

Exercise 4.4

For the following differential equations, check whether the equation is exact and obtain its general solution.

1. $(1 + e^x) dx + y dy = 0.$
2. $y dx + x(1 + y) dy = 0.$
3. $2 \cosh x dx + \sinh x dy = 0.$
4. $\sinh x \cos y dx - \cosh x \sin y dy = 0.$
5. $(3x^2y + (y/x)) dx + (x^3 + \ln x) dy = 0.$
6. $(xe^{xy} + 2y) dy + ye^{xy} dx = 0.$
7. $x dy + 2y dx = xy dy.$
8. $x dy - y dx = e^y(x^2 + y^2) dy.$
9. $x dx + y dy = 2y(x^2 + y^2) dy.$
10. $x dy - y dx + y^2 dx = 0.$
11. $y(1 + 6xy) dx + (4y - x) dy = 0.$
12. $(2x + e^y) dx + xe^y dy = 0.$
13. $(1 + x^2) dy + 2xy dx = 0.$
14. $2xy dx + (x^2 + 1) dy = 0.$
15. $(e^{2y} + 1) \cos x dx + 2e^{2y} \sin x dy = 0.$

Under what conditions, the following differential equations are exact?

16. $xy^3 dx + ax^2y^2 dy = 0.$
17. $[f(x) + g(y)] dx + [h(x) + k(y)] dy = 0.$
18. $(ax + y) dx + (kx + by) dy = 0.$
19. $(a \sinh x \cos y + b \cosh x \sin y) dx + (c \sinh x \cos y + d \cosh x \sin y) dy = 0.$

Find the integrating factor and hence solve the following differential equations

20. $(y - 1) dx - x dy = 0.$
21. $dx + e^{(y-x)} dy = 0.$
22. $(x^3 + y^3 + 1) dx + xy^2 dy = 0.$
23. $(4y + x^3) dx + x dy = 0.$