Monday, August 24, 2020 11:20 PM

Determinant

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{21}a_{12}$$

eg $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = 10 - 12 = -2$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{31}a_{23}) + a_{13}(a_{21}a_{32} - a_{31}a_{22})$$

Q The value $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$ is $2(0-20) + 3(-42-4) + 5(30-0)$

$= -28$

(a) -26 (b) -28 (c) 28 (d) 26

Minor

$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times 2}$

Minor of $a_{11} = a_{22}$

$a_{12} = +a_{21}$

$a_{21} = a_{12}$

$a_{22} = a_{11}$

Multiply minor by $(-1)^{i+j}$

Cofactor $a_{11} = (-1)^{1+1} a_{22} = +a_{22}$

$a_{12} = (-1)^{1+2} a_{21} = -a_{21}$

$a_{21} = (-1)^{2+1} a_{12} = -a_{12}$

$a_{22} = (-1)^{2+2} a_{11} = a_{11}$

$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

Cofactor of $a_{22} = (-1)^{2+2} a_{11}$

$A = \begin{bmatrix} + & - & + \\ a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ - & & \end{bmatrix}_{3\times 3}$

Minor of elements of row 1st $\begin{vmatrix} a_{22} & a_{23} \end{vmatrix}$ $\begin{vmatrix} a_{21} & a_{23} \end{vmatrix}$ $\begin{vmatrix} a_{21} & a_{22} \end{vmatrix}$

Minor of elements of row 1st $\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$, $\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{32} \end{vmatrix}$, $\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

Cofactors of elements of 1st row $+\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$, $-\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{32} \end{vmatrix}$, $+\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$

Rank → The order of largest non-zero minor of $|A|$.

① Matrix method
② Cramer's rule

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times 3}$$

$|A| = 0$ | $|A| \neq 0$

rank < 3 | rank $A = 3$

Minors of order 2×2 (a_{11}) $\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} \neq 0$ or 0

a_{12} $\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} \neq 0$ or 0

$f(A) \leq 3$

$f(A) = 2, 1$

If all minors of order 2×2 are zero then $f(A)$ or $\rho(A) < 2$

$f(A) = 1$

Note: Minimum possible rank of a non zero square matrix

Max possible rank = Same as that of order of matrix