Number Systems

DIGITS

0 , 1 , 2 , 3 , 4 , 5 , 6 , 7 , 8 , 9

NUMERAL

A group of digits, denoting a number.

2 3 6 5

Th	H	T	O
2	3	6	5

TYPES OF NUMBERS

NATURAL NUMBER

1 , 2 , 3 , 4 , 5 . . .

WHOLE NUMBER

All natural numbers including 0.

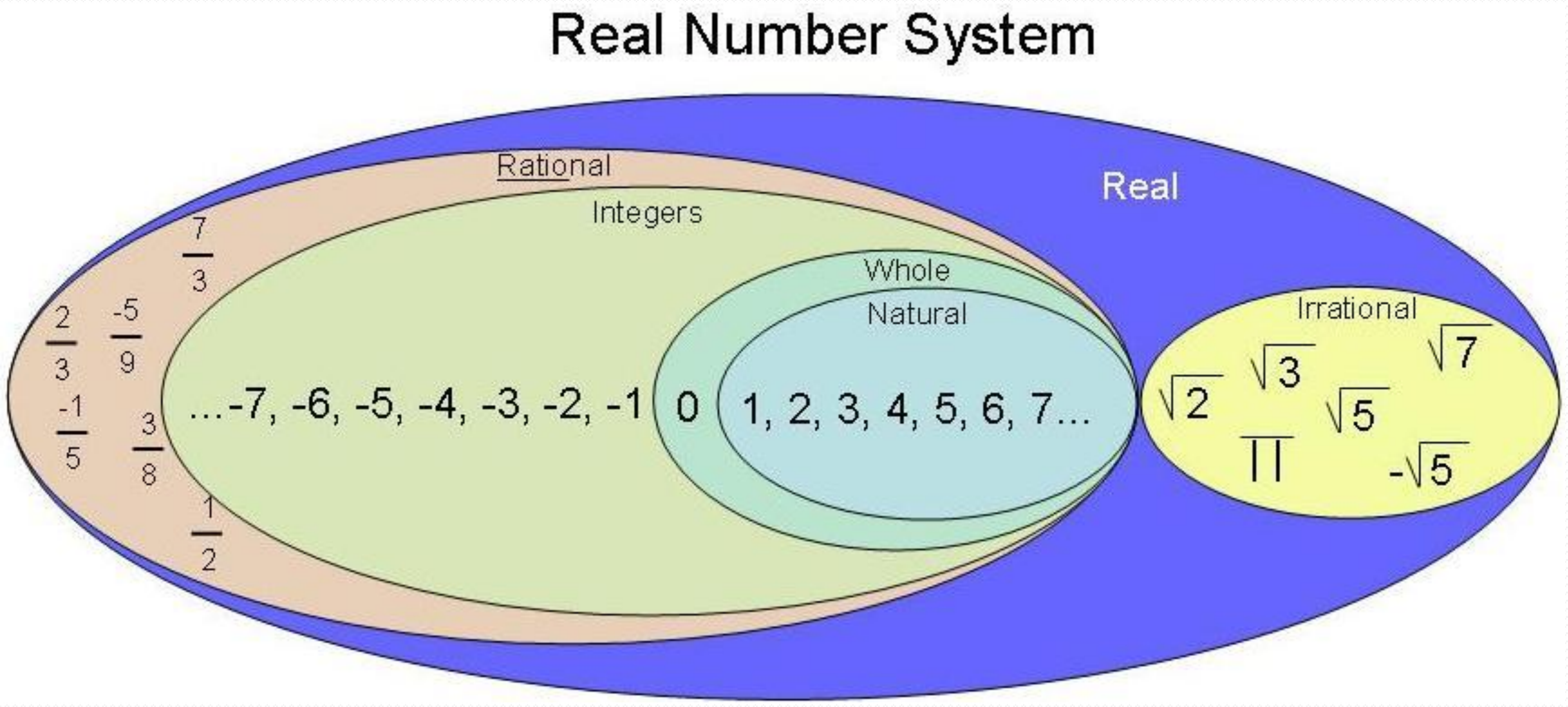
INTEGERS

All natural numbers, 0 & negative numbers

{…, -3, -2, -1, 0, 1, 2, 3, …}

- **Positive Integers** {1, 2, 3, …}
- **Negative Integers** { -1, -2, -3, …}
- Non-Positive & Non-Negative Integer is 0.

