AI-2

Exercise-1

Q1 (a) Let us assume that the ice cream seller produces

x_1 kg of choclate fudge Brownie per hour

x_2 kg of strawberry cheesecake per hour

(i) First constraint on total ice-cream production :

$\Rightarrow \boxed{x_1 + x_2 \leq 10}$ — (i)

(ii) Second constraint on total energy consump. / hour

$\Rightarrow \boxed{5x_1 + 2x_2 \leq 30}$ — (ii)

(iii) & (iv) Constraint on quantity of ice cream

~~$6 \geq x_1 \geq 0$ — (iii)~~

~~$9 \geq x_2 \geq 0$ — (iv)~~

~~$\boxed{0 \leq x_1 \leq 6}$ — (iii)~~

~~$\boxed{0 \leq x_2 \leq 9}$ — (iv)~~

$\boxed{x_1 \leq 6}$ — (iii)

$\boxed{x_2 \leq 9}$ — (iv)

$\boxed{x_1 \geq 0}$ — (v)

$\boxed{x_2 \geq 0}$ — (vi)

~~const~~ optimise the Profit

$(80 - 50)x_1 + (65 - 40)x_2$

~~$\Rightarrow$ obj fun $\underset{max}{=} 30x_1 + 25x_2$~~

OR

~~obj func $\underset{min}{=} -30x_1 - 25x_2$~~

$$\boxed{\text{obj fun} \underset{max}{=} 30x_1 + 25x_2}$$

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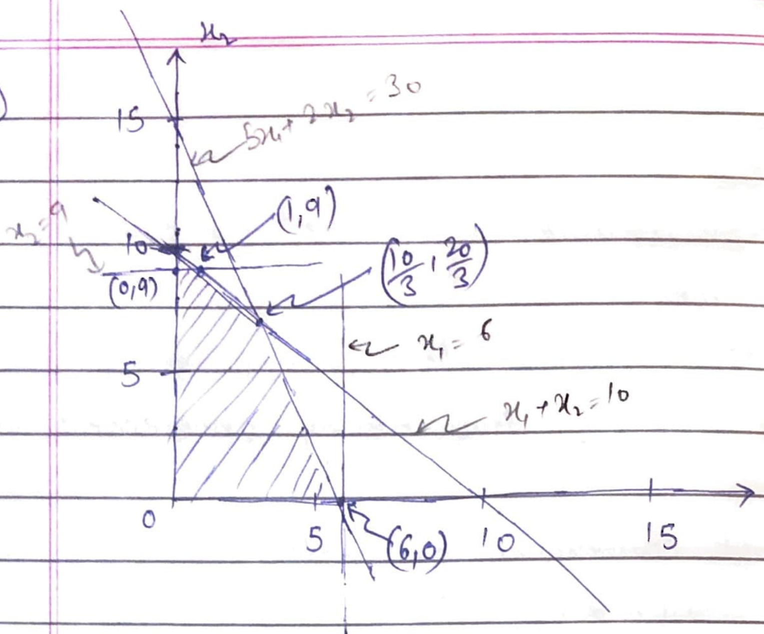
(b)



As we know the solution must be either:

(0, 9) $\Rightarrow$ obj = -225

(1, 9) $\Rightarrow$ obj = -255

$\left(\frac{10}{3}, \frac{20}{3}\right)$ $\Rightarrow$ obj = ~~266~~ $-800/3 = -266.667$

(6, 0) $\Rightarrow$ obj = -180

Hence by the graph method the solution is $\left(\frac{10}{3}, \frac{20}{3}\right)$

(c) Simplex method.

$$\max: Z = 30x_1 + 25x_2$$

$$x_1 + x_2 + S_1 = 10$$

$$5x_1 + 2x_2 + S_2 = 30$$

$$x_1 + S_3 = 6$$

$$x_2 + S_4 = 9$$

$$x_i \geq 0 \text{ \& } S_i \geq 0$$

b'	x_1	x_2	s_1	s_2	s_3	s_4	b	
$10/1 = 10$	1	1	1	0	0	0	10	s_1
Departing → $30/5 = 6$	5	2	0	1	0	0	30	s_2
~~Departing~~ $6/1 = 6$	1	0	0	0	1	0	6	s_3
(smallest non neg.) $9/0 \to \infty$	0	1	0	0	0	1	9	s_4
	-30	-25	0	0	0	0	0	

↑ entering (∵ smallest)

Current Sol = (0, 0, 10, 30, 6, 9)

~~$R_2 \to \frac{1}{5}R_1 - R_3$~~

~~$R_2 \to R_2 - 5R_3$~~

$R_5 \to R_5 + 30R_3$

$R_1 \to R_1 - \frac{1}{5}R_2$

$R_3 \to R_3 - \frac{1}{5}R_2$

$R_5 \to R_5 + 6R_2$

$R_1 \to R_1/5$

~~x_1 x_2 x_3 x_4~~

b'	x_1	x_2	s_1	s_2	s_3	s_4	b	
Departing → $4/1 = 4$	0	1	1	0	-1	0	4	s_1
0	0	2	0	1	-5	0	0	s_2
∞	1	0	0	0	1	0	6	x_1 ~~s_3~~
$9/1 = 9$	0	1	0	0	0	1	9	s_4
	0	-25	0	0	30	0	180	

↑ entering

Current Sol = (6, 0, 4, 0, 0, 9)

Since x_2 is already 1

$R_2 \to R_2 - 2R_1$

$R_4 \to R_4 - R_1$

$R_5 \to R_5 + 25R_1$

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x_1	x_2	s_1	s_2	s_3	s_4	b	
0	1	1	0	-1	0	4	x_2
0	0	-2	1	-3	0	-8	s_2
1	0	0	0	1	0	6	x_1
0	0	-1	0	1	1	5	s_4
0	0	25	0	5	0	280	

b'	x_1	x_2	s_1	s_2	s_3	s_4	b	
Departing $4 \times \frac{5}{3} = 20/3$	0	$3/5$	1	$-1/5$	0	0	4	s_1
$6 \times \frac{5}{2} = 15$	1	$2/5$	0	$1/5$	0	0	6	x_1
-0-	0	$-2/5$	0	$-1/5$	1	0	0	s_3
$9/1 = 9$	0	1	0	0	0	1	9	s_4
	0	-13	0	6	0	0	180	

↑
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$R_2 \to R_2 - \frac{2}{3} R_1$
$R_3 \to R_3 + \frac{2}{3} R_1$
$R_4 \to R_4 - \frac{5}{3} R_1$
$R_5 \to R_5 + \frac{65}{3} R_1$
$R_1 \to \frac{5}{3} R_1$.

x_1	x_2	s_1	s_2	s_3	s_4	b	
0	1	$5/3$	$-1/3$	0	0	$\boxed{20/3}$	x_2
1	0	$-2/3$	$1/3$	0	0	$\boxed{10/3}$	x_1
0	0	$2/3$	$-1/3$	1	0	$8/3$	s_3
0	0	$-5/3$	$1/3$	0	1	$7/3$	s_4
0	0	$65/3$	$5/3$	0	0	$\boxed{800/3}$	

∵ no more negative value in last row,

we have reached the optimum solution

$x_1 = \frac{10}{3}$ & $x_2 = \frac{20}{3}$

obj. fun $= \frac{800}{3}$ = max. profit