MAL7012 INTRO – MODELLING – 1 credit

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SEM-II – Assignment-I

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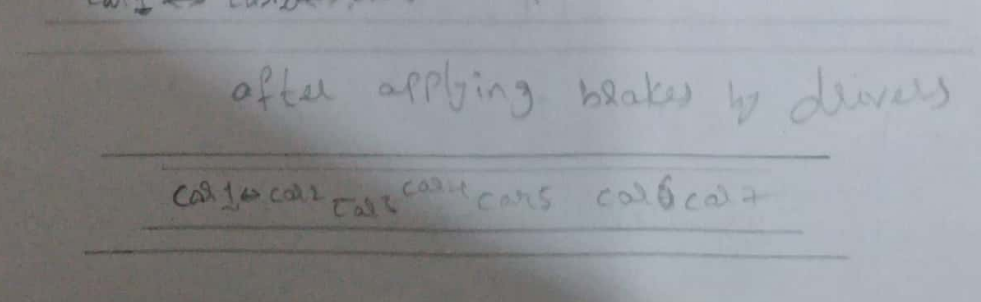
⇒ Modelling Process ⇐ Model the propagation of Breaking events in a single lane of cars, taking into A/c the reaction time of Drivers

① what is an Interesting Problem?

By modelling the propogation of Braking events in a single lane of Cars, we can better understand how the small perturbations can lead to larger discruptions in traffic flow and help inform stratergies for Reducing Congestion.

we are studying the problem of a single lane of cars & the perturbation from equillibrium that occurs when one car brakes & the braking effect travels down the line of cars, amplifying as it goes along, due to the delayed reactions time of drivers.

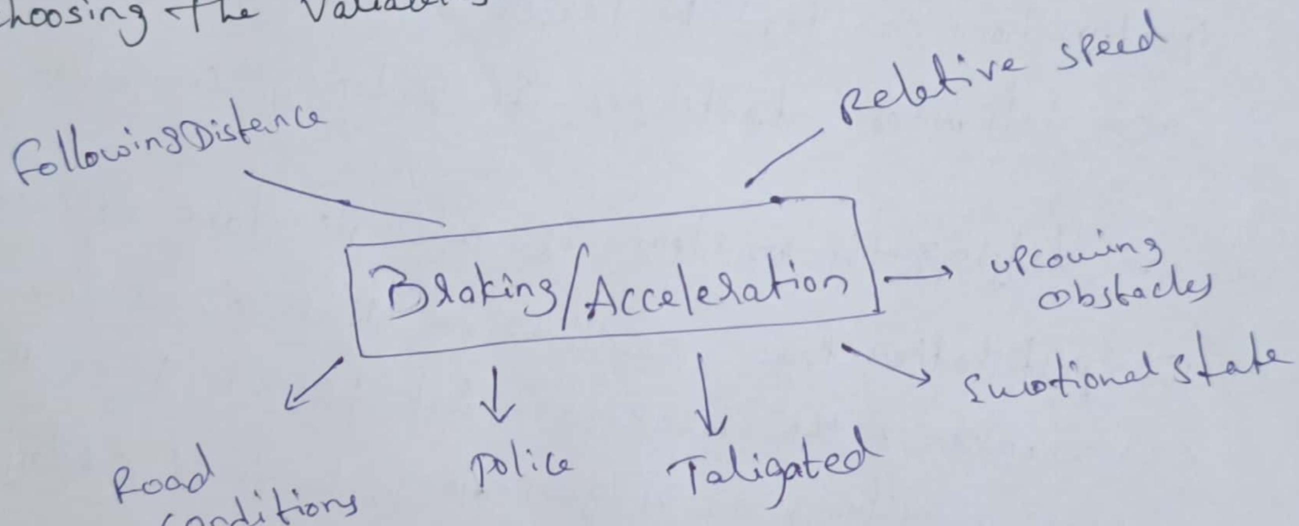
The ultimate phenomena could be pridict is that stop & go behaviour where cars dont just travel at a constant speed in rush hour, but altelnatively between accelerating & braking based on following distance & reletive velocity of the Car ahead to it.



② what are the objectives of modelling? what do you want to find?

The objectives are to understand the reaction time of Drivers affects the propogation of Braking Events in a single lane of Cars and predict the resulting Traffic Congestion

③ choosing the Variables.



Now I'm focusing on mainly 2 variables

① following distance

② Relative speed.

