*b*) its vertical component.

SOLUTION

We note:

CB exerts force **P** on *B* along *CB*, and the horizontal component of **P** is $P_x = 1200$ N:

Then



(*a*)

$$P_x = P\sin 55°$$

$$P = \frac{P_x}{\sin 55°}$$

$$= \frac{1200 \text{ N}}{\sin 55°}$$

$$= 1464.9 \text{ N}$$

$P = 1465$ N ◀

(*b*)

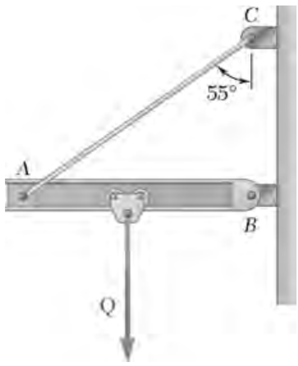
$$P_x = P_y \tan 55°$$

$$P_y = \frac{P_x}{\tan 55°}$$

$$= \frac{1200 \text{ N}}{\tan 55°}$$

$$= 840.2 \text{ N}$$

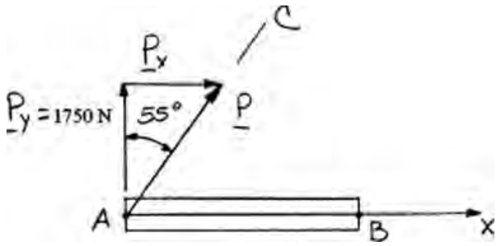
$\mathbf{P}_y = 840$ N ↓ ◀



PROBLEM 2.30

Cable *AC* exerts on beam *AB* a force **P** directed along line *AC*. Knowing that **P** must have a 1750-N vertical component, determine (*a*) the magnitude of the force **P**, (*b*) its horizontal component.

SOLUTION



(*a*)

$$P = \frac{P_y}{\cos 55°}$$

$$= \frac{1750 \text{ N}}{\cos 55°}$$

$$= 3051 \text{ N}$$

$P = 3050$ N ◀

(*b*)

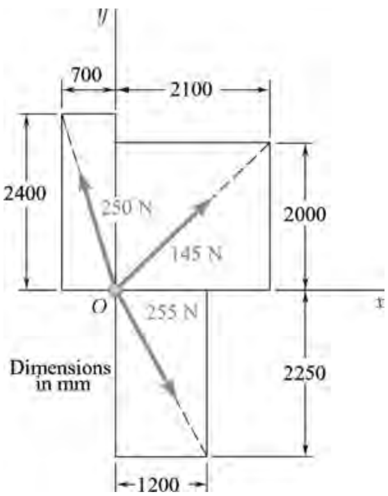
$$P_x = P \sin 55°$$

$$= (3051 \text{ N}) \sin 55°$$

$$= 2499.2 \text{ N}$$

$P_x = 2500$ N ◀

PROBLEM 2.31



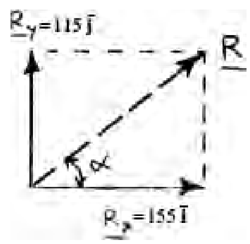
Determine the resultant of the three forces of Problem 2.22.

PROBLEM 2.22 Determine the x and y components of each of the forces shown.

SOLUTION

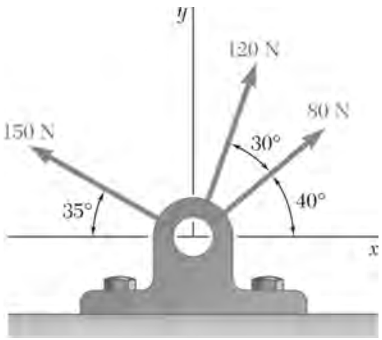
Components of the forces were determined in Problem 2.22:

Force	x Comp. (N)	y Comp. (N)
145 N	+105	+100
250 N	–70	+240
255 N	+120	–225
	$R_x = +155$	$R_y = +115$



$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j}$$
$$= (155 \text{ N})\mathbf{i} + (115 \text{ N})\mathbf{j}$$
$$\tan\alpha = \frac{R_y}{R_x}$$
$$= \frac{115}{155}$$
$$\alpha = 36.573°$$
$$R = \sqrt{(155)^2 + (115)^2}$$
$$= 193.0 \text{ N}$$

$\mathbf{R} = 193.0 \text{ N} \measuredangle 36.6°$ ◀



PROBLEM 2.32

Determine the resultant of the three forces of Problem 2.24.

PROBLEM 2.24 Determine the x and y components of each of the forces shown.

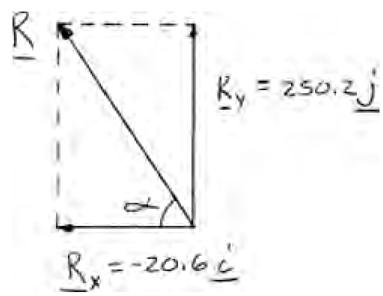
SOLUTION

Components of the forces were determined in Problem 2.24:

Force	x Comp. (N)	y Comp. (N)
80 N	+61.3	+51.4
120 N	+741.0	+112.8
150 N	–122.9	+86.0
	$R_x = -20.6$	$R_y = +250.2$

$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j}$$

$$= (-20.6\text{ N})\mathbf{i} + (250.2\text{ N})\mathbf{j}$$



$$\tan\alpha = \frac{R_y}{R_x}$$

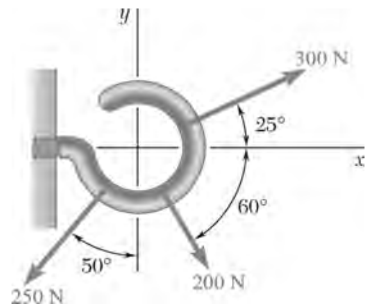
$$\tan\alpha = \frac{250.2\text{ N}}{20.6\text{ N}}$$

$$\tan\alpha = 12.1456$$

$$\alpha = 85.293°$$

$$R = \frac{250.2\text{ N}}{\sin 85.293°}$$

R = 251 N ∡ 85.3° ◀



PROBLEM 2.33

Determine the resultant of the three forces of Problem 2.23.

PROBLEM 2.23 Determine the *x* and *y* components of each of the forces shown.

SOLUTION

Force	*x* Comp. (N)	*y* Comp. (N)
200 N	+100	−173.2
250 N	−191.5	−160.7
300 N	+272	+126.8
	$R_x = +180.5$	$R_y = -207.1$

$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j}$$

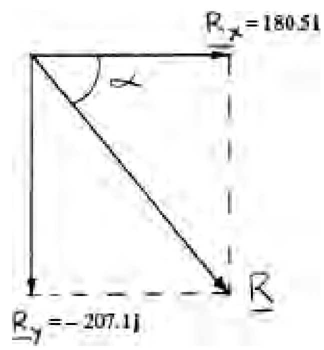
$$= +(180.5\text{ N})\mathbf{i} - (207.1\text{ N})\mathbf{j}$$

$$\tan\alpha = \frac{R_y}{R_x}$$

$$\tan\alpha = \frac{207.1\text{ N}}{180.5\text{ N}}$$

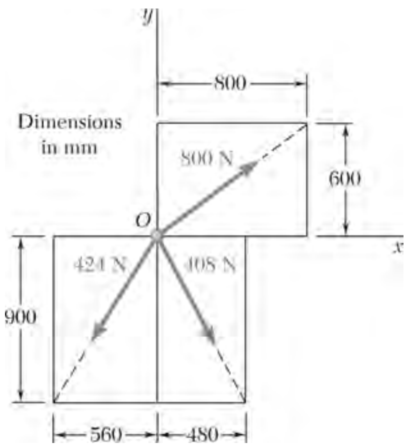
$$\tan\alpha = 1.147$$

$$\alpha = 48.9°$$



$$R = \sqrt{(180.5)^2 + (-207.1)^2} = 274.72\text{ N}$$

R = 275 N 48.9° ◀



PROBLEM 2.34

Determine the resultant of the three forces of Problem 2.21.

PROBLEM 2.21 Determine the x and y components of each of the forces shown.

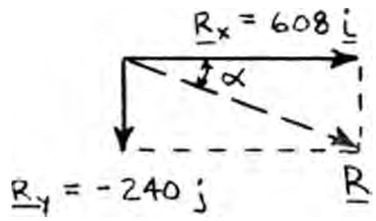
SOLUTION

Components of the forces were determined in Problem 2.21:

Force	x Comp. (N)	y Comp. (N)
800 N	+640	+480
424 N	–224	–360
408 N	+192	–360
	$R_x = +608$	$R_y = -240$

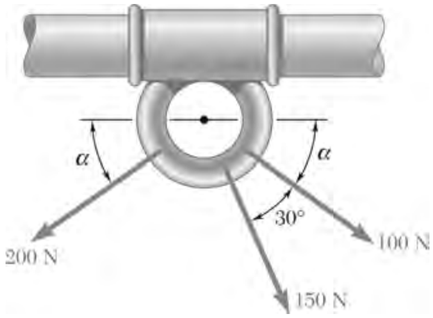
$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j}$$
$$= (608\text{ N})\mathbf{i} + (-240\text{ N})\mathbf{j}$$

$$\tan\alpha = \frac{R_y}{R_x}$$
$$= \frac{240}{608}$$
$$\alpha = 21.541°$$



$$R = \frac{240\text{ N}}{\sin(21.541°)}$$
$$= 653.65\text{ N}$$

R = 654 N ↘ 21.5° ◀



PROBLEM 2.35

Knowing that $\alpha = 35°$, determine the resultant of the three forces shown.

SOLUTION

100-N Force:

$$F_x = +(100 \text{ N})\cos 35° = +81.915 \text{ N}$$
$$F_y = -(100 \text{ N})\sin 35° = -57.358 \text{ N}$$

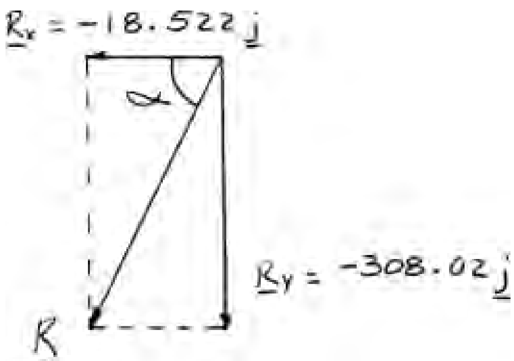
150-N Force:

$$F_x = +(150 \text{ N})\cos 65° = +63.393 \text{ N}$$
$$F_y = -(150 \text{ N})\sin 65° = -135.946 \text{ N}$$

200-N Force:

$$F_x = -(200 \text{ N})\cos 35° = -163.830 \text{ N}$$
$$F_y = -(200 \text{ N})\sin 35° = -114.715 \text{ N}$$

Force	x Comp. (N)	y Comp. (N)
100 N	+81.915	−57.358
150 N	+63.393	−135.946
200 N	−163.830	−114.715
	$R_x = -18.522$	$R_y = -308.02$



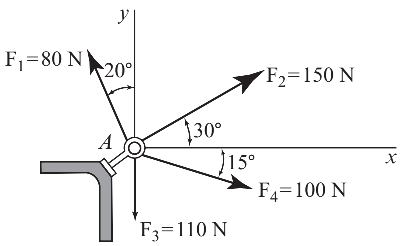
$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j}$$
$$= (-18.522 \text{ N})\mathbf{i} + (-308.02 \text{ N})\mathbf{j}$$

$$\tan\alpha = \frac{R_y}{R_x}$$
$$= \frac{308.02}{18.522}$$
$$\alpha = 86.559°$$

$$R = \frac{308.02 \text{ N}}{\sin 86.559}$$

$\mathbf{R} = 309$ N 86.6° ◄

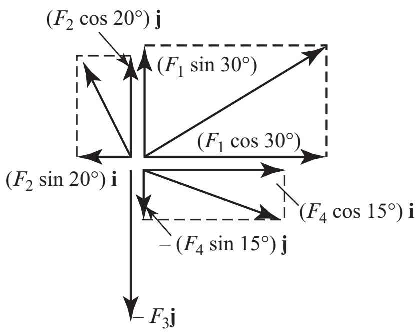
PROBLEM 2.36



Four forces act on bolt *A* as shown. Determine the resultant of the forces on the bolt.

SOLUTION

The *x* and *y* components of each force are determined by trigonometry as shown and are entered in the table below.

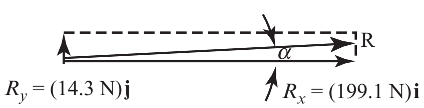


Force	Magnitude N	*x* Component N	*y* Component N
$\mathbf{F}_1$	150	+129.9	+75.0
$\mathbf{F}_2$	80	–27.4	+75.2
$\mathbf{F}_3$	110	0	–110.0
$\mathbf{F}_4$	100	+96.6	–25.9
		$R_x = +199.1$	$R_y = +14.3$

Thus the resultant **R** of the four forces is

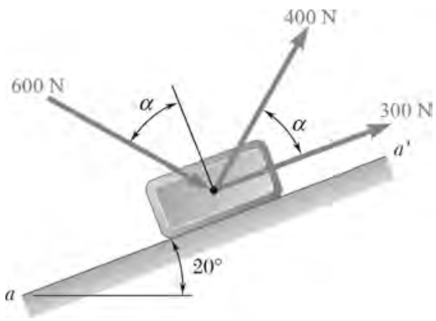
$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j} \qquad \mathbf{R} = (199.1\text{ N})\mathbf{i} + (14.3\text{ N})\mathbf{j} \blacktriangleleft$$

The magnitude and direction of the resultant may now be determined. From the triangle shown, we have



$$\tan\alpha = \frac{R_y}{R_x} = \frac{14.3\text{ N}}{199.1\text{ N}} \qquad \alpha = 4.1°$$

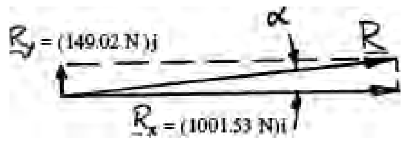
$$R = \frac{14.3\text{ N}}{\sin\alpha} = 199.6\text{ N} \qquad \mathbf{R} = 199.6\text{ N} \measuredangle\, 4.1° \blacktriangleleft$$



PROBLEM 2.37

Knowing that $\alpha = 40°$, determine the resultant of the three forces shown.

SOLUTION



300-N Force: $F_x = (300\text{ N})\cos 20° = 281.91\text{ N}$

$F_y = (300\text{ N})\sin 20° = 102.61\text{ N}$

400-N Force: $F_x = (400\text{ N})\cos 60° = 200.0\text{ N}$

$F_y = (400\text{ N})\sin 60° = 346.41\text{ N}$

600-N Force: $F_x = (600\text{ N})\cos 30° = 519.62\text{ N}$

$F_y = -(600\text{ N})\sin 30° = -300.0\text{ N}$

and $R_x = \Sigma F_x = 1001.53\text{ N}$

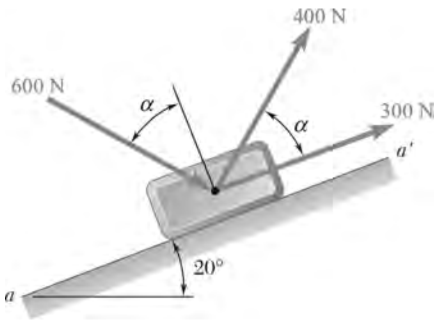
$R_y = \Sigma F_y = 149.02\text{ N}$

$$R = \sqrt{(1001.53)^2 + (149.02)^2}$$
$$= 1012.56\text{ N}$$

Further: $$\tan\alpha = \frac{149.02}{1001.53} = 0.14879$$

$$\alpha = \tan^{-1} 0.14879$$
$$= 8.463°$$

$\mathbf{R} = 1013\text{ N} \measuredangle 8.46°$ ◀



PROBLEM 2.38

Knowing that $\alpha = 75°$, determine the resultant of the forces shown.

SOLUTION

300-N Force: $F_x = (300 \text{ N})\cos 20° = 281.91 \text{ N}$

$F_y = (300 \text{ N})\sin 20° = 102.61 \text{ N}$

400-N Force: $F_x = (400 \text{ N})\cos 95° = -34.862 \text{ N}$

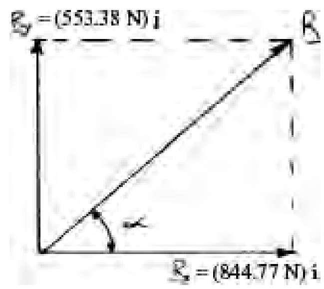
$F_y = (400 \text{ N})\sin 95° = 398.48 \text{ N}$

600-N Force: $F_x = (600 \text{ N})\cos 5° = 597.72 \text{ N}$

$F_y = (600 \text{ N})\sin 5° = 52.293 \text{ N}$

Then $R_x = \Sigma F_x = 844.77 \text{ N}$

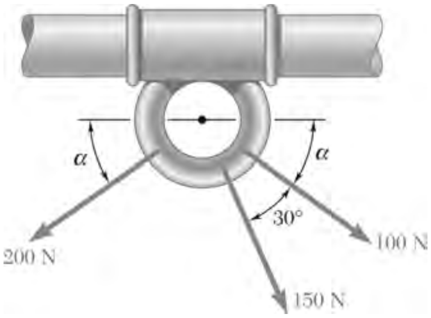
$R_y = \Sigma F_y = 553.38 \text{ N}$



and
$$R = \sqrt{(844.77)^2 + (553.38)^2}$$
$$= 1009.88 \text{ N}$$

$$\tan\alpha = \frac{553.38}{844.77}$$
$$\tan\alpha = 0.655$$
$$\alpha = 33.23°$$

$\mathbf{R} = 1010 \text{ N} \measuredangle 33.2°$ ◀



PROBLEM 2.39

For the collar of Problem 2.35, determine (*a*) the required value of α if the resultant of the three forces shown is to be vertical, (*b*) the corresponding magnitude of the resultant.

SOLUTION

$$
\begin{aligned}
R_x &= \Sigma F_x \\
&= (100\text{ N})\cos\alpha + (150\text{ N})\cos(\alpha+30°) - (200\text{ N})\cos\alpha \\
R_x &= -(100\text{ N})\cos\alpha + (150\text{ N})\cos(\alpha+30°) \qquad (1)
\end{aligned}
$$

$$
\begin{aligned}
R_y &= \Sigma F_y \\
&= -(100\text{ N})\sin\alpha - (150\text{ N})\sin(\alpha+30°) - (200\text{ N})\sin\alpha \\
R_y &= -(300\text{ N})\sin\alpha - (150\text{ N})\sin(\alpha+30°) \qquad (2)
\end{aligned}
$$

(*a*) For **R** to be vertical, we must have $R_x = 0.$ We make $R_x = 0$ in Eq. (1):

$$-100\cos\alpha + 150\cos(\alpha+30°) = 0$$

$$-100\cos\alpha + 150(\cos\alpha\cos 30° - \sin\alpha\sin 30°) = 0$$

$$29.904\cos\alpha = 75\sin\alpha$$

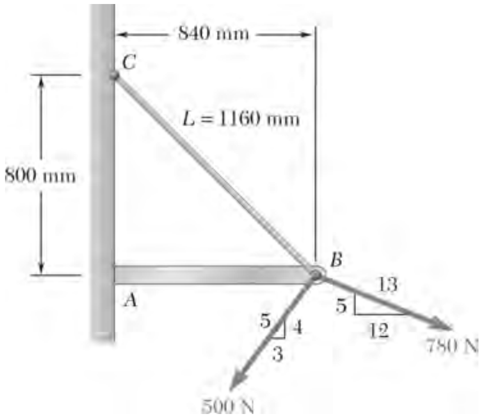
$$
\begin{aligned}
\tan\alpha &= \frac{29.904}{75} \\
&= 0.3988 \\
\alpha &= 21.74°
\end{aligned}
$$

$\alpha = 21.7°$ ◀

(*b*) Substituting for α in Eq. (2):

$$
\begin{aligned}
R_y &= -300\sin 21.74° - 150\sin 51.74° \\
&= -228.9\text{ N} \\
R &= |R_y| = 228.9\text{ N}
\end{aligned}
$$

$R = 229$ N ◀



PROBLEM 2.40

For the beam of Sample Problem 2.3, determine (*a*) the required tension in cable *BC* if the resultant of the three forces exerted at Point *B* is to be vertical, (*b*) the corresponding magnitude of the resultant.

SOLUTION

$$R_x = \Sigma F_x = -\frac{840}{1160}T_{BC} + \frac{12}{13}(780\text{ N}) - \frac{3}{5}(500\text{ N})$$

$$R_x = -\frac{21}{29}T_{BC} + 420\text{ N} \qquad (1)$$

$$R_y = \Sigma F_y = \frac{800}{1160}T_{BC} - \frac{5}{13}(780\text{ N}) - \frac{4}{5}(500\text{ N})$$

$$R_y = \frac{20}{29}T_{BC} - 700\text{ N} \qquad (2)$$

(*a*) For **R** to be vertical, we must have $R_x = 0$

Set $R_x = 0$ in Eq. (1) $\quad -\frac{21}{29}T_{BC} + 420\text{ N} = 0 \quad$ $T_{BC} = 580\text{ N}$ ◀

(*b*) Substituting for T_{BC} in Eq. (2):

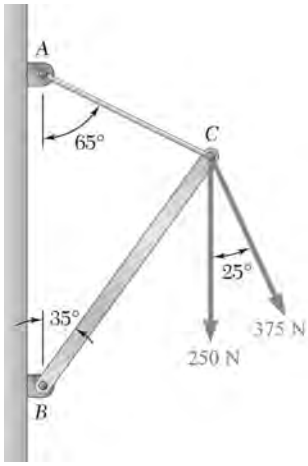
$$R_y = \frac{20}{29}(580\text{ N}) - 700\text{ N}$$

$$R_y = -300\text{ N}$$

$$R = |R_y| = 300\text{ N}$$

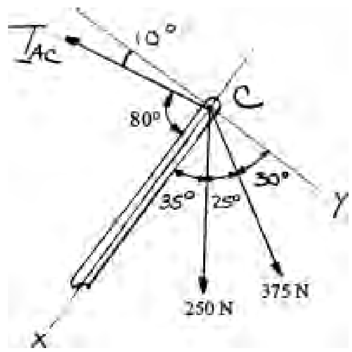
$R = 300\text{ N}$ ◀

PROBLEM 2.41



Determine (*a*) the required tension in cable *AC*, knowing that the resultant of the three forces exerted at Point *C* of boom *BC* must be directed along *BC*, (*b*) the corresponding magnitude of the resultant.

SOLUTION



Using the *x* and *y* axes shown:

$$R_x = \Sigma F_x = T_{AC}\sin 10° + (250\text{ N})\cos 35° + (375\text{ N})\cos 60°$$
$$= T_{AC}\sin 10° + 392.29\text{ N} \qquad (1)$$

$$R_y = \Sigma F_y = (250\text{ N})\sin 35° + (375\text{ N})\sin 60° - T_{AC}\cos 10°$$
$$R_y = 468.15\text{ N} - T_{AC}\cos 10° \qquad (2)$$

(*a*) Set $R_y = 0$ in Eq. (2):

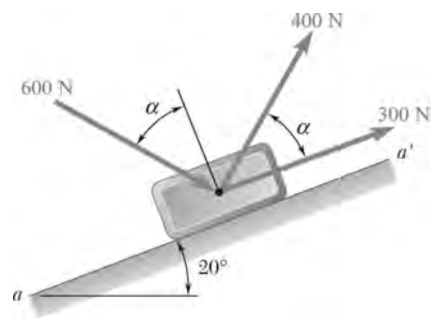
$$468.15\text{ N} - T_{AC}\cos 10° = 0$$
$$T_{AC} = 475.37\text{ N}$$

$T_{AC} = 475\text{ N}$ ◀

(*b*) Substituting for T_{AC} in Eq. (1):

$$R_x = (475.37\text{ N})\sin 10° + 392.29\text{ N}$$
$$= 474.84\text{ N}$$
$$R = R_x$$

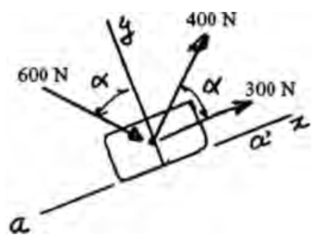
$R = 475\text{ N}$ ◀



PROBLEM 2.42

For the block of Problems 2.37, determine (*a*) the required value of α if the resultant of the three forces shown is to be parallel to the incline, (*b*) the corresponding magnitude of the resultant.

SOLUTION



Select the *x* axis to be along *a a*′.

Then

$$R_x = \Sigma F_x = (300 \text{ N}) + (400 \text{ N})\cos\alpha + (600 \text{ N})\sin\alpha \qquad (1)$$

and

$$R_y = \Sigma F_y = (400 \text{ N})\sin\alpha - (600 \text{ N})\cos\alpha \qquad (2)$$

(*a*) Set $R_y = 0$ in Eq. (2).

$$(400 \text{ N})\sin\alpha - (600 \text{ N})\cos\alpha = 0$$

Dividing each term by $\cos\alpha$ gives:

$$(400 \text{ N})\tan\alpha = 600 \text{ N}$$

$$\tan\alpha = \frac{600 \text{ N}}{400 \text{ N}}$$

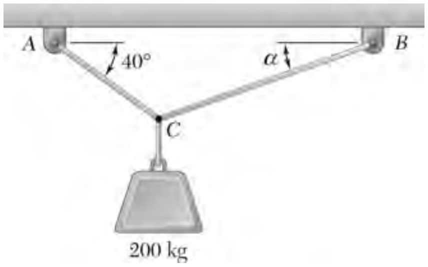
$$\alpha = 56.310^\circ$$

$\alpha = 56.3^\circ$ ◀

(*b*) Substituting for α in Eq. (1) gives:

$$R_x = 300 \text{ N} + (400 \text{ N})\cos 56.31^\circ + (600 \text{ N})\sin 56.31^\circ = 1021.11 \text{ N}$$

$R_x = 1021$ N ◀

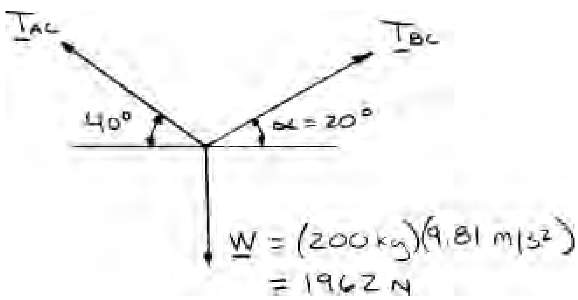


PROBLEM 2.43

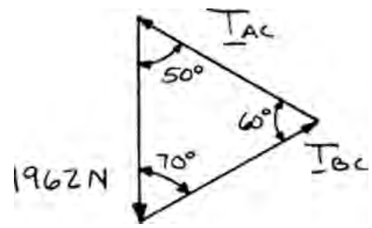
Two cables are tied together at C and are loaded as shown. Knowing that $\alpha = 20°$, determine the tension (a) in cable AC, (b) in cable BC.

SOLUTION

Free-Body Diagram



Force Triangle



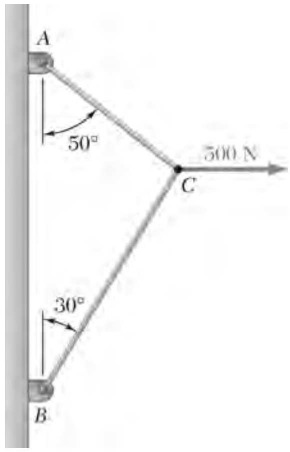
Law of sines: $$\frac{T_{AC}}{\sin 70°} = \frac{T_{BC}}{\sin 50°} = \frac{1962 \text{ N}}{\sin 60°}$$

(a) $$T_{AC} = \frac{1962 \text{ N}}{\sin 60°} \sin 70° = 2128.9 \text{ N}$$

$T_{AC} = 2.13$ kN ◀

(b) $$T_{BC} = \frac{1962 \text{ N}}{\sin 60°} \sin 50° = 1735.49 \text{ N}$$

$T_{BC} = 1.735$ kN ◀

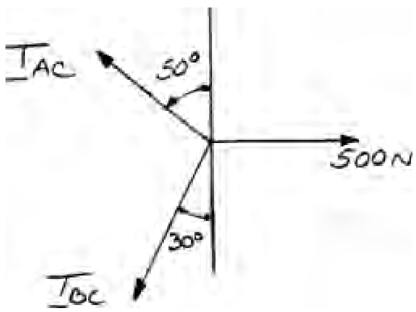


PROBLEM 2.44

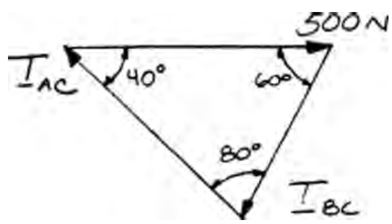
Two cables are tied together at C and are loaded as shown. Determine the tension (a) in cable AC, (b) in cable BC.

SOLUTION

Free-Body Diagram



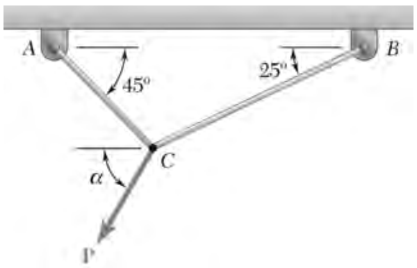
Force Triangle



Law of sines: $$\frac{T_{AC}}{\sin 60^\circ} = \frac{T_{BC}}{\sin 40^\circ} = \frac{500 \text{ N}}{\sin 80^\circ}$$

(a) $$T_{AC} = \frac{500 \text{ N}}{\sin 80^\circ} \sin 60^\circ = 439.69 \text{ N}$$ $T_{AC} = 440$ N ◀

(b) $$T_{BC} = \frac{500 \text{ N}}{\sin 80^\circ} \sin 40^\circ = 326.35 \text{ N}$$ $T_{BC} = 326$ N ◀

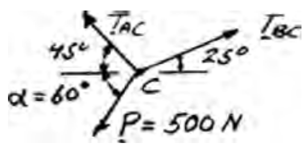


PROBLEM 2.45

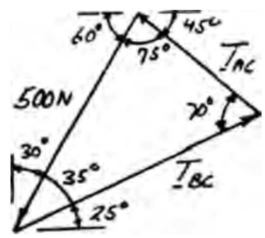
Two cables are tied together at C and are loaded as shown. Knowing that $\mathbf{P} = 500$ N and $\alpha = 60°$, determine the tension in (*a*) in cable *AC*, (*b*) in cable *BC*.

