CHAPTER

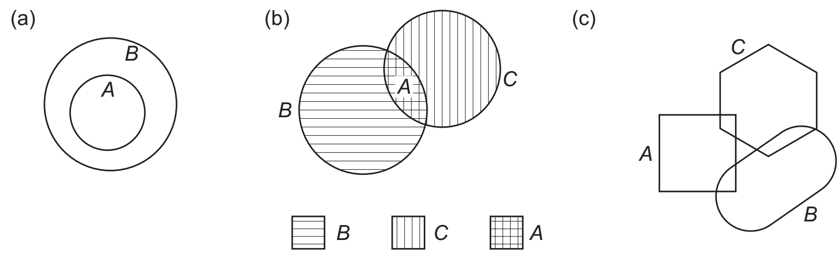
ONE

SETS AND PROPOSITIONS

1.1 (a) T (b) F (c) T
(d) T (e) F (f) F
(g) T (h) F (i) T
(j) F (k) T (l) F
(m) F (n) T

1.2 (a) $\{\phi\}$ (b) ϕ
(c) $\{a, \phi, \{\phi\}\}$ (d) $\{\phi\}$
(e) $\{a, \phi, \{\phi\}\}$ (f) $\{a, \{\phi\}\}$

1.3



1.4 $A = \phi$, $B = \{\phi\}$, $C = \{\{\phi\}\}$

1.5 (a) True
(b) False: counterexample $A = \{\phi\}$, $B = \{\{\phi\}\}$, $C = \{\{\phi\}\}$.
(c) False: counterexample $A = \phi$, $B = \{\phi\}$, $C = \{\{\phi\}\}$.
(d) False: counterexample $A = B = \{\phi\}$, $C = \{\{\phi\}\}$.

1.6 Yes, Suppose $a \in B$ and $a \notin C$. $a \in A$ leads to contradiction, so does $a \in \overline{A}$.

1.7 Let $a \in A$. If $a \in C$, then $a \in A \cap C$. Thus, $a \in B \cap C$. Thus, $a \in B$. If $a \in \overline{C}$, then $a \in A \cap \overline{C}$. Thus, $a \in B \cap \overline{C}$. Thus, $a \in B$.

1.8 (a) $P \subseteq Q$ (b) $P \supseteq Q$
(c) $Q = \phi$ (d) $P = Q$

1.9 (a) Yes, Yes.
(b) No. counterexample $W = \{a\}$, $Y = \{b\}$, $X = Z = \{a, b\}$.
No. counterexample $W = Y = \{a\}$, $X = \{a, b\}$ $Z = \{a, c\}$.

1.10 (a) No, e.g. $A = B = \{1\}$, $C = \phi$.
(b) No, e.g. $A = \phi$.
(c) Yes, Let $b \in B$.

If $b \in A$, then $b \notin A \oplus B$.
thus, $b \notin A \oplus C$.
thus, $b \in C$.
If $b \notin A$, then $b \in A \oplus B$.
thus, $b \in A \oplus C$.
thus, $b \in C$.

1.11 (a) $\{b, \{a, c\}, \phi\}$ (b) A
(c) $\{a, b, \{a, c\}\}$ (d) $\{\{a, c\}, \phi\}$
(e) $\{b, \{a, c\}, \phi\}$ (f) A
(g) $\{a, b, \phi\}$ (h) ϕ
(i) ϕ (j) ϕ
(k) $\{c\}$ (l) ϕ
(m) $\{a\}$

1.12 (a) If $a \in A - (B \cup C)$, then $a \in A$ and $a \notin B$. Thus, $a \in A - B$. Also, $a \notin C$. Thus, $a \in (A - B) - C$. Thus, $A - (B \cup C) \subseteq (A - B) - C$. If $a \in (A - B) - C$, then $a \in A - B$ and $a \notin C$. Thus, $a \in A$ and $a \notin B$. Therefore, $a \in A$ and $a \notin B \cup C$. Thus, $(A - B) - C \subseteq A - (B \cup C)$.
(b) Obvious from (a)
(c) $(A - C) - (B - C) = A - (C \cup (B - C)) = A - (B \cup C) = (A - B) - C$.

