CHAPTER SIX

MODELLING COMPUTATION

6.1 Prof, Lai lies.

6.2 (a) T (b) F (c) T
(d) T (e) T (f) F
(g) T (h) F (i) F
(j) T (k) T (l) F
(m) T (n) T (o) T

6.3 (a) ϕ (b) ϕ
(c) $\{(bc)^i | i \geq 0\}$ (d) $\{(cd)^i | i \geq 0\}$

6.4 (a) $\{xb | x \in \{a, b\}^i, i \geq 4\}$ (b) $\{ab\}$
(c) $\{abc\}$ (d) $\{a, a^3, a^4, a^5, \ldots\}$
(e) ϕ (f) ϕ

6.5 (a) Yes (b) No (c) No
(d) No (e) No (f) Yes
(g) No (h) No (i) No
(j) Yes

6.6 (a) Yes (b) Yes (c) Yes
(d) Yes (e) Yes

6.7 (a) No, No, Yes, No
(b) $\{b^i b^i c^j | i \geq 1, j \geq 1\}$

6.8 signed_integer => sign integer
=> sign digit integer
=> sign digit digit integer
=> sign digit digit digit
=> –010

6.9 sentence => Does singular-noun sing transitive-verb noun?

sentence => Do plural-noun transitive-verb noun?
sentence => Does singular-noun transitive-verb?
sentence => Do plural-noun intransitive-verb?
transitive-verb => like
intransitive-verb => come
intransitive-verb = > understand
noun => singular-noun
noun => plural-noun
singular-noun => he
singular-noun => she
singular-noun => John
singular-noun => Mary
plural-noun => You
plural-noun => 1

6.10 (a) $\{a^i b \mid i \geq 2\}$
(b) $\{(ab)^i aa \mid i \geq 0\}$
(c) $\{aab, aac, aabb, aabc, aacb, aacc\}$
(d) $\{a^i (ac) b^i \mid i \geq 0\}$
(e) $\{a^i b a^j \mid i \geq 1, j \geq 0\}$
(f) $\{a^i b \mid i \geq 0\}$
(g) $\{a^i b^j \mid i \geq 1, j \geq 1\}$
(h) $\{a^i b^i c^j \mid i \geq 1, j \geq 1\}$
(i) $\{a^i b a^j \mid i \geq 0, j \geq 1\}$
(j) $\{ab^i c^j \mid i \geq 0, j \geq 1\}$
(k) $\{a^i c b^i \mid i \geq 1\} \cup \{b^j c a^j \mid j \geq 1\}$
(l) $\{cb^i a^j \mid i \geq 0, j \geq 1\}$
(m) $\{(ab)^i c (ab)^i \mid i \geq 0\}$
(n) $\{a^i b^j a \mid i \geq 0, j \geq 1\}$
(o) $\{(aa)^i b^j \mid i \geq 1, j \geq 0\}$

6.11 (a) $\{S \to AB, A \to aaA, A \to aa, B \to bbB, B \to bb\}$
(b) $\{S \to AB, A \to abA, A \to ab, B \to ccB, B \to cc\}$
(c) $\{S \to AB, A \to aAb, A \to ab, B \to bB, B \to b\}$
(d) $\{S \to ASB, S \to AB, A \to a, B \to bb, B \to b\}$
(e) $\{S \to AB, A \to aAb, A \to ab, B \to Bc, B \to c\}$

(f) $\{S \to aSc, S \to aAc, A \to bAc, A \to bc\}$

(g) $\{S \to aaS, S \to aacA, A \to bbA, A \to bbb\}$

(h) $\{S \to ABC, A \to aAb, A \to ab, B \to cBd, B \to cd, C \to eC, C \to e\}$

6.12 (a) $\{S \to aSb, S \to ab, ab \to ba, ba \to ab\}$

(b) $\{S \to aaaSb, S \to aaab, ab \to ba, ba \to ab\}$

6.13 Let S' be the start symbol of a grammar that generates the string of 0s and ls with an equal number of 0s and 1s. Then the grammar below with S as the start symbol is a grammar for L.

$$\{S \to 0, S \to 0S, S \to 0S', S \to S'\,S\}$$

6.14 Let G_1 and G_2 be the context free grammars for the languages L_1 and L_2 respectively. Let S_1 and S_2 be the starting in G_1 and G_2 respectively.