SOLUTION

Free-Body Diagram



Force Triangle



Law of sines: $$\frac{T_{AC}}{\sin 35°} = \frac{T_{BC}}{\sin 75°} = \frac{500\text{ N}}{\sin 70°}$$

(*a*) $$T_{AC} = \frac{500\text{ N}}{\sin 70°}\sin 35°$$ $T_{AC} = 305$ N ◀

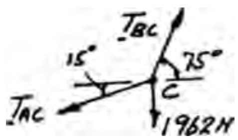
(*b*) $$T_{BC} = \frac{500\text{ N}}{\sin 70°}\sin 75°$$ $T_{BC} = 514$ N ◀

PROBLEM 2.46

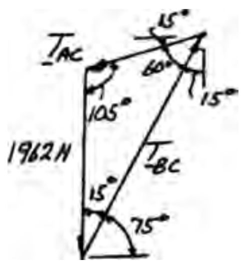
Two cables are tied together at C and are loaded as shown. Determine the tension (a) in cable AC, (b) in cable BC.

SOLUTION

Free-Body Diagram



Force Triangle



$$W = \text{mg}$$
$$= (200 \text{ kg})(9.81 \text{ m/s}^2)$$
$$= 1962 \text{ N}$$

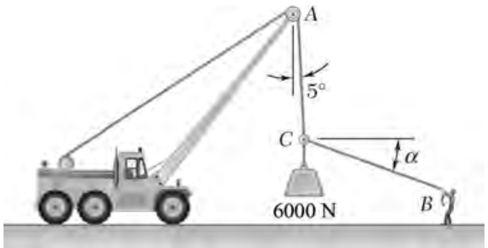
Law of sines: $$\frac{T_{AC}}{\sin 15°} = \frac{T_{BC}}{\sin 105°} = \frac{1962 \text{ N}}{\sin 60°}$$

(a) $$T_{AC} = \frac{(1962 \text{ N})\sin 15°}{\sin 60°}$$

$T_{AC} = 586$ N ◀

(b) $$T_{BC} = \frac{(1962 \text{ N})\sin 105°}{\sin 60°}$$

$T_{BC} = 2190$ N ◀

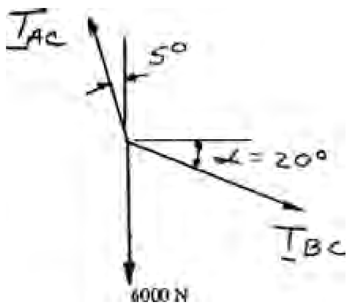


PROBLEM 2.47

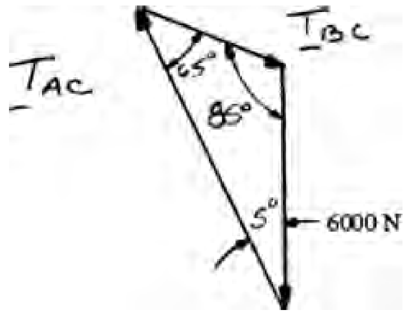
Knowing that $\alpha = 20°$, determine the tension (*a*) in cable *AC*, (*b*) in rope *BC*.

SOLUTION

Free-Body Diagram



Force Triangle

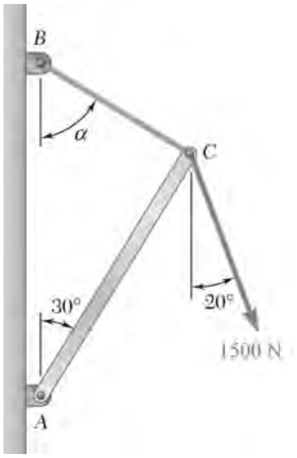


Law of sines: $$\frac{T_{AC}}{\sin 110°} = \frac{T_{BC}}{\sin 5°} = \frac{6000 \text{ N}}{\sin 65°}$$

(*a*) $$T_{AC} = \frac{6000 \text{ N}}{\sin 65°} \sin 110°$$ $T_{AC} = 6220 \text{ N}$ ◀

(*b*) $$T_{BC} = \frac{6000 \text{ N}}{\sin 65°} \sin 5°$$ $T_{BC} = 577 \text{ N}$ ◀

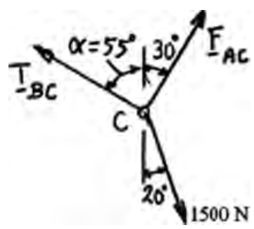
PROBLEM 2.48



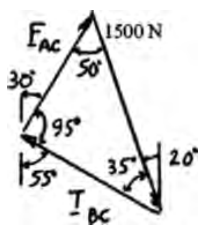
Knowing that $\alpha = 55°$ and that boom *AC* exerts on pin *C* a force directed along line *AC*, determine (*a*) the magnitude of that force, (*b*) the tension in cable *BC*.

SOLUTION

Free-Body Diagram



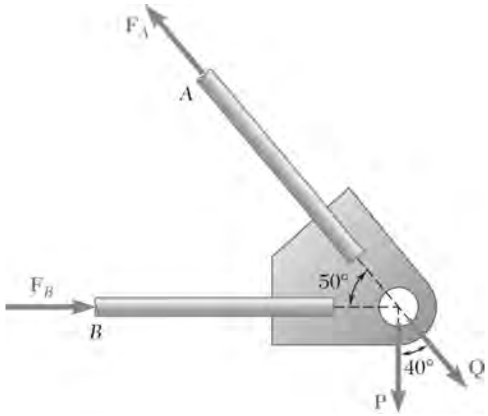
Force Triangle



Law of sines: $$\frac{F_{AC}}{\sin 35°} = \frac{T_{BC}}{\sin 50°} = \frac{1500\text{ N}}{\sin 95°}$$

(*a*) $$F_{AC} = \frac{1500\text{ N}}{\sin 95°}\sin 35°$$ $F_{AC} = 864\text{ N}$ ◀

(*b*) $$T_{BC} = \frac{1500\text{ N}}{\sin 95°}\sin 50°$$ $T_{BC} = 1153\text{ N}$ ◀

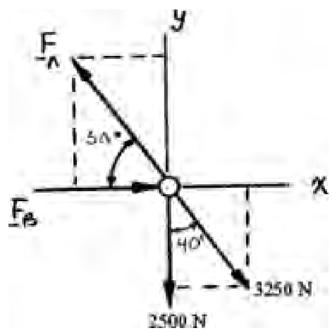


PROBLEM 2.49

Two forces **P** and **Q** are applied as shown to an aircraft connection. Knowing that the connection is in equilibrium and that $P = 2500$ N and $Q = 3250$ N, determine the magnitudes of the forces exerted on the rods A and B.

SOLUTION

Free-Body Diagram



Resolving the forces into x- and y-directions:

$$\mathbf{R} = \mathbf{P} + \mathbf{Q} + \mathbf{F}_A + \mathbf{F}_B = 0$$

Substituting components:

$$\mathbf{R} = -(2500\text{ N})\mathbf{j} + [(3250\text{ N})\cos 50°]\mathbf{i} - [(3250\text{ N})\sin 50°]\mathbf{j} + F_B\mathbf{i} - (F_A \cos 50°)\mathbf{i} + (F_A \sin 50°)\mathbf{j} = 0$$

In the y-direction (one unknown force)

$$-2500\text{ N} - (3250\text{ N})\sin 50° + F_A \sin 50° = 0$$

Thus,

$$F_A = \frac{2500\text{ N} + (3250\text{ N})\sin 50°}{\sin 50°}$$

$$= 6513.5\text{ N}$$

$F_A = 6510$ N ◀

In the x-direction:

$$(3250\text{ N})\cos 50° + F_B - F_A \cos 50° = 0$$

Thus,

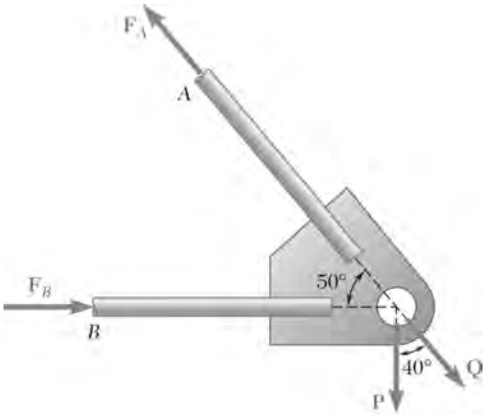
$$F_B = F_A \cos 50° - (3250\text{ N})\cos 50°$$

$$= (6513.5\text{ N})\cos 50° - (3250\text{ N})\cos 50°$$

$$= 2097.7\text{ N}$$

$F_B = 2100$ N ◀

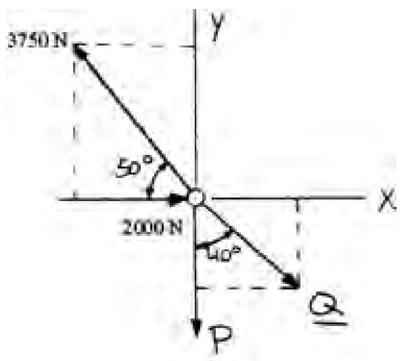
PROBLEM 2.50



Two forces **P** and **Q** are applied as shown to an aircraft connection. Knowing that the connection is in equilibrium and that the magnitudes of the forces exerted on rods *A* and *B* are $F_A = 3750$ N and $F_B = 2000$ N, determine the magnitudes of **P** and **Q**.

SOLUTION

Free-Body Diagram



Resolving the forces into *x*- and *y*-directions:

$$\mathbf{R} = \mathbf{P} + \mathbf{Q} + \mathbf{F}_A + \mathbf{F}_B = 0$$

Substituting components:

$$\mathbf{R} = -P\mathbf{j} + Q\cos 50°\mathbf{i} - Q\sin 50°\mathbf{j} - [(3750\text{ N})\cos 50°]\mathbf{i} + [(3750\text{ N})\sin 50°]\mathbf{j} + (2000\text{ N})\mathbf{i}$$

In the *x*-direction (one unknown force)

$$Q\cos 50° - [(3750\text{ N})\cos 50°] + 2000\text{ N} = 0$$

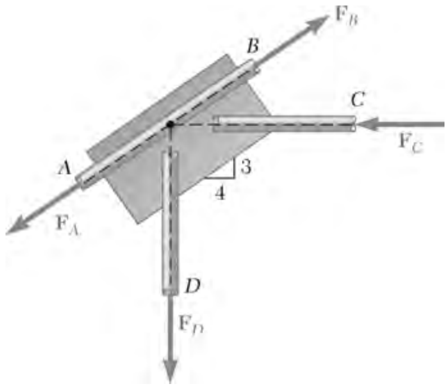
$$Q = \frac{(3750\text{ N})\cos 50° - 2000\text{ N}}{\cos 50°} = 638.55\text{ N}$$

In the *y*-direction:

$$-P - Q\sin 50° + (3750\text{ N})\sin 50° = 0$$

$$P = -Q\sin 50° + (3750\text{ N})\sin 50° = -(638.55\text{ N})\sin 50° + (3750\text{ N})\sin 50° = 2383.5\text{ N}$$

$P = 2380$ N; $Q = 639$ N ◀

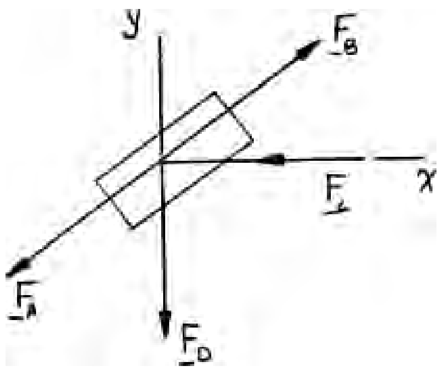


PROBLEM 2.51

A welded connection is in equilibrium under the action of the four forces shown. Knowing that $F_A = 8$ kN and $F_B = 16$ kN, determine the magnitudes of the other two forces.

SOLUTION

Free-Body Diagram of Connection



$$\Sigma F_x = 0: \quad \frac{3}{5}F_B - F_C - \frac{3}{5}F_A = 0$$

With $\quad F_A = 8$ kN

$F_B = 16$ kN

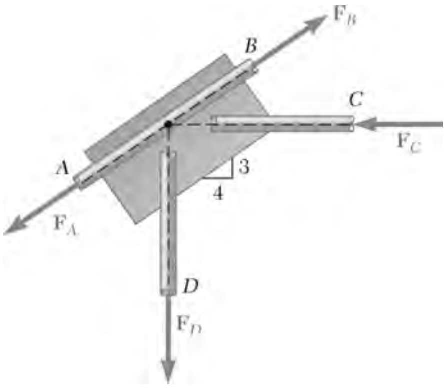
$$F_C = \frac{4}{5}(16\text{ kN}) - \frac{4}{5}(8\text{ kN})$$

$F_C = 6.40$ kN ◀

$$\Sigma F_y = 0: \quad -F_D + \frac{3}{5}F_B - \frac{3}{5}F_A = 0$$

With F_A and F_B as above: $\quad F_D = \frac{3}{5}(16\text{ kN}) - \frac{3}{5}(8\text{ kN})$

$F_D = 4.80$ kN ◀

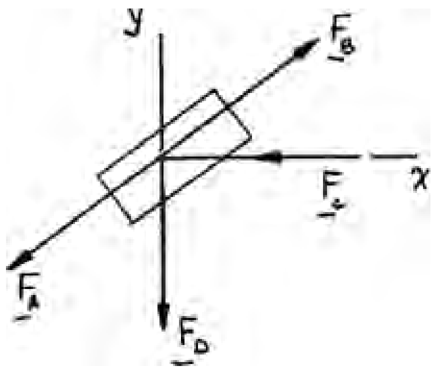


PROBLEM 2.52

A welded connection is in equilibrium under the action of the four forces shown. Knowing that $F_A = 5$ kN and $F_D = 6$ kN, determine the magnitudes of the other two forces.

SOLUTION

Free-Body Diagram of Connection



$$\Sigma F_y = 0: \quad -F_D - \frac{3}{5}F_A + \frac{3}{5}F_B = 0$$

or

$$F_B = F_D + \frac{3}{5}F_A$$

With

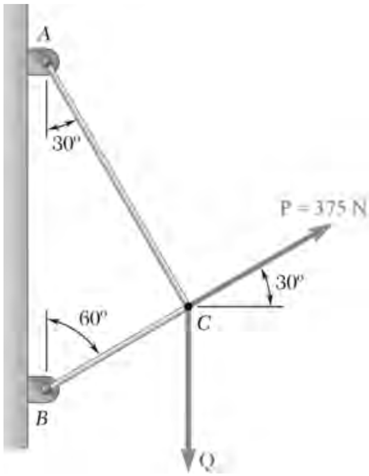
$$F_A = 5 \text{ kN}, \quad F_D = 8 \text{ kN}$$

$$F_B = \frac{5}{3}\left[6 \text{ kN} + \frac{3}{5}(5 \text{ kN})\right] \qquad F_B = 15.00 \text{ kN} \blacktriangleleft$$

$$\Sigma F_x = 0: \quad -F_C + \frac{4}{5}F_B - \frac{4}{5}F_A = 0$$

$$F_C = \frac{4}{5}(F_B - F_A)$$

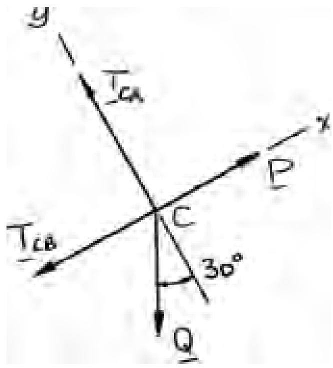
$$= \frac{4}{5}(15 \text{ kN} - 5 \text{ kN}) \qquad F_C = 8.00 \text{ kN} \blacktriangleleft$$



PROBLEM 2.53

Two cables tied together at C are loaded as shown. Knowing that $Q = 300$ N, determine the tension (a) in cable AC, (b) in cable BC.

SOLUTION



$$\Sigma F_y = 0: \quad T_{CA} - Q\cos 30° = 0$$

With $\quad Q = 300$ N

(a) $\quad T_{CA} = (300 \text{ N})(0.866) \qquad T_{CA} = 260 \text{ N}$ ◀

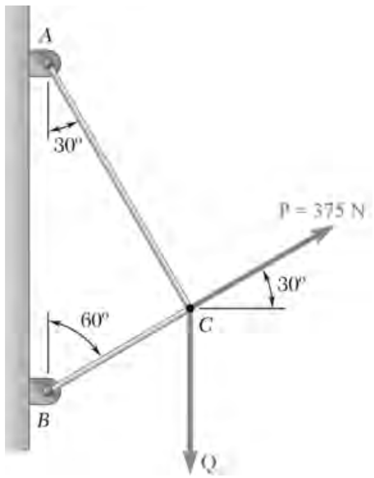
(b) $\quad \Sigma F_x = 0: \quad P - T_{CB} - Q\sin 30° = 0$

With $\quad P = 375$ N

$$T_{CB} = 375 \text{ N} - (300 \text{ N})(0.50)$$

or $T_{CB} = 225$ N ◀

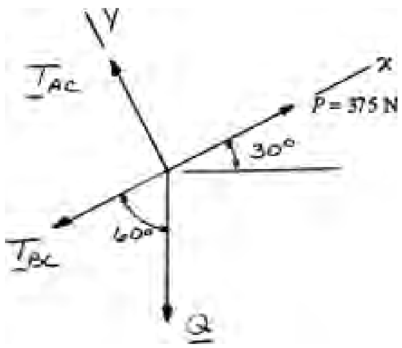
PROBLEM 2.54



Two cables tied together at C are loaded as shown. Determine the range of values of Q for which the tension will not exceed 300-N in either cable.

SOLUTION

Free-Body Diagram



$$\Sigma F_x = 0: \quad -T_{BC} - Q\cos 60° + 375 \text{ N} = 0$$

$$T_{BC} = 375 \text{ N} - Q \cos 60° \qquad (1)$$

$$\Sigma F_y = 0: \quad T_{AC} - Q\sin 60° = 0$$

$$T_{AC} = Q\sin 60° \qquad (2)$$

Requirement $\quad T_{AC} \le 300 \text{ N}$

From Eq. (2): $\quad Q\sin 60° \le 300 \text{ N}$

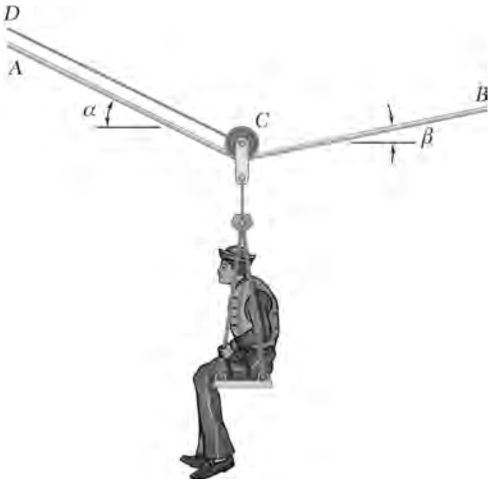
$$Q \le 346.41 \text{ N}$$

Requirement $\quad T_{BC} \le 300 \text{ N}$

From Eq. (1): $\quad 375 \text{ N} - Q\cos 60° \le 300 \text{ N}$

$$Q \ge 150 \text{ N} \qquad 150.0 \text{ N} \le Q \le 346 \text{ N} \blacktriangleleft$$

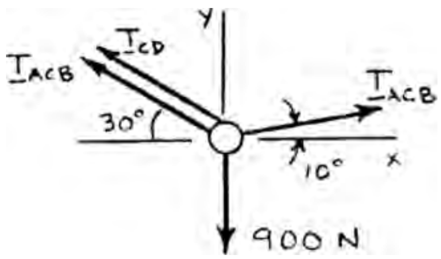
PROBLEM 2.55



A sailor is being rescued using a boatswain's chair that is suspended from a pulley that can roll freely on the support cable *ACB* and is pulled at a constant speed by cable *CD*. Knowing that $\alpha = 30°$ and $\beta = 10°$ and that the combined weight of the boatswain's chair and the sailor is 900 N, determine the tension (*a*) in the support cable *ACB*, (*b*) in the traction cable *CD*.

SOLUTION

Free-Body Diagram



$$\xrightarrow{+} \Sigma F_x = 0: \quad T_{ACB} \cos 10° - T_{ACB} \cos 30° - T_{CD} \cos 30° = 0$$

$$T_{CD} = 0.137158 T_{ACB} \qquad (1)$$

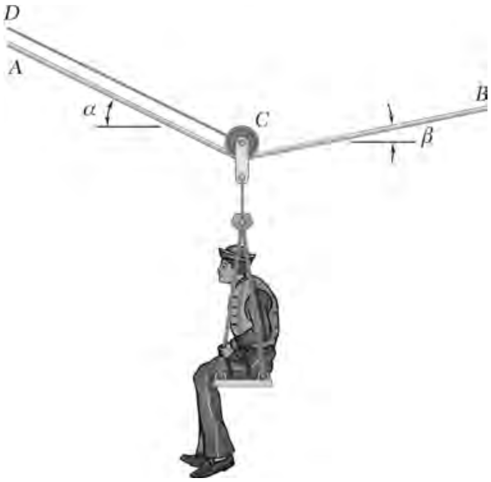
$$+\uparrow \Sigma F_y = 0: \quad T_{ACB} \sin 10° + T_{ACB} \sin 30° + T_{CD} \sin 30° - 900 = 0$$

$$0.67365 T_{ACB} + 0.5 T_{CD} = 900 \qquad (2)$$

(*a*) Substitute (1) into (2): $\quad 0.67365 T_{ACB} + 0.5(0.137158 T_{ACB}) = 900$

$$T_{ACB} = 1212.56 \text{ N} \qquad T_{ACB} = 1213 \text{ N} \blacktriangleleft$$

(*b*) From (1): $\quad T_{CD} = 0.137158(1212.56 \text{ N}) \qquad T_{CD} = 166.3 \text{ N} \blacktriangleleft$

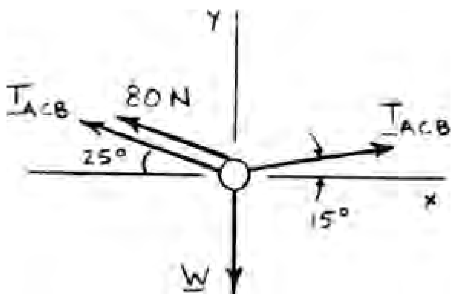


PROBLEM 2.56

A sailor is being rescued using a boatswain's chair that is suspended from a pulley that can roll freely on the support cable *ACB* and is pulled at a constant speed by cable *CD*. Knowing that $\alpha = 25°$ and $\beta = 15°$ and that the tension in cable *CD* is 80 N, determine (*a*) the combined weight of the boatswain's chair and the sailor, (*b*) in tension in the support cable *ACB*.

SOLUTION

Free-Body Diagram



$$\xrightarrow{+} \Sigma F_x = 0: \quad T_{ACB}\cos 15° - T_{ACB}\cos 25° - (80\text{ N})\cos 25° = 0$$

$$T_{ACB} = 1216.15\text{ N}$$

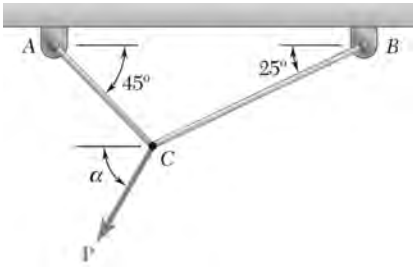
$$+\!\uparrow \Sigma F_y = 0: \quad (1216.15\text{ N})\sin 15° + (1216.15\text{ N})\sin 25°$$

$$+(80\text{ N})\sin 25° - W = 0$$

$$W = 862.54\text{ N}$$

(*a*) $W = 863\text{ N}$ ◀

(*b*) $T_{ACB} = 1216\text{ N}$ ◀

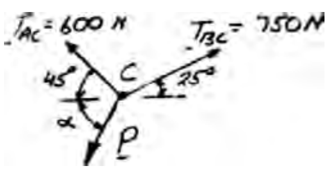


PROBLEM 2.57

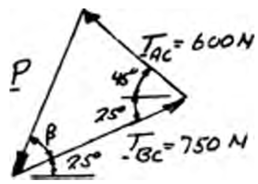
For the cables of Problem 2.45, it is known that the maximum allowable tension is 600 N in cable *AC* and 750 N in cable *BC*. Determine (*a*) the maximum force **P** that can be applied at *C*, (*b*) the corresponding value of α.

SOLUTION

Free-Body Diagram



Force Triangle



(*a*) Law of cosines

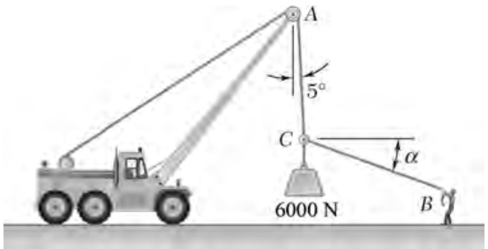
$$P^2 = (600)^2 + (750)^2 - 2(600)(750)\cos(25° + 45°)$$

$$P = 784.02 \text{ N} \qquad P = 784 \text{ N} \blacktriangleleft$$

(*b*) Law of sines

$$\frac{\sin\beta}{600 \text{ N}} = \frac{\sin(25° + 45°)}{784.02 \text{ N}}$$

$$\beta = 46.0° \qquad \alpha = 46.0° + 25° = 71.0° \blacktriangleleft$$



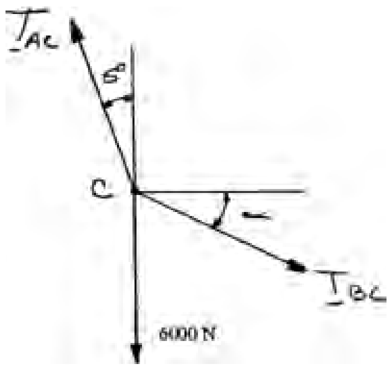
PROBLEM 2.58

For the situation described in Figure P2.47, determine (*a*) the value of α for which the tension in rope *BC* is as small as possible, (*b*) the corresponding value of the tension.

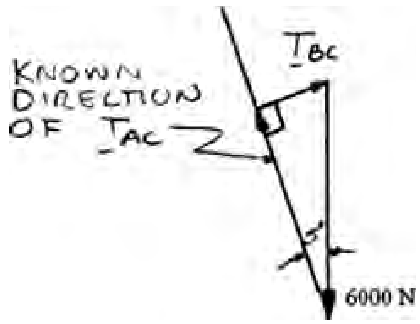
PROBLEM 2.47 Knowing that $\alpha = 20°$, determine the tension (*a*) in cable *AC*, (*b*) in rope *BC*.

SOLUTION

Free-Body Diagram



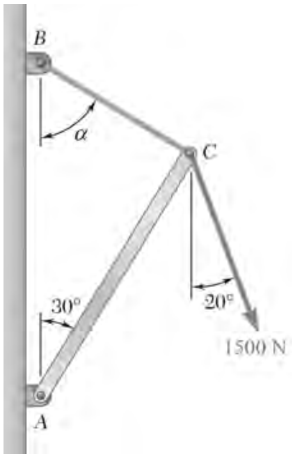
Force Triangle



To be smallest, T_{BC} must be perpendicular to the direction of T_{AC}.

(*a*) Thus, $\alpha = 5°$ $\qquad \alpha = 5.00°$ ◀

(*b*) $T_{BC} = (6000 \text{ N})\sin 5°$ $\qquad T_{BC} = 523 \text{ N}$ ◀



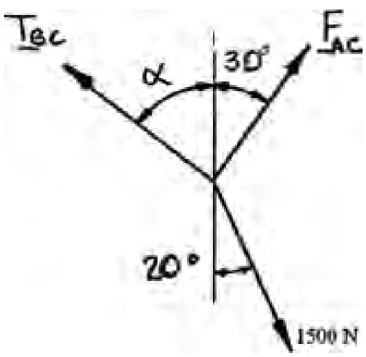
PROBLEM 2.59

For the structure and loading of Problem 2.48, determine (*a*) the value of α for which the tension in cable *BC* is as small as possible, (*b*) the corresponding value of the tension.

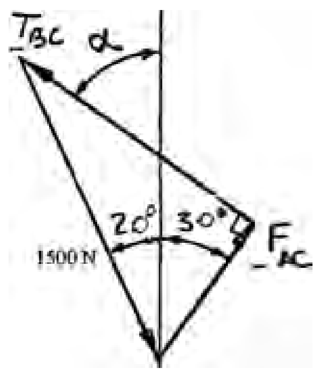
SOLUTION

T_{BC} must be perpendicular to F_{AC} to be as small as possible.

Free-Body Diagram: *C*



Force Triangle is a right triangle

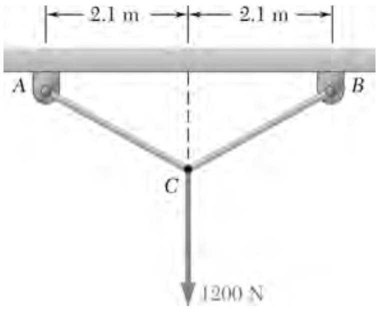


To be a minimum, T_{BC} must be perpendicular to F_{AC}.

(*a*) We observe: $\alpha = 90° - 30°$ $\qquad \alpha = 60.0°$ ◀

(*b*) $T_{BC} = (1500 \text{ N})\sin 50°$

or $T_{BC} = 1149.07 \text{ N}$ $\qquad T_{BC} = 1149 \text{ N}$ ◀



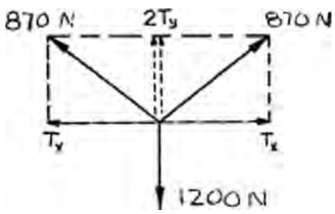
PROBLEM 2.60

Knowing that portions AC and BC of cable ACB must be equal, determine the shortest length of cable that can be used to support the load shown if the tension in the cable is not to exceed 870 N.

