# Lecture 4

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$A \to B \equiv \neg A \vee B$

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

$\neg(\neg p) \equiv p$

**(B)** $[\neg p \wedge (p \vee q)] \to q$

$[(\neg p \wedge p) \vee (\neg p \wedge q)] \to q$ By Distributive Law

$[F \vee (\neg p \wedge q)] \to q$ By Negation Law

$\underbrace{(\neg p \wedge q)}_{A} \to \underbrace{q}_{B}$ Identity Law

$\neg(\neg p \wedge q) \vee q$ By def^n of conditional statement

$(p \vee \neg q) \vee q$ By De-Morgan's Law & Double Negation Law

$p \vee (\neg q \vee q)$ Associative Law

$\equiv p \vee T$ Negation Law

$\equiv T$ Domination Law

**(C)** $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$

$[(p \vee q) \wedge \underbrace{(\neg p \vee r)}_{A \vee C} \wedge \underbrace{(\neg q \vee r)}_{B \vee C}] \to r$

$[(p \vee q) \wedge ((\neg p \wedge \neg q) \vee r)] \to r$

$[(p \vee q) \wedge (\neg(p \vee q) \vee r)] \to r$

$A \wedge (\neg A \vee B) \to r$

$[(A \wedge \neg A) \vee (A \wedge B)] \to r$

$[F \vee (A \wedge B)] \to r$

$$[F \vee (A \wedge B)] \to r$$

$$A \wedge B \to r$$

$$[(p \vee q) \wedge r] \to r$$

$$\neg((p \vee q) \wedge r) \vee r$$

$$(\underbrace{(\neg p \wedge \neg q)}_{A} \underset{\vee}{\vee} \underbrace{\neg r}_{B}) \underset{\vee}{\vee} \underbrace{r}_{C} \equiv (\neg p \wedge \neg q) \vee (\neg r \vee r)$$

$$\equiv (\neg p \wedge \neg q) \vee T$$

$$\equiv T$$

**Dual** of compound proposition with operators $\neg, \vee, \wedge$ is defined as replacing $\wedge$ by $\vee$, $\vee$ by $\wedge$, $T$ by $F$ and $F$ by $T$.

Find dual of

| | |
|---|---|
| $(p \wedge q \wedge r) \vee s$ | $(p \vee q \vee r) \wedge s$ |
| $p \wedge \neg q \wedge \neg r$ | $p \vee \neg q \vee \neg r$ |
| $(p \vee F) \wedge (q \vee T)$ | $(p \wedge T) \vee (q \wedge F)$ |

**Sheffer stroke operator** ***p NAND q***

It is denoted as $p|q$

Nand operator is false when both $p$ and $q$ have true truth values, otherwise true

**Peirce arrow operator** ***p NOR q***

It is denoted as $p \downarrow q$

**Peirce arrow operator** **$p$ NOR $q$**

It is denoted as $\boldsymbol{p \downarrow q}$

Nor operator is true when both p and q are false, otherwise False.

Q9. Show that $p \downarrow q$ is logically equivalent to $\neg(p \vee q)$.

$$p \mid q \equiv \neg(p \wedge q)$$

**Precedence of Logical Operator**

$$A \vee (B \wedge C)$$

$$\neg, \wedge, \vee, \rightarrow, \leftrightarrow$$

**Logical Equivalences involving Conditional and Biconditional statements**

1. $p \rightarrow q \equiv \neg p \vee q$
2. $p \rightarrow q \equiv \neg q \rightarrow \neg p$
3. $p \vee q \equiv \neg p \rightarrow q$
4. $p \wedge q \equiv \neg(p \rightarrow \neg q)$
5. $\neg(p \rightarrow q) \equiv p \wedge \neg q$
6. $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$
7. $(p \rightarrow r) \wedge (q \rightarrow r) \equiv (p \vee q) \rightarrow r$
8. $(p \rightarrow q) \vee (p \rightarrow r) \equiv p \rightarrow (q \vee r)$
9. $(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$
10. $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
11. $p \leftrightarrow q \equiv (\neg p \leftrightarrow \neg q)$
12. $\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$
13. $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$