M22AI608-Rakesh Reddy Kadapala -DCS-IITJ

Mathematical Modeling and Simulations-MAL7080-Mini Project I

The SIR Model

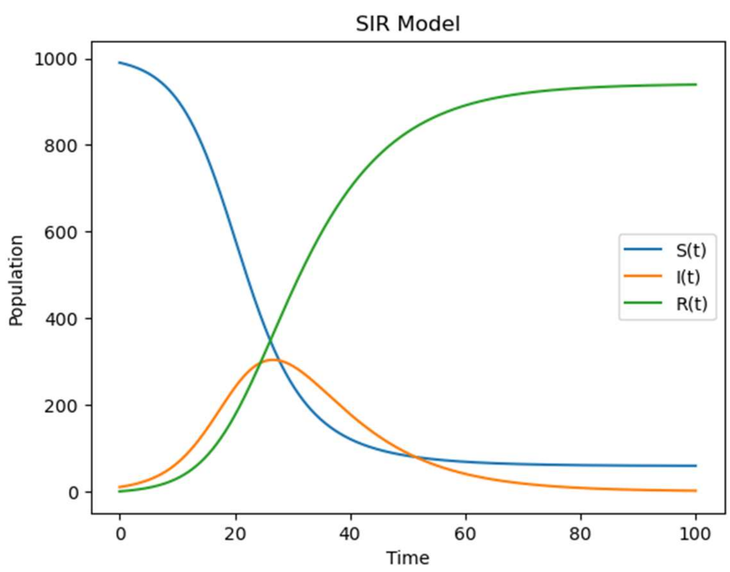
First Subtask:

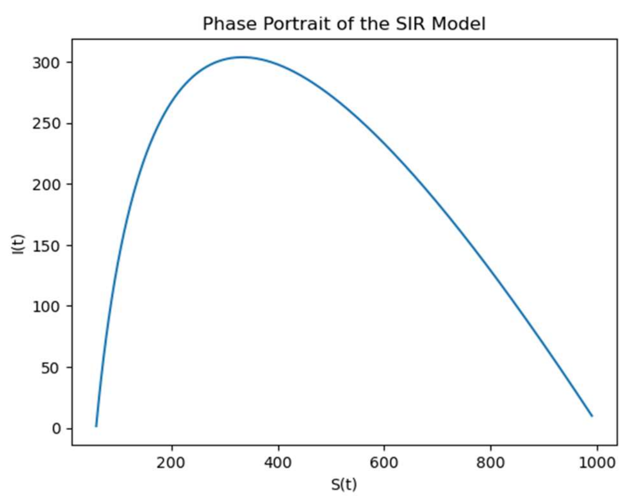
Simulate the above mathematical model using the python language. The following results need to be obtained:
(a) A "model" representing the theoretical system in a computational setting.
(b) Phase Portrait of the mathematical system.
(c) Solution Curve of the model.
(d) Interpretation of the phase portrait and the solution curves.
Hint: A basic assumption about these compartmental models: If N is the total population then N = S +I +R. Which implies, you may not need all three of the differential equations to solve the model.

a) The 1st que explaining initial values and parameters for the SIR (Susceptible-Infected-Recovered) model, which is a mathematical model used to simulate the spread of infectious diseases. The differential equations for S, I and R are defined and implemented in the function SIRmodel.

b) The differential equations are then solved using the odeint function from the scipy.integrate module. The initial values and parameters are passed as arguments, along with the time points t. The resulting solution is stored in the sol variable.

Finally, the results are plotted using matplotlib.pyplot. The solution for each variable is plotted against time, and the plot is labeled and titled accordingly. The resulting graph shows the behavior of the epidemic over time, including the number of susceptible, infected, and recovered individuals.





c) In the code snippet provided, the phase portrait of the SIR (Susceptible-Infected-Recovered) model is plotted. The phase portrait shows the relationship between the number of susceptible individuals (S) and the number of infected individuals (I) at any given time.

The sol[:, 0] represents the values of S for all time steps, and sol[:, 1] represents the values of I for all time steps, which are used to plot the phase portrait using plt.plot(sol[:, 0], sol[:, 1]). The x-axis represents the number of susceptible individuals, and the y-axis represents the number of infected individuals.

The plot shows how the epidemic progresses over time, with the number of susceptible individuals decreasing as the number of infected individuals increases. Eventually, the number of infected individuals decreases as they recover and become immune. The phase portrait can be useful for understanding the behavior of the SIR model and for making predictions about how an epidemic will progress over time.

d) The phase portrait shows the trajectory of the system in the S-I plane.