SOLUTION

Free-Body Diagram: *C*

(For $T = 725$ N)



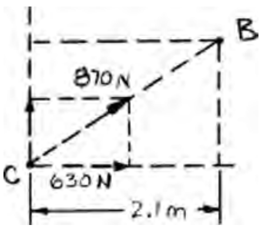
$$+\uparrow \Sigma F_y = 0: \quad 2T_y - 1200 \text{ N} = 0$$

$$T_y = 600 \text{ N}$$

$$T_x^2 + T_y^2 = T^2$$

$$T_x^2 + (600 \text{ N})^2 = (870 \text{ N})^2$$

$$T_x = 630 \text{ N}$$



By similar triangles:

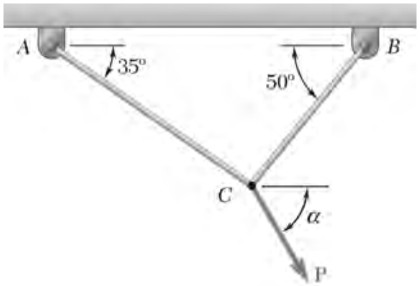
$$\frac{BC}{870 \text{ N}} = \frac{2.1 \text{ m}}{630 \text{ N}}$$

$$BC = 2.90 \text{ m}$$

$$L = 2(BC)$$

$$= 5.80 \text{ m}$$

$L = 5.80$ m ◀

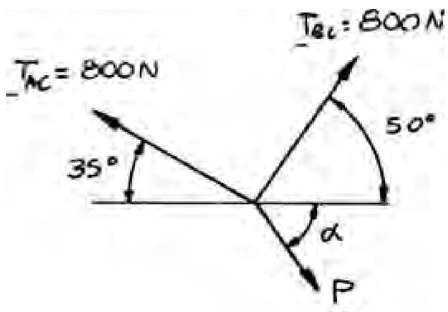


PROBLEM 2.61

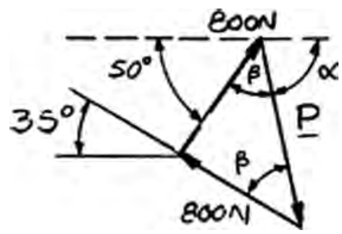
Two cables tied together at C are loaded as shown. Knowing that the maximum allowable tension in each cable is 800 N, determine (a) the magnitude of the largest force $\mathbf{P}$ that can be applied at C, (b) the corresponding value of α.

SOLUTION

Free-Body Diagram: *C*



Force Triangle



Force triangle is isosceles with

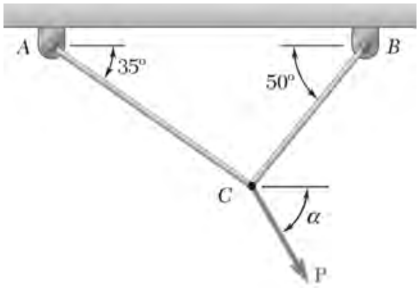
$$2\beta = 180° - 85°$$

$$\beta = 47.5°$$

(a) $$P = 2(800\text{ N})\cos 47.5° = 1081\text{ N}$$

Since $P > 0$, the solution is correct. $P = 1081\text{ N}$ ◀

(b) $$\alpha = 180° - 50° - 47.5° = 82.5°$$ $\alpha = 82.5°$ ◀

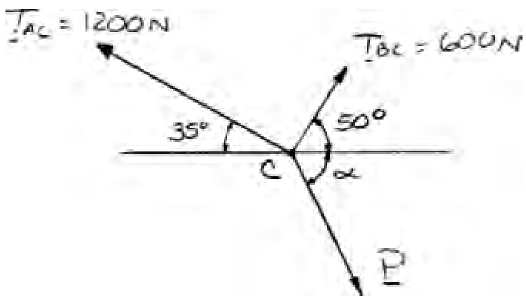


PROBLEM 2.62

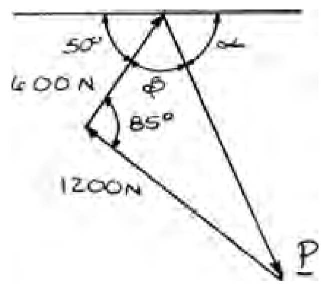
Two cables tied together at C are loaded as shown. Knowing that the maximum allowable tension is 1200 N in cable AC and 600 N in cable BC, determine (*a*) the magnitude of the largest force **P** that can be applied at C, (*b*) the corresponding value of α.

SOLUTION

Free-Body Diagram



Force Triangle



(*a*) Law of cosines:

$$P^2 = (1200\text{ N})^2 + (600\text{ N})^2 - 2(1200\text{ N})(600\text{ N})\cos 85°$$

$$P = 1294\text{ N}$$

Since $P > 1200$ N, the solution is correct.

$$P = 1294\text{ N} \blacktriangleleft$$

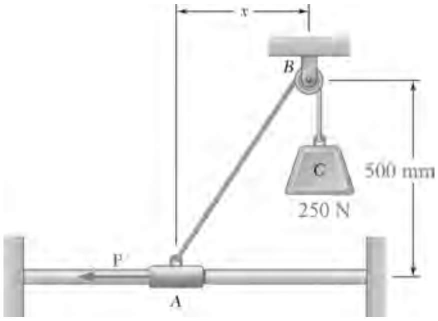
(*b*) Law of sines:

$$\frac{\sin\beta}{1200\text{ N}} = \frac{\sin 85°}{1294\text{ N}}$$

$$\beta = 67.5°$$

$$\alpha = 180° - 50° - 67.5°$$

$$\alpha = 62.5° \blacktriangleleft$$

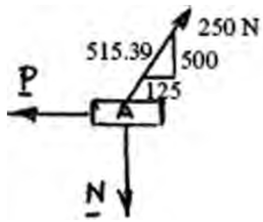


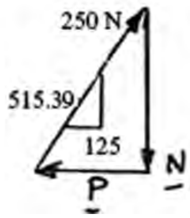
PROBLEM 2.63

Collar A is connected as shown to a 250-N load and can slide on a frictionless horizontal rod. Determine the magnitude of the force **P** required to maintain the equilibrium of the collar when (a) $x = 125$ mm, (b) $x = 375$ mm.

SOLUTION

(a) **Free Body: Collar *A***





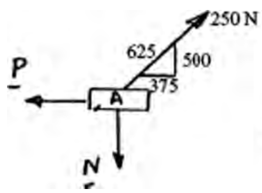
Force Triangle

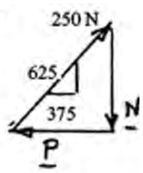
$$\frac{P}{125} = \frac{250 \text{ N}}{515.39}$$

$$P = 60.634 \text{ N}$$

$P = 60.6$ N ◀

(b) **Free Body: Collar *A***

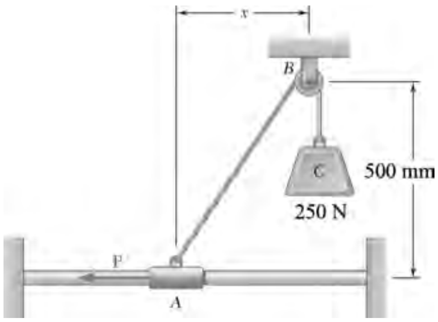




Force Triangle

$$\frac{P}{375} = \frac{250 \text{ N}}{625}$$

$P = 150.0$ N ◀

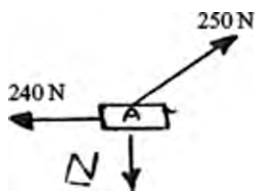


PROBLEM 2.64

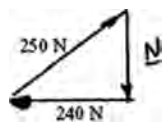
Collar A is connected as shown to a 250 N load and can slide on a frictionless horizontal rod. Determine the distance x for which the collar is in equilibrium when $P = 240$ N.

SOLUTION

Free Body: Collar *A*



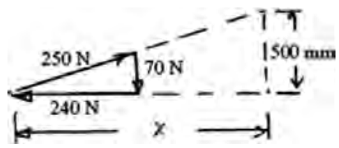
Force Triangle



$$N^2 = (250)^2 - (240)^2 = 4900$$

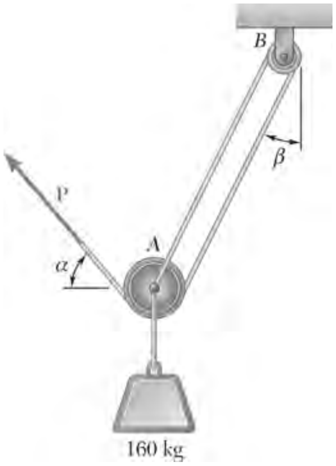
$$N = 70.0 \text{ N}$$

Similar Triangles



$$\frac{x}{500 \text{ mm}} = \frac{240 \text{ N}}{70 \text{ N}}$$

$x = 1714$ mm ◀

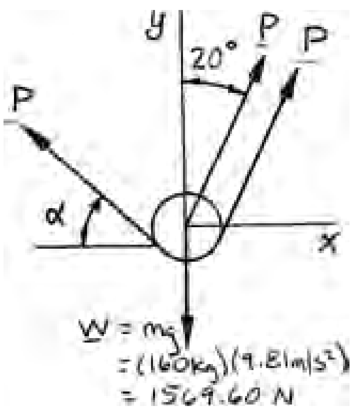


PROBLEM 2.65

A 160-kg load is supported by the rope-and-pulley arrangement shown. Knowing that $\beta = 20°,$ determine the magnitude and direction of the force **P** that must be exerted on the free end of the rope to maintain equilibrium. (*Hint:* The tension in the rope is the same on each side of a simple pulley. This can be proved by the methods of Chapter 4.)

SOLUTION

Free-Body Diagram: Pulley *A*



$$\xrightarrow{+} \Sigma F_x = 0: \quad 2P\sin 20° - P\cos\alpha = 0$$

and $\cos\alpha = 0.8452 \quad \text{or} \quad \alpha = \pm 46.840°$

$$\alpha = +46.840$$

For $+\uparrow \Sigma F_y = 0: \quad 2P\cos 20° + P\sin 46.840° - 1569.60 \text{ N} = 0$

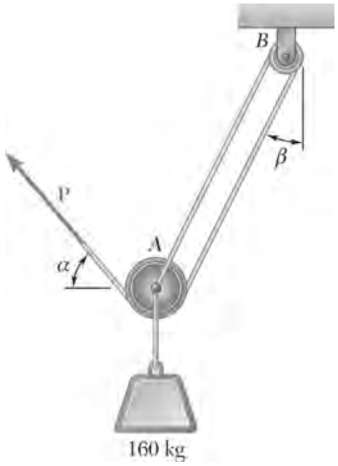
or $\mathbf{P} = 602 \text{ N} \nwarrow 46.8°$ ◀

For $\alpha = -46.840$

$$+\uparrow \Sigma F_y = 0: \quad 2P\cos 20° + P\sin(-46.840°) - 1569.60 \text{ N} = 0$$

or $\mathbf{P} = 1365 \text{ N} \swarrow 46.8°$ ◀

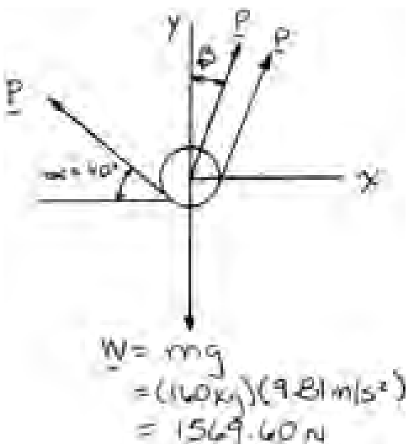
PROBLEM 2.66



A 160-kg load is supported by the rope-and-pulley arrangement shown. Knowing that $\alpha = 40°$, determine (*a*) the angle β, (*b*) the magnitude of the force **P** that must be exerted on the free end of the rope to maintain equilibrium. (See the hint for Problem 2.65.)

SOLUTION

Free-Body Diagram: Pulley *A*



(*a*) $\Sigma F_x = 0: \quad 2P \sin \sin \beta - P \cos 40° = 0$

$$\sin \beta = \frac{1}{2} \cos 40°$$

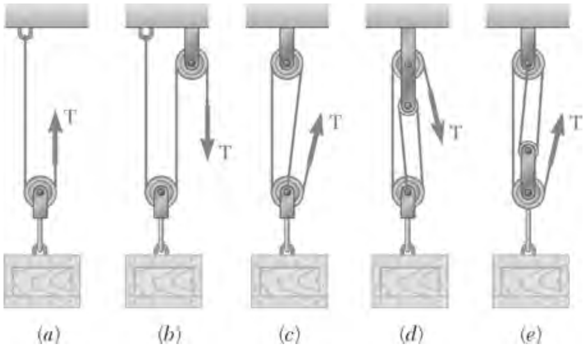
$$\beta = 22.52°$$

$$\beta = 22.5° \blacktriangleleft$$

(*b*) $\Sigma F_y = 0: \quad P \sin 40° + 2P \cos 22.52° - 1569.60 \text{ N} = 0$

$$P = 630 \text{ N} \blacktriangleleft$$

PROBLEM 2.67

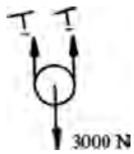


A 3000 N crate is supported by several rope-and-pulley arrangements as shown. Determine for each arrangement the tension in the rope. (See the hint for Problem 2.65.)

SOLUTION

Free-Body Diagram of Pulley

(*a*)

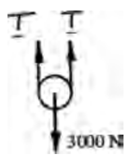


$$+\uparrow \Sigma F_y = 0: \quad 2T - (3000 \text{ N}) = 0$$

$$T = \frac{1}{2}(3000 \text{ N})$$

$$T = 1500 \text{ N} \blacktriangleleft$$

(*b*)

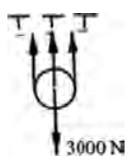


$$+\uparrow \Sigma F_y = 0: \quad 2T - (3000 \text{ N}) = 0$$

$$T = \frac{1}{2}(3000 \text{ N})$$

$$T = 1500 \text{ N} \blacktriangleleft$$

(*c*)

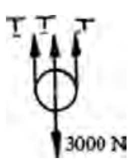


$$+\uparrow \Sigma F_y = 0: \quad 3T - (3000 \text{ N}) = 0$$

$$T = \frac{1}{3}(3000 \text{ N})$$

$$T = 1000 \text{ N} \blacktriangleleft$$

(*d*)

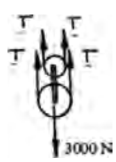


$$+\uparrow \Sigma F_y = 0: \quad 3T - (3000 \text{ N}) = 0$$

$$T = \frac{1}{3}(3000 \text{ N})$$

$$T = 1000 \text{ N} \blacktriangleleft$$

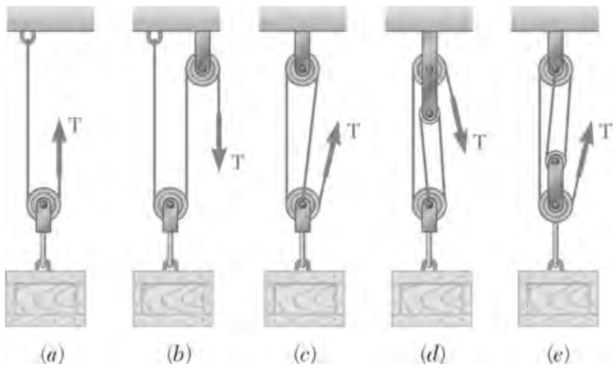
(*e*)



$$+\uparrow \Sigma F_y = 0: \quad 4T - (3000 \text{ N}) = 0$$

$$T = \frac{1}{4}(3000 \text{ N})$$

$$T = 750 \text{ N} \blacktriangleleft$$



PROBLEM 2.68

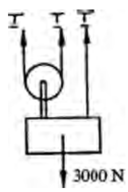
Solve Parts *b* and *d* of Problem 2.67, assuming that the free end of the rope is attached to the crate.

PROBLEM 2.67 A 3000-N crate is supported by several rope-and-pulley arrangements as shown. Determine for each arrangement the tension in the rope. (See the hint for Problem 2.65.)

SOLUTION

Free-Body Diagram of Pulley and Crate

(*b*)

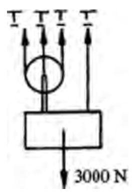


$$+\uparrow \Sigma F_y = 0: \quad 3T - (3000\text{ N}) = 0$$

$$T = \frac{1}{3}(3000\text{ N})$$

$$T = 1000\text{ N} \blacktriangleleft$$

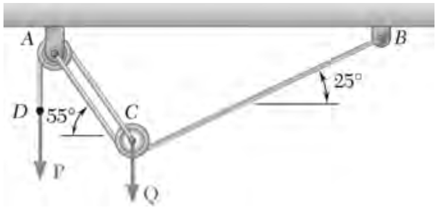
(*d*)



$$+\uparrow \Sigma F_y = 0: \quad 4T - (3000\text{ N}) = 0$$

$$T = \frac{1}{4}(3000\text{ N})$$

$$T = 750\text{ N} \blacktriangleleft$$

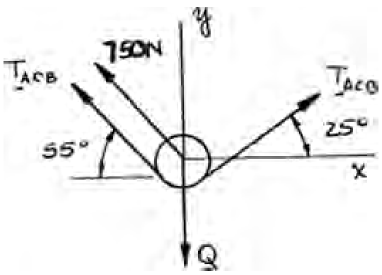


PROBLEM 2.69

A load **Q** is applied to the pulley *C*, which can roll on the cable *ACB*. The pulley is held in the position shown by a second cable *CAD*, which passes over the pulley *A* and supports a load **P**. Knowing that $P = 750$ N, determine (*a*) the tension in cable *ACB*, (*b*) the magnitude of load **Q**.

SOLUTION

Free-Body Diagram: Pulley *C*



(*a*) $\xrightarrow{+} \Sigma F_x = 0: \quad T_{ACB}(\cos 25° - \cos 55°) - (750 \text{ N})\cos 55° = 0$

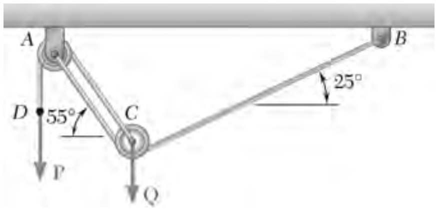
Hence: $T_{ACB} = 1292.88$ N

$T_{ACB} = 1293$ N ◀

(*b*) $+\!\uparrow \Sigma F_y = 0: \quad T_{ACB}(\sin 25° + \sin 55°) + (750 \text{ N})\sin 55° - Q = 0$

$$(1292.88 \text{ N})(\sin 25° + \sin 55°) + (750 \text{ N})\sin 55° - Q = 0$$

or $Q = 2219.8$ N $\qquad Q = 2220$ N ◀

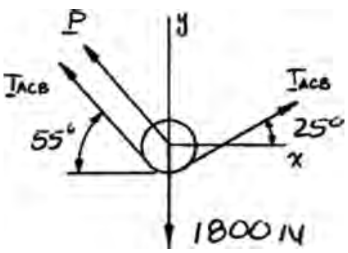


PROBLEM 2.70

An 1800-N load **Q** is applied to the pulley C, which can roll on the cable ACB. The pulley is held in the position shown by a second cable CAD, which passes over the pulley A and supports a load **P**. Determine (*a*) the tension in cable ACB, (*b*) the magnitude of load **P**.

SOLUTION

Free-Body Diagram: Pulley *C*



$$\xrightarrow{+} \Sigma F_x = 0: \quad T_{ACB}(\cos 25° - \cos 55°) - P\cos 55° = 0$$

or $$P = 0.58010 T_{ACB} \qquad (1)$$

$$+\!\uparrow \Sigma F_y = 0: \quad T_{ACB}(\sin 25° + \sin 55°) + P\sin 55° - 1800\text{ N} = 0$$

or $$1.24177 T_{ACB} + 0.81915 P = 1800\text{ N} \qquad (2)$$

(*a*) Substitute Equation (1) into Equation (2):

$$1.24177 T_{ACB} + 0.81915(0.58010 T_{ACB}) = 1800\text{ N}$$

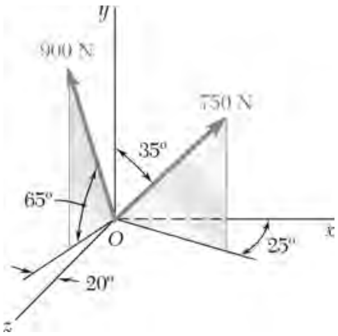
Hence: $$T_{ACB} = 1048.37\text{ N}$$

$$T_{ACB} = 1048\text{ N} \blacktriangleleft$$

(*b*) Using (1), $$P = 0.58010(1048.37\text{ N}) = 608.16\text{ N}$$

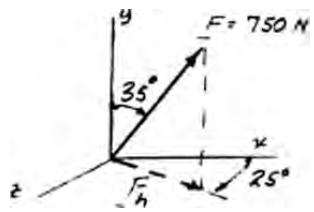
$$P = 608\text{ N} \blacktriangleleft$$

PROBLEM 2.71



Determine (*a*) the *x*, *y*, and *z* components of the 750-N force, (*b*) the angles θ_x, θ_y, and θ_z that the force forms with the coordinate axes.

SOLUTION



$$F_h = F\sin 35^\circ$$
$$= (750\text{ N})\sin 35^\circ$$
$$F_h = 430.2\text{ N}$$

(*a*)

$$F_x = F_h\cos 25^\circ \qquad F_y = F\cos 35^\circ \qquad F_z = F_h\sin 25^\circ$$

$$= (430.2\text{ N})\cos 25^\circ \qquad = (750\text{ N})\cos 35^\circ \qquad = (430.2\text{ N})\sin 25^\circ$$

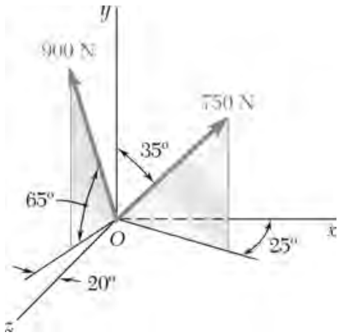
$$F_x = +390\text{ N}, \qquad F_y = +614\text{ N}, \qquad F_z = +181.8\text{ N} \quad ◀$$

(*b*)

$$\cos\theta_x = \frac{F_x}{F} = \frac{+390\text{ N}}{750\text{ N}} \qquad \theta_x = 58.7^\circ \quad ◀$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{+614\text{ N}}{750\text{ N}} \qquad \theta_y = 35.0^\circ \quad ◀$$

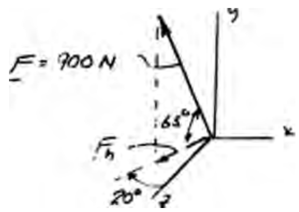
$$\cos\theta_z = \frac{F_z}{F} = \frac{+181.8\text{ N}}{750\text{ N}} \qquad \theta_z = 76.0^\circ \quad ◀$$



PROBLEM 2.72

Determine (*a*) the *x*, *y*, and *z* components of the 900-N force, (*b*) the angles θ_x, θ_y, and θ_z that the force forms with the coordinate axes.

SOLUTION



$$F_h = F\cos 65^\circ$$
$$= (900\text{ N})\cos 65^\circ$$
$$F_h = 380.4\text{ N}$$

(*a*)

$$F_x = F_h \sin 20^\circ \qquad F_y = F\sin 65^\circ \qquad F_z = F_h\cos 20^\circ$$
$$= (380.4\text{ N})\sin 20^\circ \qquad = (900\text{ N})\sin\ 65^\circ \qquad = (380.4\text{ N})\cos 20^\circ$$
$$F_x = -130.1\text{ N}, \qquad F_y = +816\text{ N}, \qquad F_z = +357\text{ N} \blacktriangleleft$$

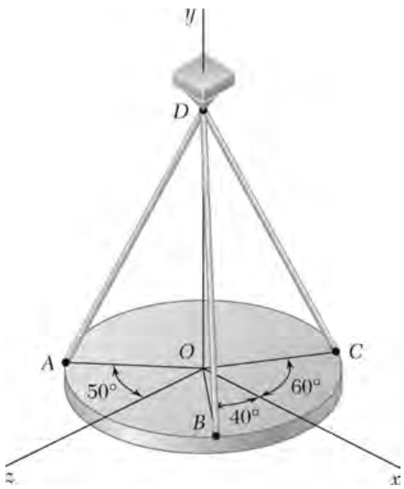
(*b*)

$$\cos\theta_x = \frac{F_x}{F} = \frac{-130.1\text{ N}}{900\text{ N}} \qquad \theta_x = 98.3^\circ \blacktriangleleft$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{+816\text{ N}}{900\text{ N}} \qquad \theta_y = 25.0^\circ \blacktriangleleft$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{+357\text{ N}}{900\text{ N}} \qquad \theta_z = 66.6^\circ \blacktriangleleft$$

PROBLEM 2.73



A horizontal circular plate is suspended as shown from three wires that are attached to a support at *D* and form 30° angles with the vertical. Knowing that the *x* component of the force exerted by wire *AD* on the plate is 110.3 N, determine (*a*) the tension in wire *AD*, (*b*) the angles θ_x, θ_y, and θ_z that the force exerted at *A* forms with the coordinate axes.

SOLUTION

(*a*)

$$F_x = F\sin 30°\sin 50° = 110.3 \text{ N} \quad \text{(Given)}$$

$$F = \frac{110.3 \text{ N}}{\sin 30° \sin 50°} = 287.97 \text{ N} \qquad F = 288 \text{ N} \blacktriangleleft$$

(*b*)

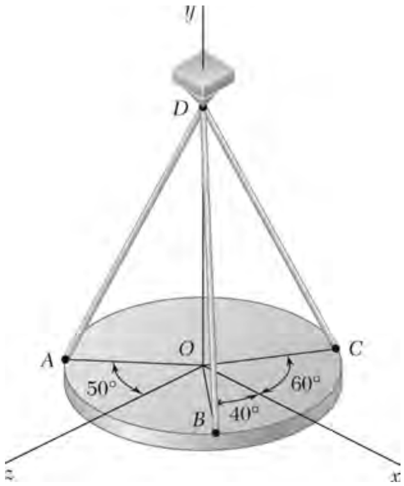
$$\cos\theta_x = \frac{F_x}{F} = \frac{110.3 \text{ N}}{287.97 \text{ N}} = 0.38303 \qquad \theta_x = 67.5° \blacktriangleleft$$

$$F_y = F\cos 30° = 249.39$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{249.39 \text{ N}}{287.97 \text{ N}} = 0.86603 \qquad \theta_y = 30.0° \blacktriangleleft$$

$$\begin{aligned} F_z &= -F\sin 30°\cos 50° \\ &= -(287.97 \text{ N})\sin 30°\cos 50° \\ &= -92.552 \text{ N} \end{aligned}$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{-92.552 \text{ N}}{287.97 \text{ N}} = -0.32139 \qquad \theta_z = 108.7° \blacktriangleleft$$



PROBLEM 2.74

A horizontal circular plate is suspended as shown from three wires that are attached to a support at *D* and form 30° angles with the vertical. Knowing that the *z* component of the force exerted by wire *BD* on the plate is –32.14 N, determine (*a*) the tension in wire *BD*, (*b*) the angles θ_x, θ_y, and θ_z that the force exerted at *B* forms with the coordinate axes.

SOLUTION

(*a*)

$$F_z = -F\sin 30°\sin 40° = 32.14 \text{ N} \quad \text{(Given)}$$

$$F = \frac{32.14}{\sin 30°\sin 40°} = 100.0 \text{ N} \qquad F = 100.0 \text{ N} \blacktriangleleft$$

(*b*)

$$\begin{aligned} F_x &= -F\sin 30°\cos 40° \\ &= -(100.0 \text{ N}) \sin 30°\cos 40° \\ &= -38.302 \text{ N} \end{aligned}$$

$$\cos\theta_x = \frac{F_x}{F} = \frac{38.302 \text{ N}}{100.0 \text{ N}} = -0.38302 \qquad \theta_x = 112.5° \blacktriangleleft$$

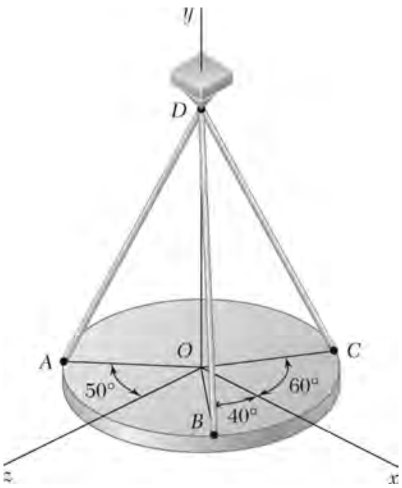
$$F_y = F\cos 30° = 86.603 \text{ N}$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{86.603 \text{ N}}{100 \text{ N}} = 0.86603 \qquad \theta_y = 30.0° \blacktriangleleft$$

$$F_z = -32.14 \text{ N}$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{-32.14 \text{ N}}{100 \text{ N}} = -0.32140 \qquad \theta_z = 108.7° \blacktriangleleft$$

PROBLEM 2.75



A horizontal circular plate is suspended as shown from three wires that are attached to a support at *D* and form 30° angles with the vertical. Knowing that the tension in wire *CD* is 300 N, determine (*a*) the components of the force exerted by this wire on the plate, (*b*) the angles θ_x, θ_y, and θ_z that the force forms with the coordinate axes.

SOLUTION

(*a*)

$$F_x = -(300\text{ N})\sin 30° \cos 60° = -75\text{ N} \qquad F_x = -75.0\text{ N} \blacktriangleleft$$

$$F_y = (300\text{ N})\cos 30° = 259.81\text{ N} \qquad F_y = +260\text{ N} \blacktriangleleft$$

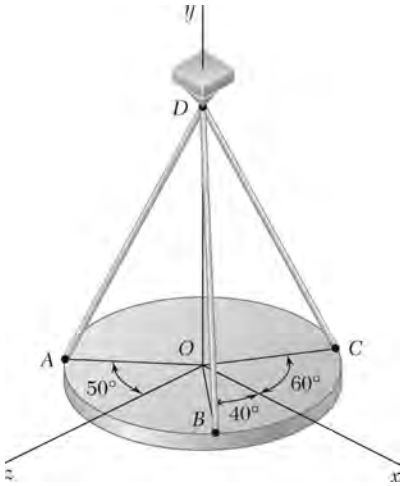
$$F_z = (300\text{ N})\sin 30° \sin 60° = 129.9\text{ N} \qquad F_z = +129.9\text{ N} \blacktriangleleft$$

(*b*)

$$\cos\theta_x = \frac{F_x}{F} = \frac{-75\text{ N}}{300\text{ N}} = -0.25 \qquad \theta_x = 104.5° \blacktriangleleft$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{259.81\text{ N}}{300\text{ N}} = 0.866 \qquad \theta_y = 30.0° \blacktriangleleft$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{129.9\text{ N}}{300\text{ N}} = 0.433 \qquad \theta_z = 64.3° \blacktriangleleft$$



PROBLEM 2.76

A horizontal circular plate is suspended as shown from three wires that are attached to a support at *D* and form 30° angles with the vertical. Knowing that the *x* component of the force exerted by wire *CD* on the plate is –100 N, determine (*a*) the tension in wire *CD*, (*b*) the angles θ_x, θ_y, and θ_z that the force exerted at *C* forms with the coordinate axes.

