Probability

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Probability is a measure of likelihood that an event will occur.

Sometimes you can measure a probability with a number: "10% chance of rain" or you can use words such as impossible, unlikely, possible, even chance, likely and certain.

**Examples**: "It is unlikely to rain tomorrow"

Many events can't be predicted with total certainty. The best we can say is how likely they are tohappen, using the idea of probability.

Tossing a coin: When a coin is tossed, there are two possible outcomes : either heads (H) or tails (T). We say that the probability of the coin landing H is ½. And the probability of the coin landing T is ½.

PROBABILITY LINE:

Probability is always between 0 and 1.

Impossible | Unlikely | Even | Likely | Certain
0 | | 1/2 | | 1
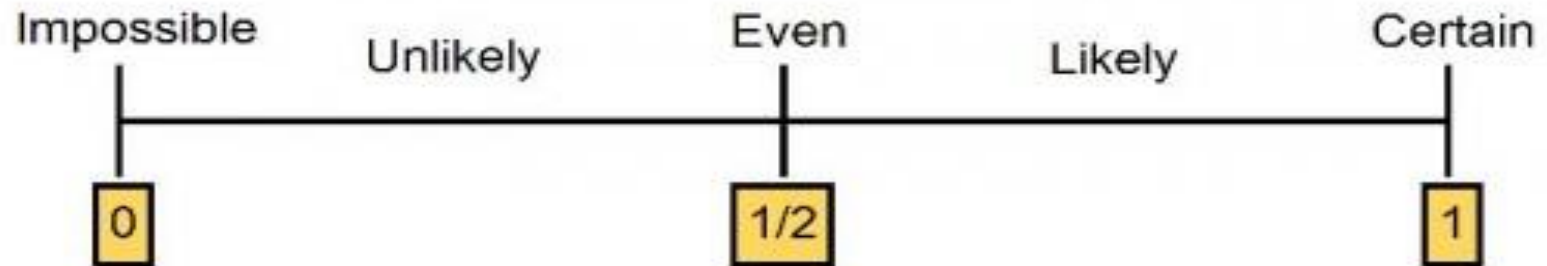
0 defines that an event can never happen. Example : the probability of getting a number greater than 6, when a single dice is thrown.
1 defines that an event can certainly happen. Example: If today is Monday. the probability that tomorrow is Tuesday is certain

Formula:

Probability is the measure of how likely an event is.

Probability of an Event = Number of Favorable Outcomes / Total Number of Possible Outcomes

Measures the likelihood of an event in the following way:

- If $P(A) > P(B)$ then event A is more likely to occur than event B.

- If $P(A) = P(B)$ then events A and B are equally likely to occur.

TERMINOLOGY

Experiment: Any planned process of data collection. It consists of a number of trials (replications) under the same condition.

Random Experiment: An experiment in which all possible outcomes are know and the exact output cannot be predicted in advance, is called a random experiment.

Examples:

Rolling an unbiased dice.

Tossing a fair coin.

Trail and Event : The performance of a random experiment is called a trial and the outcome an event. **Example :** Throwing of a dice is a trial and the result is an outcome

Exhaustive Cases: All possible outcomes of an event are known as exhaustive cases. **Example:** In a throw of single dice, the exhaustive cases are 6 but if it is 2 dice the, exhaustive cases would be 36.

Favorable Cases : The number of outcomes which result in the happening of a desired event are called favourable cases. **Example:** In a single throw of dice, the number of favourable cases of getting a multiple of 3 is two i.e; 3, 6

Mutually Exclusive Events : Two or more events are said to be mutually exclusive if both cannot occur simultaneously in the same experiment. **Example:** In a throw of single coin, either head can come or tail can come.

Equally Likely Cases: Two or more events are said to be equally likely if the possibility of their happening are equal. **Example:** In a throw of a unbiased dice , the coming up of head or dice is equally likely.

Independent Events: Two events are said to be independent of each other when the happening of one event does not affect the happening of other event and vice versa.

Example: 1. Choosing a 3 from a deck of cards, replacing it, **AND** then choosing an ace as the second card.

 2. Rolling a 4 on a single 6-sided die, **AND** then rolling a 1 on a second roll of the die.

When two events A and B are independent, the probability of both occurring is:

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

Dependent Events: Two **events** are **dependent** if the outcome or occurrence of the first affects the outcome or occurrence of the second so that the probability is changed.