TYPES OF NUMBERS



EVEN & ODD NUMBERS

EVEN NUMBER

No's divisible by 2.

ODD NUMBER

No's not divisible by 2.

Facts about Even & Odd No's

- ✓ Sum / Difference of two even numbers is an even number.
- ✓ Sum / Difference of two odd numbers is an even number.
- ✓ Sum / Difference of an even number and an odd numbers is an odd number.

TYPES OF NUMBERS – Prime No

A **Prime No** is a natural no greater than 1 that has no positive divisors other than 1 and itself.

Prime no's upto 100 are

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

A natural number greater than 1 that is not a prime number is called a **Composite No**.

Two no's a & b are said to be **Co-Primes**, if their HCF is 1.

Tests of Divisibility

A no is divisible by	If
2	Unit’s digit is divisible by 2.
3	Sum of its digits is divisible by 3.
4	No formed by last 2 digits is divisible by 4.
5	Unit’s digit is either 0 or 5.
6	It is divisible by both 2 & 3.
7	Eg 14 Double the unit digit i.e 8 subtract the remaining digit i.e 8-1 = 7 ans will be either 0 or multiple of 7
8	No formed by last 3 digits is divisible by 8.
9	Sum of its digits is divisible by 9.
10	It ends with 0.

Tests of Divisibility

A no is divisible by	If
11	The difference of sum of its digits at odd places and sum of its digits at even places, is either 0 or a no divisible by 11.
12	It is divisible by both 3 & 4.
13	Eg 26 Multiply unit digit by 4 i.e 6 X 4 = 24 add Other digit = 26 ans will be divisible by 13
14	It is divisible by both 2 & 7.
15	It is divisible by both 3 & 5.
16	The number formed by last four digits is divisible by 16.
17	Eg 34 Multiply unit digit by 5 e.g 4 X 5 =20 subtract the other digit i.e 20 – 3 = 17 ans will be divisible by 17.
18	Number must be divisible by 9 & 2 both as 9 and 2 are co prime
19	Eg 38 Multiply unit digit by 2 e.g 8 X 2 =16 add the other digit i.e 16+3 = 19 ans will be divisible by 19.

Division Algorithm

Dividend = (Divisor * Quotient) + Reminder

Basic Formulae

$(a+b)(a-b)=(a^2-b^2)$

$(a+b)^2=(a^2+\mathrm{b}^2+2ab)$

$(a-b)^2=(a^2+\mathrm{b}^2-2ab)$

$(a+b+c)^2=a^2+\mathrm{b}^2+c^2+2(ab+bc+ca)$

$(a^3+b^3)=(a+b)(a^2-ab+b^2)$

$(a^3-b^3)=(a-b)(a^2+ab+b^2)$

$(a^3+b^3+c^3-3abc)=(a+b+c)(a^2+b^2+c^2-ab-bc-ac)$

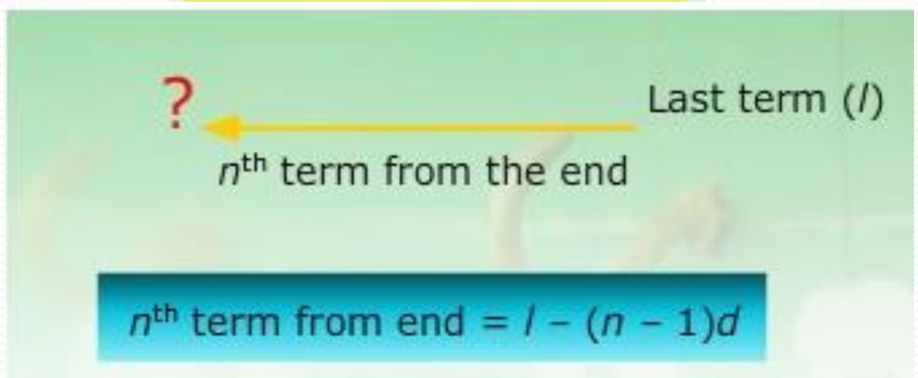
When $a+b+c=0$, then $a^3+b^3+c^3=3abc$.

