Q Solve the following system of equations

$$x - y + z = 2$$
$$2x + 3y - z = 5$$
$$x + y - z = 0$$

Solution →

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & -1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$$

Augmented Matrix $= \begin{bmatrix} 1 & -1 & 1 & 2 \\ 2 & 3 & -1 & 5 \\ 1 & 1 & -1 & 0 \end{bmatrix}$

operate $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - R_1$

$$\sim \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 5 & -3 & 1 \\ 0 & 2 & -2 & -2 \end{bmatrix}$$

operate $R_3 (1/2)$

$$\sim \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 5 & -3 & 1 \\ 0 & 1 & -1 & -1 \end{bmatrix}$$

operate $R_2 \leftrightarrow R_3$

$$\sim \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 5 & -3 & 1 \end{bmatrix}$$

operate $R_3 \to R_3 - 5R_2$

$$\sim \begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 2 & 6 \end{bmatrix} \quad \text{— (1)}$$

$\therefore \rho(K) = 3$

also $\rho(A) = 3$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$$

✓ $f(A) = f(K) =$ Number of unknowns

∴ The system has unique solution

From (1)

$x - y + z = 2$

$y - z = -1$

$2z = 6 \Rightarrow \boxed{z = 3}$

$y - 3 = -1$

$\boxed{y = 2}$

$x - 2 + 3 = 2$

$\Rightarrow \boxed{x = 1}$

∴ solution is $\boxed{x = 1, y = 2 \text{ & } z = 3}$

(a) unique
(b) Infinite

Q

$$x - 2y + 3t = -2$$
$$2x + y + z + t = -4$$
$$4x - 3y + z + 7t = -8$$

Augmented Matrix $= \left[\begin{array}{cccc|c} 1 & -2 & 0 & 3 & -2 \\ 2 & 1 & 1 & 1 & -4 \\ 4 & -3 & 1 & 7 & -8 \end{array}\right]$

operate $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 4R_1$

$\sim \left[\begin{array}{cccc|c} 1 & -2 & 0 & 3 & -2 \\ 0 & 5 & 1 & -5 & 0 \\ 0 & 5 & 1 & -5 & 0 \end{array}\right]$

operate $R_3 \to R_3 - R_2$

$\sim \left[\begin{array}{cccc|c} 1 & -2 & 0 & 3 & -2 \\ 0 & 5 & 1 & -5 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

$f(K) = 2$, $f(A) = 2$

$\rho(A) = \rho(K) = 2 <$ Nr of unknowns

$\therefore$ System will have infinite Number of solutions

Here $\begin{cases} x - 2y + 3t = -2 \\ 5y + 3 - 5t = 0 \end{cases}$

$4 - 2 = 2$

let $z = K_1$ & $t = K_2$

$\therefore$ $x - 2y = -2 - 3K_2$

$5y = 5K_2 - K_1$

$\Rightarrow \boxed{y = K_2 - \frac{K_1}{5}}$

$x = -2 - 3K_2 + 2\left(K_2 - \frac{K_1}{5}\right)$

$= -2 - 3K_2 + 2K_2 - \frac{2K_1}{5}$

$\boxed{x = -2 - K_2 - \frac{2K_1}{5}}$