PERCENTAGE

Content

1. Introduction

What is Percentage ?

Percentage

per – cent (100)

Why percentage?

To convert any base values to **100**

Application

The concept of percentage is applied in many other important topics like **Profit and Loss, Data Interpretation, Simple and Compound Interest** etc.

The shortcuts which you are going to learn will be very useful if it is applied in these topics.

Conversion of a Fraction into Percentage- To convert a fraction into percentage, multiply the fraction by 100 and put % sign.

Ex. If fraction is 1/4 then 1/4 x 100 = 25%

  If fraction is 1/8 then 1/8 x 100 = 12.5%

Conversion of a Percentage into Fraction- To convert a percentage into fraction, replace the % sign with 1/ 100 and reduce the fraction into simplest form.

Ex. 10% = 10/100 = 1/10

  75% = 75/100 = 3/4

  300% = 300/100 = 3

Relation between Fraction and Percentage

1%	=	1/100
2%	=	1/50
4%	=	1/25
5%	=	1/20
8.33%	=	1/12
10%	=	1/10
12.50%	=	1/8
16.67%	=	1/6
20%	=	1/5

25%	=	1/4
33.33%	=	1/3
37.50%	=	3/8
40%	=	2/5
50%	=	1/2
60%	=	3/5
62.50%	=	5/8
66.67%	=	2/3
75%	=	3/4

80%	=	4/5
83.33%	=	5/6
87.50%	=	7/8
100%	=	1
120%	=	6/5
125%	=	5/4
133.33%	=	4/3
150%	=	3/2
175%	=	7/4

2. Shortcuts

Shortcut 1: Splitting of values

a) 20% of 80 = ?
  100% of 80 = 80
   10% of 80 = 8
  20% of 80 = 16

Try 30% of 60

b) 15% of 80 = ?
   10% of 80= 8
    5% of 80 = 4
  15% of 80 = 12

Try 15% of 60

Shortcut 2:

How do you solve **12.5% of 80** = ?
10% + 2% + 0.5% ?

How much time will the following question take?
12.5% of 8.8 = ?

Now this
16.66% of 3.6 = ?

12.5% of 80 = ?
  1 = 100%
1/2 = 50%
1/4 = 25%
1/8 = 12.5%

∴ 12.5% of 80 = 1/8 of 80
                            = **10**

Similarly **12.5% of 8.8** = 1/8 of 8.8
$$= 1.1$$
16.66% of 3.6 = 1/6 of 3.6
$$= 0.6$$
Lets learn to convert few more fraction values

1   = 100%
1/2 = 50%
1/3 = 33.33%
1/4 = 25% (half of ½)
1/5 = 20%
1/6 = 16.66% (half of 1/3)
1/7 = 14.28%
1/8 = 12.5% (half of ¼)
1/9 = 11.11% (1/3ʳᵈ of 1/3)
1/10 = 10%
1/11 = 9.09%

Note: 1/9 x will be in the multiples of 11
1/11 x will be in the multiples of 9

Example :

Try 3.6% of 133.33

Solution for the last question:

3.6% of 133.33 = 133.33% of 3.6

$$= 100\% \text{ of } 3.6 + 33.33\% \text{ of } 3.6$$
$$= \quad 3.6 \quad + \quad 1/3^{rd} \text{ of } 3.6$$
$$= \quad 3.6 \quad + \quad 1.2$$
$$= \quad 4.8$$

3. Percentage increase and decrease

Increase and decrease

Example:

25
20

year 1   year 2
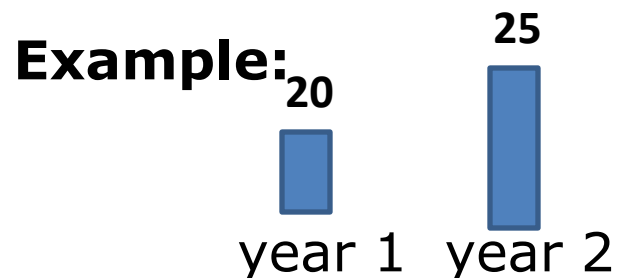
What is the percentage increase from the 1st year to the 2nd ?

