Solution manual for tutorial sheet 5

1. The position of a particle which moves along a straight line is defined by the relation $x = t^3 - 6t^2 - 15t + 40$, where x is expressed in feet and t in seconds. Determine (*a*) the time at which the velocity will be zero, (*b*) the position and distance traveled by the particle at that time, (*c*) the acceleration of the particle at that time, (*d*) the distance traveled by the particle from t = 4 s to t = 6 s.

The equations of motion are

$$x = t^3 - 6t^2 - 15t + 40 \quad (1)$$

$$v = \frac{dx}{dt} = 3t^2 - 12t - 15 \quad (2)$$

$$a = \frac{dv}{dt} = 6t - 12 \quad (3)$$

a. Time at Which $v = 0$. We set $v = 0$ in (2):

$$3t^2 - 12t - 15 = 0 \qquad t = -1 \text{ s} \qquad \text{and} \qquad t = +5 \text{ s} \blacktriangleleft$$

Only the root $t = +5$ s corresponds to a time after the motion has begun: for $t < 5$ s, $v < 0$, the particle moves in the negative direction; for $t > 5$ s, $v > 0$, the particle moves in the positive direction.

b. Position and Distance Traveled When $v = 0$. Carrying $t = +5$ s into (1), we have

$$x_5 = (5)^3 - 6(5)^2 - 15(5) + 40 \qquad x_5 = -60 \text{ ft} \blacktriangleleft$$

The initial position at $t = 0$ was $x_0 = +40$ ft. Since $v \neq 0$ during the interval $t = 0$ to $t = 5$ s, we have

$$\text{Distance traveled} = x_5 - x_0 = -60 \text{ ft} - 40 \text{ ft} = -100 \text{ ft}$$

$$\text{Distance traveled} = 100 \text{ ft in the negative direction} \blacktriangleleft$$

c. Acceleration When $v = 0$. We substitute $t = +5$ s into (3):

$$a_5 = 6(5) - 12 \qquad a_5 = +18 \text{ ft/s}^2 \blacktriangleleft$$

d. Distance Traveled from $t = 4$ s to $t = 6$ s. The particle moves in the negative direction from $t = 4$ s to $t = 5$ s and in the positive direction from $t = 5$ s to $t = 6$ s; therefore, the distance traveled during each of these time intervals will be computed separately.

From $t = 4$ s to $t = 5$ s: $\quad x_5 = -60$ ft

$$x_4 = (4)^3 - 6(4)^2 - 15(4) + 40 = -52 \text{ ft}$$

$$\begin{aligned}\text{Distance traveled} &= x_5 - x_4 = -60 \text{ ft} - (-52 \text{ ft}) = -8 \text{ ft} \\ &= 8 \text{ ft in the negative direction}\end{aligned}$$

From $t = 5$ s to $t = 6$ s: $\quad x_5 = -60$ ft

$$x_6 = (6)^3 - 6(6)^2 - 15(6) + 40 = -50 \text{ ft}$$

$$\begin{aligned}\text{Distance traveled} &= x_6 - x_5 = -50 \text{ ft} - (-60 \text{ ft}) = +10 \text{ ft} \\ &= 10 \text{ ft in the positive direction}\end{aligned}$$

Total distance traveled from $t = 4$ s to $t = 6$ s is 8 ft + 10 ft $= 18$ ft $\blacktriangleleft$

2. The motion of a particle is defined by the relation $x = 6t^4 - 2t^3 - 12t^2 + 3t + 3$, where x and t are expressed in meters and seconds, respectively. Determine the time, the position, and the velocity when a = 0.

