Lecture 24

18 October 2021 23:43

Q5. Can a simple graph exist with 15 vertices each of degree 5?

sum of degrees $= 75 = 2e$ Not Possible

Theorem 2: An undirected graph has an even number of vertices of odd degree.

$$\sum \deg = 2(\text{Edges}) = \text{Even}$$

$$\sum_{v_i \text{ is of even degree}} \deg(v_i) + \sum_{v_i \text{ is of odd degree}} \deg(v_i) = \text{Even}$$

$$\text{Even} + \sum_{v_i \text{ of odd degree}} \deg(v_i) = \text{Even}$$

$$\sum_{v_i \text{ is of odd degrees}} \deg(v_i) = \text{Even}$$

sum of odd nos is even

Even no. of vertices with odd degrees.

Is this graph possible with degrees of 5 vertices given as $1, 2, 3, 4, 5$

Not Possible.

Directed Graph

- When (u, v) is an edge of the directed graph G, u is said to be adjacent to v and v is said to be incident from u.

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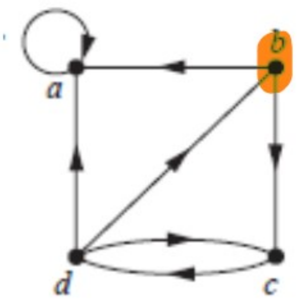
- In a graph with directed edges, the in-degree of a vertex denoted by $deg^-(v)$, is the number of edges with v as their terminal vertex. The out-degree of a vertex denoted by $deg^+(v)$, is the number of edges with v as their initial vertex.

Loop is counted once as deg^- and once as deg^+

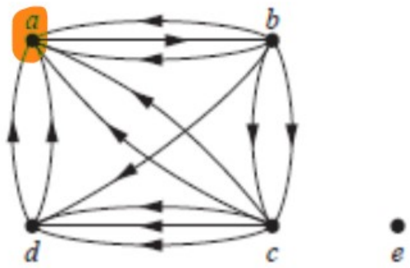
Q6. Determine the number of vertices and edges and find the in-degree and out-degree of each vertex for given directed multigraph.



No. of vertices = 4

No of edges = 7

Vertex	deg^-	deg^+
a	3	1
b	1	2
c	2	1
d	1	3
	7	7



no of vertices = 5

no of edges = 13

Vertex	deg^-	deg^+
a	6	1
b	1	5
c	2	5
d	4	2
e	0	0

a | 4 |
e | 0 | 0

Theorem 3: Let $G = (V, E)$ be a graph with directed edges, then

$$|E| = \sum_{v \in V} deg^{-}(v) = \sum_{v \in V} deg^{+}(v)$$

Q7. Show that in a simple graph with at least two vertices there must be two vertices that have the same degree.

Let G has n vertices.

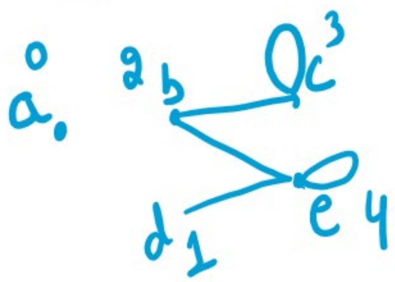
Assume that no two vertices have same degree.

deg $\geq n$ is not possible as one vertex at max can be associated with other $(n-1)$ vertices.

Then. $0, 1, 2, 3, \ldots, (n-1)$ are possible n different degrees.

Not possible simultaneously.

Can we have a simple graph with degrees as No. of odd degrees should even



(A) 2, 3, 0, 1, 1 → No

(B) 2, 3, 0, 1, 4 → No Simple graph is not possible

No two vertices with same degree.

Special Simple Graph

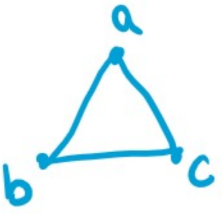
1. **Complete Graph**: It is a simple graph that contains exactly one edge between each pair of distinct vertices. It is denoted by K_n.

Each vertex is connected with other $(n-1)$ vertices.

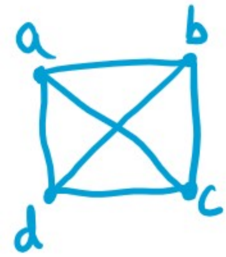
Each vertex is connected with other (n-1) vertices.