What is the increase?   5
From where it is increasing? 20

Percentage increase = 5/20 * 100
$$= ¼ * 100$$
$$= 25\%$$

What is the percentage decrease from the 2nd year to the 1st?

What is the decrease?   5
From where it is decreasing? 25

Percentage decrease = 5/25 * 100
                           = 1/5 * 100
                           = 20%

Example 2: Ram makes $50 a week from his job. He earns a raise and now makes $60 a week. What is the percent increase?
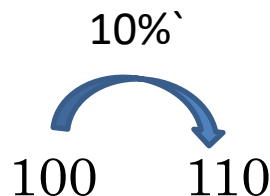
A. 16.66%
B. 20%
C. 25%
D. 50%

Solution:

Percentage increase = 10/50 * 100
$$= 20\%$$

Increase/ decrease shortcut:

If there are 100 chocolates with me and it is increased by 10%, then what will be the total number of chocolates now?
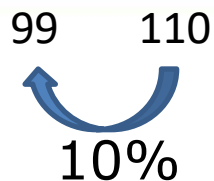
110?
Yes

10%`

100   110

Now from 110 to get back to 100, what should be the percentage decrease?
10% again?

No
Because from 110 if 10% is decreased it will become 99. (-10% of 110 = -11)

99   110

10%

To find the decrease easily lets learn a shortcut.

Between two values if the **increase** is **1/n**, then the **decrease** will be **1/n+1**.

∴ Increase % = 10% = **1/10**
   Decrease %   = **1/11** = 9.09%

∴ From 110 to 100 the decrease % should be 9.09%

Similarly for the previous question if you observe

Increase % = 25% = **1/4**
Decrease %   = **1/5** = 20%

Example 3: If A's height is 33.33% less than that of B, how much percent B's height is more than that of A?

A) 20%
B) 25%
C) 33.33%
D) 50%

Solution:

33.33% decrease = 1/3
∴   Increase = 1/2
                = 50%

Important application of 1/n shortcut:

If the price of a commodity increases by 1/n, then the consumption should be reduced by 1/n+1 to make the expenditure same.

Expenditure = Price * Consumption

                    1/n   1/n+1

This can be applied in many other topics like time speed and distance in the formula D = S * T etc.

Example 4: A person buys 750 litres of milk every year and the price of the milk is Rs 20 per litre. If the price of the milk increases to Rs 25 per litre next year, what amount of milk he/she can buy for the same expense as in the previous year?

A. 500 litres
B. 562.50 litres
C. 600 litres
D. 9000 litres

Solution: Price increase = 5/20

= ¼

∴ Consumption decrease = 1/5

1/5th of 750 litres = 150 litres should be decreased

Consumption = 750 – 150 litres
= 600 litres

Successive increase/decrease:

Example: A car is moving at some constant speed. At first it increases its speed by 25% and then again it increases its speed by 20%. What is the overall percentage increase.

Method 1:

Initial speed of the car   =   100
Speed of the car after 1st increase =   100 + 25 = 125
Speed of the car after 2nd increase =   125 + 25 = 150

Initial speed = 100
Final speed =   150

Percentage increase = 50%

Method 2:

Assume the initial speed of the car as 100kmph

Initial speed of the car   = 100
Speed of the car after the 1st increase = 125
Speed of the car after the 2nd increase = 150

