

Assignment - Fractal-3 - Machine Learning

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① Perceptron

① following Training samples are given

x_1	x_2	class
1	1	+1
-1	-1	-1
0	0.5	-1
0.1	0.5	-1
0.2	0.2	+1
0.9	0.5	+1

Table: Sample data

Assuming weight vector of initial decision boundary $w^T x = 0$ as $w = [1, 1]$
Solve following:

- <1> In how many steps Perceptron algorithm will converge?
- <2> what is the final decision boundary? show step-wise step update of weight vector using computation as well as hand-drawn plot

Assuming
The weight vector of Initial decision Boundary $w^T x = 0$ as $w = [1, 1]$

$$x_1 + x_2 = 0$$

$$b = 0$$

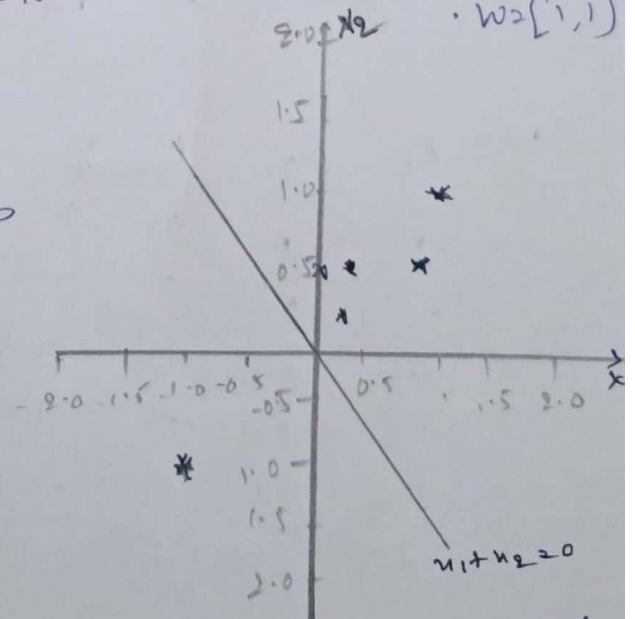
$$y_n = w^T x_n + b = w_1 x_1 + w_2 x_2 + b$$

Assume learning rate as 1.

$$y = \begin{cases} 1 & \text{if } y_n > 0 \\ 0 & \text{if } y_n = 0 \\ -1 & \text{if } y_n < 0 \end{cases}$$

$$\Delta w_1 = \alpha t x_1 \quad \Delta b = \alpha t$$

$$\Delta w_2 = \alpha t x_2$$



Initial Boundary decision & samples

<u>I</u>	x_1	x_2	class(t)	y_{in}	y	Δw_1	Δw_2	Δb	w_1	w_2	b
	1	1	+1	2	+1	0	0	0	1	1	0
	-1	-1	-1	-2	-1	0	0	0	1	1	0
	0	0.5	-1	0.5	+1	0	-0.5	-1	1	0.5	-1
	0.1	0.5	-1	-0.65	-1	0	0	0	1	0.5	-1
	0.2	0.2	+1	-0.2	-1	0.2	0.2	1	1.2	0.7	0
	0.9	0.5	+1	1.43	+1	0	0	0	1.2	0.7	0

<u>II</u>	x_1	x_2	t	y_{in}	y	Δw_1	Δw_2	Δb	w_1	w_2	b
	1	1	+1	1.9	+1	0	0	0	1.2	0.7	0
	-1	-1	-1	-1.9	-1	0	0	0	1.2	0.7	0
	0	0.5	-1	0.35	+1	0	-0.5	-1	1.2	0.2	-1
	0.1	0.5	-1	-0.78	-1	0	0	0	1.2	0.2	-1
	0.2	0.2	+1	-0.72	-1	0.2	0.2	1	1.4	0.4	0
	0.9	0.5	+1	1.46	+1	0	0	0	1.4	0.4	0

<u>III</u>	x_1	x_2	t	y_{in}	y	Δw_1	Δw_2	Δb	w_1	w_2	b
	1	1	+1	1.8	+1	0	0	0	1.4	0.4	0
	-1	-1	-1	-1.8	-1	0	0	0	1.4	0.4	0
	0	0.5	-1	0.2	+1	0	-0.5	-1	1.4	-0.1	-1
	0.1	0.5	-1	-0.81	-1	0	0	0	1.4	-0.1	-1
	0.2	0.2	+1	-0.74	-1	0.2	0.2	1	1.6	0.1	0
	0.9	0.5	+1	1.49	+1	0	0	0	1.6	0.1	0