(a) If we add the productions $S \to S_1$ and $S \to S_2$ to G_1 and G_2, then S will generate he ganguage $L_1 \cup L_2$.

(b) Similarly, If we add the production $S \to S_1 S_2$ to G_1 and G_2, then S will generate the language $L_1 L_2$.

6.15 (a) Type 0 grammar, Type 3 language.

(b) Type 1 grammar, Type 3 language.

(c) Type 2 grammar, Type 3 language.

(d) Type 3 grammar, Type 3 language.

6.16 (a) $\{a^i b^i c^i \mid i \geq 1\}$

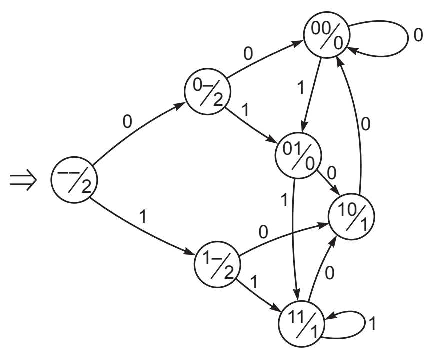
(b) $\{aa, bb\}$

(c) ϕ

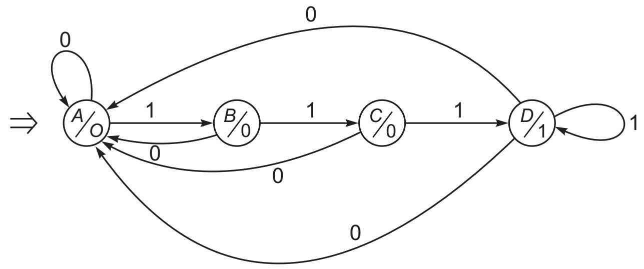
6.17 (a) $\{S \to aA, S \to bB, S \to b, A \to b, A \to bB, B \to a, B \to aA, B \to b, B \to bB\}$

(b) $\{S \to aAB, S \to aBA, S \to bAA, A \to aS, A \to a, A \to bAAA, B \to aABB, B \to aBAB, B \to aBBA, B \to bS, B \to b\}$

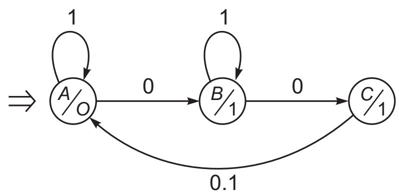
6.18



6.19



6.20



6.21 All paths from A to H are of the form A… DFH. Since the minimum-energy input sequence that take the machine from A to ABD, therefore a minimum-energy sequence is $ABDFH$ and the total energy of the sequence is 6 units.

6.22 (a) 011000

(b) 001000

(c) Let $\alpha = a_1 a_2 \ldots a_k$ be a given output sequence. Construct a graph $G = (V, E)$ where V is the set of all the states and E is defined as follows:

(i) define the sets $A_1, A_2, \ldots, A_{k+1}$ recursively. $A_1 = \{S_0\}$. A_{i+1} is the set of all states that can be reached from the states in A_i in one step with output a_i.

(ii) for every $S \in A_i$ and $T \in A_{i+1}$ such that there is a transition from S to T, add (S, T) to E and label it with an input letter that causes the transition.

(iii) if A_{k+1} is nonempty, then there exists an input sequence that will produce α and it is given by the labels along any path from S_0 to any state in A_{k+1}.

6.23 The finite state machine is:

		0	1
⇒	A	$B/0$	$B/1$
	B	$C/0$	$C/1$
	C	$A/1$	$A/0$

6.24 R is clearly symmetric and reflexive. Transitivity: $(\alpha_1, \alpha_2) \in R \Rightarrow$ both α_1 and α_2 will bring the machine from S_0 to S_i for some i. $(\alpha_2, \alpha_3) \in R \Rightarrow$ both α_2 and α_3 will bring the machine from S_0 to S_j for some j. Since α_2 brings the finite state machine from S_0 to S_i and S_j, and the machine is deterministic, therefore $S_i = S_j$. Hence both α_1 and α_3 will bring the machine from S_0 to $S_i (= S_j)$ and $(\alpha_1, \alpha_3) \in R$.

