Lecture 23

18 October 2021 00:52

Unit IV: Graphs theory I
Chapter 10

Graph Terminologies

Undirected graph $G = (V, E)$ consists of V, a non-empty set of vertices and E, a set of edges. Each edge is associated with an unordered pair of vertices $\{u, v\}$.

Simple Graph: An undirected graph in which each edge connects two different vertices and where no two edges connect the same pair of vertices, is called simple graph.

Multiple edges. Not allowed

Loops are not allowed

Multigraph: A graph having multiple edges connecting the same vertices are called multigraphs. $\{u, v\}$ is an edge of multiplicity m, when there are m different edges associated to the same unordered pair of vertices $\{u, v\}$.

Loops not allowed.

multiplicity $\{u, v\} = 3$

Pseudographs: Graphs that may include loops, connecting a vertex to itself, and multiple edges connecting the same pair of vertices are called pseudographs.

Directed graph (Digraph) $G = (V, E)$ consists of V, a non-empty set of vertices and E, a set of directed edges. Each directed edge is associated with an ordered pair of vertices. The directed edge associated with the ordered pair (u, v) is said to start at u and end at v.

(u, v)

Initial vertex

Terminal vertex.

Simple Directed Graph: A directed graph that has no loops and has no multiple directed edges is called simple directed graph.

edges is called simple directed graph.

Not allowed.

Directed Multigraph: Directed graph with multiple directed edges from a vertex to other (different or same) is called directed multigraph.

Loops are also allowed

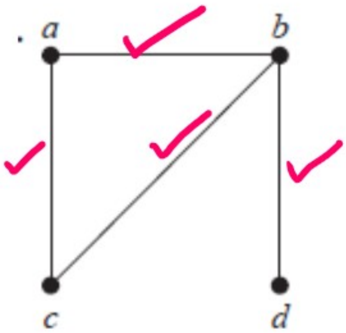
Mixed Graph: A graph with both directed and undirected edges is called mixed graph.

TABLE 1 **Graph Terminology.**

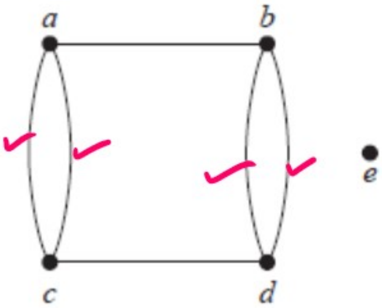
Type	*Edges*	*Multiple Edges Allowed?*	*Loops Allowed?*
Simple graph	Undirected	No	No
Multigraph	Undirected	Yes	No
Pseudograph	Undirected	Yes	Yes
Simple directed graph	Directed	No	No
Directed multigraph	Directed	Yes	Yes
Mixed graph	Directed and undirected	Yes	Yes

A B C D E

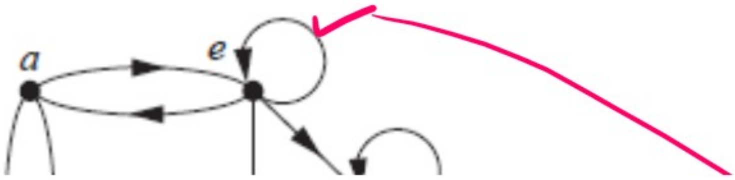
Q1. Classify as Simple/Multigraph/Pseudograph/Simple directed/Directed multigraph/Mixed?



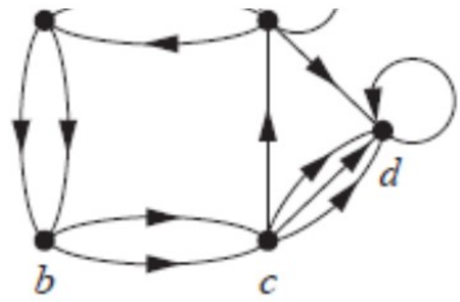
Simple graph.