When two events, A and B, are dependent, the probability of both occurring is:

$$P(A \text{ and } B) = P(A) \cdot P(B|A)$$

where **P(B|A)** is the **conditional probability** of an event B in relationship to an event A is the probability that event B occurs given that event A has already occurred.

Example: The probability of choosing a jack on the second pick given that a queen was chosen on the first pick (without replacement) is called a *conditional probability*.

Sample Space: The **sample space** of an experiment or random trial is the set of all possible outcomes or results of that experiment.

Example: If the experiment is tossing a coin, the sample space is typically the set {head, tail}.

For tossing two coins, the corresponding sample space would be {(head, head), (head, tail), (tail, head), (tail, tail)}.

For tossing a single six-sided die , the typical sample space is {1, 2, 3, 4, 5, 6} (in which the result of interest is the number of pips facing up).

PROBABILITY

Let S be the sample space and let E be the event.

PROBABILITY= n(E)/n(S)

Results on Probability:

- P(S) = 1
- $0 \leq P(E) \leq 1$
- For any events A and B we have : P(AUB) = P(A) + P(B) - P(AⴖB)
- If A' denotes (not-A), then P(A') = 1 - P(A).

Important Details

❖When we throw a coin, then either a Head (H) or a Tail (T) appears

❖A dice is a solid cube, having 6 faces, marked 1, 2, 3, 4, 5, 6 respectively. When we throw a die, the outcome is the number that appears on its upper face.

❖A pack of cards has 52 cards.
It has 13 cards of each suit, name **Spades, Clubs, Hearts and Diamonds**.
Cards of spades and clubs are **black cards**.
Cards of hearts and diamonds are **red cards**.
There are **Kings, Queens and Jacks**. These are all called **face cards**.

Odds of an event

Let E be an event associated with a random experiment. Let x outcomes are favourable to E and y outcomes are not favourable to E, then

- Odds in favour of E are x:y , i.e., x/y

- Odds against E are y:x , i.e., y/x

- P (E (in favor)) = x / x+y

- P (E^c (against)) = y / x+y

Q1. An unbiased die is tossed .find the probability of getting a multiple of 3.

A. 1/6
B. 1/3
C. 0
D. NONE OF THE ABOVE
Ans: b)

Q2. Three unbiased coins are tossed. What is the probability of getting at most two heads?

A. 3/8
B. 5/8
C. 1/8
D. 7/8

Ans: d)

Q3. Two dice are thrown simultaneously. What is the probability of getting two numbers whose product is even.

A.3 /4
B.11/36
C.23/36
D. 1/4

Ans: a)

Q4. In a lottery, there are 10 prizes and 25 blanks. A lottery is drawn at random. What is the probability of getting a prize?

A. 1/7
B. 2/7
C. 3/7
D. 4/7

Ans: b)

Q5. From a pack of 52 cards, two cards are drawn together at random. What is the probability of both the cards being kings?

A. 1/221

B. 5/221

C. 7/221

D. NONE OF THE ABOVE

Ans: a)

Q6. A bag contains 2 red, 3 green and 2 blue balls. Two balls are drawn at random. What is the probability that none of the balls drawn is blue?

A. 13/21

B. 11/21

C. 10/21

D. NONE OF THE ABOVE

Ans:c)

Q7. In a class, there are 15 boys and 10 girls. Three students are selected at random. The probability that 1 girl and 2 boys are selected, is

A. 25/117
B. 1/50
C. 21/46
D. 3/25

Ans: c)

Q8. Two cards are drawn at random from a pack of 52 cards. what is the probability that either both are black or both are queen.

A.331/1326
B.55/221
C.11/1326
D.NONE OF THE ABOVE

Q9. A bag contains 6 white and 4 black balls. 2 balls are drawn at random. Find the probability that they are of same color.

A. 7/15
B. 8/15
C. 1/5
D. 4/5

Ans: A)

Q10. A card is drawn from a pack of 52 cards. The probability of getting a queen of club or a king of heart is:

A.1/52
B.2/13
C.1/26
D.1/13

Ans: c)

Q11. A card is chosen at random from a standard deck of 52 playing cards. Without replacing it, a second card is chosen. What is the probability that the first card chosen is a queen and the second card chosen is a jack?

