Basic Knowledge on Nature, Physics, Geometry and Mathematics

First edition.

About the PDF Book

This is an unusual book. This book is not written with exams in mind. It is not meant for any course book. I do believe, however, the book will be helpful to acquire many concepts mostly useful for students.

The book is mostly collection of internet materials. I believe you will enjoy reading the book.

You are welcome to send feedback by mail to rakesh.shrestha@gmail.com

All feedback will be used to improve the next edition.

Any people who want to translate it in Nepali language also welcomed warmly.

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Table of Contents

Curiosity on the Nature....4
- Growth....4
- Material Transport....4
- Transformation....4

Perception....5
Motion to Continuity of Time....6
Space....7
Quantities....8
- Physical Classification....8
- Scalars and Vectors....8

Units, Measurements and Constants....9
Scientific Study of Shape....10
- Constructive Approach....10
- Axiomatic Approach....10
- Analytic Approach....10
- Surface Model Approach....11
- Transformational Approach....11
- Symmetry Study Approach....11
- Projective Approach....12
- Finite Approach....12
- Differential Approach....12
- Discrete Approach....14

The Axiomatic Method....15
Proof Statements....15
Reasoning and Proof....16
- Conjecture....16
- Conditional Statement....16
- Converse, Inverse, and Contrapositive....16
- Inductive Reasoning....17
- Deductive Reasoning....17
- RAA Proof....19

Principles of Math....19
- Universal Principle....19
- Equality Principles or Common Notions....19

Geometrical Postulates....20
- The Existence Postulate....20
- The Distance Postulate....20
- The Plane Postulate....20
- The Order Postulate....21
- The Plane Separation Postulate....21
- The Continuity Postulate....22

Cultural View of Mathematics and Nature....23
- Zero and One Principle....24

Modern Scientific View on Mathematics and Nature....26

Curiosity on the Nature

Change is the most fundamental observation about nature at large. It turns out that everything that happens in the world is certain types of changes. There are no exceptions. Change is also important to the human condition. It is simply the part of human experience.

Three categories of change are commonly recognized:

Growth

This category of change, is observed for animals, plants, bacteria, crystals, mountains, planets, stars and even galaxies. In the nineteenth century, changes in the population of systems, biological evolution, and in the twentieth century, changes in the size of the universe, cosmic evolution, were added to this category. Traditionally, these phenomena were studied by separate sciences. Independently they all arrived at the conclusion that growth is a combination of material transport and transformation.

Material Transport

The only type of change we call movement in everyday life is material transport, such as a person walking, a leaf falling from a tree, or a musical instrument playing. Transport is the change of position or orientation of objects, fluids included.

Early scholars differentiated types of transport by their origin. Movements such as those of the legs when walking were classified as volitional, because they are controlled by one's will, whereas movements of external objects, such as the fall of a snowflake, which cannot be influenced by willpower, were classified as passive.

The way machines work forced scholars to rethink the distinction between volitional and passive motion. Like living beings, machines are self-moving and thus mimic volitional motion. However, careful observation shows that every part in a machine is moved by another, so their motion is in fact passive.

Transformation

Another category of change groups such as the dissolution of salt in water, the formation of ice by freezing, the rotting of wood, the cooking of food, the coagulation of blood, and the melting and alloying of metals.

These changes of color, brightness, hardness, temperature and other material properties are all transformations. Transformations are changes not visibly connected with transport. To this category, a few ancient thinkers added the emission and absorption of light. In the twentieth century, these two effects were proven to be special cases of transformations, as were the newly discovered appearance and disappearance of matter, as observed in the Sun and in radioactivity. Mind change, such as change of mood a type of transformation.

Primary cause of Change

Every type of change is due to the motion of particles or physical object as whole.

Perception

Human beings enjoy perceiving. Perception starts before birth, and we continue enjoying it.

Perception is first of all the ability to distinguish. We use the basic mental act of distinguishing in almost every instant of life; for example, during childhood we first learned to distinguish familiar from unfamiliar observations. This is possible in combination with another basic ability, namely the capacity to memorize experiences. Memory gives us the ability to experience, to talk and thus to explore nature. Perceiving, classifying and memorizing together form learning. Without any one of these three abilities, we could not study anything.

Children rapidly learn to distinguish permanence from variability. They learn to recognize human faces, even though a face never looks exactly the same each time it is seen. From recognition of faces, children extend recognition to all other observations. Recognition works pretty well in everyday life; it is nice to recognize friends, even at night. The act of recognition thus always uses a form of generalization. When we observe, we always have some general idea in our mind.

In order to be able to define change or motion properly, an abstraction of the idea concerning the physical object and the environment is required, and to distinguish them from each other.

What distinguishes the physical object with the environment? In everyday life we would say: the situation or configuration of the involved entities. The situation somehow describes all those aspects that can differ from case to case. It is customary to call the list of all variable aspects of a set of objects their (physical) state of motion, or simply their state.

Two similar objects can differ, at each instant of time, in their — position,— velocity,— orientation. These properties determine the state and pinpoint the individuality of a physical system among exact copies of itself. Equivalently, the state describes the relation of an object or a system with respect to its environment. Or, again, equivalently:

The state describes all aspects of a system that depend on the observer.

In addition, physical objects are distinguished by their permanent, intrinsic properties.

Some examples are
— mass,
— shape,
— color,
— composition.

Intrinsic properties do not depend on the observer. They are permanent – at least for a certain time interval. Intrinsic properties also allow to distinguish physical systems from each other. And again, we can ask: What is the complete list of intrinsic properties in nature?

Describing nature as a collection of permanent entities and changing states is the starting point of the study of motion. Every observation of motion requires the distinction of permanent, intrinsic properties – describing the objects that move – and changing states – describing the way the objects move. Using the terms just introduced, we can say

Motion is the change of state of permanent objects.

Motion to Continuity of Time

The simplest description of motion is the one we all use throughout our everyday life:

Only one thing can be at a given spot at a given time.

This general description can be separated into three assumptions: matter is impenetrable and moves, time is made of instants, and space is made of points. Without these three assumptions (do you agree with them?) it is not possible to define motion. We thus need points embedded in continuous space and time to talk about motion.

When we throw a stone through the air, we can define a sequence of observations. Our memory and our senses give us this ability. The sense of hearing registers the various sounds during the rise, the fall and the landing of the stone. Our eyes track the location of the stone from one point to the next. All observations have their place in a sequence, with some observations preceding them, some observations simultaneous to them, and still others succeeding them. We say that observations are perceived to happen at various instants – also called 'points in time' – and we call the sequence of all instants time. An observation that is considered the smallest part of a sequence, i.e., not itself a sequence, is called an event. Events are central to the definition of time.

Sequential phenomena have an additional property known as duration. Duration expresses the idea that sequences take some interval of time. We say that a sequence takes time to express that other sequences can take place in parallel with it. Beginning at a very young age, we develop the concept of 'time' from the comparison of motions in their surroundings. Grown-ups take as a standard the motion of the Sun and call the resulting type of time local time. From the Moon they deduce a lunar calendar.

