Problems on Probability

1. What is the Chance that a leap year selected at random will contain 53 Sundays?

Solution. In a leap year (which consists of 366 days) there are 52 complete weeks and 2 days over. The following are the possible combinations for these two 'over' days:

(i) Sunday and Monday, *(ii)* Monday and Tuesday, *(iii)* Tuesday and Wednesday, *(iv)* Wednesday and Thursday, *(v)* Thursday and Friday, *(vi)* Friday and Saturday, and *(vii)* Saturday and Sunday.

In order that a leap year selected at random should contain 53 Sundays, one of the two 'over" days must be Sunday. Since out of the above 7 possibilities, 2 *viz.*, *(i)* and *(vii)*, are favourable to this event,

$$\therefore \qquad \text{Required probability} = \frac{2}{7}$$

2.A bag contains 3 red, 6 white and 7 blue balls. What is the probability that two balls drawn are white and blue?

Solution. Total number of balls = $3+6+7=16$.

Now, out of 16 balls, 2 can be drawn in ${}^{16}C_2$ ways.

$\therefore$ Exhaustive number of cases = ${}^{16}C_2 = \dfrac{16 \times 15}{2} = 120$.

Out of 6 white balls 1 ball can be drawn in ${}^{6}C_1$ ways and out of 7 blue balls 1 ball can be drawn in ${}^{7}C_1$ ways. Since each of the former cases can be associated with each of the latter cases, total number of favourable cases is : ${}^{6}C_1 \times {}^{7}C_1 = 6 \times 7 = 42$.

$\therefore$ Required probability $= \dfrac{42}{120} = \dfrac{7}{20}$.

3.What is the probability of getting 9 cards of the same suit in one hand at a game of bridge?

Solution. One hand in a game of bridge consists of 13 cards.

$\therefore$ Exhaustive number of cases $= {}^{52}C_{13}$

Number of ways in which, in one hand, a particular player gets 9 cards of one suit are ${}^{13}C_9$ and the number of ways in which the remaining 4 cards are of some other suit are ${}^{39}C_4$. Since there are 4 suits in a pack of cards, total number of favourable cases $= 4 \times {}^{13}C_9 \times {}^{39}C_4$.

$$\therefore \qquad \text{Required probability} = \frac{4 \times {}^{13}C_9 \times {}^{39}C_4}{{}^{52}C_{13}}$$

A committee of 4 people is to be appointed from 3 officers of the production department, 4 officers of the purchase department, two officers of the sales department and 1 chartered accountant. Find the probability of forming the committee in the following manner:

(i) There must be one from each category.

(ii) It should have at least one from the purchase department.

(iii) The chartered accountant must be in the committee.

ANS(i)(A)8/70 (B)9/70 (C)8/15 (D)NONE

Solution. There are 3+4+2+1=10 persons in all and a committee of 4 people can be formed out of them in ${}^{10}C_4$ ways. Hence exhaustive number of cases is

$$ {}^{10}C_4 = \frac{10 \times 9 \times 8 \times 7}{4\ !} = 210 $$

(i) Favourable number of cases for the committee to consist of 4 members, one from each category is :

$$ {}^{4}C_1 \times {}^{3}C_1 \times {}^{2}C_1 \times 1 = 4 \times 3 \times 2 = 24 $$

$$ \therefore \qquad \text{Required probability} = \frac{24}{210} = \frac{8}{70} $$

(ii) P [Committee has at least one purchase officer]

$= 1 - P$ [Committee has no purchase officer]

In order that the committee has no purchase officer, all the 4 members are to be selected from amongst officers of production department, sales department and chartered accountant, *i.e.*, out of 3+2+1=6 members and this can be done in ${}^{6}C_4 = \frac{6 \times 5}{1 \times 2} =$ 15 ways. Hence

$$ P\text{ [Committee has no purchase officer]} = \frac{15}{210} = \frac{1}{14} $$

$$ \therefore \quad P\text{ [Committee has at least one purchase officer]} = 1 - \frac{1}{14} = \frac{13}{14} $$

(iii) Favourable number of cases that the committee consists of a chartered accountant as a member and three others are :

$$1 \times {}^9C_3 = \frac{9 \times 8 \times 7}{1 \times 2 \times 3} = 84 \text{ ways},$$

since a chartered accountant can be selected out of one chartered accountant in only 1 way and the remaining 3 members can be selected out of the remaining $10 - 1 = 9$ persons in ${}^9C_3$ ways. Hence the required probability $= \frac{84}{210} = \frac{2}{5}$.

I5. If the letters of the word ‘REGULATIONS’ be arranged at random, what is the chance that there will be exactly 4 letters between R and E?

Solution. *(a)* The word '*REGULATIONS*' consists of 11 letters. The two letters R and E can occupy ${}^{11}P_2$, *i.e.*, $11 \times 10 = 110$ positions.

The number of ways in which there will be exactly 4 letters between R and E are enumerated below:

(i) R is in the 1st place and E is in the 6th place.

(ii) R is in the 2nd place and E is in the 7th place.

... ...

... ...

... ...

(vi) R is in the 6th place and E is in the 11th place.

Since R and E can interchange their positions, the required number of favourable cases is $2 \times 6 = 12$

$\therefore$ The required probability $= \dfrac{12}{110} = \dfrac{6}{55}$.

If the letters of the word ATTEMPT are written down at random, the chance that all TS are consecutive is…

(a)1/42 (b)6/7 (c)1/7 (d)1/8

The sum of two non-negative quantities is equal to $2n$. Find the chance that their product is not less than $\frac{3}{4}$ times their greatest product.

Solution. Let $x > 0$ and $y > 0$ be the given quantities so that $x + y = 2n$.

We know that the product of two positive quantities whose sum is constant is greatest when the quantities are equal. Thus the product of x and y is maximum when $x = y = n$.

$\therefore$ Maximum product $= n \cdot n = n^2$

Now $P\left[xy \not< \frac{3}{4} n^2\right] = P\left[xy \geq \frac{3}{4} n^2\right] = P\left[x(2n - x) \geq \frac{3}{4} n^2\right]$

$$= P\,[(4x^2 - 8nx + 3n^2) \leq 0]$$

$$= P\,[(2x - 3n)(2x - n) \leq 0]$$

$$= P\left[x \text{ lies between } \frac{n}{2} \text{ and } \frac{3n}{2}\right]$$

$\therefore$ Favourable range $= \frac{3n}{2} - \frac{n}{2} = n$

Total range $= 2n$

$\therefore$ Required probability $= \frac{n}{2n} = \frac{1}{2}$