Multi graph



A. Simple
B. Multigr
C. Pseudog



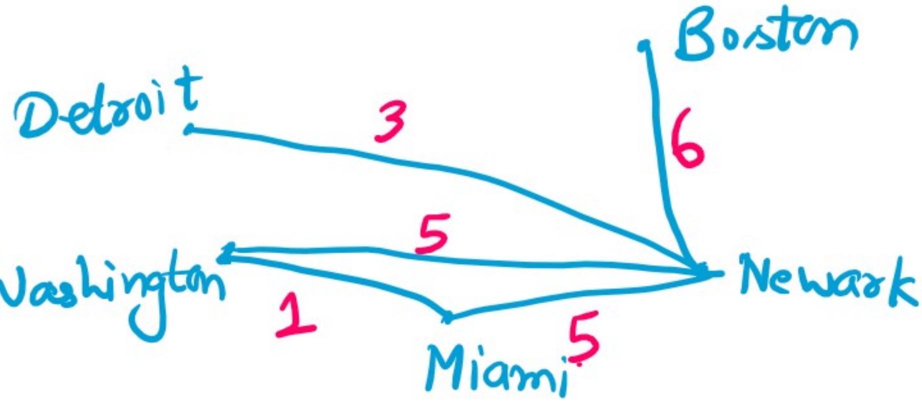
C. Pseudog
D. Simple directed
E. Multi directed

Q2.

Draw graph models, stating the type of graph (from Table 1) used, to represent airline routes where every day there are four flights from Boston to Newark, two flights from Newark to Boston, three flights from Newark to Miami, two flights from Miami to Newark, one flight from Newark to Detroit, two flights from Detroit to Newark, three flights from Newark to Washington, two flights from Washington to Newark, and one flight from Washington to Miami, with

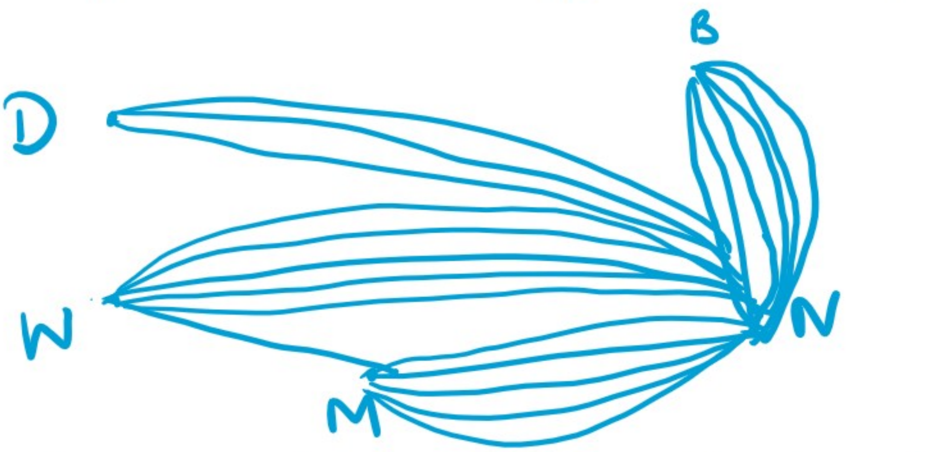
a) an edge between vertices representing cities that have a flight between them (in either direction).

Simple.



b) an edge between vertices representing cities for each flight that operates between them (in either direction).

Multi graph



c) an edge between vertices representing cities for each flight that operates between them (in either direction),

plus a loop for a special sightseeing trip that takes off and lands in Miami.

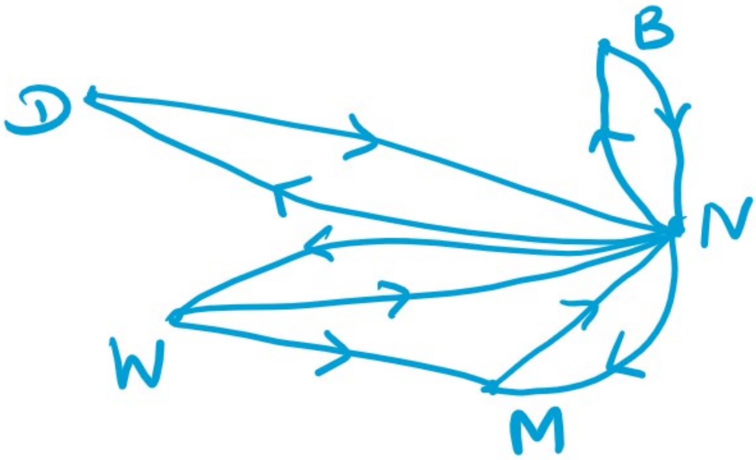
Pseudo

Same as (b) part

d) an edge from a vertex representing a city where a flight starts to the vertex representing the city where it ends.

e) an edge for each flight from a vertex representing a city where the flight begins to the vertex representing the city where the flight ends.