Time is a concept necessary to distinguish between observations. In any sequence of observations, we observe that events succeed each other smoothly, apparently without end. In this context, 'smoothly' means that observations that are not too distant tend to be not too different. Yet between two instants, as close as we can observe them, there is always room for other events.

Duration, or time intervals, measured by different people with different clocks agree in everyday life; moreover, all observers agree on the order of a sequence of events. Time is thus unique in everyday life. Time is necessary to distinguish between observations. In particular, all animal brains have internal clocks. These brain clocks allow their users to distinguish between present, recent and past data and observations.

Time is not only an aspect of observations, it is also a facet of personal experience. Even in our innermost private life, in our thoughts, feelings and dreams, we experience sequences and duration. Children learn to relate this internal experience of time with external observations, and to make use of the sequential property of events in their actions. Studies of the origin of psychological time show that it coincides – apart from its lack of accuracy – with clock time. Every living human necessarily uses in his daily life the concept of time as a combination of sequence and duration; this fact has been checked in numerous investigations. The term 'when' exists in all human languages.

All methods for the definition of time are thus based on comparisons of motions. Negating all physical object and motion, it is difficult to accept separate existence of time.

Shortest time **$5.39 \cdot 10^{-44}$ s** Human 'instant' **20 ms** Day duration **86 400.002 s**

Space

Whenever we distinguish two objects from each other, we first of all distinguish their positions. We distinguish positions with our senses of sight, touch and hearing. Position is therefore an important aspect of the physical state of an object. A position is taken by only one object at a time. Positions are limited. The set of all available positions, called (physical) space, acts as both a container and a background.

Closely related to space and position is size, the set of positions an object occupies. Small objects occupy only subsets of the positions occupied by large ones. In particular, objects can take positions in an apparently continuous manner: there indeed are more positions than can be counted. Size is captured by defining the distance between various positions, called length, or by using the field of view an object takes when touched, called its surface area. Length and area can be measured with the help of a meter stick. Meter sticks work well only if they are straight. But when humans lived in the jungle, there were no straight objects around them. The length of objects is independent of the person measuring it, of the position of the objects and of their orientation.

How do we deduce space from observations? During childhood, humans learn to bring together the various perceptions of space, namely the visual, the tactile, the auditory, etc., into one self consistent set of experiences and description. The result of this learning process is a certain concept of space in the brain. Indeed, the question ‘where?’ can be asked and answered in all languages of the world. Being more precise, adults derive space from distance measurements. The concepts of length, area, volume, angle and solid angle are all deduced with their help. Geo-meters, surveyors, architects, astronomers, carpet salesmen and producers of meter sticks base their trade on distance measurements.

Space is formed from all the position and distance relations between objects.

Like time intervals, length intervals can be described most precisely with the help of real numbers. In order to simplify communication, standard units are used, so that everybody uses the same numbers for the same length. Units allow us to explore the general properties of space experimentally: space (the container of objects) is isotropic (every direction is same), homogeneous (every part is same), three-dimensional, continuous and unbounded, flat and unique or absolute.

There are no limits to distance, length and thus to space. Experience shows us that space has three dimensions; we can define sequences of positions in precisely three independent ways. Indeed, the inner ear of (practically) all vertebrates has three semicircular canals that sense the body’s acceleration in the three dimensions of space. Similarly, each human eye is moved by three pairs of muscles.

We can precisely define motion as **change of position in space with time.**

Stating that ‘**motion is the change of position with time**’ is neither an explanation nor a definition, since both the concepts of time and position are deduced from motion itself. It is only a description of motion. But what about the question concerning to the physical object?

The shortest length **$1.62 \cdot 10^{-35}$ m**	**3** Space dimensions up to **10^{26} m**	Human hair **30 to 80 µm**
Most distant visible object **125 Ym**	Point: smallest naked eye visible object diameter **20 µm**	**1m** as a standard **26** digits of precision – Physics Law

Quantities

Physical Classification

A physical quantity is needed to describe a property of any physical object quantitatively, i.e. with a numerical value.

We look at the statement m = 5kg

Here is m a physical quantity called mass, 5 a numerical value and kg a measuring unit. We try to classify physical quantities, and this by looking at the geometrical entity they refer to:

Value refers to a point: velocity, temperature, pressure, electric potential, density,...

Value refers to a surface area: all currents: momentum current, electric current, energy current,...

Value refers to a region of space: mass, momentum, electric charge, entropy, energy,..

If a quantity refers to a point, its value can change from point to point. This is evident for temperature and for pressure. Maybe it is not so obvious that also the velocity is part of this category. This is because all points of a moving body have the same velocity, or don't they? It is sufficient to look at a rotating body to convince yourself of the opposite. In such a body, each point has a different velocity. Also the velocity of water in a river changes from height to height. Quantities that refer to a region of space are called substance-like quantities. Not all quantities fulfill this pattern, e.g. time, but also the spring constant, electric resistance and capacitance.

Scalars and Vectors

Let us look once again at the different physical quantities but this time from another perspective. We will compare two specifications at first: a temperature and a velocity. You can imagine it to be the air temperature and the wind velocity at a well-defined place and at a well-defined instant of time.

The specifications are t = 19 °C and v = 5 m/s.

Do you notice that one of the two specifications is incomplete? Although we know how fast the air is moving, i.e. 5 m/s, we do not know yet in which direction it moves. Things are clear for the temperature as the temperature has no direction. Quantities such as the temperature that are defined by a single number are called scalars. Quantities for which a direction has to be specified in addition are called vectors. Here some examples:

1. **Scalars:** energy, mass, electric charge, electric current strength, temperature, entropy
2. **Vectors:** velocity, momentum, momentum current (also called force)

The velocity is (1) a quantity that refers to a point and (2) a vector.

This means:

1. Its value can be different from point to point; it forms a distribution.
2. In each point it has a well-defined direction.

Units, Measurements and Constants

All SI units are built from seven base units. Their simplest definitions, translated from French into English, are the following ones, together with the dates of their formulation and a few comments:

1. The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom.' (1967)
2. The meter is the length of the path traveled by light in vacuum during a time interval of 1/299 792 458 of a second.' (1983)
3. The kilogram, symbol kg, is the SI unit of mass.
4. The ampere, symbol A, is the SI unit of electric current. It is defined by taking the fixed numerical value of the elementary charge e to be $1.602\ 176\ 634 \cdot 10^{-19}$ when expressed in the unit C, which is equal to A · s.' (2019).
5. The kelvin, symbol K, is the SI unit of thermodynamic temperature. It is defined by taking the fixed numerical value of the Boltzmann constant k to be $1.380649 \cdot 10^{-23}$ when expressed in the unit J · K−1.' (2019)
6. The mole, symbol mol, is the SI unit of amount of substance. One mole contains exactly $6.02214076 \cdot 10^{23}$ elementary entities.' (2019)
7. The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency $540 \cdot 10^{12}$ hertz and has a radiant intensity in that direction of (1/683) watt per steradian.'