A. 4/52
B. 1/13
C. 1/26
D. 4/663

P(queen on first pick) = 4/52

P(jack on 2nd pick given queen on 1st pick) = 4/51

P(queen and jack) = (4/52)* (4/51) = 4/ 663

Q12. Mr. Parietti needs two students to help him with a science demonstration for his class of 18 girls and 12 boys. He randomly chooses one student who comes to the front of the room. He then chooses a second student from those still seated. What is the probability that both students chosen are girls?

A. 18/30
B. 18/870
C. 51/145
D. 29/340

P(Girl 1 and Girl 2) = P(Girl 1) and P(Girl 2|Girl 1)

$$= (18/30) \cdot (17/29) = 51/145$$

Q13. In a shipment of 20 computers, 3 are defective. Three computers are randomly selected and tested. What is the probability that all three are defective if the first and second ones are not replaced after being tested?

A. 3/20
B. 13/102
C. 380/1140
D. 1/1140

P(3 defectives) = (3/20).(2/19).(1/18) = 1/1140

Q14. A bag contains 12 white and 18 black balls. Two balls are drawn in succession without replacement. What is the probability that first is white and second is black?

A.2/5
B.2/15
C.36/145
D.29/40

Ans: c)

Q15. A die is rolled and a coin is tossed, find the probability that the die shows an odd number and the coin shows a head

A. 1/13
B. 3/13
C. 1/ 4
D. 4/9

Ans: c)

What are the odds in favour getting a 1 when a die is rolled?

A. 1/6

B. 1/3

C. 1/5

D. 2/5

Ans: c)

An urn B1 contains 2 white and 3 black balls and another urn B2 contains 3 white and 4 black balls. One urn is selected at random and a ball is drawn from it. If the ball drawn is found black, find the probability that the urn chosen was B1.

A. 17/41

B. 21/41

C. 21/ 43

D. 21/50

Ans: B

Q16. One card is drawn at random from a pack of 52 cards. What is the probability that the card drawn is a face card ?

A. 1/13
B. 3/13
C. 1/ 4
D. 4/9

Ans: a)

Q17. A man and his wife appear in an interview for two vacancies in the same post. The probability of husband's selection is (1/7) and the probability of wife's selection is (1/5). What is the probability that only one of them is selected ?

A.3/ 4
B.2/7
C.4/5
D.1/7

Ans: b)

Q18. Two brother X and Y appeared for an exam. The probability of selection of X is 1/7 and that of B is 2/9. Find the probability that both of them are selected.

A. 1/9
B. 2/9
C. 1/63
D. 2/63

Q19. In a class, 30% of the students offered English, 20% offered Hindi and 10% offered both. If a student is selected at random, what is the probability that he has offered English or Hindi ?

A. 1/2

B. 3/4

C. 2/5

D. 4/5

Ans: c)

Q20. If two letters are taken at random from the word HOME, what is the probability that none of the letters would be vowels?

A. 1/2

B. 1/4

C. 1/3

D. 1/6

Q21. The blood groups of 200 people is distributed as follows: 50 have type A blood, 65 have B blood type,70 have O blood type and 15 have type AB blood. If a person from this group is selected at random, what is the probability that this person has O blood type?

A. 0.5

B. 0.35

C. 0.25

D. 0.45

Ans: b)

Q22. What is the probability that the total of two dice will be greater than 8 given that the first die is a 6?

A. 1/4

B. 1/3

C. 2/3

D. 2/5

Ans: c)

Q23Two countries - Germany and France are participating in a hockey game. Mr. Randor Guy, the famous astrologer in Germany is able to predict the winner of each match with great success. It is rumoured that in a match between 2 teams X and Y,Rando Guy picks X with the same probability as X's chances of winning. Let's assume such rumours to be true and that in a match between Germany and France, Germany the stronger team has a probability of 7/10 of winning the game. What is the probability that Randor Guy will correctly pick the winner of the Germany - France game?

A.   0.58

B.   0.68

C.   0.82

D.   0.42

Ans: a)

Solution:

- Randor Guy predicts the winner in games with the same probability of winning. --> statement I-Germany has got a chance of winning the game with 7/10 probability. France has got a chance of winning the game with 3/10 probability.
Probability of Randor Guy picking the Germany correctly is 7/10 while picking France is 3/10 (as per statement I).