1.13 (a) $B \cap C = \phi$ (b) $A \subseteq B$ and $A \subseteq C$
(c) $(B \cup C) \supseteq A$ (d) $A \cap B = A \cap C$

1.14 (a) If there exists an x such that $x \in A$ and $x \notin B$, then $A - B \neq B$. Therefore, for all x, $x \in A$ implies that $x \in B$. Thus, $A - B = \phi$. Or, $B = \phi$, $A = \phi$.
(b) If there exists an x such that $x \in A$ and $x \notin B$, then $x \in A - B$. However, $x \notin B - A$. Therefore, $x \in A$ implies $x \in B$. Similarly, $x \in B$ implies $x \in A$. It follows that $A = B$.

1.15 (a) $E \subseteq (A \cup B)$ (b) $(A \cap C) \subseteq D$
(c) $(B - C) \subseteq \overline{D}$

1.16 (a) $(B \cap D) \subseteq E$ (b) $(E \cup F) = G$
(c) $(E \cap F) = \phi$ (d) $F \subseteq (A \cup B)$
(e) $(B - (C \cup D)) \subseteq F$

1.17 (a) $\{\phi, \{a\}\}$
(b) $\{\phi, \{\{a\}\}\}$
(c) $\{\phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}\}$

1.18 (a) A (b) ϕ
(c) $\{\phi, b, \{\phi\}, \{b\}, \{\phi, b\}\}$ (d) $\{\phi\}$

1.19 $A = \{\phi\}$

$$P(A) = \{\phi, \{\phi\}\}$$
$$P(P(A)) = \{\phi, \{\{\phi\}\}, \{\phi\}, \{\phi, \{\phi\}\}\}$$

Thus all answers are yes.

1.20 $P(A) = \{\phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}\}$. Hence,
(a) T (b) T (c) T
(d) T (e) T (f) T
(g) T (h) T (i) T
(j) F

1.21 $P(A) = \{\phi, \{a\}, \{\{a\}\}, \{a, \{a\}\}\}$. Hence,
(a) T (b) T (c) T
(d) F (e) T (f) T
(g) T (h) F (i) F
(j) T

1.22 (a) F. Counterexample $A = \{a\}$
(b) F. Counterexample $A = \{a\}$
(c) T. Trivial
(d) F. Trivial
(e) F. Counterexample $A = \{\phi, a\}$
(f) F. Trivial

1.23 (a) Let $S \in P(A \cap B)$, then $S \subseteq A \cap B$.

thus, $S \subseteq A$ and $S \subseteq B$
thus, $S \in P(A)$ and $S \in P(B)$
thus, $S \in P(A) \cap P(B)$.
Hence, $P(A \cap B) \subseteq P(A) \cap P(B)$.

Similarly, it can be proved that $P(A) \cap P(B) \subseteq P(A \cap B)$.

Therefore, $P(A \cap B) = P(A) \cap P(B)$.

(b) False: counterexample $A = \{a\}$, $B = \{b\}$.

1.24 (a) – (d) countably infinite.

1.25 There are only a finite number n of words in the English language. Number the "books" $B_1, B_2, \ldots,$ in lexicographic order with shortest books first. Thus $B_1, \ldots, B_n$ are books with one word, $B_{n+1} + B_{n+2}, \ldots, B_{n+n^2}$ are books with two words, and in general $B_{A_i+1}, \ldots, B_{A_i+n}$, are books with i words where $A_i = n + n^2 + \ldots + n^{i-1}$ and $i = 1, 2, \ldots$

1.26 (a) Number them by diagonalization: (1, 1) (2, 1) (1, 2) (3, 1) (2, 2) (1, 3) (4, 1) (3, 2) (2, 3), (1, 4)… In general the point (i, j) will be numbered $\dfrac{(i + j - 1)(i + j - 2)}{2} + j$. The number of pairs is at least as great as the number of positive rationals.

(b) Set up a correspondence with the set of points with integer coordinates in the first quadrant so that (i, j) cooresponds to the ith element of the jth set. Thus the set in question is no bigger than the set of "integer points" in the first quadrant which is countably infinite by part (a).