We note that both time and length units are defined as certain properties of a standard example of motion, namely light. In other words, also the Conférence Générale des Poids et Mesures makes the point that the observation of motion is a prerequisite for the definition and construction of time and space. Motion is the fundamental of every observation and of all measurement.

All SI units are made more practical by the introduction of standard names and abbreviations for the powers of ten, the so-called prefixes.

Power Name	**Power Name**	**Power Name**	**Power Name**
10^{1} deca da	10^{-1} deci d	10^{18} Exa E	10^{-18} atto a
10^{2} hecto h	10^{-2} centi c	10^{21} Zetta Z	10^{-21} zepto z
10^{3} kilo k	10^{-3} milli m	10^{24} Yotta Y	10^{-24} yocto y
10^{6} Mega M	10^{-6} micro µ	10^{27} Xenta X	10^{-27} xenno x
10^{9} Giga G	10^{-9} nano n	10^{30} Wekta W	10^{-30} weko w
10^{12} Tera T	10^{-12} pico p	10^{33} Vendekta V	10^{-33} vendeko v
10^{15} Peta P	10^{-15} femto f	10^{36} Udekta U	10^{-36} udeko u

Scientific Study of Shape

Combining the basic terms point, line, notion of area, plane and space, we can describe shape and size of physical object and relate them with other physical properties too. The shape and size is the most fundamental property of every object.

Study of shape combines visual delights and powerful abstractions, concrete intuitions and general theories, historical perspective and contemporary applications, and surprising insights and satisfying certainty.

Spatial thinking fuses reasoning and intuition in a characteristic fashion. Mathematical insight is generally hard for students to develop. In order to develop spatial reasoning and model relations various approach has been developed as mentioned below.

Constructive Approach

Knowledge of shapes developed in all ancient cultures, involving spatial patterns and development of fine arts.

Axiomatic Approach

An axiomatic system provides an explicit foundation for a mathematical subject. Axiomatic systems include six parts: 1. axioms (postulates, conjectures, common notions), 2. the logical language, 3. rules of proof or logical steps, 4. definitions (explanations), 5. propositions or theorems, and 6. proofs of theorems.

We know little about Euclid (circa 300 B.C.E.) except his mathematics, including the most influential mathematics book of all time: the Elements [13 Chapters]. In it he organized virtually all the elementary mathematics known at the time. The Elements contains definitions, axioms, and 465 theorems and their proofs—but no explanations or applications. For centuries the format represented the ideal for mathematicians and influenced many other areas of knowledge.

Although we live in a three-dimensional world, visualizing three-dimensional figures is harder than visualizing two-dimensional figures. Euclid devoted the final three books of his Elements to three-dimension.

Archimedes (287–212 B.C.E.) was another greatest mathematician of the ancient world and its outstanding engineer and physicist.

Analytic Approach

The fruitful use of algebra to study shapes called analytic approach has become an indispensable tool for mathematicians, scientists, and those in many other fields. Although Rene Descartes ´ (1596–1650) and Pierre de Fermat (1601–1665) deserve credit for creating analytic approach, many others before and after shared in its development. And, many calculus texts include topics such as parametric equations and polar coordinates. In addition to these topics, later sections discuss Bezier curves in computer aided design and shapes in three and more dimensions.

We can realize that first and second degree equations correspond to lines and conics. We can solve some questions now considered part of calculus, such as finding maxima and minima of certain functions.

Surface Model Approach

The most common surfaces are spherical, elliptical and hyperbolic. The shortest path on the surface of a sphere connecting two points is a great circle – a circle with the same radius as the sphere. A circle is the intersection of the sphere and a plane that passes through the center of the sphere. Circles of longitude and the equator are great circles.

Transformational Approach

Moving geometric figures around is an ancient and natural approach. However, the Greek emphasis on logical or axiomatic approach and constructions and, later, the development of analytic approach overshadowed transformational thinking. Early in the twentieth century physicists realized the power of transformations, starting with Einstein's theory of relativity and then with quantum mechanics. We first investigate isometries – transformations preserving distance.

Members of the most important family of transformations, isometries, do not change the distance between points as the transformations take place or move them. Isometries are the dynamic counterpart of the notion of congruence.

Intermediate between isometries and affine transformations are similarities, the transformations corresponding to similar figures. The group of similarities includes more than the group of isometries, the transformations matching congruence. We define affine transformations by their matrix form.

Symmetry Study Approach

The rules of symmetry restrict how an artist can fit copies of a motif together to make a design. Archaeologists and anthropologists have started using symmetry in their study of designs to provide greater insight into cultures. Chemists and physicists use symmetry to organize new discoveries, to analyze empirical evidence, and to suggest fruitful lines for future inquiries.

The bodies of most animals illustrate bilateral symmetry; that is, a mirror reflection interchanges the two sides of the animal. Hunting lions as well as hunted antelopes need the ability to turn left as readily as right and to hear from each side equally well. However, feet are useful only underneath an animal, so there is no evolutionary advantage to a symmetry between up and down. Similarly, running backward isn't important for either predator or prey and we find no symmetry between the front and the back of animals. Hence the practical needs of most animals only require symmetry between right and left, but no other symmetry.

Human beings have used symmetry in art for thousands of years. Albrecht Durer (1471– ¨ 1528) and others studied symmetry in art. Group theory and transformational geometry provided the mathematics needed to study symmetry. The symmetries of a figure are the transformations under which the figure is stable, and they always form a group.

The classification of symmetry groups has supplied scientists and others with a clear understanding of the patterns that can be found in their areas. Other insights about symmetry by twentieth century mathematicians, including H. S. M. Coxeter, have affected many disciplines. Symmetric patterns, especially in physics, are often formal rather than visual. Even so, the same geometric figure intuition underlies symmetry. Questions from other disciplines have stretched the notion of symmetry and raised new mathematical questions. The beauty of the mathematics of symmetry and the beauty of symmetric objects have inspired the study of symmetry.

The beauty of crystals, especially gems, has fascinated people for centuries. However, prior to 1830 no chemical explanation existed for the regularities and other properties of crystals. The geometric classification by Hessel in 1830 of the 32 types of crystals spurred the study of geometric arrangements of atoms in crystals. Chemists sometimes say that there are 33 types; two are mirror reflections. The crystals are the three-dimensional analogs of frieze patterns and wallpaper patterns. A mathematical crystal is a discrete pattern having translations in at least three directions, not all in the same plane. The subgroup of translations takes a point to a three dimensional lattice of point.