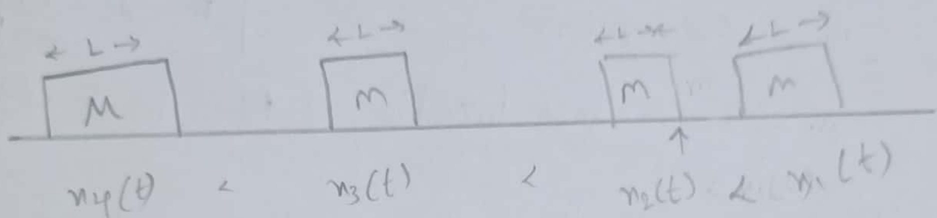
④ what is given?

Having the data on the speed and acceleration of Invidual cars in a single lane of traffic, as well as data on reaction time of drivers in similar conditions.

Data Contains the variables speed of car, drivers reaction time.

(5) what assumptions are used? How are they justified?

we have to assume reaction time of drivers is constant and speed of car and acceleration of each car modeled using a set of ordinary differential eqn's



- we have to assume All cars mass 'm' and length 'l'
- assume all cars are passing in one lane
- No crash is occured b/w cars x_i ~~$(t) < x_{i-1}$~~

if we assume cars are crashed model will get breakdown
main aim here is to prevent Bumpering / crashing b/w cars

$x_i(t) < x_{i-1}(t) + L$

$\therefore$ No cars are crashed.

- all people have response time ?

(6) How is model built? How is it solved/simulated? Explain the process?

Building the Model:

Breaking force: Here describing a force i.e. breaking force which is Newton's law of mass times acceleration.

$\boxed{m \cdot x_i''(t)}$

m = mass of car

$x_i''(t)$ = refers location of the front of the i^{th} car

We are considering 2 factors (i) Relative velocity
(ii) following Distance

Note:- larger the relative velocity > acceleration
+ smaller the following distance > acceleration

$$m \cdot x_i''(t) = c \frac{x_i'(t) - x_{(i-1)}'(t)}{|x_i(t) - x_{i-1}(t)|} \begin{matrix} \leftarrow \text{relative velocity} \\ \leftarrow \text{following distance} \end{matrix}$$

Relative velocity $(x_i'(t) - x_{(i-1)}'(t))$

∴ Breaking force is proportional to the relative velocities ($\propto$)

following distance $|x_i(t) - x_{i-1}(t)|$ ∴ The following distance That difference b/w positions of the two cars

∴ Breaking force is inversely proportional to following distance ($\propto$)

C = constant proportionality

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adding τ into the eq τ = Response time

If the 1st car applies Break, the second car driver should react and apply Breaks. so it would be difficult to apply Breaks at a time.

finally.
$$m \cdot x_i''(t+\tau) = c \frac{x_i'(t) - x_{i-1}'(t)}{|x_i(t) - x_{i-1}(t)|}$$

m & c are constants

Rewriting eq as
$$x_i''(t+\tau) = C \frac{x_i'(t) - x_{i-1}'(t)}{|x_i(t) - x_{i-1}(t)|}$$

$\int$ Integrating accelerations to relatives velocities

$$\boxed{v_i(t+\tau) = \tilde{c} \ln |x_i(t) - x_{i-1}(t)| + D_i} \quad -①$$

D_i = Constant of Integration

This is the final model it's focused on one car what does one car look forwards it looks to the car in front of it and it obeys the rules of model

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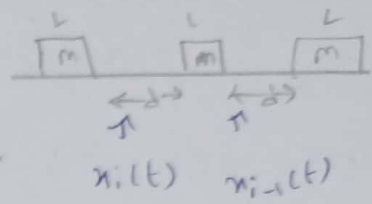
macroscopic properties and relation b/w density & velocity

Equillibrium:



- All cars same velocity v
- All cars same following distance 'd'.

Case I: cars at equillibrium

Density $\rho = \frac{\# \text{Cars}}{\text{distance}}$

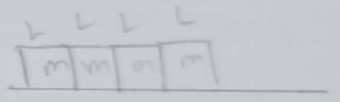
Relationship b/w density & velocity

Ans: consider 4 cars & distance 'd'

