

# CSE322 LEFT & RIGHT LINEAR REGULAR GRAMMAR

Lecture #18



### Grammars

#### Grammars



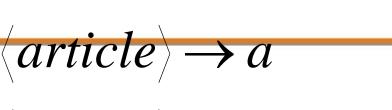
### Grammars express languages

### Example: the English language

$$\langle sentence \rangle \rightarrow \langle noun\_phrase \rangle \langle predicate \rangle$$

$$\langle noun\_phrase \rangle \rightarrow \langle article \rangle \langle noun \rangle$$

$$\langle predicate \rangle \rightarrow \langle verb \rangle$$





$$\langle article \rangle \rightarrow the$$

$$\langle noun \rangle \rightarrow boy$$
  
 $\langle noun \rangle \rightarrow dog$ 

$$\langle verb \rangle \rightarrow runs$$
  
 $\langle verb \rangle \rightarrow walks$ 



### A derivation of "the boy walks":

```
\langle sentence \rangle \Rightarrow \langle noun\_phrase \rangle \langle predicate \rangle
                        \Rightarrow \langle noun\_phrase \rangle \langle verb \rangle
                        \Rightarrow \langle article \rangle \langle noun \rangle \langle verb \rangle
                        \Rightarrow the \langle noun \rangle \langle verb \rangle
                        \Rightarrow the boy \langle verb \rangle
                        \Rightarrow the boy walks
```



### A derivation of "a dog runs":

```
\langle sentence \rangle \Rightarrow \langle noun\_phrase \rangle \langle predicate \rangle
                         \Rightarrow \langle noun\_phrase \rangle \langle verb \rangle
                         \Rightarrow \langle article \rangle \langle noun \rangle \langle verb \rangle
                         \Rightarrow a \langle noun \rangle \langle verb \rangle
                          \Rightarrow a \ dog \ \langle verb \rangle
                          \Rightarrow a \ dog \ runs
```

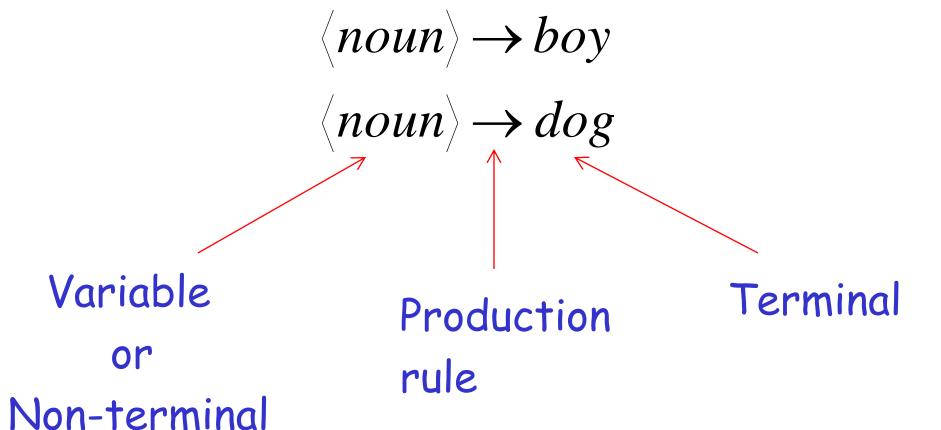


### Language of the grammar:

```
L = { "a boy runs",
     "a boy walks",
     "the boy runs",
     "the boy walks",
     "a dog runs",
     "a dog walks",
     "the dog runs",
     "the dog walks" }
```

#### Notation





### Another Example



Grammar:

$$S \rightarrow aSb$$

$$S \to \lambda$$

#### Derivation of sentence ab:

$$S \Rightarrow aSb \Rightarrow ab$$

$$S \rightarrow aSb \qquad S \rightarrow \lambda$$



Grammar: 
$$S \rightarrow aSb$$

$$S \to \lambda$$

#### Derivation of sentence aabb:

$$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aabb$$

$$S \rightarrow aSb$$

$$S \to \lambda$$



#### Other derivations:

$$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aaaSbbb \Rightarrow aaabbb$$

$$S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aaaSbbb$$

 $\Rightarrow$  aaaaSbbbb  $\Rightarrow$  aaaabbbb



### Language of the grammar

$$S \rightarrow aSb$$

$$S \to \lambda$$

$$L = \{a^n b^n : n \ge 0\}$$

#### More Notation



Grammar 
$$G = (V, T, S, P)$$

V: Set of variables

T: Set of terminal symbols

S: Start variable

P: Set of Production rules

### Example



Grammar 
$$G: S \rightarrow aSb$$

$$S \rightarrow aSb$$

$$S \to \lambda$$

$$G = (V, T, S, P)$$

$$V = \{S\}$$

$$T = \{a, b\}$$

$$P = \{S \rightarrow aSb, S \rightarrow \lambda\}$$

#### More Notation



#### Sentential Form:

A sentence that contains variables and terminals

#### Example:

 $S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aaaSbbb \Rightarrow aaabbb$ 