SOLUTION

We have $$x = 6t^4 - 2t^3 - 12t^2 + 3t + 3$$

Then $$v = \frac{dx}{dt} = 24t^3 - 6t^2 - 24t + 3$$

and $$a = \frac{dv}{dt} = 72t^2 - 12t - 24$$

When $a = 0$: $$72t^2 - 12t - 24 = 12(6t^2 - t - 2) = 0$$

or $$(3t - 2)(2t + 1) = 0$$

or $$t = \frac{2}{3}\text{ s} \quad \text{and} \quad t = -\frac{1}{2}\text{ s} \quad \text{(Reject)} \qquad t = 0.667\text{ s} \blacktriangleleft$$

At $t = \frac{2}{3}$ s: $$x_{2/3} = 6\left(\frac{2}{3}\right)^4 - 2\left(\frac{2}{3}\right)^3 - 12\left(\frac{2}{3}\right)^2 + 3\left(\frac{2}{3}\right) + 3 \quad \text{or} \quad x_{2/3} = 0.259\text{ m} \blacktriangleleft$$

$$v_{2/3} = 24\left(\frac{2}{3}\right)^3 - 6\left(\frac{2}{3}\right)^2 - 24\left(\frac{2}{3}\right) + 3 \quad \text{or} \quad v_{2/3} = -8.56\text{ m/s} \blacktriangleleft$$

3 The motion of a particle is defined by the relation $x = \frac{5}{3}t^3 - \frac{5}{2}t^2 - 30t + 8$ where x and t are expressed in meters and seconds, respectively. Determine the time, the position, and the acceleration when $v = 0$.

SOLUTION

We have $$x = \frac{5}{3}t^3 - \frac{5}{2}t^2 - 30t + 8$$

Then $$v = \frac{dx}{dt} = 5t^2 - 5t - 30$$

and $$a = \frac{dv}{dt} = 10t - 5$$

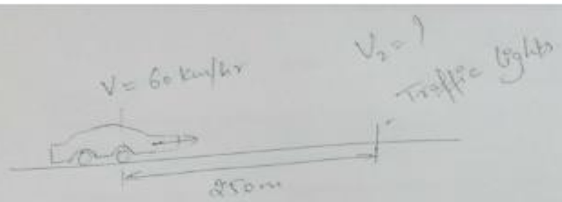
When $v = 0$: $$5t^2 - 5t - 30 = 5(t^2 - t - 6) = 0$$

or $$t = 3\text{ s} \quad \text{and} \quad t = -2\text{ s} \quad \text{(Reject)} \qquad t = 3.00\text{ s} \blacktriangleleft$$

At $t = 3$ s: $$x_3 = \frac{5}{3}(3)^3 - \frac{5}{2}(3)^2 - 30(3) + 8 \quad \text{or} \quad x_3 = -59.5\text{ m} \blacktriangleleft$$

$$a_3 = 10(3) - 5 \quad \text{or} \quad a_3 = 25.0\text{ m/s}^2 \blacktriangleleft$$

4. A driver is driving his car at 60 km/hr when he observes that a traffic light 250 m ahead turns red. The traffic light is timed to remain red for 20 seconds before it turns green. The driver wishes to pass the traffic lights without stopping to wait for it turns green. Calculate (a) the required uniform acceleration of the car, (b) the speed of the car as it passes the traffic light. [Ans: a= -0.417 m/sec2, v= 8.33 m/sec]



$t = 20 sec$

a) The required uniform acceleration of the car

$v^2 - u^2 = 2as$

Initial velocity $(u) = 60 \text{ km/hr} = 60 \times \frac{5}{18} = 16.67 \text{ m/s}$

final velocity $(v) = ?$, time $= 20 sec$, $S = 250m$

$S = ut + \frac{1}{2}at^2$

$250 = (16.67 \times 20) + \frac{1}{2} \times a \times 20^2$

$250 = (333.4) + 200a$

$250 - 333.4 = 200a$

$-83.4 = 200a$

$a = -\frac{83.4}{200} \Rightarrow \boxed{-0.417 \text{ m/s}^2}$

b) final velocity $(v) =$

$v^2 - u^2 = 2as$

$v^2 - (16.67)^2 = 2 \times (-0.417) \times 250$

$v^2 = 277.89 - 208.5$

$v = \sqrt{69.39} \Rightarrow \boxed{8.33 \text{ m/s}}$

The motion of a particle is defined by the relation $x = 6t^2 - 8 + 40 \cos \pi t$, where x and t are expressed in meters and seconds, respectively. Determine the position, the velocity, and the acceleration when $t = 6$ s.

