# Problem Formulation of PSO algorithm

**Problem:** Find the maximum of the function

$$f(x) = -x^2 + 5x + 20 \text{ with } -10 \leq x \leq 10$$

using the PSO algorithm. Use 9 particles with the initial positions $x_1 = -9.6$, $x_2 = -6$, $x_3 = -2.6$, $x_4 = -1.1$, $x_5 = 0.6$, $x_6 = 2.3$, $x_7 = 2.8$, $x_8 = 8.3$, and $x_9 = 10$. Show the detailed computations for iterations 1, 2 and 3.

Step1: Choose the number of particles: $x_1 = -9.6$ , $x_2 = -6$, $x_3 = -2.6$, $x_4 = -1.1$, $x_5 = 0.6$, $x_6 = 2.3$, $x_7 = 2.8$, $x_8 = 8.3$, and $x_9 = 10$.

The initial population (i.e. the iteration number $t = 0$) can be represented as $x_i^0, i = 1,2,3,4,5,6,7,8,9$:

$$x_1^0 = -9.6\ , x_2^0 = -6, x_3^0 = -2.6,$$
$$x_4^0 = -1.1, x_5^0 = 0.6, x_6^0 = 2.3,$$
$$x_7^0 = 2.8, \quad x_8^0 = 8.3, x_9^0 = 10.$$

Evaluate the objective function values as

$$f_1^0 = -120.16, f_2^0 = -46, f_3^0 = 0.24,$$
$$f_4^0 = 13.29, f_5^0 = 22.64\ , f_6^0 = 26.21,$$
$$f_7^0 = 26.16, f_8^0 = -7.39, f_9^0 = -30.$$

Let $c_1 = c_2 = 1$.

Set the initial velocities of each particle to zero:

$$v_i^0 = 0, i.e.\ v_1^0 = v_2^0 = v_3^0 = v_4^0 = v_5^0 = v_6^0 = v_7^0 = v_8^0 = v_9^0 = 0.$$

Step2: Set the iteration number as $t = 0 + 1 = 1$ and go to step 3.

Step3: Find the personal best for each particle by

$$P_{best,i}^{t+1} = \begin{cases} P_{best,i}^{t} & \text{if } f_i^{t+1} > P_{best,i}^{t} \\ x_i^{t+1} & \text{if } f_i^{t+1} \le P_{best,i}^{t} \end{cases}$$

So,

$P_{best,1}^{1} = -9.6$, $P_{best,2}^{1} = -6$, $P_{best,3}^{1} = -2.6$,

$P_{best,4}^{1} = -1.1$, $P_{best,5}^{1} = 0.6$, $P_{best,6}^{1} = 2.3$,

$P_{best,7}^{1} = 2.8$, $P_{best,8}^{1} = 8.3$, $P_{best,9}^{1} = 10$.

Step4: Find the global best by

$$G_{best} = \max\{P^t_{best,i}\} \text{ where } i = 1,2,3,4,5,6,7,8,9.$$

Since, the maximum personal best is $P^1_{best,6} = 2.3$ , thus $G_{best} = 2.3$.

Step5: Considering the random numbers in the range (0, 1) as $r^1_1 = 0.213$ and $r^1_2 = 0.876$, and find the velocities of the particles by

$$v^{t+1}_i = v^t_i + c_1 r^t_1 [P^t_{best,i} - x^t_i] + c_2 r^t_2 [G^t_{best} - x^t_i];\; i = 1, \ldots ,9.$$

so

$$v^1_1 = 0 + 0.213(-9.6 + 9.6) + 0.876(2.3 + 9.6) = 10.4244$$

$$v^1_2 = 7.2708,\; v^1_3 = 4.2924,\; v^1_4 = 2.9784,\; v^1_5 = 1.4892,$$

$$v^1_6 = 0,\;\; v^1_7 = -0.4380,\quad v^1_8 = 5.256,\quad v^1_9 = -6.7452.$$

Step6: Find the new values of $x_i^1, i = 1, \ldots, 9$ by

$$x_i^{t+1} = x_i^t + v_i^{t+1}$$

So

$x_1^1 = 0.8244, x_2^1 = 1.2708, x_3^1 = 1.6924,$

$x_4^1 = 1.8784, x_5^1 = 2.0892, x_6^1 = 2.3,$

$x_7^1 = 2.362, x_8^1 = 3.044, x_9^1 = 3.2548.$

Step7: Find the objective function values of $x_i^1, i = 1, \ldots, 9$:

$f_1^1 = 23.4424, f_2^1 = 24.7391, f_3^1 = 25.5978,$

$f_4^1 = 25.8636, f_5^1 = 26.0812, f_6^1 = 26.21,$

$f_7^1 = 26.231, f_8^1 = 25.9541, f_9^1 = 25.6803.$

Step 8: Stopping criterion:

If the terminal rule is satisfied, go to step 2,
Otherwise stop the iteration and output the results.

Step2: Set the iteration number as $t = 1 + 1 = 2$, and go to step 3.