$$\frac{4}{4(L+d)} = \frac{1}{L+d} \Rightarrow \frac{1}{|x_i(t) - x_{i-1}(t)|}$$

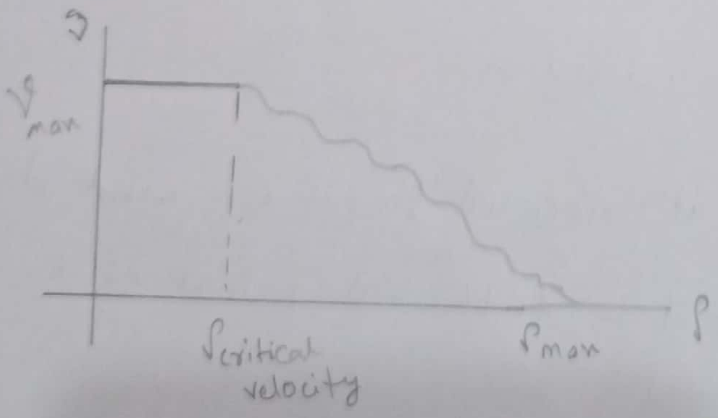
Case II: cars at no distance (or) crash

Max density: $\rho_{max} = \frac{1}{L}$



(i) velocity of cars at equillibrium related to density

∴ driver maintaining constant speed.



We hit a critical velocity which is that enough traffic on road that driver slows down car

- The curve shows declining more cars there are on road the higher their density the lower my velocity. Finally my velocity is 0 & max density when cars bumpered.

at equillibium:

$$v_i(\rho) = \tilde{c}\ln\left|\frac{1}{\rho}\right| + D_i \quad ; \quad v(\rho_{max}) = \tilde{c}\ln\left|\frac{1}{\rho_{max}}\right| + D = 0$$

velocity is same at equillibrium

$$v(\rho_{max}) = \tilde{c}\ln\left|\frac{1}{\rho}\right| - \tilde{c}\ln\left|\frac{1}{\rho_{max}}\right|$$

$$\boxed{v(\rho) = \tilde{c}\ln\left|\frac{\rho_{max}}{\rho}\right|}$$

$$v(\rho_{crit}) = \tilde{c}\ln\left|\frac{\rho_{max}}{\rho_{crit}}\right| = v_{max}$$

evaluating at critical density The density where first start going slower.

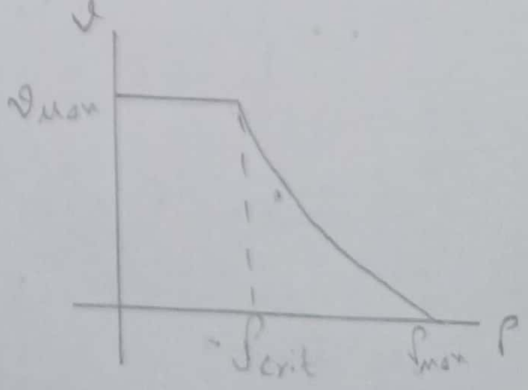
v_{max} ± going speed

$$\tilde{c} = v_{max}\left[\ln\left|\frac{\rho_{max}}{\rho_{crit}}\right|\right]^{-1}$$

finally

$$v(\rho) = v_{max}\left(\ln\left|\frac{\rho_{max}}{\rho_{crit}}\right|\right)^{-1} \cdot \ln\left|\frac{\rho_{max}}{\rho}\right|.$$

Ale to the Model we will get mode like shown in graph



finally Relation b/w density & velocity shown in below graph.

Maximizing flux & optimal density:

- Previously considered density is not most important factor

∴ ~~Now~~ ~~[illegible]~~

∴ Now explaining/considering ~~the~~ the Two streams of Traffic at Two different equilibriums.

flux: No. of cars that are going to pass a point in some unit of time

$$\text{flux} = \frac{\#\,\text{cars}}{\text{time}}$$

flux & density are related

Ext No. of cars in some unit distance some unit distance & term over time distance over time is velocity & No. of cars over distance that was notion of density

$$\text{flux} = \frac{\#\,\text{cars}}{\text{distance}} \cdot \frac{\text{distance}}{\text{time}}$$

$$\text{flux}(\rho) = \rho \cdot v(\rho)$$

$$\rho \cdot \left\{ v_{max} \left[\ln\left[\frac{\rho_{max}}{\rho_{crit}}\right]\right]^{-1} \times \ln\left|\frac{\rho_{max}}{\rho}\right| \right\}$$