SOLUTION

We have $\quad x = 6t^2 - 8 + 40 \cos \pi t$

Then $\quad v = \frac{dx}{dt} = 12t - 40\pi \sin \pi t$

and $\quad a = \frac{dv}{dt} = 12 - 40\pi^2 \cos \pi t$

At $t = 6$ s:

$x_6 = 6(6)^2 - 8 + 40 \cos 6\pi \quad$ or $\quad x_6 = 248 \text{ m}$ ◀

$v_6 = 12(6) - 40\pi \sin 6\pi \quad$ or $\quad v_6 = 72.0 \text{ m/s}$ ◀

$a_6 = 12 - 40\pi^2 \cos 6\pi \quad$ or $\quad a_6 = -383 \text{ m/s}^2$ ◀

6. A ball is tossed with a velocity of 10 m/s directed vertically upward from a window located 20 m above the ground. Knowing that the acceleration of the ball is constant and equal to 9.81 m/s2 downward, determine (*a*) the velocity *v* and elevation *y* of the ball above the ground at any time *t*, (*b*) the highest elevation reached by the ball and the corresponding value of *t*, (*c*) the time when the ball will hit the ground and the corresponding velocity. [t= 1.012 sec, y=25.1]

SOLUTION

a. Velocity and Elevation. The y axis measuring the position coordinate (or elevation) is chosen with its origin O on the ground and its positive sense upward. The value of the acceleration and the initial values of v and y are as indicated. Substituting for a in $a = dv/dt$ and noting that at $t = 0$, $v_0 = +10$ m/s, we have

$$\frac{dv}{dt} = a = -9.81 \text{ m/s}^2$$

$$\int_{v_0=10}^{v} dv = -\int_0^t 9.81\, dt$$

$$[v]_{10}^{v} = -[9.81t]_0^t$$

$$v - 10 = -9.81t$$

$$v = 10 - 9.81t \quad (1)$$

Substituting for v in $v = dy/dt$ and noting that at $t = 0$, $y_0 = 20$ m, we have

$$\frac{dy}{dt} = v = 10 - 9.81t$$

$$\int_{y_0=20}^{y} dy = \int_0^t (10 - 9.81t)\, dt$$

$$[y]_{20}^{y} = [10t - 4.905t^2]_0^t$$

$$y - 20 = 10t - 4.905t^2$$

$$y = 20 + 10t - 4.905t^2 \quad (2)$$

b. Highest Elevation. When the ball reaches its highest elevation, we have $v = 0$. Substituting into (1), we obtain

$$10 - 9.81t = 0 \qquad t = 1.019 \text{ s}$$

Carrying $t = 1.019$ s into (2), we have

$$y = 20 + 10(1.019) - 4.905(1.019)^2 \qquad y = 25.1 \text{ m}$$

c. Ball Hits the Ground. When the ball hits the ground, we have $y = 0$. Substituting into (2), we obtain

$$20 + 10t - 4.905t^2 = 0 \qquad t = -1.243 \text{ s} \quad \text{and} \quad t = +3.28 \text{ s}$$

Only the root $t = +3.28$ s corresponds to a time after the motion has begun. Carrying this value of t into (1), we have

$$v = 10 - 9.81(3.28) = -22.2 \text{ m/s} \qquad v = 22.2 \text{ m/s} \downarrow$$

The acceleration of a particle is defined by the relation $a = kt^2$. (*a*) Knowing that $v = -10$ m/s when $t = 0$ and that $v = +10$ m/s when $t = 4$ s, determine the constant k. (*b*) Write the equations of motion, knowing also that $x = 0$ when $t = 4$ s.

$$a = kt^2 \tag{1}$$

$$\frac{dv}{dt} = a = kt^2$$

$t = 0$, $v = -10$ m/s and $t = 4$ s, $v = +10$ m/s

(*a*)
$$\int_{-10}^{10} dv = \int_0^4 kt^2\, dt$$

$$10 - (-10) = \frac{1}{3}k(4)^3 \qquad k = 0.9375 \text{ m/s}^4 \blacktriangleleft$$

(*b*) Substituting $k = 0.9375 \text{ m/s}^4$ into (1)

$$\frac{dv}{dt} = a = 0.9375t^2 \qquad a = 0.9375t^2 \blacktriangleleft$$

$t = 0$, $v = -10$ m/s:
$$\int_{-10}^{v} dv = \int_0^t 0.9375t^2\, dt$$

$$v - (-10) = \frac{1}{3}0.9375(t)^3 \qquad v = (0.3125t^3 - 10) \blacktriangleleft$$