SOLUTION

(*a*)

$$F_x = -F \sin 30° \cos 60° = -100 \text{ N} \quad \text{(Given)}$$

$$F = \frac{100 \text{ N}}{\sin 30° \cos 60°} = 400 \text{ N}$$

$F = 400$ N ◀

(*b*)

$$\cos\theta_x = \frac{F_x}{F} = \frac{-100 \text{ N}}{400 \text{ N}} = -0.25$$

$\theta_x = 104.5°$ ◀

$$F_y = (400 \text{ N})\cos 30° = 346.41 \text{ N}$$

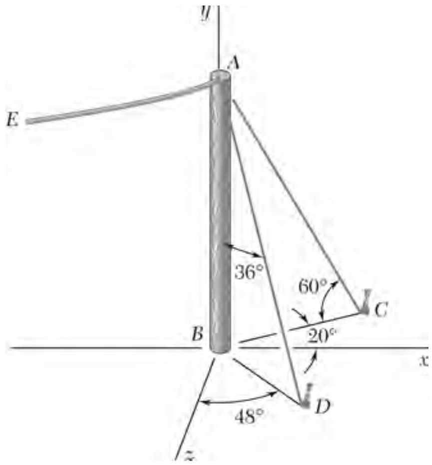
$$\cos\theta_y = \frac{F_y}{F} = \frac{346.41 \text{ N}}{400 \text{ N}} = 0.866$$

$\theta_y = 30.0°$ ◀

$$F_z = (400 \text{ N})\sin 30° \sin 60° = 173.21 \text{ N}$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{173.21 \text{ N}}{400 \text{ N}} = 0.43301$$

$\theta_z = 64.3°$ ◀



PROBLEM 2.77

The end of the coaxial cable AE is attached to the pole AB, which is strengthened by the guy wires AC and AD. Knowing that the tension in wire AC is 600 N, determine (*a*) the components of the force exerted by this wire on the pole, (*b*) the angles θ_x, θ_y, and θ_z that the force forms with the coordinate axes.

SOLUTION

(*a*)

$$F_x = (600\text{ N})\cos 60° \cos 20°$$

$$F_x = 281.91\text{ N} \qquad F_x = +282\text{ N} ◀$$

$$F_y = -(600\text{ N})\sin 60°$$

$$F_y = -519.62\text{ N} \qquad F_y = -520\text{ N} ◀$$

$$F_z = -(600\text{ N})\cos 60° \sin 20°$$

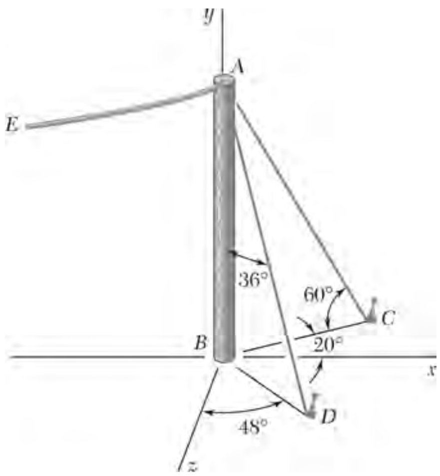
$$F_z = -102.61\text{ N} \qquad F_z = -102.6\text{ N} ◀$$

(*b*)

$$\cos\theta_x = \frac{F_x}{F} = \frac{281.91\text{ N}}{600\text{ N}} \qquad \theta_x = 62.0° ◀$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{-519.62\text{ N}}{600\text{ N}} \qquad \theta_y = 150.0° ◀$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{-102.61\text{ N}}{600\text{ N}} \qquad \theta_z = 99.8° ◀$$



PROBLEM 2.78

The end of the coaxial cable *AE* is attached to the pole *AB*, which is strengthened by the guy wires *AC* and *AD*. Knowing that the tension in wire *AD* is 425 N, determine (*a*) the components of the force exerted by this wire on the pole, (*b*) the angles θ_x, θ_y, and θ_z that the force forms with the coordinate axes.

SOLUTION

(*a*)

$$F_x = (425\text{ N})\sin 36^\circ \sin 48^\circ$$
$$= 185.64\text{ N} \qquad F_x = 185.6\text{ N} \blacktriangleleft$$

$$F_y = -(425\text{ N})\cos 36^\circ$$
$$= -343.83\text{ N} \qquad F_y = -344\text{ N} \blacktriangleleft$$

$$F_z = (425\text{ N})\sin 36^\circ \cos 48^\circ$$
$$= 167.15\text{ N} \qquad F_z = 167.2\text{ N} \blacktriangleleft$$

(*b*)

$$\cos\theta_x = \frac{F_x}{F} = \frac{185.64\text{ N}}{425\text{ N}} \qquad \theta_x = 64.1^\circ \blacktriangleleft$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{-343.83\text{ N}}{425\text{ N}} \qquad \theta_y = 144.0^\circ \blacktriangleleft$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{167.15\text{ N}}{425\text{ N}} \qquad \theta_z = 66.8^\circ \blacktriangleleft$$

PROBLEM 2.79

Determine the magnitude and direction of the force **F** = (320 N)**i** + (400 N)**j** = (250 N)**k**.

SOLUTION

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$F = \sqrt{(320 \text{ N})^2 + (400 \text{ N})^2 + (-250 \text{ N})^2} \qquad F = 570 \text{ N} \blacktriangleleft$$

$$\cos\theta_x = \frac{F_x}{F} = \frac{320 \text{ N}}{570 \text{ N}} \qquad \theta_x = 55.8° \blacktriangleleft$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{400 \text{ N}}{570 \text{ N}} \qquad \theta_y = 45.4° \blacktriangleleft$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{-250 \text{ N}}{570 \text{ N}} \qquad \theta_z = 116.0° \blacktriangleleft$$

PROBLEM 2.80

Determine the magnitude and direction of the force **F** = (240 N)**i** − (270 N)**j** + (680 N)**k**.

SOLUTION

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$F = \sqrt{(240\text{ N})^2 + (-270\text{ N})^2 + (680\text{ N})^2} \qquad F = 770\text{ N} \blacktriangleleft$$

$$\cos\theta_x = \frac{F_x}{F} = \frac{240\text{ N}}{770\text{ N}} \qquad \theta_x = 71.8° \blacktriangleleft$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{-270\text{ N}}{770\text{ N}} \qquad \theta_y = 110.5° \blacktriangleleft$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{680\text{ N}}{770\text{ N}} \qquad \theta_z = 28.0° \blacktriangleleft$$

PROBLEM 2.81

A force acts at the origin of a coordinate system in a direction defined by the angles θ_x = 70.9° and θ_y = 144.9°. Knowing that the z component of the force is –260 N, determine (a) the angle θ_z, (b) the other components and the magnitude of the force.

SOLUTION

(a) We have

$$(\cos\theta_x)^2 + (\cos\theta_y)^2 + (\cos\theta_z)^2 = 1 \Rightarrow (\cos\theta_y)^2 = 1 - (\cos\theta_y)^2 - (\cos\theta_z)^2$$

Since $F_z < 0$ we must have $\cos\theta_z < 0$

Thus, taking the negative square root, from above, we have:

$$\cos\theta_z = -\sqrt{1 - (\cos 70.9°)^2 - (\cos 144.9°)^2} = 0.47282 \qquad \theta_z = 118.2° \blacktriangleleft$$

(b) Then:

$$F = \frac{F_z}{\cos\theta_z} = \frac{260 \text{ N}}{0.47282} = 549.89 \text{ N}$$

and

$$F_x = F\cos\theta_x = (549.89 \text{ N})\cos 70.9° \qquad F_x = 179.9 \text{ N} \blacktriangleleft$$

$$F_y = F\cos\theta_y = (549.89 \text{ N})\cos 144.9° \qquad F_y = -450 \text{ N} \blacktriangleleft$$

$$F = 550 \text{ N} \blacktriangleleft$$

PROBLEM 2.82

A force acts at the origin of a coordinate system in a direction defined by the angles $\theta_y = 55°$ and $\theta_z = 45°$. Knowing that the x component of the force is -2500 N, determine (a) the angle θ_x, (b) the other components and the magnitude of the force.

SOLUTION

(a) We have

$$(\cos\theta_x)^2 + (\cos\theta_y)^2 + (\cos\theta_z)^2 = 1 \Rightarrow (\cos\theta_y)^2 = 1 - (\cos\theta_y)^2 - (\cos\theta_z)^2$$

Since $F_x < 0$ we must have $\cos\theta_x < 0$

Thus, taking the negative square root, from above, we have:

$$\cos\theta_x = -\sqrt{1-(\cos 55)^2 - (\cos 45)^2} = 0.41353 \qquad \theta_x = 114.4° \blacktriangleleft$$

(b) Then:

$$F = \frac{F_x}{\cos\theta_x} = \frac{2500\text{ N}}{0.41353} = 6045.5\text{ N} \qquad F = 6050\text{ N} \blacktriangleleft$$

and $$F_y = F\cos\theta_y = (6045.5\text{ N})\cos 55° \qquad F_y = 3470\text{ N} \blacktriangleleft$$

$$F_z = F\cos\theta_z = (6045.5\text{ N})\cos 45° \qquad F_z = 4270\text{ N} \blacktriangleleft$$

PROBLEM 2.83

A force **F** of magnitude 210 N acts at the origin of a coordinate system. Knowing that $F_x = 80$ N, $\theta_z = 151.2°$, and $F_y < 0$, determine (*a*) the components F_y and F_z, (*b*) the angles θ_x and θ_y.

SOLUTION

(*a*)

$$F_z = F\cos\theta_z = (210\text{ N})\cos 151.2°$$

$$= -184.024\text{ N} \qquad F_z = -184.0\text{ N} ◀$$

Then: $$F^2 = F_x^2 + F_y^2 + F_z^2$$

So: $$(210\text{ N})^2 = (80\text{ N})^2 + (F_y)^2 + (184.024\text{ N})^2$$

Hence: $$F_y = -\sqrt{(210\text{ N})^2 - (80\text{ N})^2 - (184.024\text{ N})^2}$$

$$= -61.929\text{ N} \qquad F_y = -62.0\text{ N} ◀$$

(*b*)

$$\cos\theta_x = \frac{F_x}{F} = \frac{80\text{ N}}{210\text{ N}} = 0.38095 \qquad \theta_x = 67.6° ◀$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{61.929\text{ N}}{210\text{ N}} = -0.29490 \qquad \theta_y = 107.2 ◀$$

PROBLEM 2.84

A force **F** of magnitude 230 N acts at the origin of a coordinate system. Knowing that $\theta_x = 32.5°$, $F_y = -60$ N, and $F_z > 0$, determine (*a*) the components F_x and F_z, (*b*) the angles θ_y and θ_z.

SOLUTION

(*a*) We have

$$F_x = F\cos\theta_x = (230\text{ N})\cos 32.5° \qquad F_x = -194.0\text{ N} \blacktriangleleft$$

Then:

$$F_x = 193.980\text{ N}$$

$$F^2 = F_x^2 + F_y^2 + F_z^2$$

So:

$$(230\text{ N})^2 = (193.980\text{ N})^2 + (-60\text{ N})^2 + F_z^2$$

Hence:

$$F_z = +\sqrt{(230\text{ N})^2 - (193.980\text{ N})^2 - (-60\text{ N})^2} \qquad F_z = 108.0\text{ N} \blacktriangleleft$$

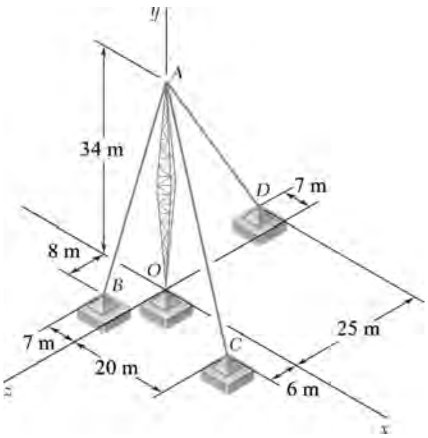
(*b*)

$$F_z = 108.036\text{ N}$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{-60\text{ N}}{230\text{ N}} = -0.26087 \qquad \theta_y = 105.1° \blacktriangleleft$$

$$\cos\theta_z = \frac{F_z}{F} = \frac{108.036\text{ N}}{230\text{ N}} = 0.46972 \qquad \theta_z = 62.0° \blacktriangleleft$$

PROBLEM 2.85



A transmission tower is held by three guy wires anchored by bolts at B, C, and D. If the tension in wire AB is 2625 N, determine the components of the force exerted by the wire on the bolt at B.

SOLUTION

$$\overrightarrow{BA} = (7 \text{ m})\mathbf{i} + (34 \text{ m})\mathbf{j} - (8 \text{ m})\mathbf{k}$$

$$BA = \sqrt{(7)^2 + (34)^2 + (8)^2}$$

$$= 35.623 \text{ m}$$

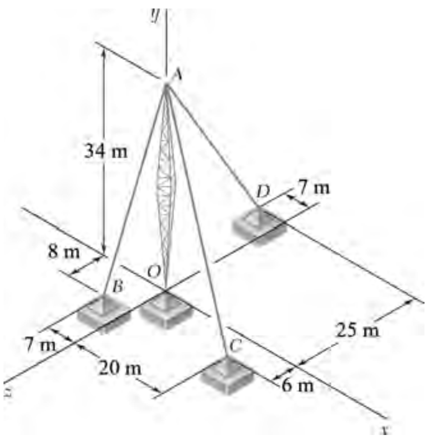
$$\mathbf{F} = F\lambda_{BA}$$

$$= F\frac{\overrightarrow{BA}}{BA}$$

$$= \frac{2625 \text{ N}}{35.623 \text{ m}}[(7 \text{ m})\mathbf{i} + (34 \text{ m})\mathbf{j} - (8 \text{ m})\mathbf{k}]$$

$$\mathbf{F} = (515.82 \text{ N})\mathbf{i} + (2505.4 \text{ N})\mathbf{j} - (589.51 \text{ N})\mathbf{k}$$

$$F_x = +516 \text{ N}, \quad F_y = +2510 \text{ N}, \quad F_z = -590 \text{ N} \blacktriangleleft$$



PROBLEM 2.86

A transmission tower is held by three guy wires anchored by bolts at B, C, and D. If the tension in wire AD is 1575 N, determine the components of the force exerted by the wire on the bolt at D.

SOLUTION

$$\overrightarrow{DA} = (7\text{ m})\mathbf{i} + (34\text{ m})\mathbf{j} + (25\text{ m})\mathbf{k}$$

$$DA = \sqrt{(7)^2 + (34)^2 + (25)^2}$$

$$= 42.778\text{ m}$$

$$\mathbf{F} = F\lambda_{DA}$$

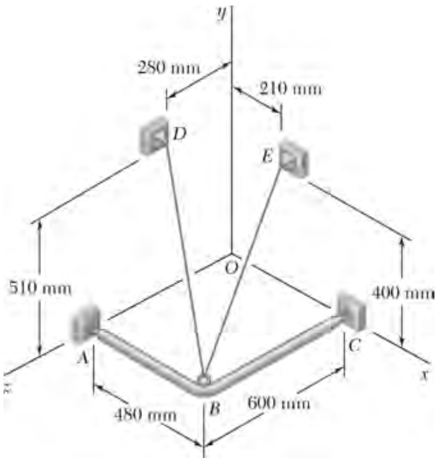
$$= F\frac{\overrightarrow{DA}}{DA}$$

$$= \frac{1575\text{ N}}{42.778\text{ m}}[(7\text{ m})\mathbf{i} + (34\text{ m})\mathbf{j} + (25\text{ m})\mathbf{k}]$$

$$\mathbf{F} = (257.73\text{ N})\mathbf{i} + (1251.81\text{ N})\mathbf{j} + (920.45\text{ N})\mathbf{k}$$

$$F_x = +258\text{ N},\quad F_y = +1252\text{ N},\quad F_z = +920\text{ N} \blacktriangleleft$$

PROBLEM 2.87



A frame *ABC* is supported in part by cable *DBE* that passes through a frictionless ring at *B*. Knowing that the tension in the cable is 385 N, determine the components of the force exerted by the cable on the support at *D*.

SOLUTION

$$\overrightarrow{DB} = (480\text{ mm})\mathbf{i} - (510\text{ mm})\mathbf{j} + (320\text{ mm})\mathbf{k}$$

$$DB = \sqrt{(480\text{ mm})^2 + (510\text{ mm}^2) + (320\text{ mm})^2}$$

$$= 770\text{ mm}$$

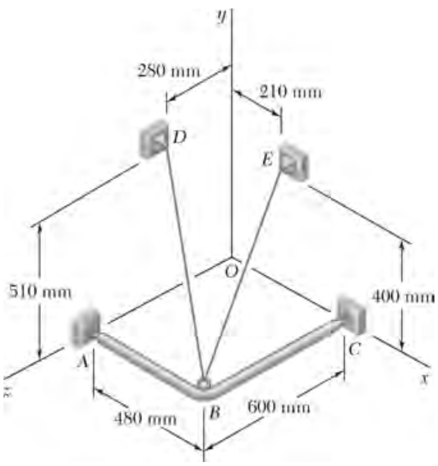
$$\mathbf{F} = F\lambda_{DB}$$

$$= F\frac{\overrightarrow{DB}}{DB}$$

$$= \frac{385\text{ N}}{770\text{ mm}}[(480\text{ mm})\mathbf{i} - (510\text{ mm})\mathbf{j} + (320\text{ mm})\mathbf{k}]$$

$$= (240\text{ N})\mathbf{i} - (255\text{ N})\mathbf{j} + (160\text{ N})\mathbf{k}$$

$$F_x = +240\text{ N},\quad F_y = -255\text{ N},\quad F_z = +160.0\text{ N} \blacktriangleleft$$



PROBLEM 2.88

For the frame and cable of Problem 2.87, determine the components of the force exerted by the cable on the support at *E*.

PROBLEM 2.87 A frame *ABC* is supported in part by cable *DBE* that passes through a frictionless ring at *B*. Knowing that the tension in the cable is 385 N, determine the components of the force exerted by the cable on the support at *D*.

SOLUTION

$$\overrightarrow{EB} = (270\text{ mm})\mathbf{i} - (400\text{ mm})\mathbf{j} + (600\text{ mm})\mathbf{k}$$

$$EB = \sqrt{(270\text{ mm})^2 + (400\text{ mm})^2 + (600\text{ mm})^2}$$

$$= 770\text{ mm}$$

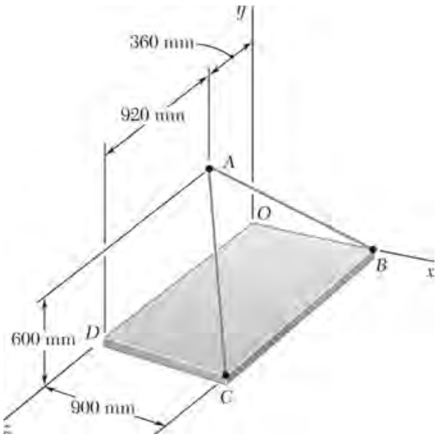
$$\mathbf{F} = F\lambda_{EB}$$

$$= F\frac{\overrightarrow{EB}}{EB}$$

$$= \frac{385\text{ N}}{770\text{ mm}}[(270\text{ mm})\mathbf{i} - (400\text{ mm})\mathbf{j} + (600\text{ mm})\mathbf{k}]$$

$$\mathbf{F} = (135\text{ N})\mathbf{i} - (200\text{ N})\mathbf{j} + (300\text{ N})\mathbf{k}$$

$$F_x = +135.0\text{ N},\quad F_y = -200\text{ N},\quad F_z = +300\text{ N} \blacktriangleleft$$



PROBLEM 2.89

Knowing that the tension in cable AB is 1425 N, determine the components of the force exerted on the plate at B.

SOLUTION

$$\overrightarrow{BA} = -(900\text{ mm})\mathbf{i} + (600\text{ mm})\mathbf{j} + (360\text{ mm})\mathbf{k}$$

$$BA = \sqrt{(900\text{ mm})^2 + (600\text{ mm})^2 + (360\text{ mm})^2}$$

$$= 1140\text{ mm}$$

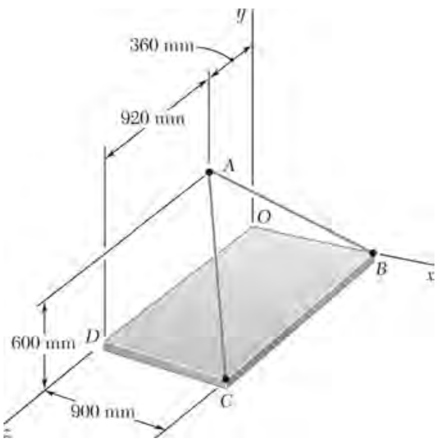
$$\mathbf{T}_{BA} = T_{BA}\lambda_{BA}$$

$$= T_{BA}\frac{\overrightarrow{BA}}{BA}$$

$$\mathbf{T}_{BA} = \frac{1425\text{ N}}{1140\text{ mm}}[-(900\text{ mm})\mathbf{i} + (600\text{ mm})\mathbf{j} + (360\text{ mm})\mathbf{k}]$$

$$= -(1125\text{ N})\mathbf{i} + (750\text{ N})\mathbf{j} + (450\text{ N})\mathbf{k}$$

$$(T_{BA})_x = -1125\text{ N},\quad (T_{BA})_y = 750\text{ N},\quad (T_{BA})_z = 450\text{ N} \blacktriangleleft$$



PROBLEM 2.90

Knowing that the tension in cable AC is 2130 N, determine the components of the force exerted on the plate at C.

SOLUTION

$$\overrightarrow{CA} = -(900 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j} - (920 \text{ mm})\mathbf{k}$$

$$CA = \sqrt{(900 \text{ mm})^2 + (600 \text{ mm})^2 + (920 \text{ mm})^2}$$

$$= 1420 \text{ mm}$$

$$\mathbf{T}_{CA} = T_{CA}\lambda_{CA}$$

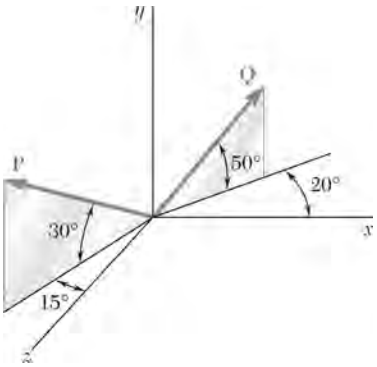
$$= T_{CA}\frac{\overrightarrow{CA}}{CA}$$

$$\mathbf{T}_{CA} = \frac{2130 \text{ N}}{1420 \text{ mm}}[-(900 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j} - (920 \text{ mm})\mathbf{k}]$$

$$= -(1350 \text{ N})\mathbf{i} + (900 \text{ N})\mathbf{j} - (1380 \text{ N})\mathbf{k}$$

$$(T_{CA})_x = -1350 \text{ N}, \quad (T_{CA})_y = 900 \text{ N}, \quad (T_{CA})_z = -1380 \text{ N} \blacktriangleleft$$

PROBLEM 2.91



Find the magnitude and direction of the resultant of the two forces shown knowing that $P = 300$ N and $Q = 400$ N.

SOLUTION

$$\mathbf{P} = (300\text{ N})[-\cos 30°\sin 15°\mathbf{i} + \sin 30°\mathbf{j} + \cos 30°\cos 15°\mathbf{k}]$$

$$= -(67.243\text{ N})\mathbf{i} + (150\text{ N})\mathbf{j} + (250.95\text{ N})\mathbf{k}$$

$$\mathbf{Q} = (400\text{ N})[\cos 50°\cos 20°\mathbf{i} + \sin 50°\mathbf{j} - \cos 50°\sin 20°\mathbf{k}]$$

$$= (400\text{ N})[0.60402\mathbf{i} + 0.76604\mathbf{j} - 0.21985\mathbf{k}]$$

$$= (241.61\text{ N})\mathbf{i} + (306.42\text{ N})\mathbf{j} - (87.939\text{ N})\mathbf{k}$$

$$\mathbf{R} = \mathbf{P} + \mathbf{Q}$$

$$= (174.367\text{ N})\mathbf{i} + (456.42\text{ N})\mathbf{j} + (163.011\text{ N})\mathbf{k}$$

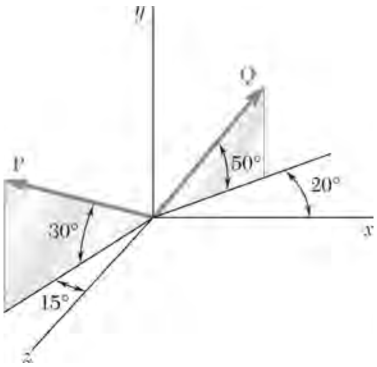
$$R = \sqrt{(174.367\text{ N})^2 + (456.42\text{ N})^2 + (163.011\text{ N})^2}$$

$$= 515.07\text{ N} \qquad R = 515\text{ N} \blacktriangleleft$$

$$\cos\theta_x = \frac{R_x}{R} = \frac{174.367\text{ N}}{515.07\text{ N}} = 0.33853 \qquad \theta_x = 70.2° \blacktriangleleft$$

$$\cos\theta_y = \frac{R_y}{R} = \frac{456.42\text{ N}}{515.07\text{ N}} = 0.88613 \qquad \theta_y = 27.6° \blacktriangleleft$$

$$\cos\theta_z = \frac{R_z}{R} = \frac{163.011\text{ N}}{515.07\text{ N}} = 0.31648 \qquad \theta_z = 71.5° \blacktriangleleft$$



PROBLEM 2.92

Find the magnitude and direction of the resultant of the two forces shown knowing that $P = 400$ N and $Q = 300$ N.

SOLUTION

$$\mathbf{P} = (400\text{ N})[-\cos 30°\sin 15°\mathbf{i} + \sin 30°\mathbf{j} + \cos 30°\cos 15°\mathbf{k}]$$

$$= -(89.678\text{ N})\mathbf{i} + (200\text{ N})\mathbf{j} + (334.61\text{ N})\mathbf{k}$$

$$\mathbf{Q} = (300\text{ N})[\cos 50°\cos 20°\mathbf{i} + \sin 50°\mathbf{j} - \cos 50°\sin 20°\mathbf{k}]$$

$$= (181.21\text{ N})\mathbf{i} + (229.81\text{ N})\mathbf{j} - (65.954\text{ N})\mathbf{k}$$

$$\mathbf{R} = \mathbf{P} + \mathbf{Q}$$

$$= (91.532\text{ N})\mathbf{i} + (429.81\text{ N})\mathbf{j} + (268.66\text{ N})\mathbf{k}$$

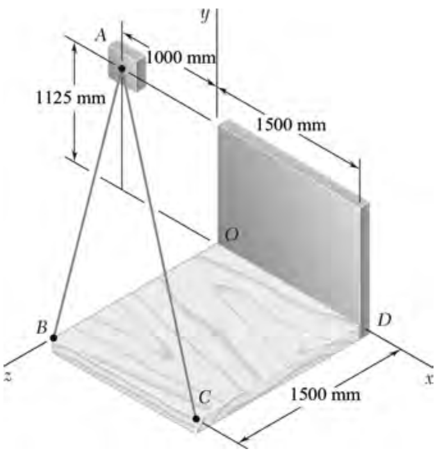
$$R = \sqrt{(91.532\text{ N})^2 + (429.81\text{ N})^2 + (268.66\text{ N})^2}$$

$$= 515.07\text{ N} \qquad R = 515\text{ N} \blacktriangleleft$$

$$\cos\theta_x = \frac{R_x}{R} = \frac{91.532\text{ N}}{515.07\text{ N}} = 0.177708 \qquad \theta_x = 79.8° \blacktriangleleft$$

$$\cos\theta_y = \frac{R_y}{R} = \frac{429.81\text{ N}}{515.07\text{ N}} = 0.83447 \qquad \theta_y = 33.4° \blacktriangleleft$$

$$\cos\theta_z = \frac{R_z}{R} = \frac{268.66\text{ N}}{515.07\text{ N}} = 0.52160 \qquad \theta_z = 58.6° \blacktriangleleft$$



PROBLEM 2.93

Knowing that the tension is 2125 N in cable *AB* and 2550 N in cable *AC*, determine the magnitude and direction of the resultant of the forces exerted at *A* by the two cables.

SOLUTION

$$\overrightarrow{AB} = (1000\text{ mm})\mathbf{i} - (1125\text{ mm})\mathbf{j} + (1500\text{ mm})\mathbf{k}$$

$$AB = \sqrt{(1000)^2 + (1125)^2 + (1500)^2} = 2125\text{ mm}$$

$$\overrightarrow{AC} = (2500\text{ mm})\mathbf{i} - (1125\text{ mm})\mathbf{j} + (1500\text{ mm})\mathbf{k}$$

$$AC = \sqrt{(2500)^2 + (1125)^2 + (1500)^2} = 3125\text{ mm}$$

$$\mathbf{T}_{AB} = T_{AB}\lambda_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = (2125\text{ N})\left[\frac{(1000\text{ mm})\mathbf{i} - (1125\text{ mm})\mathbf{j} + (1500\text{ mm})\mathbf{k}}{2125\text{ mm}}\right]$$

$$\mathbf{T}_{AB} = (1000\text{ N})\mathbf{i} - (1125\text{ N})\mathbf{j} + (1500\text{ N})\mathbf{k}$$

$$\mathbf{T}_{AC} = T_{AC}\lambda_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = (2550\text{ N})\left[\frac{(2500\text{ mm})\mathbf{i} - (1125\text{ mm})\mathbf{j} + (1500\text{ mm})\mathbf{k}}{3125\text{ mm}}\right]$$

$$\mathbf{T}_{AC} = (2040\text{ N})\mathbf{i} - (918\text{ N})\mathbf{j} + (1224\text{ N})\mathbf{k}$$

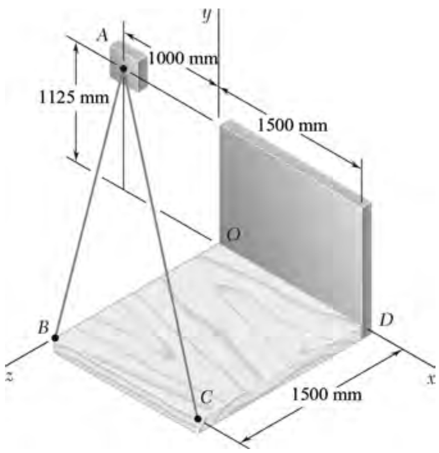
$$\mathbf{R} = \mathbf{T}_{AB} + \mathbf{T}_{AC} = (3040\text{ N})\mathbf{i} - (2043\text{ N})\mathbf{j} + (2724\text{ N})\mathbf{k} = 4564.61\text{ N}$$

Then: $R = 4564.6\text{ N}$ $\qquad R = 4560\text{ N}$ ◀

and $\cos\theta_x = \dfrac{3040\text{ N}}{4564.61\text{ N}} = 0.66599$ $\qquad \theta_x = 48.2°$ ◀

$\cos\theta_y = \dfrac{2043\text{ N}}{4564.61\text{ N}} = -0.44757$ $\qquad \theta_y = 116.6°$ ◀

$\cos\theta_z = \dfrac{2724\text{ N}}{4564.61\text{ N}} = 0.59677$ $\qquad \theta_z = 53.4°$ ◀



PROBLEM 2.94

Knowing that the tension is 2550 N in cable *AB* and 2125 N in cable *AC*, determine the magnitude and direction of the resultant of the forces exerted at *A* by the two cables.