ARITHMATIC PROGRESSION

An Arithmetic Progression (A.P.) is a sequence in which the difference between any two consecutive terms is constant.

Let a = first term, d = common difference

Then nth term $a_n = a + (n - 1)d$



Sum of an A.P.

The sum of n terms of an A.P. whose first term is a and common difference is d, is given by

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

The sum of n terms of an A.P. whose first term is a and last term is l is given by the formula:

$$S_n = \frac{n}{2}\left[a + l\right]$$

GEOMETRICAL PROGRESSION

A Geometric Progression or GP is a sequence where each new term after the first is obtained by multiplying the preceding term by a constant r, called the common ratio. If the first term of the sequence is a, then the geometric progression will be

$$a, ar, ar^2, ar^3, \ldots$$

Where the nth term is ar^{n-1}.

GEOMETRICAL PROGRESSION

If the starting value is a and the common ratio is r, then the sum of first n terms is

$$s_n = \frac{a(1-r^n)}{1-r}$$

$S_n = a(r^n - 1) / r-1$

Provided that $r \neq 1$.

If $r > 1$

The sum to infinity of a geometric progression starting with a and common ratio r is

$$s_\infty = \frac{a}{1-r}$$

Where $-1 < r < 1$

HOW TO FIND THE CYCLICITY OF A NUMBER

FOR eg NUMBER IS 2

2^1 = 2 Unit Digit = 2
2^2 = 4 Unit Digit = 4
2^3 = 8 Unit Digit = 8
2^4 = 16 Unit Digit = 6 This cycle of unit digits will repeat itself after every 4 intervals so cyclicity of number 2 is 4 .

Similarly cyclicity of number 9 is 2

HOW TO FIND THE UNIT DIGIT OF A NUMBER

Number	Cyclicit y
1	1
2	4
3	4
4	2
5	1
6	1
7	4
8	4
9	2
10	1

Question

Find the unit digit of

$$2^{95}$$

Solution

The cyclicity of 2 is 4.

$2^1 = 2$

$2^2 = 4$

$2^3 = 8$

$2^4 = 16$

Divide 95 by 4. Remainder is 3.

So, the unit digit is 8.

Practice

The largest 4 digit number exactly divisible by 88 is:

A. 9944

B. 9768

C. 9988

D. 8888

E. None of these

Solution

Answer: Option **A**

Explanation:

Largest 4-digit number = 9999

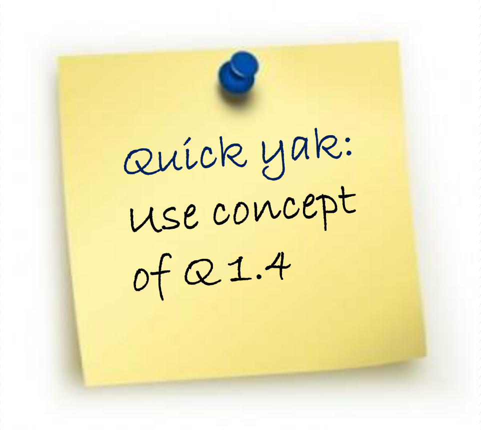
Reminder of 9999 / 88 = 55

Required number = (9999 - 55) = 9944.

Practice

Find the unit digit of

$$9^{99}$$



Solution

The cyclicity of 9 is 2.

$9^1 = 9$

$9^2 = 81$

Divide 99 by 2. Remainder is 1.

So, the unit digit is 9.

Next Class: HCF & LCM

Factors and Multiples:

If number *a* divided another number *b* exactly, we say that *a* is a ***factor*** of *b*.

In this case, *b* is called a ***multiple*** of *a*.

Highest Common Factor (H.C.F.) or Greatest Common Measure(G.C.M.) or Greatest Common Divisor (G.C.D.)

The H.C.F. of two or more than two numbers is the greatest number that divided each of them exactly.

- Now, Suppose we have to find the H.C.F. of three numbers, then,

H.C.F. of three numbers = H.C.F. of [(H.C.F. of any two) and (the third number)]

Similarly, the H.C.F. of more than three numbers may be obtained.

Least Common Multiple (L.C.M.)

The least number which is exactly divisible by each one of the given numbers is called their L.C.M.

- ✓ *Product of two numbers = Product of their H.C.F. and L.C.M.*
- ✓ *Co-primes:* Two numbers are said to be co-primes if their H.C.F. is 1.

