Financial Engineering (MAL7350)

ASSIGNMENT 2

OPTION PRICING

Submitted by: Group 12

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1. Pick a stock of your choice which is being traded in the derivative market.

We have chosen APPLE INC (AAPL). stock which is being traded in the derivative market.

Description / Introduction to chosen stock:

The global technology corporation Apple Inc. (**AAPL**) is well known for creating, producing digital content (digital content via its iCloud, Apple Music, iTunes Store, and App Store) and selling a variety of online services, softwares like iOS, macOS, watchOS, tvOS , and consumer electronics like iPhone, iPad, Mac computers, Apple Watch are among AAPL's flagship products. Apple has a sizable global presence and a solid financial position in the International Market. The company's shares, which are traded on the NASDAQ market under the ticker symbol AAPL, are widely owned by both retail and institutional investors.

Why AAPL ?

1. **Liquidity**: AAPL stock is highly liquid, meaning there are usually many buyers and sellers in the market at any given time.
2. **Volatility**: AAPL stock tends to exhibit significant price movements, especially around key events such as product launches, earnings reports, and market developments.
3. **Diversification**: Apple has a sizable global presence and a solid financial position in the International Market and are widely owned by both retail and institutional investors.

Overall, the combination of liquidity, volatility, investor interest, diversifications makes AAPL a good choice.

2. Look at the stock's and option's history on YAHOO Finance.

The below are the stock details and history of the stock chosen (AAPL).

```
Stock Details for AAPL
Name: Apple Inc.
Exchange: NMS
Currency: USD
Current Price: 165.03
Market Cap: 2548376535040
Volume: 33932186
Previous Close: 167.04
Open: 166.12
Day's Range: 164.08 - 166.4
52 Week Range: 162.8 - 199.62
```

```
                  Open        High         Low       Close   Adj Close  \
Date
2023-04-17  165.089996  165.389999  164.029999  165.229996  164.352737
2023-04-18  166.100006  167.410004  165.649994  166.470001  165.586151
2023-04-19  165.800003  168.160004  165.539993  167.630005  166.740005
2023-04-20  166.089996  167.869995  165.559998  166.649994  165.765182
2023-04-21  165.050003  166.449997  164.490005  165.020004  164.143860
2023-04-24  165.000000  165.600006  163.889999  165.330002  164.452209
2023-04-25  165.190002  166.309998  163.729996  163.770004  162.900497
2023-04-26  163.059998  165.279999  162.800003  163.759995  162.890549
2023-04-27  165.190002  168.559998  165.190002  168.410004  167.515854
2023-04-28  168.490005  169.850006  167.880005  169.679993  168.779099
2023-05-01  169.279999  170.449997  168.639999  169.589996  168.689575
2023-05-02  170.089996  170.350006  167.539993  168.539993  167.645157
2023-05-03  169.500000  170.919998  167.160004  167.449997  166.560959
2023-05-04  164.889999  167.039993  164.309998  165.789993  164.909744
2023-05-05  170.979996  174.300003  170.759995  173.570007  172.648483
2023-05-08  172.479996  173.850006  172.110001  173.500000  172.578827
2023-05-09  173.050003  173.539993  171.600006  171.770004  170.858032
2023-05-10  173.020004  174.029999  171.899994  173.559998  172.638504
2023-05-11  173.850006  174.589996  172.169998  173.750000  172.827499
2023-05-12  173.619995  174.059998  171.000000  172.570007  171.891205

               Volume
Date
2023-04-17   41516200
2023-04-18   49923000
2023-04-19   47720200
2023-04-20   52456400
2023-04-21   58337300
2023-04-24   41949600
2023-04-25   48714100
2023-04-26   45498800
2023-04-27   64902300
2023-04-28   55209200
2023-05-01   52472900
2023-05-02   48425700
2023-05-03   65136000
2023-05-04   81235400
2023-05-05  113316400
2023-05-08   55962800
2023-05-09   45326900
2023-05-10   53724500
2023-05-11   49514700
2023-05-12   45497800
```

3. Use past one-year data to estimate the annual volatility.

Annual Volatility :

The most common way to measure volatility in finance is through standard deviation. This mathematical expression says how much a variable can differ from its average.

$$\text{Annual Volatility} = \sigma \times \sqrt{T}$$

Where:

- σ is the standard deviation of the logarithmic returns.
- T is the number of trading periods in one year.