Simple directed graph

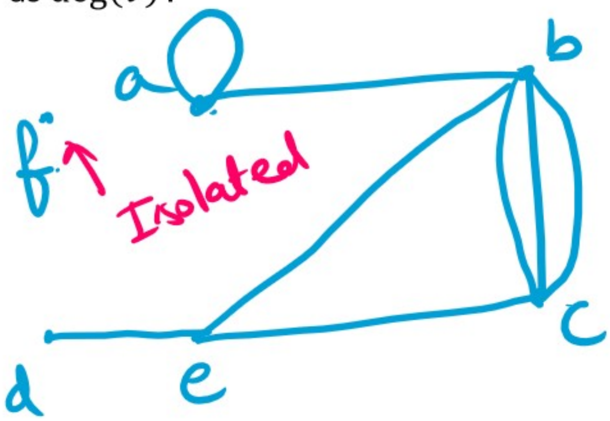


Adjacent vertices: Two vertices u, v in an undirected graph G are called adjacent if u and v are end points of an edge e of G. Such edge is called incident with the vertices u and v, and e is said to connect u and v.

Neighbourhood of a vertex: The set of all neighbours of a vertex v of $G = (V, E)$, denoted by $N(v)$, is called the neighbourhood of v. If A is a subset of V, then $N(A)$ the set of all

Neighbourhood of a vertex: The set of all neighbours of a vertex v of $G = (V, E)$, denoted by $N(v)$, is called the neighbourhood of v. If A is a subset of V, then $N(A)$ the set of all vertices in G that are adjacent to atleast one vertex in A. So $N(A) = \cup_{v \in A} N(v)$.

Degree of Vertex in an undirected graph is the number of edges incident with the vertex, except that ***a loop at a vertex contributes twice to the degree of that vertex.*** It is denoted as $\deg(v)$.



$N(a) = \{a, b\}$

$A = \{b, c\}$

$N(A) = \{a, c, e, b\}$

$\deg(a) = 3$, $\deg(b) = 5$, $\deg(c) = 4$, $\deg(e) = 3$

$\deg(f) = 0$ $\qquad$ $\deg(d) = 1$

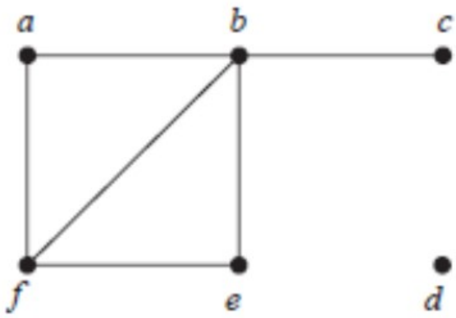
A vertex of degree zero is called Isolated.

A vertex of degree one is called Pendant

Q3.

In Exercises 1–3 find the number of vertices, the number of edges, and the degree of each vertex in the given undirected graph. Identify all isolated and pendant vertices.

1.



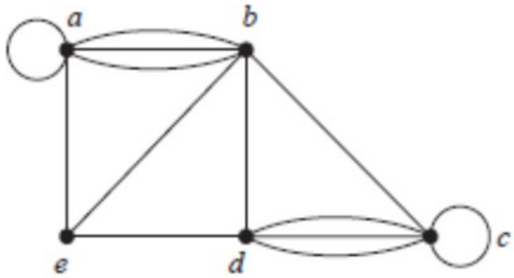
No. of vertices = 6

No of edges = 6

Vertex	degree
a	2

vertex	degree
a	2
b	4
c	1 → Pendant
d	0 → Isolated
e	2
f	3

2.

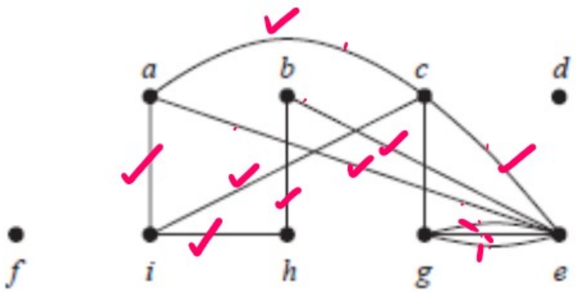


No of vertices = 5

No of edges = 13

Vertex	deg(v)
a	6
b	6
c	6
d	5
e	3

3.



No of vertices = 9

No of edges = 12

deg(a) = 3, deg(b) = 2, deg(c) = 4, deg(d) = 0

deg(e) = 6, deg(f) = 0, deg(g) = 4, deg(h)

deg(i) = 3

Theorem 1: THE HANDSHAKING THEOREM

Let $G = (V, E)$ be an undirected graph with e edges, then $2e = \sum_{v \in V} \deg(v)$

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Twice Edges = sum of degrees

Q4. For the graphs, given in Q3, verify the Handshaking theorem.