6.25 (a) $\pi_0 = \{\overline{ABCE}, \overline{DFGH}\}$

(b) $\pi_1 = \{\overline{ABCE}, \overline{D}, \overline{FGH}\}$, $\pi_2 = \{\overline{AC}, \overline{BE}, \overline{D}, \overline{FGH}\}$.

The machine with the smallest number of states:

	0	1	
$\overline{AC}$	$\overline{FGH}$	$\overline{BE}$	0
$\overline{BE}$	$\overline{D}$	$\overline{AC}$	0
$\overline{FGH}$	$\overline{AC}$	$\overline{FGH}$	1
$\overline{D}$	$\overline{BE}$	$\overline{AC}$	1

6.26 (a) $\pi_0 = \{\overline{ABCDE}, \overline{FG}\}$, $\pi_1 = \{\overline{ABCD}, \overline{E}, \overline{FG}\}$,

$\pi_2 = \{\overline{AB}, \overline{CD}, \overline{E}, \overline{FG}\}$.

$\Rightarrow$

	0	1	
$\overline{AB}$	$\overline{AB}$	$\overline{CD}$	0
$\overline{CD}$	$\overline{AB}$	$\overline{E}$	0
$\overline{E}$	$\overline{FG}$	$\overline{E}$	0
$\overline{FG}$	$\overline{AB}$	$\overline{CD}$	1

(b) $\pi_0 = \{\overline{ABCDEH}, \overline{FG}\}$, $\pi_1 = \{\overline{ACDEH}, \overline{FG}\}$,

$\pi_2 = \{\overline{ADH}, \overline{CE}, \overline{B}, \overline{FG}\}$.

$\Rightarrow$

	0	1	
$\overline{ADH}$	$\overline{ADH}$	$\overline{CE}$	0
$\overline{CE}$	$\overline{ADH}$	$\overline{B}$	0
$\overline{B}$	$\overline{FG}$	$\overline{B}$	0
$\overline{FG}$	$\overline{ADH}$	$\overline{CE}$	1

Both machines are equivalent to the following machine:

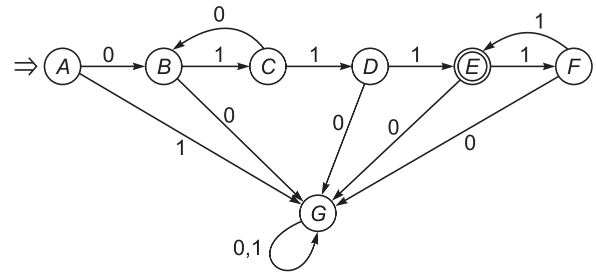
		0	1	
		0	1	
$\Rightarrow$	A	A	B	0
	B	A	C	0
	C	D	C	0
	D	A	B	1

6.27 (a) $\{\overline{BC}, \overline{AD}\}$

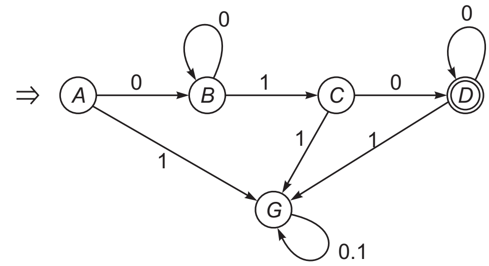
(b) $\pi_1 = \{\overline{BC}, \overline{AE}, \overline{FD}\}$, $\pi_2 = \{\overline{ACF}, \overline{BDE}\}$.