1.27 Countably infinite. We list the subsets in S as follows: First list in lexicographic order all subsets of the set $\{1\}$, then list all new (that is, have not been previously listed) subsets of the set $\{1, 2\}$, then all new subsets of $\{1, 2, 3\}$ and so on. Clearly, there is countably infinite number of such subsets.

1.28 (a) $A = \{1, 2, 3, \ldots\}$, $B = \{2, 4, 6, \ldots\}$
(b) $A = \{0, 1, 2, \ldots\}$, $B = \{0, -1, -2, \ldots\}$

1.29 If the number of such machines is countably infinite, we can list them one after another as $M_1, M_2, \ldots, M_i, \ldots$. Now cosider a machine P described as follows: for the natural number j, if M_j considers j "lucky", then P will respond with "unlucky" otherwise P will respond with "lucky". Thus P is different from all the listed machines. This leads to a contradiction.

1.30 For a postage of $3k$ cents, use k 3 cent stamps.
For a postage of $3k + 1$ cents, use $k - 3$ 3 cent stamps and two 5 cent stamps.
For a postage of $3k + 2$ cents, use $k - 1$ 3 cent stamps and one 5 cent stamp.

1.31 No, since the basis has not been established.

1.32 The induction step does not hold when $k = 1$.

1.33 Basis: $n = 0$ $\quad 1 = \dfrac{1-a^1}{1-a}$

Induction step:
$$1 + a + a^2 + \ldots + a^k + a^{k+1}$$
$$= \frac{1-a^{k+1}}{1-a} + a^{k+1}$$
$$= \frac{1-a^{k+1}+a^{k+1}-a^{k+2}}{1-a}$$
$$= \frac{1-a^{k+2}}{1-a}$$

1.34 Prove by induction.

Basis: $n = 1$ $\quad 1^3 + 2 \cdot 1 = 3$, is divisible by 3.

Induction step:
$$(k+1)^3 + 2(k+1) = k^3 + 3k^2 + 3k + 1 + 2k + 2$$
$$= k^2 + 2k + 3(k^2 + 1)$$

$k^2 + 2k$ is divisible by 3 according to the induction hypothesis.
$3(k^2 + 1)$ is clearly divisible by 3.

1.35 Basis: $n = 2$, $\quad 2^4 - 4 \cdot 2^2 = 0$, is divisible by 3.

Induction step:
$$(k+1)^4 - 4(k+1)^2$$
$$= (k^4 + 4k^3 + 6k^2 + 4k + 1) - 4(k^2 + 2k + 1)$$
$$= (k^4 - 4k^2) + 4k^3 + 6k^2 - 4k - 3$$
$$= (k^4 - 4k^2) + 3k^3 + 6k^2 - 3k - 3 + k^3 - k$$
$$= (k^4 - 4k^2) + 3(k^3 + 2k^2 - k) + (k+1)\,k(k-1)$$

The first term is divisible by induction, the second and third are clearly divisible by 3.

1.36 Basis: $n = 1$ $\quad 2 \times 2 - 1 = 3$

Induction step:
$$2^{k+1} \times 2^{k+1} - 1 = 4 \times (2^k \times 2^k) - 1$$
$$= 3 \times 2^k \times 2^k + 2^k \times 2^k - 1$$

$3 \times 2^k \times 2^k$ clearly divisible by 3.
$2^k \times 2^k - 1$ is divisible by 3 according to the induction hypothesis.