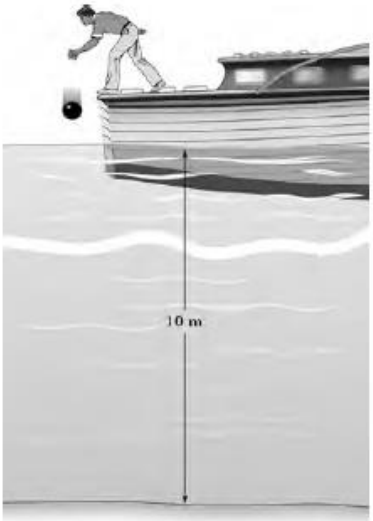
$$\frac{dx}{dt} = v = 0.3125t^3 - 10$$

$t = 4$ s, $x = 0$:
$$\int_0^x dx = \int_4^t (0.3125t^3 - 10)\, dt; \quad x = \left| 0.3125\frac{t^4}{4} - 10t \right|_4^t$$

$$x = \left[\frac{0.3125t^4}{4} - 10t\right] - \left[\frac{0.3125(4)^4}{4} - 10(4)\right]$$

$$x = \left(\frac{0.3125t^4}{4} - 10t\right) - 19.968 + 40$$

$$x = 0.078125t^4 - 10t + 20.032 \blacktriangleleft$$



PROBLEM 11.24

A bowling ball is dropped from a boat so that it strikes the surface of a lake with a speed of 8 m/s. Assuming the ball experiences a downward acceleration of $a = 3 - 0.1v^2$ when in the water, determine the velocity of the ball when it strikes the bottom of the lake.

SOLUTION

$$v_0 = 8 \text{ m/s}, \quad x - x_0 = 10 \text{ m}$$

$$a = 3 - 0.1v^2 = k(c^2 - v^2)$$

where
$$k = 0.1 \text{ m}^{-1} \quad \text{and} \quad c^2 = \frac{3}{0.1} = 30\frac{\text{m}^2}{\text{s}^2}$$

$$c = 5.4772 \text{ m/s}$$

Since $v_0 > c$, write

$$a = v\frac{dv}{dx} = -k(v^2 - c^2)$$

$$\frac{v\,dv}{v^2 - c^2} = -k\,dx$$

Integrating,
$$\frac{1}{2}\ln(v^2 - c^2)\Big|_{v_0}^{v} = -k(x - x_0)$$

$$\ln\frac{v^2 - c^2}{v_0^2 - c^2} = -2k(x - x_0)$$

$$\frac{v^2 - c^2}{v_0^2 - c^2} = e^{-2k(x - x_0)}$$

$$v^2 = c^2 + \left(v_0^2 - c^2\right)e^{-2k(x - x_0)}$$

$$= 30 + [(8)^2 - 30]\,e^{-2(0.1)(10)}$$

$$= 34.6014 \text{ m}^2/\text{s}^2$$

$v = 5.88$ m/s ◄

Example 21.12. *The equation for angular displacement of a body moving on a circular path is given by :*

$$\theta = 2t^3 + 0.5$$

where θ is in rad and t in sec. Find angular velocity, displacement and acceleration after 2 sec.

Solution. Given : Equation for angular displacement $\theta = 2t^3 + 0.5$ ...(*i*)

Angular displacement after 2 seconds

Substituting $t = 2$ in equation (*i*),

$$\theta = 2\,(2)^3 + 0.5 = 16.5 \text{ rad} \quad \textbf{Ans.}$$

Angular velocity after 2 seconds

Differentiating both sides equation (*i*) with respect to *t*,

$$\frac{d\theta}{dt} = 6\,t^2 \qquad \text{...(ii)}$$

or velocity, $\omega = 6\,t^2$ ...(*iii*)