(c) $\pi_1 \cdot \pi_2$:
a, b are in the same block in $\pi_1 \cdot \pi_2 \Rightarrow a$, b are in the same block in π_1 and also in the same block in $\pi_2 \Rightarrow f(a, i), f(b, i)$ are in the same block in π_1 and also in the same block in $\pi_2 \Rightarrow f(a, i), f(b, i)$ are in the same block in $\pi_1 \cdot \pi_2$. Hence, $\pi_1 \cdot \pi_2$ is a preserved partition.
$\pi_1 + \pi_2$:
a, b are in the same block in $\pi_1 + \pi_2 \Rightarrow$ there exists $c_1, c_2, \ldots, c_k$ such that c_i, c_{i+1} are in the same block in π_1 or $\pi_2 (i = 0, 1, 2, \ldots, k; c_0 = a, c_{k+1} = b) \Rightarrow f(c_i, j), f(c_{i+1}, j)$ are in the same block in π_1 or $\pi_2 (i = 0, 1, 2, \ldots, k) \Rightarrow f(c_i, j), f(c_{i+1}, j)$ are in the same block in $\pi_1 + \pi_2$. Hence, $\pi_1 + \pi_2$ is a preserved partition.

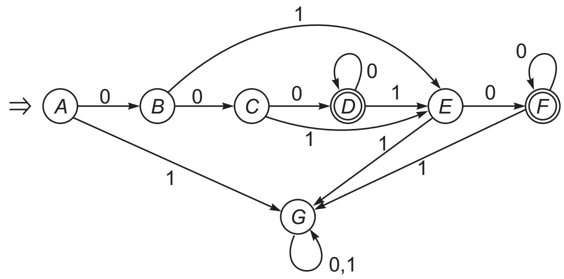
6.28 (a)



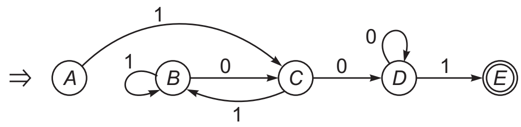
(b)



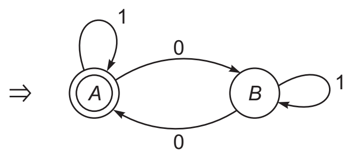
(c)


⇒
A
B
C
D
E
F
G
0
0
0
0
1
0
1
1
1
1
1
0
0,1

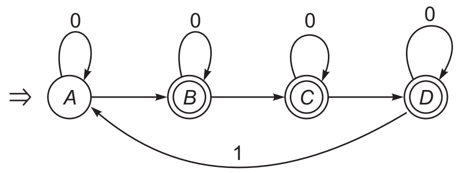
(d)


⇒
A
B
C
D
E
1
1
0
0
0
1
1

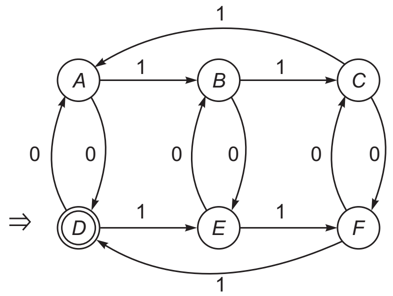
6.29 (a)


⇒
A
B
1
0
1
0

(b)


⇒
A
B
C
D
0
0
0
0
1

(c)


⇒
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B
C
D
E
F
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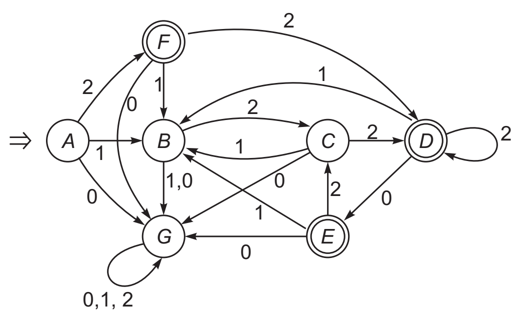
1
1
0
0
0
0
0
0
1
1
1

(d)