SOLUTION

$$\overrightarrow{AB} = (1000 \text{ mm})\mathbf{i} - (1125 \text{ mm})\mathbf{j} + (1500 \text{ mm})\mathbf{k}$$

$$AB = \sqrt{(1005)^2 + (1125)^2 + (1500)^2} = 2125 \text{ mm}$$

$$\overrightarrow{AC} = (2500 \text{ mm})\mathbf{i} - (1125 \text{ mm})\mathbf{j} + (1500 \text{ mm})\mathbf{k}$$

$$AC = \sqrt{(2500)^2 + (1125)^2 + (1500)^2} = 3125 \text{ mm}$$

$$\mathbf{T}_{AB} = T_{AB}\lambda_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = (2550 \text{ N})\left[\frac{(1000 \text{ mm})\mathbf{i} - (1125 \text{ mm})\mathbf{j} + (1500 \text{ mm})\mathbf{k}}{2125 \text{ mm}}\right]$$

$$\mathbf{T}_{AB} = (1200 \text{ N})\mathbf{i} - (1350 \text{ N})\mathbf{j} + (1800 \text{ N})\mathbf{k}$$

$$\mathbf{T}_{AC} = T_{AC}\lambda_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = (2125 \text{ N})\left[\frac{(2500 \text{ mm})\mathbf{i} - (1125 \text{ mm})\mathbf{j} + (1500 \text{ mm})\mathbf{k}}{3125 \text{ mm}}\right]$$

$$\mathbf{T}_{AC} = (1700 \text{ N})\mathbf{i} - (765 \text{ N})\mathbf{j} + (1020 \text{ N})\mathbf{k}$$

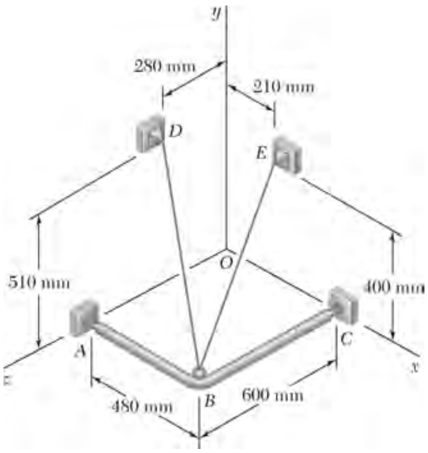
$$\mathbf{R} = \mathbf{T}_{AB} + \mathbf{T}_{AC} = (2900 \text{ N})\mathbf{i} - (2115 \text{ N})\mathbf{j} + (2820 \text{ N})\mathbf{k}$$

Then: $R = 4564.6 \text{ N}$ $\qquad R = 4560 \text{ N}$ ◄

and $\cos\theta_x = \dfrac{2900 \text{ N}}{4564.6} = 0.63532$ $\qquad \theta_x = 50.6°$ ◄

$\cos\theta_y = \dfrac{-2115 \text{ N}}{4564.6 \text{ N}} = -0.46335$ $\qquad \theta_y = 117.6°$ ◄

$\cos\theta_z = \dfrac{2820 \text{ N}}{4564.6 \text{ N}} = 0.61780$ $\qquad \theta_z = 51.8°$ ◄



PROBLEM 2.95

For the frame of Problem 2.87, determine the magnitude and direction of the resultant of the forces exerted by the cable at B knowing that the tension in the cable is 385 N.

PROBLEM 2.87 A frame ABC is supported in part by cable DBE that passes through a frictionless ring at B. Knowing that the tension in the cable is 385 N, determine the components of the force exerted by the cable on the support at D.

SOLUTION

$$\overrightarrow{BD} = -(480\text{ mm})\mathbf{i} + (510\text{ mm})\mathbf{j} - (320\text{ mm})\mathbf{k}$$

$$BD = \sqrt{(480\text{ mm})^2 + (510\text{ mm})^2 + (320\text{ mm})^2} = 770\text{ mm}$$

$$\mathbf{F}_{BD} = T_{BD}\lambda_{BD} = T_{BD}\frac{\overrightarrow{BD}}{BD}$$

$$= \frac{(385\text{ N})}{(770\text{ mm})}[-(480\text{ mm})\mathbf{i} + (510\text{ mm})\mathbf{j} - (320\text{ mm})\mathbf{k}]$$

$$= -(240\text{ N})\mathbf{i} + (255\text{ N})\mathbf{j} - (160\text{ N})\mathbf{k}$$

$$\overrightarrow{BE} = -(270\text{ mm})\mathbf{i} + (400\text{ mm})\mathbf{j} - (600\text{ mm})\mathbf{k}$$

$$BE = \sqrt{(270\text{ mm})^2 + (400\text{ mm})^2 + (600\text{ mm})^2} = 770\text{ mm}$$

$$\mathbf{F}_{BE} = T_{BE}\lambda_{BE} = T_{BE}\frac{\overrightarrow{BE}}{BE}$$

$$= \frac{(385\text{ N})}{(770\text{ mm})}[-(270\text{ mm})\mathbf{i} + (400\text{ mm})\mathbf{j} - (600\text{ mm})\mathbf{k}]$$

$$= -(135\text{ N})\mathbf{i} + (200\text{ N})\mathbf{j} - (300\text{ N})\mathbf{k}$$

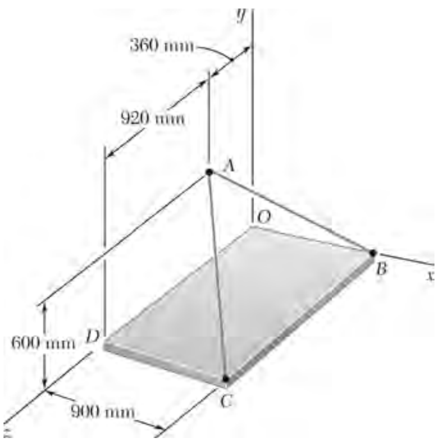
$$\mathbf{R} = \mathbf{F}_{BD} + \mathbf{F}_{BE} = -(375\text{ N})\mathbf{i} + (455\text{ N})\mathbf{j} - (460\text{ N})\mathbf{k}$$

$$R = \sqrt{(375\text{ N})^2 + (455\text{ N})^2 + (460\text{ N})^2} = 747.83\text{ N} \qquad R = 748\text{ N} \blacktriangleleft$$

$$\cos\theta_x = \frac{-375\text{ N}}{747.83\text{ N}} \qquad \theta_x = 120.1° \blacktriangleleft$$

$$\cos\theta_y = \frac{455\text{ N}}{747.83\text{ N}} \qquad \theta_y = 52.5° \blacktriangleleft$$

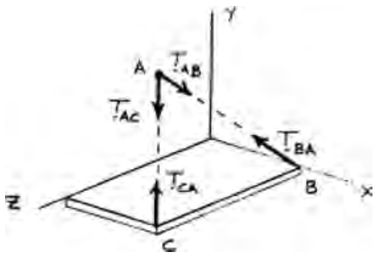
$$\cos\theta_z = \frac{-460\text{ N}}{747.83\text{ N}} \qquad \theta_z = 128.0° \blacktriangleleft$$



PROBLEM 2.96

For the cables of Problem 2.89, knowing that the tension is 1425 N in cable *AB* and 2130 N in cable *AC*, determine the magnitude and direction of the resultant of the forces exerted at *A* by the two cables.

SOLUTION



$$T_{AB} = -T_{BA} \quad \text{(use results of Problem 2.89)}$$

$$(T_{AB})_x = +1125 \text{ N} \quad (T_{AB})_y = -750 \text{ N} \quad (T_{AB})_z = -450 \text{ N}$$

$$T_{AC} = -T_{CA} \quad \text{(use results of Problem 2.90)}$$

$$(T_{AC})_x = +1350 \text{ N} \quad (T_{AC})_y = -900 \text{ N} \quad (T_{AC})_z = +1380 \text{ N}$$

Resultant:

$$R_x = \Sigma F_x = +1125 + 1350 = +2475 \text{ N}$$

$$R_y = \Sigma F_y = -750 - 900 = -1650 \text{ N}$$

$$R_z = \Sigma F_z = -450 + 1380 = +930 \text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

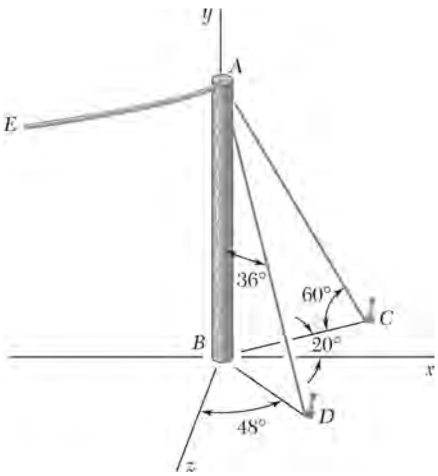
$$= \sqrt{(+2475)^2 + (-1650)^2 + (+930)^2}$$

$$= 3116.6 \text{ N} \qquad R = 3120 \text{ N} \blacktriangleleft$$

$$\cos\theta_x = \frac{R_x}{R} = \frac{+2475}{3116.6} \qquad \theta_x = 37.4° \blacktriangleleft$$

$$\cos\theta_y = \frac{R_y}{R} = \frac{-1650}{3116.6} \qquad \theta_y = 122.0° \blacktriangleleft$$

$$\cos\theta_z = \frac{R_z}{R} = \frac{+930}{3116.6} \qquad \theta_z = 72.6° \blacktriangleleft$$



PROBLEM 2.97

The end of the coaxial cable AE is attached to the pole AB, which is strengthened by the guy wires AC and AD. Knowing that the tension in AC is 750 N and that the resultant of the forces exerted at A by wires AC and AD must be contained in the xy plane, determine (a) the tension in AD, (b) the magnitude and direction of the resultant of the two forces.

SOLUTION

$$\begin{aligned}\mathbf{R} &= \mathbf{T}_{AC} + \mathbf{T}_{AD} \\ &= (750\text{ N})(\cos 60°\cos 20°\mathbf{i} - \sin 60°\mathbf{j} - \cos 60°\sin 20°\mathbf{k}) \\ &\quad + T_{AD}(\sin 36°\sin 48°\mathbf{i} - \cos 36°\mathbf{j} + \sin 36°\cos 48°\mathbf{k}) \qquad (1)\end{aligned}$$

(a) Since $R_z = 0$, The coefficient of **k** must be zero.

$$(750\text{ N})(-\cos 60°\sin 20°) + T_{AD}(\sin 36°\cos 48°) = 0$$

$$T_{AD} = 326.1\text{ N} \qquad T_{AD} = 326\text{ N} \blacktriangleleft$$

(b) Substituting for T_{AD} into Eq. (1) gives:

$$\begin{aligned}\mathbf{R} &= [(750\text{ N})\cos 60°\cos 20° + (326.1\text{ N})\sin 36°\sin 48°]\mathbf{i} \\ &\quad - [(750\text{ N})\sin 60° + (326.1\text{ N})\cos 36°]\mathbf{j} + 0\end{aligned}$$

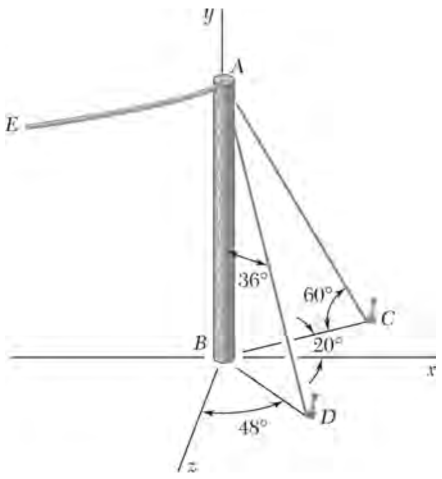
$$\mathbf{R} = (494.83\text{ N})\mathbf{i} - (913.34\text{ N})\mathbf{j}$$

$$\begin{aligned}R &= \sqrt{(494.83)^2 + (913.34)^2} \\ &= 1038.77\text{ N}\end{aligned} \qquad R = 1039\text{ N} \blacktriangleleft$$

$$\cos\theta_x = \frac{494.83\text{ N}}{1038.77\text{ N}} \qquad \theta_x = 61.6° \blacktriangleleft$$

$$\cos\theta_y = \frac{-913.34\text{ N}}{1038.76\text{ N}} \qquad \theta_y = 151.6° \blacktriangleleft$$

$$\cos\theta_z = 0 \qquad \theta_z = 90.0° \blacktriangleleft$$



PROBLEM 2.98

The end of the coaxial cable *AE* is attached to the pole *AB*, which is strengthened by the guy wires *AC* and *AD*. Knowing that the tension in *AD* is 625 N and that the resultant of the forces exerted at *A* by wires *AC* and *AD* must be contained in the *xy* plane, determine (*a*) the tension in *AC*, (*b*) the magnitude and direction of the resultant of the two forces.

SOLUTION

$$\begin{aligned}\mathbf{R} &= \mathbf{T}_{AC} + \mathbf{T}_{AD}\\ &= T_{AC}(\cos 60^\circ \cos 20^\circ \mathbf{i} - \sin 60^\circ \mathbf{j} - \cos 60^\circ \sin 20^\circ \mathbf{k})\\ &\quad + (625\text{ N})(\sin 36^\circ \sin 48^\circ \mathbf{i} - \cos 36^\circ \mathbf{j} + \sin 36^\circ \cos 48^\circ \mathbf{k}) \qquad (1)\end{aligned}$$

(*a*) Since $R_z = 0$, The coefficient of **k** must be zero.

$$T_{AC}(-\cos 60^\circ \sin 20^\circ) + (625\text{ N})(\sin 36^\circ \cos 48^\circ) = 0$$

$$T_{AC} = 1437.43\text{ N} \qquad T_{AC} = 1437\text{ N} \blacktriangleleft$$

(*b*) Substituting for T_{AC} into Eq. (1) gives:

$$\begin{aligned}\mathbf{R} &= [(1437.43\text{ N})\cos 60^\circ \cos 20^\circ + (625\text{ N})\sin 36^\circ \sin 48^\circ]\mathbf{i}\\ &\quad - [(1437.43\text{ N})\sin 60^\circ + (625\text{ N})\cos 36^\circ]\mathbf{j} + 0\end{aligned}$$

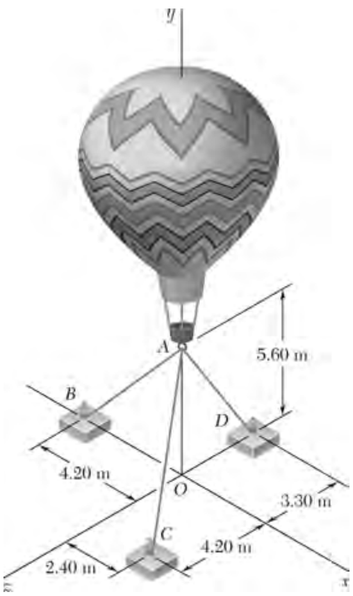
$$\mathbf{R} = (948.377\text{ N})\mathbf{i} - (1750.49\text{ N})\mathbf{j}$$

$$\begin{aligned}R &= \sqrt{(948.377)^2 + (1750.49)^2}\\ &= 1990.89\text{ N}\end{aligned} \qquad R = 1991\text{ N} \blacktriangleleft$$

$$\cos\theta_x = \frac{948.377}{1990.89} \qquad \theta_x = 61.6^\circ \blacktriangleleft$$

$$\cos\theta_y = \frac{-1750.49\text{ N}}{1990.89} \qquad \theta_y = 151.5^\circ \blacktriangleleft$$

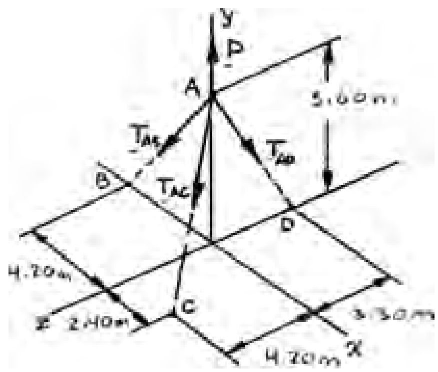
$$\cos\theta_z = 0 \qquad \theta_z = 90.0^\circ \blacktriangleleft$$



PROBLEM 2.99

Three cables are used to tether a balloon as shown. Determine the vertical force **P** exerted by the balloon at A knowing that the tension in cable AB is 259 N.

SOLUTION



The forces applied at A are: $\mathbf{T}_{AB}, \mathbf{T}_{AC}, \mathbf{T}_{AD}$, and $\mathbf{P}$

where $\mathbf{P} = P\mathbf{j}$. To express the other forces in terms of the unit vectors **i**, **j**, **k**, we write

$$\overrightarrow{AB} = -(4.20\text{ m})\mathbf{i} - (5.60\text{ m})\mathbf{j} \qquad AB = 7.00\text{ m}$$

$$\overrightarrow{AC} = (2.40\text{ m})\mathbf{i} - (5.60\text{ m})\mathbf{j} + (4.20\text{ m})\mathbf{k} \qquad AC = 7.40\text{ m}$$

$$\overrightarrow{AD} = -(5.60\text{ m})\mathbf{j} - (3.30\text{ m})\mathbf{k} \qquad AD = 6.50\text{ m}$$

and

$$\mathbf{T}_{AB} = T_{AB}\lambda_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = (-0.6\mathbf{i} - 0.8\mathbf{j})T_{AB}$$

$$\mathbf{T}_{AC} = T_{AC}\lambda_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = (0.32432 - 0.75676\mathbf{j} + 0.56757\mathbf{k})T_{AC}$$

$$\mathbf{T}_{AD} = T_{AD}\lambda_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD} = (-0.86154\mathbf{j} - 0.50769\mathbf{k})T_{AD}$$

PROBLEM 2.99 (Continued)

Equilibrium condition $\Sigma F = 0$: $\mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + P\mathbf{j} = 0$

Substituting the expressions obtained for $\mathbf{T}_{AB}$, $\mathbf{T}_{AC}$, and $\mathbf{T}_{AD}$ and factoring **i**, **j**, and **k**:

$$(-0.6T_{AB} + 0.32432T_{AC})\mathbf{i} + (-0.8T_{AB} - 0.75676T_{AC} - 0.86154T_{AD} + P)\mathbf{j}$$
$$+(0.56757T_{AC} - 0.50769T_{AD})\mathbf{k} = 0$$

Equating to zero the coefficients of **i**, **j**, **k**:

$$-0.6T_{AB} + 0.32432T_{AC} = 0 \tag{1}$$

$$-0.8T_{AB} - 0.75676T_{AC} - 0.86154T_{AD} + P = 0 \tag{2}$$

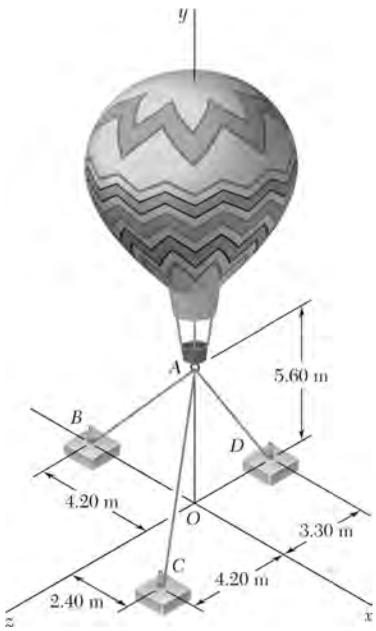
$$0.56757T_{AC} - 0.50769T_{AD} = 0 \tag{3}$$

Setting $T_{AB} = 259\text{ N}$ in (1) and (2), and solving the resulting set of equations gives

$$T_{AC} = 479.15\text{ N}$$
$$T_{AD} = 535.66\text{ N}$$

$\mathbf{P} = 1031\text{ N}\uparrow$ ◀

PROBLEM 2.100



Three cables are used to tether a balloon as shown. Determine the vertical force **P** exerted by the balloon at *A* knowing that the tension in cable *AC* is 444 N.

SOLUTION

See Problem 2.99 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-0.6T_{AB} + 0.32432T_{AC} = 0 \qquad (1)$$

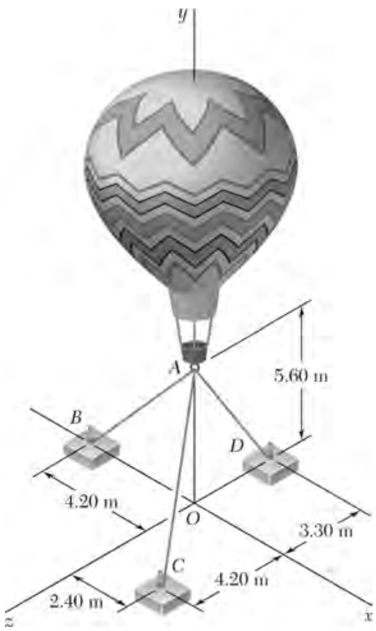
$$-0.8T_{AB} - 0.75676T_{AC} - 0.86154T_{AD} + P = 0 \qquad (2)$$

$$0.56757T_{AC} - 0.50769T_{AD} = 0 \qquad (3)$$

Substituting $T_{AC} = 444$ N in Equations (1), (2), and (3) above, and solving the resulting set of equations using conventional algorithms gives

$$T_{AB} = 240 \text{ N}$$
$$T_{AD} = 496.36 \text{ N}$$

$\mathbf{P} = 956 \text{ N} \uparrow$ ◀



PROBLEM 2.101

Three cables are used to tether a balloon as shown. Determine the vertical force **P** exerted by the balloon at A knowing that the tension in cable AD is 481 N.

SOLUTION

See Problem 2.99 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3).

$$-0.6T_{AB} + 0.32432T_{AC} = 0 \quad (1)$$

$$-0.8T_{AB} - 0.75676T_{AC} - 0.86154T_{AD} + P = 0 \quad (2)$$

$$0.56757T_{AC} - 0.50769T_{AD} = 0 \quad (3)$$

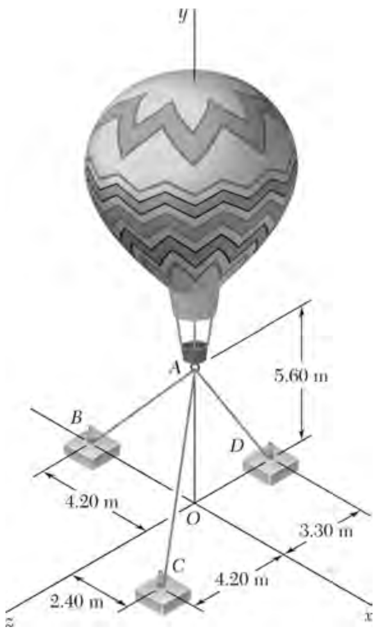
Substituting $T_{AD} = 481\text{ N}$ in Equations (1), (2), and (3) above, and solving the resulting set of equations using conventional algorithms gives

$$T_{AC} = 430.26\text{ N}$$
$$T_{AB} = 232.57\text{ N}$$

$\mathbf{P} = 926\text{ N}\uparrow$ ◀

PROBLEM 2.102

Three cables are used to tether a balloon as shown. Knowing that the balloon exerts an 800-N vertical force at A, determine the tension in each cable.



SOLUTION

See Problem 2.99 for the figure and analysis leading to the linear algebraic Equations (1), (2), and (3).

$$-0.6T_{AB} + 0.32432T_{AC} = 0 \tag{1}$$

$$-0.8T_{AB} - 0.75676T_{AC} - 0.86154T_{AD} + P = 0 \tag{2}$$

$$0.56757T_{AC} - 0.50769T_{AD} = 0 \tag{3}$$

From Eq. (1) $\quad T_{AB} = 0.54053T_{AC}$

From Eq. (3) $\quad T_{AD} = 1.11795T_{AC}$

Substituting for T_{AB} and T_{AD} in terms of T_{AC} into Eq. (2) gives:

$$-0.8(0.54053T_{AC}) - 0.75676T_{AC} - 0.86154(1.11795T_{AC}) + P = 0$$

$$2.1523T_{AC} = P; \quad P = 800\text{ N}$$

$$T_{AC} = \frac{800\text{ N}}{2.1523}$$

$$= 371.69\text{ N}$$

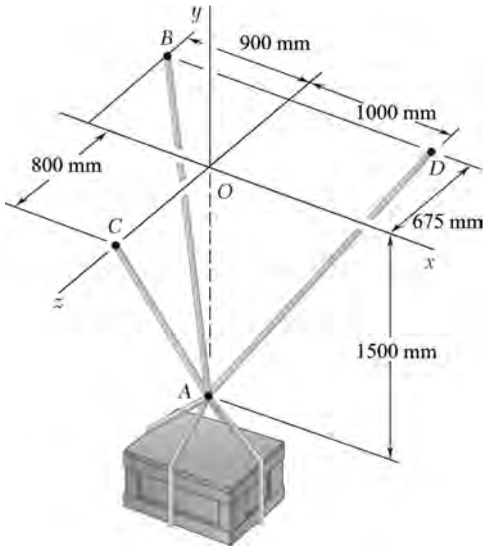
Substituting into expressions for T_{AB} and T_{AD} gives:

$$T_{AB} = 0.54053(371.69\text{ N})$$

$$T_{AD} = 1.11795(371.69\text{ N})$$

$$T_{AB} = 201\text{ N}, \quad T_{AC} = 372\text{ N}, \quad T_{AD} = 416\text{ N} \blacktriangleleft$$

PROBLEM 2.103



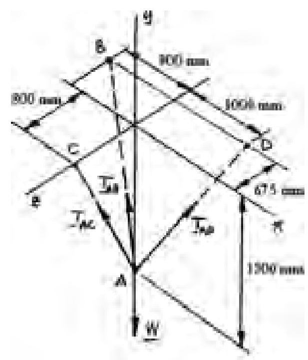
A crate is supported by three cables as shown. Determine the weight of the crate knowing that the tension in cable AB is 3750 N.

SOLUTION

The forces applied at A are:

$$\mathbf{T}_{AB}, \mathbf{T}_{AC}, \mathbf{T}_{AD} \text{ and } \mathbf{W}$$

where $\mathbf{P} = P\mathbf{j}$. To express the other forces in terms of the unit vectors **i**, **j**, **k**, we write



$$\overrightarrow{AB} = -(900 \text{ mm})\mathbf{i} + (1500 \text{ mm})\mathbf{j} - (675 \text{ mm})\mathbf{k}$$
$$AB = 1875 \text{ mm}$$
$$\overrightarrow{AC} = (1500 \text{ mm})\mathbf{j} + (800 \text{ mm})\mathbf{k}$$
$$AC = 1700 \text{ mm}$$
$$\overrightarrow{AD} = (1000 \text{ mm})\mathbf{i} + (1500 \text{ mm})\mathbf{j} - (675 \text{ mm})\mathbf{k}$$
$$AD = 1925 \text{ mm}$$

and
$$\mathbf{T}_{AB} = T_{AB}\lambda_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB}$$
$$= (-0.48\mathbf{i} + 0.8\mathbf{j} - 0.36\mathbf{k})T_{AB}$$
$$\mathbf{T}_{AC} = T_{AC}\lambda_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC}$$
$$= (0.88235\mathbf{j} + 0.47059\mathbf{k})T_{AC}$$
$$\mathbf{T}_{AD} = T_{AD}\lambda_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD}$$
$$= (0.51948\mathbf{i} + 0.77922\mathbf{j} - 0.35065\mathbf{k})T_{AD}$$

Equilibrium Condition with $\mathbf{W} = -W\mathbf{j}$

$$\Sigma F = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} - W\mathbf{j} = 0$$

PROBLEM 2.103 (Continued)

Substituting the expressions obtained for $\mathbf{T}_{AB}, \mathbf{T}_{AC}$, and $\mathbf{T}_{AD}$ and factoring **i**, **j**, and **k**:

$$(-0.48T_{AB} + 0.51948T_{AD})\mathbf{i} + (0.8T_{AB} + 0.88235T_{AC} + 0.77922T_{AD} - W)\mathbf{j}$$
$$+(-0.36T_{AB} + 0.47059T_{AC} - 0.35065T_{AD})\mathbf{k} = 0$$

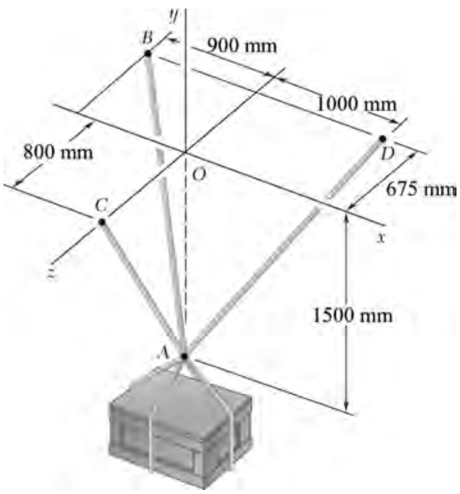
Equating to zero the coefficients of **i**, **j**, **k**:

$$-0.48T_{AB} + 0.51948T_{AD} = 0$$
$$0.8T_{AB} + 0.88235T_{AC} + 0.77922T_{AD} - W = 0$$
$$-0.36T_{AB} + 0.47059T_{AC} - 0.35065T_{AD} = 0$$

Substituting $T_{AB} = 3750$ N in Equations (1), (2), and (3) and solving the resulting set of equations, using conventional algorithms for solving linear algebraic equations, gives:

$$T_{AC} = 5450.24 \text{ N}$$
$$T_{AD} = 3465 \text{ N}$$
$$W = 10509 \text{ N}$$

$W = 10510$ N ◀



PROBLEM 2.104

A crate is supported by three cables as shown. Determine the weight of the crate knowing that the tension in cable AD is 3080 N.

