AI-2

Q1 (a) Primal :-

$$\max_{x} (c^T x)$$

s.t. $Ax \leq b$

$x \geq 0$

Dual :- $\min_{y} b^T y$

St. $A^T y \geq c$

$y \geq 0$

$\Rightarrow \max_{y} -b^T y$

st. $-A^T y \leq -c$

$y \geq 0$

Dual of Dual :-

Following changes need to be taken care ~~Prim~~

Primal to dual	Dual → Dual of Dual
① max → min	① max → min
② $c^T \to b^T$	② $(-b^T) \to (-c)^T$
③ $A \to A^T$	③ $A^T \to (A^T)^T = A$
④ $b \to c$	④ $-c \to -b$
⑤ $\leq \to \geq$	⑤ $\leq \to \geq 0$
⑥ $x \to y$	⑥ $y \to z$

Dual of Dual :-

$$\min_{z} \; -c^T z$$

$$\text{st} \quad -Az \geq -b$$
$$z \geq 0$$

$$\Rightarrow \quad \max_{z} \; c^T z$$

$$\text{st.} \quad Az \leq b$$
$$z \geq 0$$

Since z & x are variable, it can be established that the Primal & the dual of dual are same.

X ——— X ——— X ——— X

(b) Strong duality

$$c^T x^* = b^T y^* \quad -①$$

where x^* & y^* are optimum solutions of Primal & dual respectively (assumption as mentioned in ques.)

⇒ Since primal is a maximization problem i.e. $\max_{x} c^T x$

$$\Rightarrow \underline{c^T x \leq c^T x^*} \quad \forall x \in \text{feasible solution of Primal}$$

$$-②$$

since Dual is a minimization problem ie,

$\min\limits_{y} b^T y$

$\Rightarrow \underline{b^T y \geq b^T y^*}$, $\forall y \in$ feasible soln of Dual

— (3)

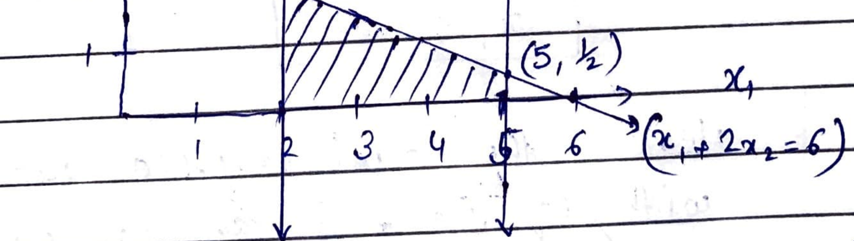
Combining eqn (1), (2) & (3)

$b^T y \geq b^T y^* = c^T x^* \geq c^T x$

$\Rightarrow \boxed{b^T y \geq c^T x}$

— x — x — x — x — x —

(c)



verticies of the ~~f.~~ feasible solution are (2,0) (5,0) (5, 1/2) & (2,2)

Obj. function value on these vertices are

(1) (2,0) : $2 \times 2 + 3 \times 0 = 4$

(2) (5,0) : $5 \times 2 + 3 \times 0 = 10$

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③ $(5, \frac{1}{2})$: $2 \times 5 + 3 \times \frac{1}{2} = 11.5$

④ $(2, 2)$: $2 \times 2 + 3 \times 2 = 10$

Hence $\boxed{(5, \frac{1}{2}) \text{ is the primal solution.}}$

Constraints to the primal problem are

$x_1 + 2x_2 \leq 6$ — ①

$x_1 \geq 2$ — ②

$x_1 \leq 5$ — ③

$x_2 \leq 4$ — ④

$\cancel{x_1 \geq 0}$

$\cancel{x_2 \geq 0}$

Since $(5, \frac{1}{2})$ lies on $x_1 + 2x_2 \leq 6$ & $x_1 \leq 5$
the carries constraints are

$$\boxed{\begin{array}{c} x_1 + 2x_2 \leq 6 \\ x_1 \leq 5 \end{array}}$$

Hence the dual variable associated with eq ① & ③ are non zero & with eq ② & ④ are zero

$\Rightarrow y_2 = y_4 = 0$

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The dual of the given problem is:

Primal

$$\max_{x_1, x_2} \; 2x_1 + 3x_2 \Rightarrow \max_{x} \underbrace{\begin{bmatrix} 2 \\ 3 \end{bmatrix}}_{=c}^{T} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

s.t. $x_1 + 2x_2 \le 6$

$x_1 \geq 2 \Rightarrow -x_1 \le -2$

$x_1 \le 5$

$x_2 \le 4$

$$\underbrace{\begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 1 & 0 \\ 0 & 4 \end{bmatrix}}_{=A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \le \underbrace{\begin{bmatrix} 6 \\ -2 \\ 5 \\ 4 \end{bmatrix}}_{=b}$$

$$A^T = \begin{bmatrix} 1 & -1 & 1 & 0 \\ 2 & 0 & 0 & 4 \end{bmatrix}$$

Dual:

$$\min_{y} \; b^T y = \min_{y} \begin{bmatrix} 6 \\ -2 \\ 5 \\ 4 \end{bmatrix}^T \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

s.t. $$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 2 & 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} \geq \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

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$$\boxed{\begin{aligned} \Rightarrow \min_{y_1, y_2, y_3, y_4} \quad & 6y_1 - 2y_2 + 5y_3 + 4y_4 \\ \text{s.t.} \quad & y_1 - y_2 + y_3 \geq 2 \quad \text{— (i)} \\ & 2y_1 + 4y_4 \geq 3 \quad \text{— (ii)} \end{aligned}}$$

From Equilibrium theorm :-

For non carries constraints, the corresponding dual variables are zero

$$\Rightarrow y_2 = y_4 = 0 \quad \text{— (a)}$$

[$\because$ eq 2 & 4 are non carries constⁿ in primal]

Also x_1 & $x_2 \neq 0$, hence the slack variable associated with dual constraint (i) & (ii) are zero.

$$\Rightarrow y_1 - y_2 + y_3 = 2 \quad \text{— (b)}$$

$$2y_1 + 4y_4 = 3 \quad \text{— (c)}$$

From (a), (b) & (c).

$$2y_1 = 3 \Rightarrow y_1 = 3/2$$

$$\frac{3}{2} + y_3 = 2 \Rightarrow y_3 = 1/2$$

Solution of Dual

$$b^T y^* = 6 \times \frac{3}{2} + (-2) \times 0 + 5 \times \frac{1}{2} + 4 \times 0 = 11.5$$

$$\Rightarrow \boxed{b^T y^* = 11.5}, \quad \boxed{y^* = \left[\tfrac{3}{2}, 0, \tfrac{1}{2}, 0\right]^T}$$