- Probability of Randor Guy picking the winning team correctly = (Probability of Germany winning the game) AND (Probability of Randor Guy picking Germany correctly) OR (Probability of France winning the game) AND (Probability of Randor Guy picking France correctly).

- Therefore, Probability of Randor Guy picking the winning team correctly = (Probability of Germany winning the game) * (Probability of Randor Guy picking Germany correctly) + (Probability of France winning the game) * (Probability of Randor Guy picking France correctly).
Probability of predicting winning team = (7/10 * 7/10) + (3/10 * 3/10)
=0.58

Q24. In an off campus placement programme, a software company recruiter interviewed 75 prospective candidates -- 10 from Civil engineering department, 5 from Bio-chemical department and rest from computer science department. If the software company finally issued offer letter to 17 candidates, what is the probability that all the selected candidates belonged to only computer science department?

A. 60C17/ 75C17

B. 65C17/75C17

C. 70C17/75C17

D. None of these

Ans: a)

Q25. There is a school were 60% are girls and 35% of the girls are poor. Students are selected at random, what is the probability of selecting a poor girl out of total strength?

A. 60/100

B. 21/100

C. 21/60

D. None of these

Ans: b)

Q26. One student to pass the subject as probability 1/4. another man to get driving licence and student passed subject probability 1/6. find the probability man get driving licence.

A. 1/24

B. 1/6

C. ¼

D. 2/3

Ans: d)

Solution:

- A , B are independent events so that
  p(A intersection B) = p(A).P(B)
  1/6 = 1/4 * x
  x = 2/3

Q27. If 3 dices are rolled and a,b,c is obtained what is the probbability that a*b+c is even?

A. 1/2

B. 1/6

C. ¼

D. 1/6

Ans: a)

Solution:

case 1: a,b,c is even

case 2: a is odd, b&c is even

case 3: a&c is even,b is odd

case 4:a,b,c is odd.

each case will have 27 outcomes

therefore, P=(27*4)/216

P=1/2

Q28. There is a set of 52 card.1 card fall down. Then we take 2 cards from the rest. That 2 cards are spade. What is the probablity of that fallen card is spade?

A. 1/4

B. 11/52

C. 13/51

D. None of these

Q29. If two dice are rolled to obtain a sum or 5 or 7 what is the probability of getting 5 before 7?

A. 1/4

B. 2/5

C. 5/36

D. None of these

Ans: b)

SOLUTION:

At each throw we have:

Probability of 5 = 4/36 = 1/9 Probability of 7 = 6/36 = 1/6

Probability of no result = 1 - (1/9 + 1/6)

= 13/18

now probability for getting 5 or 7 as sum wd gtting 5 before is

p = 1/9 + (13/18)(1/9) + (13/18)^2(1/9) + (13/18)^3(1/9) + ...

= (1/9)[1 + 13/18 + (13/18)^2 + (13/18)^3 + ...]

This is a GP with common ratio 13/18

= (1/9)/[1 -13/18]

= (1/9)/(5/18)

= 2/5

Q30. A can hit a target 6 times in 7 shots. B can hit the target 4 times in 5 shots, C can hit 3 times in 4 shots. What is the chance that the target is damaged by exactly 2 shots?

A. 18/140

B. 30/140

C. 40/140

D. 54/140

Ans: d)

Solution:

- We are supposed to find prob of hitting target by exactly two shots out of 3 shots.
  so desired combinations are
  A...B....C
  Hit hit miss
  Hit miss hit
  Miss hit hit
  person A will hit the target six out of seven times, so his hit probability is 6/7, and his miss probability is 1/7.
  B's hit probability is 4/5, and his miss probability is 1/5.
  C's hit probability is 3/4, and his miss probability is 1/4.
  So the probability that the target will be in (Miss hit hit)condition after the shooting = (1/7) * (4/5) * (3/4) = 12/140.

  prob of condition (hit miss hit) (6/7) * (1/5) * (3/4) = 18/140.
  prob of condition (hit hit miss)= (6/7) * (4/5) * (1/4) = 24/140.
  The sum of the three probabilities = prob of hitting target exactly twice 54/140