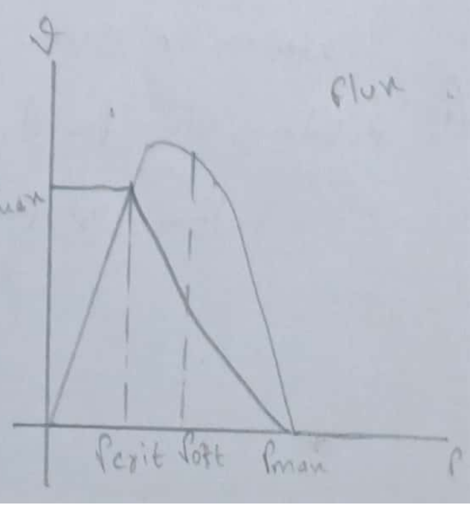
v = const

for $\rho > \rho_{crit}$

Max flux = $\frac{d}{d\rho}\,\text{flux}(\rho) = 0$

$$\boxed{\text{max at: } \rho_{opt} = \frac{\rho_{max}}{e}}$$

Initial portion ~~up~~ to row crit where v = const & flux is constant times ρ it's linear and max at optimal Density & decreases



$$\rho_{opt} = \frac{\rho_{max}}{e} = \frac{1}{eL}$$

at the equillibrium & optimal density

$$\rho_{opt} = \frac{\rho_{max}}{e} = \frac{1}{eL}$$

$$v(\rho_{opt}) = \tilde{c} \ln \frac{\rho_{max}}{\rho_{opt}}$$

$$v(\rho_{opt}) = \tilde{c} \ln \left[\frac{\rho_{max}}{\rho_{max}/e} \right]$$

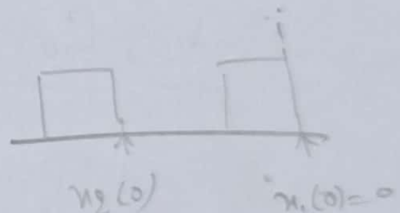
$$\Rightarrow \tilde{c} = v_{opt}$$

velocity is occur when people are driving optimal density

modelling sequence of cars :-

Trying with Transition variables x_1 time $= 0$

x_2 at time $= 0$



* imagine Transition variables at equillibrium

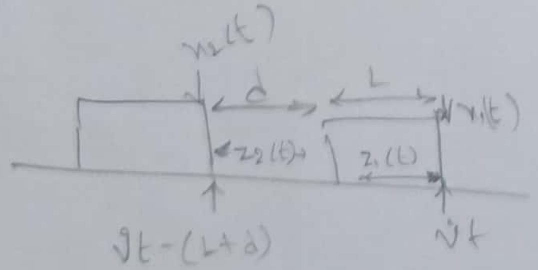
constant velocity. 1st car travelling time 't'

velocity 'v'

location 'vt'

lenght 'l'

gap b/w cars 'd'



The two displacements how much they differ while moving at equillibrium. That captures perturbation

$$z_i(t) = x_i(t) - [vt - (i-1)(d+L)]$$

$$z_i'(t) = v_i(t) - v$$

$$\boxed{z_i'(t+\tau) = v_i(t+\tau) - v}$$

$\tilde{c} = \vartheta$; $D = -\tilde{c}\ln\left|\frac{1}{S_{max}}\right|$ →.

adding values into main model

$$v_i'(t+\tau) = \vartheta\ln|x_i(t) - x_{i-1}(t)| - \vartheta\ln\left|\frac{1}{S_{max}}\right|$$

combining logarithms

$$v_i(t+\tau) = \vartheta\ln|S_{max}[x_i(t) - x_{i-1}(t)]|$$

$$\boxed{z_i'(t+\tau) = \vartheta\ln|S_{max}[d+L+z_i(t) - z_{i-1}(t)]| - \vartheta}$$

This eq describes all The Cars to which They are following behind Some other Car.

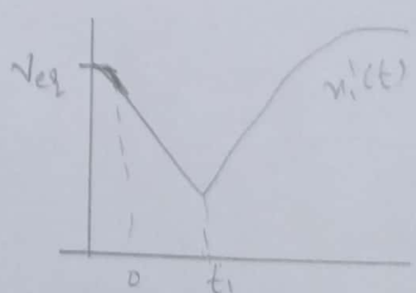
<u>finding for first car which was creates Initial Problem by braking:</u>

- starts at equilibrium velocity
- A short Braking from $t=0$ to $t=t_1$
- Then returns to Equilibrium velocity

$$x_i(t) = \begin{cases} \vartheta & t \le 0 \\ \vartheta\left(1 - kte^{\frac{t_1 - t}{t_1}}\right), & t > 0 \end{cases} \begin{matrix} \leftarrow < 0 \\ \leftarrow \text{at } 0 \end{matrix}$$

Constant velocity < 0

$$z_1(t) = -\vartheta\int_0^t kse^{\frac{t_1 - s}{t_1}}\,ds$$



~~differential de~~

Full model: Differential delay system:

It's a system of DE's one for each of car.