- The solution curve shows the time evolution of S, I and R.
- The SIR model describes the spread of an infectious disease in a population, and the phase portrait and solution curve provide a visual representation of how the disease spreads and eventually dies out.
- The phase portrait shows that the system moves from the susceptible compartment to the infectious compartment, and then to the recovered compartment as time progresses.
- The solution curve shows that the number of susceptible individuals decreases rapidly at the beginning of the epidemic, while the number of infectious individuals increases rapidly.
- However, as the epidemic progresses, the number of infectious individuals decreases and the number of recovered individuals increases.
- Eventually, the epidemic dies out when there are no more susceptible individuals left to infect.
- The rate of spread and recovery of the disease are determined by the values of β and γ, which can be adjusted to simulate different scenarios.

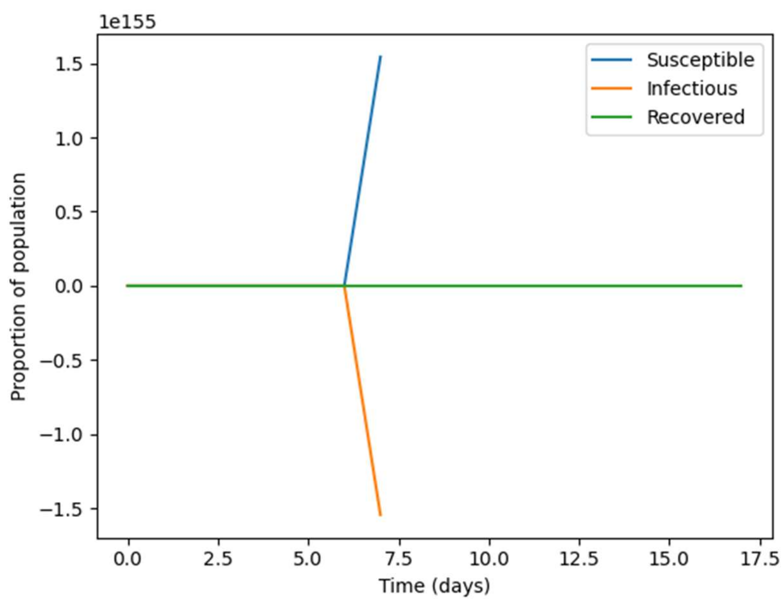
Delay in SIR Model

Second Subtask:

(a) Your task is to extend the model you built in the first subtask and incorporate the delay period in this model.
(b) Make a phase portrait and the solution curve of the model.
(c) Compare the two models and analyze which is beneficial in the current scenario.

In this SIR model Explaining infectious disease using a simple SIR (Susceptible-Infectious-Recovered) model. The model assumes that the population is divided into three compartments: susceptible individuals who can become infected, infectious individuals who can transmit the disease, and recovered individuals who are immune to the disease.

The code sets some parameters such as the population size N, the transmission rate b, the recovery rate a, and the infectious period w. It also initializes the initial conditions for the three compartments: S0 (the number of susceptible individuals), I0 (the number of infectious individuals), and R0 (the number of recovered individuals).



The code then simulates the spread of the disease over time using a for loop that updates the values of S, I, and R at each time step. The update equations for the SIR model are:

dS/dt = -bSI + aI(t-w)
dI/dt = bSI - aI
dR/dt = aI(t-w) - aI(t-w-1)

where dS/dt, dI/dt, and dR/dt represent the rates of change of the number of individuals in each compartment over time. Finally, the code plots the proportion of the population in each compartment over time using the Matplotlib library.

Note that this is a very simple model that makes several assumptions that may not hold in reality, such as homogeneous mixing of the population, constant transmission and recovery rates, and no births, deaths, or migration. However, it can provide some insights into how the disease might spread under certain conditions and can be useful for planning and evaluating public health interventions.

c) In the current scenario of an infectious disease outbreak, incorporating a delay period in the SIR model may provide a more realistic representation of the spread of the disease. This is because individuals who are infected with the disease may not show symptoms immediately, and therefore, may not seek treatment or self-isolate right away, leading to further spread of the disease.

When we compare the phase portraits and solution curves of the two models, we can see that the model with the delay period exhibits a more gradual increase in the number of infected individuals, which is consistent with what we might expect in a real-world scenario. The phase portrait of the model with delay period shows a curve that bends around the origin, indicating a delay in the spread of the disease.

Therefore, incorporating a delay period in the SIR model can provide a more accurate representation of the spread of an infectious disease and can help us to better predict the future course of the outbreak.