⇒
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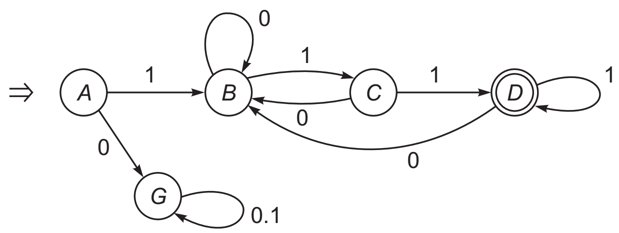
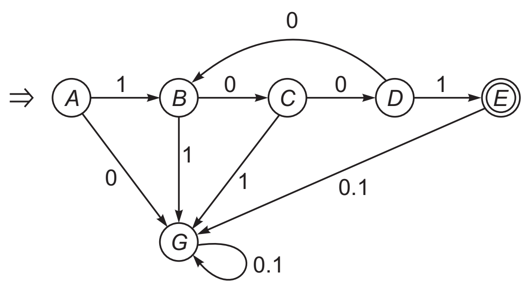
B
C
D
E
F
G
2
1
2
0
1
2
2
1
2
0
1
1,0
0
2
1
0
0,1, 2

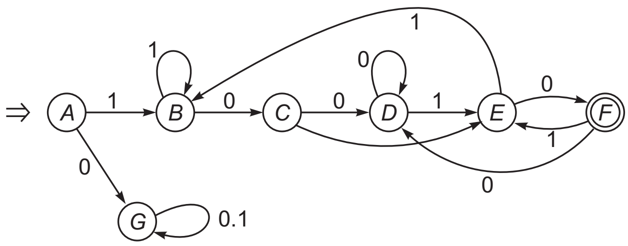
(e) $a_1 a_2 \ldots a_n$ is of the form $4k + 3$ for $k \geq 1$ iff $a_{n-1} a_n = 11$, $a_1 = 1$ and $n \geq 3$.



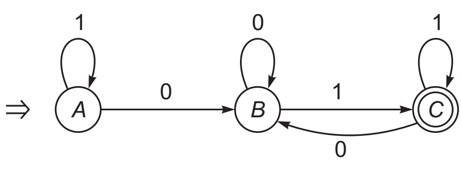
(f) $a_1 a_2 \ldots a_n$ is of the form $8^k + 1$ for $k \geq 1$ iff $a_1 = a_n = 1$, $a_i = 0$ for $i = 2, 3, \ldots, n - 1$ and $n = 3k + 1$.



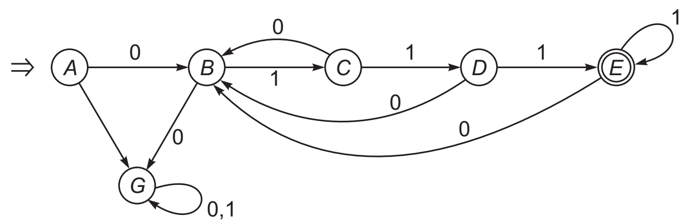
(g)



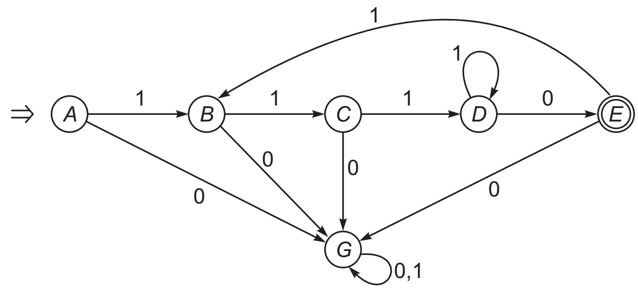
(h)



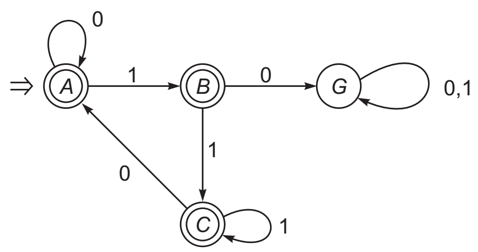
(i)



(j)



(k)



6.30 (a) $\{0(01)^n | n \geq 0\}$

(b) All binary sequences that either starts with a 0 and without consecutive 0*s* or starts with a 1 and without consecutive 1*s*.

(c) All binary sequences that end with 110.

(d) All binary sequences with total number of 0*s* $\equiv$ 3 mod 4.

6.31 (a) No.