Assuming 252 trading days in the period of one year, below are the calculated annualized volatility for the chosen stock (AAPL).

```
Annual Volatility of AAPL: 0.19594083568773518
closing price AAPL: 172.69000244140625
```

4. Use 10 years' US treasury rate or any other rate as the rate of interest.

We have considered the interest rate from the latest data as the rate of interest.

```
Risk-free Interest Rate: 0.04627999782562256
```

5. Now fix the strike price and time of maturity to evaluate the call/ put option price by using one step Binomial model.

The one-step binomial model assumes that the price of the underlying asset can fluctuate by a specific factor over a single time step, which simplifies the pricing of options.

The following are the steps used to compute the **call / put option** using one step Binomial model.

1) Compute dt:

(dt) is computed by dividing the time of maturity(T) by the number of steps.As it is a one-step model ,it is typically the same (as divided by 1 , no of steps =1).

dt = T / 1

2) Compute upward factor (u) and downward factor (d):

The upward factor and the downward factors can be computed using the below formula.

$$u = e^{\sigma\sqrt{\Delta t}}$$

Where:

- σ is the volatility of the underlying asset's price.
- Δt is the length of the time step.

$$d = \frac{1}{u}$$

Where:

- u is the upward factor calculated using the formula $u = e^{\sigma\sqrt{\Delta t}}$.

3) Compute Risk Neutral Probability (p)

The risk neutral probability can be calculated by the following formula.

$$p = \frac{e^{r\Delta t} - d}{u - d}$$

Where:

- r is the risk-free interest rate.
- Δt is the length of the time step.
- u is the upward factor calculated using the formula $u = e^{\sigma\sqrt{\Delta t}}$.
- d is the downward factor, which is the reciprocal of u, calculated as $d = \frac{1}{u}$.

4) Compute Call Option:

- For a call option:
  - $C_u = \max(0, S \times u - K)$
  - $C_d = \max(0, S \times d - K)$

Where S is the current price of the underlying asset and the K is the strike price.

$$\text{option\_price} = e^{-r\Delta t} \times (p \times C_u + (1 - p) \times C_d)$$

5) Compute Put Option:

- For a put option:
  - $P_u = \max(0, K - S \times u)$
  - $P_d = \max(0, K - S \times d)$

The computed value of option price when **call option** is considered:

```
Option Price (One-Step Binomial Model): 27.30883031423434
```

The computed value of option price when **put option** is considered:

```
Put Option Price (One-Step Binomial Model): 7.3838
```

6. Evaluate the option price by using Black Scholes Formula

A mathematical formula used to determine the theoretical price of European-style options is the Black-Scholes formula, sometimes referred to as the Black-Scholes-Merton model.The formula is especially well-known for its application to risk management and option pricing.

$$C = S\,e^{-qt}\,N(d_1) - K\,e^{-rt}\,N(d_2)$$

$$P = K \cdot e^{-rT} \cdot N(-d_2) - S \cdot N(-d_1)$$

$N(x)$ is the standard normal cumulative distribution function:

$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{\frac{-t^2}{2}} dt$$

d1 and d2

The formulas for d_1 and d_2 are:

$$d_1 = \frac{\ln(\frac{S}{K}) + t(r - q + \frac{\sigma^2}{2})}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

The value obtained after the computation is:

```
Call Option Price (Black-Scholes Formula): 25.0596
Put Option Price (Black-Scholes Formula): 5.1336
```

7. Increase the number of steps in the Binomial model to verify that the price converges to (6).

To get closer to the theoretical price found using the Black-Scholes formula, you usually increase the number of steps in the binomial model. The binomial model converges to the Black-Scholes price and gains accuracy as the number of steps increases

The value obtained after the convergence when **call option** is considered:

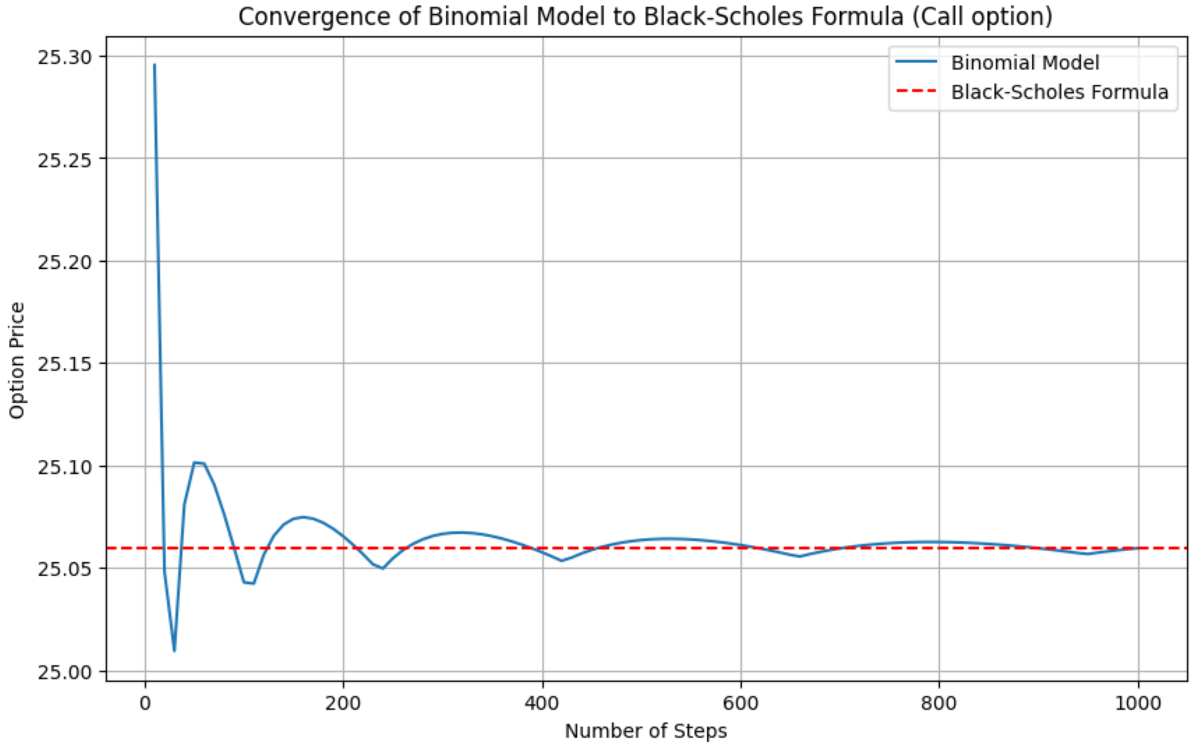
```
call Option Price (Binomial Model with 10 steps): 25.2954
call Option Price (Binomial Model with 50 steps): 25.1015
call Option Price (Binomial Model with 100 steps): 25.0430
call Option Price (Binomial Model with 500 steps): 25.0636
call Option Price (Binomial Model with 1000 steps): 25.0596
```

The values obtained after the convergence when **put option** is considered:

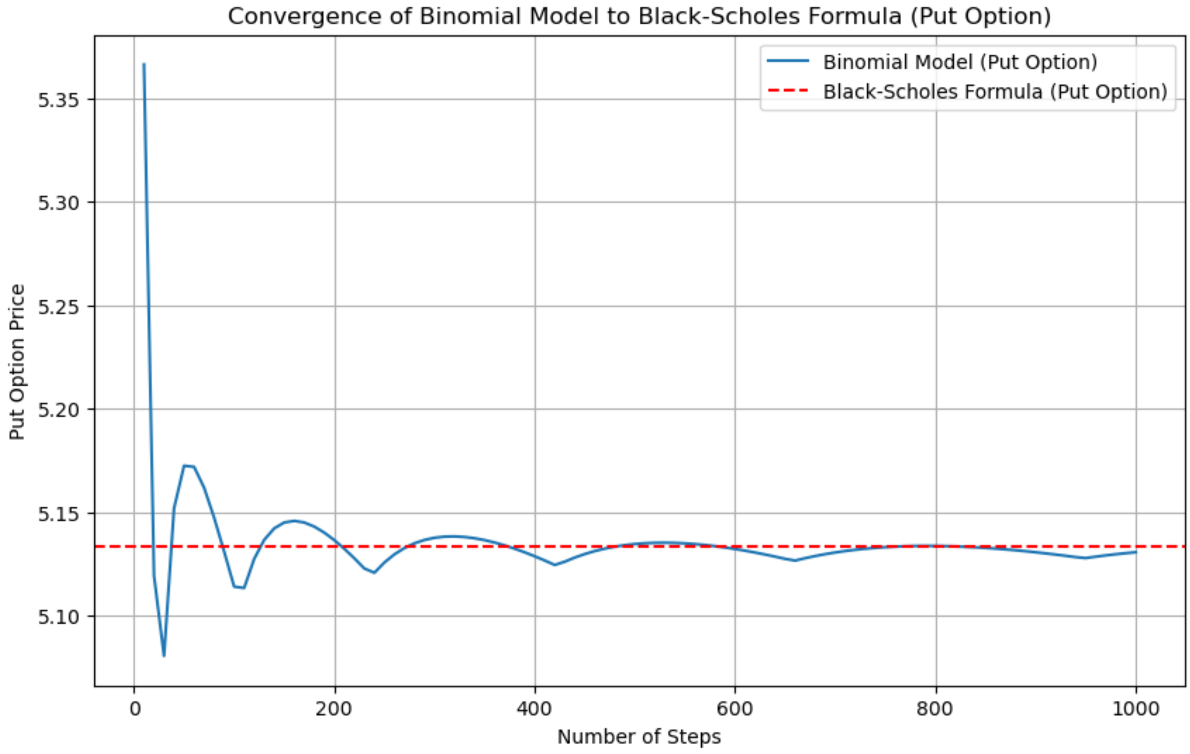
```
Put Option Price (Binomial Model with 10 steps): 5.3695
Put Option Price (Binomial Model with 50 steps): 5.1754
Put Option Price (Binomial Model with 100 steps): 5.1171
Put Option Price (Binomial Model with 500 steps): 5.1376
Put Option Price (Binomial Model with 1000 steps): 5.1336
```

8. Draw the graph to show 7.

The graph visualizing the above convergence for **call option is** :



The graph visualizing the above convergence for **put option** is :



9. How do you create a delta neutral portfolio?

Building a portfolio of financial instruments in a way that minimizes or completely eliminates changes in the value of the portfolio with minor changes in the price of the underlying asset is known as creating a delta-neutral portfolio. The term "delta" describes how sensitive the option's price is to variations in the underlying asset's price.

1. **Recognise Delta**: Delta expresses how sensitive an option's price is to variations in the price of the underlying asset.
2. **Determine Delta**: For every option position in your portfolio, find its delta.
3. **Balance Delta**: Aim for a portfolio where you can offset long positions with short positions so that the total deltas are almost equal to zero.
4. **Adjust Positions:** To reach a delta-neutral state, buy or sell options or the underlying asset.
5. **Frequent Rebalancing**: As asset prices fluctuate, keep an eye on the portfolio and make necessary adjustments to maintain delta neutrality.
6. **Examine Additional Risks**: Although delta neutrality can provide some protection against slight fluctuations in price, it is important to recognise and manage other risks, such as gamma and vega.

The delta neutral portfolio created is:

```
For call option:

Shares needed for delta-neutral portfolio: -0.7654
Options needed for delta-neutral portfolio: 1
Total delta of the portfolio: 0.0

For put option:

Shares needed for delta-neutral portfolio (put option): 0.2346
Options needed for delta-neutral portfolio (put option): 1
Total delta of the portfolio (put option): 0.0
```

10. Use numerical methods to get implied volatility? You can use Excel, Python and R for coding.

Implied Volatility:

The current market price of an option indicates the market's expectation for future price volatility of the underlying asset, which is measured as implied volatility. Stated differently, it refers to the implied level of volatility of the option based on its current market price and other specified parameters.

A crucial component of option pricing models, like the Black-Scholes model, which determine theoretical option prices, is implied volatility. Implied volatility represents the market's overall assessment of future price movements, whereas historical volatility represents past price movements of the underlying asset.