Sentential Forms

sentence



\*

We write:  $S \Rightarrow aaabbb$ 

#### Instead of:

 $S \Rightarrow aSb \Rightarrow aaSbb \Rightarrow aaaSbbb \Rightarrow aaabbb$ 



\*

### In general we write:

$$w_1 \implies w_n$$

If: 
$$w_1 \Rightarrow w_2 \Rightarrow w_3 \Rightarrow \cdots \Rightarrow w_n$$



\*

By default:

$$w \Rightarrow w$$

### Example



#### Grammar

$$S \rightarrow aSb$$

$$S \rightarrow \lambda$$

#### Derivations

$$S \Rightarrow \lambda$$

\*

$$S \Rightarrow ab$$

\*

$$S \Rightarrow aabb$$

\*

$$S \Rightarrow aaabbb$$

### Example



#### Grammar

$$S \rightarrow aSb$$

$$S \rightarrow \lambda$$

#### Derivations

$$s \Rightarrow aaSbb$$

\*  $aaSbb \Rightarrow aaaaaSbbbbb$ 

### Another Grammar Example



Grammar  $G: S \rightarrow Ab$ 

$$S \to Ab$$

$$A \rightarrow aAb$$

$$A \rightarrow \lambda$$

#### Derivations:

$$S \Rightarrow Ab \Rightarrow b$$

$$S \Rightarrow Ab \Rightarrow aAbb \Rightarrow abb$$

$$S \Rightarrow aAbb \Rightarrow aaAbbb \Rightarrow aabbb$$

### More Derivations



$$S \Rightarrow Ab \Rightarrow aAbb \Rightarrow aaAbbb \Rightarrow aaaAbbbb$$

$$\Rightarrow$$
  $aaaaAbbbbbb \Rightarrow aaaabbbbbb$ 

\*

$$S \Rightarrow aaaabbbbb$$

\*

$$S \Rightarrow aaaaaaabbbbbbbb$$

 $S \Longrightarrow a^n b^n b$ 

### Language of a Grammar



For a grammar G with start variable S:

$$L(G) = \{w: S \Longrightarrow w\}$$

String of terminals

### Example



For grammar 
$$G: S \rightarrow Ab$$

$$A \rightarrow aAb$$

$$A \rightarrow \lambda$$

$$L(G) = \{a^n b^n b: n \ge 0\}$$

Since:  $S \Rightarrow a^n b^n b$ 

### A Convenient Notation



$$\begin{array}{ccc}
A \to aAb \\
A \to \lambda
\end{array}$$

$$A \to aAb \mid \lambda$$

$$\langle article \rangle \rightarrow a$$
  
 $\langle article \rangle \rightarrow the$ 





### Linear Grammars

### Linear Grammars



Grammars with at most one variable at the right side of a production

$$S \rightarrow aSb$$

$$S \rightarrow \lambda$$

$$S \rightarrow Ab$$

$$A \rightarrow aAb$$

$$A \rightarrow \lambda$$

### A Non-Linear Grammar



Grammar 
$$G:$$

$$S \rightarrow SS$$

$$S \to \lambda$$

$$S \rightarrow aSb$$

$$S \rightarrow bSa$$

$$L(G) = \{w: n_a(w) = n_b(w)\}$$

## Another Linear Grammar

Grammar 
$$G: S \to A$$
 
$$A \to aB \mid \lambda$$
 
$$B \to Ab$$

$$L(G) = \{a^n b^n : n \ge 0\}$$

### Right-Linear Grammars



### All productions have form:

$$A \rightarrow xB$$

or

$$A \rightarrow x$$

Example: 
$$S \rightarrow abS$$

$$S \rightarrow abS$$

$$S \rightarrow a$$

### Left-Linear Grammars



### All productions have form:

$$A \rightarrow Bx$$

or

$$A \rightarrow x$$

Example: 
$$S \rightarrow Aab$$

$$A \rightarrow Aab \mid B$$

$$B \rightarrow a$$



### Regular Grammars

### Regular Grammars



# A regular grammar is any right-linear or left-linear grammar

### Examples:

 $G_1$   $G_2$   $S \rightarrow abS$   $S \rightarrow Aab$   $A \rightarrow Aab \mid B$   $B \rightarrow a$ 

#### Observation



### Regular grammars generate regular languages

### Examples:

$$G_2$$

$$G_1$$

$$S \rightarrow Aab$$

$$S \rightarrow abS$$

$$A \rightarrow Aab \mid B$$

$$S \rightarrow a$$

$$B \rightarrow a$$

$$L(G_1) = (ab) * a$$

$$L(G_2) = aab(ab) *$$



### Regular Grammars Generate Regular Languages

#### Theorem



Languages
Generated by
Regular Grammars

Regular
Languages

#### Theorem - Part 1



Languages
Generated by
Regular Grammars

Regular
Languages

Any regular grammar generates a regular language

#### Theorem - Part 2



Languages
Generated by
Regular Grammars
Regular Grammars

Any regular language is generated by a regular grammar