CSE322
CONVERTING REGULAR EXPRESSION(RE) TO REGULAR GRAMMAR(RG) & RG TO RE

Lecture #17

CONVERITNG RE TO RG

Let $M = (\{q_0, \ldots, q_n\}, \Sigma, \delta, q_0, F)$.

We have to construct a

$$G = (\{A_0, A_1, \ldots, A_n\}, \Sigma, P, A_0)$$

where P is defined by the following rules:

(i) $A_i \to aA_j$ is included in P if $\delta(q_i, a) = q_j \notin F$.

(ii) $A_i \to aA_j$ and $A_i \to a$ are included in P if $\delta(q_i, a) = q_j \in F$.

We can show that $L(G) = T(M)$ by using the construction of P. Such a construction gives

$$A_i \Rightarrow aA_j \quad \text{iff } \delta(q_i, a) = q_j$$

$$A_i \Rightarrow a \quad \text{iff } \delta(q_i, a) \in F$$

So.

$$A_0 \Rightarrow a_1A_1 \Rightarrow a_1a_2A_2 \Rightarrow \ldots \Rightarrow a_1 \ldots a_{k-1}A_k \Rightarrow a_1a_2 \ldots a_k$$

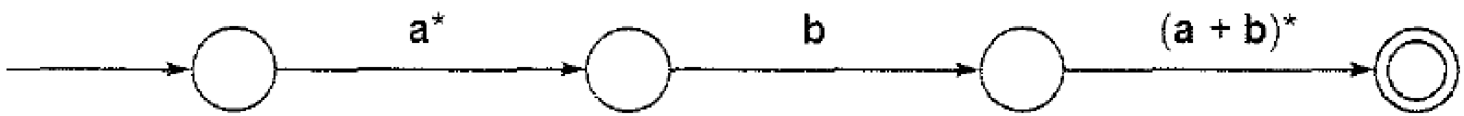
iff $\delta(q_0, a_1) = q_1, \quad \delta(q_1, a_2) = q_2, \ldots, \quad \delta(q_k, a_k) \in F$

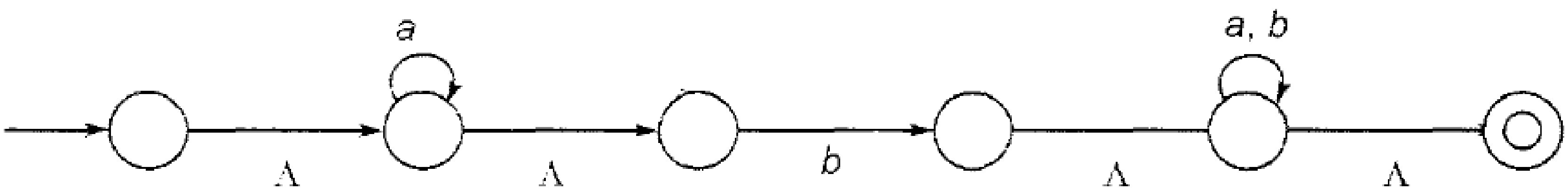
This proves that $w = a_1 \ldots a_k \in L(G)$ iff $\delta(q_0, a_1 \ldots a_k) \in F$, i.e. iff $w \in T(M)$.

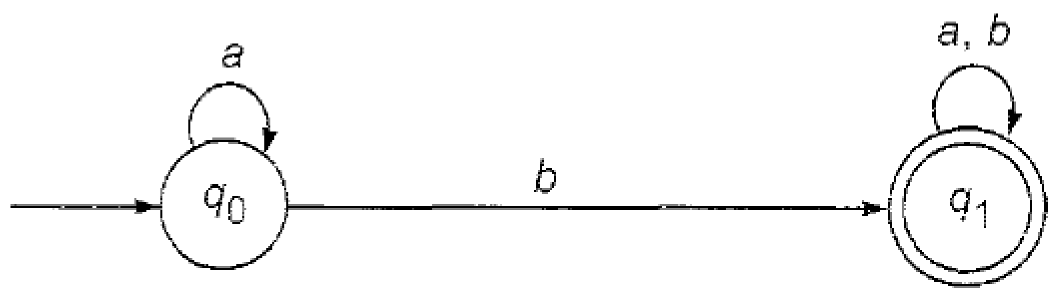
NUMERICAL

Construct a regular grammar G generating the regular set represented by $\mathbf{P} = \mathbf{a}^*\mathbf{b}(\mathbf{a} + \mathbf{b})^*$.