Projective Approach

The progress made by the pioneers was overshadowed and mostly forgotten because of the marvelous advances of analytic geometry, vector, matrix and linear algebra and then calculus, which quickly dominated mathematical thought. Led by Gaspard Monge (1746–1818), the rediscovery and investigation of projective ideas started after 1800. Study of projections include computer graphics, statistical design theory, and photogrammetry (which infers geometric properties of objects based on photographs).

Finite Approach

Finite geometries as a subject arose from the investigations of geometric axioms at the end of the nineteenth century. The advent of hyperbolic and other geometries prompted a renewed Finite Geometries interest in axioms. In particular, geometers sought models satisfying certain axioms but not others. Often the models had finitely many points, whence the term finite geometries. The first explicitly finite geometry was a three-dimensional example with fifteen points developed in 1892 by Gino Fano. Geometers soon realized that finite geometries and their axiom systems were interesting for their own sake. Looking back, they found earlier mathematical designs, results, and problems anticipating finite geometry as a subject.

Current finite geometry research benefits from the cross-fertilization of geometry, algebra, and combinatorics in mathematics and areas outside of mathematics. Transformational geometry and group theory give powerful insights about finite geometries just as they have since the nineteenth century for traditional geometries. Combinatorics provides essential insights into finite and other areas of geometry. For example, the consequences of Euler’s formula were identified through combinatorial reasoning. Error-correcting codes from coding Finite Geometries theory, are used in electronic data transmission and benefit from the interaction of these areas of mathematics.

Differential Approach

Intuitively, straight lines do not bend at all, while circles bend at constant rates depending on their radii.

In the 1730s Alexis-Claude Clairaut (1713–1765) and Leonhard Euler (1707–1783) considered space curves, such as the helix. They analyzed how much such curves twisted out of a plane, now called the torsion of a curve. More central to our focus, they and others investigated general surfaces and geodesics.

The inherent distortions of a flat map of portions of a curved earth motivated many mathematicians from Euler to Carl Friedrich Gauss (1777–1855). In fact, as part of his employment, Gauss spent some years surveying the state of Hanover, which spurred his profound investigations in differential geometry. His major paper in 1827 shaped the direction of this subject. Gauss studied a space on its own terms, not dependent on how it is embedded in flat space. For example, measurements of a portion of the surface of the earth could determine its curvature there. They link the area of a triangle in spherical and hyperbolic geometry to their angle sums. In the generalization Gauss used geodesics for the sides of the triangles. Then, as he proved, the difference between the angle sum of the triangle and π was determined by the area of the triangle and the curvature within it. The sphere has constant positive curvature and the hyperbolic plane has constant negative curvature.

In 1854 Georg Riemann (1826–1866) generalized Gauss's results to any number of dimensions and so transformed differential geometry again. Many mathematicians developed Riemann's insights into rigorous mathematics with computational methods. Albert Einstein (1879–1955) built on Riemann's profound insights to revolutionize physics with the general theory of relativity. Einstein needed a four-dimensional space built from three spatial dimensions and time, all with varying curvature corresponding to the strength of gravitational fields at different points.

Sir Issac Newton

In both mathematics and physics Sir Isaac Newton (1643–1727) stands as a giant in his own right, somewhere between Pope's intended epitaph and Newton's own words. At 18 Newton entered Cambridge University intending to study law. In his third year he started reading science and mathematics, including works of Galileo and Kepler. He realized he needed to know more geometry and set about reading a translation of Euclid's Elements. Soon after he studied Descartes' Geometry and other recent mathematics.

Newton received his bachelor's degree in 1665, just before the university had to close because of an outbreak of the plague in England. Newton went home and thought for two years, working out the essentials of what are now called calculus and Newtonian mechanics in physics. Newton delayed publishing these profound results and much of his later work for years. He returned to Cambridge University, earning a master's degree. Newton became the Lucasian professor at age 27.

An entire list of Newton's achievements in mathematics and physics would take far too much space. For this textbook, it is noteworthy that he used Euclidean geometric proofs rather than calculus in his foundational work in physics, the Principia Mathematica. He also was the first to use calculus to develop ideas now considered part of differential geometry.

Albert Einstein

His deep physical intuition helped make Albert Einstein (1879–1955) the greatest physicist since Newton. His undergraduate training was to be a mathematics and physics teacher, but he was unable to find a position when he graduated at age 22. He worked for several years in the Swiss patent office while he worked on his doctorate in physics, completing it in 1905. While still at the patent office he published groundbreaking papers on the special theory of relativity, quantum mechanics (photoelectric effect), and Brownian motion. The first two profoundly altered the study of physics, but in 1921 he received the Nobel prize in physics for the second. By this time he had published his general theory of relativity and was internationally known for his work in relativity and mostly for the formula $E=mc^2$.

Einstein became a professor in 1908 and after several posts in Switzerland, returned to Germany, where he was a professor until 1933. The rise of the Nazis caused him to renounce his German citizenship and emigrate to the United States. He was professor at Princeton and at the Institute for Advanced Study until his death. During World War II, his letter to President Franklin D. Roosevelt was instrumental in convincing Roosevelt to establish the research program resulting in the development of the atomic bomb. After the establishment of the state of Israel, he was offered the position of its first president, but declined the honor.

Einstein continued to work once he became world famous. However, neither he nor anyone since then has been able to achieve his goal of a unified field theory, which would combine quantum mechanics, relativity theory, and indeed all of physics. He doubted the standard interpretation of quantum mechanics with its inherent indeterminacy. In his efforts to counter it, he suggested a number of experiments that spurred others to important discoveries. However, all these discoveries supported the standard interpretation of quantum mechanics. While Einstein may not strictly be a mathematician, his theoretical physics depended strongly on geometry and mathematics in general. And mathematics and many mathematicians have been inspired by his profound insights.

Discrete Approach

Throughout the history of geometry mathematicians have explored a variety of questions and topics that are now considered part of discrete geometry. For example, what polygons can we use to cover the plane without gaps or overlaps? Can we always divide the interior of a polygon into triangular regions using only its vertices? If so, in how many ways? While polygons are traditional geometric objects, an answer to the questions won't be found in Euclid's text or high school geometry texts. For another example, how many different distances can a set of n points determine?

A number of mathematicians in the twentieth century focused on the preceding questions and similar ones that address combinatorial aspects of points, lines, polygons, and other configurations. Gradually the questions coalesced into a separate area of geometry, called discrete geometry. Discrete geometry tends to organize around clusters of easily understood problems, rather than a unified theory. A slight alternation of a discrete geometry question can change it from an easily solvable one to one beyond the knowledge of even experts. Part of the attraction of the field lies in how the solutions of two seemingly similar problems can range so widely in difficulty. The focus on problems and particularly the large number of unsolved problems reflect the newness of this field.

The increasing application of computers in many fields made some discrete geometry topics suddenly useful. For example, epidemiologists use Voronoi diagrams to study outbreaks of disease. However, there are often many possible sources, instead of the handful of pumps in Dr. Snow's map. With a large number of potential sources, scientists turn to computers to determine the Voronoi diagram and so look for clustering. Computational geometry arose in response to the need for efficient algorithms to implement a variety of geometric ideas using computers.

