

AI-2 Ex-6

$$Q1 @ \quad K(x, z) = (x^T z + 1)^d$$

$$\Rightarrow K(x, z) = (x^T z + 1)^2 \quad (d=1, d=2)$$

$$x = [x_1, x_2]^T$$

$$z = [z_1, z_2]^T$$

L.H.S

$$K(x, z) = \left([x_1, x_2] \cdot \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + 1 \right)^2$$

$$= (x_1 z_1 + x_2 z_2 + 1)^2$$

$$= (x_1 z_1)^2 + (x_2 z_2)^2 + 1 + 2x_1 z_1 x_2 z_2 + 2x_1 z_1 + 2x_2 z_2$$

R.H.S

$$F(x)^T F(z)$$

$$= (x_1^2, \sqrt{2}x_1 x_2, x_2^2, \sqrt{2}x_1, \sqrt{2}x_2, 1) \cdot \begin{pmatrix} z_1^2 \\ \sqrt{2}z_1 z_2 \\ z_2^2 \\ \sqrt{2}z_1 \\ \sqrt{2}z_2 \\ 1 \end{pmatrix}$$

$$= x_1^2 z_1^2 + 2x_1 x_2 z_1 z_2 + x_2^2 z_2^2 + 2x_1 z_1 + 2x_2 z_2 + 1$$

$$= (x_1 z_1)^2 + (x_2 z_2)^2 + 1 + 2x_1 z_1 x_2 z_2 + 2x_1 z_1 + 2x_2 z_2$$

$$L.H.S = R.H.S$$

(b)

$$x = (x_1, x_2)^T$$

$$z = (z_1, z_2)^T$$

$$K(x, z) = (x_1 z_1 + x_2 z_2 + 1)^3 \quad (\because V=1, d=3)$$

$$= (x_1 z_1)^3 + (x_2 z_2)^3 + 1 + 3(x_1 z_1)^2 x_2 z_2 + 3(x_1 z_1)(x_2 z_2)^2 + 3(x_1 z_1)^2 + 3x_1 z_1 + 3(x_2 z_2)^2 + 3x_2 z_2 + 6x_1 z_1 x_2 z_2$$

$$= \begin{bmatrix} x_1^3 \\ x_2^3 \\ 1 \\ \sqrt{3} x_1^2 x_2 \\ \sqrt{3} x_1 x_2^2 \\ \sqrt{3} x_1^2 \\ \sqrt{3} x_1 \\ \sqrt{3} x_2^2 \\ \sqrt{3} x_2 \\ \sqrt{6} x_1 x_2 \end{bmatrix}^T \times \begin{bmatrix} z_1^3 \\ z_2^3 \\ 1 \\ \sqrt{3} z_1^2 z_2 \\ \sqrt{3} z_1 z_2^2 \\ \sqrt{3} z_1^2 \\ \sqrt{3} z_1 \\ \sqrt{3} z_2^2 \\ \sqrt{3} z_2 \\ \sqrt{6} z_1 z_2 \end{bmatrix}$$

$$\Rightarrow f(x) = \begin{bmatrix} x_1^3, x_2^3, \sqrt{3} x_1^2 x_2, \sqrt{3} x_1 x_2^2, \sqrt{3} x_1^2, \sqrt{3} x_2^2, \sqrt{3} x_1, \sqrt{3} x_2, \sqrt{6} x_1 x_2, 1 \end{bmatrix}^T$$