

## Lecture 25

19 October 2021 10:06

How many edges are there in  $K_n$ ?

$n$  vertices, each vertex is connected with  $(n-1)$

$$\text{Sum of degrees} = n(n-1) = 2(\text{Edges})$$

$$\text{Edges in } K_n = \frac{n(n-1)}{2}$$

$$K_8, \text{ no of edges} = \frac{8(8-1)}{2} = 28 //$$

Q9. How many vertices does a regular graph of degree 4 with 10 edges have?

(A) 20 (B) 10 (C) 5 (D) 15

4-regular, edges = 10,  $n = ?$

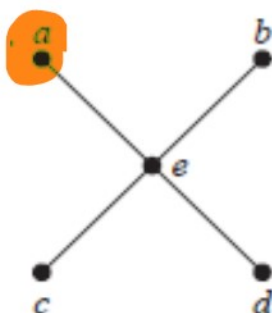
$$\sum \text{degree} = \sum_n 4 = 2(10)$$

$$4n = 20$$

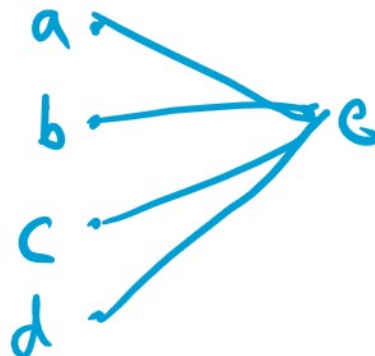
$$n = 5$$

**Bipartite Graph:** A simple graph  $G$  is called bipartite if its vertex set  $V$  can be partitioned into two disjoint sets  $V_1$  and  $V_2$  such that every edge in the graph connects a vertex in  $V_1$  and a vertex in  $V_2$  and no edge in  $G$  connects either two vertices in  $V_1$  or two vertices in  $V_2$ .

The pair  $(V_1, V_2)$  is called a bipartition of  $V$  of  $G$ .



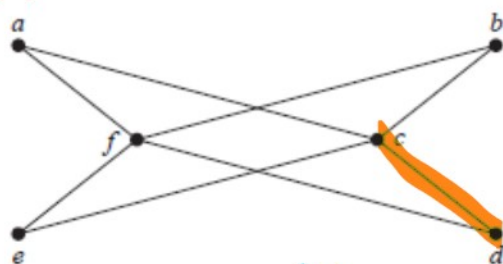
$V_1$	$V_2$
a	
b	
c	e
d	



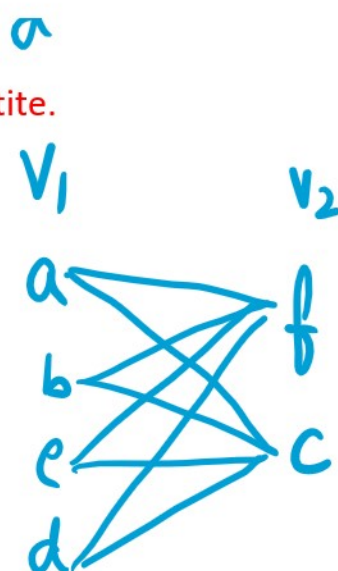
Q10. Check whether the following graph is bipartite.

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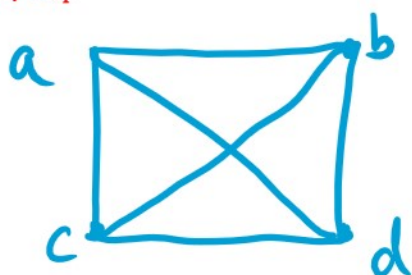
(a)



Yes Bipartite



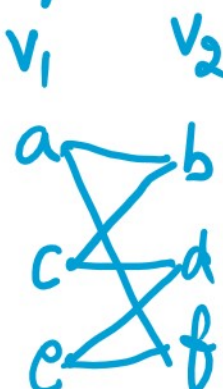
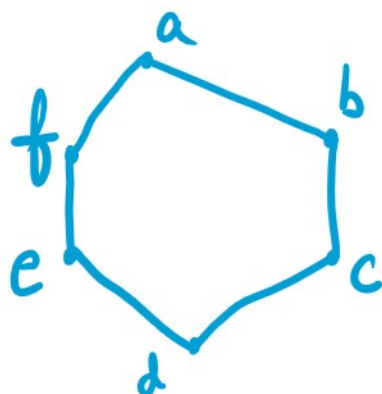
(b)  $K_4$



$K_n$  is not Bipartite for  $n > 2$   
 $K_2$  is bipartite.

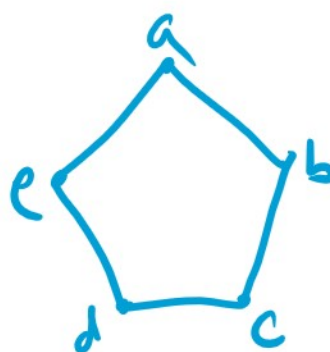


(c)  $C_6$



Bipartite

$C_5$



Not Bipartite

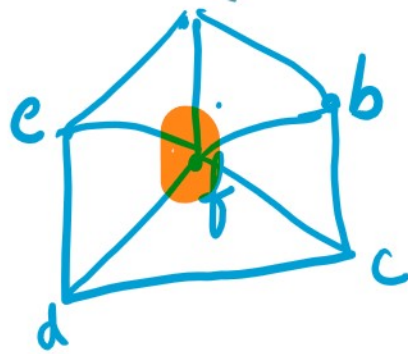
$C_n$  will be bipartite for even values of  $n$ .