The Axiomatic Method

Mathematicians make use of trial and error, computation of special cases, inspired guessing, or any other way to discover theorems. The axiomatic method is a method of proving that results are correct. Some of the most important results in mathematics were originally given only incomplete proofs (we shall see that even Euclid was guilty of this). No matter-correct proofs would be supplied later (sometimes much later) and the mathematical world would be satisfied.

So proofs give us assurance that results are correct. In many cases they also give us more general results. For example, the Egyptians and Hindus knew by experiment that if a triangle has sides of lengths 3, 4, and 5, it is a right triangle. But the Greeks proved that if a triangle has sides of lengths a, b, and c and if $a^2 + b^2 = c^2$, then the triangle is a right triangle. It would take an infinite number of experiments to check this result (and, besides, experiments only measure things approximately). Finally, proofs give us tremendous insight into relationships among different things we are studying.

What is the axiomatic method? If I wish to persuade you by pure reasoning to believe some statement S, I could show you how this statement follows logically from some other statement S1 that you may already accept. However, if you don't believe S, I would have to show you how S1 follows logically from some other statement S2. I might have to repeat this procedure several times until I reach some statement that you already accept, one I do not need to justify.

All mathematical theorems are conditional statements, statements of the form

If [hypothesis] then [conclusion].

In some cases a theorem may state only a conclusion; the axioms of the particular mathematical system are then implicit (assumed) as a hypothesis. If a theorem is not written in the conditional form, it can nevertheless be translated into that form.

Proof Statements

The following are the six types of justifications allowed for statements in proofs:

1. By definition or explanation
2. By conjectures or hypothesis
3. By postulates, common notions or axioms
4. By propositions or theorems (previously proved)
5. By steps (a previous step in the argument)
6. By rules of logic

Reasoning and Proof

Conjecture

A conjecture is an “educated guess” that is based on examples. Numerous examples may make you believe a conjecture. However, no number of examples can actually prove a conjecture. It is always possible that the next example would show that the conjecture is false. A counterexample is an example that disproves a conjecture.

Rakesh is making figures for a graphic art project. He drew polygons and some of their diagonals. Based on these examples, Rakesh made this conjecture:

If a convex polygon has n sides, then there are $n-2$ triangles drawn from any given vertex of the polygon.

Is this conjecture correct? Can you find a counter example to the conjecture?

The conjecture appears to be correct. If we draw other polygons, in every case he will be able to draw $n - 2$ triangles if the polygon has n sides. Notice that we have not proved conjecture, but only found several examples that hold true.

Conditional Statement

A conditional statement (also called an If-Then Statement) is a statement with a hypothesis followed by a conclusion. Another way to define a conditional statement is to say, “If this happens, then that will happen.”

The hypothesis is the first, or “if,” part of a conditional statement. The conclusion is the second, or “then,” part of a conditional statement. The conclusion is the result of a hypothesis.

Converse, Inverse, and Contrapositive

Consider the statement: **If the weather is nice, then I will wash the car.**

If p, then q where p = the weather is nice and q = I will wash the car. Or, $p \to q$

Converse $q \to p$ If I wash the car | {z} q , then the weather is nice | {z} p

Inverse $\sim p \to \sim q$ If the weather is not nice | {z} ~p , then I won’t wash the car | {z } ~q

Contrapositive $\sim q \to \sim p$ If I don’t wash the car | {z} ~q , then the weather is not nice | {z} ~p

If we accept “If the weather is nice, then I’ll wash the car” as true, then the converse and inverse are not necessarily true. However, if we take the original statement to be true, then the contrapositive is also true. We say that the contrapositive is logically equivalent to the original if-then statement. It is sometimes the case that a statement and its converse will both be true. These types of statements are called biconditional statements. So, $p \to q$ is true and $q \to p$ is true. It is written $p \leftrightarrow q$, with a double arrow to indicate that it does not matter if p or q is first. It is said, “p if and only if q”. Replace the “if-then” with “if and only if” in the middle of the statement. “If and only if” can be abbreviated “iff.”

Inductive Reasoning

Inductive reasoning is making conclusions based upon observations and patterns. Visual patterns and number patterns provide good examples of inductive reasoning. Let's look at some patterns to get a feel for what inductive reasoning is.

Look at the pattern: 3, 6, 12, 24, 48,. . .

a) What is the next term in the pattern? The 10th term?

b) Make a rule for the n th term.

This pattern is different than the previous two examples. Here, each term is multiplied by 2 to get the next term. Therefore, the next term will be $48 \cdot 2$ or 96.

Deductive Reasoning

Logic is the study of reasoning. Deductive reasoning draws conclusions from facts. Typically, conclusions are drawn from general statements about something more specific.

Law of Detachment

Here are two true statements:

- Every odd number is the sum of an even and an odd number.
- 5 is an odd number.

What can you conclude? Based on only these two true statements, there is one conclusion: 5 is the sum of an even and an odd number. 5 = 3+2 or 4+1. Let's change this example into symbolic form.

p : A number is odd q : It is the sum of an even and odd number

So, the first statement is $p \rightarrow q$. The second statement, "5 is an odd number," is a specific example of p. "A number" is 5. The conclusion is q. Again it is a specific example, such as 4+1 or 2+3.

The symbolic form is:

$p \rightarrow q$

p

$\therefore$ q $\therefore$ symbol for therefore

All deductive arguments that follow this pattern have a special name, the Law of Detachment. The Law of Detachment says that if $p \rightarrow q$ is a true statement and given p, then you can conclude q. Another way to say the Law of Detachment is: "If $p \rightarrow q$ is true, and p is true, then q is true."

Law of Contrapositive

The following two statements are true:

- If a student is in Geometry, then he or she has passed Algebra I.
- Raju has not passed Algebra I.

What can you conclude from these two statements?

These statements are in the form:

$p \rightarrow q$

$\sim q$

$\sim q$ is the beginning of the contrapositive ($\sim q \rightarrow \sim p$), therefore the logical conclusion is $\sim p$: Raju is not in Geometry. This example is called the Law of Contrapositive. The Law of Contrapositive says that if $p \rightarrow q$ is a true statement and given $\sim q$, then you can conclude $\sim p$. Recall that the logical equivalent to a conditional statement is its contrapositive. Therefore, the Law of Contrapositive is a logical argument.

Law of Syllogism

Determine the conclusion from the following true statements.

- If Ram is late, Shyam will be late.
- If Shyam is late, Hari will be late.

So, if Ram is late, what will happen? If Ram is late, this starts a domino effect of lateness. Shyam will be late and Hari will be late too. So, if Ram is late, then Hari will be late. Law of Syllogism says that if $p \rightarrow q$ and $q \rightarrow r$ are true, then $p \rightarrow r$ is the logical conclusion. Typically, when there are more than two linked statements, we continue to use the next letter(s) in the alphabet to represent the next statement(s); $r \rightarrow s$, $s \rightarrow t$, and so on.