SOLUTION







Let $G = (\{A0, A_1\}, \{a, b\}, P, A_0)$, where P is given by

$$A_0 \to aA_0, \qquad A_0 \to bA_1, \qquad A_0 \to b$$

$$A_1 \to aA_1, \qquad A_1 \to bA_1, \qquad A_1 \to a, \qquad A_1 \to b$$

G is the required regular grammar.

CONVERSION OG RG TO RE

CONSTRUCTION OF TRANSITION SYSTEM M ACCEPTING L(G) FOR A GIVEN REGULAR GRAMMAR G

Let $G = (\{A_0, A_1, \ldots, A_n\}, \Sigma, P, A_0)$

We define M as $(\{q_0, \ldots, q_n, q_f\}, \Sigma, \delta, q_0, \{q_f\})$ where δ is defined as follows:

(i) Each production $A_i \to aA_j$ induces a transition from q_i to q_j with label a.

(ii) Each production $A_k \to a$ induces a transition from q_k to q_f with label a.

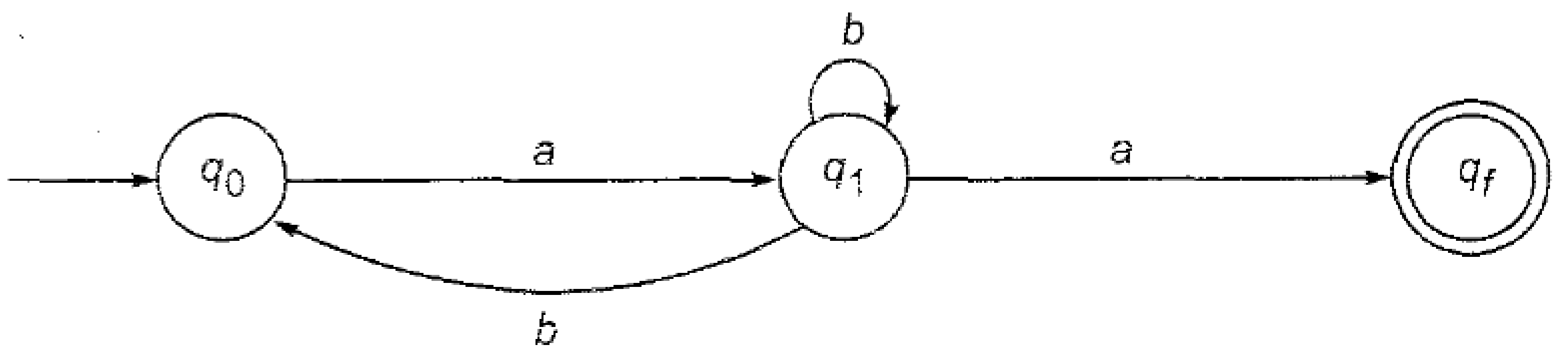
From the construction it is easy to see that $A_0 \Rightarrow a_1A_1 \Rightarrow a_1a_2A_2 \Rightarrow \ldots \Rightarrow a_1 \ldots a_{n-1}A_{n-1} \Rightarrow a_1 \ldots a_n$ is a derivation of $a_1a_2 \ldots a_n$ iff there is a path in M starting from q_0 and terminating in q_f with path value $a_1a_2 \ldots a_n$. Therefore, $L(G) = T(M)$.

NUMERICAL

Let $G = (\{A_0, A_1\}, \{a, b\}, P, A_0)$, where P consists of $A_0 \to aA_1$, $A_1 \to bA_1$, $A_1 \to a$, $A_1 \to bA_0$. Construct a transition system M accepting $L(G)$.

Solution

Let $M = (\{q_0, q_1, q_f\}, \{a, b\}, \delta, q_0, \{q_f\})$, where q_0 and q_1 correspond to A_0 and A_1, respectively and q_f is the new (final) state introduced. $A_0 \to aA_1$ induces a transition from q_0 to q_1 with label a. Similarly, $A_1 \to bA_1$ and $A_1 \to bA_0$ induce transitions from q_1 to q_1 with label b and from q_1 to q_0 with label b, respectively. $A_1 \to a$ induces a transition from q_1 to q_f with label a.



NUMERICAL