Initial speed = 100
Final speed = 150

Percentage increase = 50%

Method 3: Shortcut

If the 1st increase/ decrease is **a%** and the 2nd increase/decrease is **b%** the overall increase/decrease % will be

$$a + b + ab/100 \ \%$$

In this question **a = 25%** and **b= 20%**
Overall increase/decrease = 25 + 20 + (25)(20)/100
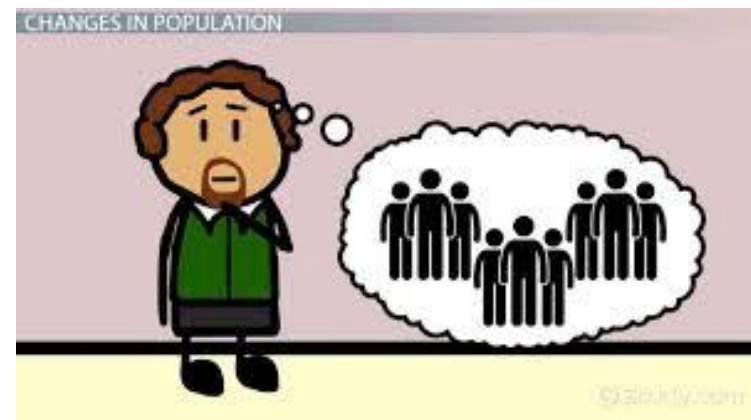= 25 + 20 + 5
= **50 %**

Note: If a or b is increase, then include +ve sign
If a or b is decrease, then include -ve sign.

Note : The final answer will be in percentage

Example 5: A city's population was 10,000 at the end of 2008. In 2009, it increased by 25% and in 2010, it decreased by 8%. What was the city's population at the end of 2010?

A. 8500

B. 11500

C. 11700

D. 13333

Solution: $25 + (-8) + (25)(-8)/100$ %

$\qquad = 25 - 8 - 200/100$ %

$\qquad = 25 - 8 - 2$ %

$\qquad = 15$ %

Important application of a + b + ab/100 % shortcut:

If the price of a commodity increases/decreases by a% and the consumption increases/reduces by b% then the expenditure will increase/decrease by **a + b + ab/100 %**

Expenditure = Price * Consumption

This can be applied in many other topics like time speed and distance in the formula D = S * T etc.

Example 6: Water tax is increased by 20% but its consumption is decreased by 20%. Then, the increase or decrease percentage in the expenditure of the money is:

a) No change
b) 5% decrease
c) 4% increase
d) 4% decrease

Solution: 20 + (-20) + (20) (-20)/100 %

$$= 20 - 20 - 400/100 \%$$

$$= 20 - 20 - 4 \%$$

$$= -4 \%$$

$$= 4\% \text{ decrease}$$

Results on Population Increase/Decrease

- If the original population is P and increase in population is at the rate of r % every year then, the population after n years will be = $P(1+r/100)^n$

- Similarly, If the original population is P and decrease in population is at the rate of r % every year then the population after n years will be =

- $P(1-r/100)^n$

- Let the population of the town be P now and suppose it increases at the rate of R% per annum, then = Population n years ago = $P / [1+(R/100)]^n$

Results on Depreciation

- Let the present value of a machine be P. Suppose it depreciates at the rate R% per annum. Then,

- Value of the machine after n years = $P[1-(R/100)]^n$

- Value of the machine n years ago = $P / [1-(R/100)]^n$

Solved examples

Q.1 Evaluate 35% of 280 + 80% of 140

A. 70
B. 140
C. 210
 D. 280

Solution:

35% of 280 + 80% of 140 = 35% of 280 + 40% of 28~
                        = 75% of 280
                        = ¾ of 280
                        = 210

Q.2 If 30% of a = b, then b% of 30 is the same as :

A. 4% of a
B. 6% of a
C. 8% of a
D. 9% of a

Solution:

Given : b = 30% of a-------1
To find : b% of 30
Divide by 100 and multiply by 30 eq 1

b% of 30 = 9% of a

Q.3 A fruit seller had some oranges. He sells 40% oranges and still has 420 oranges. How many oranges he had originally?

A. 280
B. 630
C. 700
D. 1050

Solution:

100%

(sold) 40%   60% (remaining)

to buy   to sale

420

∴ 60% = 420 oranges

No of oranges he had = 100% = ?