Computing Implied volatility:

The Newton-Raphson method is an iterative root-finding algorithm that can be used to solve for the implied volatility of an option given its market price and other parameters.

```
For call option:

Implied Volatility call option: 0.1959


For put option:

Implied Volatility: 0.2003
```

11. Calculate and interpret Option Greeks for a given option contract.

Option Greeks are metrics that are used to express how sensitive an option's price is to changes in a number of different areas, including interest rates, volatility, time to expiration, and the price of the underlying asset. They aid in the understanding of how shifts in these variables impact the worth of traders' and investors' option positions. The primary Option Greeks and their explanations are as follows:

1. **Delta (Δ)** represents the sensitivity of option price to changes in the price of the underlying asset. A higher delta indicates a more aligned price movement between the option and the underlying asset.
2. **Gamma (Γ)**: Indicates how quickly Delta changes. Greater Gamma means that even tiny changes in the underlying asset's price will cause Delta to fluctuate more quickly.
3. **Theta (Θ):** Indicates how sensitive an option's price is to time passing (time decay).
4. **Vega (v):** Measures option price sensitivity to changes in implied volatility.Higher Vega means the option's price increases more with increases in implied volatility.
5. **Rho (ρ):** Measures option price sensitivity to changes in the risk-free interest rate.Higher Rho means the option's price increases more with increases in the risk-free interest rate.

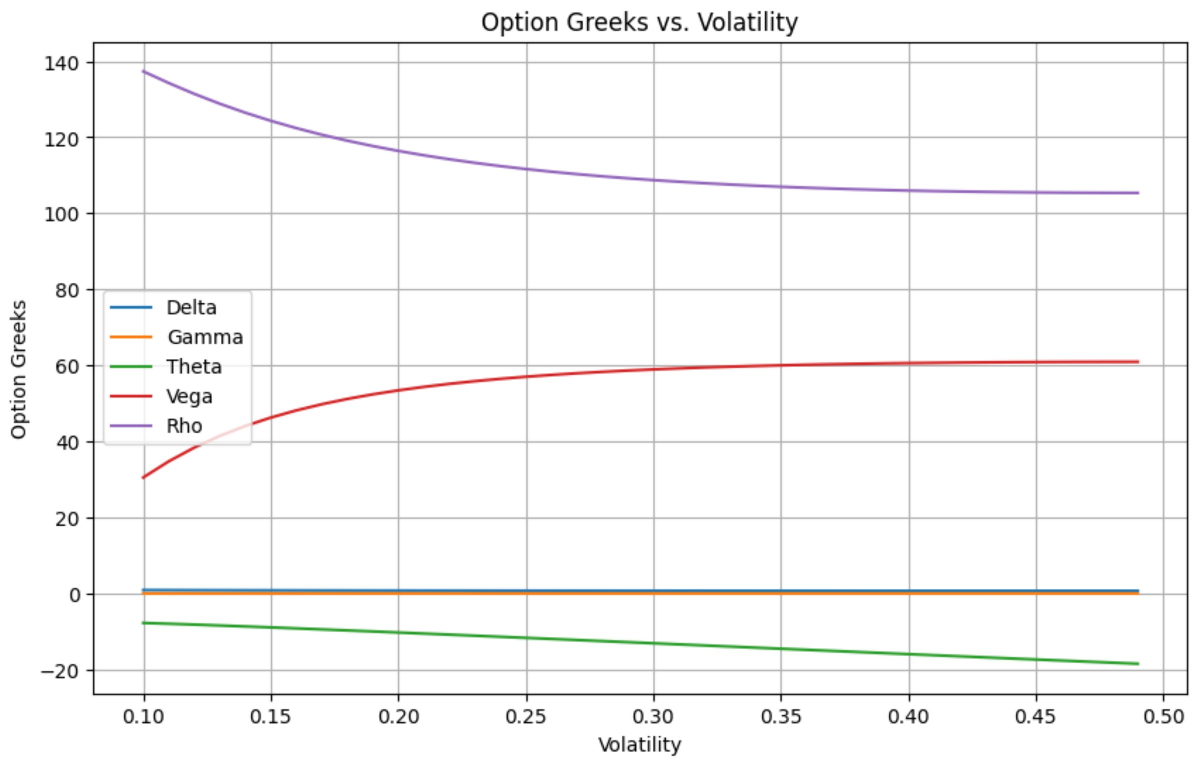
The below are the computed values of delta,gamma,theta,vega and rho for the call price.

