CSE322
ALGEBRIC METHODS USING ARDEN'S THEORM

Lecture #8

ALGEBRIC METHOD ISUNG ARDEN'S THEORM

The following method is an extension of the Arden's theorem (Theorem 5.1). This is used to find the r.e. recognized by a transition system.

The following assumptions are made regarding the transition system:

(i) The transition graph does not have Λ-moves.

(ii) It has only one initial state, say v_1.

(iii) Its vertices are $v_1 \ldots v_n$.

(iv) $\mathbf{V}_i$ the r.e. represents the set of strings accepted by the system even though v_i is a final state.

(v) α_{ij} denotes the r.e. representing the set of labels of edges from v_i to v_j. When there is no such edge, $\alpha_{ij} = \emptyset$. Consequently, we can get the following set of equations in $\mathbf{V}_1 \ldots \mathbf{V}_n$:

$$\mathbf{V}_1 = \mathbf{V}_1\alpha_{11} + \mathbf{V}_2\alpha_{21} + \cdots + \mathbf{V}_n\alpha_{n1} + \Lambda$$

$$\mathbf{V}_2 = \mathbf{V}_1\alpha_{12} + \mathbf{V}_2\alpha_{22} + \cdots + \mathbf{V}_n\alpha_{n2}$$

$$\vdots$$

$$\mathbf{V}_n = \mathbf{V}_1\alpha_{1n} + \mathbf{V}_2\alpha_{2n} + \cdots + \mathbf{V}_n\alpha_{nn}$$

By repeatedly applying substitutions and Theorem 5.1 (Arden's theorem), we can express $\mathbf{V}_i$ in terms of α_{ij}'s.

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Consider the transition system given in Fig. 5.13. Prove that the strings recognized are $(\mathbf{a} + \mathbf{a}(\mathbf{b} + \mathbf{aa})^*\mathbf{b})^* \, \mathbf{a}(\mathbf{b} + \mathbf{aa})^* \, \mathbf{a}$.

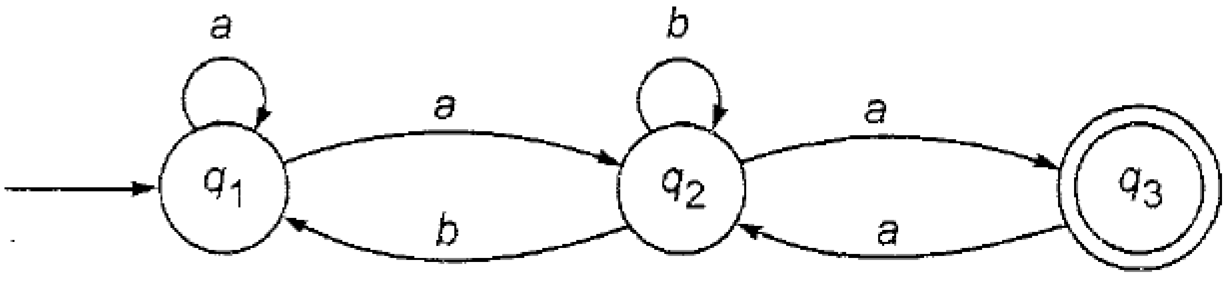

Fig. 5.13

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The three equations for q_1, q_2 and q_3 can be written as

$$\mathbf{q}_1 = \mathbf{q}_1\mathbf{a} + \mathbf{q}_2\mathbf{b} + \Lambda, \qquad \mathbf{q}_2 = \mathbf{q}_1\mathbf{a} + \mathbf{q}_2\mathbf{b} + \mathbf{q}_3\mathbf{a}, \qquad \mathbf{q}_3 = \mathbf{q}_2\mathbf{a}$$

It is necessary to reduce the number of unknowns by repeated substitution. By substituting q_3 in the q_2-equation, we get by applying Theorem 5.1

$$\begin{aligned}\mathbf{q}_2 &= \mathbf{q}_1\mathbf{a} + \mathbf{q}_2\mathbf{b} + \mathbf{q}_2\mathbf{aa}\\ &= \mathbf{q}_1\mathbf{a} + \mathbf{q}_2(\mathbf{b} + \mathbf{aa})\\ &= \mathbf{q}_1\mathbf{a}(\mathbf{b} + \mathbf{aa})^*\end{aligned}$$

Substituting $\mathbf{q}_2$ in $\mathbf{q}_1$, we get

$$\begin{aligned}\mathbf{q}_1 &= \mathbf{q}_1\mathbf{a} + \mathbf{q}_1\mathbf{a}(\mathbf{b} + \mathbf{aa})^*\mathbf{b} + \Lambda\\ &= \mathbf{q}_1(\mathbf{a} + \mathbf{a}(\mathbf{b} + \mathbf{aa})^*\mathbf{b}) + \Lambda\end{aligned}$$

Hence,

$$\begin{aligned}\mathbf{q}_1 &= \Lambda(\mathbf{a} + \mathbf{a}(\mathbf{b} + \mathbf{aa})^*\mathbf{b})^*\\ \mathbf{q}_2 &= (\mathbf{a} + \mathbf{a}(\mathbf{b} + \mathbf{aa})^*\mathbf{b})^*\ \mathbf{a}(\mathbf{b} + \mathbf{aa})^*\\ \mathbf{q}_3 &= (\mathbf{a} + \mathbf{a}(\mathbf{b} + \mathbf{aa})^*\mathbf{b})^*\ \mathbf{a}(\mathbf{b} + \mathbf{aa})^*\mathbf{a}\end{aligned}$$

Since q_3 is a final state, the set of strings recognized by the graph is given by

$$(\mathbf{a} + \mathbf{a}(\mathbf{b} + \mathbf{aa})^*\mathbf{b})^*\mathbf{a}(\mathbf{b} + \mathbf{aa})^*\mathbf{a}$$