Substituting $t = 2$ in equation (*iii*),

$$\omega = 6\,(2)^2 = 24 \text{ rad/sec} \quad \textbf{Ans.}$$

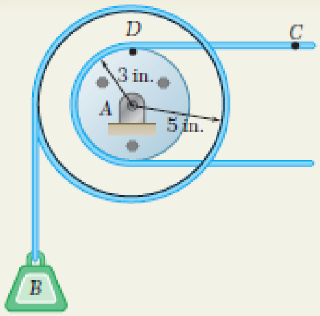
Angular acceleration after 2 seconds

Differentiating both sides of equation (*iii*) with respect to *t*,

$$\frac{d\omega}{dt} = 12t \text{ or Acceleration } \alpha = 12t$$

Now substituting $t = 2$ in above equation,

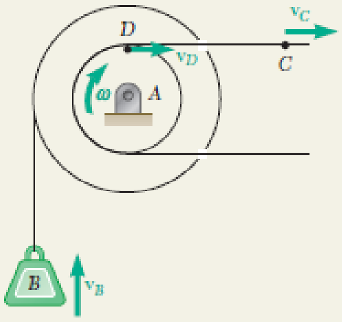
$$\alpha = 12 \times 2 = 24 \text{ rad/sec}^2 \quad \textbf{Ans.}$$



SAMPLE PROBLEM 15.1

Load B is connected to a double pulley by one of the two inextensible cables shown. The motion of the pulley is controlled by cable C, which has a constant acceleration of 9 in./s^2 and an initial velocity of 12 in./s, both directed to the right. Determine (*a*) the number of revolutions executed by the pulley in 2 s, (*b*) the velocity and change in position of the load B after 2 s, and (*c*) the acceleration of point D on the rim of the inner pulley at $t = 0$.

SOLUTION



a. Motion of Pulley. Since the cable is inextensible, the velocity of point D is equal to the velocity of point C and the tangential component of the acceleration of D is equal to the acceleration of C.

$$(\mathbf{v}_D)_0 = (\mathbf{v}_C)_0 = 12 \text{ in./s} \rightarrow \qquad (\mathbf{a}_D)_t = \mathbf{a}_C = 9 \text{ in./s}^2 \rightarrow$$

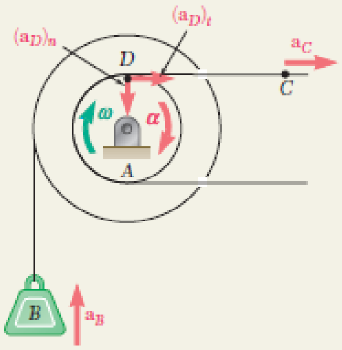
Noting that the distance from D to the center of the pulley is 3 in., we write

$$(v_D)_0 = r\omega_0 \qquad 12 \text{ in./s} = (3 \text{ in.})\omega_0 \qquad \boldsymbol{\omega}_0 = 4 \text{ rad/s} \downarrow$$
$$(a_D)_t = r\alpha \qquad 9 \text{ in./s}^2 = (3 \text{ in.})\alpha \qquad \boldsymbol{\alpha} = 3 \text{ rad/s}^2 \downarrow$$

Using the equations of uniformly accelerated motion, we obtain, for $t = 2$ s,

$$\omega = \omega_0 + \alpha t = 4 \text{ rad/s} + (3 \text{ rad/s}^2)(2 \text{ s}) = 10 \text{ rad/s}$$
$$\boldsymbol{\omega} = 10 \text{ rad/s} \downarrow$$
$$\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2 = (4 \text{ rad/s})(2 \text{ s}) + \tfrac{1}{2}(3 \text{ rad/s}^2)(2 \text{ s})^2 = 14 \text{ rad}$$
$$\theta = 14 \text{ rad} \downarrow$$

$$\text{Number of revolutions} = (14 \text{ rad})\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = 2.23 \text{ rev} \quad ◀$$