1.37 Basis: $n = 1$ $\quad 2^2 - 1 = 3 = 1 + 2.$

Induction step:
$$1 + 2 + \ldots + 2^k + 2^{k+1} = 2^{k+1} - 1 + 2^{k+1}$$
$$= 2 \cdot 2^{k+1} - 1$$
$$= 2^{k+2} - 1$$

1.38 (a) $1 + 3 + 5 + \ldots + (2k - 1) = k^2$
(b) Basis: $n = 1$ $\quad 1 = 1^2$

Induction step: $1 + 3 + \ldots + (2k - 1) + (2(k + 1) - 1)$

$= k^2 + 2(k + 1) - 1$

$= k^2 + 2k + 1$

$= (k + 1)^2$

1.39 Basis: $n = 1$ $\quad 1 \times 1! = 1 = 2! - 1$

Induction step: $1 \cdot 1! + 2 \cdot 2! + \ldots + k \cdot k! + (k + 1) \cdot (k + 1)!$

$= (k + 1)! - 1 + (k + 1) \cdot (k + 1)!$

$= (k + 2) \cdot (k + 1)! - 1$

$= (k + 2)! - 1$

1.40 Basis: $n = 1$ $\quad 1 \cdot 2 = 2 = \dfrac{1 \cdot 2 \cdot 3}{3}$

Induction step: $1 \cdot 2 + 2 \cdot 3 + \ldots + k \cdot (k + 1) + (k + 1) \cdot (k + 2)$

$$= \frac{k(k+1)(k+2)}{3} + \frac{(k+1)(k+2) \cdot 3}{3}$$

$$= \frac{(k+1)(k+2)(k+3)}{3}$$

1.41 (a) Basis: $n = 1$, $\quad 1 = (-1)^0 \cdot \dfrac{1 \cdot 2}{2} = 1$

Induction step: $1 - 2^2 + 3^2 - \ldots + (-1)^{k-1} k^2 + (-1)^k (k + 1)^2$

$$= (-1)^{k-1} \frac{k(k+1)}{2} + (-1)^k (k + 1)^2$$

$$= (-1)^k (k + 1) \left(-\frac{k}{2} + (k+1)\right)$$

$$= (-1)^k (k + 1) \cdot \frac{k+2}{2}$$

(b) $1 - 2^2 + 3^2 - 4^2 + \ldots + (-1)^{k-1} k^2$

$= 1^2 + 2^2 + 3^2 + \ldots + k^2 - 2[2^2 + 4^2 + \ldots + k^2 \vee (k - 1)^2]$

$$= \frac{k(k+1)(2k+1)}{6} - 2 \cdot 2^2 \left(1^2 + 2^2 + \ldots + \left\lfloor \frac{k}{2} \right\rfloor^2\right)$$

(if k is even)

$$= \frac{k(k+1)(2k+1)}{6} - 8 \cdot \frac{\frac{k}{2}\left(\frac{k}{2} + 1\right)(k+1)}{6}$$

$$= \frac{k(k+1)(2k+1)}{6} - \frac{k(k+2)(2k+2)}{6}$$

$$= -\frac{k}{6}(2k^2 + 6k + 4 - 2k^2 - 3k - 1)$$

$$= -\frac{k}{6}(3k + 3)$$

$$= -\frac{k(k+1)}{2}$$

Similarly if k is odd.

1.42 [Method 1]: Induction

Basis: $1^2 = \frac{1(2-1)(2+1)}{3} = 1$

Induction step: $1^2 + 3^2 + 5^2 + \dots + (2k-1)^2 + (2k+1)^2$

$$= \frac{k(2k-1)(2k+1)}{3} + (2k+1)^2$$

$$= \frac{(k+1)(2k+1)(2k+3)}{3}$$

[Method 2]: $1^2 + 2^2 + \dots + (2k-1)^2 = \frac{(2k-1)(2k)(4k-1)}{6}$

$$2^2 + 4^2 + 6^2 + \dots + (2k-2)^2 = 2^2[1^2 + 2^2 + \dots + (k-1)^2]$$

$$= 4 \cdot \frac{(k-1)(k)(2k-1)}{6}$$

Thus, $1^2 + 3^2 + 5^2 + \dots + (2k-1)^2$

$$= \frac{(2k-1)(2k)(4k-1)}{6} - \frac{4 \cdot (k-1)k(2k-1)}{6}$$

$$= \frac{k(2k-1)}{3}(4k - 1 - 2k + 2)$$

$$= \frac{k(2k-1)(2k+1)}{3}$$

1.43 [Method 1]: Induction.

[Method 2]: Note that both sums are equal to $\left[\frac{k(k+1)}{2}\right]^2$, using induction for the left side.