Consider Intial conditions which is That intial displacement for every one of the 'n' different cars at zero time.

Intial conditions: $Z_i(0) = 0$

for $1 \le i \le N$

Behavior of first car described by

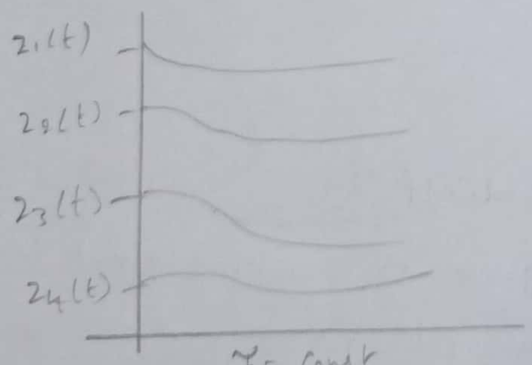
first car: $Z_1(t) = -v \int_0^t kse^{\frac{k-s}{t_1}} ds$, $t > 0$

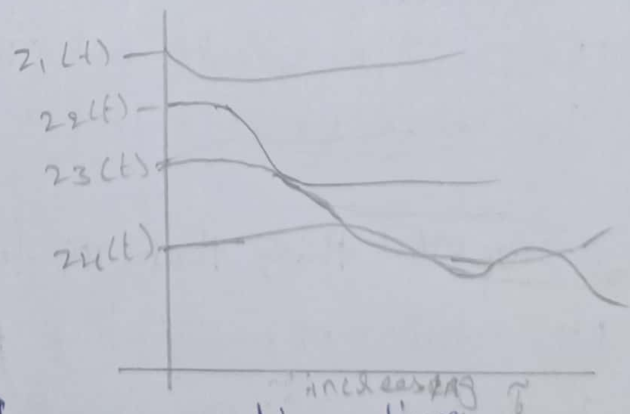
for all subsequent cars described differential delay eq:

for $2 \le i \le N$

$$Z_i'(t+\tau) = v \ln | S_{max} [d + L + Z_i(t) - Z_{i-1}(t)] | - v$$

Assesing the model:





A/c to This model Intial displacement Caused by short Brekins when it pertubs down line actually gets more delayed

τ = reaction time

- Having larger value for τ
- There lines are crossing over each other which equivalent to crash
- This is distance b/w car gets more Than value of 'd' a crash has actually occured.
- It is occured while Increasing τ.

7. How model is verified & it's Predictions validated?

The model is verified by comparing simulated Results to real world traffic data & checking for consistency with known physical laws.
The Predictions are validated by comparing them to actual traffic data such as speed & flow, and by calculating prediction accuracy metrics such as mean absolute error (MAE) or RMSE (Root Mean Squared Error)

8. Did the model Predict any new Pheonomena? How was the new Prediction is validated?

The model predict new phenomena, such as the amplification of braking events due to the reaction time of drivers, which could inform strategies for reducing congestion.

The New Predictions from model canbe validated by comparing simulated results to real world traffic data. This could involve data containing variables

$$MAE = \frac{1}{n} \sum |y_i - \hat{y}_i|$$

model can compared using metrics suchas

$$RMSE = \sqrt{\frac{1}{n} \sum (y_i - \hat{y}_i)^2}$$

y = actual value
$\hat{y}$ = Predicted value

Strengths :-

- Predict Phenomena we expect
- Informs us of ~~Potential~~ optimal density
- computable
- Limited Inputs needed

Weaknesses :-

- Assumptions are limited
- Behaviour gets non-physical

⑨ Did model require any modifications ?

The model may require modifications if the prediction accuracy is poor or if the assumptions made are found incorrect (or) we can add other variables into data can predict

⑩ was This model deployed & used ?

Model of propogation of Breaking events in single lane is used in Traffic management system to avoid Accidents by predicting the cause of breaks and Take the precaution by adding speed breaks or manageing Traffic etc.

⑪ Any other thing that you found of Intrest ?

The modelling of reaction time & the propogation of Breaking events in a single lane of cars can provide valuable Insights into the dynamics of traffic flow, which can be applied to other Transportation systems, such as trains or aircraft.