(d)  $W_5$



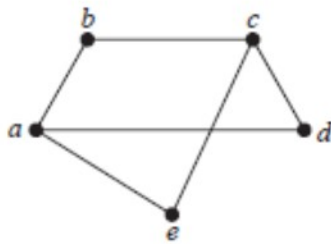
Not Bipartite

(u) v<sub>5</sub>

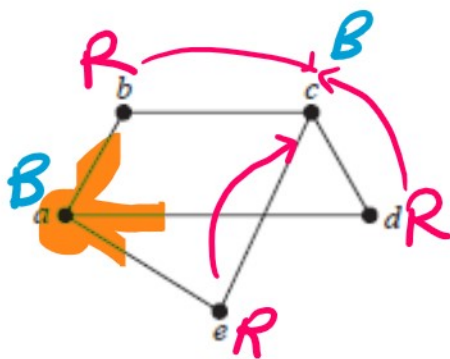


No Bipartite  
for any  $n$ .

(e)



Theorem 4: A simple graph is bipartite graph if and only if it is possible to assign one of two different colours to each vertex of the graph so that no two adjacent vertices are assigned with the same colour.

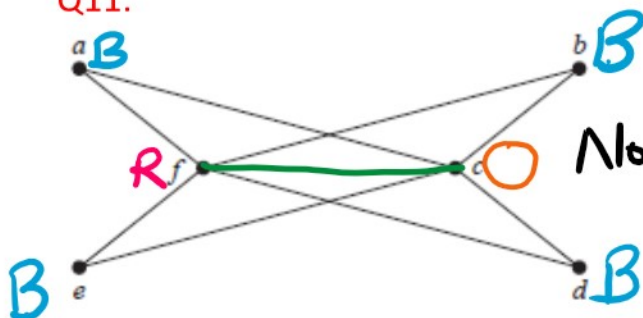


$V_1$   
a  
c

$V_2$   
b  
d  
e

Yes  
Bipartite

Q11.

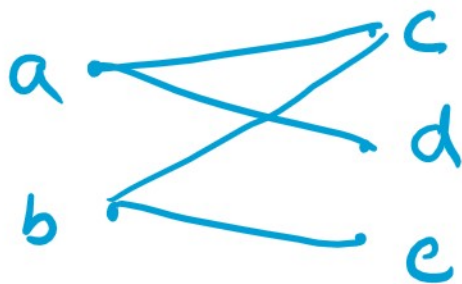


Not Bipartite

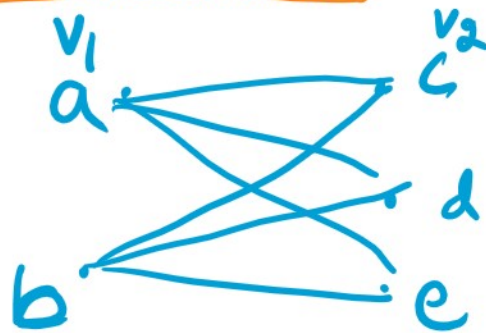
**Complete Bipartite Graph:** It is a graph that has its vertex set partitioned into two subsets of  $m$  and  $n$  vertices, respectively with an edge between two vertices if and



**Complete Bipartite Graph:** It is a graph that has its vertex set partitioned into two subsets of  $m$  and  $n$  vertices, respectively with an edge between two vertices if and only if one vertex in the first subset and the other vertex in the second subset. It is denoted as  $K_{m,n}$ .



Bipartite



Complete Bipartite

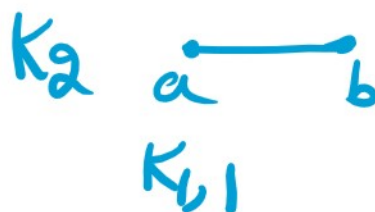
$K_{m,n}$

no. of vertices =  $m+n$

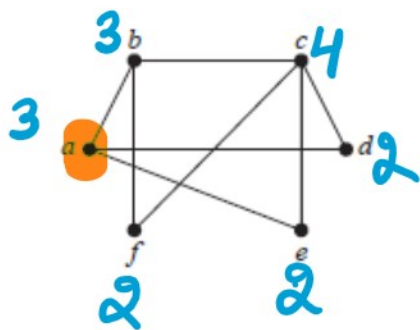
no of edges =  $mn$

degree of vertex in  $V_1 = n$

degree of vertex in  $V_2 = m$



**Degree sequence** of a graph is the sequence of the degrees of the vertices of the graph in non-increasing order.



4, 3, 3, 2, 2, 2

Q12. How many edges does a graph have if its degree sequence is 4, 3, 3, 2, 2? Draw such a graph.

4, 3, 3, 2, 2

$\sum \text{degree} = 2(E)$

\* Odd degrees  $\rightarrow$  Even  $\checkmark$

14

7

- \* Odd degrees  $\rightarrow$  Even ✓
- \* Degree repeated ✓
- \* Degree  $< n$  ✓

$$\text{Edges} = \frac{14}{2} = 7.$$

A degree sequence is called graphic if it is the degree sequence of a simple graph.