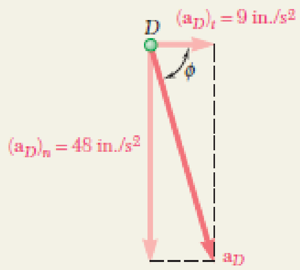


b. Motion of Load *B*. Using the following relations between linear and angular motion, with $r = 5$ in., we write

$$v_B = r\omega = (5 \text{ in.})(10 \text{ rad/s}) = 50 \text{ in./s} \qquad \mathbf{v}_B = 50 \text{ in./s} \uparrow \quad ◀$$
$$\Delta y_B = r\theta = (5 \text{ in.})(14 \text{ rad}) = 70 \text{ in.} \qquad \Delta y_B = 70 \text{ in. upward} \quad ◀$$

c. Acceleration of Point *D* at *t* = 0. The tangential component of the acceleration is

$$(\mathbf{a}_D)_t = \mathbf{a}_C = 9 \text{ in./s}^2 \rightarrow$$

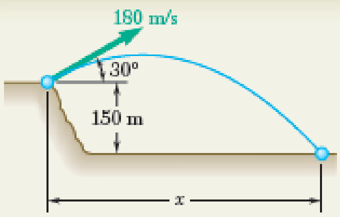


Since, at $t = 0$, $\omega_0 = 4$ rad/s, the normal component of the acceleration is

$$(a_D)_n = r_D\omega_0^2 = (3 \text{ in.})(4 \text{ rad/s})^2 = 48 \text{ in./s}^2 \qquad (\mathbf{a}_D)_n = 48 \text{ in./s}^2 \downarrow$$

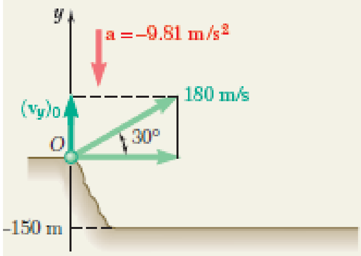
The magnitude and direction of the total acceleration can be obtained by writing

$$\tan\phi = (48 \text{ in./s}^2)/(9 \text{ in./s}^2) \qquad \phi = 79.4°$$
$$a_D \sin 79.4° = 48 \text{ in./s}^2 \qquad a_D = 48.8 \text{ in./s}^2$$
$$\mathbf{a}_D = 48.8 \text{ in./s}^2 \measuredangle 79.4° \quad ◀$$



SAMPLE PROBLEM 11.7

A projectile is fired from the edge of a 150-m cliff with an initial velocity of 180 m/s at an angle of 30° with the horizontal. Neglecting air resistance, find (*a*) the horizontal distance from the gun to the point where the projectile strikes the ground, (*b*) the greatest elevation above the ground reached by the projectile.



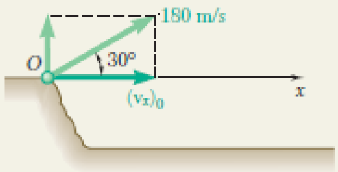
The vertical and the horizontal motion will be considered separately.

Vertical Motion. ***Uniformly Accelerated Motion.*** Choosing the positive sense of the y axis upward and placing the origin O at the gun, we have

$$(v_y)_0 = (180 \text{ m/s}) \sin 30° = +90 \text{ m/s}$$
$$a = -9.81 \text{ m/s}^2$$

Substituting into the equations of uniformly accelerated motion, we have

$$v_y = (v_y)_0 + at \qquad v_y = 90 - 9.81t \tag{1}$$
$$y = (v_y)_0 t + \tfrac{1}{2}at^2 \qquad y = 90t - 4.90t^2 \tag{2}$$
$$v_y^2 = (v_y)_0^2 + 2ay \qquad v_y^2 = 8100 - 19.62y \tag{3}$$



Horizontal Motion. ***Uniform Motion.*** Choosing the positive sense of the x axis to the right, we have

$$(v_x)_0 = (180 \text{ m/s}) \cos 30° = +155.9 \text{ m/s}$$

Substituting into the equation of uniform motion, we obtain

$$x = (v_x)_0 t \qquad x = 155.9t \tag{4}$$

a. Horizontal Distance. When the projectile strikes the ground, we have

$$y = -150 \text{ m}$$

Carrying this value into Eq. (2) for the vertical motion, we write

$$-150 = 90t - 4.90t^2 \qquad t^2 - 18.37t - 30.6 = 0 \qquad t = 19.91 \text{ s}$$