H.C.F. and L.C.M. of Fractions:

1. $$\text{H.C.F.} = \frac{\text{H.C.F. of Numerators}}{\text{L.C.M. of Denominators}}$$
2. $$\text{L.C.M.} = \frac{\text{L.C.M. of Numerators}}{\text{H.C.F. of Denominators}}$$

HCF and LCM Problem Solving

How can you tell if a word problem requires you to use Highest Common Factor or

Least Common Multiple to solve?

Q 2.1 : HCF Example

Samantha has two pieces of cloth. One piece is 72 inches wide and the other piece is 90 inches wide. She wants to cut both pieces into strips of equal width that are as wide as possible. How wide should she cut the strips?

Samantha has two pieces of cloth. One piece is 72 inches wide and the other piece is 90 inches wide. She wants to cut both pieces into strips of equal width that are as wide as possible. How wide should she cut the strips?

The pieces of cloth are 72 and 90 inches wide.

How wide should she cut the strips so that they are the largest possible equal lengths.

Samantha has two pieces of cloth. One piece is 72 inches wide and the other piece is 90 inches wide. She wants to cut both pieces into strips of equal width that are as wide as possible. How wide should she cut the strips?

This problem can be solved using Highest Common Factor because we are cutting or “dividing” the strips of cloth into smaller pieces (factor) of 72 and 90.

Find the HCF of 72 and 90

HCF Word Problem Solution

$$\begin{array}{r|l} 2 & \underline{72} \\ 2 & \underline{36} \\ 2 & \underline{18} \\ 3 & \underline{9} \\ 3 & \underline{3} \\ & 1 \end{array} \qquad \begin{array}{r|l} 2 & \underline{90} \\ 3 & \underline{45} \\ 3 & \underline{15} \\ 5 & \underline{5} \\ & 1 \end{array}$$

72 = 2 x 2 x 2 x 3 x 3

90 = 2 x 3 x 3 x 5

HCF = 2 x 3 x 3 = 18

Samantha should cut each piece to be 18 inches wide

If it is an LCM Problem

What is the question asking us?

✓Do we have an event that is or will be repeating over and over?

✓Will we have to purchase or get multiple items in order to have enough?

✓Are we trying to figure out when something will happen again at the same time?

: LCM Example

Ben exercises every 12 days and Isabel every 8 days. Ben and Isabel both exercised today. How many days will it be until they exercise together again?

Ben exercises every 12 days and Isabel every 8 days. Ben and Isabel both exercised today. How many days will it be until they exercise together again?

Ben exercises every 12 days and Isabel every 8 days and they both exercised today.

How many days is it until they will both exercise on the same day again.

Ben exercises every 12 days and Isabel every 8 days. Ben and Isabel both exercised today. How many days will it be until they exercise together again?

This problem can be solved using Least Common Multiple. We are trying to figure out when will be the next time they are exercising together.

Find the LCM of 12 and 8.

LCM Word Problem Solution

2 I 12	2 I 8
2 I 6	2 I 4
3 I 3	2 I 2
1	1

12 = 2 x 2 x 3

8 = 2 x 2 x 2

LCM = 2 x 2 x 2 x 3 = 24

Ben and Isabel would exercise on the same day every 24 days.

Practice

Rosa is making a game board that is 16 inches by 24 inches. She wants to use square tiles. What is the largest size tile she can use?

Practice

2 l 16	2 l 24
2 l 8	2 l 12
2 l 4	2 l 6
2 l 2	3 l 3
1	1

16 = 2 x 2 x 2 x 2

24 = 2 x 2 x 2 x 3

HCF = 2 x 2 x 2 = 8

The largest size tile that Rosa can use is tile of 8 inches.

Practice

Two bikers are riding a circular path. The first rider completes a round in 12 minutes. The second rider completes a round in 18 minutes. If they both started at the same place and time and go in the same direction, after how many minutes will they meet again at the starting point?

Practice

2 l 12   2 l 18

2 l 6   3 l 9

3 l 3   3 l 3

1   1

12 = 2 x 2 x 3

18 = 2 x 3 x 3

LCM = 2 x 3 x 2 x 3 = 36

Both bikers will meet after 36 minutes.

Next Class Average.