60% = 420
10% = 70
100% = 700

Q.4 An agent, gets a commission of 2.5% on the sales of cloth. If on a certain day, he gets Rs. 12.50 as commission, the cloth sold through him on that day is worth

A. 250
B. 500
C. 750
D. 1000

Solution:
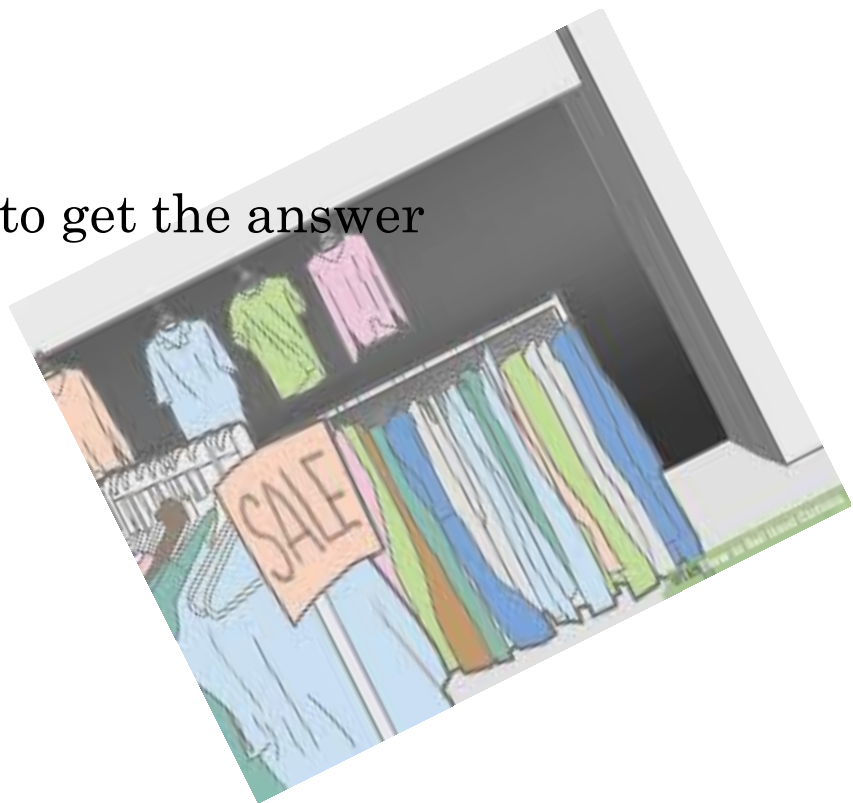
Equate percentage value with the price to get the answer

Percentage commission = 2.5%
Commission in Rs   = Rs 12.50

∴ 2.5% = Rs 12.50
    5% = Rs 25
    1% = Rs 5
100% = Rs 500

Q.5 A student has to obtain 33% of the total marks to pass. He got 125 marks and failed by 40 marks. The maximum marks are-

A. 400
B. 500
C. 600
D. 800

Solution:
Equate percentage value with the marks to get the answer

Percentage Pass mark = 33%
He got 125 marks and need 40 more marks to pass
∴ **Pass mark**   = 125 + 40 = **165**

33% = 165
Maximum marks = 100%

33% = 165
1%   = 5
100% = 500

Q.6 In a test A got 15% of the marks and failed by 7 marks whereas B got 28% and got 32 marks more than the pass mark. What was the pass mark?

A. 45
B. 52
C. 84
D. 300

Solution:

Equate percentage value with the price to get the answer

Percentage of A = 15%
Marks of A   = -7   (Deviation from pass mark)

Percentage of B = 28%
Marks of B   = + 32

Percentage difference b/w A and B = 13%
Marks difference b/w A and B   = 39

∴ 13% = 39 marks
   1% = 3 marks

Now either find 15% and add +7 to get the pass mark or find 28% and subtract 32 to get the pass mark