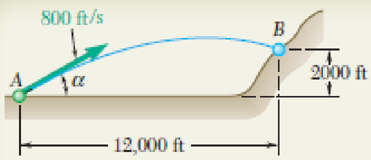
Carrying $t = 19.91$ s into Eq. (4) for the horizontal motion, we obtain

$$x = 155.9(19.91) \qquad x = 3100 \text{ m} \quad ◀$$

b. Greatest Elevation. When the projectile reaches its greatest elevation, we have $v_y = 0$; carrying this value into Eq. (3) for the vertical motion, we write

$$0 = 8100 - 19.62y \qquad y = 413 \text{ m}$$

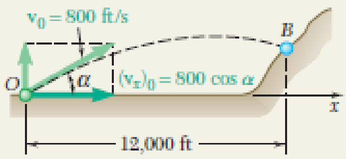
Greatest elevation above ground = 150 m + 413 m = 563 m ◀



SAMPLE PROBLEM 11.8

A projectile is fired with an initial velocity of 800 ft/s at a target B located 2000 ft above the gun A and at a horizontal distance of 12,000 ft. Neglecting air resistance, determine the value of the firing angle α.

SOLUTION



The horizontal and the vertical motion will be considered separately.

Horizontal Motion. Placing the origin of the coordinate axes at the gun, we have

$$(v_x)_0 = 800 \cos \alpha$$

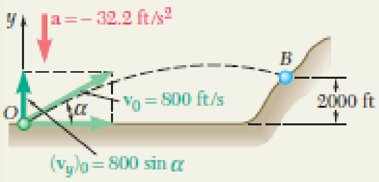
Substituting into the equation of uniform horizontal motion, we obtain

$$x = (v_x)_0 t \qquad x = (800 \cos \alpha)t$$

The time required for the projectile to move through a horizontal distance of 12,000 ft is obtained by setting x equal to 12,000 ft.

$$12{,}000 = (800 \cos \alpha)t$$

$$t = \frac{12{,}000}{800 \cos \alpha} = \frac{15}{\cos \alpha}$$



Vertical Motion

$$(v_y)_0 = 800 \sin \alpha \qquad a = -32.2 \text{ ft/s}^2$$

Substituting into the equation of uniformly accelerated vertical motion, we obtain

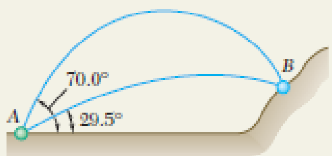
$$y = (v_y)_0 t + \tfrac{1}{2}at^2 \qquad y = (800 \sin \alpha)t - 16.1t^2$$

Projectile Hits Target. When $x = 12{,}000$ ft, we must have $y = 2000$ ft. Substituting for y and setting t equal to the value found above, we write

$$2000 = 800 \sin \alpha \frac{15}{\cos \alpha} - 16.1\left(\frac{15}{\cos \alpha}\right)^2$$

Since $1/\cos^2 \alpha = \sec^2 \alpha = 1 + \tan^2 \alpha$, we have

$$2000 = 800(15) \tan \alpha - 16.1(15^2)(1 + \tan^2 \alpha)$$
$$3622 \tan^2 \alpha - 12{,}000 \tan \alpha + 5622 = 0$$

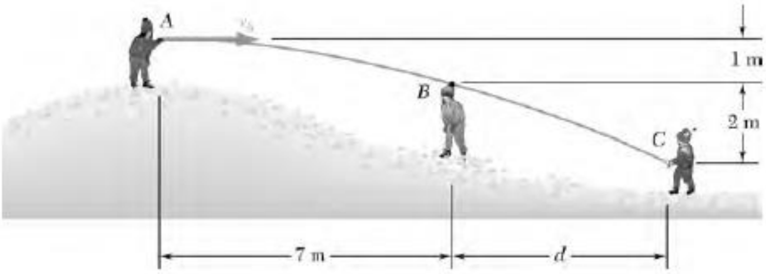


Solving this quadratic equation for $\tan \alpha$, we have

$$\tan \alpha = 0.565 \qquad \text{and} \qquad \tan \alpha = 2.75$$

$$\alpha = 29.5° \qquad \text{and} \qquad \alpha = 70.0° \quad ◀$$

The target will be hit if either of these two firing angles is used (see figure).