So far we know these symbols for logic:

- $\sim$ not (negation)
- $\rightarrow$ if-then
- $\therefore$ therefore

Two more symbols are:

- $\wedge$ and
- $\vee$ or

We would write “p and q” as $p \wedge q$ and “p or q” as $p \vee q$.

Truth tables use these symbols and are another way to analyze logic. First, let’s relate p and $\sim$ p. To make it easier, set p as: An even number. Therefore, $\sim$ p is An odd number. Make a truth table to find out if they are both true. Begin with all the “truths” of p, true (T) or false (F).

RAA Proof

There exist another type of logical step of proof most commonly not used is proof by reductio ad absurdum, abbreviated RAA. In this type of proof you want to prove a conditional statement, H ==> C, and you begin by assuming the contrary of the conclusion you seek. We call this contrary assumption the RAA hypothesis, to distinguish it from the hypothesis H. The RAA hypothesis is a temporary assumption from which we derive, by reasoning, an absurd statement ("absurd" in the sense that it denies something known to be valid). Such a statement might deny the hypothesis of the theorem or the RAA hypothesis; it might deny a previously proved theorem or an axiom. Once it is shown that the negation of C leads to an absurdity, it follows that C must be valid. This is called the RAA conclusion.

Principles of Math

Universal Principle

The rules and relation described using mathematical language is universal meaning it is true for all types of physical quantities and measurement units.

Equality Principles or Common Notions

CN1. Reflexive Property of Equality $a = a$

$25 = 25$

CN2. Symmetric Property of Equality $a = b$ and $b = a$

CN3. Transitive Property of Equality $a = b$ and $b = c$, then $a = c$

$a+4 = 10$ and $10 = 6+4$, then $a+ 4 = 6+4$

CN4. Substitution Property of Equality

If $a = b$, then b can be used in place of a and vise versa.

If $a = 9$ and $a-c = 5$, then $9-c = 5$

CN5. Addition Property of Equality If $a = b$, then $a+c = b+c$.

If $2x = 6$, then $2x+5 = 6+5$

CN6. Subtraction Property of Equality If $a = b$, then $a-c = b-c$.

If $2x = 6$, then $2x-5 = 6-5$

CN7. Multiplication Property of Equality If $a = b$, then $ac = bc$.

If $y = 8$, then $5 \cdot y = 5 \cdot 8$

CN8. Division Property of Equality If $a = b$, then $a/c = b/c$.

If $3b = 18$, then $3b/3 = 18/3$

CN9. Part Summation Property of Equality

If a, b, c, d are part of e then $a+b+c+d = e$

Geometrical Postulates

The Existence Postulate

E1. There are at least minimum two points and unique straight line connecting them exist.

More explanation:

- Through any two points we can draw one, and only one, straight line.
- Two points A and B determine a unique straight line that contains them.
- Every line contains at least two points.

The Distance Postulate

D1. Given points A and B, there is a unique real number, called the distance from A to B.

More explanation:

- If A and B are distinct points, there is a particularly useful coordinate function that assigns the coordinate 0 to $\mathbf{x_1}$ for A and a positive coordinate $\mathbf{x_2}$ for B, then distance AB or length of line AB is given by relation

  $AB = | \mathbf{x}_2 - \mathbf{x}_1 |$

- The distance from A to B satisfies the following properties:
  1. **(Symmetry)** AB = BA,
  2. **(Non-negativity)** AB ≥ 0, and
  3. **(Non-degeneracy)** AB = 0 if and only if A = B.
- For every line there exists a coordinate function.

The Plane Postulate

P1. Every plane contains at least three non-collinear points.

More explanation:

Given any three different non-collinear points, there is exactly one plane containing them.

Definition. If A, B, and C are three points not on the same line, then the system of three segments AB, BC, CA, and their endpoints is called the triangle △ABC. The three segments are called the sides of the triangle, and the three points are called the vertices.

If A, B and C are three points on the same line then according to the **CN9** or **Part Summation Property of Equality**

AC = AB + BC

P2. Space contains at least four non-coplanar points.

The Order Postulate

O1. If point B is between points A and C, then A, B, C are distinct points on the same line, and B is between C and A.

O2. For any two distinct points A and C, there exist points B and D on the line AC with B between A and C and C between A and D.

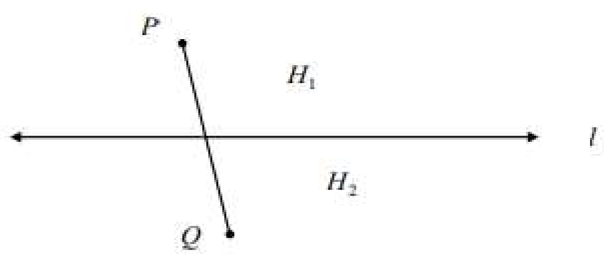
O3. If A, B, C are three distinct points on the same line, then one and only one of the points is between the other two

Definition. By the segment AB is meant the set of all points that are between A and B. Points A and B are called the endpoints of the segment. The segment AB is the same as segment BA. Line is a collection or set of points.

The Plane Separation Postulate

PS1. Give a line l, the points off l form two disjoint nonempty sets H1 and H2, called the half-planes bounded by l, with the following properties:

1. H1 and H2 are convex sets.
2. If P ∈ H1 and Q ∈ H2, then P Q cuts l



Two points A and B off line l are on the same side of l if they lie in the same half-plane bounded by l; otherwise A and B are on opposite sides of l.

More explanation:

A line l separates the points (of the plane) that are not on l into two sets such that if two points A and B are in the same set, the segment AB does not intersect l, and if A and B are in different sets, the segment AB does intersect m. In the first case A and B are said to be on the same side of m; in the second case, A and B are said to be on opposite sides of m. Actually there are only two sides to a line.

Definition. By the ray AB is meant the set of points consisting of those that are between A and B, the point B itself, and all points C such that B is between A and C. The ray AB is said to emanate from point A. A point A, on a line m, divides m into two rays such that two points are on the same ray if and only if A is not between them.

The Continuity Postulate

C1. (Archimedes Continuity) If AB and CD are two segments, there is some positive integer n such that n congruent copies of CD end to end from A along AB will pass beyond B.

C2. (Dedekind's Continuity) For every partition of the points on a line into two nonempty sets such that no point of either lies between two points of the other, there is a point of one set which lies between every other point of that set and every point of the other set.

More explanation:

When a line passes from one side to the other side of a second line, it must share a point with the second line.

There are plenty of real numbers that aren't dyadic or rational length and there are plenty of points on l that aren't dyadic points. Dedekind's construction of real numbers, essentially says that you get to any point on a line if you take enough steps but there are no gaps in a line.

