$$A = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix} \checkmark$$

(a) 1

(b) 2

✓ (c) 3

(d) 4

Q.

$$\begin{aligned} x - 2y + 3t &= -2 \\ 2x + y + z + t &= -4 \\ 4x - 3y + z + 7t &= -8 \end{aligned}$$

Augmented Matrix $= \left[\begin{array}{cccc|c} 1 & -2 & 0 & 3 & -2 \\ 2 & 1 & 1 & 1 & -4 \\ 4 & -3 & 1 & 7 & -8 \end{array}\right]$

operate $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 4R_1$

$$\sim \left[\begin{array}{cccc|c} 1 & -2 & 0 & 3 & -2 \\ 0 & 5 & 1 & -5 & 0 \\ 0 & 5 & 1 & -5 & 0 \end{array}\right]$$

$R_3 \to R_3 - R_2$

$$\sim \left[\begin{array}{cccc|c} 1 & -2 & 0 & 3 & -2 \\ 0 & 5 & 1 & -5 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \quad \text{— (1)}$$

$\rho(K) = 2$, $\rho(A) = 2$

$\rho(A) = \rho(K) = 2 <$ unknown of unknowns

The system is consistent & has infinite number of solutions.

from ① $\begin{cases} x - 2y + 3t = -2 & — ② \\ 5y + z - 5t = 0 & — ③ \end{cases}$

let $z = a$, $t = b$

$4 - 2 = 2$

∴ from ② $x - 2y = -2 - 3b$

$5y = -a + 5b$

$$\boxed{y = \frac{1}{5}(-a + 5b)}$$

∴ $x = 2y - 2 - 3b$

$= \frac{2}{5}(-a + 5b) - 2 - 3b$

$= -\frac{2a}{5} + \frac{\cancel{10}^{2} b}{\cancel{5}} - 2 - 3b$

$$\boxed{x = -2 - \frac{2a}{5} - b}$$

Q Investigate the values of λ & μ so that the equations

$$2x + 3y + 5z = 9$$
$$7x + 3y - 2z = 8$$
$$2x + 3y + \lambda z = \mu$$

have ① No solution ② a unique solution ③ an infinite number of solution

Sol →

$$\text{Augmented} = \left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 7 & 3 & -2 & 8 \\ 2 & 3 & \lambda & \mu \end{array}\right]$$

$R_2 \to R_2 - \frac{7}{2}R_1$, $R_3 \to R_3 - R_1$

$$\sim \left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 0 & -\frac{15}{2} & -\frac{39}{2} & -\frac{47}{2} \\ 0 & 0 & \lambda - 5 & \mu - 9 \end{array}\right]$$

① If $\lambda - 5 = 0$ & $\mu - 9 \neq 0$ i.e $\mu \neq 9$

$\mu - 9 = 0$

① If $\lambda - 5 = 0$ & $\mu - 9 \neq 0$ ie $\mu \neq 9$

$\rho(A) = 2$ & $\rho(K) = \underline{3}$

$\mu - 9 = 0$
$\mu - 9 \neq 3$

✓ Here $\rho(A) \neq \rho(K)$ if $\lambda = 5$ & $\mu \neq 9$

i.e. the system will have no solution

② If $\lambda - 5 \neq 0$ & μ can have any real value

ie $\lambda \neq 5$

$\rho(A) = \rho(K) =$ unknowns $= 3$

. the system will have unique solution

③ For infinite number of solution

$\rho(A) = \rho(K) <$ Number of unknowns

∴ For $\lambda = 5$ & $\mu = 9$

$\rho(A) = \rho(K) = 2 <$ Number of unknowns [3]

$$a_1x + b_1y + c_1z = 0$$
$$a_2x + b_2y + c_2z = 0$$
$$a_3x + b_3y + c_3z = 0$$

✓ $K = \begin{bmatrix} a_1 & b_1 & c_1 & 0 \\ a_2 & b_2 & c_2 & 0 \\ a_3 & b_3 & c_3 & 0 \end{bmatrix}$

✓ $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$

① $\rho(A) = \rho(K)$

i) $\rho(A) = \rho(K) =$ No of unknowns ✓

$\neq$ No of unknowns ✓