AI - 2 Ex - 05

Q ① From the graph it can be inferred that the solution is either at the boundary or where the function g is tangent to the contours of function f. Hence the possible solutions ~~are~~ is approx ~~(...), (...) or~~ $(-1.4, -1.4)$ or it is unbounded.

② Primal:

$$\max_{x_1, x} \; x_1^2 + x_2^2 + 1$$

$$\text{st.} \quad g \; x_1 + x_2 + 2\sqrt{2} = 0$$

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~~$\Lambda(x_1, x_2, \alpha)\, x_1^2 + x_2^2 + 1 + \alpha($~~

The lagrange function is :

$$\Lambda(x_1, x_2, \alpha) = x_1^2 + x_2^2 + 1 + \alpha(x_1 + x_2 + 2\sqrt{2}) \quad -(1)$$

$$\frac{\partial \Lambda}{\partial x_1} = 0 \Rightarrow 2x_1 + \alpha = 0$$

$$\Rightarrow x_1 = -\frac{\alpha}{2} \quad -(2)$$

$$\frac{\partial \Lambda}{\partial x_2} = 0 \Rightarrow 2x_2 + \alpha = 0$$

$$\Rightarrow x_2 = -\frac{\alpha}{2}$$

Substituting (2) & (3) in (1)

$$\Rightarrow \Lambda(\alpha) = \left(\frac{-\alpha}{2}\right)^2 + \left(\frac{-\alpha}{2}\right)^2 + 1 + \alpha\left(\frac{-\alpha}{2} - \frac{\alpha}{2} + 2\sqrt{2}\right)$$

$$\Rightarrow \Lambda(\alpha) = -\frac{\alpha^2}{2} + 2\sqrt{2}\alpha + 1$$

Dual:

$$\min_{\alpha} \; -\frac{\alpha^2}{2} + 2\sqrt{2}\alpha + 1$$

Solution of dual is unbounded. Hence the solution to primal is also unbounded.

Q2 From the plot it can be inferred that the maximum margin plane is only dependendent on the support vectors. Hence the dual problem

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will have alphas $\neq 0$ corresponding to these support vectors.

It is also possible that we don't find a maximum margin seperating plane because the given data is not linearly seperable.

If the primal has many constraints but only a few of those are active, which is mostly the case with support vectors, then the corresponding dual will have many alphas $= 0$, which ~~has~~ is very helpful to decrease the computational burden, hence ~~the~~ finding a ~~linear~~ max margin plane using ~~suppo~~ dual problem can often lead to reduced computational time.