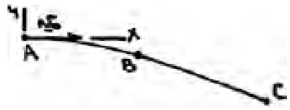


PROBLEM 11.98

Three children are throwing snowballs at each other. Child A throws a snowball with a horizontal velocity $\mathbf{v}_0$. If the snowball just passes over the head of child B and hits child C, determine (*a*) the value of v_0, (*b*) the distance d.

SOLUTION

(*a*) Vertical motion. (Uniformly accelerated motion)



$$y = 0 + (0)t - \frac{1}{2}gt^2$$

At B: $$-1\text{ m} = -\frac{1}{2}(9.81\text{ m/s}^2)t^2 \quad \text{or} \quad t_B = 0.451524\text{ s}$$

Horizontal motion (Uniform)

$$x = 0 + (v_x)_0 t$$

At B: $$7\text{ m} = v_0(0.451524\text{ s})$$

or $$v_0 = 15.5031\text{ m/s}$$

$v_0 = 15.50\text{ m/s}$ ◀

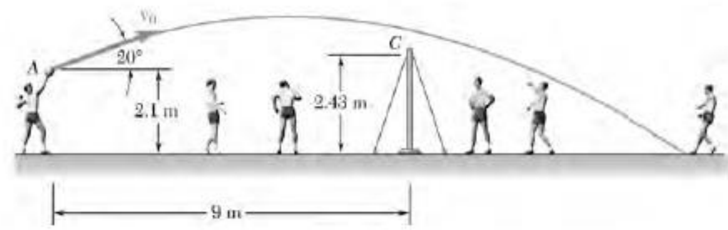
(*b*) Vertical motion: At C: $$-3\text{ m} = -\frac{1}{2}(9.81\text{ m/s}^2)t^2$$

or $$t_C = 0.782062\text{ s}$$

Horizontal motion.

At C: $$(7 + d)\text{ m} = (15.5031\text{ m/s})(0.782062\text{ s})$$

or $d = 5.12\text{ m}$ ◀



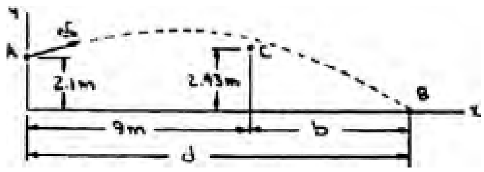
PROBLEM 11.101

A volleyball player serves the ball with an initial velocity $\mathbf{v}_0$ of magnitude 13.40 m/s at an angle of 20° with the horizontal. Determine (*a*) if the ball will clear the top of the net, (*b*) how far from the net the ball will land.

SOLUTION

First note

$$(v_x)_0 = (13.40 \text{ m/s}) \cos 20° = 12.5919 \text{ m/s}$$
$$(v_y)_0 = (13.40 \text{ m/s}) \sin 20° = 4.5831 \text{ m/s}$$



(*a*) Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t$$

At C $\quad 9 \text{ m} = (12.5919 \text{ m/s})t \quad \text{or} \quad t_C = 0.71475 \text{ s}$

Vertical motion. (Uniformly accelerated motion)

$$y = y_0 + (v_y)_0 t - \frac{1}{2}gt^2$$

At C:

$$\begin{aligned} y_C &= 2.1 \text{ m} + (4.5831 \text{ m/s})(0.71475 \text{ s}) \\ &\quad -\frac{1}{2}(9.81 \text{ m/s}^2)(0.71475 \text{ s})^2 \\ &= 2.87 \text{ m} \end{aligned}$$

$y_C > 2.43$ m (height of net) $\Rightarrow$ ball clears net ◄

(*b*) At B, $y = 0$: $\quad 0 = 2.1 \text{ m} + (4.5831 \text{ m/s})t - \frac{1}{2}(9.81 \text{ m/s}^2)t^2$

Solving $\quad t_B = 1.271175$ s (the other root is negative)

Then

$$\begin{aligned} d &= (v_x)_0 t_B = (12.5919 \text{ m/s})(1.271175 \text{ s}) \\ &= 16.01 \text{ m} \end{aligned}$$

The ball lands $\quad b = (16.01 - 9.00) \text{ m} = 7.01 \text{ m}$ from the net ◄