(b) Yes. Let G be a type-3 grammar that corresponds to the finite state machine with the nonterminals $\{S_1, S_2, \ldots, S_{n-1}\}$ being the states. Introduce a starting symbol $\tilde{S}$ and a set of additional productions $\tilde{S} \to S_0$, $\tilde{S} \to S_1, \ldots, \tilde{S} \to S_{n-1}$. These together with the productions in G forms a type-3 grammar for the language accepted by the finite state machine using the new definition. Hence the language is regular.

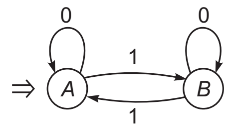
(c) No.

(d) Given a finite state machine M, obtain the corresponding finite state machine M' using the subsets construction method discussed on page 246. The initial state of M' is $\{S_0, S_1, \ldots, S_{n-1}\}$ which is the set of all the states in M. The final states of M' are $\{f_1\}$, $\{f_2\}$,… $\{f_k\}$ where the $f_i's$ are the final states of M. This machine accepts the new language and hence the language is regular.

6.32 (a) 010

(b) Let M be the finite state machine and M' be the corresponding machine constructed in 6.31(c). α is a synchronizing sequence that brings M to a state S if and only if α is a sequence that brings M' to the state $\{S\}$. Since there are $2^n - 1$ states in M', therefore α can always be reduced to a sequence of length $2^n - 2$ by the Pigeonhole principle.

(c)



All sequences from $A(B)$ to B contains an odd (even) number of 1s. Hence there are no synchronizing sequences that bring M to B (A).

6.33 The machine obtained by connecting M_1 and M_2 in series is:

		0	1	
⇒	(A,D)	(A,D)	(C,E)	0
	(A,E)	(A,E)	(C,D)	1
	(B,D)	(C,E)	(B,D)	0
	(B,E)	(C,D)	(B,E)	1
	(C,D)	(B,D)	(A,D)	1
	(C,E)	(B,E)	(A,E)	0

6.34 The machine obtained by connecting M_1 and M_2 in parallel is:

		0	1	
⇒	(A,D)	(A,D)	(C,E)	0
	(A,E)	(A,E)	(C,D)	1
	(B,D)	(C,E)	(B,E)	0
	(B,E)	(C,E)	(B,D)	1
	(C,D)	(B,D)	(A,E)	1
	(C,E)	(B,E)	(A,D)	1

6.35 If a finite state machine with n states accepts an input sequence α whose length is n or larger, then Thm. 6.2 $\Rightarrow$ α can be written as uvw s.t. v is nonempty and uv^iw is also in the language for $i \geq 0$. Since $uv^iw \neq uv^jw$ for $i \neq j$, therefore $\{uv^iw | i \geq 0\}$ is an infinite set and hence the machine accepts an infinite number of input sequences.

6.36 Suppose L is accepted by a machine with n states.

(a) $0^n 1^n \in L$. Pumping lemma $\Rightarrow$ there exists $k_1, k_2, k_3 \geq 0$, $k_2 \neq 0$ s.t. $0^n 1^n = 0^n 1^{k_1} 1^{k_2} 1^{k_3}$ and $0^n 1^{k_1+2k_2+k_3} = 0^n 1^{n+k_2} \in L$. This is a contradiction because $n + k_2 > n$.

(b) $0^n 1^n \in L$. As in (a), we can show that $0^{n+k_2} 1^n \in L$, for some $k_2 \neq 0$. This is again a contradiction.

(c) Let i be an integer s.t. $2^{i+1} - 2^i > n$. $0^{2^{i+1}} \in L$. Apply Pumping lemma to the input sequence staring at the $2^i + 1$ *th* 0. We get, $0^k \in L$ for some $2^i < k < 2^{i+1}$, which is a contradiction.

(d) $1^n 0^n 1^{2n} \in L$. Pumping lemma $\Rightarrow 1^n 0^n 1^k \in L$ for some $k < 2n$, which is a contradiction.

6.37 $10^n 10^n \in L$. Apply Pumping lemma to the last n 0s. We have $10^n 10^k \in L$ for some $k < n$, which is a contradiction.