1.44 Proof by induction:

Basis: $$\frac{1^2}{1.3} = \frac{1(2)}{2(3)}$$

Induction step: $$\frac{1^2}{1.3} + \frac{2^2}{3.5} + \ldots + \frac{k^2}{(2k-1)(2k+1)} + \frac{(k+1)^2}{(2k+1)(2k+3)}$$

$$= \frac{k(k+1)}{2(2k+1)} + \frac{(k+1)^2}{(2k+1)(2k+3)}$$

$$= \frac{k+1}{2k+1}\left[\frac{k(2k+3)+2(k+1)}{2\cdot(2k+3)}\right]$$

$$= \frac{k+1}{2k+1}\left[\frac{2k^2+5k+2}{4k+6}\right] = \frac{(k+1))(k+2)}{2(2k+3)}$$

1.45 (a) – (c) can be proved by induction as in part (d)

(d) $$\frac{1}{1\cdot(a+1)} + \frac{1}{(a+1)(2a+1)} + \ldots + \frac{1}{[a(n-1)+1](an+1)} = \frac{n}{an+1}$$

Bais: $n = 1$ $$\frac{1}{1\cdot(a+1)} = \frac{1}{a+1}$$

Induction step:

$$\frac{1}{1\cdot(a+1)} + \ldots + \frac{1}{[a(k-1)+1](ak+1)} + \frac{1}{(ak+1)[a(k+1)+1]}$$

$$= \frac{k}{ak+1} + \frac{1}{(ak+1)[a(k+1)+1]}$$

$$= \frac{k[a(k+1)+1]+1}{(ak+1)[a(k+1)+1]}$$

$$= \frac{(k+1)(ak+1)}{(ak+1)[a(k+1)+1]}$$

1.46 Basis: $1 \cdot 2 \cdot 3 = \dfrac{1\cdot 2\cdot 3\cdot 4}{4}$

Induction step: $$\sum_{i=1}^{k+1} i(i+1)(i+2)$$

$$= \sum_{i=1}^{k} i(i+1)(i+2) + (k+1)(k+2)(k+3)$$

$$= \frac{k(k+1)(k+2)(k+3)}{4} + \frac{(k+1)(k+2)(k+3)\cdot 4}{4}$$

$$= \frac{(k+1)(k+2)(k+3)(k+4)}{4}$$

1.47 $[n(n-1)+1] + [n(n-1)+3] + [n(n-1)+5] + \ldots$
$+ [n(n-1) + (2n-1)] = n^3$

Basis: $n = 1$, $[1(0) + 1] = 1 = 1^3$

Induction Step:

$[k(k+1)+1] + [k(k+1)+3] + \ldots$
$+ [k(k+1) + (2k-1)] + [k(k+1) + (2k+1)]$

$= [k(k-1) + 1 + 2k] + [k(k-1) + 3 + 2k] + \ldots$
$+ [k(k-1) + (2k-1) + 2k] + [k(k+1) + (2k+1)]$

$= [k(k-1)+1] + [k(k-1)+3] + \ldots$
$+ [k(k-1) + (2k-1)] + k(2k) + [k(k+1) + (2k+1)]$

$= k^3 + 2k^2 + k^2 + k + 2k + 1 = (k+1)^3$

1.48 Basis: $1^3 + 2^3 + 3^3 = 36$ which is divisible by 9.

Induction step: $(k+1)^3 + (k+2)^3 + (k+3)^3$
$= k^3 + (k+1)^3 + (k+2)^3 + 9k^2 + 27k + 27$

Clearly, $9k^2 + 27k + 27$ is divisible by 9.

1.49 Proof by induction:

Bais: $n = 0$, $11^2 + 12 = 133$, divisible by 133

Induction step: $(11)^{k+3} + (12)^{2k+3}$
$= 11[(11)^{k+2} + (12)^{2k+1}] + (12^2 - 11)\ 12^{2k+1}$

The first form is divisible by 133 by induction on k and $12^2 - 11 = 133$.

1.50 Let $$S_k = \frac{1}{k+1} + \frac{1}{k+2} + \ldots + \frac{1}{2k}$$

For $k = 2$, $S_2 = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$.