Step3: Find the personal best for each particle.

$$P^2_{best,1} = 0.8244, P^2_{best,2} = 1.2708, P^2_{best,3} = 1.6924,$$
$$P^2_{best,4} = 1.8784, P^2_{best,5} = 2.0892, P^2_{best,6} = 2.3,$$
$$P^2_{best,7} = 2.362, P^2_{best,8} = 3.044, P^2_{best,9} = 3.2548.$$

Step4: Find the global best.

$$G_{best} = 2.362 .$$

Step5: By considering the random numbers in the range (0, 1) as $r_1^2 = 0.113$ and $r_2^2 = 0.706$ , find the velocities of the particles by

$$v_i^{t+1} = v_i^t + c_1 r_1^t [P^t_{best,i} - x_i^t] + c_2 r_2^t [G_{best} - x_i^t]; \; i = 1, \dots, 9.$$

so

$$v_1^2 = 11.5099, v_2^2 = 8.0412, v_3^2 = 4.7651,$$
$$v_4^2 = 3.3198, v_5^2 = 1.6818, v_6^2 = 0.0438,$$
$$v_7^2 = -0.4380, v_8^2 = -5.7375, v_9^2 = -7.3755.$$

Step6: Find the new values of $x_i^2$ , $i = 1, \dots, 9$ by

$$x_i^{t+1} = x_i^t + v_i^{t+1}$$

so

$x_1^2 = 12.3343, x_2^2 = 9.312, x_3^2 = 6.4575,$

$x_4^2 = 5.1982, x_5^2 = 3.7710, x_6^2 = 2.3438,$

$x_7^2 = 1.9240, x_8^2 = -2.6935, x_9^2 = -4.1207.$

Step7: Find the objective function values of $f_i^2, i = 1, \dots, 9$:

$f_1^2 = -70.4644, f_2^2 = -20.1532, f_3^2 = 10.5882,$

$f_4^2 = 18.9696, \quad f_5^2 = 24.6346, \quad f_6^2 = 26.2256,$

$f_7^2 = 25.9182, f_8^2 = -0.7224, \quad f_9^2 = -17.5839.$

Step 8: Stopping criterion:

If the terminal rule is satisfied, go to step 2,
Otherwise stop the iteration and output the results.

Step2: Set the iteration number as $t = 1 + 1 = 3$, and go to step 3.

Step3: Find the personal best for each particle.

$$P^3_{best,1} = 0.8244, P^3_{best,2} = 1.2708, P^3_{best,3} = 1.6924,$$
$$P^3_{best,4} = 1.8784, P^3_{best,5} = 2.0892, P^3_{best,6} = 2.3438,$$
$$P^3_{best,7} = 2.362, P^3_{best,8} = 3.044, \quad P^3_{best,9} = 3.2548.$$

Step4: Find the global best.

$$G_{best} = 2.362$$

Step5: By considering the random numbers in the range (0, 1) as $r_1^3 = 0.178$ and $r_2^3 = 0.507$, find the velocities of the particles by

$$v_i^{t+1} = v_i^t + c_1 r_1^t [P^t_{best,i} - x_i^t] + c_2 r_2^t [G_{best} - x_i^t]; \; i = 1, \ldots, 9.$$

so

$$v_1^3 = 4.4052, v_2^3 = 3.0862, v_3^3 = 1.8405,$$
$$v_4^3 = 1.2909, v_5^3 = 0.6681, v_6^3 = 0.053,$$
$$v_7^3 = -0.1380, v_8^3 = -2.1531, v_9^3 = -2.7759.$$

Step6: Find the new values of $x_i^3$ , $i = 1, \ldots, 9$ by

$$x_i^{t+1} = x_i^t + v_i^{t+1}$$

so

$x_1^3 = 16.7395$, $x_2^3 = 12.3982$, $x_3^3 = 8.298$,

$x_4^3 = 6.4892$, $x_5^3 = 4.4391$, $x_6^3 = 2.3968$,

$x_7^3 = 1.786$, $x_8^3 = -4.8466$, $x_9^3 = -6.8967$.

Step7: Find the objective function values of $f_i^3$, $i = 1, \ldots, 9$:

$f_1^3 = -176.5145$, $f_2^3 = -71.7244$, $f_3^3 = -7.3673$,

$f_4^3 = 10.3367$, $f_5^3 = 22.49$, $f_6^3 = 26.2393$,

$f_7^3 = 25.7402$, $f_8^3 = -27.7222$, $f_9^3 = -62.0471$.

Step 8: Stopping criterion:

If the terminal rule is satisfied, go to step 2,
Otherwise stop the iteration and output the results.

Finally, the values of $x_i^2, i = 1,2,3,4,5,6,7,8,9$ did not converge, so we increment the iteration number as $t = 4$ and go to step 2. When the positions of all particles converge to similar values, then the method has converged and the corresponding value of $x_i^t$ is the optimum solution. Therefore the iterative process is continued until all particles meet a single value.