<u>IV</u>	x_1	x_2	t	y_{in}	y	Δw_1	Δw_2	Δb	w_1	w_2	b
	1	1	+1	1.7	+1	0	0	0	1.6	0.1	0
	-1	-1	-1	-1.7	-1	0	0	0	1.6	0.1	0
	0	0.5	-1	0.05	+1	0	-0.5	-1	1.6	-0.4	-1
	0.1	0.5	-1	-1.04	-1	0	0	0	1.6	-0.4	-1
	0.2	0.2	+1	-0.76	-1	0.2	0.2	1	1.8	-0.2	0
	0.9	0.5	+1	1.52	+1	0	0	0	1.8	-0.2	0

V

x_1	x_2	t	y_{in}	y	Δw_1	Δw_2	Δb	w_1	w_2	b
1	1	+1	1.6	+1	0	0	0	1.8	-0.2	0
-1	-1	-1	-1.6	-1	0	0	0	1.8	-0.2	0
0	0.5	+1	-0.1	-1	0	0	0	1.8	-0.2	0
0.1	0.5	-1	0.08	+1	-0.1	-0.5	-1	1.7	-0.7	-1
0.2	0.2	+1	-0.8	-1	0.2	0.2	1	1.9	-0.5	0
0.9	0.5	+1	1.46	+1	0	0	0	1.9	-0.5	0

VI

x_1	x_2	t	y_{in}	y	Δw_1	Δw_2	Δb	w_1	w_2	b
1	1	+1	1.4	+1	0	0	0	1.9	-0.5	0
-1	-1	-1	-1.4	-1	0	0	0	1.9	-0.5	0
0.01	0.5	-1	-0.25	-1	0	0	0	1.9	-0.5	0
0.4	0.5	-1	-0.06	-1	0	0	0	1.9	-0.5	0
0.2	0.2	+1	0.28	+1	0	0	0	1.9	-0.5	0
0.9	0.5	+1	1.46	+1	0	0	0	1.9	-0.5	0

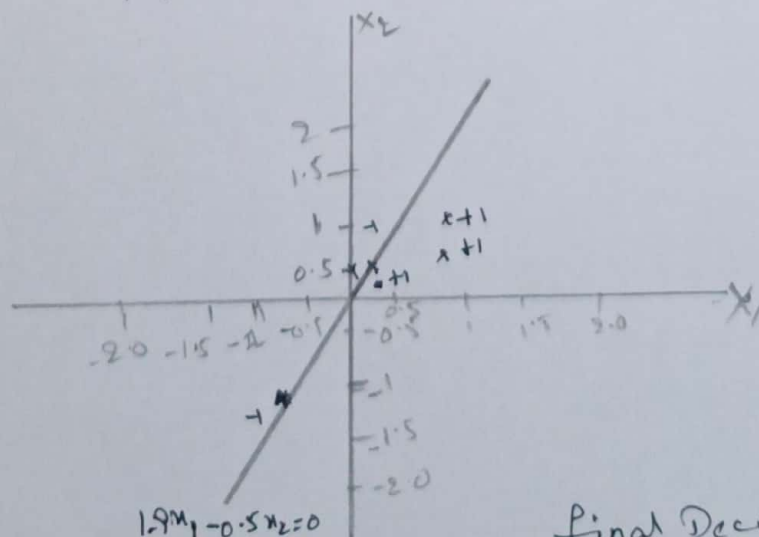
The Perceptron learning algorithm Converged in 6 steps.

The final weight vector of decision boundary is $w = [1.9, -0.5]$

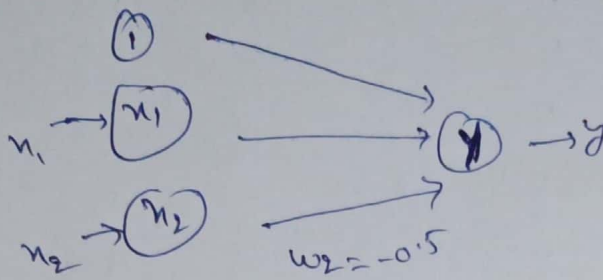
$$1.9x_1 + (-0.5x_2) = 0 \Rightarrow 1.9x_1 - 0.5x_2 = 0$$

Plotting final decision Boundary

we can see that $1.9x_1 - 0.5x_2 = 0$ line separates 2 classes correctly.



Final Decision Boundary.



neural network corresponding to Perception.