6.38 A is the starting symbol for all the grammars presented below.
Solutions for the problems in 6.30:

(a) $\{A \to 0B, A \to 1D, A \to 0, B \to 0C, B \to 1D, C \to 0D, C \to 1B, C \to 1, D \to 0D, D \to 1D\}$

(b) $\{A \to 0B, A \to 0, A \to 1D, A \to 1, B \to 0F, B \to 1C, B \to 1, C \to 0B, C \to 0, C \to 1C, C \to 1, D \to 0E, D \to 0, D \to 1F, E \to 0E, E \to 0, E \to 1D, E \to 1\}$

(c) $\{A \to 0A, A \to 1B, B \to 0A, B \to 1C, C \to 0D, C \to 0, C \to 1C, D \to 0A, D \to 1B\}$

(d) $\{A \to 0B, A \to 1A, B \to 0C, B \to 1B, C \to 0D, C \to 0, C \to 1C, D \to 0A, D \to 1D, D \to 1\}$

Solutions for the problems in 6.40:

(a) $\{A \to 0A, A \to 1B, B \to 1C, B \to 1D, B \to 1, C \to 0D, C \to 0, D \to 0A\}$

(b) $\{A \to 1D, A \to 2B, B \to 1C, C \to 0B, C \to 2E, C \to 2, D \to 1E, D \to 1, D \to 2D\}$

6.39 (a)

	0	1	
$\{A\}$	$\{B\}$	$\{A, C\}$	0
$\{B\}$	$\{C\}$	$\{A\}$	1
$\{C\}$	$\{A\}$	ϕ	0
$\{A, B\}$	$\{B, C\}$	$\{A, C\}$	1
$\{A, C\}$	$\{A, B\}$	$\{A, C\}$	0
$\{B, C\}$	$\{A, C\}$	$\{A\}$	1
$\{A, B, C\}$	$\{A, B, C\}$	$\{A, C\}$	1
ϕ	ϕ	ϕ	0

(b)

	0	1	
$\{A\}$	$\{B,C\}$	ϕ	0
$\{B\}$	$\{D\}$	$\{B\}$	0
$\{C\}$	$\{A\}$	$\{C\}$	0
$\{D\}$	$\{A\}$	$\{B,C\}$	1
$\{A,B\}$	$\{B,C,D\}$	$\{B\}$	0
$\{A,C\}$	$\{A,B,C\}$	$\{C\}$	0
$\{A,D\}$	$\{A,B,C\}$	$\{B,C\}$	1
$\{B,C\}$	$\{A,D\}$	$\{B,C\}$	0
$\{B,D\}$	$\{A,D\}$	$\{B,C\}$	1
$\{C,D\}$	$\{A\}$	$\{B,C\}$	1
$\{A,B,C\}$	$\{A,B,C,D\}$	$\{B,C\}$	1
$\{A,B,D\}$	$\{A,B,C,D\}$	$\{B,C\}$	1
$\{B,C,D\}$	$\{A,D\}$	$\{B,C\}$	1
$\{A,C,D\}$	$\{A,B,C\}$	$\{B,C\}$	1
$\{A,B,C,D\}$	$\{A,B,C,D\}$	$\{B,C\}$	1
ϕ	ϕ	ϕ	0

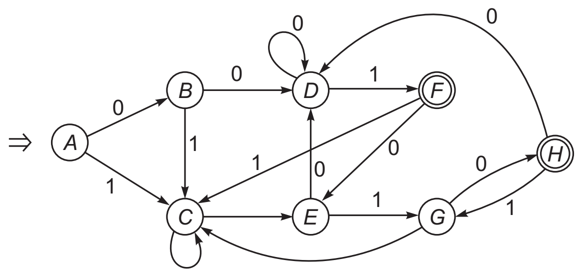
(c)

	0	1	
$\{A\}$	$\{A,C\}$	$\{A,B\}$	0
$\{B\}$	$\{E\}$	ϕ	0
$\{C\}$	ϕ	$\{E\}$	0
$\{D\}$	ϕ	ϕ	1
$\{E\}$	ϕ	ϕ	1
$\{A,B\}$	$\{A,C,E\}$	$\{A,B\}$	0
$\{A,C\}$	$\{A,C\}$	$\{A,B,E\}$	0
$\{A,B,E\}$	$\{A,C,E\}$	$\{A,B\}$	1
$\{A,C,E\}$	$\{A,C\}$	$\{A,B,E\}$	1
ϕ	ϕ	ϕ	0

6.40 (a) The set of binary sequences in each of which there are no three consecutive 1s and all of them ends with 11 or 10.