Now $$S_k = \frac{1}{k+1} + \ldots + \frac{1}{2k}$$

$$S_{k+1} = \frac{1}{k+2} + \ldots + \frac{1}{2k} + \frac{1}{2k+1} + \frac{1}{2(k+1)}$$

$$S_{k+1} - S_k = \frac{1}{2k+1} + \frac{1}{2(k+1)} - \frac{1}{k+1} = \frac{1}{2(k+1)(2k+1)} > 0$$

That is, $S_{k+1} > S_k$.

Thus, $A = \dfrac{7}{12} - \in$.

1.51 Proof by induction:

Basis: $k = 2$, $\dfrac{1}{\sqrt{1}} + \dfrac{1}{\sqrt{2}} > \sqrt{2}$

Induction step: We note first that $\left(1+\sqrt{\dfrac{k}{k+1}}\right) > 1$. That is

$$\left(\sqrt{k+1}-\sqrt{k}\right)\left(1+\sqrt{\frac{k}{k+1}}\right) > \sqrt{k+1}-\sqrt{k}$$

or $\dfrac{1}{\sqrt{k+1}} > \sqrt{k+1}-\sqrt{k}$

(This result can also be proved by induction.)

It follows that

$$\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\ldots+\frac{1}{\sqrt{k+1}} > \sqrt{k}+\sqrt{k+1}-\sqrt{k} = \sqrt{k+1}$$

1.52 Answer: n. Proof by induction:

Basis: $n = 0$ Trivially true.

Induction Step: Suppose that there are k couples. Then the number of handshakes reported must be 0, 1, 2, …, $2k$. Let A be the person who has $2k$ handshakes and B be A′s spouse. A cannot be the hostess because this means that one of the guests must have 0 handshakes. This in turn means that the hostess can only have at most $2k - 1$ handshakes. A contradiction. Thus A must be one of the guests. Now A shoke hands with everybody except B. Hence the person with 0 handshake must be B. If we disregard A and B then the number of handshakes reported by the remaining $k - 1$ couples are 0, 1, 2, …, $2k - 2$. By induction, the hostess has shaken hands with $k - 1$ hands from among these $k - 1$ couples. Furthermore, the hostess shoke hands with A but not B. Hence she has k handshakes.

1.53 (a) Assume that n_1 is the smallest natural number (larger than n_0) not in S. Since $n_1 = u^+$ for some natural number u, n_0, n_{0+1}, …, u are all in S. However, according to (2), u^+ should also be in S which is a contradiction.

(b) Directly from (a).

1.54 300 – (100 + 60 + 42 – 20 – 14 – 8 + 2) = 138.

100 – 20 – 14 + 2 = 68.

1.55 Let n be the number of horizontal segments drawn.
Furthermore, assume that the ith child will get toy P_i.

Basis: $n = 0$, Obvious that $P_i = i$, $i = 1, 2, \ldots, n$.

Induction step: Given a graph G with $k + 1$ segments, choose h to be the lowest segment. Remove h to get a graph with k horizontal segments. By induction hypothesis, the graph $G - h$ "assigns" toy P_i to the ith child. If we add h into $G - h$, and if h joins vertical lines, l_i and i_j, then in G, then h will only interchange the toys for child P_i and P_j leaving all the other assignments unaffected. Hence, it is still a permutation.

1.56 1000 – 595 – 595 – 550 + 395 + 350 + 400 – 250 = 155.

595 – 395 – 350 + 250 = 100.

1.57 No.
If that were so, then the percentage of professors who does at least one of the three things will be 60 + 50 + 70 – 20 – 30 – 40 + 20 = 110. Clearly a contradiction.

1.58 (a) 52000 – 20000 – 36000 – 12000 + 6000 + 9000 + 5000 = 4000

(b) 52000 – 6000 – 9000 – 5000 – 2(4000) = 40000

(c) Let A, B and C be the set of fans who bought bumper stickers, window decals and key rings respectively. If $|A \cup B \cup C| = 44000$, then

$$|A \cap B \cap C| = 44000 - 20000 - 36000 - 12000 + 6000 + 9000 + 5000 = -4000$$

Clealy a contradiction. Thus it is not 44000.