       1% = 3 marks
      15% = 45 marks
   15% + 7 = 52 marks

Or

       1% = 3 marks
      28% = 84 marks
   28% - 32 = 52 marks

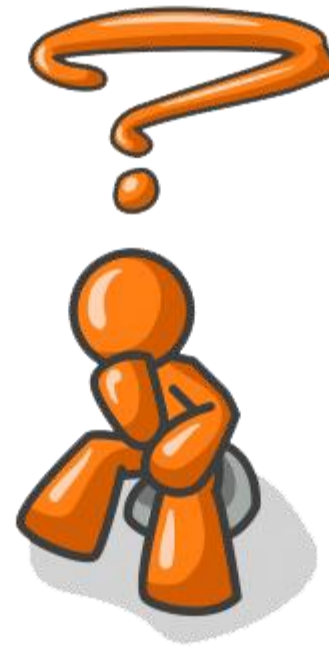# 5) Practice problems

Q. What percentage of numbers from 1 to 70 have 1 or 9 in the unit's digit?
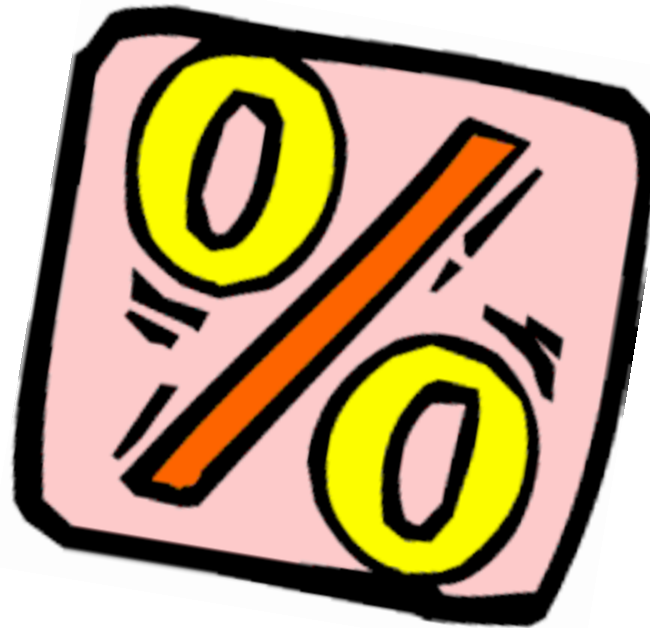
A. 1
B. 20
C. 14
D. 21

- 1,11,21,31,41,51,61,9,19,29,39,49,59,69

- 14 such numbers
- So 14/70   *100 =20%

Q.2   45 x ? = 25% of 900

A.   4
B.   5.2
C.   16
D.   5

- 45 * X = 900 /4
- X=900 / (4 * 45)=5

Q. One fourth of one third of two fifth of a number is 15. What will be 40% of that number?

A. 180
B. 360
C.   90
D. 270

- ¼ (1/3 (2/5 of x)) =15
- X=450
- 40 % of 450=40/100 *450=180

Q. If 15% of 40 is greater than 25% of a number by 2. Find the number.

A. 14
B. 18
C. 16
D. 20

- 15%of 40=15/100   * 40 = 6
- 6-25% of X=2
- 6-X/4   =2
- X=16

Q. If the price of a commodity be raised by 20% then by how much % a house holder reduce his consumption so that the expenditure does not change?

A. 8 1/3%
B. 40%
C. 20 %
D. 16 2/3%

- % increase = 1/n
- % decrease = 1 / n+1

- So Given in question % increase=20%=1/5
- % decrease=1/(5+1)=1/6=(1/6)*100 % =50/3%=16 2 /3 %

Q. Two numbers are 25% and 40% less than the third number. What % is the second of the first?

A. 40%
B. 60%
C. 80 %
D. 20%

- Let third number be 100
- First number =75
- Second number = 60
- So 60 is what % of 75?
- 60/75*100=80%

Q. In an examination it is required to get 65% of the aggregate marks to pass. A student gets 522 marks and is declared failed by 7% marks. What are the maximum aggregate marks a student can get?

A. 900
B. 1000
C. 800
D. 600

- 58%=522
- 1%=9
- 100%=900

THANKS FOR LISTENING