K_2

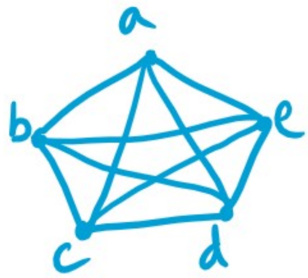
K_3



K_4



K_5



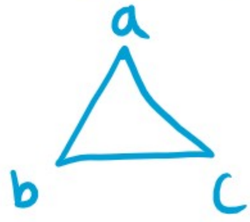
K_n, $\deg(v_i) = n-1$

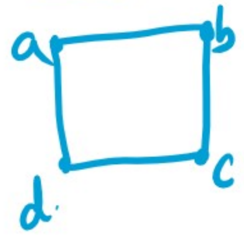
n is no. of vertices.

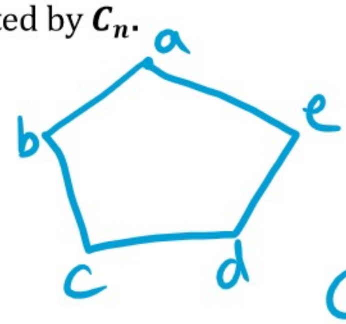
No of edges =

A simple graph for which there is at least one pair of distinct vertex not connected by an edge is called **noncomplete**.

2. **Cycle:** It consists of $n, n \geq 3$, vertices $v_1, v_2, \ldots, v_n$ and edges $\{v_1, v_2\}, \{v_2, v_3\}, \ldots, \{v_{n-1}, v_n\}$. It is denoted by $\boldsymbol{C_n}$.







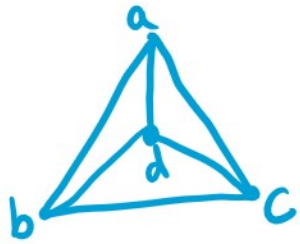
C_n

no of edges: n

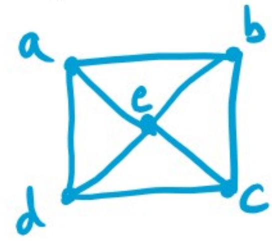
$\deg(v_i) = 2$

3. **Wheel:** When we add an additional vertex to a cycle C_n, $n \geq 3$, and connect this new vertex to each of the n vertices in C_n. It is denoted as $\boldsymbol{W_n}$.

W_3



W_4



W_n: No of vertices $= n+1$

No of edges $= 2n$

$$\deg(v_i) = \begin{cases} 3, & \text{other than additional.} \\ n, & \text{additional vertex} \end{cases}$$

4. **Regular Graph:** A simple graph in which every vertex has the same degree. A regular graph is called n −regular if every vertex has degree n.

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$C_n \to$ Regular graph $\to$ 2-regular

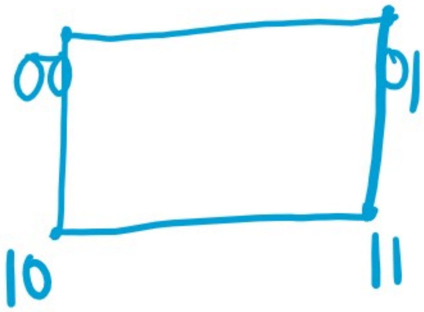
$K_n \to$ Regular graph $\to$ $(n-1)$-regular

5. $\boldsymbol{n}$ **− cubes:** A n −dimensional hypercube is a graph that has vertices representing the 2^n bit strings of length n. Two vertices are adjacent if and only if the bit strings that they differ in exactly one bit position. It is denoted as $\boldsymbol{Q_n}$.

Q_1 0 —— 1

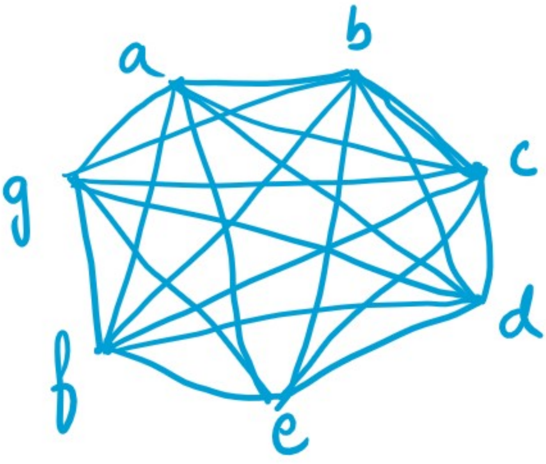
$Q_n \to$ no. of vertices $\to 2^n$.

Q_2 00 01 10 11



Q8. Draw the graphs

(a) K_7



(b) C_7

(c) W_7