SOLUTION

See Problem 2.103 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-0.48T_{AB} + 0.51948T_{AD} = 0$$

$$0.8T_{AB} + 0.88235T_{AC} + 0.77922T_{AD} - W = 0$$

$$-0.36T_{AB} + 0.47059T_{AC} - 0.35065T_{AD} = 0$$

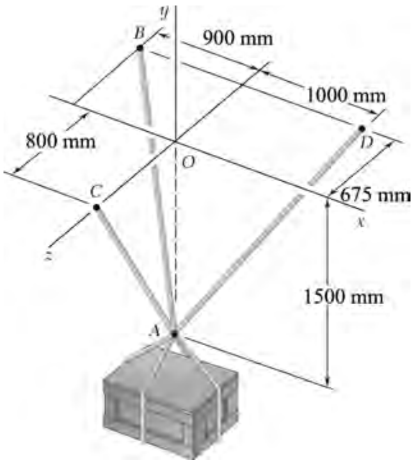
Substituting $T_{AD} = 3080\text{ N}$ in Equations (1), (2), and (3) above, and solving the resulting set of equations using conventional algorithms, gives:

$$T_{AB} = 3333.33\text{ N}$$

$$T_{AC} = 4844.66\text{ N}$$

$$W = 9341.35\text{ N}$$

$W = 9340\text{ N}$ ◀



PROBLEM 2.105

A crate is supported by three cables as shown. Determine the weight of the crate knowing that the tension in cable AC is 2720 N.

SOLUTION

See Problem 2.103 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-0.48T_{AB} + 0.51948T_{AD} = 0$$
$$0.8T_{AB} + 0.88235T_{AC} + 0.77922T_{AD} - W = 0$$
$$-0.36T_{AB} + 0.47059T_{AC} - 0.35065T_{AD} = 0$$

Substituting $T_{AC} = 2720\text{ N}$ in Equations (1), (2), and (3) above, and solving the resulting set of equations using conventional algorithms, gives:

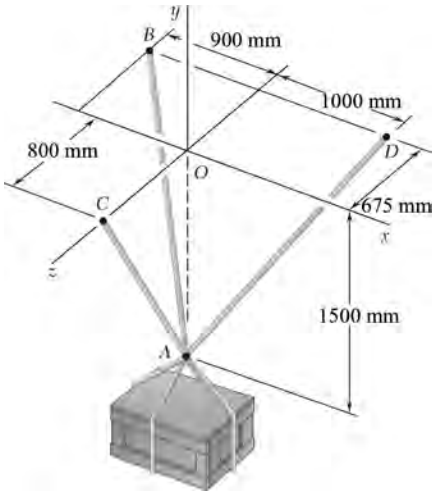
$$T_{AB} = 1871.35\text{ N}$$

$$T_{AD} = 1729.1\text{ N}$$

$$W = 5244.4\text{ N}$$

$W = 5240\text{ N}$ ◀

PROBLEM 2.106



A 8000-N crate is supported by three cables as shown. Determine the tension in each cable.

SOLUTION

See Problem 2.103 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-0.48T_{AB} + 0.51948T_{AD} = 0$$
$$0.8T_{AB} + 0.88235T_{AC} + 0.77922T_{AD} - W = 0$$
$$-0.36T_{AB} + 0.47059T_{AC} - 0.35065T_{AD} = 0$$

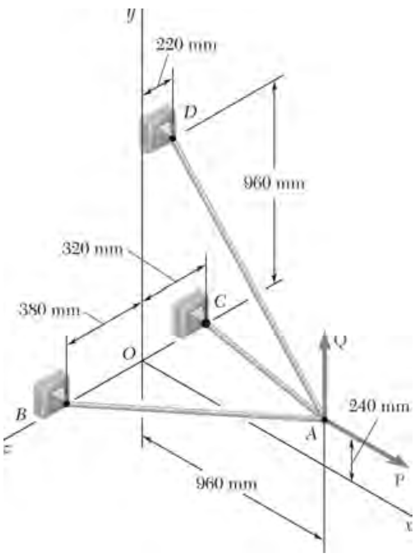
Substituting $W = 8000$ N in Equations (1), (2), and (3) above, and solving the resulting set of equations using conventional algorithms, gives:

$\mathrm{T}_{AB} = 2854.6$ N $\quad T_{AB} = 2850$ N ◀

$\mathrm{T}_{AC} = 4149.2$ N $\quad T_{AC} = 4150$ N ◀

$\mathrm{T}_{AD} = 2637.7$ N $\quad T_{AD} = 2640$ N ◀

PROBLEM 2.107



Three cables are connected at A, where the forces $\mathbf{P}$ and $\mathbf{Q}$ are applied as shown. Knowing that $Q = 0$, find the value of P for which the tension in cable AD is 305 N.

SOLUTION

$$\Sigma \mathbf{F}_A = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + \mathbf{P} = 0 \text{ where } \mathbf{P} = P\mathbf{i}$$

$$\overrightarrow{AB} = -(960 \text{ mm})\mathbf{i} - (240 \text{ mm})\mathbf{j} + (380 \text{ mm})\mathbf{k} \qquad AB = 1060 \text{ mm}$$

$$\overrightarrow{AC} = -(960 \text{ mm})\mathbf{i} - (240 \text{ mm})\mathbf{j} - (320 \text{ mm})\mathbf{k} \qquad AC = 1040 \text{ mm}$$

$$\overrightarrow{AD} = -(960 \text{ mm})\mathbf{i} + (720 \text{ mm})\mathbf{j} - (220 \text{ mm})\mathbf{k} \qquad AD = 1220 \text{ mm}$$

$$\mathbf{T}_{AB} = T_{AB}\lambda_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = T_{AB}\left(-\frac{48}{53}\mathbf{i} - \frac{12}{53}\mathbf{j} + \frac{19}{53}\mathbf{k}\right)$$

$$\mathbf{T}_{AC} = T_{AC}\lambda_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = T_{AC}\left(-\frac{12}{13}\mathbf{i} - \frac{3}{13}\mathbf{j} - \frac{4}{13}\mathbf{k}\right)$$

$$\mathbf{T}_{AD} = T_{AD}\lambda_{AD} = \frac{305 \text{ N}}{1220 \text{ mm}}[(-960 \text{ mm})\mathbf{i} + (720 \text{ mm})\mathbf{j} - (220 \text{ mm})\mathbf{k}]$$

$$= -(240 \text{ N})\mathbf{i} + (180 \text{ N})\mathbf{j} - (55 \text{ N})\mathbf{k}$$

Substituting into $\Sigma \mathbf{F}_A = 0$, factoring $\mathbf{i}, \mathbf{j}, \mathbf{k}$, and setting each coefficient equal to ϕ gives:

$$\mathbf{i}: \quad P = \frac{48}{53}T_{AB} + \frac{12}{13}T_{AC} + 240 \text{ N} \tag{1}$$

$$\mathbf{j}: \quad \frac{12}{53}T_{AB} + \frac{3}{13}T_{AC} = 180 \text{ N} \tag{2}$$

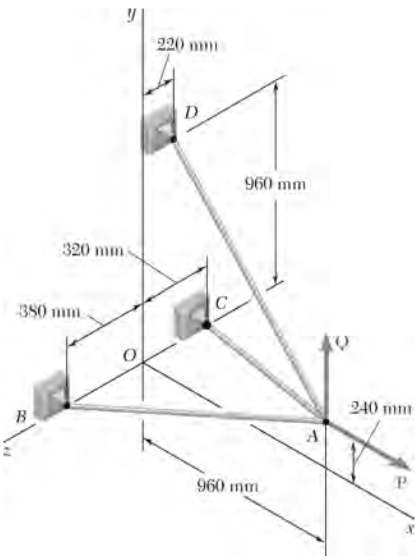
$$\mathbf{k}: \quad \frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} = 55 \text{ N} \tag{3}$$

Solving the system of linear equations using conventional algorithms gives:

$$T_{AB} = 446.71 \text{ N}$$

$$T_{AC} = 341.71 \text{ N}$$

$P = 960$ N ◀



PROBLEM 2.108

Three cables are connected at A, where the forces **P** and **Q** are applied as shown. Knowing that $P = 1200$ N, determine the values of Q for which cable AD is taut.

SOLUTION

We assume that $T_{AD} = 0$ and write $\qquad \Sigma \mathbf{F}_A = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + Q\mathbf{j} + (1200\text{ N})\mathbf{i} = 0$

$$\overrightarrow{AB} = -(960\text{ mm})\mathbf{i} - (240\text{ mm})\mathbf{j} + (380\text{ mm})\mathbf{k} \quad AB = 1060\text{ mm}$$

$$\overrightarrow{AC} = -(960\text{ mm})\mathbf{i} - (240\text{ mm})\mathbf{j} - (320\text{ mm})\mathbf{k} \quad AC = 1040\text{ mm}$$

$$\mathbf{T}_{AB} = T_{AB}\lambda_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = \left(-\frac{48}{53}\mathbf{i} - \frac{12}{53}\mathbf{j} + \frac{19}{53}\mathbf{k}\right)T_{AB}$$

$$\mathbf{T}_{AC} = T_{AC}\lambda_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = \left(-\frac{12}{13}\mathbf{i} - \frac{3}{13}\mathbf{j} - \frac{4}{13}\mathbf{k}\right)T_{AC}$$

Substituting into $\Sigma \mathbf{F}_A = 0$, factoring $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$, and setting each coefficient equal to ϕ gives:

$$\mathbf{i}: \quad -\frac{48}{53}T_{AB} - \frac{12}{13}T_{AC} + 1200\text{ N} = 0 \tag{1}$$

$$\mathbf{j}: \quad -\frac{12}{53}T_{AB} - \frac{3}{13}T_{AC} + Q = 0 \tag{2}$$

$$\mathbf{k}: \quad \frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} = 0 \tag{3}$$

Solving the resulting system of linear equations using conventional algorithms gives:

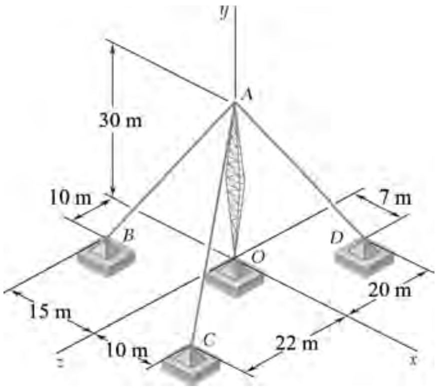
$$T_{AB} = 605.71\text{ N}$$

$$T_{AC} = 705.71\text{ N}$$

$$Q = 300.00\text{ N}$$

$0 \le Q < 300$ N ◀

Note: This solution assumes that Q is directed upward as shown $(Q \ge 0)$, if negative values of Q are considered, cable AD remains taut, but AC becomes slack for $Q = -460$ N.

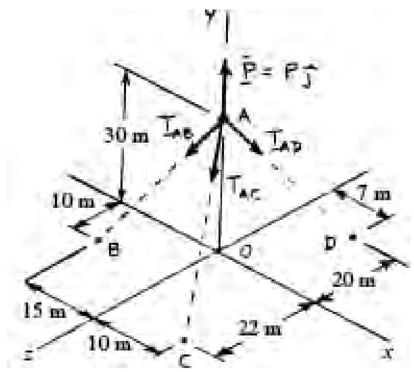


PROBLEM 2.109

A transmission tower is held by three guy wires attached to a pin at *A* and anchored by bolts at *B*, *C*, and *D*. If the tension in wire *AB* is 3150 N, determine the vertical force **P** exerted by the tower on the pin at *A*.

SOLUTION

Free Body *A*:



$$\Sigma \mathbf{F} = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + P\mathbf{j} = 0$$

$$\overrightarrow{AB} = -15\mathbf{i} - 30\mathbf{j} + 10\mathbf{k} \qquad AB = 35 \text{ m}$$

$$\overrightarrow{AC} = 10\mathbf{i} - 30\mathbf{j} + 22\mathbf{k} \qquad AC = 38.523 \text{ m}$$

$$\overrightarrow{AD} = 7\mathbf{i} - 30\mathbf{j} - 20\mathbf{k} \qquad AD = 36.729 \text{ m}$$

We write

$$\mathbf{T}_{AB} = T_{AB}\lambda_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB}$$

$$= \left(-\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right)T_{AB}$$

$$\mathbf{T}_{AC} = T_{AC}\lambda_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC}$$

$$= (0.2596\mathbf{i} - 0.7788\mathbf{j} + 0.5711\mathbf{k})T_{AC}$$

$$\mathbf{T}_{AD} = T_{AD}\lambda_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD}$$

$$= (0.19058\mathbf{i} - 0.8168\mathbf{j} - 0.5445\mathbf{k})T_{AD}$$

Substituting into the equation $\Sigma \mathbf{F} = 0$ and factoring $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$:

$$\left(-\frac{3}{7}T_{AB} + 0.2596T_{AC} + 0.19058T_{AD}\right)\mathbf{i}$$

$$+\left(-\frac{6}{7}T_{AB} - 0.7788T_{AC} - 0.8168T_{AD} + P\right)\mathbf{j}$$

$$+\left(\frac{2}{7}T_{AB} + 0.5711T_{AC} - 0.5445T_{AD}\right)\mathbf{k} = 0$$

PROBLEM 2.109 (Continued)

Setting the coefficients of **i**, **j**, **k** equal to zero:

$$\mathbf{i}: \quad -0.4285T_{AB} + 0.2596T_{AC} + 0.19058T_{AD} = 0 \qquad (1)$$

$$\mathbf{j}: \quad -0.8571T_{AB} - 0.7788T_{AC} - 0.8168T_{AD} + P = 0 \qquad (2)$$

$$\mathbf{k}: \quad 0.2857T_{AB} + 0.5711T_{AC} - 0.5445T_{AD} = 0 \qquad (3)$$

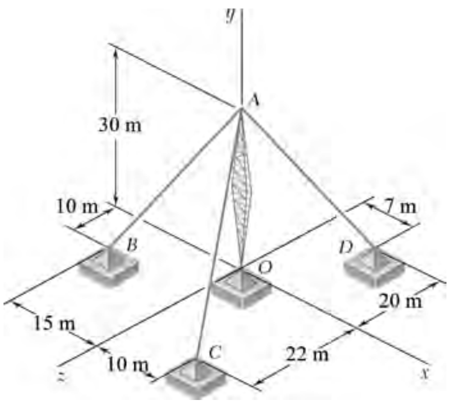
Set $T_{AB} = 3150$ N in Eqs. (1) – (3):

$$-1350 \text{ N} + 0.2596T_{AC} + 0.19058T_{AD} = 0 \qquad (1')$$

$$-2700 \text{ N} - 0.7788T_{AC} - 0.8168T_{AD} + P = 0 \qquad (2')$$

$$900 \text{ N} + 0.5711T_{AC} - 0.5445T_{AD} = 0 \qquad (3')$$

Solving, $T_{AC} = 2252.48$ N $\quad T_{AD} = 4015.41$ N $\quad P = 7734.02$ N $\qquad P = 7730 \text{ N}$ ◀



PROBLEM 2.110

A transmission tower is held by three guy wires attached to a pin at A and anchored by bolts at B, C, and D. If the tension in wire AC is 4600 N, determine the vertical force **P** exerted by the tower on the pin at A.

SOLUTION

See Problem 2.109 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$\mathbf{i}: \quad -0.4285T_{AB} + 0.2596T_{AC} + 0.19058T_{AD} = 0 \tag{1}$$

$$\mathbf{j}: \quad -0.8571T_{AB} - 0.7788T_{AC} - 0.8168T_{AD} + P = 0 \tag{2}$$

$$\mathbf{k}: \quad 0.2857T_{AB} + 0.5711T_{AC} - 0.5445T_{AD} = 0 \tag{3}$$

Substituting for $T_{AC} = 4600$ N in Equations (1), (2), and (3) above and solving the resulting set of equations using conventional algorithms gives:

$$-0.4285T_{AB} + 1194.16\text{ N} + 0.19058T_{AD} = 0 \tag{1$'$}$$

$$-0.8571T_{AB} - 3582.48\text{ N} - 0.8168T_{AD} + P = 0 \tag{2$'$}$$

$$0.2857T_{AB} + 2627.06\text{ N} - 0.5445T_{AD} = 0 \tag{3$'$}$$

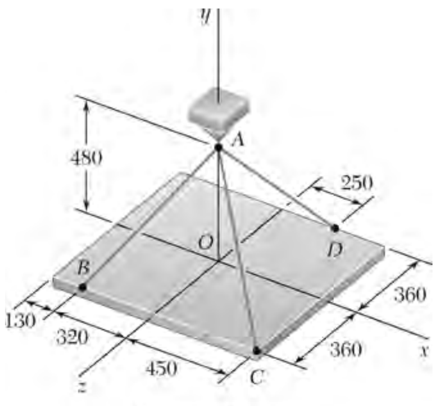
Solving,

6434.22 N

8200.76 N

15795.63 N

$P = 15800$ N ◀



Dimensions in mm

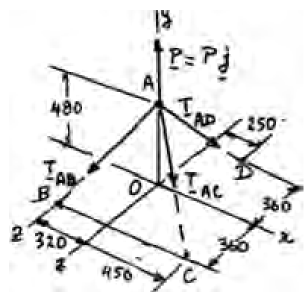
PROBLEM 2.111

A rectangular plate is supported by three cables as shown. Knowing that the tension in cable AC is 60 N, determine the weight of the plate.

SOLUTION

We note that the weight of the plate is equal in magnitude to the force **P** exerted by the support on Point A.

Free Body A:



Dimensions in mm

$$\Sigma F = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + P\mathbf{j} = 0$$

We have:

$$\overrightarrow{AB} = -(320\text{ mm})\mathbf{i} - (480\text{ mm})\mathbf{j} + (360\text{ mm})\mathbf{k} \qquad AB = 680\text{ mm}$$

$$\overrightarrow{AC} = (450\text{ mm})\mathbf{i} - (480\text{ mm})\mathbf{j} + (360\text{ mm})\mathbf{k} \qquad AC = 750\text{ mm}$$

$$\overrightarrow{AD} = (250\text{ mm})\mathbf{i} - (480\text{ mm})\mathbf{j} - (360\text{ mm})\mathbf{k} \qquad AD = 650\text{ mm}$$

Thus:

$$\mathbf{T}_{AB} = T_{AB}\lambda_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = \left(-\frac{8}{17}\mathbf{i} - \frac{12}{17}\mathbf{j} + \frac{9}{17}\mathbf{k}\right)T_{AB}$$

$$\mathbf{T}_{AC} = T_{AC}\lambda_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = (0.6\mathbf{i} - 0.64\mathbf{j} + 0.48\mathbf{k})T_{AC}$$

$$\mathbf{T}_{AD} = T_{AD}\lambda_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD} = \left(\frac{5}{13}\mathbf{i} - \frac{9.6}{13}\mathbf{j} - \frac{7.2}{13}\mathbf{k}\right)T_{AD}$$

Substituting into the equation $\Sigma F = 0$ and factoring $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$:

$$\left(-\frac{8}{17}T_{AB} + 0.6T_{AC} + \frac{5}{13}T_{AD}\right)\mathbf{i}$$

$$+\left(-\frac{12}{17}T_{AB} - 0.64T_{AC} - \frac{9.6}{13}T_{AD} + P\right)\mathbf{j}$$

$$+\left(\frac{9}{17}T_{AB} + 0.48T_{AC} - \frac{7.2}{13}T_{AD}\right)\mathbf{k} = 0$$

PROBLEM 2.111 (Continued)

Setting the coefficient of **i**, **j**, **k** equal to zero:

$$\mathbf{i}: \quad -\frac{8}{17}T_{AB} + 0.6T_{AC} + \frac{5}{13}T_{AD} = 0 \tag{1}$$

$$\mathbf{j}: \quad -\frac{12}{7}T_{AB} - 0.64T_{AC} - \frac{9.6}{13}T_{AD} + P = 0 \tag{2}$$

$$\mathbf{k}: \quad \frac{9}{17}T_{AB} + 0.48T_{AC} - \frac{7.2}{13}T_{AD} = 0 \tag{3}$$

Making $T_{AC} = 60\text{ N}$ in (1) and (3):

$$-\frac{8}{17}T_{AB} + 36\text{ N} + \frac{5}{13}T_{AD} = 0 \tag{1$'$}$$

$$\frac{9}{17}T_{AB} + 28.8\text{ N} - \frac{7.2}{13}T_{AD} = 0 \tag{3$'$}$$

Multiply (1′) by 9, (3′) by 8, and add:

$$554.4\text{ N} - \frac{12.6}{13}T_{AD} = 0 \quad T_{AD} = 572.0\text{ N}$$

Substitute into (1′) and solve for T_{AB}:

$$T_{AB} = \frac{17}{8}\left(36 + \frac{5}{13} \times 572\right) \qquad T_{AB} = 544.0\text{ N}$$

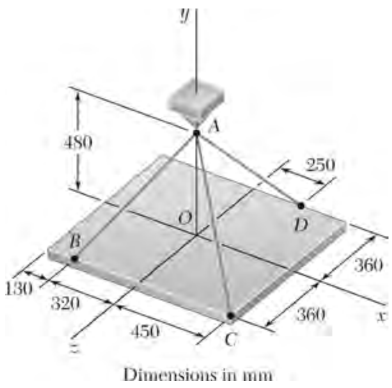
Substitute for the tensions in Eq. (2) and solve for P:

$$P = \frac{12}{17}(544\text{ N}) + 0.64(60\text{ N}) + \frac{9.6}{13}(572\text{ N})$$

$$= 844.8\text{ N}$$

Weight of plate $= P = 845\text{ N}$ ◀

PROBLEM 2.112



A rectangular plate is supported by three cables as shown. Knowing that the tension in cable AD is 520 N, determine the weight of the plate.

SOLUTION

See Problem 2.111 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-\frac{8}{17}T_{AB} + 0.6T_{AC} + \frac{5}{13}T_{AD} = 0 \tag{1}$$

$$-\frac{12}{17}T_{AB} + 0.64T_{AC} - \frac{9.6}{13}T_{AD} + P = 0 \tag{2}$$

$$\frac{9}{17}T_{AB} + 0.48T_{AC} - \frac{7.2}{13}T_{AD} = 0 \tag{3}$$

Making $T_{AD} = 520\text{ N}$ in Eqs. (1) and (3):

$$-\frac{8}{17}T_{AB} + 0.6T_{AC} + 200\text{ N} = 0 \tag{1$'$}$$

$$\frac{9}{17}T_{AB} + 0.48T_{AC} - 288\text{ N} = 0 \tag{3$'$}$$

Multiply (1′) by 9, (3′) by 8, and add:

$$9.24T_{AC} - 504\text{ N} = 0 \quad T_{AC} = 54.5455\text{ N}$$

Substitute into (1′) and solve for T_{AB}:

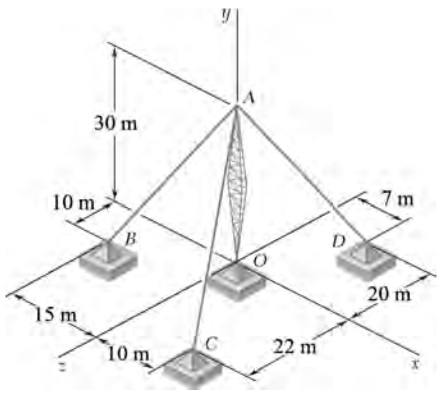
$$T_{AB} = \frac{17}{8}(0.6 \times 54.5455 + 200) \quad T_{AB} = 494.545\text{ N}$$

Substitute for the tensions in Eq. (2) and solve for P:

$$P = \frac{12}{17}(494.545\text{ N}) + 0.64(54.5455\text{ N}) + \frac{9.6}{13}(520\text{ N})$$

$$= 768.00\text{ N}$$

Weight of plate $= P = 768\text{ N}$ ◀



PROBLEM 2.113

For the transmission tower of Problems 2.109 and 2.110, determine the tension in each guy wire knowing that the tower exerts on the pin at A an upward vertical force of 10.5 kN.

SOLUTION

See Problem 2.109 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$\mathbf{i}: \quad -0.4285T_{AB} + 0.2596T_{AC} + 0.19058T_{AD} = 0 \tag{1}$$

$$\mathbf{j}: \quad -0.8571T_{AB} - 0.7788T_{AC} - 0.8168T_{AD} + P = 0 \tag{2}$$

$$\mathbf{k}: \quad 0.2857T_{AB} + 0.5711T_{AC} - 0.5445T_{AD} = 0 \tag{3}$$

Substituting for $P = 10500\text{ N}$ in Equations (1), (2), and (3) above and solving the resulting set of equations using conventional algorithms gives:

$$-0.4285T_{AB} + 0.2596T_{AC} + 0.19058T_{AD} = 0 \tag{1$'$}$$

$$-0.8571T_{AB} - 0.7788T_{AC} - 0.8168T_{AD} + 10500\text{ N} = 0 \tag{2$'$}$$

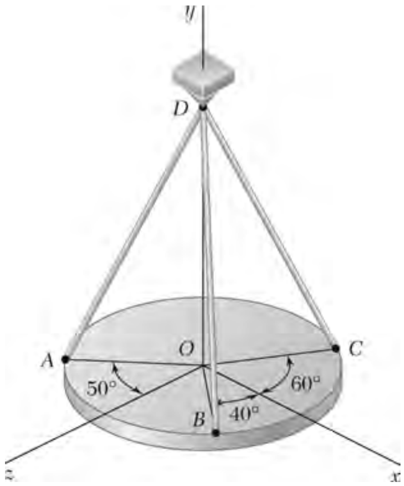
$$0.2857T_{AB} + 0.5711T_{AC} - 0.5445T_{AD} = 0 \tag{3$'$}$$

$$T_{AB} = 4277.09\text{ N}$$
$$T_{AC} = 3057.81\text{ N}$$
$$T_{AD} = 5451.38\text{ N}$$

$T_{AB} = 4280\text{ N}$ ◀

$T_{AC} = 3060\text{ N}$ ◀

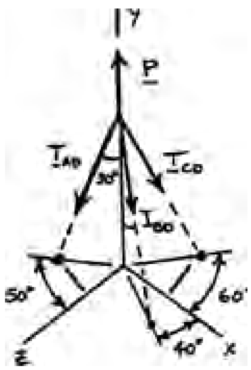
$T_{AD} = 5450\text{ N}$ ◀



PROBLEM 2.114

A horizontal circular plate weighing 300 N is suspended as shown from three wires that are attached to a support at *D* and form 30° angles with the vertical. Determine the tension in each wire.

SOLUTION



$\Sigma F_x = 0$:

$$-T_{AD}(\sin 30°)(\sin 50°) + T_{BD}(\sin 30°)(\cos 40°) + T_{CD}(\sin 30°)(\cos 60°) = 0$$

Dividing through by $\sin 30°$ and evaluating:

$$-0.76604T_{AD} + 0.76604T_{BD} + 0.5T_{CD} = 0 \qquad (1)$$

$\Sigma F_y = 0$: $\quad -T_{AD}(\cos 30°) - T_{BD}(\cos 30°) - T_{CD}(\cos 30°) + 300\text{ N} = 0$

or

$$T_{AD} + T_{BD} + T_{CD} = 346.41\text{ N} \qquad (2)$$

$\Sigma F_z = 0$: $\quad T_{AD}\sin 30°\cos 50° + T_{BD}\sin 30°\sin 40° - T_{CD}\sin 30°\sin 60° = 0$

or

$$0.64279T_{AD} + 0.64279T_{BD} - 0.86603T_{CD} = 0 \qquad (3)$$

Solving Equations (1), (2), and (3) simultaneously:

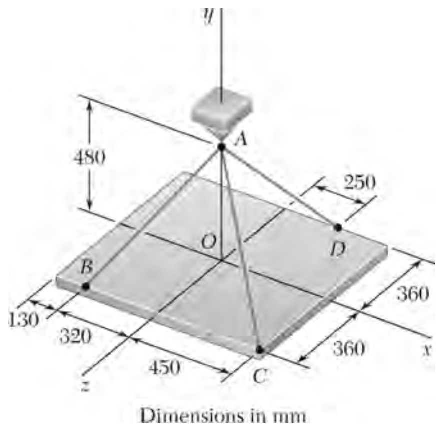
$$T_{AD} = 147.579\text{ N}$$
$$T_{BD} = 51.253\text{ N}$$
$$T_{CD} = 147.578\text{ N}$$

$T_{AD} = 147.6\text{ N}$ ◀

$T_{BD} = 51.3\text{ N}$ ◀

$T_{CD} = 147.6\text{ N}$ ◀

PROBLEM 2.115



For the rectangular plate of Problems 2.111 and 2.112, determine the tension in each of the three cables knowing that the weight of the plate is 792 N.

SOLUTION

See Problem 2.111 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below. Setting $P = 792$ N gives:

$$-\frac{8}{17}T_{AB} + 0.6T_{AC} + \frac{5}{13}T_{AD} = 0 \tag{1}$$

$$-\frac{12}{17}T_{AB} - 0.64T_{AC} - \frac{9.6}{13}T_{AD} + 792\text{ N} = 0 \tag{2}$$

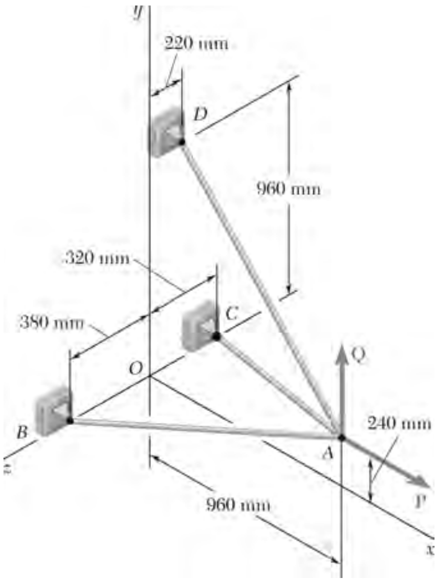
$$\frac{9}{17}T_{AB} + 0.48T_{AC} - \frac{7.2}{13}T_{AD} = 0 \tag{3}$$

Solving Equations (1), (2), and (3) by conventional algorithms gives

$T_{AB} = 510.00$ N $\qquad T_{AB} = 510$ N ◀

$T_{AC} = 56.250$ N $\qquad T_{AC} = 56.2$ N ◀

$T_{AD} = 536.25$ N $\qquad T_{AD} = 536$ N ◀



PROBLEM 2.116

For the cable system of Problems 2.107 and 2.108, determine the tension in each cable knowing that $P = 2880$ N and $Q = 0$.