Similarly, if $|A \cup B \cup C| = 60000$, then

$$|A \cap B \cap C| = 60000 - 20000 - 36000 - 12000 + 6000 + 9000 + 5000 = 12000$$

However $|A \cap B| < 12000$. Another contradiction.

1.59 (a) 130 – 60 – 51 + 30 = 49

(b) 60 – 26 – 30 + 12 = 16

(c) 130 – 60 – 51 – 54 + 30 + 26 + 21 – 12 = 30

1.60 (a) 30 = 100 – 32 – 20 – 45 + 15 + 7 + 10 – x; x = 5.

(b) 32 + 20 + 15 – 2 (15 + 7 + 10) + 3(5) = 48

1.61 5 = 30 – 17 – 16 + x; x = 8

1.62 35 children have taken exactly two rides.

Since $(x + 2 \times 35 + 3 \times 20) \times 0.50 = 70.$

$$x = 10$$

That is, 10 children have taken exactly 1 ride.

$$75 - 10 - 35 - 20 = 10.$$

Therefore, 10 children did not try any of the rides at all. Alternatively,

$$75 - 140 + (55 + 20 + 20) - 20 = 20.$$

1.63 (a) $17 = 50 - 26 - 21 + x, \quad x = 14.$
(b) $2y = 40$, $y = 20$.
$Z = 50 - 40 - 4 = 6.$

1.64 With this definition, symmetric difference is not associative,

e.g. if $P = \{a, a, a\}$, $Q = \{a, a\} = R$
then $P \oplus (Q \oplus R) = P \oplus \phi = P$,
while $(P \oplus Q) \oplus R = \{a\} \oplus R = \{a\}$.

1.65 (a) $p \vee q$ (b) $(p \vee q) \wedge (\overline{p \wedge q})$
(c) $p \wedge \overline{q}$ (d) $\overline{p \wedge r}$
(e) $(p \wedge q) \rightarrow r$ (f) $\overline{r \rightarrow (p \wedge q)}$

1.66 (a) $p \wedge q$ (b) $\overline{p} \wedge \overline{q} \wedge \overline{r}$
(c) $(\overline{p} \wedge \overline{q}) \rightarrow \overline{r}$ (d) $p \leftrightarrow q$
(e) $(p \vee \overline{q}) \wedge (\overline{p} \vee q)$

1.67 (a) $s \wedge \overline{h}$ (b) $(h \wedge \overline{r}) \rightarrow b$
(c) $\overline{m} \rightarrow g$ (d) $(c \vee \overline{r}) \rightarrow (p \leftrightarrow \overline{a})$

1.68 (a) The weaher is nice but we do not have a picnic.
(b) If the weather is nice then we have a picnic and conversely.
(c) If the weather is nice, then we have a picnic.
(d) The weather is nice but we do not have a picnic.

1.69 (a) $(\overline{p} \wedge q \wedge r) \vee (p \wedge \overline{q} \wedge r) \vee (p \wedge q \wedge \overline{r})$
(b) $\overline{p \wedge q \wedge r}$

1.70

p	(a)	(b)	(c)
T	*T*	*T*	*T*
F	*T*	*T*	*T*

p	q	(d)	(e)	(f)	(h)
T	T	T	F	F	T
T	F	T	F	T	T
F	T	T	T	F	T
F	F	T	T	F	T

p	q	r	(g)
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	T	F	T
F	F	T	T
F	F	F	T

1.71 (a) Since p is true and q is false, the formula is false.
(b) If p is false, then $\overline{p}$ is true and the formula is true.
If p is true and q is true, $p \leftrightarrow q$ is true, and the formula is true.

1.72 Statement (b) must be false, since statement (a) is clearly true. Hence statement (c) must be true.

1.73 What will your answer be if I ask you if the left branch leads to the city?

1.74 Mike dislikes snow, hence he cannot be a skier.
Furthermore, he must be either a skier or a mountain climber.
Thus, Mike is a mountain climber but not a skier.