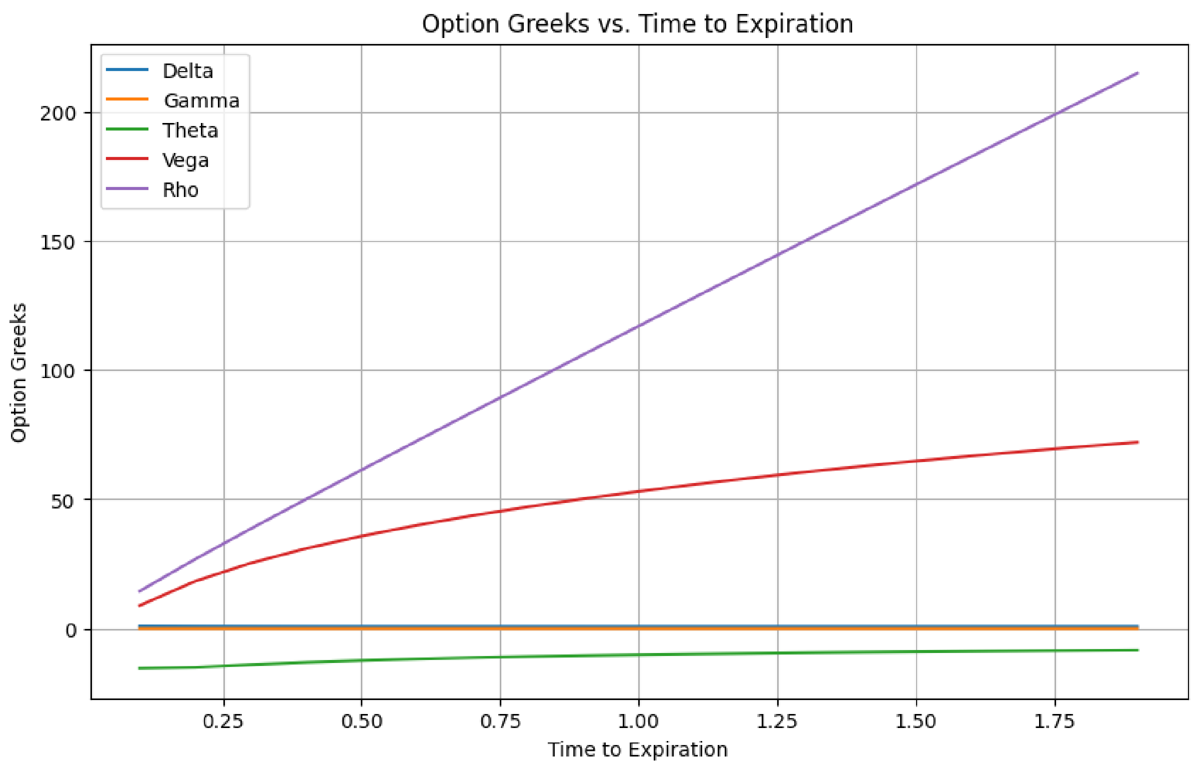
```
Call Price: 25.06
Delta: 0.7654
Gamma: 0.0091
Theta: -10.1525
Vega: 53.0132
Rho: 116.9289

Put Price: 5.13
Delta (Put): -0.2346
Gamma (Put): 0.0091
Theta (Put): -3.0796
Vega (Put): 53.0132
Rho (Put): -35.8320
```