SOLUTION

$$\Sigma\mathbf{F}_A = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + \mathbf{P} + \mathbf{Q} = 0$$

Where $\qquad \mathbf{P} = P\mathbf{i}$ and $\mathbf{Q} = Q\mathbf{j}$

$$\overrightarrow{AB} = -(960 \text{ mm})\mathbf{i} - (240 \text{ mm})\mathbf{j} + (380 \text{ mm})\mathbf{k} \quad AB = 1060 \text{ mm}$$

$$\overrightarrow{AC} = -(960 \text{ mm})\mathbf{i} - (240 \text{ mm})\mathbf{j} - (320 \text{ mm})\mathbf{k} \quad AC = 1040 \text{ mm}$$

$$\overrightarrow{AD} = -(960 \text{ mm})\mathbf{i} + (720 \text{ mm})\mathbf{j} - (220 \text{ mm})\mathbf{k} \quad AD = 1220 \text{ mm}$$

$$\mathbf{T}_{AB} = T_{AB}\lambda_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = T_{AB}\left(-\frac{48}{53}\mathbf{i} - \frac{12}{53}\mathbf{j} + \frac{19}{53}\mathbf{k}\right)$$

$$\mathbf{T}_{AC} = T_{AC}\lambda_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = T_{AC}\left(-\frac{12}{13}\mathbf{i} - \frac{3}{13}\mathbf{j} - \frac{4}{13}\mathbf{k}\right)$$

$$\mathbf{T}_{AD} = T_{AD}\lambda_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD} = T_{AD}\left(-\frac{48}{61}\mathbf{i} + \frac{36}{61}\mathbf{j} - \frac{11}{61}\mathbf{k}\right)$$

Substituting into $\Sigma\mathbf{F}_A = 0$, setting $P = (2880 \text{ N})\mathbf{i}$ and $Q = 0$, and setting the coefficients of $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ equal to 0, we obtain the following three equilibrium equations:

$$\mathbf{i}: \quad -\frac{48}{53}T_{AB} - \frac{12}{13}T_{AC} - \frac{48}{61}T_{AD} + 2880 \text{ N} = 0 \qquad (1)$$

$$\mathbf{j}: \quad -\frac{12}{53}T_{AB} - \frac{3}{13}T_{AC} + \frac{36}{61}T_{AD} = 0 \qquad (2)$$

$$\mathbf{k}: \quad \frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} - \frac{11}{61}T_{AD} = 0 \qquad (3)$$

PROBLEM 2.116 (Continued)

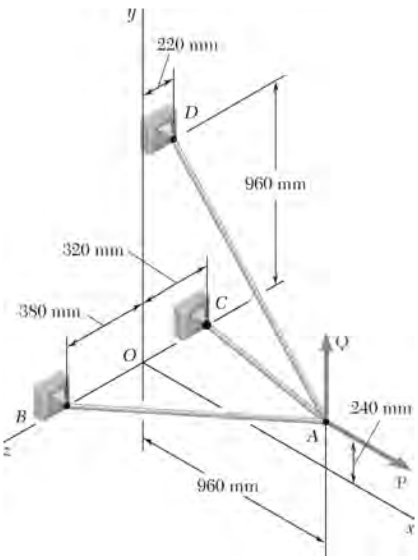
Solving the system of linear equations using conventional algorithms gives:

$$T_{AB} = 1340.14 \text{ N}$$
$$T_{AC} = 1025.12 \text{ N}$$
$$T_{AD} = 915.03 \text{ N}$$

$T_{AB} = 1340 \text{ N}$ ◀

$T_{AC} = 1025 \text{ N}$ ◀

$T_{AD} = 915 \text{ N}$ ◀



PROBLEM 2.117

For the cable system of Problems 2.107 and 2.108, determine the tension in each cable knowing that $P = 2880$ N and $Q = 576$ N.

SOLUTION

See Problem 2.116 for the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-\frac{48}{53}T_{AB} - \frac{12}{13}T_{AC} - \frac{48}{61}T_{AD} + P = 0 \tag{1}$$

$$-\frac{12}{53}T_{AB} - \frac{3}{13}T_{AC} + \frac{36}{61}T_{AD} + Q = 0 \tag{2}$$

$$\frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} - \frac{11}{61}T_{AD} = 0 \tag{3}$$

Setting $P = 2880$ N and $Q = 576$ N gives:

$$-\frac{48}{53}T_{AB} - \frac{12}{13}T_{AC} - \frac{48}{61}T_{AD} + 2880 \text{ N} = 0 \tag{1$'$}$$

$$-\frac{12}{53}T_{AB} - \frac{3}{13}T_{AC} + \frac{36}{61}T_{AD} + 576 \text{ N} = 0 \tag{2$'$}$$

$$\frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} - \frac{11}{61}T_{AD} = 0 \tag{3$'$}$$

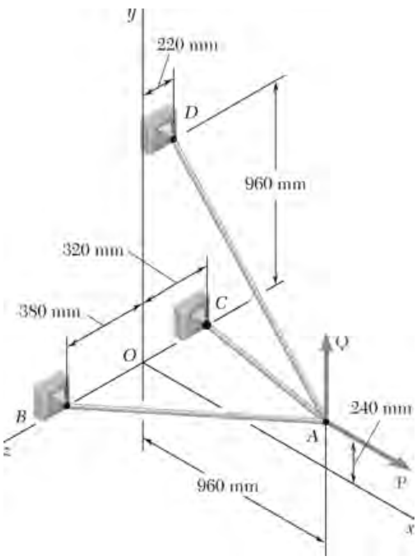
Solving the resulting set of equations using conventional algorithms gives:

$$T_{AB} = 1431.00 \text{ N}$$
$$T_{AC} = 1560.00 \text{ N}$$
$$T_{AD} = 183.010 \text{ N}$$

$T_{AB} = 1431$ N ◀

$T_{AC} = 1560$ N ◀

$T_{AD} = 183.0$ N ◀



PROBLEM 2.118

For the cable system of Problems 2.107 and 2.108, determine the tension in each cable knowing that $P = 2880$ N and $Q = -576$ N. (**Q** is directed downward).

SOLUTION

See Problem 2.116 for the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

$$-\frac{48}{53}T_{AB} - \frac{12}{13}T_{AC} - \frac{48}{61}T_{AD} + P = 0 \qquad (1)$$

$$-\frac{12}{53}T_{AB} - \frac{3}{13}T_{AC} + \frac{36}{61}T_{AD} + Q = 0 \qquad (2)$$

$$\frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} - \frac{11}{61}T_{AD} = 0 \qquad (3)$$

Setting $P = 2880$ N and $Q = -576$ N gives:

$$-\frac{48}{53}T_{AB} - \frac{12}{13}T_{AC} - \frac{48}{61}T_{AD} + 2880\text{ N} = 0 \qquad (1')$$

$$-\frac{12}{53}T_{AB} - \frac{3}{13}T_{AC} + \frac{36}{61}T_{AD} - 576\text{ N} = 0 \qquad (2')$$

$$\frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} - \frac{11}{61}T_{AD} = 0 \qquad (3')$$

Solving the resulting set of equations using conventional algorithms gives:

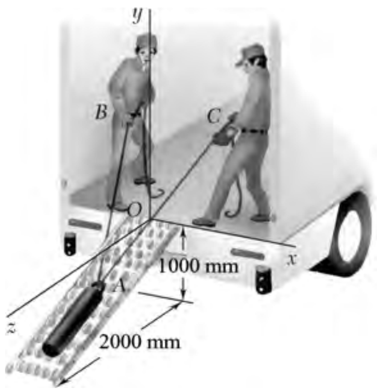
$$T_{AB} = 1249.29\text{ N}$$
$$T_{AC} = 490.31\text{ N}$$
$$T_{AD} = 1646.97\text{ N}$$

$T_{AB} = 1249$ N ◀

$T_{AC} = 490$ N ◀

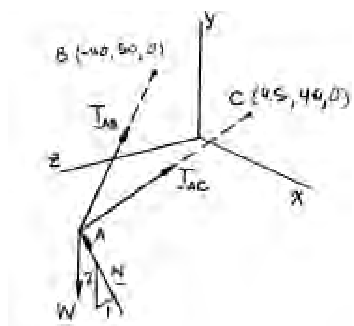
$T_{AD} = 1647$ N ◀

PROBLEM 2.119



Using two ropes and a roller chute, two workers are unloading a 1000-N cast-iron counterweight from a truck. Knowing that at the instant shown the counterweight is kept from moving and that the positions of points *A*, *B*, and *C* are, respectively, *A*(0, –500 mm, 1000 mm), *B*(–1000 mm, 1250 mm, 0), and *C*(1125 mm, 1000 mm, 0), and assuming that no friction exists between the counterweight and the chute, determine the tension in each rope. (*Hint*: Since there is no friction, the force exerted by the chute on the counterweight must be perpendicular to the chute.)

SOLUTION



From the geometry of the chute:

$$\mathbf{N} = \frac{N}{\sqrt{5}}(2\mathbf{j} + \mathbf{k})$$

$$= N(0.8944\mathbf{j} + 0.4472\mathbf{k})$$

The force in each rope can be written as the product of the magnitude of the force and the unit vector along the cable. Thus, with

$$\overrightarrow{AB} = (-1000 \text{ mm})\mathbf{i} + (1750 \text{ mm})\mathbf{j} - (1000 \text{ mm})\mathbf{k}$$

$$AB = 2250 \text{ mm}$$

$$\mathbf{T}_{AB} = T\lambda_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB}$$

$$= \frac{T_{AB}}{2250 \text{ mm}}[(-1000 \text{ mm})\mathbf{i} + (1750 \text{ mm})\mathbf{j} - (1000 \text{ mm})\mathbf{k}]$$

$$\mathbf{T}_{AB} = T_{AB}\left(-\frac{4}{9}\mathbf{i} + \frac{7}{9}\mathbf{j} - \frac{4}{9}\mathbf{k}\right)$$

and

$$\overrightarrow{AC} = (1125 \text{ mm})\mathbf{i} + (1500 \text{ mm})\mathbf{j} - (1000 \text{ mm})\mathbf{k}$$

$$AC = 2125 \text{ mm}$$

$$\mathbf{T}_{AC} = T\lambda_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC}$$

$$= \frac{T_{AC}}{2125 \text{ mm}}[(1125 \text{ mm})\mathbf{i} + (1500 \text{ mm})\mathbf{j} - (1000 \text{ mm})\mathbf{k}]$$

$$\mathbf{T}_{AC} = T_{AC}\left(\frac{9}{17}\mathbf{i} + \frac{12}{17}\mathbf{j} - \frac{8}{17}\mathbf{k}\right)$$

Then: $\Sigma\mathbf{F} = 0: \quad \mathbf{N} + \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{W} = 0$

PROBLEM 2.119 (Continued)

With $W = 1000$ N, and equating the factors of **i**, **j**, and **k** to zero, we obtain the linear algebraic equations:

$$\mathbf{i}: \quad -\frac{4}{9}T_{AB} + \frac{9}{17}T_{AC} = 0 \tag{1}$$

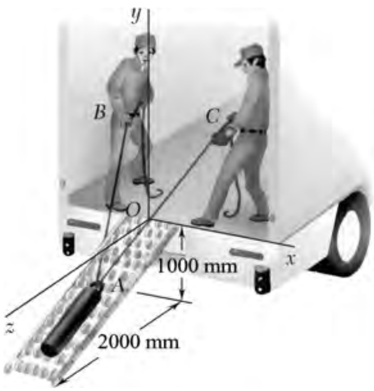
$$\mathbf{j}: \quad \frac{7}{9}T_{AB} + \frac{12}{17}T_{AC} + \frac{2}{\sqrt{5}} - 1000\text{ N} = 0 \tag{2}$$

$$\mathbf{k}: \quad -\frac{4}{9}T_{AB} - \frac{8}{17}T_{AC} + \frac{1}{\sqrt{5}}N = 0 \tag{3}$$

Using conventional methods for solving linear algebraic equations we obtain:

$\mathrm{T}_{AB} = 327.94$ N $\qquad T_{AB} = 328$ N ◀

$\mathrm{T}_{AC} = 275.30$ N $\qquad T_{AC} = 275$ N ◀



PROBLEM 2.120

Solve Problem 2.119 assuming that a third worker is exerting a force $\mathbf{P} = (-200\text{ N})\mathbf{i}$ on the counterweight.

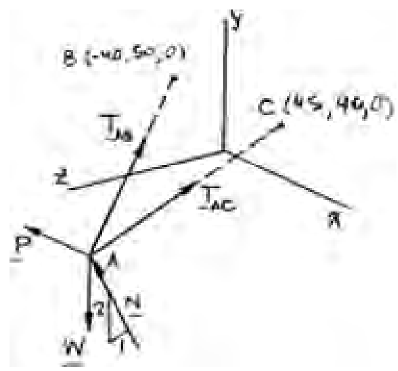
PROBLEM 2.119 Using two ropes and a roller chute, two workers are unloading a 1000-N cast-iron counterweight from a truck. Knowing that at the instant shown the counterweight is kept from moving and that the positions of Points A, B, and C are, respectively, A(0, –500 mm, 1000 mm), B(–1000 mm, 1250 mm, 0), and C(1125 mm, 1000 mm, 0), and assuming that no friction exists between the counterweight and the chute, determine the tension in each rope. (*Hint:* Since there is no friction, the force exerted by the chute on the counterweight must be perpendicular to the chute.)

SOLUTION

See Problem 2.119 for the analysis leading to the vectors describing the tension in each rope.

$$\mathbf{T}_{AB} = T_{AB}\left(-\frac{4}{9}\mathbf{i} + \frac{7}{9}\mathbf{j} - \frac{4}{9}\mathbf{k}\right)$$

$$\mathbf{T}_{AC} = T_{AC}\left(\frac{9}{17}\mathbf{i} + \frac{12}{17}\mathbf{j} - \frac{8}{17}\mathbf{k}\right)$$



Then: $\qquad \Sigma\mathbf{F}_A = 0\text{:} \quad \mathbf{N} + \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{P} + \mathbf{W} = 0$

Where $\qquad \mathbf{P} = (-200\text{ N})\mathbf{i}$

and $\qquad \mathbf{W} = (1000\text{ N})\mathbf{j}$

Equating the factors of **i**, **j**, and **k** to zero, we obtain the linear equations:

$$\mathbf{i}\text{:} \quad -\frac{4}{9}T_{AB} + \frac{9}{17}T_{AC} - 200 = 0$$

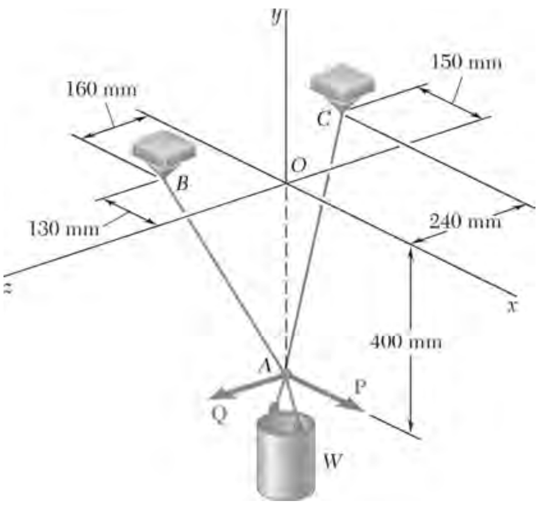
$$\mathbf{j}\text{:} \quad \frac{2}{\sqrt{5}}N + \frac{7}{9}T_{AB} + \frac{12}{17}T_{AC} - 1000 = 0$$

$$\mathbf{k}\text{:} \quad \frac{1}{\sqrt{5}}N - \frac{4}{9}T_{AB} - \frac{8}{17}T_{AC} = 0$$

Using conventional methods for solving linear algebraic equations we obtain

$$T_{AB} = 123.89\text{ N} \qquad T_{AB} = 123.9\text{ N} \blacktriangleleft$$

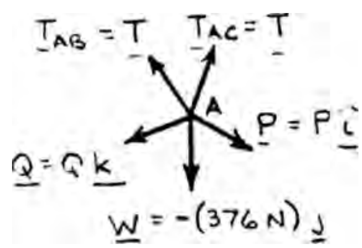
$$T_{AC} = 481.78\text{ N} \qquad T_{AC} = 482\text{ N} \blacktriangleleft$$



PROBLEM 2.121

A container of weight W is suspended from ring A. Cable BAC passes through the ring and is attached to fixed supports at B and C. Two forces $\mathbf{P} = P\mathbf{i}$ and $\mathbf{Q} = Q\mathbf{k}$ are applied to the ring to maintain the container in the position shown. Knowing that $W = 376$ N, determine P and Q. (*Hint:* The tension is the same in both portions of cable BAC.)

SOLUTION



Free Body A:

$$\mathbf{T}_{AB} = T\lambda_{AB}$$

$$= T\frac{\overrightarrow{AB}}{AB}$$

$$= T\frac{(-130\text{ mm})\mathbf{i} + (400\text{ mm})\mathbf{j} + (160\text{ mm})\mathbf{k}}{450\text{ mm}}$$

$$= T\left(-\frac{13}{45}\mathbf{i} + \frac{40}{45}\mathbf{j} + \frac{16}{45}\mathbf{k}\right)$$

$$\mathbf{T}_{AC} = T\lambda_{AC}$$

$$= T\frac{\overrightarrow{AC}}{AC}$$

$$= T\frac{(-150\text{ mm})\mathbf{i} + (400\text{ mm})\mathbf{j} + (-240\text{ mm})\mathbf{k}}{490\text{ mm}}$$

$$= T\left(-\frac{15}{49}\mathbf{i} + \frac{40}{49}\mathbf{j} - \frac{24}{49}\mathbf{k}\right)$$

$$\Sigma F = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{Q} + \mathbf{P} + \mathbf{W} = 0$$

Setting coefficients of **i**, **j**, **k** equal to zero:

$$\mathbf{i}: \quad -\frac{13}{45}T - \frac{15}{49}T + P = 0 \qquad 0.59501T = P \tag{1}$$

$$\mathbf{j}: \quad +\frac{40}{45}T + \frac{40}{49}T - W = 0 \qquad 1.70521T = W \tag{2}$$

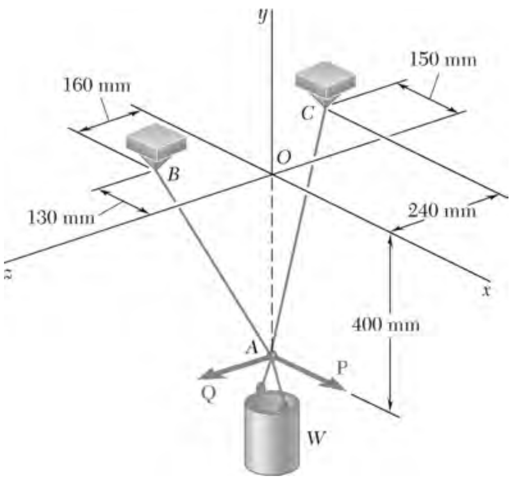
$$\mathbf{k}: \quad +\frac{16}{45}T - \frac{24}{49}T + Q = 0 \qquad 0.134240T = Q \tag{3}$$

PROBLEM 2.121 (Continued)

Data: $W = 376\text{ N} \quad 1.70521T = 376\text{ N} \quad T = 220.50\text{ N}$

$$0.59501(220.50\text{ N}) = P \qquad P = 131.2\text{ N} \blacktriangleleft$$

$$0.134240(220.50\text{ N}) = Q \qquad Q = 29.6\text{ N} \blacktriangleleft$$



PROBLEM 2.122

For the system of Problem 2.121, determine W and Q knowing that $P = 164$ N.

PROBLEM 2.121 A container of weight W is suspended from ring A. Cable BAC passes through the ring and is attached to fixed supports at B and C. Two forces $\mathbf{P} = P\mathbf{i}$ and $\mathbf{Q} = Q\mathbf{k}$ are applied to the ring to maintain the container in the position shown. Knowing that $W = 376$ N, determine P and Q. (*Hint:* The tension is the same in both portions of cable BAC.)

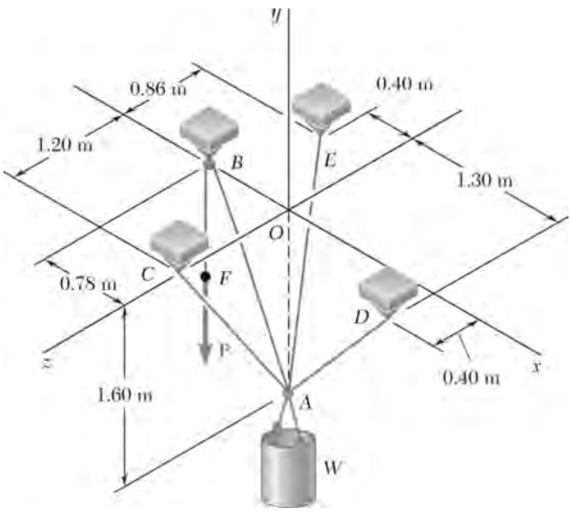
SOLUTION

Refer to Problem 2.121 for the figure and analysis resulting in Equations (1), (2), and (3) for P, W, and Q in terms of T below. Setting $P = 164$ N we have:

Eq. (1): $\quad 0.59501T = 164 \text{ N} \quad\quad T = 275.63 \text{ N}$

Eq. (2): $\quad 1.70521(275.63 \text{ N}) = W \quad\quad W = 470 \text{ N}$ ◀

Eq. (3): $\quad 0.134240(275.63 \text{ N}) = Q \quad\quad Q = 37.0 \text{ N}$ ◀



PROBLEM 2.123

A container of weight W is suspended from ring A, to which cables AC and AE are attached. A force $\mathbf{P}$ is applied to the end F of a third cable that passes over a pulley at B and through ring A and that is attached to a support at D. Knowing that $W = 1000$ N, determine the magnitude of P. (*Hint:* The tension is the same in all portions of cable $FBAD$.)

SOLUTION

The (vector) force in each cable can be written as the product of the (scalar) force and the unit vector along the cable. That is, with

$$\overrightarrow{AB} = -(0.78 \text{ m})\mathbf{i} + (1.6 \text{ m})\mathbf{j} + (0 \text{ m})\mathbf{k}$$

$$AB = \sqrt{(-0.78 \text{ m})^2 + (1.6 \text{ m})^2 + (0)^2}$$

$$= 1.78 \text{ m}$$

$$\mathbf{T}_{AB} = T\lambda_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB}$$

$$= \frac{T_{AB}}{1.78 \text{ m}}[-(0.78 \text{ m})\mathbf{i} + (1.6 \text{ m})\mathbf{j} + (0 \text{ m})\mathbf{k}]$$

$$\mathbf{T}_{AB} = T_{AB}(-0.4382\mathbf{i} + 0.8989\mathbf{j} + 0\mathbf{k})$$

and

$$\overrightarrow{AC} = (0)\mathbf{i} + (1.6 \text{ m})\mathbf{j} + (1.2 \text{ m})\mathbf{k}$$

$$AC = \sqrt{(0 \text{ m})^2 + (1.6 \text{ m})^2 + (1.2 \text{ m})^2} = 2 \text{ m}$$

$$\mathbf{T}_{AC} = T\lambda_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = \frac{T_{AC}}{2 \text{ m}}[(0)\mathbf{i} + (1.6 \text{ m})\mathbf{j} + (1.2 \text{ m})\mathbf{k}]$$

$$\mathbf{T}_{AC} = T_{AC}(0.8\mathbf{j} + 0.6\mathbf{k})$$

and

$$\overrightarrow{AD} = (1.3 \text{ m})\mathbf{i} + (1.6 \text{ m})\mathbf{j} + (0.4 \text{ m})\mathbf{k}$$

$$AD = \sqrt{(1.3 \text{ m})^2 + (1.6 \text{ m})^2 + (0.4 \text{ m})^2} = 2.1 \text{ m}$$

$$\mathbf{T}_{AD} = T\lambda_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD} = \frac{T_{AD}}{2.1 \text{ m}}[(1.3 \text{ m})\mathbf{i} + (1.6 \text{ m})\mathbf{j} + (0.4 \text{ m})\mathbf{k}]$$

$$\mathbf{T}_{AD} = T_{AD}(0.6190\mathbf{i} + 0.7619\mathbf{j} + 0.1905\mathbf{k})$$

PROBLEM 2.123 (Continued)

Finally, $\overrightarrow{AE} = -(0.4\text{ m})\mathbf{i} + (1.6\text{ m})\mathbf{j} - (0.86\text{ m})\mathbf{k}$

$$AE = \sqrt{(-0.4\text{ m})^2 + (1.6\text{ m})^2 + (-0.86\text{ m})^2} = 1.86\text{ m}$$

$$\mathbf{T}_{AE} = T\lambda_{AE} = T_{AE}\frac{\overrightarrow{AE}}{AE}$$

$$= \frac{T_{AE}}{1.86\text{ m}}[-(0.4\text{ m})\mathbf{i} + (1.6\text{ m})\mathbf{j} - (0.86\text{ m})\mathbf{k}]$$

$$\mathbf{T}_{AE} = T_{AE}(-0.2151\mathbf{i} + 0.8602\mathbf{j} - 0.4624\mathbf{k})$$

With the weight of the container $\mathbf{W} = -W\mathbf{j}$, at A we have:

$$\Sigma\mathbf{F} = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} - W\mathbf{j} = 0$$

Equating the factors of **i**, **j**, and **k** to zero, we obtain the following linear algebraic equations:

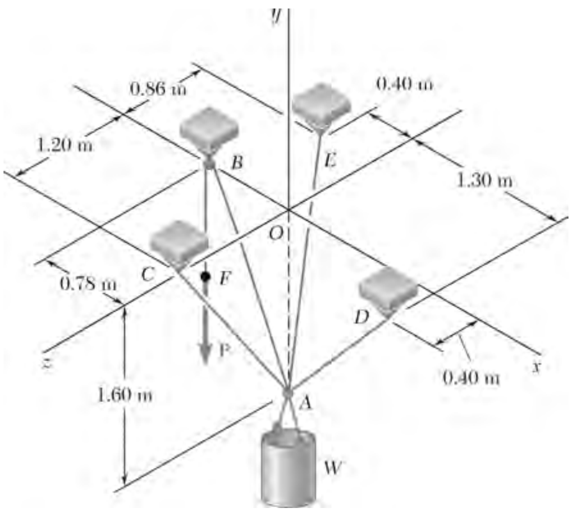
$$-0.4382T_{AB} + 0.6190T_{AD} - 0.2151T_{AE} = 0 \tag{1}$$

$$0.8989T_{AB} + 0.8T_{AC} + 0.7619T_{AD} + 0.8602T_{AE} - W = 0 \tag{2}$$

$$0.6T_{AC} + 0.1905T_{AD} - 0.4624T_{AE} = 0 \tag{3}$$

Knowing that $W = 1000$ N and that because of the pulley system at $B\,T_{AB} = T_{AD} = P$, where P is the externally applied (unknown) force, we can solve the system of linear Equations (1), (2) and (3) uniquely for P.

$P = 378$ N ◀



PROBLEM 2.124

Knowing that the tension in cable AC of the system described in Problem 2.123 is 150 N, determine (*a*) the magnitude of the force **P**, (*b*) the weight W of the container.

PROBLEM 2.123 A container of weight W is suspended from ring A, to which cables AC and AE are attached. A force **P** is applied to the end F of a third cable that passes over a pulley at B and through ring A and that is attached to a support at D. Knowing that $W = 1000$ N, determine the magnitude of P. (*Hint:* The tension is the same in all portions of cable *FBAD*.)

SOLUTION

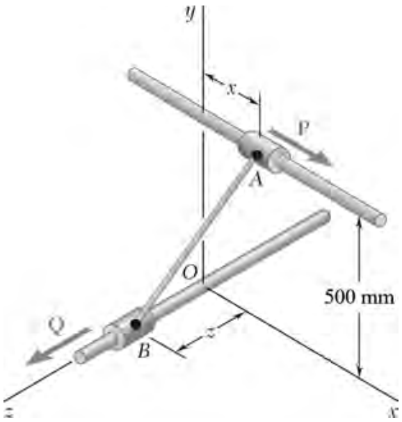
Here, as in Problem 2.123, the support of the container consists of the four cables *AE*, *AC*, *AD*, and *AB*, with the condition that the force in cables *AB* and *AD* is equal to the externally applied force P. Thus, with the condition

$$T_{AB} = T_{AD} = P$$

and using the linear algebraic equations of Problem 2.131 with $T_{AC} = 150$ N, we obtain

(*a*) $P = 454$ N ◀

(*b*) $W = 1202$ N ◀



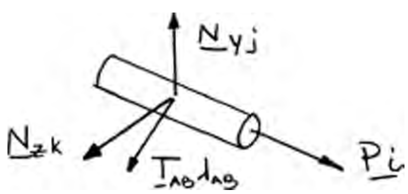
PROBLEM 2.125

Collars A and B are connected by a 625-mm long wire and can slide freely on frictionless rods. If a 300-N force **Q** is applied to collar B as shown, determine (a) the tension in the wire when $x = 225$ mm, (b) the corresponding magnitude of the force **P** required to maintain the equilibrium of the system.