Dedekind's postulate is a sort of converse to the plane separation postulate. The essence of that postulate is that any point 0 on l separates all the other points on l into those to the left of 0 and those to the right. Dedekind's postulate says that, conversely, any separation of points on l into left and right is produced by a unique point O.

Archimedes postulate says that every straight line can be made to coincide with any other straight line, and this may be done so that an arbitrarily chosen point on the first coincides with any point chosen on the second. Two figures are called congruent if one can be put on top of other, such a way that all their parts coincide; in other words, two congruent figures are one and the same figure, in two different places.

Two equal segments AB, CD can be made to coincide in two different ways; namely, the point A falling on C and B on D, or A falling on D and B on C. In other words, one can turn the segment AB around in such a way that each of the points CD takes the place of the other.

There exist only three relations in terms of size with other object:

1. **Congruent** or Equal measure
2. **Greater** than
3. **Smaller** than

If any two above mentioned relation is false, logically it must be the third one.

Any figure congruent to a straight line is itself a straight line.

Properties of Line Segment Congruence

1. **Reflexive:** Every segment is congruent to itself
2. **Symmetric:** If AB is congruent to CD then CD is congruent to AB
3. **Transitive:** If AB is congruent to CD and CD to EF then AB is congruent to EF

Cultural View of Mathematics and Nature

Let us look at the first four primary numbers in the spirit of culture. The number **ONE** can of course define a quantity; as, for example, one apple. But in its other sense, it perfectly represents the principle of absolute unity, and as such has often been used as the symbol to represent God. As a statement of form it can in one sense represent a point – it has been called the 'pointal' number, the bindu or seed in the mandala - or in another sense it can represent the perfect circle. **TWO** is a quantity, but symbolically it represents, as we have already seen, the principle of Duality, the power of multiplicity. At the same time it has its formal sense in the representation of a line, in that two points define a line. **THREE** is a quantity, but as a principle it represents the Trinity. Its formal sense is that of the triangle, which is formed from three points. With three a qualitative transition is made from the pure, abstract elements of point and line to the tangible, measurable state which is called a surface. In Tantra the triangle is called the Mother, for it is the membrane or birth channel through which all the transcendent powers of unity and its initial division into polarity must pass in order to enter into the manifest realm of surface. The triangle acts as the mother of form. But three is yet only a principle of creation, forming the passage between the transcendent and the manifest realms, whereas **FOUR** represents at last the 'first born material thing', the world of Nature, because it is the product of the procreative process, that is of multiplication: $2\times2 = 4$. As a form, four is the square, and represents materialization.

The universality of Number can be seen in another, more physical context. We learn from modern physics that from gravity to electromagnetism, light, heat, and even in what we think of as solid matter itself, the entire perceptible universe is composed of vibrations, perceived by us as wave phenomena. Waves are pure temporal patterns, that is dynamic configurations composed of amplitude, interval and frequency, and they can be defined and understood by us only through Number. Thus our whole universe is reducible to Number. Every living body physically vibrates, all elemental or inanimate matter vibrates molecularly or atomically, and every vibrating body emits a sound. The study of sound, as the ancients intuited, provides a key to the understanding of the universe.

Music is pervaded by the fundamental law of reciprocity; changes in frequency and wave length are reciprocal. Rising or falling tones, as reciprocal arithmetic ratios, are applied to string-lengths. 'Major' and 'minor' are reciprocal tonal patterns. As Ernest McClain points out in The Myth of Invariance, Plato conceived the World-Soul as constituted of reciprocal ratios identical with those which, in **Agamic** mythology, create the musical '**Drum of Shiva**', the pulsating instrument of creation.

Whether the product of an eastern or a western culture, the circular mandala or sacred diagram is a familiar and pervasive image throughout the history of art. Nepal, Tibet, India, Islam and medieval Europe have all produced them in abundance, and most tribal cultures employ them as well, either in the form of paintings or buildings or dances. Such diagrams are often based on the division of the circle into four quarters, and all the parts and elements involved are interrelated into a unified design. They are most often in some way cosmological; that is, they represent in symbol what is thought to be the essential structure of the universe: for example, the four spatial directions, the four elements, the four seasons, various divinities and often man himself. But what is most consistently striking about this form of diagram is that it expresses the notion of cosmos, that is of reality conceived as an organized, unified whole.

One of the most striking uses of the mandala is dome architecture. The square represents the earth held in fourfold embrace by the circular vault of the sky and hence subject to the ever-flowing wheel of time. When the incessant movement of the universe, depicted by the circle, yields to comprehensible order, one finds the square. The square then presupposes the circle and results from it. The relationship of form and movement, space and time, is evoked by the mandala.

The Sri Yantra is drawn from nine triangles, four pointed downward and five pointed upward, thus forming 42 (6×7) triangular fragments around a central triangle. There is probably no other set of triangles which interlock with such integrational perfection.

Zero and One Principle

I would like to consider in some detail two symbol, **Zero and One**, because they provide an exceptional example of how mathematical concepts are the prototypes for the dynamics of thought, of **action** and of **structuring**.

We know now that we exist in groups, determined by various levels of energetic affinities, repelling, exchanging and absorbing through interconnected, subtle energetic communications. And our being extends outward through various energy fields to connect with larger fields. We have had to learn that there is nowhere that we can dispose of the things we have finished using – that there is no zero drain in our sink; there is no factory pipe or hole in the ground that does not lead somewhere. Everything remains here with us; the cycles of growth, utilization and decay are unbroken. With zero we have at the beginning of modern mathematics a number concept which is philosophically misleading and one which creates a separation between our system of numerical symbols and the structure of the natural world. On the other hand, the notion of Unity remains, literally, unthinkable; simply because in order for anything to be, to exist, it must, in the very positive affirmation of itself, negate that which it is not. Cold is only cold because it is the negation of heat. For a thing to be, its opposite must also be. Everybody knows it is symbolically represented in **Yin-Yang Principle** or **Shiva-Shakti Principle**.

The origin and concept of the symbol **zero or 0** as a place value begins with **ABACUS** the first calculating device. In **Yogic** doctrines we can find its expression and zero is accepted as reality. This school laid exclusive emphasis on the goal of obtaining personal transcendence and escape from karma through renunciation of the natural world, even to the extent of mortification of the physical body. The goal of this highly ascetic pursuit was the attainment of an utterly impersonal reality, a total cessation of movement within **I-ego** centric consciousness. A description of it attributed to **Nagarjuna** is '**a state of incognizable, imperishable, selfless absence**'. This single aspect or possibility of meditative experience was held to be the ultimate goal of the created Universe as well as the goal of all individual spiritual development. Rich spiritual heritage of **Yoga**, tradition which upholds a spiritual significance in both the manifested and the unmanifested expressions of God, and whose tantric and yogic practices work towards an intensification of the relationship and harmonization between matter and spirit. From this time that the concept of zero took on a new tangibility and presence. The result is that it achieved a specific name and symbol in both metaphysics and mathematics. In mathematics it came to be considered just like the other numbers, as a symbol which can be operated upon and calculated with. The name given to this concept in Sanskrit was sunya, meaning 'empty'. Hence zero provided a framework for the development of atheism or negation of absolute reality of material world.