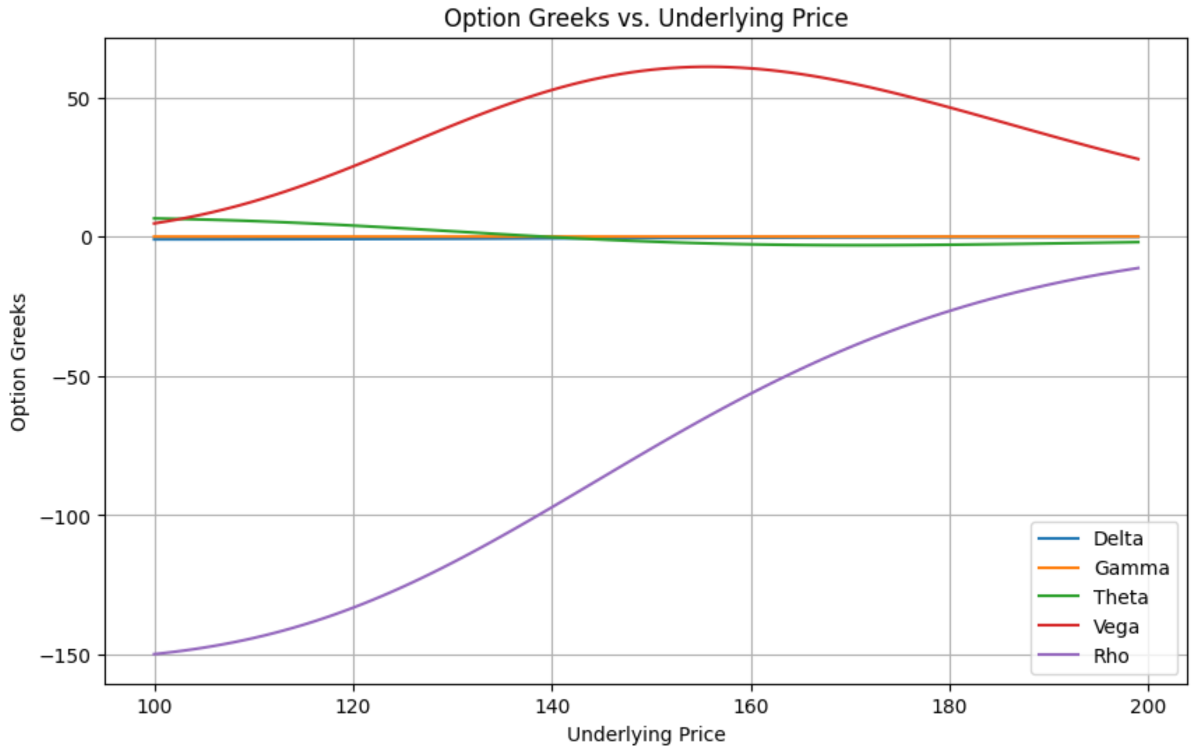
12. Visualize the impact of changes in underlying price, volatility, and time to expiration on Option Greeks.

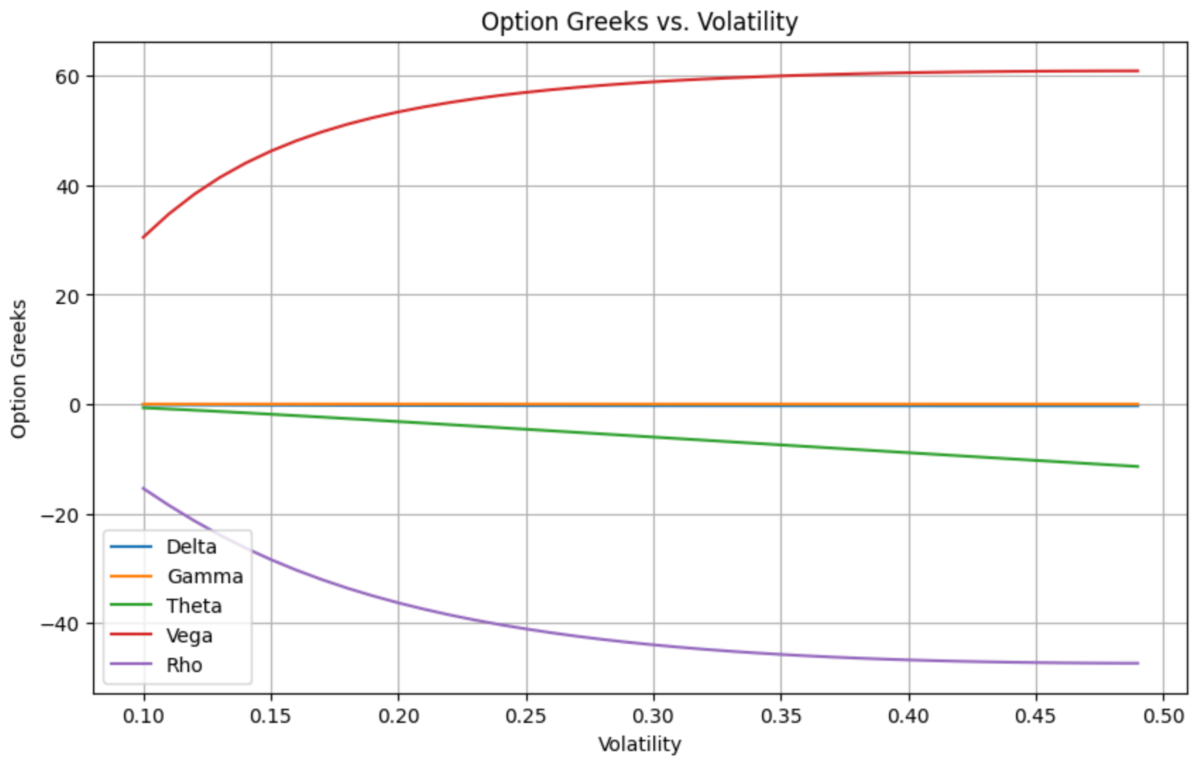
For call option:

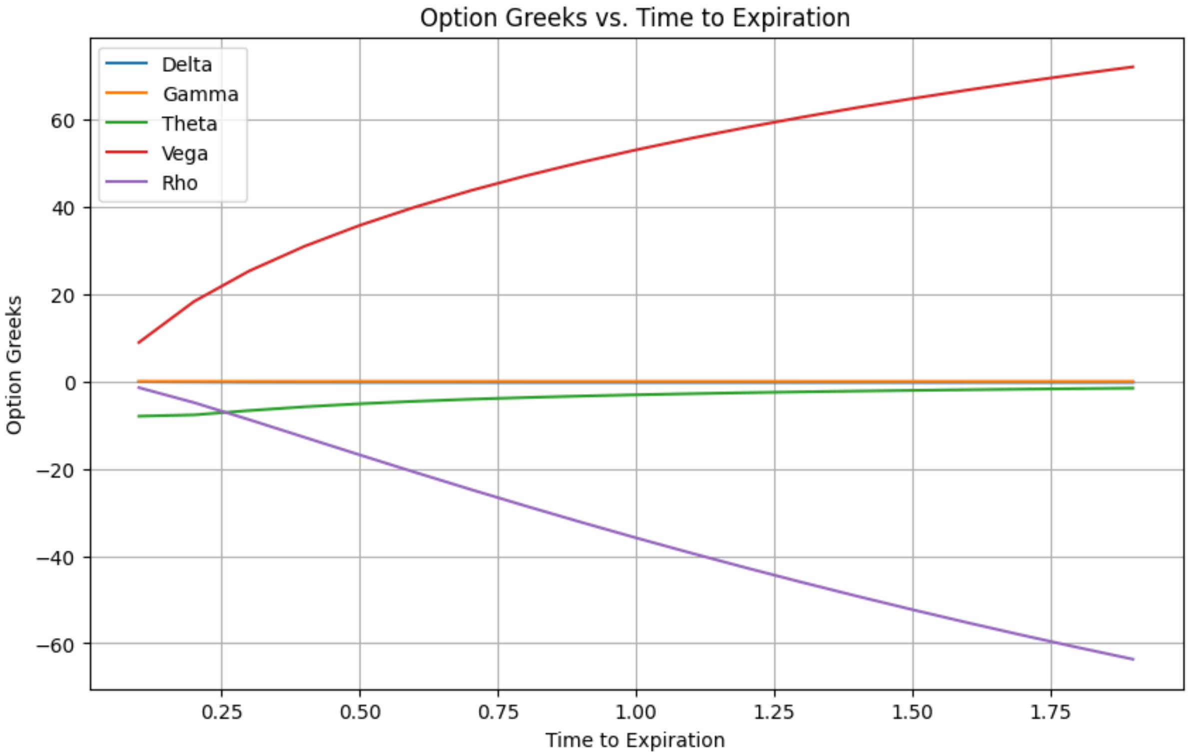

Option Greeks vs. Volatility
140
120
100
80
60
40
20
0
−20
Option Greeks
Delta
Gamma
Theta
Vega
Rho
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
Volatility


Option Greeks vs. Time to Expiration
200
150
100
50
0
Option Greeks
Delta
Gamma
Theta
Vega
Rho
0.25
0.50
0.75
1.00
1.25
1.50
1.75
Time to Expiration

For put option:






Option Greeks vs. Time to Expiration
Delta
Gamma
Theta
Vega
Rho
60
40
20
0
−20
−40
−60
Option Greeks
0.25
0.50
0.75
1.00
1.25
1.50
1.75
Time to Expiration