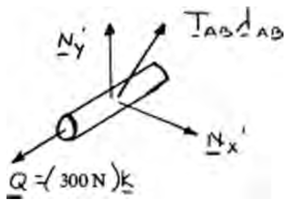
SOLUTION

Free Body Diagrams of Collars:

***A*:**



***B*:**



$$\lambda_{AB} = \frac{\overrightarrow{AB}}{AB} = \frac{-x\mathbf{i} - (500 \text{ mm})\mathbf{j} + z\mathbf{k}}{625 \text{ mm}}$$

Collar A: $\qquad \Sigma\mathbf{F} = 0: \quad P\mathbf{i} + N_y\mathbf{j} + N_z\mathbf{k} + T_{AB}\lambda_{AB} = 0$

Substitute for λ_{AB} and set coefficient of **i** equal to zero:

$$P - \frac{T_{AB}x}{625 \text{ mm}} = 0 \qquad (1)$$

Collar B: $\qquad \Sigma\mathbf{F} = 0: \quad (300 \text{ N})\mathbf{k} + N'_x\mathbf{i} + N'_y\mathbf{j} - T_{AB}\lambda_{AB} = 0$

Substitute for λ_{AB} and set coefficient of **k** equal to zero:

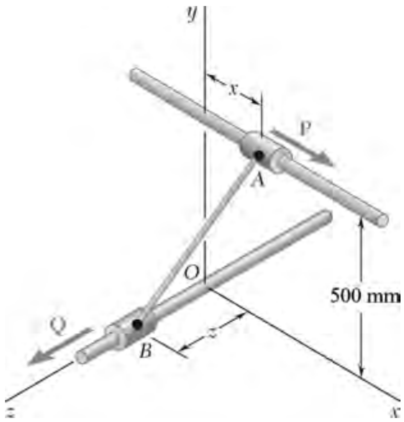
$$300 \text{ N} - \frac{T_{AB}z}{625 \text{ mm}} = 0 \qquad (2)$$

(a) $\qquad x = 225$ mm $\qquad (225 \text{ mm})^2 + (500 \text{ mm})^2 + z^2 = (625 \text{ mm})^2$

$$z = 300 \text{ mm}$$

From Eq. (2): $\qquad 300 \text{ N} - \frac{T_{AB}(300 \text{ mm})}{625 \text{ mm}} = 0 \qquad T_{AB} = 625 \text{ N}$ ◀

(b) From Eq. (1): $\qquad P = \frac{(625 \text{ N})(225 \text{ mm})}{625 \text{ mm}} \qquad P = 225 \text{ N}$ ◀



PROBLEM 2.126

Collars A and B are connected by a 625-mm long wire and can slide freely on frictionless rods. Determine the distances x and z for which the equilibrium of the system is maintained when $P = 600$ N and $Q = 300$ N.

SOLUTION

See Problem 2.125 for the diagrams and analysis leading to Equations (1) and (2) below:

$$P = \frac{T_{AB}x}{625 \text{ mm}} \tag{1}$$

$$300 \text{ N} - \frac{T_{AB}z}{625 \text{ mm}} = 0 \tag{2}$$

For $P = 600$ N, Eq. (1) yields $\quad T_{AB}x = (625 \text{ mm})(600 \text{ N}) \quad (1')$

From Eq. (2) $\quad T_{AB}z = (625 \text{ mm})(300 \text{ N}) \quad (2')$

Dividing Eq. (1′) by (2′): $\quad \dfrac{x}{z} = 2 \quad (3)$

Now write $\quad x^2 + z^2 + (500 \text{ mm})^2 = (625 \text{ mm})^2 \quad (4)$

Solving (3) and (4) simultaneously

$$4z^2 + z^2 + 250000 = 390625$$

$$z^2 = 167$$

$$z = 167.71 \text{ mm}$$

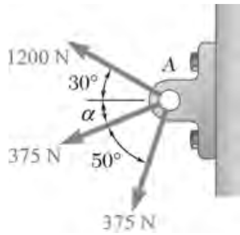
From Eq. (3)

$$x = 2z$$

$$= 335.4 \text{ mm}$$

$x = 167.7$ mm, $\quad z = 335$ mm ◀

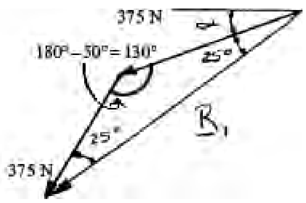
PROBLEM 2.127



The direction of the 375-N forces may vary, but the angle between the forces is always 50°. Determine the value of α for which the resultant of the forces acting at A is directed horizontally to the left.

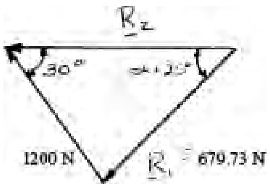
SOLUTION

We must first replace the two 375 N forces by their resultant $\mathbf{R}_1$ using the triangle rule.



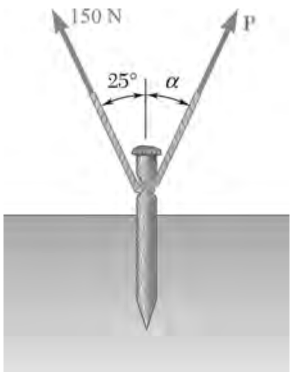
$$\mathbf{R}_1 = 2(375\text{ N})\cos 25°$$
$$= 679.73\text{ N}$$
$$\mathbf{R}_1 = 679.73\text{ N} \;\swarrow\; \alpha + 25°$$

Next we consider the resultant $\mathbf{R}_2$ of $\mathbf{R}_1$ and the 1200 N force where $\mathbf{R}_2$ must be horizontal and directed to the left. Using the triangle rule and law of sines,



$$\frac{\sin(\alpha + 25°)}{1200\text{ N}} = \frac{\sin(30°)}{679.73\text{ N}}$$
$$\sin(\alpha + 25°) = 0.88270$$
$$\alpha + 25° = 61.970°$$
$$\alpha = 36.970°$$

$\alpha = 37.0°$ ◀

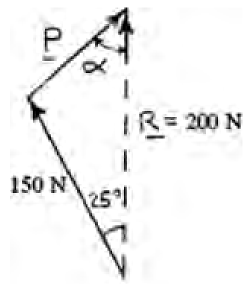


PROBLEM 2.128

A stake is being pulled out of the ground by means of two ropes as shown. Knowing the magnitude and direction of the force exerted on one rope, determine the magnitude and direction of the force **P** that should be exerted on the other rope if the resultant of these two forces is to be a 200-N vertical force.

SOLUTION

Triangle rule:



Law of cosines: $P^2 = (150)^2 + (200)^2 - 2(150)(200)\cos 25°$

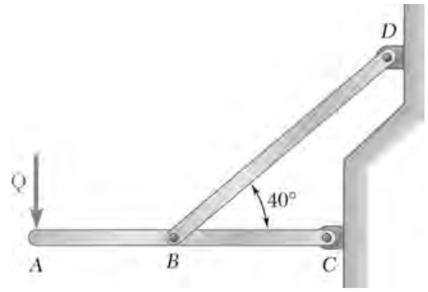
$P = 90.1195 \text{ N}$

Law of sines: $\dfrac{\sin\alpha}{150 \text{ N}} = \dfrac{\sin 25°}{90.1195 \text{ N}}$

$\alpha = 44.703°$

$90° - \alpha = 45.297°$

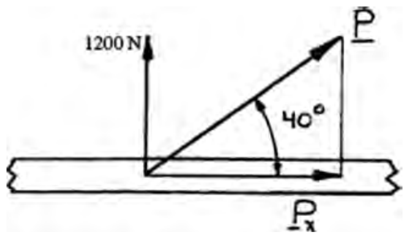
$\mathbf{P} = 90.1 \text{ N} \measuredangle 45.3°$ ◀



PROBLEM 2.129

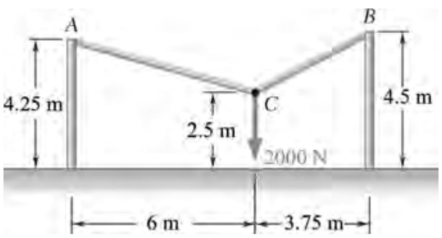
Member *BD* exerts on member *ABC* a force **P** directed along line *BD*. Knowing that **P** must have a 1200 N vertical component, determine (*a*) the magnitude of the force **P**, (*b*) its horizontal component.

SOLUTION



(*a*) $$P = \frac{P_y}{\sin 35°} = \frac{1200 \text{ N}}{\sin 40°} = 1866.9 \text{ N}$$ or $P = 1867$ N ◀

(*b*) $$P_x = \frac{P_y}{\tan 40°} = \frac{1200 \text{ N}}{\tan 40°} = 1430.1 \text{ N}$$ or $P_x = 1430$ N ◀

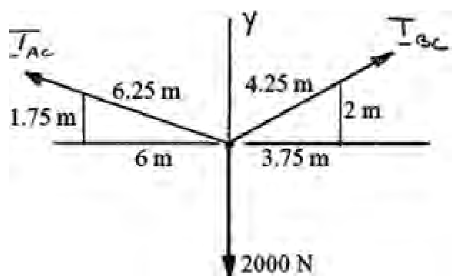


PROBLEM 2.130

Two cables are tied together at *C* and loaded as shown. Determine the tension (*a*) in cable *AC*, (*b*) in cable *BC*.

SOLUTION

Free Body Diagram at *C*:



$$\Sigma \mathbf{F}_x = 0: \quad -\frac{6\text{ m}}{6.25\text{ m}}T_{AC} + \frac{3.75\text{ m}}{4.25\text{ m}}T_{BC} = 0$$

$$T_{BC} = 1.08800T_{AC}$$

$$\Sigma \mathbf{F}_y = 0: \quad \frac{1.75\text{ m}}{6.25\text{ m}}T_{AC} + \frac{2\text{ m}}{4.25\text{ m}}T_{BC} - 2000\text{ N} = 0$$

(*a*)

$$\frac{1.75\text{ m}}{6.25\text{ m}}T_{AC} + \frac{2\text{ m}}{4.25\text{ m}}(1.08800T_{AC}) - 2000\text{ N} = 0$$

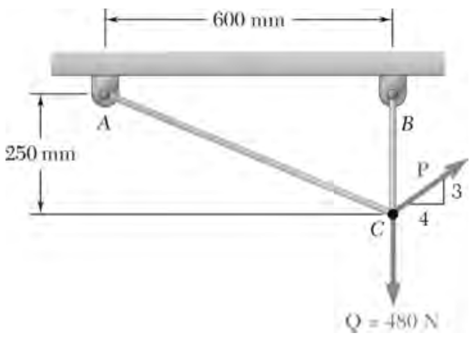
$$(0.28000 + 0.51200)T_{AC} = 2000\text{ N}$$

$$T_{AC} = 2525.25\text{ N} \qquad T_{AC} = 2530\text{ N} \blacktriangleleft$$

(*b*)

$$T_{BC} = (1.08800)(2525.25\text{ N})$$

$$= 2747.5\text{ N} \qquad T_{BC} = 2750\text{ N} \blacktriangleleft$$

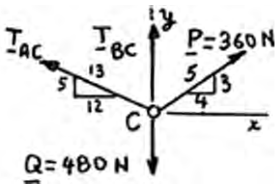


PROBLEM 2.131

Two cables are tied together at C and loaded as shown. Knowing that $P = 360$ N, determine the tension (a) in cable AC, (b) in cable BC.

SOLUTION

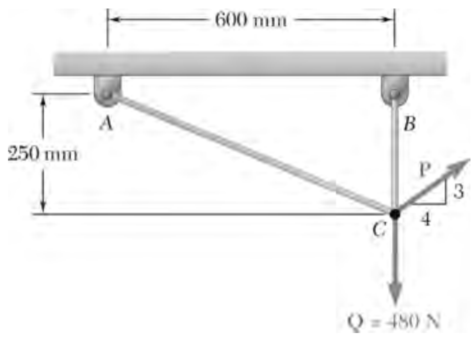
Free Body: *C*



(a) $\Sigma F_x = 0: \quad -\dfrac{12}{13}T_{AC} + \dfrac{4}{5}(360 \text{ N}) = 0$ $\qquad T_{AC} = 312 \text{ N}$ ◀

(b) $\Sigma F_y = 0: \quad \dfrac{5}{13}(312 \text{ N}) + T_{BC} + \dfrac{3}{5}(360 \text{ N}) - 480 \text{ N} = 0$

$T_{BC} = 480 \text{ N} - 120 \text{ N} - 216 \text{ N}$ $\qquad T_{BC} = 144 \text{ N}$ ◀

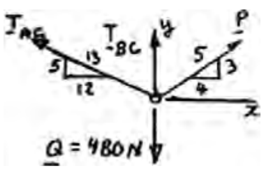


PROBLEM 2.132

Two cables are tied together at C and loaded as shown. Determine the range of values of P for which both cables remain taut.

SOLUTION

Free Body: *C*



$$\Sigma F_x = 0: \quad -\frac{12}{13}T_{AC} + \frac{4}{5}\mathbf{P} = 0$$

$$T_{AC} = \frac{13}{15}P \qquad (1)$$

$$\Sigma F_y = 0: \quad \frac{5}{13}T_{AC} + T_{BC} + \frac{3}{5}P - 480\text{ N} = 0$$

Substitute for T_{AC} from (1):

$$\left(\frac{5}{13}\right)\left(\frac{13}{15}\right)P + T_{BC} + \frac{3}{5}P - 480\text{ N} = 0$$

$$T_{BC} = 480\text{ N} - \frac{14}{15}P \qquad (2)$$

From (1), $T_{AC} > 0$ requires $P > 0$.

From (2), $T_{BC} > 0$ requires $\frac{14}{15}P < 480\text{ N}, \quad P < 514.29\text{ N}$

Allowable range: $0 < P < 514\text{ N}$ ◀

PROBLEM 2.133

A force acts at the origin of a coordinate system in a direction defined by the angles $\theta_x = 69.3°$ and $\theta_z = 57.9°$. Knowing that the *y* component of the force is –870 N, determine (*a*) the angle θ_y, (*b*) the other components and the magnitude of the force.

SOLUTION

(*a*) To determine θ_y, use the relation

$$\cos^2\theta_y + \cos^2\theta_y + \cos^2\theta_z = 1$$

or

$$\cos^2\theta_y = 1 - \cos^2\theta_x - \cos^2\theta_y$$

Since $F_y < 0$, we must have $\cos\theta_y < 0$

$$\cos\theta_y = -\sqrt{1-\cos^2 69.3° - \cos^2 57.9°}$$

$$= -0.76985 \qquad \theta_y = 140.3° \blacktriangleleft$$

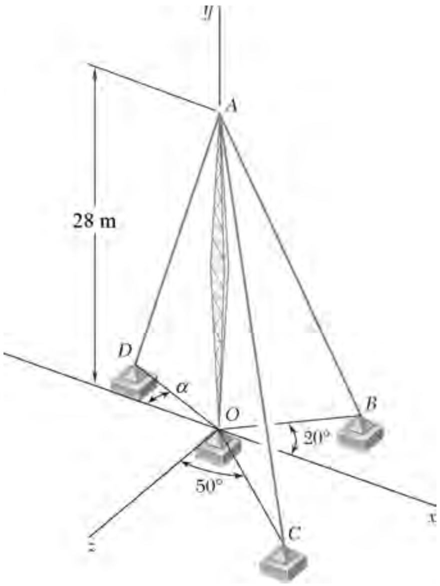
(*b*)

$$F = \frac{F_y}{\cos\theta_y} = \frac{-870\text{ N}}{-0.76985} = 1130.09\text{ N} \qquad F = 1130\text{ N} \blacktriangleleft$$

$$F_x = F\cos\theta_x = (1130.09\text{ N})\cos 69.3° \qquad F_x = 399\text{ N} \blacktriangleleft$$

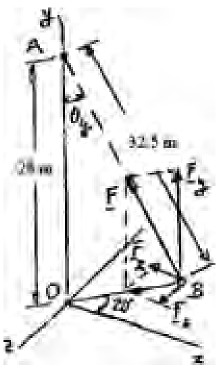
$$F_z = F\cos\theta_z = (1130.09\text{ N})\cos 57.9° \qquad F_z = 601\text{ N} \blacktriangleleft$$

PROBLEM 2.134



Cable AB is 32.5 m long, and the tension in that cable is 20 kN. Determine (*a*) the *x*, *y*, and *z* components of the force exerted by the cable on the anchor *B*, (*b*) the angles θ_x, θ_y, and θ_z defining the direction of that force.

SOLUTION



From triangle AOB:

$$\cos\theta_y = \frac{28 \text{ m}}{32.5 \text{ m}}$$
$$= 0.86154$$
$$\theta_y = 30.51°$$

(a) $$F_x = -F \sin\theta_y \cos 20°$$
$$= -(20000 \text{ N}) \sin 30.51° \cos 20° = 9541.43 \text{ N}$$

$$F_x = -9.54 \text{ kN} \blacktriangleleft$$

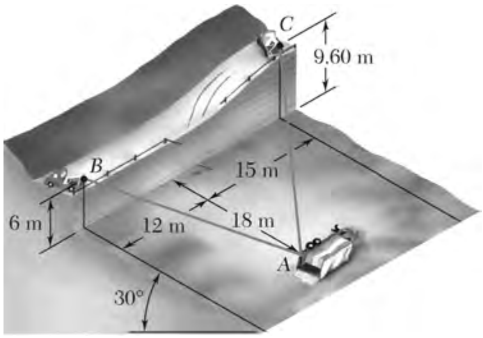
$$F_y = +F \cos\theta_y = (20000 \text{ N})(0.86154) = 17231 \text{ N} \qquad F_y = +17.23 \text{ kN} \blacktriangleleft$$

$$F_z = +(20000 \text{ N}) \sin 30.51° \sin 20° = 3472.8 \text{ N} \qquad F_z = +3.47 \text{ kN} \blacktriangleleft$$

(b) $$\cos\theta_x = \frac{F_x}{F} = \frac{-9541.43 \text{ N}}{20000 \text{ N}} = -0.4771 \qquad \theta_x = 118.5° \blacktriangleleft$$

From above: $\theta_y = 30.51°$ $$\theta_y = 30.5° \blacktriangleleft$$

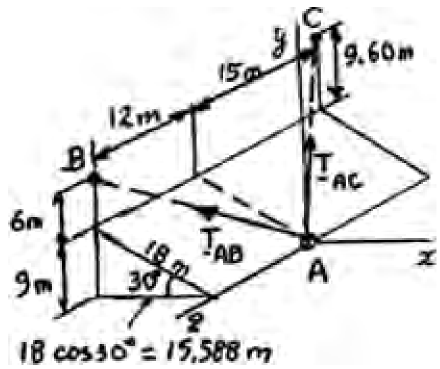
$$\cos\theta_z = \frac{F_z}{F} = +\frac{3472.8 \text{ N}}{20000 \text{ N}} = +0.1736 \qquad \theta_z = 80.0° \blacktriangleleft$$



PROBLEM 2.135

In order to move a wrecked truck, two cables are attached at A and pulled by winches B and C as shown. Knowing that the tension is 10 kN in cable AB and 7.5 kN in cable AC, determine the magnitude and direction of the resultant of the forces exerted at A by the two cables.

SOLUTION



$$\overrightarrow{AB} = -15.588\mathbf{i} + 15\mathbf{j} + 12\mathbf{k}$$

$$AB = 24.739 \text{ m}$$

$$\overrightarrow{AC} = -15.588\mathbf{i} + 18.60\mathbf{j} - 15\mathbf{k}$$

$$AC = 28.530 \text{ m}$$

$$\mathbf{T}_{AB} = T_{AB}\lambda_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB}$$

$$\mathbf{T}_{AB} = (10 \text{ kN})\frac{-15.588\mathbf{i} + 15\mathbf{j} + 12\mathbf{k}}{24.739}$$

$$\mathbf{T}_{AB} = (6.301 \text{ kN})\mathbf{i} + (6.063 \text{ kN})\mathbf{j} + (4.851 \text{ kN})\mathbf{k}$$

$$\mathbf{T}_{AC} = T_{AC}\lambda_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC}(7.5 \text{ kN})\frac{-15.588\mathbf{i} + 18.60\mathbf{j} - 15\mathbf{k}}{28.530}$$

$$\mathbf{T}_{AC} = -(4.098 \text{ kN})\mathbf{i} + (4.890 \text{ kN})\mathbf{j} - (3.943 \text{ kN})\mathbf{k}$$

$$\mathbf{R} = \mathbf{T}_{AB} + \mathbf{T}_{AC} = -(10.399 \text{ kN})\mathbf{i} + (10.953 \text{ kN})\mathbf{j} + (0.908 \text{ kN})\mathbf{k}$$

$$R = \sqrt{(10.399)^2 + (10.953)^2 + (0.908)^2}$$

$$= 15.130 \text{ kN}$$

$R = 15.13$ kN ◀

$$\cos\theta_x = \frac{R_x}{R} = \frac{-10.399 \text{ kN}}{15.130 \text{ kN}} = -0.6873$$

$\theta_z = 133.4°$ ◀

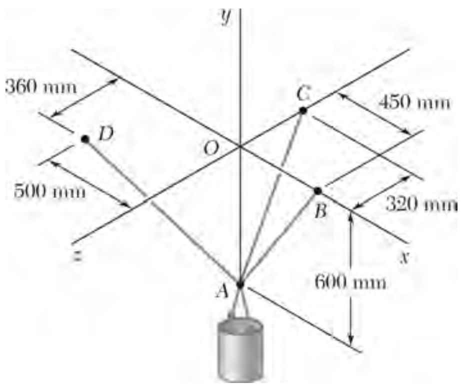
$$\cos\theta_y = \frac{R_y}{R} = \frac{10.953 \text{ kN}}{15.130 \text{ kN}} = 0.7239$$

$\theta_y = 43.6°$ ◀

$$\cos\theta_z = \frac{R_z}{R} = \frac{0.908 \text{ kN}}{15.130 \text{ kN}} = 0.0600$$

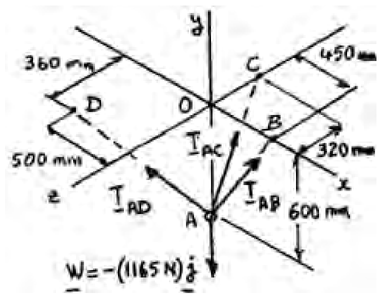
$\theta_z = 86.6°$ ◀

PROBLEM 2.136



A container of weight $W = 1165$ N is supported by three cables as shown. Determine the tension in each cable.

SOLUTION



Free Body: ***A*** $\quad \Sigma\mathbf{F} = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + \mathbf{W} = 0$

$$\overrightarrow{AB} = (450 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j}$$

$$AB = 750 \text{ mm}$$

$$\overrightarrow{AC} = (600 \text{ mm})\mathbf{j} - (320 \text{ mm})\mathbf{k}$$

$$AC = 680 \text{ mm}$$

$$\overrightarrow{AD} = (500 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j} + (360 \text{ mm})\mathbf{k}$$

$$AD = 860 \text{ mm}$$

We have:

$$\mathbf{T}_{AB} = T_{AB}\lambda_{AB} = T_{AB}\frac{\overrightarrow{AB}}{AB} = \left(\frac{450}{750}\mathbf{i} + \frac{600}{750}\mathbf{j}\right)T_{AB}$$

$$= (0.6\mathbf{i} + 0.8\mathbf{j})T_{AB}$$

$$\mathbf{T}_{AC} = T_{AC}\lambda_{AC} = T_{AC}\frac{\overrightarrow{AC}}{AC} = \left(\frac{600}{680}\mathbf{j} - \frac{320}{680}\mathbf{k}\right)\mathrm{T}_{AC} = \left(\frac{15}{17}\mathbf{j} - \frac{8}{17}\mathbf{k}\right)\mathrm{T}_{AC}$$

$$\mathbf{T}_{AD} = T_{AD}\lambda_{AD} = T_{AD}\frac{\overrightarrow{AD}}{AD} = \left(-\frac{500}{860}\mathbf{i} + \frac{600}{860}\mathbf{j} + \frac{360}{860}\mathbf{k}\right)T_{AD}$$

$$\mathbf{T}_{AD} = \left(-\frac{25}{43}\mathbf{i} + \frac{30}{43}\mathbf{j} + \frac{18}{43}\mathbf{k}\right)T_{AD}$$

Substitute into $\Sigma\mathbf{F} = 0$, factor $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$, and set their coefficient equal to zero:

$$0.6T_{AB} - \frac{25}{43}T_{AD} = 0 \qquad T_{AB} = 0.96899T_{AD} \tag{1}$$

$$0.8T_{AB} + \frac{15}{17}T_{AC} + \frac{30}{43}T_{AD} - 1165 \text{ N} = 0 \tag{2}$$

$$-\frac{8}{17}T_{AC} + \frac{18}{43}T_{AD} = 0 \qquad T_{AC} = 0.88953T_{AD} \tag{3}$$

PROBLEM 2.136 (Continued)

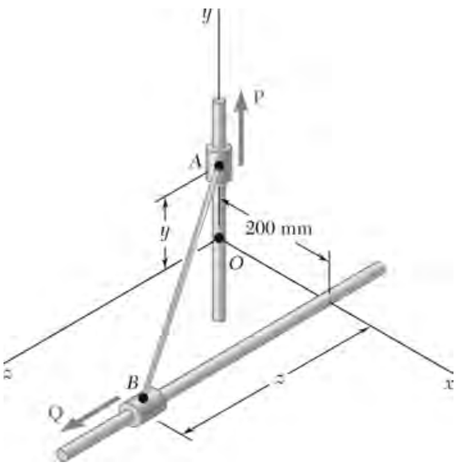
Substitute for T_{AB} and T_{AC} from (1) and (3) into (2):

$$\left(0.8\times 0.96899+\frac{15}{17}\times 0.88953+\frac{30}{53}\right)T_{AD}-1165\text{ N}=0$$

$$2.2578T_{AD}-1165\text{ N}=0 \qquad T_{AD}=516\text{ N} \blacktriangleleft$$

From (1): $T_{AB}=0.96899(516\text{ N})$ $\qquad T_{AB}=500\text{ N} \blacktriangleleft$

From (3): $T_{AC}=0.88953(516\text{ N})$ $\qquad T_{AC}=459\text{ N} \blacktriangleleft$



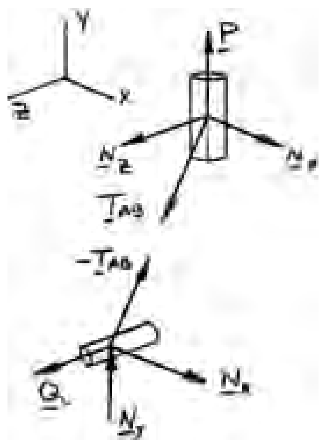
PROBLEM 2.137

Collars A and B are connected by a 525-mm-long wire and can slide freely on frictionless rods. If a force $\mathbf{P} = (341\text{ N})\mathbf{j}$ is applied to collar A, determine (a) the tension in the wire when $y = 155$ mm, (b) the magnitude of the force $\mathbf{Q}$ required to maintain the equilibrium of the system.

SOLUTION

For both Problems 2.137 and 2.138:

Free Body Diagrams of Collars:



$$(AB)^2 = x^2 + y^2 + z^2$$

Here $\quad (0.525\text{ m})^2 = (.20\text{ m})^2 + y^2 + z^2$

or $\quad y^2 + z^2 = 0.23563\text{ m}^2$

Thus, why y given, z is determined,

Now
$$\lambda_{AB} = \frac{\overrightarrow{AB}}{AB}$$
$$= \frac{1}{0.525\text{ m}}(0.20\mathbf{i} - y\mathbf{j} + z\mathbf{k})\text{m}$$
$$= 0.38095\mathbf{i} - 1.90476y\mathbf{j} + 1.90476z\mathbf{k}$$

Where y and z are in units of meters, m.

From the F.B. Diagram of collar A: $\quad \Sigma\mathbf{F} = 0: \quad N_x\mathbf{i} + N_z\mathbf{k} + P\mathbf{j} + T_{AB}\lambda_{AB} = 0$

Setting the $\mathbf{j}$ coefficient to zero gives: $\quad P - (1.90476y)T_{AB} = 0$

With $\quad P = 341\text{ N}$

$$T_{AB} = \frac{341\text{ N}}{1.90476y}$$

Now, from the free body diagram of collar B: $\quad \Sigma\mathbf{F} = 0: \quad N_x\mathbf{i} + N_y\mathbf{j} + Q\mathbf{k} - T_{AB}\lambda_{AB} = 0$

Setting the $\mathbf{k}$ coefficient to zero gives: $\quad Q - T_{AB}(1.90476z) = 0$

And using the above result for T_{AB} we have $\quad Q = T_{AB}z = \dfrac{341\text{ N}}{(1.90476)y}(1.90476z) = \dfrac{(341\text{ N})(z)}{y}$

PROBLEM 2.137 (Continued)

Then, from the specifications of the problem, $y = 155 \text{ mm} = 0.155 \text{ m}$

$$z^2 = 0.23563 \text{ m}^2 - (0.155 \text{ m})^2$$
$$z = 0.46 \text{ m}$$

and

(*a*)
$$T_{AB} = \frac{341 \text{ N}}{0.155(1.90476)}$$
$$= 1155.00 \text{ N}$$

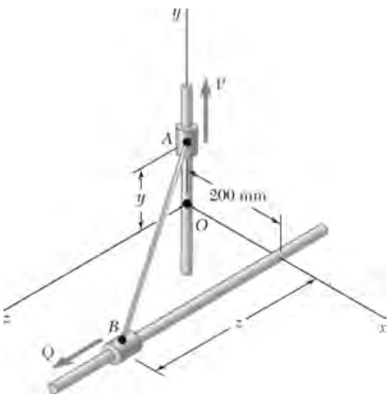
or $T_{AB} = 1.155 \text{ kN}$ ◀

and

(*b*)
$$Q = \frac{341 \text{ N}(0.46 \text{ m})(0.866)}{(0.155 \text{ m})}$$
$$= (1012.00 \text{ N})$$

or $Q = 1.012 \text{ kN}$ ◀

PROBLEM 2.138



Solve Problem 2.137 assuming that $y = 275$ mm.

PROBLEM 2.137 Collars *A* and *B* are connected by a 525-mm-long wire and can slide freely on frictionless rods. If a force $\mathbf{P} = (341\text{ N})\mathbf{j}$ is applied to collar *A*, determine (*a*) the tension in the wire when $y = 155$ mm, (*b*) the magnitude of the force **Q** required to maintain the equilibrium of the system.

SOLUTION

From the analysis of Problem 2.137, particularly the results:

$$y^2 + z^2 = 0.23563\text{ m}^2$$

$$T_{AB} = \frac{341\text{ N}}{1.90476y}$$

$$Q = \frac{341\text{ N}}{y}z$$

With $y = 275\text{ mm} = 0.275\text{ m},$ we obtain:

$$z^2 = 0.23563\text{ m}^2 - (0.275\text{ m})^2$$

$$z = 0.40\text{ m}$$

and

(*a*) $$T_{AB} = \frac{341\text{ N}}{(1.90476)(0.275\text{ m})} = 651.00$$

or $$T_{AB} = 651\text{ N} \blacktriangleleft$$

and

(*b*) $$Q = \frac{341\text{ N}(0.40\text{ m})}{(0.275\text{ m})}$$

or $$Q = 496\text{ N} \blacktriangleleft$$