In India after Brahmagupta the zero is treated as a tangible entity, as a number. Aristotle and other Greek teachers had talked about the concept of zero philosophically, but Greek mathematics, fortified as it was by the Pythagorean teaching from Egypt, resisted the incorporation of zero into its system. Maya civilization used a sort of egg-shaped symbol. In Babylonia the empty space would be designated by two marks like this //.

The idea of the unknowable Unity at the beginning has been the basis of many philosophies and mythological systems. According to Shankharacharya the main stream of **Vedanta** has always rested on the notion of the **One**, the Divine, who divided himself within himself to form his own self-created opposite, the manifested universe. Within the divine self-regard, three qualities of himself became distinguished: **Sat** (immobile being), **Chit** (consciousness-force) and **Ananda** (bliss). The original unity, represented by a circle, is then restated in the concept of the Real Idea, the thought of God, called the **bindu** or seed, what we call the geometrical point.

The point, according to the **Shiva Sutra Vimarshini** Commentaries, forms the limit between the manifest and non-manifest, between the spatial and the non-spatial. The **bindu** corresponds to the '**seed-sound idea**' of the Tantras. The Divine transforms himself into sound vibration (**nada**), and proliferates the universe, which is not different from himself, by giving form or verbal expression to this self-idea. Ramakrishna summarized the scripture by saying, '**The Universe is nothing but the Divine uttering his own name to himself.'** Thus the universe springs forth from the Word.

This transcendent Word is only a vibration (a materialization) of the Divine thought which gives rise to the fractioning of unity which is creation. The Word (**sabda** in Sanskrit, the **logos** of the Christians and **Gnostics**), whose nature is pure vibration, represents the essential nature of all that exists. Concentric vibrational waves span outward from innumerable centers and their overlappings (interference patterns) form **nodules** (generic term derived from node) of trapped energy which become the whirling, fiery bodies of the heavens. The Real-Idea, the Purusha, the inaudible and invisible point of the sound-idea remains fixed and immutable. Its names, however, can be investigated through geometry and number.

This emitted sound, the naming of God's idea, is what the Pythagorean would call the Music of the Spheres. In ancient Egypt the primordial vibrational field (called **nada** in India) is called **Nun**, the primal ocean. It is the One imaged as undifferentiated cosmic substance, the source of all creation. Submerged within this primal ocean is **Atum**, the creator, who must first distinguish himself from **Nun** in order for creation to begin. **Atum** is masculine, and analogous to **Chit** (consciousness-force) of the Yogic mythology. **Atum** is pictured in a state of total self-absorbed bliss. He coughed and spit out **Shu** and **Tefnut**, who, together with himself, form the first triad of the nine great **Neteru** or principles of creation.

Thus the most recent scientific model of creation is allied to the image given in ancient mythology, and both acknowledge an absolute singularity or Unity at the beginning. In terms of the orthodoxy of ancient mathematics, the symbols of mathematics should reflect the realities they describe. With zero and the army of merely mental and statistical signs which followed from it, we are not very far from having a system of mathematical symbols which corresponds to the pure order of Nature.

The word Nature means 'that which is born', and all birth into nature requires this crossing of opposites. So the square came to represent the earth, and as such symbolized the conscious experience of finite existence, of what is born into Nature. This brings us to the problem of whether the sides of the square are curved or straight: if the overall reality of the universe is an endless curvature, an endless movement, there is yet a consciousness which is capable of temporarily arresting, both conceptually and perceptually, segments of the universal continuum. This objective consciousness might be seen as a reduced velocity of the universal consciousness, and has as its instrument the cerebral cortex in man.

Modern Scientific View on Mathematics and Nature

In science today the fundamental nature of the material world is knowable only through its underlying patterns of wave forms. The point of view of modern force-field theory and wave mechanics corresponds to the ancient geometric-harmonic vision of universal order as being an interwoven configuration of wave patterns.

In biology, the fundamental role of geometry and proportion becomes even more evident when we consider that moment by moment, year by year, every atom of every molecule of both living and inorganic substance is being changed and replaced. Every one of us within the next five to seven years will have a completely new body, down to the very last atom. Amid this constancy of change, where can we find the basis for all that which appears to be consistent and stable? Biologically we may look to our ideas of genetic coding as the vehicle of replication and continuity, but this coding does not lie in the particular atoms (or carbon, hydrogen, oxygen and nitrogen) of which the gene substance, DNA, is composed; these are all also subject to continual change and replacement. Thus the carrier of continuity is not only the molecular composition of the DNA, but also its helix form. This form is responsible for the replicating power of the DNA. The helix, which is a special type from the group of regular spirals, results from sets of fixed geometric proportions. These proportions can be understood to exist a priori, without any material counterpart, as abstract, geometric relationships. The architecture of bodily existence is determined by an invisible, immaterial world of pure form and geometry.

Modern biology increasingly recognizes the importance of the form and the bonding relationships of the few substances which comprise the molecular body of living organisms. Plants, for example, can carry out the process of photosynthesis only because the carbon, hydrogen, nitrogen and magnesium of the chlorophyll molecule are arranged in a complex twelve fold symmetrical pattern. It seems that the same constituents in any other arrangement cannot transform the radiant energy of light into life substance. The specialization of cells in the body's tissue is determined in part by the spatial position of each cell in relation to other cells in its region, as well as by an informational image of the totality to which it belongs. This spatial awareness on a cellular level may be thought of as the innate geometry of life. All our sense organs function in response to the geometrical or proportional – not quantitative – differences inherent in the stimuli they receive. For example, when we smell a rose we are not responding to the chemical substances of its perfume, but instead to the geometry of their molecular construction. That is to say, any chemical substance that is bonded together in the same geometry as that of the rose will smell as sweet. Similarly, we do not hear simple quantitative differences in sound wave frequencies, but rather the logarithmic, proportional differences between frequencies, logarithmic expansion being the basis of the geometry of spirals.

Our visual sense differs from our sense of touch only because the nerves of the retina are not tuned to the same range of frequencies as are the nerves embedded in our skin. If our tactile or haptic sensibilities were responsive to the same frequencies as our eyes, then all material objects would be perceived to be as ethereal as projections of light and shadow. Our different perceptual faculties such as sight, hearing, touch and smell are a result then of various proportioned reductions of one vast spectrum of vibratory frequencies. We can understand these proportional relationships as a sort of geometry of perception. Within the human consciousness is the unique ability to perceive the transparency between absolute, permanent relationships, contained in the insubstantial forms of a geometric order, and the transitory, changing forms of our actual world. The content of our experience results from an immaterial, abstract, geometric architecture which is composed of harmonic waves of energy, nodes of relationality, melodic forms springing forth from the eternal realm of geometric proportion.