(b) $L = \{21(01)^k 2 | k \geq 0\} \cup \{12^k 1 | k \geq 0\}$

6.41 (a)



(b) All binary sequences ending with either 1010 or 001.

	0	1	
$\{A\}$	$\{A, B\}$	$\{A, E\}$	0
$\{B\}$	$\{C\}$	ϕ	0
$\{C\}$	ϕ	$\{D\}$	0
$\{D\}$	ϕ	ϕ	1
$\{E\}$	$\{F\}$	ϕ	0
$\{F\}$	ϕ	$\{G\}$	0
$\{G\}$	$\{H\}$	ϕ	0
$\{H\}$	ϕ	ϕ	1
$\{A, B\}$	$\{A, B, C\}$	$\{A, E\}$	0
$\{A, E\}$	$\{A, B, F\}$	$\{A, E\}$	0
$\{A, B, C\}$	$\{A, B, C\}$	$\{A, E, D\}$	0
$\{A, E, D\}$	$\{A, B, F\}$	$\{A, E\}$	1
$\{A, B, F\}$	$\{A, B\}$	$\{A, E, G\}$	0
$\{A, E, G\}$	$\{A, B, F, H\}$	$\{A, E, G\}$	0
$\{A, B, F, H\}$	$\{A, B\}$	$\{A, E, G\}$	1
ϕ	ϕ	ϕ	0

6.42 (a) 110, 11010, 10110 are accepted by the machine. The set of sequences that starts with a 1, and then follow by a sequence of k_1 01s ($k_1 \geq 0$), and then follow by a sequence of k_2 10s ($k_2 \geq 1$).

(b) 101 is accepted by the machine in (b) but not by the machine in (a).

(c) (i) For every pair of states S and T in M: for all the paths from S to T with one and only one transition corresponds to an input letter i and the rest are λ-arrows, join S to T and label it with i.

(ii) Remove all λ-arrows.

6.43 Let G_1, G_2 be two type-3 grammars for L_1, L_2 respectively. Introduce a new starting symbol S_0 and the productions $\{S_0 \to S_1, S_0 \to S_2\}$, where S_1, S_2 are the starting symbols of G_1, G_2 respectively. These together with the productions in G_1 and G_2 forms a type-3 grammar for $L_1 \cup L_2$.

6.44 Let M be a finite state machine with initial state S_0 and finite states $f_1, f_2, \ldots, f_k$ that accepts L. A nondeterministic finite state machine M' with λ-arrows (prob. 7.26) can be constructed to accept the language L^R as follows:

(1) Reverse all the directions of the arrows.

(2) Add in an extra state q_0 and k λ-arrows from q_0 to $f_1, f_2, \ldots, f_k$.

(3) The initial and final states for M' are q_0, S_0 respectively.

6.45 L^R is a language specified by a grammar in which productions are of the forms $A \to a$ and $A \to aB$. Hence L^R is a finite state language. Prob. 7.27 $\Rightarrow L = (L^R)^R$ is a finite state language.

6.46 It is sufficient to show that: Given a grammar for L in which the productions are of the forms $A \to \gamma$ and $A \to \gamma B$, we can find an equivalent grammar for L in which the productions are of the forms $A \to a$ and $A \to aB$. In fact, $A \to a_1a_2 \ldots a_k$ can be replaced by $\{A \to a_1A_1, A_1 \to a_2A_2, \ldots, A_{k-2} \to a_{k-1} A_{k-1}, A_{k-1} \to a_k\}$, and $A \to a_1a_2 \ldots a_kB$ can be replaced by $\{A \to a_1A_1, A_1 \to a_2A_2, \ldots, A_{k-2} \to a_{k-1} A_{k-1}, A_{k-1} \to a_kB\}$.